



FOURTH EDITION

Design of Fluid Thermal Systems

William S. Janna

Design of Fluid Thermal Systems

Fourth Edition

William S. Janna
The University of Memphis



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*To Him who is our source of grace,
our source of love, and our source of knowledge,
And to Marla, whose love is a source of joy.*

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Preface to the Fourth Edition

The course for which this book is intended is a capstone type of course in the energy systems (or thermal sciences) area that corresponds to the machine design course in the mechanical systems area. This text is written for seniors in engineering who intend to practice fluid/thermal design. Fluid mechanics is a prerequisite. Heat transfer is a prerequisite or at least should be taken with this course.

Contents

The text is organized into two major sections. The first is on piping systems, blended with the economics of pipe size selection and the sizing of pumps for piping systems. The second is on heat exchangers, or, more generally, devices available for the exchange of heat between two process streams. The list of topics that can be added is almost endless.

The text begins with an introductory chapter, that provides examples of fluid/thermal systems. A pump and piping system, a household air conditioner, a baseboard heater, a water slide, and a vacuum cleaner are such examples. Also presented are dimensions and unit systems used in conventional engineering practice (i.e., Engineering and British Gravitational systems). The SI unit system is also presented. The student is expected to know about unit systems, which are presented in Chapter 1 to introduce conversion factor tables in the Appendix and to familiarize the reader with the notation in this text. Chapter 1 also contains a description of the design process. A design project example is given, and the steps involved in completing it are presented. These steps include the bid process, project management, construction of a bar chart of project activities, written and oral reports, internal documentation, and evaluation and assessment of results.

Chapter 2 is a review chapter on the properties of fluids and the equations of fluid mechanics. This chapter is included to familiarize the student with the tables of fluid properties in the Appendix. This chapter can be omitted from a one semester course if students are confident in their ability to solve problems in fluid mechanics. Viscosity data of various commonly encountered foodstuffs (catsup, peanut butter, etc.) is included to stimulate the student's interest.

Chapter 3 is about piping systems. It is expected that by the time students take this course, they will have learned about piping systems in a first course in fluid mechanics. Here, however, the subject of piping systems is covered in greater detail and depth. Specifications for pipes and tubes are discussed. Circular, square, rectangular, and annular cross sections are presented. Laminar and turbulent flow in each of these cross sections is modeled.

Chapter 4 begins with a new section on optimization. Various types of problems are covered to illustrate how system optimization is achieved. This provides a lead-in to the economics of pipe size selection, where the least annual cost method is introduced and developed. The next section is on the equivalent length of fittings, presented as an alternative to the minor loss presentation in Chapter 3. Chapter 4 also contains ANSI standards on how piping systems are to be drawn in isometric views. System behavior, in which flow rate through a given piping system is determined as a function of the driving force, is also presented. Both Chapters 3 and 4 contain modified pipe friction diagrams useful in solving special types of problems.

Chapter 5 is on selected topics in fluid mechanics. This chapter begins with a section on flow in pipe networks, focusing specifically on the Hardy Cross method of solution. The next section is on flow in parallel piping systems. Next is a section on the measurement of flow rate in closed conduits where venturi, orifice, turbine-type, variable area, and elbow meters are all described. The chapter continues with equations for modeling unsteady flow in draining tank problems. Chapter 5 is included as a reference chapter and can be omitted in a one semester course.

Chapter 6 is about pumps. Types of machines are discussed, and testing methods for centrifugal pumps are presented. Typical charts that one might find in manufacturers' catalogs are described and are used to illustrate the steps in sizing a pump for a piping system. Fans and fan sizing are also discussed. At the conclusion of studying this chapters, an engineer should be able to design an optimized piping system; that is, given a pipe layout and desired flow rate, the student can select the most economical pipe size, pipe material, pipe fittings, pump, hangers, and hanger spacing.

Chapter 7 provides an introduction to heat transfer basics in order to present the appropriate heat transfer properties and the heat transfer tables in the Appendix. Conduction and convection are both described, but radiation is not. This chapter is intended as an introduction to heat exchangers which are found in the following chapter. Chapter 7 demonstrates how the general heat transfer problem that includes conduction and convection can be modeled successfully.

Chapter 8 is about double pipe heat exchangers. The Log Mean Temperature Difference (LMTD) method is derived and used to analyze existing exchangers. The Effectiveness-NTU method is also derived and used for analysis. Design considerations, namely sizing a double pipe heat exchanger, is covered, and a procedure is developed. Chapter 9 continues with Shell and Tube Heat Exchangers. Again, the LMTD and Effectiveness-NTU methods are used to analyze existing exchangers. Also included here are methods used to increase the amount of heat that can be transferred in such exchangers. The optimum water outlet temperature for minimum cost is also presented.

Chapter 10 is about the plate and frame heat exchanger, as well as the cross flow heat exchanger. Both of these exchangers are analyzed using

traditional methods. Design considerations are also presented. The emphasis in the heat exchanger chapters is on design and selection.

Most chapters contain a section entitled “Show and Tell.” Students are asked to provide very brief presentations on selected topics. For example, in Chapter 3, one Show and Tell requires that the student give a presentation on various types of valves that are commonly used. The valves that are available are brought to class and taken apart (or cut in half prior to class) to illustrate how each works. A Show and Tell assignment on this and many other topics is far more effective than a photograph, and gives the student some practice in making an oral presentation.

Chapter 11 is an introduction to the projects. The course for which this text is intended requires the students to complete term projects. Each project has associated with it a project description that begins with a few introductory comments and concludes with several tasks that are to be completed. Each project has an estimate of the number of engineers required to finish it in the given school term. The students are responsible for selecting project partners and, as a group, deciding on which projects they would like to work. Each group elects its own project manager or leader.

Projects

With regard to the projects, the instructor is like a general contractor who has a number of projects/problems that need to be solved. The student groups are like small consulting companies, and it must be decided who gets what project. The awarding of projects is done on a “lowest bidder” basis. All group members earn the same salary (e.g., \$55,000 per year or as assigned). All group leaders likewise earn the same salary (slightly more than the group members’). Based on each group’s estimate of the number of person-hours required to complete the tasks of the project, a personnel cost is calculated. Other costs include benefits, fees for experts, computer time, and overhead.

Each group fills out a bid sheet (see Chapter 1 for an example) for every project that the group is interested in—usually no less than three. The bids are sealed in envelopes, and one class period is spent in a “bid opening ceremony.” The lowest bidder for each project then has the option of accepting (or not accepting) that project to work on, keeping in mind that a project for which a group is the lowest bidder might not be the one that group would most like to work on. Each group will then have one project to devote the entire school term to completing.

The projects must be managed to ensure that all the work is done before the last one or two weeks of classes, when quality will suffer because of the frantic, last-minute pace. Each group must complete a task planning sheet. Each task is shown on the sheet, along with who is to complete that task and when it is to be completed. Each and every student is to keep a spiral ring (or equivalent) notebook in which everything, from actual design work

to a mere phone call, that the student does on the project and time spent is recorded. The project leaders make sure that the responsible group member completes his/her assigned task on schedule, per the task planning sheet. The task planning sheet can be changed during the school term, but an equitable division of labor must be adhered to and individual tasks must be completed. At the end of the term, students tally their hours and compare the actual cost of completing the project to the estimated cost at time of bid.

Project reports are to be given in two forms: written and oral. The written report should detail the solution to all phases of the project as outlined in the original description or as modified in discussion sessions with the instructor. The oral report should summarize the findings and give recommendations; it should be limited in time.

It must be emphasized that this text does not provide a complete description in any one area. The objective here is to provide some design concepts currently used by practicing engineers in the area of fluid/thermal systems. The student should remember that actual design details of various systems can be found in textbooks, reference books, and periodicals.

Fourth Edition Modifications

The fourth edition of this text contains a number of additions and modifications made in response to comments from reviewers.

- New information in Chapter 1 includes details on the system approach versus the individual approach to modeling a fluid thermal system.
- Chapter 2 additions include viscosity data of non-Newtonian fluids. Chapter 3 is basically unchanged from the second edition except for reorganizing a few sections and topics.
- Chapter 4 has an expanded section on optimization in which many example problems have been added.
- Chapter 5 has been reorganized, containing information on pipe networks, parallel piping systems, and measurement of flow rate in a pipeline.
- Chapter 6 now contains more details on typical pump curves found in catalogs from manufacturers. An expanded section on cavitation has been added.
- Chapter 7 contains a review of heat transfer and has been expanded, adding example problems.
- Chapters 8, 9, and 10 are basically the same as those in the previous edition.
- Chapter 11 provides descriptions of design projects. Many new projects have been added, and an organizational table has been updated. Report writing is discussed in Chapter 1 and is reiterated in this chapter.

Example and practice problems have been added where appropriate, and the end-of-chapter problems are still separated according to sections. The reader can thus easily locate or review problems that relate to a particular topic.

A number of design problems have been added. Many topics have been expanded upon, and portions of the text have been reorganized.

Instructor's Guide and Solutions Manual

An *Instructor's Guide and Solutions Manual* is available to accompany this text. The *Guide* provides solutions to the problems in the text and gives a detailed outline of the course. The outline is laid out in a twelve-week plan showing problem assignments, Show and Tell assignments, and project scheduling. The *Guide* is available to all adopters of the text. Please send a written request on school letterhead to the publisher (Attn: Engineering Editor) in order to obtain your copy.

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William S. Janna

Nomenclature

Symbol	Definition	Unit	
		SI	Engineering
A	area	m^2	ft^2
a	acceleration	m/s^2	ft/s^2
C_p	specific heat	$\text{J}/(\text{kg}\cdot\text{K})$	$\text{BTU}/(\text{lbm}\cdot^\circ\text{R})$
C	ratio of capacitances	—	—
C_o	orifice coefficient	—	—
C_v	venturi coefficient	—	—
D	diameter	m	ft
$D_h = 4A/P$	hydraulic diameter	m	ft
D_e	heat transfer characteristic dimension	m	ft
D_{eff}	effective diameter	m	ft
F	force	N	lbf
g	gravitational acceleration	m/s^2	ft/s^2
g_c	conversion factor	—	$32.17 \text{ lbm}\cdot\text{ft}/(\text{lbf}\cdot\text{s}^2)$
h	enthalpy	J/kg	BTU/lbm
h_c	convection coefficient	$\text{W}/(\text{m}^2\cdot\text{K})$	$\text{BTU}/(\text{ft}^2\cdot\text{hr}\cdot^\circ\text{R})$
k_f	thermal conductivity	$\text{W}/(\text{m}\cdot\text{K})$	$\text{BTU}/(\text{ft}\cdot\text{hr}\cdot^\circ\text{R})$
L	length	m	ft
m	mass	kg	lbm
$\dot{m}$	mass flow rate	kg/s	lbm/s
Nu	Nusselt number	—	—
N	number of transfer units	—	—
P_T	pitch of tube bank	m	ft
P	perimeter	m	ft
Pr	Prandtl number	—	—
p	pressure	$\text{Pa} = \text{N}/\text{m}^2$	lbf/in^2
Q	volume flow rate	m^3/s	ft^3/s
Q_{ac}	actual flow rate	m^3/s	ft^3/s
Q_{th}	theoretical flow rate	m^3/s	ft^3/s
q	heat transferred	W	BTU/hr
q''	heat transferred/area	W/m^2	$\text{BTU}/(\text{ft}^2\cdot\text{hr})$

(continued)

Nomenclature (continued)

Symbol	Definition	Unit	
		SI	Engineering
R	gas constant	—	—
R	radius	m	ft
R	ratio of capacitances	—	—
R_h	hydraulic radius	m	ft
r	radius or radial coord	m	ft
Ra	Rayleigh number	—	—
Re	Reynolds number	—	—
T	temperature	K or °C	°R or °F
t	time	s	s
U	overall heat transfer coefficient	W/(m ² ·K)	BTU/(ft ² ·hr·°R)
V	velocity	m/s	ft/s
∇	volume	m ³	ft ³
dW/dt	power	J/s	ft-lbf/s or HP
Greek Letters			
$\alpha = k_f/\rho C_p$	thermal diffusivity	m ² /s	ft ² /s
η	efficiency	—	—
μ	viscosity	N·s/m ²	lbf·s/ft ²
$\nu = \mu g_c/\rho$	kinematic viscosity	m ² /s	ft ² /s
ρ	density	kg/m ³	lbm/ft ³
σ	surface tension	N/m	lbf/ft

CHAPTER 1 Introduction

Fluid thermal systems is a very broad term that refers to many designs and devices. A pump and pipe combination is an example of a fluid system in which fluid is being conveyed. An air conditioner is a device in which a fluid is conveyed, so it is an example of a fluid system. Moreover, because heat transfer effects are important in the air conditioner, we can consider it a fluid thermal system.

For purposes of illustration, suppose that in a food processing operation, one seeks to move peas from one location to a place where they will be packaged and frozen. This feat can be accomplished through use of a *freight pipeline*. A sketch of such a system is shown in Figure 1.1. Air is moved through a piping system by a fan, and a feed hopper will drop peas into the moving flow of air. The air/pea combination ultimately makes its way to a separator, where the air is discharged, and the peas accumulate.

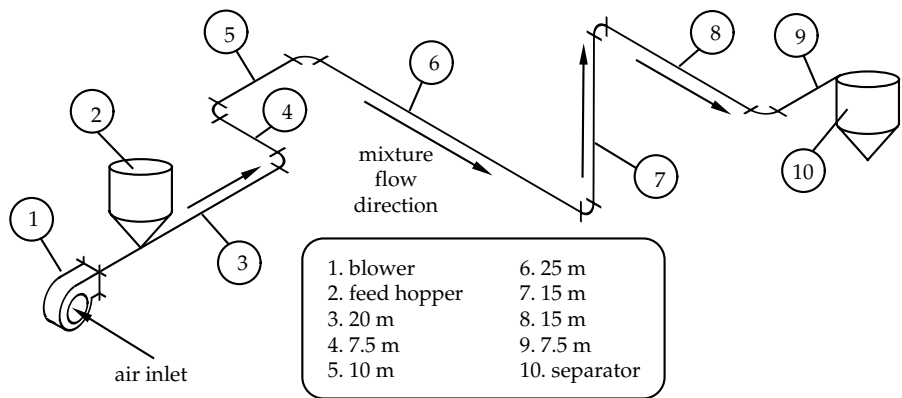


FIGURE 1.1. Sketch of a freight pipeline.

In designing this system, it will be necessary to know the physical properties of peas: range of diameters, weight of a volume of peas, and their density. It will be necessary to size the pipeline, paying strict attention to regulations regarding health and safety issues (e.g., stainless

steel must be used for foodstuffs). The fan, the feed hopper, and the separator must be selected. Once the design is completed, hangers and supports for the pipeline have to be selected. The entire design has to be checked and re-checked to be sure that it will work to deliver the volume of peas needed at the separator. Because the investment in such an operation will be sizeable, the overall cost of the system must be kept to an affordable limit. Initial and operating costs, as well as the life expectancy of the installation must also be considered. The design of this unusual fluid system is not trivial, but requires careful planning. These are the concerns of the engineer in designing such a system.

Let us next consider an air conditioning, or refrigeration unit. Figure 1.2 is a sketch of such a device. The fluid within, known as a *refrigerant*, undergoes a cycle as it moves throughout the system. The fluid is compressed by the compressor and leaves as a superheated vapor. The vapor enters what is called a *heat exchanger* (like the radiator of a car). A fan moves atmospheric air over the coils or tubes of the condenser. Heat is transferred from the refrigerant within the tubes to the air outside the tubes. During this process, the refrigerant condenses. The liquid refrigerant next goes to a receiver tank (not shown), where the liquid is separated from any remaining vapor by gravity. Liquid is drawn off from the bottom of this tank and moves through a capillary tube: a long tube of very small diameter. Liquid refrigerant passing through a capillary tube experiences a significant loss of pressure and, correspondingly, a decrease in temperature. The cold liquid refrigerant is then piped to an *evaporator*, a device similar to the condenser. Air moving past the outside of the evaporator coils loses energy to the refrigerant inside. The refrigerant gains enough energy to vaporize. Once past the evaporator, the refrigerant goes to an accumulator tank (not shown) where liquid and vapor are separated by gravity. Vapor is drawn off from the top of this tank and returned to the compressor. The cycle is repeated.

When this system is used to cool the air in a house or a refrigerator, the evaporator is located within the house or refrigerator and inside air is moved past the coils. The condenser and compressor are usually located outside and ambient air is moved past the condenser coils. Thus the refrigerant transfers energy from the evaporator within the house, as well as from the compressor, to the condenser.

As indicated in the above discussion, the compressor moves the fluid throughout the system. The fluid itself undergoes a change in phase at places within the system and effects an energy transfer from the evaporator to the condenser. The compressor power must be determined, the fluid conveying lines must be sized, the heat exchangers must be selected, the entire system must be housed, and the fluid itself must be chosen from among many fluids available, requiring strict attention to guidelines regarding the environment. Moreover, the overall cost of the system must be kept to within competitive and affordable limits. Its initial cost, operating cost,

and life expectancy must also be considered. Obviously, the design of this common fluid thermal system is not trivial, but instead requires careful thought and extensive planning. These are the concerns of the engineer in designing such a system.

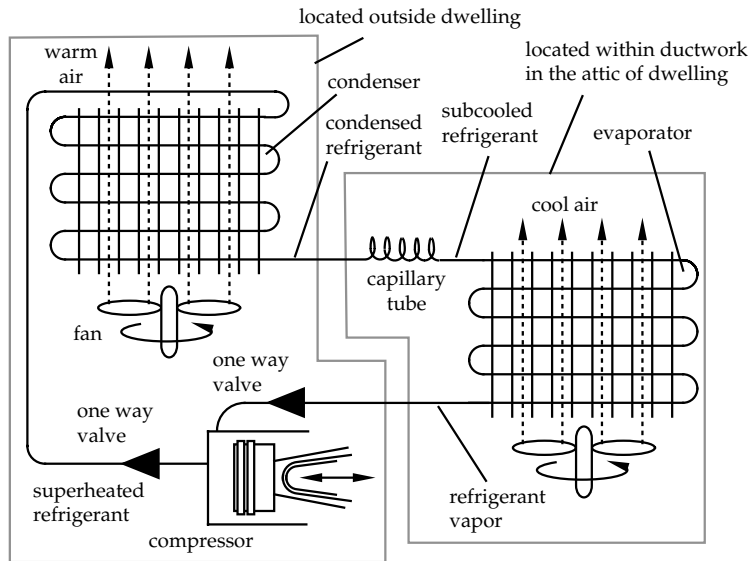


FIGURE 1.2. *Sketch of an air conditioning unit.*

Next, consider the operation of a power plant. In conventional systems, steam is produced and passes through a turbine. Downstream of the turbine is a heat exchanger, whose function is to condense the steam to liquid water. The heat from the steam is transferred to water that is taken from a nearby river or lake. However, due to environmental concerns, it may be desired to use a cooling pond rather than a nearby river to dissipate the heat rejected from the system. A cooling pond is a human-made pond, roughly the size of a small lake, that contains sprayers. The sprayers float on the water surface and spray water upward. A portion of the sprayed water vaporizes and transfers heat to the air above the pond. Other modes of heat transfer may also be present.

Figure 1.3 shows a plan view of a power plant condenser and a cooling pond. For whatever air temperature exists, it is desired to cool the water to as low a temperature as possible with the cooling pond. Decisions that need to be made include: the amount of heat that is to be rejected, the temperature of the water in the pond as well as the wet bulb temperature of the air, the proximity of the pond to the power plant, the amount of land available for the pond, the size of the pond, the size of the pump required to move water from the condenser to the pond and back, and among other

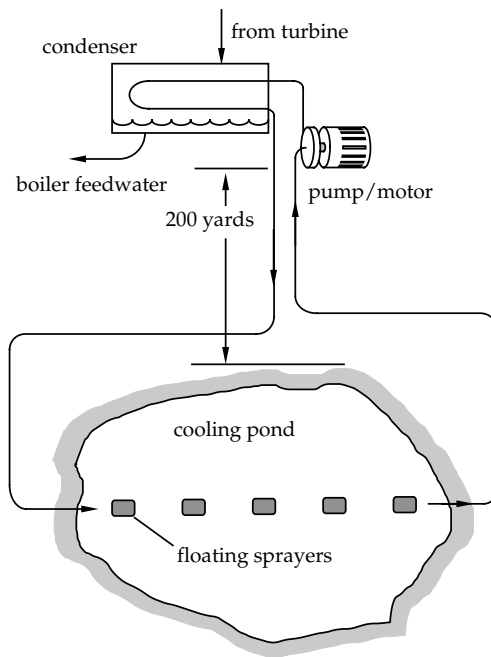


FIGURE 1.3. *Sketch of a cooling pond installation.*

things the pipe sizes required. The design engineer will make these decisions when designing this fluid thermal system.

The freight pipeline, the air conditioner, and the cooling pond are three examples of the many fluid thermal systems that exist and that must be designed. Other examples include:

- A layout of a piping system to deliver ink to various locations in a printing shop.
- A sand blaster that uses ice instead of sand in order to minimize health hazards and make cleanup easy.
- A meter that gives an instantaneous reading of miles/gallon for an automobile.
- A funnel that signals the user to stop pouring before an overflow occurs.
- A system for recovering heat from a conventional fireplace.
- A piping system to provide sufficient heat removal to create an ice rink.
- A system for testing the efficiency of ceiling fan blades.
- A ventilation system for mines.
- A device for producing hot lather for shaving.
- A system useful for measuring thrust developed by a diver who is testing swim fins (or flippers).
- An apparatus for testing proposed designs of pulsating shower heads.

- The design of an amusement park water slide.
- The design of a water-oil separation tank to help with cleanup of oil spills.

The list can be expanded to include many more examples. Each system requires considerable design work, extensive refining, inevitable redesigning, and an economic analysis. During this process, there will be meetings and discussions, and records will be kept of all deliberations.

The objective in this text is to discuss some of the concepts learned in engineering and in economics courses, and to synthesize these concepts into a coherent presentation in which practical applications are given great consideration. Fundamental concepts of fluid mechanics, thermodynamics, heat transfer, material science, manufacturing methods, and economics are combined in order to illustrate how devices and systems are designed. Hopefully, this text will provide the engineer with ideas and design concepts that will enhance his or her future practice.

1.1 The Design Process

The design process (ranging from accepting a “job” to producing a final report) involves more than merely finding a solution. So in this section, we will discuss several aspects associated with obtaining a solution to a design project. It is prudent to note that in engineering design work, there may be many possible solutions to a design problem. We will discuss the nature of engineering design, the bidding process, project management, and evaluation and assessment of performance.

Nature of Engineering Design

The design activity can include looking at drawings, making decisions, gathering information, attending meetings, considering alternatives, and much more. Design is not necessarily a single task but an entire process. An engineer goes through this process to determine how best to use resources to accomplish a required job. Engineers design systems or devices that could be of interest to the public, or to satisfy the desires of a single client.

An unfortunate aspect of design is that, in most cases, what the client wants may be unclear to both the engineer and to the client. Problem statements are often filled with uncertainty and are poorly articulated. For this reason a good design engineer will spend considerable time defining the problem and planning the way it will be solved. Work on a project should not be delayed until the last minute when failure to meet an unanticipated requirement leaves no time for correction. Project work requires careful planning and sound management methods. Otherwise, deadlines will be

missed, and a workable solution may not be obtained. Thus, design involves the use of engineering methods to bring about the “best” change in a poorly understood situation. The change must be brought about with the available resources and by the deadline.

One unique feature of design problems is that there is no one “correct answer.” For example, in sizing a heat exchanger to provide specific outlet temperatures, one would find that several heat exchangers will work. Each solution will have good and poor design aspects associated with it. A rather large group of interrelated and complex factors must usually be considered, and some good points may have to be neglected to satisfy other needs.

The needs that must be considered in a design are referred to as **constraints**. Besides engineering considerations, constraints can include effects on the environment, effects on the health of individuals who may have to work with the design, economic factors including initial and operating costs, manufacturability, sustainability, and effects of public opinion on the outcome.

Design Phases

The design process encompasses many phases including: recognizing a need, identifying the problem, synthesizing a solution, re-designing (if necessary) for optimizing the design, evaluating the design, and communicating the results. Figure 1.4 illustrates one (of many) ways that the steps in a design can be synthesized.

Design begins when a client recognizes a need and begins working on satisfying that need. The need can be something obvious or merely a sense that something is “not right.” Recognition of the need may be triggered by an adverse circumstance.

Recognizing a need and identifying or defining the problem are different things. We might recognize the need for cleaner air in a building, and the problem might be an inadequate filtering system. Defining the problem must include all specifications for the system to be designed. This includes its dimensions, characteristics, location, costs, expected life, operating conditions and limitations. Restrictions often encountered include available manufacturing processes, labor skills, materials to be used, and sizes in stock.

An optimum solution can be sought once the problem has been defined and its constraints have been identified. Synthesis of the optimum solution requires analysis and optimization. The design must comply with the specifications and if it is not optimum, a re-design is necessary. This part of the process is iterative in nature and continues until the “best” solution is found.

Evaluation of the design is a significant aspect of the design process. Evaluation is proof that the design is successful.

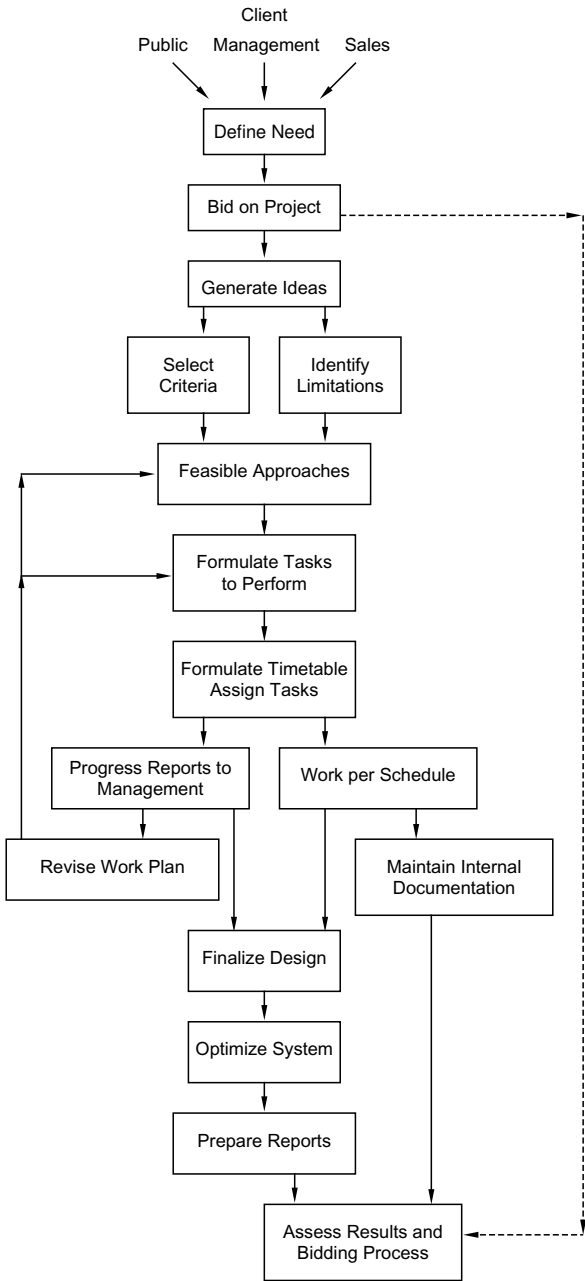


FIGURE 1.4. *Design process from defining a need to assessing results.*

Communicating the result is the final step in the design process. Communication is done orally and/or by means of a written, detailed report. Presenting the results is a selling job in which the engineer tries to convince the client that this solution is the “best” one. Selling must be done successfully, or the effort is wasted. An engineer who is repeatedly successful in selling results will usually be successful in the profession. An engineer must have effective written and oral communication skills. These include writing, speaking, and drawing, which can be developed and improved with *guided* practice.

A competent engineer should not be afraid of failing to sell an idea. An occasional failure is to be expected, and much is learned from failure. Some great gains can be obtained by someone willing to risk defeat. The real failure is in not trying. A failure in the traditional sense should be viewed as feedback needed to make improvements in a future cycle of designing and selling.

In many instances, a rational mathematical approach is abandoned in favor of knowing what the client likes to make selling easier. Using oversized bolts or frames, for example, might create an impression of durability and strength, which is a good selling point. Attractive styling might also be something a client would like to see. While these factors are cosmetic, they should not interfere with the sound operation of the design itself.

Codes and Standards

A **code** is a specification for the analysis, design, or construction of something that specifies the minimum acceptable level of safety for constructed objects. For example, each locality has a code for the size of tubing to use in the plumbing of a house. The purpose of a code is to guarantee a certain degree of safety, performance, and quality. *Absolute* safety is not necessarily assured by a code, but a reasonable level can be met. There are many established standards and codes, enacted by the appropriate authority. Each organization deals with a specific area, such as plumbing, construction, parking lots, and pedestrian walkways. Codes and standards are sometimes established by manufacturers, and sometimes by engineers who work in the industry. Establishment of a code or standard in almost all cases is in response to a perceived need.

A **standard** is a specification for sizes of parts, types of materials, or manufacturing processes. The purpose of a standard is to provide the public or the customer with uniformity in size and quality. A bolt standard, for example, is $\frac{1}{4}$ -20. All $\frac{1}{4}$ -20 bolts and nuts have the same thread specifications or standards. There are standards for clothing sizes, paper sizes, wire sizes, shoe sizes, can sizes, furniture sizes, newspaper sizes, as well as bolts and nuts.

Economics

Cost considerations play an important role in the design process. Unfortunately, costs are sometimes unpredictable. The designer may have missed a hidden cost. Furthermore, some costs change from year to year depending on the economy of the nation. Cost must be considered, however, as thoroughly as possible in the design process.

Using standard sizes is almost a necessity in keeping costs low. A good example is pipe and tubing, both of which are available in discrete sizes. Pumps, motors, fasteners, and the like., are all manufactured to certain standards, and using stock sizes is an excellent idea.

Product Safety

The engineer should make every effort to ensure that the design is safe and has no defects. An engineer or manufacturer can be held liable for unforeseen defects, even those that surface years after the design was finalized. Public safety is the engineer's chief concern.

1.2 The Bid Process

The majority of design and construction contracts are awarded through a competitive bidding process. A bid is an offer by a firm (a **contractor**) to perform work requested by a client (**contracting agency**). The objective of going through a bidding process is to locate the firm that will do the work for the least cost. There are two major areas to consider: bidding to do work for a private owner, or bidding to do work for the government.

The client (or contracting agency) initiates the bidding process by issuing an invitation for bids. In the private sector, the invitation can be in the form of notices sent to individual contractors describing the work to be performed and soliciting bids to complete it. In addition, the contracting agency can place an advertisement in trade journals or in the local newspaper. However governmental solicitations are subject to comprehensive regulations regarding the solicitation of bids.

After a contractor has decided to submit a bid, the contractor must determine the cost of completing the work. This may take several weeks of preparation in reviewing specifications, determining the number of person-hours required, calculating overhead and profit, and so on. Once complete, the contractor prepares bid documents as required by the client or contracting agency. The bid documents are placed in a sealed envelope or container and submitted to the client, usually by some advertised closing date for bid submission. Bids are opened and reviewed at a bid-opening ceremony. Each contractor who submitted a bid will want to have a representative at the bid opening. Any bid submitted after the closing date can be rejected.

All bids are opened and reviewed by the client or client's agent. The cost estimates are then made public and all contractors will know what has been submitted. In the private sector, the client can select any contractor to do the work regardless of whether the contractor is the lowest bidder. The government, however, is usually bound to grant the contract to the lowest bidder unless there is some justifiably compelling reason not to do so.

Exceptions to competitive bidding on the part of the government can exist in a number of circumstances: when it is impractical to have open competition; when only one source and no other supplies services that satisfy the requirements; when there is unusual and compelling urgency; when precluded by agreement; when authorized by law; or when open competition is not in the best interest of the public.

Before awarding a contract, it is wise for the contracting agency (client) to review the background of the prospective contractor. Things to consider in trying to determine whether the contractor can successfully complete the job are:

- Is the contractor responsible?
- Is there a work related legal action against the contractor?
- Is the contractor financially stable?
- What ongoing work is the contractor involved in that might cause interference?
- How has the contractor performed on previous jobs?
- Can the contractor meet deadlines?
- Does the contractor have integrity and an ethical standard?
- Does the contractor have the technical skills to complete the project?

Once a bid has been awarded and accepted, the contractor becomes liable for living up to the terms of the contract.

In some instances, a contractor may wish to withdraw a bid due to a technical or to a clerical error. There are established procedures for withdrawing a bid or correcting it in these cases. Errors of judgment, however, are not correctable. Such errors include failure to accurately estimate length of performance, overhead costs, profit, and manner of performance. (Information from "Competitive Bidding" by I. Genberg, *Construction Business Review*, Sept.-Oct. 1993, pp. 31-34.)

1.3 Approaches to Engineering Design

There are two approaches to solving a design problem. One is the *systems approach* and the other is the *individual approach*. The systems approach involves writing an **objective function** for the problem at hand. The objective function in engineering problems in some cases is an equation

for the total cost of a system. The total cost will include an initial cost (for equipment as an example) and operating costs (for electricity or fuel). The initial cost is modified in order to make it an annual cost. Then the annualized initial cost and the operating cost are added to obtain the total annual cost. In a “good” design, we would seek to minimize the total cost, so we would differentiate the total cost expression and set it equal to zero. We would thus obtain an equation that we could solve for a particular parameter. For example, suppose we wish to minimize the cost of a pipeline. We might express all cost parameters in terms of pipeline diameter. We differentiate the total cost expression with respect to diameter and solve for the optimum diameter which is the one that would give us the minimum total annual cost for the system.

For a design problem involving many devices, the total cost function can be quite complex. Usually other equations are required in order to solve the differentiated cost equation. These other equations are called **constraint equations**. They could consist of continuity and energy equations written for every device in the system. Once the objective function and constraint equations are written, we end up with a system of equations that must be solved simultaneously. A number of methods can be used to solve these equations. The systems approach is used in Chapter 4 for pipe sizing. (For a description of the systems approach applied to a number of problems, and solution methods, see *Design of Thermal Systems* by W. F. Stoecker, McGraw-Hill Co., 2nd ed., 1980.)

The other approach to design problems is to consider each device in a system individually. The cost of each device is minimized, thereby minimizing the total cost of the entire system. The advantage of this method is that the equations to solve are simplified.

1.4 Design Project Example

We illustrate some of the points made in the preceding sections with an example and some specific ideas. Consider that we are interested in working on a problem that involves the recovery of waste heat in a manufacturing facility. The problem is stated as follows.

Heat Recovery in a Sheetrock Plant

One of the components needed in the manufacture of sheetrock is water. The process requires 70 gpm of water at a temperature of about 85°F. During summer months, the city water supply provides water whose temperature can be as high as 90°F. During other months, the average temperature of water supplied by the city is about 45°F. This water must be heated so that

it can be used successfully for the process. The water is heated by natural gas burners while it is in a storage tank.

One of the final phases of sheetrock production is the drying stage. Heated air is moved by a fan around the sheetrock in an oven. The air is then exhausted. It is desired to recover energy from the warm, humid exhaust air and use the energy to pre-heat the incoming city water from 45°F (worst case) to as warm as possible. The energy recovered would reduce the need for natural gas to be used as the main heating medium. Conditions indicate that the heat recovery system will be in operation for 24 hours per day (six days per week) for eight months.

Figure 1.5 shows the position of the drying oven and of the holding tank. As shown, the water tank is 300 ft from the oven.

Suppose that this Heat Recovery Project arises in a plant that does not have enough engineers available to work on it. Management has decided to allow an outside engineering consulting company to solve the problem or at least to see if it is cost effective. Management will contact any number of consultants and invite them publicly to bid on the project. That is, each consultant is invited to submit to management (the client in this example) a proposal that outlines what is to be done and how much the consulting company will charge to perform *only* the design work. Actual construction or installation might also be part of the bid, depending on what the client requests of the bidders.

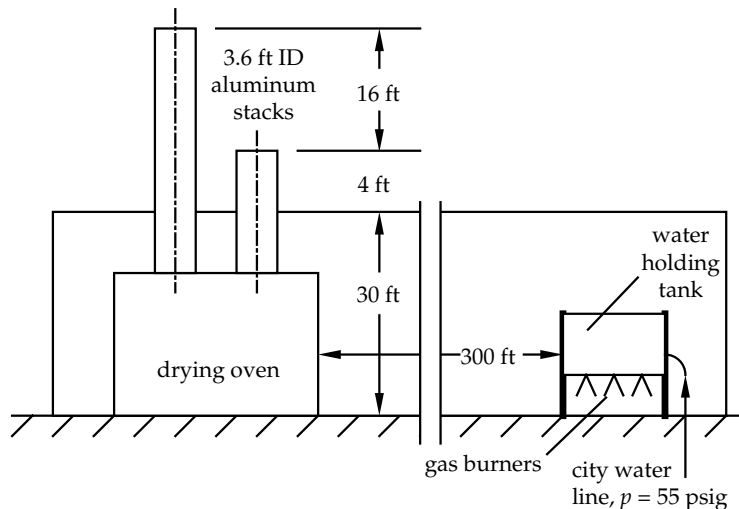


FIGURE 1.5. Layout showing drying oven and holding tank.

“Our” company has been invited to submit a bid to do the design work on this project. Before we can prepare a bid, however, we must have some idea of what needs to be done. So we review the problem statement assuming

that it is all we have been given by the client. We examine a number of things with reference to the comments made earlier.

Insufficient Information

The problem statement seems clear enough, but there is information which is missing that we will need. At first glance, we might wish to place a heat exchanger in each stack. To size the heat exchangers, however, we must know the temperature of the exhaust gases in the stacks and the flow rate of air through them. The client might have no idea what these are, and we may be required to have access to the roof, taking a thermocouple, a digital thermometer, a pitot-static tube and a differential pressure meter. We need to know the ceiling height between the holding tank and the stacks, in case we wish to install the pipe with hangers along the ceiling, or along a wall, rather than on the floor. In order to perform an economic analysis, we must have information on the natural gas usage and city water temperature on a monthly basis for at least one (and preferably three) years. Certainly, there is insufficient information at this point, and one possible reason for this is that the person(s) providing the problem statement do not know what an engineer needs to solve it.

No Unique Solution

Our company might design a system having one heat exchanger in only one stack. Another company might propose a heat exchanger in each stack. A separate fluid system containing ethylene glycol and water could be used to transfer heat from a heat exchanger in the stack to water in the holding tank. Alternatively, city water can be run to the heat exchanger in the stack first and then to the holding tank. Obviously this design problem has many solutions.

Constraints

Finding the “best” solution to this problem from the many that will work is a matter of trying out several (on paper) and determining which satisfies the constraints. Does the client wish to save as much money as possible by installing this system? Has the Environmental Protection Agency put a limit on the temperature of exhaust gases from this facility? The objectives and constraints need to be identified.

Submitting a Bid

We have decided that our company has the required expertise, time, and skill to solve this problem, and now we would like to submit a bid. We

must consider more specifically the tasks involved. We have obtained the additional information we need from the contracting agency.

It is apparent that we have to design the piping system from the tank to one of the stacks and back. We have to specify a pipe size, determine the routing of the line itself, select a pump for the job, and size a heat exchanger to place in one of the stacks. More specifically, the things that must be completed are:

1. Appropriate heat exchanger type, size, and material of construction.
2. Pump (if necessary) size, location, and material of construction.
3. Piping, pipe fittings, size, routing, and material. (Consider that a flowmeter and/or pipe insulation may be desirable.)
4. Total cost of system including installation, operation, and maintenance.
5. Payback period on the investment.
6. Use stock sizes everywhere possible.
7. Investigate local codes and adhere to them.
8. Analyze the system for safety considerations.

This list of items makes up the engineering phase of the project, which is only a small portion of the entire design process.

Bidding

We have identified what we think needs to be done, based on our concept of how this problem should be solved. It may have taken several days to draw up preliminary sketches, attend meetings, visit the plant, and so on. We are now ready to prepare our bid documents. In many cases, these documents are provided by the client and we merely fill them out. In some cases, however, each bidder can complete an in-house form and submit that to the client.

In this example, suppose we are working with a very simplified form such as that in Figure 1.6. Note carefully what is requested on this form. First, the project title is required along with an estimated bid amount. The estimated bid amount is calculated by completing this form.

Notice that there is a column appended to this form labeled “Actual.” This column is added for internal record-keeping and would not be submitted to the client.

We must remember that this bid is to be submitted by the closing date which appears on the second line of the bid sheet. The due date is when we agree to have our work completed. The hours required to complete it is our estimate of the person-hours that must be devoted to the project. The list of persons in the design group who will work on this project is given in Part A along with person-hours and salary. At best, the number of person-hours is an estimate, but an experienced bidder can make an accurate appraisal. **Fringe Benefits** for each employee working on the project are paid directly from the project budget. These include insurance, medical benefits,

				Estimated	Actual
Title of Project				Bid Amount	
Closing Date		Hrs Req'd to			
Due Date		Complete			
Project Director					
A. Personnel					
Name	Telephone	Person-hrs	Salary		
1.			\$	\$	
2.			\$	\$	
3.			\$	\$	
4.			\$	\$	
5.			\$	\$	
6. Subtotal			\$	\$	
B. Fringe Benefits (35% of A.6)			\$	\$	
C. Total Salaries, Wages, & Fringe Benefits (A.6 + B)			\$	\$	
D. Miscellaneous Costs					
1. Materials & Supplies			\$	\$	
2. Other			\$	\$	
3. Subtotal			\$	\$	
E. Travel			\$	\$	
F. Consultant Services					
1.			\$	\$	
2.			\$	\$	
3. Subtotal			\$	\$	
G. Total Direct Costs (C + D.3 + E + F.3)			\$	\$	
H. Indirect Costs (50% of G)			\$	\$	
I. Amount of this Bid (G + H)			\$	\$	
Signatures of Engineers		Date		Initials	
1.					
2.					
3.					
4.					
5.					

FIGURE 1.6. Example of a budget-bid sheet.

retirement benefits, and the like. **Miscellaneous Costs** (including materials, supplies if a model is to be built, etc.) are charged to the project. **Travel** by the group (to the facility for example) is charged to the project. It may be necessary for the design group to use the services of an expert (**Consultant Services**) and payment to the expert for his or her services is also charged to the project. The **Indirect Costs** (including overhead to pay for utilities, office space, secretarial help, profit, etc.) are also a part of the project cost. The total of these items is the cost that we, the contractor, propose to charge the client to complete the design—not to build it, but merely design it. Note that most of the items are tied directly to the person-hours estimated in the beginning, so an accurate estimate is highly critical.

1.5 Project Management

Suppose now that the Heat Recovery Project has been awarded to us because our company is the lowest bidder (or because we personally know the plant manager!). Completion of the job requires an organized and well-managed effort. The project must be divided into several smaller jobs that are finally synthesized into the overall solution. This phase involves identifying the smaller jobs, assigning the completion of each small job to an individual or individuals, and requiring each small job to be completed at a certain time. This breakdown can be done by the **Project Manager** or **Project Director**, who is ultimately responsible for ensuring that the job is finished on time and within budget. Thus, a Project Director must be selected at this point, and his or her job will be to manage the personnel in the group so that they complete the project on time.

It is convenient for the Project Director to compose a **bar chart** of project activities that outlines the tasks, or smaller jobs, to be performed in completing the projects. The bar chart is much like a graph in which time is laid out on a horizontal axis and project activities appear on the vertical axis. The advantages of such a layout are that all activities are mapped out and assigned, that the order of the activities can be readily seen, and that an overall readable picture with the expected completion time is on hand. One disadvantage of such a chart is that it will probably need updating, which can require much time.

A bar chart for the Heat Recovery Project is shown in Figure 1.7. It is a “first draft” that includes all of the tasks that we could identify. They are listed as **activities** in order on the left. The chart shows which activities or tasks require the completion of another task beforehand. The entire project is mapped out over an eight-week period with *estimates* of how long each activity will take. Also, letters appearing in each shaded rectangle represent the initials of the engineer(s) who is (are) responsible for completing the corresponding task. The shaded rectangles are connected

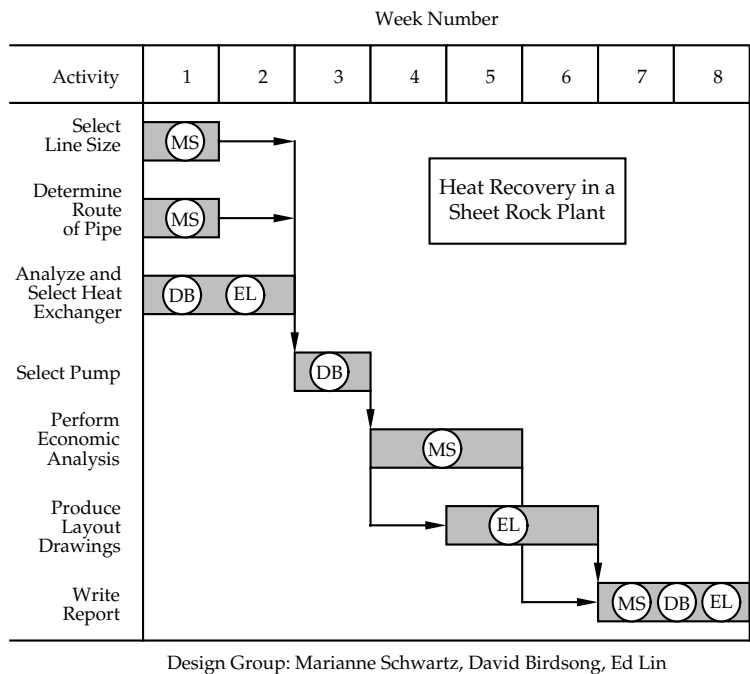


FIGURE 1.7. Bar chart of small jobs to perform in completing the Heat Recovery Project.

with lines and arrows that indicate a succession of events. Thus, before a pump is selected, for example, the line size, its route, and the heat exchanger must first be specified.

Suppose that after some time has passed, we think of (or are assigned) several other tasks to perform, or some tasks were completed before their target completion date. It is advisable to rework the chart to add the new event(s), assign a responsibility to it (them), and update the completion of the finished smaller jobs. Suppose also that it appears as if the project will be finished earlier (or later) than what was originally scheduled. This is brought out in the modified chart as well.

Say that after much study, we find we must use an exchanger in both stacks in order to recover the required energy. Figure 1.8 shows a modified chart. Note that the new activities have been added in the appropriate positions showing their relationship to other tasks.

The Project Director is also responsible for handling the budget allowed for completion of the project. This would include signing all requests for payment and keeping track of how the project budget funds are expended.

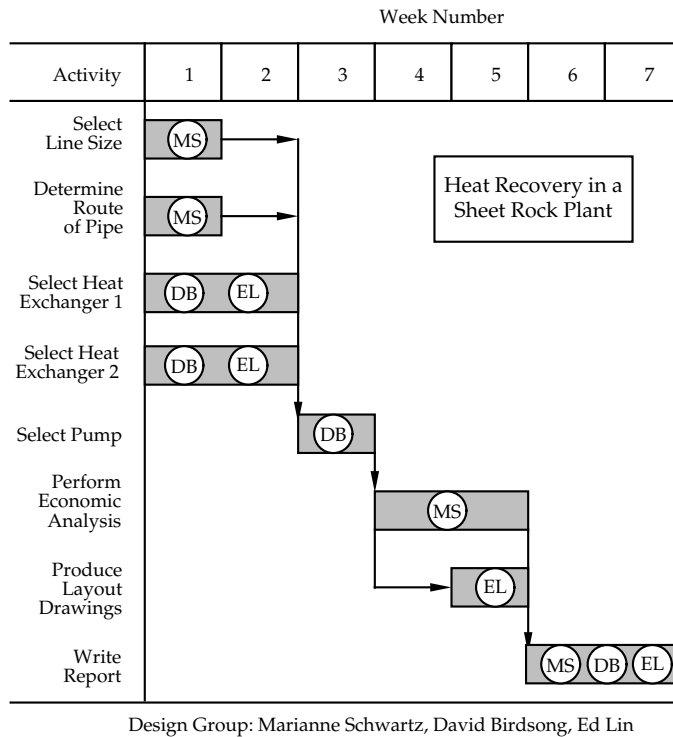


FIGURE 1.8. *Modified bar chart of small jobs performed in the Heat Recovery Project.*

The Project Director will meet frequently on an as needed basis with the group members to offer assistance if necessary.

Project management has been the subject of much study. As a result, it has been found that some of the more important tasks of a design group relate to how the group members interact or deal with each other, rather than how the engineering work is completed. People skills, like technical expertise, require continual refining. With regard to these comments, it is prudent to remember that the primary functions of the Project Director and the group members are as follows:

- Always keep in mind the objective of the entire project,
- Know exactly how each group member will contribute to the overall success of the design effort (i.e., each group member will know without question exactly what his/her responsibility is),
- Identify any and all obstacles that prevent a group member from completing a task,
- Remove the obstacles,

- Remember that all group members are “being paid” to maintain an effective working relationship with the others.

To these ends, the Project Director should schedule regular meetings of the group and should prepare extensive minutes of the meeting. Minutes should include extensive details, such as the agenda and who is responsible for presenting information. The Project Director should also prepare oral and/or written progress reports on a weekly basis for the client. The client must be kept informed regularly on the progress being made.

Internal Documentation

The design process described above listed various activities that will eventually result in a report. The report is then provided to the client. The consulting company, however, will need to keep on file much more information about the project than is included in the written report.

Once an engineer begins work on a project, the engineer is to obtain a notebook and keep track of all things performed in association with the project (in ink), especially including dates and time spent. Even the most seemingly trivial contribution (such as a phone call) should be recorded. Nothing is to be erased or eradicated from the notebook. The notebook should also contain all the engineering work and calculations done on the project. The notebook is a diary. Errors are “removed” from the diary by drawing a straight line through them but they must still be readable. Each member of the group will have his or her own notebook for each project. The progress made by the group member can be ascertained by reviewing his or her diary.

The copy of the final report that stays within the consulting company files should contain the budget-bid sheet originally submitted. At the time that the project is finished, the column labeled “Actual” on the budget-bid sheet is completed to show the actual costs of items requested as part of the project. These include the person-hours expended and the profit earned on those person-hours. Remember that we are in “business” to make money and that performance on the project will be evaluated in proportion to the actual profit realized.

The engineers’ notebooks, final report, and completed budget-bid sheet make up the documentation that the consulting company will want to keep on file for future reference. Should the project need to be reviewed in the future, the necessary details will be available.

The Reports

Next, suppose that the engineering phase of the project has been completed and it is now necessary to communicate the results. Usually a **written report** and an **oral presentation** are given by the consultant to the

client. The written report will contain several items; a *suggested* example format is given in Figure 1.9. Each item is described as follows:

Letter of Transmittal—Written to the client stating that the project has been completed and that the results are presented in the accompanying report.

Title Page—Lists project title, finished project due date, engineers who worked on the project, and the name of the consulting company. Note that all pages of the report need to be numbered, dated, and identified somehow with the consulting company.

Problem Statement—Reiterates succinctly the problem, included so that all concerned will know what project was completed.

Summary of Findings—Summarizes the details of the solution. This section might present a list showing, for example, and what pump to buy, what line size to use, where to route the pipe, what heat exchanger to use, suggested suppliers, and costs for all components. Drawings of the system would also be included. The summary should be complete enough so that the client could submit it to a contractor who could complete the installation of all components.

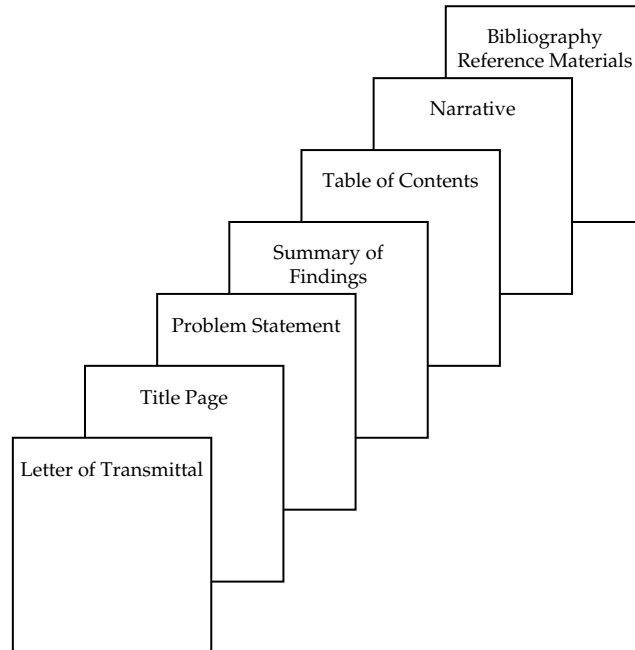


FIGURE 1.9. *Elements of the written report.*

Table of Contents—Refers the reader to any section of the report.

Narrative—Presents the details of all components specified in the summary and why each component was selected. For example, the details of how the pump was selected would be included here. Enough written detail must be included so that the reader can follow *every* step of the development. The organization of the narrative and titles of all sections will vary from writer to writer. However: *If your audience has read your report and does not understand all that you wrote, then you have not expressed yourself clearly enough.* Write for the audience.

Bibliography/Reference Materials—Shows text titles and publications used to arrive at the specifics of the design. This section should also include information from catalogs of suppliers, such as pump performance curves, if appropriate.

The written report should appear professional in every way. A well-written presentation will show that the writer is meticulous and convince the client that a great deal of care went into completing the job. Text, graphs, drawings, and charts are done by computer with nothing drawn freehand. The entire report should be bound, and the client should be provided with more than one copy.

The oral report should be short and it need not be detailed. The oral report consists of the problem statement and a summary of the findings, which includes initial and operating costs. If questions arise, the presenter can refer to details found in the narrative. Therefore, the presenter should be prepared to give details of the *entire* study but present only the problem statement and the summary.

Evaluation and Assessment of Results

The work performed on a project must be evaluated if possible. The two items of importance are: Will the system work as designed, and have we made a profit by delivering a good product?

In some cases, the client will not construct the system for a number of months or even years. Moreover, it is unlikely that the installed system will contain the necessary instrumentation to evaluate performance (e.g., thermocouples, flow meters, etc.). Even after installation, it may take years to determine if the system works as designed. In the Heat Recovery Project of this chapter, it is necessary to wait for one year after installation to see if there is any savings in natural gas expenditures.

Suppose that the system for some reason does not work as designed and some of those who worked on it do not remember all of the details, or have

accepted employment elsewhere. We can refer to the saved diaries of the engineers and work with the client appropriately to correct the problem.

Whether a good product was delivered might never be assessed. Profit realized can be assessed, however, and is usually measured by an elaborate accounting system outside the scope of engineering.

It will become evident that the engineering phase of any project requires a relatively minor amount of total time expended. Equally important is the time and effort spent in documenting activities and especially in communications.

1.6 Dimensions and Units

The unit systems used in this text are primarily the British Gravitational, the Engineering or U.S. Customary system, and the SI unit system. Fundamental dimensions and units in each of these systems are listed in Table 1.1. Also shown are dimensions and units in other systems that have been developed, namely, the British absolute and the CGS absolute.

When using U.S. Customary units, a conversion factor between force and mass units must be used. This conversion factor is

$$g_c = 32.2 \frac{\text{lbm} \cdot \text{ft}}{\text{lbf} \cdot \text{s}^2} \quad (\text{U.S. Customary}) \quad (1.1)$$

In the other unit systems listed in Table 1.1, the conversion g_c is not necessary nor is it used. The equations of this text will contain g_c and if U.S. Customary units are not used, the reader is advised to either ignore g_c or set it equal to

$$g_c = 1 \frac{\text{mass unit} \cdot \text{length unit}}{\text{force unit} \cdot \text{time unit}^2} \quad (\text{Other unit systems}) \quad (1.2)$$

When solving problems, a unit system must be selected for use. All equations that we write must be dimensionally consistent. Therefore all parameters we substitute into the equations must be in proper units. The proper units are the fundamental units in each system.

With the huge volume of parameters that have acquired specialized units (e.g., horsepower, ton of air conditioning, BTU, tablespoon, etc.), we see that the process of converting to fundamental units is a never-ending but ever-present necessity. To ease this burden, Appendix A provides a set of conversion factor tables as well as prefixes that are used when working in SI units.

TABLE 1.1. *Conventional unit systems.*

Dimension	British Gravitational	SI	Engineering	British Absolute	CGS Absolute
Mass (M) (derived)	slug				
Mass (M) (fundamental)		kg	lbm	lbm	gram
Force (F) (derived)		N		poundal	dyne
Force (F) (fundamental)	lbf		lbf		
Length (L)	ft	m	ft	ft	cm
Time (T)	s	s	s	s	s
Temperature (t)	°R °F	K [†] °C	°R °F	°R °F	°K °C
Conversion Factor g_c	—	—	32.2 $\frac{\text{lbm}\cdot\text{ft}}{\text{lbf}\cdot\text{s}^2}$	—	—

[†]Note that in SI, the degree Kelvin is properly written without the ° symbol.

1.7 Summary

In this chapter, we have examined some fluid thermal systems and indirectly defined them. We have also discussed the design process, including the nature of design, design phases, codes and standards, economics, product safety, and the bid process. Furthermore, we have described project management methods, as well as report writing and evaluation of results. These topics are amplified in the questions and problems that follow.

We also briefly discussed unit systems including SI, U.S. Customary, and British Gravitational systems. Mention was made of some specialized units that have arisen in industry and that fundamental units should be used when solving problems.

1.8 Questions for Discussion

The following questions should be addressed by groups of four or five individuals who spend 10 minutes on each assigned question. At the end of the discussions, the conclusions should be shared with the other groups in the form of a three-minute or shorter oral report.

1. Discuss the properties of a plastic that is to be used for cassettes or CDs.

2. What factors influence the decision on how large to make a hand calculator?
3. What are the desirable properties of a material used as a bathtub?
4. Discuss the desirable properties of a tube or tubes used in a solar collector. The tubes are to convey water that is heated by the sun.
5. What are the desirable properties of a material that is to be made into an inflatable float for people to use at a swimming pool?
6. What are the desirable properties of an automobile bumper that can withstand the impact of a 5 mph accident?
7. What should be the properties of a material used as brake lining in a conventional automobile?
8. Discuss the factors that contribute to the decision on how much weight a ladder should be able to support and what material it should be made of.
9. Discuss the factors that contribute to the decision on how tight a manufacturer should make the threaded top on a jar of mayonnaise.
10. Silverware refers generally to eating utensils. However, certain types of such utensils are made of silver and other types made of stainless steel. Which of these materials is the better choice for producing eating utensils? Why?
11. What are the expected properties of a paint that is used on streets as a lane marker?
12. Discuss the properties of a material that is to be used for a balloon—the inflatable type that would be used for parties.
13. Discuss the desirable properties of a tank that is to be used to store liquid oxygen.
14. How long should the soles of dress shoes last? What should be the properties of shoe soles?
15. Is it always appropriate for a job to be awarded to the lowest bidder? List exceptions and give reasons why or why not. Is it fair for a contractor to be awarded a bid merely because he “personally knows the plant manager”? Is it necessary to be fair in the private sector? Is it necessary to be fair when the government is involved? Define “fair.”
16. Should profit always be a motive in the consulting business?
17. Consider the question “Have we made a profit by delivering a good product?” Define a “good” product with regard to the Heat Recovery Project.
18. A manufacturer of vacuum cleaners says his product is “twice as good” as any on the market. With regard to this claim,

- a. Determine how to evaluate the performance of vacuum cleaners, taking into account as many factors as appropriate.
 - b. Determine in what way a vacuum cleaner can be “twice as good” as another, if possible.
19. What makes a detergent “better” than that marketed by other companies? How would you evaluate the performance of laundry detergent? What makes a laundry detergent “good”? In very specific terms, describe how a detergent can be “better” than another. Does this mean that the detergent cleans “twice as good” as another? If so, what does “twice as good” mean in this context?
 20. Car wax is an interesting product and is available from many manufacturers. You want to check out a car wax before you make a buying decision. What makes a car wax “better” than any other? Determine how to evaluate a car wax. That is, what makes a car wax a “good” car wax?
 21. You have been contacted as an expert by a college friend who has gone into business for herself. She is a chemist and is making and marketing hair color products. She believes she can enlist your expert services for a reasonable fee. She wishes to claim that the hair color product she makes is “better” than any other on the market. She would like you to help plan a testing program. Determine how to evaluate a hair color and a way to conduct a testing program. What are the desirable properties of a substance marketed as something to use to color hair?
 22. Determine how to evaluate a chemical that imparts “stain resistance.” What is stain “resistance” versus something that does not stain at all? Determine a testing program for defining and evaluating stain resistance. Would a number scale be appropriate for such chemicals?
 23. You wish to determine how effective certain color combinations are with respect to human eyesight and reaction times. Data on this sort of thing is important to sign companies and to the government when making signs to help signal motorists. For example, is black lettering on a white background better than yellow on green? If signs are made with these (or other) color combinations, which of them will be recognized more quickly by an observer? Determine a testing method to measure color contrast that can be used on signs having various color combinations.

1.9 Show and Tell

The answers to the following questions should be addressed and presented by individual students in the form of an oral report.

1. What instrumentation is necessary in the Heat Recovery Project if an evaluation of system performance is to be conducted? Explain the function of each instrument and what calculations need to be made using them.
2. What are the human safety factors that should be considered in the Heat Recovery Project of this chapter? Is there a local safety code?
3. Obtain a set of bid documents from a contractor or a contracting agency. Give a presentation on the items contained.
4. Locate a local code that applies to plumbing or electrical work. Give a presentation on what it contains.
5. Give several examples of standards used in industry.

1.10 Problems

Conversions and Unit Systems

1. Consult an appropriate source to determine the conversion factors associated with the following conversions:
 - a. number of *dashes* per teaspoon
 - b. number of *teaspoons* per tablespoon
 - c. number of *tablespoons* per cup
 - d. number of *cups* per quart
 - e. number of *quarts* per gallon
2. Consult an appropriate source to determine the meaning of the following units (remember to look up the proper pronunciation as well):
 - a. coomb
 - b. scruple
 - c. cord of wood
 - d. ream of paper
3. Consult an appropriate source to determine the meaning of the following units:
 - a. gill
 - b. degree-day
 - c. ton versus metric ton
 - d. long ton versus short ton
4. How many “hins” are in a “bath”? How many “baths” are in a gallon?
5. What is the relationship between an ounce (16 ounces per pound) and a fluid ounce (16 ounces equals one pint)?
6. What is the origin of the “horsepower”? Why would anyone wish to express power in the unit of horsepower? How many watts are in one horsepower?
7. The unit for volume flow rate is gallons per minute, but cubic feet per second is preferred. Use the conversion factor tables in Appendix A to obtain a conversion between these two units.
8. Which is heavier—a grain or a dram? Express both in the appropriate English fundamental unit.
9. How many years does a furlong designate? Furthermore, if a furlong is a linear measurement, what does this have to do with “years”? Why?
10. What is the difference between troy weight and avoirdupois weight? Express a pound in each of these systems in terms of grains.
11. Is something called a “log” a unit of measure for liquid or dry goods? What is the conversion between a log and the appropriate SI unit?
12. In the plastics industry, what is a gaylord?
13. A shotgun gauge refers to what dimension of the firearm?

14. Nine gage wire and eleven gage wire are both used extensively in making chain link fence material. What are the diameters of these wires? What other wire diameters exist, and what is the standard used in sizing wires?
15. Sheet metal screws are sized according to what standard? What is the difference between a machine screw and a sheet metal screw?

Measurement Scales

16. What is the Forel Ule scale used to measure?
17. What is the Snellen Fraction?
18. The Scoville Unit is a measure of what?
19. What is the Shore Hardness scale used to measure?
20. The Shade number is used as a measure of what?
21. The Beaufort scale measures what?
22. What scale is used to measure the “strength” of an earthquake? What is actually being measured?
23. What is the Numeric Rating Scale (NRS-11) used to measure?
24. To what does “Carat Purity” refer?
25. Canned foods are quite common. How are can sizes measured or expressed by can manufacturers? What are the differences in can size specifications used in the United States? What is used in the metric system?

Miscellaneous Measurements

26. Why is a mile 5 280 ft?
27. By what must a quart be multiplied to obtain a bushel?
28. What is a “hat trick”?
29. How long is the circumference of a racetrack; i.e., how many laps must be made in a one mile race?
30. What is the significance of an acre, and how many square feet are contained in one? Which is larger, an acre or an arpent?
31. What is the definition of the body mass index? What is yours?
32. How many barleycorns are in an inch?
33. How many inches are in a fistmele?

34. Use the conversion factor tables in Appendix A to develop a conversion factor between gallons and cubic feet.
35. What is the equation for calculating the heat index?
36. How long is a “handbreadth”? How long is a “finger”? How many fingers are in a handbreadth?
37. Consult an appropriate source to determine the conversion factors that apply to
 - a. parsecs per mile
 - b. stadia per mile
38. The quantity called a “measure” is used in measuring the capacity of dry goods as well as liquid goods. What is the conversion between a measure and a bushel (dry goods)? What is the conversion between a measure and a gallon?
39. How is blood pressure expressed? Why isn’t the result expressed in psig or kPa?
40. How are shoes sized; i.e., what is the relationship between shoe sizes and any other commonly used unit?

CHAPTER 2 Fluid Properties and Basic Equations

In this chapter, we review some fundamental principles of fluid mechanics. Fluid properties are briefly defined in an effort to refresh the reader's memory and to make the reader familiar with the notation used in this text. The equations of continuity, momentum, and energy and the Bernoulli equation are stated but not derived. Review problems requiring the application of these equations are also provided.

2.1 Fluid Properties

The fluid properties discussed in this section include density, specific gravity, specific weight, absolute or dynamic viscosity, kinematic viscosity, specific heat, internal energy, enthalpy, and bulk modulus. We will also examine some of the common techniques used for measuring selected properties.

Density, Specific Gravity, and Specific Weight

Density of a fluid is defined as its mass per unit volume and is denoted by the letter ρ . Density has dimensions of M/L^3 (lbm/ft^3 or kg/m^3).

Specific gravity of a fluid is the ratio of its density to the density of water at 4°C :

$$\text{Sp. Gr.} = \frac{\rho}{\rho_w} \quad (2.1)$$

where ρ_w is the density of water. Values of specific gravity for various fluids appear in the property tables of the Appendix. It is customary to take water density to be

$$\rho_w = 62.4 \text{ lbm/ft}^3 = 1.94 \text{ slug/ft}^3 = 1\,000 \text{ kg/m}^3$$

Specific weight is a useful quantity related to density. While density is a *mass* per unit volume, specific weight is a *force* per unit volume. Density and specific weight are related by

$$\text{SW} = \frac{\rho g}{g_c} \quad (2.2)$$

The dimension of specific weight is F/L^3 (lbf/ft^3 or N/m^3).

For a liquid, density can be measured directly by weighing a known volume. Specific weight can be determined by submerging an object of known volume into the liquid. The weight of the object in air minus its weight measured while it is submerged gives the buoyant force exerted on the object by the liquid. The buoyant force divided by the volume of the object is the specific weight of the liquid.

In the petroleum industry, specific gravity of a fuel oil is expressed as Sp. Gr. $60^\circ\text{F}/60^\circ\text{F}$. This nomenclature indicates that specific gravity is the ratio of oil density at 60°F to water density at 60°F . *API gravity* is the standard used. The API gravity is related to specific gravity by

$$\text{Sp. Gr. } 60^\circ\text{F}/60^\circ\text{F} = \frac{141.5}{131.5 + ^\circ\text{API}} \quad (2.3)$$

where $^\circ\text{API}$ is read as “degrees API.”

For a gas, specific gravity can be found using any of a number of tests. One such method is the direct weighing procedure in which a volume of gas and an equal volume of air (both at standard conditions for which air properties are known) are collected and weighed. The weight differential allows for calculating the specific gravity of the gas. Other methods involve variations on the theme of measuring a differential weight or mass.

Viscosity

The viscosity of a fluid is a measure of the fluid’s resistance to motion under the action of an applied shear stress. Consider the sketch of Figure 2.1, in which a liquid layer of thickness Δy is between two parallel plates. The lower plate is stationary while the upper plate is being pulled to the right by a force F . The area of contact between the moving plate and the liquid is A , so the applied shear stress is $\tau = F/A$. The liquid continuously deforms under the action of the applied shear stress. The continuous deformation is expressed in terms of a strain rate, which

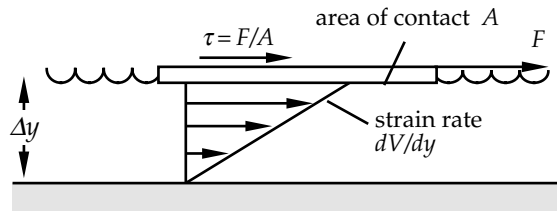


FIGURE 2.1. Definition sketch for viscosity determination.

physically is the slope of the resultant linear velocity distribution within the liquid. At the stationary surface, the liquid velocity is zero, while at the moving plate, the liquid velocity equals the plate velocity. This apparent adhering of the liquid to solid boundaries is known as the **non-slip condition**. For each applied shear stress there will correspond only one strain rate. A series of measurements of forces versus resultant strain rates would yield data for a graph of shear stress versus strain rate. Such a plot is called a **rheological diagram**, an example of which is provided in Figure 2.2. If the liquid between the plates is water or oil, for example, then the line labeled “Newtonian” in Figure 2.2 results. The slope of that line is known as the **dynamic** or **absolute viscosity** of the liquid. For a Newtonian fluid, we have

$$\tau = \mu \frac{dV}{dy} \quad (\text{Newtonian}) \quad (2.4)$$

where μ is the absolute viscosity, τ is the applied shear stress, and dV/dy is the strain rate. The dimension of viscosity is $F \cdot T / L^2$ (lbf·s/ft² or N·s/m²). Other Newtonian fluids are air, oxygen, nitrogen, and glycerine, to name a few.

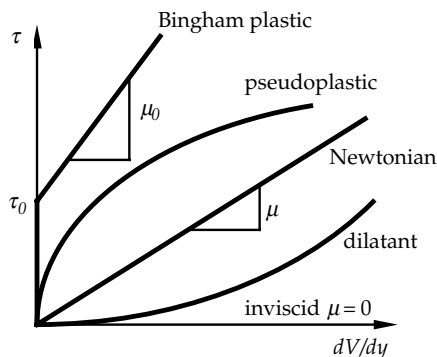


FIGURE 2.2. A rheological diagram characterizing various fluids.

Also shown in Figure 2.2 is the curve for a fluid known as a “Bingham plastic.” Such fluids behave as solids until what is known as the **initial yield stress** of the fluid τ_0 is exceeded. That is, if the fluid in Figure 2.1 is a Bingham plastic, the plate will not move unless the applied shear stress τ exceeds the initial yield stress of the fluid. The initial yield stress is a property just as viscosity is a property. As an example, suppose a jar of peanut butter is inverted. By experience, we know that the peanut butter will not flow out of the jar. The force of gravity does not exert a great enough shear stress on the fluid such that the initial yield stress of the peanut butter is exceeded. If the applied shear stress does exceed the initial shear stress, however, the Bingham plastic behaves like a Newtonian fluid, or in a number of cases, like a pseudoplastic fluid. The Bingham plastics depicted in Figure 2.2 may be described by

$$\tau = \tau_0 + \mu_0 \frac{dV}{dy} \quad (\text{Bingham plastic}) \quad (2.5)$$

where μ_0 is the apparent viscosity. Chocolate mixtures, some greases, paints, paper pulp, drilling muds, soap, and toothpaste are examples of Bingham plastics.

Fluids that exhibit a decrease in viscosity with increasing shear stress are known as **pseudoplastic fluids**. If the fluid in Figure 2.1 is pseudoplastic, then the resistance to motion (e.g., viscosity) will decrease with increasing shear stress. Some greases, mayonnaise, and starch suspensions are examples of such fluids. A power law equation, called the Ostwald-deWaele equation, describes the curve as

$$\tau = K \left(\frac{dV}{dy} \right)^n \quad (n < 1 \text{ pseudoplastic}) \quad (2.6)$$

where K is called a **consistency index** with dimensions of $\text{F} \cdot \text{T}^n / \text{L}^2$ ($\text{lbf} \cdot \text{s}^n / \text{ft}^2$ or $\text{N} \cdot \text{s}^n / \text{m}^2$) and n is a dimensionless **flow behavior index**. As mentioned earlier, a pseudoplastic fluid exhibits less resistance to flow as shear stress increases. So pumping a pseudoplastic fluid at a high flow rate (corresponding to a high shear stress) would involve smaller frictional effects than a low flow rate would.

Fluids that exhibit an increase in viscosity with increasing shear stress are known as **dilatant fluids**. Wet beach sand, starch in water, and water solutions containing a high concentration of powder are examples of such fluids. Again, a power law equation describes the curve as

$$\tau = K \left(\frac{dV}{dy} \right)^n \quad (n > 1 \text{ dilatant}) \quad (2.7)$$

A dilatant fluid's resistance to flow increases with flow rate or rate of shear, just opposite to the behavior of pseudoplastic fluids.

Other types of fluids not shown in Figure 2.2 are rheopectic, thixotropic, and viscoelastic fluids. For a **rheopectic fluid**, the applied shear stress would have to increase with time to maintain a constant strain rate—a gypsum suspension is an example. With a **thixotropic fluid**, the applied shear stress would decrease with time to maintain a constant strain rate—liquid foods and shortening are examples. A **viscoelastic fluid** exhibits elastic and viscous properties. Such fluids partly recover from deformations caused during flow—flour dough is an example.

Kinematic Viscosity

In many equations of fluid mechanics, the term $\mu g_c / \rho$ appears frequently. This ratio is called the kinematic viscosity ν which has dimensions of L^2/T (ft^2/s or m^2/s).

EXAMPLE 2.1. Tomato paste was tested in a viscometer, and the following data were obtained. Determine if the fluid is Newtonian and its descriptive equation.

τ	(N/m ²)*	51	71.6	90.8	124.0	162.0
dV/dy	(rad/s)	0.95	4.7	12.3	40.6	93.5

Solution: A plot of these data appears in Figure 2.3. The fluid is pseudoplastic, based on its shear stress-strain rate curve, so we assume that Equation 2.6 applies:

$$\tau = K \left(\frac{dV}{dy} \right)^n$$

It is possible to follow a statistical approach (i.e., the least squares method) and use a calculator to determine the values of K and n . If this is done, the resulting equation is:

$$\tau = 49.3 \left(\frac{dV}{dy} \right)^{0.257}$$

Alternatively, the shear stress-strain rate data can be entered into a

* Data from *Fundamentals of Food Engineering* by Stanley E. Charm, 2nd ed., Avi Publishing Co., Westport, Conn., 1971, p. 62.

spreadsheet (preferred method). A trend line and power law equation can be obtained in moments. The results are:

$$\tau = 49.93 \left(\frac{dV}{dy} \right)^{0.250} \quad (\text{spreadsheet})$$

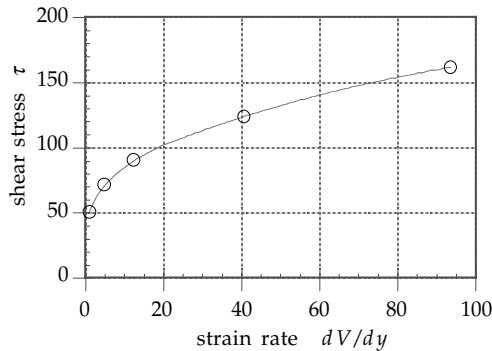


FIGURE 2.3. Graph of tomato paste data.

Compressibility Factor

The compressibility factor is a property that describes the change in density experienced by a liquid during a change in pressure. The compressibility factor is given by

$$\beta = -\frac{1}{\mathcal{V}} \left(\frac{\partial \mathcal{V}}{\partial p} \right)_T \quad (2.8)$$

where $\mathcal{V}$ is the volume, $\partial \mathcal{V} / \partial p$ is the change in volume with respect to pressure, and the subscript indicates that the process is to occur at constant temperature. Liquids in general are incompressible. Water, for example, experiences only a 1% change in density for a corresponding tenfold pressure increase.

Internal Energy

Internal energy (represented by the letter U) is the energy associated with the motion of the molecules of a substance. An increase in the internal energy of a substance is manifested usually as an increase in temperature. Internal energy has dimensions of F·L. Internal energy is often expressed on a per unit mass basis ($u = U/m$) and has dimensions of F·L/M (ft·lbf/lbm or BTU/lbm or N·m/kg).

Enthalpy

Enthalpy is defined as the sum of internal energy and flow work:

$$H = U + pV$$

On a per unit mass basis, we have

$$h = u + pv = u + \frac{p}{\rho}$$

where $u = U/m$ is specific internal energy, $h = H/m$ is specific enthalpy, and $v = V/m$ is specific volume. The sum of internal energy and flow work appears in the energy equation and the introduction of enthalpy is a simplification. Note that enthalpy is merely a combination of known properties. Enthalpy per unit mass has the same dimensions as internal energy $F \cdot L / M$ (ft·lbf/lbm, or BTU/lbm, or N·m/kg).

Pressure

Pressure at a point is a time-averaged normal force exerted by molecules impacting a unit surface. The area must be small, but large enough, compared to molecular distances, to be consistent with the continuum approach. Thus, pressure is defined as

$$p = \lim_{A \rightarrow A^*} \frac{F}{A}$$

where A^* is a small area experiencing enough molecular collisions to be representative of the fluid bulk, and F is the time-averaged normal force. Note that if A^* were to shrink to zero, then it is possible that no molecules would strike it, yielding a zero normal force and a definition of pressure that has little physical significance. The dimensions of pressure are F/L^2 (lbf/ft² or psi in the English system and Pa = 1 N/m² in SI).

2.2 Measurement of Viscosity

Viscosity can be measured with a number of commercially available viscometers or viscosimeters. Each works on basically the same principle. A laminar motion of the fluid is caused and suitable measurements are taken. For the created laminar conditions, an exact solution to the equation of motion will exist that relates the viscosity to the geometry of the device and to the data obtained.

One such device consists of two concentric cylinders, one of which is free to rotate (see Figure 2.4). Liquid is placed in the annulus between the two

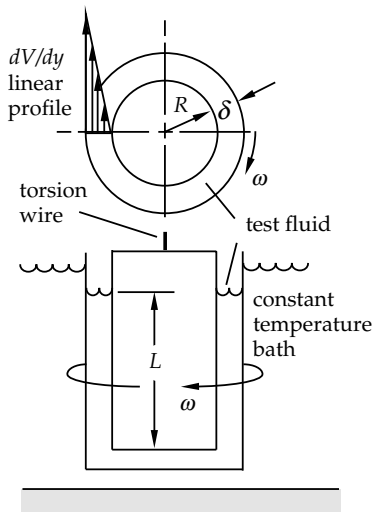


FIGURE 2.4. A narrow gap-rotating-cup viscometer.

cylinders. The outer cylinder is rotated at a known and carefully controlled angular velocity, and the momentum of the outer cylinder is transported through the liquid. In turn, a torque is exerted on the inner cylinder. The inner cylinder might be suspended by a torsion wire or a spring that measures the angular deflection caused by the fluid. The dynamic viscosity μ is proportional to the torque transmitted. If the gap between the cylinders is very narrow, no more than 0.1 of the inner cylinder radius, then the velocity distribution of the fluid in the annulus is approximately linear. Otherwise, the velocity distribution is better described as a parabola. For a linear velocity profile, we have

$$\begin{aligned}\tau &= \mu \frac{dV}{dy} = \mu \frac{V}{\delta} \\ \tau &= \frac{\mu(R + \delta)\omega}{\delta}\end{aligned}\tag{2.9}$$

where ω is the rotational speed. The torque exerted on the inner cylinder is

$$T = \text{shear force} \times \text{distance} = \text{shear stress} \times \text{area} \times \text{distance}$$

$$T = \tau(2\pi RL)R$$

The shear stress in terms of torque then is

$$\tau = \frac{T}{2\pi R^2 L}$$

Substitution into Equation 2.9 gives

$$\frac{T}{2\pi R^2 L} = \frac{\mu(R + \delta)\omega}{\delta}$$

Solving for viscosity, we get

$$\mu = \frac{T\delta}{2\pi R^2(R + \delta)L\omega} \quad (2.10)$$

All parameters on the right-hand side of the preceding equation, except torque T and rotational speed ω , are geometric terms. Torque and rotational speed are the only dynamic measurements required.

Another device for measuring viscosity is called the **capillary viscometer** (see Figure 2.5). This device consists of a glass tube of small diameter etched at three locations. By applying a vacuum to the right side, the liquid level is raised until it just reaches the uppermost etched line. At this point the liquid is released and allowed to flow under the action of gravity. The time required for the liquid level to fall from the middle to the lowest etched line is measured. Obviously, this problem is unsteady; nevertheless, acceptable results are obtained.

The volume of liquid involved is called the **efflux volume** $\mathcal{V}$ which is carefully measured for the experiment. The average flow rate through the capillary tube is

$$Q = \frac{\mathcal{V}}{t}$$

where t is the time recorded for the fluid to flow through the tube. For laminar flow conditions,

$$Q = \left(-\frac{dp}{dz}\right) \frac{\pi R^4}{8\mu} \quad (2.11)$$

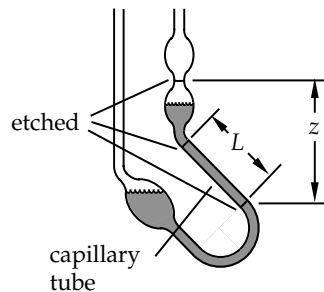


FIGURE 2.5. A capillary tube viscometer.

where R is the tube radius and $(-dp/dz)$ is a positive pressure drop

$$\left(-\frac{dp}{dz}\right) = \frac{p_2 - p_1}{L}$$

The pressure drop equals the available hydrostatic head, which contains the gravity term (gravity is the driving force):

$$p_2 - p_1 = \frac{\rho g}{g_c} z$$

Substituting into the volume flow equation gives

$$\frac{V}{t} = \frac{\rho g}{g_c} \frac{z}{L} \frac{\pi R^4}{8\mu}$$

Solving for viscosity, we get

$$\frac{\mu g_c}{\rho} = \nu = \left(\frac{z \pi R^4 g}{8 L V} \right) t \quad (2.12)$$

For a given viscometer, the quantity in parentheses (a geometric quantity) is a constant. So kinematic viscosity is proportional to the time.

EXAMPLE 2.2. Figure 2.6 shows another version of the capillary tube viscometer. In this case, the tube is horizontally oriented, and it is instrumented. Pressure is measured at two points a distance L apart, and flow rate is measured with a flow meter. For the following conditions, determine the viscosity of the liquid flowing through the tube:

$$\begin{aligned} p_1 - p_2 &= 1.4 \text{ kPa} & L &= 15 \text{ cm} & D &= 0.775 \text{ cm} \\ \rho &= 1100 \text{ kg/m}^3 & Q &= 5 \times 10^{-5} \text{ m}^3/\text{s} \end{aligned}$$

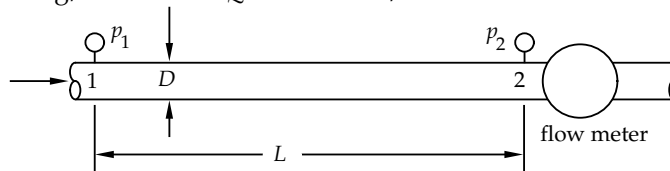


FIGURE 2.6. Flow through a small diameter tube.

Solution: If laminar conditions exist in the tube, then Equation 2.11 applies:

$$Q = \left(-\frac{dp}{dz} \right) \frac{\pi R^4}{8\mu}$$

Rearranging, solving for μ , and substituting gives

$$\mu = \frac{\Delta p}{L} \frac{\pi R^4}{8Q} = \frac{1\,400}{0.15} \frac{\pi(0.007\,75/2)^4}{8(5 \times 10^{-5})}$$

or $\mu = 1.65 \times 10^{-2} \text{ N}\cdot\text{s}/\text{m}^2$

This result is valid only if

$$\frac{\rho V D}{\mu g_c} < 2\,100$$

The volume flow rate through the tube is divided by area to obtain average velocity

$$V = \frac{Q}{A} = \frac{4Q}{\pi D^2} = \frac{4(5 \times 10^{-5})}{\pi(0.007\,75)^2}$$

or $V = 1.06 \text{ m/s}$

Substituting,

$$\frac{\rho V D}{\mu g_c} = \frac{1\,100(1.06)(0.007\,75)}{1.65 \times 10^{-2}} = 547$$

which is less than 2 100, so our result is valid. Thus

$$\mu = 1.65 \times 10^{-2} \text{ N}\cdot\text{s}/\text{m}^2$$

A **falling sphere viscometer** is an inexpensive device that can be used to measure the viscosity of a transparent or semi-transparent liquid. It consists of a vertically oriented tube sealed at its bottom and filled with the liquid. A sphere is dropped into the liquid, and the time it takes for the sphere to travel a pre-determined distance is measured.

When a sphere that starts from rest falls through a fluid (liquid or gas), the sphere accelerates and eventually reaches a constant speed known as the **terminal velocity**. With the falling sphere viscometer, what is actually being measured is the terminal velocity (= predetermined distance divided by the time).

Figure 2.7 shows a free-body diagram of a non-accelerating, submerged

sphere. The forces acting are due to gravity, buoyancy, and friction (or drag). A force balance gives

$$\text{Weight} - \text{Buoyancy} - \text{Drag} = 0$$

$$\frac{mg}{g_c} - \frac{\rho \mathcal{V} g}{g_c} - 3\pi D V \mu = 0 \quad (2.13)$$

where $\mathcal{V}$ is the sphere volume, $\rho \mathcal{V}$ is the mass of fluid displaced by the sphere, V is the terminal velocity, and D is the sphere diameter. The expression for drag in the preceding equation is valid only if

$$\frac{\rho V D}{\mu g_c} < 1$$

Writing the weight of the sphere in terms of its density ρ_s , we have $mg = \rho_s \mathcal{V} g$; the volume of a sphere is $\pi D^3/6$. Substituting into Equation 2.13, rearranging and solving for viscosity gives

$$\begin{aligned} \rho_s \frac{\pi D^3}{6} \frac{g}{g_c} - \rho \frac{\pi D^3}{6} \frac{g}{g_c} - 3\pi D V \mu &= 0 \\ \mu &= \left(\frac{\rho_s}{\rho} - 1 \right) \frac{\rho g D^2}{g_c 18 V} \end{aligned} \quad (2.14)$$

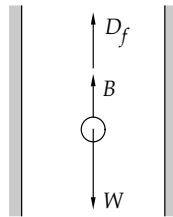


FIGURE 2.7. Forces acting on sphere falling at terminal velocity.

EXAMPLE 2.3. A falling sphere viscometer is used to measure the viscosity of honey, whose specific gravity is 1.18. The viscometer consists of a 4 in. inside diameter, vertically oriented plastic cylinder that is filled with honey. A 1/4 in. ball bearing (Sp. Gr. = 7.9) is dropped into the liquid and the time required for it to fall 3 ft is 7.5 s. Determine the viscosity of the honey.

Solution: Equation 2.14 applies if $\rho V D / \mu g_c < 1$. This condition must be checked after the viscosity is calculated. Equation 2.14 is

$$\mu = \left(\frac{\rho_s}{\rho} - 1 \right) \frac{\rho g}{g_c} \frac{D^2}{18V}$$

The terminal velocity is

$$V = \frac{3 \text{ ft}}{7.5 \text{ s}} = 0.4 \text{ ft/s}$$

Substituting,

$$\mu = \left(\frac{7.9(1.94)}{1.18(1.94)} - 1 \right) (1.18)(1.94)(32.2) \frac{(0.25/12)^2}{18(0.4)}$$

Solving, the viscosity then is

$$\mu = 2.53 \times 10^{-2} \text{ lbf}\cdot\text{s}/\text{ft}^2$$

We next calculate

$$\frac{\rho V D}{\mu g_c} = \frac{1.18(1.94)(0.4)(0.25/12)}{2.53 \times 10^{-2}} = 0.754$$

which is less than 1, so our result is acceptable. Thus,

$$\mu = 2.53 \times 10^{-2} \text{ lbf}\cdot\text{s}/\text{ft}^2$$

There are other types of viscometers, such as the **cone and plate viscometer** and the **wide-gap concentric-cylinder viscometer**. In addition to these types, other types of viscometers have been devised. Various industries have adopted different standards and methods for measuring viscosity. We examine two types—the Saybolt viscometer and the Stormer viscometer.

Saybolt Viscometer

In the oil industry, liquid viscosity is measured with a Saybolt viscometer, illustrated schematically in Figure 2.8. Oil (a Newtonian fluid) to be tested is placed in the central cup, which is surrounded by an oil bath. The oil bath controls the test oil temperature. When the desired temperature is reached, a stopper in the bottom of the cup is removed and test oil flows out. The time required for 60 ml of oil to pass through a

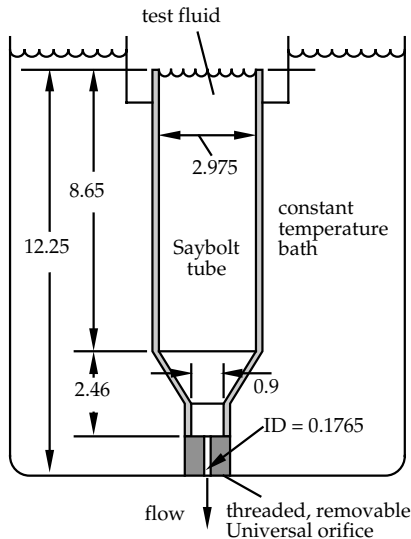


FIGURE 2.8. Schematic of a Saybolt viscometer. (Dimensions in cm.)

standard orifice into a calibrated flask is measured. The viscosity is then expressed in terms of **Saybolt Universal Seconds (SUS)**. An orifice known as a **Universal-type orifice** is used for lighter oils, and a larger orifice called a **Furol orifice** is used for heavier oils. A reading in Saybolt Universal seconds can be converted into units of kinematic viscosity by the following equations:

a) $34 < \text{SUS} < 115$

$$\nu \text{ (m}^2\text{/s)} = 0.224 \times 10^{-6} (\text{SUS}) - \frac{185 \times 10^{-6}}{(\text{SUS})} \quad (2.15a)$$

b) $115 < \text{SUS} < 215$

$$\nu \text{ (m}^2\text{/s)} = 0.223 \times 10^{-6} (\text{SUS}) - \frac{155 \times 10^{-6}}{(\text{SUS})} \quad (2.15b)$$

c) $\text{SUS} > 215$

$$\nu \text{ (m}^2\text{/s)} = 0.2158 \times 10^{-6} (\text{SUS}) \quad (2.15c)$$

Stormer Viscometer

Figure 2.9 is a schematic of the Stormer viscometer, which is used in the paint industry to measure the viscosity of paint (actually consistency). A stainless steel rod with two paddles on it is submerged into a 1 pint of

paint. The rod attaches to a gear box. The rotation of the rod is caused by a falling weight that pulls a string which, in turn, rotates a drum. The drum rotation is connected to the rod through the gear box. A counter on top of the gear box registers the number of rotations made by the rod. The method of obtaining data is to allow the weight to fall and cause the rod to rotate for 100 revolutions while submerged in the paint. The time for 100 revolutions is then used with a pre-determined scale to determine the viscosity, which is expressed in what is known as **Krebs units**. Determining the viscosity in terms of $\text{N}\cdot\text{s}/\text{m}^2$ is not important in using this device. Consistency and repeatability are the factors that are essential in evaluating paints and related coatings. A number of recent modifications have been made to this device. These include replacing the falling weight/pulley system with a constant speed motor and a digital measurement of the torque. The results are calibrated to give the viscosity in Krebs units or in units of kinematic viscosity.

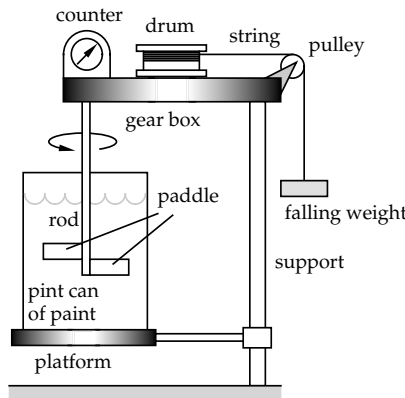


FIGURE 2.9. *Schematic of Stormer viscometer.*

2.3 Measurement of Pressure

We will now examine classical methods and devices available for measuring pressure: pressure gages and manometers.

A pressure gage (Figure 2.10) consists of a housing with a fitting for attaching it to a pressure vessel. Inside the housing is a curved elliptical tube called a Bourdon tube. This tube is connected to the fitting at one end and to a rack and pinion assembly at its other end. When exposed to high pressure, the tube tends to straighten, in turn pulling the rack and rotating the pinion gear. The shaft of the pinion gear extends through the face of the gage. A needle is pressed or bolted onto the shaft. When the gage is exposed to atmospheric pressure, the face is calibrated to read 0 (zero) because the gage itself really measures the pressure difference from inside the tube to the outside. The reading from a gage is appropriately termed *gage pressure*.

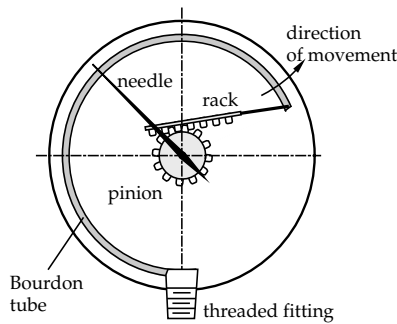


FIGURE 2.10. *Schematic of a pressure gage.*

Absolute pressure, on the other hand, would read zero only in a complete vacuum. Thus, gage and absolute pressures are related by

$$\text{Absolute pressure} = \text{Gage pressure} + \text{Atmospheric pressure}$$

In engineering units, to denote that gage pressure is being reported, the notation “psig” (pounds per square inch gage) is used. The unit “psia” (pounds per square inch absolute) is used when reporting absolute pressure. In SI, when gage pressure is being reported, the phrase “a gage pressure of...” is used, with a similar phrase for reporting absolute pressure.

Atmospheric pressure can be measured with a barometer, which consists of a sufficiently long tube that is inverted while submerged and full of liquid. A vacuum is created above the liquid column in the tube. (See Figure 2.11.) The height of liquid above the reservoir surface is related to atmospheric pressure by the hydrostatic equation

$$p_{\text{atm}} = \frac{\rho g}{g_c} z \quad (\text{Figure 2.11})$$

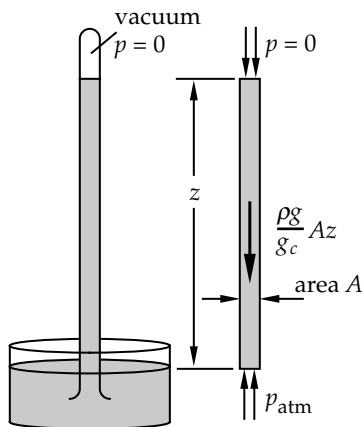


FIGURE 2.11. *A barometer.*

Atmospheric pressure in this text is taken to be 14.7 psia or 101.3 kPa (absolute).

Pressure differences can be measured with vertical columns of liquid. One device that can be used for this measurement is called a *manometer*. Manometers can be set up in a number of configurations depending on the application and on how tall the columns of liquid can be. Figure 2.12 illustrates a U-tube manometer configuration. One leg of the manometer is attached to a pressure vessel, and the other leg is open to the atmosphere. Applying the hydrostatic equation ($p = \rho g \Delta z / g_c$) to each leg of the manometer gives

$$p_A + \frac{\rho_1 g}{g_c} z_1 = p_B$$

$$p_B = p_C$$

$$\text{and} \quad p_C = p_D + \frac{\rho_2 g}{g_c} z_2 = p_{\text{atm}} + \frac{\rho_2 g}{g_c} z_2$$

Combining the preceding equations, we get

$$p_A - p_{\text{atm}} = (\rho_2 z_2 - \rho_1 z_1) \frac{g}{g_c} \quad (\text{Figure 2.12}) \quad (2.16)$$

Another manometer configuration is shown in Figure 2.13. In this case, a U-tube manometer is used to measure the pressure difference between two vessels. Applying the hydrostatic equation, we obtain

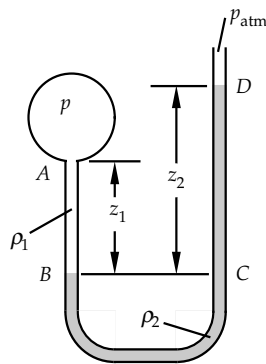


FIGURE 2.12. U-tube manometer for measuring pressure in a vessel.

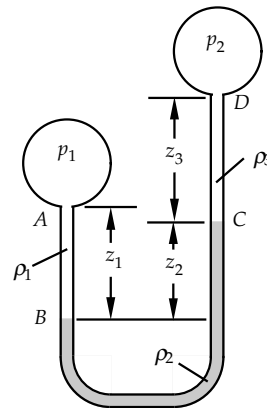


FIGURE 2.13. U-tube manometer for measuring pressure difference between two vessels.

$$p_A + \frac{\rho_1 g}{g_c} z_1 = p_B$$

$$p_D + \frac{\rho_3 g}{g_c} z_3 + \frac{\rho_2 g}{g_c} z_2 = p_B$$

Combining, we get

$$p_A - p_D = (\rho_3 z_3 + \rho_2 z_2 - \rho_1 z_1) \frac{g}{g_c} \quad (\text{Figure 2.13}) \quad (2.17)$$

A third manometer configuration is shown in Figure 2.14. An inverted U-tube is used to measure the pressure difference between two vessels. Applying the hydrostatic equation, we write

$$p_B + \frac{\rho_1 g}{g_c} z_1 = p_A$$

and
$$p_B + \frac{\rho_2 g}{g_c} z_2 + \frac{\rho_3 g}{g_c} z_3 = p_D$$

Combining and simplifying, we find

$$p_A - p_D = (\rho_1 z_1 - \rho_2 z_2 - \rho_3 z_3) \frac{g}{g_c} \quad (\text{Figure 2.14}) \quad (2.18)$$

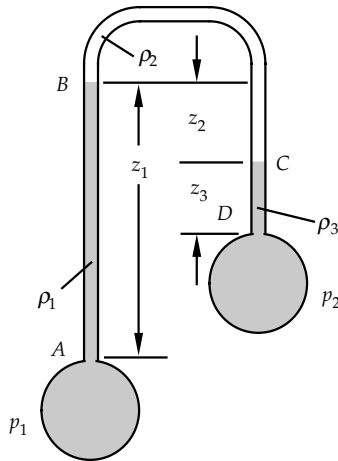


FIGURE 2.14. An inverted U-tube differential manometer.

EXAMPLE 2.4. Figure 2.15 shows a venturi meter with an air-over-liquid manometer attached. The meter is inserted into a pipeline that conveys a

liquid of density ρ . Determine the pressure drop from location 1 to location 2 (where the manometer legs attach).

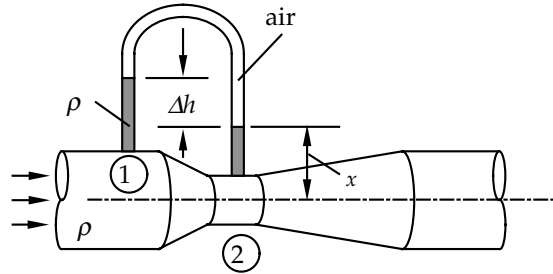


FIGURE 2.15. *Venturi meter with inverted U-tube manometer attached.*

Solution: For the required derivation, we define a distance x from the meter centerline to the lowest liquid interface in either leg. We make all vertical measurements from the meter centerline and apply the hydrostatic equation to obtain

$$p_1 - \frac{\rho g}{g_c} x - \frac{\rho g}{g_c} \Delta h = p_2 - \frac{\rho g}{g_c} x - \frac{\rho_{\text{air}} g}{g_c} \Delta h$$

We see that the terms containing x cancel so that x is arbitrary. In addition, the density of air is assumed to be much smaller than that of the liquid, so the term containing ρ_{air} is negligible. Rearranging and solving for pressure drop gives

$$\underline{p_1 - p_2 = \frac{\rho g}{g_c} \Delta h}$$

EXAMPLE 2.5. Figure 2.16 shows several manometers containing different fluids and attached together. For the configuration shown, determine the pressure in the water tank at A. (All dimensions are in inches.)

Solution: We can apply the hydrostatic equation directly beginning at A and ending at atmospheric pressure:

$$p_A + \rho_{\text{H}_2\text{O}} g (6/12) = p_B$$

$$p_B = p_C + \rho_{\text{oil}} g (7/12)$$

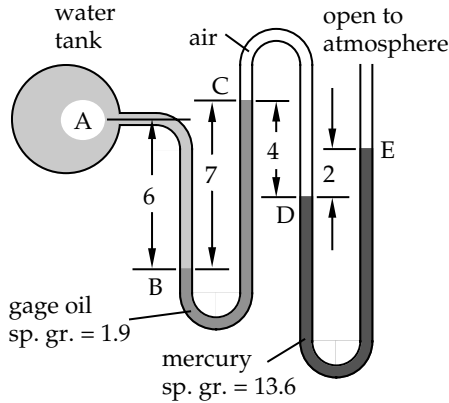


FIGURE 2.16. Several manometers attached together.

$$p_D = p_C + \rho_{\text{air}} g(4/12)$$

$$p_D = p_E + \rho_{\text{Hg}} g(2/12)$$

Combining gives

$$p_A + \rho_{\text{H}_2\text{O}} g(6/12) = p_E + \rho_{\text{Hg}} g(2/12) - \rho_{\text{air}} g(4/12) + \rho_{\text{oil}} g(7/12)$$

Noting that $p_E = p_{\text{atm}} = (14.7 \text{ lbf/in}^2)(144 \text{ in}^2/\text{ft}^2)$ and that the air density ρ_{air} is negligible compared to the other fluid densities, the preceding equation becomes

$$p_A = 14.7(144) - \rho_{\text{H}_2\text{O}} g(6/12) + \rho_{\text{Hg}} g(2/12) + \rho_{\text{oil}} g(7/12)$$

Substituting, we get

$$p_A = 14.7(144) - 1.94(32.2)(6/12) + 13.6(1.94)(32.2)(2/12) + 1.9(1.94)(32.2)(7/12)$$

or $\underline{p_A = 2296 \text{ lbf/ft}^2 = 16.0 \text{ psia}}$

2.4 Basic Equations of Fluid Mechanics

In this section, we will discuss the definitions associated with fluid flow. We will also write the equations of fluid mechanics including continuity, momentum, energy, and the Bernoulli equation.

Flows can be characterized according to their geometries. **Closed conduit flows** are those that are completely enclosed by a restraining solid surface, such as flow in a pipe. **Open channel flows** are those that have a surface exposed to atmospheric pressure, such as flow in a river. **Unbounded flows** are those in which the fluid is not in contact with any solid surface, such as the flow that issues from a can of spray paint.

Flows can be classified according to how we mathematically describe gradients in the flow field. If the velocity of the fluid is constant at any cross section normal to the flow, or if the velocity is represented by an average value, the flow is said to be **one dimensional**. Although the velocity is constant, there will exist a driving force that changes with the flow direction. In pipe flow, for example, a pressure gradient dp/dz exists in the axial (or z) direction. The pressure gradient is what causes the fluid to flow. A pressure gradient and a constant velocity at any cross section are considered one dimensional (only one gradient). A pressure gradient dp/dz with a velocity profile that varies with only one space variable is a two-dimensional flow (two gradients, $p(z)$ and $V(r)$, for example). The definition is easily extended to three-dimensional flows.

Flows can be described as being **steady**, **unsteady**, or **quasi steady**. Steady flows have conditions that do not vary with time. Unsteady flows have conditions that do vary with time. Quasi steady flows are actually unsteady but because they proceed so slowly, they can be treated mathematically as if they were steady.

A fluid while flowing can be subjected to variations in pressure. If fluid density changes significantly as a result of pressure variations, then the fluid is considered to be **compressible**. If density remains practically unchanged with variations in pressure, then the fluid is **incompressible**. Usually, gases and vapors are compressible while liquids are incompressible. These two cases are treated differently mathematically.

We will use the **control volume approach** to model problems. We will select a region of study within the flow field and apply equations to that region. The control volume is bounded by what is called the **control surface**. Everything outside the control volume is the **surroundings**. Where to place the boundary of the control volume to best advantage is largely a matter of experience, but general guidelines will be presented where appropriate.

Continuity Equation

The continuity equation is a statement of conservation of mass. For a control volume we can write

$$\left(\begin{array}{c} \text{rate of} \\ \text{mass in} \end{array} \right) = \left(\begin{array}{c} \text{rate of} \\ \text{mass out} \end{array} \right) + \left(\begin{array}{c} \text{rate of} \\ \text{mass stored} \end{array} \right)$$

or

$$0 = \left(\begin{array}{c} \text{rate of} \\ \text{mass stored} \end{array} \right) + \left(\begin{array}{c} \text{net mass out} \\ \text{out minus in} \end{array} \right)$$

In equation form, we have

$$0 = \frac{\partial m}{\partial t} \Big|_{\text{CV}} + \int_{\text{CS}} \rho V_n dA \quad (2.19)$$

where $\partial m / \partial t$ is the time rate of change of mass within the control volume per unit time, the integral term is to be applied at places where mass crosses the control surface, ρ is the fluid density, and V_n is the velocity normal to the control surface integrated over the area dA . For a one-dimensional ($V_n = \text{a constant}$), steady ($\partial m / \partial t = 0$) flow, we have

$$\sum_{\text{inlets}} \rho AV = \sum_{\text{outlets}} \rho AV \quad (2.20)$$

The product ρAV is often called the **mass flow rate** $\dot{m}$ with dimensions of M/T (lbm/s or kg/s). Furthermore, if the flow is incompressible, Equation 2.20 becomes

$$\sum_{\text{inlets}} AV = \sum_{\text{outlets}} AV \quad (2.21)$$

The product AV is called the **volume flow rate** Q with dimensions of L³/T (ft³/s or m³/s).

EXAMPLE 2.6. Benzene flows through a converging duct, as indicated in Figure 2.17. The diameters at sections 1 and 2 are 7 cm and 3 cm, respectively. For a mass flow rate of benzene equal to 1 kg/s, determine the velocity at 1 and 2.

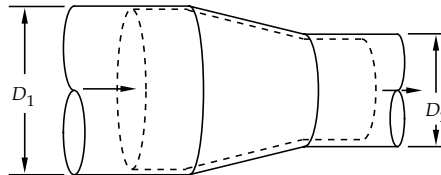


FIGURE 2.17. Flow through a converging duct.

Solution: The specific gravity of benzene is 0.876 from Appendix Table B.1. For a one-dimensional, steady flow through a system, we write

$$\dot{m} = (\rho AV)_1 = (\rho AV)_2$$

At section 1, the cross-sectional area is

$$A_1 = \frac{\pi D^2}{4} = \frac{\pi(0.07)^2}{4} = 3.84 \times 10^{-3} \text{ m}^2$$

Similarly,

$$A_2 = \frac{\pi(0.03)^2}{4} = 7.07 \times 10^{-4} \text{ m}^2$$

The velocity at section 1 then is

$$V_1 = \frac{\dot{m}}{\rho A_1} = \frac{1}{0.876(1\,000)(3.84 \times 10^{-3})}$$

$$\underline{V_1 = 0.3 \text{ m/s}}$$

At section 2,

$$V_2 = \frac{\dot{m}}{\rho A_2} = \frac{1}{0.876(1\,000)(7.07 \times 10^{-4})}$$

$$\underline{V_2 = 1.61 \text{ m/s}}$$

EXAMPLE 2.7. Figure 2.18 shows a tank being filled with kerosene issuing from two pipes, while a third pipe is simultaneously draining the tank. The pipe at A (ID = 0.1723 ft) discharges kerosene at 6 ft/s. The pipe at B (ID = 0.08742 ft) discharges kerosene into the tank at a velocity of 6.5 ft/s. The kerosene velocity in the discharge pipe (ID = 0.1342 ft) is 1 ft/s. Under these conditions, determine the time it takes for the kerosene level to change from a depth of 1 ft to 4 ft. The tank diameter is 10 ft.

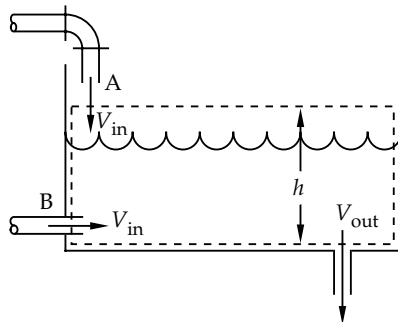


FIGURE 2.18. A tank with two inlets and one drain.

Solution: From Table Appendix B.1, the specific gravity of kerosene is 0.823. The unsteady form of the continuity equation applies in this case:

$$0 = \frac{\partial m}{\partial t} \Big|_{CV} + \int_{CS} \rho V_n dA$$

We apply this equation to the control volume of Figure 2.18 such that at places where mass crosses the control surface, it does so at a right angle. Thus the velocities entering and leaving the control volume are normal to the control surface. The mass of kerosene in the tank at any time is

$$m = \rho V$$

The volume of liquid in the tank in terms of depth is

$$V = \frac{\pi D^2}{4} h = \frac{\pi (10)^2}{4} h$$

$$V = 78.5h$$

and the mass of kerosene is

$$m = 0.823(1.94)(78.5h) = 125.4h$$

The unsteady term in the continuity equation becomes

$$\frac{\partial m}{\partial t} \Big|_{CV} = 125.4 \frac{dh}{dt}$$

The integral term in the continuity equation is evaluated next:

$$\int_{CS} \rho V_n dA = \int_{out} \rho V_n dA - \int_{in} \rho V_n dA$$

At the outlet pipe, density is constant; the velocity normal to the control surface V_n is constant at 1 ft/s. We therefore have

$$\int_{out} \rho V_n dA = \rho V_n \int_{out} dA = (\rho A V)_{outlet}$$

Substituting,

$$(\rho AV)_{\text{outlet}} = 0.823(1.94) \frac{\pi(0.1342)^2}{4} (1)$$

$$(\rho AV)_{\text{outlet}} = 0.0226 \text{ slug/s}$$

Following similar lines of reasoning, at A we have

$$(\rho AV)_A = 0.823(1.94) \frac{\pi(0.1723)^2}{4} (6) = 0.223 \text{ slug/s}$$

At B,

$$(\rho AV)_B = 0.823(1.94) \frac{\pi(0.08742)^2}{4} (6.5) = 0.0623 \text{ slug/s}$$

Substituting these values into the unsteady continuity equation gives

$$0 = 125.4 \frac{dh}{dt} + 0.0226 - (0.223 + 0.0623)$$

Rearranging and simplifying,

$$\frac{dh}{dt} = 2.09 \times 10^{-3}$$

Separating variables and integrating from $h = 1$ to 4, corresponding to $t = 0$ to t , we have

$$\int_1^4 dh = 2.09 \times 10^{-3} \int_0^t dt$$

$$4 - 1 = 2.09 \times 10^{-3} t$$

Solving,

$$t = 1432 \text{ s} = 23.9 \text{ min} = 0.4 \text{ hr}$$

EXAMPLE 2.8. An air compressor is used to pressurize a tank. The tank volume is 30 ft³ and the air temperature in the tank is a constant at 75°F. The air supply line has an inside diameter of 0.08742 ft. The compressor

output provides air at 35 psia and 75°F (with the use of an intercooler). Calculate the time required for the tank pressure to change from 10 psia to 20 psia if the velocity of air in the inlet line is 5 ft/s.

Solution: The control volume we select for analysis is the tank itself. The unsteady form of the continuity equation is

$$0 = \frac{\partial m}{\partial t} \Big|_{\text{CV}} + \int_{\text{CS}} \rho V_n dA$$

The mass of air in the tank at any time can be estimated with the ideal gas law

$$m = \frac{pV}{RT}$$

For constant volume and temperature, the unsteady term in the continuity equation becomes

$$\frac{\partial m}{\partial t} \Big|_{\text{CV}} = \frac{V}{RT} \frac{dp}{dt}$$

The integral term in the continuity is evaluated next:

$$\int_{\text{CS}} \rho V_n dA = \int_{\text{out}} \rho V_n dA - \int_{\text{in}} \rho V_n dA$$

No mass leaves the tank. Entering the tank is air at a constant temperature and pressure. The preceding equation becomes

$$\int_{\text{CS}} \rho V_n dA = 0 - \int_{\text{in}} \frac{p}{RT} V_n dA$$

For constant temperature and pressure, we have

$$\int_{\text{CS}} \rho V_n dA = \frac{p_{\text{in}}}{RT_{\text{in}}} V_{\text{in}} A_{\text{in}}$$

where p_{in} equals the pressure in the inlet line. Substituting into the unsteady continuity equation, we obtain

$$\frac{\cancel{V}}{RT} \frac{dp}{dt} = \frac{p_{\text{in}}}{RT_{\text{in}}} V_{\text{in}} A_{\text{in}}$$

With $RT_{\text{in}} = RT$, we rearrange to get

$$dp = \frac{p_{\text{in}}}{\cancel{V}} V_{\text{in}} A_{\text{in}} dt$$

Substituting,

$$dp = \frac{35(144)}{30} (5) \frac{\pi(0.08742)^2}{4} dt$$

$$\text{or } dp = 5.042 dt$$

Integrating,

$$\int_{10(144)}^{20(144)} dp = 5.042 \int_0^t dt$$

$$20(144) - 10(144) = 5.042t$$

Solving,

$$\underline{t = 285.6 \text{ s} = 4.76 \text{ min}}$$

Momentum Equation

The momentum equation is a statement of conservation of linear momentum. For a control volume in Cartesian coordinates, we have

$$\Sigma F_x = \frac{1}{g_c} \frac{d(mV)_x}{dt} \Big|_{\text{system}} = \frac{1}{g_c} \frac{\partial(mV)_x}{\partial t} \Big|_{\text{CV}} + \frac{1}{g_c} \iint_{\text{CS}} V_x \rho V_n dA \quad (2.22a)$$

$$\Sigma F_y = \frac{1}{g_c} \frac{d(mV)_y}{dt} \Big|_{\text{system}} = \frac{1}{g_c} \frac{\partial(mV)_y}{\partial t} \Big|_{\text{CV}} + \frac{1}{g_c} \iint_{\text{CS}} V_y \rho V_n dA \quad (2.22b)$$

$$\Sigma F_z = \frac{1}{g_c} \frac{d(mV)_z}{dt} \Big|_{\text{system}} = \frac{1}{g_c} \frac{\partial(mV)_z}{\partial t} \Big|_{\text{CV}} + \frac{1}{g_c} \iint_{\text{CS}} V_z \rho V_n dA \quad (2.22c)$$

The ΣF term represents all external forces applied to the control volume. The first term after the second equal sign is the rate of storage of linear

momentum within the control volume (CV). The last term represents the rate of linear momentum out of the control volume minus the rate of linear momentum in. For a steady, one-dimensional flow, the equations become

$$\Sigma F_i = \frac{1}{g_c} \int \int_{CS} V_i \rho V_n dA \quad (2.23)$$

which can be applied to any direction i . For a steady flow system in which one uniform fluid stream enters (*in*) and one leaves (*out*) the control volume, we have for the i direction:

$$\Sigma F_i = \frac{V_{\text{out}}(\rho AV)_{\text{out}}}{g_c} \Big|_i - \frac{V_{\text{in}}(\rho AV)_{\text{in}}}{g_c} \Big|_i$$

$$\text{or } \Sigma F_i = \frac{\dot{m}}{g_c} (V_{\text{out}} - V_{\text{in}}) \Big|_i$$

EXAMPLE 2.9. A water jet impacts a flat surface, as shown in Figure 2.19. At section 1, the liquid jet has a diameter of 1 in. and a velocity of 15 ft/s. Determine the force exerted on the surface by the jet.

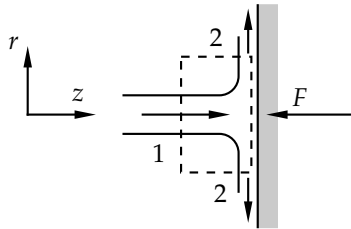


FIGURE 2.19. A jet of liquid impacting a flat plate.

Solution: The equation that relates forces to property changes of a fluid is the momentum equation. Before applying it, however, it is necessary to select a coordinate system, which is shown in the figure. It is also necessary to select a control volume for analysis. This too is shown in the figure. Note that where mass crosses the control surface, it does so at a right angle. If the surface is stationary, then it exerts a force F on the jet, as indicated. In this case, the one-dimensional momentum equation applies in the z direction:

$$\Sigma F_z = \frac{\dot{m}}{g_c} (V_{\text{out}} - V_{\text{in}}) \Big|_z$$

The only force exerted on the control volume is the restraining force F acting

in the negative z direction. This force is equal in magnitude but opposite in direction to the force exerted on the flat plate by the liquid stream. The V_{out} term is zero because there is no mass leaving the control volume in the z -direction. The V_{in} term is given as 15 ft/s. The mass flow rate is

$$\dot{m} = \rho AV$$

applied anywhere that the properties are known. With D_1 given,

$$A_1 = \frac{\pi D_1^2}{4} = \frac{\pi(1/12)^2}{4} = 5.45 \times 10^{-3} \text{ ft}^2$$

The density of water is taken to be 1.94 slug/ft³. Thus, we calculate

$$\dot{m} = (1.94)(5.45 \times 10^{-3})(15) = 0.159 \text{ slug/s}$$

The momentum equation after substitution becomes

$$-F = 0.159(0 - 15) = -2.38 \text{ slug}\cdot\text{ft/s}^2$$

or $F = 2.38 \text{ lbf}$

where by definition, 1 lbf = 1 slug·ft/s². The result is positive, which indicates that the direction we assumed for F was correct.

EXAMPLE 2.10. A jet hits a stationary vane, as shown in Figure 2.20. At the inlet, the jet makes an angle of θ_1 with the horizontal, while at the outlet, the angle is θ_2 . For the conditions shown, determine an equation for the ratio F_x/F_y of the reaction forces.

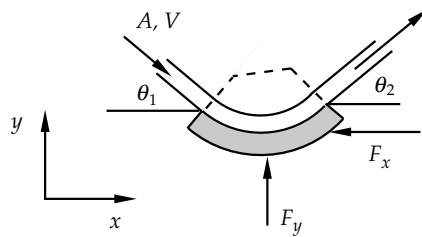


FIGURE 2.20. A jet of liquid impacting a curved plate.

Solution: We select a coordinate system and a control volume. Both are shown in the figure. The control volume was drawn such that where mass crosses the control surface, it does so at a right angle. We identify section 1 as the location where the liquid enters the control volume, and section 2

where the jet exits. The magnitude of the velocity at section 1 equals that at section 2 if there is no frictional losses as the jet moves past the plate.

The forces exerted by the jet of liquid are balanced by the external forces F_x and F_y necessary to keep the plate from moving. The one-dimensional momentum equation applies in the x -direction:

$$\text{or } \Sigma F_x = \frac{\dot{m}}{g_c} (V_{\text{out}} - V_{\text{in}}) \Big|_x$$

The only x -directed external (to the control volume) force exerted is the restraining force F_x acting in the negative x -direction. This force is equal in magnitude but opposite in direction to the force exerted on the flat plate by the liquid stream.

The V_{out} term is $V \cos \theta_2$, and the V_{in} term is $V \cos \theta_1$. The mass flow rate is

$$\dot{m} = \rho AV$$

applied anywhere (sections 1 or 2) that the properties are known. Substituting into the momentum equation gives

$$-F_x = \frac{\rho AV}{g_c} (V \cos \theta_2 - V \cos \theta_1) = \frac{\rho AV^2}{g_c} (\cos \theta_2 - \cos \theta_1)$$

Similarly, the y -directed momentum equation is

$$\Sigma F_y = \frac{\dot{m}}{g_c} (V_{\text{out}} - V_{\text{in}}) \Big|_y$$

Substituting,

$$F_y = \frac{\rho AV}{g_c} [(V \sin \theta_2) - (-V \sin \theta_1)] = \frac{\rho AV^2}{g_c} (\sin \theta_2 + \sin \theta_1)$$

The ratio of forces asked for in the problem statement is

$$\frac{F_x}{F_y} = - \frac{(\rho AV^2) (\cos \theta_2 - \cos \theta_1)}{(\rho AV^2) (\sin \theta_2 + \sin \theta_1)}$$

$$\text{or } \frac{F_x}{F_y} = \frac{(\cos \theta_1 - \cos \theta_2)}{(\sin \theta_2 + \sin \theta_1)}$$

Energy Equation

The energy equation is known also as the First Law of Thermodynamics. It allows us to make calculations describing the transformation of energy from one form to another and includes the effects of work and heat transfer. The energy equation states that

$$\left\{ \begin{array}{c} \text{total rate of} \\ \text{change of energy} \\ \text{within system} \end{array} \right\} = \left\{ \begin{array}{c} \text{rate of} \\ \text{energy} \\ \text{stored} \end{array} \right\} + \left\{ \begin{array}{c} \text{rate of energy out} \\ \text{minus} \\ \text{rate of energy in} \end{array} \right\}$$

In equation form, we have

$$\left. \frac{dE}{dt} \right|_{\text{system}} = \left. \frac{\partial E}{\partial t} \right|_{\text{CV}} + \int_{\text{CS}} e \rho V_n dA \quad (2.24)$$

where E is the total energy of a system and e is the total energy per unit mass. The total energy is traditionally considered to consist of internal, kinetic, and potential energies:

$$E = U + KE + PE$$

and

$$e = \frac{E}{m} = u + \frac{V^2}{2g_c} + \frac{gz}{g_c} \quad (2.25)$$

Experimental observations of devices and mathematical models of their behavior have led to the following relation between energy, heat transfer, and work:

$$\left\{ \begin{array}{c} \text{total rate of} \\ \text{change of energy} \\ \text{within system} \end{array} \right\} = \left\{ \begin{array}{c} \text{rate of} \\ \text{heat transferred} \\ \text{out of system} \end{array} \right\} - \left\{ \begin{array}{c} \text{rate of work} \\ \text{done by} \\ \text{system} \end{array} \right\}$$

or

$$\left. \frac{dE}{dt} \right|_{\text{system}} = \frac{\partial \tilde{Q}}{\partial t} - \frac{\partial W'}{\partial t} \quad (2.26)$$

where $\tilde{Q}$ is the heat transferred to the system and W' is all forms of work done by the system. Combining Equations 2.24, 2.25, and 2.26 gives

$$\left. \frac{dE}{dt} \right|_{\text{system}} = \frac{\partial \tilde{Q}}{\partial t} - \frac{\partial W'}{\partial t} = \left. \frac{\partial E}{\partial t} \right|_{\text{CV}} + \int_{\text{CS}} e \rho V_n dA$$

or

$$\frac{\partial \tilde{Q}}{\partial t} - \frac{\partial W'}{\partial t} = \left. \frac{\partial E}{\partial t} \right|_{\text{CV}} + \int_{\text{CS}} \left(u + \frac{V^2}{2g_c} + \frac{gz}{g_c} \right) \rho V_n dA \quad (2.27)$$

The work term W' consists of all forms of work crossing the boundary of the control volume including electric, magnetic, viscous shear or friction, flow work, and shaft work. Flow work is done by or on the system when mass crosses the control surface at entrances or exits. It is customary to divide the work term W' into two components: shaft work W and flow work W_f . Thus, we can write:

$$\frac{\partial W'}{\partial t} = \frac{\partial W}{\partial t} + \frac{\partial W_f}{\partial t}$$

The flow work is given by

$$\frac{\partial W_f}{\partial t} = \int_{\text{CS}} \frac{p}{\rho} \rho V_n dA$$

Combining the preceding equations with Equation 2.27 and rearranging,

$$\frac{\partial \tilde{Q}}{\partial t} - \frac{\partial W}{\partial t} = \left. \frac{\partial E}{\partial t} \right|_{\text{CV}} + \int_{\text{CS}} \left(\frac{p}{\rho} + u + \frac{V^2}{2g_c} + \frac{gz}{g_c} \right) \rho V_n dA$$

Recall that enthalpy is defined as $h = u + p/\rho$. Substituting into the preceding equation gives

$$\frac{\partial \tilde{Q}}{\partial t} - \frac{\partial W}{\partial t} = \left. \frac{\partial E}{\partial t} \right|_{\text{CV}} + \int_{\text{CS}} \left(h + \frac{V^2}{2g_c} + \frac{gz}{g_c} \right) \rho V_n dA \quad (2.28)$$

For the case of steady, one-dimensional flow, the preceding equation becomes

$$\frac{\partial \tilde{Q}}{\partial t} - \frac{\partial W}{\partial t} = \left[\left(h + \frac{V^2}{2g_c} + \frac{gz}{g_c} \right)_{\text{out}} - \left(h + \frac{V^2}{2g_c} + \frac{gz}{g_c} \right)_{\text{in}} \right] \rho V A \quad (2.29)$$

For an adiabatic process, the heat transferred is zero and the preceding equation reduces to

$$-\frac{\partial W}{\partial t} = \left\{ \left(h + \frac{V^2}{2g_c} + \frac{gz}{g_c} \right)_{\text{out}} - \left(h + \frac{V^2}{2g_c} + \frac{gz}{g_c} \right)_{\text{in}} \right\} \rho V A \quad (2.30)$$

EXAMPLE 2.11. The system shown in Figure 2.21 contains a pump that conveys propylene from a tank to some location downstream. The pipe at the pump exit has an internal diameter of 7.792 cm and the velocity in the exit line is 2 m/s. The gage in the outlet line reads 120 kPa, and the vertical distance from the reference datum to the gage 1.4 m ($= z_2$). For a liquid depth of 2 m ($= z_1$), what is the energy delivered to the liquid (the pump power)? Neglect frictional effects.

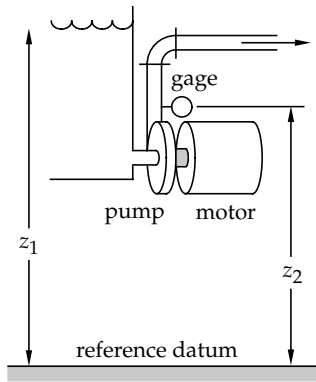


FIGURE 2.21. Propylene being pumped from a tank.

Solution: The following equation applies for a pump, fan, or compressor:

$$-\frac{\partial W}{\partial t} = \left\{ \left(\frac{p}{\rho} + \frac{V^2}{2g_c} + \frac{gz}{g_c} \right)_{\text{out}} - \left(\frac{p}{\rho} + \frac{V^2}{2g_c} + \frac{gz}{g_c} \right)_{\text{in}} \right\} \rho V A$$

The pump power $\partial W/\partial t$ is what we are seeking. We apply this equation to any two sections that bound the pump. We select the free surface of the liquid as “in” and the location of the pressure gage as “out.” We now evaluate properties at these sections:

$$V_{\text{out}} = 2 \text{ m/s} \quad z_{\text{out}} = 1.4 \text{ m} \quad p_{\text{out}} = 120\,000 \text{ Pa (gage)}$$

At the free surface, we have

$$p_{\text{in}} = p_{\text{atm}} = 0 \quad V_{\text{in}} = 0 \quad z_{\text{in}} = 2 \text{ m}$$

The density of propylene is (from Appendix Table B.1) $0.516(1\,000) \text{ kg/m}^3$.

The outlet area is calculated to be

$$A_{\text{out}} = \frac{\pi(0.07792)^2}{4} = 4.77 \times 10^{-3} \text{ m}^2$$

The mass flow rate can be calculated at any section where properties are known. At the outlet gage,

$$\dot{m} = \rho AV = 516(4.77 \times 10^{-3})(2)$$

$$\text{or } \dot{m} = 4.92 \text{ kg/s}$$

Substituting into the energy equation, we get

$$-\frac{\partial W}{\partial t} = \left(\frac{120\,000}{516} + \frac{2^2}{2} + 9.81(1.4) - [0 + 0 + 9.81(2)] \right) (4.92)$$

$$-\frac{\partial W}{\partial t} = (2.33 \times 10^2 + 2 + 13.73 - 19.62)(4.92)$$

We see that much of the input power from the pump goes into increasing the pressure in the outlet line. Changes in kinetic and potential energies are smaller. Solving,

$$-\frac{\partial W}{\partial t} = 1125 \text{ W} = \frac{1125}{746} = 1.51 \text{ HP}$$

EXAMPLE 2.12. A water turbine is located in a dam as shown in Figure 2.22. The volume flow rate through the system is 50,000 gpm. The exit pipe diameter is 4 ft. Calculate the work done by (or power received from) the water as it flows through the dam.

The density of water is (from Appendix Table B.1) 1.94 slug/ft³. The following equation applies for the water turbine of this example:

$$-\frac{\partial W}{\partial t} = \left\{ \left(\frac{p}{\rho} + \frac{V^2}{2g_c} + \frac{gz}{g_c} \right)_{\text{out}} - \left(\frac{p}{\rho} + \frac{V^2}{2g_c} + \frac{gz}{g_c} \right)_{\text{in}} \right\} \rho VA$$

The power $\partial W/\partial t$ is what we are seeking, and we can apply this equation to any two sections that bound the system. We select the free surface of the liquid as “in” and the location of the outlet pipe (where properties are known) as “out.” We will need to find the outlet area:

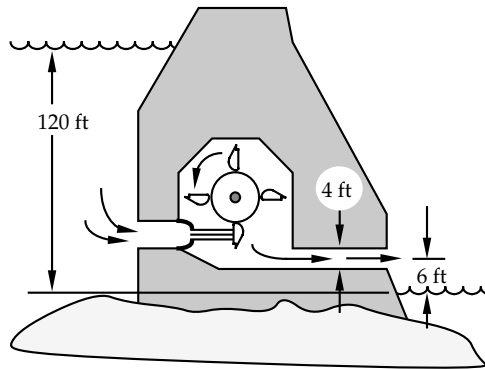


FIGURE 2.22. Flow through a water turbine.

$$A_{\text{out}} = \frac{\pi(4)^2}{4} = 12.6 \text{ ft}^2$$

The volume flow rate of water is given as 50,000 gpm, which is 111.4 ft³/s. The mass flow rate of water is

$$\dot{m} = \rho Q = 1.94(111.4) = 216.1 \text{ slug/s.}$$

The outlet velocity then is

$$V_{\text{out}} = \frac{Q}{A} = \frac{111.4}{12.6} = 8.8 \text{ ft/s}$$

We also have

$$z_{\text{out}} = 6 \text{ ft} \quad p_{\text{in}} = p_{\text{out}} = p_{\text{atm}}$$

At the free surface,

$$V_{\text{in}} = 0 \quad z_{\text{in}} = 120 \text{ ft}$$

Substituting into the energy equation, we get

$$+ \frac{\partial W}{\partial t} = \left((0) + (0) + 32.2(120) - \left[0 + \frac{8.8^2}{2} + 32.2(6) \right] \right) (216.1)$$

Note the sign change on the power. The potential energy in 120 ft of water is being converted to power, and to exit kinetic and potential energies. Continuing,

$$+ \frac{\partial W}{\partial t} = (3864 - 39.3 - 193.2) \frac{\text{ft}^2}{\text{s}^2} (216.1 \text{ slug/s}) = 7.8 \times 10^5 \text{ slug}\cdot\text{ft}^2/\text{s}^3$$

The conversion that applies to these units is 1 slug·ft/(lbf·s²). Solving,

$$+ \frac{\partial W}{\partial t} = 7.8 \times 10^5 \text{ ft}\cdot\text{lbf/s} = \frac{7.8 \times 10^5}{550} = 1427 \text{ HP}$$

EXAMPLE 2.13. A window fan is located in a 24 x 24 x 6 in. housing as indicated in Figure 2.23. The fan moves air at a velocity of 20 ft/s. Determine the pressure rise across the fan for an input power of 1/4 HP.

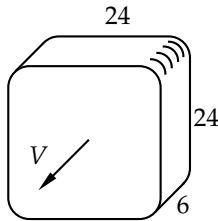


FIGURE 2.23. Flow through a window fan.

Solution: In this example, we assume that the air behaves like an ideal gas and that the temperature rise across the fan is negligible. Equation 2.30 applies with enthalpies replaced by pressure terms:

$$- \frac{\partial W}{\partial t} = \left\{ \left(\frac{p}{\rho} + \frac{V^2}{2g_c} + \frac{gz}{g_c} \right)_{\text{out}} - \left(\frac{p}{\rho} + \frac{V^2}{2g_c} + \frac{gz}{g_c} \right)_{\text{in}} \right\} \rho V A$$

The pressure rise $p_{\text{out}} - p_{\text{in}}$ is what we are seeking. The continuity equation is

$$Q = A_{\text{in}} V_{\text{in}} = A_{\text{out}} V_{\text{out}}$$

With no area change, the preceding equation indicates that $V_{\text{in}} = V_{\text{out}}$. Moreover, with a horizontal configuration, $z_{\text{in}} = z_{\text{out}}$. The air density can be taken to be (from Table C.1):

$$\rho = 0.0735 \text{ lbm/ft}^3$$

The power input is

$$- \frac{\partial W}{\partial t} = \frac{1}{4} \text{ HP} [550 \text{ ft}\cdot\text{lbf}/(\text{s}\cdot\text{HP})] = 137.5 \text{ ft}\cdot\text{lbf/s}$$

Rearranging the energy equation and substituting, we get

$$-\frac{\partial W}{\partial t} = \frac{p_{\text{out}} - p_{\text{in}}}{\rho} \rho V A = (p_{\text{out}} - p_{\text{in}}) A V$$

$$137.5 = (p_{\text{out}} - p_{\text{in}})(24/12)(24/12)(20)$$

Solving,

$$(p_{\text{out}} - p_{\text{in}}) = 1.72 \text{ lbf/ft}^2$$

When dealing with an air flow system, it is customary to express pressure changes in terms of a column of liquid, specifically, in cm or in. of water. Thus, we are seeking

$$\Delta h = \frac{p_{\text{out}} - p_{\text{in}}}{\rho_{\text{H}_2\text{O}}} \frac{g_c}{g} = \frac{1.72}{62.4} \frac{32.2}{32.2}$$

$$\Delta h = 0.0275 \text{ ft of H}_2\text{O} = 0.33 \text{ in of H}_2\text{O}$$

The Bernoulli Equation

The Bernoulli Equation relates velocity, elevation, and pressure in a flow field. This equation results from the energy equation (2.29) for adiabatic, one-dimensional flow with no work and negligible change in internal energy. The Bernoulli equation also results from applying the momentum equation to a streamline in the flow field. Thus, under special conditions, the momentum and the energy equations reduce to the same equation, which is why the Bernoulli equation is often called the **mechanical energy equation**. The Bernoulli equation is written as

$$\int_1^2 \frac{dp}{\rho} + \frac{(V_2^2 - V_1^2)}{2g_c} + \frac{g(z_2 - z_1)}{g_c} = 0$$

For an incompressible fluid for which density ρ is a constant, the preceding equation becomes

$$\frac{p_2 - p_1}{\rho} + \frac{(V_2^2 - V_1^2)}{2g_c} + \frac{g(z_2 - z_1)}{g_c} = 0 \quad (2.31)$$

or

$$\frac{p}{\rho} + \frac{V^2}{2g_c} + \frac{gz}{g_c} = \text{a constant} \quad (2.32)$$

The Bernoulli equation does not account for frictional effects or shaft work.

EXAMPLE 2.14. A water jet issues from a faucet and falls vertically downward. The water flow rate is such that it will fill a 250 ml cup in 8 seconds. The faucet is 28 cm above the sink, and at the faucet exit, the jet diameter is 0.35 cm. What is the jet diameter at the point of impact on the sink surface?

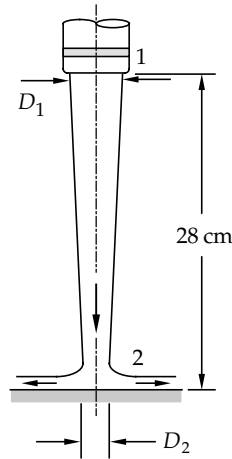


FIGURE 2.24. *Jet of liquid impacting a sink.*

Solution: Figure 2.24 shows a jet exiting a faucet and impacting a flat surface. We locate section 1 at the faucet exit and section 2 at the sink. The continuity equation is

$$Q = A_1 V_1 = A_2 V_2$$

assuming one-dimensional, steady flow. The volume flow rate is

$$Q = \frac{0.250 \text{ l}}{8 \text{ s}} = 0.03125 \text{ l/s} = 0.03125 \times 10^{-3} \text{ m}^3/\text{s}$$

Substituting for flow rate and area gives the velocity at each section as

$$V_1 = \frac{Q}{A_1} = \frac{4Q}{\pi D_1^2} = \frac{4(0.03125 \times 10^{-3})}{\pi(0.0035)^2} = 3.25 \text{ m/s}$$

Also,

$$V_2 = \frac{Q}{A_2} = \frac{4Q}{\pi D_2^2} = \frac{4(0.03125 \times 10^{-3})}{\pi D_2^2} = \frac{3.98 \times 10^{-5}}{D_2^2}$$

The Bernoulli equation applied to these two sections is

$$\frac{p_1}{\rho} + \frac{V_1^2}{2g_c} + \frac{gz_1}{g_c} = \frac{p_2}{\rho} + \frac{V_2^2}{2g_c} + \frac{gz_2}{g_c}$$

Evaluating properties:

$$p_1 = p_2 = p_{\text{atm}}$$

$$z_1 = 0.28 \text{ m} \quad z_2 = 0$$

The Bernoulli equation becomes, after simplification,

$$\frac{V_1^2}{2g} + z_1 = \frac{V_2^2}{2g}$$

Substituting,

$$\frac{3.25^2}{2(9.81)} + 0.28 = \left(\frac{3.98 \times 10^{-5}}{D_2^2} \right)^2 \frac{1}{2(9.81)}$$

which becomes

$$0.538 + 0.28 = \frac{8.07 \times 10^{-11}}{D_2^4}$$

Solving,

$$D_2^4 = 9.86 \times 10^{-11}$$

and

$$\underline{D_2 = 3.15 \times 10^{-3} \text{ m} = 0.315 \text{ cm}}$$

2.5 Summary

In this chapter, we examined fluid properties and wrote equations of fluid mechanics without derivation. This chapter was intended as a brief review and much detail has been omitted. The reader is referred to any text on Fluid Mechanics for more information on any of the points addressed here.

2.6 Show and Tell

Obtain a catalog from the appropriate manufacturer(s) and give an oral report on the following viscometer(s) as assigned by the instructor. In all cases present the theoretical basis for the operation of the device and, if available, demonstrate its operation.

1. The cone and plate viscometer.
2. The falling sphere viscometer.
3. The wide-gap concentric-cylinder viscometer.
4. Saybolt viscometer.
5. Stormer viscometer for measuring viscosity of paint.
6. Any other type of viscometer you encounter that is not mentioned in this chapter.

Obtain a catalog of the appropriate type and give an oral report on the following devices useful for measuring pressure. In all cases, present the theoretical basis for the operation of the device.

7. A pitot tube and a pitot-static tube.
8. Pressure transducers.

2.7 Problems

Density, Specific Gravity, Specific Weight

1. What is the specific gravity of 38°API oil?
2. The specific gravity of manometer gage oil is 0.826. What are its density and its °API rating?
3. What is the difference in density between a 50°API oil and a 40°API oil?
4. A 35°API oil has a viscosity of $0.825 \text{ N}\cdot\text{s}/\text{m}^2$. Express its viscosity in Saybolt Universal Seconds (SUS).
5. Air is collected in a 1.2 m^3 container and weighed using a balance as indicated in Figure P2.5. On the other end of the balance arm is 1.2 m^3 of CO_2 . The air and the CO_2 are at 27°C and atmospheric pressure. What is the difference in weight between these two volumes?

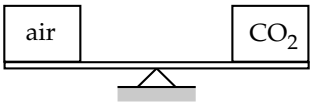


FIGURE P2.5.

6. A container of castor oil is used to measure the density of a solid. The solid is cubical in shape, 3 cm x 3 cm x 3 cm, and weighs 10 N in air. While submerged, the object weighs 7 N. What is the density of the liquid?
7. A brass cylinder (Sp. Gr. = 8.5) has a diameter of 1 in. and a length of 4 in. It is submerged in a liquid of unknown density, as indicated in Figure P2.7. While submerged, the weight of the cylinder is measured as 0.8 lbf. Determine the density of the liquid.

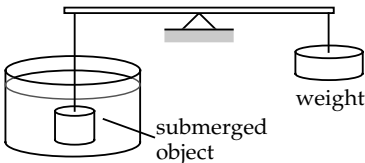


FIGURE P2.7.

Viscosity

8. Actual tests on vaseline yielded the following data:

τ in N/m ²	0	200	600	1 000
dV/dy in 1/s	0	500	1 000	1 200

Determine the fluid type and the proper descriptive equation.

9. A popular mayonnaise is tested with a viscometer and the following data were obtained:

τ in g/cm ²	40	100	140	180
dV/dy in rev/s	0	3	7	15

Determine the fluid type and the proper descriptive equation.

10. A cod-liver oil emulsion is tested with a viscometer and the following data were obtained:

τ in lbf/ft ²	0	40	60	80	120
dV/dy in rev/s	0	0.5	1.7	3	6

Graph the data and determine the fluid type. Derive the descriptive equation.

11. A rotating cup viscometer has an inner cylinder diameter of 2.00 in., and the gap between cups is 0.2 in. The inner cylinder length is 2.50 in. The viscometer is used to obtain viscosity data on a Newtonian liquid. When the inner cylinder rotates at 10 rev/min, the torque on the inner cylinder is measured to be 0.00011 in-lbf. Calculate the viscosity of the fluid. If the fluid density is 850 kg/m³, calculate the kinematic viscosity.

12. A rotating cup viscometer has an inner cylinder whose diameter is 3.8 cm and whose length is 8 cm. The outer cylinder has a diameter of 4.2 cm. The viscometer is used to measure the viscosity of a liquid. When the outer cylinder rotates at 12 rev/min, the torque on the inner cylinder is measured to be 4×10^{-6} N·m. Determine the kinematic viscosity of the fluid if its density is $1\,000\text{ kg/m}^3$.
13. A rotating cup viscometer has an inner cylinder diameter of 2.25 in. and an outer cylinder diameter of 2.45 in. The inner cylinder length is 3.00 in. When the inner cylinder rotates at 15 rev/min, what is the expected torque reading if the fluid is propylene glycol?
14. A capillary tube viscometer is used to measure the viscosity of water (density is 62.4 lbm/ft^3 , viscosity is $0.89 \times 10^{-3}\text{ N}\cdot\text{s/m}^2$) for calibration purposes. The capillary tube inside diameter must be selected so that laminar flow conditions (i.e., $VD/\nu < 2\,100$) exist during the test. For values of $L = 3\text{ in.}$ and $z = 10\text{ in.}$, determine the maximum tube size permissible.
15. A Saybolt viscometer is used to measure oil viscosity and the time required for 60 ml of oil to pass through a standard orifice is 180 SUS. The specific gravity of the oil is found as 44°API . Determine the absolute viscosity of the oil.
16. A 10 cm^3 capillary tube viscometer is used to measure the viscosity of a liquid. For values of $L = 4\text{ cm}$, $z = 25\text{ cm}$, and $D = 0.8\text{ mm}$, determine the viscosity of the liquid. The time recorded for the experiment is 12 seconds.
17. A Saybolt viscometer is used to obtain oil viscosity data. The time required for 60 ml of oil to pass through the orifice is 70 SUS. Calculate the kinematic viscosity of the oil. If the specific gravity of the oil is 35°API , find also its absolute viscosity.
18. A 2-mm diameter ball bearing is dropped into a container of glycerine. How long will it take the bearing to fall a distance of 1 m?
19. A 1/8-in. diameter ball bearing is dropped into a viscous oil. The terminal velocity of the sphere is measured as 2 ft/15 s. What is the kinematic viscosity of the oil if its specific gravity is 0.8?

Pressure and Its Measurement

20. A mercury manometer is used to measure pressure at the bottom of a tank containing acetone, as shown in Figure P2.20. The manometer is to be replaced with a gage. What is the expected reading in psig if $\Delta h = 5\text{ in.}$ and $x = 2\text{ in.}$
21. Referring to Figure P2.21, determine the pressure of the water at the point where the manometer attaches to the vessel. All dimensions are in inches and the problem is to be worked using Engineering or British Gravitational units.

22. Figure P2.22 shows a portion of a pipeline that conveys benzene. A gage attached to the line reads 150 kPa. It is desired to check the gage reading with a benzene-over-mercury U-tube manometer. Determine the expected reading Δh on the manometer.

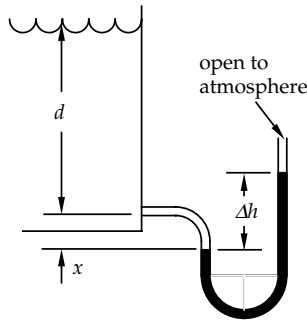


FIGURE P2.20.

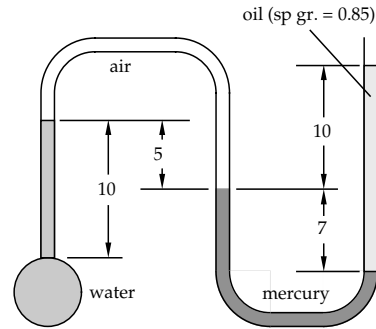


FIGURE P2.21.

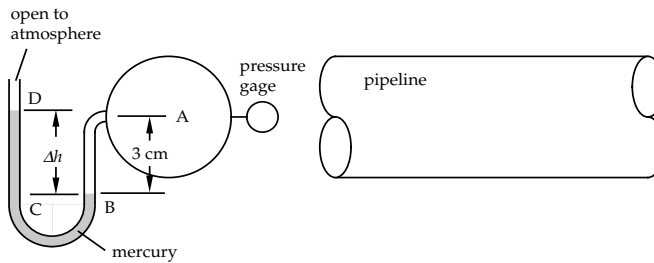


FIGURE P2.22.

23. An unknown fluid is in the manometer of Figure P2.23. The pressure difference between the two air chambers is 700 kPa and the manometer reading Δh is 6 cm. Determine the density and specific gravity of the unknown fluid.
24. A U-tube manometer is used to measure the pressure difference between two air chambers, as shown in Figure P2.24. If the reading Δh is 6 in., determine the pressure difference. The manometer fluid is water.

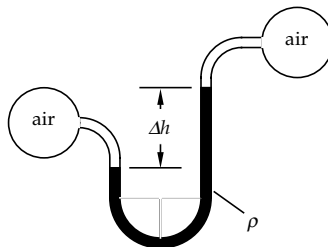


FIGURE P2.23, P2.24.

25. A manometer containing mercury is used to measure the pressure increase experienced by a water pump as shown in Figure P2.25. Calculate the pressure rise if Δh is 7 cm of mercury (as shown). All dimensions are in cm.

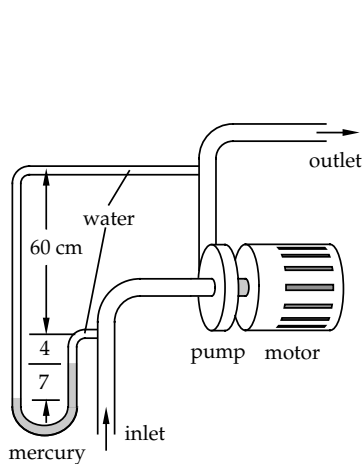


FIGURE P2.25.

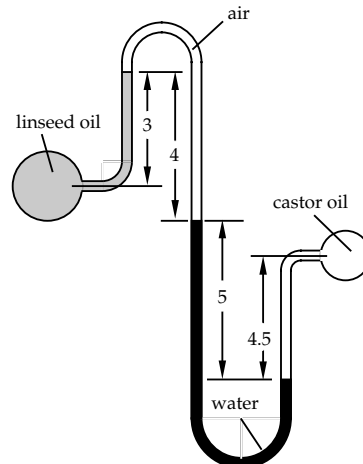


FIGURE P2.26.

26. Determine the pressure difference between the linseed and castor oils of Figure P2.26. (All dimensions are in inches.)
27. For the system of Figure P2.27, determine the pressure of the air in the tank.

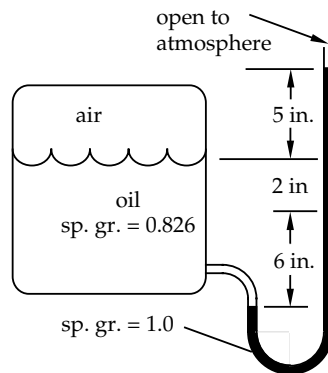


FIGURE P2.27.

Continuity Equation

28. Figure P2.28 shows a reducing bushing. A liquid leaves the bushing at a velocity of 4 m/s. Calculate the inlet velocity. What effect does the fluid density have?

29. Figure P2.29 shows a reducing bushing. Flow enters the bushing at a velocity of 0.5 m/s. Calculate the outlet velocity.

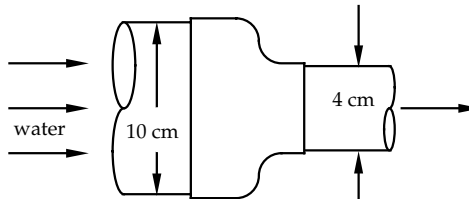


FIGURE P2.28, P2.29.

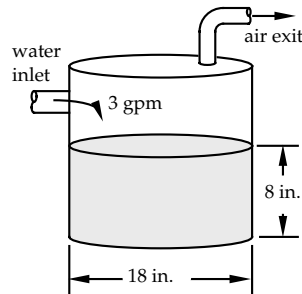


FIGURE P2.30.

30. Three gallons per minute of water enters the tank of Figure P2.30. The inlet line is 2-1/2 in. in diameter. The air vent is 1.5 in. in diameter. Determine the air exit velocity at the instant shown.
31. An air compressor is used to pressurize a tank of volume 3 m³. Simultaneously, air leaves the tank and is used for some process downstream. At the inlet, the pressure is 350 kPa, the temperature is 20°C, and the velocity is 2 m/s. At the outlet, the temperature is 20°C, the velocity is 0.5 m/s, and the pressure is the same as that in the tank. Both flow lines (inlet and outlet) have internal diameters of 2.7 cm. The temperature of the air in the tank is a constant at 20°C. If the initial tank pressure is 200 kPa, what is the pressure in the tank after 5 minutes?
32. Figure P2.32 shows a cross-flow heat exchanger used to condense Freon-12. Freon-12 vapor enters the unit at a flow rate of 0.065 kg/s. Freon-12 leaves the exchanger as a liquid (Sp. Gr. = 1.915) at room temperature and pressure. Determine the exit velocity of the liquid.

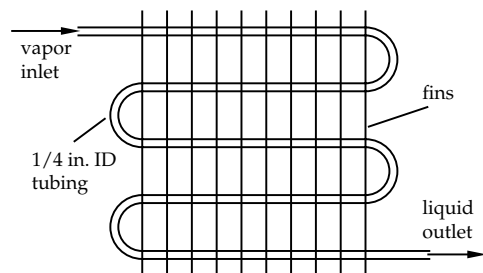


FIGURE P2.32.

33. Nitrogen enters a pipe at a flow rate of 0.2 lbm/s. The pipe has an inside diameter of 4 in. At the inlet, the nitrogen temperature is 540°R ($\rho = 0.073$ lbm/ft³) and at the outlet, the nitrogen temperature is 1800°R ($\rho = 0.0213$ lbm/ft³). Calculate the inlet and outlet velocities of the nitrogen. Are they equal? Should they be?

Momentum Equation

34. A garden hose is used to squirt water at someone who is protecting herself with a garbage can lid. Figure P2.34 shows the jet in the vicinity of the lid. Determine the restraining force F for the conditions shown.

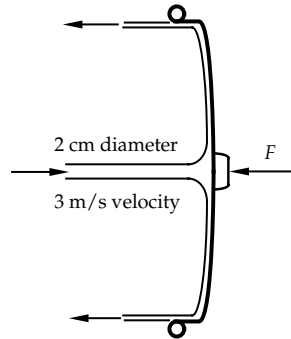


FIGURE P2.34

35. A two-dimensional, liquid jet strikes a concave semicircular object, as shown in Figure P2.35. Calculate the restraining force F .
36. A two-dimensional, liquid jet strikes a concave semicircular object, as shown in Figure P2.36. Calculate the restraining force F .

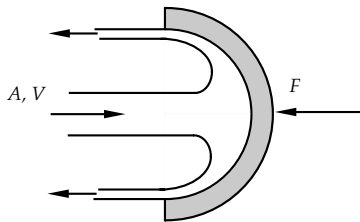


FIGURE P2.35.

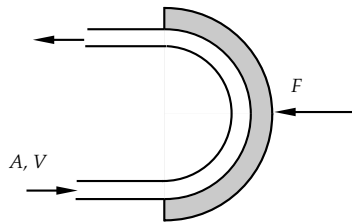


FIGURE P2.36.

37. A two-dimensional liquid jet is turned through an angle θ ($0^\circ < \theta < 90^\circ$) by a curved vane, as shown in Figure P2.37. The forces are related by $F_2 = 3F_1$. Determine the angle θ through which the liquid jet is turned.
38. A two-dimensional liquid jet is turned through an angle θ ($0^\circ < \theta < 90^\circ$) by a curved vane as shown in Figure P2.38. The forces are related by $F_1 = 2F_2$. Determine the angle θ through which the liquid jet is turned.

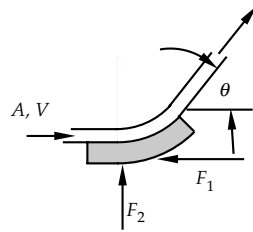


FIGURE P2.37.

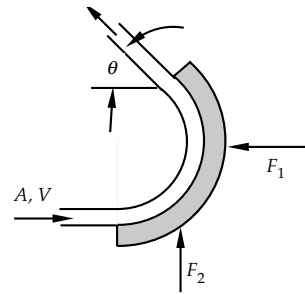


FIGURE 2.38.

Energy Equation

39. Figure P2.39 shows a water turbine located in a dam. The volume flow rate through the system is 5000 gpm. The exit pipe diameter is 4 ft. Calculate the work done by (or power received from) the water as it flows through the dam. (Compare to the results of the example problem in this chapter.)

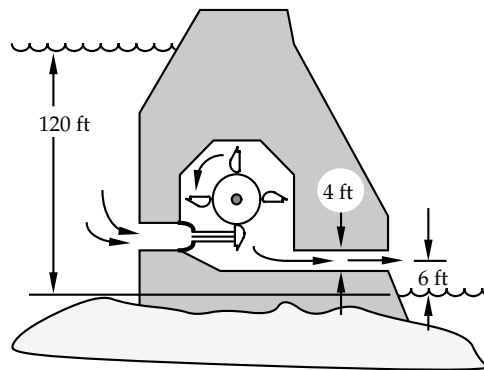


FIGURE P2.39.

40. Air flows through a compressor at a mass flow rate of 0.003 slug/s. At the inlet, the air velocity is negligible. At the outlet, air leaves through an exit pipe of diameter 2 in. The inlet properties are 14.7 psia and 75°F. The outlet pressure is 120 psia. For an isentropic (reversible and adiabatic) compression process, we have

$$\frac{T_2}{T_1} = \left\{ \frac{p_2}{p_1} \right\}^{(\gamma-1)/\gamma}$$

Determine the outlet temperature of the air and the power required. Assume that air behaves as an ideal gas ($dh = c_p dT$, $du = c_v dT$, and $\rho = p/RT$).

41. An air turbine is used with a generator to generate electricity. Air at the turbine inlet is at 700 kPa and 25°C. The turbine discharges air to the atmosphere at a temperature of 11°C. Inlet and outlet air velocities are 100 m/s and 2 m/s, respectively. Determine the work per unit mass delivered to the turbine from the air.
42. A pump moving hexane is illustrated in Figure P2.42. The flow rate is 0.02 m³/s; inlet and outlet gage pressure readings are -4 kPa and 190 kPa, respectively. Determine the required power input to the fluid as it flows through the pipe.

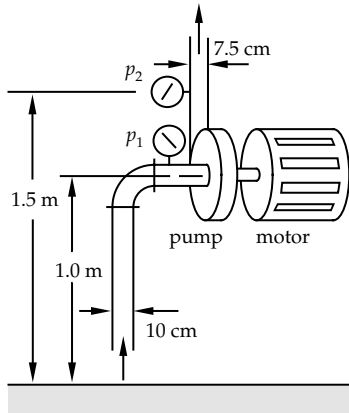


FIGURE P2.42.

Bernoulli Equation

43. Figure 2.15 shows a venturi meter. Show that the Bernoulli and continuity equations when applied combine to become

$$Q = A_2 \sqrt{\frac{2g\Delta h}{1 - (D_2^4/D_1^4)}}$$

44. A jet of water issues from a kitchen faucet and falls vertically downward at a flow rate of 1.5 fluid ounces per second. At the faucet, which is 14 in. above the sink bottom, the jet diameter is 5/8 in. Determine the diameter of the jet where it strikes the sink.
45. A jet of water issues from a valve and falls vertically downward at a flow rate of 30 cm³/s. The valve exit is 5 cm above the ground; the jet diameter at the ground is 5 mm. Determine the diameter of the jet at the valve exit.
46. A garden hose is used as a siphon to drain a pool, as shown in Figure P2.46. The garden hose has a 3/4-in. inside diameter (ID). Assuming no friction, calculate the flow rate of water through the hose if the hose is 25 ft long.

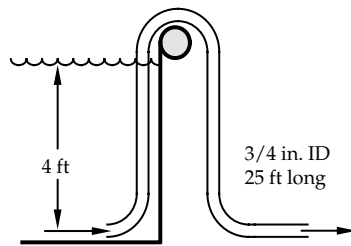


FIGURE P2.46.

Miscellaneous Problems

47. A pump draws castor oil from a tank, as shown in Figure P2.47. A venturi meter with a throat diameter of 2 in. is located in the discharge line. For the conditions shown, calculate the expected reading on the manometer of the meter. Assume that frictional effects are negligible and that the pump delivers 0.25 HP to the liquid. If all that is available is a 6-ft tall manometer, can it be used in the configuration shown? If not, suggest an alternative way to measure pressure difference. (All measurements are in inches.)

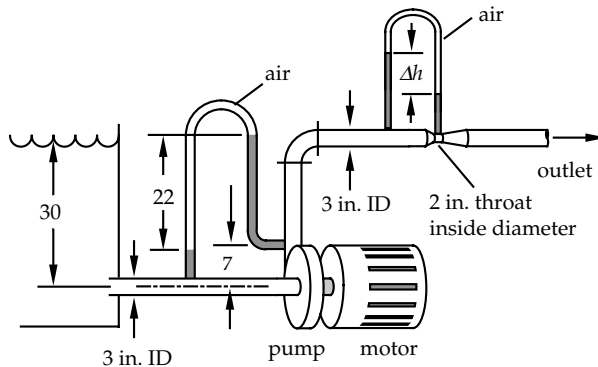


FIGURE P2.47.

48. A 4.2-cm ID pipe is used to drain a tank, as shown in Figure P2.48. Simultaneously, a 5.2-cm ID inlet line fills the tank. The velocity in the inlet line is 1.5 m/s. Determine the equilibrium height h of the liquid in the tank if it is octane. How does the height change if the liquid is ethyl alcohol? Assume in both cases that frictional effects are negligible and that z is 4 cm.

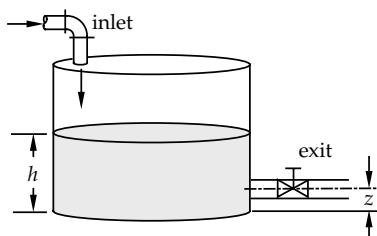


FIGURE P2.48.

Computer Problems

49. One of the examples in this chapter dealt with the following impact problem, with the result that the ratio of forces is given by:

$$\frac{F_x}{F_y} = \frac{(\cos \theta_1 - \cos \theta_2)}{(\sin \theta_2 + \sin \theta_1)}$$

For an angle of $\theta_1 = 0$, produce a graph of the force ratio as a function of the angle θ_2 .

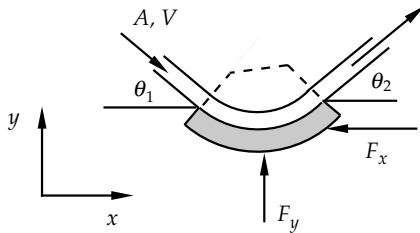


FIGURE P2.49

50. One of the examples in this chapter involved calculations made to determine the power output of a turbine in a dam (see Figure P2.50). When the flow through the turbine was 50,000 gpm, and the upstream height was 120 ft, the power was found to be 1427 hp. The relationship between the flow through the turbine and the upstream height is linear. Calculate the work done by (or power received from) the water as it flows through the dam for upstream heights that range from 60 to 120 ft.

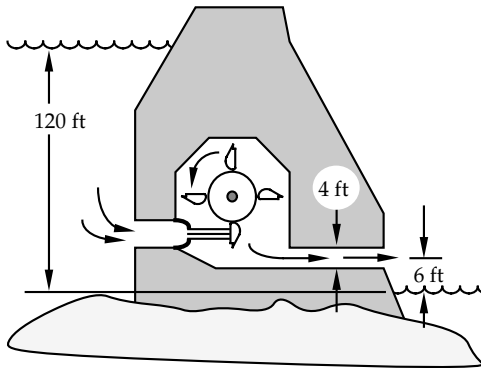


FIGURE P2.50

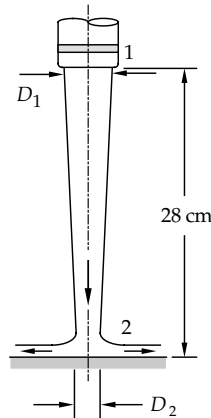


FIGURE P2.51

51. One of the examples in this chapter dealt with a water jet issuing from a faucet. The water flow rate was 250 ml per 8 seconds, the jet diameter at faucet exit is 0.35 cm, and the faucet is 28 cm above the sink. Calculations were made to find the jet diameter at impact on the sink surface. Repeat the calculations for volumes per time that range from 0.1 liters/8 seconds to 0.5 liters/8 seconds, and graph jet diameter at 2 as a function of the volume flow rate.

CHAPTER 3 Piping Systems I

In this chapter, we will review some of the basic concepts associated with piping systems. We will discuss effective and hydraulic diameters and present equations of motion for modeling flow in closed conduits. We will also examine minor losses in detail, discuss flow in noncircular cross sections, and conclude with a description of series piping systems.

Flow in closed conduits is an extremely important area of study because it is the most common way of transporting liquids. Crude oil and its components are moved about in a refinery or across the country by pumping them through pipes. Water in the home is transported to various parts of the house through tubing. Heated and air-conditioned air are distributed to all parts of a dwelling in circular and/or rectangular ducts. Examples of flow in closed conduits are everywhere.

It is important to recall that flow in a duct can be either **laminar** or **turbulent**. When laminar flow exists, the fluid flows smoothly through the duct in layers called *laminae*. A fluid particle in one layer stays in that layer. When turbulent flow exists, flowing fluid particles move about the cross section. Eddies and vortices are responsible for the mixing action; such eddies and vortices do not exist in laminar flow.

The criterion for distinguishing between laminar and turbulent flow is the observed mixing action. Experiments have shown that laminar flow exists when the **Reynolds number** is less than 2 100:

$$\text{Re} = \frac{\rho V D}{\mu g_c} = \frac{V D}{\nu} < 2\,100 \quad (\text{laminar flow}) \quad (3.1)$$

where V is the *average velocity* of the flow and D is a characteristic dimension of the duct cross section. For circular ducts, D is usually taken to be the inside diameter. For noncircular cross sections, D is usually taken to be the *hydraulic diameter* (discussed later in this chapter).

3.1 Pipe and Tubing Standards

Pipes and tubes are made of many materials. Pipes can be cast or, like tubes, can be extruded. Sizes for pipes and tubes are standardized and so are tolerances on their dimensions.

Pipe Specifications and Attachment Methods

Pipes are specified by a **nominal diameter** and a **schedule number**—for example, “2-nominal schedule 40.” The nominal diameter does not necessarily equal the inside or outside diameter of the pipe. Each nominal diameter will specify one and only one outside diameter. The schedule of the pipe specifies the wall thickness such that the larger the schedule number, the thicker the pipe wall. Appendix Table D.1 gives dimensions of pipe sizes that vary from 1/8-nominal to 40-nominal in English and in SI units. Schedule 40 pipe (or the standard size) is used in common engineering applications. Schedule 80 and pipes with thicker walls may be used in applications where there is a considerable elevated pressure.

Most pipe materials are manufactured to the dimensions given in Appendix Table D.1. Thus fittings made of steel may be used with pipe made of PVC. It should be noted that stainless steel pipe is available in thinner pipe wall sizes due to its strength; for example, it is available in schedule 20 sizes.

Pipes can be attached together or to fittings in various ways. Pipe ends can be threaded, which is done primarily in the smaller sizes (less than 4-nominal). The number of threads per inch as well as the thread profile are standardized for each pipe size. The threaded end of the pipe is usually tapered as well. Regardless of pipe schedule, all pipe having the same nominal diameter [i.e., outside diameter (OD)] will have the same thread specification. Before threaded pipes are attached to fittings, the threads are coated with a viscous compound or wrapped with special tape. The thread preparation and the wedging action of tapered threads together help to ensure a fluid-tight connection.

Pipes can be welded together or to fittings if the material is weldable. Welding is more commonly done with the larger pipe sizes.

Pipe ends can be threaded into or welded to flanges. Flanges are then bolted together. Usually a rubber or cork gasket is installed between the flanges to ensure that the connection is leakproof. Flanges are made in many pipe sizes, and standards have been established for their construction details including the minimum number of bolt holes, and their placement.

Plastic pipe, such as polyvinyl chloride (PVC), can be attached to a fitting by threading or by using an adhesive. Plastic pipe is specified in the same manner as other pipe.

Water Tubing Specifications and Attachment Methods

Water tubing is specified by giving a **standard diameter** and a **type**—for example, “1-standard type K.” The standard size does not necessarily equal the inside or outside diameter of the tube. Each standard size corresponds to one and only one outside diameter. The type (K, L, or M) specifies the wall thickness. Type K is used for underground service and

general plumbing. Type L is used primarily for interior plumbing, while type M is made for use with soldered fittings. Appendix Table D.2 gives dimensions of copper water-tubing sizes that vary from 1/4-std to 12-std in English and in SI units.

Note that copper can be a *tubing* material or a *pipe* material. If used as a pipe material, then its dimensions follow pipe specifications. One difference between pipe and tubing is that tubing has a thinner wall and cannot sustain the high fluid pressures that pipe can.

Another type of tubing commonly used in air conditioners and heat pumps is called refrigeration tubing. It is specified in the same manner as copper water tubing and is usually made of a copper alloy. The difference is that copper water tubing is quite rigid, while refrigeration tubing is rather ductile (it can be bent by hand).

Tubes are also used in heat exchangers, and such tubes are referred to as condenser tubes. They are manufactured in sizes that are not the same as copper water tubing. Condenser tubes are discussed in the chapters on heat exchangers.

Tubing can be attached to fittings in a number of ways. A tube end can be **flared** and attached to a **flared end fitting**; the tube end is flared outward uniformly with a **flaring tool**. Another attachment method involves use of a **compression fitting**, in which the tube end is inserted through a snugly fitting ring that is furnished as part of the fitting itself. When the fitting nut is tightened, it compresses the ring and causes the copper tube end to expand against the inside wall of the fitting. A third joining technique involves brazing or soldering. The tube end is inserted into a fitting that fits like a sleeve. The joint is *fluxed* to remove the oxide, and the fitting and tube are soldered or brazed (commonly referred to as **sweating**).

In Europe where the SI unit system is used, there is a pipe sizing system that differs from that used in the U.S.

Bell and Spigot Pipe

Another type of pipe is known as bell and spigot pipe. In this system, one end of each pipe length is enlarged enough to accept another pipe. These pipes are specified according to a size, which equals the inside diameter of the pipe. When two pipes are joined together by placing one end into the expanded end of another, a gasket type of material is placed between them to ensure that a fluid-tight connection is obtained. Fittings are available for this type of pipe as well. Typically, bell and spigot pipes are made of cast iron or of PVC. PVC tubes of this type are used extensively in underground sprinkler systems.

3.2 Equivalent Diameters for Noncircular Ducts

Noncircular ducts are found in a number of fluid conveying systems. Rectangular and square cross sections are used for air-conditioning or heating ducts and for gutters and downspouts. Annular cross sections are found in double-pipe heat exchangers where one tube is placed within another; the section between the tubes is annular. The question arises as to what should be used for the characteristic dimension of the noncircular duct. Three different choices for the characteristic dimension have been proposed: hydraulic radius, effective diameter, and hydraulic diameter.

The **hydraulic radius** R_h is used widely for flow in open channels. The hydraulic radius is defined as

$$R_h = \frac{\text{area of flow}}{\text{wetted perimeter}} = \frac{A}{P} \quad (3.2)$$

This definition is entirely satisfactory for flow in an open channel but leads to an undesirable result when modeling close conduits. For a circular duct flowing full, we find

$$R_h = \frac{A}{P} = \frac{\pi D^2/4}{\pi D} = \frac{D}{4}$$

Thus, the hydraulic radius for flow in a pipe is one-fourth of its diameter. Traditionally, diameter D is preferred to represent a circular duct rather than $D/4$, so we tend to not use hydraulic radius.

The **effective diameter** D_{eff} is the diameter of a circular duct that has the same area as the noncircular duct of interest. Thus,

$$\frac{\pi D_{\text{eff}}^2}{4} = A_{\text{noncirc duct}} \quad (3.3)$$

Consider, for example, a rectangular duct of dimensions $h \times w$. The effective diameter is found with

$$\frac{\pi D_{\text{eff}}^2}{4} = hw$$

or

$$D_{\text{eff}} = 2\sqrt{hw/\pi}$$

The third equivalent diameter we will define is called the **hydraulic diameter** D_h :

$$D_h = \frac{4 \cdot \text{area of flow}}{\text{wetted perimeter}} = \frac{4A}{P} \quad (3.4)$$

For a circular duct flowing full, we calculate

$$D_h = \frac{4A}{P} = \frac{4\pi D^2/4}{\pi D} = D$$

which gives us an acceptable result. For a rectangular duct of dimensions $h \times w$,

$$D_h = \frac{4hw}{2h + 2w} = \frac{2hw}{h + w}$$

This equation gives results that are entirely different from Equation 3.3 and it should be noted that the hydraulic and effective diameters cannot be made equal for any value of w given h .

The hydraulic diameter arises when the momentum equation is applied to flow in a closed conduit. Traditionally, the hydraulic diameter is used more widely than the effective diameter. We shall use the hydraulic diameter in this text except in a few selected exercises.

EXAMPLE 3.1. Figure 3.1 is a sketch of the cross section of a heat exchanger. A warm fluid flows through the center tube and a cooler fluid flows through the annulus. The annulus is bounded by 4 standard type K tubing (outside) and 2 standard type K inside. Determine the hydraulic radius, the effective diameter, and the hydraulic diameter of the annular flow area.

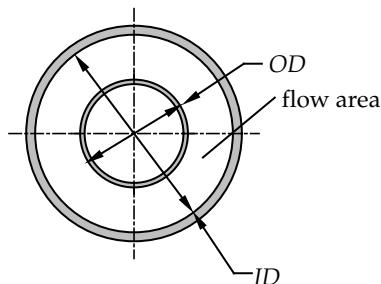


FIGURE 3.1. Annular cross section bounded by two tubes.

Solution: From the Appendix tables, we obtain the following data:

4 std type K	ID = 9.8 cm
2 std type K	OD = 5.398 cm

The annular flow area is

$$A = \frac{\pi(ID)^2}{4} - \frac{\pi(OD)^2}{4} = \frac{\pi}{4} (9.8^2 - 5.398^2)$$

$$A = 52.54 \text{ cm}^2$$

The wetted perimeter of this cross section is the length associated with both tubes:

$$P = \pi(ID) + \pi(OD) = \pi(9.8 + 5.398)$$

$$P = 47.75 \text{ cm}$$

The hydraulic radius is calculated as

$$R_h = \frac{A}{P} = \frac{52.54}{47.75}$$

$$\underline{R_h = 1.1 \text{ cm}}$$

The effective diameter D_{eff} is the diameter of a circular duct that has the same area as the noncircular duct of interest:

$$\frac{\pi D_{\text{eff}}^2}{4} = A$$

or

$$D_{\text{eff}} = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{4(52.54)}{\pi}}$$

$$\underline{D_{\text{eff}} = 8.18 \text{ cm}}$$

The hydraulic diameter D_h is found as

$$D_h = \frac{4A}{P} = \frac{4(52.54)}{47.75}$$

$$\underline{D_h = 4.4 \text{ cm}}$$

3.3 Equation of Motion for Flow in a Duct

In this section, we develop an expression for pressure loss in a conduit due to frictional effects. Figure 3.2 illustrates flow in a closed conduit. A circular cross section is illustrated, but the results remain general until geometry terms for a specific cross section are introduced into the equations.

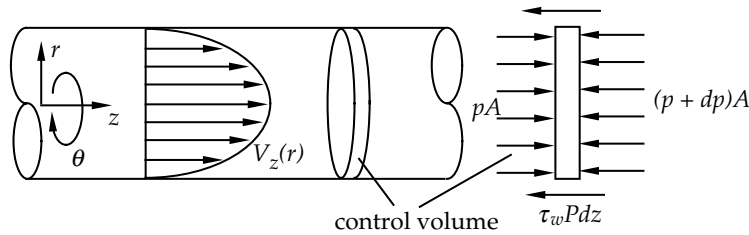


FIGURE 3.2. Laminar flow in a circular duct.

Shown is a coordinate system with the z direction along the axis of the duct. Also shown is a control volume in the shape of a disk whose diameter equals the inside diameter of the duct. The forces acting on the control volume include pressure and friction. The force due to friction is a wall shear stress that acts over the surface area of the control volume. Gravity forces are neglected. The momentum equation applied to the control volume is

$$\Sigma F_z = \frac{1}{g_c} \int \int_{CS} V_z \rho V_n dA$$

We note that the z -directed velocity out of the control volume equals that into the control volume, making the right-hand side of the momentum equation equal to zero. The left hand side will include the forces acting on the control volume that we wish to consider: pressure and friction. Thus the preceding equation becomes

$$pA - \tau_w Pd z - (p + dp)A = 0 \quad (3.5)$$

The term A is the cross-sectional area, and Pdz is perimeter times axial distance, which equals the surface area over which the wall shear stress τ_w acts. Equation 3.5 becomes

$$\frac{dp}{dz} = -\tau_w \frac{P}{A} = -\tau_w \frac{4P}{4A}$$

In terms of hydraulic diameter,

$$\frac{dp}{dz} = -\frac{4\tau_w}{D_h} \quad (3.6)$$

The pressure change per unit length (dp/dz) is thus a function of the wall shear stress and the hydraulic diameter of the duct. Equation 3.6 applies to *any* cross section.

We now introduce a **friction factor** f customarily defined as

$$f = \frac{4\tau_w g_c}{\rho V^2 / 2} \quad (3.7)$$

where V is the average velocity of the flow in the conduit. The preceding definition is the **Darcy-Weisbach friction factor**. The form of this equation has force (per unit area) in the numerator and kinetic energy in the denominator.

The **Fanning friction factor**, used in some texts, is defined as

$$f' = \frac{\tau_w g_c}{\rho V^2 / 2}$$

Both definitions for friction factor are commonly used. The Darcy-Weisbach definition is conveniently applied when hydraulic diameter is the characteristic length. The Fanning friction factor is used in formulations where hydraulic radius is the characteristic length, typically in open channel flow applications. We will use the Darcy-Weisbach definition of Equation 3.7.

Solving Equation 3.7 for $4\tau_w$ and substituting into Equation 3.6 gives

$$dp = -\frac{\rho V^2}{2g_c} \frac{f dz}{D_h} \quad (3.8a)$$

Integrating from section 1 to section 2, where section 2 is a distance L downstream gives

$$p_2 - p_1 = -\frac{\rho V^2}{2g_c} \frac{fL}{D_h} \quad (3.8b)$$

Equations 3.8a and b give the pressure drop in a duct due to friction. Again, these equations are independent of duct cross section.

To model flow in a duct, we use the Bernoulli equation. As developed in the last chapter, it is apparent that the Bernoulli equation does not account for frictional effects. For flow in a duct, friction is manifested as a loss in

pressure with axial distance as shown in Equation 3.8b. So to use the Bernoulli equation for flow in a duct, we must first modify it by combining it with Equation 3.8b. The result is

$$\frac{p_1 g_c}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2 g_c}{\rho g} + \frac{V_2^2}{2g} + z_2 + \frac{fL}{D_h} \frac{V^2}{2g} \quad (3.9)$$

The above equation is actually an energy balance performed for two points a distance L apart within a duct. The equation states that

$$\left(\begin{array}{c} \text{pressure} \\ \text{head} \end{array} + \text{KE} + \text{PE} \right)_1 = \left(\begin{array}{c} \text{pressure} \\ \text{head} \end{array} + \text{KE} + \text{PE} \right)_2 + \left(\begin{array}{c} \text{energy loss} \\ \text{due to friction} \end{array} \right)$$

The head loss is expressed as a product of a friction term (fL/D_h) and the kinetic energy of the flow. Note that Equation 3.9 can be applied to any cross section as long as the appropriate hydraulic diameter is used.

3.4 Friction Factor and Pipe Roughness

In this section, we will present methods of evaluating friction factor for a circular duct under laminar and under turbulent flow conditions. Results for a variety of noncircular ducts are presented later in this chapter.

Laminar Flow of a Newtonian Fluid in a Circular Duct

Our interest in this area is in having an equation for the velocity profile and for the average velocity. Figure 3.2 illustrates laminar flow in a duct as well as the polar coordinate system we will use in our formulation. The z -directed instantaneous velocity is

$$V_z = \left(-\frac{dp}{dz} \right) \left(\frac{R^2}{4\mu} \right) \left[1 - \left(\frac{r}{R} \right)^2 \right] \quad \left(\begin{array}{c} \text{laminar flow} \\ \text{circular duct} \end{array} \right) \quad (3.10)$$

This equation is derived by applying the momentum equation to a control volume within the duct. (See the problem section at the end of this chapter for a step-by-step procedure.) Note that as axial distance z increases, the pressure p decreases. Therefore, dp/dz is a negative quantity and the term $(-dp/dz)$ in Equation 3.10 is actually positive. Moreover, $(-dp/dz)$, which is the pressure drop per unit length is a constant.

When Equation 3.10 is integrated over the cross-sectional area (as per the continuity equation), the **volume flow rate** Q results:

$$Q = \int_{CS} \int V_n dA = \int_0^{2\pi} \int_0^R \left(-\frac{dp}{dz} \right) \left(\frac{R^2}{4\mu} \right) \left[1 - \left(\frac{r}{R} \right)^2 \right] r dr d\theta$$

We note that the terms $(-dp/dz)$ and $R^2/4\mu$ are both constant. Integrating and solving, we get

$$Q = \frac{\pi R^4}{8\mu} \left(-\frac{dp}{dz} \right) \quad (3.11)$$

The average velocity is given by

$$V = \frac{Q}{A} = \frac{R^2}{8\mu} \left(-\frac{dp}{dz} \right) \quad (3.12)$$

Recall Equation 3.8a,

$$dp = -\frac{\rho V^2}{2g_c} \frac{f dz}{D_h} \quad (3.8a)$$

We now equate Equations 3.12 and 3.8a. Eliminating the pressure drop term and solving for friction factor, we find

$$f = \frac{32\mu g_c}{\rho V R} = \frac{64\mu g_c}{\rho V D}$$

$$\text{or } f = \frac{64}{\text{Re}} \quad (\text{laminar flow circular duct}) \quad (3.13)$$

where the diameter D has been substituted for hydraulic diameter D_h .

Turbulent Flow in a Circular Duct

For turbulent flow, we rely on experimental methods to develop a relationship between the pertinent variables. Based on results of many tests performed using artificially roughened pipe walls, it has been determined that the friction factor is dependent upon Reynolds number Re and relative roughness ε/D :

$$f = f(\text{Re}, \varepsilon/D) \quad (3.14)$$

where ε is a characteristic linear dimension representing the roughness of the inside surface area of the conduit wall.

Extensive research on surface roughness has been performed and

correlated. Sand particles of known dimensions or diameter (sizes separated by sieving) were attached with an adhesive to the inside surface of a pipe. The pipe was then tested; that is, pressure drop versus volume flow rate data were obtained for fluid pumped through the pipe. Tests were repeated with many pipe sizes and many sand particle diameters. When tests were then performed on commercially available pipes (e.g., pressure drop versus flow rate), a comparison could be made. For example, a commercial steel pipe exhibits the same or similar pressure drop versus flow rate behavior as a pipe coated with sand particles of size $\varepsilon = 0.00015$ ft (0.0046 cm). Some texts call ε an “equivalent sand roughness factor.” Values of ε for various materials are provided in Table 3.1.

A graph of the data to predict the friction factor f given the Reynolds number $Re (= \rho V D / \mu g_c)$ and the relative roughness (ε/D) is customarily known as the **Moody diagram**. (“Friction Factors in Pipe Flow,” by L. Moody; in *Transactions of ASME*, 1944, **68**, 672.) Exhaustive amounts of data were compiled and consolidated into this graph. Figure 3.3 is a graph of the Moody diagram.

A number of equations have been written to curve fit the Moody diagram. The older equations (e.g., Colebrook equation) are known to involve an iterative process when trying to calculate friction factor f given Reynolds number Re and relative roughness ε/D . Recently published equations, however, overcome this difficulty. The Chen equation, the Churchill equation, the Haaland equation, and the Swamee-Jain equation all solve for f explicitly in terms of Re and ε/D . So when Re and ε/D are known, these equations allow for calculating f directly just as with the Moody Diagram.

The **Chen equation** is valid for $Re \geq 2100$ and is written as

$$f = \left[-2.0 \log \left\{ \frac{\varepsilon}{3.7065D} - \frac{5.0452}{Re} \log \left(\frac{1}{2.8257} \left[\frac{\varepsilon}{D} \right]^{1.1098} + \frac{5.8506}{Re^{0.8981}} \right) \right\} \right]^{-2} \quad (3.15)$$

The **Churchill equation**, also valid for $Re \geq 2100$, is

$$f = 8 \left\{ \left[\frac{8}{Re} \right]^{12} + \frac{1}{(B + C)^{1.5}} \right\}^{\frac{1}{12}} \quad (3.16)$$

where

$$B = \left[2.457 \ln \frac{1}{(7/Re)^{0.9} + (0.27\varepsilon/D)} \right]^{16} \quad \text{and} \quad C = \left(\frac{37530}{Re} \right)^{16}$$

TABLE 3.1. *Roughness factor for various pipe materials.*

Pipe Material	ϵ , ft	ϵ , cm
Steel		
Commercial	0.00015	0.004 6
Corrugated	0.003–0.03	0.09–0.9
Riveted	0.003–0.03	0.09–0.9
Galvanized	0.0002–0.0008	0.006–0.025
Mineral		
Brick sewer	0.001–0.01	0.03–0.3
Cement-asbestos		
Clays		
Concrete		
Wood stave	0.0006–0.003	0.018–0.09
Cast iron	0.00085	0.025
Asphalt coated	0.0004	0.012
Bituminous lined	0.000008	0.000 25
Cement lined	0.000008	0.000 25
Centrifugally spun	0.00001	0.000 31
Drawn tubing	0.000005	0.000 15
Miscellaneous		
Brass	0.000005	0.000 15
Copper		
Glass		
Lead		
Plastic		
Tin	0.0002–0.0008	0.006–0.025
Galvanized		
Wrought iron	0.00015	0.004 6
PVC	Smooth	Smooth

The **Haaland Equation** is

$$f = \left\{ -0.782 \ln \left[\frac{6.9}{\text{Re}} + \left(\frac{\epsilon}{3.7D} \right)^{1.11} \right] \right\}^{-2} \quad (3.17)$$

Finally, the **Swamee-Jain Equation** is

$$f = \frac{0.250}{\left\{ \log \left[\frac{\epsilon}{3.7D} + \frac{5.74}{\text{Re}^{0.9}} \right] \right\}^2} \quad (3.18)$$

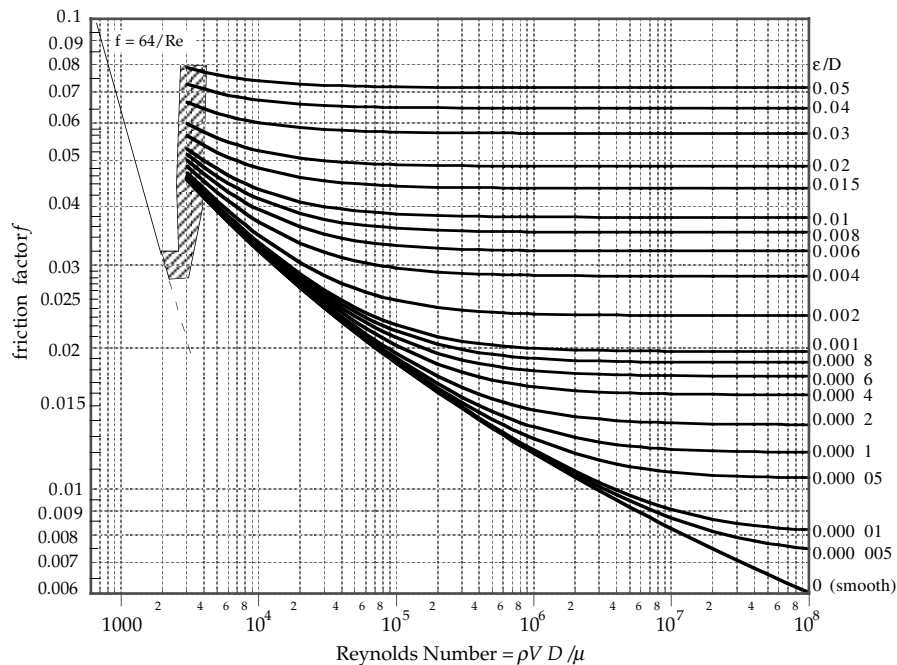


FIGURE 3.3. *Moody diagram constructed with Chen equation.*

Figure 3.3 is a version of the Moody diagram that was generated by using the Chen equation. The Reynolds number appears on the horizontal axis and varies from just under 1 000 to 100 000 000. The relative roughness is an independent variable and ranges from 0 to 0.05. The friction factor appears on the vertical axis and varies to 0.1. Note that the friction factor of Figure 3.3 is the Darcy-Weisbach friction factor, which can be seen by the label on the laminar flow friction factor: $f = 64/\text{Re}$.

Other forms of the Moody diagram have been developed in order to simplify calculations in problems where iterative methods (or trial and error) are required (i.e., volume flow rate Q unknown, diameter D unknown). Consider that in a piping problem, six variables can enter the problem: Δp (or Δh), Q , D , v , L , and ϵ . Usually in the traditional type of problem, five of these variables are known and the sixth is to be found. When pressure drop Δp (or head loss $\Delta h = \Delta p g_c / \rho g$) is unknown, then the problem can be solved in a straightforward manner using the Moody diagram, Figure 3.3. When volume flow rate Q is unknown, use of the Moody diagram requires a trial-and-error procedure to obtain a solution. If a graph of f versus $\text{Re}\sqrt{f}$ is available, however, then the unknown Q problem can be solved in a straightforward manner. Such a graph is provided in Figure 3.4.

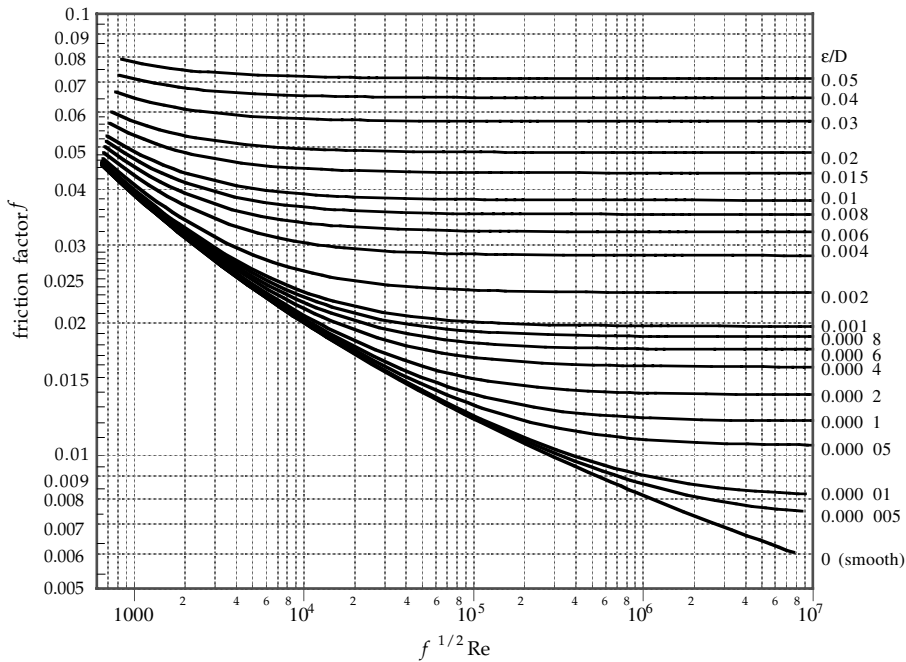


FIGURE 3.4. *Modified pipe friction diagram for solving volume flow rate unknown problems.*

Equation 3.15, the Chen equation, was used to generate the f versus $f^{1/2}Re$ graph of Figure 3.4. A value of ϵ/D was selected, and the Reynolds number was allowed to vary from 2×10^3 to 10^8 . The friction factor was found, and $f^{1/2}Re$ was calculated. Values generated were graphed, the result of which is provided in Figure 3.4.

When diameter D is unknown, use of the Moody diagram again requires a trial-and-error procedure, unless a graph of f versus $f^{1/5}Re$ is available. Such a graph is given in Figure 3.5. The trial-and-error process required when the diameter D is unknown can be eliminated only with a change of independent variable, ϵ/D . This is due to the fact that the relative roughness term contains diameter D , which is unknown. In studies involving economics of pipe size selection, a new variable is introduced to rid the roughness term of diameter. The new parameter is called the **roughness number** and is defined as:

$$Ro = \frac{\epsilon/D}{Re} = \frac{(\epsilon/D)\mu g_c}{\rho V D}$$

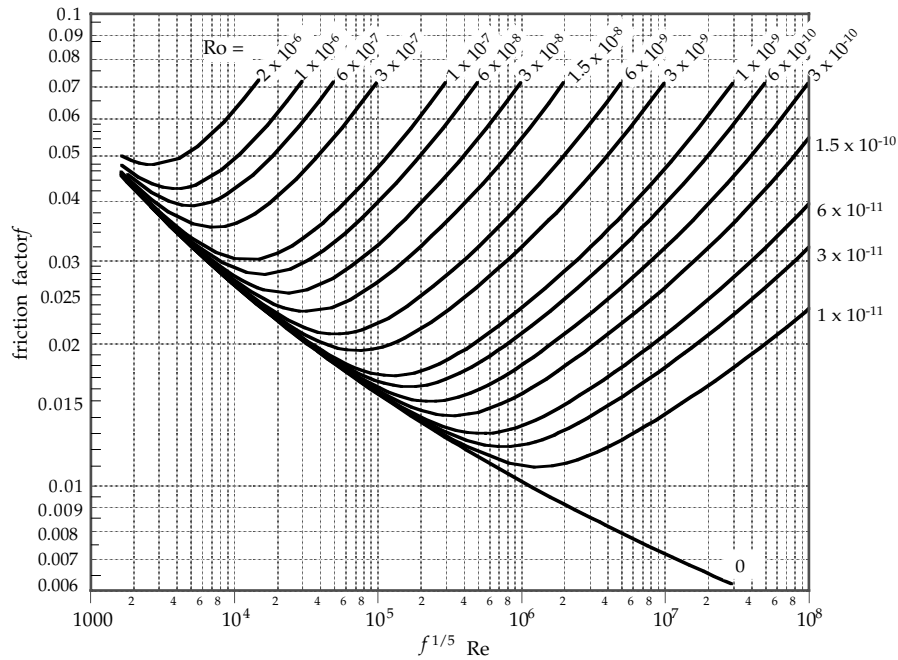


FIGURE 3.5. Modified pipe friction diagram for solving diameter unknown problems.

It is advantageous in these problems to express velocity in terms of flow rate and diameter; for a circular duct, we have

$$V = \frac{Q}{A} = \frac{4Q}{\pi D^2}$$

Using this equation, the roughness number becomes

$$Ro = \frac{\pi \epsilon \mu g_c}{4 \rho Q} = \frac{\epsilon/D}{Re} \quad (3.19)$$

For the diameter unknown problem, it is desirable to have a graph of f versus $f^{1/5}Re$ with $Ro [= (\epsilon/D)/Re]$ as an independent variable. This graph is provided as Figure 3.5, which was generated with Equation 3.15, the Chen equation. A value of ϵ/D was selected as was a single value of Re . The friction factor was found; the Roughness number $Ro (= \epsilon/D/Re)$ and $f^{1/5}Re$ were calculated. The next value of ϵ/D was selected in harmony with the next Re such that Ro was held constant. The objective was to generate lines of constant Ro . Values of f , $f^{1/5}Re$, and Ro were then graphed and the result is provided in Figure 3.5.

For the graphs in Figures 3.4 and 3.5, the exponent of f is selected as being $1/2$ or $1/5$. The reason for choosing these values arises from the solution of the equations for specific problems. How the graphs are used to solve the traditional pipe flow problems will be illustrated by example.

EXAMPLE 3.2. A 4-nominal schedule 40 pipe conveys castor oil at a flow rate of $0.01 \text{ m}^3/\text{s}$. The pipe is made of commercial steel and is 250 m long. Determine the pressure drop experienced by the fluid.

Solution: From the various property tables, we read

castor oil	$\rho = 960 \text{ kg/m}^3$	$\mu = 650 \times 10^{-3} \text{ N}\cdot\text{s/m}^2$	[App. Table B.1]
4-nom sch 40	$D = 10.23 \text{ cm}$	$A = 82.19 \text{ cm}^2$	[App. Table D.1]

commercial steel	$\varepsilon = 0.0046 \text{ cm}$	[Table 3.1]

The conservation of mass equation written for this system is

$$Q = A_1 V_1 = A_2 V_2$$

where subscript “1” refers to the pipe inlet and “2” refers to the outlet. Because the pipe area does not change, $A_1 = A_2$, then the velocity at the inlet equals that at the outlet: $V_1 = V_2$. The Bernoulli equation with friction is

$$\frac{p_1 g_c}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2 g_c}{\rho g} + \frac{V_2^2}{2g} + z_2 + \frac{fL}{D} \frac{V^2}{2g}$$

It is assumed that the inlet and the outlet are at the same elevation (nothing specific was given in the problem statement). So $z_1 = z_2$ and the Bernoulli equation becomes

$$\frac{p_1 g_c}{\rho g} - \frac{p_2 g_c}{\rho g} = \frac{fL}{D} \frac{V^2}{2g}$$

$$\text{or } p_1 - p_2 = \frac{fL}{D} \frac{\rho V^2}{2g_c}$$

The average velocity is found as

$$V = \frac{Q}{A} = \frac{0.01}{82.19 \times 10^{-4}} = 1.22 \text{ m/s}$$

The Reynolds number then becomes

$$\text{Re} = \frac{\rho V D}{\mu g_c} = \frac{960(1.22)(0.1023)}{650 \times 10^{-3}}$$

$$\text{or } \text{Re} = 184$$

The flow is laminar because the Reynolds number is less than 2 100. The friction factor f is calculated to be (laminar flow in a circular duct)

$$f = \frac{64}{\text{Re}} = \frac{64}{184}$$

$$f = 0.348$$

Substituting into the Bernoulli equation gives

$$p_1 - p_2 = \frac{fL}{D} \frac{\rho V^2}{2g_c} = \frac{0.348(250)}{0.1023} \frac{960(1.22)^2}{2}$$

$$p_1 - p_2 = 6.08 \times 10^5 \text{ N/m}^2 = 608 \text{ kPa}$$

This result is independent of pipe material because the flow is laminar.

EXAMPLE 3.3. Chloroform flows at a rate of $0.01 \text{ m}^3/\text{s}$ through a 4-nominal schedule 40 wrought iron pipe. The pipe is laid out horizontally and is 250 m long. Calculate the pressure drop of the chloroform.

Solution: From the property tables, we read

$$\text{chloroform} \quad \rho = 1.47(1\,000) \text{ kg/m}^3 \quad \mu = 0.53 \times 10^{-3} \text{ N}\cdot\text{s/m}^2 \quad [\text{App. Table B.1}]$$

$$4\text{-nom sch 40} \quad \text{ID} = D = 10.23 \text{ cm} \quad A = 82.19 \text{ cm}^2 \quad [\text{App. Table D.1}]$$

$$\text{wrought iron} \quad \varepsilon = 0.0046 \text{ cm} \quad [\text{Table 3.1}]$$

The continuity equation for incompressible steady flow through the pipe is

$$A_1 V_1 = A_2 V_2$$

Because $A_1 = A_2$, then $V_1 = V_2$. The Bernoulli equation applies:

$$\frac{p_1 g_c}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2 g_c}{\rho g} + \frac{V_2^2}{2g} + z_2 + \frac{fL}{D_h} \frac{V^2}{2g}$$

where points 1 and 2 are $L = 250$ m apart, $z_1 = z_2$ for a horizontal pipe, and $p_1 - p_2$ is sought. The above equation reduces to

$$p_1 - p_2 = \frac{fL}{D_h} \frac{\rho V^2}{2g_c}$$

The flow velocity is

$$V = \frac{Q}{A} = \frac{0.01}{82.19 \times 10^{-4}} = 1.22 \text{ m/s}$$

The Reynolds number is calculated to be

$$\text{Re} = \frac{\rho V D}{\mu g_c} = \frac{1.47(1\,000)(1.22)(0.102\,3)}{0.53 \times 10^{-3}} = 3.46 \times 10^5$$

The flow is therefore turbulent. Thus,

$$\left. \begin{array}{l} \text{Re} = 3.46 \times 10^5 \\ \text{Also } \frac{\varepsilon}{D} = \frac{0.004\,6}{10.23} = 0.000\,45 \end{array} \right\} f = 0.018 \quad (\text{Figure 3.3})$$

The pressure loss is

$$p_1 - p_2 = \frac{fL}{D_h} \frac{\rho V^2}{2g_c} = \frac{0.018(250)}{0.102\,3} \frac{1.47(1\,000)(1.22)^2}{2}$$

$$\underline{p_1 - p_2 = 48\,120 \text{ N/m}^2 = 48.1 \text{ kPa}}$$

This and the previous example are identical except for fluid properties. In the previous example, the flow is laminar and the pressure drop is 608 kPa. In this example, the flow is turbulent and the pressure drop is 48.1 kPa. The frictional loss in the first example is large due to the high viscosity of the fluid.

EXAMPLE 3.4. Water flows in an asphalt coated cast iron pipe that is 100 m long. The pressure drop over this length is 685 N/m². The pipe itself is 2¹/₂ nominal schedule 80. Determine the volume flow rate under these

conditions.

Solution: From the property tables, we read

water $\rho = 1\,000\text{ kg/m}^3$ $\mu = 0.89 \times 10^{-3}\text{ N}\cdot\text{s/m}^2$ [App. Table B.1]

$2\frac{1}{2}$ -nom sch 80 ID = $D = 5.901\text{ cm}$ $A = 27.35\text{ cm}^2$ [App. Table D.1]

asphalted cast iron $\varepsilon = 0.012\text{ cm}$ [Table 3.1]

The continuity equation for incompressible, steady flow is

$$A_1 V_1 = A_2 V_2$$

Because $A_1 = A_2$, then $V_1 = V_2$. The Bernoulli equation with friction is

$$\frac{p_1 g_c}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2 g_c}{\rho g} + \frac{V_2^2}{2g} + z_2 + \sum \frac{fL}{D_h} \frac{V^2}{2g}$$

where length L and $(p_1 - p_2)$ are both given. With $z_1 = z_2$, the preceding equation becomes:

$$p_1 - p_2 = \Delta p = \frac{fL}{D_h} \frac{\rho V^2}{2g_c}$$

Rearranging and solving for velocity, we obtain

$$V = \sqrt{\frac{2D\Delta p g_c}{\rho f L}}$$

Substituting,

$$V = \sqrt{\frac{2(0.05901)(685)}{1\,000f(100)}} = \frac{0.028\,43}{\sqrt{f}} \quad (\text{i})$$

The Reynolds number of the flow is

$$\text{Re} = \frac{\rho V D}{\mu g_c} = \frac{1\,000 V (0.05901)}{0.89 \times 10^{-3}}$$

or

$$\underline{\text{Re} = 6.63 \times 10^4 V} \quad (\text{ii})$$

and

$$\frac{\varepsilon}{D} = \frac{0.012}{5.901} = 0.002 \quad (\text{iii})$$

The operating point here is somewhere on this ε/D line, as indicated in the abbreviated Moody diagram of Figure 3.6. We begin by assuming that the friction factor f will correspond to the fully turbulent value of $\varepsilon/D = 0.002$ (Figure 3.6). (This step is not always possible, especially for smaller values of ε/D . Alternatively, we could start with a randomly selected value of f , which will also work.) We read $f = 0.024$.

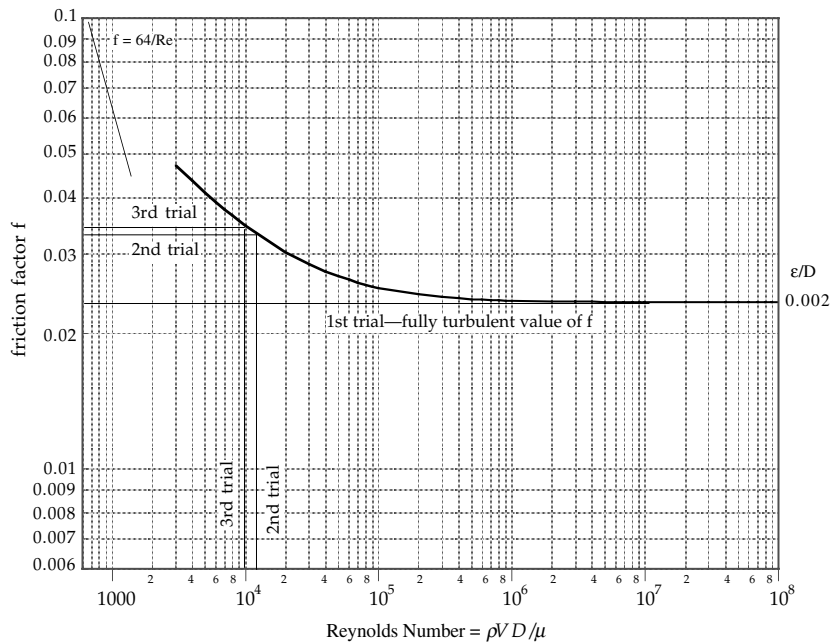


FIGURE 3.6. *Abbreviated Moody diagram to illustrate the trial and error procedure for the volume flow rate unknown problem.*

With this value for the friction factor, the velocity is

$$V = 0.02843/\sqrt{0.024} = 0.183 \text{ m/s}$$

The Reynolds number then is

$$\left. \begin{aligned} \text{Re} &= 6.63 \times 10^4 (0.183) = 1.2 \times 10^4 \\ \frac{\varepsilon}{D} &= 0.002 \end{aligned} \right\} f = 0.034$$

Repeating the calculations with this new value of f gives

$$f = 0.034; V = 0.154; \text{Re} = 1.02 \times 10^4; f \approx 0.035 \text{ close enough}$$

$$f = 0.035; V = 0.152 \text{ m/s}$$

The velocity is thus 0.152 m/s. The volume flow rate is calculated as

$$Q = AV = (27.35 \times 10^{-4} \text{ m}^3/\text{s})(0.152 \text{ m/s})$$

or

$$\underline{Q = 4.16 \times 10^{-4} \text{ m}^3/\text{s}}$$

Summary of calculations

$$\text{1st trial: } f = 0.024; V = 0.183; \text{Re} = 1.2 \times 10^4$$

$$\text{2nd trial: } f = 0.034; V = 0.154; \text{Re} = 1.02 \times 10^4 \quad f \approx 0.035 \text{ close enough}$$

$$\text{3rd trial: } f = 0.035; V = 0.152 \text{ m/s}$$

Suppose we wish to use Figure 3.4 and avoid the trial-and-error procedure. We set up the calculations and arrive at the following [Equations i, ii and iii]:

$$V = \frac{0.02848}{\sqrt{f}} \quad (\text{i})$$

$$\text{Re} = 6.63 \times 10^4 V \quad (\text{ii})$$

$$\frac{\varepsilon}{D} = 0.002 \quad (\text{iii})$$

We combine Equations i and ii to eliminate velocity V and obtain

$$\text{Re} = 6.63 \times 10^4 (0.02843/\sqrt{f})$$

or

$$\left. \begin{array}{l} f^{1/2} \text{Re} = 1.9 \times 10^3 \\ \frac{\varepsilon}{D} = 0.002 \end{array} \right\} \quad f = 0.035 \quad (\text{Figure 3.4})$$

Substituting into Equation i, we get

$$p_1 - p_2 = \Delta p = \frac{fL}{D} \frac{\rho}{2\pi^2 D^4} \frac{16Q^2}{g_c}$$

Rearranging and solving for diameter D , we get

$$D = \sqrt[5]{\frac{8\rho Q^2 f L}{\pi^2 \Delta p g_c}}$$

Substituting, we have

$$D = \sqrt[5]{\frac{8(1.1)(1\,000)(50 \times 10^{-3})^2 f (2\,000)}{\pi^2 (200\,000)}}$$

or

$$D = 0.467 f^{1/5} \quad (\text{i})$$

In terms of flow rate, the Reynolds number becomes

$$\text{Re} = \frac{\rho V D}{\mu g_c} = \frac{4\rho Q}{\pi D \mu g_c}$$

Substituting,

$$\text{Re} = \frac{4(1.1)(1\,000)(50 \times 10^{-3})}{\pi D (16.2 \times 10^{-3})} = 4.3 \times 10^3 / D \quad (\text{ii})$$

In order to use the Moody diagram of Figure 3.3, we assume a friction factor to initiate the trial-and-error method. Assume

$$f = 0.025 \quad (\text{randomly selected})$$

then

$$D = 0.467(0.025)^{1/5} = 0.223 \text{ m}$$

and

$$\left. \begin{aligned} \text{Re} &= 4.3 \times 10^3 / 0.223 = 1.93 \times 10^4 \\ \frac{\varepsilon}{D} &= \text{“smooth”} \end{aligned} \right\} \quad f = 0.026 \quad (\text{Figure 3.3})$$

For the second trial,

$$f = 0.026; \quad D = 0.225; \quad \text{Re} = 1.9 \times 10^4; \quad f \approx 0.026 \quad (\text{close enough})$$

The diameter we select then is 0.225 m. Referring to Appendix Table D.1 and recalling that the problem statement requested schedule 40 pipe, we find that the size we select falls between 8-nominal schedule 40 and 10-nominal schedule 40. The smaller pipe can deliver the required flow rate, but at a pressure drop that will exceed 200 kPa, so we specify

$D = 10\text{-nominal schedule 40 pipe}$

Suppose we elect to avoid the trial-and-error procedure and use Figure 3.5. We arrive at Equations i and ii in the usual way, and then combine them to eliminate D ; we obtain

$$\left. \begin{aligned} \text{Re} &= 4.3 \times 10^3 / 0.467f^{1/5} \\ \text{or} \quad f^{1/5} \text{Re} &= 9.2 \times 10^3 \\ \text{Ro} = \frac{\pi \varepsilon \mu g_c}{4 \rho Q} &= 0 = \text{“smooth”} \end{aligned} \right\} \quad f = 0.026 \quad (\text{Figure 3.5})$$

The diameter is calculated as

$$D = 0.467(0.026)^{1/5} = 0.225 \text{ m}$$

which is the same result obtained before.

3.5 Minor Losses

The term **minor losses** refers to pressure losses encountered by a fluid as it flows through a fitting or a valve in a piping system. Fittings and valves are used to direct the flow, to connect conduits together, to re-route the fluid, and to control the flow rate. Fittings are an integral part of any piping system, and how their presence affects the fluid is the subject of this section.

As fluid flows through a fitting, the fluid may undergo an abrupt change in area (increase or decrease). The fluid may also have to negotiate a sharp curve and might do so by forming a separation region within the fitting. The fluid will encounter a loss in pressure. We treat this loss mathematically by assigning to each fitting a loss factor K . The pressure loss is then expressed as a multiple of the kinetic energy of the flow:

$$p_1 - p_2 = \sum K \frac{\rho V^2}{2g_c} \quad (3.20)$$

The Bernoulli equation when written to include the effects of friction and of minor losses becomes

$$\frac{p_1 g_c}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2 g_c}{\rho g} + \frac{V_2^2}{2g} + z_2 + \frac{fL}{D_h} \frac{V^2}{2g} + \sum K \frac{V^2}{2g} \quad (3.21)$$

We refer to Equation 3.21 as the **modified Bernoulli equation**.

Loss coefficients for a number of fittings are provided in Table 3.2. Most of the information in that table is a result of measurements made on fittings. A number of the fittings in the table have a constant value of the loss coefficient K and a corresponding equation. When performing calculations by hand, it is convenient to use a constant value, which is why they are provided. When using a computer, on the other hand, it is easy to use an equation for the loss coefficient, also provided in the table. Note that loss coefficient varies with pipe diameter for many of the fittings listed.

Table 3.2 also gives minor loss coefficients for several types of valves. Valves are available in a variety of types and sizes, and selecting the right valve for the job should receive due attention. A wrong valve can have disastrous consequences, so it is desirable to have information on valve selection. Table 3.3 (in the Summary section of this chapter) gives general guidelines on selecting the proper valve for a given application. Included are valve characteristics, advantages, and disadvantages.

Before proceeding to solve piping problems, it is worthwhile to review the concept of the control volume and how to apply the modified Bernoulli equation correctly. The first step in formulating the solution to a problem is to determine where the boundaries of the control volume are to be located. In so doing, we identify the cross sections where mass crosses the control surface. The pressure p , velocity V , and height z terms of the modified Bernoulli equation apply only to cross sections where mass crosses the boundary. These terms are to be applied to nothing outside or inside the control volume. The friction term fL/D_h and the minor loss coefficient K apply to what is happening within the piping system. These terms do not apply to anything that happens outside the control volume.

As an example, consider the system shown in Figure 3.7. A pipe is connected to two tanks. We will examine five different control volumes applied to this same pipe, and write the modified Bernoulli equation for each case. In Figure 3.7a, the control volume includes all the fluid in the pipe and the fluid in both tanks. Section 1 is the free surface of the tank on the left and section 2 is the free surface of the tank on the right. We now evaluate each property at both sections:

TABLE 3.2. *Loss coefficients for pipe fittings: inlets, exits, and elbows.*

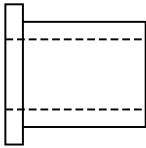
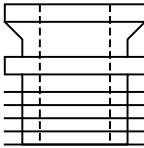
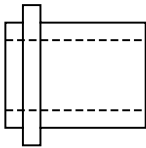
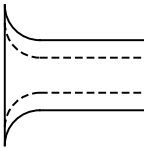
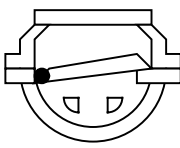
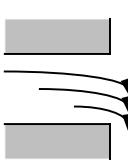
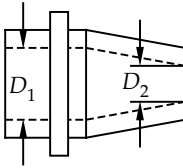
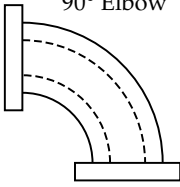
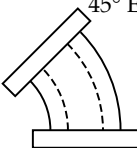
	Square edged inlet $K = 0.5$		Basket strainer $K = 1.3$
	Re-entrant inlet or inward projecting pipe $K = 1.0$		Well rounded inlet or a bell mouth inlet $K = 0.05$
	Foot valve $K = 0.8$		Exit $K = 1.0$
 Convergent outlet or nozzle $K = 0.1(1 - D_2/D_1)$ D_2/D_1 from 0.5 to 0.9			
 90° Elbow	threaded	flanged, welded, glued, bell & spigot	
	regular $K = 1.4$ $K = 1.4(ID)^{-0.53}$ ID from 0.3 to 4 in long radius $K = 0.75$ $K = 0.75(ID)^{-0.81}$ ID from 0.3 to 4 in	regular $K = 0.31$ $K = 0.44(ID)^{-0.23}$ ID from 1 to 25 in long radius $K = 0.22$ $K = 0.51(ID)^{-0.58}$ ID from 1 to 23 in	
 45° Elbow	regular $K = 0.35$ $K = 0.35(ID)^{-0.14}$ ID from 0.3 to 4 in	long radius $K = 0.17$ $K = 0.22(ID)^{-0.14}$ ID from 1 to 23 in	

TABLE 3.2 continued. *Loss coefficients for pipe fittings: elbows, T-joints, and couplings*

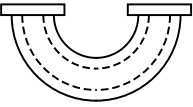
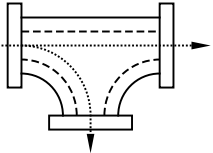
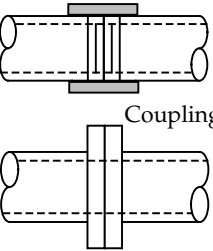
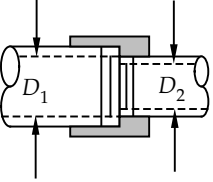
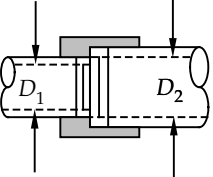
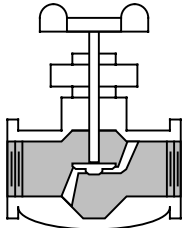
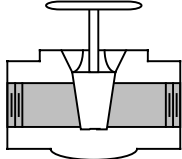
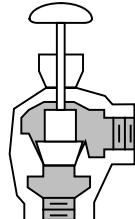
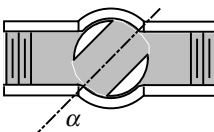
	threaded	flanged, welded, glued, bell & spigot
Return bend 	regular $K = 1.5$ $K = 1.5(ID)^{-0.57}$ ID from 0.3 to 4 in	regular $K = 0.3$ $K = 0.43(ID)^{-0.26}$ ID from 1 to 23 in long radius $K = 0.2$ $K = 0.43(ID)^{-0.53}$ ID from 1 to 23 in
T joint 	line flow $K = 0.9$ all sizes ID from 0.3 to 4 in branch flow $K = 1.9$ $K = 1.9(ID)^{-0.38}$ ID from 0.3 to 4 in	line flow $K = 0.14$ $K = 0.27(ID)^{-0.46}$ ID from 1 to 20 in branch flow $K = 0.69$ $K = 1.0(ID)^{-0.29}$ ID from 1 to 20 in
 Coupling	$K = 0.08$ $K = 0.083(ID)^{-0.69}$ ID from 0.4 to 4 in	$K = 0.08$ ID from 0.3 to 23 in
 Reducing bushing	$K = 0.5 - 0.167(D_2/D_1) - 0.125(D_2/D_1)^2 - 0.208(D_2/D_1)^3$ $0.25 < D_2/D_1 < 1$	
 Sudden expansion	$K = ((D_2/D_1)^2 - 1)^2$ $1 < D_2/D_1 < 5$	

TABLE 3.2 continued. *Loss coefficients for pipe fittings: valves.*

	threaded	flanged, welded, glued, bell & spigot																		
Globe valve 	fully open $K = 10$ $K = \exp\{2.158 - 0.459 \ln(ID) + 0.259[\ln(ID)]^2 - 0.123[\ln(ID)]^3\}$ ID from 0.3 to 4 in	fully open $K = 10$ $K = \exp\{2.565 - 0.916 \ln(ID) + 0.339[\ln(ID)]^2 - 0.01416[\ln(ID)]^3\}$ ID from 0.3 to 4 in																		
Gate Valve 	fully open $K = 0.15$ $K = 0.24(ID)^{-0.47}$ ID from 0.3 to 4 in	fully open $K = 0.15$ $K = 0.78(ID)^{-1.14}$ ID from 1 to 20 in																		
	All sizes																			
	<table><tr><td>Fraction closed</td><td>0</td><td>1/4</td><td>3/8</td><td>1/2</td><td>5/8</td><td>3/4</td><td>7/8</td></tr><tr><td>$K = 0.15$</td><td>0.26</td><td>0.81</td><td>2.06</td><td>5.52</td><td>17.0</td><td>97.8</td><td></td></tr></table>		Fraction closed	0	1/4	3/8	1/2	5/8	3/4	7/8	$K = 0.15$	0.26	0.81	2.06	5.52	17.0	97.8			
Fraction closed	0	1/4	3/8	1/2	5/8	3/4	7/8													
$K = 0.15$	0.26	0.81	2.06	5.52	17.0	97.8														
	fully open $K = 2.0$ $K = 4.5(ID)^{-1.08}$ ID from 0.6 to 4 in	fully open $K = 2.0$ $K = \exp\{1.569 - 1.43 \ln(ID) + 0.8[\ln(ID)]^2 - 0.137[\ln(ID)]^3\}$ ID from 1 to 20 in																		
Angle Valve																				
	All sizes																			
	<table><tr><td>$\alpha : 0$</td><td>10</td><td>20</td><td>30</td><td>40</td><td>50</td><td>60</td><td>70</td><td>80</td></tr><tr><td>$K = 0.05$</td><td>0.29</td><td>1.56</td><td>5.47</td><td>17.3</td><td>25.6</td><td>206</td><td>485</td><td>∞</td></tr></table>		$\alpha : 0$	10	20	30	40	50	60	70	80	$K = 0.05$	0.29	1.56	5.47	17.3	25.6	206	485	∞
$\alpha : 0$	10	20	30	40	50	60	70	80												
$K = 0.05$	0.29	1.56	5.47	17.3	25.6	206	485	∞												
Ball Valve																				
Check Valves Swing Type Ball Type Lift Type	$K = 2.5$ $K = 70.0$ $K = 12.0$	$K = 2.5$ $K = 70.0$ $K = 12.0$																		

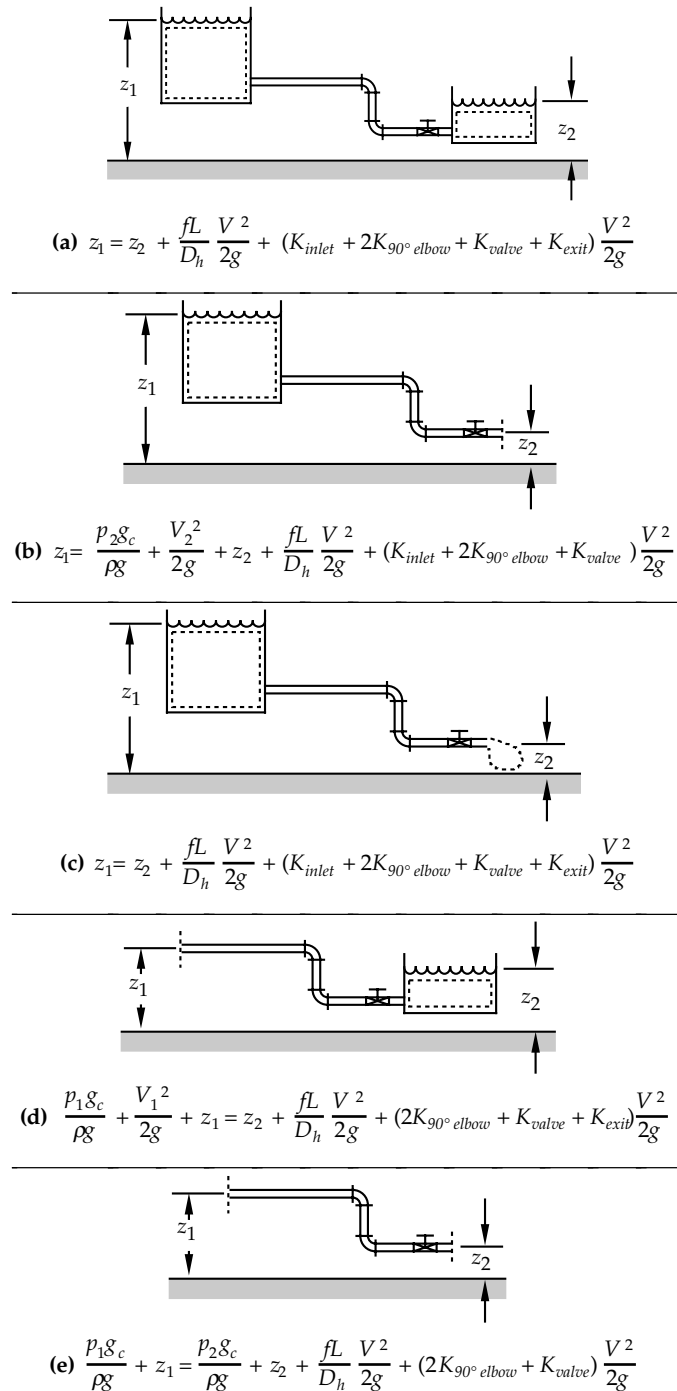


FIGURE 3.7. Modified Bernoulli equation written for various systems.

$$p_1 = p_2 = p_{\text{atm}} = 0$$

$$V_1 = \text{surface velocity} \approx 0 \text{ (compared to velocity in pipe); } V_2 \approx 0$$

$$z_1 = \text{height at section 1; } z_2 = \text{height at section 2}$$

$$\frac{fL}{D} = \text{friction term applied to piping system}$$

$$\Sigma K = \text{minor losses encountered by a fluid particle in traveling from section 1 to section 2; inlet, two } 90^\circ \text{ elbows, valve, and exit}$$

The modified Bernoulli equation (3.21) is

$$\frac{p_1 g_c}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2 g_c}{\rho g} + \frac{V_2^2}{2g} + z_2 + \frac{fL}{D_h} \frac{V^2}{2g} + \Sigma K \frac{V^2}{2g} \quad (3.21)$$

For this application, we get for Figure 3.7a

$$z_1 = z_2 + \frac{fL}{D_h} \frac{V^2}{2g} + (K_{\text{inlet}} + 2K_{90^\circ \text{ elbow}} + K_{\text{valve}} + K_{\text{exit}}) \frac{V^2}{2g} \quad (3.22)$$

This result accompanies the figure.

In Figure 3.7b, the control volume includes the fluid in the tank at the left and all the fluid in the pipe. Section 1 is the free surface of the liquid in the tank and section 2 is just at the end of the pipe. After section 2, the liquid could be discharged to the atmosphere, to another tank, or to another pump. Its destination after section 2 is of no concern with regard to the analysis we formulate. The properties are:

$$p_1 = p_{\text{atm}} = 0; p_2 = \text{pressure at section 2} \neq p_{\text{atm}}$$

$$V_1 = \text{surface velocity} \approx 0 \text{ (compared to velocity in pipe)}$$

$$V_2 = \text{velocity in the pipe}$$

$$z_1 = \text{height at section 1; } z_2 = \text{height at section 2}$$

$$\frac{fL}{D} = \text{friction term applied to piping system}$$

$$\Sigma K = \text{minor losses encountered by a fluid particle in traveling from section 1 to section 2; inlet, two } 90^\circ \text{ elbows, and a valve}$$

The exit loss is not accounted for in Figure 3.7b because the pressure loss in a fitting is realized by the fluid only *after* it passes through it. The Modified Bernoulli equation (for Figure 3.7b) reduces to

$$z_1 = \frac{p_2 g_c}{\rho g} + \frac{V_2^2}{2g} + z_2 + \frac{fL}{D_h} \frac{V^2}{2g} + (K_{inlet} + 2K_{90^\circ elbow} + K_{valve}) \frac{V^2}{2g} \quad (3.23)$$

where the exit velocity V_2 equals the velocity in the pipe V . This result accompanies the figure.

Figure 3.7c shows the right tank removed as does Figure 3.7b. In Figure 3.7c, however, our control volume does not end abruptly with the end of the pipe. Instead, we use a large surface area as section 2. The pressure at the exit of the pipe is usually not equal to atmospheric pressure, so we allow the fluid to expand until its pressure does equal p_{atm} . Thus section 2 is assumed to be the location where the liquid pressure has become equal to atmospheric pressure. Moreover, because the area at section 2 is so large, the velocity of the liquid (or its kinetic energy) is reduced to a negligible value (compared to the velocity in the pipe). In other words at section 2, the pressure equals atmospheric pressure and the kinetic energy of the liquid has dissipated. The properties are:

$$p_1 = p_2 = p_{atm} = 0 \quad V_2 \approx 0$$

(The conditions $p_2 = p_{atm}$, $V_2 = 0$, and a nonzero exit loss *must* all be taken simultaneously for this case.)

$$V_1 = \text{surface velocity} \approx 0 \text{ (compared to velocity in pipe)}$$

$$z_1 = \text{height at section 1}; \quad z_2 = \text{height at section 2}$$

$$\frac{fL}{D} = \text{friction term applied to piping system}$$

$$\sum K = \text{minor losses encountered by fluid particle traveling from section 1 to section 2; inlet, two } 90^\circ \text{ elbows, valve, and exit}$$

The modified Bernoulli equation applied to Figure 3.7c is

$$z_1 = z_2 + \frac{fL}{D_h} \frac{V^2}{2g} + (K_{inlet} + 2K_{90^\circ elbow} + K_{valve} + K_{exit}) \frac{V^2}{2g} \quad (3.24)$$

This result accompanies the figure.

In Figure 3.7d, we have the same pipe leading to a tank. The inlet of the pipe could be fed from a reservoir, by a pump, or by another pipeline. It makes no difference in our analysis. Section 1 is at the pipe inlet and section 2 is the free surface of the liquid in the tank. The properties are

p_1 = pressure at section 1; $p_2 = p_{\text{atm}} = 0$

V_1 = velocity at section 1 = velocity in the pipe = V

V_2 = surface velocity ≈ 0 (compared to velocity in pipe)

z_1 = height at section 1; z_2 = height at section 2

$\frac{fL}{D}$ = friction term applied to piping system

ΣK = minor losses encountered by a fluid particle in traveling from section 1 to section 2; two 90° elbows, valve, and exit

For Figure 3.7d, the modified Bernoulli equation reduces to

$$\frac{p_1 g_c}{\rho g} + \frac{V_1^2}{2g} + z_1 = z_2 + \frac{fL}{D_h} \frac{V^2}{2g} + (2K_{90^\circ \text{ elbow}} + K_{\text{valve}} + K_{\text{exit}}) \frac{V^2}{2g} \quad (3.25)$$

in which $V_1 = V$. This result accompanies the figure.

Figure 3.7e shows the pipe without tanks attached. The liquid source, or its ultimate destination, does not affect our analysis. The locations of section 1 and section 2 are shown. The properties are

p_1 = pressure at section 1; p_2 = pressure at section 2

V_1 = velocity at section 1 = velocity in the pipe = V

V_2 = velocity at section 2 = velocity in the pipe = V

z_1 = height at section 1; z_2 = height at section 2

$\frac{fL}{D}$ = friction term applied to piping system

ΣK = minor losses encountered by a fluid particle in traveling from section 1 to section 2; two 90° elbows and valve

For Figure 3.7e, the modified Bernoulli equation reduces to

$$\frac{p_1 g_c}{\rho g} + z_1 = \frac{p_2 g_c}{\rho g} + z_2 + \frac{fL}{D_h} \frac{V^2}{2g} + (2K_{90^\circ \text{ elbow}} + K_{\text{valve}}) \frac{V^2}{2g} \quad (3.26)$$

This result is shown in the figure. As indicated in the previous discussion, it is extremely important to clearly define the boundary of the control volume.

We are now equipped to model piping problems. Again, we examine problems in which pressure drop Δp (or Δh), volume flow rate Q , or diameter D is unknown.

EXAMPLE 3.6. Figure 3.8 shows a portion of a piping system used to convey 750 gpm of ethyl alcohol. The system contains 180 ft of 12-nominal schedule 40 commercial steel pipe. All fittings are of the long radius type and are flanged. Calculate the pressure drop over this portion of the pipeline if $z_1 = z_2$.

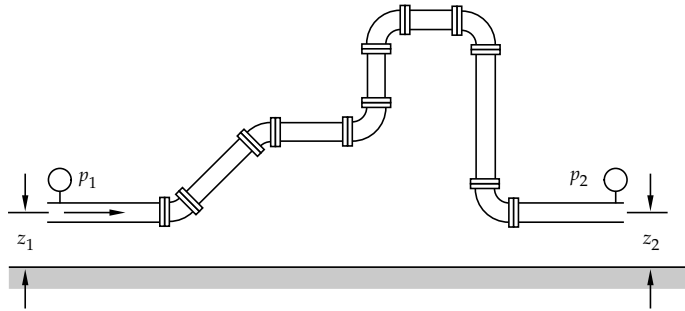


FIGURE 3.8. The piping system of Example 3.6.

Solution: The control volume we select includes all the liquid in the pipe and extends to each pressure gage. The calculation procedure is as follows.

ethyl alcohol $\rho = 0.787(62.4) \text{ lbm/ft}^3$ $\mu = 2.29 \times 10^{-5} \text{ lbf}\cdot\text{s/ft}^2$
[App. Table B.1]

12-nom sch 40 $D = 0.9948 \text{ ft}$ $A = 0.773 \text{ ft}^2$ [App. Table D.1]

commercial steel $\varepsilon = 0.00015 \text{ ft}$ [Table 3.1]

Modified Bernoulli equation (3.21):

$$\frac{p_1 g_c}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2 g_c}{\rho g} + \frac{V_2^2}{2g} + z_2 + \frac{fL}{D_h} \frac{V^2}{2g} + \sum K \frac{V^2}{2g} \quad (3.21)$$

Property evaluation:

$$V_1 = V_2; \quad z_1 = z_2; \quad L = 180 \text{ ft}$$

$$\Sigma K = 2K_{45^\circ \text{ elbow}} + 4K_{90^\circ \text{ elbow}} = 2(0.17) + 4(0.22) = 1.22$$

The Modified Bernoulli equation reduces to:

$$\frac{p_1 g_c}{\rho g} = \frac{p_2 g_c}{\rho g} + \left(\frac{fL}{D_h} + \Sigma K \right) \frac{V^2}{2g}$$

We now work toward evaluating the remaining terms. The volume flow rate is

$$Q = 750 \text{ gpm} = 1.67 \text{ ft}^3/\text{s}$$

Average velocity:

$$V = \frac{Q}{A} = \frac{1.67}{0.773} = 2.16 \text{ ft/s}$$

Reynolds number

$$\text{Re} = \frac{\rho V D}{\mu g_c} = \frac{0.787(62.4)(2.16)(0.9948)}{2.29 \times 10^{-5} (32.2)}$$

Relative roughness and friction factor:

$$\left. \begin{array}{l} \text{Re} = 1.43 \times 10^5 \\ \frac{\varepsilon}{D} = \frac{0.00015}{0.9948} = 0.00015 \end{array} \right\} f = 0.018 \quad (\text{Figure 3.3})$$

Substituting into the equation of motion:

$$\frac{p_1 g_c}{\rho g} = \frac{p_2 g_c}{\rho g} + \left(\frac{0.018(180)}{0.9948} + 1.22 \right) \frac{(2.16)^2}{2(32.2)}$$

or

$$\frac{(p_1 - p_2) g_c}{\rho g} = 0.324 \text{ ft of ethyl alcohol}$$

Thus if we attached an air-over-ethyl alcohol, inverted U-tube manometer from section 1 to 2, it would read $\Delta h = 0.324 \text{ ft}$. The pressure drop is now calculated as

$$p_1 - p_2 = \frac{0.324(0.787)(62.4)(32.2)}{32.2} = 15.9 \text{ lbf/ft}^2$$

or

$$p_1 - p_2 = 0.11 \text{ psi}$$

EXAMPLE 3.7. A huge water reservoir is drained with a 2-nominal schedule 40 galvanized steel pipe that is 60 m long. The piping system is shown in Figure 3.9. The fittings are regular and threaded. Determine the volume flow rate through the system if $z_1 = 5 \text{ m}$, and $z_2 = 2 \text{ m}$.

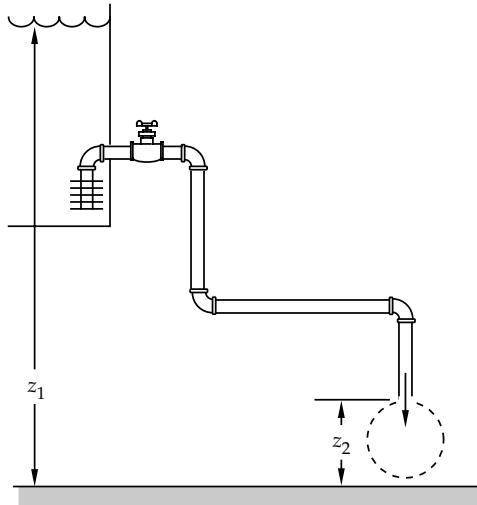


FIGURE 3.9. The piping system of Example 3.7.

Solution: The control volume we select includes the water in the reservoir and in the piping system. Section 1 is the free surface of the water in the reservoir, and section 2 is located such that $p_2 = p_{\text{atm}}$. The calculations are:

water $\rho = 1\,000 \text{ kg/m}^3$ $\mu = 0.89 \times 10^{-3} \text{ N}\cdot\text{s/m}^2$ [App. Table B.1]

2-nom sch 40 $D = 5.252 \text{ cm}$ $A = 21.66 \text{ cm}^2$ [App. Table D.1]

galvanized steel $\varepsilon = 0.0155 \text{ cm}$ (average value) [Table 3.1]

Modified Bernoulli equation (3.21):

$$\frac{p_1 g_c}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2 g_c}{\rho g} + \frac{V_2^2}{2g} + z_2 + \frac{fL}{D_h} \frac{V^2}{2g} + \sum K \frac{V^2}{2g} \quad (3.21)$$

Property evaluation:

$$p_1 = p_2 = p_{\text{atm}} \quad V_1 \approx 0 \quad V_2 = 0$$

$$z_1 = 5 \text{ m} \quad z_2 = 2 \text{ m} \quad L = 60 \text{ m}$$

$$\Sigma K = K_{\text{basket strainer}} + 4K_{90^\circ \text{ elbow}} + K_{\text{globe valve}} + K_{\text{exit}}$$

$$\Sigma K = 1.3 + 4(1.4) + 10 + 1.0 = 17.9$$

The Modified Bernoulli equation reduces to:

$$z_1 = z_2 + \frac{fL}{D_h} \frac{V^2}{2g} + \Sigma K \frac{V^2}{2g}$$

or

$$5 = 2 + \frac{60f}{0.05252} \frac{V^2}{2(9.81)} + 17.9 \frac{V^2}{2(9.81)}$$

Rearranging and solving for velocity, we get

$$3 = (58.23f + 0.912)V^2$$

or

$$V = \sqrt{\frac{3}{58.23f + 0.912}} \quad (\text{i})$$

Reynolds number $\text{Re} = \rho VD/\mu g_c$:

$$\text{Re} = \frac{1\,000(V)(0.097\,18)}{0.89 \times 10^{-3}} = 1.09 \times 10^5 V \quad (\text{ii})$$

$$\frac{\varepsilon}{D} = \frac{0.015\,5}{5.252} = 0.000\,03 \quad (\text{iii})$$

A trial-and-error process involving Equations i, ii, and iii is required. First we assume a value of the friction factor corresponding to the fully developed turbulent flow value for which $\varepsilon/D = 0.000\,015$:

1st trial:	$f = 0.009$;	$V = 1.45$;	$\text{Re} = 8.53 \times 10^4$	$f = 0.019$
2nd trial:	$f = 0.019$;	$V = 1.22$;	$\text{Re} = 7.19 \times 10^4$	$f \approx 0.019\,5$
(close enough)				

The velocity is thus 1.22 m/s. The volume flow rate then is

$$Q = AV = 21.66 \times 10^{-4} (1.22)$$

$$\text{or } Q = 0.0026 \text{ m}^3/\text{s} = 2.7 \text{ l/s}$$

EXAMPLE 3.8. Figure 3.10 shows a piping system that consists of a line connected to two branches. When the bypass branch is closed off with its valve, the flow line is to deliver $0.3 \text{ ft}^3/\text{s}$ of benzene with a pressure drop of $p_1 - p_2$ of 8.5 psi. Select a suitable size for the pipe if it is made of uncoated cast iron and has regular threaded fittings. The length of pipe required is 700 ft. Due to cost considerations, it is desirable to use schedule 40 pipe.

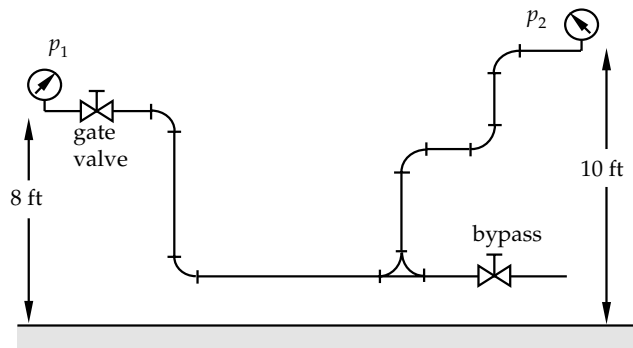


FIGURE 3.10. The piping system of Example 3.8.

Solution: The control volume we select includes all the fluid in the pipe from the gage at section 1 to the gage at section 2, excluding the bypass. The method of solution is

$$\text{benzene} \quad \rho = 0.876(62.4) \text{ lbfm/ft}^3 \quad \mu = 1.26 \times 10^{-5} \text{ lbf}\cdot\text{s/ft}^2 \quad [\text{App. Table B.1}]$$

$$\text{uncoated cast iron} \quad \varepsilon = 0.00085 \text{ ft} \quad [\text{Table 3.1}]$$

Continuity equation:

$$Q = A_1 V_1 = A_2 V_2 \quad A_1 = A_2, \text{ therefore } V_1 = V_2.$$

Modified Bernoulli equation 3.21):

$$\frac{p_1 g_c}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2 g_c}{\rho g} + \frac{V_2^2}{2g} + z_2 + \frac{fL}{D_h} \frac{V^2}{2g} + \sum K \frac{V^2}{2g} \quad (3.21)$$

Property evaluation:

$$\frac{(p_1 - p_2)g_c}{\rho g} = \frac{8.5(144)(32.2)}{0.876(62.4)(32.2)} = 22.4 \text{ ft of benzene}$$

$$V_1 = V_2 \quad z_1 = 8 \text{ ft} \quad z_2 = 10 \text{ ft} \quad L = 700 \text{ ft}$$

$$\Sigma K = K_{\text{gate valve}} + 5K_{90^\circ \text{ elbow}} + K_{T\text{-joint}}$$

$$\Sigma K = 0.15 + 5(1.4) + 1.9 = 9.05$$

Flow velocity $V = Q/A$:

$$V = \frac{4Q}{\pi D^2} = \frac{4(0.3)}{\pi D^2} = \frac{0.382}{D^2}$$

Equation of motion:

$$\frac{p_1 g_c}{\rho g} - \frac{p_2 g_c}{\rho g} + z_1 - z_2 = \left(\frac{fL}{D_h} + \Sigma K \right) \frac{V^2}{2g}$$

Substituting,

$$22.4 + 8 - 10 = \left(\frac{700f}{D} + 9.05 \right) \frac{(0.382)^2}{2D^4(32.2)}$$

or

$$9003 = \frac{700f}{D^5} + \frac{9.05}{D^4}$$

Rearranging and simplifying, we obtain

$$f = 12.86D^5 - 0.01293D \quad (\text{i})$$

Reynolds number $\text{Re} = \rho V D / \mu g_c$:

$$\text{Re} = \frac{0.876(62.4)(0.382/D^2)D}{1.26 \times 10^{-5} (32.2)}$$

$$\text{Re} = \frac{5.15 \times 10^4}{D} \quad (\text{ii})$$

Relative roughness:

$$\frac{\varepsilon}{D} = \frac{0.00085}{D} \quad (\text{iii})$$

The solution method involves a trial-and-error procedure, which begins by assuming a diameter:

$$\text{1st trial:} \quad D = 1/4 = 0.25 \text{ ft}; \quad f = 0.00933 \quad (\text{Eq. i})$$

then

$$\left. \begin{aligned} \text{Re} &= 5.15 \times 10^4 / D = 2.06 \times 10^5 \\ \frac{\varepsilon}{D} &= \frac{0.00085}{0.25} = 0.0034 \end{aligned} \right\} \quad f = 0.028 \quad (\text{Figure 3.3})$$

$$\text{2nd trial:} \quad D = 1/3 = 0.333 \text{ ft}; \quad f = 0.0484$$

then

$$\left. \begin{aligned} \text{Re} &= 5.15 \times 10^4 / D = 1.54 \times 10^5 \\ \frac{\varepsilon}{D} &= \frac{0.00085}{0.333} = 0.00255 \end{aligned} \right\} \quad f = 0.026 \quad (\text{Figure 3.3})$$

The diameter we seek falls between 0.25 ft and 0.333 ft. Continuing,

$$\text{3rd trial:} \quad D = 0.3 \text{ ft}; \quad f = 0.027$$

then

$$\left. \begin{aligned} \text{Re} &= 5.15 \times 10^4 / D = 1.72 \times 10^5 \\ \frac{\varepsilon}{D} &= \frac{0.00085}{0.3} = 0.0028 \end{aligned} \right\} \quad f = 0.027 \quad (\text{Figure 3.3})$$

which is close enough. Thus $D = 0.3$ ft. Referring to Appendix Table D.1, we read:

$$\begin{array}{ll} 3^{1/2}\text{-nom sch 40} & D = 0.2957 \text{ ft} \\ 4\text{-nom sch 40} & D = 0.3355 \text{ ft} \end{array}$$

The question now arises as to which of these pipes to select. The smaller size will deliver the required flow rate but at a pressure drop that will

exceed that which was specified. So we use 4-nominal schedule 40 pipe. We must now go back and calculate the actual pressure drop. What was given in the problem statement ($\Delta p = 8.5$ psi and $Q = 0.3$ ft³/s) will be satisfied only with a diameter of 0.3 ft. The diameter we are using, however, is 0.3355 ft, which is larger than necessary.

At $Q = 0.3$ ft³/s, we calculate the actual pressure drop with the equation of motion:

$$\frac{p_1 g_c}{\rho g} - \frac{p_2 g_c}{\rho g} + z_1 - z_2 = \left(\frac{fL}{D_h} + \sum K \right) \frac{V^2}{2g}$$

For $D = 0.3355$ ft, $A = \pi D^2/4 = 0.08840$ ft². The average velocity then is

$$V = \frac{Q}{A} = \frac{0.3}{0.08840} = 3.39 \text{ ft/s}$$

and

$$\text{Re} = 1.53 \times 10^5 \quad \epsilon/D = 0.00253 \quad f = 0.027$$

Substituting,

$$\frac{(p_1 - p_2)32.2}{0.876(62.4)(32.2)} + 8 - 10 = \left(\frac{0.027(700)}{0.3355} + 9.05 \right) \frac{(3.39)^2}{2(32.2)}$$

Solving, we get

$$\underline{p_1 - p_2 = 5.1 \text{ psi}}$$

This pressure drop is less than the 8.5 psi that was specified. Had we used 3¹/₂-nominal schedule 40, the pressure drop would have been 8.9 psi.

3.6 Series Piping Systems

A number of closed conduit piping problems are more complex than those considered in previous sections. These complex piping problems include systems with more than one line size, systems with parallel piping arrangements or networks, and systems with multiple tanks draining simultaneously. These and other complex problems are modeled with the information provided in this chapter. Here, however, we consider only pipes in series.

Two or more pipes of different sizes or even of different roughnesses connected together form a series piping system. The velocity within different size pipes in series is different, as is the Reynolds number and hence the friction factor. Consequently, such systems need special consideration.

Two types of problems can be encountered in series systems. The “Type I” problem is one in which volume flow rate is known and pressure drop is to be determined. The “Type II” problem is where the pressure drop is known and volume flow rate is sought. The Type II problem is considerably more difficult than the Type I problem. Both are handled by starting with the modified Bernoulli equation and proceeding in the usual way. The procedure is illustrated by the following examples.

EXAMPLE 3.9. A 2-nominal schedule 40 pipe that is 70 ft long is attached to 60 ft of 3-nominal schedule 40 pipe in series, as shown in Figure 3.11. The 2-nominal pipe contains a gate valve. For a volume flow rate of 60 gpm, determine the pressure drop $p_1 - p_2$. The fluid is hexane, and the pipes are made of galvanized steel.

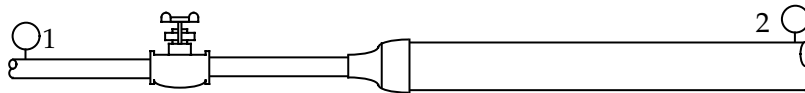


FIGURE 3.11. A series piping system.

Solution: We proceed in the usual way; from the various property tables, we read

$$\text{hexane} \quad \rho = 0.657(1.94) \text{ slug/ft}^3 \quad \mu = 0.622 \times 10^{-5} \text{ lbf}\cdot\text{s/ft}^2 \quad [\text{App. Table B.1}]$$

$$\begin{array}{lll} \text{2-nom sch 40} & D = 0.1723 \text{ ft} & A = 0.02330 \text{ ft}^2 \\ \text{3-nom sch 40} & D = 0.2557 \text{ ft} & A = 0.05134 \text{ ft}^2 \end{array} \quad [\text{App. Table D.1}]$$

$$\text{galvanized steel} \quad \varepsilon = 0.0005 \text{ ft} \quad [\text{Table 3.1}]$$

The modified Bernoulli equation applied to the pipeline is

$$\frac{p_1 g_c}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2 g_c}{\rho g} + \frac{V_2^2}{2g} + z_2 + \left(\frac{fL}{D} + \Sigma K \right) \frac{V_1^2}{2g} + \left(\frac{fL}{D} + \Sigma K \right) \frac{V_2^2}{2g}$$

where subscript 1 refers to properties at the inlet, and subscript 2 to properties at the outlet. The volume flow rate through the system is

$$Q = 60 \text{ gal/min} = 0.134 \text{ ft}^3/\text{s}$$

The velocity in each line is

$$V_{2\text{-nom}} = \frac{Q}{A} = \frac{0.134}{0.02330} = 5.7 \text{ ft/s}$$

$$V_{3\text{-nom}} = \frac{0.134}{0.05134} = 2.6 \text{ ft/s}$$

The minor losses include a gate valve and an expansion fitting:

$$K_{\text{gate valve}} = 0.15$$

$$K_{\text{exp fitting}} = \left[\frac{(D_{3\text{-nom}})^2}{(D_{2\text{-nom}})^2} - 1 \right]^2 = \left[\frac{A_{3\text{-nom}}}{A_{2\text{-nom}}} - 1 \right]^2$$

Substituting,

$$K_{\text{exp fitting}} = \left[\frac{0.05134}{0.02330} - 1 \right]^2 = 1.448$$

where this loss will be based on the downstream kinetic energy. Summarizing, we have

$$V_1 = 5.7 \text{ ft/s} \quad V_2 = 2.6 \text{ ft/s} \quad z_1 = z_2$$

$$L_1 = 70 \text{ ft} \quad L_2 = 60 \text{ ft}$$

$$\Sigma K_{2\text{-nom}} = 0.15 \quad \Sigma K_{3\text{-nom}} = 1.448$$

The Reynolds number for each line is

$$\text{Re}_{2\text{-nom}} = \frac{\rho V D}{\mu g_c} = \frac{0.657(1.94)(5.7)(0.1723)}{0.622 \times 10^{-5}}$$

or

$$\left. \begin{aligned} \text{Re}_{2\text{-nom}} &= 2.0 \times 10^5 \\ \frac{\varepsilon}{D} &= \frac{0.0005}{0.1723} = 0.0029 \end{aligned} \right\} f_{2\text{-nom}} = 0.027$$

Similarly,

$$\left. \begin{aligned} \text{Re}_{3\text{-nom}} &= 1.36 \times 10^5 \\ \frac{\varepsilon}{D} &= \frac{0.0005}{0.2557} = 0.002 \end{aligned} \right\} \quad f_{3\text{-nom}} = 0.0245$$

Substituting into the Bernoulli equation gives

$$\begin{aligned} \frac{p_1 g_c}{\rho g} + \frac{5.7^2}{2(32.2)} + 0 &= \frac{p_2 g_c}{\rho g} + \frac{2.6^2}{2(32.2)} + 0 + \left(\frac{0.027(70)}{0.1723} + 0.15 \right) \frac{5.7^2}{2(32.2)} + \\ &\quad \left(\frac{0.0245(60)}{0.2557} + 1.448 \right) \frac{2.6^2}{2(32.2)} \\ \frac{(p_1 - p_2) g_c}{\rho g} &= -0.505 + 0.105 + 5.610 + 0.755 = 5.97 \end{aligned}$$

Solving for pressure drop, we get

$$p_1 - p_2 = 0.657(1.94)(32.2)(5.97)$$

and, finally,

$$\underline{p_1 - p_2 = 244.8 \text{ lbf/ft}^2 = 1.7 \text{ psi}}$$

The method is straightforward because flow rates, and hence velocities, are known.

EXAMPLE 3.10. Copper tubing is used to convey methyl alcohol in a lubricating system. The system consists of 2 m of 3/4 std type M tubing attached in series to 3 m of 1/2 std type M copper tubing, as indicated in Figure 3.12. The pressure drop over the 5 m length is 100 kPa. Determine the flow rate through the system.

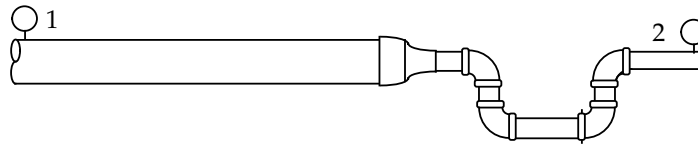


FIGURE 3.12. A series piping system with flow rate unknown.

Solution: This type of problem (flow rate Q unknown) is somewhat more involved than the other type (Δp unknown). The procedure is familiar, however:

$$V_1 = \frac{A_2}{A_1} V_2 = \frac{1.642}{3.333} V_2$$

$$V_1 = 0.493 V_2 \quad (\text{i})$$

After rearranging and substituting for V_1 , the modified Bernoulli equation becomes

$$\frac{(p_1 - p_2)g_c}{\rho g} = \left(\frac{fL}{D} + \Sigma K - 1 \right) \frac{V_1^2}{2g} + \left(\frac{fL}{D} + \Sigma K + 1 \right) \frac{V_2^2}{2g}$$

$$\frac{(p_1 - p_2)g_c}{\rho g} = \left(\frac{fL}{D} + \Sigma K - 1 \right) \frac{0.243 V_2^2}{2g} + \left(\frac{fL}{D} + \Sigma K + 1 \right) \frac{V_2^2}{2g}$$

Factoring the kinetic energy term gives

$$\frac{(p_1 - p_2)g_c}{\rho g} = \frac{V_2^2}{2g} \left\{ 0.243 \frac{f_1 L_1}{D_1} + 0.243 \Sigma K_1 - 0.243 + \frac{f_2 L_2}{D_2} + \Sigma K_2 + 1 \right\}$$

Substituting known quantities,

$$\frac{30\,000}{789(9.81)} = \frac{V_2^2}{2(9.81)} \left\{ 0.243 \frac{f_1(2)}{0.020\,60} + 0 - 0.243 + \frac{f_2(3)}{0.013\,40} + 1.490 + 1 \right\}$$

which reduces to

$$253.5 = V_2^2(17.37f_1 + 223.9f_2 + 2.311)$$

$$\text{or } V_2 = \left(\frac{253.5}{17.37f_1 + 223.9f_2 + 2.311} \right)^{1/2} \quad (\text{ii})$$

The Reynolds number for each line is

$$\text{Re}_{3/4 \text{ std}} = \frac{\rho V D}{\mu g_c} = \frac{789 V_1 (0.020\,6)}{0.56 \times 10^{-3}}$$

$$\text{Re}_{3/4 \text{ std}} = 2.9 \times 10^4 V_1$$

We also calculate

$$\frac{\varepsilon}{D} = \frac{0.00015}{2.060} = 0.000073$$

Similarly,

$$\text{Re}_{1/2 \text{ std}} = 2.04 \times 10^4 V_2$$

and

$$\frac{\varepsilon}{D} = 0.0001$$

1st trial: Assuming the fully turbulent values for each friction factor gives

$$\begin{array}{ll} f_1 = 0.0115 & (3/4 \text{ std}) \\ f_2 = 0.012 & (1/2 \text{ std}) \end{array}$$

Substituting into Equation ii of this example gives

$$\text{or } V_2 = \left(\frac{253.5}{17.37(0.0115) + 223.9(0.012) + 2.311} \right)^{1/2}$$

$$V_2 = 7.1 \text{ m/s}$$

Substituting into Equation i, we get

$$V_1 = 0.493(7.1) = 3.5 \text{ m/s}$$

The Reynolds numbers and friction factors become

$$\left. \begin{array}{l} \text{Re}_{3/4 \text{ std}} = 2.9 \times 10^4 V_1 = 2.9 \times 10^4 \\ \frac{\varepsilon}{D} = 0.000073 \end{array} \right\} f_1 = 0.016$$

and,

$$\left. \begin{array}{l} \text{Re}_{1/2 \text{ std}} = 2.04 \times 10^4 \\ \frac{\varepsilon}{D} = 0.0001 \end{array} \right\} f_2 = 0.02$$

2nd trial: Using these friction factors, we calculate new velocities:

$$V_2 = 6.11 \text{ m/s} \quad \text{and} \quad V_1 = 3.01 \text{ m/s}$$

With these new values, we repeat the calculations to get

$$\left. \begin{array}{l} \text{Re}_{3/4 \text{ std}} = 2.9 \times 10^4 V_1 = 7.28 \times 10^4 \\ \frac{\epsilon}{D} = 0.000\,073 \end{array} \right\} f_1 = 0.0163$$

and

$$\left. \begin{array}{l} \text{Re}_{1/2 \text{ std}} = 6.14 \times 10^4 \\ \frac{\epsilon}{D} = 0.000\,1 \end{array} \right\} f_2 = 0.02$$

3rd trial: The velocities are now found as

$$V_2 = 6.11 \text{ m/s} \quad \text{and} \quad V_1 = 3.01 \text{ m/s (close enough)}$$

The volume flow rate then is

$$Q = A_1 V_1 = A_2 V_2$$

$$Q = 3.333 \times 10^{-4} (3.01)$$

$$\text{or } \underline{Q = 1.0 \times 10^{-3} \text{ m}^3/\text{s}}$$

3.7 Flow Through Noncircular Cross Sections

Flow through noncircular cross sections can become quite complicated because of geometry factors for various shapes that sometimes are difficult to express mathematically. Numerous noncircular cross sections can be modeled, but we consider some of the more common cases here. These include: annulus, rectangular duct, circular sectors, triangles, and the finned annulus.

Laminar Flow of a Newtonian Fluid in an Annulus

Flow through an annulus is illustrated in Figure 3.13. The annular flow area is bounded by the outside diameter of the inner duct (OD_p) and the inside diameter of the outer duct (ID_a). Also shown in the figure is one half of the control volume we use for study. The forces acting on the control

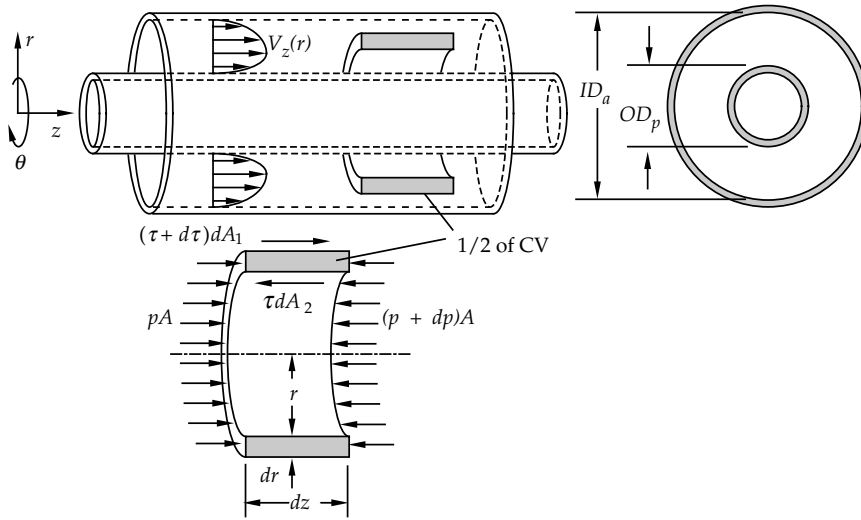


FIGURE 3.13. Laminar flow in an annulus.

volume are due to pressure and viscosity. Gravity is neglected. We are seeking an equation for the velocity distribution V_z that we can integrate over the cross section to obtain average velocity. The results can be combined with Equation 3.8 to find an equation for friction factor just as is done with the circular duct. By applying the momentum equation, it can be shown that the z -directed velocity is given by

$$V_z = \left(-\frac{dp}{dz} \right) \left(\frac{R^2}{4\mu} \right) \left[1 - \left(\frac{r}{R} \right)^2 - \frac{1 - \kappa^2}{\ln(\kappa)} \ln \frac{r}{R} \right] \quad \left(\begin{array}{l} \text{laminar flow} \\ \text{annular duct} \end{array} \right) \quad (3.29)$$

where $\kappa = OD_p/ID_a$. (See the problems section for a detailed derivation of this equation.) The volume flow rate is found by integrating Equation 3.29 over the cross section:

$$Q = \int_0^{2\pi} \int_{\kappa R}^R V_z r dr d\theta$$

which becomes

$$Q = \int_0^{2\pi} \int_{\kappa R}^R \left(-\frac{dp}{dz} \right) \left(\frac{R^2}{4\mu} \right) \left[1 - \left(\frac{r}{R} \right)^2 - \frac{1 - \kappa^2}{\ln(\kappa)} \ln \frac{r}{R} \right] r dr d\theta$$

Integrating gives the volume flow rate as

$$Q = \left(-\frac{dp}{dz} \right) \left(\frac{\pi R^4 (1 - \kappa^2)}{8\mu} \right) \left(1 + \kappa^2 + \frac{1 - \kappa^2}{\ln(\kappa)} \right) \quad (3.28)$$

The average velocity is

$$V = \frac{Q}{A} = \frac{Q}{\pi(R^2 - \kappa^2 R^2)} = \frac{Q}{\pi R^2 (1 - \kappa^2)}$$

or

$$V = \left(-\frac{dp}{dz} \right) \left(\frac{R^2}{8\mu} \right) \left(1 + \kappa^2 + \frac{1 - \kappa^2}{\ln(\kappa)} \right) \quad (3.29)$$

The hydraulic diameter for the annular flow section is

$$D_h = \frac{4A}{P} = \frac{4(\pi(ID_a/2)^2 - \pi(OD_p/2)^2)}{\pi(ID_a) + \pi(OD_p)}$$

which simplifies to

$$D_h = ID_a - OD_p \quad (3.30)$$

Equation 3.8a relates the pressure drop to the friction factor:

$$dp = - \frac{\rho V^2}{2g_c} \frac{f dz}{D_h} \quad (3.8a)$$

Combining Equations 3.8a, 3.29, and 3.30, and after considerable simplification, we get

$$\frac{1}{f} = \frac{\text{Re}}{64} \left(\frac{1 + \kappa^2}{(1 - \kappa)^2} + \frac{1 + \kappa}{(1 - \kappa) \ln(\kappa)} \right) \quad (3.31)$$

where

$$\text{Re} = \frac{VD}{\nu} = \frac{V(ID_a - OD_p)}{\nu}$$

Turbulent Flow Through an Annulus

For turbulent flow through an annulus, we cannot derive an equation for the velocity profile and continue as we did for the laminar case. Instead, we rely on experimental results. When $\kappa (= OD_p/ID_a)$ is less than 0.75, the

Moody diagram can be used with little error to find the friction factor for flow in an annulus. However, the characteristic dimension $D_h (= ID_a - OD_p)$ must be used in the Reynolds number equation ($Re = VD_h/\nu$) and in the relative roughness (ε/D_h).

EXAMPLE 3.11. An axial flow pump is sketched in Figure 3.14. The pump impeller is submerged in kerosene, and as the impeller rotates, it moves kerosene upward through an annular flow passage. The annulus is bounded by an inner casing (2-nominal schedule 40 pipe) and an outer casing (6-nominal schedule 40 pipe), both made of wrought iron. In order to determine pumping power, it is necessary to calculate the pressure change from section 1 to section 2, which are a distance of 33 ft ($= H$) apart. Determine the pressure drop $p_1 - p_2$ if the average velocity in the annulus is 6 ft/s.

Solution: We proceed in the usual way by first obtaining values from the various property tables:

kerosene

$$\rho = 0.823(1.94 \text{ slug/ft}^3)$$

$$\mu = 3.42 \times 10^{-5} \text{ lbf}\cdot\text{s/ft}^2$$

pipe sizes [App. Table D.2]

$$6\text{-nom sch 40; } ID_a = 0.5054 \text{ ft}$$

$$2\text{-nom sch 40; } OD_p = 0.1723 \text{ ft}$$

wrought iron [App. Table B.1]

$$\varepsilon = 0.00015 \text{ ft} \quad [\text{Table 3.1}]$$

The continuity equation is written as

$$Q = A_1 V_1 = A_2 V_2$$

With $A_1 = A_2$, we conclude that $V_1 = V_2$. The Bernoulli equation applied to this system is

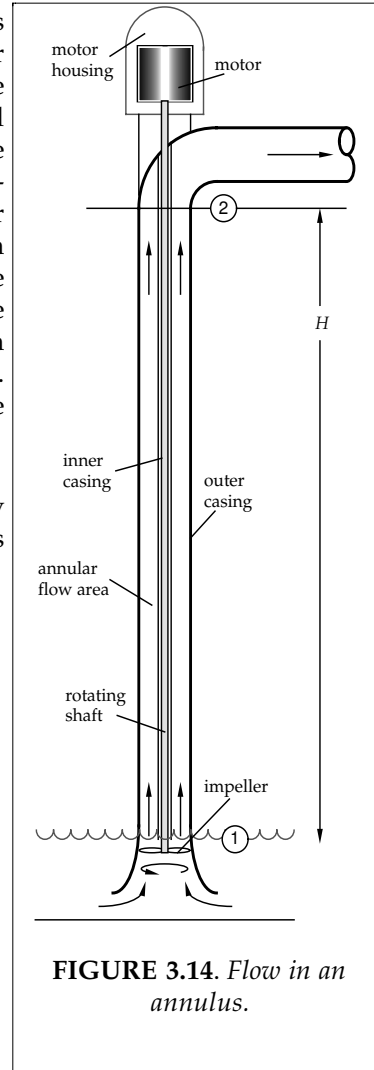


FIGURE 3.14. Flow in an annulus.

$$\frac{p_1 g_c}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2 g_c}{\rho g} + \frac{V_2^2}{2g} + z_2 + \frac{fL}{D_h} \frac{V^2}{2g}$$

With no change in kinetic energy, the equation becomes:

$$\frac{(p_1 - p_2)g_c}{\rho g} = z_2 - z_1 + \frac{fL}{D_h} \frac{V^2}{2g}$$

or

$$p_1 - p_2 = (z_2 - z_1) \frac{\rho g}{g_c} + \frac{fL}{D_h} \frac{\rho V^2}{2g_c}$$

Evaluating terms, we have

$$H = 33 \text{ ft} \quad V = 6 \text{ ft/s}$$

$$D_h = ID_a - OD_p = 0.5054 - 0.1723$$

$$D_h = 0.3331 \text{ ft}$$

$$z_2 = 33 \text{ ft} \quad z_1 = 0$$

The Reynolds number is calculated as

$$Re = \frac{\rho V D_h}{\mu g_c} = \frac{0.823(1.94)(6)(0.3331)}{3.42 \times 10^{-5}} = 9.33 \times 10^4$$

Thus, the flow is turbulent. We find the friction factor to be

$$\text{Also } \left. \begin{array}{l} Re = 9.33 \times 10^4 \\ \frac{\varepsilon}{D} = \frac{0.00015}{0.3331} = 0.00045 \end{array} \right\} f = 0.021 \quad (\text{Figure 3.3})$$

Substituting into the equation of motion,

$$p_1 - p_2 = (z_2 - z_1) \frac{\rho g}{g_c} + \frac{fL}{D_h} \frac{\rho V^2}{2g_c}$$

we get

$$p_1 - p_2 = (33 - 0)(0.823)(1.94)(32.2) + \frac{0.021(33)}{0.3331} \frac{0.823(1.94)(6)^2}{2}$$

$$p_1 - p_2 = 1696 \text{ psf} + 59.8 \text{ psf}$$

$$p_1 - p_2 = 1756 \text{ lbf/ft}^2$$

or $p_1 - p_2 = 12.2 \text{ psi}$

EXAMPLE 3.12. A double-pipe heat exchanger consists of a tube inside of another tube. It is made of drawn copper water tubing with fittings that are soldered. One exchanger is 12 ft long. Eight of these exchangers are connected together in a series configuration and laid out in a horizontal plane. The tubes are 4 std and 2 std, both type M. Acetone flows through the annulus and is subjected to a 0.5 psi pressure drop in the flow direction. Determine the volume flow rate through the exchanger.

Solution: We proceed in the usual way by first obtaining values from the various property tables:

acetone $\rho = 0.787(62.4 \text{ lbm/ft}^3)$ $\mu = 0.659 \times 10^{-5} \text{ lbf}\cdot\text{s/ft}^2$
[App. Table B.1]

4 std type M $ID_a = 0.3279 \text{ ft}$
 2 std type M $OD_p = 2.125/12 = 0.177 \text{ ft}$ [App. Table D.2]

drawn tubing $\epsilon = 0.000005 \text{ ft}$ [Table 3.1]

We calculate the annular flow area and hydraulic diameter:

$$A = \pi(ID_a^2 - OD_p^2) / 4 = 0.0598 \text{ ft}^2$$

$$D_h = ID_a - OD_p = 0.151 \text{ ft}$$

The continuity equation is written as

$$Q = A_1 V_1 = A_2 V_2$$

With $A_1 = A_2$, we conclude that $V_1 = V_2$. The Bernoulli equation applied to this system is

$$\frac{p_1 g_c}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2 g_c}{\rho g} + \frac{V_2^2}{2g} + z_2 + \frac{fL}{D_h} \frac{V^2}{2g}$$

The pressure drop is

$$p_1 - p_2 = 0.5(144) = 72 \text{ lbf/ft}^2$$

For a horizontal exchanger,

$$z_1 = z_2$$

Simplifying the Bernoulli equation and substituting, we obtain

$$\frac{(p_1 - p_2)g_c}{\rho g} = \frac{fL}{D_h} \frac{V^2}{2g}$$

$$\frac{72(32.2)}{0.787(62.4)(32.2)} = \frac{96f}{0.151} \frac{V^2}{2(32.2)}$$

or

$$1.47 = 9.87fV^2$$

$$\text{and } V = \frac{0.386}{\sqrt{f}}$$

The Reynolds number is calculated as

$$\text{Re} = \frac{\rho V D_h}{\mu g_c} = \frac{0.787(62.4)(V)(0.151)}{0.659 \times 10^{-5} (32.2)}$$

or

$$\text{Re} = 3.49 \times 10^4 V$$

The relative roughness is found to be

$$\frac{\epsilon}{D_h} = \frac{0.000005}{0.151} = 0.000033$$

A trial-and-error process is required in order to solve this problem using Figure 3.3. For our first trial, we use the fully turbulent value of friction factor corresponding to the relative roughness calculated above. Thus,

$$\text{1st trial: } f = 0.0095 \quad \left(\text{corresponding to } \frac{\epsilon}{D_h} = 0.000033\right)$$

Then

$$V = 0.386/\sqrt{f} = 0.386/\sqrt{0.0095} = 3.96 \text{ ft/s}$$

$$\text{Re} = 3.49 \times 10^4 (3.96)$$

or

$$\left. \begin{array}{l} \text{Re} = 1.39 \times 10^5 \\ \frac{\varepsilon}{D} = 0.000033 \end{array} \right\} \quad f = 0.016 \quad (\text{Figure 3.3})$$

$$\begin{array}{llll} \text{2nd trial: } f = 0.016 & V = 3.05 & \text{Re} = 1.06 \times 10^5 & f = 0.02 \\ \text{3rd trial: } f = 0.02 & V = 2.73 & \text{Re} = 9.5 \times 10^4 & f \approx 0.02 \text{ (close enough)} \end{array}$$

Therefore

$$V = 2.73 \text{ ft/s}$$

and

$$Q = AV = 0.0598(2.73)$$

$$\underline{Q = 0.163 \text{ ft}^3/\text{s} = 73 \text{ gpm}}$$

Alternatively, we could use Figure 3.4. By combining the velocity and Reynolds number equations above, we get

$$\text{Re} = 3.49 \times 10^4 \quad V = 3.49 \times 10^4 (0.386/\sqrt{f})$$

or

$$\left. \begin{array}{l} f^{1/2} \text{Re} = 1.35 \times 10^4 \\ \frac{\varepsilon}{D} = 0.000033 \end{array} \right\} \quad f = 0.02 \quad (\text{Figure 3.4})$$

which is the same result obtained with Figure 3.3.

Laminar Flow of a Newtonian Fluid in a Rectangular Duct

Flow through a rectangular duct is illustrated in Figure 3.15. The cross section is assumed to be very wide compared to its height. Flow is in the z -direction, and the control volume we are working with does not extend to the wall surfaces. Applying the momentum equation to the control volume gives a result that can be used to determine the velocity profile. The z -directed velocity is given by

$$V_z = \left(-\frac{dp}{dz} \right) \left(\frac{h^2}{2\mu} \right) \left(\frac{1}{4} - \frac{y^2}{h^2} \right) \quad \left(\begin{array}{l} \text{laminar flow} \\ \text{2-D rectangular duct} \end{array} \right) \quad (3.32)$$

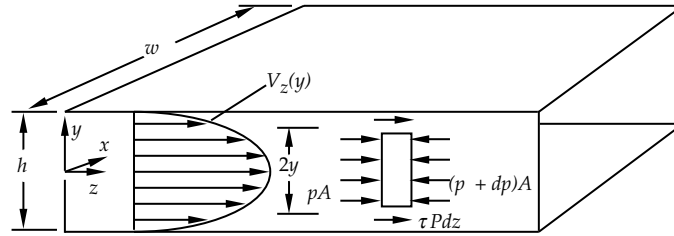


FIGURE 3.15. Laminar flow through a rectangular duct.

(For a detailed procedure, see the Problems section.) The volume flow rate is found by integrating the velocity V_z over the cross-sectional area

$$Q = \int_0^w \int_{-h/2}^{+h/2} \left(-\frac{dp}{dz} \right) \left(\frac{h^2}{2\mu} \right) \left(\frac{1}{4} - \frac{y^2}{h^2} \right) dy dx$$

Integrating and simplifying, we find

$$Q = \frac{h^3 w}{12\mu} \left(-\frac{dp}{dz} \right) \quad (3.33)$$

The average velocity then becomes

$$V = \frac{Q}{A} = \frac{h^2}{12\mu} \left(-\frac{dp}{dz} \right) \quad (3.34)$$

From Equation 3.8a, which relates the pressure loss to the average velocity in a duct for any cross section, we have

$$dp = - \frac{\rho V^2}{2g_c} \frac{f dz}{D_h} \quad (3.8a)$$

Also, for a two-dimensional duct, the hydraulic diameter is

$$D_h = \frac{4A}{P} = \frac{4hw}{2h + 2w} \approx \frac{4hw}{2w}$$

$$\text{or } D_h = 2h \quad (3.35)$$

Combining Equations 3.8a, 3.34, and 3.35 gives

$$f = \frac{96\mu g_c}{\rho V D_h} = \frac{96\mu g_c}{\rho V (2h)}$$

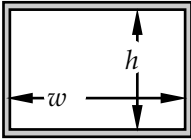
$$\text{or } f = \frac{96}{\text{Re}} \quad (3.36)$$

where

$$\text{Re} = \frac{\rho V (2h)}{\mu g_c}$$

Thus for laminar flow of a Newtonian fluid in a two-dimensional duct, the friction factor $\times$ Reynolds number product is 96. For other rectangular ducts, the friction factor $\times$ Reynolds number product is found as a function of the height/width ratio h/w . Results are provided in Table 3.4.

TABLE 3.4. Friction factor–Reynolds number product for laminar flow of a Newtonian fluid in a rectangular duct.

Rectangular Duct	h/w	$f \cdot \text{Re}$	
 $A = wh$ $P = 2w + 2h$	0	96	2-D duct
	0.05	89.81	
	0.1	84.68	
	0.125	82.34	
	0.167	78.81	
	0.25	72.93	
	0.4	65.47	square
	0.5	62.19	
	0.75	57.89	
	1	56.91	

Turbulent Flow in a Rectangular Duct

For turbulent flow through a rectangular duct, we cannot develop an equation for velocity and proceed as we did for the laminar case. Experience has shown, however, that we can use the Moody diagram. The only restriction is that the characteristic dimension to be used is the hydraulic diameter

$$D_h = \frac{4A}{P} = \frac{2hw}{h + w} \quad (3.37)$$

The characteristic dimension is substituted into the Reynolds number equation $Re = VD_h/\nu$ and into the relative roughness ε/D_h .

EXAMPLE 3.13. Air flows through a horizontal mine shaft that is 30 m long. The duct is rectangular (3 m x 1.5 m) and is made of clay. Fresh air flows through the mine shaft at a velocity of 20 ft/s. Determine the pressure drop.

Solution: Air is a compressible fluid, but compressibility effects are negligible at velocities less than several hundred feet per second. So at the given velocity, we can model this problem as if the air were incompressible. From the various property tables, we get

$$\text{air} \quad \rho = 1.17 \text{ kg/m}^3 \quad \mu = 18 \times 10^{-6} \text{ N}\cdot\text{s/m}^2 \quad [\text{App. Table C.1}]$$

$$\text{clay} \quad \varepsilon = \frac{0.03 + 0.3}{2} \text{ cm} = 0.165 \text{ cm} = 0.00165 \text{ m} \quad (\text{avg value}) \quad [\text{Table 3.1}]$$

For the rectangular duct,

$$A = hw = 3(1.5) = 4.5 \text{ m}^2$$

$$D_h = \frac{2hw}{h + w} = \frac{2(3)(1.5)}{3 + 1.5} = 2 \text{ m}$$

The flow velocity is given as

$$V = 20 \text{ ft/s} = 6.1 \text{ m/s}$$

The continuity equation applied over the length of duct is

$$Q = A_1 V_1 = A_2 V_2$$

With $A_1 = A_2$, we conclude that $V_1 = V_2$. The Bernoulli equation applied to this system is

$$\frac{p_1 g_c}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2 g_c}{\rho g} + \frac{V_2^2}{2g} + z_2 + \frac{fL}{D_h} \frac{V^2}{2g}$$

With $V_1 = V_2$ and $z_1 = z_2$, the preceding equation simplifies to

$$\frac{(p_1 - p_2) g_c}{\rho g} = \frac{fL}{D_h} \frac{V^2}{2g} \quad (\text{i})$$

All parameters are known except the friction factor f , which we now determine. The Reynolds number is

$$\text{Re} = \frac{\rho V D_h}{\mu g_c} = \frac{1.17(6.1)(2)}{18 \times 10^{-6}}$$

or

$$\text{Re} = 7.93 \times 10^5$$

$$\left. \begin{array}{l} \text{Also } \frac{\varepsilon}{D_h} = \frac{0.00165}{2} = 0.00083 \end{array} \right\} f = 0.019 \quad (\text{Figure 3.3})$$

Substituting into the right-hand side of Equation i gives

$$\frac{(p_1 - p_2)g_c}{\rho g} = \frac{0.019(30)}{2} \frac{(6.1)^2}{2(9.81)} = 0.545 \text{ m} \quad (\text{ii})$$

The pressure drop then is

$$p_1 - p_2 = 1.17(9.81)(0.545)$$

$$\underline{p_1 - p_2 = 6.2 \text{ N/m}^2 = 6.2 \text{ Pa}}$$

The problem asks for the pressure drop; however, it is customary when dealing with air flows to express the pressure loss in terms of a head of water. Thus,

$$\Delta h_{\text{H}_2\text{O}} = \frac{(p_1 - p_2)g_c}{\rho_{\text{H}_2\text{O}}g} = \frac{6.2}{1000(9.81)}$$

$$\Delta h_{\text{H}_2\text{O}} = 0.00064 \text{ m of H}_2\text{O} = 0.064 \text{ cm of H}_2\text{O}$$

Miscellaneous Cross Sections

Table 3.5 gives the friction factor-Reynolds number product for laminar flow of a Newtonian fluid through a variety of cross sections. Shown are a circular segment, a circular sector, an isosceles triangle, and a right triangle. The friction factor-Reynolds number product is a function of the geometry of the section, notably an angle or half angle and associated linear dimensions. Also provided are expressions for area and perimeter, useful for calculating the hydraulic diameter of the cross section.

For turbulent flow through these noncircular cross sections, the Moody diagram can be used to obtain a reasonable estimate of the friction factor.

The method requires that hydraulic diameter be used for the characteristic dimension in the Reynolds number expression ($= \rho V D_h / \mu g_c$) and in the relative roughness ($= \varepsilon / D_h$). Given the Reynolds number and relative roughness, the Moody diagram, or any of the curve fit equations, can be used to determine the friction factor.

The Finned Annulus

Figure 3.17 shows the cross section of a finned annulus. This type of duct can be used as a heat exchanger in which one fluid flows through the center pipe, and another fluid of different temperature flows through the annular section between the fins. The fins provide a greater surface area between the two fluids and should enhance the heat transfer characteristics over that of an unfinned system.

It can be seen that each of the chambers of the finned annulus is similar in shape to a circular sector (Table 3.5). This is also illustrated in Figure 3.17.

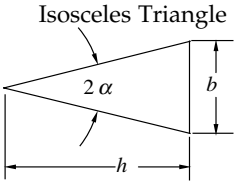
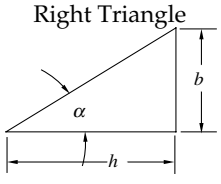
Laminar flow through a finned annulus has been modeled successfully. Equations are available for the laminar flow velocity profile, the volume flow rate, and the friction factor. The solution for friction factor is in the form of an infinite series, but its derivation is beyond the scope of this text. A graph of the equation, however, is provided in Figure 3.17. On the horizontal axis is r_1/r_2 , which ranges from 0.25 to 0.95. On the vertical axis is the $f \text{ Re}$ product, which ranges from 13 to 96. Lines on this graph are for various angles θ_o . The friction factor for laminar flow through a number of different finned annuli can be found from this graph.

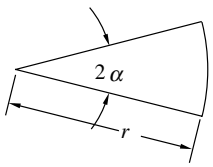
As with the other noncircular cross sections, for turbulent flow through a finned annulus, the Moody diagram can be used to obtain a reasonable estimate of the friction factor if hydraulic diameter is used for the characteristic dimension in the Reynolds number expression ($= \rho V D_h / \mu g_c$) and in the relative roughness ($= \varepsilon / D_h$).

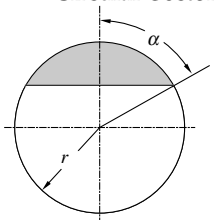
3.8 Summary

In this chapter, we have examined pipe and tubing standards, and discussed the current specifications that apply to them. We have stated three definitions of characteristic dimensions used to represent noncircular cross sections. Equations for velocity, flow rate, Reynolds number, and friction factor were provided for circular and various noncircular cross sections. The Moody diagram was discussed, and two modified versions of it were also provided. Sample problems were given to illustrate the use of these charts. Minor losses were discussed, and recommended procedures for accounting for them were given. Series piping systems were also defined and discussed.

TABLE 3.5. Friction factor-Reynolds number product for laminar flow through various noncircular cross sections.

	 Isosceles Triangle $A = bh/2$ $P = \tan^{-1}(b/2h)$	 Right Triangle $A = bh/2$ $P = b + h + (b + h)^{1/2}$
α	$f \cdot \text{Re}$	$f \cdot \text{Re}$
0	48.0	48.0
10	51.6	49.9
20	52.9	51.2
30	53.3	52.0
40	52.9	52.4
50	52.0	52.4
60	51.1	52.0
70	49.5	51.2
80	48.3	49.9
90	48.0	48.0

	 Circular Segment $A = \alpha r^2$ $P = 2r(1 + \alpha)$
α	$f \cdot \text{Re}$
0	62.2
10	62.2
20	62.3
30	62.4
40	62.5
60	62.8
90	63.1
120	63.3
150	63.7
180	64.0

	 Circular Sector $A = r^2(\alpha - \sin \alpha \cos \alpha)$ $P = 2r(\alpha + \sin \alpha)$
α	$f \cdot \text{Re}$
0	48.0
10	51.8
20	54.5
30	56.7
40	58.4
50	59.7
60	60.8
70	61.7
80	62.5
90	63.1

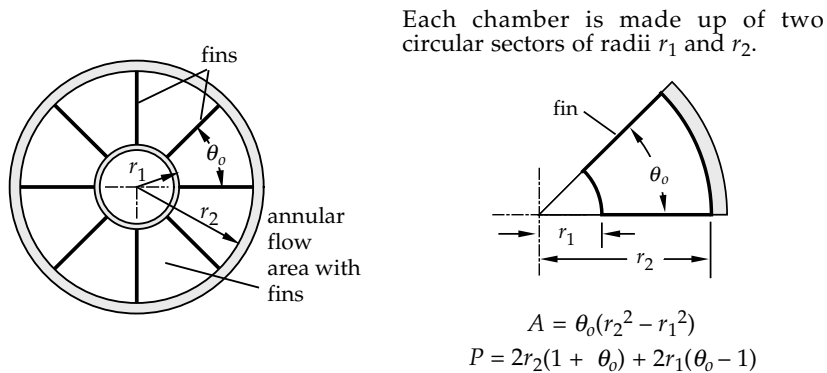


FIGURE 3.16. Definition sketch for a finned annular cross section.

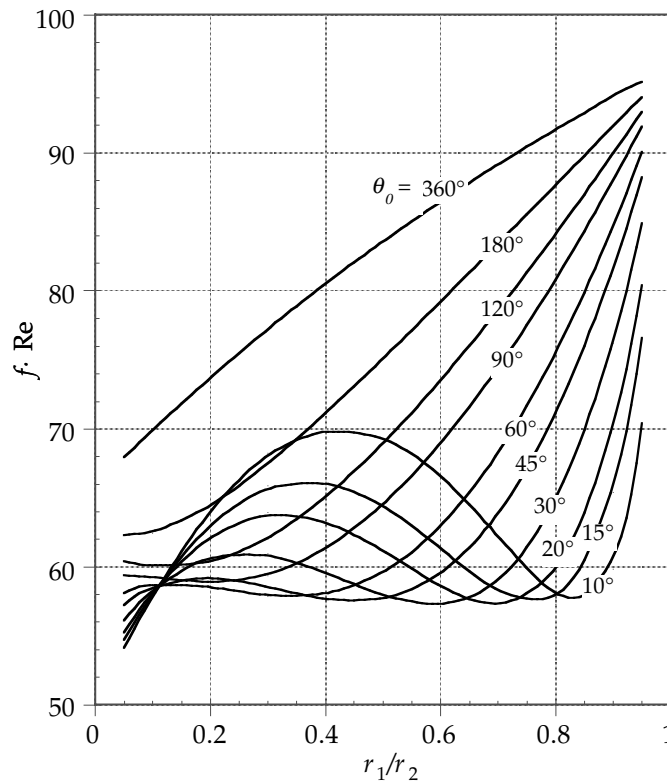


FIGURE 3.17. Friction factor-Reynolds number product for laminar flow through a finned annulus. (Obtained by using Mathematica to solve the series solution given in "Laminar Flow and Pressure Drop in Internally Finned Annular Ducts," by E. M. Sparrow, et al. *Int J Heat Mass Transfer*, vol. 7, no. 5, May 1964, pp. 583–585.)

TABLE 3.3. *Valve selection guide. (Information from “Selecting the Proper Valve—Parts 1 and 2” by J. L. Lyons and C. Askland, Design News, December 1974, pg. 56.)*

Valve Type	Applications	Advantages	Disadvantages
Ball Valve Ported sphere within housing Rotation of sphere by 90° changes from fully open to fully closed Variety of sizes	Flow control Pressure control Shutoff Can be used at high pressures and temperatures Fluids: common, corrosive, cryogenic, viscous and slurries	Low pressure drop Low leakage rate Small size/weight ratio Rapid opening Insensitive to contamination	Seats of ball subject to wear if used as throttle Fluid trapped in ball when closed Quick opening may cause surges or water hammer
Butterfly Valve Disc within housing Disc rotates about a shaft Disc closes against ring seal	Low pressure systems Leakage unimportant Large diameter lines Fluids: common	Low pressure drop Light weight Small face-to-face dimension	Leakage fairly high Seals often damaged by high velocity Require high actuation forces Limited to low pressure systems
Gate Valve Sliding disc or gate Moves perpendicular to flow Not used as a throttle High temperature High pressures	Stop valves—fully open or fully closed Tight seal when fully closed Insensitive to contamination Fluids: common	Low pressure drop when fully open	Prone to vibration Subject to disc and seat wear Slow response characteristics Require high actuation forces Not suited for steam
Globe Valve Closure member travels in direction perpendicular to flow	Throttling purposes Power & process piping General purpose control Fluids: common	Faster to open than gate valve Seating surface less subject to wear Pressure control	High pressure drop Require considerable power to operate Often heavier than other valves
Poppet Valve Closure member moves parallel to fluid flow and perpendicular to sealing surface Closure element can be flat, conical, or spherical	Safety & relief functions Pressure control Check valve Fluids: common	Excellent leakage control Low pressure drop	Subject to pressure imbalances that may cause chattering Some seat surfaces subject to contamination
Swing Valve Similar to butterfly valves Disc hinged at one end	Check valve Unidirectional flow control	Low pressure drop Lightweight Low cost	May have high leakage Seal may erode Introduces turbulence at low flow rates

3.9 Show and Tell

1. Prepare a demonstration that illustrates the following (dependent upon availability of tools and equipment):
 - a. taper of pipe threads
 - b. application of “pipe dope” or tape prior to attaching a fitting to a pipe
 - c. cutting and threading of a pipe using a hand die and/or a machine
2. Prepare a demonstration that illustrates the following (dependent upon availability of tools and equipment):
 - a. use of a flaring tool to flare the end of a tube
 - b. attachment of the tube to a flared end fitting
 - c. installation of a compression fitting
 - d. sweating of a tubing joint
3. Prepare a demonstration that illustrates different types of bell and spigot pipe. Show how the joints are made using seals.
4. Give a presentation on the Victaulic pipe joining system.
5. Prepare a presentation on the difference between conventional pipe sizes for steel and those for stainless steel.
6. Prepare a demonstration that illustrates how the valves mentioned in this chapter operate, as assigned by the instructor and according to availability.

3.10 Problems

Laminar and Turbulent Flows

1. Six gallons per minute of methyl alcohol flows in a 2-nominal schedule 40 pipe. Is the flow laminar or turbulent?
2. Fifteen liters per second of propane flows through 4 standard type M copper tubing. Is the flow laminar or turbulent?
3. Turpentine flows through a 12-nominal schedule 40 pipe. What is the flow rate that corresponds to a Reynolds number of 2000?
4. Air at standard conditions flows through a 2 standard type M copper tube. What is the maximum velocity allowable for laminar flow conditions to exist? (At standard conditions, air can be treated as incompressible.)
5. One hundred cubic feet per minute of air flows through an 2 standard type M copper tube. Is the flow laminar or turbulent?

Pressure Drop Unknown; No Minor Losses

6. Acetone flows at a volume flow rate of 50 gpm through a 2 nominal schedule 40 commercial steel pipe. The pipe is laid out horizontally and is 50 ft long. Calculate the pressure drop.

7. A 4 nominal schedule 80 asphalt coated cast iron pipe conveys benzene at a rate of 10 l/s. The pipe is laid out horizontally and is 60 m long. Calculate the pressure drop.
8. Carbon disulfide flows through a 1 standard type K tube at a rate of 1 l/s. The pipe is laid out horizontally and is 300 m long. Calculate the pressure drop of the carbon disulfide.
9. Castor Oil flows at a rate of 500 gpm through a 6 nominal schedule 40 galvanized steel pipe. The pipe is laid out horizontally and is 25 ft long. Calculate the pressure drop.
10. A 2^{1/2} nominal schedule 40 commercial steel pipe conveys ether at a flow rate of 6 l/s. The pipe is laid out horizontally and is 50 m long. Calculate the pressure drop.
11. A 1^{1/8} nominal schedule 40 wrought iron pipe conveys ethylene glycol at a rate of 0.1 l/s. The pipe is laid out horizontally and is 30 m long. Calculate the pressure drop.
12. A 10 nominal schedule 40 PVC pipe conveys heptane at a rate of 100 l/s. The pipe is laid out horizontally and is 75 m long. Calculate the pressure drop.
13. Hexane flows at 2 gpm through a 1/4 standard type K copper tube. The tube is laid out horizontally and is 450 ft long. Calculate the pressure drop.
14. A 16 nominal schedule 160 commercial steel pipe conveys methyl alcohol at 150 l/s. The pipe is laid out horizontally and is 50 m long. Calculate the pressure drop.
15. A 4 standard type L copper tube conveys propyl alcohol at 15 l/s. The pipe is laid out horizontally. It is 60 m long. Calculate the pressure drop.
16. Kerosene flows at a rate of 8 gpm through a 3/4 standard type K copper tube. The tube is laid out horizontally and is 1400 ft long. Calculate the pressure drop.
17. Octane flows at a rate of 5 gpm through a 1/2 standard type K copper tube. The tube is laid out horizontally and is 190 ft long. Calculate the pressure drop.
18. Propylene flows at 250 gpm through a 4 nominal schedule 40 commercial steel pipe. The pipe is laid out horizontally and is 405 ft long. Calculate the pressure drop.
19. Turpentine flows at 720 gpm through an 8 nominal schedule 80 wrought iron pipe. Calculate the pressure drop if the pipe is laid out horizontally and is 150 ft long.
20. A 2 nominal schedule 40 PVC pipe conveys hydraulic oil at a rate of 3 l/s. The pipe is laid out horizontally and is 10 m long. Calculate the pressure drop of the oil. (Hydraulic oil properties: $\rho = 0.888(1\,000) \text{ kg/m}^3$, $\mu = 0.799 \times 10^{-3} \text{ N}\cdot\text{s/m}^2$.)

Volume Flow Rate Unknown; No Minor Losses

21. A garden hose is used to siphon water as shown in Figure P3.21. The hose is made of a rubber material ("smooth") and is 50 ft long. For the configuration shown, determine the volume flow rate through the hose if (a) frictional effects are neglected; and (b) if friction is accounted for. The inside diameter of the hose is 5/8 in. Neglect minor losses.

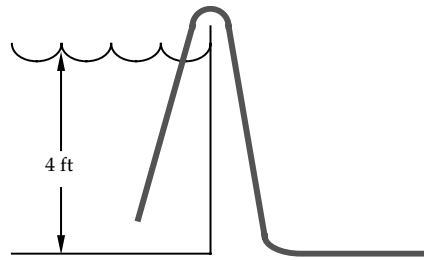


FIGURE P3.21.

22. A 6 nominal schedule 40 galvanized steel pipe is 25 ft long. It is to convey castor oil. The available pump can provide a pressure drop of 1.64 psi. Determine the expected flow rate of castor oil in the pipe.
23. A 1/4 standard type K copper tube is 450 ft long. It is to convey hexane. The available pump can provide a pressure drop of 133.0 psi. Determine the expected flow rate of hexane.
24. A 2 standard type K copper tube is 15 ft long, and is to convey linseed oil. The pressure drop is measured at 1.05 psi. Determine the expected flow rate of linseed oil in the pipe.
25. Kerosene flows through 3/4 standard type K drawn copper tube. The pressure drop measured at two points 50 m apart is 130 kPa. Determine the flow rate of the kerosene.
26. A 2 nominal schedule 40 PVC pipe is 75 ft long. It is to convey water. The available pump can provide a pressure drop of 1.84 psi. Determine the expected flow rate of water in the pipe.
27. A 4 standard type L copper tube is 200 ft long. It is to convey propyl alcohol. The available pump can provide a pressure drop of 2.79 psi. Determine the expected flow rate of propyl alcohol.
28. A 4 nominal schedule 40 commercial steel pipe is 405 ft long. It is to convey propylene, at a corresponding a pressure drop of 2.80 psi. Determine the expected flow rate of propylene in the pipe.
29. A 3 nominal schedule 160 galvanized steel pipe is 110 ft long, and conveys propylene glycol, with a pressure drop of 5.36 psi. Determine the expected flow rate of propylene glycol in the pipe.
30. An 8 nominal schedule 80 wrought iron pipe is 150 ft long. It is to convey turpentine. The available pump can provide a pressure drop of 0.59 psi. Determine the expected flow rate of turpentine in the pipe.

Diameter Unknown; No Minor Losses

31. A fuel line is to convey octane over a distance of 35 ft. The required flow rate is $0.3 \text{ ft}^3/\text{s}$ and the allowable pressure drop is 75 kPa. Select an appropriate line size if drawn copper is used.
32. Linseed oil is to be pumped at a flow rate of 12 gpm over a distance of 18 m. The allowable pressure drop is 15 psi. Centrifugally spun cast iron is to be used as a pipe material. Determine the appropriate line size.

33. A fuel line is to convey 80 l/s of propane with an allowable pressure drop of 13 psi over a 10 m length. Select a suitable diameter if drawn copper tubing is to be used.
34. A galvanized steel pipe conveys methyl alcohol at a rate of $0.05 \text{ m}^3/\text{s}$ a distance of 4 000 m. The available pump can overcome a pressure drop of 300 kPa. Select a suitable pipe size for the installation, assuming that either a schedule 40 or a schedule 80 size is to be used.
35. Acetone flows through a 20 m long pipe at a flow rate of 10 gpm. The pipe is made of commercial steel and should be a schedule 40 size. The pressure drop is 250 psf. Select a suitable schedule 40 size for the installation.

Miscellaneous Problems with Minor Losses

36. Propyl alcohol flows at $0.0005 \text{ m}^3/\text{s}$ through the piping system of Figure P3.29. The system is made of 1/2-nominal schedule 40 commercial steel. Determine the pressure drop from section 1 to section 2, if the pipe length is 15 m, and the exit is 1 m lower than the inlet. All fittings are regular and threaded.

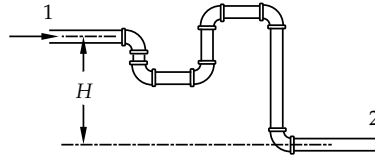


FIGURE P3.36.

37. A pressurized tank and piping system are shown in Figure P3.37. The tank pressure is maintained at 175 kPa. The line is made of 12 m of 1 std type M copper tubing and it conveys gasoline (octane). What is the expected flow rate through the line? All fittings are soldered (same as flanged) and regular.

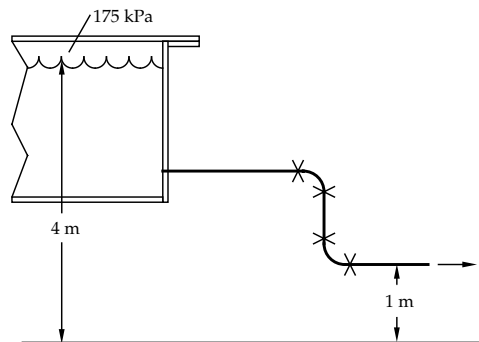


FIGURE P3.37.

38. Castor oil flows through the piping system of Figure P3.38. The pipe is made of 6-nominal schedule 40 galvanized steel. All fittings are flanged and are of the long radius type. Calculate the flow rate of liquid through the pipe if it is 250 ft long.
39. Suppose the receiver tank and discharge end of the pipe in Problem 38 are changed to the configuration shown in Figure P3.39. Rework the problem with this new setup and compare the following details between the two problems (noting of course the differences in control volume selection):
 - a. continuity equation
 - b. modified Bernoulli equation after simplification and before substitution

- of numbers
 c. minor losses
 d. Reynolds numbers and friction factors
 e. solution

Assume that the pipe is 5 ft longer in this problem than in Problem 3.38.

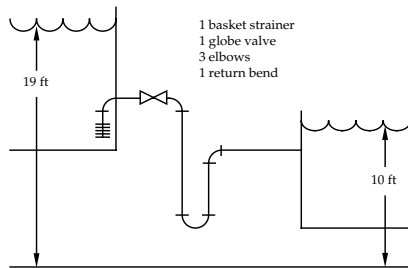


FIGURE P3.38.

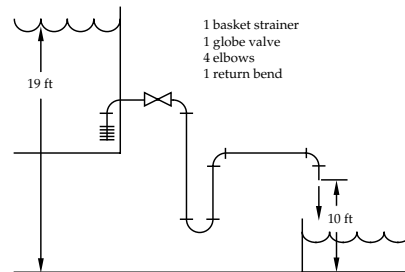


FIGURE P3.39.

40. The tubing arrangement of a cross-flow heat exchanger is given schematically in Figure P3.40. It consists of type M drawn copper tubing with fins attached. There are 2 elbows and 7 return bends, all regular. The tube is to convey 0.4 l/s of propylene glycol. The allowable pressure drop in the system (inlet to outlet) is 85 kPa, and the tube length is 10 m. Select a suitable line diameter. All fittings are regular and soldered (same minor loss as flanged).

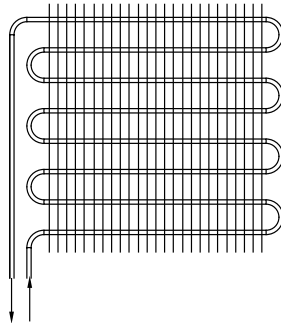


FIGURE P3.40.

41. A commercial steel pipeline is 200 ft long and is to convey 400 gpm of water. The system will contain three couplings and eight-90° elbows. The flow will be controlled by a gate valve. The inlet is fed by a pump, and the pressure there is 125 psia. The outlet height is exactly the same height as the inlet, and water is discharged to the atmosphere. Select a suitable line size. It is desirable to use threaded fittings if the diameter is 2-nominal or smaller and flanged fittings if the diameter is larger than 2-nominal. All fittings are to be regular. Schedule 40 is preferable.
42. A piping system is used to drain a tank as indicated in Figure P3.42. Water enters the tank while it is being drained so that the liquid level remains at a constant depth of 2 m ($= d$) above the outlet at tank bottom. The piping system is made of PVC and contains a square-edged inlet, five elbows (regular, flanged), and one ball-type check valve. The water is discharged to the atmosphere such that $H = 3$ m. The volume flow rate through the system is 0.005 m³/s. If the total length of PVC pipe is 25 m, select a suitable line size using a schedule 40 pipe with glued fittings.

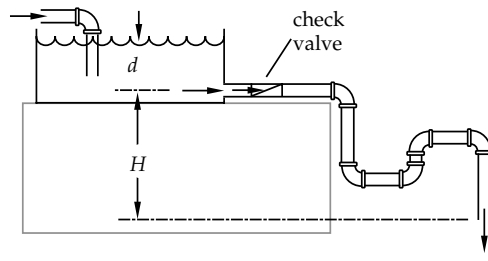


FIGURE P3.42.

Pipes in Series

43. A piping system is shown in Figure P3.43. It consists of 6 m of 6-std type K and 12 m of 4-std type K, both drawn copper tubing. The system conveys ethylene glycol at a rate of $0.013 \text{ m}^3/\text{s}$. The pressure drop from section 1 to section 2 is to be calculated. All fittings are soldered (same minor loss as flanged) and regular.

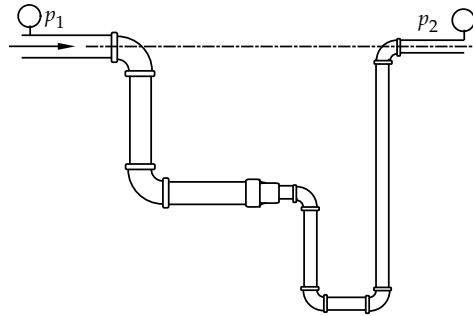


FIGURE P3.43.

44. A series piping system consists of 100 ft of 2-nominal pipe and 50 ft of $2\frac{1}{2}$ -nominal pipe, both schedule 40 galvanized steel. The water velocity through the 2-nominal pipe is 6 ft/s. Calculate the pressure drop through the system for equal inlet and outlet heights.
45. A series piping system conveys methyl alcohol. The system consists of 70 m of 1-nominal pipe followed by 50 m of 2-nominal pipe, both schedule 40 commercial steel. The 1-nominal pipe contains three elbows (regular) and a fully open gate valve, all threaded. The pressure drop through the system is 150 kPa. Determine the volume flow rate for a system that is horizontally laid.

Derivation of Velocity Profiles

46. Refer to Figure P3.46 and derive the equation of velocity for laminar flow in a circular duct by following the steps outlined below
- Perform a force balance on the control volume in the figure and verify that

$$pA + \tau dA_p - (p + dp)A = 0$$

where A is cross sectional area and dA_p is perimeter times axial length.

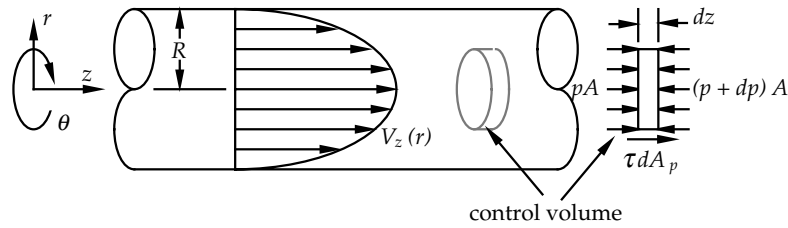


FIGURE P3.46.

- b. Substitute $dA_p = 2\pi r dz$ and $A = \pi r^2$ and show that

$$\frac{dp}{dz} = \frac{2\tau}{r}$$

- c. Assuming a Newtonian fluid with constant properties, let

$$\tau = \mu \frac{dV_z}{dr}$$

and verify that

$$\frac{dV_z}{dr} = \frac{r}{2\mu} \frac{dp}{dz}$$

- d. Verify that the boundary condition $r = R, V_z = 0$ is correct. Integrate the equation for dV_z/dr above and apply the boundary condition. Show that

$$V_z = \left(-\frac{dp}{dz} \right) \left(\frac{R^2}{4\mu} \right) \left[1 - \left(\frac{r}{R} \right)^2 \right] \quad \left(\begin{array}{l} \text{laminar flow} \\ \text{circular duct} \end{array} \right)$$

47. Start with Equation 3.10 and verify that Equation 3.12 is correct.
48. Combine Equations 3.8a and 3.12 to derive Equation 3.13.
49. Referring to Figure 3.12, derive the equation for velocity for laminar flow in an annulus by following the steps outlined below:
- a. The momentum equation applied to the control volume of Figure 3.12 is

$$\Sigma F_z = \frac{1}{g_c} \int \int_{cs} V_z \rho V_n dA$$

Show that this equation becomes

$$pA + (\tau + d\tau)dA_1 - \tau dA_2 - (p + dp)A = 0$$

- b. Simplify the preceding equation to obtain

$$(\tau + d\tau)dA_1 - \tau dA_2 - Adp = 0$$

- c. The surface areas over which the shear stresses act are evaluated as

$$dA_1 = 2\pi(r + dr)dz \quad \text{and} \quad dA_2 = 2\pi r dz$$

The cross sectional area is

$$A = \pi(r + dr)^2 - \pi r^2 = 2\pi r dr$$

Substitute these areas into the momentum equation and show that

$$(\tau + d\tau) 2\pi(r + dr)dz - \tau 2\pi r dz - 2\pi r dr dp = 0$$

- d. Simplify the preceding equation, and neglect the $dr d\tau$ term as being small compared to the others. Show that

$$\frac{\tau}{r} + \frac{d\tau}{dr} = \frac{dp}{dz}$$

which becomes

$$\frac{1}{r} \frac{d}{dr} (r\tau) = \frac{dp}{dz}$$

- e. For a Newtonian fluid,

$$\tau = \mu \frac{dV_z}{dr}$$

Combine with the preceding equation to get

$$\frac{d}{dr} \left(r \frac{dV_z}{dr} \right) = \frac{r}{\mu} \frac{dp}{dz} \quad (i)$$

- f. Verify that the boundary conditions are

$$1. r = OD_p/2, V_z = 0$$

$$2. r = ID_a/2, V_z = 0$$

The boundary conditions can be expressed in a slightly different and ultimately more convenient way. Define

$$R = ID_a/2 \quad \text{and} \quad \kappa = \frac{OD_p/2}{ID_a/2} = \frac{OD_p}{ID_a}$$

Show that the boundary conditions can now be written as

$$1. r = R, V_z = 0$$

$$2. r = \kappa R, V_z = 0$$

- g. Integrate Equation i and apply the boundary conditions; show that the velocity profile is

$$V_z = \left(-\frac{dp}{dz} \right) \left(\frac{R^2}{4\mu} \right) \left[1 - \left(\frac{r}{R} \right)^2 - \frac{1 - \kappa^2}{\ln(\kappa)} \ln \frac{r}{R} \right] \quad \left(\begin{array}{l} \text{laminar flow} \\ \text{annular duct} \end{array} \right)$$

50. a. The volume flow rate for flow through an annulus is found by integrating the velocity profile over the cross section:

$$Q = \int_0^{2\pi} \int_{\kappa R}^R V_z r dr d\theta$$

- b. Substitute the velocity profile for laminar flow through an annulus, namely,

$$V_z = \left(-\frac{dp}{dz} \right) \left(\frac{R^2}{4\mu} \right) \left[1 - \left(\frac{r}{R} \right)^2 - \frac{1 - \kappa^2}{\ln(\kappa)} \ln \frac{r}{R} \right] \quad \left(\begin{array}{l} \text{laminar flow} \\ \text{annular duct} \end{array} \right)$$

into the volume flow rate equation and integrate. Show that

$$Q = \left(-\frac{dp}{dz} \right) \left(\frac{\pi R^4 (1 - \kappa^2)}{8\mu} \right) \left(1 + \kappa^2 + \frac{1 - \kappa^2}{\ln(\kappa)} \right)$$

- c. With the average velocity found with

$$V = \frac{Q}{A}$$

show that

$$V = \left(-\frac{dp}{dz} \right) \left(\frac{R^2}{8\mu} \right) \left(1 + \kappa^2 + \frac{1 - \kappa^2}{\ln(\kappa)} \right)$$

51. The hydraulic diameter for an annular flow section is

$$D_h = ID_a - OD_p$$

In terms of the ratio of diameters, we define

$$\kappa = \frac{OD_p/2}{ID_a/2} = \frac{OD_p}{ID_a}$$

Equation 3.8a relates the pressure drop to the friction factor:

$$dp = - \frac{\rho V^2}{2g_c} \frac{f dz}{D_h}$$

The velocity profile for laminar flow in an annulus is given by

$$V = \left(-\frac{dp}{dz} \right) \left(\frac{R^2}{8\mu} \right) \left(1 + \kappa^2 + \frac{1 - \kappa^2}{\ln(\kappa)} \right)$$

Combine the preceding equations and show that:

$$\frac{1}{f} = \frac{\text{Re}}{64} \left(\frac{1 + \kappa^2}{(1 - \kappa)^2} + \frac{1 + \kappa}{(1 - \kappa) \ln(\kappa)} \right)$$

where

$$\text{Re} = \frac{VD}{\nu} = \frac{2RV(1 - \kappa)}{\nu}$$

52. Flow through a rectangular duct is illustrated in Figure 3.13. The cross section is assumed to be very wide compared to its height. Flow is in the z -direction and the control volume we are working with does not extend to the wall surfaces. The momentum equation applied to the control volume is

$$\Sigma F_z = \frac{1}{g_c} \int \int_{\text{cs}} V_z \rho V_n dA$$

- a. Considering forces due to pressure and friction only, show that the above equation when applied to Figure 3.13 becomes

$$pA + \tau P dz - (p + dp)A = 0$$

- b. For a rectangular duct, the area and perimeter are

$$A = 2xy$$

$$P = 2y + x + 2y + x = 2x + 4y$$

The x dimension ($\sim w$) is much larger than the y dimension ($\sim 2h$) of the duct. The perimeter term can therefore be reduced to

$$P \approx 2x$$

Show that the momentum equation reduces to

$$\frac{dp}{dz} = \frac{\tau}{y}$$

- c. For a Newtonian fluid, $\tau = \mu dV_z/dy$. Combine with the preceding equation and rearrange to obtain

$$\frac{dV_z}{dy} = \frac{y}{\mu} \frac{dp}{dz}$$

- d. Verify that the boundary conditions are

$$\begin{aligned} 1. \quad y = \pm h/2 \quad V_z &= 0 \\ 2. \quad y = 0 \quad \frac{\partial V_z}{\partial y} &= 0 \end{aligned}$$

- e. Integrate the momentum equation and apply the boundary conditions to show that

$$V_z = \left(-\frac{dp}{dz} \right) \left(\frac{h^2}{2\mu} \right) \left(\frac{1}{4} - \frac{y^2}{h^2} \right) \quad \left(\begin{array}{c} \text{laminar flow} \\ \text{2-D rectangular duct} \end{array} \right)$$

53. The volume flow rate for laminar flow through a two-dimensional duct is found by integrating the velocity V_z over the cross-sectional area

$$Q = \int_0^w \int_{-h/2}^{+h/2} \left(-\frac{dp}{dz} \right) \left(\frac{h^2}{2\mu} \right) \left(\frac{1}{4} - \frac{y^2}{h^2} \right) dy dx$$

Show that

$$Q = \frac{h^3 w}{12\mu} \left(-\frac{dp}{dz} \right)$$

54. The average velocity for laminar flow through a two-dimensional rectangular duct is found as

$$V = \frac{Q}{A} = \frac{h^2}{12\mu} \left(-\frac{dp}{dz} \right)$$

Equation 3.8a relates the pressure loss to the average velocity in a duct for any cross section

$$dp = - \frac{\rho V^2}{2g_c} \frac{f dz}{D_h}$$

Also, for a two-dimensional duct, the hydraulic diameter is

$$D_h = \frac{4A}{P} = \frac{4hw}{2h + 2w} \approx \frac{4hw}{2w} = 2h$$

Combine the preceding equations and show that the friction factor is

$$f = \frac{96}{\text{Re}}$$

where

$$\text{Re} = \frac{\rho V (2h)}{\mu g_c}$$

Hydraulic Diameter, Hydraulic Radius, Effective Diameter

55. Determine the hydraulic radius of a two-dimensional rectangular duct in which the width is much greater than the height ($w \gg h$).
56. Determine the effective diameter of a two-dimensional rectangular duct in which the width is much greater than the height ($w \gg h$).
57. A rectangular duct has dimensions of $h \times w$. The height h is 4 cm. Determine the width w if the hydraulic and effective diameters are equal, if possible.

58. An annular duct consists of two tubes. The inner tube is 2 std and the outer tube is 4 std, both type K. Calculate the hydraulic radius, hydraulic diameter, and effective diameter of the flow passage.
59. A flow passage is bounded by the outside area of a 1 std type L copper tube and the inside of a 6 in. x 4 in. square duct. Calculate the hydraulic radius, hydraulic diameter, and effective diameter of the flow passage.
60. Figure P3.60 shows a cross section of a shell and tube heat exchanger. The outer tube is 8-std type K copper tubing. Inside are four smaller tubes, each made of 1 std also type K copper tubing. Determine the hydraulic radius, the effective diameter and the hydraulic diameter of the flow area, which is bounded by the ID of the 8-std tube and the OD of the 1 std tubes.

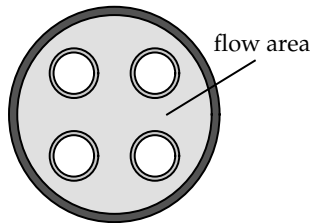


FIGURE P3.60.

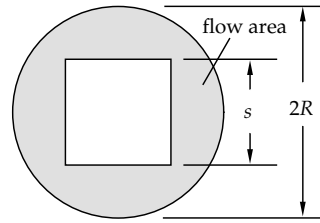


FIGURE P3.61.

61. The cross section of a flow conduit is bounded on the exterior by a circular tube of inside radius R , and the outside surface of a square of side s (See Figure P3.61). For this cross section and for the special case where $R = s$, what is the
 - a. hydraulic radius?
 - b. the effective diameter?
 - c. the hydraulic diameter?
62. A circular segment has a half angle α of 40° and a radius of 6 in. What is the hydraulic diameter of the cross section?
63. A duct in the shape of a circular segment has a half angle α of 20° and a radius of 10 cm. A duct in the shape of an isosceles triangle has a half angle also of 20° . If it has the same hydraulic diameter as the circular segment, what must its height h be?
64. A circular sector has an angle α of 45° and a radius of 4 ft. Determine its hydraulic diameter.
65. A duct with the shape of a right triangle has an angle α of 50° and a height h of 1 m. Determine its effective and hydraulic diameters.
66. A flow passage is bounded by the outside area of a 1-std type L copper tube and the inside of a 4 in. x 4 in. square duct. Calculate the hydraulic radius, hydraulic diameter, and effective diameter of the flow passage.
67. A duct with the shape of a right triangle has an angle α of 50° and a height h of 1 m. Determine its hydraulic diameter.

Noncircular Cross Sections—Miscellaneous Problems

68. An annular flow passage is 25 m long and is formed by placing a 2-nominal pipe within a 4-nominal pipe (both schedule 40 and made of uncoated cast iron). The

flow passage is to convey 8.5 l/s of carbon disulfide. Calculate the pressure drop over the 5 m length.

69. An annular flow area is formed by 3-std type M copper tube and 1-std type L copper tube, both 6 ft long. Glycerine is pumped through the annulus. Attached pressure gages show that the pressure drop is 3 psi. Determine the volume flow rate of glycerine.
70. As part of a heat exchanger, an annulus is to convey ethyl alcohol at a flow rate of 10 l/s. The inner tube is 3/4 std type M copper tubing and the size of the outer tube is to be determined. The available pump can overcome a pressure drop of 5 psi, and the annular flow passage is 8 ft long. Select a suitable outer tube size.
71. An asphalt-coated, 6-m long rectangular duct has internal dimensions of 0.5 m x 1.5 m. It conveys air at a flow rate of 1.5 m³/s. Calculate the pressure drop.
72. A portion of the railway that leads to Union Station in Chicago is underground, and for health reasons, it is necessary to provide fresh air to the area. This is accomplished by fans that move air through a system of ducts. Consider one such duct that is rectangular, 35 ft long, and made of galvanized sheet metal. The duct is 3 ft wide by 6 ft tall, and delivers cooled air ($T \approx 60^\circ\text{F}$) with an allowable pressure drop is 0.01 in. of water. Calculate the flow rate of air assuming it to be an ideal gas.

Miscellaneous Problems

73. Equation 3.8b was derived from Equation 3.5 using the Darcy-Weisbach definition of friction factor and the hydraulic diameter. Begin again with Equation 3.5 but use the Fanning friction factor and the hydraulic radius to derive an equation analogous to Equation 3.8b.
74. Benzene flows through a 200-ft long 4-nominal schedule 80 pipe at a rate of 250 gpm. The corresponding pressure drop is 6 psi. Determine the value of the surface roughness ϵ for this pipe.
75. Water flows through a portion of a pipe that contains a valve as shown in Figure P3.75. An air-over-liquid manometer attached to the pipeline measures the head loss. The pipe is made of 3-nominal schedule 160 galvanized steel and conveys water at a rate of 0.25 ft³/s. Determine the value of K for a head loss Δh of 6 in. of water.

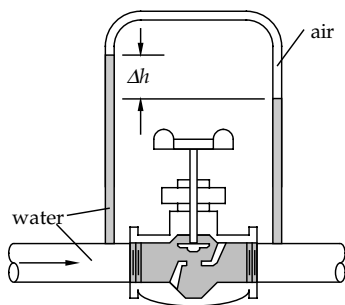


FIGURE P3.75.

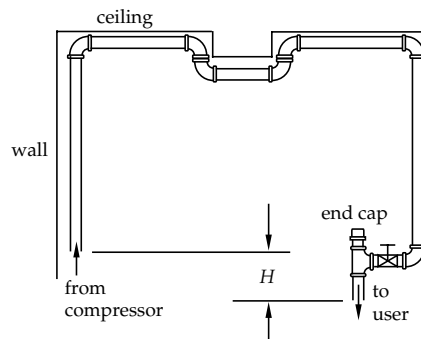


FIGURE P3.76.

Computer Problems

76. Figure P3.76 shows a piping system made of 1-standard type K drawn copper tubing. It conveys air and the pressure drop over the 150 m of drawn tubing is 175 kPa. The fittings are all regular and soldered; the valve is a globe valve. Given that $H = 0.5$ m, determine the flow rate through the tube for air temperatures that range from 0°C to 100°C .
77. Select five values of Reynolds number and ε/D together. Then use the Moody diagram curve fit equations given in this chapter to determine f . Compare the results.
78. Figure P3.78 shows a piping system made of 4-nominal schedule 40 galvanized steel pipe. The system is used to drain a tank of water. Determine the flow rate through the pipe for water temperatures that range from 5°C to 95°C . The elbows are all regular and welded; the check valve is a swing-type. With $H = 2$ m and $L = 25$ m, determine how velocity varies with temperature.
79. Figure P3.79 shows a sketch of the pipeline from one of the examples in this chapter. The pipeline was used to drain a tank that contained water. When $z_1 = 5$ m, and $z_2 = 2$ m, the flow rate through the line was 2.7 l/s. The pipe is made of 2 nominal schedule 40 galvanized steel, and the fittings are regular and threaded. Determine the volume flow rate through the system for z_1 that ranged from 3 to 10 m. Graph flow rate as a function of z_1 .

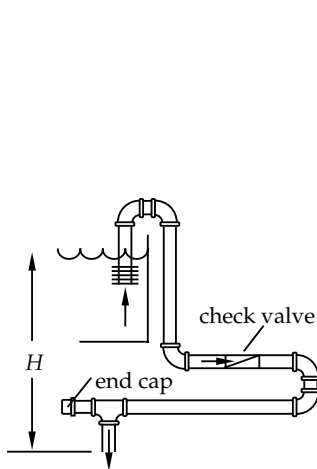


FIGURE P3.78.

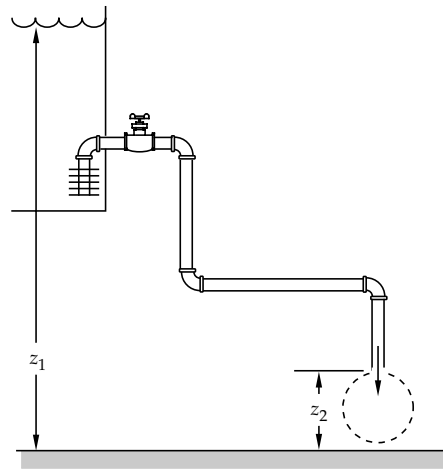


FIGURE P3.79

80. A number of years ago (in the pre-computer, pre-internet era), engineers used nomographs and tables as aids in making design decisions regarding pipe sizes, flow rates, and so on. A compilation of numerous aids was published by the Crane Company as Technical Paper No. 410, which is still available as a downloadable pdf file. Tech Paper 410 included among many other things a chart that related flow through a pipeline in gpm to the pressure drop in psi/100 ft experienced by the fluid. The fluid was water, the pipe was made of schedule 40 steel, and the pipe length was 100 ft. The chart included line sizes that varied from 1/8 to 24 inch (page 8-14 of Tech Paper 410). The equation that was solved in producing the chart was:

$$p_1 - p_2 = \frac{fL}{D_h} \frac{\rho V^2}{2g_c}$$

The following table is such a chart, and it was produced using a spreadsheet. The fluid is water, and the line size is 1 in-schedule 40 steel. The pressure drop is listed as psi, but it should be remembered that the actual unit is psi/100 ft. This was done so that the user could easily calculate the pressure drop over any length of pipeline. Data used in producing the chart are:

$$\begin{aligned} D &= 1 \text{ in.} = 0.08742 \text{ ft} & A &= 0.006002 \text{ ft}^2 \\ \rho &= 1.94 \text{ slug/ft}^3 & L &= 100 \text{ ft} \\ \mu &= 0.000019 \text{ lbf}\cdot\text{s/ft}^2 & \varepsilon &= 0.00015 \text{ ft} \end{aligned}$$

As assigned by your instructor, produce a similar chart for any of the following combinations:

- Line size other than 1 in.
- Fluid other than water.
- SI units of l/s and kPa.
- Pipe material other than steel.
- Flow rate range different from 1 to 100 gpm.

Q , gpm	Q , ft ³ /s	V , ft/s	Re	f	Δp , psi
1	0.002228	0.371	3313	0.044	0.047
2	0.004456	0.742	6627	0.037	0.155
3	0.006684	1.11	9940	0.033	0.317
4	0.008912	1.48	13253	0.031	0.530
5	0.01114	1.86	16567	0.030	0.794
6	0.01337	2.23	19880	0.029	1.11
8	0.01782	2.97	26507	0.028	1.88
10	0.02228	3.71	33133	0.027	2.85
15	0.03342	5.57	49700	0.026	6.12
20	0.04456	7.42	66266	0.025	10.6
25	0.05570	9.28	82833	0.024	16.3
30	0.06684	11.1	99400	0.024	23.1
35	0.07798	13.0	115966	0.024	31.2
40	0.08912	14.8	132533	0.024	40.4
45	0.1003	16.7	149099	0.024	50.9
50	0.1114	18.6	165666	0.024	62.5
60	0.1337	22.3	198799	0.023	89.4
70	0.1560	26.0	231932	0.023	121
80	0.1782	29.7	265065	0.023	158
100	0.2228	37.1	331332	0.023	245

CHAPTER 4 Piping Systems II

In the last chapter, a study of piping systems was introduced and most of the material was reproduced from elementary fluid mechanics. In this chapter, we continue our study of piping systems by combining results of the last chapter with an economic analysis to develop a new method of pipe sizing. In order to do so, we must first examine the concept of optimization as it pertains to fluid thermal systems. The optimization process involves deriving an equation for modeling a system subject to certain constraints, and then taking the derivative of that equation in order to minimize a pressure drop, for example, or to minimize the installed cost of a pipeline.

Equations for the least annual cost method of economic pipe diameter selection are then derived. The equations include economic and pipe friction parameters. The derivations lead to a new format for the traditional pipe friction diagram (the Moody diagram). Three new graphs are presented that aid in determining the economic diameter when economic parameters and power costs are known. A new dimensionless group has been developed by combining the relative roughness and the Reynolds number. An example problem is provided to illustrate use of the graphs and the method in general. The method can be used successfully to select the economic diameter and satisfy least cost (first plus operating) requirements.

Next, the concept of equivalent length of minor losses is discussed. The equivalent length is defined and calculated for a fitting, to illustrate the definition. Methods of graphically represented piping systems are also discussed. ANSI piping symbols are given as well. The behavior of a system is also described in this chapter. A system curve is defined to show how frictional effects influence the volume flow rate. We conclude with a section about conventional hardware available for physically supporting a piping system.

4.1 The Optimization Process

One important calculus applications is in the concept of optimization. In such problems, some quantity must be maximized or minimized. Examples of optimization problems abound in many areas. For example, an airline must decide on how many flights to schedule between two cities, the objective being to optimize its profits. A manufacturer needs to determine how often

to replace equipment in order to minimize maintenance and replacement costs. We might have a piping system in which we seek to minimize the pressure drop by using an optimum pipe diameter. We may wish to locate a filter within a duct such that the installed cost is a minimum. Any number of optimization problems can be devised. We will examine various optimization examples in order to demonstrate the method.

An optimization problem is one in which we seek to minimize (or maximize) a specific variable that helps to describe a system. The formulation of an optimization problem involves several features. First, we will have to derive what is known as an **objective function** or equation. This equation will be differentiated with respect to one of the variables. The objective function in many cases contains terms that are interrelated, and it may be necessary to have additional equations in order to solve the problem. These additional equations are referred to as **constraint equations**. The objective function is said to be solved subject to the constraints. In some problems, there are no constraints, and so we would have an **unconstrained optimization problem**.

Suppose we wish to determine the minimum value of a function given as:

$$f(x) = 2x^3 - 15x^2 + 24x + 19$$

in the range $x \geq 0$. A graph of this equation is provided in Figure 4.1. The lowest point on the graph is at (4, 3), and the minimum value of the function f is 3. We can obtain this value by differentiating the function and setting it equal to zero, to obtain:

$$\frac{df}{dx} = 6x^2 - 30x + 24 = 0$$

Simplifying,

$$x^2 - 5x + 4 = 0$$

with solution

$$(x - 4)(x - 1) = 0$$

$$x = 4 \quad \text{and} \quad x = 1$$

Thus the slope of the graph of the function f is zero at these two points, and both are within the range of $x \geq 0$. To determine which is minimum, we obtain the second derivative:

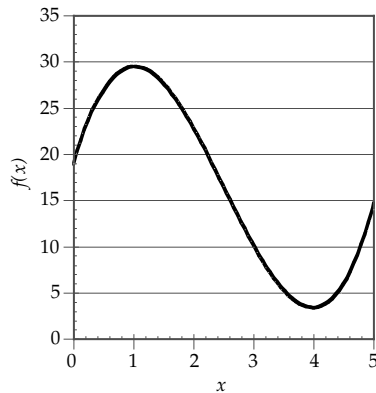


FIGURE 4.1. Graph of the function $f(x) = 2x^3 - 15x^2 + 24x + 19$.

$$\frac{d^2f}{dx^2} = 12x - 30$$

At $x = 1$, $d^2f/dx^2 < 0$, so it is a maximum. At $x = 4$, $d^2f/dx^2 > 0$, so it is a minimum and $x = 4$ is the solution.

EXAMPLE 4.1. A manufacturer wants to cordon off an area within one of its buildings as a tool crib. The area is to be rectangular in shape, and located along an existing wall, as indicated in Figure 4.2. Determine the dimensions of the largest area that can be enclosed if 40 ft of fencing material is to be used.

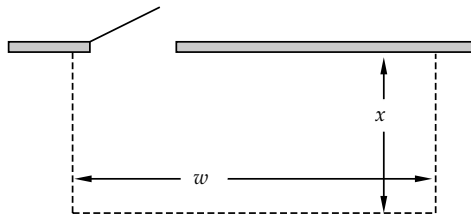


FIGURE 4.2. Tool crib enclosure.

Solution. The area has dimensions of x and width w . The total fencing is 40 ft, so we write:

$$2x + w = L = 40 \text{ ft} \quad (\text{i})$$

The area A is to be maximized:

$$A = wx \quad (\text{ii})$$

We solve Equation i for w :

$$w = L - 2x \quad (\text{iii})$$

and substitute into Equation ii:

$$A = (L - 2x)x = xL - 2x^2$$

Differentiating and setting the result equal to zero, we get

$$\frac{dA}{dx} = L - 4x = 0$$

Solving,

$$x = \frac{L}{4} = \frac{40 \text{ ft}}{4}$$

$$\underline{x = 10 \text{ ft}}$$

The width is found with Equation iii:

$$w = 40 - 2(10)$$

$$\underline{w = 20 \text{ ft}}$$

We have two equations in this problem. Equation i is the *objective function*, and it has many solutions. Equation ii is the constraint equation, and it places a limit or a constraint of the way w and x may vary. This problem is referred to as a *constrained optimization* problem. The essential step in the solution was to solve the constraint equation, and substitute into the objective function to obtain the area in terms of only one variable. This procedure, however, is not always possible.

EXAMPLE 4.2. A manufacturer of galvanized steel playground equipment stores material in a fenced in area outdoors when it is ready to ship. The area is to be 55 m², with three sides built of redwood fencing. The fourth side is made of concrete blocks, as indicated in Figure 4.3. The fencing costs \$21 per m of length, and the blocks cost \$42/m. Determine the dimensions of the fenced in area that minimizes the total cost.

Solution. The dimensions of the area are x by y , as shown in Figure 4.3. The constraint equation is:

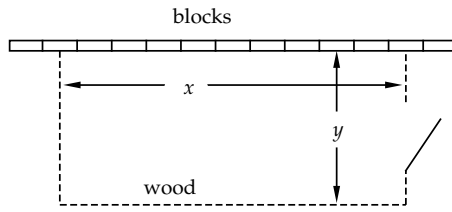


FIGURE 4.3. *Outdoor storage area.*

$$xy = A = 55 \text{ m}^2 \quad (\text{i})$$

The objective function is the total cost of the materials; that is,

$$[\text{Cost of redwood}] = [\text{cost per meter}] \bullet [\text{length}] = (\$21) (x + 2y)$$

or

$$C_{rw} = 21x + 42y \quad (\text{ii})$$

Likewise,

$$[\text{Cost of blocks}] = C_b = [\text{cost per meter}] \bullet [\text{length}] = (42)x$$

The total cost (the objective function) then is

$$C_{total} = 21x + 42y + 42x = 63x + 42y \quad (\text{iii})$$

We solve Equation i for y (or x) to obtain:

$$y = \frac{A}{x}$$

Substituting into Equation iii,

$$C_{total} = 63x + 42 \frac{A}{x}$$

Differentiating, and setting the result equal to zero, we get

$$\frac{dC_{total}}{dx} = 63 - \frac{42A}{x^2} = 0$$

Rearranging,

$$x^2 = \frac{42A}{63} = \frac{42(55 \text{ m}^2)}{63} = 36.67 \text{ m}^2$$

and

$$\underline{x = 6.05 \text{ m}}$$

$$\underline{y = \frac{55}{6.05} = 9.08 \text{ m}}$$

A graph of the objective function is provided in Figure 4.4. The cost equation in this example yields a family of curves, and the constraint equation denotes which of those curves applies.

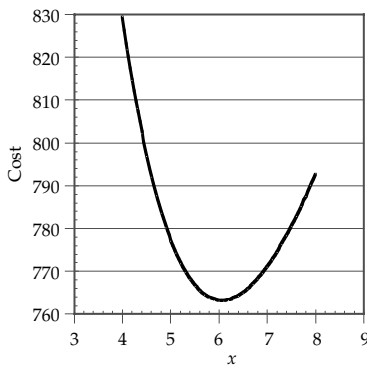


FIGURE 4.4. *The objective function of Example 4.2.*

EXAMPLE 4.3. A company produces small parts that are to be shipped to various locations. The manager of the packaging department has determined a way to reduce storage, shipping, and container costs. These costs can be reduced if the packages are made of cardboard, cylindrical in shape, and have a length plus circumference total of no more than 200 cm. Find the dimensions of the cylindrical package that has the largest volume.

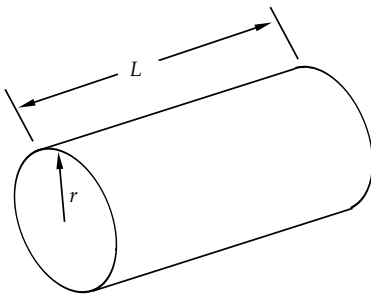


FIGURE 4.5. *Cylindrically shaped shipping container.*

Solution: Figure 4.5 is a sketch of the container, with L its length and r its

radius. The volume of the package is the objective function, which is to be maximized:

$$V = \pi r^2 L \quad (\text{i})$$

The constraint equation is given by

$$L + 2\pi r = 2 \text{ m} \quad (\text{ii})$$

Solving the constraint equation for L and substituting into the objective function gives

$$L = 2 - 2\pi r$$

$$V = \pi r^2(2 - 2\pi r) = 2\pi r^2 - 2\pi^2 r^3$$

A graph of volume versus r is given in Figure 4.6 for $L = 0.68 \text{ m}$. Differentiating with respect to r , we obtain

$$\frac{dV}{dr} = 4\pi r - 6\pi^2 r^2 = 0$$

Simplifying and solving,

$$r = \frac{0.66}{\pi}$$

$$\underline{r = 0.21 \text{ m} = 21 \text{ cm}}$$

$$\underline{L = 2 - 2\pi(0.21) = 0.68 \text{ m} = 68 \text{ cm}}$$

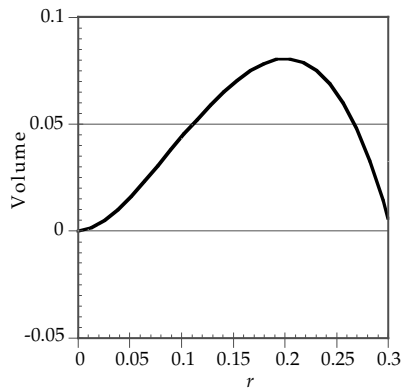


FIGURE 4.6. Optimization curve for the cylindrical container.

The preceding examples demonstrate the method used to find the required maximum or minimum in a problem. The following examples have greater application in the areas covered in this text.

EXAMPLE 4.4. We would like to install insulation around a pipe that is carrying a heated fluid, as illustrated in Figure 4.7. Due to space limitations, the outside diameter of the insulation D_2 cannot exceed 12 cm. On the one hand, we would like to install as large a pipe as possible so that the cost of pumping the fluid is not excessive. On the other hand, we would like to use as thick an insulation as possible to reduce the heat loss. The cost of pumping the fluid through a pipe is given by

$$C_p = \frac{3 \times 10^{-6}}{D_1^5}$$

where D_1 is in m, and the cost is in \$/year. The cost of heating the fluid is given by

$$C_h = \frac{9}{2t}$$

in which t is the insulation thickness ($2t = D_2 - D_1$), in meters, and cost is again in \$/year.

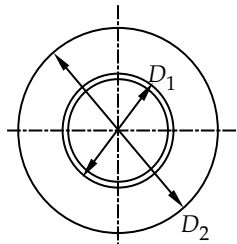


FIGURE 4.7 *The insulated pipe of Example 4.4.*

- Write the equation for total cost; the constraint is $D_2 = 12$ cm.
- Differentiate the cost equation and set it equal to zero.
- Solve for the diameter D_1 .
- Graph total cost versus diameter and verify that the results are correct.

Solution: The total cost is the sum of the pumping and heating costs:

$$C_T = \frac{3 \times 10^{-6}}{D_1^5} + \frac{9}{2t}$$

Substituting $2t = D_2 - D_1 = 0.12 - D_1$,

$$C_T = \frac{3 \times 10^{-6}}{D_1^5} + \frac{9}{0.12 - D_1} \quad (\text{i})$$

A graph of this equation is given in Figure 4.8.

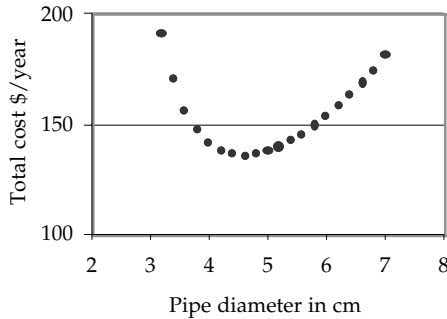


FIGURE 4.8. Graph of cost versus diameter for Example 4.4.

Differentiating with respect to D_1 , we get

$$\begin{aligned} \frac{dC_T}{dD_1} &= -5(3 \times 10^{-6})D_1^{-6} - \frac{9(-1)}{(0.12 - D_1)^2} = 0 \\ \frac{1.5 \times 10^{-5}}{D_1^6} &= \frac{9}{(0.12 - D_1)^2} \end{aligned}$$

This equation can be solved in an iterative fashion by rewriting it as

$$D_1 = (1.67 \times 10^{-6}(0.12 - D_1)^2)^{1/6}$$

We first assume a value of D_1 . Then, substituting into the right-hand side gives the value of D_1 (left hand side) to be used in a second calculation. For example,

$$D_1 = 0.01 \text{ m}; \quad (0.12 - D_1) = 0.11; \quad (1.67 \times 10^{-6}(0.12 - D_1)^2)^{1/6} = 0.052$$

Using 0.052 in the right-hand side of our equation gives $D_1 = 0.044$. Continuing, we see that the solution quickly converges to 0.046 m. The solution then is

$$\underline{D_1 = 0.046 \text{ m} = 4.6 \text{ cm}}$$

EXAMPLE 4.5. Consider the air handling duct shown in Figure 4.9. Flow enters the system at A. Some of the flow is discharged at 2 and some at 3. Our objective is to size the ductwork so that the pressure drop is minimized. That is, we desire to determine the diameters D_1 and D_2 so that the pressure losses due to friction are minimized. We must make several assumptions at this point. First, we assume that the transition between L_1 and L_2 is coincident with the diameter change. Further, we will assume a value for the friction factor in both ducts to be a constant and for purposes of illustration, equal to 0.02.

To demonstrate the calculation procedure, we take the flow rates to be $Q_1 = 0.7 \text{ m}^3/\text{s}$, and $Q_2 = 0.4 \text{ m}^3/\text{s}$. The tube lengths are $L_1 = 14 \text{ m}$ and $L_2 = 16 \text{ m}$. With air density equal to 1.2 kg/m^3 , the pressure drop due only to friction from A to B is written as

$$\Delta p = \frac{f_1 L_1}{D_1} \frac{\rho V_1^2}{2} + \frac{f_2 L_2}{D_2} \frac{\rho V_2^2}{2}$$

This is our objective function, and we seek to make it a minimum. To do so, we will have to express all the parameters on the right-hand side in terms of D_1 (or D_2) and differentiate that expression with respect to D_1 (or D_2).

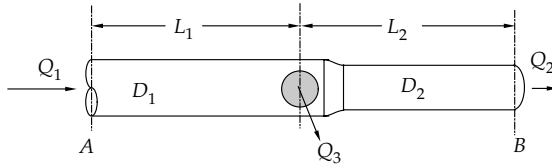


FIGURE 4.9. An air handling system.

The volume flow rates were given as $Q_1 = 0.7 \text{ m}^3/\text{s}$, and $Q_2 = 0.4 \text{ m}^3/\text{s}$. The velocity in each section of the duct may be written as

$$V_1 = \frac{Q_1}{A_1} = \frac{4Q_1}{\pi D_1^2} = \frac{4(0.7)}{\pi D_1^2} = \frac{0.891}{D_1^2}$$

$$V_2 = \frac{Q_2}{A_2} = \frac{4Q_2}{\pi D_2^2} = \frac{4(0.4)}{\pi D_2^2} = \frac{0.509}{D_2^2}$$

We can now calculate

$$V_1^2 = \frac{0.794}{D_1^4}$$

$$V_2^2 = \frac{0.259}{D_2^4}$$

Substituting into the pressure drop expression and simplifying,

$$\Delta p = \frac{0.02(14)}{D_1} \frac{1.2}{2} \frac{0.794}{D_1^4} + \frac{0.02(16)}{D_2} \frac{1.2}{2} \frac{0.259}{D_2^4}$$

$$\Delta p = \frac{0.133}{D_1^5} + \frac{0.0498}{D_2^5}$$

The derivative of this expression with respect to D_1 is

$$\frac{d\Delta p}{dD_1} = -5(0.133)D_1^{-6} - 5(0.0498)D_2^{-6} \frac{dD_2}{dD_1} = 0 \quad (i)$$

To evaluate the pressure drop and obtain a value for the diameter D_1 , we will need a second equation—a constraint equation. Specifically, we will need some relationship between the diameters D_2 and D_1 . We have already used the continuity equation when we wrote the velocities in terms of the flow rates.

We can make up one of several constraints. The duct itself is usually made of sheet metal that is cut, rolled into cylinders, and soldered or riveted together. Suppose, for example, that the sheet metal area is a constant and equal to 40 m², a constraint that could be profit motivated. The following equation becomes our constraint:

$$\pi D_1 L_1 + \pi D_2 L_2 = 40$$

Substituting known quantities,

$$\pi D_1(14) + \pi D_2(16) = 40$$

Rearranging and solving for D_2 ,

$$14D_1 + 16D_2 = 12.7$$

$$D_2 = 0.796 - 0.875D_1$$

and so

$$\frac{dD_2}{dD_1} = -0.875 \quad [= -\text{length ratio} = -\frac{L_1}{L_2}, \text{ independent of the } 40 \text{ m}^2 \text{ area}]$$

Combining with the differentiated pressure drop equation (i) and setting the result equal to zero, we have

$$\frac{d\Delta p}{dD_1} = -5(0.133)D_1^{-6} - 5(0.0498)(0.796 - 0.875D_1)^{-6} (-0.875) = 0$$

Simplifying and rearranging, we have

$$\frac{0.133}{D_1^6} = \frac{(0.0498)(0.875)}{(0.796 - 0.875D_1)^6}$$

$$\frac{D_1^6}{(0.796 - 0.875D_1)^6} = 3.05$$

Taking the sixth root,

$$\frac{D_1}{(0.796 - 0.875D_1)} = 1.204$$

$$\text{or } D_1 = 0.959 - 1.05D_1$$

Solving,

$$\underline{D_1 = 0.466 \text{ m}}$$

$$\underline{D_2 = 0.796 - 0.875(0.466) = 0.387 \text{ m}}$$

The pressure drop is calculated to be

$$\Delta p = \frac{0.133}{D_1^5} + \frac{0.0498}{D_2^5} = 11.7 \text{ Pa}$$

4.2 Economic Pipe Diameter

Engineers typically learn about piping systems in a first course in fluid mechanics. Three types of pipe flow problems are usually discussed: pressure drop Δp unknown, volume flow rate Q unknown, or inside diameter D unknown. In all cases, six variables enter the problem (L = pipe length, ϵ = surface roughness, ν = kinematic viscosity of the fluid, Δp = pressure drop, Q = volume flow rate, and D = inside diameter). In any of the three types of problems, five variables are known and the sixth one is solved for.

In a real design problem, however, the value of five variables is usually not known. Suppose a tank contains liquid, for example, that is to be pumped to a bottling machine of given capacity (flow rate specified). The length of pipe, the surface roughness, and kinematic viscosity would be known. The pressure drop allowable must be determined, and the size of the pipe must be selected. Usually, a number of different pipe sizes can be used, and each will have an associated pressure drop. Thus, with only four parameters known, additional criteria must be used to solve the problem.

It is reasonable to use cost figures in the above-cited problem as a selection guide. On the one hand, the larger the pipe diameter, the greater the initial cost, which suggests that a small diameter should be selected. On the other hand, fluid flowing through a small diameter pipe undergoes a large friction loss and thus a larger pump is required. A larger pump means greater initial and operating costs. In general, there exists a diameter that minimizes the total cost (initial cost plus operating costs) of the pump, the pipe, and the fittings. This diameter is called the **optimum economic diameter**, D_{opt} , and solving for this diameter is an optimization problem.

Here, we present what is traditionally known as the **least annual cost method** of economic pipe diameter selection, and we derive appropriate dimensionless groups that arise in this method. Results are applicable to gravity flow situations and can be used whether or not pumps are present. An equation for the optimum economic diameter D_{opt} is derived, but solving it using the classical Moody Diagram requires a trial-and-error procedure. To avert trial and error, three graphs are presented of the Darcy–Weisbach friction factor f versus $f^x \text{Re}^y$ with $(\varepsilon/D)/\text{Re}$ as an independent variable. The values used for x and y will become evident in later sections.

Analysis

The optimum economic diameter is the diameter that minimizes the *total* cost of a piping system. The total cost will consist of fixed plus operating costs. The fixed costs include those for the pipe, the fittings, the hangers or supports, the pump, and the installation. The fixed costs are a function of the size of the pipe. The operating costs include those associated with pumping power requirements, which could be in the form of electricity or engine fuel. The power in such a system is that which is needed to overcome friction losses, changes in elevation, and changes in pressure, if any. We will formulate an equation for the initial and operating costs of the pipe, fittings, installation, and pump, and express the result on a cost-per-year basis. Next we differentiate the expression with respect to diameter to obtain the desired result—minimum cost. Exactly what costs to use can vary from one formulation to another, but the method is still the same.

If we are to formulate a least annual cost analysis, then the initial cost of an entire system must first be converted to an equivalent annual cost. We can do this by assuming that the capital is borrowed from a loan institution at an annual interest rate i , and that it must be repaid or **amortized**, with m yearly payments. The annual cost (or annuity) to repay a loan of, say, \$1 over m years is given by

$$a = \frac{i}{1 - \left[\frac{1}{1 + i} \right]^m} \quad (4.1)$$

The parameter a is known as the **amortization rate**.

The initial cost C_I of a piping system (or any system) can be converted to an annual cost C_A with the following equation:

$$C_A = aC_I = \frac{iC_I}{1 - \left[\frac{1}{1 + i} \right]^m}$$

As an example, consider that we are installing a pipeline with fittings, a pump, and so on, for \$10,000. Suppose further that we fund the installation with someone else's money borrowed at 9% interest, to be repaid after seven years; that is, seven annual payments ($= m$). The annual cost is

$$C_A = \frac{0.09}{1 - \left[\frac{1}{1 + 0.09} \right]^7} 10,000 = 0.1987(10,000)$$

$$C_A = \$1987 \text{ per year}$$

The amortization rate is 0.1987. The annual cost C_A reflects not only the initial investment of \$10,000 but also the interest charges. The money that is repaid after seven years is $7(\$1,987) = \$13,909$. In the analysis that we formulate, our initial cost will include that for the pump, the piping system, fittings, supports for the pipe, as well as installation.

Table 4.1 lists pipe costs for various grades. The grades themselves are ANSI designations and refer to strength properties of the material. (For more information, see Annex A, ASME B16.47.1990.) The costs in Table 4.1 are installed costs and are expressed in dollars per foot. Figure 4.10 is a graph of the Table 4.1 cost versus pipe size data.

Data in Table 4.1 have been updated from a source document published in 1982. The costs have been converted to today's dollars (at the time of this writing) using what is known as the **consumer price index**. Based on government figures, something that costs \$100 in 1982 would cost \$244 in 2013.

For our analysis in determining optimum pipe sizes, we will be fitting equations to the data of Table 4.1. The only effect that we need to consider with regard to the consumer price index is how the cost of 12-nominal pipe has changed.

TABLE 4.1. *Installed pipeline costs for various sizes in 2013 dollars/ft. (Data taken from “Direct Determination of Optimum Economic Pipe Diameter for Non-Newtonian Fluids” by R. Darby and J. D. Melson, J of Pipelines, v. 2, (1982), pp. 11–21.)*

Nominal Diameter in Inches	ANSI Designation				
	300 #	400 #	600#	900#	1500 #
4	17	17	20	27	34
6	24	24	27	34	46
8	32	32	39	49	69
10	42	44	51	69	98
12	54	56	69	88	130
14	64	66	78	95	142
16	73	76	93	117	181
18	86	88	110	144	220
20	95	98	135	169	250
22	105	110	159	196	316
24	115	120	181	232	360
26	132	149	198	274	399
30	161	181	237	330	
32	171	191	264	382	
34	193	213	299	396	
36	210	235	316		

In order to implement a least annual cost analysis, we will need to derive a curve fit equation for the pipe cost data. As can be discerned from Figure 4.4, each curve is parabolic in shape, and thus, a curve fit equation would take the form

$$\frac{\text{Cost}}{\text{Length}} = B_0 + B_1 D + B_2 D^2$$

where the coefficients (B_0 , B_1 , and B_2) have to be determined independently for each curve. Alternatively, the same data could be plotted on a log-log graph, as shown in Figure 4.11. These curves are approximately linear. The equation of the curves shown in Figure 4.11 is

$$C_p = C_1 D^n \quad (4.2)$$

where C_p is the pipe cost in monetary units per length = MU/L (\$/ft or \$/m), C_1 is the cost of a reference size (MU/Lⁿ⁺¹), and n is the (dimensionless) exponent.

To investigate Equation 4.2 in more detail, consider that it represents a straight line on a log-log graph. Taking the natural logarithm of this equation gives

$$\ln C_p = \ln C_1 + n \ln D$$

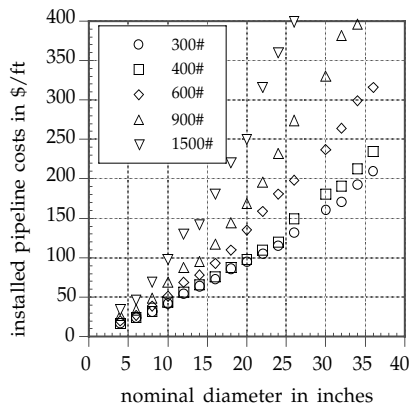


FIGURE 4.10. Installed pipeline costs as a function of nominal pipe size.

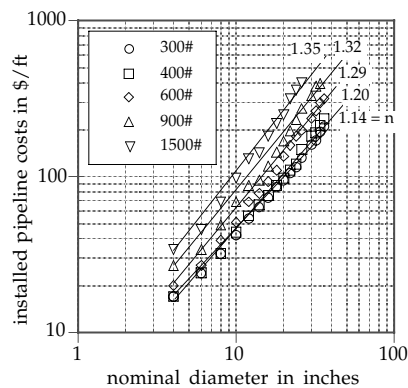


FIGURE 4.11. Log-log graph of the data of Table 4.1. (See Table 4.1 for data source.)

This equation is of the form

$$y = d + gx$$

where g is the slope of the line (corresponding to n) and d is the intercept obtained by setting x equal to 0. Now, $x = 0$ exists on a Cartesian graph, and it will correspond to $x = 1$ on a log axis [$\ln(1) = 0$]. Therefore, referring to Figure 4.11, we select C_1 in Equation 4.2 to be the cost of 12-inch pipe [12 in./ (12 in./ft)], which corresponds to a nominal diameter of 1 ft in engineering units, or to a diameter of 1 m in SI.

The value of the exponent n varies from 1.0 to about 1.4. If we apply the consumer price index calculation to the data source of Table 4.1, the slope of each line remains unchanged from year to year. On the other hand, the value of C_1 (corresponding to 12 nominal pipe) typically varies from $\$22/\text{ft}^{n+1}$ to about $\$55/\text{ft}^{n+1}$ in 1982 dollars. Converted to 2013 dollars, C_1 varies from $\$54/\text{ft}^{n+1}$ to about $\$130/\text{ft}^{n+1}$.

So with regard to a curve fit equation, we could use the parabolic form, but it is easier to use Equation 4.2, which is what we elect to do.

Next, suppose we express the cost of fittings, valves, supports, pump(s), and installation as a multiplier F of the pipe costs. We get

$$C_F = FC_P = FC_1 D^n$$

where C_F is the cost of fittings, and the like. in MU/L, and F is a multiplier that ranges typically from 6 to 7 (see Table 4.1 for reference source). The total cost (pipe, fittings, supports, installation) is the sum of Equation 4.2 and the preceding equation:

$$C_{PF} = C_P + C_F = C_1 D^n + F C_1 D^n = (1 + F) C_1 D^n \quad (4.3)$$

where C_{PF} has dimensions of MU/L, and this is the initial or first cost of the system.

The annualized amortization rate a of the cost of the system is a fraction of the cost of the pipe and fittings. The equation we wrote for the amortization rate included an interest rate i and a series of regular payments m . These may not be known for each and every case, so we will use an approximation rather than the equation given for a . We will take the amortization rate a to be the reciprocal of the expected life of the system in years. In other words, the initial cost expressed in Equation 4.3 may be converted to an annual cost by multiplying by a , and a is taken to be the reciprocal of the life of the system in years. Thus the (converted) annual cost of the pipe plus fittings is given by:

$$C_{PF} = a(1 + F) C_1 D^n$$

In addition to amortizing the initial cost, we wish to include maintenance of the installed system. The annual maintenance cost is a fraction of the cost of the pipe and fittings, denoted as b . Thus, the total amortized, installed cost of the piping system and its maintenance is

$$C_{PT} = (a + b)(1 + F) C_1 D^n \quad (4.4)$$

in which C_{PT} is the total annualized cost of the piping system in MU/(L·T) [\$/ (ft·yr) or \$/(m·yr)]. The total cost of the piping system (including a pump) is now expressed on a yearly basis.

The second factor in the total annual cost analysis is the cost of moving fluid through the pipe. This cost can reflect the cost associated with overcoming friction and/or changes in kinetic and potential energies. In the most general case, we elect to include all these factors in our model. The energy required per unit mass of fluid to pump the fluid through the pipeline is found with the energy equation written for a general system.

Consider the pump and piping system of Figure 4.12. We identify section 1 to be at the pipe inlet; section 2 is at the pipe outlet; section 3 is just upstream of the pump; and section 4 is just downstream of the pump. We write the modified Bernoulli equation (with friction) from 1 to 3 as

$$\frac{p_1 g_c}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_3 g_c}{\rho g} + \frac{V_3^2}{2g} + z_3 + \frac{fL}{D} \frac{V^2}{2g} + \sum K \frac{V^2}{2g}$$

We write the energy equation across the pump, section 3 to section 4, as

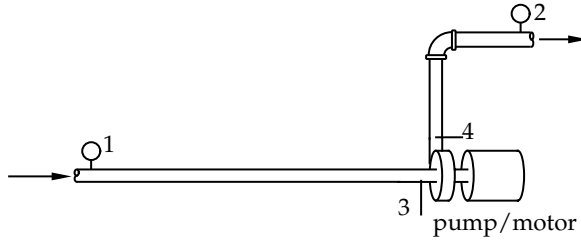


FIGURE 4.12. A generalized piping system.

$$\frac{p_3 g_c}{\rho g} + \frac{V_3^2}{2g} + z_3 = \frac{p_4 g_c}{\rho g} + \frac{V_4^2}{2g} + z_4 + \frac{g_c}{\dot{m} g} \frac{dW}{dt}$$

The modified Bernoulli equation for the pump outlet pipe, section 4 to 2, is written as

$$\frac{p_4 g_c}{\rho g} + \frac{V_4^2}{2g} + z_4 = \frac{p_2 g_c}{\rho g} + \frac{V_2^2}{2g} + z_2 + \frac{fL}{D} \frac{V^2}{2g} + \Sigma K \frac{V^2}{2g}$$

Adding the preceding equations gives for the pump and piping system combination:

$$\begin{aligned} \frac{p_1 g_c}{\rho g} + \frac{V_1^2}{2g} + z_1 = & \frac{p_2 g_c}{\rho g} + \frac{V_2^2}{2g} + z_2 + \left(\frac{fL}{D} \frac{V^2}{2g} + \Sigma K \frac{V^2}{2g} \right)_{\text{inlet pipe}} \\ & + \left(\frac{fL}{D} \frac{V^2}{2g} + \Sigma K \frac{V^2}{2g} \right)_{\text{outlet pipe}} + \frac{g_c}{\dot{m} g} \frac{dW}{dt} \end{aligned} \quad (4.5)$$

We define the **head** or **energy** H as

$$H = \left(\frac{p g_c}{\rho g} + \frac{V^2}{2g} + z \right)$$

and in terms of H , Equation 4.5 becomes

$$\begin{aligned} H_1 = H_2 + & \left(\frac{fL}{D} \frac{V^2}{2g} + \Sigma K \frac{V^2}{2g} \right)_{\text{inlet pipe}} \\ & + \left(\frac{fL}{D} \frac{V^2}{2g} + \Sigma K \frac{V^2}{2g} \right)_{\text{outlet pipe}} + \frac{g_c}{\dot{m} g} \frac{dW}{dt} \end{aligned}$$

The preceding equation can be simplified in a number of ways. For this analysis, we assume that the minor losses are either negligible or that they

can be combined in some way with other friction terms. In addition, we further assume that the entire pipeline consists of only one size of pipe. Many industrial pumps have inlet line sizes that are larger than the outlet lines, but we will not be concerned with that fact at this point. Rearranging and solving for power, we therefore obtain

$$\frac{dW}{dt} = -\dot{m} \left((H_2 - H_1) \frac{g}{g_c} + \frac{fL}{D} \frac{V^2}{2g_c} \right) \quad (4.6)$$

It is convenient to rewrite the velocity in Equation 4.6 in terms of the mass flow rate using continuity:

$$V = \frac{Q}{A} = \frac{\dot{m}}{\rho A} = \frac{4\dot{m}}{\rho \pi D^2} \quad (4.7)$$

Substituting Equation 4.7 into Equation 4.6 and simplifying, we have

$$-\frac{dW}{dt} = \dot{m} \left((H_2 - H_1) \frac{g}{g_c} + \frac{8fL\dot{m}^2}{\pi^2 \rho^2 D^5 g_c} \right) \quad (4.8)$$

Now dW/dt is the power that must be supplied to the fluid to overcome head changes and frictional effects. The actual motor size is $(dW/dt)/\eta$, where η is the pump efficiency.

The cost of operating the pump on a yearly basis is given by

$$C_{OP} = \frac{C_2 t (-dW/dt)}{\eta} \quad (4.9)$$

in which C_{OP} is the annualized cost in MU/T (\$/yr), C_2 is the cost of energy in MU/(F·L) (\$/(kW·hr)), t is the time during which the system operates per year (hr/yr), and η , as mentioned previously, is the efficiency of the pump (dimensionless).

The initial cost of the pump varies with size. The larger the pump, the greater the cost. For pumping stations placed in remote locations and used to pump fluids over many miles, initial costs can vary to \$6 × 10⁶ for an installation of 4000 horsepower. On the other hand, for a small installation of 100 HP or less, the cost is a few thousand dollars. The initial pump cost can be accounted for in this analysis in one of two ways: separate term(s) where cost is expressed as a function of diameter; or included in the pipe cost Equation 4.4 as a part of F . In this analysis, the pump cost is included in the factor F .

The total annual cost associated with the piping system (initial + maintenance + operating + pumping) with an amortization rate of a is given by the sum of Equations 4.4 and 4.9:

$$C_T = LC_{PT} + C_{OP} = (a + b)(1 + F)C_1 D^n L + C_2 t (-dW/dt) / \eta$$

Substituting Equation 4.8 for $-dW/dt$, the cost becomes

$$\begin{aligned} C_T &= LC_{PT} + C_{OP} \\ &= (a + b)(1 + F)C_1 D^n L + \frac{\dot{m} C_2 t}{\eta} (H_2 - H_1) \frac{g}{g_c} + \frac{8fL\dot{m}^3}{\pi^2 \rho^2 D^5 g_c} \frac{C_2 t}{\eta} \end{aligned} \quad (4.10)$$

The optimum economic diameter is the one that minimizes Equation 4.10 for total cost. The minimum is found by differentiating Equation 4.10 with respect to diameter (holding all other variables constant) and setting the result equal to zero:

$$\frac{\partial C_T}{\partial D} = n(a + b)(1 + F)C_1 D^{(n-1)}L - 5 \left(\frac{8fL\dot{m}^3}{\pi^2 \rho^2 D^6 g_c} \frac{C_2 t}{\eta} \right) = 0$$

This result is an example of an unconstrained optimization problem; that is, we do not need a constraint equation to solve for the optimum diameter. Rearranging and solving for diameter gives

$$D^{n+5} = \frac{40f\dot{m}^3 C_2 t}{n(a + b)(1 + F)C_1 \eta \pi^2 \rho^2 g_c} \quad (4.11)$$

$$\text{or } D_{\text{opt}} = \left[\frac{40f\dot{m}^3 C_2 t}{n(a + b)(1 + F)C_1 \eta \pi^2 \rho^2 g_c} \right]^{\frac{1}{n+5}} \quad (4.12)$$

where the parameters in Equation 4.12 are defined in Table 4.2, which also gives some typical values. Although not trivial to show, Equation 4.12 is dimensionally homogeneous.

Several important features are noticeable in Equation 4.12:

- Pipe length does not appear in the equation.
- Viscosity of the fluid does not appear but the density does. Viscosity influences the Reynolds number, which in turn affects the friction factor f .
- Diameter is unknown, and so a trial-and-error solution will be required if the Moody diagram is used because diameter is given in terms of friction factor f .
- Head loss (ΔH) does not appear in the equation.
- If there were no frictional effects (i.e., $f = 0$), an optimum diameter could not be calculated.

TABLE 4.2. *Factors in the optimum economic diameter analysis.*

Symbol	Definition	Dimensions (Units)	Typical Values
D_{opt}	the optimum economic diameter	L (ft or m)	—
$\dot{m}$	mass flow rate	M / T	—
f	friction factor	—	—
C_2	cost of energy	MU/(F·L) [\$/(kW·hr)]	\$0.05/(kW·hr) or \$0.05/(738 ft·lbf·hr)
t	time during which system operates per year	(hr/yr)	7 880 hr/yr (10% downtime)
n	exponent of D in curve fit of pipe cost data	—	1.0 to 1.4
a	amortization rate	1/T (1/yr)	1/7 to 1/20
b	yearly maintenance cost fraction	1/T (1/yr)	0.01
F	multiplier of pipe cost representing cost of fittings, pump, installation, etc.	—	6 to 7
C_1	constant in curve fit of pipe cost data	MU/L ⁿ⁺¹	\$54/ft ⁿ⁺¹ to \$130/ft ⁿ⁺¹ \$627/m ⁿ⁺¹ to \$1 570/m ⁿ⁺¹
η	efficiency of pump	—	0.6 to 0.9
ρ	density of liquid	M/L ³ (lbm/ft ³ or kg/m ³)	—

$$D_{\text{opt}} = \left[\frac{40f\dot{m}^3 C_2 t}{n(a+b)(1+F)C_1 \eta \pi^2 \rho^2 g_c} \right]^{\frac{1}{n+5}} \quad \text{Ro} = \frac{\pi \epsilon \mu g_c}{4\dot{m}}$$

$$(f(\text{Re})^{n+5})^{1/6} = \left[\frac{128}{5\pi^3 g_c^4} \frac{\dot{m}^2}{\mu^5} \left(\frac{4\dot{m}}{\pi \mu g_c} \right)^n \left(\frac{n(a+b)(1+F)C_1 \eta \rho^2}{C_2 t} \right) \right]^{1/6}$$

By appropriate manipulation of Equation 4.12, dimensionless groups can be derived to obtain a new correlation that can then be used as a graphical scaling parameter. The objective is to rid the right-hand side of Equation 4.12 of the friction factor f . The reciprocal of Equation 4.12 is

$$\frac{1}{D_{\text{opt}}} = \left[\frac{n(a+b)(1+F)C_1\eta\pi^2\rho^2g_c}{40f\dot{m}^3C_2t} \right]^{\frac{1}{n+5}}$$

Multiplying both sides by $4\dot{m}/\pi\mu g_c$ gives

$$\frac{4\dot{m}}{\pi\mu g_c D_{\text{opt}}} = \left[\frac{n(a+b)(1+F)C_1\eta\pi^2\rho^2g_c}{40f\dot{m}^3C_2t} \frac{4^{n+5}\dot{m}^{n+5}}{\pi^{n+5}\mu^{n+5}g_c^{n+5}} \right]^{\frac{1}{n+5}}$$

$$\text{or } \left(\frac{4\dot{m}}{\pi\mu g_c D_{\text{opt}}} \right)^{n+5} = \frac{256}{10\pi^3g_c^4} \frac{\dot{m}^2}{\mu^5} \left(\frac{4\dot{m}}{\pi\mu g_c} \right)^n \left(\frac{n(a+b)(1+F)C_1\eta\rho^2}{fC_2t} \right) \quad (4.13)$$

The term in parentheses in the left-hand side is recognized as the Reynolds number. Multiplying both sides by the friction factor f and taking the sixth root rids the right-hand side of friction factor f and gives

$$(f(\text{Re})^{n+5})^{1/6} = \left[\frac{128}{5\pi^3g_c^4} \frac{\dot{m}^2}{\mu^5} \left(\frac{4\dot{m}}{\pi\mu g_c} \right)^n \left(\frac{n(a+b)(1+F)C_1\eta\rho^2}{C_2t} \right) \right]^{1/6} \quad (4.14)$$

Because the friction factor f and the Reynolds number are dimensionless groups, their product is also dimensionless and can be used as a scaling parameter.

Referring to a Moody diagram, we know that ε/D is a significant group; but before it can be evaluated, diameter must be known. This difficulty can be overcome by introducing another new group, called the roughness number:

$$\text{Ro} = \frac{\varepsilon/D}{\text{Re}} = \frac{\varepsilon}{D} \left(\frac{\pi D \mu g_c}{4\dot{m}} \right)$$

$$\text{or } \text{Ro} = \frac{\pi \varepsilon \mu g_c}{4\dot{m}} \quad (4.15)$$

For the optimum economic diameter problem, it is convenient to have a

graph f versus $(f(\text{Re})^{n+5})^{1/6}$ with Ro as an independent parameter. As indicated in Equation 4.1, n is the exponent of diameter in the pipe cost expression. The exponent n varies from 1.0 to 1.4. Three graphs for the results of this formulation have therefore been developed:

1. f versus $(f(\text{Re})^6)^{1/6}$ with Ro as an independent variable ($n = 1.0$)
2. f versus $(f(\text{Re})^{6.2})^{1/6}$ with Ro as an independent variable ($n = 1.2$)
3. f versus $(f(\text{Re})^{6.4})^{1/6}$ with Ro as an independent variable ($n = 1.4$)

These three graphs were constructed by using the Chen equation (presented in Chapter 3). The Chen equation solves for friction factor f in terms of Reynolds number Re and ε/D and is valid in the transition and turbulent regimes. Reynolds number Re and ε/D values for the new graphs were selected, and friction factor f was calculated. For each Re and ε/D , one value each of f , $(f(\text{Re})^{n+5})^{1/6}$, and $\text{Ro} (= \varepsilon/D/\text{Re})$ were calculated. Successive values of Reynolds number Re and ε/D were selected in harmony so that Ro remained constant. Graphs were then prepared.

Figures 4.13, 4.14, and 4.15 are the graphs prepared as a result of the analysis. The graphs are similar in appearance; and for each, friction factor is plotted on the vertical axis. In all cases, the vertical axis varies to 0.1. The horizontal axis in Figure 4.4 ranges from 10^3 to 10^8 , while in Figures 4.5 and 4.6, it ranges from 10^3 to 10^9 . The roughness number Ro in all graphs ranges from 0 (smooth wall) to 2.0×10^{-6} . All graphs span the transition and turbulent flow regimes.

EXAMPLE 4.5. Linseed oil is to be pumped from a tank to a bottling machine. The machine can fill and cap 30 two-liter bottles in one minute. Determine the optimum size for the installation. Use the following parameters:

$$\begin{array}{ll}
 C_2 = \$0.05/(\text{kW} \cdot \text{hr}) = (\$0.05 \text{ s})/(738 \text{ ft} \cdot \text{lbf} \cdot \text{hr}) & \\
 C_1 = \$70/\text{ft}^{2.2} & t = 7000 \text{ hr/yr} \\
 F = 6.75 & n = 1.2 \\
 a = 1/(10 \text{ yr}) & b = 0.01 \\
 \eta = 75\% = 0.75 & \text{PVC schedule 40 pipe}
 \end{array}$$

Solution: We now work toward calculating the optimum diameter using Equation 4.12 and the Moody Diagram. We begin by obtaining properties from the appropriate table:

$$\begin{array}{lll}
 \text{linseed oil} & \rho = 0.93(62.4) \text{ lbm/ft}^3 & \mu = 69 \times 10^{-5} \text{ lbf} \cdot \text{s/ft}^2 \\
 & & [\text{App. Table B.1}]
 \end{array}$$

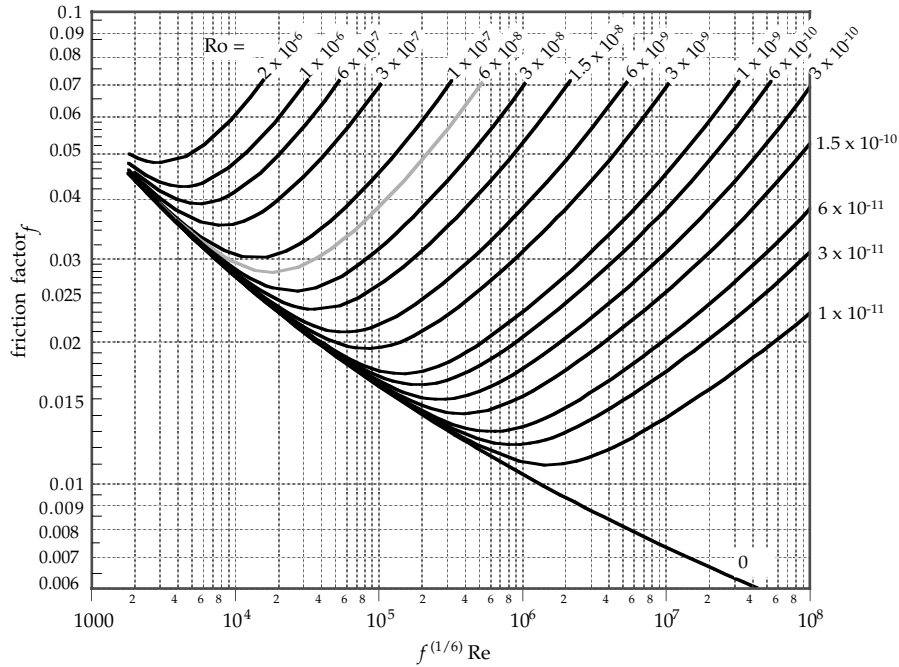


FIGURE 4.13. Friction factor graph for $n = 1$.

The volume and mass flow rates are

$$Q = 30(2) \text{ l/min} = 60 \text{ l/min} = 1 \text{ l/s} = 3.53 \times 10^{-2} \text{ ft}^3/\text{s}$$

$$\dot{m} = \rho Q = 0.93(62.4)(3.53 \times 10^{-2}) = 2.05 \text{ lbm/s}$$

Substituting into Equation 4.12 gives

$$D_{\text{opt}} = \left[\frac{40f\dot{m}^3 C_2 t}{n(a+b)(1+F)C_1 \eta \pi^2 \rho^2 g_c} \right]^{\frac{1}{n+5}}$$

$$D_{\text{opt}} = \left[\frac{40f(2.05)^3(0.05/738)(7000)}{1.2(1/10 + 0.01)(1 + 6.75)(70)(0.75)\pi^2 (0.93(62.4))^2(32.2)} \right]^{\frac{1}{6.2}}$$

(Note that the cost of \$0.05 **must** be divided by 738 in English units or by 1 000 in SI.) Solving,

$$D_{\text{opt}} = 0.127f^{0.161} \quad (\text{i})$$

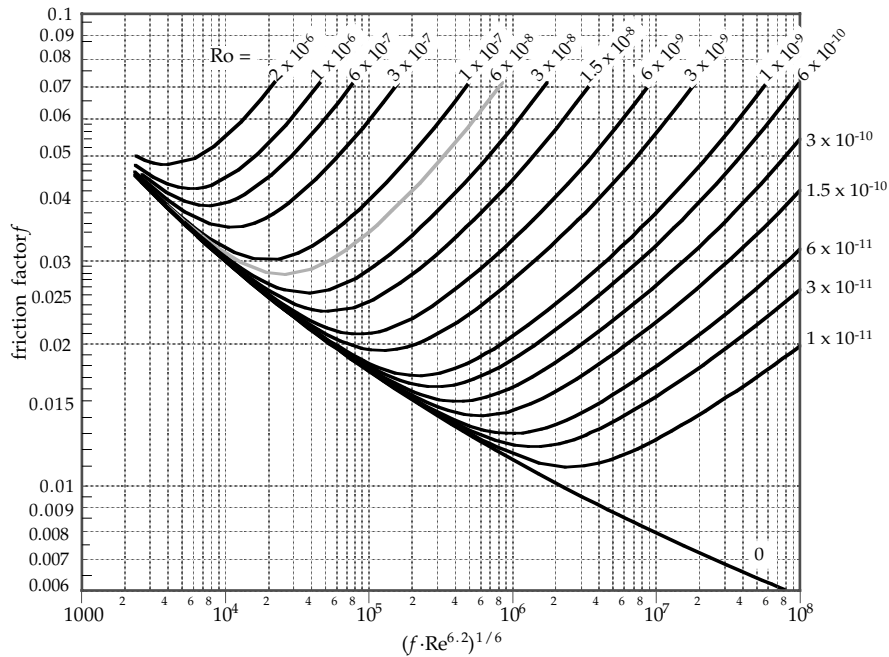


FIGURE 4.14. Friction factor graph for $n = 1.2$.

The Reynolds number of the flow is

$$\text{Re} = \frac{\rho V D}{\mu g_c} = \frac{4 \rho Q}{\pi D \mu g_c}$$

Substituting,

$$\text{Re} = \frac{4(2.05)}{\pi D (69 \times 10^{-5}(32.2))} = \frac{117}{D}$$

For PVC pipe, we use the “smooth” curve ($\varepsilon \approx 0$) on the Moody diagram. Ordinarily, we now select values of diameter, calculate a Reynolds number, determine friction factor, and find a new value for diameter. Here, however, because the numerator in the Reynolds number equation (117) is less than 2 100, it would be prudent to begin by assuming that laminar flow exists. For laminar flow of a Newtonian fluid in a circular duct, we have

$$f = \frac{64}{\text{Re}} = \frac{64D}{117} = 0.545D$$

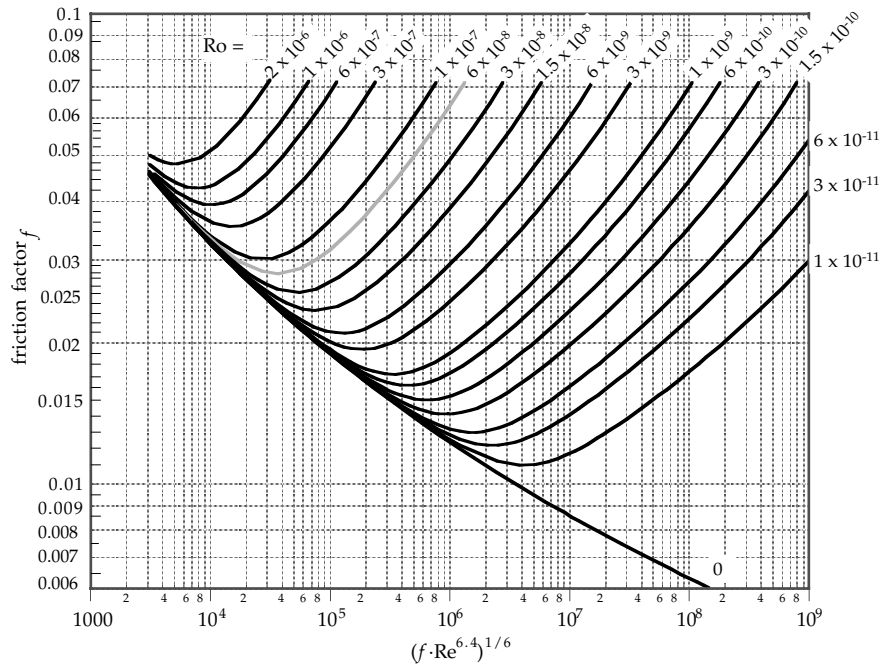


FIGURE 4.15. Friction factor graph for $n = 1.4$.

Substituting into Equation i found earlier, we get

$$D_{\text{opt}} = 0.127f^{0.161} = 0.127(0.545D_{\text{opt}})^{0.161} = 0.115D_{\text{opt}}^{0.161}$$

Solving yields

$$D_{\text{opt}}^{(1-0.161)} = 0.115$$

$$D_{\text{opt}} = 0.076 \text{ ft} = 0.91 \text{ in.}$$

As a check on the laminar flow assumption, we calculate

$$\text{Re} = \frac{117}{D} = \frac{117}{0.076} = 1540$$

$$f = \frac{64}{\text{Re}} = \frac{64}{1540} = 0.042$$

and

$$D_{\text{opt}} = 0.127f^{0.161} = 0.127(0.042)^{0.161} = 0.076 \text{ ft}$$

Examination of Appendix Table D.1 shows that this diameter falls between 3/4 nominal and 1 nominal, both schedule 40 sizes; because nothing more specific was requested, we use schedule 40. The smaller size will not deliver the required flow rate without an increase in the pressure drop. The larger size will deliver the specified flow rate and will require less power. So the correct size to use is

$$D_{\text{opt}} = 1\text{-nominal schedule 40 pipe} \quad (D = 0.076 \text{ ft})$$

EXAMPLE 4.6. Water is to be conveyed at a flow rate of 3.8 l/s in a commercial steel pipeline. Determine the optimum economic pipe size for the installation given that:

$$C_2 = \$0.04/(\text{kW}\cdot\text{hr}) = \$0.04/(1\,000 \text{ W}\cdot\text{hr})$$

$$C_1 = \$700/\text{m}^{2.2}$$

$$t = 6\,000 \text{ hr/yr}$$

$$F = 7.0$$

$$n = 1.2$$

$$a = 1/(7 \text{ yr})$$

$$b = 0.01$$

$$\eta = 75\% = 0.75$$

Solution: In the last example, we worked with Equation 4.12 directly in order to find the optimum diameter D_{opt} . The same procedure will be used here. The appropriate economic diameter selection graph will also be used in order to illustrate the method and to compare the results. We begin by obtaining properties from the appropriate tables:

$$\text{water} \quad \rho = 1\,000 \text{ kg/m}^3 \quad \mu = 0.89 \times 10^{-3} \text{ N}\cdot\text{s/m}^2 \quad [\text{Appendix Table B.1}]$$

$$\text{commercial steel} \quad \varepsilon = 0.0046 \text{ cm} = 0.000046 \text{ m} \quad [\text{Table 3.1}]$$

Next, we substitute all known parameters into Equation 4.12 to formulate the trial-and-error procedure. The volume flow rate is

$$Q = 3.8 \text{ l/s} = 0.0038 \text{ m}^3/\text{s}$$

The mass flow rate is calculated as

$$\dot{m} = \rho Q = 1\,000(0.0038) = 3.8 \text{ kg/s}$$

Equation 4.12 is

$$D_{\text{opt}} = \left[\frac{40f\dot{m}^3 C_2 t}{n(a+b)(1+F)C_1 \eta \pi^2 \rho^2 g_c} \right]^{\frac{1}{n+5}}$$

Substituting gives

$$D_{\text{opt}} = \left[\frac{40f(3.8)^3(0.04/1\,000)(6\,000)}{1.2(1/7 + 0.01)(1 + 7.0)(700)(0.75)\pi^2(1\,000)^2} \right]^{1/6.2}$$

$$\text{or } D_{\text{opt}} = 0.070f^{0.161} \quad (\text{i})$$

The Reynolds number of the flow is

$$\begin{aligned} \text{Re} &= \frac{\rho V D}{\mu g_c} = \frac{4\rho Q}{\pi D \mu g_c} = \frac{4\dot{m}}{\pi D \mu g_c} \\ \text{Re} &= \frac{4(3.8)}{\pi D(0.89 \times 10^{-3})} = \frac{5.44 \times 10^3}{D} \end{aligned}$$

We start by assuming a diameter selected from a table of pipe sizes (such as Appendix Table D.1) or a diameter selected at random. Here we choose the latter.

1st trial: $D = 4$ cm; then

$$\left. \begin{aligned} \text{Re} &= 5.44 \times 10^3 / 0.04 = 1.36 \times 10^5 \\ \frac{\varepsilon}{D} &= \frac{0.0046}{4} = 0.00115 \end{aligned} \right\} f = 0.022 \text{ (Moody diagram)}$$

Using Equation i of this example,

$$D_{\text{opt}} = 0.070(0.022)^{0.161} = 0.0378 \text{ m}$$

2nd trial: $D = 0.0378$ m; then

$$\left. \begin{aligned} \text{Re} &= 5.44 \times 10^3 / 0.0378 = 1.44 \times 10^5 \\ \frac{\varepsilon}{D} &= \frac{0.0046}{3.78} = 0.0012 \end{aligned} \right\} f \approx 0.022 \text{ (Moody Diagram)}$$

which equals our assumed value. So

$$\underline{D_{\text{opt}} = 0.0378 \text{ m}} \quad (\text{close enough})$$

In order to avert trial and error, we can use Figure 4.5 to solve this problem. We now work toward calculating $(f(\text{Re})^{n+5})^{1/6} = (f(\text{Re})^{6.2})^{1/6}$ using Equation

4.14. Substituting into Equation 4.14 gives

$$(f(\text{Re})^{n+5})^{1/6} = \left[\frac{128}{5\pi^3 g_c^4} \frac{\dot{m}^2}{\mu^5} \left(\frac{4\dot{m}}{\pi\mu g_c} \right)^n \left(\frac{n(a+b)(1+F)C_1\eta\rho^2}{C_2 t} \right) \right]^{1/6} \quad (4.14)$$

$$(f(\text{Re})^{6.2})^{1/6} = \left[\frac{128}{5\pi^3 (1)^4} \frac{3.8^2}{(0.89 \times 10^{-3})^5} \left(\frac{4(3.8)}{\pi(0.89 \times 10^{-3})(1)} \right)^{1.2} \right. \\ \left. \times \left(\frac{1.2(1/7 + 0.01)(1+7)(700)(0.75)(1\,000)^2}{(0.04/1\,000)(6\,000)} \right) \right]^{1/6}$$

Solving,

$$(f(\text{Re})^{6.2})^{1/6} = 1.13 \times 10^5$$

Also, the roughness number is calculated to be

$$\text{Ro} = \frac{\pi\epsilon\mu g_c}{4\dot{m}} = \frac{\pi(0.000\,046)(0.89 \times 10^{-3})(1)}{4(3.8)} = 8.46 \times 10^{-9}$$

With these values,

$$\left. \begin{aligned} (f(\text{Re})^{6.2})^{1/6} &= 1.13 \times 10^5 \\ \text{Ro} &= 8.46 \times 10^{-9} \end{aligned} \right\} \quad f \approx 0.022 \quad (\text{Figure 4.5})$$

Therefore,

$$(f(\text{Re})^{6.2})^{1/6} = (0.022(\text{Re})^{6.2})^{1/6} = 1.13 \times 10^5$$

and the Reynolds number is calculated to be

$$\text{Re} = \left(\frac{(1.13 \times 10^5)^6}{0.022} \right)^{1/6.2} = 1.44 \times 10^5$$

From the definition of Reynolds number,

$$\text{Re} = \frac{4\dot{m}}{\pi D \mu g_c}$$

we write

$$D = \frac{4\dot{m}}{\pi \text{Re} \mu g_c} = \frac{4(3.8)}{\pi(1.44 \times 10^5)(0.89 \times 10^{-3})(1)}$$

or $D = 3.78 \times 10^{-2} \text{ m} = 3.78 \text{ cm}$

Using Equation 4.12 for D_{opt} , we obtained 3.78 cm. Any discrepancy that might have resulted in these two results is due to roundoff errors and to errors in reading the graphs. Use of the dimensionless graphs has now been illustrated.

EXAMPLE 4.7. Use the preceding result (3.78 cm) in order to select a schedule 40 pipe size for the application given in the problem statement. Determine the pipe size that should be used.

Solution: Referring to a table of pipe sizes (Appendix Table D.1), it is apparent that 3.78 cm does not appear explicitly as a schedule 40 pipe size. The required diameter falls within the following range of diameters:

$1\frac{1}{4}$ -nominal schedule 40	$D = 3.504 \text{ cm}$
$1\frac{1}{2}$ -nominal schedule 40	$D = 4.09 \text{ cm}$

At first glance, it would appear that $1\frac{1}{4}$ -nominal schedule 40 is the size to use, because 3.504 cm is closer to the results obtained. To be sure, however, recall that we are seeking the diameter that corresponds to minimum cost. So we will calculate the actual cost using Equation 4.10 which is

$$\begin{aligned} C_T &= LC_{PT} + C_{OP} \\ &= (a + b)(1 + F)C_1 D^n L + \frac{\dot{m} C_2 t}{\eta} (H_2 - H_1) \frac{g}{g_c} + \frac{8fL\dot{m}^3}{\pi^2 \rho^2 D^5 g_c} \frac{C_2 t}{\eta} \end{aligned}$$

At this point $H_2 - H_1$ is unknown, so cost calculated with the preceding equation will have to be done assuming that $H_2 - H_1 = 0$. Furthermore, length is unspecified, so cost per unit length will be evaluated for the two pipe sizes. Equation 4.10 becomes

$$\frac{C_T}{L} = (a + b)(1 + F)C_1 D^n + \frac{8f\dot{m}^3}{\pi^2 \rho^2 D^5 g_c} \frac{C_2 t}{\eta}$$

For $1\frac{1}{4}$ -nominal schedule 40 pipe,

$$\frac{C_T}{L} = (1/7 + 0.01)(1 + 7)(700)(0.035\ 04)^{1.2} + \frac{8(0.023)(3.8)^3}{\pi^2(1\ 000)^2(0.035\ 04)^5(1)} \frac{(0.04/1\ 000)(6\ 000)}{0.75}$$

$$\text{or } \frac{C_T}{L} = \$21.26/(\text{yr} \cdot \text{m of pipe}) \quad \left(\begin{array}{c} 1^{1/4}\text{-nominal schedule 40} \\ D = 3.504\ \text{cm} \end{array} \right)$$

For $1^{1/2}$ -nominal schedule 40 pipe,

$$\frac{C_T}{L} = (1/7 + 0.01)(1 + 7)(700)(0.040\ 9)^{1.2} + \frac{8(0.023)(3.8)^3}{\pi^2(1\ 000)^2(0.040\ 9)^5(1)} \frac{(0.04/1\ 000)(6\ 000)}{0.75}$$

$$\text{or } \frac{C_T}{L} = \$21.20/(\text{yr} \cdot \text{m of pipe}) \quad \left(\begin{array}{c} 1^{1/2}\text{-nominal schedule 40} \\ D = 4.09\ \text{cm} \end{array} \right)$$

Comparing the two results leads to the conclusion that the least cost pipe is

$$1^{1/2}\text{-nominal schedule 40} \quad D = 4.09\ \text{cm}$$

The preceding calculations were made assuming that friction factor is the same for both pipe sizes ($f = 0.022$ from the last example) conveying the same flow rate. Calculations made using $1^{1/4}$ - and $1^{1/2}$ -nominal schedule 40 pipe yielded the following:

Nominal Size	ID	Re	ϵ/D	f
$1^{1/4}$ -nom sch 40	0.035 04 m	1.55×10^4	0.001 31	0.023
$1^{1/2}$ -nom sch 40	0.040 9 m	1.33×10^5	0.001 12	0.022

Thus friction factor remains nearly constant over the range of interest. In fact, for most calculations of this type, friction factor varies only from 5% to 30% for all cases, even for non-Newtonian fluids.

Figure 4.16 is a graph of the cost data in this example, provided to illustrate the behavior of the cost equations. Diameter, ranging from 0.02 to 0.1 m, appears on the horizontal axis. Cost per length (per year) is on the vertical axis and three curves are shown. The pipe cost curve is calculated using the data of the example with

$$C_{PT} = (a + b)(1 + F)C_1 D^n = (1/7 + 0.01)(1 + 7)(700)D^{1.2} = 856D^{1.2}$$

The operating cost curve is calculated (assuming $H_2 - H_1 = 0$ and $f = 0.022$) as

$$\frac{C_{OP}}{L} = \frac{8fm^3}{\pi^2 \rho^2 D^5 g_c} \frac{C_2 t}{\eta} = \frac{8(0.022)(3.8)^3}{\pi^2 (1\,000)^2 D^5 (1)} \frac{(0.04/1\,000)(6\,000)}{0.75}$$

$$\text{or } \frac{C_{OP}}{L} = \frac{3.12 \times 10^{-7}}{D^5}$$

The total cost curve is determined by summing C_{PT} and C_{OP}/L at each diameter. As seen in the figure, the total cost is a minimum at $D \approx 0.037$ m.

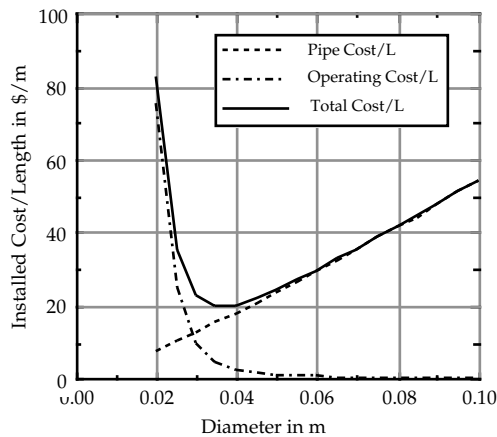


FIGURE 4.16. Graph of the cost data of the example.

In this example, the data gave optimum diameter results that were close to both pipe sizes. A slight change in reading the Moody diagram (or in one of the given parameters) could have easily pointed to using the smaller diameter. For example, if $t = 4\,000$ rather than $6\,000$ hr/yr, then the lower cost would have been the $1\frac{1}{4}$ pipe.

It should be mentioned that operating costs (costs to run pumps, compressors, etc.) are never expected to decrease. When operating costs increase, the total cost curve shifts to the right; that is, the minimum cost will be associated with a larger diameter. Thus, as a rule of thumb, when the optimum diameter falls between two pipe sizes, it is prudent to select the larger size.

The preceding equations and derivations for determining the optimum economic diameter have two obvious shortcomings: the problem of minor losses; and the difficulty in applying the results to noncircular cross

sections. These shortcomings can be easily overcome in both cases with a slightly different approach to the problem. For a given set of parameters, the optimum economic diameter is first calculated for a straight run of a constant-diameter circular pipe. When diameter is known, cross-sectional area is calculated and divided into the flow rate. The result is what is known as the **optimum economic velocity**. The optimum economic velocity is then used as necessary to size the dimensions of a noncircular duct, and fittings can be added to the pipeline as appropriate.

For example, the data of the previous two examples can be used to calculate the optimum economic velocity; we have a water density of 1 000 kg/m³, a mass flow rate of 3.8 kg/s, and a diameter of 4.09 cm. The optimum velocity is

$$V_{\text{opt}} = \frac{\dot{m}}{\rho A} = \frac{4\dot{m}}{\rho \pi D^2} = \frac{4(3.8)}{1\,000\pi(0.0409)^2} = 2.9 \text{ m/s}$$

So for the conditions stated, if the water velocity is maintained at or near 2.9 m/s through fittings or through noncircular cross sections, the minimum cost requirements will be met.

4.3 Equivalent Length of Fittings

As seen in the last chapter, minor losses can consume significant amounts of energy in the form of a pressure loss when the length of pipe is relatively short. Also, recall that the inclusion of minor losses in the modified Bernoulli equation can make the iterative (or trial-and-error) type problems exceedingly less popular than if minor losses could be ignored. Consequently, efforts have been made to represent minor losses in a different way using the concept of what is called an **equivalent length**.

Consider for the moment the modified Bernoulli equation:

$$\frac{p_1 g_c}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2 g_c}{\rho g} + \frac{V_2^2}{2g} + z_2 + \sum \frac{fL}{D_h} \frac{V^2}{2g} + \sum K \frac{V^2}{2g} \quad (4.16)$$

The friction and minor-loss terms are

$$\frac{fL}{D_h} \frac{V^2}{2g} + \sum K \frac{V^2}{2g} = \left(\frac{fL}{D_h} + \sum K \right) \frac{V^2}{2g}$$

The concept of equivalent length allows us to replace the minor-loss term with

$$\Sigma K = \frac{fL_{eq}}{D_h} \quad (4.17)$$

where f is the friction factor that applies to the entire pipe, D is the pipe diameter (characteristic length), and L_{eq} is the equivalent length. Physically, we are calculating the length of pipe (of the original material, size, and schedule) that we can “replace” the fitting(s) with to obtain the same pressure loss. The following example shows how to calculate equivalent length from data on loss coefficient K .

EXAMPLE 4.8. A horizontal pipeline made of 12-nominal schedule 40 commercial steel pipe of length 180 ft conveys ethyl alcohol at a rate of 750 gpm. The pipeline contains two 45° elbows and two 90° elbows. Determine the equivalent length of the fittings.

Solution: We begin by obtaining properties from the appropriate tables:

$$\text{ethyl alcohol} \quad \rho = 0.787(62.4) \text{ lbm/ft}^3 \quad \mu = 2.29 \times 10^{-5} \text{ lbf}\cdot\text{s/ft}^2$$

[App. Table B.1]

$$\text{12-nom sch 40} \quad D = 0.9948 \text{ ft} \quad A = 0.773 \text{ ft}^2$$

[App. Table D.1]

$$\text{commercial steel} \quad \varepsilon = 0.00015 \text{ ft}$$

[Table 3.1]

Flow velocity $V = Q/A$:

$$Q = 750 \text{ gpm} = 1.67 \text{ ft}^3/\text{s} \quad V = \frac{1.67}{0.773} = 2.16 \text{ ft/s}$$

Reynolds number $Re = \rho VD/\mu g_c$:

$$Re = \frac{0.787(62.4)(2.16)(0.9948)}{2.29 \times 10^{-5}(32.2)}$$

or

$$\left. \begin{array}{l} Re = 1.43 \times 10^5 \\ \frac{\varepsilon}{D} = \frac{0.00015}{0.9948} = 0.00015 \end{array} \right\} \quad f = 0.018 \quad (\text{Moody Diagram})$$

Minor losses:

$$\Sigma K = 2K_{45^\circ \text{ elbow}} + 2K_{90^\circ \text{ elbow}} = 2(0.17) + 2(0.22) = 0.78$$

At this point, we replace the minor-loss value (0.78) and use Equation 4.17 to solve for equivalent length:

$$\sum K = \frac{fL_{\text{eq}}}{D_h}$$

$$\text{or } 0.78 = \frac{0.018L_{\text{eq}}}{0.9948}$$

where we use the friction factor and diameter for the pipe itself. Solving for equivalent length, we get

$$\underline{L_{\text{eq}} = 43.1 \text{ ft}}$$

Thus, if we were to replace the two 45° elbows and the two 90° elbows with a 43.1-ft length of 12-nominal schedule 40 commercial steel pipe (to make a straight run of length $180 + 43.1 = 223.1$ ft), we would obtain the same pressure loss as would be obtained in the original configuration with the fittings. See Figure 4.17.

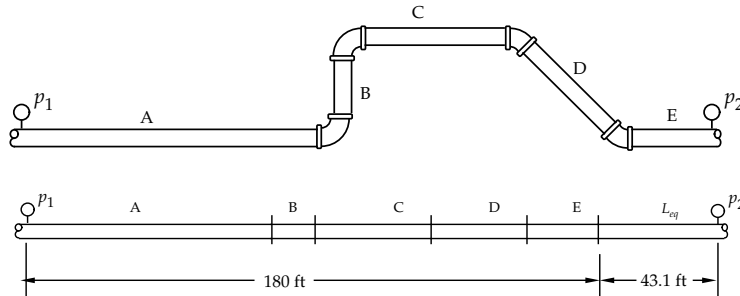


FIGURE 4.17. The concept of equivalent length applied to Example 4.6.

Some minor-loss tables provide equivalent length data rather than K values. It would not be uncommon, for example, to find that a 90° elbow has an equivalent length of 30 diameters ($= 30D$) as its loss factor.

Table 4.3 provides data for several fittings in the form of a loss coefficient K and a length-to-diameter ratio. Using values from the table is relatively straightforward. In the modified Bernoulli equation the minor-loss term $\sum K$ would be replaced with fL_{eq}/D , where f is the same friction factor as in the pipeline and L_{eq}/D would be obtained from the table.

TABLE 4.3. Loss coefficient K and equivalent length-to-diameter ratio L_{eq}/D for various fittings.

Fitting	Loss Coefficient K	Equivalent Length-to-Diameter Ratio L_{eq}/D
Reentrant inlet	1.0	
Basket strainer	1.3	
Foot valve	0.8	
90° elbow, threaded		
regular	1.4	30
long radius	0.75	20
90° elbow, flanged		
regular	0.31	
long radius	0.22	
45° elbow, threaded, regular	0.35	16
45° elbow, flanged, long radius	0.17	
Return bend, threaded, regular	1.5	50
Return bend, flanged		
regular	0.30	
long radius	0.20	
T-joint, threaded		
through flow	0.9	20
branch flow	1.9	60
T-joint, flanged		
through flow	0.14	
branch flow	0.69	
Coupling or union	0.08	
Globe valve, fully open	10.0	340
Gate valve, fraction open:		
1/4	17.0	900
1/2	2.06	160
3/4	0.26	35
fully open	0.15	13
Angle valve	2.0	145
Ball valve, fully open	0.05	
Butterfly valve, fully open		40
Check valves		
swing type	2.5	135
ball type	70.0	
lift type	12.0	
Outlet		1.0

Sources: Loss coefficient data from pages 77 & 78 of the *Engineering Data Book*, 2nd ed., ©1990 by The Hydraulic Institute. Reprinted by permission. Equivalent length/diameter ratio data obtained from "Technical Paper No. 410—Flow of Fluids," 11th printing, © Crane Co.

EXAMPLE 4.9. Propyl alcohol flows at a rate of $0.01 \text{ m}^3/\text{s}$ through a 2-nominal schedule 40 wrought iron pipe. The pipe is laid out horizontally and is 250 m long. It contains four regular, threaded 90° elbows and a globe valve. Calculate the pressure drop of the propyl alcohol using loss coefficients, and again using the equivalent length.

Solution: From the property tables, we read

$$\begin{array}{lll} \text{propyl alcohol} & \rho = 0.802(1\,000) \text{ kg/m}^3 & \mu = 1.92 \times 10^{-3} \text{ N}\cdot\text{s/m}^2 \\ & & [\text{App. Table B.1}] \\ \text{2-nom sch 40} & \text{ID} = D = 5.252 \text{ cm} & A = 21.66 \text{ cm}^2 \quad [\text{App. Table D.1}] \\ \text{wrought iron} & \varepsilon = 0.0046 \text{ cm} & [\text{Table 3.1}] \end{array}$$

The continuity equation for incompressible steady flow through the pipe is

$$A_1 V_1 = A_2 V_2$$

Because $A_1 = A_2$, then $V_1 = V_2$. The Bernoulli equation applies:

$$\frac{p_1 g_c}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2 g_c}{\rho g} + \frac{V_2^2}{2g} + z_2 + \frac{fL}{D_h} \frac{V^2}{2g} + \sum K \frac{V^2}{2g}$$

where points 1 and 2 are $L = 250$ m apart, $z_1 = z_2$ for a horizontal pipe, and $p_1 - p_2$ is sought. The above equation reduces to

$$p_1 - p_2 = \frac{fL}{D_h} \frac{\rho V^2}{2g_c} + \sum K \frac{V^2}{2g}$$

The flow velocity is

$$V = \frac{Q}{A} = \frac{0.01}{21.66 \times 10^{-4}} = 4.62 \text{ m/s}$$

The Reynolds number is calculated to be

$$\text{Re} = \frac{\rho V D}{\mu g_c} = \frac{0.802(1\,000)(4.62)(0.052\,52)}{0.53 \times 10^{-3}} = 1.01 \times 10^5$$

The flow is therefore turbulent. Thus,

$$\left. \begin{array}{l} \text{Re} = 1.01 \times 10^5 \\ \text{Also } \frac{\varepsilon}{D} = \frac{0.0046}{5.252} = 0.00088 \end{array} \right\} f = 0.022 \quad (\text{Figure 3.3})$$

For 4 elbows and 1 globe valve, we read values from Table 4.3:

$$K = 4(1.4) + 10 = 15.6$$

and

$$\frac{L_{eq}}{D} = 4(30) + 340 = 460$$

Using the K values, the pressure loss is

$$p_1 - p_2 = \left(\frac{fL}{D_h} + K \right) \frac{\rho V^2}{2g_c} = \left(\frac{0.022(250)}{0.05252} + 15.6 \right) \frac{0.802(1000)(4.62)^2}{2}$$

$$\underline{p_1 - p_2 = 1\,022\,912 \text{ N/m}^2 = 1\,022 \text{ kPa}} \quad (\text{Using } K)$$

With the concept of equivalent length, we write

$$p_1 - p_2 = \left(\frac{fL}{D_h} + \frac{fL_{eq}}{D_h} \right) \frac{\rho V^2}{2g_c}$$

$$p_1 - p_2 = \left(\frac{0.022(250)}{0.05252} + 0.022(460) \right) \frac{0.802(1000)(4.62)^2}{2}$$

$$\underline{p_1 - p_2 = 975\,594 \text{ N/m}^2 = 976 \text{ kPa}} \quad (\text{Using } L_{eq}/D)$$

4.4 Graphical Symbols for Piping Systems

Because of the large number of fittings available for piping systems and the various ways of joining pipes to fittings, standards for representing piping systems have been developed. The American National Standards Institute (ANSI) provides charts (ANSI Z32.2.3) showing graphical symbols that are accepted standards in industry. Table 4.4 shows representations of some common fittings and of the usual attachment methods.

Table 4.4 shows graphic symbols used in single-line drawings of piping systems. There are established conventions for double-line drawings as well. For example, the piping system sketched in Figure 4.18 shows three drawings of the same system: a double line; a single-line representation, and an isometric drawing. The drawings indicate that flanged fittings are used. In an isometric drawing, three-dimensional effects can be more clearly shown, although this advantage is not apparent in Figure 4.18.

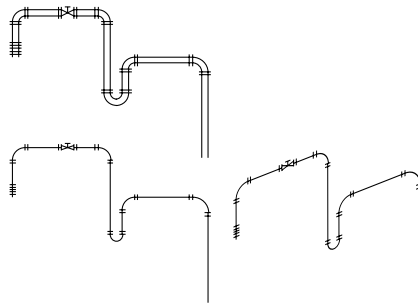


FIGURE 4.18. *Methods for drawing a piping system.*

Figure 4.19 shows a piping system that has pipes and fittings that are not all in one plane. Plan and profile views of the piping system are provided. These two views alone do not seem to be adequate in representing the system. Consequently, an isometric representation is provided in addition, and it definitely provides a better picture. Note the compass direction provided in the isometric. Workers can use a properly labeled isometric (including length of each run of pipe, fitting descriptions, installation notes, etc.) to piece together the piping system. A detailed and well-labeled isometric drawing is called a **spool drawing**.

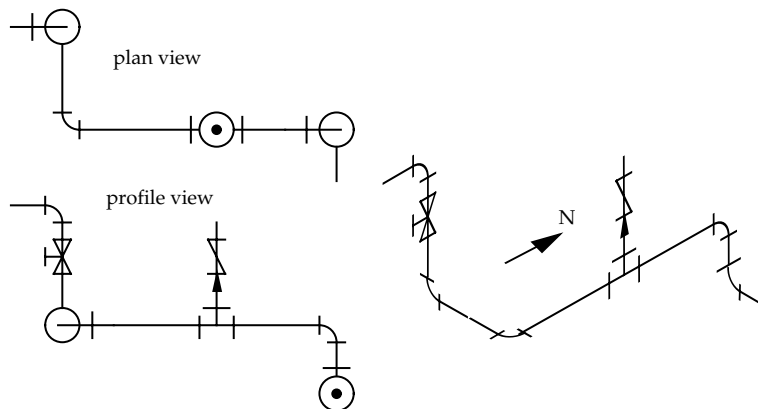


FIGURE 4.19. *Schematic representation of a piping system.*

4.5 System Behavior

It is often necessary to know how the flow rate through a given piping system varies with the pressure drop or equivalent head loss. When the head loss is graphed versus flow rate, we have what is called a **system curve**. A system curve is useful for predicting the off-design behavior of the system or, in many instances, for sizing a pump.

TABLE 4.4. *Graphical symbols for piping system.*
(Condensed from ANSI Z32.2.3.)

	threaded	flanged	bell & spigot	welded	soldered
joint					
elbow					
long radius elbow					
45° elbow					
reducing elbow					
union					
elbow facing up					
elbow facing down					
T-joint					
T-facing up					
T-facing down					
concentric reducer					
eccentric reducer					
gate valve					
globe valve					
check valve					
safety valve					

To obtain the customary form for the system curve, we first refer to the modified Bernoulli equation:

$$\frac{p_1 g_c}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2 g_c}{\rho g} + \frac{V_2^2}{2g} + z_2 + \sum \frac{fL}{D_h} \frac{V^2}{2g} + \sum K \frac{V^2}{2g}$$

Recall the definition of head, or energy H , at any section:

$$H = \frac{p g_c}{\rho g} + \frac{V^2}{2g} + z$$

where the dimension of H is L (ft or m). In terms of H , the modified Bernoulli equation for a piping system of constant diameter becomes

$$H_1 - H_2 = \left(\sum \frac{fL}{D_h} + \sum K \right) \frac{V^2}{2g} \quad (4.18)$$

Equation 4.18 contains velocity, and our interest is in the volume flow rate. We therefore substitute for velocity from the continuity equation

$$V = \frac{Q}{A} = \frac{4Q}{\pi D^2}$$

to obtain

$$\Delta H = H_1 - H_2 = \left(\sum \frac{fL}{D} + \sum K \right) \frac{16Q^2}{2\pi^2 D^4 g}$$

or

$$\Delta H = Q^2 \left(\frac{8(\sum fL/D + \sum K)}{\pi^2 D^4 g} \right) \quad (4.19)$$

This is the equation of a parabola (ΔH versus Q) and can be graphed for any system in which diameter is known or has been selected as a trial value.

EXAMPLE 4.10. Figure 4.20 shows a piping system made of 3-nominal schedule 40 PVC pipe that conveys water from a tank. The tank level is variable, and so it is desired to have information on how the flow rate will vary through the system. It is proposed that the tank be replaced with a pump, and before such a decision is acted upon, a system curve must be drawn. Generate a system curve of ΔH versus Q for the setup shown assuming the tank liquid level z can vary from 1 to 8 ft. The pipe length is 45 ft, and the distance from pipe exit to tank bottom is 3 ft.

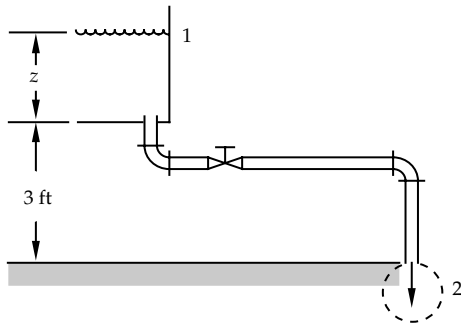


FIGURE 4.20. The piping system of Example 4.10.

Solution: The drawing indicates that the fittings are threaded. (With PVC, the fittings could be attached to the pipe with an adhesive, and in such a case, minor losses are the same as for flanged fittings.) The control volume we select includes all the water in the tank and in the piping system. Notice that at the exit, the area A_2 approaches infinity, and correspondingly: the pressure there is atmospheric, the velocity is zero, and we must include a minor loss at the exit.

We proceed in the usual way, by first obtaining properties:

water	$\rho = 62.4 \text{ lbm/ft}^3$	$\mu = 1.9 \times 10^{-5} \text{ lbf}\cdot\text{s/ft}^2$
		[Appendix Table B.1]

3-nom sch 40	$ID = 0.2557 \text{ ft}$	$A = 0.05134 \text{ ft}^2$
		[Appendix Table D.1]

PVC	$\varepsilon = \text{"smooth"} \approx 0$

Modified Bernoulli equation:

$$\frac{p_1 g_c}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2 g_c}{\rho g} + \frac{V_2^2}{2g} + z_2 + \frac{fL}{D_h} \frac{V^2}{2g} + \Sigma K \frac{V^2}{2g}$$

or

$$H_1 = H_2 + Q^2 \left(\frac{8(fL/D + \Sigma K)}{\pi^2 D^4 g} \right) \quad (\text{i})$$

Property Evaluation:

$$p_1 = p_2 = p_{\text{atm}} = 0; \quad V_1 = V_2 \approx 0 \text{ compared to the velocity in the pipe}$$

$$z_1 = 3 + z; \quad 1 \leq z \leq 8 \text{ ft} \quad z_2 = 0 \quad L = 45 \text{ ft}$$

$$\Sigma K = K_{\text{reentrant inlet}} + 2K_{90^\circ \text{ elbow}} + K_{\text{globe valve}} + K_{\text{exit}}$$

$$\Sigma K = 1.0 + 2(1.4) + 10 + 1 = 14.8 \text{ (threaded, regular)}$$

The head at section 1 is

$$H_1 = \frac{p_1 g_c}{\rho g} + \frac{V_1^2}{2g} + z_1 = 0 + 0 + 3 + z$$

At section 2,

$$H_2 = \frac{p_2 g_c}{\rho g} + \frac{V_2^2}{2g} + z_2 = 0$$

The change in head then becomes

$$\Delta H = H_1 - H_2 = 3 + z$$

Substituting into Equation i gives

$$3 + z = Q^2 \left(\frac{8(f(45)/(0.2557) + 14.8)}{\pi^2 (0.2557)^4 (32.2)} \right)$$

or

$$\Delta H = 3 + z = Q^2 [5.88(176f + 14.8)] \quad (\text{ii})$$

Flow velocity $V = Q/A$:

$$V = \frac{4Q}{\pi D^2}$$

Reynolds number $\text{Re} = \rho V D / \mu g_c = 4\rho Q / \pi D \mu g_c$

$$\text{Re} = \frac{4(62.4)Q}{\pi(0.2557)(1.9 \times 10^{-5})(32.2)} = 5.07 \times 10^5 Q \quad (\text{iii})$$

We now select values of flow rate, then calculate Reynolds number and find the friction factor from the Moody diagram using the “smooth” curve. Friction factor and flow rate are then substituted into Equation ii and ΔH is calculated. The procedure is repeated until the desired range of ΔH has been solved for. (It is customary in engineering units to express flow rate in gallons per minute, gpm.) A summary of the calculations is given in Table 4.5. Note that when calculations were made, the range of Q was not

known. So in the first column of Table 4.5, the assumed values of flow rate were selected in order to narrow down the range. It was sought to identify flow rates that yielded values for z that were within the required limits ($1 \leq z \leq 8$ ft).

TABLE 4.5. *Summary of calculations for Example 4.10.*

Q , ft ³ /s	Re	f (Fig. 3.6)	ΔH , ft	z , ft	Q gpm
0.01	5080	0.038	0.01	< 0	too low
0.1	50800	0.021	1.09	< 0	too low
1	508000	0.013	100.7	97.7	too high
0.5	254000	0.015	25.63	22.6	too high
0.3	152000	0.0164	9.37	6.37	135
0.32	162000	0.0162	10.64	7.64	143
0.33	168000	0.0161	11.31	8.31	147
0.25	127000	0.017	6.55	3.55	112
0.2	102000	0.018	4.22	1.22	90
0.23	117000	0.017	5.56	2.56	103

Graphs of the results are provided in Figure 4.21. The graph of ΔH versus Q (or z versus Q) is called a *system curve*. Customarily, a system curve is drawn with volume flow rate Q on the horizontal axis.

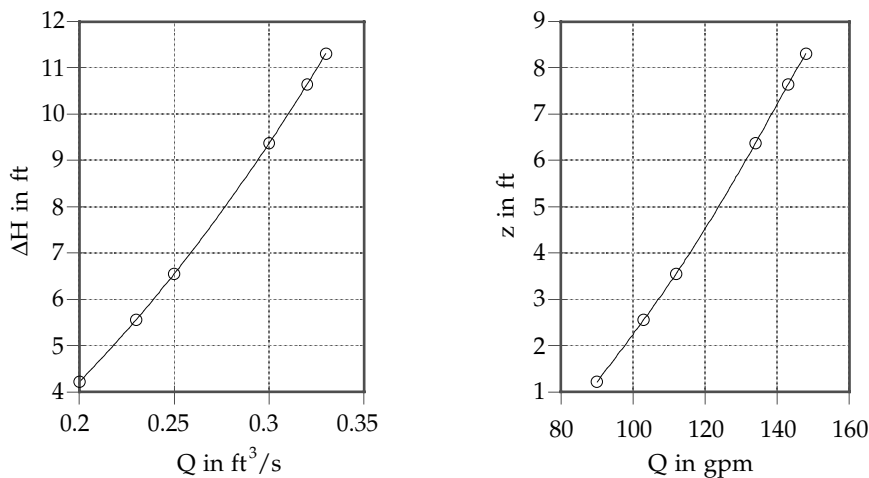


FIGURE 4.21. *System curve for the piping arrangement of Example 4.10.*

4.6 Support Systems for Pipes

Unless pipes and tubes are located underground, they must be supported with a system that is practical and economical. Piping supports are usually called **pipe hangers**. The primary function of hangers is to provide support for the piping loads and movements (expansions or contractions) and to allow the building structure to safely accommodate them.

A number of companies manufacture components that can be pieced together to form a pipe support system. Figure 4.22 shows different methods of supporting pipes. When pipes are subject to expansion and contraction while in service, a hanger such as that shown in Figure 4.22a will be adequate. The pipe rests on a cylinder whose shape cradles the pipe in its center. The cylinder is attached with bar stock to an overhead support in a trapeze-style configuration. In Figure 4.22b, the pipe has a thin and narrow piece of sheet metal wrapped around it that in turn is attached with a single piece of bar stock to an overhead support. Figure 4.22c shows another trapeze-style installation, in which clamps are bolted around the pipe. The bottom part of the clamps fit tightly into a U-shaped or square cross-sectioned channel. Thus, in Figure 4.22c, pipes are rigidly clamped in position and are not as free to move as in Figure 4.22a. Usually, threaded bar stock is used in these hangers. Note that hangers like those in Figure 4.22a and 4.22b are made for an individual pipe, while those in Figure 4.22c can be made part of a system that supports many pipes.

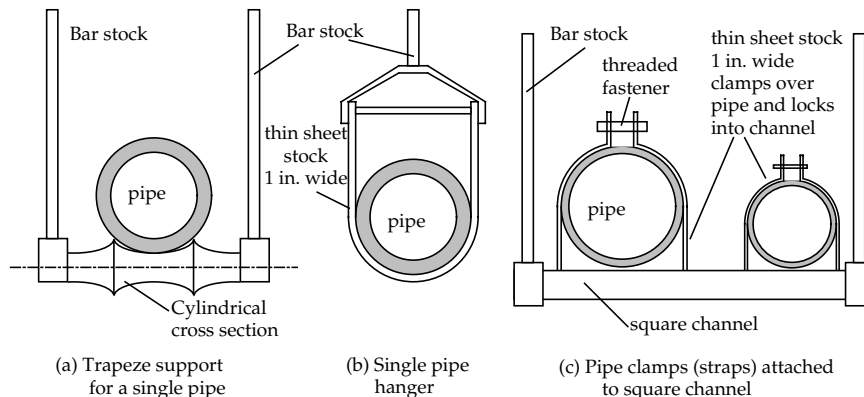


FIGURE 4.22. Examples of pipe support hardware.

The question now arises as to how far apart hangers should be spaced for adequate support. For economic reasons, we wish to use larger spaces, but we must consider safety of the installation. Based on many tests and calculations made for piping systems, the chart in Table 4.6 has been

TABLE 4.6. *Maximum horizontal spacing between supports of pipes. (Data from Manufacturers' Standardization Society SP-69, Table 3, used with permission.)*

Nominal Diameter	Steel Pipe				Copper Tubing			
	Water		Vapor		Water		Vapor	
	ft	m	ft	m	ft	m	ft	m
1/4	7	2.1	8	2.4	5	1.5	5	1.5
3/8	7	2.1	8	2.4	5	1.5	6	1.8
1/2	7	2.1	8	2.4	5	1.5	6	1.8
3/4	7	2.1	9	2.7	5	1.5	7	2.1
1	7	2.1	9	2.7	6	1.8	8	2.4
1 1/4	7	2.1	9	2.7	7	2.1	9	2.7
1 1/2	9	2.7	12	3.7	8	2.4	10	3.0
2	10	3.0	13	4.0	8	2.4	11	3.4
2 1/2	11	3.4	14	4.3	9	2.7	13	4.0
3	12	3.7	15	4.6	10	3.0	14	4.3
3 1/2	13	4.0	16	4.9	11	3.4	15	4.6
4	14	4.3	17	5.2	12	3.7	16	4.9
5	16	4.9	19	5.8	13	4.0	18	5.5
6	17	5.2	21	6.4	14	4.3	20	6.1
8	19	5.8	24	7.3	16	4.9	23	7.0
10	22	6.1	26	7.9	18	5.5	25	7.6
12	23	7.0	30	9.1	19	5.8	28	8.5
14	25	7.6	32	9.8				
16	27	8.2	35	10.7				
18	28	8.5	37	11.3				
20	30	9.1	39	11.9				
24	32	9.8	42	12.8				
30	33	10.1	44	13.4				

Additional hangers required at concentrated loads between supports.

Plastic Pipe	Follow Pipe Manufacturers' Recommendations for Spacing and Service Conditions
Fiberglass Reinforced Pipe	
Asbestos Cement Pipe	
Glass Pipe	Follow Pipe Manufacturers' Recommendations Use 8-ft (2.4-m) Max Spacing
Fire Protection	Follow Requirements of National Fire Protection Assoc.

devised. As indicated, space between hangers is given as a function of nominal pipe size and the type of fluid being conveyed. Remember that the hangers support the weight of the pipe *and* the fluid inside.

4.7 Summary

In this chapter, we have considered optimization problems, and the economics of pipe size selection. Specifically, we examined how costs can be minimized in the selection of a pipe size. We discussed the concept of equivalent length and graphical symbols used for drawing piping systems.

We also obtained a system curve for a given piping system. Finally, we looked at commercial hardware for physically supporting a piping system.

4.8 Show and Tell

1. We have been using a nominal diameter and a schedule to specify pipe sizes. Table 4.1, however, cites what is known as an ANSI designation. To what does ANSI designation refer? Is there something similar for tubing? Give a report on this alternative designation. (See ANSI/AWWA C150.)
2. Determine how to convert cost figures from one year to another; for example, 1990 dollars per foot converted to 2000 dollars per foot. Prepare an explanation of how costs like these are converted, and graph dollars per foot as a function of date.
3. Conduct an interview with a piping engineer and report on how pipe sizing is done by a practicing engineer. Is economics important?
4. There exists a fitting called a “union”; what is a union fitting and why is such a fitting necessary to have? Why is there no need for such a fitting in bell and spigot piping systems?
5. What is a “sleeve fitting”? With what type of pipe or tubing is it used? What is its function? Give a report on this type of fitting.
6. Obtain the appropriate catalog(s) and give a report on the variety of systems available for supporting pipes.

4.9 Problems

Optimization Problems

1. Find two positive numbers whose sum is 100, and whose product is as large as possible.
2. Find two positive numbers whose product is 100, and whose sum is as small as possible.
3. An open rectangular box has a square base, x by x . Find the height h of the box if its volume is 50 cm^3 , and the material needed to construct it is a minimum.
4. A window is shown in Figure P4.4. Find the value of x such that the perimeter of the window is 3 m, and the area of the window is as large as possible.
5. A rectangular area used to enclose machinery is to be fenced off using 300 m of material. What are the dimensions of such a rectangular enclosure if the area is maximized?
6. A large soup can is designed to hold 28 fluid ounces of soup. The can diameter is r , and its height is h . Find the values of r and h , such that the amount of metal needed is as small as possible.

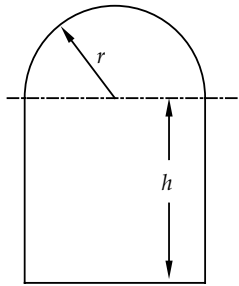


FIGURE P4.4

7. Figure P4.7 shows a pump and tank at A and a tank at B. It is desired to install a pipeline from A to B in the shortest possible distance. The pipeline is to be buried, and it will have to cross a roadway whose width is w . However, the roadway is used extensively by forklifts and other material handling equipment. So to minimize the disruption involved, it is necessary for the pipeline to cross the roadway at a right angle, and the pipeline will therefore take the path indicated in the figure. Determine the minimum pipeline length from A to B; that is, determine the value of x that minimizes the distance between A and B.
8. A production facility produces metal fasteners, and realizes a profit of \$800 per machine per month for 10 machines. Each machine requires cooling water to keep it running effectively. Management wishes to install more machinery, but the cooling water available cannot keep up with unlimited demand, and adding more machines would cause them all to run more slowly. Each additional machine reduces profit by \$25 per month. Thus with 10 machines, the profit is \$8000 ($= 800 \times 10$). For 11 machines the profit is \$8525 [$= (800 - 25) \times 11$]. Likewise for 12 machines, the profit is \$9000 [$= (800 - 2(25)) \times 12$]. Determine the optimum number of machines that should be installed to maximize profit.

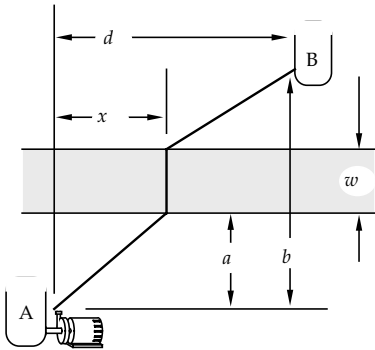


FIGURE P4.7

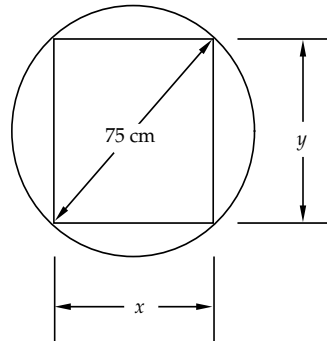


FIGURE P4.8

9. The strength S of a rectangular beam is proportional to its width x and the square of its depth y , so that $S = kxy^2$ where k is a positive constant. The beam is to be cut from a circular log whose diameter is 75 cm. What are the dimensions of the strongest beam that can be cut from such a log? (See Figure P4.8.)
10. Figure P4.10 shows a rectangular open channel conveying a liquid. We wish to find a relationship among the variables that will yield a hydraulically optimum cross section, which is one that provides maximum conveyance or volume carrying capacity for a given flow area.

The Manning equation for flow rate in an open channel is

$$Q = AV = \frac{AR_h^{2/3}S^{1/2}}{n} = \frac{A^{5/3}S^{1/2}}{P^{2/3}n}$$

where A is the cross-sectional area, P is the perimeter, S is the channel slope, and n is a friction term. The flow rate Q can be maximized if the perimeter is minimized. The flow area is given by $A = bz$. The perimeter is given by $P = 2z + b$.

Solve the area equation for b and substitute into the equation for perimeter. Differentiate the resulting equation for perimeter with respect to z and set the result equal to zero. Show that for a hydraulically optimum cross section

$$z = \frac{b}{2}$$

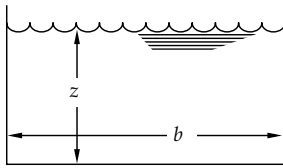


FIGURE P4.10

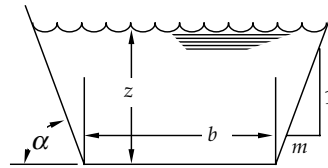


FIGURE P4.11

11. Figure P4.11 shows a trapezoidal open channel conveying a liquid. We wish to find a relationship among the variables that will yield a hydraulically optimum cross section, which is one that provides maximum conveyance or volume carrying capacity for a given flow area.

The Manning equation for flow rate in an open channel is

$$Q = AV = \frac{AR_h^{2/3}S^{1/2}}{n} = \frac{A^{5/3}S^{1/2}}{P^{2/3}n}$$

where A is the cross-sectional area, P is the perimeter, S is the channel slope, and n is a friction term.

The flow rate Q can be maximized if the perimeter is minimized. The flow area is given by $A = bz + z^2 \cot \alpha$. The perimeter is given by

$$P = b + \frac{2z}{\sin \alpha}$$

Solve the area equation for b and substitute into the equation for perimeter. Differentiate the resulting equation for perimeter with respect to z and set the result equal to zero. Next take the derivative with respect to α , and set that result equal to zero. Show that for a hydraulically optimum cross section

$$b = \frac{2\sqrt{3}}{3} z$$

12. Figure P4.12 shows a heat exchanger that consists of a shell of inside diameter ID , and length L . Inside the shell are a number of tubes. One fluid flows through the tubes and another in the shell outside the tubes. Calculations on such an exchanger yield results about the surface area required to exchange heat between the two fluids; specifically, how many tubes and how large a shell are needed to transfer the required heat?

The cost of the shell of inside diameter D and length L (both in m) is given by

$$C_s = \$1,800D^{2.5}L \quad (i)$$

Typically, to obtain the preceding equation, we would need data on heat exchanger costs and derive the cost equation by using curve-fitting techniques. This method is not difficult, but gathering and obtaining the necessary data could be cumbersome.

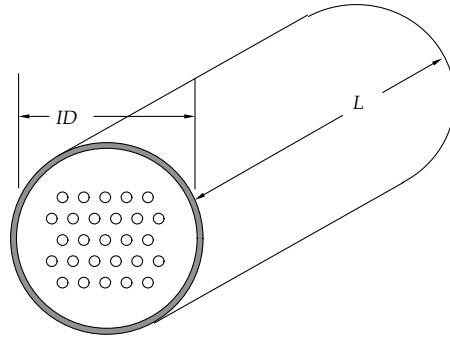


FIGURE P4.12

One problem with such exchangers is that the tube and shell surfaces will become fouled. They will have deposits of minerals or other substances on the surfaces that retard the flow of heat. The heat exchanger will of necessity have to be cleaned annually. The cost of cleaning is an annual cost, but the manufacturer will clean the exchanger for an initial one time fee given by

$$C_f = \$350DL \quad (ii)$$

The first cost of the heat exchanger tubing that will be contained within the exchanger is given by

$$C_t = \$2,000 \quad (iii)$$

Thus, the first cost of the exchanger is given by the sum of Equations (i), (ii) and (iii):

$$C = 1,800D^{2.5}L + 350DL + 2,000$$

This cost equation is to be minimized in this problem.

Now tube spacing within a shell-and-tube heat exchanger is dictated by safety concerns as well as performance requirements. The more tubes we can fit into the exchanger, the greater the surface area and, correspondingly, the greater the heat transfer rate. However, the tubes must fit together into a leakproof configuration without weakening the structure of the heat exchanger itself. Ordinarily, we would have an idea of the required tube surface area, and hence tube diameter, that must be used for the heat transfer requirements. From such calculations, suppose it is known that a total tube length of 150 m must be used. That is, the shell area $\times$ tube length per area should equal 150 m. Thus

$$(A_s L \text{ m}^3)(200 \text{ tubes/m}^2) = 150 \text{ m}$$

In terms of shell diameter, the preceding equation (our constraint) becomes

$$\frac{\pi D^2}{4} L (200) = 150$$

Determine the diameter D and length L of the heat exchanger shell for minimum first cost conditions.

13. Shown in Figure P4.13 are two pumps connected in parallel that move water from one tank up to another. The total volume flow rate to be delivered is a known quantity, $Q = 0.01 \text{ m}^3/\text{s}$. To deliver this flow rate, calculations indicate that the pressure rise to be provided by each pump is given by

$$\Delta p_1 = C_1 Q_1^2 \quad \text{and} \quad \Delta p_2 = C_2 Q_2^2 \quad (\text{i})$$

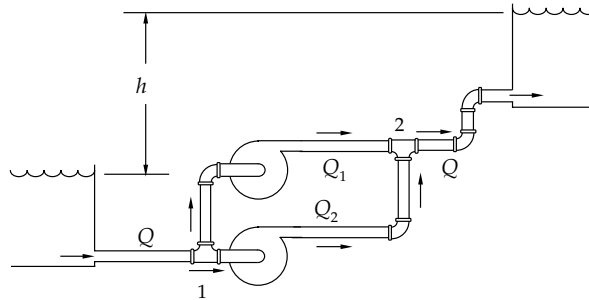


FIGURE P4.13

where Δp is in Pa, and the flow rates are in m^3/s for lines 1 and 2, respectively. The two pump/motor combinations are nearly identical. The efficiency of both pumps is $\eta = 0.75$. Determine an equation for the volume flow rates required to minimize the total power by completing the following steps:

- a. The power delivered by either pump is given by

$$-\frac{\partial W}{\partial t} = \rho A V \left\{ \left[\frac{p}{\rho} + \frac{V^2}{2g_c} + \frac{gz}{g_c} \right] \Big|_{\text{out}} - \left[\frac{p}{\rho} + \frac{V^2}{2g_c} + \frac{gz}{g_c} \right] \Big|_{\text{in}} \right\}$$

Assuming no changes in kinetic or in potential energies, verify that the power equation reduces to

$$\frac{dW}{dt} = \Delta p Q$$

Accounting for the pump efficiency, we have

$$\frac{dW_a}{dt} = \frac{\Delta p Q}{\eta}$$

Substituting for the pressures, show that the total power is given by:

$$\frac{dW_a}{dt} = \frac{C_1 Q_1^3}{\eta} + \frac{C_2 Q_2^3}{\eta}$$

which is our objective function. The total flow Q was given so that

$$Q_2 = Q - Q_1$$

which is the constraint. Substitute the constraint equation into the total power equation. Differentiate the resulting equation, and set the result equal to zero. Show that Q_1 is given by

$$Q_1 = \frac{-C_2Q \pm \sqrt{C_2^2Q^2 + (C_1 - C_2)C_2Q^2}}{(C_1 - C_2)}$$

- b. If $C_1 = 2.5 \times 10^{10}$ and $C_2 = 3.4 \times 10^{10}$, calculate the flow rates required for minimum power. Show that for minimum power,

$$Q_1 = 0.0054 \text{ m}^3/\text{s} \quad \text{and} \quad Q_2 = 0.0046 \text{ m}^3/\text{s}$$

- c. Show that the minimum power is 9.6 kW.

14. The inlet to a wind tunnel is in the shape of a converging duct, as shown in Figure P4.14. The duct is configured such that the x axis is coincident with the axial direction. The area of the duct at any location is given by

$$A = 3.5 - 2.5x/3 \quad (\text{in m}^2) \quad (\text{i})$$

It is desired to locate the place where flow straighteners will be placed. The cost of the straighteners is proportional to the flow area and is given by

$$C = C_1 A^n$$

where A is area, C_1 is a constant, and n is another constant. The annual cost of the straighteners is found by multiplying the preceding equation by an amortization rate a :

$$C_S = aC_1 A^n$$

The pressure drop across the straighteners is given by

$$\Delta p = K \frac{\rho V^2}{2g_c}$$

where K is the minor loss associated with the straighteners.

- a. Express velocity in terms of flow rate and area and pressure drop in terms of Δh ($= \Delta p g_c / \rho g$). Show that

$$\Delta h = \frac{KQ^2}{2A^2g}$$

The pumping cost associated with this loss is

$$C_P = C_2 \Delta h = C_2 \frac{KQ^2}{2A^2g}$$

where C_2 is a constant. Verify that the total cost is given by

$$C_T = aC_1 A^n + C_2 \frac{KQ^2}{2A^2g}$$

- b. Differentiate this expression with respect to area A and solve for the optimized area A_{opt} .
- c. Substitute Equation i for area and solve for x .
- d. For the following conditions, determine a numerical value for x :

$$Q = 30 \text{ m}^3/\text{s} \quad a = 0.5/\text{yr} \quad C_1 = \$200/(\text{m}^2)^n \quad C_2 = \$20.80/\text{m}$$

$$K = 0.75 \quad n = 1.1$$

15. The inlet in an open channel that feeds a pump is in the shape of a converging duct as shown in Figure P4.15. The duct is configured such that the x axis is coincident with the axial direction. The area of the duct at any location is given by

$$A = 30 - 11x/2 + x^2/2 \quad (\text{in ft}^2) \quad (\text{i})$$

where x varies from 0 to 6 ft. It is desired to locate the place where flow straighteners will be placed. The cost of the straighteners is proportional to the flow area and is given by

$$C = C_1 A^n$$

where A is area, C_1 is a constant, and n is another constant. The annual cost of the straighteners is found by multiplying the preceding equation by an amortization rate a :

$$C_S = aC_1 A^n$$

The pressure drop across the straighteners is given by

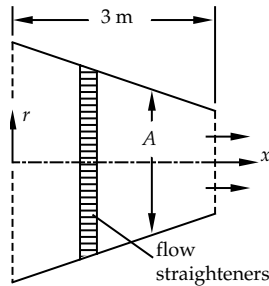


FIGURE P4.14.

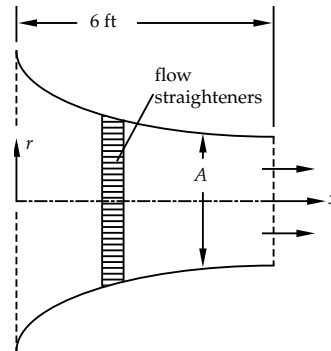


FIGURE P4.15.

$$\Delta p = K \frac{\rho V^2}{2g_c}$$

where K is the minor loss associated with the straighteners. If we express velocity in terms of flow rate and area and pressure drop in terms of Δh ($= \Delta p g_c / \rho g$), we obtain

$$\Delta h = \frac{KQ^2}{2A^2g}$$

The pumping cost associated with this loss is

$$C_P = C_2 \Delta h = C_2 \frac{KQ^2}{2A^2g}$$

where C_2 is a constant.

- a. Verify that the total cost is given by

$$C_T = aC_1A^n + C_2 \frac{KQ^2}{2A^2g}$$

- b. Differentiate this expression with respect to area A and solve for the optimized area A_{opt} .
- c. Substitute Equation i for area and solve for x .
- d. For the following conditions, determine a numerical value for x :

$$Q = 380 \text{ ft}^3/\text{s} \quad a = 0.2/\text{yr} \quad C_1 = \$20/(\text{ft}^2)^n \quad C_2 = \$600/\text{ft}$$

$$K = 0.10 \quad n = 1.8$$

16. The development regarding optimum economic diameter was formulated using pipe costs expressed as

$$C = C_1 D^n$$

Suppose instead that we use a parabolic curve fit:

$$C = B_0 + B_1 D + B_2 D^2$$

Following the development in the text, formulate an equation for the pipe costs; differentiate it and set it equal to zero. Show that the condition for minimum cost is given by

$$D_{\text{opt}} = \left[\frac{40fm^3C_2t}{(a+b)(1+F)(2B_2D_{\text{opt}} + B_1)\eta\pi^2\rho^2g_c} \right]^{1/6}$$

Economic Diameter

17. Using the information in Table 4.2, verify that Equation 4.10 is dimensionally consistent.
18. Beginning with Equation 4.10, derive Equation 4.12.
19. Verify that Equation 4.12 is dimensionally consistent.
20. Verify that the derivation of Equation 4.14 is correct, beginning with Equation 4.12.
21. Verify that Equation 4.14 is dimensionally correct.
22. Shown in the accompanying chart are data on the cost of PVC plastic pipe obtained from the classified section of a newspaper:

Nominal Diameter (schedule 40) in Inches	Cost per Foot (clearance prices)
2	\$0.56/ft
3	\$1.35/ft
4	\$1.56/ft
6	\$3.99/ft
8	\$7.96/ft

- Construct a graph of the data on linear paper.
- Construct a graph of the data on log-log paper.
- Determine the parameters of the following equation:

$$C_p = C_1 D^n$$

where C_p has dimensions of MU/L, C_1 has dimensions of MU/Lⁿ⁺¹ evaluated numerically for 12-nominal pipe, and D has dimensions of L.

- Repeat Problem 4.22 for the following data, which are of PVC high-pressure plastic pipe:

Nominal Diameter (schedule 40) in Inches	Cost per meter (clearance prices)
4	\$9.92/m
8	\$26.12/m
10	\$39.24/m
12	\$52.36/m

- A galvanized steel pipeline conveys ethyl alcohol at a rate of 40 l/s. Determine the optimum economic pipe size for the installation given that:

$$\begin{aligned} C_2 &= \$0.05/(1\,000W \cdot \text{hr}) & n &= 1.2 \\ C_1 &= \$1100/\text{m}^{n+1} & a &= 1/10 = 0.10 \\ t &= 6000 \text{ hr/yr} & b &= 0.01 \\ F &= 6.75 & \eta &= 0.75 \end{aligned}$$

- A commercial steel pipeline conveys ethylene glycol at a rate of 15.0 l/s. Determine the optimum economic pipe size for the installation given that:

$$\begin{aligned} C_2 &= \$0.04/(1\,000W \cdot \text{hr}) & n &= 1.2 \\ C_1 &= \$1050/\text{m}^{n+1} & a &= 1/7 = 0.14 \\ t &= 7000 \text{ hr/yr} & b &= 0.01 \\ F &= 6.5 & \eta &= 0.75 \end{aligned}$$

- A commercial steel pipeline conveys kerosene at a rate of 3.0 gpm. Determine the optimum economic pipe size for the installation given that:

$$\begin{aligned} C_2 &= \$0.01 \text{ s}/(738 \text{ ft} \cdot \text{lbf} \cdot \text{hr}) & n &= 1.4 \\ C_1 &= \$115/\text{ft}^{n+1} & a &= 1/7 = 0.14 \\ t &= 7800 \text{ hr/yr} & b &= 0.01 \\ F &= 6.5 & \eta &= 0.8 \end{aligned}$$

27. A PVC pipeline conveys heptane at a rate of 100.0 l/s. Determine the optimum economic pipe size for the installation given that:

$$\begin{array}{ll} C_2 = \$0.05/(1\,000\text{W}\cdot\text{hr}) & n = 1.2 \\ C_1 = \$1300/\text{m}^{n+1} & a = 1/10 = 0.10 \\ t = 4000\text{ hr/yr} & b = 0.01 \\ F = 6.75 & \eta = 0.75 \end{array}$$

28. A drawn tubing pipeline conveys linseed oil at a rate of 0.0010 m³/s. Determine the optimum economic pipe size for the installation given that:

$$\begin{array}{lll} C_2 = \$0.04/(1\,000\text{W}\cdot\text{hr}) & C_1 = \$700/\text{m}^{n+1} & \\ t = 7000\text{ hr/yr} & F = 6.75 & n = 1.2 \\ a = 1/10 = 0.10 & b = 0.01 & \eta = 0.75 \end{array}$$

29. A commercial steel pipeline conveys methyl alcohol at a rate of 240.0 l/s. Determine the optimum economic pipe size for the installation given that:

$$\begin{array}{ll} C_2 = \$0.05/(1\,000\text{W}\cdot\text{hr}) & n = 1 \\ C_1 = \$700/\text{m}^{n+1} & a = 1/10 = 0.10 \\ t = 750\text{ hr/yr} & b = 0.01 \\ F = 6.75 & \eta = 0.65 \end{array}$$

30. Octane flows at a rate of 5.0 gpm through a pipeline made of drawn tubing. Determine the optimum economic pipe size for the installation given that:

$$\begin{array}{ll} C_2 = \$0.05\text{ s}/(738\text{ ft}\cdot\text{lbf}\cdot\text{hr}) & n = 1.2 \\ C_1 = \$55/\text{ft}^{n+1} & a = 1/5 = 0.20 \\ t = 4000\text{ hr/yr} & b = 0.01 \\ F = 6.75 & \eta = 0.75 \end{array}$$

31. A drawn tubing pipeline conveys propane at a rate of 150.0 l/s. Determine the optimum economic pipe size for the installation given that:

$$\begin{array}{ll} C_2 = \$0.05/(1\,000\text{W}\cdot\text{hr}) & n = 1.2 \\ C_1 = \$850/\text{m}^{n+1} & a = 1/6 = 0.17 \\ t = 5000\text{ hr/yr} & b = 0.01 \\ F = 6.75 & \eta = 0.65 \end{array}$$

32. A galvanized steel pipeline conveys propylene glycol at a rate of 100.0 l/s. Determine the optimum economic pipe size for the installation given that:

$$\begin{array}{ll} C_2 = \$0.05/(1\,000\text{W}\cdot\text{hr}) & n = 1.2 \\ C_1 = \$1000/\text{m}^{n+1} & a = 1/10 = 0.10 \\ t = 4000\text{ hr/yr} & b = 0.01 \\ F = 6.75 & \eta = 0.75 \end{array}$$

33. A galvanized steel pipeline conveys castor oil at a rate of 0.0120 m³/s. Determine the optimum economic pipe size for the installation given that:

$$\begin{array}{lll} C_2 = \$0.05/(1\,000\text{W}\cdot\text{hr}) & C_1 = 850/\text{m}^{n+1} & \\ t = 5000\text{ hr/yr} & F = 6.75 & n = 1.4 \\ a = 1/7 = 0.14 & b = 0.01 & \eta = 0.60 \end{array}$$

34. A wrought iron pipeline conveys turpentine at a rate of 75.0 l/s. Determine the optimum economic pipe size for the installation given that:

$$\begin{array}{ll}
 C_2 = \$0.05/(1\,000W \cdot \text{hr}) & n = 1.4 \\
 C_1 = \$1100/\text{m}^{n+1} & a = 1/5 = 0.20 \\
 t = 4000 \text{ hr/yr} & b = 0.01 \\
 F = 6.75 & \eta = 0.75
 \end{array}$$

35. A commercial steel pipeline conveys propylene at a rate of 250 gpm. Determine the optimum economic pipe size for the installation given that:

$$\begin{array}{ll}
 C_2 = \$0.05 \text{ s}/(738 \text{ ft} \cdot \text{lbf} \cdot \text{hr}) & n = 1.2 \\
 C_1 = \$70/\text{ft}^{n+1} & a = 1/9 = 0.11 \\
 t = 3000 \text{ hr/yr} & b = 0.01 \\
 F = 6.75 & \eta = 0.75
 \end{array}$$

36. A commercial steel pipeline conveys methyl alcohol at a rate of 240.0 l/s. Determine the optimum economic pipe size for the installation given that:

$$\begin{array}{ll}
 C_2 = \$0.05/(1\,000W \cdot \text{hr}) & n = 1 \\
 C_1 = \$700/\text{m}^{n+1} & a = 1/10 = 0.10 \\
 t = 750 \text{ hr/yr} & b = 0.01 \\
 F = 6.75 & \eta = 0.65
 \end{array}$$

37. A wrought iron pipeline conveys turpentine at a rate of 700 gpm. Determine the optimum economic pipe size for the installation given that:

$$\begin{array}{ll}
 C_2 = \$0.05 \text{ s}/(738 \text{ ft} \cdot \text{lbf} \cdot \text{hr}) & n = 1.2 \\
 C_1 = \$80/\text{ft}^{n+1} & a = 1/10 = 0.10 \\
 t = 4000 \text{ hr/yr} & b = 0.01 \\
 F = 6.75 & \eta = 0.75
 \end{array}$$

38. A commercial steel pipeline conveys ethylene glycol at a rate of 15.0 l/s. Determine the optimum economic pipe size for the installation given that:

$$\begin{array}{ll}
 C_2 = \$0.04/(1\,000W \cdot \text{hr}) & n = 1.2 \\
 C_1 = \$1050/\text{m}^{n+1} & a = 1/7 = 0.14 \\
 t = 7000 \text{ hr/yr} & b = 0.01 \\
 F = 6.5 & \eta = 0.75
 \end{array}$$

39. A PVC pipeline conveys glycerine at a rate of $0.0100 \text{ m}^3/\text{s}$. Determine the optimum economic pipe size for the installation given that:

$$\begin{array}{lll}
 C_2 = \$0.045/(1\,000W \cdot \text{hr}) & C_1 = \$1200/\text{m}^{n+1} & \\
 t = 4000 \text{ hr/yr} & F = 6.75 & n = 1 \\
 a = 1/5 = 0.20 & b = 0.01 & \eta = 0.70
 \end{array}$$

40. Water flows through a commercial steel pipeline at a 60.0 gpm. Determine the optimum economic pipe size for the installation given that:

$$\begin{array}{ll}
 C_2 = \$0.045 \text{ s}/(738 \text{ ft} \cdot \text{lbf} \cdot \text{hr}) & n = 1 \\
 C_1 = \$85/\text{ft}^{n+1} & a = 1/6 = 0.17 \\
 t = 4000 \text{ hr/yr} & b = 0.01 \\
 F = 7 & \eta = 0.75
 \end{array}$$

46. Figure P4.46 shows a conduit made of 2-nominal schedule 40 commercial steel that conveys ethylene glycol at a flow rate of 4 l/s. All fittings are regular and threaded. (a) Calculate the pressure drop through the system using minor-loss values (K). (b) Calculate the pressure drop through the system using equivalent length values. Compare the results. Take the piping system to be in a horizontal plane and the check valve to be a swing type.
47. Figure P4.47 shows a conduit made of 1-nominal schedule 40 galvanized steel that conveys water. The pressure drop from beginning to end is 2.5 psi. All fittings are regular and threaded. (a) Calculate the flow rate through the system using minor-loss values (K). (b) Calculate the flow rate through the system using equivalent length values. Compare the results. Calculate the total length of pipe by adding values given in the drawing. Is there any advantage to using equivalent length in this type of problem?

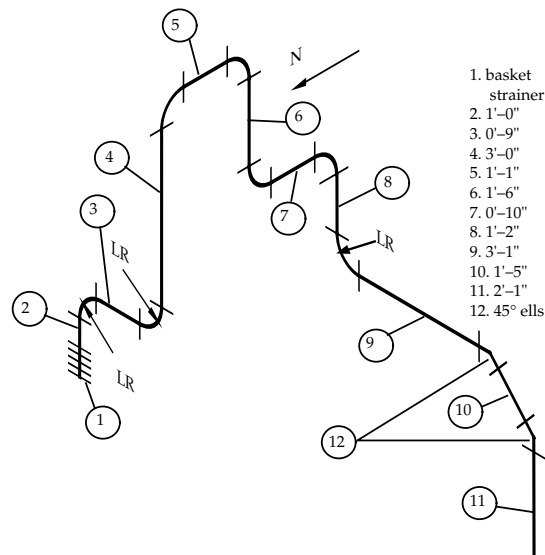
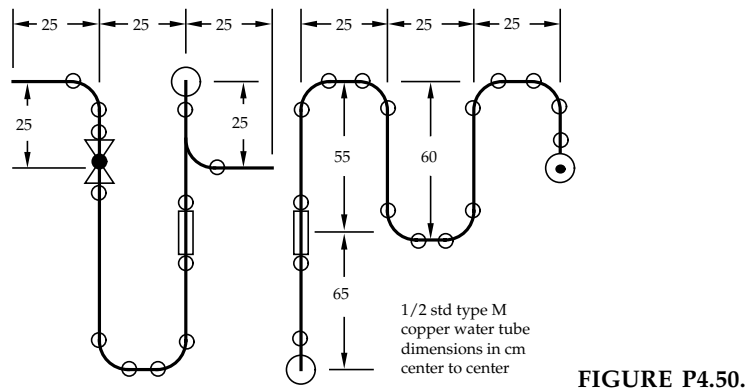
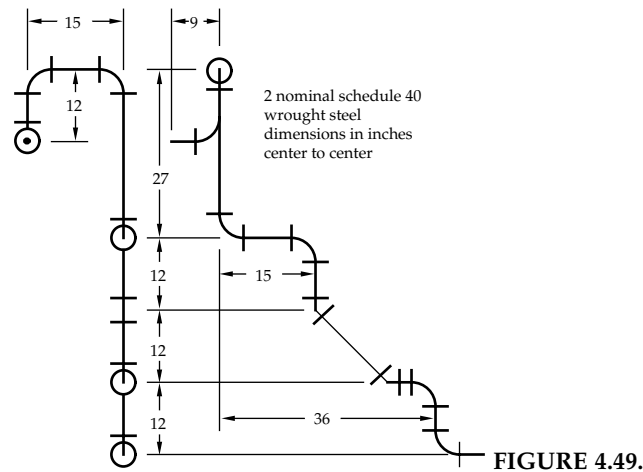


FIGURE P4.47, P4.48,
P4.58

Piping System Graphics

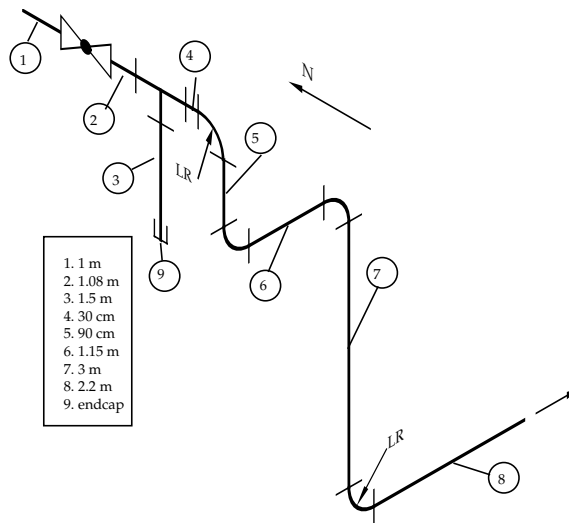
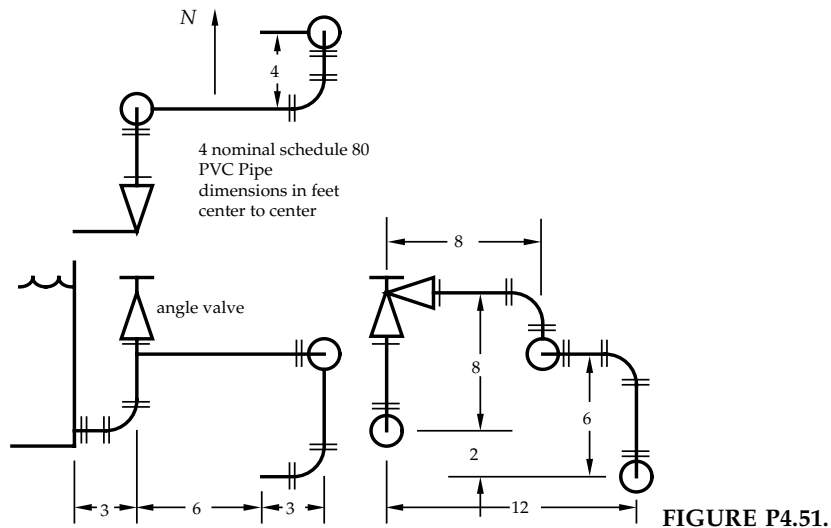
48. Sketch front and profile views of the pipeline of Figure P4.47.
49. Figure P4.49 shows front and side views of a piping system. Draw an isometric view of the system, and compose a list of all fittings and attachment methods. The pipe is made of 2-nominal schedule 40 wrought steel.
50. Figure P4.50 shows front and side views of a piping system. Draw an isometric view of the system and compose a list of all fittings and attachment methods. The pipe is made of $1/2$ -standard type M copper water tubing.
51. Figure P4.51 shows plan, front, and side views of a piping system. Draw an isometric view of the system. List all fittings and attachment methods if the pipe is made of 4-nominal schedule 80 PVC.
52. List all fittings and attachment methods for the piping system of Figure P4.52. The conduit is made up of $2\frac{1}{2}$ -nominal schedule 40 galvanized pipe. Also, prepare a frontal view (as seen by someone facing east) and a profile view (as seen by someone facing north) of the piping system.



53. Sketch to scale what someone facing north would see while looking at the pipeline of Figure P4.53.
54. Sketch to scale what someone facing in the north direction would see while looking at the pipeline of Figure P4.54.
55. Sketch to scale what someone facing in the north direction would see while looking at the pipeline of Figure P4.55.

System Behavior

56. Water flows through the piping system of the figure (P4.56). The conduit is made up of 2¹/₂-nominal schedule 40 galvanized pipe. Construct a system curve for the water, with flow rate in gpm graphed as a function of pressure drop from inlet to exit in psi. Let velocity vary from 4 to 10 ft/s.



57. The piping system of Figure P4.57 conveys glycerine at a flow rate that varies from 0.3 to 1.2 l/s. Calculate the pressure drop from inlet to outlet and generate a system curve of volume flow rate in l/s vs pressure drop in kPa. The pipe material is 1-std type M drawn copper tubing.
58. The piping system of Figure P4.58 is attached to a tank containing linseed oil. The free surface of the oil varies from 2 to 8 ft above the basket strainer. Generate a system curve of oil height above the basket strainer ($8 \text{ ft} \leq h \leq 2 \text{ ft}$) as a function of volume flow rate expressed in gpm. The pipe is made up of $3/4$ -nominal schedule 40 commercial steel. Assume that the linseed oil exits to atmospheric pressure.

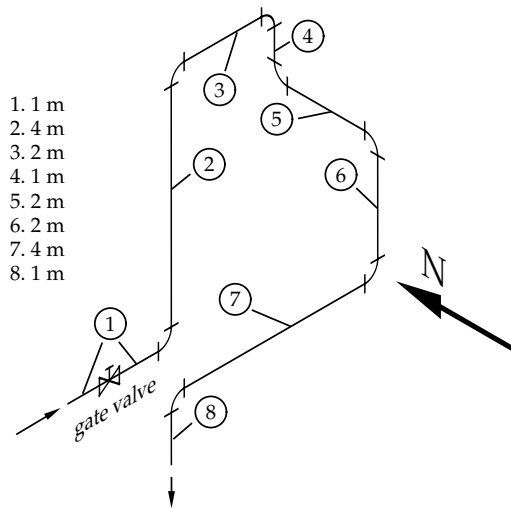


FIGURE P4.53

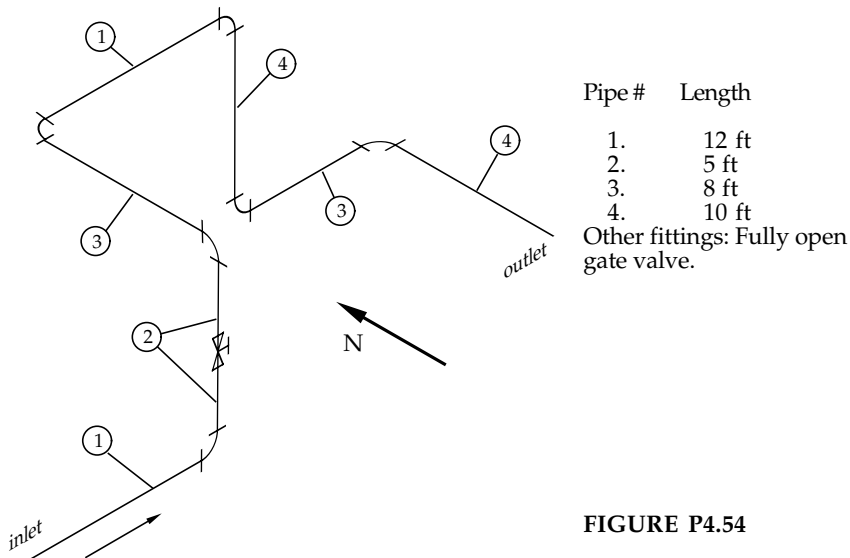
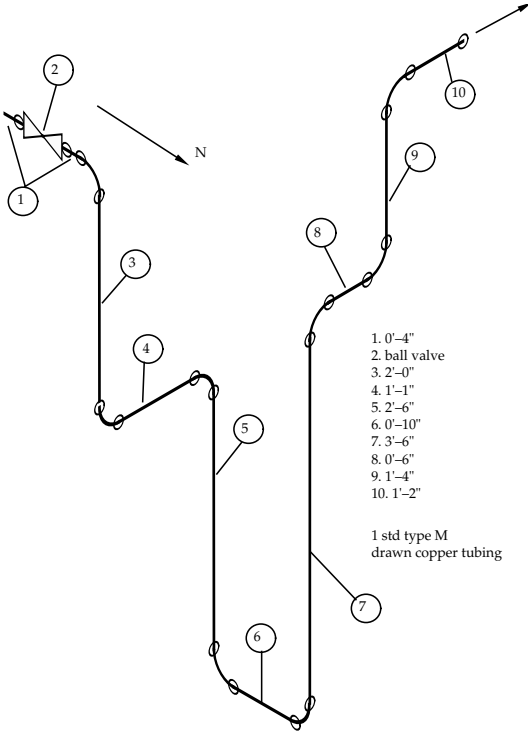
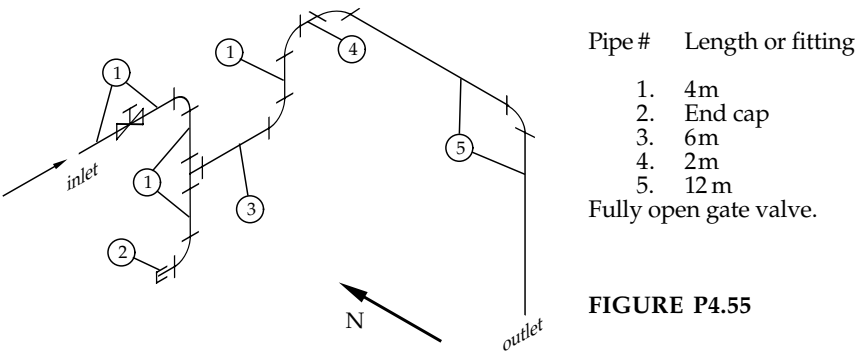


FIGURE P4.54

59. Draw a system curve for the piping system of Figure P4.59. Let the flow rate be controlled by various settings of the ball valve, and graph volume flow rate in liters per second as a function of the valve angle. The system is made up of 1-std type M copper tubing. The pressure drop in all cases is 400 kPa and the fluid is ethylene glycol.



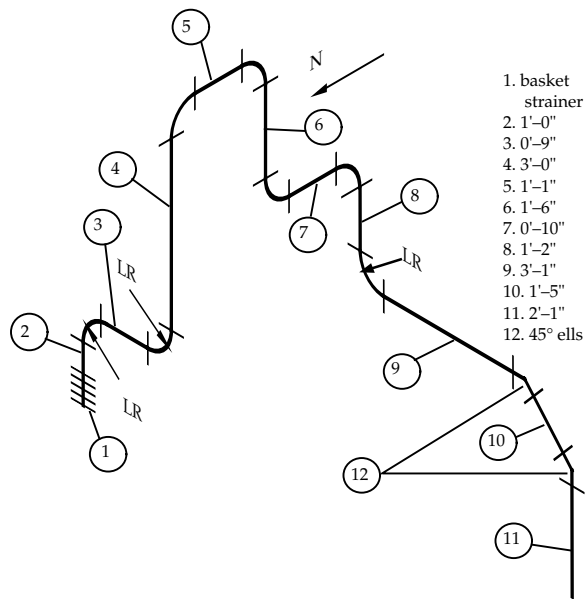


FIGURE P4.58

CHAPTER 5 Selected Topics in Fluid Mechanics

In this chapter, we consider various topics in fluid mechanics: flow in pipe networks, flow through parallel piping systems, measurement of flow rate in closed conduits, and unsteady draining tank problems.

The section on flow in pipe networks provides a background for the analysis of such systems. We develop a method of identifying the components of a network, and we review the traditional approach to solving network problems; namely the Hardy Cross method. We next formulate a solution method for modeling flow in a parallel piping system.

We also consider flow meters used in piping systems: the turbine-type meter, the rotameter, the venturi meter, the orifice meter, and the elbow meter. Other types of meters exist, of course, and these types are described in detail on the Internet.

The chapter continues with a section that models draining tank problems. Equations that describe draining tanks can present difficulties when trying to solve them. A numerical solution technique is described, and its use solving draining tank problems is demonstrated.

5.1 Flow in Pipe Networks

A piping network is an assembly of connected pipes (or tubes) used to distribute fluid to users in a specific area. The area could be a subdivision of residences with an underground piping network that distributes water to each home. It could also be a group of printing presses with an overhead piping network that distributes liquid ink to each machine. There are many examples of such systems.

A piping network consists of pipes or tubes of various sizes, geometric orientation, and frictional characteristics. The system could contain pumps, valves, fittings, and the like. Figure 5.1 shows a plan view of a piping network. There are places where the fluid enters and exits the piping

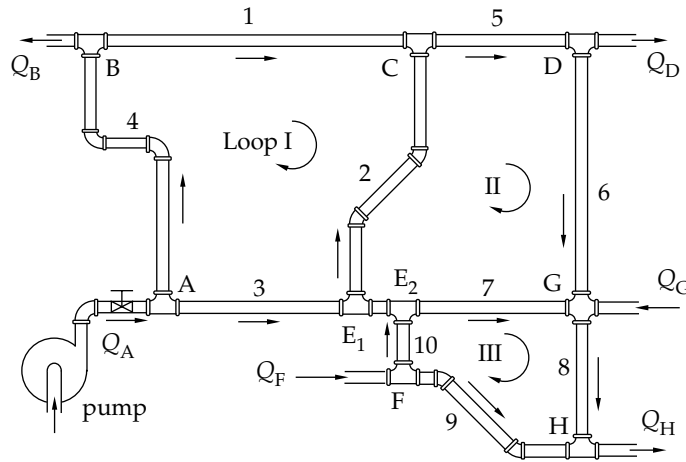


FIGURE 5.1. Plan view of a piping network.

system. The objective of an analysis of this network is to determine the volume flow rate in each pipe. For networks, this is not an easy task, but it can be done if a systematic procedure is formulated and followed.

For purposes of analysis, we will introduce a notational scheme that is used to identify various components of the system. Every junction of the network of Figure 5.1 is labeled with a letter, A through H. Each pipe is labeled with a number; 4 for example, is the line (including elbows) that connects A to B. Note that the joints labeled E_1 and E_2 are so close that they can be considered a single joint.

We have two general equations we use to analyze the dynamics of this system: the continuity equation and the modified Bernoulli equation. For the entire network, all flows entering must equal all flows leaving; that is, we write:

$$\sum Q_{\text{in}} = \sum Q_{\text{out}}$$

$$\text{or } Q_A + Q_F + Q_G = Q_H + Q_D + Q_B$$

We also write similar equations for each joint; before doing so, however, we first assume a flow direction within each pipe. These are indicated in the figure. Thus, for the joint labeled A, we have

$$\sum Q_{\text{in}} = \sum Q_{\text{out}}$$

$$Q_A = Q_3 + Q_4$$

For the joint labeled E, we write

$$Q_3 + Q_{10} = Q_2 + Q_7$$

We would write an equation for every junction and end up with eight equations (one for each junction). Only seven of these equations are independent, but they are all linear.

Next, we identify “loops” within the network and select a direction we define as positive. For example, the pipes labeled 1-2-3-4 make up what we will call Loop I; 5-6-7-2 we call Loop II; and, 7-8-9-10 is Loop III. In all loops, as indicated in the figure, the clockwise direction is positive.

These definitions, along with the modified Bernoulli equation, are used to obtain three equations, one for each loop. For pipe #1, going from B to C (in the flow direction), the modified Bernoulli equation is

$$\frac{p_B g_c}{\rho g} + \frac{V_B^2}{2g} + z_B = \frac{p_C g_c}{\rho g} + \frac{V_C^2}{2g} + z_C + \frac{f_1 L_1}{D_1} \frac{V_1^2}{2g} + \sum K \frac{V_1^2}{2g} \quad (5.1)$$

If the elevation at B equals that at C, then $z_B = z_C$. For a constant diameter, $V_B = V_C$. With no minor losses, $\sum K = 0$. The modified Bernoulli equation reduces to

$$\frac{p_B g_c}{\rho g} - \frac{p_C g_c}{\rho g} = \frac{f_1 L_1}{D_1} \frac{V_1^2}{2g}$$

$$\text{or } \Delta p_{BC} = \frac{f_1 L_1}{D_1} \frac{\rho V_1^2}{2g_c} \quad (5.2)$$

This pressure drop is positive because it is coincident with the clockwise direction. The flow rate is what we are interested in finding, so we substitute for velocity

$$V_1 = \frac{Q_1}{A_1} = \frac{4Q_1}{\pi D_1^2}$$

Substituting into Equation 5.2 gives

$$\Delta p_{BC} = \frac{f_1 L_1}{D_1} \frac{\rho}{2g_c} \frac{16Q_1^2}{\pi^2 D^4}$$

$$\Delta p_{BC} = \frac{8\rho L_1}{\pi^2 D_1^5 g_c} f_1 Q_1^2$$

$$\text{or } \Delta p_{BC} = C_1 f_1 Q_1^2 \quad (5.3)$$

where the constant C_1 is given by

$$C_1 = \frac{8\rho L_1}{\pi^2 D_1^5 g_c}$$

with dimensions of $\text{F}\cdot\text{T}^2/\text{L}^8$. We have thus separated out the constants associated with pipe #1. For purposes of illustration, let us write equations for the other pressure drops in Loop I assuming no minor losses. We have results similar to Equation 5.3:

$$\Delta p_{CE} = -C_2 f_2 Q_2^2 \quad (5.4)$$

$$\Delta p_{EA} = -C_3 f_3 Q_3^2 \quad (5.5)$$

and

$$\Delta p_{AB} = C_4 f_4 Q_4^2 \quad (5.6)$$

The negative values correspond to flow directions that are counterclockwise. The pressure drops in all the pipes making up each loop should total zero. Thus for Loop I,

$$\Delta p_{BC} + \Delta p_{CE} + \Delta p_{EA} + \Delta p_{AB} = 0$$

$$C_1 f_1 Q_1^2 - C_2 f_2 Q_2^2 - C_3 f_3 Q_3^2 + C_4 f_4 Q_4^2 = 0$$

This procedure will give one equation for each loop for a total of three independent nonlinear equations.

The procedure of applying continuity and the modified Bernoulli equation to Figure 5.1 yields seven independent linear equations (from continuity) and three nonlinear equations. These ten equations must be solved simultaneously to obtain values for the ten unknown flow rates. The method involves a trial-and-error approach requiring several iterations.

There are a number of ways to solve the equations. The earliest published method is called the **Hardy Cross method**. This technique is a special case of the **Newton method**. Another method involves a linearization process. We will focus here on the Hardy Cross method with solution using a spreadsheet. It should be mentioned that computer programs are available for solving for any number of loops in a piping network (see *Analysis of Flow in Pipe Networks* by R. W. Jeppson; Ann Arbor Science; Ann Arbor Michigan; 1982). Solution methods are available on the Internet as well.

The Hardy Cross Method

Suppose we have set up our equations for a particular network. For a system with N junctions, we obtain from continuity $N - 1$ independent linear equations. With M loops, we obtain M nonlinear equations. The equations must be solved simultaneously to determine the volume flow rate in each pipe. The nonlinear equations present tremendous difficulties that make a solution elusive using traditional means. The Hardy Cross method has been developed in order to help solve the equations.

The method begins by assuming reasonable values for several of the unknown flow rates. These assumed values are substituted into the equations, and the pressure drops are calculated and summed. These results are then used to find a correction ΔQ , which is then applied to the assumed values to obtain improved estimates. This procedure is repeated until the correction ΔQ is acceptably small; that is, until the sum of the pressure drops in each loop add to zero. The iterative technique demonstrated here is called the Hardy Cross method.

To illustrate the steps involved, consider the network illustrated in Figure 5.2. Shown are two loops each of which contains four pipes. The joints or junctions are labeled A through F, and the pipes are labeled 1 through 7. Next to each pipe are its diameter and length. Pipe #2 is common to both loops. All units are in SI.

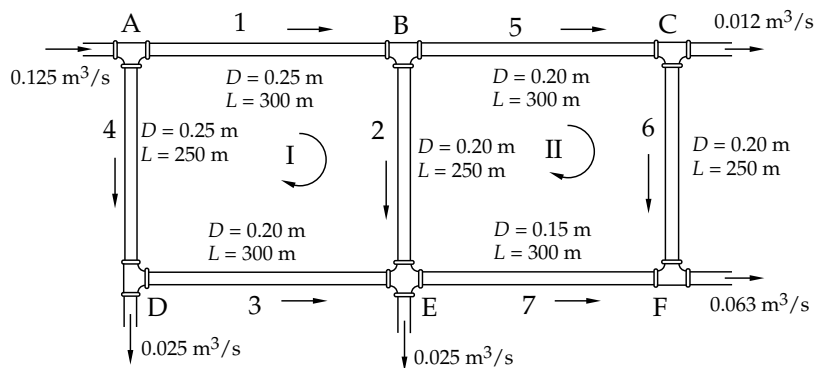


FIGURE 5.2. Two loop network used to illustrate the Hardy Cross method.

Step 1: Assume a flow direction in each pipe.

If a wrong direction is assumed, the calculations will provide the correction. Arrows next to each Roman numeral in Figure 5.2 show the assumed flow directions.

Step 2: Check continuity for the entire network.

Referring to the figure, the sum of all flows entering the network must equal all flows leaving. Thus

$$Q_{\text{in}} = 0.125 \text{ m}^3/\text{s}$$

$$Q_{\text{out}} = 0.012 + 0.063 + 0.025 + 0.025 = 0.125 \text{ m}^3/\text{s}$$

Flow into the system must equal flow out.

Step 3: Write the continuity equation for each joint.

This is done with reference to Figure 5.3. For each junction, we write

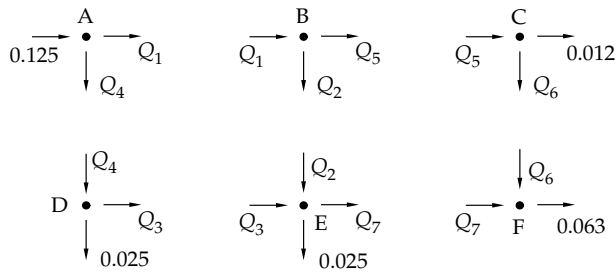


FIGURE 5.3. Auxiliary diagram to determine flows at each joint.

$$\begin{array}{ll}
 \text{A:} & 0.125 = Q_1 + Q_4 \\
 \text{B:} & Q_1 = Q_2 + Q_5 \\
 \text{C:} & Q_5 = 0.012 + Q_6 \\
 \text{D:} & Q_4 = Q_3 + 0.025 \\
 \text{E:} & Q_2 + Q_3 = Q_7 + 0.025 \\
 \text{F:} & Q_7 + Q_6 = 0.063
 \end{array} \quad (5.7)$$

We obtained six ($= N$ junctions) linear equations, only five ($= N - 1$) of which are independent. Thus, any one of the equations could be obtained by some linear combination of the others.

Step 4: Identify each loop and assign a positive direction.

This is indicated in the figure. Both Loops I and II have the clockwise direction as positive.

Step 5: Write the pressure-drop equation for each loop.

In the absence of minor losses, we write for each loop:

$$\text{I:} \quad \Delta p_{AB} + \Delta p_{BE} - \Delta p_{ED} - \Delta p_{DA} = 0$$

$$\text{II:} \quad \Delta p_{BC} + \Delta p_{CF} - \Delta p_{FE} - \Delta p_{EB} = 0$$

$$\begin{aligned} \text{or} \quad C_1 f_1 Q_1^2 + C_2 f_2 Q_2^2 - C_3 f_3 Q_3^2 - C_4 f_4 Q_4^2 &= 0 \\ C_5 f_5 Q_5^2 + C_6 f_6 Q_6^2 - C_7 f_7 Q_7^2 - C_2 f_2 Q_2^2 &= 0 \end{aligned}$$

where the constant $C = 8\rho L/(\pi^2 D^5 g_c)$ for each pipe. The positive values are those that correspond to a “clockwise” direction in the loop and vice versa.

It is these nonlinear equations that keep us from substituting the continuity relations and solving directly. If minor losses were included, the appropriate pressure-drop terms would contain a $\sum K$ term written with flow rate Q rather than the velocity V . The preceding two equations and the continuity equations must now be solved simultaneously.

Step 6: Simplify the continuity equations.

We do this by solving in terms of two flow rates, so we solve for all flow rates in terms of Q_1 and Q_2 (selected arbitrarily). We get

$$\left. \begin{aligned} Q_4 &= 0.125 - Q_1 \\ Q_5 &= Q_1 - Q_2 \\ Q_6 &= Q_1 - Q_2 - 0.012 \\ Q_3 &= 0.1 - Q_1 \\ \text{and } Q_7 &= Q_2 - Q_1 - 0.075 \end{aligned} \right\} \quad (5.8)$$

If we assume flow rate values for Q_1 and Q_2 , all the others can be determined with these equations.

Step 7: Set up a solution table to summarize the calculations.

It will simplify matters greatly if we set up a solution table, which is conveniently done using a spreadsheet; see Table 5.1. Fluid properties of density ρ and viscosity μ are those for water. The roughness is that for cast iron. The physical data for each pipe (although not shown in the table) can also be included: diameter D , length L , the constant C , and the relative roughness ϵ/D .

The calculations are made separately for each loop. For the first iteration, “reasonable” estimates were made for Q_1 ($= 0.060 \text{ m}^3/\text{s}$) and Q_2 ($= 0.020 \text{ m}^3/\text{s}$). The rest of the flow rates were calculated with the continuity equations, Equation 5.8. Table 5.1 represents calculations made for the first attempt at finding a solution; that is, the first iteration. Column 2 shows pipe number, and column 3 is flow rate.

We are working toward calculating pressure drops, which require values for the friction factors. These in turn require the Reynolds number, which is defined as

TABLE 5.1. *Fluid properties, data table, and first iteration table for the network of Figure 5.2.*

Fluid Properties $\rho = 1\,000\text{ kg/m}^3$ $\mu = 0.89 \times 10^{-3}\text{ N}\cdot\text{s/m}^2$
 Cast Iron $\varepsilon = 0.00025\text{ m}$

a) Data table:

Pipe	D, m	L, m	$C, \text{N}\cdot\text{s}^2/\text{m}^8$	ε/D
1	0.250	300	2.49E+08	0.000 184
2	0.200	250	6.33E+08	0.000 230
3	0.200	300	7.60E+08	0.000 230
4	0.250	250	2.08E+08	0.000 184
5	0.200	300	7.60E+08	0.000 230
6	0.200	250	6.33E+08	0.000 230
7	0.150	300	3.20E+09	0.000 307

b) First iteration for each loop:

Loop	Pipe	$Q\text{ m}^3/\text{s}$	Re	f	$\Delta p\text{ N/m}^2$	$\Delta p/Q$
I	1	0.060	3.43×10^5	0.021	1.85×10^4	3.09×10^5
	2	0.020	1.43×10^5	0.023	5.73×10^3	2.86×10^5
	3	-0.040	2.86×10^5	0.022	-2.65×10^4	6.64×10^5
	4	-0.065	3.72×10^5	0.021	-1.81×10^4	2.78×10^5
$\Sigma =$					-2.04×10^4	1.54×10^6
$\Delta Q = -(-2.04 \times 10^4 / (2(1.54 \times 10^6)))$					$\Delta Q =$	0.006 6

II	5	0.040	2.86×10^5	0.022	2.65×10^4	6.64×10^5
	6	0.028	2.00×10^5	0.022	1.10×10^4	3.93×10^5
	7	-0.035	3.34×10^5	0.023	-9.09×10^4	2.60×10^6
	2	-0.020	1.43×10^5	0.023	-5.73×10^3	2.86×10^5
$\Sigma =$					-5.91×10^4	3.94×10^6
$\Delta Q = -(-5.91 \times 10^4 / (2(3.94 \times 10^6)))$					$\Delta Q =$	0.007 5

$$\text{Re} = \frac{\rho V D}{\mu g_c}$$

With average velocity $V = Q/A = 4Q/\pi D^2$, the Reynolds number becomes

$$\text{Re} = \frac{4\rho Q}{\pi D \mu g_c}$$

Calculations made with this equation for both loops are shown in the fourth column of the tables.

The friction factor for each pipe was calculated with the Chen equation which is

$$f = \left[-2.0 \log \left\{ \frac{\varepsilon}{3.7065D} - \frac{5.0452}{\text{Re}} \log \left(\frac{1}{2.8257} \left[\frac{\varepsilon}{D} \right]^{1.1098} + \frac{5.8506}{\text{Re}^{0.8981}} \right) \right\} \right]^{-2}$$

All flows in this network are turbulent; alternatively, the laminar expression $f = 64/\text{Re}$ would have been used for laminar flows. Pressure drop in each pipe is calculated with

$$\Delta p = CfQ^2$$

and displayed in the next column. When these pressure drops are summed for each loop, we get $\Sigma \Delta p_I = -2.04 \times 10^4$ and $\Sigma \Delta p_{II} = -5.91 \times 10^4$ (column 6 of the table). The values we seek for each of these sums is zero.

Step 8: Determine an improved value for the flow rates; begin the next iteration.

The question now arises as to how we should modify our assumed flow rates so that we can make another iteration. Moreover, in this and in subsequent iterations, it is necessary that successive assumed flow rates converge to some value that satisfies the equations. So now we develop a method for making a correction to the assumed flow rates.

Consider the following equation for flow rate:

$$Q_{2\text{nd}} = Q_{1\text{st}} + \Delta Q$$

where $Q_{2\text{nd}}$ is the improved value that we would use for a second iteration, $Q_{1\text{st}}$ is the value used in the first iteration, and ΔQ is the modification we are seeking. We define a function F and apply it to the preceding equation:

$$F(Q_{2\text{nd}}) = F(Q_{1\text{st}} + \Delta Q)$$

Eventually we will let $F = \Sigma \Delta p$. Expanding the function F in a Taylor series gives

$$F(Q_{2\text{nd}}) = F(Q_{1\text{st}} + \Delta Q) = F(Q_{1\text{st}}) + \Delta Q \frac{dF}{dQ_{1\text{st}}} + (\Delta Q)^2 \frac{d^2F}{dQ_{1\text{st}}^2} + \dots = 0$$

The $(\Delta Q_{1st})^2$ and other higher order terms are assumed negligible. Using the first and second terms, we write

$$F(Q_{1st}) + \Delta Q \frac{dF}{dQ_{1st}} = 0$$

Solving for ΔQ ,

$$\Delta Q = - \frac{F(Q_{1st})}{(dF/dQ_{1st})} \quad (5.9)$$

We now evaluate the numerator and denominator of this expression as applied to the network problem. For each loop, we let the function F be the sum of the pressure drops in each loop:

$$F(Q_{1st}) = \Sigma \Delta p \quad (5.10)$$

and because $\Sigma \Delta p = \Sigma C f Q^2$ for each loop, we also have

$$F(Q_{1st}) = \Sigma C f Q_{1st}^2$$

Then, assuming a constant friction factor (or one experiencing small changes),

$$\frac{dF}{dQ_{1st}} = 2 \Sigma C f Q_{1st} \quad (5.11)$$

We can simplify this equation by again using

$$\Sigma \Delta p = \Sigma C f Q_{1st}^2$$

Dividing by Q_{1st} , we get

$$\Sigma (\Delta p / Q_{1st}) = \Sigma C f Q_{1st}$$

Equation 5.11 becomes

$$\frac{dF}{dQ_{1st}} = 2 \Sigma (\Delta p / Q_{1st}) \quad (5.12)$$

By substitution into Equation 5.9, we finally obtain for the correction

$$\Delta Q = -\frac{\Sigma \Delta p}{2 \Sigma (\Delta p / Q_{1st})} = \frac{\text{Equation 5.10}}{\text{Equation 5.12}} \quad (5.13)$$

So the flow rates we use for the second iteration are those we used for the first iteration plus the correction ΔQ , and ΔQ is calculated with Equation 5.13.

This correction must be applied to each flow rate in all the loops to obtain an improved value for the second and subsequent iterations. The terms in this correction are dependent on the sign.

Referring to Table 5.1, column 6 shows the pressure drops Δp that have been calculated, as well as $\Delta p / Q$ in column 7. The sign of the Δp values depends on the sign of the flow rate, while $\Delta p / Q$ is always positive. Using the data of the table,

$$\Delta Q = -\frac{\Sigma \Delta p}{2 \Sigma \Delta p / Q}$$

or for both loops,

$$\Delta Q = -\frac{-2.04 \times 10^4}{2(1.54 \times 10^6)} = 0.0066 \quad (\text{Loop I})$$

$$\Delta Q = -\frac{-5.91 \times 10^4}{2(3.94 \times 10^6)} = 0.0075 \quad (\text{Loop II})$$

So for the second iteration,

$$Q_{\text{pipe \#1}} = Q_{1st \text{ iteration}} + \Delta Q = 0.060 + 0.0066 = 0.067$$

For Q_2 , however, the modification must include a correction from both loops because pipe #2 is common to both loops. Thus,

$$Q_{\text{pipe \#2}} = Q_{1st \text{ iteration}} + \Delta Q = 0.020 + 0.0066 - 0.0075 = 0.019$$

where 0.0075 is negative because the flow rate in pipe #2 in Loop II is counterclockwise. The remainder of the flow rates are determined from Equation 5.8. Calculations are now repeated for the second iteration. The results of this second iteration are shown in Table 5.2.

Step 9: Continue the calculation procedure until a solution is obtained.

An improved value for these flow rates is again determined and the calculations are repeated as needed until convergence is achieved, that is, until $\Sigma \Delta p$ ($\sim \Delta Q$) in each loop becomes acceptably small. Calculation

TABLE 5.2. *Second iteration solution table for the network of Figure 5.2.*

Loop	Pipe	$Q \text{ m}^3/\text{s}$	Re	f	$\Delta p \text{ N/m}^2$	$\Delta p/Q$
I	1	0.067	3.81×10^5	0.021	2.28×10^4	3.42×10^5
	2	0.019	1.37×10^5	0.023	5.25×10^3	2.75×10^5
	3	-0.033	2.39×10^5	0.022	-1.86×10^4	5.58×10^5
	4	-0.058	3.34×10^5	0.021	-1.46×10^4	2.51×10^5
$\Sigma =$					-5.24×10^3	1.43×10^6
$\Delta Q = 5.24 \times 10^3 / (2(1.43 \times 10^6))$					$\Delta Q =$	0.001 8
II	5	0.047	3.40×10^5	0.022	3.72×10^4	7.83×10^5
	6	0.035	2.54×10^5	0.022	1.75×10^4	4.93×10^5
	7	-0.028	2.62×10^5	0.023	-5.65×10^4	2.06×10^6
	2	-0.019	1.37×10^5	0.023	-5.25×10^3	2.75×10^5
$\Sigma =$					-7.07×10^3	3.61×10^6
$\Delta Q = 7.07 \times 10^3 / (2(3.61 \times 10^6))$					$\Delta Q =$	0.001 0

TABLE 5.3. *Third iteration table for the network of Figure 5.2.*

Loop	Pipe	$Q \text{ m}^3/\text{s}$	Re	f	$\Delta p \text{ N/m}^2$	$\Delta p/Q$
I	1	0.068	3.92×10^5	0.021	2.40×10^4	3.51×10^5
	2	0.020	1.43×10^5	0.023	5.71×10^3	2.86×10^5
	3	-0.032	2.26×10^5	0.022	-1.67×10^4	5.28×10^5
	4	-0.057	3.24×10^5	0.021	-1.38×10^4	2.43×10^5
$\Sigma =$					-6.81×10^2	1.41×10^6
$\Delta Q = 6.81 \times 10^2 / (2(1.41 \times 10^6))$					$\Delta Q =$	0.000 2
II	5	0.048	3.47×10^5	0.022	3.87×10^4	7.99×10^5
	6	0.036	2.61×10^5	0.022	1.85×10^4	5.06×10^5
	7	-0.027	2.53×10^5	0.023	-5.26×10^4	1.98×10^6
	2	-0.020	1.43×10^5	0.023	-5.71×10^3	2.86×10^5
$\Sigma =$					-1.15×10^3	3.58×10^6
$\Delta Q = 1.51 \times 10^3 / (2(3.58 \times 10^6))$					$\Delta Q =$	0.000 2

results for the next iteration are shown in Table 5.3. The solution in that table yields a very small correction, which is acceptable. The solution then for each pipe is found to be

$$Q_1 = 0.069 \text{ m}^3/\text{s}$$

$$Q_4 = 0.056 \text{ m}^3/\text{s}$$

$$Q_7 = 0.026 \text{ m}^3/\text{s}$$

$$Q_2 = 0.020 \text{ m}^3/\text{s}$$

$$Q_5 = 0.049 \text{ m}^3/\text{s}$$

$$Q_3 = 0.031 \text{ m}^3/\text{s}$$

$$Q_6 = 0.037 \text{ m}^3/\text{s}$$

Several comments should be made at this point. With a spreadsheet, the user can go back to the first step and insert the results of the second iteration into the first step and in effect make several more iterations. Also, many spreadsheet programs contain a provision for automatically making successive iterations, so that a very high accuracy can be obtained in one table, without creating three tables. Summarizing, the procedure for the Hardy Cross Method is:

Step 1: Assume a flow direction in each pipe.

Step 2: Check continuity for the entire network.

Step 3: Write the continuity equation for each joint.

Step 4: Identify each loop and assign a positive direction.

Step 5: Write the pressure-drop equation for each loop.

Step 6: Simplify the continuity equations.

Step 7: Set up a solution table to summarize the calculations.

Step 8: Determine an improved values for the flow rates and begin the next iteration.

Step 9: Continue the calculation procedure until a solution is obtained.

5.2 Pipes in Parallel

As we have seen, some of the more complex problems in the study of pipeline design involve multiple-pipe systems. These might include two different sizes of pipes connected in series, several pipes connected in parallel, or hundreds of pipes in a network. In all of these cases, the equation of motion is the same, although the method of solving the equation(s) will be different. In this discussion, we examine pipes in parallel, which can be modeled as a pipe network with only one loop. Here, however, we examine an alternative method of solution.

Consider a pipeline installation in which it is desired to increase the volume flow rate from one point to another. One way of doing this is to install a larger pipe; however in some applications, it may be impossible to do so. Another method is to lay a pipeline parallel to the existing line for all or a portion of its length, as shown in Figure 5.4. This is called *looping*. The objective of analyzing such a system would be to determine the flow rate through each pipeline for a given pressure drop. It should be mentioned that a system can consist of any number of parallel pipes, and that there are computer programs available for analyzing such systems. In this

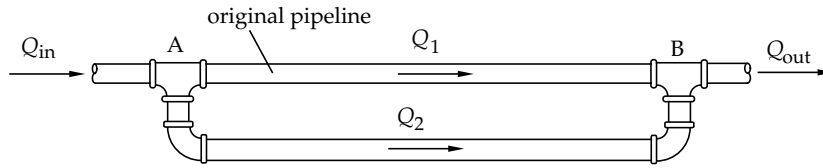


FIGURE 5.4. *Flow through parallel pipes.*

section, we examine only a two-pipe system to illustrate the calculation method.

Applying the continuity equation to the junction at A (or at B) in Figure 5.4 gives

$$Q_{\text{in}} = Q_{\text{out}} = Q_1 + Q_2$$

where Q_1 and Q_2 are the flow rates in each line. For a parallel-pipe system, the pressure drop along both pipes should be equal; that is,

$$\Delta p_1 = \Delta p_2$$

where Δp_1 is the pressure drop in the line where the flow rate is Q_1 . Applying the modified Bernoulli equation to line 1 gives

$$\frac{p_A g_c}{\rho g} + \frac{V_A^2}{2g} + z_A = \frac{p_B g_c}{\rho g} + \frac{V_B^2}{2g} + z_B + \frac{f_1 L_1}{D_1} \frac{V_1^2}{2g} + \sum K_1 \frac{V_1^2}{2g}$$

For inlet and outlet lines of the same diameter, $V_A = V_B$, and for points A and B at the same elevation, $z_A = z_B$. Assuming no (or negligible) minor losses, $\sum K = 0$. The modified Bernoulli equation reduces to

$$\frac{p_A g_c}{\rho g} - \frac{p_B g_c}{\rho g} = \frac{\Delta p_1 g_c}{\rho g} = \frac{f_1 L_1}{D_1} \frac{V_1^2}{2g} \quad (5.14)$$

This equation can be used directly, or, alternatively, pressure drop can be expressed in terms of flow rate. With $V_1 = Q_1/A_1 = 4Q_1/\pi D_1^2$, the preceding equation becomes

$$\Delta p_1 = \frac{f_1 L_1}{D_1} \frac{\rho V_1^2}{2g_c} = \frac{f_1 L_1}{D_1} \frac{\rho}{2g_c} \frac{16Q_1^2}{\pi^2 D_1^4}$$

$$\text{or } \Delta p_1 = \frac{8\rho L_1}{\pi^2 D_1^5 g_c} f_1 Q_1^2 = C_1 f_1 Q_1^2$$

where C_1 is a constant. Similarly, for the loop,

$$\Delta p_2 = \frac{8\rho L_2}{\pi^2 D_2^5 g_c} f_2 Q_2^2 = C_2 f_2 Q_2^2$$

Applying these equations to a parallel-pipe system is not difficult. Solving the equations simultaneously is sometimes frustrating, however. The type of problem encountered is usually one in which pressure drop is known and flow rate in the parallel lines is to be calculated. Once these flow rates are known, it may be desired or necessary to change the pressure drop so that power remains constant. Determining the flow rates under these new conditions is then required.

One method used to solve such a problem is called the *percentage method*, in which a selected pressure drop is applied to each pipe in the system and the corresponding flow rates are determined. It is assumed that for any other pressure drop, the ratio of these flow rates will remain the same. This method gives good results for turbulent flow, and it provides at least a starting point for flows near transition. In another method, the calculations are made and an iteration scheme is set up. Convergence is achieved usually after three trials for each pipe in the system.

As mentioned earlier, application of the equations to a parallel piping system allows for finding the flow rate in each line. The method is illustrated in the next example.

EXAMPLE 5.1. A line made of 6-nominal schedule 40 galvanized pipe conveys water. The pipeline is 1000 ft long and the pressure drop over this length is 8 psi. In order to increase the flow rate through this system, a 4-nominal schedule 40 galvanized pipe is added in parallel to the 6-in. line. (a) Determine the flow rate through the 6-nominal pipe in the original configuration. (b) For a combined flow rate through both lines of $1.25 \text{ ft}^3/\text{s}$ ($= Q_1 + Q_2$), determine the flow rate in each line. (See Figure 5.4.)

Solution: We approach the problem in the usual way, by obtaining properties from the appropriate tables:

water	$\rho = 1.94 \text{ slug/ft}^3$	$\mu = 1.9 \times 10^{-5} \text{ lbf}\cdot\text{s/ft}^2$	[App. Table B.1]
6-nom sch 40	$D = 0.5054 \text{ ft}$	$A = 0.2006 \text{ ft}^2$	
4-nom sch 40	$D = 0.3355 \text{ ft}$	$A = 0.08841 \text{ ft}^2$	[App. Table D.1]

galvanized surface

$$\varepsilon = 0.0005 \text{ ft}$$

[Table 3.1]

In part a of this problem, we are dealing only with the original pipeline, 6-nominal schedule 40. Referring to Figure 5.4, we write the modified Bernoulli equation from A to B along the Q_1 line as

$$\frac{p_A g_c}{\rho g} + \frac{V_A^2}{2g} + z_A = \frac{p_B g_c}{\rho g} + \frac{V_B^2}{2g} + z_B + \frac{f_1 L_1}{D_1} \frac{V_1^2}{2g} + \sum K_1 \frac{V_1^2}{2g}$$

Property evaluation:

$$\Delta p = 8(144) = 1152 \text{ psf}$$

$$V_A = V_B$$

$$z_A = z_B$$

$$L = 1000 \text{ ft}$$

$$\sum K = 0$$

where the minor losses are assumed negligible in this problem. Thus, the equation of motion reduces to

$$\Delta p = \frac{f_1 L_1}{D_1} \frac{\rho V_1^2}{2g_c}$$

Substituting,

$$1152 = \frac{f(1000)}{0.5054} \frac{1.94V^2}{2}$$

where the “1” subscript has been dropped. Rearranging and solving for velocity gives

$$fV^2 = 0.6$$

or

$$V = \sqrt{\frac{0.6}{f}} \quad (\text{i})$$

The Reynolds number is

$$\text{Re} = \frac{\rho V D}{\mu g_c} = \frac{1.94V(0.5054)}{1.9 \times 10^{-5}} = 5.16 \times 10^4 V \quad (\text{ii})$$

Relative roughness:

$$\frac{\varepsilon}{D} = \frac{0.0005}{0.5054} = 0.000989$$

We begin by assuming a friction factor corresponding to fully developed turbulent flow for $\varepsilon/D = 0.000989$:

$$\begin{array}{lll} \text{1st trial:} & f = 0.02 & V = 5.48 \text{ ft/s (Eq. i)} & \text{Re} = 2.82 \times 10^5 \text{ (Eq. ii)} \\ \text{2nd trial:} & f = 0.022 & V = 5.22 \text{ ft/s} & \text{Re} = 2.7 \times 10^5 \\ & f = 0.022 \text{ close enough} & & \end{array}$$

The velocity is therefore 5.22 ft/s. The volume flow rate then is

$$Q = AV = 0.2006(5.22)$$

$$(a) \quad \underline{Q = 1.05 \text{ ft}^3/\text{s} = 470 \text{ gpm}} \quad (\text{for the 6-nom line @ 1152 psf})$$

In part a, we were given a pressure drop in the 6-nominal line, and we found the volume flow rate. We use this same pressure drop to calculate a flow rate in the 4-nominal line. Proceeding in the same manner as before, we begin with the reduced equation of motion:

$$\Delta p = \frac{f_2 L_2}{D_2} \frac{\rho V_2^2}{2g_c}$$

Substituting,

$$1152 = \frac{f(1000)}{0.3355} \frac{1.94V^2}{2} = 2891fV^2$$

where the subscript on f and on V has been dropped. Rearranging and solving for velocity gives

$$V = \sqrt{\frac{0.398}{f}} \quad (\text{iii})$$

Also,

$$\text{Re} = \frac{\rho VD}{\mu g_c} = \frac{1.94V(0.3355)}{1.9 \times 10^{-5}} = 3.42 \times 10^4 V$$

We assume a friction factor corresponding to fully developed turbulent flow for $\varepsilon/D = 0.0005/0.3355 = 0.0015$:

$$\begin{array}{lll} \text{1st trial:} & f = 0.022 & V = 4.25 \text{ ft/s} & \text{Re} = 1.45 \times 10^5 \\ \text{2nd trial:} & f = 0.023 & V = 4.15 \text{ ft/s} & \text{Re} = 1.42 \times 10^5 \\ & f = 0.023 \text{ close enough} & & \end{array}$$

The velocity is therefore 4.15 ft/s. The volume flow rate then is

$$Q = AV = 0.08841(4.15)$$

$$(b) \quad \underline{Q = 0.367 \text{ ft}^3/\text{s}} \quad (\text{for the 4-nom line @ 1152 psf})$$

So when the pressure drop is 1152 psf (= 8 psi), the flow rates are

$$Q_1 = 1.05 \text{ ft}^3/\text{s} \quad \text{and} \quad Q_2 = 0.367 \text{ ft}^3/\text{s}$$

By adding the 4 in. line, the flow rate has increased from 1.05 ft³/s to 1.417 ft³/s (= 1.05 + 0.367), if there is a power source in the line, and if it can provide this increase.

In the percentage method, we use a proportionality to obtain the corrected flow rate in the pipe lines for a different configuration. We assume that these flow rates are in the same ratio for other pressure drops and for which turbulent flow exists. We calculate the total flow rate as

$$Q_{\text{total}} = Q_1 + Q_2 = 1.05 + 0.367 = 1.417$$

Dividing the preceding equation by Q_{total} gives

$$1 = \frac{Q_1}{Q_{\text{total}}} + \frac{Q_2}{Q_{\text{total}}}$$

At the modified condition, where $Q_{\text{new}} = 1.25 \text{ ft}^3/\text{s}$, we multiply the preceding equation by Q_{new} to get

$$Q_{\text{new}} = Q_{\text{new}} \frac{Q_1}{Q_{\text{total}}} + Q_{\text{new}} \frac{Q_2}{Q_{\text{total}}} = Q_{1\text{new}} + Q_{2\text{new}}$$

So the new flow rates are

$$Q_{1\text{new}} = 1.25 \frac{1.05}{1.417}$$

$$\underline{Q_{1\text{new}} = 0.926 \text{ ft}^3/\text{s}}$$

and

$$Q_{2\text{new}} = 1.25 \frac{0.367}{1.417}$$

$$\underline{Q_{2\text{new}} = 0.324 \text{ ft}^3/\text{s}}$$

As a check on these results, we calculate the pressure drop in each line, and they should be equal:

$$\Delta p_1 = \frac{f_1 L_1}{D_1} \frac{\rho V_1^2}{2g_c} = \frac{0.023(1000)}{0.5054} \frac{1.94(5.22)^2}{2}$$

$$\underline{\Delta p_1 = 1203 \text{ psf}} \quad (6\text{-nominal})$$

Similarly, we calculate

$$\Delta p_2 = \frac{0.023(1000)}{0.3355} \frac{1.94(4.15)^2}{2}$$

$$\underline{\Delta p_2 = 1145 \text{ psf}} \quad (4\text{-nominal})$$

These are close enough; the error is less than 5% and could be attributed to errors in reading the Moody diagram.

The percentage method works well if the flows are turbulent and one iteration is sufficient. For all cases, however, Equations ii and iii of the previous example can be used in an iteration scheme. The procedure is to assume a value for the friction factors corresponding to fully developed flow at the correct ε/D ratio. Then, Equations ii and iii are used to find velocity; the Reynolds number is calculated, and an improved value of the friction factor is determined. The process is repeated until convergence is achieved.

5.3 Measurement of Flow Rate in Closed Conduits

Flow rate in a conduit can be measured by using a **rate meter**. A rate meter is a device that is inserted into a pipe (or tube), which allows for determining the volume rate of flow in the line. The meters we discuss in this section are the **turbine meter**, the **rotameter** (or **variable-area meter**), the **venturi meter**, the **orifice meter**, and the **elbow meter**. Other meters, such as the **nozzle type** and the **totalizing meter**, operate on similar principles but will not be discussed here.

Turbine Meter

Figure 5.5 is a sketch of a turbine meter. It consists of a pipe or a tube with appropriate pipe fittings (not shown) at each end. Inside the tube are flow straighteners on both sides of a propeller or turbine. Flow entering the meter passes through the straighteners and causes the propeller to rotate at an angular velocity that is proportional to the flow rate. A magnetic pickup senses blade passages and transmits a signal to a readout device

that totals the pulses. Turbine meters are usually made of stainless steel or brass, but special metals are available. Typically turbine meters are accurate to within $\pm 1\%$.

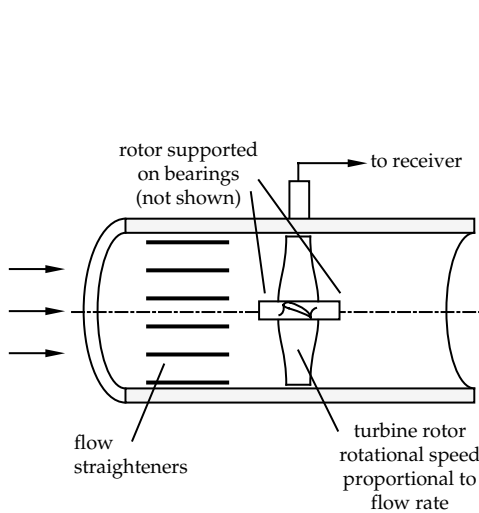


FIGURE 5.5. Sketch of a turbine meter.

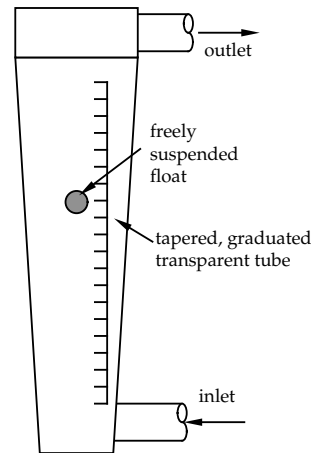


FIGURE 5.6. Schematic of a rotameter or variable-area meter.

Rotameter or Variable-Area Meter

Figure 5.6 is a schematic of a rotameter, also called a variable-area meter. It consists of a tapered, graduated, vertically oriented, transparent tube (usually glass or plexiglas). Appropriate pipe fittings are at each end of the tube. Inside is a float that is free to move. The float can be spherical or cylindrical with its axis vertical. Flow enters the meter at the bottom and raises the float to an equilibrium position. The higher the float position, the larger the annular area between the float and the tube. When the forces due to drag, buoyancy, and gravity are all balanced, the float reaches an equilibrium position. Flow rate is determined by reading the scale at the float position. Alternatively, an electronic sensor can transmit a signal to a remote readout device. Accuracy of a rotameter is usually within $\pm 1\%$ on expensive units and within $\pm 5\%$ on less expensive ones.

Venturi Meter

Another type of rate meter is one that introduces a flow constriction, which in turn causes a change in one of the measurable properties of the flow. The measured property change is then related to the flow rate

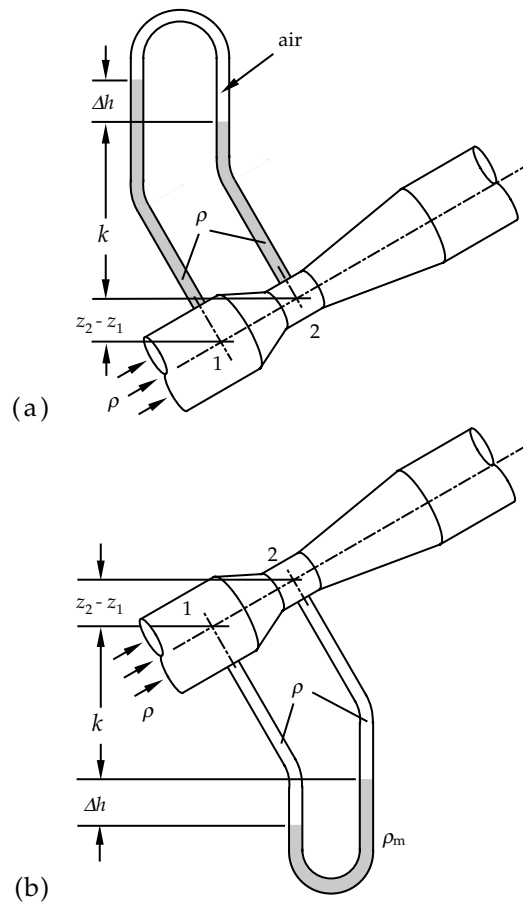


FIGURE 5.7. A venturi meter shown in two configurations.

through the meter. A venturi meter is an example of this kind of meter, and is shown in two configurations in Figure 5.7.

The venturi meter consists of an upstream section, a convergent section leading to a throat, and a divergent section. The upstream section is the same diameter as the pipe line and attaches to it. At the upstream section and at the throat are static pressure taps that can be connected to two legs of a manometer. It is considered good practice to provide at least 10 diameters of approach piping upstream of the meter to ensure that the flow is fully developed and uniform at the meter entrance. The size of the venturi meter is usually specified by a pipe and a throat size. For example, a 6 × 4 meter attaches to a 6-nominal pipeline and has a throat diameter corresponding to a 4-nominal pipe.

When fluid flows through the meter, there is an increase in the velocity from the upstream section to the throat. There is a corresponding pressure drop also from upstream to the throat. The pressure drop increases with increasing flow rate through the meter. A graph of pressure drop versus flow rate for the meter is referred to as a **calibration curve**. Often, a calibration curve must be determined experimentally; that is, flow through the meter must be collected over a certain time interval to find the actual volume flow rate. The corresponding pressure drop must also be measured to find the actual flow rate Q_{ac} as a function of pressure drop Δp or head loss Δh .

Standards for venturi meters (and other constriction meters) can be found in various publications. (*Fluid Meters—Their Theory and Application*, 6th ed., published by ASME, 1971, is especially recommended.) Such publications show construction details and recommendations that allow the user to successfully select or design a meter for a specific application.

Discharge Coefficient

When consulting a meters handbook or a similar reference text, the information provided usually consists of meter construction details and a dimensionless graph of **discharge coefficient** versus Reynolds number. The discharge coefficient relates the actual flow rate through the meter to the theoretical flow rate predicted by the Bernoulli equation. Construction details contain dimensions that are expressed in terms of pipe diameters. For example, the upstream static pressure tap might be located one-half pipe diameter ($D_1/2$) from the edge of the convergent section. The meter length might be expressed as 10 pipe diameters ($10D_1$); and so on. Figure 5.8 shows some typical details on pressure tap locations for a venturi meter. When constructed according to the recommended dimensions, the applicable discharge coefficient versus Reynolds number relationship (graph or equation) can then be used to obtain a calibration curve for the meter, *without experimentally calibrating the meter*.

To investigate this point further, we derive equations for both configurations shown in Figure 5.7. For an incompressible fluid flowing through either configuration, the continuity equation is

$$Q = A_1 V_1 = A_2 V_2$$

Because the throat area A_2 is less than the upstream area A_1 , continuity predicts that $V_1 < V_2$. Thus the fluid velocity must increase at the throat. For frictionless flow through the meter, the Bernoulli equation is

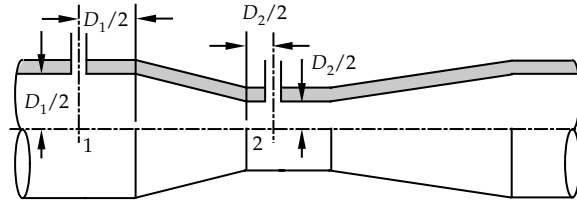


FIGURE 5.8. Section view of a venturi meter showing tap locations.

$$\frac{p_1 g_c}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2 g_c}{\rho g} + \frac{V_2^2}{2g} + z_2 \quad (5.15)$$

Rearranging and substituting $V = Q/A$ gives

$$\frac{(p_1 - p_2)g_c}{\rho g} + z_1 - z_2 = \frac{Q^2}{2g} \left(\frac{1}{A_2^2} - \frac{1}{A_1^2} \right) = \frac{Q^2}{2gA_2^2} \left(1 - \frac{A_2^2}{A_1^2} \right)$$

Solving for flow rate, we get

$$Q = A_2 \sqrt{\frac{2g \{ [(p_1 - p_2)g_c / \rho g] + (z_1 - z_2) \}}{1 - A_2^2 / A_1^2}}$$

Noting that $A_2^2 / A_1^2 = D_2^4 / D_1^4$ and that this is the theoretical equation for the venturi meter, assuming frictionless, incompressible flow, we write:

$$Q_{th} = A_2 \sqrt{\frac{2g \{ [(p_1 - p_2)g_c / \rho g] + (z_1 - z_2) \}}{1 - D_2^4 / D_1^4}} \quad (5.16)$$

where the “th” subscript refers to a theoretical value for the flow rate. For the configuration of Figure 5.7a, we have the following for the manometer:

$$p_1 - \frac{\rho g}{g_c} [(z_2 - z_1) + k + \Delta h] = p_2 - \frac{\rho g}{g_c} k - \frac{\rho_{air} g}{g_c} \Delta h$$

The density of air is small compared to the liquid density, so the term containing ρ_{air} can be neglected. Rearranging and simplifying gives

$$\frac{(p_1 - p_2)g_c}{\rho g} + z_1 - z_2 = \Delta h$$

Substituting into Equation 5.16 yields

$$Q_{th} = A_2 \sqrt{\frac{2g\Delta h}{1 - D_2^4/D_1^4}} \quad \left(\begin{array}{c} \text{air over liquid} \\ \text{manometer} \end{array} \right) \quad (5.17)$$

Thus, the theoretical flow rate through the meter is related to the manometer reading such that the meter orientation is not important (because $z_2 - z_1$ no longer appears in Equation 5.17). The same equation results whether the meter is horizontal, inclined, or vertical.

For the two-fluid manometer of Figure 5.7b, we write

$$p_1 + \frac{\rho g}{g_c} (k + \Delta h) = p_2 + \frac{\rho g}{g_c} [(z_2 - z_1) + k] + \frac{\rho_m g}{g_c} \Delta h$$

Rearranging,

$$\frac{(p_1 - p_2)g_c}{\rho g} + z_1 - z_2 = \Delta h \frac{\rho_m - \rho}{\rho} = \Delta h \left(\frac{\rho_m}{\rho} - 1 \right)$$

Substituting into Equation 5.16 gives

$$Q_{th} = A_2 \sqrt{\frac{2g\Delta h(\rho_m/\rho - 1)}{1 - D_2^4/D_1^4}} \quad \left(\begin{array}{c} \text{two-liquid} \\ \text{manometer} \end{array} \right) \quad (5.18)$$

This equation differs from that of Equation 5.17 by the $(\rho_m/\rho - 1)$ term, which results from having a two-liquid manometer, as indicated in Figure 5.7b.

For a given meter, liquid, and manometer fluid, the variables D_1 , D_2 , A_2 , ρ , and ρ_m are all known. Therefore a curve of theoretical flow rate Q_{th} versus head loss Δh using Equation 5.17 or 5.18 can be constructed. Consider the line labeled Q_{th} in Figure 5.9 as such a curve. Suppose next that this same meter is taken to the laboratory and actual measurements of Q_{ac} versus Δh are made (i.e., a calibration curve is determined). Typical data are also plotted in Figure 5.9, giving the line labeled as Q_{ac} .

Now for any pressure drop Δh_i , there are two corresponding flow rates: Q_{ac} and Q_{th} . The ratio of these flow rates is the venturi discharge coefficient C_v , defined as

$$C_v = \frac{Q_{ac}}{Q_{th}} \quad (5.19)$$

The coefficient C_v can be calculated for many Δh values and will vary over the entire range. The flow rate Q_{ac} will always be less than Q_{th} because of frictional effects, which are not accounted for in the Bernoulli equation. For each C_v calculated, we can also calculate a throat Reynolds number, defined as

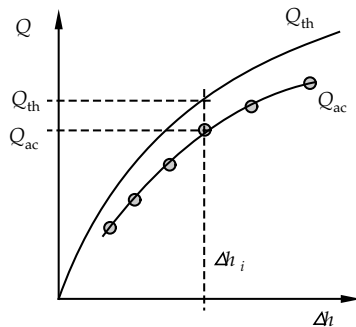


FIGURE 5.9. Flow rate as a function of pressure drop for a venturi meter.

$$\text{Re} = \frac{V_2 D_2}{\nu} = \frac{4\rho Q_{ac}}{\pi D_2 \mu g_c} \quad (5.20)$$

The Reynolds number is based on the actual flow rate Q_{ac} and on the throat diameter D_2 . The discharge coefficient for a venturi meter with a rough cast inlet varies from 0.95 to 0.984 for a Reynolds number that varies from 3×10^4 to 2×10^5 . Beyond 2×10^5 , the coefficient C_v is a constant and equal to 0.984. For a precisely machined meter, the discharge coefficient can be as high as 0.995.

The procedure for generating a calibration curve (Q_{ac} versus Δh) without taking data is simple but tedious:

1. Obtain a recommended configuration for the meter from a handbook (or another appropriate source).
2. Make physical measurements on the meter and determine liquid density as well as manometer fluid density.
3. Construct a graph of Q_{th} versus Δh .
4. Refer to discharge coefficient information provided with the meter.
5. At various flow rates, calculate Δh , Re , and C_v .
6. Use the C_v numbers and Q_{th} to determine Q_{ac} .
7. Graph Q_{ac} versus Δh .

The procedure for calibrating a venturi meter is illustrated in the following example.

EXAMPLE 5.2. A pipeline company is responsible for pumping hexane from a manufacturer to a distributor. The company wishes to install a meter in the pipeline to monitor the flow rate. A fluid meters reference book was consulted, and it has been decided to install a 6 × 4 venturi meter in the line. The discharge coefficient for the meter is a constant at 0.984 for a Reynolds number greater than 2×10^5 , provided that pressure taps are located as indicated in Figure 5.8. The meter is to be installed in an upward-sloping configuration that makes an angle of 20° with the horizontal, as shown in

Figure 5.10. It is proposed to use a hexane-over-mercury manometer. Generate a calibration curve for the meter for a flow rate up to $0.15 \text{ m}^3/\text{s}$.

Solution: From the Appendix we read:

hexane	$\rho = 0.657(1\,000) \text{ kg/m}^3$	$\mu = 0.297 \times 10^{-3} \text{ N}\cdot\text{s/m}^2$	
mercury	$\rho_m = 13.6(1\,000) \text{ kg/m}^3$		[Appendix Table B.1]

6-nom sch 40	$D_1 = 15.41 \text{ cm}$	$A_1 = 186.50 \text{ cm}^2$	
4-nom sch 40	$D_2 = 10.23 \text{ cm}$	$A_2 = 82.19 \text{ cm}^2$	[Appendix Table C.1]

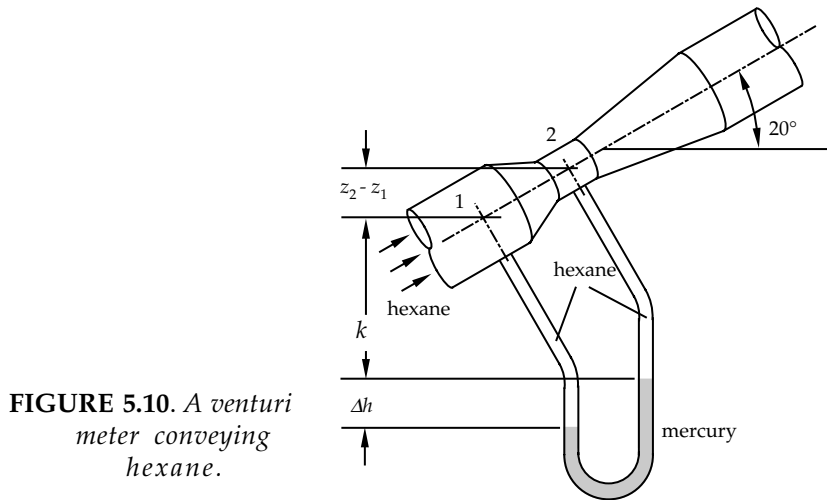


FIGURE 5.10. A venturi meter conveying hexane.

For the manometer, we have

$$\frac{(p_1 - p_2)g_c}{\rho g} + z_1 - z_2 = \Delta h \left(\frac{\rho_m}{\rho} - 1 \right) = \Delta h \left(\frac{13.6(1\,000)}{0.657(1\,000)} - 1 \right) = 19.7 \Delta h$$

Substituting into Equation 5.18 gives

$$Q_{th} = A_2 \sqrt{\frac{2g\Delta h(\rho_m/\rho - 1)}{1 - D_2^4/D_1^4}} \quad \left(\begin{array}{l} \text{two-liquid} \\ \text{manometer} \end{array} \right) \quad (5.18)$$

$$Q_{th} = 82.19 \times 10^{-4} \left(\frac{2(9.81)(19.7)\Delta h}{1 - \left(\frac{10.23}{15.41} \right)^4} \right)^{1/2}$$

or $Q_{th} = 0.18 \sqrt{\Delta h}$

A tabulation of Q_{th} versus Δh is provided in Table 5.4.

The Reynolds number for the flowing hexane is

$$Re = \frac{V_2 D_2}{\nu} = \frac{4 \rho Q_{ac}}{\pi D_2 \mu g_c} = \frac{4(0.657)(1\,000)C_v Q_{th}}{\pi(0.102\,3)(0.297 \times 10^{-3})}$$

or $\frac{Re}{C_v} = 2.75 \times 10^7 Q_{th}$

Results of calculations made with this equation are also provided in Table 5.4. We see that for all Δh values greater than 0.2, the ratio Re/C_v is greater than 2×10^5 . Now because C_v is always less than one, the Reynolds number will be greater than 2×10^5 . So for the calculations of this example, $C_v = 0.984$, which is listed in the table. The actual flow rate Q_{ac} ($= C_v Q_{th}$) is also listed. A graph of flow rate versus Δh is provided in Figure 5.11.

TABLE 5.4. Results of calculations made for Example 5.2.

Δh m	Q_{th} m^3/s	Re/C_v	C_v	$Q_{ac} = C_v Q_{th}$ m^3/s
0	0	0	—	0
0.2	0.081	2.21×10^6	0.984	0.08
0.4	0.114	3.13×10^6	0.984	0.112
0.6	0.139	3.83×10^6	0.984	0.137
0.8	0.161	4.43×10^6	0.984	0.158

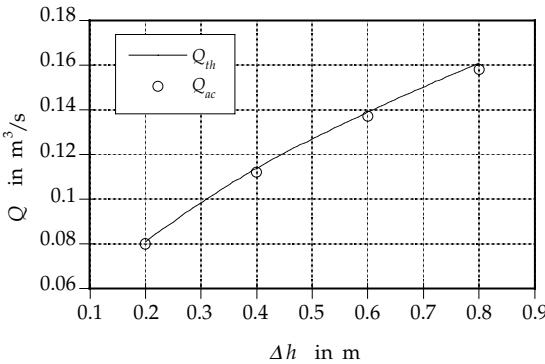


FIGURE 5.11. Solution to Example 5.2 shown graphically.

Orifice Meter

Another type of constriction device is the orifice meter, shown schematically in Figure 5.12. The orifice meter consists of a flat plate with a hole that is inserted in a pipeline, conveniently between flanges. The hole can be drilled so that the orifice is either sharp edged or square edged. Flow through the plate follows a pattern similar to that shown in Figure 5.12. Downstream of the plate, the flow reaches a point of minimum area called a **vena contracta**.

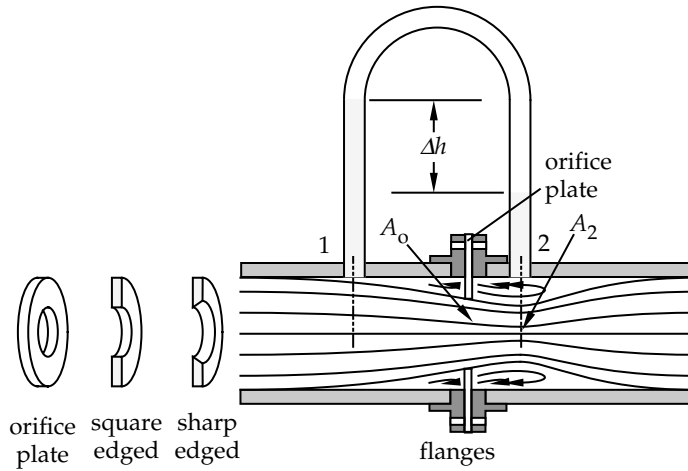


FIGURE 5.12. Orifice plates and streamlines of flow through an orifice meter.

Pressure taps are attached to the meter at various recommended locations, as shown in Figure 5.13: $1D$ and $\frac{1}{2}D$ locations; or $2\frac{1}{2}D$ and $8D$ locations; or flange taps where holes are drilled through the flanges; or corner taps where holes are drilled through the pipe wall leading to the very edge of the plate.

Referring to Figure 5.12, we can identify three different areas associated with the orifice meter:

A_1 = upstream area corresponding to pipeline diameter

A_2 = flow area at the vena contracta

A_0 = orifice hole area (calculated with the orifice diameter)

The area at section 2, where pressure p_2 is measured (the vena contracta), is unknown, but it can be expressed in terms of the orifice area:

$$A_2 = C_c A_0$$

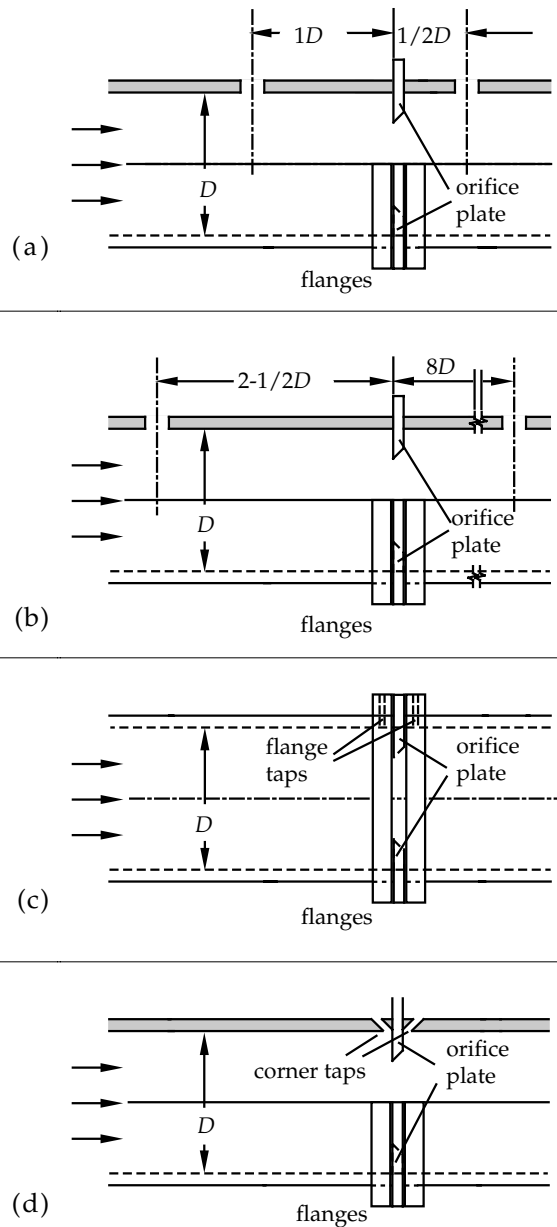


FIGURE 5.13. Recommended locations for pressure taps for an orifice meter.

where C_c is a *contraction coefficient*.

Applying Bernoulli's equation to the points where the manometer attaches (i.e., A_1 and A_2) yields the same result as for the venturi meter:

$$Q_{th} = C_c A_o \sqrt{\frac{2(p_1 - p_2)g_c}{\rho(1 - D_2^4/D_1^4)}} \quad \left(\begin{array}{c} \text{air-over-liquid} \\ \text{manometer} \end{array} \right) \quad (5.21)$$

where $C_c A_o$ has been substituted for A_2 . The actual flow rate through the meter is considerably less than the theoretical flow rate. We define a discharge coefficient as

$$C = \frac{Q_{ac}}{Q_{th}}$$

and combine with Equation 5.21 to obtain

$$Q_{ac} = CC_c A_o \sqrt{\frac{2(p_1 - p_2)g_c}{\rho(1 - D_2^4/D_1^4)}} \quad (5.22)$$

To simplify this formulation, we rewrite Equations 5.21 and 5.22 to get

$$Q_{th} \approx A_o \sqrt{\frac{2(p_1 - p_2)g_c}{\rho(1 - D_o^4/D_1^4)}} \quad \left(\begin{array}{c} \text{air-over-liquid} \\ \text{manometer} \end{array} \right) \quad (5.23)$$

$$Q_{ac} = C_o A_o \sqrt{\frac{2(p_1 - p_2)g_c}{\rho(1 - D_o^4/D_1^4)}} \quad \left(\begin{array}{c} \text{air-over-liquid} \\ \text{manometer} \end{array} \right) \quad (5.24)$$

The area of significance in these equations is A_o , the orifice area, rather than A_2 , where the pressure p_2 is measured. In addition, an orifice coefficient is defined as

$$C_o = CC_c = \frac{Q_{ac}}{Q_{th}}$$

Remember that the static pressure p_2 is not measured at the orifice area A_o , but this discrepancy and the losses encountered by the fluid are accounted for in the overall coefficient C_o . Tests on many meters have resulted in an equation (called the **Stolz equation**) for the loss coefficient:

$$C_o = 0.5959 + 0.0312\beta^{2.1} - 0.184\beta^8 + 0.0029\beta^{2.5} \left(\frac{10^6}{Re\beta} \right)^{0.75} \\ + 0.09L_1 \left(\frac{\beta^4}{1 - \beta^4} \right) - L_2 (0.00337\beta^3) \quad (5.25)$$

$$\text{where } \text{Re} = \frac{\rho V_o D_o}{\mu g_c} = \frac{4\rho Q_{ac}}{\pi D_o \mu g_c} \quad \beta = \frac{D_o}{D_1}$$

$$\begin{aligned} L_1 &= 0 && \text{(for corner taps)} \\ L_1 &= 1/D_1 && \text{(for flange taps)} \\ L_1 &= 1 && \text{(for } 1D \text{ \& } \tfrac{1}{2}D \text{ taps)} \end{aligned}$$

and if $L_1 \geq 0.433$, the coefficient of the $\left(\frac{\beta^4}{1 - \beta^4}\right)$ term becomes 0.039.

$$\begin{aligned} L_2 &= 0 && \text{(for corner taps)} \\ L_2 &= 1/D_1 && \text{(for flange taps)} \\ L_2 &= 0.5 - E/D_1 && \text{(for } 1D \text{ \& } \tfrac{1}{2}D \text{ taps) and} \end{aligned}$$

E = orifice plate thickness (nominally 0.25 in. = 0.635 cm)

There are other equations, as well as graphs and tables, for the orifice coefficient C_o , but Equation 5.25 is one of the simplest to use. Note that the Stolz equation is not recommended for $\frac{1}{2}D$ and $8D$ taps.

As with the venturi meter, it is considered good practice to allow at least a 10-diameter approach section upstream of the meter.

EXAMPLE 5.3. An orifice meter is set up in a horizontal flow line with $1D$ and $\frac{1}{2}D$ taps. As water flows through the meter, pressure transducers attached to the taps read $p_1 = 17.7$ psia and $p_2 = 14.6$ psia. If the flow line is 2-nominal schedule 40 and the orifice hole diameter is 1.2 in., determine the actual flow rate through the meter.

Solution: From the various property tables, we read

$$\text{water} \quad \rho = 1.94 \text{ slug/ft}^3 \quad \mu = 1.9 \times 10^{-5} \text{ lbf}\cdot\text{s/ft}^2 \quad [\text{Appendix Table B.1}]$$

$$2\text{-nom sch 40} \quad D_1 = 0.1723 \text{ ft} \quad A_1 = 0.02330 \text{ ft}^2 \quad [\text{Appendix Table D.1}]$$

Also, we were given $D_o = 1.2 \text{ in.} = 0.1 \text{ ft}$. The theoretical flow rate is found with

$$Q_{th} \approx A_o \sqrt{\frac{2(p_1 - p_2)g_c}{\rho(1 - D_o^4/D_1^4)}}$$

Evaluating terms,

$$A_o = \frac{\pi D_o^2}{4} = \frac{\pi(0.1)^2}{4} = 0.00785 \text{ ft}^2$$

$$\frac{(p_1 - p_2)g_c}{\rho} = \frac{(17.7 - 14.6)(144)}{1.94} = 230 \text{ ft}^2/\text{s}^2$$

$$1 - \frac{D_o^4}{D_1^4} = 1 - \frac{(0.1)^4}{(0.1723)^4} = 0.8865$$

Substituting,

$$Q_{th} = 0.00785 \sqrt{\frac{2(230)}{0.8865}} = 0.179 \text{ ft}^3/\text{s}$$

In order to find the actual flow rate, we must first calculate the orifice coefficient C_o using the Stolz equation. The terms in that equation are evaluated as:

$$\beta = \frac{D_o}{D_1} = \frac{0.1}{0.1723} = 0.5804$$

$$L_1 = 1 \geq 0.4333 \text{ so the coefficient of } \left(\frac{\beta^4}{1 - \beta^4} \right) \text{ becomes } 0.039$$

$$E = 0.25 \text{ in.} = 0.0208 \text{ ft (no plate thickness given, so assume the nominal size of } \frac{1}{4} \text{ in.)}$$

$$L_2 = 0.5 - \frac{E}{D_1} = 0.5 - \frac{0.0208}{0.1723} = 0.3791$$

$$\text{Re} = \frac{V_o D_o}{\nu} = \frac{4\rho Q_{ac}}{\pi D_o \mu g_c} = \frac{4(1.94)(0.179)C_o}{\pi(0.1)(1.9 \times 10^{-5})} = 2.33 \times 10^5 C_o$$

In terms of Reynolds number, the Stolz equation becomes, after substitution,

$$\begin{aligned} C_o = & 0.5959 + 0.0312(0.5804)^{2.1} - 0.184(0.5804)^8 \\ & + 0.0029(0.5804)^{2.5} \left(\frac{10^6}{\text{Re}(0.5804)} \right)^{0.75} \\ & + 0.039 \left(\frac{(0.5804)^4}{1 - (0.5804)^4} \right) - 0.3791[0.00337(0.5804)^3] \end{aligned}$$

which becomes

$$C_o = 0.5959 + 0.009954 - 0.002369 + \frac{35.39}{\text{Re}^{0.75}} + 0.004992 - 0.0002498$$

$$\text{or } C_o = 0.6082 + \frac{35.39}{\text{Re}^{0.75}}$$

The procedure for solving this problem begins by assuming a value for C_o , calculating the Reynolds number, and substituting into the Stolz equation. The calculations are repeated until convergence is achieved. Assume:

$$\text{1st trial: } C_o = 0.6; \quad Re = 2.33 \times 10^5(0.6) = 1.40 \times 10^5$$

$$\text{and} \quad C_o = 0.6082 + \frac{35.39}{(1.40 \times 10^5)^{0.75}} = 0.6083$$

$$\text{2nd trial: } C_o = 0.6083; \quad Re = 1.422 \times 10^5 \quad \text{and} \quad C_o = 0.6083$$

The method converges rapidly. The actual flow rate then is

$$Q_{ac} = C_o Q_{th} = 0.6083(0.179)$$

$$\underline{Q_{ac} = 0.109 \text{ ft}^3/\text{s}}$$

Elbow Meter

Another type of meter—known as an **elbow meter**—can also be used to measure flow rate in a pipe. For an existing pipeline containing elbows, an elbow meter is perhaps the easiest meter to set up. All that would be required is to drill and tap a couple of holes in the elbow, and attach them to a device for measuring pressure drop. A sketch of an elbow meter installation is shown in Figure 5.14. The pressure taps *must* be aligned.

The internal dimensions of R and D must be known for the elbow of interest. Holes are drilled in the specific locations shown to accept 1/8th nominal pipe threads or something different if desired, and they must be perfectly aligned. To calibrate the meter, pressure measurements are made to determine the pressure drop that exists as well as the corresponding actual flow rate. The actual flow rate through the meter can be calculated with

$$Q_{ac} = A K \sqrt{\frac{R}{D} \frac{\Delta p}{\rho}}$$

where A = cross sectional area = $\pi D^2/4$ and K is a correction factor given by

$$K = 1 - \frac{6.5}{\sqrt{Re}} = \text{correction factor}$$

and valid over the Reynolds number range

$$10^4 \leq \text{Re} = \rho V D / \mu \leq 10^6$$

The pipeline diameter D is easily found, but note the dependence of K on the radius centerline elbow R . Values of the elbow radius can be found in the table accompanying Figure 5.14.

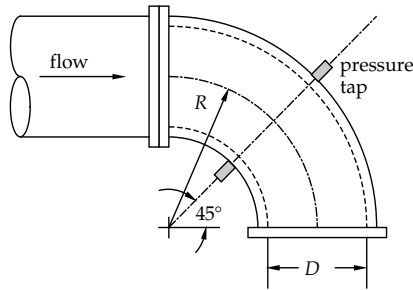


FIGURE 5.14. Schematic of an elbow meter showing pressure tap locations.

Pipe Size Schedule 40	Inside Diameter D		Short Radius Elbow R	
	cm	ft	cm	inches
1	2.664	0.08742	2.54	1
1 $\frac{1}{2}$	4.090	0.1342	3.81	1.5
2	5.252	0.1723	5.08	2
2 $\frac{1}{2}$	6.271	0.2058	6.35	2.5
4	10.23	0.3355	10.2	4
6	15.41	0.5054	15.2	6
8	20.64	0.6771	20.3	8
10	26.04	0.8542	25.4	10
12	31.12	1.021	30.5	12

EXAMPLE 5.4. The flow rate of water in a 4-nominal schedule 80 pipe is designed to be 150 gpm. It is desired to obtain a measurement as a check. Rather than use a venturi or an orifice meter, a short radius 90° elbow in the pipeline has been located, and a manometer has been attached. If the flow rate is indeed 150 gpm, what is the expected pressure drop?

Solution: For a 4-nominal, short radius elbow, we have

$$R = 4 \text{ in.} = 0.333 \text{ ft} \quad (\text{Figure 5.14})$$

$$D = 0.3198 \text{ ft} \quad (\text{Appendix Table D.1})$$

We calculate

$$A = \frac{\pi D^2}{4} = \frac{\pi (0.3198)^2}{4} = 0.08 \text{ ft}^2$$

For water,

$$\rho = 1.94 \text{ slug/ft}^3 \quad \mu = 1.9 \times 10^{-5} \text{ lbf}\cdot\text{s/ft}^2$$

A calibration curve is one that relates the actual volume flow rate through the meter to the pressure drop over a large range of values, using whatever units are convenient. We are calculating the *expected* calibration results for only one data point in such a curve. In this example, because we are using a manometer, we will express pressure drop in terms of inches of water. The calculation for a flow rate of 150 gpm is

$$Q = 150 \text{ gpm} (2.229 \times 10^{-3}) = 0.334 \text{ ft}^3/\text{s}$$

The flow velocity then is

$$V = \frac{Q}{A} = \frac{0.334}{0.0803} = 4.16 \text{ ft/s}$$

To find Δp , we must first find K , which in turn depends on Reynolds number:

$$\text{Re} = \frac{\rho V D}{\mu} = \frac{1.94(4.16)(0.3198)}{1.9 \times 10^{-5}} = 1.36 \times 10^5$$

Then

$$K = 1 - \frac{6.5}{\sqrt{\text{Re}}} = 1 - \frac{6.5}{\sqrt{1.36 \times 10^5}} = 0.982$$

For an elbow meter,

$$Q_{\text{ac}} = A K \sqrt{\frac{R}{D} \frac{\Delta p}{\rho}}$$

Rearranging and solving for Δp gives

$$\Delta p = \rho \left(\frac{Q_{\text{ac}}}{A K} \right)^2 \frac{D}{R}$$

Substituting

$$\Delta p = (1.94) \left(\frac{0.334}{0.0803(0.982)} \right)^2 \frac{0.3198}{0.333}$$

or $\Delta p = 33.4 \text{ lbf/ft}^2$

Now in terms of a column of water,

$$\Delta h = \frac{\Delta p}{\rho g} = \frac{33.4}{1.94(32.2)} = 0.534 \text{ ft of water}$$

or $\Delta h = 6.41 \text{ in. of water}$

For calculations of this type on an elbow that has not been tested in the laboratory, the result is accurate to within $\pm 4\%$. That is, for a reading Δh of 6.41 in. of water, the flow rate can be as high as 156 gpm or as low as 144 gpm. Note that a manometer is not readable to the nearest hundredth of an inch. Typically a reading will be to the nearest tenth of an inch. Also note the dependence of the results on having an equation for the correction factor K .

So based on the results here, one data point on the calibration curve is

$$\Delta h = 6.4 \text{ in. of water} \quad Q = 150 \text{ gpm} \pm 4\%$$

Compressible Flow Through a Meter

When a compressible fluid (vapor or gas) flows through a meter, compressibility effects must be accounted for. This is done by introducing a **compressibility factor**, which can be determined analytically for some meters (venturi). For an orifice meter, on the other hand, the compressibility factor must be measured.

The equations and formulation developed thus far were for incompressible flow through a meter. For compressible flows, the derivation is somewhat different. When the fluid flows through a meter and encounters a change in area, the velocity changes as does the pressure. When pressure changes, the density of the fluid changes and this effect must be accounted for in order to obtain accurate results. To account for compressibility, we will rewrite the descriptive equations.

Consider isentropic, subsonic, steady flow of an ideal gas through a venturi meter (Figure 5.8). The continuity equation is

$$\rho_1 A_1 V_1 = \rho_2 A_2 V_2 = \dot{m}_{\text{isentropic}} = \dot{m}_s \quad (5.26)$$

Neglecting changes in potential energy (negligible compared to changes in enthalpy), the energy equation is

$$h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2} \quad (5.27)$$

The enthalpy change can be found by assuming that the compressible fluid is ideal:

$$h_1 - h_2 = C_p(T_1 - T_2)$$

Substituting this equation and Equation 5.27 into Equation 5.26 and rearranging, we get

$$C_p T_1 + \frac{\dot{m}_s^2}{2\rho_1^2 A_1^2} = C_p T_2 + \frac{\dot{m}_s^2}{2\rho_2^2 A_2^2}$$

or

$$\dot{m}_s^2 \left(\frac{1}{\rho_2^2 A_2^2} - \frac{1}{\rho_1^2 A_1^2} \right) = 2C_p(T_1 - T_2) = 2C_p T_1 \left(1 - \frac{T_2}{T_1} \right) \quad (5.28)$$

If we assume an isentropic compression process through the meter, then we can write

$$\frac{p_2}{p_1} = \left(\frac{T_2}{T_1} \right)^{\frac{\gamma}{\gamma-1}}$$

where γ is the ratio of specific heats ($\gamma = C_p/C_v$). Also, recall that for an ideal gas,

$$C_p = \frac{R\gamma}{\gamma-1}$$

Substituting the preceding two equations into Equation 5.28, rearranging and simplifying, we get

$$\frac{\dot{m}_s^2}{\rho_2^2 A_2^2} \left(1 - \frac{\rho_2^2 A_2^2}{\rho_1^2 A_1^2} \right) = 2 \frac{R\gamma}{\gamma-1} T_1 \left[1 - \left(\frac{p_2}{p_1} \right)^{\frac{\gamma-1}{\gamma}} \right] \quad (5.29)$$

For an ideal gas, we write $\rho = p/RT$. Substituting for the RT_1 term in the preceding equation yields

$$\frac{\dot{m}_s^2}{A_2^2} = 2\rho_2^2 \frac{\gamma}{\gamma-1} \left(\frac{p_1}{\rho_1} \right) \frac{1 - (p_2/p_1)^{(\gamma-1)/\gamma}}{1 - (\rho_2^2 A_2^2 / \rho_1^2 A_1^2)} \quad (5.30)$$

For an isentropic process, we can also write

$$\frac{p_1}{\rho_1^\gamma} = \frac{p_2}{\rho_2^\gamma}$$

$$\text{or } \rho_2 = \left(\frac{p_2}{p_1} \right)^{1/\gamma} \rho_1$$

from which we obtain

$$\rho_2^2 = \left(\frac{p_2}{p_1} \right)^{2/\gamma} \rho_1^2$$

Substituting into Equation 5.30,

$$\frac{\dot{m}_s^2}{A_2^2} = \frac{2\rho_1^2 (p_2/p_1)^{2/\gamma} [\gamma/(\gamma-1)] (p_1/\rho_1) [1 - (p_2/p_1)^{(\gamma-1)/\gamma}]}{1 - (p_2/p_1)^{2/\gamma} (A_2^2/A_1^2)} \quad (5.31)$$

from which we finally have

$$\dot{m}_s = A_2 \left\{ \frac{2p_1\rho_1 (p_2/p_1)^{2/\gamma} [\gamma/(\gamma-1)] [1 - (p_2/p_1)^{(\gamma-1)/\gamma}]}{1 - (p_2/p_1)^{2/\gamma} (D_2^4/D_1^4)} \right\}^{1/2} \quad (5.32)$$

Thus, for compressible flow through a venturi meter, the measurements needed are p_1 , p_2 , T_1 , the venturi dimensions, and the fluid properties. By introducing the venturi discharge coefficient C_v , the actual flow rate through the meter is determined to be

$$\begin{aligned} \dot{m}_{ac} &= C_v \dot{m}_s \\ \dot{m}_{ac} &= C_v A_2 \left\{ \frac{2p_1\rho_1 (p_2/p_1)^{2/\gamma} [\gamma/(\gamma-1)] [1 - (p_2/p_1)^{(\gamma-1)/\gamma}]}{1 - (p_2/p_1)^{2/\gamma} (D_2^4/D_1^4)} \right\}^{1/2} \end{aligned} \quad (5.33)$$

It would be convenient if we could rewrite Equation 5.33 in such a way that the compressibility effects could be consolidated into one term. We attempt this by using the flow rate equation for the incompressible case multiplied by another coefficient called the **compressibility factor** Y ; we therefore write

$$\dot{m}_{ac} = C_v Y \rho_1 A_2 \sqrt{\frac{2g_c(p_1 - p_2)}{\rho_1(1 - D_2^4/D_1^4)}} \quad (5.34)$$

We now set Equation 5.34 equal to Equation 5.33 and solve for Y . We obtain

$$Y = \sqrt{\frac{\gamma}{\gamma - 1} \frac{[(p_2/p_1)^{2/\gamma} - (p_2/p_1)^{(\gamma+1)/\gamma}](1 - D_2^4/D_1^4)}{[1 - (D_2^4/D_1^4)(p_2/p_1)^{2/\gamma}](1 - p_2/p_1)}} \quad (5.35)$$

The ratio of specific heats γ will be known for a given compressible fluid, and so Equation 5.35 could be plotted as compressibility factor Y versus pressure ratio p_2/p_1 for various values of D_2/D_1 . The advantage of using Equation 5.34 over 5.33 is that a pressure drop term appears just as with the incompressible case, which is convenient if a manometer is used to measure pressure. Moreover, the compressibility effect has been isolated into one factor, Y .

EXAMPLE 5.5. A 10 × 6 venturi meter is used in a flow line that conveys air at 5 kg/s. The line pressure is 150 kPa (gage pressure), and the air temperature is 25°C. (a) Determine the expected pressure reading if a gage were attached to the throat. (b) Determine the expected reading if we erroneously neglected compressibility effects. Take atmospheric pressure to be 101.3 kPa.

Solution: From the appendix tables we read

air	$R = 286.8 \text{ J}/(\text{kg} \cdot \text{K})$	$\mu = 18 \times 10^{-6} \text{ N} \cdot \text{s}/\text{m}^2$	
	$C_p = 1005 \text{ J}/(\text{kg} \cdot \text{K})$	$\gamma = 1.4$	[App. Table C.1]

10-nom sch 40	$D = 25.46 \text{ cm}$	$A = 509.1 \text{ cm}^2$	
6-nom sch 40	$D = 15.41 \text{ cm}$	$A = 186.5 \text{ cm}^2$	[App. Table D.1]

The actual mass flow rate through the meter is given by

$$\dot{m}_{ac} = C_v Y \rho_1 A_2 \sqrt{\frac{2g_c(p_1 - p_2)}{\rho_1(1 - D_2^4/D_1^4)}}$$

We now evaluate each term. The actual flow rate is 5 kg/s. For an ideal gas,

$$\rho_1 = \frac{p_1}{RT_1} = \frac{150\,000 + 101\,300}{286.8(273 + 25)} = 2.94 \text{ kg}/\text{m}^3$$

The average velocity in the pipeline is found as

$$V_1 = \frac{\dot{m}_{ac}}{\rho_1 A_1} = \frac{5}{2.94(509.1 \times 10^{-4})} = 33.4 \text{ m/s}$$

The Reynolds number then is

$$\text{Re} = \frac{\rho_1 V_1 D_1}{\mu g_c} = \frac{2.94(33.4)(0.254 \text{ m})}{18 \times 10^{-6}} = 1.39 \times 10^6$$

The flow is thus highly turbulent, and so the venturi discharge coefficient is $C_v = 0.984$. The diameter ratio is

$$\frac{D_2}{D_1} = \frac{15.41}{25.46} = 0.605$$

Substituting into the mass flow equation gives

$$5 = 0.984Y(2.94)(186.5 \times 10^{-4}) \sqrt{\frac{2(150\,000 + 101\,300 - p_2)}{2.94(1 - 0.605^4)}}$$

Rearranging and solving for the pressure at the throat, we get

$$p_2 = 251\,300 - \frac{1.093 \times 10^4}{Y^2} \quad (\text{i})$$

The compressibility factor Y is unknown and, because it depends on p_2 , an iterative solution method is required. We first assume a value for p_2 , then calculate the pressure ratio p_2/p_1 , and Y from Equation 5.35. Next, we substitute into Equation i of this example and find a new value of p_2 . The process is repeated until the solution converges. Substituting into Equation 5.35 gives

$$Y = \sqrt{\frac{1.4}{1.4 - 1} \frac{[(p_2/p_1)^{2/1.4} - (p_2/p_1)^{(1.4 + 1)/1.4}](1 - 0.605^4)}{[1 - (0.605^4)(p_2/p_1)^{2/1.4}](1 - p_2/p_1)}} \quad (5.35)$$

The results are

p_2	p_2/p_1	Y (Eq 5.35)	p_2 (Eq i)
160 000	0.636	0.754	232 115
232 115	0.923	0.950	239 198
239 198	0.952	0.969	239 653
239 653	0.954	0.970	239 682
239 682	0.954	0.970	239 683

Thus,

$$p_2 = 239\,700 \text{ kPa (absolute)}$$

$$\text{or } p_2 = 239\,700 - 101\,300$$

$$(a) \quad \underline{p_2 = 138\,400 \text{ Pa (gage)}}$$

If we neglect compressibility, then $Y = 1$, and from Equation i,

$$p_2 = 240\,400 \text{ kPa (absolute)}$$

$$\text{or } p_2 = 240\,400 - 101\,300$$

$$(b) \quad \underline{p_2 = 139\,100 \text{ Pa (gage)}}$$

The preceding example shows that if compressibility is neglected, the difference is less than 0.5%, which is typical for a venturi meter. For an orifice meter, however, the compressibility factor is much lower, and neglecting compressibility yields a much higher error. The compressibility factor for an orifice meter cannot be derived but instead must be measured. Results of such tests have yielded the Buckingham equation:

$$Y = 1 - (0.41 + 0.35\beta^4) \frac{(1 - p_2/p_1)}{\gamma} \quad (5.36)$$

which is valid for any manometer connection system except $2^{1/2} D \times 8D$.

Energy Cost

In all meter installations, there will be a permanent energy loss due to the presence of the meter, just like a minor loss associated with the presence of a fitting. The additional energy cost may influence what type of meter might be selected for a given application.

The energy lost due to the presence of a meter is found with

$$\left. \frac{dW}{dt} \right|_{\text{lost}} = \frac{\rho g Q h_m}{g_c} \quad (5.37)$$

where h_m is the head loss due to the meter, which can be calculated with the appropriate equation found in Table 5.5.

TABLE 5.5. *Loss equations for various meters. (From Flow Measurement Handbook, by R. W. Miller, published by McGraw-Hill Book Co., 1983.)*

Meter	Loss equation (liquids)
Turbine meter	$h_m = 0.00577 \frac{V^2}{g}$
Venturi meter	$h_m = (0.436 - 0.86\beta + 0.59\beta^2)\Delta h$
Orifice meter	$h_m = (1 - 0.24\beta - 0.52\beta^2 - 0.16\beta^3)\Delta h$
Notes:	$\Delta h = \text{meter reading} \quad \beta = \frac{\text{throat diameter}}{\text{upstream diameter}}$

Meter Selection Guide

Flowmeter selection depends on a number of variables, so the final decision is a matter of judgment based on experience. To help in the decision-making process, consider the following when selecting a meter (information from *Flow Measurement Handbook*, by R. W. Miller, published by McGraw-Hill Book Co., 1983.):

- Fluid type—Liquid, gas, vapor, slurry, clean, dirty, corrosive.
- Limitations—Temperature, pressure, velocity.
- Installation conditions—Line size, Reynolds number, upstream approach length, vibration problems, steady or unsteady flow.
- Performance—Required accuracy, flow rate range.
- Economics—Initial cost, operating cost, reliability, availability of parts.
- Relative initial cost—High cost (venturi); medium cost (turbine); low cost (elbow, variable-area meter or rotameter, orifice meter).

EXAMPLE 5.6. Example 5.4 dealt with an orifice meter having the following data:

$$\begin{aligned} \beta &= 0.5804 & p_1 &= 17.7 \text{ psia} & p_2 &= 14.6 \text{ psia} \\ \rho &= 1.94 \text{ slug/ft}^3 & Q_{ac} &= 0.109 \text{ ft}^3/\text{s} \end{aligned}$$

If energy costs \$0.05/(kW·hr), determine the energy cost per year associated with this meter.

Solution: For the orifice meter, Table 5.5 shows the loss as

$$h_m = (1 - 0.24\beta - 0.52\beta^2 - 0.16\beta^3)\Delta h$$

The pressure drop is found with

$$\frac{(p_1 - p_2)g_c}{\rho g} = \frac{(17.7 - 14.6)(144)}{1.94(32.2)} = 7.15 \text{ ft}$$

Substituting gives

$$h_m = (1 - 0.24(0.5804) - 0.52(0.5804)^2 - 0.16(0.5804)^3)(7.15)$$

$$h_m = 0.65(7.15) = 4.65 \text{ ft of water}$$

The energy lost due to the presence of the meter is

$$\left. \frac{dW}{dt} \right|_{\text{lost}} = \frac{\rho g Q h_m}{g_c} = (1.94)(32.2)(0.109)(4.65)$$

$$\left. \frac{dW}{dt} \right|_{\text{lost}} = 31.7 \text{ ft} \cdot \text{lbf/s} = 42.9 \text{ W}$$

The cost of energy is

$$\text{Energy cost} = (42.9 \text{ W}) \left(\frac{\$0.05}{1\,000 \text{ W} \cdot \text{hr}} \right) (8760 \text{ hr/yr})$$

$$\underline{\text{Energy cost} = \$18.79/\text{yr}}$$

5.4 The Unsteady Draining Tank Problem

Consider a tank of liquid with an attached pipe as indicated in Figure 5.15. Liquid drains from this tank and encounters friction within the pipe. In the draining tank problems usually considered, the liquid surface velocity in the tank is assumed negligible. In this analysis, however, it is our desire to determine how the velocity in the pipe and how the depth of liquid in the tank both vary with time. We consider unsteady flow from the draining tank, and we wish to determine how long it will take for the tank to drain.

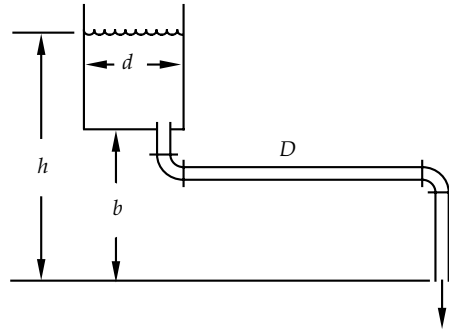


FIGURE 5.15. *The unsteady draining tank problem.*

As indicated in Figure 5.15, the tank is circular and has a diameter d . The pipe diameter is D , and the distance from the pipe exit to the free surface of the liquid in the tank is h . The distance from tank bottom to pipe exit is b , which is a constant. We identify the free surface of liquid in the tank as section 1, and the pipe exit as section 2. The equation relating areas and velocities at any time is

$$A_1 V_1 = A_2 V_2$$

So at any time and for any depth, the velocity of the liquid surface at 1 (V_1) in terms of the velocity in the pipe is given by

$$V_1 = \frac{A_2}{A_1} V_2 \quad (5.38)$$

The modified Bernoulli equation written from 1 (free surface in tank) to 2 (pipe exit) is

$$\frac{p_1 g_c}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2 g_c}{\rho g} + \frac{V_2^2}{2g} + z_2 + \frac{fL}{D_h} \frac{V_2^2}{2g} + \Sigma K \frac{V_2^2}{2g}$$

Evaluating properties, we have

$$p_1 = p_2 = p_{\text{atm}} \quad z_2 = 0 \quad z_1 = h \quad V_2 = V$$

Substituting these values and Equation 5.38, the modified Bernoulli equation becomes

$$\left(\frac{A_2}{A_1} \right)^2 \frac{V^2}{2g} + h = \frac{V^2}{2g} + \left(\frac{fL}{D} + \Sigma K \right) \frac{V^2}{2g}$$

which simplifies to

$$h = \frac{V^2}{2g} \left[1 - \left(\frac{A_2}{A_1} \right)^2 + \frac{fL}{D} + \Sigma K \right]$$

Solving for velocity,

$$V = \left[\frac{2gh}{1 - (A_2/A_1)^2 + fL/D + \Sigma K} \right]^{1/2} \quad (5.39)$$

This equation is the equation of motion relating depth of liquid in the tank to efflux velocity. [It is interesting to compare this result to that obtained using various simplifications. For example, assuming that the velocity at section 1 (V_1) is much smaller than the velocity in the exit pipe (V_2), the area ratio is negligible. Moreover, for frictionless flow, f and ΣK vanish. Incorporating both assumptions gives the familiar result: $V = \sqrt{2gh}$.]

Next, we will write the unsteady continuity equation for the tank liquid:

$$0 = \frac{\partial m}{\partial t} + \int \int_{cs} \rho V_n dA \quad (5.40)$$

where $\partial m / \partial t$ is the change of mass in the tank. The integral term must be evaluated at locations where mass leaves the tank and where mass enters the tank. The mass of liquid in the tank at any time is

$$m = \rho V = \rho \frac{\pi d^2}{4} (h - b) = \frac{\pi d^2}{4} \rho h - \frac{\pi d^2}{4} \rho b$$

The change of mass in the tank with respect to time is

$$\frac{\partial m}{\partial t} = \frac{\pi d^2}{4} \rho \frac{dh}{dt} \quad (5.41)$$

The integral term becomes

$$\int \int_{cs} \rho V_n dA = \int \int_{out} \rho V_n dA - \int \int_{in} \rho V_n dA$$

With no fluid entering the tank, the preceding equation reduces to

$$\int_{cs} \rho V_n dA = \rho \frac{\pi D^2}{4} V - 0$$

Substituting this equation and Equation 5.41 into Equation 5.40, we get

$$0 = \frac{\pi d^2}{4} \rho \frac{dh}{dt} + \rho \frac{\pi D^2}{4} V$$

or

$$0 = A_1 \frac{dh}{dt} + A_2 V$$

Rearranging,

$$\frac{dh}{dt} = -\frac{A_2}{A_1} V \quad (5.42)$$

For the draining tank problem, then, Equation 5.39 must be solved simultaneously with Equation 5.42. Equation 5.42, however, varies with time; the friction factor might not be constant over the range of interest of h , the depth of liquid, and this is why a solution cannot be obtained directly. Note that Equation 5.39 is independent of fluid properties. Moreover, the only place where fluid properties has an influence is in calculating the Reynolds number, which is used to find the friction factor in Equation 5.39.

In order to solve an unsteady draining tank problem with a friction factor that is not constant, the solution procedure involves rewriting Equation 5.27 in difference form:

$$\Delta h = -\frac{A_2}{A_1} V \Delta t \quad (5.43)$$

The solution is obtained by starting with the initial height h and substituting into Equation 5.39. The Reynolds number and relative roughness are calculated and, by trial and error, the friction factor as well as velocity are found. A suitably small time increment Δt is selected, and these parameters are substituted into Equation 5.43. The value of Δh is calculated and added to the initial height h to find a new height. This new height is then substituted into Equation 5.39 and the procedure is repeated. Eventually, the final value of h is reached and the calculations are concluded. The results provide values of height h and velocity in the pipe V versus time t .

EXAMPLE 5.7. The 20-cm-diameter ($= d$) tank of Figure 5.15 contains chloroform, and it is being drained by the attached piping system. The pipe

itself is made of 3-nominal schedule 40 commercial steel and it is 25 m long. Neglecting minor losses, determine the variation of velocity and height with time if the height h is allowed to vary from an initial value of 2 m to 1.2 m.

Solution: We begin by determining properties from the appropriate tables:

$$\text{chloroform} \quad \rho = 1\,470 \text{ kg/m}^3 \quad \mu = 0.53 \times 10^{-3} \text{ N}\cdot\text{s/m}^2 \quad [\text{App. Table B.1}]$$

$$\text{3-nom sch 40} \quad D = 7.792 \text{ cm} \quad A = 47.69 \text{ cm}^2 \quad [\text{App. Table D.1}]$$

$$\text{commercial steel} \quad \varepsilon = 0.0046 \text{ cm} \quad [\text{Table 3.1}]$$

With the tank diameter of 0.2 m, we calculate

$$A_1 = \frac{\pi d^2}{4} = \frac{\pi(0.2)^2}{4} = 0.03142 \text{ m}^2$$

The area ratio then is

$$\left(\frac{A_2}{A_1}\right)^2 = \left(\frac{47.69 \times 10^{-4}}{0.03142}\right)^2 = 0.023$$

Equation 5.43 becomes

$$\Delta h = - \left(\frac{47.69 \times 10^{-4}}{0.03142} \right) V \Delta t$$

or

$$\Delta h = -0.1518 V \Delta t \quad (\text{i})$$

Substituting known quantities into Equation 5.39, we get

$$V = \left[\frac{2gh}{1 - (A_2/A_1)^2 + fL/D + \sum K} \right]^{1/2}$$

$$V = \left[\frac{2(9.81)h}{1 - 0.023 + f(25)/0.07792 + 0} \right]^{1/2}$$

or

$$V = \left(\frac{19.62h}{320.8f + 0.977} \right)^{1/2} \quad (\text{ii})$$

The Reynolds number in terms of velocity is

$$\text{Re} = \frac{\rho V D}{\mu g_c} = \frac{1.47(1\,000)V(0.077\,92)}{0.53 \times 10^{-3}}$$

$$\text{Re} = 2.16 \times 10^5 V$$

Also

$$\frac{\varepsilon}{D} = \frac{0.004\,6}{7.792} = 0.000\,6$$

We begin the calculations by letting $h = 2$ m, as given in the problem statement. Thus,

1st step: $h = 2$ m, then from Equation ii,

$$V = \left(\frac{19.62(2)}{320.8f + 0.977} \right)^{1/2}$$

Assume	$f = 0.018$	$V = 2.4$ m/s	$\text{Re} = 5.2 \times 10^5$
	$f = 0.018$	close enough	

Next, we substitute into Equation i:

$$\Delta h = -0.151\,8V\Delta t = -0.151\,8(2.4)\Delta t$$

Before proceeding, we must select a value for Δt . Without rigorous proof, we select a value for Δt that will result in at least 10 time intervals. Selecting Δt is sometimes a matter of trial and error. Suppose for example, that we select $\Delta t = 1$ second. Then we find $\Delta h = -0.151\,8(2.4)(1) = -0.36$ m. For our second step, the value of h we use is the previous value plus Δh : $h = 2 - 0.36 = 1.64$ m. By the second or third step, the value of h will have reached the 1.2 m given in the problem statement, so $\Delta t = 1$ s is too large. A better selection is $\Delta t = 0.2$ s. Thus, we have

$$\Delta h = -0.151\,8(2.4)(0.2) = -0.073$$

Then $h = 2 - 0.073 = 1.93$ m; and

2nd step: $h = 1.93$ m

Now we proceed by using Equation ii to find velocity V and Equation i to find Δh . We continue in this way until the desired Δh is reached. The solution is provided in Table 5.6. The steps are easily completed using a spreadsheet.

TABLE 5.6. *Solution table for the unsteady draining tank; $\Delta t = 0.2$ s.*

h	f	V	Re	Δh	t
2	0.018	2.399	5.19×10^5	-0.0728	0.0
1.93	0.018	2.398	5.09×10^5	-0.0728	0.2
1.85	0.018	2.397	4.99×10^5	-0.0728	0.4
1.78	0.018	2.396	4.89×10^5	-0.0727	0.6
1.71	0.018	2.395	4.79×10^5	-0.0727	0.8
1.64	0.018	2.394	4.68×10^5	-0.0727	1.0
1.56	0.018	2.393	4.58×10^5	-0.0727	1.2
1.49	0.018	2.392	4.47×10^5	-0.0726	1.4
1.42	0.018	2.391	4.35×10^5	-0.0726	1.6
1.35	0.018	2.389	4.24×10^5	-0.0725	1.8
1.27	0.018	2.388	4.12×10^5	-0.0725	2.0
1.20	0.018	2.386	4.00×10^5	-0.0724	2.2

We follow this stepwise solution because the friction factor f may vary with time. In this example, however, friction factor was constant, because the flow in the pipe is highly turbulent. If we operate in the transition region, however, then f would not be a constant, and the numerical method is the only way to obtain a solution.

As mentioned in the previous example, a stepwise solution method must be used in problems where friction factor f is not constant, but when friction factor is constant, we can formulate another solution. For the draining tank problem, the modified Bernoulli equation and the unsteady continuity equation, derived earlier, would become

$$V = \left[\frac{2gh}{1 - (A_2/A_1)^2 + fL/D + \Sigma K} \right]^{1/2} \quad (5.44)$$

$$\Delta h = -\frac{A_2}{A_1} V \Delta t \quad (5.45)$$

To simplify the algebra, Equation 5.44 is rewritten as

$$V = \left[\frac{2gh}{C} \right]^{1/2} \quad (5.46)$$

where the constant C is defined as

$$C = 1 - (A_2/A_1)^2 + fL/D + \Sigma K \quad (5.47)$$

Combining Equation 5.46 with Equation 5.42, we get

$$\frac{dh}{dt} = - \left(\frac{A_2}{A_1} \sqrt{\frac{2g}{C}} \right) h^{1/2}$$

Separating variables,

$$\frac{dh}{h^{1/2}} = - \left(\frac{A_2}{A_1} \sqrt{\frac{2g}{C}} \right) dt$$

We can integrate this equation from time = 0, corresponding to height = h_o , to a future time = t , corresponding to a liquid height h :

$$\int_{h_o}^h \frac{dh}{h^{1/2}} = - \int_0^t \left(\frac{A_2}{A_1} \sqrt{\frac{2g}{C}} \right) dt$$

Integrating,

$$2(h^{1/2} - h_o^{1/2}) = - \left(\frac{A_2}{A_1} \sqrt{\frac{2g}{C}} \right) t$$

Rearranging and solving for the height h yields

$$h = \left[h_o^{1/2} - \left(\frac{A_2}{2A_1} \sqrt{\frac{2g}{C}} \right) t \right]^2 \quad (5.48)$$

This equation gives the height h in the tank at any time for a friction factor that is constant, with C given by Equation 5.47. So h versus t is found with Equation 5.48, and V versus t is found with Equation 5.46.

It is interesting to compare results calculated with Equation 5.48 to those obtained with the stepwise solution method. This is done in the next example.

EXAMPLE 5.8. Determine the time it takes for the tank of the previous

example to drain from a height h of 2 m to 1.2 m, assuming that the constant friction factor formulation can be applied.

Solution: Having completed the calculations of the previous example, we know that the friction factor is constant. Suppose, however, that we did not know this. The question arises as to how should we proceed. First we arrive at Equation ii in the usual way:

$$V = \left(\frac{19.62h}{320.8f + 0.977} \right)^{1/2} \quad (\text{Equation ii, Example 5.7})$$

We then substitute $h = 2$ m and $h = 1.2$ m (the limits on h given in the problem statement), to calculate velocity and friction factor. Doing so shows that $f = 0.018$ during the time interval of interest. Next, we would calculate

$$C = \left[1 - \left(\frac{A_2}{A_1} \right)^2 + \frac{fL}{D} + \Sigma K \right] = \left[1 - 0.023 + \frac{0.018(25)}{0.07792} + 0 \right]$$

$$C = 6.752$$

Substituting into Equation 5.48, we find

$$h = \left[h_o^{1/2} - \left(\frac{A_2}{2A_1} \sqrt{\frac{2g}{C}} \right) t \right]^2$$

$$h = \left[2^{1/2} - \left(\frac{0.1518}{2} \sqrt{\frac{2(9.81)}{6.752}} \right) t \right]^2$$

or

$$h = (1.414 - 0.1294t)^2$$

We can use this equation to obtain the height h in the tank at any time t .

The stepwise solution indicates that when $h = 1.2$ m, the time required is 2.2 s. The preceding equation is used to make this same calculation:

$$1.2 = (1.414 - 0.1294t)^2$$

Solving for time, we get

$$\underline{t = 2.46 \text{ s}}$$

The error is about 10%, which can be attributed to the approximations made in the stepwise method.

5.5 Summary

In this chapter, we considered the flow in a pipe network, and we developed a method for solving network problems. We also developed a method for analyzing flows in parallel pipes, and studied various fluid meters. We concluded the chapter by examining unsteady flow in a draining tanks.

5.6 Show and Tell

1. Locate computer programs or spreadsheet solutions for networks on the Internet and report on your findings.
2. Find examples of pipe networks and provide sketches; report on your findings.
3. Another method of modeling flow in a pipe network involves use of the Hazen-Williams equation. What is this equation, and how is the approach different from that in this chapter?
4. Locate computer programs or spreadsheet solutions for parallel pipes on the Internet, and report on your findings.

Obtain a flowmeters handbook or reference text and give a brief presentation on the following meters, showing recommended construction details and the applicable loss coefficient. Include information for liquids, vapors, and gases.

8. venturi meter
9. orifice meter
10. nozzle-type meter
11. elbow meter

Give a presentation on how the following meters operate and various details about them; e.g., fluid type, materials of construction, expected energy losses, etc.

12. nutating disk meter
13. target flowmeter
14. magnetic flowmeter
15. vortex meter
16. rotameter
17. It was mentioned in the chapter that there is an ASME fluid meters handbook. Refer to various measurement textbooks and determine if other organizations publish recommendations on various flowmeters. Give a brief presentation on your findings.
18. The Instrument Society of America (ISA) publishes information on flowmeters. Obtain information from ISA about meter calibration, and report on your findings.

5.7 Problems

Pipe Networks

1. For the piping system shown in Figure P5.1, determine the flow rate in each line. Water is the fluid moving through the system, and the pipe is made of commercial steel, all schedule 40. Neglect minor losses. Data are as follows

Pipe #	Nominal Diameter	L in m	Flow Rates m^3/s
1	3	15	$Q_{\text{in}1} = 3.300$
2	2	12	$Q_{\text{in}2} = 3.300$
3	2	15	$Q_{\text{out}2} = 0.891$
4	3	12	$Q_{\text{out}2} = 5.100$
5	2	15	$Q_{\text{out}3} = 0.669$
6	2	12	
7	1	15	

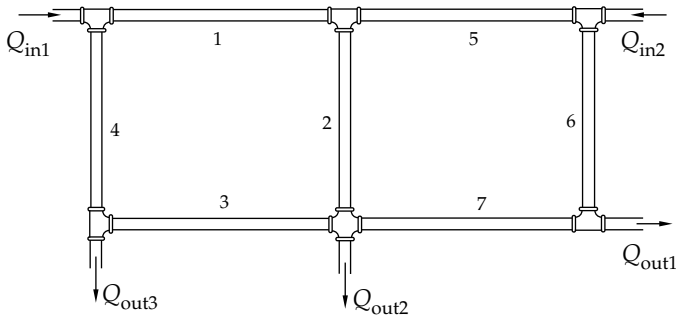


FIGURE P5.1

2. For the piping system shown in Figure P5.2, determine the flow rate in each line. Kerosene is the fluid moving through the system, and the pipe is made of commercial steel, all schedule 40. Neglect minor losses. Data are as follows

Pipe #	Nominal Diameter	L in m	Flow Rates m^3/s
1	4	30	$Q_{\text{in}1} = 0.2$
2	2	160	$Q_{\text{in}2} = 0.15$
3	3	100	$Q_{\text{out}1} = 0.15$
4	4	110	$Q_{\text{out}2} = 0.1$
5	4	100	$Q_{\text{out}3} = 0.1$
6	3	110	
7	3	30	

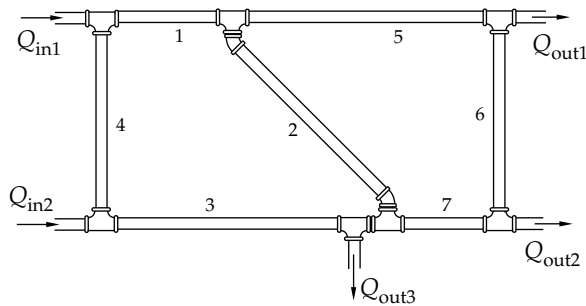


FIGURE P5.2

3. For the piping system shown in Figure P5.3, determine the flow rate in each line. Carbon tetrachloride is the fluid moving through the system, and the pipe is made of commercial steel, all schedule 40. Neglect minor losses. Data are as follows

Pipe #	Nominal Diameter	L in m
1	1	10
2	1/2	20
3	1	10
4	1	20
5	3/4	30
6	3/4	15
7	3/4	15

Flow Rates m^3/s
$Q_{\text{in}1} = 0.4$
$Q_{\text{out}1} = 0.1$
$Q_{\text{out}2} = 0.1$
$Q_{\text{out}3} = 0.1$
$Q_{\text{out}4} = 0.1$

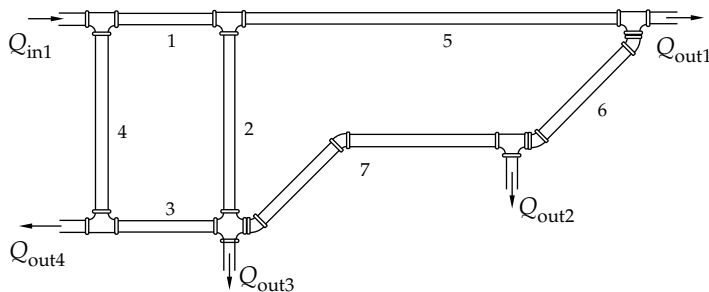


FIGURE P5.3

4. For the piping system shown in Figure P5.3, determine the flow rate in each line. Castor oil is the fluid moving through the system, and the pipe is made of commercial steel, all schedule 40. Neglect minor losses. Data are as follows

Pipe #	Nominal Diameter	L in m
1	4	10
2	4	20
3	4	10
4	4	20
5	4	30
6	4	15
7	4	15

Flow Rates m^3/s
$Q_{\text{in}1} = 0.004$
$Q_{\text{out}1} = 0.001$
$Q_{\text{out}2} = 0.001$
$Q_{\text{out}3} = 0.001$
$Q_{\text{out}4} = 0.001$

5. For the piping system shown in Figure P5.5, determine the flow rate in each line. Ethylene glycol is the fluid moving through the system, and the pipe is made of commercial steel. Neglect minor losses. Data are as follows

Pipe #	Diameter	<i>L</i> in m	Flow Rates m ³ /s
1	0.02664	30	$Q_{in1} = 0.050$
2	0.02093	30	$Q_{out1} = 0.010$
3	0.02664	30	$Q_{out2} = 0.020$
4	0.02664	30	$Q_{out3} = 0.020$
5	0.02093	20	
6	0.02093	30	
7	0.02093	20	

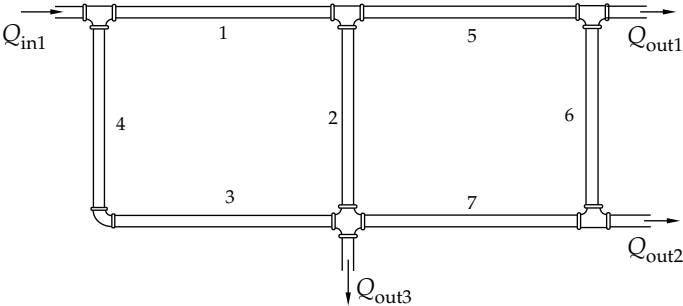


FIGURE P5.5

6. For the piping system shown in Figure P5.6, determine the flow rate in each line. Acetone is the fluid moving through the system, and the pipe is made of commercial steel. Neglect minor losses. Data are as follows

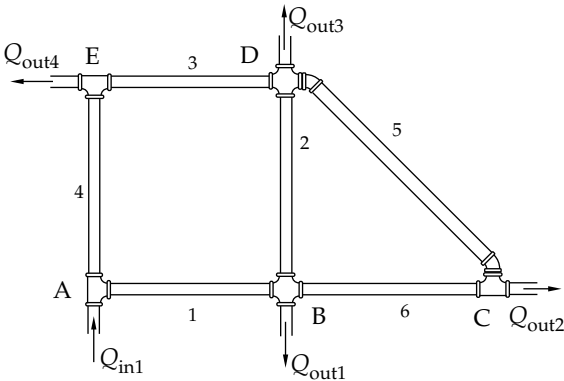


FIGURE P5.6

Pipe #	Diameter	<i>L</i> in m	Flow Rates m ³ /s
1	0.05252	50	$Q_{in1} = 0.100$
2	0.04090	30	$Q_{out1} = 0.040$
3	0.05252	50	$Q_{out2} = 0.010$
4	0.04090	30	$Q_{out3} = 0.010$
5	0.02093	42	$Q_{out4} = 0.040$
6	0.05252	30	

7. For the piping system shown in Figure P5.7, determine the flow rate in each line. Benzene is the fluid moving through the system, and the pipe is made of commercial steel. Neglect minor losses. Data are as follows

Pipe #	Diameter	L in m
1	0.02664	10
2	0.02093	30
3	0.02664	10
4	0.02664	30
5	0.02093	40
6	0.02093	30
7	0.02093	40

Flow Rates	m^3/s
$Q_{\text{in}1}$	$= 0.100$
$Q_{\text{out}1}$	$= 0.040$
$Q_{\text{out}2}$	$= 0.025$
$Q_{\text{out}3}$	$= 0.020$
$Q_{\text{out}4}$	$= 0.015$

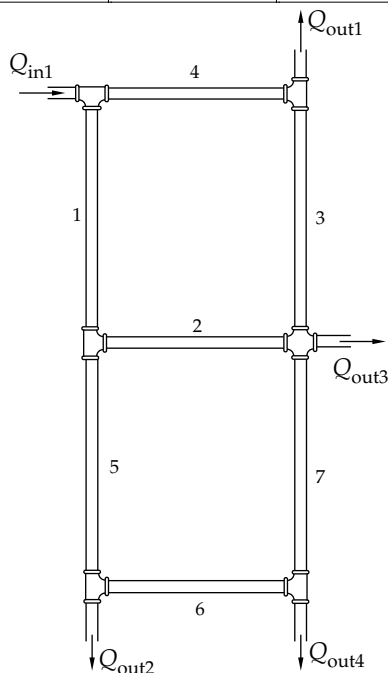


FIGURE P5.7

8. For the piping system shown in Figure P5.8, determine the flow rate in each line. Octane is the fluid moving through the system, and the pipe is made of commercial steel, all schedule 40. Neglect minor losses. Data are as follows

Pipe #	Nominal Diameter	L in m
1	6	100
2	4	80
3	6	100
4	6	80
5	6	30
6	2	80
7	4	30

Flow Rates	m^3/s
$Q_{\text{in}1}$	$= 0.5$
$Q_{\text{out}1}$	$= 0.2$
$Q_{\text{out}2}$	$= 0.1$
$Q_{\text{out}3}$	$= 0.1$
$Q_{\text{out}4}$	$= 0.05$
$Q_{\text{out}5}$	$= 0.05$

8	4	70
9	4	80
10	4	70

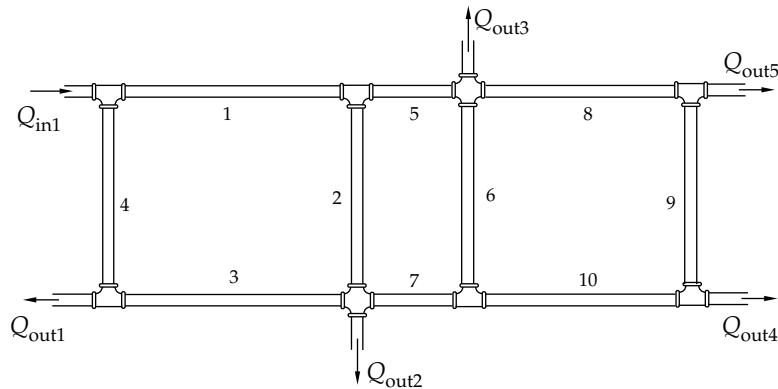


FIGURE P5.8

Parallel Pipes

9. Figure P5.9 shows a parallel piping system that delivers turpentine to a mixing tank, where it will be used to prepare paint thinner. The turpentine must be supplied at a rate of 100 gpm ($= Q_{out}$). Line 1 is made of 2-nominal pipe that is 150 ft long from A to B. Line 2 is $1\frac{1}{2}$ -nominal pipe that is 125 ft long. Both pipes are schedule 40 stainless steel. The valve in line 1 is a fully open globe valve, and all fittings are threaded. The pressure at B must be 10 psig as required by the mixing process. Determine the flow rate in each line and the pressure at A.

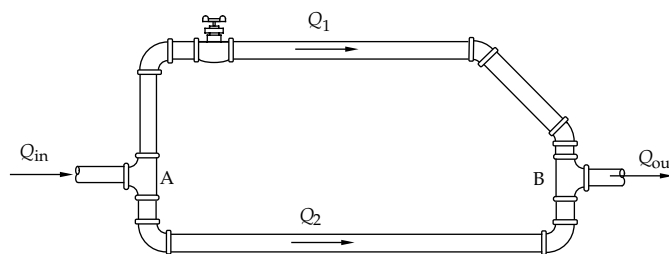


FIGURE P5.9

10. Figure P5.10 shows a parallel piping system that delivers oil which is used for cooling in a machining operation. The oil (same properties as kerosene) must be supplied at a rate of 2 l/s ($= Q_{out}$). Line 1 is made of 1 standard copper tubing, and line 2 is $\frac{3}{4}$ standard copper tubing, both type L. Line 1 is 50 m long and line 2 is 30 m. The valve in line 1 is a fully open globe valve, and the valve in line 2 is a ball check valve. All fittings are soldered. If the pressure at B is 200 kPa, determine the flow rate in each line and the pressure at A.

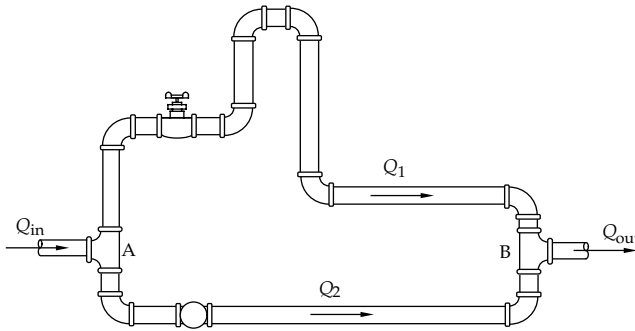


FIGURE P5.10

11. An elevated tank containing oil (Sp. Gr. = 0.888, $\nu = 9 \times 10^{-4} \text{ m}^2/\text{s}$) drains through a flow line that splits into two other lines. (See Figure P5.11.) Each line provides oil to bearings of a rotating shaft within a machine; the bearings must be lubricated continuously. Downstream of the bearings, the flow lines join together and lead to a second tank. The flow lines are made of $1/2$ standard type M drawn-copper tubing, with regular fittings, all soldered together. Both bearings have a loss coefficient of 10. The flow line on the left (from A to B) is 30 ft long and, due to the presence of various machine components, takes the path shown. The line on the right (A to B) is only 15 ft long, and the distance z is 12 ft. The valves are both ball valves. Determine the flow rate of oil delivered to each bearing. Also, redraw the piping system showing soldered fittings. Take the distance from the upper tank to A to be 1 ft and from B to the lower tank to be 10 in.
12. Figure P5.12 shows a spraying system for rinsing portions of a car in a car wash. At section 1, the pressure is measured by a gage to be 50 psig. The water is routed through a gate valve, then through a meter ($K = 6$) that records total flow delivered. Downstream of the meter, the flow line splits at section A. One line leads to a nozzle at B ($K = 30$), while the other line is routed up to the ceiling and back down to a nozzle ($K = 30$) at C, which is at the same elevation as B. The nozzles both expand the water to atmospheric pressure in the form of sprays. The flow line is $3/4$ standard type M drawn-copper tubing, with all fittings soldered. The tube lengths are: 1 to A, 2.5 m; A to B, 1 m; and A to C, 7 m. Determine the flow rate of water delivered to each nozzle. Re-draw the system showing soldered fittings.

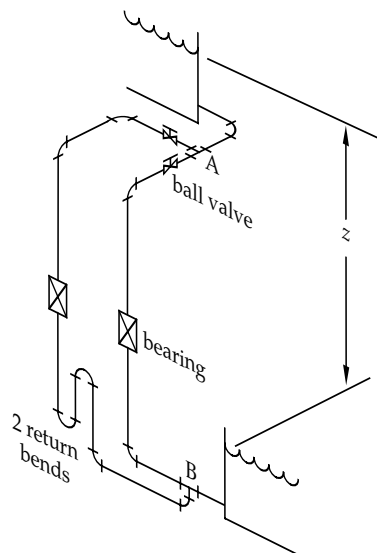


FIGURE P5.11

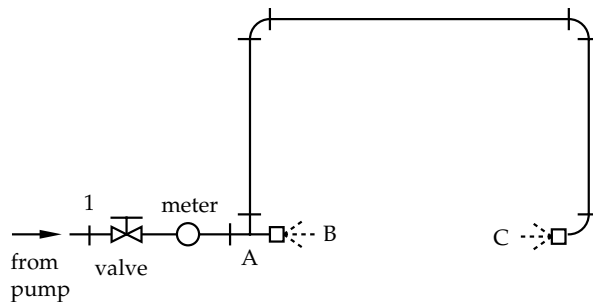


FIGURE P5.12

13. Figure P5.13 shows two views (a and b) of a pipeline connecting two tanks. The original configuration is as shown in (a). It is desired to increase the flow rate from one tank to the other, and so it has been proposed to add a loop as indicated in (b). The original line is 4-nominal schedule 40 commercial steel. The loop is 3-nominal schedule 40 commercial steel. (a) Determine the flow rate from tank to tank in the original system. (b) Determine the flow rate from tank to tank in the modified system. Has adding a second line increased the flow rate? (c) The pipeline is drawn as if threaded fittings were used. If so, it would have to be assembled with union fittings. (d) Where would they be placed? (e) Redraw the systems showing welded fittings. Are union fittings required with welded fittings? Other information: turpentine is the liquid; original line has a length of 250 m; a basket strainer; 2 elbows, and a fully open gate valve. Loop: from T-joint to T-joint, the 4-nominal line is 150 m long; the 3-nominal line is only 100 m long. Length from tank upstream to the first T-joint is 90 m. Loop line contains two 45° elbows, fully open gate valve, and one 90° elbow. The distance h is 1 m.

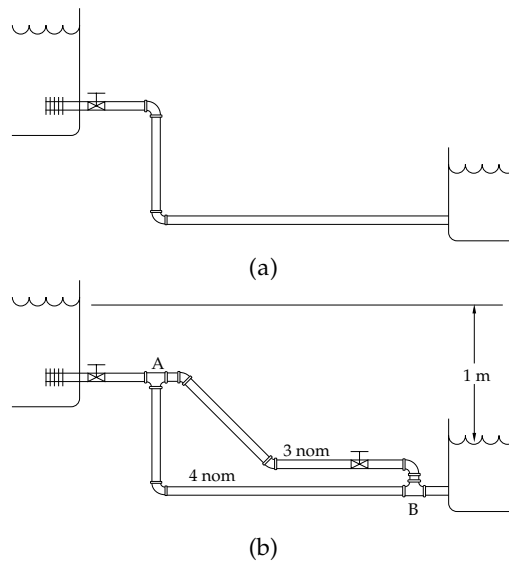
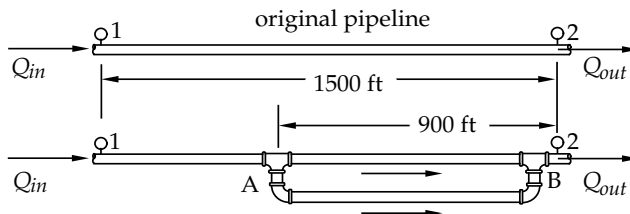


FIGURE P5.13

14. A horizontally laid 2 standard type K copper tube is 1500 ft long and conveys kerosene at a flow rate of 65 gpm. (a) Determine the corresponding pressure drop. The pipeline is modified by adding a loop made of $1\frac{1}{2}$ standard type K copper tubing that is only 900 ft long. (b) What is the expected increase in flow rate through the system for the same pressure drop as in the original pipeline? Neglect minor losses. (See Figure P5.15.)



Fluid Meters

15. A 12 x 10 (both schedule 20) venturi meter is placed in a horizontal flow line that conveys linseed oil. A mercury manometer is attached to the meter. Determine the expected reading on the manometer for a volume flow rate of $1.5 \text{ m}^3/\text{s}$. Will a 1-m-tall manometer work, or should pressure gages be used instead?

16. Carbon tetrachloride is flowing in a line that contains a 10 x 8 (both schedule 40) venturi meter. The meter is inclined at an angle of 30° with the horizontal, with flow in the downhill direction. A mercury manometer attached to the meter reads 12.7 cm. For a discharge coefficient of 0.984, determine the flow rate through the meter.
17. A venturi meter ($1 \times 1/2$) is calibrated in the laboratory using water as the working fluid and an air-over-water, inverted U-tube manometer. Data obtained are as follows:

Q_{ac} in ft^3/s	Δh in in. of H_2O	Q_{ac} in ft^3/s	Δh in in. of H_2O
2.00×10^{-3}	0.4	8.47×10^{-3}	2.7
4.01×10^{-3}	0.9	10.0×10^{-3}	3.8
5.57×10^{-3}	1.3	11.1×10^{-3}	4.6
6.46×10^{-3}	1.6	13.4×10^{-3}	5.8
7.80×10^{-3}	2.3		

- a. Plot the actual flow rate versus Δh .
 b. Plot the theoretical flow rate versus Δh on the same axes of part a.
 c. Plot $C_v (= Q_{ac}/Q_{th})$ versus $Re (= 4\rho Q_{ac}/\pi D_2 \mu g_c)$ on semilog paper.
18. An orifice meter ($D_1 = 1.025$ in. and $D_o = 0.625$ in.) is calibrated in the laboratory with water as the working fluid and with an air-over-water, inverted U-tube manometer. Data are as follows:

Q_{ac} in gpm	Δh in in. of H_2O	Q_{ac} in gpm	Δh in in. of H_2O
0.9	0.3	3.8	5.9
1.8	1.5	4.5	8.0
2.5	2.3	5.0	9.8
2.9	3.3	6.0	11.6
3.5	4.5		

- a. Plot the actual flow rate versus Δh .
 b. On the same axes of part a, plot the theoretical flow rate versus Δh .
 c. Plot $C_o (= Q_{ac}/Q_{th})$ versus $Re (= 4\rho Q_{ac}/\pi D_o \mu g_c)$ on semilog paper.
19. A 10-nominal schedule 40 pipe contains an orifice meter with a hole diameter of 6.0 in. Heptane flows through the meter, and the pressure drop measured with an air-over-heptane manometer is 6 ft.
- Determine the actual flow rate through the meter if flange taps are used.
 - Determine the actual flow rate through the meter if corner taps are used.
20. Octane flows through a horizontal line containing an orifice meter with corner taps. The flow line is 6-nominal schedule 40 and the bore diameter of the orifice plate is 10.0 cm. For a flow rate of $0.03 \text{ m}^3/\text{s}$, determine the expected pressure drop in kPa.
21. A horizontal water main is made of 12-nominal schedule 160 pipe and conveys water at 750 gpm. An orifice meter is placed in the line to measure the flow rate. The desired pressure drop for the installation is to be no more than 1.5 psi. What should the hole diameter be in the orifice plate to meet this condition? Use $1D$ and $1/2D$ taps.
22. Repeat Problem 5.21 using flange taps.

23. Repeat Problem 5.21 using corner taps.
24. Figure P5.24 shows a nozzle meter placed in a flow line with pipe wall taps. One (of many) equation(s) for the discharge coefficient is

$$C_n = 0.194\,36 + 0.152\,884(\ln Re) - 0.009\,778\,5(\ln Re)^2 + 0.000\,209\,03(\ln Re)^3$$

where $Re = 4\rho Q_{ac}/\pi D_2 \mu g_c$ and D_2 = nozzle throat diameter. For the following flow parameters, determine the diameter D_2 required to meet the conditions:

12-nominal schedule 160 pipe
water at 750 gpm
pressure drop is to be no more than 1.5 psi

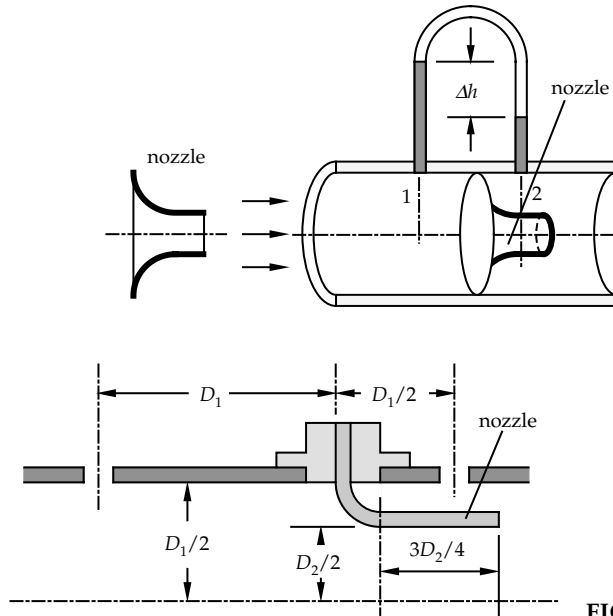


FIGURE P5.24.

25. On semilog paper, construct a graph of the orifice coefficient C_o versus Re for $\beta = 0.2, 0.3, 0.4, 0.5$, and 0.6 using the Stolz equation. Let the Reynolds number vary from 10^3 to 10^7 , and assume that corner taps are used.
26. On semilog paper, construct a graph of the nozzle coefficient C_n versus Re using the following equation (same as in Problem 5.24):

$$C_n = 0.194\,36 + 0.152\,884(\ln Re) - 0.009\,778\,5(\ln Re)^2 + 0.000\,209\,03(\ln Re)^3$$

Let the Reynolds number vary from 10^3 to 10^7 .

27. Air flows through a flow line that contains a $2 \times 1^{1/2}$ venturi meter. The pressure upstream of the meter is 15 psig, the air temperature is 90°F , and the velocity is 100 ft/s. Calculate the expected pressure at the throat.

28. Oxygen flows through a 6-nominal schedule 40 flow line, and the flow rate is measured with an orifice meter. The meter diameter ratio is 0.8. Gages attached about the meter give readings of 150 kPa and 140 kPa. The line temperature is 25°C. What is the flow rate through the meter?
29. A 4-nominal schedule 40 elbow meter is placed in a horizontal flow line that conveys linseed oil. A U-tube (air over oil) manometer is attached to the meter. Determine the expected reading on the manometer for a volume flow rate of $0.015 \text{ m}^3/\text{s}$.
30. Acetone is flowing in a 1-nominal schedule 40 line that contains an elbow meter. A U-tube (air over acetone) manometer attached to the meter reads 12.7 cm. Determine the flow rate through the meter.
31. Octane flows through a line containing an elbow meter. The flow line is 8-nominal schedule 40 pipe. For a flow rate of 500 gpm, determine the expected pressure drop within the meter in psf.
32. A horizontal water main is made of 12-nominal schedule 160 pipe and conveys water at 750 gpm. An elbow meter is placed in the line to measure the flow rate. Find the pressure drop for this installation.
33. On semilog paper, construct a graph of K versus Reynolds number for an elbow meter. Let the Reynolds number vary from 1×10^3 to 1×10^7 .

Draining Tank Problems

34. Ethyl alcohol is in a 30-cm-diameter tank as indicated in Figure P5.34. The ethyl alcohol exits the tank through an attached tube (square-edged inlet) made of $1/2$ standard type K drawn copper tubing that is 12 m long. Determine the variation of efflux velocity with time for a liquid height h that varies from 1 m initially to 0.3 m.

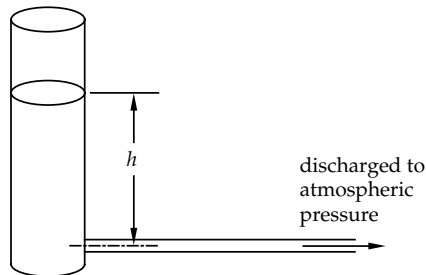


FIGURE P5.34

35. Figure P5.35 shows a 1-gallon tank that contains gasoline (assume same properties as octane). The tank has a cross section of $5\frac{1}{2} \times 8\frac{1}{2}$ in. and it is $5\frac{1}{2}$ in. tall. Soldered to the side of this tank is $1/4$ in.-ID drawn copper tubing that is 2 ft long (squared-edged inlet). Gasoline drains through this tube and is discharged to the atmosphere. Determine the variation of efflux velocity with time for a liquid height h that varies from 4 in. to 0.5 in.

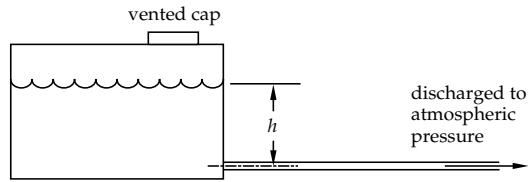


FIGURE P5.35

36. Water flows from a large tank (cross section is 12 ft x 12 ft) through a piping system set up as shown in Figure P5.36. The heights h_1 and h_2 are 10 ft and 12 ft, respectively, and the pipe is made of 1-nominal schedule 40 commercial steel. The gate valve is fully open, and the nozzle at the exit has an internal diameter of 0.5 in. The pipe attaches to the tank with a square-edged inlet, and all fittings are threaded and regular. Determine the variation of velocity with time for a liquid height h_1 that varies from 10 ft to 6 in. The pipe length is 15 ft.

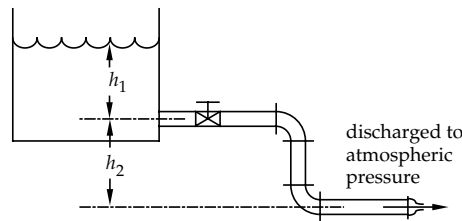


FIGURE P5.36

37. Figure P5.37 shows a 3-gallon tank positioned 6 ft ($= h_2$) above a reference plane. Such tanks were used extensively in the 19th century with toilets. When the tank plug was pulled up, water drained out through a 6-ft- (approximately) long tube to the toilet. The tank is 18 in. wide ($= w$) by 6 in. into the page. Determine the variation of velocity with liquid tank depth for a height h_1 that varies from 9 in. to 2 in. Neglect minor losses, and take the tube size to be $1\frac{1}{4}$ standard type K and made of drawn copper.

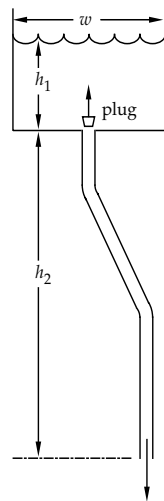


FIGURE P5.37.

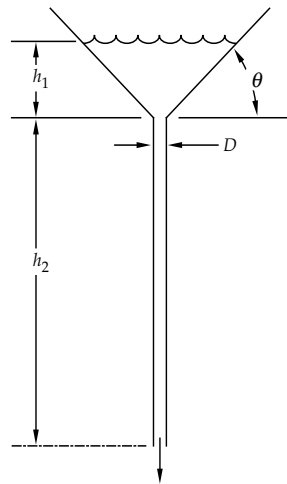


FIGURE P5.38

38. A 45° ($= \theta$) funnel (see Figure P5.38) contains linseed oil that is draining out of the attached tube. The tube itself has an inside diameter of 1 cm ($= D$) and is 60 cm ($= h_2$) long. The funnel and the tube are one-piece cast plastic (ϵ same as drawn tubing). Determine the variation of the efflux velocity with time for a height h_1 that varies from 16 cm to 4 cm.

CHAPTER 6 Pumps and Piping Systems

Pumps are devices used to move fluid through a pipeline. There are many types of pumps designed for different applications. We will examine the types of pumps available and present guidelines useful in selecting a type of pump for a particular job. We will further discuss pump testing methods and focus exclusively on centrifugal pumps. We will show how pump test results are used to size a pump for a given piping configuration. We will cover the concept of cavitation and how it is avoided with design procedures that include calculation of net positive suction head. We conclude with a section on current design practices.

6.1 Types of Pumps

The two general categories of pumps are dynamic and positive displacement. Dynamic pumps usually have a rotating component that imparts energy to the fluid in the form of high velocity, high pressure, or high temperature. Positive-displacement pumps have fixed-volume chambers that take in and discharge the pumped fluid.

Dynamic pumps are usually classified according to the direction the fluid flows through them with respect to the axis of rotation. Fluid flows through an **axial flow pump** in a direction that is parallel to the axis of rotation of moving parts. Fluid passes through a **radial flow pump** in a direction normal to the axis of pump rotation. In a **mixed flow pump**, the fluid flow direction is neither purely axial nor purely radial but some combination of the two.

An axial flow pump (known also as a **propeller pump** or **turbine pump**) is used in low-lift (short vertical pumping distance) applications. An electrically driven motor or an engine can be used to power these pumps. The motor or engine rotates a shaft onto which the impeller is attached. The rotating shaft is enclosed in a housing. The flow passage downstream of the impeller is bounded by this housing and by an outside casing. In a radial flow pump (known also as a **centrifugal pump**), flow passes through the casing, where fluid enters and exits the rotating impeller in the radial direction.

In some pump designs, the discharge of one impeller immediately enters another. A multistage turbine pump operates in this way to pump water upward. The discharge from the first or lowest impeller casing enters the second, and so forth. The impeller casings are bolted together and can consist of any number of desired stages. Also available for any of the pumps just mentioned is a number of different impeller designs, including what are known as **semiopen impellers** and **enclosed impellers**.

There are a number of designs of **positive displacement pumps** that have various uses. A **reciprocating pump**, for example, is made for pumping mud or cement. A reciprocating piston draws in fluid on an intake stroke and moves that fluid out on the discharge stroke. One-way valves in the flow lines control the flow direction.

A **rotary gear pump** is another of the positive-displacement type. It consists of two meshed gears that rotate within a housing. Fluid enters the region between the two gears, and as the gears rotate, fluid is drawn into the volumes between adjacent teeth and the housing. Fluid is discharged on the other side of the housing.

Details regarding the design of pumps are the responsibility of pump manufacturers. Our purpose here is to examine how such pumps are tested and sized for a given application.

6.2 Pump Testing Methods

In this section, we examine a method for testing pumps, and we will use a centrifugal pump for illustrative purposes. Figure 6.1 shows a pump and piping system. The pump contains an impeller within its housing. The impeller is attached to the shaft of the motor, and the motor is mounted so that it is free to rotate, within limits. As the motor rotates and the impeller moves liquid through the pump, the motor housing tends to rotate in the opposite direction from that of the impeller. Weights are placed on the weight hanger so that, at any rotational speed, the motor is kept at an equilibrium position. The amount of weight needed to balance the motor multiplied by the distance from the motor axis to the weight hanger gives the *torque* exerted by the motor.

The *rotational speed* of the motor is obtained with any number of devices available. The product of rotational speed and torque is the *input power* to the impeller from the motor.

Gages in the inlet and outlet lines about the pump give the corresponding *pressures* in gage pressure units. The gages are located at known *heights* from a reference plane. The flowmeter gives a reading of the *volume flow rate* of liquid through the pump.

The valve in the outlet line is used to control the volume flow rate. As far as the pump is concerned, the resistance offered by the valve simulates a piping system with a controllable friction loss. Thus, for any

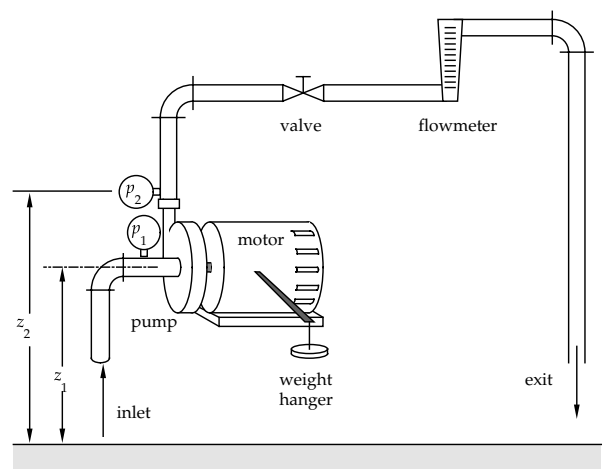


FIGURE 6.1. Centrifugal pump testing setup.

TABLE 6.1. Pump testing parameters.

Raw Data				
Parameter	Symbol	Dimensions	Units	
Torque	T	$F \cdot L$	$J = N \cdot m$	lbf·ft
Rotational speed	ω	$1/T$	rad/s	rad/s
Inlet pressure	p_1	F/L^2	kPa	psi (lbf/ft ²)
Outlet pressure	p_2	F/L^2	kPa	psi (lbf/ft ²)
Volume flow rate	Q	L^3/T	m ³ /s	ft ³ /s (gpm)

valve position in terms of % closed, the following data can be obtained: torque, rotational speed, inlet pressure, outlet pressure, and volume flow rate. These parameters are summarized in Table 6.1.

The parameters used to characterize the pump are calculated with the raw data obtained from the test (listed above) and are as follows: input power to the pump, the total head difference (as outlet minus inlet), the power imparted to the liquid, and the efficiency. These parameters are summarized in Table 6.2.

The raw data are manipulated to obtain the reduced data, which in turn are used to characterize the performance of the pump. The input power to the pump from the motor is the product of torque and rotational speed:

TABLE 6.2. *Pump characterization parameters.*

Reduced Data				
Parameter	Symbol	Dimensions	Units	
input power	dW_a/dt	F·L/T	W = J/s	ft·lbf/s (hp)
total head diff	ΔH	L	m	ft
power to liquid	dW/dt	F·L/T	W	ft·lbf/s (hp)
efficiency	η	—	—	—

$$-\frac{dW_a}{dt} = T\omega \quad (6.1)$$

where the negative sign is added as a matter of convention. The total head at section 1, where the inlet pressure is measured (see Figure 6.1), is defined as

$$H_1 = \frac{p_1 g_c}{\rho g} + \frac{V_1^2}{2g} + z_1$$

where ρ is the liquid density and $V_1 (= Q/A_1)$ is the velocity in the inlet line. Similarly, the total head at position 2, where the outlet pressure is measured, is

$$H_2 = \frac{p_2 g_c}{\rho g} + \frac{V_2^2}{2g} + z_2$$

The *total head difference* is given by

$$\Delta H = H_2 - H_1 = \frac{p_2 g_c}{\rho g} + \frac{V_2^2}{2g} + z_2 - \left(\frac{p_1 g_c}{\rho g} + \frac{V_1^2}{2g} + z_1 \right) \quad (6.2)$$

The dimension of the head H is L (ft or m). The power imparted to the liquid is calculated with the steady flow energy equation applied from section 1 to section 2:

$$-\frac{dW}{dt} = \frac{\dot{m}g}{g_c} \left[\left(\frac{p_2 g_c}{\rho g} + \frac{V_2^2}{2g} + z_2 \right) - \left(\frac{p_1 g_c}{\rho g} + \frac{V_1^2}{2g} + z_1 \right) \right] \quad (6.3)$$

In terms of total head H , we have

$$-\frac{dW}{dt} = \frac{\dot{m}g}{g_c} (H_2 - H_1) = \frac{\dot{m}g}{g_c} \Delta H \quad (6.4)$$

The efficiency is determined with

$$\eta = \frac{dW/dt}{dW_a/dt} = \frac{\text{power imparted to liquid—Equation 6.3}}{\text{input power to impeller—Equation 6.1}} \quad (6.5)$$

How raw data are manipulated to obtain the pump parameters is illustrated in the following example.

EXAMPLE 6.1. A pump is tested as in Figure 6.1, and for one setting of the valve in the discharge line, the following data were read:

Torque	= $T = 3 \text{ ft}\cdot\text{lbf}$
Rotational speed	= $\omega = 1800 \text{ rpm}$
Inlet pressure	= $p_1 = 3 \text{ psig}$
Outlet pressure	= $p_2 = 20 \text{ psig}$
Volume flow rate	= $Q = 6 \text{ gpm}$
Height to inlet gage	= $z_1 = 2 \text{ ft}$
Height to outlet gage	= $z_2 = 3 \text{ ft}$
Inlet flow line	= 2-nominal schedule 40
Outlet flow line	= 1½-nominal schedule 40
Fluid	= water

Calculate the pertinent pump parameters.

Solution: The power transmitted from the motor is

$$-\frac{dW_a}{dt} = T\omega$$

$$-\frac{dW_a}{dt} = 3 \text{ ft}\cdot\text{lbf} \left(1800 \frac{\text{rev}}{\text{min}} \right) \left(2\pi \frac{\text{rad}}{\text{rev}} \right) \left(\frac{1 \text{ min}}{60 \text{ s}} \right)$$

in which it is noted that radians per second rather than revolutions per minute are the proper units to use for rotational speed. Solving,

$$-\frac{dW_a}{dt} = 565.5 \text{ ft}\cdot\text{lbf/s} = \frac{565.5}{550} = 1.03 \text{ hp}$$

In order to calculate the change in head, we must first determine the liquid velocity in the inlet and outlet lines. For 2-nominal schedule 40 pipe, we read from Table D.1

$$ID_1 = 0.1723 \text{ ft} \quad A = 0.02330 \text{ ft}^2$$

Also, for 1½-nominal schedule 40,

$$ID_2 = 0.1342 \text{ ft} \quad A = 0.01414 \text{ ft}^2$$

The volume flow rate was measured to be

$$Q = 55 \frac{\text{gal}}{\text{min}} = 0.123 \text{ ft}^3/\text{s}$$

The inlet velocity is

$$V_1 = \frac{Q}{A_1} = \frac{0.123}{0.02330} = 5.28 \text{ ft/s}$$

The outlet velocity is

$$V_2 = \frac{Q}{A_2} = \frac{0.123}{0.01414} = 8.67 \text{ ft/s}$$

The density of water is taken here to be 1.94 slug/ft³, and so g_c is 1 in the equations. The total head at section 1 is

$$H_1 = \frac{p_1 g_c}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{3(144)}{1.94(32.2)} + \frac{5.28^2}{2(32.2)} + 2$$

or

$$H_1 = 9.35 \text{ ft}$$

At section 2,

$$H_2 = \frac{p_2 g_c}{\rho g} + \frac{V_2^2}{2g} + z_2 = \frac{20(144)}{1.94(32.2)} + \frac{8.67^2}{2(32.2)} + 3$$

or

$$H_2 = 50.27 \text{ ft}$$

Gage pressures were used in the preceding calculations. It makes no difference whether gage or absolute pressures are used, however, because our interest is in the head difference. The head difference would be the

same for either absolute or gage pressures. The total head difference is given by

$$\Delta H = H_2 - H_1 = 50.27 - 9.35 = 40.92 \text{ ft}$$

The power imparted to the liquid (as evidenced by its change in pressure, velocity, and height) is calculated with

$$-\frac{dW}{dt} = \frac{\dot{m}g}{g_c} (H_2 - H_1) = \frac{\rho Q g}{g_c} (H_2 - H_1)$$

Substituting,

$$-\frac{dW}{dt} = 1.94(0.123)(32.2)(40.92) = 314.4 \text{ ft}\cdot\text{lbf/s}$$

The efficiency is determined with

$$\eta = \frac{dW/dt}{dW_o/dt} = \frac{314.4}{565.5}$$

or $\eta = 0.55 = 55\%$

The experimental technique used in obtaining data depends on the desired method of expressing performance characteristics. For example, data could be taken on only one impeller-casing-motor combination. One data point would first be taken at a certain valve setting and at a preselected rotational speed. The valve setting would then be changed, and the speed control on the motor (not shown in Figure 6.1) is adjusted if necessary so that the rotational speed remains constant. The objective in all tests is to show how the pump operates over all possible variations of conditions.

Figure 6.2 illustrates a graph of data obtained from tests performed on a centrifugal pump. On the horizontal axis is the volume flow rate through the pump. On the vertical axis is the head difference ΔH , defined as

$$\Delta H = H_2 - H_1$$

where

$$H = \frac{pg_c}{\rho g} + \frac{V^2}{2g} + z$$

and from Equation 6.3,

$$\Delta H = - \frac{g_c}{\dot{m}g} \frac{dW}{dt}$$

Figure 6.2 is known as a **performance map**, which essentially is a graph of power ($\propto \Delta H$) versus flow rate. The data are for a specific pump impeller-housing combination operated at four different rotational speeds. The head difference ΔH for each of the lines tends to decrease with increasing volume flow rate Q . The lines were drawn through discrete data points, which customarily are not shown in the figure. Lines have also been drawn through data points of equal efficiency to yield **iso-efficiency** curves. The objective of the entire test is to locate the region of maximum efficiency for the pump, which is now easily identified.

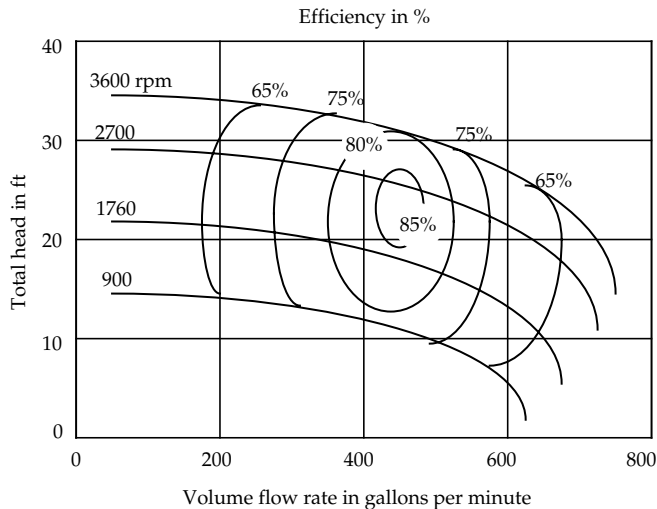


FIGURE 6.2. Performance map of one impeller-casing-motor combination obtained at four different rotational speeds.

Figure 6.3 represents an alternative method of obtaining data and expressing results. One pump casing-motor combination is used with four different impeller sizes. All data are obtained at only one rotational speed, however. Data for one impeller would be taken at a certain valve setting. The valve setting is then changed, and motor control is adjusted if necessary, so that the rotational speed is maintained constant (in this case, 1760 rpm). Again, the objective is to show how the pump operates over all possible variations. The tendency for the head difference ΔH to decrease with increasing volume flow rate Q is apparent. Iso-efficiency lines have

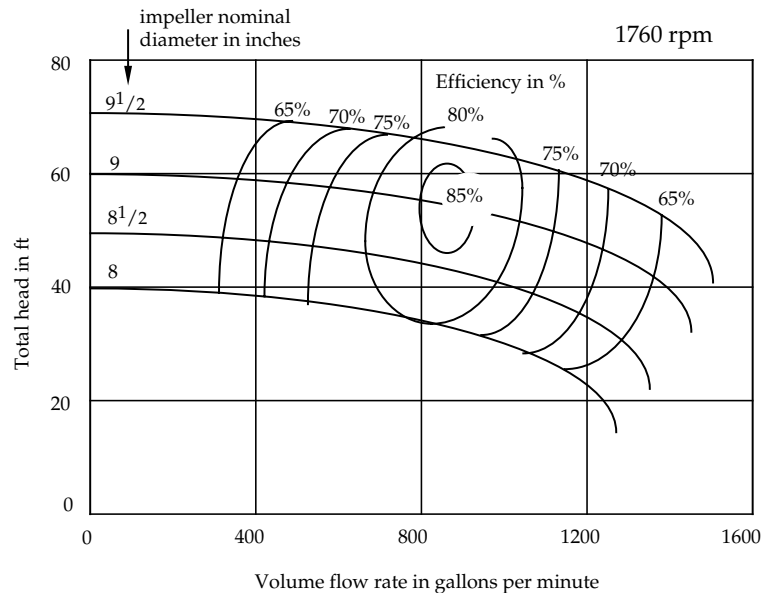


FIGURE 6.3. Performance map of one motor-casing combination and four different impellers obtained at one rotational speed.

been drawn through discrete data points in order to locate the region of maximum efficiency.

A manufacturer might produce over a dozen different housings. For each housing, four or five different impellers could be used. Each combination would require testing and the production of a performance map. The reasoning is that if we were to use the pump represented in Figure 6.2 (or any pump), we would prefer to operate it in the maximum efficiency region. So with regard to all the pumps one company might produce, all that the manufacturer would need to supply to potential users is a summary or composite display of all the maximum efficiency regions.

Figure 6.4 is a graph of ΔH versus Q for a number of pumps showing only the maximum efficiency region for each. Within each region is a number that corresponds to the pump whose maximum efficiency region is represented. Figures like this are used to select a pump for a particular application. Such graphs can be produced for virtually any of the pumps discussed in this chapter. The following example shows how Figure 6.4 is used to select a pump for a given piping system.

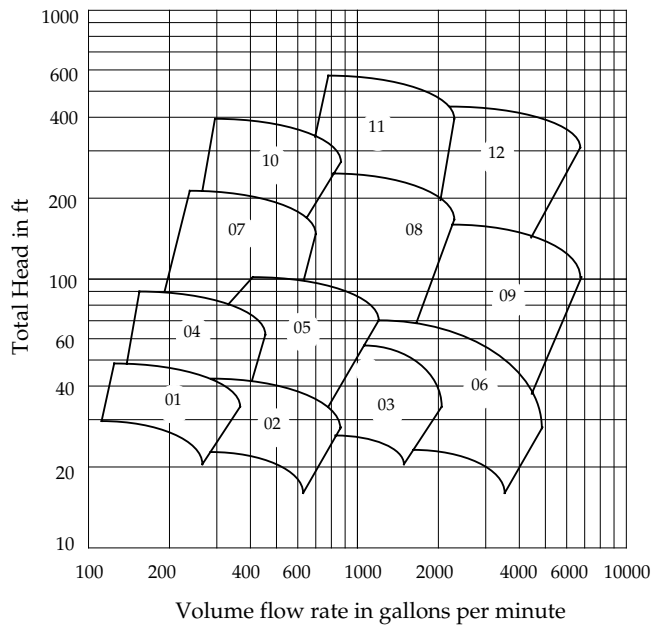


FIGURE 6.4. Composite graph of total head difference versus volume flow rate showing maximum efficiency regions for 20 pumps.

EXAMPLE 6.2. Figure 6.5 shows a pipeline that conveys water to an elevated tank at a campsite. The elevated tank supplies water to people taking showers. The 40-ft-long pipeline is attached at A with a re-entrant inlet. The line contains three elbows and one ball check valve (at B) and is made of 6-nominal schedule 40 PVC pipe. The pump must deliver 250 gpm. Use Figure 6.4 to select a pump for the system, and calculate the pumping power if $\Delta z = 30$ ft.

Solution: We will need to calculate the total head difference and enter Figure 6.4 at the given flow rate and calculated ΔH . Proceeding in the usual way,

$$\text{water } \rho = 62.4 \text{ lbm/ft}^3 \quad \mu = 1.9 \times 10^{-5} \text{ lbf}\cdot\text{s/ft}^2 \quad [\text{Appendix Table B.1}]$$

$$6\text{-nom sched 40 } ID = 0.5054 \text{ ft } A = 0.2006 \text{ ft}^2 \quad [\text{Appendix Table D.1}]$$

For PVC, we use the $\varepsilon/D = \text{“smooth”}$ curve on the Moody Diagram. Referring to Figure 6.5, we note that the pump must lift water 30 ft and overcome minor losses and friction in a 40-ft length of pipe. The steady-flow energy equation including friction for the piping system of Figure 6.5 is

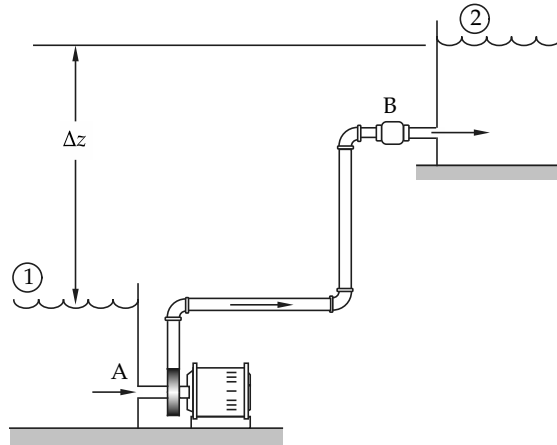


FIGURE 6.5. *The piping system of Example 6.2.*

$$\frac{p_1 g_c}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2 g_c}{\rho g} + \frac{V_2^2}{2g} + z_2 + \sum \frac{fL}{D_h} \frac{V^2}{2g} + \sum K \frac{V^2}{2g} + \frac{g_c}{\dot{m}g} \frac{dW}{dt} \quad (6.6)$$

in which we take section 1 to be the free surface in the sump tank and section 2 to be at the free surface of the elevated tank. The method involves solving Equation 6.6 for power (dW/dt) and then finding ΔH with Equation 6.4. Evaluating properties,

$$p_1 = p_2 = p_{\text{atm}} = 0 \quad V_1 = V_2 \quad z_1 = 0 \quad z_2 = 30 \text{ ft}$$

$$Q = 250 \text{ gal/min} = 0.555 \text{ ft}^3/\text{s} \quad V = \frac{Q}{A} = \frac{0.555}{0.2006} = 2.76 \text{ ft/s}$$

Reynolds number $\text{Re} = \rho V D / \mu g_c$:

$$\text{Re} = \frac{62.4(2.76)(0.5054)}{1.9 \times 10^{-5} (32.2)}$$

and so

$$\left. \begin{array}{l} \text{Re} = 1.43 \times 10^5 \\ \frac{\varepsilon}{D} = \text{"smooth"} \end{array} \right\} \quad f = 0.0165 \quad (\text{Moody Diagram})$$

Minor losses [Table 3.3]:

$$\Sigma K = 3K_{90^\circ \text{ elbow}} + K_{\text{re-entrant inlet}} + K_{\text{ball check valve}} + K_{\text{exit}}$$

$$\Sigma K = 3(0.31) + 1.0 + 70 + 1.0 = 72.9$$

Equation 6.6 reduces to

$$0 = z_2 + \left(\Sigma \frac{fL}{D_h} + \Sigma K \right) \frac{V^2}{2g} + \frac{g_c}{\dot{m}g} \frac{dW}{dt}$$

Substituting gives

$$0 = 30 + \left(\frac{0.0165(40)}{0.5054} + 72.9 \right) \frac{2.76^2}{2(32.2)} + \frac{g_c}{\dot{m}g} \frac{dW}{dt}$$

Solving, we get

$$-\frac{g_c}{\dot{m}g} \frac{dW}{dt} = \Delta H = 38.8 \text{ ft}$$

Although the following step was not asked for in the problem statement, we can also find the power:

$$-\frac{dW}{dt} = \rho Q \frac{g}{g_c} (38.8) = 62.4(0.555)(32.2/32.2)(38.8)$$

$$-\frac{dW}{dt} = 1344 \text{ ft}\cdot\text{lbf/s} = 1344/550$$

$$-\frac{dW}{dt} = 2.44 \text{ hp} \approx 2.5 \text{ hp}$$

Thus, the total head difference found as 38.8 ft includes lifting the water 30 ft and overcoming friction and minor losses. At 38.8 ft and 250 gpm, Figure 6.4 shows that the pump corresponding to the region labeled "01" would be suitable for this application.

6.3 Cavitation and Net Positive Suction Head

The suction line of a centrifugal pump contains liquid at a pressure that may be much lower than atmospheric pressure. If this suction pressure is sufficiently low, the liquid will begin to boil at the local temperature. For example, water boils at 33°C (92°F) if its pressure is lowered to 5.1 kPa (0.75 psia). Boiling itself involves vapor bubble formation, and when this phenomenon occurs in a pump it is called **cavitation**.

In a cavitating centrifugal pump, vapor bubbles usually form at the eye of the impeller; as they move radially through the impeller with the liquid, the bubbles encounter a high-pressure region. It is here where they collapse and send pressure waves outward. The pressure waves have an erosive effect on the impeller and housing, known as **cavitation erosion**. As indicated in the preceding discussion, when cavitation occurs, the impeller is no longer moving an all-liquid fluid through the housing. As a result, the efficiency of the pump falls drastically. If the situation is not corrected, the pump may eventually fail due to metal erosion and fatigue of shaft bearings and/or seals.

Cavitation is not a problem that should be corrected after installation of the system. Instead, the inception of cavitation is predictable, and the engineer should ensure that cavitation will not occur when the system is designed. Pump manufacturers perform tests on pumps and provide information useful for predicting when cavitation will occur.

Net Positive Suction Head

Consider the centrifugal pump and inlet configurations of Figure 6.6. Illustrated in both cases is a centrifugal pump moving liquid from a tank. Two configurations are shown: suction lift, when the liquid level in the tank lies below the pump impeller centerline; and suction head, in which the liquid level in the tank is above the pump impeller centerline.

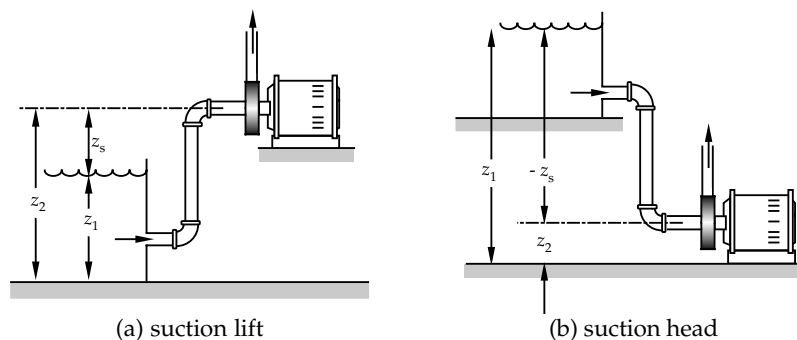


FIGURE 6.6. *Illustration of suction lift and suction head at pump inlet.*

We will now apply the modified Bernoulli equation from section 1 (free surface of the tank liquid) to section 2 (inlet to the pump housing) for the case when the pump in Figure 6.6a is operating. Our objective is to determine the pressure at the pump inlet and compare it to the vapor pressure of the liquid. If the pressure at the pump inlet is less than the vapor pressure of the liquid at the local temperature, then the liquid will boil at the impeller; thus the pump will cavitate. The modified Bernoulli equation is

$$\frac{p_1 g_c}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2 g_c}{\rho g} + \frac{V_2^2}{2g} + z_2 + \Sigma \frac{fL}{D_h} \frac{V^2}{2g} + \Sigma K \frac{V^2}{2g} \quad (6.7)$$

Although p_1 is atmospheric, we will not set it equal to zero. This will allow our final equation to account for an overpressure, expressed in absolute units, on the liquid surface in the tank. Evaluating properties yields

$$V_1 = 0 \quad z_2 - z_1 = +z_s \quad V_2 = V = \text{velocity in the pipe}$$

Rearranging Equation 6.7 and solving for $p_2 g_c / \rho g$, we obtain

$$\frac{p_2 g_c}{\rho g} = \frac{p_1 g_c}{\rho g} - z_s - \left(\Sigma \frac{fL}{D_h} + \Sigma K + 1 \right) \frac{V^2}{2g}$$

Next we subtract the vapor pressure from both sides of the preceding equation, and rearrange slightly to obtain

$$\frac{p_2 g_c}{\rho g} - \frac{p_v g_c}{\rho g} = \frac{p_1 g_c}{\rho g} - z_s - \left(\Sigma \frac{fL}{D_h} + \Sigma K + 1 \right) \frac{V^2}{2g} - \frac{p_v g_c}{\rho g} \quad (6.8a)$$

or

$$\text{NPSH}_a = \frac{p_2 g_c}{\rho g} - \frac{p_v g_c}{\rho g} = \frac{p_1 g_c}{\rho g} - z_s - \left(\Sigma \frac{fL}{D_h} + \Sigma K + 1 \right) \frac{V^2}{2g} - \frac{p_v g_c}{\rho g}$$

(Figure 6.6a)
(suction lift)

where the left-hand side of the preceding equation is defined as the **net positive suction head available**, NPSH_a . Equation 6.8a applies to the inlet configuration of Figure 6.6a. For Figure 6.6b, we would apply Equation 6.7 as before to obtain:

$$\frac{p_2 g_c}{\rho g} - \frac{p_v g_c}{\rho g} = \frac{p_1 g_c}{\rho g} + z_s - \left(\Sigma \frac{fL}{D_h} + \Sigma K + 1 \right) \frac{V^2}{2g} - \frac{p_v g_c}{\rho g} \quad (6.8b)$$

$$\text{NPSH}_a = \frac{p_2 g_c}{\rho g} - \frac{p_v g_c}{\rho g} = \frac{p_1 g_c}{\rho g} + z_s - \left(\sum \frac{fL}{D_h} + \sum K + 1 \right) \frac{V^2}{2g} - \frac{p_v g_c}{\rho g}$$

(Figure 6.6b)
(suction head)

Manufacturers perform tests on pumps and report values of **net positive suction head required**, NPSH_r . Cavitation is prevented when the available net positive suction head is greater than the required net positive suction head, or

$$\text{NPSH}_a > \text{NPSH}_r \quad (\text{cavitation prevention}) \quad (6.9)$$

Data on net positive suction head for a particular pump are often shown on the pump performance map or are provided separately.

In order to make calculations on net positive suction head, it is necessary to have data on vapor pressure. Figure 6.7 is a graph of vapor pressure versus temperature for various liquids.

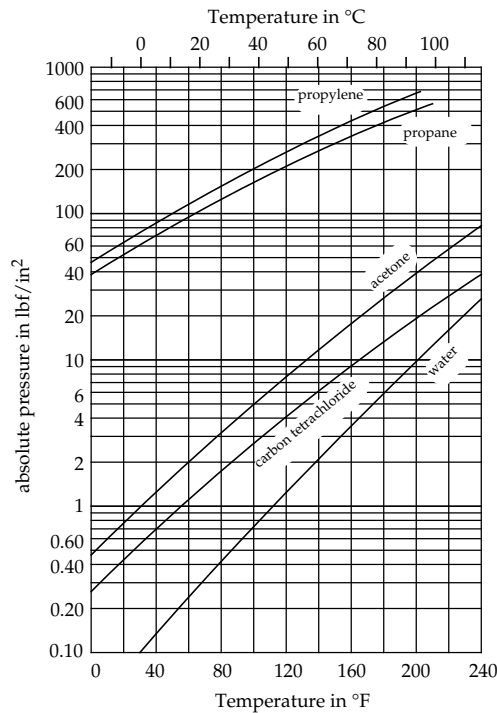


FIGURE 6.7. Vapor pressure versus temperature for various liquids.
(Data from several sources.)

The dimension of the cavitation equations is L (ft or m of liquid), and so each term can be represented as a head of liquid. Equation 6.8a in some texts is written as

$$h_{p_2} - h_{vp} = \text{NPSH}_a = h_{p_1} - h_{z_2} - h_f - h_{vp} \quad (6.10)$$

EXAMPLE 6.3. A certain pump delivers 900 gpm of water from a tank at a head difference ΔH of 8 ft. The net positive suction head required is 10 ft. Determine where the pump inlet should be with respect to the level of water in the tank. The water surface is exposed to atmospheric pressure. Neglect frictional effects and take the water temperature to be 90°F.

Solution: Here we apply Equation 6.8a, assuming that we have a suction lift as in Figure 6.6a:

$$\text{NPSH}_a = \frac{p_1 g_c}{\rho g} - z_s - \left(\sum \frac{fL}{D_h} + \sum K + 1 \right) \frac{V^2}{2g} - \frac{p_v g_c}{\rho g} \quad \left(\begin{array}{l} \text{Figure 6.6a} \\ \text{suction lift} \end{array} \right)$$

With $\text{NPSH}_r = 10$ ft, we have

$$\text{NPSH}_a = \frac{p_2 g_c}{\rho g} - \frac{p_v g_c}{\rho g} = 10 \text{ ft}$$

This means that the pressure of the liquid at the casing inlet at the impeller should exceed the vapor pressure of the liquid by an amount equal to or greater than 10 ft in order to prevent cavitation. Proceeding with the calculations,

$$\text{water} \quad \rho = 62.4 \text{ lbm/ft}^3 \quad [\text{Appendix Table B.1}]$$

$$p_v = 0.55 \text{ lbf/in}^2 \quad [\text{Figure 6.7 @ 90°F, water}]$$

Property evaluation:

$$p_1 = p_{atm} = 14.7 \text{ lbf/in}^2 \quad \text{Frictional effects} = 0$$

Substituting into Equation 6.8a gives

$$10 = \frac{14.7(144)(32.2)}{62.4(32.2)} - z_s - \frac{0.55(144)(32.2)}{62.4(32.2)}$$

Rearranging and solving, we find

$$-z_s = 10 - 33.9 + 1.27$$

or $\underline{z_s = 22.63 \text{ ft}}$

Thus, to prevent cavitation, $z_s (= z_2 - z_1)$ would have to be equal to or less than 22.6 ft in the configuration of Figure 6.6a.

6.4 Dimensional Analysis of Pumps

A dimensional analysis can be performed for pumps, and the result can be used as an aid in selecting a pump type (e.g., centrifugal versus mixed versus axial). With regard to the flow of an incompressible fluid through a pump, we wish to relate three of the variables introduced thus far to the flow parameters. The three variables of interest here are the efficiency η , the energy transfer rate $g \Delta H$, and the power dW/dt . These three parameters are presumed to be functions of fluid properties density ρ and viscosity μ , volume flow rate through the machine Q , rotational speed ω , and a characteristic dimension (usually impeller diameter) D . We therefore write three functional dependencies:

$$\eta = f_1(\rho, \mu, Q, \omega, D, g_c) \quad (6.11)$$

$$g \Delta H = f_2(\rho, \mu, Q, \omega, D, g_c) \quad (6.12)$$

$$\frac{dW}{dt} = f_3(\rho, \mu, Q, \omega, D, g_c) \quad (6.13)$$

Beginning with Equation 6.11, we assume a relationship of the form

$$\eta = a \rho^b \mu^c Q^d \omega^e D^f g_c^g \quad (6.14)$$

Next we substitute dimensions into Equation 6.14 for each parameter to obtain

$$0 = a \left(\frac{\text{M}}{\text{L}^3} \right)^b \left(\frac{\text{F} \cdot \text{T}}{\text{L}^2} \right)^c \left(\frac{\text{L}^3}{\text{T}} \right)^d \left(\frac{1}{\text{T}} \right)^e (\text{L})^f \left(\frac{\text{M} \cdot \text{L}}{\text{F} \cdot \text{T}^2} \right)^g$$

We can now write an equation for each dimension in the equation:

$$\text{M:} \quad 0 = b + g$$

$$\text{F:} \quad 0 = c - g$$

$$\text{L:} \quad 0 = -3b - 2c + 3d + f + g$$

$$\text{T:} \quad 0 = c - d - e - 2g$$

Solving simultaneously gives

$$b = -g \quad c = g \quad d = -e - g \quad f = 3e + g$$

Substituting into Equation 6.14 gives

$$\eta = a \frac{1}{\rho^g} \mu^g \frac{1}{Q^e Q^g} \omega^e D^{3e} D^g g_c^g$$

Grouping terms with like exponents yields

$$\eta = a \left(\frac{D g_c \mu}{\rho Q} \right)^g \left(\frac{\omega D^3}{Q} \right)^e \quad (6.15)$$

Although the above groups are dimensionless and the equation itself is valid, current practice calls for a slight rearrangement of the result. The groups on the right-hand side can be combined to yield a more easily recognizable result:

$$\frac{\rho Q}{D g_c \mu} \cdot \frac{\omega D^3}{Q} = \frac{\rho \omega D^2}{\mu g_c}$$

Equation 6.15 can now be rewritten in functional form as

$$\eta = f_1 \left(\frac{\rho \omega D^2}{\mu g_c}, \frac{Q}{\omega D^3} \right) \quad (6.16)$$

Likewise, Equations 6.12 and 6.13 become

$$\frac{g \Delta H}{\omega^2 D^2} = f_2 \left(\frac{\rho \omega D^2}{\mu g_c}, \frac{Q}{\omega D^3} \right) \quad (6.17)$$

$$\frac{g_c (dW/dt)}{\rho \omega^3 D^5} = f_3 \left(\frac{\rho \omega D^2}{\mu g_c}, \frac{Q}{\omega D^3} \right) \quad (6.18)$$

where

$$\frac{g \Delta H}{\omega^2 D^2} = \text{energy transfer coefficient}$$

$$\frac{Q}{\omega D^3} = \text{volumetric flow coefficient}$$

$$\frac{\rho \omega D^2}{\mu g_c} = \text{rotational Reynolds number}$$

$$\frac{g_c (dW/dt)}{\rho \omega^3 D^5} = \text{power coefficient}$$

Experiments conducted with pumps show that the rotational Reynolds number ($\rho \omega D^2 / \mu g_c$) has a smaller effect on the dependent variables than does the flow coefficient. So for incompressible flow through pumps, Equations 6.16, 6.17, and 6.18 reduce to

$$\eta \approx f_1 \left(\frac{Q}{\omega D^3} \right) \quad (6.19)$$

$$\frac{g \Delta H}{\omega^2 D^2} \approx f_2 \left(\frac{Q}{\omega D^3} \right) \quad (6.20)$$

$$\frac{g_c (dW/dt)}{\rho \omega^3 D^5} \approx f_3 \left(\frac{Q}{\omega D^3} \right) \quad (6.21)$$

The significance of these ratios for a centrifugal pump is apparent when performance data are being modeled. Suppose, for example, that performance data are available for a particular pump operating under certain conditions. The data can be used to predict the performance of the pump when something has been changed, such as rotational speed, impeller diameter, volume flow rate, or fluid density. The above equations are known as **similarity laws** or **affinity laws** for pumps. The method for using them is illustrated in the following example.

EXAMPLE 6.4. Actual performance data on a centrifugal pump are as follows:

$$\begin{aligned} \text{Rotational speed} &= \omega = 3500 \text{ rpm} \\ \text{Total head difference} &= \Delta H = 80 \text{ ft} \\ \text{Volume flow rate} &= Q = 50 \text{ gpm} \\ \text{Impeller diameter} &= D = 5^{1/8} \text{ in.} \\ \text{Fluid} &= \text{water} \end{aligned}$$

It is desired to change the rotational speed to 1750 rpm and the impeller diameter to $4\frac{5}{8}$ in. Determine how the new configuration will affect the pump performance with water as the working fluid.

Solution: We will use similarity to determine how the new parameters will affect the pump performance. We will refer to the original configuration of the pump with a subscript of “1” and the new configuration with a “2.” When we use similarity laws, we assume that the dimensionless ratios apply to both pumps. Equations 6.19–6.21 contain the dimensionless groups needed; the flow coefficient applied to both pumps is

$$\left. \frac{Q}{\omega D^3} \right|_1 = \left. \frac{Q}{\omega D^3} \right|_2$$

Substituting and noting that conversion factors would cancel if used gives

$$\frac{50}{3500(5.125)^3} = \frac{Q_2}{1750(4.625)^3}$$

Solving, we find the flow rate in the new configuration to be

$$\underline{Q_2 = 18.4 \text{ gpm}}$$

The head coefficient in Equation 6.20 can also be applied:

$$\left. \frac{g \Delta H}{\omega^2 D^2} \right|_1 = \left. \frac{g \Delta H}{\omega^2 D^2} \right|_2$$

Substituting and noting that gravity is constant yields

$$\frac{80}{3500^2(5.125)^2} = \frac{\Delta H_2}{1750^2(4.625)^2}$$

Solving for the total head in the new configuration gives

$$\underline{\Delta H_2 = 16.3 \text{ ft}}$$

If necessary, the power can be found with Equation 6.4:

$$-\frac{dW}{dt} = \frac{\dot{m}g}{g_c} \Delta H = \frac{\rho Qg}{g_c} \Delta H \quad (6.4)$$

or with the power coefficient $[g_c(dW/dt)/\rho\omega^3D^5]$ of Equation 6.21 applied to both configurations. The efficiency remains nearly constant for small changes.

For the pump of this example, data on the original configuration were taken from an actual pump performance curve. For purposes of comparison, the actual data (from a manufacturer) on the new configuration are

For $Q \approx 20$ gpm, $\Delta H \approx 17$ ft, compared to 16.3 ft calculated earlier

Also from the pump performance graphs, the efficiency of the original configuration is 57%, compared to 48% in the new one.

As seen in the previous example, pump affinity laws might be rewritten for two similar pumps as:

$$\frac{Q_1}{\omega_1 D_1^3} = \frac{Q_2}{\omega_2 D_2^3}$$

$$\frac{\Delta H_1}{\omega_1^2 D_1^2} = \frac{\Delta H_2}{\omega_2^2 D_2^2}$$

$$\frac{(dW/dt)_1}{\rho_1 \omega_1^3 D_1^5} = \frac{(dW/dt)_2}{\rho_2 \omega_2^3 D_2^5}$$

In industry, pump affinity laws are written as

$$\frac{Q_1}{D_1} = \frac{Q_2}{D_2} \quad \text{or} \quad \frac{Q_1}{\omega_1} = \frac{Q_2}{\omega_2}$$

$$\frac{\Delta H_1}{\omega_1^2} = \frac{\Delta H_2}{\omega_2^2} \quad \text{or} \quad \frac{\Delta H_1}{D_1^2} = \frac{\Delta H_2}{D_2^2}$$

and $\frac{(dW/dt)_1}{\omega_1^3} = \frac{(dW/dt)_2}{\omega_2^3} \quad \text{or} \quad \frac{(dW/dt)_1}{D_1^3} = \frac{(dW/dt)_2}{D_2^3}$

6.5 Specific Speed and Pump Types

With the many types of pumps available, it is necessary to have some criteria regarding the type of pump to use for a specific application: axial, mixed, or radial flow. A dimensionless group known as **specific speed** is used in the decision-making process. Specific speed is defined as the speed (in

rpm) of a geometrically similar pump (to the one being studied) that will deliver one gallon per minute against one foot of head. Specific speed is found by combining head coefficient and flow coefficient in order to eliminate characteristic length D :

$$\omega_{ss} = \left(\frac{Q}{\omega D^3} \right)^{1/2} \left(\frac{\omega^2 D^2}{g \Delta H} \right)^{3/4}$$

or $\omega_{ss} = \frac{\omega Q^{1/2}}{(g \Delta H)^{3/4}} \quad [\text{dimensionless}] \quad (6.22)$

Exponents other than 1/2 and 3/4 could be used (to eliminate D), but 1/2 and 3/4 are customarily selected for modeling pumps. Another definition for specific speed is given by

$$\omega_s = \frac{\omega Q^{1/2}}{\Delta H^{3/4}} \quad \left[\text{rpm} = \frac{\text{rpm}(\text{gpm})^{1/2}}{\text{ft}^{3/4}} \right] \quad (6.23)$$

in which the rotational speed ω is expressed in rpm, volume flow rate Q is in gpm, total head ΔH is in ft of liquid, and specific speed ω_s is arbitrarily assigned the unit of rpm. Equation 6.22 for specific speed ω_{ss} is dimensionless, whereas Equation 6.23 for ω_s is not. Specific speed found with Equation 6.22 can be applied using any unit system, but Equation 6.23 is defined exclusively in engineering units. Moreover, results of calculations made with these equations differ by a large magnitude.

The specific speed of a pump can be calculated at any operating point. It is advantageous, however, to calculate specific speed for a pump only at its maximum efficiency. Then specific speed data can be used to select a pump with the knowledge that the pump will be operating at its maximum efficiency point. To illustrate these concepts, refer to the pump performance curve of Figure 6.2. At maximum efficiency, we read

$$Q \approx 450 \text{ gpm} \quad \Delta H \approx 24 \text{ ft} \quad \omega \approx 2500 \text{ rpm} \quad \eta \approx 85\%$$

A flow rate of 450 gpm equals 1.004 ft³/s, and 2500 rpm equals 262 rad/s. Substituting into Equation 6.22 gives

$$\omega_{ss} = \frac{262(1.004)^{1/2}}{(32.2(24))^{3/4}} = 1.791$$

Equation 6.23, on the other hand, yields

$$\omega_s = \frac{2500(450)^{1/2}}{24^{3/4}} = 4890 \text{ rpm}$$

After manipulation of the conversion factors, we find that

$$\omega_s \approx 2736\omega_{ss}$$

With data obtained from tests on many types of pumps (including axial, mixed, and radial or centrifugal), the data of Table 6.3 have been produced. When this table is used, a machine that operates at or near its maximum efficiency is being selected.

TABLE 6.3. *Pump selection chart for finding efficiency in %.*

ω_s rpm	Flow Rate in Gallons per Minute, gpm							ω_{ss}	Pump Type
	100	200	500	1000	3000	10,000	>10,000		
500	46	54						0.18	Centrifugal or radial flow type
600	52	58						0.22	
700	55	62	70					0.26	
800	58	64	72	76				0.29	
900	61	66	74	77	81			0.33	
1000	62	67	75	78	82			0.37	
1500	67	72	79	82	85	87	89	0.55	Mixed flow type
2000	70	75	81	83	87	89	92	0.73	
3000			81	83	87	89	92	1.10	
4000			79	81	85	88	90	1.46	
5000					84	87	90	1.83	
6000					83	85	87	2.19	
7000					82	84	86	2.56	
8000					80	84	86	2.92	
9000						83	85	3.29	
10,000						82	84	3.65	Axial flow type
15,000						77	80	5.48	

Equations: $\omega_{ss} = \frac{\omega Q^{1/2}}{(g\Delta H)^{3/4}}$ [dimensionless] (6.22)

$$\omega_s = \frac{\omega Q^{1/2}}{\Delta H^{3/4}} \left[\text{rpm} = \frac{\text{rpm}(\text{gpm})^{1/2}}{\text{ft}^{3/4}} \right] \quad (6.23)$$

Note on reading the table: For a specific speed of 5000 rpm and a flow rate of 3000 gpm, the maximum efficiency (any pump) that can be expected is approximately 84% and the recommended pump is a mixed flow type.

EXAMPLE 6.5. Determine the type of pump best suited for pumping 300 gpm ($= 0.67 \text{ ft}^3/\text{s}$) of water with a corresponding head of 25 ft. The motor to be used has a rotational speed of 1150 rpm.

Solution: We calculate the specific speed using Equation 6.23:

$$\omega_s = \frac{\omega Q^{1/2}}{\Delta H^{3/4}} \quad \left[\text{rpm} = \frac{\text{rpm}(\text{gpm})^{1/2}}{\text{ft}^{3/4}} \right] \quad (6.23)$$

$$\omega_s = \frac{1150(300)^{1/2}}{25^{3/4}} = 1782 \text{ rpm}$$

We enter Table 6.3 at 1700 rpm (approximately). We could interpolate to obtain the exact value at 300 gpm, but it is not necessary to do so because these values are approximate. We read an efficiency of ~75%. Moving to the far right column, we find that the impeller shape suggested as best for this application is that for a mixed flow pump. Thus, the table indicates that we should be able to obtain a mixed flow pump for this application, and that the efficiency we can expect is approximately 75%.

We can in addition calculate the power using the energy equation, which is

$$\frac{dW}{dt} = \frac{\rho g Q \Delta H}{g_c} = \frac{62.4(32.2)(0.67)(25)}{32.2}$$

$$\text{or } \frac{dW}{dt} = 1044 \text{ ft}\cdot\text{lbf/s} = 1044/550 = 1.9 \text{ hp}$$

The above value is the power actually transmitted to the fluid. The power required to be transferred from the motor is

$$\frac{dW_a}{dt} = \frac{dW/dt}{\eta} = \frac{1044}{0.75}$$

where $\eta = 75\%$ was obtained from Table 6.3. Solving, we get

$$\frac{dW_a}{dt} = 1392 \text{ ft}\cdot\text{lbf/s} = 2.53 \text{ hp} \quad (\text{mixed flow impeller shape})$$

6.6 Piping System Design Practices

Transporting a fluid from one location to another does not enhance its value. Yet a piping system can be one of the greatest cost items in an installation. For example, in a processing plant, the piping system is usually 15–20% of the total plant cost. Consequently, piping should be designed to meet minimum cost requirements and still be adequate for meeting operational requirements. Fortunately, however, pipes are available in a few discrete sizes, so the economic analysis is somewhat simplified. It is the responsibility of the engineer to ensure that the optimum system is designed.

Another viewpoint to consider is that of the pump manufacturer. Pumps are made to last for a long time. Loss of a sale to a pump manufacturer is a permanent loss. A manufacturer that gains a sale will have income in future sales of replacement parts such as seals or bearings.

In light of the facts associated with the design of a piping system, following now are design practices that will help an engineer in the decision-making process.

As shown in Chapter 4, the economic line size is the one that minimizes the total cost of a pipeline, including fittings, hangers, pumps, valves, and installation. Charts were provided that should aid in determining the economic line size. Not mentioned previously, there is one limitation that should be observed. At pressure drops greater than 25–30 psi per 1000 ft (175–200 kPa per 300 m) of pipe for liquids and 10–15 psi per 1000 ft (70–104 kPa per 300 m) of pipe for gases, excessive and objectionable vibrations in the system will result. For low-pressure steam, the pressure drop should be limited to 4 psi (28 kPa) per 100 ft (30 m).

The engineer is responsible for recommending the most economic line size, unless unusual conditions exist for the system of interest. It is generally worthwhile to make a complete and thorough analysis because a bad decision during the design phase can lead to years of unnecessarily high costs that are wasteful and not recoverable. When the economic diameter is calculated, the result usually falls between two nominal sizes. Selecting the smaller size results in a lower initial capital investment. Selecting the larger size results in a lower operating cost. From an economics viewpoint, the total yearly cost is nearly the same for either size selected. From an engineering standpoint, the larger size leads to more design flexibility. In addition, selecting the larger size allows for any late changes in the specifications for volume carrying capacity or for errors. Lower operating costs is also an advantage, as power costs are never expected to decrease. Furthermore, the smaller-size pipe could have a pressure drop that is excessive, leading to vibrations. Finally, the smaller pipe size is subject to corrosion and/or sediment deposition on the surface of the conduit to a greater degree than the larger size would be.

The proper design of the system contains many aspects. The economic size should be determined. The pressure drop and the power should also be calculated. If a pump is required, then it too should be selected and the net positive suction head should always be determined so that cavitation is avoided. An inadequate suction line can cause problems for an entire plant, especially if downstream conditions need liquid to be pumped at a rate that is higher than a cavitating pump can deliver.

It is necessary to determine conditions of the fluid upstream and downstream of the piping system. Specifically, pressure and/or fluid level in a container should be known. The minimum operating pressure upstream and the maximum operating pressure downstream should be calculated. When dealing with tanks, the minimum liquid level in the upstream tank (usually 2 ft, or 60 cm, above pipe inlet) and the maximum level in the downstream tank (usually full) should be calculated. Making such calculations is vital to avoiding problems after installation.

If the engineer believes that the line will someday be required to convey a flow rate that is greater than the design value, the engineer should not modify the design. Allowances for future increases in volume flow rate should not be considered unless specifically spelled out as part of the design. Determining extra-capacity requirements is the responsibility of management. Notifying management that this practice may be of some value is the responsibility of the engineer.

In gravity-fed systems (where gravity rather than a pump is the driving power), a line size larger than the economic diameter might be required in order to meet volume flow rate requirements. In some cases where gravity might be used, it could be more economical to install a pump and a smaller line size. Thus, the optimum economic diameter selection process might not apply.

In all piping systems, it is possible that air will be trapped somewhere in the line. It is advisable to lay out pipelines with a slight grade upward in the flow direction so that air will tend not to remain in the line. Where this is not possible, a small valve should be installed at places where air (or vapor) might tend to accumulate.

Piping System Design—Suggested Procedure

1. Given the economic parameters associated with the system, determine the economic line size. If the cross section is noncircular and/or contains fittings, perform calculations for a straight run of circular pipe. Use the calculated economic diameter to find the optimum economic velocity. Use the economic velocity to complete the details of the system design, including the placement of hangers. Table 6.4 gives results of calculations of economic or reasonable velocity ranges for many fluids.
2. Calculate the pump power required for the system using the optimum

economic line size. Check to ensure that the pressure drop is not excessive, which leads to objectionable vibrations. Prepare a system curve of ΔH versus Q .

TABLE 6.4. *Reasonable velocities for various fluids, calculated by using optimum economic diameter equations.*

Fluid	Economic Velocity Range	
	ft/s	m/s
Acetone	4.9–9.8	1.5–3.0
Ethyl Alcohol	4.8–9.6	1.5–3.0
Methyl Alcohol	4.8–9.6	1.5–3.0
Propyl Alcohol	4.7–9.4	1.4–2.8
Benzene	4.6–9.2	1.4–2.8
Carbon Disulfide	4.2–8.4	1.3–2.6
Carbon Tetrachloride	3.9–7.8	1.2–2.4
Castor Oil	1.6–3.2	0.5–1.0
Chloroform	4.0–8.0	1.2–2.4
Decane	4.9–9.8	1.5–3.0
Ether	5.0–10.0	1.5–3.0
Ethylene Glycol	3.9–7.8	1.2–2.4
R-11	4.0–8.0	1.2–2.4
Glycerine	1.4–2.8	0.43–0.86
Heptane	5.1–10.2	1.5–3.0
Hexane	5.2–10.4	1.6–3.2
Kerosene	4.7–9.4	1.4–2.8
Linseed Oil	4.9–9.8	1.5–3.0
Mercury	2.1–4.2	0.64–1.3
Octane	5.0–10.0	1.5–3.0
Propane	5.6–11.2	1.7–3.4
Propylene	5.5–11.0	1.7–3.4
Propylene Glycol	4.5–9.0	1.4–2.8
Turpentine	4.6–9.2	1.4–2.8
Water	4.4–8.8	1.4–2.8

3. In systems where the exit is lower (physically) than the inlet and where friction plus minor losses are small, the pressure at the exit might be calculated to be greater than that at the inlet for the specified flow rate. This means the fluid will flow under the action of gravity and a pump may not be needed. Further, it might be impossible to satisfy optimum velocity conditions as well. However, if a pump is to be used, determine from the appropriate chart which pump should be selected. Refer to the pump performance map if available, and superimpose the system curve on it to find the exact operating point. Use NPSH data to specify the exact location of the pump.
4. If tanks are present, specify the minimum and maximum liquid heights in them.

5. Prepare a drawing for the system and a summary of specifications sheet that lists results of calculations only. Attach the calculations to the summary sheet.

While the preceding list is not comprehensive, it does provide a useful checklist of things that should be done during the design phase of laying out a piping system.

EXAMPLE 6.6. Figure 6.8 shows a pump and piping system that is to convey 600 gpm of propylene glycol from a tank to atmospheric conditions. Follow the suggested design procedure and make recommendations about the piping system.

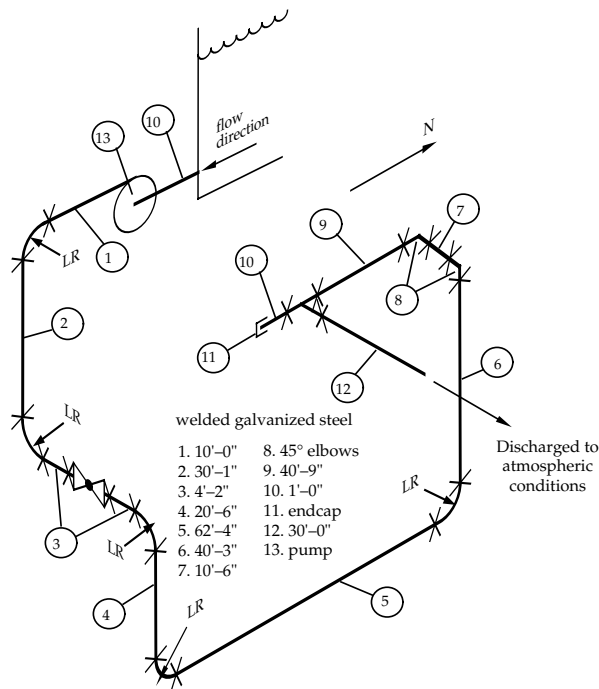


FIGURE 6.8. The piping system of Example 6.6.

Solution: We follow the design procedure in a step-by-step fashion.

1. Economic Line Size

Table 6.4 lists the economic velocity range for propylene glycol as

$$4.5 \leq V_{\text{opt}} \leq 9.0 \text{ ft/s}$$

Values in the table were prepared using current and projected economic parameters to determine economic diameters. The calculated diameters were divided into flow rate to determine economic velocity. We can use any value inside the range, but the velocity we use will depend on what pipe sizes are appropriate for the job. We will make initial calculations for both 4.5 and 9.0 ft/s to determine the sizes available. For a volume flow rate of 600 gpm ($= 1.34 \text{ ft}^3/\text{s}$), we calculate two flow areas:

$$A_{\text{upper limit}} = \frac{Q}{V} = \frac{1.34}{4.5} = 0.298 \text{ ft}^2$$

$$A_{\text{lower limit}} = \frac{1.34}{9} = 0.150 \text{ ft}^2$$

We now refer to a table of pipe sizes. The only standard size that falls within this range of flow areas is 6-nominal schedule 40, which is the size we use for the remainder of the calculations.

Economic line size = 6-nom sch 40

2. Pump Power

In order to calculate pumping power, we must first obtain properties, and eventually substitute into the modified Bernoulli equation. We proceed in the usual way:

propylene glycol $\rho = 0.968(1.94) \text{ slug/ft}^3$ $\mu = 88 \times 10^{-5} \text{ lbf}\cdot\text{s/ft}^2$
[Appendix Table B.1]

6-nominal schedule 40 $ID = 0.5054 \text{ ft}$ $A = 0.2006 \text{ ft}^2$
[Appendix Table D.1]

galvanized surface $\varepsilon = 0.0005 \text{ ft}$ (the average value)
[Table 3.1]

The modified Bernoulli equation with pump power is written as

$$\frac{p_1 g_c}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2 g_c}{\rho g} + \frac{V_2^2}{2g} + z_2 + \sum \frac{fL}{D_h} \frac{V^2}{2g} + \sum K \frac{V^2}{2g} + \frac{g_c}{\dot{m}g} \frac{dW}{dt}$$

We define section 1 as the free surface of the propylene glycol in the tank and section 2 as the outlet of the pipe labeled “12.” Recall the convention that if we take the pressure at section 2 to be atmospheric and the velocity there to be zero, then we must include an exit loss in the minor-loss calculation. Evaluating properties, we have

$$p_1 = p_2 = p_{\text{atm}} \qquad V_1 = V_2 = 0$$

The velocity in the pipeline is

$$V = \frac{Q}{A} = \frac{1.34}{0.2006} = 6.68 \text{ ft/s}$$

Making height measurements from length #5, we write

$$z_1 = 3' + 20'6'' + 30'1'' = 53.6 \text{ ft}$$

where the 3' was added to account for the *minimum* depth of liquid in the tank. We also have

$$z_2 = 40'3'' + 10'6'' \sin 45^\circ = 47.7 \text{ ft}$$

Also,

$$L = 1' + 10'0'' + 30'1'' + 4'2'' + 4'2'' + 20'6'' + 62'4'' + 40'3'' + 10'6'' + 40'9'' + 30'0''$$

or $L = 253.8 \text{ ft}$

The minor-loss values from Table 3.3 include

$$\Sigma K = K_{\text{inlet}} + 5K_{\substack{\text{long radius} \\ 90^\circ \text{ ells}}} + K_{\substack{\text{globe} \\ \text{valve}}} + 2K_{\substack{45^\circ \text{ ells} \\ T\text{-joint}}} + K_{\text{branch}} + K_{\text{exit}}$$

and $\Sigma K = 0.5 + 5(0.22) + 10 + 2(0.17) + 0.69 + 1.0$

where we have assumed a square-edged inlet, because this is typically what is installed in a welded system. We have also assumed that the inlet to the pump from the tank is the same diameter as the outlet pipe. The minor losses then are

$$\Sigma K = 13.63$$

The Reynolds number is calculated as

$$\text{Re} = \frac{\rho V D}{\mu g_c} = \frac{0.968(1.94)(6.68)(0.5054)}{88 \times 10^{-5}}$$

and so

$$\left. \begin{array}{l} \text{Re} = 7.21 \times 10^3 \\ \text{and } \varepsilon/D = \frac{0.0005}{0.5054} = 0.00099 \end{array} \right\} f = 0.036 \quad (\text{Moody diagram})$$

The modified Bernoulli equation becomes

$$z_1 = z_2 + \frac{fL}{D_h} \frac{V^2}{2g} + \Sigma K \frac{V^2}{2g} + \frac{g_c}{\dot{m}g} \frac{dW}{dt}$$

From the pump power equation,

$$-\frac{g_c}{\dot{m}g} \frac{dW}{dt} = \Delta H$$

Substituting into the modified Bernoulli equation, we get

$$\Delta H = z_2 - z_1 + \left(\frac{fL}{D_h} + \Sigma K \right) \frac{V^2}{2g} \quad (\text{i})$$

Substituting,

$$\Delta H = (47.7 - 53.6) + \left[\frac{0.036(253.8)}{0.5054} + 13.63 \right] \frac{6.68^2}{2(32.2)}$$

$$\Delta H = -5.9 + 21.97$$

or $\Delta H = 16.0 \text{ ft @ } Q = 600 \text{ gpm}$

3. System Curve

The system curve in this case is a graph of power (ΔH) versus flow rate (Q). We generate data for this curve by using Equation i above, written in terms of friction factor and velocity:

$$\Delta H = (47.7 - 53.6) + \left[\frac{f(253.8)}{0.5054} + 13.63 \right] \frac{V^2}{2(32.2)}$$

or $\Delta H = -5.9 + (7.8f + 0.212)V^2$

We now select values of flow rate, calculate velocity and Reynolds number, determine friction factor, and find ΔH . Calculation results are summarized in the following table, and the system curve is shown in Figure 6.9.

Q , gpm	Q , ft ³ /s	$V = Q/A$, ft/s	Re	f	ΔH , ft
100	0.223	1.11	1199	0.062	-5.04
200	0.446	2.22	2398	0.049	-2.97
300	0.669	3.33	3597	0.043	0.19
400	0.892	4.45	4796	0.040	4.39
500	1.12	5.56	5995	0.037	9.61
600	1.34	6.67	7194	0.035	15.8
700	1.56	7.78	8393	0.034	23.0
800	1.78	8.89	9592	0.033	31.2
900	2.01	10.0	10791	0.032	40.4
1000	2.23	11.1	11990	0.031	50.5

4. Excessive Vibrations

Next we determine the pressure drop that corresponds to the flow conditions of this problem and see if it leads to excessive vibrations. A pressure drop of 25–30 psi per 1000 ft is the lower limit of when excessive vibrations leading to objectionable noise occur. (If noise is not a problem, this calculation can be skipped.) We will use 30 psi per 1000 ft arbitrarily. To determine the pressure drop, we select two points in the pipeline; for the flow rate specified, we calculate the pressure drop. We make calculations for a 1000-ft length of horizontal pipe and write

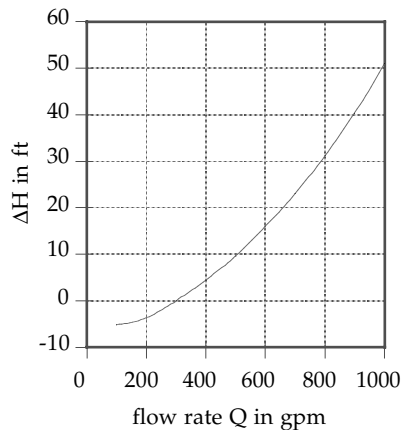


FIGURE 6.9. System curve for the layout of Figure 6.8.

$$\frac{p_1 g_c}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2 g_c}{\rho g} + \frac{V_2^2}{2g} + z_2 + \sum \frac{fL}{D_h} \frac{V^2}{2g}$$

where sections 1 and 2 are at opposite ends of this pipe. We have

$$V_1 = V_2 \quad z_1 = z_2 \quad f = 0.036 \quad V = 6.68 \text{ ft/s}$$

Simplifying and substituting,

$$\Delta p = \frac{fL}{D_h} \frac{V^2}{2g} = \frac{0.036(1000)}{0.5054} \frac{6.68^2}{2(32.2)}$$

$$\text{or } \Delta p = 49.36 \text{ psf} = 0.343 \text{ psi}$$

which is far below the limit of 30 psi per 1000 ft.

5. Pump Selection

We first determine the type of pump that is best suited for the job. We use Table 6.3, which requires a calculation of specific speed. Equation 6.23 defines specific speed as

$$\omega_s = \frac{\omega Q^{1/2}}{\Delta H^{3/4}}$$

The specific speed should be calculated at the rotational speed of the pump-motor combination being considered. For example, if a manufacturer sells pump-motor combinations with rotational speeds of 3600 and 1800 rpm, then the specific speed should be calculated with these values. For this example, we assume that the manufacturer has provided us with rotational speeds that correspond to those listed in Figure 6.2. Thus, we will calculate specific speed for 3600, 2700, 1760, and 900 rpm. For 3600 rpm, we have

$$\omega_{s1} = \frac{(3600)(600)^{1/2}}{(16.0)^{3/4}} = 11,020 \text{ rpm}$$

Likewise,

$$\omega_{s2} = \frac{(2700)(600)^{1/2}}{(16.0)^{3/4}} = 8267 \text{ rpm}$$

$$\omega_{s3} = \frac{(1760)(600)^{1/2}}{(16.0)^{3/4}} = 5389 \text{ rpm}$$

$$\omega_{s4} = \frac{(900)(600)^{1/2}}{(16.0)^{3/4}} = 2756 \text{ rpm}$$

Trying to locate these values in Table 6.3 shows that the first three specific speeds (11,020, 8267, and 5389 rpm) corresponding to rotational speeds of 3600, 2700, and 1760 rpm do not appear. The fourth value of specific speed (2756 rpm) coupled with the given flow rate of 600 gpm is on the chart, however. We read from Table 6.3:

$$\left. \begin{array}{l} \omega_{s4} = 2756 \text{ rpm} \\ Q = 600 \text{ gpm} \end{array} \right\} \eta \approx 81\% \quad \text{Mixed flow type}$$

If two or more of the specific speed values did appear, then the pump to select would depend on some other parameter, such as cost, local availability, or just an arbitrary decision. Continuing with the calculations, we next find the power required using ΔH found previously:

$$\frac{dW}{dt} = \frac{\rho Q g \Delta H}{g_c} = 0.968(1.94)(1.34)(32.2)(16.0)$$

$$\text{or } \frac{dW}{dt} = 1296 \text{ ft}\cdot\text{lbf/s} = 2.4 \text{ hp}$$

The above power is that which must be transferred to the fluid. The power that is needed from the motor is

$$\frac{dW_a}{dt} = \frac{dW/dt}{\eta} = \frac{2.4}{0.81}$$

$$\text{or } \frac{dW_a}{dt} = 2.9 \text{ hp}$$

(As mentioned: Motors are not available in any desired size, but, like pipe and tubing, they are available in discrete sizes. Here we might use 3 hp.)

For purposes of illustration, suppose that we have consulted a composite graph like that of Figure 6.4 and specified the appropriate mixed flow pump. We would then locate the corresponding performance map and superimpose our system curve on that performance map. This is illustrated in Figure 6.10. The point of intersection is the actual operating

point of the system, which is indicated in the figure by the dotted lines. Further, the pump manufacturer will specify NPSH data, and it is appropriate to make calculations at this point to make sure that cavitation will not be a problem for the pump selected.

6. Summary of Results

Economic line size = 6-nominal schedule 40

Layout = Figure 6.8

System curve = Figure 6.9

Specific speed = 2756 rpm at 600 gpm

Expected pump eff = ~81%

Pump type = Mixed flow

Motor power $\approx$ 3 hp

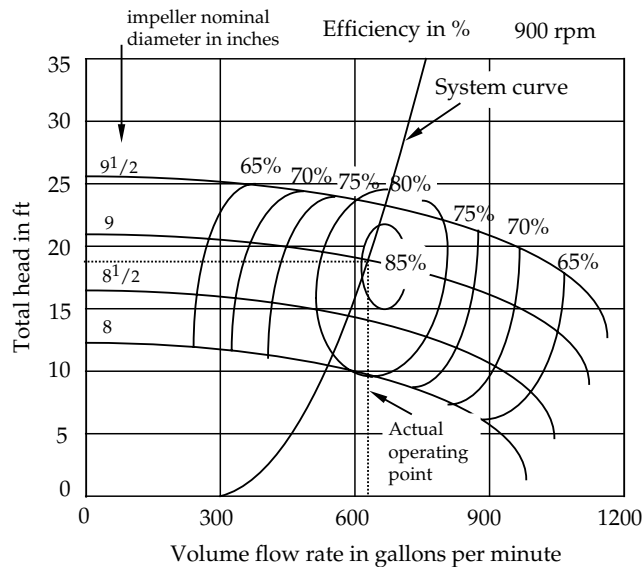


FIGURE 6.10. System curve superimposed on performance map to find the actual operating point.

Comment Regarding Practice

We see that the *desired* flow rate of 600 gpm will be slightly exceeded by the pump selected. We refer to the system curve and read the actual expected flow rate directly. Some companies will install this system with a valve in the outlet line, just downstream of the pump. The valve will be slightly closed so that the 600 gpm requirement will be met. The pump

therefore works against a partially closed valve, in essence wasting energy to meet a requirement that may or may not be critical. This practice should be avoided if possible.

EXAMPLE 6.7. The accompanying figure shows a pump and piping system that is to convey 120 gpm (minimum) of water at 75°F from a tank to a heat exchanger ($K = 10$) and back to the tank. The tank itself contains cooling coils (not shown) that keep the tank water at the desired temperature. Follow the suggested design procedure and select a pipe size, and a pump.

Solution: We follow the design procedure in a step-by-step fashion.

1. Economic Line Size

Table 6.4 lists the economic velocity range for water as

$$4.4 \leq V_{opt} \leq 8.8 \text{ ft/s}$$

We will make initial calculations for both 4.4 and 8.8 ft/s to determine the sizes that might be used. For a volume flow rate of 120 gpm ($= 0.2676 \text{ ft}^3/\text{s}$), we calculate two flow areas:

$$A_{\text{upper limit}} = \frac{Q}{V} = \frac{0.2676}{4.4} = 0.0608 \text{ ft}^2$$

$$A_{\text{lower limit}} = \frac{0.2676}{8.8} = 0.0304 \text{ ft}^2$$

We now refer to a table of pipe sizes, and we read for schedule 40 pipe:

- $A = 0.03325 \text{ ft}^2$ for $2^{1/2}$ nominal
- $A = 0.0514 \text{ ft}^2$ for 3 nominal

We select the larger size, which is the size we use for the remainder of the calculations.

Economic Line Size = 3-nom sch 40

2. Pump Power

In order to calculate pumping power, we must first obtain properties, and eventually substitute into the modified Bernoulli equation. We proceed in the usual way:

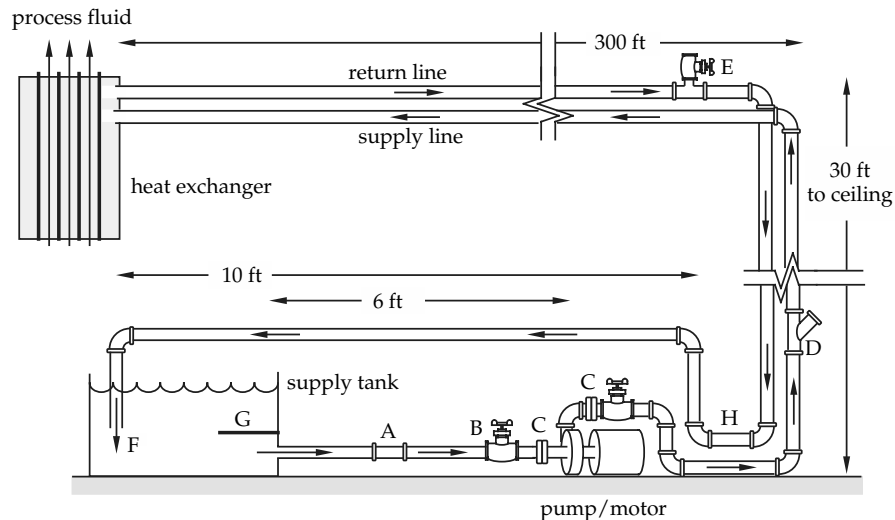


FIGURE 6.11. The piping system of Example 6.7.

Notes:

- A T-joint with plug; to be used as a drain if necessary
- B Valve in the pump inlet and pump outlet lines to isolate the fluid in the piping system if the pump is to be removed
- C Union fittings for pump removal
- D Y-strainer to filter out any solids remaining from the installation
- E Air outlet valve
- F Return from the heat exchanger to be positioned as far as practical from the pump inlet line
- G Flat plate welded to the interior of the tank to prevent a vortex from forming when the pump is operating
- H The pipelines are shown not to scale, but should be installed next to each other where possible

Fittings:

- threaded
- square edged inlet
- pipe diameter at pump inlet = pipe diameter at pump outlet
- Y-strainer has a loss coefficient of 1
- pipe lengths are 20 ft nominally; union fittings required every 20 ft

The return line to the supply tank must be insulated

Heat exchanger is a finned tube type

Pipe supports every 7 ft in both 30 ft lines

Pipe supports every 7 ft in both 300 ft lines

water $\rho = 1.94 \text{ slug/ft}^3$ $\mu = 1.9 \times 10^{-5} \text{ lbf}\cdot\text{s/ft}^2$ [Appendix Table B.1]

3-nom sch 40 $ID = 0.2557 \text{ ft}$ $A = 0.0514 \text{ ft}^2$ [Appendix Table D.1]

We select galvanized pipe because no special need was specified:

Galvanized surface $\varepsilon = 0.0005 \text{ ft}$ (average value) [Table 3.1]

The modified Bernoulli equation with pump power is written as

$$\frac{p_1 g_c}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2 g_c}{\rho g} + \frac{V_2^2}{2g} + z_2 + \sum \frac{fL}{D_h} \frac{V^2}{2g} + \sum K \frac{V^2}{2g} + \frac{g_c}{\dot{m}g} \frac{dW}{dt}$$

We define both section 1 and section 2 as the free surface of the water in the tank. Evaluating properties, we have

$$p_1 = p_2 = p_{\text{atm}} \quad V_1 = V_2 = 0$$

The velocity in the pipeline is

$$V = \frac{Q}{A} = \frac{0.2676}{0.0514} = 5.21 \text{ ft/s}$$

Also, $z_1 = z_2$. The pipe length is (approximately)

$$L = 300 + 300 + 30 + 30 + 35$$

or $L = 695 \text{ ft}$

The minor-loss values from Table 3.3 for threaded fittings include:

$\sum K$ = square edged inlet + 2 straight T fittings (one at A and one at E) +
2 globe valves + 34 unions + 9 regular ells + Y strainer + heat
exchanger + exit

$$\begin{aligned} \sum K &= 0.5 + 2(0.9) + 2(10) + 34(0.08) + 9(1.4) + 1 + 10 + 1 \\ \sum K &= 49.62 \end{aligned}$$

The Reynolds number is calculated as

$$\text{Re} = \frac{\rho V D}{\mu g_c} = \frac{(1.94)(5.21)(0.2557)}{1.9 \times 10^{-5}}$$

and so

$$\left. \begin{array}{l} \text{Re} = 1.36 \times 10^5 \\ \text{and } \varepsilon/D = \frac{0.0005}{0.2557} = 0.0002 \end{array} \right\} \quad f = 0.025 \quad (\text{Moody Diagram})$$

The modified Bernoulli equation becomes

$$0 = 0 + \frac{fL}{D_h} \frac{V^2}{2g} + \Sigma K \frac{V^2}{2g} + \frac{g_c}{\dot{m}g} \frac{dW}{dt}$$

From the pump power equation,

$$-\frac{g_c}{\dot{m}g} \frac{dW}{dt} = \Delta H$$

Rearranging the modified Bernoulli equation, we get

$$\Delta H = \left(\frac{fL}{D_h} + \Sigma K \right) \frac{V^2}{2g} \quad (\text{i})$$

Substituting,

$$\Delta H = \left[\frac{0.025(695)}{0.2557} + 49.62 \right] \frac{5.21^2}{2(32.2)}$$

$$\text{or } \underline{\Delta H = 49.3 \text{ ft @ } Q = 120 \text{ gpm}}$$

3. System Curve

The system curve in this case is a graph of power (ΔH) versus flow rate (Q). We generate data for this curve by using Equation i, written in terms of friction factor and velocity:

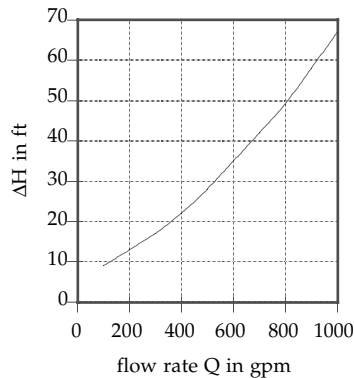
$$\Delta H = \left[\frac{f(695)}{0.2557} + 49.62 \right] \frac{V^2}{2(32.2)}$$

$$\text{or } \underline{\Delta H = (42.21f + 0.770)V^2}$$

We now select values of flow rate, calculate velocity and Reynolds number, determine friction factor (Swamee-Jain equation), and find ΔH . The system curve is shown in Figure 6.12, with calculated data provided in the accompanying table.

TABLE 6.5. *Results of system curve calculations.*

Q gpm	Q ft ³ /s	V = Q/A ft/s	Re	f	Power ΔH , ft
50	0.112	2.17	56,774	0.026	9
60	0.134	2.61	68,129	0.026	13
70	0.156	3.04	79,484	0.026	17
80	0.179	3.48	90,839	0.025	22
90	0.201	3.91	102,194	0.025	28
100	0.226	5.21	113,549	0.025	35
110	0.246	4.78	124,904	0.025	42
120	0.268	5.22	136,259	0.025	49
130	0.290	5.65	147,613	0.025	58
140	0.313	6.09	158,968	0.025	67

**FIGURE 6.12.** *System curve for the layout of Figure 6.11.*

4. Pump Selection

We first determine the type of pump that is best suited for the job. We use Table 6.3, which requires a calculation of specific speed. Equation 6.23 defines specific speed as

$$\omega_s = \frac{\omega Q^{1/2}}{\Delta H^{3/4}}$$

The specific speed should be calculated at the rotational speed of the pump-motor combination being considered. For example, if a manufacturer sells pump-motor combinations with rotational speeds of 3600 rpm and 1800 rpm, then the specific speed should be calculated with these values. For this example, we tentatively select a pump (for purposes of illustration) that can be operated at rotational speeds of 3500 rpm, 1750 rpm, or 1150 rpm. The same pump (same housing, same impeller, etc.) might be used but

operated at one of these speeds.

Next, we will calculate specific speed for 3500, 1750, and 1150 rpm. For 3500 rpm, we have

$$\omega_{s1} = \frac{(3500)(120)^{1/2}}{(49.3)^{3/4}} = 2060 \text{ rpm}$$

Likewise,

$$\omega_{s2} = \frac{(1750)(120)^{1/2}}{(49.3)^{3/4}} = 1030 \text{ rpm}$$

$$\omega_{s3} = \frac{(1150)(120)^{1/2}}{(49.3)^{3/4}} = 676 \text{ rpm}$$

Trying to locate these values in Table 6.3 shows that the first specific speed (2060 rpm @ 120 gpm) corresponding to a rotational speeds of 3500 does not appear. The second specific speed (1030 rpm @ 120 gpm) lies somewhere between 1000 and 1500. If we use such a pump, it would be a centrifugal pump with an expected efficiency of about 62%. The third specific speed (676 rpm @ 120 gpm) also appears, and if we use this pump, its expected efficiency is about 54%. For any pump that would work in this application, its estimated efficiency ranges $54 \leq \eta \leq 62\%$.

Continuing with the calculations, we next find the power required using ΔH found previously:

$$\frac{dW}{dt} = \frac{\rho Q g \Delta H}{g_c} = (1.94)(0.2676)(32.2)(49.3)$$

$$\text{or } \frac{dW}{dt} = 824.5 \text{ ft}\cdot\text{lbf/s} = 1.5 \text{ hp}$$

The preceding calculated power is that which must be transferred to the fluid. The power that is needed from the motor is (assuming the higher efficiency pump is selected):

$$\frac{dW_a}{dt} = \frac{dW/dt}{\eta} = \frac{1.5}{0.65}$$

$$\text{or } \frac{dW_a}{dt} = 2.3 \text{ hp}$$

Recall that motors are not available in any desired size, but, like pipe and tubing, they are available in discrete sizes. Here we might use 2.5 hp; however, the motor size will be specified by the pump manufacturer.

We now consult a composite graph from the manufacturer (like that of Figure 6.4) and we find pumps that satisfy flow rate and efficiency calculations. A typical performance graph follows, Figure 6.13. The graph contains several curves, four of which correspond to specific impeller diameter (e.g., 11 indicates an 11 in. impeller). The bottom curve is used to predict when cavitation would occur. Efficiencies are shown, and we are expecting an efficiency of between 55 and 65%. Data on this graph would have been obtained at a single rotational speed. We note that the desired operating point (given in the problem statement) is 120 gpm. If we use the pump of Figure 6.13, the operating point will be closer to 125 gpm, and the corresponding ΔH is roughly 50 ft. This pump, therefore should work.

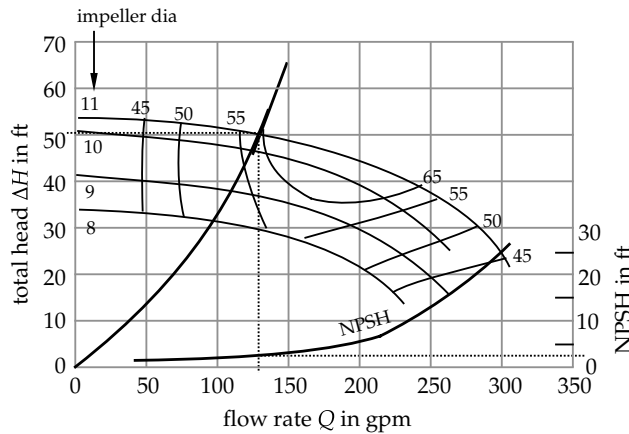


FIGURE 6.13. Pump performance graph with system curve imposed.

5. Cavitation

With regard to cavitation, the performance map shows that the NPSH required is approximately 2.5 ft. We will now apply the modified Bernoulli equation from section 1 (which we identify as the free surface of the tank liquid) to section 2 (which is the inlet to the pump housing) for the case where we have a suction head. Section 2 in the subsequent calculation is *not* the same as that for our initial analysis. Our objective here is to determine the pressure at the pump inlet (section 2) and compare it to the vapor pressure of the liquid. If the pressure at the pump inlet is less than the vapor pressure of the liquid at the local temperature, then the liquid will boil at the impeller; thus the pump will cavitate. The modified Bernoulli equation is

$$\frac{p_1 g_c}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2 g_c}{\rho g} + \frac{V_2^2}{2g} + z_2 + \sum \frac{fL}{D_h} \frac{V^2}{2g} + \sum K \frac{V^2}{2g}$$

Evaluating properties yields

$$V_1 = 0 \qquad z_2 - z_1 = -z_s \qquad V_2 = V = \text{velocity in the pipe}$$

Rearranging and solving for $p_2 g_c / \rho g$, we obtain

$$\frac{p_2 g_c}{\rho g} = \frac{p_1 g_c}{\rho g} + z_s - \left(\sum \frac{fL}{D_h} + \sum K + 1 \right) \frac{V^2}{2g}$$

Next we subtract the vapor pressure from both sides of the preceding equation and rearrange slightly to obtain

$$\frac{p_2 g_c}{\rho g} - \frac{p_v g_c}{\rho g} = \frac{p_1 g_c}{\rho g} + z_s - \left(\sum \frac{fL}{D_h} + \sum K + 1 \right) \frac{V^2}{2g} - \frac{p_v g_c}{\rho g}$$

or for suction head,

$$\text{NPSH}_a = \frac{p_2 g_c}{\rho g} - \frac{p_v g_c}{\rho g} = \frac{p_1 g_c}{\rho g} + z_s - \left(\sum \frac{fL}{D_h} + \sum K + 1 \right) \frac{V^2}{2g} - \frac{p_v g_c}{\rho g}$$

The left hand side of this equation is 2.5 ft obtained from the performance map. The vapor pressure of water at 75°F is about 0.35 psia. The length of the inlet line is about 6 ft; the minor losses are the squared edged inlet, the straight T, the globe valve and the union fitting. These add up to be

$$\sum K_{\text{inlet pipe}} = 0.5 + 0.9 + 10 + 0.08 = 11.48$$

Substituting into the NPSH equation gives

$$2.5 = \frac{14.7(144)}{1.94(32.2)} + z_s - \left(\frac{0.025(6)}{0.2557} + 11.48 + 1 \right) \frac{5.21^2}{2(32.2)} - \frac{0.35(144)}{1.94(32.2)}$$

$$2.5 = 33.8 + z_s - 5.51 - 0.807$$

Solving,

$$z_s = -25.0 \text{ ft}$$

For our installation, z_s is positive, so cavitation will not be a problem.

6. Summary of Results

“Economic” line size = 3-nominal schedule 40
Layout = given
System curve = Figure 6.6b
Specific speed = 807 rpm at 120 gpm
Expected pump eff = ~65%
Pump type = Radial flow
Motor power $\approx$ 1.5 hp

6.7 Fans and Fan Performance

Fans, blowers, and compressors are devices used to move compressible fluids (vapors and gases) through a duct. A fan imparts a comparatively small pressure increase to a fluid. A blower provides a larger pressure increase, while a compressor imparts a very large increase. A comprehensive discussion of these machines requires much space, so here we describe fans and limit the discussion to only two types.

Fans are classified according to how the fluid flows through them with respect to the axis of rotation. In an axial flow fan (such as a window fan), the fluid flows through the machine parallel to the axis of rotation of the moving parts. In a radial flow fan or blower, fluid passes through the machine perpendicular to the axis of rotation. A radial flow fan is also called a centrifugal fan, of which there are many designs. One type is often referred to as a “squirrel cage” fan.

Fan Testing Methods

There are standard test procedures for evaluating the performance of fans. The Air Moving and Conditioning Association (AMCA) led the way in this area with recommended testing procedures that apply to all aspects of fan performance, including sound. The American Society of Heating, Refrigerating and Air Conditioning Engineers (ASHRAE) also has a fan test code. Test codes provide a starting point for performance specifications, and test results provide data that are used to size a fan for a particular installation.

Figure 6.14 shows two (of several) possible test setups for measuring fan performance. Figure 6.14a shows a fan with only an outlet duct. Air enters the system just upstream of the fan and is discharged through the duct. The duct itself contains flow straighteners (not shown) and, at section 3, a device called a **pitot-static tube** for measuring velocity profile. This device is used to obtain data from which a velocity profile at section 3 is determined. The velocity profile is then integrated numerically to yield

the average velocity and hence the volume flow rate. The movable plug is positioned so as to increase or decrease the flow rate.

Figure 6.14b is an inlet duct apparatus. The inlet duct here contains flow straighteners and, at section 3, a pitot-static tube. The objective again is to determine the flow rate for various positions of the movable plug. The velocity profile is measured at section 3 with a pitot-static tube.

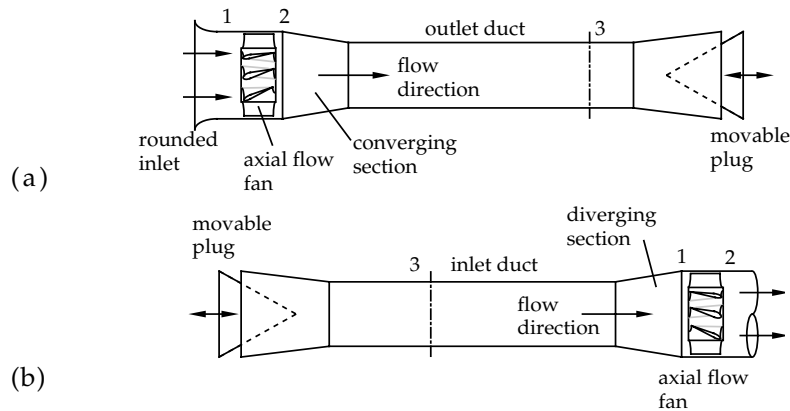


FIGURE 6.14. Two possible fan test setups.

Pitot-Static Tube

When a fluid flows through a pipe, it exerts pressure that is made up of static and dynamic components. The **static pressure** is indicated by a measuring device that moves with the flow or that causes no velocity change in the flow. Usually, to measure static pressure, a small hole perpendicular to the flow is drilled through the container wall and connected to a manometer (or pressure gage), as indicated in Figure 6.15.

The **dynamic pressure** is due to the movement of the fluid. The dynamic pressure and the static pressure together make up the **total, or stagnation, pressure**. The stagnation pressure can be measured in the flow with a **pitot tube**. The pitot tube is an open-ended tube facing the flow directly. Figure 6.15 presents a sketch of the measurement of stagnation pressure.

The **pitot-static tube** combines the effects of static and stagnation pressure measurement into one device. Figure 6.16 is a schematic of the pitot-static tube. It consists of a tube within a tube that is placed in the duct, facing upstream. The pressure tap that faces the flow directly gives a measurement of the stagnation pressure, while the tap that is perpendicular to the flow gives the static pressure.

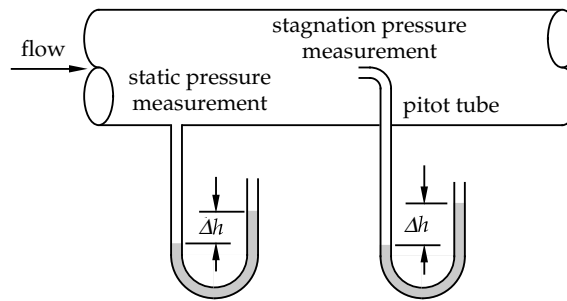


FIGURE 6.15. *Measurement of static and stagnation pressures.*

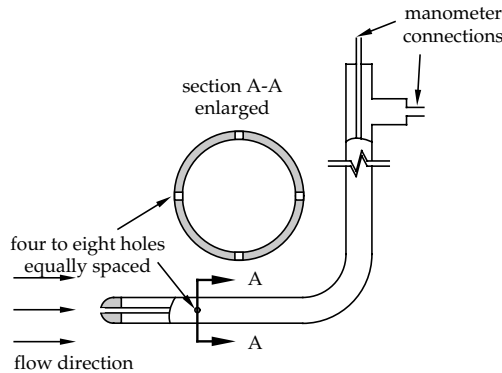


FIGURE 6.16. *Schematic of a pitot-static tube.*

When the pitot-static tube is immersed in the flow of a fluid, the pressure difference (stagnation minus static) can be read directly using a manometer and connecting the pressure taps to each leg. Applying the Bernoulli equation between the two pressure taps yields

$$\frac{p_1 g_c}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2 g_c}{\rho g} + \frac{V_2^2}{2g} + z_2$$

where state “1” is the stagnation state (which will be changed to subscript “t”), and state “2” is the static state (no subscript). Elevation differences are negligible; and at the point where stagnation pressure is measured, the velocity is zero. The Bernoulli equation thus reduces to

$$\frac{p_t g_c}{\rho g} = \frac{p_g g_c}{\rho g} + \frac{V^2}{2g}$$

Next, we rearrange the preceding equation and solve for velocity:

$$V = \sqrt{\frac{2(p_t - p)}{\rho}}$$

A manometer connected to the pitot-static tube would provide head loss readings Δh given by

$$\Delta h = \frac{p_t - p}{\rho g}$$

where the density is that of the flowing fluid. So velocity in terms of head loss is

$$V = \sqrt{2g\Delta h}$$

Note that this equation applies only to incompressible flows. Compressibility effects are not accounted for. Furthermore, Δh is the head loss in terms of the flowing fluid and not in terms of the reading on the manometer. The manometer, for instance, may be reading in inches of water, while the preceding equation requires a reading of inches of air.

For flow in a duct, manometer readings are to be taken at a number of locations within the cross section of the flow. The velocity profile is then plotted using the results. Velocities at specific points are then determined from these profiles. The objective here is to obtain data, graph a velocity profile, and then determine the average velocity.

Average Velocity

The average velocity is related to the flow rate through a duct as

$$V = \frac{Q}{A}$$

where Q is the volume flow rate and A is the cross-sectional area of the duct. We can divide the flow area into five equal areas, as shown in Figure 6.17. The velocity is to be obtained at those locations labeled in the figure. The chosen positions divide the cross section into five equal concentric areas. The flow rate through each area, labeled from 1 to 5, is found as

$$Q_1 = A_1 V_1 \qquad Q_2 = A_2 V_2$$

$$Q_3 = A_3 V_3 \qquad Q_4 = A_4 V_4$$

$$Q_5 = A_5 V_5$$

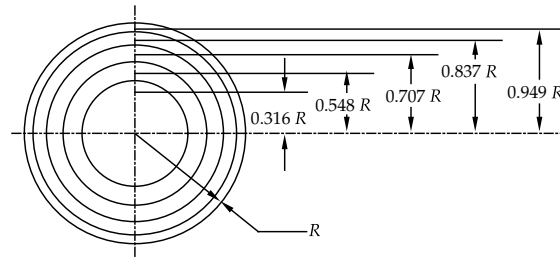


FIGURE 6.17. Five positions within the cross section where velocity is to be determined.

The total flow rate through the entire cross section is the sum of these:

$$Q_{\text{total}} = \sum_1^5 Q_i = A_1 V_1 + A_2 V_2 + A_3 V_3 + A_4 V_4 + A_5 V_5$$

or $Q_{\text{total}} = A_1(V_1 + V_2 + V_3 + V_4 + V_5)$

The total area A_{total} is $5A_1$ and so

$$V = \frac{Q_{\text{total}}}{A_{\text{total}}} = \frac{(A_{\text{total}}/5)(V_1 + V_2 + V_3 + V_4 + V_5)}{A_{\text{total}}}$$

The average velocity then becomes

$$V = \frac{(V_1 + V_2 + V_3 + V_4 + V_5)}{5}$$

The importance of the five chosen radial positions for measuring V_1 through V_5 is now evident.

In the test setups of Figure 6.14, it is necessary to obtain static pressure measurements. In Figure 6.15a, the static pressure increase across the fan must be determined. Upstream of the fan, the gage pressure is taken to be zero ($0 \text{ psig} = p_{\text{atm}}$). The static pressure must be measured downstream at section 2. The static pressure rise is $p_2 - p_{\text{atm}}$. Thus, for one setting of the movable plug, pressure increase across the fan and volume flow rate through the fan are determined simultaneously. Similar measurements are made using the setup of Figure 6.15b.

With regard to fans, current practice defines three pressures associated with moving air: total pressure p_t , static pressure p , and velocity pressure $\rho V^2/2g_c$ (recognized as kinetic energy of the flow). These are related by

$$p_t = p + \frac{\rho V^2}{2g_c} \quad (6.24)$$

The average velocity is found from continuity, $V = Q/A$, and the flow rate Q is calculated with data obtained with the pitot-static tube.

Like pumps, fans exert a torque as they move air through a duct. The product of torque and rotational speed is the power

$$\frac{dW_a}{dt} = T\omega$$

This is the input power from motor to fan. The power imparted to the fluid can be found by applying the energy equation to the fan:

$$\frac{dW}{dt} = \Delta p_t Q$$

where Δp_t is the rise in total pressure (or total energy) across the fan. (See Problem 6.45 for a derivation.) The efficiency is defined as

$$\eta = \frac{dW/dt}{dW_a/dt} = \frac{\Delta p_t Q}{T\omega} \quad (6.25)$$

The user of a fan is ultimately interested in how it will perform in the application of interest. Performance parameters of importance include volume flow rate (also called volume or capacity), pressure, and power. Figure 6.18 is a typical performance graph for an axial flow fan. Plotted on the horizontal axis is volume flow rate (usually expressed in ft^3/min , abbreviated cfm in industry). There are two vertical axes—one of total pressure p_t (actually total pressure rise) and one of power. Units used for pressure rise are inches of water gage (abbreviated **in. of wg**). Horespower or watts are the units used for power.

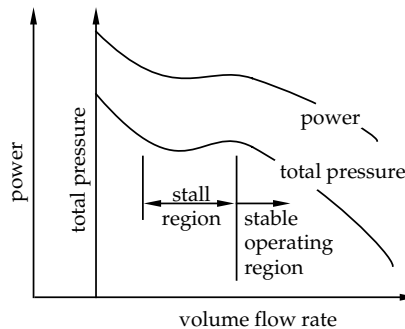
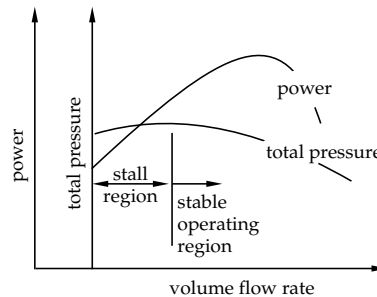


FIGURE 6.18. Typical performance graph for an axial flow fan.

Examination of Figure 6.18 reveals several features of note. The power and pressure curves follow the same trends. The pressure curve is a maximum at zero flow. Then as the flow rate increases, the fan enters a “stall” region during which the flow separates from the fan blades. This yields a drop in pressure before the curve increases again. Operation in the stall region is to be avoided, because a fan performing in this region gives a pulsating flow accompanied by a high noise level.

Figure 6.19 is a typical performance plot for centrifugal fans. (It should be noted that compared to a centrifugal fan, the axial flow fan usually provides a higher velocity for comparable power input.) Examination of this figure shows several interesting features. The power and pressure curves follow different trends. The pressure curve increases slightly (at zero flow rate) to a maximum, after which pressure decreases with further increases in flow rate. The power curve starts at a low value (at zero flow rate) and increases with further increases in flow rate. Eventually, a maximum is reached and power then decreases with further increases in flow rate. Although a stall region is not identified, a centrifugal fan can still experience air separation from the blades, during which the pressure decreases with an accompanying high sound level. Stall occurs in the region to the left of maximum on the pressure curve. Operation in this region is to be avoided.

FIGURE 6.19. Typical performance graph for a centrifugal fan.



Axial flow fans usually have blades that can be adjusted, known commercially as **variable pitch blades**. The blades can be rotated at the point where they attach to the hub so that the angle between the incoming air and the blades can be adjusted to any desired setting.

Tests performed on a fan with one blade setting yields a curve like that of Figure 6.20. Each setting of the blades yields one performance graph. A composite graph of pressure and power versus flow rate for various configurations of the same fan is given in Figure 6.20. Shown are performance lines for four different blade settings (01, 02, 03, and 04). Note that there is a line for pressure and for power corresponding to each other, and that each pair spans the same flow rate range. Shown also on this graph is a shaded region that identifies efficiencies of 80% or greater for the fan.

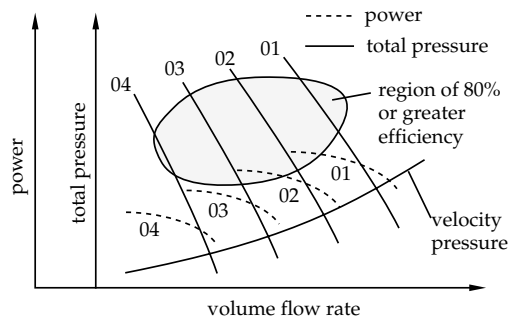


FIGURE 6.20. Composite performance graph for an axial flow fan.

One manufacturer of fans might produce dozens of fans, and each has operating characteristics that must be measured. Once found, the optimum efficiency regions can be identified and graphed separately. A selection chart showing maximum efficiency for many fans is then prepared by the manufacturer; such a graph can be used to select an appropriate fan for a specific application.

Figure 6.21 is an optimum efficiency chart (note log-log axes), which is drawn typically for axial flow fans. The horizontal axis is of volume flow rate. The vertical axis is of total pressure rise. Shown are efficiency regions that identify fans by a catalog designation. Knowing volume flow rate and pressure rise, a fan that will work successfully can be selected. Once the “best” fan is identified, its performance graph is located; on the same set of axes, the system curve should be plotted. This procedure is the same as that for pumps. The only tricky part of the selection procedure is in converting what is known into what is needed to use the graph.

EXAMPLE 6.8. An axial flow fan is to be installed in a 1-m-inside diameter duct. The fan is to deliver 400 m³/min of air. The Bernoulli equation applied to the duct shows that the static pressure drop over the total length (expressed in column of liquid units) is 3.8 cm of water. Determine the appropriate fan for this installation using Figure 6.21.

Solution: As the figure is in engineering units, we will convert the given data. Properties of air at room temperature conditions are:

$$\rho = 0.0023 \text{ slug/ft}^3$$

We also have

$$D = 1 \text{ m} = 39.37 \text{ in.} = 3.28 \text{ ft}$$

$$Q = 400 \text{ m}^3/\text{min} = 14,130 \text{ ft}^3/\text{min} = 14,130 \text{ cfm}$$

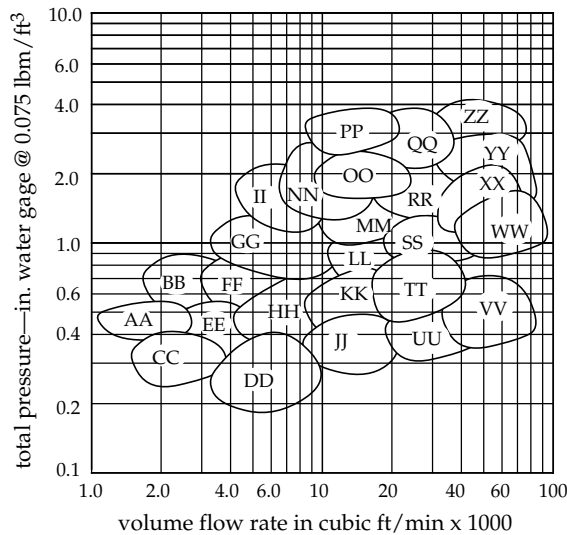


FIGURE 6.21. Fan selection chart.

(In some installations, the volume flow rate is specified in terms of air properties at **standard temperature and pressure**. This is sometimes done because flow rate is sensitive to ambient properties. For example, 14,000 cfm of air at 10 psia, 45°F is not the same as 14,000 cfm at 14.7 psia, 90°F. Manufacturers often provide correction factors to convert desired conditions to standard conditions.)

The static pressure drop in the piping system is given in cm of water, so we must convert it to something with dimensions of F/L². Thus

$$\frac{\Delta p g_c}{\rho_{\text{H}_2\text{O}} g} = 3.8 \text{ cm of H}_2\text{O} = 1.5 \text{ in. of H}_2\text{O} = 0.125 \text{ ft of H}_2\text{O}$$

$$p = \frac{\rho_{\text{H}_2\text{O}} g h}{g_c} = 1.94(32.2)(0.125) = 7.79 \text{ lbf/ft}^2$$

The average air velocity through the system is

$$V = \frac{Q}{A} = \frac{14,130}{\pi(3.28)^2/4} = 1672 \text{ ft/min} = 27.9 \text{ ft/s}$$

(The unit ft/min is often abbreviated as fpm.) The velocity pressure (= kinetic energy of the flow) then is

$$\frac{\rho_{\text{air}} V^2}{2 g_c} = \frac{0.0023(27.9)^2}{2} = 0.895 \text{ lbf/ft}^2$$

The fan is required to take air at atmospheric pressure and zero velocity and provide enough energy to accelerate the air to 27.9 ft/s while overcoming a static pressure drop of 7.79 lbf/ft². So upstream of the fan, we have $p_t = 0$. Downstream of the fan, we have

$$p_t = p + \frac{\rho_{\text{air}} V^2}{2g_c} = 7.79 + 0.895 = 8.69 \text{ lbf/ft}^2$$

The change in total pressure then is $8.69 - 0 = 8.69 \text{ lbf/ft}^2$. Now, to use Figure 6.20, we must convert this pressure rise to units of inches of water. We write

$$\Delta H = \frac{\Delta p_t g_c}{\rho_{\text{H}_2\text{O}} g} = \frac{8.69}{1.94(32.2)} = 0.139 \text{ ft of H}_2\text{O}$$

or $\Delta H = 1.67 \text{ in. of H}_2\text{O}$

Note carefully where water and air densities are used in the preceding equations.

Now, at a flow rate of 14,130 ft³/min and a total pressure rise ΔH of 1.67 in. of water, Figure 6.21 shows that the fan to use is labeled NN (or OO; either will perform satisfactorily) with an efficiency equal to or greater than 80%.

The power imparted to the fluid is calculated as

$$\frac{dW}{dt} = \Delta p_t Q = (8.69 \text{ lbf/ft}^2)(14,130 \text{ ft}^3/\text{min})(1 \text{ min}/60 \text{ s}) = 2046 \text{ ft}\cdot\text{lbf/s}$$

The motor power required for an 80% efficiency is

$$\frac{dW_a}{dt} = \frac{dW/dt}{\eta} = \frac{2046}{0.8} = 2558 \text{ ft}\cdot\text{lbf/s}$$

$$\text{or } \frac{dW_a}{dt} = 4.65 \text{ HP}$$

This calculation of power is only an estimate and that the actual power curve for the fan should be consulted.

6.8 Summary

In this chapter, we have examined commercially available pumps. We discussed pump testing and looked over methods and results obtained from

testing centrifugal pumps. We saw how such results are used to select a pump. Pump performance maps were discussed as well as series and parallel operation, cavitation, and net positive suction head. Dimensional analysis was performed and dimensionless ratios for pumps were derived. Specific speed was defined and used to determine the type of pump most suitable for a given configuration. Design guidelines were provided so that a piping system could be designed and the necessary details determined. The chapter concluded with a brief discussion of fan performance and selection procedures.

6.9 Show and Tell

1. Obtain a catalog (or an actual model) of **axial flow pumps** and give a presentation on their operation, test methods used in evaluating their performance, and typical performance curves.
2. Obtain a catalog (or an actual model) of **mixed flow pumps** and give a presentation on their operation, test methods used in evaluating their performance, and typical performance curves.
3. Obtain a catalog (or an actual model) of **radial flow** or **centrifugal pumps** and give a presentation on their operation, test methods used in evaluating their performance, and typical performance curves.
4. Obtain a catalog (or an actual model) of **reciprocating positive-displacement pumps** and give a presentation on their operation as well as test methods used in evaluating their performance. Discuss typical performance curves.
5. Obtain a catalog (or an actual model) of **gear pumps** and give a presentation on their operation, test methods used in evaluating their performance, and typical performance curves.
6. Obtain a centrifugal pump catalog and locate graphs like those presented in this chapter.
7. Obtain an impeller and/or pump housing (perhaps from a manufacturer) that has been subject to cavitation erosion and prepare a presentation about it. Locate positions where the erosive effect is greatest and explain its formation.
8. What is a **vane pump**? Give a report on how a vane pump operates, and cite applications where it would be useful.
9. Obtain catalogs of centrifugal fans and give a presentation on their operation, test procedures, and typical performance curves.
10. Obtain catalogs of axial flow fans and give a presentation on their operation, test procedures, and typical performance curves.
11. Obtain information about blowers and give a presentation discussing the differences between the performance of a blower and a fan.

12. Obtain information about compressors and give a presentation discussing the differences between the performance of a compressor and that of a fan.

6.10 Problems

Pump Performance and Selection

1. A water pump is tested in the laboratory and the following data were obtained:

$$\begin{aligned}
 \text{Torque} &= T = 2 \text{ N}\cdot\text{m} \\
 \text{Rotational speed} &= \omega = 1760 \text{ rpm} \\
 \text{Inlet pressure} &= p_1 = 75 \text{ kPa} \\
 \text{Outlet pressure} &= p_2 = 210 \text{ kPa} \\
 \text{Volume flow rate} &= Q = 0.002 \text{ m}^3/\text{s} \\
 \text{Height to inlet gage} &= z_1 = 0.4 \text{ m} \\
 \text{Height to outlet gage} &= z_2 = 1 \text{ m} \\
 \text{Inlet flow line} &= 2^{1/2}\text{-nominal schedule 40} \\
 \text{Outlet flow line} &= 2\text{-nominal schedule 40}
 \end{aligned}$$

Determine the pump efficiency.

2. A pump is tested in the laboratory and the following data were obtained:

$$\begin{aligned}
 \text{Torque} &= T = 35 \text{ ft}\cdot\text{lbf} \\
 \text{Rotational speed} &= \omega = 3600 \text{ rpm} \\
 \text{Inlet pressure} &= p_1 = 2.5 \text{ psia} \\
 \text{Outlet pressure} &= p_2 = 30 \text{ psia} \\
 \text{Volume flow rate} &= Q = 1000 \text{ gpm} \\
 \text{Height to inlet gage} &= z_1 = 2 \text{ ft} \\
 \text{Height to outlet gage} &= z_2 = 5 \text{ ft} \\
 \text{Inlet flow line} &= 10\text{-nominal schedule 40} \\
 \text{Outlet flow line} &= 8\text{-nominal schedule 40} \\
 \text{Fluid} &= \text{octane}
 \end{aligned}$$

Determine the pump efficiency.

3. A pump is tested in the laboratory and the following data were obtained:

$$\begin{aligned}
 \text{Torque} &= T = 6 \text{ ft}\cdot\text{lbf} \\
 \text{Rotational speed} &= \omega = 1800 \text{ rpm} \\
 \text{Inlet pressure} &= p_1 = -3.5 \text{ psia} \\
 \text{Outlet pressure} &= p_2 = 8 \text{ psia} \\
 \text{Velocity in the inlet line} &= V = 6 \text{ ft/s} \\
 \text{Height to inlet gage} &= z_1 = 2 \text{ ft} \\
 \text{Height to outlet gage} &= z_2 = 5 \text{ ft} \\
 \text{Inlet flow line} &= 4\text{-nominal schedule 40} \\
 \text{Outlet flow line} &= 3^{1/2}\text{-nominal schedule 40} \\
 \text{Fluid} &= \text{Octane}
 \end{aligned}$$

Determine the pump efficiency.

4. A pump is tested in the laboratory and the following data were obtained:

$$\begin{aligned}
 \text{Torque} &= T = 5 \text{ ft}\cdot\text{lbf} \\
 \text{Rotational speed} &= \omega = 1200 \text{ rpm} \\
 \text{Inlet pressure} &= p_1 = 10 \text{ psia} \\
 \text{Outlet pressure} &= p_2 = 28 \text{ psia} \\
 \text{Velocity in the inlet line} &= V = 8 \text{ ft/s} \\
 \text{Height to inlet gage} &= z_1 = 2 \text{ ft} \\
 \text{Height to outlet gage} &= z_2 = 5 \text{ ft} \\
 \text{Inlet flow line} &= 2\text{-nominal schedule 40} \\
 \text{Outlet flow line} &= 2\text{-nominal schedule 40} \\
 \text{Fluid} &= \text{water}
 \end{aligned}$$

- a) Determine the pump efficiency.
- b) Consult an actual pump catalog and select a pump for these conditions.

5. A pump is tested in the laboratory and the following data were obtained:

$$\begin{aligned}
 \text{Torque} &= T = 2 \text{ ft}\cdot\text{lbf} \\
 \text{Rotational speed} &= \omega = 1800 \text{ rpm} \\
 \text{Inlet pressure} &= p_1 = 0 \text{ psia} \\
 \text{Outlet pressure} &= p_2 = 9.5 \text{ psia} \\
 \text{Velocity in the inlet line} &= V = 4.0 \text{ ft/s} \\
 \text{Height to inlet gage} &= z_1 = 0 \text{ ft} \\
 \text{Height to outlet gage} &= z_2 = 0.1 \text{ ft} \\
 \text{Inlet flow line} &= 3\text{-nominal schedule 40} \\
 \text{Outlet flow line} &= 2^{1/2}\text{-nominal schedule 40} \\
 \text{Fluid} &= \text{water}
 \end{aligned}$$

- a) Estimate the power delivered to the fluid.
- b) Estimate the efficiency.
- c) Estimate the motor power required.

6. A pump is tested in the laboratory and the following data were obtained:

$$\begin{aligned}
 \text{Torque} &= T = 3 \text{ ft}\cdot\text{lbf} \\
 \text{Rotational speed} &= \omega = 1760 \text{ rpm} \\
 \text{Inlet pressure} &= p_1 = 0 \text{ psia} \\
 \text{Outlet pressure} &= p_2 = 7.6 \text{ psia} \\
 \text{Velocity in the inlet line} &= V = 6 \text{ ft/s} \\
 \text{Height to inlet gage} &= z_1 = 0 \text{ ft} \\
 \text{Height to outlet gage} &= z_2 = 1.4 \text{ ft} \\
 \text{Inlet flow line} &= 3\text{-nominal schedule 40} \\
 \text{Outlet flow line} &= 2^{1/2}\text{-nominal schedule 40} \\
 \text{Fluid} &= \text{water}
 \end{aligned}$$

- a) Estimate the efficiency.
- b) Estimate the motor power required.

7. A pump is tested in the laboratory and the following data were obtained:

$$\begin{aligned}
 \text{Torque} &= T = 8 \text{ ft}\cdot\text{lbf} \\
 \text{Rotational speed} &= \omega = 1760 \text{ rpm} \\
 \text{Inlet pressure} &= p_1 = 4 \text{ psia} \\
 \text{Outlet pressure} &= p_2 = 12 \text{ psia} \\
 \text{Velocity in the inlet line} &= V = 8 \text{ ft/s} \\
 \text{Height to inlet gage} &= z_1 = 2 \text{ ft} \\
 \text{Height to outlet gage} &= z_2 = 5 \text{ ft} \\
 \text{Inlet flow line} &= 4\text{-nominal schedule 40} \\
 \text{Outlet flow line} &= 3^{1/2}\text{-nominal schedule 40} \\
 \text{Fluid} &= \text{water}
 \end{aligned}$$

Estimate the efficiency.

8. The inlet of a centrifugal pump is 8-nominal schedule 40, while the discharge line is 6-nominal schedule 40. Inlet and outlet pressures are 4.35 psig and 36.2 psig, respectively. Volume flow rate through the pump is 40 l/s, and the pump efficiency is 0.8. Determine the pump power. Elevation differences are negligible, and the fluid is water.
9. Consult an actual pump catalog and select a pump for the pump of Problem 6.8.
10. A motor operates a pump whose efficiency is 65%. The inlet and outlet lines are 1-1/2 nominal and 1 nominal, respectively, both schedule 40 pipe. The pressure increase across the pump amounts to 20 ft of ethylene glycol. The volume flow rate of ethylene glycol through the pump is 0.05 ft³/s. The pressure gage at the outlet is 0.4 m higher than the one in the inlet. Find the pump and motor power required.
11. Consult an actual pump catalog and select a pump for the pump of Problem 6.10.
12. A pump operates at 1800 rpm. The inlet line is 1-1/2 nominal and the outlet is 1-nominal, both schedule 40 PVC pipe. The pressure in the inlet line is 12 kPa, while the outlet line pressure is 200 kPa. The outlet pressure gage is 0.7 m higher than the inlet gage. The torque exerted is measured to be 4 N·m, and the fluid is water. If the efficiency is 0.57, determine the expected volume flow rate through the pump.
13. A pump operating at 3600 rpm has a measured input torque of 0.5 ft·lbf. The volume flow rate through the pump is 45 gpm, and the outlet pressure gage is located 17 in. above the inlet pressure gage. Inlet and outlet pipes are 1^{1/2}-nominal and 1-nominal schedule 40 pipes, respectively. The fluid being pumped is water. Determine the pressure rise across the pump for an efficiency of 75%.
14. The following data were taken on a centrifugal pump operating under conditions where the efficiency is known to be 76%:

$$\begin{aligned}
 \text{Torque} &= T = 1 \text{ N}\cdot\text{m} \\
 \text{Rotational speed} &= \omega = 1800 \text{ rpm} \\
 \text{Inlet pressure} &= p_1 = 21 \text{ kPa (absolute)} \\
 \text{Volume flow rate} &= Q = 0.0015 \text{ m}^3/\text{s}
 \end{aligned}$$

$$\begin{aligned}
 \text{Height to inlet gage} &= z_1 = 0.3 \text{ m} \\
 \text{Height to outlet gage} &= z_2 = 1 \text{ m} \\
 \text{Inlet flow line} &= 1\frac{1}{2}\text{-nominal schedule 40} \\
 \text{Outlet flow line} &= 1\text{-nominal schedule 40} \\
 \text{Fluid} &= \text{water}
 \end{aligned}$$

Determine the expected reading on the outlet pressure gage.

15. A centrifugal pump has an efficiency of 71% at a rotational speed of 1200 rpm. Inlet and outlet pressure gage readings are 7 and 20 psia, respectively. The volume flow rate through the pump is 20 gpm. Inlet and outlet pipes are 2- and 1 $\frac{1}{2}$ -nominal schedule 40, respectively, galvanized steel. The elevation of the outlet gage over that of the inlet gage is 2 ft. What is the required input torque if the fluid is carbon tetrachloride?
16. A pump was tested at 3450 rpm. The input torque was 20 ft-lbf. Inlet and outlet pressures were measured to be 7 psia and 20 psia, respectively. The inlet line is 12-nominal schedule 40 and the outlet line is 10-nominal schedule 40. The volume flow rate was 1200 gpm. The outlet pressure gage is 2 ft higher than the inlet gage. Determine the pump efficiency if the fluid is (a) water; (b) propane. Comment on how density affects efficiency.

Cavitation and NPSH

17. Derive Equation 6.8b for net positive suction head as applied to the configuration of Figure 6.6b.
18. Repeat Example 6.3 for acetone rather than water, assuming all other conditions are unchanged.
19. The inlet line of a pump is 4-nominal schedule 40 PVC pipe that is 30 ft long and includes one elbow and one well-rounded inlet. The pump conveys 0.5 ft³/s of water at 60°F from a tank. The net positive suction head required by the pump is 8 ft. Determine where the location of the water level in the tank should be with respect to the pump impeller shaft to prevent cavitation.
20. The inlet line of a centrifugal pump is made of 3-nominal schedule 40 commercial steel pipe 2 m long and includes a basket strainer. The pump delivers 8 l/s of carbon tetrachloride at 70°F. The net positive suction head required by the pump is 1 m of water. Determine where the location of the water level in the tank should be with respect to the pump impeller shaft to prevent cavitation.

Dimensional Analysis of Pumps

21. Beginning with Equation 6.12, derive Equation 6.17.
22. Beginning with Equation 6.13, derive Equation 6.18.
23. Verify that the energy transfer coefficient is dimensionless:

$$\frac{g \Delta H}{\omega^2 D^2} = \text{energy transfer coefficient}$$

24. Verify that the volumetric flow coefficient is dimensionless:

$$\frac{Q}{\omega D^3} = \text{volumetric flow coefficient}$$

25. Verify that the rotational Reynolds number is dimensionless:

$$\frac{\rho \omega D^2}{\mu g_c} = \text{rotational Reynolds number}$$

26. Verify that the power coefficient is dimensionless:

$$\frac{g_c (dW/dt)}{\rho \omega^3 D^5} = \text{power coefficient}$$

27. Actual performance data on a centrifugal pump are as follows:

Rotational speed	= 2400 rpm
Head difference	= 70 ft
Volume flow rate	= 100 gpm
Impeller diameter	= 9 in
Fluid	= water

The rotational speed of this pump is changed to 1760 rpm. What is the expected volume flow rate and head difference at the new speed?

28. Actual performance data on a pump are as follows:

Rotational speed	= 3600 rpm
Head difference	= 10 m
Volume flow rate	= 20 l/s
Impeller diameter	= 20 cm

The pump is next operated at 2400 rpm with an impeller whose diameter is 18 cm. What are the expected flow rate and the expected head difference for the new configuration?

29. A pump is tested in the laboratory and the following data were obtained:

Fluid	= water
Rotational speed	= 300 rpm
Head difference	= 8 ft
Volume flow rate	= 10 gpm
Impeller diameter	= 4 in.

The impeller is changed to one whose diameter is 3-1/2 in., and the pump is run at 900 rpm. Find the new volume flow rate and head difference. Calculate also the power for both configurations.

30. Tests on a pump yielded the following:

Fluid	= water
Rotational speed	= 1750 rpm
Volume flow rate	= 8 l/s
Head difference	= 15 ft
Impeller diameter	= 6 in.

It is desired to run the pump with a 5 in. impeller. Determine its operating characteristics when the head difference is 20 ft. Find the new volume flow rate.

Specific Speed

31. What are the actual dimensions of specific speed in Equation 6.23? (Recall that it is *arbitrarily* given the units of rpm.)
32. Calculate the specific speed for the pump of Figure 6.3.
33. Calculate the specific speed for the pump of Figure 6.10.
34. Determine the type of pump and power required for the pump corresponding to that of Figure 6.3 when operated in its maximum efficiency region.
35. Determine the type of pump and power required for the pump represented by Figure 6.10 when operated in its maximum efficiency region.
36. A 20 HP pump moves water at 500 gpm through a pipeline. The pump rotational speed is 1800 rpm. Determine its specific speed in rpm.

Fans and Fan Power

37. An axial flow fan is to move air through a duct whose inside diameter is 24 in. The air velocity is 30 ft/s. Calculations on the piping system indicate that the static pressure drop is 1.5 in. of water. Select an appropriate fan for this installation, and estimate the power required for an efficiency of 80%.
38. An underground mine shaft is rectangular in cross section, 8 ft wide by 7 ft tall, approximately. Air is to be supplied to this shaft for the workers inside. The air flow rate is 33,000 ft³/min. The mine shaft is straight and is 1 mile long. Calculate the fan power and select a fan to use in this application. (Note that the roughness is anyone's guess. So in order to solve this problem, an estimate of the roughness must be made.)
39. An axial flow fan moves air through a rectangular duct 1 m x 80 cm. The fan delivers 4 m³/s of air. The duct serves as a header that supplies several rooms with ventilation. The static pressure that the fan must overcome is 1 cm of water. Determine the appropriate fan for this installation, and calculate the fan power for an efficiency of 80%.
40. An axial flow fan with a 6-HP motor has an efficiency of 80%. The fan delivers 10,000 cfm of air through a duct having a 3 ft inside diameter. Calculate the static pressure rise associated with this fan.
41. Verify that Equation 6.25 has the correct dimensions.
42. Air at 200 kPa, 25°C enters a 2-std type K copper tube that is used to supply air to various laboratories in a building. The tube is 125 m long. At the inlet, the air flow is 0.02 m³/s. Determine the pressure drop experienced by the air; express it in cm of water. Using a manufacturer's catalog, select a fan that will work in this installation.
43. For a fan, it was stated that the power can be determined with

$$\frac{dW}{dt} = \Delta p_t Q$$

We derive this equation in this problem. Figure P6.43 shows a generalized machine. The control volume from section 1 to section 2 includes all the fluid inside the fan and the exit duct. The inlet is labeled as section 1 and has an area (indicated by the dotted line) so huge that the velocity at section 1 is negligible compared to the velocity at section 2. The pressure at 1 equals atmospheric pressure. The fan thus accelerates the flow from a velocity of 0 to a velocity we identify as V_2 . The continuity equation is

$$\dot{m}_1 = \dot{m}_2$$

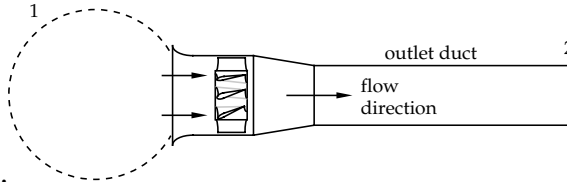


FIGURE P6.43 .

The energy equation is

$$0 = -\frac{dW}{dt} + \dot{m}_1 \left(h_1 + \frac{V_1^2}{2g_c} \right) - \dot{m}_2 \left(h_2 + \frac{V_2^2}{2g_c} \right)$$

- a. Combine the previous equations with the definition of enthalpy ($h = u + pv$) and show that

$$\frac{dW}{dt} = \dot{m}(u_1 - u_2) + \dot{m} \left\{ \left[\frac{p_1}{\rho} + \frac{V_1^2}{2g_c} \right] - \left[\frac{p_2}{\rho} + \frac{V_2^2}{2g_c} \right] \right\}$$

- b. Assuming ideal gas behavior, we write

$$u_1 - u_2 = c_v(T_1 - T_2)$$

With a fan, however, we assume an isothermal process, so that $T_1 \approx T_2$ and $\rho_1 \approx \rho_2$. With $\dot{m} = \rho AV$ (evaluated at the outlet, section 2), the energy equation becomes

$$\frac{dW}{dt} = \rho A_2 V_2 \left\{ \left[\frac{p_1}{\rho} + \frac{V_1^2}{2g_c} \right] - \left[\frac{p_2}{\rho} + \frac{V_2^2}{2g_c} \right] \right\}$$

Recall that in this analysis, we set up our control volume so the inlet velocity $V_1 = 0$; actually $V_1 \ll V_2$. Thus

$$\left\{ \left[\frac{p_1}{\rho} + \frac{V_1^2}{2g_c} \right] - \left[\frac{p_2}{\rho} + \frac{V_2^2}{2g_c} \right] \right\} \approx \left[\frac{p_1}{\rho} - \frac{p_2}{\rho} - \frac{V_2^2}{2g_c} \right]$$

in which $p_1 - p_2$ is the pressure drop associated with the piping system. The quantity in the previous equation is the change in total pressure $\Delta p_t / \rho$.

Verify that

$$\frac{dW}{dt} = \Delta p_t Q$$

44. The equations given in this chapter for predicting fan performance are based on assuming incompressible flow through the fan (see Problem 6.43). That is, for fans, compressibility effects can be assumed negligible; changes in kinetic energy are comparatively low, pressure rise is relatively small, and the process is assumed to be isothermal. In this problem, however, we model flow through a blower or a compressor, where compressibility effects cannot be ignored. Figure P6.43 shows a generalized sketch of a blower. The control volume from section 1 to 2 includes all the fluid inside the blower and the exit duct. The inlet is labeled as section 1 and has an area (indicated by the dotted line) so huge that the velocity at section 1 is negligible compared to the velocity at section 2. Applying the continuity equation, we get

$$\dot{m}_1 = \dot{m}_2 = \dot{m}$$

- a. Neglecting changes in potential and kinetic energies, verify that the energy equation applied to the system is

$$0 = -\frac{dW}{dt} + \dot{m}_1 h_1 - \dot{m}_2 h_2$$

Combine with the continuity equation and show that

$$\dot{m} = \frac{dW/dt}{h_1 - h_2}$$

- b. For an ideal gas, the enthalpy change is $\Delta h = c_p \Delta T$. We also have $\dot{m} = \rho A V$ (evaluated at the outlet, section 2). Show that the preceding equation becomes

$$V_2 = \frac{dW/dt}{\rho_2 A_2 c_p (T_1 - T_2)}$$

- c. For an ideal gas, $\rho = p/RT$ and $c_p = \gamma R/(\gamma - 1)$, where γ is the ratio of specific heats c_p/c_v . Using these definitions, show that

$$V_2 = \frac{dW/dt}{\frac{p_2 A_2 \gamma}{\gamma - 1} \left(\frac{T_1}{T_2} - 1 \right)}$$

- d. For an isentropic compression from sections 1 to 2, we have

$$\frac{T_1}{T_2} = \left(\frac{p_1}{p_2} \right)^{(\gamma - 1)/\gamma}$$

Show that the exit velocity (or the power) can be predicted with

$$V_2 = \frac{dW/dt}{\frac{p_2 A_2 \gamma}{\gamma - 1} \left\{ \left(\frac{p_1}{p_2} \right)^{(\gamma - 1)/\gamma} - 1 \right\}}$$

Thus, the exit velocity (or the power) is a function of the pressure ratio.

45. A commercial blower (radial flow fan) used to clean up lawn clippings is shown schematically in Figure P6.45. Air at 14.7 psia, 70°F is drawn into the inlet and is discharged at 13.9 psia, 70°F. The outlet tube has an ID of 2-1/2 in. The blower housing has a huge sticker on it boldly boasting “1 HP.” Assuming that this is the motor horsepower and that the fan efficiency is 80%, determine the velocity in the exit duct. Use the results of Problem 6.43 to obtain a result. Take $\gamma = 1.4$.

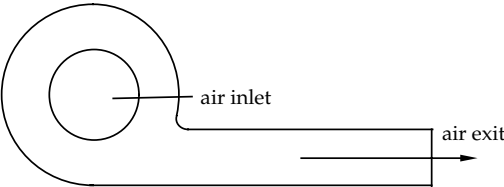


FIGURE P6.45.

6.11 Group Problems

- G1. A series of tests was conducted on a centrifugal pump to verify some iso-efficiency test results. The pump itself has a 12-nominal schedule 40 inlet line and a 10-nominal schedule 40 outlet line. Water was the liquid used, and the pressure gage in the outlet line was 2 ft higher than the inlet pressure gage. Following is a chart of data obtained from the pump, all at 75% efficiency. Fill in the blank spaces with the expected value:

Torque ft·lbf	Rotational Speed, rpm	Inlet Pressure, psia	Outlet Pressure, psia	Volume Flow Rate, gpm
30.0	1200	_____	16	800
25.5	_____	7	17	850
22.6	1800	8	_____	900
16.4	2400	6	16	_____
12.9	_____	5	15	775
_____	3600	5	15	625

- G2. A pump was tested in the laboratory to obtain data and construct a performance map. The pump had 1-nominal schedule 40 inlet and outlet lines. The inlet pressure valve was 4 in. below the outlet pressure valve. The data are provided in the following table.
- Construct a performance map for the data given in the table. Plot ΔH versus Q for all impeller diameters. Show iso-efficiency lines for 30%, 35%, 40%, 43%, 45%, and 46%. The pump operates at 1750 rpm, and all pressures are in psi.

Q , gpm	Impeller Diameter									
	4-7/8 in.		4-1/2 in.		4 in.		3-1/2 in.		3 in.	
	Δp	Torque	Δp	Torque	Δp	Torque	Δp	Torque	Δp	Torque
0	11.4	0.000	9.5	0.000	7.5	0.000	5.6	0.000	3.7	0.000
4	11.2	0.313	9.3	0.242	7.4	0.191	5.5	0.154	3.5	0.121
8	10.6	0.425	9.1	0.335	6.9	0.262	5.2	0.214	3.2	0.152
12	10.0	0.487	8.4	0.394	6.3	0.293	4.5	0.239	2.7	0.178
16	9.1	0.554	7.6	0.452	5.4	0.337	3.5	0.256	1.4	0.136
20	7.8	0.635	6.2	0.503	4.0	0.360	1.9	0.244		
24	6.2	0.683	4.8	0.572	2.6	0.405				
28	4.5	0.743	2.8	0.460						

- G3.** For all data in the table given in Problem G2, calculate values of the following dimensionless groups:

$$\frac{g \Delta H}{\omega^2 D^2} = \text{energy transfer coefficient}$$

$$\frac{Q}{\omega D^3} = \text{volumetric flow coefficient}$$

$$\frac{\rho \omega D^2}{\mu g_c} = \text{rotational Reynolds number}$$

$$\frac{g_c (dW/dt)}{\rho \omega^3 D^3} = \text{power coefficient}$$

$$\eta = \text{efficiency}$$

Construct graphs of: (a) energy transfer coefficient versus volumetric flow coefficient; (b) energy transfer coefficient versus rotational Reynolds number; (c) power coefficient versus volumetric flow coefficient; (d) power coefficient versus rotational Reynolds number; (e) efficiency versus volumetric flow coefficient; (f) efficiency versus rotational Reynolds number. Take the fluid to be water in all cases.

- G4.** It is often preferable when working with noncircular cross sections and with minor losses to have information on economic velocity. The economic velocity will vary with the economic parameters, and so minimum values should be selected for calculation purposes. The values that yield the minimum economic velocity are as follows:

$$\begin{aligned} C_2 &= \$0.10/(\text{kW} \cdot \text{hr}) = \$0.10/(738 \text{ ft} \cdot \text{lbf} \cdot \text{hr}); \\ C_1 &= \$300/\text{m}^{2.2} & n &= 1.4 \\ t &= 7880 \text{ hr/yr} & a &= 1/(7 \text{ yr}) \\ F &= 6 & b &= 0.01 \\ \eta &= 75\% = 0.75 & \varepsilon &= 0.00025 \end{aligned}$$

For five fluids assigned by the instructor, calculate (a) economic diameter and (b) economic velocity by using the continuity equation. Using the above data

will provide a minimum for the economic velocity. In all cases, take the volume flow rate to be 1 l/s. Explain how to modify the results without recalculating if the volume flow rate in an actual case is different from 1 l/s.

- G5.** A pipeline that is 600 ft long is to convey chloroform at a flow rate of 850 gpm. The line is to be suspended 2 ft (or so) from a ceiling. Determine: (a) the economic line size and piping material; (b) the system curve to 1000 gpm; (c) the pump power requirements; (d) the expected pump efficiency; (e) an appropriate pump to use (refer to a catalog rather than a curve from this chapter); and, (f) the location and configuration of pipe hangers. The pipeline is horizontal, and minor losses can be neglected.
- G6.** A pipeline conveying ether is 80 m long. The flow rate of the ether is $0.1 \text{ m}^3/\text{s}$. The line is horizontal and contains three regular elbows; all other minor losses can be neglected. The first 70 m of the line are a straight run and should be elevated a distance of 3 ft from the ground. Determine: (a) the economic line size and piping material; (b) the system curve to $0.15 \text{ m}^3/\text{s}$; (c) the pump power requirements; (d) the expected pump efficiency; (e) an appropriate pump to use (refer to a catalog rather than a curve from this chapter); and (f) the placement and configuration of pipe supports for the first 70 m of the line. Take inlet and outlet pressures to be equal.
- G7.** A pipeline is to be designed to convey 5 gpm of glycerine a distance of 25 ft. The line contains six elbows and a globe valve (fully open). Determine: (a) the economic line size and piping material; (b) the system curve to 10 gpm; (c) the pump power requirements; (d) the expected pump efficiency; and, (e) an appropriate pump to use (refer to a catalog rather than a curve from this chapter). The pressures at inlet and outlet are equal.
- G8.** Turpentine is stored in a 25-ft-diameter tank. Inside the storage tank, the turpentine depth is 18 ft. A piping system containing a pump is to move the turpentine to a holding tank having a very large diameter. The turpentine depth in the holding tank is constant at 6 ft. The pipeline contains a basket strainer, three elbows, and a ball valve. At the exit, the end of the pipeline is submerged 5 ft below the free surface of the turpentine. For a volume flow rate of 50 gpm, determine: (a) the economic line size and piping material; (b) the system curve to 75 gpm; (c) the pump power requirements; (d) the expected pump efficiency; and (e) an appropriate pump to use (refer to a catalog rather than a curve from this chapter). The pipeline is 108 ft long and is horizontal, with inlet and exit at the same elevation.
- G9.** A pipeline is to convey water uphill. The pipeline is 20 m long and is buried 18 in. underground. The inlet of the pipeline is to be submerged in water and should contain a basket strainer. The exit of the pipeline is submerged also in water at an elevation of 3 m above the inlet. The pipeline should convey the water at 25 gpm. The pipeline contains eight elbows, a ball check valve, and a gate valve. Determine: (a) the economic line size and piping material; (b) the system curve to 30 gpm; (c) the pump power requirements; (d) the expected pump efficiency; and (e) an appropriate pump to use (refer to a catalog rather than a curve from this chapter).

Open-Ended Design Problems

The following problems are taken from actual machines or devices. The problem statements purposely do not contain all of the information needed to determine what assumptions to make in order to obtain a solution, and the solution might not be unique.

- G10.** For the piping system of Figure P6.G10, perform calculations to design the piping system following the suggested procedure in the design practice section. Use the following economic parameters:

$$\begin{aligned}
 C_2 &= \$0.04/(\text{kW}\cdot\text{hr}) = \$0.04/(738 \text{ ft}\cdot\text{lb}\cdot\text{ft}\cdot\text{hr}); \\
 C_1 &= \$400/\text{m}^{2.2} & \text{PVC schedule 40 pipe} \\
 t &= 7880 \text{ hr/yr} & F = 7 \\
 n &= 1.2 & a = 1/(7 \text{ yr}) \\
 b &= 0.01 & \eta = 75\% = 0.75 \\
 \text{fluid} &= \text{octane} & Q = 30 \text{ gpm}
 \end{aligned}$$

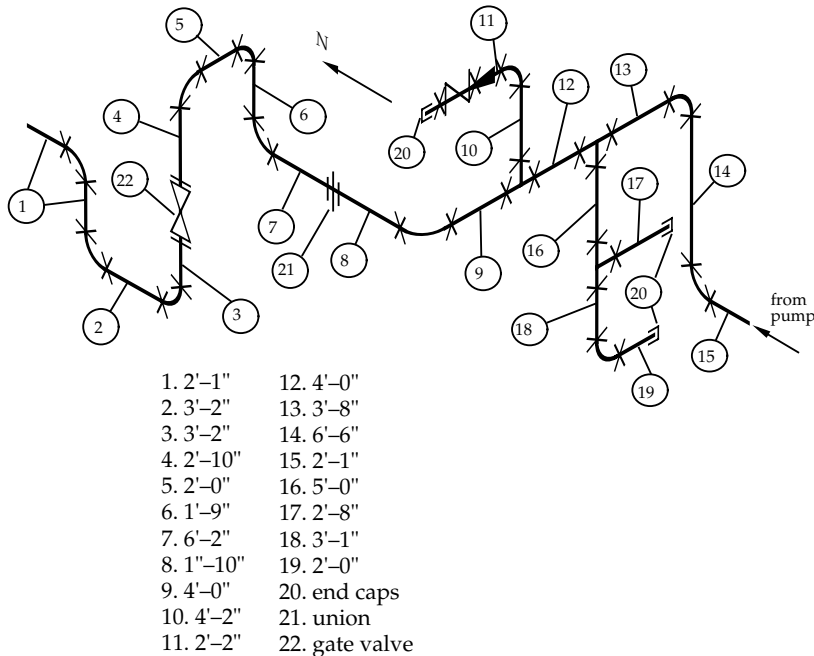


FIGURE P6.G10.

There is a pump in the system that has to be selected and whose position must be determined. Select a pump from a catalog, and ensure that it does not cavitate.

- G11** You work for a machine shop that just bought a used milling machine. Due to unfortunate misuse by the previous owner, the machine needs a new cooling system. Figure P6.G11 shows a side view of the milling machine. At the base of the machine is a well filled with SAE 10 oil. A submersible pump moves oil through a piping system up to the head of the machine, where the oil is discharged at the cutter and used as a coolant for the milling operation. The oil flow rate should be 2 ounces per second, or at least laminar when it is discharged. The height h is 4 ft, and the tube length is 20 ft. After the coolant is discharged, it drains into a collection trough and then through a return line to the well.

1. Select a tube or pipe to use for the coolant line from pump to discharge. Should it be made of stainless steel, or is copper tubing good enough?
2. Select a tube or pipe to use for the return line. What material should be used?
3. Select a material to use for the trough and design its cross section. Oil that collects in the trough should never fill the trough to overflowing.
4. Use a manufacturer's catalog and select a pump.

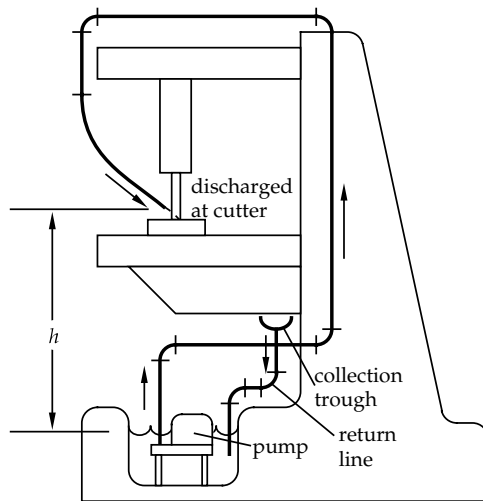


FIGURE P6.G11.

- G12.** (This problem is based on a design found in a delicatessen in San Francisco.) You have been hired by an architectural firm to finish the design of a piping system that is proposed to install in a restaurant. Figure P6.G12a shows a setup designed to produce a rain-like water shower, and as indicated, the system consists of a number of components. Embedded within the concrete floor are two pools, each lined with stainless steel sheet metal and separated by a distance of 3 ft. The pools are 6 ft x 3 ft (into the page) and 10 ft x 3 ft. The water depth in the pools is 8 in. Piping from the bottom of each pool leads to a centrifugal pump. The pump discharges water into a pipe concealed behind a wall. The pipe leads over the top of a dropped ceiling. Piping goes through the ceiling to flexible tubing (labeled as B in the drawing) that discharges water to three identical troughs.

One trough is separate, and the other two are joined together. The troughs are suspended 6 in. below the ceiling, although this is not indicated in the figure. The troughs are 4 ft long by 18 in. wide, and are made of Plexiglas, as shown in the trough detail drawing, Figure P6.G12b. The bottom of the troughs is in the shape of inverted roofs and has a number of holes drilled in them. So when water is discharged into the troughs, it drains through the holes and falls in separate streams. The sound is like rainfall as the streams hit the pool water. The troughs are centered over the pools. The ceiling-to-floor height is 12 ft, and the distance from the right edge of the 10-ft long pool to the wall is 10 ft. Such a system adds atmosphere to what might be an otherwise ordinary luncheon environment.

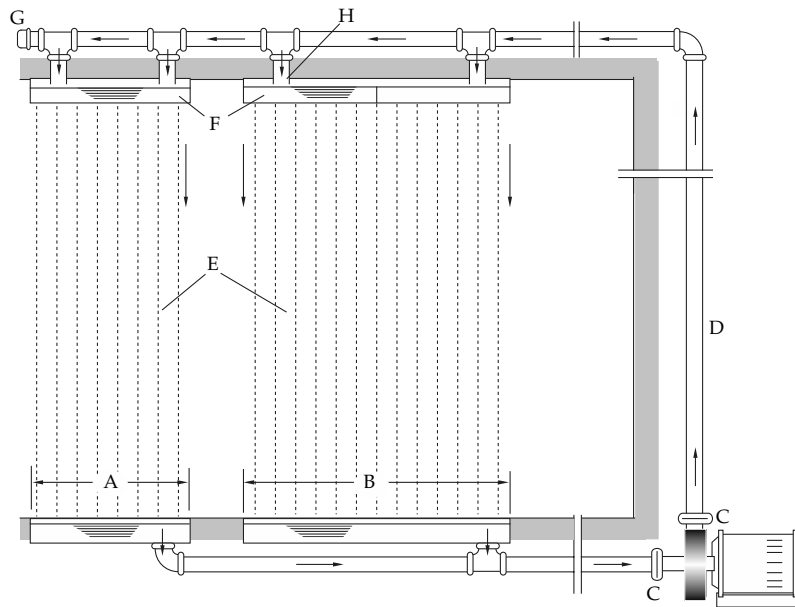


FIGURE P6.G12a.

Notes:

A	6 ft	D	13 ft
B	10 ft	E	Rainlike falling water
C	Union fittings about the pump for easy replacement if necessary	F	Troughs
		G	End cap
		H	Flexible tubing

Pipe lengths total 30 ft; square edged inlets at the pools; total pipe length from trough A to pump is 6 ft

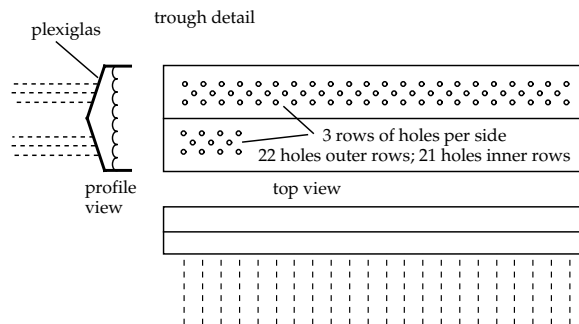


FIGURE P6.G12b.

1. Select a tube or pipe size and a pipe material for pump inlet and discharge lines.

2. Keep in mind: Children will probably be in this restaurant while the system is working; children might be tempted to throw coins or even food in the pools; and some adults will behave like children. Make recommendations on how the inlet piping should be attached to the pools so as to avoid problems with litter.
3. Select a diameter to use for drilling holes in the troughs and design its cross section (e.g., angle and expected water depth in the troughs).
4. Use a manufacturer's catalog and select a pump.
5. When the system is operating, the water depth in the pools might fall below 8 in., but it should not, in order to prevent air from being entrained in the pump inlet line. How is this problem to be alleviated?

G13. You work for a consulting company that has been hired to modify the piping system used in a dairy. The piping system shown in Figure P6.G13 contains a pump that conveys milk from one tank to another, the second tank being a feed tank to a bottling machine. The system consists of 7 m of 2-nominal schedule 40 stainless steel pipe from pump to downstream tank. The inlet line from the tank on the left to the pump is 2½-nominal schedule 40 stainless steel. The vertical distance from the reference datum to the pressure gage at the pump inlet ($= z_2$) is 1.4 m. The venturi meter has approach and exit line diameters of 2-nominal schedule 40 pipe, while at the throat the diameter corresponds to 1½-nominal schedule 40. The venturi meter has an air-over-milk inverted U-tube manometer attached. The liquid height in the upstream tank ($= z_1$) must not be less than 2 m, and due to the bottling machine requirements, the height z_3 should always be equal to or greater than 3 m.

1. Explain why the piping, the tanks, and the pump should all be stainless steel.
2. The bottling machine can fill and cap a number of bottles per minute. The flow rate requirement can be met if the milk velocity in the 2-nominal line is 1.5 m/s. Under this condition, what is the expected reading on the pressure gage at pump outlet?
3. Calculate the pump power required.
4. An air-over-milk inverted U-tube manometer is entirely unsatisfactory. Determine a better way to measure the pressure difference at the meter. What is the expected pressure difference? Is there a better meter to use than a venturi?

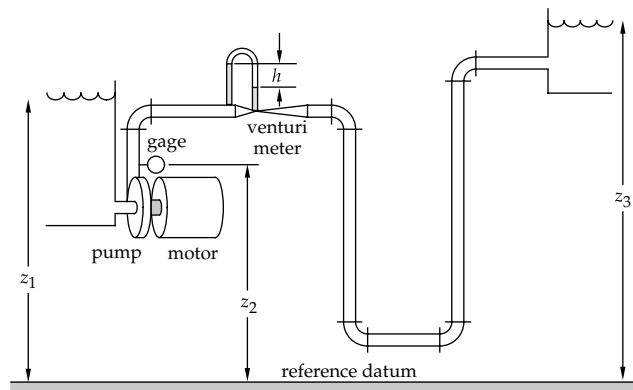


FIGURE P6.G13.

- G14.** You work for a manufacturer of hydraulic presses and have been assigned the enviable task of analyzing the proposed piping system of Figure P6.G14. A pump moves hydraulic oil from a supply tank to the press. The press needs at least 20 gpm to operate properly. The valve in the pump outlet line is to be used to reduce the flow rate if more than 30 gpm is provided. The pipe sizes have already been selected, not based on a minimum cost analysis, but because “we have lots of these pipe sizes in stock.” The inlet line to the pump is 2-nominal schedule 40 PVC, 2.5 m long. The outlet line from pump to press is 1½-nominal schedule 40 PVC, 6 m long. The pressure of the hydraulic oil delivered to the press must be 30 psia.

After being used in the press, the oil is discharged to an elevated tank at the top of the press. The return line from this tank to the supply tank is 12 m long, and its diameter has not been selected, but it too should be made of PVC. The heights are $H = 6$ ft, $h = 8$ in., and $d = 12$ in.

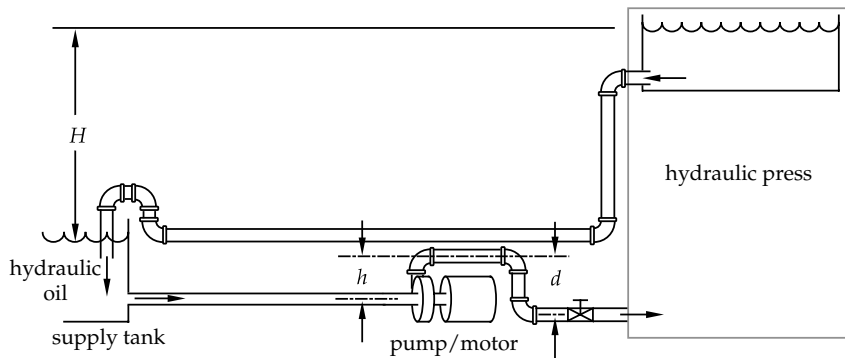


FIGURE P6.G14.

1. Size the pump. For this application, a positive-displacement pump is to be used. Select one from a manufacturer's catalog.
2. The elevated tank has a cross-sectional area of 2.5 ft^2 . Due to structural considerations, the oil in this tank should not have more than 10 gallons of oil in it. Size the return line based on this requirement. Do you think it is a good idea to modify the inlet in the return line to make it an overflow inlet and thus maintain 10 gallons or so of oil in the elevated tank? (This modification is given consideration in order to keep air from entering the return line.)
3. Does the return line convey oil fast enough? Is a pump necessary in the return line, or is gravity a good enough “prime mover”?
4. What are the diameters that you would recommend based on a minimum cost analysis?
5. What kind of valve are you going to specify, and why?

- G15.** Figure P6.G15 shows a pump and piping system that is to drain a 12,000 gallon tank in 3 hours. The tank is used to prepare a liquid, and once this has been done, the pump is to move the liquid to a storage tank or to a tank truck (the exit is to atmospheric pressure). The liquid has the same properties as kerosene. The total length of pipe in the system is 60 ft. The tank itself contains heating coils (not shown) that can be used to warm the fluid prior to pumping it. Follow the suggested design procedure and select a pump for the case when $z_1 = 6$ ft and $z_2 = 12$ ft.

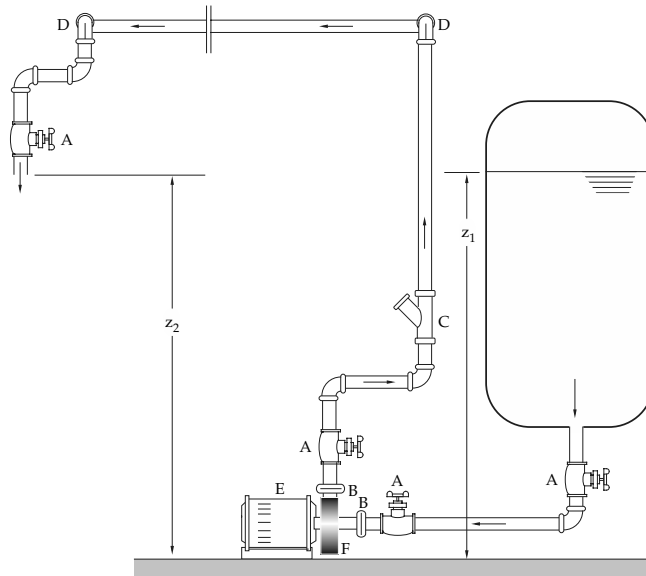


FIGURE P6G15

Notes:

- A Globe valves (4 of them)
- B Union fittings (4 with 2 not shown)
- C Y-strainer ($K = 1$)
- D Elbow facing away, with pipe attached (into the page)
(2 more obscured by the elbows at D)
- 9 Elbows total
- E Motor
- F Pump

The pipe is all wrought iron, with threaded fittings, schedule 40.

Pipe lengths total 60 ft.

The tank contains a chemical, which is prepared, mixed, and stored until it is to be shipped or stored in another tank. All tanks vented to atmosphere.

The liquid has the same properties as kerosene.

The tank volume is 12,000 gallons.

The tank is to be emptied in 3 hours.

Square edged inlet at the tank.

Total pipe length from tank to pump is 6 ft.

G16. Figure P6.G16 shows a pump and piping system. The pump is to move ethylene glycol at a flow rate of 50 gpm. Follow the suggested design procedure and select a pump for the case when $z_1 = 4$ ft and $z_2 = 12$ ft.

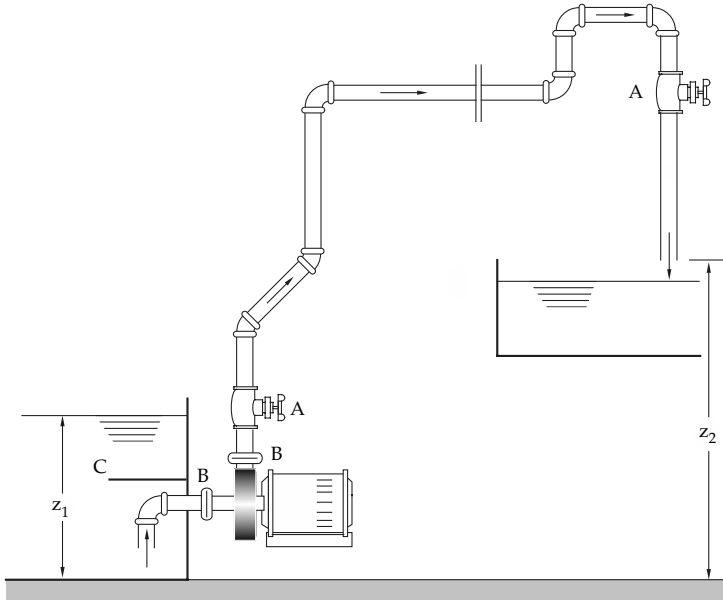


FIGURE P6.G16

Notes:

- A Globe valves (2 of them)
- B Union fittings (4 with 2 not shown)
- C Plate welded to tank to suppress air entrainment in pump inlet
- D 5-90° Elbows
2-45° Elbows

The pipe is all wrought iron, with threaded fittings, schedule 40.

Pipe lengths total 116 ft.

The fluid is ethylene glycol.

The flow rate required is 50 gpm.

Total inlet pipe length from tank to pump is 4 ft.

$z_1 = 4$ ft and $z_2 = 12$ ft.

G17. Figure P6.G17 shows a pump and piping system. The pump is to move ethylene glycol at a flow rate of 50 gpm. Follow the suggested design procedure and select a pump for the case when $z_1 = 4$ ft and $z_2 = 12$ ft.

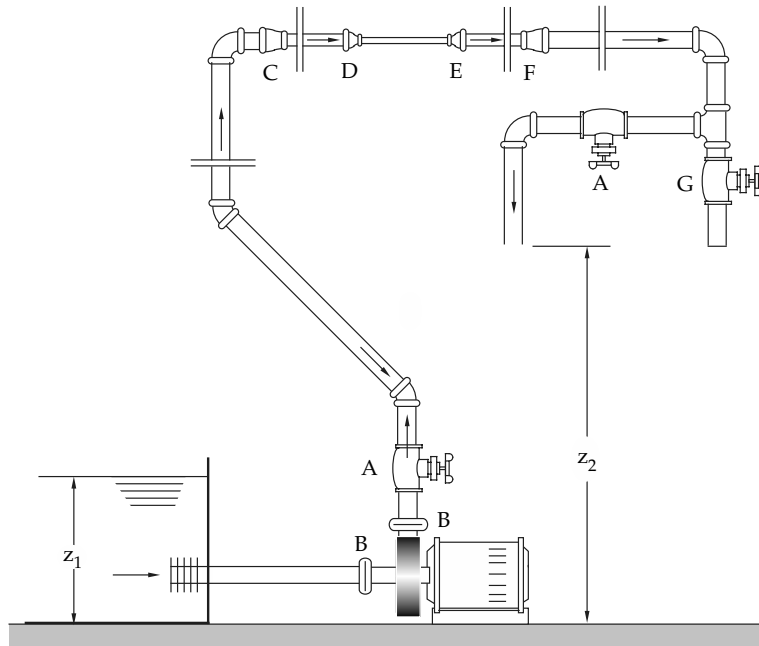


FIGURE P6.G17

Notes:

A	Globe valves
B	Union fittings
C, F	3 x 2 concentric reducer
D, E	2 x 1 ¹ / ₅ concentric reducer
G	Globe valve closed

Pipe & Fittings	All pipe is schedule 10, stainless steel Welded fittings except for the unions at B

Inlet to pump 11 ft of 3 nominal, Basket strainer, 1 union fitting

Exit from pump to reducer at C: 89 ft of 3 nominal, 1-90° elbow, 2-45° elbows
1 union fitting, 1 Globe valve

Reducer at C to reducer at D: 15 ft of 2 nominal,
3 x 2 concentric reducer

Reducer at D to reducer at E: 2 ft of 1¹/₅ nominal,
2 concentric reducers 2 x 1¹/₅

Reducer at E to reducer at F: 597 ft of 2 nominal, 20-90° elbows (not shown)

Reducer at F to exit at z_2 : 10 ft of 3 nominal, 2-90° elbows, 1 globe valve,
1 T-joint

- G18.** It may appear to the discerning individual that the line size selected in Example 6.7 (3 nominal) is too large, based on the area limits in Step 1. Rework the problem using the smaller size (2¹/₂ nominal). Complete the following chart:

Item		Example 6.7	Your results
"Economic" line size	=	3-nom sch 40	2 ¹ / ₂ -nom sch 40
Layout	=	given	given
System curve	=		
Specific speed	=		
Expected pump eff	=		
Pump type	=		
Motor power	≈		

- G19.** Figure P6.G.19 shows a tubing system used as a heat exchanger. The exchanger has fins attached (as shown) and is made of 1/2-std type M copper tubing. There are 40 ft of tubing and, in order to transfer the required amount of heat, the liquid velocity can be no less than 2 m/s. If the liquid is benzene, determine the pump power required. All fittings are soldered together, and the pressure drop from inlet to exit is 4 psi.

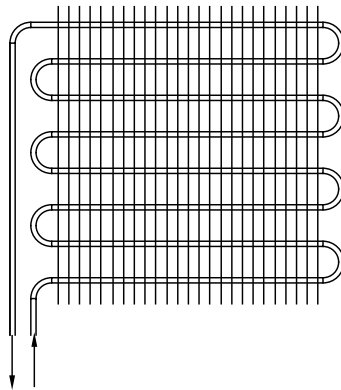


FIGURE P6.G.19.

CHAPTER 7 Some Heat Transfer Fundamentals

Heat is transferred from a source to a receiver in three distinct ways: conduction, convection, and radiation. Most engineering applications involve identifying one or two dominant modes and applying simplifying assumptions in order to solve the problem at hand. In this chapter, we review some of the fundamental concepts of heat transfer and define pertinent properties of substances. Solution methods for simple conduction and convection problems are illustrated. Equations are presented for various types of convection problems. Radiation heat transfer concepts are not discussed. The main purpose of the chapter, however, is to call attention to the heat transfer properties of fluids and to the solution methods in convection heat transfer. The objective is to lay a foundation for the modeling of heat exchangers that follows in subsequent chapters.

7.1 Conduction of Heat Through a Plane Wall

Figure 7.1 shows a planar wall in contact with a heat source on the left (a bank of heaters) and a heat sink on the right (a water jacket). Also shown imposed on the geometry of the wall are temperature T versus x axes. The heaters provide a constant heat flow per unit area per unit time, q_x'' . Once this system reaches steady state, temperature within the material is measured and graphed on the axes. As shown, temperature T varies linearly with x , and the slope of the temperature profile is written as dT/dx . The flow of heat per unit area (normal to the heat flow direction) per unit time is proportional to the temperature gradient:

$$q_x'' \propto \left(-\frac{dT}{dx} \right) \quad (7.1)$$

Note that as distance x increases, temperature T decreases, so $-dT/dx$ is actually a positive quantity. To make Equation 7.1 an equality, we introduce a proportionality constant:

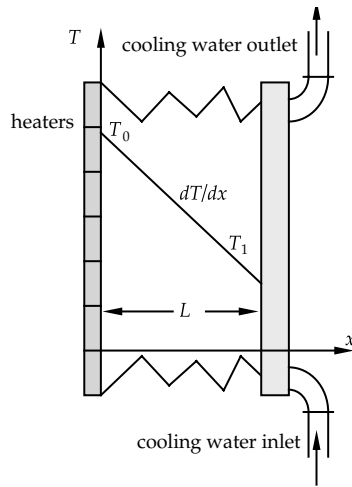


FIGURE 7.1. *Conduction of heat through a plane wall.*

$$q_x = kA \left(-\frac{dT}{dx} \right) \quad (7.2)$$

in which k is known as the **thermal conductivity** of the substance, with dimensions of $\text{F}\cdot\text{L}/(\text{T}\cdot\text{L}\cdot\text{t})$ [$\text{BTU}/(\text{hr}\cdot\text{ft}\cdot^\circ\text{R})$ or $\text{W}/(\text{m}\cdot\text{K})$]. Thermal conductivity k is a property of substances and in general is evaluated experimentally. Values of thermal conductivity for various metals and building materials are provided in Table 7.1. (More extensive data are provided in various reference books.) Values of thermal conductivity for selected liquids and gases are found in the property tables of the Appendix: Tables B.1–B.5 for liquids and Tables C.1–C.6 for gases.

For the planar wall of Figure 7.1, Equation 7.2 can be integrated directly to obtain

$$q_x = kA \frac{T_0 - T_1}{L} \quad (7.3)$$

where $(T_0 - T_1)$ is the temperature difference over the wall thickness L . Equation 7.3 can be rewritten as

$$q_x = \frac{T_0 - T_1}{L/kA} = \frac{T_0 - T_1}{R} \quad (7.4)$$

in which $L/kA = R$ is introduced as a resistance through which heat q_x is transferred under steady conditions due to the imposed temperature difference $(T_0 - T_1)$. The resistance R has dimensions of $\text{t}\cdot\text{T}/(\text{F}\cdot\text{L})$ ($\text{hr}\cdot^\circ\text{R}/\text{BTU}$ or K/W).

TABLE 7.1. *Thermal properties of selected metals and alloys.*

Material	Specific Gravity	Specific Heat C_p		Thermal Conductivity, k		Diffusivity	
		$\frac{\text{J}}{\text{kg}\cdot\text{K}}$	$\frac{\text{BTU}}{\text{lbm}\cdot^\circ\text{R}}$	$\frac{\text{W}}{\text{mK}}$	$\frac{\text{BTU}}{\text{hr}\cdot\text{ft}\cdot^\circ\text{R}}$	$\text{m}^2/\text{s} \times 10^6$	$\text{ft}^2/\text{s} \times 10^3$
<i>Selected Metals at 293 K = 20°C = 528°R = 68°F</i>							
Aluminum	2.702	896	0.214	236	163	97.5	1.05
Bronze	8.666	343	0.0819	26	15.0	8.59	0.0925
Brass	8.522	385	0.0920	111	64.1	34.12	0.3673
Cast iron	7.272	420	0.100	52	30.0	17.02	0.1832
Copper	8.933	383	0.0915	399	231	116.6	1.26
Wrought iron	7.849	460	0.110	59	34.1	16.26	0.1750
Carbon steel	7.801	473	0.113	43	24.8	11.72	0.1262
Chrome steel	7.865	460	0.110	61	35.2	16.65	0.1792
Silicon steel	7.769	460	0.110	42	24.3	11.64	0.1254
Stainless steel	7.817	461	0.110	14.3	8.26	3.87	0.0417

Material	Specific Gravity	$\frac{\text{J}}{\text{kg}\cdot\text{K}}$	$\frac{\text{BTU}}{\text{lbm}\cdot^\circ\text{R}}$	$\frac{\text{W}}{\text{mK}}$	$\frac{\text{BTU}}{\text{hr}\cdot\text{ft}\cdot^\circ\text{R}}$	$\text{m}^2/\text{s} \times 10^5$	$\text{ft}^2/\text{s} \times 10^6$
<i>Selected Building Materials at 293 K = 20°C = 528°R = 68°F</i>							
Asbestos	0.383	816	0.195	0.113	0.0653	0.036	3.88
Asphalt	2.120			0.698	0.403		
Brick							
Common	1.800	840	0.201	0.45	0.26	0.031	3.33
Masonry	1.700	837	0.200	0.658	0.38	0.046	5.0
Silica	1.9			1.07	0.618		
Cardboard				0.25	0.14		
Concrete	0.500	837	0.200	0.128	0.074	0.049	5.3
Cork	0.120	1880	0.449	0.042	0.0243	0.03	3.15
Ebonite				0.163	0.0093		
Glass fiber	0.220			0.035	0.02		
Glass wool				0.040	0.023		
Glass							
(window)	2.800	800	0.191	0.81	0.47	0.034	3.66
Ice at 0°C	0.913	1830	0.437	2.22	1.28	0.124	13.3
Kapok	0.025			0.035	0.02		
Wood							
Fir, pine, spruce	0.444	2720	0.650	0.15	0.087	0.0124	1.33
Oak	0.705	2390	0.571	0.19	0.11	0.0113	1.22
Wool	0.200			0.038	0.022		

Notes: Data from several sources; see references at end of text.

Density = ρ = specific gravity $\times$ 62.4 lbm/ft³ = specific gravity $\times$ 1 000 kg/m³
 = specific gravity $\times$ 1.94 slug/ft³.

Diffusivity of aluminum = α = $97.5 \times 10^{-6} \text{ m}^2/\text{s}$ = $1.05 \times 10^{-3} \text{ ft}^2/\text{s}$

EXAMPLE 7.1. Figure 7.2 shows a cross section of a **guarded hot-plate** apparatus. It is used to measure the thermal conductivity of a planar shaped material, such as plywood, insulation, sheet rock, etc. The guarded hot-plate apparatus consists of a main heater and a guard heater. The guard heater completely surrounds the main heater. Two samples of the material to be tested are required. One sample is placed on each side of the heaters. Cooling water jackets are made to contact the samples. To operate the device, both heaters are activated. Heat from the main heater is made to flow in one dimension through each sample to the cooling water jackets. The guard heater supplies energy to the outer perimeter of the samples so that heat flowing from the main heater will not flow in any lateral direction. Thus, a one-dimensional flow of heat from the main heater to the cooling water jackets is set up.

Both heaters are heated electrically, so readings of voltage and amperage on the wires to the main heater provide data from which input power to *both* samples can be calculated. Thermocouples are used to make temperature readings, which are needed for several purposes. To ensure that one-dimensional heat flow exists, temperature at surface 1 (Figure 7.2) of the samples must be the same at the main and the guard heater once steady state is achieved. At that time, the temperature at surface 1 of the main heater and at surfaces 2 and 3 of the cooling water jackets are recorded.

A guarded hot-plate apparatus is used to measure the thermal conductivity of 3/8-in.-thick plywood. The electrical input to the main heater is 110 V $\times$ 1 A. The temperature at the main heater surface is 210°F, while the surface temperature of the cooling jackets is 80°F. The cross-sectional area through which heat flows is 0.75 ft². (a) Determine the thermal conductivity of the plywood, and (b) calculate the value of the resistance as defined in Equation 7.4.

Solution: Half the electrical power input goes into each piece of plywood. The heat flow into each piece is given by Equation 7.4:

$$q_x = \frac{T_0 - T_1}{L/kA} = \frac{T_0 - T_1}{R} \quad (7.4)$$

in which q_x is calculated to be

$$q_x = 0.5(110)(1) = 55 \text{ W}/0.2928 = 187 \text{ BTU/hr}$$

The conversion factor was obtained from Appendix Table A.2. Rearranging Equation 7.4 to solve for thermal conductivity and substituting, we have

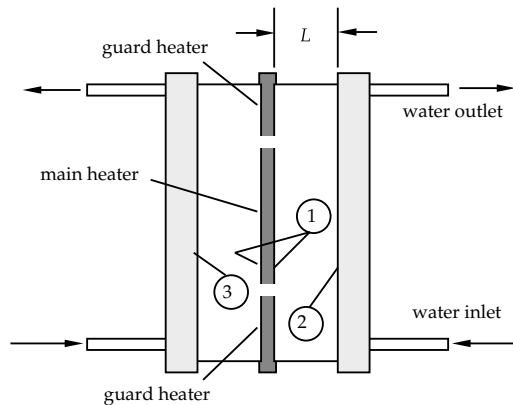


FIGURE 7.2. Schematic of a guarded hot-plate heater for measuring thermal conductivity.

$$k = \frac{Lq_x}{A(T_0 - T_1)} = \frac{3/(8(12))(187)}{0.75(210 - 80)}$$

(a) $k = 0.060 \text{ BTU}/(\text{hr} \cdot \text{ft} \cdot ^\circ\text{R})$

The resistance is found from the definition

$$R = \frac{L}{kA} = \frac{3/(8(12))}{0.060(0.75)}$$

(b) $R = 0.692 \text{ hr} \cdot ^\circ\text{R}/\text{BTU}$

The concept of resistance to heat flow through a planar material can be used to apply the one-dimensional heat flow equation to materials in series. Consider the composite wall of Figure 7.3. As shown, heat flows from left to right through three substances of different thermal conductivities in contact with one another. For the entire wall, we can write

$$q_x = \frac{T_0 - T_3}{R_{03}} \quad (7.5)$$

The heat flow through all the materials is equal and is identical to the heat flow through the wall; so in addition to Equation 7.5, we can write

$$q_x = \frac{T_0 - T_1}{R_{01}} = \frac{T_1 - T_2}{R_{12}} = \frac{T_2 - T_3}{R_{23}} = \frac{T_0 - T_3}{R_{03}}$$

We therefore conclude that the overall resistance equals the sum of the individual resistances:

$$R_{03} = R_{01} + R_{12} + R_{23} \quad (7.6a)$$

or

$$R_{03} = \frac{L_{01}}{Ak_{01}} + \frac{L_{12}}{Ak_{12}} + \frac{L_{23}}{Ak_{23}} \quad (7.6b)$$

Substituting into Equation 7.5, we find that the heat flow through the wall can therefore be written as

$$q_x = \frac{T_0 - T_3}{R_{03}} = \frac{T_0 - T_3}{\frac{L_{01}}{Ak_{01}} + \frac{L_{12}}{Ak_{12}} + \frac{L_{23}}{Ak_{23}}} \quad (7.7)$$

EXAMPLE 7.2. An oven wall consists of three layers of brick arranged as in Figure 7.3. The inside wall is made of silica brick, 4 in. thick, covered with masonry brick, 8 in. thick, while the outside layer is of common brick, 6 in. thick. During operation, the inside oven wall temperature reaches 1000°F, and the outside surface temperature is 130°F. Calculate the heat transferred through the wall per square foot. Determine also the interface temperatures.

Solution: We can apply the equations of the preceding section. We will calculate the resistance offered by each layer and ultimately solve for the heat transferred.

Assumptions:

1. The system is at steady state.
2. The thermal properties of the materials are constant (although it is known that they vary with temperature).

From Table 7.1, we read the following values for thermal conductivity:

silica brick	$k_{01} = 0.618 \text{ BTU/hr}\cdot\text{ft}\cdot^\circ\text{R}$
masonry brick	$k_{12} = 0.38 \text{ BTU/hr}\cdot\text{ft}\cdot^\circ\text{R}$
common brick	$k_{23} = 0.26 \text{ BTU/hr}\cdot\text{ft}\cdot^\circ\text{R}$

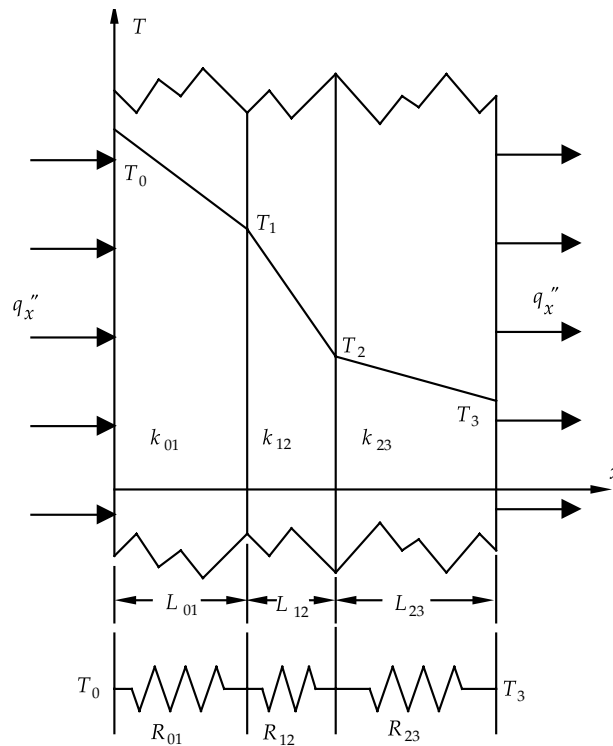


FIGURE 7.3. Heat flow through a composite wall.

We now calculate the resistance offered by each layer assuming a cross-sectional area A of 1 ft²:

silica brick $R_{01} = L_{01}/Ak_{01} = (4/12)/(1(0.618)) = 0.539 \text{ hr} \cdot ^\circ\text{R}/\text{BTU}$

masonry brick $R_{12} = L_{12}/Ak_{12} = (8/12)/(1(0.38)) = 1.75 \text{ hr} \cdot ^\circ\text{R}/\text{BTU}$

common brick $R_{23} = L_{23}/Ak_{23} = (6/12)/(1(0.26)) = 1.92 \text{ hr} \cdot ^\circ\text{R}/\text{BTU}$

The total resistance for the three layers is the sum of these:

$$R_{03} = R_{01} + R_{12} + R_{23} = 0.539 + 1.75 + 1.92 \quad (7.6a)$$

or $R_{03} = 4.21 \text{ hr} \cdot ^\circ\text{R}/\text{BTU}$

The heat loss per square foot of wall cross section is

$$q_x = \frac{T_0 - T_3}{R_{03}} = \frac{1000 - 130}{4.21} \quad (7.5)$$

or $\underline{q_x = 207 \text{ BTU/hr}}$

For the first layer, we can write

$$q_x = \frac{T_0 - T_1}{R_{01}}$$

Rearranging and solving for T_1 , we obtain

$$T_1 = T_0 - q_x R_{01} = 1000 - 207(0.539)$$

Solving,

$$\underline{T_1 = 888^\circ\text{F}}$$

Similarly,

$$q_x = \frac{T_1 - T_2}{R_{12}}$$

and $T_2 = T_1 - q_x R_{12} = 888 - 207(1.75)$

Solving,

$$\underline{T_2 = 526^\circ\text{F}}$$

7.2 Conduction of Heat Through a Cylindrical Wall

As seen in the last section, the cross-sectional area through which heat flows in a planar geometry is constant. The model developed involved the concept of resistance to heat flow in order to make the appropriate calculations. We now extend the discussion to a cylindrical geometry.

Figure 7.4 shows a circular cylinder with inside and outside radii of R_1 and R_2 , respectively. The temperature of the inside and outside surfaces are T_1 and T_2 . The cylinder length is L . The figure contains a sketch of temperature versus radius (T versus r) as well as the resistance R_{12} that corresponds to the cross section. In order to determine an equation for R_{12} , we begin with Equation 7.2 written in the r direction, which is

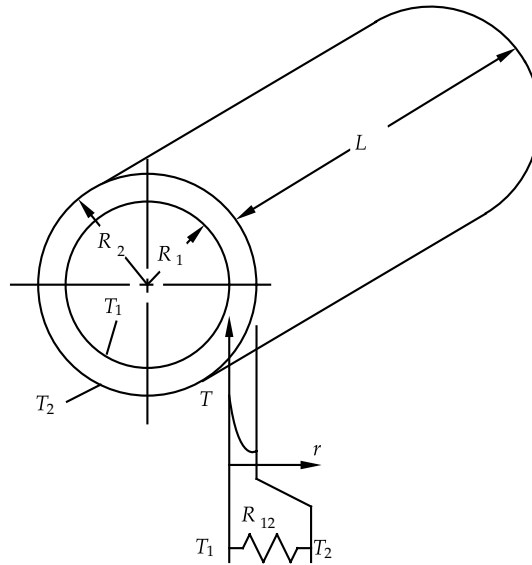


FIGURE 7.4. Heat flow through a cylindrical wall.

$$q_r = kA \left(-\frac{dT}{dr} \right) \quad (7.2)$$

where q_r represents heat flow in the radial direction and $-dT/dr$ is the temperature gradient in the pipe wall. At any radius $R_1 < r < R_2$, the cross-sectional area is given by

$$A = 2\pi rL$$

Substituting into Equation 7.2 and separating variables for integration, we get

$$\int_{T_1}^{T_2} dT = \int_{R_1}^{R_2} -\frac{1}{2\pi kL} \frac{q_r dr}{r}$$

Integrating gives

$$T_2 - T_1 = -\frac{q_r}{2\pi kL} \ln \frac{R_2}{R_1}$$

$$\text{or } T_1 - T_2 = \frac{q_r}{2\pi k L} \ln \frac{R_2}{R_1} \quad (7.8)$$

Equation 7.8 can be rearranged in order to solve for a resistance as in the plane wall problem:

$$q_r = \frac{T_1 - T_2}{R_{12}} = \frac{T_1 - T_2}{\frac{1}{2\pi k L} \ln \frac{R_2}{R_1}}$$

We therefore conclude that in a cylindrical geometry the resistance to heat flow is given by

$$R_{ij} = \frac{1}{2\pi k L} \ln \frac{R_j}{R_i}$$

in which the subscripts i and j refer to surface locations.

In many practical cases, such as an insulated pipe, the geometry of interest will consist of two cylinders in series. This is illustrated in Figure 7.5. In calculating heat flow through such a geometry, we use the same approach as was done for a plane wall, namely, the concept of resistance to heat flow. From the inside surface to the outside surface, we have

$$q_r = \frac{T_1 - T_3}{R_{13}} \quad (7.9)$$

In addition, for each material,

$$q_r = \frac{T_1 - T_2}{R_{12}} = \frac{T_2 - T_3}{R_{23}}$$

Therefore,

$$R_{13} = R_{12} + R_{23}$$

or

$$R_{13} = \frac{1}{2\pi k_{12} L} \ln \frac{R_2}{R_1} + \frac{1}{2\pi k_{23} L} \ln \frac{R_3}{R_2} \quad (7.10)$$

Equation 7.9 then becomes

$$q_r = \frac{T_1 - T_3}{\frac{1}{2\pi k_{12} L} \ln \frac{R_2}{R_1} + \frac{1}{2\pi k_{23} L} \ln \frac{R_3}{R_2}} \quad (7.11)$$

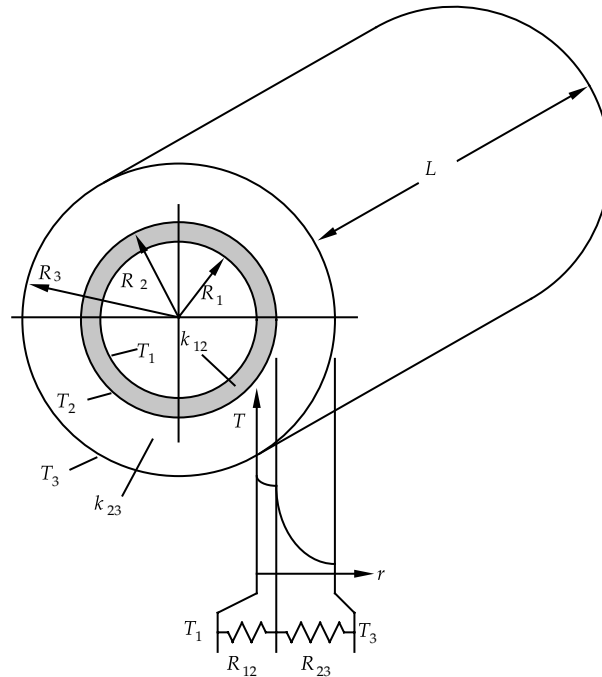


FIGURE 7.5. Resistances for two cylinders in series.

EXAMPLE 7.3. A steel pipe [$k = 40 \text{ W}/(\text{m}\cdot\text{K})$] is insulated with kapok insulation, similar in cross section to the sketch of Figure 7.5. The pipe carries a fluid that maintains the inside surface at 100°C . The outside surface of the insulation is at 25°C . The pipe is 4-nom sch 40 and the insulation is 6 cm thick. Determine the heat transferred per unit length through the cylindrical wall, and the temperature at the interface between the two materials.

Solution: We can apply the equations of the preceding section. We will calculate the resistance offered by each layer and ultimately solve for the heat transferred.

Assumptions:

1. The system is at steady state.
2. The thermal properties of the materials are constant (although it is known that they vary with temperature).

From Appendix Table D.1, we read the following dimensions of 4-nom sch 40 pipe:

$$OD = 11.43 \text{ cm} \quad ID = 10.23 \text{ cm}$$

From Table 7.1, the thermal conductivity of kapok is $0.035 \text{ W}/(\text{m}\cdot\text{K})$. In terms of the notation of Figure 7.5, we have for each radius

$$R_1 = 10.23/2 = 5.12 \text{ cm}$$

$$R_2 = 11.43/2 = 5.72 \text{ cm}$$

$$R_3 = R_2 + 6 = 11.72 \text{ cm}$$

Also, for each material,

$$\text{steel} \quad k_{12} = 40 \text{ W}/(\text{m}\cdot\text{K})$$

$$\text{kapok} \quad k_{23} = 0.035 \text{ W}/(\text{m}\cdot\text{K})$$

Assuming a unit length, the resistances are calculated to be

$$\text{steel} \quad R_{12} = \frac{1}{2\pi k_{12}L} \ln \frac{R_2}{R_1} = \frac{1}{2\pi (40)(1)} \ln \frac{5.72}{5.12} = 0.00044 \text{ K/W}$$

$$\text{kapok} \quad R_{23} = \frac{1}{2\pi k_{23}L} \ln \frac{R_3}{R_2} = \frac{1}{2\pi (0.035)(1)} \ln \frac{11.72}{5.72} = 3.26 \text{ K/W}$$

As seen from these figures, the insulation offers a much greater resistance to the flow of heat than does steel. The total resistance then is

$$R_{13} = R_{12} + R_{23} = 3.26 \text{ K/W}$$

The heat transfer rate becomes

$$q_r = \frac{T_1 - T_3}{R_{13}} = \frac{100 - 25}{3.26} \quad (7.9)$$

$$\text{or} \quad \underline{q_r = 23 \text{ W}}$$

In order to find the interface temperature, we apply the heat flow equation to either material. For the steel,

$$q_r = \frac{T_1 - T_2}{R_{12}}$$

Rearranging and solving for the interface temperature, we obtain

$$T_2 = T_1 - q_r R_{12} = 100 - 23(0.00044)$$

Solving,

$$\underline{T_2 \approx 100^\circ\text{C}}$$

The temperature drop across the steel is virtually negligible. In many practical problems, temperature within a metal is often assumed to be a constant throughout.

7.3 Convection Heat Transfer—The General Problem

Heat transfer by convection occurs when a solid surface is in contact with a moving fluid and a temperature difference exists between the two. We identify two different ways convection heat transfer takes place: **forced convection** and **natural convection**. Forced convection occurs when the fluid motion is due to an external motive force. Natural convection (also traditionally known as **free convection**) occurs if fluid motion is induced by the transfer of heat.

The heat transferred by convection is calculated by use of a **convection coefficient** $\bar{h}$. The dimensions of the convection coefficient are $\text{F}\cdot\text{L}/(\text{T}\cdot\text{L}^2\cdot\text{t})$ [$\text{W}/(\text{m}^2\cdot\text{K})$ or $\text{BTU}/(\text{hr}\cdot\text{ft}^2\cdot^\circ\text{R})$]. The overbar denotes that the convection coefficient for the problem of interest is an average value, valid over the entire surface or geometry. In this text, the overbar is not used, in order to simplify the notation and because the average value of the convection coefficient h is all that will be used here.

Measurements of heat transfer rates and temperatures must be made in order to calculate the convection coefficient. It has been found that the convection coefficient is a function of temperature difference and of actual temperatures. Therefore, the convection coefficient (also called the **surface coefficient**) cannot be calculated except by trial-and-error methods. Once the convection coefficient h is known, the heat transfer rate can be found with

$$q = hA(T_s - T_\infty) \quad (7.12)$$

The convection coefficient in Equation 7.12 relies on surface and free-stream temperatures T_s and T_∞ , respectively. Other temperature differences can be and have been devised to yield alternatives to Equation 7.12.

By rewriting Equation 7.12 in a form similar to that of Equation 7.9, we can define a resistance to convection heat transfer and treat the convection

problem like the conduction problem. Equation 7.12 becomes

$$q = \frac{T_s - T_\infty}{R_{s\infty}} = \frac{T_s - T_\infty}{1/hA}$$

So the resistance to heat transfer at a convective surface is given by

$$R_{s\infty} = \frac{1}{hA} \quad (7.13)$$

Heat Transfer Properties of Fluids

In previous chapters, the emphasis was on fluid mechanics types of problems. Fluid properties were discussed, but these were isothermal properties. It is known that fluid properties do indeed vary with temperature. Although the Appendix Tables B.1 (*Properties of liquids at room temperature and pressure*) and C.1 (*Physical properties of gases at room temperature and pressure*) are sufficient for isothermal problems, it is desirable to have more extensive data available for the solution of heat transfer problems. Such data are provided in the Appendix for several fluids. Tables B.1-B.5 in the appendix show properties of liquids, and Tables C.1-C.6 show properties of gases (specific gravity, thermal conductivity, kinematic viscosity, thermal diffusivity, and Prandtl number).

7.4 Convection Heat Transfer Problems: Formulation and Solution

Exhaustive amounts of research have been performed in order to develop relations for the convection coefficient h in various geometries. Results of such measurements are usually given in terms of dimensionless ratios. For example, the convection coefficient is traditionally represented by the **Nusselt number**, defined as

$$\text{Nu} = \frac{hL}{k_f} \quad (7.14)$$

where L is a characteristic length appropriate to the geometry of interest and k_f is the thermal conductivity of the fluid. Table 7.2 gives a list of some dimensionless groups encountered in fluid mechanics and heat transfer problems. Calculation of heat transfer rate using a convection coefficient equation is illustrated in the following example.

TABLE 7.2. *Some commonly encountered dimensionless groups.*

Ratio	Symbol	Name
hL/k	Bi	Biot number
$\mu V^2/[k_f(T_s - T_\infty)]$	Br	Brinkman number
$2D_f g_c/\rho V^2 D^2$	C_D	Drag coefficient
$2 \Delta p D/\rho V^2 L$	f	Friction factor
V^2/gL	Fr	Froude number
$g\beta(T_s - T_\infty)L^3/\nu^2$	Gr	Grashof number
hL/k_f	Nu	Nusselt number
VL/α	Pe = Re · Pr	Peclet number
$C_p \mu/k_f = \nu/\alpha$	Pr	Prandtl number
$2 \Delta p g_c/\rho V^2$	C_p	Pressure coefficient
$g\beta(T_s - T_\infty)L^3/\nu\alpha$	Ra = Gr · Pr	Rayleigh number
$\rho VD/\mu g_c$	Re	Reynolds number
$h/\rho V C_p$	$\text{St} = \frac{\text{Nu}}{\text{Re} \cdot \text{Pr}}$	Stanton number
$\rho V^2 L/\sigma g_c$	We	Weber number

EXAMPLE 7.4. It is desired to determine how much heat is transferred through a vertical wall of sheetrock that is 1/2 in. thick. The sheet rock has a paper surface on both of its sides. As indicated in Figure 7.6, the left side of the material (i.e., paper) is maintained at 75°C, while the right side is exposed to air at 25°C. The sheetrock properties are (Type X):

$$\rho = 711 \text{ kg/m}^3 \qquad C_p = 1\,089 \text{ J/(kg}\cdot\text{K)} \quad k = 0.258 \text{ W/(m}\cdot\text{K)}$$

Determine the heat transferred through the sheetrock.

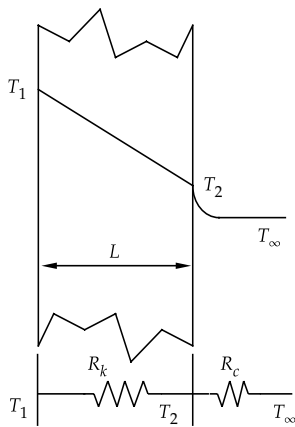


FIGURE 7.6. Heat flow through a wall.

Solution: Figure 7.6 shows a cross section of the wall, as well as a temperature profile and the resistances to the flow of heat. Heat is conducted through the wall, and convected from the outside surface. For the outside surface, heat transfer occurs by natural convection. Heat is also transferred to the surroundings by radiation.

Assumptions:

1. The system is at steady state.
 2. Properties of the materials are constant.
 3. Air properties are constant and are evaluated at the appropriate temperatures.
 4. Radiation heat transfer is neglected.
 5. Resistance to heat flow by the paper cladding is negligible.
- We can write the following equation for the heat transferred through the wall:

$$q = \frac{T_1 - T_\infty}{R_k + R_c} \quad (i)$$

Each resistance is found as

$$R_k = R_{12} = \frac{L_{12}}{k_{12}A}$$

$$R_c = \frac{1}{hA}$$

The sheet rock thickness is 1/2 in. = 1.27 cm = 0.012 7 m. The area is taken to be 1 m², and so the results will be expressed on a per square meter basis. The resistances then are

$$R_{12} = \frac{L_{12}}{k_{12}A} = \frac{0.0127}{0.258(1)} = 0.04922 \text{ K/W}$$

$$R_c = \frac{1}{hA} = \frac{1}{h}$$

Combining with Equation i of this example, and after substituting for the temperatures, we get

$$q = \frac{75 - 25}{0.04922 + 1/h} \quad (\text{i})$$

The difficulty at this point is in finding the convection coefficient. It is usually determined (and defined) as a function of the surface temperature, T_2 in this example, which is unknown. The convection coefficient is also unknown.

We rely on experimental results obtained through previously performed research. We use the experimentally determined Churchill-Chu equation:

$$\text{Nu} = \frac{hL}{k_f} = 0.68 + \frac{0.67 \text{ Ra}^{1/4}}{\left(1 + \left[\frac{0.492}{\text{Pr}}\right]^{9/16}\right)^{4/9}}$$

where the Rayleigh and Prandtl numbers are given, respectively, as

$$\text{Ra} = \frac{g\beta(T_s - T_\infty)L^3}{\nu\alpha} < 10^9;$$

$$0 < \text{Pr} = \frac{\nu}{\alpha} = < \infty$$

and

$$\beta = 1/T_\infty = \text{coefficient of thermal expansion}$$

It is necessary to keep in mind that the length L in this equation is the vertical length (or height) of the wall. The length in the conductive resistance equation refers to the sheet rock thickness. It is important to not confuse the two.

For convection heat transfer on the right side of the sheet rock, we will need air properties to calculate the Rayleigh and Prandtl numbers. The properties of air vary with temperature, so we must first decide on the temperature to use for this case. For the right hand side (Figure 1.6), the air temperature is 25°C ($+ 273 = 298 \text{ K}$), but at the surface of the sheet rock, the temperature is known to be somewhat higher. So for the right-hand side,

which is at 298 K, we evaluate properties at 300 K (a guess):

$$\begin{aligned}\rho &= 1.177 \text{ kg/m}^3 & k_f &= 0.02624 \text{ W/(m}\cdot\text{K)} \\ \nu &= 15.68 \times 10^{-6} \text{ m}^2/\text{s} & C_p &= 1005.7 \text{ J/(kg}\cdot\text{K)} \\ \alpha &= 0.22160 \times 10^{-4} \text{ m}^2/\text{s} & \text{Pr} &= 0.708\end{aligned}$$

The Rayleigh number is calculated with

$$\text{Ra} = \frac{g\beta(T_2 - T_\infty)L^3}{\nu\alpha}$$

where $\beta = 1/T_\infty$, in which temperature is in absolute units, and the wall height L is taken to be 1 m. We have $\beta = 1/T_\infty = 1/(25 + 273) = 0.003356/\text{K}$. The Rayleigh number is now calculated as

$$\begin{aligned}\text{Ra} &= \frac{(9.81)(0.003356)(T_2 - 25)}{(15.68 \times 10^{-6})(0.22160 \times 10^{-4})} \\ &= 9.475 \times 10^7 (T_2 - 25)\end{aligned}\quad (\text{ii})$$

To find the convection coefficients we substitute into the Churchill-Chu equation:

$$\begin{aligned}\text{Nu} &= \frac{hL}{k_f} = 0.68 + \frac{0.67 \text{ Ra}^{1/4}}{\left(1 + \left[\frac{0.492}{\text{Pr}}\right]^{9/16}\right)^{4/9}} \\ h &= \frac{0.02624}{1} \left\{ 0.68 + \frac{0.67 \text{ Ra}^{1/4}}{\left(1 + \left[\frac{0.492}{0.708}\right]^{9/16}\right)^{4/9}} \right\}\end{aligned}$$

or

$$h = 0.01784 + 0.01349 \text{ Ra}^{1/4} \quad (\text{iii})$$

Now because the Rayleigh number is in terms of T_2 , it will be necessary to develop an iterative scheme to solve for it and for the convection coefficient. In addition to Equation i, we can write another equation for the heat transferred through the wall in terms of the surface temperature:

$$q = \frac{T_2 - T_\infty}{R_c}$$

Rearranging to solve for the surface temperature gives

$$T_2 = T_\infty + q (R_c) = 25 + \frac{q}{h} \quad (\text{iv})$$

The surface temperature can be determined with this equation when heat transfer rate is known.

We next formulate an iterative procedure to determine the heat transferred through the wall. The steps are as follows:

- Assume a temperature for T_2 . Calculate:
- Rayleigh number with Equations ii;
- Convection coefficient h , with Equation iii;
- Heat transfer rate q with Equation i;
- Refined values of T_2 with Equation iv;
- Calculate the new surface temperature

The procedure is repeated until convergence within a tolerable limit is achieved. The following summarizes the results of these calculations:

$$\text{Ra} = 9.475 \times 10^7 (T_2 - 25) \quad (\text{ii})$$

$$h = 0.01784 + 0.01349 \text{ Ra}^{1/4} \quad (\text{iii})$$

$$q = \frac{75 - 25}{0.04922 + 1/h} \quad (\text{i})$$

$$T_2 = T_\infty + q (R_c) = 25 + \frac{q}{h} \quad (\text{iv})$$

Trials:

	T_2 °C	Ra	h W/(m·K)	q W	T_2 °C
1st	30	4.7×10^8	2	91.4	70
2nd	70	4.2×10^9	3.46	148	67.7
3rd	67.7	4.1×10^9	3.42	146	67.8

The solution then is:

$$\underline{q = 146 \text{ W}} \quad (\text{for each m}^2 \text{ of surface})$$

EXAMPLE 7.5. A 1-m-tall vertical wall of a kitchen oven consists of three materials placed in series—sheet metal, insulation, and sheet metal. The sheet metal pieces are made of carbon steel and are 1 mm thick, while the (glass fiber) insulation is 4 cm thick. Inside the oven, the air temperature is 250°C and heat is convected to the wall. Determine the heat transferred through the wall if the outside surface is in contact with air at 25°C.

Solution: Figure 7.6 shows a cross section of the wall, as well as a temperature profile and the resistances to the flow of heat. Heat is

convected to one surface, conducted through the wall, and convected from the outside surface. For both outside surfaces, the heat transfer process is one of natural convection. Heat is also transferred to the surroundings by radiation.

Assumptions:

1. The system is at steady state.
2. Properties of the materials are constant.
3. Air properties are constant and are evaluated at the appropriate temperatures.
4. Radiation heat transfer is neglected.
5. Resistance to heat flow within the sheet metal pieces is negligible, so the temperature of each is uniform throughout.

Reference to a heat transfer textbook shows that a number of equations are available to determine a natural convection coefficient for a vertical wall. Here we use the experimentally determined Churchill-Chu equation:

$$\text{Nu} = \frac{hL}{k_f} = 0.68 + \frac{0.67 \text{ Ra}^{1/4}}{\left(1 + \left[\frac{0.492}{\text{Pr}}\right]^{9/16}\right)^{4/9}} \quad (7.15)$$

where

$$\text{Ra} = \frac{g\beta(T_s - T_\infty)L^3}{\nu\alpha} < 10^9; \quad 0 < \text{Pr} = \frac{\nu}{\alpha} < \infty$$

and

$$\beta = 1/T_\infty = \text{coefficient of thermal expansion}$$

For convection heat transfer on the outside of each piece of sheet metal, we will need air properties. The properties of air vary with temperature, so we must first decide on the temperature to use for both cases. For the left-hand side (Figure 7.7), the air temperature is 250°C (+ 273 = 523 K), but near the surface of the sheet metal, the temperature is known to be somewhat lower. So for the left-hand side, we *intuitively* elect to use properties evaluated at 500 K. Similarly for the right-hand side, we have an air temperature of 25°C (= 298 K). We evaluate properties at 300 K. From Appendix Table C.2, we read for air at 500 K:

$$\begin{array}{ll} \rho = 0.705 \text{ kg/m}^3 & C_p = 1.0295 \text{ J/(kg}\cdot\text{K)} \\ k_f = 0.04038 \text{ W/(m}\cdot\text{K)} & \alpha = 0.5564 \times 10^{-4} \text{ m}^2/\text{s} \\ \nu = 37.90 \times 10^{-6} \text{ m}^2/\text{s} & \text{Pr} = 0.68 \end{array}$$

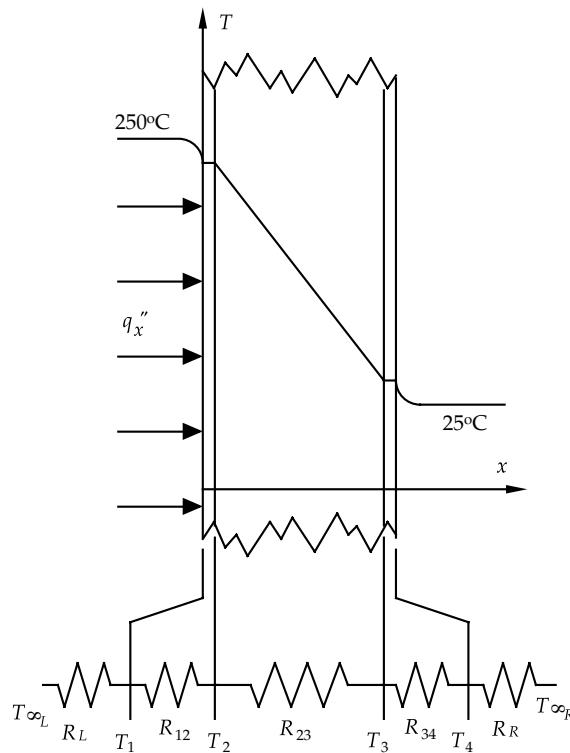


FIGURE 7.7. Heat transfer through a vertical wall.

For air at 300 K, we read

$\rho = 1.177 \text{ kg/m}^3$	$C_p = 1\,005.7 \text{ J/(kg}\cdot\text{K)}$
$k_f = 0.026\,24 \text{ W/(m}\cdot\text{K)}$	$\alpha = 0.221\,60 \times 10^{-4} \text{ m}^2/\text{s}$
$\nu = 15.68 \times 10^{-6} \text{ m}^2/\text{s}$	$\text{Pr} = 0.708$

Table 7.1 gives thermal conductivities of

sheet metal	$k = 43 \text{ W/(m}\cdot\text{K)}$
glass fiber	$k = 0.035 \text{ W/(m}\cdot\text{K)}$

The material thicknesses are

sheet metal	$L = 0.001 \text{ m}$
glass fiber	$L = 0.04 \text{ m}$

Heat is transferred through the wall, so we can write

$$q = \frac{T_{\infty L} - T_{\infty R}}{R_L + R_{12} + R_{23} + R_{34} + R_R} \quad (\text{i})$$

Each resistance is found from

$$\begin{aligned} R_L &= \frac{1}{h_L A} & R_{12} &= \frac{L_{12}}{k_{12} A} & R_{23} &= \frac{L_{23}}{k_{23} A} \\ R_{34} &= \frac{L_{34}}{k_{34} A} & R_R &= \frac{1}{h_R A} \end{aligned}$$

It is apparent that the temperature within the sheet metal pieces is uniform due to how thin they are; therefore

$$T_1 \approx T_2 \quad T_3 \approx T_4 \quad R_{12} = 0 \quad R_{34} = 0$$

Each resistance contains a cross-sectional area term A ; because area is unspecified, we assume an area of 1 m^2 and perform the calculations on a per-square-meter basis. The resistances are now determined as

$$R_L = \frac{1}{h_L} \quad (\text{evaluated with Equation 7.15}) \quad (\text{ii})$$

$$R_{12} = 0 \quad R_{23} = \frac{L_{23}}{k_{23} A} = \frac{0.04}{0.035(1)} = 1.143 \text{ K/W}$$

$$R_{34} = 0$$

$$R_R = \frac{1}{h_R} \quad (\text{evaluated with Equation 7.15}) \quad (\text{iii})$$

To find the convection coefficients, we start with the Churchill-Chu equation:

$$\text{Nu} = \frac{hL}{k_f} = 0.68 + \frac{0.67 \text{ Ra}^{1/4}}{\left(1 + \left[\frac{0.492}{\text{Pr}}\right]^{9/16}\right)^{4/9}} \quad (7.15)$$

The length term in Equation 7.15 does not refer to a wall thickness but instead to a wall height. This was given as $L = 1 \text{ m}$. So for the left side of the wall,

$$h_L = \frac{0.040\ 38}{1} \left\{ 0.68 + \frac{0.67\ \text{Ra}^{1/4}}{\left(1 + \left[\frac{0.492}{0.68} \right]^{9/16} \right)^{4/9}} \right\}$$

$$\text{or } h_L = 0.027\ 46 + 0.020\ 66\ \text{Ra}^{1/4} \quad (\text{iv})$$

Likewise for the right side,

$$h_R = \frac{0.026\ 24}{1} \left\{ 0.68 + \frac{0.67\ \text{Ra}^{1/4}}{\left(1 + \left[\frac{0.492}{0.708} \right]^{9/16} \right)^{4/9}} \right\}$$

$$\text{or } h_R = 0.017\ 84 + 0.013\ 49\ \text{Ra}^{1/4} \quad (\text{v})$$

The Rayleigh number is calculated with

$$\text{Ra} = \frac{g\beta(T_s - T_\infty)L^3}{\nu\alpha}$$

where $\beta = 1/T_\infty$.

For the left side, $\beta = 1/T_{\infty L} = 1/(250 + 273) = 0.001\ 912/\text{K}$, and for the right side, $\beta = 1/T_{\infty R} = 1/(25 + 273) = 0.003\ 356/\text{K}$. The Rayleigh numbers are now calculated as

$$\text{Ra}_L = \frac{(9.81)(0.001\ 912)(250 - T_1)}{(37.90 \times 10^{-6})(0.556\ 4 \times 10^{-4})} = 8.895 \times 10^6(250 - T_1) \quad (\text{vi})$$

$$\text{Ra}_R = \frac{(9.81)(0.003\ 356)(T_4 - 25)}{(15.68 \times 10^{-6})(0.221\ 60 \times 10^{-4})} = 9.475 \times 10^7(T_4 - 25) \quad (\text{vii})$$

In addition to Equation i, we can write other equations for the heat transferred through the wall as

$$q = \frac{T_{\infty L} - T_2}{R_L + R_{12}} \qquad q = \frac{T_3 - T_{\infty R}}{R_{34} + R_R}$$

Rearranging these equations gives

$$T_2 = T_{\infty L} - q(R_L + R_{12}) \quad (\text{viii})$$

$$T_3 = T_{\infty R} + q(R_{34} + R_R) \quad (\text{ix})$$

Interface temperatures can be determined with the above equations when the heat transfer rate is known.

We now formulate an iterative procedure to determine the heat transferred through the wall. The steps are as follows:

1. Assume temperatures $T_1 (= T_2)$ and $T_3 (= T_4)$.
2. Rayleigh numbers Ra_L and Ra_R with Equations (vi) and (vii).
3. Convection coefficients h_L and h_R with Equations (iv) and (v).
4. Resistances R_L and R_R with Equations (ii) and (iii).
5. Heat transfer rate q with Equation (i).
6. Refined values of T_2 and T_4 with Equations viii and ix.
7. Repeat the calculations with the new interface temperatures.

The procedure is repeated until convergence within a tolerable limit is achieved. The following tables summarize the results of these calculations.

$T_1 = T_2$ (assumed)	Ra_L (Eq. vi)	h_L (Eq. iv)	R_L (Eq. ii)	q (Eq. i)	T_2 (Eq. vii)
225°C	2.224×10^8	2.550 W/(m ² ·K)	0.392 1 K/W	115.1 W	205°C
205	4.003×10^8	2.950	0.339 0	127.4	207
207	3.825×10^8	2.917	0.342 9	125.6	207
207	3.825×10^8	2.917	0.342 9	126.0	206.8

$T_3 = T_4$ (assumed)	Ra_R (Eq. vii)	h_R (Eq. v)	R_R (Eq. iii)	q (Eq. i)	T_3 (Eq. ix)
35°C	9.475×10^8	2.385 W/(m ² ·K)	0.419 4 K/W	115.1 W	73.3°C
73.3	4.576×10^9	3.527	0.238 6	127.4	61.1
61.1	3.420×10^9	3.280	0.304 9	125.6	63.3
63.3 (close enough)	3.630×10^9	3.329	0.300 4	126.0	62.8

The solution then is:

$$\underline{q = 126 \text{ W}} \quad (\text{for each m}^2 \text{ of surface})$$

EXAMPLE 7.5. A horizontally laid 2-nom sch 40 steel pipe ($k = 25$ BTU/hr·ft·°R) is lagged with fiberglass insulation ($k = 0.02$ BTU/hr·ft·°R) that is 1 in. thick. The pipe conveys steam that maintains the inside surface temperature at 250°F. Air outside the insulation is at 80°F. Determine the heat loss through the pipe and insulation.

Solution: Figure 7.8 shows a cross section of the insulated pipe, as well as a temperature profile and the appropriate resistances to the flow of heat. Heat is transferred by conduction through the pipe wall and through the insulation. Heat is transferred by natural convection from the outside surface of the insulation to the surrounding air. Heat is also transferred to the surroundings by radiation.

Assumptions:

1. The system is at steady state.
2. Properties of the materials are constant.
3. Air properties are constant and are evaluated at 80°F.
4. Radiation heat transfer is neglected.

Reference to a heat transfer textbook shows that a number of equations are available to determine a natural convection coefficient for a horizontal cylinder. Here we use another experimentally determined equation developed by Churchill-Chu:

$$\text{Nu} = \frac{hD}{k_f} = \left\{ 0.60 + \frac{0.387 \text{ Ra}^{1/6}}{\left(1 + \left[\frac{0.559}{\text{Pr}} \right]^{9/16} \right)^{8/27}} \right\}^2 \quad (7.16)$$

where

$$10^{-5} < \text{Ra} = \frac{g\beta(T_s - T_\infty)L^3}{\nu\alpha} < 10^{12}; \quad 0 < \text{Pr} = \frac{\nu}{\alpha} < \infty$$

and

$$\beta = 1/T_\infty = \text{coefficient of thermal expansion}$$

From Appendix Table D.1, we read the following for 2-nom sch 40 pipe:

$$OD = 2.375 \text{ in.} \quad ID = 0.1723 \text{ ft}$$

In terms of the notation of this problem,

$$\begin{aligned} R_1 &= 0.1723 \text{ ft}/2 \\ R_2 &= (2.375/12)/2 = 0.198 \text{ ft}/2 \\ R_3 &= ((2.375 + 2)/12)/2 = 0.365 \text{ ft}/2 \end{aligned}$$

The properties of air at 540°R = (80°F + 460) are obtained from Appendix Table C.2:

$$\begin{aligned} \rho &= 0.0735 \text{ lbm/ft}^3 & C_p &= 0.240 \text{ BTU/lbm}\cdot^\circ\text{R} \\ k_f &= 0.01516 \text{ BTU/hr}\cdot\text{ft}\cdot^\circ\text{R} & \alpha &= 0.859 \text{ ft}^2/\text{hr} \\ \nu &= 16.88 \times 10^{-5} \text{ ft}^2/\text{s} & \text{Pr} &= 0.708 \end{aligned}$$

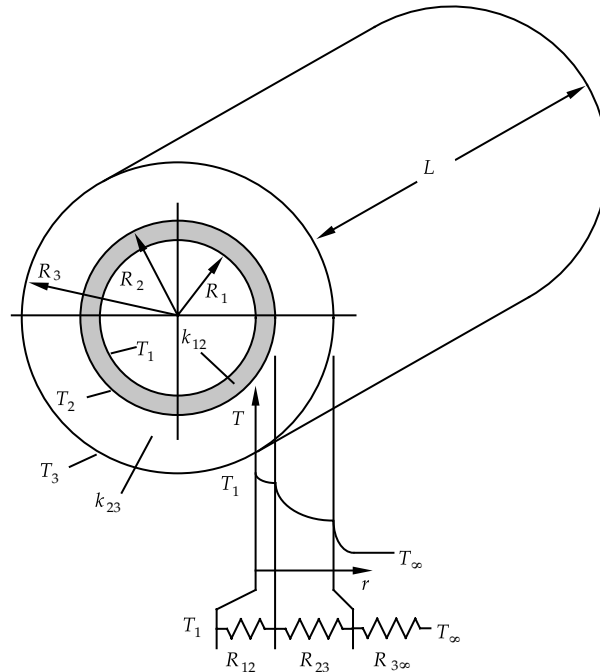


FIGURE 7.8. Heat transfer from an insulated pipe with convection.

We calculate $\beta = 1/(80 + 460) = 0.00185/^{\circ}\text{R}$. The total heat transferred is

$$q = \frac{T_1 - T_{\infty}}{R_{12} + R_{23} + R_{3\infty}} \quad (\text{i})$$

In addition, we write

$$q = \frac{T_1 - T_3}{R_{12} + R_{23}} \quad (\text{ii})$$

For conduction through the solid materials, we have

$$R_{ij} = \frac{1}{2\pi k L} \ln \frac{R_j}{R_i}$$

Assuming a unit length, we substitute to get

$$\text{steel} \quad R_{12} = \frac{1}{2\pi (25)(1)} \ln \frac{0.198/2}{0.1723/2} = 0.000885 \text{ hr} \cdot ^\circ\text{R}/\text{BTU}$$

$$\text{fiberglass} \quad R_{23} = \frac{1}{2\pi (0.02)(1)} \ln \frac{0.365/2}{0.198/2} = 4.87 \text{ hr} \cdot ^\circ\text{R}/\text{BTU}$$

For convection from the outside surface of the insulation, we calculate the resistance using

$$R_{3\infty} = \frac{1}{hA}$$

where the convection coefficient h is to be found with Equation 7.16 and the outside surface area of the insulation is

$$A = \pi(2R_3)L = \pi(0.365)(1) = 1.147 \text{ ft}^2$$

Equation 7.16, however, depends on the outside surface temperature of the insulation (T_3), which is unknown at this point. We must therefore resort to an iterative procedure to find the convection coefficient h and ultimately the heat transferred q . We begin by substituting all known quantities into the parameters of Equation 7.16. The Rayleigh number is

$$\text{Ra} = \frac{g\beta(T_s - T_\infty)L^3}{\nu\alpha}$$

Now, length L in the above equation for Rayleigh number refers to the axial length of insulated pipe. Because L was not specified, we assume a unit length (1 ft), and so our results apply on a per-foot basis. Substituting gives

$$\text{Ra} = \frac{32.2(0.00185)(T_3 - 80)(1)^3}{16.88 \times 10^{-5} (0.859/3600)} = 1.48 \times 10^6 (T_3 - 80) \quad (\text{iii})$$

The convection coefficient is found by rearranging Equation 7.16 slightly:

$$\text{Nu} = \frac{hD}{k_f} = \left\{ 0.60 + \frac{0.387 \text{ Ra}^{1/6}}{\left(1 + \left[\frac{0.559}{\text{Pr}} \right]^{9/16} \right)^{8/27}} \right\}^2 \quad (7.16)$$

$$h = \frac{k_f}{2R_3} \left\{ 0.60 + \frac{0.387 \text{ Ra}^{1/6}}{\left(1 + \left[\frac{0.559}{\text{Pr}} \right]^{9/16} \right)^{8/27}} \right\}^2$$

Substituting gives

$$h = \frac{0.01516}{0.365} \left\{ 0.60 + \frac{0.387 \text{ Ra}^{1/6}}{\left(1 + \left[\frac{0.559}{0.708} \right]^{9/16} \right)^{8/27}} \right\}^2$$

$$\text{or } h = 0.0415 (0.6 + 0.321 \text{ Ra}^{1/6})^2 \quad (\text{iv})$$

The iterative procedure is as follows:

1. Assume T_3 ; then calculate the following
2. Rayleigh number Ra from Equation iii.
3. Convection coefficient h from Equation iv,
4. Resistance $R_{3\infty} = 1/hA = 1/(1.147h)$.
5. Total resistance $R_{1\infty} = R_{12} + R_{23} + R_{3\infty}$
 $R_{1\infty} = 0.000885 + 4.87 + R_{3\infty} = 4.87 + R_{3\infty}$.
6. Heat transferred $q = (T_1 - T_\infty)/R_{1\infty} = (250 - 80)/R_{1\infty} = 170/R_{1\infty}$.
7. Refined value of the surface temperature from Equation ii
 $T_3 = T_1 - q(R_{12} + R_{23}) = 250 - 4.87q$.
8. Repeat the calculations until convergence is achieved.

The following table summarizes the results:

T_3	Ra (Eq. iii)	h (Eq. iv)	$R_{3\infty} = 1/hA$	$R_{1\infty}$	q	T_3 (Eq. ii)
100°F	2.96×10^7	1.62	0.538	5.41	31.4	96.9
96.9	2.50×10^7	1.54	0.566	5.44	31.3	97.7
98	2.66×10^7	1.57	0.556	5.43	31.3	97.4

(close enough)

These calculations show that the heat transfer rate q is comparatively insensitive to large changes in temperature. For example, if the assumed value of T_3 is 200°F, then the heat transfer rate is 32.8 BTU/hr. For this example, the solution is

$$\underline{q = 31.3 \text{ BTU/hr}} \quad (\text{for each ft of pipe})$$

7.5 Optimum Thickness of Insulation

We can extend the results of the previous section to the problem of finding an optimum thickness of insulation for an insulated pipe or tube. The optimum thickness can be determined by straightforward calculation with suitable cost data. The procedure is to calculate heat loss for various insulation thicknesses. The annual cost of the heat loss for each thickness is expressed in terms of monetary units per heat loss unit (\$/BTU or \$/J). The annual installed cost of the insulation is found from the initial cost and the annual depreciation rate. The results are very similar to those developed earlier for optimum pipe diameter.

Figure 7.9 is a graph of costs for the optimum insulation thickness problem. The fixed charges increase with increasing insulation thickness. The cost associated with heat loss through the insulation decreases with increasing thickness. The total cost is the sum of these two costs, and it appears to have a minimum value that defines the optimum insulation thickness.

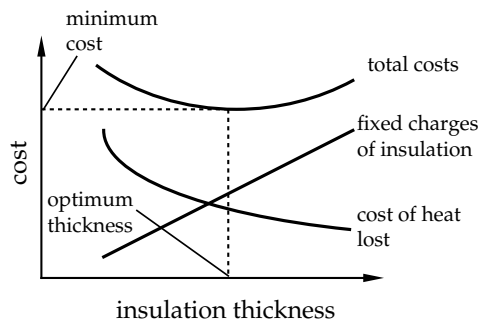


FIGURE 7.9. *Costs associated with the optimum insulation thickness problem.*

Critical Radius

Under certain conditions, adding insulation to a heated pipe actually increases the heat transfer loss. As insulation is added, the conduction resistance increases, but so does the surface area. The increased surface area causes more heat to be transferred away by convection. We investigate this effect by an example.

Consider a heated 1-nominal pipe ($OD = 3.340$ cm; $R = 1.67$ cm) covered with kapok insulation [$k = 0.035$ W/(m·K)]. The outside diameter of the pipe is maintained at 100°C . The insulation transfers heat to the environment, which is at 20°C , and the convection coefficient is assumed constant at 1.7 W/($\text{m}^2\cdot\text{K}$). We will determine the heat transferred to the ambient for various insulation thicknesses.

Figure 7.10 illustrates the insulated pipe. We define T_2 as the temperature of the surface exposed to the ambient. For no insulation, the heat loss is given by

$$q = \frac{T_2 - T_\infty}{1/hA_2} \quad (7.17a)$$

With $A_2 = 2\pi R_2 L$, we substitute to obtain the heat transfer per unit length as

$$\frac{q}{L} = \frac{T_2 - T_\infty}{1/h(2\pi R_2)} = 2\pi R_2 h(T_2 - T_\infty) \quad (7.17b)$$

Substituting,

$$\frac{q}{L} = 2\pi(1.67 \times 10^{-2})(1.7)(100 - 20)$$

$$\text{or } \frac{q}{L} = 14.3 \text{ W/m} \quad (\text{no insulation})$$

If insulation is added, we write

$$q = \frac{T_2 - T_\infty}{\frac{\ln(R_3/R_2)}{2\pi kL} + \frac{1}{hA_3}} \quad (7.18)$$

With $A_3 = 2\pi R_3 L$, Equation 7.18 becomes

$$q = \frac{T_2 - T_\infty}{\frac{\ln(R_3/R_2)}{2\pi kL} + \frac{1}{h2\pi R_3 L}} \quad (7.19a) \text{ or}$$

$$\frac{q}{L} = \frac{2\pi(T_2 - T_\infty)}{\frac{\ln(R_3/R_2)}{k} + \frac{1}{hR_3}} \quad (7.19b)$$

Substituting gives

$$\frac{q}{L} = \frac{2\pi(100 - 20)}{\frac{\ln(R_3/0.0167)}{0.035} + \frac{1}{1.7R_3}} \quad (7.20)$$

The outer radius R_3 has been left as a variable. Now the insulation thickness is

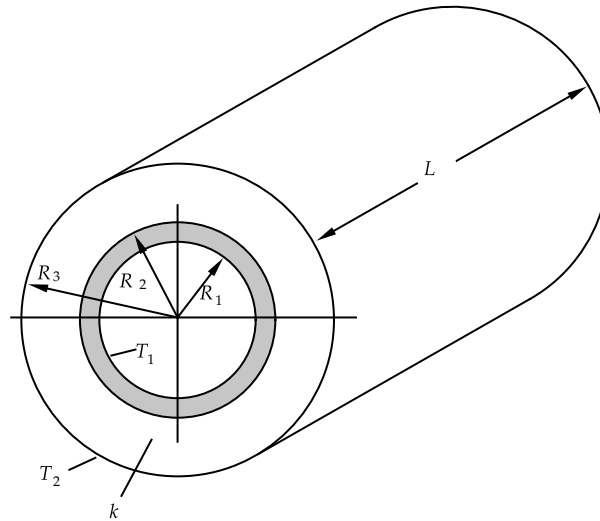


FIGURE 7.10. Sketch of an insulated pipe transferring heat to the environment.

$$t = R_3 - R_2 = R_3 - 0.0167 \quad (7.21)$$

or $R_3 = t + 0.0167$

We select values for the insulation thickness and calculate R_3 and q/L using Equation 7.20. The results are given in the following table and graphed in Figure 7.11.

As shown in the figure, adding insulation with a thickness of less than 1 cm causes an increase in the heat transfer rate. At 1 cm, the heat transfer rate just about equals that for no insulation. Beyond 1 cm thickness, the heat transfer rate decreases steadily. The maximum q/L occurs at an insulation thickness of about 0.5 cm.

To obtain a more general relationship between insulation thickness and heat transfer rate, we refer to Equation 7.19b:

$$\frac{q}{L} = \frac{2\pi(T_2 - T_\infty)}{\frac{\ln(R_3/R_2)}{k} + \frac{1}{hR_3}} \quad (7.19b)$$

Differentiating with respect to R_3 and setting the result equal to zero will give an equation for what is defined as the critical value of R_3 for which q/L is a maximum. This radius is called the **critical radius**. Differentiating Equation 7.19b, we get

thickness t in m	R_3 in m	q/L in W/m
0	0.016 7	14.3
0.005	0.021 7	14.5
0.01	0.026 7	14.2
0.015	0.031 7	13.6
0.02	0.036 7	13.0
0.025	0.041 7	12.5

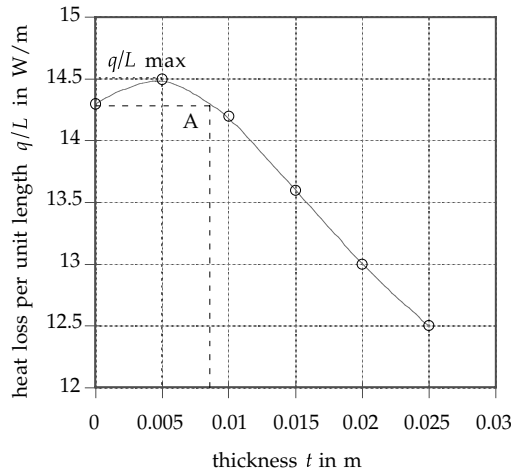


FIGURE 7.11. Heat transfer variation with insulation thickness.

$$\frac{d(q/L)}{dR_3} = - \frac{2\pi(T_2 - T_\infty) \left(\frac{1}{k} \frac{1}{R_3} - \frac{1}{hR_3^2} \right)}{\left(\frac{\ln(R_3/R_2)}{k} + \frac{1}{hR_3} \right)^2} = 0$$

Solving for the critical radius R_{cr} gives

$$R_{cr} = \frac{k}{h} \quad (7.22)$$

This is the value of R_3 (the outside radius of the insulation) that gives a maximum value for the heat transferred. For the example just discussed,

$$R_{\text{cr}} = \frac{0.035}{1.7} = 0.0206 \text{ m} = 2.06 \text{ cm}$$

The critical insulation thickness is

$$t_{\text{cr}} = R_{\text{cr}} - R_2 = 0.0206 - 0.0167 = 0.0039 \text{ m}$$

or $t_{\text{cr}} = 0.39 \text{ cm}$

The corresponding heat transfer per unit length is

$$\left. \frac{q}{L} \right|_{\text{max}} = \frac{2\pi(100 - 20)}{\frac{\ln(0.0206/0.0167)}{0.035} + \frac{1}{1.7(0.0206)}}$$

$$\left. \frac{q}{L} \right|_{\text{max}} = 14.5 \text{ W/m}$$

If the outer radius of insulation is less than k/h , then adding insulation increases the heat transferred. Conversely, if the outer radius of insulation is greater than k/h , then adding insulation decreases the heat transferred.

A more practical way to look at this problem is to find the insulation thickness that corresponds to the case of no insulation. Mathematically, the critical value of R_3 is found as k/h . Practically, however, we would rather know the thickness of insulation required for q/L to equal the no insulation case. Referring to Figure 7.10, we are seeking the insulation thickness that corresponds to point A. To find this value, we set Equation 7.17b equal to Equation 7.19a:

$$q = \frac{T_2 - T_\infty}{1/h2\pi R_2 L} = \frac{T_2 - T_\infty}{\frac{\ln(R_3/R_2)}{2\pi k L} + \frac{1}{h2\pi R_3 L}}$$

Canceling $2\pi L$ and the temperature difference gives

$$\frac{1}{hR_2} = \frac{\ln(R_3/R_2)}{k} + \frac{1}{hR_3}$$

Clearing fractions,

$$kR_3 = R_2 R_3 h \ln(R_3/R_2) + kR_2$$

Rearranging and solving for R_3 ,

$$R_3 = \frac{R_2}{1 - (R_2 h/k) \ln (R_3/R_2)} \quad (7.23)$$

For the example previously discussed,

$$R_3 = \frac{0.0167}{1 - (0.0167(1.7)/0.035)[\ln (R_3/0.0167)]}$$

At first glance, it would appear that this equation could be solved iteratively by substituting a value for R_3 into the right-hand side to obtain an improved value, which again is substituted, and the procedure repeated until convergence is achieved. Though elegant, this method does not work. A more successful approach is merely to assume a value of R_3 and substitute it into the right-hand side. The result is compared to the left-hand side and the difference is taken. This procedure is repeated until the difference is zero or very small. The following table summarizes the calculations:

Trial	Assumed R_3 in m	RHS in m	$R_3 - \text{RHS}$
1	0.03	0.0318	− 0.0018
2	0.029	0.0302	−0.0012
3	0.028	0.0287	−0.0007
4	0.026	0.0273	0.000056
5	0.025	0.0260	0.00017
6	0.0255	0.0248	0.0006
7	0.0257	0.0254	0.0002
8	0.0258	0.0258	0.000004 (close enough)

This method takes only a few iterations, and it gives us the radius of insulation required so that q/L with insulation exceeds q/L for no insulation. We define this radius as the **practical radius**. The corresponding thickness for this example then is

$$t = R_3 - R_2 = 0.0258 - 0.0167 = 0.0091 \text{ m}$$

or $t = 0.91 \text{ cm}$

It is this thickness that must be exceeded to yield a decrease in heat loss.

The effect of increasing heat transfer by adding insulation occurs usually for small pipe diameters and low values of the convection coefficient. For steam flowing in a pipe, it is usually not desirable to have a loss of energy through the pipe wall. Therefore, the critical (or practical) thickness should be calculated and the pipe insulated properly. On the

other hand, current flowing through a wire could generate heat and raise the temperature of the wire. Insulation in this case could be added in order to help dissipate the heat and to keep the wire from overheating and from short-circuiting.

7.6 Summary

In this chapter, we have reviewed simple, one-dimensional conduction heat transfer in planar and cylindrical coordinates. We defined the concept of resistance to heat transfer and derived an equation for the conduction problem. Convection heat transfer was introduced, as was the pertinent fluid properties for convection problems. The concept of a resistance to heat transfer was extended to convection problems, and several example problems were solved.

7.7 Problems

One-Dimensional Planar Conduction

1. An outdoor grill is made of masonry brick. Consider one wall of the grill made of a single layer of brick that is 4 in. thick. During operation, the inside surface temperature reaches 180°F and the outside surface temperature reaches 80°F. Determine the heat transferred through the brick wall.
2. A safe is made of stainless steel with walls that are 6 cm thick. It is to be designed so that, under conditions of fire, the heat transferred through the wall is to be no greater than 100 BTU/(hr·ft²). The temperature of the outside surface of the safe will reach 400°C. Under these conditions, what will the temperature of the inside surface be?
3. A furnace wall is to be made of two materials placed in series—common brick and masonry brick. The heat loss through the wall is to be reduced to 1 000 W/m². The common brick is 8 in. thick, and its left surface will reach 1500°F. The masonry brick is placed next to the common brick, and the right surface of the masonry brick will reach 100°F. Determine the thickness of the masonry brick required.
4. Figure P7.4 shows the one-dimensional profile of two materials. The temperature of the left face is the same for both materials, the heat flow rate in both cases is identical, and the thicknesses are equal. Which material has the higher thermal conductivity? Why?
5. Referring to the composite wall of Figure 7.3, which material has the highest thermal conductivity for constant heat flux through the wall?
6. A wall of a kitchen oven is made of three materials placed in series—sheet metal, insulation, and plywood. The sheet metal is 0.040 in. thick. The plywood is 3/4 in. thick, and the insulation thickness is to be determined. For economic reasons, the oven wall should transfer about 60 BTU/(hr·ft²) of heat when the inside surface temperature of the sheet metal is 500°F. For safety reasons, the

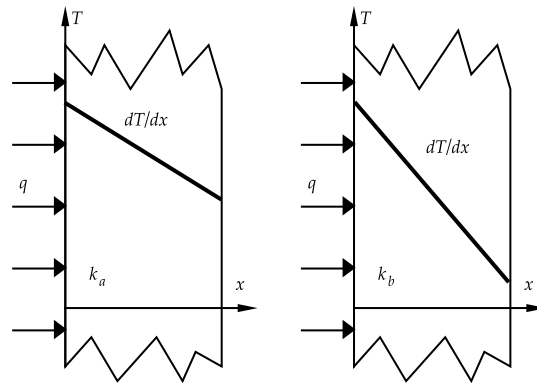


FIGURE P7.4.

plywood outside surface temperature should not exceed 100°F. What is the minimum thickness of fiberglass insulation required to meet these criteria?

7. A furnace wall has an inner layer of silica brick [$k = 1.07 \text{ W}/(\text{m}\cdot\text{K})$] and an outer layer of masonry brick [$k = 0.66 \text{ W}/(\text{m}\cdot\text{K})$], as indicated in Figure P7.7. Furnace gas conditions are $T_i = 320^\circ\text{C}$, $h_i = 45 \text{ W}/(\text{m}^2\cdot\text{K})$, and the outside air has $T_o = 25^\circ\text{C}$, $h_o = 25 \text{ W}/(\text{m}^2\cdot\text{K})$. It is desired to limit the heat loss to $800 \text{ W}/\text{m}^2$, and the interface temperature T_2 should be no more than 150°C . Calculate the thickness L required to do this.
8. Figure P7.8 shows a temperature profile for heat transfer through a 12-cm-thick piece of aluminum [$k = 164 \text{ W}/(\text{m}^2\cdot\text{K})$]. The convection coefficient on the right face is $250 \text{ W}/(\text{m}^2\cdot\text{K})$. Determine the temperature T_∞ .

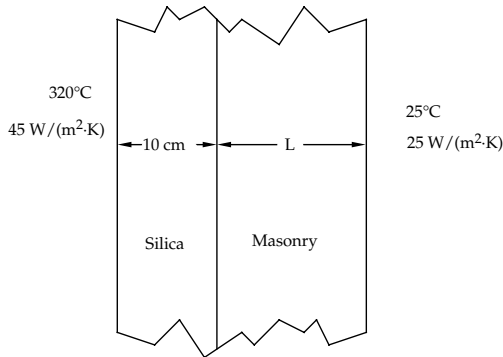


FIGURE P7.7.

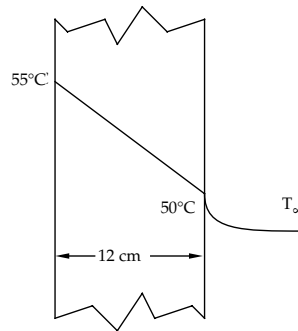


FIGURE P7.8.

One-Dimensional Cylindrical Conduction

9. A 4-nom sch 40 steel pipe conveys steam that maintains the pipe inside surface temperature at 450°F . The pipe has a 1-in.-thick layer of kapok insulation. The outside surface temperature of the kapok is 70°F . Determine the heat flow through the pipe.

10. An aluminum rod is 1 cm in diameter and is used as a handle for a grill cover. The aluminum rod temperature reaches a uniform 80°C. The rod is to be covered with a high-temperature plastic, such that the outside surface of the plastic is 20°C for a heat flow rate of 50 W. If the thickness of the plastic is 5 mm, determine the thermal conductivity that the plastic must have.
11. Suppose that the pipe of Example 7.5 is not insulated. Rework the problem for no insulation, and compare the heat flow rate of the insulated to the uninsulated pipe.
12. Example 7.5 was solved for a pipe that is laid horizontally. Suppose the same pipe is oriented vertically. Rework the problem for a vertical configuration, and compare the results of the two solutions. For a heated vertical cylinder losing heat to the environment, we have

$$\text{Nu} = 0.6 \left(\text{Ra} \frac{D}{L} \right)^{0.25} \quad \text{Ra} \frac{D}{L} \geq 10^4$$

$$\text{Nu} = 1.37 \left(\text{Ra} \frac{D}{L} \right)^{0.16} \quad 0.05 \leq \text{Ra} \frac{D}{L} \leq 10^4$$

$$\text{Nu} = 0.93 \left(\text{Ra} \frac{D}{L} \right)^{0.05} \quad \text{Ra} \frac{D}{L} \leq 0.05$$

$$\text{Ra} = \frac{g\beta(T_s - T_\infty)L^3}{\nu\alpha}; \quad \text{Pr} = \frac{\nu}{\alpha}$$

and $\beta = 1/T_\infty = \text{coefficient of thermal expansion}$.

13. A steam pipe is made of 2-nominal schedule 40 low carbon steel. It is covered by glass fiber insulation that is 1 in. thick. The glass fiber is covered with aluminum foil that is 0.008 in. thick. The steam temperature inside the pipe is 300°F, while the outside temperature is 80°F. The convection coefficient between the steam and the pipe is 25 BTU/(hr·ft²·°R), and the convection coefficient between the aluminum foil and the air is 12 BTU/(hr·ft²·°R). The pipe is 30 ft long. Determine the heat loss in BTU/hr.

Conduction-Convection Problems

14. Consider a vertical wall made of 2-in.-thick stainless steel. The left face of the stainless steel is maintained at a temperature of 300°F. The right face convects heat away to the surrounding air, whose temperature is 75°F. Determine the heat transferred through the wall and the temperature of the right face.
15. A vertical wall is made up of fiberglass (5 cm thick) attached to common brick (10 cm thick). The uninsulated side of the brick is at -40°C during winter months. The insulation receives energy by convection from the surrounding air, whose temperature is 20°C. Determine the interface temperature between the two materials under these conditions.
16. A vertical pane of window glass of a house is 1/2 in. thick and 3 ft tall. On the outside, the air temperature is 95°F, while inside the air temperature is 70°F. Determine the heat flow through the glass for these conditions.

17. A single pane glass window of a heated house is shown in cross section in the accompanying figure. Outside the window, the ambient temperature is 0°C . Inside the dwelling, the temperature is 22°C . A temperature profile is given in the figure. The glass thickness is 5 mm and the window height is 60 cm. Determine the heat lost through the glass per unit area. Use the following properties for air on either side of the glass:

$$\begin{aligned}\rho &= 1.177 \text{ kg/m}^3 & k_f &= 0.02624 \text{ W/(m}\cdot\text{K)} \\ c_p &= 1005.7 \text{ J/(kg}\cdot\text{K)} & \nu &= 15.68 \times 10^{-6} \text{ m}^2/\text{s} \\ \beta &= 1/(295 \text{ K}) = 0.00338/\text{K}\end{aligned}$$

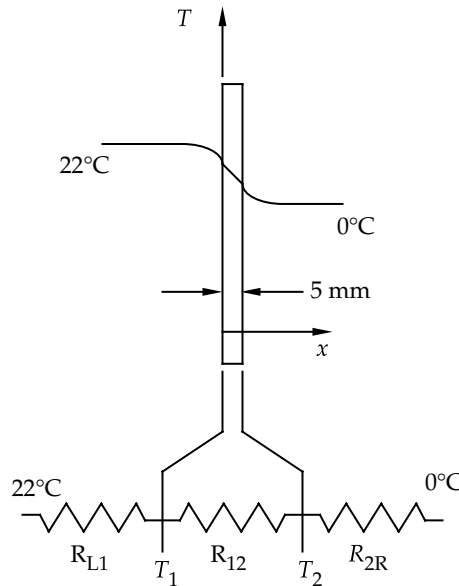


FIGURE P7.17.

18. A new building material made of plywood with styrofoam insulation glued to it is to be tested. The plywood is 2 cm thick, and the styrofoam is 7 cm thick. The test is to be performed while the sample is in a vertical configuration. On the plywood side, the sample is exposed to air at 35°C . On the styrofoam side, the sample is in contact with air at 0°C . Calculate the interface temperature between the two materials and the heat transferred through the wall.
19. A refrigerator wall consists of low carbon steel [$k = 43 \text{ W/(m}\cdot\text{K)}$] 1 mm thick on the outside, 2-mm-thick polystyrene [$k = 0.157 \text{ W/(m}\cdot\text{K)}$] inside, with 2-cm thick expanded cork [$k = 0.036 \text{ W/(m}\cdot\text{K)}$] in between as an insulation. Outside the refrigerator, the air temperature is 25°C , while inside, the air temperature is 10°C . Determine the heat flow rate through the wall per unit area. Use the following equation (known as the Churchill-Chu equation) to calculate Nusselt number and convection coefficient, applicable to both sides of the wall:

$$\text{Nu} = \frac{hL}{k_f} = 0.68 + \frac{0.67 \text{ Ra}^{1/4}}{\left[1 + \left(\frac{0.492}{\text{Pr}}\right)^{9/16}\right]^{4/9}}$$

where $Ra = Gr Pr = \frac{g\beta(T_w - T_\infty)L^3}{\nu\alpha}$

Use the following properties for air on either side of the wall:

$$\begin{aligned}\rho &= 1.177 \text{ kg/m}^3 & k_f &= 0.02624 \text{ W/(m}\cdot\text{K)} \\ c_p &= 1005.7 \text{ J/(kg}\cdot\text{K)} & \nu &= 15.68 \times 10^{-6} \text{ m}^2/\text{s} \\ \beta &= 1/(295 \text{ K}) = 0.00338/\text{K}\end{aligned}$$

20. Oak plywood that is 2.5 cm thick is used temporarily in a window to replace broken glass. On one side of the plywood, the air is at 0°C . On the other side, the wood surface is maintained at 25°C . The descriptive sketch for this system is shown in the accompanying diagram. What is the temperature of the plywood on the 0°C side?

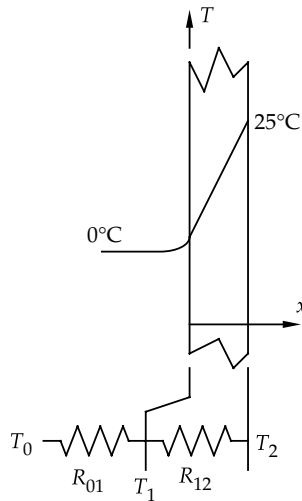


FIGURE P7.20.

21. A 3-nom sch 40 stainless steel pipe carries high temperature oil from a cracking tower, where crude oil is separated into components, to a packaging operation. The oil maintains the inside surface of the pipe at 180°F . The air temperature surrounding the pipe is at 60°F . Determine the heat flow through the pipe wall for the following: (a) the pipe is uninsulated, and (b) the pipe is insulated with a 1-in. thickness of kapok.
22. A horizontal pipe made of stainless steel (2-nom sch 40) carries oil (same properties as unused engine oil) whose temperature is 100°C and whose velocity is 0.1 m/s . Heat is convected from the steam to the inside surface of the stainless steel. The pipe is insulated with glass fiber that is 2 cm thick. Air surrounding the pipe is at 25°C . Determine the heat transferred from the oil through the pipe and insulation to the air. For laminar flow of fluid through a cylinder, we have

$$Nu = \frac{hD}{k_f} = 1.86 \left(\frac{D Re Pr}{L} \right)^{1/3}$$

$$\text{Re} = \frac{VD}{\nu} < 2\,200$$

$$0.48 < \text{Pr} = \frac{\nu}{\alpha} < 16\,700 \quad \mu \text{ changes moderately with temperature}$$

Critical Radius

23. Calculate the critical radius for the insulation of Example 7.5.
24. A pipe made of 1½-nominal steel conveys steam. Its outside surface is maintained at 240°F. The pipe is to be insulated with a material whose thermal conductivity is the same as that for cardboard. The outside air is at 70°F, and the convection coefficient between the insulation and the air is constant at 1.5 BTU/(hr·ft²·°R). Graph heat loss as a function of insulation thickness.
25. A 1-nominal pipe is covered with glass wool insulation. The outside wall temperature of the pipe is 120°C, and the insulation is exposed to air at 25°C. The convection coefficient between the insulation and the air is 1 W/(m²·K). Graph q/L as a function of the insulation thickness.
26. A current carrying wire is 1 mm in diameter. It carries 250 amps at 115 V and is made of copper (assume properties the same as pure copper). The wire is insulated with a material whose properties are the same as those of ebonite rubber. The ambient air is at 18°C. For maximum heat transfer to the ambient, how thick should the insulation be on this wire? Assume a convection coefficient of 1 W/(m²·K).
27. A 1½-nominal pipe is covered with an insulation made of a blend of fibers whose properties are the same as asbestos. The outside surface area of the pipe is at 150°F, and the convection coefficient between the insulation and the air is 1.2 BTU/(hr·ft²·°R). Determine the critical radius and practical insulation thickness if the ambient temperature is 40°F.
28. How does the value of R_3 affect the heat transfer rate in Example 7.3? Re-solve that example for insulation thicknesses that vary from 0 to 6 cm. Construct a graph of q/L versus insulation thickness.

CHAPTER 8 Double Pipe Heat Exchangers

A heat exchanger is a device used to transfer heat from one fluid to another. There are many different types of heat exchangers, including **double pipe**, **shell and tube**, **cross flow**, and **plate and frame**. In this text, we will examine these types and write equations to predict their performance. The methods presented here can be applied to the performance appraisal of any heat exchanger.

8.1 The Double Pipe Heat Exchanger

A **double pipe heat exchanger** (also known as a **concentric tube heat exchanger**) consists of two concentric, different-diameter tubes with fluid flowing in each, as indicated in Figures 8.1 and 8.2. If the two fluids travel in opposite directions, as illustrated in Figure 8.1, the exchanger is a **counterflow** type. If the fluids travel in the same direction, as shown in Figure 8.2, **parallel flow** (or **unidirectional flow**) exists. The same apparatus is used for either flow configuration. Also shown in Figures 8.1 and 8.2 are temperature versus distance graphs. These temperature profiles will be discussed in this chapter.

The objective in using a heat exchanger is to transfer as much heat as possible for as small a cost as necessary. In many cases involving the sizing or selection of a particular exchanger, all that will be known are the physical properties of the fluids and their inlet temperatures. If the tube sizes and areas (surface and cross-sectional areas) are known, then the amount of heat transferred can be readily calculated. Conversely, if a heat transfer rate is specified, then the required surface area can be determined.

As indicated in Figures 8.1 and 8.2, a double pipe heat exchanger will bring together two fluid streams. As each fluid passes through, its temperature changes. Moreover, as temperature changes, the *local* convection coefficient between either fluid and the wall changes. Our interest here, however, is in the *overall* heat transfer coefficient and not necessarily with the instantaneous or local values.

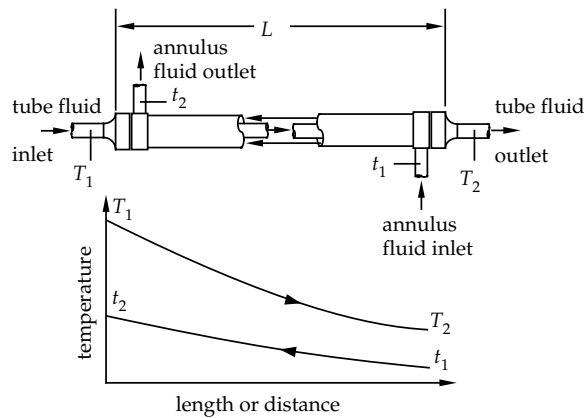


FIGURE 8.1. A double pipe heat exchanger set up in counterflow and the corresponding temperature profile.

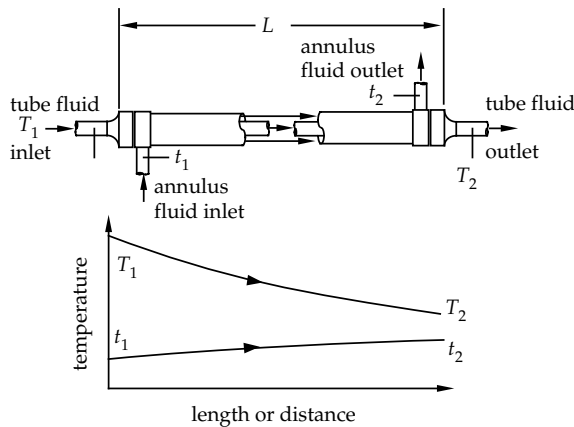


FIGURE 8.2. A double pipe heat exchanger set up in parallel flow and the corresponding temperature profile.

Figure 8.3 shows a cross section of the double pipe heat exchanger and the associated temperature variation at any axial location. For purposes of discussion, we assume that heat is transferred from the fluid within the tube to the fluid within the annulus. Also shown in Figure 8.3 are the resistances through which heat passes. The sum of the resistances is

$$\Sigma R = R_{12} + R_{23} + R_{34}$$

$$\text{or } \Sigma R = \frac{1}{h_i A_i} + \frac{1}{2\pi k L} \ln \frac{OD_p}{ID_p} + \frac{1}{h_o A_o} \quad (8.1)$$

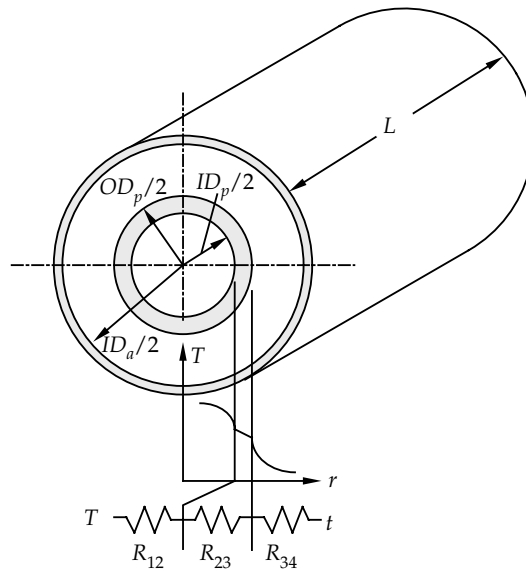


FIGURE 8.3. Temperature profile and resistances for heat flow within a double pipe heat exchanger.

where h_i is the convection coefficient between the fluid in the tube and the tube wall and h_o applies between the fluid in the annulus and the tube. As shown in the last chapter, temperature drop across a thin-walled metal pipe is virtually negligible. This is true also for a tube. The implication here is that the second resistance $[(1/2\pi kL)(\ln (OD_p/ID_p))]$ may be neglected in Equation 8.1, with small error. Also, the area associated with the convection coefficient h_i is the *inside* surface area of the tube A_i . The area associated with the convection coefficient h_o is the *outside* surface area of the tube A_o . It is necessary in the analysis that h_i and h_o be referred to the same surface. The surface areas are:

$$A_i = \pi ID_p L$$

$$A_o = \pi OD_p L$$

It is standard practice to base the resistance on the outside surface area A_o . Multiplying Equation 8.1 by A_o , we get

$$A_o \Sigma R = \frac{1}{U_o} = \frac{\pi OD_p L}{h_i \pi ID_p L} + \frac{1}{h_o}$$

$$A_o \Sigma R = \frac{1}{U_o} = \frac{OD_p}{h_i ID_p} + \frac{1}{h_o} \quad (8.2)$$

in which the overall heat transfer coefficient U_o based on A_o has been introduced. The overall heat transfer coefficient has the same dimensions as h , namely, $[F \cdot T / (T \cdot L^2 \cdot t)]$ $[BTU / hr \cdot ft^2 \cdot ^\circ R]$ or $W / (m^2 \cdot K)$. (Note also that $A_i U_i = A_o U_o$, and the development could proceed using U_i .)

The heat transferred within the heat exchanger equals the product of the overall heat transfer coefficient U_o , the outside surface area of the inner tube A_o , and a temperature difference. Thus

$$q = U_o A_o \Delta t \quad (\text{or } q = U_i A_i \Delta t) \quad (8.3)$$

The overall heat transfer coefficient can be calculated if the film coefficients are known, and they can be calculated by using the appropriate correlations for forced convection.

However, Equation 8.3 contains Δt , which is an as-yet-undetermined temperature difference that applies to the entire exchanger. We turn our attention to evaluating Δt . At any z -location within the exchanger (Figures 8.1 and 8.2), the temperature difference is $T - t$. This difference varies throughout the exchanger. It is advantageous to use inlet and outlet temperatures to evaluate Δt rather than use local values, because these temperatures are easily measured. So we seek an expression for Δt in terms of inlet and outlet temperatures rather than local values.

Although heat is transferred in either parallel or counterflow configurations, the temperature difference Δt is not the same for both cases. We will begin with a counterflow arrangement and derive an expression for the temperature difference. We assume the following:

1. The overall heat transfer coefficient U_o (based on A_o) is constant over the entire length of the exchanger.
2. Fluid properties are constant.
3. Steady flow exists.
4. There are no phase changes in the exchanger.
5. There are no heat losses. All heat lost by the warmer fluid is gained by the cooler fluid.

The magnitude of the heat transferred at any location along the length of the heat exchanger (Figure 8.1) is given by

$$dq = U_o (T - t) dA_o \quad (8.4)$$

The heat lost by the warmer fluid over a differential length is

$$dq = \dot{m}_w C_{pw} dT \quad (8.5)$$

where $\dot{m}_w$ is the mass flow rate of the warmer fluid and C_{pw} is its specific heat. For the cooler fluid, the magnitude of the heat gained is

$$dq = \dot{m}_c C_{pc} dt \quad (8.6)$$

We next set Equations 8.5 and 8.6 equal to each other and integrate from the cold fluid inlet end of the exchanger to any z location within:

$$\int_{T_2}^T \dot{m}_w C_{pw} dT = \int_{t_1}^t \dot{m}_c C_{pc} dt$$

Note that at the cold fluid inlet where the temperature is t_1 , the warmer fluid temperature is T_2 . Integrating,

$$\dot{m}_w C_{pw} (T - T_2) = \dot{m}_c C_{pc} (t - t_1)$$

Rearranging and solving for the temperature of the warmer fluid, we have

$$T = T_2 + \frac{\dot{m}_c C_{pc}}{\dot{m}_w C_{pw}} (t - t_1) \quad (8.7a)$$

If we integrated to the entire length L of the exchanger, we would get

$$T_1 = T_2 + \frac{\dot{m}_c C_{pc}}{\dot{m}_w C_{pw}} (t_2 - t_1) \quad (8.7b)$$

Substituting Equation 8.7a into Equation 8.4, we get

$$dq = U_o \left(T_2 + \frac{\dot{m}_c C_{pc}}{\dot{m}_w C_{pw}} (t - t_1) - t \right) dA_o$$

Having eliminated the warmer fluid temperature T as a variable, we next set the preceding equation equal to Equation 8.6 for the cooler fluid:

$$\dot{m}_c C_{pc} dt = U_o \left(T_2 + \frac{\dot{m}_c C_{pc}}{\dot{m}_w C_{pw}} (t - t_1) - t \right) dA_o$$

The cooler fluid temperature t and the surface area are the only differential areas. Separating terms and integrating,

$$\int_0^{A_o} \frac{U_o dA_o}{\dot{m}_c C_{pc}} = \int_{t_1}^{t_2} \frac{dt}{\left(T_2 + \frac{\dot{m}_c C_{pc}}{\dot{m}_w C_{pw}} (t - t_1) - t \right)}$$

or

$$\frac{U_o A_o}{\dot{m}_c C_{pc}} = \left[\frac{1}{(\dot{m}_c C_{pc} / \dot{m}_w C_{pw}) - 1} \right] \ln \left[\frac{T_2 - \frac{\dot{m}_c C_{pc}}{\dot{m}_w C_{pw}} t_1 + \left(\frac{\dot{m}_c C_{pc}}{\dot{m}_w C_{pw}} - 1 \right) t_2}{T_2 - \frac{\dot{m}_c C_{pc}}{\dot{m}_w C_{pw}} t_1 + \left(\frac{\dot{m}_c C_{pc}}{\dot{m}_w C_{pw}} - 1 \right) t_1} \right]$$

Solving Equation 8.7b for T_2 , substituting into the previous equation, and (after a lengthy algebraic exercise) simplifying, we have

$$\frac{U_o A_o}{\dot{m}_c C_{pc}} = \left[\frac{1}{(\dot{m}_c C_{pc} / \dot{m}_w C_{pw}) - 1} \right] \ln \frac{T_1 - t_2}{T_2 - t_1} \quad (8.8)$$

Rearranging Equation 8.7b, we get

$$\frac{\dot{m}_c C_{pc}}{\dot{m}_w C_{pw}} = \frac{T_1 - T_2}{t_2 - t_1}$$

Substituting into Equation 8.8,

$$\begin{aligned} \frac{U_o A_o}{\dot{m}_c C_{pc}} &= \frac{1}{[(T_1 - T_2)/(t_2 - t_1) - 1]} \ln \frac{T_1 - t_2}{T_2 - t_1} \\ &= \frac{t_2 - t_1}{(T_1 - t_2) - (T_2 - t_1)} \ln \frac{T_1 - t_2}{T_2 - t_1} \end{aligned}$$

Rearranging,

$$\dot{m}_c C_{pc}(t_2 - t_1) = U_o A_o \frac{(T_1 - t_2) - (T_2 - t_1)}{\ln [(T_1 - t_2)/(T_2 - t_1)]} \quad (8.9a)$$

The left-hand side is recognized as the heat gained by the cooler fluid. So

$$q = U_o A_o \frac{(T_1 - t_2) - (T_2 - t_1)}{\ln [(T_1 - t_2)/(T_2 - t_1)]} \quad (\text{counterflow}) \quad (8.9b)$$

Comparing to Equation 8.3

$$q = U_o A_o \Delta t \quad (8.3)$$

we see that the temperature difference or driving potential for heat transfer in counterflow within a double pipe heat exchanger is

$$\Delta t = \frac{(T_1 - t_2) - (T_2 - t_1)}{\ln [(T_1 - t_2)/(T_2 - t_1)]} = LMTD \quad (\text{counterflow}) \quad (8.10)$$

Equation 8.10 is often called the *log mean temperature difference*, abbreviated as LMTD. It is sometimes written as

$$LMTD = \frac{\Delta t_2 - \Delta t_1}{\ln (\Delta t_2 / \Delta t_1)}$$

in which Δt_2 is the temperature difference between the two fluids at one end of the exchanger, and Δt_1 is the difference at the other end. For counterflow,

$$\left. \begin{array}{l} \Delta t_2 = T_1 - t_2 \\ \Delta t_1 = T_2 - t_1 \end{array} \right\} \quad \text{counterflow}$$

We can repeat the preceding derivation for a parallel flow arrangement. The basic equations are the same as those just written. The results are

$$q = U_o A_o \frac{(T_1 - t_1) - (T_2 - t_2)}{\ln [(T_1 - t_1)/(T_2 - t_2)]} \quad (\text{parallel flow}) \quad (8.11)$$

$$LMTD = \frac{\Delta t_2 - \Delta t_1}{\ln (\Delta t_2 / \Delta t_1)} = \frac{(T_1 - t_1) - (T_2 - t_2)}{\ln [(T_1 - t_1)/(T_2 - t_2)]} \quad (8.12)$$

$$\left. \begin{array}{l} \Delta t_1 = T_1 - t_1 \\ \Delta t_2 = T_2 - t_2 \end{array} \right\} \quad \text{parallel flow}$$

Note the difference between these results and those for counterflow. In these and in the equations that follow, uppercase “ T ” signifies the warmer fluid, and a “1” subscript denotes an inlet condition. Lowercase “ t ” refers to the cooler fluid, and a “2” subscript denotes an exit condition.

Comparison of Counterflow and Parallel Flow Configurations

At first glance, it appears that counterflow and parallel flow arrangements should yield equal heat transfer rates. To investigate this point, it is instructive to use the equations to calculate the log mean temperature difference for several cases.

EXAMPLE 8.1. A fluid with a temperature of 100°C enters a double pipe heat exchanger and is cooled to 75°C by a second fluid entering at 25°C and heated to 40°C. Calculate the log mean temperature difference for counterflow and parallel flow.

Solution: Given the following temperatures,

$$\begin{array}{ll} T_1 = 100^\circ\text{C} & t_1 = 25^\circ\text{C} \\ T_2 = 75^\circ\text{C} & t_2 = 40^\circ\text{C} \end{array}$$

we substitute into Equation 8.10 to obtain

$$LMTD = \frac{(100 - 40) - (75 - 25)}{\ln [(100 - 40)/(75 - 25)]} = 54.8^\circ\text{C} \quad (\text{counterflow})$$

Also,

$$LMTD = \frac{(100 - 25) - (75 - 40)}{\ln [(100 - 25)/(75 - 40)]} = 52.5^\circ\text{C} \quad (\text{parallel flow})$$

In an exchanger with these temperatures, we would use Equation 8.3 to calculate the heat transfer rate. If the overall heat transfer coefficient is constant for both flow arrangements, then the parallel flow configuration requires a greater surface area A_o than the counterflow configuration to transfer the same energy.

EXAMPLE 8.2. A double pipe heat exchanger is to be used to exchange heat between two fluids such that their outlet temperatures are equal; specifically, a warmer fluid is cooled from 300°F to 200°F, while the cooler fluid is heated from 150°F to 200°F. Calculate the log mean temperature difference for counterflow and for parallel flow.

Solution: Given the following temperatures,

$$\begin{array}{ll} T_1 = 300^\circ\text{F} & t_1 = 150^\circ\text{F} \\ T_2 = 200^\circ\text{F} & t_2 = 200^\circ\text{F} \end{array}$$

we substitute into Equation 8.10 to get

$$LMTD = \frac{(300 - 200) - (200 - 150)}{\ln [(300 - 200)/(200 - 150)]} = 72.5^\circ\text{F} \quad (\text{counterflow})$$

Likewise, Equation 8.12 gives

$$LMTD = \frac{(300 - 150) - (200 - 200)}{\ln [(300 - 150)/(200 - 200)]} = 0^\circ\text{F} \quad (\text{parallel flow})$$

So, in parallel flow, the surface area of the heat exchanger would have to be infinite in order to make the outlet temperatures equal (as per Equation 8.3, $q = U_o A_o \Delta t$). This is not feasible, so we conclude that there is a distinct thermal disadvantage to using parallel flow. Consequently, unless specified otherwise, all calculations made on double pipe heat exchangers will be performed using counterflow. Counterflow is considered the “best possible” configuration.

EXAMPLE 8.3. Steam, at saturated vapor conditions and with a temperature of 100°C , enters a double pipe heat exchanger. The steam is condensed by a second fluid that enters at 25°C . The condensation process occurs isothermally, so the outlet temperature of the condensate is also at 100°C . The outlet temperature of the coolant is 50°C . Calculate the log mean temperature difference for counterflow and for parallel flow.

Solution: Given the following temperatures,

$$\begin{array}{ll} T_1 = 100^\circ\text{C} & t_1 = 25^\circ\text{C} \\ T_2 = 100^\circ\text{C} & t_2 = 50^\circ\text{C} \end{array}$$

we substitute into Equation 8.10 to get

$$LMTD = \frac{(100 - 50) - (100 - 25)}{\ln [(100 - 50)/(100 - 25)]} = 61.7^\circ\text{F} \quad (\text{counterflow})$$

Likewise,

$$LMTD = \frac{(100 - 25) - (100 - 50)}{\ln [(100 - 25)/(100 - 50)]} = 61.7^\circ\text{F} \quad (\text{parallel flow})$$

So, when one of the fluids in the exchanger is changing phase and has equal inlet and outlet temperatures (i.e., no superheat or subcooling), then the log mean temperature difference for counterflow equals that for parallel flow.

Referring to Figures 8.1 and 8.2, note that the outlet temperatures in parallel flow can only approach each other. For counterflow, the outlet temperature of the cooler fluid can be made to exceed the outlet temperature of the warmer fluid. The counterflow apparatus has a much greater ability to transfer heat than does the parallel flow apparatus.

A double pipe heat exchanger is easily made using standard pipe or tubing with appropriate fittings, or one can be purchased. It provides an inexpensive way to transfer heat between two fluids having relatively low flow rates. Double pipe heat exchangers can be assembled in any desired length, and often two are used in what is known as a “hairpin” arrangement, as sketched in Figure 8.4. Many of these hairpins can be assembled to yield a very large surface area for the transfer of heat.

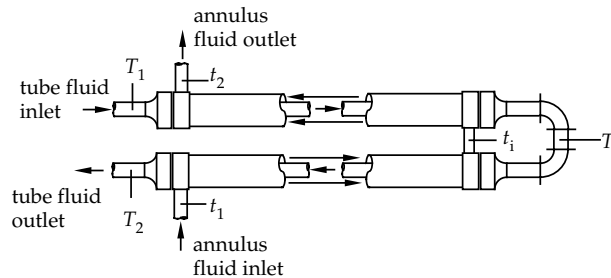


FIGURE 8.4. Schematic of a hairpin heat exchanger.

8.2 Analysis of Double Pipe Heat Exchangers

Equation 8.3 relates the heat transferred in a double pipe heat exchanger to the overall heat transfer coefficient, the outside surface area of the inner tube, and to the log mean temperature difference (LMTD):

$$q = U_o A_o \Delta t \quad (8.3)$$

Equation 8.2 defines the heat transfer coefficient U_o in terms of convection coefficients h_i and h_o for the inside and outside surfaces of the inner tube. Convection coefficients are calculated from equations for Nusselt numbers. The equations to be used in the analysis were developed for flow in a circular duct. In order to find the convection coefficient for flow in an annulus, we use the same equations but modify them for the change in geometry. Modifications will involve the use of a characteristic length to replace diameter in the equations. For friction factor calculations, we defined hydraulic diameter as

$$D_h = \frac{4 \times \text{flow area}}{\text{friction perimeter}} = \frac{4\pi(ID_a^2 - OD_p^2)}{4\pi(ID_a + OD_p)}$$

$$\text{or } D_h = ID_a - OD_p \quad (8.13)$$

where the diameters ID_a and OD_p are defined in Figure 8.6.

The above definition of hydraulic diameter applies to friction calculations. In the same way, we define an **equivalent diameter** D_e as

$$D_e = \frac{4 \times \text{flow area}}{\text{heat transfer perimeter}} = \frac{4\pi(ID_a^2 - OD_p^2)}{4\pi OD_p}$$

$$\text{or } D_e = \frac{ID_a^2 - OD_p^2}{OD_p} \quad (8.14)$$

The heat transfer perimeter is the outside surface area of the inner tube. Note carefully the difference in hydraulic and equivalent diameters. Once these diameters are calculated, they can be used in equations for Reynolds and Nusselt numbers. The equations are:

Sieder-Tate Equation for Laminar Flow

$$Nu = \frac{hD}{k_f} = 1.86 \left(\frac{DRePr}{L} \right)^{1/3} \quad (8.15)$$

$$Re = \frac{VD}{\nu} < 2\,200 \quad D = ID_p \text{ if cross section is tubular}$$

$$D = D_e \text{ if cross section is annular}$$

$$0.48 < Pr = \frac{\nu}{\alpha} < 16\,700 \quad \mu \text{ changes moderately with temperature}$$

Properties evaluated at average fluid temperature $[= (\text{inlet} + \text{outlet})/2]$

Modified Dittus-Boelter Equation for Turbulent Flow

$$Nu = \frac{hD}{k_f} = 0.023(Re)^{4/5} Pr^n \quad (8.16)$$

$$n = 0.4 \text{ if fluid is being heated} \quad D = ID_p \text{ if cross section is tubular}$$

$$n = 0.3 \text{ if fluid is being cooled} \quad D = D_e \text{ if cross section is annular}$$

$$Re = \frac{VD}{\nu} \geq 10\,000; \quad 0.7 \leq Pr = \frac{\nu}{\alpha} \leq 160; \quad L/D \geq 60$$

Properties evaluated at average fluid temperature $[= (\text{inlet} + \text{outlet})/2]$

Transitional Flow

When the Reynolds number falls between 2 200 and 10 000, then interpolation can be used to find the Nusselt number. Equation 8.15 is used to

find Nu at $Re = 2\,200$. Equation 8.16 is used to find Nu at $Re = 10\,000$. Interpolation gives a usable *estimate* of the Nusselt number for the transition Reynolds number. It is recommended, however, that transitional flow be avoided in practice due to the uncertainty in modeling it.

A quantity known as the **mass velocity** is widely used in heat exchanger analyses. The mass velocity G is defined as

$$G = \frac{\dot{m}}{A} = \rho V$$

and has dimensions of $M/(L^2 \cdot T)$ [$lbm/(ft^2 \cdot s)$ or $kg/(m^2 \cdot s)$].

In Equations 8.15 and 8.16, equivalent diameter D_e would substitute for D when applying these equations to an annular duct. Note that when the Reynolds number is being calculated for evaluating the friction factor, D_h is used. When heat transfer effects are being modeled, D_e is used. Thus, for flow in an annulus, there will be two Reynolds numbers: one based on hydraulic diameter for finding friction factor, and one based on equivalent diameter for calculating heat transfer rates.

Outlet Temperature Equations

When a specific size of heat exchanger is selected, then the geometry is fixed. The only controls the operator has are flow rate and perhaps inlet temperature. In order to determine how effective the heat exchanger operates, it is necessary to calculate outlet temperature. The equations for calculating outlet temperature are derived here. For the cooler fluid, we write

$$q = \dot{m}_c C_{pc} (t_2 - t_1) \quad (8.17)$$

where $\dot{m}_c$ is the mass flow rate of the cooler fluid and C_{pc} is its specific heat. Assuming counterflow, we write

$$q = U_o A_o \Delta t = U_o A_o \frac{(T_1 - t_2) - (T_2 - t_1)}{\ln [(T_1 - t_2)/(T_2 - t_1)]} \quad (8.9b)$$

Setting these two expressions equal to one another and rearranging, we get

$$\ln \frac{(T_1 - t_2)}{(T_2 - t_1)} = \frac{U_o A_o}{\dot{m}_c C_{pc}} \left(\frac{T_1 - T_2}{t_2 - t_1} - 1 \right) \quad (8.18)$$

For the warmer fluid,

$$q = \dot{m}_w C_{pw} (T_1 - T_2) \quad (8.19)$$

where $\dot{m}_w$ is the mass flow rate of the warmer fluid and C_{pw} is its specific heat. Assuming all heat lost by the warmer fluid is gained by the cooler fluid, we set Equation 8.17 equal to Equation 8.19:

$$\dot{m}_w C_{pw} (T_1 - T_2) = \dot{m}_c C_{pc} (t_2 - t_1)$$

Rearranging and introducing a new variable, R , we have

$$\frac{\dot{m}_c C_{pc}}{\dot{m}_w C_{pw}} = \frac{T_1 - T_2}{t_2 - t_1} = R \quad (8.20)$$

Substituting into Equation 8.18 and removing logarithms, we obtain

$$\frac{T_1 - t_2}{T_2 - t_1} = \exp \left[\frac{U_o A_o}{\dot{m}_c C_{pc}} (R - 1) \right] = E_c \quad (8.21)$$

where the notation E_c has been introduced and defined. Equation 8.20 can be rearranged to give an equation for the outlet temperature of the cooler fluid:

$$t_2 = t_1 + \frac{T_1 - T_2}{R} \quad (8.22)$$

Substituting into Equation 8.21, we get an equation for the outlet temperature of the warmer fluid as

$$T_2 = \frac{(1 - R)T_1 + (1 - E_c)Rt_1}{1 - RE_c} \quad (\text{counterflow}) \quad (8.23)$$

Equation 8.23 allows us to calculate the outlet temperature of the warmer fluid knowing its flow rate, fluid properties, and only the inlet temperatures. Once the outlet temperature of the warmer fluid T_2 is known, then Equation 8.22 can be used to find the outlet temperature for the cooler fluid t_2 .

A similar analysis can be performed for parallel flow. We begin by defining:

$$E_p = \exp \left[\frac{U_o A_o}{\dot{m}_c C_{pc}} (R + 1) \right] \quad (8.24)$$

Following the same line of reasoning as in the counterflow case, the outlet

temperature of the warmer fluid becomes

$$T_2 = \frac{(R + E_p)T_1 + (E_p - 1)Rt_1}{(R + 1)E_p} \quad (\text{parallel flow}) \quad (8.25)$$

Again, once the outlet temperature of the warmer fluid T_2 is known, the outlet temperature of the cooler fluid t_2 is found with Equation 8.22.

Fouling Factors

When a heat exchanger has been in service for a certain amount of time, scale and dirt will deposit on the surfaces of the tubes. These deposits reduce the rate of heat transfer between the fluids by increasing the resistance to heat flow through the inner tube wall. Figure 8.5 shows a cross section of the double pipe heat exchanger with these additional resistances.

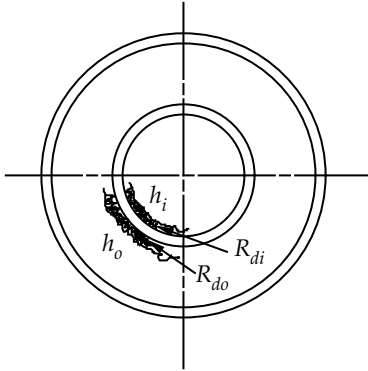


FIGURE 8.5. Cross section of double pipe heat exchanger showing additional resistances due to fouling.

The additional resistances on the inside and outside surfaces are identified as R_{di} and R_{do} , respectively. They affect the overall heat transfer coefficient defined earlier in Equation 8.2 as

$$\frac{1}{U_o} = \frac{OD_p}{h_i ID_p} + \frac{1}{h_o} = \frac{1}{h_p} + \frac{1}{h_o} \quad (8.2)$$

Equation 8.2 applies when the heat exchanger is new and the tubes are clean. To reflect the added resistances due to surface deposits, we define a *dirty* or *design* coefficient U as

$$\frac{1}{U} = \frac{1}{U_o} + R_{di} + R_{do} \quad (8.26a)$$

or

$$\frac{1}{U} = \frac{OD_p}{h_i ID_p} + \frac{1}{h_o} + R_{di} + R_{do} \quad (8.26b)$$

Values of the resistances for various fluids have been measured as a result of years of experience and are provided in Table 8.1. Note that the resistances are actually referred to specific areas. Because the resistance values are only best estimates, an area correction need not be used with R_{di} and R_{do} in Equation 8.26. The fouling factors represent the fouling that would build up in the course of approximately one year.

TABLE 8.1. *Values of fouling factors for various fluids.*

Fluid	R_d	
	(m ² ·K/W)	ft ² ·hr·°R/BTU
Air	0.000 4	0.002
Brine	0.000 2	0.001
Alcohol vapor	0.000 1	0.0005
Diesel engine exhaust	0.002	0.01
Engine oil	0.000 2	0.001
Organic vapors	0.000 1	0.0005
Organic liquids	0.000 2	0.001
Refrigerant liquid	0.000 2	0.001
Refrigerant vapors	0.000 4	0.002
Steam	0.000 1	0.0005
Vegetable oil	0.000 6	0.003
Water		
City water	0.000 2–0.000 4	0.001–0.002
Distilled	0.000 1–0.000 2	0.0005–0.001
Seawater	0.000 1–0.000 2	0.0005–0.001
Well water	0.000 2–0.000 4	0.001–0.002

Pressure Drop in Pipes and Annuli

The pressure drop for flow in a tube is easily calculated with methods of previous chapters. For flow in a tube, we write

$$\Delta p_t = \frac{fL}{D_h} \frac{\rho V^2}{2g_c} = \frac{fL}{ID_p} \frac{\rho V^2}{2g_c} \quad (\text{tube flow}) \quad (8.27)$$

where friction factor f is obtained from Reynolds number and ε/ID_p data.

We use the same equation for finding pressure drop in an annulus, except we substitute hydraulic diameter for the characteristic dimension in the Reynolds number and relative roughness expressions. In addition, we account for losses in the inlet and outlet fittings by use of a minor-loss term; that is,

$$\Delta p_a = \left(\frac{fL}{D_h} + 1 \right) \frac{\rho V^2}{2g_c} \quad (\text{annular flow}) \quad (8.28)$$

For good design, the pressure drop for either stream (tube fluid or annular fluid) should be less than 10 psi (70 kPa).

The equations for the analysis of a double pipe heat exchanger have been stated and are summarized in a suggested order of calculations procedure, as follows.

ANALYSIS OF DOUBLE PIPE HEAT EXCHANGERS, SUGGESTED ORDER OF CALCULATIONS

Problem	Complete problem statement.
Discussion	Potential heat losses; other sources of difficulties.
Assumptions	1. Steady-state conditions exist. 2. Fluid properties remain constant and are evaluated at a temperature of ____ .
Nomenclature	1. T refers to the temperature of the warmer fluid. 2. t refers to the temperature of the cooler fluid. 3. w subscript refers to the warmer fluid. 4. h subscript refers to hydraulic diameter. 5. c subscript refers to the cooler fluid. 6. a subscript refers to the annular flow area or dimension. 7. p subscript refers to the tubular flow area or dimension. 8. 1 subscript refers to an inlet condition. 9. 2 subscript refers to an outlet condition. 10. e subscript refers to equivalent diameter.

A. Fluid Properties

$\dot{m}_w$	=	T_1	=
ρ	=	C_p	=
k_f	=	α	=
ν	=	Pr	=
$\dot{m}_c$	=	t_1	=
ρ	=	C_p	=
k_f	=	α	=
ν	=	Pr	=

B. Tubing Sizes

ID_a	=	OD_p	=
ID_p	=		

C. Flow Areas $A_p = \pi ID_p^2/4 =$
 $A_a = \pi(ID_a^2 - OD_p^2)/4 =$

D. Fluid Velocities [Route the fluid with the higher flow rate through the flow cross section with the greater area.]

$$V_p = \dot{m}/\rho A = \quad G_p = \dot{m}/A =$$

$$V_a = \dot{m}/\rho A = \quad G_a = \dot{m}/A =$$

E. Annulus Equivalent Diameters

Friction $D_h = ID_a - OD_p =$

Heat Transfer $D_e = (ID_a^2 - OD_p^2)/OD_p =$

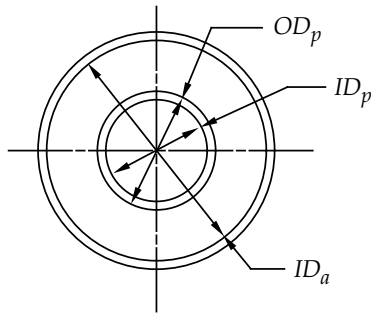


FIGURE 8.6. Definition sketch of diameters associated with an annulus.

F. Reynolds Numbers

$$Re_p = V_p ID_p / \nu =$$

$$Re_a = V_a D_e / \nu =$$

G. Nusselt Numbers

Modified Sieder-Tate Equation for Laminar Flow:

$$Nu = \frac{hD}{k_f} = 1.86 \left(\frac{D Re Pr}{L} \right)^{1/3}$$

$$Re = \frac{VD}{\nu} < 2\,200 \quad \begin{array}{l} D = ID_p \text{ if cross section is tubular} \\ D = D_e \text{ if cross section is annular} \end{array}$$

$$0.48 < Pr = \frac{\nu}{\alpha} < 16\,700$$

μ changes moderately with temperature
 Properties evaluated at the average fluid
 temperature $[= (\text{inlet} + \text{outlet})/2]$

Modified Dittus-Boelter Equation for Turbulent Flow:

$$\text{Nu} = \frac{hD}{k_f} = 0.023(\text{Re})^{4/5} \text{Pr}^n$$

$n = 0.4$ if fluid is being heated

$n = 0.3$ if fluid is being cooled

$$\text{Re} = \frac{VD}{\nu} \geq 10\,000 \quad \begin{array}{l} D = ID_p \text{ if cross section is tubular} \\ D = D_e \text{ if cross section is annular} \end{array}$$

$$0.7 \leq \text{Pr} = \frac{\nu}{\alpha} \leq 160$$

$$L/D \geq 60$$

Properties evaluated at the average fluid temperature $[= (\text{inlet} + \text{outlet})/2]$

$$\text{Nu}_p =$$

$$\text{Nu}_a =$$

H. Convection Coefficients

$$h_i = \text{Nu}_p k_f / ID_p =$$

$$h_p = h_i ID_p / OD_p =$$

$$h_a = \text{Nu}_a k_f / D_e =$$

I. Exchanger Coefficient

$$\frac{1}{U_o} = \frac{1}{h_p} + \frac{1}{h_a}$$

$$U_o =$$

J. Outlet Temperature Calculations (Exchanger length $L =$)

$$R = \frac{\dot{m}_c C_{pc}}{\dot{m}_w C_{pw}} =$$

$$A_o = \pi OD_p L =$$

Counterflow $E_{\text{counter}} = \exp [U_o A_o (R - 1) / \dot{m}_c C_{pc}] =$

$$T_2 = \frac{T_1(R - 1) - R t_1(1 - E_{\text{counter}})}{R E_{\text{counter}} - 1}$$

$$t_2 = t_1 + \frac{T_1 - T_2}{R}$$

$$\text{Parallel Flow } E_{\text{para}} = \exp[U_o A_o (R + 1) / \dot{m}_c C_{pc}] =$$

$$T_2 = \frac{(R + E_{\text{para}})T_1 + R t_1 (E_{\text{para}} - 1)}{(R + 1)E_{\text{para}}}$$

$$t_2 = t_1 + \frac{T_1 - T_2}{R}$$

$$T_2 =$$

$$t_2 =$$

K. Log Mean Temperature Difference

$$\text{Counterflow } LMTD = \frac{(T_1 - t_2) - (T_2 - t_1)}{\ln [(T_1 - t_2) / (T_2 - t_1)]} =$$

$$\text{Parallel Flow } LMTD = \frac{(T_1 - t_1) - (T_2 - t_2)}{\ln [(T_1 - t_1) / (T_2 - t_2)]} =$$

L. Heat Balance

$$q_w = \dot{m}_w C_{pw} (T_1 - T_2) =$$

$$q_c = \dot{m}_c C_{pc} (t_2 - t_1) =$$

$$q = U_o A_o LMTD = \quad \quad \quad (\text{clean})$$

M. Fouling Factors and Design Coefficient

$$R_{di} = \quad \quad \quad R_{do} =$$

$$\frac{1}{U} = \frac{1}{U_o} + R_{di} + R_{do} \quad \quad U =$$

N. Heat Transfer Area and Tube Length (unless already known)

$$A_o = \frac{q}{U (LMTD)} =$$

$$L = \frac{A_o}{\pi (OD_p)} =$$

O. Friction Factors

$$\left. \begin{aligned} \text{Re}_p &= V_p ID_p / \nu = \\ \frac{\varepsilon}{ID_p} &= \end{aligned} \right\} f_p =$$

$$\left. \begin{aligned} \text{Re}_a &= V_a D_h / \nu = \\ \frac{\varepsilon}{D_h} &= \end{aligned} \right\} f_a =$$

Laminar Flow Equations

$$\text{Laminar flow in a tube} \quad f_p = \frac{64}{\text{Re}_p} \quad \text{Re}_p = \frac{V_p ID_p}{\nu} \leq 2\,200$$

$$\text{Laminar flow in an annulus} \quad \kappa = \frac{OD_p}{ID_a} \quad \text{Re}_a = \frac{V_a D_h}{\nu} \leq 10\,000$$

$$\frac{1}{f_a} = \frac{\text{Re}_a}{64} \left[\frac{1 + \kappa^2}{(1 - \kappa)^2} + \frac{1 + \kappa}{(1 - \kappa) \ln(\kappa)} \right]$$

Turbulent Flow Equations

$$D = ID_p \quad (\text{if cross section is tubular})$$

$$D = D_h \quad (\text{if cross section is annular})$$

Chen Equation

$$\frac{1}{\sqrt{f}} = -2.0 \log \left\{ \frac{\varepsilon}{3.7065D} - \frac{5.0452}{\text{Re}} \log \left[\frac{1}{2.8257} \left(\frac{\varepsilon}{D} \right)^{1.1098} + \frac{5.8506}{\text{Re}^{0.8981}} \right] \right\}$$

Churchill Equation

$$f = 8 \left[\left(\frac{8}{\text{Re}} \right)^{12} + \frac{1}{(B + C)^{1.5}} \right]^{1/12}$$

$$\text{where } B = \left[2.457 \ln \frac{1}{(7/\text{Re})^{0.9} + (0.27\varepsilon/D)} \right]^{16}$$

$$\text{and } C = \left(\frac{37\,530}{\text{Re}} \right)^{16}$$

P. Pressure Drop Calculations

$$\Delta p_p = \frac{f_p L}{ID_p} \frac{\rho_p V_p^2}{2g_c} =$$

$$\Delta p_a = \left(\frac{f_a L}{D_h} + 1 \right) \frac{\rho_a V_a^2}{2g_c} =$$

Q. Summary of Information Requested in Problem Statement

EXAMPLE 8.4. Water at a temperature of 175°F and a mass flow rate of 5,000 lbm/hr is to be used to heat ethylene glycol. The ethylene glycol is available at 90°F with a mass flow rate of 30,000 lbm/hr. A double pipe heat exchanger consisting of a 1¹/₄-standard type M copper tubing inside of 2-standard type M copper tubing is to be used. The exchanger consists of two double hairpin exchangers, each 6 ft long. The flow configuration is such that both fluids travel in series throughout. Determine the outlet temperature of both fluids using counterflow and again using parallel flow.

Solution. Water loses energy only to the ethylene glycol, and as heat is transferred, fluid properties change with temperature changes. Outlet temperatures are unknown, so in order to evaluate properties, we use either the inlet temperatures or the average of both inlet temperatures. The fluid with the higher flow rate should be placed in the passage (annular or tubular) having the greater cross-sectional area so that pressure losses are minimized. With the piping arrangement all in series, the system is equivalent to having one 24-ft-long double pipe heat exchanger.

Assumptions

1. Steady-state conditions exist.
2. Fluid properties remain constant and as a first attempt, are evaluated at 132°F [$\approx (175 + 90)/2$].

Nomenclature

1. T refers to the temperature of the warmer fluid.
2. t refers to the temperature of the cooler fluid.
3. w subscript refers to the warmer fluid.
4. h subscript refers to hydraulic diameter.
5. c subscript refers to the cooler fluid.
6. a subscript refers to the annular flow area or dimension.
7. p subscript refers to the tubular flow area or dimension.
8. 1 subscript refers to an inlet condition.
9. 2 subscript refers to an outlet condition.
10. e subscript refers to equivalent diameter.

A. Fluid Properties (interpolated from the property tables in the appendix)

Water	$\dot{m}_w = 5000 \text{ lbm/hr}$	$T_1 = 175^\circ\text{F}$
132°F	$\rho = 61.6 \text{ lbm/ft}^3$	$C_p = 0.999 \text{ BTU/lbm}\cdot^\circ\text{R}$
	$k_f = 0.373 \text{ BTU/ (hr}\cdot\text{ft}\cdot^\circ\text{R)}$	$\alpha = 6.06 \times 10^{-3} \text{ ft}^2/\text{hr}$
	$\nu = 5.57 \times 10^{-6} \text{ ft}^2/\text{s}$	$\text{Pr} = 3.31$
Ethylene Glycol	$\dot{m}_c = 30,000 \text{ lbm/hr}$	$t_1 = 90^\circ\text{F}$
132°F	$\rho = 68.0 \text{ lbm/ft}^3$	$C_p = 0.607 \text{ BTU/lbm}\cdot^\circ\text{R}$
	$k_f = 0.150 \text{ BTU/ (hr}\cdot\text{ft}\cdot^\circ\text{R)}$	$\alpha = 3.6 \times 10^{-3} \text{ ft}^2/\text{hr}$
	$\nu = 6.05 \times 10^{-5} \text{ ft}^2/\text{s}$	$\text{Pr} = 60.2$

B. Tubing Sizes

2-std M	$ID_a = 0.1674 \text{ ft}$	
1¹/₄-std M	$ID_p = 0.1076 \text{ ft}$	$OD_p = 1.375/12 = 0.1146 \text{ ft}$

C. Flow Areas

$$A_p = \pi ID_p^2 / 4 = 0.00909 \text{ ft}^2$$

$$A_a = \pi (ID_a^2 - OD_p^2) / 4 = 0.0117 \text{ ft}^2$$

D. Fluid Velocities [Because $A_a > A_p$, we route the ethylene glycol (the fluid with the higher flow rate) through the annulus.]

Water	$V_p = \dot{m}_w / \rho A_p = 2.48 \text{ ft/s}$	$G_p = \dot{m}_w / A_p = 153 \text{ lbm/ (ft}^2\cdot\text{s)}$
Ethylene Glycol	$V_a = \dot{m}_c / \rho A_a = 10.47 \text{ ft/s}$	$G_a = \dot{m}_c / A_a = 712 \text{ lbm/ (ft}^2\cdot\text{s)}$

E. Annulus Equivalent Diameters

Friction	$D_h = ID_a - OD_p = 0.0528 \text{ ft}$
Heat Transfer	$D_e = (ID_a^2 - OD_p^2) / OD_p = 0.1299 \text{ ft}$

F. Reynolds Numbers

Water	$\text{Re}_p = V_p ID_p / \nu = 4.8 \times 10^4$
Ethylene Glycol	$\text{Re}_a = V_a D_e / \nu = 2.25 \times 10^5$

G. Nusselt Numbers

Water	$\text{Nu}_p = 0.023(\text{Re}_p)^{4/5} \text{Pr}^{0.3} = 182$
Ethylene Glycol	$\text{Nu}_a = 0.023(\text{Re}_a)^{4/5} \text{Pr}^{0.4} = 359$

H. Convection Coefficients

Water	$h_i = \text{Nu}_p k_f / ID_p = 634$	$h_p = h_i ID_p / OD_p = 595$
Ethylene Glycol	$h_a = \text{Nu}_a k_f / D_e = 413 \text{ BTU/ (hr}\cdot\text{ft}^2\cdot^\circ\text{R)}$	

I. Exchanger Coefficient

$$\frac{1}{U_o} = \frac{1}{h_p} + \frac{1}{h_a} \quad U_o = 244 \text{ BTU}/(\text{hr} \cdot \text{ft}^2 \cdot ^\circ\text{R})$$

J. Outlet Temperature Calculations (Exchanger length $L = 24 \text{ ft}$)

$$R = \frac{\dot{m}_c C_{pc}}{\dot{m}_w C_{pw}} = 3.65 \quad A_o = \pi OD_p L = 8.639 \text{ ft}^2$$

$$\text{Counterflow} \quad E_{\text{counter}} = \exp [U_o A_o (R - 1) / \dot{m}_c C_{pc}] = 1.358$$

$$T_2 = \frac{T_1(R - 1) - R t_1(1 - E_{\text{counter}})}{R E_{\text{counter}} - 1} = 146.9^\circ\text{F}$$

$$t_2 = t_1 + \frac{T_1 - T_2}{R} = 97.7^\circ\text{F}$$

$$\text{Parallel Flow} \quad E_{\text{para}} = \exp [U_o A_o (R + 1) / \dot{m}_c C_{pc}] = 1.711$$

$$T_2 = \frac{(R + E_{\text{para}})T_1 + R t_1(E_{\text{para}} - 1)}{(R + 1)E_{\text{para}}} = 147.3^\circ\text{F}$$

$$t_2 = t_1 + \frac{T_1 - T_2}{R} = 97.6^\circ\text{F}$$

At this point, we should re-evaluate the fluid properties at new temperatures. So for the water and for the ethylene glycol, we find the average of inlet and outlet temperatures as

$$\text{Water} \quad T = \frac{175 + 146.9}{2} = 160.9^\circ\text{F}$$

$$\text{Ethylene Glycol} \quad t = \frac{90 + 97.7}{2} = 93.9^\circ\text{F}$$

The system is now re-analyzed using properties evaluated at these temperatures. If necessary, a third iteration should be made so that a better estimate of the outlet temperatures is obtained. (With a spreadsheet, many iterations can be made in a very short time.) If this is done, the temperatures we use to find properties are (after several iterations):

$$\text{Water} \quad T = 162.0^\circ\text{F}$$

$$\text{Ethylene Glycol} \quad t = 93.7^\circ\text{F}$$

K. Requested Information

Overall Coefficient	$U_0 = 222 \text{ BTU}/(\text{hr}\cdot\text{ft}^2\cdot^\circ\text{R})$	(new)
Water	$T_2 = 148.9^\circ\text{F}$	(counterflow)
Ethylene Glycol	$t_2 = 97.4^\circ\text{F}$	(counterflow)
Water	$T_2 = 149.2^\circ\text{F}$	(parallel Flow)
Ethylene Glycol	$t_2 = 97.3^\circ\text{F}$	(parallel Flow)

L. Heat Balance (as a check on the results)*Counterflow*

Water	$q_w = \dot{m}_w C_{pw}(T_1 - T_2) = 36.2 \text{ BTU/s}$
Ethylene Glycol	$q_c = \dot{m}_c C_{pc}(t_2 - t_1) = 36.2 \text{ BTU/s}$
Overall	$q = UA(LMTD) = 36.2 \text{ BTU/s}$

M. Fouling Factors and Design Coefficient ($\text{ft}^2\cdot\text{hr}\cdot^\circ\text{R}/\text{BTU}$)

Water	$R_{di} = 0.00075$	(avg value for distilled water)
Ethylene Glycol	$R_{do} = 0.001$	(organic liquid)
Overall	$\frac{1}{U} = \frac{1}{U_0} + R_{di} + R_{do}$	$U = 160 \text{ BTU}/(\text{hr}\cdot\text{ft}^2\cdot^\circ\text{R})$

We return to step J and calculate the outlet temperatures that correspond to the design value of the overall heat transfer coefficient. The results for counterflow) are:

Overall Coefficient	$U = 160 \text{ BTU}/(\text{hr}\cdot\text{ft}^2\cdot^\circ\text{R})$	(1 year)
Water	$T_2 = 155.1^\circ\text{F}$	(counterflow)
Ethylene Glycol	$t_2 = 95.7^\circ\text{F}$	(counterflow)
Water	$q_w = \dot{m}_w C_{pw}(T_1 - T_2) = 27.6 \text{ BTU/s}$	
Ethylene Glycol	$q_c = \dot{m}_c C_{pc}(t_2 - t_1) = 27.6 \text{ BTU/s}$	
Overall	$q = UA(LMTD) = 27.6 \text{ BTU/s}$	

Summary of Performance of Heat Exchanger (Counterflow)

New	After 1 year
$U_0 = 222 \text{ BTU}/(\text{hr}\cdot\text{ft}^2\cdot^\circ\text{R})$	$U = 160 \text{ BTU}/(\text{hr}\cdot\text{ft}^2\cdot^\circ\text{R})$
$T_2 = 148.9^\circ\text{F}$ (water)	$T_2 = 155.1^\circ\text{F}$ (water)
$t_2 = 97.4^\circ\text{F}$ (E.G.)	$t_2 = 95.7^\circ\text{F}$ (E.G.)
$q = 36.2 \text{ BTU/s}$	$q = 27.6 \text{ BTU/s}$

Comments on the Results:

- The results show that little difference exists between parallel flow and counterflow for this example. This is not always the case, however. Counterflow is usually the preferred flow configuration.
- Outlet temperatures for the first iteration were: water $T = 146.9^\circ\text{F}$ and ethylene glycol $t = 97.7^\circ\text{F}$ for counterflow. When the properties were re-evaluated at the new temperatures, the final results were (after several iterations): water $T = 148.9^\circ\text{F}$ and ethylene glycol $t = 97.4^\circ\text{F}$.
- The temperature at which properties were evaluated had a comparatively small influence on the outlet temperatures. This occurred for these fluids under the conditions of the problem statement.
- For some fluids, however (such as oils), properties change more drastically with temperature and evaluating properties at a reasonable temperature becomes very important.
- When fouling effects were taken into account, the overall heat transfer coefficient was reduced from $222 \text{ BTU}/(\text{hr}\cdot\text{ft}^2\cdot^\circ\text{R})$ to $160 \text{ BTU}/(\text{hr}\cdot\text{ft}^2\cdot^\circ\text{R})$. Likewise, the outlet temperatures were also affected. The fouling factors apply to the performance that will exist after one year.
- The performance of this (and any) exchanger should be evaluated at the design value. Suppose that this exchanger is to provide an outlet temperature of T_2 of 155°F . When new, this exchanger will perform better than expected by giving $T_2 = 149^\circ\text{F}$.

Temperature Profile

Calculations made using the data of the previous example reveal a number of important details worth noting. In order to examine the performance of the exchanger, we allow the length to increase and we calculate outlet temperatures for both parallel and counterflow configurations. The results are graphed in Figures 8.7 and 8.8.

Figure 8.7 shows temperature variation with length for a counterflow arrangement. Length on the horizontal axis has been allowed to vary to

112 ft, selected because it is the distance required for the fluids to have equal outlet temperatures. The temperature of the warm fluid at inlet is 175°F; and as length increases, the temperature decreases, following a curve that is concave downward. The cooler fluid enters the exchanger at 90°F; as length increases, temperature increases, following a curve that is also concave downward. Throughout the exchanger, the cooler fluid must travel a comparatively considerable distance for even a small temperature increase, whereas the warmer fluid experiences a greater temperature change. This is due to what is known as the **capacitances** (i.e., $\dot{m}C_p$ product) of the fluids. For the fluids shown, $\dot{m}_c C_{pc} > \dot{m}_w C_{pw}$, which means that $(t_2 - t_1) < (T_1 - T_2)$, because

$$\dot{m}_c C_{pc}(t_2 - t_1) = \dot{m}_w C_{pw}(T_1 - T_2) = q$$

The curve shapes are typical for this condition.

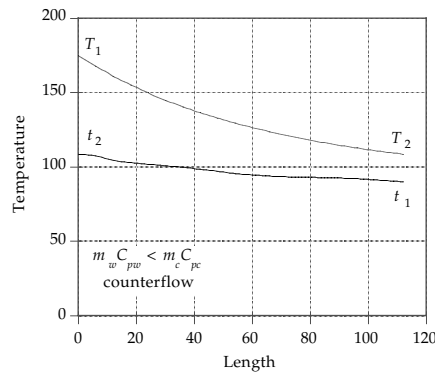


FIGURE 8.7. Temperature (in °F) variation with length (in ft) for the double pipe heat exchanger of Example 8.3 in counterflow.

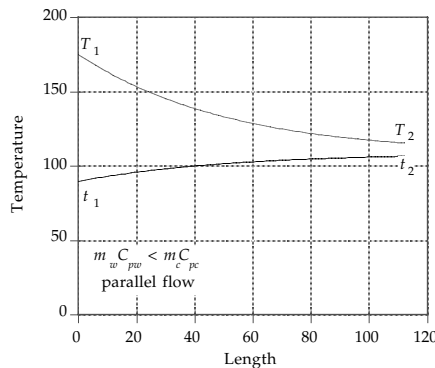


FIGURE 8.8. Temperature (in °F) variation with length (in ft) for the double pipe heat exchanger of Example 8.3 in parallel flow.

Figure 8.8 shows temperature profiles for the same exchanger operating in parallel (or unidirectional) flow. Note that even after 112 ft, the outlet temperatures are not equal, nor can they be made equal, theoretically, unless the surface area (proportional to length) is infinite. (Actually, for this exchanger, outlet temperatures are equal at a length of 620 ft.) Again, the temperature changes for each fluid follow the same trends as in the counterflow case. Calculations and temperature profiles presented indicate that the stream with the smaller **thermal capacity** or **capacitance** $\dot{m}C_p$ limits the amount of heat that can be transferred.

Other cases of interest are shown in Figures 8.9 and 8.10. Figure 8.9 shows temperature versus length (or area) profiles for an exchanger having $\dot{m}_c C_{pc} < \dot{m}_w C_{pw}$; that is, the warmer fluid has the larger capacitance. Counterflow and parallel flow cases are shown. Figure 8.9a shows a counterflow arrangement, and as indicated, the temperature profiles of both fluids are concave upward. The fluid with the largest temperature change (the cooler fluid) has the minimum capacitance. Figure 8.9b shows the same exchanger in a parallel flow configuration. Trends are just the opposite of the those illustrated in Figures 8.7 and 8.8.

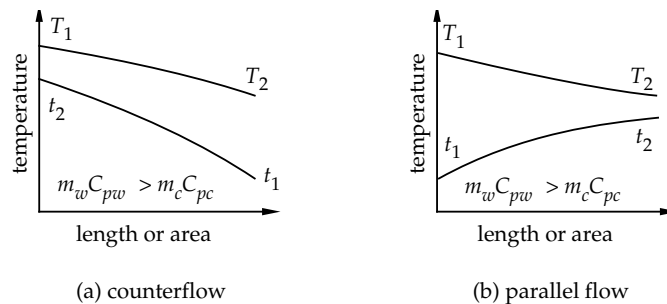


FIGURE 8.9. Temperature versus length graphs for a double pipe heat exchanger in counterflow and in parallel flow arrangements. The cooler fluid having the minimum capacitance.

Figure 8.10 shows temperature versus length profiles for the case where the heat exchanger is used as a condenser or as an evaporator. As shown, one fluid has a constant temperature throughout, as the phase change occurs isothermally. Figure 8.10a illustrates the profiles for a condenser with the warmer fluid changing phase. Its temperature is shown as constant throughout. Superheating or subcooling is assumed not to occur, although such is possible in a real situation. Figure 8.10b shows a typical temperature profile for the case where the exchanger is used as an evaporator. Note that in either case, the fluid that is changing phase has a capacitance that is infinite: $\dot{m}_w C_{pw}$ in Figure 8.10a and $\dot{m}_c C_{pc}$ in Figure 8.10b.

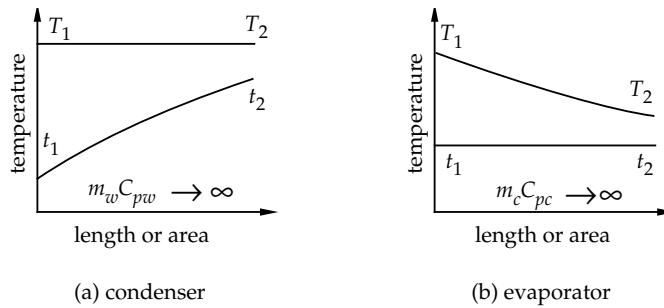


FIGURE 8.10. Temperature versus length graphs for a double pipe heat exchanger used as a condenser or as an evaporator. The fluid changing phase has an infinite capacitance.

8.3 Effectiveness-NTU Analysis

The analysis presented in the preceding section (the **LMTD method**) is rather traditional. A seemingly more popular (but equivalent) method of analysis is called the **effectiveness-Number of Transfer Units method (effectiveness-NTU)**. As indicated in the previous section, the stream with the smaller thermal capacity $\dot{m}C_p$ limits the amount of heat that can be transferred. We use this limitation in defining effectiveness.

The effectiveness is dependent on which of the two fluids has the minimum mass flow rate $\times$ specific heat product; that is, the minimum **capitance**. The effectiveness E is defined as

$$E = \frac{t_2 - t_1}{T_1 - t_1} \quad (\text{if } \dot{m}_c C_{pc} < \dot{m}_w C_{pw}) \quad (8.29a)$$

$$E = \frac{T_1 - T_2}{T_1 - t_1} \quad (\text{if } \dot{m}_w C_{pw} < \dot{m}_c C_{pc}) \quad (8.29b)$$

(Effectiveness E is not to be confused with E_c or E_p defined earlier in Equations 8.21 and 8.24, respectively.) A consequence of these definitions is the following equation:

$$E = \frac{q}{q_{\max}}$$

where q is the actual heat transferred, given by

$$q = E (\dot{m}C_p)_{\min}(T_1 - t_1) \quad (8.30)$$

and $q_{\max}$ is based on the minimum capacitance and on the maximum temperature difference, which is the temperature difference between both fluids as they enter the exchanger. Thus $q_{\max}$ is defined as

$$q_{\max} = (\dot{m}C_p)_{\min}(T_1 - t_1) \quad (8.31)$$

These equations look odd, but they are really just a rewritten version of what is already known. Consider, for example, Equation 8.30. If $(\dot{m}C_p)_{\min} = \dot{m}_c C_{pc}$, then Equation 8.30 becomes

$$q = \frac{t_2 - t_1}{T_1 - t_1} \dot{m}_c C_{pc} (T_1 - t_1)$$

$$\text{or } q = \dot{m}_c C_{pc} (t_2 - t_1)$$

which is just the energy gained by the cooler fluid.

Several important features of these definitions should be noted:

- The denominator in Equations 8.29 is the maximum temperature difference (inlet temperatures) associated with the exchanger.
- The numerator in Equations 8.29 is the temperature difference of the fluid having the minimum capacitance.
- Effectiveness E ranges from 0 to 1.
- The definitions of effectiveness are independent of exchanger type and so can be applied to double pipe, shell-and-tube, cross flow heat exchangers, and so on.

If effectiveness is known for the heat exchanger of interest, then the heat transfer rate can be found with Equation 8.30. All that is needed in this development is an equation or graph of effectiveness E for the exchanger that is to be analyzed.

Effectiveness equations have been derived for many types of heat exchangers, and usually contain a term called the **number of transfer units**, N , defined as

$$N = \frac{UA}{(\dot{m}C_p)_{\min}} \quad (8.32)$$

We also define what is known as the **ratio of capacitances** C , which is always less than 1:

$$C = \frac{(\dot{m}C_p)_{\min}}{(\dot{m}C_p)_{\max}} < 1$$

This definition is similar to, but not to be confused with, the definition of R given in Equation 8.20.

Using these definitions, we can derive an equation for the effectiveness of a double pipe heat exchanger. Consider a counterflow arrangement, for which Equation 8.18 applies:

$$\ln \frac{(T_1 - t_2)}{(T_2 - t_1)} = \frac{UA}{\dot{m}_c C_{pc}} \left(\frac{T_1 - T_2}{t_2 - t_1} - 1 \right) \quad (8.18)$$

Now if $(\dot{m}C_p)_{\min} = \dot{m}_c C_{pc}$, then Equation 8.18 becomes

$$\ln \frac{(T_1 - t_2)}{(T_2 - t_1)} = N \left(\frac{(\dot{m}C_p)_{\min}}{(\dot{m}C_p)_{\max}} - 1 \right) = N(C - 1)$$

or $\ln \frac{(T_1 - t_2)}{(T_2 - t_1)} = -N(1 - C)$

Exponentiating both sides,

$$\frac{(T_1 - t_2)}{(T_2 - t_1)} = \exp [-N(1 - C)] \quad (8.33)$$

The heat balance equation is

$$\dot{m}_c C_{pc} (t_2 - t_1) = \dot{m}_w C_{pw} (T_1 - T_2) = q$$

Rearranging and solving for t_2 , we have

$$t_2 = t_1 + \frac{\dot{m}_w C_{pw}}{\dot{m}_c C_{pc}} (T_1 - T_2) = t_1 + \frac{T_1 - T_2}{C}$$

or $t_2 = \frac{Ct_1 + T_1 - T_2}{C}$

Substituting into Equation 8.33, we get

$$\frac{T_1 - \frac{Ct_1 + T_1 - T_2}{C}}{T_2 - t_1} = \exp [-N(1 - C)]$$

Simplifying,

$$\frac{CT_1 - Ct_1 - T_1 + T_2}{C(T_2 - t_1)} = \exp [-N(1 - C)]$$

$$\text{or } \frac{T_1 - t_1}{T_2 - t_1} - \frac{T_1 - T_2}{C(T_2 - t_1)} = \exp [-N(1 - C)]$$

After lengthy but diligent algebraic manipulations, the left-hand side is found to be equal to

$$\frac{1}{1 - EC} - \frac{1}{C} \frac{EC}{1 - EC} = \exp [-N(1 - C)]$$

Combining,

$$\frac{1 - E}{1 - EC} = \exp [-N(1 - C)]$$

Our objective is to solve for the effectiveness. Rearranging and solving,

$$E = \frac{1 - \exp [-N(1 - C)]}{1 - C \exp [-N(1 - C)]} \quad (\text{counterflow}) \quad (8.34)$$

Equation 8.34 was derived by assuming that $(\dot{m}C_p)_{\min} = \dot{m}_c C_{pc}$. If we proceeded by assuming instead that $(\dot{m}C_p)_{\min} = \dot{m}_w C_{pw}$, the same equation would have resulted.

For parallel flow, we can perform a similar analysis and show that

$$E = \frac{1 - \exp [-N(1 + C)]}{1 + C} \quad (\text{parallel flow}) \quad (8.35)$$

(For a derivation of this equation, see the Problems section.)

The summary table at the end of the chapter (Table 8.4) lists effectiveness equations in terms of number of transfer units N and ratio of capacitances C for a number of heat exchanger types.

When analyzing a heat exchanger in the way that was done in Example 8.4, the effectiveness can be used to calculate the outlet temperatures. The procedure is the same as in the example, except that effectiveness E , rather than E_c or E_p , is calculated in Step J. Once effectiveness is known, outlet temperatures are found with

$$\left. \begin{aligned} t_2 &= t_1 + E(T_1 - t_1) \\ T_2 &= T_1 - C(t_2 - t_1) \end{aligned} \right\} \quad \left(\text{if } \dot{m}_c C_{pc} < \dot{m}_w C_{pw} \text{ with } E = \frac{t_2 - t_1}{T_1 - t_1} \right)$$

or in the opposite case

$$\left. \begin{aligned} T_2 &= T_1 - E(T_1 - t_1) \\ t_2 &= t_1 + C(T_1 - T_2) \end{aligned} \right\} \left(\text{if } \dot{m}_w C_{pw} < \dot{m}_c C_{pc} \text{ with } E = \frac{T_1 - T_2}{T_1 - t_1} \right)$$

When analyzing double pipe heat exchangers, neither method (i.e., the LMTD method or the effectiveness-NTU method) is superior to the other. Both give identical results for outlet temperature.

EXAMPLE 8.5. Oil is to be heated in a double pipe heat exchanger from 90°F to 100°F. The oil flow rate is 400 lbm/hr. Water at 150°F is available at 5000 lbm/hr. The heat exchanger is made of 2 x 1¹/₄ std type M copper tubing that is 15 ft long. Analyze the proposed setup completely. Use the effectiveness-NTU method, and place the oil in the annulus. Oil properties at 95°F are

$$\begin{aligned} \rho &= 54.8 \text{ lbm/ft}^3 & C_p &= 0.464 \text{ BTU/(lbm}\cdot^\circ\text{R)} \\ \nu &= 4.27 \times 10^{-3} \text{ ft}^2/\text{s} & \alpha &= 0.00327 \text{ ft}^2/\text{hr} \end{aligned}$$

Solution. For a liquid to liquid heat exchange, we use a double pipe heat exchanger. We obtain tube dimensions from the appropriate table in the Appendix. Oil properties are given in the problem statement. Water properties are obtained from the property tables, and are interpolated at the average of inlet and outlet temperatures. Several iterations are necessary. The numbers that appear in the following are those calculated after the final iteration.

Assumptions

1. Steady-state conditions exist.
2. Water properties remain constant and are evaluated at a temperature of 150°F for the water.

Nomenclature

1. T refers to the temperature of the warmer fluid.
2. t refers to the temperature of the cooler fluid.
3. w subscript refers to the warmer fluid.
4. h subscript refers to hydraulic diameter.
5. c subscript refers to the cooler fluid.
6. a subscript refers to the annular flow area or dimension.
7. p subscript refers to the tubular flow area or dimension.
8. 1 subscript refers to an inlet condition.
9. 2 subscript refers to an outlet condition.
10. e subscript refers to equivalent diameter.

A. Fluid Properties

Water	$\dot{m}_w = 1.39 \text{ lbm/s}$	$T_1 = 150^\circ\text{F}$
	$\rho = 61.3 \text{ lbm/ft}^3$	$C_p = 1.00 \text{ BTU}/(\text{lbm}\cdot^\circ\text{R})$
	$k_f = 0.3787 \text{ BTU}/(\text{hr}\cdot\text{ft}\cdot^\circ\text{R})$	$\alpha = 0.00618 \text{ ft}^2/\text{hr}$
	$\nu = 4.81 \times 10^{-6} \text{ ft}^2/\text{s}$	$\text{Pr} = 2.8$
Oil	$\dot{m}_c = 0.11 \text{ lbm/s}$	$t_1 = 90^\circ\text{F}$
	$\rho = 54.8 \text{ lbm/ft}^3$	$C_p = 0.464 \text{ BTU}/(\text{lbm}\cdot^\circ\text{R})$
	$k_f = 0.0832 \text{ BTU}/(\text{hr}\cdot\text{ft}\cdot^\circ\text{R})$	$\alpha = 0.003269 \text{ ft}^2/\text{hr}$
	$\nu = 4.27 \times 10^{-3} \text{ ft}^2/\text{s}$	$\text{Pr} = 4699$

B. Tubing Sizes

$$ID_a = 0.1674 \text{ ft}$$

$$ID_p = 0.1076 \text{ ft} \qquad OD_p = 0.1146 \text{ ft}$$

C. Flow Areas

$$A_p = \pi ID_p^2 / 4 = 0.00909 \text{ ft}^2$$

$$A_a = \pi (ID_a^2 - OD_p^2) / 4 = 0.01170 \text{ ft}^2$$

D. Fluid Velocities

Water $V_p = \dot{m} / \rho A = 2.49 \text{ ft/s}$

Oil $V_a = \dot{m} / \rho A = 0.173 \text{ ft/s}$

E. Annulus Equivalent Diameters

Friction $D_h = ID_a - OD_p = 0.0528 \text{ ft}$

Heat Transfer $D_e = (ID_a^2 - OD_p^2) / OD_p = 0.1300 \text{ ft}$

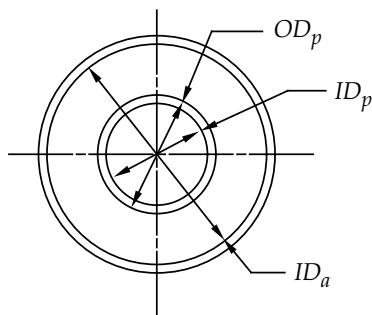


FIGURE 8.6. Definition sketch of diameters associated with an annulus.

F. Reynolds Numbers

Water $\text{Re}_p = V_p ID_p / \nu = 55,782$

Oil $\text{Re}_a = V_a D_e / \nu = 5$ (use laminar flow equations)

G. Nusselt Numbers

Water $Nu_p = 196$

Oil $Nu_a = 11$

H. Convection Coefficients (in BTU/(hr·ft²·°R))

Water $h_i = Nu_p k_f / ID_p = 691$ $h_p = h_i ID_p / OD_p = 649$

Oil $h_a = Nu_a k_f / D_e = 7$

I. Exchanger Coefficient

$$\frac{1}{U_o} = \frac{1}{h_p} + \frac{1}{h_a} \qquad U_o = 7 \text{ BTU}/(\text{hr} \cdot \text{ft}^2 \cdot ^\circ\text{R})$$

J. Outlet Temperature Calculations (Exchanger length $L = 15$ ft)

$$C = \frac{\dot{m}_c C_{pc}}{\dot{m}_w C_{pw}} = 0.037 \qquad A_o = \pi OD_p L = 5.4 \text{ ft}^2$$

$$N = \frac{UA}{(\dot{m}C_p)_{\min}} = 0.205$$

Counterflow Effectiveness (counterflow)

$$E = \frac{1 - \exp[-N(1 - C)]}{1 - C \exp[-N(1 - C)]} = 0.1849$$

$$t_2 = t_1 + E(T_1 - t_1) \qquad \dot{m}_c C_{pc} < \dot{m}_w C_{pw}$$

$$T_2 = T_1 - C(t_2 - t_1)$$

Water $T_2 = 150^\circ\text{F}$

Oil $t_2 = 101^\circ\text{F}$

K. Log Mean Temperature Difference

$$\text{Counterflow} \quad LMTD = \frac{(T_1 - t_2) - (T_2 - t_1)}{\ln [(T_1 - t_2)/(T_2 - t_1)]} = 54.1^\circ\text{F}$$

L. Heat Balance

Water $q_w = \dot{m}_w C_{pw} (T_1 - T_2) = 0.572 \text{ BTU/s}$

Oil $q_c = \dot{m}_c C_{pc} (t_2 - t_1) = 0.572 \text{ BTU/s}$

$$q = U_o A_o LMTD = 0.572 \text{ BTU/s}$$

M. Fouling Factors and Design Coefficient (assumed for the oil)

$$R_{di} = 0.0002 \quad R_{do} = 0.0002$$

$$\frac{1}{U} = \frac{1}{U_o} + R_{di} + R_{do} \quad U = 7 \text{ BTU}/(\text{hr} \cdot \text{ft}^2 \cdot ^\circ\text{R})$$

N. Heat Transfer Area and Tube Length

$$A_o = \frac{q}{U(LMTD)} = 5.415 \text{ ft}^2$$

$$L = \frac{A_o}{\pi(OD_p)} = 16 \text{ ft}$$

O. Friction Factors

$$\left. \begin{aligned} \text{Re}_p &= \frac{V_p ID_p}{\nu} = 55 \\ \frac{\varepsilon}{ID_p} &= 0.000047 \end{aligned} \right\} f_p = 0.021$$

$$\left. \begin{aligned} \text{Re}_a &= \frac{V_a D_h}{\nu} = 2.14 \\ \frac{\varepsilon}{D_h} &= 0.000095 \end{aligned} \right\} f_a = 29.9$$

P. Pressure Drop Calculations

$$\text{Water} \quad \Delta p_p = \frac{f_p L}{ID_p} \frac{\rho_p V_p^2}{2g_c} = 0.126 \text{ psi}$$

$$\text{Oil} \quad \Delta p_a = \left(\frac{f_a L}{D_h} + 1 \right) \frac{\rho_a V_a^2}{2g_c} = 1.61 \text{ psi}$$

Q. Summary of Information Requested in Problem Statement

The outlet temperatures are 150°F for the water and 101°F for the oil. Pressure drops are 0.126 psi for the water and 1.61 psi for the oil, both are under 10 psi, which is the maximum allowable. The length given was 15 ft. The length calculated in Step N includes the effects of the dirt factors. Because the overall heat transfer coefficient does not change the outlet temperatures will be the same after one year. The exchanger selected should be 16 ft long.

8.4 Double Pipe Heat Exchanger Design Considerations

In many of the problems discussed thus far, only a minimum number of variables are left as unknown. The problems are therefore easy to solve and require only a few assumptions to obtain a solution. When this is not the case, we must resort to cost information to determine the particular design that will yield the lowest total cost per year. The optimum design could involve specification of tube length within an exchanger or the modification of the fluid flow rates. In addition, the apparatus is usually designed to meet certain safety codes, such as those established by ASME and by an organization called the Tubular Exchanger Manufacturers Association (TEMA).

The exchanger of interest in this chapter is the double pipe heat exchanger. In the following chapters, we consider shell-and-tube, cross-flow, and plate-and-frame heat exchangers. Each is suitable for certain applications. A double pipe heat exchanger is used for low to moderate flow rates and low to moderate heat transfer rates. The fluids are usually liquid to liquid or vapor to vapor or gas to gas. A shell-and-tube heat exchanger is used for high flow rates and high heat transfer rates (flow rates over 10 times those in double pipe heat exchangers). The most suitable fluid combinations are liquid to liquid, gas to gas, or vapor to vapor. The cross-flow heat exchanger can be sized to exchange heat at low, medium, or high flow rates and low, medium, or high heat transfer rates. It is suited for liquid to vapor, liquid to gas, gas to gas, and vapor to vapor heat exchange. Any of the above-mentioned heat exchangers can be used as condensers or evaporators, although some devices are better suited for such service. Many other types of heat exchangers are commercially available; they are not discussed here, however, for reasons of space. The methods of analysis are identical.

Consider a problem in which the inlet and outlet temperatures and mass flow rates are specified, and it is desired to use an exchanger that will transfer the required heat load and that will minimize costs. When mass flow rates are known, it is advisable to fix the fluid velocities at the optimum values. Optimum velocity values for various fluids were presented in Chapter 6 and, for convenience, are given again in Table 8.2. It is to be remembered that when the optimum velocity is used, the total cost (first costs plus operating costs) of moving the fluid are minimized. When flow rate and velocity are known, the required cross-sectional area can be easily calculated.

Consider next a double pipe heat exchanger that we wish to size for a given service. In such a problem, a chart showing geometry factors for various tubing combinations is a labor-saving device. A chart of this type is given in Table 8.3.

In view of the preceding discussion, the problem we seek to solve is as follows: When inlet temperatures and flow rates are known, and certain outlet temperatures are desired, what size double pipe heat exchanger will transfer the required energy for a minimum cost? Without a reformulation of all the appropriate economic parameters, this problem can be solved with information already available. The method is illustrated in the next example, following a slightly modified version of the suggested order of calculations for double pipe heat exchangers given earlier. (An economic analysis has been formulated and can be found in “Thermoeconomically Optimum Counterflow Heat Exchanger Effectiveness” by D. K. Edwards and R. Matavosian; *J Heat Transfer*, v. **104**, pp. 191–193, 1982.)

TABLE 8.2. *Reasonable velocities for various fluids.*

Fluid	Economic Velocity Range	
	ft/s	m/s
Acetone	4.9–9.8	1.5–3.0
Ethyl alcohol	4.8–9.6	1.5–3.0
Methyl alcohol	4.8–9.6	1.5–3.0
Propyl alcohol	4.7–9.4	1.4–2.8
Benzene	4.6–9.2	1.4–2.8
Carbon disulfide	4.2–8.4	1.3–2.6
Carbon tetrachloride	3.9–7.8	1.2–2.4
Castor oil	1.6–3.2	0.5–1.0
Chloroform	4.0–8.0	1.2–2.4
Decane	4.9–9.8	1.5–3.0
Ether	5.0–10.0	1.5–3.0
Ethylene glycol	3.9–7.8	1.2–2.4
R-11	4.0–8.0	1.2–2.4
Glycerine	1.4–2.8	0.43–0.86
Heptane	5.1–10.2	1.5–3.0
Hexane	5.2–10.4	1.6–3.2
Kerosene	4.7–9.4	1.4–2.8
Linseed oil	4.9–9.8	1.5–3.0
Mercury	2.1–4.2	0.64–1.3
Octane	5.0–10.0	1.5–3.0
Propane	5.6–11.2	1.7–3.4
Propylene	5.5–11.0	1.7–3.4
Propylene glycol	4.5–9.0	1.4–2.8
Turpentine	4.6–9.2	1.4–2.8
Water	4.4–8.8	1.4–2.8

TABLE 8.3. Double pipe heat exchanger tube combinations.

Size	ID_a ft	Type M Tubing (Engineering Units)				D_h ft	D_e ft
		ID_p ft	OD_p ft	A_p ft ²	A_a ft ²		
2 x 1 ¹ / ₄	0.1674	0.1076	0.1146	0.009093	0.01169	0.0528	0.1299
2 ¹ / ₂ x 1 ¹ / ₄	0.2079	0.1076	0.1146	0.009093	0.02363	0.0933	0.2625
3 x 2	0.2484	0.1674	0.1771	0.02201	0.02382	0.0713	0.1713
4 x 3	0.3279	0.2484	0.2604	0.04846	0.03118	0.0675	0.1524

Size	ID_a m	Type M Tubing (SI Units)				D_h m	D_e m
		ID_p m	OD_p m	A_p m ²	A_a m ²		
2 x 1 ¹ / ₄	0.051 02	0.032 79	0.034 93	0.000 844 4	0.001 086	0.016 09	0.039 59
2 ¹ / ₂ x 1 ¹ / ₄	0.063 38	0.032 79	0.034 93	0.000 844 4	0.002 196	0.028 45	0.080 07
3 x 2	0.075 72	0.051 02	0.053 98	0.002 044	0.002 214	0.021 74	0.052 23
4 x 3	0.099 98	0.075 72	0.079 38	0.004 503	0.002 901	0.020 6	0.046 54

EXAMPLE 8.6 Benzene is used for the manufacture of detergent. A double pipe heat exchanger must be sized to exchange heat between benzene and water. The benzene flow rate is 10,000 lbm/hr, and it is to be heated from 75°F to 100°F. The water is available at 140°F. Select an appropriate heat exchanger and determine the required water flow rate.

Benzene properties:

T in °F	sp. gr.	C_p in BTU/(lbm·°R)	ν in ft ² /s	k_f in BTU/(hr·ft·°R)
153	0.828	0.440	4.74E-06	0.0757
81	0.872	0.413	7.64E-06	0.0827

Other properties are calculated using $\alpha = k_f/\rho C_p$ and $Pr = \nu/\alpha$.

Solution. For liquid-to-liquid heat exchange, we can use a double pipe, a shell-and-tube or a plate-and-frame heat exchanger. The benzene flow rate of 10,000 lbm/hr is small enough so that a double pipe heat exchanger will probably work. (If we did not know this by experience, we would begin by trying to size a double pipe heat exchanger first. If one could not be made to work, then another type of exchanger would be tried next.)

The flow rate of benzene is known, and the optimum velocity is found from Table 8.2. If not listed, the optimum velocity can be determined from the equations of Chapter 4. Knowing optimum velocity, it is possible to calculate the cross-sectional area required for minimum cost conditions.

For benzene, the optimum velocity range is 4.6 to 9.2 ft/s, while for water it is 4.4 to 8.8 ft/s. Moreover, the specific heat of water is greater than that of benzene, and so the water will not experience as great a temperature change as will the benzene for equal flow rates.

Because a range of velocities is given, several choices need to be made. For example, it is possible to try to operate near the maximum velocity without exceeding the maximum permissible pressure drop of 10 psi (72.4 kPa). A high velocity results in a high Reynolds number, a high Nusselt number, and a high convection coefficient. At the same time, it is prudent to remember that the velocity ranges given are merely guides, and so staying at or near the median value might be a better choice.

The fouling factors for benzene and water will influence the choice of which stream to place in the pipe and which to place in the annulus. For benzene (an organic liquid) the fouling factor from Table 8.1 is 0.001 ft²·hr·°R/BTU, while for distilled water the fouling factor is taken to be 0.00075 ft²·hr·°R/BTU [= (0.0005 + 0.001)/2]. Benzene therefore has a greater tendency to cause fouling on surfaces it contacts, which affects the pressure drop experienced by the fluid. If placed in the annulus, benzene will form deposits on the outside surface of the inner tube *and* on the inside surface of the outer tube. So our intent at this point is to route the benzene through the pipe, or the inner tube, of the double pipe heat exchanger we select.

- Assumptions**
1. Steady-state conditions exist.
 2. Benzene properties are evaluated at $(100 + 75)/2 = 87.5^{\circ}\text{F}$.
Water properties are evaluated at the average of inlet and outlet temperatures. The calculations were done iteratively, and the final results appear in what follows.

- Nomenclature**
1. T refers to the temperature of the warmer fluid.
 2. t refers to the temperature of the cooler fluid.
 3. w subscript refers to the warmer fluid.
 4. h subscript refers to hydraulic diameter.
 5. c subscript refers to the cooler fluid.
 6. a subscript refers to the annular flow area or dimension.
 7. p subscript refers to the tubular flow area or dimension.
 8. 1 subscript refers to an inlet condition.
 9. 2 subscript refers to an outlet condition.
 10. e subscript refers to equivalent diameter.

A. Fluid Properties

H₂O	$\dot{m}_w$ = TO BE SELECTED	T_1 = 140°F
@ 136.7°F	ρ = 61.5 lbm/ft ³	C_p = 0.9993 BTU/lbm·°R
	k_f = 0.375 BTU/ (hr·ft·°R)	α = 6.02 x 10 ⁻³ ft ² /hr
	ν = 0.532 x 10 ⁻⁵ ft ² /s	Pr = 3.14
	$4.4 \leq V_{\text{opt}} \leq 8.8$ ft/s	

Benzene	$\dot{m}_c$ = 10,000 lbm/hr = 2.78 lbm/s	t_1 = 75°F
@ 88°F	ρ = 54.2 lbm/ft ³	C_p = 0.416 BTU/lbm·°R
	k_f = 0.082 BTU/ (hr·ft·°R)	α = 3.63 x 10 ⁻³ ft ² /hr
	ν = 7.37 x 10 ⁻⁶ ft ² /s	Pr = 7.28
	$4.6 \leq V_{\text{opt}} \leq 9.2$ ft/s	t_2 = 100°F required

Benzene Min flow area = $\dot{m}_c / \rho V_{\text{max}} = 2.78 / [54.3(9.2)] = 0.0056$ ft²

Benzene Max flow area = $\dot{m}_c / \rho V_{\text{min}} = 2.78 / [54.3(4.6)] = 0.0113$ ft²

Referring to Table 8.3 and assuming the given sizes are all that are available, we see that the maximum flow area corresponds roughly to A_p for a 2 x 1¹/₄ or a 2¹/₂ x 1¹/₄ double pipe heat exchanger. We select the 2 x 1¹/₄ size because it is smaller, and proceed with the calculations on a first trial basis.

B. Tubing Sizes

2 x 1¹/₄	ID_a = 0.1674 ft	
	ID_p = 0.1076 ft	OD_p = 0.1146 ft

C. Flow Areas $A_p = \pi ID_p^2 / 4 = 0.009093$ ft²
 $A_a = \pi (ID_a^2 - OD_p^2) / 4 = 0.01169$ ft²

D. Fluid Velocities [Route the benzene through the pipe or inner tube.]

Benzene $V_p = \dot{m} / \rho A = 5.6$ ft/s

H₂O $V_a = \dot{m} / \rho A = 6$ ft/s

Water velocity is arbitrarily selected within the correct range; mass flow rate is now calculated as:

$$\dot{m}_w = \rho A_a V_a = 0.985(62.4)(0.01169)(6) = 4.33 \text{ lbm/s}$$

E. Annulus Equivalent Diameters

Friction $D_h = ID_a - OD_p = 0.0528$ ft

Heat Trans $D_e = (ID_a^2 - OD_p^2) / OD_p = 0.1299$ ft

F. Reynolds Numbers

$$\text{Benzene} \quad \text{Re}_p = V_p ID_p / \nu = 8.2 \times 10^4$$

$$\text{H}_2\text{O} \quad \text{Re}_a = V_a D_e / \nu = 1.47 \times 10^5$$

G. Nusselt Numbers

$$\text{Benzene} \quad \text{Nu}_p = 357$$

$$\text{H}_2\text{O} \quad \text{Nu}_a = 495$$

H. Convection Coefficients

$$\text{Benzene} \quad h_i = \text{Nu}_p k_f / ID_p = 272 \quad h_p = h_i ID_p / OD_p = 256$$

$$\text{H}_2\text{O} \quad h_a = \text{Nu}_a k_f / D_e = 1428 \text{ BTU}/(\text{hr} \cdot \text{ft}^2 \cdot ^\circ\text{R})$$

I. Exchanger Coefficient

$$\frac{1}{U_o} = \frac{1}{h_p} + \frac{1}{h_a} \quad U_o = 217 \text{ BTU}/(\text{hr} \cdot \text{ft}^2 \cdot ^\circ\text{R}) \text{ (new)}$$

J. Heat Balance

$$\text{Benzene} \quad q_c = \dot{m}_c C_{pc} (t_2 - t_1) = 28.9 \text{ BTU/s} = 1.04 \times 10^5 \text{ BTU/hr}$$

$$\text{H}_2\text{O} \quad q_w = \dot{m}_w C_{pw} (T_1 - T_2)$$

$$\text{with } \dot{m}_w = 4.33 \text{ lbm/s, we find}$$

$$T_2 = 133.3^\circ\text{F}$$

K. Outlet Temperature Calculations (exchanger length L is to be found)

$$\text{H}_2\text{O} \quad T_2 = 133.3^\circ\text{F}$$

$$\text{Benzene} \quad t_2 = 100^\circ\text{F} \quad (\text{required})$$

L. Log Mean Temperature Difference

$$\text{Counterflow} \quad \text{LMTD} = \frac{(T_1 - t_2) - (T_2 - t_1)}{\ln [(T_1 - t_2) / (T_2 - t_1)]} = 48.6^\circ\text{F}$$

M. Fouling Factors and Design Coefficient

$$R_{di} = 0.001 \quad R_{do} = 0.00075 \text{ ft}^2 \cdot \text{hr} / \text{BTU}$$

$$\frac{1}{U} = \frac{1}{U_o} + R_{di} + R_{do} \quad U = 157 \text{ BTU}/(\text{hr} \cdot \text{ft}^2 \cdot ^\circ\text{R}) \quad (\text{design})$$

N. Heat Transfer Area and Tube Length (unless already known)

$$A_o = \frac{q}{U(LMTD)} = \frac{1.04 \times 10^5}{157(48.6)} = 13.61 \text{ ft}^2$$

$$L = \frac{A_o}{\pi(OD_p)} = 37.8 \text{ ft (3 heat exchangers, each 15 ft long)}$$

O. Friction Factors

Benzene	$Re_p = V_p ID_p / \nu = 8.2 \times 10^4$ $\frac{\epsilon}{ID_p} = \text{smooth}$	}	$f_p = 0.019$
H₂O	$Re_a = V_a D_h / \nu = 6.0 \times 10^4$ $\frac{\epsilon}{D_h} = \text{smooth}$	}	$f_a = 0.020$

P. Pressure Drop Calculations

$$\text{Benzene} \quad \Delta p_p = \frac{f_p L}{ID_p} \frac{\rho_p V_p^2}{2g_c} = 179 \text{ psf} = 1.24 \text{ psi}$$

$$\text{H}_2\text{O} \quad \Delta p_a = \left(\frac{f_a L}{D_h} + 1 \right) \frac{\rho_a V_a^2}{2g_c} = 543 \text{ psf} = 3.77 \text{ psi}$$

Q. Summary of Information Requested in Problem Statement

All that is needed to heat the benzene from 75°F to 100°F is a length of 37.8 ft under the conditions of this problem, notably the arbitrary selection of 6 ft/s for the water velocity. Note that if the flow rate of water is increased, the length required is decreased but not by much. For example, if the flow rate is increased from 4.33 lbm/s to 6.4 lbm/s; this would yield the maximum permissible velocity for water in the optimum range. However, the length of exchanger required would decrease from 37.8 ft to 36 ft. The corresponding pressure drop would increase to over 7 psi.

If, however, we are required to use three of the 2 × 1¹/₄ double pipe heat exchangers to give an overall length of 45 ft, the water velocity can be as low as 3 ft/s.

Note that the length of 37.8 ft calculated in this example is based on U , the design coefficient. The fouling factors are estimates of buildup on the tube walls that will exist after one year. Therefore, the design length will deliver the required outlet temperatures after one year of operation. When the exchanger is new, however, it will perform better than required. Its

performance is expected to deteriorate over the one-year period until the design condition is achieved. At this point, it should be disassembled, cleaned, and put back into service (or replaced).

Route the benzene through the inner tube and the water through the annulus. Set the water flow rate at 4.33 lbm/s or 15,600 lbm/hr.

Summary of Performance of Heat Exchanger (Counterflow)

New	After 1 year
$U_0 = 217 \text{ BTU}/(\text{hr}\cdot\text{ft}^2\cdot^\circ\text{R})$	$U = 157 \text{ BTU}/(\text{hr}\cdot\text{ft}^2\cdot^\circ\text{R})$
$T_2 = 133.3^\circ\text{F}$ (water)	$T_2 = 132.4^\circ\text{F}$
$t_2 = 110.2^\circ\text{F}$ (benzene)	$t_2 = 103.4^\circ\text{F}$
$q_w = 40.8 \text{ BTU/s}$	$q = 32.9 \text{ BTU/s}$

8.5 Summary

In this chapter, we have described double pipe heat exchangers in counterflow and in parallel flow arrangements. The LMTD method of analysis was presented, and temperature profiles for various flow configurations were given. The effectiveness-NTU method was also described. Design considerations associated with the sizing of double pipe heat exchangers were discussed.

TABLE 8.4. Effectiveness equations for various heat exchangers. (From Janna, W. S., *Engineering Heat Transfer 3rd edition*, CRC Press, 2009.)

Double Pipe	Counterflow	$E = \frac{1 - \exp [-N(1 - C)]}{1 - C \exp [-N(1 - C)]}$
	Parallel flow	$E = \frac{1 - \exp [-N(1 + C)]}{1 + C}$
Shell and Tube 1 shell pass; 2, 4, 6, etc., tube passes		
$E = 2 \left\{ 1 + C + \frac{1 + \exp [-N(1 + C^2)^{1/2}]}{1 - \exp [-N(1 + C^2)^{1/2}]} (1 + C^2)^{1/2} \right\}^{-1}$		

Cross flow	mixed–unmixed with ($\dot{m}C_p$) _{min} unmixed	$E = C\{1 - \exp[-C(1 - \exp[-N])]\}$
Cross flow	mixed–unmixed with ($\dot{m}C_p$) _{max} unmixed	$E = 1 - \exp[-C(1 - \exp[-N \cdot C])]$
Cross flow	unmixed–unmixed	$E \approx 1 - \exp[CN^{0.22}\{\exp[-CN^{0.78}] - 1\}]$

$$N = \frac{UA}{(\dot{m}C_p)_{\min}}$$

$$C = \frac{(\dot{m}C_p)_{\min}}{(\dot{m}C_p)_{\max}} < 1$$

8.6 Show and Tell

1. Obtain two different sizes of copper tubing and the appropriate fittings and show how a double pipe heat exchanger would be constructed.
2. Obtain a double pipe heat exchanger and illustrate how it operates.
3. Obtain a catalog of double pipe heat exchangers and discuss the various designs that have been implemented.

8.7 Problems

Wall Resistance

1. In many problems, the resistance of the tube (or pipe) wall is considered negligible when calculating the overall heat transfer coefficient. For the following data, determine whether this is a good assumption:

$$h_i = 1\,200 \text{ W}/(\text{m}^2 \cdot \text{K}) \quad h_o = 1\,100 \text{ W}/(\text{m}^2 \cdot \text{K})$$

2-nominal schedule 40 stainless steel pipe

2. In many problems, the resistance of the tube (or pipe) wall is considered negligible when calculating the overall heat transfer coefficient. For the following data, determine whether this is a good assumption:

$$h_i = 175 \text{ BTU}/(\text{hr} \cdot \text{ft}^2 \cdot ^\circ\text{R}) \quad h_o = 200 \text{ BTU}/(\text{hr} \cdot \text{ft}^2 \cdot ^\circ\text{R})$$

3-std type K copper tubing

3. In many problems, the resistance of the tube (or pipe) wall is considered negligible when calculating the overall heat transfer coefficient. For the following data, determine whether this is a good assumption:

$$h_i = 8\,500 \text{ W}/(\text{m}^2 \cdot \text{K}) \quad h_o = 500 \text{ W}/(\text{m}^2 \cdot \text{K})$$

2-std type K copper tubing

4. Calculate the overall heat transfer coefficient for the following:

$$h_i = 1\,100 \text{ W}/(\text{m}^2\cdot\text{K}) \quad h_o = 200 \text{ W}/(\text{m}^2\cdot\text{K})$$

1^{1/4}-standard type M copper tube

Make the calculations assuming that the wall resistance is negligible and again assuming that it is not. Compare the results.

5. Calculate the overall heat transfer coefficient for the following:

$$h_i = 200 \text{ BTU}/\text{hr}\cdot\text{ft}^2\cdot^\circ\text{R} \quad h_o = 300 \text{ BTU}/\text{hr}\cdot\text{ft}^2\cdot^\circ\text{R}$$

2-std type M copper tubing

Make the calculations assuming that the wall resistance is negligible and again assuming that it is not. Compare the results.

6. Calculate the overall heat transfer coefficient for the following cases:

$$\begin{array}{ll} \text{a. } h_i = 1\,500 \text{ W}/(\text{m}^2\cdot\text{K}) & h_o = 1\,200 \text{ W}/(\text{m}^2\cdot\text{K}) \\ \text{b. } h_i = 1\,500 \text{ W}/(\text{m}^2\cdot\text{K}) & h_o = 200 \text{ W}/(\text{m}^2\cdot\text{K}) \end{array}$$

What conclusions can be drawn about these two cases, specifically with regard to how close the coefficients are in part a, and to how different they are in part b? Use 2-std type M copper tube dimensions.

7. Calculate the overall heat transfer coefficient for the following cases:

$$\begin{array}{ll} \text{a. } h_i = 400 \text{ BTU}/\text{hr}\cdot\text{ft}^2\cdot^\circ\text{R} & h_o = 290 \text{ BTU}/\text{hr}\cdot\text{ft}^2\cdot^\circ\text{R} \\ \text{b. } h_i = 400 \text{ BTU}/\text{hr}\cdot\text{ft}^2\cdot^\circ\text{R} & h_o = 75 \text{ BTU}/\text{hr}\cdot\text{ft}^2\cdot^\circ\text{R} \end{array}$$

What conclusions can be drawn about these two cases, specifically with regard to how close the coefficients are in part a, and to how different they are in part b? Use 2 std type M copper tube dimensions.

Log Mean Temperature Difference

8. A hot fluid at 65°C is cooled in a double pipe heat exchanger to 30°C. A cool fluid is warmed in the exchanger from 15°C to 20°C. Calculate log mean temperature difference for (a) counterflow and (b) parallel flow.
9. A cool fluid enters a double pipe heat exchanger and is heated from 65°F to 150°F. A warmer fluid is cooled in the heat exchanger from 200°F to 180°F. Calculate LMTD for (a) counterflow and for (b) parallel flow.
10. A fluid passes through a double pipe heat exchanger and changes phase but does not change temperature. A warm fluid at a temperature of 220°F enters a double pipe heat exchanger and leaves still at a temperature of 220°F. A cooler fluid enters the exchanger at 100°F and is heated to 150°F. Determine the LMTD for (a) counterflow and for (b) parallel flow.
11. Repeat Problem 8.10 for a phase change fluid temperature of 5°C and a warmer fluid temperature change of 45°C (inlet) to 10°C (outlet).
12. Calculate the LMTD for (a) counterflow and for (b) parallel flow for the following fluid temperatures:

Warmer Fluid	Cooler Fluid
$T_1 = 110^\circ\text{C}$	$t_1 = 25^\circ\text{C}$
$T_2 = 70^\circ\text{C}$	$t_2 = 40^\circ\text{C}$

13. Beginning with Equation 8.10, derive Equation 8.17.
14. Beginning with Equation 8.10, derive Equation 8.19.
15. For counterflow in a double pipe heat exchanger, the following equations were derived:

$$T_2 = \frac{(1 - R)T_1 + (1 - E_c)Rt_1}{1 - RE_c}$$

$$\text{where } R = \frac{\dot{m}_c C_{pc}}{\dot{m}_w C_{pw}} = \frac{T_1 - T_2}{t_2 - t_1}$$

$$\text{and } E_c = \exp \left[\frac{UA_o}{\dot{m}_c C_{pc}} (R - 1) \right]$$

How are these equations affected when one of the fluids in the exchanger changes phase so that its temperature is the same at inlet and outlet? Simplify these equations for the case where $T_1 = T_2$.

16. For parallel flow in a double pipe heat exchanger, the following equations apply:

$$T_2 = \frac{(R + E_p)T_1 + (E_p - 1)Rt_1}{(R + 1)E_p}$$

$$\text{where } R = \frac{\dot{m}_c C_{pc}}{\dot{m}_w C_{pw}} = \frac{T_1 - T_2}{t_2 - t_1}$$

$$\text{and } E_p = \exp \left[\frac{UA_o}{\dot{m}_c C_{pc}} (R + 1) \right]$$

How are these equations affected when one of the fluids in the exchanger changes phase so that its temperature is the same at inlet and outlet? Simplify these equations for the case where $T_1 = T_2$.

17. The parameter E_c was derived as

$$E_c = \exp \left[\frac{UA_o}{\dot{m}_c C_{pc}} (R - 1) \right]$$

and in terms of temperature,

$$E_c = \frac{T_1 - t_2}{T_2 - t_1}$$

The parameter E_p is

$$E_p = \exp \left[\frac{UA_o}{\dot{m}_c C_{pc}} (R + 1) \right]$$

Show that E_p in terms of temperatures is $(T_1 - t_1)/(T_2 - t_2)$.

LMTD and Effectiveness

18. Measured inlet and outlet temperatures of the fluids flowing through a heat exchanger are

$$\begin{array}{ll} T_1 = 170^\circ\text{F} & T_2 = 70^\circ\text{F} \\ t_1 = 50^\circ\text{F} & t_2 = 100^\circ\text{F} \end{array}$$

- Which fluid has the higher capacitance?
- Calculate E_c .
- Calculate the effectiveness.

19. Measured inlet and outlet temperatures of the fluids flowing through a heat exchanger are

$$\begin{array}{ll} T_1 = 80^\circ\text{C} & T_2 = 60^\circ\text{C} \\ t_1 = 40^\circ\text{C} & t_2 = 50^\circ\text{C} \end{array}$$

- Which fluid has the higher capacitance?
- Calculate E_c .
- Calculate the effectiveness.

20. Measured inlet and outlet temperatures of the fluids flowing through a heat exchanger are

$$\begin{array}{ll} T_1 = 90^\circ\text{C} & T_2 = 90^\circ\text{C} \\ t_1 = 25^\circ\text{C} & t_2 = 40^\circ\text{C} \end{array}$$

- Which fluid has the higher capacitance?
- Calculate E_c .
- Calculate the effectiveness.

21. A double pipe heat exchanger consists of four hairpin exchangers that are each 6 ft long. The inner tube of these exchangers is $1^{1/4}$ -standard tubing, while the outer tube is 2-standard, both type M. The inlet temperatures are 100°F and 200°F . For the following conditions, determine the expected outlet temperatures:

$$\begin{array}{ll} \text{Capacitance Ratio} & = 1.5 \\ \text{Capacitance for Cooler Fluid} & = 10,000 \text{ BTU}/(\text{hr}\cdot^\circ\text{R}) \\ \text{Overall Heat Transfer Coefficient} & = 185 \text{ BTU}/(\text{hr}\cdot\text{ft}^2\cdot^\circ\text{R}) \end{array}$$

Calculate the effectiveness of the exchanger.

22. A double pipe heat exchanger consists of two hairpin exchangers that are 6 ft long. The inner tube of these exchangers is $1^{1/4}$ -standard tubing, while the outer

tube is 2-standard, both type M. The inlet temperatures are 90°F and 180°F. For the following conditions, determine the expected outlet temperatures:

Capacitance Ratio = 3.5
 Capacitance for Cooler Fluid = 18,000 BTU/(hr·°R)
 Overall Heat Transfer Coefficient = 225 BTU/(hr·ft²·°R)

23. For the heat exchanger described in Example 8.4, calculate the following parameters:
- Effectiveness
 - The number of transfer units
 - The ratio of capacitances

Heat Exchanger Calculations (Use the LMTD or the E-NTU method.)

24. Water is used to cool ethylene glycol in a 60-ft-long double pipe heat exchanger made of 4-std and 2-std (both type M) copper tubing. The water inlet temperature is 60°F and the ethylene glycol inlet temperature is 180°F.
 The flow rate of the ethylene glycol is 20 lbm/s, while that for the water is 30 lbm/s. Calculate the expected outlet temperature of the ethylene glycol and determine the pressure drop expected for both streams. Assume counterflow, and place the ethylene glycol in the inner tube. Compose a Summary of Performance chart.
25. Reverse the direction of either fluid in Problem 8.24 and repeat the calculations for parallel flow, again with ethylene glycol in the inner tube. Compose a Summary of Performance chart.
26. Problem 8.24 was solved with water in the annulus and ethylene glycol in the inner tube. Repeat the calculations placing water in the pipe and ethylene glycol in the annulus. Compose a Summary of Performance chart.
27. Four 2-m-long double pipe heat exchangers made of 4-std type M and 3-std type M copper tubing are connected in series to form one 8-m-long exchanger. It is used to cool (unused) engine oil. The exchanger takes in water at 20°C at a flow rate of 30 kg/s and oil at a temperature of 140°C with a flow rate of 0.2 kg/s. Determine the expected outlet temperature of the oil and the pressure drop encountered by both streams. Assume counterflow. Compose a Summary of Performance chart.
28. Ammonia is used in liquid form for a process and is needed at a temperature of 15°C. It is available at 0°C. Liquid carbon dioxide is used rather than water as a heat source (using water may lead to trouble). Liquid carbon dioxide is available at 5°C and a mass flow rate of 1.25 kg/s. A 15-m-long double pipe heat exchanger, made of 2¹/₂ × 1¹/₄ type K copper tubing, is available for this service. For an ammonia flow rate of 1.2 kg/s, calculate outlet temperatures and pressure drops. Will this setup work? Compose a Summary of Performance chart.
29. In an air separation plant, air is cooled and its components are separated from the mixture. Cooled oxygen at a temperature of 20°C must be heated to a temperature of 30°C for accurate metering. The oxygen flow rate at 30°C is 0.01 kg/s. Air (available at 35°C and 0.015 kg/s) is used as the heating medium. A number of 3 × 2 schedule 40 double pipe heat exchangers that are 2 m long and made of galvanized steel are available. Determine how many are required. Calculate outlet temperatures and pressure drops. Compose a Summary of Performance chart.
30. A 4 × 3 double pipe heat exchanger, 15 ft long and made of type M copper tubing, is used to cool air having an inlet temperature of 120°F and a flow rate of 0.07 lbm/s. Methyl chloride is the cooling medium, and it is available at 15°F and

a flow rate of 0.05 lbm/s. Calculate outlet temperatures and pressure drops. Compose a Summary of Performance chart.

31. An earth-coupled heat pump consists of a heat pump apparatus that transfers heat from a well (dug into the ground) to a home. Within this apparatus is a double pipe heat exchanger that exchanges heat between water and refrigerant-22. When operating at steady state, the refrigerant enters the heat exchanger as a saturated liquid at 5°C and leaves at the same temperature but as a saturated vapor. Water enters the exchanger at 18°C. The mass flow rate of the refrigerant is 0.1 kg/s and the heat exchanger is made of 4 × 3 type M copper tubing. The water flow rate is 48.5 kg/s. The heat exchanger length is 0.5 m, and the enthalpy of vaporization of the R-22 is taken to be 200 kJ/kg.
- Sketch the expected temperature versus length diagram for this exchanger.
 - Calculate the overall heat transfer coefficient.
 - What are the pressure drops for each fluid stream?
- Compose a Summary of Performance chart.

Temperature Profiles

32. Figure 8.7 shows temperature versus length for a counterflow heat exchanger having $\dot{m}_c C_{pc} > \dot{m}_w C_{pw}$. In this problem, we develop a temperature versus length profile for the case where $\dot{m}_c C_{pc} < \dot{m}_w C_{pw}$. Ethylene glycol enters a heat exchanger at 30°C and a mass flow rate of 5 kg/s. Water enters the heat exchanger at 10°C and a flow rate of 2 kg/s. The heat exchanger is made of 2¹/₂ × 1¹/₄ type M copper tubing. Determine outlet temperatures versus length up to the point where the outlet temperatures are equal. Graph temperature versus length and compare the results to those of Figure 8.7. Use counterflow.
33. Figure 8.8 shows temperature versus length for a parallel flow heat exchanger having $\dot{m}_c C_{pc} > \dot{m}_w C_{pw}$. In this problem, we develop a temperature versus length profile for the case where $\dot{m}_c C_{pc} < \dot{m}_w C_{pw}$. Ethylene glycol enters a heat exchanger at 30°C and a mass flow rate of 5 kg/s. Water enters the heat exchanger at 10°C and a flow rate of 2 kg/s. The heat exchanger is made of 2¹/₂ × 1¹/₄ type M copper tubing. Determine outlet temperatures versus length for lengths that vary to 100 m. Graph temperature versus length and compare the results to those of Figure 8.7. Use parallel flow.

Effectiveness-NTU Derivations

34. The derivation of effectiveness for a double pipe heat exchanger operating in counterflow proceeded to the point where the following equation was derived:

$$\frac{T_1 - t_1}{T_2 - t_1} - \frac{T_1 - T_2}{C(T_2 - t_1)} = \exp[-N(1 - C)]$$

The text then says that the left-hand side of this equation is

$$\frac{1}{1 - EC} - \frac{1}{C} \frac{EC}{1 - EC} = \exp[-N(1 - C)]$$

We investigate this claim in this problem. For counterflow, Equation 8.28 is

$$E = \frac{1 - \exp[-N(1 - C)]}{1 - C \exp[-N(1 - C)]} = \frac{t_2 - t_1}{T_1 - t_1} \quad \text{if } \dot{m}_c C_{pc} < \dot{m}_w C_{pw}$$

Also,

$$C = \frac{(\dot{m}C_p)_c}{(\dot{m}C_p)_w} = \frac{T_1 - T_2}{t_2 - t_1}$$

- Evaluate EC in terms of temperature.
- Evaluate $1 - EC$.
- Evaluate $1/(1 - EC)$.
- Evaluate $EC/(1 - EC)$.

Were the substitutions correct?

- The effectiveness of a counterflow heat exchanger was derived in the text. Derive a similar equation for parallel flow by completing the following steps:
 - Set the heat balance equations equal to one another; namely,

$$q = UA(LMTD) = \dot{m}_c C_{pc}(t_2 - t_1)$$

and show that for parallel flow

$$\ln \frac{T_1 - t_1}{T_2 - t_2} = \frac{UA}{\dot{m}_c C_{pc}} \left(\frac{T_1 - T_2}{t_2 - t_1} + 1 \right)$$

- With $(\dot{m}C_p)_{\min}$ being that for the cooler fluid, substitute

$$N = \frac{UA}{\dot{m}_c C_{pc}}$$

and

$$C = \frac{\dot{m}_c C_{pc}}{\dot{m}_w C_{pw}} = \frac{T_1 - T_2}{t_2 - t_1}$$

Show that

$$\frac{T_2 - t_2}{T_1 - t_1} = \exp [-N(1 + C)]$$

- Solve the equation for C (given in part b) for T_2 , and substitute into the numerator of the preceding equation. Rearrange and show that

$$\frac{(T_1 - t_1) - (1 + C)(t_2 - t_1)}{(T_1 - t_1)} = \exp [-N(1 + C)]$$

(Hint: Add and subtract t_1 from the numerator of the left-hand side after substituting for T_2 .)

- The definition of effectiveness for this system is

$$E = \frac{t_2 - t_1}{T_1 - t_1} \quad \text{if } \dot{m}_c C_{pc} < \dot{m}_w C_{pw}$$

Substitute into the result of part c and simplify. Show that

$$E = \frac{1 - \exp[-N(1 + C)]}{1 + C} \quad (\text{parallel flow})$$

36. Under what conditions does C , the ratio of capacitances in the effectiveness-NTU method, equal R in the LMTD method?
37. A **balanced** heat exchanger is one in which the capacitances for both fluids are equal; i.e., $\dot{m}_c C_{pc} = \dot{m}_w C_{pw}$. How is the effectiveness affected? Show that the effectiveness for a balanced exchanger in counterflow is

$$E = \frac{N}{1 + N}$$

38. A **balanced** heat exchanger is one in which the capacitances for both fluids are equal; that is, $\dot{m}_c C_{pc} = \dot{m}_w C_{pw}$. How is the effectiveness affected? Show that the effectiveness for a balanced exchanger in parallel flow is:

$$E = \frac{1}{2} (1 - \exp(-2N))$$

39. Referring to Table 8.3, we want to add a 6 × 4 double pipe heat exchanger to the list. Make all calculations for each column in the table for this new combination.

Design Problems

40. Raw water is used to cool the distilled water of a small electrical generating facility by using a double pipe heat exchanger. The raw water is taken from a nearby stream and is available at a temperature that ranges from 32°F to 55°F over the course of one year. The distilled water is to be cooled from 210°F to as cold as possible. The raw water flow rate is 8500 lbm/hr, while the distilled water flow rate is 8000 lbm/hr. The exchanger is 18 ft long and is made of 2-std and 1¹/₄-std, both type M copper tubing. Predict outlet temperatures over the course of one year, i.e., for raw water inlet temperatures that range 32°F to 55°F.
41. Water at a temperature of 150°F is to be used to heat glycerin in a 4 × 3-std type M copper tubing heat exchanger that is 10 ft long. The water flow rate is 10 lbm/s. Glycerin is available at a temperature of 75°F. Determine water and glycerin outlet temperatures as a function of glycerin flow rate, which ranges from 0.01 to 1 lbm/s.
42. Kerosene [$\rho = 0.73(1\,000) \text{ kg/m}^3$, $C_p = 2\,470 \text{ J/(kg}\cdot\text{K)}$, $\mu = 0.40 \text{ cp}$, $k_f = 0.132 \text{ W/(m}\cdot\text{K)}$, and $1.5 \leq V_{\text{optimum}} \leq 3 \text{ m/s}$] is to be preheated in a double pipe heat exchanger before being pumped to a distillation facility. The kerosene flow rate is 8 000 kg/hr and it is to be heated from 24 to 35°C. Water is available from the condensed exhaust of a small steam turbine, and its flow rate can be controlled. The water is available at 95°C. Select an appropriate heat exchanger.
43. Crude oil is stored in a tank and maintained at 30°C. It is to be pumped to a distillation column for separation into usable products. Kerosene leaves the column at 200°C, and it is proposed to transfer heat from the kerosene to the crude oil so that the cost associated with heating the crude oil by other means is reduced. The crude oil has a flow rate of 12 000 kg/hr, and a significant savings will be realized if it can be heated at least to 50°C. Determine an appropriate heat exchanger to use and analyze it completely. Use the following properties for both

fluids:

Kerosene		Crude Oil
42	°API	34
0.73	sp. gr.	0.83
0.4 cp	viscosity	3.6 cp [†]
2 470 J/(kg·K)	spec heat	2 050 J/(kg·K)
0.132 W/(m·K)	therm con	0.133 W/(m·K)
1.5–3 m/s	opt velocity	1.50–3 m/s

[†]cp is an abbreviation for centipoise, a unit of viscosity.

44. A phosphate solution [Sp. Gr. = 1.3, $C_p = 0.757$ BTU/(lbm·°R), $\mu_{g_c} = 2.9$ lbm/(ft·hr), and $k_f = 0.3$ BTU/(hr·ft·°R)] is used in the production of fertilizer. The solution is to be cooled from 150°F to 140°F using well water, available at 55°F. The phosphate solution flows at a rate of 7000 lbm/hr and has a dirt factor of 0.001 hr·ft·°R/BTU. Select an appropriate exchanger and analyze it completely.
45. Engine oil is used as a lubricant, and due to an unforeseen “hot spot” in the system, the oil temperature reaches 95°C. The oil has a flow rate of 500 kg/hr, and it should be cooled to 75°C. Raw water is available at a temperature of 25°C. Select an appropriate heat exchanger for this service and analyze it completely.
46. Acetone is used as a solvent for greases and other petroleum products. It is pumped to a machine for bottling, which can fill 9,000 sixteen-ounce bottles per hour. The oil is stored in tanks and is maintained at 45°C. A flowmeter in the pipeline from tank to machine requires the oil to be at 30°C for accurate metering. City water is available at 15°C for cooling the acetone. Select an appropriate heat exchanger and analyze it completely.
47. The process of printing an advertising brochure involves the use of four colors of ink. The ink itself is actually a mixture of solvent (toluene) and ink solids (pigment). Ordinarily, the ink is stored in tanks and fed directly to printing presses, but a problem has arisen. If the ink mixture is left unattended in a tank, the solids tend to settle out, which results in a non-uniform color being fed to the presses. Using a mixer in the ink tank has been ruled out as an acceptable solution to the problem. Instead, the ink is circulated about the building via a pump-and-piping system and is then returned to the tank. This movement warms the ink in the tank to about 130°F. The toluene tends to vaporize too quickly, however, when the ink temperature exceeds 120°F, thus causing a health hazard. Furthermore, the printing process itself requires that the ink mix be supplied at a flow rate of 60,000 lbm/hr. Water at 65°F is available for cooling the ink. Determine the type of heat exchanger to be used to cool the ink to 110°F and analyze it completely. Take the properties of ink to be the same as those for toluene:

$$\begin{aligned}
 \rho &= 54.3 \text{ lbm/ft}^3 & C_p &= 0.44 \text{ BTU/(lbm·°R)} \\
 \mu &= 0.41 \text{ cp} & k_f &= 0.085 \text{ BTU/(hr·ft·°R)} \\
 4.4 &\leq V_{\text{optimum}} \leq 9.5 \text{ ft/s}
 \end{aligned}$$

CHAPTER 9 Shell and Tube Heat Exchangers

In this chapter, we continue the discussion begun in the last chapter by considering shell and tube heat exchangers. They are described and methods of modeling their performance are discussed. The LMTD and the effectiveness-NTU methods are both used in the analysis. Design methods are then presented, and they include the sizing and selection of shell and tube heat exchangers. An analysis for determining the optimum outlet temperature is given, and it is shown how to use it to minimize the costs associated with sizing a heat exchanger.

9.1 The Shell and Tube Heat Exchanger

A double pipe heat exchanger consists of two concentric tubes with a heat transfer area equal to the outer surface area of the inner tube. The flow and heat transfer rates in these exchangers are moderate because the apparatus is comparatively small. For large flow rates and in applications where great heat transfer rates are required, larger cross-sectional and surface areas are necessary. These conditions are addressed through the use of what is known as a **shell and tube heat exchanger**.

Figure 9.1 is a sketch of a shell and tube heat exchanger. It consists of a **shell** that in essence is a cylinder of diameter ranging from less than 12 in. (30 cm) to over 39 in. (100 cm). The shell can be made as long as necessary to deliver the required heat transfer rate. The shell is used to house a number of tubes, called a **tube bundle** (up to 1200 in a 39-in. inside-diameter shell), which are pressed into what are called **tube sheets**. The tube sheets hold the tubes in position, and the tube-to-tube-sheet connection must be made leakproof. Attached to the ends of the shell are **channels**, and within the shell are **baffles** to control the flow of the fluid that passes through the shell and around the tubes.

Shells

The shell of a shell and tube exchanger is usually made of wrought iron or steel pipe, but special metals can be used when corrosion might be a

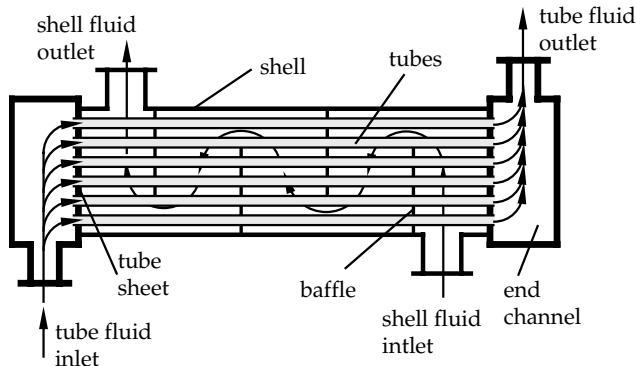


FIGURE 9.1. Sketch of a shell and tube heat exchanger.

problem. Operating pressures greatly influence the wall thickness required. Appendix Table D.1 gives pipe dimensions that apply to shells. In some applications, the inside surface of the shell is machined.

Tubes

The tubes used in a shell and tube heat exchanger are specified differently from water tubing. Specifications of **heat exchanger tubes**, known also as **condenser tubes**, follow a standard called the **Birmingham Wire Gage**, abbreviated BWG. A 1-in. condenser tube will have an outside diameter of 1 in. Table 9.1 presents dimensions of condenser tubes. Heat exchanger tubes are available in a variety of metals.

The tubes are held in position within the shell by holes drilled in the tube sheets. Tube holes cannot be drilled too close to one another because the tube sheet becomes structurally weakened, although it is desirable to use as many tubes as possible. The distance between adjacent tube centers, called the **tube pitch**, has been standardized. Tubes are laid out so that adjacent tube centers form square or triangular patterns.

Figure 9.2 shows tubes laid out on a square pitch pattern, while Figure 9.3 shows a triangular configuration. Common square layouts are made up of 3/4 in.-OD tubes on a 1 in.-square pitch, and 1 in.-OD on a 1-1/4 in. square pitch. Common triangular layouts are 3/4 in.-OD on a 15/16 in. triangular pitch, 3/4 in.-OD on a 1 in.-triangular pitch, and 1 in.-OD on a 1-1/4 in. triangular pitch.

Tube Counts

Table 9.2 presents what are called **tube counts**. For a given shell diameter, the tube count is the maximum number of tubes that can be placed within the shell and not significantly weaken the tube sheet.

TABLE 9.1. *Physical dimensions of condenser tubes in terms of BWG.*
 (From Process Heat Transfer by D. Q. Kern, McGraw-Hill Book Co., 1950, p. 843.)

Tube OD		BWG	Tube ID	
inches	cm		inches	cm
3/4	1.91	10	0.482	1.22
		11	0.510	1.29
		12	0.532	1.35
		13	0.560	1.42
		14	0.584	1.48
		15	0.606	1.54
		16	0.620	1.57
		17	0.634	1.61
		18	0.652	1.66
1	2.54	8	0.670	1.70
		9	0.704	1.79
		10	0.732	1.86
		11	0.760	1.93
		12	0.782	1.99
		13	0.810	2.06
		14	0.834	2.12
		15	0.856	2.17
		16	0.870	2.21
		17	0.884	2.25
		18	0.902	2.29

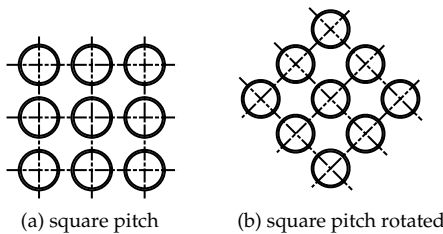


FIGURE 9.2. *Square pitch layout.*

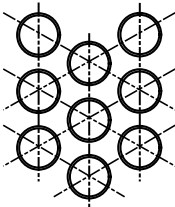


FIGURE 9.3. *Triangular pitch layout.*

Flow Configuration

Also shown in Figure 9.1 are fluid flow lines. The two fluid streams transfer heat within the exchanger. One of the fluids, referred to as the **tube fluid**, enters an end channel and is routed through all the tubes. The tube fluid then exits through the other end channel. The second fluid, referred to as the **shell fluid**, enters at the shell fluid inlet and is routed

TABLE 9.2. *Maximum number of tubes (tube counts) for shell and tube equipment.*

3/4 in. OD on a 1 in. square pitch					
Shell ID inches	1-P	2-P	4-P	6-P	8-P
8	32	26	20	20	
10	52	52	40	36	
12	81	76	68	68	60
13 ¹ / ₄	97	90	82	76	70
15 ¹ / ₄	137	124	116	108	108)
17 ¹ / ₄	177	166	158	150	142
19 ¹ / ₄	224	220	204	192	188
21 ¹ / ₄	277	270	246	240	234
23 ¹ / ₄	341	324	308	302	292
25	413	394	470	356	346
27	481	460	432	420	408
29	553	526	480	468	456
31	657	640	600	580	560
33	749	718	688	676	648
35	845	824	780	766	748
37	934	914	886	866	838
39	1049	1024	982	968	948

3/4 in. OD on a 1 ¹ / ₄ in. square pitch					
Shell ID inches	1-P	2-P	4-P	6-P	8-P
8	21	16	14		
10	32	32	26	24	
12	48	45	40	38	36
13 ¹ / ₄	61	56	52	48	44
15 ¹ / ₄	81	76	68	68	64
17 ¹ / ₄	112	112	96	90	82
19 ¹ / ₄	138	132	128	122	116
21 ¹ / ₄	177	166	158	152	148
23 ¹ / ₄	213	208	192	184	184
25	260	252	238	226	222
27	300	288	278	268	260
29	341	326	300	294	286
31	406	398	380	368	358
33	465	460	432	420	414
35	522	518	488	484	472
37	596	574	562	544	532
39	665	644	624	612	600

3/4 in. OD on a ¹⁵ / ₁₆ in. triangular pitch					
Shell ID inches	1-P	2-P	4-P	6-P	8-P
8	36	32	26	24	18
10	62	56	47	42	36
12	109	98	86	82	78
13 ¹ / ₄	127	114	96	90	86
15 ¹ / ₄	170	160	140	136	128
17 ¹ / ₄	239	224	194	188	178
19 ¹ / ₄	301	282	252	244	234
21 ¹ / ₄	361	342	314	306	290
23 ¹ / ₄	442	420	386	378	364
25	532	506	468	446	434
27	637	602	550	536	524
29	721	692	640	620	594
31	847	822	766	722	720
33	974	938	878	852	826
35	1102	1068	1004	988	958
37	1240	1200	1144	1104	1072
39	1377	1330	1258	1248	1212

3/4 in. OD on a 1 in. triangular pitch					
Shell ID inches	1-P	2-P	4-P	6-P	8-P
8	37	30	24	24	
10	61	52	40	36	
12	92	82	76	74	70
13 ¹ / ₄	109	106	86	82	74
15 ¹ / ₄	151	138	122	118	110
17 ¹ / ₄	203	196	178	172	166
19 ¹ / ₄	262	250	226	216	210
21 ¹ / ₄	316	302	278	272	260
23 ¹ / ₄	384	376	352	342	328
25	470	452	422	394	382
27	559	534	488	474	464
29	630	604	556	538	508
31	745	728	678	666	640
33	856	830	774	760	732
35	970	938	882	864	848
37	1074	1044	1012	986	870
39	1206	1176	1128	1100	1078

TABLE 9.2 continued. *Maximum number of tubes (tube counts) for shell and tube equipment.*

1 in. OD on a 1 ¹ / ₄ in. triangular pitch					
Shell ID inches	1-P	2-P	4-P	6-P	8-P
8	21	16	16	14	
10	32	32	26	24	
12	55	52	48	46	44
13 ¹ / ₄	68	66	58	54	50
15 ¹ / ₄	91	86	80	74	72
17 ¹ / ₄	131	118	106	104	94
19 ¹ / ₄	163	152	140	136	128
21 ¹ / ₄	199	188	170	164	160
23 ¹ / ₄	241	232	212	212	202
25	294	282	256	252	242
27	349	334	302	296	286
29	397	376	338	334	316
31	472	454	430	424	400
33	538	522	486	470	454
35	608	592	562	546	532
37	674	664	632	614	598
39	766	736	700	688	672

around the exterior of the tubes by the baffles. The exchanger can be set up so that the back-and-forth motion of the shell fluid can be side to side or up and down. The shell fluid is made to make multiple passes over the tubes by the presence of what are known as **segmental baffles**.

Baffles

Baffles are placed within the shell of the heat exchanger in order to direct the flow of the shell fluid and to support the tubes. The distance between adjacent baffles is usually a constant and is called the **baffle pitch** or **baffle spacing**. It is typically never greater than the shell diameter or less than 1/5th the shell diameter. Baffles are held securely by baffle spacers (not shown in Figure 9.1). Figures 9.4, 9.5, and 9.6 show three types of baffles: the segmental baffle, the disc and doughnut baffle, and the orifice baffle. Our analysis will be for the segmental baffle only. (Much of the information known about shell and tube heat exchangers is proprietary. Manufacturers obtain their own data or have an outside consultant obtain data. Results are usually kept confidential.)

Figure 9.1 is of a counterflow exchanger. The shell fluid passes through only once, and the tube fluid passes through only once. The exchanger would traditionally be referred to as a **1-1 shell and tube heat exchanger**. Figure 9.7 shows a similar exchanger with modified end channels. A partition has been placed in the left end channel. The shell fluid passes through only

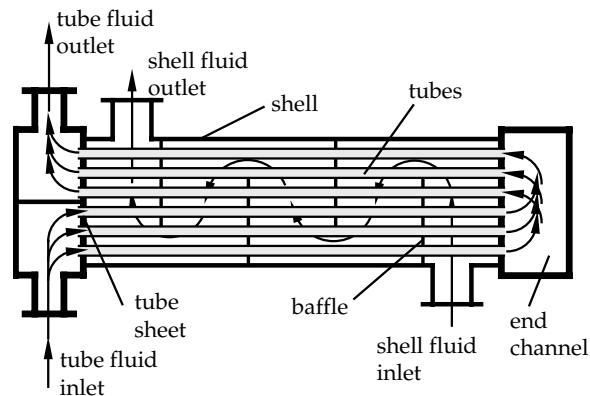
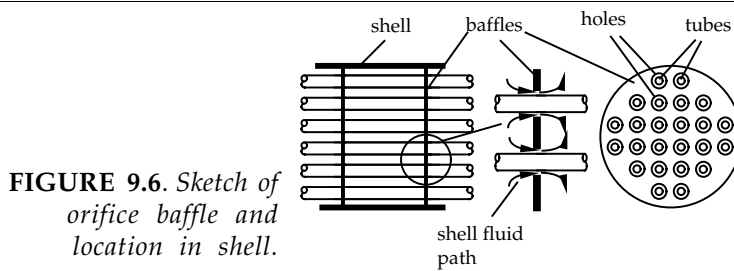
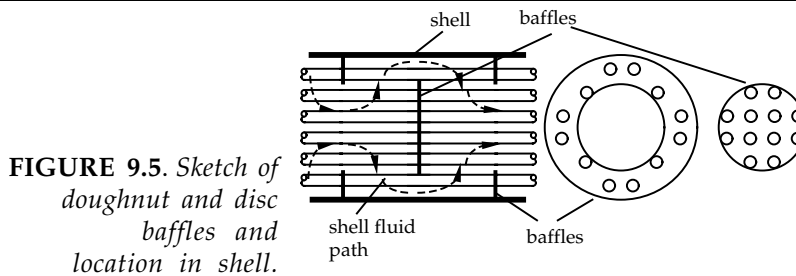
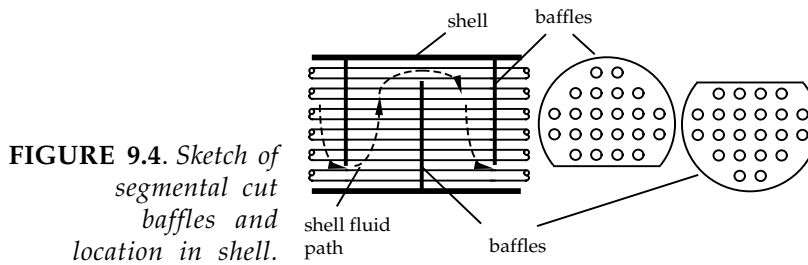


FIGURE 9.7. *Sketch of 1-2 shell and tube heat exchanger.*

once. The tube fluid enters one of the end channels and is routed into only half the tubes. At the other end channel, the tube fluid turns and is routed through the other half of the tubes. The fluid then exits through the same end channel it entered, and so the tube fluid has passed through the exchanger twice. This exchanger would then be called a **1-2 shell and tube heat exchanger**. Modifications to the end channels control the number of times the tube fluid passes through the exchanger. Accordingly, there are 1-4, 1-6, and 1-8 exchangers. An odd number of tube passes is seldom used. The tube count charts, Table 9.2, list the maximum number of tubes that can be placed in an exchanger. At the top of the columns appears “1-P,” “2-P,” etc. These designations refer to the number of tube fluid passes. As the number of passes increases, the tube count decreases.

It is likely that, in some applications, the tubes might like to expand more than the shell would. Consequently, it is necessary to use modified forms of the shell and tube heat exchanger to accommodate thermal expansion effects. Figures 9.8 and 9.9 illustrate only two of the many alternative designs that have been used successfully to solve this problem.

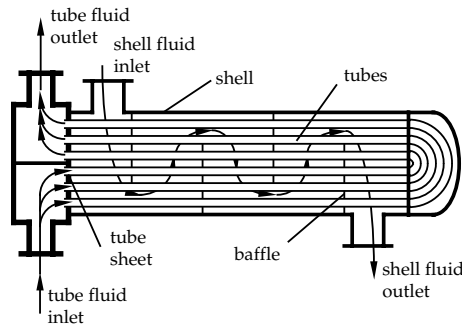


FIGURE 9.8. Sketch of 1-2, U-bend shell and tube heat exchanger.

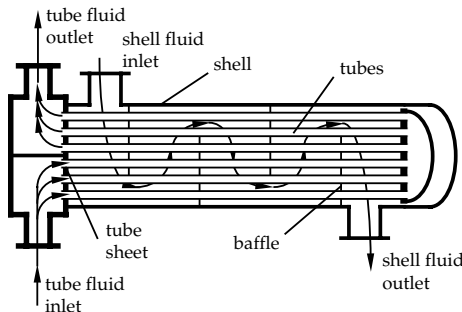


FIGURE 9.9. Sketch of 1-2, pull-through, floating-head shell and tube heat exchanger.

9.2 Analysis of Shell and Tube Heat Exchangers

Figures 9.10 and 9.11 show the temperature variation of two fluids as they flow through a 1-2 shell and tube heat exchanger. The amount of heat exchanged is given by

$$q = U_o A_o \Delta t = \dot{m}_w C_{pw} (T_1 - T_2) = \dot{m}_c C_{pc} (t_2 - t_1) \quad (9.1)$$

where U_o is the overall heat transfer coefficient, A_o is the outside surface area of all the tubes, and Δt is the temperature difference that applies to the 1-2 shell and tube heat exchanger. If the flow through the exchanger is entirely counterflow or parallel flow, then Δt would be the log mean temperature difference for counterflow or parallel flow, respectively.

The 1-2 shell and tube heat exchanger is a combination of both flows, however, as indicated in Figures 9.10 and 9.11. The established method of analysis involves the use of the log mean temperature difference (LMTD) for counterflow, as a “best possible” case, and a correction factor, F . An equation for the correction factor rather than its derivation will be given here.

We begin by introducing a new parameter, called the **temperature factor**, defined as

$$S = \frac{t_2 - t_1}{T_1 - t_1} \quad (9.2)$$

Notice that the denominator of S is the maximum temperature difference associated with the exchanger. Recall the definition of R given earlier as

$$R = \frac{\dot{m}_c C_{pc}}{\dot{m}_w C_{pw}} = \frac{(T_1 - T_2)}{(t_2 - t_1)} \quad (9.3)$$

Next we state (without derivation) the **correction factor** that involves S and R , given as

$$F = \frac{\sqrt{R^2 + 1} \ln [(1 - S)/(1 - RS)]}{(R - 1) \ln \left[\frac{2 - S(R + 1 - \sqrt{R^2 + 1})}{2 - S(R + 1 + \sqrt{R^2 + 1})} \right]} \quad (9.4)$$

The temperature difference of Equation 9.1 now becomes

$$\Delta t = F (LMTD_{\text{counterflow}}) = F \frac{(T_1 - t_2) - (T_2 - t_1)}{\ln [(T_1 - t_2)/(T_2 - t_1)]} \quad (9.5)$$

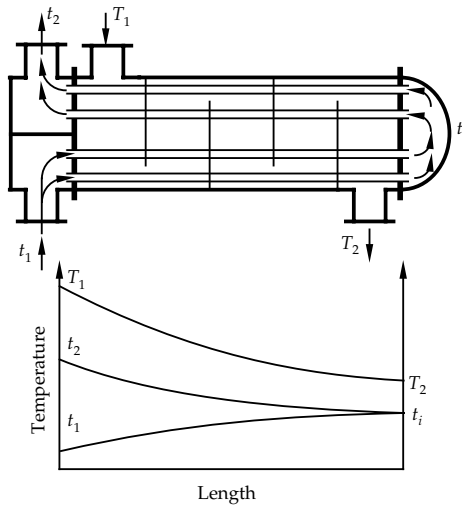


FIGURE 9.10. Temperature variation with length for fluids traveling through a 1-2 shell and tube heat exchanger.

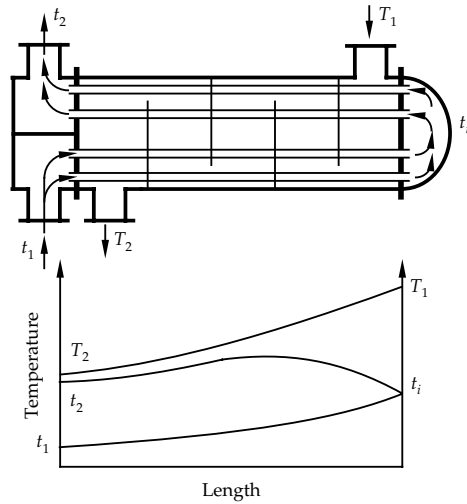


FIGURE 9.11. Temperature variation with length for fluids traveling through a 1-2 shell and tube heat exchanger; an alternative configuration to Figure 9.10.

The correction factor F is graphed in Figure 9.12 as a function of S , with R as an independent parameter. Figure 9.12 applies to shell and tube heat exchangers with 1 shell pass and 2 or more tube passes. From a practical viewpoint, the correction factor F is indicative of how efficient the exchanger is thermally. If F is less than 0.75, then the exchanger is operating in a costly mode and correspondingly low efficiency. Thus, for good practice, $F \geq 0.75$.

The Overall Heat Transfer Coefficient

The equation for heat transfer within a 1-2 shell and tube heat exchanger is written as

$$q = U_o A_o F \frac{(T_1 - t_2) - (T_2 - t_1)}{\ln [(T_1 - t_2)/(T_2 - t_1)]} \quad (9.6)$$

The overall heat transfer coefficient is found with

$$\frac{1}{U_o} = \frac{OD_t}{h_i ID_t} + \frac{1}{h_o} = \frac{1}{h_t} + \frac{1}{h_o} \quad (9.7)$$

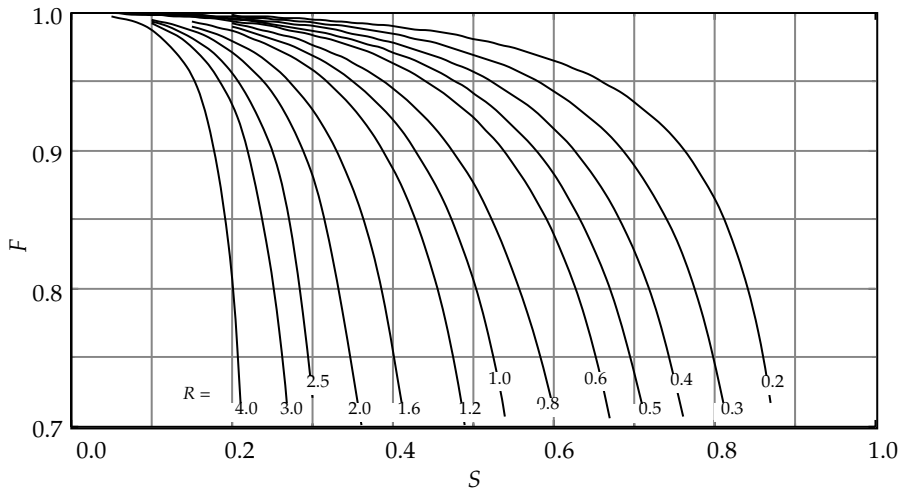


FIGURE 9.12. Correction factor graph for a shell and tube heat exchanger with 1 shell pass and 2 or more tube passes.

where OD_t and ID_t are the outside and inside diameters, respectively, of the tubes used.

Tube Side Convection Coefficient and Pressure Drop

In order to calculate the overall heat transfer coefficient in Equation 9.7, we need equations for h_i and h_o . These surface coefficients are found with the equations written earlier for double pipe heat exchangers, and found in Chapter 8.

The pressure drop encountered by the fluid making N_p passes through the exchanger is a multiple of the kinetic energy of the flow:

$$\Delta p_{\text{tubes}} = N_p \frac{fL}{ID_t} \frac{\rho V^2}{2g_c} \quad (9.8)$$

where f is the friction factor and L is the tube length. In addition, the tube fluid experiences a pressure loss when it is forced to return through the exchanger. The tube fluid encounters a sudden expansion and a sudden contraction. The return pressure loss for the fluid making N_p passes is treated like a minor loss and is given by

$$\Delta p_{\text{return}} = 4N_p \frac{\rho V^2}{2g_c} \quad (9.9)$$

where the 4 is found from empirical measurements. The tube fluid therefore experiences a pressure loss given as

$$\Delta p_t = \Delta p_{\text{tubes}} + \Delta p_{\text{return}} = N_p \left(\frac{fL}{ID_t} + 4 \right) \frac{\rho V^2}{2g_c} \quad (9.10)$$

Shell Side Convection Coefficient and Pressure Drop

The velocity of the fluid passing through the shell varies continuously because the flow area is not constant. The shell fluid travels around tubes and baffles, and although it is not constant, we wish to identify a single representative velocity. In order to do this, we must use a single characteristic length or dimension for the geometry of the shell.

Figure 9.13 shows a cross section of a square pitch layout. The tube pitch P_T and the clearance between adjacent tubes C are both defined. We now develop an equation for the equivalent length, as was done for the double pipe heat exchanger:

$$D_e = \frac{4 \times \text{area}}{\text{heat transfer perimeter}}$$

The area in the above equation is that of a square minus the area of four quarter circles; that is, the shaded area of Figure 9.13. The heat transfer perimeter is that of four quarter circles. Thus, for the square pitch,

$$D_e = \frac{4(P_T^2 - \pi OD_t^2/4)}{\pi OD_t}$$

$$\text{or } D_e = \frac{4P_T^2}{\pi OD_t} - OD_t \quad (\text{square pitch}) \quad (9.11)$$

Figure 9.14 shows a cross section of a triangular pitch layout. By substituting into the expression for equivalent diameter, we obtain

$$D_e = \frac{3.46P_T^2}{\pi OD_t} - OD_t \quad (\text{triangular pitch}) \quad (9.12)$$

We next define a characteristic flow area for the shell geometry. It is remembered that the shell area is not constant, but defining an area is useful in finding the film coefficient. The characteristic area A_s of the shell is defined as

$$A_s = \frac{D_s C B}{P_T} \quad (9.13)$$

where D_s is the inside diameter of the shell, C is the clearance between

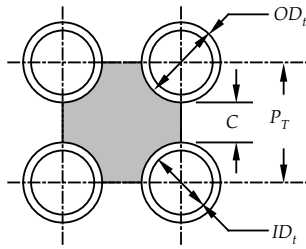


FIGURE 9.13. Square pitch layout.

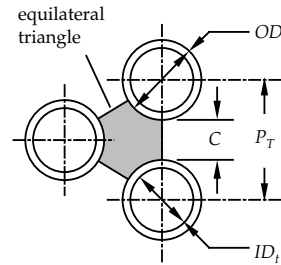


FIGURE 9.14. Triangular pitch layout.

adjacent tubes (not to be confused with the ratio of capacitances), B is the baffle spacing, and P_T is tube pitch. The shell fluid velocity is found with

$$V_s = \frac{\dot{m}}{\rho A_s} \quad (9.14)$$

In addition, the mass velocity of the shell fluid is given by

$$G = \frac{\dot{m}}{A_s} = \rho V_s \quad (9.15)$$

The Nusselt number for the shell fluid is given by an equation that is based on experimental results obtained on a number of heat exchanger tests as

$$\text{Nu} = \frac{h_o D_e}{k_f} = 0.36 \text{Re}^{0.55} \text{Pr}^{1/3} \quad (9.16)$$

valid for $2 \times 10^3 \leq \text{Re}_s = V_s D_o / \nu \leq 1 \times 10^6$ $\text{Pr} = \nu / \alpha > 0$
 μ changes moderately with temperature and
 properties evaluated at the average fluid temperature
 [= (inlet + outlet)/2]

The shell fluid experiences a pressure drop as it passes through the exchanger, over tubes, and around baffles. If the shell fluid nozzles (inlet and outlet ports) are on the same side of the exchanger, then the shell fluid makes an even number of tube bundle crossings. If the shell nozzles are on opposite sides, then the shell fluid makes an odd number of bundle crossings. The number of bundle crossings influences the pressure drop. Based on experiment, the pressure drop experienced by the shell fluid is

$$\Delta p_s = f(N_b + 1) \frac{D_s}{D_e} \frac{\rho V_s^2}{2g_c} \quad (9.17)$$

where N_b is the number of baffles and $N_b + 1$ is the number of times that the shell fluid crosses the tube bundle. The friction factor in Equation 9.17 is given by

$$f = \exp (0.576 - 0.19 \ln \text{Re}_s) \quad (9.18)$$

valid for $400 \leq \text{Re}_s = V_s D_o / \nu \leq 1 \times 10^6$

Equations 9.17 and 9.18 are formulated to include inlet and exit losses experienced by the shell fluid. For a well-designed heat exchanger, the pressure drop experienced by either stream should be less than 10 psi (70 kPa).

Outlet Temperature Calculation

In a number of applications where a shell and tube heat exchanger can be used, the inlet temperatures and flow rates would be known and the outlet temperatures must be calculated. This can be done in two ways: the traditional **LMTD method** (given here) and the **effectiveness-NTU method** (given in the next section).

In the LMTD method, we proceed by stating an equation (without derivation) that was developed to predict outlet temperatures in terms of the previously defined quantities R and S :

$$\frac{UA_o}{\dot{m}_c C_{pc}} = \frac{1}{\sqrt{R^2 + 1}} \ln \left[\frac{2 - S(R + 1 - \sqrt{R^2 + 1})}{2 - S(R + 1 + \sqrt{R^2 + 1})} \right] \quad (9.19a)$$

where (as defined earlier in this section):

$$S = \frac{t_2 - t_1}{T_1 - t_1} \quad (9.2)$$

$$R = \frac{\dot{m}_c C_{pc}}{\dot{m}_w C_{pw}} = \frac{T_1 - T_2}{t_2 - t_1} \quad (9.3)$$

The parameter S must be determined to calculate the outlet temperatures. We first define the following quantities:

$$C_1 = \exp \left[\frac{UA_o}{\dot{m}_c C_{pc}} \sqrt{R^2 + 1} \right]$$

$$C_2 = (R + 1 - \sqrt{R^2 + 1})$$

$$C_3 = (R + 1 + \sqrt{R^2 + 1})$$

Rearrangng Equation 9.19a we solve for S to obtain:

$$S = \frac{2(1 - C_1)}{C_2 - C_1 C_3} \quad (9.19b)$$

The outlet temperature of the cooler fluid is then determined by solving for S using 9.19b, and then combining the result with Equation 9.2, which is rearranged to give:

$$t_2 = (T_1 - t_1)S + t_1$$

The outlet temperature of the warmer fluid is finally calculated with Equation 9.3, rearranged to yield

$$T_2 = T_1 - R(t_2 - t_1) \quad (9.20)$$

Like double pipe heat exchangers, shell and tube heat exchangers are subject to mineral deposits on the tube and shell surfaces. The effect is that heat must be transferred through additional resistances. We define a “dirty” or “design” overall heat transfer coefficient as

$$\frac{1}{U} = \frac{1}{U_o} + R_{di} + R_{do} \quad (9.21)$$

The design coefficient U is used when determining the area required to transfer heat.

The preceding equations have been organized into a suggested order, which now follows.

SUGGESTED ORDER OF CALCULATIONS FOR A SHELL AND TUBE HEAT EXCHANGER

Problem	Complete problem statement.
Discussion	Potential heat losses; other sources of difficulties
Assumptions	1. Steady-state conditions exist. 2. Fluid properties remain constant and are evaluated at a temperature of _____.
Nomenclature	1. T refers to the temperature of the warmer fluid. 2. t refers to the temperature of the cooler fluid.

3. w subscript refers to the warmer fluid.
4. h subscript refers to hydraulic diameter.
5. c subscript refers to the cooler fluid.
6. s subscript refers to the shell flow area or dimension.
7. t subscript refers to the tubular flow area or dimension.
8. 1 subscript refers to an inlet condition.
9. 2 subscript refers to an outlet condition.
10. e subscript refers to equivalent diameter.

A. Fluid Properties

$$\begin{array}{ll}
 \dot{m}_w = & T_1 = \\
 \rho = & C_p = \\
 k_f = & \alpha = \\
 \nu = & \text{Pr} =
 \end{array}$$

$$\begin{array}{ll}
 \dot{m}_c = & t_1 = \\
 \rho = & C_p = \\
 k_f = & \alpha = \\
 \nu = & \text{Pr} =
 \end{array}$$

B. Tubing Sizes

$$\begin{array}{ll}
 ID_t = & OD_t = \\
 N_t = \text{no. of tubes} = & N_p = \text{no. of passes} =
 \end{array}$$

C. Shell Data

$$\begin{array}{ll}
 D_s = \text{shell inside diameter} & = \\
 B = \text{baffle spacing} & = \\
 N_b = \text{number of baffles} & = \\
 P_T = \text{tube pitch} & = \\
 C = P_T - OD_t = \left(\begin{array}{l} \text{clearance between} \\ \text{adjacent tubes} \end{array} \right) & =
 \end{array}$$

$$\begin{array}{l}
 \text{D. Flow Areas} \quad A_t = N_t \pi (ID_t^2) / 4N_p = \\
 \quad A_s = D_s C B / P_T =
 \end{array}$$

E. Fluid Velocities [Route the fluid with the higher flow rate through the flow cross section with the greater area.]

$$\begin{array}{ll}
 V_t = \dot{m} / \rho A = & G_t = \dot{m} / A = \\
 V_s = \dot{m} / \rho A = & G_s = \dot{m} / A =
 \end{array}$$

F. Shell Equivalent Diameter

$$D_e = \frac{4P_T^2 - \pi OD_t^2}{\pi OD_t} = \left(\begin{array}{c} \text{square} \\ \text{pitch} \end{array} \right)$$

$$D_e = \frac{3.46P_T^2 - \pi OD_t^2}{\pi OD_t} = \left(\begin{array}{c} \text{triangular} \\ \text{pitch} \end{array} \right)$$

G. Reynolds Numbers

$$Re_t = V_t ID_t / \nu =$$

$$Re_s = V_s D_e / \nu =$$

H. Nusselt NumbersTube Side

Modified Sieder-Tate equation for Laminar Flow:

$$Nu_t = \frac{h_i ID_t}{k_f} = 1.86 \left(\frac{ID_t Re_t Pr}{L} \right)^{1/3}$$

$$Re_t < 2\,200 \quad 0.48 < Pr = \nu/\alpha < 16\,700$$

Modified Dittus-Boelter equation for Turbulent Flow:

$$Nu_t = \frac{h_i ID_t}{k_f} = 0.023 Re_t^{4/5} Pr^n$$

$n = 0.4$ if fluid is being heated

$n = 0.3$ if fluid is being cooled

$$Re_t > 10\,000; \quad 0.7 < Pr = \nu/\alpha < 160; \quad L/D > 60$$

Conditions:

μ changes moderately with temperature

Properties evaluated at the average fluid

temperature [= (inlet + outlet)/2]

Shell Side

$$Nu_s = \frac{h_o D_e}{k_f} = 0.36 Re_s^{0.55} Pr^{1/3}$$

$$2 \times 10^3 < Re_s = V_s D_e / \nu < 1 \times 10^6 \quad Pr = \nu/\alpha > 0$$

μ changes moderately with temperature

Properties evaluated at the average fluid

temperature [= (inlet + outlet)/2]

$$\text{Nu}_t =$$

$$\text{Nu}_s =$$

I. Convection Coefficients

$$h_i = \text{Nu}_t k_f / ID_t = \quad h_t = h_i ID_t / OD_t =$$

$$h_o = \text{Nu}_s k_f / D_e =$$

J. Exchanger Coefficient

$$\frac{1}{U_o} = \frac{1}{h_t} + \frac{1}{h_o} \quad U_o =$$

K. Outlet Temperatures Calculations (Exchanger length $L =$)

$$R = \frac{\dot{m}_c C_{pc}}{\dot{m}_w C_{pw}} = \quad A_o = N_t \pi OD_t L =$$

$$\frac{U_o A_o}{\dot{m}_c C_{pc}} = \quad S = \quad (\text{Equation 9.19b})$$

$$t_2 = S(T_1 - t_1) + t_1 =$$

$$T_2 = T_1 - R(t_2 - t_1) =$$

L. Log Mean Temperature Difference

$$\text{Counterflow} \quad LMTD = \frac{(T_1 - t_2) - (T_2 - t_1)}{\ln [(T_1 - t_2)/(T_2 - t_1)]} =$$

M. Heat Balance for Fluids

$$q_w = \dot{m}_w C_{pw} (T_1 - T_2) =$$

$$q_c = \dot{m}_c C_{pc} (t_2 - t_1) =$$

N. Overall Heat Balance for the Exchanger

$$F = \quad (\text{Figure 9.12})$$

$$q = U_o A_o F (LMTD) =$$

O. Fouling Factors and Design Coefficient

$$R_{di} = \quad R_{do} =$$

$$\frac{1}{U} = \frac{1}{U_o} + R_{di} + R_{do} = \quad U =$$

P. Area Required to Transfer Heat

$$A_o = \frac{q}{UF (LMTD)} =$$

$$L = \frac{A_o}{N_t \pi OD_t} =$$

Q. Friction FactorsTube Side

Laminar Flow in a Tube:

$$f_t = \frac{64}{Re_t} \quad Re_t < 2\,200 \text{ (Step G above)}$$

Turbulent Flow in a Tube:

Swamee-Jain Equation

$$f = \frac{0.250}{\left\{ \log \left[\frac{\varepsilon}{3.7D} - \frac{5.74}{(Re)^{0.9}} \right] \right\}^2}$$

Shell Side

$$f_s = \exp(0.576 - 0.19 \ln Re_s) \quad Re_s \text{ from step G}$$

$$\left. \begin{array}{l} Re_t = \\ \frac{\varepsilon}{ID_t} = \end{array} \right\} \quad f_t =$$

$$Re_s = \quad f_s =$$

R. Pressure Drop Calculations

$$\Delta p_t = \frac{\rho V_t^2}{2g_c} \left(\frac{f_t L}{ID_t} + 4 \right) N_p =$$

$$\Delta p_s = \frac{\rho V_s^2}{2g_c} \frac{D_s}{D_e} f_s (N_b + 1) =$$

S. Summary of Information Requested in Problem Statement

EXAMPLE 9.1. In a facility where electricity is generated, condensed (distilled) water is to be cooled by means of a shell and tube heat exchanger. Distilled water enters the exchanger at 110°F at a flow rate of 170,000 lbm/hr, and it is desired to cool the water to 95°F. Heat will be transferred to raw water (from a nearby lake), which is available at 65°F and 150,000 lbm/hr. Preliminary calculations indicate that it may be appropriate to use a heat exchanger that has a 17-1/4-in.-inside-diameter shell, and 3/4-in.-OD, 18 BWG tubes that are 16 ft long. The tubes are laid out on a 1-in. triangular pitch, and the tube fluid will make two passes. The exchanger contains baffles that are spaced 1 ft apart. Analyze the proposed heat exchanger to determine its suitability. Route the distilled water through the tubes.

Solution. The flow rates are greater here than with double pipe heat exchangers. It is essential in this problem to evaluate properties at the average of inlet and outlet temperatures. This has been done in the results that follow. Accordingly, numbers in this example were obtained after several iterations.

- Assumptions**
1. Steady-state conditions exist.
 2. Raw and distilled water properties can be obtained from the same property table.
 3. Raw water properties are evaluated at 68°F initially, and after several iterations, properties are evaluated at 76.7°F.
 4. Distilled water properties are evaluated initially at 104°F, and after several iterations, at 99.8°F.
 5. All heat lost by the distilled water is transferred to the raw water.

- Nomenclature**
1. T refers to the temperature of the warmer fluid.
 2. t refers to the temperature of the cooler fluid.
 3. w subscript refers to the warmer fluid.
 4. h subscript refers to hydraulic diameter.
 5. c subscript refers to the cooler fluid.
 6. s subscript refers to the shell flow area or dimension.
 7. t subscript refers to the tubular flow area or dimension.
 8. 1 subscript refers to an inlet condition.

9. 2 subscript refers to an outlet condition.
 10. e subscript refers to equivalent diameter.

A. Fluid Properties

Dist Water at 99.8°F	$\dot{m}_w$	= 170,000 lbm/hr	T_1	= 110°F
	ρ	= 62.1 lbm/ft ³	C_p	= 0.998 BTU/lbm·°R
	k_f	= 0.361 BTU/hr·ft·°R	α	= 5.82×10^{-3} ft ² /hr
	ν	= 0.759×10^{-5} ft ² /s	Pr	= 4.69

Raw Water at 76.7°F	$\dot{m}_c$	= 150,000 lbm/hr	t_1	= 65°F
	ρ	= 62.3 lbm/ft ³	C_p	= 0.9985 BTU/lbm·°R
	k_f	= 0.350 BTU/hr·ft·°R	α	= 5.62×10^{-3} ft ² /hr
	ν	= 0.986×10^{-5} ft ² /s	Pr	= 6.32

B. Tubing Sizes

ID_t	= 0.0543 ft	OD_t	= 0.06255 ft
N_t	= no. of tubes = 196	N_p	= no. of passes = 2

C. Shell Data

D_s	= shell inside diameter	= 17.25 in. = 1.438 ft
B	= baffle spacing	= 1 ft
N_b	= number of baffles	= 15
P_T	= tube pitch	= 1 in. = 0.08333 ft
C	= $\left(\begin{array}{c} \text{clearance between} \\ \text{adjacent tubes} \end{array} \right)$	= $P_T - OD_t = 0.02078$ ft

D. Flow Areas $A_t = N_t \pi (ID_t^2) / 4N_p = 0.2269$ ft²
 $A_s = D_s C B / P_T = 0.3586$ ft² $A_s > A_t$

E. Fluid Velocities

Dist Water	$V_t = \dot{m} / \rho A = 3.35$ ft/s	$G_t = \dot{m} / A = 208$ lbm/ft ² ·s
Raw Water	$V_s = \dot{m} / \rho A = 1.87$ ft/s	$G_s = \dot{m} / A = 117$ lbm/ft ² ·s

F. Shell Equivalent Diameter

$$D_e = \frac{3.46 P_T^2 - \pi OD_t^2}{\pi OD_t} = 0.05901 \text{ ft} \quad \left(\begin{array}{c} \text{triangular} \\ \text{pitch} \end{array} \right)$$

G. Reynolds Numbers

Dist Water	$Re_t = V_t ID_t / \nu = 23,995$
Raw Water	$Re_s = V_s D_e / \nu = 11,163$

H. Nusselt Numbers

Dist Water $Nu_t = 117$

Raw Water $Nu_s = 127$

I. Convection Coefficients

Dist Water $h_i = Nu_t k_f / ID_t = 775$ $h_t = h_i ID_t / OD_t = 673$

Raw Water $h_o = Nu_s k_f / D_e = 751 \text{ BTU}/(\text{hr} \cdot \text{ft}^2 \cdot ^\circ\text{R})$

J. Exchanger Coefficient

$$\frac{1}{U_o} = \frac{1}{h_t} + \frac{1}{h_o} \quad U_o = 355 \text{ BTU}/(\text{hr} \cdot \text{ft}^2 \cdot ^\circ\text{R})$$

K. Outlet Temperatures Calculations (Exchanger length $L = 16 \text{ ft}$)

$$R = \frac{\dot{m}_c C_{pc}}{\dot{m}_w C_{pw}} = 0.8831 \quad A_o = N_t \pi OD_t L = 616.2 \text{ ft}^2$$

$$\frac{U_o A_o}{\dot{m}_c C_{pc}} = 1.460 \quad S = 0.546 \quad (\text{Equation 9.19b})$$

Dist Water $T_2 = T_1 - R(t_2 - t_1) = 88.3^\circ\text{F} \quad (\text{new})$

Raw Water $t_2 = S(T_1 - t_1) + t_1 = 89.6^\circ\text{F} \quad (\text{new})$

L. Log Mean Temperature Difference

$$\text{Counterflow} \quad LMTD = \frac{(T_1 - t_2) - (T_2 - t_1)}{\ln [(T_1 - t_2)/(T_2 - t_1)]} = 21.8^\circ\text{F}$$

M. Heat Balance for Fluids

Dist Water $q_w = \dot{m}_w C_{pw}(T_1 - T_2) = 3.67 \times 10^6 \text{ BTU/hr}$

Raw Water $q_c = \dot{m}_c C_{pc}(t_2 - t_1) = 3.67 \times 10^6 \text{ BTU/hr}$

N. Overall Heat Balance for the Exchanger

$$F = 0.772 \quad (\text{Equation 9.4})$$

$$q = U_o A_o F (LMTD) = 3.67 \times 10^6 \text{ BTU/hr}$$

O. Fouling Factors and Design Coefficient

$$R_{di} = 0.00075 \text{ hr}\cdot\text{ft}^2\cdot^\circ\text{R}/\text{BTU} \quad R_{do} = 0.00075$$

$$\frac{1}{U} = \frac{1}{U_o} + R_{di} + R_{do} \quad U = 232 \text{ BTU}/(\text{hr}\cdot\text{ft}^2\cdot^\circ\text{R})$$

P. Outlet Temperatures for the Design Coefficient

Without detailing the calculations, the outlet temperatures for $U = 232$ BTU/(hr·ft²·°R) are calculated to be

Dist Water $T_2 = T_1 - R(t_2 - t_1) = 91.3^\circ\text{F}$ (after 1 year)

Raw Water $t_2 = S(T_1 - t_1) + t_1 = 86.2^\circ\text{F}$ (after 1 year)

Q. Friction Factors

Dist Water $\left. \begin{array}{l} \text{Re}_t = 2.4 \times 10^4 \\ \frac{\varepsilon}{ID_t} = \text{smooth} \end{array} \right\} f_t = 0.025$

Raw Water $\text{Re}_s = 1.11 \times 10^4 \quad f_s = 0.303$

R. Pressure Drop Calculations

Dist Water $\Delta p_t = \frac{\rho V_t^2}{2g_c} \left(\frac{f_t L}{ID_t} + 4 \right) N_p = 1.7 \text{ psi}$

Raw Water $\Delta p_s = \frac{\rho V_s^2}{2g_c} \frac{D_s}{D_e} f_s (N_b + 1) = 2.76 \text{ psi}$

S. Summary of Performance

$$T_2 = 95^\circ\text{F} \text{ (required)}$$

$T_1 = 110^\circ\text{F}$ (distilled)	$t_1 = 65^\circ\text{F}$ (raw)
$\Delta p = 1.7 \text{ psi}$ (distilled)	$\Delta p = 2.8 \text{ psi}$ (raw)
$R_d = 0.00075$ (distilled)	$R_d = 0.00075$ (raw)
New	After 1 year
$U_o = 355 \text{ BTU}/(\text{hr}\cdot\text{ft}^2\cdot^\circ\text{R})$	$U = 232 \text{ BTU}/(\text{hr}\cdot\text{ft}^2\cdot^\circ\text{R})$
$T_2 = 88.3^\circ\text{F}$ (distilled)	$T_2 = 91.3^\circ\text{F}$ (distilled)
$t_2 = 89.6^\circ\text{F}$ (raw)	$t_2 = 86.2^\circ\text{F}$ (raw)
$q = 3.67 \times 10^6 \text{ BTU/hr}$	$q = 3.17 \times 10^6 \text{ BTU/hr}$

The required outlet temperature is 95°F ($=T_2$), and this exchanger exceeds that requirement when fouled. The exchanger is suitable: both pressure drops are less than 10 psi, and F is greater than 0.75.

9.3 Effectiveness-NTU Analysis

As with double pipe heat exchangers, the **effectiveness-NTU method** can be applied to shell and tube exchangers. With double pipe heat exchangers, the effectiveness-NTU method was as easy to use as the LMTD method. With shell and tube heat exchangers, however, the effectiveness-NTU method offers a definite advantage.

In the previous example, calculation of outlet temperature required finding the temperature factor S first. Now S is found with Equation 9.19b, which was stated earlier as

$$S = \frac{2(1 - C_1)}{C_2 - C_1 C_3} \quad (9.19b)$$

The outlet temperature of the cooler fluid is then determined with Equation 9.2, which is rearranged to give:

$$t_2 = (T_1 - t_1)S + t_1$$

The outlet temperature of the warmer fluid is finally calculated with Equation 9.3, rearranged to yield

$$T_2 = T_1 - R(t_2 - t_1) \quad (9.20)$$

The effectiveness-NTU method offers an alternative way to evaluate outlet temperatures for the shell and tube exchanger. The equations for this method, first stated in Chapter 7, are repeated here.

The effectiveness E is defined as

$$E = \frac{t_2 - t_1}{T_1 - t_1} \quad (\text{if } \dot{m}_c C_{pc} < \dot{m}_w C_{pw}) \quad (9.22)$$

$$E = \frac{T_1 - T_2}{T_1 - t_1} \quad (\text{if } \dot{m}_w C_{pw} < \dot{m}_c C_{pc}) \quad (9.23)$$

and

$$E = \frac{q}{q_{\max}}$$

where q is the actual heat transferred, given by

$$q = E (\dot{m}C_p)_{\min} (T_1 - t_1) \quad (9.24)$$

and $q_{\max}$ is

$$q_{\max} = (\dot{m}C_p)_{\min} (T_1 - t_1) \quad (9.25)$$

If effectiveness is known for the heat exchanger of interest, then the heat transfer rate can be found with Equation 9.24. Effectiveness equations have been derived for many types of heat exchangers and usually contain a term called the **number of transfer units**, N , defined as

$$N = \frac{UA}{(\dot{m}C_p)_{\min}} \quad (9.26)$$

We also define what is known as the **ratio of capacitances** C , which is always less than 1:

$$C = \frac{(\dot{m}C_p)_{\min}}{(\dot{m}C_p)_{\max}} < 1$$

Referring to Table 8.4, the effectiveness for a 1-2 shell and tube heat exchanger is given by:

$$E = 2 \left\{ 1 + C + \frac{1 + \exp[-N(1 + C^2)^{1/2}]}{1 - \exp[-N(1 + C^2)^{1/2}]} (1 + C^2)^{1/2} \right\}^{-1} \quad (9.27)$$

This equation applies to an exchanger having 1 shell pass, and 2, 4, 6, and so on, tube passes. Figure 9.15 is a graph of this equation, with the number of transfer units N on the horizontal axis. The effectiveness, which ranges from 0 to 1, is graphed on the vertical axis.

When analyzing a heat exchanger in the way that was done in Example 9.1, the effectiveness can be used to calculate the outlet temperatures. The procedure is the same as in the example, except that effectiveness E is calculated before S ; once effectiveness is known, outlet temperatures are found with

$$\left. \begin{aligned} t_2 &= t_1 + E(T_1 - t_1) \\ T_2 &= T_1 - C(t_2 - t_1) \end{aligned} \right\} \quad \left(\text{if } \dot{m}_c C_{pc} < \dot{m}_w C_{pw} \text{ with } E = \frac{t_2 - t_1}{T_1 - t_1} \right)$$

or in the opposite case

$$\left. \begin{aligned} T_2 &= T_1 - E(T_1 - t_1) \\ t_2 &= t_1 + C(T_1 - T_2) \end{aligned} \right\} \quad \left(\text{if } \dot{m}_w C_{pw} < \dot{m}_c C_{pc} \text{ with } E = \frac{T_1 - T_2}{T_1 - t_1} \right)$$

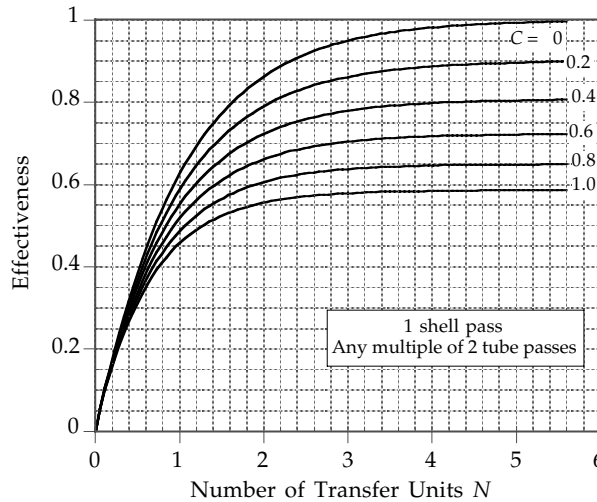


FIGURE 9.15. *Effectiveness- N graph for a shell and tube heat exchanger having one shell pass and any multiple of two tube passes.*

Finding S is still necessary in order to evaluate the correction factor F , but after outlet temperatures are known, we can use $S = (t_2 - t_1)/(T_1 - t_1)$.

As stated earlier, the LMTD method is difficult (but not impossible) to apply due to the nature of Equation 9.19, which cannot be solved in closed form for S . The effectiveness-NTU method is more convenient to use because Equation 9.27 for effectiveness is easier to apply. Both give identical results for outlet temperature.

EXAMPLE 9.2. In a facility where detergent is manufactured, benzene is to be cooled from 125°F by using water, which is available at 65°F. Process conditions require a benzene flow rate of 150,000 lbm/hr. The water flow rate is 180,000 lbm/hr. Available for this service is a shell and tube exchanger that has a shell ID of 17¹/₄ in. and contains ³/₄ OD, 13 BWG tubes that are 16 ft long. The tubes are laid out on a 1-in. triangular pitch, and the tube fluid will make two passes. The exchanger contains 10 evenly spaced baffles. Analyze the proposed system, and predict outlet temperatures as well as pressure drops. Route the benzene through the tubes, and use the following properties for benzene:

116.6°F	$\rho = 0.8504(62.4) \text{ lbm/ft}^3$	$C_p = 0.4254 \text{ BTU/lbm}\cdot^\circ\text{R}$
	$k_f = 0.07476 \text{ BTU/hr}\cdot\text{ft}\cdot^\circ\text{R}$	$\nu = 5.74 \times 10^{-6} \text{ ft}^2/\text{s}$
80.6°F	$\rho = 0.876(62.4) \text{ lbm/ft}^3$	$C_p = 0.4135 \text{ BTU/lbm}\cdot^\circ\text{R}$
	$k_f = 0.08268 \text{ BTU/hr}\cdot\text{ft}\cdot^\circ\text{R}$	$\nu = 7.22 \times 10^{-6} \text{ ft}^2/\text{s}$

Solution. The flow rates are greater here than with double pipe heat exchangers. It is essential in this problem to evaluate properties at the average of inlet and outlet temperatures. This has been done in what follows, which reflects the result of several iterations.

- Assumptions**
1. Steady-state conditions exist.
 2. Benzene properties are evaluated after several iterations at 105°F.
 4. Water properties are evaluated after several iterations at 72°F.
 5. All heat lost by the benzene is transferred to the water.

- Nomenclature**
1. T refers to the temperature of the warmer fluid.
 2. t refers to the temperature of the cooler fluid.
 3. w subscript refers to the warmer fluid.
 4. h subscript refers to hydraulic diameter.
 5. c subscript refers to the cooler fluid.
 6. s subscript refers to the shell flow area or dimension.
 7. t subscript refers to the tubular flow area or dimension.
 8. 1 subscript refers to an inlet condition.
 9. 2 subscript refers to an outlet condition.
 10. e subscript refers to equivalent diameter.

A. Fluid Properties

Benzene at 105°F	$\dot{m}_w$	= 150,000 lbm/hr	T_1	= 125°F
	ρ	= 53.59 lbm/ft ³	C_p	= 0.421 BTU/lbm·°R
	k_f	= 0.0773 BTU/hr·ft·°R	α	= 3.42×10^{-3} ft ² /hr
	ν	= 6.22×10^{-6} ft ² /s	Pr	= 6.54

Water at 72°F	$\dot{m}_c$	= 180,000 lbm/hr	t_1	= 65°F
	ρ	= 62.4 lbm/ft ³	C_p	= 0.9987 BTU/lbm·°R
	k_f	= 0.347 BTU/hr·ft·°R	α	= 5.77×10^{-3} ft ² /hr
	ν	= 1.04×10^{-5} ft ² /s	Pr	= 6.72

B. Tubing Sizes	ID_t	= 0.0543 ft	OD_t	= 0.06255 ft
	N_t	= no. of tubes = 196	N_p	= no. of passes = 2

C. Shell Data	D_s	= shell inside diameter	= 17.25 in. = 1.438 ft
	B	= baffle spacing	= 1.46 ft
	N_b	= number of baffles	= 10
	P_T	= tube pitch	= 1 in. = 0.08333 ft
	C	= $\left(\begin{array}{c} \text{clearance between} \\ \text{adjacent tubes} \end{array} \right)$	= $P_T - OD_t = 0.02078$ ft

D. Flow Areas $A_t = N_t \pi (ID_t^2) / 4N_p = 0.2269 \text{ ft}^2$
 $A_s = D_s CB / P_T = 0.538 \text{ ft}^2$

E. Fluid Velocities

Benzene $V_t = \dot{m} / \rho A = 3.42 \text{ ft/s}$ $G_t = \dot{m} / A = 183 \text{ lbm/ft}^2 \cdot \text{s}$

Raw Water $V_s = \dot{m} / \rho A = 1.49 \text{ ft/s}$ $G_s = \dot{m} / A = 93 \text{ lbm/ft}^2 \cdot \text{s}$

F. Shell Equivalent Diameter

$$D_e = \frac{3.46 P_T^2 - \pi OD_t^2}{\pi OD_t} = 0.05901 \text{ ft} \quad \left(\begin{array}{c} \text{triangular} \\ \text{pitch} \end{array} \right)$$

G. Reynolds Numbers

Benzene $Re_t = V_t ID_t / \nu = 29933$

Water $Re_s = V_s D_e / \nu = 8453$

H. Nusselt Numbers

Benzene $Nu_t = 154$

Water $Nu_s = 111$

I. Convection Coefficients

Benzene $h_t = Nu_t k_f / ID_t = 219$ $h_t = h_t ID_t / OD_t = 190$

Water $h_o = Nu_s k_f / D_e = 656 \text{ BTU/hr} \cdot \text{ft}^2 \cdot ^\circ\text{R}$

J. Exchanger Coefficient

$$\frac{1}{U_o} = \frac{1}{h_t} + \frac{1}{h_o} \quad U_o = 147 \text{ BTU/hr} \cdot \text{ft}^2 \cdot ^\circ\text{R}$$

K. Outlet Temperatures Calculations (Exchanger length $L = 16 \text{ ft}$)

$$R = \frac{\dot{m}_c C_{pc}}{\dot{m}_w C_{pw}} = 2.843 \quad A_o = N_t \pi OD_t L = 616.2 \text{ ft}^2$$

$$C = \frac{(\dot{m} C_p)_{\min}}{(\dot{m} C_p)_{\max}} = 0.352 \quad N = \frac{UA}{(\dot{m} C_p)_{\min}} = 1.44$$

$$E = 2 \left\{ 1 + C + \frac{1 + \exp [-N(1 + C^2)^{1/2}]}{1 - \exp [-N(1 + C^2)^{1/2}]} (1 + C^2)^{1/2} \right\}^{-1} = 0.666$$

$$\dot{m}_w C_{pw} < \dot{m}_c C_{pc} \quad T_2 = T_1 - E(T_1 - t_1)$$

$$t_2 = t_1 + C(T_1 - T_2)$$

Benzene $T_2 = 85^\circ\text{F}$

Water $t_2 = 79^\circ\text{F}$

L. Log Mean Temperature Difference

Counterflow $LMTD = \frac{(T_1 - t_2) - (T_2 - t_1)}{\ln [(T_1 - t_2)/(T_2 - t_1)]} = 31.2^\circ\text{F}$

M. Heat Balance for Fluids

Benzene $q_w = \dot{m}_w C_{pw} (T_1 - T_2) = 7.02 \times 10^2 \text{ BTU/s}$

Water $q_c = \dot{m}_c C_{pc} (t_2 - t_1) = 7.02 \times 10^2 \text{ BTU/s}$

N. Overall Heat Balance for the Exchanger

$$S = \frac{t_2 - t_1}{T_1 - t_1} = 0.234$$

$$F = 0.772 \quad (\text{Figure 9.12})$$

$$q = U_o A_o F (LMTD) = 7.02 \times 10^2 \text{ BTU/s}$$

O. Fouling Factors and Design Coefficient

$$R_{di} = 0.001 \text{ hr}\cdot\text{ft}^2\cdot^\circ\text{R}/\text{BTU} \quad R_{do} = 0.00075$$

$$\frac{1}{U} = \frac{1}{U_o} + R_{di} + R_{do} \quad U = 117 \text{ BTU}/(\text{hr}\cdot\text{ft}^2\cdot^\circ\text{R})$$

Repeating the outlet temperature calculations at the design coefficient, the outlet temperatures are calculated to be:

Benzene $T_2 = T_1 - R(t_2 - t_1) = 89^\circ\text{F} \quad (\text{after 1 year})$

Raw Water $t_2 = S(T_1 - t_1) + t_1 = 78^\circ\text{F} \quad (\text{after 1 year})$

P. Area Required to Transfer Heat—NA

Q. Friction Factors

$$\left. \begin{array}{l} \text{Benzene} \quad \text{Re}_t = 2.99 \times 10^4 \\ \frac{\epsilon}{ID_t} = \text{smooth} \end{array} \right\} \quad f_t = 0.024$$

$$\text{Water} \quad \text{Re}_s = 8453 \quad f_s = 0.319$$

R. Pressure Drop Calculations

$$\text{Benzene} \quad \Delta p_t = \frac{\rho V_t^2}{2g_c} \left(\frac{f_t L}{ID_t} + 4 \right) N_p = 1.48 \text{ psi}$$

$$\text{Water} \quad \Delta p_s = \frac{\rho V_s^2}{2g_c} \frac{D_s}{D_e} f_s (N_b + 1) = 1.28 \text{ psi}$$

S. Summary of Information

$T_1 = 125^\circ\text{F}$ (benzene)	$t_1 = 65^\circ\text{F}$ (water)
$\Delta p = 1.5 \text{ psi}$ (benzene)	$\Delta p = 1.3 \text{ psi}$ (water)
$R_d = 0.001$ (benzene)	$R_d = 0.00075$ (water)
New	After 1 year
$U_0 = 147 \text{ BTU}/(\text{hr}\cdot\text{ft}^2\cdot^\circ\text{R})$	$U = 117 \text{ BTU}/(\text{hr}\cdot\text{ft}^2\cdot^\circ\text{R})$
$T_2 = 85^\circ\text{F}$ (benzene)	$T_2 = 89^\circ\text{F}$ (benzene)
$t_2 = 79^\circ\text{F}$ (water)	$t_2 = 78^\circ\text{F}$ (water)
$q = 2.53 \times 10^6 \text{ BTU/hr}$	$q = 2.29 \times 10^6 \text{ BTU/hr}$

Exchanger is suitable. Both pressure drops are less than 10 psi. F is greater than 0.75.

9.4 Increased Heat Recovery in Shell and Tube Heat Exchangers

In analyzing heat exchangers, the terms **approach** and **cross** are frequently used. These terms refer to the relationship between the outlet temperatures of both fluids. The **approach** is defined as the difference in the outlet temperatures, warmer minus cooler $T_2 - t_2$, and has significance if $T_2 > t_2$. We say that as the fluids flow through the exchanger, the fluid outlet temperatures “approach” each other. On the other hand, if $t_2 > T_2$, then $t_2 - T_2$ is called the **temperature cross**; that is, as the fluid flows through the heat exchanger, the fluid outlet temperatures (actually temperature profiles) “cross” each other.

Calculations performed on shell and tube equipment show that the correction factor F decreases with decreasing approach. Thus, the closer the outlet temperatures are to each other, the smaller the correction factor F

will be. This is an important point when considering that F should be equal to or greater than 0.75 for an efficient operation. Generally, F will equal 0.75 for a temperature cross in the range of 0° to 10°C . The following example illustrates the effect that approach and cross have on the correction factor.

EXAMPLE 9.3. Determine the correction factor F for the following cases:

- a. $T_1 = 270^\circ\text{C}$ $t_1 = 20^\circ\text{C}$
 $T_2 = 170^\circ\text{C}$ $t_2 = 120^\circ\text{C}$
 (approach is $170 - 120 = 50^\circ\text{C}$)
- b. $T_1 = 220^\circ\text{C}$ $t_1 = 20^\circ\text{C}$
 $T_2 = 120^\circ\text{C}$ $t_2 = 120^\circ\text{C}$
 (approach is $120 - 120 = 0^\circ\text{C}$)
- c. $T_1 = 200^\circ\text{C}$ $t_1 = 20^\circ\text{C}$
 $T_2 = 100^\circ\text{C}$ $t_2 = 120^\circ\text{C}$
 (cross is $120 - 100 = 20^\circ\text{C}$)

Solution: Note that in all cases, the cooler fluid temperatures are maintained the same while the warmer fluid temperatures are modified. Furthermore, the differences for each fluid (i.e., $T_1 - T_2$ and $t_1 - t_2$) are 100°C , again for all cases. In order to find the correction factor, we first calculate R and S . For the given temperatures, we have

- a. 50°C temperature approach

$$\left. \begin{aligned} R &= \frac{T_1 - T_2}{t_2 - t_1} = \frac{100}{100} = 1.0 \\ S &= \frac{t_2 - t_1}{T_1 - t_1} = \frac{100}{250} = 0.4 \end{aligned} \right\} \quad \underline{F = 0.925} \quad (\text{Figure 9.12})$$

- b. 0°C temperature approach

$$\left. \begin{aligned} R &= 1.0 \\ S &= 0.5 \end{aligned} \right\} \quad \underline{F = 0.8} \quad (\text{Figure 9.12})$$

- c. 20°C temperature cross

$$\left. \begin{aligned} R &= 1.0 \\ S &= 0.556 \end{aligned} \right\} \quad \underline{F = 0.64} \quad (\text{Equation 9.4})$$

The factor R is constant for the above combinations, but the temperature factor S is not. The difference in inlet temperatures $T_1 - t_1$ was changed in the calculations so that effects of temperature approach on F could be investigated. We see that, as S increased, F decreased; primarily, as the approach decreased, the correction factor decreased. We conclude that the efficiency of a 1-2 shell and tube heat exchanger increases with increasing inlet temperature difference or decreasing temperature approach.

When a 1-2 shell and tube heat exchanger has a correction factor F that is less than 0.75, then it is operating in a less than desirable application. It is possible to improve the performance by connecting two of the exchangers in series, as illustrated schematically in Figure 9.16.

The configuration of Figure 9.16 shows that the shell fluid passes through the *combined* exchangers twice, while the tube fluid passes through them four times. This configuration is called a 2-4 shell and tube heat exchanger. The analysis of such a combination of exchangers is identical to that for the 1-2 exchanger, except that the correction factor F is different. Figure 9.17 shows an alternative configuration for a 2-4 shell and tube heat exchanger using a single shell. The presence of a longitudinal baffle causes the shell fluid to pass through twice. Modifications to the end channels cause the tube fluid to pass through four times. Figure 9.18 is a graph of correction factor F for the 2-4 exchanger. It is similar to that of Figure 9.12. When a 1-2 shell and tube exchanger has a correction factor F that is too low, then a 2-4 exchanger will usually be acceptable.

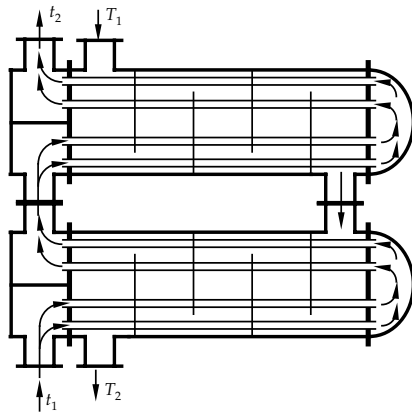


FIGURE 9.16. Two 1-2 shell and tube heat exchangers connected in series to form a 2-4 heat exchanger.

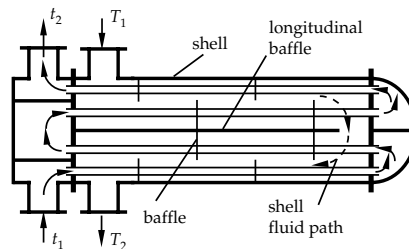


FIGURE 9.17. A 2-4 shell and tube heat exchanger using a single shell.

To model a 2-4 shell and tube heat exchanger using the effectiveness-NTU method, an equation for effectiveness is needed. For such an arrangement, we have

$$E_{2-4} = \left[\left(\frac{1 - E_{1-2}C}{1 - E_{1-2}} \right)^2 - 1 \right] \left[\left(\frac{1 - E_{1-2}C}{1 - E_{1-2}} \right)^2 - C \right]^{-1} \quad (9.28)$$

where E_{2-4} is the effectiveness of the 2-4 exchanger and E_{1-2} is the effectiveness of the 1-2 exchanger, as given in Equation 9.27. Figure 9.19 is a graph of Equation 9.28.

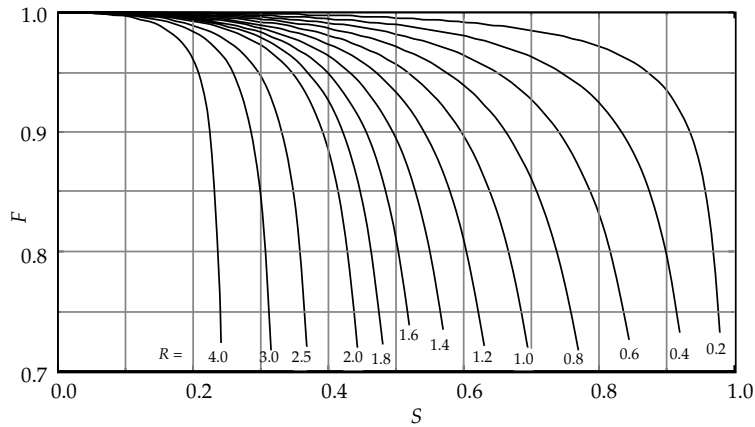


FIGURE 9.18. Correction factor chart for a 2-4 shell and tube heat exchanger.

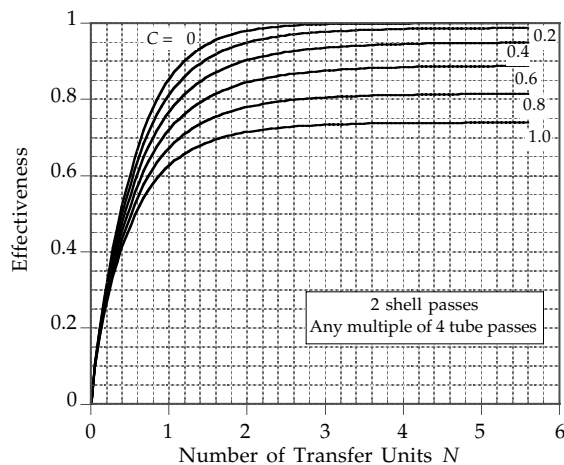


FIGURE 9.19. Effectiveness-NTU chart for a shell and tube heat exchanger having two shell passes and any multiple of four tube passes.

EXAMPLE 9.4. Repeat the calculations in Example 9.3 to find the correction factor F for a 2-4 shell and tube heat exchanger. The data from that example are as follows:

- a. $T_1 = 270^\circ\text{C}$ $t_1 = 20^\circ\text{C}$
 $T_2 = 170^\circ\text{C}$ $t_2 = 120^\circ\text{C}$
 (approach is $170 - 120 = 50^\circ\text{C}$)
- b. $T_1 = 220^\circ\text{C}$ $t_1 = 20^\circ\text{C}$
 $T_2 = 120^\circ\text{C}$ $t_2 = 120^\circ\text{C}$
 (approach is $120 - 120 = 0^\circ\text{C}$)
- c. $T_1 = 200^\circ\text{C}$ $t_1 = 20^\circ\text{C}$
 $T_2 = 100^\circ\text{C}$ $t_2 = 120^\circ\text{C}$
 (cross is $120 - 100 = 20^\circ\text{C}$)

Solution: The results for each case are:

a. 50°C temperature approach

$$\left. \begin{aligned} R &= \frac{T_1 - T_2}{t_2 - t_1} = \frac{100}{100} = 1.0 \\ S &= \frac{t_2 - t_1}{T_1 - t_1} = \frac{100}{250} = 0.4 \end{aligned} \right\} \quad \underline{F = 0.98} \quad (\text{Figure 9.19})$$

b. 0°C temperature approach

$$\left. \begin{aligned} R &= 1.0 \\ S &= 0.5 \end{aligned} \right\} \quad \underline{F = 0.96} \quad (\text{Figure 9.19})$$

c. 20°C temperature cross

$$\left. \begin{aligned} R &= 1.0 \\ S &= 0.556 \end{aligned} \right\} \quad \underline{F = 0.93} \quad (\text{Figure 9.19})$$

As shown, the correction factor is greater for the 2-4 exchanger than for a 1-2 exchanger for the same temperatures.

9.5 Shell and Tube Heat Exchanger Design Considerations

Shell and tube heat exchanger calculations performed in the last section were made for an existing exchanger. Such problems are relatively easy to solve by following the suggested calculation procedure. There exists

another class of problems in which a heat exchanger must be sized to perform a given task. For example, given inlet temperatures, flow rates, and *desired* outlet temperatures, what size heat exchanger is required to perform the task? The calculations for such a problem can proceed as in the last chapter by assuming a certain size exchanger and evaluating its performance. It should be noted, however, that several exchangers can do the job, and analyzing via trial and error to find one can cost considerable time. A structured trial-and-error method can save time and is presented here.

Tube Side Considerations

Higher flow rates within a 1-2 shell and tube heat exchanger give a greater heat transfer rate (higher velocity $\rightarrow$ higher Reynolds number $\rightarrow$ higher Nusselt number $\rightarrow$ higher convection coefficient). As flow rate increases, however, so does the pressure drop. From Chapter 8, the Nusselt number equation for turbulent flow in a circular duct is

$$\text{Nu}_t = \frac{h_i D_t}{k_f} = 0.023 \text{Re}^{4/5} \text{Pr}^n \quad (9.29)$$

where $\text{Re} = VD/\nu$ and $\text{Pr} = \nu/\alpha$. So the convection coefficient varies with $V^{0.8}$. The pressure drop within the tube is given by

$$\Delta p_t = \frac{\rho V^2}{2g_c} \times (\text{geometry factors}) \quad (9.30)$$

The pressure drop varies with the square of the average velocity in the duct V^2 . An increase in velocity will increase the convection coefficient, which is accompanied by a greater increase in the pressure drop.

The number of tube fluid passes can be varied from one to eight, although one tube pass is seldom used. In larger shells, the variation can range to as high as 16 passes. In 1-2 shell and tube heat exchangers, the worst performance is obtained with maximum baffle spacing and two tube passes. As indicated in the previous equations, the tube fluid convection coefficient and pressure drop for turbulent flow vary according to

$$h_i \propto V_t^{0.8}$$

$$\Delta p_t \propto V_t^2 L$$

For purposes of comparison, we calculate the ratio of convection coefficients for eight tube fluid passes to two fluid passes as

$$\frac{h_{i(8 \text{ passes})}}{h_{i(2 \text{ passes})}} = \frac{[8V_t]^{0.8}}{[2V_t]^{0.8}} = 3.03$$

Similarly, for the pressure drop,

$$\frac{\Delta p_{t(8 \text{ passes})}}{\Delta p_{t(2 \text{ passes})}} = \frac{[8V_t]^2 \cdot 8}{[2V_t]^2 \cdot 2} = 64$$

Thus by increasing from two to eight tube passes, the convection coefficient for the tube fluid increases by a factor of 3, while the pressure drop increases by a factor of 64.

Another factor to consider when sizing an exchanger is the tube length. Tubing is available in a number of sizes, but standard sizes (8, 12, or 16 ft) should be used. Exchanger size is dictated by costs associated with cleaning, by available space, and by what sizes are commonly used in other exchangers at the facility.

Shell Side Considerations

Based on experience with shell and tube equipment, the baffle spacing varies according to:

$$\left. \begin{array}{ll} \text{Maximum baffle spacing} & B = ID \text{ of shell} \\ \text{Minimum baffle spacing} & B = \frac{ID \text{ of shell}}{5} \\ & B = 2.25 \text{ in} \end{array} \right\} \begin{array}{l} \text{(whichever} \\ \text{is larger)} \end{array}$$

Thus baffle spacing can be altered by a factor of 5 between minimum and maximum values. At wide baffle spacing, the shell fluid tends to be more axial in its flow direction rather than across the tube bundle. At closer spacing, there will exist excessive leakage between baffle and shell and between baffle and tubes.

Consider also the variation in the shell side convection coefficient. From the equations for Nusselt number, the shell side convection coefficient and pressure drop vary according to

$$h_o \propto V_s^{0.55}$$

$$\Delta p_s \propto V_s^2 (N_b + 1)$$

where N_b is the number of baffles and $N_b + 1$ is the number of times the shell fluid crosses the tube bundle and the number of spaces between baffles from end to end within the exchanger. For variation in baffle spacing between the maximum and the minimum, we calculate

$$\frac{h_{o \min}}{h_{o \max}} = \frac{[5V_s]^{0.55}}{[V_s]^{0.55}} = 2.24$$

Similarly, for the pressure drop,

$$\frac{\Delta p_{s \min}}{\Delta p_{s \max}} = \frac{[5V_s]^2 \cdot 5}{[V_s]^2 \cdot 1} = 125$$

Thus, by changing the baffle spacing from its minimum to its maximum value, the shell side pressure drop is increased by a factor of 125, while the convection coefficient is changed by only a factor of 2.24.

Miscellaneous Factors and Procedures

Other factors enter into the process of specifying an exchanger for a given duty. For example, if one of the fluids tends to foul surfaces more than the other, then the more rapidly fouling fluid should be routed through the tubes. If routed through the shell, it will foul the outside surfaces of the tubes and the inside surface of the shell. When routed through the tubes, it will foul only the inside surface of the tubes, and these are more readily cleaned. Larger tube sizes should be used for fluids that foul tubes rapidly. Similarly, a corrosive fluid requiring a special metal should be routed through the tubes; otherwise, special metal must be used for the shell as well. If both fluids are nonfouling, the higher-pressure fluid should be routed through the tubes, avoiding the need for and expense of a thicker-walled shell.

The sizing method is started by assuming a trial size for the heat exchanger. The trial size should be rather small, with the objective being to select the smallest heat exchanger that will work subject to the following constraints:

- The exchanger should be as **small** as possible.
- The **correction factor** F should be equal to or greater than 0.75.
- The **velocity** of the tube fluid should be within the **optimum** range as calculated with the methods of Chapter 4 or from Table 8.3.
- The exchanger when fouled should still **deliver the required energy exchange**—therefore, the clean overall coefficient will be greater than the design value. So when the exchanger is new, it will exceed the required performance (e.g., the outlet temperatures will be better than expected). When fouled, it will perform as designed or better. The fouling factor table values give best approximations for fouling that will exist after 1 year.
- The **overall pressure drop** for both streams should be less than 10 psi (70 kPa).

Finding an exchanger that will satisfy these criteria involves a trial-and-error procedure. When performing calculations by hand, the procedure could involve significant amounts of time. A spreadsheet, however, can be used to great advantage in this application. The method is illustrated in the following example.

EXAMPLE 9.5. Gasoline [$\rho = 0.701(62.4)$ lbm/ft³, $\mu = 1.07 \times 10^{-5}$ lbf·s/ft², $C_p = 0.5$ BTU/lbm·°R, and $k_f = 0.08$ BTU/hr·hr·ft·°R] flowing at 70,000 lbm/hr is to be cooled from 200°F to 120°F using city water at an inlet temperature of 80°F. Determine the specifications of a heat exchanger that will perform this service.

Solution: The water flow rate is needed before we proceed to solve this problem. We can obtain an estimate by performing a heat balance as a preliminary calculation.

Heat Balance for Fluids

$$\text{Gasoline} \quad q_w = \dot{m}_w C_{pw} (T_1 - T_2) = 70,000(0.5)(200 - 120) = 2.8 \times 10^6$$

$$\text{H}_2\text{O} \quad q_c = \dot{m}_c C_{pc} (t_2 - t_1) = \dot{m}_c (0.998)(t_2 - 80) = 2.8 \times 10^6 \text{ BTU/hr}$$

A number of combinations of $\dot{m}_c$ and t_2 can be selected to satisfy the above equation. We rely on the correction factor to aid in finding t_2 . We first calculate the ratios R and S :

$$R = \frac{\dot{m}_c C_{pc}}{\dot{m}_w C_{pw}} = \frac{T_1 - T_2}{t_2 - t_1} \quad S = \frac{t_2 - t_1}{T_1 - t_1}$$

Substituting,

$$R = \frac{80}{t_2 - 80} \quad S = \frac{t_2 - 80}{200 - 80}$$

Referring to Figure 9.12, we can compose the following chart for various values of the outlet temperature t_2 of the cooler fluid:

t_2	R	S	F
90	8	0.0833	~0.98
100	4	0.1667	~0.95
110	2.67	0.25	~0.88
120	2	0.333	~0.83

Values of F are obtained with Figure 9.12 or Equation 9.4. For this trial, we arbitrarily use an outlet temperature of the water of $t_2 = 96^\circ\text{F}$, and this is done merely to obtain a usable value for the water flow. We would like to avoid a small-temperature approach. The flow rate of the cooling water then becomes

$$\dot{m}_c = \frac{2.8 \times 10^6}{0.998(96 - 80)} = 175,400 \text{ lbm/hr}$$

Some designers make the mass velocity ($G = \rho V$) of both fluids equal in order to obtain an estimate of the water flow rate. Using 175,400 lbm/hr, we can now proceed with the calculations.

Shell ID and Fluid Routing

We begin by assuming a size for the shell, selected at random. In this example, we start with a $13\frac{1}{4}\text{-ID}$ shell. If it is too large, then the outlet temperature of 120°F will be greatly exceeded and we can reject it. If the exchanger is too small, then the outlet temperature requirement will not be met. We select the number of passes based on the pressure drop of the tube fluid. Gasoline (an organic liquid) and water both foul the surfaces at about the same rate of $0.001 \text{ hr}\cdot\text{ft}^2\cdot^\circ\text{R}/\text{BTU}$. We arbitrarily route the gasoline through the tubes, although we could have routed the water through the tubes. The optimum velocity for the gasoline is assumed to be approximately 6 ft/s.

Tube Size

For this first trial, we use $\frac{3}{4}\text{-in.-ID}$, 13 BWG tubes on a 1-in. square pitch. This is the smallest diameter in the tables (smaller tubing sizes are available commercially). The 13 BWG is a common size, and 1-in. square pitch uses fewer tubes than a triangular pitch.

Number of Tubes

We consult the tube count tables for a $13\frac{1}{4}\text{-ID}$ shell. For a first try, we select four passes for a tube count of 82. Calculations show that the tube fluid pressure drop will exceed 10 psi, so we use two passes instead. The tube count becomes 90 for a tube fluid pressure drop of 2.9 psi.

Exchanger Length and Number of Baffles

We select the number of baffles based on the length of the exchanger. As a first try, we use the shortest standard tube length of 8 ft. Lengths of 8, 12, and 16 ft are standard, although any length can be purchased. The maximum number of baffles we use is that which makes the baffle spacing equal to or less than the shell ID. In this case, we use seven baffles, which gives a baffle spacing of 1 ft (shell ID is $13\frac{1}{4} = 1.104 \text{ ft}$).

Once the exchanger is selected, calculations are made to determine

outlet temperatures, correction factor, and pressure drops. We calculate that, for this setup, the outlet temperature of the gasoline is 141°F, the correction factor is 0.982, pressure drops are both less than 10 psi, and the gasoline (tube fluid) velocity is 5.8 ft/s. So this exchanger is rejected because it cannot meet the outlet temperature requirement.

We proceed by next assuming a 12 ft length, then we try 16 ft as the longest exchanger with this shell diameter. We proceed to the next larger shell size, and try it for 8, 12, and 16 ft lengths. We continue in this fashion until all requirements are met. Table 9.3 summarizes all the trials and gives the final solution. Calculations were made with a spreadsheet, and property values were interpolated for all cases using an average of the inlet and outlet temperatures for each fluid.

Comments on the Results

Referring to Table 9.3, we see that the sixth exchanger satisfies almost all criteria, except that after fouling occurs, the outlet temperature is 121°F (120°F is required). This is an unfortunate circumstance.

The exchanger of Trial 8 satisfies all criteria. Now, when new, this exchanger will deliver the gasoline at an outlet temperature of 106°F with an overall heat transfer coefficient of 181 BTU/(hr·ft²·°R). After one year, fouling has occurred, and the overall heat transfer coefficient becomes 133 BTU/(hr·ft²·°R). The outlet temperature of the gasoline then becomes 116°F.

TABLE 9.3. *Solution of Example 9.5.*

Trial	D_s in.	N_t	N_p	L ft	N_b	T_2 °F	F	Δp_t psi	Δp_s psi	suit- able
1	13.25	90	2	8	7	141	0.98	2.9	1.6	R1
2	13.25	90	2	12	10	126	0.961	3.8	1.8	R1
3	13.25	90	2	16	14	115	0.931	4.6	2.6	R2
4	15.25	124	2	8	6	138	0.979	1.6	0.9	R1
5	15.25	124	2	12	9	122	0.954	2.0	1.2	R1
6	15.25	124	2	16	12	112	0.919	2.5	1.5	R2
7	17.25	158	4	8	5	119	0.946	2.5	0.52	R2
8	17.25	158	4	12	8	106	0.882	9.6	0.8	S

Notes: All trials with $3/4$ -in.-ID, 13 BWG tubes on a 1-in. square pitch. $R_d = 0.001$ hr·ft²·°R/BTU for both liquids. Suitability column: R1 = does not meet temperature requirement. R2 = works when new but not after fouling occurs. S = satisfactory.

It is interesting to consider the case where we select too large an exchanger. For example, suppose we started with a 31-in.-ID shell, three baffles, eight passes, 560 tubes, and a length of 8 ft. The outlet temperature of the gasoline, when the exchanger is new, would be 98°F, with a correction factor F of 0.779. After one year, the gasoline outlet temperature would be

101°F. The pressure drops are both under 10 psi. Thus, this exchanger satisfies all criteria as well. It is, however, larger than we need. The 17¹/₄ ID exchanger is smaller and works satisfactorily without the added expense of 402 more tubes and a larger shell.

Summary of Information Requested in Problem Statement

Exchanger	Shell diameter = 17 ¹ / ₄ in.
Specifications	Shell fluid passes = 1
	Tube fluid passes = 4
	Number of tubes = 158, 12 ft long
	Number of baffles = 8
	Tubing = ³ / ₄ -in. OD, 13 BWG
	1-in. sq. pitch
Mass flow of cooling water	= 175,400 lbm/hr

Performance Prediction	New	Parameter	After 1 year
	106°F	T_2	116°F
	0.882	F	0.935
	9.6 psi	Δp_t	9.14 psi
	0.8 psi	Δp_s	0.8 psi
	99°F	t_2	97°F

9.6 Optimum Water Outlet Temperature Analysis

Consider a conventional power plant that makes use of raw water from a nearby source as a cooling medium. This application is quite common, and so it is desirable to formulate a model to optimize such a system. Although the model will be for a problem in which water is the cooling medium, the results can be applied to other fluids if cooling fluid costs are known. The optimization process becomes one of minimizing costs and determining the optimum cooling water (or fluid) temperature. This concept is illustrated in Figure 9.20.

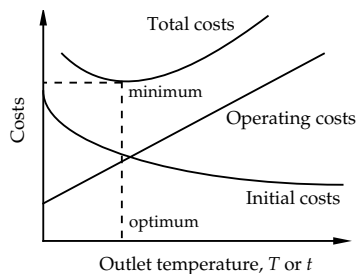


FIGURE 9.20. Cost relationships in shell and tube equipment as a function of outlet temperature.

It is possible, on the one hand, to use a great quantity of cooling water to obtain a small temperature increase. If this is done, then less surface area is required and so a smaller heat exchanger can be used. Therefore, the original investment involves smaller cost, but we also have a greater operating cost. On the other hand, a small flow rate of cooling water will require a greater surface area, and so a larger heat exchanger must be used. Therefore, the original investment is greater, but the operating costs will be reduced. In view of these two extremes, we conclude that there must be an optimum operating point that minimizes the initial plus operating costs.

The model we formulate will yield an annual cost for the exchanger. The total annual cost is the sum of the annual cost of water (cooling fluid), the fixed costs, maintenance, depreciation, amortization, and so on. With regard to the discussion above, the trade-off is between surface area A_o and mass flow rate $\dot{m}_c$. It is appropriate to include these quantities as primary variables in the analysis. So for the cooling water,

$$q = \dot{m}_c C_{pc}(t_2 - t_1) = UA_o F (LMTD) \quad (9.31)$$

From Equation 9.31,

$$\dot{m}_c = \frac{q}{C_{pc}(t_2 - t_1)}$$

and

$$A_o = \frac{q}{UF(LMTD)}$$

The cost of water per year is given as

$$C_{H_2O} = C_W \dot{m}_c t \quad (9.32)$$

where: C_{H_2O} is the cost of water per year, with dimensions of MU/yr (\$/yr);
 C_W is the cost of water per unit mass with dimensions of MU/M (\$/lbm or \$/kg);
 $\dot{m}_c$ is the mass flow rate of water (the cooling medium) with dimensions of M/T (lbm/s or kg/s); and,
 t is the number of hours per year that the exchanger is in operation (hr/yr).

The annual or operating cost of the exchanger will include amortization, pumping costs, and maintenance. These quantities can all be included in a single term that is expressed on a per-unit area basis. Thus,

$$C_A = C_F A_o \quad (9.33)$$

where: C_A is the cost of the exchanger per year, with dimensions of MU/yr (\$/yr);
 C_F is the annual cost of the exchanger on a per-unit square-foot basis with dimensions of MU/L² (\$/ft² or \$/m²); and,
 A_o is the heat transfer surface area of the exchanger (the outside surface area of all the tubes) with dimensions of L² (ft² or m²).

The total cost of the heat exchanger per year then becomes

$$C_T = C_{H_2O} + C_A = C_W \dot{m}_c t + C_F A_o$$

Substituting for mass flow rate and area gives

$$C_T = \frac{C_W t q}{C_{pc}(t_2 - t_1)} + \frac{C_F q}{UF(LMTD)}$$

In terms of inlet and outlet temperatures, the preceding equation becomes

$$C_T = \frac{C_W t q}{C_{pc}(t_2 - t_1)} + \frac{C_F q}{UF \left(\frac{(T_1 - t_2) - (T_2 - t_1)}{\ln [(T_1 - t_2)/(T_2 - t_1)]} \right)} \quad (9.34)$$

The next step involves differentiating this expression with respect to the outlet temperature t_2 of the coolant (water) in order to minimize the total cost. Taking the partial derivative $\partial C_T / \partial t_2$ and setting the results equal to zero gives, after considerable manipulation,

$$\frac{UF t C_W}{C_F C_{pc}} \left(\frac{(T_1 - t_2) - (T_2 - t_1)}{t_2 - t_1} \right)^2 = \ln \frac{T_1 - t_2}{T_2 - t_1} - \left(1 - \frac{T_2 - t_1}{T_1 - t_2} \right) \quad (9.35)$$

This equation is graphed in Figure 9.21 (see Problem 9.11 for the procedure). The ratio $UF t C_W / C_F C_{pc}$ is plotted on the horizontal axis, which ranges from 0.1 to 10. The temperature ratio $(T_1 - t_2) / (T_2 - t_1)$, which ranges from 0.1 to 10, is plotted on the vertical axis, with $(T_1 - T_2) / (T_2 - t_1)$ appearing on the graph as an independent variable. It is easier to use the graph of Figure 9.21 to find the optimum water outlet temperature than to solve for it (trial-and-error style) with Equation 9.35.

EXAMPLE 9.6. A shell and tube heat exchanger uses water as a cooling medium. Data on this particular heat exchanger are given below. Use the

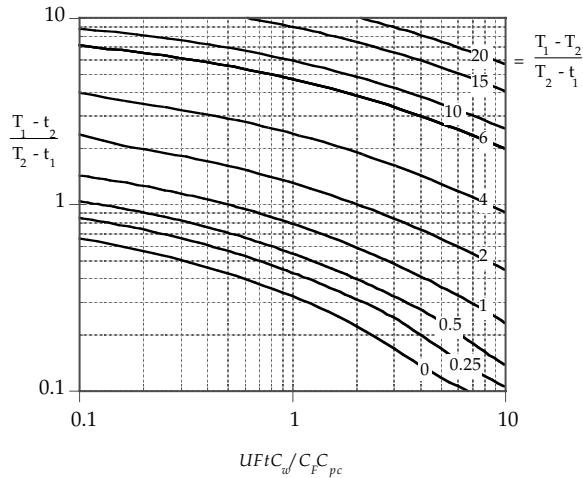


FIGURE 9.21. Graph for finding optimum cooling fluid outlet temperature.

data to calculate the optimum cooling water outlet temperature. Assume that the exchanger is in operation 7800 hr/yr, that water costs \$0.05/(1000 gallons), and that the annual cost of operating the exchanger amounts to \$20/(ft² of surface area·yr).

$$\begin{array}{ll}
 A_o &= 1074.2 \text{ ft}^2 & T_1 &= 200^\circ\text{F} \\
 C_{pc} &= 0.9980 \text{ BTU/lbm}\cdot^\circ\text{R} & T_2 &= 100^\circ\text{F} \\
 U &= 108 \text{ BTU/hr}\cdot\text{ft}^2\cdot^\circ\text{R} & t_1 &= 80^\circ\text{F} \\
 F &= 0.817 & \rho &= 0.994(62.4) \text{ lbm/ft}^3
 \end{array}$$

Solution: The water cost is given as \$0.05/(1000 gallons), which must be converted from monetary units per volume to monetary units per unit mass. Thus,

$$C_W = \frac{\$0.05}{1000 \text{ gal}} \frac{2.831 \times 10^{-2} \text{ gal}}{3.785 \times 10^{-3} \text{ ft}^3} \frac{\text{ft}^3}{(0.994)62.4 \text{ lbm}}$$

$$\text{or } C_W = \$6.03 \times 10^{-6} / \text{lbm}$$

We now calculate the dimensionless ratio:

$$\frac{U F t C_W}{C_F C_{pc}} = \frac{108(0.817)(7800)(6.03 \times 10^{-6})}{20(0.9980)} = 0.208$$

The temperature ratio we need is found as

$$\frac{T_1 - T_2}{T_2 - t_1} = \frac{200 - 100}{100 - 80} = 5$$

We read from Figure 9.21

$$\frac{T_1 - t_2}{T_2 - t_1} \approx 5.0$$

The optimum outlet temperature of the water then is

$$t_2 = T_1 - 5.0(T_2 - t_1) = 200 - 5.0(100 - 80)$$

or $\underline{t_2 = 100^\circ\text{F}}$

It is instructive at this point to calculate the correction factor F , as a check on the results. We first determine S and R :

$$S = \frac{t_2 - t_1}{T_1 - t_1} = \frac{100 - 80}{200 - 80} = 0.1667$$

$$R = \frac{\dot{m}_c C_{pc}}{\dot{m}_w C_{pw}} = \frac{(T_1 - T_2)}{(t_2 - t_1)} = \frac{200 - 100}{100 - 80} = 5$$

For a 1-2 shell and tube exchanger, Equation 9.4 gives

$$F \approx 0.817$$

9.7 Show and Tell

1. Obtain a catalog of shell and tube heat exchangers and give a presentation on the various designs that are available. Discuss especially designs that have been used to alleviate expansion problems.
2. How are shell and tube heat exchangers cleaned? Give a presentation on devices that are used for this purpose.
3. Several manufacturers (such as Dow Chemical and Monsanto) market what are known commercially as "heat transfer fluids." What are they, and what are they designed to be used for? Give a presentation on heat transfer fluids and the properties they have.

9.8 Problems

Analysis of Existing Exchangers

- During one phase of the separation of crude oil into its components, the oil is to be heated by water in a 1-4 shell and tube heat exchanger. The oil has a flow rate of 110,000 lbm/hr, and it enters the heat exchanger at 120°F. Water enters the exchanger at a flow rate of 66,000 lbm/hr and a temperature of 180°F. It is proposed to use an exchanger having a shell inside diameter of 23¹/₄-in. and containing 1-in.-OD tubes, 13 BWG, laid out on a 1¹/₄-in. square pitch. The 192 tubes are 12-ft long, and the exchanger contains six baffles. Determine expected outlet temperatures and pressure drops when the exchanger is new. Route the oil through the tubes, and assume that oil has the following properties:

$$C_p = 2\,050 \text{ J/(kg}\cdot\text{K)}$$

$$\mu = 3.4 \text{ cp}$$

$$\rho = 740 \text{ kg/m}^3$$

$$k_f = 0.132 \text{ W/(m}\cdot\text{K)}$$

- A 12-in.-ID 1-2 shell and tube heat exchanger is used with water as a cooling medium. Kerosene [$C_p = 2\,530 \text{ J/(kg}\cdot\text{K)}$, $\rho = 800 \text{ kg/m}^3$, $\mu = 0.4 \text{ cp}$, and $k_f = 0.133 \text{ W/(m}\cdot\text{K)}$] at an inlet temperature of 220°F and a flow rate of 100,000 lbm/hr is to be cooled with water available at 70°F and a flow rate of 96,000 lbm/hr. The exchanger contains 45 1-in.-OD tubes, 6 ft long, 13 BWG, laid out on a 1¹/₄-in. square pitch. Analyze the heat exchanger completely and determine if it is suitable for this service. The exchanger contains six evenly spaced baffles. Determine expected outlet temperatures and pressure drops when the exchanger is new. Route the water through the tubes.
- Liquid carbon dioxide at a flow rate of 110 000 kg/hr is to be heated from 0°C to 20°C in a 1-2 shell and tube heat exchanger. Water is available at a flow rate of 112 500 kg/hr and a temperature of 40°C. A 25-in.-ID 1-2 shell and tube exchanger having 3³/₄-in., 10 BWG tubes laid out on a 1-in. triangular pitch is available. The tubes are 2 m long and the exchanger contains three baffles. Determine expected outlet temperatures and pressure drops when the exchanger is new. Route the carbon dioxide through the tubes and take the CO₂ properties to be

$$\text{At } 0^\circ\text{C: } C_p = 2\,470 \text{ J/(kg}\cdot\text{K)}$$

$$k_f = 0.104\,5 \text{ W/(m}\cdot\text{K)}$$

$$\text{Sp. Gr.} = 0.926$$

$$\nu = 1.08 \times 10^{-7} \text{ m}^2/\text{s}$$

$$\text{At } 10^\circ\text{C: } C_p = 3\,140 \text{ J/(kg}\cdot\text{K)}$$

$$k_f = 0.097\,1 \text{ W/(m}\cdot\text{K)}$$

$$\text{Sp. Gr.} = 0.860$$

$$\nu = 1.01 \times 10^{-7} \text{ m}^2/\text{s}$$

- Kerosene at 200°F flows at a rate of 150,000 lbm/hr and must be cooled to 190°F. Crude oil is available at 70°F and flows at a rate of 150,000 lbm/hr. It is proposed to use a 21¹/₄-in.-ID shell and tube heat exchanger containing 1-in.-OD tubes, 13 BWG laid out on a 1¹/₄-in. square pitch. The 166 tubes are 15 ft long. The baffles are spaced 12 in. apart, and the tube fluid will make four passes through the exchanger. Determine expected outlet temperatures and pressure drops when the exchanger is new. Route the crude oil through the tubes, and take the fluid properties to be

Kerosene:

$$\begin{array}{ll} C_p = 0.59 \text{ BTU/lbm}\cdot^\circ\text{R} & \rho = 0.73(62.4) \text{ lbm/ft}^3 \\ k_f = 0.0765 \text{ BTU/hr}\cdot\text{ft}\cdot^\circ\text{R} & \mu = 8.35 \times 10^{-6} \text{ lbf}\cdot\text{s/ft}^2 \end{array}$$

Crude Oil:

$$\begin{array}{ll} C_p = 0.49 \text{ BTU/lbm}\cdot^\circ\text{R} & \rho = 0.83(62.4) \text{ lbm/ft}^3 \\ k_f = 0.077 \text{ BTU/hr}\cdot\text{ft}\cdot^\circ\text{R} & \mu = 7.52 \times 10^{-5} \text{ lbf}\cdot\text{s/ft}^2 \end{array}$$

5. A sugar solution ($\rho = 1\,080 \text{ kg/m}^3$, $C_p = 3\,601 \text{ J/(kg}\cdot\text{K)}$, $k_f = 0.576 \text{ W/(m}\cdot\text{K)}$, and $\mu = 1.3 \times 10^{-3} \text{ N}\cdot\text{s/m}^2$) flows at a rate of 60 000 kg/hr and is to be heated from 25°C to 40°C. Water at 95°C is available at a flow rate of 75 000 kg/hr. It is proposed to use a 12-in.-ID, 1-2 shell and tube heat exchanger containing 3/4-in.-OD, 16 BWG tubes, 1 m long and laid out on a 1-in. square pitch. The exchanger contains three baffles spaced evenly. Will the exchanger be suitable? If not, can it be made to work by doubling the flow rate of the water? Find outlet temperatures for the configuration that works when the exchanger is new.
6. Measured inlet and outlet temperatures of the fluids flowing through a heat exchanger are

$$\begin{array}{ll} T_1 = 180^\circ\text{F} & T_2 = 80^\circ\text{F} \\ t_1 = 50^\circ\text{F} & t_2 = 100^\circ\text{F} \end{array}$$

Which fluid has the higher capacitance? Calculate E_c for this exchanger.

7. Measured inlet and outlet temperatures of the fluids flowing through a heat exchanger are

$$\begin{array}{ll} T_1 = 95^\circ\text{F} & T_2 = 90^\circ\text{F} \\ t_1 = 25^\circ\text{F} & t_2 = 40^\circ\text{F} \end{array}$$

Determine the correction factor F if these temperatures exist in the following heat exchangers:

- a. A double pipe heat exchanger operating in counterflow.
- b. A 1-2 shell-and-tube heat exchanger.

Derivations

8. Using the definition of equivalent diameter D_e , show that for a triangular pitch layout,

$$D_e = \frac{3.46 P_T^2}{\pi OD_t} - OD_t$$

9. For turbulent flow, the development in Section 9.5 showed that the convection coefficient increases by a factor of 3.03 when increasing from two to eight tube passes in a 1-2 shell and tube heat exchanger. The corresponding pressure drop increases by a factor of 64. Repeat these calculations for laminar flow conditions.
10. Start with Equation 9.34 and derive Equation 9.35.

11. The equation for optimum cooling water outlet temperature was derived as

$$\frac{UFtC_W}{C_F C_{pc}} \left(\frac{(T_1 - t_2) - (T_2 - t_1)}{t_2 - t_1} \right)^2 = \ln \frac{T_1 - t_2}{T_2 - t_1} - \left(1 - \frac{T_2 - t_1}{T_1 - t_2} \right)$$

Given certain operating conditions and economic parameters, this equation can be solved for t_2 . Now t_2 appears on both sides of this equation, and it is difficult to solve for it directly. However, a graph that allows us to solve for t_2 would be convenient. The task becomes one of identifying ratios to use for the parameters of the graph. For the horizontal axis, we define

$$x = \frac{UFtC_W}{C_F C_{pc}}$$

For the vertical axis, we define

$$y = \frac{T_1 - t_2}{T_2 - t_1}$$

- a. Verify that the equation to solve becomes

$$x \left(\frac{(T_1 - t_2) - (T_2 - t_1)}{t_2 - t_1} \right)^2 = \ln y - (1 - 1/y)$$

If we can express the term in parentheses on the left-hand side with a linear expression in terms of y and some other parameter, then we can determine the scales for a graph. We define a new independent term that does not contain t_2 (found by trial and error) as

$$A = \frac{T_1 - T_2}{T_2 - t_1}$$

- b. Show that

$$\frac{(T_1 - t_2) - (T_2 - t_1)}{t_2 - t_1} = \frac{y - 1}{A - y + 1}$$

Thus, A can be used as an independent parameter, with x and y , to produce a graph.

Optimum Cooling Water Outlet Temperature

12. Calculate the optimum water outlet temperature for the conditions given below. Assume that the exchanger is in operation for 7800 hr/yr, that water costs \$0.05/(1000 gallons), and that the annual fixed charges amount to \$20/(ft²·yr).

A_o	= 616.2 ft ²	T_1	= 130°F
C_{pc}	= 0.9988 BTU/lbm·°R	T_2	= 98.4°F
U	= 224.9 BTU/hr·ft ² ·°R	t_1	= 65°F
F	= 0.775	ρ	= 62.4 lbm/ft ³

13. If the exchanger in Problem 9.12 is in operation for only 4000 hr/yr, how does the outlet temperature of the cooler fluid t_2 change?

14. Calculate the optimum water outlet temperature for the conditions given below. Assume that the exchanger is in operation for 7800 hr/yr, that water costs \$0.05/(4 m³), and that the annual fixed charges amount to \$150/(m²·yr).

$$\begin{array}{ll} A_o = 6.6 \text{ m}^2 & T_1 = 121^\circ\text{C} \\ C_{pc} = 4179 \text{ J/(kg}\cdot\text{K)} & T_2 = 103^\circ\text{C} \\ U = 1073 \text{ W/(m}^2\cdot\text{K)} & t_1 = 24^\circ\text{C} \\ F = 0.995 & \rho = 1000 \text{ kg/m}^3 \end{array}$$

15. Calculate the optimum water outlet temperature for the conditions given below. Assume that the exchanger is in operation for 7800 hr/yr, that water costs \$0.04/(4 m³), and that the annual fixed charges amount to \$160/(m²·yr).

$$\begin{array}{ll} A_o = 23.1 \text{ m}^2 & T_1 = 93^\circ\text{C} \\ C_{pc} = 4179 \text{ J/(kg}\cdot\text{K)} & T_2 = 48^\circ\text{C} \\ U = 1030 \text{ W/(m}^2\cdot\text{K)} & t_1 = 27^\circ\text{C} \\ F = 0.946 & \rho = 1000 \text{ kg/m}^3 \end{array}$$

Design Problems

In most of the following problems, properties of fluids are not given. It is therefore necessary to locate a reference text that contains the properties needed.

16. Octane at 70°F must be heated to 110°F by using ethylene glycol, which is available at 165°F. The flow rate of both fluids is 110,000 lbm/hr. Size a heat exchanger for this service; remember to account for fouling effects.
17. Warm methanol is used as a solvent in a cleaning operation. The methanol is at 25°C and it must be heated to 40°C at a flow rate of 50 000 kg/hr. Water at 80°C and a flow rate of 40 000 kg/hr is available to heat the methanol. Select a heat exchanger for this service. Do not neglect effects of fouling, and assume that tube lengths of 1, 2, 3, 4, or 5 m are available.
18. Steam is often used as a heating medium. Some companies, however, manufacture and market what they refer to as “heat transfer fluids” that work as effectively as steam and at a lower cost. One such fluid (from Monsanto) is called “Therminol,” which is available in many different formulas. Each formula is designed to work over a specific temperature range.

Acetone is to be heated with Therminol 60 in a manufacturing facility. The acetone has a temperature of 10°C and it is to be heated to 30°C, at a flow rate of 80 000 kg/hr. Therminol 60 (an organic liquid) is used throughout the facility absorbing reject-heat in one area and providing heat in another. Heated Therminol 60 available at 100°C and 100 000 kg/hr is available to heat the acetone. Select a heat exchanger for this service, and take into account the effects of fouling. Tube lengths available are 1, 2, 3, and 4 m.

Therminol 60 properties:

At 80°C:

$$\begin{array}{ll} C_p = 1830 \text{ J/(kg}\cdot\text{K)} & \text{Sp. Gr.} = 0.958 \\ k_f = 0.125 \text{ W/(m}\cdot\text{K)} & \mu = 2.05 \times 10^{-3} \text{ N}\cdot\text{s/m}^2 \end{array}$$

At 100°C:

$$\begin{array}{ll} C_p = 1900 \text{ J/(kg}\cdot\text{K)} & \text{Sp. Gr.} = 0.944 \\ k_f = 0.123 \text{ W/(m}\cdot\text{K)} & \mu = 1.52 \times 10^{-3} \text{ N}\cdot\text{s/m}^2 \end{array}$$

19. Ammonia is used in a system that produces chilled water. Ammonia at 4°C must be heated to at least 40°C using steam. The steam is a saturated vapor at 15 kPa and should leave the exchanger as a saturated liquid. The steam flow rate is variable. Size a heat exchanger for this task and take into account fouling effects. What is the required steam flow rate when the exchanger is new, and what is it after 1 year has passed? The ammonia flow rate is 150 000 kg/hr.
20. Saturated steam vapor at 5 psia enters a condenser at a flow rate of 35,000 lbm/hr. At the condenser exit, the fluid leaves as a saturated liquid. Cooling water at 55°F from a nearby river is used to condense the steam. For environmental reasons, it is desirable to return the water to the river at no warmer than 80°F , if possible. Size the heat exchanger for this service, taking fouling effects into account.

Group Problems

A water-to-water system is used to test the effects of changing tube length, baffle spacing, tube pitch, pitch layout, and tube diameter. Cold water at 25°C and 100 000 kg/hr is heated by hot water at 100°C , also at 100 000 kg/hr. The exchanger has a 12-in.-ID shell. Perform calculations on this 1-2 shell and tube heat exchanger for the following conditions, as outlined in the problems, and put together an overall comparison chart. Express trends that are substantiated by the calculations. For example, if tube length increases: tube fluid pressure drop increases; outlet temperature of the cooler fluid increases; outlet temperature of the warmer fluid decreases, etc.

- GP1. 3/4-in.-OD tubes, 16 BWG laid out on a 1-in. triangular pitch; three baffles per meter of tube length. Analyze the exchanger for tube lengths of 1, 1.5, 2, 2.5 and 3 m.
- GP2. 3/4-in.-OD tubes, 16 BWG laid out on a 1-in. triangular pitch; 2 m long. Analyze for 2 baffles, 3 baffles, 4 baffles, and 5 baffles.
- GP3. 3/4-in.-OD tubes, 16 BWG; 2 m long; and four baffles. Analyze the exchanger for tube layouts of 15/16-in. triangular pitch, 1 in. triangular pitch, and 1-in. square pitch.
- GP4. 1-in.-OD tubes, 16 BWG, 2 m long; and 4 baffles. Analyze the exchanger for tube layouts of $1\frac{1}{4}$ -in. triangular pitch, and $1\frac{1}{4}$ -in. square pitch.
- GP5. 3/4-in.-OD tubes, 16 BWG laid out on a 1-in. triangular pitch; 2 m long; and four baffles. Analyze the exchanger for 2, 4, 6, and 8 tube passes, and compare to the case of true counterflow (i.e., one tube pass).

CHAPTER 10 Plate and Frame and Cross Flow Heat Exchangers

In this chapter, the discussion of heat exchangers is continued by considering the plate and frame heat exchanger and the cross flow heat exchanger. Each exchanger is described, as are applications where these exchangers might be used. The effectiveness-NTU method is applied to the analysis of these exchangers, and example problems are provided to show how they are modeled.

10.1 The Plate and Frame Heat Exchanger

The plate and frame heat exchanger was originally introduced in the 1930s and is used extensively in the food industries. These heat exchangers have many characteristics that are of significance to process industries as well.

A plate and frame heat exchanger consists of several metal sheets with corrugated surfaces that are clamped together. Figure 10.1 shows a frontal view of a plate with a herringbone pattern; other patterns are also in use (such as one referred to as a “washboard”). In the herringbone pattern, the angle made between adjacent ribs and the vertical is called the **chevron angle**, θ . Plates can be made with small chevron angles (low- θ plates) or large angles (high- θ plates). Performance of the heat exchanger is a function of the chevron angle. High- θ plates provide high heat transfer rates with high pressure losses. The converse is also true.

The plates have rubber gaskets glued to them in the pattern shown in the figure or in some similar way. Also shown in Figure 10.1 is a profile view that indicates how the two fluids flow about adjacent plates. Figure 10.2 shows how the plates are arranged and how two fluids are routed as they pass through the heat exchanger itself. As indicated in these two figures, each sheet separates the cold and warm fluids, which can be made to flow in either a countercurrent (Figure 10.1) or a parallel flow pattern (Figure 10.2).

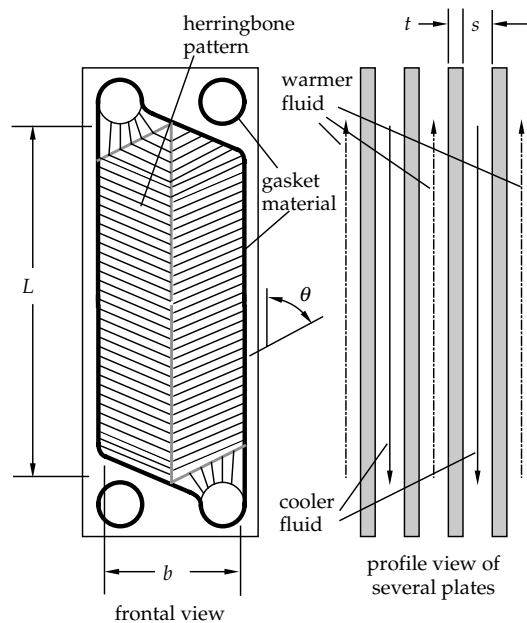


FIGURE 10.1. *Frontal view of one plate and profile views of several plates showing a countercurrent flow configuration. (Countercurrent flow is represented.)*

Plate exchangers are well suited for liquid-liquid flows under turbulent conditions. These exchangers can also be used as condensing units operating at moderate flow rates and pressures (up to 60 psia). Large-volume flow rates in a condenser application, however, are better handled by shell and tube heat exchangers. The greatest advantages of a plate and frame exchanger is that it can be modified readily and is small in size. If an existing exchanger cannot transfer the required amount of heat, for example, one or more plates can easily be added on an as-needed basis.

Plate Construction and Materials

The surface of each plate is important because it is the heat transfer area. Corrugating the plates imparts a certain degree of stiffness and provides contact points between adjacent plates when they are clamped together. Furthermore, a dimpled or corrugated surface on each plate causes turbulent mixing to occur that will enhance the heat transferred. Plate thicknesses can be as small as 0.6 mm (0.024 in.).

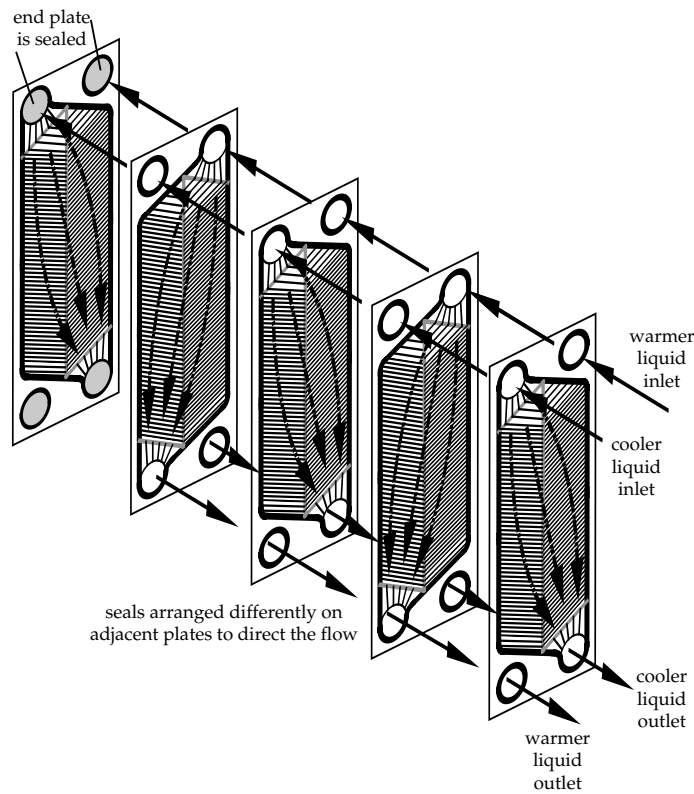
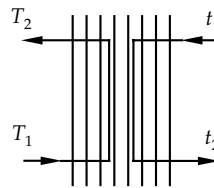


FIGURE 10.2a. Flow through a plate and frame heat exchanger. (Parallel flow is represented.)

FIGURE 10.2b. Schematic representation of a plate and frame heat exchanger. (Counterflow configuration)



Little or no welding is involved in producing a plate. They are often merely stampings, so any metal that can be cold-formed can be used as a plate material. Stainless steel (18/10 same as AISI 304 or 18/2/2.5 Mo same as AISI 316) is probably the most commonly used material. Titanium is also popular for systems containing chloride solutions or brackish cooling water because of titanium's resistance to corrosion. Other materials used include nickel alloys, copper alloys, aluminum alloys, brass alloys, zirconium alloys, as well as pure metals such as copper, aluminum, nickel, and silver.

Gaskets

At least two gaskets separate the fluids. If there is a gasket failure, the leaking fluid is discharged to the atmosphere and the two fluids seldom mix. Gasket materials include natural rubber styrene, resin-cured nitrile, and silicone and butyl rubbers. Neoprene and compressed asbestos are also used.

Frames

The sheet metal plates are clamped together with nuts and long bolts in a frame that contains pipe connections for both fluids. Frames are usually free standing; because of the way they are constructed, the entire heat exchanger can be taken apart in a very short time. Frames are usually made of carbon steel, which is painted or coated to protect against corrosion. Connections are usually made of the same material as the plates in order to prevent electrochemical problems.

Limitations

The maximum allowable pressure of the fluids is determined by frame strength, plate deformation limits, or gasket retainment ability. Frame strength is usually the limiting parameter. Operating pressures typically vary from 85 psig to over 200 psig, and in some cases to as high as 300 psig.

Operating temperatures are usually limited by gasket strength or a tendency to decompose. A styrene gasket is usually good to 70°C (160°F), whereas compressed asbestos fiber can be used to 200°C (390°F).

10.2 Analysis of Plate and Frame Heat Exchangers

Figure 10.3 shows a profile view of a plate within an exchanger and the associated resistances to heat transfer. For the sake of discussion, the warmer fluid is on the left, and heat is transferred through the plate to the cooler fluid. The resistances include a convection resistance on the warm side, a conduction resistance through the plate, and a convection resistance on the cooler side. The *heat transfer* area A_o is the same as the surface area, and equals plate width b times height L . The sum of the resistances is

$$\Sigma R = R_{12} + R_{23} + R_{34}$$

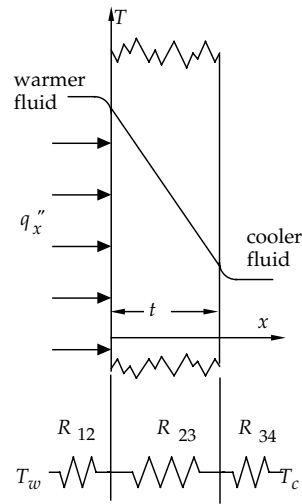


FIGURE 10.3. Profile view of one plate and the resistances to heat transfer.

which becomes

$$\Sigma R = \frac{1}{h_i A_o} + \frac{t}{k A_e} + \frac{1}{h_o A_o} \quad (10.1)$$

where t is the thickness of the plate, h_i is the convection coefficient between the warmer fluid and the plate, and h_o applies between the cooler fluid and the plate. Usually, the temperature drop across a thin-walled metal is virtually negligible, but for the plate and frame exchanger, such may not be the case. The convection coefficients are often so high that the conduction resistance is the same order of magnitude as the convection resistances. With this in mind, we define an overall heat transfer coefficient U_o that will exist when the exchanger is new:

$$A_o \Sigma R = \frac{1}{U_o} = \frac{1}{h_i} + \frac{t}{k} + \frac{1}{h_o} \quad (\text{one plate}) \quad (10.2)$$

in which the overall heat transfer coefficient U_o is based on area $A_o = bL$. As before, the overall heat transfer coefficient has the same dimensions as h , namely, $[F \cdot T / (T \cdot L^2 \cdot t)]$ $[BTU / \text{hr} \cdot \text{ft}^2 \cdot ^\circ R]$ or $W / (\text{m}^2 \cdot K)$.

The heat transferred within the heat exchanger equals the product of the overall heat transfer coefficient U_o , the total surface area of N_s plates, which is $A_o N_s$, and a temperature difference. The amount of heat exchanged is given by

$$q = U_o A_o N_s \Delta t = \dot{m}_w C_{pw} (T_1 - T_2) = \dot{m}_c C_{pc} (t_2 - t_1) \quad (10.3)$$

where Δt is the temperature difference that applies to the plate and frame

heat exchanger. If we were to examine the temperature profile of the fluids as they pass through the exchanger, we would find that pure counterflow does not exist. If the flow through the exchanger is entirely counterflow or parallel flow, then Δt would be the log mean temperature difference for counterflow or parallel flow, respectively. The plate and frame exchanger has features of both flows, however. The established method of analysis involves the use of the log mean temperature difference for counterflow, as a “best possible” case, and a correction factor, F . An equation for the correction factor that applies to the plate and frame exchanger (rather than its derivation) will be given here. We begin with the **number of transfer units** N , defined as

$$N = \frac{U_o A_o N_s}{(\dot{m} C_p)_{\min}} \quad (10.4)$$

where $(\dot{m} C_p)_{\min}$ is the minimum mass flow rate-specific heat product for either fluid. A number of flow configurations could be used with a plate and frame heat exchanger. Several can be linked together in various ways. For example, consider four exchangers with the warm fluid passing through them all in a series pattern while the cooler fluid passes through in parallel. We would call such an arrangement a 4/1 pass system. Correction factor equations and charts are available for 1/1, 2/1, 3/1, 2/2, 3/3, and 4/4 pass systems. For our purposes, we consider only a 1/1 pass system. The correction factor for a 1/1 plate and frame exchanger is given in terms of the number of transfer units as

$$F \approx 1 - 0.0166N \quad (10.5)$$

The driving temperature difference in the heat balance equation then becomes

$$\Delta t = F (LMTD_{\text{counterflow}}) = F \frac{(T_1 - t_2) - (T_2 - t_1)}{\ln [(T_1 - t_2)/(T_2 - t_1)]}$$

From a practical viewpoint, the correction factor F is indicative of how efficient the exchanger is thermally. The equation for heat transfer within a plate and frame heat exchanger is written as

$$q = U_o A_o N_s F \frac{(T_1 - t_2) - (T_2 - t_1)}{\ln [(T_1 - t_2)/(T_2 - t_1)]}$$

Convection Coefficient and Pressure Drop

In order to calculate the overall heat transfer coefficient U_o in Equation 10.3, we need equations for h_i and h_o . These surface coefficients are

found with equations that have been developed by experimental means. Before stating them, however, it is important to consider the geometry of the flow and to develop an equation for the characteristic length. Plates are spaced close together to form what is in essence a two-dimensional flow channel for each fluid. The flow passage, then, is bounded by the distance between the plates (s) and the plate width (b , gasket to gasket). The hydraulic diameter for a rectangular flow section is

$$D_h = \frac{4 \times \text{area}}{\text{perimeter}} = \frac{4sb}{2s + 2b}$$

where the area is that which is normal to the flow direction between adjacent plates. For a two-dimensional flow passage in which $b \gg s$, the hydraulic diameter becomes

$$D_h \approx \frac{4sb}{2b}$$

or

$$D_h = 2s \quad (10.6)$$

The spacing s between plates varies typically from 2 to 5 mm.

For laminar flow through a plate and frame heat exchanger, the Sieder-Tate equation can be used:

$$\text{Nu} = \frac{hD_h}{k_f} = 1.86 \left(\frac{D_h \text{RePr}}{L} \right)^{1/3} \quad (10.7)$$

where $\text{Re} = VD_h/\nu$. The difficulty in applying Equation 10.7 is that the transition from laminar to turbulent flow occurs over a range of Reynolds numbers that vary from 10 to 400. For our purposes, we assume transition to occur at $\text{Re} = 100$.

In general for turbulent flow, one of the most widely used relationships is

$$\text{Nu} = \frac{hD_h}{k_f} = 0.374 \text{Re}^{0.668} \text{Pr}^{1/3} \quad (10.8)$$

valid for $100 \leq \text{Re} = VD_h/\nu$ $\text{Pr} = \nu/\alpha > 0$

μ changes moderately with temperature

properties evaluated at the average fluid temperature

$[= (\text{inlet} + \text{outlet})/2]$

The pressure drop encountered by the fluids as they flow through the exchanger is a multiple of the kinetic energy of the flow:

$$\Delta p_{\text{plates}} = \frac{fL}{D_h} \frac{\rho V^2}{2g_c} \quad (10.9)$$

where f is the friction factor and L is the plate length. The friction factor varies over the range of Reynolds numbers according to the following table:

Reynolds Number Range	Darcy-Weisbach Friction Factor
1–10	$f = \frac{280}{\text{Re}}$
10–100	$f = \frac{100}{\text{Re}^{0.589}}$
> 100	$f = \frac{12}{\text{Re}^{0.183}}$

The velocity between plates will equal the mass flow rate divided by density, by the flow area $A = sb$, and by the number of flow passages for the fluid of interest. If there is an odd number of plates, then there will be an equal number of flow passages; that is, the cooler and the warmer fluids will go through the same number of passages. If there is an even number of plates, then one of the fluids will go through one more flow passage than the other. For an odd number of plates, the flow velocity between plates is given by

$$V = \frac{\dot{m}/\rho A}{(N_s + 1)/2} \quad (10.10)$$

where N_s is the number of plates. So for an exchanger with, say, three plates, the exchanger is divided into $N_s + 1 = 4$ flow passages. One fluid will flow through only two of those passages; thus, the velocity of that fluid through one passage is the total flow divided by 2:

$$V = \frac{\dot{m}/\rho A}{(N_s + 1)/2} = \frac{\dot{m}/\rho A}{(3 + 1)/2} = \frac{\dot{m}/\rho A}{2}$$

This equation would apply to both fluids.

For an even number of plates, one of the fluids will have a velocity given by

$$V = \frac{\dot{m}/\rho A}{N_s/2} \quad (10.11a)$$

The other fluid will pass through one more passage, and its velocity is found with

$$V = \frac{\dot{m}/\rho A}{(N_s + 2)/2} \quad (10.11b)$$

For an exchanger with an even number of plates, say, four, the exchanger is divided into $N_s + 1 = 5$ flow passages. One of the fluids will flow through two of those passages, and its total flow will be divided in half:

$$V = \frac{\dot{m}/\rho A}{N_s/2} = \frac{\dot{m}/\rho A}{4/2} = \frac{\dot{m}/\rho A}{2}$$

Now, the other fluid will flow through three of the passages, so its total flow will be divided into thirds. Equation 10.11b gives

$$V = \frac{\dot{m}/\rho A}{(N_s + 2)/2} = \frac{\dot{m}/\rho A}{(4 + 2)/2} = \frac{\dot{m}/\rho A}{3}$$

We need the velocities in order to calculate Reynolds number and pressure losses.

Fluids will be entering and exiting the heat exchanger through standard piping connections, and the loss associated with these sudden changes in geometry is treated as a minor loss, called the **port loss**. The port loss is calculated as a loss coefficient ($K = 1.3$ typically) multiplied by the kinetic energy of the flow in the port (inlet or outlet) itself:

$$\Delta p_{\text{port}} = 1.3 \frac{\rho V_p^2}{2g_c} \quad (10.12)$$

where V_p is the velocity in the port, or inlet/outlet connection. The total pressure loss associated with the flow of either stream through the exchanger, then, is the sum of Δp_{plates} and Δp_{port} .

Outlet Temperature Calculation

In many applications, inlet temperatures and flow rates would be known and the outlet temperatures must be calculated. Ordinarily, we would need a separate analysis for a plate and frame heat exchanger in order to predict outlet temperature. However, because the correction factor F is very near 1, it is possible to use the equation developed for a counterflow double pipe heat exchanger to predict outlet temperature. The only modification is that the correction factor F must be included in the equation. Again we use the parameter R , first defined in Chapter 8:

$$R = \frac{\dot{m}_c C_{pc}}{\dot{m}_w C_{pw}} = \frac{T_1 - T_2}{t_2 - t_1} \quad (10.13)$$

The parameter E_c will contain the correction factor F , modified from the equation given originally in Chapter 7:

$$E_c = \frac{T_1 - t_2}{T_2 - t_1} = \exp \left[\frac{U_o A_o N_s F}{\dot{m}_c C_{pc}} (R - 1) \right] \quad (10.14)$$

Rearranging Equation 10.13 gives an equation for the outlet temperature of the cooler fluid:

$$t_2 = t_1 + \frac{T_1 - T_2}{R} \quad (10.15)$$

Substituting into another equation borrowed from Chapter 8, we get an equation for the outlet temperature of the warmer fluid:

$$T_2 = \frac{(1 - R)T_1 + (1 - E_c)Rt_1}{1 - RE_c} \quad (10.16)$$

Fouling Factors

Like other heat exchangers, plate and frame heat exchangers are subject to fouling on the plate surfaces. The effect is that heat must be transferred through additional resistances. We define a “dirty” or “design” overall heat transfer coefficient as

$$\frac{1}{U} = \frac{1}{U_o} + R_{di} + R_{do} \quad (10.17)$$

The design coefficient U is used when determining the area required to transfer heat.

Compared to other heat exchangers, plate and frame exchangers have smaller fouling factors for a number of reasons:

1. A high degree of turbulence is maintained in the flow, which tends to keep solids in suspension. Solids thus have little tendency to become deposited on the plate surfaces.
2. The plate surfaces are quite smooth, and so there is an absence of deposition sites for minerals or other solids.
3. There are no stagnation or “dead” spaces within the exchanger.

4. High convection coefficients are accompanied by lower plate surface temperatures where the cold fluid contacts the plates; it is the cooler fluid that has the greater tendency to leave deposits on a surface.

The plate and frame heat exchanger is simple to clean, and chemical cleaning methods are rapid and highly effective. Mechanical cleaning is easily performed because the exchanger can be readily taken apart.

Values of the resistances for various fluids have been measured as a result of years of experience and are provided in Table 10.1. Note that these resistances are different from those in Table 8.1. The resistances in Table 10.1 apply only to plate and frame heat exchangers.

TABLE 10.1. *Values of fouling factors for various fluids for a plate and frame heat exchanger.*

Fluid	R_d	
	$\text{m}^2\cdot\text{K}/\text{W}$	$\text{ft}^2\cdot\text{hr}\cdot^\circ\text{R}/\text{BTU}$
Engine oil	4×10^{-6} – 1×10^{-5}	0.00002–0.00005
Organic liquids	2×10^{-6} – 6×10^{-6}	0.00001–0.00003
Process fluids, in general	2×10^{-6} – 1.2×10^{-5}	0.00001–0.00006
Steam	2×10^{-6}	0.00001
Vegetable oil	4×10^{-6} – 1.2×10^{-5}	0.00002–0.00006
Water		
City water	4×10^{-6} – 1×10^{-5}	0.00002–0.00005
Distilled or mineral	2×10^{-6}	0.00001
Seawater	6×10^{-6} – 1×10^{-5}	0.00003–0.00005
Well water	1×10^{-5}	0.00005

The equations for the analysis of a plate and frame heat exchanger have been stated and are summarized in a suggested order of calculations procedure, which now follows.

SUGGESTED ORDER OF CALCULATIONS FOR A PLATE AND FRAME HEAT EXCHANGER

Problem	Complete problem statement.
Discussion	Potential heat losses; other sources of difficulties.
Assumptions	1. Steady-state conditions exist. 2. Fluid properties remain constant and are evaluated at a temperature of _____ .

- Nomenclature**
1. T refers to the temperature of the warmer fluid.
 2. t refers to the temperature of the cooler fluid.
 3. w subscript refers to the warmer fluid.
 4. h subscript refers to hydraulic diameter.
 5. c subscript refers to the cooler fluid.
 6. 1 subscript refers to an inlet condition.
 7. 2 subscript refers to an outlet condition.

A. Fluid Properties

$\dot{m}_w$	T_1
ρ	C_p
k_f	α
ν	Pr
$\dot{m}_c$	t_1
ρ	C_p
k_f	α
ν	Pr

B. Plate Dimensions and Properties

b	= plate width =
L	= plate height =
s	= plate spacing =
t	= plate thickness =
A_o	= plate surface area = bL =
A	= flow area = sb =
D_h	= hydraulic diameter of flow passage = $2s$ =
N_s	= number of plates =
k	= thermal conductivity of plate =
Plate material =	

C. Fluid Velocities

Odd number of plates

$$V = \frac{\dot{m}/\rho A}{(N_s + 1)/2} \quad (\text{for both fluids})$$

For an even number of plates: one fluid will have a velocity given by

$$V = \frac{\dot{m}/\rho A}{N_s/2} \quad (\text{for one fluid})$$

and

$$V = \frac{\dot{m}/\rho A}{(N_s + 2)/2} \quad (\text{for the other fluid})$$

D. Reynolds Numbers

$$\text{Re}_w = V_w D_h / \nu =$$

$$\text{Re}_c = V_c D_h / \nu =$$

E. Nusselt Numbers

Modified Sieder-Tate Equation for laminar flow:

$$\text{Nu} = \frac{h D_h}{k_f} = 1.86 \left(\frac{D_h \text{Re} \text{Pr}}{L} \right)^{1/3}$$

$$\text{Re} < 100 \quad 0.48 < \text{Pr} = \nu/\alpha < 16\,700$$

Modified Dittus-Boelter Equation for turbulent flow:

$$\text{Nu} = \frac{h D_h}{k_f} = 0.374 \text{Re}^{0.668} \text{Pr}^{1/3}$$

$$\text{Re} > 100; \quad \text{Pr} = \nu/\alpha > 0;$$

Conditions:

μ changes moderately with temperature
Properties evaluated at the average fluid
temperature [= (inlet + outlet)/2]

$$\text{Nu}_w =$$

$$\text{Nu}_c =$$

F. Convection Coefficients

$$h_i = \text{Nu}_w k_f / D_h =$$

$$h_o = \text{Nu}_c k_f / D_h =$$

G. Exchanger Coefficient

$$\frac{1}{U_o} = \frac{1}{h_i} + \frac{t}{k} + \frac{1}{h_o} \quad U_o =$$

H. Capacitances

$$(\dot{m} C_p)_w =$$

$$(\dot{m} C_p)_c =$$

$$(\dot{m}C_p)_{\min} =$$

I. Number of Transfer Units and Correction Factor

$$N = \frac{U_o A_o N_s}{(\dot{m}C_p)_{\min}} =$$

$$F = 1 - 0.0166 N =$$

J. Outlet Temperature Calculations

$$R = \frac{\dot{m}_c C_{pc}}{\dot{m}_w C_{pw}} =$$

$$E_{\text{counter}} = \exp [U_o A_o N_s F (R - 1) / \dot{m}_c C_{pc}] =$$

$$T_2 = \frac{T_1 (R - 1) - R t_1 (1 - E_{\text{counter}})}{R E_{\text{counter}} - 1} =$$

$$t_2 = (T_1 - T_2) / R + t_1 =$$

K. Log Mean Temperature Difference

$$\text{Counterflow} \quad LMTD = \frac{(T_1 - t_2) - (T_2 - t_1)}{\ln [(T_1 - t_2) / (T_2 - t_1)]} =$$

L. Heat Balance for Fluids

$$q_w = \dot{m}_w C_{pw} (T_1 - T_2) =$$

$$q_c = \dot{m}_c C_{pc} (t_2 - t_1) =$$

N. Overall Heat Balance for the Exchanger

$$q = U_o A_o N_s F (LMTD) =$$

O. Fouling Factors and Design Coefficient

$$R_{di} =$$

$$R_{do} =$$

$$\frac{1}{U} = \frac{1}{U_o} + R_{di} + R_{do} =$$

$$U =$$

P. Area Required to Transfer Heat (Determination of Plate Area)

$$A_o = \frac{q}{U F N_s (LMTD)} =$$
$$A_o = bL = \qquad \qquad \qquad b = \qquad \qquad \qquad L =$$

Q. Friction Factors

Reynolds Number Range	Darcy-Weisbach Friction Factor
1–10	$f = \frac{280}{Re}$
10–100	$f = \frac{100}{Re^{0.589}}$
> 100	$f = \frac{12}{Re^{0.183}}$

$$f_w =$$

$$f_c =$$

R. Pressure Drop Calculations

$$\Delta p_w = \frac{f_w L}{D_h} \frac{\rho_w V_w^2}{2g_c} + 1.3 \frac{\rho_w V_p^2}{2g_c}$$
$$\Delta p_c = \frac{f_c L}{D_h} \frac{\rho_c V_c^2}{2g_c} + 1.3 \frac{\rho_c V_p^2}{2g_c}$$

S. Summary of Information Requested in Problem Statement

EXAMPLE 10.1. A cereal manufacturing facility uses two huge, hollow rollers in order to flatten moistened grains into cereal “flakes,” as indicated in Figure 10.4. The two rollers rotate at different rotational speeds to provide a flattening as well as a stretching effect on the food particles.

Friction at the point of contact between the two rollers generates much heat, and so a cooling system has been set up inside each roller. The cooling system consists of a series of nozzles that spray water onto the inner surface of the roller at the point of contact. The original plan was to feed city water to the nozzles and merely discharge the water to the city drainage system. This plan was discarded in favor of a plan to recirculate the cooling

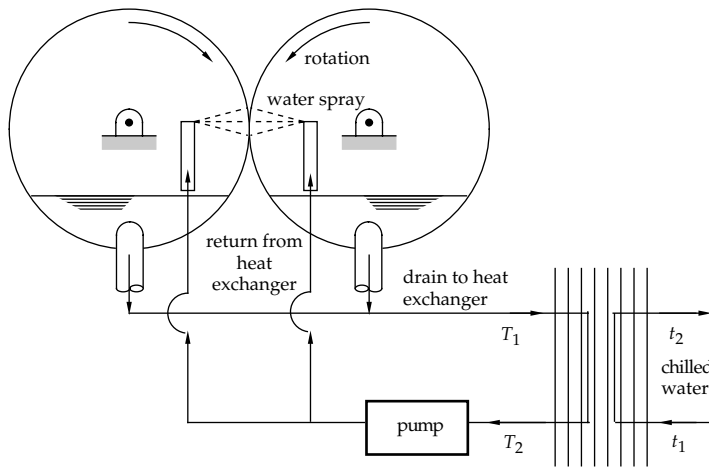


FIGURE 10.4. *Rollers used to flatten and stretch cereal particles into “flakes.”*

water. Thus, the cooling water will spray the inside of the rollers and then be pumped through a plate and frame heat exchanger, where chilled water will absorb the energy of the cooling water.

After leaving the rollers, cooling water enters the plate and frame heat exchanger at $T_1 = 75^\circ\text{F}$ with a flow rate of 7740 lbm/hr. It is to be cooled to a temperature of $T_2 = 60^\circ\text{F}$ (or less). Chilled water is available at 45°F ($= t_1$) at a flow rate of 7800 lbm/hr. The exchanger to be used has a plate width (b) of 18 in. and a plate height (L) of 36 in. The plates are spaced at 0.2 in. (s) and are 0.04 in. thick (t). The thermal conductivity of the plate material is 8.26 BTU/(hr·ft·°R). Preliminary calculations show that seven plates are needed. Determine the outlet temperatures of both fluids if this exchanger is used.

Solution. The calculations for this problem are straightforward following the suggested procedure. Fluid properties are evaluated at the average of inlet and outlet temperatures for each fluid. Several iterations are required, and what follows is the final result.

The resistance to heat flow offered by the plate will be taken into account, as will fouling effects. The fouling coefficient for both fluids is taken to be 0.00002 ft²·hr·°R/BTU.

- Assumptions**
1. Steady-state conditions exist.
 2. Heat lost by the warm water is transferred entirely to the cool water.

- Nomenclature**
1. T refers to the temperature of the warmer fluid.
 2. t refers to the temperature of the cooler fluid.
 3. w subscript refers to the warmer fluid.
 4. h subscript refers to hydraulic diameter.
 5. c subscript refers to the cooler fluid.
 6. 1 subscript refers to an inlet condition.
 7. 2 subscript refers to an outlet condition.

A. Fluid Properties

Warm H₂O @ 59.8°F	$\dot{m}_w$	= 2.16 lbm/s	T_1	= 75°F
	ρ	= 62.4 lbm/ft ³	C_p	= 0.999 BTU/(lbm·°R)
	k_f	= 0.343 BTU/(hr·ft·°R)	α	= 0.00551 ft ² /hr
	ν	= 1.0×10^{-5} ft ² /hr	Pr	= 7.34

Cool H₂O @ 50.1°F	$\dot{m}_c$	= 2.18 lbm/s	t_1	= 45°F
	ρ	= 63.4 lbm/ft ³	C_p	= 1.00 BTU/(lbm·°R)
	k_f	= 0.334 BTU/(hr·ft·°R)	α	= 0.00526 ft ² /hr
	ν	= 1.0×10^{-5} ft ² /hr	Pr	= 9.7

B. Plate Dimensions and Properties

b	= plate width	= 1.5 ft
L	= plate height	= 3 ft
s	= plate spacing	= 0.0167 ft
t	= plate thickness	= 0.003333 ft
A_o	= plate surface area	= bL = 4.5 ft ²
A	= flow area	= sb = 0.0208 ft ²
D_h	= hydraulic diameter of flow passage	= $2s$ = 0.0333 ft
N_s	= number of plates	= 7
k	= thermal conductivity of plate	= 8.26 BTU/(hr·ft·°R)
Plate material = metal		

C. Fluid Velocities

$$\text{Odd number of plates } (N_s = 7) \quad V = \frac{\dot{m} / \rho A}{(N_s + 1) / 2}$$

Warm H₂O	V_w = 0.395 ft/s
Cool H₂O	V_c = 0.395 ft/s

D. Reynolds Numbers

$$\text{Re}_w = V_w D_h / \nu = 1170$$

$$\text{Re}_c = V_c D_h / \nu = 927$$

E. Nusselt Numbers

Modified Dittus-Boelter Equation for turbulent flow:

$$\text{Nu} = \frac{hD_h}{k_f} = 0.374\text{Re}^{0.668} \text{Pr}^{1/3}$$

$$\text{Re} > 100; \quad \text{Pr} = \nu/\alpha > 0$$

Warm H₂O $\text{Nu}_w = 81.5$

Cool H₂O $\text{Nu}_c = 76.5$

F. Convection Coefficients

Warm H₂O $h_i = \text{Nu}_w k_f / D_h = 840 \text{ BTU}/(\text{hr} \cdot \text{ft}^2 \cdot ^\circ\text{R})$

Cool H₂O $h_o = \text{Nu}_c k_f / D_h = 768 \text{ BTU}/(\text{hr} \cdot \text{ft}^2 \cdot ^\circ\text{R})$

G. Exchanger Coefficient

$$\frac{1}{U_o} = \frac{1}{h_i} + \frac{t}{k} + \frac{1}{h_o} \quad U_o = 345 \text{ BTU}/(\text{hr} \cdot \text{ft}^2 \cdot ^\circ\text{R})$$

H. Capacitances

Warm H₂O $(\dot{m}C_p)_w = 2.154 \text{ BTU}/(\text{s} \cdot ^\circ\text{R})$

Cool H₂O $(\dot{m}C_p)_c = 2.195 \text{ BTU}/(\text{s} \cdot ^\circ\text{R})$

$$(\dot{m}C_p)_{\min} = 2.154 \text{ BTU}/(\text{s} \cdot ^\circ\text{R})$$

I. Number of Transfer Units, and Correction Factor

$$N = \frac{U_o A_o N_s}{(\dot{m}C_p)_{\min}} = 1.403$$

$$F = 1 - 0.0166N = 0.977$$

J. Outlet Temperature Calculations

$$R = \frac{\dot{m}_c C_{pc}}{\dot{m}_w C_{pw}} = 1.019$$

$$E_{\text{counter}} = \exp [U_o A_o N_s F(R - 1) / \dot{m}_c C_{pc}] = 1.026$$

$$\text{Warm H}_2\text{O} \quad T_2 = \frac{T_1(R - 1) - Rt_1(1 - E_{\text{counter}})}{RE_{\text{counter}} - 1} = 57.4^\circ\text{F}$$

$$\text{Cool H}_2\text{O} \quad t_2 = (T_1 - T_2)/R + t_1 = 66.2^\circ\text{F}$$

K. Log Mean Temperature Difference

$$\text{Counterflow} \quad LMTD = \frac{(T_1 - t_2) - (T_2 - t_1)}{\ln [(T_1 - t_2)/(T_2 - t_1)]} = 12.6^\circ\text{F}$$

L. Heat Balance for Fluids

$$\text{Warm H}_2\text{O} \quad q_w = \dot{m}_w C_{pw}(T_1 - T_2) = 37.8 \text{ BTU/s}$$

$$\text{Cool H}_2\text{O} \quad q_c = \dot{m}_c C_{pc}(t_2 - t_1) = 37.8 \text{ BTU/s}$$

N. Overall Heat Balance for the Exchanger

$$q = U_o A_o N_s F (LMTD) = 37.2 \text{ BTU/s (close enough)}$$

O. Fouling Factors and Design Coefficient

$$R_{di} = 0.00002 \text{ hr}^2 \cdot \text{ft} \cdot ^\circ\text{R}/\text{BTU} \quad R_{do} = 0.00002 \text{ hr}^2 \cdot \text{ft} \cdot ^\circ\text{R}/\text{BTU}$$

$$\frac{1}{U} = \frac{1}{U_o} + R_{di} + R_{do} \quad U = 345 \text{ BTU}/(\text{hr} \cdot \text{ft}^2 \cdot ^\circ\text{R})$$

P. Area Required to Transfer Heat (Determination of Plate Area) *Not Required*

Q. Friction Factors

$$\text{Warm H}_2\text{O} \quad f_w = 3.29$$

$$\text{Cool H}_2\text{O} \quad f_c = 3.44$$

R. Pressure Drop Calculations (with $V_p = 0$)

$$\text{Warm H}_2\text{O} \quad \Delta p_w = \frac{f_w L}{D_h} \frac{\rho_w V_w^2}{2g_c} + 1.3 \frac{\rho_w V_p^2}{2g_c} = 0.311 \text{ psi}$$

$$\text{Cool H}_2\text{O} \quad \Delta p_c = \frac{f_c L}{D_h} \frac{\rho_c V_c^2}{2g_c} + 1.3 \frac{\rho_c V_p^2}{2g_c} = 0.330 \text{ psi}$$

S. Summary of Information Requested in Problem Statement

$$\text{Warm H}_2\text{O} \quad T_2 = 57.4^\circ\text{F}$$

$$\text{Cool H}_2\text{O} \quad t_2 = 62.2^\circ\text{F}$$

These are the outlet temperatures when the exchanger is new. After it has become fouled (one year), the design coefficient is unchanged from the new coefficient; that is, 345 BTU/(hr·ft²·°R). The exchanger is to deliver an outlet temperature T_2 of 60°F or less, and it does. So the exchanger should work.

10.3 Cross-Flow Heat Exchangers

A cross-flow heat exchanger, like all other types, brings two fluids together so that energy is transferred from the warmer to the cooler fluid. The smaller exchangers of this type are often referred to as *compact heat exchangers*. In many such exchangers, the heat transfer surface area is increased by the addition of fins, and there are many design variations.

Cross-flow heat exchangers are widely used in industry. They can be used in applications such as gas-to-gas, gas-to-vapor, gas-to-liquid, liquid-to-liquid. Examples are automotive radiators, condensers and evaporators in air conditioning and refrigeration systems, oil coolers, air heaters, intercoolers on compressors, electronics, cryogenic processes, and more.

The objective in the design of a compact heat exchanger is to produce a unit that transfers heat at minimum cost and minimum space. This is why fins are usually found in such exchangers.

The term “cross flow” means that the fluids flow at right angles to each other as they pass through the exchanger. Each fluid stream can pass through and remain **unmixed** or **mixed**. Figure 10.5 illustrates the definitions of mixed versus unmixed flows through similar ducts. In an unmixed-flow situation, the flow channel or passageway would contain internal channels (e.g., tubes or walls) that restrict lateral fluid motion. In a mixed-flow passageway, internal channels are not present and the fluid particles are free to move about (and mix) over the cross section. Figure 10.6 shows examples of cross-flow heat exchangers that contain mixed- and unmixed-flow passageways.

It is important to be able to model the energy transfer that occurs within a cross-flow heat exchanger. Tests on many cross-flow heat exchangers have been performed (see *Compact Heat Exchangers* by W. M. Kays and A. L. London, McGraw-Hill Book Co., 1964). Results of heat transfer and friction characteristics have been reported for many exchangers and many designs. Due to space limitations here, however, it is not convenient to reproduce the results of such tests. We will therefore focus on only one type of cross-flow heat exchanger, and make illustrative calculations to show generally how cross-flow heat exchangers can be analyzed.

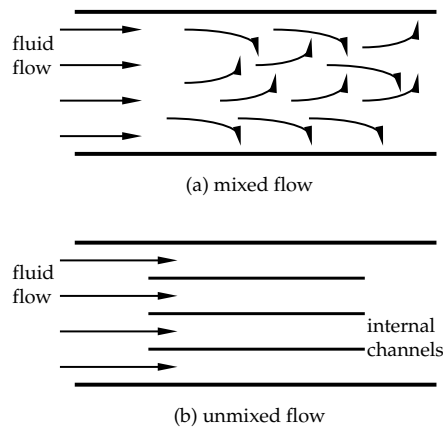


FIGURE 10.5. *Mixed- and unmixed-flow passages.*

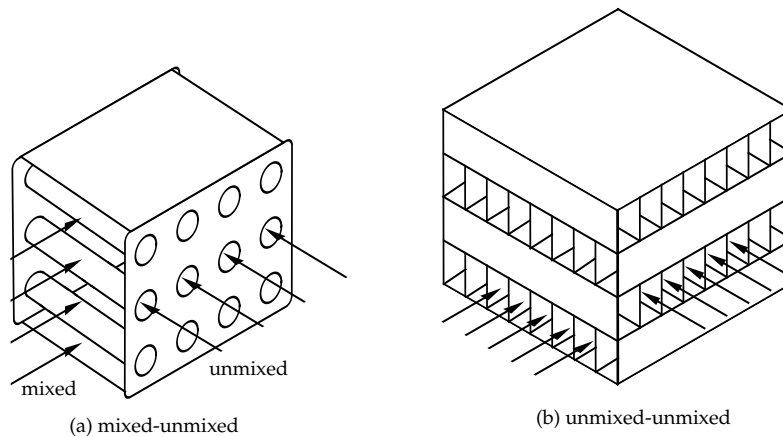


FIGURE 10.6. *Cross-flow heat exchangers with mixed and unmixed passages.*

Figure 10.7 shows an unmixed-unmixed cross-flow heat exchanger in which the metals used are made into a shape reminiscent of a sine wave. This type is used primarily in some oil coolers. Figure 10.8 shows a finned tube type of design in which tubes (usually copper) are pressed through thin metal fins. The fins act to increase the heat transfer surface area. The fins can be attached to the tubes in a variety of ways (e.g., welding, soldering). Figure 10.9a shows the frontal and profile views of the same exchanger. The tubes in the profile view are staggered, with the centers of three adjacent tubes forming an isosceles triangle. An alternative design is shown in Figure 10.9b. The flow passages in this profile view are flattened tubes, and they are aligned.

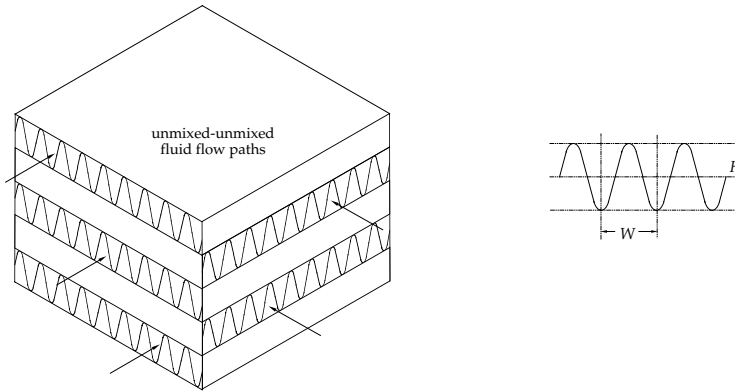


FIGURE 10.7. *Cross-flow heat exchanger made with metals fashioned with a sine-wave type of design.*

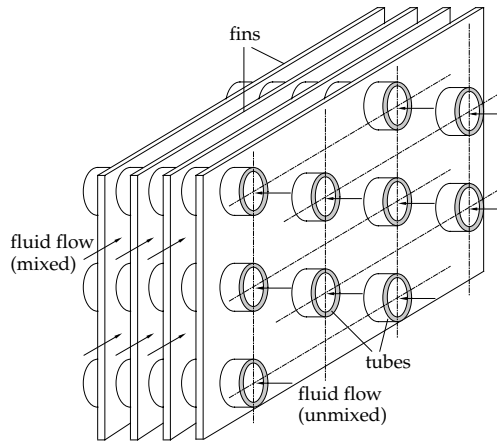


FIGURE 10.8. *Finned tube cross-flow heat exchanger.*

Heat transfer and friction data have been gathered for many types of cross flow heat exchangers. The heat transfer data for this type of exchanger are expressed in terms of a parameter known as the Colburn modulus j as a function of the Reynolds number. Likewise, the friction losses are provided also as friction factor versus Reynolds number.

The Colburn modulus is defined as

$$j = \frac{h}{G C_p} \text{Pr}^{2/3} \quad (10.18)$$

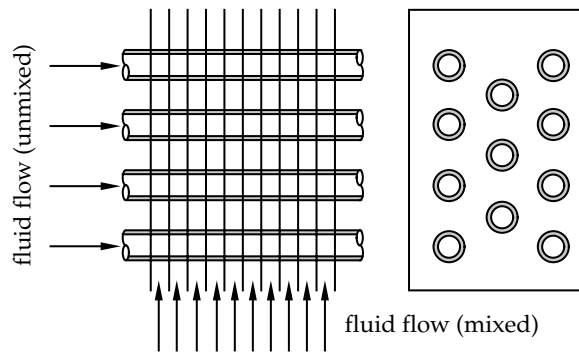


FIGURE 10.9a. Frontal and profile views of a finned tube cross-flow heat exchanger—circular tubes in a staggered configuration.

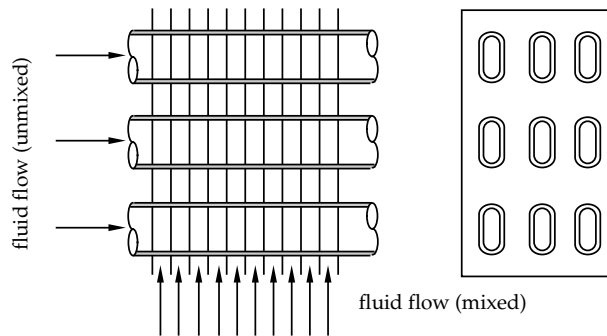


FIGURE 10.9b. Frontal and profile views of a finned tube cross-flow heat exchanger—flattened tubes in an aligned configuration.

where G was introduced in Chapter 8 as the mass flux:

$$G = \rho V$$

The Colburn modulus is dimensionless and may be manipulated to form more familiar dimensionless groups by multiplying the first term on the right-hand side of Equation 10.18 as follows:

$$\frac{h}{\rho V C_p} \frac{D}{D} \frac{\mu}{\mu} \frac{\rho}{\rho} \frac{k}{k} = \left(\frac{h D}{k} \right) \left(\frac{\mu g_c}{\rho V D} \right) \left(\frac{k}{\rho C_p} \right) \left(\frac{\rho}{\mu g_c} \right) = \text{Nu} \frac{1}{\text{Re}} \propto \frac{1}{v}$$

which becomes

$$\frac{h}{\rho V C_p} = \frac{\text{Nu}}{(\text{Re})(\text{Pr})} = \text{St}$$

This ratio of dimensionless groups is recognized as the Stanton number. The Colburn modulus then becomes

$$j = \frac{h}{G C_p} \text{Pr}^{2/3} = \frac{\text{Nu}}{(\text{Re})(\text{Pr})} \text{Pr}^{2/3} = \frac{\text{Nu}}{\text{Re} \text{Pr}^{1/3}}$$

The Colburn modulus for numerous gas-to-liquid heat exchangers has been obtained experimentally and catalogued for use in predicting the performance of such exchangers. We will focus on the finned tube type illustrated in Figure 10.8, for which many surface geometries can be manufactured.

Consider one such exchanger with the following parameters (from *Compact Heat Exchangers* 3rd edition, by W. M. Kays and A. L. London, McGraw-Hill Book Co., 1984, and identified as 8.0-3/8T):

Typical Cross-Flow Heat Exchanger Data—Figure 10.8

Triangular pitch

Fin pitch or number of fins per cm = 3.15 fins/cm

Fin thickness = 0.33 mm

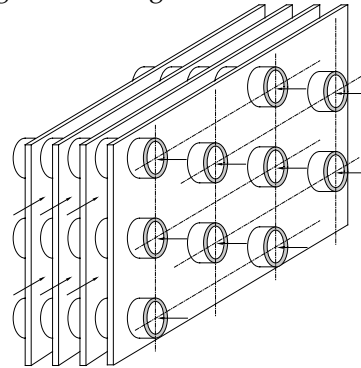
Free flow area/frontal area = $A/A_f = 0.534$

Air passage hydraulic diameter = $D_h = 0.3633$

Heat transfer area/total volume = $587 \text{ m}^2/\text{m}^3$

Outside diameter of tubes = $OD_t = 1.02 \text{ cm}$

Fin area/total area = 0.839



The Colburn modulus for this exchanger is given in graphical form, and the experimental results are given as:

$$j = \frac{h}{G C_p} \text{Pr}^{2/3} = \frac{\text{Nu}}{\text{Re} \text{Pr}^{1/3}} = 0.178 \text{ Re}^{-0.4085}$$

or

$$\text{Nu} \approx 0.178 \text{ Re}^{0.59} \text{Pr}^{1/3} \quad (10.19)$$

This equation allows for finding the convection coefficient for the air side of the heat exchanger. The air side heat transfer coefficient is usually much smaller than that for the liquid side. So the contribution of the liquid coefficient to the overall heat transfer coefficient is negligible.

The friction factor for the air coefficient is also given in graphical form and the corresponding curve fit equation is

$$f = 0.121 \operatorname{Re}^{-0.205} \quad (10.20)$$

The Reynolds number in these equations is based on a maximum velocity attained by the air as it flows through the exchanger:

$$\operatorname{Re} = \frac{V_{\max} D_h}{\nu}$$

where D_h is the hydraulic diameter of the flow passage. The presence of tubes in the exchanger reduces the flow area, and causes the velocity to increase around the tubes. The maximum velocity is found by applying the continuity equation to the region upstream of the exchanger to the location where the area is a minimum:

$$(\rho A_f V_{\text{free stream}}) = (\rho A V_{\max})$$

Assuming constant density, the previous equation can be solved for the maximum velocity as

$$V_{\max} = \frac{A_f}{A} V_{\text{free stream}}$$

For the exchanger of Figure 10.8, the reciprocal of the area ratio is 0.534.

The procedure for analyzing an air-to-liquid cross flow heat exchanger is the same as with the others. We use the appropriate equations for finding the convection coefficients, and then calculate the overall heat transfer coefficient for the exchanger. The heat transfer rate is calculated with

$$q = UAF (\text{LMTD}) \quad (10.21)$$

in which U is the overall heat transfer coefficient with dimensions of $[\text{F} \cdot \text{L} / (\text{T} \cdot \text{L}^2 \cdot \text{t})]$; $\text{BTU} / (\text{hr} \cdot \text{ft}^2 \cdot ^\circ\text{R})$ or $\text{W} / (\text{m}^2 \cdot \text{K})$; A is the heat transfer area of the heat exchanger; F is a correction factor; and LMTD is the log mean temperature difference for counterflow

$$\text{LMTD} = \frac{(T_1 - t_2) - (T_2 - t_1)}{\ln [(T_1 - t_2) / (T_2 - t_1)]} \quad (10.22)$$

The heat transfer area of a cross-flow heat exchanger is extremely difficult to determine because of the construction methods used. It is not uncommon to report only the UA product. In almost all applications, it is necessary to calculate the size of an exchanger required to transfer a certain heat load or to deliver a desired outlet temperature. So in this case, the *frontal area* (as opposed to the heat transfer area A) is more convenient to use.

Equations for the correction factor have been derived for cross-flow

heat exchangers (see *Process Heat Transfer* by D. Q. Kern, McGraw-Hill Book Co., 1950, Chapter 16). As before, the correction factor is a function of the capacitance ratio R and the temperature factor S :

$$R = \frac{T_1 - T_2}{t_2 - t_1} = \frac{\dot{m}_c C_{pc}}{\dot{m}_w C_{pw}} \quad (10.23)$$

and

$$S = \frac{t_2 - t_1}{T_1 - t_1} \quad (10.24)$$

Figure 10.10 is a sketch of the temperature variation of both fluids as they pass through a cross-flow heat exchanger. As indicated, we use the lower case ' t ' to denote the cooler fluid, and the uppercase ' T ' refers to the warmer fluid. Also, the ' 1 ' subscript denotes an inlet condition and a ' 2 ' refers to an outlet condition.

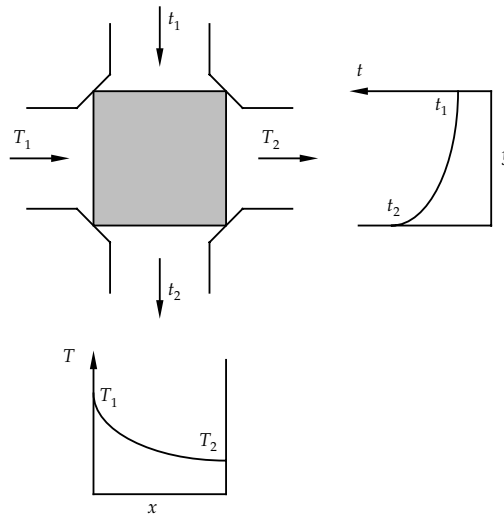


FIGURE 10.10. Temperature variation of the cooler and warmer fluids as they flow through a cross-flow heat exchanger.

When the performance of a cross-flow heat exchanger is evaluated, the results show that a counterflow, double pipe heat exchanger is more efficient. The standard of comparison, then, is the counterflow heat exchanger, which is why LMTD in Equation 10.22 is for counterflow.

Figure 10.11 is a graph of correction factor F as a function of S for various values of R for a mixed-unmixed cross-flow heat exchanger. Figure 10.12 is a graph of correction factor F versus S for various values of R for an unmixed-unmixed cross-flow heat exchanger. Figure 10.11 was graphed from the original equation, while Figure 10.12 was graphed by applying a scale

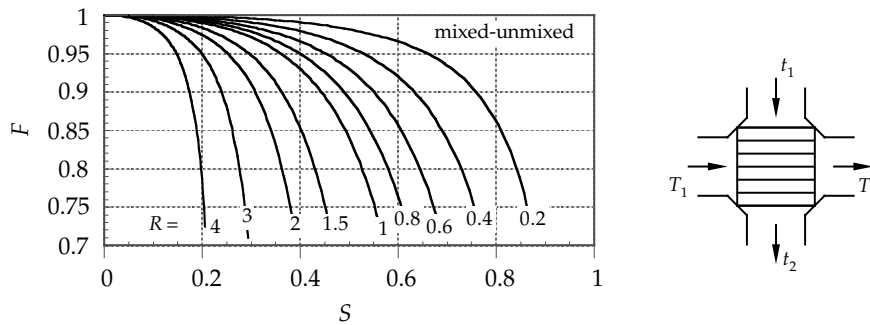


FIGURE 10.11. Correction factor graph for a *mixed-unmixed* cross flow heat exchanger.

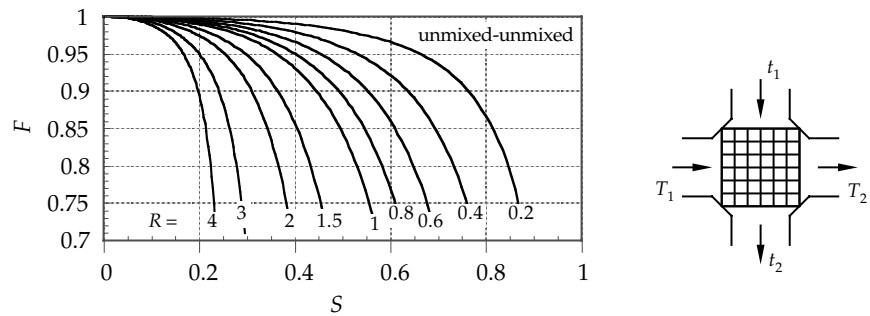


FIGURE 10.12. Correction factor graph for an *unmixed-unmixed* cross-flow heat exchanger.

factor to Figure 10.11. Figure 10.12 is thus only an approximation (within 5%) of the graph obtained from the corresponding original equation for unmixed-unmixed flows.

Outlet Temperature Calculation

In many applications, only fluid inlet temperatures are known and it is necessary to calculate outlet temperatures for a given heat exchanger. This can be done in several ways but the **effectiveness-NTU** method presented here is the most convenient. The effectiveness E depends on which of the two fluids has the minimum mass flow rate $\times$ specific heat product; that is, the minimum **capacitance**. The effectiveness E and corresponding outlet temperature equations are

$$\left. \begin{aligned} E &= \frac{t_2 - t_1}{T_1 - t_1} \\ t_2 &= t_1 + E(T_1 - t_1) \\ T_2 &= T_1 - C(t_2 - t_1) \end{aligned} \right\} \quad (\text{if } \dot{m}_c C_{pc} < \dot{m}_w C_{pw}) \quad (10.25a)$$

and

$$\left. \begin{aligned} E &= \frac{T_1 - T_2}{T_1 - t_1} \\ T_2 &= T_1 - E(T_1 - t_1) \\ t_2 &= t_1 + C(T_1 - T_2) \end{aligned} \right\} \quad (\text{if } \dot{m}_w C_{pw} < \dot{m}_c C_{pc}) \quad (10.25b)$$

If effectiveness is known for the heat exchanger of interest, then the heat transfer rate can be found with

$$q = E(\dot{m}C_p)_{\min} (T_1 - t_1) \quad (10.26)$$

Effectiveness equations have been derived for many types of heat exchangers, and are written in terms of the **number of transfer units**, N , defined for cross-flow heat exchangers as

$$N = \frac{UA}{(\dot{m}C_p)_{\min}} \quad (10.27)$$

The **ratio of capacitances** C defined as

$$C = \frac{(\dot{m}C_p)_{\min}}{(\dot{m}C_p)_{\max}} < 1 \quad (10.28)$$

The effectiveness of the mixed-unmixed and unmixed-unmixed heat exchangers are:

Cross flow, mixed-unmixed with $(\dot{m}C_p)_{\min}$ unmixed (see Figure 10.13a)

$$E = C\{1 - \exp[-C(1 - \exp[-N])]\}$$

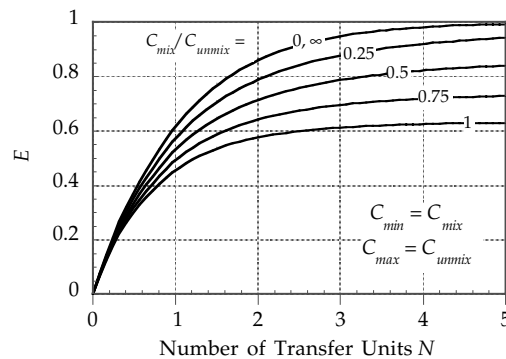


FIGURE 10.13a. Effectiveness as a function of number of transfer units for a *mixed-unmixed* cross-flow heat exchanger. Fluid with minimum capacitance is mixed.

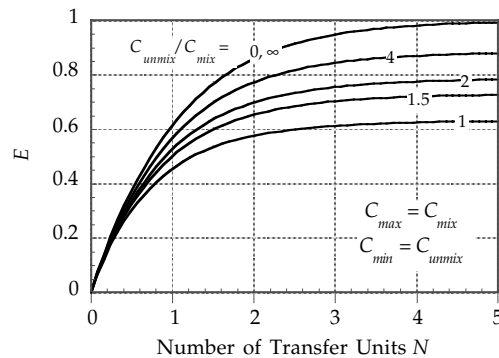


FIGURE 10.13b. Effectiveness as a function of number of transfer units for a *mixed-unmixed* cross-flow heat exchanger. Fluid with minimum capacitance is unmixed.

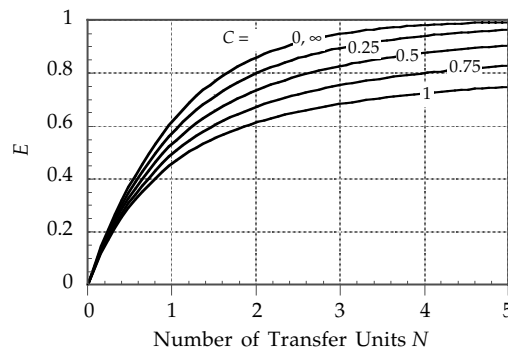


FIGURE 10.14. Effectiveness as a function of number of transfer units for an *unmixed-unmixed* cross-flow heat exchanger.

Cross flow, mixed-unmixed with $(\dot{m}C_p)_{\max}$ unmixed (See Figure 10.13b)

$$E = 1 - \exp [-C(1 - \exp [-N \cdot C])]$$

Cross flow, unmixed-unmixed (See Figure 10.14)

$$E \approx 1 - \exp [C(N)^{0.22} \{\exp [-C(N)^{0.78}] - 1\}]$$

Figures 10.13a and 10.13b are graphs of the first two of these equations (E versus N) for mixed-unmixed exchangers. Figure 10.13a applies when the fluid with the minimum capacitance is mixed, and Figure 10.13b applies when the fluid with minimum capacitance is unmixed. Figure 10.14 is a graph of effectiveness E versus number of transfer units N for unmixed-unmixed cross-flow heat exchangers.

Like other types, cross-flow heat exchangers are subject to fouling but it is difficult to accurately apply fouling factor data to them. We will therefore not consider fouling effects in such exchangers. Equally difficult to determine are pressure drops of the fluid streams as they pass through the exchanger. In cases where it is possible to calculate it, the pressure drop can be treated as a minor loss in the overall fluid flow system. The minor loss can vary tremendously for both fluid streams. For our purposes, we assume a minor loss of $K = 10$ for each stream.

It is important to know the frontal area (i.e., for the air stream) required to transfer a required amount of energy. Figure 10.15 is an empirical graph relating frontal area to the UA product for a cross-flow heat exchanger. This graph is a result of calculations made on a number of exchangers.

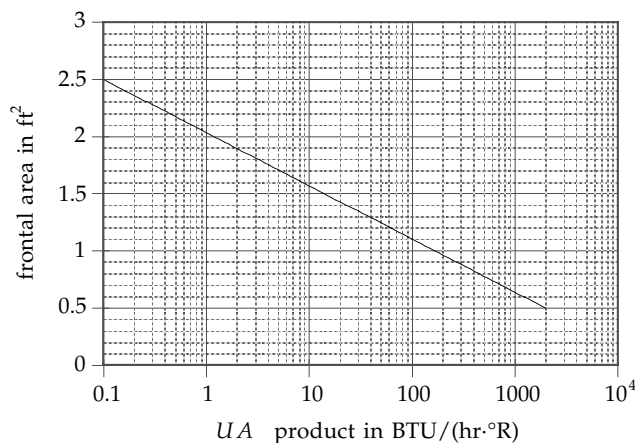


FIGURE 10.15. Empirical relationship between frontal area (normal to airflow) and the UA product for an air-to-liquid cross-flow heat exchanger.

The method of analysis for cross-flow heat exchangers has been organized into a suggested order for performing calculations, which now follows.

SUGGESTED ORDER OF CALCULATIONS FOR ANALYSIS OF CROSS-FLOW HEAT EXCHANGERS

Problem	Complete problem statement.
Discussion	Potential heat losses; other sources of difficulties.
Assumptions	1. Steady-state conditions exist. 2. Fluid properties remain constant and are evaluated at a temperature of ____.
Nomenclature	1. T refers to the temperature of the warmer fluid. 2. t refers to the temperature of the cooler fluid. 3. w subscript refers to the warmer fluid. 4. h subscript refers to hydraulic diameter. 5. c subscript refers to the cooler fluid. 6. 1 subscript refers to an inlet condition. 7. 2 subscript refers to an outlet condition.

A. Fluid Properties

$\dot{m}_w =$	$T_1 =$
$\rho =$	$C_p =$
$k_f =$	$\alpha =$
$\nu =$	$\text{Pr} =$
$\dot{m}_c =$	$t_1 =$
$\rho =$	$C_p =$
$k_f =$	$\alpha =$
$\nu =$	$\text{Pr} =$

B. Heat Balance

$$q_w = \dot{m}_w C_{pw} (T_1 - T_2) =$$

$$q_c = \dot{m}_c C_{pc} (t_2 - t_1) =$$

C. Log Mean Temperature Difference

$$\text{Counterflow} \quad LMTD = \frac{(T_1 - t_2) - (T_2 - t_1)}{\ln [(T_1 - t_2) / (T_2 - t_1)]} =$$

D. Correction Factor (if needed)

$$\left. \begin{aligned} S &= \frac{t_2 - t_1}{T_1 - t_1} = \\ R &= \frac{T_1 - T_2}{t_2 - t_1} = \frac{\dot{m}_c C_{pc}}{\dot{m}_w C_{pw}} = \end{aligned} \right\} F = \begin{pmatrix} \text{Figure 10.11 mixed-unmixed} \\ \text{Figure 10.12 unmixed-unmixed} \end{pmatrix}$$

E. UA Product

$$UA = \frac{q}{F (LMTD)} =$$

F. Capacitances

	<i>Mixed-Unmixed</i>
	$(\dot{m}C_p)_{\text{mixed}} =$
	$(\dot{m}C_p)_{\text{unmixed}} =$
	$C = \frac{(\dot{m}C_p)_{\text{mixed}}}{(\dot{m}C_p)_{\text{unmixed}}} =$
	<i>Unmixed-Unmixed</i>
	$(\dot{m}C_p)_w =$
	$(\dot{m}C_p)_c =$
	$C = \frac{(\dot{m}C_p)_{\text{min}}}{(\dot{m}C_p)_{\text{max}}} =$

G. Number of Transfer Units

$$N = \frac{UA}{(\dot{m}C_p)_{\text{min}}} =$$

H. Effectiveness, E

$$\left. \begin{aligned} N &= \\ C &= \end{aligned} \right\} E = \begin{pmatrix} \text{Figure 10.13 mixed-unmixed} \\ \text{Figure 10.14 unmixed-unmixed} \end{pmatrix}$$

I. Outlet Temperature Calculations

$$\begin{array}{l}
 E = \frac{t_2 - t_1}{T_1 - t_1} \\
 t_2 = t_1 + E(T_1 - t_1) \\
 T_2 = T_1 - C(t_2 - t_1)
 \end{array}
 \left. \vphantom{\begin{array}{l} E = \frac{t_2 - t_1}{T_1 - t_1} \\ t_2 = t_1 + E(T_1 - t_1) \\ T_2 = T_1 - C(t_2 - t_1) \end{array}} \right\} \quad (\text{if } \dot{m}_c C_{pc} < \dot{m}_w C_{pw})$$

or

$$\begin{array}{l}
 E = \frac{T_1 - T_2}{T_1 - t_1} \\
 T_2 = T_1 - E(T_1 - t_1) \\
 t_2 = t_1 + C(T_1 - T_2)
 \end{array}
 \left. \vphantom{\begin{array}{l} E = \frac{T_1 - T_2}{T_1 - t_1} \\ T_2 = T_1 - E(T_1 - t_1) \\ t_2 = t_1 + C(T_1 - T_2) \end{array}} \right\} \quad (\text{if } \dot{m}_w C_{pw} < \dot{m}_c C_{pc})$$

$$T_2 =$$

$$t_2 =$$

J. Velocities in Connecting Tubes/Ducts

$$V_w =$$

$$V_c =$$

K. Pressure Drop Calculations (Use $K = 10$)

$$\Delta p_w = K \frac{\rho_w V_w^2}{2g_c} =$$

$$\Delta p_c = K \frac{\rho_c V_c^2}{2g_c} =$$

L. Frontal Area Required

$$UA = \quad \quad \quad A = \quad \quad \quad (\text{Figure 10.15})$$

Q. Summary of Information Requested in Problem Statement

EXAMPLE 10.2. A cross-flow heat exchanger (unmixed-unmixed) is used to cool water at the test stand of a diesel engine. The water enters the

exchanger at 200°F; air in the room (~70°F) is used as the cooling fluid. The fan available can move the air at 2.5 lbm/hr. The frontal area (i.e., normal to the air flow) of the heat exchanger is 2 ft². If the flow rate of the water is 0.7 lbm/hr, determine the expected outlet temperatures of both fluids.

Solution. The equations for the overall heat transfer coefficient are unavailable unless some very specific information is available about the exchanger; however, we can obtain the UA product for this exchanger by using Figure 10.15. For a given UA product, the number of transfer units can be calculated, and the effectiveness determined. With effectiveness known, the outlet temperatures are then calculated.

Assumptions

1. Steady-state conditions exist.
2. Fluid properties remain constant and are evaluated at average temperatures (inlet + outlet)/2 for each fluid. Several iterations are required, and what follows are the final results.

Nomenclature

1. T refers to the temperature of the warmer fluid.
2. t refers to the temperature of the cooler fluid.
3. w subscript refers to the warmer fluid.
4. h subscript refers to hydraulic diameter.
5. c subscript refers to the cooler fluid.
6. 1 subscript refers to an inlet condition.
7. 2 subscript refers to an outlet condition.

A. Fluid Properties

H₂O at 184.1°F	$\dot{m}_w = 0.7 \text{ lbm/hr}$	$T_1 = 200^\circ\text{F}$
	$\rho = 60.9 \text{ lbm/ft}^3$	$C_p = 1.002 \text{ BTU/lbm}\cdot^\circ\text{R}$
	$k_f = 0.385 \text{ BTU/(hr}\cdot\text{ft}\cdot^\circ\text{R)}$	$\alpha = 6.32 \times 10^{-3} \text{ ft}^2/\text{hr}$
	$\nu = 0.399 \times 10^{-5} \text{ ft}^2/\text{s}$	$\text{Pr} = 2.27$

Air at 94.0°F	$\dot{m}_c = 2.5 \text{ lbm/hr}$	$t_1 = 70^\circ\text{F}$
	$\rho = 0.070 \text{ lbm/ft}^3$	$C_p = 0.240 \text{ BTU/lbm}\cdot^\circ\text{R}$
	$k_f = 0.016 \text{ BTU/hr}\cdot\text{ft}\cdot^\circ\text{R}$	$\alpha = 0.951 \text{ ft}^2/\text{hr}$
	$\nu = 18.8 \times 10^{-5} \text{ ft}^2/\text{s}$	$\text{Pr} = 0.71$

B. Capacitances—Unmixed-Unmixed

$$\text{H}_2\text{O} \quad (\dot{m}C_p)_w = 0.9(1.002) = 0.701 \text{ BTU}/(\text{hr}\cdot^\circ\text{R})$$

$$\text{Air} \quad (\dot{m}C_p)_c = 2.5(0.240) = 0.601 \text{ BTU}/(\text{hr}\cdot^\circ\text{R})$$

$$C = \frac{(\dot{m}C_p)_{\min}}{(\dot{m}C_p)_{\max}} = 0.856$$

C. UA product (Figure 10.15 at $A_{\text{frontal}} = 2 \text{ ft}^2$) = 1 BTU/(hr·°R)

D. Number of Transfer Units

$$N = \frac{UA}{(\dot{m}C_p)_{\min}} = 1.665$$

E. Effectiveness, E (unmixed-unmixed)

$$\left. \begin{array}{l} N = 1.665 \\ C = 0.856 \end{array} \right\} E = 0.494$$

F. Outlet Temperatures $\dot{m}_c C_{pc} < \dot{m}_w C_{pw}$

$$\text{H}_2\text{O} \quad T_2 = T_1 - C(t_2 - t_1) = 145^\circ\text{F}$$

$$\text{Air} \quad t_2 = (T_1 - t_1)E + t_1 = 134^\circ\text{F}$$

G. Log Mean Temperature Difference

$$\text{Counterflow} \quad LMTD = \frac{(T_1 - t_2) - (T_2 - t_1)}{\ln [(T_1 - t_2)/(T_2 - t_1)]} = 70.3^\circ\text{F}$$

K. Summary of Information Requested

$$\text{H}_2\text{O} \quad T_2 = 145^\circ\text{F}$$

$$\text{Air} \quad t_2 = 134^\circ\text{F}$$

10.4 Summary

In this chapter, we have described plate and frame heat exchangers as well as cross-flow heat exchangers. An analysis was developed for both of these types, and example problems were provided. Design graphs useful for sizing these exchangers were also given.

10.5 Show and Tell

1. Obtain a catalog of plate and frame heat exchangers and give a presentation on them. Show specifically how they are assembled and explain the advantages they have over other exchangers.
2. Obtain the parts (from a manufacturer, perhaps) of an automobile radiator. Give a presentation on how it is made and whether the flows are mixed or unmixed.

3. Obtain the parts of a finned tube heat exchanger, like that in an air conditioning unit. Give a presentation on how it is made and whether the flows are mixed or unmixed.

10.6 Problems

Plate and Frame Heat Exchangers

1. A plate and frame heat exchanger is used to exchange heat in a water-to-water flow system. Hot water enters the exchanger at 90°F with a flow rate of 200 gpm. Cold water also at 200 gpm enters the exchanger at 55°F. The exchanger contains 27 plates. The plate spacing is 0.2 in., the plate material is 304SS (stainless steel), the plate thickness is 0.04 in., the plate width is 15 in., and the plate height is 3 ft. Determine the outlet temperatures of both liquid streams.
2. A plate and frame heat exchanger has 15 plates made of stainless steel (316SS) that are 1 m tall. The plates are 1 mm thick x 0.6 m wide and are spaced 4.8 mm apart. The exchanger is used to cool a weak solution of sulfuric acid (assume the same properties as water). The volume flow rate of this liquid is 0.011 m³/s, and its inlet temperature is 30°C. Water is the cooling medium, and it enters the exchanger at 6°C with a volume flow rate of 0.02 m³/s. What outlet temperatures can be expected under these conditions?
3. Hot water at 100 gpm, 180°F, enters a plate and frame heat exchanger and is cooled to 135°F. Cold water enters this same exchanger at 120 gpm, 100°F. The plates are made of stainless steel (316SS) with a heat transfer area of 3.4 ft² per plate. The plate spacing is 0.2 in., the plate width is 15 in., and the plate thickness is 0.04 in. Determine the number of plates required for this exchanger to work.
4. Several solar collectors are used to obtain energy from the sun to heat ethylene glycol. The ethylene glycol flows at 0.006 m³/s from the collectors to a plate and frame heat exchanger. The ethylene glycol enters the heat exchanger at 24°C, where it transfers heat to fresh water. The water enters the exchanger at 0.005 m³/s, with a temperature of 10°C. The plate height is 1.5 m, and a plate spacing of 4.3 mm. There are 15 plates made of 304SS (stainless steel) that are 0.8 mm thick x 0.5 m wide. Determine the outlet temperature of both fluids.
5. Oil (assume engine oil) used in a quenching operation is to be cooled with a plate and frame heat exchanger. The oil flows into the heat exchanger at 20 gpm, 175°F, and must be cooled to 160°F. Water enters the heat exchanger at 90°F and can exit at a temperature that is no warmer than 105°F. The heat exchanger contains plates (6 in. wide) made of 304SS (stainless steel), 0.024 in. thick, and spaced 0.14 in. apart. The heat transfer area of one plate is 0.94 ft². How many plates would be needed to cool the oil to the required temperature? Is the condition on the water outlet temperature met by this exchanger? Take the flow rate of water to be 15 gpm.
6. Waste water at 0.01 m³/s enters a plate and frame heat exchanger at 60°C. Fresh water enters this same exchanger at 0.017 m³/s, 2°C. The exchanger has 19 plates (0.609 m x 1.24 m tall) made of 316SS (stainless steel), 0.6 mm thick, that are spaced 4 mm apart. Determine the expected outlet temperatures of both fluids.

Cross-Flow Heat Exchangers

7. Suppose the cross-flow heat exchanger of Example 10.2 is the mixed-unmixed type. How are the outlet temperatures affected?
8. A cross-flow heat exchanger (mixed-unmixed) is used to recover heat from the exhaust of a diesel engine. The exhaust gases enter the exchanger at 400°F. Water is available at 50°F, 1 lbm/hr, and is not of much use unless it can be heated to 80°F. Determine the size of the exchanger required if the mass flow rate of diesel exhaust is 4 lbm/hr. Assume that diesel exhaust gases have the same properties as carbon dioxide and that the water is unmixed.
9. A cross-flow heat exchanger (unmixed-unmixed) is marketed as an oil cooler (same properties as unused engine oil). Oil enters the cooler at a flow rate of 8 lbm/hr and a temperature of 170°F. The oil should be cooled to 150°F. Air is used to cool the oil. The air inlet velocity is 80 ft/min and the frontal area through which the air flows is 10 in. $\times$ 10 in. The air inlet temperature is 70°F. Is this exchanger large enough?
10. Water enters a cross flow heat exchanger at a temperature of 85°C and a flow rate of 2.5 kg/s. Air enters the exchanger at 20°C, leaves at 40°C, and has a flow rate of 12 kg/s. Which type of cross flow heat exchanger will have the greater effectiveness—mixed-unmixed or unmixed-unmixed? Does it make a difference which fluid is mixed?
11. For the information given in Problem 10.10, which type of cross flow heat exchanger will need to have the larger frontal area—mixed-unmixed or unmixed-unmixed?

CHAPTER 11 Project Descriptions

Each of the projects described in this chapter is designed to be solved over the course of a semester by an engineering team. The projects are listed with a recommended number of team members.

Students who desire to work on these projects are referred to Chapter 1 for a review of the design process, the nature of engineering design, design phases, codes and standards, economics, product safety, the bid process, and especially project management, project planning, internal documentation, and report preparation. The student is encouraged to go through each of these topics and apply them appropriately in the solution process.

Consideration should also be given to other parameters not discussed in Chapter 1, because they are important. These include production processes, economic factors, safety, reliability, aesthetics, ethical issues, and social impact.

It is also important to consider the total life cycle of the design. Specifically, how will the design operate after 1 or 5 or 20 years? How will it be maintained? How will it be used? How will it be marketed, and how will customer satisfaction be determined? These things should be given attention as part of the solution.

The Written Report

Next, suppose that the engineering phase of the project has been completed and it is now necessary to communicate the results. Usually a **written report** and an **oral presentation** are given by the consultant to the client. The written report will contain several items; a *suggested* example format is given in the accompanying figure. Each item is described as follows:

Letter of Transmittal—Written to the client stating that the project has been completed and the results are presented in the accompanying report.

Title Page—Lists project title, finished project due date, engineers who worked on the project, and the name of the consulting company. Note that all pages of the report need to be numbered, dated, and identified somehow with the consulting company.

Problem Statement—Succinctly reiterates the problem; included so that all concerned will know what project was completed.

Summary of Findings—Summarizes the details of the solution. This section might present a list showing, for example, what pump to buy, what line size to use, where to route the pipe, what heat exchanger to use, etc., suggested suppliers, and costs for all components. Drawings of the system would also be included. The summary should be complete enough so that the client could submit it to a contractor who could complete the installation of all components.

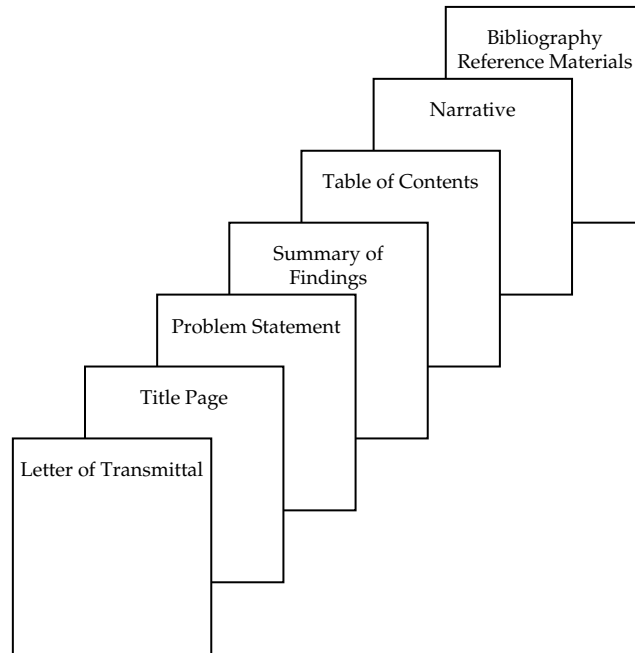


FIGURE 11.1. *Elements of the written report.*

Table of Contents—Refers the reader to any section of the report.

Narrative—Presents the details of all components specified in the summary and why each component was selected. For example, the details of how the pump was selected would be included here. Enough written detail must be included so that the reader can follow *every* step of the development. The organization of the narrative and titles of all sections will vary from writer to writer. However: *If your audience has read your report and does not*

understand all that you wrote, then you have not expressed yourself clearly enough. Write for the audience.

Current topics that should receive special consideration are as follows. Not all of these topics apply to every design project. However, the topics that apply to your project should be addressed at the end of the narrative:

- Constraints
- Economic Factors
- Environmental Factors
- Sustainability
- Manufacturability
- Ethical Issues
- Health and Safety Issues
- Social Issues
- Political Issues

Bibliography/Reference Materials—Shows text titles and publications used to arrive at the specifics of the design. This section should also include information from catalogs of suppliers, such as pump performance curves, if appropriate.

The written report should appear professional in every way. A well written presentation will show that the writer is meticulous and convince the client that a great deal of care went into completing the job. Text, graphs, drawings, and charts are done by computer, with nothing drawn freehand. The entire report should be bound, and the client should be provided with more than one copy.

The Oral Report

The oral report should be short and it need not be detailed. The oral report consists of the problem statement and a summary of the findings, which includes initial and operating costs. If questions arise, the presenter can refer to details found in the narrative. Therefore, the presenter should be prepared to give details of the *entire* study but present only the problem statement and the summary.

Titles of Projects

Projects with Three Engineers		Fluid Mechanics	Heat Transfer
1.	Bicycle Dynamometer	√	
2.	Tire Pressure Readout	√	
3.	Automatic Lawn Sprinkler System	√	
4.	Domestic In-Wall Vacuum Cleaning System	√	
5.	Dehumidified Air Supply	√	
6.	Signaling Funnel	√	
7.	An Artist's Palette	√	
8.	Vacuum System for Cleaning Buses	√	
9.	Ventilation System for a Spraying Booth	√	
10.	Pneumatic Seat	√	
11.	Pneumatic Nut Cracker	√	
12.	Fireplace Heat Recovery	√	√
13.	Heat Recovery in a Sheetrock Plant	√	√
14.	Design of a Baseboard Heater	√	√
15.	A Sandblaster That Uses Ice	√	√
16.	Warming Plate for Food Services	√	√
17.	Shaved-Ice Maker	√	√

Projects with Four Engineers		Fluid Mechanics	Heat Transfer
18.	Inflatable Dike	√	
19.	Measurement of Flipper Thrust	√	
20.	Vacuum Cleaner Efficiency Ratings	√	
21.	Paddle Wheel Design	√	
22.	Bicycle Frame Finishing Process	√	
23.	Ceiling Fan Blade Design	√	
24.	Mileage Meter for a Conventional Automobile	√	
25.	Design of a Portable Car-Washing System	√	
26.	Solar Assisted Composter	√	
27.	Underground-Stored Barricade	√	
28.	Controlled Environment for Special Metals	√	
29.	Elevator Buffer Design	√	
30.	Slab-Heated Warehouse	√	√
31.	Hot-Water Dispenser	√	√
32.	Personal Ice Maker	√	√
33.	Greenhouse Climatic Conditions	√	√
34.	Solar-Heated Swimming Pool	√	√
35.	Economics of Fuel Tank Insulation	√	√

36.	A Three-Ton Source of Cool Air	√	√
37.	Heated Driveway Slab	√	√
38.	A Portable Source of Cool Air for Golf Carts	√	√

Projects with Five Engineers	Fluid Mechanics	Heat Transfer
39. Finishing System for Wood Fences	√	
40. Pneumatic Freight Pipeline	√	
41. Windmill Sizing and Feasibility Study	√	
42. Water Feature	√	
43. A Toy Vertical Wind Tunnel	√	
44. Water Oil Separation Tank—Analysis & Testing	√	
45. High-Pressure Hose Tester	√	
46. Determination of Valve Coefficient	√	
47. Cooling System for a Series of Heat Sources	√	
48. Eccentric Guy's Pond	√	
49. The Cat's Meow: A Cat Washer	√	
50. Amusement Park Water Slide	√	
51. Flash Freezing of Chicken	√	√
52. Automatic Solar Hot Dog Cooker	√	√
53. Cooling Tower for an Internal Combustion Engine	√	√
54. Human-Powered Distillation Unit	√	√
55. Hot Lather Machine	√	√
56. Apparatus for Illustrating the First Law of Thermodynamics	√	√
57. Shower-Head Testing Unit	√	√
58. Ice Skating Rink	√	√
59. Analysis of "Energy-Efficient" Methods in Home Construction	√	√
60. Earth-Coupled Air Conditioning Unit	√	√
61. A Heating System for a Tank Car	√	√
62. Energy Recovery from an Electrical Generating Facility	√	√
63. Cooling Pond for a Power Plant	√	√
64. Heating of a Portable Review Platform	√	√
65. Analysis of a Piping Network	√	√
66. Steam and Air Supply Systems	√	√

1. Bicycle Dynamometer (3 engineers)

A dynamometer is a device for measuring the output power of a system. Dynamometers have been used extensively for measuring power from an internal combustion engine, from a turbine, and from an automobile. Dynamometers can be of the electric type, in which the output power is used to produce electric power, or of the fluid type, in which the output power is used to pump a liquid (usually water) or move air.

On a smaller scale (than IC engines), it is desirable to measure the output of a person pedaling a bicycle. Results of such tests are of interest in physical education studies of human power output and endurance, and to manufacturers of bicycles. Consequently, there is a need to have a dynamometer onto which a complete bicycle can be attached and pedaled, and from which output power can be calculated.

To be designed, selected, or determined:

1. A system for supporting the weight of the bicycle and the rider while attached to the dynamometer.
 2. A fluid system that is used to measure the output power of the rider over a broad range of conditions.
 3. Materials for construction and design of the entire device.
 4. Costs of building such a device.
-

2. Tire Pressure Readout (3 engineers)

Tractor-trailer trucks are used extensively to haul goods over long distances. A trucker may be driving for many miles before he or she stops for fuel and inspects the vehicle. Tire pressure, however, is critical because at high speed, a loss of tire pressure can be dangerous to the driver and the vehicle. Consequently, a need exists for a system that allows tire pressure to be known at times when the vehicle is moving. Such a system can be used for buses as well.

To be designed, selected, or determined:

1. A sensor that will have an output that varies with the pressure in the tire.
2. A sending system that transmits the sensor signal to the tractor.
3. Total cost of the sensor and transmitter.

Notes: Assume that an electronics box in the tractor will accept the signal and display a digital readout in psig or kPa gage pressure. Your design and costs will exclude the electronics box.

If such devices have been designed, your own design must not infringe on the patent rights of others.

3. Automatic Lawn Sprinkler System (3 engineers)

The plan view of a lawn and single-story home will be provided by the instructor. It is desired to install an underground automatic lawn sprinkler system. At appropriate times, the system will distribute the proper amount of water or water/fertilizer mix uniformly over the grassy areas and over the shrub and flower areas. Rainwater is collected by the homeowner in two 55-gallon steel drums, and provision must be made for using rainwater rather than city water. The house already exists, and this is a retrofit installation. (Alternatively, the house is yet to be built, and this is a new installation, to be decided by the instructor.)

To be designed, selected, or determined:

1. Location, size, and type of pump if necessary. (Take the city water pressure to be 65 psig.)
2. Routing of underground pipes or tubes and the materials of construction.
3. Provision for the user to add liquid or dissolved fertilizer to the lawn (at times specified by you) via the sprinkler system. (The grass is St. Augustine.)
4. Provision for the system to distribute collected rainwater rather than city water. When there is no rainwater, the system is to use city water and must automatically make the changeover.
5. How much water to provide for the grass, shrubs, and flowers, and when to do so.
6. Sprinkler nozzle type(s), number, and location.
7. Automatic regulating device for proper system operation.
8. Total cost, including installation, expected life, and maintenance.

Note: There are manufacturers and installers of such systems. So before consulting any of those available, perform a preliminary design to express your thoughts on and to develop your intuition for designing such a system.

4. Domestic In-Wall Vacuum Cleaning System (3 engineers)

The plan view of a single-story home will be provided by the instructor. It is desired to install an in-wall vacuum system. A centrally located vacuum cleaner motor provides power for cleaning. A number of hoses connect the cleaner motor to various wall outlets throughout the dwelling. The user attaches a hose and floor nozzle (supplied by a

manufacturer of portable vacuum cleaners) to any desired outlet connector and then vacuums the carpet. The house already exists and this is a retrofit installation. (Alternatively, the house is yet to be built and this is a new installation, to be decided by the instructor.)

To be designed, selected, or determined:

1. Location, size, and type of vacuum motor. (Consider that the motor collects dirt into a bag and that the bag must be changed with regular use of the system.)
2. Wall outlet locations (paying strict attention to locations where accessibility is not convenient, such as where a sofa or couch might be placed).
3. Piping size, material, and routing from the motor to the wall outlets. (Must all the piping be the same diameter?)
4. Hose length and floor nozzle attachments. (Also, how will the hose and nozzles be stored?)
5. Total cost of the system, including installation. Compare the cost to that of a portable vacuum cleaner, and determine whether the in-wall system is cost effective.

Note: There are manufacturers and installers of such systems. So before consulting any of those available, perform a preliminary design to express your thoughts on and to develop your intuition for designing such a system.

5. Dehumidified Air Supply (3 engineers)

A 150-HP internal combustion engine is to be tested to determine its operating characteristics in a dry environment, such as a desert. The main requirement is that a steady supply of dehumidified air be made available to the engine. Moreover, the dehumidified air must be supplied over a range of temperatures for a proper simulation of desert-like conditions.

To be designed, selected, or determined:

1. The air consumption of a 150-HP, four-stroke engine.
 2. A dehumidifier to remove as much moisture as possible from inlet air, to simulate desert conditions.
 3. What are desert conditions?
 4. A means for controlling the temperature of the dehumidified air.
 5. How the air is to be made available (e.g., in a steady-flow situation or from a storage tank).
 6. Total cost of system, including all components that make it ready for attachment to the inlet of the engine.
-

6. Signaling Funnel (3 engineers)

People who have used funnels to fill tanks often experience the problem of overfilling the container and spilling liquid over the top. This is especially common in the filling of lawn mower tanks. The user's view of the inside of the tank is obstructed by the funnel and, when the tank is full, gasoline spills out, creating a safety hazard as well as an unpleasant mess. It is proposed to design a funnel that signals the user to stop pouring when the tank is full.

To be designed, selected, or determined:

1. The proper material for a funnel designed for use with gasoline.
 2. The signaling device or method.
 3. An adjustment method for the signal to accommodate differing reaction times, since one individual might require less time to stop than another.
 4. Total cost to manufacture the signaling funnel.
-

7. An Artist's Palette

Consider an artist's palette. It contains 7 cavities or wells into which 7 different color mediums are to be placed. In this case, it is desire to have a palette with 8 cavities, so that melted crayons can be used by the artist. The crayons are made by Crayola[®], which come in an 8 pack.

One crayon color is to be placed in each cavity, and the palette is to be heated to melt all the crayons simultaneously. The artist may then dip a brush (or sponge or any applicator) into the color of choice, and apply that color to paper (or canvas, etc.). The palette must be portable, and it is desired to use batteries in some way to melt the crayons, and keep them melted for as long as possible.

To be designed selected or determined:

1. At what temperature will the crayons melt? Will all colors melt at the same temperature? What color crayons are provided in the 8 pack? Why are these colors chosen? (It is advisable to measure the melting temperature rather than use published data.)
2. Design a system that can be used to melt the crayons with batteries. Does the palette have to be made of a metal for the design to be effective? Can it be made of plastic? Will plastic or metal be a better choice for heat transfer purposes? If the crayons have to be maintained at some elevated temperature, will the user have to wear a glove to use it?
3. Design a palette with 8 cavities.

4. Dipping a room temperature brush into a melted crayon pool might tend to solidify the crayon. To alleviate this problem, should the brush be heated in some way beforehand? If so, design a method for heating the brush.
 5. A 7 cavity palette made of plastic lists for \$3.75. A palette with 9 cavities sells for \$5.25, and one with 11 cavities sells for \$6.75. Why are there odd numbers of cavities in these palettes?
 6. Determine the cost of the palette in your design.
-

8. Vacuum System for Cleaning Buses (3 engineers)

The local transit authority is responsible for operating and maintaining vehicles used in public transportation, specifically buses. One of the maintenance tasks includes cleaning the buses every evening when they are returned to the garage. Currently, buses are cleaned by an individual who walks through, picks up large refuse items, and vacuums the floor. This procedure is time consuming and costly in person-hours.

It is proposed to use a huge air-moving system to clean the buses. Two huge air pipes are attached to a bus. One pipe delivers high-velocity, high-pressure air, while the other pipe takes in air exhausted from the passenger compartment. Use of such a system eliminates the need for someone to walk through the bus to clean it and would take far less time to complete the work.

To be designed, selected, or determined:

1. Total design of the system.
 2. The size of the air pipes, the material of construction, and the method of attachment.
 3. The size of the air-moving unit and source of purchase.
 4. The total cost of the system.
 5. The savings in time and labor charges (if any) with a statement about cost effectiveness.
-

9. Ventilation System for a Spraying Booth (3 engineers)

A manufacturer of small furniture items has decided to install a spraying booth in the shop so that wood items can be given finish coats of varnish by spraying rather than by using a brush. The room allotted is 12 ft x 12 ft x 10 ft tall.

The varnish is to be applied by an operator using a spray gun. Airborne droplets of varnish cause a health problem, however, and so the booth must be adequately ventilated. Moreover, the ventilation must in no way inhibit

the application process or the drying process.

To be designed, selected, or determined:

1. Whether the size of the booth is adequate.
 2. The ducting system for introducing air to and exhausting air from the spray booth.
 3. The fan required to move the air.
 4. The total cost of the ducting, fittings, and fan.
 5. The effect that the airflow through the booth has on the application and drying process.
-

10. Pneumatic Seat (3 engineers)

Elderly people and those with certain disabilities often have difficulty standing up from a seated position. It is proposed to design an inflatable device that a person would sit on and would help lift the person up from the seat. The device is to be inflated with a small compressor and should be comfortable to sit on. The device can be part of a chair or be something portable and placed on the chair.

To be designed, selected, or determined:

1. The inflatable seat itself.
 2. The material to be used for it.
 3. A suitable compressor and connecting tubes.
 4. The total cost of the system.
-

11. Pneumatic Nut Cracker (3 engineers)

During cooler months of the year, various types of nuts are harvested and sold in grocery stores. The consumer buys them and uses a nutcracker to open the shell in order to separate the edible meat from the nut. Most commercial nut crackers are manually operated. In one design, the nutcracker has two legs attached by hinge pins to a common bridge. The nut is placed between the legs, and the legs are squeezed until the shell cracks open. The user then works diligently and sometimes impatiently to get the meat out of the shell.

In another design, the nut is placed at the end of a track against a metal stop. A movable piece of metal rides on the track and is pushed against the nut by a hinged lever arm. The force exerted is enough to crack the shell. Alternatively, the metal piece is accelerated toward the nut by a spring. The metal piece is moved away from the nut to compress a spring. When

released, the spring pushes the metal toward the nut, and the impact force is strong enough to crack the shell.

A marketing firm has decided that it would be desirable to market a pneumatic nut cracker. Rather than use the potential energy in a spring, the potential energy in an air chamber containing compressed air is used to accelerate a metal piece toward the nut. Now, the marketing firm (and engineers who might work on this project) realize that this device is probably not the most efficient of nut crackers. However, it is believed that the idea is novel, that the design should be eye-catching enough, and that the impulsive buyer would purchase one due to its unique features. Thus, appearance and novelty are extremely important in this design.

To be designed, selected, or determined:

1. A method for determining the force required to crack various nuts of the type available in the fall.
 2. A pneumatic nutcracker showing design details and calculations indicating the impact forces generated.
 3. A description showing the unique and attractive nature of the design.
 4. Total cost of the pneumatic nut cracker.
-

12. Fireplace Heat Recovery (3 engineers)

Sheet metal fireplaces can be added to a room after the structure is built; that is, the fireplace need not be built when the home is. In order to enhance the usefulness of a sheet metal fireplace, it has been proposed to devise a means for recovering more of the heat that would ordinarily be discharged up the stack with the exhaust gases. It is believed that the most effective way to transfer more of the heat from combustion is by convection, so more air in the room is heated.

To be designed, selected, or determined:

1. A sheet metal fireplace for your study (with the approval of your instructor).
 2. A heat exchanger for it. (If one has already been designed, your design must be substantially different. Moreover, you are also required to analyze the existing unit.)
 3. The total cost of the device.
-

13. Heat Recovery in a Sheetrock Plant (3 engineers)

One of the components needed in the manufacture of Sheetrock is water. The process requires 70 gpm of water at a temperature of about 85°F. During summer months, the city water supply provides water whose temperature can be as high as 90°F. During other months, the average temperature of water supplied by the city is about 45°F. This water must be heated so that it can be used successfully for the process. The water is heated by natural gas burners while it is in a storage tank.

One of the final phases of Sheetrock production is the drying stage. Heated air is moved by a fan around the Sheetrock in an oven. The air is then exhausted. It is desired to recover energy from the warm, humid exhaust air and use the energy to heat the incoming city water from 45°F (worst case) to as warm as possible. The energy recovered would reduce the need for natural gas to be used as the main heating medium. Conditions indicate that the heat recovery system will be in operation for 24 hrs per day (6-day week) for 8 months.

Figure 11.2 shows the position of the drying oven and of the holding tank. As shown, the water tank is 300 ft from the oven. The volume flow rate of air in each stack is measured as 14,000 cfm, and at the outlet of each stack the air temperature is about 230°F.

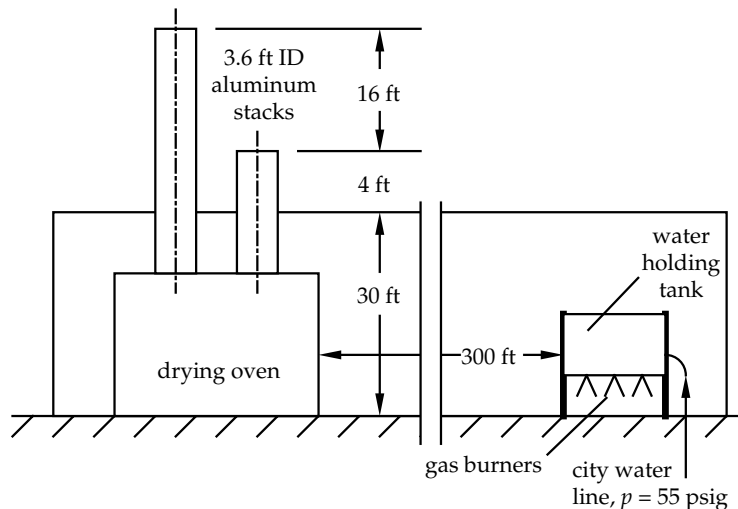


FIGURE 11.2. Layout showing drying oven and holding tank.

To be designed, selected, or determined:

1. Appropriate heat exchanger type, size, location, and material of construction.

2. Pump (if necessary), size, location, and material of construction.
 3. Piping, pipe fittings, size, routing, and material. (Consider that a flow meter and/or pipe insulation may be desirable.)
 4. Total system cost including installation, operation, and maintenance.
 5. Payback period on the investment.
-

14. Design of a Baseboard Heater (3 engineers)

A baseboard heater consists of a fluid-tight length of copper tubing (it could be another metal) with fins attached. Hot water or oil is made to pass through the tube, and the fins help to transfer heat from the liquid to the surrounding air by natural convection. A manufacturer of heating devices wishes to expand its line of items by adding three of these types of heaters. Rather than circulate a fluid through the tube, however, it has been decided to fill the tube with a liquid, attach an electric heater of some sort to the tube, and seal the tube itself. Thus, a user can purchase the size heater needed, attach it to the wall at the baseboard, and provide 110 V AC, and the heater will convert the electric power into heat. The manufacturer desires to market 1000-, 2000-, and 3000-BTU/hr heaters, because these are commonly used in conventional homes.

To be designed, selected, or determined:

1. The size of the tube needed for each heater.
2. Whether to buy finned tubes “off the shelf” or construct them.
3. The size and type of heating device needed for each heater.
4. The protective yet decorative housing available for each heater.
5. The type of heat exchange liquid (or gas) with which to fill the tube.
6. The total cost of each heater.

Such devices have been designed and are currently available. Your own design must not infringe on the patent rights of others.

15. A Sandblaster That Uses Ice (3 engineers)

Sandblasting is an operation in which fine sand particles are propelled by a jet of high-velocity air at an object such as a brick wall or painted metal. The sand acts as an abrasive that removes a portion of the surface and, in effect, refinishes the object. Sand then permeates the air, along with the sanded material, around the workplace (unless it is enclosed), and this is objectionable from a safety viewpoint. Furthermore, sand accumulates on the ground and presents a cleanup problem.

It is proposed to use ice particles as a substitute for the sand. The ice

will refinish most surfaces, and with the passage of time, the ice melts into water, which then drains away.

To be designed, selected, or determined:

1. A system for producing an “ice” blaster.
 2. The proper shape for the ice particles, and how they are to be produced.
 3. The method for propelling the ice particles. (Is there an optimum velocity?)
 4. Whether or not ice can be substituted for sand in a conventional sand blaster.
 5. Total cost of system.
 6. A comparison to a conventional sandblaster in performance and cost.
-

16. Warming Plate for Food Services(3 engineers)

Figure 11.3 is a sketch of a warming plate. Such plates are used in food service applications to warm meats without cooking them. Steaks, for example, might be stored in a freezer in a restaurant. When an enthusiastic consumer of meats wants a steak, the meat should be thawed before it is cooked. The objective in this project is to design an apparatus that will thaw meat without cooking it. Figure 11.3 shows a preliminary design of such a device.

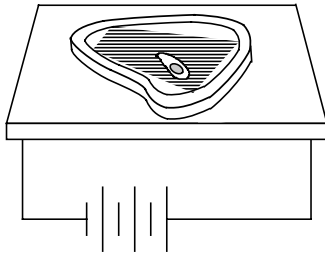


FIGURE 11.3. *A warming plate.*

To be designed, selected, or determined:

1. The design of the power source for a warming plate.
2. The material to be used. Must it be stainless steel?
3. The “appropriate” temperature of the surface of the warming plate. It should warm the object without cooking it.
4. Other foodstuffs that might be used with this apparatus. Should meats and, say, vegetables be used simultaneously with the warming plate?
5. How large should the warming surface be?
6. Design the device and determine how much it would cost.

17. Shaved-Ice Maker (3 engineers)

In many southern cities, a “snowball” is a popular summertime treat. A snowball (sometimes called a snow cone or a slush) consists of 8 or 12 ounces of ice over which a flavored syrup is poured. The syrup is a mixture of water, sugar, and a flavoring, such as strawberry, chocolate, or lemon.

A manufacturer of food products wishes to market a device that produces shave or crushed ice that is suitable for making a snowball. Typically, a 1 × 1 × 1-ft block of ice is delivered, and the machine will shave it or reduce it in some way to crushed ice.

To be designed, selected, or determined:

1. Design a machine that produces shaved or crushed ice. How much power is required to shave ice? What kind of “blades” are used, if any? Specify all materials of construction.
 2. How is the quality of shaved or crushed ice determined? (Quality does not refer to a thermodynamic property, but to how well the ice is shaved.) Establish a quality scale by defining high versus medium versus low quality of shaved ice.
 3. Is there an ASTM standard concerning shaved or crushed ice?
 4. Based on your (or some other) defined standard of quality, what should be the quality of ice that is used for a snowball?
 5. How much does your ice-shaving/crushing machine cost? What is the cost of other machines currently on the market?
-

18. Inflatable Dike (4 engineers)

Flooding in low-elevation areas is disastrous and costly. Homeowners who are victims of flooding are faced with enormous cleaning and rebuilding costs. Moreover, insurance companies are hesitant to provide flood insurance policies to homeowners whose dwellings are below sea level. Consequently, some sort of protection against flooding is most desirable.

It is proposed that a properly formed inflatable material be buried around a house (that was built on a slab), and when flooding by rising groundwater occurs, the buried device is inflated. (See Figure 11.4.) The dike must extend completely around the house and provide ample protection from floodwater that is expected to rise 10 in. above the slab, based on worst-case past experiences. In addition, when not in use, the dike must be able to be deflated and buried under the ground. The topsoil will then be landscaped, and no trace of the dike is to be seen.

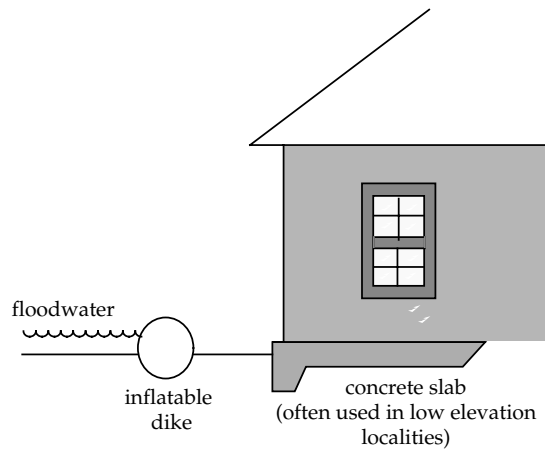


FIGURE 11.4. *View of inflatable dike concept.*

To be designed, selected, or determined:

1. The design of an inflatable dike that extends completely around the house. (Must the dike material extend under the slab? Must the material be inflated with air, or will a liquid be satisfactory?)
2. The material to be used for the dike, and how it is to be constructed.
3. The method for inflating the dike (or filling it with liquid).
4. Total installed cost of the entire system and its operation.

19. Measurement of Flipper Thrust (4 engineers)

A company that manufactures flippers is interested in modifying its traditional design and marketing a pair of flippers that provides greater thrust. It is first necessary, however, to be able to measure the thrust developed by a diver using flippers so that a comparison can be made. It has been suggested that an open-channel water tunnel be constructed and appropriately instrumented so that a diver can go through the motions of underwater swimming, although remaining stationary, and the force developed by the flippers is measured.

To be designed, selected, or determined:

1. A facility for a diver (real or mechanical) to swim underwater while not moving forward. If mechanical, then the mechanical device must also be designed.
2. The instrumentation necessary to obtain appropriate measurements.
3. The materials for construction of the facility, its design, layout, and

operation.

4. Total cost of building the entire facility, and cost to operate.
-

20. Vacuum Cleaner Efficiency Ratings (4 engineers)

Many devices that are manufactured for the consumer are rated. For example, refrigerators are rated by manufacturers by putting them through standard tests that reveal the cost of running them per year. Automobiles are rated in standard tests, and miles-per-gallon figures are reported by the manufacturer. The key to developing ratings is in the establishment of standard tests. Such tests are usually developed with the aid or supervision of professional and technical societies or, alternatively, by engineers who work and have gained considerable experience in the industry.

Vacuum cleaner manufacturers have decided to allow a consulting company design standardized methods for testing vacuum cleaners. The parameters deemed important are to be selected by the consultants. The procedures are to be written up in an acceptable or traditional format.

To be designed, selected, or determined:

1. Study existing methods and techniques for testing vacuum cleaners.
 2. Develop a method and procedure for evaluating vacuum cleaner performance.
 3. Consult ASTM standards and write your procedures in the same style.
 4. Following your own procedures, select three different vacuum cleaners and evaluate them. Express the results in a performance factor rating that you have developed.
-

21. Paddle Wheel Design (4 engineers)

At one time, steamboats were used extensively to transport goods and passengers. Steam-driven pistons provided power to rotate huge paddle wheels that propelled the ships over the water. The paddles are traditionally just flat boards bolted onto a frame. As each board was rotated under the water, thrust was developed. The shape of the boards was flat, but it is felt that a different shape or a different method of rotation under water would provide a greater transfer of power.

To be designed, selected, or determined:

1. Analysis of the traditional “blade” design, including material, typical

- rotational speeds, thrust developed by a wheel, and efficiency.
2. Proposed new design, including material, number of blades, expected thrust, and efficiency. (Rotational speed should be the same as for the traditional style.)
 3. Current costs of a traditional wheel.
 4. Current costs of the newly designed wheel.
-

22. Bicycle Frame Finishing Process (4 engineers)

A manufacturer of bicycle frames that previously sent the frames out for finishing has decided to set up a painting facility right in the manufacturing plant. The objective of the facility is to take frames that have been treated and given a primer coat of paint and to apply a finish coat. It has been decided that enamel-type paints are satisfactory for finishing frames. “Orange Peel” (a finish coat that is reminiscent of the surface of an orange rind) is unacceptable.

To be designed, selected, or determined:

1. The operations required to produce an attractive, durable, rust-inhibiting finish coat on the (steel) bicycle frames.
 2. Whether the finish operations should be performed in a batch-type process or an assembly-line-type process.
 3. The systems required for each operation. (For example, a chemical-treatment operation performed in a tank requires design of the tank and selection of the chemicals. A paint spray booth requires design of the booth and selection of the spraying equipment.)
 4. The safety requirements (adequate ventilation, for example, when a spray booth is used).
 5. Total cost of the facility, including operating and maintenance costs.
-

23. Ceiling Fan Blade Design (4 engineers)

Ceiling fans have been used for a number of years to cause air to circulate within a room and make the inhabitants more comfortable. The fans consist of a motor and four or more blades. The blades themselves are usually flat and wider at the tip than at the root. A performance rating of the air moving ability of conventional fans does not yet exist because a standardized method for making the needed measurements has not been devised.

To be designed, selected, or determined:

1. Compose a proposed description for a standardized test of conventional ceiling fans.
 2. Select a fan and perform an analysis of the blades as marketed.
 3. Determine if the blade design can be improved upon (i.e., if the same power can move more air merely by changing blade design).
-

24. Mileage Meter for a Conventional Automobile (4 engineers)

Automobile mileage estimates are useful for comparative purposes when selecting a car. However, many drivers would like to have an inexpensive meter that gives an instantaneous reading of mileage in digital form, so that while driving, the operator will know immediately how many miles (or kilometers) are obtained per gallon (cubic meter) of fuel. A driving method can then be developed to yield a savings in fuel dollars spent for the auto owner.

To be designed, selected, or determined:

1. A technique for measuring the (mass or volume) flow rate of the fuel. (An obstruction in the flow line could lead to an excessive pressure drop. Any obstructions therefore should be selected and positioned with care.)
 2. A method for transmitting the measured flow rate to the “electronics box.” (Assume that the design of the electronics required to process input signals will be performed elsewhere. It is sufficient to say that the microprocessor involved can add, subtract, multiply, and/or divide any two analog or digital input signals and deliver a digital readout. Moreover, the display can read in miles/gallon or km/m³, and this conversion is also handled electronically.)
 3. A technique for measuring and transmitting to the electronics box the speed of the automobile.
 4. Materials, placement, and construction of the add-on devices.
 5. Total cost of all components (excluding microprocessor unit), including initial, operating, and maintenance costs.
-

25. Design of a Portable Car-Washing System (4 engineers)

Washing a car by hand with a garden hose and drying it with perhaps a chamois is somewhat tedious and requires a certain amount of time. The method is inexpensive, however. Alternatively, car owners many times

employ commercial establishments to perform the task. This too requires a certain amount of time, sometimes longer than if done by hand, and it is costly. Between these two extremes is the fund-raising operation that costs little but is many times disorganized and, again, takes a certain amount of time.

It has been proposed that a portable system be set up that fund-raising groups or homeowners can use. It should consist of: (a) a framework of some sort with a garden hose connection for rinsing the car; (b) a method of applying a cleaning agent and washing the car; and (c) a system for drying the car that uses a high-velocity-output blower, the type used in cleaning sidewalks and driveways. The portable system thus uses a garden hose to connect to a water supply and a garden blower for drying, assuming that these items are readily found in most households. It is further assumed that the actual cost of using this system is comparable to that of hand washing. The objective of this system, however, is to save time without incurring too much additional operating cost.

To be designed, selected, or determined:

1. A water distribution framework on wheels that can move around and that can be disassembled.
2. The hose connections and water application system for the frame.
3. A method of applying a cleaning agent (perhaps another frame that dispenses soap and water) and of washing the vehicle.
4. A system for attaching a blower for drying the vehicle.
5. Total cost of the system, including parts, operation, and maintenance.
6. Total time involved in using the system compared to hand washing.

26. Solar Assisted Composter

An enterprising, environmentally conscious individual wishes to make a composter for her personal use. The apparatus is to consist of a barrel to receive the appropriate waste materials, and a system for rotating the barrel to keep what is inside well mixed. The barrel should have a top to allow the user to put waste material inside. The barrel should also have provision for removing the compost after it is ready.

As mentioned, the barrel will need to be rotated or turned and this should be done using solar energy. A solar collector will convert sunlight to electrical energy that may be used to keep a battery charged, and the battery will power a motor that will rotate the barrel as necessary.

To be designed, selected, or determined:

1. How large should the barrel or drum be? Can a 55 gallon metal drum be

- used or should something smaller made of plastic be used?
2. Should the interior of the barrel have vanes or some other obstruction whose function is to keep refuse well mixed? Does heating the mix have an effect?
 3. How frequently should the barrel be rotated?
 4. Design a method for putting waste products in, and for removing compost.
 5. What are the waste products that can be accommodated with this apparatus?
 6. Design a method for rotating the barrel.
 7. What size motor is required to rotate the barrel, and what should its rotational speed be?
 8. How large a solar collector and battery is required? Is a battery required?
 9. Design an apparatus capable of achieving the project objectives.
 10. In addition to writing a final report, write an instruction manual for the user of your apparatus.

Note: If such an apparatus exists and is now on the market, your apparatus should not in any way be a copy of it.

Submitted by Prof. Paul J. Palazolo

27. Underground-Stored Barricade (4 engineers)

Some city streets are used for automobile traffic during business hours and only for pedestrians in the evening. In order to keep automobiles from using such a street, posts are placed in holes in the middle of the street. To move the posts and store them elsewhere during business hours is cumbersome and requires manual labor. It is proposed that underground posts be used instead. When the post is not to restrict traffic, it “retreats” to a vertical cavity under the street surface. When it is to be up, it is raised with either an air or a hydraulic system, as indicated in Figure 11.5. Air or hydraulic fluid fills and expands the expandable chamber, causing the post to move upward. Note that instead of an expanding chamber, a telescoping cylinder might be used.

To be designed, selected, or determined:

1. The post material and dimensions for it to be an effective barricade. (How much weight should the post be able to support in the stored position and in the up position?)
2. The expandable chamber material and mode of operation.
3. The dimensions of the cavity. (Should the cavity be lined with a pipe

or some other material?)

4. The location of the device used to inflate or pump up the expandable chamber. (Note that if an oil is used, leakage of oil from where it is stored and from the chamber into the ground is not permitted.)
5. Total cost of one set of posts for a two-lane street one block long, including all parts and installation.

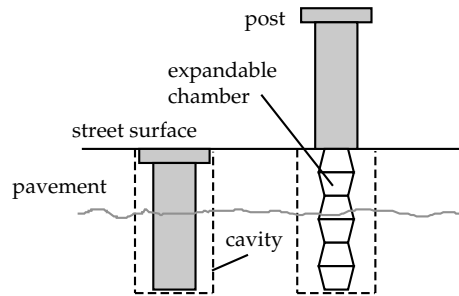


FIGURE 11.5. An underground-stored barricade.

28. Controlled Environment—Special Metals (4 engineers)

A museum has recently inherited a number of timepieces made during the 19th century. It has been decided to display these timepieces in a glass-sided and -topped case of internal dimensions 6 ft long x 2 ft tall x 2 ft wide. It is necessary, however, to control the gaseous environment about the timepieces in order to maximize their life span. For example, is dehumidifying the air sufficient for keeping the timepieces in good condition, or is maintaining a vacuum about them a better solution? The difficulty is that the composition of the metals used by the makers of the timepieces is unknown. Moreover, museum management will not permit destructive testing of any sort or magnitude to be performed on any of the timepieces to determine the composition of the metals.

To be designed, selected, or determined:

1. The probable composition of the timepieces. (Is there a nondestructive test for determining their exact composition?)
2. The most favorable environment for the timepieces.
3. A means for maintaining this environment in the 6 x 2 x 2 ft glass housing.
4. A suitable material to use as the base within the housing to set the timepieces on (felt on wood, for example).
5. Total cost of the system used to maintain the environment within the

- cabinet, including ductwork (if any, it should be hidden from view), fans, etc., and estimated life of this system. (If a partial or total vacuum is to be maintained within the glass housing, determine how to seal the glass pieces at the edges and corners. Also, assume that the glass is properly braced.)
6. Estimate the life of the timepieces within the cabinet and if additional care (such as cleaning at regular intervals) is required for an extended life.
-

29. Elevator Buffer Design (4 engineers)

Elevator safety codes require the use of a device called a “buffer” that is to be located at the bottom of an elevator shaft (in a region called the “pit”). The buffer should be capable of stopping a fully loaded (or nearly empty) elevator car falling at full speed into the landing. The deceleration rate should be less than 1 g. A number of buffer designs exist, and these designs have been in use for over 40 years. An elevator company wishes to re-design buffers using modern “high-tech” methods.

To be designed, selected, or determined:

1. Review the appropriate elevator codes regarding buffers.
 2. Examine the existing designs of buffers and become familiar with how they work.
 3. Design a buffer, and analyze how it works throughout its entire operating phase.
 4. Compose a test procedure for the buffer to determine whether it will meet code requirements.
-

30. Slab-Heated Warehouse (4 engineers)

A warehouse is to be constructed and will consist of a concrete slab with a sheet metal structure on it. The slab is 8 in. thick and measures 50 ft x 100 ft. The metal structure bolts to the slab with anchors and will be well insulated. The walls (12 ft tall) will have 4-in. insulation, while the ceiling will have 6 in. of insulation. The interior of the warehouse is box-like—12 ft x 50 ft x 100 ft (nominally).

The warehouse is located in a region where daytime temperatures are 65°F and nighttime temperatures are 40°F. These temperatures are the same throughout the year. Paint is to be stored in the warehouse and, accordingly, it is desirable that the temperature inside be kept at 70°F at all times. It is proposed to use a heating method in the building that

involves embedding tubing in the slab; that is, the tubes are laid out and the cement mix is poured directly over the tubes. Warm fluid can then be circulated through the tubing, thereby heating the slab by a convective effect, which in turn will heat the warehouse interior by convection and by radiation.

To be designed, selected, or determined:

1. The heating load for the warehouse, assuming it is 60% (by volume) occupied by 5-gallon cans of paint (accounting for aisle space, etc.).
 2. An appropriate layout for the tubing in the slab. (Is an 8-in. slab thick enough? Should a serpentine flow arrangement be used, or some other layout?)
 3. The heat transfer fluid and how it is to be heated.
 4. The method of circulating the fluid through the tubing.
 5. The total cost of the installation. (What additional costs are incurred by using this method of heating over other methods?)
 6. Desirability of this method over other conventional methods (e.g., radiant heaters, forced convection heaters, baseboard heaters).
-

31. Hot Water Dispenser (4 engineers)

Instant coffee, hot chocolate, broth, and tea are examples of drinks that require hot water for their preparation. The hot water is prepared by heating it on a stove or in a microwave oven—both require finite amounts of time. It is proposed that an in-line heater be placed in a water line so that hot water is available from a tap at the kitchen sink. One merely places instant coffee, for example, in a cup and adds the hot water, and the drink is ready for consumption.

To be designed, selected, or determined:

1. The optimum temperature (with justification) of the hot water—if there is one.
 2. An in-line heater to heat the water from an expected low temperature that ranges from 55°F to 70°F to the proper temperature. (Should the pipe be insulated? Is a thermostat needed to control the temperature of the delivered hot water?)
 3. A means for dispensing the water at a selected flow rate.
 4. Safety considerations to avoid burns.
 5. Total installed cost of the system and operating costs.
-

32. Personal Ice Maker (4 engineers)

People who travel long distances by car usually like to carry coolers along in order to keep food and drinks cold. Evenings are spent in motels, and motel owners often do not permit guests to have ice for their coolers. Consequently, it seems that there is a need for an alternative supply of ice. One source is in gas stations or markets. Another possible source is with the use of a portable ice maker.

The portable ice maker seems to be an attractive alternative. Upon arriving at the motel, the guest would fill the ice maker with water and provide it with electricity. By morning, the ice maker would have produced a quantity of ice sufficient for the needs of the user.

To be designed, selected, or determined:

1. A portable ice-making machine to produce and store enough ice (in the form of cubes) to fill a 1 ft x 1 ft x 1 ft volume.
 2. All the internal workings, including (if appropriate) compressor, evaporator, and condenser.
 3. The housing and mounting of all components.
 4. The material used in the housing, and so on.
 5. The cost of producing an ice maker.
 6. The cost of running the ice maker to make ice.
-

33. Greenhouse Climatic Conditions (4 engineers)

The owners of a greenhouse desire to control the application of water (or water-fertilizer mix) and wintertime temperature within the structure to protect and feed the plants. The greenhouse is 10 m wide by 30 m long. The wall height is 2.5 m, and the lengthwise sloping roof is set at a pitch of 40° with the horizontal. Tall, potted plants can be set on the floor. Short, potted plants are set on 3-ft-high tables. The greenhouse is to produce plants sold at wholesale prices to retailers and not the general public. Thus, only a few different types of plants are in the greenhouse at any time.

To be designed, selected, or determined:

1. A system for irrigating the plants, preferably automatic. (Consider that the entire floor area must be able to receive water or a water-fertilizer mix. Also, it may be desired to be able to have some areas that receive little or no water, as deemed necessary by the operator.)
2. Piping (if any), piping material, and routing. (Consider that, if a water-fertilizer mix is used, the fertilizer may have a corrosive effect on certain materials.)

3. Pump size (if any), type, and location. (Assume that city water is available at 55 psig.)
 4. A drainage system (if necessary) that can be put into the slab before it is poured.
 5. A system for heating the greenhouse in winter during times when the temperature inside might fall below 40°F.
 6. Fan, ductwork, and heat source for heating purposes (or alternative heating technique).
 7. Total cost of all items, including installation and expected life.
-

34. Solar-Heated Swimming Pool (4 engineers)

The swimming pool of a motel is currently outdoors. Management wants to erect a structure over the pool and use solar energy to heat the water. The pool dimensions are 10 m long x 7 m wide, with a water depth that varies in the lengthwise direction from 1 m to 3 m. It is proposed to have flat-plate collectors receive energy from the sun and use the energy to maintain the water at a comfortable temperature year-round.

There exists a manufacturer of hot-water heaters that markets swimming pool heaters. These heaters warm the water with natural gas burners.

To be designed, selected, or determined:

1. A comfortable water temperature for the pool.
 2. The number of solar collectors required. (Assume that the motel is in a city to be specified by the instructor.)
 3. The placement of these collectors, their orientation, location, and number required.
 4. Pump, piping, and materials of construction.
 5. Total cost of system, including initial, operating, and maintenance costs.
 6. Compare these costs to those associated with the use of a natural-gas water heating system.
-

35. Economics of Fuel Tank Insulation (4 engineers)

Fuel tanks contain heated fuel oil or crude oil. The liquid is heated in order to lower its viscosity and make it less costly to pump. There are costs associated with keeping the liquid heated in a storage tank. These costs are investigated in this project.

It is desired to analyze heat transfer characteristics from uninsulated tanks and how these tanks would be affected if they were insulated. Three tanks are to be considered. Two types of insulation will be compared.

An uninsulated tank containing a heated liquid loses heat by conduction through its bottom to the ground, by natural convection through its sides and roof to the ambient air, and by radiation. Referring to Figure 11.6, we see that the total heat loss is made up of:

- A = heat lost from liquid through side wall
- B = heat lost from vapor through side wall
- C = heat lost from vapor through roof
- D = heat lost to the ground

The tank wall is neglected as being a significant insulator that presents a high resistance to heat transfer.

To be designed, selected, or determined:

1. The heat loss through each tank described below, uninsulated.
2. The heat loss with a 2-in.-thick layer of urethane added to the side wall.
3. The heat loss with a 2-in.-thick layer of fiberglass added to the side wall.
4. Based on a 20-year life span, calculate the rate of return on investment, including first cost, maintenance costs, and savings in fuel costs for each type of insulation. (Use a fuel cost of \$2.30 for each 10^6 BTU.)
5. Make a recommendation.

Tank Descriptions

Tank #1 is 170 ft in diameter and 40 ft high. It contains #6 fuel oil maintained at 140°F. Assume that the tank on the average is 0.85 (x height of tank) full. Further assume a surface emissivity of 0.9 and a wind velocity of 10 mph. The ambient temperature is 70°F.

Tank #2 is 70 ft in diameter and 40 ft high. It contains #6 fuel oil maintained at 140°F. Assume that the tank on the average is 0.85 (x height) full of liquid, that the surface emissivity is 0.9, and that the wind velocity is 10 mph. Again the ambient temperature is 70°F.

Tank #3 is 100 ft in diameter and 40 ft high. It contains paraxylene maintained at 80°F. The tank on the average is 0.85 (x height) full of liquid, and its surface emissivity is 0.9. The average wind velocity is 10 mph, and the ambient air temperature is 70°F.

Insulation Information

Urethane has a thermal conductivity of 0.14 BTU/hr·ft·°R. Fiberglass has a thermal conductivity of 0.27 BTU/hr·ft·°R. Urethane costs \$2.55/ft² and

must be resurfaced after 10 years at a cost of 1.50/ft². The fiberglass, although it has a higher thermal conductivity, needs no resurfacing every 10 years, and it has an initial cost of \$3.25/ft². The urethane and fiberglass initial costs are based on a 2-in. thickness, which is what your calculations should be based on. (For further information and the calculations involved in this project, see “Insulation Saves Energy” by R. Hughes and V. Deumaga, *Chemical Engineering*, May 27, 1974, pp. 95–100.)

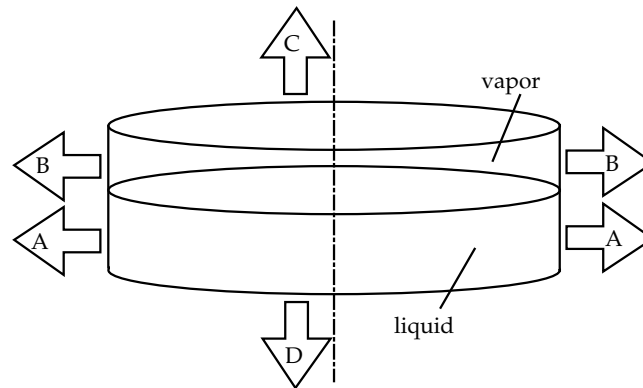


FIGURE 11.6. Sketch of heat losses from a fuel tank.

36. A Three-Ton Source of Cool Air (4 engineers)

A remotely located office-shop building has been converted into a church. A one-ton air conditioner originally used to cool just the offices is the only system available for making the entire building comfortable for a few hours on Sunday mornings. It is believed that finances do not permit the purchase and installation of an additional unit or units. It has been determined that 3 tons of cooling are required on Sundays to accommodate the current attendance level.

It has been suggested that the air conditioner be made to run all week to form a huge block of ice or to cool some other “storage” facility. Then on Sunday mornings, air can be moved past the ice and the cooled air used to make the occupants comfortable.

It is desired to design an appropriate “storage” facility using minimum space that can later be used to provide cool air. Ice is not necessarily the answer, although water is inexpensive.

To be designed, selected, or determined:

1. A facility (liquid, gas, solid) that can be cooled during a 6¹/₂-day period by a one-ton air conditioner to yield 3 tons of cooling for half a day.

2. Dimensions of the required facility (should be as small as possible).
 3. Total cost of the facility (assuming space within the building is available). Cost should include all items required, including insulation, if any, and operation of the one-ton unit.
 4. Compare these costs to those associated with the purchase of the proper-size cooling system (including installation and operation).
 5. Make a recommendation based on your calculations.
-

37. Heated Driveway Slab (4 engineers)

It is not uncommon to see concrete driveways laid out on an incline. Snow and ice accumulate on driveway surfaces during winter months. If the snow and ice remain unattended, it becomes difficult to control an automobile on the driveway. The owner can manually clear the driveway or install a heating system within the slab when it is poured. A proposed heating system is the subject of this project.

Consider a driveway that is 20 ft wide and 40 ft long. It is desired to embed within the driveway a tubing system through which water is circulated. The water provides heat to the concrete driveway so that ice and snow melt and flow away. Embedded tubes must not detract from the strength of the driveway.

To be designed, selected, or determined:

1. A layout for the tubes over which the concrete is poured, including braces if needed.
 2. The tubing material and size and joining method. (Should refrigeration tubing be used so that fewer joints are needed?)
 3. The method used to move water through the tubing. (Is a pump preferable?)
 4. The source of the water. (Should water be pumped through and discharged to a drain, or should water be pumped from and back to a sump tank?)
 5. The heat source needed to warm the water, if any is necessary.
 6. The appropriate fluid to use. (Water has been suggested above, but would a mix of water and antifreeze be better? Is oil a better choice?)
 7. Determine the total cost of the system, including installation.
-

38. Portable Source of Cool Air for Golf Carts (4 engineers)

A golf cart manufacturer wants to market a device that will mount on the back of a golf cart and produce cool air for the comfort of the golfers while they are in the cart. An air conditioner would be desirable to have,

but the compressor uses too much power to make its use feasible. It has been suggested that an insulated ice chest or cooler be used with a fan to produce cool air. The ice chest would be filled with ice, and air or liquid would be circulated throughout. As the fluid is cooled, it would be used to cool the air that would be directed at the golfers. A loose-fitting cover over the golf cart would help keep the cooled air within the cart. The entire unit should be self-contained. If possible, the interior of the chest should have enough additional room to store beverages.

To be designed, selected, or determined:

1. A cooler to store the ice.
 2. A method for using the melting ice to exchange heat with air that is to be cooled. (Should the interior of the ice chest contain tubes to circulate water or air throughout? If so, size the tubes and the fan or pump appropriately.)
 3. Size the fan that will be used to direct cool air toward the golfers.
 4. Determine the power source that is needed for this service. Note that it is desirable for the system to operate satisfactorily for the time it takes to finish nine holes of golf. More ice can be added if necessary.
 5. Determine the cost of this device.
 6. Conduct a survey of golfers and determine how many would purchase the device at the agreed upon price.
-

39. Finishing System for Wood Fences (5 engineers)

Wood fences are popular with residential homeowners. The fences are easy to install, effective, and attractive. A wood fence can be left in its natural, unfinished state, or a protective finish with or without a stain can be applied. The subject of this project is the design of a device that will prepare the wood surface for finishing and of a device that will apply the protective coating. Such devices would be of value to manufacturers of wood finishes, to painting contractors, and to fence installation companies.

The surface of a wood fence must be cleaned before a finish is applied to ensure that the finish will adhere satisfactorily to the wood. One finish manufacturer recommends that the fence be cleaned using a stiff brush and a mixture of water, bleach, and trisodium phosphate. (It should be noted that a fence constructed of “green”—recently harvested—wood contains a certain amount of moisture and may take a year or so to dry out sufficiently. The dried wood shrinks slightly. If the new fence boards are put up touching each other, there will be a small space between adjacent boards after they dry out. Once dried, the fence can be cleaned and finished.) After cleaning, the fence is rinsed well and allowed to dry for at least three days. The finish is then applied using a spray gun, a roller, or a paint pad.

The cleaning and finishing operations can be done by hand, but this becomes quite a tedious chore. It is desired to design an automatic system for accomplishing these tasks for the type of fence shown in Figure 11.7. This figure shows 1 x 6 (typically cedar or pine) fence boards nailed to 2 x 4's. The 2 x 4's in turn are nailed to 4 x 4's, which are cemented into the ground. It is desired to design a device that will clean the wood surfaces, and a second device that will apply the wood finish. However, the same device can be used for both jobs if it is advantageous to do so.

For the cleaning operation: The operator of the automatic device that cleans the wood surface would prepare the cleaning solution and make sure that it is continuously fed to the cleaning brushes that scrub the fence. Rinsing can be done manually.

For the finishing operation: Later, the operator provides containers of the wood finish to the device that automatically applies the finish to the fence. Figure 11.7 shows spray nozzles and a *suggested* rope-and-pulley system for moving them. Thus, in the case of cleaning or finishing, the operator provides whatever the device(s) needs and makes sure the systems remain in adjustment during operation.

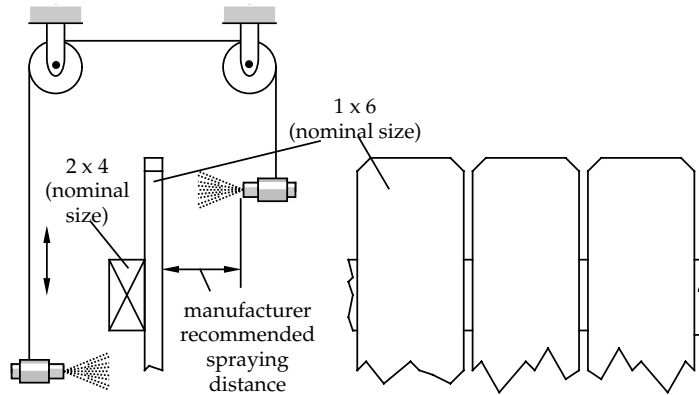


FIGURE 11.7. Profile and frontal views of fence boards.

To be designed, selected, or determined:

1. Current (manual or automatic) methods for cleaning and finishing wood fences.
2. The design of an automatic device for cleaning the front and the back of a fence of the type shown in Figure 11.7.
3. The design of an automatic device for applying finish to the front and the back of a fence of the type shown in Figure 11.7.
4. Recommended procedures and instruction "manual" for using the devices.

5. Cost of constructing the devices.
6. Approximate cost of renting them to the public by an equipment rental firm; expected payback on the investment should be within five years.
7. Recommendations regarding how cost effective these devices are.

40. Pneumatic Freight Pipeline (5 engineers)

A freight pipeline involves the use of a fluid moving through a pipeline to transport a solid or a liquid substance over a certain distance; in other words, the moving fluid conveys “freight.” The fluid itself in many cases is merely discharged at the destination.

Consider the freight pipeline sketched in Figure 11.8. At the inlet, air is moved into the pipeline by a blower. Just downstream is a hopper from which the substance to be conveyed is fed into the moving air stream. The mixture then flows through the pipeline until it encounters a separator. The air is discharged to the atmosphere, and the freight is collected at the bottom of the separator, where it can be discharged to a receiver tank.

The system of Figure 11.8 is to convey green peas to a packaging machine. Once packaged, the peas will be frozen and sent to market. The peas are to be conveyed at a rate of 3 000 kg/hr, which is the maximum capacity of the packaging machine. The path to be followed is as indicated in the figure.

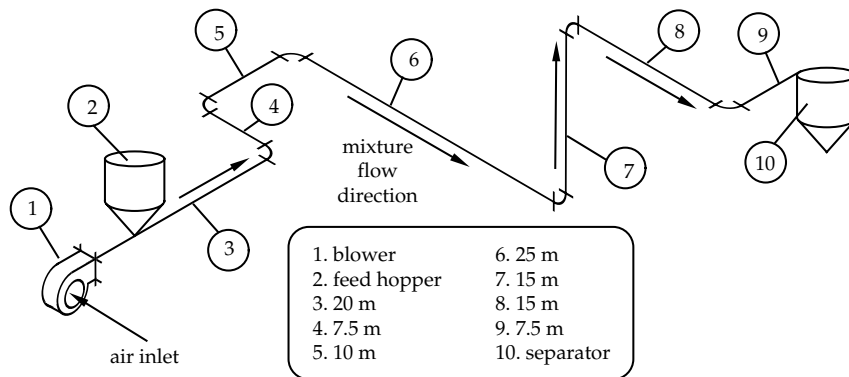


FIGURE 11.8. A freight pipeline using air to convey peas.

To be designed, selected, or determined:

1. The properties (particle density and average particle size) of peas.
2. The material the pipeline should be made of.
3. The air flow rate required to convey the peas at maximum capacity.

4. The total pressure drop of the system.
5. The power requirements of the blower, and the blower type (positive displacement, or rotary, or centrifugal, etc.).
6. The feed hopper and the separator.
7. The configuration of the elbows; i.e., the radius of each elbow. If the elbows have too sharp a radius, then a relatively high pressure drop will be encountered.

(Reference: *Pneumatic Conveying of Solids* by R. D. Marcus, L. S. Leung, G. E. Klinzing, and F. Rizk, Chapman and Hall, New York, 1990.)

41. Windmill Sizing and Feasibility Study (5 engineers)

Advances in the technology of capturing useful energy from the wind have led to new and better windmill designs. It may now be possible to generate significant amounts of electrical energy from the wind in order to supplement energy provided by the local utility company. The one overriding requirement is that the cost of wind-generated power be low enough to justify such an installation.

A windmill feasibility study will essentially consist of four parts: determine the wind characteristics (i.e., wind velocity and direction) at the proposed installation site; study a number of commercially available wind energy systems; perform an energy recovery analysis to determine how much energy can be generated by the systems of interest; and perform an economic analysis. Based on the results, significant conclusions can be drawn and meaningful recommendations can be made.

To be designed, selected, or determined:

1. Unless available otherwise, obtain wind velocity and direction data from a nearby airport. (It will be assumed that such data are valid for whatever site is selected for the windmill installation.) It is anticipated that the data available will contain wind velocity and direction taken every hour on the hour. It will be necessary to obtain hourly data for a three-year period.
2. Reduce the data using an appropriate averaging scheme to produce wind summary tables. Such tables will show the percentages of wind direction and speed over various speed intervals. A table should be produced for each of the three years selected.
3. From the wind summary tables, produce a graph showing average wind speed versus month. The average speed will include all three years.
4. Study the characteristics of four windmills of 10-kW size or smaller.
5. Using the wind summary tables and the power output curves for the

- windmills, determine how much energy would be produced (average energy, that is) by each machine if installed at the proposed site.
6. Obtain economic data (such as inflation rate, loan interest values, cost of electricity) and determine how long each machine would take to pay for itself in energy savings.
 7. Consult the local utility company to learn about how windmills can be made to fit into their generation scheme. If the windmill generates more power than is needed by the user, can the excess energy be sold to the local utility company?
 8. Make recommendations based on this study, and be prepared to answer the following questions:

Should an individual homeowner (3000 square feet, family of four) consider buying a windmill and installing it in his/her back yard in order to generate electricity? Is it cost effective?

Should a community of 100 or so individuals with homes of 3000 square feet (average) consider pooling their financial resources and purchase a huge windmill that all can obtain energy from? Is this a cost-effective purchase?

Should the local utility company consider purchasing several huge windmills for electricity generation to supplement what they now sell to their customers? Is this a cost-effective venture?

With regard to any of these three installations, what would the cost of electricity have to be in order to make a windmill cost effective?

References: *Wind Power for Farms, Homes, and Small Industry* by J. Park and D. Schwind, National Technical Information Service, U.S. Dept. of Commerce, 5285 Port Royal Rd, Springfield VA 22151, 1978.

Siting Handbook for Small Wind Energy Conversion Systems by H. L. Wegley, M. M. Orgill and R. L. Drake, National Technical Information Service, U.S. Dept. of Commerce, 5285 Port Royal Rd, Springfield VA 22151, 1978.

Winds and Wind Systems Performance by C. G. Justus, Franklin Institute Press, 1978.

42. Water Feature[†] (5 engineers)

Landscape architects and garden designers use the term “water feature” to refer to a full range of pools, ponds, cascades, streams, waterfalls, and fountains. These devices historically were gravity powered, but in more recent designs, pumps are being used to move the water.

Examples of water features can be found in airports (e.g., Detroit Metro), amusement parks, or in shopping malls (e.g., Oak Court Mall—Memphis, TN). Their function is to provide an attractive environment for the casual observer.

The objective of this project is to design a decorative combination of

structure and flowing water that is currently popular for homes and offices. Examples include: water dripping from platform to platform, water running over rocks, and a sheet of falling water; generally, something that would dress-up a departmental office.

To be designed, selected, or determined:

1. Identify and visit at least three devices that are considered “water features.”
2. Analyze the designs to determine (a) how the water is moved through the system; (b) what makes each attractive or appealing to the public; and (c) the approximate cost, including labor, of the installation.
3. Produce a brief written report for each device, detailing the information put together in response to the previous question. This is to be written for a nontechnical audience.
3. Design an inviting water feature that incorporates the things identified as appealing or attractive.
4. Determine how the water is to be moved throughout the system.
5. Size the pump or other prime mover, if there is one.
6. Determine the cost of the installation.

[†]Submitted by Professor John I. Hochstein.

43. A Toy Vertical Wind Tunnel[†] (5 engineers)

The objective of this project is to design a vertical wind tunnel (VWT) that can be used as a toy. A company that manufactures toys wishes to market a vertical wind tunnel to use with their other products such as those advertised as “hero action figures.” The action figure would be placed on a starting position, air would be directed upward to move the action figure to a maximum height of 6 ft, and then the air would be turned off. The action figure would then begin falling. A parachute would open, and the figure would be brought slowly to the ground.

Thus, the design would have the following constraints; the VWT should:

- Be as small as possible.
- Present NO safety hazards, as it is to be used by children.
- Be operable outside in cold weather as well as in warm weather.
- Not make excessive noise (if possible).
- Offer the operator a fine adjustment of air velocity.
- Not be costly.
- Be maintenance free for at least one year.
- Be able to levitate an action figure to a height of 6 ft.

To be designed, selected, or determined:

1. Design a device that can be used as a vertical wind tunnel to levitate a hero action figure. Does the action figure have to be oriented in a certain way; i.e., with arms and legs extended as in a skydiving configuration?
2. Select a fan for moving air through the system. What type of fan is best for this purpose? What type of fan is better suited for this purpose: a high flow rate fan or a high pressure fan? Prove that your system will work as designed by using engineering calculations.
3. What velocity is required to move the action figure? How does velocity change with height as the air moves upward? If your system contains a duct attached to a fan, for example, should the duct have a constant diameter, or should it be conical in shape?
4. What safety issues must be addressed in your design? Show what can be done to minimize the safety hazards.
5. How large must the unit be?
6. Determine the cost to build the device.
7. Determine how much can be charged to the consumer for your toy vertical wind tunnel.

†Submitted by Professor Julie Mathis.

44. Water Oil Separation Tank—Analysis and Testing (5 engineers)

A water-oil separation tank performs as the name implies. A water-oil mixture (or any two immiscible liquids of different density) is pumped into one end of the tank; water exits at one location and oil exits at another. Such tanks are available in various sizes ranging from 1000 to 20,000 gallons. The accompanying figure is a sketch of a separation tank.

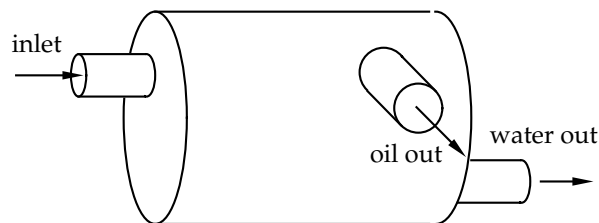


FIGURE 11.9. *A water oil separation tank.*

Separation tanks can operate on a continuous basis, but batch operations are more prevalent. For example, consider an area where there are a number of activities occurring and where oil drums are stored. A fork lift accidentally punctures a 55 gallon drum and oil leaks out. Someone with a water hose rinses down the area and washes the oil into a drain. The drain leads the mix to a tank where most of the oil is recovered and separated from the water.

The tank first receives a water-oil mixture that could be of any proportion (50% oil and 50% water, for instance). Then as rinsing continues, the tank will receive a mix that is almost entirely water.

To be designed, selected, or determined:

1. Compose a procedure for testing water-oil separation tanks that accurately reflects the scenario described in the preceding paragraph. Are there other scenarios of cleanup that should be taken into account?
2. Are there any ASTM standards that apply to the testing of these tanks?
3. If possible, perform tests on any tanks that are available.
4. How good is the separation? If drinking water is used in the cleanup, is water leaving the tank fit for drinking? Consult a chemist about this aspect.
5. What is considered a good separation? How is the purity of the water exiting the tank determined?
6. Locate catalogs and performance statistics of at least three tanks from different manufacturers. Explain how these tanks work. Put together a comparison chart for the tanks.
7. Are these tanks cost effective? How much is a tank, and how many cleanups must it be used for to pay for itself? Provide case studies of cost savings.
8. How much oil, kerosene, fuel oil, gasoline, etc., is spilled annually?

45. High Pressure Hose Tester (5 engineers)

A company makes flexible hose to be used in automotive air conditioning systems. Hose sizes can be 1/2 in., 3/4 in., or 7/8 in. ID. The hose material itself is cut to length (typically 3 ft), and an aluminum fitting is pressed onto each end. The fitting has male pipe threads at one end and female at the other, 1/2 NPT.

The company would like to be sure that their product will not fail at the hose-fitting connection. To test a hose, it is attached to a machine that will cyclically pump hydraulic oil through it. The oil will be pumped through at high pressure for 15 seconds. Then the direction of flow of oil

will be reversed, and the oil will again be pumped through again for 15 seconds.

To be designed, selected, or determined:

1. Design a machine that the hose can be attached to and that will move fluid through it as described. The machine should be designed with safety considerations in mind.
2. What pressure should be used during testing? What pressures exist in an automotive air conditioning application? Should the test pressure be greater than this pressure? Why or why not?
3. Is hydraulic oil the “right” fluid to move through the hose? Is there a better fluid to use?
4. If the company produces 100,000 hoses per year, how many hoses should be tested? How long will a test take? How many cycles are necessary before a hose is determined to be acceptable?
5. Is it a good idea to test the hose under vacuum conditions?
6. Is 15 seconds a “good” length of time for the testing time interval? Design the machine to have a variable time testing interval from some low value to 60 seconds.
7. Determine the cost of the entire system.

46. Determination of Valve Coefficient (5 engineers)

Valves are used in piping systems to control flow (on-off) or to control flow rate. Globe valves are good for controlling flow rate in a more precise way than ball valves, but globe valves cause a high pressure loss when placed in a pipeline. Moreover, closing a globe valve from fully open requires turning its handle 7 or 8 revolutions.

Ball valves, on the other hand, have a very low loss coefficient, and they can be shut off with a 90° rotation of the valve handle. However, ball valves are not good for controlling flow rates.

There is always a question of how to size a valve. In industry, valves are sized by using what is known as a valve coefficient C_v (not to be confused with a venturi meter coefficient C_v). The valve coefficient is defined as the flow rate through the valve (expressed in gallons per minute, gpm) divided by the square root of the pressure drop (expressed in psi). The valve coefficient is to be obtained experimentally by setting up a flow system such that the pressure drop across the valve is 1 psi. The corresponding flow rate is then equal to the valve coefficient. This value is reported in catalogs and is designed to be used by engineers when sizing a valve for a system.

Clearly, the valve coefficient is not dimensionless, but instead has units of $\text{gpm}/\sqrt{\text{psi}}$. Although this is not academically acceptable, it is very

useful when used as is.

Many ball valves have been tested to determine their valve coefficients, but limited testing has been done on three-way valves.

To be designed, selected, or determined:

1. Search the literature and review how the valve coefficient came into being and how it is used to size a valve.
 2. What are typical values of the valve coefficient for various types of valves? Show by an example how the valve coefficient is used to size a valve for a given piping system.
 3. Are there extensive published data for three way valves?
 4. Describe in detail the experimental method developed for measuring the valve coefficient.
 4. Design an apparatus for determining the valve coefficient. The apparatus should be able to accommodate measurements of valve coefficients for 1/2, 3/4, 1, 1.5, and 2 in. valves.
 5. Evaluate the usefulness of the valve coefficient.
 6. Is there a dimensionless number related to the valve coefficient?
 7. Is there a better way to size a valve for a pipeline?
-

47. Cooling System for a Series of Heat Sources (5 engineers)

Figure 11.10 figure shows a piping system and a number of pumps. The piping system consists of four injection molding machines (labeled as **process heat source**), through which SAE 10W-40 oil is pumped. The oil keeps the machines cool and, in so doing, is heated by 10°C. The oil flow rate through each machine is 25 gpm.

A pump moves the oil through the process heat source and then to a heat exchanger. Water is also circulated through the heat exchangers. The function of the water is to cool the oil.

Water is fed by a main pump into a pipeline labeled as the **supply header**. The water is piped to the heat exchangers and is routed back to a **return header**. The heated water is routed to an outside **heat rejection unit**.

The system is designed to remove heat from the process heat source and to discharge the heat outside.

To be designed, selected, or determined:

1. Size the pumps used to move oil through the process heat source and the heat exchanger.
2. Size the flow lines used to convey the oil (assume they are 35 ft in length). Is this particular oil a good one to use in this application, or is

there something better?

3. Size the pump that moves the water-ethylene glycol mix (assume it to be a 50-50 mixture).
4. Size the supply and return headers. (Should they be the same size? Is the flow arrangement shown the “best possible,” or is there a better layout for the headers?) The headers are 100 ft long with a distance of 25 ft between adjacent machines.
5. Determine the type of heat exchangers to be used for the water-mix-to-oil heat exchange.

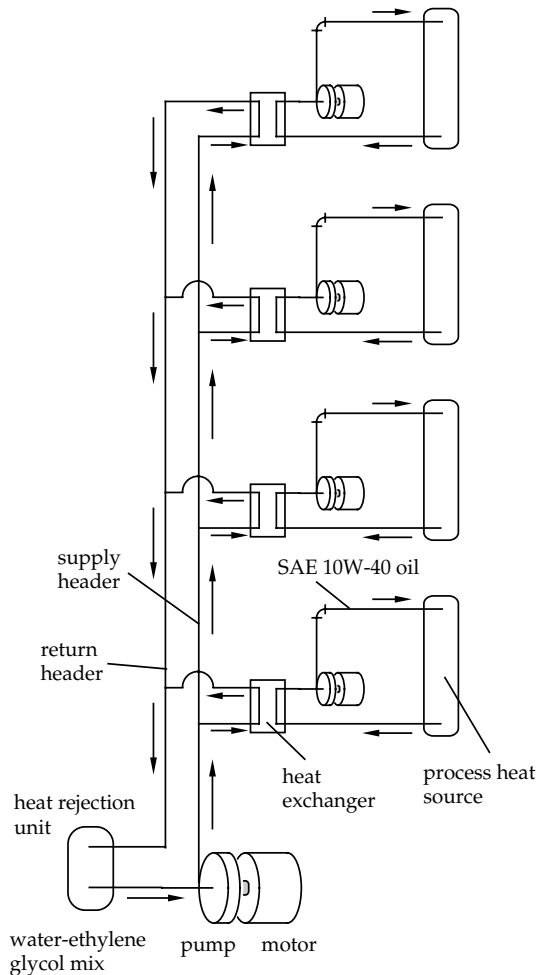


FIGURE 11.10. Sketch of the cooling system for a series of heat sources.

6. Determine what should be used for an outside (of the building) heat rejection unit.
 7. Determine the cost of the entire system.
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48. “Eccentric Guy’s” Pond[†] (5 engineers)

Consider a somewhat eccentric individual who would like to stock a flesh-eating fish in a pond that would be located on his property. The owner is wealthy, and can have this pond located anywhere in North America.

The fish would have to be fed on a regular basis and be in a supportive environment that would keep them alive. They must be able to thrive, reproduce, and be a source of joy to the owner. The fish should be capable of feeding on locally available meat, and the owner should not have to import any special food for their support.

The pond environment should be a place that reminds one of the locale where the fish were originally found. It should be lush and green, and have a park-like atmosphere. It should have benches where one might sit and enjoy sunshine.

To be designed, selected, or determined:

1. Design an inviting environment for such an installation.
2. What is a good fish (or animal) to have living in this pond? Are piranha the ideal fish for this purpose? How about sharks or eels? What laws apply to having and breeding such fish?
3. Size the pond required for this service. Should the pond be any particular depth? Suppose someone falls into the pond? Should the depth be such that the person can walk on the bottom while his or her head is above water?
4. How wide should the pond be? Remember that it should be wide enough so that the wildlife can be comfortable and thrive.
5. What eco-system should exist for the natural support of the wildlife you plan to stock in the pond? What sort of plants should be found floating on it, if any?
6. Should the water be salt- or fresh-water? Does the water have to be pumped through a filter on a regular basis?
7. Determine the safety features that an installation of this sort should have.
8. Determine the cost of the entire system.

[†]An altered form of a project that was submitted by Professor Julie Mathis.

49. The Cat's Meow: A Cat Washer[†] (5 engineers)

Consider a cat washer that would be targeted for use by large commercial pet stores, such as “PetCo.” The cat washer would do more than merely wash a cat. A more inclusive list of objectives would be to design a device that would:

- Wash a cat with water and liquid shampoo.
- Apply up to two post-wash water soluble additives such as conditioner/deodorizer/flea killer.
- Remove loose hair from the cat's coat.
- Dry the cat.

The cat washer should be adaptable to any size of cat and should minimize trauma to the animal. The device should optimally clean all parts of the cat.

The cat washer should be reliable and rugged, as well as inexpensive. It should operate quietly and be easy to clean and maintain. Ideally, the device should operate automatically to minimize any actions required of an operator.

If electricity is needed, the device should operate at 120 V AC. The washer must be safe for both the cat and the operator and should be designed for indoor use.

To be designed, selected, or determined:

1. Design a device that can be used effectively for washing a cat, and for applying post-wash additives. Assuming that tap water is used, does it have to be heated or cooled? What is a “comfortable” water temperature for the animal? Is a storage tank needed for water or shampoo or additives?
2. Specify the fluid/thermal components of the entire apparatus.
3. How large should the unit be? Should it be mounted on a truck or pulled on a trailer?
4. Determine the cost of the cat washer, as well as the number of cats it can wash in a typical 8-hour day.
5. How much should be charged to wash/condition/deodorize/de-flea a cat?

[†]Submitted by Professor Julie Mathis.

50. Amusement Park Water Slide (5 engineers)

It is desired to construct a water slide in an amusement park. People of all ages and body shapes will climb stairs or a ramp to the top of the slide

and ride down on a rubber or rubber-substitute mat. One acre of land can be devoted to the slide and its associated operation (e.g., ticket-selling booth, steps, or ramp).

To be designed, selected, or determined:

1. Slide height, width, path, and material of construction.
 2. Support structure and material of construction, including ground support.
 3. “Catch” basin or method for individuals coming off the slide at the bottom.
 4. Pump size, type, and location.
 5. Piping, piping material, and routing.
 6. Additives required to maintain the water (consider also that water may freeze during winter months).
 7. Mat size, material, color, and number to be ordered.
 8. Layout of the slide, and its mode of operation on the acre of land.
 9. Total cost of the slide, its expected life span, and an estimate of the portion of the cost of a ticket that will cover this cost over the expected lifespan.
-

51. Flash Freezing of Chicken (5 engineers)

Freshly slaughtered chicken are cut up and stored in boxes that hold 40 lbs of chicken (gross weight). The chicken meat in the box must be frozen within 24 hours to meet FDA regulations, but the vendor wants them frozen within 16 hours, so they can be marketed in a shorter time. A blast freezer is to be used for initial freezing, and afterward, the boxes of chicken will be stored in a freezer at -20°F . The boxes have tops so that they can be stacked.

To be designed, selected, or determined:

1. A method for freezing the boxes in the desired time period. Is it better to freeze the boxes as they move on a conveyor, for example, or after they are stacked in a warehouse?
 2. A system that will provide the necessary cooling.
 3. How much cooling is required? A piece of raw chicken at room temperature is placed in a freezer. What is the temperature versus time history that will result? Can this be predicted with equations, or must it be measured?
 4. Materials for construction and design of the device.
 5. Costs of building such a device.
 6. An exact statement of FDA requirements regarding this procedure.
-

52. Automatic Solar Hot Dog Cooker (5 engineers)

Solar cookers that will cook hot dogs have been on the market for some time. These devices consist of a solar reflector that reflects and concentrates energy from the sun. The concentrated energy is directed at a location where the user would place a hot dog. The energy cooks the meat, and within minutes the cook becomes a consumer.

In this project, it is desired to build a modified version of a solar cooker. The solar hot dog cooker is to have an automated feature. The user puts raw hot dogs in one end of the device and removes cooked hot dogs from the other. The cooking rate should be one hot dog every 15 seconds—a maximum speed. (Fifteen seconds is the estimated time needed to put condiments on a hot dog bun.) Furthermore, the apparatus that moves the meat through must also be solar operated. It can contain a battery or other energy storage device, but the device must be charged by solar energy. Finally, the solar cooker must have a method for warming the hot dog buns, and this too must be automated and run using energy from the sun.

To be designed, selected, or determined:

1. Search the literature and review existing designs of solar hot dog cookers.
2. What center temperature is required to be reached before the hot dog is considered “cooked”?
3. What is the required shape for the solar reflector? How long does it take to cook a hot dog with the sun’s energy? What physical dimensions are required for the reflector to cook at the rate required?
4. How will the cooker be supported? Must it be above ground by an appreciable amount?
5. What method is to be used to move the meat through the reflector?
6. How are the buns going to be heated, and to what temperature?
7. Is there a safety problem; i.e., is there a burn danger for the user?
8. Determine the cost of the entire system.

53. Cooling Tower for an Internal Combustion Engine[†] (5 engineers)

An internal combustion engine is set up in a laboratory, and it is necessary during operation of the engine to reject heat to the environment. A common radiator (a cross-flow heat exchanger) is part of the system, and it will reject heat to the air. The power developed by the engine is absorbed by a water brake dynamometer (similar to a pump), in which water is moved under pressure through an elaborate system of vanes, pipes, and

valves. The water will gain energy in the form of heat, and this heat must be rejected safely to the environment. It is proposed to reject this heat by means of a cooling tower.

A cooling tower will cool the water to near-ambient temperature. As indicated in Figure 11.11, a cooling tower is an evaporative cooler that causes the evaporation of a small portion of the water to transfer heat from the system. Cooling towers can be small rooftop units or very large structures.

Cooling towers can be classified in a number of ways: according to size, or according to how air is made to flow through the system duty, or how the air and water flow paths align with respect to each other.

Wet cooling towers transfer heat to the surrounding air by the evaporation of a small fraction of the water flowing through. *Dry cooling towers* contain a heat exchanger in which the water and air are kept separate. Heat is transferred through the exchanger surfaces. In a *natural draft* cooling tower, a tall chimney is used to enhance the movement of a warmed current of air. The air moves due to buoyant effects, as opposed to that in a *mechanical draft* tower where the air is moved by a fan.

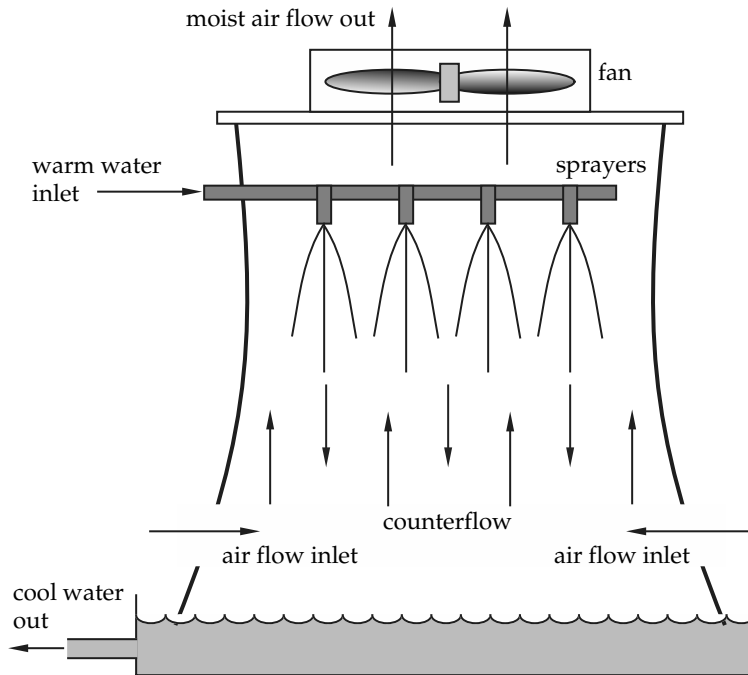


FIGURE 11.11. Sketch of a cooling tower.

To be designed, selected, or determined:

1. Prepare a written description for the nontechnical reader of how a wet cooling tower works.
2. What is “splash fill”?
3. What possible malfunctions are encountered if a cooling tower is used during cold weather? What safety concerns exist where cooling towers are used?
4. Design a mechanical draft, counterflow cooling tower to discharge heat from an internal combustion engine test stand to the environment. The engine specifications will be provided by the instructor.
5. Use commonly available materials wherever possible.
6. Determine the cost of such a device.
7. Is a pump necessary to move the water through the tower? What is the flow rate of the water?
8. Is a fan necessary to move air through the tower? What is the flow rate of the air?

[†]Submitted by Professor John I. Hochstein.

54. Human-Powered Distillation Unit[†] (5 engineers)

The boiling of water can be used to separate it from other substances that can be found mixed or dissolved in the water itself. If the boiled water is then condensed, purified water is obtained. Figure 11.12 shows how such a system might be configured. Water is heated to the boiling temperature in a boiler and vaporizes. Substances that have a lower boiling point remain in the boiler. The steam leaves the boiler and is directed to a cooling device where it condenses. The condensate is purified water, and the process itself is referred to as distillation.

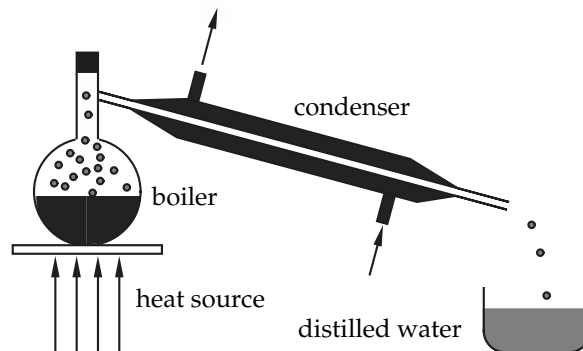


FIGURE 11.12. *Sketch of the distillation process.*

Distillation is an effective method of obtaining purified water, and obviously, it can be done by using any heat source: fire, electricity, solar, or human power. Any pot or container can be used to hold the water while it is being heated. The condensing unit can be one of a variety of devices; for example, a long copper tube bent into a spiral, provided there is something available to condense the steam.

The distillation process removes water from almost any other substance such as heavy metals, poisons, bacteria, and viruses. Substances with lower boiling points will not be removed. These include oils, alcohol, and in general, petroleum products. Also, those substances that do not vaporize remain in the boiler, and so it will need to be cleaned periodically. The subject of this project involves the use of human power as a heat source for boiling the water.

To be designed, selected, or determined:

1. Review a sampling of distillation devices and determine a relationship between the power input required to boil water and the volume of distillate obtained.
2. Design a distillation unit consisting of a boiler and a condenser. The accompanying sketch is of a system that produces batches of distilled water. Instead of a batch process, should the design be of a system that produces distilled water on a continuous basis?
3. Design a human-powered energy source that can be used for boiling the liquid.
4. Determine how much power a human being is capable of producing, and how much of that power will go into the boiling of the liquid.
5. Write an instruction manual for the operation of the device. This is to be written for a nontechnical audience. Pay particular attention to safety issues.
6. Determine the expected volume per unit time of distilled liquid that can be obtained.
7. If the human body is unable to provide the energy required, it would be appropriate to add solar assistance to the design.
8. Determine the cost of the system.

[†]Submitted by Professor John I. Hochstein.

55. Hot Lather Machine (5 engineers)

Shaving is sometimes a chore that can be made more pleasant by the use of heated lather. A device for heating lather is to be designed. The device should be capable of being attached to the top of *any* commercially

available container. Also, the lather should be heated in the device without decreasing (by too much) the volume flow rate from the container.

To be designed, selected, or determined:

1. What is the “typical” flow rate of lather from a container? What pressure exists within a new container, and how does pressure vary with the amount of lather discharged? What size containers are available commercially?
2. What is a “comfortable” temperature for the heated lather? How quickly does heated lather cool down to the ambient temperature? Is it necessary to know the physical properties of lather to design such a device?
3. Is the use of an electrical heater a good way to warm the lather? How much power is required to warm the lather to the “comfortable” temperature?
4. Design the system and produce working drawings that can be used to construct a prototype.
5. Calculate the expected cost (to the consumer) of the device. Who would likely use such a device?

Such devices have been designed and are currently available. Your own design must not infringe on the patent rights of others.

56. Apparatus for Illustrating the First Law of Thermodynamics (5 engineers)

Some companies manufacture and market laboratory equipment for engineering schools. Consider one such company interested in marketing an apparatus for illustrating the first law of thermodynamics by using some common industrial equipment.

It is proposed to construct an experiment in which students can investigate the performance of an air motor and of a gear pump. The analysis of the air motor involves the application of the First Law of Thermodynamics for an ideal gas, while for the gear pump, the First Law is applied to an incompressible fluid.

Figure 11.13 is a sketch of the apparatus. It consists of an air motor, a torque meter/tachometer, and a gear pump. Air from an existing compressor is routed through a pressure regulator to some predetermined pressure p_0 . The air then goes through a flowmeter and on to the air motor. After passing through the air motor, the air is discharged to the atmosphere.

The air-motor power output is monitored by a torque meter/tachometer. The torque meter provides a readout of torque exerted by the air motor and

its rotational speed. This power is used to rotate a gear pump, which pumps water from a tank, through a flowmeter and a valve, and then back to the tank.

The system is appropriately instrumented. Readings of pressure, temperature, and flow rate are obtained so that power calculations can be made.

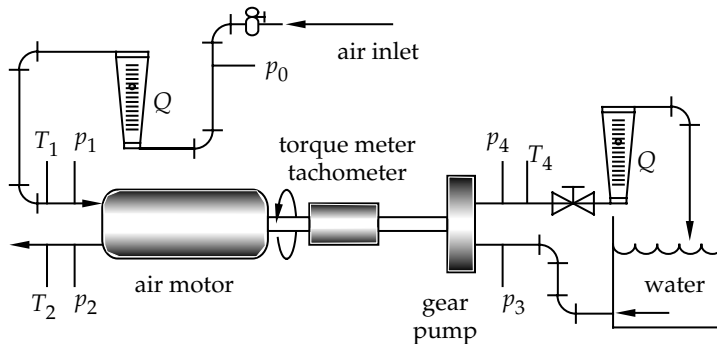


FIGURE 11.13. Sketch of the First Law of Thermodynamics experiment.

To be designed, selected, or determined:

1. Apply the first law of thermodynamics to the air motor and write an equation for the power out in terms of flow rate and enthalpies. Assuming an ideal gas, simplify the equation to obtain power in terms of temperatures. Write an equation for power in terms of torque and rotational speed. In light of these equations, does the sketch show sufficient instrumentation attached to the system in order to make the necessary power calculations? What is the efficiency of the air motor in terms of these power equations?
2. Apply the first law of thermodynamics to the pump and write an equation for the power out in terms of flow rate and energy changes. Assuming an incompressible fluid, simplify the equation to obtain power in terms of changes in pressure, kinetic energy, and potential energy. The power measured at the torque meter is the input power to the pump, written previously in terms of torque and rotational speed. In light of these equations, does the sketch show sufficient instrumentation attached to the system in order to make the necessary power calculations? What is the efficiency of the gear pump in terms of these power equations?
3. Perform a dimensional analysis for the air motor and for the gear pump. What dimensionless groups apply to these turbomachines?
4. Design the apparatus. Then with the necessary instructor approval, secure the components and have the apparatus built.

5. Obtain data from the apparatus, enough to make meaningful calculations. Determine power for each device as well as efficiency. Construct graphs of the dimensionless groups derived earlier.
 6. Keep a record of all purchases. Produce written and oral reports.
-

57. Shower-Head Testing Unit (5 engineers)

Shower heads found in homes or in institutional shower stalls are manufactured to specific standards. However, each design must be tested to be sure that it operates properly.

Pulsating shower heads consist of the same housing as an ordinary shower head. The pulsations are caused by a spinning disk inside just upstream of the outlet holes. The spinning disk effectively plugs $1/3$ to $1/2$ of the holes at any one position. The water streams exiting the unplugged holes can be seen with a strobe light. The streams appear as moving strings of a finite length.

In order to evaluate a design, the shower head must be installed in a testing unit. The testing unit should have several features: ability to install and remove a shower head quickly; a source of hot water; a strobe light for viewing the pulsating streams of water; and a way to control the volume flow rate of water through the shower head.

To be designed, selected, or determined:

1. Determine a comfortable water temperature for a person taking a shower. Also determine how long a typical shower takes.
 2. Obtain at least three pulsating shower heads. Disassemble each and determine how they work, specifically noting the method by which the pulsations are produced.
 3. Determine the flow rate of water through a conventional shower head. What is the water pressure at the shower head?
 4. Design a system for testing the shower heads. This can be a tank containing heated water, a system that heats water on demand, or some other method.
 5. Can the shower head be installed and removed quickly?
 6. Determine the source of hot water, noting the time required and water temperature typically encountered.
 7. Determine how the pulsations will be seen by the naked eye.
 8. Determine the cost of the device.
 9. Write an instruction manual for using the apparatus, which is to be written for a nontechnical audience.
-

58. Ice Skating Rink (5 engineers)

An ice skating rink is to be constructed and is to occupy a floor area of 25 m x 15 m. The skating rink must be removable so that the floor space can be used for other purposes.

To be designed, selected, or determined:

1. The cooling system for producing an ice skating rink.
 2. All components of the system (such as piping), including materials of construction.
 3. The substructure of the floor to support the rink.
 4. A means for draining water when the rink is to be taken down. (Consider that a simple drain may not be sufficient and that a pump might be desirable.)
 5. Total cost of the entire setup, including any special flooring required.
-

59. Analysis of “Energy-Efficient” Methods in Home Construction (5 engineers)

An individual who desires to construct a new home will want to investigate the costs associated with operating it once it is built. Operating costs must be balanced with initial costs to provide an overall cost analysis for the structure. A number of “energy-efficient” devices and methods have become available or recommended in recent years. In this project, cost comparisons are to be made for a number of items to determine the payback period or rate of return on investment. The comparison is between conventional construction techniques and modifications and methods designed to conserve energy. A floor plan of a proposed home and its location will be provided by the instructor. The owner has decided to investigate the modifications listed in Table 11.1.

To be designed, selected, or determined:

1. Make a cost comparison for each of the items listed in Table 11.1.
 2. Determine the additional initial cost associated with each of the proposed modifications.
 3. Determine the reduction in operating cost associated with each proposed modification.
 4. Where the modification provides a cost savings, perform an economic analysis showing the rate of return on the proposed investment.
-

TABLE 11.1. *Modifications to be investigated.*

<u>Conventional</u>		<u>Proposed Modification</u>
All exterior walls made with 2 x 4 wall studs spaced 16 in. center to center filled with 4 in. (nominal) of insulation.	(1)	All exterior walls made with 2 x 6 wall studs spaced 24 in. center to center filled with 6 in. (nominal) of insulation.
Ceiling made with 2 x 10 studs spaced 24 in. center to center filled with 4 in. (nominal) of insulation.	(2)	Ceiling made with 2 x 12 studs spaced 24 in. center to center and filled with 12 in. (nominal) of insulation.
8-ft-tall ceilings.	(3)	9-, 10-, or 12-ft tall ceilings.
Air-cooled central air conditioner for summer and gas heater (in one unit) for winter.	(4)	Water-cooled air conditioner using lake water as a sink for summer and heat pump (in one unit) using lake water as a source for winter.
40-gallon gas water heater.	(5)	"Tankless" gas or electric water heater.
—	(6)	A roof water-spraying system to keep the roof from becoming excessively warm.
—	(7)	Roof vent(s).

60. Earth-Coupled Air Conditioning Unit (5 engineers)

Dwellings are cooled during summer months by an air conditioner. Figure 11.14 is a sketch of a conventional air conditioning unit. The fluid within the system is refrigerant. It leaves the compressor as a superheated vapor and enters the condenser. A fan moves outside air past the condenser, where the air absorbs energy from the refrigerant, causing the refrigerant to condense. The pressure of the refrigerant drops only slightly (due to wall friction). The refrigerant passes through an expansion valve or a capillary tube (throttling devices), after which the refrigerant pressure is greatly decreased. At this point, the liquid refrigerant is quite cold (35°F is not unusual). The refrigerant next passes through the evaporator and absorbs energy from inside air. The refrigerant vaporizes due to this energy (or heat) addition. It also experiences only a slight decrease in pressure (because of friction). The low-pressure, vaporized refrigerant now returns to the compressor and completes another cycle. While we have followed the

path taken by a finite mass of refrigerant, it should be remembered that the process is continuous.

In many conventional units, the condenser is actually a refrigerant-to-air cross-flow heat exchanger. Recently, it has been proposed to replace the condenser with a double pipe heat exchanger in which the refrigerant flows in the inner tube and water flows in the annulus. The water would absorb energy (heat) from the refrigerant. The water is then pumped underground, and by heat transfer to the soil, the water loses the energy it gained from the refrigerant. Figure 11.15a shows the water-cooled system. A number of manufacturers produce and market such systems, commonly referred to as *earth-coupled air conditioners*. They can operate in reverse (by changing the direction of the flow of refrigerant) and be used to heat a dwelling during winter months.

Figure 11.15b shows an alternative configuration for burying the water pipe. The advantage of the setup in Figure 11.15b (over Figure 11.15a) is that the well depth theoretically need not be so great. It is desired to perform an analysis of the system in Figure 11.15a (or b, as assigned).

To be designed, selected, or determined:

1. Pump, size, and material of construction.
2. Pipe or tubing size and material. (Allow at least 25 ft of pipe for each water line from the double pipe heat exchanger to the ground surface.)
3. Depth of well required for the system of Figure 11.14.
4. Diameter and depth of well required if the system of Figure 11.15 is used.

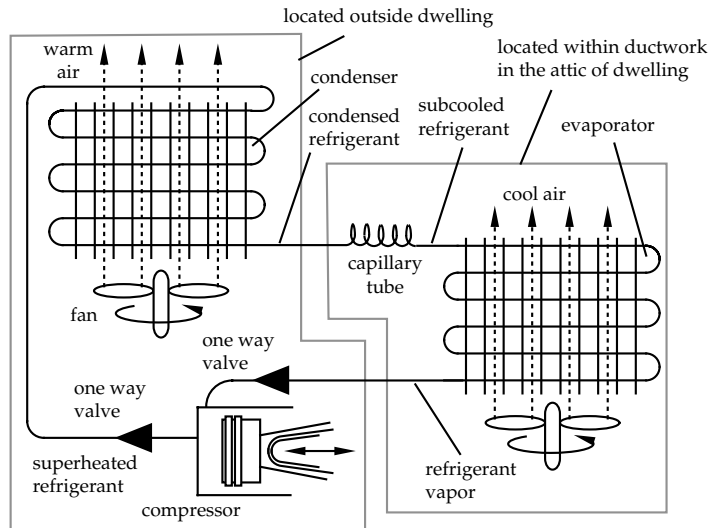


FIGURE 11.14. A conventional air conditioning system.

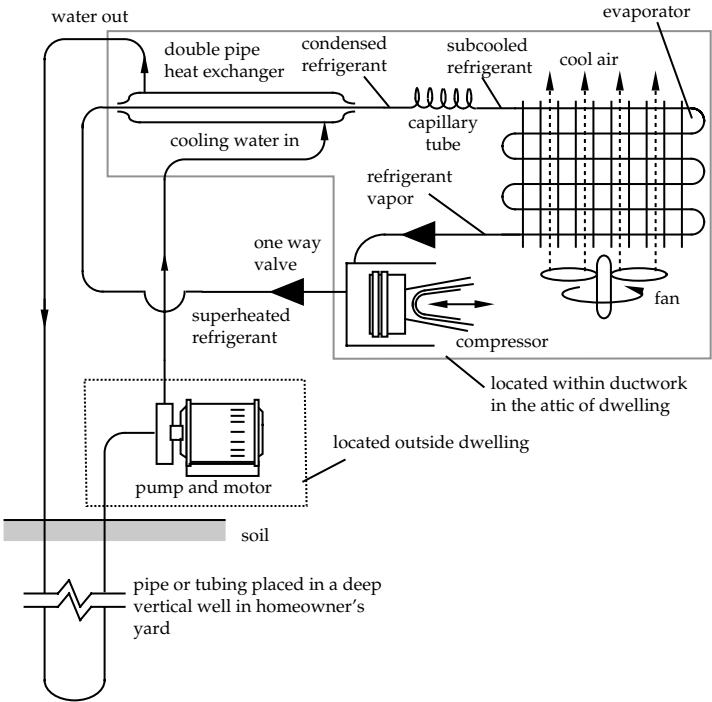


FIGURE 11.15a An earth-coupled air conditioning system.

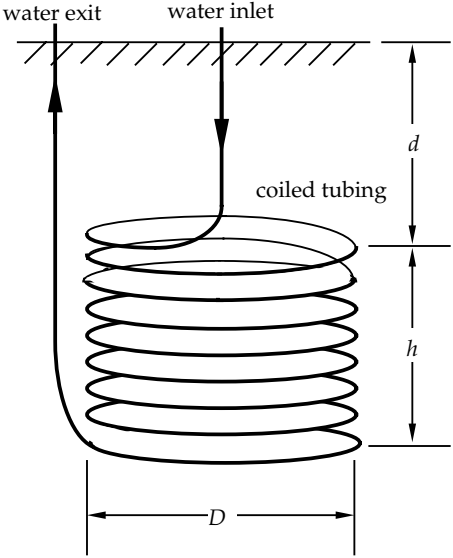


FIGURE 11.15b. Alternative configuration to the deep vertical well.

5. Double pipe heat exchanger size and material of construction. (Use a commercially available one if possible.)
 6. Total cost of system, including exchanger, pump, piping, maintenance, and operation. Compare costs to those encountered in Figure 11.18.
 7. Perform the analysis for a 3-ton air conditioner.
-

61. A Heating System for a Tank Car (5 engineers)

Conventional railroad tank cars are used to transport glucose from a supplier to a company that makes jellies, jams, and preserves. Upon arrival, the glucose in the tank car must be heated to make it easier to pump out. Heating can be done by using steam or electricity. These two methods are compared in this project.

If the glucose is to be heated with steam, then the tank car must be manufactured with tubes inside. When the car is filled with glucose, the tubes will be completely submerged. An external steam source is then connected to the tube inlet at the car, and steam is pumped inside. The tubes must be completely sealed so that steam cannot leak inside.

If the glucose is to be heated electrically, then electric resistance heaters are to be installed in the tank car at the time it is manufactured. The resistance heaters will be submerged within the glucose. An external power supply is then connected to the heaters and the glucose is heated.

To be designed, selected, or determined:

1. A layout for the steam tubes within the car.
 2. A tube diameter.
 3. Materials to be used for the tubing.
 4. Steam flow rate and inlet temperature. (After passing through, should the steam be discharged to atmosphere or collected and reheated?)
 5. Cost of the steam heating system (tubing cost, labor associated with fluid tight installation, and operating costs).
 6. A layout or positioning of the immersion heater(s).
 7. Electric power required.
 8. Cost of electric heating system (heating, labor associated with installation, and operating costs).
 9. The optimum temperature at which glucose should be maintained, as dictated by minimum cost analysis.
 10. Perform all calculations for a conventional 40-ft tank car. Should the tank be insulated? What are its exact dimensions?
-

62. Energy Recovery from an Electrical Generating Facility (5 engineers)

Four 250-HP diesel engines are used to generate electricity at a facility that is located where power from the utility company is very expensive. It may be cost effective for the facility to generate its own power, and so it is desirable to have as little waste as possible. The proposed layout of the system is shown in Figure 11.16.

As indicated, the engine-generator units are spaced 2 m apart, although this spacing can be changed if necessary. All are fed from a common header that brings fuel from a tank located at an unknown (at this time) distance L away.

The exhaust gases from these engines are to be used to heat water to as high a temperature as possible. The water flow rate is that typically supplied by the city, although a pump can be used if a higher flow rate is desirable. Any type of heat exchanger can be used.

It has also been suggested that the waste heat at the radiators be recovered as well. The water-antifreeze mixture that circulates through the four radiators would be used to heat water in one or more heat exchangers, again to as high a temperature as possible. The water being heated at the radiators can be combined in any way with the water being heated by the exhaust gases.

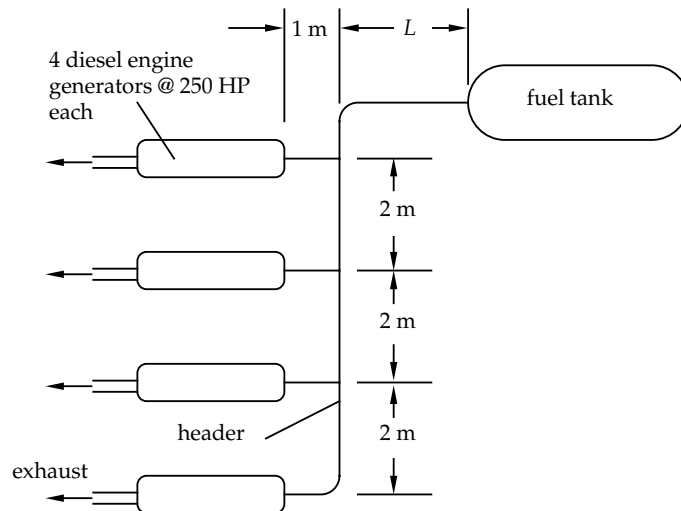


FIGURE 11.16. Layout of diesel engines for the heat recovery project.

To be designed, selected, or determined:

1. What typically is the fuel requirement for a 250-HP diesel engine? What is the power delivered by a 250-HP engine? Design the manifold and fuel inlet line to each engine. Specify the material to be used.
 2. How much of the energy in the fuel is used to generate electricity? How much energy is exhausted from these engines, and at what temperature? How much energy is rejected at the radiators and at what temperature?
 3. The heat available in the exhaust from the four engines can be transferred to water in a parallel or series flow arrangement within one or more heat exchangers. The same can be said of the heat rejected by the engines at their radiators. Select flow arrangements for increased heat recovery and determine an optimum water heating scheme. Keep in mind that the objective is to obtain the highest possible outlet temperature of the water. City water is to be used with or without a pump. What is the typical pressure and flow rate delivered by the city and at what cost?
 4. Calculate the cost of the fuel piping system and the cost of the heat exchangers. Devise a calculation for estimating the savings involved in the heat recovery system.
-

63. Cooling Pond for a Power Plant (5 engineers)

The design of a power plant is in its preliminary stages, and due to environmental concerns, it is desired to use a cooling pond rather than a nearby river to dissipate the heat rejected from the system. A cooling pond is a man-made pond, roughly the size of a small lake, that contains sprayers. The sprayers float on the water surface and spray water upward. A portion of the sprayed water vaporizes, which transfers heat to the air above the pond. Other modes of heat transfer may also be present.

Figure 11.17 shows a plan view of a power plant condenser and a cooling pond. The condenser is the heat source for the water that the cooling pond is to cool. The condenser must condense 30,000 lbm/hr of treated water that is used in the power plant. The water entering the condenser is saturated vapor at 0.01 MPa. The ambient air temperature varies over the course of one year from a typical low of 35°F to 95°F. For whatever air temperature exists, it is desired to cool the water to as low a temperature as possible with the cooling pond.

The cooling pond can be of any sensible and feasible dimensions. It is to be located no closer than 200 yards from the condenser.

To be designed, selected, or determined:

1. What is the lowest temperature we can cool the water to with a cooling

pond? Is it a function of ground temperature or of air temperature or both?

2. Size the condenser; that is, design a unit that can condense the required 30,000 lbm/hr. Using your design, determine the amount of heat that the pond must dissipate by analyzing the condenser over a range of cooling water inlet temperatures. Calculate the expected temperature of the cooling water after it leaves the condenser and goes to the cooling pond.

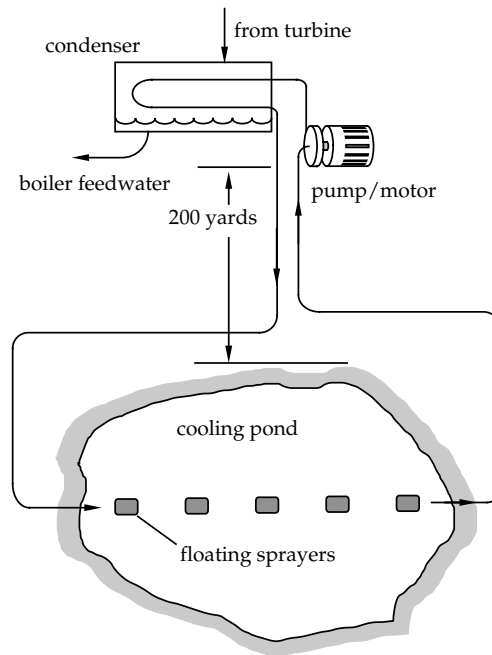


FIGURE 11.17. *Sketch of a cooling pond installation.*

3. What are the dimensions of the cooling pond (length x width x depth)? Should it be shaped like, say, a swimming pool (e.g., rectangular)? How many sprayers are required? Is the amount of water vaporized while operating so significant that makeup water must be supplied to the pool? Should the water be treated before it goes to the condenser?
4. Lay out the piping system to and from the pond. What pipe size should be used, and what material should it be made of? The sketch shows one pump. Are two pumps necessary? Size the pump(s).
5. Is anything to be gained by using a finned tube for the flow line from the condenser to the pond? Should the lines be buried or supported above the ground? Should there be two lines to the pond?

6. Evaluate the total cost, including digging out the pond.

64. Heating of a Portable Review Platform[†] (5 engineers)

A parade review platform is a seating structure where people sit and watch a passing parade. When the weather is cold, however, remaining approximately motionless (e.g., sitting or standing and waving) does little to raise one's comfort level. A company that manufactures portable platforms wishes to market an optional heating system with their product.

Figure 11.18 shows a profile view of a four-tiered seating platform, but the modular design allows for any number of tiers, up to and including eight. The seats are benches without backs made of extruded aluminum 8 ft long. (The 8-ft length used is due to storage and handling considerations, which are functions of portability.) A typical platform contains two 8-ft sections with a 3-ft-wide aisle between them. The platform seats are supported by steel framework that is skirted by 1/2-in. plywood. The flooring for each bench seat is 3/4-in. plywood, and the difference in height between two adjacent seats is 16 in. A canvas canopy to protect people from rain is available as an option as well. It is suggested that a finned tube be placed somewhere underneath each bench seat and that warm fluid (such as steam or hot water) be circulated through the tubes. This would warm the fun-seekers' feet, and rising warm air would provide further comfort. (This method will be referred to as the **natural convection arrangement**.)

Another suggestion is to use a warm-air forced convection system. A heat source would provide warm air produced, for example, within a "box." The warmed air would be moved to tubes located underneath each bench seat. The tubes have holes in them (i.e., perforated at selected locations) so that warm air is delivered through these holes to the spectators. A fan would move the warm air to these tubes. (This method will be referred to as the **forced convection arrangement**.)

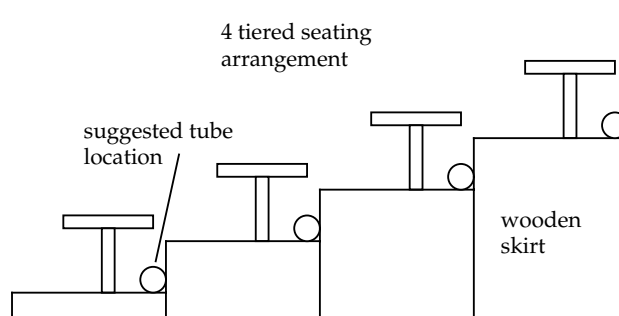


FIGURE 11.18. Profile view of a four-tiered seating platform.

To be designed, selected, or determined:

Preliminaries

1. Determine the heat load for this platform. How many people can sit comfortably in such a seating platform? How much heat should be supplied? Define comfort with regard to the amount of heat required. Should the amount of heat delivered be a function of ambient temperature?

Natural Convection Arrangement

1. Design a system of finned tubes to be placed on the platform, specifying their precise locations. Often the fins on such tubes are made of very thin aluminum, which is quite delicate. Should the use of this type of fin be avoided?
2. Select a heat source (e.g., a steam generator, a hot water heater, or an electric heater), and determine what fluid is to be circulated through the tubes. Is a pump necessary in such a system? If so, size an appropriate pump. The heating system must be located in the space underneath the platform, concealed by the platform skirt.
3. Must a canopy be used with this design? What about insulation? How much is needed, and where should it be placed?
4. Determine the cost of this system.

Forced Convection Arrangement

1. Design a system of perforated tubes to be placed on the platform, specifying their precise locations. Where should these perforations be located; i.e., how many holes and what size?
2. Select an appropriate air heating method, and determine the flow rate of air to be discharged through the tubes. Size the box where the heat source is to be located. Obviously a fan is necessary in such a system; size an appropriate fan. Where should the fan be located with respect to the box? The heating system must be located in the space underneath the platform, concealed by the platform skirt.
3. Must a canopy be used with this design? What about insulation? How much is needed, and where should it be placed?
4. Determine the cost of this system.

Conclusion

Which heating method do you recommend, and why? Or do you recommend abandoning this idea?

[†]Suggested by Professor Emeritus Prasanna V. Kadaba, Georgia Institute of Technology.

hazardous to the environment? What are the properties (density, viscosity, etc.) of these inks? Are all inks solvent based or are there water based inks on the market?

3. Working only with black ink, size the pipelines in Figure 11.19b, using the optimum diameter to minimize costs. Must a special metal be used for the pipe? Determine the flow rate in each of the interior lines.
4. Are there other modifications that should be considered and analyzed?

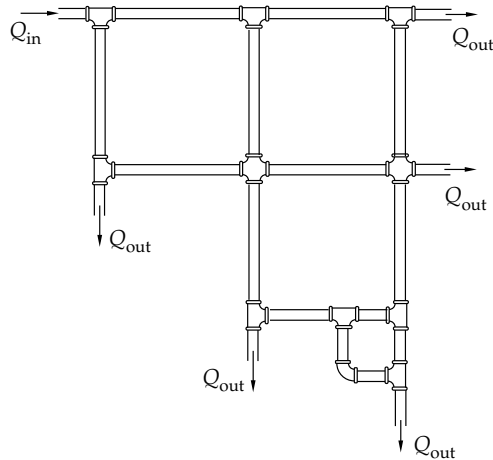


FIGURE 11.19b. *Plan view of modified piping network for conveying ink.*

5. What is the power that has to be supplied at the inlet? What pressure is required?
6. In continuously moving the ink through the network, the ink temperature is raised by about 10°C. Size or select a heat exchanger that will transfer the heat to water, whose inlet temperature is 25°C. Select heat exchanger type, flow rates of both fluids required, and predict outlet temperatures.

66. Steam and Air Supply Systems (5 engineers)

A small manufacturing plant has a number of batch and continuous operations that require steam and air to be supplied to various locations within the building. Figure 11.20 shows the plan view of the building. A steam generator and a compressor are located within an isolated “mechanical room.” Pipelines from each of these sources must be laid out so that steam and air are distributed to where they are needed. The locations are as follows:

Location	Fluid	Condition	Minimum flow rate
1	Dry steam	125°C	0.01 kg/s
2	Dry steam	125°C	0.005 kg/s
3	Dry steam	125°C	0.007 kg/s
4	Dry air	30 psig	0.004 kg/s
5	Dry air	25 psig	0.004 kg/s
6	Dry air	30 psig	0.003 kg/s
7	Dry air	30 psig	0.002 kg/s

These locations are along walls and should have valves where the lines terminate. Management does not want any walls to be drilled through, so all pipelines must go around all walls. The ceiling is 12 ft high throughout the structure. Suggested piping layouts for both fluids are provided in Figures 11.21 and 11.22.

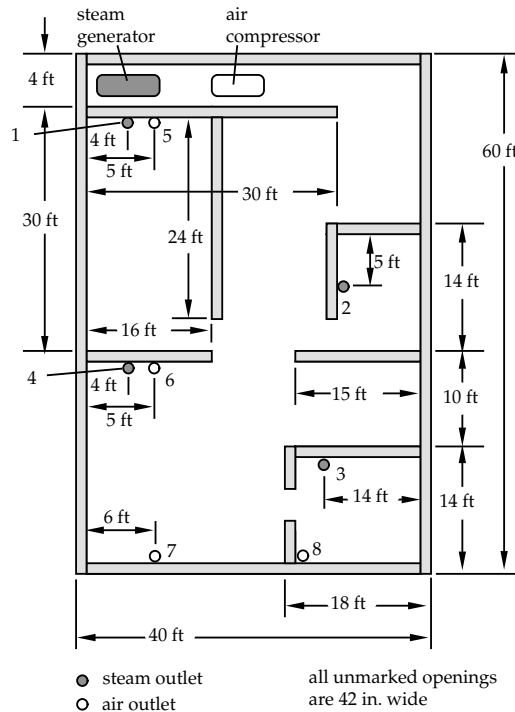


FIGURE 11.20. Floor plan of a small manufacturing facility.

To be designed, selected, or determined:

1. Size the steam generator and the flow lines to each location. What is a “steam trap”? Does this installation need one? Are there other devices that should be installed in a steam line?

2. Size the compressor and the flow lines to each location. What is a pressure regulator? How is air “dried” (the specifications require dry air)? Are there other devices that should be installed in an air line? How are the air and steam lines going to be distinguished?
3. How is each pipeline to be supported? Are wall hangers preferable to ceiling hangers?
4. What is the cost of the entire installation for both the steam and air?
5. Size the pipelines according to optimum diameter requirements, if possible.

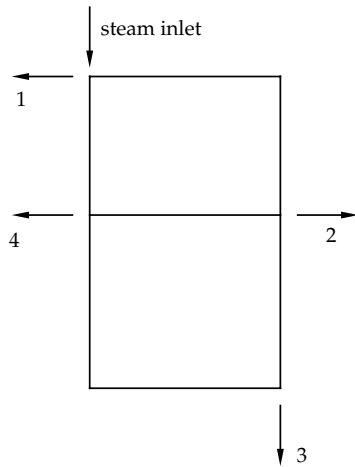


FIGURE 11.21. *Steam piping network*

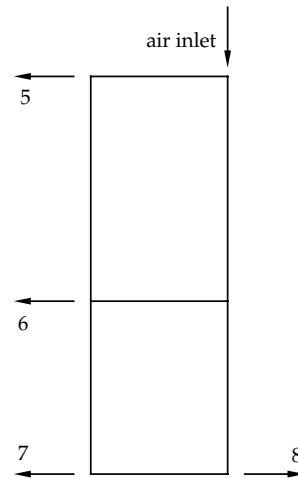


FIGURE 11.22. *Air piping network*

Appendix Tables

TABLE A.1. *Prefixes applicable to units in SI.*

TABLE A.2. *Factors for converting to SI units, listed by physical quantity.*

TABLE A.3. *Miscellaneous conversion factors.*

TABLE A.4. *Temperature conversions.*

TABLE A.5. *The Greek alphabet.*

TABLE B.1. *Properties of liquids at room temperature and pressure.*

TABLE B.2. *Properties of saturated liquids: unused **engine oil**.*

TABLE B.3. *Properties of saturated liquids: **ethylene glycol**
 $C_2H_4(OH)_2$.*

TABLE B.4. *Properties of saturated liquids: **glycerin** $C_3H_5(OH)_3$.*

TABLE B.5. *Properties of saturated liquids: **water** H_2O .*

TABLE C.1. *Physical properties of gases at room temperature and pressure.*

TABLE C.2. *Properties of gases at atmospheric pressure (101.3 kPa = 14.7 psia). **Air** (gas constant = 286.8 J/(kg·K) = 53.3 ft·lbf/(lbm·°R); $\gamma = C_p/C_v = 1.4$)*

TABLE C.3. *Properties of gases at atmospheric pressure (101.3 kPa = 14.7 psia). **Carbon Dioxide** (gas constant = 188.9 J/(kg·K) = 35.11 ft·lbf/(lbm·°R); $\gamma = C_p/C_v = 1.3$)*

TABLE C.4. *Properties of gases at atmospheric pressure (101.3 kPa = 14.7 psia). **Nitrogen** (gas constant = 296.8 J/(kg·K) = 55.16 ft·lbf/(lbm·°R); $\gamma = C_p/C_v = 1.4$)*

TABLE C.5. *Properties of gases at atmospheric pressure (101.3 kPa = 14.7 psia). **Oxygen** (gas constant = 260 J/(kg·K) = 48.3 ft·lbf/(lbm·°R); $\gamma = C_p/C_v = 1.4$)*

TABLE C.6. *Properties of gases at atmospheric pressure (101.3 kPa = 14.7 psia). **Water Vapor or Steam** (gas constant = 461.5 J/(kg·K) = 85.78 ft·lbf/(lbm·°R); $\gamma = C_p/C_v = 1.333$)*

TABLE D.1. *Pipe dimensions.*

TABLE D.2. *Dimensions of seamless copper tubing.*

TABLE A.1. *Prefixes applicable to units in SI.*

Factor by which unit is multiplied	Prefix	Symbol
10^{12}	tera	T
10^9	giga	G
10^6	mega	M
10^3	kilo	k
10^2	hecto	h
10	deka	da
10^{-1}	deci	d
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p
10^{-15}	femto	f
10^{-18}	atto	a

Source: E. A. Mechtly, NASA SP 7012, 1973.

TABLE A.2. *Factors for converting to SI units, listed by physical quantity.*

To convert from	To	Multiply by
<i>Acceleration (L/T²)</i>		
foot/second ²	*meter/second ²	3.048×10^{-1}
inch/second ²	*meter/second ²	2.54×10^{-2}
<i>Area (L²)</i>		
foot ²	*meter ²	$9.290\,304 \times 10^{-2}$
inch ²	*meter ²	6.4516×10^{-4}
mile ² (U.S. statute)	*meter ²	$2.589\,988\,110\,336 \times 10^6$
yard ²	*meter ²	$8.361\,273\,6 \times 10^{-1}$
<i>Convection Coefficient or Film Conductance [F·L/(T·L²·t)]</i>		
BTU/hour·ft ² ·°R	W/(m ² ·K)	5.678×10^0
<i>Density (M/L³)</i>		
gram/centimeter ³	*kilogram/meter ³	1.00×10^3
lbm/inch ³	kilogram/meter ³	$2.767\,990\,5 \times 10^4$
lbm/foot ³	kilogram/meter ³	$1.601\,846\,3 \times 10^1$
slug/inch ³	kilogram/meter ³	$8.912\,929\,4 \times 10^5$
slug/foot ³	kilogram/meter ³	$5.153\,79 \times 10^2$

* Indicates an exact conversion. Otherwise, conversions are approximate representations of definitions or are results of physical measurements. Source: E. A. Mechtly, NASA SP 7012, 1973.

TABLE A.2. *Factors for converting to SI units, continued.*

To convert from	To	Multiply by
<i>Energy (F·L)</i>		
BTU	joule	$1.054\,350 \times 10^3$
calorie	*joule	4.184×10^0
foot-pound force	joule	$1.355\,817\,9 \times 10^0$
kilocalorie	*joule	4.184×10^3
kilowatt hour	*joule	3.60×10^6
watt hour	*joule	3.60×10^3
<i>Energy/(area·time) or Heat Flux [F·L/(L²·T)]</i>		
BTU/(ft ² second)	watt/meter ²	$1.134\,893\,1 \times 10^4$
BTU/(foot ² minute)	watt/meter ²	$1.891\,488\,5 \times 10^2$
BTU/(foot ² hour)	watt/meter ²	$3.152\,480\,8 \times 10^0$
BTU/(inch ² second)	watt/meter ²	$1.634\,246\,2 \times 10^6$
calorie/(cm ² minute)	watt/meter ²	$6.973\,333\,3 \times 10^2$
watt/centimeter ²	*watt/meter ²	1.00×10^4
<i>Force (F)</i>		
lbf	*newton	$4.448\,221\,615\,260\,5 \times 10^0$
<i>Length (L)</i>		
foot	*meter	3.048×10^{-1}
inch	*meter	2.54×10^{-2}
mile (U.S. statute)	*meter	$1.609\,344 \times 10^3$
mile (U.S. nautical)	*meter	1.852×10^3
yard	*meter	9.144×10^{-1}
<i>Mass (M)</i>		
gram	*kilogram	1.00×10^{-3}
lbm (pound mass)	*kilogram	$4.535\,923\,7 \times 10^{-1}$
slug	kilogram	$1.459\,390\,29 \times 10^1$
<i>Power or Heat Flow Rate (F·L/T)</i>		
BTU/second	watt	$1.054\,350\,264\,488 \times 10^3$
BTU/hour	watt	$2.928\,750\,7 \times 10^{-1}$
calorie/second	*watt	4.184×10^0
calorie/minute	watt	$6.973\,333\,3 \times 10^{-2}$
foot lbf/hour	watt	$3.766\,161\,0 \times 10^{-4}$
foot lbf/minute	watt	$2.259\,696\,6 \times 10^{-2}$
foot lbf/second	watt	$1.355\,817\,9 \times 10^0$
horsepower	watt	$7.456\,998\,7 \times 10^2$
kilocalorie/minute	watt	$6.973\,333\,3 \times 10^1$

* Indicates an exact conversion. Otherwise, conversions are approximate representations of definitions or are results of physical measurements. Source: E. A. Mechtly, NASA SP 7012, 1973.

TABLE A.2. *Factors for converting to SI units, continued.*

To convert from	To	Multiply by
<i>Pressure (F/L²)</i>		
atmosphere	*newton/meter ²	1.013 25 × 10 ⁵
bar	*newton/meter ²	1.00 × 10 ⁵
centimeter Hg (0°C)	*newton/meter ²	1.333 22 × 10 ³ *
centimeter H ₂ O (4°C)	newton/meter ²	9.806 38 × 10 ¹
dyne/centimeter ²	*newton/meter ²	1.00 × 10 ¹
foot of H ₂ O (39.2°F)	newton/meter ²	2.988 98 × 10 ³
inch of Hg (32°F)	newton/meter ²	3.386 389 × 10 ³
inch of Hg (60°F)	newton/meter ²	3.376 85 × 10 ³
inch of water (39.2°F)	newton/meter ²	2.490 82 × 10 ²
inch of water (60°F)	newton/meter ²	2.488 4 × 10 ²
lbf/foot ²	newton/meter ²	4.788 025 8 × 10 ¹
lbf/inch ² (psi)	newton/meter ²	6.894 757 2 × 10 ³
pascal	*newton/meter ²	1.00 × 10 ⁰
torr (0°C)	newton/meter ²	1.333 22 × 10 ²
<i>Specific Heat Capacity [F·L/(M·t)]</i>		
BTU/(lbm·°F)	joule/(kilogram·K)	4.187 × 10 ³
<i>Speed or Velocity (L/T)</i>		
foot/hour	meter/second	8.466 666 6 × 10 ⁻⁵
foot/minute	*meter/second	5.08 × 10 ⁻³
foot/second	*meter/second	3.048 × 10 ⁻¹
inch/second	*meter/second	2.54 × 10 ⁻¹
kilometer/hour	meter/second	2.777 777 8 × 10 ⁻¹
knot (international)	meter/second	5.144 444 444 × 10 ⁻¹
mile/hour	*meter/second	4.470 4 × 10 ⁻¹
mile/minute	*meter/second	2.682 24 × 10 ¹
mile/second	*meter/second	1.609 344 × 10 ³
<i>Temperature (t)</i>		
Celsius	Kelvin	$t_K = t_C + 273.15$
Fahrenheit	Kelvin	$t_K = (5/9)(t_F + 459.67)$
Fahrenheit	Celsius	$t_C = (5/9)(t_F - 32)$
Rankine	Kelvin	$t_K = (5/9)t_R$
<i>Thermal Conductivity [F·L/(T·L·t)]</i>		
BTU/(hour·foot·°R)	watt/(meter·K)	1.731 × 10 ⁰
<i>Thermal Diffusivity (L²/T)</i>		
foot ² /second	meter ² /second	9.29 × 10 ⁻²
foot ² /hour	meter ² /second	2.581 × 10 ⁻⁵

* Indicates an exact conversion. Otherwise, conversions are approximate representations of definitions or are results of physical measurements. Source: E. A. Mechtly, NASA SP 7012, 1973.

TABLE A.2. *Factors for converting to SI units, continued.*

To convert from	To	Multiply by
<i>Thermal Resistance</i> [T·t/(F·L)]		
hour·°R/BTU	K/watt	1.895 8 × 10 ⁰
<i>Time</i> (T)		
day	*second	8.64 × 10 ⁴
hour	*second	3.60 × 10 ³
minute	*second	6.00 × 10 ¹
month	*second	2.628 × 10 ⁶
year	*second	3.153 6 × 10 ⁷
<i>Viscosity–kinematic</i> (L ² /T)		
centistoke	*meter ² /second	1.00 × 10 ⁻⁶
stoke	*meter ² /second	1.00 × 10 ⁻⁴
foot ² /second	*meter ² /second	9.290 304 × 10 ⁻²
<i>Viscosity–absolute</i> (F·T/L ²)		
centipoise	*newton-second/meter ²	1.00 × 10 ⁻³
lbm/(foot-second)	newton-second/meter ²	1.488 163 9 × 10 ⁰
lbf-second/foot ²	newton-second/meter ²	4.788 025 8 × 10 ¹
poise	*newton-second/meter ²	1.00 × 10 ⁻¹
<i>Volume</i> (L ³)		
acre-foot	meter ³	1.233 481 9 × 10 ⁻³
barrel (42 gallons)	meter ³	1.589 873 × 10 ⁻¹
cup	*meter ³	2.365 882 365 × 10 ⁻⁴
foot ³	*meter ³	2.831 684 659 2 × 10 ⁻²
gallon (U.S. dry)	*meter ³	4.404 883 770 86 × 10 ⁻³
gallon (U.S. liquid)	*meter ³	3.785 411 784 × 10 ⁻³
inch ³	*meter ³	1.638 706 4 × 10 ⁻⁵
liter	*meter ³	1.00 × 10 ⁻³
ounce (U.S. fluid)	*meter ³	2.957 352 956 25 × 10 ⁻⁵
pint (U.S. dry)	*meter ³	5.506 104 713 575 × 10 ⁻⁴
pint (U.S. liquid)	*meter ³	4.731 764 73 × 10 ⁻⁴
quart (U.S. dry)	*meter ³	1.101 220 942 715 × 10 ⁻³
quart (U.S. liquid)	meter ³	9.463 529 5 × 10 ⁻⁴
tablespoon	*meter ³	1.478 676 478 125 × 10 ⁻⁵
teaspoon	*meter ³	4.928 921 593 75 × 10 ⁻⁶
yard ³	*meter ³	7.645 548 579 84 × 10 ⁻¹

* Indicates an exact conversion. Otherwise, conversions are approximate representations of definitions or are results of physical measurements. Source: E. A. Mechty, NASA SP 7012, 1973.

TABLE A.3. *Miscellaneous conversion factors.*

To convert from	To	Multiply by
acres	square feet	4.356×10^4
acres	square miles	1.562×10^{-2}
atmospheres	cm of Hg	7.6×10
BTU	horsepower-hours	3.931×10^{-4}
BTU	kilowatt-hours	2.928×10^{-4}
BTU/hr	watts	2.928×10^{-1}
calories	BTU	3.968×10^{-3}
centimeters	feet	3.281×10^{-2}
centimeters	inches	3.937×10^{-1}
chain	inches	7.92×10^2
gallons per minute	cubic feet per second	2.228×10^{-3}

TABLE A.4. *Temperature conversions.*

K	°C	°R	°F
250	-23.15	450	-10
260	-13.15	468	8
270	-3.15	486	26
280	6.85	504	44
290	16.85	522	62
295	21.85	531	71
300	26.85	540	80
310	36.85	558	98
320	46.85	576	116
330	56.85	594	134
340	66.85	612	152
350	76.85	630	170
360	86.85	648	188
370	96.85	666	206
380	106.85	684	224
390	116.85	702	242
400	126.85	720	260
410	136.85	738	278
420	146.85	756	296

TABLE A.5. *The Greek alphabet.*

English Spelling	Greek Capital Letters	Greek Lowercase Letters	English Spelling	Greek Capital Letters	Greek Lowercase Letters
Alpha	A	α	Nu	N	ν
Beta	B	β	Xi	Ξ	ξ
Gamma	Γ	γ	Omicron	O	o
Delta	Δ	δ	Pi	Π	π
Epsilon	E	ϵ	Rho	P	ρ
Zeta	Z	ζ	Sigma	Σ	σ
Eta	H	η	Tau	T	τ
Theta	Θ	θ	Upsilon	Y	υ
Iota	I	ι	Phi	ϑ	ϕ
Kappa	K	κ	Chi	X	χ
Lambda	Λ	λ	Psi	Ψ	ψ
Mu	M	μ	Omega	Ω	ω

TABLE B.1. *Properties of liquids at room temperature and pressure.*

Fluid	Specific Gravity	Absolute Viscosity		Surface Tension (N/m) x 10 ³
		(N·s/m ²) x 10 ³	(lbf·s/ft ²) x 10 ⁵	
Acetone	0.787	0.316	0.659	23.1
Benzene	0.876	0.601	1.26	28.18
Carbon disulfide	1.265	0.36	0.752	32.33
Carbon tetrachloride	1.59	0.91	1.90	26.3
Castor oil	0.96	650	1356	—
Chloroform	1.47	0.53	1.11	27.14
Decane	0.728	0.859	1.79	23.43
Ether	0.715	0.223	0.466	16.42
Ethyl alcohol	0.787	1.095	2.29	22.33
Ethylene glycol	1.100	16.2	33.8	48.2
Glycerine	1.263	950	1983	63.0
Heptane	0.681	0.376	0.786	19.9
Hexane	0.657	0.297	0.622	18.0
Kerosene	0.823	1.64	3.42	—
Linseed oil	0.93	33.1	69.0	—
Mercury	13.6	1.53	3.20	484
Methyl alcohol	0.789	0.56	1.17	22.2
Octane	0.701	0.51	1.07	21.14
Propane	0.495	0.11	0.23	6.6
Propyl alcohol	0.802	1.92	4.01	23.5
Propylene	0.516	0.09	0.19	7.0
Propylene glycol	0.968	42	88	36.3
Turpentine	0.87	1.375	2.87	—
Water	1.00	0.89	1.9	71.97

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Notes on reading the table:

Density ρ = (Sp.Gr. x 1.94) slug/ft³ = (Sp.Gr. x 1 000) kg/m³ = (Sp.Gr. x 62.4) lbfm/ft³

Viscosity of acetone: $\mu \times 10^3 = 0.316 \text{ N}\cdot\text{s}/\text{m}^2$; $\mu = 0.316 \times 10^{-3} \text{ N}\cdot\text{s}/\text{m}^2$

TABLE B.2. *Properties of saturated liquids: unused engine oil.*

Temp °C	°F	Specific		Kinematic Viscosity ν m ² /s x 10 ⁴ ft ² /s x 10 ⁴	Thermal		Thermal Diffusivity α m ² /s x 10 ⁸ ft ² /hr x 10 ³	Prandtl Number Pr			
		Specific Gravity	Heat C_p $\frac{\text{J}}{\text{kg}\cdot\text{K}}$ $\frac{\text{BTU}}{\text{lbm}\cdot^\circ\text{R}}$		Conductivity k $\frac{\text{W}}{\text{m}\cdot\text{K}}$ $\frac{\text{BTU}}{\text{hr}\cdot\text{ft}\cdot^\circ\text{R}}$						
0	32	0.899	1 796	0.429	42.8	46.1	0.147	0.085	9.11	3.53	47 100
20	68	0.888	1 880	0.449	9.0	9.7	0.145	0.084	8.72	3.38	10 400
40	104	0.876	1 964	0.469	2.4	2.6	0.144	0.083	8.34	3.23	2 870
60	140	0.864	2 047	0.489	0.839	0.903	0.140	0.081	8.00	3.10	1 050
80	176	0.852	2 131	0.509	0.375	0.404	0.138	0.080	7.69	2.98	490
100	212	0.840	2 219	0.530	0.203	0.219	0.137	0.079	7.38	2.86	276
120	248	0.828	2 307	0.551	0.124	0.133	0.135	0.078	7.10	2.75	175
140	284	0.816	2 395	0.572	0.080	0.086	0.133	0.077	6.86	2.66	116
160	320	0.805	2 483	0.593	0.056	0.060	0.132	0.076	6.63	2.57	84
$\beta = 0.70 \times 10^{-3}/\text{K} = 0.39 \times 10^{-3}/^\circ\text{R}$										Example of reading values: kinematic viscosity at 0°C is $\nu = 42.8 \times 10^{-4} \text{ m}^2/\text{s}$	

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TABLE B.3. *Properties of saturated liquids: ethylene glycol C₂H₄(OH)₂.*

Temp °C	°F	Specific		Kinematic		Thermal		Thermal Diffusivity α m ² /s × 10 ⁸	Prandtl Number Pr
		Specific Gravity	Heat C_p J/kg·K	Viscosity ν m ² /s × 10 ⁶	Viscosity ν ft ² /s × 10 ⁶	Conductivity k W/m·K	Conductivity k BTU/hr·ft·°R		
0	32	1.130	2 294	57.53	61.92	0.242	0.140	9.34	615
20	68	1.116	2 382	19.18	20.64	0.249	0.144	9.39	204
40	104	1.101	2 474	8.69	9.35	0.256	0.148	9.39	93
60	140	1.087	2 562	4.75	5.11	0.260	0.150	9.32	51
80	176	1.077	2 650	2.98	3.21	0.261	0.151	9.21	32.4
100	212	1.058	2 742	2.03	2.18	0.263	0.152	9.08	22.4

$\beta = 0.65 \times 10^{-3} / \text{K} = 0.36 \times 10^{-3} / ^\circ\text{R}$ Example of reading values: kinematic viscosity at 0°C is $\nu = 57.53 \times 10^{-6} \text{ m}^2/\text{s}$

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TABLE B.4. *Properties of saturated liquids: **glycerin** $C_3H_5(OH)_3$.*

Temp °C	°F	Specific		Kinematic Viscosity ν $m^2/s \times 10^3$	Thermal		Thermal Diffusivity α $m^2/s \times 10^8$	Prandtl Number Pr
		Specific Gravity	Heat C_p $J/kg \cdot K$		Conductivity k $W/m \cdot K$	$BTU/hr \cdot ft \cdot ^\circ R$		
0	32	1.276	2 261	8.31	0.282	0.163	9.83	84 700
10	50	1.270	2 319	3.00	0.284	0.164	9.65	31 000
20	68	1.264	2 386	1.18	0.286	0.156	9.47	12 500
30	86	1.258	2 445	0.50	0.286	0.165	9.29	5 380
40	104	1.252	2 512	0.22	0.286	0.165	9.14	2 450
50	122	1.244	2 583	0.15	0.287	0.166	8.93	1 630
$\beta = 0.50 \times 10^{-3}/K = 0.28 \times 10^{-3}/^\circ R$				Example of reading values: kinematic viscosity at $0^\circ C$ is $\nu = 8.31 \times 10^{-3} m^2/s$				

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TABLE B.5. *Properties of saturated liquids: water H₂O.*

Temp °C	°F	Specific		Kinematic Viscosity ν $\text{m}^2/\text{s} \times 10^7$	Thermal		Thermal Diffusivity α $\text{m}^2/\text{s} \times 10^7$	Prandtl Number Pr			
		Specific Gravity	Heat C_p $\frac{\text{J}}{\text{kg} \cdot \text{K}}$		Conductivity k $\frac{\text{W}}{\text{m} \cdot \text{K}}$	$\frac{\text{BTU}}{\text{hr} \cdot \text{ft} \cdot ^\circ\text{R}}$					
0	32	1.002	4 217	1.0074	17.88	19.25	0.552	0.319	1.308	5.07	13.6
20	68	1.000	4 181	0.9988	10.06	10.83	0.597	0.345	1.430	5.54	7.02
40	104	0.994	4 178	0.9980	6.58	7.08	0.628	0.363	1.512	5.86	4.34
60	140	0.985	4 184	0.9994	4.78	5.14	0.651	0.376	1.554	6.02	3.02
80	176	0.974	4 196	1.0023	3.64	3.92	0.668	0.386	1.636	6.34	2.22
100	212	0.960	4 216	1.0070	2.94	3.16	0.680	0.393	1.680	6.51	1.74
120	248	0.945	4 250	1.015	2.47	2.66	0.685	0.396	1.708	6.62	1.446
140	284	0.928	4 283	1.023	2.14	2.30	0.684	0.395	1.724	6.68	1.241
160	320	0.909	4 342	1.037	1.90	2.04	0.670	0.393	1.729	6.70	1.099
180	356	0.889	4 417	1.055	1.73	1.86	0.675	0.390	1.724	6.68	1.004
200	392	0.866	4 505	1.076	1.60	1.72	0.665	0.384	1.706	6.61	0.937
220	428	0.842	4 610	1.101	1.50	1.61	0.572	0.377	1.680	6.51	0.891
240	464	0.815	4 756	1.136	1.43	1.54	0.635	0.367	1.639	6.35	0.871
260	500	0.785	4 949	1.182	1.37	1.48	0.611	0.353	1.577	6.11	0.874
280	537	0.752	5 208	1.244	1.35	1.45	0.580	0.335	1.481	5.74	0.910
300	572	0.714	5 728	1 368	1.35	1.45	0.540	0.312	1.324	5.13	1.109

$\beta = 0.18 \times 10^{-3} / \text{K} = 0.10 \times 10^{-3} / ^\circ\text{R}$
Example of reading values: kinematic viscosity at 0°C is $\nu = 17.88 \times 10^{-7} \text{ m}^2/\text{s}$

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TABLE C.1. *Physical properties of gases at room temperature and pressure.*

Gas	Density ρ		Specific Heat C_p		Kinematic Viscosity ν		Gas Constant R	
	kg/m ³	lbm/ft ³	$\frac{\text{J}}{\text{kg}\cdot\text{K}}$	$\frac{\text{BTU}}{\text{lbm}\cdot^\circ\text{R}}$	m ² /s $\times 10^6$	ft ² /s $\times 10^5$	$\frac{\text{J}}{\text{kg}\cdot\text{K}}$	$\frac{\text{ft}\cdot\text{lb}_f}{\text{lbm}\cdot^\circ\text{R}}$ $\gamma = C_p/C_v$
Air	1.177	0.0735	1 005.7	0.240	15.68	16.88	0.026 24	0.01516
Carbon dioxide	1.797	0.1122	871	0.208	8.321	8.957	0.016 572	0.009575
Nitrogen	1.142	0.0713	1 040.8	0.2486	15.63	16.82	0.026 20	0.01514
Oxygen	1.300	0.0812	920.3	0.2198	15.86	17.07	0.026 76	0.01546

Example of reading values: kinematic viscosity of air is $\nu = 15.68 \times 10^{-6} \text{ m}^2/\text{s}$

MW = molecular weight (engineering units) = molecular mass (SI units)

Gas constant $R = R_u/MW$; $R_u = 1545 \text{ ft}\cdot\text{lb}_f/\text{lbmol}\cdot^\circ\text{R} = 49,700 \text{ ft}\cdot\text{lb}_f/\text{slugmol}\cdot^\circ\text{R} = 8\,312 \text{ N}\cdot\text{m}/(\text{mol}\cdot\text{K})$

1 BTU = 778 ft·lb_f

Source: Data taken from a number of sources; see Bibliography at the end of the text.

TABLE C.2. *Properties of gases at atmospheric pressure (101.3 kPa = 14.7 psia).**Air (gas constant = 286.8 J/(kg·K) = 53.3 ft·lbf/(lbm·°R); $\gamma = C_p/C_v = 1.4$)*

Temp K	°R	Density ρ kg/m ³	Specific		Kinematic Viscosity ν m ² /s x 10 ⁶	Thermal		Thermal Diffusivity α m ² /s x 10 ⁴	Prandtl Number Pr
			Density ρ lbm/ft ³	Heat C_p J/kg·K		Heat C_p BTU/lbm·°R	Conductivity k W/m·K		
100	180	3.601	0.225	1 026.6	1.923	2.070	0.009 246	0.005342	0.0969
150	270	3.268	0.148	1 009.9	4.343	4.674	0.013 735	0.007936	0.223
200	360	1.768	0.110	1 006.1	7.490	8.062	0.018 09	0.01045	0.394
250	450	1.413	0.0882	1 005.3	9.49	10.2	0.022 27	0.01287	0.510
300	540	1.177	0.0735	1 005.7	15.68	16.88	0.026 24	0.01516	0.859
350	630	0.998	0.0623	1 009.0	20.76	22.35	0.030 03	0.01735	1.156
400	720	0.883	0.0551	1 104.0	25.90	27.88	0.033 65	0.01944	1.457
450	810	0.783	0.0489	1 020.7	28.86	31.06	0.037 07	0.02142	1.636
500	900	0.705	0.0440	1 029.5	37.90	40.80	0.040 38	0.02333	2.156
550	990	0.642	0.0401	1 039.2	44.34	47.73	0.043 60	0.02519	2.531
600	1080	0.589	0.0367	1 055.1	51.34	55.26	0.046 59	0.02692	2.911
650	1170	0.543	0.0339	1 063.5	58.51	62.98	0.049 53	0.02862	3.324
700	1260	0.503	0.0314	1 075.2	66.25	71.31	0.052 30	0.03022	3.748
750	1350	0.471	0.0294	1 085.6	73.91	79.56	0.055 09	0.03183	4.175
800	1440	0.441	0.0275	1 097.8	82.29	88.58	0.057 79	0.03339	4.631
850	1530	0.415	0.0259	1 109.5	90.75	97.68	0.060 28	0.03483	5.075
900	1620	0.393	0.0245	1 121.2	99.3	107.0	0.062 79	0.03628	5.530
950	1710	0.372	0.0232	1 132.1	108.2	116.5	0.065 25	0.03770	6.010
1 000	1800	0.352	0.0220	1 141.7	117.8	126.8	0.067 52	0.03901	6.502
1 100	1980	0.320	0.0120	1 160	138.6	149.2	0.073 2	0.0423	7.630

Example of reading values: kinematic viscosity at 100 K is $\nu = 1.923 \times 10^{-6} \text{ m}^2/\text{s}$

TABLE C.2. Properties of gases at atmospheric pressure (101.3 kPa = 14.7 psia), continued.*Air* (gas constant = 286.8 J/(kg·K) = 53.3 ft·lbf/(lbm·°R); $\gamma = C_p/C_v = 1.4$)

Temp K	°R	Density ρ kg/m ³	Specific		Kinematic Viscosity ν m ² /s x 10 ⁶	Thermal			Prandtl Number Pr		
			$\frac{J}{kg \cdot K}$	Heat C_p BTU lbm·°R		Conductivity k $\frac{BTU}{hr \cdot ft \cdot ^\circ R}$	Diffusivity α m ² /s x 10 ⁴				
1 200	2160	0.295	0.0184	1 179	0.282	159.1	171.3	0.078 2	2.251	8.723	0.707
1 300	2340	0.271	0.0169	1 197	0.286	182.1	196.0	0.083 7	2.583	10.01	0.705
1 400	2520	0.252	0.0157	1 214	0.290	205.5	221.2	0.089 1	2.920	11.32	0.705
1 500	2700	0.236	0.0147	1 230	0.294	229.1	246.6	0.094 6	3.262	12.64	0.705
1 600	2880	0.221	0.0138	1 248	0.298	254.5	273.9	0.100	3.609	13.98	0.705
1 700	3060	0.208	0.0130	1 267	0.303	280.5	301.9	0.105	3.977	15.41	0.705
1 800	3240	0.197	0.0123	1 287	0.307	308.1	331.6	0.111	4.379	16.97	0.704
1 900	3420	0.186	0.0115	1 309	0.313	338.5	364.4	0.117	4.811	18.64	0.704
2 000	3600	0.176	0.0110	1 338	0.320	369.0	397.2	0.124	5.260	20.38	0.702
2 100	3780	0.168	0.0105	1 372	0.328	399.6	430.1	0.131	5.715	22.15	0.700
2 200	3960	0.160	0.0100	1 419	0.339	432.6	465.6	0.139	6.120	23.72	0.707
2 300	4140	0.154	0.00955	1 482	0.354	464.0	499.4	0.149	6.540	25.34	0.710
2 400	4320	0.146	0.00905	1 574	0.376	504.0	542.5	0.161	7.020	27.20	0.718
2 500	4500	0.139	0.00868	1 688	0.403	543.5	585.0	0.175	7.441	28.83	0.730

Example of reading values: kinematic viscosity at 100 K is $\nu = 159.1 \times 10^{-6} \text{ m}^2/\text{s}$

Example of reading values: kinematic viscosity at 100 K is $\nu = 159.1 \times 10^{-6} \text{ m}^2/\text{s}$

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TABLE C.3. Properties of gases at atmospheric pressure (101.3 kPa = 14.7 psia).
Carbon Dioxide (gas constant = 188.9 J/(kg·K) = 35.11 ft·lbf/(lbm·°R); $\gamma = C_p/C_v = 1.3$)

Temp K	°R	Density ρ kg/m ³	Specific		Kinematic Viscosity ν m ² /s x 10 ⁶	Thermal		Thermal Diffusivity α ft ² /hr	Prandtl Number Pr			
			Heat C_p $\frac{\text{J}}{\text{kg}\cdot\text{K}}$	$\frac{\text{BTU}}{\text{lbm}\cdot\text{°R}}$		Conductivity k $\frac{\text{W}}{\text{m}\cdot\text{K}}$	$\frac{\text{BTU}}{\text{hr}\cdot\text{ft}\cdot\text{°R}}$					
220	396	2.473	0.1544	783	0.187	4.490	4.833	0.010 805	0.006243	0.059 20	0.2294	0.818
250	450	2.165	0.1352	804	0.192	5.813	6.257	0.012 884	0.007444	0.074 01	0.2868	0.793
300	540	1.797	0.1122	871	0.208	8.321	8.957	0.016 572	0.009575	0.105 88	0.4103	0.770
350	630	1.536	0.0959	900	0.215	11.19	12.05	0.020 47	0.01183	0.148 08	0.5738	0.755
400	720	1.342	0.0838	942	0.225	14.39	15.49	0.024 61	0.01422	0.194 63	0.7542	0.738
450	810	1.191	0.0744	980	0.234	17.90	19.27	0.028 97	0.01674	0.248 13	0.9615	0.721
500	900	1.073	0.067	1 013	0.242	21.67	23.33	0.033 52	0.01937	0.308 4	1.195	0.702
550	990	0.973	0.0608	1 047	0.250	25.74	27.72	0.038 21	0.02208	0.375 0	1.453	0.685
600	1080	0.893	0.0558	1 076	0.257	30.02	32.31	0.043 11	0.02491	0.448 3	1.737	0.668

Example of reading values: kinematic viscosity at 100 K is $\nu = 4.490 \times 10^{-6} \text{ m}^2/\text{s}$

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TABLE C.4. *Properties of gases at atmospheric pressure (101.3 kPa = 14.7 psia).***Nitrogen** (*gas constant* = 296.8 J/(kg·K) = 55.16 ft·lbf/(lbm·°R); $\gamma = C_p/C_v = 1.4$)

Temp K	°R	Density ρ kg/m ³	Specific		Kinematic Viscosity ν m ² /s × 10 ⁶	Thermal		Thermal Diffusivity α m ² /s × 10 ⁴	Prandtl Number Pr
			Density ρ lbm/ft ³	Heat C_p J/kg·K		Heat C_p BTU/lbm·°R	Conductivity k W/m·K		
100	180	3.480	0.2173	1 072.2	0.2571	1.971	2.122	0.009 450	0.005460
200	360	1.710	0.1068	1 042.9	0.2491	7.568	8.146	0.018 24	0.01054
300	540	1.142	0.0713	1 040.8	0.2486	15.63	16.82	0.026 20	0.01514
400	720	0.853	0.0533	1 045.9	0.2498	25.74	27.71	0.033 35	0.01927
500	900	0.682	0.0426	1 055.5	0.2521	37.66	40.54	0.039 84	0.02302
600	1080	0.568	0.0355	1 075.6	0.2569	51.19	55.10	0.045 80	0.02646
700	1260	0.493	0.0308	1 096.9	0.2620	65.13	70.10	0.051 23	0.02960
800	1440	0.427	0.0267	1 122.5	0.2681	81.46	87.68	0.056 09	0.03241
900	1620	0.379	0.0237	1 146.4	0.2738	91.06	98.02	0.060 70	0.03507
1 000	1800	0.341	0.0213	1 167.7	0.2789	117.2	126.2	0.064 75	0.03741
1 100	1980	0.310	0.0194	1 185.7	0.2382	136.0	146.4	0.068 50	0.03958
1 200	2160	0.285	0.0178	1 203.7	0.2875	156.1	168.0	0.071 84	0.04151

Example of reading values: kinematic viscosity at 100 K is $\nu = 1.971 \times 10^{-6}$ m²/sSource: *Data taken from Analysis of Heat and Mass Transfer, by E. R. G. Eckert and R. M. Drake, Jr., Taylor & Francis Group—Hemisphere Publishing Corp., New York, © 1987. Used with permission of the publisher.*

TABLE C.5. Properties of gases at atmospheric pressure (101.3 kPa = 14.7 psia).
Oxygen (gas constant = 260 J/(kg·K) = 48.3 ft·lbf/(lbm·°R); $\gamma = C_p/C_v = 1.4$)

Temp K	Temp °R	Specific		Density ρ kg/m ³	Heat C_p $\frac{\text{J}}{\text{kg}\cdot\text{K}}$	Kinematic		Thermal		Thermal Diffusivity α m ² /s x 10 ⁴	Prandtl Number Pr	
		$\frac{\text{J}}{\text{kg}\cdot\text{K}}$	$\frac{\text{BTU}}{\text{lbm}\cdot^{\circ}\text{R}}$			Viscosity ν m ² /s x 10 ⁶	$\frac{W}{\text{m}\cdot\text{K}}$	Conductivity k $\frac{\text{BTU}}{\text{hr}\cdot\text{ft}\cdot^{\circ}\text{R}}$				
100	180	3.992	0.2492	947.9	0.2264	1.946	2.095	0.009 03	0.00522	0.023 876	0.09252	0.815
150	270	2.619	0.1635	917.8	0.2192	4.387	4.722	0.013 67	0.00790	0.056 88	0.2204	0.773
200	360	1.955	0.1221	913.1	0.2181	7.593	8.173	0.018 24	0.01054	0.102.14	0.3958	0.745
250	450	1.561	0.0975	915.7	0.2187	11.45	12.32	0.022 59	0.01305	0.157 94	0.6120	0.725
300	540	1.300	0.0812	920.3	0.2198	15.86	17.07	0.026 76	0.01546	0.223 53	0.8662	0.709
350	630	1.113	0.0695	929.1	0.2219	20.80	22.39	0.030 70	0.01774	0.296 8	1.150	0.702
400	720	0.975	0.0609	942.0	0.2250	26.18	28.18	0.034 61	0.02000	0.376 8	1.460	0.695
450	810	0.868	0.0542	956.7	0.2285	31.99	34.43	0.038 28	0.02212	0.460 9	1.786	0.694
500	900	0.780	0.0487	972.2	0.2322	38.34	41.27	0.041 73	0.02411	0.550 2	2.132	0.697
550	990	0.709	0.0443	988.1	0.2360	45.05	48.49	0.045 17	0.02610	0.644 1	2.496	0.700
600	1080	0.650	0.0406	1 004.4	0.2399	52.15	56.13	0.048 82	0.02792	0.739 9	2.867	0.704

Example of reading values: kinematic viscosity at 100 K is $\nu = 1.946 \times 10^{-6} \text{ m}^2/\text{s}$

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TABLE C.6. Properties of gases at atmospheric pressure (101.3 kPa = 14.7 psia).
Water Vapor or Steam (gas constant = 461.5 J/(kg·K) = 85.78 ft·lbf/(lbm·°R); $\gamma = C_p/C_v = 1.333$)

Temp K	°R	Density ρ kg/m ³	Specific		Kinematic Viscosity ν m ² /s x 10 ⁵	Thermal		Thermal Diffusivity α ft ² /hr	Prandtl Number Pr			
			Heat C_p $\frac{\text{J}}{\text{kg}\cdot\text{K}}$	$\frac{\text{BTU}}{\text{lbm}\cdot\text{°R}}$		Conductivity k $\frac{\text{W}}{\text{m}\cdot\text{K}}$	$\frac{\text{BTU}}{\text{hr}\cdot\text{ft}\cdot\text{°R}}$					
380	684	0.586	0.0366	2 060	0.492	2.16	2.33	0.024 6	0.0142	0.293 6	0.789	1.060
400	720	0.554	0.0346	2 014	0.481	2.42	2.61	0.026 1	0.0151	0.233 8	0.906	1.040
450	810	0.490	0.0306	1 980	0.473	3.11	3.35	0.029 9	0.0173	0.307	1.19	1.010
500	900	0.440	0.0275	1 985	0.474	3.86	4.16	0.033 9	0.0196	0.386	1.50	0.996
550	990	0.400	0.0250	1 997	0.477	4.70	5.06	0.037 9	0.0219	0.475	1.84	0.991
600	1080	0.365	0.0228	2 026	0.484	5.66	6.09	0.042 2	0.0244	0.573	2.22	0.986
650	1170	0.338	0.0211	2 056	0.491	6.64	7.15	0.046 4	0.0268	0.666	2.58	0.995
700	1260	0.314	0.0196	2 085	0.498	7.72	8.31	0.050 5	0.0292	0.772	2.99	1.000
750	1350	0.293	0.0183	2 119	0.506	8.88	9.56	0.054 9	0.0317	0.883	3.42	1.005
800	1440	0.274	0.0171	2 152	0.514	10.20	10.98	0.059 2	0.0342	1.001	3.88	1.010
850	1530	0.258	0.0161	2 186	0.522	11.52	12.40	0.063 7	0.0368	1.130	4.38	1.019

Example of reading values: kinematic viscosity at 380 K is $\nu = 2.16 \times 10^{-5}$ m²/s

Source: Data taken from a number of references.

TABLE D.1 *Pipe dimensions.*

Nominal Diameter	Outside Diameter in. (ft)	Diameter cm	Schedule	Inside Diameter ft	Diameter cm	Flow Area ft ²	Area cm ²
1/8	0.405 (0.03375)	1.029	40 (std) 80 (xs)	0.02242 0.01792	0.683 0.547	0.0003947 0.0002522	0.366 4 0.235 0
1/4	0.540 (0.045)	1.372	40 (std) 80 (xs)	0.03033 0.02517	0.924 0.768	0.0007227 0.0004974	0.670 6 0.463 2
3/8	0.675 (0.05625)	1.714	40 (std) 80 (xs)	0.04108 0.03525	1.252 1.074	0.001326 0.0009759	1.233 9 0.905 9
1/2	0.840 (0.070)	2.134	40 (std) 80 (xs) 160 (xxs)	0.05183 0.04550 0.03867 0.02100	1.580 1.386 1.178 0.640	0.002110 0.001626 0.001174 0.0003464	1.961 1.508 1.090 0.321 7
3/4	1.050 (0.0875)	2.667	40 (std) 80 (xs) 160 (xxs)	0.06867 0.06183 0.05100 0.03617	2.093 1.883 1.555 1.103	0.003703 0.003003 0.002043 0.001027	3.441 2.785 1.898 0.955 5
1	1.315 (0.1095)	3.340	40 (std) 80 (xs) 160 (xxs)	0.08742 0.07975 0.06792 0.04992	2.664 2.430 2.070 1.522	0.006002 0.004995 0.003623 0.001957	5.574 5.083 3.365 1.815
1 ¹ / ₄	1.660 (0.1383)	4.216	40 (std) 80 (xs) 160 (xxs)	0.1150 0.1065 0.09667 0.07467	3.504 3.246 2.946 2.276	0.01039 0.008908 0.007339 0.004379	9.643 8.275 6.816 4.069
1 ¹ / ₂	1.900 (0.1583)	4.826	40 (std) 80 (xs) 160 (xxs)	0.1342 0.1250 0.1115 0.09167	4.090 3.810 3.398 2.794	0.01414 0.01227 0.009764 0.007700	13.13 11.40 9.068 6.131

Notes: std implies standard; xs is extra strong; xxs is double extra strong.

TABLE D.1 continued. *Pipe dimensions.*

Nominal Diameter	Outside Diameter in. (ft)	cm	Schedule	Inside Diameter ft	cm	Flow Area ft ²	cm ²
2	2.375 (0.1979)	6.034	40 (std)	0.1723	5.252	0.02330	21.66
			80 (xs)	0.1616	4.926	0.02051	19.06
			160	0.1406	4.286	0.01552	14.43
			(xxs)	0.1253	3.820	0.01232	11.46
2 ¹ / ₂	2.875 (0.2396)	7.303	40 (std)	0.2058	6.271	0.03325	30.89
			80 (xs)	0.1936	5.901	0.02943	27.35
			160	0.1771	5.397	0.02463	22.88
			(xxs)	0.1476	4.499	0.01711	15.90
3	3.500 (0.2917)	8.890	40 (std)	0.2557	7.792	0.05134	47.69
			80 (xs)	0.2417	7.366	0.04587	42.61
			160	0.2187	6.664	0.03755	34.88
			(xxs)	0.1917	5.842	0.02885	26.80
3 ¹ / ₂	4.000 (0.3333)	10.16	40 (std)	0.2957	9.012	0.06866	63.79
			80 (xs)	0.2803	8.544	0.06172	57.33
4	4.500 (0.375)	11.43	40 (std)	0.3355	10.23	0.08841	82.19
			80 (xs)	0.3198	9.718	0.07984	74.17
			120	0.3020	9.204	0.07163	66.54
			160	0.2865	8.732	0.06447	59.88
			(xxs)	0.2626	8.006	0.05419	50.34
5	5.563 (0.4636)	14.13	40 (std)	0.4206	12.82	0.1389	129.10
			80 (xs)	0.4011	12.22	0.1263	117.30
			120	0.3803	11.59	0.1136	105.50
			160	0.3594	10.95	0.1015	94.17
			(xxs)	0.3386	10.32	0.09004	83.65
6	6.625 (0.5521)	16.83	40 (std)	0.5054	15.41	0.2006	186.50
			80 (xs)	0.4801	14.64	0.1810	168.30
			120	0.4584	13.98	0.1650	153.50
			160	0.4823	13.18	0.1467	136.40
			(xxs)	0.4081	12.44	0.1308	121.50

Notes: std implies standard; xs is extra strong; xxs is double extra strong.

TABLE D.1 continued. *Pipe dimensions.*

Nominal Diameter	Outside Diameter in. (ft)	Diameter cm	Schedule	Inside Diameter ft	Diameter cm	Flow Area ft ²	Area cm ²
8	8.625 (0.7188)	21.91	20	0.6771	20.64	0.3601	334.60
			30	0.6726	20.50	0.3553	330.10
			40 (std)	0.6651	20.27	0.3474	322.70
			60	0.6511	19.85	0.3329	309.50
			80 (xs)	0.6354	19.37	0.3171	294.70
			100	0.6198	18.89	0.3017	280.30
			120	0.5989	18.26	0.2817	261.90
			140	0.5834	17.79	0.2673	248.60
			(xxs)	0.5729	17.46	0.2578	239.40
			160	0.5678	17.31	0.2532	235.30
10	10.750 (0.8958)	27.31	20	0.8542	26.04	0.5730	332.60
			30	0.8446	25.75	0.5604	520.80
			40 (std)	0.8350	25.46	0.5476	509.10
			60 (xs)	0.8125	24.77	0.5185	481.90
			80	0.7968	24.29	0.4987	463.40
			100	0.7760	23.66	0.4730	439.70
			120	0.7552	23.02	0.4470	416.20
			140 (xxs)	0.7292	22.23	0.4176	388.10
			160	0.7083	21.59	0.3941	366.10
12	12.750 (1.058)	32.39	20	1.021	31.12	0.8185	760.60
			30	1.008	30.71	0.7972	740.71
			(std)	1.000	30.48	0.7854	729.70
			40	0.9948	30.33	0.773	722.50
			(xs)	0.9792	29.85	0.7530	699.80
			60	0.9688	29.53	0.7372	684.90
			80	0.9478	28.89	0.7056	655.50
			100	0.9218	28.10	0.6674	620.20
			120 (xxs)	0.8958	27.31	0.6303	585.80
			140	0.8750	26.67	0.6013	558.60
			160	0.8438	25.72	0.5592	519.60
14	14.000 (1.1667)	35.57	30 (std)	1.104	33.65	0.9575	889.30
			160	0.9323	28.42	0.6827	634.40

Notes: std implies standard; xs is extra strong; xxs is double extra strong.

TABLE D.1 continued. *Pipe dimensions.*

Nominal Diameter	Outside Diameter in. (ft)	cm	Schedule	Inside Diameter ft	cm	Flow Area ft ²	cm ²
16	16.000 (1.333)	40.64	30 (std) 160	1.271 1.068	38.73 32.54	1.268 0.8953	1 178.00 831.60
18	18.000 (1.500)	45.72	(std) 160	1.438 1.203	43.81 36.67	1.623 1.137	1 507.00 1 056.00
20	20.000 (1.6667)	50.80	20 (std) 160	1.604 1.339	48.89 40.80	2.021 1.407	1 877.00 1 307.00
22	22.000 (1.8333)	55.88	20 (std) 160	1.771 1.479	53.97 45.08	2.463 1.718	2 288.00 1 596.00
24	24.000 (2.00)	60.96	20 (std) 160	1.938 1.609	59.05 49.05	2.948 2.034	2 739.00 1 890.00
26	26.000 (2.167)	66.04	(std)	2.104	64.13	3.477	3 230.00
28	28.000 (2.333)	71.12	(std)	2.271	69.21	4.050	3 762.00
30	30.000 (2.500)	76.20	(std)	2.438	74.29	4.666	4 335.00
32	32.000 (2.667)	81.28	(std)	2.604	79.34	5.326	4 944.00
34	34.000 (2.833)	86.36	(std)	2.771	84.45	6.030	5 601.00
36	36.000 (3.000)	91.44	(std)	2.938	89.53	6.777	6 295.00
38	38.000 (3.167)	96.52	—	3.104	94.61	7.568	7 030.00
40	40.000 (3.333)	101.6	—	3.271	99.69	8.403	7 805.00

Notes: std implies standard; xs is extra strong; xxs is double extra strong.

Source: *Dimensions in English units obtained from ANSI B36.10–79, American National Standard Wrought Steel and Wrought Iron Pipes.* Reprinted by permission of the publisher, The American Society of Mechanical Engineers.

TABLE D.2.*Dimensions of seamless copper tubing.*

Standard Size	Outside Diameter in. (ft)	Diameter cm	Type	Inside Diameter ft	Diameter cm	Flow ft ²	Area cm ²
1/4	0.375 (0.03125)	0.953	K L	0.02542 0.02625	0.775 0.801	0.0005074 0.0005412	0.471 7 0.503 9
3/8	0.500 (0.04167)	1.270	K L M	0.03350 0.03583 0.03750	1.022 1.092 1.142	0.0008814 0.001008 0.001104	0.820 3 0.936 6 1.024
1/2	0.625 (0.05208)	1.588	K L M	0.04392 0.04542 0.04742	1.340 1.384 1.446	0.001515 0.001620 0.001766	1.410 1.505 1.642
5/8	0.750 (0.0625)	1.905	K L	0.05433 0.05550	1.657 1.691	0.002319 0.002419	2.156 2.246
3/4	0.875 (0.0729)	2.222	K L M	0.06208 0.06542 0.06758	1.892 1.994 2.060	0.003027 0.003361 0.003587	2.811 3.123 3.333
1	1.125 (0.09375)	2.858	K L M	0.08292 0.08542 0.08792	2.528 2.604 2.680	0.005400 0.005730 0.006071	5.019 5.326 5.641
1 ¹ / ₄	1.375 (0.1146)	3.493	K L M	0.1038 0.1054 0.1076	3.163 3.213 3.279	0.008454 0.008728 0.009090	7.858 8.108 8.444
1 ¹ / ₂	1.625 (0.1354)	4.128	K L M	0.1234 0.1254 0.1273	3.762 3.824 3.880	0.01196 0.01235 0.01272	11.12 11.48 11.82
2	2.125 (0.1771)	5.398	K L M	0.1633 0.1654 0.1674	4.976 5.042 5.102	0.02093 0.02149 0.02201	11.95 19.97 20.44
2 ¹ / ₂	2.625 (0.21875)	6.668	K L M	0.2029 0.2054 0.2079	6.186 6.262 6.338	0.03234 0.03314 0.03395	30.05 30.80 40.17

TABLE D.2.continued. *Dimensions of seamless copper tubing.*

Standard Size	Outside Diameter in. (ft)	cm	Type	Inside Diameter ft	cm	Flow Area ft ²	cm ²
3	3.125 (0.2604)	7.938	K	0.2423	7.384	0.04609	42.82
			L	0.2454	7.480	0.04730	43.94
			M	0.2484	7.572	0.04847	45.03
$\frac{3}{2}$	3.625 (0.3021)	9.208	K	0.2821	8.598	0.06249	58.06
			L	0.2854	8.700	0.06398	59.45
			M	0.2883	8.786	0.06523	60.63
4	4.125 (0.34375)	10.48	K	0.3214	9.800	0.08114	75.43
			L	0.3254	9.922	0.08317	77.32
			M	0.3279	9.998	0.08445	78.51
5	5.125 (0.4271)	13.02	K	0.4004	12.21	0.1259	117.10
			L	0.4063	12.38	0.1296	120.50
			M	0.4089	12.47	0.1313	112.10
6	6.125 (0.5104)	15.56	K	0.4784	14.58	0.1798	167.00
			L	0.4871	14.85	0.1863	173.20
			M	0.4901	14.93	0.1886	175.30
8	8.125 (0.6771)	20.64	K	0.6319	19.26	0.3136	291.50
			L	0.6438	19.62	0.3255	302.50
			M	0.6488	19.78	0.3306	307.20
10	10.125 (0.84375)	25.72	K	0.7874	24.00	0.4870	452.50
			L	0.8021	24.45	0.5053	469.50
			M	0.8084	24.64	0.5133	476.80
12	12.125 (1.010)	30.80	K	0.9429	28.74	0.6983	648.80
			L	0.9638	29.38	0.7295	677.90
			M	0.9681	29.51	0.7361	684.00

Note: Type K is for underground service and general plumbing; type L is for interior plumbing; type M is for use only with soldered fittings.

Source: *Dimensions in English units obtained from ANSI/ASTM B88-58 Standard Specifications for Seamless Copper Water Tube*. Copyright ASTM. Reprinted with permission.

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