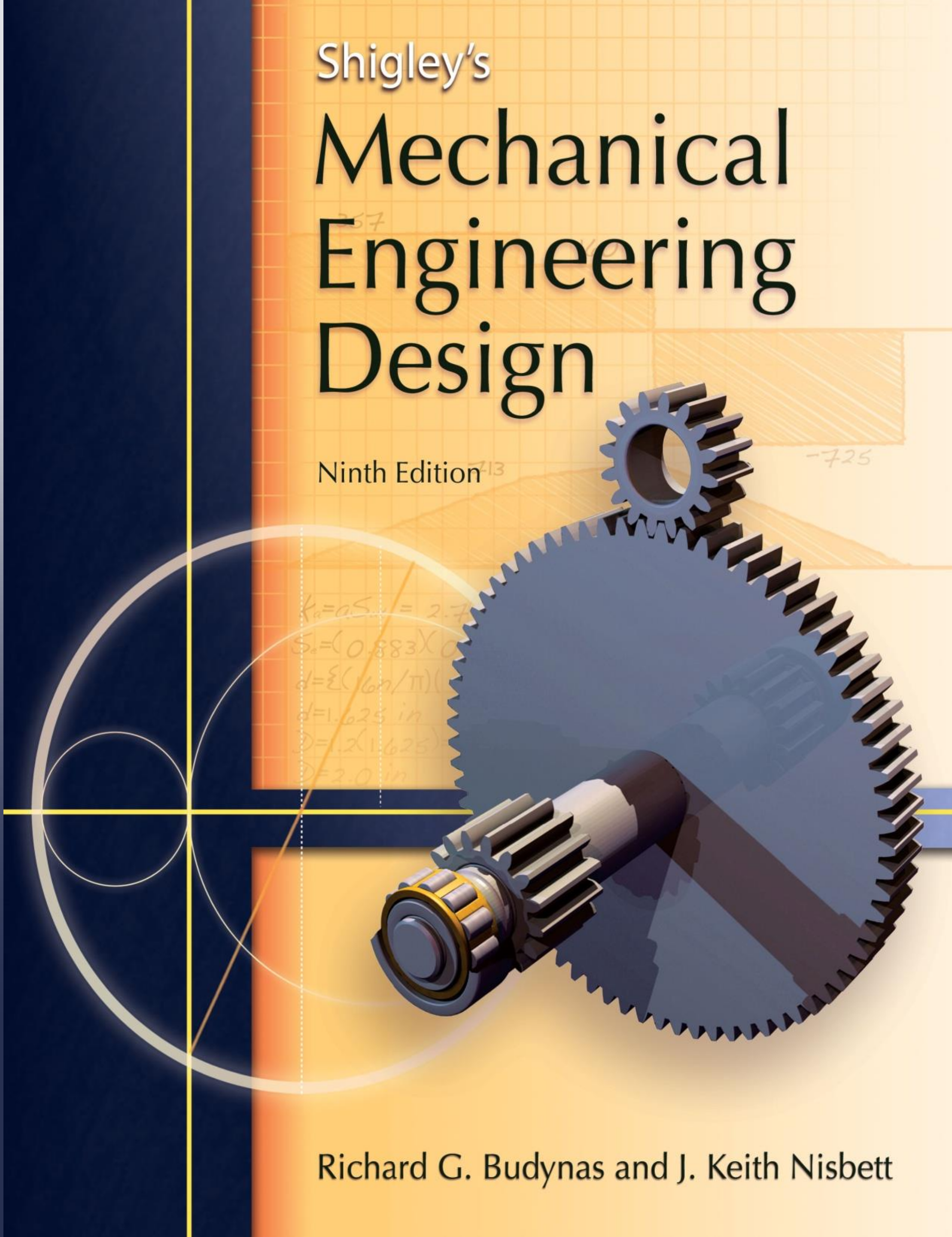
Lecture Slides

Chapter 2

Materials



Chapter Outline

Example 1-2

A rod with a cross-sectional area of A and loaded in tension with an axial force of $P =$ 2000 lbf undergoes a stress of $\sigma = P/A$. Using a material strength of 24 kpsi and a *design factor* of 3.0, determine the minimum diameter of a solid circular rod. Using Table A–17, select a preferred fractional diameter and determine the rod's *factor of safety*.

Solution

Since $A = \pi d^2/4$, $\sigma = P/A$, and from Eq. (1–3), $\sigma = S/n_d$, then

$$\sigma = \frac{P}{A} = \frac{P}{\pi d^2/4} = \frac{S}{n_d}$$

Solving for d yields

$$d = \left(\frac{4Pn_d}{\pi S}\right)^{1/2} = \left(\frac{4(2000)3}{\pi(24\,000)}\right)^{1/2} = 0.564 \text{ in}$$ **Answer**

From Table A–17, the next higher preferred size is $\frac{5}{8}$ in $= 0.625$ in. Thus, when n_d is replaced with n in the equation developed above, the factor of safety n is

$$n = \frac{\pi S d^2}{4P} = \frac{\pi(24\,000)0.625^2}{4(2000)} = 3.68$$ **Answer**

Thus rounding the diameter has increased the actual design factor.

Standard Tensile Test

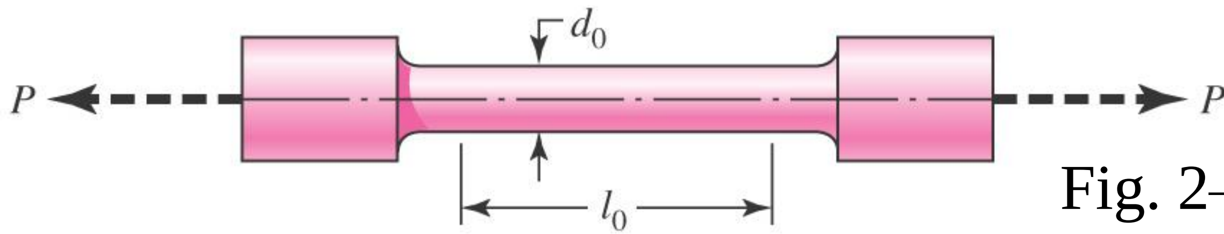


Fig. 2–1

- Used to obtain material characteristics and strengths
- Loaded in tension with slowly increasing *P*
- Load and deflection are recorded

Stress and Strain

The *stress* is calculated from

$$\sigma = \frac{P}{A_0} \tag{2–1}$$

where $A_0 = \frac{1}{4}\pi d_0^2$ is the original cross-sectional area.

The *normal strain* is calculated from

$$\epsilon = \frac{l - l_0}{l_0} \tag{2–2}$$

where l_0 is the original gauge length and l is the current length corresponding to the current P.

Stress-Strain Diagram

- Plot stress vs. normal strain
- Typically linear relation until the *proportional limit, pl*
- No permanent deformation until the *elastic limit, el*
- *Yield strength,* S_y *,* defined at point where significant plastic deformation begins, or where permanent set reaches a fixed amount, usually 0.2% of the original gauge length
- *Ultimate strength,* S_u*,* defined as the maximum stress on the diagram

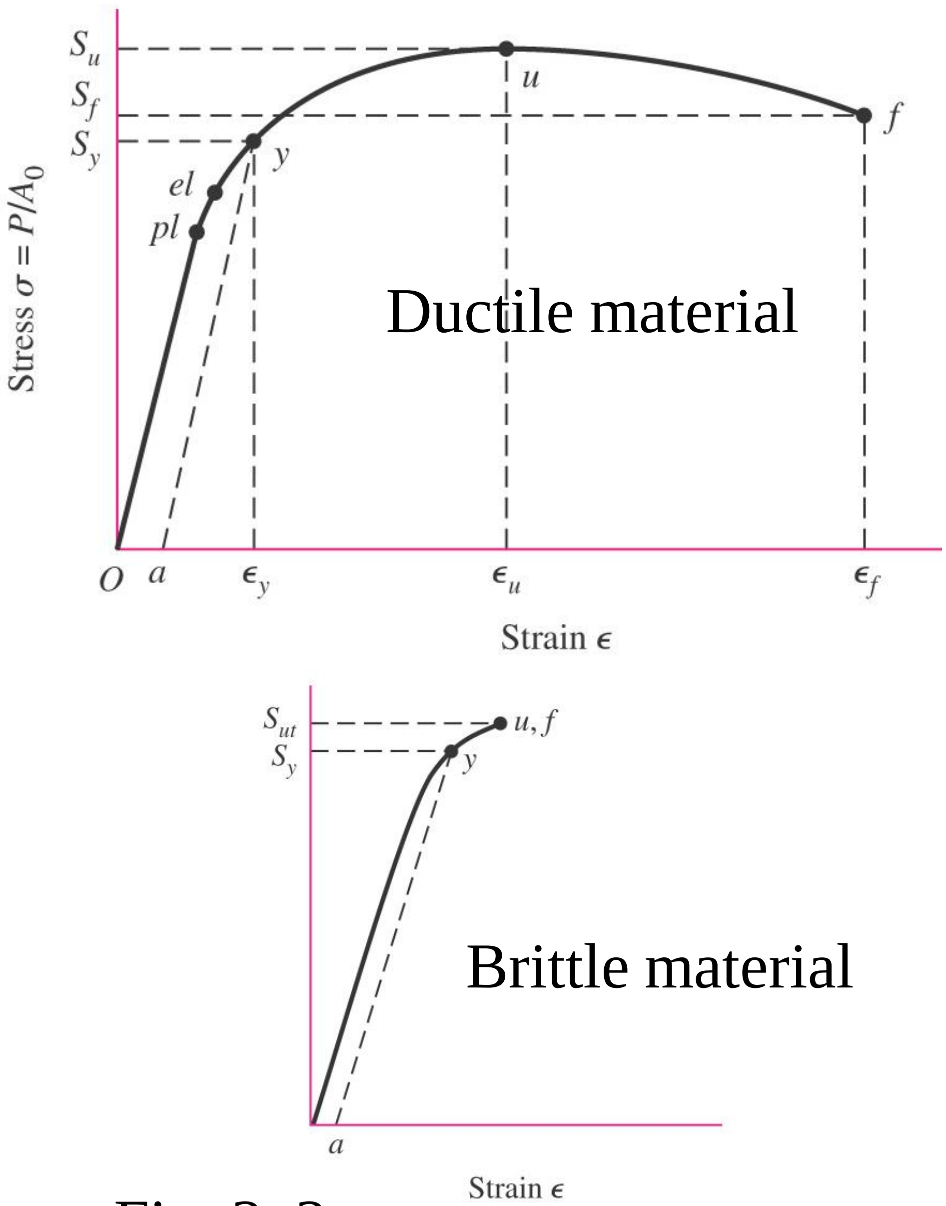


Fig. 2–2

Elastic Relationship of Stress and Strain

- Slope of linear section is *Young's Modulus*, or *modulus of elasticity, E*
- *Hooke's law*

$$\sigma = E\epsilon$$

- *E* is relatively constant for a given type of material (e.g. steel, copper, aluminum)
- See Table A-5 for typical values
- Usually independent of heat treatment, carbon content, or alloying

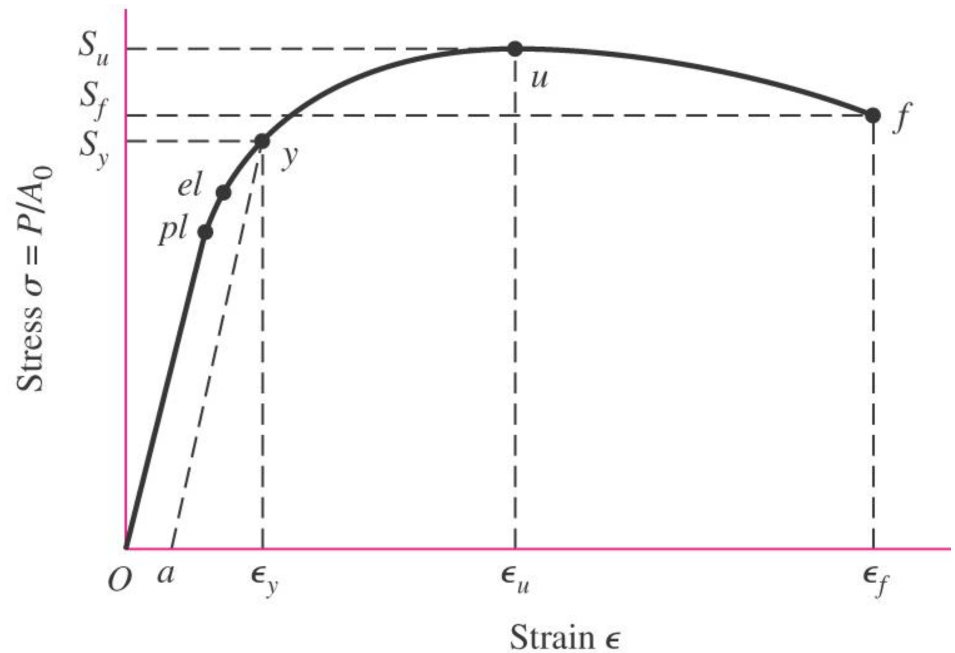
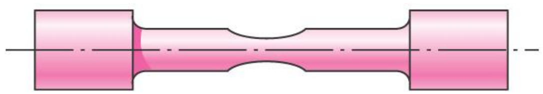


Fig. 2–2 (*a*)

True Stress-Strain Diagram

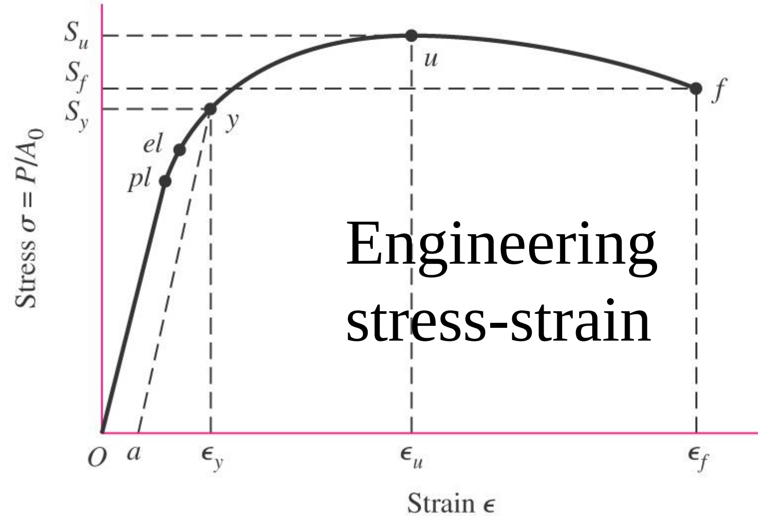
- *Engineering* stress-strain diagrams (commonly used) are based on original area.
- Area typically reduces under load, particularly during “necking” after point *u*.

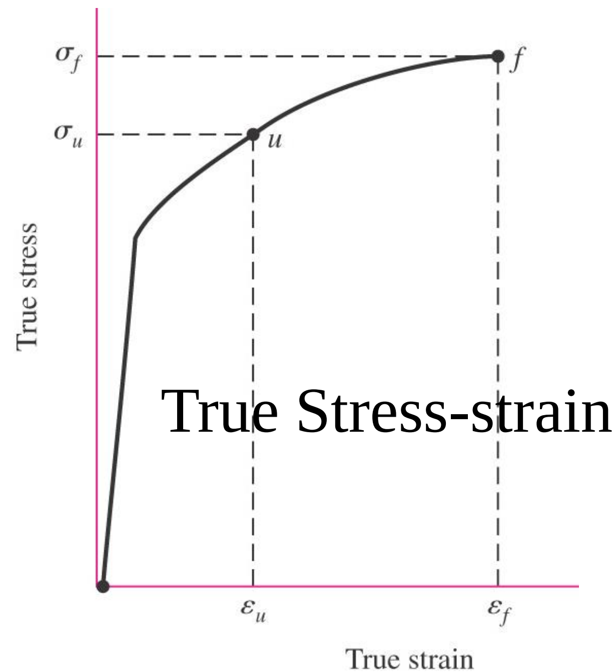


- *True stress* is based on actual area corresponding to current *P*.
- *True strain* is the sum of the incremental elongations divided by the *current* gauge length at load *P*.

$$\varepsilon = \int_{l_0}^{l} \frac{dl}{l} = \ln \frac{l}{l_0} \qquad \text{(2-4)}$$

- Note that true stress continually increases all the way to fracture.





Compression Strength

- Compression tests are used to obtain compressive strengths.
- Buckling and bulging can be problematic.
- For ductile materials, compressive strengths are usually about the same as tensile strengths, $S_{uc} = S_{ut}$.
- For brittle materials, compressive strengths, S_{uc} , are often greater than tensile strengths, S_{ut} .

Torsional Strengths

- Torsional strengths are found by twisting solid circular bars.
- Results are plotted as a *torque-twist diagram*.
- Shear stresses in the specimen are linear with respect to the radial location – zero at the center and maximum at the outer radius.
- Maximum shear stress is related to the angle of twist by

$$\tau_{\max} = \frac{Gr}{l_0}\theta \qquad (2\text{–}5)$$

  - q is the angle of twist (in radians)
  - r is the radius of the bar
  - l_0 is the gauge length
  - G is the material stiffness property called the *shear modulus* or *modulus of rigidity*.

Torsional Strengths

- Maximum shear stress is related to the applied torque by

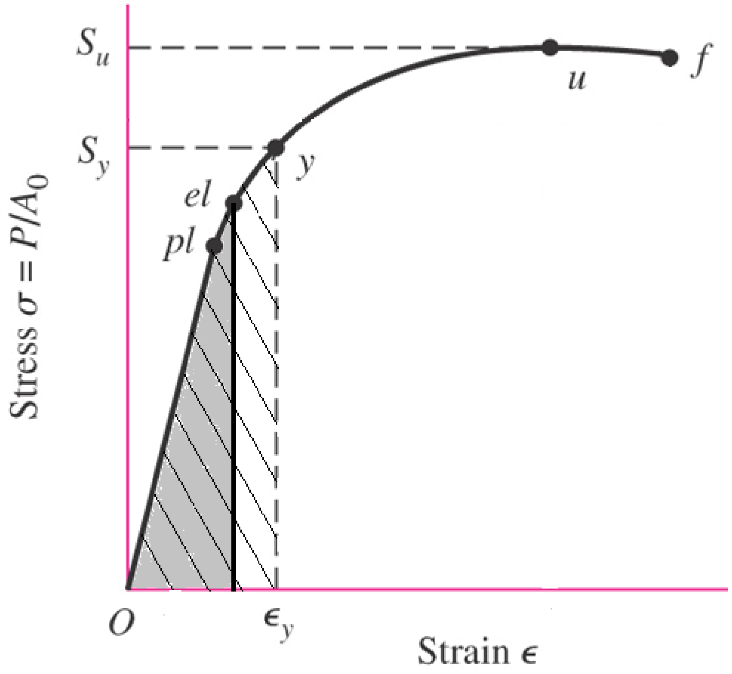
$$\tau_{max} = \frac{Tr}{J} \qquad (2\text{–}6)$$

  - J is the polar second moment of area of the cross section
  - For round cross section, $J = \frac{1}{2}\pi r^4$
- *Torsional yield strength*, S_{sy} corresponds to the maximum shear stress at the point where the torque-twist diagram becomes significantly non-linear
- *Modulus of rupture*, S_{su} corresponds to the torque T_u at the maximum point on the torque-twist diagram

$$S_{su} = \frac{T_u r}{J} \qquad (2\text{–}7)$$

Resilience

- *Resilience* – Capacity of a material to absorb energy within its elastic range
- *Modulus of resilience, u_R*
  - Energy absorbed per unit volume without permanent deformation
  - Equals the area under the stress-strain curve up to the elastic limit
  - Elastic limit often approximated by yield point



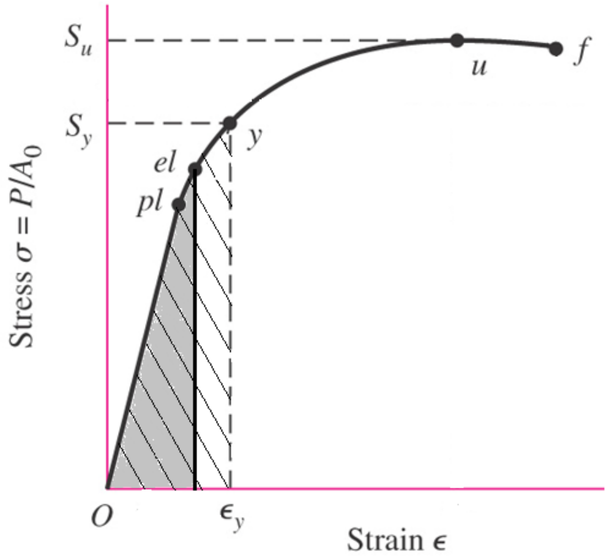
Resilience

- Area under curve to yield point gives approximation

$$u_R \cong \int_0^{\epsilon_y} \sigma\, d\epsilon \qquad (2\text{–}8)$$

- If elastic region is linear,

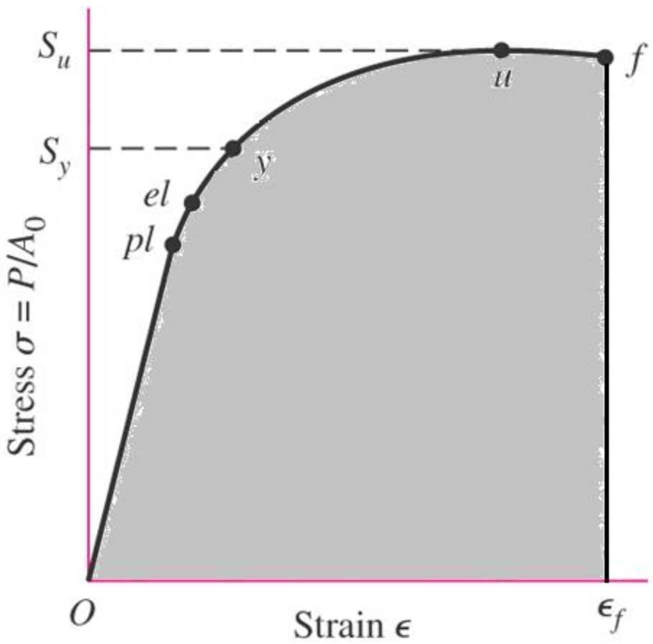
$$u_R \cong \frac{1}{2} S_y \epsilon_y = \frac{1}{2}(S_y)(S_y/E) = \frac{S_y^2}{2E} \qquad (2\text{–}9)$$



- For two materials with the same yield strength, the less stiff material (lower *E*) has greater resilience

Toughness

- *Toughness* – capacity of a material to absorb energy without fracture
- *Modulus of toughness,* u_T
  - Energy absorbed per unit volume without fracture
  - Equals area under the stress-strain curve up to the fracture point



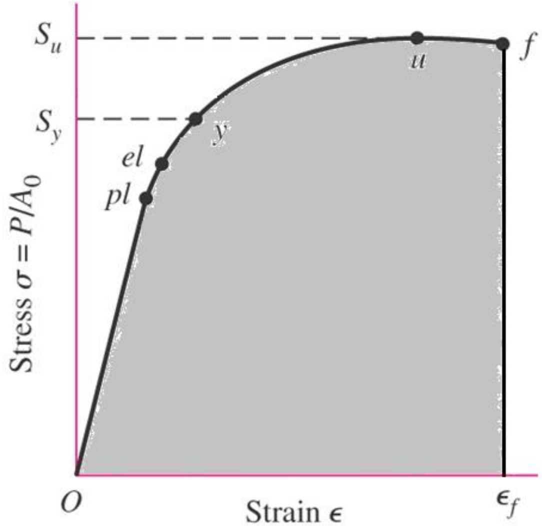
Toughness

- Area under curve up to fracture point

$$u_T = \int_0^{\epsilon_f} \sigma \, d\epsilon \qquad (2\text{–}10)$$

- Often estimated graphically from stress-strain data
- Approximated by using the average of yield and ultimate strengths and the strain at fracture

$$u_T \cong \left(\frac{S_y + S_{ut}}{2}\right)\epsilon_f \qquad (2\text{–}11)$$



Resilience and Toughness

- Measures of energy absorbing characteristics of a material
- Units are energy per unit volume
  - $lbf \cdot in/in^3$ or J/m^3
- Assumes low strain rates
- For higher strain rates, use impact methods (See Sec. 2-5)

Statistical Significance of Material Properties

- Strength values are obtained from testing many nominally identical specimens
- Strength, a material property, is distributional and thus statistical in nature
- Example – Histographic report for maximum stress of 1000 tensile tests on 1020 steel

Class Frequency f_i	2	18	23	31	83	109	138	151	139	130	82	49	28	11	4	2
Class Midpoint x_i, kpsi	56.5	57.5	58.5	59.5	60.5	61.5	62.5	63.5	64.5	65.5	66.5	67.5	68.5	69.5	70.5	71.5

Example for Statistical Material Property

- Histographic report for maximum stress of 1000 tensile tests on 1020 steel

Class Frequency f_i	2	18	23	31	83	109	138	151	139	130	82	49	28	11	4	2
Class Midpoint x_i, kpsi	56.5	57.5	58.5	59.5	60.5	61.5	62.5	63.5	64.5	65.5	66.5	67.5	68.5	69.5	70.5	71.5

- *Probability density* – number of occurrences divided by the total sample number
- Histogram of probability density for 1020 steel

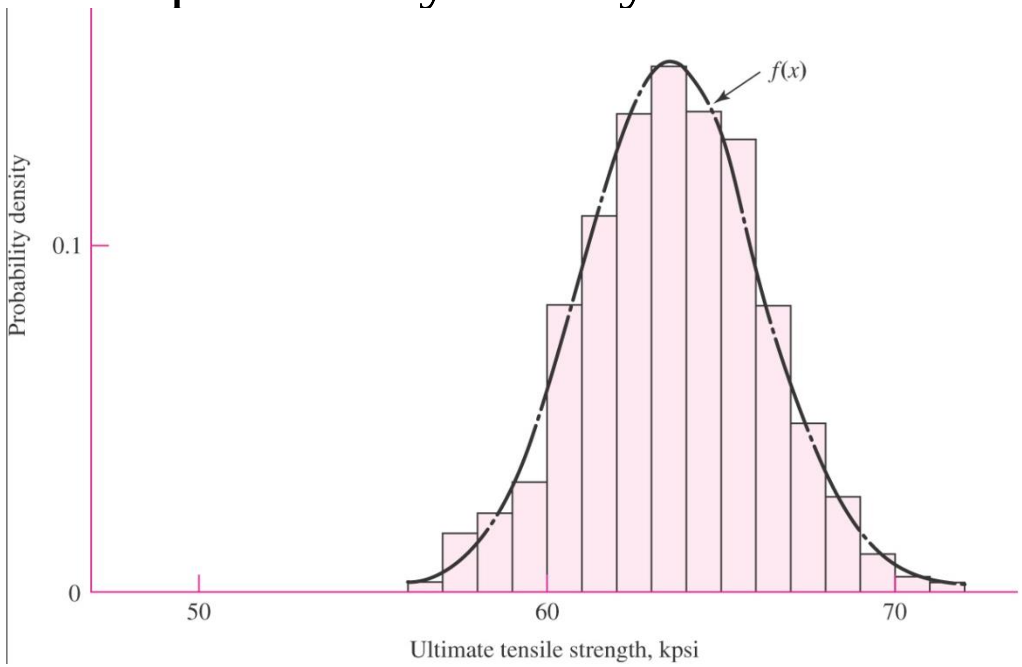


Fig. 2–5

Example for Statistical Material Property

- Probability density function (See Ex. 20-4)

$$f(x) = \frac{1}{2.594\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{x - 63.62}{2.594}\right)^2\right]$$

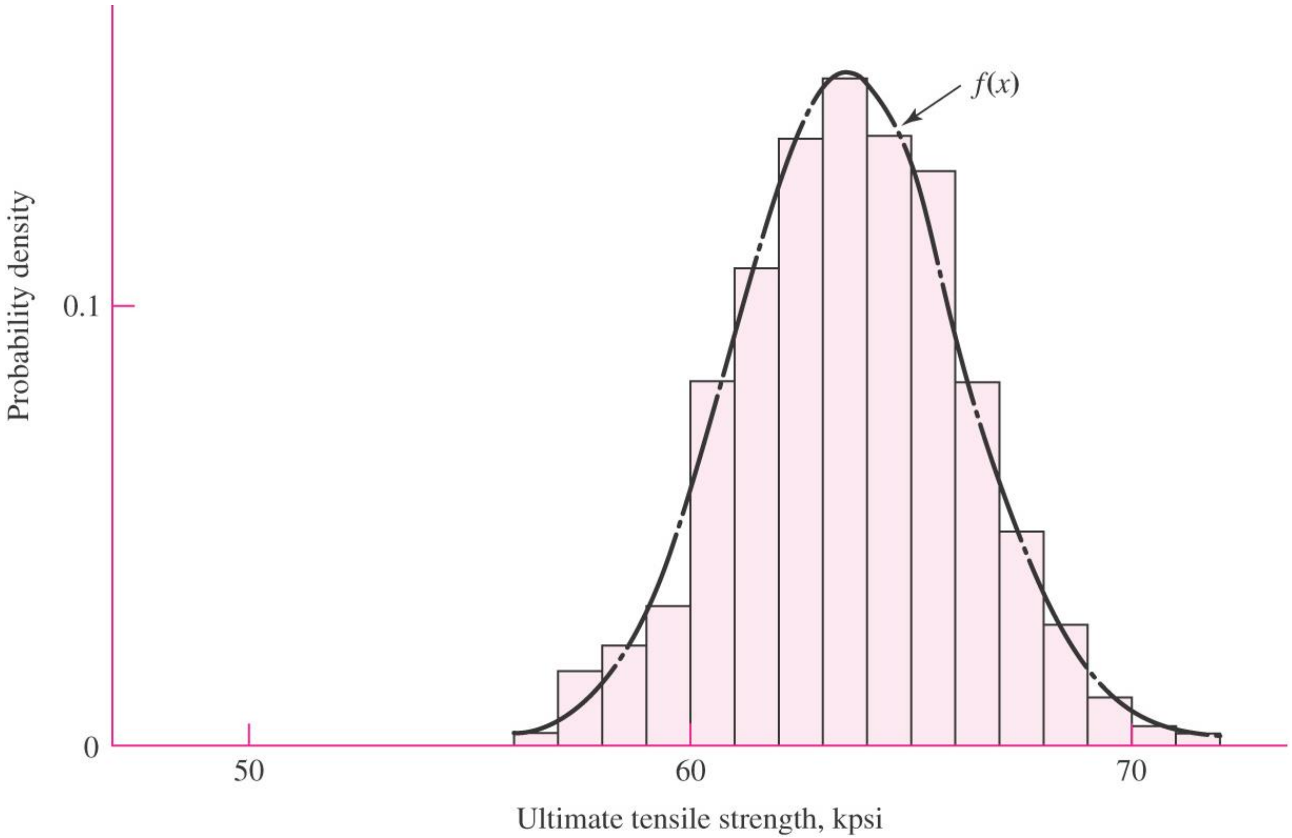
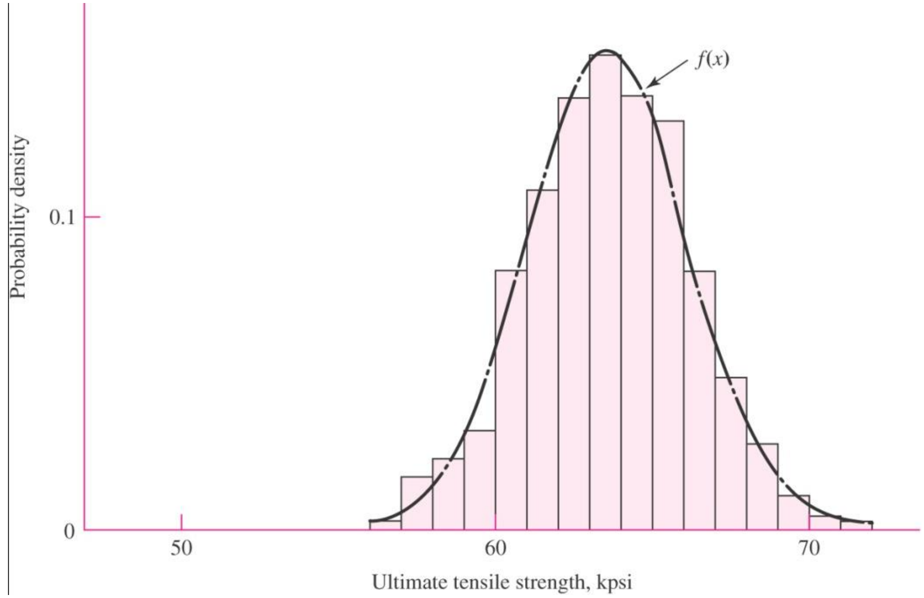


Fig. 2–5

Statistical Quantity

- Statistical quantity described by mean, standard deviation, and distribution type
- From 1020 steel example:
  - Mean stress = 63.62 kpsi
  - Standard deviation = 2.594 kpsi
  - Distribution is *normal*
  - Notated as $\mathbf{S}_{ut} = \mathbf{N}(63.62, 2.594)$ kpsi



Strengths from Tables

- Property tables often only report a single value for a strength term
- Important to check if it is mean, minimum, or some percentile
- Common to use 99% minimum strength, indicating 99% of the samples exceed the reported value

Cold Work

- *Cold work* – Process of plastic straining below recrystallization temperature in the plastic region of the stress-strain diagram
- Loading to point *i* beyond the yield point, then unloading, causes permanent plastic deformation, ϵ_p
- Reloading to point *i* behaves elastically all the way to *i*, with additional elastic strain ϵ_e

$$\epsilon = \epsilon_p + \epsilon_e \qquad \epsilon_e = \frac{\sigma_i}{E}$$

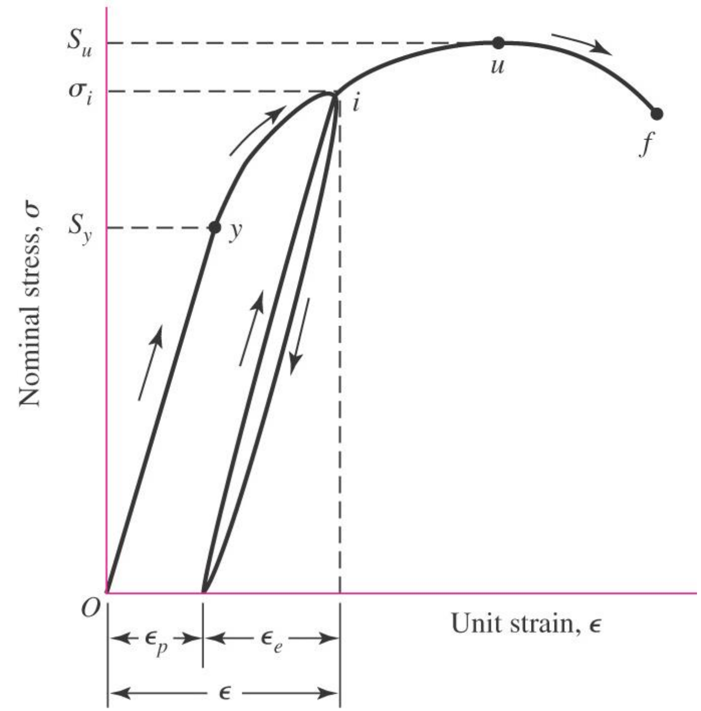


Fig. 2–6 (*a*)

Cold Work

- The yield point is effectively increased to point *i*
- Material is said to have been *cold worked*, or *strain hardened*
- Material is less ductile (more brittle) since the plastic zone between yield strength and ultimate strength is reduced
- Repeated strain hardening can lead to brittle failure

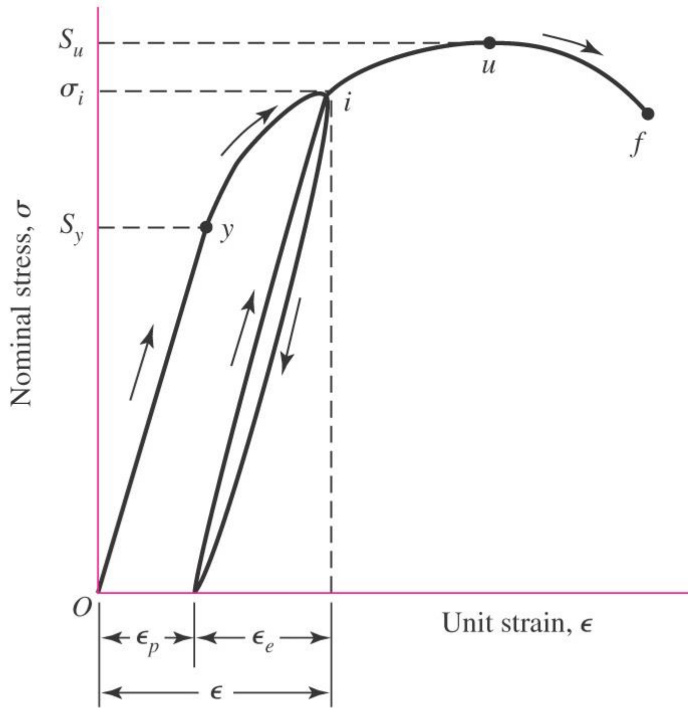


Fig. 2–6 (*a*)

Reduction in Area

- Plot load P vs. Area Reduction
- *Reduction in area* corresponding to load P_f at fracture is

$$R = \frac{A_0 - A_f}{A_0} = 1 - \frac{A_f}{A_0} \qquad \text{(2-12)}$$

- R is a measure of *ductility*
- Ductility represents the ability of a material to absorb overloads and to be cold-worked

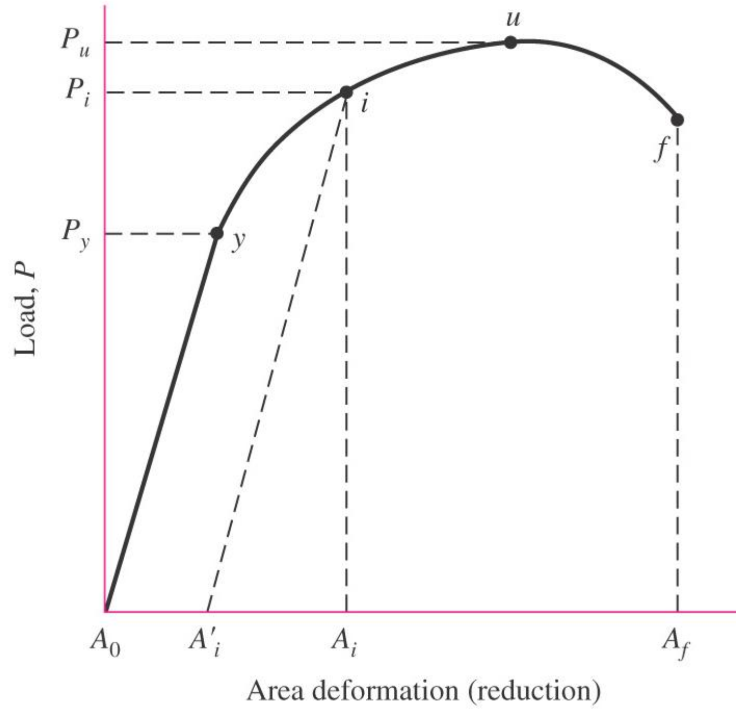


Fig. 2–6 (*b*)

Cold-work Factor

- *Cold-work factor W* – A measure of the quantity of cold work

$$W = \frac{A_0 - A_i'}{A_0} \approx \frac{A_0 - A_i}{A_0} \qquad (2\text{–}13)$$

$$A_i' = A_0(1 - W) \qquad (2\text{–}14)$$

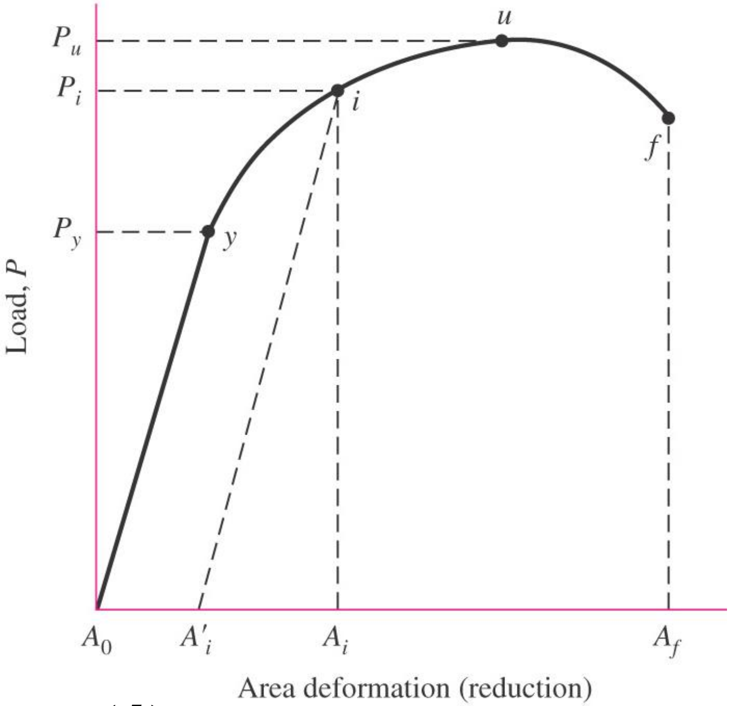


Fig. 2–6 (*b*)

Equations for Cold-worked Strengths

$$\sigma = \sigma_0 \varepsilon^m \qquad (2\text{–}15)$$

$$m = \varepsilon_u \qquad (2\text{–}16)$$

$$\varepsilon = \ln \frac{l}{l_0} = \ln \frac{A_0}{A} \qquad (2\text{–}17)$$

$$S'_y = \frac{P_i}{A'_i} = \sigma_0 \varepsilon_i^m \qquad P_i \le P_u \qquad (2\text{–}18)$$

$$S'_u = \frac{S_u A_0}{A_0(1-W)} = \frac{S_u}{1-W} \qquad \varepsilon_i \le \varepsilon_u \qquad (2\text{–}19)$$

$$S'_u \doteq S'_y \doteq \sigma_0 \varepsilon_i^m \qquad \varepsilon_i > \varepsilon_u \qquad (2\text{–}20)$$

Example 2-1

An annealed AISI 1018 steel (see Table A–22) has $S_y = 32.0$ kpsi, $S_u = 49.5$ kpsi, $\sigma_f = 91.1$ kpsi, $\sigma_0 = 90$ kpsi, $m = 0.25$, and $\varepsilon_f = 1.05$ in/in. Find the new values of the strengths if the material is given 15 percent cold work.

Solution

From Eq. (2–16), we find the true strain corresponding to the ultimate strength to be

$$\varepsilon_u = m = 0.25$$

The ratio A_0/A_i is, from Eq. (2–13),

$$\frac{A_0}{A_i} = \frac{1}{1 - W} = \frac{1}{1 - 0.15} = 1.176$$

Example 2-1 (Continued)

The true strain corresponding to 15 percent cold work is obtained from Eq. (2–17).

$$\varepsilon_i = \ln \frac{A_0}{A_i} = \ln 1.176 = 0.1625$$

Since $\varepsilon_i < \varepsilon_u$, Eqs. (2–18) and (2–19) apply. Therefore,

$$S'_y = \sigma_0 \varepsilon_i^m = 90(0.1625)^{0.25} = 57.1 \text{ kpsi} \qquad \text{Answer}$$

$$S'_u = \frac{S_u}{1 - W} = \frac{49.5}{1 - 0.15} = 58.2 \text{ kpsi} \qquad \text{Answer}$$

Hardness

- *Hardness* – The resistance of a material to penetration by a pointed tool
- Two most common hardness-measuring systems
  - Rockwell
    - A, B, and C scales
    - Specified indenters and loads for each scale
    - Hardness numbers are relative
  - Brinell
    - Hardness number H_B is the applied load divided by the spherical surface area of the indentation

Strength and Hardness

- For many materials, relationship between ultimate strength and Brinell hardness number is roughly linear
- For steels

$$S_u = \begin{cases} 0.5 H_B & \text{kpsi} \\ 3.4 H_B & \text{MPa} \end{cases} \qquad (2\text{–}21)$$

- For cast iron

$$S_u = \begin{cases} 0.23 H_B - 12.5 \text{ kpsi} \\ 1.58 H_B - 86 \text{ MPa} \end{cases} \qquad (2\text{–}22)$$

Example 2-2

It is necessary to ensure that a certain part supplied by a foundry always meets or exceeds ASTM No. 20 specifications for cast iron (see Table A–24). What hardness should be specified?

Solution

From Eq. (2–22), with $(S_u)_{\min} = 20$ kpsi, we have

$$H_B = \frac{S_u + 12.5}{0.23} = \frac{20 + 12.5}{0.23} = 141$$

Answer

If the foundry can control the hardness within 20 points, routinely, then specify $145 < H_B < 165$. This imposes no hardship on the foundry and assures the designer that ASTM grade 20 will always be supplied at a predictable cost.

Impact Properties

- Charpy notched-bar test used to determine brittleness and impact strength
- Specimen struck by pendulum
- Energy absorbed, called *impact value*, is computed from height of swing after fracture

Effect of Temperature on Impact

- Some materials experience a sharp transition from ductile to brittle at a certain temperature

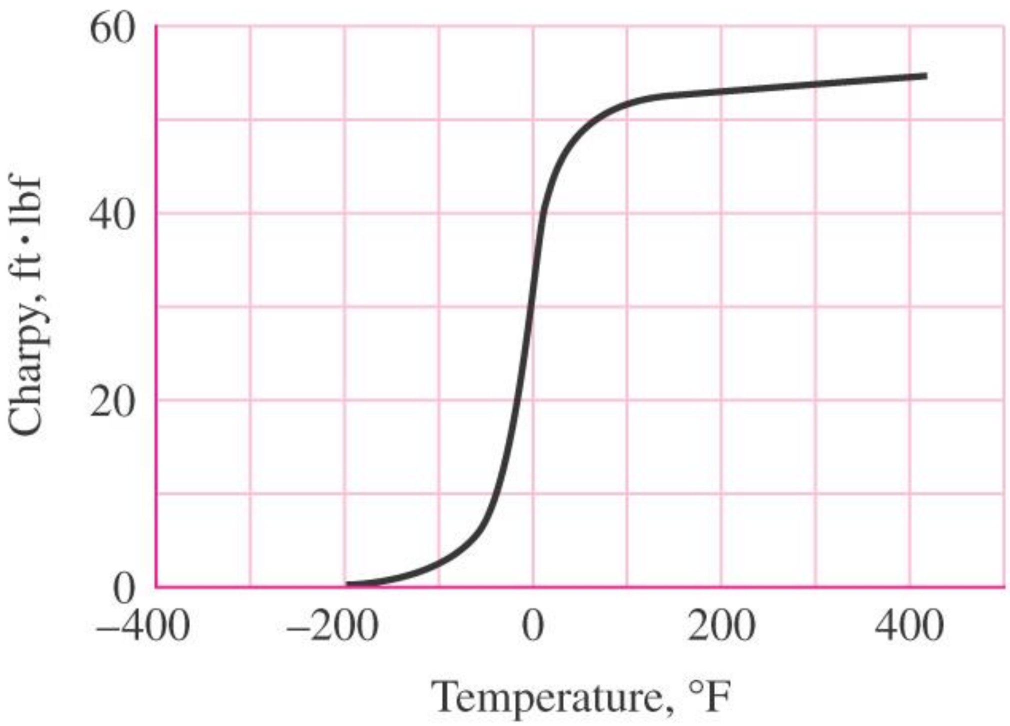


Fig. 2–7

Effect of Strain Rate on Impact

- Average strain rate for stress-strain diagram is 0.001 in/(in·s)
- Increasing strain rate increases strengths
- Due to yield strength approaching ultimate strength, a mild steel could be expected to behave elastically through practically its entire strength range under impact conditions

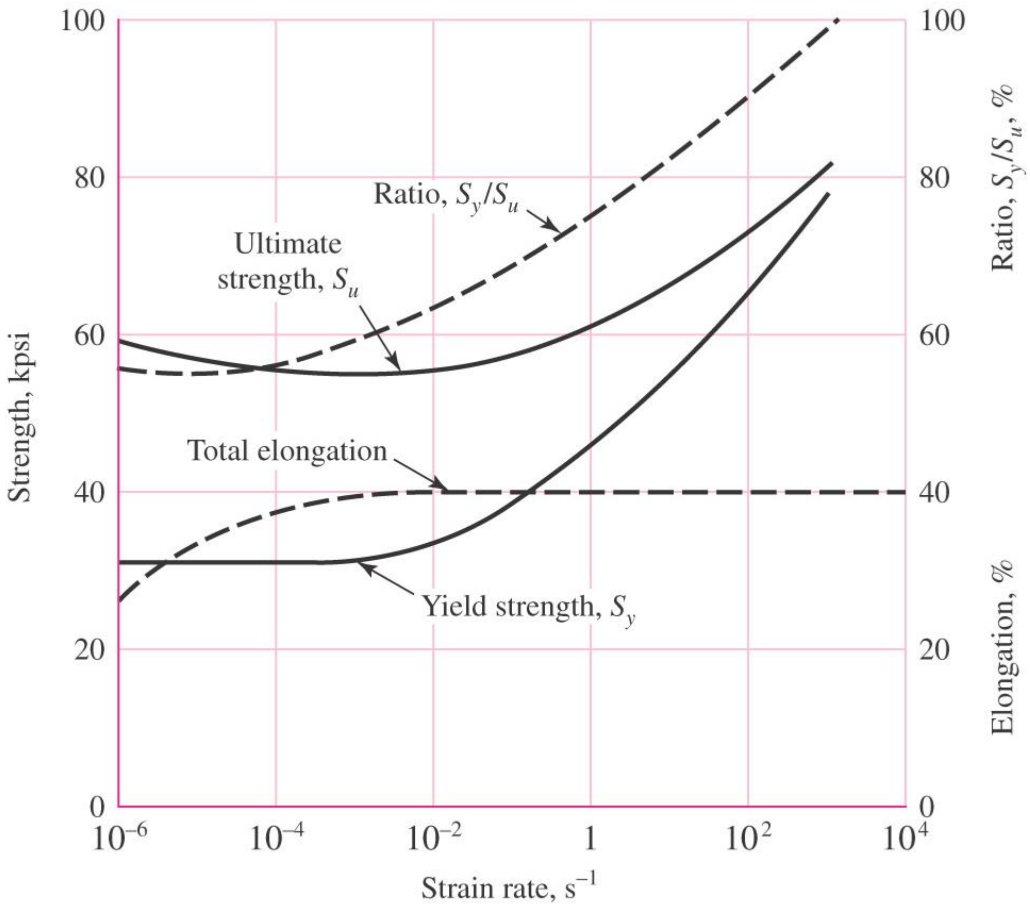


Fig. 2–8

Temperature Effects on Strengths

- Plot of strength vs. temperature for carbon and alloy steels
- As temperature increases above room temperature
  - S_{ut} increase slightly, then decreases significantly
  - S_y decreases continuously
  - Results in increased ductility

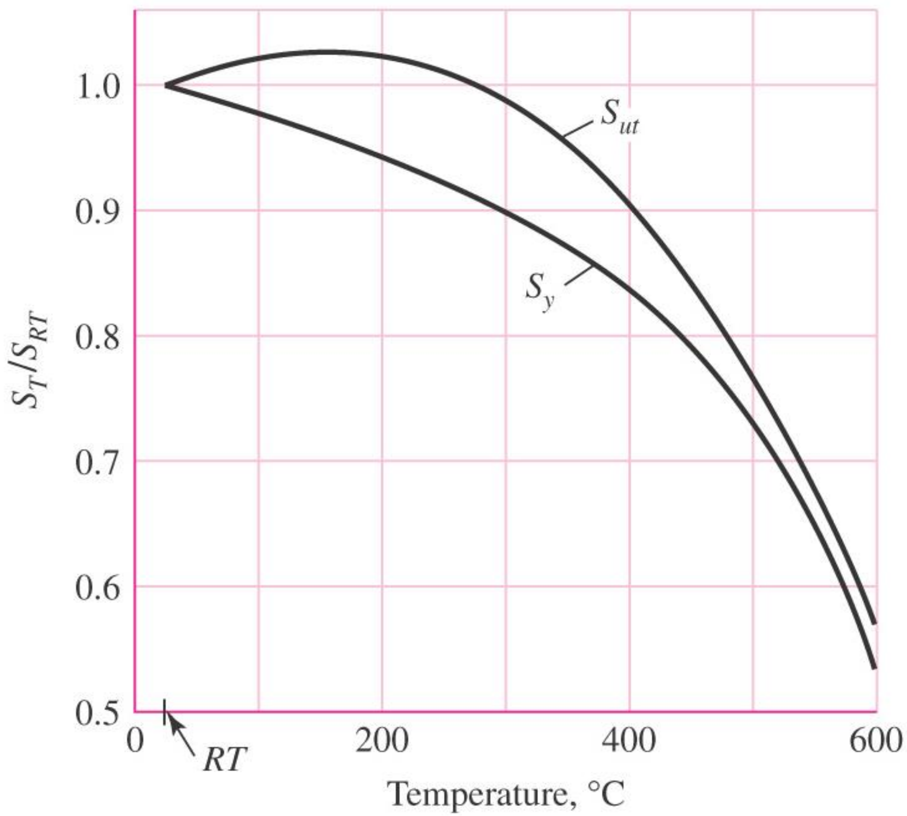


Fig. 2–9

Creep

- *Creep* – a continuous deformation under load for long periods of time at elevated temperatures
- Often exhibits three stages
  - 1st stage: elastic and plastic deformation; decreasing creep rate due to strain hardening
  - 2nd stage: constant minimum creep rate caused by the annealing effect
  - 3rd stage: considerable reduction in area; increased true stress; higher creep rate leading to fracture

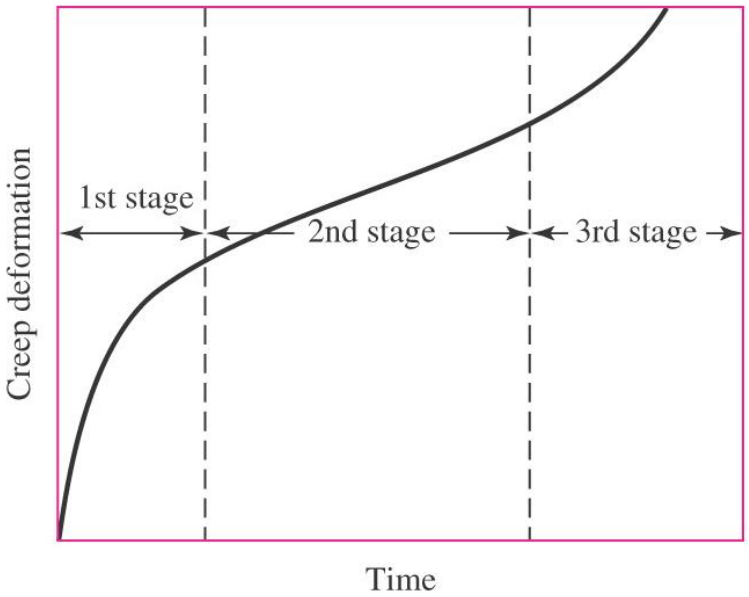


Fig. 2–10

Material Numbering Systems

- Common numbering systems
  - Society of Automotive Engineers (SAE)
  - American Iron and Steel Institute (AISI)
  - Unified Numbering System (UNS)
  - American Society for Testing and Materials (ASTM) for cast irons

UNS Numbering System

- UNS system established by SAE in 1975
- Letter prefix followed by 5 digit number
- Letter prefix designates material class
  - G – carbon and alloy steel
  - A – Aluminum alloy
  - C – Copper-based alloy
  - S – Stainless or corrosion-resistant steel

UNS for Steels

- For steel, letter prefix is G
- First two numbers indicate composition, excluding carbon content

G10	Plain carbon	G46	Nickel-molybdenum
G11	Free-cutting carbon steel with more sulfur or phosphorus	G48	Nickel-molybdenum
G13	Manganese	G50	Chromium
G23	Nickel	G51	Chromium
G25	Nickel	G52	Chromium
G31	Nickel-chromium	G61	Chromium-vanadium
G33	Nickel-chromium	G86	Chromium-nickel-molybdenum
G40	Molybdenum	G87	Chromium-nickel-molybdenum
G41	Chromium-molybdenum	G92	Manganese-silicon
G43	Nickel-chromium-molybdenum	G94	Nickel-chromium-molybdenum

- Second pair of numbers indicates carbon content in hundredths of a percent by weight
- Fifth number is used for special situations
- Example: G52986 is chromium alloy with 0.98% carbon

Some Casting Processes

- Sand Casting
- Shell Molding
- Investment Casting
- Powder-Metallurgy Process

Hot-working Processes

- Process in which metal is formed while heated above recrystallization temperature
- Refined grain size
- Rough surface finish
- Rolling, forging, extrusion, pressing
- Common bar cross-sections from hot-rolling

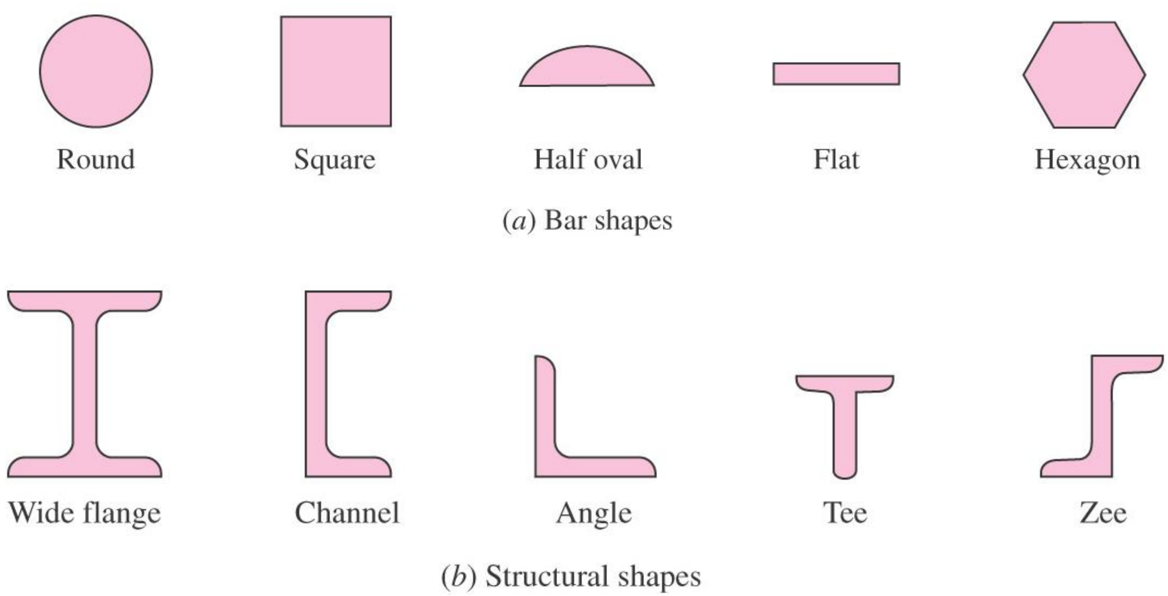


Fig. 2–11

Cold-working Processes

- Forming of metal without elevating temperature
- Strain hardens, resulting in increase in yield strength
- Increases hardness and ultimate strength, decreases ductility
- Produces bright, smooth, reasonably accurate finish
- Cold-rolling used to produce wide flats and sheets
- Cold-drawing draws a hot-rolled bar through a smaller die

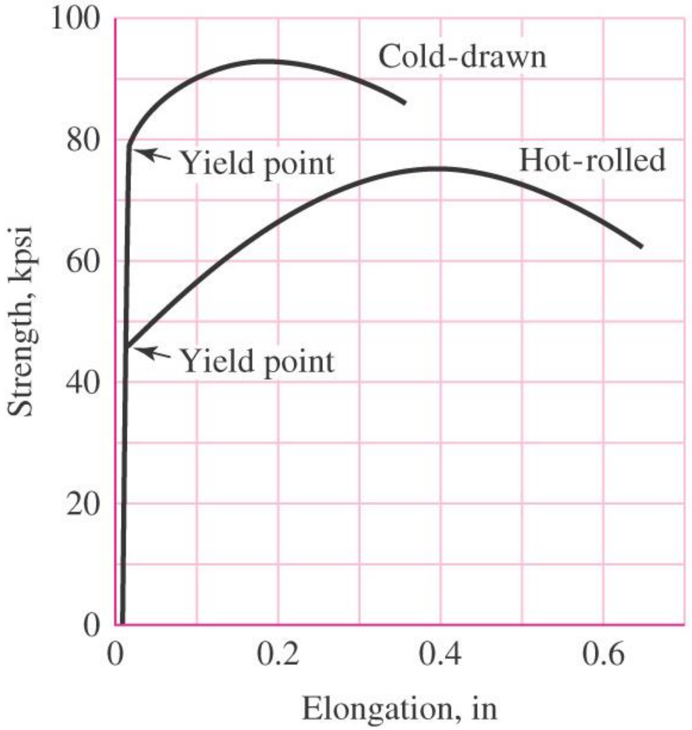


Fig. 2–12

Heat Treatment of Steel

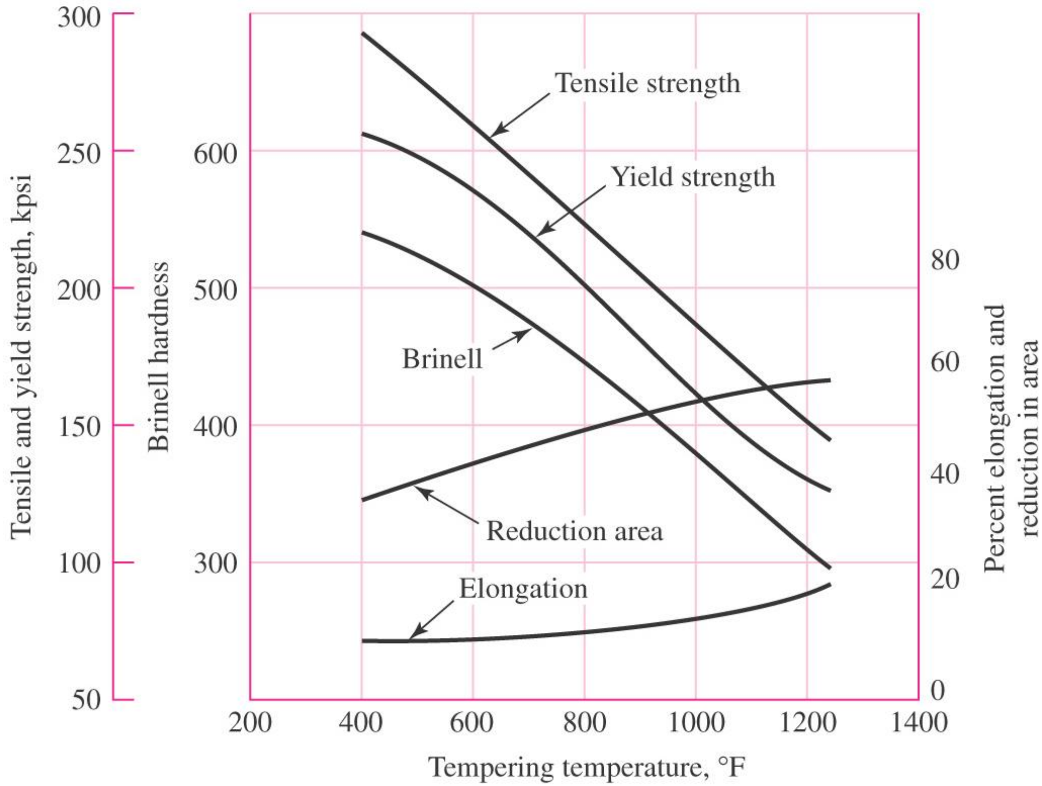
- Time and temperature controlled processes that modifies material properties
- *Annealing*
  - Heated above critical temperature, held, then slowly cooled
  - Refines grain structure, softens, increases ductility
  - Erases memory of prior operations
  - *Normalizing* provides partial annealing by adjusting time and temperature
- *Quenching*
  - Controlled cooling rate prevents full annealing
  - Less pearlite, more martensite and/or bainite
  - Increased strength, hardness, brittleness

Heat Treatment of Steel

- *Tempering*
  - Reheat after quenching to a temperature below the critical temperature
  - Relieves internal stresses
  - Increases ductility, slight reduction in strength and hardness

Effects of Heat Treating

Fig. 2–13



Condition	Tensile strength, kpsi	Yield strength, kpsi	Reduction in area, %	Elongation in 2 in, %	Brinell hardness, Bhn
Normalized	200	147	20	10	410
As rolled	190	144	18	9	380
Annealed	120	99	43	18	228

Case Hardening

- Process to increase hardness on outer surface, while retaining ductility and toughness in the core
- Addition of carbon to outer surface by exposure to high carbon solid, liquid, or gas at elevated temperature
- Can also achieve case hardening by heat treating only the outer surface, e.g. induction hardening or flame hardening

Alloy Steels

- Chromium
- Nickel
- Manganese
- Silicon
- Molybdenum
- Vanadium
- Tungsten

Corrosion-Resistant Steels

- *Stainless steels*
  - Iron-base alloys with at least 12 % chromium
  - Resists many corrosive conditions
- Four types of stainless steels
  - Ferritic chromium
  - Austenitic chromium-nickel
  - Martensitic
  - Precipitation-hardenable

Casting Materials

- Gray Cast Iron
- Ductile and Nodular Cast Iron
- White Cast Iron
- Malleable Cast Iron
- Alloy Cast Iron
- Cast Steel

Nonferrous Metals

- Aluminum
- Magnesium
- Titanium
- Copper-based alloys
  - Brass with 5 to 15 percent zinc
    - Gilding brass, commercial bronze, red brass
  - Brass with 20 to 36 percent zinc
    - Low brass, cartridge brass, yellow brass
    - Low-leaded brass, high-leaded brass (engraver's brass), free-cutting brass
    - Admiralty metal
    - Aluminum brass
  - Brass with 36 to 40 percent zinc
    - Muntz metal, naval brass
  - Bronze
    - Silcon bronze, phosphor bronze, aluminum bronze, beryllium bronze

Plastics

- *Thermoplastic* – any plastic that flows or is moldable when heat is applied
- *Thermoset* – a plastic for which the polymerization process is finished in a hot molding press where the plastic is liquefied under pressure

Thermoplastic Properties (Table 2-2)

Name	S_u kpsi	E Mpsi	Hardness Rockwell	Elongation %	Dimensional Stability	Heat Resistance	Chemical Resistance	Processing
ABS group	2–8	0.10–0.37	60–110R	3–50	Good	*	Fair	EMST
Acetal group	8–10	0.41–0.52	80–94M	40–60	Excellent	Good	High	M
Acrylic	5–10	0.20–0.47	92–110M	3–75	High	*	Fair	EMS
Fluoroplastic group	0.50–7	. . .	50–80D	100–300	High	Excellent	Excellent	MPR†
Nylon	8–14	0.18–0.45	112–120R	10–200	Poor	Poor	Good	CEM
Phenylene oxide	7–18	0.35–0.92	115R, 106L	5–60	Excellent	Good	Fair	EFM
Polycarbonate	8–16	0.34–0.86	62–91M	10–125	Excellent	Excellent	Fair	EMS
Polyester	8–18	0.28–1.6	65–90M	1–300	Excellent	Poor	Excellent	CLMR
Polyimide	6–50	. . .	88–120M	Very low	Excellent	Excellent	Excellent†	CLMP
Polyphenylene sulfide	14–19	0.11	122R	1.0	Good	Excellent	Excellent	M
Polystyrene group	1.5–12	0.14–0.60	10–90M	0.5–60	. . .	Poor	Poor	EM
Polysulfone	10	0.36	120R	50–100	Excellent	Excellent	Excellent†	EFM
Polyvinyl chloride	1.5–7.5	0.35–0.60	65–85D	40–450	. . .	Poor	Poor	EFM

*Heat-resistant grades available.

†With exceptions.

C Coatings L Laminates R Resins E Extrusions M Moldings S Sheet F Foams P Press and sinter methods T Tubing

Thermoset Properties (Table 2-3)

Name	S_u kpsi	E Mpsi	Hardness Rockwell	Elongation %	Dimensional Stability	Heat Resistance	Chemical Resistance	Processing
Alkyd	3–9	0.05–0.30	99M*	. . .	Excellent	Good	Fair	M
Allylic	4–10	. . .	105–120M	. . .	Excellent	Excellent	Excellent	CM
Amino group	5–8	0.13–0.24	110–120M	0.30–0.90	Good	Excellent*	Excellent*	LR
Epoxy	5–20	0.03–0.30*	80–120M	1–10	Excellent	Excellent	Excellent	CMR
Phenolics	5–9	0.10–0.25	70–95E	. . .	Excellent	Excellent	Good	EMR
Silicones	5–6	. . .	80–90M	. . .	. . .	Excellent	Excellent	CLMR

*With exceptions.

C Coatings L Laminates R Resins E Extrusions M Moldings S Sheet F Foams P Press and sinter methods T Tubing

Composite Materials

- Formed from two or more dissimilar materials, each of which contributes to the final properties
- Materials remain distinct from each other at the macroscopic level
- Usually amorphous and non-isotropic
- Often consists of *laminates* of *filler* to provide stiffness and strength and a *matrix* to hold the material together
- Common filler types:

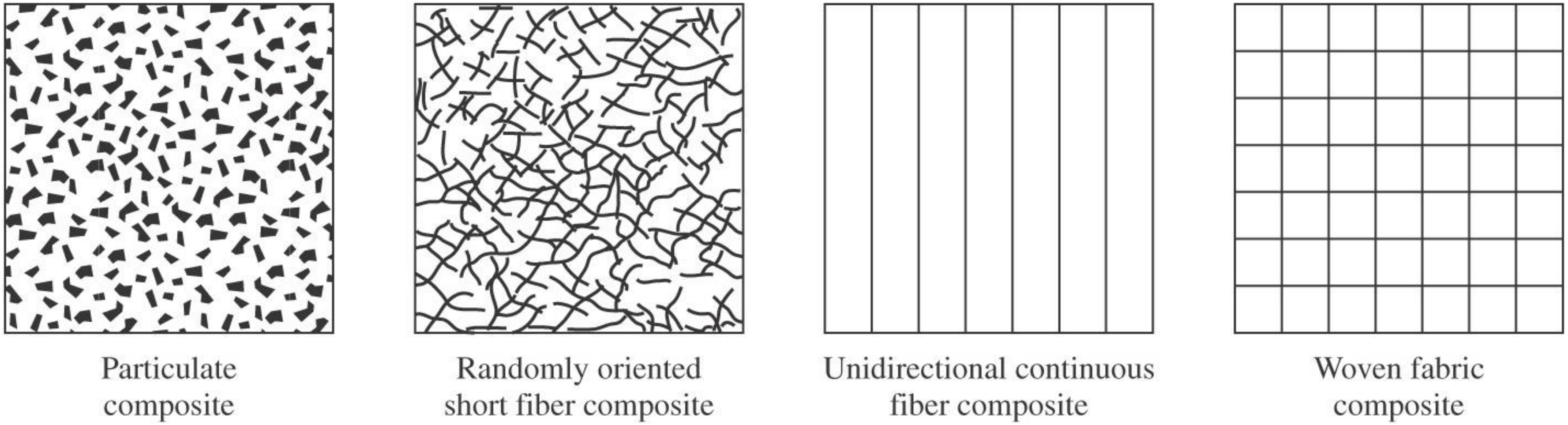


Fig. 2–14

Material Families and Classes (Table 2-4)

Family	Classes	Short Name
Metals (the metals and alloys of engineering)	Aluminum alloys	Al alloys
	Copper alloys	Cu alloys
	Lead alloys	Lead alloys
	Magnesium alloys	Mg alloys
	Nickel alloys	Ni alloys
	Carbon steels	Steels
	Stainless steels	Stainless steels
	Tin alloys	Tin alloys
	Titanium alloys	Ti alloys
	Tungsten alloys	W alloys
	Lead alloys	Pb alloys
	Zinc alloys	Zn alloys

(continued)

Material Families and Classes (Table 2-4)

Family	Classes	Short Name
Ceramics	Alumina	Al_2O_3
Technical ceramics (fine ceramics capable of load-bearing application)	Aluminum nitride	AlN
	Boron carbide	B_4C
	Silicon carbide	SiC
	Silicon nitride	Si_3N_4
	Tungsten carbide	WC
Nontechnical ceramics (porous ceramics of construction)	Brick	Brick
	Concrete	Concrete
	Stone	Stone
Glasses	Soda-lime glass	Soda-lime glass
	Borosilicate glass	Borosilicate glass
	Silica glass	Silica glass
	Glass ceramic	Glass ceramic

(continued)

Material Families and Classes (Table 2-4)

Family	Classes	Short Name
Polymers (the thermoplastics and thermosets of engineering)	Acrylonitrile butadiene styrene	ABS
	Cellulose polymers	CA
	Ionomers	Ionomers
	Epoxies	Epoxy
	Phenolics	Phenolics
	Polyamides (nylons)	PA
	Polycarbonate	PC
	Polyesters	Polyester
	Polyetheretherkeytone	PEEK
	Polyethylene	PE
	Polyethylene terephalate	PET or PETE
	Polymethylmethacrylate	PMMA
	Polyoxymethylene(Acetal)	POM
	Polypropylene	PP
	Polystyrene	PS
	Polytetrafluorethylene	PTFE
	Polyvinylchloride	PVC

(continued)

Material Families and Classes (Table 2-4)

Family	Classes	Short Name
Elastomers (engineering rubbers, natural and synthetic)	Butyl rubber	Butyl rubber
	EVA	EVA
	Isoprene	Isoprene
	Natural rubber	Natural rubber
	Polychloroprene (Neoprene)	Neoprene
	Polyurethane	PU
	Silicon elastomers	Silicones
Hybrids		
Composites	Carbon-fiber reinforced polymers	CFRP
	Glass-fiber reinforced polymers	GFRP
	SiC reinforced aluminum	Al-SiC
Foams	Flexible polymer foams	Flexible foams
	Rigid polymer foams	Rigid foams
Natural materials	Cork	Cork
	Bamboo	Bamboo
	Wood	Wood

Young's Modulus for Various Materials

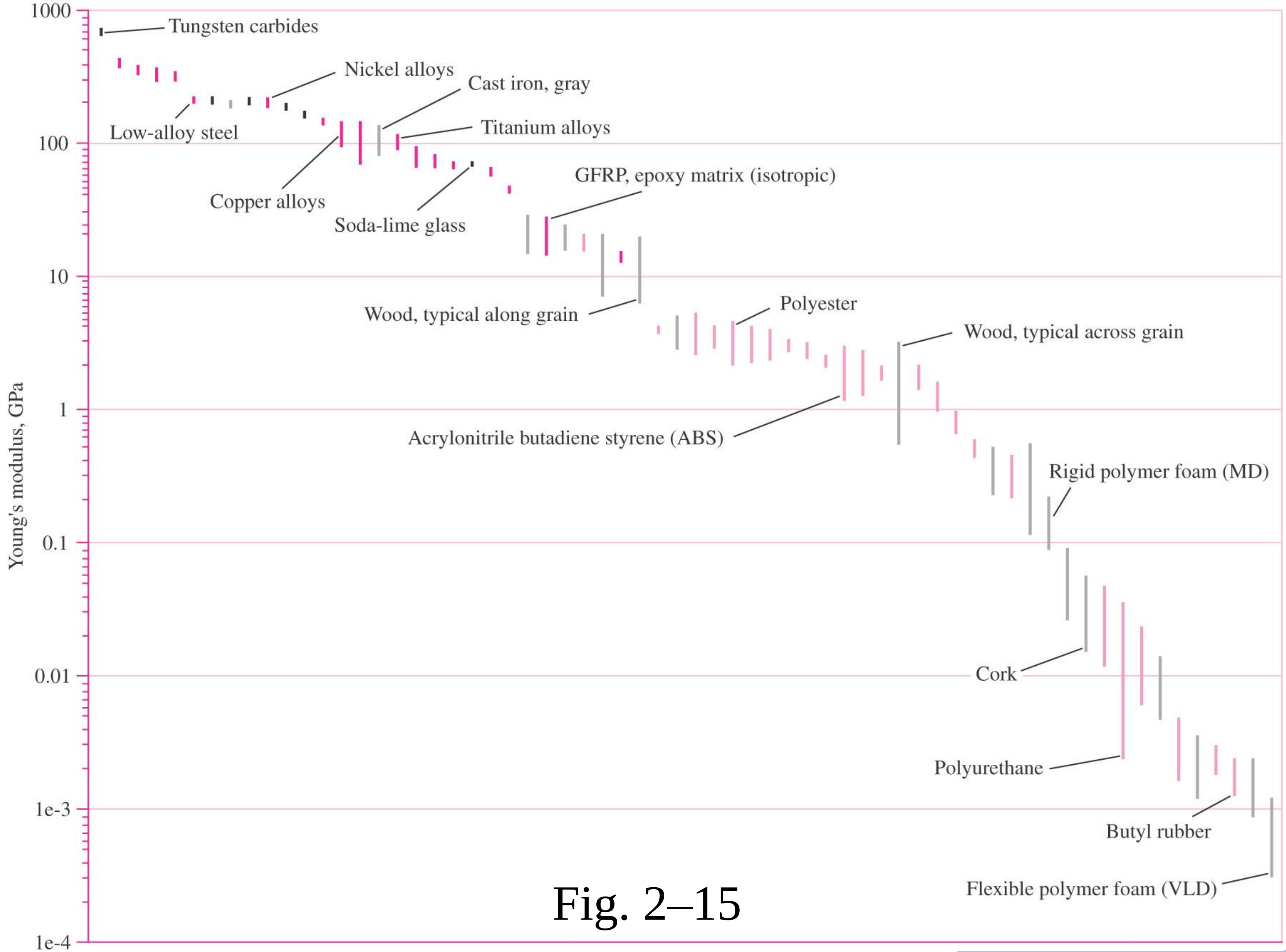


Fig. 2–15

Young's Modulus vs. Density

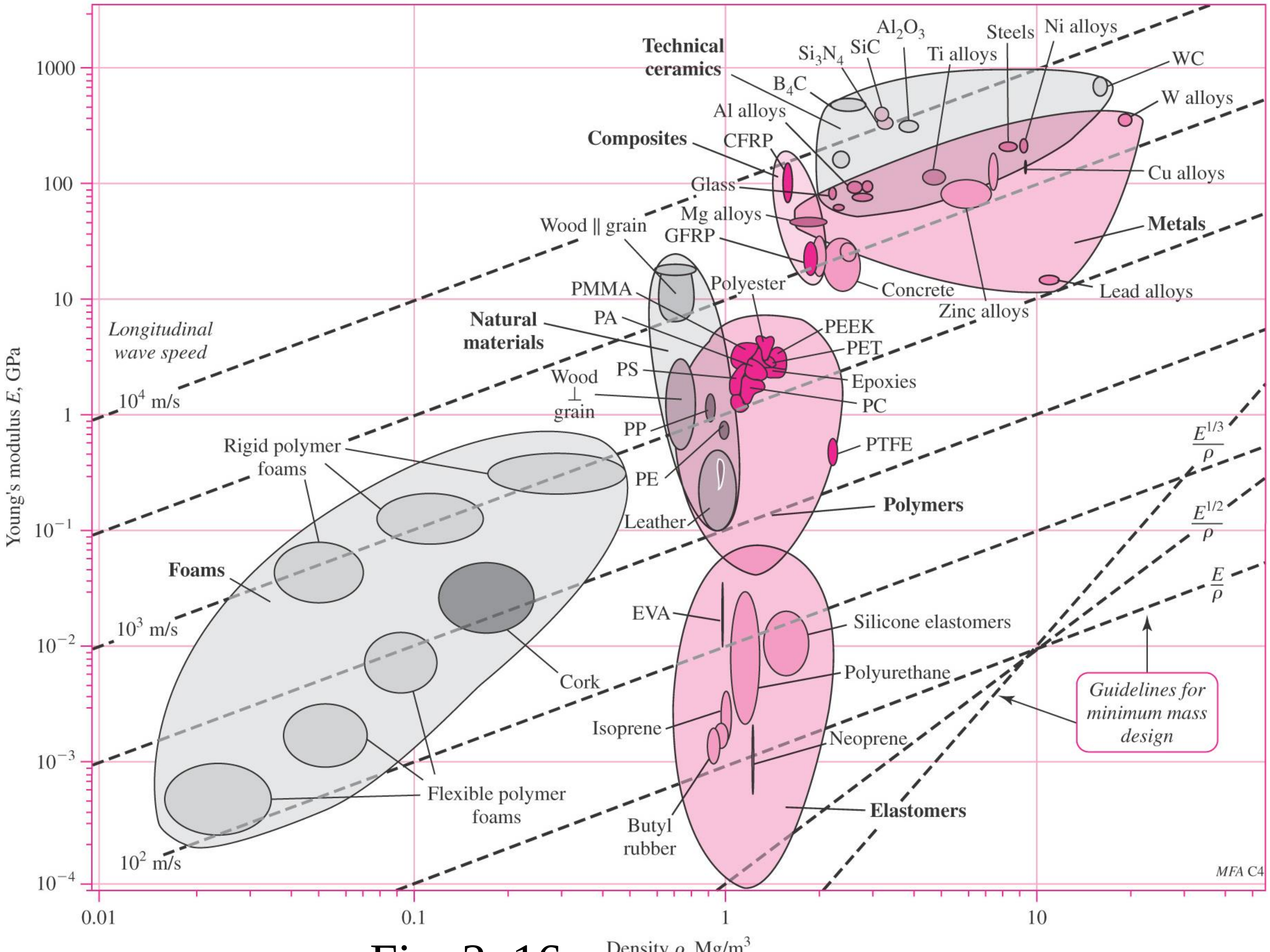


Fig. 2–16

Specific Modulus

- *Specific Modulus* – ratio of Young's modulus to density, E/ρ
- Also called *specific stiffness*
- Useful to minimize weight with primary design limitation of deflection, stiffness, or natural frequency
- Parallel lines representing different values of E/ρ allow comparison of specific modulus between materials

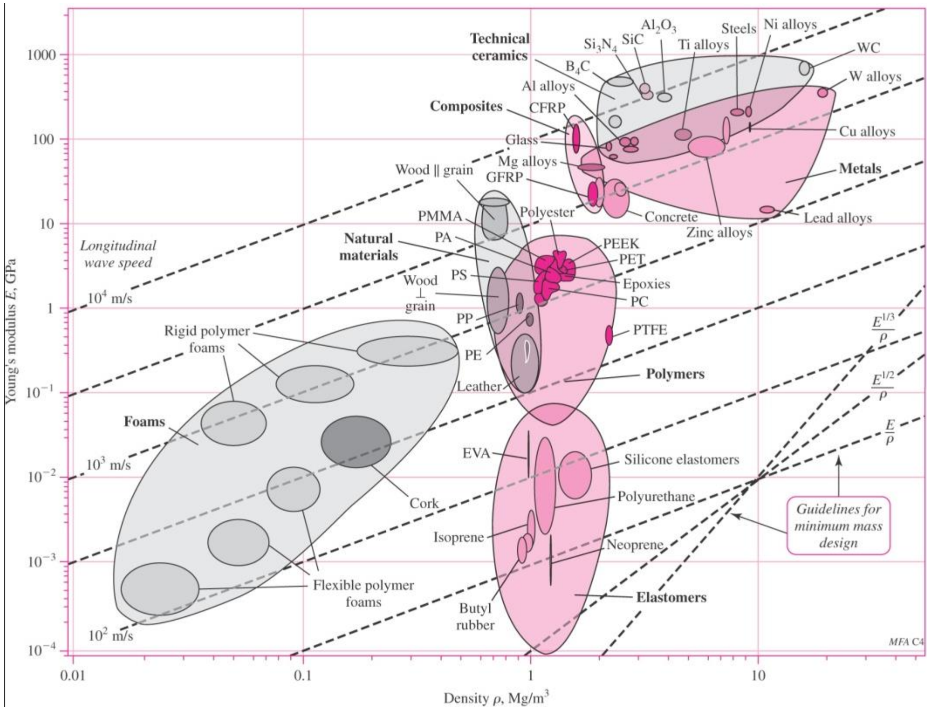


Fig. 2–16

Minimum Mass Guidelines for Young's Modulus-Density Plot

- Guidelines plot constant values of E^{β}/ρ
- β depends on type of loading
- $\beta = 1$ for axial
- $\beta = 1/2$ for bending

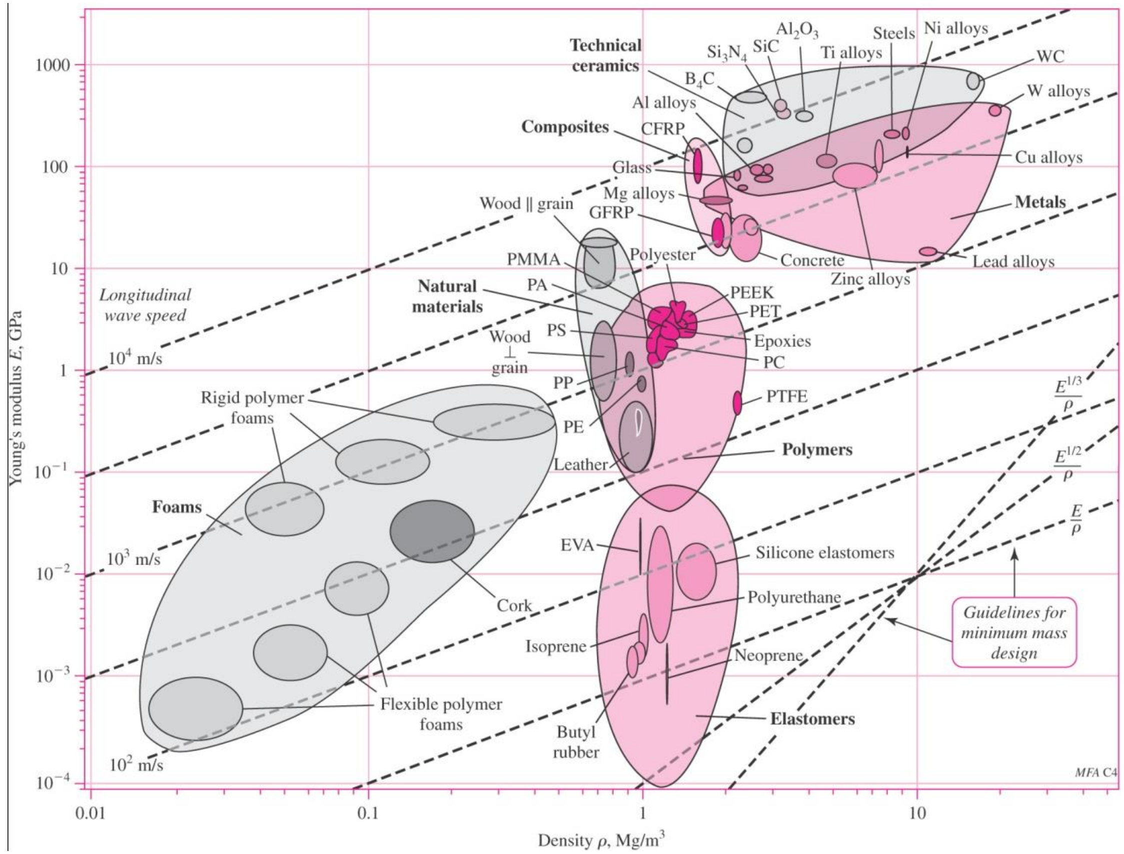


Fig. 2–16

Example, for axial loading,

$$k = AE/l \quad \Rightarrow \quad A = kl/E$$

$$m = Al\rho = (kl/E)\, l\rho = kl^2 \rho / E$$

Thus, to minimize mass, maximize $E/\rho \quad (\beta = 1)$

The Performance Metric

The *performance metric* depends on (1) the functional requirements, (2) the geometry, and (3) the material properties.

$$P = \left[\begin{pmatrix}\text{functional} \\ \text{requirements } F\end{pmatrix}, \begin{pmatrix}\text{geometric} \\ \text{parameters } G\end{pmatrix}, \begin{pmatrix}\text{material} \\ \text{properties } M\end{pmatrix}\right]$$

$$P = f(F, G, M)$$

The function is often separable,

$$P = f_1(F) \cdot f_2(G) \cdot f_3(M)$$

$f_3(M)$ is called the *material efficiency coefficient.*

Maximizing or minimizing $f_3(M)$ allows the material choice to be used to optimize P.

Performance Metric Example

- Requirements: light, stiff, end-loaded cantilever beam with circular cross section
- Mass *m* of the beam is chosen as the performance metric to minimize
- Stiffness is functional requirement
- Stiffness is related to material and geometry

$$k = \frac{F}{\delta}$$

Performance Metric Example

From beam deflection table, $\delta = \dfrac{Fl^3}{3EI}$

$$k = \frac{F}{\delta} = \frac{3EI}{l^3} \qquad (2\text{–}25)$$

$$I = \frac{\pi D^4}{64} = \frac{A^2}{4\pi} \qquad (2\text{–}26)$$

Sub Eq. (2-26) into Eq. (2-25) and solve for A

$$A = \left(\frac{4\pi k l^3}{3E}\right)^{1/2} \qquad (2\text{–}27)$$

The performance metric is

$$m = Al\rho \qquad (2\text{–}28)$$

Sub Eq. (2-27) into Eq. (2-28),

$$m = 2\sqrt{\frac{\pi}{3}}(k^{1/2})(l^{5/2})\left(\frac{\rho}{E^{1/2}}\right) \qquad (2\text{–}29)$$

Performance Metric Example

$$m = 2\sqrt{\frac{\pi}{3}}(k^{1/2})(l^{5/2})\left(\frac{\rho}{E^{1/2}}\right) \qquad (2\text{–}29)$$

Separating into the form of Eq. (2-24),

$$P = f_1(F) \cdot f_2(G) \cdot f_3(M) \qquad (2\text{–}24)$$

$$f_1(F) = 2\sqrt{\pi/3}(k^{1/2})$$

$$f_2(G) = (l^{5/2})$$

$$f_3(M) = \frac{\rho}{E^{1/2}} \qquad (2\text{–}30)$$

To minimize m, need to minimize $f_3(M)$, or maximize

$$M = \frac{E^{1/2}}{\rho} \qquad (2\text{–}31)$$

Performance Metric Example

- *M* is called *material index*
- For this example, $\beta = ½$
- Use guidelines parallel to $E^{1/2}/\rho$
- Increasing *M*, move up and to the left
- Good candidates for this example are certain woods, composites, and ceramics

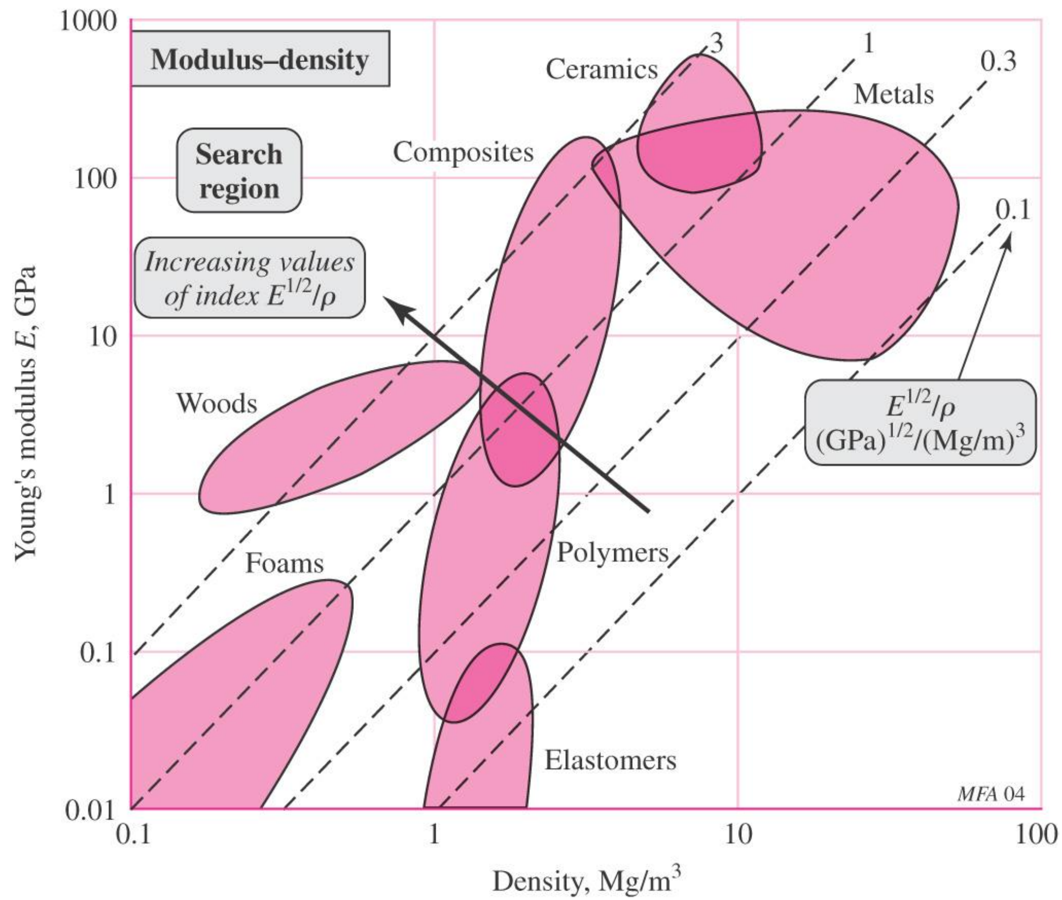


Fig. 2–17

Performance Metric Example

- Additional constraints can be added as needed
- For example, if it is desired that $E > 50$ GPa, add horizontal line to limit the solution space
- Wood is eliminated as a viable option

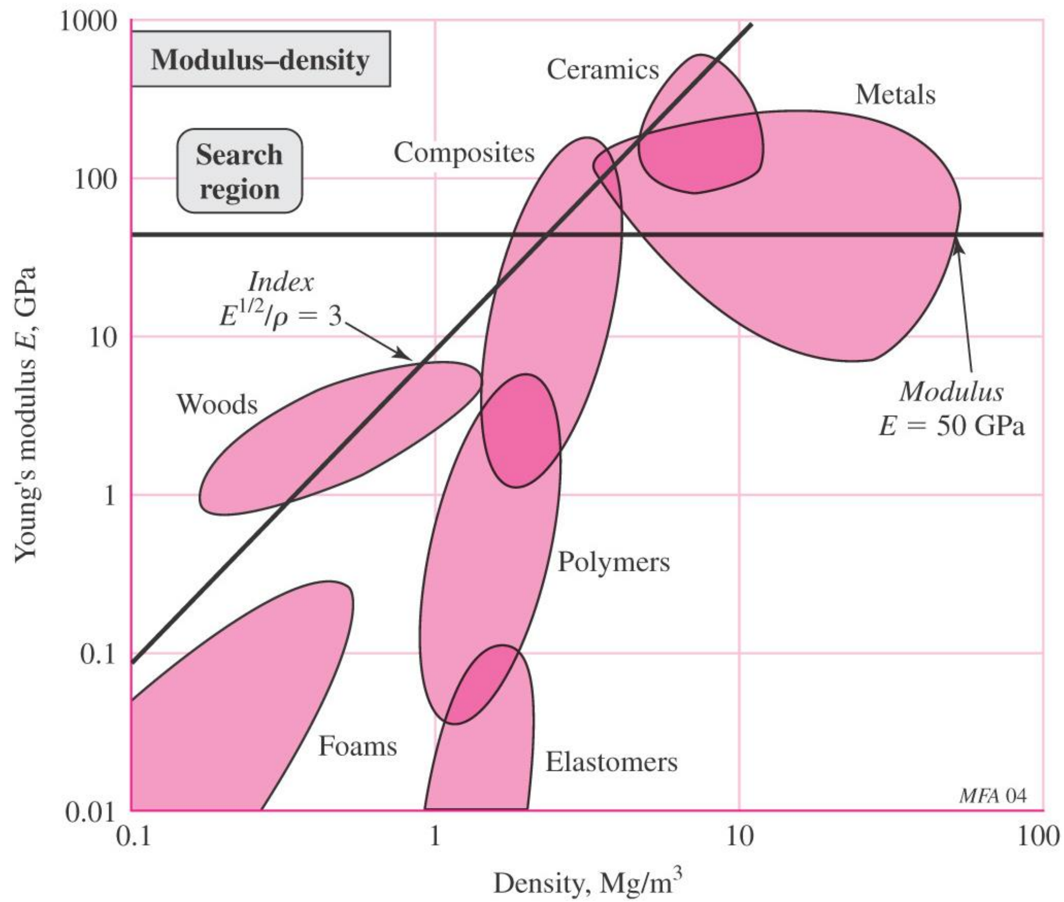


Fig. 2–18

Strength vs. Density

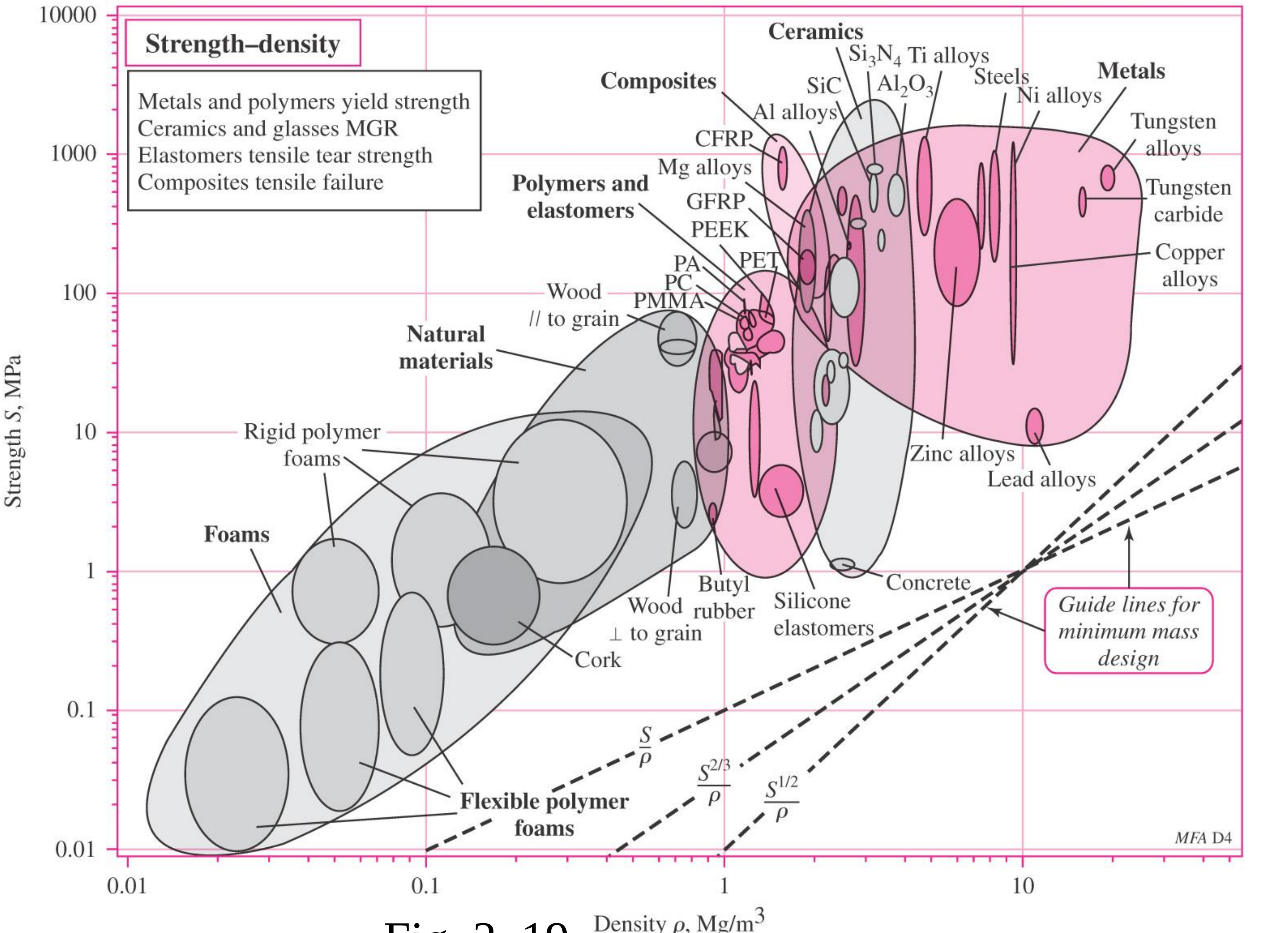


Fig. 2–19

Specific Modulus

- *Specific Strength* – ratio of strength to density, S / ρ
- Useful to minimize weight with primary design limitation of strength
- Parallel lines representing different values of S / ρ allow comparison of specific strength between materials

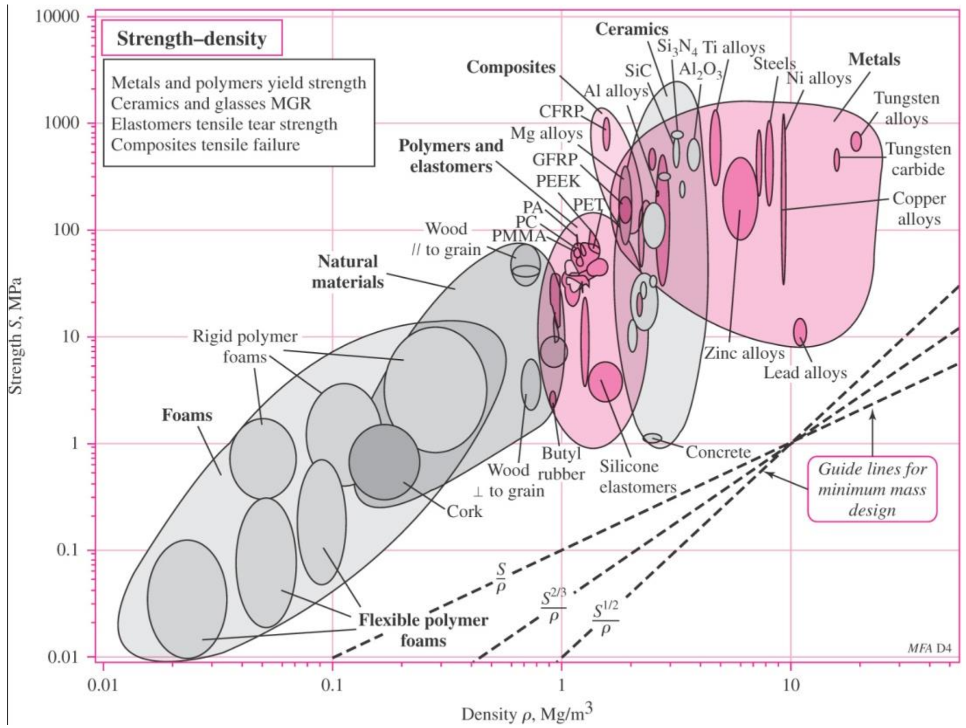
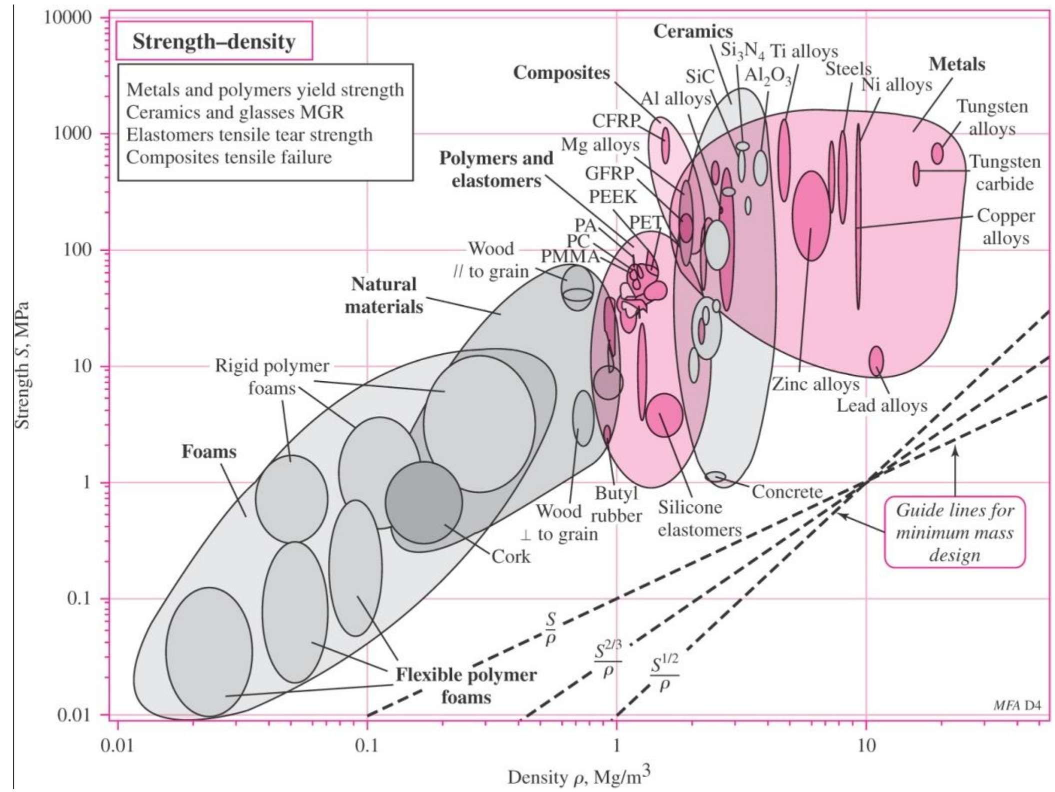


Fig. 2–19

Minimum Mass Guidelines for Strength-Density Plot

- Guidelines plot constant values of S^{β}/ρ
- β depends on type of loading
- β = 1 for axial
- β = 2/3 for bending



Example, for axial loading,

Fig. 2–19

$\sigma = F/A = S \quad \Rightarrow \quad A = F/S$

$m = Al\rho = (F/S)\ l\rho$

Thus, to minimize m, maximize $S/\rho \quad (\beta = 1)$