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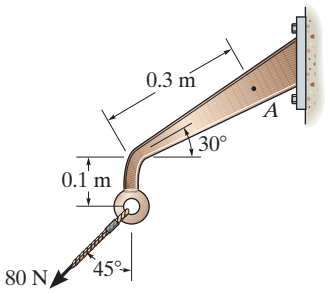
1–1. A force of 80 N is supported by the bracket as shown. Determine the resultant internal loadings acting on the section through point A.



SOLUTION

Equations of Equilibrium:

$+\nearrow \Sigma F_{x'} = 0;$ $\quad N_A - 80\cos 15° = 0$

$N_A = 77.3\ \text{N}$ **Ans.**

$\nwarrow^{+} \Sigma F_{y'} = 0;$ $\quad V_A - 80\sin 15° = 0$

$V_A = 20.7\ \text{N}$ **Ans.**

$\circlearrowleft + \Sigma M_A = 0;$ $\quad M_A + 80\cos 45°(0.3\cos 30°)$

$-80\sin 45°(0.1 + 0.3\sin 30°) = 0$

$M_A = -0.555\ \text{N}\cdot\text{m}$ **Ans.**

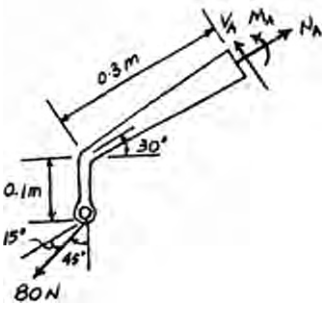
or

$\circlearrowleft + \Sigma M_A = 0;$ $\quad M_A + 80\sin 15°(0.3 + 0.1\sin 30°)$

$-80\cos 15°(0.1\cos 30°) = 0$

$M_A = -0.555\ \text{N}\cdot\text{m}$ **Ans.**

Negative sign indicates that M_A acts in the opposite direction to that shown on FBD.



These solutions represent a preliminary version of the Instructors' Solutions Manual (ISM). It is possible and even likely that at this preliminary stage of preparing the ISM there are some omissions and errors in the draft solutions. These will be corrected and this manual will be republished.

Ans:

$N_A = 77.3\ \text{N}, V_A = 20.7\ \text{N}, M_A = -0.555\ \text{N}\cdot\text{m}$

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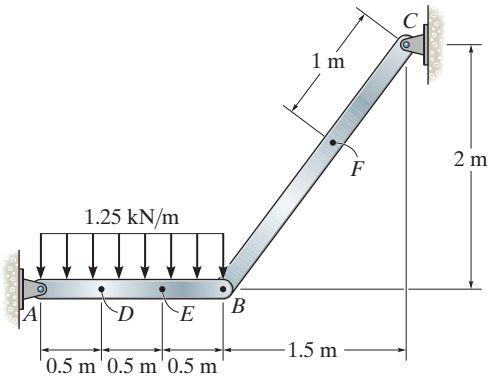
1–2.

Determine the resultant internal loadings on the cross section at point D.



SOLUTION

Support Reactions: Member BC is the two force member.

$$\zeta+\Sigma M_A = 0; \quad \frac{4}{5}F_{BC}(1.5) - 1.875(0.75) = 0$$

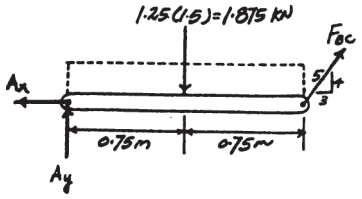
$$F_{BC} = 1.1719\ \text{kN}$$

$$+\uparrow\Sigma F_y = 0; \quad A_y + \frac{4}{5}(1.1719) - 1.875 = 0$$

$$A_y = 0.9375\ \text{kN}$$

$$\pm\rightarrow \Sigma F_x = 0; \quad \frac{3}{5}(1.1719) - A_x = 0$$

$$A_x = 0.7031\ \text{kN}$$



Equations of Equilibrium: For point D

$$\pm\rightarrow \Sigma F_x = 0; \quad N_D - 0.7031 = 0$$

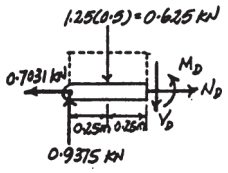
$$N_D = 0.703\ \text{kN} \qquad \textbf{Ans.}$$

$$+\uparrow\Sigma F_y = 0; \quad 0.9375 - 0.625 - V_D = 0$$

$$V_D = 0.3125\ \text{kN} \qquad \textbf{Ans.}$$

$$\zeta+\Sigma M_D = 0; \quad M_D + 0.625(0.25) - 0.9375(0.5) = 0$$

$$M_D = 0.3125\ \text{kN}\cdot\text{m} \qquad \textbf{Ans.}$$



Ans:
$N_D = 0.703$ kN,
$V_D = 0.3125$ kN,
$M_D = 0.3125\ \text{kN}\cdot\text{m}$

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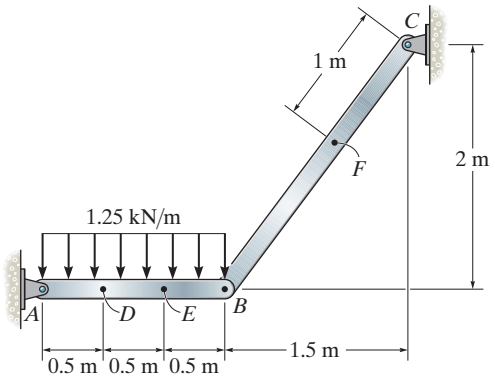
1–3.

Determine the resultant internal loadings at cross sections at points E and F on the assembly.



SOLUTION

Support Reactions: Member BC is the two-force member.

$$\circlearrowleft +\Sigma M_A = 0; \quad \frac{4}{5}F_{BC}(1.5) - 1.875(0.75) = 0$$

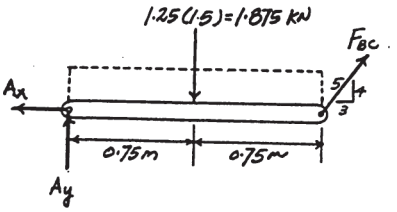
$$F_{BC} = 1.1719 \text{ kN}$$

$$+\uparrow \Sigma F_y = 0; \quad A_y + \frac{4}{5}(1.1719) - 1.875 = 0$$

$$A_y = 0.9375 \text{ kN}$$

$$\pm \Sigma F_x = 0; \quad \frac{3}{5}(1.1719) - A_x = 0$$

$$A_x = 0.7031 \text{ kN}$$



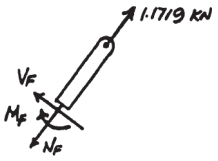
Equations of Equilibrium: For point F

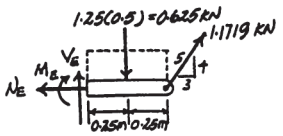
$$+\swarrow \Sigma F_{x'} = 0; \quad N_F - 1.1719 = 0$$

$$N_F = 1.17 \text{ kN} \quad \textbf{Ans.}$$

$$\nwarrow +\Sigma F_{y'} = 0; \quad V_F = 0 \quad \textbf{Ans.}$$

$$\circlearrowleft +\Sigma M_F = 0; \quad M_F = 0 \quad \textbf{Ans.}$$





Equations of Equilibrium: For point E

$$\pm \Sigma F_x = 0; \quad N_E - \frac{3}{5}(1.1719) = 0$$

$$N_E = 0.703 \text{ kN} \quad \textbf{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \quad V_E - 0.625 + \frac{4}{5}(1.1719) = 0$$

$$V_E = -0.3125 \text{ kN} \quad \textbf{Ans.}$$

$$\circlearrowleft +\Sigma M_E = 0; \quad -M_E - 0.625(0.25) + \frac{4}{5}(1.1719)(0.5) = 0$$

$$M_E = 0.3125 \text{ kN} \cdot \text{m} \quad \textbf{Ans.}$$

Negative sign indicates that $\mathbf{V}_E$ acts in the opposite direction to that shown on FBD.

Ans:
$N_F = 1.17$ kN,
$V_F = 0$,
$M_F = 0$,
$N_E = 0.703$ kN,
$V_E = -0.3125$ kN,
$M_E = 0.3125$ kN · m

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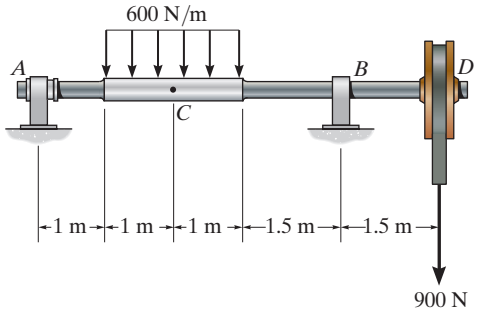
***1–4.**

The shaft is supported by a smooth thrust bearing at A and a smooth journal bearing at B. Determine the resultant internal loadings acting on the cross section at C.



SOLUTION

Support Reactions: We will only need to compute $\mathbf{B}_y$ by writing the moment equation of equilibrium about A with reference to the free-body diagram of the entire shaft, Fig. *a*.

$$\circlearrowleft + \Sigma M_A = 0; \qquad B_y(4.5) - 600(2)(2) - 900(6) = 0 \qquad B_y = 1733.33 \text{ N}$$

Internal Loadings: Using the result of $\mathbf{B}_y$, section CD of the shaft will be considered. Referring to the free-body diagram of this part, Fig. *b*,

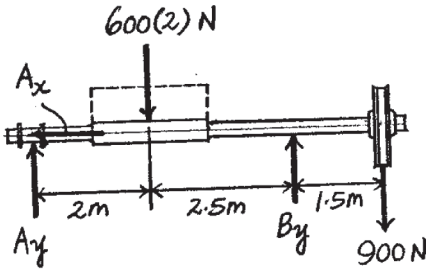
$$\pm \rightarrow \Sigma F_x = 0; \qquad N_C = 0 \qquad \textbf{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \qquad V_C - 600(1) + 1733.33 - 900 = 0 \qquad V_C = -233 \text{ N} \qquad \textbf{Ans.}$$

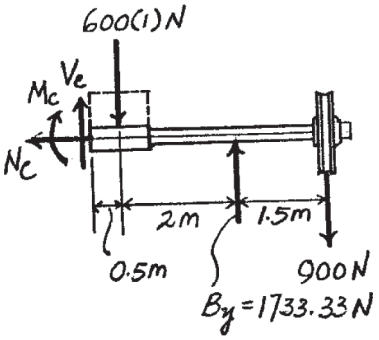
$$\circlearrowleft + \Sigma M_C = 0; \qquad 1733.33(2.5) - 600(1)(0.5) - 900(4) - M_C = 0$$

$$M_C = 433 \text{ N} \cdot \text{m} \qquad \textbf{Ans.}$$

The negative sign indicates that $\mathbf{V}_C$ acts in the opposite sense to that shown on the free-body diagram.



(a)



(b)

Ans:
$N_C = 0$,
$V_C = -233$ N,
$M_C = 433$ N · m

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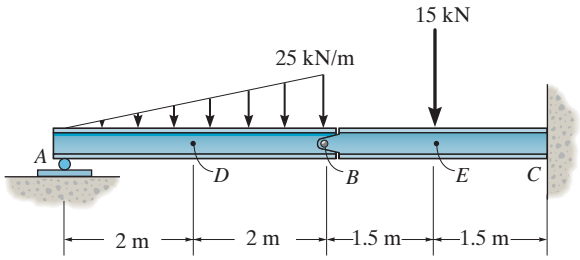
1–5. Determine the resultant internal loadings in the beam at cross sections through points D and E. Point E is just to the right of the 15-kN load.



SOLUTION

Support Reactions: For member AB

$\circlearrowleft + \Sigma M_B = 0; \quad 50(4/3) - A_y(4) = 0 \quad A_y = 16.67 \text{ kN}$

$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad B_x = 0$

$+\uparrow \Sigma F_y = 0; \quad B_y + 16.67 - 50 = 0 \quad B_y = 33.33 \text{ kN}$

Equations of Equilibrium: For point D

$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad N_D = 0$ **Ans.**

$+\uparrow \Sigma F_y = 0; \quad 16.67 - 12.5 - V_D = 0$

$V_D = 4.17 \text{ kN}$ **Ans.**

$\circlearrowleft + \Sigma M_D = 0; \quad M_D + 12.25(\tfrac{2}{3}) - 16.67(2) = 0$

$M_D = 25.17 \text{ kN} \cdot \text{m}$ **Ans.**

Equations of Equilibrium: For point E

$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad N_E = 0$ **Ans.**

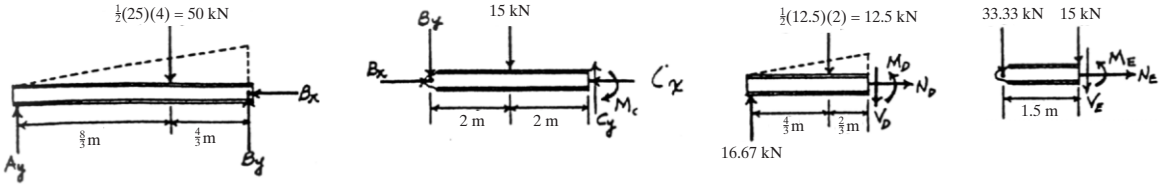
$+\uparrow \Sigma F_y = 0; \quad -33.33 - 15 - V_E = 0$

$V_E = -48.33 \text{ kN}$ **Ans.**

$\circlearrowleft + \Sigma M_E = 0; \quad M_E + 33.33(1.5) = 0$

$M_E = -50.00 \text{ kN} \cdot \text{m}$ **Ans.**

Negative signs indicate that M_E and V_E act in the opposite direction to that shown on FBD.



Ans:
$N_D = 0, V_D = 4.17 \text{ kN},$
$M_D = 25.0 \text{ kN} \cdot \text{m}, N_E = 0, V_E = -48.3 \text{ kN},$
$M_E = -50.0 \text{ kN} \cdot \text{m}$

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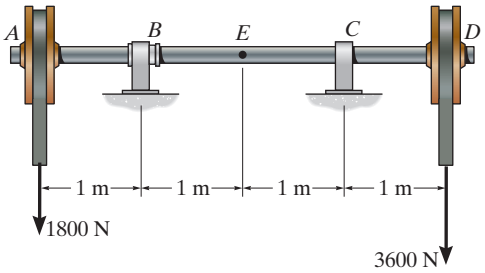
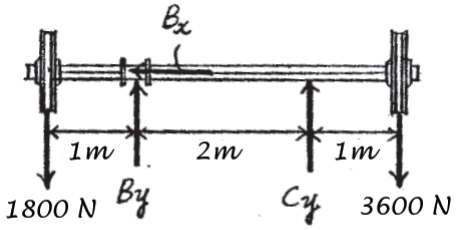
1–6. The shaft is supported by a smooth thrust bearing at B and a journal bearing at C. Determine the resultant internal loadings acting on the cross section at E.



SOLUTION

Support Reactions: We will only need to compute $\mathbf{C}_y$ by writing the moment equation of equilibrium about B with reference to the free-body diagram of the entire shaft, Fig. *a*.

$$\circlearrowleft + \Sigma M_B = 0; \qquad C_y(2) + 1800(1) - 3600(3) = 0 \qquad C_y = 4500\ \text{N}$$



(a)

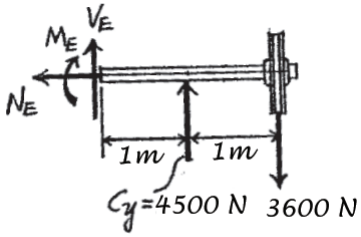
Internal Loadings: Using the result for $\mathbf{C}_y$, section DE of the shaft will be considered. Referring to the free-body diagram, Fig. *b*,

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad N_E = 0 \qquad \textbf{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \qquad V_E + 4500 - 3600 = 0 \qquad V_E = -900\ \text{N} \qquad \textbf{Ans.}$$

$$\circlearrowleft + \Sigma M_E = 0;\ 4500(1) - 3600(2) - M_E = 0$$

$$M_E = -2700\ \text{N}\cdot\text{m} = -2.70\ \text{kN}\cdot\text{m} \qquad \textbf{Ans.}$$



(b)

The negative signs indicates that $\mathbf{V}_E$ and $\mathbf{M}_E$ act in the opposite sense to that shown on the free-body diagram.

Ans:
$N_E = 0,\ V_E = -900\ \text{N},\ M_E = -2.7\ \text{kN}\cdot\text{m}$

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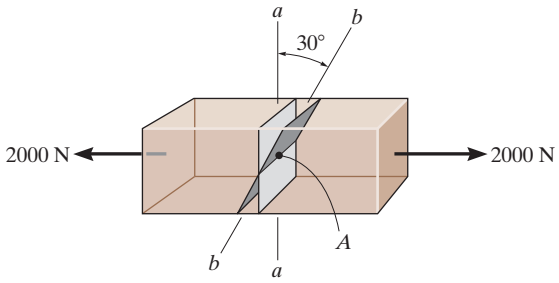
1–7. Determine the resultant internal normal and shear force in the member at (a) section *a–a* and (b) section *b–b*, each of which passes through point *A*. The 2000-N load is applied along the centroidal axis of the member.

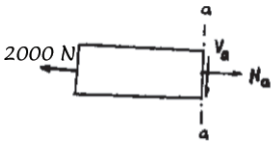


(a)

$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad N_a - 2000 = 0$

$N_a = 2000 \text{ N}$ **Ans.**

$+\downarrow \Sigma F_y = 0; \quad V_a = 0$ **Ans.**



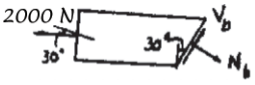
(b)

$\overset{+}{\searrow} \Sigma F_x = 0; \quad N_b - 2000 \cos 30° = 0$

$N_b = 1732 \text{ N}$ **Ans.**

$\overset{+}{\nearrow} \Sigma F_y = 0; \quad V_b - 2000 \sin 30° = 0$

$V_b = 1000 \text{ N}$ **Ans.**



Ans:
$N_a = 2000 \text{ N}, V_a = 0,$
$N_b = 1732 \text{ N}, V_b = 1000 \text{ N}$

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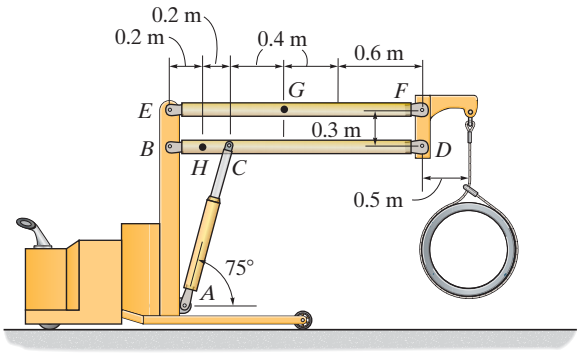
***1–8.**

The floor crane is used to lift a 600-kg concrete pipe. Determine the resultant internal loadings acting on the cross section at G.



SOLUTION

Support Reactions: We will only need to compute $\mathbf{F}_{EF}$ by writing the moment equation of equilibrium about D with reference to the free-body diagram of the hook, Fig. *a*.

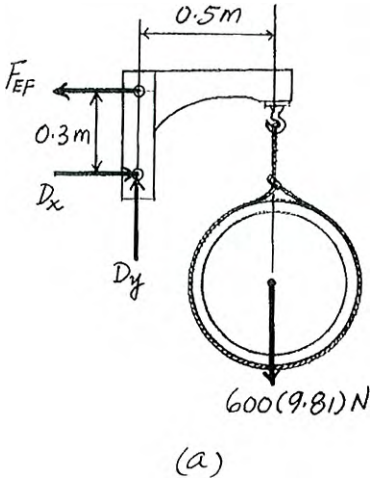
$$\curvearrowleft + \Sigma M_D = 0; \qquad F_{EF}(0.3) - 600(9.81)(0.5) = 0 \qquad F_{EF} = 9810\ \text{N}$$

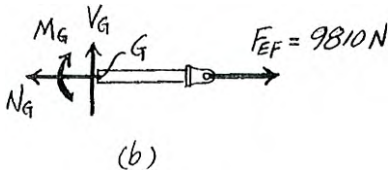
Internal Loadings: Using the result for $\mathbf{F}_{EF}$, section FG of member EF will be considered. Referring to the free-body diagram, Fig. *b*,

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 9810 - N_G = 0 \qquad N_G = 9810\ \text{N} = 9.81\ \text{kN} \qquad \textbf{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \qquad V_G = 0 \qquad \textbf{Ans.}$$

$$\curvearrowleft + \Sigma M_G = 0; \qquad M_G = 0 \qquad \textbf{Ans.}$$





Ans:

$N_G = 9.81\ \text{kN},\ V_G = 0,\ M_G = 0$

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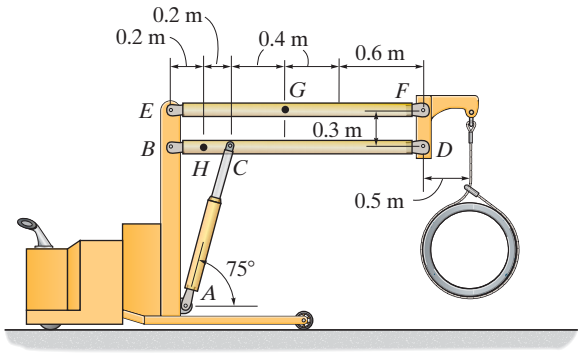
1–9. The floor crane is used to lift a 600-kg concrete pipe. Determine the resultant internal loadings acting on the cross section at H.



SOLUTION

Support Reactions: Referring to the free-body diagram of the hook, Fig. *a*.

$\zeta + \Sigma M_F = 0; \quad D_x(0.3) - 600(9.81)(0.5) = 0 \quad D_x = 9810\ \text{N}$

$+\uparrow \Sigma F_y = 0; \quad D_y - 600(9.81) = 0 \quad D_y = 5886\ \text{N}$

Subsequently, referring to the free-body diagram of member BCD, Fig. *b*,

$\zeta + \Sigma M_B = 0; \quad F_{AC} \sin 75°(0.4) - 5886(1.8) = 0 \quad F_{AC} = 27\,421.36\ \text{N}$

$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad B_x + 27\,421.36 \cos 75° - 9810 = 0 \quad B_x = 2712.83\ \text{N}$

$+\uparrow \Sigma F_y = 0; \quad 27\,421.36 \sin 75° - 5886 - B_y = 0 \quad B_y = 20\,601\ \text{N}$

Internal Loadings: Using the results of $\mathbf{B}_x$ and $\mathbf{B}_y$, section BH of member BCD will be considered. Referring to the free-body diagram of this part shown in Fig. *c*,

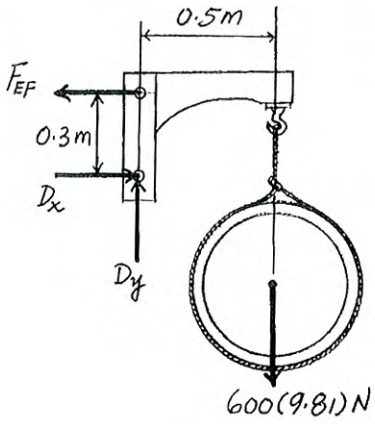
$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad N_H + 2712.83 = 0 \quad N_H = -2712.83\ \text{N} = -2.71\ \text{kN}$ **Ans.**

$+\uparrow \Sigma F_y = 0; \quad -V_H - 2060 = 0 \quad V_H = -20601\ \text{N} = -20.6\ \text{kN}$ **Ans.**

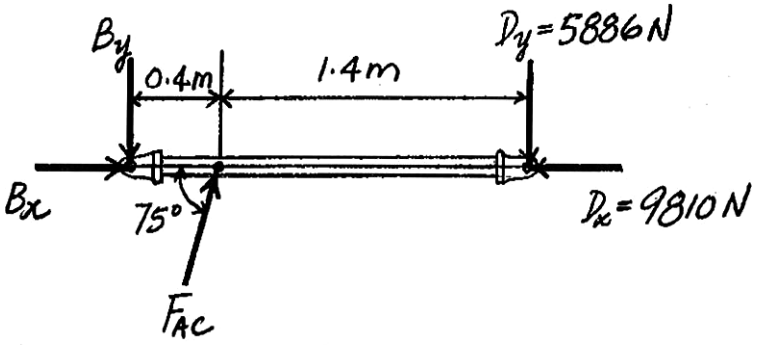
$\zeta + \Sigma M_D = 0; \quad M_H + 20601(0.2) = 0 \quad M_H = -4120.2\ \text{N} \cdot \text{m}$

$= -4.12\ \text{kN} \cdot \text{m}$ **Ans.**

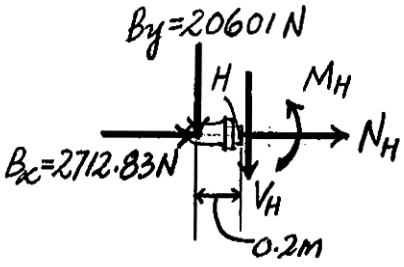
The negative signs indicates that N_H, V_H, and M_H act in the opposite sense to that shown on the free-body diagram.



(a)



(b)



(C)

Ans:

$N_H = -2.71$ kN, $V_H = -20.6$ kN,
$M_H = -4.12\ \text{kN} \cdot \text{m}$

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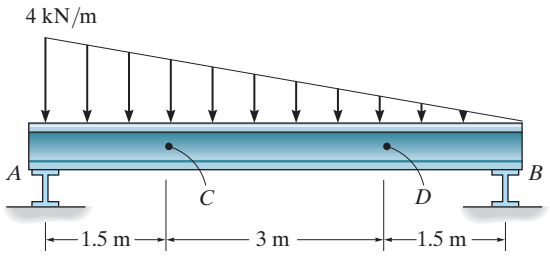
1–10.

The beam supports the distributed load shown. Determine the resultant internal loadings acting on the cross section at point C. Assume the reactions at the supports A and B are vertical.



SOLUTION

Support Reactions: Referring to the FBD of the entire beam, Fig. *a*,

$$\zeta+\Sigma M_A = 0; \qquad B_y(6) - \frac{1}{2}(4)(6)(2) = 0 \qquad B_y = 4.00\ \text{kN}$$

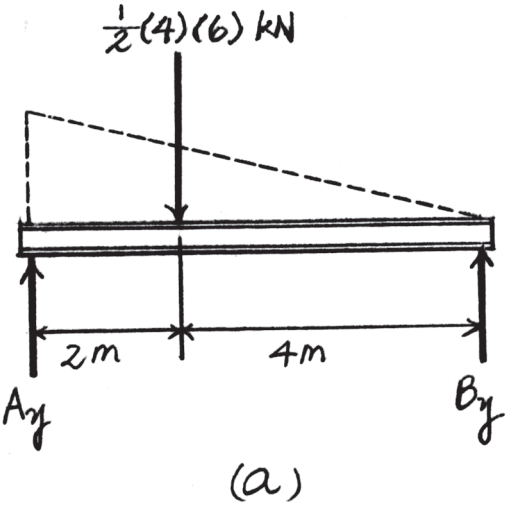
Internal Loadings: Referring to the FBD of the right segment of the beam sectioned through C, Fig. *b*,

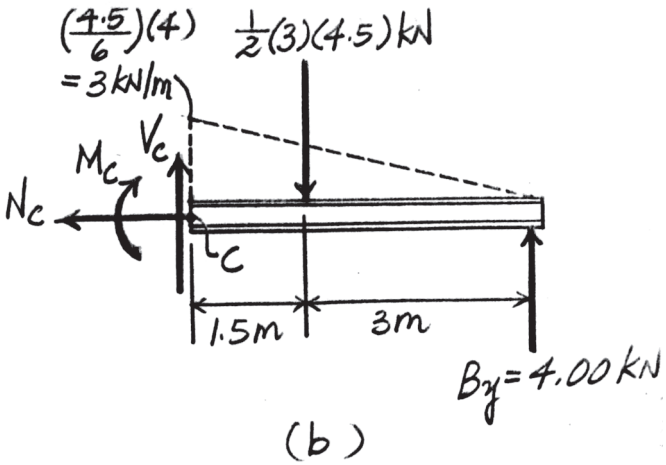
$$\pm\rightarrow \Sigma F_x = 0; \qquad N_C = 0 \qquad \textbf{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \qquad V_C + 4.00 - \frac{1}{2}(3)(4.5) = 0 \qquad V_C = 2.75\ \text{kN} \qquad \textbf{Ans.}$$

$$\zeta+\Sigma M_C = 0; \qquad 4.00(4.5) - \frac{1}{2}(3)(4.5)(1.5) - M_C = 0$$

$$M_C = 7.875\ \text{kN}\cdot\text{m} \qquad \textbf{Ans.}$$





Ans:
$N_C = 0$,
$V_C = 2.75$ kN,
$M_C = 7.875\ \text{kN}\cdot\text{m}$

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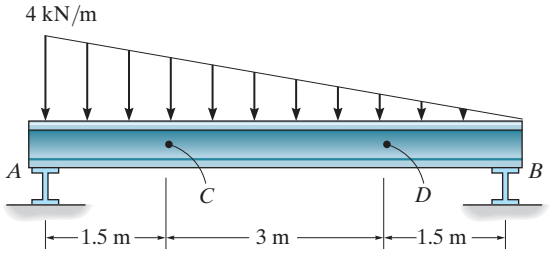
1–11.

The beam supports the distributed load shown. Determine the resultant internal loadings acting on the cross section at point D. Assume the reactions at the supports A and B are vertical.



SOLUTION

Support Reactions: Referring to the FBD of the entire beam, Fig. *a*,

$$\zeta+\Sigma M_A = 0; \qquad B_y(6) - \frac{1}{2}(4)(6)(2) = 0 \qquad B_y = 4.00\ \text{kN}$$

Internal Loadings: Referring to the FBD of the right segment of the beam sectioned through D, Fig. *b*,

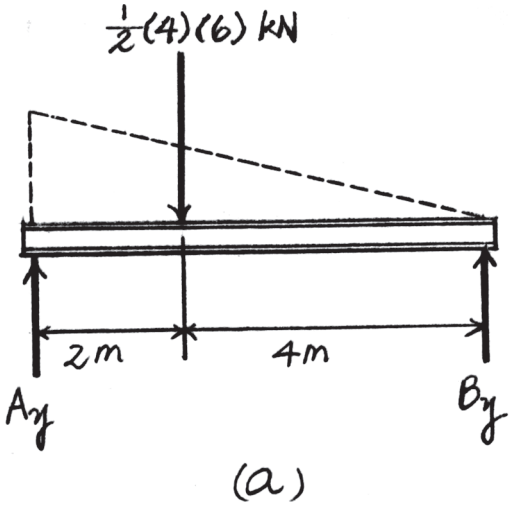
$$\pm\rightarrow \Sigma F_x = 0; \qquad N_D = 0 \qquad \textbf{Ans.}$$

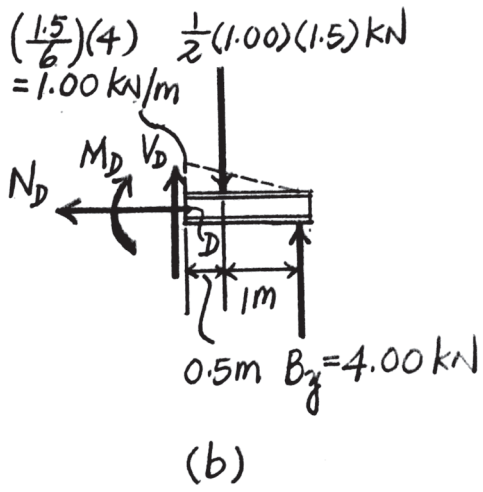
$$+\uparrow \Sigma F_y = 0; \qquad V_D + 4.00 - \frac{1}{2}(1.00)(1.5) = 0 \qquad V_D = -3.25\ \text{kN} \qquad \textbf{Ans.}$$

$$\zeta+\Sigma M_D = 0; \qquad 4.00(1.5) - \frac{1}{2}(1.00)(1.5)(0.5) - M_D = 0$$

$$M_D = 5.625\ \text{kN}\cdot\text{m} \qquad \textbf{Ans.}$$

The negative sign indicates that $\mathbf{V}_D$ acts in the sense opposite to that shown on the FBD.





Ans:
$N_D = 0$,
$V_D = -3.25$ kN,
$M_D = 5.625\ \text{kN}\cdot\text{m}$

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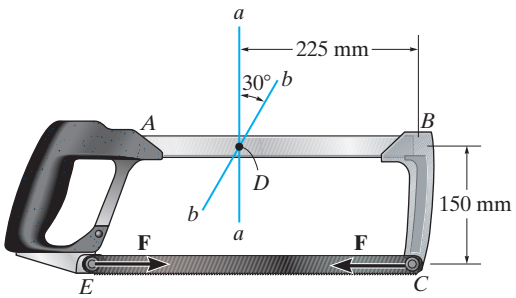
***1–12.**

The blade of the hacksaw is subjected to a pretension force of $F = 100$ N. Determine the resultant internal loadings acting on section *a–a* that passes through point *D*.



SOLUTION

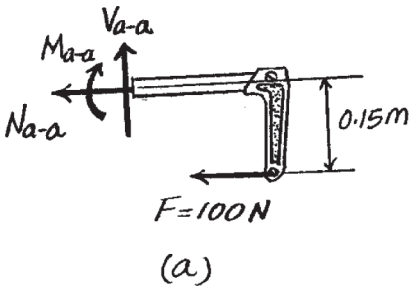
Internal Loadings: Referring to the free-body diagram of the section of the hacksaw shown in Fig. *a*,

$\pm \Sigma F_x = 0;$ $\quad N_{a-a} + 100 = 0$ $\quad N_{a-a} = -100$ N **Ans.**

$+\uparrow \Sigma F_y = 0;$ $\quad V_{a-a} = 0$ **Ans.**

$\circlearrowleft + \Sigma M_D = 0;$ $\quad -M_{a-a} - 100(0.15) = 0$ $\quad M_{a-a} = -15$ N·m **Ans.**

The negative sign indicates that $\mathbf{N}_{a-a}$ and $\mathbf{M}_{a-a}$ act in the opposite sense to that shown on the free-body diagram.



(a)

Ans:
$N_{a-a} = -100$ N, $V_{a-a} = 0$, $M_{a-a} = -15$ N·m

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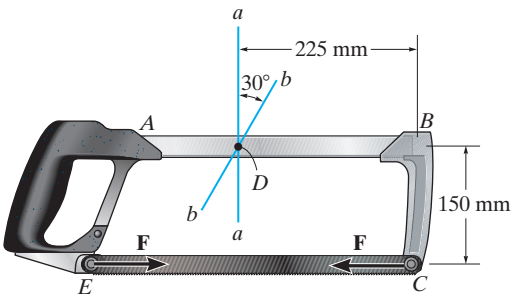
1–13.

The blade of the hacksaw is subjected to a pretension force of $F = 100$ N. Determine the resultant internal loadings acting on section b–b that passes through point D.



SOLUTION

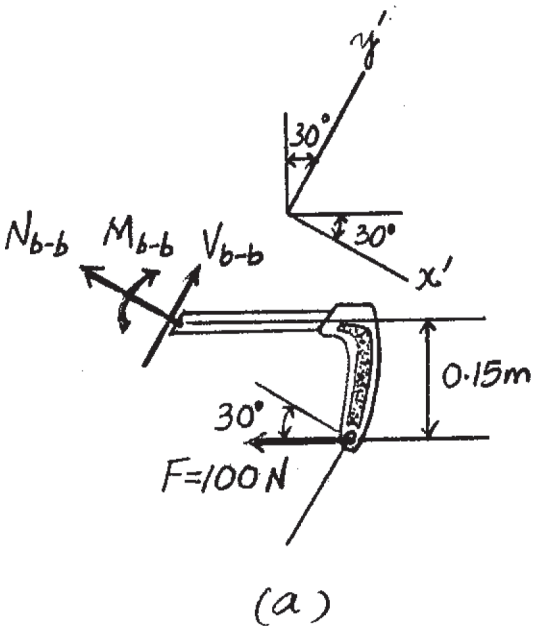
Internal Loadings: Referring to the free-body diagram of the section of the hacksaw shown in Fig. *a*,

$\Sigma F_{x'} = 0;$ $N_{b-b} + 100 \cos 30° = 0$ $N_{b-b} = -86.6$ N **Ans.**

$\Sigma F_{y'} = 0;$ $V_{b-b} - 100 \sin 30° = 0$ $V_{b-b} = 50$ N **Ans.**

$\zeta + \Sigma M_D = 0;$ $-M_{b-b} - 100(0.15) = 0$ $M_{b-b} = -15$ N · m **Ans.**

The negative sign indicates that $\mathbf{N}_{b-b}$ and $\mathbf{M}_{b-b}$ act in the opposite sense to that shown on the free-body diagram.



(a)

Ans:
$N_{b-b} = -86.6$ N, $V_{b-b} = 50$ N,
$M_{b-b} = -15$ N · m

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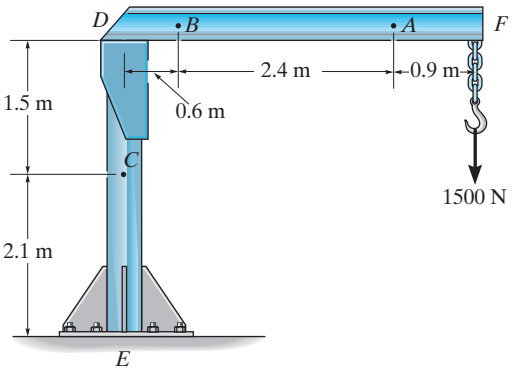
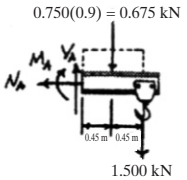
1–14. The boom DF of the jib crane and the column DE have a uniform wieght of 750 N/m. If the hoist and load weigh 1500 N, determine the resultant internal loadings in the crane on cross sections through points A, B, and C.



SOLUTION

Equations of Equilibrium: For point A



$\overset{+}{\leftarrow} \Sigma F_x = 0; \quad N_A = 0$ **Ans.**

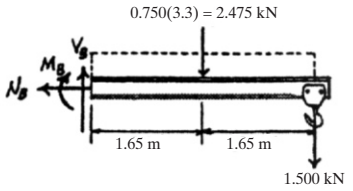
$+\uparrow \Sigma F_y = 0; \quad V_A - 0.675 - 1.500 = 0$

$V_A = 2.175 \text{ kN}$ **Ans.**

$\circlearrowleft + \Sigma M_A = 0; \quad -M_A - 0.675(0.45) - 1.500(0.9) = 0$

$M_A = -1.65 \text{ kN} \cdot \text{m}$ **Ans.**

Negative sign indicates that M_A acts in the opposite direction to that shown on FBD.



Equations of Equilibrium: For point B

$\overset{+}{\leftarrow} \Sigma F_x = 0; \quad N_B = 0$ **Ans.**

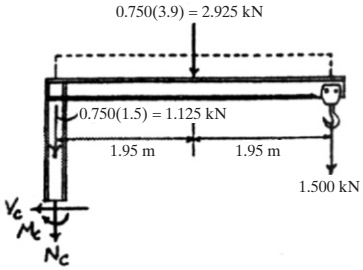
$+\uparrow \Sigma F_y = 0; \quad V_B - 2.475 - 1.5 = 0$

$V_B = 3.975 \text{ kN}$ **Ans.**

$\circlearrowleft + \Sigma M_B = 0; \quad -M_B - 2.475(1.65) - 1.500(3.3) = 0$

$M_B = -9.03 \text{ kN} \cdot \text{m}$ **Ans.**

Negative sign indicates that M_B acts in the opposite direction to that shown on FBD.



Equations of Equilibrium: For point C

$\overset{+}{\leftarrow} \Sigma F_x = 0; \quad V_C = 0$ **Ans.**

$+\uparrow \Sigma F_y = 0; \quad -N_C - 1.125 - 2.925 - 1.500 = 0$

$N_C = -5.55 \text{ kN}$ **Ans.**

$\circlearrowleft + \Sigma M_C = 0; \quad -M_C - 2.925(1.95) - 1.500(3.9) = 0$

$M_C = -11.6 \text{ kN} \cdot \text{m}$ **Ans.**

Negative signs indicate that N_C and M_C act in the opposite direction to that shown on FBD.

Ans:

$N_A = 0, V_A = 2.175 \text{ kN}, M_A = -1.65 \text{ kN} \cdot \text{m},$
$N_B = 0, V_B = 3.975 \text{ kN}, M_B = -9.03 \text{ kN} \cdot \text{m},$
$V_C = 0, N_C = -5.55 \text{ kN}, M_C = -11.6 \text{ kN} \cdot \text{m}$

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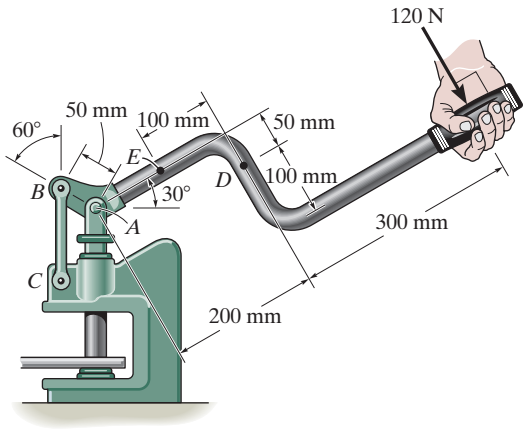
1–15.

The metal stud punch is subjected to a force of 120 N on the handle. Determine the magnitude of the reactive force at the pin A and in the short link BC. Also, determine the resultant internal loadings acting on the cross section at point D.



SOLUTION

Member:

$\zeta+\Sigma M_A = 0;$ $\quad F_{BC}\cos 30°(50) - 120(500) = 0$

$F_{BC} = 1385.6\text{ N} = 1.39\text{ kN}$ **Ans.**

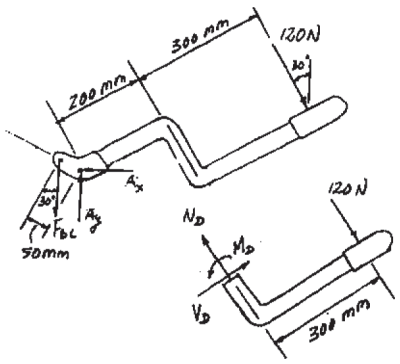
$+\uparrow\Sigma F_y = 0;$ $\quad A_y - 1385.6 - 120\cos 30° = 0$

$A_y = 1489.56\text{ N}$

$\pm\Sigma F_x = 0;$ $\quad A_x - 120\sin 30° = 0;$ $\quad A_x = 60\text{ N}$

$F_A = \sqrt{1489.56^2 + 60^2}$

$= 1491\text{ N} = 1.49\text{ kN}$ **Ans.**



Segment:

$\nwarrow^+\Sigma F_{x'} = 0;$ $\quad N_D - 120 = 0$

$N_D = 120\text{ N}$ **Ans.**

$+\nearrow\Sigma F_{y'} = 0;$ $\quad V_D = 0$ **Ans.**

$\zeta+\Sigma M_D = 0;$ $\quad M_D - 120(0.3) = 0$

$M_D = 36.0\text{ N}\cdot\text{m}$ **Ans.**

Ans:
$F_{BC} = 1.39\text{ kN}, F_A = 1.49\text{ kN}, N_D = 120\text{ N},$
$V_D = 0, M_D = 36.0\text{ N}\cdot\text{m}$

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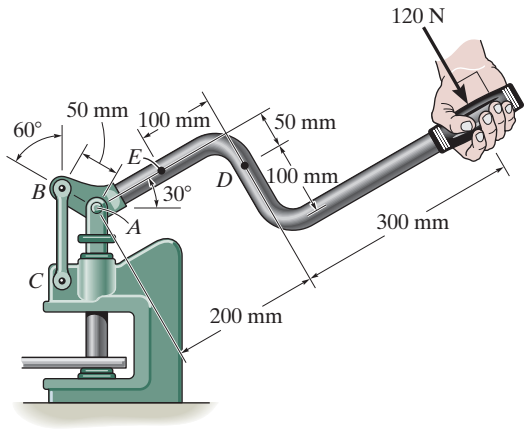
***1–16.**

Determine the resultant internal loadings acting on the cross section at point E of the handle arm, and on the cross section of the short link BC.



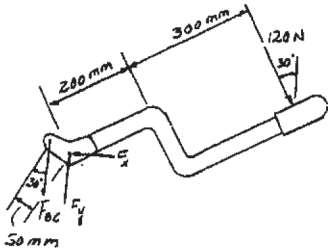
SOLUTION

Member:

$\zeta+\Sigma M_A = 0; \qquad F_{BC}\cos 30°(50) - 120(500) = 0$

$F_{BC} = 1385.6\ \text{N} = 1.3856\ \text{kN}$

Segment:

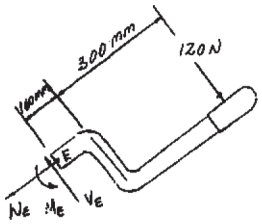


$+\swarrow\Sigma F_{x'} = 0; \qquad N_E = 0$ **Ans.**

$\nwarrow+\Sigma F_{y'} = 0; \qquad V_E - 120 = 0; \qquad V_E = 120\ \text{N}$ **Ans.**

$\zeta+\Sigma M_E = 0; \qquad M_E - 120(0.4) = 0; \qquad M_E = 48.0\ \text{N}\cdot\text{m}$ **Ans.**

Short link:



$\pm\Sigma F_x = 0; \qquad V = 0$ **Ans.**

$+\uparrow\Sigma F_y = 0; \qquad 1.3856 - N = 0; \qquad N = 1.39\ \text{kN}$ **Ans.**

$\zeta+\Sigma M_H = 0; \qquad M = 0$ **Ans.**

Ans:
$N_E = 0, V_E = 120\ \text{N}, M_E = 48.0\ \text{N}\cdot\text{m}$,
Short link: $V = 0, N = 1.39\ \text{kN}, M = 0$

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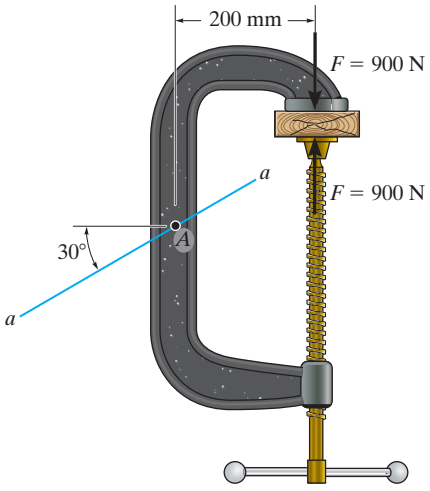
1–17. The forged steel clamp exerts a force of $F = 900$ N on the wooden block. Determine the resultant internal loadings acting on section a–a passing through point A.



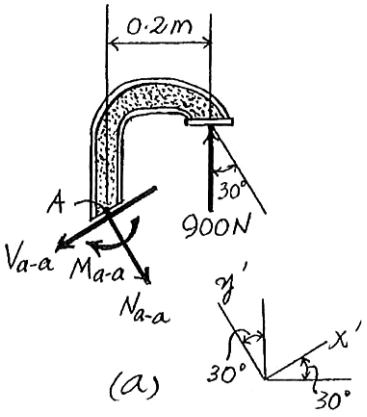
SOLUTION

Internal Loadings: Referring to the free-body diagram of the section of the clamp shown in Fig. a,

$\Sigma F_{y'} = 0;$ $\quad 900 \cos 30° - N_{a-a} = 0$ $\quad N_{a-a} = 779$ N **Ans.**

$\Sigma F_{x'} = 0;$ $\quad V_{a-a} - 900 \sin 30° = 0$ $\quad V_{a-a} = 450$ N **Ans.**

$\zeta + \Sigma M_A = 0;$ $\quad 900(0.2) - M_{a-a} = 0$ $\quad M_{a-a} = 180$ N · m **Ans.**



Ans:
$N_{a-a} = 779$ N, $V_{a-a} = 450$ N,
$900(0.2) - M_{a-a} = 0$, $M_{a-a} = 180$ N · m

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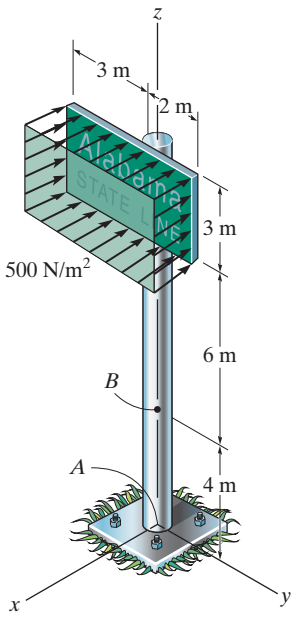
1–18. Determine the resultant internal loadings acting on the cross section through point B of the signpost. The post is fixed to the ground and a uniform pressure of 500 N/m^2 acts perpendicular to the face of the sign.



SOLUTION

$\Sigma F_x = 0; \quad (V_B)_x - 7500 = 0; \quad (V_B)_x = 7500\ \text{N} = 7.5\ \text{kN}$ **Ans.**

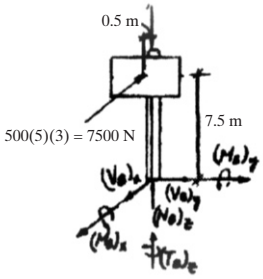
$\Sigma F_y = 0; \quad (V_B)_y = 0$ **Ans.**

$\Sigma F_z = 0; \quad (N_B)_z = 0$ **Ans.**

$\Sigma M_x = 0; \quad (M_B)_x = 0$ **Ans.**

$\Sigma M_y = 0; \quad (M_B)_y - 7500(7.5) = 0; \quad (M_B)_y = 56250\ \text{N} \cdot \text{m} = 56.25\ \text{kN} \cdot \text{m}$ **Ans.**

$\Sigma M_z = 0; \quad (T_B)_z - 7500(0.5) = 0; \quad (T_B)_z = 3750\ \text{N} \cdot \text{m} = 3.75\ \text{kN} \cdot \text{m}$ **Ans.**



Ans:

$(V_B)_x = 7.5\ \text{kN}, (V_B)_y = 0, (N_B)_z = 0,$
$(M_B)_x = 0, (M_B)_y = 56.25\ \text{kN} \cdot \text{m},$
$(T_B)_z = 3.75\ \text{kN} \cdot \text{m}$

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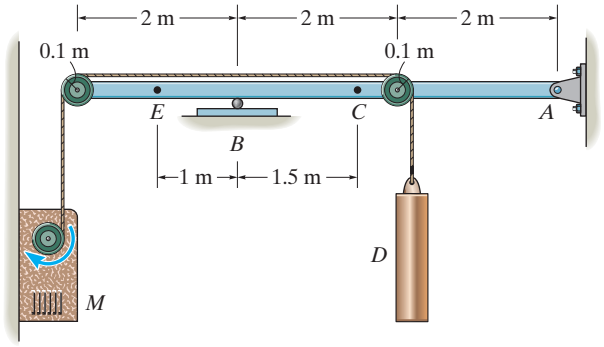
1–19.

Determine the resultant internal loadings acting on the cross section at point C in the beam. The load D has a mass of 300 kg and is being hoisted by the motor M with constant velocity.



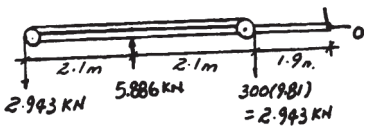
SOLUTION

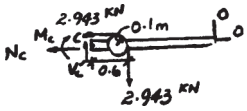
$\stackrel{+}{\leftarrow}\Sigma F_x = 0; \quad N_C + 2.943 = 0; \quad N_C = -2.94\ \text{kN}$ **Ans.**

$+\uparrow\Sigma F_y = 0; \quad V_C - 2.943 = 0; \quad V_C = 2.94\ \text{kN}$ **Ans.**

$\zeta+\Sigma M_C = 0; \quad -M_C - 2.943(0.6) + 2.943(0.1) = 0$

$M_C = -1.47\ \text{kN}\cdot\text{m}$ **Ans.**





Ans:

$N_C = -2.94\ \text{kN}$,
$V_C = 2.94\ \text{kN}$,
$M_C = -1.47\ \text{kN}\cdot\text{m}$

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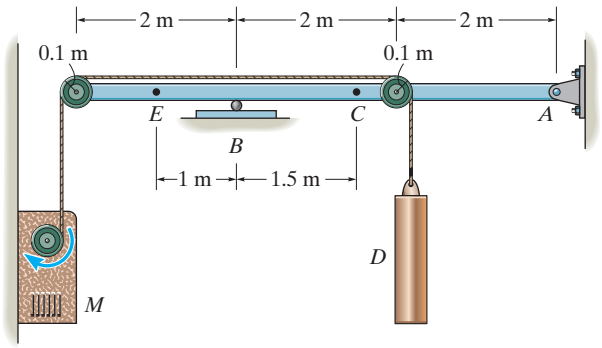
***1–20.**

Determine the resultant internal loadings acting on the cross section at point E. The load D has a mass of 300 kg and is being hoisted by the motor M with constant velocity.



SOLUTION

$\stackrel{+}{\rightarrow}\Sigma F_x = 0;\quad N_E + 2943 = 0$

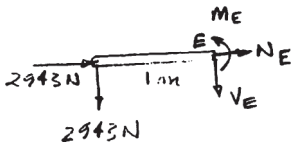
$N_E = -2.94\ \text{kN}$ **Ans.**

$+\uparrow\Sigma F_y = 0;\quad -2943 - V_E = 0$

$V_E = -2.94\ \text{kN}$ **Ans.**

$\circlearrowleft+\Sigma M_E = 0;\quad M_E + 2943(1) = 0$

$M_E = -2.94\ \text{kN}\cdot\text{m}$ **Ans.**



Ans:

$N_E = -2.94\ \text{kN}$,
$V_E = -2.94\ \text{kN}$,
$M_E = -2.94\ \text{kN}\cdot\text{m}$

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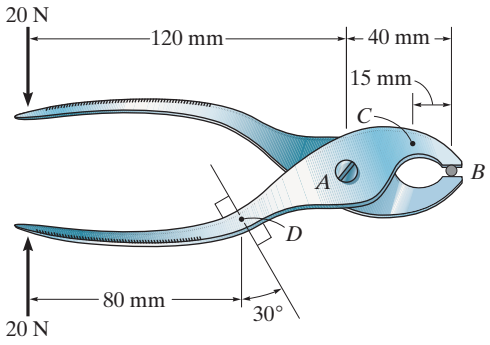
1–21. Determine the resultant internal loading on the cross section through point C of the pliers. There is a pin at A, and the jaws at B are smooth.

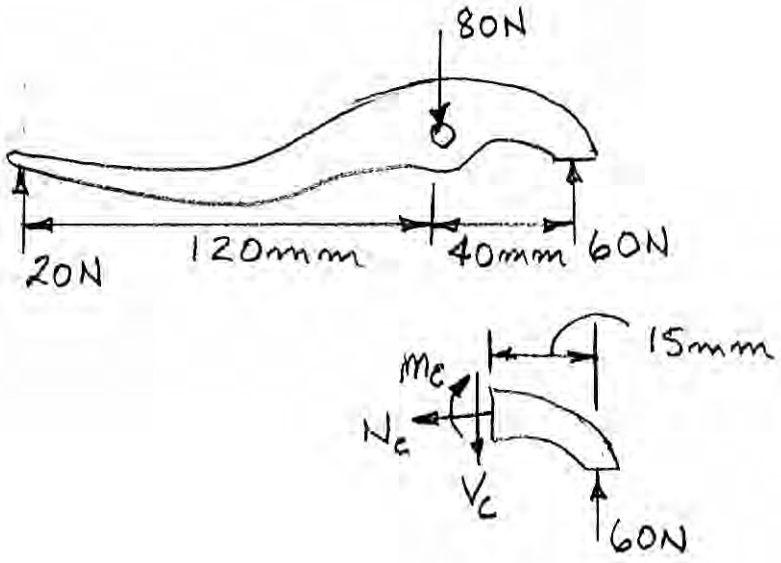


SOLUTION

$+\uparrow \Sigma F_y = 0; \quad -V_C + 60 = 0; \quad V_C = 60\,\text{N}$ **Ans.**

$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad N_C = 0$ **Ans.**

$+\circlearrowleft \Sigma M_C = 0; \quad -M_C + 60(0.015) = 0; \quad M_C = 0.9\,\text{N.m}$ **Ans.**



Ans:

$V_C = 60\,\text{N}, N_C = 0, M_C = 0.9\,\text{N}\cdot\text{m}$

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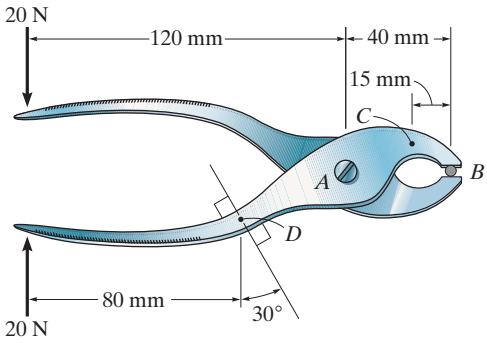
1–22. Determine the resultant internal loading on the cross section through point *D* of the pliers.

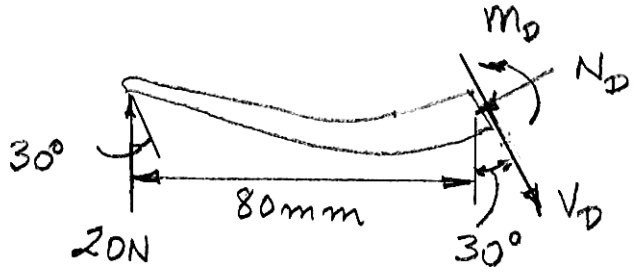


SOLUTION

$\searrow+\Sigma F_y = 0;\quad V_D - 20\cos 30° = 0;\quad V_D = 17.3\text{ N}$ **Ans.**

$+\swarrow\Sigma F_x = 0;\quad N_D - 20\sin 30° = 0;\quad N_D = 10\text{ N}$ **Ans.**

$+\circlearrowleft\Sigma M_D = 0;\quad M_D - 20(0.08) = 0;\quad M_D = 1.60\text{ N.m}$ **Ans.**



Ans:

$V_D = 17.3\text{ N}, N_D = 10\text{ N}, M_D = 1.60\text{ N}\cdot\text{m}$

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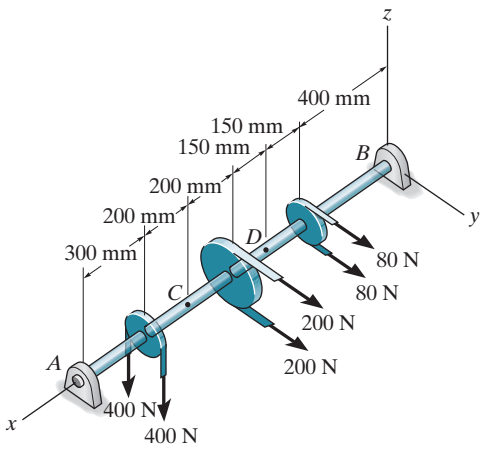
1–23.

The shaft is supported at its ends by two bearings A and B and is subjected to the forces applied to the pulleys fixed to the shaft. Determine the resultant internal loadings acting on the cross section at point C. The 400-N forces act in the $-z$ direction and the 200-N and 80-N forces act in the $+y$ direction. The journal bearings at A and B exert only y and z components of force on the shaft.



SOLUTION

Support Reactions:

$\Sigma M_z = 0; \quad 160(0.4) + 400(0.7) - A_y(1.4) = 0$

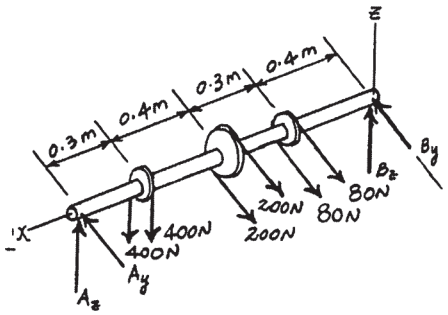
$A_y = 245.71 \text{ N}$

$\Sigma F_y = 0; \quad -245.71 - B_y + 400 + 160 = 0$

$B_y = 314.29 \text{ N}$

$\Sigma M_y = 0; \quad 800(1.1) - A_z(1.4) = 0 \qquad A_z = 628.57 \text{ N}$

$\Sigma F_z = 0; \quad B_z + 628.57 - 800 = 0 \qquad B_z = 171.43 \text{ N}$



Equations of Equilibrium: For point C

$\Sigma F_x = 0; \quad (N_C)_x = 0$ **Ans.**

$\Sigma F_y = 0; \quad -245.71 + (V_C)_y = 0$

$(V_C)_y = -246 \text{ N}$ **Ans.**

$\Sigma F_z = 0; \quad 628.57 - 800 + (V_C)_z = 0$

$(V_C)_z = -171 \text{ N}$ **Ans.**

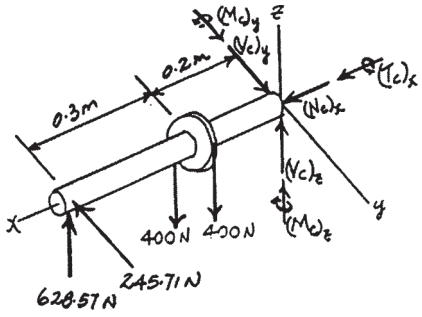
$\Sigma M_x = 0; \quad (T_C)_x = 0$ **Ans.**

$\Sigma M_y = 0; \quad (M_C)_y - 628.57(0.5) + 800(0.2) = 0$

$(M_C)_y = -154 \text{ N} \cdot \text{m}$ **Ans.**

$\Sigma M_z = 0; \quad (M_C)_z - 245.71(0.5) = 0$

$(M_C)_z = -123 \text{ N} \cdot \text{m}$ **Ans.**



Ans:
$(N_C)_x = 0,$
$(V_C)_y = -246 \text{ N},$
$(V_C)_z = -171 \text{ N},$
$(T_C)_x = 0,$
$(M_C)_y = -154 \text{ N} \cdot \text{m},$
$(M_C)_z = -123 \text{ N} \cdot \text{m}$

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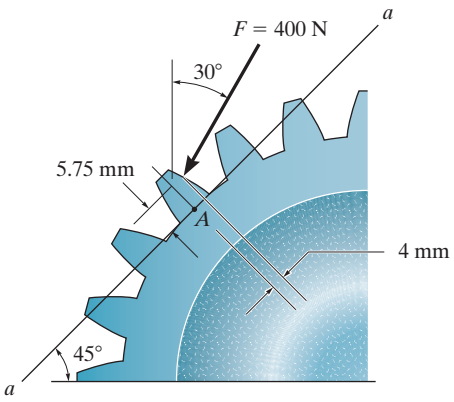
*** 1.24.**

The force $F = 400$ N acts on the gear tooth. Determine the resultant internal loadings on the root of the tooth, i.e., at the centroid point A of section a–a.



SOLUTION

Equations of Equilibrium: For section a–a

$+\nearrow \Sigma F_{x'} = 0; \quad V_A - 400 \cos 15° = 0$

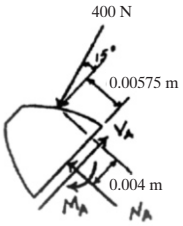
$$V_A = 386.37 \text{ N} \quad \textbf{Ans.}$$

$\nwarrow^{+} \Sigma F_{y'} = 0; \quad N_A - 400 \sin 15° = 0$

$$N_A = 103.53 \text{ N} \quad \textbf{Ans.}$$

$\circlearrowleft + \Sigma M_A = 0; \quad -M_A - 400 \sin 15°(0.004) + 400 \cos 15°(0.00575) = 0$

$$M_A = 1.808 \text{ N} \cdot \text{m} \quad \textbf{Ans.}$$



Ans:
$V_A = 386.37$ N, $N_A = 103.53$ N,
$M_A = 1.808$ N · m

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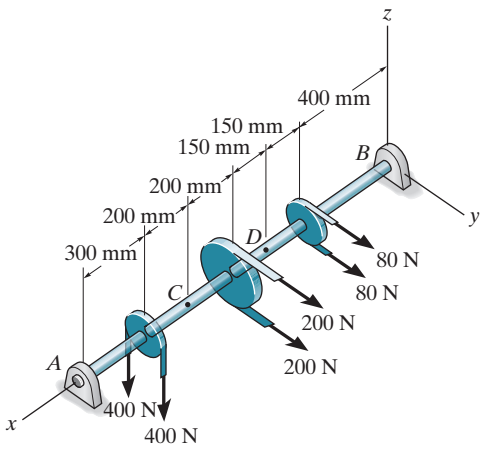
1–25.

The shaft is supported at its ends by two bearings A and B and is subjected to the forces applied to the pulleys fixed to the shaft. Determine the resultant internal loadings acting on the cross section at point D. The 400-N forces act in the $-z$ direction and the 200-N and 80-N forces act in the $+y$ direction. The journal bearings at A and B exert only y and z components of force on the shaft.



SOLUTION

Support Reactions:

$\Sigma M_z = 0; \quad 160(0.4) + 400(0.7) - A_y(1.4) = 0$

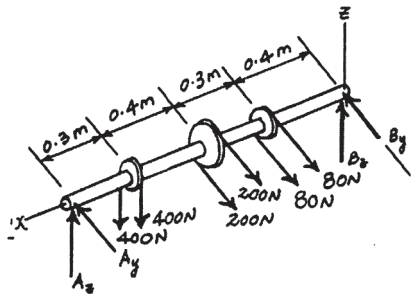
$A_y = 245.71 \text{ N}$

$\Sigma F_y = 0; \quad -245.71 - B_y + 400 + 160 = 0$

$B_y = 314.29 \text{ N}$

$\Sigma M_y = 0; \quad 800(1.1) - A_z(1.4) = 0 \qquad A_z = 628.57 \text{ N}$

$\Sigma F_z = 0; \quad B_z + 628.57 - 800 = 0 \qquad B_z = 171.43 \text{ N}$



Equations of Equilibrium: For point D

$\Sigma F_x = 0; \quad (N_D)_x = 0$ **Ans.**

$\Sigma F_y = 0; \quad (V_D)_y - 314.29 + 160 = 0$

$(V_D)_y = 154 \text{ N}$ **Ans.**

$\Sigma F_z = 0; \quad 171.43 + (V_D)_z = 0$

$(V_D)_z = -171 \text{ N}$ **Ans.**

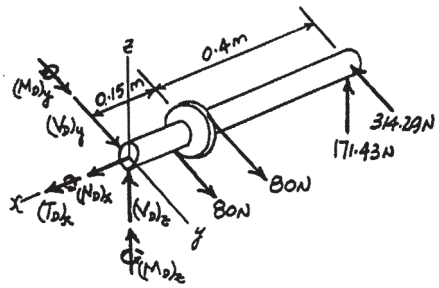
$\Sigma M_x = 0; \quad (T_D)_x = 0$ **Ans.**

$\Sigma M_y = 0; \quad 171.43(0.55) + (M_D)_y = 0$

$(M_D)_y = -94.3 \text{ N} \cdot \text{m}$ **Ans.**

$\Sigma M_z = 0; \quad 314.29(0.55) - 160(0.15) + (M_D)_z = 0$

$(M_D)_z = -149 \text{ N} \cdot \text{m}$ **Ans.**



Ans:

$(N_D)_x = 0,$
$(V_D)_y = 154 \text{ N},$
$(V_D)_z = -171 \text{ N},$
$(T_D)_x = 0,$
$(M_D)_y = -94.3 \text{ N} \cdot \text{m},$
$(M_D)_z = -149 \text{ N} \cdot \text{m}$

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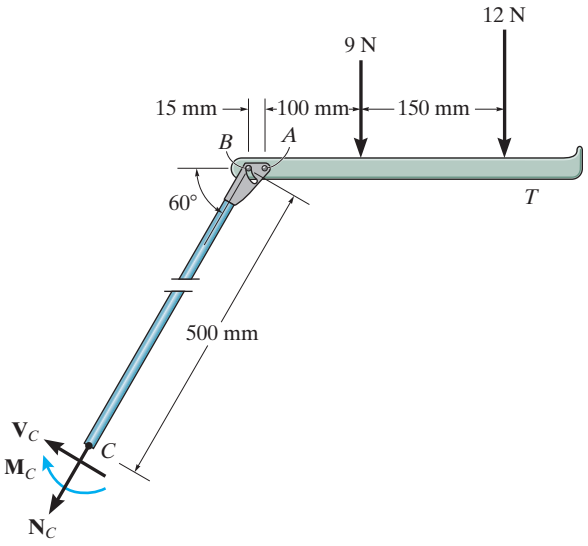
1–26. The serving tray T used on an airplane is supported on *each side* by an arm. The tray is pin connected to the arm at A, and at B there is a smooth pin. (The pin can move within the slot in the arms to permit folding the tray against the front passenger seat when not in use.) Determine the resultant internal loadings acting on the cross section of the arm through point C when the tray arm supports the loads shown.

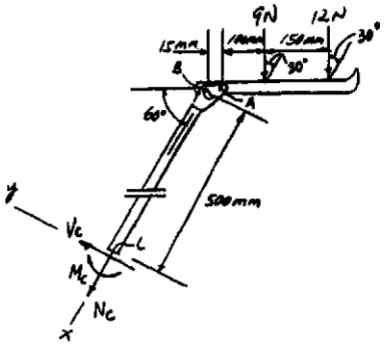


$\swarrow + \Sigma F_x = 0;$ $N_C + 9\cos 30° + 12\cos 30° = 0;$ $N_C = -18.2$ N **Ans.**

$\nwarrow + \Sigma F_y = 0;$ $V_C - 9\sin 30° - 12\sin 30° = 0;$ $V_C = 10.5$ N **Ans.**

$\circlearrowleft + \Sigma M_C = 0;$ $-M_C - 9(0.5\cos 60° + 0.115) - 12(0.5\cos 60° + 0.265) = 0$

$M_C = -9.46$ N · m **Ans.**



Ans:
$N_C = -18.2$ N
$V_C = 10.5$ N
$M_C = -9.46$ N · m

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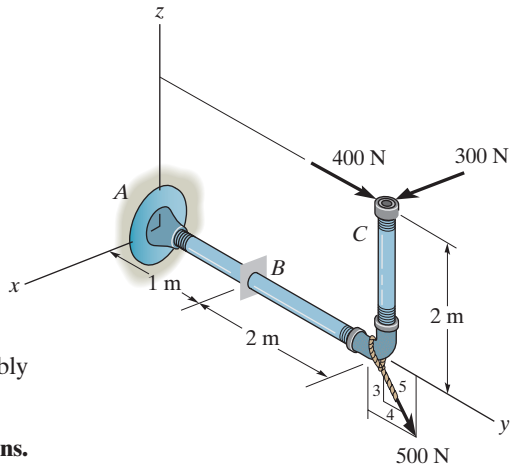
1–27.

The pipe has a mass of 12 kg/m. If it is fixed to the wall at A, determine the resultant internal loadings acting on the cross section at B.



SOLUTION

Internal Loadings: Referring to the FBD of the right segment of the pipe assembly sectioned through B, Fig. a,

$\Sigma F_x = 0;\quad (V_B)_x + 300 = 0 \qquad (V_B)_x = -300\ \text{N}$ **Ans.**

$\Sigma F_y = 0;\quad (N_B)_y + 400 + 500\left(\frac{4}{5}\right) = 0 \qquad (N_B)_y = -800\ \text{N}$ **Ans.**

$\Sigma F_z = 0;\quad (V_B)_z - 2\left[12(2)(9.81)\right] - 500\left(\frac{3}{5}\right) = 0$

$(V_B)_z = 770.88\ \text{N} = 771\ \text{N}$ **Ans.**

$\Sigma M_x = 0;\quad (M_B)_x - 12(2)(9.81)(1) - 12(2)(9.81)(2) - 500\left(\frac{3}{5}\right)(2)$

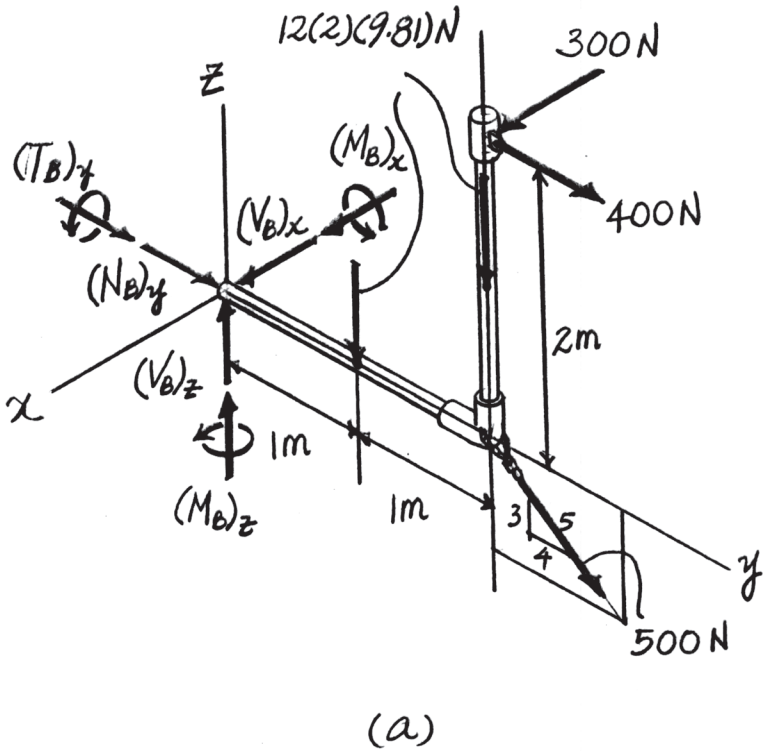
$- 400(2) = 0$

$(M_B)_x = 2106.32\ \text{N}\cdot\text{m} = 2.11\ \text{kN}\cdot\text{m}$ **Ans.**

$\Sigma M_y = 0;\quad (T_B)_y + 300(2) = 0 \qquad (T_B)_y = -600\ \text{N}\cdot\text{m}$ **Ans.**

$\Sigma M_z = 0;\quad (M_B)_z - 300(2) = 0 \qquad (M_B)_z = 600\ \text{N}\cdot\text{m}$ **Ans.**

The negative signs indicates that $(\mathbf{V}_B)_x$, $(\mathbf{N}_B)_y$, and $(\mathbf{T}_B)_y$ act in the sense opposite to those shown in the FBD.



(a)

Ans:
$(V_B)_x = -300\ \text{N}$,
$(N_B)_y = -800\ \text{N}$,
$(V_B)_z = 771\ \text{N}$,
$(M_B)_x = 2.11\ \text{kN}\cdot\text{m}$,
$(T_B)_y = -600\ \text{N}\cdot\text{m}$,
$(M_B)_z = 600\ \text{N}\cdot\text{m}$

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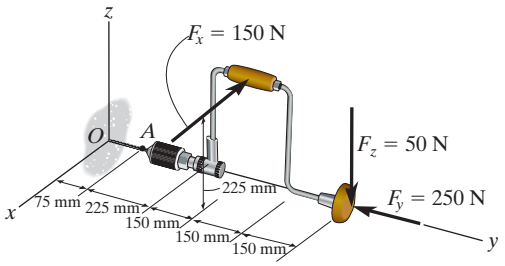
***1–28.** The brace and drill bit is used to drill a hole at O. If the drill bit jams when the brace is subjected to the forces shown, determine the resultant internal loadings acting on the cross section of the drill bit at A.



SOLUTION

Internal Loading: Referring to the free-body diagram of the section of the drill and brace shown in Fig. *a*,

$\Sigma F_x = 0;$ $\quad (V_A)_x - 150 = 0$ $\quad (V_A)_x = 150$ N **Ans.**

$\Sigma F_y = 0;$ $\quad (N_A)_y - 250 = 0$ $\quad (N_A)_y = 250$ N **Ans.**

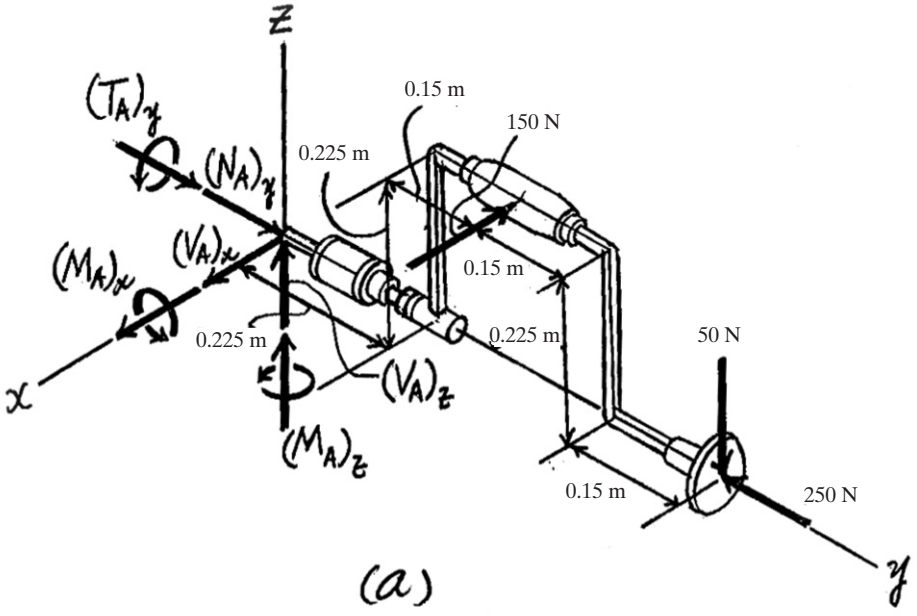
$\Sigma F_z = 0;$ $\quad (V_A)_z - 50 = 0$ $\quad (V_A)_z = 50$ N **Ans.**

$\Sigma M_x = 0;$ $\quad (M_A)_x - 50(0.675) = 0$ $\quad (M_A)_x = 33.75$ N · m **Ans.**

$\Sigma M_y = 0;$ $\quad (T_A)_y - 150(0.225) = 0$ $\quad (T_A)_y = 33.75$ N · m **Ans.**

$\Sigma M_z = 0;$ $\quad (M_A)_z + 150(0.375) = 0$ $\quad (M_A)_z = -56.25$ N · m **Ans.**

The negative sign indicates that $(M_A)_Z$ acts in the opposite sense to that shown on the free-body diagram.



Ans:
$(V_A)_x = 150$ N, $(N_A)_y = 250$ N, $(V_A)_z = 50$ N,
$(M_A)_x = 33.75$ N · m, $(T_A)_y = 33.75$ N · m,
$(M_A)_z = -56.25$ N · m

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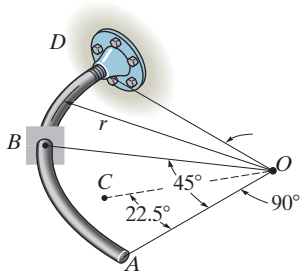
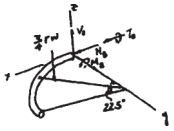
1–29.

The curved rod AD of radius r has a weight per length of w. If it lies in the horizontal plane, determine the resultant internal loadings acting on the cross section at point B. *Hint:* The distance from the centroid C of segment AB to point O is $CO = 0.9745r$.

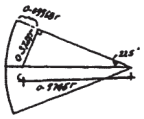


SOLUTION



$\Sigma F_z = 0; \quad V_B - \frac{\pi}{4} rw = 0; \quad V_B = 0.785\, w\, r$ **Ans.**

$\Sigma F_x = 0; \quad N_B = 0$ **Ans.**



$\Sigma M_x = 0; \quad T_B - \frac{\pi}{4} rw(0.09968r) = 0; \quad T_B = 0.0783\, w\, r^2$ **Ans.**

$\Sigma M_y = 0; \quad M_B + \frac{\pi}{4} rw(0.3729\, r) = 0; \quad M_B = -0.293\, w\, r^2$ **Ans.**

Ans:
$V_B = 0.785wr$,
$N_B = 0$,
$T_B = 0.0783wr^2$,
$M_B = -0.293wr^2$

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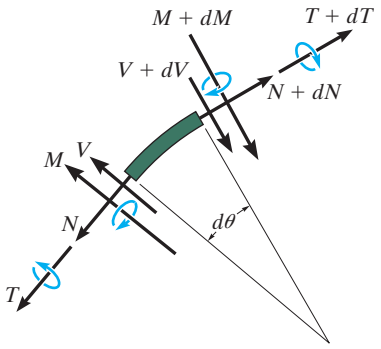
1–30.

A differential element taken from a curved bar is shown in the figure. Show that $dN/d\theta = V$, $dV/d\theta = -N$, $dM/d\theta = -T$, and $dT/d\theta = M$.



SOLUTION

$\Sigma F_x = 0;$

$$N\cos\frac{d\theta}{2} + V\sin\frac{d\theta}{2} - (N + dN)\cos\frac{d\theta}{2} + (V + dV)\sin\frac{d\theta}{2} = 0 \qquad \textbf{(1)}$$

$\Sigma F_y = 0;$

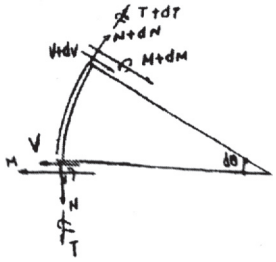
$$N\sin\frac{d\theta}{2} - V\cos\frac{d\theta}{2} + (N + dN)\sin\frac{d\theta}{2} + (V + dV)\cos\frac{d\theta}{2} = 0 \qquad \textbf{(2)}$$

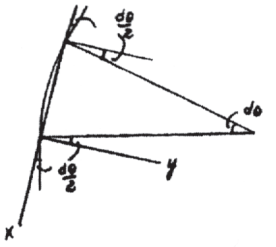
$\Sigma M_x = 0;$

$$T\cos\frac{d\theta}{2} + M\sin\frac{d\theta}{2} - (T + dT)\cos\frac{d\theta}{2} + (M + dM)\sin\frac{d\theta}{2} = 0 \qquad \textbf{(3)}$$

$\Sigma M_y = 0;$

$$T\sin\frac{d\theta}{2} - M\cos\frac{d\theta}{2} + (T + dT)\sin\frac{d\theta}{2} + (M + dM)\cos\frac{d\theta}{2} = 0 \qquad \textbf{(4)}$$





Since $\frac{d\theta}{2}$ is can add, then $\sin\frac{d\theta}{2} = \frac{d\theta}{2}$, $\cos\frac{d\theta}{2} = 1$

Eq. (1) becomes $Vd\theta - dN + \frac{dVd\theta}{2} = 0$

Neglecting the second order term, $Vd\theta - dN = 0$

$\frac{dN}{d\theta} = V$ QED

Eq. (2) becomes $Nd\theta + dV + \frac{dNd\theta}{2} = 0$

Neglecting the second order term, $Nd\theta + dV = 0$

$\frac{dV}{d\theta} = -N$ QED

Eq. (3) becomes $Md\theta - dT + \frac{dMd\theta}{2} = 0$

Neglecting the second order term, $Md\theta - dT = 0$

$\frac{dT}{d\theta} = M$ QED

Eq. (4) becomes $Td\theta + dM + \frac{dTd\theta}{2} = 0$

Neglecting the second order term, $Td\theta + dM = 0$

$\frac{dM}{d\theta} = -T$ QED

Ans:
N/A

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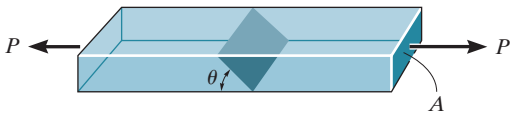
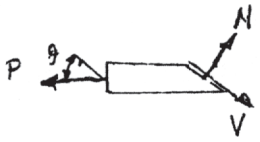
1–31.

The bar has a cross-sectional area A and is subjected to the axial load P. Determine the average normal and average shear stresses acting over the shaded section, which is oriented at θ from the horizontal. Plot the variation of these stresses as a function of θ $(0 \le \theta \le 90°)$.



SOLUTION



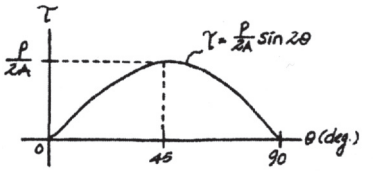
Equations of Equilibrium:

$$\searrow + \Sigma F_x = 0; \qquad V - P\cos\theta = 0 \qquad V = P\cos\theta$$

$$\nearrow + \Sigma F_y = 0; \qquad N - P\sin\theta = 0 \qquad N = P\sin\theta$$

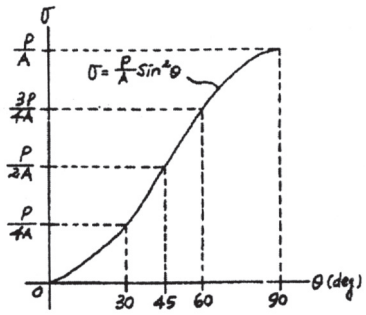
Average Normal Stress and Shear Stress: Area at θ plane, $A' = \dfrac{A}{\sin\theta}$.

$$\sigma_{avg} = \frac{N}{A'} = \frac{P\sin\theta}{\frac{A}{\sin\theta}} = \frac{P}{A}\sin^2\theta$$ **Ans.**



$$\tau_{avg} = \frac{V}{A'} = \frac{P\cos\theta}{\frac{A}{\sin\theta}}$$

$$= \frac{P}{A}\sin\theta\cos\theta = \frac{P}{2A}\sin 2\theta$$ **Ans.**



Ans:

$$\sigma_{avg} = \frac{P}{A}\sin^2\theta, \quad \tau_{avg} = \frac{P}{2A}\sin 2\theta$$

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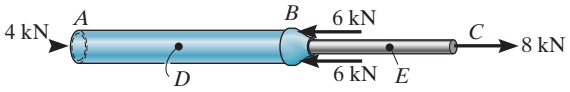
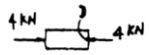
***1–32.**



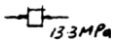
The built-up shaft consists of a pipe AB and solid rod BC. The pipe has an inner diameter of 20 mm and outer diameter of 28 mm. The rod has a diameter of 12 mm. Determine the average normal stress at points D and E and represent the stress on a volume element located at each of these points.

SOLUTION

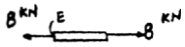
At D:



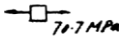
$$\sigma_D = \frac{P}{A} = \frac{4(10^3)}{\frac{\pi}{4}(0.028^2 - 0.02^2)} = 13.3 \text{ MPa} \quad \text{(C)}$$ **Ans.**



At E:



$$\sigma_E = \frac{P}{A} = \frac{8(10^3)}{\frac{\pi}{4}(0.012^2)} = 70.7 \text{ MPa} \quad \text{(T)}$$ **Ans.**



Ans:

$\sigma_D = 13.3$ MPa, $\sigma_E = 70.7$ MPa

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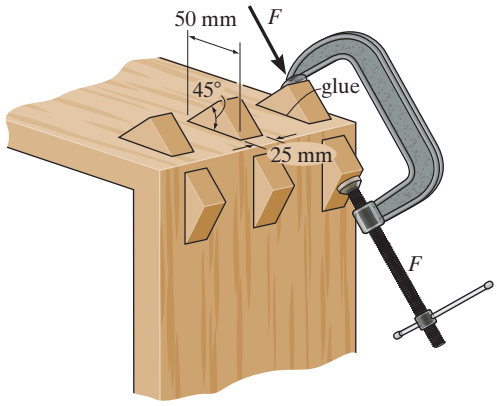
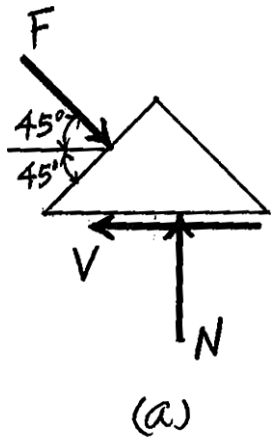
1–33. The triangular blocks are glued along each side of the joint. A C-clamp placed between two of the blocks is used to draw the joint tight. If the glue can withstand a maximum average shear stress of 800 kPa, determine the maximum allowable clamping force **F**.



SOLUTION

Internal Loadings: The shear force developed on the glued shear plane can be obtained by writing the force equation of equilibrium along the x axis with reference to the free-body diagram of the triangular block, Fig. *a*.

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad F\cos 45° - V = 0 \qquad V = \frac{\sqrt{2}}{2}F$$



Average Normal and Shear Stress: The area of the glued shear plane is $A = 0.05(0.025) = 1.25(10^{-3})\text{m}^2$. We obtain

$$\tau_{\text{avg}} = \frac{V}{A}; \qquad 800(10^3) = \frac{\frac{\sqrt{2}}{2}F}{1.25(10^{-3})}$$

$$F = 1414\ \text{N} = 1.41\ \text{kN} \qquad \textbf{Ans.}$$

Ans:

$F = 1.41$ kN

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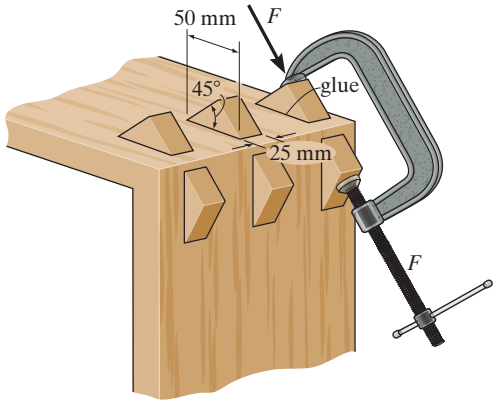
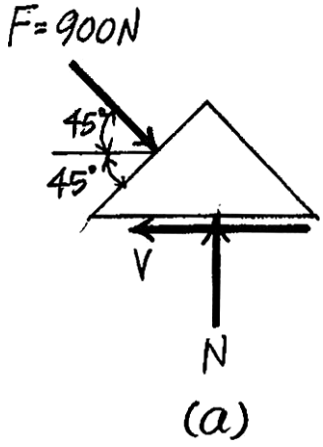
1–34. The triangular blocks are glued along each side of the joint. A C-clamp placed between two of the blocks is used to draw the joint tight. If the clamping force is $F = 900$ N, determine the average shear stress developed in the glued shear plane.



SOLUTION

Internal Loadings: The shear force developed on the glued shear plane can be obtained by writing the force equation of equilibrium along the x axis with reference to the free-body diagram of the triangular block, Fig. *a*.

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 900 \cos 45° - V = 0 \qquad V = 636.40 \text{ N}$$



Average Normal and Shear Stress: The area of the glued shear plane is $A = 0.05(0.025) = 1.25(10^{-3})\text{m}^2$. We obtain

$$\tau_{\text{avg}} = \frac{V}{A} = \frac{636.40}{1.25(10^{-3})} = 509 \text{ kPa} \qquad \textbf{Ans.}$$

Ans.

$V = 636.40$ N, $\tau_{\text{avg}} = 509$ kPa

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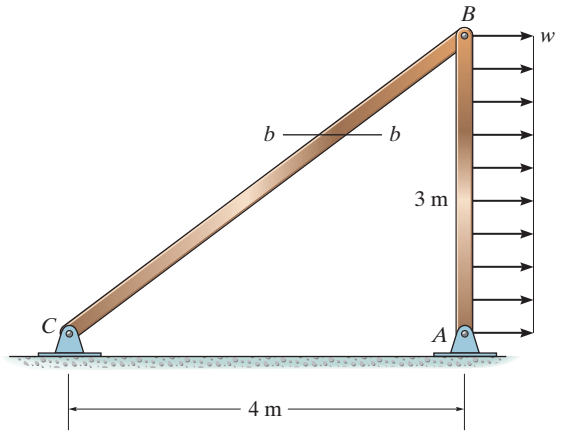
1–35.

Determine the largest intensity w of the uniform loading that can be applied to the frame without causing either the average normal stress or the average shear stress at section b–b to exceed $\sigma = 15$ MPa and $\tau = 16$ MPa, respectively. Member CB has a square cross section of 30 mm on each side.



SOLUTION

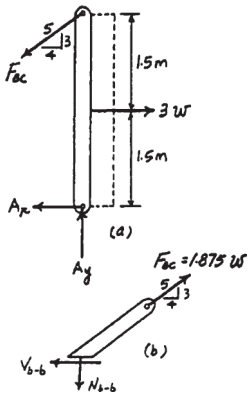
Support Reactions: FBD(a)

$$\zeta+\Sigma M_A = 0; \qquad \frac{4}{5}F_{BC}(3) - 3w(1.5) = 0 \qquad F_{BC} = 1.875w$$

Equations of Equilibrium: For section b–b, FBD(b)

$$\pm\rightarrow \Sigma F_x = 0; \qquad \frac{4}{5}(1.875w) - V_{b-b} = 0 \qquad V_{b-b} = 1.50w$$

$$+\uparrow \Sigma F_y = 0; \qquad \frac{3}{5}(1.875w) - N_{b-b} = 0 \qquad N_{b-b} = 1.125w$$



Average Normal Stress and Shear Stress: The cross-sectional area of section b–b, $A' = \frac{5A}{3}$; where $A = (0.03)(0.03) = 0.90(10^{-3})\ \text{m}^2$.

Then $A' = \frac{5}{3}(0.90)(10^{-3}) = 1.50(10^{-3})\ \text{m}^2$.

Assume failure due to normal stress.

$$(\sigma_{b-b})_{\text{Allow}} = \frac{N_{b-b}}{A'}; \qquad 15(10^6) = \frac{1.125w}{1.50(10^{-3})}$$

$$w = 20000\ \text{N/m} = 20.0\ \text{kN/m}$$

Assume failure due to shear stress.

$$(\tau_{b-b})_{\text{Allow}} = \frac{V_{b-b}}{A'}; \qquad 16(10^6) = \frac{1.50w}{1.50(10^{-3})}$$

$$w = 16000\ \text{N/m} = 16.0\ \text{kN/m}\ \textbf{(}\textbf{\textit{Controls !}}\textbf{)} \qquad \textbf{Ans.}$$

Ans:
$w = 16.0\ \text{kN/m}$ **(*Controls !*)**

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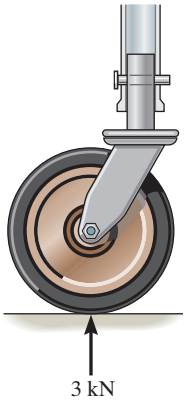
***1–36.**

The supporting wheel on a scaffold is held in place on the leg using a 4-mm-diameter pin. If the wheel is subjected to a normal force of 3 kN, determine the average shear stress in the pin. Assume the pin only supports the vertical 3-kN load.



SOLUTION

$+\uparrow \Sigma F_y = 0; \qquad 3\text{ kN} \cdot 2V = 0; \qquad V = 1.5\text{ kN}$

$$\tau_{\text{avg}} = \frac{V}{A} = \frac{1.5(10^3)}{\frac{\pi}{4}(0.004)^2} = 119\text{ MPa}$$ **Ans.**

Ans:
$\tau_{\text{avg}} = 119\text{ MPa}$

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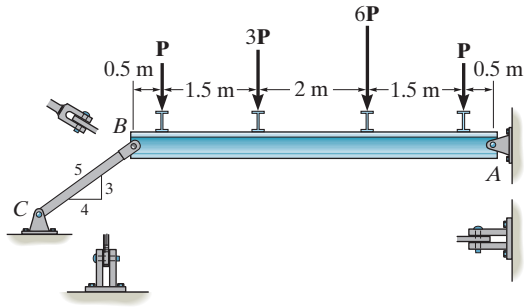
1–37.

If $P = 5$ kN, determine the average shear stress in the pins at A, B, and C. All pins are in double shear, and each has a diameter of 18 mm.



SOLUTION

Support Reactions: Referring to the FBD of the entire beam, Fig. *a*,

$$\zeta+\Sigma M_A = 0; \quad 5(0.5) + 30(2) + 15(4) + 5(5.5) - F_{BC}\left(\frac{3}{5}\right)(6) = 0$$

$$F_{BC} = 41.67 \text{ kN}$$

$$\zeta+\Sigma M_B = 0; \quad A_y(6) - 5(0.5) - 15(2) - 30(4) - 5(5.5) = 0 \quad A_y = 30.0 \text{ kN}$$

$$\pm\rightarrow \Sigma F_x = 0; \quad 41.67\left(\frac{4}{5}\right) - A_x = 0 \quad A_x = 33.33 \text{ kN}$$

Thus,

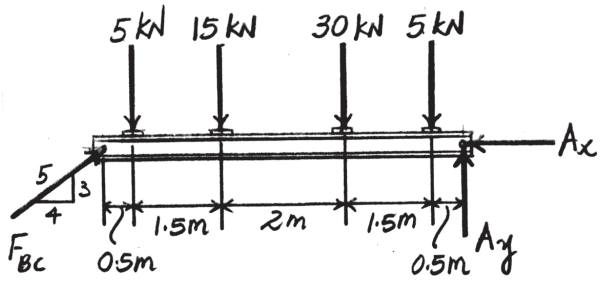
$$F_A = \sqrt{A_x^2 + A_y^2} = \sqrt{33.33^2 + 30.0^2} = 44.85 \text{ kN}$$

Average Shear Stress: Since all the pins are subjected to double shear, $V_B = V_C = \frac{F_{BC}}{2} = \frac{41.67}{2}\text{ kN} = 20.83$ kN (Fig. *b*) and $V_A = 22.42$ kN (Fig. *c*)

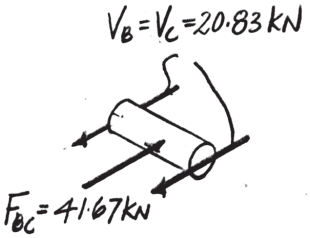
For pins B and C

$$\tau_B = \tau_C = \frac{V_C}{A} = \frac{20.83(10^3)}{\frac{\pi}{4}(0.018^2)} = 81.87 \text{ MPa} = 81.9 \text{ MPa} \qquad \textbf{Ans.}$$

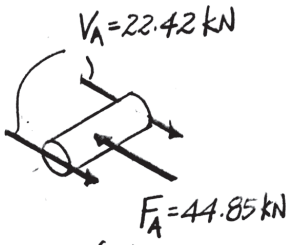
$$\tau_A = \frac{V_A}{A} = \frac{22.42(10^3)}{\frac{\pi}{4}(0.018^2)} = 88.12 \text{ MPa} = 88.1 \text{ MPa} \qquad \textbf{Ans.}$$



(a)



(b)



(c)

Ans:
$\tau_B = \tau_C = 81.9$ MPa,
$\tau_A = 88.1$ MPa

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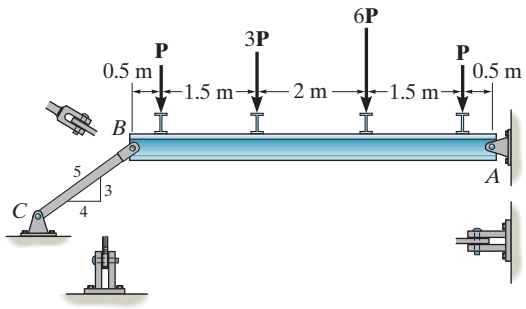
1–38.

Determine the maximum magnitude P of the loads the beam can support if the average shear stress in each pin is not to exceed 80 MPa. All pins are in double shear, and each has a diameter of 18 mm.



SOLUTION

Support Reactions: Referring to the FBD of the entire beam, Fig. *a*,

$$\zeta+\Sigma M_A = 0; \qquad P(0.5) + 6P(2) + 3P(4) + P(5.5) - F_{BC}\left(\frac{3}{5}\right)(6) = 0$$

$$F_{BC} = 8.3333P$$

$$\zeta+\Sigma M_B = 0; \qquad A_y(6) - P(0.5) - 3P(2) - 6P(4) - P(5.5) = 0 \quad A_y = 6.00P$$

$$\pm\rightarrow \Sigma F_x = 0; \qquad 8.3333P\left(\frac{4}{5}\right) - A_x = 0 \qquad A_x = 6.6667P$$

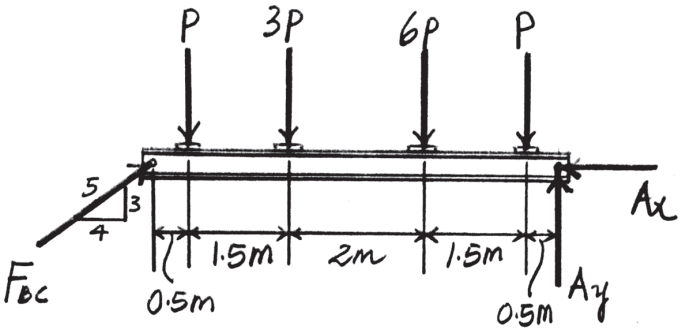
Thus,

$$F_A = \sqrt{A_x^2 + A_y^2} = \sqrt{(6.6667P)^2 + (6.00P)^2} = 8.9691P$$

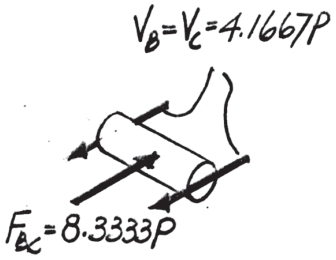
Average Shear Stress: Since all the pins are subjected to double shear, $V_B = V_C = \frac{F_{BC}}{2} = \frac{8.3333P}{2} = 4.1667P$ (Fig. *b*) and $V_A = 4.4845P$ (Fig. *c*). Since pin A is subjected to a larger shear force, it is critical. Thus

$$\tau_{\text{allow}} = \frac{V_A}{A}; \qquad 80(10^6) = \frac{4.4845P}{\frac{\pi}{4}(0.018^2)}$$

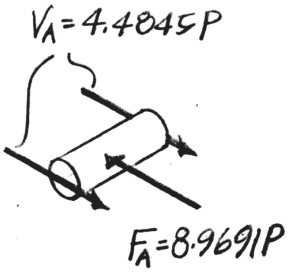
$$P = 4.539(10^3)\,\text{N} = 4.54\,\text{kN} \qquad \textbf{Ans.}$$



(a)



(b)



(c)

Ans:
$P = 4.54$ kN

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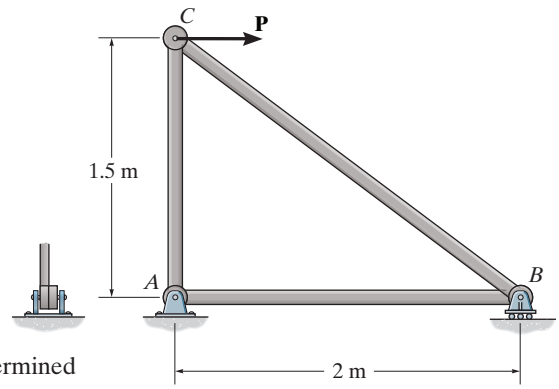
1–39.

Determine the average normal stress in each of the 20-mm-diameter bars of the truss. Set $P = 40$ kN.



SOLUTION

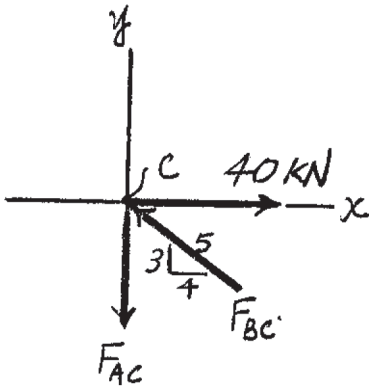
Internal Loadings: The force developed in each member of the truss can be determined by using the method of joints. First, consider the equilibrium of joint C, Fig. a,

$$\stackrel{+}{\rightarrow}\Sigma F_x = 0; \qquad 40 - F_{BC}\left(\frac{4}{5}\right) = 0 \qquad F_{BC} = 50 \text{ kN (C)}$$

$$+\uparrow \Sigma F_y = 0; \qquad 50\left(\frac{3}{5}\right) - F_{AC} = 0 \qquad F_{AC} = 30 \text{ kN (T)}$$

Subsequently, the equilibrium of joint B, Fig. b, is considered

$$\stackrel{+}{\rightarrow}\Sigma F_x = 0; \qquad 50\left(\frac{4}{5}\right) - F_{AB} = 0 \qquad F_{AB} = 40 \text{ kN (T)}$$



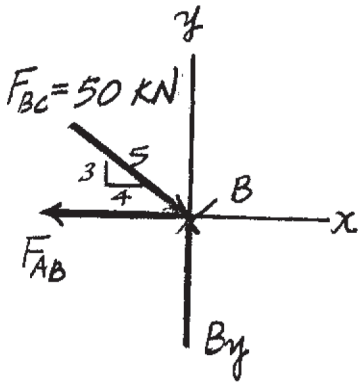
(a)

Average Normal Stress: The cross-sectional area of each of the bars is $A = \frac{\pi}{4}(0.02^2) = 0.3142(10^{-3})\ \text{m}^2$. We obtain,

$$(\sigma_{\text{avg}})_{BC} = \frac{F_{BC}}{A} = \frac{50(10^3)}{0.3142(10^{-3})} = 159 \text{ MPa} \qquad \textbf{Ans.}$$

$$(\sigma_{\text{avg}})_{AC} = \frac{F_{AC}}{A} = \frac{30(10^3)}{0.3142(10^{-3})} = 95.5 \text{ MPa} \qquad \textbf{Ans.}$$

$$(\sigma_{\text{avg}})_{AB} = \frac{F_{AB}}{A} = \frac{40(10^3)}{0.3142(10^{-3})} = 127 \text{ MPa} \qquad \textbf{Ans.}$$



(b)

Ans:
$(\sigma_{\text{avg}})_{BC} = 159$ MPa,
$(\sigma_{\text{avg}})_{AC} = 95.5$ MPa,
$(\sigma_{\text{avg}})_{AB} = 127$ MPa

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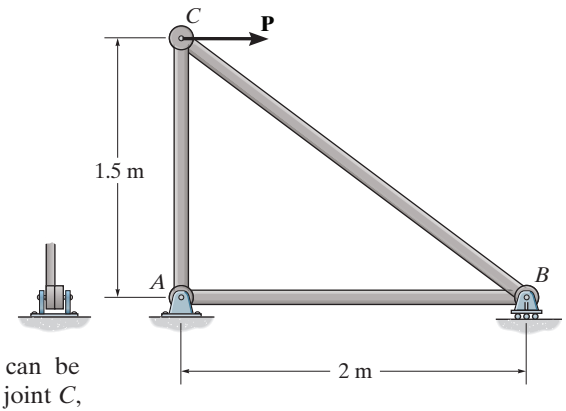
***1–40.**

If the average normal stress in each of the 20-mm-diameter bars is not allowed to exceed 150 MPa, determine the maximum force **P** that can be applied to joint *C*.



SOLUTION

Internal Loadings: The force developed in each member of the truss can be determined by using the method of joints. First, consider the equilibrium of joint *C*, Fig. *a*.

$$\stackrel{+}{\rightarrow}\Sigma F_x = 0; \qquad P - F_{BC}\left(\frac{4}{5}\right) = 0 \qquad F_{BC} = 1.25P(\text{C})$$

$$+\uparrow\Sigma F_y = 0; \qquad 1.25P\left(\frac{3}{5}\right) - F_{AC} = 0 \qquad F_{AC} = 0.75P(\text{T})$$

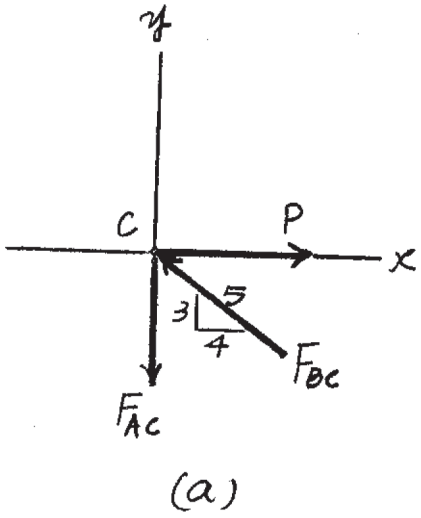
Subsequently, the equilibrium of joint *B*, Fig. *b*, is considered.

$$\stackrel{+}{\rightarrow}\Sigma F_x = 0; \qquad 1.25P\left(\frac{4}{5}\right) - F_{AB} = 0 \qquad F_{AB} = P(\text{T})$$

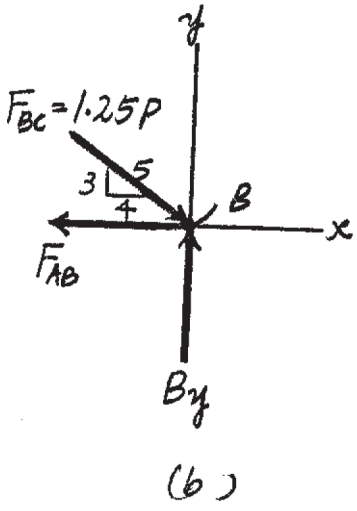
Average Normal Stress: Since the cross-sectional area and the allowable normal stress of each bar are the same, member *BC*, which is subjected to the maximum normal force, is the critical member. The cross-sectional area of each of the bars is $A = \frac{\pi}{4}(0.02^2) = 0.3142(10^{-3})\ \text{m}^2$. We have,

$$(\sigma_{\text{avg}})_{\text{allow}} = \frac{F_{BC}}{A}; \quad 150(10^6) = \frac{1.25P}{0.3142(10^{-3})}$$

$$P = 37\,699\ \text{N} = 37.7\ \text{kN} \qquad \textbf{Ans.}$$



(a)



(b)

Ans:
$P = 37.7$ kN

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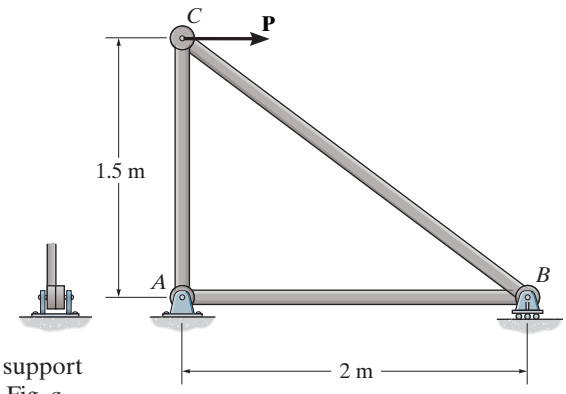
1–41.

Determine the maximum average shear stress in pin A of the truss. A horizontal force of $P = 40$ kN is applied to joint C. Each pin has a diameter of 25 mm and is subjected to double shear.



SOLUTION

Internal Loadings: The forces acting on pins A and B are equal to the support reactions at A and B. Referring to the free-body diagram of the entire truss, Fig. a,

$\Sigma M_A = 0;$ $\quad B_y(2) - 40(1.5) = 0$ $\quad B_y = 30$ kN

$\pm\rightarrow \Sigma F_x = 0;$ $\quad 40 - A_x = 0$ $\quad A_x = 40$ kN

$+\uparrow \Sigma F_y = 0;$ $\quad 30 - A_y = 0$ $\quad A_y = 30$ kN

Thus,

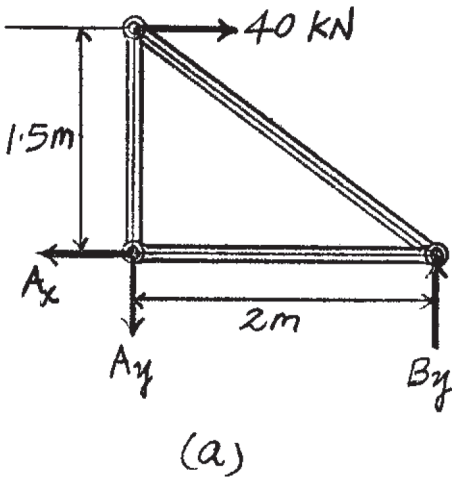
$$F_A = \sqrt{A_x^2 + A_y^2} = \sqrt{40^2 + 30^2} = 50 \text{ kN}$$

Since pin A is in double shear, Fig. b, the shear forces developed on the shear planes of pin A are

$$V_A = \frac{F_A}{2} = \frac{50}{2} = 25 \text{ kN}$$

Average Shear Stress: The area of the shear plane for pin A is $A_A = \frac{\pi}{4}(0.025^2) = 0.4909(10^{-3})\ \text{m}^2$. We have

$$(\tau_{\text{avg}})_A = \frac{V_A}{A_A} = \frac{25(10^3)}{0.4909(10^{-3})} = 50.9 \text{ MPa}$$ **Ans.**



(a)

Pin A

(b)

Ans:
$(\tau_{\text{avg}})_A = 50.9$ MPa

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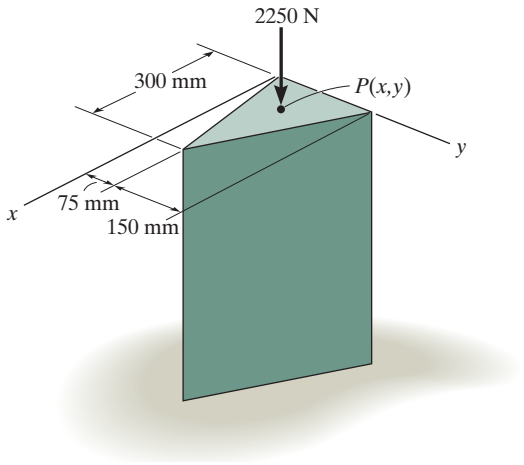
1–42. The pedestal has a triangular cross section as shown. If it is subjected to a compressive force of 2250 N, specify the x and y coordinates for the location of point $P(x, y)$, where the load must be applied on the cross section, so that the average normal stress is uniform. Compute the stress and sketch its distribution acting on the cross section at a location removed from the point of load application.

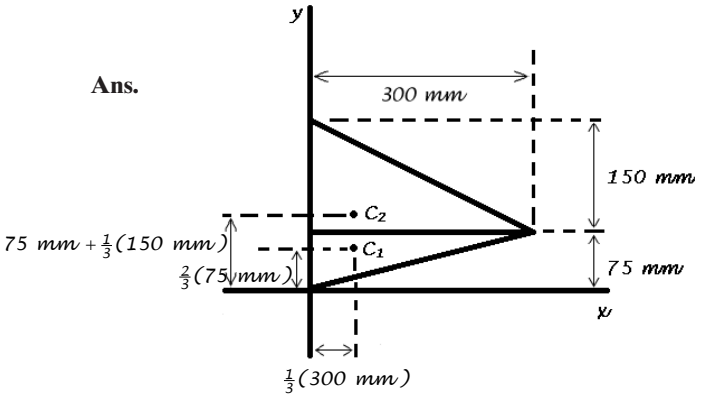


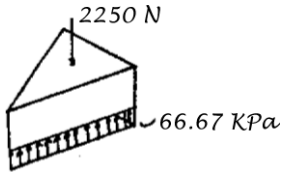
SOLUTION

$$x = \frac{\left[\frac{2}{3}(75)\right]\left[\frac{1}{2}(75)(300)\right] + \left[75 + \frac{1}{3}(150)\right]\left[\frac{1}{2}(150)(300)\right]}{\frac{1}{2}(75)(300) + \frac{1}{2}(150)(300)} = 100 \text{ mm}$$ **Ans.**

$$y = \tfrac{1}{3}(300 \text{ mm}) = 100 \text{ mm}$$ **Ans.**

$$\sigma = \frac{P}{A} = \frac{2250}{\frac{1}{2}(0.225)(0.3)}$$
$$= 66.67(10^3) \text{ N/m}^2$$
$$= 66.7 \text{ kPa}$$ **Ans.**





Ans:

$x = 100$ mm, $y = 100$ mm, $\sigma = 66.7$ kPa

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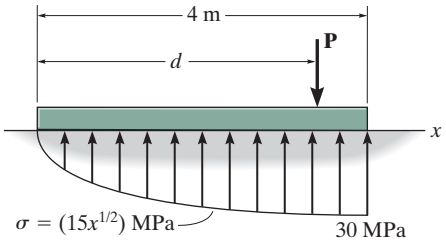
1–43.

The plate has a width of 0.5 m. If the stress distribution at the support varies as shown, determine the force **P** applied to the plate and the distance d to where it is applied.



SOLUTION

The resultant force dF of the bearing pressure acting on the plate of area $dA = b\,dx = 0.5\,dx$, Fig. a,

$$dF = \sigma_b\,dA = (15x^{\frac{1}{2}})(10^6)(0.5dx) = 7.5(10^6)x^{\frac{1}{2}}\,dx$$

$$+\uparrow \Sigma F_y = 0; \qquad \int dF - P = 0$$

$$\int_0^{4\,\text{m}} 7.5(10^6)x^{\frac{1}{2}}\,dx - P = 0$$

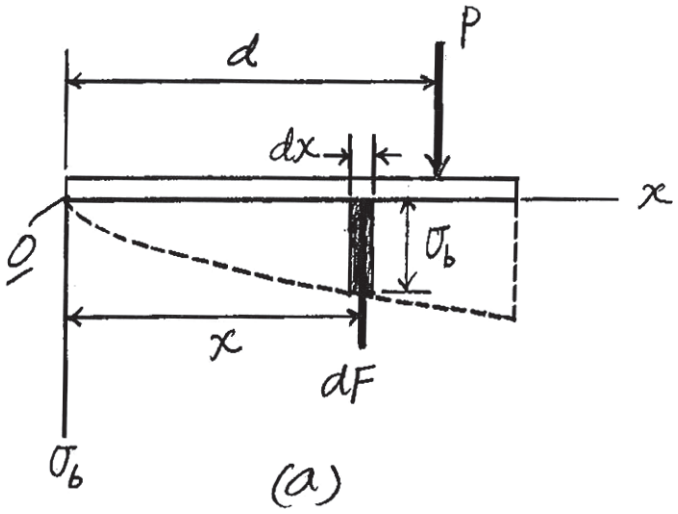
$$P = 40(10^6)\ \text{N} = 40\ \text{MN} \qquad \textbf{Ans.}$$

Equilibrium requires

$$\circlearrowleft +\Sigma M_O = 0; \qquad \int x\,dF - Pd = 0$$

$$\int_0^{4\,\text{m}} x[7.5(10^6)x^{\frac{1}{2}}\,dx] - 40(10^6)\,d = 0$$

$$d = 2.40\ \text{m} \qquad \textbf{Ans.}$$



Ans:
$P = 40$ MN, $d = 2.40$ m

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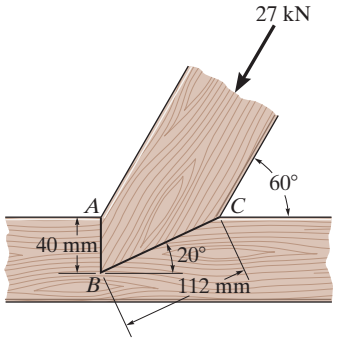
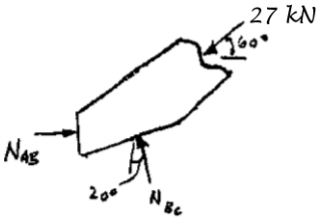
***1–44.**

The joint is subjected to the axial member force of 27 kN. Determine the average normal stress acting on sections *AB* and *BC*. Assume the member is smooth and is 40 mm thick.



SOLUTION



$+\uparrow \Sigma F_y = 0;$ $\quad -27 \sin 60° + N_{BC} \cos 20° = 0$

$N_{BC} = 24.88 \text{ kN}$

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad N_{AB} - 27 \cos 60° - 24.88 \sin 20° = 0$

$N_{AB} = 22.01 \text{ kN}$

$$\sigma_{AB} = \frac{N_{AB}}{A_{AB}} = \frac{22.01\,(10^3)}{(0.04)(0.04)} = 13.76(10^6)\ \text{N/m}^2 = 13.8\ \text{MPa}$$ **Ans.**

$$\sigma_{BC} = \frac{N_{BC}}{A_{BC}} = \frac{24.88\,(10^3)}{(0.04)(0.112)} = 5.554(10^6)\ \text{N/m}^2 = 5.55\ \text{MPa}$$ **Ans.**

Ans:
$\sigma_{AB} = 13.8$ MPa, $\sigma_{BC} = 5.55$ MPa

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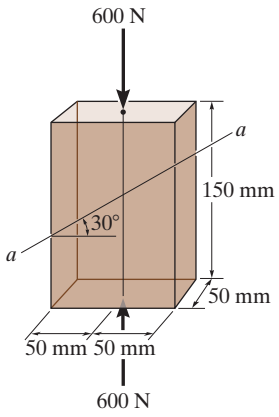
1–45.

The plastic block is subjected to an axial compressive force of 600 N. Assuming that the caps at the top and bottom distribute the load uniformly throughout the block, determine the average normal and average shear stress acting along section *a–a*.



SOLUTION

Along *a–a*:

$$+\swarrow \Sigma F_x = 0; \quad V - 600 \sin 30° = 0$$

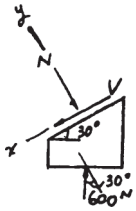
$$V = 300 \text{ N}$$

$$+\searrow \Sigma F_y = 0; \quad -N + 600 \cos 30° = 0$$

$$N = 519.6 \text{ N}$$

$$\sigma_{a-a} = \frac{519.6}{(0.05)\left(\frac{0.1}{\cos 30°}\right)} = 90.0 \text{ kPa} \qquad \textbf{Ans.}$$

$$\tau_{a-a} = \frac{300}{(0.05)\left(\frac{0.1}{\cos 30°}\right)} = 52.0 \text{ kPa} \qquad \textbf{Ans.}$$



Ans:
$\sigma_{a-a} = 90.0$ kPa,
$\tau_{a-a} = 52.0$ kPa

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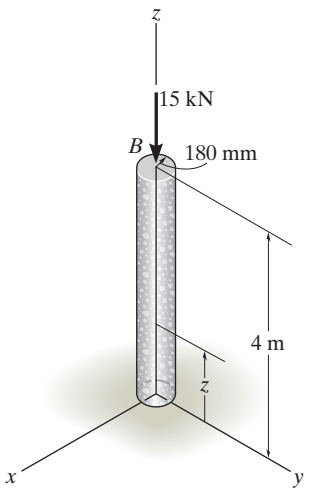
1–46.

The column is made of concrete having a density of 2.30 Mg/m^3. At its top B it is subjected to an axial compressive force of 15 kN. Determine the average normal stress in the column as a function of the distance z measured from its base.

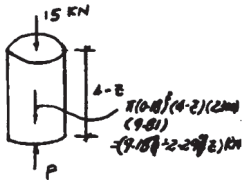


SOLUTION

$+\uparrow \Sigma F_y = 0 \qquad P - 15 - 9.187 + 2.297z = 0$

$$P = 24.187 - 2.297z$$

$$\sigma = \frac{P}{A} = \frac{24.187 - 2.297z}{\pi(0.18)^2} = (238 - 22.6z)\ \text{kPa}$$ **Ans.**



Ans:
$\sigma = (238 - 22.6z)$ kPa

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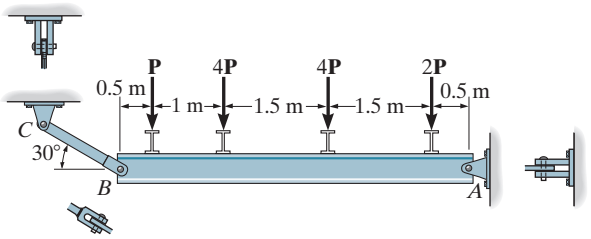
1–47.

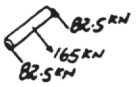
If $P = 15$ kN, determine the average shear stress in the pins at A, B, and C. All pins are in double shear, and each has a diameter of 18 mm.



SOLUTION

For pins B and C:

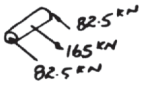
$$\tau_B = \tau_C = \frac{V}{A} = \frac{82.5\,(10^3)}{\frac{\pi}{4}\left(\frac{18}{1000}\right)^2} = 324 \text{ MPa}$$ **Ans.**



For pin A:

$$F_A = \sqrt{(82.5)^2 + (142.9)^2} = 165 \text{ kN}$$

$$\tau_A = \frac{V}{A} = \frac{82.5\,(10^3)}{\frac{\pi}{4}\left(\frac{18}{1000}\right)^2} = 324 \text{ MPa}$$ **Ans.**



Ans:
$\tau_B = 324$ MPa,
$\tau_A = 324$ MPa

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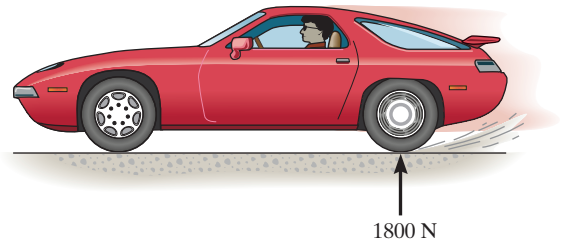
* **1–48.** The driver of the sports car applies his rear brakes and causes the tires to slip. If the normal force on each rear tire is 1800 N and the coefficient of kinetic friction between the tires and the pavement is $\mu_k = 0.5$, determine the average shear stress developed by the friction force on the tires. Assume the rubber of the tires is flexible and each tire is filled with an air pressure of 225 K P



SOLUTION

From the tire pressure,

$$p = \frac{N}{A}; \qquad 225(10^3) = \frac{1800}{A} \qquad A = 0.008\ \text{m}^2$$

The friction is

$$F = \mu_k N = 0.5(1800) = 900\ \text{N}$$

Then the shear stress is

$$\tau_{\text{avg}} = \frac{F}{A} = \frac{900}{0.008} = 112.5(10^3)\ \text{N/m}^2 = 112.5\ \text{KPa}$$ **Ans.**

Ans:
$\tau_{\text{avg}} = 112.5$ KPa

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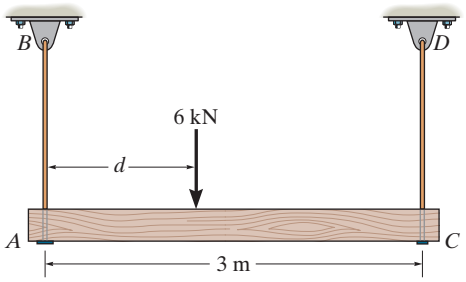
1–49.

The beam is supported by two rods AB and CD that have cross-sectional areas of 12 mm^2 and 8 mm^2, respectively. If $d = 1$ m, determine the average normal stress in each rod.



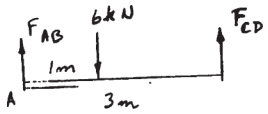
SOLUTION

$\zeta+\Sigma M_A = 0; \qquad F_{CD}(3) - 6(1) = 0$

$F_{CD} = 2$ kN

$+\uparrow \Sigma F_y = 0; \qquad F_{AB} - 6 + 2 = 0$

$F_{AB} = 4$ kN



$$\sigma_{AB} = \frac{F_{AB}}{A_{AB}} = \frac{4(10^3)}{12(10^{-6})} = 333 \text{ MPa}$$ **Ans.**

$$\sigma_{CD} = \frac{F_{CD}}{A_{CD}} = \frac{2(10^3)}{8(10^{-6})} = 250 \text{ MPa}$$ **Ans.**

Ans:
$\sigma_{AB} = 333$ MPa,
$\sigma_{CD} = 250$ MPa

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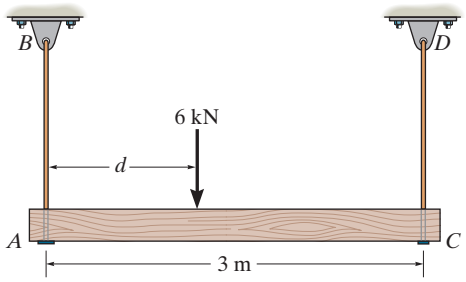
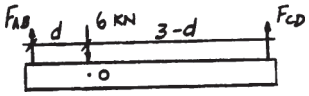
1–50.

The beam is supported by two rods AB and CD that have cross-sectional areas of 12 mm^2 and 8 mm^2, respectively. Determine the position d of the 6-kN load so that the average normal stress in each rod is the same.



SOLUTION

$\circlearrowleft+\Sigma M_O = 0; \qquad F_{CD}(3 - d) - F_{AB}(d) = 0$ **(1)**



$$\sigma = \frac{F_{AB}}{12} = \frac{F_{CD}}{8}$$

$F_{AB} = 1.5\,F_{CD}$ **(2)**

From Eqs. (1) and (2),

$F_{CD}(3 - d) - 1.5\,F_{CD}(d) = 0$

$F_{CD}(3 - d - 1.5\,d) = 0$

$3 - 2.5\,d = 0$

$d = 1.20$ m **Ans.**

Ans:
$d = 1.20$ m

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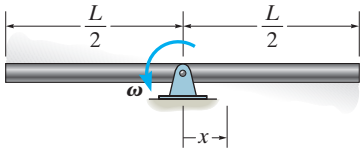
1–51.

The uniform bar, having a cross-sectional area of A and mass per unit length of m, is pinned at its center. If it is rotating in the horizontal plane at a constant angular rate of ω, determine the average normal stress in the bar as a function of x.

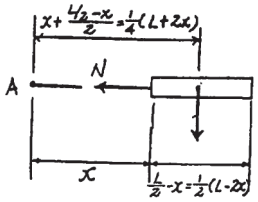


SOLUTION

Equation of Motion:

$$\overleftarrow{+}\Sigma F_x = ma_N; \qquad N = m\left[\frac{1}{2}(L - 2x)\right]\omega^2\left[\frac{1}{4}(L + 2x)\right]$$

$$= \frac{m\omega^2}{8}(L^2 - 4x^2)$$



Average Normal Stress:

$$\sigma = \frac{N}{A} = \frac{m\omega^2}{8A}(L^2 - 4x^2)$$ **Ans.**

Ans:

$$\sigma = \frac{m\omega^2}{8A}(L^2 - 4x^2)$$

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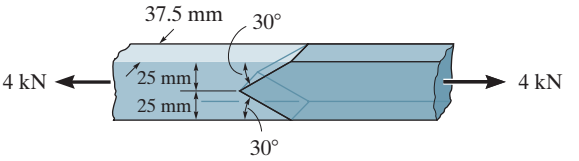
***1–52.**

The two members used in the construction of an aircraft fuselage are joined together using a 30° fish-mouth weld. Determine the average normal and average shear stress on the plane of each weld. Assume each inclined plane supports a horizontal force of 2 kN.



SOLUTION

$\nwarrow + \Sigma F_y = 0;$ $\quad N - 2\sin 30° = 0;$ $\quad N = 1$ kN

$+\nearrow \Sigma F_x = 0;$ $\quad 2\cos 30° - V = 0;$ $\quad V = 1.732$ kN

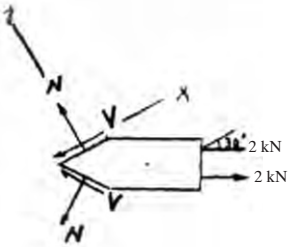
$$A' = \frac{(0.0375)(0.025)}{\sin 30°} = 1.875(10^{-3})\ \text{m}^2$$

$$\sigma = \frac{N}{A'} = \frac{1(10^3)}{1.875(10^{-3})\ \text{m}^2} = 533\ \text{kPa}$$

Ans.

$$\tau = \frac{V}{A'} = \frac{1.732(10^3)}{1.875(10^{-3})\ \text{m}^2} = 924\ \text{kPa}$$

Ans.



Ans:

$\sigma = 533$ kPa

$\tau = 924$ kPa

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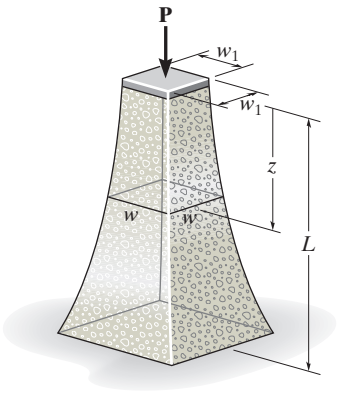
1–53.

The pier is made of material having a specific weight γ. If it has a square cross section, determine its width w as a function of z so that the average normal stress in the pier remains constant. The pier supports a constant load **P** at its top where its width is w_1.



SOLUTION

Assume constant stress σ_1, then at the top,

$$\sigma_1 = \frac{P}{{w_1}^2} \qquad \textbf{(1)}$$

For an increase in z the area must increase,

$$dA = \frac{dW}{\sigma_1} = \frac{\gamma A\, dz}{\sigma_1} \qquad \text{or} \qquad \frac{dA}{A} = \frac{\gamma}{\sigma_1} dz$$

For the top section:

$$\int_{A_1}^{A} \frac{dA}{A} = \frac{\gamma}{\sigma_1} \int_0^z dz$$

$$\text{In}\, \frac{A}{A_1} = \frac{\gamma}{\sigma_1} z$$

$$A = A_1 e^{\left(\frac{\gamma}{\sigma_1}\right) z}$$

$$A = w^2$$

$$A_1 = w_1^2$$

$$w = w_1\, e^{\left(\frac{\gamma}{2\sigma_1}\right) z}$$

From Eq. (1),

$$w = w_1\, e^{\left[\frac{w_1^2 \gamma}{2P}\right] z} \qquad \textbf{Ans.}$$

Ans:

$$w = w_1\, e^{\left[\frac{w_1^2 \gamma}{2P}\right] z}$$

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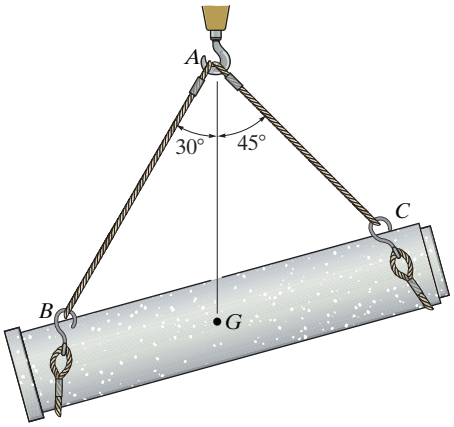
1–54.

The 2-Mg concrete pipe has a center of mass at point G. If it is suspended from cables AB and AC, determine the average normal stress in the cables. The diameters of AB and AC are 12 mm and 10 mm, respectively.

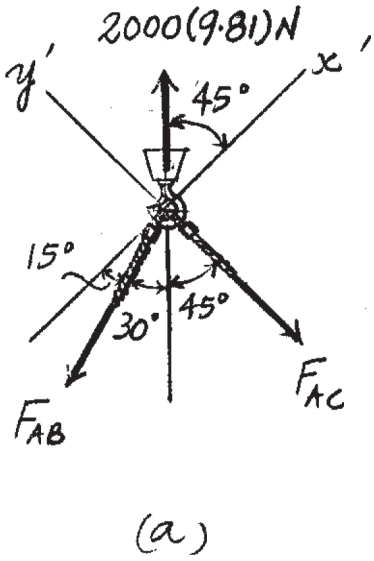


SOLUTION

Internal Loadings: The normal force developed in cables AB and AC can be determined by considering the equilibrium of the hook for which the free-body diagram is shown in Fig. a.

$\Sigma F_{x'} = 0; \quad 2000(9.81)\cos 45° - F_{AB}\cos 15° = 0 \quad F_{AB} = 14\,362.83\ \text{N (T)}$

$\Sigma F_{y'} = 0; \quad 2000(9.81)\sin 45° - 14\,362.83\sin 15° - F_{AC} = 0 \quad F_{AC} = 10\,156.06\ \text{N (T)}$



(a)

Average Normal Stress: The cross-sectional areas of cables AB and AC are $A_{AB} = \frac{\pi}{4}(0.012^2) = 0.1131(10^{-3})\ \text{m}^2$ and $A_{AC} = \frac{\pi}{4}(0.01^2) = 78.540(10^{-6})\ \text{m}^2$. We have

$$\sigma_{AB} = \frac{F_{AB}}{A_{AB}} = \frac{14\,362.83}{0.1131(10^{-3})} = 127\ \text{MPa} \qquad \textbf{Ans.}$$

$$\sigma_{AC} = \frac{F_{AC}}{A_{AC}} = \frac{10\,156.06}{78.540(10^{-6})} = 129\ \text{MPa} \qquad \textbf{Ans.}$$

Ans:
$\sigma_{AB} = 127$ MPa, $\sigma_{AC} = 129$ MPa

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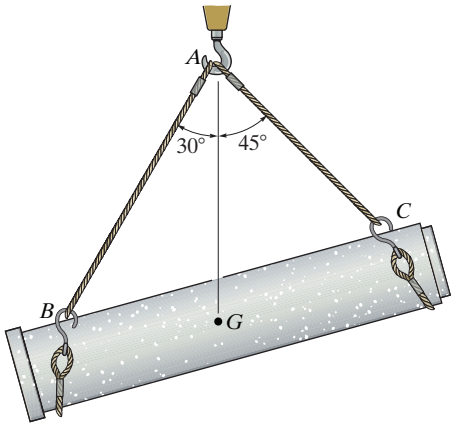
1–55.

The 2-Mg concrete pipe has a center of mass at point G. If it is suspended from cables AB and AC, determine the diameter of cable AB so that the average normal stress in this cable is the same as in the 10-mm-diameter cable AC.

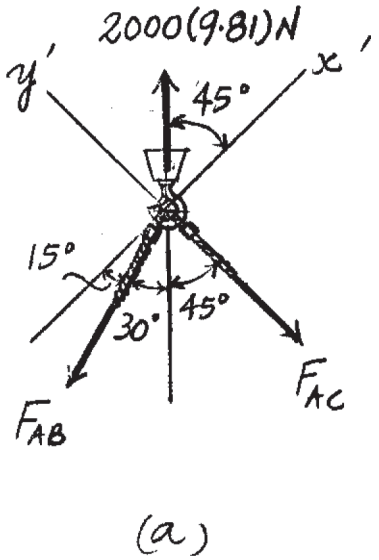


SOLUTION

Internal Loadings: The normal force in cables AB and AC can be determined by considering the equilibrium of the hook for which the free-body diagram is shown in Fig. *a*.

$\Sigma F_{x'} = 0;\ 2000(9.81)\cos 45° - F_{AB}\cos 15° = 0 \qquad F_{AB} = 14\,362.83\ \text{N (T)}$

$\Sigma F_{y'} = 0;\ 2000(9.81)\sin 45° - 14\,362.83\sin 15° - F_{AC} = 0 \quad F_{AC} = 10\,156.06\ \text{N (T)}$



(a)

Average Normal Stress: The cross-sectional areas of cables AB and AC are $A_{AB} = \frac{\pi}{4}d_{AB}^2$ and $A_{AC} = \frac{\pi}{4}(0.01^2) = 78.540(10^{-6})\ \text{m}^2$.

Here, we require

$$\sigma_{AB} = \sigma_{AC}$$

$$\frac{F_{AB}}{A_{AB}} = \frac{F_{AC}}{A_{AC}}$$

$$\frac{14\,362.83}{\frac{\pi}{4}d_{AB}^2} = \frac{10\,156.06}{78.540(10^{-6})}$$

$$d_{AB} = 0.01189\ \text{m} = 11.9\ \text{mm}$$ **Ans.**

Ans:
$d_{AB} = 11.9\ \text{mm}$

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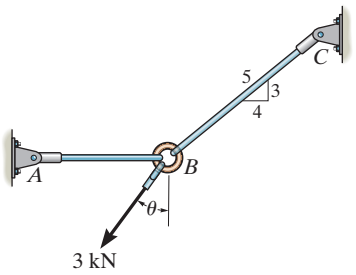
***1–56.**

Rods AB and BC have diameters of 4 mm and 6 mm, respectively. If the 3 kN force is applied to the ring at B, determine the angle θ so that the average normal stress in each rod is equivalent. What is this stress?

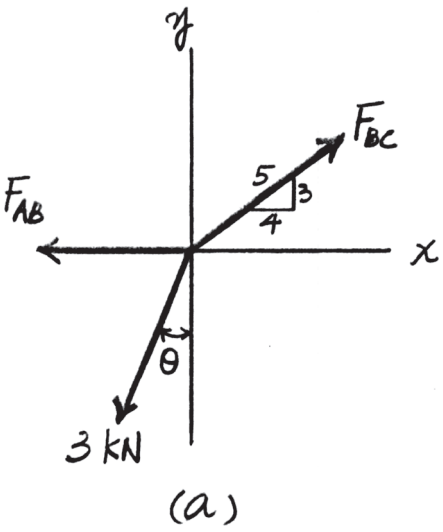


SOLUTION

Method of Joints: Referring to the FBD of joint B, Fig. a,

$$+\uparrow \Sigma F_y = 0; \qquad F_{BC}\left(\frac{3}{5}\right) - 3\cos\theta = 0 \qquad F_{BC} = 5\cos\theta \text{ kN}$$

$$\pm\rightarrow \Sigma F_x = 0; \qquad (5\cos\theta)\left(\frac{4}{5}\right) - 3\sin\theta - F_{AB} = 0 \qquad F_{AB} = (4\cos\theta - 3\sin\theta) \text{ kN}$$



Average Normal Stress:

$$\sigma_{AB} = \frac{F_{AB}}{A_{AB}} = \frac{[4\cos\theta - 3\sin\theta](10^3)}{\frac{\pi}{4}(0.004)^2} = \frac{250(10^6)}{\pi}(4\cos\theta - 3\sin\theta)$$

$$\sigma_{BC} = \frac{F_{BC}}{A_{BC}} = \frac{(5\cos\theta)(10^3)}{\frac{\pi}{4}(0.006)^2} = \left[\frac{555.56(10^6)}{\pi}\right]\cos\theta$$

It is required that

$$\sigma_{AB} = \sigma_{BC}$$

$$\frac{250(10^6)}{\pi}(4\cos\theta - 3\sin\theta) = \left[\frac{555.56(10^6)}{\pi}\right]\cos\theta$$

$$1.7778\cos\theta - 3\sin\theta = 0$$

$$\tan\theta = \frac{1.7778}{3}$$

$$\theta = 30.65° = 30.7° \qquad \textbf{Ans.}$$

Then

$$\sigma = \sigma_{BC} = \left[\frac{555.56(10^6)}{\pi}\right]\cos 30.65° = 152.13(10^6) \text{ Pa} = 152 \text{ MPa} \qquad \textbf{Ans.}$$

Ans:
$\theta = 30.7°$,
$\sigma = 152$ MPa

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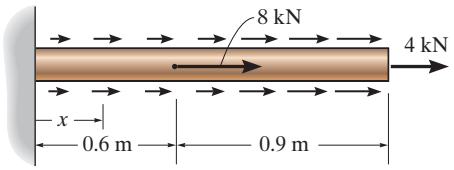
1–57.

The bar has a cross-sectional area of $400(10^{-6})$ m^2. If it is subjected to a triangular axial distributed loading along its length which is 0 at $x = 0$ and 9 kN/m at $x = 1.5$ m, and to two concentrated loads as shown, determine the average normal stress in the bar as a function of x for $0 \le x < 0.6$ m.



SOLUTION

Internal Loading: Referring to the FBD of the right segment of the bar sectioned at x, Fig. a,

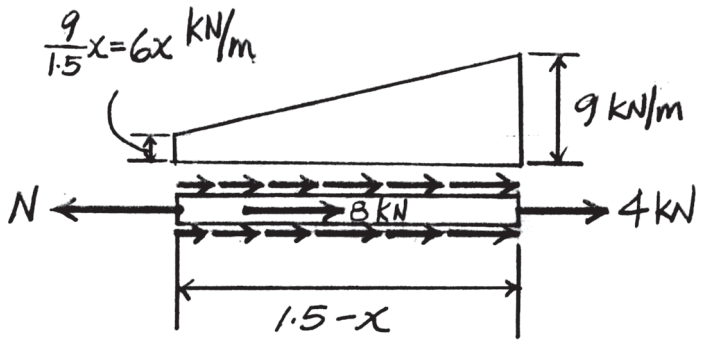
$$\stackrel{+}{\rightarrow} \Sigma F_x = 0; \qquad 8 + 4 + \frac{1}{2}(6x + 9)(1.5 - x) = 0$$

$$N = \left\{18.75 - 3x^2\right\} \text{ kN}$$

Average Normal Stress:

$$\sigma = \frac{N}{A} = \frac{\left(18.75 - 3x^2\right)\left(10^3\right)}{400\left(10^{-6}\right)}$$

$$= \left\{46.9 - 7.50x^2\right\} \text{ MPa} \qquad \textbf{Ans.}$$



(a)

Ans:
$\sigma = \left\{46.9 - 7.50x^2\right\}$ MPa

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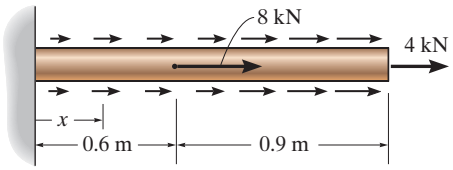
1–58.

The bar has a cross-sectional area of $400(10^{-6})\ \text{m}^2$. If it is subjected to a uniform axial distributed loading along its length of 9 kN/m, and to two concentrated loads as shown, determine the average normal stress in the bar as a function of x for $0.6\ \text{m} < x \leq 1.5\ \text{m}$.



SOLUTION

Internal Loading: Referring to a FBD of the right segment of the bar sectioned at x,

$$\stackrel{+}{\rightarrow} \Sigma F_x = 0; \qquad 4 + 9\,(1.5 - x) - N = 0$$

$$N = \left\{17.5 - 9x\right\}\ \text{kN}$$

Average Normal Stress:

$$\sigma = \frac{N}{A} = \frac{(17.5 - 9x)(10^3)}{400(10^{-6})}$$

$$= \left\{43.75 - 22.5x\right\}\ \text{MPa} \qquad \textbf{Ans.}$$

Ans:
$\sigma = \left\{43.75 - 22.5x\right\}\ \text{MPa}$

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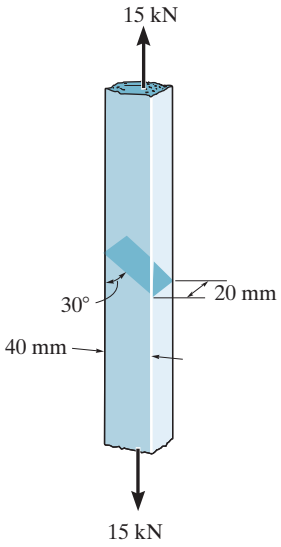
1–59.

The two steel members are joined together using a 30° scarf weld. Determine the average normal and average shear stress resisted in the plane of the weld.



SOLUTION

Internal Loadings: Referring to the FBD of the upper segment of the member sectioned through the scarf weld, Fig. *a*,

$\Sigma F_x = 0; \quad N - 15 \sin 30° = 0 \quad N = 7.50 \text{ kN}$

$\Sigma F_y = 0; \quad V - 15 \cos 30° = 0 \quad V = 12.99 \text{ kN}$

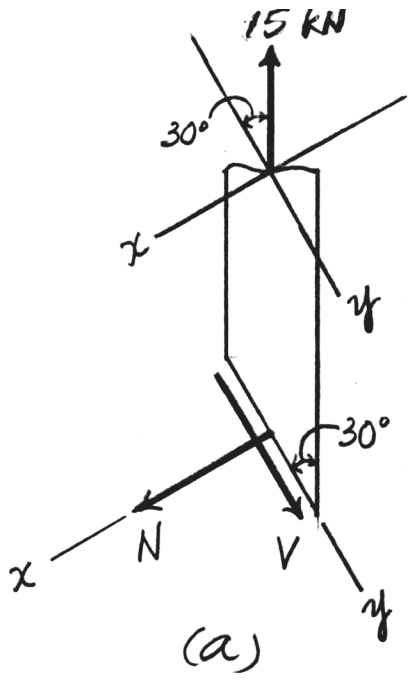
Average Normal and Shear Stress: The area of the scarf weld is

$$A = 0.02\left(\frac{0.04}{\sin 30°}\right) = 1.6\left(10^{-3}\right) \text{ m}^2$$

Thus,

$$\sigma = \frac{N}{A_n} = \frac{7.50\left(10^3\right)}{1.6\left(10^{-3}\right)} = 4.6875\left(10^6\right) \text{ Pa} = 4.69 \text{ MPa} \qquad \textbf{Ans.}$$

$$\tau = \frac{V}{A_v} = \frac{12.99\left(10^3\right)}{1.6\left(10^{-3}\right)} = 8.119\left(10^6\right) \text{ Pa} = 8.12 \text{ MPa} \qquad \textbf{Ans.}$$



Ans:
$\sigma = 4.69$ MPa,
$\tau = 8.12$ MPa

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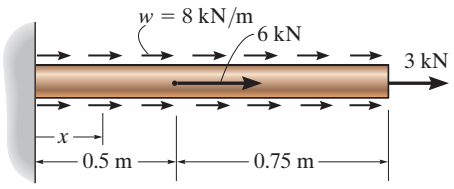
***1–60.**

The bar has a cross-sectional area of $400(10^{-6})$ m^2. If it is subjected to a uniform axial distributed loading along its length and to two concentrated loads, determine the average normal stress in the bar as a function of x for $0 < x \leq 0.5$ m.

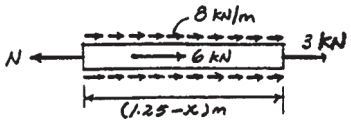


SOLUTION

Equation of Equilibrium:

$$\stackrel{+}{\rightarrow} \Sigma F_x = 0; \qquad -N + 3 + 6 + 8(1.25 - x) = 0$$

$$N = (19.0 - 8.00x) \text{ kN}$$



Average Normal Stress:

$$\sigma = \frac{N}{A} = \frac{(19.0 - 8.00x)(10^3)}{400(10^{-6})}$$

$$= (47.5 - 20.0x) \text{ MPa} \qquad \textbf{Ans.}$$

Ans:
$\sigma = (47.5 - 20.0x)$ MPa

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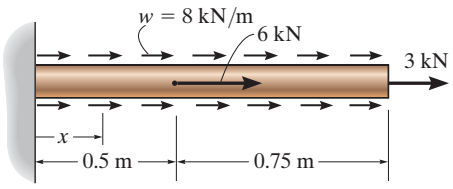
1–61.

The bar has a cross-sectional area of $400(10^{-6})$ m^2. If it is subjected to a uniform axial distributed loading along its length and to two concentrated loads, determine the average normal stress in the bar as a function of x for $0.5\text{ m} < x \leq 1.25\text{ m}$.

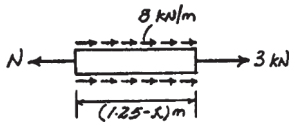


SOLUTION

Equation of Equilibrium:

$$\stackrel{+}{\rightarrow}\Sigma F_x = 0; \qquad -N + 3 + 8(1.25 - x) = 0$$

$$N = (13.0 - 8.00x)\text{ kN}$$



Average Normal Stress:

$$\sigma = \frac{N}{A} = \frac{(13.0 - 8.00x)(10^3)}{400(10^{-6})}$$

$$= (32.5 - 20.0x)\text{ MPa} \qquad \textbf{Ans.}$$

Ans:
$\sigma = (32.5 - 20.0x)$ MPa

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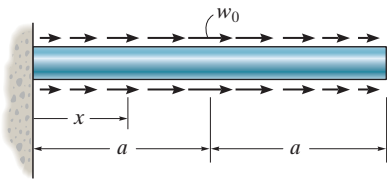
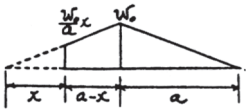
1–62.

The prismatic bar has a cross-sectional area A. If it is subjected to a distributed axial loading that increases linearly from $w = 0$ at $x = 0$ to $w = w_0$ at $x = a$, and then decreases linearly to $w = 0$ at $x = 2a$, determine the average normal stress in the bar as a function of x for $0 \le x < a$.



SOLUTION

Equation of Equilibrium:

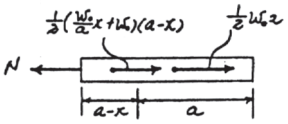


$$\stackrel{+}{\rightarrow} \Sigma F_x = 0; \qquad -N + \frac{1}{2}\left(\frac{w_0}{a}x + w_0\right)(a - x) + \frac{1}{2}w_0 a = 0$$

$$N = \frac{w_0}{2a}\left(2a^2 - x^2\right)$$

Average Normal Stress:

$$\sigma = \frac{N}{A} = \frac{\frac{w_0}{2a}\left(2a^2 - x^2\right)}{A} = \frac{w_0}{2aA}\left(2a^2 - x^2\right) \qquad \textbf{Ans.}$$



Ans:

$$\sigma = \frac{w_0}{2aA}\left(2a^2 - x^2\right)$$

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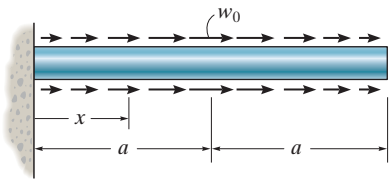
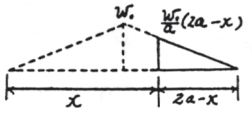
1–63.

The prismatic bar has a cross-sectional area A. If it is subjected to a distributed axial loading that increases linearly from $w = 0$ at $x = 0$ to $w = w_0$ at $x = a$, and then decreases linearly to $w = 0$ at $x = 2a$, determine the average normal stress in the bar as a function of x for $a < x \leq 2a$.



SOLUTION

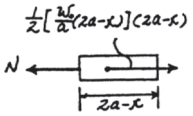


Equation of Equilibrium:

$$\stackrel{+}{\rightarrow} \Sigma F_x = 0; \qquad -N + \frac{1}{2}\left[\frac{w_0}{a}(2a - x)\right](2a - x) = 0$$

$$N = \frac{w_0}{2a}(2a - x)^2$$

Average Normal Stress:



$$\sigma = \frac{N}{A} = \frac{\frac{w_0}{2a}(2a - x)^2}{A} = \frac{w_0}{2aA}(2a - x)^2$$ **Ans.**

Ans:

$$\sigma = \frac{w_0}{2aA}(2a - x)^2$$

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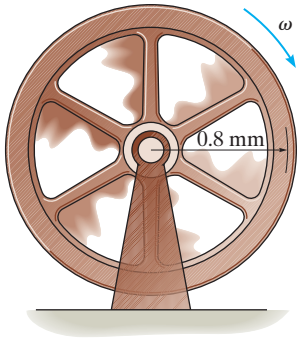
***1–64.**

Determine the greatest constant angular velocity ω of the flywheel so that the average normal stress in its rim does not exceed $\sigma = 15$ MPa. Assume the rim is a thin ring having a thickness of 3 mm, width of 20 mm, and a mass of 30 kg/m. Rotation occurs in the horizontal plane. Neglect the effect of the spokes in the analysis. *Hint:* Consider a free-body diagram of a semicircular segment of the ring. The center of mass for this segment is located at $\hat{r} = 2r/\pi$ from the center.



SOLUTION

$+\downarrow \Sigma F_n = m(a_G)_n;$ $\quad 2T = m(\bar{r})\omega^2$

$$2\sigma A = m\left(\frac{2r}{\pi}\right)\omega^2$$

$$2\left(15\left(10^6\right)\right)(0.003)(0.020) = \pi(0.8)(30)\left(\frac{2(0.8)}{\pi}\right)\omega^2$$

$$\omega = 6.85 \text{ rad/s}$$ **Ans.**

Ans:
$\omega = 6.85$ rad/s

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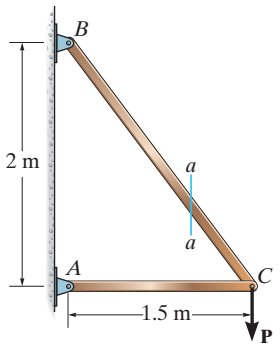
1–65.

Determine the largest load **P** that can be applied to the frame without causing either the average normal stress or the average shear stress at section *a–a* to exceed $\sigma = 150$ MPa and $\tau = 60$ MPa, respectively. Member *CB* has a square cross section of 25 mm on each side.



SOLUTION

Analyze the equilibrium of joint *C* using the FBD shown in Fig. *a*,

$$+\uparrow \Sigma F_y = 0; \quad F_{BC}\left(\frac{4}{5}\right) - P = 0 \quad F_{BC} = 1.25P$$

Referring to the FBD of the cut segment of member *BC* Fig. *b*.

$$\stackrel{+}{\rightarrow} \Sigma F_x = 0; \qquad N_{a-a} - 1.25P\left(\frac{3}{5}\right) = 0 \qquad N_{a-a} = 0.75P$$

$$+\uparrow \Sigma F_y = 0; \qquad 1.25P\left(\frac{4}{5}\right) - V_{a-a} = 0 \qquad V_{a-a} = P$$

The cross-sectional area of section *a–a* is $A_{a-a} = (0.025)\left(\frac{0.025}{3/5}\right) = 1.0417(10^{-3})\ \text{m}^2$. For Normal stress,

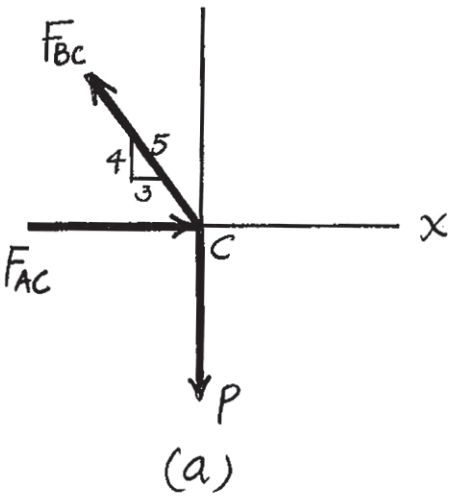
$$\sigma_{\text{allow}} = \frac{N_{a-a}}{A_{a-a}}; \quad 150(10^6) = \frac{0.75P}{1.0417(10^{-3})}$$

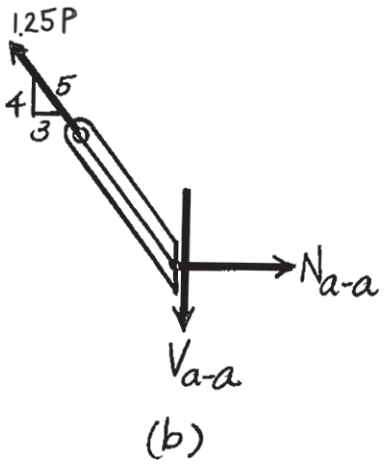
$$P = 208.33(10^3)\ \text{N} = 208.33\ \text{kN}$$

For Shear Stress

$$\tau_{\text{allow}} = \frac{V_{a-a}}{A_{a-a}}; \quad 60(10^6) = \frac{P}{1.0417(10^{-3})}$$

$P = 62.5(10^3)$ N $= 62.5$ kN (Controls!) **Ans.**





Ans:
$P = 62.5$ kN

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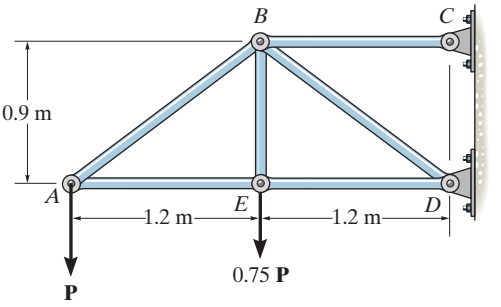
1–66. The bars of the truss each have a cross-sectional area of 780 mm^2. Determine the average normal stress in each member due to the loading P = 40 kN. State whether the stress is tensile or compressive.

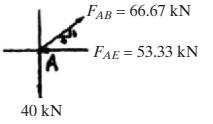


SOLUTION

Joint A:

$$\sigma_{AB} = \frac{F_{AB}}{A_{AB}} = \frac{66.67(10^3)}{780(10^{-6})} = 85.5 \text{ MPa} \quad \text{(T)} \qquad \textbf{Ans.}$$

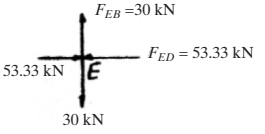
$$\sigma_{AE} = \frac{F_{AE}}{A_{AE}} = \frac{53.33(10^3)}{780(10^{-6})} = 68.4 \text{ MPa} \quad \text{(C)} \qquad \textbf{Ans.}$$



Joint E:

$$\sigma_{ED} = \frac{F_{ED}}{A_{ED}} = \frac{53.33(10^3)}{780(10^{-6})} = 68.4 \text{ MPa} \quad \text{(C)} \qquad \textbf{Ans.}$$

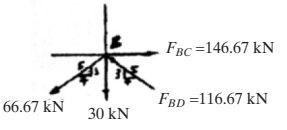
$$\sigma_{EB} = \frac{F_{EB}}{A_{EB}} = \frac{30(10^3)}{780(10^{-6})} = 38.5 \text{ MPa} \quad \text{(T)} \qquad \textbf{Ans.}$$



Joint B:

$$\sigma_{BC} = \frac{F_{BC}}{A_{BC}} = \frac{146.67(10^3)}{780(10^{-6})} = 188 \text{ MPa} \quad \text{(T)} \qquad \textbf{Ans.}$$

$$\sigma_{BD} = \frac{F_{BD}}{A_{BD}} = \frac{116.67(10^3)}{780(10^{-6})} = 150 \text{ MPa} \quad \text{(C)} \qquad \textbf{Ans.}$$



Ans:

Joint A: σ_{AB} = 85.5 MPa (T),
σ_{AE} = 68.4 MPa (C)
Joint E: σ_{ED} = 68.4 MPa (C),
σ_{EB} = 38.5 MPa (T)
Joint B: σ_{BC} = 188 MPa (T),

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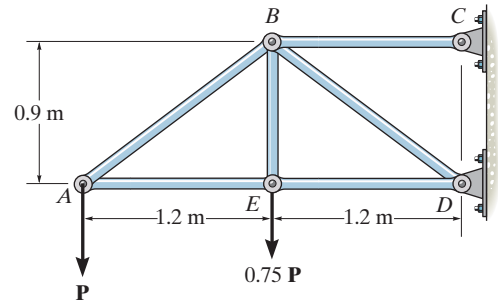
1–67. The bars of the truss each have a cross-sectional area of 780 mm^2. If the maximum average normal stress in any bar is not to exceed 140 MPa, determine the maximum magnitude P of the loads that can be applied to the truss.



SOLUTION

Joint A:

$$+\uparrow \Sigma F_y = 0; \quad -P + \frac{3}{5} F_{AB} = 0$$

$$F_{AB} = 1.6667P \quad \text{(T)}$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad -F_{AE} + (1.6667P)\left(\frac{4}{5}\right) = 0$$

$$F_{AE} = 1.3333P \quad \text{(C)}$$

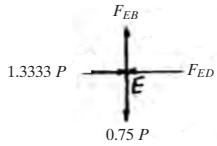
Joint E:

$$+\uparrow \Sigma F_y = 0; \quad F_{EB} - 0.75P = 0$$

$$F_{EB} = 0.75P \quad \text{(T)}$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad 1.3333P - F_{ED} = 0$$

$$F_{ED} = 1.3333P \quad \text{(C)}$$



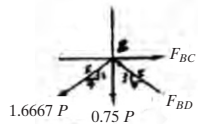
Joint B:

$$+\uparrow \Sigma F_y = 0; \quad \frac{3}{5}F_{BD} - 0.75P - (1.6667P)\left(\frac{3}{5}\right) = 0$$

$$F_{BD} = 2.9167P \quad \text{(C)}$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad F_{BC} - (2.9167P)\left(\frac{4}{5}\right) - (1.6667P)\left(\frac{4}{5}\right) = 0$$

$$F_{BC} = 3.6667P \quad \text{(T)}$$



The highest stressed member is BC:

$$\sigma_{BC} = \sigma_{\max}; \quad \frac{3.6667P}{780(10^{-6})} = 140(10^6)$$

$$P = 29.78(10^3)\ \text{N} = 29.8\ \text{kN} \quad \textbf{Ans.}$$

Ans:
$P = 29.8$ kN

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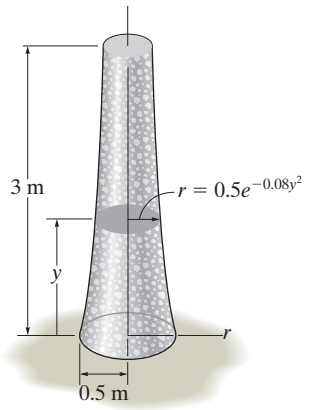
***1–68.**

The radius of the pedestal is defined by $r = (0.5e^{-0.08y^2})$ m, where y is in meters. If the material has a density of 2.5 Mg/m^3, determine the average normal stress at the support.



SOLUTION

$A = \pi(0.5)^2 = 0.7854 \text{ m}^2$

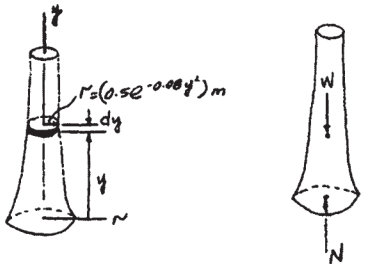
$dV = \pi(r^2)\, dy = \pi(0.5)^2 \left(e^{-0.08y^2}\right)^2$

$V = \int_0^3 \pi(0.5)^2 \left(e^{-0.08y^2}\right)^2 dy = 0.7854 \int_0^3 \left(e^{-0.08y^2}\right)^2 dy$

$W = \rho g\, V = (2500)(9.81)(0.7854) \int_0^3 \left(e^{-0.08y^2}\right)^2 dy$

$W = 19.262\left(10^3\right) \int_0^3 \left(e^{-0.08y^2}\right)^2 dy = 38.849 \text{ kN}$

$\sigma = \dfrac{W}{A} = \dfrac{38.849}{0.7854} = 49.5 \text{ kPa}$ **Ans.**



Ans:
$\sigma = 49.5$ kPa

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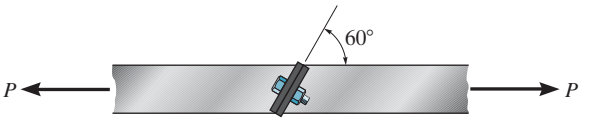
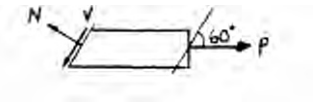
1–69. The tension member is fastened together using *two* bolts, one on each side of the member as shown. Each bolt has a diameter of 7.5 mm. Determine the maximum load P that can be applied to the member if the allowable shear stress for the bolts is $\tau_{allow} = 84$ MPa and the allowable average normal stress is $\sigma_{allow} = 140$ MPa.



SOLUTION



$\nwarrow + \Sigma F_y = 0; \quad N - P \sin 60° = 0$

$N = 0.8660\,P$

$\swarrow + \Sigma F_x = 0; \quad V - P \cos 60° = 0$

$V = 0.5P$

Assume failure due to shear:

$$\tau_{allow} = \frac{V}{2A_b}; \quad 84(10^6) = \frac{0.5P}{2\left[\frac{\pi}{4}(0.0075^2)\right]}$$

$P = 14.84(10^3)$ N = 14.84 kN

Assume failure due to normal force:

$$\sigma_{allow} = \frac{N}{2A_b}; \quad 140(10^6) = \frac{0.8660P}{2\left[\frac{\pi}{4}(0.0075^2)\right]}$$

$P = 14.28(10^3)$ N = 14.3 kN (controls) **Ans.**

Ans:
$P = 14.3$ kN (controls)

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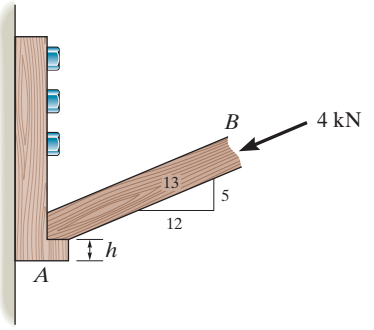
1–70. Member B is subjected to a compressive force of 4 kN. If A and B are both made of wood and are 10 mm thick, determine to the nearest multiples of 5 mm the smallest dimension h of the horizontal segment so that it does not fail in shear. The average shear stress for the segment is $\tau_{allow} = 2.1$ MPa.

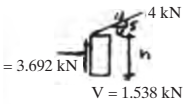


SOLUTION

$$\tau_{allow} = \frac{V}{A};\quad 2.1(10^6) = \frac{1.538(10^3)}{(0.01)h}$$

$$h = 0.07326 \text{ m} = 73.26 \text{ mm}$$

Use $h = 75$ mm **Ans.**



Ans:

$h = 0.07326$ m, use $h = 75$ mm

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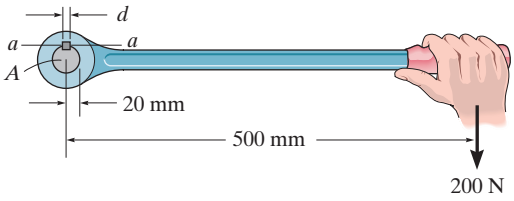
1–71.

The lever is attached to the shaft A using a key that has a width d and length of 25 mm. If the shaft is fixed and a vertical force of 200 N is applied perpendicular to the handle, determine the dimension d if the allowable shear stress for the key is $\tau_{\text{allow}} = 35$ MPa.



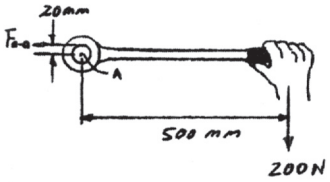
SOLUTION

$\zeta+\Sigma M_A = 0;$ $\quad F_{a-a}(20) - 200(500) = 0$

$F_{a-a} = 5000$ N

$\tau_{\text{allow}} = \dfrac{F_{a-a}}{A_{a-a}};$ $\quad 35(10^6) = \dfrac{5000}{d(0.025)}$

$d = 0.00571$ m $= 5.71$ mm **Ans.**



Ans:
$d = 5.71$ mm

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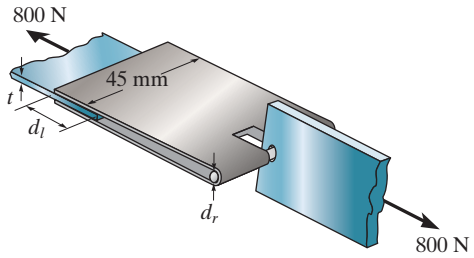
***1–72.** The lapbelt assembly is to be subjected to a force of 800 N. Determine (a) the required thickness t of the belt if the allowable tensile stress for the material is $(\sigma_t)_{\text{allow}} = 10\,\text{MPa}$, (b) the required lap length d_l if the glue can sustain an allowable shear stress of $(\tau_{\text{allow}})_g = 0.75\,\text{MPa}$, and (c) the required diameter d_r of the pin if the allowable shear stress for the pin is $(\tau_{\text{allow}})_p = 30\,\text{MPa}$.

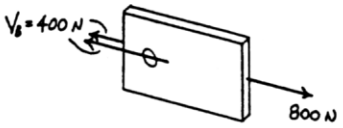


SOLUTION

Allowable Normal Stress: Design of belt thickness.

$$(\sigma_t)_{\text{allow}} = \frac{P}{A}; \qquad 10\left(10^6\right) = \frac{800}{(0.045)t}$$

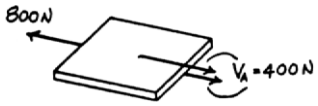
$$t = 0.001778\,\text{m} = 1.78\,\text{mm} \qquad \textbf{Ans.}$$



Allowable Shear Stress: Design of lap length.

$$(\tau_{\text{allow}})_g = \frac{V_A}{A}; \qquad 0.750\left(10^6\right) = \frac{400}{(0.045)\,d_t}$$

$$d_t = 0.01185\,\text{m} = 11.9\,\text{mm} \qquad \textbf{Ans.}$$



Allowable Shear Stress: Design of pin size.

$$(\tau_{\text{allow}})_P = \frac{V_B}{A}; \qquad 30\left(10^6\right) = \frac{400}{\frac{\pi}{4}\,d_r^2}$$

$$d_r = 0.004120\,\text{m} = 4.12\,\text{mm} \qquad \textbf{Ans.}$$

Ans:

$t = 1.78\,\text{mm}$, $d_t = 11.9\,\text{mm}$,
$d_r = 4.12\,\text{mm}$

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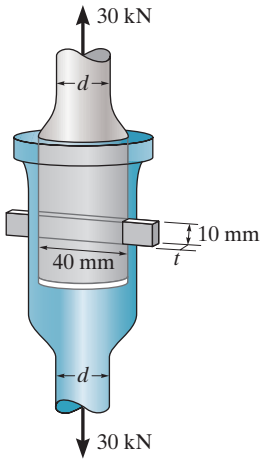
1–73.

The cotter is used to hold the two rods together. Determine the smallest thickness t of the cotter and the smallest diameter d of the rods. All parts are made of steel for which the failure normal stress is $\sigma_{\text{fail}} = 500$ MPa and the failure shear stress is $\tau_{\text{fail}} = 375$ MPa. Use a factor of safety of $(\text{F.S.})_t = 2.50$ in tension and $(\text{F.S.})_s = 1.75$ in shear.



SOLUTION

Allowable Normal Stress: Design of rod size

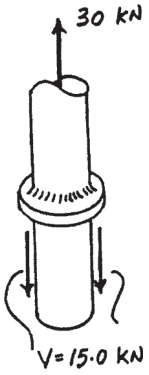
$$\sigma_{\text{allow}} = \frac{\sigma_{\text{fail}}}{\text{F.S}} = \frac{P}{A}; \qquad \frac{500(10^6)}{2.5} = \frac{30(10^3)}{\frac{\pi}{4}d^2}$$

$$d = 0.01382\ \text{m} = 13.8\ \text{mm} \qquad \textbf{Ans.}$$

Allowable Shear Stress: Design of cotter size.

$$\tau_{\text{allow}} = \frac{\tau_{\text{fail}}}{\text{F.S}} = \frac{V}{A}; \qquad \frac{375(10^6)}{1.75} = \frac{15.0(10^3)}{(0.01)t}$$

$$t = 0.0070\ \text{m} = 7.00\ \text{mm} \qquad \textbf{Ans.}$$



Ans:
$d = 13.8$ mm,
$t = 7.00$ mm

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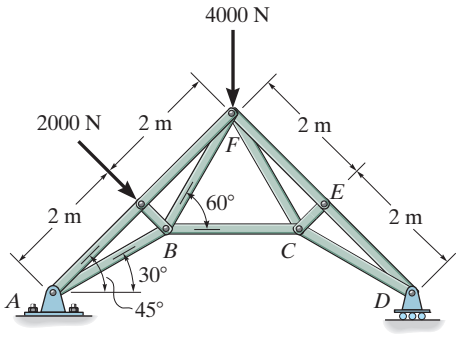
1–74. The truss is used to support the loading shown Determine the required cross-sectional area of member BC if the allowable normal stress is $\sigma_{\text{allow}} = 165$ MPa.

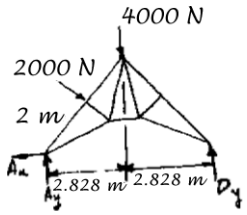


SOLUTION

For the entire truss,

$$\circlearrowleft + \Sigma M_A = 0; \quad -2000(2) - 4000(2.8284) + 2(2.8284)(D_y) = 0$$

$$D_y = 2707.11 \text{ N}$$



For method of section,

$$\circlearrowleft + \Sigma M_F = 0; \quad 2707.11(2.8284) - F_{BC}(1.7932) = 0$$

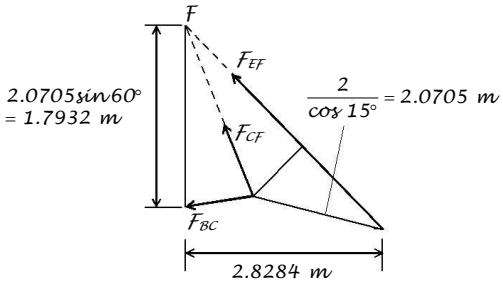
$$F_{BC} = 4270.06 \text{ N}$$

Using the given allowable stress,

$$\sigma_{\text{allow}} = \frac{F_{BC}}{A}; \quad 165(10^6) = \frac{4270.06}{A}$$

$$A = 25.88(10^{-6}) \text{ m}^2 = 25.9 \text{ mm}^2$$

Ans.



Ans:

$A = 25.9 \text{ mm}^2$

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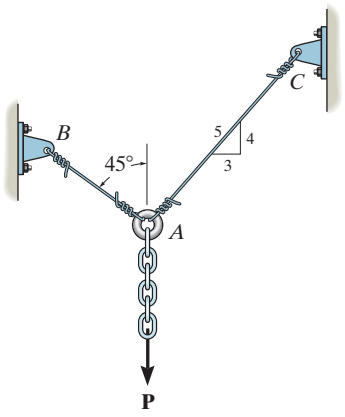
1–75.

If the allowable tensile stress for wires AB and AC is $\sigma_{\text{allow}} = 200$ MPa, determine the required diameter of each wire if the applied load is $P = 6$ kN.



SOLUTION

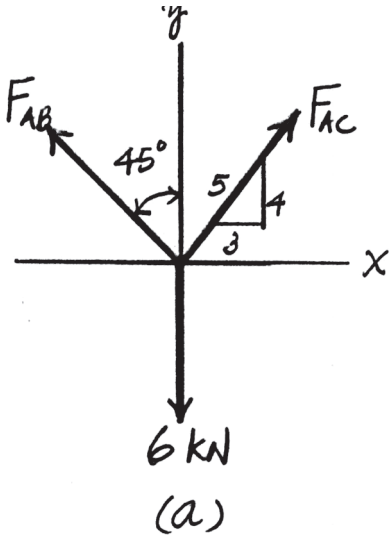
Normal Forces: Analyzing the equilibrium of joint A, Fig. a,

$$\stackrel{+}{\rightarrow}\Sigma F_x = 0; \qquad F_{AC}\left(\frac{3}{5}\right) - F_{AB}\sin 45° = 0 \qquad \textbf{(1)}$$

$$+\uparrow \Sigma F_y = 0; \qquad F_{AC}\left(\frac{4}{5}\right) + F_{AB}\cos 45° - 6 = 0 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2)

$$F_{AC} = 4.2857 \text{ kN} \qquad F_{AB} = 3.6365 \text{ kN}$$



Average Normal Stress: For wire AB,

$$\sigma_{\text{allow}} = \frac{F_{AB}}{A_{AB}}; \qquad 200(10^6) = \frac{3.6365(10^3)}{\frac{\pi}{4}d_{AB}^2}$$

$$d_{AB} = 0.004812 \text{ m} = 4.81 \text{ mm} \qquad \textbf{Ans.}$$

For wire AC,

$$\sigma_{\text{allow}} = \frac{F_{AC}}{A_{AC}}; \qquad 200(10^6) = \frac{4.2857(10^3)}{\frac{\pi}{4}d_{AC}^2}$$

$$d_{AC} = 0.005223 \text{ m} = 5.22 \text{ mm} \qquad \textbf{Ans.}$$

Ans:
$d_{AB} = 4.81$ mm,
$d_{AC} = 5.22$ mm

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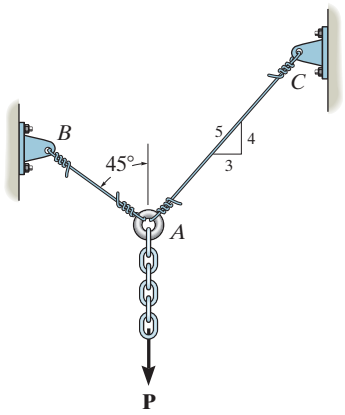
***1–76.**

If the allowable tensile stress for wires AB and AC is $\sigma_{\text{allow}} = 180$ MPa, and wire AB has a diameter of 5 mm and AC has a diameter of 6 mm, determine the greatest force P that can be applied to the chain.



SOLUTION

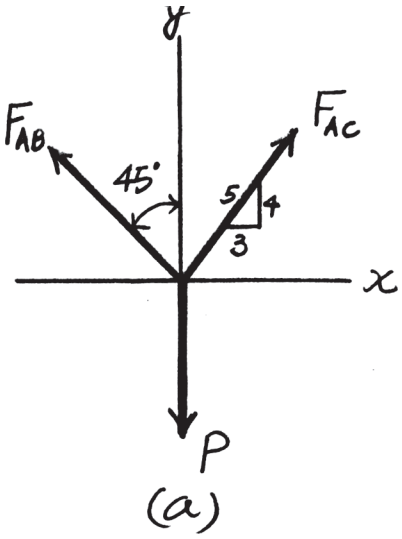
Normal Forces: Analyzing the equilibrium of joint A, Fig. a,

$$\stackrel{+}{\rightarrow}\Sigma F_x = 0; \qquad F_{AC}\left(\frac{3}{5}\right) - F_{AB}\sin 45° = 0 \qquad \textbf{(1)}$$

$$+\uparrow \Sigma F_y = 0; \qquad F_{AC}\left(\frac{4}{5}\right) + F_{AB}\cos 45° - P = 0 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2)

$$F_{AC} = 0.7143P \qquad F_{AB} = 0.6061P$$



Average Normal Stress: Assuming failure of wire AB,

$$\sigma_{\text{allow}} = \frac{F_{AB}}{A_{AB}}; \qquad 180(10^6) = \frac{0.6061P}{\frac{\pi}{4}(0.005^2)}$$

$$P = 5.831(10^3)\text{ N} = 5.83\text{ kN}$$

Assume the failure of wire AC,

$$\sigma_{\text{allow}} = \frac{F_{AC}}{A_{AC}}; \qquad 180(10^6) = \frac{0.7143P}{\frac{\pi}{4}(0.006^2)}$$

$$P = 7.125(10^3)\text{ N} = 7.13\text{ kN}$$

Choose the smaller of the two values of P,

$$P = 5.83\text{ kN} \qquad \textbf{Ans.}$$

Ans:
$P = 5.83$ kN

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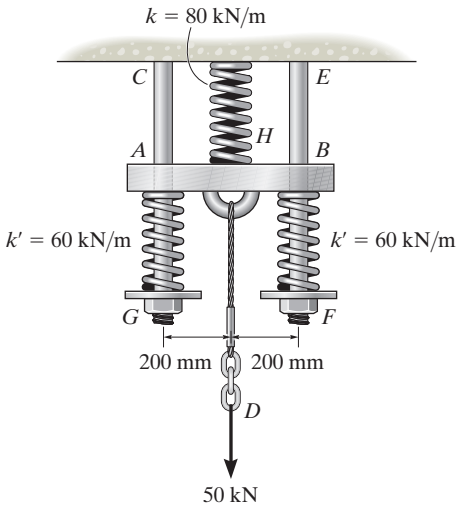
1–77.

The spring mechanism is used as a shock absorber for a load applied to the drawbar AB. Determine the force in each spring when the 50-kN force is applied. Each spring is originally unstretched and the drawbar slides along the smooth guide posts CG and EF. The ends of all springs are attached to their respective members. Also, what is the required diameter of the shank of bolts CG and EF if the allowable stress for the bolts is $\sigma_{\text{allow}} = 150$ MPa?



SOLUTION

Equations of Equilibrium:

$$\zeta + \Sigma M_H = 0; \qquad -F_{BF}(200) + F_{AG}(200) = 0$$

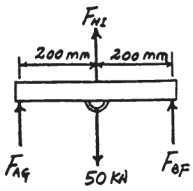
$$F_{BF} = F_{AG} = F$$

$$+\uparrow \Sigma F_y = 0; \qquad 2F + F_H - 50 = 0 \qquad \textbf{(1)}$$

Required,

$$\Delta_H = \Delta_B; \qquad \frac{F_H}{80} = \frac{F}{60}$$

$$F = 0.75\,F_H \qquad \textbf{(2)}$$



Solving Eqs. (1) and (2) yields,

$$F_H = 20.0 \text{ kN} \qquad \textbf{Ans.}$$

$$F_{BF} = F_{AG} = F = 15.0 \text{ kN} \qquad \textbf{Ans.}$$

Allowable Normal Stress: Design of bolt shank size.

$$\sigma_{\text{allow}} = \frac{P}{A}; \qquad 150(10^6) = \frac{15.0(10^3)}{\frac{\pi}{4}d^2}$$

$$d = 0.01128 \text{ m} = 11.3 \text{ mm}$$

$$d_{EF} = d_{CG} = 11.3 \text{ mm} \qquad \textbf{Ans.}$$

Ans:
$F_H = 20.0$ kN,
$F_{BF} = F_{AG} = 15.0$ kN,
$d_{EF} = d_{CG} = 11.3$ mm

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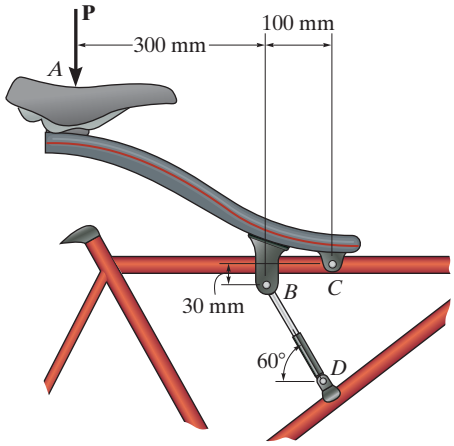
1–78. The soft-ride suspension system of the mountain bike is pinned at C and supported by the shock absorber BD. If it is designed to support a load $P = 1500$ N, determine the required minimum diameter of pins B and C. Use a factor of safety of 2 against failure. The pins are made of material having a failure shear stress of $\tau_{fail} = 150$ MPa, and each pin is subjected to double shear.



SOLUTION

Internal Loadings: The forces acting on pins B and C can be determined by considering the equilibrium of the free-body diagram of the soft-ride suspension system shown in Fig. *a*.

$\circlearrowleft + \Sigma M_C = 0;$ $\quad 1500(0.4) - F_{BD}\sin 60°(0.1) - F_{BD}\cos 60°(0.03) = 0$

$F_{BD} = 5905.36$ N

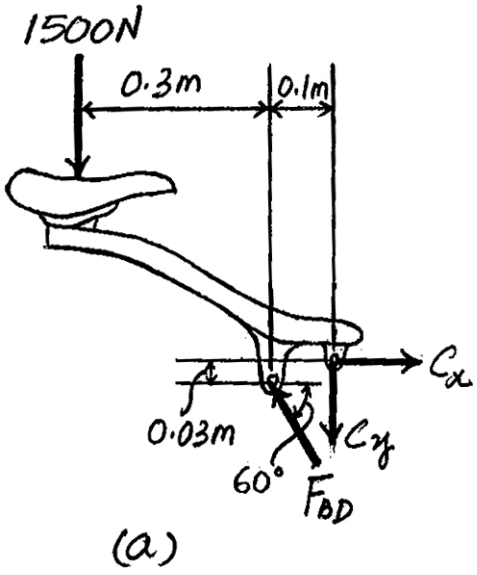
$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad C_x - 5905.36\cos 60° = 0$ $\quad C_x = 2952.68$ N

$+\uparrow \Sigma F_y = 0;$ $\quad 5905.36\sin 60° - 1500 - C_y = 0$ $\quad C_y = 3614.20$ N

Thus,

$F_B = F_{BD} = 5905.36$ N $\quad F_C = 2\sqrt{C_x^2 + C_y^2} = 2\sqrt{2952.68^2 + 3614.20^2}$

$= 4666.98$ N



Since **both** pins are in double shear,

$V_B = \frac{F_B}{2} = \frac{5905.36}{2} = 2952.68$ N $\quad V_C = \frac{F_C}{2} = \frac{4666.98}{2} = 2333.49$ N

Allowable Shear Stress:

$\tau_{allow} = \frac{\tau_{fail}}{\text{F.S.}} = \frac{150}{2} = 75$ MPa

Using this result,

$\tau_{allow} = \frac{V_B}{A_B};$ $\quad 75(10^6) = \frac{2952.68}{\frac{\pi}{4}d_B^2}$

$d_B = 0.007080$ m $= 7.08$ mm **Ans.**

$\tau_{allow} = \frac{V_C}{A_C};$ $\quad 75(10^6) = \frac{2333.49}{\frac{\pi}{4}d_C^2}$

$d_C = 0.006294$ m $= 6.29$ mm **Ans.**

Ans:

$d_B = 7.08$ mm, $d_C = 6.29$ mm

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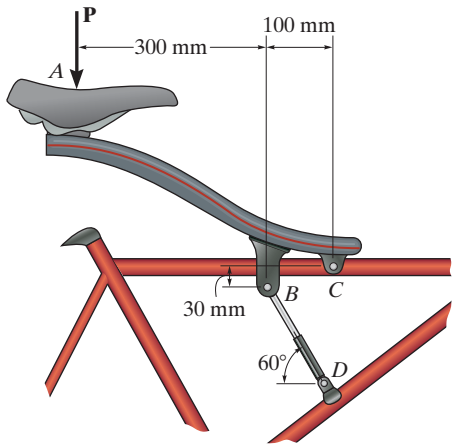
1–79. The soft-ride suspension system of the mountain bike is pinned at C and supported by the shock absorber BD. If it is designed to support a load of $P = 1500$ N, determine the factor of safety of pins B and C against failure if they are made of a material having a shear failure stress of $\tau_{fail} = 150$ MPa. Pin B has a diameter of 7.5 mm, and pin C has a diameter of 6.5 mm. Both pins are subjected to double shear.



SOLUTION

Internal Loadings: The forces acting on pins B and C can be determined by considerning the equilibrium of the free-body diagram of the soft-ride suspension system shown in Fig. *a*.

$+\Sigma M_C = 0;$ $\quad 1500(0.4) - F_{BD}\sin 60°(0.1) - F_{BD}\cos 60°(0.03) = 0$

$F_{BD} = 5905.36$ N

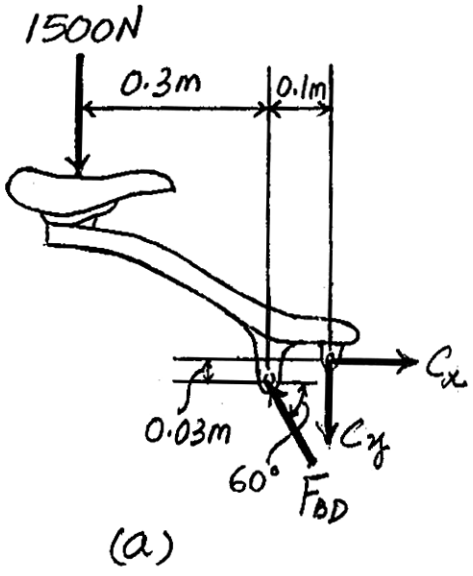
$\overset{+}{\rightarrow}\Sigma F_x = 0;$ $\quad C_x - 5905.36\cos 60° = 0$ $\quad C_x = 2952.68$ N

$+\uparrow\Sigma F_y = 0;$ $\quad 5905.36\sin 60° - 1500 - C_y = 0$ $\quad C_y = 3614.20$ N

Thus,

$F_B = F_{BD} = 5905.36$ N $\quad F_C = 2\sqrt{C_x^2 + C_y^2} = 2\sqrt{2952.68^2 + 3614.20^2}$

$= 4666.98$ N



Since both pins are in double shear,

$$V_B = \frac{F_B}{2} = \frac{5905.36}{2} = 2952.68\text{N} \qquad V_C = \frac{F_C}{2} = \frac{4666.98}{2} = 2333.49\text{ N}$$

Allowable Shear Stress: The areas of the shear plane for pins B and C are $A_B = \frac{\pi}{4}(0.0075^2) = 44.179(10^{-6})\text{m}^2$ and $A_C = \frac{\pi}{4}(0.0065^2) = 33.183(10^{-6})\text{m}^2$. We obtain

$$(\tau_{avg})_B = \frac{V_B}{A_B} = \frac{2952.68}{44.179(10^{-6})} = 66.84\text{ MPa}$$

$$(\tau_{avg})_C = \frac{V_C}{A_C} = \frac{2333.49}{33.183(10^{-6})} = 70.32\text{ MPa}$$

Using these results,

$$(\text{F.S.})_B = \frac{\tau_{fail}}{(\tau_{avg})_B} = \frac{150}{66.84} = 2.24 \qquad \textbf{Ans.}$$

$$(\text{F.S.})_C = \frac{\tau_{fail}}{(\tau_{avg})_C} = \frac{150}{70.32} = 2.13 \qquad \textbf{Ans.}$$

Ans:

$(\text{F.S.})_B = 2.24$, $(\text{F.S.})_C = 2.13$

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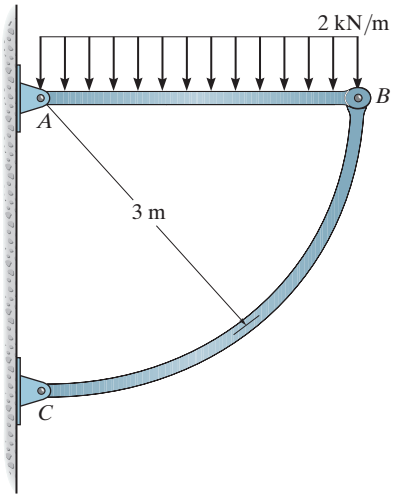
***1–80.**

Determine the required diameter of the pins at A and B if the allowable shear stress for the material is $\tau_{\text{allow}} = 100$ MPa. Both pins are subjected to double shear.



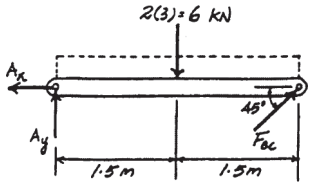
SOLUTION

Support Reactions: Member BC is a two force member.

$$\curvearrowright +\Sigma M_A = 0; \qquad F_{BC}\sin 45°(3) - 6(1.5) = 0$$
$$F_{BC} = 4.243 \text{ kN}$$

$$+\uparrow \Sigma F_y = 0; \qquad A_y + 4.243\sin 45° - 6 = 0$$
$$A_y = 3.00 \text{ kN}$$

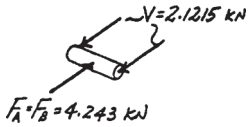
$$\pm \Sigma F_x = 0; \qquad A_x - 4.243\cos 45° = 0$$
$$A_x = 3.00 \text{ kN}$$



Allowable Shear Stress: Pin A and pin B are subjected to double shear.

$F_A = \sqrt{3.00^2 + 3.00^2} = 4.243$ kN and
$F_B = F_{BC} = 4.243$ kN.
Therefore,

$$V_A = V_B = \frac{4.243}{2} = 2.1215 \text{ kN}$$



$$\tau_{\text{allow}} = \frac{V}{A}; \qquad 100(10^6) = \frac{2.1215(10^3)}{\frac{\pi}{4}d^2}$$
$$d = 0.005197 \text{ m} = 5.20 \text{ mm}$$
$$d_A = d_B = d = 5.20 \text{ mm} \qquad \textbf{Ans.}$$

Ans:
$d_A = d_B = 5.20$ mm

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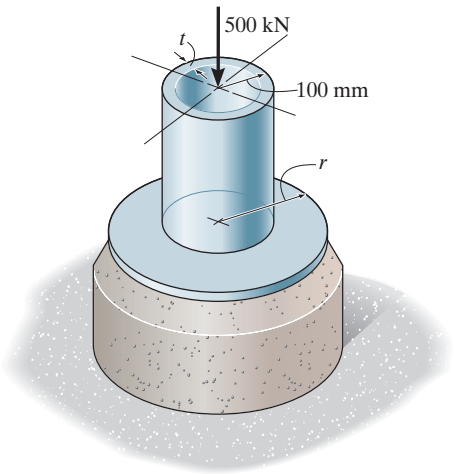
1–81.

The steel pipe is supported on the circular base plate and concrete pedestal. If the thickness of the pipe is $t = 5$ mm and the base plate has a radius of 150 mm, determine the factors of safety against failure of the steel and concrete. The applied force is 500 kN, and the normal failure stresses for steel and concrete are $(\sigma_{fail})_{st} = 350$ MPa and $(\sigma_{fail})_{con} = 25$ MPa, respectively.



SOLUTION

Average Normal and Bearing Stress: The cross-sectional area of the steel pipe and the bearing area of the concrete pedestal are $A_{st} = \pi(0.1^2 - 0.095^2) = 0.975(10^{-3})\pi$ m^2 and $(A_{con})_b = \pi(0.15^2) = 0.0225\pi$ m^2. We have

$$(\sigma_{avg})_{st} = \frac{P}{A_{st}} = \frac{500(10^3)}{0.975(10^{-3})\pi} = 163.24 \text{ MPa}$$

$$(\sigma_{avg})_{con} = \frac{P}{(A_{con})_b} = \frac{500(10^3)}{0.0225\pi} = 7.074 \text{ MPa}$$

Thus, the factor of safety against failure of the steel pipe and concrete pedestal are

$$(\text{F.S.})_{st} = \frac{(\sigma_{fail})_{st}}{(\sigma_{avg})_{st}} = \frac{350}{163.24} = 2.14 \qquad \textbf{Ans.}$$

$$(\text{F.S.})_{con} = \frac{(\sigma_{fail})_{con}}{(\sigma_{avg})_{con}} = \frac{25}{7.074} = 3.53 \qquad \textbf{Ans.}$$

Ans:
$(\text{F.S.})_{st} = 2.14, (\text{F.S.})_{con} = 3.53$

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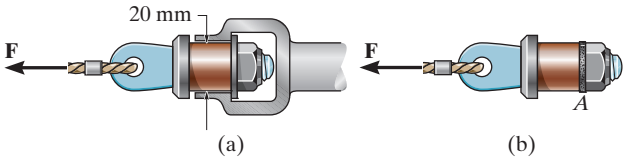
1–82. The steel swivel bushing in the elevator control of an airplane is held in place using a nut and washer as shown in Fig. (*a*). Failure of the washer *A* can cause the push rod to separate as shown in Fig. (*b*). If the average shear stress is $\tau_{avg} = 145$ MPa, determine the force **F** that must be applied to the bushing that will cause this to happen. The washer is 1.5 mm thick.



SOLUTION

From the given average shear stress,

$$\tau_{avg} = \frac{V}{A}; \qquad 145(10^6) = \frac{F}{2\pi(0.01)(0.0015)}$$

$$F = 13.67(10^3)$$

$$= 13.7 \text{ kN} \qquad \textbf{Ans.}$$

Ans:
$F = 13.7$ kN

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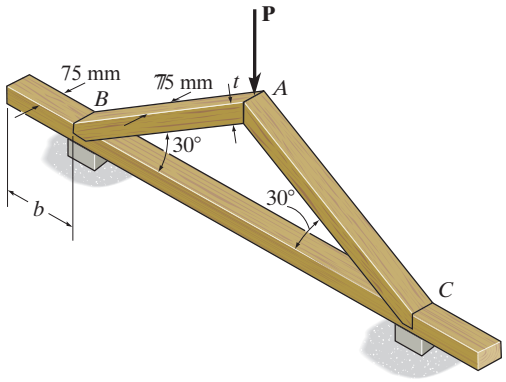
1–83. Determine the required minimum thickness t of member AB and edge distance b of the frame if $P = 40$ kN and the factor of safety against failure is 2. The wood has a normal failure stress of $\sigma_{fail} = 42$ MPa, a nd shear failure stress of $\tau_{fail} = 10.5$ MPa.



SOLUTION

Internal Loadings: The normal force developed in member AB can be determined by considering the equilibrium of joint A. Fig. a.

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad F_{AB}\cos 30° - F_{AC}\cos 30° = 0$ $\quad F_{AC} = F_{AB}$

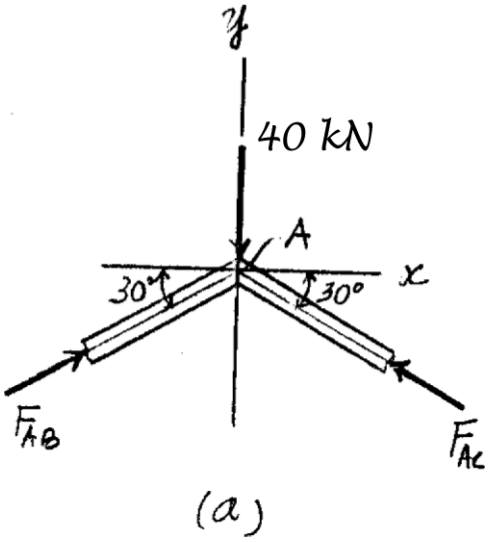
$+\uparrow \Sigma F_y = 0;$ $\quad 2F_{AB}\sin 30° - 40 = 0$ $\quad F_{AB} = 40$ kN

Subsequently, the horizontal component of the force acting on joint B can be determined by analyzing the equilibrium of member AB, Fig. b.

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad (F_B)_x - 40\cos 30° = 0$ $\quad (F_B)_x = 34.64$ kN

Referring to the free-body diagram shown in Fig. c, the shear force developed on the shear plane a–a is

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad V_{a-a} - 34.64 = 0$ $\quad V_{a-a} = 34.64$ kN



(a)

Allowable Normal Stress:

$$\sigma_{allow} = \frac{\sigma_{fail}}{\text{F.S.}} = \frac{42}{2} = 21 \text{ MPa}$$

$$\tau_{allow} = \frac{\tau_{fail}}{\text{F.S.}} = \frac{10.5}{2} = 5.25 \text{ MPa}$$

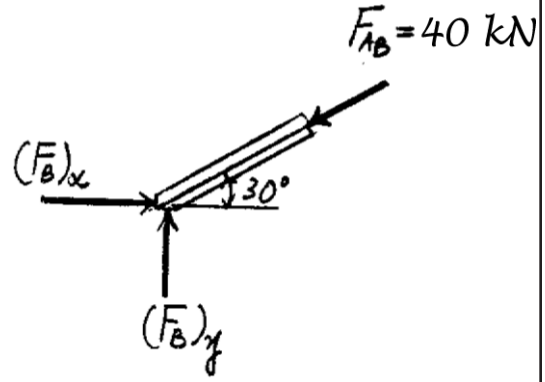
Using these results,

$$\sigma_{allow} = \frac{F_{AB}}{A_{AB}}; \qquad 21(10^6) = \frac{40(10^3)}{(0.075)t}$$

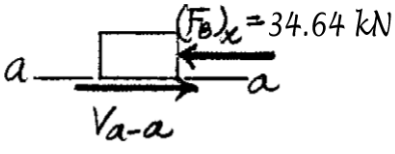
$t = 0.02540$ m $= 25.4$ mm **Ans.**

$$\tau_{allow} = \frac{V_{a-a}}{A_{a-a}}; \qquad 5.25(10^6) = \frac{34.64(10^3)}{(0.075)b}$$

$b = 0.08798$ m $= 88.0$ mm **Ans.**



(b)



(c)

Ans:

$t = 25.4$ mm, $b = 88.0$ mm

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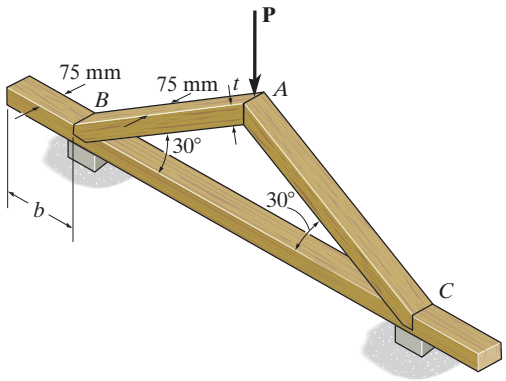
***1–84.** Determine the maximum allowable load **P** that can be safely supported by the frame if $t = 30$ mm and $b = 90$ mm. The wood has a normal failure stress of $\sigma_{fail} = 42$ MPa, and shear failure stress of $\tau_{fail} = 10.5$ MPa. Use a factor of safety against failure of 2.



SOLUTION

Internal Loadings: The normal force developed in member AB can be determined by considering the equilibrium of joint A. Fig. a.

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad F_{AB}\cos 30° - F_{AC}\cos 30° = 0$ $\quad F_{AC} = F_{AB}$

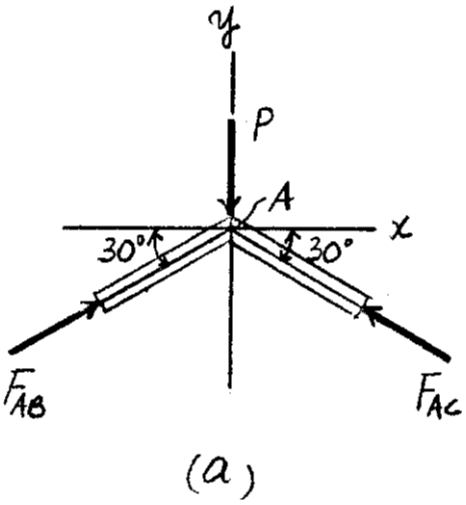
$+\uparrow \Sigma F_y = 0;$ $\quad 2F_{AB}\sin 30° - P = 0$ $\quad F_{AB} = P$

Subsequently, the horizontal component of the force acting on joint B can be determined by analyzing the equilibrium of member AB, Fig. b.

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad (F_B)_x - P\cos 30° = 0$ $\quad (F_B)_x = 0.8660P$

Referring to the free-body diagram shown in Fig. c, the shear force developed on the shear plane a–a is

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad V_{a-a} - 0.8660P = 0$ $\quad V_{a-a} = 0.8660P$



(a)

Allowable Normal and Shear Stress:

$$\sigma_{allow} = \frac{\sigma_{fail}}{\text{F.S.}} = \frac{42}{2} = 21 \text{ MPa}$$

$$\tau_{allow} = \frac{\tau_{fail}}{\text{F.S.}} = \frac{10.5}{2} = 5.25 \text{ MPa}$$

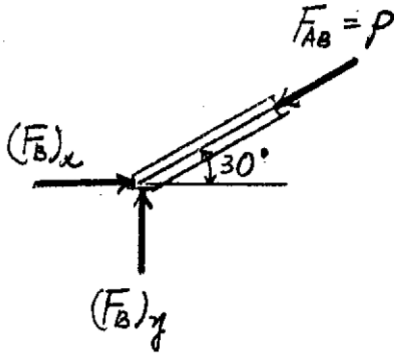
Using these results,

$$\sigma_{allow} = \frac{F_{AB}}{A_{AB}}; \qquad 21(10^6) = \frac{P}{0.075(0.03)}$$

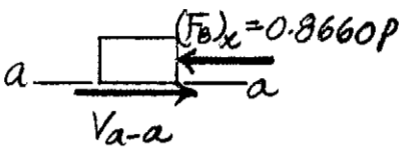
$$P = 47.25(10^3) \text{ N} = 47.25 \text{ kN}$$

$$\tau_{allow} = \frac{V_{a-a}}{A_{a-a}}; \qquad 5.25(10^6) = \frac{0.8660P}{0.075(0.09)}$$

$$P = 40.92(10^3) \text{ N} = 40.9 \text{ kN (controls)}$$ **Ans.**



(b)



(c)

Ans:

$P = 40.9$ kN

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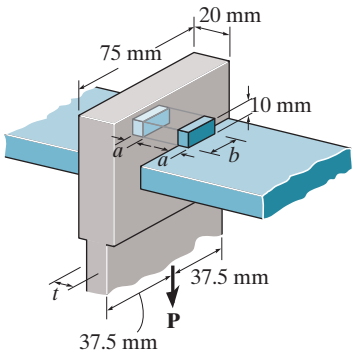
1–85.

The hanger is supported using the rectangular pin. Determine the magnitude of the allowable suspended load **P** if the allowable bearing stress is $(\sigma_b)_{\text{allow}} = 220$ MPa, the allowable tensile stress is $(\sigma_t)_{\text{allow}} = 150$ MPa, and the allowable shear stress is $\tau_{\text{allow}} = 130$ MPa. Take $t = 6$ mm, $a = 5$ mm and $b = 25$ mm.

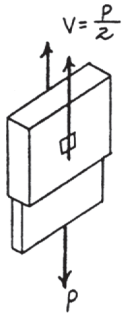


SOLUTION

Allowable Normal Stress: For the hanger

$$(\sigma_t)_{\text{allow}} = \frac{P}{A}; \qquad 150\left(10^6\right) = \frac{P}{(0.075)(0.006)}$$

$$P = 67.5 \text{ kN}$$



Allowable Shear Stress: The pin is subjected to double shear. Therefore, $V = \frac{P}{2}$

$$\tau_{\text{allow}} = \frac{V}{A}; \qquad 130\left(10^6\right) = \frac{P/2}{(0.01)(0.025)}$$

$$P = 65.0 \text{ kN}$$

Allowable Bearing Stress: For the bearing area

$$(\sigma_b)_{\text{allow}} = \frac{P}{A}; \qquad 220\left(10^6\right) = \frac{P/2}{(0.005)(0.025)}$$

$$P = 55.0 \text{ kN } (\textit{Controls!}) \qquad \textbf{Ans.}$$

Ans:
$P = 55.0$ kN

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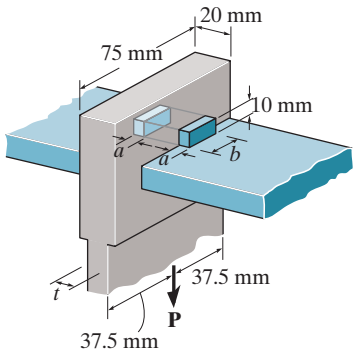
1–86.

The hanger is supported using the rectangular pin. Determine the required thickness t of the hanger, and dimensions a and b if the suspended load is $P = 60$ kN. The allowable tensile stress is $(\sigma_t)_{\text{allow}} = 150$ MPa, the allowable bearing stress is $(\sigma_b)_{\text{allow}} = 290$ MPa, and the allowable shear stress is $\tau_{\text{allow}} = 125$ MPa.

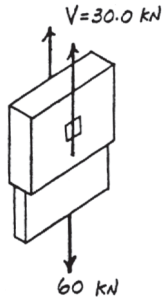


SOLUTION

Allowable Normal Stress: For the hanger

$$(\sigma_t)_{\text{allow}} = \frac{P}{A}; \qquad 150\left(10^6\right) = \frac{60(10^3)}{(0.075)t}$$

$$t = 0.005333 \text{ m} = 5.33 \text{ mm} \qquad \textbf{Ans.}$$



Allowable Shear Stress: For the pin

$$\tau_{\text{allow}} = \frac{V}{A}; \qquad 125\left(10^6\right) = \frac{30(10^3)}{(0.01)b}$$

$$b = 0.0240 \text{ m} = 24.0 \text{ mm} \qquad \textbf{Ans.}$$

Allowable Bearing Stress: For the bearing area

$$(\sigma_b)_{\text{allow}} = \frac{P}{A}; \qquad 290\left(10^6\right) = \frac{30(10^3)}{(0.0240)\,a}$$

$$a = 0.00431 \text{ m} = 4.31 \text{ mm} \qquad \textbf{Ans.}$$

Ans:

$t = 5.33$ mm, $b = 24.0$ mm, $a = 4.31$ mm

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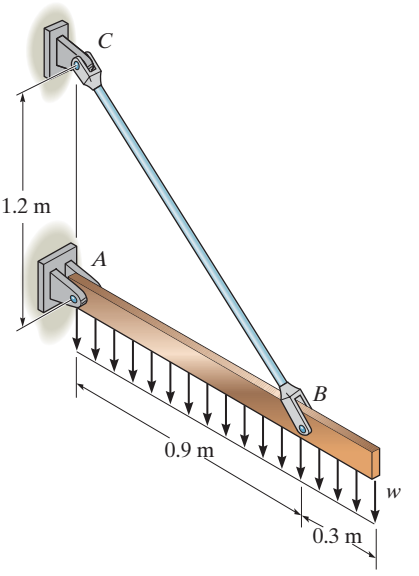
1-87. The assembly is used to support the distributed loading of w = 10 kN/m. Determine the factor of safety with respect to yielding for the steel rod BC and the pins at A and B if the yield stress for the steel in tension is σ_y = 250 MPa and in shear τ_y = 125 MPa. The rod has a diameter of 13 mm, and the pins each have a diameter of 10 mm.



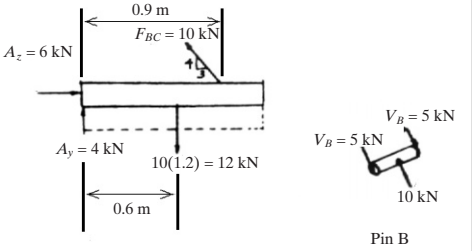
SOLUTION

For rod BC:

$$\sigma = \frac{P}{A} = \frac{10(10^3)}{\frac{\pi}{4}(0.013^2)} = 75.34 \text{ MPa}$$

$$\text{F. S.} = \frac{\sigma_y}{\sigma} = \frac{250}{75.34} = 3.32 \qquad \textbf{Ans.}$$

For pin B:



Pin B

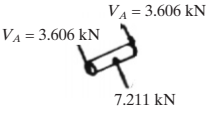
$$\tau_B = \frac{V_B}{A} = \frac{5(10^3)}{\frac{\pi}{4}(0.01^2)} = 63.66 \text{ MPa}$$

$$\text{F. S.} = \frac{\tau_Y}{\tau_B} = \frac{125}{63.66} = 1.96 \qquad \textbf{Ans.}$$

For pin A:

$$F_A = \sqrt{4^2 + 6^2} = 7.211 \text{ kN}$$

$$V_A = \frac{7.211 \text{ kN}}{2} = 3.606 \text{ kN}$$



Pin A

$$\tau_A = \frac{V_A}{A} = \frac{3.606(10^3)}{\frac{\pi}{4}(0.01^2)} = 45.91 \text{ MPa}$$

$$\text{F. S.} = \frac{\tau_Y}{\tau_A} = \frac{125}{45.91} = 2.72 \qquad \textbf{Ans.}$$

Ans:

$(\text{F.S.})_{rod} = 3.32$,
$(\text{F.S.})_{pinB} = 1.96$
$(\text{F.S.})_{pinA} = 2.72$

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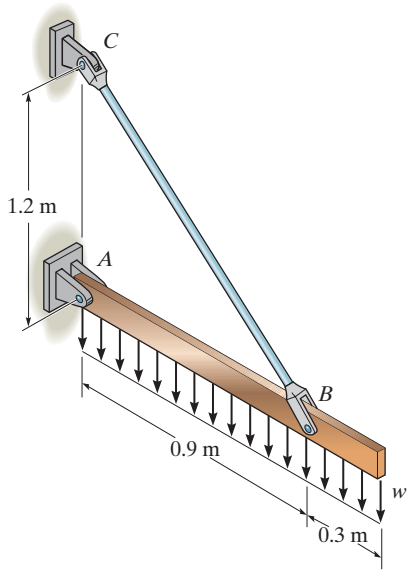
***1–88. If** the allowable shear stress for each of the 10-mm-diameter steel pins at A, B, and C is $\tau_{allow} = 90$ MPa, and the allowable normal stress for the 13-mm-diameter rod is $\sigma_{allow} = 150$ MPa, determine the largest intensity w of the uniform distributed load that can be suspended from the beam.



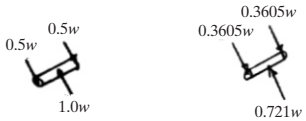
SOLUTION

Assume failure of pins B and C:

$$\tau_{allow} = 90 = \frac{0.5w(10^3)}{\frac{\pi}{4}(10^2)}$$

$w = 14.14$ kN/m (controls)

Ans.



Assume failure of pins A:

$$F_A = \sqrt{(0.6w)^2 + (0.4w)^2} = 0.721w$$

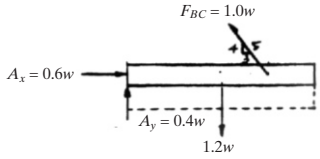
$$\tau_{allow} = 90 = \frac{0.3605w(10^3)}{\frac{\pi}{4}(10^2)}$$

$w = 19.61$ kN/m

Assume failure of rod BC:

$$\sigma_{allow} = 150 = \frac{1.0w(10^3)}{\frac{\pi}{4}(13^2)}$$

$w = 19.91$ kN/m



Ans:

$w = 14.14$ kN/m (controls)

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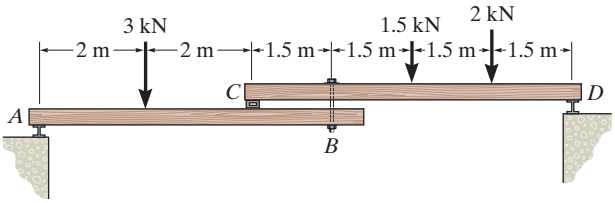
1–89.

The compound wooden beam is connected together by a bolt at B. Assuming that the connections at A, B, C, and D exert only vertical forces on the beam, determine the required diameter of the bolt at B and the required outer diameter of its washers if the allowable tensile stress for the bolt is $(\sigma_t)_{\text{allow}} = 150\,\text{MPa}$ and the allowable bearing stress for the wood is $(\sigma_b)_{\text{allow}} = 28\,\text{MPa}$. Assume that the hole in the washers has the same diameter as the bolt.

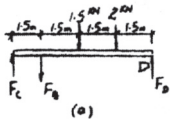


SOLUTION

From FBD (a):

$$\zeta + \Sigma M_D = 0; \qquad F_B(4.5) + 1.5(3) + 2(1.5) - F_C(6) = 0$$

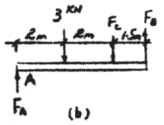
$$4.5\,F_B - 6\,F_C = -7.5 \qquad \textbf{(1)}$$



From FBD (b):

$$\zeta + \Sigma M_A = 0; \qquad F_B(5.5) - F_C(4) - 3(2) = 0$$

$$5.5\,F_B - 4\,F_C = 6 \qquad \textbf{(2)}$$



Solving Eqs. (1) and (2) yields

$$F_B = 4.40\,\text{kN}; \qquad F_C = 4.55\,\text{kN}$$

For bolt:

$$\sigma_{\text{allow}} = 150(10^6) = \frac{4.40(10^3)}{\frac{\pi}{4}(d_B)^2}$$

$$d_B = 0.00611\,\text{m}$$

$$= 6.11\,\text{mm} \qquad \textbf{Ans.}$$

For washer:

$$\sigma_{\text{allow}} = 28\,(10^4) = \frac{4.40(10^3)}{\frac{\pi}{4}(d_w^2 - 0.00611^2)}$$

$$d_w = 0.0154\,\text{m} = 15.4\,\text{mm} \qquad \textbf{Ans.}$$

Ans:

$d_B = 6.11\,\text{mm}, d_w = 15.4\,\text{mm}$

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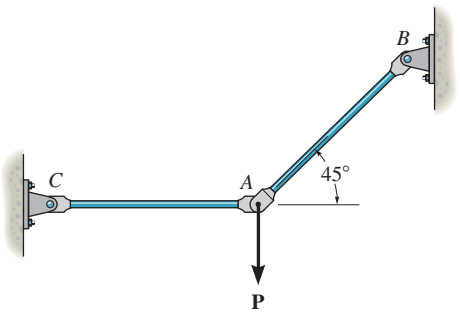
1–90.

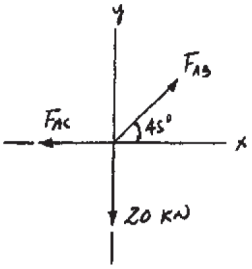
The two aluminum rods support the vertical force of $P = 20$ kN. Determine their required diameters if the allowable tensile stress for the aluminum is $\sigma_{\text{allow}} = 150$ MPa.



SOLUTION

$+\uparrow \Sigma F_y = 0; \qquad F_{AB} \sin 45° - 20 = 0; \qquad F_{AB} = 28.284 \text{ kN}$

$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 28.284 \cos 45° - F_{AC} = 0; \qquad F_{AC} = 20.0 \text{ kN}$



For rod AB:

$$\sigma_{\text{allow}} = \frac{F_{AB}}{A_{AB}}; \qquad 150(10^6) = \frac{28.284(10^3)}{\frac{\pi}{4} d_{AB}^2}$$

$d_{AB} = 0.0155 \text{ m} = 15.5 \text{ mm}$ **Ans.**

For rod AC:

$$\sigma_{\text{allow}} = \frac{F_{AC}}{A_{AC}}; \qquad 150(10^6) = \frac{20.0(10^3)}{\frac{\pi}{4} d_{AC}^2}$$

$d_{AC} = 0.0130 \text{ m} = 13.0 \text{ mm}$ **Ans.**

Ans:
$d_{AB} = 15.5$ mm, $d_{AC} = 13.0$ mm

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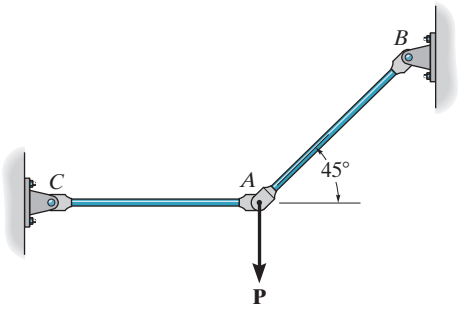
1–91.

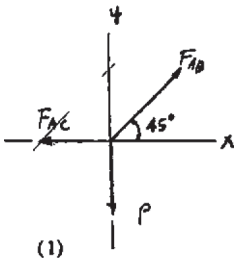
The two aluminum rods AB and AC have diameters of 10 mm and 8 mm, respectively. Determine the largest vertical force **P** that can be supported. The allowable tensile stress for the aluminum is $\sigma_{\text{allow}} = 150$ MPa.



SOLUTION

$+\uparrow \Sigma F_y = 0;$ $\quad F_{AB}\sin 45° - P = 0;$ $\quad P = F_{AB}\sin 45°$ **(1)**

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad F_{AB}\cos 45° - F_{AC} = 0$ **(2)**



Assume failure of rod AB:

$$\sigma_{\text{allow}} = \frac{F_{AB}}{A_{AB}}; \quad 150(10^6) = \frac{F_{AB}}{\frac{\pi}{4}(0.01)^2}$$

$F_{AB} = 11.78$ kN

From Eq. (1),

$P = 8.33$ kN

Assume failure of rod AC:

$$\sigma_{\text{allow}} = \frac{F_{AC}}{A_{AC}}; \quad 150(10^6) = \frac{F_{AC}}{\frac{\pi}{4}(0.008)^2}$$

$F_{AC} = 7.540$ kN

Solving Eqs. (1) and (2) yields:

$F_{AB} = 10.66$ kN; $\quad P = 7.54$ kN

Choose the smallest value

$P = 7.54$ kN **Ans.**

Ans:
$P = 7.54$ kN

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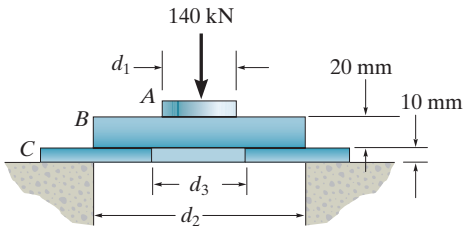
***1–92.**

The assembly consists of three disks A, B, and C that are used to support the load of 140 kN. Determine the smallest diameter d_1 of the top disk, the largest diameter d_2 of the opening, and the largest diameter d_3 of the hole in the bottom disk. The allowable bearing stress for the material is $(\sigma_b)_{\text{allow}} = 350$ MPa and allowable shear stress is $\tau_{\text{allow}} = 125$ MPa.



SOLUTION

Allowable Shear Stress: Assume shear failure for disk C.

$$\tau_{\text{allow}} = \frac{V}{A};\quad 125(10^6) = \frac{140(10^3)}{\pi d_2(0.01)}$$

$$d_2 = 0.03565 \text{ m} = 35.7 \text{ mm} \qquad \textbf{Ans.}$$

Allowable Bearing Stress: Assume bearing failure for disk C.

$$(\sigma_b)_{\text{allow}} = \frac{P}{A};\quad 350(10^6) = \frac{140(10^3)}{\frac{\pi}{4}(0.03565^2 - d_3^2)}$$

$$d_3 = 0.02760 \text{ m} = 27.6 \text{ mm} \qquad \textbf{Ans.}$$

Allowable Bearing Stress: Assume bearing failure for disk B.

$$(\sigma_b)_{\text{allow}} = \frac{P}{A};\quad 350(10^6) = \frac{140(10^3)}{\frac{\pi}{4}d_1^2}$$

$$d_1 = 0.02257 \text{ m} = 22.6 \text{ mm}$$

Since $d_3 = 27.6 \text{ mm} > d_1 = 22.6 \text{ mm}$, disk B might fail due to shear.

$$\tau = \frac{V}{A} = \frac{140(10^3)}{\pi(0.02257)(0.02)} = 98.7 \text{ MPa} < \tau_{\text{allow}} = 125 \text{ MPa} \ (\textbf{\textit{O.K!}})$$

Therefore, $d_1 = 22.6 \text{ mm}$ **Ans.**

Ans:
$d_2 = 35.7$ mm,
$d_3 = 27.6$ mm,
$d_1 = 22.6$ mm

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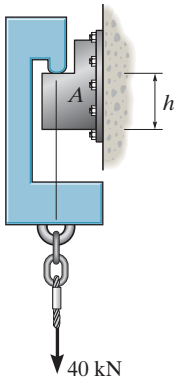
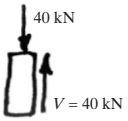
1–93. The aluminium bracket A is used to support the centrally applied load of 40 kN. If it has a constant thickness of 12 mm, determine the smallest height h in order to prevent a shear failure. The failure shear stress is $\tau_{fail} = 160$ MPa. Use a factor of safety for shear of F.S. = 2.5.



SOLUTION

Equation of Equilibrium:

$+\uparrow \Sigma F_y = 0; \qquad V - 40 = 0 \qquad V - 40 \text{ kN}$



Allowable Shear Stress: Design of the support size

$$\tau_{allow} = \frac{\tau_{fail}}{\text{F.S}} = \frac{V}{A}; \qquad \frac{160(10^6)}{2.5} = \frac{40(10^3)}{h(0.012)}$$

$h = 0.05208 \text{ m} = 52.1 \text{ mm}$ **Ans.**

Ans:

$h = 52.1$ mm

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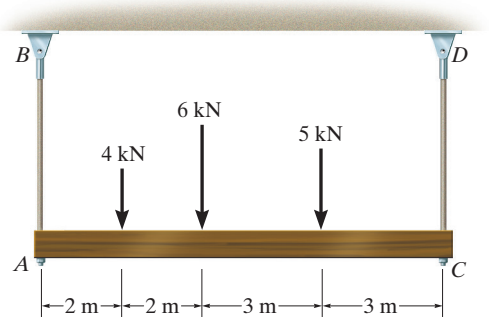
1–94.

The rods AB and CD are made of steel. Determine their smallest diameter so that they can support the dead loads shown. The beam is assumed to be pin connected at A and C. Use the LRFD method, where the resistance factor for steel in tension is $\phi = 0.9$, and the dead load factor is $\gamma_D = 1.4$. The failure stress is $\sigma_{\text{fail}} = 345\,\text{MPa}$.



SOLUTION

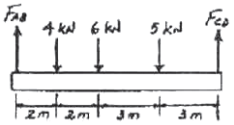
Support Reactions:

$$\zeta+\Sigma M_A = 0; \qquad F_{CD}(10) - 5(7) - 6(4) - 4(2) = 0$$

$$F_{CD} = 6.70\,\text{kN}$$

$$\zeta+\Sigma M_C = 0; \qquad 4(8) + 6(6) + 5(3) - F_{AB}(10) = 0$$

$$F_{AB} = 8.30\,\text{kN}$$



Factored Loads:

$$F_{CD} = 1.4(6.70) = 9.38\,\text{kN}$$

$$F_{AB} = 1.4(8.30) = 11.62\,\text{kN}$$

For rod *AB*

$$0.9[345(10^6)]\,\pi\left(\frac{d_{AB}}{2}\right)^2 = 11.62(10^3)$$

$$d_{AB} = 0.00690\,\text{m} = 6.90\,\text{mm} \qquad \textbf{Ans.}$$

For rod *CD*

$$0.9[345(10^6)]\,\pi\left(\frac{d_{CD}}{2}\right)^2 = 9.38(10^3)$$

$$d_{CD} = 0.00620\,\text{m} = 6.20\,\text{mm} \qquad \textbf{Ans.}$$

Ans:
$d_{AB} = 6.90\,\text{mm}$, $d_{CD} = 6.20\,\text{mm}$

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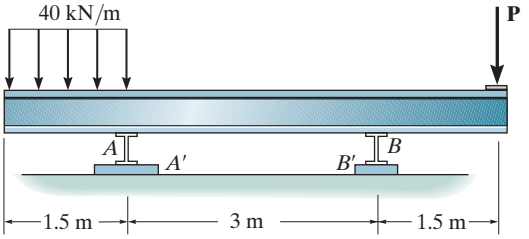
1–95. If the allowable bearing stress for the material under the supports at A and B is $(\sigma_b)_{\text{allow}} = 1.5$ MPa, determine the size of *square* bearing plates A' and B' required to support the load. Dimension the plates to the nearest mm. The reactions at the supports are vertical. Take $P = 100$ kN.

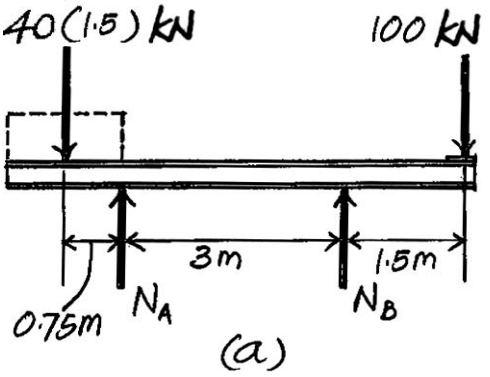


SOLUTION

Referring to the FBD of the bean, Fig. *a*

$\zeta + \Sigma M_A = 0; \quad N_B(3) + 40(1.5)(0.75) - 100(4.5) = 0 \quad N_B = 135$ kN

$\zeta + \Sigma M_B = 0; \quad 40(1.5)(3.75) - 100(1.5) - N_A(3) = 0 \quad N_A = 25.0$ kN



For plate A',

$$(\sigma_b)_{\text{allow}} = \frac{N_A}{A_{A'}}; \qquad 1.5(10^6) = \frac{25.0(10^3)}{a_{A'}^2}$$

$$a_{A'} = 0.1291 \text{ m} = 130 \text{ mm}$$

Ans.

For plate B',

$$\sigma_{\text{allow}} = \frac{N_B}{A_{B'}}; \qquad 1.5(10^6) = \frac{135(10^3)}{a_{B'}^2}$$

$$a_{B'} = 0.300 \text{ m} = 300 \text{ mm}$$

Ans.

Ans:

$a_{A'} = 130$ mm, $a_{B'} = 300$ mm

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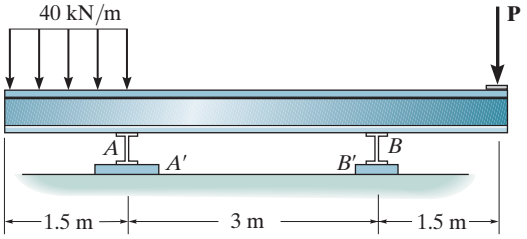
***1–96.** If the allowable bearing stress for the material under the supports at A and B is $(\sigma_b)_{\text{allow}} = 1.5$ MPa, determine the maximum load P that can be applied to the beam. The bearing plates A' and B' have square cross sections of 150 mm × 150 mm and 250 mm × 250 mm, respectively.



SOLUTION

Referring to the FBD of the beam, Fig. *a*,

$$\zeta + \Sigma M_A = 0; \qquad N_B(3) + 40(1.5)(0.75) - P(4.5) = 0 \qquad N_B = 1.5P - 15$$

$$\zeta + \Sigma M_B = 0; \qquad 40(1.5)(3.75) - P(1.5) - N_A(3) = 0 \qquad N_A = 75 - 0.5P$$

For plate A',

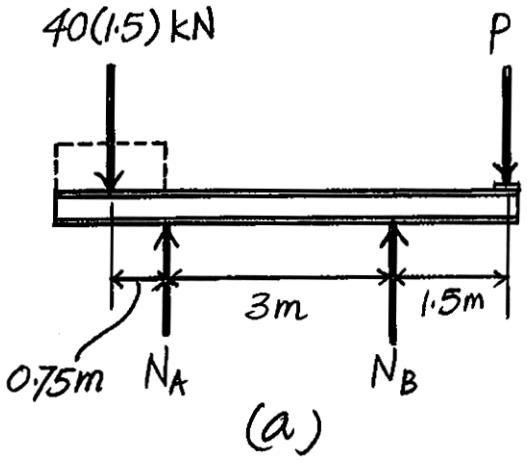
$$(\sigma_b)_{\text{allow}} = \frac{N_A}{A_{A'}}; \qquad 1.5(10^6) = \frac{(75 - 0.5P)(10^3)}{0.15(0.15)}$$

$$P = 82.5 \text{ kN}$$

For plate B',

$$(\sigma_b)_{\text{allow}} = \frac{N_B}{A_{B'}}; \qquad 1.5(10^6) = \frac{(1.5P - 15)(10^3)}{0.25(0.25)}$$

$$P = 72.5 \text{ kN} \quad \text{(Controls!)}$$



Ans.

Ans:

$P = 72.5$ kN (Controls!)

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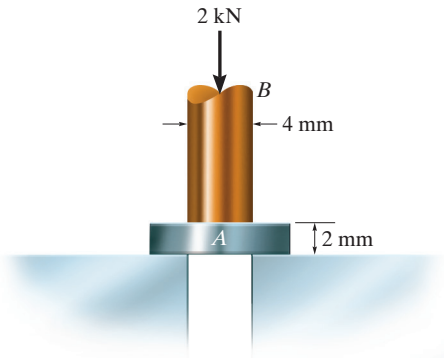
R1–1.

The circular punch B exerts a force of 2 kN on the top of the plate A. Determine the average shear stress in the plate due to this loading.



SOLUTION

Average Shear Stress: The shear area $A = \pi(0.004)(0.002) = 8.00(10^{-6})\pi \text{ m}^2$

$$\tau_{\text{avg}} = \frac{V}{A} = \frac{2(10^3)}{8.00(10^{-6})\pi} = 79.6 \text{ MPa} \qquad \textbf{Ans.}$$

Ans:
$\tau_{\text{avg}} = 79.6$ MPa

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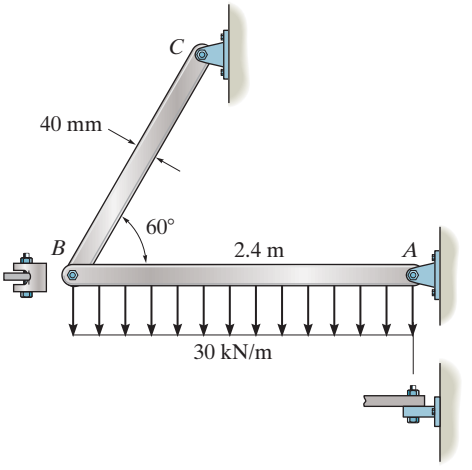
R1–2.

Determine the required thickness of member BC and the diameter of the pins at A and B if the allowable normal stress for member BC is $\sigma_{\text{allow}} = 200$ MPa and the allowable shear stress for the pins is $\tau_{\text{allow}} = 70$ MPa.



SOLUTION

Referring to the FBD of member AB, Fig. a,

$\zeta + \Sigma M_A = 0;\quad 30(2.4)(1.2) - F_{BC}\sin 60°\,(2.4) = 0 \qquad F_{BC} = 41.57$ kN

$\overset{+}{\rightarrow} \Sigma F_x = 0;\quad 41.57\cos 60° - A_x = 0 \qquad A_x = 20.785$ kN

$+\uparrow \Sigma F_y = 0;\quad 41.57\sin 60° - 30(2.4) + A_y = 0 \qquad A_y = 36.00$ kN

Thus, the force acting on pin A is

$$F_A = \sqrt{A_x^2 + A_y^2} = \sqrt{20.785^2 + 36.00^2} = 41.57 \text{ kN}$$

Pin A is subjected to single shear, Fig. c, while pin B is subjected to double shear, Fig. b.

$$V_A = F_A = 41.57 \text{ kN} \qquad V_B = \frac{F_{BC}}{2} = \frac{41.57}{2} = 20.785 \text{ kN}$$

For member BC

$$\sigma_{\text{allow}} = \frac{F_{BC}}{A_{BC}}; \qquad 200(10^6) = \frac{41.57(10^3)}{(0.04)t} \qquad t = 0.005196 \text{ m} = 5.196 \text{ mm}$$

Use $t = 6$ mm **Ans.**

For pin A,

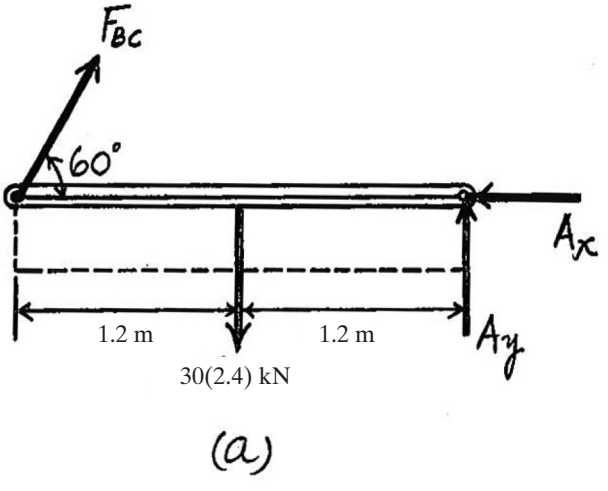
$$\tau_{\text{allow}} = \frac{V_A}{A_A}; \qquad 70(10^6) = \frac{41.57(10^3)}{\frac{\pi}{4}d_A^2} \qquad d_A = 0.02750 \text{ m} = 27.50 \text{ mm}$$

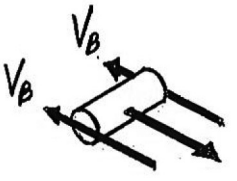
Use $d_A = 28$ mm **Ans.**

For pin B,

$$\tau_{\text{allow}} = \frac{V_B}{A_B}; \qquad 70(10^6) = \frac{20.785(10^3)}{\frac{\pi}{4}d_B^2} \qquad d_B = 0.01944 \text{ m} = 19.44 \text{ mm}$$

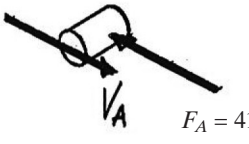
Use $d_B = 20$ mm **Ans.**





$F_{BC} = 41.57$ kN

(b)



$F_A = 41.57$ kN

(c)

Ans:
$t = 6$ mm, $d_A = 28$ mm, $d_B = 20$ mm

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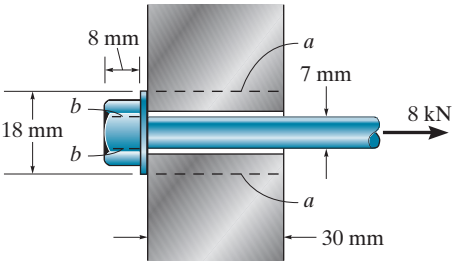
R1–3.

The long bolt passes through the 30-mm-thick plate. If the force in the bolt shank is 8 kN, determine the average normal stress in the shank, the average shear stress along the cylindrical area of the plate defined by the section lines *a–a*, and the average shear stress in the bolt head along the cylindrical area defined by the section lines *b–b*.



SOLUTION

$$\sigma_s = \frac{P}{A} = \frac{8\,(10^3)}{\frac{\pi}{4}(0.007)^2} = 208 \text{ MPa}$$ **Ans.**

$$(\tau_{\text{avg}})_a = \frac{V}{A} = \frac{8\,(10^3)}{\pi\,(0.018)(0.030)} = 4.72 \text{ MPa}$$ **Ans.**

$$(\tau_{\text{avg}})_b = \frac{V}{A} = \frac{8\,(10^3)}{\pi\,(0.007)(0.008)} = 45.5 \text{ MPa}$$ **Ans.**

Ans:
$\sigma_s = 208$ MPa, $(\tau_{\text{avg}})_a = 4.72$ MPa,
$(\tau_{\text{avg}})_b = 45.5$ MPa

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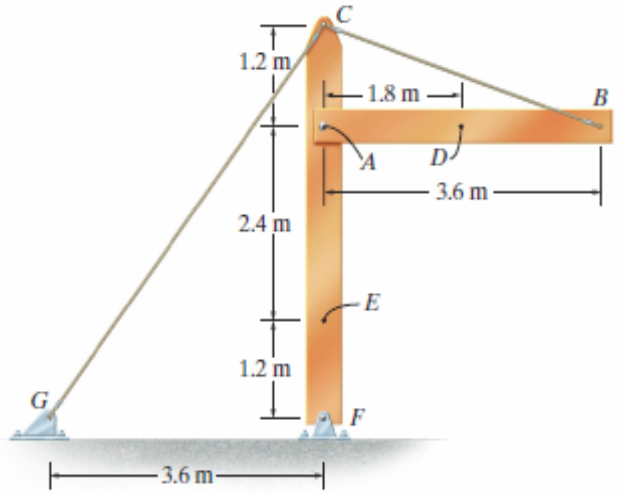
***R1–4.**

The beam *AB* is pin supported at *A* and supported by a cable *BC*. A *separate* cable *CG* is used to hold up the frame. If *AB* weighs 2.0 kN/m and the column *FC* has a weight of 3.0 kN/m, determine the resultant internal loadings acting on cross sections located at points *D* and *E*. Neglect the thickness of both the beam and column in the calculation.



Given:

$w_b := 2.0\frac{kN}{m}$ $L := 3.6m$ $d := 1.8m$

$w_c := 3.0\frac{kN}{m}$ $H := 4.8m$ $e := 1.2m$

$a := 3.6m$ $b := 3.6m$ $c := 1.2m$

SOLUTION

Beam *AB*: $L_c := \sqrt{L^2 + c^2}$ $v_b := \frac{c}{L_c}$ $h_b := \frac{L}{L_c}$

$\circlearrowleft + \Sigma M_A = 0;$ $B_y \cdot (L) - w_{AB} \cdot (L)(0.5 \cdot L) = 0$

$B_y := w_b \cdot (L) \cdot \left(0.5\frac{L}{L}\right)$

$B_y = 7936.64\ \frac{m}{s^2}\ lb$

$+\downarrow \Sigma F_y = 0;$ $-A_y - B_y + w_b \cdot (L) = 0$ $A_y := -B_y + w_b \cdot (L)$ $A_y = 3.6\,kN$

$B_y = F_{BC} \cdot (v_b)$ $F_{BC} := \frac{B_y}{v_b}$ $F_{BC} = 11.38\,kN$

$\stackrel{+}{\rightarrow} \Sigma F_x = 0;$ $-F_{BC} \cdot (h) + A_x = 0$ $A_x := F_{BC} \cdot (h_b)$ $A_x = 10.8\,kN$

Segment *AD*:

$\stackrel{+}{\rightarrow} \Sigma F_x = 0;$ $N_D + A_x = 0$ $N_D := -A_x$ $N_D = -10.8\,kN$ **Ans.**

$+\downarrow \Sigma F_y = 0;$ $-A_y + w_b \cdot (d) + V_D = 0$ $V_D := A_y - w_b \cdot (d)$ $V_D = 0\,kN$ **Ans.**

$\circlearrowleft + \Sigma M_D = 0;$ $M_D + \left[w_b \cdot (d)\right] \cdot (0.5 \cdot d) - A_y \cdot (d) = 0$

$M_D := -\left[w_b \cdot (d)\right] \cdot (0.5 \cdot d) + A_y \cdot (d)$ $M_D = 3.24\,kN \cdot m$ **Ans.**

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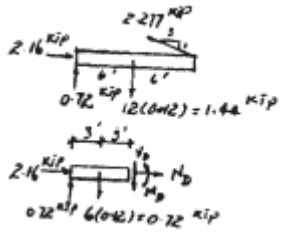
∗**R1–4. Continued**

Member *CG*: $H_b := \sqrt{H^2 + b^2}$ $v_c := \frac{H}{H_b}$ $h_c := \frac{b}{H_b}$

Column *FC*:

$\circlearrowleft + \Sigma M_C = 0;$ $F_x \cdot (H) - A_x \cdot c = 0$ $F_x := A_x \cdot \left(\frac{c}{H}\right)$

$F_x = 2.7\,\text{kN}$



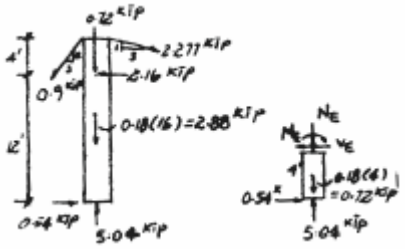
$\pm\rightarrow \Sigma F_x = 0;$ $F_{BC} \cdot (h_b) - A_x + F_x - F_{CG} \cdot (h_c) = 0$

$+\downarrow \Sigma F_y = 0;$ $F_{CG} := \frac{F_{BC} \cdot (h_b) - A_x + F_x}{h_c}$ $F_{CG} = 4.5\,\text{kN}$

$-F_y + B_y + w_c \cdot (H) + F_{BC} \cdot (v_b) + F_{CG} \cdot (v_c) = 0$

$F_y := B_y + w_c \cdot (H) + F_{BC} \cdot (v_b) + F_{CG} \cdot (v_c)$

$F_y = 25.2\,\text{kN}$



Segment *FE*:

$\pm\leftarrow \Sigma F_x = 0;$ $V_E - F_x = 0$ $V_E := F_x$ $V_E = 2.7\,\text{kN}$ **Ans.**

$+\downarrow \Sigma F_y = 0;$ $N_E + w_c \cdot (e) - F_y = 0$ $N_E := -w_c \cdot (e) + F_y$ $N_E = 21.6\,\text{kN}$ **Ans.**

$\circlearrowleft + \Sigma M_E = 0;$ $-M_E + F_y \cdot (e) = 0$ $M_E := F_y \cdot (e)$ $M_E = 30.24\,\text{kN}\cdot\text{m}$ **Ans.**

Ans:

$N_D = -10.8$ kN, $V_D = 0$ kN,
$M_D = 3.24\ \text{kN} \cdot \text{m}$
$N_E = 21.6$ kN, $V_E = 2.7$ kN,
$M_E = 30.24\ \text{kN} \cdot \text{m}$

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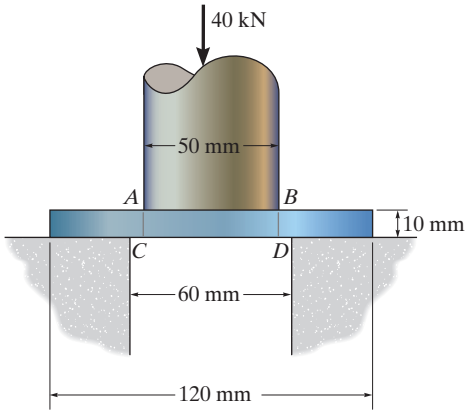
R1–5.

Determine the average punching shear stress the circular shaft creates in the metal plate through section AC and BD. Also, what is the average bearing stress developed on the surface of the plate under the shaft?



SOLUTION

Average Shear and Bearing Stress: The area of the shear plane and the bearing area on the punch are $A_V = \pi(0.05)(0.01) = 0.5(10^{-3})\pi\ \text{m}^2$ and $A_b = \frac{\pi}{4}(0.12^2 - 0.06^2) = 2.7(10^{-3})\pi\ \text{m}^2$. We obtain

$$\tau_{\text{avg}} = \frac{P}{A_V} = \frac{40(10^3)}{0.5(10^{-3})\pi} = 25.5\ \text{MPa} \qquad \textbf{Ans.}$$

$$\sigma_b = \frac{P}{A_b} = \frac{40(10^3)}{2.7(10^{-3})\pi} = 4.72\ \text{MPa} \qquad \textbf{Ans.}$$

Ans:
$\tau_{\text{avg}} = 25.5$ MPa, $\sigma_b = 4.72$ MPa

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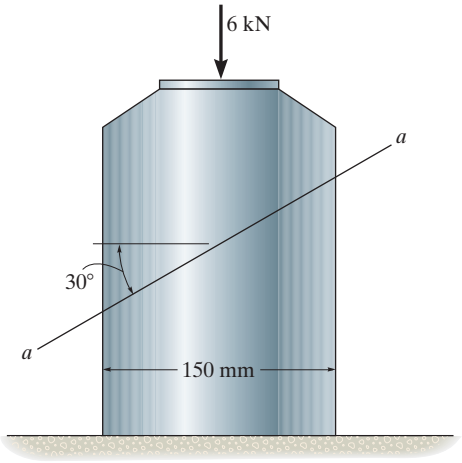
R1–6.

The 150 mm by 150 mm block of aluminum supports a compressive load of 6 kN. Determine the average normal and shear stress acting on the plane through section *a–a*. Show the results on a differential volume element located on the plane.



SOLUTION

Equation of Equilibrium:

$+\nearrow \Sigma F_x = 0; \quad V_{a-a} - 6\cos 60° = 0 \quad V_{a-a} = 3.00 \text{ kN}$

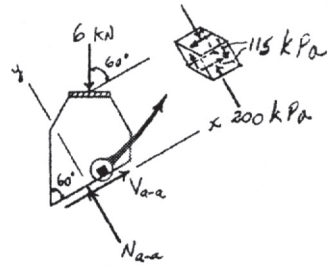
$\nwarrow + \Sigma F_y = 0; \quad N_{a-a} - 6\sin 60° = 0 \quad N_{a-a} = 5.196 \text{ kN}$

Average Normal Stress and Shear Stress: The cross sectional Area at section *a–a* is

$$A = \left(\frac{0.15}{\sin 60°}\right)(0.15) = 0.02598 \text{ m}^2.$$

$$\sigma_{a-a} = \frac{N_{a-a}}{A} = \frac{5.196(10^3)}{0.02598} = 200 \text{ kPa}$$ **Ans.**

$$\tau_{a-a} = \frac{V_{a-a}}{A} = \frac{3.00(10^3)}{0.02598} = 115 \text{ kPa}$$ **Ans.**



Ans:
$\sigma_{a-a} = 200$ kPa, $\tau_{a-a} = 115$ kPa

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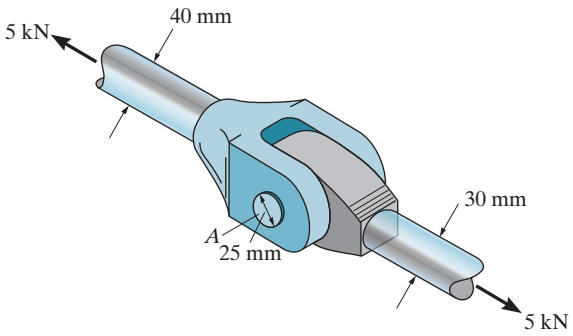
R1–7.

The yoke-and-rod connection is subjected to a tensile force of 5 kN. Determine the average normal stress in each rod and the average shear stress in the pin A between the members.



SOLUTION

For the 40 - mm - dia. rod:

$$\sigma_{40} = \frac{P}{A} = \frac{5\,(10^3)}{\frac{\pi}{4}(0.04)^2} = 3.98 \text{ MPa}$$ **Ans.**

For the 30 - mm - dia. rod:

$$\sigma_{30} = \frac{V}{A} = \frac{5\,(10^3)}{\frac{\pi}{4}(0.03)^2} = 7.07 \text{ MPa}$$ **Ans.**

Average shear stress for pin A:

$$\tau_{\text{avg}} = \frac{P}{A} = \frac{2.5\,(10^3)}{\frac{\pi}{4}(0.025)^2} = 5.09 \text{ MPa}$$ **Ans.**

Ans:
$\sigma_{40} = 3.98$ MPa, $\sigma_{30} = 7.07$ MPa,
$\tau_{\text{avg}} = 5.09$ MPa

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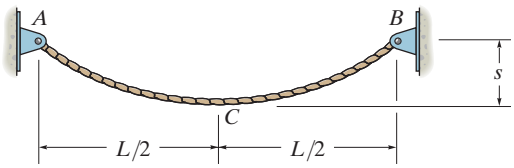
***R1–8.**

The cable has a specific weight γ (weight/volume) and cross-sectional area A. Assuming the sag s is small, so that the cable's length is approximately L and its weight can be distributed uniformly along the horizontal axis, determine the average normal stress in the cable at its lowest point C.

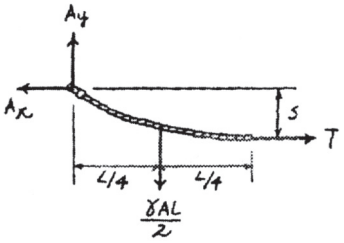


SOLUTION

Equation of Equilibrium:

$$\zeta+\Sigma M_A = 0; \qquad Ts - \frac{\gamma AL}{2}\left(\frac{L}{4}\right) = 0$$

$$T = \frac{\gamma AL^2}{8\,s}$$



Average Normal Stress:

$$\sigma = \frac{T}{A} = \frac{\frac{\gamma AL^2}{8\,s}}{A} = \frac{\gamma L^2}{8\,s}$$

Ans.

Ans:

$$\sigma = \frac{\gamma L^2}{8s}$$

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2–1.

An air-filled rubber ball has a diameter of 150 mm. If the air pressure within it is increased until the ball's diameter becomes 175 mm, determine the average normal strain in the rubber.

SOLUTION

$d_0 = 150\text{ mm}$

$d = 175\text{ mm}$

$$\varepsilon = \frac{\pi d - \pi d_0}{\pi d_0} = \frac{175 - 150}{150} = 0.167\text{ mm/mm}$$ **Ans.**

These solutions represent a preliminary version of the Instructors' Solutions Manual (ISM). It is possible and even likely that at this preliminary stage of preparing the ISM there are some omissions and errors in the draft solutions. These will be corrected and this manual will be republished.

Ans:

$\varepsilon = 0.167\text{ mm/mm}$

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2–2.

A thin strip of rubber has an unstretched length of 375 mm. If it is stretched around a pipe having an outer diameter of 125 mm, determine the average normal strain in the strip.

SOLUTION

$L_0 = 375 \text{ mm}$

$L = \pi(125 \text{ mm})$

$$\varepsilon = \frac{L - L_0}{L_0} = \frac{125\pi - 375}{375} = 0.0472 \text{ mm/mm}$$ **Ans.**

Ans.

$\varepsilon = 0.0472 \text{ mm/mm}$

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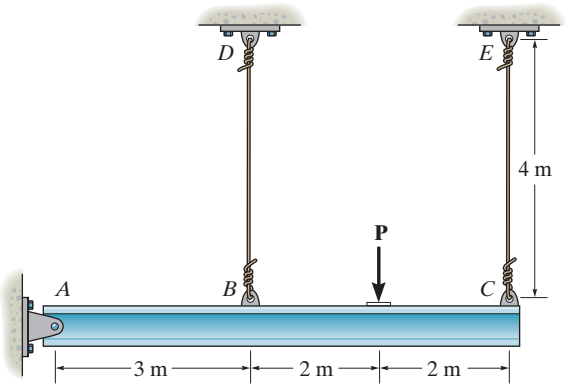
2–3.

If the load **P** on the beam causes the end *C* to be displaced 10 mm downward, determine the normal strain in wires *CE* and *BD*.



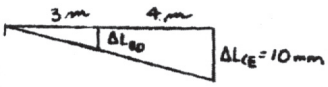
SOLUTION

$$\frac{\Delta L_{BD}}{3} = \frac{\Delta L_{CE}}{7}$$

$$\Delta L_{BD} = \frac{3\,(10)}{7} = 4.286 \text{ mm}$$

$$\epsilon_{CE} = \frac{\Delta L_{CE}}{L} = \frac{10}{4000} = 0.00250 \text{ mm/mm}$$ **Ans.**

$$\epsilon_{BD} = \frac{\Delta L_{BD}}{L} = \frac{4.286}{4000} = 0.00107 \text{ mm/mm}$$ **Ans.**



Ans:
$\epsilon_{CE} = 0.00250$ mm/mm, $\epsilon_{BD} = 0.00107$ mm/mm

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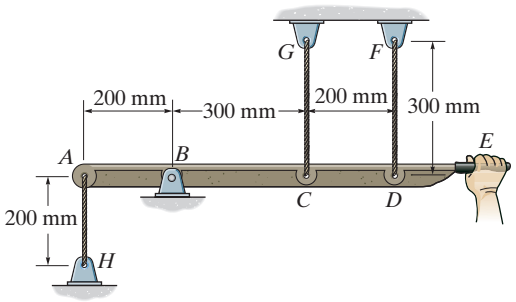
***2–4.**

The force applied at the handle of the rigid lever causes the lever to rotate clockwise about the pin B through an angle of 2°. Determine the average normal strain in each wire. The wires are unstretched when the lever is in the horizontal position.



SOLUTION

Geometry: The lever arm rotates through an angle of $\theta = \left(\frac{2°}{180}\right)\pi \text{ rad} = 0.03491 \text{ rad}$.

Since θ is small, the displacements of points A, C, and D can be approximated by

$$\delta_A = 200(0.03491) = 6.9813 \text{ mm}$$

$$\delta_C = 300(0.03491) = 10.4720 \text{ mm}$$

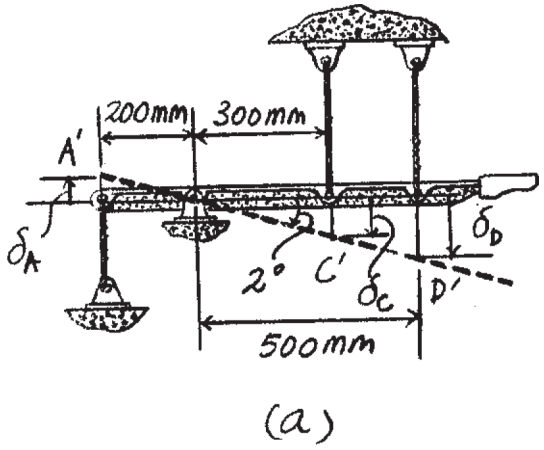
$$\delta_D = 500(0.03491) = 17.4533 \text{ mm}$$

Average Normal Strain: The unstretched length of wires AH, CG, and DF are $L_{AH} = 200$ mm, $L_{CG} = 300$ mm, and $L_{DF} = 300$ mm. We obtain

$$(\epsilon_{\text{avg}})_{AH} = \frac{\delta_A}{L_{AH}} = \frac{6.9813}{200} = 0.0349 \text{ mm/mm} \qquad \textbf{Ans.}$$

$$(\epsilon_{\text{avg}})_{CG} = \frac{\delta_C}{L_{CG}} = \frac{10.4720}{300} = 0.0349 \text{ mm/mm} \qquad \textbf{Ans.}$$

$$(\epsilon_{\text{avg}})_{DF} = \frac{\delta_D}{L_{DF}} = \frac{17.4533}{300} = 0.0582 \text{ mm/mm} \qquad \textbf{Ans.}$$



(a)

Ans:
$(\epsilon_{\text{avg}})_{AH} = 0.0349 \text{ mm/mm}$
$(\epsilon_{\text{avg}})_{CG} = 0.0349 \text{ mm/mm}$
$(\epsilon_{\text{avg}})_{DF} = 0.0582 \text{ mm/mm}$

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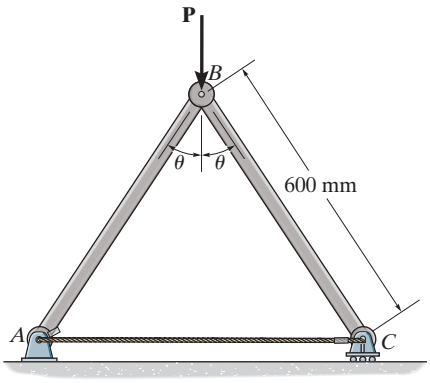
2–5.

The pin-connected rigid rods AB and BC are inclined at $\theta = 30°$ when they are unloaded. When the force **P** is applied θ becomes 30.2°. Determine the average normal strain in wire AC.



SOLUTION

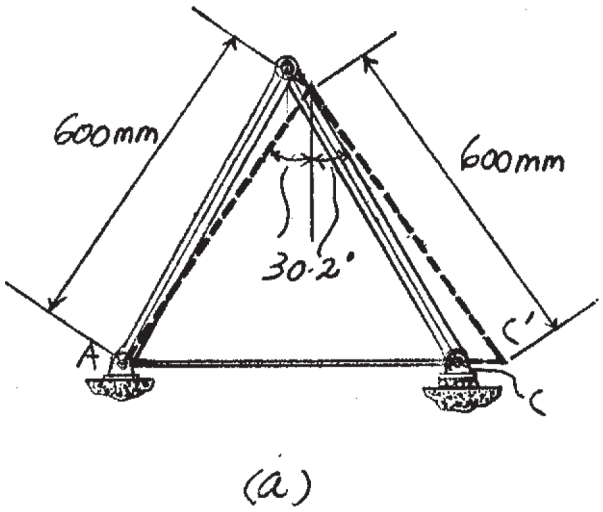
Geometry: Referring to Fig. *a*, the unstretched and stretched lengths of wire AD are

$$L_{AC} = 2(600 \sin 30°) = 600 \text{ mm}$$

$$L_{AC'} = 2(600 \sin 30.2°) = 603.6239 \text{ mm}$$

Average Normal Strain:

$$(\epsilon_{\text{avg}})_{AC} = \frac{L_{AC'} - L_{AC}}{L_{AC}} = \frac{603.6239 - 600}{600} = 6.04(10^{-3}) \text{ mm/mm} \qquad \textbf{Ans.}$$



(a)

Ans:
$(\epsilon_{\text{avg}})_{AC} = 6.04(10^{-3}) \text{ mm/mm}$

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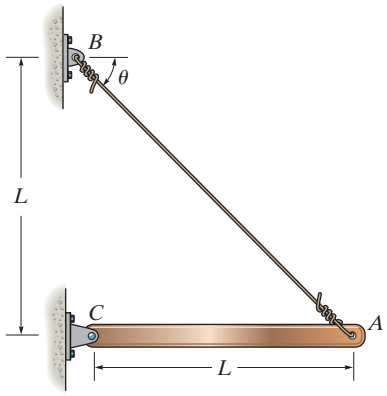
2–6.

The wire AB is unstretched when $\theta = 45°$. If a load is applied to the bar AC, which causes θ to become 47°, determine the normal strain in the wire.



SOLUTION

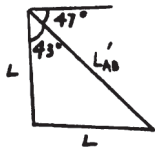
$$L^2 = L^2 + L_{AB}'^2 - 2LL'_{AB}\cos 43°$$

$$L'_{AB} = 2L\cos 43°$$

$$\epsilon_{AB} = \frac{L'_{AB} - L_{AB}}{L_{AB}}$$

$$= \frac{2L\cos 43° - \sqrt{2}L}{\sqrt{2}L}$$

$$= 0.0343 \qquad \textbf{Ans.}$$



Ans:
$\epsilon_{AB} = 0.0343$

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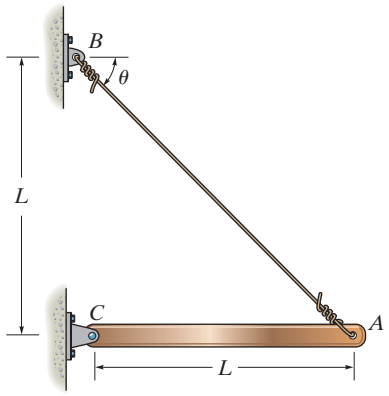
2–7.

If a horizontal load applied to the bar AC causes point A to be displaced to the right by an amount ΔL, determine the normal strain in the wire AB. Originally, $\theta = 45°$.



SOLUTION

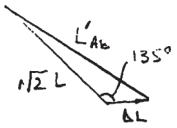
$$L'_{AB} = \sqrt{(\sqrt{2}L)^2 + \Delta L^2 - 2(\sqrt{2}L)(\Delta L)\cos 135°}$$

$$= \sqrt{2L^2 + \Delta L^2 + 2L\Delta L}$$

$$\epsilon_{AB} = \frac{L'_{AB} - L_{AB}}{L_{AB}}$$

$$= \frac{\sqrt{2L^2 + \Delta L^2 + 2L\Delta L} - \sqrt{2}L}{\sqrt{2}L}$$

$$= \sqrt{1 + \frac{\Delta L^2}{2L^2} + \frac{\Delta L}{L}} - 1$$



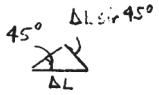
Neglecting the higher-order terms,

$$\epsilon_{AB} = \left(1 + \frac{\Delta L}{L}\right)^{\frac{1}{2}} - 1$$

$$= 1 + \frac{1}{2}\frac{\Delta L}{L} + ... - 1 \qquad \text{(binomial theorem)}$$

$$= \frac{0.5\Delta L}{L} \qquad \textbf{Ans.}$$

Also,



$$\epsilon_{AB} = \frac{\Delta L \sin 45°}{\sqrt{2}L} = \frac{0.5\,\Delta L}{L} \qquad \textbf{Ans.}$$

Ans:

$$\epsilon_{AB} = \frac{0.5\Delta L}{L}$$

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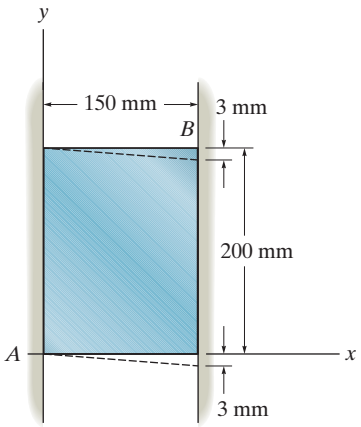
***2–8.**

The rectangular plate is subjected to the deformation shown by the dashed line. Determine the average shear strain γ_{xy} in the plate.



SOLUTION

Geometry:

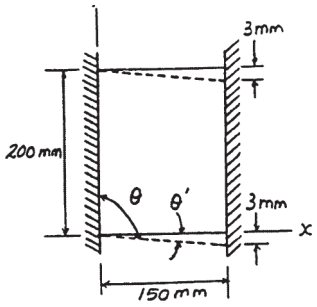
$$\theta' = \tan^{-1}\frac{3}{150} = 0.0200 \text{ rad}$$

$$\theta = \left(\frac{\pi}{2} + 0.0200\right) \text{rad}$$

Shear Strain:

$$\gamma_{xy} = \frac{\pi}{2} - \theta = \frac{\pi}{2} - \left(\frac{\pi}{2} + 0.0200\right)$$

$$= -0.0200 \text{ rad} \qquad \textbf{Ans.}$$



Ans:
$\gamma_{xy} = -0.0200$ rad

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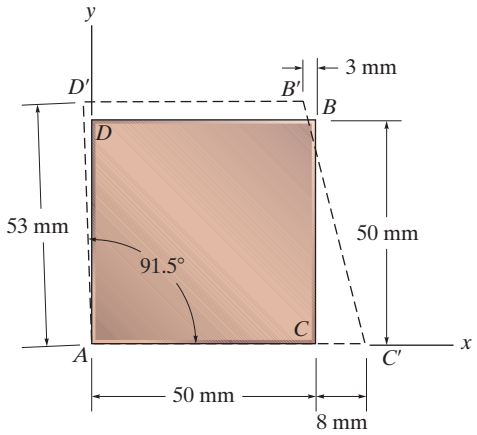
2–9.

The square deforms into the position shown by the dashed lines. Determine the shear strain at each of its corners, A, B, C, and D, relative to the x, y axes. Side $D'B'$ remains horizontal.



SOLUTION

Geometry:

$$B'C' = \sqrt{(8 + 3)^2 + (53 \sin 88.5°)^2} = 54.1117 \text{ mm}$$

$$C'D' = \sqrt{53^2 + 58^2 - 2(53)(58) \cos 91.5°}$$

$$= 79.5860 \text{ mm}$$

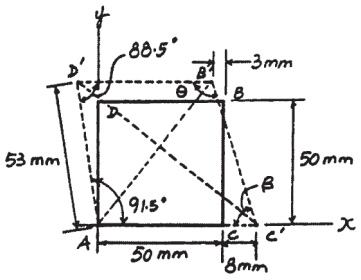
$$B'D' = 50 + 53 \sin 1.5° - 3 = 48.3874 \text{ mm}$$

$$\cos \theta = \frac{(B'D')^2 + (B'C')^2 - (C'D')^2}{2(B'D')(B'C')}$$

$$= \frac{48.3874^2 + 54.1117^2 - 79.5860^2}{2(48.3874)(54.1117)} = -0.20328$$

$$\theta = 101.73°$$

$$\beta = 180° - \theta = 78.27°$$



Shear Strain:

$$(\gamma_A)_{xy} = \frac{\pi}{2} - \pi\left(\frac{91.5°}{180°}\right) = -0.0262 \text{ rad}$$ **Ans.**

$$(\gamma_B)_{xy} = \frac{\pi}{2} - \theta = \frac{\pi}{2} - \pi\left(\frac{101.73°}{180°}\right) = -0.205 \text{ rad}$$ **Ans.**

$$(\gamma_C)_{xy} = \beta - \frac{\pi}{2} = \pi\left(\frac{78.27°}{180°}\right) - \frac{\pi}{2} = -0.205 \text{ rad}$$ **Ans.**

$$(\gamma_D)_{xy} = \pi\left(\frac{88.5°}{180°}\right) - \frac{\pi}{2} = -0.0262 \text{ rad}$$ **Ans.**

Ans:
$(\gamma_A)_{xy} = -0.0262$ rad
$(\gamma_B)_{xy} = -0.205$ rad
$(\gamma_C)_{xy} = -0.205$ rad
$(\gamma_D)_{xy} = -0.0262$ rad

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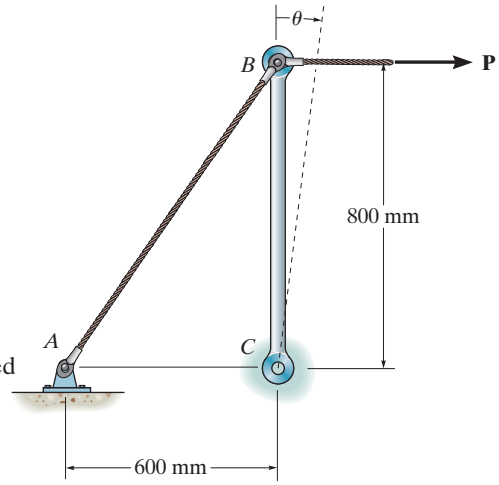
2–10.

Part of a control linkage for an airplane consists of a rigid member CB and a flexible cable AB. If a force is applied to the end B of the member and causes it to rotate by $\theta = 0.5°$, determine the normal strain in the cable. Originally the cable is unstretched.



SOLUTION

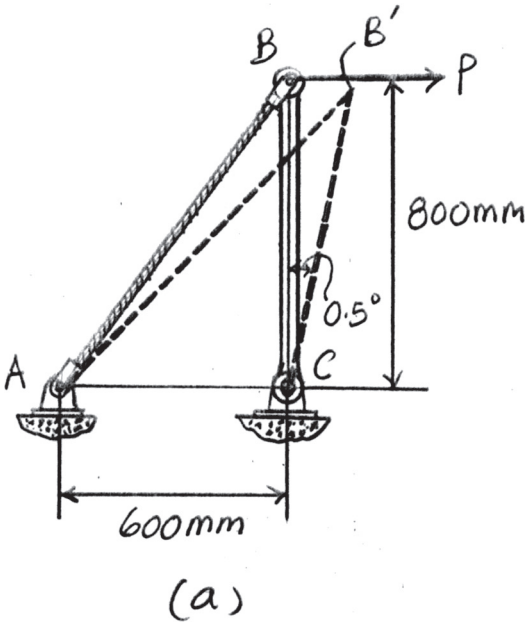
Geometry: Referring to the geometry shown in Fig. *a*, the unstretched and stretched lengths of cable AB are

$$L_{AB} = \sqrt{600^2 + 800^2} = 1000 \text{ mm}$$

$$L_{AB'} = \sqrt{600^2 + 800^2 - 2(600)(800)\cos 90.5°} = 1004.18 \text{ mm}$$

Average Normal Strain:

$$\epsilon_{AB} = \frac{L_{AB'} - L_{AB}}{L_{AB}} = \frac{1004.18 - 1000}{1000} = 0.00418 \text{ mm/mm} \qquad \textbf{Ans.}$$



(a)

Ans:
$\epsilon_{AB} = 0.00418 \text{ mm/mm}$

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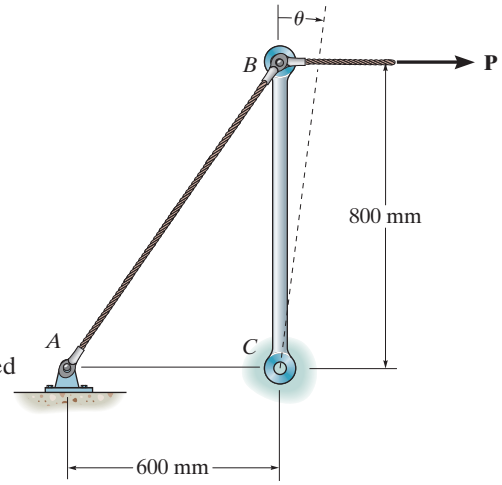
2–11.

Part of a control linkage for an airplane consists of a rigid member CB and a flexible cable AB. If a force is applied to the end B of the member and causes a normal strain in the cable of 0.004 mm/mm, determine the displacement of point B. Originally the cable is unstretched.



SOLUTION

Geometry: Referring to the geometry shown in Fig. *a*, the unstretched and stretched lengths of cable AB are

$$L_{AB} = \sqrt{600^2 + 800^2} = 1000 \text{ mm}$$

$$L_{AB'} = \sqrt{600^2 + 800^2 - 2(600)(800)\cos(90° + \theta)}$$

$$L_{AB'} = \sqrt{1(10^6) - 0.960(10^6)\cos(90° + \theta)}$$

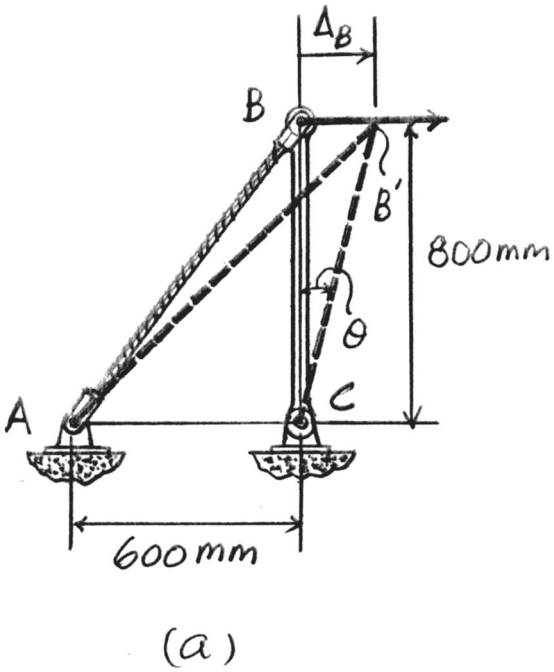
Average Normal Strain:

$$\epsilon_{AB} = \frac{L_{AB'} - L_{AB}}{L_{AB}}; \quad 0.004 = \frac{\sqrt{1(10^6) - 0.960(10^6)\cos(90° + \theta)} - 1000}{1000}$$

$$\theta = 0.4784°\left(\frac{\pi}{180°}\right) = 0.008350 \text{ rad}$$

Thus,

$$\Delta_B = \theta L_{BC} = 0.008350(800) = 6.68 \text{ mm}$$ **Ans.**



(a)

Ans:
$\Delta_B = 6.68$ mm

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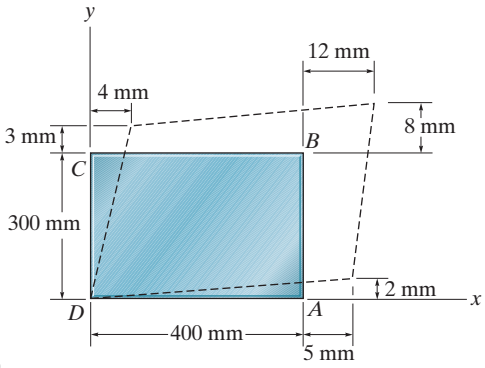
***2–12.**

Determine the shear strain γ_{xy} at corners A and B if the plastic distorts as shown by the dashed lines.



SOLUTION

Geometry: Referring to the geometry shown in Fig. *a*, the small-angle analysis gives

$$\alpha = \psi = \frac{7}{306} = 0.022876 \text{ rad}$$

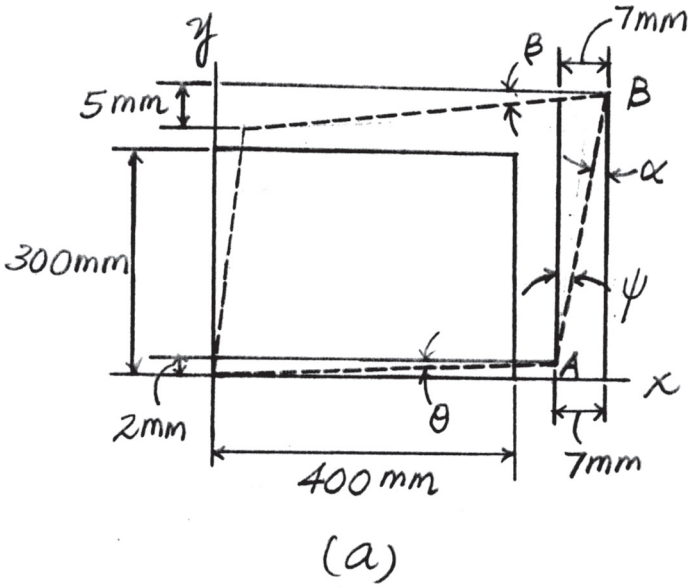
$$\beta = \frac{5}{408} = 0.012255 \text{ rad}$$

$$\theta = \frac{2}{405} = 0.0049383 \text{ rad}$$

Shear Strain: By definition,

$$(\gamma_A)_{xy} = \theta + \psi = 0.02781 \text{ rad} = 27.8(10^{-3}) \text{ rad} \quad \textbf{Ans.}$$

$$(\gamma_B)_{xy} = \alpha + \beta = 0.03513 \text{ rad} = 35.1(10^{-3}) \text{ rad} \quad \textbf{Ans.}$$



(a)

Ans:
$(\gamma_A)_{xy} = 27.8(10^{-3})$ rad
$(\gamma_B)_{xy} = 35.1(10^{-3})$ rad

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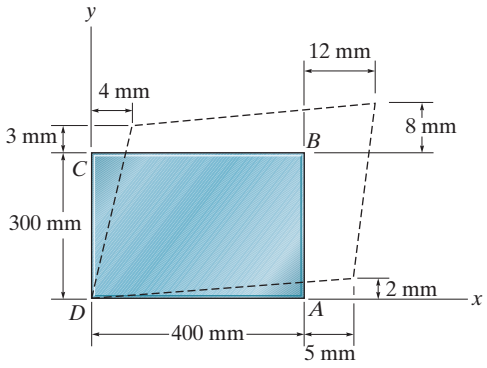
2–13.

Determine the shear strain γ_{xy} at corners D and C if the plastic distorts as shown by the dashed lines.



SOLUTION

Geometry: Referring to the geometry shown in Fig. *a*, the small-angle analysis gives

$$\alpha = \psi = \frac{4}{303} = 0.013201 \text{ rad}$$

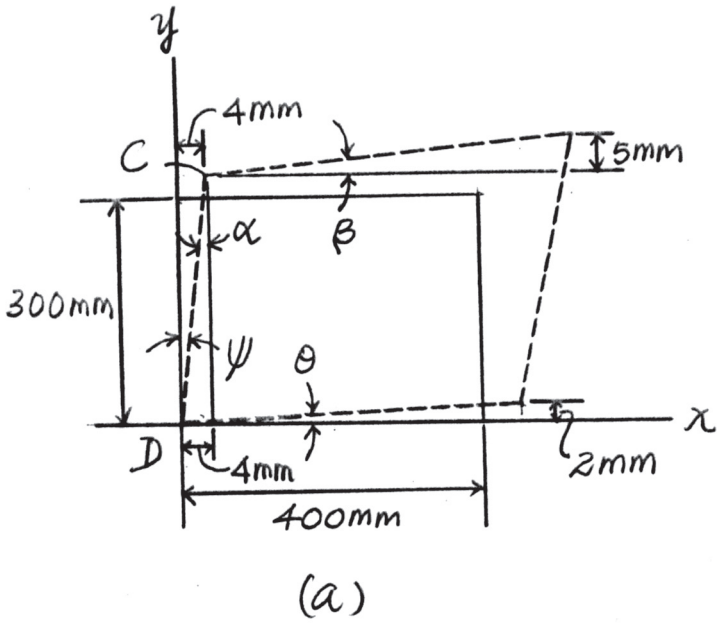
$$\theta = \frac{2}{405} = 0.0049383 \text{ rad}$$

$$\beta = \frac{5}{408} = 0.012255 \text{ rad}$$

Shear Strain: By definition

$$(\gamma_{xy})_C = \alpha + \beta = 0.02546 \text{ rad} = 25.5(10^{-3}) \text{ rad} \quad \textbf{Ans.}$$

$$(\gamma_{xy})_D = \theta + \psi = 0.01814 \text{ rad} = 18.1(10^{-3}) \text{ rad} \quad \textbf{Ans.}$$



(a)

Ans:
$(\gamma_{xy})_C = 25.5(10^{-3})$ rad
$(\gamma_{xy})_D = 18.1(10^{-3})$ rad

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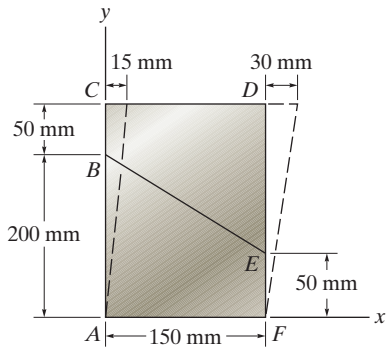
2–14.

The material distorts into the dashed position shown. Determine the average normal strains ϵ_x, ϵ_y and the shear strain γ_{xy} at A, and the average normal strain along line BE.



SOLUTION

Geometry: Referring to the geometry shown in Fig. *a*,

$$\tan\theta = \frac{15}{250}; \qquad \theta = (3.4336°)\left(\frac{\pi}{180°}\text{ rad}\right) = 0.05993\text{ rad}$$

$$L_{AC'} = \sqrt{15^2 + 150^2} = \sqrt{62725}\text{ mm}$$

$$\frac{BB'}{15} = \frac{200}{250}; \qquad BB' = 12\text{ mm} \qquad \frac{EE'}{30} = \frac{50}{250}; \qquad EE' = 6\text{ mm}$$

$$x' = 150 + EE' - BB' = 150 + 6 - 12 = 144\text{ mm}$$

$$L_{BE} = \sqrt{150^2 + 150^2} = 150\sqrt{2}\text{ mm} \quad L_{B'E'} = \sqrt{144^2 + 150^2} = \sqrt{43236}\text{ mm}$$

Average Normal and Shear Strain: Since no deformation occurs along x axis,

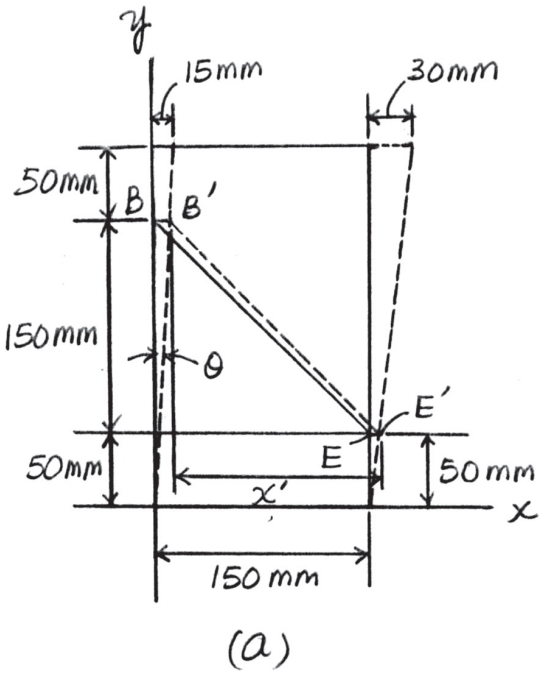
$$(\epsilon_x)_A = 0 \qquad \textbf{Ans.}$$

$$(\epsilon_y)_A = \frac{L_{AC'} - L_{AC}}{L_{AC}} = \frac{\sqrt{62725} - 250}{250} = 1.80(10^{-3})\text{ mm/mm} \qquad \textbf{Ans.}$$

By definition,

$$(\gamma_{xy})_A = \theta = 0.0599\text{ rad} \qquad \textbf{Ans.}$$

$$\epsilon_{BE} = \frac{L_{B'E'} - L_{BE}}{L_{BE}} = \frac{\sqrt{43236} - 150\sqrt{2}}{150\sqrt{2}} = -0.0198\text{ mm/mm} \qquad \textbf{Ans.}$$



(a)

Ans:
$(\epsilon_x)_A = 0$
$(\epsilon_y)_A = 1.80(10^{-3})$ mm/mm
$(\gamma_{xy})_A = 0.0599$ rad
$\epsilon_{BE} = -0.0198$ mm/mm

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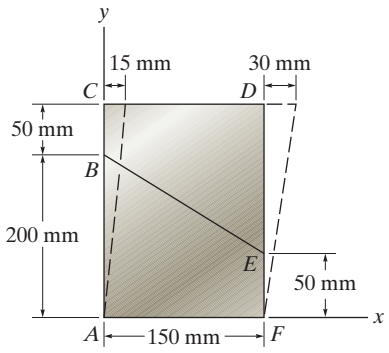
2–15.

The material distorts into the dashed position shown. Determine the average normal strains along the diagonals *AD* and *CF*.



SOLUTION

Geometry: Referring to the geometry shown in Fig. *a*,

$$L_{AD} = L_{CF} = \sqrt{150^2 + 250^2} = \sqrt{85000}\,\text{mm}$$

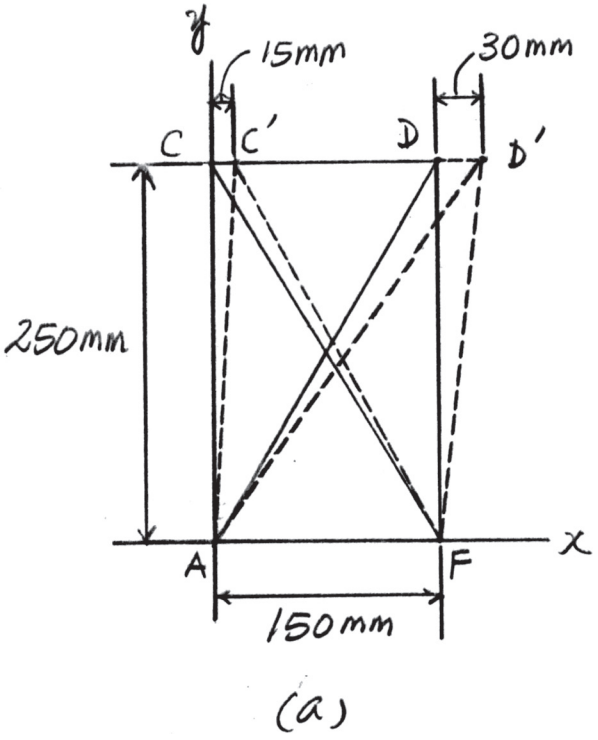
$$L_{AD'} = \sqrt{(150 + 30)^2 + 250^2} = \sqrt{94900}\,\text{mm}$$

$$L_{C'F} = \sqrt{(150 - 15)^2 + 250^2} = \sqrt{80725}\,\text{mm}$$

Average Normal Strain:

$$\epsilon_{AD} = \frac{L_{AD'} - L_{AD}}{L_{AD}} = \frac{\sqrt{94900} - \sqrt{85000}}{\sqrt{85000}} = 0.0566\,\text{mm/mm} \qquad \textbf{Ans.}$$

$$\epsilon_{CF} = \frac{L_{C'F} - L_{CF}}{L_{CF}} = \frac{\sqrt{80725} - \sqrt{85000}}{\sqrt{85000}} = -0.0255\,\text{mm/mm} \qquad \textbf{Ans.}$$



(a)

Ans:
$\epsilon_{AD} = 0.0566\,\text{mm/mm}$
$\epsilon_{CF} = -0.0255\,\text{mm/mm}$

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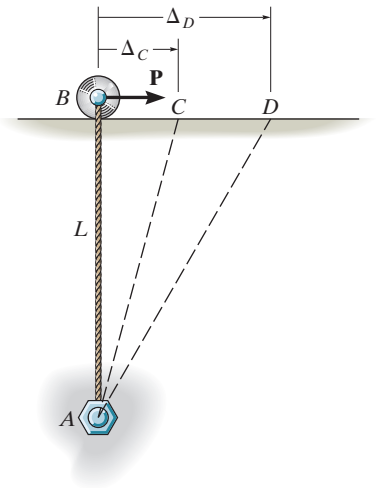
***2–16.**

The nylon cord has an original length L and is tied to a bolt at A and a roller at B. If a force **P** is applied to the roller, determine the normal strain in the cord when the roller is at C, and at D. If the cord is originally unstrained when it is at C, determine the normal strain ϵ'_D when the roller moves to D. Show that if the displacements Δ_C and Δ_D are small, then $\epsilon'_D = \epsilon_D - \epsilon_C$.



SOLUTION

$$L_C = \sqrt{L^2 + \Delta_C^2}$$

$$\epsilon_C = \frac{\sqrt{L^2 + \Delta_C^2} - L}{L}$$

$$= \frac{L\sqrt{1 + \left(\frac{\Delta_C^2}{L^2}\right)} - L}{L} = \sqrt{1 + \left(\frac{\Delta_C^2}{L^2}\right)} - 1$$

For small Δ_C,

$$\epsilon_C = 1 + \frac{1}{2}\left(\frac{\Delta_C^2}{L^2}\right) - 1 = \frac{1}{2}\frac{\Delta_C^2}{L^2} \qquad \textbf{Ans.}$$

In the same manner,

$$\epsilon_D = \frac{1}{2}\frac{\Delta_D^2}{L^2} \qquad \textbf{Ans.}$$

$$\epsilon_D{}' = \frac{\sqrt{L^2 + \Delta_D^2} - \sqrt{L^2 + \Delta_C^2}}{\sqrt{L^2 + \Delta_C^2}} = \frac{\sqrt{1 + \frac{\Delta_D^2}{L^2}} - \sqrt{1 + \frac{\Delta_C^2}{L^2}}}{\sqrt{1 + \frac{\Delta_C^2}{L^2}}}$$

For small Δ_C and Δ_D,

$$\epsilon_D{}' = \frac{\left(1 + \frac{1}{2}\frac{\Delta_C^2}{L^2}\right) - \left(1 + \frac{1}{2}\frac{\Delta_D^2}{L^2}\right)}{\left(1 + \frac{1}{2}\frac{\Delta_C^2}{L^2}\right)} = \frac{\frac{1}{2L^2}\left(\Delta_C^2 - \Delta_D^2\right)}{\frac{1}{2L^2}\left(2L^2 + \Delta_C^2\right)}$$

$$\epsilon_D{}' = \frac{\Delta_C^2 - \Delta_D^2}{2L^2 - \Delta_C^2} = \frac{1}{2L^2}\left(\Delta_C^2 - \Delta_D^2\right) = \epsilon_C - \epsilon_D \qquad \textbf{QED}$$

Also this problem can be solved as follows:

$$A_C = L\sec\theta_C; \qquad A_D = L\sec\theta_D$$

$$\epsilon_C = \frac{L\sec\theta_C - L}{L} = \sec\theta_C - 1$$

$$\epsilon_D = \frac{L\sec\theta_D - L}{L} = \sec\theta_D - 1$$

Expanding $\sec\theta$

$$\sec\theta = 1 + \frac{\theta^2}{2!} + \frac{5\,\theta^4}{4!} \ldots\ldots$$

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***2–16. Continued**

For small θ neglect the higher order terms

$$\sec\theta = 1 + \frac{\theta^2}{2}$$

Hence,

$$\epsilon_C = 1 + \frac{{\theta_C}^2}{2} - 1 = \frac{{\theta_C}^2}{2}$$

$$\epsilon_D = 1 + \frac{{\theta_D}^2}{2} - 1 = \frac{{\theta_D}^2}{2}$$

$$\epsilon_D{}' = \frac{L\sec\theta_D - L\sec\theta_C}{L\sec\theta_C} = \frac{\sec\theta_D}{\sec\theta_C} - 1 = \sec\theta_D\cos\theta_C - 1$$

Since $\cos\theta = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} .....$

$$\sec\theta_D\cos\theta_C = \left(1 + \frac{{\theta_D}^2}{2}.....\right)\left(1 - \frac{{\theta_C}^2}{2}.....\right)$$

$$= 1 - \frac{\theta_C^2}{2} + \frac{\theta_D^2}{2} - \frac{\theta_C^2\theta_D^2}{4}$$

Neglecting the higher order terms

$$\sec\theta_D\cos\theta_C = 1 + \frac{\theta_D^2}{2} - \frac{\theta_C^2}{2}$$

$$\epsilon_D{}' = \left[1 + \frac{{\theta_2}^2}{2} - \frac{{\theta_1}^2}{2}\right] - 1 = \frac{{\theta_D}^2}{2} - \frac{{\theta_C}^2}{2}$$

$$= \epsilon_D - \epsilon_C$$ **QED**

Ans:

$$\epsilon_C = \frac{1}{2}\frac{\Delta_C^2}{L^2}$$

$$\epsilon_D = \frac{1}{2}\frac{\Delta_D^2}{L^2}$$

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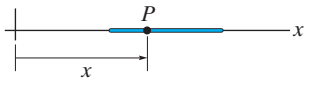
2–17.

A thin wire, lying along the x axis, is strained such that each point on the wire is displaced $\Delta x = kx^2$ along the x axis. If k is constant, what is the normal strain at any point P along the wire?



SOLUTION

$$\epsilon = \frac{d(\Delta x)}{dx} = 2\,k\,x$$ **Ans.**

Ans:
$\epsilon = 2kx$

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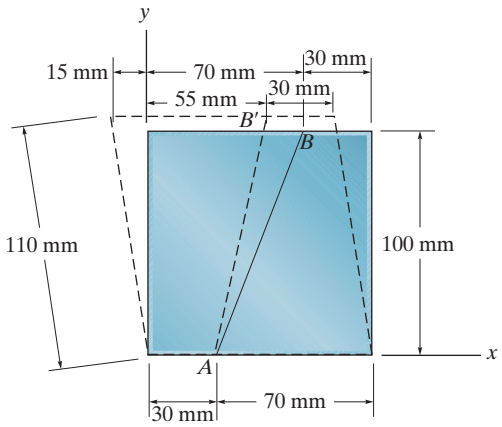
2–18.

The block is deformed into the position shown by the dashed lines. Determine the average normal strain along line AB.



SOLUTION

Geometry:

$$AB = \sqrt{100^2 + (70 - 30)^2} = 107.7033 \text{ mm}$$

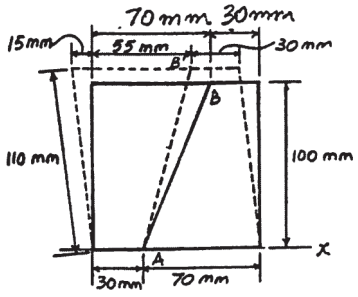
$$AB' = \sqrt{(70 - 30 - 15)^2 + \left(110^2 - 15^2\right)} = 111.8034 \text{ mm}$$

Average Normal Strain:

$$\epsilon_{AB} = \frac{AB' - AB}{AB}$$

$$= \frac{111.8034 - 107.7033}{107.7033}$$

$$= 0.0381 \text{ mm/mm} = 38.1\left(10^{-3}\right) \text{ mm}$$ **Ans.**



Ans:
$\epsilon_{AB} = 38.1\left(10^{-3}\right)$ mm

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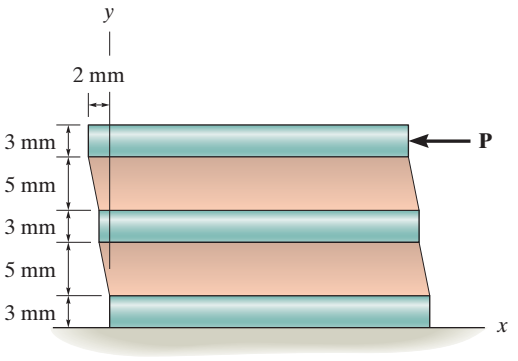
2–19. Nylon strips are fused to glass plates. When moderately heated the nylon will become soft while the glass stays approximately rigid. Determine the average shear strain in the nylon due to the load **P** when the assembly deforms as indicated.



SOLUTION

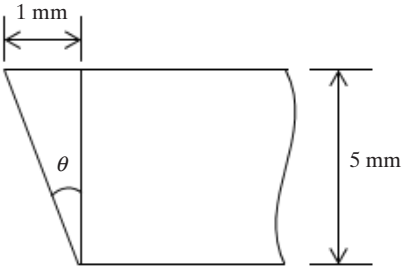
From the geometry shown in Fig. a

$$\theta = \tan^{-1}\left(\frac{1}{5}\right) = 11.31^\circ = 0.1974 \text{ rad}$$

The shear strain is

$$\gamma = \frac{\pi}{2} - \left(\frac{\pi}{2} + \theta\right) = -0.1974 \text{ rad} = -0.197 \text{ rad}$$

Ans.



(a)

Ans:

$\gamma = -0.197$ rad

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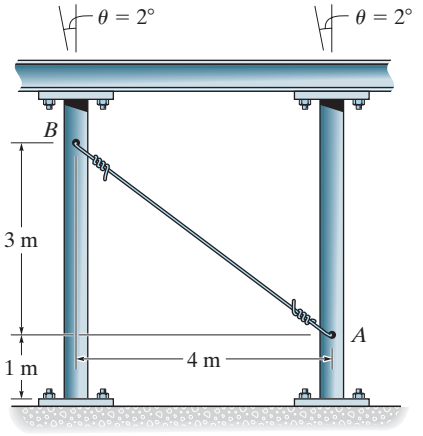
***2–20.**

The guy wire AB of a building frame is originally unstretched. Due to an earthquake, the two columns of the frame tilt $\theta = 2°$. Determine the approximate normal strain in the wire when the frame is in this position. Assume the columns are rigid and rotate about their lower supports.



SOLUTION

Geometry: The vertical displacement is negligible

$$x_A = (1)\left(\frac{2°}{180°}\right)\pi = 0.03491 \text{ m}$$

$$x_B = (4)\left(\frac{2°}{180°}\right)\pi = 0.13963 \text{ m}$$

$$x = 4 + x_B - x_A = 4.10472 \text{ m}$$

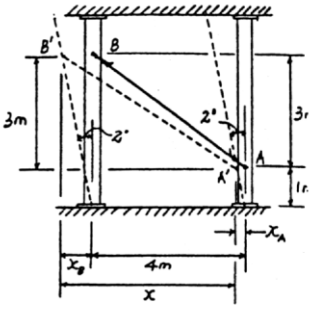
$$A'B' = \sqrt{3^2 + 4.10472^2} = 5.08416 \text{ m}$$

$$AB = \sqrt{3^2 + 4^2} = 5.00 \text{ m}$$

Average Normal Strain:

$$\varepsilon_{AB} = \frac{A'B' - AB}{AB}$$

$$= \frac{5.08416 - 5}{5} = 16.8\left(10^{-3}\right) \text{ m/m}$$



Ans.

Ans:

$A'B' = 5.08416 \text{ m}$

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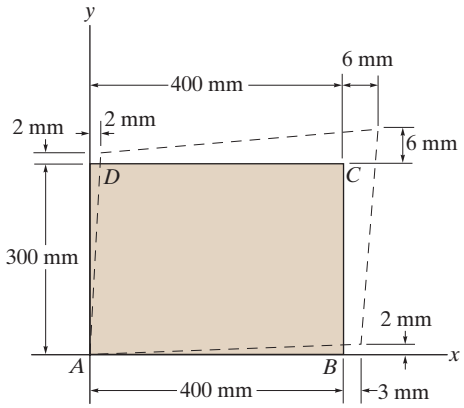
2–21.

The rectangular plate is deformed into the shape shown by the dashed lines. Determine the average normal strain along diagonal AC, and the average shear strain at corner A relative to the x, y axes.



SOLUTION

Geometry: The unstretched length of diagonal AC is

$$L_{AC} = \sqrt{300^2 + 400^2} = 500 \text{ mm}$$

Referring to Fig. *a*, the stretched length of diagonal AC is

$$L_{AC'} = \sqrt{(400 + 6)^2 + (300 + 6)^2} = 508.4014 \text{ mm}$$

Referring to Fig. *a* and using small angle analysis,

$$\phi = \frac{2}{300 + 2} = 0.006623 \text{ rad}$$

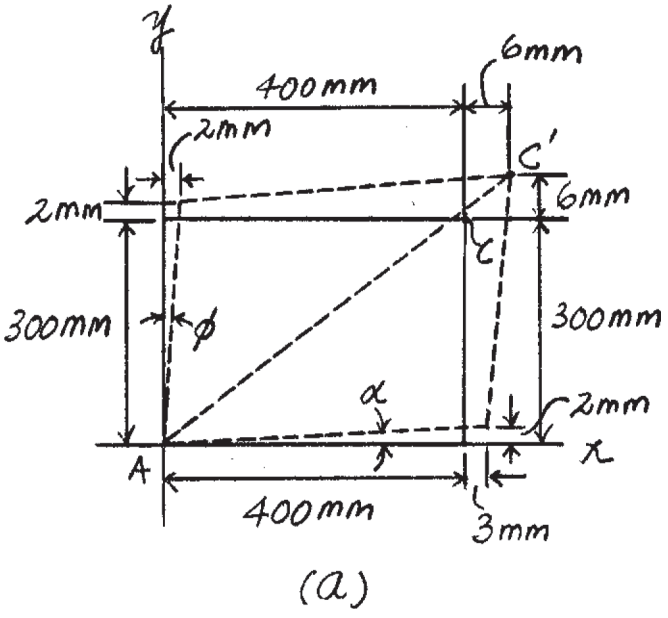
$$\alpha = \frac{2}{400 + 3} = 0.004963 \text{ rad}$$

Average Normal Strain: Applying Eq. 2,

$$(\epsilon_{\text{avg}})_{AC} = \frac{L_{AC'} - L_{AC}}{L_{AC}} = \frac{508.4014 - 500}{500} = 0.0168 \text{ mm/mm} \qquad \textbf{Ans.}$$

Shear Strain: Referring to Fig. *a*,

$$(\gamma_A)_{xy} = \phi + \alpha = 0.006623 + 0.004963 = 0.0116 \text{ rad} \qquad \textbf{Ans.}$$



(a)

Ans:
$(\epsilon_{\text{avg}})_{AC} = 0.0168 \text{ mm/mm}$, $(\gamma_A)_{xy} = 0.0116 \text{ rad}$

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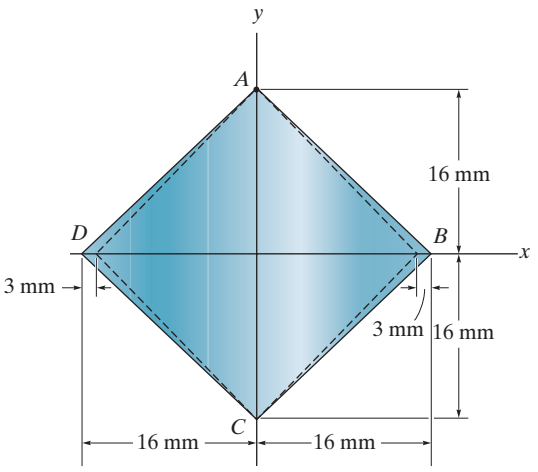
2–22.

The corners B and D of the square plate are given the displacements indicated. Determine the shear strains at A and B.



SOLUTION

Applying trigonometry to Fig. a

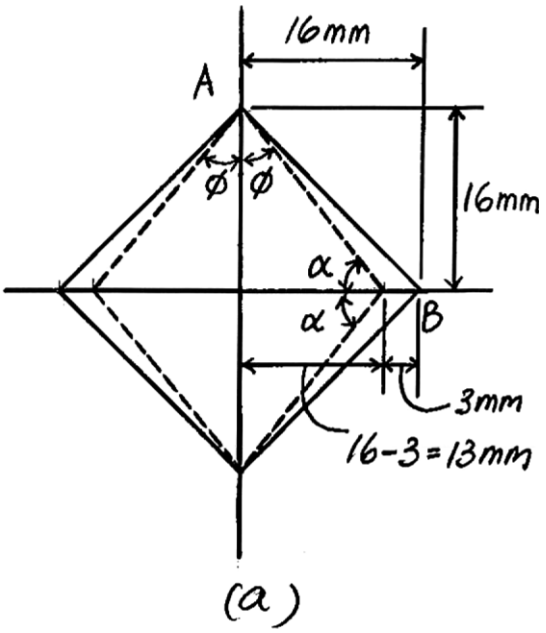
$$\phi = \tan^{-1}\left(\frac{13}{16}\right) = 39.09^\circ\left(\frac{\pi \text{ rad}}{180^\circ}\right) = 0.6823 \text{ rad}$$

$$\alpha = \tan^{-1}\left(\frac{16}{13}\right) = 50.91^\circ\left(\frac{\pi \text{ rad}}{180^\circ}\right) = 0.8885 \text{ rad}$$

By the definition of shear strain,

$$\left(\gamma_{xy}\right)_A = \frac{\pi}{2} - 2\phi = \frac{\pi}{2} - 2(0.6823) = 0.206 \text{ rad}$$ **Ans.**

$$\left(\gamma_{xy}\right)_B = \frac{\pi}{2} - 2\alpha = \frac{\pi}{2} - 2(0.8885) = -0.206 \text{ rad}$$ **Ans.**



Ans:

$\left(\gamma_{xy}\right)_A = 0.206$ rad

$\left(\gamma_{xy}\right)_B = -0.206$ rad

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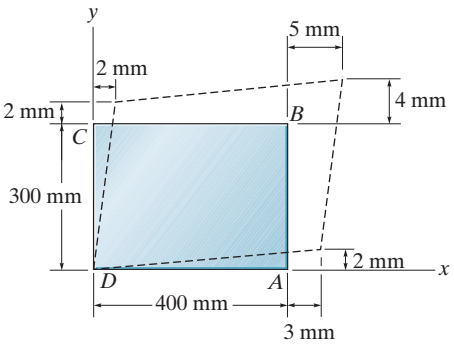
2–23.

Determine the shear strain γ_{xy} at corners A and B if the plate distorts as shown by the dashed lines.

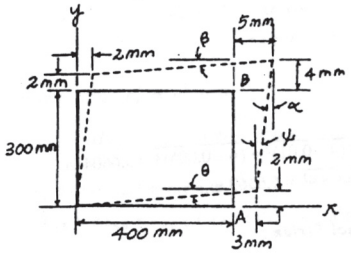


SOLUTION

Geometry: For small angles,

$$\alpha = \psi = \frac{2}{302} = 0.00662252 \text{ rad}$$

$$\beta = \theta = \frac{2}{403} = 0.00496278 \text{ rad}$$



Shear Strain:

$$(\gamma_B)_{xy} = \alpha + \beta$$

$$= 0.0116 \text{ rad} = 11.6\left(10^{-3}\right) \text{ rad} \quad \textbf{Ans.}$$

$$(\gamma_A)_{xy} = \theta + \psi$$

$$= 0.0116 \text{ rad} = 11.6\left(10^{-3}\right) \text{ rad} \quad \textbf{Ans.}$$

Ans:
$(\gamma_B)_{xy} = 11.6(10^{-3})$ rad,
$(\gamma_A)_{xy} = 11.6(10^{-3})$ rad

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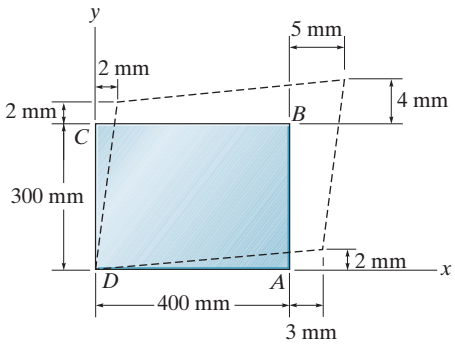
***2–24.**

Determine the shear strain γ_{xy} at corners D and C if the plate distorts as shown by the dashed lines.

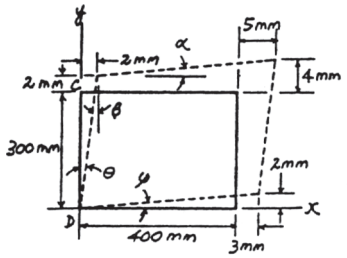


SOLUTION

Geometry: For small angles,

$$\alpha = \psi = \frac{2}{403} = 0.00496278 \text{ rad}$$

$$\beta = \theta = \frac{2}{302} = 0.00662252 \text{ rad}$$



Shear Strain:

$$(\gamma_C)_{xy} = \alpha + \beta$$

$$= 0.0116 \text{ rad} = 11.6\left(10^{-3}\right) \text{ rad} \quad \textbf{Ans.}$$

$$(\gamma_D)_{xy} = \theta + \psi$$

$$= 0.0116 \text{ rad} = 11.6\left(10^{-3}\right) \text{ rad} \quad \textbf{Ans.}$$

Ans:
$(\gamma_C)_{xy} = 11.6(10^{-3})$ rad,
$(\gamma_D)_{xy} = 11.6(10^{-3})$ rad

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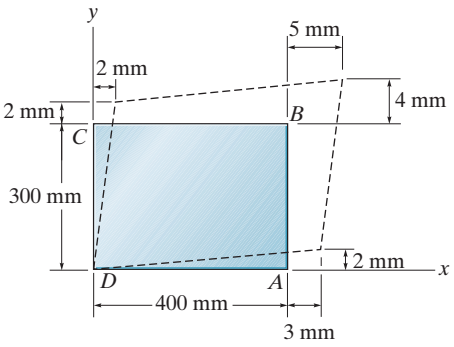
2–25.

Determine the average normal strain that occurs along the diagonals AC and DB.



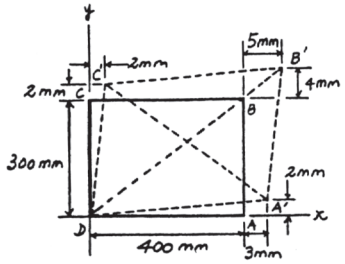
SOLUTION

Geometry:

$AC = DB = \sqrt{400^2 + 300^2} = 500\text{ mm}$

$DB' = \sqrt{405^2 + 304^2} = 506.4\text{ mm}$

$A'C' = \sqrt{401^2 + 300^2} = 500.8\text{ mm}$



Average Normal Strain:

$$\epsilon_{AC} = \frac{A'C' - AC}{AC} = \frac{500.8 - 500}{500}$$

$$= 0.00160\text{ mm/mm} = 1.60\left(10^{-3}\right)\text{ mm/mm} \qquad \textbf{Ans.}$$

$$\epsilon_{DB} = \frac{DB' - DB}{DB} = \frac{506.4 - 500}{500}$$

$$= 0.0128\text{ mm/mm} = 12.8\left(10^{-3}\right)\text{ mm/mm} \qquad \textbf{Ans.}$$

Ans:
$\epsilon_{AC} = 1.60\left(10^{-3}\right)\text{ mm/mm}$
$\epsilon_{DB} = 12.8\left(10^{-3}\right)\text{ mm/mm}$

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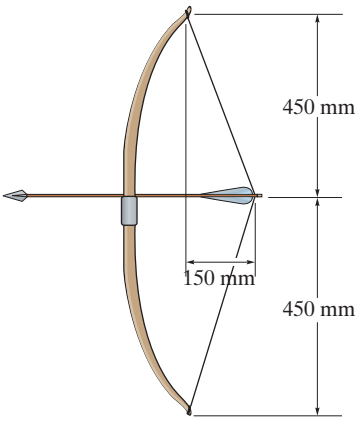
2–26.

If the unstreched length of the bowstring is 887.5 mm, determine the average normal strain in the string when it is streched to the position shown.

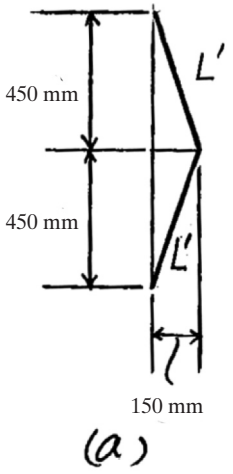


Geometry: Referring to Fig. *a*, the stretched length of the string is

$$L = 2L' = 2\sqrt{450^2 + 150^2} = 948.68 \text{ mm}$$

Average Normal Strain:

$$\varepsilon_{avg} = \frac{L - L_0}{L_0} = \frac{948.68 - 887.5}{887.5} = 0.0689 \text{ mm/mm}$$ **Ans.**



(a)

Ans:

$\varepsilon_{avg} = 0.0689$ mm/mm

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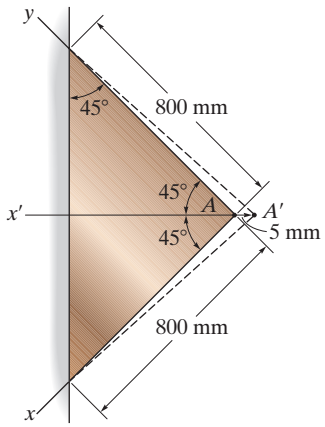
2–27.

The triangular plate is fixed at its base, and its apex A is given a horizontal displacement of 5 mm. Determine the shear strain, γ_{xy}, at A.



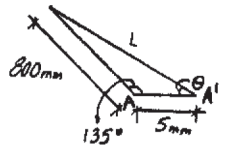
SOLUTION

$$L = \sqrt{800^2 + 5^2 - 2(800)(5)\cos 135°} = 803.54 \text{ mm}$$

$$\frac{\sin 135°}{803.54} = \frac{\sin\theta}{800}; \qquad \theta = 44.75° = 0.7810 \text{ rad}$$

$$\gamma_{xy} = \frac{\pi}{2} - 2\theta = \frac{\pi}{2} - 2(0.7810)$$

$$= 0.00880 \text{ rad} \qquad \textbf{Ans.}$$



Ans:
$\gamma_{xy} = 0.00880$ rad

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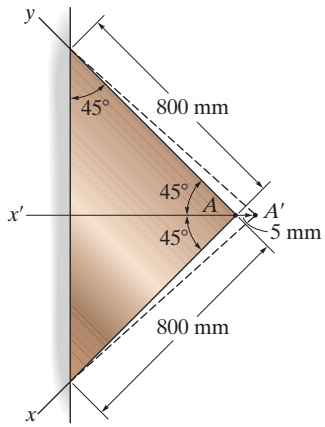
***2–28.**

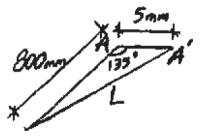
The triangular plate is fixed at its base, and its apex A is given a horizontal displacement of 5 mm. Determine the average normal strain ϵ_x along the x axis.



SOLUTION

$$L = \sqrt{800^2 + 5^2 - 2(800)(5)\cos 135°} = 803.54 \text{ mm}$$

$$\epsilon_x = \frac{803.54 - 800}{800} = 0.00443 \text{ mm/mm}$$ **Ans.**



Ans:
$\epsilon_x = 0.00443 \text{ mm/mm}$

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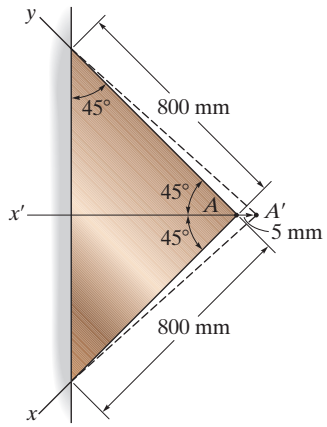
2–29.

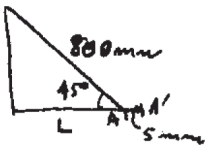
The triangular plate is fixed at its base, and its apex A is given a horizontal displacement of 5 mm. Determine the average normal strain $\epsilon_{x'}$ along the x' axis.



SOLUTION

$L = 800 \cos 45° = 565.69 \text{ mm}$

$$\epsilon_{x'} = \frac{5}{565.69} = 0.00884 \text{ mm/mm}$$ **Ans.**



Ans:
$\epsilon_{x'} = 0.00884 \text{ mm/mm}$

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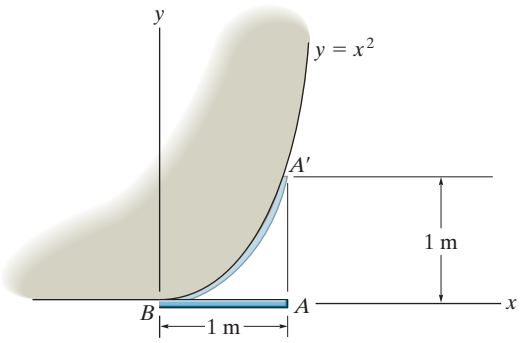
2–30.

The rubber band AB has an unstretched length of 1 m. If it is fixed at B and attached to the surface at point A', determine the average normal strain in the band.The surface is defined by the function $y = (x^2)$ m, where x is in meters.



SOLUTION

Geometry:

$$L = \int_0^{1\text{ m}} \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

However $y = x^2$ then $\dfrac{dy}{dx} = 2x$

$$L = \int_0^{1\text{ m}} \sqrt{1 + 4x^2}\, dx$$

$$= \frac{1}{4}\left[2x\sqrt{1 + 4x^2} + \ln\left(2x + \sqrt{1 + 4x^2}\right)\right]\Big|_0^{1\text{ m}}$$

$$= 1.47894 \text{ m}$$

Average Normal Strain:

$$\varepsilon_{avg} = \frac{L - L_0}{L_0} = \frac{1.47894 - 1}{1} = 0.479 \text{ m/m}$$ **Ans.**

Ans:

$\varepsilon_{avg} = 0.479$ m/m

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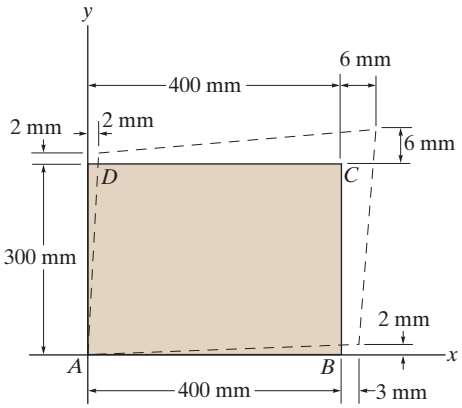
2–31.

The rectangular plate is deformed into the shape shown by the dashed lines. Determine the average normal strain along diagonal BD, and the average shear strain at corner B relative to the x, y axes.



SOLUTION

Geometry: The unstretched length of diagonal BD is

$$L_{BD} = \sqrt{300^2 + 400^2} = 500 \text{ mm}$$

Referring to Fig. a, the stretched length of diagonal BD is

$$L_{B'D'} = \sqrt{(300 + 2 - 2)^2 + (400 + 3 - 2)^2} = 500.8004 \text{ mm}$$

Referring to Fig. a and using small angle analysis,

$$\phi = \frac{2}{403} = 0.004963 \text{ rad}$$

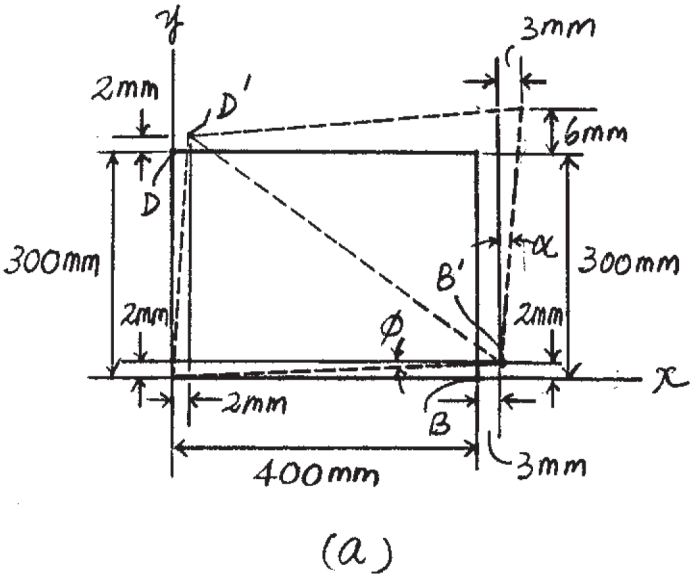
$$\alpha = \frac{3}{300 + 6 - 2} = 0.009868 \text{ rad}$$

Average Normal Strain: Applying Eq. 2,

$$(\epsilon_{\text{avg}})_{BD} = \frac{L_{B'D'} - L_{BD}}{L_{BD}} = \frac{500.8004 - 500}{500} = 1.60(10^{-3}) \text{ mm/mm} \quad \textbf{Ans.}$$

Shear Strain: Referring to Fig. a,

$$(\gamma_B)_{xy} = \phi + \alpha = 0.004963 + 0.009868 = 0.0148 \text{ rad} \quad \textbf{Ans.}$$



(a)

Ans:
$(\epsilon_{\text{avg}})_{BD} = 1.60(10^{-3})$ mm/mm,
$(\gamma_B)_{xy} = 0.0148$ rad

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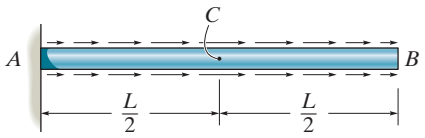
***2–32.**

The nonuniform loading causes a normal strain in the shaft that can be expressed as $\epsilon_x = k \sin\left(\frac{\pi}{L}x\right)$, where k is a constant. Determine the displacement of the center C and the average normal strain in the entire rod.



SOLUTION

$$\epsilon_x = k \sin\left(\frac{\pi}{L}x\right)$$

$$(\Delta x)_C = \int_0^{L/2} \epsilon_x dx = \int_0^{L/2} k \sin\left(\frac{\pi}{L}x\right) dx$$

$$= -k\left(\frac{L}{\pi}\right) \cos\left(\frac{\pi}{L}x\right)\Big|_0^{L/2} = -k\left(\frac{L}{\pi}\right)\left(\cos\frac{\pi}{2} - \cos 0\right)$$

$$= \frac{kL}{\pi} \qquad \textbf{Ans.}$$

$$(\Delta x)_B = \int_0^{L} k \sin\left(\frac{\pi}{L}x\right) dx$$

$$= -k\left(\frac{L}{\pi}\right) \cos\left(\frac{\pi}{L}x\right)\Big|_0^{L} = -k\left(\frac{L}{\pi}\right)(\cos\pi - \cos 0) = \frac{2kL}{\pi}$$

$$\epsilon_{\text{avg}} = \frac{(\Delta x)_B}{L} = \frac{2k}{\pi} \qquad \textbf{Ans.}$$

Ans:

$$(\Delta x)_C = \frac{kL}{\pi}$$

$$\epsilon_{\text{avg}} = \frac{2k}{\pi}$$

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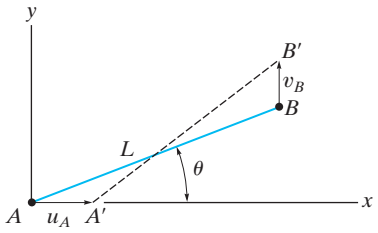
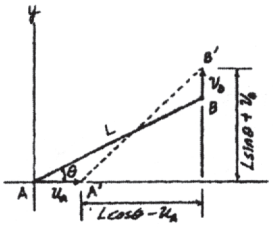
2–33.

The fiber AB has a length L and orientation θ. If its ends A and B undergo very small displacements u_A and v_B respectively, determine the normal strain in the fiber when it is in position $A'B'$.



SOLUTION



Geometry:

$$L_{A'B'} = \sqrt{(L\cos\theta - u_A)^2 + (L\sin\theta + v_B)^2}$$

$$= \sqrt{L^2 + u_A^2 + v_B^2 + 2L(v_B\sin\theta - u_A\cos\theta)}$$

Average Normal Strain:

$$\epsilon_{AB} = \frac{L_{A'B'} - L}{L}$$

$$= \sqrt{1 + \frac{u_A^2 + v_B^2}{L^2} + \frac{2(v_B\sin\theta - u_A\cos\theta)}{L}} - 1$$

Neglecting higher terms u_A^2 and v_B^2

$$\epsilon_{AB} = \left[1 + \frac{2(v_B\sin\theta - u_A\cos\theta)}{L}\right]^{\frac{1}{2}} - 1$$

Using the binomial theorem:

$$\epsilon_{AB} = 1 + \frac{1}{2}\left(\frac{2v_B\sin\theta}{L} - \frac{2u_A\cos\theta}{L}\right) + \ldots - 1$$

$$= \frac{v_B\sin\theta}{L} - \frac{u_A\cos\theta}{L}$$

Ans.

Ans:

$$\epsilon_{AB} = \frac{v_B\sin\theta}{L} - \frac{u_A\cos\theta}{L}$$

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2–34.

If the normal strain is defined in reference to the final length $\Delta s'$, that is,

$$\epsilon' = \lim_{\Delta s' \to 0} \left(\frac{\Delta s' - \Delta s}{\Delta s'} \right)$$

instead of in reference to the original length, Eq. 2–2, show that the difference in these strains is represented as a second-order term, namely, $\epsilon - \epsilon' = \epsilon\,\epsilon'$.

SOLUTION

$$\epsilon = \frac{\Delta s' - \Delta s}{\Delta s}$$

$$\epsilon - \epsilon' = \frac{\Delta s' - \Delta s}{\Delta s} - \frac{\Delta s' - \Delta s}{\Delta s'}$$

$$= \frac{\Delta s'^2 - \Delta s \Delta s' - \Delta s' \Delta s + \Delta s^2}{\Delta s \Delta s'}$$

$$= \frac{\Delta s'^2 + \Delta s^2 - 2\Delta s' \Delta s}{\Delta s \Delta s'}$$

$$= \frac{(\Delta s' - \Delta s)^2}{\Delta s \Delta s'} = \left(\frac{\Delta s' - \Delta s}{\Delta s} \right) \left(\frac{\Delta s' - \Delta s}{\Delta s'} \right)$$

$$= \epsilon\,\epsilon'$$ **(Q.E.D)**

Ans:
N/A

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3–1. A tension test was performed on a steel specimen having an original diameter of 12.5 mm and gauge length of 50 mm. The data is listed in the table. Plot the stress–strain diagram and determine approximately the modulus of elasticity, the yield stress, the ultimate stress, and the rupture stress. Use a scale of 25 mm = 140 MPa and 25 mm= 0.05 mm/mm. Redraw the elastic region, using the same stress scale but a strain scale of 25mm = 0.001 mm/mm.

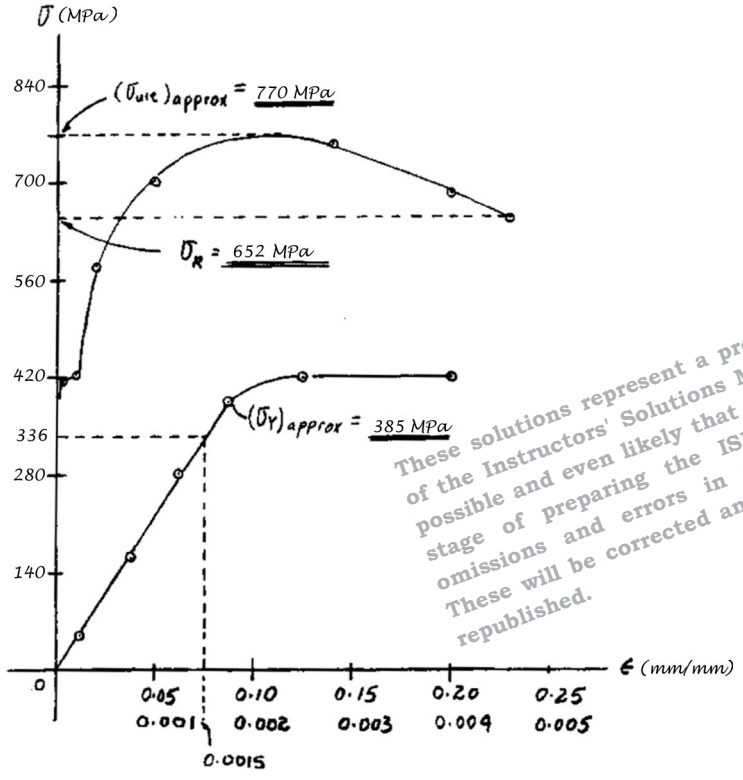
Load (kN)	Elongation (mm)
0	0
7.0	0.0125
21.0	0.0375
36.0	0.0625
50.0	0.0875
53.0	0.125
53.0	0.2
54.0	0.5
75.0	1.0
90.0	2.5
97.0	7.0
87.8	10.0
83.3	11.5

$$A = \frac{1}{4}\pi(0.0125^2) = 122.72(10^{-6})\ \text{m}^2$$

$L = 50$ mm.

σ(MPa)	ε(mm/mm)
0	0
57.07	0.00025
171.21	0.00075
293.51	0.00125
407.66	0.00175
432.12	0.0025
432.12	0.0040
440.27	0.010
611.49	0.020
733.79	0.050
790.86	0.140
715.85	0.200
679.16	0.230

$$E_{\text{approx}} = \frac{336(10^6)}{0.0015} = 224(10^9)\ \text{N/m}^2 = 224\ \text{GPa}$$ **Ans.**



These solutions represent a preliminary version of the Instructors' Solutions Manual (ISM). It is possible and even likely that at this preliminary stage of preparing the ISM there are some omissions and errors in the draft solutions. These will be corrected and this manual will be republished.

Ans:
$(\sigma_{\text{ult}})_{\text{approx}} = 770$ MPa, $(\sigma_R)_{\text{approx}} = 652$ MPa,
$(\sigma_Y)_{\text{approx}} = 385$ MPa, $E_{\text{approx}} = 224$ GPa

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3–2. Data taken from a stress–strain test for a ceramic are given in the table. The curve is linear between the origin and the first point. Plot the diagram, and determine the modulus of elasticity and the modulus of resilience.

σ (MPa)	ϵ (mm/mm)
0	0
232.4	0.0006
318.5	0.0010
345.8	0.0014
360.5	0.0018
373.8	0.0022

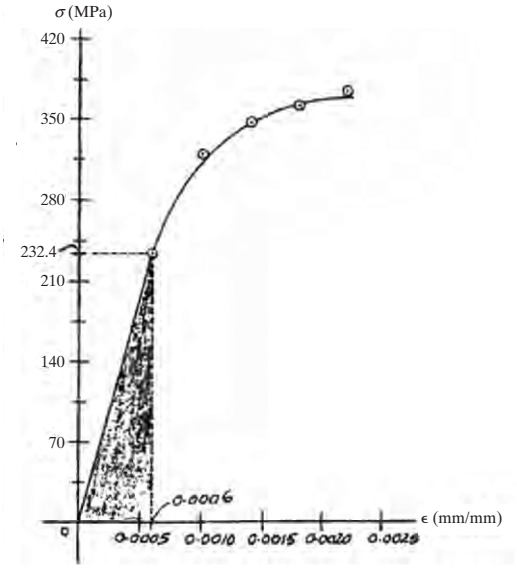
SOLUTION

Modulus of Elasticity: From the stress–strain diagram

$$E = \frac{232.4(10^6) - 0}{0.0006 - 0} = 387.33(10^9)\ \text{N/m}^2 = 387\ \text{GPa}$$ **Ans.**

Modulus of Resilience: The modulus of resilience is equal to the area under the *linear portion* of the stress–strain diagram (shown shaded).

$$(U_i)_r = \frac{1}{2}\left[232.4(10^6)\ \frac{\text{N}}{\text{m}^2}\right]\left(0.0006\ \frac{\text{m}}{\text{m}}\right) = 69.72(10^3)\ \frac{\text{N}\cdot\text{m}}{\text{m}^3} = 69.7\ \text{kJ/m}^3$$ **Ans.**



Ans:

$E = 387$ GPa, $u_r = 69.7\ \text{kJ/m}^3$

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3–3. Data taken from a stress–strain test for a ceramic are given in the table. The curve is linear between the origin and the first point. Plot the diagram, and determine approximately the modulus of toughness. The rupture stress is $\sigma_r = 373.8$ MPa

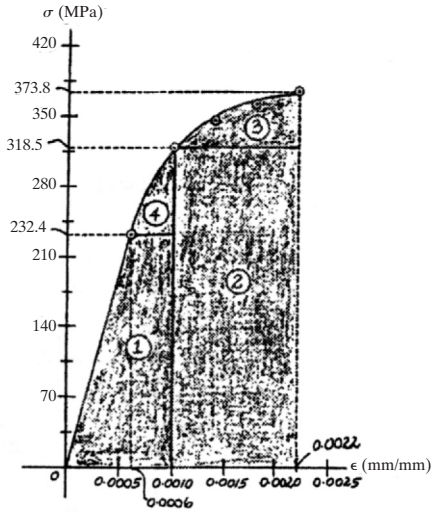
σ (MPa)	ϵ (mm/mm)
0	0
232.4	0.0006
318.5	0.0010
345.8	0.0014
360.5	0.0018
373.8	0.0022

SOLUTION

Modulus of Toughness: The modulus of toughness is equal to the area under the stress–strain diagram (shown shaded).

$$(U_i)_t = \frac{1}{2}\left[232.4(10^6)\ \frac{\text{N}}{\text{m}^2}\right]\left[(0.0004 + 0.0010)\ \frac{\text{m}}{\text{m}}\right] + \left[318.5(10^6)\ \frac{\text{N}}{\text{m}^2}\right]\left(0.0012\ \frac{\text{m}}{\text{m}}\right) + \frac{1}{2}\left[55.3(10^6)\ \frac{\text{N}}{\text{m}^2}\right]\left(0.0012\ \frac{\text{m}}{\text{m}}\right) + \frac{1}{2}\left[86.1(10^6)\ \frac{\text{N}}{\text{m}^2}\right]\left(0.0004\ \frac{\text{m}}{\text{m}}\right)$$

$$= 595.28(10^3)\ \frac{\text{N}\cdot\text{m}}{\text{m}^3} = 595\ \text{kJ/m}^3$$ **Ans.**



Ans:
$(u_i)_t = 595\ \text{kJ/m}^3$

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***3–4.**

The stress–strain diagram for a metal alloy having an original diameter of 12 mm and a gauge length of 50 mm is given in the figure. Determine approximately the modulus of elasticity for the material, the load on the specimen that causes yielding, and the ultimate load the specimen will support.

SOLUTION

From the stress–strain diagram, Fig. *a*,

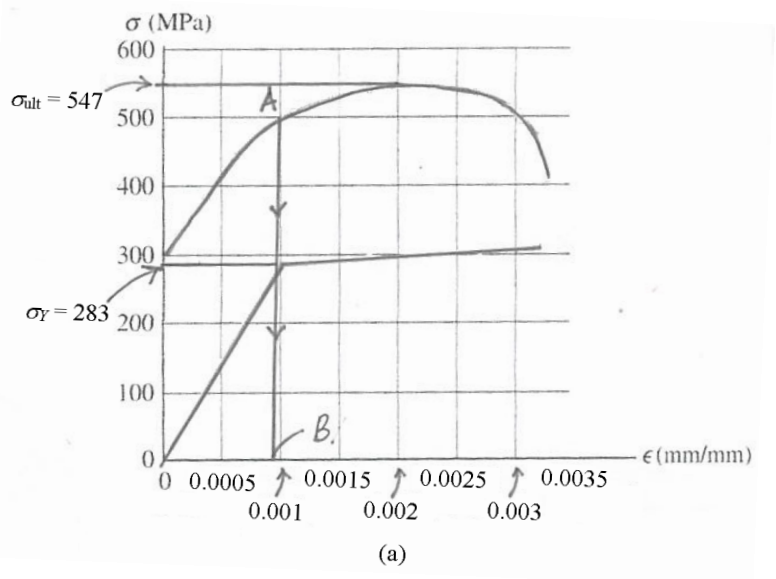
$$E = \frac{283(10^6)}{0.001} = 283(10^9)\ \text{N/m}^2 = 283\ \text{GPa} \qquad \textbf{Ans.}$$

$$\sigma_Y = 283\ \text{MPa} \qquad \sigma_{u/t} = 547\ \text{GPa}$$

Thus,

$$P_Y = \sigma_Y A = \left[283(10^6)\right]\left[\tfrac{\pi}{4}(0.012^2)\right] = 32.01(10^3)\ \text{N} = 32.0\ \text{kN} \qquad \textbf{Ans.}$$

$$P_{u/t} = \sigma_{u/t} A = \left[547(10^6)\right]\left[\tfrac{\pi}{4}(0.012^2)\right] = 61.86(10^3)\ \text{N} = 61.9\ \text{kN} \qquad \textbf{Ans.}$$



(a)

Ans:
$E = 283$ GPa, $P_Y = 32.0$ kN, $P_{ult} = 61.9$ kN

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3–5.

The stress–strain diagram for a steel alloy having an original diameter of 12 mm and a gauge length of 50 mm is given in the figure. If the specimen is loaded until it is stressed to 500 MPa, determine the approximate amount of elastic recovery and the increase in the gauge length after it is unloaded.

SOLUTION

From the stress–strain diagram Fig. *a*, the modulus of elasticity for the steel alloy is

$$E = \frac{283(10^6)}{0.001} = 283(10^9)\ \text{N/m}^2 = 283\ \text{GPa}$$

when the specimen is unloaded, its normal strain recovered along line AB, Fig. *a*, which has a gradient of E. Thus

$$\text{Elasticity Recovery} = \frac{\sigma}{E} = \frac{500(10^6)}{283(10^9)} = 0.001767\ \text{mm/mm}$$

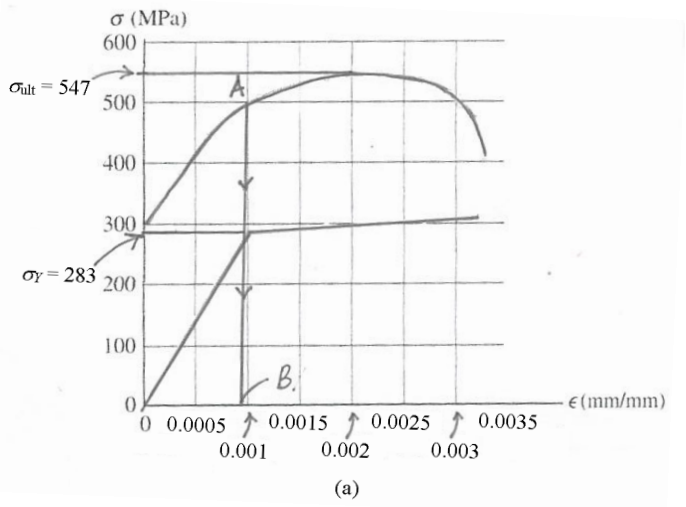
$$\text{Amount of Elastic Recovery} = (0.001767\ \text{mm/mm})(50\ \text{mm})$$
$$= 0.08834\ \text{mm} = 0.0883\ \text{mm} \qquad \textbf{Ans.}$$

Thus, the permanent set is

$$\epsilon_P = 0.08 - 0.001767 = 0.07823\ \text{mm/mm}$$

Then, the increase in gauge length is

$$\Delta L = \epsilon_P L = 0.07823(50\ \text{mm}) = 3.9137\ \text{mm} = 3.91\ \text{mm} \qquad \textbf{Ans.}$$



(a)

Ans:
Elastic Recovery = 0.0883 mm
ΔL = 3.91 mm

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3-6.

The stress–strain diagram for a steel alloy having an original diameter of 12 mm and a gauge length of 50 mm is given in the figure. Determine approximately the modulus of resilience and the modulus of toughness for the material.

SOLUTION

The Modulus of resilience is equal to the area under the stress–strain diagram up to the proportional limit.

$$\sigma_{PL} = 283 \text{ MPa} \qquad \epsilon_{PL} = 0.001 \text{ m/m}$$

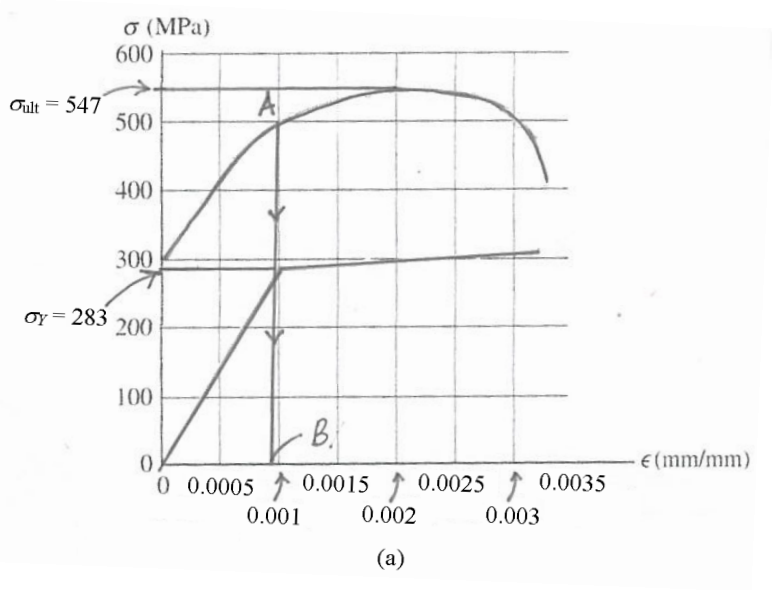
Thus,

$$(U_i)_r = \frac{1}{2}\sigma_{PL}\epsilon_{PL} = \frac{1}{2}\left[283(10^6)\ \frac{\text{N}}{\text{m}^2}\right]\left(0.001\ \frac{\text{m}}{\text{m}}\right) = 141.5(10^3)\ \frac{\text{N}\cdot\text{m}}{\text{m}^3}$$

$$= 141.5 \text{ kJ/m}^3 \qquad \textbf{Ans.}$$

The modulus of toughness is equal to the area under the entire stress–strain diagram. This area can be approximated by counting the number of squares. The total number is 32. Thus,

$$\left[(U_i)_{ult}\right]_{aprox} = 32\left[100(10^6)\ \frac{\text{N}}{\text{m}^2}\right]\left(0.04\ \frac{\text{m}}{\text{m}}\right) = 128(10^6)\ \frac{\text{N}\cdot\text{m}}{\text{m}^3} = 128 \text{ MJ/m}^3 \qquad \textbf{Ans.}$$



(a)

Ans:

$(u_i)_r = 141.5 \text{ kJ/m}^3$, $[(U_i)_{ult}]_{\text{approx}} = 128 \text{ MJ/m}^3$

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3–7. A specimen is originally 300 mm long, has a diameter of 12 mm, and is subjected to a force of 2.5 kN. When the force is increased from 2.5 kN to 9 kN,, the specimen elongates 0.225 mm. Determine the modulus of elasticity for the material if it remains linear elastic.

SOLUTION

Normal Stress and Strain: Applying $\sigma = \frac{P}{A}$ and $\varepsilon = \frac{\delta L}{L}$.

$$\sigma_1 = \frac{2.5(10^3)}{\frac{\pi}{4}(0.012^2)} = 22.10(10^6)\ \text{N/m}^2 = 22.10\ \text{MPa}$$

$$\sigma_2 = \frac{9(10^3)}{\frac{\pi}{4}(0.012^2)} = 79.58(10^6)\ \text{N/m}^2 = 79.58\ \text{MPa}$$

$$\Delta\varepsilon = \frac{0.225}{300} = 0.000750\ \text{mm/mm}$$

Modulus of Elasticity:

$$E = \frac{\Delta\sigma}{\Delta\varepsilon} = \frac{(79.58 - 22.10)(10^6)}{0.000750} = 76.63(10^9)\ \text{Pa} = 76.6\ \text{GPa} \qquad \textbf{Ans.}$$

Ans:

$E = 76.6$ GPa

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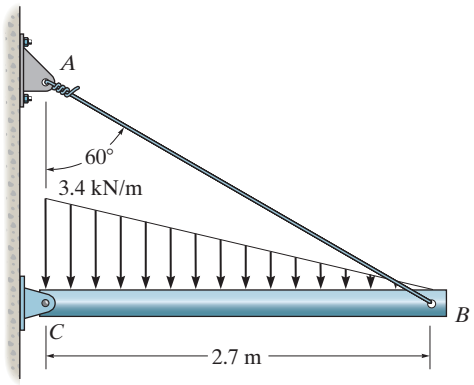
***3–8.** The strut is supported by a pin at C and an A-36 steel guy wire AB. If the wire has a diameter of 5 mm, determine how much it stretches when the distributed load acts on the strut.



SOLUTION

Here, we are only interested in determining the force in wire AB.

$$\zeta + \Sigma M_C = 0; \qquad F_{AB}\cos 60°(2.7) - \frac{1}{2}(3.4)(2.7)(0.9) = 0 \quad F_{AB} = 3.06 \text{ kN}$$

The normal stress the wire is

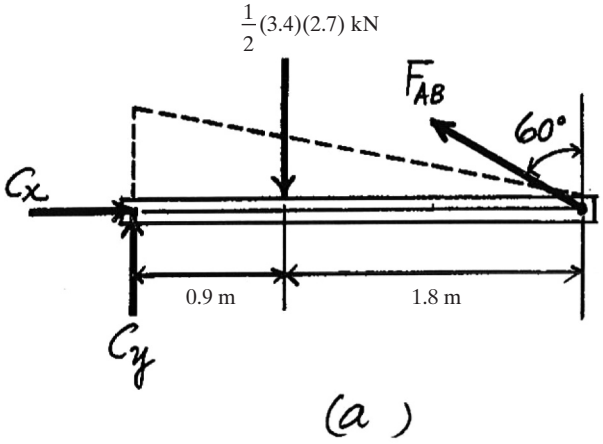
$$\sigma_{AB} = \frac{F_{AB}}{A_{AB}} = \frac{3.06(10^3)}{\frac{\pi}{4}(0.005^2)} = 155.84(10^6) \text{ N/m}^2 = 155.84 \text{ MPa}$$

Since $\sigma_{AB} < \sigma_Y = 250$ MPa, Hooke's Law can be applied to determine the strain in wire.

$$\sigma_{AB} = E\epsilon_{AB}; \qquad 155.84(10^6) = 200(10^9)\epsilon_{AB}$$
$$\epsilon_{AB} = 0.7792(10^{-3}) \text{ mm/mm}$$

The unstretched length of the wire is $L_{AB} = \dfrac{2.7(10^3)}{\sin 60°} = 3117.69$ mm. Thus, the wire stretches

$$\delta_{AB} = \epsilon_{AB} L_{AB} = 0.7792(10^{-3})(3117.69)$$
$$= 2.429 \text{ mm} = 2.43 \text{ mm} \qquad \textbf{Ans.}$$



Ans:
$\delta_{AB} = 2.43$ mm

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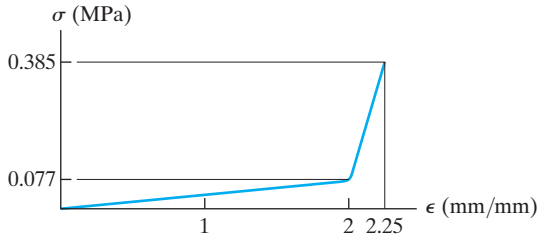
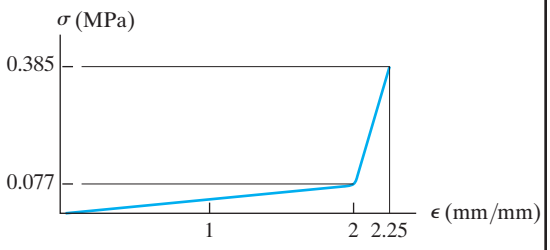
3–9. The $\sigma-\epsilon$ diagram for elastic fibers that make up human skin and muscle is shown. Determine the modulus of elasticity of the fibers and estimate their modulus of toughness and modulus of resilience.



SOLUTION



$$E = \frac{0.077}{2} = 0.0385 \text{ MPa}$$ **Ans.**

$$(U_i)_r = \frac{1}{2}\left[0.077(10^6)\ \text{N/m}^2\right](2\ \text{m/m})$$

$$= 77.0(10^3)\ \text{N}\cdot\text{m/m}^3 = 77.0\ \text{kJ/m}^3$$ **Ans.**

$$(U_i)_t = \frac{1}{2}\left[0.077(10^6)\ \text{N/m}^2\right](2\ \text{m/m})$$

$$+\frac{1}{2}\left\{\left[0.077(10^6) + 0.385(10^6)\right]\ \text{N/m}^2\right\}\left[(2.25-2)\ \text{m/m}\right]$$

$$= 134.75\ \text{N}\cdot\text{m/m}^3 = 135\ \text{kJ/m}^3$$ **Ans.**

Ans:

$E = 0.0385$ MPa, $(u_i)r = 77.0$ kJ/m^3,
$(u_i)t = 135$ kJ/m^3

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3–10.

A structural member in a nuclear reactor is made of a zirconium alloy. If an axial load of 20 kN is to be supported by the member, determine its required cross-sectional area. Use a factor of safety of 3 relative to yielding. What is the load on the member if it is 1 m long and its elongation is 0.5 mm? $E_{zr} = 100$ GPa, $\sigma_Y = 400$ MPa. The material has elastic behavior.

SOLUTION

Allowable Normal Stress:

$$\text{F.S.} = \frac{\sigma_y}{\sigma_{\text{allow}}}$$

$$3 = \frac{400}{\sigma_{\text{allow}}}$$

$$\sigma_{\text{allow}} = 133.33 \text{ MPa}$$

$$\sigma_{\text{allow}} = \frac{P}{A}$$

$$133.33 = \frac{20(10^3)}{A}$$

$$A = 150 \text{ mm}^2 \qquad \textbf{Ans.}$$

Stress–Strain Relationship: Applying Hooke's law with

$$\varepsilon = \frac{\delta}{L} = \frac{0.5}{1(10^3)} = 0.0005 \text{ mm/mm}$$

$$\sigma = E\varepsilon = 100(10^3)(0.0005) = 50 \text{ MPa}$$

Normal Force: Applying equation $\sigma = \frac{P}{A}$.

$$P = \sigma A = 50(150) = 7500 \text{ N} = 7.5 \text{ kN} \qquad \textbf{Ans.}$$

Ans:

$A = 150 \text{ mm}^2, P = 7.5 \text{ kN}$

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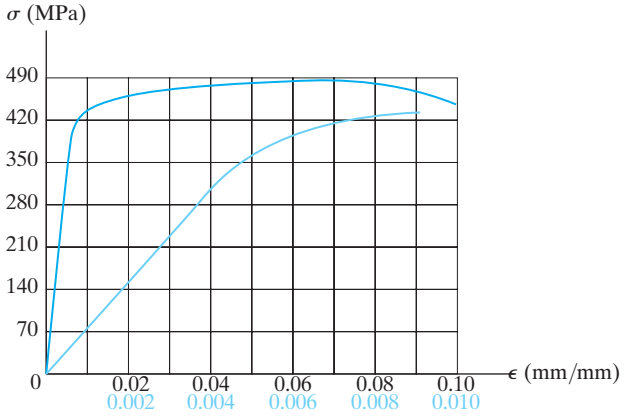
3–11.

A tension test was performed on an aluminum 2014-T6 alloy specimen. The resulting stress–strain diagram is shown in the figure. Estimate (a) the proportional limit,(b) the modulus of elasticity, and (c) the yield strength based on a 0.2% strain offset method.



SOLUTION

Proportional Limit and Yield Strength: From the stress–strain diagram, Fig. *a*,

$$\sigma_{pl} = 308 \text{ MPa} \qquad \textbf{Ans.}$$

$$\sigma_Y = 420 \text{ MPa} \qquad \textbf{Ans.}$$

Modulus of Elasticity: From the stress–strain diagram, the corresponding strain for $\sigma_{PL} = 308$ MPa is $\varepsilon_{pl} = 0.004$ mm/mm. Thus,

$$E = \frac{308 - 0}{0.004 - 0} = 77.0(10^3) \text{ MPa} = 77.0 \text{ GPa} \qquad \textbf{Ans.}$$

Ans:
$\sigma_{pl} = 308$ MPa
$\sigma_Y = 420$ MPa
$E = 77.0$ GPa

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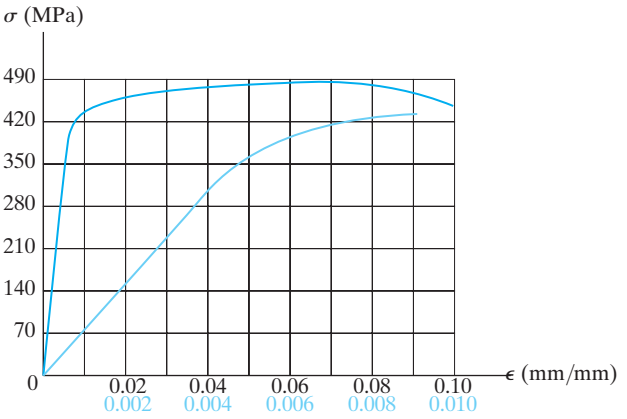
***3–12.**

A tension test was performed on an aluminum 2014-T6 alloy specimen. The resulting stress–strain diagram is shown in the figure. Estimate (a) the modulus of resilience; and (b) modulus of toughness.



SOLUTION

Modulus of Resilence The modulus of resilence is equal to the area under the stress–strain diagram up to the propotional limit. From the stress–strain diagram,

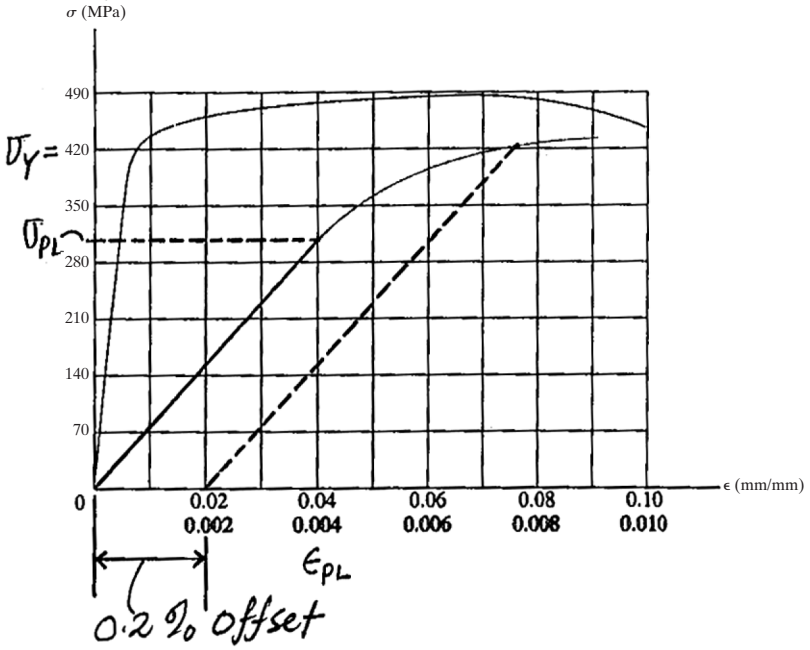
$$\sigma_{pl} = 308 \text{ MPa} \qquad \varepsilon_{pl} = 0.004 \text{ mm/mm}$$

Thus,

$$(U_i)_r = \frac{1}{2}\sigma_{pl}\varepsilon_{pl} = \frac{1}{2}[308(10^6)](0.004) = 0.616 \text{ MJ/m}^3 \qquad \textbf{Ans.}$$

Modulus of Toughness: The modulus of toughness is equal to the area under the entire stress–strain diagram. This area can be approximated by counting the number of squares. The total number of squares is 65. Thus,

$$[(U_i)_t]_{\text{approx}} = 65[70(10^6)](0.01) = 45.5 \text{ MJ/m}^3 \qquad \textbf{Ans.}$$



Ans:

$(U_i)_r = 0.616 \text{ MJ/m}^3$

$[(U_i)_t]_{\text{approx}} = 45.5 \text{ MJ/m}^3$

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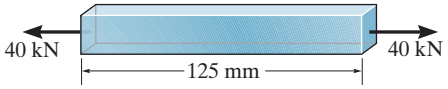
3.13.



A bar having a length of 125 mm and cross-sectional area of 437.5 mm^2 is subjected to an axial force of 40 kN. If the bar streches 0.05 mm, determine the modulus of elasticity of the material. The material has linear-elastic behavior.

SOLUTION

Normal Stress and Strain:

$$\sigma = \frac{P}{A} = \frac{40(10^3)}{437.5(10^{-6})} = 91.43(10^6)\ \text{N/m}^2 = 91.43\ \text{MPa}$$

$$\varepsilon = \frac{\delta L}{L} = \frac{0.05}{125} = 0.000400\ \text{mm/mm}$$

Modulus of Elasticity:

$$E = \frac{\sigma}{\varepsilon} = \frac{91.43(10^6)}{0.000400} = 228.57(10^9)\ \text{N/m}^2 = 229\ \text{GPa}$$ **Ans.**

Ans:

$E = 229$ GPa

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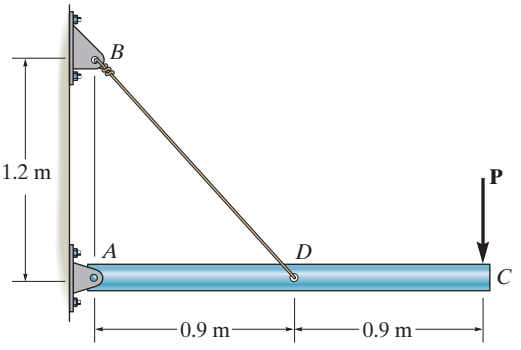
3–14. The rigid pipe is supported by a pin at A and an A-36 steel guy wire BD. If the wire has a diameter of 6.5 mm, determine how much it streches when a load of $P = 3$ kN acts on the pipe.



SOLUTION

Here, we are only interested in determining the force in wire BD. Referring to the FBD in Fig. *a*

$$\zeta + \Sigma M_A = 0; \qquad F_{BD}\left(\tfrac{4}{5}\right)(0.9) - 3(1.8) = 0 \qquad F_{BD} = 7.50 \text{ kN}$$

The normal stress developed in the wire is

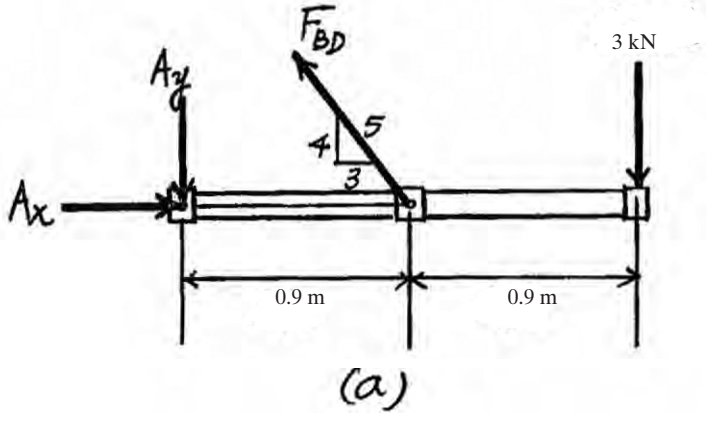
$$\sigma_{BD} = \frac{F_{BD}}{A_{BD}} = \frac{7.50(10^3)}{\frac{\pi}{4}(0.0065^2)} = 226.02(10^6) \text{ N/m}^2 = 226.02 \text{ MPa}$$

Since $\sigma_{BD} < \sigma_y = 250$ MPa, Hooke's Law can be applied to determine the strain in the wire.

$$\sigma_{BD} = E\epsilon_{BD}; \qquad 226.02(10^6) = 200(10^9)\,\epsilon_{AB}$$
$$\epsilon_{BD} = 1.1301(10^{-3}) \text{ mm/mm}$$

The unstretched length of the wire is $L_{BD} = \sqrt{0.9^2 + 1.2^2} = 1.5$ m. Thus, the wire stretches

$$\delta_{BD} = \epsilon_{BD} L_{BD} = 1.1301(10^{-3})[1.5(10^3)]$$
$$= 1.6951 \text{ mm} = 1.70 \text{ mm} \qquad \textbf{Ans.}$$



Ans:

$\delta_{BD} = 1.70$ mm

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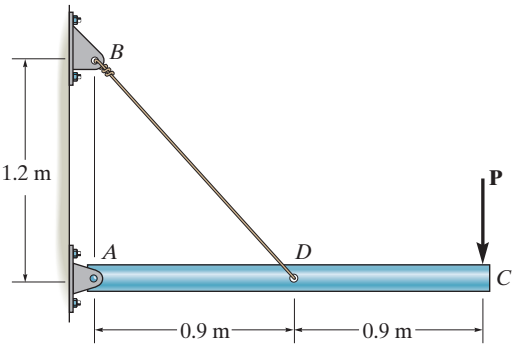
3–15. The rigid pipe is supported by a pin at A and an A-36 guy wire BD. If the wire has a diameter of 6 mm, determine the load P if the end C is displaced 1.875 mm downward.



SOLUTION

Here, we are only interested in determining the force in wire BD. Referring to the FBD in Fig. *a*

$$\zeta + \Sigma M_A = 0; \qquad F_{BD}\left(\tfrac{4}{5}\right)(0.9) - P(1.8) = 0 \qquad F_{BD} = 2.50\,P$$

The unstretched length for wire BD is $L_{BD} = \sqrt{0.9^2 + 1.2^2} = 1.5$ m. From the geometry shown in Fig. *b*, the stretched length of wire BD is

$$L_{BD'} = \sqrt{1500^2 + 1.675^2 - 2(1500)(1.675)\cos 143.13^\circ} = 1501.3403 \text{ mm}$$

Thus, the normal strain is

$$\epsilon_{BD} = \frac{L_{BD'} - L_{BD}}{L_{BD}} = \frac{1501.3403 - 1500}{1500} = 0.8936(10^{-3}) \text{ mm/mm}$$

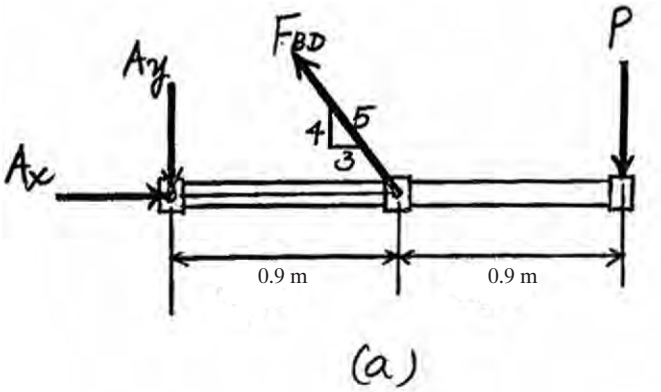
Then, the normal stress can be obtain by applying Hooke's Law.

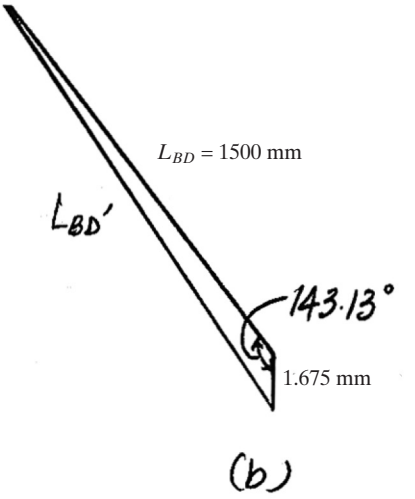
$$\sigma_{BD} = E\epsilon_{BD} = 200(10^9)[0.8936(10^{-3})] = 178.71(10^6) \text{ N/m}^2 = 178.71 \text{ MPa}$$

Since $\sigma_{BD} < \sigma_y = 250$ MPa, the result is valid.

$$\sigma_{BD} = \frac{F_{BD}}{A_{BD}}; \qquad 178.71(10^6) = \frac{2.50P}{\frac{\pi}{4}(0.0065^2)}$$

$$P = 2.3721(10^3) \text{ N} = 2.37 \text{ kN} \qquad \textbf{Ans.}$$





Ans:
$P = 2.37$ kN

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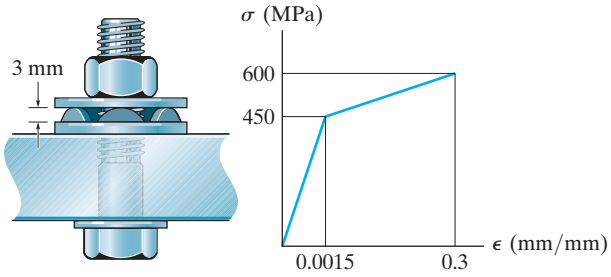
***3–16.**

Direct tension indicators are sometimes used instead of torque wrenches to ensure that a bolt has a prescribed tension when used for connections. If a nut on the bolt is tightened so that the six 3-mm high heads of the indicator are strained 0.1 mm/mm, and leave a contact area on each head of 1.5 mm^2, determine the tension in the bolt shank. The material has the stress–strain diagram shown.



SOLUTION

Stress–Strain Relationship: From the stress–strain diagram with

$\epsilon = 0.1 \text{ mm/mm} > 0.0015 \text{ mm/mm}$

$$\frac{\sigma - 450}{0.1 - 0.0015} = \frac{600 - 450}{0.3 - 0.0015}$$

$$\sigma = 499.497 \text{ MPa}$$

Axial Force: For each head

$$P = \sigma A = 499.4971(10^6)(1.5)(10^{-6}) = 749.24 \text{ N}$$

Thus, the tension in the bolt is

$$T = 6P = 6(749.24) = 4495 \text{ N} = 4.50 \text{ kN}$$ **Ans.**

Ans:
$T = 4.50$ kN

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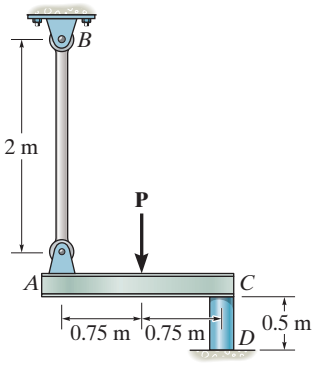
3–17.

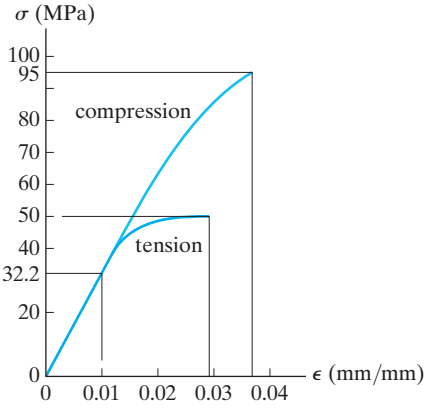
The stress–strain diagram for a polyester resin is given in the figure. If the rigid beam is supported by a strut *AB* and post *CD*, both made from this material, and subjected to a load of $P = 80$ kN, determine the angle of tilt of the beam when the load is applied. The diameter of the strut is 40 mm and the diameter of the post is 80 mm.



SOLUTION

From the stress–strain diagram,

$$E = \frac{32.2(10)^6}{0.01} = 3.22(10^9)\ \text{Pa}$$



Thus,

$$\sigma_{AB} = \frac{F_{AB}}{A_{AB}} = \frac{40(10^3)}{\frac{\pi}{4}(0.04)^2} = 31.83\ \text{MPa}$$

$$\varepsilon_{AB} = \frac{\sigma_{AB}}{E} = \frac{31.83(10^6)}{3.22(10^9)} = 0.009885\ \text{mm/mm}$$

$$\sigma_{CD} = \frac{F_{CD}}{A_{CD}} = \frac{40(10^3)}{\frac{\pi}{4}(0.08)^2} = 7.958\ \text{MPa}$$

$$\varepsilon_{CD} = \frac{\sigma_{CD}}{E} = \frac{7.958(10^6)}{3.22(10^9)} = 0.002471\ \text{mm/mm}$$

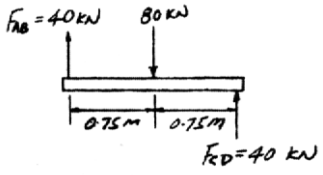
$$\delta_{AB} = \varepsilon_{AB} L_{AB} = 0.009885(2000) = 19.771\ \text{mm}$$

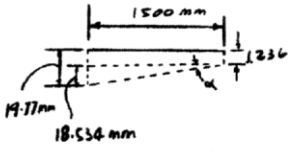
$$\delta_{CD} = \varepsilon_{CD} L_{CD} = 0.002471(500) = 1.236\ \text{mm}$$

Angle of tilt α:

$$\tan\alpha = \frac{18.535}{1500};\qquad \alpha = 0.708^\circ$$

Ans.





Ans.

$\alpha = 0.708^\circ$

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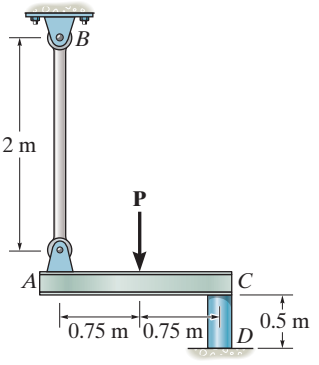
3–18.

The stress–strain diagram for a polyester resin is given in the figure. If the rigid beam is supported by a strut AB and post CD made from this material, determine the largest load P that can be applied to the beam before it ruptures. The diameter of the strut is 12 mm and the diameter of the post is 40 mm.



SOLUTION

Rupture of strut AB:

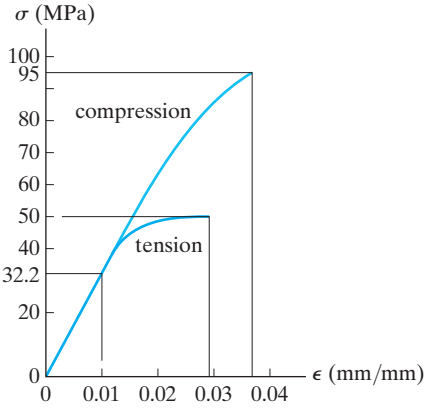
$$\sigma_R = \frac{F_{AB}}{A_{AB}}; \qquad 50(10^6) = \frac{P/2}{\frac{\pi}{4}(0.012)^2};$$

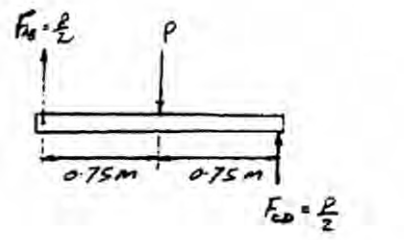
$$P = 11.3 \text{ kN (controls)}$$ **Ans.**

Rupture of post CD:

$$\sigma_R = \frac{F_{CD}}{A_{CD}}; \qquad 95(10^6) = \frac{P/2}{\frac{\pi}{4}(0.04)^2}$$

$$P = 239 \text{ kN}$$





Ans.

$P = 11.3$ kN (controls)

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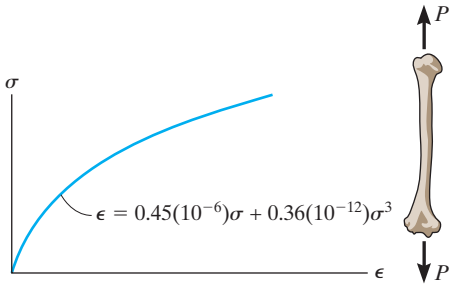
3–19.

The stress–strain diagram for a bone is shown, and can be described by the equation $\epsilon = 0.45(10^{-6})\,\sigma + 0.36(10^{-12})\,\sigma^3$, where σ is in kPa. Determine the yield strength assuming a 0.3% offset.

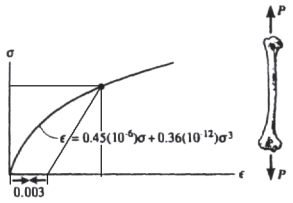


SOLUTION

$$\epsilon = 0.45(10^{-6})\sigma + 0.36(10^{-12})\sigma^3,$$

$$d\epsilon = \left(0.45(10^{-6}) + 1.08(10^{-12})\,\sigma^2\right)d\sigma$$

$$E = \left.\frac{d\sigma}{d\epsilon}\right|_{\sigma=0} = \frac{1}{0.45(10^{-6})} = 2.22(10^6)\text{ kPa} = 2.22\text{ GPa}$$



The equation for the recovery line is $\sigma = 2.22(10^6)(\epsilon - 0.003)$.

This line intersects the stress–strain curve at $\sigma_{YS} = 2027\text{ kPa} = 2.03\text{ MPa}$ **Ans.**

Ans:
$\sigma_{YS} = 2.03\text{ MPa}$

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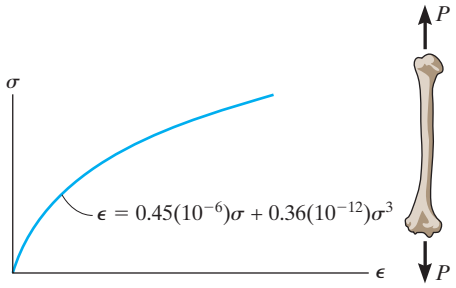
***3–20.**

The stress–strain diagram for a bone is shown and can be described by the equation $\epsilon = 0.45(10^{-6})\,\sigma + 0.36(10^{-12})\,\sigma^3$, where σ is in kPa. Determine the modulus of toughness and the amount of elongation of a 200-mm-long region just before it fractures if failure occurs at $\epsilon = 0.12$ mm/mm.



SOLUTION

When $\epsilon = 0.12$

$$120(10^{-3}) = 0.45\,\sigma + 0.36(10^{-6})\sigma^3$$

Solving for the real root:

$$\sigma = 6873.52\ \text{kPa}$$

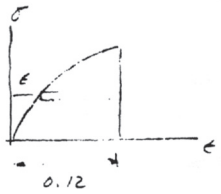
$$u_t = \int_A dA = \int_0^{6873.52} (0.12 - \epsilon)\,d\sigma$$

$$u_t = \int_0^{6873.52} (0.12 - 0.45(10^{-6})\sigma - 0.36(10^{-12})\sigma^3)\,d\sigma$$

$$= 0.12\,\sigma - 0.225(10^{-6})\sigma^2 - 0.09(10^{-12})\sigma^4\Big|_0^{6873.52}$$

$$= 613\ \text{kJ/m}^3 \qquad \textbf{Ans.}$$

$$\delta = \epsilon L = 0.12(200) = 24\ \text{mm} \qquad \textbf{Ans.}$$



Ans:
$u_t = 613\ \text{kJ/m}^3$,
$\delta = 24\ \text{mm}$

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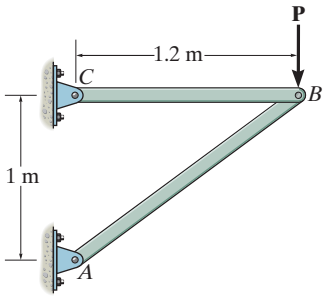
3–21. The two bars are made of polystyrene, which has the stress–strain diagram shown. If the cross-sectional area of bar AB is 975 mm^2 and BC is 2600 mm^2, determine the largest force P that can be supported before any member ruptures. Assume that buckling does not occur.



SOLUTION

$+\uparrow \Sigma F_y = 0; \quad \frac{5}{\sqrt{61}} F_{AB} - P = 0; \qquad F_{AB} = \frac{\sqrt{61}}{5} P \quad \text{(C)} \quad (1)$

$\overset{+}{\leftarrow} \Sigma F_x = 0; \quad F_{BC} = \left(\frac{\sqrt{61}}{5} P\right)\left(\frac{6}{\sqrt{61}}\right) = 0 \qquad F_{BC} = 1.20P \quad \text{(T)} \quad (2)$

Assuming failure of bar BC:

From the stress–strain diagram $(\sigma_R)_t = 35$ MPa

$$\sigma = \frac{F_{BC}}{A_{BC}}; \qquad 35(10^6) = \frac{F_{BC}}{2600(10^{-6})}; \qquad F_{BC} = 91.0(10^3)\ \text{N} = 91.0\ \text{kN}$$

From Eq. (2), $P = 75.83$ kN

Assuming failure of bar AB:

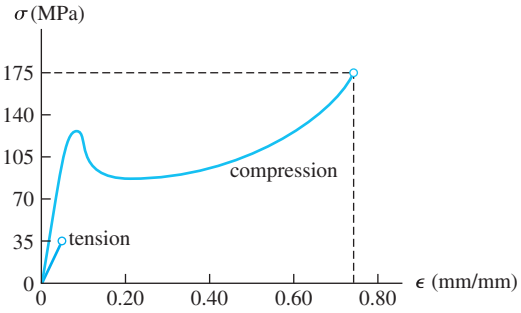
From stress–strain diagram $(\sigma_R)_c = 175$ MPa

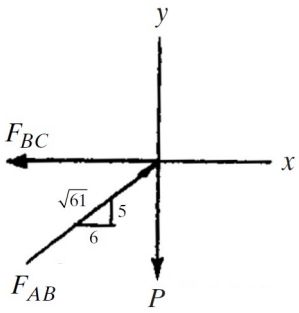
$$\sigma = \frac{F_{AB}}{A_{AB}}; \qquad 175(10^6) = \frac{F_{AB}}{975(10^{-6})}; \qquad F_{AB} = 170.625(10^3)\ \text{N} = 170.625\ \text{kN}$$

From Eq. (1), $P = 109.23$ kN

Choose the smallest value

$$P = 75.8\ \text{kN} \qquad \textbf{Ans.}$$





Ans:
$P = 75.8$ kN

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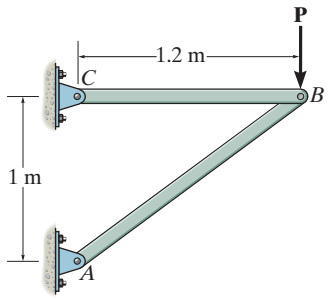
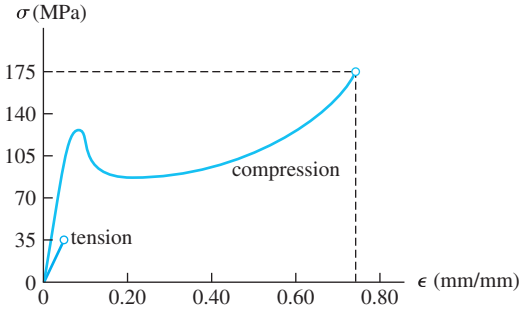
3–22. The two bars are made of polystyrene, which has the stress–strain diagram shown. Determine the cross-sectional area of each bar so that the bars rupture simultaneously when the load $P = 13.5$ kN. Assume that buckling does not occur.



SOLUTION



$$+\uparrow \Sigma F_y = 0; \qquad F_{AB} = \left(\frac{5}{\sqrt{61}}\right) - 13.5 = 0 \qquad F_{AB} = 21.09 \text{ kN}$$

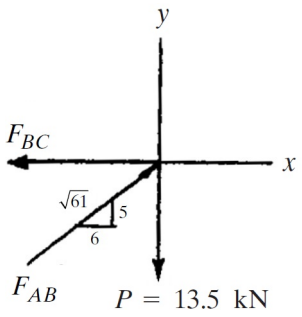
$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad -F_{BC} + 21.06\left(\frac{6}{\sqrt{61}}\right) = 0 \qquad F_{BC} = 16.2 \text{ kN}$$

For member BC:

$$(\sigma_{\max})_t = \frac{F_{BC}}{A_{BC}}; \qquad 35(10^6) = \frac{16.2(10^3)}{A_{BC}};$$

$$A_{BC} = 0.4629(10^{-3}) \text{ m}^2 = 462.9 \text{ mm}^2 = 463 \text{ mm}^2 \qquad \textbf{Ans.}$$

For member BA:



$$(\sigma_{\max})_c = \frac{F_{BA}}{A_{BA}}; \qquad 175(10^6) = \frac{21.09(10^3)}{A_{AB}};$$

$$A_{AB} = 0.1205(10^{-3}) \text{ m}^2 = 120.5 \text{ mm}^2 = 121 \text{ mm}^2 \qquad \textbf{Ans.}$$

Ans:
$A_{BC} = 463 \text{ mm}^2, A_{BA} = 121 \text{ mm}^2$

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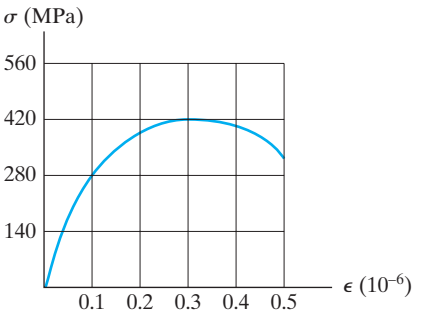
3–23. The stress–strain diagram for many metal alloys can be described analytically using the Ramberg-Osgood three parameter equation $\epsilon = \sigma/E + k\sigma^n$, where E, k, and n are determined from measurements taken from the diagram. Using the stress–strain diagram shown in the figure, take $E = 210$ GPa and determine the other two parameters k and n and thereby obtain an analytical expression for the curve.



SOLUTION

Choose,

$$\sigma = 280 \text{ MPa}, \quad \varepsilon = 0.1$$

$$\sigma = 420 \text{ MPa}, \quad \varepsilon = 0.3$$

$$0.1 = \frac{280}{210(10^3)} + k(280)^n$$

$$0.3 = \frac{420}{210(10^3)} + k(420)^n$$

$$0.098667 = k(280)^n$$

$$0.29800 = k(420)^n$$

$$0.3310962 = (0.6667)^n$$

$$\ln(0.3310962) = n \ln(0.6667)$$

$$n = 2.73 \quad \textbf{Ans.}$$

$$k = 21.0(10^{-9}) \quad \textbf{Ans.}$$

Ans:

$n = 2.73$

$k = 21.0(10^{-9})$

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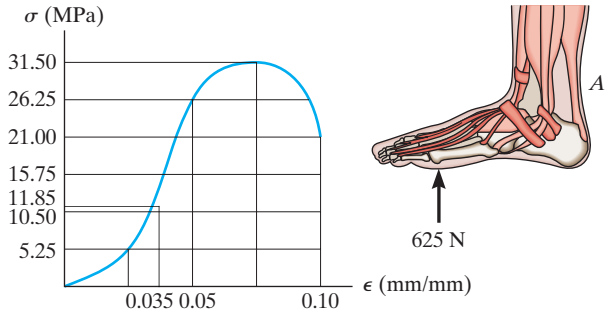
***3–24.** The $\sigma-\epsilon$ diagram for a collagen fiber bundle from which a human tendon is composed is shown. If a segment of the Achilles tendon at A has a length of 165 mm and an approximate cross-sectional area of 145 mm^2, determine its elongation if the foot supports a load of 625 N, which causes a tension in the tendon of 1718.75 N.



SOLUTION

$$\sigma = \frac{P}{A} = \frac{1718.75}{145(10^{-6})} = 11.85 \text{ MPa}$$

From the graph $\varepsilon = 0.035$ mm/mm

$$\delta = \varepsilon L = 0.035(165) = 5.775 \text{ mm}$$ **Ans.**

Ans:
$\delta = 5.775$ mm

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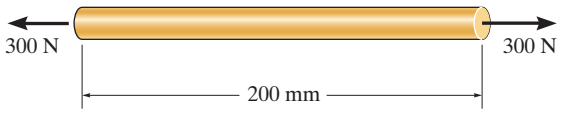
3–25.

The acrylic plastic rod is 200 mm long and 15 mm in diameter. If an axial load of 300 N is applied to it, determine the change in its length and the change in its diameter. $E_p = 2.70$ GPa, $\nu_p = 0.4$.



SOLUTION

$$\sigma = \frac{P}{A} = \frac{300}{\frac{\pi}{4}(0.015)^2} = 1.678 \text{ MPa}$$

$$\epsilon_{long} = \frac{\sigma}{E} = \frac{1.678(10^6)}{2.70(10^9)} = 0.0006288$$

$$\delta = \epsilon_{long} L = 0.0006288(200) = 0.126 \text{ mm}$$ **Ans.**

$$\epsilon_{lat} = -\nu\epsilon_{long} = -0.4(0.0006288) = -0.0002515$$

$$\Delta d = \epsilon_{lat} d = -0.0002515(15) = -0.00377 \text{ mm}$$ **Ans.**

Ans:
$\delta = 0.126$ mm, $\Delta d = -0.00377$ mm

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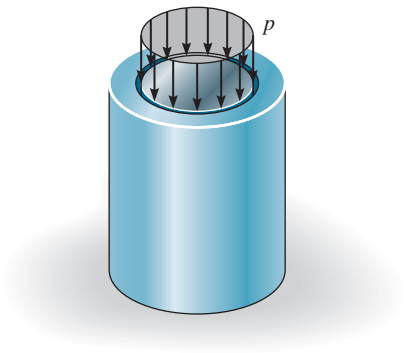
3–26.

The plug has a diameter of 30 mm and fits within a rigid sleeve having an inner diameter of 32 mm. Both the plug and the sleeve are 50 mm long. Determine the axial pressure p that must be applied to the top of the plug to cause it to contact the sides of the sleeve. Also, how far must the plug be compressed downward in order to do this? The plug is made from a material for which $E = 5$ MPa, $\nu = 0.45$.



SOLUTION

$$\epsilon_{\text{lat}} = \frac{d' - d}{d} = \frac{32 - 30}{30} = 0.06667 \text{ mm/mm}$$

$$\nu = -\frac{\epsilon_{\text{lat}}}{\epsilon_{\text{long}}}; \qquad \epsilon_{\text{long}} = -\frac{\epsilon_{\text{lat}}}{\nu} = -\frac{0.06667}{0.45} = -0.1481 \text{ mm/mm}$$

$$p = \sigma = E\,\epsilon_{\text{long}} = 5(10^6)(0.1481) = 741 \text{ kPa} \qquad \textbf{Ans.}$$

$$\delta = |\epsilon_{\text{long}} L| = |-0.1481(50)| = 7.41 \text{ mm} \qquad \textbf{Ans.}$$

Ans:
$p = 741$ kPa,
$\delta = 7.41$ mm

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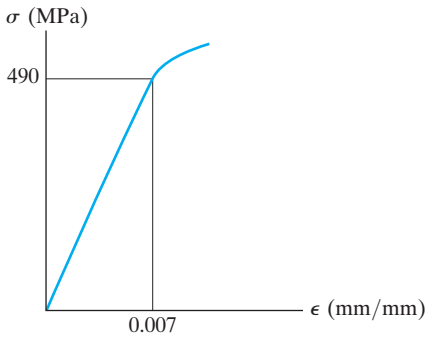
3–27.

The elastic portion of the stress–strain diagram for an aluminum alloy is shown in the figure. The specimen from which it was obtained has an original diameter of 12.7 mm and a gage length of 50.8 mm. When the applied load on the specimen is 50 kN, the diameter is 12.67494 mm. Determine Poisson's ratio for the material.



SOLUTION

Average Normal Stress:

$$\sigma = \frac{N}{A} = \frac{50(10^3)}{\frac{\pi}{4}(0.0127^2)} = 394.71(10^6)\ \text{Pa} = 394.71\ \text{MPa}$$

Average Normal Strain: Referring to the stress–strain diagram, the modulus of elasticity is $E = \frac{490(10^6)}{0.007} = 70.0(10^9)\ \text{Pa} = 70.0\ \text{GPa}$.

$$\epsilon_{\text{long}} = \frac{\sigma}{E} = \frac{394.71(10^6)}{70.0(10^9)} = 0.0056386\ \text{mm/mm}$$

$$\epsilon_{\text{lat}} = \frac{d - d_0}{d_0} = \frac{12.67494 - 12.7}{12.7} = -0.0019732$$

Poisson's Ratio: The lateral and longitudinal strain can be related using Poisson's ratio, that is

$$\nu = -\frac{\epsilon_{\text{lat}}}{\epsilon_{\text{long}}} = -\frac{(-0.019732)}{0.0056386} = 0.350 \qquad \textbf{Ans.}$$

Ans:
$\nu = 0.350$

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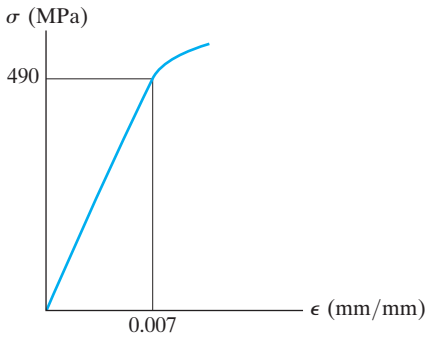
***3–28.**

The elastic portion of the stress–strain diagram for an aluminum alloy is shown in the figure. The specimen from which it was obtained has an original diameter of 12.7 mm and a gage length of 50.8 mm. If a load of $P = 60$ kN is applied to the specimen, determine its new diameter and length. Take $\nu = 0.35$.



SOLUTION

Average Normal Stress:

$$\sigma = \frac{N}{A} = \frac{60(10^3)}{\frac{\pi}{4}(0.0127^2)} = 473.65(10^6)\text{ Pa} = 473.65\text{ MPa}$$

Average Normal Strain: Referring to the stress–strain diagram, the modulus of elasticity is $E = \dfrac{490(10^6)}{0.007} = 70.0(10^9)\text{ Pa} = 70.0\text{ GPa}$.

$$\epsilon_{\text{long}} = \frac{\sigma}{E} = \frac{473.65(10^6)}{70.0(10^9)} = 0.0067664\text{ mm/mm}$$

Thus,

$$\delta L = \epsilon_{\text{long}} L_0 = 0.0067664(50.8) = 0.34373\text{ mm}$$

Then

$$L = L_0 + \delta L = 50.8 + 0.34373 = 51.1437\text{ mm}$$ **Ans.**

Poisson's Ratio: The lateral strain can be related to the longitudinal strain using Poisson's ratio.

$$\epsilon_{\text{lat}} = -\nu\epsilon_{\text{long}} = -0.35(0.0067664) = -0.0023682\text{ mm/mm}$$

Thus,

$$\delta d = \epsilon_{\text{lat}}\, d = -0.0023682(12.7) = -0.030077\text{ mm}$$

Then

$$d = d_0 + \delta d = 12.7 + (-0.030077) = 12.66992\text{ mm}$$
$$= 12.67\text{ mm}$$ **Ans.**

Ans:
$L = 51.1437$ mm,
$d = 12.67$ mm

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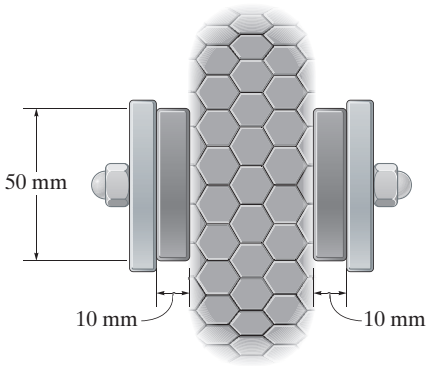
3–29.

The brake pads for a bicycle tire are made of rubber. If a frictional force of 50 N is applied to each side of the tires, determine the average shear strain in the rubber. Each pad has cross-sectional dimensions of 20 mm and 50 mm. $G_r = 0.20$ MPa.



SOLUTION

Average Shear Stress: The shear force is $V = 50$ N.

$$\tau = \frac{V}{A} = \frac{50}{0.02(0.05)} = 50.0 \text{ kPa}$$

Shear-Stress – Strain Relationship: Applying Hooke's law for shear

$$\tau = G\gamma$$

$$50.0(10^3) = 0.2(10^6)\gamma$$

$$\gamma = 0.250 \text{ rad} \qquad \textbf{Ans.}$$

Ans:
$\gamma = 0.250$ rad

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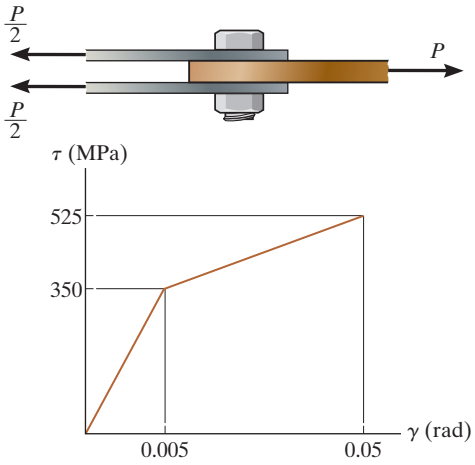
3–30.

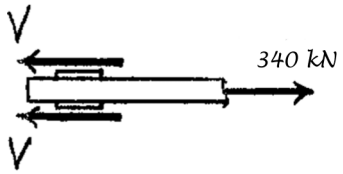
The lap joint is connected together using a 30 mm diameter bolt. If the bolt is made from a material having a shear stress–strain diagram that is approximated as shown, determine the shear strain developed in the shear plane of the bolt when $P = 340$ kN.



SOLUTION

Internal Loadings: The shear force developed in the shear planes of the bolt can be determined by considering the equilibrium of the free-body diagram shown in Fig. *a*.

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad 340 - 2V = 0 \qquad V = 170 \text{ kN}$$



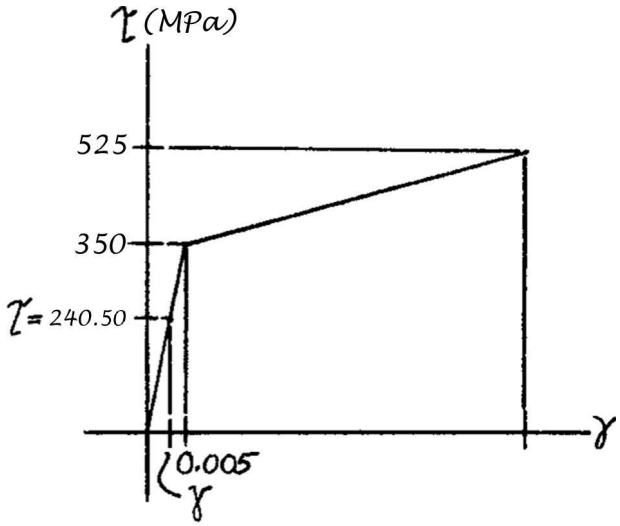
(a)

Shear Stress and Strain:

$$\tau = \frac{V}{A} = \frac{170(10^3)}{\frac{\pi}{4}(0.03^2)} = 240.50(10^6) \text{ N/m}^2 = 240.50 \text{ MPa}$$

Using this result, the corresponding shear strain can be obtained from the shear stress–strain diagram, Fig. *b*.

$$\frac{240.50}{\gamma} = \frac{350}{0.005}; \qquad \gamma = 3.4357(10^{-3}) \text{ rad} = 3.44(10^{-3}) \text{ rad}$$ **Ans.**



(b)

Ans:
$\gamma = 3.44(10^{-3})$ rad

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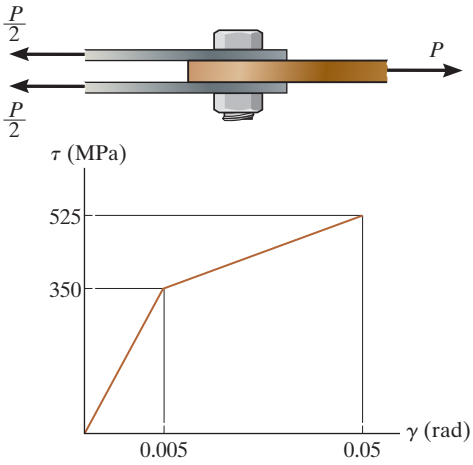
3–31

The lap joint is connected together using a 30 mm diameter bolt. If the bolt is made from a material having a shear stress–strain diagram that is approximated as shown, determine the permanent shear strain in the shear plane of the bolt when the applied force $P = 680$ kN is removed.



SOLUTION

Internal Loadings: The shear force developed in the shear planes of the bolt can be determined by considering the equilibrium of the free-body diagram shown in Fig. *a*.

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad 680 - 2V = 0 \qquad V = 340\ \text{kN}$$

Shear Stress and Strain:

$$\tau = \frac{V}{A} = \frac{340(10^3)}{\frac{\pi}{4}(0.03^2)} = 481.00(10^6)\ \text{N/m}^2 = 481.00\ \text{MPa}$$

Using this result, the corresponding shear strain can be obtained from the shear stress–strain diagram, Fig. *b*.

$$\frac{481.00 - 350}{\gamma - 0.005} = \frac{525 - 350}{0.05 - 0.005}; \qquad \gamma = 0.03869\ \text{rad}$$

When force **P** is removed, the shear strain recovers linearly along line *BC*, Fig. *b*, with a slope that is the same as line *OA*. This slope represents the shear modulus.

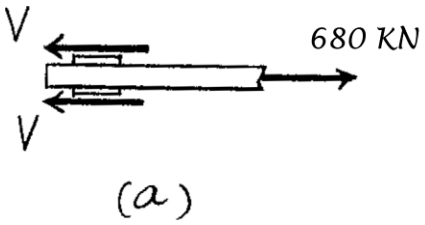
$$G = \frac{350(10^3)}{0.005} = 700.0(10^9)\ \text{Pa} = 70.0\ \text{GPa}$$

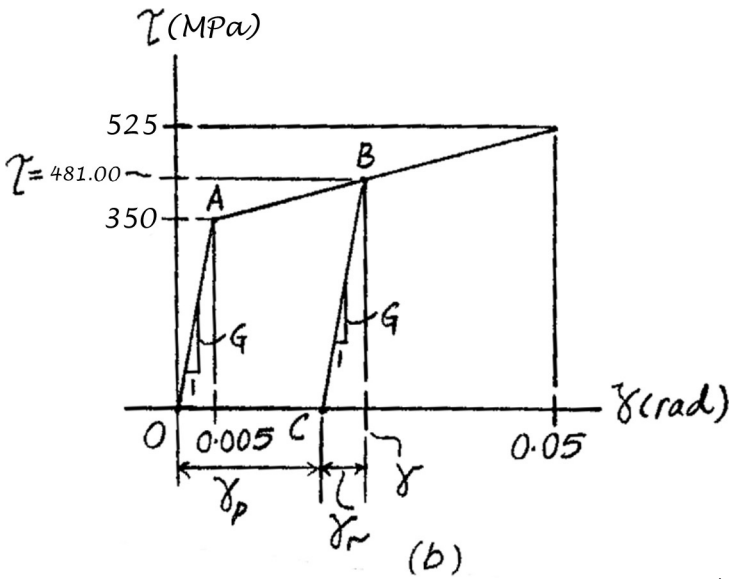
Thus, the elastic recovery of shear strain is

$$\tau = G\gamma_r; \qquad 481.00(10^6) = 70.0(10^9)\gamma_r \qquad \gamma_r = 6.8715(10^{-3})\ \text{rad}$$

And the permanent shear strain is

$$\gamma_P = \gamma - \gamma_r = 0.03869 - 6.874(10^{-3}) = 0.031815\ \text{rad} = 0.0318\ \text{rad} \qquad \textbf{Ans.}$$





Ans:

$\gamma_P = 0.0318$ rad

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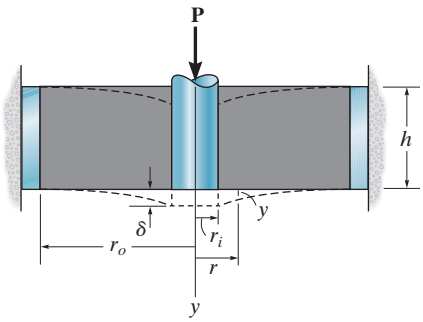
***3–32.** A shear spring is made by bonding the rubber annulus to a rigid fixed ring and a plug. When an axial load **P** is placed on the plug, show that the slope at point y in the rubber is $dy/dr = -\tan\gamma = -\tan(P/(2\pi hGr))$. For small angles we can write $dy/dr = -P/(2\pi hGr)$. Integrate this expression and evaluate the constant of integration using the condition that $y = 0$ at $r = r_o$. From the result compute the deflection $y = \delta$ of the plug.



Shear Stress–Strain Relationship: Applying Hooke's law with $\tau_A = \dfrac{P}{2\pi\, r\, h}$.

$$\gamma = \frac{\tau_A}{G} = \frac{P}{2\pi\, h\, G\, r}$$

$$\frac{dy}{dr} = -\tan\gamma = -\tan\left(\frac{P}{2\pi\, h\, G\, r}\right) \qquad \text{(Q.E.D)}$$

If γ is small, then $\tan\gamma = \gamma$. Therefore,

$$\frac{dy}{dr} = -\frac{P}{2\pi\, h\, G\, r}$$

$$y = -\frac{P}{2\pi\, h\, G}\int\frac{dr}{r}$$

$$y = -\frac{P}{2\pi\, h\, G}\ln r + C$$

At $r = r_o$, $\quad y = 0$

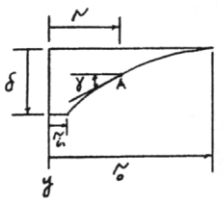
$$0 = -\frac{P}{2\pi\, h\, G}\ln r_o + C$$

$$C = \frac{P}{2\pi\, h\, G}\ln r_o$$

Then, $y = \dfrac{P}{2\pi\, h\, G}\ln\dfrac{r_o}{r}$

At $r = r_i$, $\quad y = \delta$

$$\delta = \frac{P}{2\pi\, h\, G}\ln\frac{r_o}{r_i} \qquad \textbf{Ans.}$$



Ans.

$$\delta = \frac{P}{2\pi\, h\, G}\ln\frac{r_o}{r_i}$$

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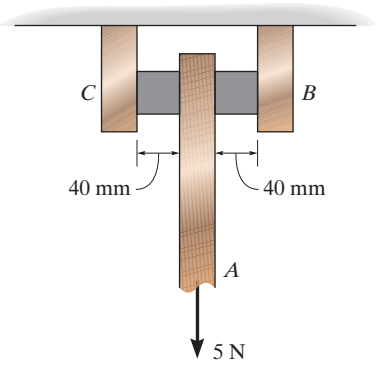
3–33. The support consists of three rigid plates, which are connected together using two symmetrically placed rubber pads. If a vertical force of 5 N is applied to plate A, determine the approximate vertical displacement of this plate due to shear strains in the rubber. Each pad has cross-sectional dimensions of 30 mm and 20 mm. $G_r = 0.20$ MPa.

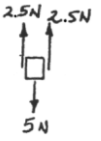


SOLUTION

$$\tau_{avg} = \frac{V}{A} = \frac{2.5}{(0.03)(0.02)} = 4166.7 \text{ Pa}$$

$$\gamma = \frac{\tau}{G} = \frac{4166.7}{0.2(10^6)} = 0.02083 \text{ rad}$$

$$\delta = 40(0.02083) = 0.833 \text{ mm}$$ **Ans.**



Ans.

$\delta = 0.833$ mm

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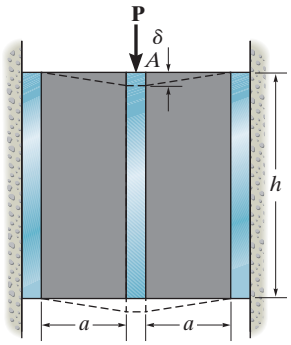
3–34.

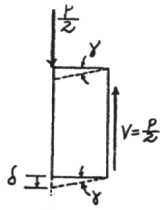
A shear spring is made from two blocks of rubber, each having a height h, width b, and thickness a. The blocks are bonded to three plates as shown. If the plates are rigid and the shear modulus of the rubber is G, determine the displacement of plate A when the vertical load **P** is applied. Assume that the displacement is small so that $\delta = a \tan \gamma \approx a\gamma$.



SOLUTION

Average Shear Stress: The rubber block is subjected to a shear force of $V = \frac{P}{2}$.

$$\tau = \frac{V}{A} = \frac{\frac{P}{2}}{bh} = \frac{P}{2bh}$$



Shear Strain: Applying Hooke's law for shear

$$\gamma = \frac{\tau}{G} = \frac{\frac{P}{2bh}}{G} = \frac{P}{2bhG}$$

Thus,

$$\delta = a\gamma = \frac{Pa}{2bhG} \qquad \textbf{Ans.}$$

Ans:

$$\delta = \frac{Pa}{2bhG}$$

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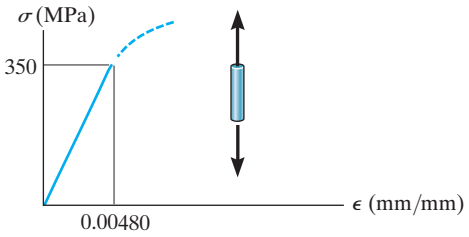
R3–1.

The elastic portion of the tension stress–strain diagram for an aluminum alloy is shown in the figure. The specimen used for the test has a gauge length of 50 mm and a diameter of 12.5 mm. When the applied load is 45 kN, the new diameter of the specimen is 12.4780 mm. Calculate the shear modulus G_{al} for the aluminum.



SOLUTION

From the stress–strain diagram,

$$E_{al} = \frac{\sigma}{\varepsilon} = \frac{350(10^6)}{0.00480} = 72.92(10^9)\ \text{N/m}^2 = 72.92\ \text{GPa}$$

When specimen is loaded with a 45-kN load,

$$\sigma = \frac{P}{A} = \frac{45(10^3)}{\frac{\pi}{4}(0.0125^2)} = 366.69(10^6)\ \text{N/m}^2 = 366.69\ \text{MPa}$$

$$\varepsilon_{\text{long}} = \frac{\sigma}{E_{al}} = \frac{366.69(10^6)}{72.92(10^9)} = 5.0289(10^{-3})\ \text{mm/mm}$$

$$\varepsilon_{\text{lat}} = \frac{d' - d}{d} = \frac{12.4780 - 12.5}{12.5} = -1.76(10^{-3})\ \text{mm/mm}$$

$$V = \frac{\varepsilon_{\text{lat}}}{\varepsilon_{\text{long}}} = \frac{-1.76(10^{-3})}{5.0289(10^{-3})} = 0.3500$$

$$G_{al} = \frac{E_{at}}{2(1+\nu)} = \frac{72.92(10^9)}{2(1+0.3500)} = 27.00(10^9)\ \text{N/m}^2 = 27.0\ \text{GPa}$$ **Ans.**

Ans:

$G_{al} = 27.0$ GPa

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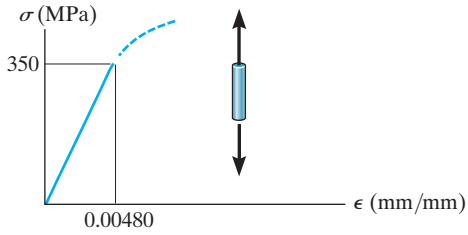
R3–2.

The elastic portion of the tension stress–strain diagram for an aluminum alloy is shown in the figure. The specimen used for the test has a gauge length of 50 mm and a diameter of 12.5 mm. If the applied load is 40 kN, determine the new diameter of the specimen. The shear modulus is $G_{al} = 27$ GPa.



SOLUTION

$$\sigma = \frac{P}{A} = \frac{40(10^3)}{\frac{\pi}{4}(0.0125^2)} = 325.95(10^6)\ \text{N/m}^2 = 325.95\ \text{MPa}$$

From the stress–strain diagram,

$$E = \frac{350(10^6)}{0.00480} = 72.92(10^9)\ \text{N/m}^2 = 72.92\ \text{GPa}$$

$$\varepsilon_{\text{long}} = \frac{\sigma}{E} = \frac{325.95(10^6)}{72.92(10^9)} = 4.4702(10^{-3})\ \text{mm/mm}$$

$$G = \frac{E}{2(1+\nu)};\quad 27(10^9) = \frac{72.92(10^9)}{2(1+\nu)};\quad \nu = 0.3503$$

$$\varepsilon_{\text{lat}} = -\nu\varepsilon_{\text{long}} = -0.3503[4.4702(10^{-3})] = -1.5659(10^{-3})\ \text{mm/mm}$$

$$\Delta d = \varepsilon_{\text{lat}} d = [-1.5659(10^{-3})](12.5) = -0.01957\ \text{mm}$$

$$d' = d + \Delta d = 12.5 - 0.01957 = 12.4804\ \text{mm}$$ **Ans.**

Ans:
$d' = 12.4804$ mm

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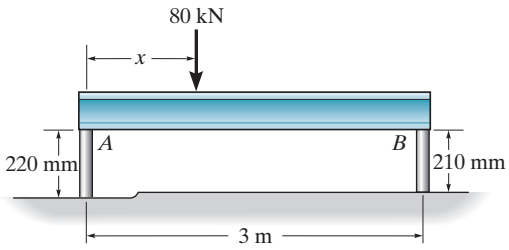
R3–3.

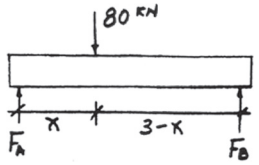
The rigid beam rests in the horizontal position on two 2014-T6 aluminum cylinders having the *unloaded* lengths shown. If each cylinder has a diameter of 30 mm, determine the placement x of the applied 80-kN load so that the beam remains horizontal. What is the new diameter of cylinder A after the load is applied? $\nu_{al} = 0.35$.



SOLUTION

$$\circlearrowleft +\Sigma M_A = 0; \quad F_B(3) - 80(x) = 0; \quad F_B = \frac{80x}{3} \qquad \textbf{(1)}$$

$$\circlearrowleft +\Sigma M_B = 0; \quad -F_A(3) + 80(3 - x) = 0; \quad F_A = \frac{80(3 - x)}{3} \qquad \textbf{(2)}$$



Since the beam is held horizontally, $\delta_A = \delta_B$

$$\sigma = \frac{P}{A}; \quad \epsilon = \frac{\sigma}{E} = \frac{\frac{P}{A}}{E}$$

$$\delta = \epsilon L = \left(\frac{\frac{P}{A}}{E}\right) L = \frac{PL}{AE}$$

$$\delta_A = \delta_B; \quad \frac{\frac{80(3 - x)}{3}(220)}{AE} = \frac{\frac{80x}{3}(210)}{AE}$$

$$80(3 - x)(220) = 80x(210)$$

$$x = 1.53 \text{ m} \qquad \textbf{Ans.}$$

From Eq. (2),

$$F_A = 39.07 \text{ kN}$$

$$\sigma_A = \frac{F_A}{A} = \frac{39.07(10^3)}{\frac{\pi}{4}(0.03^2)} = 55.27 \text{ MPa}$$

$$\epsilon_{long} = \frac{\sigma_A}{E} = -\frac{55.27(10^6)}{73.1(10^9)} = -0.000756$$

$$\epsilon_{lat} = -\nu\epsilon_{long} = -0.35(-0.000756) = 0.0002646$$

$$d'_A = d_A + d\,\epsilon_{lat} = 30 + 30(0.0002646) = 30.008 \text{ mm} \qquad \textbf{Ans.}$$

Ans:
$x = 1.53$ m, $d'_A = 30.008$ mm

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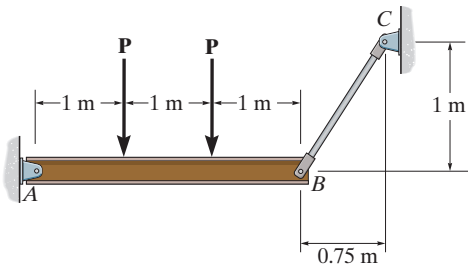
***R3–4.**

When the two forces are placed on the beam, the diameter of the A-36 steel rod BC decreases from 40 mm to 39.99 mm. Determine the magnitude of each force **P**.



SOLUTION

Equations of Equilibrium: The force developed in rod BC can be determined by writing the moment equation of equilibrium about A with reference to the free-body diagram of the beam shown in Fig. *a*.

$$\zeta + \Sigma M_A = 0; \qquad F_{BC}\left(\frac{4}{5}\right)(3) - P(2) - P(1) = 0 \qquad F_{BC} = 1.25P$$

Normal Stress and Strain: The lateral strain of rod BC is

$$\epsilon_{\text{lat}} = \frac{d - d_0}{d_0} = \frac{39.99 - 40}{40} = -0.25(10^{-3})\ \text{mm/mm}$$

$$\epsilon_{\text{lat}} = -\nu\epsilon_a; \qquad -0.25(10^{-3}) = -(0.32)\epsilon_a$$

$$\epsilon_a = 0.78125(10^{-3})\ \text{mm/mm}$$

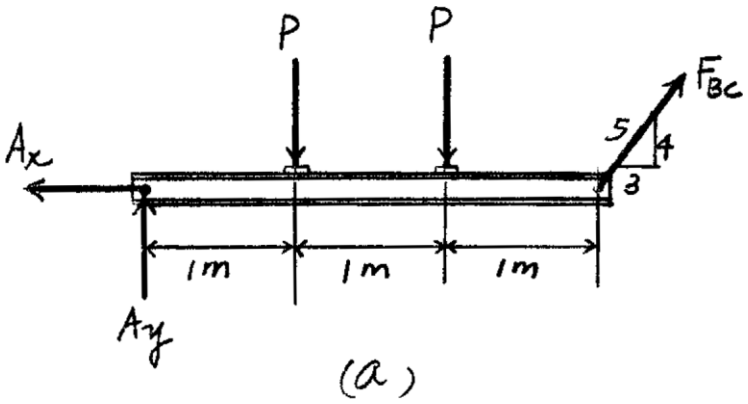
Assuming that Hooke's Law applies,

$$\sigma_{BC} = E\epsilon_a; \qquad \sigma_{BC} = 200(10^9)(0.78125)(10^{-3}) = 156.25\ \text{MPa}$$

Since $\sigma < \sigma_Y$, the assumption is correct.

$$\sigma_{BC} = \frac{F_{BC}}{A_{BC}}; \qquad 156.25(10^6) = \frac{1.25P}{\frac{\pi}{4}\left(0.04^2\right)}$$

$$P = 157.08(10^3)\text{N} = 157\ \text{kN} \qquad \textbf{Ans.}$$



Ans:
$P = 157\ \text{kN}$

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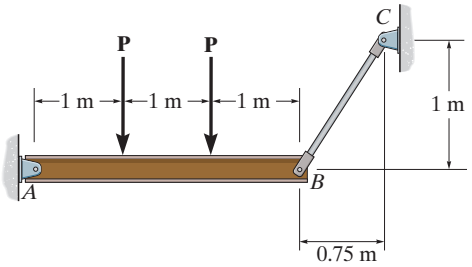
R3–5.

If $P = 150$ kN, determine the elastic elongation of rod BC and the decrease in its diameter. Rod BC is made of A-36 steel and has a diameter of 40 mm.



SOLUTION

Equations of Equilibrium: The force developed in rod BC can be determined by writing the moment equation of equilibrium about A with reference to the free-body diagram of the beam shown in Fig. *a*.

$$\zeta + \Sigma M_A = 0; \qquad F_{BC}\left(\frac{4}{5}\right)(3) - 150(2) - 150(1) = 0 \qquad F_{BC} = 187.5 \text{ kN}$$

Normal Stress and Strain: The lateral strain of rod BC is

$$\sigma_{BC} = \frac{F_{BC}}{A_{BC}} = \frac{187.5(10^3)}{\frac{\pi}{4}(0.04^2)} = 149.21 \text{ MPa}$$

Since $\sigma < \sigma_Y$, Hooke's Law can be applied. Thus,

$$\sigma_{BC} = E\epsilon_{BC}; \qquad 149.21(10^6) = 200(10^9)\epsilon_{BC}$$

$$\epsilon_{BC} = 0.7460(10^{-3}) \text{ mm/mm}$$

The unstretched length of rod BC is $L_{BC} = \sqrt{750^2 + 1000^2} = 1250$ mm. Thus the elongation of this rod is given by

$$\delta_{BC} = \epsilon_{BC} L_{BC} = 0.7460(10^{-3})(1250) = 0.933 \text{ mm} \qquad \textbf{Ans.}$$

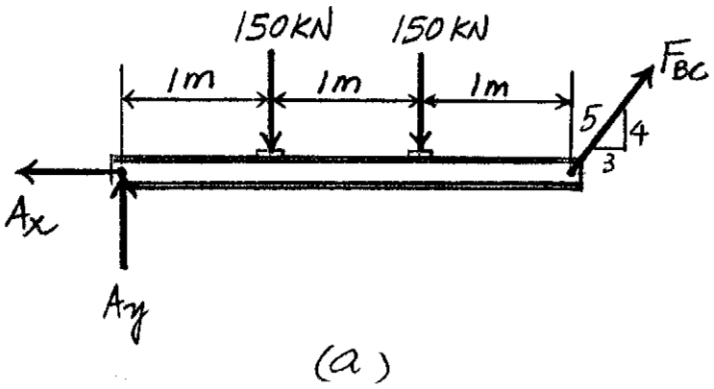
We obtain,

$$\epsilon_{\text{lat}} = -\nu\epsilon_a; \qquad \epsilon_{\text{lat}} = -(0.32)(0.7460)(10^{-3})$$

$$= -0.2387(10^{-3}) \text{ mm/mm}$$

Thus,

$$\delta d = \epsilon_{\text{lat}}\, d_{BC} = -0.2387(10^{-3})(40) = -9.55(10^{-3}) \text{ mm} \qquad \textbf{Ans.}$$



Ans.

$\delta_{BC} = 0.933$ mm

$\delta d = -9.55(10^{-3})$ mm

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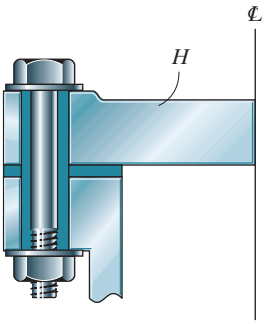
R3–6.

The head H is connected to the cylinder of a compressor using six steel bolts. If the clamping force in each bolt is 4 kN, determine the normal strain in the bolts. Each bolt has a diameter of 5 mm. If σ_Y = 280 MPa and E_{st} = 200 GPa, what is the strain in each bolt when the nut is unscrewed so that the clamping force is released?



SOLUTION

Normal Stress:

$$\sigma = \frac{P}{A} = \frac{4(10^3)}{\frac{\pi}{4}(0.005^2)} = 203.72(10^6)\ \text{N/m}^2 = 203.72\ \text{MPa} < \sigma_\gamma = 280\ \text{MPa}$$

Normal Strain: Since $\sigma < \sigma_\gamma$, Hooke's law is still valid.

$$\varepsilon = \frac{\sigma}{E} = \frac{203.72(10^6)}{200(10^9)} = 1.0186(10^{-3})\ \text{mm/mm} = 1.02(10^{-3})\ \text{mm/mm}$$ **Ans.**

If the nut is unscrewed, the load is zero. Also, the nut is not stressed beyond σ_γ and so no permanent strain will be developed. Therefore, the strain $\varepsilon = 0$ **Ans.**

Ans:

$\varepsilon = 0.0010186$ mm/mm, $\varepsilon_{unscr} = 0$

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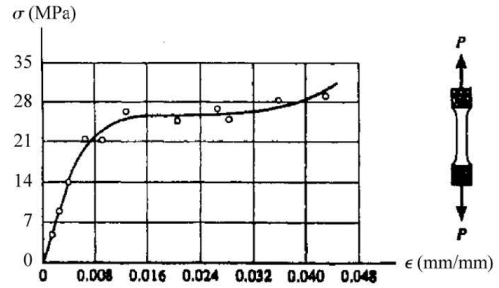
R3–7.

The stress–strain diagram for polyethylene, which is used to sheath coaxial cables, is determined from testing a specimen that has a gauge length of 250 mm. If a load P on the specimen develops a strain of $\epsilon = 0.024$ mm/mm, determine the approximate length of the specimen, measured between the gauge points, when the load is removed. Assume the specimen recovers elastically.



SOLUTION

From the graph $\sigma = 14$ Mpa, $\epsilon = 0.004$ mm/mm,

$$E = \frac{\sigma}{\epsilon} = \frac{14(10^6)}{0.004} = 3.50(10^9)\text{ Pa} = 3.50\text{ GPa}$$

$\epsilon = 0.024$ mm/mm, $\sigma = 25.67$ Mpa

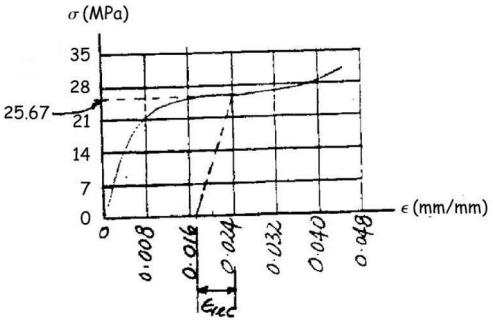
$L' = 250\text{ mm} + 0.024(250) = 256\text{ mm}$

Elastic strain recovery:

$$\epsilon_{rec} = \frac{\sigma}{E} = \frac{25.67(10^6)}{3.50(10^9)} = 7.3333(10^{-3})\text{ mm/mm}$$

$$\delta = \epsilon_{rec}L = [7.3333(10^{-3})](250)\text{ mm/mm} = 1.8333\text{ mm}$$

$$L = L' - \delta = 256\text{ mm} - 1.8333\text{ mm} = 254.167\text{ mm}$$ **Ans.**



Ans:
254.167 mm

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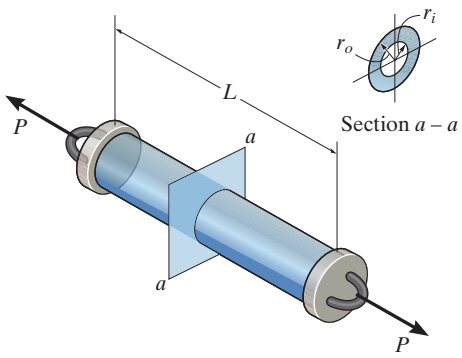
***R3–8.**

The pipe with two rigid caps attached to its ends is subjected to an axial force P. If the pipe is made from a material having a modulus of elasticity E and Poisson's ratio ν, determine the change in volume of the material.



SOLUTION

Normal Stress: The rod is subjected to uniaxial loading. Thus, $\sigma_{\text{long}} = \dfrac{P}{A}$ and $\sigma_{\text{lat}} = 0$.

$$\delta V = A\delta L + 2\pi rL\delta r$$

$$= A\epsilon_{\text{long}} L + 2\pi rL\epsilon_{\text{lat}} r$$

Using Poisson's ratio and noting that $AL = \pi r^2 L = V$,

$$\delta V = \epsilon_{\text{long}} V - 2\nu\epsilon_{\text{long}} V$$

$$= \epsilon_{\text{long}} (1 - 2\nu)V$$

$$= \frac{\sigma_{\text{long}}}{E}(1 - 2\nu)V$$

Since $\sigma_{\text{long}} = P/A$,

$$\delta V = \frac{P}{AE}(1 - 2\nu)AL$$

$$= \frac{PL}{E}(1 - 2\nu) \qquad \textbf{Ans.}$$

Ans:

$$\delta V = \frac{PL}{E}(1 - 2\nu)$$

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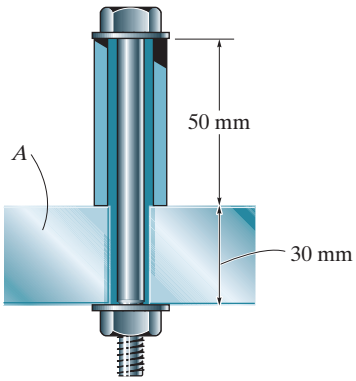
R3–9.

The 8-mm-diameter bolt is made of an aluminum alloy. It fits through a magnesium sleeve that has an inner diameter of 12 mm and an outer diameter of 20 mm. If the original lengths of the bolt and sleeve are 80 mm and 50 mm, respectively, determine the strains in the sleeve and the bolt if the nut on the bolt is tightened so that the tension in the bolt is 8 kN. Assume the material at A is rigid. $E_{al} = 70$ GPa, $E_{mg} = 45$ GPa.



SOLUTION

Normal Stress:

$$\sigma_b = \frac{P}{A_b} = \frac{8(10^3)}{\frac{\pi}{4}(0.008^2)} = 159.15 \text{ MPa}$$

$$\sigma_s = \frac{P}{A_s} = \frac{8(10^3)}{\frac{\pi}{4}(0.02^2 - 0.012^2)} = 39.79 \text{ MPa}$$

Normal Strain: Applying Hooke's Law

$$\epsilon_b = \frac{\sigma_b}{E_{al}} = \frac{159.15(10^6)}{70(10^9)} = 0.00227 \text{ mm/mm} \qquad \textbf{Ans.}$$

$$\epsilon_s = \frac{\sigma_s}{E_{mg}} = \frac{39.79(10^6)}{45(10^9)} = 0.000884 \text{ mm/mm} \qquad \textbf{Ans.}$$

Ans:
$\epsilon_b = 0.00227$ mm/mm, $\epsilon_s = 0.000884$ mm/mm

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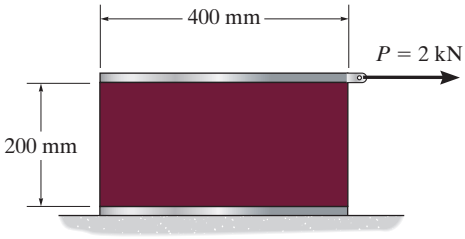
R3–10.

An acetal polymer block is fixed to the rigid plates at its top and bottom surfaces. If the top plate displaces 2 mm horizontally when it is subjected to a horizontal force $P = 2$ kN, determine the shear modulus of the polymer. The width of the block is 100 mm. Assume that the polymer is linearly elastic and use small angle analysis.



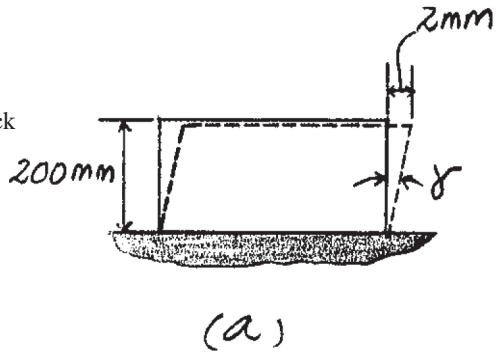
SOLUTION

Normal and Shear Stress:

$$\tau = \frac{V}{A} = \frac{2(10^3)}{0.4(0.1)} = 50 \text{ kPa}$$

Referring to the geometry of the undeformed and deformed shape of the block shown in Fig. *a*,

$$\gamma = \frac{2}{200} = 0.01 \text{ rad}$$



Applying Hooke's Law,

$\tau = G\gamma; \quad 50(10^3) = G(0.01)$

$$G = 5 \text{ MPa}$$ **Ans.**

Ans:
$G = 5$ MPa

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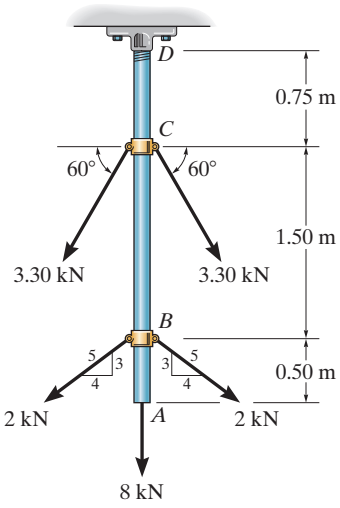
4–1. The A992 steel rod is subjected to the loading shown. If the cross-sectional area of the rod is 60 mm^2, determine the displacement of B and A, Neglect the size of the couplings at B, C, and D.

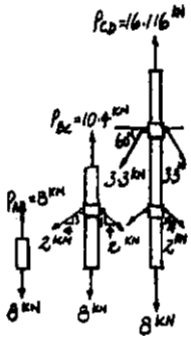


SOLUTION

$$\delta_B = \Sigma\frac{PL}{AE} = \frac{16.116(10^3)(0.75)}{60(10^{-6})(200)(10^9)} + \frac{10.4(10^3)(1.50)}{60(10^{-6})(200)(10^9)}$$

$$= 0.00231 \text{ m} = 2.31 \text{ mm} \quad \textbf{Ans.}$$

$$\delta_A = \delta_B + \frac{8(10^3)(0.5)}{60(10^{-6})(200)(10^9)} = 0.00264 \text{ m} = 2.64 \text{ mm} \quad \textbf{Ans.}$$



These solutions represent a preliminary version of the Instructors' Solutions Manual (ISM). It is possible and even likely that at this preliminary stage of preparing the ISM there are some omissions and errors in the draft solutions. These will be corrected and this manual will be republished.

Ans:
$\delta_B = 2.31$ mm, $\delta_A = 2.64$ mm

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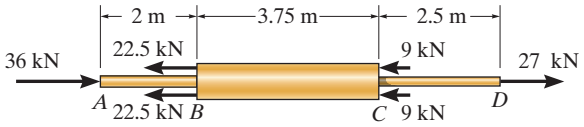
4–2. The copper shaft is subjected to the axial loads shown. Determine the displacement of end A with respect to end D if the diameters of each segment are $d_{AB} = 20$ mm, $d_{BC} = 25$ mm, and $d_{CD} = 12$ mm. Take $E_{cu} = 126$ GPa.

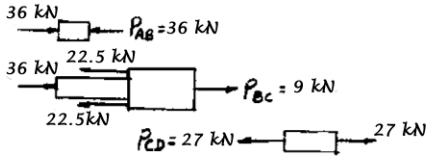


SOLUTION

$$\delta_{A/D} = \Sigma \frac{PL}{AE} = \frac{-36(10^3)(2000)}{\left[\frac{\pi}{4}(0.02^2)\right]\left[126(10^9)\right]} + \frac{9(10^3)(3750)}{\left[\frac{\pi}{4}(0.025^2)\right]\left[126(10^9)\right]} + \frac{27(10^3)(2500)}{\left[\frac{\pi}{4}(0.012^2)\right]\left[126(10^9)\right]}$$

$$= 3.4635 \text{ mm} = 3.46 \text{ mm}$$ **Ans.**

The positive sign indicates that end A moves away from end D.



Ans:

$\delta_{A/D} = 3.46$ mm away from end D.

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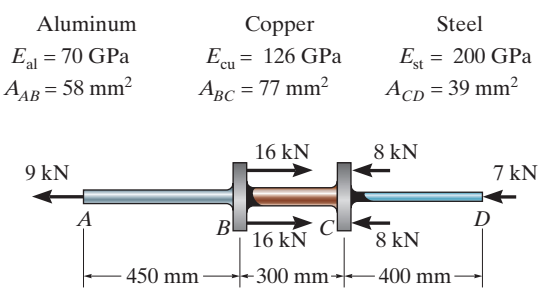
4–3. The composite shaft, consisting of aluminum, copper, and steel sections, is subjected to the loading shown. Determine the displacement of end A with respect to end D and the normal stress in each section. The cross-sectional area and modulus of elasticity for each section are shown in the figure. Neglect the size of the collars at B and C.

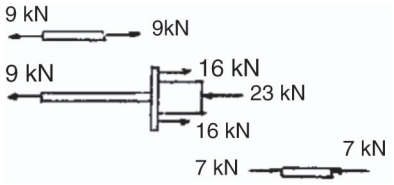


SOLUTION

$$\sigma_{AB} = \frac{P_{AB}}{A_{AB}} = \frac{9(10^3)}{58(10^{-6})} = 155.17(10^6)\ \text{N/m}^2 = 155\ \text{MPa} \quad \text{(T)} \quad \textbf{Ans.}$$

$$\sigma_{BC} = \frac{P_{BC}}{A_{BC}} = \frac{23(10^3)}{77(10^{-6})} = 298.70(10^6)\ \text{N/m}^2 = 299\ \text{MPa} \quad \text{(C)} \quad \textbf{Ans.}$$

$$\sigma_{CD} = \frac{P_{BC}}{A_{BC}} = \frac{7(10^3)}{39(10^{-6})} = 179.49(10^6)\ \text{N/m}^2 = 179\ \text{MPa} \quad \text{(C)} \quad \textbf{Ans.}$$



$$\delta_{ND} = \Sigma\frac{PL}{AE} = \frac{9(10^3)(450)}{\left[58(10^{-6})\right]\left[70(10^9)\right]} + \frac{-23(10^3)(300)}{\left[77(10^{-6})\right]\left[126(10^9)\right]} + \frac{-7(10^3)(400)}{\left[39(10^{-6})\right]\left[200(10^9)\right]}$$

$$= -0.07263\ \text{mm} = -0.0726\ \text{mm} \quad \textbf{Ans.}$$

The negative sign indicates end A moves towards end D.

Ans:
$\sigma_{AB} = 155$ MPa (T), $\sigma_{BC} = 299$ MPa (C), $\sigma_{CD} = 179$ MPa (C), $\delta_{A/D} = 0.0726$ mm towards end D.

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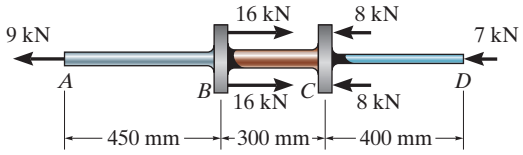
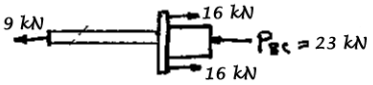
***4–4.** Determine the displacement of B with respect to C of the composite shaft in Prob. 4–3.

Aluminum	Copper	Steel
$E_{al} = 70$ GPa	$E_{cu} = 126$ GPa	$E_{st} = 200$ GPa
$A_{AB} = 58$ mm^2	$A_{BC} = 77$ mm^2	$A_{CD} = 39$ mm^2



$$\delta_{B/C} = \frac{PL}{AE} = \frac{-23(10^3)(300)}{\left[77(10^{-6})\right]\left[126(10^9)\right]} = -0.7112 \text{ mm} = -0.711 \text{ mm}$$

Ans.



The negative sign indicates end B moves towards end C.

Ans.

$\delta_{B/C} = -0.711$ mm

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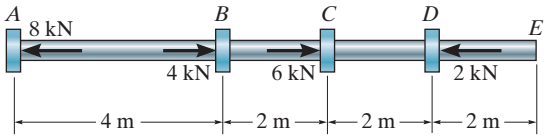
4–5.

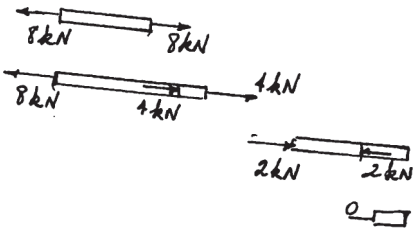
The 2014-T6 aluminium rod has a diameter of 30 mm and supports the load shown. Determine the displacement of end A with respect to end E. Neglect the size of the couplings.



SOLUTION

$$\delta_{A/E} = \Sigma\frac{PL}{AE} = \frac{1}{AE}[8(4) + 4(2) - 2(2) + 0(2)]\left(10^3\right)$$

$$= \frac{36\left(10^3\right)}{\frac{\pi}{4}(0.03)^2(73.1)\left(10^9\right)} = 0.697\left(10^{-3}\right) = 0.697\text{ mm}$$ **Ans.**



Ans:
$\delta_{A/E} = 0.697$ mm

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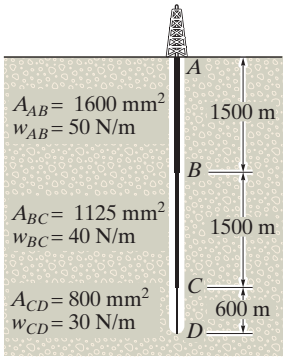
4–6. The A992 steel drill shaft of an oil well extends 3600 m into the ground. Assuming that the pipe used to drill the well is suspended freely from the derrick at A, determine the maximum average normal stress in each pipe segment and the elongation of its end D with respect to the fixed end at A. The shaft consists of three different sizes of pipe, AB, BC, and CD, each having the length, weight per unit length, and cross-sectional area indicated.

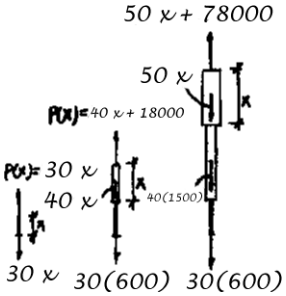


SOLUTION

$$\sigma_A = \frac{P}{A} = \frac{50(1500) + 78000}{1600(10^{-6})} = 95.625(10^6)\ \text{N-m}^2 = 95.6\ \text{MPa} \qquad \textbf{Ans.}$$

$$\sigma_B = \frac{P}{A} = \frac{40(1500) + 18000}{1125(10^{-6})} = 69.33(10^6)\ \text{N-m}^2 = 69.3\ \text{MPa} \qquad \textbf{Ans.}$$

$$\sigma_C = \frac{P}{A} = \frac{30(600)}{800(10^{-6})} = 22.5(10^6)\ \text{N-m}^2 = 22.5\ \text{MPa} \qquad \textbf{Ans.}$$



$$\delta_D = \Sigma \int \frac{P(x)\,dx}{A(x)\,E} = \int_0^{600\ \text{m}} \frac{30x\,dx}{\left[800(10^{-6})\right]\left[200(10^9)\right]} + \int_0^{1500\ \text{m}} \frac{(40x + 18000)dx}{\left[1125(10^{-6})\right]\left[200(10^9)\right]} + \int_0^{1500\ \text{m}} \frac{(50x + 78000)dx}{\left[1600(10^{-6})\right]\left[200(10^9)\right]}$$

$$= 0.8952\ \text{m} = 0.895\ \text{m} \qquad \textbf{Ans.}$$

Ans.

$\sigma_A = 95.6$ MPa, $\sigma_B = 69.3$ MPa,
$\sigma_C = 22.5$ MPa, $\delta_D = 0.895$ m

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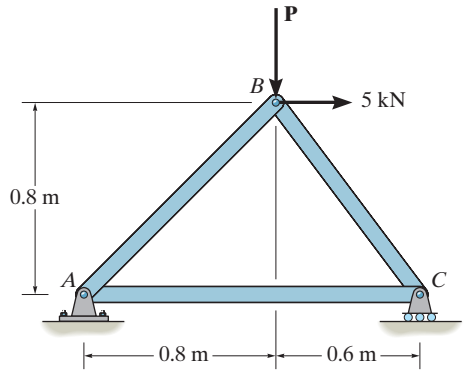
4–7.

The truss is made of three A-36 steel members, each having a cross-sectional area of 400 mm^2. Determine the horizontal displacement of the roller at C when $P = 8$ kN.

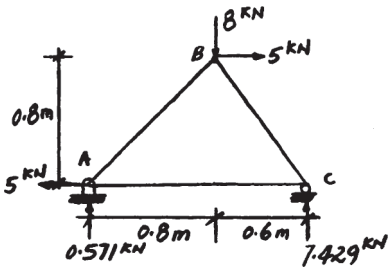


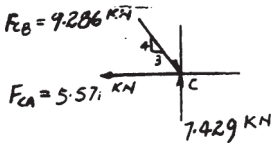
SOLUTION

By observation the horizontal displacement of roller C is equal to the displacement of point C obtained from member AC.

$F_{CA} = 5.571$ kN

$$\delta_C = \frac{F_{CA}L}{AE} = \frac{5.571(10^3)(1.40)}{(400)(10^{-6})(200)(10^6)} = 0.0975 \text{ mm} \rightarrow$$ **Ans.**





Ans:
$\delta_C = 0.0975$ mm $\rightarrow$

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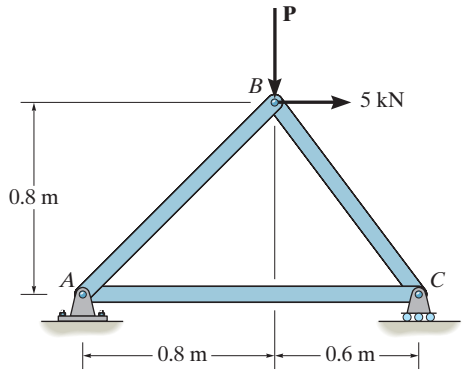
***4–8.**

The truss is made of three A-36 steel members, each having a cross-sectional area of 400 mm^2. Determine the magnitude P required to displace the roller to the right 0.2 mm.



SOLUTION

$$\circlearrowleft +M_A = 0; \qquad -P(0.8) - 5(0.8) + C_y(1.4) = 0$$

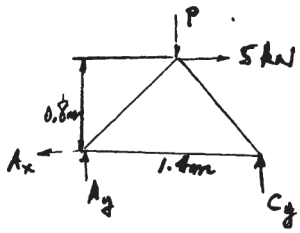
$$C_y = 0.5714\,P + 2.857$$

$$+\uparrow \Sigma F_y = 0; \qquad C_y - F_{BC}\left(\frac{4}{5}\right) = 0$$

$$F_{BC} = 1.25\,C_y$$

$$\pm \Sigma F_x = 0; \qquad -F_{AC} + 1.25\,C_y\,(0.6) = 0$$

$$F_{AC} = 0.75\,C_y = 0.4286\,P + 2.14286$$



Require,

$$\delta_{C_A} = 0.0002 = \frac{(0.4286\,P + 2.14286)(10^3)(1.4)}{(400)(10^{-6})(200)(10^9)}$$

$$P = 21.7 \text{ kN} \qquad \textbf{Ans.}$$

Ans:
$P = 21.7$ kN

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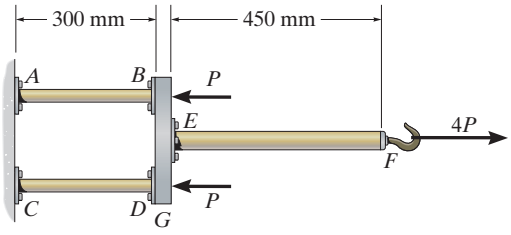
4–9.

The assembly consists of two 10-mm diameter red brass C83400 copper rods *AB* and *CD*, a 15-mm diameter 304 stainless steel rod *EF*, and a rigid bar *G*. If $P = 5$ kN, determine the horizontal displacement of end *F* of rod *EF*.



SOLUTION

Internal Loading: The normal forces developed in rods *EF*, *AB*, and *CD* are shown on the free-body diagrams in Figs. *a* and *b*.

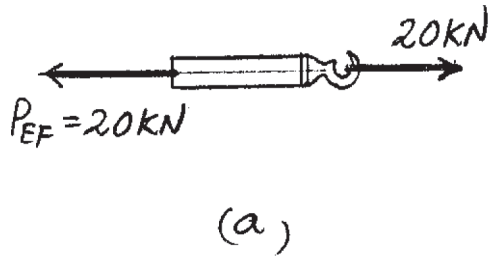
Displacement: The cross-sectional areas of rods *EF* and *AB* are $A_{EF} = \frac{\pi}{4}(0.015^2) = 56.25(10^{-6})\pi \text{ m}^2$ and $A_{AB} = \frac{\pi}{4}(0.01^2) = 25(10^{-6})\pi \text{ m}^2$.

$$\delta_F = \Sigma\frac{PL}{AE} = \frac{P_{EF}L_{EF}}{A_{EF}E_{\text{st}}} + \frac{P_{AB}L_{AB}}{A_{AB}E_{\text{br}}}$$

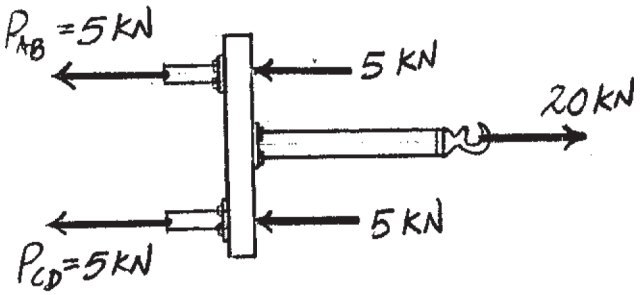
$$= \frac{20(10^3)(450)}{56.25(10^{-6})\pi(193)(10^9)} + \frac{5(10^3)(300)}{25(10^{-6})\pi(101)(10^9)}$$

$$= 0.453 \text{ mm}$$ **Ans.**

The positive sign indicates that end *F* moves away from the fixed end.



(a)



(b)

Ans:
$\delta_F = 0.453$ mm

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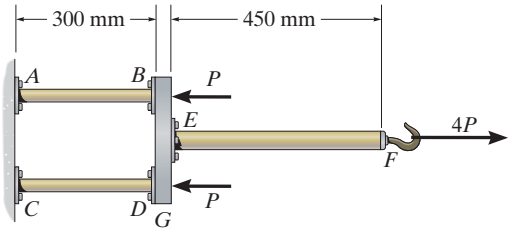
4–10.

The assembly consists of two 10-mm diameter red brass C83400 copper rods *AB* and *CD*, a 15-mm diameter 304 stainless steel rod *EF*, and a rigid bar *G*. If the horizontal displacement of end *F* of rod *EF* is 0.45 mm, determine the magnitude of *P*.



SOLUTION

Internal Loading: The normal forces developed in rods *EF*, *AB*, and *CD* are shown on the free-body diagrams in Figs. *a* and *b*.

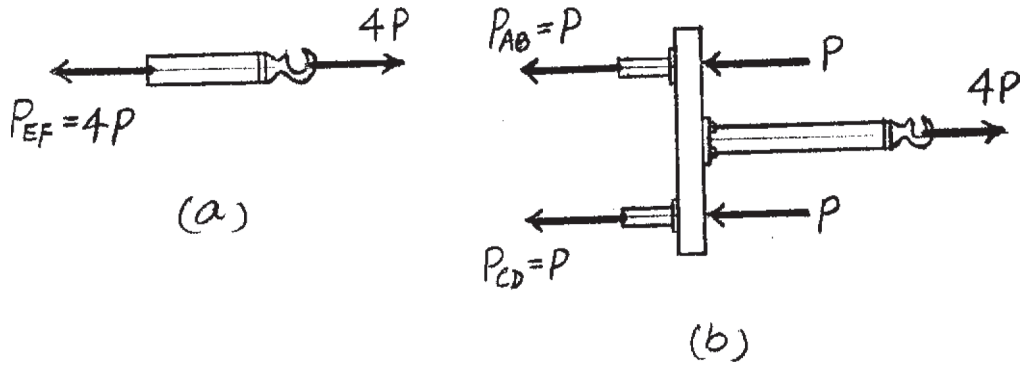
Displacement: The cross-sectional areas of rods *EF* and *AB* are $A_{EF} = \frac{\pi}{4}(0.015^2) = 56.25(10^{-6})\pi \text{ m}^2$ and

$$A_{AB} = \frac{\pi}{4}(0.01^2) = 25(10^{-6})\pi \text{ m}^2.$$

$$\delta_F = \Sigma\frac{PL}{AE} = \frac{P_{EF}L_{EF}}{A_{EF}E_{\text{st}}} + \frac{P_{AB}L_{AB}}{A_{AB}E_{\text{br}}}$$

$$0.45 = \frac{4P(450)}{56.25(10^{-6})\pi(193)(10^9)} + \frac{P(300)}{25(10^{-6})\pi(101)(10^9)}$$

$$P = 4967 \text{ N} = 4.97 \text{ kN}$$ **Ans.**



Ans:
$P = 4.97$ kN

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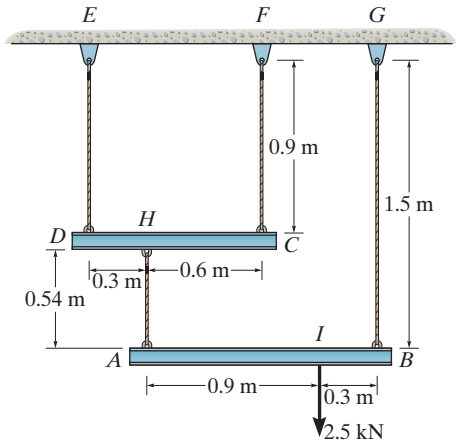
4–11. The load is supported by the four 304 stainless steel wires that are connected to the rigid members *AB* and *DC*. Determine the vertical displacement of the 2.5-kN load if the members were originally horizontal when the load was applied. Each wire has a cross-sectional area of 16 mm^2.



SOLUTION

Internal Forces in the wires:

FBD (b)

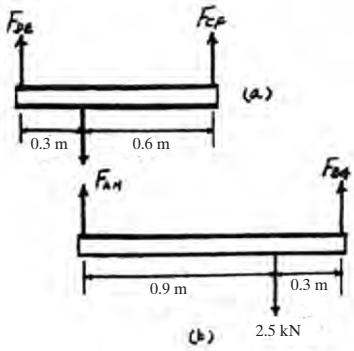
$\circlearrowleft +\Sigma M_A = 0;$ $F_{BG}(1.2) - 2.5(0.9) = 0$ $F_{BC} = 1.875$ kN

$+\uparrow \Sigma F_y = 0;$ $F_{AH} + 1.875 - 2.5 = 0$ $F_{AH} = 0.625$ kN

FBD (a)

$\circlearrowleft +\Sigma M_D = 0;$ $F_{CF}(0.9) - 0.625(0.3) = 0$ $F_{CF} = 0.2083$ kN

$+\uparrow \Sigma F_y = 0;$ $F_{DE} + 0.2083 - 0.625 = 0$ $F_{DE} = 0.4167$ kN



Displacement:

$$\delta_D = \frac{F_{DE}L_{DE}}{A_{DE}E} = \frac{0.4167(10^3)(0.9)(10^3)}{[16(10^{-6})][193(10^9)]} = 0.121438 \text{ mm}$$

$$\delta_C = \frac{F_{CF}L_{CF}}{A_{CF}E} = \frac{0.2083(10^3)(0.9)(10^3)}{[16(10^{-6})][193(10^9)]} = 0.060719 \text{ mm}$$

$$\frac{\delta'_H}{0.6} = \frac{0.060719}{0.9}; \quad \delta'_H = 0.040479 \text{ mm}$$

$$\delta_H = 0.040479 + 0.060719 = 0.101198 \text{ mm}$$

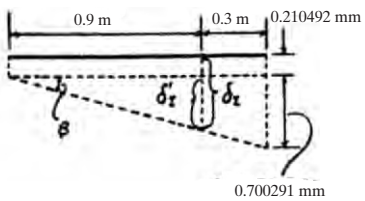
$$\delta_{A/H} = \frac{F_{AH}L_{AH}}{A_{AH}E} = \frac{0.625(10^3)(0.54)(10^3)}{[16(10^{-6})][193(10^9)]} = 0.109294 \text{ mm}$$

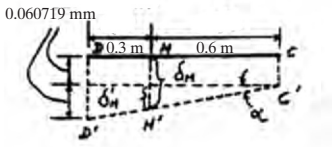
$$\delta_A = \delta_H + \delta_{A/H} = 0.101198 + 0.109294 = 0.210492 \text{ mm}$$

$$\delta_B = \frac{F_{BG}L_{BG}}{A_{BG}E} = \frac{1.875(10^3)(1.5)(10^3)}{[16(10^{-6})][193(10^9)]} = 0.910784 \text{ mm}$$

$$\frac{\delta'_I}{0.9} = \frac{0.700291}{1.2}; \quad \delta'_I = 0.525219 \text{ mm}$$

$$\delta_I = 0.210492 + 0.525219 = 0.7357 \text{ mm} = 0.736 \text{ mm}$$ **Ans.**





Ans:
$\delta_l = 0.736$ mm

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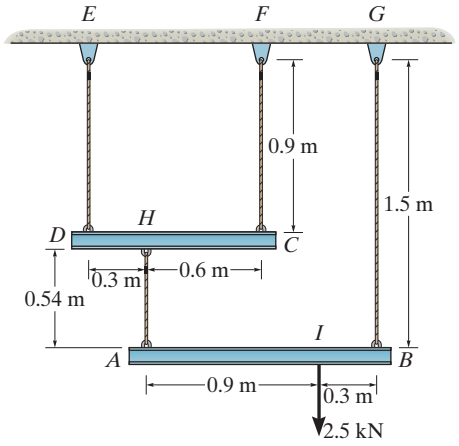
***4–12.** The load is supported by the four 304 stainless steel wires that are connected to the rigid members *AB* and *DC*. Determine the angle of tilt of each member after the 2.5-kN load is applied. The members were originally horizontal, and each wire has a cross-sectional area of 16 mm^2.



SOLUTION

Internal Forces in the wires:

FBD (b)

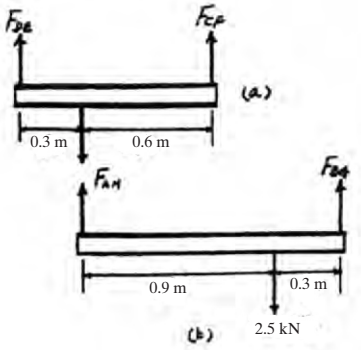
$\zeta + \Sigma M_A = 0;$ $F_{BG}(1.2) - 2.5(0.9) = 0$ $F_{BG} = 1.875$ kN

$+\uparrow \Sigma F_y = 0;$ $F_{AH} + 1.875 - 2.5 = 0$ $F_{AH} = 0.625$ kN

FBD (a)

$\zeta + \Sigma M_D = 0;$ $F_{CF}(0.9) - 0.625(0.3) = 0$ $F_{CF} = 0.2083$ kN

$+\uparrow \Sigma F_y = 0;$ $F_{DE} + 0.2083 - 0.625 = 0$ $F_{DE} = 0.4167$ kN



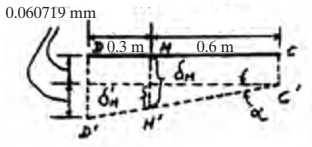
Displacement:

$$\delta_D = \frac{F_{DE}L_{DE}}{A_{DE}E} = \frac{0.4167(10^3)(0.9)(10^3)}{[16(10^{-6})][193(10^9)]} = 0.121438 \text{ mm}$$

$$\delta_C = \frac{F_{CF}L_{CF}}{A_{CF}E} = \frac{0.2083(10^3)(0.9)(10^3)}{[16(10^{-6})][193(10^9)]} = 0.060719 \text{ mm}$$

$$\frac{\delta'_H}{0.6} = \frac{0.060719}{0.9}; \quad \delta'_H = 0.040479 \text{ mm}$$

$$\delta_H = \delta'_H + \delta_C = 0.040479 + 0.060719 = 0.101198 \text{ mm}$$



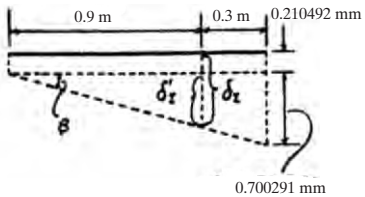
$$\tan\alpha = \frac{0.060719}{0.9(10^3)}; \quad \alpha = 0.003865° = 0.00387°$$ **Ans.**

$$\delta_{A/H} = \frac{F_{AH}L_{AH}}{A_{AH}E} = \frac{0.625(10^3)(0.54)(10^3)}{[16(10^{-6})][193(10^9)]} = 0.109294 \text{ mm}$$

$$\delta_A = \delta_H + \delta_{A/H} = 0.101198 + 0.109294 = 0.210492 \text{ mm}$$

$$\delta_B = \frac{F_{BG}L_{BG}}{A_{BG}E} = \frac{1.875(10^3)(1.5)(10^3)}{[16(10^{-6})][193(10^9)]} = 0.910784 \text{ mm}$$

$$\tan\beta = \frac{0.700291}{1.2(10^3)}; \quad \beta = 0.03344° = 0.0334°$$



Ans.
$\alpha = 0.00387°$
$\beta = 0.0334°$

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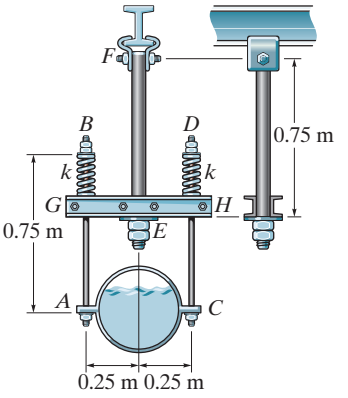
4–13. A spring-supported pipe hanger consists of two springs which are originally unstretched and have a stiffness of $k = 60$ kN/m, three 304 stainless steel rods, AB and CD, which have a diameter of 5 mm, and EF, which has a diameter of 12 mm, and a rigid beam GH. If the pipe and the fluid it carries have a total weight of 4 kN, determine the displacement of the pipe when it is attached to the support.



SOLUTION

Internal Force in the Rods:

FBD (a)

$\zeta + \Sigma M_A = 0; \quad F_{CD}(0.5) - 4(0.25) = 0 \quad F_{CD} = 2.00 \text{ kN}$

$+\uparrow \Sigma F_y = 0; \quad F_{AB} + 2.00 - 4 = 0 \quad F_{AB} = 2.00 \text{ kN}$

FBD (b)

$+\uparrow \Sigma F_y = 0; \quad F_{EF} - 2.00 - 2.00 = 0 \quad F_{EF} = 4.00 \text{ kN}$

Displacement:

$$\delta_D = \delta_E = \frac{F_{EF}L_{EF}}{A_{EF}E} = \frac{4.00(10^3)(750)}{\frac{\pi}{4}(0.012)^2(193)(10^9)} = 0.1374 \text{ mm}$$

$$\delta_{A/B} = \delta_{C/D} = \frac{P_{CD}L_{CD}}{A_{CD}E} = \frac{2(10^3)(750)}{\frac{\pi}{4}(0.005)^2(193)(10^9)} = 0.3958 \text{ mm}$$

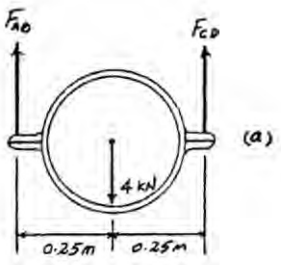
$$\delta_C = \delta_D + \delta_{C/D} = 0.1374 + 0.3958 = 0.5332 \text{ mm}$$

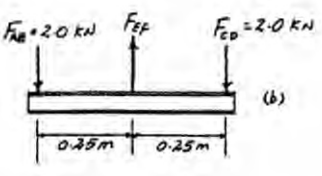
Displacement of the spring

$$\delta_{sp} = \frac{F_{sp}}{k} = \frac{2.00}{60} = 0.0333333 \text{ m} = 33.3333 \text{ mm}$$

$$\delta_{\text{lat}} = \delta_C + \delta_{sp}$$

$$= 0.5332 + 33.3333 = 33.9 \text{ mm}$$ **Ans.**





Ans.
$\delta_D = 0.1374$ mm, $\delta_{A/B} = 0.3958$ mm,
$\delta_C = 0.5332$ mm, $\delta_{\text{tot}} = 33.9$ mm

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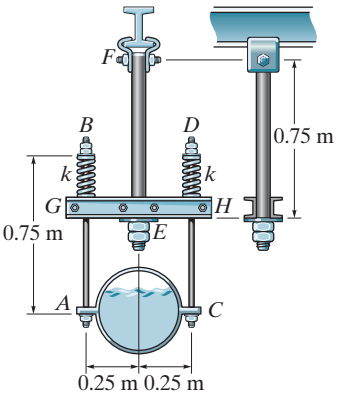
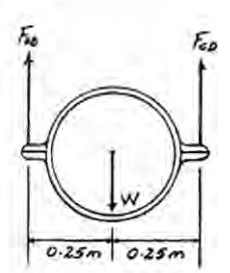
4–14. A spring-supported pipe hanger consists of two springs, which are originally unstretched and have a stiffness of $k = 60$ kN/m, three 304 stainless steel rods, AB and CD, which have a diameter of 5 mm, and EF, which has a diameter of 12 mm, and a rigid beam GH. If the pipe is displaced 82 mm when it is filled with fluid, determine the weight of the fluid.



SOLUTION

Internal Force in the Rods:

FBD (a)



$$\circlearrowleft + \Sigma M_A = 0; \qquad F_{CD}(0.5) - W(0.25) = 0 \qquad F_{CD} = \frac{W}{2}$$

$$+\uparrow \Sigma F_y = 0; \qquad F_{AB} + \frac{W}{2} - W = 0 \qquad F_{AB} = \frac{W}{2}$$

FBD (b)

$$+\uparrow \Sigma F_y = 0; \qquad F_{EF} - \frac{W}{2} - \frac{W}{2} = 0 \qquad F_{EF} = W$$

Displacement:

$$\delta_D = \delta_E = \frac{F_{EF}L_{EF}}{A_{EF}E} = \frac{W(750)}{\frac{\pi}{4}(0.012)^2(193)(10^9)}$$

$$= 34.35988(10^{-6})\,W$$

$$\delta_{A/B} = \delta_{C/D} = \frac{F_{CD}L_{CD}}{A_{CD}E} = \frac{\frac{W}{2}(750)}{\frac{\pi}{4}(0.005)^2(193)(10^9)}$$

$$= 98.95644(10^{-6})\,W$$

$$\delta_C = \delta_D + \delta_{C/D}$$

$$= 34.35988(10^{-6})\,W + 98.95644(10^{-6})\,W$$

$$= 0.133316(10^{-3})\,W$$

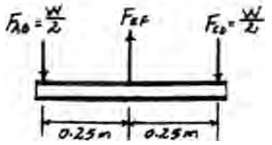
Displacement of the spring

$$\delta_{sp} = \frac{F_{sp}}{k} = \frac{\frac{W}{2}}{60(10^3)}(1000) = 0.008333\,W$$

$$\delta_{\text{lat}} = \delta_C + \delta_{sp}$$

$$82 = 0.133316(10^{-3})\,W + 0.008333W$$

$$W = 9685\text{ N} = 9.69\text{ kN} \qquad \textbf{Ans.}$$



Ans.
$W = 9.69$ kN

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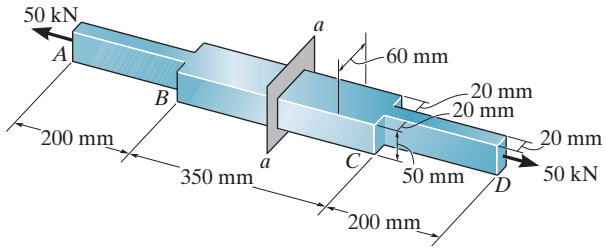
4–15.

The steel bar has the original dimensions shown in the figure. If it is subjected to an axial load of 50 kN, determine the change in its length and its new cross-sectional dimensions at section *a–a*. $E_{st} = 200$ GPa, $\nu_{st} = 0.29$.



SOLUTION

$$\delta_{A/D} = \Sigma \frac{PL}{AE} = \frac{2(50)(10^3)(200)}{(0.02)(0.05)(200)(10^9)} + \frac{50(10^3)(350)}{(0.06)(0.05)(200)(10^9)}$$

$$= 0.129 \text{ mm} \qquad \textbf{Ans.}$$

$$\delta_{B/C} = \frac{PL}{AE} = \frac{50(10^3)(350)}{(0.06)(0.05)(200)(10^9)} = 0.02917 \text{ mm}$$

$$\epsilon_{BC} = \frac{\delta_{B/C}}{L_{BC}} \frac{0.02917}{350} = 0.00008333$$

$$\epsilon_{\text{lat}} = -\nu\, \epsilon_{\text{long}} = -(0.29)(0.00008333) = -0.00002417$$

$$h' = 50 - 50(0.00002417) = 49.9988 \text{ mm} \qquad \textbf{Ans.}$$

$$w' = 60 - 60(0.00002417) = 59.9986 \text{ mm} \qquad \textbf{Ans.}$$

Ans:
$\delta_{A/D} = 0.129$ mm,
$h' = 49.9988$ mm,
$w' = 59.9986$ mm

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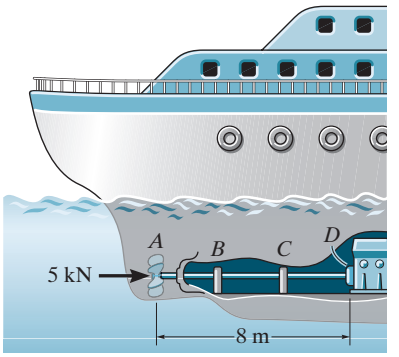
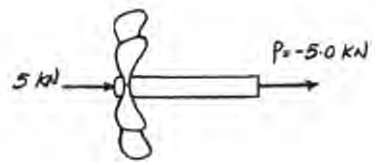
***4–16.** The ship is pushed through the water using an A-36 steel propeller shaft that is 8 m long, measured from the propeller to the thrust bearing D at the engine. If it has an outer diameter of 400 mm and a wall thickness of 50 mm, determine the amount of axial contraction of the shaft when the propeller exerts a force on the shaft of 5 kN. The bearings at B and C are journal bearings.



SOLUTION

Internal Force: As shown on *FBD*.



Displacement:

$$\delta_A = \frac{PL}{AE} = \frac{-5.00\,(10^3)(8)}{\frac{\pi}{4}\,(0.4^2 - 0.3^2)\,200(10^9)}$$

$$= -3.638(10^{-6})\ \text{m}$$

$$= -3.64\left(10^{-3}\right)\ \text{mm} \qquad \textbf{Ans.}$$

Negative sign indicates that end A moves towards end D.

Ans.
$\delta_A = -3.64(10^{-3})$ mm

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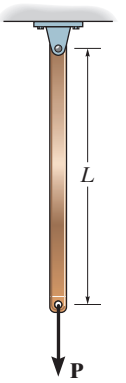
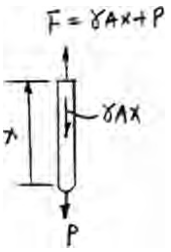
4–17. The bar has a length L and cross-sectional area A. Determine its elongation due to the force **P** and its own weight. The material has a specific weight γ (weight/volume) and a modulus of elasticity E.



SOLUTION

$$\delta = \int \frac{P(x)\,dx}{A(x)\,E} = \frac{1}{AE}\int_0^L (\gamma A x + P)\,dx$$

$$= \frac{1}{AE}\left(\frac{\gamma A L^2}{2} + PL\right) = \frac{\gamma L^2}{2E} + \frac{PL}{AE}$$ **Ans.**



Ans.

$$\delta = \frac{1}{AE}\int_0^L (\gamma A x + P)\,dx = \frac{\gamma L^2}{2E} + \frac{PL}{AE}$$

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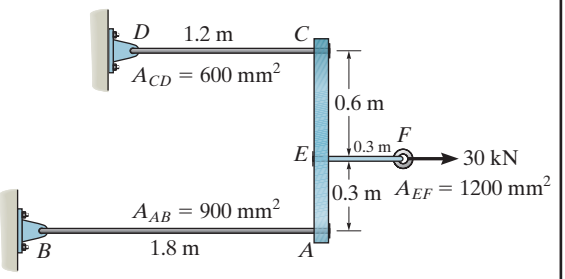
4–18. The assembly consists of three titanium (Ti-6A1-4V) rods and a rigid bar *AC*. The cross-sectional area of each rod is given in the figure. If a force of 30 kN is applied to the ring *F*, determine the horizontal displacement of point *F*.

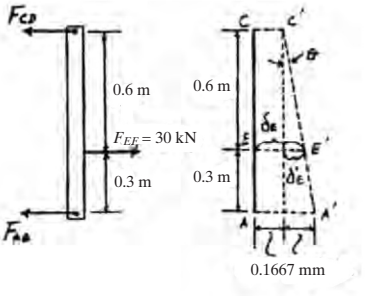


SOLUTION

Internal Force in the Rods:

$\zeta + \Sigma M_A = 0;\quad F_{CD}(0.9) - 30(0.3) = 0 \quad F_{CD} = 10\ \text{kN}$

$\stackrel{+}{\rightarrow} \Sigma F_x = 0;\quad 30 - 10 - F_{AB} = 0 \quad F_{AB} = 20\ \text{kN}$



Displacement:

$$\delta_C = \frac{F_{CD}L_{CD}}{A_{CD}E} = \frac{10(10^3)(1.2)(10^3)}{[600(10^{-6})][120(10^9)]} = 0.1667\ \text{mm}$$

$$\delta_A = \frac{F_{AB}L_{AB}}{A_{AB}E} = \frac{20(10^3)(1.8)(10^3)}{[900(10^{-6})][120(10^9)]} = 0.3333\ \text{mm}$$

$$\delta_{F/E} = \frac{F_{EF}L_{EF}}{A_{EF}E} = \frac{30(10^3)(0.3)(10^3)}{[1200(10^{-6})][120(10^9)]} = 0.0625\ \text{mm}$$

$$\frac{\delta'_E}{0.6} = \frac{0.1667}{0.9}, \quad \delta'_E = 0.1111\ \text{mm}$$

$$\delta_E = \delta_C + \delta'_E = 0.1667 + 0.1111 = 0.2778\ \text{mm}$$

$$\delta_F = \delta_E + \delta_{F/E}$$
$$= 0.2778 + 0.0625 = 0.3403\ \text{mm} = 0.340\ \text{mm}$$

Ans.

Ans.
$\delta_F = 0.3403\ \text{mm}$

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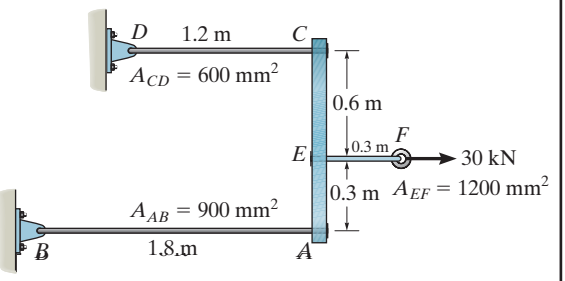
4–19.

The assembly consists of three titanium (Ti-6A1-4V) rods and a rigid bar AC. The cross-sectional area of each rod is given in the figure. If a forceof 30 kN is applied to the ring F, determine the angle of tilt of bar AC.



SOLUTION

Internal Force in the Rods:

$\zeta + \Sigma M_A = 0;$ $\quad F_{CD}(0.9) - 30(0.3) = 0$ $\quad F_{CD} = 10$ kN

$\pm\rightarrow \Sigma F_x = 0;$ $\quad 30 - 10 - F_{AB} = 0$ $\quad F_{AB} = 20$ kN

Displacement:

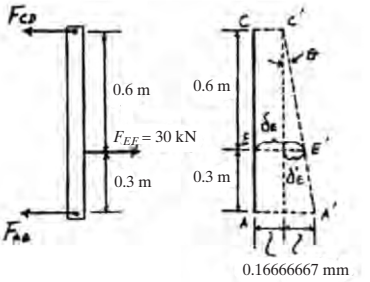
$$\delta_C = \frac{F_{CD}L_{CD}}{A_{CD}E} = \frac{10(10^3)(1.2)(10^3)}{[600(10^{-6})][120(10^9)]} = 0.1667 \text{ mm}$$

$$\delta_A = \frac{F_{AB}L_{AB}}{A_{AB}E} = \frac{20(10^3)(1.8)(10^3)}{[900(10^{-6})][120(10^9)]} = 0.3333 \text{ mm}$$

$$\theta = \tan^{-1}\frac{\delta_A - \delta_C}{L_{AC}} = \tan^{-1}\frac{0.3333 - 0.1667}{0.9(10^3)}$$

$$= 0.01061° = 0.0106°$$

Ans.



Ans.

$\theta = 0.0106°$

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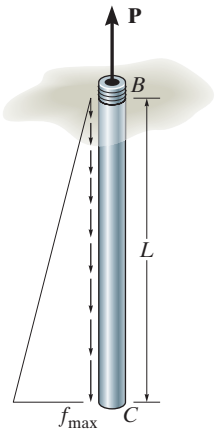
***4–20.**

The pipe is stuck in the ground so that when it is pulled upward the frictional force along its length varies linearly from zero at B to $f_{\max}$ (force/length) at C. Determine the initial force P required to pull the pipe out and the pipe's elongation just before it starts to slip. The pipe has a length L, cross-sectional area A, and the material from which it is made has a modulus of elasticity E.

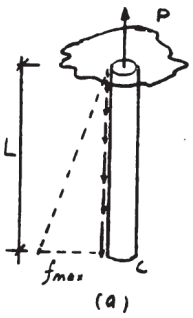


SOLUTION

From FBD (a)

$$+\uparrow \Sigma F_y = 0; \qquad P - \frac{1}{2}(F_{\max} L) = 0$$

$$P = \frac{F_{\max} L}{2} \qquad \textbf{Ans.}$$



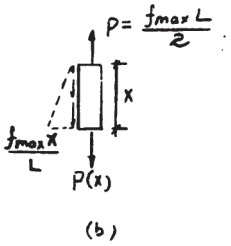
From FBD (b)

$$+\downarrow \Sigma F_y = 0; \qquad P(x) + \frac{1}{2}\left(\frac{F_{\max} x}{L}\right)x - \frac{F_{\max} L}{2} = 0$$

$$P(x) = \frac{F_{\max} L}{2} - \frac{F_{\max} x^2}{2L}$$

$$\delta = \int_0^L \frac{P(x)\,dx}{A(x)E} = \int_0^L \frac{F_{\max} L}{2AE} dx - \int_0^L \frac{F_{\max} x^2}{2AEL} dx$$

$$= \frac{F_{\max} L^2}{3AE} \qquad \textbf{Ans.}$$



Ans:

$$P = \frac{F_{\max} L}{2},$$

$$\delta = \frac{F_{\max} L^2}{3AE}$$

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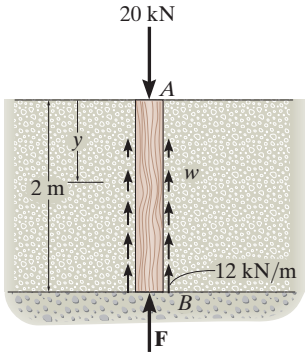
4–21.

The post is made of Douglas fir and has a diameter of 100 mm. If it is subjected to the load of 20 kN and the soil provides a frictional resistance distributed around the post that is triangular along its sides; that is, it varies from $w = 0$ at $y = 0$ to $w = 12$ kN/m at $y = 2$ m, determine the force **F** at its bottom needed for equilibrium. Also, what is the displacement of the top of the post A with respect to its bottom B? Neglect the weight of the post.



SOLUTION

Equation of Equilibrium: Referring to the FBD of the entire post, Fig. *a*,

$$+\uparrow \Sigma F_y = 0; \qquad F + \frac{1}{2}(12)(2) - 20 = 0 \qquad F = 8.00 \text{ kN} \qquad \textbf{Ans.}$$

Normal Force: Referring to the FBD of the upper segment of the post sectioned at arbitrary distance y, Fig. *b*

$$+\uparrow \Sigma F_y = 0; \qquad \frac{1}{2}(6y)(y) - 20 - P(y) = 0 \qquad Py = \left(3y^2 - 20\right) \text{kN}$$

Displacement: For Douglas Fir, $E = 13.1$ GPa

$$\delta_{A/B} = \int_0^L \frac{N(y)dy}{A(y)E} = \frac{1}{AE}\int_0^{2 \text{ meters}} \left(3y^2 - 20\right)dy$$

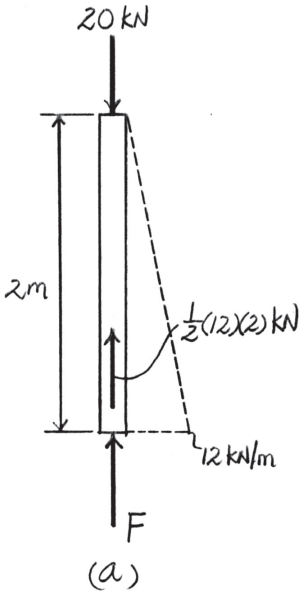
$$= \frac{1}{AE}\left(y^3 - 20y\right)\Big|_0^{2 \text{ meters}}$$

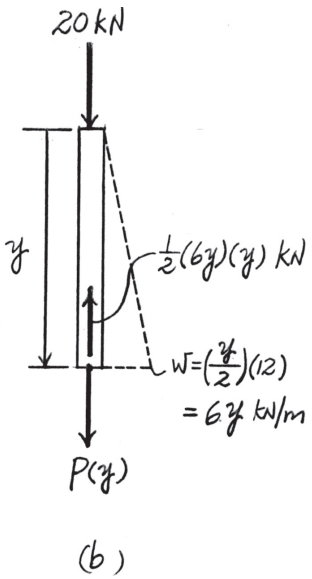
$$= -\frac{32 \text{ kN} \cdot \text{m}}{AE}$$

$$= -\frac{32\left(10^3\right)}{\frac{\pi}{4}\left(0.1^2\right)\left[13.1\left(10^9\right)\right]}$$

$$= -0.3110\left(10^{-3}\right) \text{ m} = -0.311 \text{ mm} \qquad \textbf{Ans.}$$

The sign indicates that end A moves toward end B.





Ans:
$F = 8.00$ kN,
$\delta_{A/B} = -0.311$ mm

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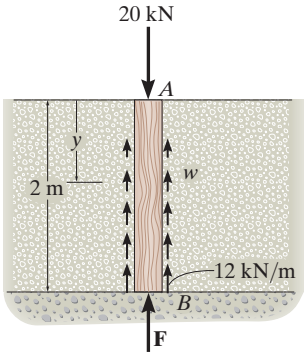
4–22.

The post is made of Douglas fir and has a diameter of 100 mm. If it is subjected to the load of 20 kN and the soil provides a frictional resistance that is distributed along its length and varies linearly from $w = 4$ kN/m at $y = 0$ to $w = 12$ kN/m at $y = 2$ m, determine the force **F** at its bottom needed for equilibrium. Also, what is the displacement of the top of the post A with respect to its bottom B? Neglect the weight of the post.



SOLUTION

Equation of Equilibrium: Referring to the FBD of the entire post, Fig. *a*,

$$+\uparrow \Sigma F_y = 0; \quad F + \frac{1}{2}(4 + 12)(2) - 20 = 0 \qquad F = 4.00 \text{ kN} \qquad \textbf{Ans.}$$

Normal Force: Referring to the FBD of the upper segment of the post sectioned at arbitrary distance y, Fig. *b*,

$$+\uparrow \Sigma F_y = 0; \quad (4 + 2y)y - 20 - P(y) = 0 \qquad P(y) = \left(2y^2 + 4y - 20\right) \text{kN}$$

Displacement: For Douglas Fir, $E = 13.1$ GPa

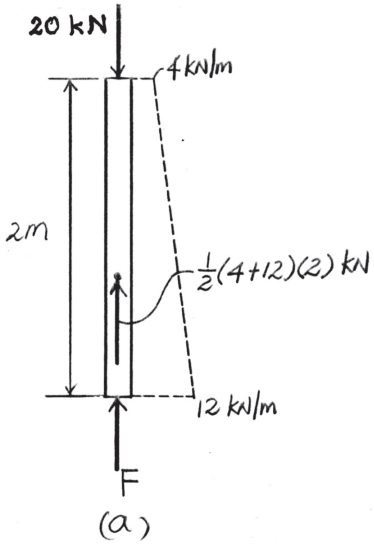
$$\delta_{A/B} = \int_0^L \frac{N(y)dy}{A(y)E} = \frac{1}{AE}\int_0^{2\text{ m}} \left(2y^2 + 4y - 20\right)dy$$

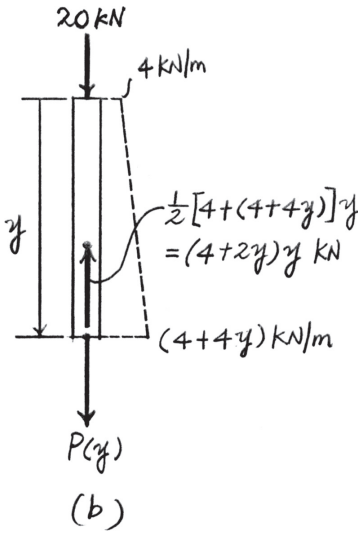
$$= \frac{1}{AE}\left(\frac{2}{3}y^3 + 2y^2 - 20y\right)\Big|_0^{2\text{ m}}$$

$$= -\frac{80 \text{ kN} \cdot \text{m}}{3\,AE}$$

$$= -\frac{80(10^3)}{3\left[\frac{\pi}{4}(0.1^2)\right][13.1(10^9)]}$$

$$= -0.2592(10^{-3}) \text{ m} = -0.259 \text{ mm} \qquad \textbf{Ans.}$$





Ans:
$F = 4.00$ kN,
$\delta_{A/B} = -0.259$ mm

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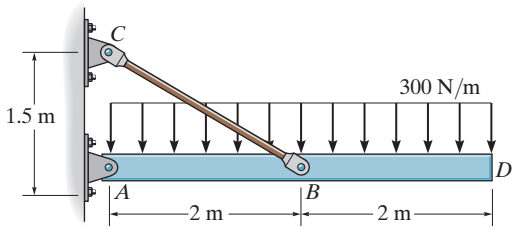
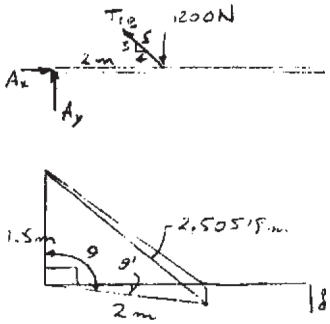
4–23.

The rigid bar is supported by the pin-connected rod *CB* that has a cross-sectional area of 14 mm^2 and is made from 6061-T6 aluminum. Determine the vertical deflection of the bar at *D* when the distributed load is applied.



SOLUTION



$\circlearrowleft + \Sigma M_A = 0; \quad 1200(2) - T_{CB}(0.6)(2) = 0$

$$T_{CB} = 2000 \text{ N}$$

$$\delta_{B/C} = \frac{PL}{AE} = \frac{(2000)(2.5)}{14(10^{-6})(68.9)(10^9)} = 0.0051835$$

$$(2.5051835)^2 = (1.5)^2 + (2)^2 - 2(1.5)(2)\cos\theta$$

$$\theta = 90.248°$$

$$\theta = 90.248° - 90° = 0.2478° = 0.004324 \text{ rad}$$

$$\delta_D = \theta\, r = 0.004324(4000) = 17.3 \text{ mm}$$ **Ans.**

Ans:
$\delta_D = 17.3$ mm

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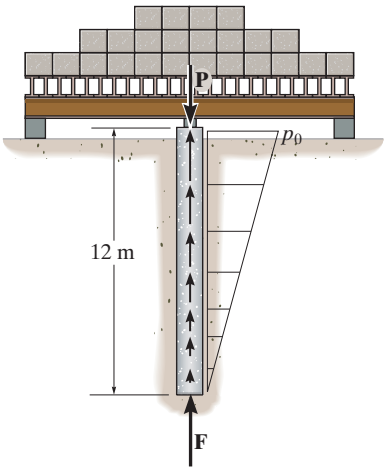
***4–24.** The weight of the kentledge exerts an axial force of $P = 1500$ kN on the 300-mm diameter high-strength concrete bore pile. If the distribution of the resisting skin friction developed from the interaction between the soil and the surface of the pile is approximated as shown, determine the resisting bearing force **F** for equilibrium. Take $p_0 = 180$ kN/m. Also, find the corresponding elastic shortening of the pile. Neglect the weight of the pile.



SOLUTION

Internal Loading: By considering the equilibrium of the pile with reference to its entire free-body diagram shown in Fig. *a*. We have

$$+\uparrow \Sigma F_y = 0; \qquad F + \frac{1}{2}(180)(12) - 1500 = 0 \qquad F = 420 \text{ kN} \qquad \textbf{Ans.}$$

Also,

$$p(y) = \frac{180}{12}y = 15y \text{ kN/m}$$

The normal force developed in the pile as a function of y can be determined by considering the equilibrium of the sectional of the pile with reference to its free-body diagram shown in Fig. *b*.

$$+\uparrow \Sigma F_y = 0; \qquad \frac{1}{2}(15y)y + 420 - P(y) = 0 \qquad P(y) = (7.5y^2 + 420) \text{ kN}$$

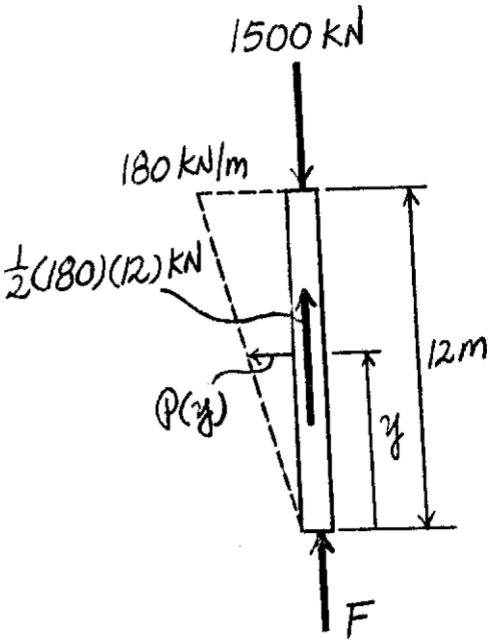
Displacement: The cross-sectional area of the pile is $A = \frac{\pi}{4}(0.3^2) = 0.0225\pi \text{ m}^2$. We have

$$\delta = \int_0^L \frac{P(y)dy}{A(y)E} = \int_0^{12\text{ m}} \frac{(7.5y^2 + 420)(10^3)dy}{0.0225\pi(29.0)(10^9)}$$

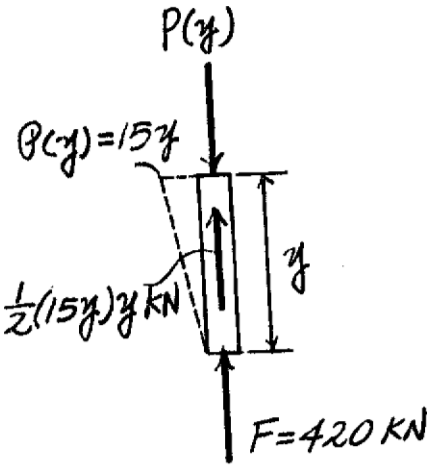
$$= \int_0^{12\text{ m}} \left[3.6587(10^{-6})y^2 + 0.2049(10^{-3})\right]dy$$

$$= \left[1.2196(10^{-6})y^3 + 0.2049(10^{-3})y\right]_0^{12\text{ m}}$$

$$= 4.566(10^{-3}) \text{ m} = 4.57 \text{ mm} \qquad \textbf{Ans.}$$



(a)



(b)

Ans:

$F = 420$ kN, $\delta = 4.57$ mm

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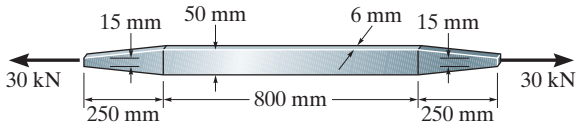
4–25.

Determine the elongation of the aluminum strap when it is subjected to an axial force of 30 kN. $E_{al} = 70$ GPa.



SOLUTION

$$\delta = (2)\frac{Ph}{Et(d_2 - d_1)}\ln\frac{d_2}{d_1} + \frac{PL}{AE}$$

$$= \frac{2(30)(10^3)(250)}{(70)(10^9)(0.006)(0.05 - 0.015)}\left(\ln\frac{50}{15}\right) + \frac{30(10^3)(800)}{(0.006)(0.05)(70)(10^9)}$$

$$= 2.37 \text{ mm}$$ **Ans.**

Ans:
$\delta = 2.37$ mm

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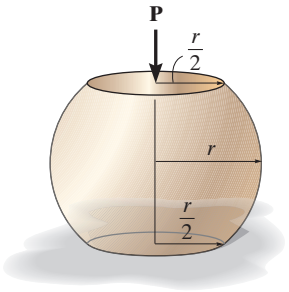
4–26.

The ball is truncated at its ends and is used to support the bearing load **P**. If the modulus of elasticity for the material is E, determine the decrease in the ball's height when the load is applied.



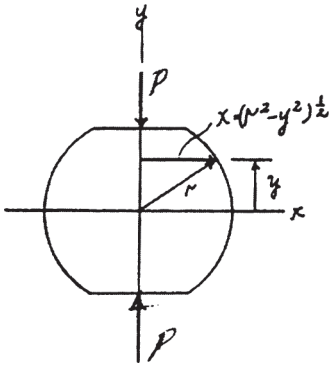
SOLUTION

Displacement:

Geometry:

$$A(y) = \pi x^2 = \pi(r^2 - y^2)$$

Displacement: When $x = \frac{r}{2}$, $\quad y = \pm\frac{\sqrt{3}}{2}r$



$$\delta = \int_0^L \frac{P(y)\,dy}{A(y)\,E}$$

$$= \frac{P}{\pi E}\int_{-\frac{\sqrt{3}}{2}r}^{\frac{\sqrt{3}}{2}r} \frac{dy}{r^2 - y^2}$$

$$= \frac{P}{\pi E}\left[\frac{1}{2r}\ln\frac{r + y}{r - y}\right]\Bigg|_{-\frac{\sqrt{3}}{2}r}^{\frac{\sqrt{3}}{2}r}$$

$$= \frac{P}{2\pi r E}[\ \ln 13.9282 - \ln 0.07180\ ]$$

$$= \frac{2.63\,P}{\pi r E}$$ **Ans.**

Ans:

$$\delta = \frac{2.63\,P}{\pi r E}$$

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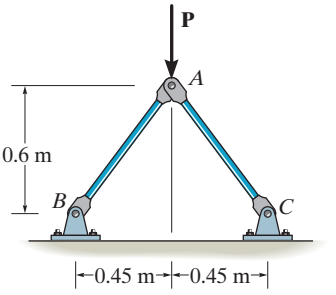
4–27. The linkage is made of two pin-connected A-36 steel members, each having a cross-sectional area of 1000 mm^2. If a vertical force of P = 250 kN is applied to point A, determine its vertical displacement at A.



SOLUTION

Analysing the equilibrium of Joint A by referring to its FBD, Fig. a,

$$\stackrel{+}{\rightarrow}\Sigma F_x = 0; \qquad F_{AC}\left(\frac{3}{5}\right) - F_{AB}\left(\frac{3}{5}\right) = 0 \qquad F_{AC} = F_{AB} = F$$

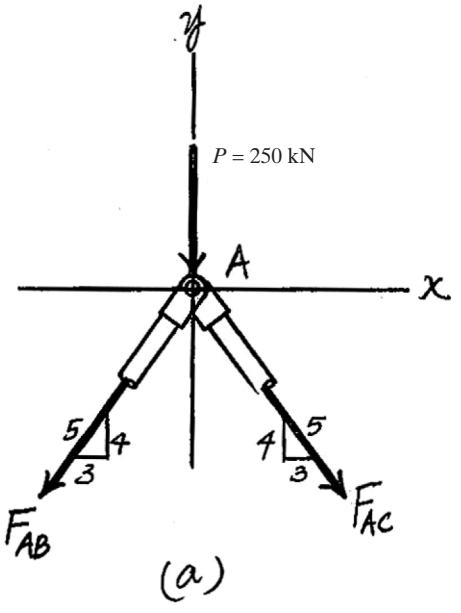
$$+\uparrow \Sigma F_y = 0 \qquad -2F\left(\frac{4}{5}\right) - 250 = 0 \qquad F = -156.25 \text{ kN}$$

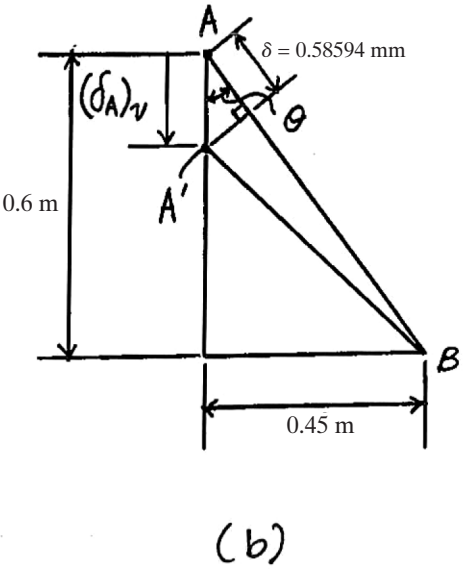
The initial length of members AB and AC is $L = \sqrt{0.45^2 + 0.6^2} = 0.75$ m. The axial deformation of members AB and AC is

$$\delta = \frac{FL}{AE} = \frac{(-156.25)(10^3)(750)}{[1000(10^{-6})][200(10^9)]} = -0.58594 \text{ mm}$$

The negative sign indicates that end A moves toward B and C. From the geometry shown in Fig. b, $\theta = \tan^{-1}\left(\frac{0.45}{0.6}\right) = 36.87°$. Thus,

$$(\delta_A)_v = \frac{\delta}{\cos\theta} = \frac{0.58594}{\cos 36.87°} = 0.7324 \text{ mm} = 0.732 \text{ mm} \downarrow \qquad \textbf{Ans.}$$





Ans.
$(\delta_A)_v = 0.732$ mm ↓

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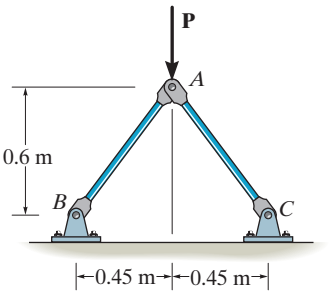
*4–28. The linkage is made of two pin-connected A-36 steel members, each having a cross-sectional area of 1000 mm^2. Determine the magnitude of the force **P** needed to displace point A 0.625 mm downward.



SOLUTION

Analysing the equilibrium of joint A by referring to its *FBD*, Fig. *a*

$$\stackrel{+}{\rightarrow}\Sigma F_x = 0; \qquad F_{AC}\left(\frac{3}{5}\right) - F_{AB}\left(\frac{3}{5}\right) = 0 \qquad F_{AC} = F_{AB} = F$$

$$+\uparrow \Sigma F_y = 0; \qquad -2F\left(\frac{4}{5}\right) - P = 0 \qquad F = -0.625\,P$$

The initial length of members AB and AC are $L = \sqrt{0.45^2 + 0.6^2} = 0.75$ m. The axial deformation of members AB and AC is

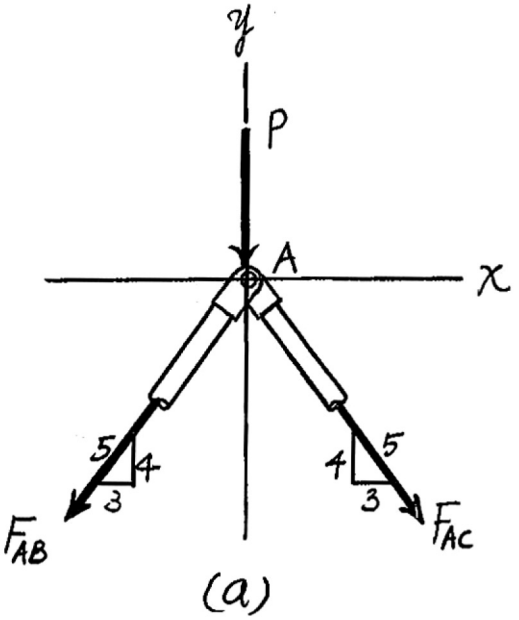
$$\delta = \frac{FL}{AE} = \frac{-0.625P(750)}{[1000(10^{-6})][200(10^{9})]} = -2.34375(10^{-6})P$$

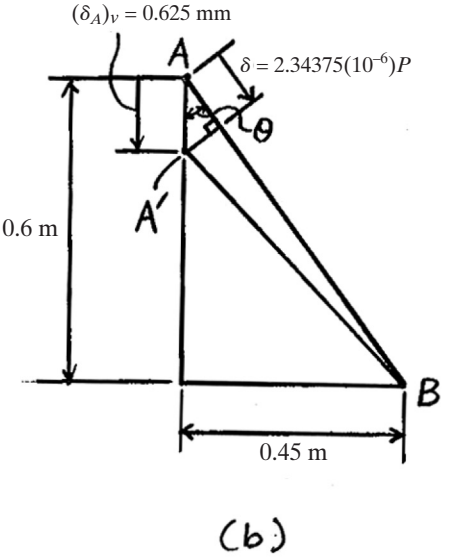
The negative sign indicates that end A moves toward B and C. From the geometry shown in Fig. *b*, we obtain $\theta = \tan^{-1}\left(\frac{0.45}{0.6}\right) = 36.87°$. Thus,

$$(\delta_A)_v = \frac{\delta}{\cos\theta}$$

$$0.625 = \frac{2.34375(10^{-6})P}{\cos 36.87°}$$

$$P = 213.33(10^{3})\ \text{N} = 213\ \text{kN} \qquad \textbf{Ans.}$$





Ans.
$P = 213$ kN

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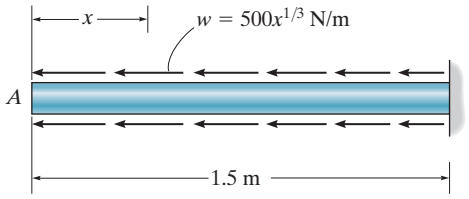
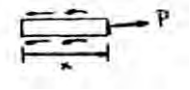
4–29. The bar has a cross-sectional area of 1800 mm^2, and E = 250 GPa. Determine the displacement of its end A when it is subjected to the distributed loading.



SOLUTION



$$P(x) = \int_0^x w\,dx = 500\int_0^x x^{\frac{1}{3}}\,dx = \left(375x^{4/3}\right)\text{N}$$

$$\delta_A = \int_0^L \frac{P(x)\,dx}{AE} = \int_0^{1.5\text{ m}} \frac{\left(375x^{4/3}\right)dx}{[1800(10^{-6})][250(10^9)]} = 0.8333(10^{-6})\left(\frac{3}{7}x^{7/3}\right)\Big|_0^{1.5\text{ m}} = 0.9199(10^{-6})\text{ m} = 0.920\text{ mm}$$

$\delta_A = 9.199(10^{-7})$ m **Ans.**

= 0.00092 mm

Ans:
δ_A = 2.990 mm

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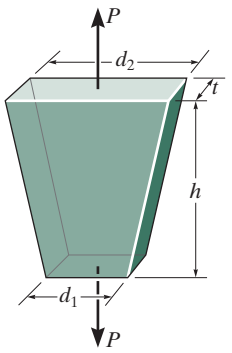
4–30.

Determine the relative displacement of one end of the tapered plate with respect to the other end when it is subjected to an axial load P.



SOLUTION

$$w = d_1 + \frac{d_2 - d_1}{h}x = \frac{d_1 h + (d_2 - d_1)x}{h}$$

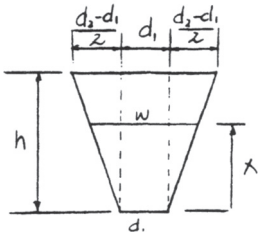
$$\delta = \int \frac{P(x)\,dx}{A(x)E} = \frac{P}{E}\int_0^h \frac{dx}{\frac{[d_1 h + (d_2 - d_1)x]t}{h}}$$

$$= \frac{Ph}{E\,t}\int_0^h \frac{dx}{d_1 h + (d_2 - d_1)x}$$

$$= \frac{Ph}{E\,t\,d_1 h}\int_0^h \frac{dx}{1 + \frac{d_2 - d_1}{d_1 h}x} = \frac{Ph}{E\,t\,d_1 h}\left(\frac{d_1 h}{d_2 - d_1}\right)\left[\ln\left(1 + \frac{d_2 - d_1}{d_1 h}x\right)\right]_0^h$$

$$= \frac{Ph}{E\,t(d_2 - d_1)}\left[\ln\left(1 + \frac{d_2 - d_1}{d_1}\right)\right] = \frac{Ph}{E\,t(d_2 - d_1)}\left[\ln\left(\frac{d_1 + d_2 - d_1}{d_1}\right)\right]$$

$$= \frac{Ph}{E\,t(d_2 - d_1)}\left[\ln\frac{d_2}{d_1}\right] \qquad \textbf{Ans.}$$



Ans:

$$\delta = \frac{Ph}{E\,t(d_2 - d_1)}\left[\ln\frac{d_2}{d_1}\right]$$

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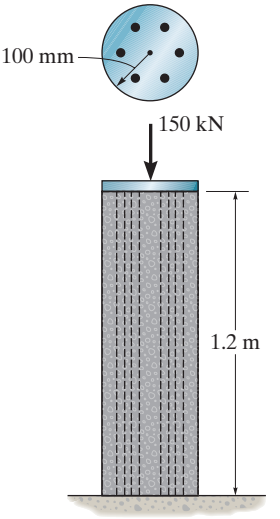
4–31. The column is constructed from high-strength concrete and six A-36 steel reinforcing rods. If it is subjected to an axial force of 150 kN, determine the average normal stress in the concrete and in each rod. Each rod has a diameter of 20 mm.



SOLUTION

Equations of Equilibrium:

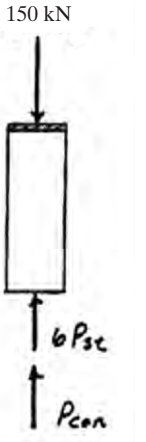
$$+\uparrow \Sigma F_y = 0; \qquad 6P_{st} + P_{\text{con}} - 150 = 0 \qquad \textbf{[1]}$$

Compatibility:

$$\delta_{st} = \delta_{\text{con}}$$

$$\frac{P_{st}(1.2)}{\left[\frac{\pi}{4}(0.02^2)\right][200(10^9)]} = \frac{P_{\text{con}}(1.2)}{\left[\frac{\pi}{4}(0.2^2) - 6\left(\frac{\pi}{4}\right)(0.02^2)\right][29.0(10^9)]}$$

$$P_{st} = 0.073368\, P_{\text{con}} \qquad \textbf{[2]}$$



Solving Eqs. [1] and [2] yields:

$$P_{st} = 7.641 \text{ kN} \qquad P_{\text{con}} = 104.152 \text{ kN}$$

Average Normal Stress:

$$\sigma_{st} = \frac{P_{st}}{A_{st}} = \frac{7.641(10^3)}{\frac{\pi}{4}(0.02^2)} = 24.32(10^6) \text{ N/m}^2 = 24.3 \text{ MPa} \qquad \textbf{Ans.}$$

$$\sigma_{\text{con}} = \frac{P_{\text{con}}}{A_{\text{con}}} = \frac{104.15(10^3)}{\frac{\pi}{4}(0.2^2) - 6\left(\frac{\pi}{4}\right)(0.02^2)} = 3.527(10^6) \text{ N/m}^2 = 3.53 \text{ MPa} \qquad \textbf{Ans.}$$

Ans.
$\sigma_{\text{st}} = 24.3$ MPa, $\sigma_{\text{con}} = 3.53$ MPa

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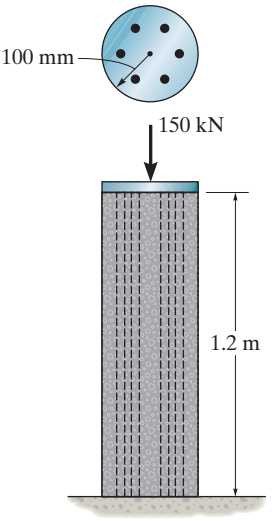
***4–32.** The column is constructed from high-strength concrete and six A-36 steel reinforcing rods. If it is subjected to an axial force of 150 kN, determine the required diameter of each rod so that one-fourth of the load is carried by the concrete and three-fourths by the steel.



SOLUTION

Equilibrium: The force of 150 kN is required to distribute in such a manner that 3/4 of the force is carried by steel and 1/4 of the force is carried by concrete. Hence

$$P_{st} = \frac{3}{4}(150) = 112.5 \text{ kN} \qquad P_{\text{con}} = \frac{1}{4}(150) = 37.5 \text{ kN}$$

Compatibility:

$$\delta_{st} = \delta_{\text{con}}$$

$$\frac{P_{st}L}{A_{st}E_{st}} = \frac{P_{\text{con}}L}{A_{\text{con}}E_{\text{con}}}$$

$$A_{st} = \frac{P_{st}A_{\text{con}}E_{\text{con}}}{P_{\text{con}}E_{st}}$$

$$6\left(\frac{\pi}{4}\right)d^2 = \frac{(112.5)\left[\frac{\pi}{4}(0.2^2) - 6\left(\frac{\pi}{4}\right)(0.02^2)\right]\left[29(10^9)\right]}{(37.5)\left[200(10^9)\right]}$$

$$d = 0.05221 \text{ m} = 52.2 \text{ mm}$$ **Ans.**

Ans.
$d = 52.2$ mm

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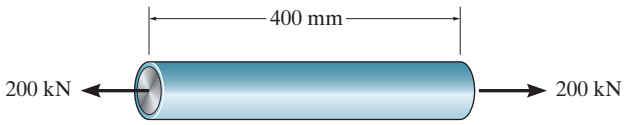
4–33.

The A-36 steel pipe has a 6061-T6 aluminum core. It is subjected to a tensile force of 200 kN. Determine the average normal stress in the aluminum and the steel due to this loading. The pipe has an outer diameter of 80 mm and an inner diameter of 70 mm.



SOLUTION

Equations of Equilibrium:

$$\overset{+}{\leftarrow} \Sigma F_x = 0; \qquad P_{al} + P_{st} - 200 = 0 \qquad \textbf{(1)}$$

Compatibility:

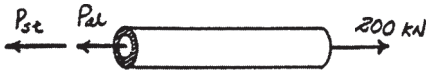
$$\delta_{al} = \delta_{st}$$

$$\frac{P_{al}(400)}{\frac{\pi}{4}(0.07^2)(68.9)(10^9)} = \frac{P_{st}(400)}{\frac{\pi}{4}(0.08^2 - 0.07^2)(200)(10^9)}$$

$$P_{al} = 1.125367\, P_{st} \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields:

$$P_{st} = 94.10 \text{ kN} \qquad P_{al} = 105.90 \text{ kN}$$



Average Normal Stress:

$$\sigma_{al} = \frac{P_{al}}{A_{al}} = \frac{105.90(10^3)}{\frac{\pi}{4}(0.07^2)} = 27.5 \text{ MPa} \qquad \textbf{Ans.}$$

$$\sigma_{st} = \frac{P_{st}}{A_{st}} = \frac{94.10(10^3)}{\frac{\pi}{4}(0.08^2 - 0.07^2)} = 79.9 \text{ MPa} \qquad \textbf{Ans.}$$

Ans:
$\sigma_{al} = 27.5$ MPa,
$\sigma_{st} = 79.9$ MPa

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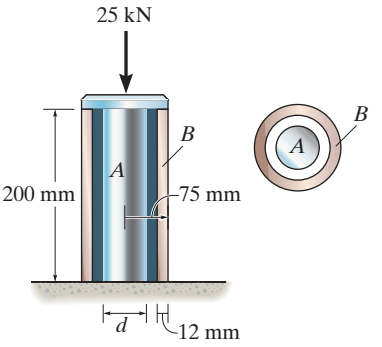
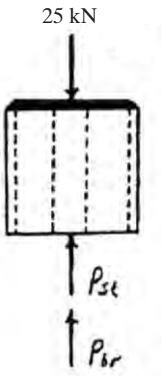
4–34. The 304 stainless steel post A has a diameter of $d = 50$ mm and is surrounded by a red brass C83400 tube B. Both rest on the rigid surface. If a force of 25 kN is applied to the rigid cap, determine the average normal stress developed in the post and the tube.



SOLUTION

Equations of Equilibrium:

$$+\uparrow \Sigma F_y = 0; \qquad P_{st} + P_{br} - 25 = 0 \qquad \textbf{[1]}$$



Compatibility:

$$\delta_{st} = \delta_{br}$$

$$\frac{P_{st}(200)}{\left[\frac{\pi}{4}(0.05^2)\right][193(10^9)]} = \frac{P_{b\pi}(200)}{\left[\frac{\pi}{4}(0.15^2 - 0.126^2)\right][101(10^9)]}$$

$$P_{st} = 0.72120\, P_{br} \qquad \textbf{[2]}$$

Solving Eqs. [1] and [2] yields:

$$P_{br} = 14.525 \text{ kN} \qquad P_{st} = 10.475 \text{ kN}$$

Average Normal Stress:

$$\sigma_{br} = \frac{P_{br}}{A_{br}} = \frac{14.525(10^3)}{\frac{\pi}{4}(0.15^2 - 0.126^2)} = 2.792(10^6) \text{ N/m}^2 = 2.79 \text{ MPa} \qquad \textbf{Ans.}$$

$$\sigma_{st} = \frac{P_{st}}{A_{st}} = \frac{10.475(10^3)}{\frac{\pi}{4}(0.05^2)} = 5.335(10^6) \text{ N/m}^2 = 5.34 \text{ MPa} \qquad \textbf{Ans.}$$

Ans.
$\sigma_{br} = 2.79$ MPa, $\sigma_{st} = 5.34$ MPa

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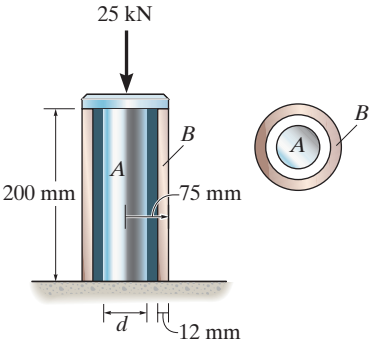
4–35. The 304 stainless steel post *A* is surrounded by a red brass C83400 tube *B*. Both rest on the rigid surface. If a force of 25 kN is applied to the rigid cap, determine the required diameter *d* of the steel post so that the load is shared equally between the post and tube.



SOLUTION

Equilibrium: The force of 25 kN is shared equally by the brass and steel. Hence

$$P_{st} = P_{br} = P = 12.5 \text{ kN}$$

Compatibility:

$$\delta_{st} = \delta_{br}$$

$$\frac{PL}{A_{st}E_{st}} = \frac{PL}{A_{br}E_{br}}$$

$$A_{st} = \frac{A_{br}E_{br}}{E_{st}}$$

$$\left(\frac{\pi}{4}\right)d^2 = \frac{\left[\frac{\pi}{4}(0.15^2 - 0.126^2)\right]\left[101(10^9)\right]}{193(10^9)}$$

$$d = 0.05888 \text{ m} = 58.9 \text{ mm} \qquad \textbf{Ans.}$$

Ans.
$d = 58.9$ mm

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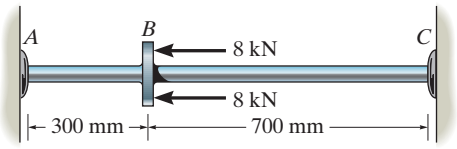
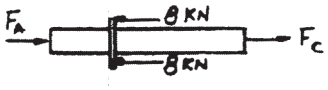
***4–36.**

The A-36 steel pipe has an outer radius of 20 mm and an inner radius of 15 mm. If it fits snugly between the fixed walls before it is loaded, determine the reaction at the walls when it is subjected to the load shown.



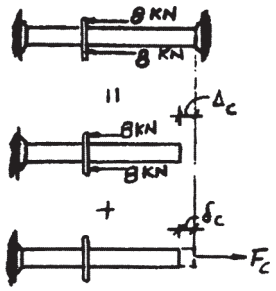
SOLUTION

$$\stackrel{+}{\rightarrow} \Sigma F_x = 0; \qquad F_A + F_C - 16 = 0 \qquad \textbf{(1)}$$



By superposition:

$$\left(\stackrel{+}{\rightarrow}\right) \qquad 0 = -\Delta_C + \delta_C$$



$$0 = \frac{-16\,(300)}{AE} + \frac{F_C\,(1000)}{AE}$$

$$F_C = 4.80 \text{ kN} \qquad \textbf{Ans.}$$

From Eq. (1),

$$F_A = 11.2 \text{ kN} \qquad \textbf{Ans.}$$

Ans:
$F_C = 4.80$ kN,
$F_A = 11.2$ kN

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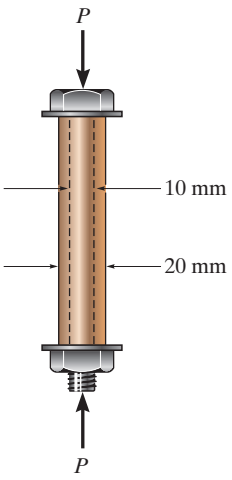
4–37.

The 10-mm-diameter steel bolt is surrounded by a bronze sleeve. The outer diameter of this sleeve is 20 mm, and its inner diameter is 10 mm. If the yield stress for the steel is $(\sigma_Y)_{st} = 640$ MPa, and for the bronze $(\sigma_Y)_{br} = 520$ MPa, determine the magnitude of the largest elastic load P that can be applied to the assembly. $E_{st} = 200$ GPa, $E_{br} = 100$ GPa.



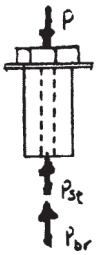
SOLUTION

$+\uparrow \Sigma F_y = 0; \qquad P_{st} + P_{br} - P = 0 \qquad$ **(1)**

Assume failure of bolt:

$$P_{st} = (\sigma_Y)_{st}(A) = 640(10^6)\left(\frac{\pi}{4}\right)(0.01^2)$$

$$= 50265.5 \text{ N}$$



$$\delta_{st} = \delta_{br}$$

$$\frac{P_{st}L}{\frac{\pi}{4}(0.01^2)(200)(10^9)} = \frac{P_{br}L}{\frac{\pi}{4}(0.02^2 - 0.01^2)(100)(10^9)}$$

$$P_{st} = 0.6667\, P_{br}$$

$$50\,265.5 = 0.6667\, P_{br}$$

$$P_{br} = 75398.2 \text{ N}$$

From Eq. (1)

$$P = 50265.5 + 75398.2$$

$$= 125663.7 \text{ N} = 126 \text{ kN} \qquad \text{(controls)} \qquad \textbf{Ans.}$$

Assume failure of sleeve:

$$P_{br} = (\sigma_Y)_{br}(A) = 520(10^6)\left(\frac{\pi}{4}\right)(0.02^2 - 0.01^2) = 122\,522.11 \text{ N}$$

$$P_{st} = 0.6667\, P_{br}$$

$$= 0.6667(122\,522.11)$$

$$= 81\,681.4 \text{ N}$$

From Eq. (1),

$$P = 122\,522.11 + 81\,681.4$$

$$= 204\,203.52 \text{ N}$$

$$= 204 \text{ kN}$$

Ans:
$P = 126$ kN

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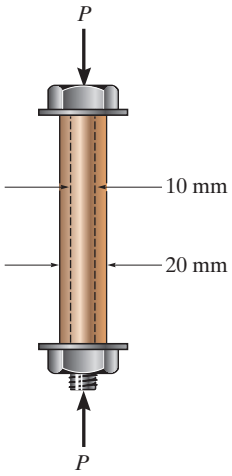
4–38.

The 10-mm-diameter steel bolt is surrounded by a bronze sleeve. The outer diameter of this sleeve is 20 mm, and its inner diameter is 10 mm. If the bolt is subjected to a compressive force of $P = 20$ kN, determine the average normal stress in the steel and the bronze. $E_{st} = 200$ GPa, $E_{br} = 100$ GPa.



SOLUTION

$$+\uparrow \Sigma F_y = 0; \qquad P_{st} + P_{br} - 20 = 0 \qquad \textbf{(1)}$$

$$\delta_{st} = \delta_{br}$$

$$\frac{P_{st}L}{\frac{\pi}{4}(0.01^2)(200)(10^9)} = \frac{P_{br}L}{\frac{\pi}{4}(0.02^2 - 0.01^2)(100)(10^9)}$$

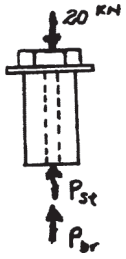
$$P_{st} = 0.6667\, P_{br} \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields

$$P_{st} = 8\text{ kN} \qquad P_{br} = 12\text{ kN}$$

$$\sigma_{st} = \frac{P_{st}}{A_{st}} = \frac{8(10^3)}{\frac{\pi}{4}(0.01^2)} = 102\text{ MPa} \qquad \textbf{Ans.}$$

$$\sigma_{br} = \frac{P_{br}}{A_{br}} = \frac{12(10^3)}{\frac{\pi}{4}(0.02^2 - 0.01^2)} = 50.9\text{ MPa} \qquad \textbf{Ans.}$$



Ans:
$\sigma_{st} = 102$ MPa,
$\sigma_{br} = 50.9$ MPa

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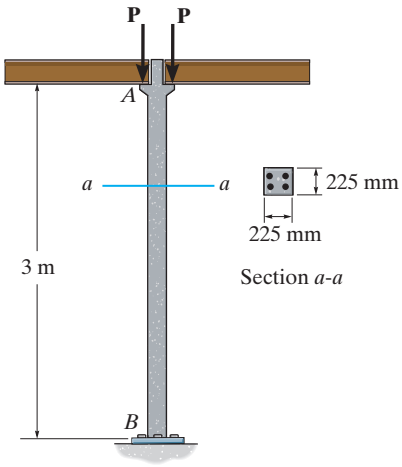
4–39. If column AB is made from high strength pre-cast concrete and reinforced with four 20 mm diameter A-36 steel rods, determine the average normal stress developed in the concrete and in each rod. Set $P = 350$ kN.



SOLUTION

Equation of Equilibrium: Referring to the free-body diagram of the cut part of the concrete column shown in Fig. *a*,

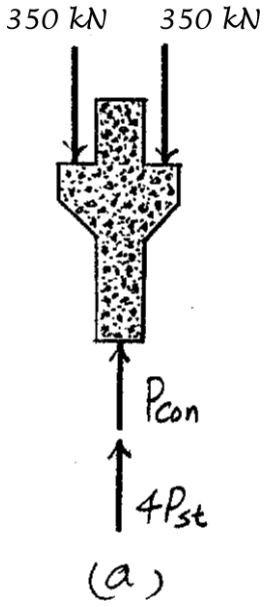
$$+\uparrow \Sigma F_y = 0; \qquad P_{con} + 4P_{st} - 2(350) = 0 \tag{1}$$

Compatibility Equation: Since the steel bars and the concrete are firmly bonded, their deformation must be the same. Thus,

$$\delta_{con} = \delta_{st}$$

$$\frac{P_{con}(3)}{\left[0.225(0.225) - 4\left(\frac{\pi}{4}\right)(0.02^2)\right][29.0(10^9)]} = \frac{P_{st}(3)}{\left[\frac{\pi}{4}(0.2^2)\right][200(10^9)]}$$

$$P_{con} = 22.7859P_{st} \tag{2}$$



Solving Eqs. (1) and (2),

$$P_{st} = 26.13 \text{ kN} \qquad P_{con} = 595.47 \text{ kN}$$

Normal Stress: Applying Eq. (1-6),

$$\sigma_{con} = \frac{P_{con}}{A_{con}} = \frac{595.47(10^3)}{0.225(0.225) - 4\left(\frac{\pi}{4}\right)(0.02^2)} = 12.06(10^6) \text{ N/m}^2 = 12.1 \text{ Mpa} \quad \textbf{Ans.}$$

$$\sigma_{st} = \frac{P_{st}}{A_{st}} = \frac{26.13(10^3)}{\left(\frac{\pi}{4}\right)(0.02^2)} = 83.18(10^6) \text{ N/m}^2 = 83.2 \text{ Mpa} \quad \textbf{Ans.}$$

Ans:
$\sigma_{con} = 12.1$ MPa, $\sigma_{st} = 83.2$ MPa

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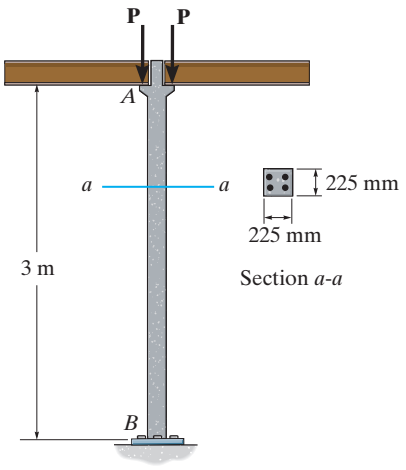
***4–40.** If column AB is made from high strength pre-cast concrete and reinforced with four 20 mm diameter A-36 steel rods, determine the maximum allowable floor loadings **P**. The allowable normal stress for the high strength concrete and the steel are $(\sigma_{allow})_{con} = 18$ MPa and $(\sigma_{allow})_{st} = 170$ MPa, respectively.



SOLUTION

Equation of Equilibrium: Referring to the free-body diagram of the cut part of the concrete column shown in Fig. *a*,

$$+\uparrow \Sigma F_y = 0; \qquad P_{con} + 4P_{st} - 2P = 0 \tag{1}$$

Compatibility Equation: Since the steel bars and the concrete are firmly bonded, their deformation must be the same. Thus,

$$\delta_{con} = \delta_{st}$$

$$\frac{P_{con}(3)}{\left[0.225(0.225) - 4\left(\frac{\pi}{4}\right)(0.02^2)\right][29.0(10^9)]} = \frac{P_{st}(3)}{\left[\frac{\pi}{4}(0.2^2)\right][200(10^9)]}$$

$$P_{con} = 22.7859P_{st} \tag{2}$$

Solving Eqs. (1) and (2),

$$P_{st} = 0.07467P \qquad P_{con} = 1.7013P$$

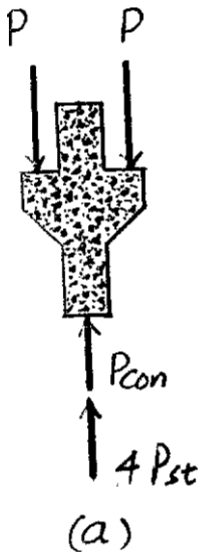
Allowable Normal Stress:

$$(\sigma_{con})_{allow} = \frac{P_{con}}{A_{con}}; \qquad 18(10^6) = \frac{1.7013P}{0.225(0.225) - 4\left(\frac{\pi}{4}\right)(0.02^2)}$$

$$P = 522.31(10^3)\ \text{N} = 522\ \text{kN (controls)} \qquad \textbf{Ans.}$$

$$(\sigma_{st})_{allow} = \frac{P_{st}}{A_{st}}; \qquad 170(10^6) = \frac{0.07467P}{\frac{\pi}{4}(0.02^2)}$$

$$P = 715.28(10^3)\ \text{N} = 715.28\ \text{kN}$$



Ans:
$P = 522$ kN

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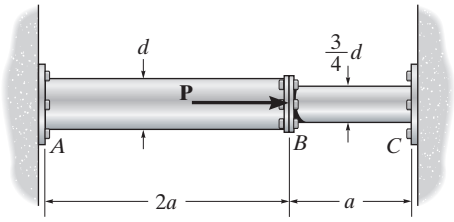
4–41.

Determine the support reactions at the rigid supports A and C. The material has a modulus of elasticity of E.



SOLUTION

Equation of Equilibrium: Referring to the free-body diagram of the assembly shown in Fig. *a*,

$$\stackrel{+}{\rightarrow}\Sigma F_x = 0; \qquad P - F_A - F_C = 0 \qquad \textbf{(1)}$$

Compatibility Equation: Using the method of superposition, Fig. *b*,

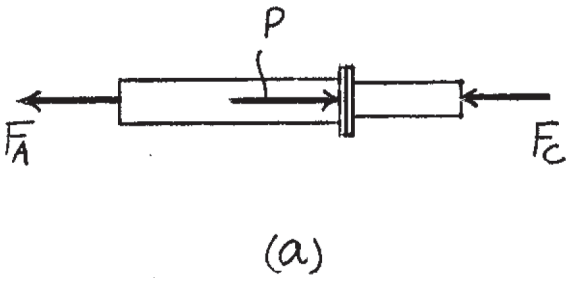
$$(\stackrel{+}{\rightarrow}) \qquad \delta = \delta_P - \delta_{F_C}$$

$$0 = \frac{P(2a)}{\left(\frac{\pi}{4}d^2\right)E} - \left[\frac{F_C a}{\frac{\pi}{4}\left(\frac{3}{4}d\right)^2 E} + \frac{F_C(2a)}{\left(\frac{\pi}{4}d^2\right)E}\right]$$

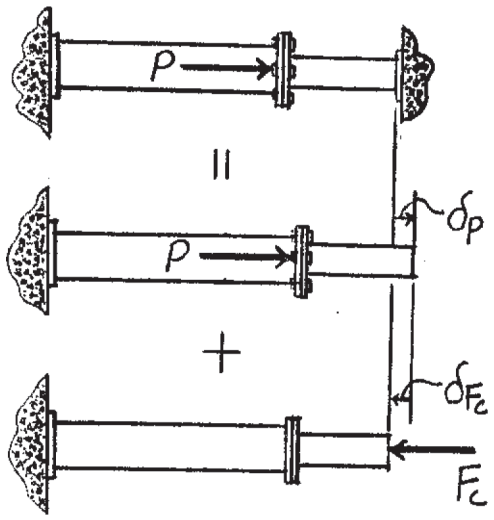
$$F_C = \frac{9}{17}P \qquad \textbf{Ans.}$$

Substituting this result into Eq. (1),

$$F_A = \frac{8}{17}P \qquad \textbf{Ans.}$$



(a)



(b)

Ans:

$F_C = \frac{9}{17}P,$

$F_A = \frac{8}{17}P$

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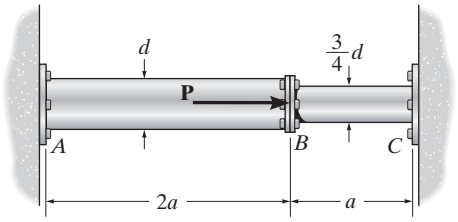
4–42.

If the supports at A and C are flexible and have a stiffness k, determine the support reactions at A and C. The material has a modulus of elasticity of E.



SOLUTION

Equation of Equilibrium: Referring to the free-body diagram of the assembly shown in Fig. *a*,

$$\stackrel{+}{\rightarrow} \Sigma F_x = 0; \qquad P - F_A - F_C = 0 \qquad \textbf{(1)}$$

Compatibility Equation: Using the method of superposition, Fig. *b*,

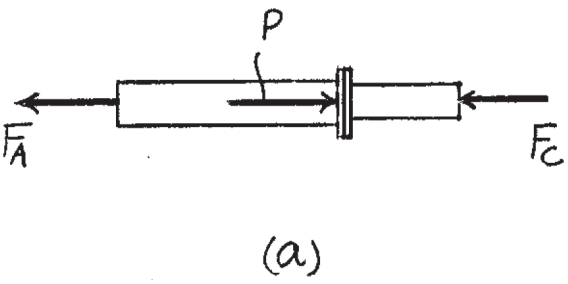
$$(\stackrel{+}{\rightarrow}) \qquad \delta_C = \delta_P - \delta_{F_C}$$

$$\frac{F_C}{k} = \left[\frac{P(2a)}{\left(\frac{\pi}{4}d^2\right)E} + \frac{P}{k}\right] - \left[\frac{F_C a}{\frac{\pi}{4}\left(\frac{3}{4}d\right)^2 E} + \frac{F_C(2a)}{\left(\frac{\pi}{4}d^2\right)E} + \frac{F_C}{k}\right]$$

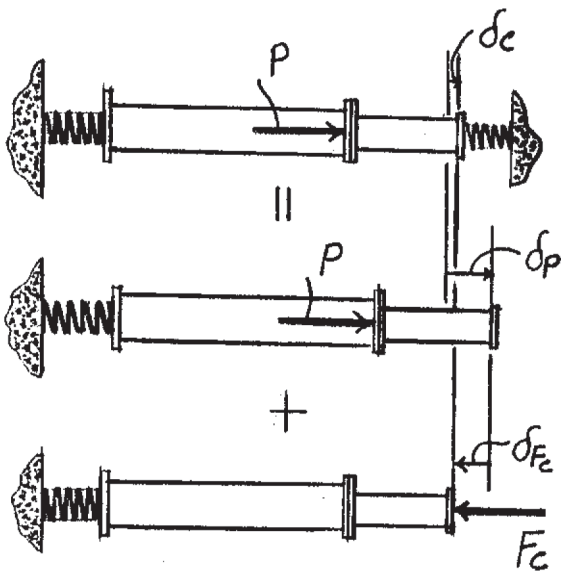
$$F_C = \left[\frac{9(8ka + \pi d^2 E)}{136ka + 18\pi d^2 E}\right]P \qquad \textbf{Ans.}$$

Substituting this result into Eq. (1),

$$F_A = \left(\frac{64ka + 9\pi d^2 E}{136ka + 18\pi d^2 E}\right)P \qquad \textbf{Ans.}$$



(a)



(b)

Ans:

$$F_C = \left[\frac{9(8ka + \pi d^2 E)}{136ka + 18\pi d^2 E}\right]P,$$

$$F_A = \left(\frac{64ka + 9\pi d^2 E}{136ka + 18\pi d^2 E}\right)P$$

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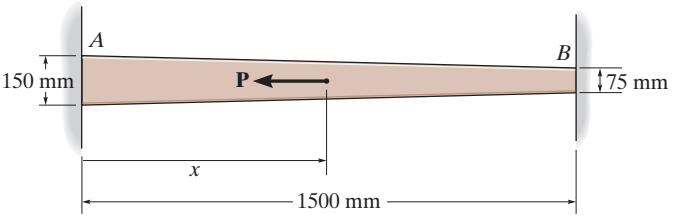
4–43. The tapered member is fixed connected at its ends A and B and is subjected to a load $P = 35$ kN at $x = 750$ mm Determine the reactions at the supports. The material is 50 mm thick and is made froom 2014-T6 aluminum.



SOLUTION

Equilibrium. Refering to the FBD of the member, Fig. a

$$\stackrel{+}{\rightarrow} \Sigma F_x = 0; \qquad F_A + F_B - 35 = 0 \qquad \textbf{(1)}$$

Compatibility. From the geometry shown in Fig. b

$$y = 2\left[0.0375 + \left(\frac{1.5 - x}{1.5}\right)(0.0375)\right] = [0.05(3 - x)]\ \text{m}$$

Thus, the cross-sectional area of the member as a function of x is

$$A = (0.05)[0.05(3 - x)] = [0.0025(3 - x)]\ \text{m}^2$$

It is required that

$$\delta_{A/B} = 0$$

$$-\int_0^{0.75\ \text{m}} \frac{F_A\, dx}{0.0025(3 - x)(E)} + \int_{0.75\ \text{m}}^{1.5\ \text{m}} \frac{F_B\, dx}{0.0025(3 - x)(E)} = 0$$

$$-\int_0^{0.75\ \text{m}} \frac{F_A\, dx}{3 - x} + \int_{0.75\ \text{m}}^{1.5\ \text{m}} \frac{F_B dx}{3 - x} = 0$$

$$F_A \ln(3 - x)\Big|_0^{0.75\ \text{m}} - F_B \ln(3 - x)\Big|_{0.75\ \text{m}}^{1.5\ \text{m}} = 0$$

$$F_A \ln\left(\frac{2.25}{3}\right) - F_B \ln\left(\frac{1.5}{2.25}\right) = 0$$

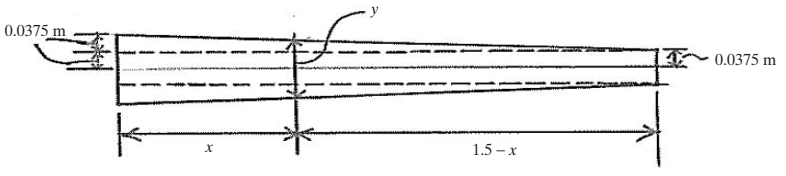
$$F_A = 1.4094 F_B \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2)

$$F_A = 20.47 = 20.5\ \text{kN} \qquad \textbf{Ans.}$$

$$F_B = 14.53 = 14.5\ \text{kN} \qquad \textbf{Ans.}$$

(a)



(b)

Ans.
$y = 75 - 0.025x$, $F_A = 20.47$ kN, $F_B = 14.53$ kN

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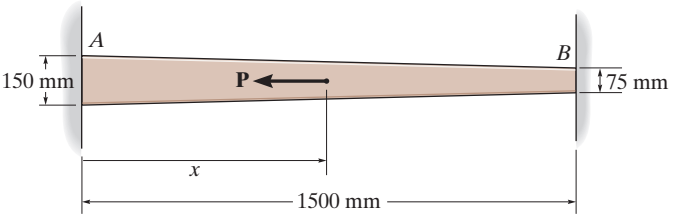
***4–44.** The tapered member is fixed connected at its ends A and B and is subjected to a load **P**. Determine the location x of the load and its greatest magnitude so that the average normal stress in the bar does not exceed $\sigma_{allow} = 28$ MPa. The number is 50 mm thick.



SOLUTION

Equilibrium. Refering to the FBD of the member, Fig. a

$$\stackrel{+}{\rightarrow} \Sigma F_x = 0; \qquad F_A + F_B - P = 0 \qquad \textbf{(1)}$$

Compatibility. From the geometry shown in Fig. b

$$y = 2\left[0.0375 + \left(\frac{1.5 - x}{1.5}\right)(0.0375)\right] = [0.05(3 - x)]\ \text{m}$$

Thus, the cross-sectional area of the member as a function of x is

$$A = (0.05)[0.05(3 - x)] = [0.0025(3 - x)]\ \text{m}^2$$

It is required that

$$\delta_{A/B} = 0$$

$$-\int_0^x \frac{F_A\, dx}{0.0025(3 - x)(E)} + \int_x^{1.5\ \text{m}} \frac{F_B\, dx}{0.0025(3 - x)(E)} = 0$$

$$-\int_0^x \frac{F_A\, dx}{3 - x} + \int_x^{1.5\ \text{m}} \frac{F_B dx}{3 - x} = 0$$

$$F_A \ln(3 - x)\Big|_0^x - F_B \ln(3 - x)\Big|_x^{1.5\ \text{m}} = 0$$

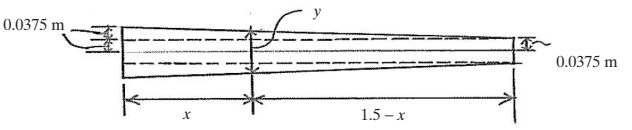
$$F_A \ln\left(\frac{3 - x}{3}\right) - F_B \ln\left(\frac{1.5}{3 - x}\right) = 0 \qquad \textbf{(2)}$$

The maximum normal stress in segment AC occurs at position x and in segment BC occurs at B. Then

$$\sigma_{allow} = \frac{F_A}{A}; \quad 28(10^6) = \frac{F_A}{0.0025(3 - x)} \qquad F_A = [70(10^3)(3 - x)]\ \text{N}$$

$$\sigma_{allow} = \frac{F_B}{A}; \quad 28(10^6) = \frac{F_B}{(0.05)(0.075)} \qquad F_B = 105(10^3)\ \text{N}$$

(a)



(b)

Substitute these results into Eq. (2)

$$70(10^3)(3 - x)\ln\left(\frac{3 - x}{3}\right) - 105(10^3)\ln\left(\frac{1.5}{3 - x}\right) = 0$$

$$\ln\left(\frac{3 - x}{3}\right)^{3 - x} = \ln\left(\frac{1.5}{3 - x}\right)^{1.5}$$

$$\left(\frac{3 - x}{3}\right)^{3 - x} = \left(\frac{1.5}{3 - x}\right)^{1.5}$$

Solving numerically,

$x = 0.721698$ m $= 722$ m **Ans.**

Then,

$$F_A = [70(10^3)(3 - 0.7216980) = 159.48(10^3)\ \text{N}$$

Substitute the results of F_A and F_B into Eq. (1)

$$159.48(10^3) + 105(10^3) - P = 0$$

$$P = 264.48(10^3) = 264\ \text{N} \qquad \textbf{Ans.}$$

Ans.
$x = 722$ m, $P = 264$ N

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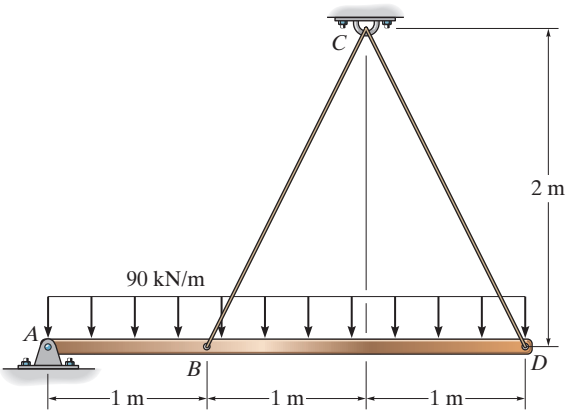
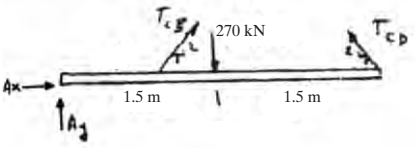
4–45. The rigid bar supports the uniform distributed load of 90 kN/m. Determine the force in each cable if each cable has a cross-sectional area of 36 mm^2, and E = 200 GPa.



SOLUTION

Equilibrium:

$$\zeta + \Sigma M_A = 0; \qquad T_{CB}\left(\frac{2}{\sqrt{5}}\right)(1) - 270(1.5) + T_{CD}\left(\frac{2}{\sqrt{5}}\right)(3) = 0 \qquad \textbf{(1)}$$



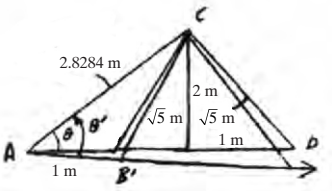
Geometry and compatibility:

$$\theta = \tan^{-1}\frac{2}{2} = 45°$$

$$L_{B'C}^2 = (1)^2 + (2.8284)^2 - 2(1)(2.8284)\cos\theta'$$

Also,

$$L_{D'C}^2 = (3)^2 + (2.8284)^2 - 2(3)(2.8284)\cos\theta' \qquad \textbf{(2)}$$



Thus, eliminating $\cos\theta'$.

$$-L_{B'C}^2(0.176778) + 1.590978 = -L_{D'C}^2(0.058926) + 1.001735$$

$$-L_{B'C}^2(0.176778) = 0.058926L_{D'C}^2 + 0.589243$$

$$-L_{B'C}^2 = 0.3333L_{D'C}^2 + 3.333$$

But,

$$L_{B'C} = \sqrt{5} + \delta_{BC}, \qquad L_{D'C} = \sqrt{5} + \delta_{DC},$$

Neglect squares or δ'_B since small strain occurs.

$$L_{B'C}^2 = (\sqrt{5} + \delta_{BC})^2 = 5 + 2\sqrt{5}\,\delta_{BC}$$

$$L_{D'C}^2 = (\sqrt{5} + \delta_{DC})^2 = 5 + 2\sqrt{5}\,\delta_{DC}$$

$$5 + 2\sqrt{5}\delta_{BC} = 0.3333(5 + 2\sqrt{5}\delta_{DC}) + 3.333$$

$$2\sqrt{5}\delta_{BC} = 0.3333(2\sqrt{5}\delta_{DC})$$

$$\delta_{DC} = 3\delta_{BC}$$

Thus,

$$\frac{T_{CD}\sqrt{5}}{AE} = 3\frac{T_{CB}\sqrt{5}}{AE}$$

$$T_{CD} = 3\,T_{CB}$$

From Eq. (1).

T_{CD} = 135.84 kN = 136 kN **Ans.**

T_{CB} = 45.28 kN = 45.3 kN **Ans.**

Ans.
T_{CD} = 136 kN, T_{CB} = 45.3 kN

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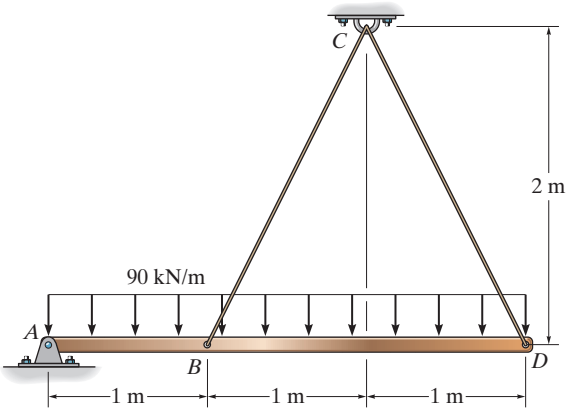
4–46. The rigid bar is originally horizontal and is supported by two cables each having a cross-sectional area of 36 mm^2, and E = 200 GPa. Determine the slight rotation of the bar when the uniform load is applied.



SOLUTION

See solution of Prob. 4-51.

$$T_{CD} = 135.84 \text{ kN}$$

$$\delta_{DC} = \frac{T_{CD}L_{CD}}{AE} = \frac{(135.84)(10^3)\sqrt{5}}{[36(10^{-6})][200(10^9)]} = 0.04219 \text{ m}$$

Using Eq. (2) of Prob. 4-51,

$$(\sqrt{5} + 0.04219)^2 = (3)^2 + (2.8284)^2 - 2(3)(2.8284)\cos\theta'$$

$$\theta' = 45.9023°$$

Thus,

$$\Delta\theta = 45.9023° - 45° = 0.9023° = 0.902°$$ **Ans.**

Ans.
$\Delta\theta = 0.902°$

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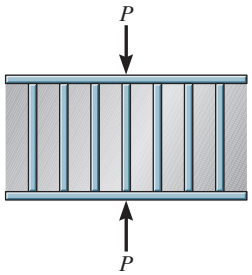
4–47.

The specimen represents a filament-reinforced matrix system made from plastic (matrix) and glass (fiber). If there are n fibers, each having a cross-sectional area of A_f and modulus of E_f, embedded in a matrix having a cross-sectional area of A_m and modulus of E_m, determine the stress in the matrix and in each fiber when the force P is applied on the specimen.

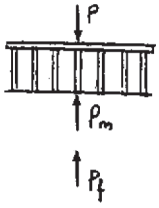


SOLUTION

$$+\uparrow \Sigma F_y = 0; \qquad -P + P_m + P_f = 0 \tag{1}$$

$$\delta_m = \delta_f$$

$$\frac{P_m L}{A_m E_m} = \frac{P_f L}{n A_f E_f}; \qquad P_m = \frac{A_m E_m}{n A_f E_f} P_f \tag{2}$$



Solving Eqs. (1) and (2) yields

$$P_m = \frac{A_m E_m}{n A_f E_f + A_m E_m} P; \qquad P_f = \frac{n A_f E_f}{n A_f E_f + A_m E_m} P$$

Normal stress:

$$\sigma_m = \frac{P_m}{A_m} = \frac{\left(\dfrac{A_m E_m}{n A_f E_f + A_m E_m} - P\right)}{A_m} = \frac{E_m}{n A_f E_f + A_m E_m} P \qquad \textbf{Ans.}$$

$$\sigma_f = \frac{P_f}{n A_f} = \frac{\left(\dfrac{n A_f E_f}{n A_f E_f + A_m E_m} P\right)}{n A_f} = \frac{E_f}{n A_f E_f + A_m E_m} P \qquad \textbf{Ans.}$$

Ans:

$$\sigma_m = \frac{E_m}{n A_f E_f + A_m E_m} P,$$

$$\sigma_f = \frac{E_f}{n A_f E_f + A_m E_m} P$$

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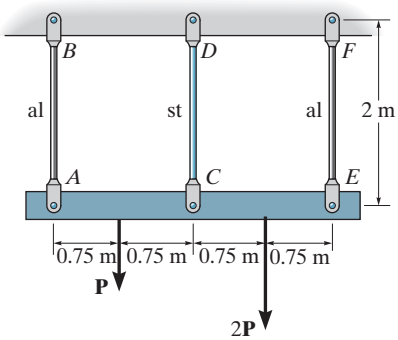
***4–48.**

The rigid beam is supported by the three suspender bars. Bars *AB* and *EF* are made of aluminum and bar *CD* is made of steel. If each bar has a cross-sectional area of 450 mm^2, determine the maximum value of *P* if the allowable stress is $(\sigma_{allow})_{st} = 200$ MPa for the steel and $(\sigma_{allow})_{al} = 150$ MPa for the aluminum. $E_{st} = 200$ GPa, $E_{al} = 70$ GPa.



SOLUTION

Equation of Equilibrium: Referring to the FBD of the rigid beam Fig. *a*,

$$\circlearrowleft+\Sigma M_A = 0; \quad F_{CD}(1.5) + F_{EF}(3) - P(0.75) - 2P(2.25) = 0$$

$$1.5F_{CD} + 3F_{EF} = 5.25P \quad \textbf{(1)}$$

$$\circlearrowleft+\Sigma M_E = 0; \quad 2P(0.75) + P(2.25) - F_{CD}(1.5) - F_{AB}(3) = 0$$

$$1.5F_{CD} + 3F_{AB} = 3.75P \quad \textbf{(2)}$$

Compatibility: Referring to the displacement diagram of the rigid beam, Fig. *b*,

$$\frac{\delta_{CD} - \delta_{AB}}{1.5} = \frac{\delta_{EF} - \delta_{AB}}{3}$$

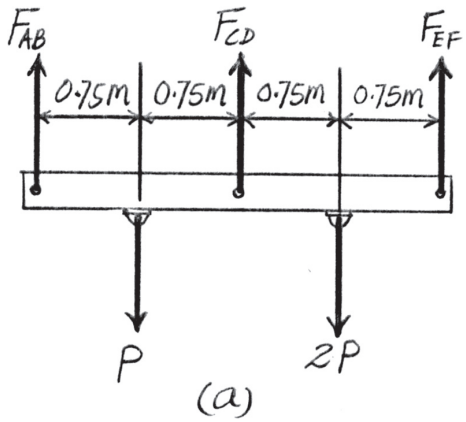
$$2\delta_{CD} = \delta_{EF} + \delta_{AB}$$

$$2\left(\frac{F_{CD}L}{A[200(10^9)]}\right) = \frac{F_{EF}L}{A[70(10^9)]} + \frac{F_{AB}L}{A[70(10^9)]}$$

$$F_{CD} = \frac{10}{7}(F_{EF} + F_{AB}) \quad \textbf{(3)}$$

Solving Eqs. (1), (2) and (3),

$$F_{EF} = 0.8676P \qquad F_{AB} = 0.3676P \qquad F_{CD} = 1.7647P$$



(a)

Assuming that bar *EF* fails. Then

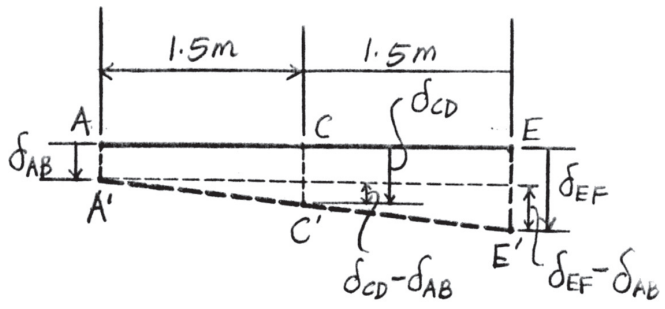
$$(\sigma_{allow})_{al} = \frac{F_{EF}}{A}; \quad 150(10^6) = \frac{0.8676P}{450(10^{-6})}$$

$$P = 77.80(10^3)\text{ N} = 77.80\text{ kN}$$

Assuming that bar *CD* fails. Then

$$(\sigma_{allow})_{st} = \frac{F_{CD}}{A}; \quad 200(10^6) = \frac{1.7647P}{450(10^{-6})}$$

$$P = 51.0(10^3)\text{ N} = 51.0\text{ kN}$$ (control!) **Ans.**



(b)

Ans:
$P = 51.0$ kN

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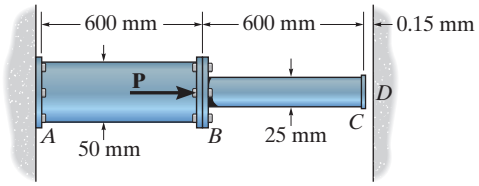
4–49.

If the gap between C and the rigid wall at D is initially 0.15 mm, determine the support reactions at A and D when the force $P = 200$ kN is applied. The assembly is made of solid A-36 steel cylinders.



SOLUTION

Equation of Equilibrium: Referring to the free-body diagram of the assembly shown in Fig. *a*,

$$\stackrel{+}{\rightarrow}\Sigma F_x = 0; \qquad 200(10^3) - F_D - F_A = 0 \qquad \textbf{(1)}$$

Compatibility Equation: Using the method of superposition, Fig. *b*,

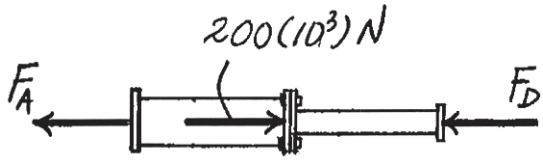
$$(\stackrel{+}{\rightarrow}) \qquad \delta = \delta_P - \delta_{F_D}$$

$$0.15 = \frac{200(10^3)(600)}{\frac{\pi}{4}(0.05^2)(200)(10^9)} - \left[\frac{F_D\,(600)}{\frac{\pi}{4}(0.05^2)(200)(10^9)} + \frac{F_D\,(600)}{\frac{\pi}{4}(0.025^2)(200)(10^9)}\right]$$

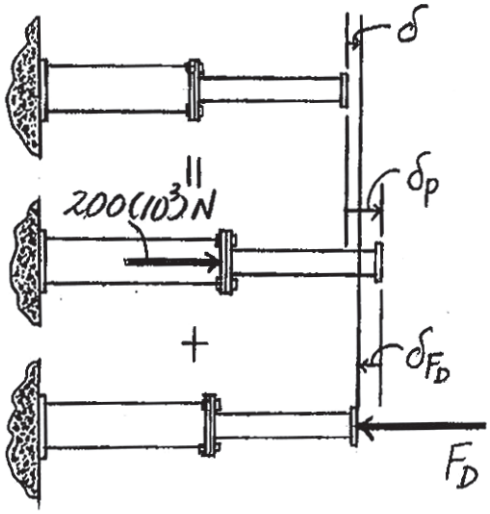
$$F_D = 20\,365.05\ \text{N} = 20.4\ \text{kN} \qquad \textbf{Ans.}$$

Substituting this result into Eq. (1),

$$F_A = 179\,634.95\ \text{N} = 180\ \text{kN} \qquad \textbf{Ans.}$$



(a)



(b)

Ans:
$F_D = 20.4$ kN, $F_A = 180$ kN

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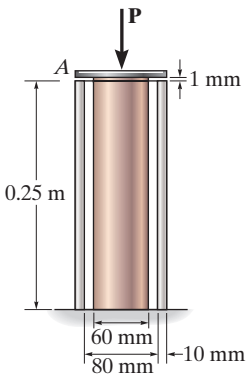
4–50.

The support consists of a solid red brass C83400 copper post surrounded by a 304 stainless steel tube. Before the load is applied the gap between these two parts is 1 mm. Given the dimensions shown, determine the greatest axial load that can be applied to the rigid cap A without causing yielding of any one of the materials.

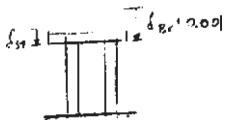


SOLUTION

Require,

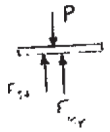
$$\delta_{st} = \delta_{br} + 0.001$$

$$\frac{F_{st}(0.25)}{\pi[(0.05)^2 - (0.04)^2]193(10^9)} = \frac{F_{br}(0.25)}{\pi(0.03)^2(101)(10^9)} + 0.001$$



$$0.45813\,F_{st} = 0.87544\,F_{br} + 10^6 \qquad \textbf{(1)}$$

$$+\uparrow \Sigma F_y = 0; \qquad F_{st} + F_{br} - P = 0 \qquad \textbf{(2)}$$



Assume brass yields, then

$$(F_{br})_{max} = \sigma_\gamma A_{br} = 70(10^6)(\pi)(0.03)^2 = 197\,920.3\ \text{N}$$

$$(\epsilon_\gamma)_{br} = \sigma_\gamma / E = \frac{70.0(10^6)}{101(10^9)} = 0.6931(10^{-3})\ \text{mm/mm}$$

$$\delta_{br} = (\epsilon_\gamma)_{br} L = 0.6931(10^{-3})(0.25) = 0.1733\ \text{mm} < 1\ \text{mm}$$

Thus only the brass is loaded.

$$P = F_{br} = 198\ \text{kN} \qquad \textbf{Ans.}$$

Ans:
$P = 198$ kN

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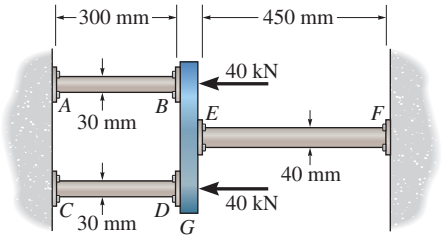
4–51.

The assembly consists of two red brass C83400 copper rods *AB* and *CD* of diameter 30 mm, a stainless 304 steel alloy rod *EF* of diameter 40 mm, and a rigid cap *G*. If the supports at *A*, *C*, and *F* are rigid, determine the average normal stress developed in the rods.



SOLUTION

Equation of Equilibrium: Due to symmetry, $F_{AB} = F_{CD} = F$. Referring to the free-body diagram of the assembly shown in Fig. *a*,

$$\stackrel{+}{\rightarrow} \Sigma F_x = 0; \qquad 2F + F_{EF} - 2\left[40(10^3)\right] = 0 \qquad \textbf{(1)}$$

Compatibility Equation: Using the method of superposition, Fig. *b*,

$$\left(\stackrel{+}{\rightarrow}\right) \quad 0 = -\delta_P + \delta_{EF}$$

$$0 = -\frac{40(10^3)(300)}{\frac{\pi}{4}(0.03^2)(101)(10^9)} + \left[\frac{F_{EF}(450)}{\frac{\pi}{4}(0.04^2)(193)(10^9)} + \frac{\left(F_{EF}/2\right)(300)}{\frac{\pi}{4}(0.03^2)(101)(10^9)}\right]$$

$$F_{EF} = 42\,483.23 \text{ N}$$

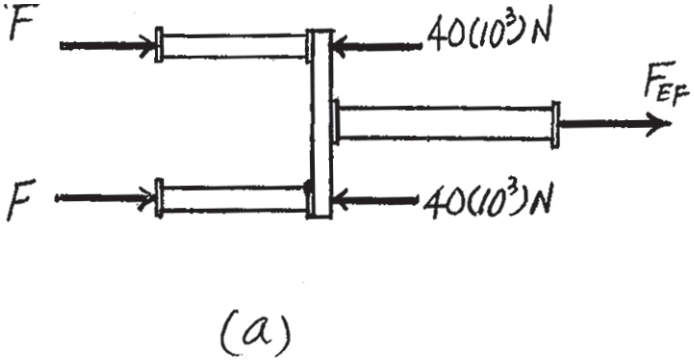
Substituting this result into Eq. (1),

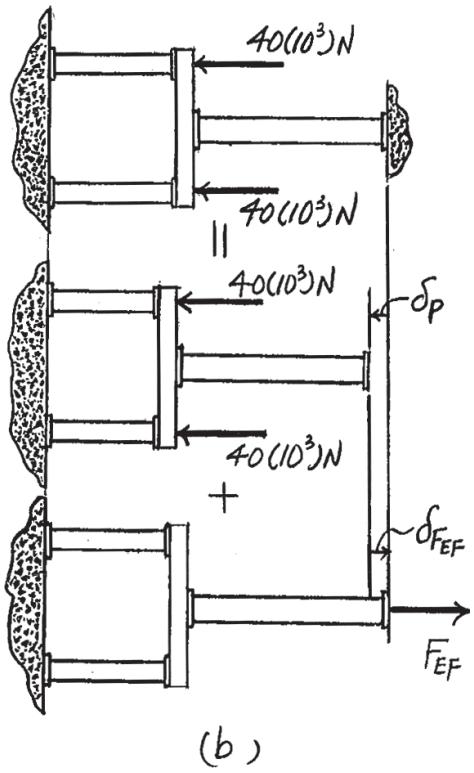
$$F = 18\,758.38 \text{ N}$$

Normal Stress: We have,

$$\sigma_{AB} = \sigma_{CD} = \frac{F}{A_{CD}} = \frac{18\,758.38}{\frac{\pi}{4}(0.03^2)} = 26.5 \text{ MPa} \qquad \textbf{Ans.}$$

$$\sigma_{EF} = \frac{F_{EF}}{A_{EF}} = \frac{42\,483.23}{\frac{\pi}{4}(0.04^2)} = 33.8 \text{ MPa} \qquad \textbf{Ans.}$$





Ans:
$\sigma_{AB} = \sigma_{CD} = 26.5$ MPa, $\sigma_{EF} = 33.8$ MPa

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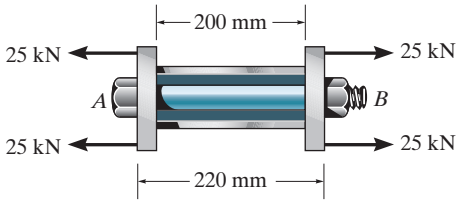
***4–52.**

The bolt AB has a diameter of 20 mm and passes through a sleeve that has an inner diameter of 40 mm and an outer diameter of 50 mm. The bolt and sleeve are made of A-36 steel and are secured to the rigid brackets as shown. If the bolt length is 220 mm and the sleeve length is 200 mm, determine the tension in the bolt when a force of 50 kN is applied to the brackets.

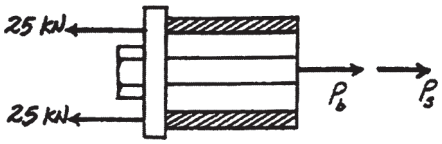


SOLUTION

Equation of Equilibrium:

$$\stackrel{+}{\rightarrow} \Sigma F_x = 0; \qquad P_b + P_s - 25 - 25 = 0$$

$$P_b + P_s - 50 = 0 \qquad \textbf{(1)}$$



Compatibility:

$$\delta_b = \delta_s$$

$$\frac{P_b(220)}{\frac{\pi}{4}(0.02^2)200(10^9)} = \frac{P_s(200)}{\frac{\pi}{4}(0.05^2 - 0.04^2)(200)(10^9)}$$

$$P_b = 0.40404\, P_s \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields:

$$P_s = 35.61 \text{ kN}$$

$$P_b = 14.4 \text{ kN} \qquad \textbf{Ans.}$$

Ans:
$P_b = 14.4$ kN

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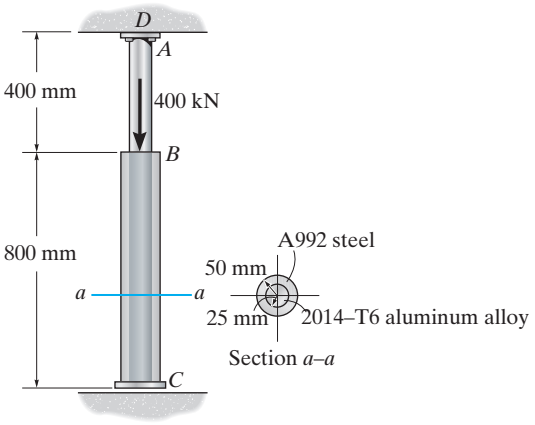
4–53.

The 2014-T6 aluminum rod *AC* is reinforced with the firmly bonded A992 steel tube *BC*. If the assembly fits snugly between the rigid supports so that there is no gap at *C*, determine the support reactions when the axial force of 400 kN is applied. The assembly is attached at *D*.



SOLUTION

Equation of Equilibrium: Referring to the free-body diagram of the assembly shown in Fig. *a*,

$$+\uparrow \Sigma F_y = 0; \qquad F_D + (F_C)_{al} + (F_C)_{st} - 400(10^3) = 0 \qquad \textbf{(1)}$$

Compatibility Equation: Using the method of superposition, Fig. *b*,

$$(+\downarrow) \qquad 0 = \delta_p - \delta_{FC}$$

$$0 = +\frac{400(10^3)(400)}{\pi(0.025^2)(73.1)(10^9)} - \left[\frac{(F_C)_{al}(800)}{\pi(0.025^2)(73.1)(10^9)} + \frac{[(F_C)_{al} + (F_C)_{st}](400)}{\pi(0.025^2)(73.1)(10^9)}\right]$$

$$400(10^3) = 3(F_C)_{al} + (F_C)_{st} \qquad \textbf{(2)}$$

Also, since the aluminum rod and steel tube of segment *BC* are firmly bonded, their deformation must be the same. Thus,

$$(\delta_{BC})_{st} = (\delta_{BC})_{al}$$

$$\frac{(F_C)_{st}(800)}{\pi(0.05^2 - 0.025^2)(200)(10^9)} = \frac{(F_C)_{al}(800)}{\pi(0.025^2)(73.1)(10^9)}$$

$$(F_C)_{st} = 8.2079(F_C)_{al} \qquad \textbf{(3)}$$

Solving Eqs. (1) and (2),

$$(F_C)_{al} = 35.689 \text{ kN} \qquad (F_C)_{st} = 292.93 \text{ kN}$$

Substituting these results into Eq. (1),

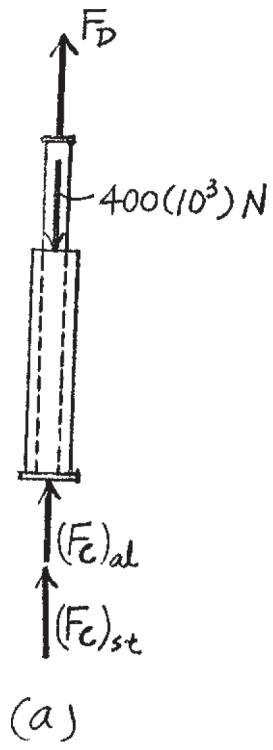
$$F_D = 71.4 \text{ kN} \qquad \textbf{Ans.}$$

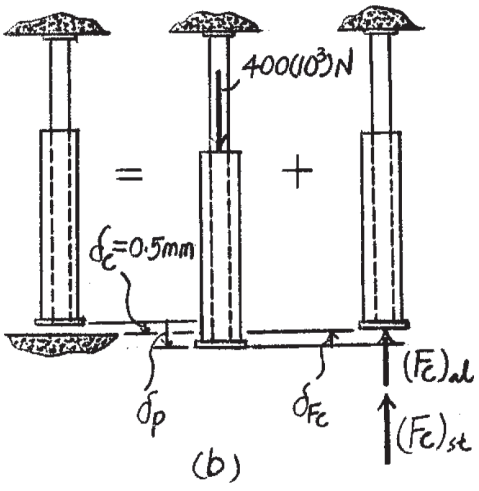
Also,

$$F_C = (F_C)_{st} + (F_C)_{al}$$

$$= 35.689 + 292.93$$

$$= 329 \text{ kN} \qquad \textbf{Ans.}$$





Ans:
$F_D = 71.4$ kN, $F_C = 329$ kN

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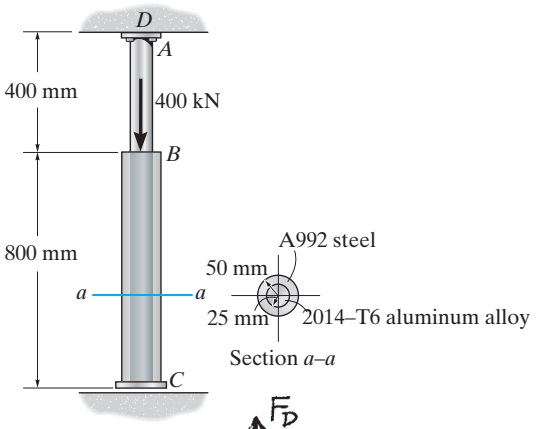
4–54.

The 2014-T6 aluminum rod *AC* is reinforced with the firmly bonded A992 steel tube *BC*. When no load is applied to the assembly, the gap between end *C* and the rigid support is 0.5 mm. Determine the support reactions when the axial force of 400 kN is applied.



SOLUTION

Equation of Equilibrium: Referring to the free-body diagram of the assembly shown in Fig. *a*,

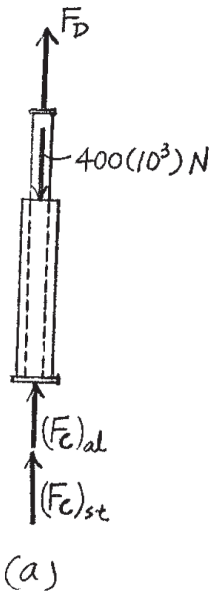
$$+\uparrow \Sigma F_y = 0; \qquad F_D + (F_C)_{al} + (F_C)_{st} - 400(10^3) = 0 \qquad \textbf{(1)}$$

Compatibility Equation: Using the method of superposition, Fig. *b*,

$$(+\downarrow) \qquad \delta_C = \delta_P - \delta_{FC}$$

$$0.5 = + \frac{400(10^3)(400)}{\pi(0.025^2)(73.1)(10^9)} - \left[\frac{(F_C)_{al}(800)}{\pi(0.025^2)(73.1)(10^9)} + \frac{[(F_C)_{al} + (F_C)_{st}](400)}{\pi(0.025^2)(73.1)(10^9)} \right]$$

$$220.585(10^3) = 3(F_C)_{al} + (F_C)_{st} \qquad \textbf{(2)}$$



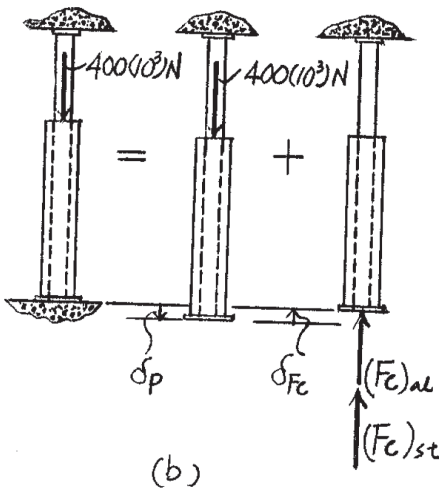
(a)

Also, since the aluminum rod and steel tube of segment *BC* are firmly bonded, their deformation must be the same. Thus,

$$(\delta_{BC})_{st} = (\delta_{BC})_{al}$$

$$\frac{(F_C)_{st}(800)}{\pi(0.05^2 - 0.025^2)(200)(10^9)} = \frac{(F_C)_{al}(800)}{\pi(0.025^2)(73.1)(10^9)}$$

$$(F_C)_{st} = 8.2079(F_C)_{al} \qquad \textbf{(3)}$$



(b)

Solving Eqs. (2) and (3),

$$(F_C)_{al} = 19.681 \text{ kN} \qquad (F_C)_{st} = 161.54 \text{ kN}$$

Substituting these results into Eq. (1),

$$F_D = 218.777 \text{ kN} = 219 \text{ kN} \qquad \textbf{Ans.}$$

Also,

$$F_C = (F_C)_{al} + (F_C)_{st}$$

$$= 19.681 + 161.54$$

$$= 181 \text{ kN} \qquad \textbf{Ans.}$$

Ans:
$F_D = 219$ kN, $F_C = 181$ kN

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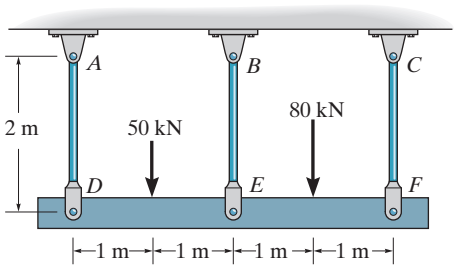
4–55.

The three suspender bars are made of A992 steel and have equal cross-sectional areas of 450 mm^2. Determine the average normal stress in each bar if the rigid beam is subjected to the loading shown.



SOLUTION

Referring to the *FBD* of the rigid beam, Fig. *a*,

$$+\uparrow \Sigma F_y = 0; \quad F_{AD} + F_{BE} + F_{CF} - 50(10^3) - 80(10^3) = 0 \qquad \textbf{(1)}$$

$$\circlearrowleft + \Sigma M_D = 0; \quad F_{BE}(2) + F_{CF}(4) - 50(10^3)(1) - 80(10^3)(3) = 0 \qquad \textbf{(2)}$$

Referring to the geometry shown in Fig. *b*,

$$\delta_{BE} = \delta_{AD} + \left(\frac{\delta_{CF} - \delta_{AD}}{4}\right)(2)$$

$$\delta_{BE} = \frac{1}{2}\left(\delta_{AD} + \delta_{CF}\right)$$

$$\frac{F_{BE}\cancel{L}}{\cancel{AE}} = \frac{1}{2}\left(\frac{F_{AD}\cancel{L}}{\cancel{AE}} + \frac{F_{CF}\cancel{L}}{\cancel{AE}}\right)$$

$$F_{AD} + F_{CF} = 2\,F_{BE} \qquad \textbf{(3)}$$

Solving Eqs. (1), (2), and (3) yields

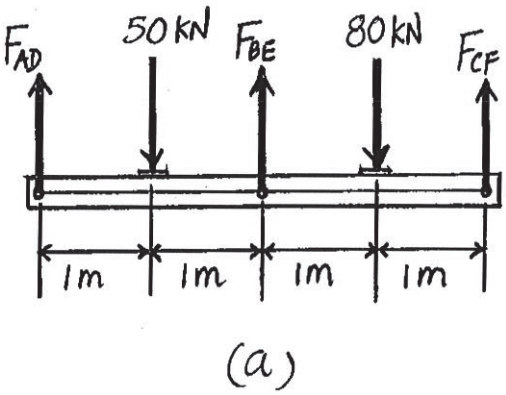
$$F_{BE} = 43.33(10^3)\ \text{N} \qquad F_{AD} = 35.83(10^3)\ \text{N} \qquad F_{CF} = 50.83(10^3)\ \text{N}$$

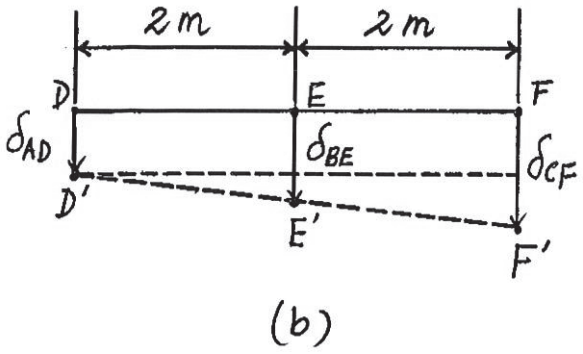
Thus,

$$\sigma_{BE} = \frac{F_{BE}}{A} = \frac{43.33(10^3)}{0.45(10^{-3})} = 96.3\ \text{MPa} \qquad \textbf{Ans.}$$

$$\sigma_{AD} = \frac{F_{AD}}{A} = \frac{35.83(10^3)}{0.45(10^{-3})} = 79.6\ \text{MPa} \qquad \textbf{Ans.}$$

$$\sigma_{CF} = 113\ \text{MPa} \qquad \textbf{Ans.}$$





Ans:
$\sigma_{BE} = 96.3$ MPa, $\sigma_{AD} = 79.6$ MPa,
$\sigma_{CF} = 113$ MPa

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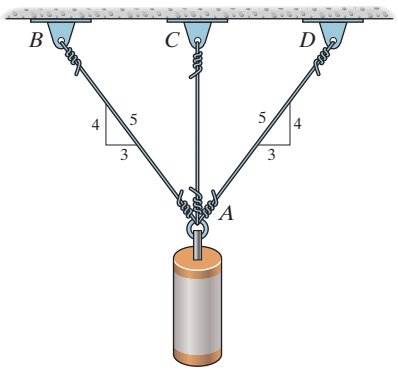
***4–56.**

The three A-36 steel wires each have a diameter of 2 mm and unloaded lengths of $L_{AC} = 1.60$ m and $L_{AB} = L_{AD} = 2.00$ m. Determine the force in each wire after the 150-kg mass is suspended from the ring at A.



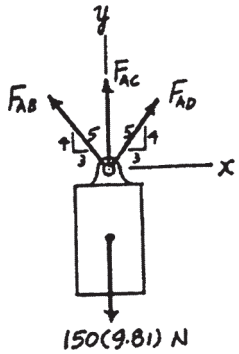
SOLUTION

Equations of Equilibrium:

$$\stackrel{+}{\rightarrow}\Sigma F_x = 0; \qquad \frac{3}{5}F_{AD} - \frac{3}{5}F_{AB} = 0 \qquad F_{AD} = F_{AB} = F$$

$$+\uparrow \Sigma F_y = 0; \qquad 2\left(\frac{4}{5}F\right) + F_{AC} - 150(9.81) = 0$$

$$1.6F + F_{AC} - 1471.5 = 0 \qquad \textbf{(1)}$$



Compatibility:

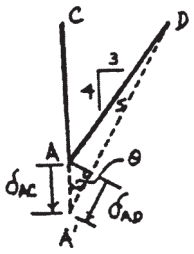
$$\delta_{AD} = \delta_{AC}\cos\theta$$

Since the displacement is very small, $\cos\theta = \frac{4}{5}$

$$\delta_{AD} = \frac{4}{5}\delta_{AC}$$

$$\frac{F(2)}{AE} = \frac{4}{5}\left[\frac{F_{AC}(1.6)}{AE}\right]$$

$$F = 0.640\,F_{AC} \qquad \textbf{(2)}$$



Solving Eqs. (1) and (2) yields:

$$F_{AC} = 727\text{ N} \qquad \textbf{Ans.}$$

$$F_{AB} = F_{AD} = F = 465\text{ N} \qquad \textbf{Ans.}$$

Ans:
$F_{AC} = 727$ N,
$F_{AB} = 465$ N

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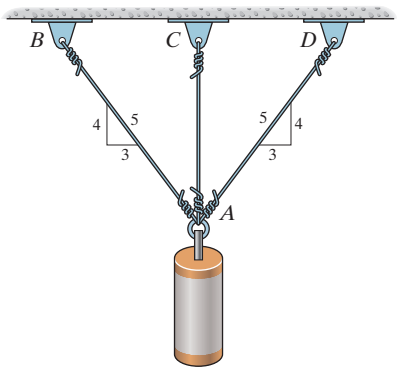
4–57.

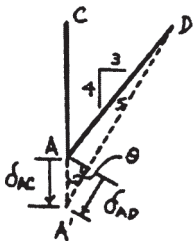
The A-36 steel wires AB and AD each have a diameter of 2 mm and the unloaded lengths of each wire are $L_{AC} = 1.60$ m and $L_{AB} = L_{AD} = 2.00$ m. Determine the required diameter of wire AC so that each wire is subjected to the same force when the 150-kg mass is suspended from the ring at A.



SOLUTION

Equations of Equilibrium: Each wire is required to carry the same amount of load. Hence

$$F_{AB} = F_{AC} = F_{AD} = F$$



Compatibility:

$$\delta_{AD} = \delta_{AC} \cos\theta$$

Since the displacement is very small, $\cos\theta = \dfrac{4}{5}$

$$\delta_{AD} = \frac{4}{5}\delta_{AC}$$

$$\frac{F(2)}{\frac{\pi}{4}(0.002^2)E} = \frac{F(1.6)}{\frac{\pi}{4}d_{AC}^2 E}$$

$$d_{AC} = 0.001789 \text{ m} = 1.79 \text{ mm} \qquad \textbf{Ans.}$$

Ans:
$d_{AC} = 1.79$ mm

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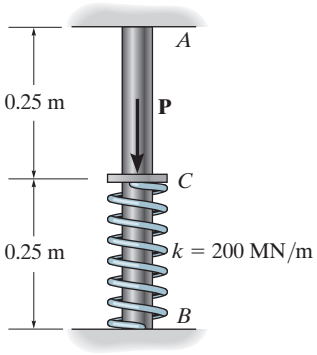
4–58.

The post is made from 606l-T6 aluminum and has a diameter of 50 mm. It is fixed supported at A and B, and at its center C there is a coiled spring attached to the rigid collar. If the spring is originally uncompressed, determine the reactions at A and B when the force $P = 40$ kN is applied to the collar.



SOLUTION

Equations of Equilibrium:

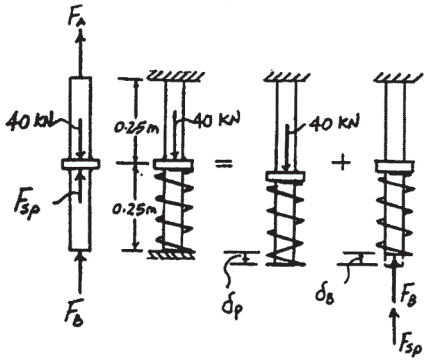
$$+\uparrow \Sigma F_y = 0; \qquad F_A + F_B + F_{sp} - 40(10^3) = 0 \qquad \textbf{(1)}$$

Compatibility:

$$0 = \delta_p - \delta_B$$

$$0 = \frac{40(10^3)(0.25)}{\frac{\pi}{4}(0.05^2)68.9(10^9)} - \left[\frac{(F_B + F_{sp})(0.25)}{\frac{\pi}{4}(0.05^2)68.9(10^9)} + \frac{F_B + F_{sp}}{\frac{\frac{\pi}{4}(0.05^2)68.9(10^9)}{0.25} + 200(10^6)}\right]$$

$$F_B + F_{sp} = 23119.45 \qquad \textbf{(2)}$$



Also,

$$\delta_{sp} = \delta_{BC}$$

$$\frac{F_{sp}}{200(10^6)} = \frac{F_B + F_{sp}}{\frac{\frac{\pi}{4}(0.05^2)68.9(10^9)}{0.25} + 200(10^6)}$$

$$F_B = 2.7057\, F_{sp} \qquad \textbf{(3)}$$

Solving Eq. (2) and (3) yields

$$F_{sp} = 6238.9 \text{ N}$$

$$F_B = 16880.6 \text{ N} = 16.9 \text{ kN} \qquad \textbf{Ans.}$$

Substitute the results into Eq. (1)

$$F_A = 16880.6 \text{ N} = 16.9 \text{ kN} \qquad \textbf{Ans.}$$

Ans:
$F_B = 16.9$ kN,
$F_A = 16.9$ kN

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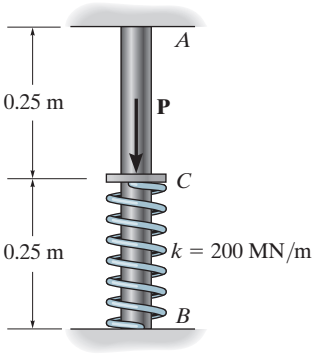
4–59.

The post is made from 606l-T6 aluminum and has a diameter of 50 mm. It is fixed supported at A and B, and at its center C there is a coiled spring attached to the rigid collar. If the spring is originally uncompressed, determine the compression in the spring when the load of $P = 50$ kN is applied to the collar.



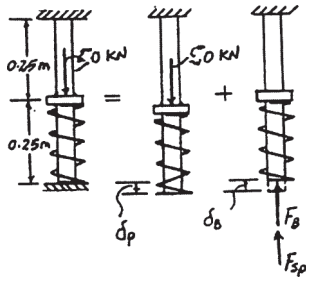
SOLUTION

Compatibility:

$$0 = \delta_p - \delta_B$$

$$0 = \frac{50(10^3)(0.25)}{\frac{\pi}{4}(0.05^2)68.9(10^9)} - \left[\frac{(F_B + F_{sp})(0.25)}{\frac{\pi}{4}(0.05^2)68.9(10^9)} + \frac{F_B + F_{sp}}{\frac{\frac{\pi}{4}(0.05^2)68.9(10^9)}{0.25} + 200(10^6)}\right]$$

$$F_B + F_{sp} = 28899.31 \qquad \textbf{(1)}$$



Also,

$$\delta_{sp} = \delta_{BC}$$

$$\frac{F_{sp}}{200(10^6)} = \frac{F_B + F_{sp}}{\frac{\frac{\pi}{4}(0.05^2)68.9(10^9)}{0.25} + 200(10^6)}$$

$$F_B = 2.7057\, F_{sp} \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yield

$$F_{sp} = 7798.6 \text{ N} \qquad F_B = 21100.7 \text{ N}$$

Thus,

$$\delta_{sp} = \frac{F_{sp}}{k} = \frac{7798.6}{200(10^6)}$$

$$= 0.0390(10^{-3}) \text{ m} = 0.0390 \text{ mm} \qquad \textbf{Ans.}$$

Ans:
$\delta_{sp} = 0.0390$ mm

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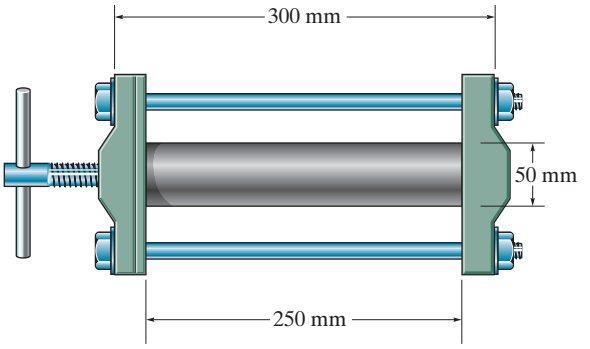
***4–60.** The press consists of two rigid heads that are held together by the two A-36 steel 12-mm-diameter rods. A 6061-T6-solid-aluminum cylinder is placed in the press and the screw is adjusted so that it just presses up against the cylinder. If it is then tightened one-half turn, determine the average normal stress in the rods and in the cylinder. The single-threaded screw on the bolt has a lead of 0.25 mm *Note*: The lead represents the distance the screw advances along its axis for one complete turn of the screw.



SOLUTION

Equilibrium:

$$\stackrel{+}{\rightarrow} \Sigma F_x = 0; \qquad 2F_{st} - F_{al} = 0$$

Compatibility:

$$\delta_{st} = 0.125(10^{-3}) - \delta_{al}$$

$$\frac{F_{st}(0.3)}{\left[\frac{\pi}{4}(0.012^2)\right][200(10^9)]} = 0.125(10^{-3}) - \frac{F_{al}(0.25)}{\left[\frac{\pi}{4}(0.05^2)\right][68.9(10^9)]}$$

$$13.2629F_{st} = 125(10^3) - 1.8480F_{al}$$

Solving,

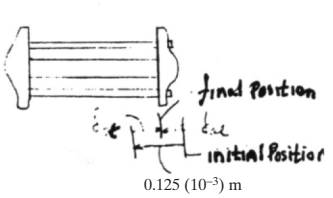
$$F_{st} = 7.3708(10^3)\ \text{N}$$

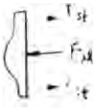
$$F_{al} = 14.7416(10^3)\ \text{N}$$

Normal stress:

$$\sigma_{rod} = \frac{F_{st}}{A_{st}} = \frac{7.3708(10^3)}{\left(\frac{\pi}{4}\right)(0.012^2)} = 65.17(10^6)\ \text{N/m}^2 = 65.2\ \text{MPa}$$ **Ans.**

$$\sigma_{cyl} = \frac{F_{al}}{A_{al}} = \frac{14.7416(10^3)}{\left(\frac{\pi}{4}\right)(0.05^2)} = 7.508(10^6)\ \text{N/m}^2 = 7.51\ \text{MPa}$$ **Ans.**





Ans.
$\sigma_{rod} = 65.2$ MPa, $\sigma_{cyl} = 7.51$ MPa

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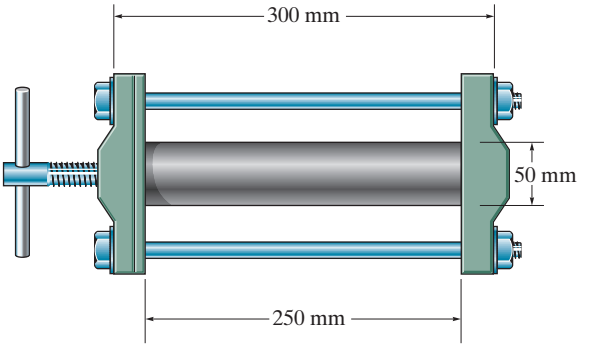
4–61. The press consists of two rigid heads that are held together by the two A-36 steel 12-mm-diameter rods. A 6061-T6-solid-aluminum cylinder is placed in the press and the screw is adjusted so that it just presses up against the cylinder. Determine the angle through which the screw can be turned before the rods or the specimen begin to yield. The single-threaded screw on the bolt has a lead of 0.25 mm. *Note*: The lead represents the distance the screw advances along its axis for one complete turn of the screw.



SOLUTION

Equilibrium:

$$\stackrel{+}{\rightarrow} \Sigma F_x = 0; \qquad 2F_{st} - F_{al} = 0$$

Compatibility:

$$\delta_{st} = d - \delta_{al}$$

$$\frac{F_{st}(0.3)}{\left[\frac{\pi}{4}(0.012^2)\right][200(10^9)]} = d - \frac{F_{al}(0.25)}{\left[\frac{\pi}{4}(0.05^2)\right][68.9(10^9)]}$$

$$13.2629F_{st} = (10^9)d - 1.8480F_{al} \qquad \textbf{(1)}$$

Assume steel yields first,

$$\sigma_Y = 250(10^6) = \frac{F_{st}}{\left(\frac{\pi}{4}\right)(0.012^2)}; \qquad F_{st} = 28.27(10^3)\ \text{N}$$

Then $F_{al} = 56.55(10^3)$ N

$$\sigma_{al} = \frac{56.55(10^3)}{\left(\frac{\pi}{4}\right)(0.05^2)} = 28.80(10^6)\ \text{N/m}^2 = 28.80\ \text{MPa}$$

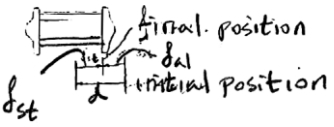
28.80 MPa < 255 MPa, steel yields first as assumed. From Eq. (1),

$$d = 0.4795(10^{-3})\ \text{m} = 0.4795\ \text{mm}$$

Thus,

$$\frac{\theta}{360^\circ} = \frac{0.4795}{0.25}$$

$$\theta = 690.48^\circ = 690^\circ \qquad \textbf{Ans.}$$



Ans.
$\theta = 690^\circ$

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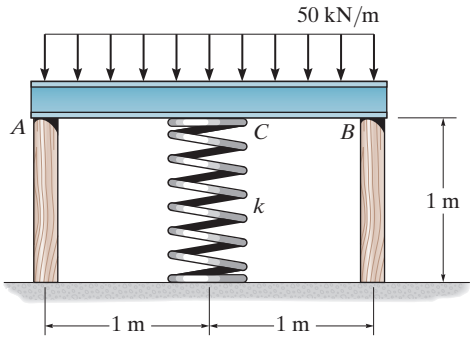
4–62.

The rigid bar is supported by the two short white spruce wooden posts and a spring. If each of the posts has an unloaded length of 1 m and a cross-sectional area of 600 mm^2, and the spring has a stiffness of $k = 2$ MN/m and an unstretched length of 1.02 m, determine the force in each post after the load is applied to the bar.



SOLUTION

Equations of Equilibrium:

$$\circlearrowleft +\Sigma M_C = 0; \qquad F_B(1) - F_A(1) = 0 \qquad F_A = F_B = F$$

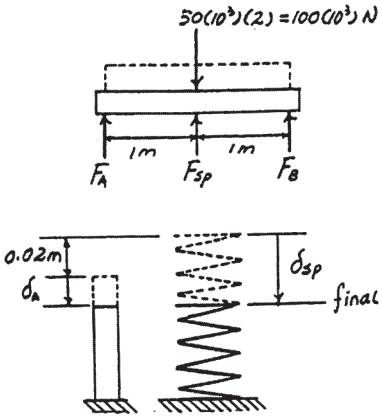
$$+\uparrow \Sigma F_y = 0; \qquad 2F + F_{sp} - 100\left(10^3\right) = 0 \qquad \textbf{(1)}$$

Compatibility:

$$(+\downarrow) \qquad \delta_A + 0.02 = \delta_{sp}$$

$$\frac{F(1)}{600\left(10^{-6}\right)9.65\left(10^9\right)} + 0.02 = \frac{F_{sp}}{2.0\left(10^6\right)}$$

$$0.1727F + 20\left(10^3\right) = 0.5\,F_{sp} \qquad \textbf{(2)}$$



Solving Eqs. (1) and (2) yields:

$$F_A = F_B = F = 25581.7\text{ N} = 25.6\text{ kN} \qquad \textbf{Ans.}$$

$$F_{sp} = 48\,836.5\text{ N}$$

Ans:
$F_A = F_B = 25.6$ kN

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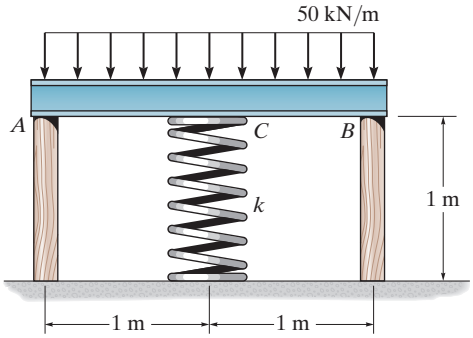
4–63.

The rigid bar is supported by the two short white spruce wooden posts and a spring. If each of the posts has an unloaded length of 1 m and a cross-sectional area of 600 mm^2, and the spring has a stiffness of $k = 2$ MN/m and an unstretched length of 1.02 m, determine the vertical displacement of A and B after the load is applied to the bar.

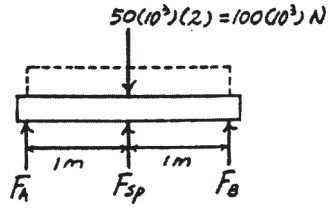


SOLUTION

Equations of Equilibrium:

$$\zeta+\Sigma M_C = 0; \qquad F_B(1) - F_A(1) = 0 \qquad F_A = F_B = F$$

$$+\uparrow \Sigma F_y = 0; \qquad 2F + F_{sp} - 100(10^3) = 0 \qquad \textbf{(1)}$$

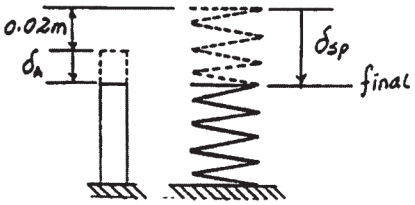


Compatibility:

$$(+\downarrow) \qquad \delta_A + 0.02 = \delta_{sp}$$

$$\frac{F(1)}{600(10^{-6})9.65(10^9)} + 0.02 = \frac{F_{sp}}{2.0(10^6)}$$

$$0.1727F + 20(10^3) = 0.5\,F_{sp} \qquad \textbf{(2)}$$



Solving Eqs. (1) and (2) yields:

$$F = 25\,581.7 \text{ N} \qquad F_{sp} = 48\,836.5 \text{ N}$$

Displacement:

$$\delta_A = \delta_B = \frac{FL}{AE}$$

$$= \frac{25\,581.7(1000)}{600(10^{-6})(9.65)(10^9)} = 4.42 \text{ mm} \qquad \textbf{Ans.}$$

Ans:
$\delta_A = \delta_B = 4.42$ mm

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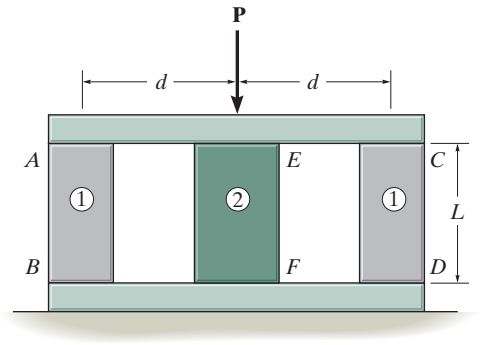
***4–64.**

The assembly consists of two posts AB and CD each made from material 1 having a modulus of elasticity of E_1 and a cross-sectional area A_1, and a central post made from material 2 having a modulus of elasticity E_2 and cross-sectional area A_2. If a load **P** is applied to the rigid cap, determine the force in each material.

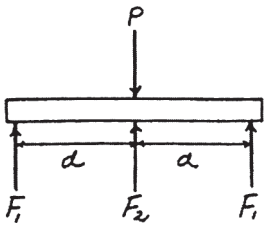


SOLUTION

Equilibrium:

$$+\uparrow \Sigma F_y = 0; \qquad 2F_1 + F_2 - P = 0 \qquad \textbf{(1)}$$

Compatibility:



$$\delta = \delta_1 = \delta_2$$

$$\frac{F_1 L}{A_1 E_1} = \frac{F_2 L}{A_2 E_2} \qquad F_1 = \left(\frac{A_1 E_1}{A_2 E_2}\right) F_2 \qquad \textbf{(2)}$$

Solving Eq. (1) and (2) yields:

$$F_1 = \left(\frac{A_1 E_1}{2\,A_1 E_1 + A_2 E_2}\right) P \qquad \textbf{Ans.}$$

$$F_2 = \left(\frac{A_2 E_2}{2A_1 E_1 + A_2 E_2}\right) P \qquad \textbf{Ans.}$$

Ans:

$$F_1 = \left(\frac{A_1 E_1}{2\,A_1 E_1 + A_2 E_2}\right) P,$$

$$F_2 = \left(\frac{A_2 E_2}{2A_1 E_1 + A_2 E_2}\right) P$$

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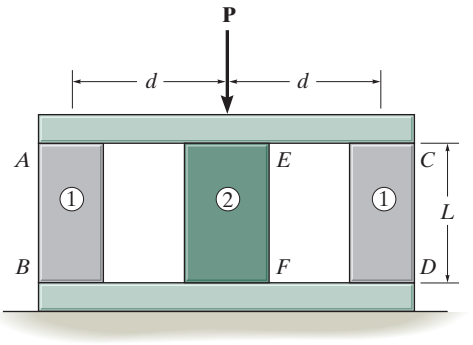
4–65.

The assembly consists of two posts AB and CD each made from material 1 having a modulus of elasticity of E_1 and a cross-sectional area A_1, and a central post EF made from material 2 having a modulus of elasticity E_2 and a cross-sectional area A_2. If posts AB and CD are to be replaced by those having a material 2, determine the required cross-sectional area of these new posts so that both assemblies deform the same amount when loaded.



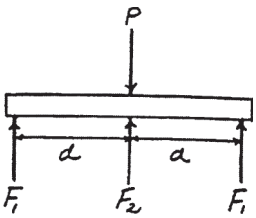
SOLUTION

$$+\uparrow \Sigma F_y = 0; \qquad 2F_1 + F_2 - P = 0 \qquad \textbf{(1)}$$

Compatibility:

$$\delta_{in} = \delta_1 = \delta_2$$

$$\frac{F_1 L}{A_1 E_1} = \frac{F_2 L}{A_2 E_2} \qquad F_1 = \left(\frac{A_1 E_1}{A_2 E_2}\right) F_2 \qquad \textbf{(2)}$$



Solving Eq. (1) and (2) yields:

$$F_1 = \left(\frac{A_1 E_1}{2A_1 E_1 + A_2 E_2}\right) P \qquad F_2 = \left(\frac{A_2 E_2}{2A_1 E_1 + A_2 E_2}\right) P$$

$$\delta_{in} = \frac{F_2 L}{A_2 E_2} = \frac{\left(\frac{A_2 E_2}{2A_1 E_1 + A_2 E_2}\right) PL}{A_2 E_2} = \frac{PL}{2A_1 E_1 + A_2 E_2}$$

Compatibility: When material 1 has been replaced by material 2 for two side posts, then

$$\delta_{final} = \delta_1 = \delta_2$$

$$\frac{F_1 L}{A_1' E_2} = \frac{F_2 L}{A_2 E_2} \qquad F_1 = \left(\frac{A_1'}{A_2}\right) F_2 \qquad \textbf{(3)}$$

Solving for F_2 from Eq. (1) and (3)

$$F_2 = \left(\frac{A_2}{2A_1' + A_2}\right) P$$

$$\delta_{final} = \frac{F_2 L}{A_2 E_2} = \frac{\left(\frac{A_2}{2A_1' + A_2}\right) PL}{A_2 E_2} = \frac{PL}{E_2(2A_1' + A_2)}$$

Requires, $\delta_{in} = \delta_{final}$

$$\frac{PL}{2A_1 E_1 + A_2 E_2} = \frac{PL}{E_2(2A_1' + A_2)}$$

$$A_1' = \left(\frac{E_1}{E_2}\right) A_1 \qquad \textbf{Ans.}$$

Ans:

$$A_1' = \left(\frac{E_1}{E_2}\right) A_1$$

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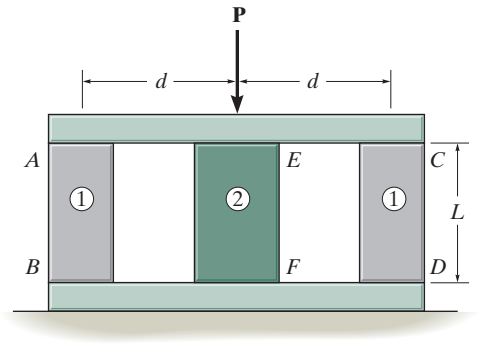
4–66.

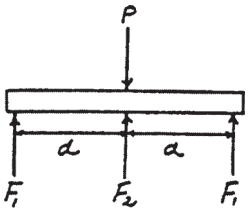
The assembly consists of two posts AB and CD each made from material 1 having a modulus of elasticity of E_1 and a cross-sectional area A_1, and a central post EF made from material 2 having a modulus of elasticity E_2 and a cross-sectional area A_2. If post EF is to be replaced by one having a material 1, determine the required cross-sectional area of this new post so that both assemblies deform the same amount when loaded.



SOLUTION

$$+\uparrow \Sigma F_y = 0; \qquad 2F_1 + F_2 - P = 0 \qquad \textbf{(1)}$$

Compatibility:



$$\delta_{in} = \delta_1 = \delta_2$$

$$\frac{F_1 L}{A_1 E_1} = \frac{F_2 L}{A_2 E_2} \qquad F_1 = \left(\frac{A_1 E_1}{A_2 E_2}\right) F_2 \qquad \textbf{(2)}$$

Solving Eq. (1) and (2) yields:

$$F_1 = \left(\frac{A_1 E_1}{2A_1 E_1 + A_2 E_2}\right) P \qquad F_2 = \left(\frac{A_2 E_2}{2A_1 E_1 + A_2 E_2}\right) P$$

$$\delta_{in} = \frac{F_2 L}{A_2 E_2} = \frac{\left(\frac{A_2 E_2}{2A_1 E_1 + A_2 E_2}\right) P}{A_2 E_2} = \frac{PL}{2A_1 E_1 + A_2 E_2}$$

Compatibility: When material 2 has been replaced by material 1 for central posts, then

$$\delta_{final} = \delta_1 = \delta_2$$

$$\frac{F_1 L}{A_1 E_1} = \frac{F_2 L}{A_2' E_1} \qquad F_2 = \left(\frac{A_2'}{A_2}\right) F_1 \qquad \textbf{(3)}$$

Solving for F_1 from Eq. (1) and (3)

$$F_1 = \left(\frac{A_1}{2A_1 + A_2'}\right) P$$

$$\delta_{final} = \frac{F_1 L}{A_1 E_1} = \frac{\left(\frac{A_1}{2A_1 + A_2'}\right) PL}{A_1 E_1} = \frac{PL}{E_1 (2A_1 + A_2')}$$

Requires, $\delta_{in} = \delta_{final}$

$$\frac{PL}{2A_1 E_1 + A_2 E_2} = \frac{PL}{E_1 (2A_1 + A_2')}$$

$$A_2' = \left(\frac{E_2}{E_1}\right) A_2 \qquad \textbf{Ans.}$$

Ans:

$$A_2' = \left(\frac{E_2}{E_1}\right) A_2$$

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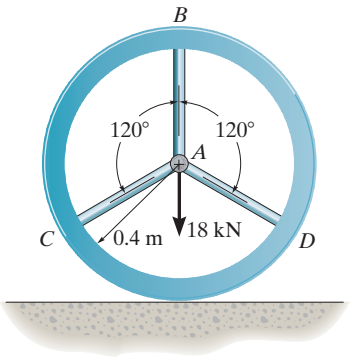
4–67.

The wheel is subjected to a force of 18 kN from the axle. Determine the force in each of the three spokes. Assume the rim is rigid and the spokes are made of the same material, and each has the same cross-sectional area.



SOLUTION

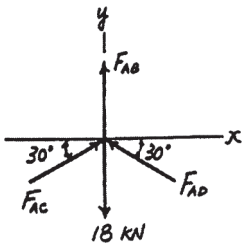
Equations of Equilibrium:

$$\stackrel{+}{\rightarrow}\Sigma F_x = 0; \qquad F_{AC}\cos 30° - F_{AD}\cos 30° = 0$$

$$F_{AC} = F_{AD} = F$$

$$+\uparrow \Sigma F_y = 0; \qquad F_{AB} + 2F\sin 30° - 18 = 0$$

$$F_{AB} + F = 18 \qquad \textbf{(1)}$$

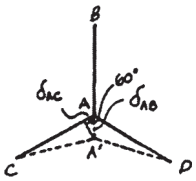


Compatibility:

$$\delta_{AC} = \delta_{AB}\cos 60°$$

$$\frac{F(0.4)}{AE} = \frac{F_{AB}(0.4)}{AE}\cos 60°$$

$$F = 0.5F_{AB} \qquad \textbf{(2)}$$



Solving Eq. (1) and (2) yields:

$$F_{AB} = 12.0\text{ kN (T)} \qquad \textbf{Ans.}$$

$$F_{AC} = F_{AD} = F = 6.00\text{ kN (C)} \qquad \textbf{Ans.}$$

Ans:
$F_{AB} = 12.0$ kN (T)
$F_{AC} = F_{AD} = 6.00$ kN (C)

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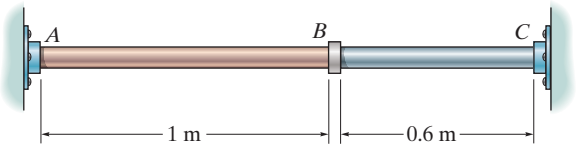
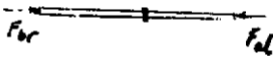
***4–68.** The C83400-red-brass rod AB and 2014-T6-aluminum rod BC are joined at the collar B and fixed connected at their ends. If there is no load in the members when $T_1 = 10°C$, determine the average normal stress in each member when $T_2 = 45°C$. Also, how far will the collar be displaced? The cross-sectional area of each member is 1130 mm^2.



SOLUTION

Equilibrium:

$\Sigma F_x = 0; \qquad F_{br} = F_{al} = F$



Compatibility:

$$\delta_{A/C} = 0$$

$$-\frac{F_{br} L_{AB}}{A_{AB} E_{br}} + \alpha_{br} \Delta T \, L_{AB} - \frac{F_{al} L_{BC}}{A_{BC} E_{al}} + \alpha_{al} \Delta T \, L_{BC} = 0$$

$$-\frac{F(1)}{[(1130)(10^{-6})][101(10^9)]} + 18(10^{-6})(45 - 10)(1)$$

$$-\frac{F(0.6)}{[(1130)(10^{-6})][73.1(10^9)]} + 23(10^{-6})(45 - 10)(0.6) = 0$$

$$F = 69.45(10^3) \text{ N}$$

$$\sigma_{br} = \sigma_{al} = \frac{69.45(10^3)}{1130(10^{-6})} = 61.46(10^6) \text{ N/m}^2 = 61.5 \text{ MPa} \qquad \textbf{Ans.}$$

$61.46 \text{ MPa} < (\sigma\gamma)_{al}$ and $(\sigma\gamma)_{br}$ **OK**

$$\delta_B = -\frac{[69.45(10^3)][(1)(10^3)]}{[1130(10^{-6})][101(10^9)]} + 18(10^{-6})(45 - 10)[1(10^3)]$$

$$\delta_B = 0.02147 \text{ mm} = 0.0215 \text{ mm} \rightarrow \qquad \textbf{Ans.}$$

Ans.
$\sigma_{br} = 61.5$ MPa
$\delta_B = 0.0215$ mm

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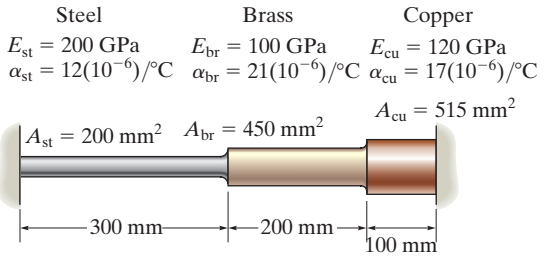
4–69. Three bars each made of different materials are connected together and placed between two walls when the temperature is $T_1 = 12°C$. Determine the force exerted on the (rigid) supports when the temperature becomes $T_2 = 18°C$. The material properties and cross-sectional area of each bar are given in the figure.

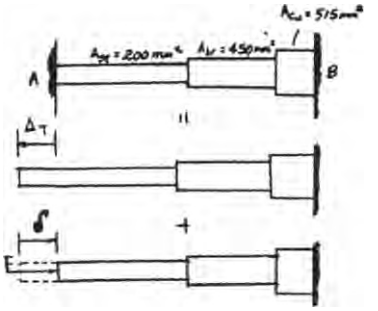


SOLUTION

$$(\overset{+}{\leftarrow}) \qquad 0 = \Delta_T - \delta$$

$$0 = 12(10^{-6})(6)(0.3) + 21\,(10^{-6})(6)(0.2) + 17\,(10^{-6})(6)(0.1) - \frac{F(0.3)}{200(10^{-6})(200)(10^{9})} - \frac{F(0.2)}{450(10^{-6})(100)(10^{9})} - \frac{F(0.1)}{515(10^{-6})(120)(10^{9})}$$

$$F = 4203\text{ N} = 4.20\text{ kN} \qquad \textbf{Ans.}$$



Ans.
$0 = \Delta_T - \delta, F = 4.20\text{ kN}$

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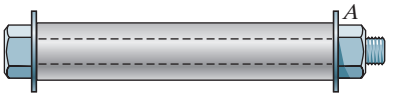
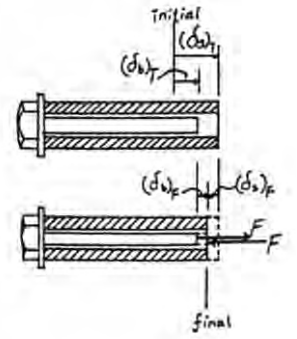
4–70. The steel bolt has a diameter of 7 mm and fits through an aluminum sleeve as shown. The sleeve has an inner diameter of 8 mm and an outer diameter of 10 mm. The nut at A is adjusted so that it just presses up against the sleeve. If the assembly is originally at a temperature of $T_1 = 20°C$ and then is heated to a temperature of $T_2 = 100°C$, determine the average normal stress in the bolt and the sleeve. $E_{st} = 200$ GPa, $E_{al} = 70$ GPa, $\alpha_{st} = 14(10^{-6})/°C$, $\alpha_{al} = 23(10^{-6})/°C$.





Compatibility:

$$(\delta_s)_T - (\delta_b)_T = (\delta_s)_F + (\delta_b)_F$$

$$23(10^{-6})(100 - 20)L - 14(10^{-6})(100 - 20)L = \frac{FL}{\frac{\pi}{4}(0.01^2 - 0.008^2)70(10^9)} + \frac{FL}{\frac{\pi}{4}(0.007^2)200(10^9)}$$

$$F = 1133.54 \text{ N}$$

Average Normal Stress:

$$\sigma_s = \frac{F}{A_s} = \frac{1133.54}{\frac{\pi}{4}(0.01^2 - 0.008^2)} = 40.1 \text{ MPa} \qquad \textbf{Ans.}$$

$$\sigma_b = \frac{F}{A_b} = \frac{1133.54}{\frac{\pi}{4}(0.007^2)} = 29.5 \text{ MPa} \qquad \textbf{Ans.}$$

Ans.
$\sigma_s = 40.1$ MPa, $\sigma_b = 29.5$ MPa

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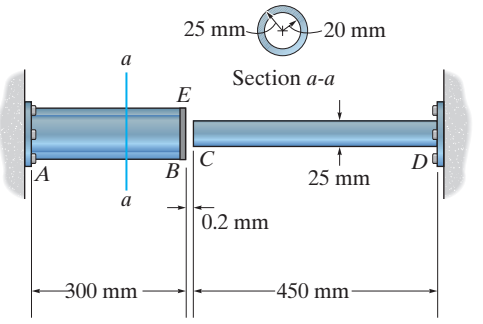
4–71. The AM1004-T61 magnesium alloy tube AB is capped with a rigid plate E. The gap between E and end C of the 6061-T6 aluminum alloy solid circular rod CD is 0.2 mm when the temperature is at 30° C. Determine the normal stress developed in the tube and the rod if the temperature rises to 80° C. Neglect the thickness of the rigid cap.



SOLUTION

Compatibility Equation: If tube AB and rod CD are unconstrained, they will have a free expansion of $(\delta_T)_{AB} = \alpha_{mg}\Delta T L_{AB} = 26(10^{-6})(80 - 30)(300) = 0.39$ mm and $(\delta_T)_{CD} = \alpha_{al}\Delta T L_{CD} = 24(10^{-6})(80 - 30)(450) = 0.54$ mm. Referring to the deformation diagram of the tube and the rod shown in Fig. *a*,

$$\delta = \left[(\delta_T)_{AB} - (\delta_F)_{AB}\right] + \left[(\delta_T)_{CD} - (\delta_F)_{CD}\right]$$

$$0.2 = \left[0.39 - \frac{F(300)}{\pi(0.025^2 - 0.02^2)(44.7)(10^9)}\right] + \left[0.54 - \frac{F(450)}{\frac{\pi}{4}(0.025^2)(68.9)(10^9)}\right]$$

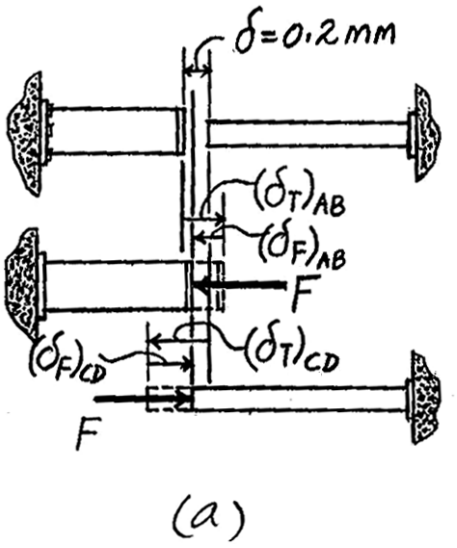
$$F = 32\,017.60 \text{ N}$$

Normal Stress:

$$\sigma_{AB} = \frac{F}{A_{AB}} = \frac{32\,017.60}{\pi(0.025^2 - 0.02^2)} = 45.3 \text{ MPa}$$ **Ans.**

$$\sigma_{CD} = \frac{F}{A_{CD}} = \frac{32\,017.60}{\frac{\pi}{4}(0.025^2)} = 65.2 \text{ MPa}$$ **Ans.**

$$F = 107\,442.47 \text{ N}$$



(a)

Ans.
$\sigma_{AB} = 45.3$ MPa, $\sigma_{CD} = 65.2$ MPa,
$F = 107{,}442.47$ N

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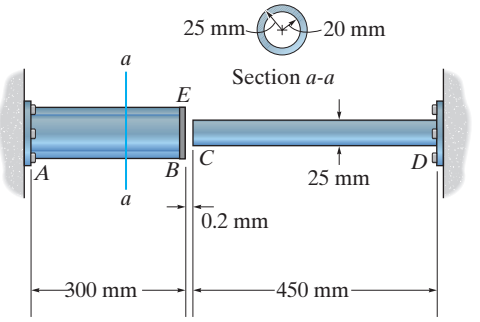
***4–72.** The AM1004-T61 magnesium alloy tube AB is capped with a rigid plate. The gap between E and end C of the 6061-T6 aluminum alloy solid circular rod CD is 0.2 mm when the temperature is at 30° C. Determine the highest temperature to which it can be raised without causing yielding either in the tube or the rod. Neglect the thickness of the rigid cap.



SOLUTION

Then

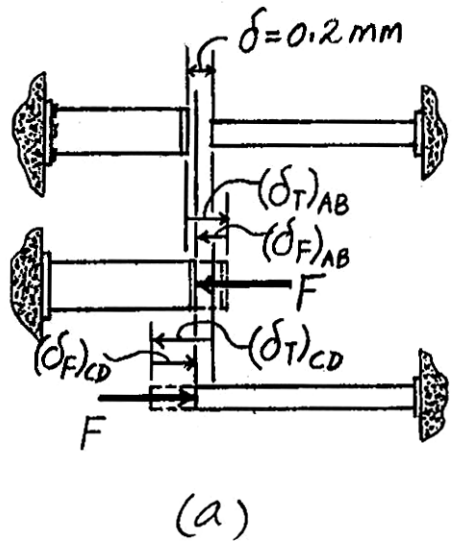
$$\sigma_{CD} = \frac{F}{A_{CD}} = \frac{107\,442.47}{\frac{\pi}{4}\left(0.025^2\right)} = 218.88\text{MPa} < (\sigma_Y)_{\text{al}} \qquad \textbf{(O.K.!)}$$

Compatibility Equation: If tube AB and rod CD are unconstrained, they will have a free expansion of $\left(\delta_T\right)AB = \alpha_{\text{mg}}\,\Delta T L_{AB} = 26(10^{-6})(T - 30)(300) = 7.8(10^{-6})(T - 30)$ and $\left(\delta_T\right)_{CD} = \alpha_{\text{al}}\,\Delta T L_{CD} = 24(10^{-6})(T - 30)(450) = 0.0108(T - 30)$. Referring to the deformation diagram of the tube and the rod shown in Fig. a,

$$\delta = \left[\left(\delta_T\right)_{AB} - \left(\delta_F\right)_{AB}\right] + \left[\left(\delta_T\right)_{CD} - \left(\delta_F\right)_{CD}\right]$$

$$0.2 = \left[7.8(10^{-3})(T - 30) - \frac{107\,442.47(300)}{\pi\left(0.025^2 - 0.02^2\right)(44.7)(10^9)}\right] + \left[0.0108(T - 30) - \frac{107\,442.47(450)}{\frac{\pi}{4}\left(0.025^2\right)(68.9)(10^9)}\right]$$

$T = 172°$ C **Ans.**



(a)

Ans.
$T = 172°$ C

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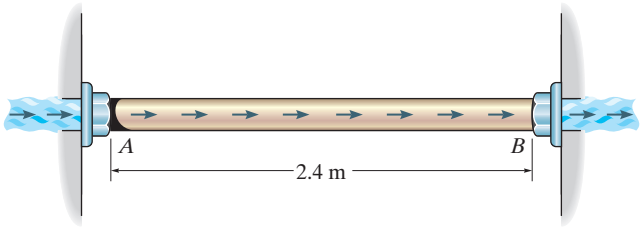
4–73. The pipe is made of A992 steel and is connected to the collars at A and B. When the temperature is 15°C, there is no axial load in the pipe. If hot gas traveling through the pipe causes its temperature to vary along the pipe defined by $T = (35 + 30x)$°C, where x is in meter, determine the average normal stress in the pipe. The inner diameter is 50 mm, the wall thickness is 4 mm.



SOLUTION

Compatibility:

The cross-sectional area is $A = \frac{\pi}{4}(.,.38^2 - .,.3^2) = 216(10^{-6})\ \pi\ \text{m}^2$.

$0 = \delta_T - \delta_F$ Where $\delta_T = \int_0^L \alpha\ \Delta T\ dx$ and ΔT ; &13) 1.x' 1 /3 ; &0.) 1.x'°C.

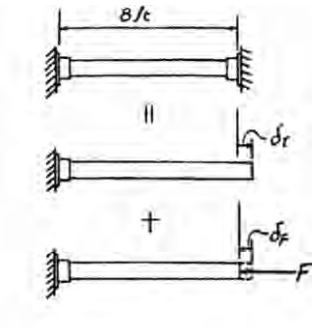
$$0 = 12(10^{-6})\int_0^{2.4\text{m}} (20 + 30x)\ dx - \frac{F(2.4)}{[216(10^{-6})\pi][200(10^9)]}$$

$$0 = 12(10^{-6})\left[20(2.4) + \frac{30(2.4)^2}{2}\right] - \frac{F(2.4)}{[216(10^{-6})\pi][200(10^9)]}$$

$F = 91.20(10^3)$ N

Average Normal Stress:

$$\sigma = \frac{91.20(10^3)}{216(10^{-6})\pi} = 134.40(10^6)\ \text{N/m}^2 = 134\ \text{MPa}$$ **Ans.**



Ans:
$\sigma = 134$ MPa

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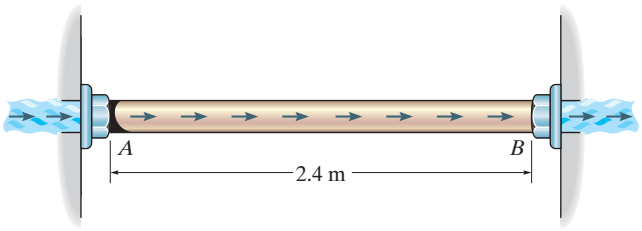
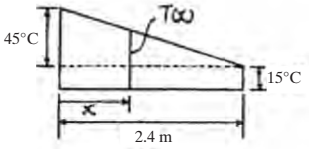
4–74. The bronze C86100 pipe has an inner radius of 12.5 mm and a wall thickness of 5 mm. If the gas flowing through it changes the temperature of the pipe uniformly from $T_A = 60°C$ at A to $T_B = 15°C$ at B, determine the axial force it exerts on the walls. The pipe was fitted between the walls when $T = 15°C$.



SOLUTION

Temperature Gradient:

$$T(x) = 15 + \left(\frac{2.4 - x}{2.4}\right)(45) = 60 - 18.75x$$



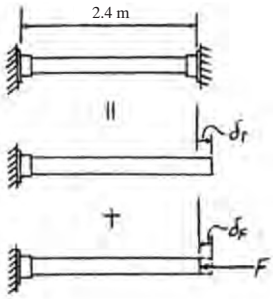
Compatibility:

$0 = \delta_T - \delta_F$ Where $\delta_T = \int \alpha \Delta T\, dx$

$$0 = 17(10^{-6})\int_0^{2.4\text{m}} [(60 - 18.75x) - 15]\, dx - \frac{F(2.4)}{\left[\frac{\pi}{4}(0.035^2 - 0.025^2)\right][103(10^9)]}$$

$$0 = 17(10^{-6})\int_0^{2.4\text{m}} (45 - 18.75x)\, dx - \frac{F(2.4)}{\left[\frac{\pi}{4}(0.035^2 - 0.025^2)\right][103(10^9)]}$$

$F = 18.57(10^3)\text{ N} = 18.6\text{ kN}$ **Ans.**



Ans:
$F = 18.6$ kN

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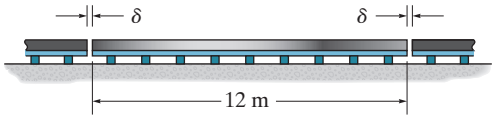
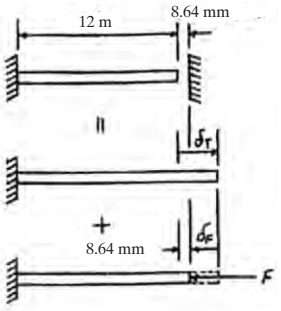
4–75. The 12-m-long A-36 steel rails on a train track are laid with a small gap between them to allow for thermal expansion. Determine the required gap δ so that the rails just touch one another when the temperature is increased from $T_1 = -30°C$ to $T_2 = 30°C$. Using this gap, what would be the axial force in the rails if the temperature were to rise to $T_3 = 40°C$? The cross-sectional area of each rail is 3200 mm^2.



SOLUTION

Thermal Expansion: Note that since adjacent rails expand, each rail will be required to expand $\frac{\delta}{2}$ on each end, or δ for the entine rail.



$$\delta = \alpha \Delta T L = 12(10^{-6})[30 - (-30)](12)(10^3)$$

$$= 8.64 \text{ mm} \qquad \textbf{Ans.}$$

Compatibility:

$$\left(\stackrel{+}{\rightarrow}\right) \quad 8.64 \text{ mm} = \delta_T - \delta_F$$

$$8.64 = 12(10^{-6})[40 - (-30)]12(10^3) - \frac{F(12)(10^3)}{[3200(10^{-6})][200(10^9)]}$$

$$F = 76.8(10^3) \text{ N} = 76.8 \text{ kN} \qquad \textbf{Ans.}$$

Ans:
$\delta = 8.64$ mm, $F = 76.8$ kN

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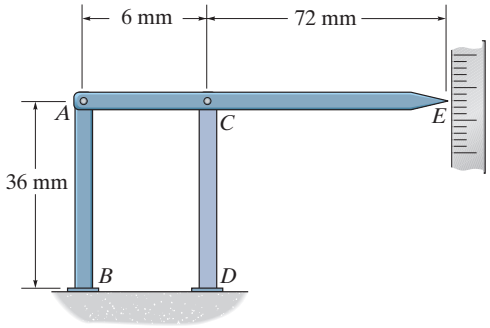
***4–76.** The device is used to measure a change in temperature. Bars AB and CD are made of A-36 steel and 2014-T6 aluminum alloy respectively. When the temperature is at 40°C, ACE is in the horizontal position. Determine the vertial displacement of the pointer at E when the temperature rises to 80°C.



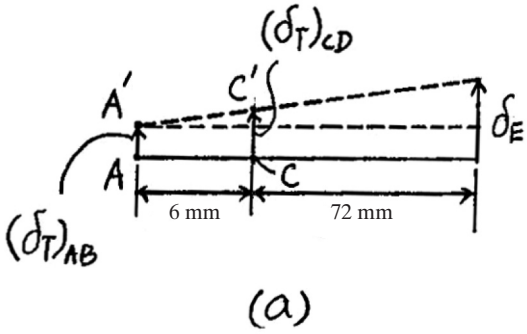
SOLUTION

Thermal Expansion:

$$\left(\delta_T\right)_{CD} = \alpha_{al}\Delta T L_{CD} = 23(10^{-6})(80-40)(36) = 33.12(10^{-3}) \text{ mm}$$

$$\left(\delta_T\right)_{AB} = \alpha_{st}\Delta T L_{AB} = 12(10^{-6})(80-40)(36) = 17.28(10^{-3}) \text{ mm}$$

From the geometry of the deflected bar AE shown Fig. b,



(a)

$$\delta_E = \left(\delta_T\right)_{AB} + \left[\frac{\left(\delta_T\right)_{CD} - \left(\delta_T\right)_{AB}}{6}\right](78)$$

$$= 17.28(10^{-3}) + \left[\frac{32.12(10^{-3}) - 17.28(10^{-3})}{6}\right](78)$$

$$= 0.2102 \text{ mm}$$ **Ans.**

Ans.
$\delta_E = 0.2102$ mm

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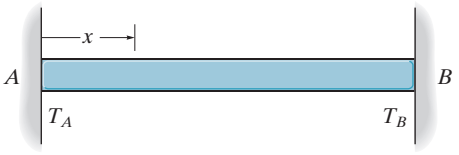
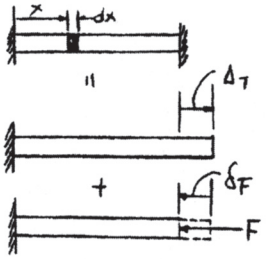
4–77.

The bar has a cross-sectional area A, length L, modulus of elasticity E, and coefficient of thermal expansion α. The temperature of the bar changes uniformly along its length from T_A at A to T_B at B so that at any point x along the bar $T = T_A + x(T_B - T_A)/L$. Determine the force the bar exerts on the rigid walls. Initially no axial force is in the bar and the bar has a temperature of T_A.



SOLUTION

$$\stackrel{+}{\rightarrow} \quad 0 = \Delta_T - \delta_F \qquad \textbf{(1)}$$



However,

$$d\Delta_T = \alpha \Delta_T \, dx = \alpha\left(T_A + \frac{T_B - T_A}{L}x - T_A\right)dx$$

$$\Delta_T = \alpha \int_0^L \frac{T_B - T_A}{L} x \, dx = \alpha\left[\frac{T_B - T_A}{2L}x^2\right]\Big|_0^L$$

$$= \alpha\left[\frac{T_B - T_A}{2}L\right] = \frac{\alpha L}{2}(T_B - T_A)$$

From Eq. (1).

$$0 = \frac{\alpha L}{2}(T_B - T_A) - \frac{FL}{AE}$$

$$F = \frac{\alpha AE}{2}(T_B - T_A) \qquad \textbf{Ans.}$$

Ans:

$$F = \frac{\alpha AE}{2}(T_B - T_A)$$

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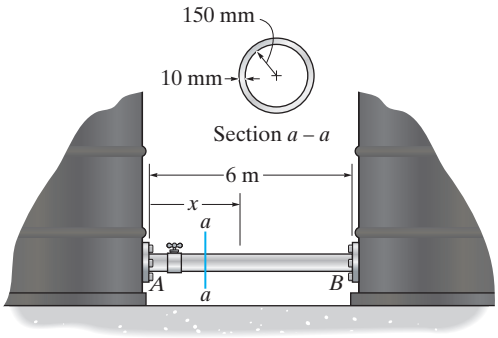
4–78.

When the temperature is at 30°C, the A-36 steel pipe fits snugly between the two fuel tanks. When fuel flows through the pipe, the temperatures at ends A and B rise to 130°C and 80°C, respectively. If the temperature drop along the pipe is linear, determine the average normal stress developed in the pipe. Assume each tank provides a rigid support at A and B.



SOLUTION

Temperature Gradient: Since the temperature varies linearly along the pipe, Fig. *a*, the temperature gradient can be expressed as a function of x as

$$T(x) = 80 + \frac{50}{6}(6 - x) = \left(130 - \frac{50}{6}x\right)°\text{C}$$

Thus, the change in temperature as a function of x is

$$\Delta T = T(x) - 30° = \left(130 - \frac{50}{6}x\right) - 30 = \left(100 - \frac{50}{6}x\right)°\text{C}$$

Compatibility Equation: If the pipe is unconstrained, it will have a free expansion of

$$\delta_T = \alpha \int \Delta T dx = 12(10^{-6}) \int_0^{6\text{m}} \left(100 - \frac{50}{6}x\right) dx = 0.0054 \text{ m} = 5.40 \text{ mm}$$

Using the method of superposition, Fig. *b*,

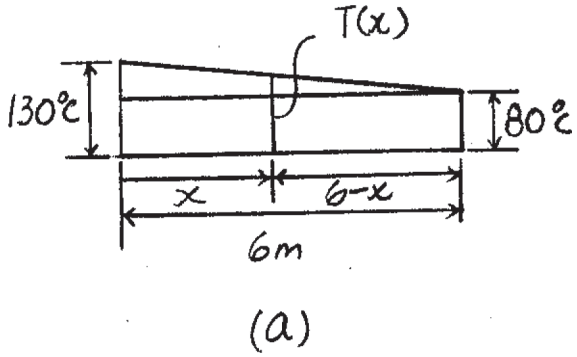
$$(\overset{+}{\rightarrow}) \qquad 0 = \delta_T - \delta_F$$

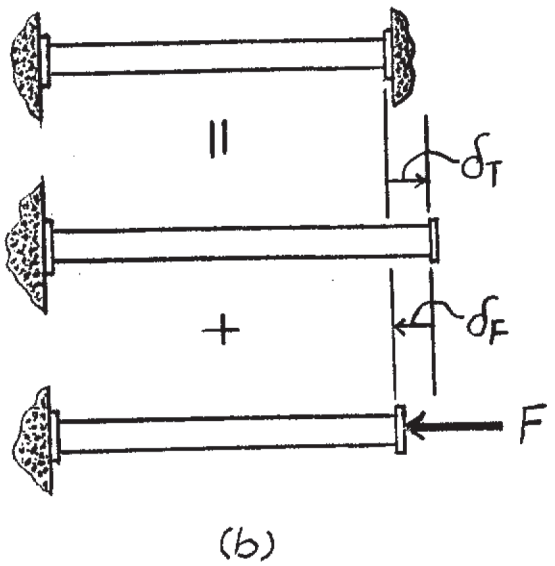
$$0 = 5.40 - \frac{F(6000)}{\pi(0.16^2 - 0.15^2)(200)(10^9)}$$

$$F = 1\,753\,008 \text{ N}$$

Normal Stress:

$$\sigma = \frac{F}{A} = \frac{1\,753\,008}{\pi(0.16^2 - 0.15^2)} = 180 \text{ MPa} \qquad \textbf{Ans.}$$





Ans:
$\sigma = 180$ MPa

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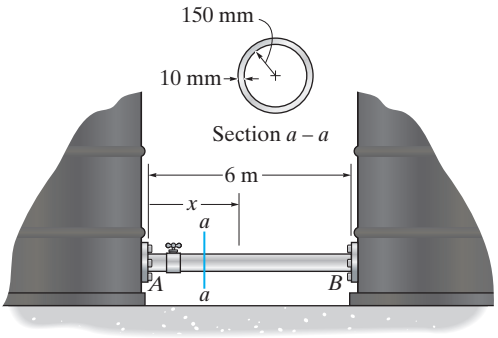
4–79.

When the temperature is at 30°C, the A-36 steel pipe fits snugly between the two fuel tanks. When fuel flows through the pipe, the temperatures at ends A and B rise to 130°C and 80°C, respectively. If the temperature drop along the pipe is linear, determine the average normal stress developed in the pipe. Assume the walls of each tank act as a spring, each having a stiffness of $k = 900\ \text{MN/m}$.



SOLUTION

Temperature Gradient: Since the temperature varies linearly along the pipe, Fig. *a,* the temperature gradient can be expressed as a function of x as

$$T(x) = 80 + \frac{50}{6}(6 - x) = \left(130 - \frac{50}{6}x\right)°\text{C}$$

Thus, the change in temperature as a function of x is

$$\Delta T = T(x) - 30° = \left(130 - \frac{50}{6}x\right) - 30 = \left(100 - \frac{50}{6}x\right)°\text{C}$$

Compatibility Equation: If the pipe is unconstrained, it will have a free expansion of

$$\delta_T = \alpha \int \Delta T dx = 12(10^{-6}) \int_0^{6\text{ m}} \left(100 - \frac{50}{6}x\right) dx = 0.0054\text{ m} = 5.40\text{ mm}$$

Using the method of superposition, Fig. *b*,

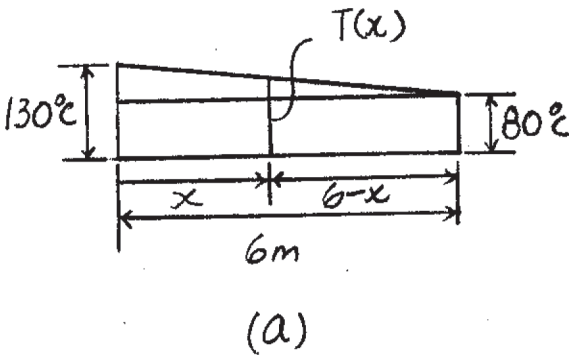
$$(\overset{+}{\rightarrow}) \qquad \delta = \delta_T - \delta_F$$

$$\frac{F}{900(10^6)}(1000) = 5.40 - \left[\frac{F(6000)}{\pi(0.16^2 - 0.15^2)(200)(10^9)} + \frac{F}{900(10^6)}(1000)\right]$$

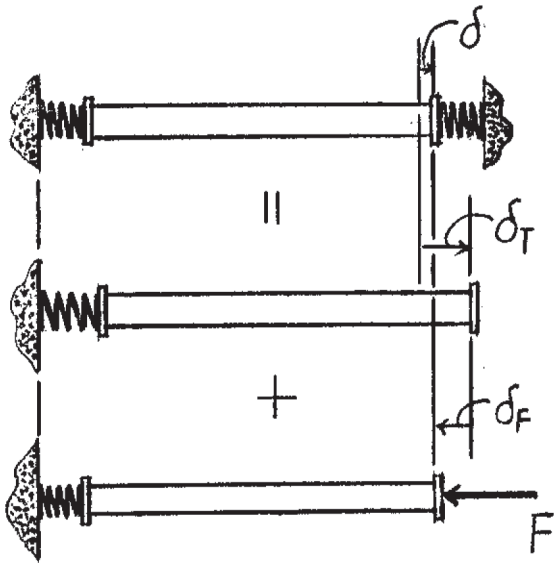
$$F = 1\,018\,361\text{ N}$$

Normal Stress:

$$\sigma = \frac{F}{A} = \frac{1\,018\,361}{\pi(0.16^2 - 0.15^2)} = 105\text{ MPa} \qquad \textbf{Ans.}$$



(a)



(b)

Ans:
$\sigma = 105\text{ MPa}$

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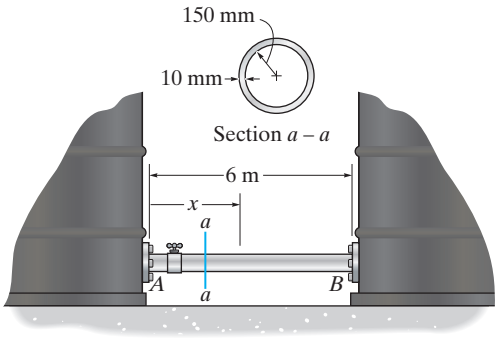
***4–80.**

When the temperature is at 30°C, the A-36 steel pipe fits snugly between the two fuel tanks. When fuel flows through the pipe, it causes the temperature to vary along the pipe as $T = (\frac{5}{3}x^2 - 20x + 120)°C$, where x is in meters. Determine the normal stress developed in the pipe. Assume each tank provides a rigid support at A and B.



SOLUTION

Compatibility Equation: The change in temperature as a function of x is $\Delta T = T - 30° = \left(\frac{5}{3}x^2 - 20x + 120\right) - 30 = \left(\frac{5}{3}x^2 - 20x + 90\right)°C$. If the pipe is unconstrained, it will have a free expansion of

$$\delta_T = \alpha \int \Delta T dx = 12(10^{-6}) \int_0^{6\text{ m}} \left(\frac{5}{3}x^2 - 20x + 90\right) dx = 0.0036\text{ m} = 3.60\text{ mm}$$

Using the method of superposition, Fig. b,

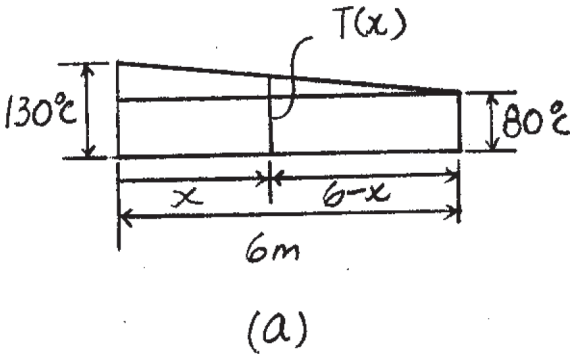
$$(\overset{+}{\rightarrow}) \qquad 0 = \delta_T - \delta_F$$

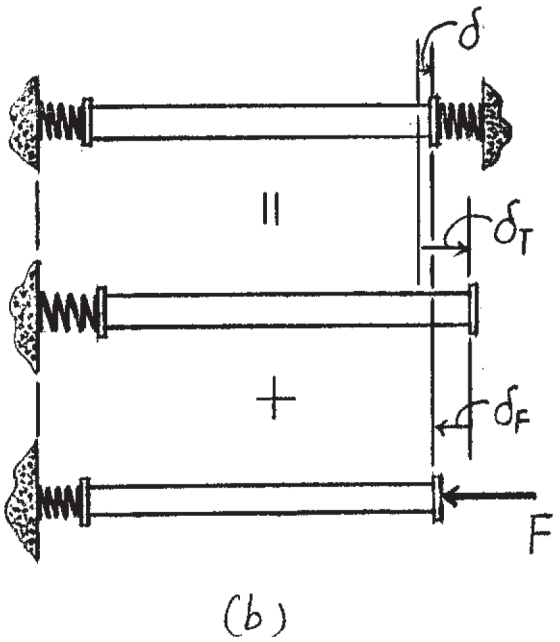
$$0 = 3.60 - \frac{F(6000)}{\pi(0.16^2 - 0.15^2)(200)(10^9)}$$

$$F = 1\,168\,672.47\text{ N}$$

Normal Stress:

$$\sigma = \frac{F}{A} = \frac{1\,168\,672.47}{\pi(0.16^2 - 0.15^2)} = 120\text{ MPa} \qquad \textbf{Ans.}$$





Ans:
$\sigma = 120$ MPa

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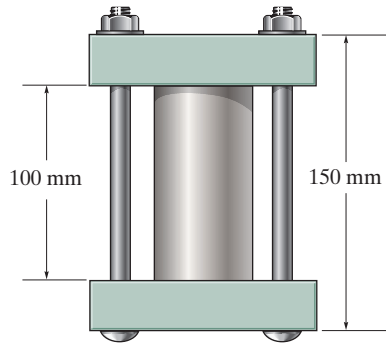
4–81.

The 50-mm-diameter cylinder is made from Am 1004-T61 magnesium and is placed in the clamp when the temperature is $T_1 = 20°$ C. If the 304-stainless-steel carriage bolts of the clamp each have a diameter of 10 mm, and they hold the cylinder snug with negligible force against the rigid jaws, determine the force in the cylinder when the temperature rises to $T_2 = 130°$C.

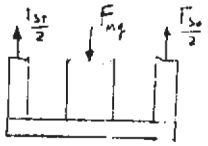


SOLUTION

$+\uparrow \Sigma F_y = 0; \qquad F_{st} = F_{mg} = F$

$$\delta_{mg} = \delta_{st}$$

$$\alpha_{mg} L_{mg} \Delta T - \frac{F_{mg} L_{mg}}{E_{mg} A_{mg}} = \alpha_{st} L_{st} \Delta T + \frac{F_{st} L_{st}}{E_{st} A_{st}}$$



$$26(10^{-6})(0.1)(110) - \frac{F(0.1)}{44.7(10^9)\frac{\pi}{4}(0.05)^2} = 17(10^{-6})(0.150)(110) + \frac{F(0.150)}{193(10^9)(2)\frac{\pi}{4}(0.01)^2}$$

$F = 904$ N **Ans.**

Ans:
$F = 904$ N

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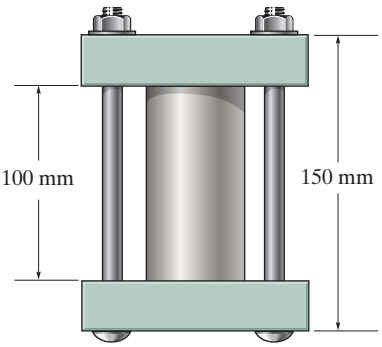
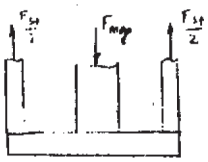
4–82.

The 50-mm-diameter cylinder is made from Am 1004-T61 magnesium and is placed in the clamp when the temperature is $T_1 = 15°C$. If the two 304-stainless-steel carriage bolts of the clamp each have a diameter of 10 mm, and they hold the cylinder snug with negligible force against the rigid jaws, determine the temperature at which the average normal stress in either the magnesium or the steel first becomes 12 MPa.



SOLUTION



$+\uparrow \Sigma F_y = 0; \qquad F_{st} = F_{mg} = F$

$$\delta_{mg} = \delta_{st}$$

$$\alpha_{mg} L_{mg} \Delta T - \frac{F_{mg} L_{mg}}{E_{mg} A_{mg}} = \alpha_{st} L_{st} \Delta T + \frac{F_{st} L_{st}}{E_{st} A_{st}}$$

$$26(10^{-6})(0.1)(\Delta T) - \frac{F(0.1)}{44.7(10^9)\frac{\pi}{4}(0.05)^2} = 17(10^{-6})(0.150)(\Delta T) + \frac{F(0.150)}{193(10^9)(2)\frac{\pi}{4}(0.01)^2}$$

The steel has the smallest cross-sectional area.

$$F = \sigma A = 12(10^6)(2)\left(\frac{\pi}{4}\right)(0.01)^2 = 1885.0\ \text{N}$$

Thus,

$$\Delta T = 229°$$

$$T_2 = 229° + 15° = 244°\ \text{C} \qquad \textbf{Ans.}$$

Ans:
$T_2 = 244°\ \text{C}$

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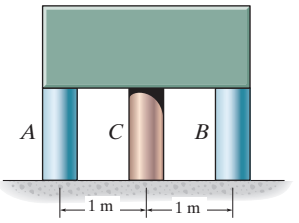
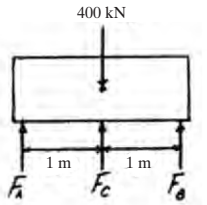
4–83. The rigid block has a weight of 400 kN and is to be supported by posts A and B, which are made of A-36 steel, and the post C, which is made of C83400 red brass. If all the posts have the same original length before they are loaded, determine the average normal stress developed in each post when post C is heated so that its temperature is increased by 10°C. Each post has a cross-sectional area of 5000 mm^2.



SOLUTION

Equations of Equilibrium:



$\circlearrowleft + \Sigma M_C = 0; \quad F_B(1) - F_A(1) = 0 \quad F_A = F_B = F$

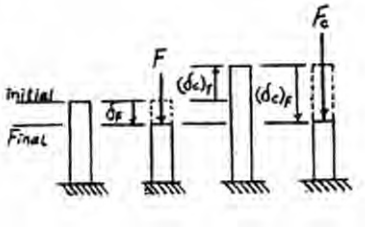
$+\uparrow \Sigma F_y = 0; \quad 2F + F_C - 400(10^3) = 0$ **[1]**

Compatibility:

$(+\downarrow) \qquad (\delta_C)_F - (\delta_C)_T = \delta_F$

$$\frac{F_C L}{\left[5000(10^{-6})\right][101(10^9)]} - 18(10^{-6})(10)L = \frac{FL}{\left[5000(10^{-6})\right][200(10^9)]}$$

$$1.9802F_C - F = 180(10^3) \quad \textbf{[2]}$$



Solving Eqs. [1] and [2] yields:

$F = 123.393(10^3)$ N $\qquad F_C = 153.214(10^3)$ N

average Normal Sress:

$$\sigma_A = \sigma_B = \frac{F}{A} = \frac{123.393(10^3)}{5000(10^{-6})} = 24.68(10^6)\ \text{N/m}^2 = 24.7\ \text{MPa} \quad \textbf{Ans.}$$

$$\sigma_C = \frac{F_C}{A} = \frac{153.214(10^3)}{5000(10^{-6})} = 30.64(10^6)\ \text{N/m}^2 = 30.6\ \text{MPa} \quad \textbf{Ans.}$$

Ans.
$\sigma_A = \sigma_B = 24.7$ MPa, $\sigma_C = 30.6$ MPa

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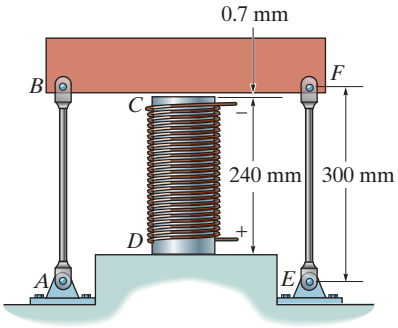
***4–84.**

The cylinder CD of the assembly is heated from $T_1 = 30°C$ to $T_2 = 180°C$ using electrical resistance. At the lower temperature T_1 the gap between C and the rigid bar is 0.7 mm. Determine the force in rods AB and EF caused by the increase in temperature. Rods AB and EF are made of steel, and each has a cross-sectional area of 125 mm^2. CD is made of aluminum and has a cross-sectional area of 375 mm^2. $E_{st} = 200$ GPa, $E_{al} = 70$ GPa, and $\alpha_{al} = 23(10^{-6})/°C$.



SOLUTION

$$\delta_{st} = (\delta_\gamma)_{al} - \delta_{al} - 0.0007$$

$$\frac{F_{st}(0.3)}{(125)(10^{-6})(200)(10^{9})} = 23(10^{-6})(150)(0.24) - \frac{F(0.24)}{(375)(10^{-6})(70)(10^{9})} - 0.0007$$

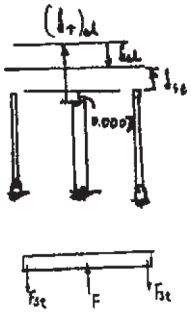
$$12F_{st} = 128\,000 - 9.1428F \qquad \textbf{(1)}$$

$$+\uparrow \Sigma F_y = 0; \qquad F - 2F_{st} = 0 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields,

$$F_{AB} = F_{EF} = F_{st} = 4.23 \text{ kN}$$

$$F_{CD} = F = 8.45 \text{ kN} \qquad \textbf{Ans.}$$



Ans:
$F_{CD} = 8.45$ kN

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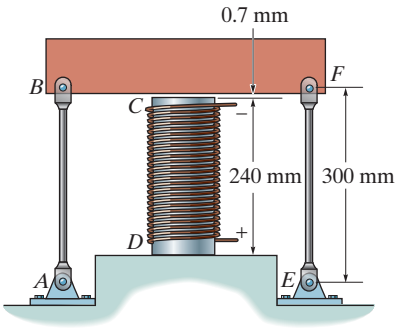
4–85.

The cylinder CD of the assembly is heated from $T_1 = 30°C$ to $T_2 = 180°C$ using electrical resistance. Also, the two end rods AB and EF are heated from $T_1 = 30°C$ to $T_2 = 50°C$. At the lower temperature T_1 the gap between C and the rigid bar is 0.7 mm. Determine the force in rods AB and EF caused by the increase in temperature. Rods AB and EF are made of steel, and each has a cross-sectional area of 125 mm^2. CD is made of aluminum and has a cross-sectional area of 375 mm^2. $E_{st} = 200$ GPa, $E_{al} = 70$ GPa, $\alpha_{st} = 12(10^{-6})/°C$, and $\alpha_{al} = 23(10^{-6})/°C$.



SOLUTION

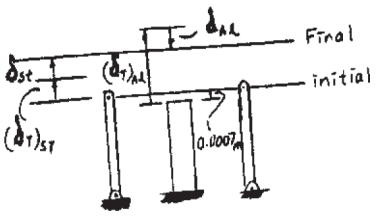
$$\delta_{st} + (\delta_T)_{st} = (\delta_T)_{al} - \delta_{al} - 0.0007$$

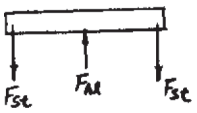
$$\frac{F_{st}(0.3)}{(125)(10^{-6})(200)(10^{9})} + 12(10^{-6})(50 - 30)(0.3)$$

$$= 23(10^{-6})(180 - 30)(0.24) - \frac{F_{al}(0.24)}{375(10^{-6})(70)(10^{9})} - 0.0007$$

$$12.0F_{st} + 9.14286F_{al} = 56000 \qquad \textbf{(1)}$$

$$+\uparrow \Sigma F_y = 0; \qquad F_{al} - 2F_{st} = 0 \qquad \textbf{(2)}$$





Solving Eqs. (1) and (2) yields:

$$F_{AB} = F_{EF} = F_{st} = 1.85 \text{ kN} \qquad \textbf{Ans.}$$

$$F_{CD} = F_{al} = 3.70 \text{ kN}$$

Ans:
$F_{AB} = F_{EF} = 1.85$ kN

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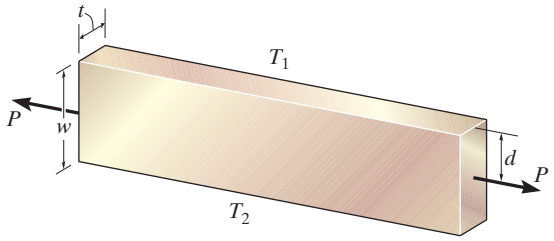
4–86.

The metal strap has a thickness t and width w and is subjected to a temperature gradient T_1 to T_2 ($T_1 < T_2$). This causes the modulus of elasticity for the material to vary linearly from E_1 at the top to a smaller amount E_2 at the bottom. As a result, for any vertical position y, measured from the top surface, $E = [(E_2 - E_1)/w]y + E_1$. Determine the position d where the axial force P must be applied so that the bar stretches uniformly over its cross section.



SOLUTION

$\epsilon = \text{constant} = \epsilon_0$

$$\epsilon_0 = \frac{\sigma}{E} = \frac{\sigma}{\left(\left(\frac{E_2 - E_1}{w}\right)y + E_1\right)}$$

$$\sigma = \epsilon_0\left(\frac{E_2 - E_1}{w}y + E_1\right)$$

$$\pm\rightarrow \Sigma F_x = 0: \quad P - \int_A \sigma\, dA = 0$$

$$P = \int_0^w \sigma t\, dy = \int_0^m \epsilon_0\left(\frac{E_2 - E_1}{w}y + E_1\right) t\, dy$$

$$P = \epsilon_0 t\left(\frac{E_2 - E_1}{2} + E_1 w\right) = \epsilon_0 t\left(\frac{E_2 + E_1}{2}\right) w$$

$$\circlearrowleft + \Sigma M_0 = 0: \quad P(d) - \int_A y\, \sigma dA = 0$$

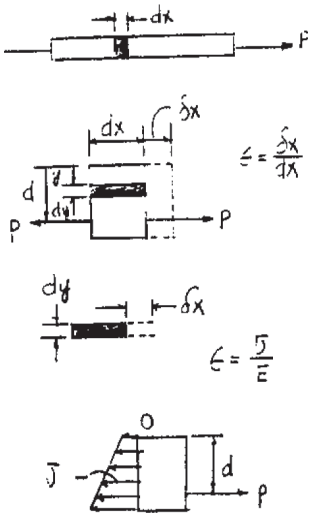
$$\epsilon_0 t\left(\frac{E_2 + E_1}{2}\right) wd = \int_0^w \epsilon_0\left(\left(\frac{E_2 - E_1}{w}\right)y^2 + E_1 y\right) t\, dy$$

$$\epsilon_0 t\left(\frac{E_2 + E_1}{2}\right) wd = \epsilon_0 t\left(\frac{E_2 - E_1}{3}w^2 + \frac{E_1}{2}w^2\right)$$

$$\left(\frac{E_2 + E_1}{2}\right) d = \frac{1}{6}(2E_2 + E_1) w$$

$$d = \left(\frac{2\,E_2 + E_1}{3(E_2 + E_1)}\right) w$$

Ans.



Ans:

$$d = \left(\frac{2E_2 + E_1}{3(E_2 + E_1)}\right) w$$

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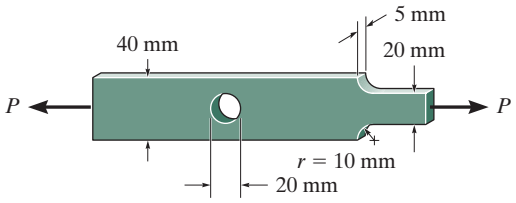
4–87.

Determine the maximum normal stress developed in the bar when it is subjected to a tension of $P = 8$ kN.



SOLUTION

For the fillet:

$$\frac{w}{h} = \frac{40}{20} = 2 \qquad \frac{r}{h} = \frac{10}{20} = 0.5$$

From Fig. 4–23, $K = 1.4$

$$\sigma_{max} = K\sigma_{avg}$$

$$= 1.4\left(\frac{8\,(10^3)}{0.02\,(0.005)}\right)$$

$$= 112 \text{ MPa}$$

For the hole:

$$\frac{2r}{w} = \frac{20}{40} = 0.5$$

From Fig. 4–24, $K = 2.1$

$$\sigma_{max} = K\sigma_{avg}$$

$$= 2.1\left(\frac{8\,(10^3)}{(0.04 - 0.02)(0.005)}\right)$$

$$= 168 \text{ MPa}$$ **Ans.**

Ans:
$\sigma_{max} = 168$ MPa

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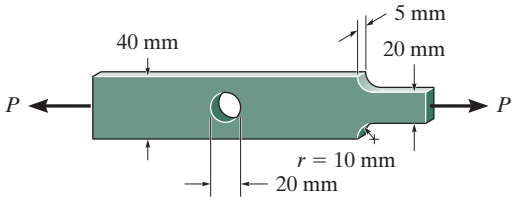
***4–88.**

If the allowable normal stress for the bar is $\sigma_{\text{allow}} = 120\,\text{MPa}$, determine the maximum axial force P that can be applied to the bar.



SOLUTION

Assume failure of the fillet.

$$\frac{w}{h} = \frac{40}{20} = 2; \qquad \frac{r}{h} = \frac{10}{20} = 0.5$$

From Fig. 4–23. $K = 1.4$

$$\sigma_{\text{allow}} = \sigma_{\max} = K\sigma_{\text{avg}}$$

$$120\,(10^6) = 1.4\left(\frac{P}{0.02\,(0.005)}\right)$$

$$P = 8.57\ \text{kN}$$

Assume failure of the hole.

$$\frac{2r}{w} = \frac{20}{40} = 0.5$$

From Fig. 4–24. $K = 2.1$

$$\sigma_{\text{allow}} = \sigma_{\max} = K\sigma_{\text{avg}}$$

$$120\,(10^4) = 2.1\left(\frac{P}{(0.04 - 0.02)\,(0.005)}\right)$$

$$P = 5.71\ \text{kN (controls)} \qquad \textbf{Ans.}$$

Ans:
$P = 5.71\ \text{kN}$

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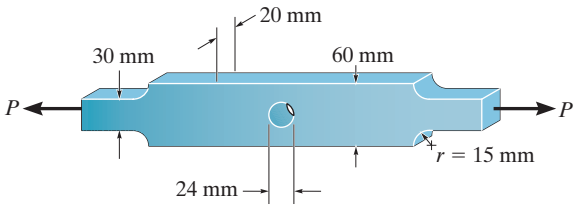
4–89.

The steel bar has the dimensions shown. Determine the maximum axial force P that can be applied so as not to exceed an allowable tensile stress of $\sigma_{allow} = 150$ MPa.



SOLUTION

Assume failure occurs at the fillet:

$$\frac{w}{h} = \frac{60}{30} = 2 \quad \text{and} \quad \frac{r}{h} = \frac{15}{30} = 0.5$$

From the text, $K = 1.4$

$$\sigma_{max} = \sigma_{allow} = K\sigma_{avg}$$

$$150\,(10^6) = 1.4\left[\frac{P}{0.03\,(0.02)}\right]$$

$$P = 64.3 \text{ kN}$$

Assume failure occurs at the hole:

$$\frac{2r}{w} = \frac{24}{60} = 0.4$$

From the text, $K = 2.2$

$$\sigma_{max} = \sigma_{allow} = K\sigma_{avg}$$

$$150\,(10^6) = 2.2\left[\frac{P}{(0.06 - 0.024)\,(0.02)}\right]$$

$$P = 49.1 \text{ kN (controls!)} \qquad \textbf{Ans.}$$

Ans:
$P = 49.1$ kN

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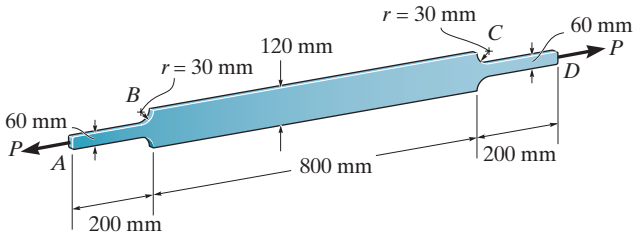
4–90.

The A-36 steel plate has a thickness of 12 mm. If $\sigma_{allow} = 150$ MPa, determine the maximum axial load P that it can support. Calculate its elongation, neglecting the effect of the fillets.



SOLUTION

Maximum Normal Stress at fillet:

$$\frac{r}{h} = \frac{30}{60} = 0.5 \quad \text{and} \quad \frac{w}{h} = \frac{120}{60} = 2$$

From the text, $K = 1.4$

$$\sigma_{max} = \sigma_{allow} = K\sigma_{avg}$$

$$150(10^6) = 1.4\left[\frac{P}{0.06(0.012)}\right]$$

$$P = 77142.86\ \text{N} = 77.1\ \text{kN} \qquad \textbf{Ans.}$$

Displacement:

$$\delta = \Sigma\frac{PL}{AE}$$

$$= \frac{77142.86(400)}{(0.06)(0.012)(200)(10^9)} + \frac{77142.86(800)}{(0.12)(0.012)(200)(10^9)}$$

$$= 0.429\ \text{mm} \qquad \textbf{Ans.}$$

Ans:
$P = 77.1$ kN,
$\delta = 0.429$ mm

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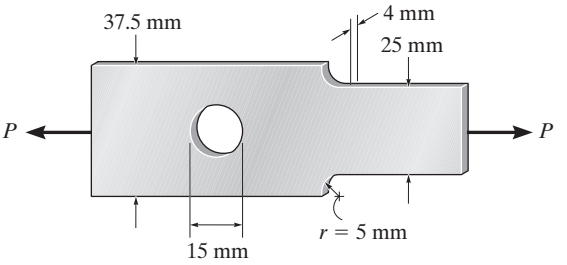
4–91. Determine the maximum axial force P that can be applied to the bar. The bar is made from steel and has an allowable stress of $\sigma_{allow} = 147$ MPa.



SOLUTION

Assume failure of the fillet.

$$\frac{r}{h} = \frac{5}{25} = 0.2 \qquad \frac{w}{h} = \frac{37.5}{25} = 1.5$$

From Fig. 4-24, $K = 1.73$

$$\sigma_{allow} = \sigma_{max} = K\sigma_{avg}$$

$$147(10^6) = 1.73\left[\frac{P}{(0.025)(0.004)}\right]$$

$$P = 8.497(10^3)\ \text{N}$$

Assume failure of the hole.

$$\frac{r}{w} = \frac{7.5}{37.5} = 0.20$$

From Fig. 4-25, $K = 2.45$

$$\sigma_{allow} = \sigma_{max} = K\sigma_{avg}$$

$$147(10^6) = 2.45\left[\frac{P}{(0.0375 - 0.015)(0.004)}\right]$$

$$P = 5.400(10^3)\ \text{N} \quad \text{(controls)} \qquad \textbf{Ans.}$$

$$= 5.40\ \text{kN} \qquad \textbf{Ans.}$$

Ans.
$P = 5.40$ kN

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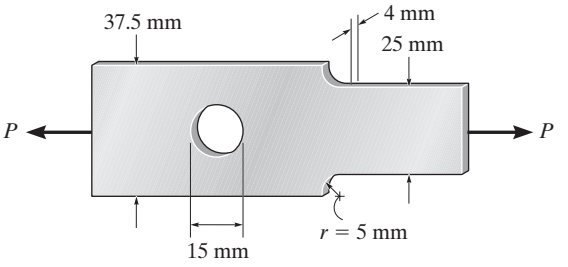
*4–92.

Determine the maximum normal stress developed in the bar when it is subjected to a tension of $P = 8$ kN.



SOLUTION

At fillet:

$$\frac{r}{h} = \frac{5}{25} = 0.2 \qquad \frac{w}{h} = \frac{37.5}{25} = 1.5$$

From Fig. 4-24, $K = 1.73$

$$\sigma_{\max} = K\left(\frac{P}{A}\right) = 1.73\left[\frac{8(10^3)}{(0.025)(0.004)}\right] = 138.4(10^6)\ \text{N/m}^2 = 138\ \text{MPa}$$

At hole:

$$\frac{r}{w} = \frac{7.5}{37.5} = 0.20$$

From Fig. 4-25, $K = 2.45$

$$\sigma_{\max} = 2.45\left[\frac{8(10^3)}{(0.0375 - 0.015)(0.004)}\right] = 217.78(10^6)\ \text{N/m}^2 = 218\ \text{MPa}\ \text{(Controls)}$$

Ans.

Ans.

$\sigma_{\max} = 2.45$ MPa

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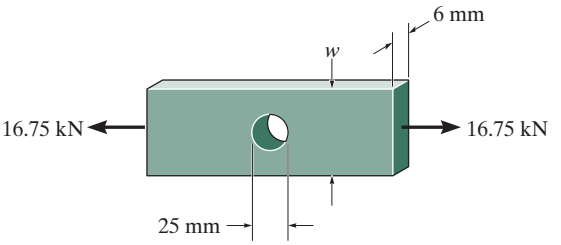
4–93. The member is to be made from a steel plate that is 6 mm thick. If a 25-mm hole is drilled through its center, determine the approximate width w of the plate so that it can support an axial force of 16.75 kN. The allowable stress is $\sigma_{allow} = 150$ MPa.



SOLUTION

$$\sigma_{allow} = \sigma_{max} = K\sigma_{avg}$$

$$150(10^6) = K\left[\frac{16.75(10^3)}{(w - 0.025)(0.006)}\right]$$

$$w = 0.025 + 0.01861K$$

By trial and error, from Fig. 4-25, choose $\frac{r}{w} = 0.17$; $K = 2.50$

$$w = 0.025 + 0.01861(2.50) = 0.07153 \text{ m} = 71.5 \text{ mm}$$ **Ans.**

Since $\frac{r}{w} = \frac{12.5}{71.53} = 0.17$ OK

Ans.
$K = 2.50, w = 71.5$ mm

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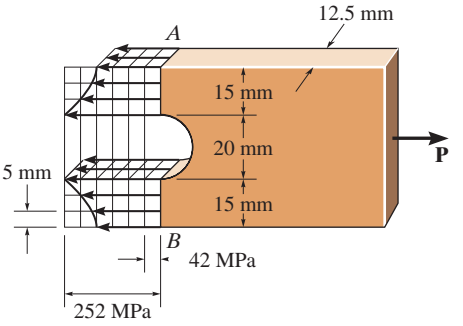
4–94.

The resulting stress distribution along section AB for the bar is shown. From this distribution, determine the approximate resultant axial force P applied to the bar. Also, what is the stress-concentration factor for this geometry?



SOLUTION

Number of squares = 28

$$P = 28[42(10^6)](0.005)(0.0125) = 73.5(10^3)\ \text{N} = 73.5\ \text{kN}$$ **Ans.**

$$\sigma_{avg} = \frac{P}{A} = \frac{73.5(10^3)}{2(0.015)(0.0125)} = 196(10^6)\ \text{N/m}^2 = 196\ \text{MPa}$$

$$K = \frac{\sigma_{max}}{\sigma_{avg}} = \frac{252}{196} = 1.286 = 1.29$$ **Ans.**

Ans.

$P = 73.5\ \text{kN}, K = 1.29$

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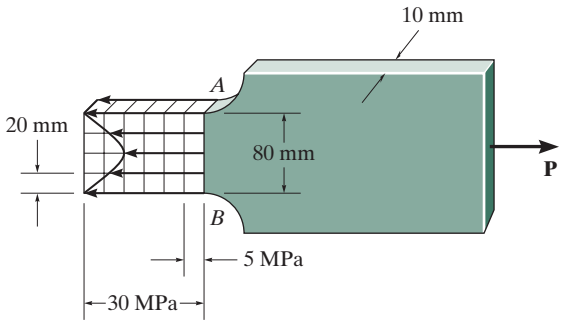
4–95.

The resulting stress distribution along section AB for the bar is shown. From this distribution, determine the approximate resultant axial force P applied to the bar. Also, what is the stress concentration factor?



SOLUTION

Number of squares = 19

$$P = 19(5)(10^6)(0.02)(0.01) = 19\ \text{kN}$$ **Ans.**

$$\sigma_{\text{avg}} = \frac{P}{A} = \frac{19(10^3)}{0.08(0.01)} = 23.75\ \text{MPa}$$

$$K = \frac{\sigma_{\text{max}}}{\sigma_{\text{avg}}} = \frac{30\ \text{MPa}}{23.75\ \text{MPa}} = 1.26$$ **Ans.**

Ans:
$P = 19$ kN,
$K = 1.26$

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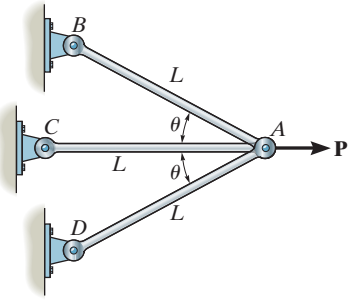
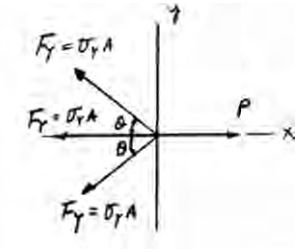
***4–96.**

The three bars are pinned together and subjected to the load **P**. If each bar has a cross-sectional area A, length L, and is made from an elastic perfectly plastic material, for which the yield stress is σ_Y, determine the largest load (ultimate load) that can be supported by the bars, i.e., the load P that causes all the bars to yield. Also, what is the horizontal displacement of point A when the load reaches its ultimate value? The modulus of elasticity is E.



SOLUTION

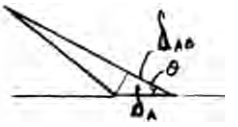


When all bars yield, the force in each bar is, $F_Y = \sigma_Y A$

$\stackrel{+}{\rightarrow} \Sigma F_x = 0; \qquad P - 2\sigma_Y A \cos\theta - \sigma_Y A = 0$

$P = \sigma_Y A(2\cos\theta + 1)$ **Ans.**

Bar AC will yield first followed by bars AB and AD.



$$\delta_{AB} = \delta_{AD} = \frac{F_Y(L)}{AE} = \frac{\sigma_Y AL}{AE} = \frac{\sigma_Y L}{E}$$

$$\delta_A = \frac{\delta_{AB}}{\cos\theta} = \frac{\sigma_Y L}{E\cos\theta}$$ **Ans.**

Ans.

$$\delta_A = \frac{\delta_{AB}}{\cos\theta} = \frac{\sigma_Y L}{E\cos\theta}$$

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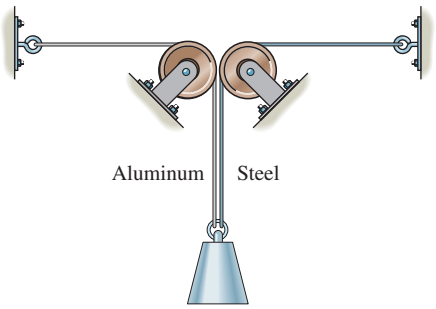
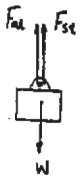
4–98.

The weight is suspended from steel and aluminum wires, each having the same initial length of 3 m and cross-sectional area of 4 mm^2. If the materials can be assumed to be elastic perfectly plastic, with $(\sigma_Y)_{st} = 120$ MPa and $(\sigma_Y)_{al} = 70$ MPa, determine the force in each wire if the weight is (a) 600 N and (b) 720 N. $E_{al} = 70$ GPa, $E_{st} = 200$ GPa.



SOLUTION

$+\uparrow \Sigma F_y = 0; \quad F_{al} + F_{st} - W = 0$ **(1)**



Assume both wires behave elastically.

$$\delta_{al} = \delta_{st}; \quad \frac{F_{al}L}{A(70)} = \frac{F_{st}L}{A(200)}$$

$F_{al} = 0.35\,F_{st}$ **(2)**

(a) When $W = 600$ N, solving Eqs. (1) and (2) yields:

$F_{st} = 444.44$ N $= 444$ N **Ans.**

$F_{al} = 155.55$ N $= 156$ N **Ans.**

$$\sigma_{al} = \frac{F_{al}}{A_{st}} = \frac{155.55}{4(10^{-6})} = 38.88 \text{ MPa} < (\sigma_\gamma)_{al} = 70 \text{ MPa}$$ **OK**

$$\sigma_{st} = \frac{F_{st}}{A_{st}} = \frac{444.44}{4(10^{-6})} = 111.11 \text{ MPa} < (\sigma_\gamma)_{st} = 120 \text{ MPa}$$ **OK**

The elastic analysis is valid for both wires.

(b) When $W = 720$ N, solving Eqs. (1) and (2) yields:

$F_{st} = 533.33$ N: $\quad F_{st} = 186.67$ N

$$\sigma_{al} = \frac{F_{al}}{A_{al}} = \frac{186.67}{4(10^{-6})} = 46.67 \text{ MPa} < (\sigma_\gamma)_{al} = 70 \text{ MPa}$$ **OK**

$$\sigma_{st} = \frac{F_{st}}{A_{st}} = \frac{533.33}{4(10^{-6})} = 133.33 \text{ MPa} > (\sigma_\gamma)_{st} = 120 \text{ MPa}$$

Therefore, the steel wire yields. Hence,

$F_{st} = (\sigma_\gamma)_{st}A_{st} = 120(10^6)(4)(10^{-6}) = 480$ N **Ans.**

From Eq. (1), $F_{al} = 240$ N **Ans.**

$$\sigma_{al} = \frac{240}{4(10^{-6})} = 60 \text{ MPa} < (\sigma_\gamma)_{al}$$ **OK**

Ans:
(a) $F_{st} = 444$ N, $F_{al} = 156$ N,
(b) $F_{st} = 480$ N, $F_{al} = 240$ N

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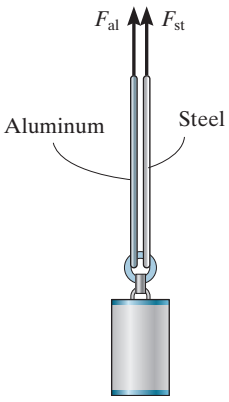
4–99.

The weight is suspended from steel and aluminum wires, each having the same initial length of 3 m and cross-sectional area of 4 mm^2. If the materials can be assumed to be elastic perfectly plastic, with $(\sigma_Y)_{st} = 120$ MPa and $(\sigma_Y)_{al} = 70$ MPa, determine the force in each wire if the weight is (a) 600 N and (b) 720 N. $E_{al} = 70$ GPa, $E_{st} = 200$ GPa.



SOLUTION

Equations of Equilibrium:

$$+\uparrow \Sigma F_y = 0; \qquad F_{al} + F_{st} - w = 0 \qquad \textbf{(1)}$$

Elastic Analysis: Assume both wires behave elastically.

$$\delta_{al} = \delta_{st}$$

$$\frac{F_{al} L}{A(70)(10^9)} = \frac{F_{st} L}{A(200)(10^9)}$$

$$F_{al} = 0.350\, F_{st} \qquad \textbf{(2)}$$

a) When $W = 600$ N, solving Eq. (1) and (2) yields:

$$F_{st} = 444.44 \text{ N} = 444 \text{ N} \qquad \textbf{Ans.}$$

$$F_{al} = 155.55 \text{ N} = 156 \text{ N} \qquad \textbf{Ans.}$$

Average Normal Stress:

$$\sigma_{al} = \frac{F_{al}}{A_{sl}} = \frac{155.55}{4.00(10^{-6})} = 38.88 \text{ MPa} < (\sigma_\gamma)_{al} = 70.0 \text{ MPa} \qquad \textbf{(OK!)}$$

$$\sigma_{st} = \frac{F_{st}}{A_{st}} = \frac{444.44}{4.00(10^{-6})} = 111.11 \text{ MPa} < (\sigma_\gamma)_{st} = 120 \text{ MPa} \qquad \textbf{(OK!)}$$

The average normal stress for both wires do not exceed their respective yield stress. Therefore, the elastic analysis is vaild for both wires

b) When $W = 720$ N, solving Eq. (1) and (2) yields:

$$F_{st} = 533.33 \text{ N} \qquad F_{al} = 186.67 \text{ N}$$

Average Normal Stress:

$$\sigma_{al} = \frac{F_{al}}{A_{al}} = \frac{186.67}{4.00(10^{-6})} = 46.67 \text{ MPa} < (\sigma_\gamma)_{al} = 70.0 \text{ MPa} \qquad \textbf{(OK!)}$$

$$\sigma_{st} = \frac{F_{st}}{A_{st}} = \frac{533.33}{4.00(10^{-6})} = 133.33 \text{ MPa} > (\sigma_\gamma)_{st} = 120 \text{ MPa}$$

Therefore, the steel wire yields. Hence,

$$F_{st} = (\sigma_\gamma)_{st} A_{st} = 120(10^6)(4.00)(10^{-6}) = 480 \text{ N} \qquad \textbf{Ans.}$$

From Eq. (1), $F_{al} = 240$ N **Ans.**

$$\sigma_{al} = \frac{240}{4.00(10^{-6})} = 60.00 \text{ MPa} < (\sigma_\gamma)_{al} \qquad \textbf{(OK!)}$$

Ans:
$F_{st} = 444$ N,
$F_{al} = 156$ N,
$F_{st} = 480$ N,
$F_{al} = 240$ N

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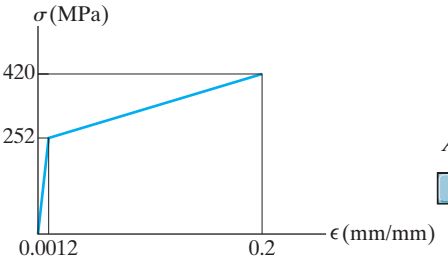
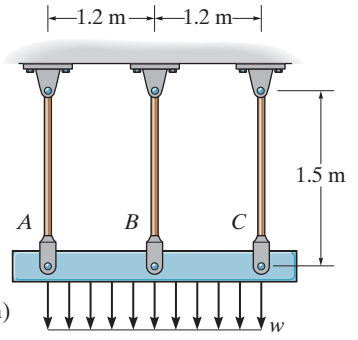
***4–100.**

The distributed loading is applied to the rigid beam, which is supported by the three bars. Each bar has a cross-sectional area of 780 mm^2 and is made from a material having a stress–strain diagram that can be approximated by the two line segments shown. If a load of w = 400 kN/m is applied to the beam, determine the stress in each bar and the vertical displacement of the beam.





SOLUTION

$\zeta + \Sigma M_B = 0; \quad F_C(1.2) - F_A(1.2) = 0;$

$F_A = F_C = F$

$+\uparrow \Sigma F_y = 0; \quad 2F + F_B - 960 = 0$ **(1)**

Since the loading and geometry are symmetrical, the bar will remain horizontal. Therefore, the displacement of the bars is the same and hence, the force in each bar is the same. From Eq. (1).

$F = F_B = 320$ kN

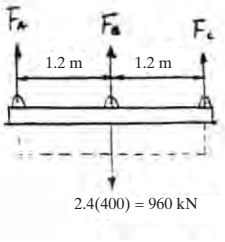
Thus,

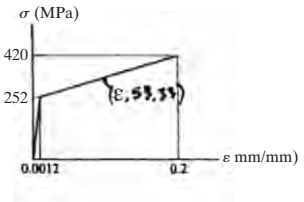
$$\sigma_A = \sigma_B = \sigma_C = \frac{320(10^3)}{780(10^{-6})} = 410.26(10^6)\ \text{N-m}^2 = 410\ \text{MPa}$$ **Ans.**

From the stress-strain diagram:

$$\frac{410.26 - 252}{\varepsilon - 0.0012} = \frac{420 - 252}{0.2 - 0.0012}; \quad \varepsilon = 0.18847\ \text{mm-mm}$$

$$\delta = \varepsilon L = 0.18847(1.5)(10^3) = 282.71\ \text{mm} = 283\ \text{mm}$$ **Ans.**





Ans.
$\delta = 283$ mm

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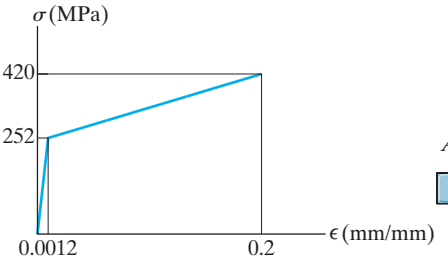
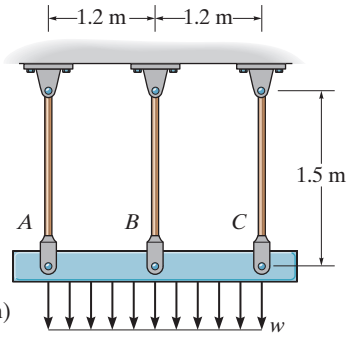
4–101.

The distributed loading is applied to the rigid beam, which is supported by the three bars. Each bar has a cross-sectional area of 468 mm^2 and is made from a material having a stress–strain diagram that can be approximated by the two line segments shown. Determine the intensity of the distributed loading w needed to cause the beam to be displaced downard 37.5 mm.





SOLUTION

$\zeta + \Sigma M_B = 0; \quad F_C(1.2) - F_A(1.2) = 0; \quad F_A = F_C = F$

$+\uparrow \Sigma F_y = 0; \quad 2F + F_B - 2.4w = 0$ **(1)**

Since the system and the loading are symmetrical, the bar will remain horizontal. Hence the displacement of the bars is the same and the force supported by each bar is the same.

From Eq. (1),

$F_B = F = 0.8\,w$ **(2)**

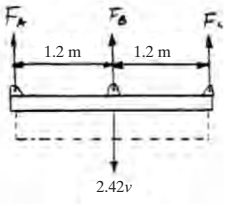
From the stress-strain diagram:

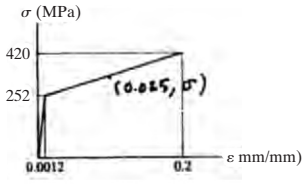
$$\varepsilon = \frac{37.5}{1.5(10^3)} = 0.025 \text{ mm/mm}$$

$$\frac{\sigma - 252}{0.025 - 0.0012} = \frac{420 - 252}{0.2 - 0.0012}; \quad \sigma = 272.11 \text{ MPa}$$

Hence $F = \sigma A = [272.11(10^6)][468(10^{-6})] = 127.35(10^3)$ N

From Eq. (2), $w = 159.19(10^3)$ N/m = 159 kN/m **Ans.**





Ans.
$w = 159$ kN/m

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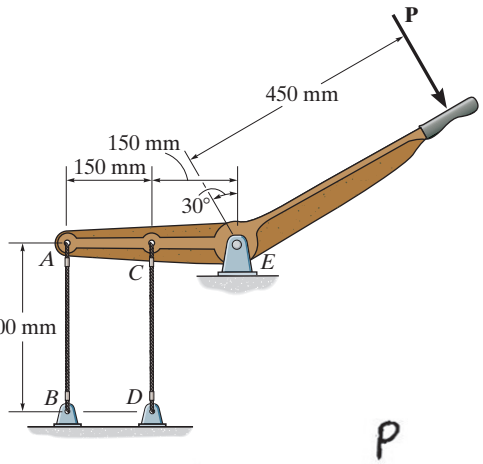
4–102.

The rigid lever arm is supported by two A-36 steel wires having the same diameter of 4 mm. Determine the smallest force **P** that will cause (a) only one of the wires to yield; (b) both wires to yield. Consider A-36 steel as an elastic perfectly plastic material.



SOLUTION

Equation of Equilibrium: Referring to the free-body diagram of the lever arm shown in Fig. *a*,

$$\zeta+\Sigma M_E = 0; \qquad F_{AB}(300) + F_{CD}(150) - P(450) = 0$$

$$2F_{AB} + F_{CD} = 3P \qquad \textbf{(1)}$$

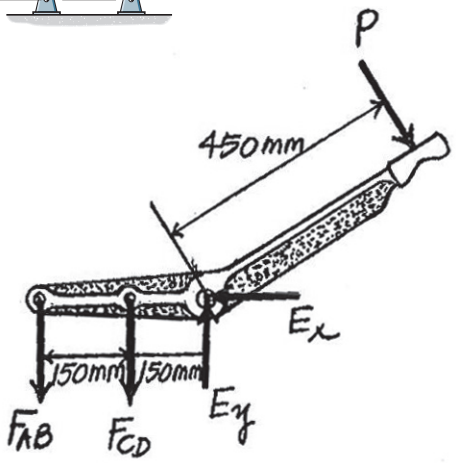
(a) Elastic Analysis: The compatibility equation can be written by referring to the geometry of Fig. *b*.

$$\delta_{AB} = \left(\frac{300}{150}\right)\delta_{CD}$$

$$\delta_{AB} = 2\delta_{CD}$$

$$\frac{F_{AB}L}{AE} = 2\left(\frac{F_{CD}L}{AE}\right)$$

$$F_{CD} = \frac{1}{2}F_{AB} \qquad \textbf{(2)}$$



(a)

Assuming that wire *AB* is about to yield first,

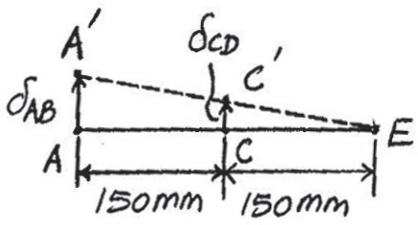
$$F_{AB} = (\sigma_Y)_{st}\,A_{AB} = 250\left(10^6\right)\left[\frac{\pi}{4}\left(0.004^2\right)\right] = 3141.59\,\text{N}$$

From Eq. (2),

$$F_{CD} = \frac{1}{2}(3141.59) = 1570.80\,\text{N}$$

Substituting the result of F_{AB} and F_{CD} into Eq. (1),

$$P = 2618.00\,\text{N} = 2.62\,\text{kN} \qquad \textbf{Ans.}$$



(b)

(b) Plastic Analysis: Since both wires *AB* and *CD* are required to yield,

$$F_{AB} = F_{CD} = (\sigma_Y)_{st}\,A = 250\left(10^6\right)\left[\frac{\pi}{4}\left(0.004^2\right)\right] = 3141.59\,\text{N}$$

Substituting this result into Eq. (1),

$$P = 3141.59\ \text{N} = 3.14\ \text{kN} \qquad \textbf{Ans.}$$

Ans:
(a) $P = 2.62\,\text{kN}$, (b) $P = 3.14\ \text{kN}$

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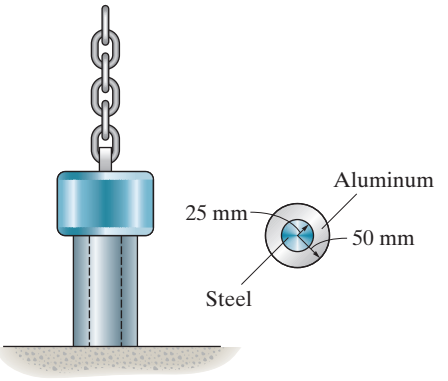
4–103. The 1500-kN weight is slowly set on the top of a post made of 2014-T6 aluminum with an A-36 steel core. If both materials can be considered elastic perfectly plastic, determine the stress in each material.

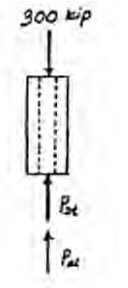


SOLUTION

Equations of Equilibrium:

$$+\uparrow \Sigma F_y = 0; \qquad P_{st} + P_{al} - 1500 = 0 \qquad \textbf{[1]}$$

Elastic Analysis: Assume both materials still behave elastically under the load.



$$\delta_{st} = \delta_{al}$$

$$\frac{P_{st}L}{\left[\frac{\pi}{4}(0.05^2)\right][200(10^9)]} = \frac{P_{al}L}{\left[\frac{\pi}{4}(0.1^2 - 0.05^2)\right][73.1(10^9)]}$$

$$P_{st} = 0.9120 P_{al}$$

Solving Eqs. [1] and [2] yields:

$$P_{al} = 784.52 \text{ kN} \qquad P_{st} = 715.48 \text{ kN}$$

Average Normal Stress:

$$\sigma_{al} = \frac{P_{al}}{A_{al}} = \frac{784.52(10^3)}{\frac{\pi}{4}(0.1^2 - 0.05^2)} = 133.18 \text{ MPa} < (\sigma_\gamma)_{al} = 414 \text{ MPa} \qquad \textbf{(OK!)}$$

$$\sigma_{st} = \frac{P_{st}}{A_{st}} = \frac{715.48(10^3)}{\frac{\pi}{4}(0.05^2)} = 364.39 \text{ MPa} > (\sigma_\gamma)_{st} = 250 \text{ MPa}$$

Therefore, the steel core yields and so the elastic analysis is invalid. The stress in the steel is

$$\sigma_{st} = (\sigma_\gamma)_{st} = 250 \text{ MPa} \qquad \textbf{Ans.}$$

$$P_{st} = (\sigma_\gamma)_{st} A_{st} = \left[250(10^6)\right]\left[\frac{\pi}{4}(0.05^2)\right] = 490.87(10^3) \text{ N} = 490.87 \text{ kN}$$

From Eq. [1] $P_{al} = 1009.13$ kN

$$\sigma_{al} = \frac{P_{al}}{A_{al}} = \frac{1009.13(10^3)}{\frac{\pi}{4}(0.1^2 - 0.05^2)} = 171.31(10^6) \text{ N-m}^2 = 171.31 \text{ MPa} < (\sigma_\gamma)_{al} = 414 \text{ MPa}$$

Then $\sigma_{al} = 171.31 \text{ MPa} = 171 \text{ MPa}$ **Ans.**

Ans.
$\sigma_{st} = 250$ MPa, $\sigma_{al} = 171$ MPa

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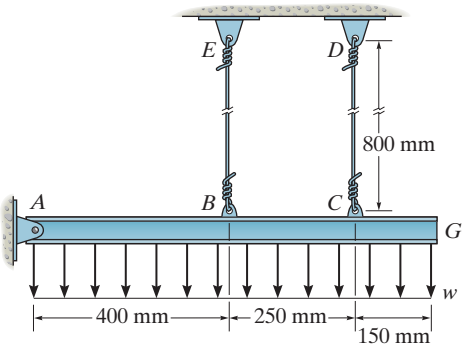
***4–104.** The rigid bar is supported by a pin at A and two steel wires, each having a diameter of 4 mm. If the yield stress for the wires is $\sigma_Y = 530$ MPa, and $E_{st} = 200$ GPa, determine (a) the intensity of the distributed load w that can be placed on the beam that will cause only one of the wires to start to yield and (b) the smallest intensity of the distributed load that will cause both wires to yield. For the calculation, assume that the steel is elastic perfectly plastic.



SOLUTION

Equations of Equilibrium:

$$\zeta + \Sigma M_A = 0; \qquad F_{BE}(0.4) + F_{CD}(0.65) - 0.8w\,(0.4) = 0$$

$$0.4\,F_{BE} + 0.65\,F_{CD} = 0.32w \qquad \textbf{[1]}$$

(a) By observation, wire CD will yield first.

Then $F_{CD} = \sigma_\gamma A = 530\left(10^6\right)\left(\frac{\pi}{4}\right)\left(0.004^2\right) = 6.660$ kN.

From the geometry

$$\frac{\delta_{BE}}{0.4} = \frac{\delta_{CD}}{0.65}; \qquad \delta_{CD} = 1.625\delta_{BE}$$

$$\frac{F_{CD}L}{AE} = 1.625\,\frac{F_{BE}L}{AE}$$

$$F_{CD} = 1.625\,F_{BE} \qquad \textbf{[2]}$$

Using $F_{CD} = 6.660$ kN and solving Eqs. [1] and [2] yields:

$$F_{BE} = 4.099 \text{ kN}$$

$$w = 18.7 \text{ kN/m} \qquad \textbf{Ans.}$$

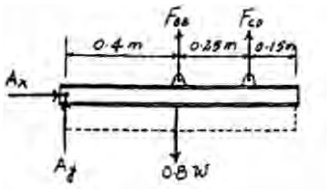
(b) When both wires yield

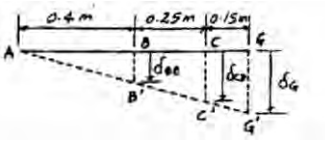
$$F_{BE} = F_{CD} = (\sigma_\gamma)A$$

$$= 530\left(10^6\right)\left(\frac{\pi}{4}\right)\left(0.004^2\right) = 6.660 \text{ kN}$$

Substituting the results into Eq. [1] yields:

$$w = 21.9 \text{ kN/m} \qquad \textbf{Ans.}$$





Ans.
$w = 21.9$ kN/m

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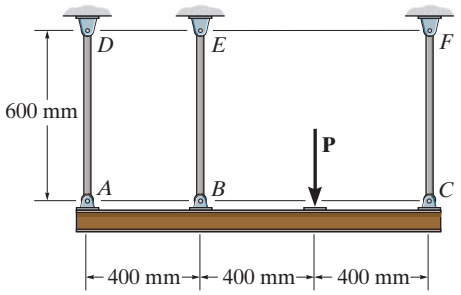
4–105.

The rigid beam is supported by three 25-mm diameter A-36 steel rods. If the beam supports the force of $P = 230$ kN, determine the force developed in each rod. Consider the steel to be an elastic perfectly plastic material.



SOLUTION

Equation of Equilibrium: Referring to the free-body diagram of the beam shown in Fig. *a*,

$$+\uparrow \Sigma F_y = 0; \qquad F_{AD} + F_{BE} + F_{CF} - 230(10^3) = 0 \qquad \textbf{(1)}$$

$$\circlearrowleft + \Sigma M_A = 0; \qquad F_{BE}(400) + F_{CF}(1200) - 230(10^3)(800) = 0$$

$$F_{BE} + 3F_{CF} = 460(10^3) \qquad \textbf{(2)}$$

Elastic Analysis: Referring to the deflection diagram of the beam shown in Fig. *b*, the compatibility equation can be written as

$$\delta_{BE} = \delta_{AD} + \left(\frac{\delta_{CF} - \delta_{AD}}{1200}\right)(400)$$

$$\delta_{BE} = \frac{2}{3}\delta_{AD} + \frac{1}{3}\delta_{CF}$$

$$\frac{F_{BE}L}{AE} = \frac{2}{3}\left(\frac{F_{AD}L}{AE}\right) + \frac{1}{3}\left(\frac{F_{CF}L}{AE}\right)$$

$$F_{BE} = \frac{2}{3}F_{AD} + \frac{1}{3}F_{CF} \qquad \textbf{(3)}$$

Solving Eqs. (1), (2), and (3)

$F_{CF} = 131\,428.57$ N $\qquad F_{BE} = 65\,714.29$ N $\qquad F_{AD} = 32\,857.14$ N

Normal Stress:

$$\sigma_{CF} = \frac{F_{CF}}{A_{CF}} = \frac{131428.57}{\frac{\pi}{4}(0.025^2)} = 267.74\ \text{MPa} > (\sigma_Y)_{\text{st}} \qquad \textbf{(N.G.)}$$

$$\sigma_{BE} = \frac{F_{BE}}{A_{BE}} = \frac{65714.29}{\frac{\pi}{4}(0.025^2)} = 133.87\ \text{MPa} < (\sigma_Y)_{\text{st}} \qquad \textbf{(O.K.)}$$

$$\sigma_{AD} = \frac{F_{AD}}{A_{AD}} = \frac{32857.14}{\frac{\pi}{4}(0.025^2)} = 66.94\ \text{MPa} < (\sigma_Y)_{\text{st}} \qquad \textbf{(O.K.)}$$

Since rod *CF* yields, the elastic analysis is not valid. The solution must be reworked using

$$F_{CF} = (\sigma_Y)_{\text{st}} A_{CF} = 250\,(10^6)\left[\frac{\pi}{4}(0.025^2)\right] = 122\,718.46\,\text{N} = 123\,\text{kN} \qquad \textbf{Ans.}$$

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4–105. Continued

Substituting this result into Eq. (2),

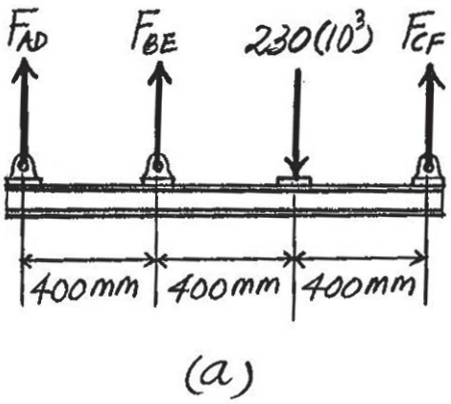
$$F_{BE} = 91844.61\ \text{N} = 91.8\ \text{kN}$$ **Ans.**

Substituting the result for F_{CF} and F_{BE} into Eq. (1),

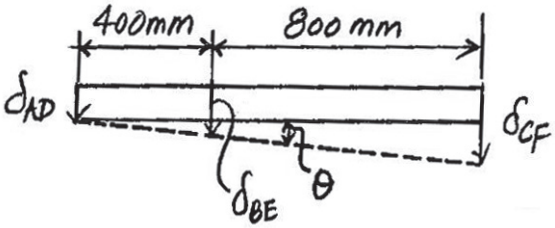
$$F_{AD} = 15436.93\ \text{N} = 15.4\ \text{kN}$$ **Ans.**

$$\sigma_{BE} = \frac{F_{BE}}{A_{BE}} = \frac{91844.61}{\frac{\pi}{4}(0.025^2)} = 187.10\ \text{MPa} < (\sigma_Y)_{\text{st}}$$ **(O.K.)**

$$\sigma_{AD} = \frac{F_{AD}}{A_{AD}} = \frac{15436.93}{\frac{\pi}{4}(0.025^2)} = 31.45\ \text{MPa} < (\sigma_Y)_{\text{st}}$$ **(O.K.)**



(a)



(b)

Ans:
$F_{CF} = 123$ kN,
$F_{BE} = 91.8$ kN,
$F_{AD} = 15.4$ kN

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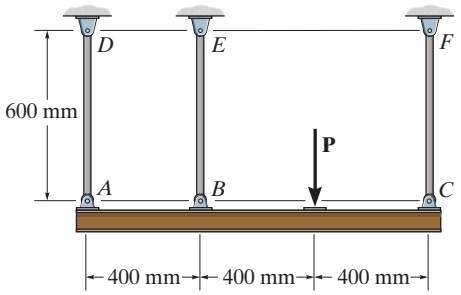
4–106.

The rigid beam is supported by three 25-mm diameter A-36 steel rods. If the force of $P = 230$ kN is applied on the beam and removed, determine the residual stresses in each rod. Consider the steel to be an elastic perfectly plastic material.



SOLUTION

Equation of Equilibrium: Referring to the free-body diagram of the beam shown in Fig. *a*,

$$+\uparrow \Sigma F_y = 0; \qquad F_{AD} + F_{BE} + F_{CF} - 230(10^3) = 0 \tag{1}$$

$$\circlearrowleft + \Sigma M_A = 0; \qquad F_{BE}(400) + F_{CF}(1200) - 230(10^3)(800) = 0$$

$$F_{BE} + 3F_{CF} = 460(10^3) \tag{2}$$

Elastic Analysis: Referring to the deflection diagram of the beam shown in Fig. *b*, the compatibility equation can be written as

$$\delta_{BE} = \delta_{AD} + \left(\frac{\delta_{CF} - \delta_{AD}}{1200}\right)(400)$$

$$\delta_{BE} = \frac{2}{3}\delta_{AD} + \frac{1}{3}\delta_{CF} \tag{3}$$

$$\frac{F_{BE}L}{AE} = \frac{2}{3}\left(\frac{F_{AD}L}{AE}\right) + \frac{1}{3}\left(\frac{F_{CF}L}{AE}\right)$$

$$F_{BE} = \frac{2}{3}F_{AD} + \frac{1}{3}F_{CF} \tag{4}$$

Solving Eqs. (1), (2), and (4)

$$F_{CF} = 131428.57 \text{ N} \qquad F_{BE} = 65714.29 \text{ N} \quad F_{AD} = 32857.14 \text{ N}$$

Normal Stress:

$$\sigma_{CF} = \frac{F_{CF}}{A_{CF}} = \frac{131428.57}{\frac{\pi}{4}(0.025^2)} = 267.74 \text{ MPa (T)} > (\sigma_Y)_{\text{st}} \qquad \textbf{(N.G.)}$$

$$\sigma_{BE} = \frac{F_{BE}}{A_{BE}} = \frac{65714.29}{\frac{\pi}{4}(0.025^2)} = 133.87 \text{ MPa (T)} < (\sigma_Y)_{\text{st}} \qquad \textbf{(O.K.)}$$

$$\sigma_{AD} = \frac{F_{AD}}{A_{AD}} = \frac{32857.14}{\frac{\pi}{4}(0.025^2)} = 66.94 \text{ MPa (T)} < (\sigma_Y)_{\text{st}} \qquad \textbf{(O.K.)}$$

Since rod *CF* yields, the elastic analysis is not valid. The solution must be reworked using

$$\sigma_{CF} = (\sigma_Y)_{\text{st}} = 250 \text{ MPa (T)}$$

$$F_{CF} = \sigma_{CF} A_{CF} = 250(10^6)\left[\frac{\pi}{4}(0.025^2)\right] = 122718.46 \text{ N}$$

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4–106. Continued

Substituting this result into Eq. (2),

$$F_{BE} = 91844.61 \text{ N}$$

Substituting the result for F_{CF} and F_{BE} into Eq. (1),

$$F_{AD} = 15436.93 \text{ N}$$

$$\sigma_{BE} = \frac{F_{BE}}{A_{BE}} = \frac{91844.61}{\frac{\pi}{4}(0.025^2)} = 187.10 \text{ MPa (T)} < (\sigma_Y)_{st} \qquad \textbf{(O.K.)}$$

$$\sigma_{AD} = \frac{F_{AD}}{A_{AD}} = \frac{15436.93}{\frac{\pi}{4}(0.025^2)} = 31.45 \text{ MPa (T)} < (\sigma_Y)_{st} \qquad \textbf{(O.K.)}$$

Residual Stresses: The process of removing **P** can be represented by applying the force **P**′, which has a magnitude equal to that of **P** but is opposite in sense. Since the process occurs in a linear manner, the corresponding normal stress must have the same magnitude but opposite sense to that obtained from the elastic analysis. Thus,

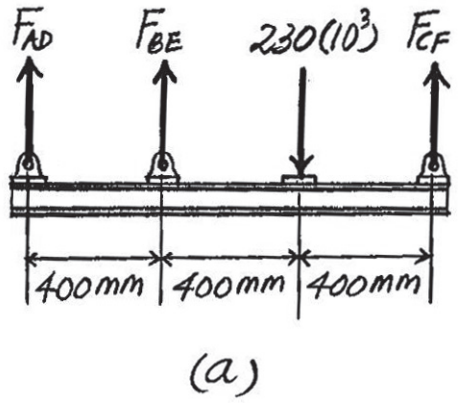
$\sigma'_{CF} = 267.74$ MPa (C) $\quad \sigma'_{BE} = 133.87$ MPa (C) $\quad \sigma'_{AD} = 66.94$ MPa (C)

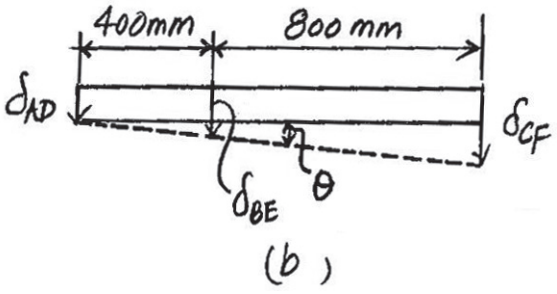
Considering the tensile stress as positive and the compressive stress as negative,

$(\sigma_{CF})_r = \sigma_{CF} + \sigma'_{CF} = 250 + (-267.74) = -17.7 \text{ MPa} = 17.7 \text{ MPa (C)}$ **Ans.**

$(\sigma_{BE})_r = \sigma_{BE} + \sigma'_{BE} = 187.10 + (-133.87) = 53.2 \text{ MPa (T)}$ **Ans.**

$(\sigma_{AD})_r = \sigma_{AD} + \sigma'_{AD} = 31.45 + (-66.94) = -35.5 \text{ MPa} = 35.5 \text{ MPa (C)}$ **Ans.**





Ans:
$(\sigma_{CF})_r = 17.7$ MPa (C),
$(\sigma_{BE})_r = 53.2$ MPa (T),
$(\sigma_{AD})_r = 35.5$ MPa (C)

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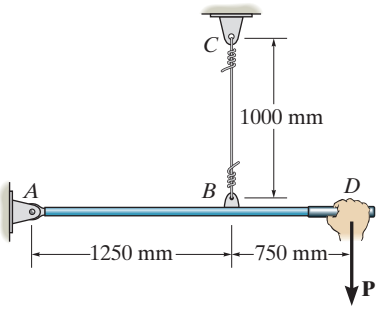
4–107.

The wire BC has a diameter of 3.4 mm and the material has the stress–strain characteristics shown in the figure. Determine the vertical displacement of the handle at D if the pull at the grip is slowly increased and reaches a magnitude of (a) $P = 2250$ N, (b) $P = 3000$ N.

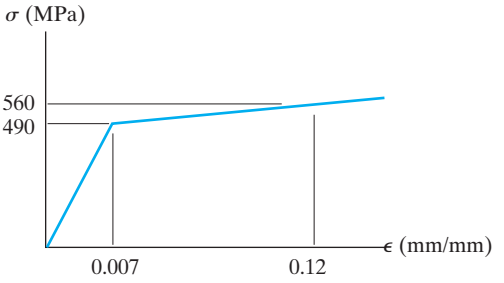


SOLUTION

Equations of Equilibrium:

$$\zeta + \Sigma M_A = 0; \qquad F_{BC}(1250) - P(2000) = 0 \qquad [1]$$

(a) From Eq. [1] when $P = 2250$ N, $F_{BC} = 3600$ N

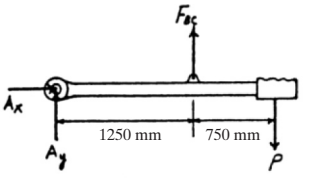


Average Normal Stress and Strain:

$$\sigma_{BC} = \frac{F_{BC}}{A_{BC}} = \frac{3600}{\frac{\pi}{4}(0.0034^2)} = 396.51(10^6)\ \text{N/m}^2 = 396.51\ \text{MPa}$$

From the Stress–Strain diagram

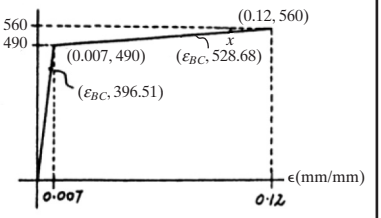
$$\frac{396.51}{\varepsilon_{BC}} = \frac{490}{0.007}; \qquad \varepsilon_{BC} = 0.005664\ \text{mm/mm}$$



Displacement:

$$\delta_{BC} = \varepsilon_{BC} L_{BC} = 0.005664(1000) = 5.6644\ \text{mm}$$

$$\frac{\delta_D}{2000} = \frac{\delta_{BC}}{1250}; \qquad \delta_D = (1.6)(5.664) = 9.0631\ \text{mm} = 9.06\ \text{mm}$$ **Ans.**



(b) From Eq. [1] when $P = 3000$ N, $F_{BC} = 4800$ N

Average Normal Stress and Strain:

$$\sigma_{BC} = \frac{F_{BC}}{A_{BC}} = \frac{4800}{\frac{\pi}{4}(0.0034^2)} = 528.68(10^6)\ \text{N/m}^2 = 528.68\ \text{MPa}$$

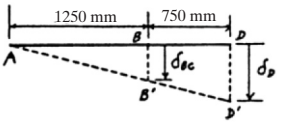
From Stress–Strain diagram

$$\frac{528.68 - 490}{\varepsilon_{BC} - 0.007} = \frac{560 - 490}{0.12 - 0.007} \qquad \varepsilon_{BC} = 0.06944\ \text{mm/mm}$$

Displacement:

$$\delta_{BC} = \varepsilon_{BC} L_{BC} = 0.06944(1000) = 69.44\ \text{mm}$$

$$\frac{\delta_D}{2000} = \frac{\delta_{BC}}{1250}; \qquad \delta_D = (1.6)(69.44) = 111.11\ \text{mm} = 111\ \text{mm}$$ **Ans.**



Ans.

(a) $\delta_D = 9.06$ mm, (b) $\delta_D = 111$ mm

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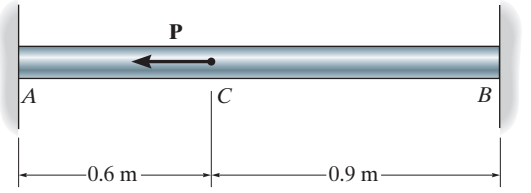
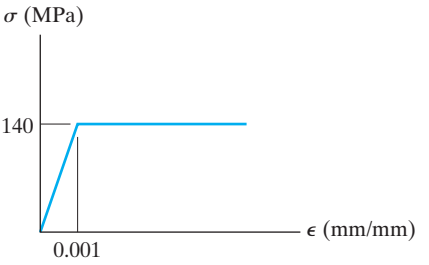
***4–108.**

The bar having a diameter of 50 mm is fixed connected at its ends and supports the axial load **P**. If the material is elastic perfectly plastic as shown by the stress–strain diagram, determine the smallest load P needed to cause segment CB to yield. If this load is released, determine the permanent displacement of point C.





SOLUTION

When P is increased, region AC will become plastic first, then CB will become plastic. Thus,

$$F_A = F_B = \sigma A = \left[140(10^6)\right]\left[\tfrac{\pi}{4}(0.05^2)\right] = 274.89(10^3)\ \text{N} = 274.89\ \text{kN}$$

$$\stackrel{+}{\rightarrow} \Sigma F_x = 0; \qquad F_A + F_B - P = 0 \qquad \textbf{(1)}$$

$$P = 2(274.89) = 549.78\ \text{kN}$$

$$P = 550\ \text{kN} \qquad \textbf{Ans.}$$

The deflection of point C is,

$$\delta_C = \varepsilon L = (0.001)(0.9)(10^3) = 0.9\ \text{mm} \leftarrow$$

Consider the reverse of P on the bar.

$$\frac{F_B'(0.6)}{AE} = \frac{F_B'(0.9)}{AE}$$

$$F_A' = 1.5\,F_B'$$

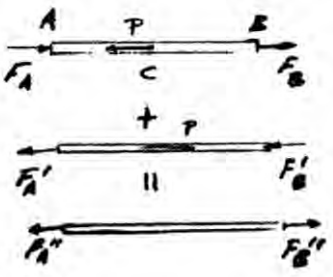
So that from Eq. (1)

$$F_B' = 0.4P$$

$$F_A' = 0.6P$$

$$\delta_C' = \frac{F_{B'}L}{AE} = \frac{0.4\left[549.78(10^3)\right]\left[0.9(10^3)\right]}{\left[\tfrac{\pi}{4}(0.05^2)\right]\left[140(10^6)/0.001\right]} = 0.720\ \text{mm} \rightarrow$$

$$\Delta\delta = 0.9 - 0.720 = 0.180\ \text{mm} \leftarrow \qquad \textbf{Ans.}$$



Ans.

$P = 550\ \text{kN},\ \Delta\delta = 0.180\ \text{mm}. \leftarrow$

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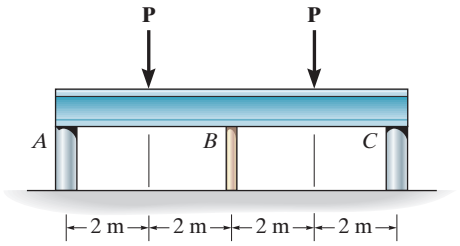
4–109.

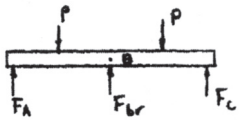
The rigid beam is supported by the three posts A, B, and C of equal length. Posts A and C have a diameter of 75 mm and are made of a material for which $E = 70$ GPa and $\sigma_Y = 20$ MPa. Post B has a diameter of 20 mm and is made of a material for which $E' = 100$ GPa and $\sigma_Y' = 590$ MPa. Determine the smallest magnitude of **P** so that (a) only rods A and C yield and (b) all the posts yield.



SOLUTION

$\Sigma M_B = 0; \quad F_A = F_C = F_{\text{al}}$

$+\uparrow \Sigma F_y = 0; \quad F_{at} + 2F_{at} - 2P = 0$ **(1)**



(a) Post A and C will yield,

$$F_{\text{al}} = (\sigma_t)_{\text{al}} A$$

$$= 20(10^4)\left(\frac{\pi}{a}\right)(0.075)^2$$

$$= 88.36 \text{ kN}$$

$$(E_{\text{al}})_r = \frac{(\sigma_r)_{\text{al}}}{E_{\text{al}}} = \frac{20(10^4)}{70(10^4)} = 0.0002857$$

Compatibility condition:

$$\delta_{\text{br}} = \delta_{\text{al}}$$

$$= 0.0002857(L)$$

$$\frac{F_{\text{br}}(L)}{\frac{\pi}{4}(0.02)^2(100)(10^4)} = 0.0002857\, L$$

$$F_{\text{br}} = 8.976 \text{ kN}$$

$$\sigma_{\text{br}} = \frac{8.976(10^3)}{\frac{\pi}{4}(0.02^3)} = 28.6 \text{ MPa} < \sigma_\gamma$$ **OK.**

From Eq. (1),

$$8.976 + 2(88.36) - 2P = 0$$

$$P = 92.8 \text{ kN}$$ **Ans.**

(b) All the posts yield:

$$F_{\text{br}} = (\sigma_\gamma)_{\text{br}} A$$

$$= (590)(10^4)\left(\frac{\pi}{4}\right)(0.02^2)$$

$$= 185.35 \text{ kN}$$

$$F_{\text{al}} = 88.36 \text{ kN}$$

From Eq. (1); $185.35 + 2(88.36) - 2P = 0$

$$P = 181 \text{ kN}$$ **Ans.**

Ans:
$P = 92.8$ kN,
$P = 181$ kN

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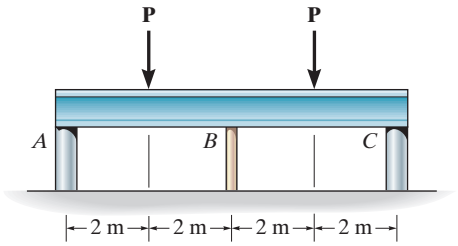
4–110.

The rigid beam is supported by the three posts *A*, *B*, and *C*. Posts *A* and *C* have a diameter of 60 mm and are made of a material for which $E = 70$ GPa and $\sigma_Y = 20$ MPa. Post *B* is made of a material for which $E' = 100$ GPa and $\sigma_Y' = 590$ MPa. If $P = 130$ kN, determine the diameter of post *B* so that all three posts are about to yield.



SOLUTION

$+\uparrow \Sigma F_y = 0;\quad 2(F_\gamma)_{al} + F_{br} - 260 = 0$ **(1)**

$$(F_{al})_\gamma = (\sigma_\gamma)_{al} A$$

$$= 20(10^6)\left(\frac{\pi}{4}\right)(0.06)^2 = 56.55 \text{ kN}$$

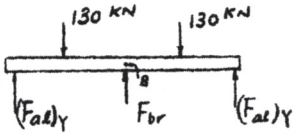
From Eq. (1),

$$2(56.55) + F_{br} - 260 = 0$$

$$F_{br} = 146.9 \text{ kN}$$

$$(\sigma_\gamma)_{br} = 590(10^6) = \frac{146.9(10^3)}{\frac{\pi}{4}(d_B)^3}$$

$$d_B = 0.01779 \text{ m} = 17.8 \text{ mm}$$ **Ans.**



Ans:
$d_B = 17.8$ mm

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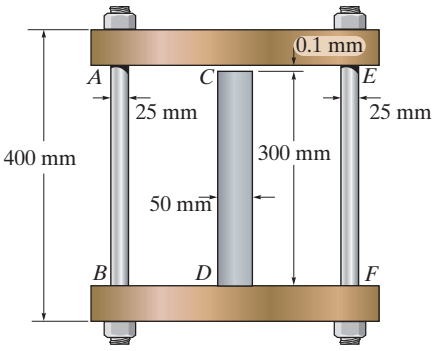
R4–1.

The assembly consists of two A992 steel bolts AB and EF and an 6061-T6 aluminum rod CD. When the temperature is at 30° C, the gap between the rod and rigid member AE is 0.1 mm. Determine the normal stress developed in the bolts and the rod if the temperature rises to 130° C. Assume BF is also rigid.



SOLUTION

Equation of Equilibrium: Referring to the free-body diagram of the rigid cap shown in Fig. *a*,

$$+\uparrow \Sigma F_y = 0; \quad F_r - 2F_b = 0 \qquad \textbf{(1)}$$

Compatibility Equation: If the bolts and the rod are unconstrained, they will have a free expansion of $(\delta_T)_b = \alpha_{st}\Delta T L_b = 12(10^{-6})(130 - 30)(400) = 0.48$ mm and $(\delta\gamma)_r = \alpha_{al}\Delta T L_r = 24(10^{-6})(130 - 30)(300) = 0.72$ mm. Referring to the initial and final position of the assembly shown in Fig. *b*,

$$(\delta_T)_r - \delta_{Fr} - 0.1 = (\delta_T)_b + \delta_{Fb}$$

$$0.72 - \frac{F_r(300)}{\frac{\pi}{4}(0.05^2)(68.9)(10^9)} - 0.1 = 0.48 + \frac{F_b(400)}{\frac{\pi}{4}(0.025^2)(200)(10^9)}$$

$$F_b + 0.5443F_r = 34361.17 \qquad \textbf{(2)}$$

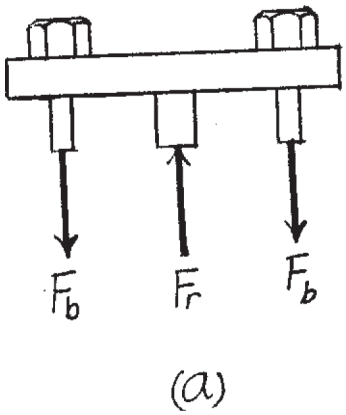
Solving Eqs. (1) and (2).

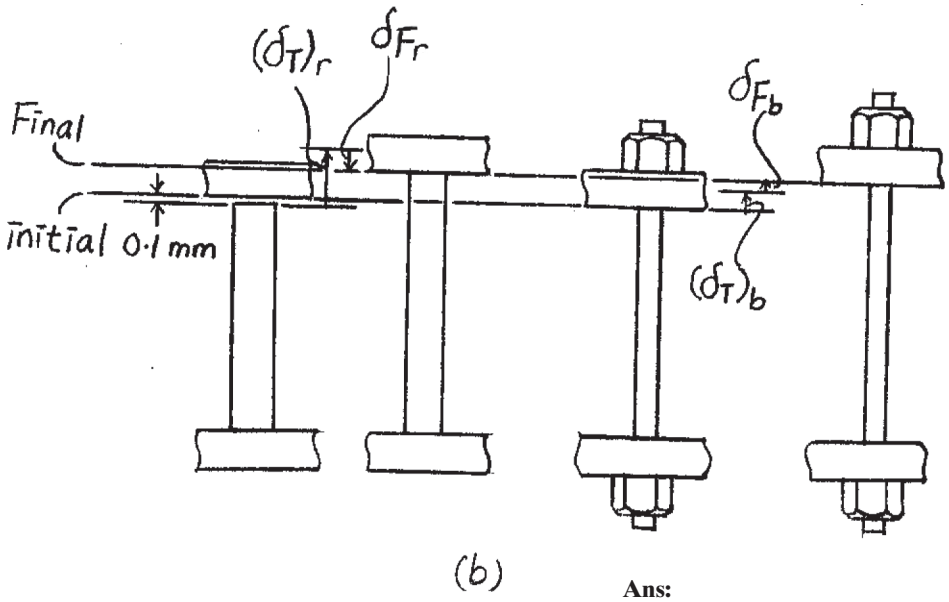
$$F_b = 16\,452.29 \text{ N} \qquad F_r = 32\,904.58 \text{ N}$$

Normal Stress:

$$\sigma_b = \frac{F_b}{A_b} = \frac{16\,452.29}{\frac{\pi}{4}(0.025^2)} = 33.5 \text{ MPa} \qquad \textbf{Ans.}$$

$$\sigma_r = \frac{F_r}{A_r} = \frac{32\,904.58}{\frac{\pi}{4}(0.05^2)} = 16.8 \text{ MPa} \qquad \textbf{Ans.}$$





Ans:
$\sigma_b = 33.5$ MPa, $\sigma_r = 16.8$ MPa

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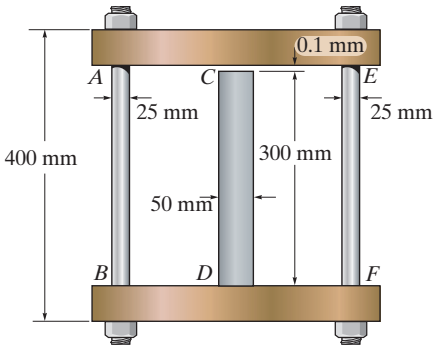
R4–2.

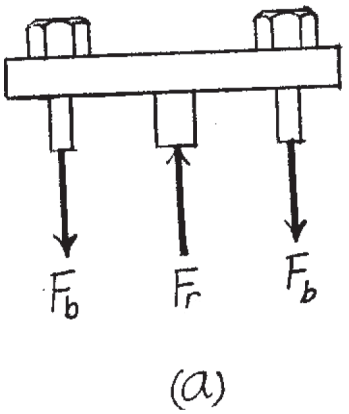
The assembly shown consists of two A992 steel bolts *AB* and *EF* and an 6061-T6 aluminum rod *CD*. When the temperature is at 30° C, the gap between the rod and rigid member *AE* is 0.1 mm. Determine the highest temperature to which the assembly can be raised without causing yielding either in the rod or the bolts. Assume *BF* is also rigid.



SOLUTION

Equation of Equilibrium: Referring to the free-body diagram of the rigid cap shown in Fig. *a*,

$$+\uparrow \Sigma F_y = 0; \quad F_p - 2F_b = 0 \qquad \textbf{(1)}$$



(a)

Normal Stress: Assuming that the steel bolts yield first, then

$$F_b = (\sigma\gamma)_{st} A_b = 250(10^6)\left[\frac{\pi}{4}(0.025^2)\right] = 122\,718.46 \text{ N}$$

Substituting this result into Eq. (1),

$$F_p = 245\,436.93 \text{ N}$$

Then,

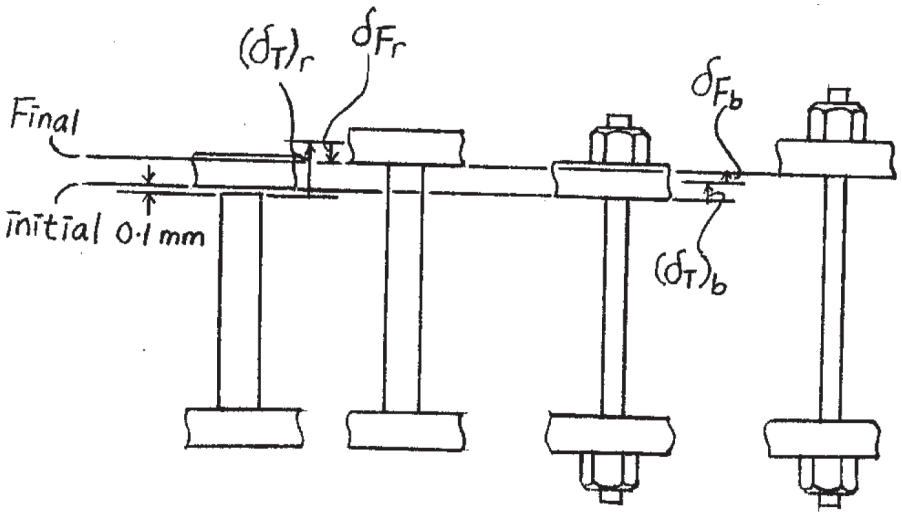
$$\sigma_p = \frac{F_p}{A_p} = \frac{245\,436.93}{\frac{\pi}{4}(0.05^2)} = 125 \text{ MPa} < (\sigma\gamma)_{al} \qquad \textbf{(O.K!)}$$

Compatibility Equation: If the assembly is unconstrained, the bolts and the post will have free expansion of $(\delta_T)_b = \alpha_{st}\Delta T L_b = 12(10^{-6})(T-30)(400) = 4.8(10^{-3})(T-30)$ and $(\delta_T)_p = \alpha_{al}\Delta T L_p = 24(10^{-6})T - 30)(300) = 7.2(10^{-3})(T - 30)$. Referring to the initial and final position of the assembly shown in Fig. *b*,

$$(\delta_T)_p - \delta_{Fp} - 0.1 = (\delta_T)_b + \delta_{Fb}$$

$$7.2(10^{-3})(T-30) - \frac{245\,436.93(300)}{\frac{\pi}{4}(0.05^2)(68.9)(10^9)} - 0.1 = 4.8(10^{-3})(T-30) + \frac{122\,718.46(400)}{\frac{\pi}{4}(0.025^2)(200)(10^9)}$$

$$T = 506.78° \text{ C} = 507°\text{C} \qquad \textbf{Ans.}$$



(b)

Ans:
$T = 507°\text{C}$

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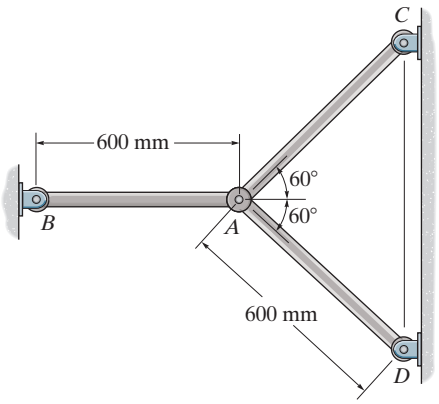
R4–3.

The rods each have the same 25-mm diameter and 600-mm length. If they are made of A992 steel, determine the forces developed in each rod when the temperature increases by 50° C.



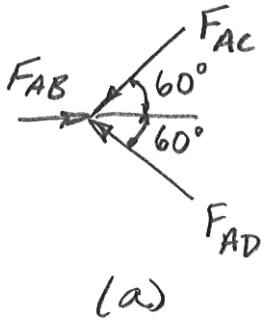
SOLUTION

Equation of Equilibrium: Referring to the free-body diagram of joint A shown in Fig. a,

$$+\uparrow \Sigma F_x = 0; \qquad F_{AD}\sin 60° - F_{AC}\sin 60° = 0 \qquad F_{AC} = F_{AD} = F$$

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad F_{AB} - 2F\cos 60° = 0$$

$$F_{AB} = F \qquad \textbf{(1)}$$



Compatibility Equation: If AB and AC are unconstrained, they will have a free expansion of $(\delta_T)_{AB} = (\delta_T)_{AC} = \alpha_{st}\Delta TL = 12(10^{-6})(50)(600) = 0.36$ mm. Referring to the initial and final position of joint A,

$$\delta_{F_{AB}} - (\delta_T)_{AB} = \left(\delta_T{}'\right)_{AC} - \delta_{F_{AC}}{}'$$

Due to symmetry, joint A will displace horizontally, and $\delta_{AC}{}' = \dfrac{\delta_{AC}}{\cos 60°} = 2\delta_{AC}$. Thus, $\left(\delta_T{}'\right)_{AC} = 2(\delta_T)_{AC}$ and $\delta_{F_{AC}}{}' = 2\delta_{F_{AC}}$. Thus, this equation becomes

$$\delta_{F_{AB}} - (\delta_T)_{AB} = 2(\delta_T)_{AC} - 2\delta_{AC}$$

$$\frac{F_{AB}(600)}{\frac{\pi}{4}(0.025^2)(200)(10^9)} - 0.36 = 2(0.36) - 2\left[\frac{F(600)}{\frac{\pi}{4}(0.025^2)(200)(10^9)}\right]$$

$$F_{AB} + 2F = 176\,714.59 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2),

$$F_{AB} = F_{AC} = F_{AD} = 58\,904.86\,\text{N} = 58.9\,\text{kN (C)} \qquad \textbf{Ans.}$$

Ans:
$F_{AB} = F_{AC} = F_{AD} = 58.9$ kN (C)

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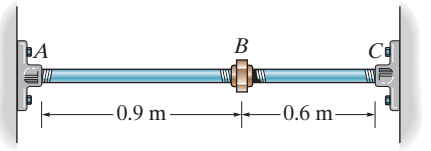
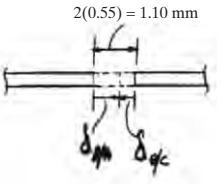
***R4–4.**



Two A-36 steel pipes, each having a cross-sectional area of 200 mm^2, are screwed together using a union at B as shown. Originally the assembly is adjusted so that no load is on the pipe. If the union is then tightened so that its screw, having a lead of 0.55 mm, undergoes two full turns, determine the average normal stress developed in the pipe. Assume that the union at B and couplings at A and C are rigid. Neglect the size of the union. *Note:* The lead would cause the pipe, when *unloaded*, to shorten shorten 0.55 mm when the union is rotated one revolution.



The loads acting on both segments AB and BC are the same since no external load acts on the system.

$$1.10 = \delta_{B/A} + \delta_{B/C}$$

$$1.10 = \frac{P(0.9)(1000)}{[200(10^{-6})][200(10^{9})]} + \frac{P(0.6)(1000)}{[200(10^{-6})][200(10^{9})]}$$

$$P = 29.33(10^{3}) = 29.33 \text{ kN}$$

$$\sigma_{AB} = \sigma_{BC} = \frac{P}{A} = \frac{29.33(10^{3})}{200(10^{-6})} = 146.67(10^{6}) \text{ N/m}^2 = 147 \text{ MPa} \qquad \textbf{Ans.}$$

Ans.

$\sigma_{AB} = \sigma_{BC} = 147$ MPa

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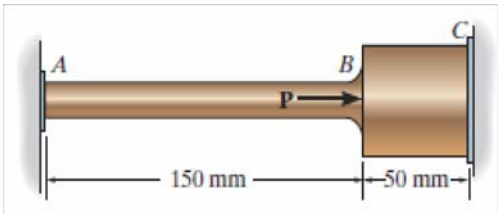
R4-5.

The force **P** is applied to the bar, which is composed of an elastic perfectly plastic material. Construct a graph to show how the force in each section *AB* and *BC* (ordinate) varies as *P* (abscissa) is increased. The bar has cross-sectional areas of 625 mm^2 in region *AB* and 2500 mm^2 in region *BC*, and σ_Y = 210 MPa.

Given: $L_{AB} := 150mm \quad A_{AB} := 625mm^2$

$L_{BC} := 50mm \quad A_{BC} := 2500mm^2$

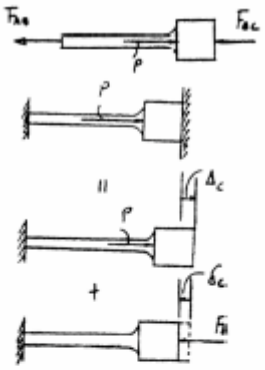
$\sigma_Y := 210MPa$



Solution:

Equations of equilibrium:

$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad P - F_A - F_C = 0$ **[1]**



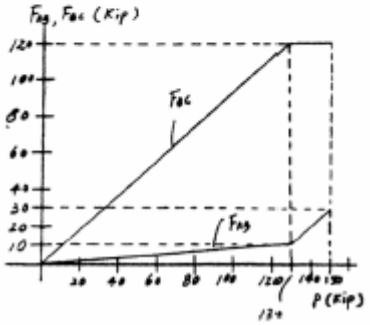
Elastic behavior:

$\overset{+}{\rightarrow} \quad 0 = \Delta_C - \delta_C$

$$0 = \frac{(P)\cdot L_{AB}}{E\cdot A_{AB}} - \left[\frac{(F_C)\cdot L_{BC}}{E\cdot A_{BC}} + \frac{(F_C)\cdot L_{AB}}{E\cdot A_{AB}}\right]$$

$0 = 6P - (F_C)(0.5 + 6) \qquad F_C = \frac{12}{13}P$ **[2]**

Substituting **[2]** into **[1]**: $F_A = \frac{1}{13}P$ **[3]**



By comparison, segment *BC* will yield first. Hence,

$F_C := (\sigma_Y)\cdot A_{BC} \qquad F_C = 525\,kN$

From **[2]**: $P := \frac{13}{12}\cdot F_C \qquad P = 568.75\,kN$

From **[3]**: $F_A := \frac{1}{13}\cdot P \qquad F_A = 43.75\,kN$

When segment *AB* yields,

$F_A := (\sigma_Y)\cdot A_{AB} \qquad F_A = 131.25\,kN$

$F_C := (\sigma_Y)\cdot A_{BC} \qquad F_C = 525\,kN$

From **[1]**: $P := F_A + F_C \qquad P = 656.25\,kN$

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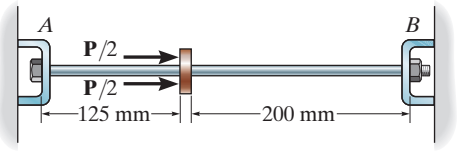
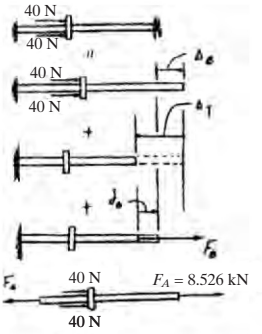
R4–6. The 2014-T6 aluminum rod has a diameter of 12 mm and is lightly attached to the rigid supports at A and B when $T_1 = 25°C$. If the temperature becomes $T_2 = -20°$, and an axial force of $P = 80$ N is applied to the rigid collar as shown, determine the reactions at A and B.



SOLUTION

Compatibility:



$$\stackrel{+}{\rightarrow} 0 = \Delta_B - \Delta_T + \delta_B$$

$$0 = \frac{80(125)}{\left[\frac{\pi}{4}(0.012^2)\right][73.1(10^9)]} - 23(10^{-6})[25° - (-20°)](325) + \frac{F_B(325)}{\left[\frac{\pi}{4}(0.012^2)\right][73.1(10^9)]}$$

$$F_B = 8.526(10^3) = 8.53 \text{ kN} \qquad \textbf{Ans.}$$

$$\stackrel{+}{\rightarrow} \Sigma F_x = 0; \qquad 2(0.040) + 8.526 - F_A = 0$$

$$F_A = 8.606 \text{ kN} = 8.61 \text{ kN} \qquad \textbf{Ans.}$$

Ans.
$F_B = 8.53$ kN, $F_A = 8.61$ kN

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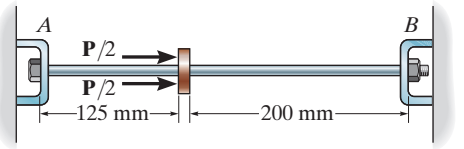
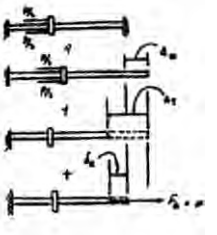
R4–7. The 2014-T6 aluminum rod has a diameter of 12 mm and is lightly attached to the rigid supports at A and B when $T_1 = 40°C$. Determine the force P that must be applied to the collar so that, when $T = 0°C$, the reaction at B is zero.



SOLUTION



$$\stackrel{+}{\rightarrow} \quad 0 = \Delta_B - \Delta_T + \delta_B$$

$$0 = \frac{P(125)}{\left[\frac{\pi}{4}(0.012^2)\right][73.1(10^9)]} - 23(10^{-6})(40)(325) + 0$$

$$P = 19.776(10^3)\ \text{N} = 19.8\ \text{kN}$$ **Ans.**

Ans.
$P = 19.8$ kN

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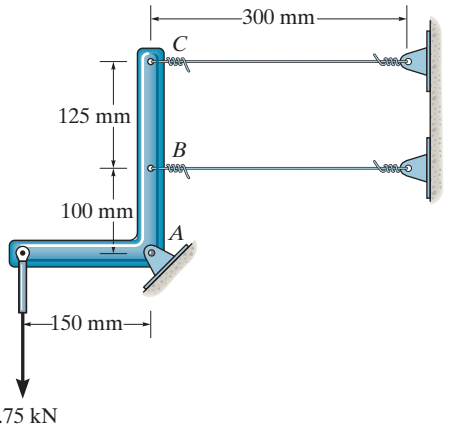
***R4–8.** The rigid link is supported by a pin at A and two A-36 steel wires, each having an unstretched length of 300 mm and cross-sectional area of 7.8 mm^2. Determine the force developed in the wires when the link supports the vertical load of 1.75 kN.



SOLUTION

Equations of Equilibrium:

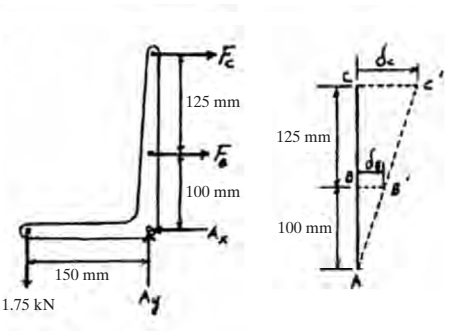
$$\circlearrowleft + \Sigma M_A = 0; \qquad -F_C(225) - F_B(100) + 1.75(150) = 0 \qquad \textbf{[1]}$$

Compatibility:

$$\frac{\delta_B}{100} = \frac{\delta_C}{225}$$

$$\frac{F_B(L)}{100AE} = \frac{F_C(L)}{225AE}$$

$$2.25F_B - F_C = 0, \qquad \textbf{[2]}$$



Solving Eqs. [1] and [2] yields:

$$F_B = 0.4330 \text{ kN} = 433 \text{ N} \qquad \textbf{Ans.}$$

$$F_C = 0.9742 \text{ kN} = 974 \text{ N} \qquad \textbf{Ans.}$$

Ans.
$F_B = 433$ N, $F_C = 974$ N,

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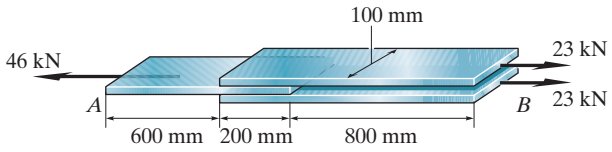
R4–9.

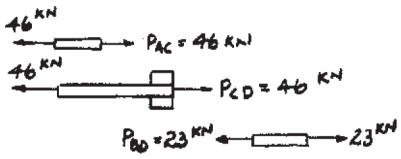
The joint is made from three A992 steel plates that are bonded together at their seams. Determine the displacement of end A with respect to end B when the joint is subjected to the axial loads. Each plate has a thickness of 5 mm.



SOLUTION

$$\delta_{A/B} = \Sigma\frac{PL}{AE} = \frac{46(10^3)(600)}{(0.005)(0.1)(200)(10^9)} + \frac{46(10^3)(200)}{3(0.005)(0.1)(200)(10^9)} + \frac{23(10^3)(800)}{(0.005)(0.1)(200)(10^9)}$$

$$= 0.491 \text{ mm} \qquad \textbf{Ans.}$$



Ans:
$\delta_{A/B} = 0.491$ mm

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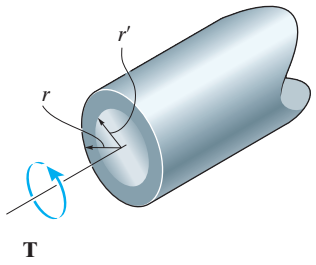
5–1.

The solid shaft of radius r is subjected to a torque **T**. Determine the radius r' of the inner core of the shaft that resists one-half of the applied torque $(T/2)$. Solve the problem two ways: (a) by using the torsion formula, (b) by finding the resultant of the shear-stress distribution.



SOLUTION

a) $\tau_{\max} = \dfrac{Tc}{J} = \dfrac{Tr}{\frac{\pi}{2}r^4} = \dfrac{2T}{\pi r^3}$

$$\tau = \frac{\left(\frac{T}{2}\right)r'}{\frac{\pi}{2}(r')^4} = \frac{T}{\pi(r')^3}$$

Since $\tau = \dfrac{r'}{r}\tau_{\max}$; $\quad \dfrac{T}{\pi(r')^3} = \dfrac{r'}{r}\left(\dfrac{2T}{\pi r^3}\right)$

$$r' = \frac{r}{2^{\frac{1}{4}}} = 0.841r \qquad \textbf{Ans.}$$

b) $\displaystyle\int_0^{\frac{r}{2}} dT = 2\pi \int_0^{r'} \tau\rho^2\, d\rho$

$$\int_0^{\frac{r}{2}} dT = 2\pi \int_0^{r'} \frac{\rho}{r}\tau_{\max}\rho^2\, d\rho$$

$$\int_0^{\frac{r}{2}} dT = 2\pi \int_0^{r'} \frac{\rho}{r}\left(\frac{2T}{\pi r^3}\right)\rho^2\, d\rho$$

$$\frac{T}{2} = \frac{4T}{r^4}\int_0^{r'} \rho^3\, d\rho$$

$$r' = \frac{r'}{2^{\frac{1}{4}}} = 0.841r \qquad \textbf{Ans.}$$

These solutions represent a preliminary version of the Instructors' Solutions Manual (ISM). It is possible and even likely that at this preliminary stage of preparing the ISM there are some omissions and errors in the draft solutions. These will be corrected and this manual will be republished.

Ans:
$r' = 0.841r$

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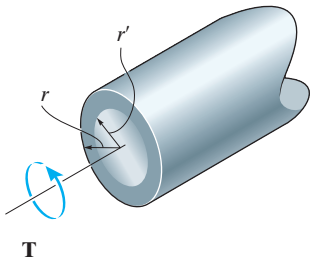
5–2.

The solid shaft of radius r is subjected to a torque **T**. Determine the radius r' of the inner core of the shaft that resists one-quarter of the applied torque $(T/4)$. Solve the problem two ways: (a) by using the torsion formula, (b) by finding the resultant of the shear-stress distribution.

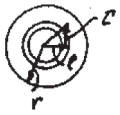


SOLUTION

a) $\tau_{\max} = \dfrac{Tc}{J} = \dfrac{T(r)}{\frac{\pi}{2}(r^4)} = \dfrac{2T}{\pi r^3}$

Since $\tau = \dfrac{r'}{r}\tau_{\max} = \dfrac{2Tr'}{\pi r^4}$

$\tau' = \dfrac{T'c'}{J'}; \quad \dfrac{2Tr'}{\pi r^4} = \dfrac{(\frac{T}{4})r'}{\frac{\pi}{2}(r')^4}$



$r' = \dfrac{r}{4^{\frac{1}{4}}} = 0.707\, r$ **Ans.**

b) $\tau = \dfrac{\rho}{c}\tau_{\max} = \dfrac{\rho}{r}\left(\dfrac{2T}{\pi r^3}\right) = \dfrac{2T}{\pi r^4}\rho; \qquad dA = 2\pi\rho\, d\rho$

$dT = \rho\tau\, dA = \rho\left[\dfrac{2T}{\pi r^4}\rho\right](2\pi\rho\, d\rho) = \dfrac{4T}{r^4}\rho^3 d\rho$

$\displaystyle\int_0^{\frac{T}{4}} dT = \frac{4T}{r^4}\int^{r'} \rho^3 d\rho$

$\dfrac{T}{4} = \dfrac{4T}{r^4}\dfrac{\rho^4}{4}\Big|_0^{r'}; \qquad \dfrac{1}{4} = \dfrac{(r')^4}{r^4}$

$r' = 0.707\, r$ **Ans.**

Ans:
$r' = 0.707\, r$

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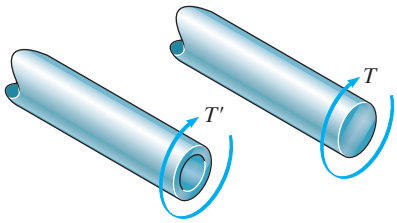
5–3.

A shaft is made of an aluminum alloy having an allowable shear stress of $\tau_{allow} = 100$ MPa. If the diameter of the shaft is 100 mm, determine the maximum torque **T** that can be transmitted. What would be the maximum torque **T**′ if a 75-mm-diameter hole were bored through the shaft? Sketch the shear-stress distribution along a radial line in each case.



SOLUTION

Allowable Shear Stress: Torsion formula can be applied. For the solid shaft,

$$\tau_{max} = \tau_{allow} = \frac{Tc}{J};\ 100(10^6) = \frac{T(0.05)}{\frac{\pi}{2}(0.05^4)}$$

$$T = 19.63\,(10^3)\ \text{N}\cdot\text{m} = 19.6\ \text{kN}\cdot\text{m} \qquad \textbf{Ans.}$$

For the hollow shaft,

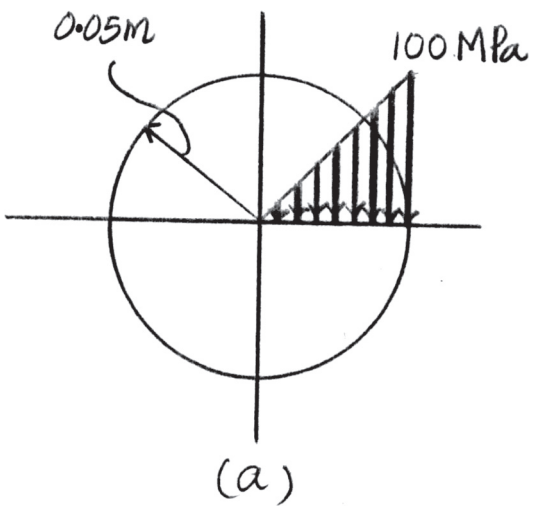
$$\tau_{max} = \tau_{allow} = \frac{Tc}{J};\ 100(10^6) = \frac{T'(0.05)}{\frac{\pi}{2}(0.05^4 - 0.0375^4)}$$

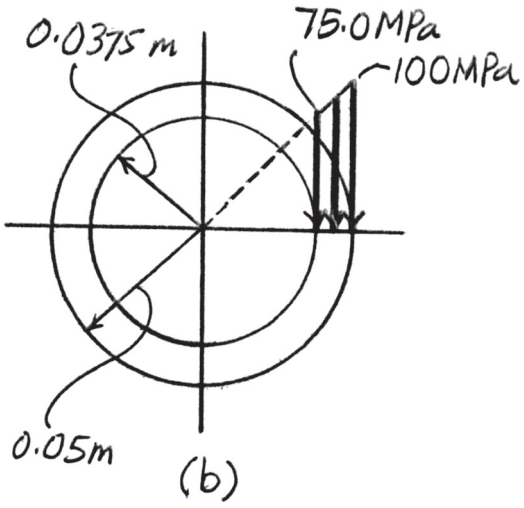
$$T' = 13.42\,(10^3)\ \text{N}\cdot\text{m} = 13.4\ \text{kN}\cdot\text{m} \qquad \textbf{Ans.}$$

The shear stress at the inner surface where $\rho = 0.0375$ m is

$$\tau_\rho = 0.0375\ \text{m} = \frac{T'\rho}{\text{J}} = \frac{13.42(10^3)\,(0.0375)}{\frac{\pi}{2}(0.05^4 - 0.0375^4)} = 75.0(10^6)\ \text{Pa} = 75.0\ \text{MPa}$$

The shear stress distribution along the radius of the cross-section of the solid and hollow shafts are shown in Figs. *a* and *b* respectively.





Ans:
$T = 19.6\ \text{kN}\cdot\text{m}$,
$T' = 13.4\ \text{kN}\cdot\text{m}$

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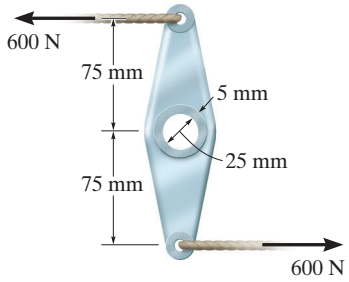
***5–4.**

The link acts as part of the elevator control for a small airplane. If the attached aluminum tube has an inner diameter of 25 mm and a wall thickness of 5 mm, determine the maximum shear stress in the tube when the cable force of 600 N is applied to the cables. Also, sketch the shear-stress distribution over the cross section.

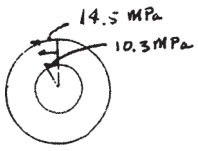


SOLUTION

$T = 600(0.15) = 90\ \text{N} \cdot \text{m}$

$$\tau_{\max} = \frac{Tc}{J} = \frac{90(0.0175)}{\frac{\pi}{2}\left[(0.0175)^4 - (0.0125)^4\right]} = 14.5\ \text{MPa}$$ **Ans.**

$$\tau_i = \frac{T\rho}{J} = \frac{90(0.0125)}{\frac{\pi}{2}\left[(0.0175)^4 - (0.0125)^4\right]} = 10.3\ \text{MPa}$$



Ans:
$\tau_{\max} = 14.5\ \text{MPa}$

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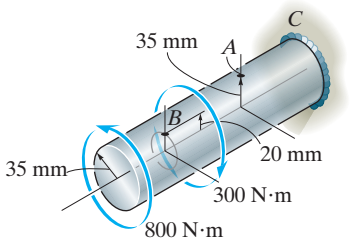
5–5.

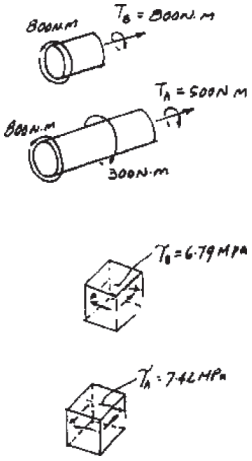
The solid shaft is fixed to the support at C and subjected to the torsional loadings. Determine the shear stress at points A and B on the surface, and sketch the shear stress on volume elements located at these points.



SOLUTION

$$\tau_B = \frac{T_B \rho}{J} = \frac{800(0.02)}{\frac{\pi}{2}(0.035^4)} = 6.79 \text{ MPa}$$ **Ans.**

$$\tau_A = \frac{T_A c}{J} = \frac{500(0.035)}{\frac{\pi}{2}(0.035^4)} = 7.42 \text{ MPa}$$ **Ans.**



Ans:
$\tau_B = 6.79$ MPa, $\tau_A = 7.42$ MPa

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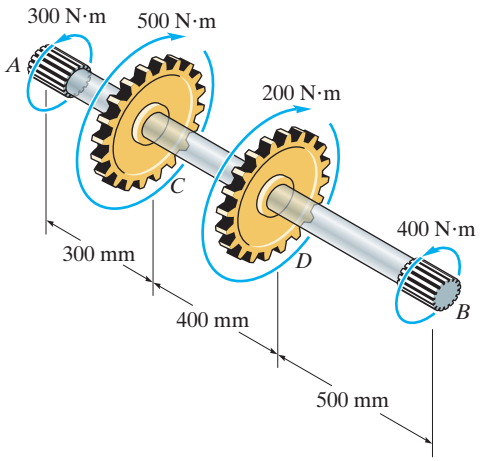
5–6.

The solid 30-mm-diameter shaft is used to transmit the torques applied to the gears. Determine the absolute maximum shear stress in the shaft.



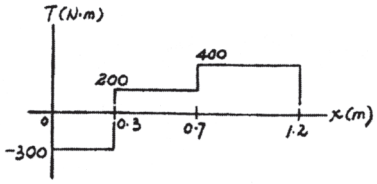
SOLUTION

Internal Torque: As shown on torque diagram.

Maximum Shear Stress: From the torque diagram $T_{\max} = 400\ \text{N} \cdot \text{m}$. Then, applying torsion formula,

$$\tau_{\substack{\text{abs}\\\text{max}}} = \frac{T_{\max}\, c}{J}$$

$$= \frac{400(0.015)}{\frac{\pi}{2}(0.015^4)} = 75.5\ \text{MPa} \qquad \textbf{Ans.}$$



Ans:
$\tau_{\substack{\text{abs}\\\text{max}}} = 75.5\ \text{MPa}$

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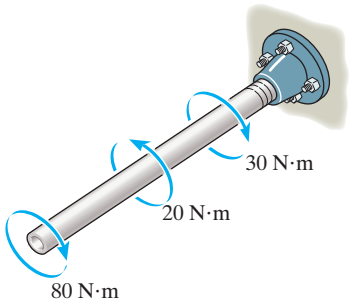
5–7.

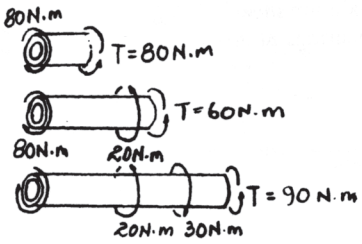
The copper pipe has an outer diameter of 40 mm and an inner diameter of 37 mm. If it is tightly secured to the wall and three torques are applied to it, determine the absolute maximum shear stress developed in the pipe.



SOLUTION

$$\tau_{\max} = \frac{T_{\max} c}{J} = \frac{90(0.02)}{\frac{\pi}{2}(0.02^4 - 0.0185^4)}$$

$$= 26.7 \text{ MPa}$$ **Ans.**



Ans:
$\tau_{\max} = 26.7$ MPa

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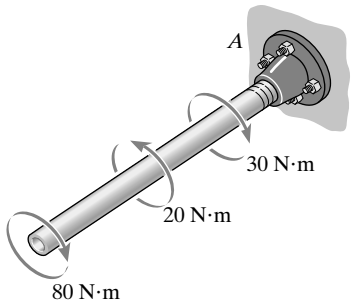
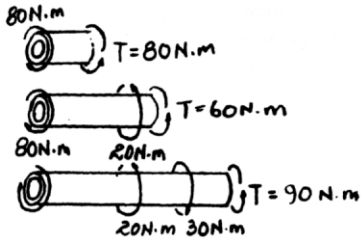
***5–8.** The copper pipe has an outer diameter of 40 mm and an inner diameter of 37 mm. If it is tightly secured to the wall at A and three torques are applied to it as shown, determine the absolute maximum shear stress developed in the pipe.



SOLUTION

$$\tau_{\max} = \frac{T_{\max}\, c}{J} = \frac{90(0.02)}{\frac{\pi}{2}\,(0.02^4 - 0.0185^4)}$$

$$= 26.7 \text{ MPa}$$ **Ans..**



Ans:
$\tau_{\max} = 26.7$ MPa

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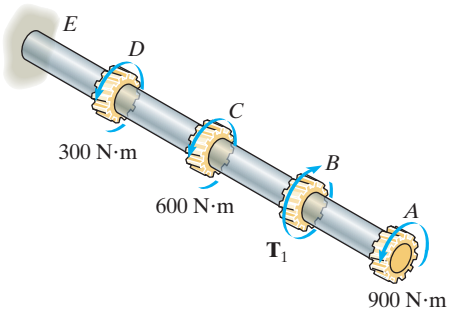
5–9.

The solid aluminum shaft has a diameter of 50 mm and an allowable shear stress of $\tau_{allow} = 60$ MPa. Determine the largest torque T_1 that can be applied to the shaft if it is also subjected to the other torsional loadings. It is required that $\mathbf{T}_1$ act in the direction shown. Also, determine the maximum shear stress within regions CD and DE.



SOLUTION

Internal Torque: Assuming that failure occurs at region BC of the shaft, where the torque will be greatest. Referring to the FBD of the right segment of the shaft sectioned through region BC, Fig. a

$$\Sigma M_x = 0; \qquad T_1 - 900 - T_{BC} = 0 \qquad T_{BC} = T_1 - 900$$

Maximum Shear Stress: Applying the torsion formula,

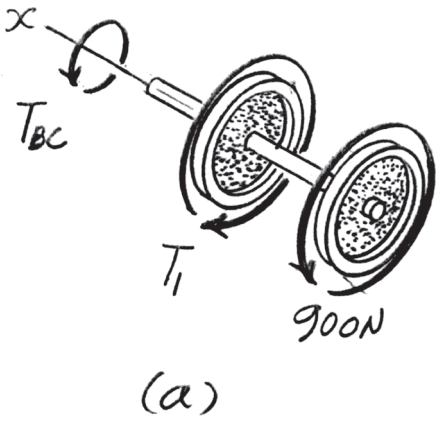
$$\tau_{max} = \tau_{allow} = \frac{T_{BC}\, c}{J}; \qquad 60(10^6) = \frac{(T_1 - 900)(0.025)}{\frac{\pi}{2}(0.025^4)}$$

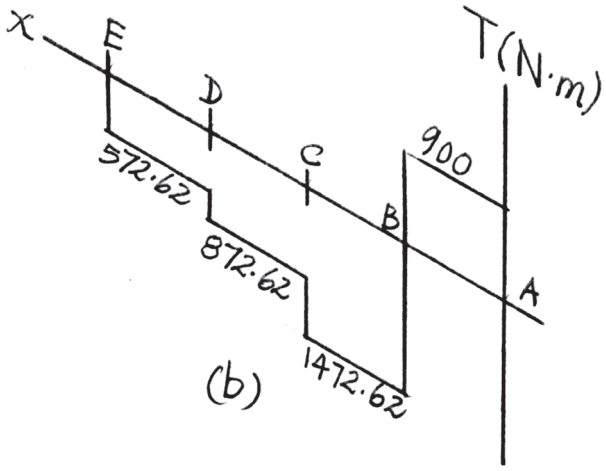
$$(T_1)_{max} = T_1 = 2372.62\ \text{N}\cdot\text{m} = 2.37\ \text{kN}\cdot\text{m} \qquad \textbf{Ans.}$$

using this result the torque diagram shown in Fig. b can be plotted. This indicates that region BC indeed is subjected to maximum internal torque, thus, the critical region. From the torque diagram, the internal torques in regions CD and DE are $T_{CD} = 872.62\ \text{N}\cdot\text{m}$ and $T_{DE} = 572.62\ \text{N}\cdot\text{m}$ respectively.

$$(\tau_{max})_{CD} = \frac{T_{CD}\, C}{J} = \frac{872.62\ (0.025)}{\frac{\pi}{2}(0.025^4)} = 35.55\ (10^6)\ \text{Pa} = 35.6\ \text{MPa} \qquad \textbf{Ans.}$$

$$(\tau_{max})_{DE} = \frac{T_{DC}\, C}{J} = \frac{572.62\ (0.025)}{\frac{\pi}{2}(0.025^4)} = 23.33\ (10^6)\ \text{Pa} = 23.3\ \text{MPa} \qquad \textbf{Ans.}$$





Ans:
$(T_1)_{max} = 2.37\ \text{kN}\cdot\text{m}$,
$(\tau_{max})_{CD} = 35.6\ \text{MPa}$,
$(\tau_{max})_{DE} = 23.3\ \text{MPa}$

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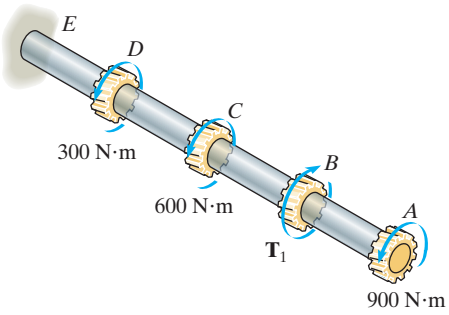
5–10.

The solid aluminum shaft has a diameter of 50 mm. Determine the absolute maximum shear stress in the shaft and sketch the shear-stress distribution along a radial line of the shaft where the shear stress is maximum. Set $T_1 = 2000\ \text{N} \cdot \text{m}$.

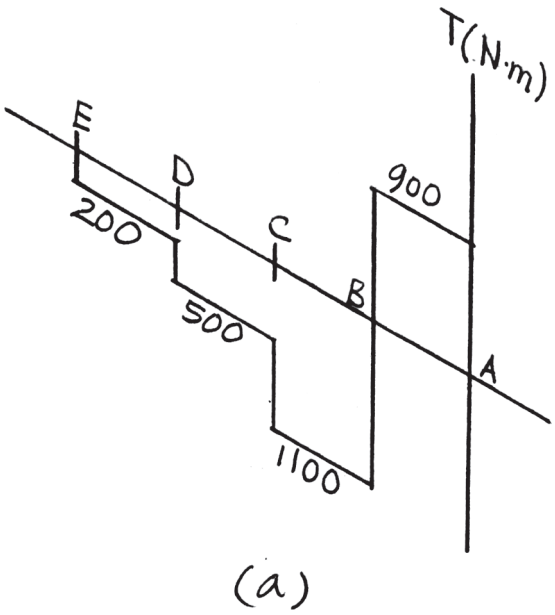


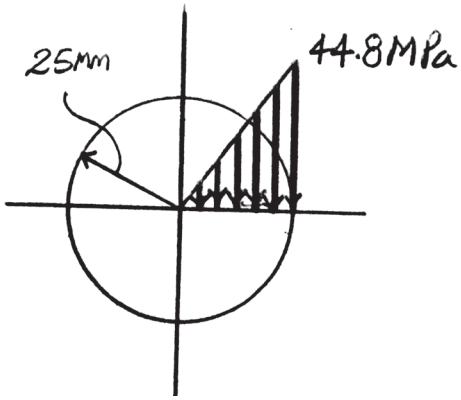
SOLUTION

Internal Torque: The torque diagram plotted in Fig. *a* indicates that region *BC* of the shaft is subjected to the greatest internal torque; thus, the critical region where the absolute maximum shear stress occurs. Here $T_{BC} = 1100\ \text{N} \cdot \text{m}$. Applying the torsion formula,

$$\tau_{\substack{\text{abs}\\\text{max}}} = \frac{T_{BC}\, c}{J} = \frac{1100\ (0.025)}{\frac{\pi}{2}\left(0.025^4\right)} = 44.82\left(10^6\right)\ \text{Pa} = 44.8\ \text{MPa} \qquad \textbf{Ans.}$$

The shear stress distribution along the radius of the cross-section of the shaft is shown in Fig. *b*.





Ans:
$\tau_{\substack{\text{abs}\\\text{max}}} = 44.8\ \text{MPa}$

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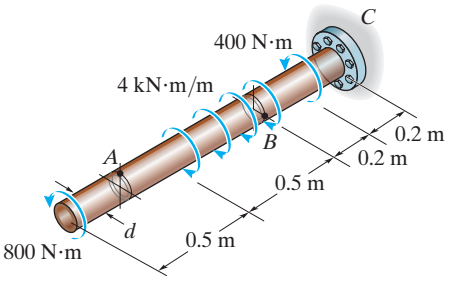
5–11.

The 60-mm-diameter solid shaft is subjected to the distributed and concentrated torsional loadings shown. Determine the absolute maximum and minimum shear stresses on the shaft's surface and specify their locations, measured from the free end.



SOLUTION

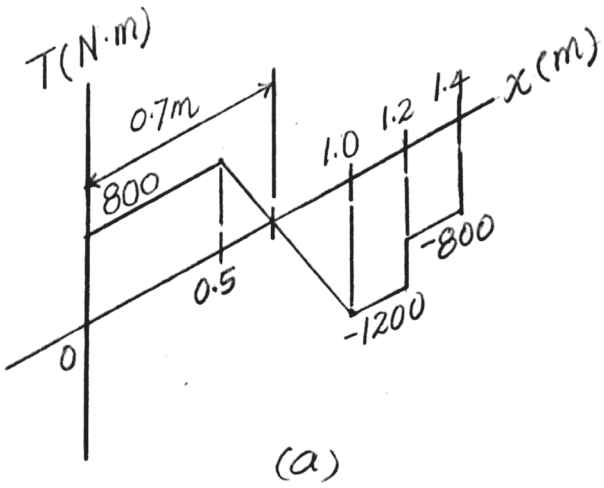
Internal Torque: The torque diagram plotted in Fig. *a* indicates that region $1.0\text{ m} < x < 1.2\text{ m}$ of the shaft is subjected to the greatest internal torque where $T_{\max} = 1200\text{ N}\cdot\text{m}$, whereas the minimum internal torque, $T_{\min} = 0$, occurs at $x = 0.7\text{ m}$.

Shear Stress: Applying the torsion formula,

$$\tau_{\max}^{\text{abs}} = \frac{T_{\max} C}{J} = \frac{1200\,(0.03)}{\frac{\pi}{2}(0.03^4)} = 28.29\,(10^6)\text{ Pa} = 28.3\text{ MPa} \qquad \textbf{Ans.}$$

occurs within the region $1.0\text{ m} < x < 1.2\text{ m}$ **Ans.**

$$\tau_{\min}^{\text{abs}} = \frac{T_{\min} C}{J} \qquad \text{occurs at } x = 0.700\text{ m} \qquad \textbf{Ans.}$$



Ans:
$\tau_{\max}^{\text{abs}} = 28.3\text{ MPa}$,
for $1.0\text{ m} < x < 1.2\text{ m}$,
$\tau_{\min}^{\text{abs}} = 0$,
at $x = 0.700\text{ m}$

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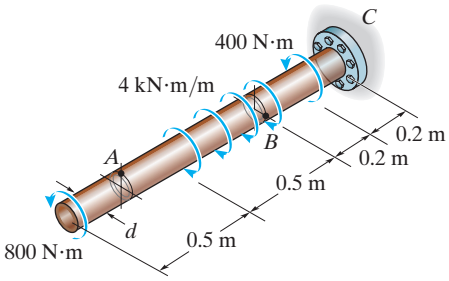
***5–12.**

The solid shaft is subjected to the distributed and concentrated torsional loadings shown. Determine the required diameter d of the shaft if the allowable shear stress for the material is $\tau_{allow} = 60$ MPa.



SOLUTION

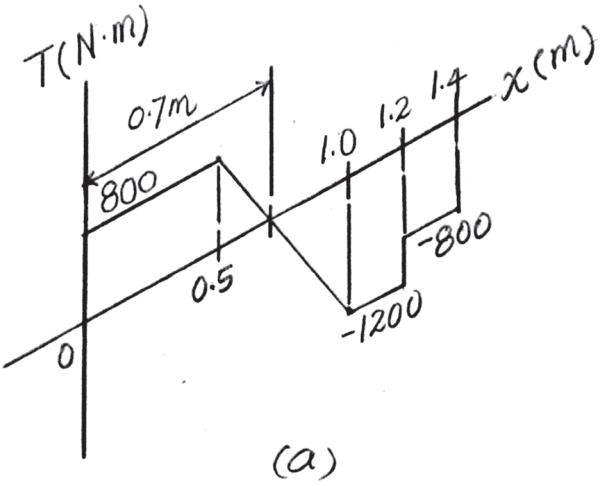
Internal Torque: The torque diagram plotted in Fig. *a* indicates that region $1.0\text{ m} < x < 1.2\text{ m}$ of the shaft is subjected to the greatest internal torque $T_{max} = 1200\text{ N}\cdot\text{m}$; thus, that is the critical region where the absolute maximum shear stress occurs.

Allowable Shear Stress: Applying the torsion formula,

$$\tau_{max}^{abs} = \tau_{allow} = \frac{T_{max}C}{J}$$

$$60\,(10^6) = \frac{1200\,(\frac{d}{2})}{\frac{\pi}{2}(\frac{d}{2})^4}$$

$$d = 0.04670\text{ m} = 46.7\text{ mm}$$ **Ans.**



(a)

Ans:
$d = 46.7$ mm

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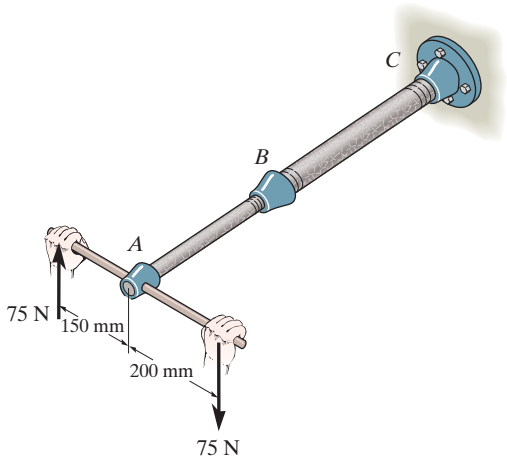
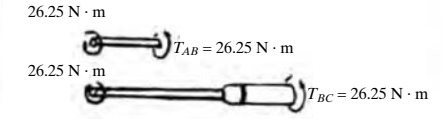
5–13. The assembly consists of two sections of galvanized steel pipe connected together using a reducing coupling at *B*. The smaller pipe has an outer diameter of 18.75 mm and an inner diameter of 17 mm, whereas the larger pipe has an outer diameter of 25 mm and an inner diameter of 21.5 mm. If the pipe is tightly secured to the wall at *C*, determine the maximum shear stress developed in each section of the pipe when the couple shown is applied to the handles of the wrench.



SOLUTION

$$\tau_{AB} = \frac{Tc}{J} = \frac{26.25(0.009375)}{\frac{\pi}{2}(0.009375^4 - 0.0085^4)} = 62.54(10^6)\ \text{N/m}^2 = 62.5\ \text{MPa}$$ **Ans.**

$$\tau_{BC} = \frac{Tc}{J} = \frac{26.25(0.0125)}{\frac{\pi}{2}(0.0125^4 - 0.01075^4)} = 18.89(10^6)\ \text{N/m}^2 = 18.9\ \text{MPa}$$ **Ans.**



Ans:

$\tau_{AB} = 62.5$ MPa, $\tau_{BC} = 18.9$ MPa

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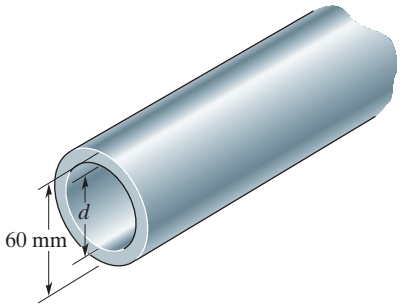
5–14. A steel tube having an outer diameter of 60 mm is used to transmit 6.75 kW when turning at 27 rev/min. Determine the inner diameter d of the tube to the nearest mm if the allowable shear stress is $\tau_{\text{allow}} = 70$ MPa.



SOLUTION

$$\omega = \left(27\frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right) = 0.9\pi \text{ rad/s}$$

$$P = T\omega$$

$$6.75(10^3) = T(0.9\pi)$$

$$T = 2.387(10^3) \text{ N} \cdot \text{m}$$

$$\tau_{\text{max}} = \tau_{\text{allow}} = \frac{Tc}{J}$$

$$70(10^6) = \frac{2.387(10^3)(0.03)}{\frac{\pi}{2}\left(0.03^4 - c_i^4\right)}$$

$$c_i = 0.01996 \text{ m}$$

$$d = 2c_i = 2(0.01996) = 0.03992 \text{ m} = 39.92 \text{ mm}$$

Use $d = 40$ mm **Ans.**

Ans:

Use $d = 40$ mm

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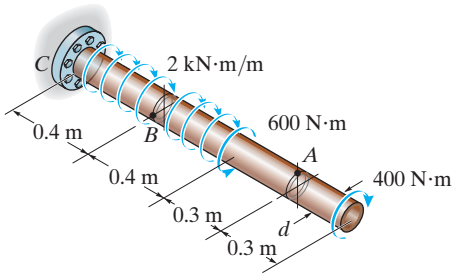
5–15.

The 60-mm-diameter solid shaft is subjected to the distributed and concentrated torsional loadings shown. Determine the shear stress at points A and B, and sketch the shear stress on volume elements located at these points.



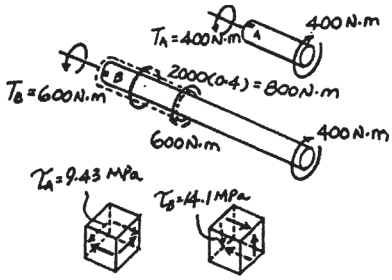
SOLUTION

Internal Torque: As shown on FBD.

Maximum Shear Stress: Applying the torsion formula

$$\tau_A = \frac{T_A\, c}{J} = \frac{400(0.03)}{\frac{\pi}{2}(0.03^4)} = 9.43 \text{ MPa} \qquad \textbf{Ans.}$$

$$\tau_B = \frac{T_B\, c}{J} = \frac{600(0.03)}{\frac{\pi}{2}(0.03^4)} = 14.1 \text{ MPa} \qquad \textbf{Ans.}$$



Ans:
$\tau_A = 9.43$ MPa,
$\tau_B = 14.1$ MPa

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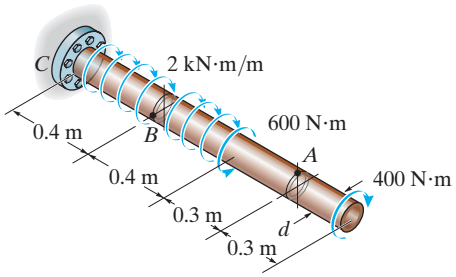
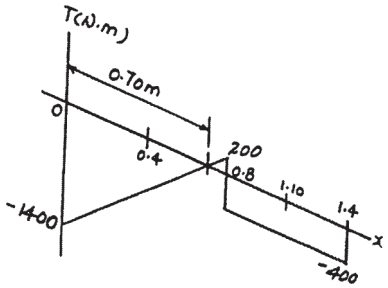
***5–16.**

The 60-mm-diameter solid shaft is subjected to the distributed and concentrated torsional loadings shown. Determine the absolute maximum and minimum shear stresses on the shaft's surface, and specify their locations, measured from the fixed end *C*.



SOLUTION

Internal Torque: From the torque diagram, the maximum torque $T_{max} = 1400\ \text{N}\cdot\text{m}$ occurs at the fixed support and the minimum torque $T_{min} = 0$ occurs at $x = 0.700$ m.



Shear Stress: Applying the torsion formula

$$\tau_{\substack{\text{abs}\\\text{min}}} = \frac{T_{min}\,c}{J} = 0 \qquad \text{occurs at} \qquad x = 0.700\ \text{m} \qquad \textbf{Ans.}$$

$$\tau_{\substack{\text{abs}\\\text{max}}} = \frac{T_{max}\,c}{J} = \frac{1400(0.03)}{\frac{\pi}{2}(0.03^4)} = 33.0\ \text{MPa} \qquad \textbf{Ans.}$$

$$\text{occurs at} \qquad x = 0 \qquad \textbf{Ans.}$$

According to Saint-Venant's principle, application of the torsion formula should be at points sufficiently removed from the supports or points of concentrated loading. Therefore, $\tau_{\substack{\text{abs}\\\text{max}}}$ obtained *is not valid*.

Ans:

$\tau_{\substack{\text{abs}\\\text{max}}} = 0$ occurs at $x = 0.700$ m,

$\tau_{\substack{\text{abs}\\\text{min}}} = 33.0$ MPa occurs at $x = 0$

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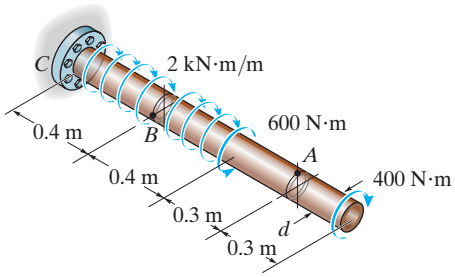
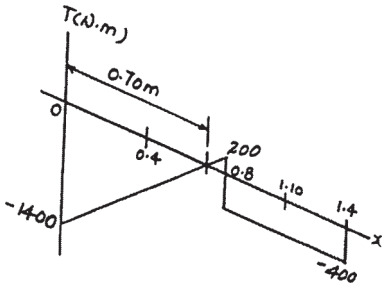
5–17.

The solid shaft is subjected to the distributed and concentrated torsional loadings shown. Determine the required diameter d of the shaft if the allowable shear stress for the material is $\tau_{allow} = 1.6$ MPa.



SOLUTION

Internal Torque: From the torque diagram, the maximum torque $T_{max} = 1400\ \text{N} \cdot \text{m}$ occurs at the fixed support.



Allowable Shear Stress: Applying the torsion formula

$$\tau_{\substack{abs \\ max}} = \tau_{allow} = \frac{T_{max}\, c}{J}$$

$$175(10^6) = \frac{1400\left(\frac{d}{2}\right)}{\frac{\pi}{2}\left(\frac{d}{2}\right)^4}$$

$$d = 0.03441\ \text{m} = 34.4\ \text{mm} \qquad \textbf{Ans.}$$

According to Saint-Venant's principle, application of the torsion formula should be at points sufficiently removed from the supports or points of concentrated loading. Therefore, the above analysis *is not valid*.

Ans:
$d = 34.4$ mm

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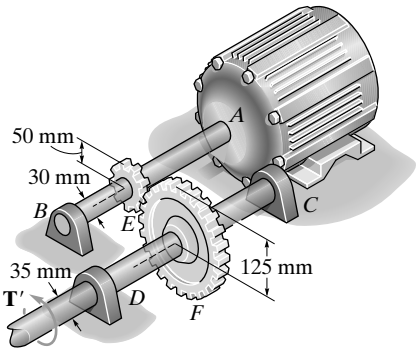
5–18. The motor delivers a torque of 50 N · m to the shaft *AB*. This torque is transmitted to shaft *CD* using the gears at *E* and *F*. Determine the equilibrium torque **T′** on shaft *CD* and the maximum shear stress in each shaft. The bearings *B*, *C*, and *D* allow free rotation of the shafts.



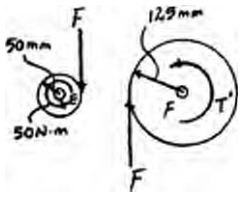
SOLUTION

Equilibrium:

$\zeta + \Sigma M_E = 0; \quad 50 - F(0.05) = 0 \quad F = 1000\ \text{N}$

$\zeta + \Sigma M_F = 0; \quad T' - 1000(0.125) = 0$

$$T' = 125\ \text{N} \cdot \text{m} \qquad \textbf{Ans.}$$

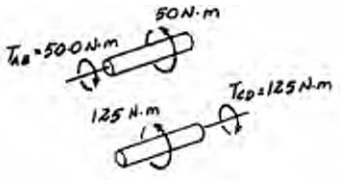


Internal Torque: As shown on FBD.

Maximum Shear Stress: Applying torsion Formula.

$$(\tau_{AB})_{\max} = \frac{T_{AB}\, c}{J} = \frac{50.0(0.015)}{\frac{\pi}{2}(0.015^4)} = 9.43\ \text{MPa} \qquad \textbf{Ans.}$$

$$(\tau_{CD})_{\max} = \frac{T_{CD} c}{J} = \frac{125(0.0175)}{\frac{\pi}{2}(0.0175^4)} = 14.8\ \text{MPa} \qquad \textbf{Ans.}$$



Ans.
$(\tau_{AB})_{\max} = 9.43\ \text{MPa}$
$(\tau_{CD})_{\max} = 14.8\ \text{MPa}$

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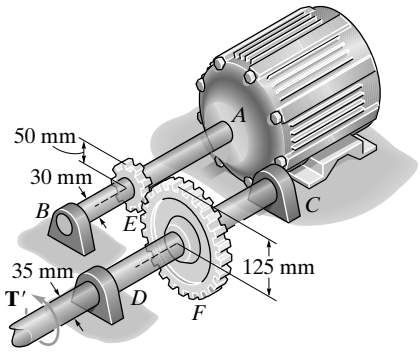
5–19. If the applied torque on shaft CD is $T' = 75\ \text{N}\cdot\text{m}$, determine the absolute maximum shear stress in each shaft. The bearings B, C, and D allow free rotation of the shafts, and the motor holds the shafts fixed from rotating.



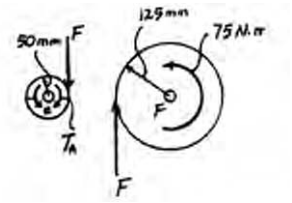
SOLUTION

Equilibrium:

$$\zeta + \Sigma M_F = 0; \qquad 75 - F(0.125) = 0; \qquad F = 600\ \text{N}$$

$$\zeta + \Sigma M_E = 0; \qquad 600(0.05) - T_A = 0$$

$$T_A = 30.0\ \text{N}\cdot\text{m}$$

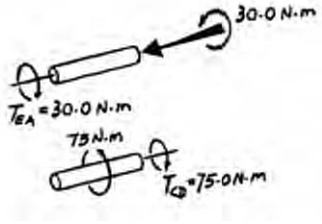


Internal Torque: As shown on FBD.

Maximum Shear Stress: Applying the torsion formula

$$(\tau_{EA})_{\max} = \frac{T_{EA}\,c}{J} = \frac{30.0(0.015)}{\frac{\pi}{2}(0.015^4)} = 5.66\ \text{MPa} \qquad \textbf{Ans.}$$

$$(\tau_{CD})_{\max} = \frac{T_{CD}c}{J} = \frac{75.0(0.0175)}{\frac{\pi}{2}(0.0175^4)} = 8.91\ \text{MPa} \qquad \textbf{Ans.}$$



Ans.
$(\tau_{EA})_{\max} = 5.66\ \text{MPa}$
$(\tau_{CD})_{\max} = 8.91\ \text{MPa}$

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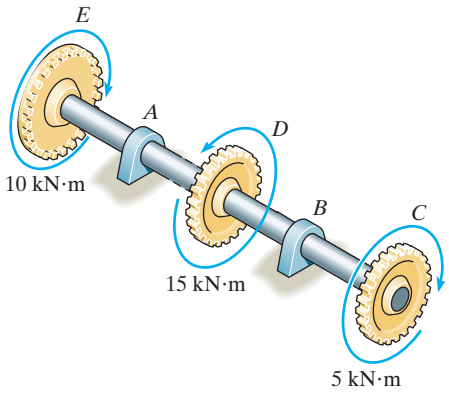
***5–20.**

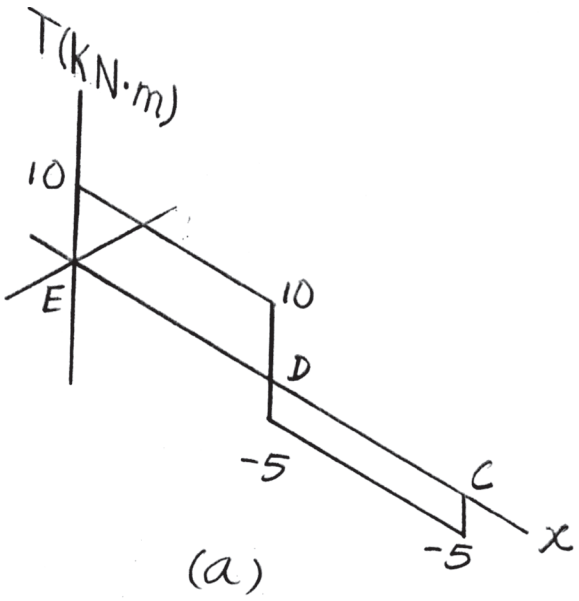
The shaft has an outer diameter of 100 mm and an inner diameter of 80 mm. If it is subjected to the three torques, determine the absolute maximum shear stress in the shaft. The smooth bearings A and B do not resist torque.



SOLUTION

Maximum Shear Stress: Referring to the torque diagram shown in Fig. a, region ED is subjected to the largest internal torque, which is $T_{\max} = 10\ \text{kN} \cdot \text{m}$. Thus, the absolute maximum shear stress occurs in this region. Applying the torsion formula,

$$\tau_{\substack{\text{abs}\\\max}} = \frac{T_{\max}\, C}{J} = \frac{10\,(10^3)(0.05)}{\frac{\pi}{2}\,(0.05^4 - 0.04^4)} = 86.26\,(10^6)\ \text{Pa} = 86.3\ \text{MPa} \qquad \textbf{Ans.}$$



Ans:
$\tau_{\substack{\text{abs}\\\max}} = 86.3\ \text{MPa}$

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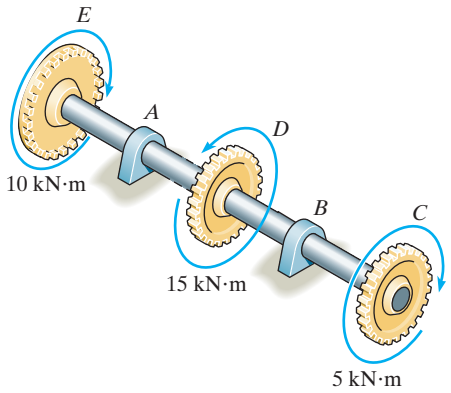
5–21.

The shaft has an outer diameter of 100 mm and an inner diameter of 80 mm. If it is subjected to the three torques, plot the shear stress distribution along a radial line for the cross section within region *CD* of the shaft. The smooth bearings at *A* and *B* do not resist torque.



SOLUTION

Shear Stress: Referring to the torque diagram shown in Fig. *a*, Region *CD* of the shaft is subjected to an internal torque of $T_{CD} = 5\ \text{kN}\cdot\text{m}$. The torsion formula will be applied. The maximum shear stress is

$$(\tau_{\max})_{CD} = \frac{T_{CD}\,C}{J} = \frac{5(10^3)(0.05)}{\frac{\pi}{2}\left(0.05^4 - 0.04^4\right)} = 43.13\,(10^6)\ \text{Pa} = 43.1\ \text{MPa}$$

The shear stress at the inner surface of the hollow shaft is

$$(\tau_p = 0.04\ \text{m})_{CD} = \frac{T_{CD}\rho}{J} = \frac{5(10^3)(0.04)}{\frac{\pi}{2}\left(0.05^4 - 0.04^4\right)} = 34.51\,(10^6)\ \text{Pa} = 34.5\ \text{MPa}$$

Also,

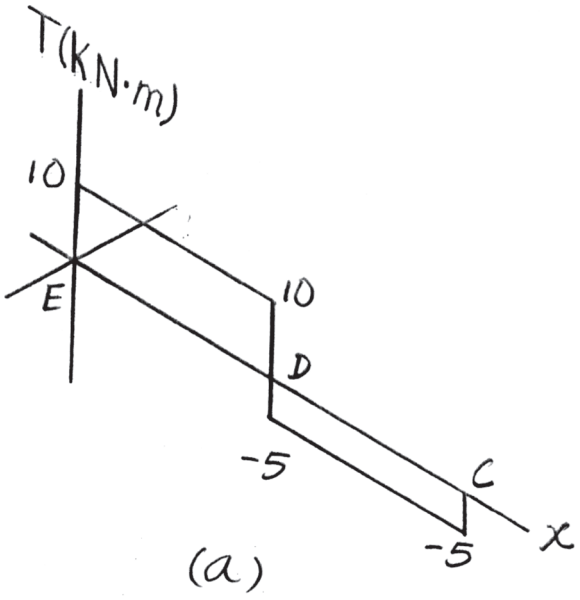
$$\frac{\tau_{\max}}{C} = \frac{\tau_p}{\rho};\quad (\tau_{p=0.04\ \text{m}})_{CD} = \left(\frac{\rho}{C}\right)\tau_{\max}$$

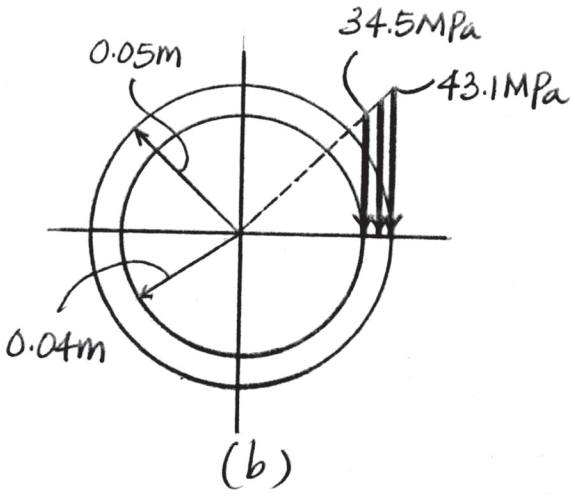
$$= \left(\frac{0.04}{0.05}\right)(43 \cdot 13)$$

$$= 34.51\ \text{MPa}$$

$$= 34.5\ \text{MPa}$$

The shear stress distribution along the radius of the cross-section of the shaft is shown in Fig. *b*.





Ans:
N/A

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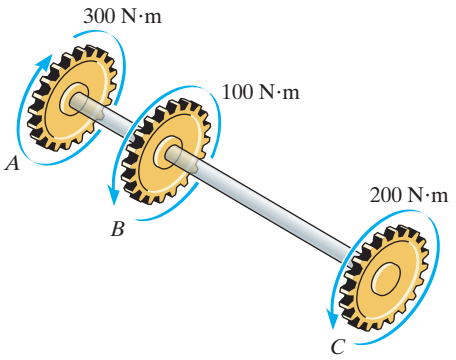
5–22.

If the gears are subjected to the torques shown, determine the maximum shear stress in the segments AB and BC of the A-36 steel shaft. The shaft has a diameter of 40 mm.



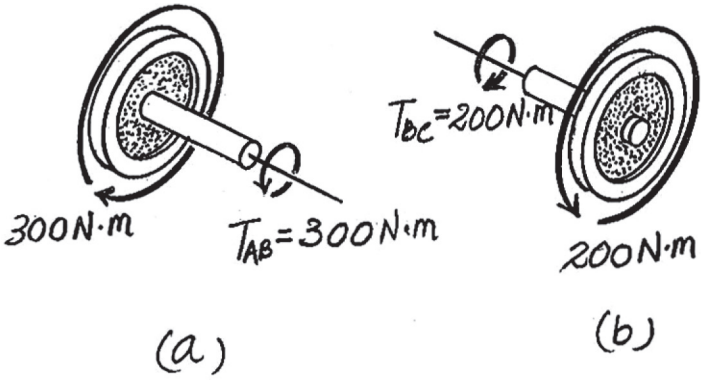
SOLUTION

The internal torque developed in segments AB and BC are shown in their respective FBDs, Figs. a and b.

The polar moment of inertia of the shaft is $J = \frac{\pi}{2}(0.02^4) = 80(10^{-9})\pi \text{ m}^4$. Thus,

$$(\tau_{AB})_{\max} = \frac{T_{AB}c}{J} = \frac{300(0.02)}{80(10^{-9})\pi} = 23.87(10^6)\text{Pa} = 23.9 \text{ MPa} \qquad \textbf{Ans.}$$

$$(\tau_{BC})_{\max} = \frac{T_{BC}c}{J} = \frac{200(0.02)}{80(10^{-9})\pi} = 15.92(10^6)\text{Pa} = 15.9 \text{ MPa} \qquad \textbf{Ans.}$$



Ans:
$(\tau_{AB})_{\max} = 23.9$ MPa,
$(\tau_{BC})_{\max} = 15.9$ MPa

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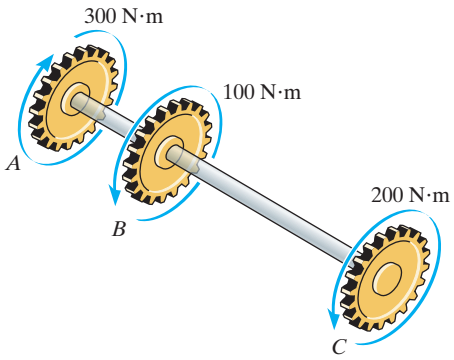
5–23.

If the gears are subjected to the torques shown, determine the required diameter of the A-36 steel shaft to the nearest mm if $\tau_{allow} = 60$ MPa.



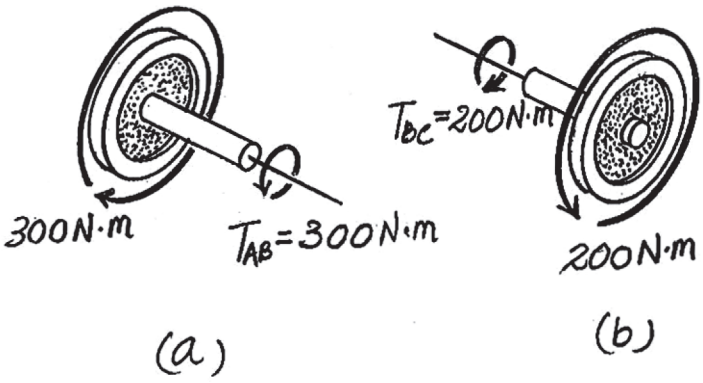
SOLUTION

The internal torque developed in segments AB and BC are shown in their respective FBDs, Fig. *a* and *b*.

Here, segment AB is critical since its internal torque is the greatest. The polar moment of inertia of the shaft is $J = \frac{\pi}{2}\left(\frac{d}{2}\right)^4 = \frac{\pi d^4}{32}$. Thus,

$$\tau_{allow} = \frac{T_C}{J}; \quad 60(10^6) = \frac{300(d/2)}{\pi d^4/32}$$

$$d = 0.02942 \text{ m} = 30 \text{ mm} \qquad \textbf{Ans.}$$



Ans:
$d = 30$ mm

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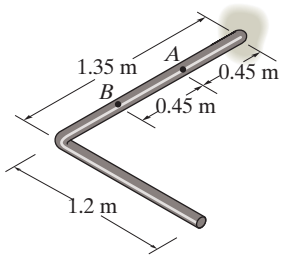
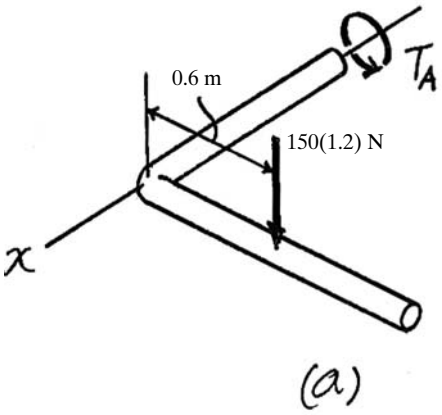
***5–24.** The rod has a diameter of 25 mm and a weight of 150 N/m. Determine the maximum torsional stress in the rod at a section located at A due to the rod's weight.



SOLUTION

Here, we are only interested in the internal torque. Thus, other components of the internal loading are not indicated in the FBD of the cut segment of the rod, Fig. a.

$$\Sigma M_x = 0; \qquad T_A - 150(1.2)(0.6) = 0 \qquad T_A = 108\ \text{N} \cdot \text{m}$$



(a)

The polar moment of inertia of the cross section at A is

$J = \frac{\pi}{2}(0.0125^4) = 38.35(10^{-9})\ \text{m}^4$. Thus

$$\tau_{\max} = \frac{T_A c}{J} = \frac{108(0.0125)}{38.35(10^{-9})} = 35.20(10^6)\ \text{N/m}^2 = 35.2\ \text{MPa}$$ **Ans.**

Ans:
$\tau_{\max} = 35.2$ MPa

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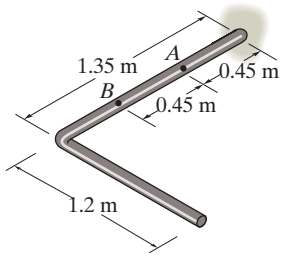
5–25. The rod has a diameter of 25 mm and a weight of 225 N/m. Determine the maximum torsional stress in the rod at a section located at *B* due to the rod's weight.



SOLUTION

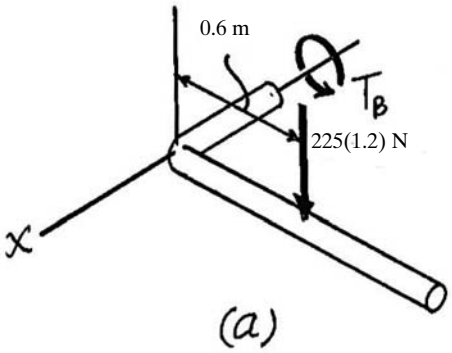
Here, we are only interested in the internal torque. Thus, other components of the internal loading are not indicated in the FBD of the cut segment of the rod, Fig. a.

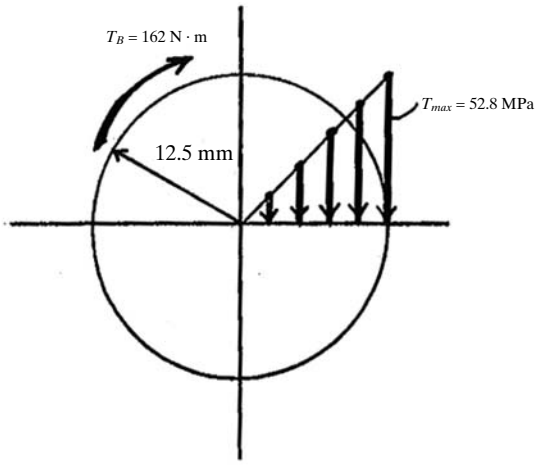
$$\Sigma M_x = 0; \qquad T_B - 225(1.2)(0.6) = 0 \qquad T_B = 162\ \text{N} \cdot \text{m}$$

The polar moment of inertia of the cross-section at *B* is

$J = \frac{\pi}{2}(0.0125^4) = 38.35(10^{-9})\ \text{m}^4$. Thus,

$$\tau_{\max} = \frac{T_B\, c}{J} = \frac{162(0.0125)}{38.35(10^{-9})} = 52.80(10^6)\ \text{N/m}^2 = 52.8\ \text{MPa} \qquad \textbf{Ans.}$$





Ans:

$\tau_{\max} = 52.8\ \text{MPa}$

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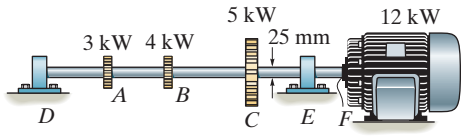
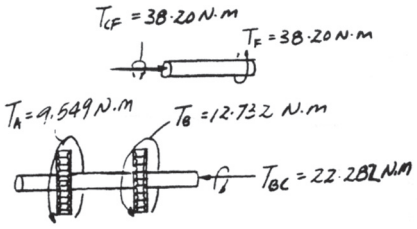
5–26.

The solid steel shaft DF has a diameter of 25 mm and is supported by smooth bearings at D and E. It is coupled to a motor at F, which delivers 12 kW of power to the shaft while it is turning at 50 rev/s. If gears A, B, and C remove 3 kW, 4 kW, and 5 kW respectively, determine the maximum shear stress in the shaft within regions CF and BC. The shaft is free to turn in its support bearings D and E.



SOLUTION



$$\omega = 50\,\frac{\text{rev}}{\text{s}}\left[\frac{2\pi\ \text{rad}}{\text{rev}}\right] = 100\,\pi\ \text{rad/s}$$

$$T_F = \frac{P}{\omega} = \frac{12(10^3)}{100\,\pi} = 38.20\ \text{N}\cdot\text{m}$$

$$T_A = \frac{P}{\omega} = \frac{3(10^3)}{100\,\pi} = 9.549\ \text{N}\cdot\text{m}$$

$$T_B = \frac{P}{\omega} = \frac{4(10^3)}{100\,\pi} = 12.73\ \text{N}\cdot\text{m}$$

$$(\tau_{\max})_{CF} = \frac{T_{CF}\,c}{J} = \frac{38.20(0.0125)}{\frac{\pi}{2}(0.0125^4)} = 12.5\ \text{MPa}$$ **Ans.**

$$(\tau_{\max})_{BC} = \frac{T_{BC}\,c}{J} = \frac{22.282(0.0125)}{\frac{\pi}{2}(0.0125^4)} = 7.26\ \text{MPa}$$ **Ans.**

Ans:
$(\tau_{\max})_{CF} = 12.5$ MPa, $(\tau_{\max})_{BC} = 7.26$ MPa

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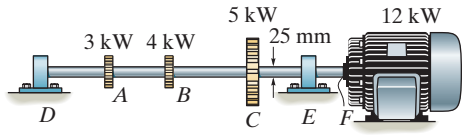
5–27.

The solid steel shaft DF has a diameter of 25 mm and is supported by smooth bearings at D and E. It is coupled to a motor at F, which delivers 12 kW of power to the shaft while it is turning at 50 rev/s. If gears A, B, and C remove 3 kW, 4 kW, and 5 kW respectively, determine the absolute maximum shear stress in the shaft.



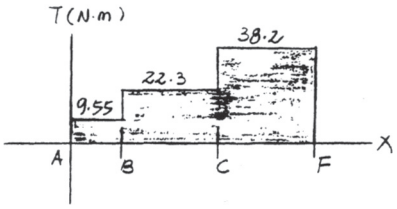
SOLUTION

$$\omega = 50\,\frac{\text{rev}}{\text{s}}\left[\frac{2\pi\ \text{rad}}{\text{rev}}\right] = 100\,\pi\ \text{rad/s}$$

$$T_F = \frac{P}{\omega} = \frac{12(10^3)}{100\pi} = 38.20\ \text{N}\cdot\text{m}$$

$$T_A = \frac{P}{\omega} = \frac{3(10^3)}{100\pi} = 9.549\ \text{N}\cdot\text{m}$$

$$T_B = \frac{P}{\omega} = \frac{4(10^3)}{100\pi} = 12.73\ \text{N}\cdot\text{m}$$



From the torque diagram,

$$T_{\max} = 38.2\ \text{N}\cdot\text{m}$$

$$\tau_{\substack{\text{abs}\\\text{max}}} = \frac{Tc}{J} = \frac{38.2(0.0125)}{\frac{\pi}{2}(0.0125^4)} = 12.5\ \text{MPa}$$ **Ans.**

Ans:

$$\tau_{\substack{\text{abs}\\\text{max}}} = 12.5\ \text{MPa}$$

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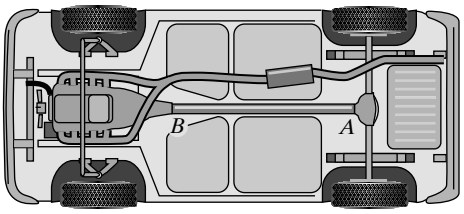
***5–28.** The drive shaft AB of an automobile is made of a steel having an allowable shear stress of $\tau_{allow} = 56$ MPa. If the outer diameter of the shaft is 62.5 mm and the engine delivers 165 kW to the shaft when it is turning at 1140 rev/min, determine the minimum required thickness of the shaft's wall.



SOLUTION

$$\omega = \left(1140\frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right) = 38\pi \text{ rad/s}$$

$$P = T\omega$$

$$165(10^3) = T(38\pi)$$

$$T = 1.382(10^3) \text{ N} \cdot \text{m}$$

$$\tau_{allow} = \frac{Tc}{J}$$

$$56(10^6) = \frac{[1.382(10^3)](0.03125)}{\frac{\pi}{2}(0.03125^4 - r_i^4)}$$

$$r_i = 0.02608 \text{ m}$$

$$t = r_0 - r_i = 0.03125 - 0.02608 = 0.005170 \text{ m} = 5.17 \text{ mm}$$ **Ans.**

Ans.
$t = 5.17$ mm

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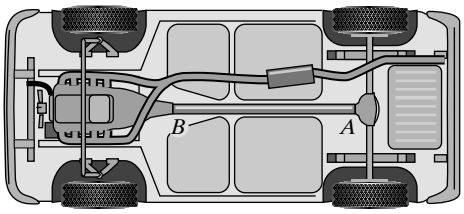
5–29. The drive shaft AB of an automobile is to be designed as a thin-walled tube. The engine delivers 125 kW when the shaft is turning at 1500 rev/min. Determine the minimum thickness of the shaft's wall if the shaft's outer diameter is 62.5 mm. The material has an allowable shear stress of $\tau_{\text{allow}} = 50$ MPa.



SOLUTION

$$\omega = \left(1500\frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right) = 50\pi \text{ rad/s}$$

$$P = T\omega$$

$$125(10^3) = T(50\pi)$$

$$T = 795.77 \text{ N} \cdot \text{m}$$

$$\tau_{\text{allow}} = \frac{Tc}{J}$$

$$50(10^6) = \frac{795.77(0.03125)}{\frac{\pi}{2}(0.03125^4 - r_i^4)}$$

$$r_i = 0.02825 \text{ m}$$

$$t = r_0 - r_i = 0.03125 - 0.02825 = 0.002998 \text{ m} = 3.00 \text{ mm}$$ **Ans.**

Ans.
$t = 3.00$ mm

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5–30. A ship has a propeller drive shaft that is turning at 1500 rev/min while developing 1500 kW. If it is 2.4 m long and has a diameter of 100 mm, determine the maximum shear stress in the shaft caused by torsion.

SOLUTION

Internal Torque:

$$\omega = \left(1500\frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right) = 50.0\pi \text{ rad/s}$$

$$P = 1500 \text{ kW} = 1500(10^3) \text{ N} \cdot \text{m/s}$$

$$T = \frac{P}{\omega} = \frac{1500(10^3)}{50.0\pi} = 9549.30 \text{ N} \cdot \text{m}$$

Maximum Shear Stress: Applying torsion formula

$$\tau_{\max} = \frac{Tc}{J} = \frac{9549.30(0.05)}{\frac{\pi}{2}(0.05^4)} = 48.63(10^3) \text{ N/m}^2 = 48.6 \text{ MPa}$$ **Ans.**

Ans.

$\tau_{\max} = 48.6$ MPa

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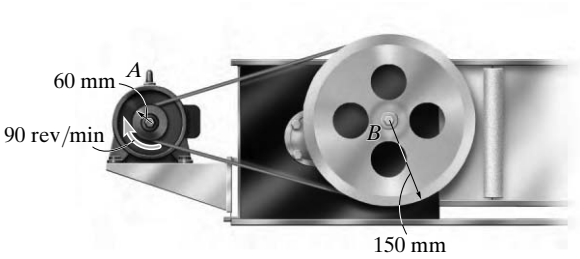
5–31. The motor A develops a power of 300 W and turns its connected pulley at 90 rev/min. Determine the required diameters of the steel shafts on the pulleys at A and B if the allowable shear stress is $\tau_{\text{allow}} = 85$ MPa.



SOLUTION

Internal Torque: For shafts A and B

$$\omega_A = 90\,\frac{\text{rev}}{\text{min}}\left(\frac{2\pi\ \text{rad}}{\text{rev}}\right)\frac{1\ \text{min}}{60\ \text{s}} = 3.00\pi\ \text{rad/s}$$

$$P = 300\ \text{W} = 300\ \text{N}\cdot\text{m/s}$$

$$T_A = \frac{P}{\omega_A} = \frac{300}{3.00\pi} = 31.83\ \text{N}\cdot\text{m}$$

$$\omega_B = \omega_A\left(\frac{r_A}{r_B}\right) = 3.00\pi\left(\frac{0.06}{0.15}\right) = 1.20\pi\ \text{rad/s}$$

$$P = 300\ \text{W} = 300\ \text{N}\cdot\text{m/s}$$

$$T_B = \frac{P}{\omega_B} = \frac{300}{1.20\pi} = 79.58\ \text{N}\cdot\text{m}$$

Allowable Shear Stress: For shaft A

$$\tau_{\text{max}} = \tau_{\text{allow}} = \frac{T_A\,c}{J}$$

$$85\left(10^6\right) = \frac{31.83\left(\frac{d_A}{2}\right)}{\frac{\pi}{2}\left(\frac{d_A}{2}\right)^4}$$

$$d_A = 0.01240\ \text{m} = 12.4\ \text{mm}$$ **Ans.**

For shaft B

$$\tau_{\text{max}} = \tau_{\text{allow}} = \frac{T_B\,c}{J}$$

$$85\left(10^6\right) = \frac{79.58\left(\frac{d_B}{2}\right)}{\frac{\pi}{2}\left(\frac{d_B}{2}\right)^4}$$

$$d_B = 0.01683\ \text{m} = 16.8\ \text{mm}$$ **Ans.**

Ans.
$d_A = 12.4$ mm, $d_B = 16.8$ mm

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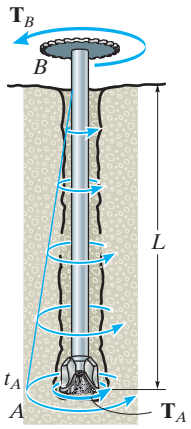
***5–32.**

When drilling a well at constant angular velocity, the bottom end of the drill pipe encounters a torsional resistance T_A. Also, soil along the sides of the pipe creates a distributed frictional torque along its length, varying uniformly from zero at the surface B to t_A at A. Determine the minimum torque T_B that must be supplied by the drive unit to overcome the resisting torques, and calculate the maximum shear stress in the pipe. The pipe has an outer radius r_o and an inner radius r_i.



SOLUTION

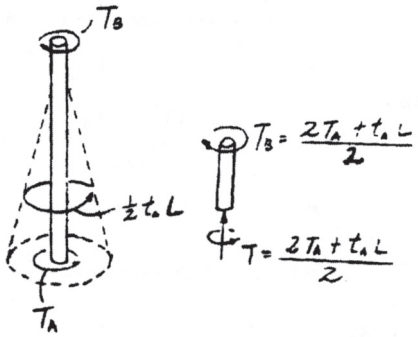
$$T_A + \frac{1}{2}t_A L - T_B = 0$$

$$T_B = \frac{2T_A + t_A L}{2} \qquad \textbf{Ans.}$$

Maximum shear stress: The maximum torque is within the region above the distributed torque.

$$\tau_{\max} = \frac{Tc}{J}$$

$$\tau_{\max} = \frac{\left[\frac{(2T_A + t_A L)}{2}\right](r_o)}{\frac{\pi}{2}(r_o^4 - r_i^4)} = \frac{(2T_A + t_A L)r_o}{\pi(r_o^4 - r_i^4)} \qquad \textbf{Ans.}$$



Ans:

$$T_B = \frac{2T_A + t_A L}{2},$$

$$\tau_{\max} = \frac{(2T_A + t_A L)r_o}{\pi(r_o^4 - r_i^4)}$$

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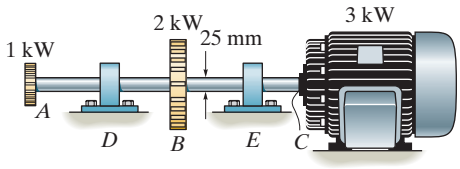
5–33.

The solid steel shaft AC has a diameter of 25 mm and is supported by smooth bearings at D and E. It is coupled to a motor at C, which delivers 3 kW of power to the shaft while it is turning at 50 rev/s. If gears A and B remove 1 kW and 2 kW, respectively, determine the maximum shear stress in the shaft within regions AB and BC. The shaft is free to turn in its support bearings D and E.



SOLUTION

$$T_C = \frac{P}{\omega} = \frac{3(10^3)}{50(2\pi)} = 9.549\ \text{N}\cdot\text{m}$$

$$T_A = \frac{1}{3}T_C = 3.183\ \text{N}\cdot\text{m}$$

$$(\tau_{AB})_{\max} = \frac{T_C}{J} = \frac{3.183\ (0.0125)}{\frac{\pi}{2}(0.0125^4)} = 1.04\ \text{MPa}$$ **Ans.**

$$(\tau_{BC})_{\max} = \frac{T_C}{J} = \frac{9.549\ (0.0125)}{\frac{\pi}{2}(0.0125^4)} = 3.11\ \text{MPa}$$ **Ans.**

Ans:
$(\tau_{AB})_{\max} = 1.04$ MPa, $(\tau_{BC})_{\max} = 3.11$ MPa

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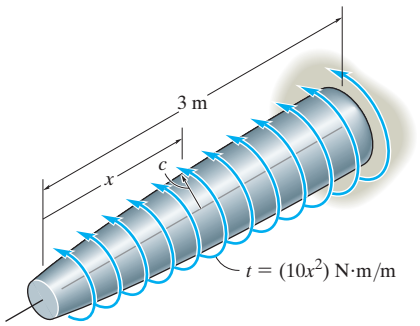
5–34.

The shaft is subjected to a distributed torque along its length of $t = (10x^2)\ \text{N}\cdot\text{m/m}$, where x is in meters. If the maximum stress in the shaft is to remain constant at 80 MPa, determine the required variation of the radius c of the shaft for $0 \le x \le 3$ m.



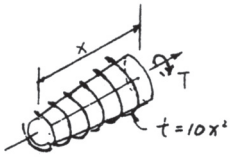
SOLUTION

$$T = \int t\,dx = \int_0^x 10\,x^2 dx = \frac{10}{3}x^3$$

$$\tau = \frac{Tc}{J};\quad 80(10^6) = \frac{(\frac{10}{3})x^3\,c}{\frac{\pi}{2}c^4}$$

$$c^3 = 26.526(10^{-9})\,x^3$$

$$c = (2.98\,x)\ \text{mm}$$ **Ans.**



Ans:
$c = (2.98\,x)$ mm

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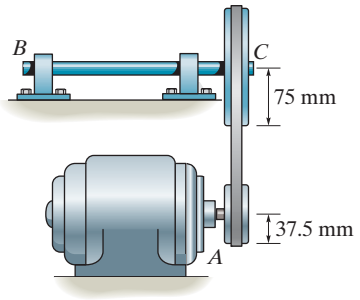
5–35. The motor delivers 12 kW to the pulley A while turning at a constant rate of 1800 rpm. Determine to the nearest multiples of 5 mm the smallest diameter of shaft BC if the allowable shear stress is $\tau_{allow} = 84$ MPa. The belt does not slip on the pulley.



SOLUTION

The angular velocity of shaft BC can be determined using the pulley ratio that is

$$\omega_{BC} = \left(\frac{r_A}{r_C}\right)\omega_A = \left(\frac{0.0375}{0.075}\right)\left(1800\,\frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right) = 30\pi\text{ rad/s}$$

The power is

$$P = 12\text{ kW} = 12(10^3)\text{ N}\cdot\text{m/s}$$

Thus,

$$T = \frac{P}{\omega} = \frac{12(10^3)}{30\pi} = 127.32\text{ N}\cdot\text{m}$$

The polar moment of inertia of the shaft is $J = \frac{\pi}{2}\left(\frac{d}{2}\right)^4 = \frac{\pi d^4}{32}$. Thus,

$$\tau_{allow} = \frac{Tc}{J};\qquad 84(10^6) = \frac{127.32(d/2)}{\pi d^4/32}$$

$$d = 0.01976\text{ m} = 19.76\text{ mm}$$

Use $d = 20$ mm **Ans.**

Ans.
$d = 20$ mm

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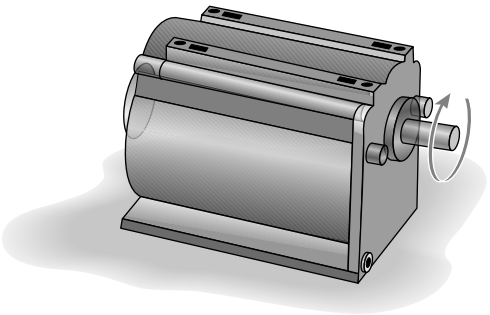
***5–36.** The gear motor can develop 1.6 kW when it turns at 450 rev/min. If the shaft has a diameter of 25 mm, determine the maximum shear stress developed in the shaft.



SOLUTION

The angular velocity of the shaft is

$$\omega = \left(450\,\frac{\cancel{\text{rev}}}{\cancel{\text{min}}}\right)\left(\frac{2\pi\text{ rad}}{1\,\cancel{\text{rev}}}\right)\left(\frac{1\,\cancel{\text{min}}}{60\text{ s}}\right) = 15\pi\text{ rad/s}$$

and the power is

$$P = 1.6\text{ kW} = 1.6(10^3)\text{ N}\cdot\text{m/s}$$

Then

$$T = \frac{P}{\omega} = \frac{1.6(10^3)}{15\pi} = 33.95\text{ N}\cdot\text{m}$$

The polar moment of inertia of the shaft is $J = \frac{\pi}{2}(0.0125^4) = 38.35(10^{-9})\text{ m}^4$. Thus,

$$\tau_{\max} = \frac{T\,c}{J} = \frac{33.95(0.0125)}{38.35(10^{-9})} = 11.07(10^6)\text{ N/m}^2 = 11.1\text{ MPa}$$ **Ans.**

Ans.
$\tau_{\max} = 11.1$ MPa

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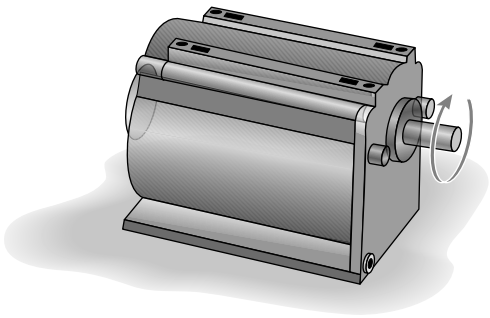
5–37. The gear motor can develop 2.4 kW when it turns at 150 rev/min. If the allowable shear stress for the shaft is $\tau_{allow} = 84$ MPa, determine the smallest diameter of the shaft to the nearest multiples of 5 mm that can be used.



SOLUTION

The angular velocity of the shaft is

$$\omega = \left(150\,\frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right) = 5\pi\text{ rad/s}$$

and the power is

$$P = 2.4\text{ kW} = 2.4(10^3)\text{ N}\cdot\text{m/s}$$

Then

$$T = \frac{P}{\omega} = \frac{2.4(10^3)}{5\pi} = 152.79\text{ N}\cdot\text{m}$$

The polar moment of inertia of the shaft is $J = \frac{\pi}{2}\left(\frac{d}{2}\right)^4 = \frac{\pi d^4}{32}$. Thus,

$$\tau_{allow} = \frac{T\,c}{J};\qquad 84(10^6) = \frac{152.79(d/2)}{\pi d^4/32}$$

$$d = 0.02100\text{ m} = 21.00\text{ mm}$$ **Ans.**

use $d = 25$ mm

Ans.
$d = 21.00$ mm

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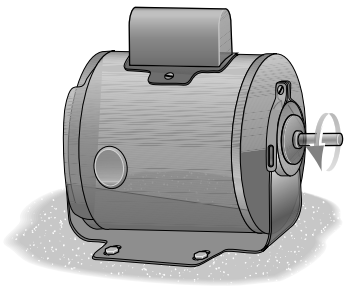
5–38. The 25-mm-diameter shaft on the motor is made of a material having an allowable shear stress of $\tau_{allow} = 75$ MPa. If the motor is operating at its maximum power of 5 kW, determine the minimum allowable rotation of the shaft.



SOLUTION

Allowable Shear Stress: The polar moment of inertia of the shaft is $J = \frac{\pi}{2}\left(0.0125^4\right) = 38.3495(10^{-9})\ \text{m}^4$.

$$\tau_{allow} = \frac{Tc}{J}; \qquad 75(10^6) = \frac{T(0.0125)}{38.3495(10^{-9})}$$

$$T = 230.10\ \text{N}\cdot\text{m}$$

Internal Loading:

$$T = \frac{P}{\omega}; \qquad 230.10 = \frac{5(10^3)}{\omega}$$

$$\omega = 21.7\ \text{rad/s} \qquad \textbf{Ans.}$$

Ans.

$\omega = 21.7$ rad/s

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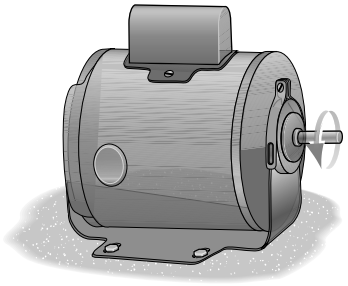
5–39. The drive shaft of the motor is made of a material having an allowable shear stress of $\tau_{allow} = 75$ MPa. If the outer diameter of the tubular shaft is 20 mm and the wall thickness is 2.5 mm, determine the maximum allowable power that can be supplied to the motor when the shaft is operating at an angular velocity of 1500 rev/min.



SOLUTION

Internal Loading: The angular velocity of the shaft is

$$\omega = \left(1500\,\frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right) = 50\pi\text{ rad/s}$$

We have

$$T = \frac{P}{\omega} = \frac{P}{50\pi}$$

Allowable Shear Stress: The polar moment of inertia of the shaft is $J = \frac{\pi}{2}\left(0.01^4 - 0.0075^4\right) = 10.7379(10^{-9})\text{ m}^4$.

$$\tau_{allow} = \frac{Tc}{J}; \qquad 75(10^6) = \frac{\left(\frac{P}{50\pi}\right)(0.01)}{10.7379(10^{-9})}$$

$$P = 12\,650.25\text{ W} = 12.7\text{ kW} \qquad \textbf{Ans.}$$

Ans.

$P = 12.7$ kW

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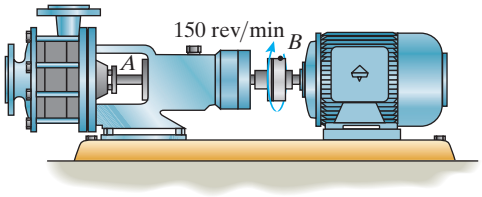
***5–40.**

The pump operates using the motor that has a power of 85 W. If the impeller at B is turning at 150 rev/min, determine the maximum shear stress in the 20-mm-diameter transmission shaft at A.



SOLUTION

Internal Torque:

$$\omega = 150\,\frac{\text{rev}}{\text{min}}\left(\frac{2\pi\ \text{rad}}{\text{rev}}\right)\frac{1\ \text{min}}{60\ \text{s}} = 5.00\pi\ \text{rad/s}$$

$$P = 85\ \text{W} = 85\ \text{N}\cdot\text{m/s}$$

$$T = \frac{P}{\omega} = \frac{85}{5.00\pi} = 5.411\ \text{N}\cdot\text{m}$$

Maximum Shear Stress: Applying torsion formula

$$\tau_{\max} = \frac{T\,c}{J}$$

$$= \frac{5.411\ (0.01)}{\frac{\pi}{2}(0.01^4)} = 3.44\ \text{MPa}$$ **Ans.**

Ans:
$\tau_{\max} = 3.44$ MPa

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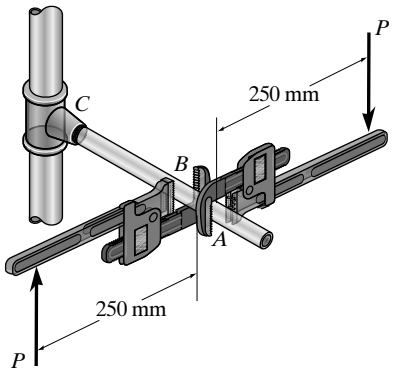
5–41. Two wrenches are used to tighten the pipe. If P = 300 N is applied to each wrench, determine the maximum torsional shear stress developed within regions AB and BC. The pipe has an outer diameter of 25 mm and inner diameter of 20 mm. Sketch the shear stress distribution for both cases.



SOLUTION

Internal Loadings: The internal torque developed in segments AB and BC of the pipe can be determined by writing the moment equation of equilibrium about the x axis by referring to their respective free - body diagrams shown in Figs. a and b.

$$\Sigma M_x = 0;\ T_{AB} - 300(0.25) = 0 \qquad T_{AB} = 75\,\text{N}\cdot\text{m}$$

And

$$\Sigma M_x = 0;\ T_{BC} - 300(0.25) - 300(0.25) = 0 \qquad T_{BC} = 150\,\text{N}\cdot\text{m}$$

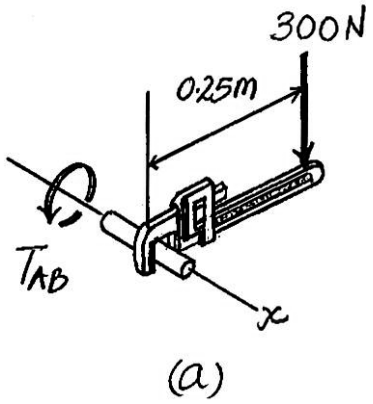
Allowable Shear Stress: The polar moment of inertia of the pipe is $J = \frac{\pi}{2}\left(0.0125^4 - 0.01^4\right) = 22.642(10^{-9})\text{m}^4$.

$$\left(\tau_{\max}\right)_{AB} = \frac{T_{AB}\,c}{J} = \frac{75(0.0125)}{22.642(10^{-9})} = 41.4\ \text{MPa} \qquad \textbf{Ans.}$$

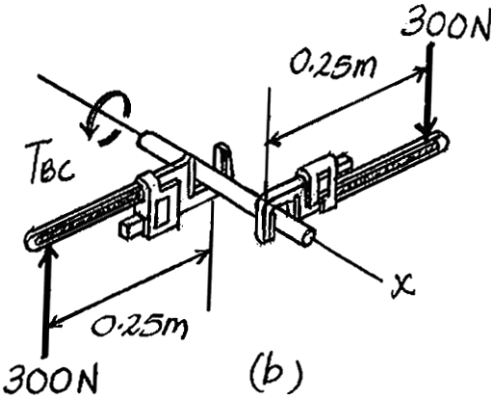
$$\left(\tau_{AB}\right)_{\rho=0.01\ \text{m}} = \frac{T_{AB}\,\rho}{J} = \frac{75(0.01)}{22.642(10^{-9})} = 33.1\ \text{MPa}$$

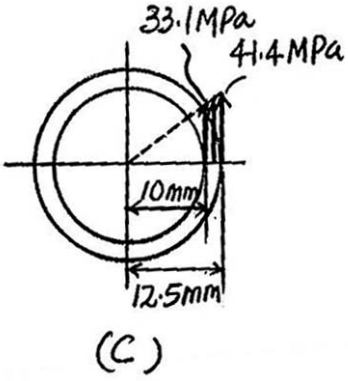
$$\left(\tau_{\max}\right)_{BC} = \frac{T_{BC}\,c}{J} = \frac{150(0.0125)}{22.642(10^{-9})} = 82.8\ \text{MPa} \qquad \textbf{Ans.}$$

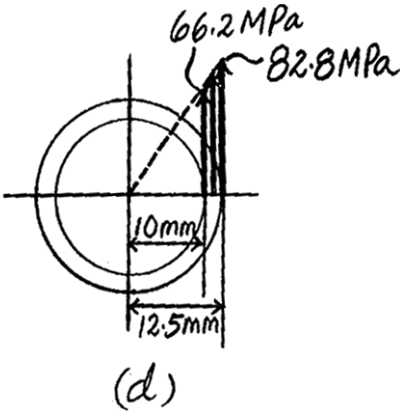
$$\left(\tau_{BC}\right)_{\rho=0.01\ \text{m}} = \frac{T_{BC}\,\rho}{J} = \frac{150(0.01)}{22.642(10^{-9})} = 66.2\ \text{MPa}$$



The shear stress distribution along the radial line of segments AB and BC of the pipe is shown in Figs. c and d, respectively.







Ans:
$(\tau_{\max})_{AB} = 41.4$ MPa, $(\tau_{\max})_{BC} = 82.8$ MPa

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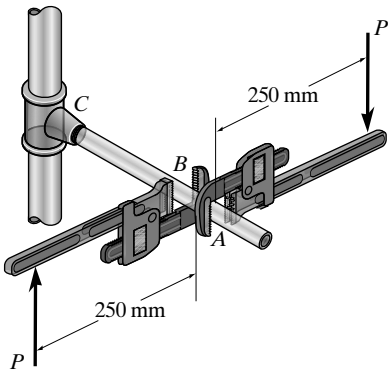
5–42. Two wrenches are used to tighten the pipe. If the pipe is made from a material having an allowable shear stress of $\tau_{\text{allow}} = 85$ MPa, determine the allowable maximum force **P** that can be applied to each wrench. The pipe has an outer diameter of 25 mm and inner diameter of 20 mm.



SOLUTION

Internal Loading: By observation, segment BC of the pipe is critical since it is subjected to a greater internal torque than segment AB. Writing the moment equation of equilibrium about the x axis by referring to the free-body diagram shown in Fig. a, we have

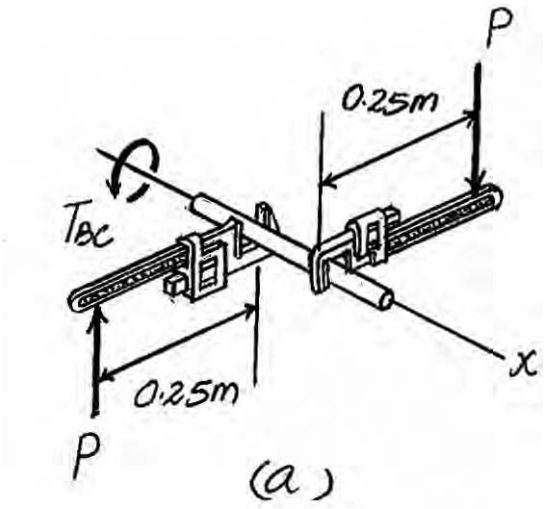
$$\Sigma M_x = 0;\ T_{BC} - P(0.25) - P(0.25) = 0 \qquad T_{BC} = 0.5P$$

Allowable Shear Stress: The polar moment of inertia of the pipe is

$$J = \frac{\pi}{2}\left(0.0125^4 - 0.01^4\right) = 22.642(10^{-9})\text{m}^4$$

$$\tau_{\text{allow}} = \frac{T_{BC}\,c}{J}; \qquad 85(10^6) = \frac{0.5P(0.0125)}{22.642(10^{-9})}$$

$$P = 307.93\text{N} = 308\,\text{N} \qquad \textbf{Ans.}$$



Ans.
$P = 308\,\text{N}$

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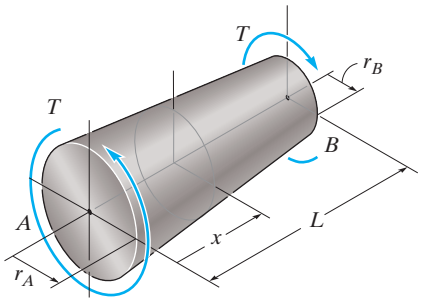
5–43.

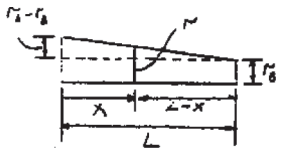
The solid shaft has a linear taper from r_A at one end to r_B at the other. Derive an equation that gives the maximum shear stress in the shaft at a location x along the shaft's axis.



SOLUTION

$$r = r_B + \frac{r_A - r_B}{L}(L - x) = \frac{r_B L + (r_A - r_B)(L - x)}{L}$$

$$= \frac{r_A(L - x) + r_B x}{L}$$



$$\tau_{\max} = \frac{Tc}{J} = \frac{Tr}{\frac{\pi}{2}r^4} = \frac{2T}{\pi r^3}$$

$$= \frac{2T}{\pi\left[\frac{r_A(L - x) + r_B x}{L}\right]^3} = \frac{2TL^3}{\pi[r_A(L - x) + r_B x]^3}$$

Ans.

Ans:

$$\tau_{\max} = \frac{2TL^3}{\pi[r_A(L - x) + r_B x]^3}$$

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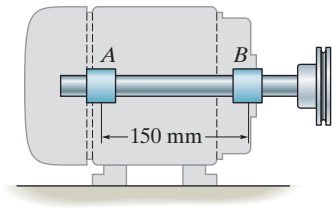
***5–44.** A motor delivers 375 kW to the shaft, which is tubular and has an outer diameter of 50 mm. If it is rotating at 200 rad/s, determine its largest inner diameter to the nearest mm if the allowable shear stress for the material is $\tau_{allow} = 175$ MPa.



SOLUTION

$P = 375(10^3)\ \text{N} \cdot \text{m/s}$

$$T = \frac{P}{\omega} = \frac{375(10^3)}{200} = 1.875(10^3)\ \text{N} \cdot \text{m}$$

$$\tau_{max} = \tau_{allow} = \frac{Tc}{J}$$

$$175(10^6) = \frac{1.875(10^3)(0.025)}{\frac{\pi}{2}\left(0.025^4 - c_i^4\right)}$$

$c_i = 0.02166$ m

$d = 2c_i = 2(0.02166) = 0.04332\ \text{m} = 43.32\ \text{mm}$

Use $d_i = 43$ mm **Ans.**

Ans:

Use $d_i = 43$ mm

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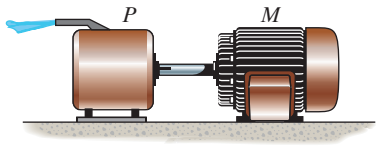
5–45.

The A-36 steel tubular shaft is 2 m long and has an outer diameter of 50 mm. When it is rotating at 40 rad/s, it transmits 25 kW of power from the motor M to the pump P. Determine the smallest thickness of the tube if the allowable shear stress is $\tau_{\text{allow}} = 80$ MPa.



SOLUTION

The internal torque in the shaft is

$$T = \frac{P}{\omega} = \frac{25(10^3)}{40} = 625\ \text{N}\cdot\text{m}$$

The polar moment of inertia of the shaft is $J = \frac{\pi}{2}(0.025^4 - c_i{}^4)$. Thus,

$$\tau_{\text{allow}} = \frac{Tc}{J}; \qquad 80(10^6) = \frac{625(0.025)}{\frac{\pi}{2}(0.025^4 - c_i{}^4)}$$

$$c_i = 0.02272\ \text{m}$$

So that

$$t = 0.025 - 0.02272$$

$$= 0.002284\ \text{m} = 2.284\ \text{mm} = 2.28\ \text{mm} \qquad \textbf{Ans.}$$

Ans:
$t = 2.28$ mm

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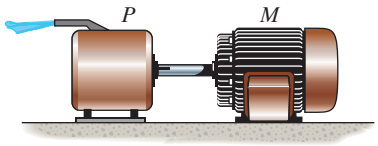
5–46.

The A-36 solid steel shaft is 2 m long and has a diameter of 60 mm. It is required to transmit 60 kW of power from the moor M to the pump P. Determine the smallest angular velocity the shaft if the allowable shear stress is $\tau_{\text{allow}} = 80$ MPa.



SOLUTION

The polar moment of inertia of the shaft is $J = \frac{\pi}{2}(0.03^4) = 0.405(10^{-6})\pi \text{ m}^4$. Thus,

$$\tau_{\text{allow}} = \frac{Tc}{J}; \qquad 80(10^6) = \frac{T(0.03)}{0.405(10^{-6})\pi}$$

$$T = 3392.92 \text{ N} \cdot \text{m}$$

$$P = T\omega; \qquad 60(10^3) = 3392.92\,\omega$$

$$\omega = 17.68 \text{ rad/s} = 17.7 \text{ rad/s} \qquad \textbf{Ans.}$$

Ans:
$\omega = 17.7$ rad/s

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5–47.

The propellers of a ship are connected to an A-36 steel shaft that is 60 m long and has an outer diameter of 340 mm and inner diameter of 260 mm. If the power output is 4.5 MW when the shaft rotates at 20 rad/s, determine the maximum torsional stress in the shaft and its angle of twist.

SOLUTION

$$T = \frac{P}{\omega} = \frac{4.5(10^6)}{20} = 225(10^3)\ \text{N} \cdot \text{m}$$

$$\tau_{\max} = \frac{Tc}{J} = \frac{225(10^3)(0.170)}{\frac{\pi}{2}[(0.170)^4 - (0.130)^4]} = 44.3\ \text{MPa} \qquad \textbf{Ans.}$$

$$\phi = \frac{TL}{JG} = \frac{225\left(10^3\right)(60)}{\frac{\pi}{2}[(0.170)^4 - (0.130)^4)75(10^9)} = 0.2085\ \text{rad} = 11.9^\circ \qquad \textbf{Ans.}$$

Ans:
$\tau_{\max} = 44.3\ \text{MPa}$, $\phi = 11.9^\circ$

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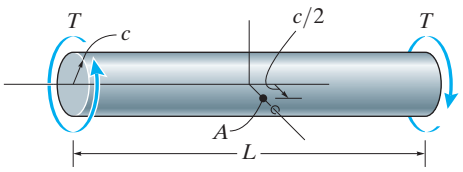
***5–48.**

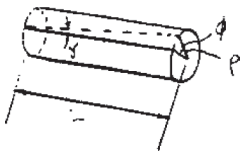
The solid shaft of radius c is subjected to a torque **T** at its ends. Show that the maximum shear strain in the shaft is $\gamma_{max} = Tc/JG$. What is the shear strain on an element located at point A, $c/2$ from the center of the shaft? Sketch the shear strain distortion of this element.



SOLUTION

From the geometry:

$$\gamma L = \rho\,\phi; \qquad \gamma = \frac{\rho\,\phi}{L}$$

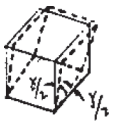


Since $\phi = \dfrac{TL}{JG}$, then

$$\gamma = \frac{T\rho}{JG} \qquad \textbf{(1)}$$

However the maximum shear strain occurs when $\rho = c$

$$\gamma_{max} = \frac{Tc}{JG} \qquad \textbf{QED}$$



Shear strain when $\rho = \dfrac{c}{2}$ is from Eq. (1),

$$\gamma = \frac{T(c/2)}{JG} = \frac{Tc}{2\,JG} \qquad \textbf{Ans.}$$

Ans:

$$\gamma = \frac{Tc}{2JG}$$

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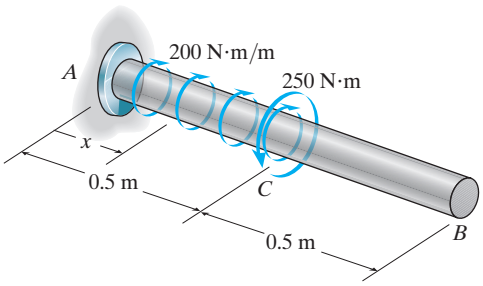
5–49.

The A-36 steel shaft has a diameter of 50 mm and is subjected to the distributed and concentrated loadings shown. Determine the absolute maximum shear stress in the shaft and plot a graph of the angle of twist of the shaft in radians versus x.



SOLUTION

Internal Torque: As shown on FBD.

Maximum Shear Stress: The maximum torque occurs at $x = 0.5$ m where $T_{max} = 150 + 200(0.5) = 250\ \text{N}\cdot\text{m}$.

$$\tau_{\substack{\max \\ \text{ABS}}} = \frac{T_{\max}\, c}{J} = \frac{250(0.025)}{\frac{\pi}{2}(0.025^4)} = 10.2\ \text{MPa} \qquad \textbf{Ans.}$$

Angle of Twist:

For $0 \le x < 0.5$ m

$$\phi(x) = \int_0^L \frac{T(x)\, dx}{JG}$$

$$= \int_0^x \frac{(150 + 200x)\, dx}{JG}$$

$$= \frac{150x + 100x^2}{JG}$$

$$= \frac{150x + 100x^2}{\frac{\pi}{2}\left(0.025^4\right) 75.0\left(10^9\right)}$$

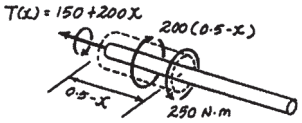
$$= \left[3.26x + 2.17x^2\right]\left(10^{-3}\right)\ \text{rad}$$

At $x = 0.5$ m, $\quad \phi = \phi_C = 0.00217$ rad

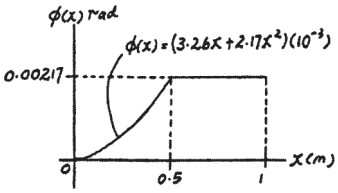
For $0.5\ \text{m} < x < 1\ \text{m}$ $\quad$ Since $T(x) = 0$, then

$$\phi(x) = \phi_C = 0.00217\ \text{rad}$$

For $0.5\text{m} < x \le 1\text{m}$



For $0 \le x < 0.5$ m



Ans:

$\tau_{\substack{\max \\ \text{ABS}}} = 10.2$ MPa

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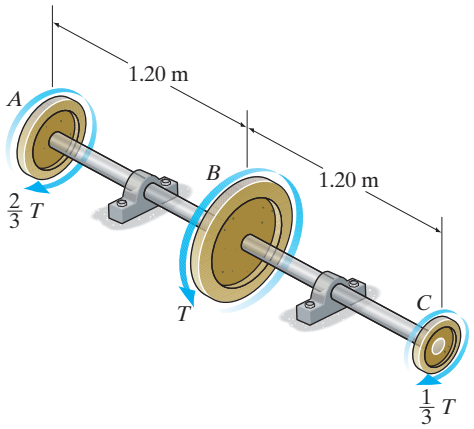
5–50.

The 60-mm-diameter shaft is made of 6061-T6 aluminum having an allowable shear stress of $\tau_{allow} = 80$ MPa. Determine the maximum allowable torque **T**. Also, find the corresponding angle of twist of disk *A* relative to disk *C*.



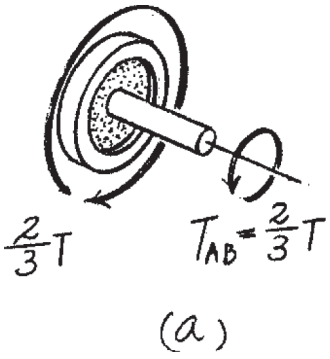
SOLUTION

Internal Loading: The internal torques developed in segments *AB* and *BC* of the shaft are shown in Figs. *a* and *b*, respectively.

Allowable Shear Stress: Segment *AB* is critical since it is subjected to a greater internal torque. The polar moment of inertia of the shaft is $J = \frac{\pi}{2}(0.03^4) = 0.405(10^{-6})\pi\ \text{m}^4$. We have

$$\tau_{allow} = \frac{T_{AB}\,c}{J}; \qquad 80(10^6) = \frac{(\frac{2}{3}T)(0.03)}{0.405(10^{-6})\pi}$$

$$T = 5089.38\ \text{N}\cdot\text{m} = 5.09\ \text{kN}\cdot\text{m} \qquad \textbf{Ans.}$$



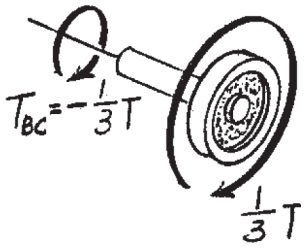
(a)

Angle of Twist: The internal torques developed in segments *AB* and *BC* of the shaft are $T_{AB} = \frac{2}{3}(5089.38) = 3392.92\ \text{N}\cdot\text{m}$ and $T_{BC} = -\frac{1}{3}(5089.38) = -1696.46\ \text{N}\cdot\text{m}$. We have

$$\phi_{A/C} = \Sigma\frac{T_i L_i}{J_i G_i} = \frac{T_{AB}L_{AB}}{JG_{al}} + \frac{T_{BC}L_{BC}}{JG_{al}}$$

$$\phi_{A/C} = \frac{3392.92(1.20)}{0.405(10^{-6})\pi(26)(10^9)} + \frac{-1696.46(1.20)}{0.405(10^{-6})\pi(26)(10^9)}$$

$$= 0.06154\ \text{rad} = 3.53° \qquad \textbf{Ans.}$$



(b)

Ans:
$T = 5.09\ \text{kN}\cdot\text{m}$, $\phi_{A/C} = 3.53°$

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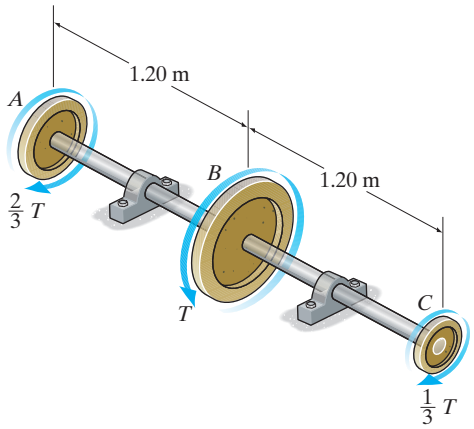
5–51.

The 60-mm-diameter shaft is made of 6061-T6 aluminum. If the allowable shear stress is $\tau_{allow} = 80$ MPa, and the angle of twist of disk A relative to disk C is limited so that it does not exceed 0.06 rad, determine the maximum allowable torque **T**.



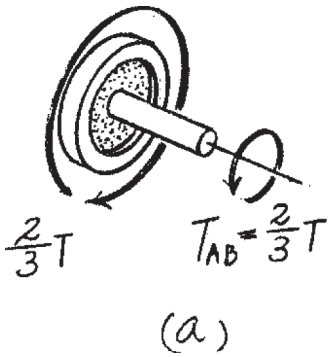
SOLUTION

Internal Loading: The internal torques developed in segments AB and BC of the shaft are shown in Figs. a and b, respectively.

Allowable Shear Stress: Segment AB is critical since it is subjected to a greater internal torque. The polar moment of inertia of the shaft is $J = \frac{\pi}{2}(0.03^4) = 0.405(10^{-6})\pi\ \text{m}^4$. We have

$$\tau_{\text{allow}} = \frac{T_{AB}\,c}{J}; \qquad 80(10^3) = \frac{(\frac{2}{3}T)(0.03)}{0.405(10^{-6})\pi}$$

$$T = 5089.38\ \text{N}\cdot\text{m} = 5.089\ \text{kN}\cdot\text{m}$$

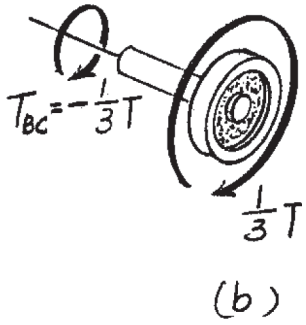


Angle of Twist: It is required that $\phi_{A/C} = 0.06$ rad. We have

$$\phi_{A/C} = \Sigma\frac{T_i L_i}{J_i G_i} = \frac{T_{AB}L_{AB}}{JG_{al}} + \frac{T_{BC}L_{BC}}{JG_{al}}$$

$$0.06 = \frac{(\frac{2}{3}T)(1.2)}{0.405(10^{-6})\pi(26)(10^9)} + \frac{(-\frac{1}{3}T)(1.2)}{0.405(10^{-6})\pi(26)(10^9)}$$

$$T = 4962.14\ \text{N}\cdot\text{m} = 4.96\ \text{kN}\cdot\text{m (controls)} \qquad \textbf{Ans.}$$



Ans:
$T = 4.96\ \text{kN}\cdot\text{m}$

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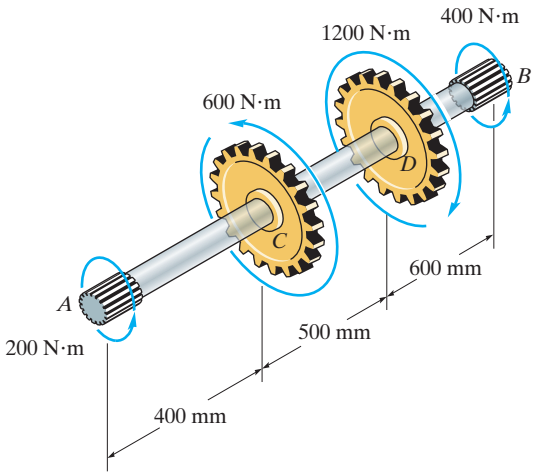
***5–52.**

The splined ends and gears attached to the A992 steel shaft are subjected to the torques shown. Determine the angle of twist of end B with respect to end A. The shaft has a diameter of 40 mm.



SOLUTION

Internal Torque: The torque diagram shown in Fig. *a* can be plotted. From this diagram, $T_{AC} = 200\ \text{N}\cdot\text{m}$, $T_{CD} = 800\ \text{N}\cdot\text{m}$ and $T_{DB} = 400\ \text{N}\cdot\text{m}$.

Angle of Twist:

$$\phi_{B/A} = \Sigma \frac{TL}{JG}$$

$$= \frac{1}{JG}(T_{AC}L_{AC} + T_{CD}L_{CD} + T_{DB}L_{DB})$$

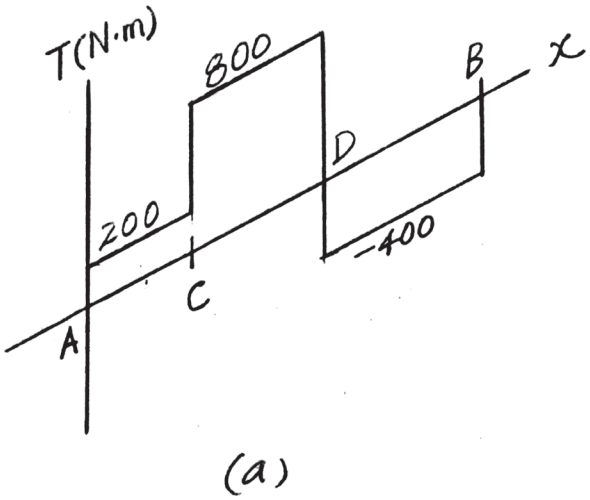
$$= \frac{1}{JG}[200(0.4) + 800(0.5) + (-400)(0.6)]$$

$$= \frac{240\ \text{N}\cdot\text{m}^2}{JG}$$

For A992 steel, $G = 75$ GPa. Then

$$\phi_{B/A} = \frac{240}{\frac{\pi}{2}(0.02^4)[75(10^9)]}$$

$$= (0.01273\ \text{rad})\left(\frac{180^\circ}{\pi\ \text{rad}}\right) = 0.730^\circ \text{ ⟲}$$ **Ans.**



(a)

Ans:
$\phi_{B/A} = 0.730^\circ$ ⟲

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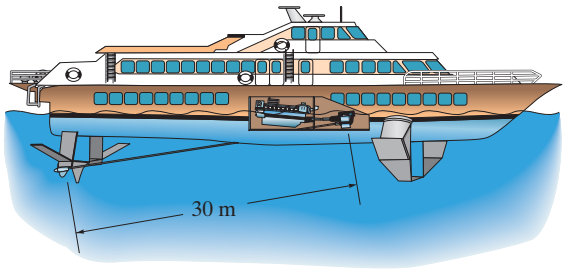
5–53. The hydrofoil boat has an A-36 steel propeller shaft that is 30 m long. It is connected to an in-line diesel engine that delivers a maximum power of 2000 kW and causes the shaft to rotate at 1700 rpm. If the outer diameter of the shaft is 200 mm and the wall thickness is 10 mm, determine the maximum shear stress developed in the shaft. Also, what is the "wind up," or angle of twist in the shaft at full power?



SOLUTION

Internal Torque:

$$\omega = \left(1700\frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi\ \text{rad}}{1\ \text{rev}}\right)\left(\frac{1\ \text{min}}{60\ \text{s}}\right) = 56.667\pi\ \text{rad/s}$$

$$P = 2000\ \text{kW} = 2000(10^3)\ \text{N}\cdot\text{m/s}$$

$$T = \frac{P}{\omega} = \frac{2000(10^3)}{56.667\pi} = 11.23(10^3)\ \text{N}\cdot\text{m}$$

Maximum Shear Stress: Applying torsion Formula.

$$\tau_{\max} = \frac{T\,c}{J}$$

$$= \frac{11.23(10^3)(0.1)}{\frac{\pi}{2}(0.1^4 - 0.09^4)} = 20.80(10^6)\ \text{N/m}^2 = 20.8\ \text{MPa} \qquad \textbf{Ans.}$$

Angle of Twist:

$$\phi = \frac{TL}{JG} = \frac{11.23(10^3)(30)}{[\frac{\pi}{2}(0.1^4 - 0.09^4)][75(10^9)]}$$

$$= 0.08319\ \text{rad} = 4.77° \qquad \textbf{Ans.}$$

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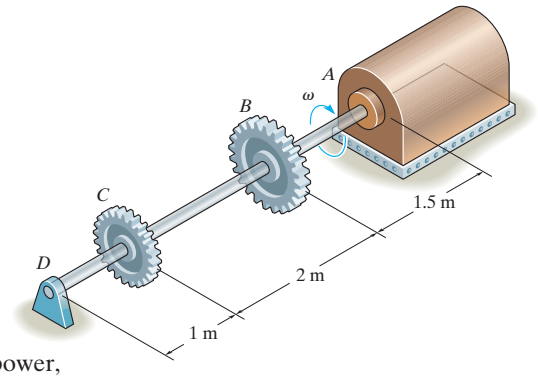
5–54.

The turbine develops 300 kW of power, which is transmitted to the gears such that both B and C receive an equal amount. If the rotation of the 100-mm-diameter A992 steel shaft is $\omega = 600$ rev/min., determine the absolute maximum shear stress in the shaft and the rotation of end D of the shaft relative to A. The journal bearing at D allows the shaft to turn freely about its axis.



SOLUTION

External Applied Torque: Gears B and C withdraw equal amount of power, $P_B = P_C = P = \frac{300}{2} = 150$ kw. Here, the angular velocity of the shaft is $\omega = \left(600\,\frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right) = 20\pi$ rad/s. Then, the torque exerted on gears B and C can be determined from

$$T_B = T_C\frac{p}{\omega} = \frac{150(10^3)}{20\pi} = \frac{7500}{\pi}\ \text{N}\cdot\text{m}$$

Using this result, the torque diagram shown in Fig. a can be plotted.

Maximum Shear Stress: From the torque diagram, we notice that the maximum torque $T_{\max} = \frac{15000}{\pi}$ N · m occurs at region BA. Thus it is the critical region where the absolute maximim shear stress occurs. Applying the torsion formula,

$$\tau_{\substack{\text{abs}\\\text{max}}} = \frac{T_{\max}C}{J} = \frac{\frac{15000}{\pi}(0.05)}{\frac{\pi}{2}(0.05^4)} = 24.32\,(10^6)\text{ Pa} = 24.3\text{ MPa}$$ **Ans.**

Angle of twist: From the torque diagram, $T_{DC} = 0$, $T_{CB} = \frac{7500}{\pi}$ N · m and $T_{BA} = \frac{15000}{\pi}$ N · m

$$\phi_{D/A} = \Sigma\frac{TL}{JG} = \frac{1}{JG}(T_{DC}L_{DC} + T_{CB}L_{CB} + T_{BA}L_{BA})$$

$$= \frac{1}{JG}\left[0 + \left(\frac{7500}{\pi}\right)(2) + \left(\frac{15000}{\pi}\right)(1.5)\right]$$

$$= \frac{37500\text{ N}\cdot\text{m}^2}{\pi JG}$$

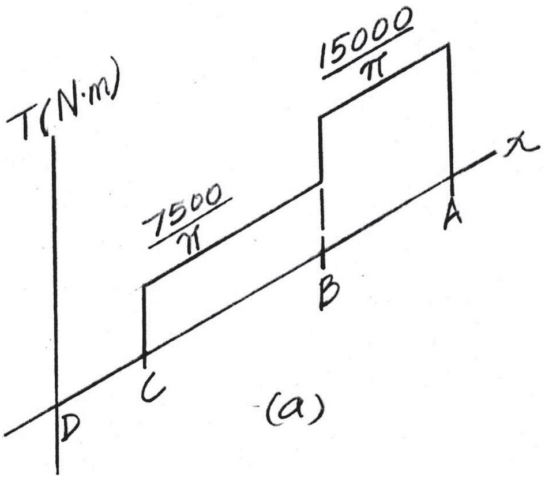
For A992 steel, $G = 75$ GPa. Thus

$$\phi_{D/A} = \frac{37500}{\pi\left[\frac{\pi}{2}(0.05^4)\right][75(10^9)]}$$

$$= (0.016211\text{ rad})\left(\frac{180^\circ}{\pi\text{ rad}}\right)$$

$$= 0.9288^\circ$$

$$= 0.929^\circ$$ **Ans.**



Ans:
$\tau_{\substack{\text{abs}\\\text{max}}} = 24.3$ MPa
$\phi_{D/A} = 0.929^\circ$

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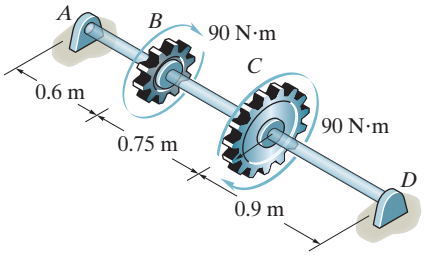
5–55. The shaft is made of A992 steel. It has a diameter of 25 mm and is supported by bearings at A and D, which allow free rotation. Determine the angle of twist of B with respect to D.



SOLUTION

The internal torques developed in segments BC and CD are shown in Figs. a and b.

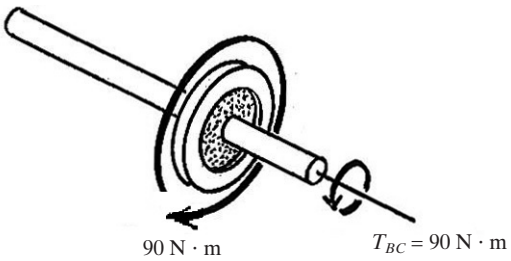
The polar moment of inertia of the shaft is $J = \frac{\pi}{2}(0.0125^4) = 38.35(10^{-9})\ \text{m}^4$. Thus,

$$\phi_{B/D} = \sum \frac{T_i L_i}{J_i G_i} = \frac{T_{BC} L_{BC}}{J\, G_{st}} + \frac{T_{CD} L_{CD}}{J\, G_{st}}$$

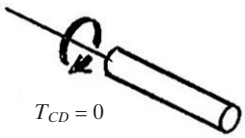
$$= \frac{90(0.75)}{[38.35(10^{-9})]75(10^6)} + 0$$

$$= 0.02347 \text{ rad} = 1.34°$$

Ans.



(a)



(b)

Ans:

$\phi_{B/D} = 1.34°$

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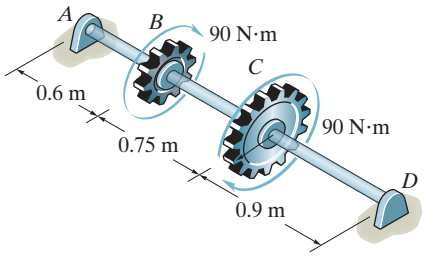
5–56. The shaft is made of A-36 steel. It has a diameter of 25 mm and is supported by bearings at A and D, which allow free rotation. Determine the angle of twist of gear C with respect to B.



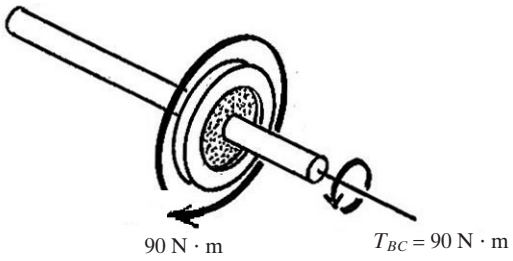
SOLUTION

The internal torque developed in segment BC is shown in Fig. a

The polar moment of inertia of the shaft is $J = \frac{\pi}{2}(0.0125^4) = 38.35(10^{-9})\ \text{m}^4$. Thus,

$$\phi_{C/B} = \frac{T_{BC}\,L_{BC}}{J\,G_{st}} = \frac{90(0.75)}{[38.35(10^{-9})]75(10^6)}$$

$$= 0.02347\ \text{rad} = 1.34° \qquad \textbf{Ans.}$$



(a)

Ans:
$\phi_{C/B} = 1.34°$

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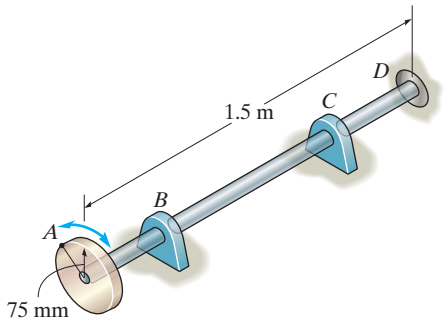
5–57.

The rotating flywheel-and-shaft, when brought to a sudden stop at D, begins to oscillate clockwise-counter-clockwise such that a point A on the outer edge of the fly-wheel is displaced through a 6-mm arc. Determine the maximum shear stress developed in the tubular A-36 steel shaft due to this oscillation. The shaft has an inner diameter of 24 mm and an outer diameter of 32 mm. The bearings at B and C allow the shaft to rotate freely, whereas the support at D holds the shaft fixed.



SOLUTION

$s = r\,\theta$

$6 = 75\,\phi \qquad \phi = 0.08$ rad

$$\phi = \frac{TL}{JG}$$

$$0.08 = \frac{T(1.5)}{J(75)(10^9)}$$

$$T = 4(10^9)\,J$$

$$\tau_{\max} = \frac{T_C}{J}$$

$$= \frac{4(10^9)(J)(0.016)}{J}$$

$= 64.0$ MPa **Ans.**

Ans:
$\tau_{\max} = 64.0$ MPa

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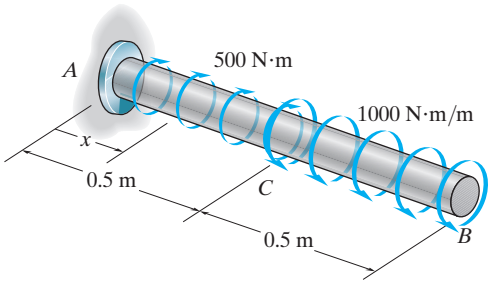
5–58.

The A992 steel shaft has a diameter of 50 mm and is subjected to the distributed loadings shown. Determine the absolute maximum shear stress in the shaft and plot a graph of the angle of twist of the shaft in radians versus x.



SOLUTION

Internal Torque: Referring to the FBD of the right segment of the shaft shown in Fig. a ($0.5\text{ m} < x \leq 1\text{ m}$)

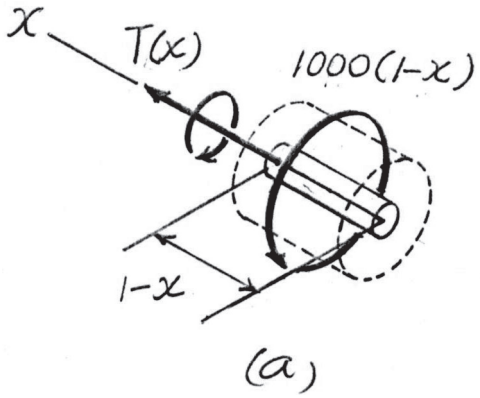
$$\Sigma M_x = 0; \quad T(x) - 1000\,(1-x) = 0 \qquad T(x) = \{1000 - 1000x\}\,\text{N}\cdot\text{m}$$

And Fig. b ($0 \leq x < 0.5\text{ m}$)

$$\Sigma M_x = 0; \quad T(x) + 500\,(0.5-x) - 1000\,(0.5) = 0 \quad T_{Ac} = \{500x + 250\}\,\text{N}\cdot\text{m}$$

Maximum Shear Stress: The maximum torque at $x = 0.5$ m, where $T_{\max} = 1000 - 1000(0.5) = 500\,\text{N}\cdot\text{m}$. Applying the torsion formula,

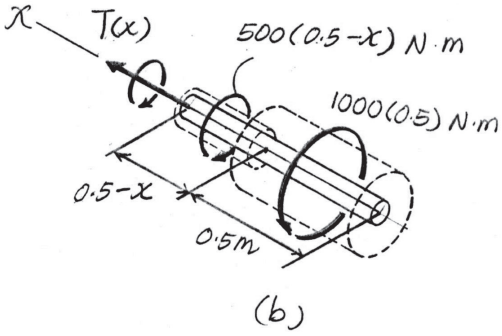
$$\tau_{\max}^{\text{abs}} = \frac{T_{\max}C}{J} = \frac{500\,(0.025)}{\frac{\pi}{2}(0.025^4)} = 20.37\,(10^6)\,P_a = 20.4\,\text{MPa} \qquad \textbf{Ans.}$$



Angle of Twist: For A992 steel $G = 75$ GPa. For region $0 \leq x < 0.5$ m (between B and C).

$$\phi(x) = \int \frac{T(x)dx}{JG} = \frac{1}{\left[\frac{\pi}{2}(0.025^4)\right]\left[75\,(10^9)\right]}\int_o^x (500\,x + 250)\,dx$$

$$= \left\{0.005432\,(x^2 + x)\right\}\,\text{rad} \qquad \textbf{Ans.}$$

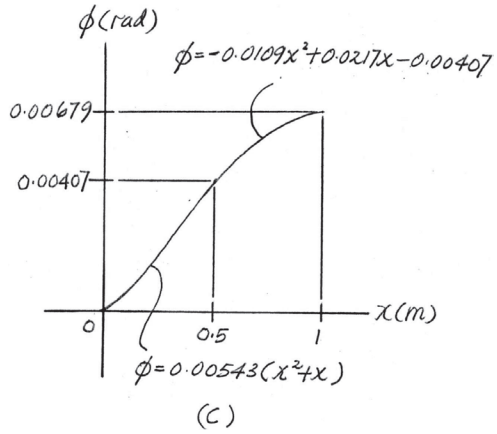


At $x = 0.5$ m, $\phi = \phi_C = 0.004074$ rad. For region $0.5\text{ m} < x \leq 1\text{ m}$

$$\phi\,(x) = \phi_C + \int \frac{T(x)\,dx}{JG}$$

$$= 0.004074 + \frac{1}{\left[\frac{\pi}{2}(0.025^4)\left[75\,(10^9)\right]\right]}\int_{0.5\text{ m}}^x (1000 - 1000x)\,dx$$

$$= \{-0.01086x^2 + 0.02173\,x - 0.004074\}\,\text{rad} \qquad \textbf{Ans.}$$



The maximum ϕ occurs at where $\dfrac{d\phi}{dx} = 0$.

$$\frac{d\phi}{dx} = -0.02173x + 0.02173 = 0$$

$$x = 1\text{ m}$$

Thus, $\phi_{\max} = \phi_B = -0.01086\,(1^2) + 0.02173(1) - 0.004074 = 0.006791$ rad
The plot of ϕ vs x is as shown in Fig. c.

Ans:
$\tau_{\max}^{\text{abs}} = 20.4$ MPa
For $0 \leq x < 0.5$ m,
$\phi(x) = \left\{0.005432\,(x^2 + x)\right\}$ rad
For $0.5\text{ m} < x \leq 1$ m,
$\phi(x) = \{-0.01086x^2 + 0.02173\,x - 0.004074\}$ rad

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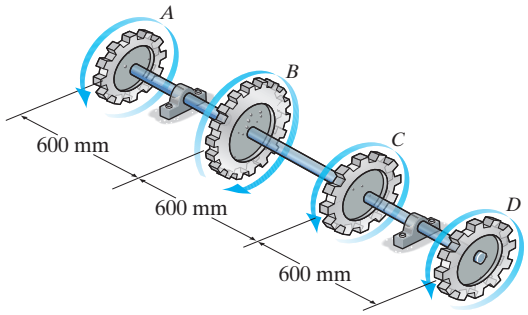
5–59.

The shaft is made of A992 steel with the allowable shear stress of $\tau_{allow} = 75$ MPa. If gear B supplies 15 kW of power, while gears A, C and D withdraw 6 kW, 4 kW and 5 kW, respectively, determine the required minimum diameter d of the shaft to the nearest millimeter. Also, find the corresponding angle of twist of gear A relative to gear D. The shaft is rotating at 600 rpm.



SOLUTION

Internal Loading: The angular velocity of the shaft is

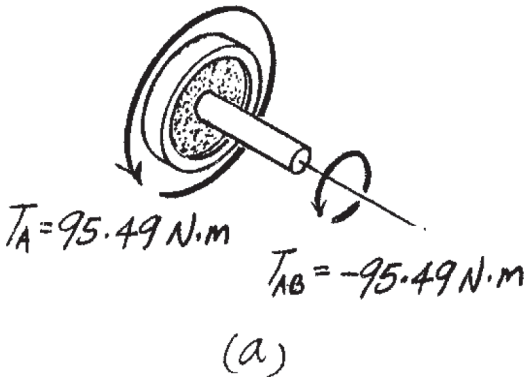
$$\omega = \left(600\,\frac{\text{rev}}{\text{min}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right)\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right) = 20\pi\,\text{rad/s}$$

Thus, the torque exerted on gears A, C, and D are

$$T_A = \frac{P_A}{\omega} = \frac{6(10^3)}{20\pi} = 95.49\,\text{N}\cdot\text{m}$$

$$T_C = \frac{P_C}{\omega} = \frac{4(10^3)}{20\pi} = 63.66\,\text{N}\cdot\text{m}$$

$$T_D = \frac{P_D}{\omega} = \frac{5(10^3)}{20\pi} = 79.58\,\text{N}\cdot\text{m}$$



(a)

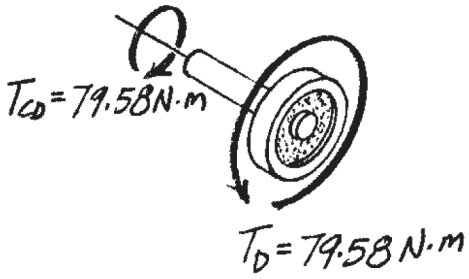
The internal torque developed in segments AB, CD, and BC of the shaft are shown in Figs. a, b, and c, respectively.

Allowable Shear Stress: Segment BC of the shaft is critical since it is subjected to a greater internal torque.

$$\tau_{\text{allow}} = \frac{T_{BC}\,c}{J}; \qquad 75(10^6) = \frac{143.24\left(\frac{d}{2}\right)}{\frac{\pi}{2}\left(\frac{d}{2}\right)^4}$$

$$d = 0.02135\text{ m} = 21.35\text{ mm}$$

Use $d = 22$ mm **Ans.**



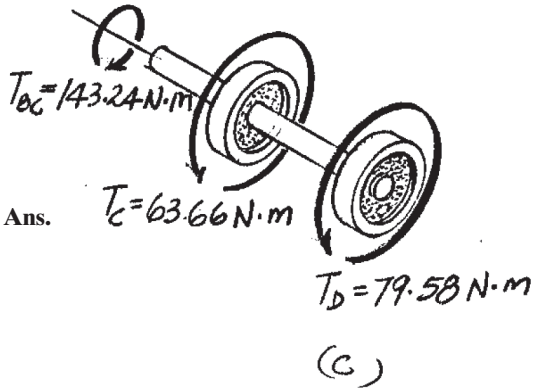
(b)

Angle of Twist: The polar moment of inertia of the shaft is $J = \frac{\pi}{2}(0.011^4) = 7.3205(10^{-9})\pi\text{ m}^4$. We have

$$\phi_{A/D} = \Sigma\frac{T_i L_i}{J_i G_i} = \frac{T_{AB}L_{AB}}{JG_{st}} + \frac{T_{BC}L_{BC}}{JG_{st}} + \frac{T_{CD}L_{CD}}{JG_{st}}$$

$$\phi_{A/D} = \frac{0.6}{7.3205(10^{-9})\pi(75)(10^9)}(-95.49 + 143.24 + 79.58)$$

$$= 0.04429\text{ rad} = 2.54°$$ **Ans.**



(c)

Ans:
Use $d = 22$ mm, $\phi_{A/D} = 2.54°$

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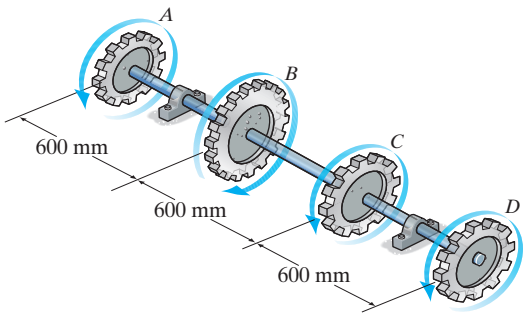
***5–60.**

Gear B supplies 15 kW of power, while gears A, C and D withdraw 6 kW, 4 kW and 5 kW, respectively. If the shaft is made of steel with the allowable shear stress of $\tau_{\text{allow}} = 75$ MPa, and the relative angle of twist between any two gears cannot exceed 0.05 rad, determine the required minimum diameter d of the shaft to the nearest millimeter. The shaft is rotating at 600 rpm.



SOLUTION

Internal Loading: The angular velocity of the shaft is

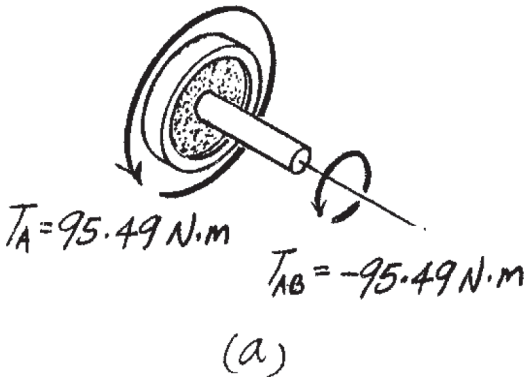
$$\omega = \left(600\,\frac{\text{rev}}{\text{min}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right)\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right) = 20\pi\text{ rad/s}$$

Thus, the torque exerted on gears A, C, and D are

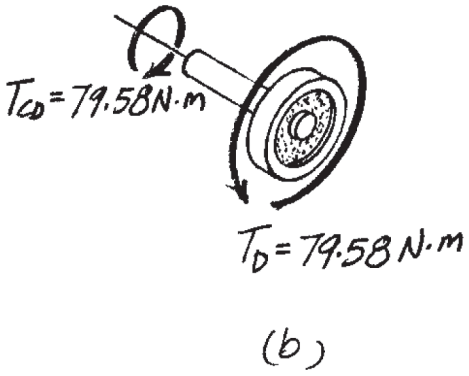
$$T_A = \frac{P_A}{\omega} = \frac{6(10^3)}{20\pi} = 95.49\,\text{N}\cdot\text{m}$$

$$T_C = \frac{P_C}{\omega} = \frac{4(10^3)}{20\pi} = 63.66\,\text{N}\cdot\text{m}$$

$$T_D = \frac{P_D}{\omega} = \frac{5(10^3)}{20\pi} = 79.58\,\text{N}\cdot\text{m}$$



The internal torque developed in segments AB, CD, and BC of the shaft are shown in Figs. a, b, and c, respectively.



Allowable Shear Stress: Segment BC of the shaft is critical since it is subjected to a greater internal torque.

$$\tau_{\text{allow}} = \frac{T_{BC}\,c}{J}; \qquad 75(10^6) = \frac{143.24\left(\frac{d}{2}\right)}{\frac{\pi}{2}\left(\frac{d}{2}\right)^4}$$

$$d = 0.02135\text{ m} = 21.35\text{ mm}$$

Angle of Twist: By observation, the relative angle of twist of gear D with respect to gear B is the greatest.

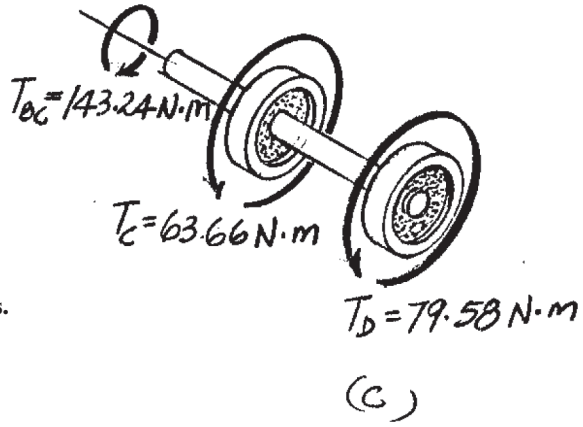
Thus, the requirement is $\phi_{D/B} = 0.05$ rad.

$$\phi_{D/B} = \Sigma\frac{T_i L_i}{J_i G_i} = \frac{T_{BC}L_{BC}}{JG_{st}} + \frac{T_{CD}L_{CD}}{JG_{st}} = 0.05$$

$$\frac{0.6}{\frac{\pi}{2}(\frac{d}{2})^4(75)(10^9)}(143.24 + 79.58) = 0.05$$

$$d = 0.02455\text{ m} = 24.55\text{ mm} = 25\text{ mm (controls!)}$$

Use $d = 25$ mm **Ans.**



Ans:
Use $d = 25$ mm

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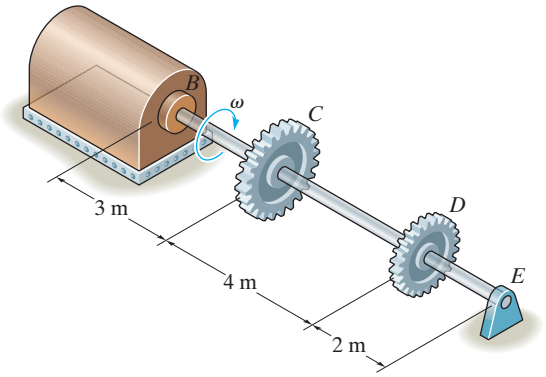
5–61.

The turbine develops 150 kW of power, which is transmitted to the gears such that C receives 70% and D receives 30%. If the rotation of the 100-mm-diameter A-36 steel shaft is $\omega = 800$ rev/min., determine the absolute maximum shear stress in the shaft and the angle of twist of end E of the shaft relative to B. The journal bearing at E allows the shaft to turn freely about its axis.



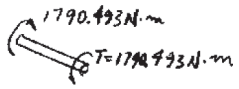
SOLUTION

$$P = T\omega; \qquad 150(10^3)\ \text{W} = T\left(800\frac{\text{rev}}{\text{min}}\right)\left(\frac{1\ \text{min}}{60\ \text{sec}}\right)\left(\frac{2\pi\ \text{rad}}{1\ \text{rev}}\right)$$

$$T = 1790.493\ \text{N}\cdot\text{m}$$

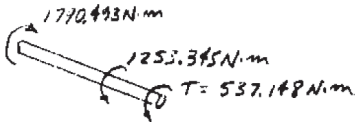
$$T_C = 1790.493(0.7) = 1253.345\ \text{N}\cdot\text{m}$$

$$T_D = 1790.493(0.3) = 537.148\ \text{N}\cdot\text{m}$$



Maximum torque is in region BC.

$$\tau_{\max} = \frac{T_C}{J} = \frac{1790.493(0.05)}{\frac{\pi}{2}(0.05)^4} = 9.12\ \text{MPa}$$ **Ans.**



$$\phi_{E/B} = \Sigma\left(\frac{TL}{JG}\right) = \frac{1}{JG}[1790.493(3) + 537.148(4) + 0]$$

$$= \frac{7520.171}{\frac{\pi}{2}(0.05)^4(75)(10^9)} = 0.0102\ \text{rad} = 0.585^\circ$$ **Ans.**

Ans:
$\tau_{\max} = 9.12$ MPa, $\phi_{E/B} = 0.585^\circ$

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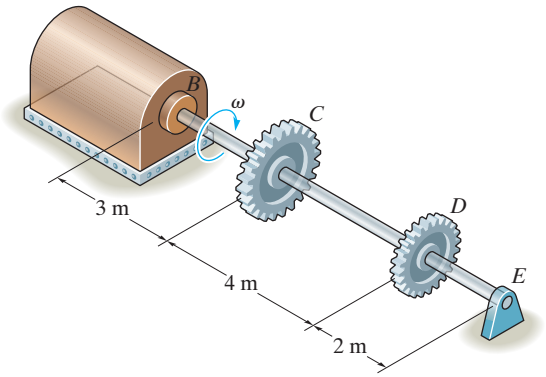
5–62.

The turbine develops 150 kW of power, which is transmitted to the gears such that both C and D receive an equal amount. If the rotation of the 100-mm-diameter A-36 steel shaft is $\omega = 500\ \text{rev/min}$., determine the absolute maximum shear stress in the shaft and the rotation of end B of the shaft relative to E. The journal bearing at E allows the shaft to turn freely about its axis.



SOLUTION

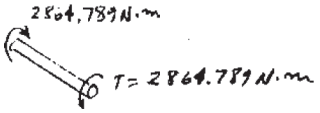
$$P = T\omega; \qquad 150(10^3)\ \text{W} = T\left(500\frac{\text{rev}}{\text{min}}\right)\left(\frac{1\ \text{min}}{60\ \text{sec}}\right)\left(\frac{2\pi\ \text{rad}}{1\ \text{rev}}\right)$$

$$T = 2864.789\ \text{N}\cdot\text{m}$$

$$T_C = T_D = \frac{T}{2} = 1432.394\ \text{N}\cdot\text{m}$$

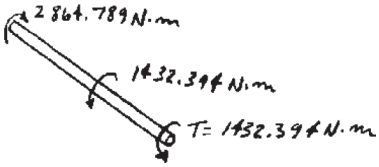
Maximum torque is in region BC.

$$\tau_{\max} = \frac{T_C}{J} = \frac{2864.789(0.05)}{\frac{\pi}{2}(0.05)^4} = 14.6\ \text{MPa}$$ **Ans.**



$$\phi_{B/E} = \Sigma\left(\frac{TL}{JG}\right) = \frac{1}{JG}[2864.789(3) + 1432.394(4) + 0]$$

$$= \frac{14323.945}{\frac{\pi}{2}(0.05)^4(75)(10^9)} = 0.0195\ \text{rad} = 1.11^\circ$$ **Ans.**



Ans:
$\tau_{\max} = 14.6\ \text{MPa}$, $\phi_{B/E} = 1.11^\circ$

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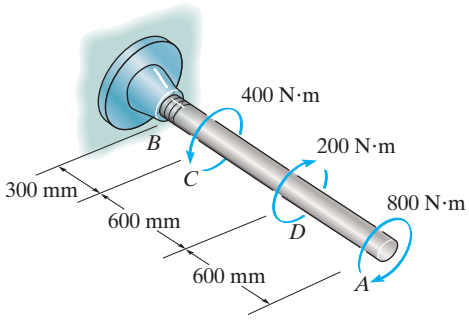
5–63.

The 50-mm-diameter A992 steel shaft is subjected to the torques shown. Determine the angle of twist of the end A.



SOLUTION

Internal Torque: The torque diagram shown in Fig. *a* can be plotted. From this diagram, $T_{AD} = -800\ \text{N}\cdot\text{m}$, $T_{DC} = -1000\ \text{N}\cdot\text{m}$ and $T_{CB} = -600\ \text{N}\cdot\text{m}$.

Angle of Twist:

$$\phi_{B/A} = \Sigma \frac{TL}{JG}$$

$$= \frac{1}{JG}(T_{AD}\,L_{AD} + T_{DC}\,L_{DC} + T_{CB}\,L_{CB})$$

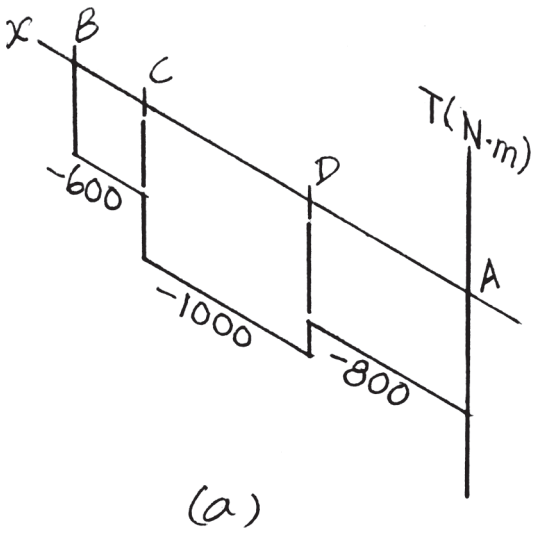
$$= \frac{1}{JG}[(-800)(0.6) + (-1000)(0.6) + (-600)(0.3)]$$

$$= -\frac{1260\ \text{N}\cdot\text{m}^2}{JG}$$

For A992 steel, $G = 75$ GPa. Then

$$\phi_A = \frac{1260}{\frac{\pi}{2}(0.025^4)[75(10^9)]}$$

$$= (-0.02738\ \text{rad})\left(\frac{180^\circ}{\pi\ \text{rad}}\right) = -1.569^\circ = 1.57^\circ \circlearrowleft$$ **Ans.**



Ans:
$\phi_A = 1.57^\circ \circlearrowleft$

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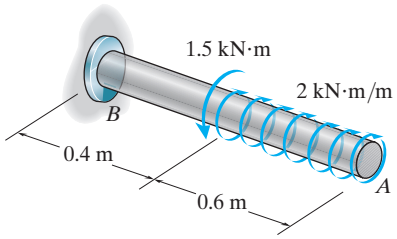
***5–64.**

The 60-mm-diameter solid shaft is made of 2014-T6 aluminum and is subjected to the distributed and concentrated torsional loadings shown. Determine the angle of twist at the free end A of the shaft.



SOLUTION

Internal Torque: Referring to the FBD of the right segments of the shaft shown in Fig. *a* and *b*

$\Sigma M_x = 0; \quad T_{AC} + 2000x = 0 \quad T_{AC} = (-2000x)\ \text{N}\cdot\text{m}$

And

$\Sigma M_x = 0; \quad T_{CB} + 2000\,(0.6) - 1500 = 0 \quad T_{CB} = 300\ \text{N}\cdot\text{m}$

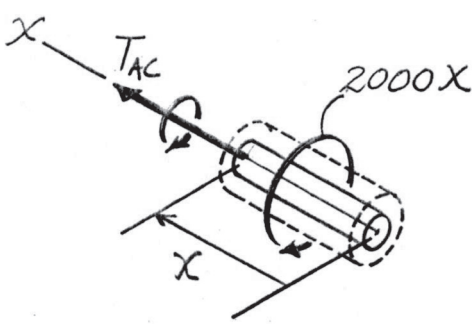
Angle of Twist: For 2014 - T6 Aluminum, $G = 27$ GPa.

$$\phi_A = \Sigma \frac{TL}{JG} = \frac{1}{JG}\left[\int_0^x T_{AC}\,dx + T_{CB}\,L_{CB}\right]$$

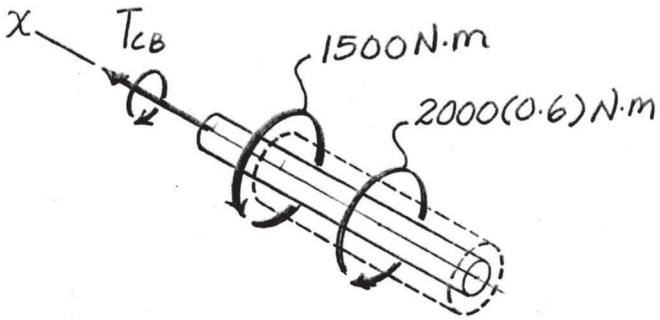
$$= \frac{1}{\left[\frac{\pi}{2}(0.03)^4\right]\left[27(10^9)\right]}\left[\int_0^{0.6\text{ m}} (-2000\,x)dx + 300\,(0.4)\right]$$

$$= (-0.006986\text{ rad})\left(\frac{180^\circ}{\pi\text{ rad}}\right)$$

$$= -0.400^\circ = 0.400^\circ \circlearrowright$$ **Ans.**



(a)



(b)

Ans:
$\phi_A = 0.400^\circ \circlearrowright$

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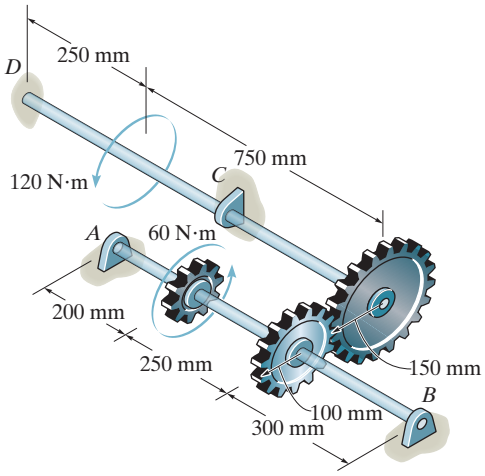
5–65. The two shafts are made of A-36 steel. Each has a diameter of 25 mm, and they are supported by bearings at *A*, *B*, and *C*, which allow free rotation. If the support at *D* is fixed, determine the angle of twist of end *A* when the torques are applied to the assembly as shown.



SOLUTION

Internal Torque: As shown on FBD.

Angle of Twist:

$$\phi_E = \sum \frac{TL}{JG}$$

$$= \frac{1}{[\frac{\pi}{2}(0.0125^4)][75(10^9)]}\{-90(0.75) + 30(0.25)\}$$

$$= -0.020861 = 0.020861 \text{ rad}$$

$$\phi_F = \left(\frac{150}{100}\right)\phi_E = \left(\frac{150}{100}\right)(0.020861) = 0.031291 \text{ rad}$$

$$\phi_{A/F} = \frac{T_{GF}\, L_{GF}}{JG}$$

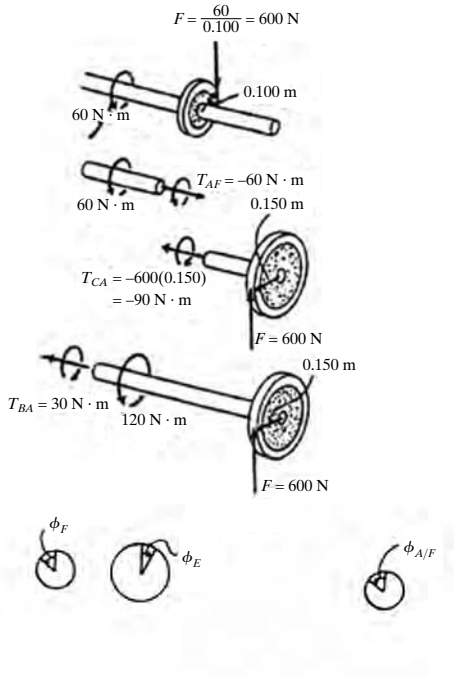
$$= \frac{-60(0.25)}{[\frac{\pi}{2}(0.0125^4)][75(10^9)]}$$

$$= -0.0052152 \text{ rad} = 0.0052152 \text{ rad}$$

$$\phi_A = \phi_F + \phi_{A/F}$$

$$= 0.031291 + 0.0052152$$

$$= 0.036506 \text{ rad} = 2.09°$$



Ans.

Ans.
$\phi_A = 2.09°$

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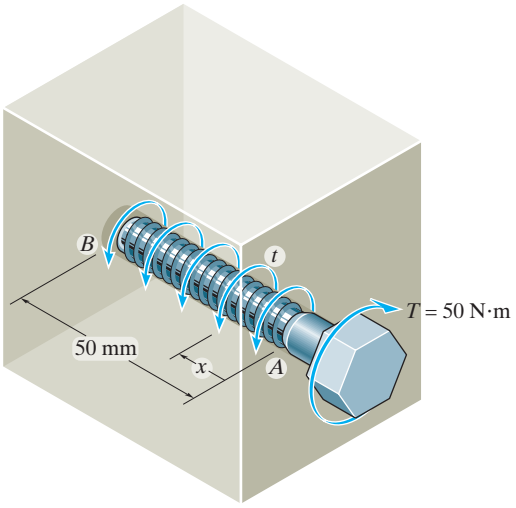
5–66.

The A-36 steel bolt is tightened within a hole so that the reactive torque on the shank AB can be expressed by the equation $t = (kx^2)\ \mathrm{N \cdot m/m}$, where x is in meters. If a torque of $T = 50\ \mathrm{N \cdot m}$ is applied to the bolt head, determine the constant k and the amount of twist in the 50-mm length of the shank. Assume the shank has a constant radius of 4 mm.



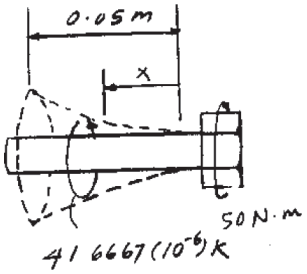
SOLUTION

$dT = t\,dx$

$$T = \int_0^{0.05\ \mathrm{m}} kx^2 dx = k\frac{x^3}{3}\Big|_0^{0.05} = 41.667(10^{-6})k$$

$$50 - 41.6667(10^{-6})\,k = 0$$

$$k = 1.20(10^6)\ \mathrm{N/m^2} \qquad \textbf{Ans.}$$



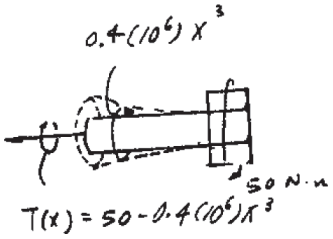
In the general position, $T = \int_0^x 1.20(10^6)x^2 dx = 0.4(10^6)x^3$

$$\phi = \int \frac{T(x)dx}{JG} = \frac{1}{JG}\int_0^{0.05\ \mathrm{m}} [50 - 0.4(10^6)x^3]dx$$

$$= \frac{1}{JG}\left[50x - \frac{0.4(10^6)x^4}{4}\right]_0^{0.05\ \mathrm{m}}$$

$$= \frac{1.875}{JG} = \frac{1.875}{\frac{\pi}{2}(0.004^4)(75)(10^9)}$$

$$= 0.06217\ \mathrm{rad} = 3.56^\circ \qquad \textbf{Ans.}$$



Ans:
$k = 1.20(10^6)\ \mathrm{N/m^2}$, $\phi = 3.56^\circ$

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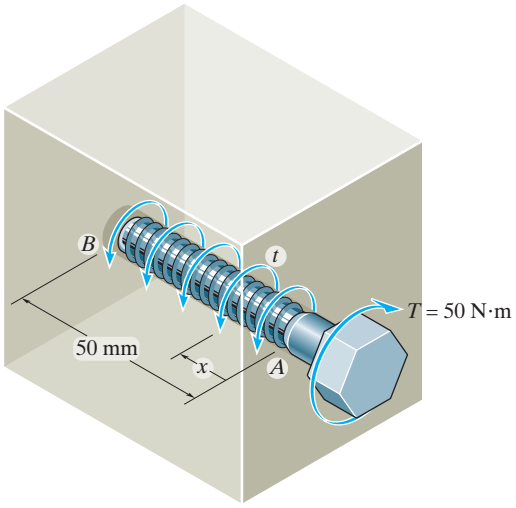
5–67.

The A-36 steel bolt is tightened within a hole so that the reactive torque on the shank AB can be expressed by the equation $t = (kx^{2/3})\,\mathrm{N\cdot m/m}$, where x is in meters. If a torque of $T = 50\,\mathrm{N\cdot m}$ is applied to the bolt head, determine the constant k and the amount of twist in the 50-mm length of the shank. Assume the shank has a constant radius of 4 mm.



SOLUTION

$$dT = t\,dx$$

$$T = \int_0^{0.05} kx^{\frac{2}{3}}\,dx = k\frac{3}{5}x^{\frac{5}{3}}\Big|_0^{0.05} = (4.0716)(10^{-3})\,k$$

$$50 - 4.0716\,(10^{-3})\,k = 0$$

$$k = 12.3(10^3)\,\mathrm{N/m}^{(\frac{2}{3})} \qquad \textbf{Ans.}$$

In the general position,

$$T = \int_0^x 12.28(10^3)x^{\frac{2}{3}}\,dx = 7.368(10^3)\,x^{\frac{5}{3}}$$

Angle of twist:

$$\phi = \int \frac{T(x)\,dx}{JG} = \frac{1}{JG}\int_0^{0.05\ \mathrm{m}} [50 - 7.3681(10^3)x^{\frac{8}{3}}]dx$$

$$= \frac{1}{JG}\left[50x - 7.3681(10^3)\left(\frac{3}{8}\right)x^{\frac{8}{3}}\right]\Big|_0^{0.05\ \mathrm{m}}$$

$$= \frac{1.5625}{\frac{\pi}{2}(0.004^4)(75)(10^9)} = 0.0518\ \mathrm{rad} = 2.97° \qquad \textbf{Ans.}$$

Ans:
$k = 12.3(10^3)\,\mathrm{N/m}^{2/3}$, $\phi = 2.97°$

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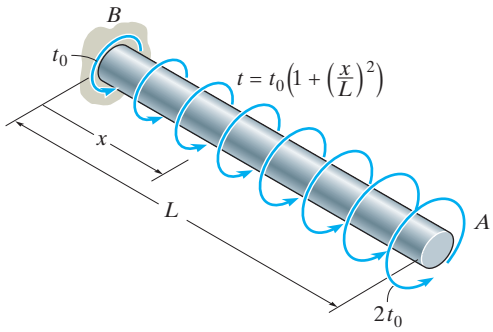
***5–68.**

The shaft of radius c is subjected to a distributed torque t, measured as torque/length of shaft. Determine the angle of twist at end A. The shear modulus is G.

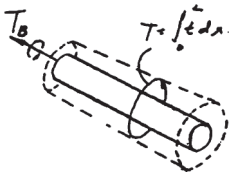


SOLUTION

$$T_B - \int t\,dx = 0$$

$$T_B = \int t\,dx = t_0 \int \left(1 + \frac{x^2}{L^2}\right) dx$$

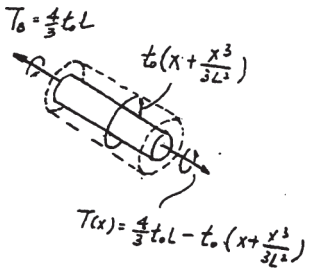
$$= t_0\left[x + \frac{x^3}{3L^2}\right]\Big|_0^L = t_0\left(L + \frac{L}{3}\right) = \frac{4}{3}t_0 L$$



$$\phi = \int \frac{T(x)\,dx}{JG}$$

$$= \frac{1}{JG}\int_0^L \left[\frac{4}{3}t_0 L - t_0\left(x + \frac{x^3}{3L^2}\right)\right] dx$$

$$= \frac{t_0}{JG}\left[\frac{4}{3}Lx - \left(\frac{x^2}{2} + \frac{x^4}{12L^2}\right)\right]\Big|_0^L = \frac{7\,t_0\,L^2}{12\,JG}$$



However $J = \frac{\pi}{2}c^4$,

$$\phi = \frac{7\,t_0\,L^2}{6\,\pi\,c^4 G}$$ **Ans.**

Ans:

$$\phi = \frac{7\,t_0\,L^2}{6\,\pi\,c^4 G}$$

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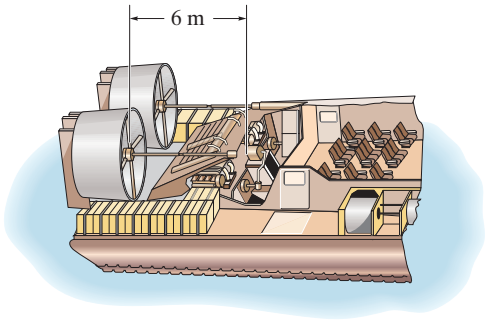
5–69.

The tubular drive shaft for the propeller of a hovercraft is 6 m long. If the motor delivers 4 MW of power to the shaft when the propellers rotate at 25 rad/s, determine the required inner diameter of the shaft if the outer diameter is 250 mm. What is the angle of twist of the shaft when it is operating? Take $\tau_{allow} = 90$ MPa and $G = 75$ GPa.



SOLUTION

Internal Torque:

$$P = 4\left(10^6\right)\text{ W} = 4\left(10^6\right)\text{ N}\cdot\text{m/s}$$

$$T = \frac{P}{\omega} = \frac{4\left(10^6\right)}{25} = 160\left(10^3\right)\text{ N}\cdot\text{m}$$

Maximum Shear Stress: Applying torsion formula.

$$\tau_{max} = \tau_{allow} = \frac{Tc}{J}$$

$$90\left(10^6\right) = \frac{160\left(10^3\right)\left(0.125\right)}{\frac{\pi}{2}\left[0.125^4 - \left(\frac{d_t}{2}\right)^4\right]}$$

$$d_t = 0.2013\text{ m} = 201\text{ mm} \qquad \textbf{Ans.}$$

Angle of Twist:

$$\phi = \frac{TL}{JG} = \frac{160\left(10^3\right)\left(6\right)}{\frac{\pi}{2}\left(0.125^4 - 0.10065^4\right)75\left(10^9\right)}$$

$$= 0.0576\text{ rad} = 3.30° \qquad \textbf{Ans.}$$

Ans:
$d_t = 201$ mm,
$\phi = 3.30°$

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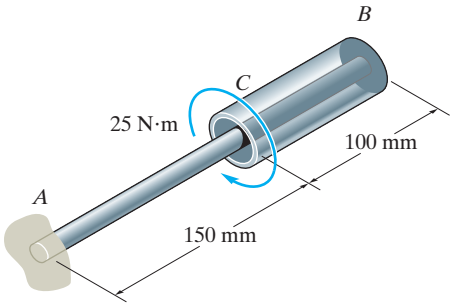
5–70. The A-36 steel assembly consists of a tube having an outer radius of 25 mm and a wall thickness of 3 mm. Using a rigid plate at B, it is connected to the solid 25-mm-diameter shaft AB. Determine the rotation of the tube's end C if a torque of 25 N · m is applied to the tube at this end. The end A of the shaft is fixed supported.



SOLUTION

$$\phi_B = \frac{T_{AB}L}{JG} = \frac{25(0.25)}{[\frac{\pi}{2}(0.0125^4)][75(10^9)]} = 0.0021730\ \pi\text{ad}$$

$$\phi_{C/B} = \frac{T_{CB}L}{JG} = \frac{-25(0.1)}{[\frac{\pi}{2}(0.025^4 - 0.022^4)][75(10^9)]}$$

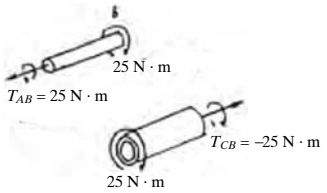
$$= -0.0001357\ \pi\text{ad} = 0.0001357\ \pi\text{ad}\ \curvearrowleft$$

$$(+\curvearrowleft)\quad \phi_C = \phi_B + \phi_{C/B}$$

$$= 0.0021730 + 0.0001357$$

$$= 0.0023087\ \text{rad} = 0.132°$$

Ans.



Ans.
$\phi_C = 0.113°$

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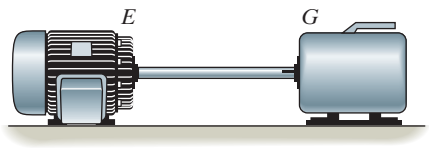
5–71.

The A-36 hollow steel shaft is 2 m long and has an outer diameter of 40 mm. When it is rotating at 80 rad/s, it transmits 32 kW of power from the engine E to the generator G. Determine the smallest thickness of the shaft if the allowable shear stress is $\tau_{\text{allow}} = 140$ MPa and the shaft is restricted not to twist more than 0.05 rad.



SOLUTION

$$P = T\omega$$

$$32(10^3) = T(80)$$

$$T = 400\ \text{N}\cdot\text{m}$$

Shear stress failure

$$\tau = \frac{Tc}{J}$$

$$\tau_{\text{allow}} = 140\,(10^6) = \frac{400\,(0.02)}{\frac{\pi}{2}(0.02^4 - r_i{}^4)}$$

$$r_i = 0.01875\ \text{m}$$

Angle of twist limitation:

$$\phi = \frac{TL}{JG}$$

$$0.05 = \frac{400(2)}{\frac{\pi}{2}(0.02^4 - r_i{}^4)(75)(10^9)}$$

$$r_i = 0.01247\ \text{m} \qquad \text{(controls)}$$

$$t = r_o - r_i = 0.02 - 0.01247$$

$$= 0.00753\ \text{m}$$

$$= 7.53\ \text{mm} \qquad \textbf{Ans.}$$

Ans:
$t = 7.53$ mm

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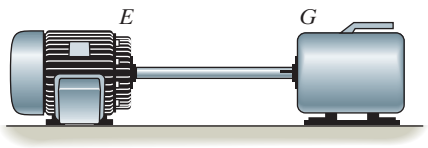
***5–72.**

The A-36 solid steel shaft is 3 m long and has a diameter of 50 mm. It is required to transmit 35 kW of power from the engine E to the generator G. Determine the smallest angular velocity of the shaft if it is restricted not to twist more than 1°.



SOLUTION

$$\phi = \frac{TL}{JG}$$

$$\frac{1°(\pi)}{180°} = \frac{T(3)}{\frac{\pi}{2}(0.025^4)(75)(10^9)}$$

$$T = 267.73\ \text{N}\cdot\text{m}$$

$$P = T\omega$$

$$35(10^3) = 267.73(\omega)$$

$$\omega = 131\ \text{rad/s}$$ **Ans.**

Ans:
$\omega = 131$ rad/s

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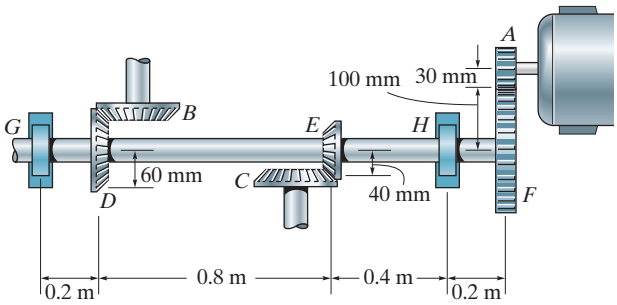
5–73.

The motor produces a torque of $T = 20\,\text{N}\cdot\text{m}$ on gear A. If gear C is suddenly locked so it does not turn, yet B can freely turn, determine the angle of twist of F with respect to E and F with respect to D of the L2-steel shaft, which has an inner diameter of 30 mm and an outer diameter of 50 mm. Also, calculate the absolute maximum shear stress in the shaft. The shaft is supported on journal bearings at G at H.

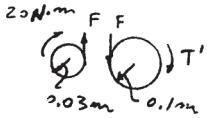


SOLUTION

$F(0.03) = 20$

$F = 666.67\,\text{N}$

$T' = (666.67)(0.1) = 66.67\,\text{N}\cdot\text{m}$



Since shaft is held fixed at C, the torque is only in region EF of the shaft.

$$\phi_{F/E} = \frac{TL}{JG} = \frac{66.67(0.6)}{\frac{\pi}{2}\left[(0.025)^4 - (0.015)^4\right]75(10^9)} = 0.999(10)^{-3}\,\text{rad}$$ **Ans.**

Since the torque in region ED is zero,

$\phi_{F/D} = 0.999(10)^{-3}\,\text{rad}$ **Ans.**

$$\tau_{\max} = \frac{Tc}{J} = \frac{66.67(0.025)}{\frac{\pi}{2}\left((0.025)^4 - (0.015)^4\right)}$$

$= 3.12\,\text{MPa}$ **Ans.**

Ans:
$\phi_{F/E} = 0.999(10)^{-3}\,\text{rad}$,
$\phi_{F/D} = 0.999(10)^{-3}\,\text{rad}$,
$\tau_{\max} = 3.12\,\text{MPa}$

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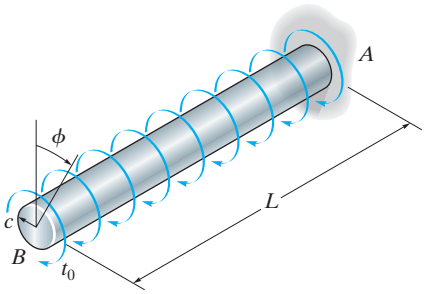
5–74.

The shaft has a radius c and is subjected to a torque per unit length of t_0, which is distributed uniformly over the shaft's entire length L. If it is fixed at its far end A, determine the angle of twist ϕ of end B. The shear modulus is G.

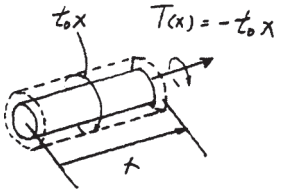


SOLUTION

$$\phi = \int \frac{T(x)\,dx}{JG} = \frac{-t_0}{JG}\int_0^L x\,dx$$

$$= \frac{-t_0}{JG}\left[\frac{x^2}{2}\right]\Big|_0^L = \frac{-t_0}{JG}\,\frac{L^2}{2}$$

$$= \frac{-t_0 L^2}{2JG}$$



However, $J = \frac{\pi}{2}c^4$

$$\phi = \frac{-t_0 L^2}{\pi c^4 G} = \frac{t_0 L^2}{\pi c^4 G}$$

Ans.

Ans:

$$\phi = \frac{t_0 L^2}{\pi c^4 G}$$

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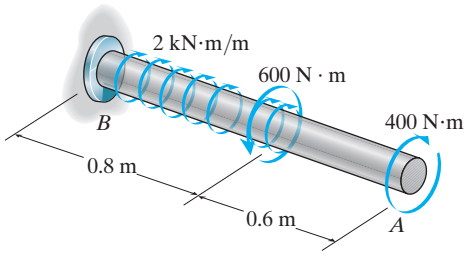
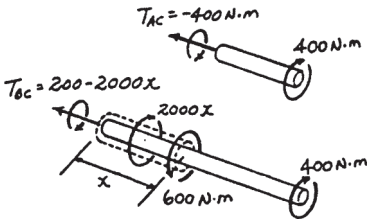
5–75.

The 60-mm-diameter solid shaft is made of A-36 steel and is subjected to the distributed and concentrated torsional loadings shown. Determine the angle of twist at the free end A of the shaft due to these loadings.



SOLUTION

Internal Torque: As shown on FBD.



Angle of Twist:

$$\phi_A = \Sigma \frac{TL}{JG}$$

$$= \frac{-400(0.6)}{\frac{\pi}{2}(0.03^4)75.0(10^9)} + \int_0^{0.8\,\text{m}} \frac{(200 - 2000x)dx}{\frac{\pi}{2}(0.03^4)75.0(10^9)}$$

$$= -0.007545 \text{ rad} = 0.432^\circ \text{ ↻}$$ **Ans.**

Ans:
$\phi_A = 0.432^\circ$ ↻

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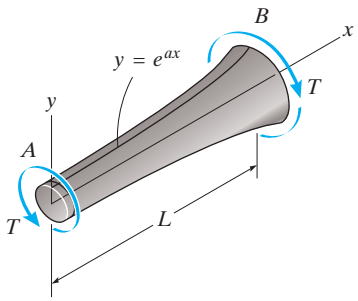
***5–76.**

The contour of the surface of the shaft is defined by the equation $y = e^{ax}$, where a is a constant. If the shaft is subjected to a torque T at its ends, determine the angle of twist of end A with respect to end B. The shear modulus is G.



SOLUTION

$$\phi = \int \frac{T\,dx}{J(x)G} \qquad \text{where, } J(x) = \frac{\pi}{2}(e^{ax})^4$$

$$= \frac{2T}{\pi G}\int_0^L \frac{dx}{e^{4ax}} = \frac{2T}{\pi G}\left(-\frac{1}{4\,a\,e^{4ax}}\right)\Big|_0^L$$

$$= \frac{2T}{\pi G}\left(-\frac{1}{4\,a\,e^{4aL}} + \frac{1}{4a}\right) = \frac{T}{2\,a\,\pi\,G}\left(\frac{e^{4aL} - 1}{e^{4aL}}\right)$$

$$= \frac{T}{2\,a\,\pi\,G}(1 - e^{-4aL}) \qquad \textbf{Ans.}$$

Ans:

$$\phi = \frac{T}{2a\pi G}(1 - e^{-4aL})$$

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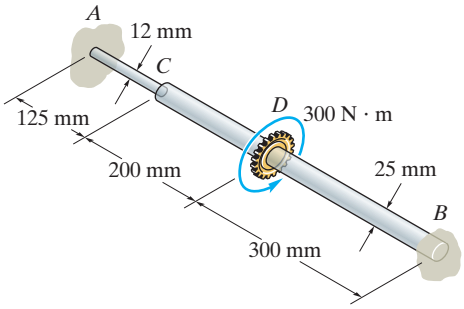
5–77. The steel shaft is made from two segments: AC has a diameter of 12 mm, and CB has a diameter of 25 mm. If it is fixed at its ends A and B and subjected to a torque of 300 N · m, determine the maximum shear stress in the shaft. $G_{st} = 75$ GPa.

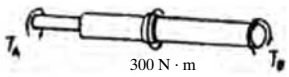


SOLUTION

Equilibrium:

$$T_A + T_B - 300 = 0 \qquad \textbf{(1)}$$

Compatibility condition:



$$\phi_{D/A} = \phi_{D/B}$$

$$\frac{T_A(0.125)}{[\frac{\pi}{2}(0.006^4)]G} + \frac{T_A(0.2)}{[\frac{\pi}{2}(0.0125^4)]G} = \frac{T_B(0.3)}{[\frac{\pi}{2}(0.0125^4)]G}$$

$$T_A = 0.11743T_B \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields

$$T_A = 31.53\ \text{N}\cdot\text{m} \qquad T_B = 268.47\ \text{N}\cdot\text{m}$$

$$(\tau_{AC})_{\max} = \frac{T_C}{J} = \frac{31.53(0.06)}{\frac{\pi}{2}(0.006^4)} = 92.92(10^6)\ \text{N/m}^2 = 92.9\ \text{MPa (Max)} \qquad \textbf{Ans.}$$

$$(\tau_{DB})_{\max} = \frac{T_C}{J} = \frac{268.47(0.0125)}{\frac{\pi}{2}(0.0125^4)} = 87.51(10^6)\ \text{N/m}^2 = 87.5\ \text{MPa}$$

Ans:
$(\tau_{AC})_{\max} = 92.9$ MPa

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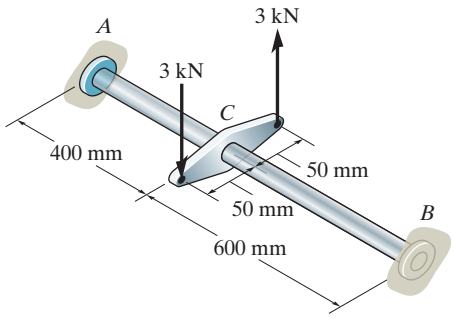
5–78.

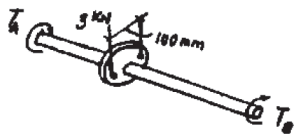
The steel shaft has a diameter of 40 mm and is fixed at its ends A and B. If it is subjected to the couple, determine the maximum shear stress in regions AC and CB of the shaft. $G_{st} = 75$ GPa.



SOLUTION

Equilibrium:

$$T_A + T_B - 3000(0.1) = 0 \qquad \textbf{(1)}$$



Compatibility condition:

$$\phi_{C/A} = \phi_{C/B}$$

$$\frac{T_A(400)}{JG} = \frac{T_B(600)}{JG}$$

$$T_A = 1.5\,T_B \qquad \textbf{(2)}$$

Solving Eqs (1) and (2) yields:

$$T_B = 120\ \text{N}\cdot\text{m}$$

$$T_A = 180\ \text{N}\cdot\text{m}$$

$$(\tau_{AC})_{\max} = \frac{Tc}{J} = \frac{180(0.02)}{\frac{\pi}{2}(0.02^4)} = 14.3\ \text{MPa} \qquad \textbf{Ans.}$$

$$(\tau_{CB})_{\max} = \frac{Tc}{J} = \frac{120(0.02)}{\frac{\pi}{2}(0.02^4)} = 9.55\ \text{MPa} \qquad \textbf{Ans.}$$

Ans:
$(\tau_{AC})_{\max} = 14.3$ MPa, $(\tau_{CB})_{\max} = 9.55$ MPa

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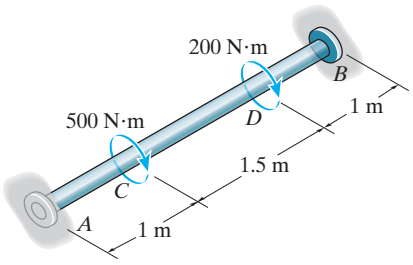
5–79.

The A992 steel shaft has a diameter of 60 mm and is fixed at its ends A and B. If it is subjected to the torques shown, determine the absolute maximum shear stress in the shaft.



SOLUTION

Referring to the FBD of the shaft shown in Fig. *a*,

$$\Sigma M_x = 0; \quad T_A + T_B - 500 - 200 = 0 \qquad \textbf{(1)}$$

Using the method of superposition, Fig. *b*

$$\phi_A = (\phi_A)_{T_A} - (\phi_A)_T$$

$$0 = \frac{T_A\,(3.5)}{JG} - \left[\frac{500\,(1.5)}{JG} + \frac{700\,(1)}{JG}\right]$$

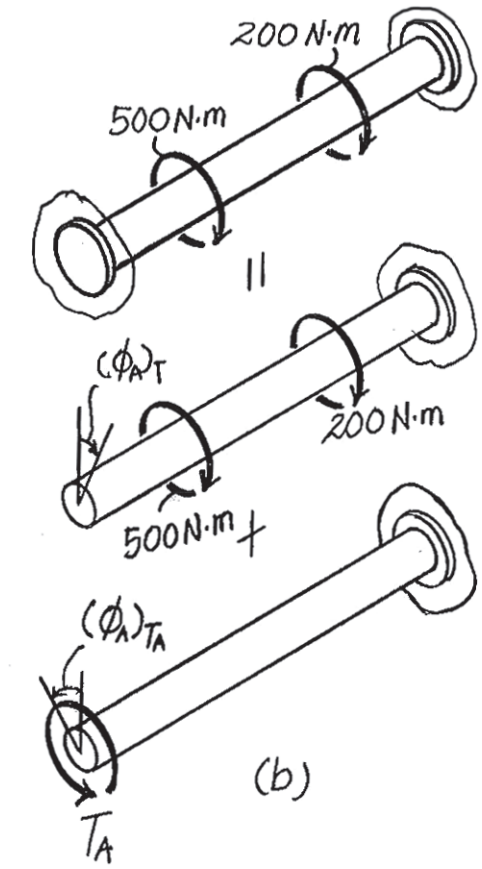
$$T_A = 414.29\ \text{N}\cdot\text{m}$$

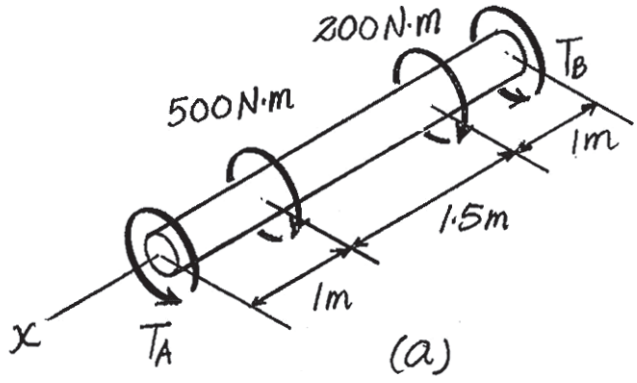
Substitute this result into Eq (1),

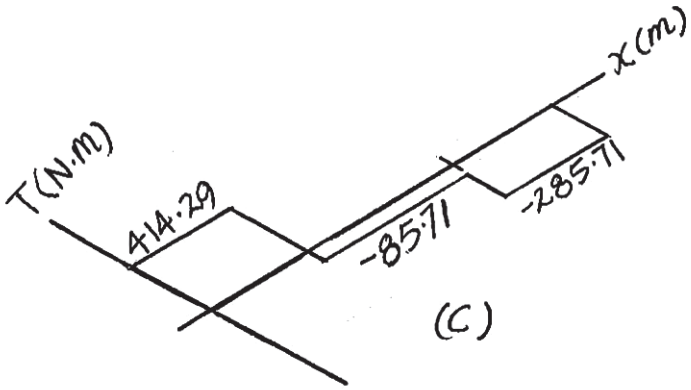
$$T_B = 285.71\ \text{N}\cdot\text{m}$$

Referring to the torque diagram shown in Fig. *c*, segment AC is subjected to maximum internal torque. Thus, the absolute maximum shear stress occurs here.

$$\tau_{\substack{\text{abs}\\\text{max}}} = \frac{T_{AC}\,c}{J} = \frac{414.29\,(0.03)}{\frac{\pi}{2}(0.03)^4} = 9.77\ \text{MPa} \qquad \textbf{Ans.}$$







Ans:

$\tau_{\substack{\text{abs}\\\text{max}}} = 9.77\ \text{MPa}$

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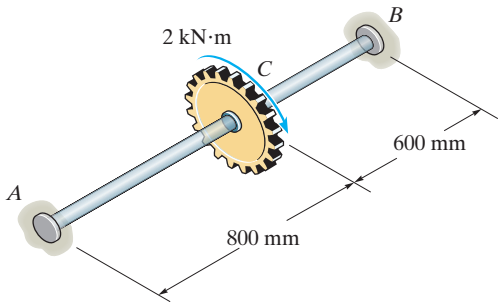
***5–80.**

The shaft is made of L2 tool steel, has a diameter of 40 mm, and is fixed at its ends A and B. If it is subjected to the torque, determine the maximum shear stress in regions AC and CB.



SOLUTION

Equilibrium: Referring to the *FBD* of the shaft, Fig. *a*,

$$\Sigma M_x = 0; \qquad T_A + T_B - 2000 = 0 \qquad \textbf{(1)}$$

Compatibility: It is required that

$$\phi_{CA} = \phi_{CB}$$

$$\frac{T_A L_{CA}}{JG} = \frac{T_B L_{CB}}{JG}$$

$$\frac{T_A (0.8)}{JG} = \frac{T_B (0.6)}{JG}$$

$$T_A = 0.75\, T_B \qquad \textbf{(2)}$$

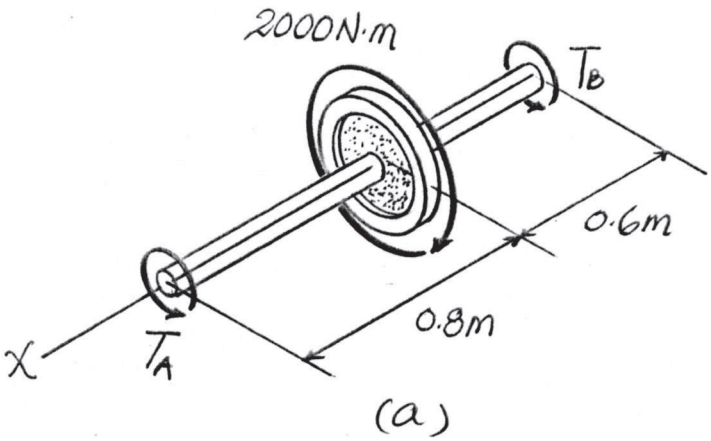
Solving Eqs (1) and (2)

$$T_B = 1142.86\ \text{N}\cdot\text{m} \qquad T_A = 857.14\ \text{N}\cdot\text{m}$$

Maximum Shear Stress: Applying the torsion formula,

$$(\tau_{\max})_{AC} = \frac{T_A c}{J} = \frac{857.14\,(0.02)}{\frac{\pi}{2}(0.02^4)} = 68.21(10^6)\ \text{Pa} = 68.2\ \text{MPa} \qquad \textbf{Ans.}$$

$$(\tau_{\max})_{BC} = \frac{T_B c}{J} = \frac{1142.86\,(0.02)}{\frac{\pi}{2}(0.02^4)} = 90.946(10^6)\ \text{Pa} = 90.9\ \text{MPa} \qquad \textbf{Ans.}$$



Ans:
$(\tau_{\max})_{AC} = 68.2$ MPa,
$(\tau_{\max})_{BC} = 90.9$ MPa

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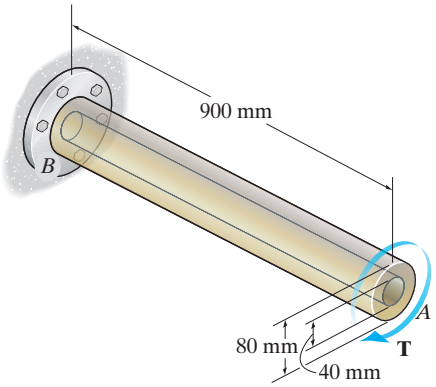
5–81.

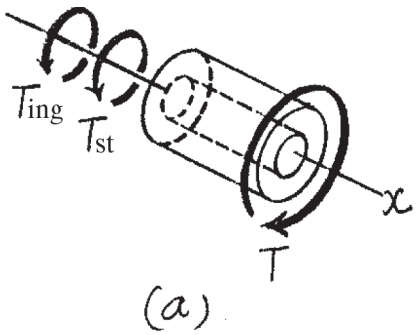
The Am1004-T61 magnesium tube is bonded to the A-36 steel rod. If the allowable shear stresses for the magnesium and steel are $(\tau_{allow})_{mg} = 45$ MPa and $(\tau_{allow})_{st} = 75$ MPa, respectively, determine the maximum allowable torque that can be applied at A. Also, find the corresponding angle of twist of end A.



SOLUTION

Equilibrium: Referring to the free-body diagram of the cut part of the assembly shown in Fig. *a*,

$$\Sigma M_x = 0; \quad T_{mg} + T_{st} - T = 0 \tag{1}$$



Compatibility Equation: Since the steel rod is bonded firmly to the magnesium tube, the angle of twist of the rod and the tube must be the same. Thus,

$$(\phi_{st})_A = (\phi_{mg})_A$$

$$\frac{T_{st}L}{\frac{\pi}{2}(0.02^4)(75)(10^9)} = \frac{T_{mg}L}{\frac{\pi}{2}(0.04^4 - 0.02^4)(18)(10^9)}$$

$$T_{st} = 0.2778T_{mg} \tag{2}$$

Solving Eqs. (1) and (2),

$$T_{mg} = 0.7826T \qquad T_{st} = 0.2174T$$

Allowable Shear Stress:

$$(\tau_{allow})_{mg} = \frac{T_{mg}c}{J}; \; 45(10^6) = \frac{0.7826T(0.04)}{\frac{\pi}{2}(0.04^4 - 0.02^4)}$$

$$T = 5419.25 \text{ N} \cdot \text{m}$$

$$(\tau_{allow})_{st} = \frac{T_{st}c}{J}; \quad 75(10^6) = \frac{0.2174T(0.02)}{\frac{\pi}{2}(0.02^4)}$$

$$T = 4335.40 \text{ N} \cdot \text{m} = 4.34 \text{ kN} \cdot \text{m (control!)} \quad \textbf{Ans.}$$

Angle of Twist: Using the result of T, $T_{st} = 942.48$ N · m. We have

$$\phi_A = \frac{T_{st}L}{J_{st}G_{st}} = \frac{942.48(0.9)}{\frac{\pi}{2}(0.02^4)(75)(10^9)} = 0.045 \text{ rad} = 2.58^\circ \quad \textbf{Ans.}$$

Ans:
$T = 4.34$ kN · m,
$\phi_A = 2.58°$

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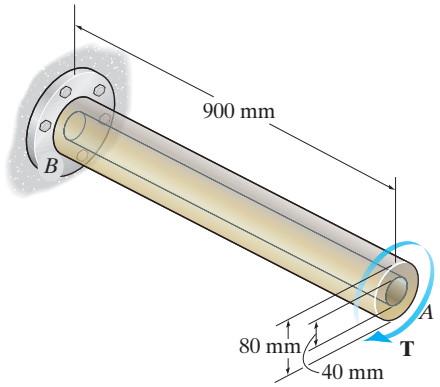
5–82.

The Am1004-T61 magnesium tube is bonded to the A-36 steel rod. If a torque of $T = 5$ kN · m is applied to end A, determine the maximum shear stress in each material. Sketch the shear stress distribution.



SOLUTION

Equilibrium: Referring to the free-body diagram of the cut part of the assembly shown in Fig. *a*,

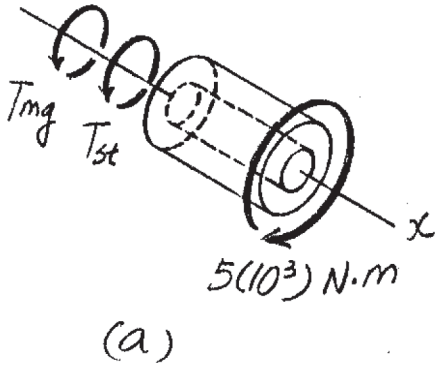
$$\Sigma M_x = 0; \quad T_{mg} + T_{st} - 5(10^3) = 0 \qquad \textbf{(1)}$$

Compatibility Equation: Since the steel rod is bonded firmly to the magnesium tube, the angle of twist of the rod and the tube must be the same. Thus,

$$(\phi_{st})_A = (\phi_{mg})_A$$

$$\frac{T_{st}L}{\frac{\pi}{2}(0.02^4)(75)(10^9)} = \frac{T_{mg}L}{\frac{\pi}{2}(0.04^4 - 0.02^4)(18)(10^9)}$$

$$T_{st} = 0.2778T_{mg} \qquad \textbf{(2)}$$



(a)

Solving Eqs. (1) and (2),

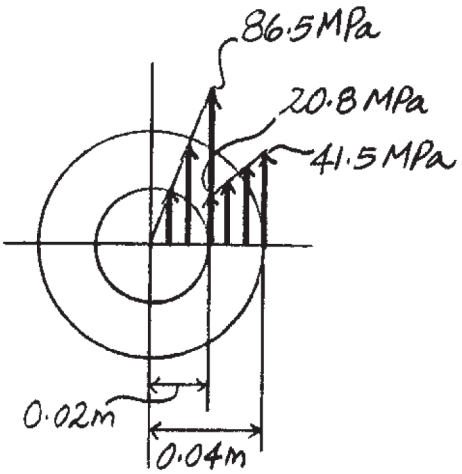
$$T_{mg} = 3913.04 \text{ N} \cdot \text{m} \qquad T_{st} = 1086.96 \text{ N} \cdot \text{m}$$

Maximum Shear Stress:

$$(\tau_{st})_{max} = \frac{T_{st}c_{st}}{J_{st}} = \frac{1086.96(0.02)}{\frac{\pi}{2}(0.02^4)} = 86.5 \text{ MPa} \qquad \textbf{Ans.}$$

$$(\tau_{mg})_{max} = \frac{T_{mg}c_{mg}}{J_{mg}} = \frac{3913.04(0.04)}{\frac{\pi}{2}(0.04^4 - 0.02^4)} = 41.5 \text{ MPa} \qquad \textbf{Ans.}$$

$$(\tau_{mg})\big|_{\rho=0.02\text{ m}} = \frac{T_{mg}\rho}{J_{mg}} = \frac{3913.04(0.02)}{\frac{\pi}{2}(0.04^4 - 0.02^4)} = 20.8 \text{ MPa} \qquad \textbf{Ans.}$$



(b)

Ans:
$(\tau_{st})_{max} = 86.5$ MPa, $(\tau_{mg})_{max} = 41.5$ MPa,
$(\tau_{mg})\big|_{\rho=0.02\text{m}} = 20.8$ MPa

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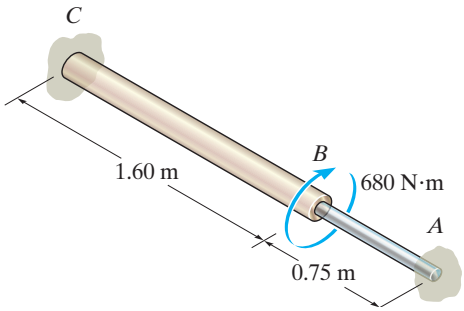
5–83.

A rod is made from two segments: AB is steel and BC is brass. It is fixed at its ends and subjected to a torque of $T = 680\ \text{N}\cdot\text{m}$. If the steel portion has a diameter of 30 mm, determine the required diameter of the brass portion so the reactions at the walls will be the same. $G_{st} = 75$ GPa, $G_{br} = 39$ GPa.



SOLUTION

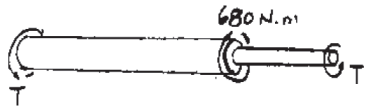
Compatibility Condition:

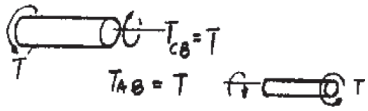
$$\phi_{B/C} = \phi_{B/A}$$

$$\frac{T(1.60)}{\frac{\pi}{2}(c^4)(39)(10^9)} = \frac{T(0.75)}{\frac{\pi}{2}(0.015^4)(75)(10^9)}$$

$$c = 0.02134\ \text{m}$$

$$d = 2c = 0.04269\ \text{m} = 42.7\ \text{mm}$$ **Ans.**





Ans:
$d = 42.7$ mm

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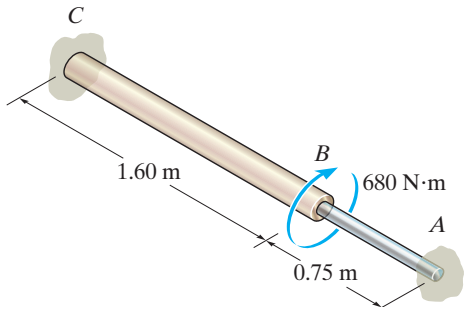
***5–84.**

Determine the absolute maximum shear stress in the shaft of Prob. 5–88.

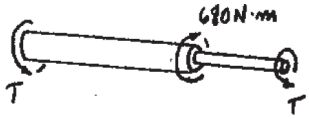


SOLUTION

Equilibrium,

$$2T = 680$$

$$T = 340\ \text{N} \cdot \text{m}$$



$\tau_{\substack{\text{abs}\\\text{max}}}$ occurs in the steel. See solution to Prob. 5–88.

$$\tau_{\substack{\text{abs}\\\text{max}}} = \frac{Tc}{J} = \frac{340(0.015)}{\frac{\pi}{2}(0.015)^4}$$

$$= 64.1\ \text{MPa}$$ **Ans.**

Ans:

$\tau_{\substack{\text{abs}\\\text{max}}} = 64.1$ MPa

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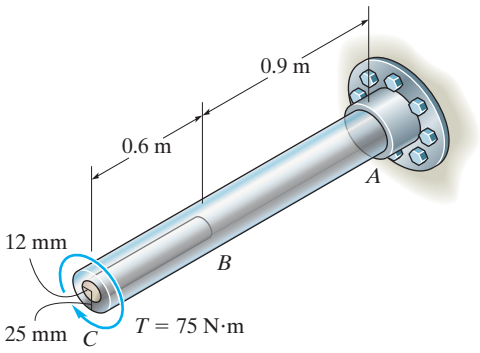
5–85. The shaft is made from a solid steel section AB and a tubular portion made of steel and having a brass core. If it is fixed to a rigid support at A, and a torque of $T = 75\ \text{N} \cdot \text{m}$ is applied to it at C, determine the angle of twist that occurs at C and compute the maximum shear stress and maximum shear strain in the brass and steel. Take $G_{st} = 75\ \text{GPa}$, $G_{br} = 38\ \text{GPa}$.



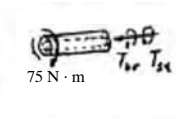
SOLUTION

Equilibrium:

$$T_{br} + T_{st} - 75 = 0 \qquad \textbf{(1)}$$

Both the steel tube and brass core undergo the same angle of twist $\phi_{C/B}$

$$\phi_{C/B} = \frac{TL}{JG} = \frac{T_{br}(0.6)}{[\frac{\pi}{2}(0.0125^4)][38(10^9)]} = \frac{T_{st}(0.6)}{[\frac{\pi}{2}(0.025^4 - 0.0125^4)][75(10^9)]}$$



$$T_{br} = 0.033778\ T_{st} \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields:

$$T_{st} = 72.549\ \text{N} \cdot \text{m}; \qquad T_{br} = 2.451\ \text{N} \cdot \text{m}$$

$$\phi_C = \Sigma\frac{TL}{JG} = \frac{2.451(0.6)}{[\frac{\pi}{2}(0.0125^4)][38(10^9)]} + \frac{75(0.9)}{[\frac{\pi}{2}(0.025^4)][75(10^9)]}$$

$$= 0.002476\ \text{rad} = 0.142^\circ \qquad \textbf{Ans.}$$

$$(\tau_{st})_{\max\ AB} = \frac{T_{AB}c}{J} = \frac{75(0.025)}{\frac{\pi}{2}(0.025^4)} = 3.056(10^6)\ \text{N/m}^2 = 3.056\ \text{MPa}$$

$$(\tau_{st})_{\max\ BC} = \frac{T_{st}\,c}{J} = \frac{72.549(0.025)}{[\frac{\pi}{2}(0.025^4 - 0.0125^4)]}$$

$$= 3.153(10^6)\ \text{N/m}^2 = 3.15\ \text{MPa (Max)} \qquad \textbf{Ans.}$$

$$(\gamma_{st})_{\max} = \frac{(\tau_{st})_{\max}}{G} = \frac{3.153(10^6)}{75(10^9)} = 42.0(10^{-6})\ \text{rad} \qquad \textbf{Ans.}$$

$$(\tau_{br})_{\max} = \frac{T_{br}\,c}{J} = \frac{2.451(0.0125)}{\frac{\pi}{2}(0.0125^4)} = 0.7988(10^6)\ \text{N/m}^2 = 0.799\ \text{MPa} \qquad \textbf{Ans.}$$

$$(\gamma_{br})_{\max} = \frac{(\tau_{br})_{\max}}{G} = \frac{0.7988(10^6)}{38(10^9)} = 21.0(10^{-6})\ \text{rad} \qquad \textbf{Ans.}$$

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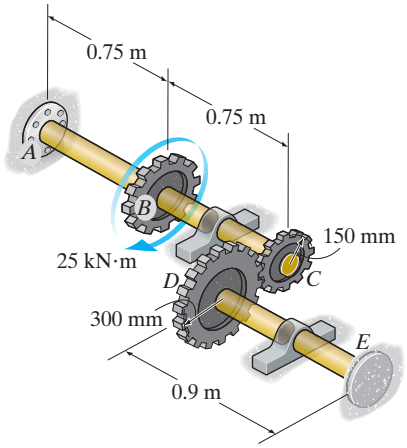
5–86. The shafts are made of A-36 steel and have the same diameter of 100 mm. If a torque of 25 kN · m is applied to gear B, determine the absolute maximum shear stress developed in the shaft.



SOLUTION

Equilibrium: Referring to the free - body diagrams of shafts ABC and DE shown in Figs. a and b, respectively, we have

$$\Sigma M_x = 0; \quad T_A + F(0.15) - 25 = 0 \tag{1}$$

and

$$\Sigma M_x = 0; \quad F(0.3) - T_E = 0 \tag{2}$$

Internal Loadings: The internal torques developed in segments AB and BC of shaft ABC and shaft DE are shown in Figs. c, d, and e, respectively.

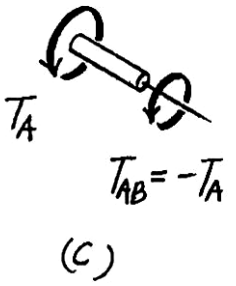
Compatibility Equation:

$$\phi_C r_C = \phi_D r_D$$

$$\left(\frac{T_{AB}L_{AB}}{JG_{st}} + \frac{T_{BC}L_{BC}}{JG_{st}}\right)r_C = \left(\frac{T_{DE}\,L_{DE}}{JG_{st}}\right)r_D$$

$$[-T_A(0.75) + F(0.15)(0.75)](0.15) = -T_E(0.9)(0.3)$$

$$T_A - 0.15F = 2.4T_E \tag{3}$$



(c)

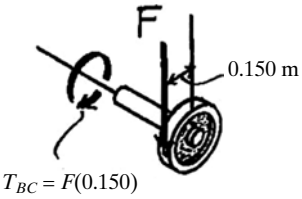
Solving Eqs. (1), (2), and (3), we have

$F = 24.51$ kN $\qquad T_E = 7.353$ kN · m $\qquad T_A = 21.32$ kN · m

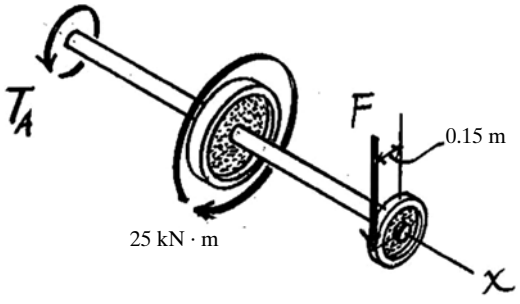
Maximum Shear Stress: By inspection, segment AB of shaft ABC is subjected to the greater torque.

$$(\tau_{\max})_{abs} = \frac{T_{ABC}}{J_{st}} = \frac{[21.32(10^3)](0.05)}{\frac{\pi}{2}(0.05^4)}$$

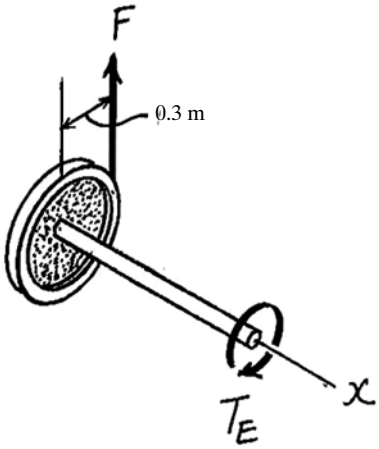
$$= 108.60(10^6)\ \text{N/m}^2 = 109\ \text{MPa} \qquad \textbf{Ans.}$$



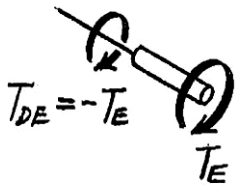
(d)



(a)



(b)



(e)

Ans.

$(\tau_{\max})_{abs} = 109$ MPa

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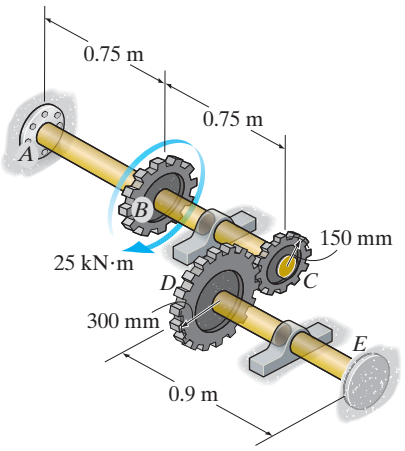
5–87. The shafts are made of A-36 steel and have the same diameter of 100 mm. If a torque of 25 kN · m is applied to gear B, determine the angle of twist of gear B.



SOLUTION

Equilibrium: Referring to the free - body diagrams of shafts ABC and DE shown in Figs. a and b, respectively,

$$\Sigma M_x = 0; \quad T_A + F(0.15) - 25 = 0 \tag{1}$$

and

$$\Sigma M_x = 0; \quad F(0.3) - T_E = 0 \tag{2}$$

Internal Loadings: The internal torques developed in segments AB and BC of shaft ABC and shaft DE are shown in Figs. c, d, and e, respectively.

Compatibility Equation: It is required that

$$\phi_C r_C = \phi_D r_D$$

$$\left(\frac{T_{AB}\,L_{AB}}{JG_{st}} + \frac{T_{BC}\,L_{BC}}{JG_{st}}\right) r_C = \left(\frac{T_{DE}\,L_{DE}}{JG_{st}}\right) r_D$$

$$[-T_A(0.75) + F(0.15)(0.75)](0.15) = -T_E(0.9)(0.3)$$

$$T_A - 0.15F = 2.4T_E \tag{3}$$

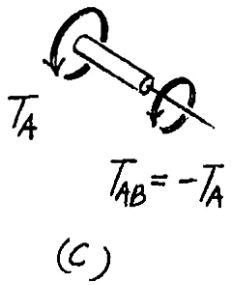
Solving Eqs. (1), (2), and (3),

$F = 24.51$ kN $\qquad T_E = 7.353$ kN · m $\qquad T_A = 21.32$ kN · m

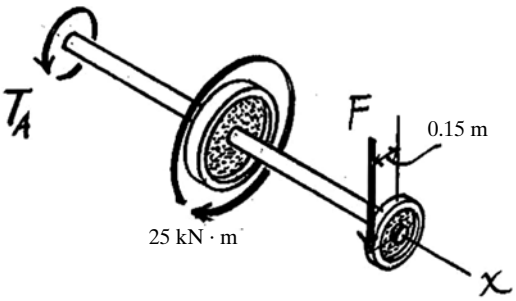
Angle of Twist: Here, $T_{AB} = -T_A = -21.32$ kN · m

$$\phi_B = \frac{T_{AB}\,L_{AB}}{JG_{st}} = \frac{[-21.32(10^3)](0.75)}{[\frac{\pi}{2}(0.05^4)][75(10^9)]}$$

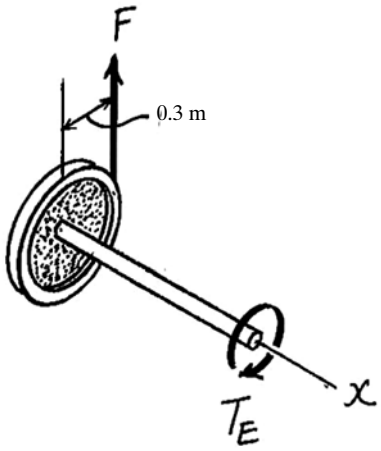
$$= -0.02172 \text{ rad} = 1.24^\circ \circlearrowleft \qquad \textbf{Ans.}$$



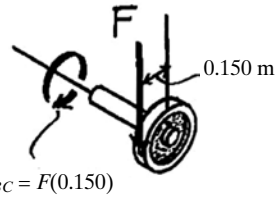
(c)



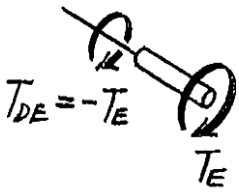
(a)



(b)



(d)



(e)

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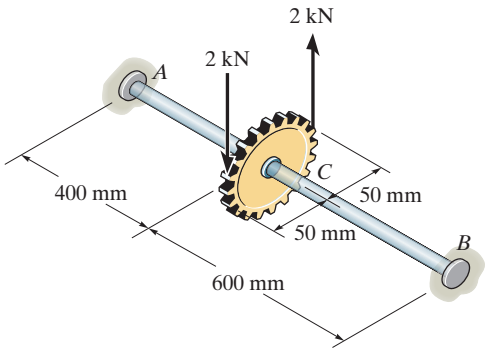
***5–88.**

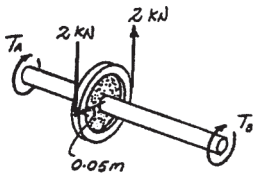
The shaft is made of L2 tool steel, has a diameter of 40 mm, and is fixed at its ends A and B. If it is subjected to the couple, determine the maximum shear stress in regions AC and CB.



SOLUTION

Equilibrium:

$$T_A + T_B - 2(0.1) = 0 \qquad \textbf{(1)}$$



Compatibility:

$$\phi_{C/A} = \phi_{C/B}$$

$$\frac{T_A(0.4)}{JG} = \frac{T_B(0.6)}{JG}$$

$$T_A = 1.50T_B \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields:

$$T_B = 0.080\ \text{kN}\cdot\text{m} \qquad T_A = 0.120\ \text{kN}\cdot\text{m}$$

Maximum Shear Stress:

$$(\tau_{AC})_{\max} = \frac{T_A\, c}{J} = \frac{0.12(10^3)(0.02)}{\frac{\pi}{2}(0.02^4)} = 9.55\ \text{MPa} \qquad \textbf{Ans.}$$

$$(\tau_{CB})_{\max} = \frac{T_B\, c}{J} = \frac{0.08(10^3)(0.02)}{\frac{\pi}{2}(0.02^4)} = 6.37\ \text{MPa} \qquad \textbf{Ans.}$$

Ans:
$(\tau_{AC})_{\max} = 9.55$ MPa,
$(\tau_{CB})_{\max} = 6.37$ MPa

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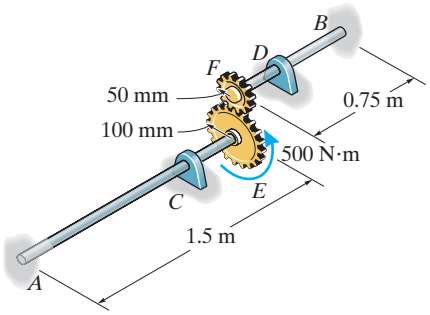
5–89.

The two shafts are made of A-36 steel. Each has a diameter of 25 mm and they are connected using the gears fixed to their ends. Their other ends are attached to fixed supports at *A* and *B*. They are also supported by journal bearings at *C* and *D*, which allow free rotation of the shafts along their axes. If a torque of 500 N · m is applied to the gear at *E*, determine the reactions at *A* and *B*.

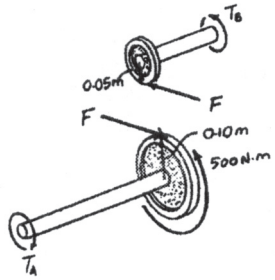


SOLUTION

Equilibrium:

$$T_A + F(0.1) - 500 = 0 \quad \textbf{(1)}$$

$$T_B - F(0.05) = 0 \quad \textbf{(2)}$$



From Eqs. (1) and (2)

$$T_A + 2T_B - 500 = 0 \quad \textbf{(3)}$$

Compatibility:

$$0.1\phi_E = 0.05\phi_F$$

$$\phi_E = 0.5\phi_F$$

$$\frac{T_A(1.5)}{JG} = 0.5\left[\frac{T_B(0.75)}{JG}\right]$$

$$T_A = 0.250T_B \quad \textbf{(4)}$$

Solving Eqs. (3) and (4) yields:

$$T_B = 222\ \text{N} \cdot \text{m} \quad \textbf{Ans.}$$

$$T_A = 55.6\ \text{N} \cdot \text{m} \quad \textbf{Ans.}$$

Ans:
$T_B = 22.2\ \text{N} \cdot \text{m}$, $T_A = 55.6\ \text{N} \cdot \text{m}$

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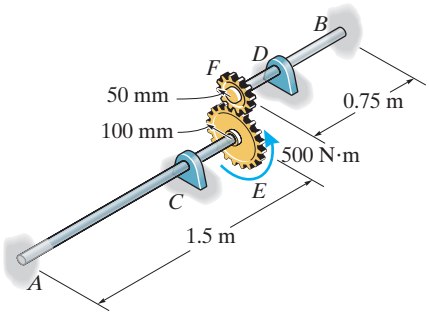
5–90.

The two shafts are made of A-36 steel. Each has a diameter of 25 mm and they are connected using the gears fixed to their ends. Their other ends are attached to fixed supports at *A* and *B*. They are also supported by journal bearings at *C* and *D*, which allow free rotation of the shafts along their axes. If a torque of 500 N · m is applied to the gear at *E*, determine the rotation of this gear.

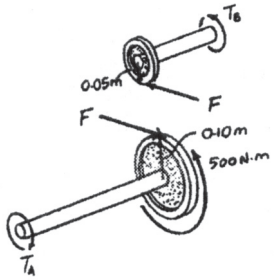


SOLUTION

Equilibrium:

$$T_A + F(0.1) - 500 = 0 \qquad \textbf{(1)}$$

$$T_B - F(0.05) = 0 \qquad \textbf{(2)}$$



From Eqs. (1) and (2)

$$T_A + 2T_B - 500 = 0 \qquad \textbf{(3)}$$

Compatibility:

$$0.1\phi_E = 0.05\phi_F$$

$$\phi_E = 0.5\phi_F$$

$$\frac{T_A(1.5)}{JG} = 0.5\left[\frac{T_B(0.75)}{JG}\right]$$

$$T_A = 0.250T_B \qquad \textbf{(4)}$$

Solving Eqs. (3) and (4) yields:

$$T_B = 222.22\ \text{N} \cdot \text{m} \qquad T_A = 55.56\ \text{N} \cdot \text{m}$$

Angle of Twist:

$$\phi_E = \frac{T_A L}{JG} = \frac{55.56(1.5)}{\frac{\pi}{2}(0.0125^4)(75.0)(10^9)}$$

$$= 0.02897\ \text{rad} = 1.66^\circ \qquad \textbf{Ans.}$$

Ans:

$\phi_E = 1.66^\circ$

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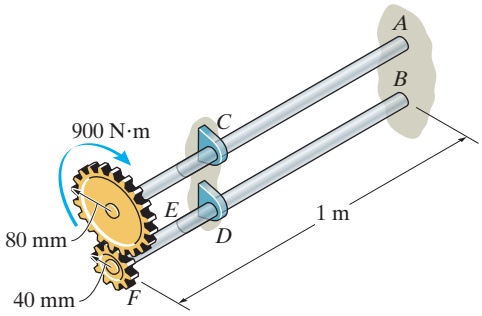
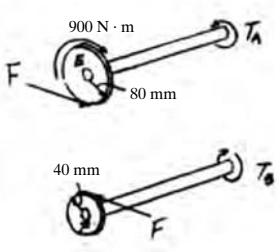
5–91. The two 1-m-long shafts are made of 2014-T6 aluminum. Each has a diameter of 30 mm and they are connected using the gears fixed to their ends. Their other ends are attached to fixed supports at A and B. They are also supported by bearings at C and D, which allow free rotation of the shafts along their axes. If a torque of 900 N · m is applied to the top gear as shown, determine the maximum shear stress in each shaft.



SOLUTION

$$T_A + F(0.08) - 900 = 0 \qquad \textbf{(1)}$$

$$T_B - F(0.04) = 0 \qquad \textbf{(2)}$$



From Eqs. (1) and (2)

$$T_A + 2T_B - 900 = 0 \qquad \textbf{(3)}$$

$$80(\phi_E) = 40(\phi_F); \quad \phi_E = 0.5\phi_F$$

$$\frac{T_A L}{JG} = 0.5\left(\frac{T_B L}{JG}\right); \quad T_A = 0.5T_B \qquad \textbf{(4)}$$

Solving Eqs. (3) and (4) yields:

$$T_B = 360 \text{ N} \cdot \text{m}; \quad T_A = 180 \text{ N} \cdot \text{m}$$

$$(\tau_{BD})_{\max} = \frac{T_B c}{J} = \frac{360(0.015)}{\frac{\pi}{2}(0.015^4)} = 67.91(10^6) \text{ N/m}^2 = 67.9 \text{ MPa} \qquad \textbf{Ans.}$$

$$(\tau_{AC})_{\max} = \frac{T_A c}{J} = \frac{180(0.015)}{\frac{\pi}{2}(0.015^4)} = 33.95(10^6) \text{ N/m}^2 = 34.0 \text{ MPa} \qquad \textbf{Ans.}$$

Ans.
$(\tau_{BD})_{\max} = 67.9$ MPa, $(\tau_{AC})_{\max} = 34.0$ MPa

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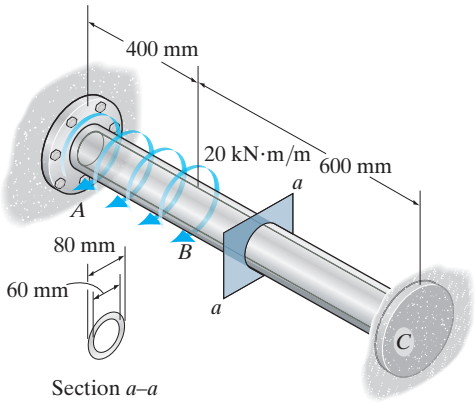
***5–92.**

If the shaft is subjected to a uniform distributed torque of $t = 20\ \text{kN}\cdot\text{m/m}$, determine the maximum shear stress developed in the shaft. The shaft is made of 2014-T6 aluminum alloy and is fixed at A and C.



SOLUTION

Equilibrium: Referring to the free-body diagram of the shaft shown in Fig. *a*, we have

$$\Sigma M_x = 0; \quad T_A + T_C - 20(10^3)(0.4) = 0 \qquad \textbf{(1)}$$

Compatibility Equation: The resultant torque of the distributed torque within the region x of the shaft is $T_R = 20(10^3)x\ \text{N}\cdot\text{m}$. Thus, the internal torque developed in the shaft as a function of x when end C is free is $T(x) = 20(10^3)x\ \text{N}\cdot\text{m}$, Fig. *b*. Using the method of superposition, Fig. c,

$$\phi_C = (\phi_C)_t - (\phi_C)_{T_C}$$

$$0 = \int_0^{0.4\text{ m}} \frac{T(x)dx}{JG} - \frac{T_C L}{JG}$$

$$0 = \int_0^{0.4\text{ m}} \frac{20(10^3)xdx}{JG} - \frac{T_C(1)}{JG}$$

$$0 = 20(10^3)\left(\frac{x^2}{2}\right)\Big|_0^{0.4\text{ m}} - T_C$$

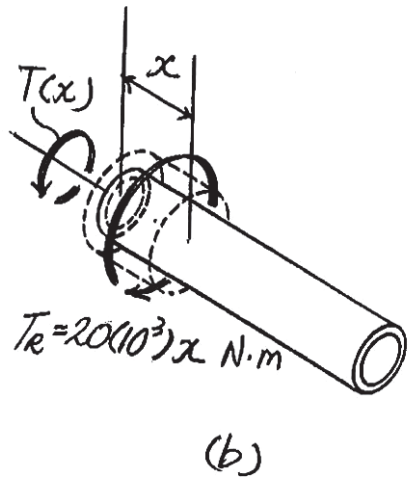
$$T_C = 1600\ \text{N}\cdot\text{m}$$

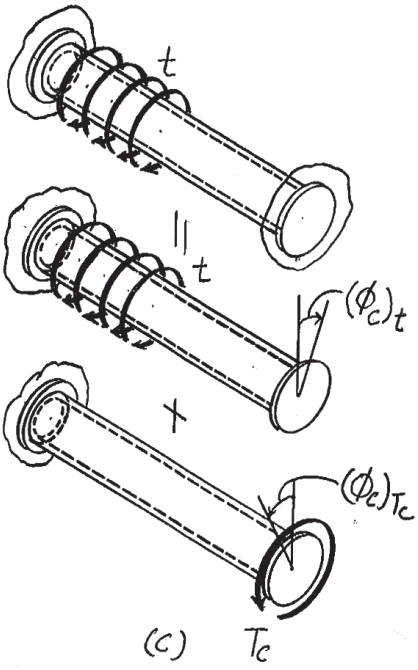
Substituting this result into Eq. (1),

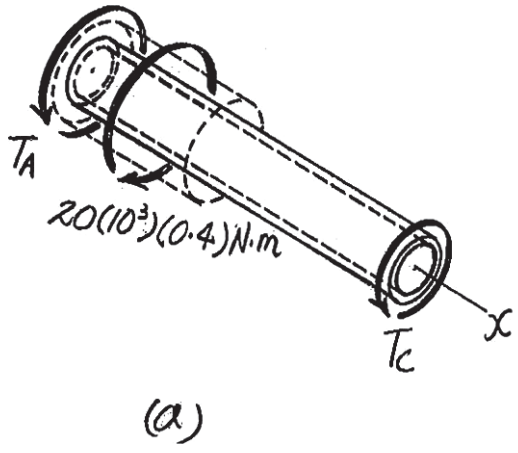
$$T_A = 6400\ \text{N}\cdot\text{m}$$

Maximum Shear Stress: By inspection, the maximum internal torque occurs at support A. Thus,

$$(\tau_{\max})_{\text{abs}} = \frac{T_A\, c}{J} = \frac{6400(0.04)}{\frac{\pi}{2}\left(0.04^4 - 0.03^4\right)} = 93.1\ \text{MPa} \qquad \textbf{Ans.}$$







Ans:
$\tau_{\substack{\text{abs}\\\text{max}}} = 93.1\ \text{MPa}$

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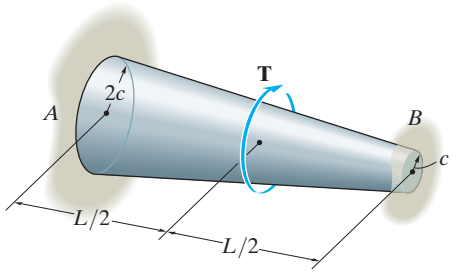
5–93.

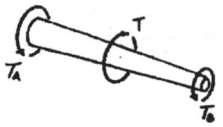
The tapered shaft is confined by the fixed supports at A and B. If a torque $\mathbf{T}$ is applied at its mid-point, determine the reactions at the supports.



SOLUTION

Equilibrium:

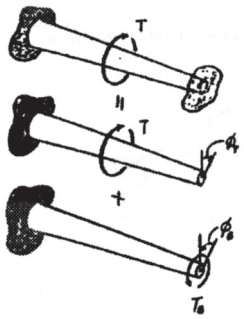
$$T_A + T_B - T = 0 \qquad (1)$$



Section Properties:

$$r(x) = c + \frac{c}{L}x = \frac{c}{L}(L + x)$$

$$J(x) = \frac{\pi}{2}\left[\frac{c}{L}(L + x)\right]^4 = \frac{\pi c^4}{2L^4}(L + x)^4$$



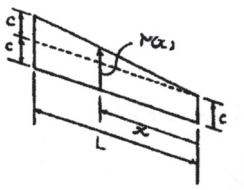
Angle of Twist:

$$\phi_T = \int \frac{T dx}{J(x)G} = \int_{\frac{L}{2}}^{L} \frac{T dx}{\frac{\pi c^4}{2L^4}(L + x)^4 G}$$

$$= \frac{2TL^4}{\pi c^4 G}\int_{\frac{L}{2}}^{L} \frac{dx}{(L + x)^4}$$

$$= -\frac{2TL^4}{3\pi c^4 G}\left[\frac{1}{(L + x)^3}\right]\Bigg|_{\frac{L}{2}}^{L}$$

$$= \frac{37TL}{324\,\pi c^4 G}$$



$$\phi_B = \int \frac{T dx}{J(x)G} = \int_0^L \frac{T_B dx}{\frac{\pi c^4}{2L^4}(L + x)^4 G}$$

$$= \frac{2T_B L^4}{\pi c^4 G}\int_0^L \frac{dx}{(L + x)^4}$$

$$= -\frac{2T_B L^4}{3\pi c^4 G}\left[\frac{1}{(L + x)^3}\right]\Bigg|_0^L$$

$$= \frac{7T_B L}{12\pi c^4 G}$$

Compatibility:

$$0 = \phi_T - \phi_B$$

$$0 = \frac{37TL}{324\pi c^4 G} - \frac{7T_B L}{12\pi c^4 G}$$

$$T_B = \frac{37}{189}T \qquad \textbf{Ans.}$$

Substituting the result into Eq. (1) yields:

$$T_A = \frac{152}{189}T \qquad \textbf{Ans.}$$

Ans:

$$T_B = \frac{37}{189}T,\ T_A = \frac{152}{189}T$$

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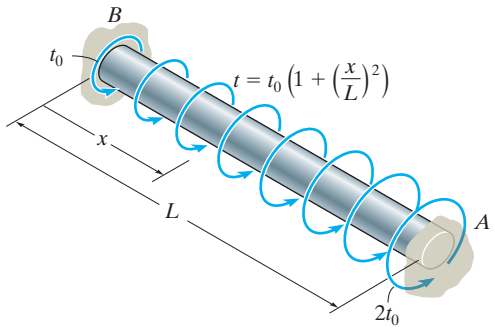
5–94.

The shaft of radius c is subjected to a distributed torque t, measured as torque/length of shaft. Determine the reactions at the fixed supports A and B.



SOLUTION

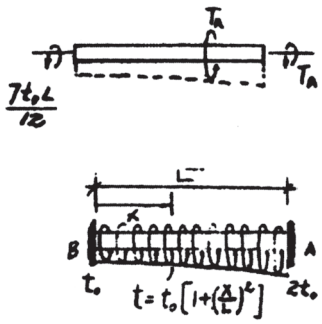
$$T(x) = \int_0^x t_0\left(1 + \frac{x^2}{L^2}\right) dx = t_0\left(x + \frac{x^3}{3L^2}\right) \quad \textbf{(1)}$$

By superposition:

$$0 = \phi - \phi_B$$

$$0 = \int_0^L \frac{t_0\left(x + \frac{x^3}{3L^2}\right) dx}{JG} - \frac{T_B(L)}{JG} = \frac{7t_0L^2}{12} - T_B(L)$$

$$T_B = \frac{7t_0L}{12} \quad \textbf{Ans.}$$

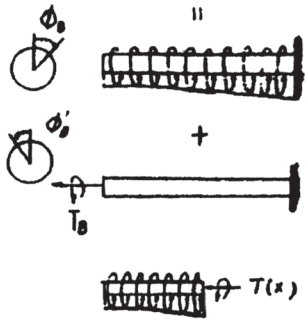


From Eq. (1),

$$T = t_0\left(L + \frac{L^3}{3L^2}\right) = \frac{4t_0L}{3}$$

$$T_A + \frac{7t_0L}{12} - \frac{4t_0L}{3} = 0$$

$$T_A = \frac{3t_0L}{4} \quad \textbf{Ans.}$$



Ans:

$$T_B = \frac{7t_0L}{12}, T_A = \frac{3t_0L}{4}$$

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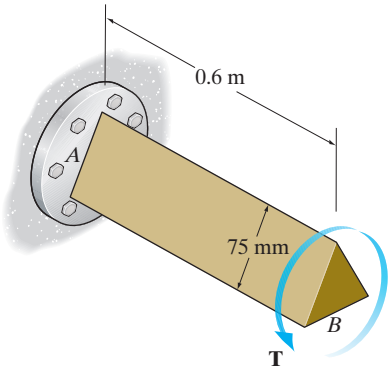
5–95. If end B of the shaft, which has an equilateral triangle cross section, is subjected to a torque of $T = 1200\ \text{N} \cdot \text{m}$, determine the maximum shear stress developed in the shaft. Also, find the angle of twist of end B. The shaft is made from 6061-T1 aluminum.



SOLUTION

Maximum Shear Stress:

$$\tau_{\max} = \frac{20T}{a^3} = \frac{20(1200)}{0.075^3} = 56.89(10^6)\ \text{N/m}^2 = 56.9\ \text{MPa} \qquad \textbf{Ans.}$$

Angle of Twist:

$$\phi = \frac{46TL}{a^4 G}$$

$$= \frac{46(1200)(0.6)}{(0.075^4)[26(10^9)]}$$

$$= 0.04026\ \text{rad} = 2.31° \qquad \textbf{Ans.}$$

Ans.
$\tau_{\max} = 56.9\ \text{MPa}$
$\phi = 2.31°$

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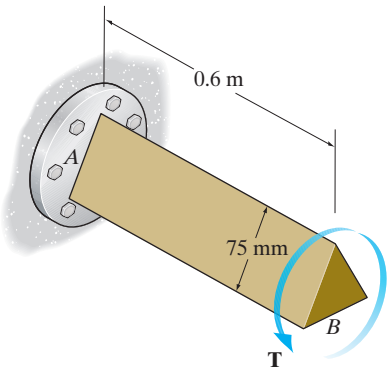
***5–96.** If the shaft has an equilateral triangle cross section and is made from 606 |-T| aluminum alloy that has an allowable shear stress of $\tau_{allow} = 84$ MPa, determine the maximum allowable torque **T** that can be applied to end *B*. Also, find the corresponding angle of twist of end *B*.



SOLUTION

Allowable Shear Stress:

$$\tau_{allow} = \frac{20T}{a^3}; \qquad 84(10^6) = \frac{20T}{0.075^3}$$

$$T = 1.772(10^3)\ \text{N}\cdot\text{m} = 1.77\ \text{kN}\cdot\text{m} \qquad \textbf{Ans.}$$

Angle of Twist:

$$\phi = \frac{46TL}{a^4G}$$

$$= \frac{46[1.772(10^3)](0.6)}{(0.075^4)[26(10^9)]}$$

$$= 0.05945\ \text{rad} = 3.41° \qquad \textbf{Ans.}$$

Ans:

$T = 1.77\ \text{kN}\cdot\text{m},\ \phi = 3.41°$

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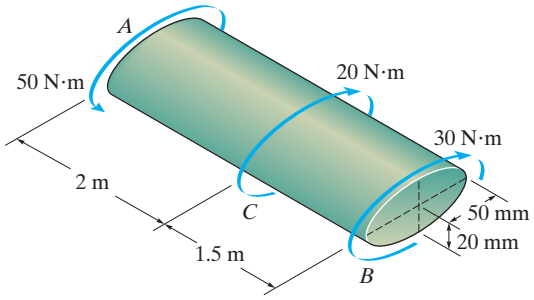
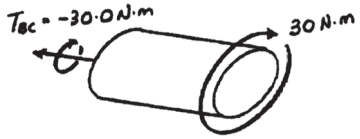
5–97.

The shaft is made of red brass C83400 and has an elliptical cross section. If it is subjected to the torsional loading, determine the maximum shear stress within regions AC and BC, and the angle of twist ϕ of end B relative to end A.



SOLUTION

Maximum Shear Stress:



$$(\tau_{BC})_{max} = \frac{2T_{BC}}{\pi\, a\, b^2} = \frac{2(30.0)}{\pi(0.05)(0.02^2)}$$

$$= 0.955\ \text{MPa} \qquad \textbf{Ans.}$$

$$(\tau_{AC})_{max} = \frac{2T_{AC}}{\pi\, a\, b^2} = \frac{2(50.0)}{\pi(0.05)(0.02^2)}$$

$$= 1.59\ \text{MPa} \qquad \textbf{Ans.}$$

Angle of Twist:

$$\phi_{B/A} = \sum \frac{(a^2 + b^2)T\,L}{\pi\, a^3 b^3\, G}$$

$$= \frac{(0.05^2 + 0.02^2)}{\pi(0.05^3)(0.02^3)(37.0)(10^9)}[(-30.0)(1.5) + (-50.0)(2)]$$

$$= -0.003618\ \text{rad} = 0.207° \qquad \textbf{Ans.}$$

Ans:
$(\tau_{BC})_{max} = 0.955$ MPa, $(\tau_{AC})_{max} = 1.59$ MPa,
$\phi_{B/A} = 0.207°$

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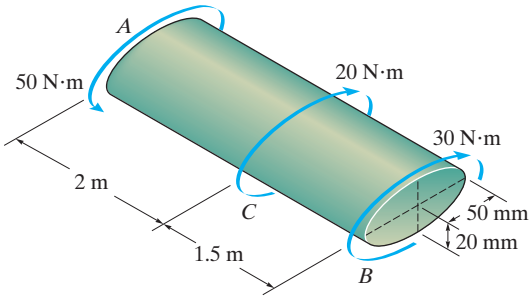
5–98.

Solve Prob. 5–98 for the maximum shear stress within regions AC and BC, and the angle of twist ϕ of end B relative to C.

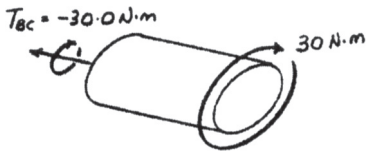


SOLUTION

Maximum Shear Stress:

$$(\tau_{BC})_{\max} = \frac{2T_{BC}}{\pi\, a\, b^2} = \frac{2(30.0)}{\pi(0.05)(0.02^2)}$$

$$= 0.955 \text{ MPa} \qquad \textbf{Ans.}$$



$$(\tau_{AC})_{\max} = \frac{2T_{AC}}{\pi\, a\, b^2} = \frac{2(50.0)}{\pi(0.05)(0.02^2)}$$

$$= 1.59 \text{ MPa} \qquad \textbf{Ans.}$$

Angle of Twist:

$$\phi_{B/C} = \frac{(a^2 + b^2)\, T_{BC}\, L}{\pi\, a^3\, b^3\, G}$$

$$= \frac{(0.05^2 + 0.02^2)(-30.0)(1.5)}{\pi(0.05^3)(0.02^3)(37.0)(10^9)}$$

$$= -0.001123 \text{ rad} = 0.0643^\circ \qquad \textbf{Ans.}$$

Ans:
$(\tau_{BC})_{\max} = 0.955$ MPa, $(\tau_{AC})_{\max} = 1.59$ MPa,
$\phi_{B/C} = 0.0643^\circ$

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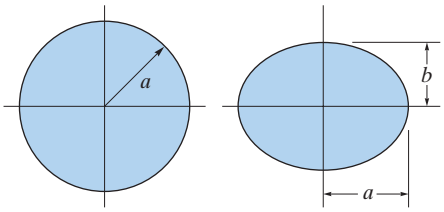
5–99.

If $a = 25$ mm and $b = 15$ mm, determine the maximum shear stress in the circular and elliptical shafts when the applied torque is $T = 80\ \text{N}\cdot\text{m}$. By what percentage is the shaft of circular cross section more efficient at withstanding the torque than the shaft of elliptical cross section?



SOLUTION

For the circular shaft:

$$(\tau_{\max})_c = \frac{T\,c}{J} = \frac{80(0.025)}{\frac{\pi}{2}(0.025^4)} = 3.26\ \text{MPa}$$ **Ans.**

For the elliptical shaft:

$$(\tau_{\max})_e = \frac{2T}{\pi\, a\, b^2} = \frac{2(80)}{\pi(0.025)(0.015^2)} = 9.05\ \text{MPa}$$ **Ans.**

$$\%\ \text{more efficient} = \frac{(\tau_{\max})_e - (\tau_{\max})_c}{(\tau_{\max})_c}(100\%)$$

$$= \frac{9.05 - 3.26}{3.26}(100\%) = 178\%$$ **Ans.**

Ans:
$(\tau_{\max})_c = 3.26$ MPa,
$(\tau_{\max})_e = 9.05$ MPa,
% more efficient $= 178\%$

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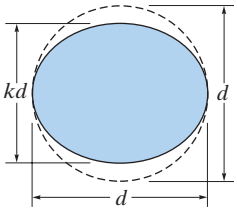
***5–100.**

It is intended to manufacture a circular bar to resist torque; however, the bar is made elliptical in the process of manufacturing, with one dimension smaller than the other by a factor k as shown. Determine the factor by which the maximum shear stress is increased.



SOLUTION

For the circular shaft:

$$(\tau_{\max})_c = \frac{Tc}{J} = \frac{T\left(\frac{d}{2}\right)}{\frac{\pi}{2}\left(\frac{d}{2}\right)^4} = \frac{16T}{\pi\, d^3}$$

For the elliptical shaft:

$$(\tau_{\max})_e = \frac{2T}{\pi\, a\, b^2} = \frac{2T}{\pi\left(\frac{d}{2}\right)\left(\frac{kd}{2}\right)^2} = \frac{16T}{\pi\, k^2\, d^3}$$

$$\text{Factor of increase in maximum shear stress} = \frac{(\tau_{\max})_e}{(\tau_{\max})_c} = \frac{\frac{16T}{\pi\, k^2 d^3}}{\frac{16T}{\pi\, d^3}}$$

$$= \frac{1}{k^2} \qquad \textbf{Ans.}$$

Ans:

$$\text{Factor of increase} = \frac{1}{k^2}$$

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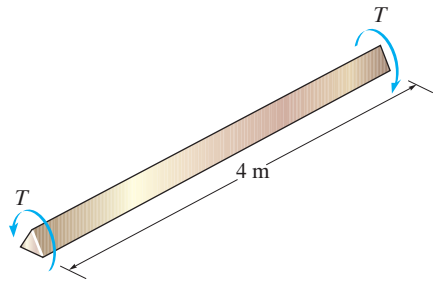
5–101.

The brass wire has a triangular cross section, 2 mm on a side. If the yield stress for brass is $\tau_Y = 205$ MPa, determine the maximum torque T to which it can be subjected so that the wire will not yield. If this torque is applied to the 4-m-long segment, determine the greatest angle of twist of one end of the wire relative to the other end that will not cause permanent damage to the wire. $G_{br} = 37$ GPa.



SOLUTION

Allowable Shear Stress:

$$\tau_{max} = \tau_\gamma = \frac{20T}{a^3}$$

$$205(10^6) = \frac{20T}{0.002^3}$$

$$T = 0.0820\ \text{N}\cdot\text{m} \qquad \textbf{Ans.}$$

Angle of Twist:

$$\phi = \frac{46TL}{a^4G} = \frac{46(0.0820)(4)}{(0.002^4)(37)(10^9)}$$

$$= 25.5\ \text{rad} \qquad \textbf{Ans.}$$

Ans:
$T = 0.0820\ \text{N}\cdot\text{m}$,
$\phi = 25.5$ rad

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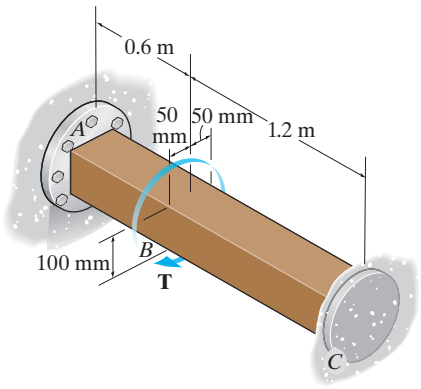
5–102. If the solid shaft is made from red brass C83400 copper having an allowable shear stress of $\tau_{allow} = 28$ MPa, determine the maximum allowable torque **T** that can be applied at *B*.



SOLUTION

Equilibrium: Referring to the free-body diagram of the square bar shown in Fig. *a*, we have

$$\Sigma M_x = 0; \quad T_A + T_C - T = 0 \tag{1}$$

Compatibility Equation: Here, it is required that

$$\phi_{B/A} = \phi_{B/C}$$

$$\frac{7.10T_A(0.6)}{a^4G} = \frac{7.10T_C(1.2)}{a^4G}$$

$$T_A = 2T_C \tag{2}$$

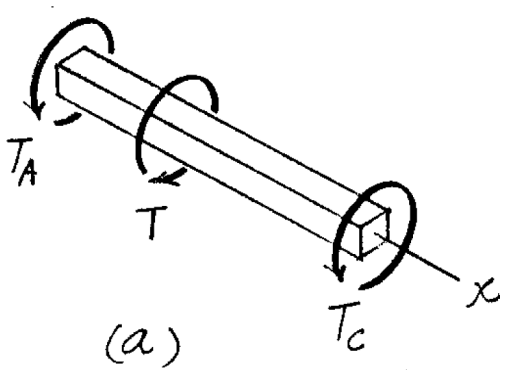
Solving Eqs. (1) and (2),

$$T_C = \frac{1}{3}T \qquad T_A = \frac{2}{3}T$$

Allowable Shear Stress: Segment *AB* is critical since it is subjected to the greater internal torque.

$$\tau_{allow} = \frac{4.81T_A}{a^3}; \qquad 28(10^6) = \frac{4.81\left(\frac{2}{3}T\right)}{0.1^3}$$

$$T = 8.732(10^3)\ \text{N}\cdot\text{m} = 8.73\ \text{kN}\cdot\text{m} \qquad \textbf{Ans.}$$



Ans:

$T = 8.73\ \text{kN}\cdot\text{m}$

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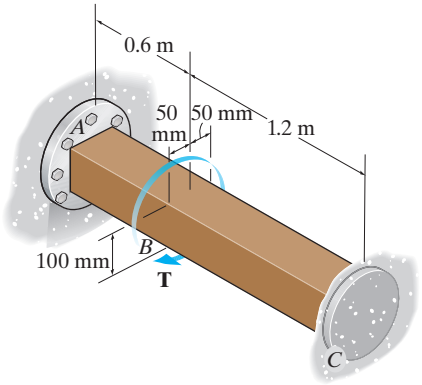
5–103. If the solid shaft is made from red brass C83400 copper and it is subjected to a torque $T = 8$ kN · m at B, determine the maximum shear stress developed in segments AB and BC.



SOLUTION

Equilibrium: Referring to the free-body diagram of the square bar shown in Fig. *a*, we have

$$\Sigma M_x = 0; \quad T_A + T_C - 8 = 0 \tag{1}$$

Compatibility Equation: Here, it is required that

$$\phi_{B/A} = \phi_{B/C}$$

$$\frac{7.10T_A(0.6)}{a^4G} = \frac{7.10T_C(1.2)}{a^4G}$$

$$T_A = 2T_C \tag{2}$$

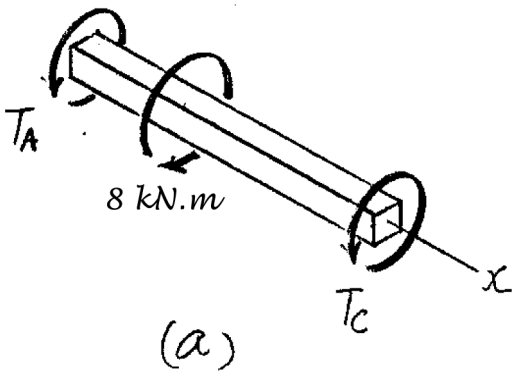
Solving Eqs. (1) and (2),

$$T_C = 2.667 \text{ kN} \cdot \text{m} \qquad T_A = 5.333 \text{ kN} \cdot \text{m}$$

Maximum Shear Stress:

$$(\tau_{\max})_{AB} = \frac{4.81T_A}{a^3} = \frac{4.81[5.333(10^3)]}{0.1^3} = 25.65(10^6) \text{ N/m}^2 = 25.7 \text{ MPa} \quad \textbf{Ans.}$$

$$(\tau_{\max})_{BC} = \frac{4.81T_C}{a^3} = \frac{4.81[2.667(10^3)]}{0.1^3} = 12.83(10^6) \text{ N/m}^2 = 12.8 \text{ MPa} \quad \textbf{Ans.}$$



Ans.
$(\tau_{\max})_{AB} = 25.7$ MPa, $(\tau_{\max})_{BC} = 12.8$ MPa

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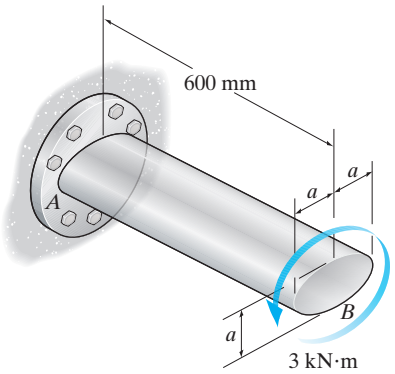
***5–104.**

If the shaft is subjected to the torque of 3 kN · m, determine the maximum shear stress developed in the shaft. Also, find the angle of twist of end B. The shaft is made from A-36 steel. Set $a = 50$ mm.



SOLUTION

Maximum Shear Stress:

$$\tau_{\max} = \frac{2T}{\pi ab^2} = \frac{2(3)(10^3)}{\pi(0.05)(0.025^2)} = 61.1 \text{ MPa} \qquad \textbf{Ans.}$$

Angle of Twist:

$$\phi = \frac{(a^2 + b^2)TL}{\pi a^3 b^3 G}$$

$$= \frac{(0.05^2 + 0.025^2)(3)(10^3)(0.6)}{\pi(0.05^3)(0.025^3)(75)(10^9)}$$

$$= 0.01222 \text{ rad} = 0.700° \qquad \textbf{Ans.}$$

Ans:
$\tau_{\max} = 61.1$ MPa, $\phi_B = 0.700°$

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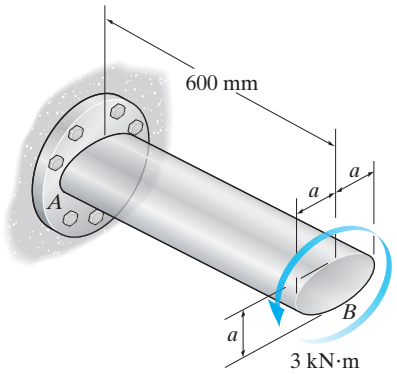
5–105.

If the shaft is made from A-36 steel having an allowable shear stress of $\tau_{allow} = 75$ MPa, determine the minimum dimension a for the cross section to the nearest millimeter. Also, find the corresponding angle of twist at end B.



SOLUTION

Allowable Shear Stress:

$$\tau_{allow} = \frac{2T}{\pi ab^2}; \qquad 75(10^6) = \frac{2(3)(10^3)}{\pi(a)(\frac{a}{2})^2}$$

$$a = 0.04670 \text{ m}$$

Use $a = 47$ mm **Ans.**

Angle of Twist:

$$\phi = \frac{(a^2 + b^2)TL}{\pi a^3 b^3 G}$$

$$= \frac{\left[0.047^2 + \left(\frac{0.047}{2}\right)^2\right](3)(10^3)(0.6)}{\pi(0.047^3)\left(\frac{0.047}{2}\right)^3(75)(10^9)}$$

$$= 0.01566 \text{ rad} = 0.897° \quad \textbf{Ans.}$$

Ans:
Use $a = 47$ mm, $\phi_B = 0.897°$

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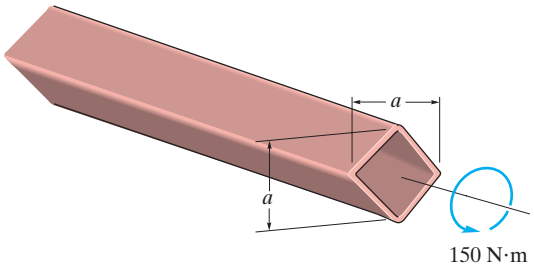
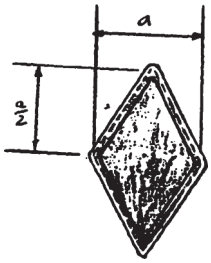
5–106.

The plastic tube is subjected to a torque of 150 N · m. Determine the mean dimension a of its sides if the allowable shear stress is $\tau_{allow} = 60$ MPa. Each side has a thickness of $t = 3$ mm.



SOLUTION



$$A_m = 4\left[\frac{1}{2}\left(\frac{a}{2}\right)\left(\frac{a}{2}\right)\right] = \frac{a^2}{2}$$

$$\tau_{avg} = \frac{T}{2\,tA_m}; \qquad 60(10^6) = \frac{150}{2(0.003)\,\frac{1}{2}\,a^2}$$

$$a = 0.0289\ \text{m} = 28.9\ \text{mm}$$ **Ans.**

Ans:
$a = 28.9$ mm

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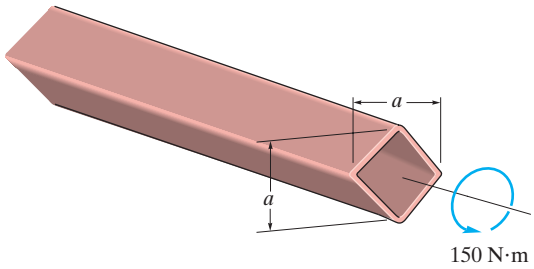
5–107.

The plastic tube is subjected to a torque of 150 N · m. Determine the average shear stress in the tube if the mean dimension $a = 200$ mm. Each side has a thickness of $t = 3$ mm.



SOLUTION

$$A_m = 4\left[\frac{1}{2}\left(\frac{0.2}{2}\right)\left(\frac{0.2}{2}\right)\right] = 0.02\ \text{m}^2$$

$$\tau_{\text{avg}} = \frac{T}{2\,t\,A_m} = \frac{150}{2(0.003)(0.02)}$$

$$= 1.25\ \text{MPa}$$ **Ans.**

Ans:
$\tau_{\text{avg}} = 1.25$ MPa

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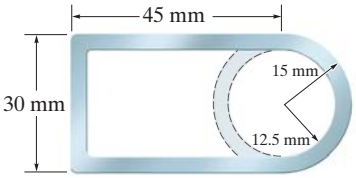
***5–108.** For a given maximum shear stress, determine the factor by which the torque carrying capacity is increased if the half-circular section is reversed from the dashed-line position to the section shown. The tube is 2.5 mm thick.



SOLUTION

$$A_m = (0.0275)(0.04375) - \frac{\pi(0.01375^2)}{2} = 0.9061(10^{-3})\ \text{m}^2$$

$$A_m' = (0.0275)(0.04375) + \frac{\pi(0.01375^2)}{2} = 1.5001(10^{-3})\ \text{m}^2$$

$$\tau_{\max} = \frac{T}{2t\,A_m}$$

$$T = 2\,t\,A_m\,\tau_{\max}$$

$$\text{Factor} = \frac{2t\,A_m'\,\tau_{\max}}{2t\,A_m\,\tau_{\max}}$$

$$= \frac{A_m'}{A_m} = \frac{1.5001(10^{-3})}{0.9061(10^{-3})} = 1.655 = 1.66 \qquad \textbf{Ans.}$$

Ans:
Factor = 1.66

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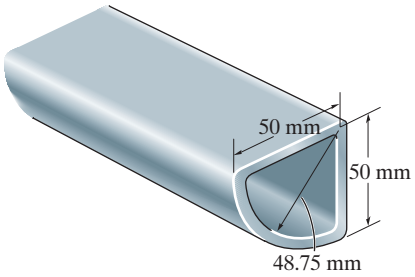
5–109. A torque of 200 N · m is applied to the tube. If the wall thickness is 2.5 mm, determine the average shear stress in the tube.



SOLUTION

$$A_m = \frac{\pi(0.04875^2)}{4} = 1.8665(10^{-3})\ \text{m}^2$$

$$\tau_{\text{avg}} = \frac{T}{2t\,A_m} = \frac{200}{2(0.0025)[1.8665(10^{-3})]} = 21.43(10^6)\ \text{N/m}^2 = 21.4\ \text{MPa} \qquad \textbf{Ans.}$$

Ans:
$\tau_{\text{avg}} = 21.44\ \text{MPa}$

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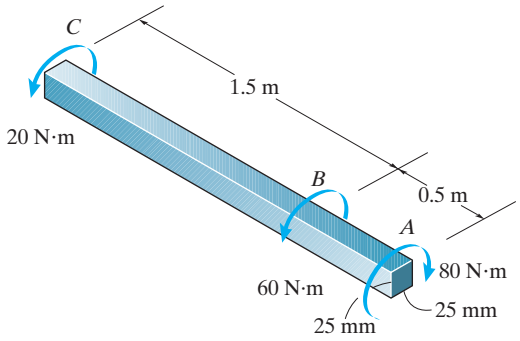
5–110.

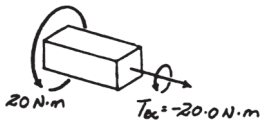
The 6061-T6 aluminum bar has a square cross section of 25 mm by 25 mm. If it is 2 m long, determine the maximum shear stress in the bar and the rotation of one end relative to the other end.



SOLUTION

Maximum Shear Stress:

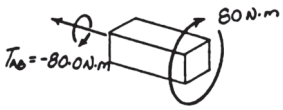
$$\tau_{\max} = \frac{4.81 T_{\max}}{a^3} = \frac{4.81(80.0)}{(0.025^3)} = 24.6 \text{ MPa} \qquad \textbf{Ans.}$$



Angle of Twist:

$$\phi_{A/C} = \sum \frac{7.10TL}{a^4 G} = \frac{7.10(-20.0)(1.5)}{(0.025^4)(26.0)(10^9)} + \frac{7.10(-80.0)(0.5)}{(0.025^4)(26.0)(10^9)}$$

$$= -0.04894 \text{ rad} = 2.80° \qquad \textbf{Ans.}$$



Ans:
$\tau_{\max} = 24.6$ MPa,
$\phi_{A/C} = 2.80°$

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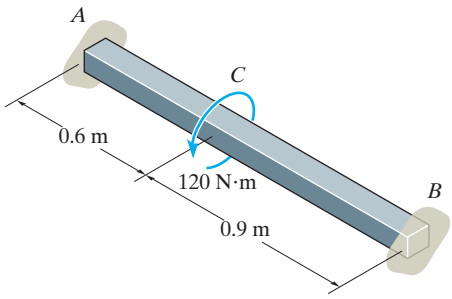
5–111. The aluminum strut is fixed between the two walls at A and B. If it has a 50 mm by 50 mm square cross section, and it is subjected to the torque of 120 N · m at C, determine the reactions at the fixed supports. Also, what is the angle of twist at C? $G_{al} = 27$ GPa.



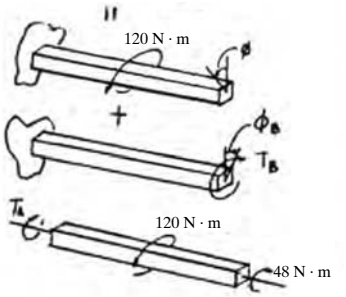
SOLUTION

By superposition:

$$0 = \phi - \phi_B$$

$$0 = \frac{7.10(120)(0.6)}{a^4 G} - \frac{7.10(T_B)(1.5)}{a^4 G}$$

$$T_B = 48 \text{ N} \cdot \text{m} \qquad \textbf{Ans.}$$



$$T_A + 48 - 120 = 0$$

$$T_A = 72 \text{ N} \cdot \text{m} \qquad \textbf{Ans.}$$

$$\phi_C = \frac{7.10(48)(0.9)}{(0.05^4)[27(10^9)]} = 0.0018176 \text{ rad} = 0.104° \qquad \textbf{Ans.}$$

Ans:

$T_B = 48 \text{ N} \cdot \text{m}$,
$T_A = 72 \text{ N} \cdot \text{m}$,
$\phi_C = 0.104°$

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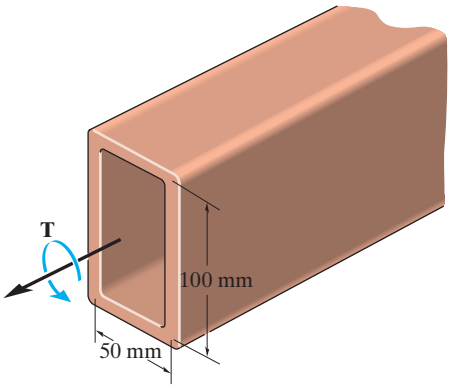
***5–112.** Determine the constant thickness of the rectangular tube if average stress is not to exceed 84 MPa when a torque of $T = 2$ kN · m is applied to the tube. Neglect stress concentrations at the corners. The mean dimensions of the tube are shown.



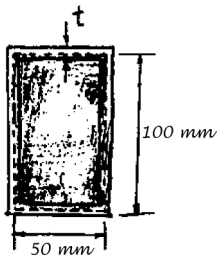
SOLUTION

$A_m = (0.05)(0.1) = 0.005\ \text{m}^2$

$\tau_{\text{avg}} = \dfrac{T}{2t\,A_m}$

$84(10^6) = \dfrac{2(10^3)}{2t(0.005)}$

$t = 0.002381\ \text{m} = 2.38\ \text{mm}$ **Ans.**



Ans:
$t = 2.38$ mm

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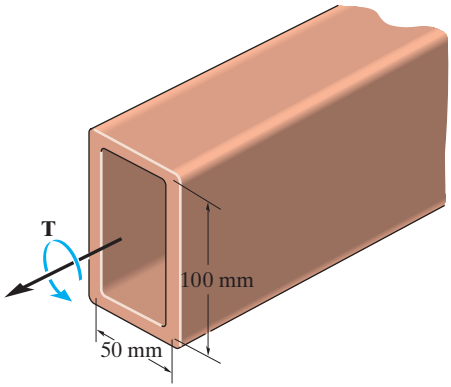
5–113. Determine the torque T that can be applied to the rectangular tube if the average shear stress is not to exceed 84 MPa. Neglect stress concentrations at the corners. The mean dimensions of the tube are shown and the tube has a thickness of 3 mm.



SOLUTION

$A_m = (0.05)(0.1) = 0.005\ \text{m}^2$

$\tau_{\text{avg}} = \dfrac{T}{2t\,A_m}; \qquad 84(10^6) = \dfrac{2(10^3)}{2(0.003)(0.005)}$

$t = 2.52(10^3)\ \text{N}\cdot\text{m} = 2.52\ \text{kN}\cdot\text{m}$ **Ans.**

Ans:

$T = 2.52\ \text{kN}\cdot\text{m}$

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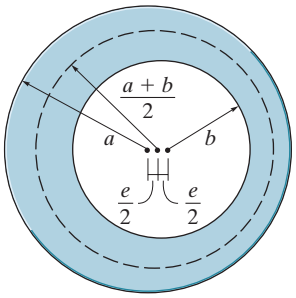
5–114.

Due to a fabrication error the inner circle of the tube is eccentric with respect to the outer circle. By what percentage is the torsional strength reduced when the eccentricity e is one-fourth of the difference in the radii?



SOLUTION

Average Shear Stress:

For the aligned tube

$$\tau_{\text{avg}} = \frac{T}{2\,t\,A_m} = \frac{T}{2(a-b)(\pi)\left(\frac{a+b}{2}\right)^2}$$

$$T = \tau_{\text{avg}}\,(2)(a-b)(\pi)\left(\frac{a+b}{2}\right)^2$$

For the eccentric tube

$$\tau_{\text{avg}} = \frac{T'}{2\,tA_m}$$

$$t = a - \frac{e}{2} - \left(\frac{e}{2} + b\right) = a - e - b$$

$$= a - \frac{1}{4}(a-b) - b = \frac{3}{4}(a-b)$$

$$T' = \tau_{\text{avg}}\,(2)\left[\frac{3}{4}(a-b)\right](\pi)\left(\frac{a+b}{2}\right)^2$$

$$\text{Factor} = \frac{T'}{T} = \frac{\tau_{\text{avg}}\,(2)\left[\frac{3}{4}(a-b)\right](\pi)\left(\frac{a+b}{2}\right)^2}{\tau_{\text{avg}}\,(2)(a-b)(\pi)\left(\frac{a+b}{2}\right)^2} = \frac{3}{4}$$

$$\text{Percent reduction in strength} = \left(1 - \frac{3}{4}\right) \times 100\% = 25\%$$ **Ans.**

Ans:
Percent reduction in strength = 25%

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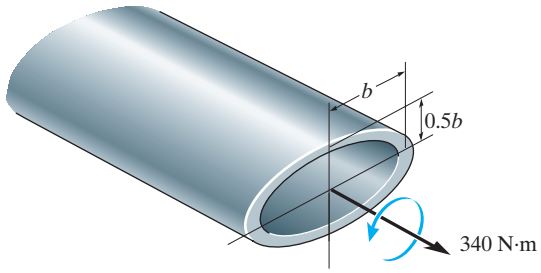
5–115. The steel tube has an elliptical cross section of mean dimensions shown and a constant thickness of $t = 5$ mm. If the allowable shear stress is $\tau_{allow} = 56$ MPa, and the tube is to resist a torque of $T = 340\ \text{N} \cdot \text{m}$, determine the necessary dimension b. The mean area for the ellipse is $A_m = \pi b(0.5b)$.



SOLUTION

$$\tau_{avg} = \tau_{allow} = \frac{T}{2tA_m}$$

$$56(10^6) = \frac{340}{2(0.005)[\pi b(0.5b)]}$$

$b = 0.01966\ \text{m} = 19.7\ \text{mm}$ **Ans.**

Ans:
$b = 19.7$ mm

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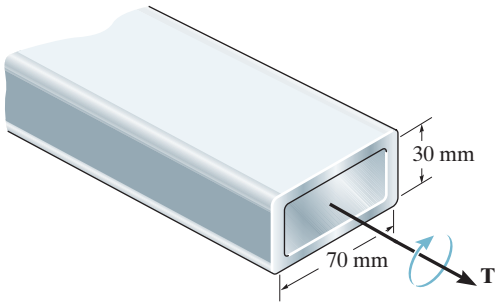
***5–116.**

The 304 stainless steel tube has a thickness of 10 mm. If the allowable shear stress is $\tau_{\text{allow}} = 80$ MPa, determine the maximum torque T that it can transmit. Also, what is the angle of twist of one end of the tube with respect to the other if the tube is 4 m long? The mean dimensions are shown.

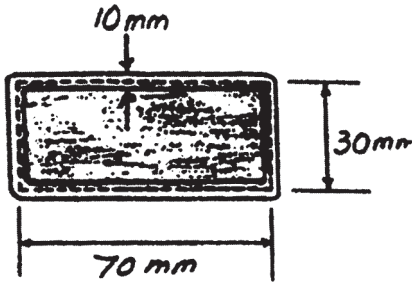


SOLUTION

Section Properties:

$$A_m = 0.07(0.03) = 0.00210\ \text{m}^2$$

$$\int ds = 2(0.07) + 2(0.03) = 0.200\ \text{m}$$



Allowable Average Shear Stress:

$$\tau_{\text{avg}} = \tau_{\text{allow}} = \frac{T}{2\,t\,A_m}$$

$$80(10^6) = \frac{T}{2(0.01)(0.00210)}$$

$$T = 3360\ \text{N}\cdot\text{m} = 3.36\ \text{kN}\cdot\text{m} \qquad \textbf{Ans.}$$

Angle of Twist:

$$\phi = \frac{TL}{4\,A_m^2 G}\int \frac{ds}{t} = \frac{3360(4)(0.200)}{4\left(0.00210^2\right)(75.0)\left(10^9\right)(0.01)}$$

$$= 0.2032\ \text{rad} = 11.6^\circ \qquad \textbf{Ans.}$$

Ans:
$T = 3.36\ \text{kN}\cdot\text{m}$,
$\phi = 11.6^\circ$

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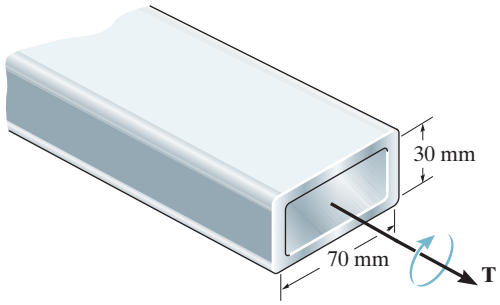
5–117.

The 304 stainless steel tube has a thickness of 10 mm. If the applied torque is $T = 50\ \text{N} \cdot \text{m}$, determine the average shear stress in the tube. The mean dimensions are shown.

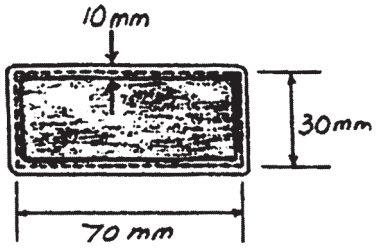


SOLUTION

Section Properties:

$$A_m = 0.07(0.03) = 0.00210\ \text{m}^2$$

Average Shear Stress:



$$\tau_{\text{avg}} = \frac{T}{2\,t\,A_m}$$

$$= \frac{50}{2(0.01)(0.00210)}$$

$$= 1.19\ \text{MPa}$$ **Ans.**

Ans:
$\tau_{\text{avg}} = 1.19\ \text{MPa}$

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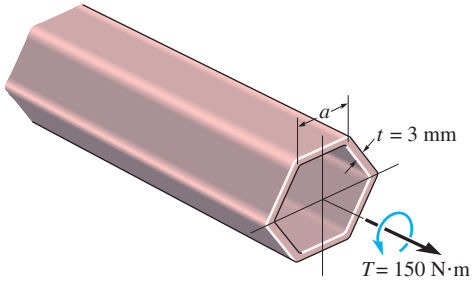
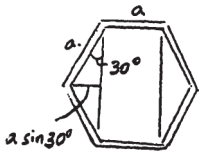
5–118.

The plastic hexagonal tube is subjected to a torque of 150 N · m. Determine the mean dimension a of its sides if the allowable shear stress is $\tau_{\text{allow}} = 60$ MPa. Each side has a thickness of $t = 3$ mm.



SOLUTION

$$A_m = 4\left[\frac{1}{2}(a\cos 30°)(a\sin 30°)\right] + (a)(2a)\cos 30° = 2.5981\,a^2$$



$$\tau_{\text{avg}} = \tau_{\text{allow}} = \frac{T}{2\,t\,A_m}$$

$$60(10^6) = \frac{150}{(2)(0.003)(2.5981\,a^2)}$$

$$a = 0.01266\text{ m} = 12.7\text{ mm}$$ **Ans.**

Ans:
$a = 12.7$ mm

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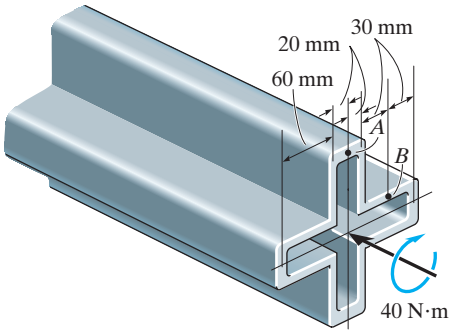
5–119.

The symmetric tube is made from a high-strength steel, having the mean dimensions shown and a thickness of 5 mm. If it is subjected to a torque of $T = 40\ \text{N}\cdot\text{m}$, determine the average shear stress developed at points A and B. Indicate the shear stress on volume elements located at these points.

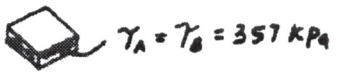


SOLUTION

$$A_m = 4(0.04)(0.06) + (0.04)^2 = 0.0112\ \text{m}^2$$

$$\tau_{\text{avg}} = \frac{T}{2\,t A_m}$$

$$(\tau_{\text{avg}})_A = (\tau_{\text{avg}})_B = \frac{40}{2(0.005)(0.0112)} = 357\ \text{kPa}$$ **Ans.**



Ans:
$(\tau_{\text{avg}})_A = (\tau_{\text{avg}})_B = 357\ \text{kPa}$

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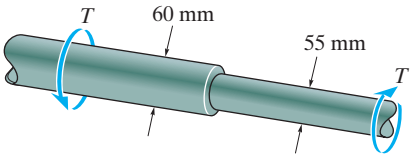
***5–120.**

The stepped shaft is subjected to a torque **T** that produces yielding on the surface of the larger diameter segment. Determine the radius of the elastic core produced in the smaller diameter segment. Neglect the stress concentration at the fillet.



SOLUTION

Maximum Elastic Torque: For the larger diameter segment

$$\tau_Y = \frac{T_Y c}{J}$$

$$T_Y = \frac{\tau_Y J}{c}$$

$$= \frac{\tau_Y \left(\frac{\pi}{2}\right)\left(0.03^4\right)}{0.03}$$

$$= 13.5\left(10^{-6}\right)\pi\tau_Y$$

Elastic - Plastic Torque: For the smaller diameter segment
$T_P = \frac{2\pi}{3}\tau_Y c^3 = \frac{2\pi}{3}\tau_Y\left(0.0275^3\right) = 13.86\left(10^{-6}\right)\pi\tau_Y > 13.5\left(10^{-6}\right)\pi\tau_Y$.
Applying Eq. 5–26 of the text, we have

$$T = \frac{\pi\,\tau_Y}{6}\left(4\,c^3 - \rho_Y^3\right)$$

$$13.5\left(10^{-6}\right)\pi\tau_Y = \frac{\pi\,\tau_Y}{6}\left[4\left(0.0275^3\right) - \rho_Y^3\right]$$

$$\rho_Y = 0.01298\text{ m} = 13.0\text{ mm}$$ **Ans.**

Ans:
$\rho_Y = 13.0$ mm

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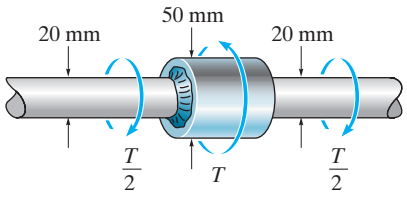
5–121.

The steel step shaft has an allowable shear stress of $\tau_{allow} = 8$ MPa. If the transition between the cross sections has a radius $r = 4$ mm, determine the maximum torque T that can be applied.



SOLUTION

Allowable Shear Stress:

$$\frac{D}{d} = \frac{50}{20} = 2.5 \qquad \text{and} \qquad \frac{r}{d} = \frac{4}{20} = 0.20$$

From the text, $K = 1.25$

$$\tau_{max} = \tau_{allow} = K\frac{Tc}{J}$$

$$8(10^6) = 1.25\left[\frac{\frac{T}{2}(0.01)}{\frac{\pi}{2}(0.01^4)}\right]$$

$$T = 20.1\ \text{N}\cdot\text{m} \qquad \textbf{Ans.}$$

Ans:
$T = 20.1\ \text{N}\cdot\text{m}$

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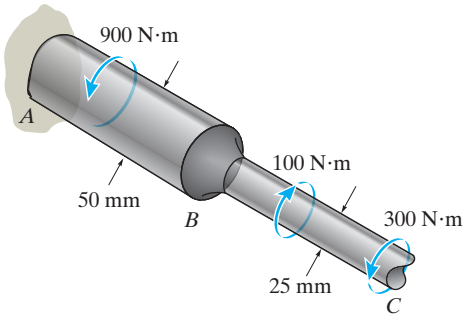
5–122.

The shaft is fixed to the wall at A and is subjected to the torques shown. Determine the maximum shear stress in the shaft. A fillet weld having a radius of 2.75 mm is used to connect the shafts at B.



SOLUTION

Internal Torque: The internal torque in regions CD, BD and AE are indicated in their respective FBD, Fig. a,

Maximum Shear Stress: For region CD,

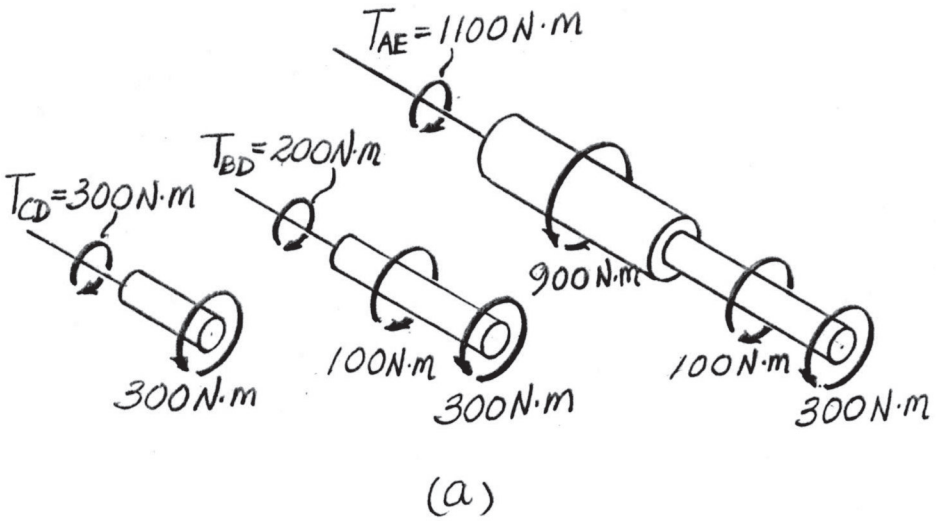
$$(\tau_{\max})_{CD} = \frac{T_{CD}C}{J} = \frac{300(0.0125)}{\frac{\pi}{2}(0.0125^4)} = 97.78(10^6)\ \text{Pa} = 97.8\ \text{MPa}\ (\textbf{Max.}) \quad \textbf{Ans.}$$

For Region AE,

$$(\tau_{\max})_{AE} = \frac{T_{AE}C}{J} = \frac{1100(0.025)}{\frac{\pi}{2}(0.025^4)} = 44.82(10^6)\ \text{Pa} = 44.8\ \text{MPa}$$

For the fillet, enter $\frac{D}{d} = \frac{50}{25} = 2$ and $\frac{r}{d} = \frac{2.75}{25} = 0.11$ into Fig. 5-32, we get $K = 1.375$.

$$(\tau_{\max})_f = K\left(\frac{T_{BD}C}{J}\right) = 1.375\left[\frac{200(0.0125)}{\frac{\pi}{2}(0.0125^4)}\right] = 89.64(10^6)\text{Pa} = 89.6\ \text{MPa}$$



(a)

Ans:
$(\tau_{\max}) = 97.8\ \text{MPa}$

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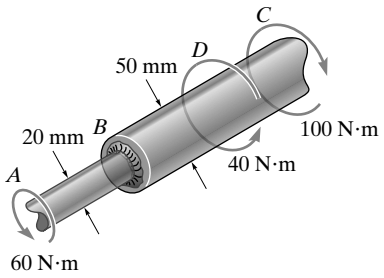
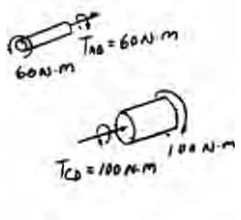
5–123. The steel shaft is made from two segments: AB and BC, which are connected using a fillet weld having a radius of 2.8 mm. Determine the maximum shear stress developed in the shaft.



SOLUTION

$$(\tau_{\max})_{CD} = \frac{T_{CD}c}{J} = \frac{100(0.025)}{\frac{\pi}{2}(0.025^4)}$$

$$= 4.07 \text{ MPa}$$



For the fillet:

$$\frac{D}{d} = \frac{50}{20} = 2.5; \qquad \frac{r}{d} = \frac{2.8}{20} = 0.14$$

From Fig. 5-32, $K = 1.325$

$$(\tau_{\max})_f = K\frac{T_{AB}c}{J} = 1.325\left[\frac{60(0.01)}{\frac{\pi}{2}(0.01^4)}\right]$$

$$= 50.6 \text{ MPa (max)} \qquad \textbf{Ans.}$$

Ans.
$(\tau_{\max})_f = 50.6$ MPa (max)

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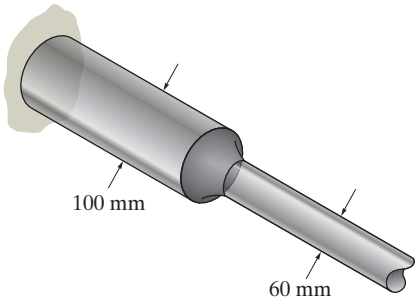
***5–124.**

The built-up shaft is to be designed to rotate at 450 rpm while transmitting 230 kW of power. Is this possible? The allowable shear stress is $\tau_{\text{allow}} = 150$ MPa.



SOLUTION

Internal Torque: Here,

$$\omega = \left(450\,\frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right) = 15\pi\text{ rad/s}$$

Then,

$$T = \frac{P}{\omega} = \frac{230\left(10^3\right)}{15\,\pi} = 4880.75\text{ N}\cdot\text{m}$$

Maximum Shear Stress: At the fillet,

$$\tau_{\max} = \tau_{\text{allow}} = K\left(\frac{T_C}{J}\right)$$

$$150\left(10^6\right) = K\left[\frac{4880.75\,(0.03)}{\frac{\pi}{2}\left(0.03^4\right)}\right]$$

$$K = 1.30$$

Enter this value and $\frac{D}{d} = \frac{100}{60} = 1.67$ into Fig. 5-32, we get $\frac{r}{d} = 0.14$. Thus,

$$\frac{r}{60} = 0.14\,; \qquad r = 8.4\text{ mm}$$

Since $\frac{D-d}{2} = \frac{100-60}{2} = 20\text{ mm} > r = 8.4\text{ mm}$, **it is possible to construct the transition.** **Ans.**

Ans:
It is possible.

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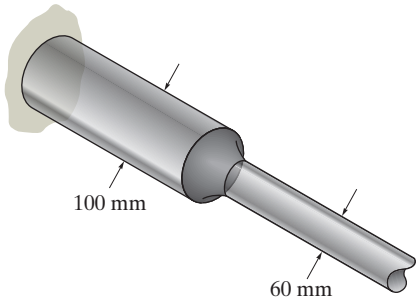
5–125.

The built-up shaft is designed to rotate at 450 rpm. If the radius of the fillet weld connecting the shafts is $r = 13.2$ mm, and the allowable shear stress for the material is $\tau_{\text{allow}} = 150$ MPa, determine the maximum power the shaft can transmit.



SOLUTION

Internal Torque: Here,

$$\omega = \left(450\,\frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right) = 15\pi\text{ rad/s}$$

Then,

$$T = \frac{P}{\omega} = \frac{P}{15\,\pi}$$

Maximum Shear Stress: Enter $\frac{D}{d} = \frac{100}{60} = 1.67$ and $\frac{r}{d} = 0.22$ into Fig 5-32, we get $K = 1.2$. Then

$$\tau_{\max} = \tau_{\text{allow}} = K\left(\frac{T_C}{J}\right)$$

$$150\left(10^6\right) = 1.2\left[\frac{\frac{P}{15\,\pi}(0.03)}{\frac{\pi}{2}\left(0.03^4\right)}\right]$$

$$P = 249.82\left(10^3\right)\text{W} = 250\text{ kW}$$ **Ans.**

Ans:
$P = 250$ kW

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5–126.

A solid shaft has a diameter of 40 mm and length of 1 m. It is made from an elastic-plastic material having a yield stress of $\tau_Y = 100$ MPa. Determine the maximum elastic torque T_Y and the corresponding angle of twist. What is the angle of twist if the torque is increased to $T = 1.2T_Y$? $G = 80$ GPa.

SOLUTION

Maximum elastic torque T_Y,

$$\tau_Y = \frac{T_Y c}{J}$$

$$T_Y = \frac{\tau_Y J}{c} = \frac{100(10^6)\left(\frac{\pi}{2}\right)(0.02^4)}{0.02} = 1256.64\ \text{N}\cdot\text{m} = 1.26\ \text{kN}\cdot\text{m} \qquad \textbf{Ans.}$$

Angle of twist:

$$\gamma_Y = \frac{\tau_Y}{G} = \frac{100(10^6)}{80(10^9)} = 0.00125\ \text{rad}$$

$$\phi = \frac{\gamma_Y}{\rho_Y} L = \frac{0.00125}{0.02}(1) = 0.0625\ \text{rad} = 3.58^\circ \qquad \textbf{Ans.}$$

Also,

$$\phi = \frac{T_Y L}{JG} = \frac{1256.64(1)}{\frac{\pi}{2}(0.02^4)(80)(10^9)} = 0.0625\ \text{rad} = 3.58^\circ$$

From Eq. 5–26 of the text,

$$T = \frac{\pi\tau_Y}{6}\left(4c^3 - \rho_Y^3\right); \qquad 1.2(1256.64) = \frac{\pi(100)(10^6)}{6}\left[4\left(0.02^3\right) - \rho_Y^3\right]$$

$$\rho_Y = 0.01474\ \text{m}$$

$$\phi' = \frac{\gamma_Y}{\rho_Y} L = \frac{0.00125}{0.01474}(1) = 0.0848\ \text{rad} = 4.86^\circ \qquad \textbf{Ans.}$$

Ans:
$T_Y = 1.26\ \text{kN}\cdot\text{m}$,
$\phi = 3.58^\circ$,
$\phi' = 4.86^\circ$

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5–127.

Determine the torque needed to twist a short 2-mm-diameter steel wire through several revolutions if it is made from steel assumed to be elastic perfectly plastic and having a yield stress of $\tau_Y = 50$ MPa. Assume that the material becomes fully plastic.

SOLUTION

Fully plastic torque is applied.

$$T_P = \frac{2\pi}{3}\tau_Y c^3 = \frac{2\pi}{3}(50)(10^6)(0.001^3) = 0.105\ \text{N}\cdot\text{m}$$ **Ans.**

Ans:
$T_P = 0.105\ \text{N}\cdot\text{m}$

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5–128. A solid shaft is subjected to the torque T, which causes the material to yield. If the material is elastic plastic, show that the torque can be expressed in terms of the angle of twist ϕ of the shaft as $T = \frac{4}{3}T_Y(1 - \phi^3{}_Y/4\phi^3)$, where T_Y and ϕ_Y are the torque and angle of twist when the material begins to yield.

SOLUTION

$$\phi = \frac{\gamma L}{\rho} = \frac{\gamma_Y}{\rho_Y}L$$

$$\rho_Y = \frac{\gamma_Y L}{\phi} \qquad \textbf{(1)}$$

When $\rho_Y = c, \phi = \phi_Y$

From Eq. (1),

$$c = \frac{\gamma_Y L}{\phi_Y} \qquad \textbf{(2)}$$

Dividing Eq. (1) by Eq. (2) yields:

$$\frac{\rho_Y}{c} = \frac{\phi_Y}{\phi} \qquad \textbf{(3)}$$

Use Eq. 5-26 from the text.

$$T = \frac{\pi\,\tau_Y}{6}(4\,c^3 - \rho_Y^3) = \frac{2\pi\,\tau_Y c^3}{3}\left(1 - \frac{\rho_Y^3}{4\,c^3})\right)$$

Use Eq. 5-24, $T_Y = \frac{\pi}{2}\tau_Y c^3$ from the text and Eq. (3)

$$T = \frac{4}{3}T_Y\left(1 - \frac{\phi_Y^3}{4\,\phi^3}\right) \qquad \textbf{QED}$$

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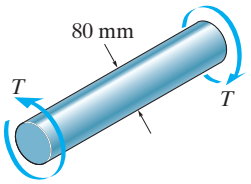
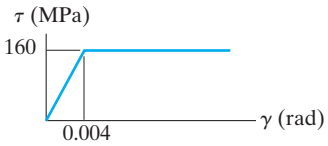
5–129.

The solid shaft is made of an elastic perfectly plastic material. Determine the torque T needed to form an elastic core in the shaft having a radius of $\rho_Y = 20$ mm. If the shaft is 3 m long, through what angle does one end of the shaft twist with respect to the other end? When the torque is removed, determine the residual stress distribution in the shaft and the permanent angle of twist.





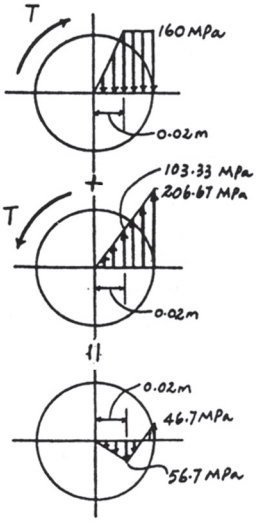
SOLUTION

Elastic-Plastic Torque:

$$T = \frac{\pi\,\tau_Y}{6}\left(4c^3 - \rho_Y^3\right)$$

$$= \frac{\pi(160)(10^6)}{6}\left[4\left(0.04^3\right) - 0.02^3\right]$$

$$= 20776.40\ \text{N}\cdot\text{m} = 20.8\ \text{kN}\cdot\text{m} \qquad \textbf{Ans.}$$



Angle of Twist:

$$\phi = \frac{\gamma_Y}{\rho_Y}L = \left(\frac{0.004}{0.02}\right)(3) = 0.600\ \text{rad} = 34.4^\circ \qquad \textbf{Ans.}$$

When the reverse $T = 20776.4\ \text{N}\cdot\text{m}$ is applied,

$$G = \frac{160(10^6)}{0.004} = 40\ \text{GPa}$$

$$\phi' = \frac{TL}{JG} = \frac{20776.4(3)}{\frac{\pi}{2}(0.04^4)(40)(10^9)} = 0.3875\ \text{rad}$$

The permanent angle of twist is,

$$\phi_r = \phi - \phi'$$

$$= 0.600 - 0.3875 = 0.2125\ \text{rad} = 12.2^\circ \qquad \textbf{Ans.}$$

Residual Shear Stress:

$$(\tau')_{\rho=c} = \frac{Tc}{J} = \frac{20776.4(0.04)}{\frac{\pi}{2}(0.04^4)} = 206.67\ \text{MPa}$$

$$(\tau')_{\rho=0.02\ \text{m}} = \frac{Tc}{J} = \frac{20776.4(0.02)}{\frac{\pi}{2}(0.04^4)} = 103.33\ \text{MPa}$$

$$(\tau_r)_{\rho=c} = -160 + 206.67 = 46.7\ \text{MPa}$$

$$(\tau_r)_{\rho=0.02\ \text{m}} = -160 + 103.33 = -56.7\ \text{MPa} \qquad \textbf{Ans.}$$

Ans:
$T = 20.8\ \text{kN}\cdot\text{m}$, $\phi = 34.4^\circ$, $(\tau_r)_{\max} = 56.7\ \text{MPa}$, $\phi_r = 12.2^\circ$

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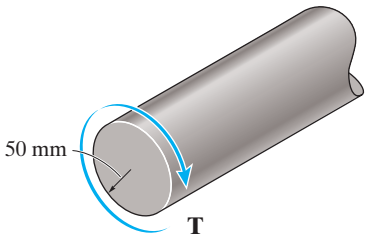
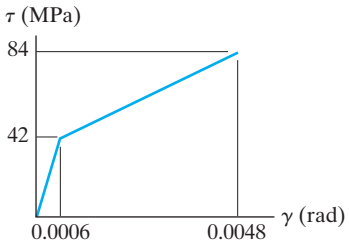
5–130. The shaft is subjected to a maximum shear strain of 0.0048 rad. Determine the torque applied to the shaft if the material has strain hardening as shown by the shear stress–strain diagram.





SOLUTION

From the shear - strain diagram,

$$\frac{\rho_Y}{0.0006} = \frac{0.05}{0.0048}; \qquad \rho_Y = 0.00625 \text{ m}$$

From the shear stress–strain diagram,

$$\tau_1 = \left[\frac{42(10^6)}{0.00625}\right]\rho = 6.72(10^9)\rho$$

$$\frac{\tau_2 - 42(10^6)}{\rho - 0.00625} = \frac{84(10^6) - 42(10^6)}{0.05 - 0.00625}$$

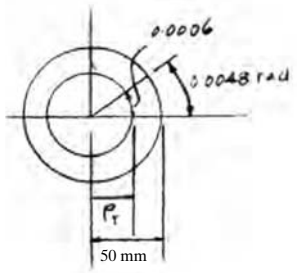
$$\tau_2 = 0.96(10^9)\rho + 36(10^6)$$

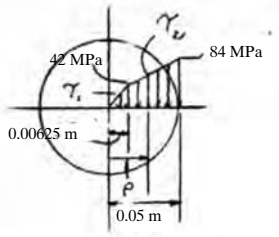
$$T = 2\pi\int_0^c \tau\rho^2 d\rho$$

$$= 2\pi\int_0^{0.00625\text{ m}} [6.72(10^9)\rho]\rho^2 d\rho + 2\pi\int_{0.00625\text{ m}}^{0.05\text{ m}} [0.96(10^9)\rho + 36(10^6)]\rho^2 d\rho$$

$$= 2\pi[1.68(10^9)\rho^4]\Big|_0^{0.00625\text{ m}} + 2\pi[0.24(10^9)\rho^4 + 12(10^6)\rho^3]\Big|_{0.00625\text{ m}}^{0.05\text{ m}}$$

$$= 18.84(10^3)\text{ N}\cdot\text{m} = 18.8\text{ kN}\cdot\text{m}$$ **Ans.**





Ans:

$T = 18.8\text{ kN}\cdot\text{m}$

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5–131. A solid shaft having a diameter of 50 mm is made of elastic-plastic material having a yield stress of $\tau_Y = 112$ MPa and shear modulus of $G = 84$ GPa. Determine the torque required to developo an elastic core in the shaft having a diameter of 25 mm. Also, what is the plasstic torque?

SOLUTION

Use Eq. 5-26 from the text:

$$T = \frac{\pi \tau_Y}{6}(4c^3 - \rho_Y{}^3) = \frac{\pi[112(10^6)]}{6}[4(0.025^3) - 0.0125^3]$$

$$= 3.551(10^3)\ \text{N}\cdot\text{m} = 3.55\ \text{kN}\cdot\text{m} \qquad \textbf{Ans.}$$

Use Eq. 5-27 from the text:

$$T_P = \frac{2\pi}{3}\tau_Y c^3 = \frac{2\pi}{3}[112(10^6)](0.025^3)$$

$$= 3.665(10^3)\ \text{N}\cdot\text{m} = 3.67\ \text{kN}\cdot\text{m} \qquad \textbf{Ans.}$$

Ans.
$T = 3.55\ \text{kN}\cdot\text{m}$, $T_P = 3.67\ \text{kN}\cdot\text{m}$

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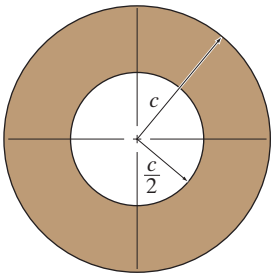
***5–132.**

The hollow shaft has the cross section shown and is made of an elastic perfectly plastic material having a yield shear stress of τ_Y. Determine the ratio of the plastic torque T_p to the maximum elastic torque T_Y.



SOLUTION

Maximum Elastic Torque: In this case, the torsion formula is still applicable.

$$\tau_Y = \frac{T_Y c}{J}$$

$$T_Y = \frac{J}{c}\tau_Y$$

$$= \frac{\frac{\pi}{2}\left[c^4 - \left(\frac{c}{2}\right)^4\right]\tau_Y}{c}$$

$$= \frac{15}{32}\pi c^3 \tau_Y$$

Plastic Torque: Using the general equation, with $\tau = \tau_Y$,

$$T_P = 2\pi\tau_Y \int_{c/2}^{c} \rho^2 d\rho$$

$$= 2\pi\tau_Y \left(\frac{\rho^3}{3}\right)\Big|_{c/2}^{c}$$

$$= \frac{7}{12}\pi c^3 \tau_Y$$

The ratio is

$$\frac{T_P}{T_Y} = \frac{\frac{7}{12}\pi c^3 \tau_Y}{\frac{15}{32}\pi c^3 \tau_Y} = 1.24 \qquad \textbf{Ans.}$$

Ans:
$\frac{T_P}{T_Y} = 1.24$

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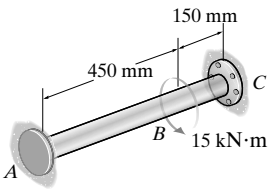
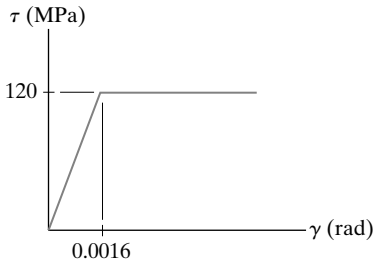
5–133. The hollow shaft has inner and outer diameters of 60 mm and 80 mm, respectively. If it is made of an elastic-perfectly plastic material, which has the $\tau-\gamma$ diagram shown, determine the reactions at the fixed supports A and C.





SOLUTION

Equation of Equilibrium. Refering to the free - body diagram of the shaft shown in Fig. *a*,

$$\Sigma M_x = 0;\ T_A + T_C - 15\left(10^3\right) = 0 \qquad \textbf{(1)}$$

Elastic Analysis. It is required that $\phi_{B/A} = \phi_{B/C}$. Thus, the compatibility equation is

$$\phi_{B/A} = \phi_{B/C}$$

$$\frac{T_A L_{AB}}{JG} = \frac{T_C L_{BC}}{JG}$$

$$T_A(0.45) = T_C(0.15)$$

$$T_C = 3T_A \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2),

$$T_A = 3750\ \text{N}\cdot\text{m} \qquad T_C = 11\,250\text{N}\cdot\text{m}$$

The maximum elastic torque and plastic torque in the shaft can be determined from

$$T_Y = \frac{J}{c}\tau_Y = \left[\frac{\frac{\pi}{2}\left(0.04^4 - 0.03^4\right)}{0.04}\right](120)\left(10^6\right) = 8246.68\,\text{N}\cdot\text{m}$$

$$T_P = 2\pi\tau_Y \int_{c_i}^{c_o} \rho^2 d\rho$$

$$= 2\pi(120)\left(10^6\right)\left(\frac{\rho^3}{3}\right)\Bigg|_{0.03\text{ m}}^{0.04\text{ m}} = 9299.11\,\text{N}\cdot\text{m}$$

Since $T_C > T_Y$, the results obtained using the elastic analysis are not valid.

Plastic Analysis. Assuming that segment BC is fully plastic,

$$T_C = T_P = 9299.11\ \text{N}\cdot\text{m} = 9.3\ \text{kN}\cdot\text{m} \qquad \textbf{Ans.}$$

Substituting this result into Eq. (1),

$$T_A = 5700\ \text{N}\cdot\text{m} = 5.70\ \text{kN}\cdot\text{m} \qquad \textbf{Ans.}$$

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5–133. Continued

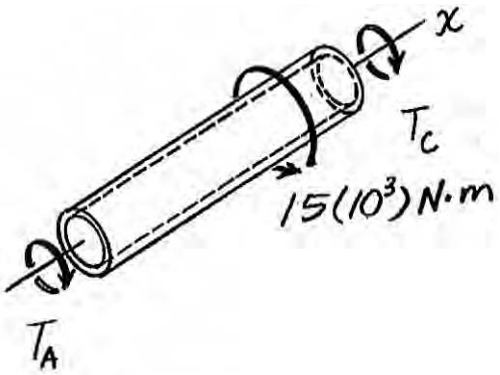
Since $T_A < T_Y$, segment AB of the shaft is still linearly elastic. Here, $G = \frac{120(10^6)}{0.0016} = 75\text{GPa}$.

$$\phi_{B/C} = \phi_{B/A} = \frac{T_A L_{AB}}{JG} = \frac{5700.89(0.45)}{\frac{\pi}{2}\left(0.04^4 - 0.03^4\right)(75)(10^9)} = 0.01244 \text{ rad}$$

$$\phi_{B/C} = \frac{\gamma_i}{c_i} L_{BC}; \qquad 0.01244 = \frac{\gamma_i}{0.03}(0.15)$$

$$\gamma_i = 0.002489 \text{ rad}$$

Since $\gamma_i > \gamma_Y$, segment BC of the shaft is indeed fully plastic.



(a)

Ans.

$T_A = 5.70 \text{ kN} \cdot \text{m}, \; T_C = 9.3 \text{ kN} \cdot \text{m}$

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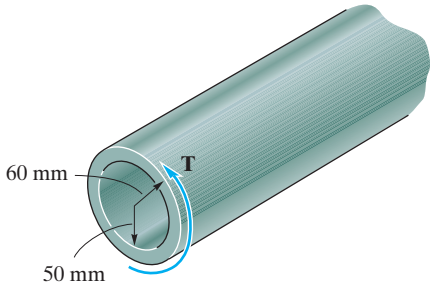
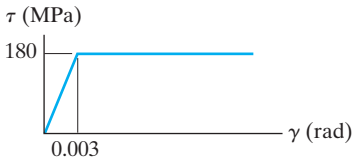
5–134.

The 2-m-long tube is made of an elastic perfectly plastic material as shown. Determine the applied torque T, which subjects the material at the tube's outer edge to a shear strain of $\gamma_{max} = 0.006$ rad. What would be the permanent angle of twist of the tube when this torque is removed? Sketch the residual stress distribution in the tube.



SOLUTION



Plastic Torque: The tube will be fully plastic if $\gamma_i \geq \gamma_Y = 0.003$ rad. From Fig. *a*,

$$\frac{\gamma_i}{0.05} = \frac{0.006}{0.06}; \qquad \gamma_i = 0.005 \text{ rad} > \gamma_Y$$

The tube indeed is fully plastic. Then

$$T_p = 2\pi \int_{c_i}^{c_0} \tau_Y \rho^2 dp$$

$$= \frac{2}{3}\pi\tau_Y \left(C_0^3 - C_i^3\right)$$

$$= \frac{2}{3}\pi\left[180(10^6)\right](0.06^3 - 0.05^3)$$

$$= 34.306(10^3)\text{N}\cdot\text{m} = 34.3 \text{ kN}\cdot\text{m} \qquad \textbf{Ans.}$$

Angle of Twist:

$$\phi_P = \frac{\gamma_{max}}{c_0} L = \left(\frac{0.006}{0.06}\right)(2) = 0.2 \text{ rad}$$

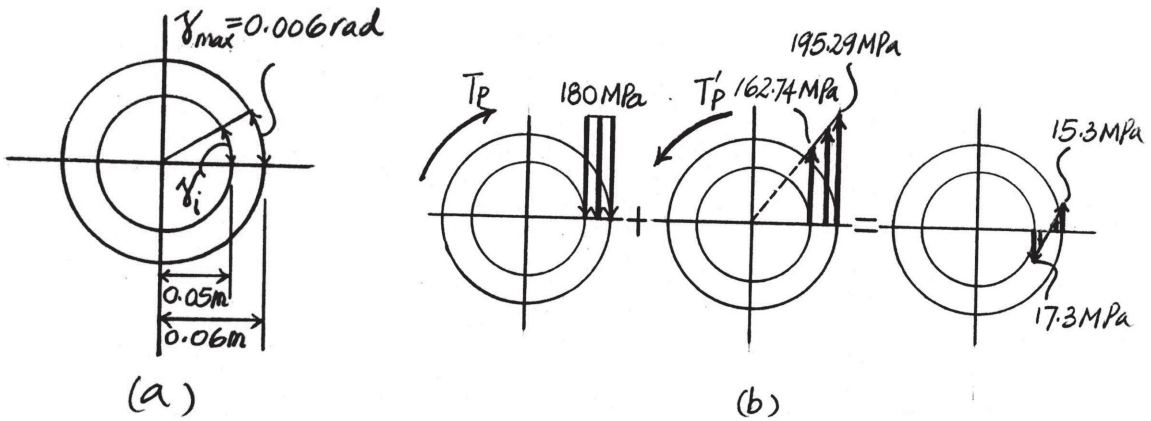
Here, $G = \dfrac{\tau_Y}{\gamma_Y} = \dfrac{180(10^6)}{0.003} = 60(10^9)$Pa. When a reverse torque of

$T'_P = 34.306(10^3)$ N · m is applied

$$\phi'_P = \frac{T_P' L}{JG} = \frac{34.306(10^3)(2)}{\frac{\pi}{2}(0.06^4 - 0.05^4)\left[60(10^9)\right]} = 0.1085 \text{ rad}$$

Thus, the permanent angle of twist is

$$\phi_r = \phi_p - \phi'_p = 0.2 - 0.1085 = (0.09151 \text{ rad})\left(\frac{180^\circ}{\pi \text{ rad}}\right) = 5.24^\circ \qquad \textbf{Ans.}$$



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5–134. Continued

Residual Shear Stress:

$$(\tau'_P)_o = \frac{T'_P C}{J} = \frac{34.306(10^3)(0.06)}{\frac{\pi}{2}(0.06^4 - 0.05^4)} = 195.29(10^6)\text{Pa} = 195.29\text{ MPa}$$

$$(\tau'_P)_o = \frac{T'_P \rho}{J} = \frac{34.306(10^3)(0.05)}{\frac{\pi}{2}(0.06^4 - 0.05^4)} = 162.74(10^6)\text{Pa} = 162.74\text{ MPa}$$

Thus

$$(\tau_r)_o = -\tau_Y + (\tau'_p)_o = -180 + 195.29 = 15.3\text{ MPa}$$ **Ans.**

$$(\tau_r)_i = -\tau_Y + (\tau'_P)_i = -180 + 162.74 = -17.3\text{ MPa}$$ **Ans.**

The sketch of the residual shear stress distribution is shown in Fig. *b*.

Ans:
$T_P = 34.3\text{ kN}\cdot\text{m}$,
$\phi_r = 5.24°$,
$(\tau_r)_o = 15.3\text{ MPa}$,
$(\tau_r)_i = -17.3\text{ MPa}$

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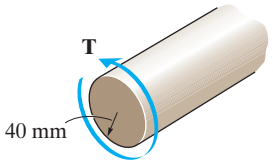
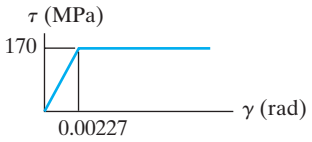
5–135.

The shaft is made of an elastic perfectly plastic material as shown. Plot the shear-stress distribution acting along a radial line if it is subjected to a torque of $T = 20\ \text{kN}\cdot\text{m}$. What is the residual stress distribution in the shaft when the torque is removed?





SOLUTION

Elastic - Plastic Torque: The maximum elastic torque is

$$T_Y = \frac{\pi}{2}\tau_Y C^3 = \frac{\pi}{2}\left[170(10^6)\right](0.04^3) = 17.09\,(10^3)\text{N}\cdot\text{m}$$

And the plastic torque is

$$T_p = \frac{2}{3}\pi\,\tau_Y C^3 = \frac{2}{3}\pi\left[170(10^6)\right](0.04^3) = 22.79\,(10^3)\text{N}\cdot\text{m}$$

Since $T_Y < T < T_P$, the shaft exhibits elastic-plastic behavior.

$$T = \frac{\pi\tau_Y}{6}(4C^3 - \rho_Y^3)$$

$$20(10^3) = \frac{\pi[170(10^6)]}{6}[4(0.04^3) - \rho_Y^3]$$

$$\rho_Y = 0.03152\ \text{m} = 31.5\ \text{mm}$$

Residual shear stress: When the reverse torque $T' = 20\ \text{kN}\cdot\text{m}$ is applied,

$$\tau'_C = \frac{T'C}{J} = \frac{20(10^3)(0.04)}{\frac{\pi}{2}(0.04^4)} = 198.94(10^6)\text{Pa} = 198.94\ \text{MPa}$$

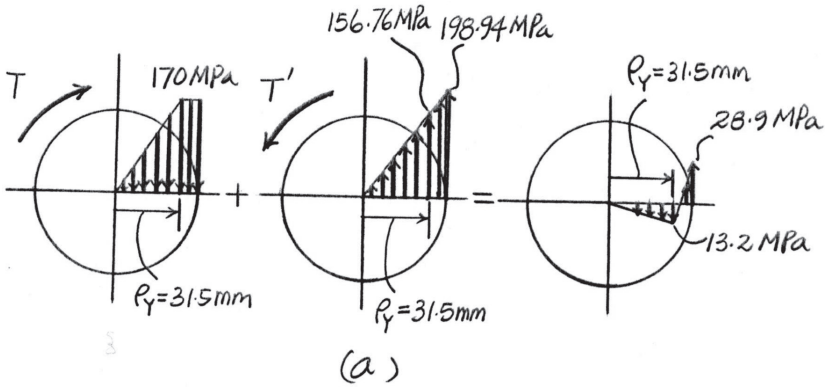
$$\tau'_{\rho Y} = \frac{T'\rho_Y}{J} = \frac{20(10^3)(0.03152)}{\frac{\pi}{2}(0.04^4)} = 156.76(10^6)\text{Pa} = 156.76\ \text{MPa}$$

Thus,

$$(\tau_r)_c = -\tau_Y + \tau'_C = -170 + 198.94 = 28.9\ \text{MPa}$$ **Ans.**

$$(\tau_r)_{\rho_Y} = -\tau_Y + \tau'_{\rho_Y} = -170 + 156.76 = -13.2\ \text{MPa}$$ **Ans.**

The sketch of the residual shear stress distribution is shown in Fig. *b*.



Ans:
$(\tau_r)_c = 28.9\ \text{MPa}$,
$(\tau_r)\rho_Y = -13.2\ \text{MPa}$

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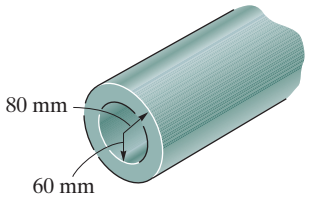
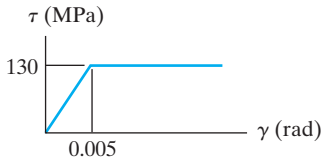
***5–136.**

The tube has a length of 2 m and is made of an elastic perfectly plastic material as shown. Determine the torque needed to just cause the material to become fully plastic. What is the permanent angle of twist of the tube when this torque is removed?





SOLUTION

Plastic Torque:

$$T_p = 2\pi \int_{c_i}^{c_o} \tau_Y \rho^2 dp$$

$$= \frac{2}{3}\pi\tau_Y \left(C_0^3 - C_i^3\right)$$

$$= \frac{2}{3}\pi\left[130\left(10^6\right)\right]\left(0.08^3 - 0.06^3\right)$$

$$= 80.592\left(10^3\right)\text{N}\cdot\text{m} = 80.6\ \text{KN}\cdot\text{m} \qquad \textbf{Ans.}$$

Angle of Twist: Here, $\rho_Y = C_i = 0.06$ m.

$$\phi_P = \frac{\gamma_Y}{\rho_Y} L = \left(\frac{0.005}{0.06}\right)(2) = 0.16667 \text{ rad}$$

Here, $G = \dfrac{\tau_Y}{\gamma_Y} = \dfrac{130\left(10^6\right)}{0.005} = 26\left(10^9\right)$ Pa. When a reverse torque of $T'_P = 80.592\left(10^3\right)$ N · m is applied,

$$\phi'_P = \frac{T'_P L}{JG} = \frac{80.592\left(10^3\right)(2)}{\frac{\pi}{2}\left(0.08^4 - 0.06^4\right)\left[26\left(10^9\right)\right]} = 0.14095 \text{ rad}$$

Thus, the permanent angle of twist is

$$\phi_r = \phi_p - \phi'_p = 0.16667 - 0.14095$$

$$= (0.02571 \text{ rad})\left(\frac{180^\circ}{\pi \text{ rad}}\right) = 1.47^\circ \qquad \textbf{Ans.}$$

Ans:
$T_P = 80.6\ \text{kN}\cdot\text{m}$,
$\phi_r = 1.47^\circ$

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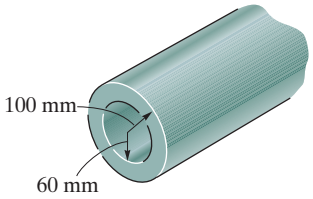
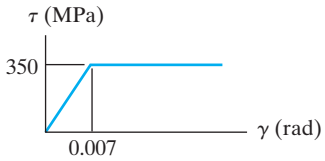
5–137.

The tube has a length of 2 m and is made of an elastic perfectly plastic material as shown. Determine the torque needed to just cause the material to become fully plastic. What is the permanent angle of twist of the tube when this torque is removed?





SOLUTION

Plastic Torque:

$$T_P = 2\pi \int_{c_i}^{c_o} \tau_Y \rho^2 \, d\rho$$

$$= \frac{2\pi\, \tau_Y}{3}\left(c_o^3 - c_i^3\right)$$

$$= \frac{2\pi(350)(10^6)}{3}\left(0.05^3 - 0.03^3\right)$$

$$= 71837.75\ \text{N}\cdot\text{m} = 71.8\ \text{kN}\cdot\text{m} \qquad \textbf{Ans.}$$

Angle of twist:

$$\phi_\rho = \frac{\gamma_Y}{\rho_Y} L \qquad \text{Where } \rho_Y = c_i = 0.03\ \text{m}$$

$$= \left(\frac{0.007}{0.03}\right)(2) = 0.4667\ \text{rad}$$

When a reverse $T_P = 71837.75\ \text{N}\cdot\text{m}$ is applied.

$$G = \frac{350(10^6)}{0.007} = 50\ \text{GPa}$$

$$\phi_P' = \frac{T_P L}{JG} = \frac{71837.75(2)}{\frac{\pi}{2}\left(0.05^4 - 0.03^4\right)50(10^9)} = 0.3363\ \text{rad}$$

Permanent angle of twist:

$$\phi_r = \phi_P - \phi_P' = 0.4667 - 0.3363$$

$$= 0.1304\ \text{rad} = 7.47° \qquad \textbf{Ans.}$$

Ans:
$T_P = 71.8\ \text{kN}\cdot\text{m}$,
$\phi_r = 7.47°$

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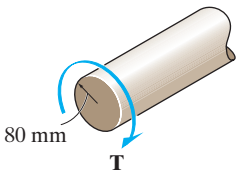
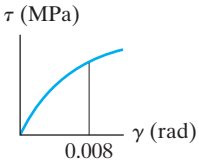
5–138.

A torque is applied to the shaft having a radius of 80 mm. If the material obeys a shear stress–strain relation of $\tau = 500\ \gamma^{1/4}$ MPa, determine the torque that must be applied to the shaft so that the maximum shear strain becomes 0.008 rad.





SOLUTION

τ-ρ Function: First, relate γ to ρ.

$$\gamma = \frac{\rho}{c}\gamma_{\max} = \left(\frac{\rho}{0.08}\right)(0.008) = 0.1\rho$$

Then

$$\tau = 500(10^6)(0.1\rho)^{1/4} = 281.17(10^6)\rho^{1/4}$$

Torque:

$$T = 2\pi\int_o^c \tau\rho^2\,dp$$

$$= 2\pi\int_o^{0.08m} 281.17(10^6)\rho^{\frac{9}{4}}\,dp$$

$$= 1.7666(10^9)\left(\frac{4}{13}\rho^{13/4}\right)\Big|_0^{0.08\text{ m}}$$

$$= 148.02(10^3)\text{ N}\cdot\text{m}$$

$$= 148\text{ kN}\cdot\text{m}$$ **Ans.**

Ans:
$T = -148\text{ kN}\cdot\text{m}$

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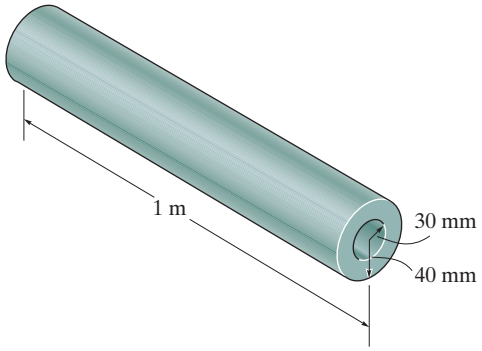
5–139.

A tubular shaft has an inner diameter of 60 mm, an outer diameter of 80 mm, and a length of 1 m. It is made of an elastic perfectly plastic material having a yield stress of $\tau_Y = 150$ MPa. Determine the maximum torque it can transmit. What is the angle of twist of one end with respect to the other end if the inner surface of the tube is about to yield? $G = 75$ GPa.



SOLUTION

Plastic Torque: The plastic torque is the maximum torque a circular shaft can transmit.

$$T_p = 2\pi \int_{C_i}^{C_o} \tau_Y p^2 dp$$

$$= 2\pi\, \tau_Y \left(\frac{p^3}{3}\right)\Big|_{C_i}^{C_o}$$

$$= \frac{2}{3}\pi\, \tau_Y (C_0^3 - C_i^3)$$

$$= \frac{2}{3}\pi \left[150(10^6)\right](0.04^3 - 0.03^3)$$

$$= 11.624\,(10^3)\ \text{N}\cdot\text{m}$$

$$= 11.6\ \text{kN}\cdot\text{m} \qquad \textbf{Ans.}$$

Angle of Twist: The yield strain at the inner surface of the tube can be determined from

$$\tau_Y = G\,\gamma_Y; \qquad 150(10^6) = 75(10^9)\,\gamma_Y$$

$$\gamma_Y = 0.002\ \text{rad}$$

Then, the angle of twist is

$$\phi = \frac{\gamma_Y}{\rho_Y} L = \left(\frac{0.002}{0.03}\right)(1)$$

$$= (0.06667\ \text{rad})\left(\frac{180^\circ}{\pi\ \text{rad}}\right) = 3.82^\circ \qquad \textbf{Ans.}$$

Ans:
$T_P = 11.6\ \text{kN}\cdot\text{m}$,
$\phi = 3.82^\circ$

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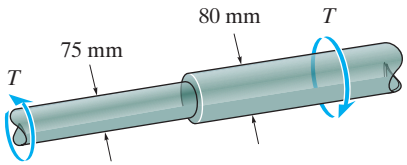
***5–140.**

The stepped shaft is subjected to a torque **T** that produces yielding on the surface of the larger diameter segment. Determine the radius of the elastic core produced in the smaller diameter segment. Neglect the stress concentration at the fillet.



SOLUTION

Maximum Elastic Torque: For the larger diameter segment, the torsion formula can be applied.

$$\tau_Y = \frac{T_Y c}{J}$$

$$T = \frac{\tau_Y J}{C} = \frac{\tau_Y\left[\frac{\pi}{2}\left(0.04^4\right)\right]}{0.04} = 32.0\left(10^{-6}\right)\pi\,\tau_Y$$

Elastic - Plastic Torque: For the smaller diameter segment, the plastic torque can be determined from

$$T_p = 2\pi\int_0^c \tau_Y\,\rho^2\,dp$$

$$= \frac{2\pi}{3}\tau_Y c^3$$

$$= \frac{2\pi}{3}\,\tau_Y\left(0.0375^3\right)$$

$$= 35.16\left(10^{-6}\right)\pi\,\tau_Y$$

Since $T < T_P$, the smaller segment is not fully plastic

$$T = \frac{\pi\,\tau_Y}{6}\left(4c^3 - \rho_Y^3\right)$$

$$32.0\left(10^{-6}\right)\pi\,\tau_Y = \frac{\pi\,\tau_Y}{6}\left[4\left(0.0375^3\right) - \rho_Y^3\right]$$

$$\rho_Y = 0.02665\text{ m}$$

$$= 26.7\text{ mm}$$ **Ans.**

Ans:
$\rho_Y = 26.7$ mm

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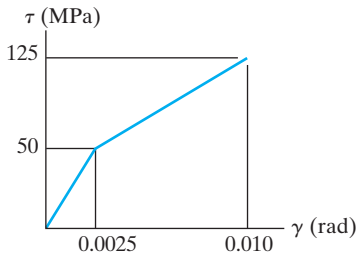
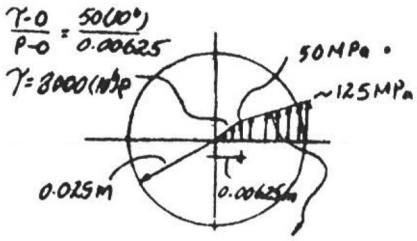
5–141.

The shear stress–strain diagram for a solid 50-mm-diameter shaft can be approximated as shown in the figure. Determine the torque required to cause a maximum shear stress in the shaft of 125 MPa. If the shaft is 3 m long, what is the corresponding angle of twist?



SOLUTION



$$\gamma = \frac{\rho}{c}\gamma_{max}$$

$$\gamma_{max} = 0.01$$

When $\gamma = 0.0025$

$$\rho = \frac{c\gamma}{\gamma_{max}}$$

$$= \frac{0.025(0.0025)}{0.010} = 0.00625$$

$$\frac{\tau - 0}{\rho - 0} = \frac{50(10^6)}{0.00625}$$

$$\tau = 8000(10^6)(\rho)$$

$$\frac{\tau - 50(10^6)}{\rho - 0.00625} = \frac{125(10^6) - 50(10^6)}{0.025 - 0.00625}$$

$$\tau = 4000(10^6)(\rho) + 25(10^6)$$

$$T = 2\pi \int_0^c \tau\rho^2\, d\rho$$

$$= 2\pi \int_0^{0.00625} 8000(10^6)\rho^3\, d\rho$$

$$+ 2\pi \int_{0.00625}^{0.025} [4000(10^6)\rho + 25(10^6)]\rho^2\, d\rho$$

$$T = 3269\ \text{N}\cdot\text{m} = 3.27\ \text{kN}\cdot\text{m}$$ **Ans.**

$$\phi = \frac{\gamma_{max}}{c}L = \frac{0.01}{0.025}(3)$$

$$= 1.20\ \text{rad} = 68.8°$$ **Ans.**

Ans:
$T = 3.27\ \text{kN}\cdot\text{m}$, $\phi = 68.8°$

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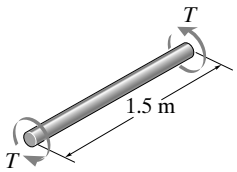
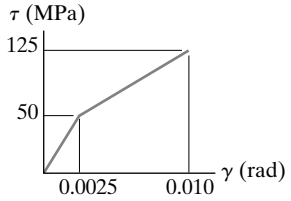
5–142. The shear stress–strain diagram for a solid 50-mm-diameter shaft can be approximated as shown in the figure. Determine the torque T required to cause a maximum shear stress in the shaft of 125 MPa. If the shaft is 1.5 m long, what is the corresponding angle of twist?





SOLUTION

Strain Diagram:

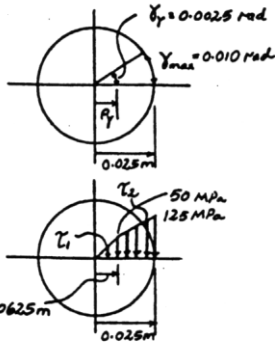
$$\frac{\rho_\gamma}{0.0025} = \frac{0.025}{0.01}; \qquad \rho_\gamma = 0.00625 \text{ m}$$

Stress Diagram:

$$\tau_1 = \frac{50(10^6)}{0.00625}\,\rho = 8(10^9)\,\rho$$

$$\frac{\tau_2 - 50(10^6)}{\rho - 0.00625} = \frac{125(10^6) - 50(10^6)}{0.025 - 0.00625}$$

$$\tau_2 = 4\left(10^9\right)\,\rho + 25\left(10^6\right)$$



The Ultimate Torque:

$$T = 2\pi \int_0^c \tau\,\rho^2 d\rho$$

$$= 2\pi \int_0^{0.00625\text{ m}} 8\left(10^9\right)\rho^3\,d\rho + 2\pi \int_{0.00625\text{ m}}^{0.025\text{ m}} \left[4\left(10^9\right)\rho + 25\left(10^6\right)\right]\rho^2 d\rho$$

$$= 2\pi\left[2\left(10^9\right)\rho^4\right]\Big|_0^{0.00625\text{m}} + 2\pi\left[1\left(10^9\right)\rho^4 + \frac{25(10^6)\rho^3}{3}\right]\Bigg|_{0.00625\text{ m}}^{0.025\text{ m}}$$

$$= 3269.30\ \text{N}\cdot\text{m} = 3.27\ \text{kN}\cdot\text{m} \qquad \textbf{Ans.}$$

Angle of Twist:

$$\phi = \frac{\gamma_{\max}}{c}L = \left(\frac{0.01}{0.025}\right)(1.5) = 0.60\ \text{rad} = 34.4^\circ \qquad \textbf{Ans.}$$

Ans.
$T = 3.27\ \text{kN}\cdot\text{m},\ \phi = 34.4^\circ$

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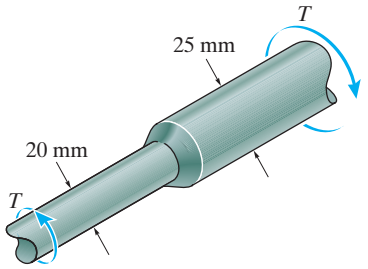
5–143. The shaft consists of two sections that are rigidly connected. If the material is elastic plastic as shown, determine the largest torque T that can be applied to the shaft. Also, draw the shear-stress distribution over a radial line for each section. Neglect the effect of stress concentration.



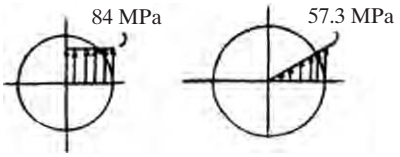
SOLUTION

20-mm diameter segment will be fully plastic. From Eq. 5-27 of the text:

$$T = T_p = \frac{2\pi}{3}\tau_Y c^3$$

$$= \frac{2\pi}{3}[84(10^6)](0.01^3)$$

$$= 175.93\ \text{N}\cdot\text{m} = 176\ \text{N}\cdot\text{m}$$ **Ans.**



For 25-mm diameter segment:

$$\tau_{\max} = \frac{Tc}{J} = \frac{175.93(0.0125)}{\frac{\pi}{2}(0.0125^4)} = 57.34(10^6)\ \text{N/m}^2$$

$$= 57.3\ \text{MPa} < \tau_Y$$

Ans.
$T = T_p = 176\ \text{N}\cdot\text{m}$

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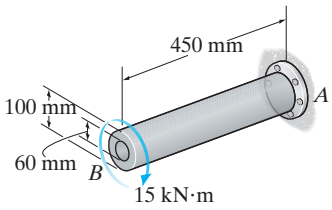
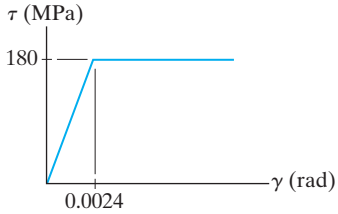
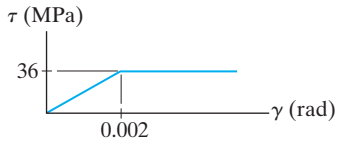
***5–144.**

A steel alloy core is bonded firmly to the copper alloy tube to form the shaft shown. If the materials have the $\tau-\gamma$ diagrams shown, determine the torque resisted by the core and the tube.





Steel Alloy



Copper Alloy

SOLUTION

Equation of Equilibrium. Referring to the free-body diagram of the cut part of the assembly shown in Fig. *a*,

$$\Sigma M_x = 0; \quad T_c + T_t - 15\left(10^3\right) = 0 \tag{1}$$

Elastic Analysis. The shear modulus of steel and copper are $G_{st} = \dfrac{180\left(10^6\right)}{0.0024} = 75 \text{ GPa}$ and $G_{\infty} = \dfrac{36\left(10^6\right)}{0.002} = 18 \text{ GPa}$. Compatibility requires that

$$\phi_C = \phi_t$$

$$\frac{T_c L}{J_c G_{st}} = \frac{T_t L}{J_t G_{\infty}}$$

$$\frac{T_c}{\frac{\pi}{2}\left(0.03^4\right)(75)\left(10^9\right)} = \frac{T_t}{\frac{\pi}{2}\left(0.05^4 - 0.03^4\right)(18)\left(10^9\right)}$$

$$T_c = 0.6204 T_t \tag{2}$$

Solving Eqs. (1) and (2),

$$T_t = 9256.95 \text{ N} \cdot \text{m} \qquad T_c = 5743.05 \text{ N} \cdot \text{m}$$

The maximum elastic torque and plastic torque of the core and the tube are

$$(T_Y)_c = \frac{1}{2}\pi c^3 (\tau_Y)_{st} = \frac{1}{2}\pi\left(0.03^3\right)(180)\left(10^6\right) = 7634.07 \text{ N} \cdot \text{m}$$

$$(T_P)_c = \frac{2}{3}\pi c^3 (\tau_Y)_{st} = \frac{2}{3}\pi\left(0.03^3\right)(180)\left(10^6\right) = 10\,178.76 \text{ N} \cdot \text{m}$$

and

$$(T_Y)_t = \frac{J}{c}\tau_Y = \left[\frac{\frac{\pi}{2}\left(0.05^4 - 0.03^4\right)}{0.05}\right]\left[(36)\left(10^6\right)\right] = 6152.49 \text{ N} \cdot \text{m}$$

$$(T_P)_t = 2\pi(\tau_Y)_{\infty}\int_{c_i}^{c_o}\rho^2 d\rho = 2\pi(36)\left(10^6\right)\left(\frac{\rho^3}{3}\right)\Big|_{0.03 \text{ m}}^{0.05 \text{ m}} = 7389.03 \text{ N} \cdot \text{m}$$

Since $T_t > (T_Y)_t$, the results obtained using the elastic analysis are not valid.

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5–144. Continued

Plastic Analysis. Assuming that the tube is fully plastic,

$$T_t = (T_P)_t = 7389.03\ \text{N}\cdot\text{m} = 7.39\ \text{kN}\cdot\text{m}$$ **Ans.**

Substituting this result into Eq. (1),

$$T_c = 7610.97\ \text{N}\cdot\text{m} = 7.61\ \text{kN}\cdot\text{m}$$ **Ans.**

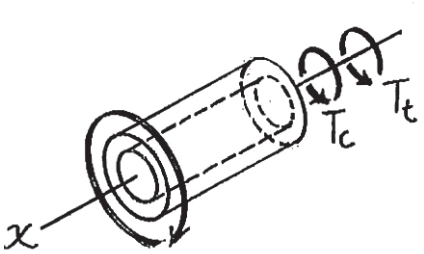
Since $T_c < (T_Y)_c$, the core is still linearly elastic. Thus,

$$\phi_t = \phi_{tc} = \frac{T_c L}{J_c G_{st}} = \frac{7610.97(0.45)}{\frac{\pi}{2}(0.03^4)(75)(10^9)} = 0.03589\ \text{rad}$$

$$\phi_t = \frac{\gamma_i}{c_i} L; \qquad 0.3589 = \frac{\gamma_i}{0.03}(0.45)$$

$$\gamma_i = 0.002393\ \text{rad}$$

Since $\gamma_i > (\gamma_Y)_\infty = 0.002$ rad, the tube is indeed fully plastic.



(a)

Ans:
$T_t = 7.39\ \text{kN}\cdot\text{m}$, $T_c = 7.61\ \text{kN}\cdot\text{m}$

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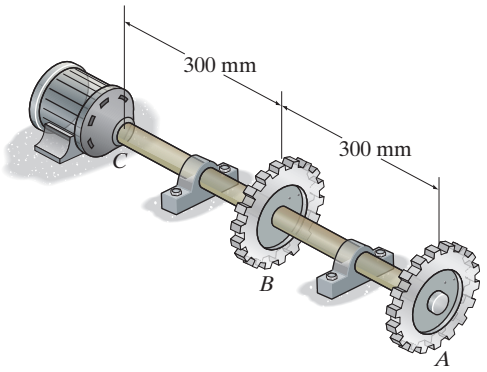
R5–1.

The shaft is made of A992 steel and has an allowable shear stress of $\tau_{\text{allow}} = 75$ MPa. When the shaft is rotating at 300 rpm, the motor supplies 8 kW of power, while gears A and B withdraw 5 kW and 3 kW, respectively. Determine the required minimum diameter of the shaft to the nearest millimeter. Also, find the rotation of gear A relative to C.



SOLUTION

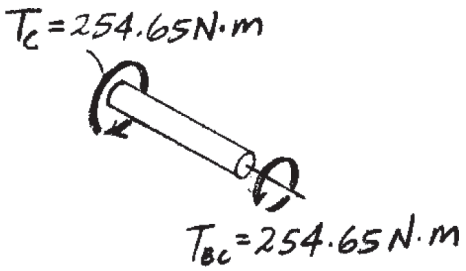
Applied Torque: The angular velocity of the shaft is

$$\omega = \left(300\frac{\text{rev}}{\text{min}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right)\left(\frac{2\pi\text{rad}}{1\text{ rev}}\right) = 10\pi\text{ rad/s}$$

Thus, the torque at C and gear A are

$$T_C = \frac{P_C}{\omega} = \frac{8(10^3)}{10\pi} = 254.65\text{ N}\cdot\text{m}$$

$$T_A = \frac{P_A}{\omega} = \frac{5(10^3)}{10\pi} = 159.15\text{ N}\cdot\text{m}$$



(a)

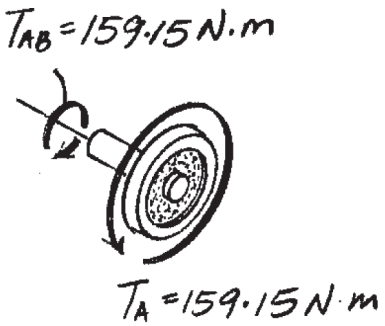
Internal Loading: The internal torque developed in segment BC and AB of the shaft are shown in Figs. a and b, respectively.

Allowable Shear Stress: By inspection, segment BC is critical.

$$\tau_{\text{allow}} = \frac{T_{BC}c}{J}; \qquad 75(10^6) = \frac{254.65(\frac{d}{2})}{\frac{\pi}{2}(\frac{d}{2})^4}$$

$$d = 0.02586\text{ m}$$

Use $d = 26$ mm **Ans.**



(b)

Angle of Twist: Using $d = 26$ mm,

$$\phi_{A/C} = \Sigma\frac{T_iL_i}{J_iG_i} = \frac{T_{AB}L_{AB}}{JG} + \frac{T_{BC}L_{BC}}{JG}$$

$$= \frac{0.3}{\frac{\pi}{2}(0.013^4)(75)(10^9)}(159.15 + 254.65)$$

$$= 0.03689\text{ rad} = 2.11°$$ **Ans.**

Ans:
Use $d = 26$ mm, $\phi_{A/C} = 2.11°$

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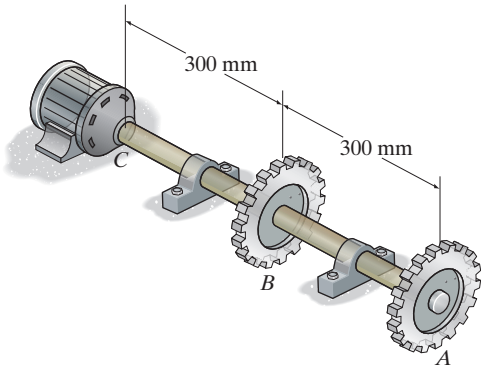
R5–2.

The shaft is made of A992 steel and has an allowable shear stress of $\tau_{\text{allow}} = 75$ MPa. When the shaft is rotating at 300 rpm, the motor supplies 8 kW of power, while gears A and B withdraw 5 kW and 3 kW, respectively. If the angle of twist of gear A relative to C is not allowed to exceed 0.03 rad, determine the required minimum diameter of the shaft to the nearest millimeter.



SOLUTION

Applied Torque: The angular velocity of the shaft is

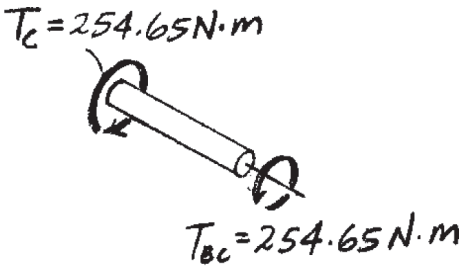
$$\omega = \left(300\frac{\text{rev}}{\text{min}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right)\left(\frac{2\pi\text{rad}}{1\text{ rev}}\right) = 10\pi\text{ rad/s}$$

Thus, the torque at C and gear A are

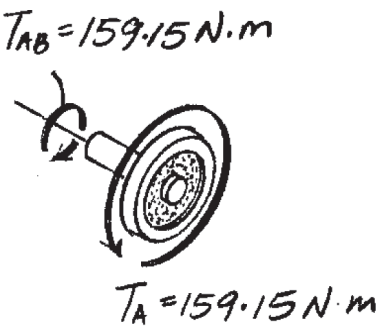
$$T_C = \frac{P_C}{\omega} = \frac{8(10^3)}{10\pi} = 254.65\text{ N}\cdot\text{m}$$

$$T_A = \frac{P_A}{\omega} = \frac{5(10^3)}{10\pi} = 159.15\text{ N}\cdot\text{m}$$

Internal Loading: The internal torque developed in segment BC and AB of the shaft are shown in Figs. a and b, respectively.



(a)



(b)

Allowable Shear Stress: By inspection, segment BC is critical.

$$\tau_{\text{allow}} = \frac{T_{BC}c}{J}; \qquad 75(10^3) = \frac{254.65(\frac{d}{2})}{\frac{\pi}{2}(\frac{d}{2})^4}$$

$$d = 0.02586$$

Angle of Twist:

$$\phi_{A/C} = \Sigma\frac{T_iL_i}{J_iG_i} = \frac{T_{AB}L_{AB}}{JG} + \frac{T_{BC}L_{BC}}{JG}$$

$$0.03 = \frac{0.3}{\frac{\pi}{2}(\frac{d}{2})^4(75)(10^9)}(159.15 + 254.65)$$

$$d = 0.02738\text{ m(controls)}$$

Use $d = 28$ mm **Ans.**

Ans:
Use $d = 28$ mm

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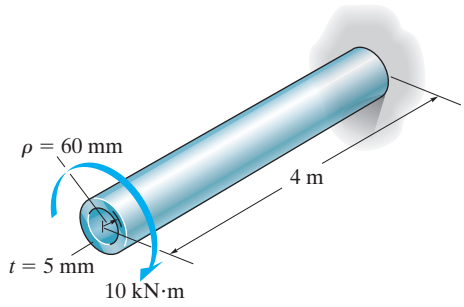
R5–3.

The A-36 steel circular tube is subjected to a torque of 10 kN · m. Determine the shear stress at the mean radius $\rho = 60$ mm and calculate the angle of twist of the tube if it is 4 m long and fixed at its far end. Solve the problem using Eqs. 5–7 and 5–15 and by using Eqs. 5–18 and 5–20.



SOLUTION

We show that two different methods give similar results:

Shear Stress:

Applying Eq. 5-7,

$$r_o = 0.06 + \frac{0.005}{2} = 0.0625 \text{ m} \qquad r_i = 0.06 - \frac{0.005}{2} = 0.0575 \text{ m}$$

$$\tau_{\rho=0.06\text{ m}} = \frac{T\rho}{J} = \frac{10(10^3)(0.06)}{\frac{\pi}{2}(0.0625^4 - 0.0575^4)} = 88.27 \text{ MPa}$$

Applying Eq. 5-18,

$$\tau_{\text{avg}} = \frac{T}{2\,t\,A_m} = \frac{10(10^3)}{2(0.005)(\pi)(0.06^2)} = 88.42 \text{ MPa}$$

Angle of Twist:

Applying Eq. 5-15,

$$\phi = \frac{TL}{JG}$$

$$= \frac{10(10^3)(4)}{\frac{\pi}{2}(0.0625^4 - 0.0575^4)(75.0)(10^9)}$$

$$= 0.0785 \text{ rad} = 4.495°$$

Applying Eq. 5-20,

$$\phi = \frac{TL}{4A_m^2 G} \int \frac{ds}{t}$$

$$= \frac{TL}{4A_m^2 G\,t} \int ds \qquad \text{Where} \qquad \int ds = 2\pi\rho$$

$$= \frac{2\pi TL\rho}{4A_m^2 G\,t}$$

$$= \frac{2\pi(10)(10^3)(4)(0.06)}{4[(\pi)(0.06^2)]^2(75.0)(10^9)(0.005)}$$

$$= 0.0786 \text{ rad} = 4.503°$$

Rounding to three significant figures, we find

$\tau = 88.3$ MPa **Ans.**

$\phi = 4.50°$ **Ans.**

Ans:
$\tau = 88.3$ MPa, $\phi = 4.50°$

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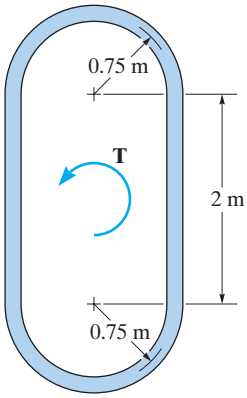
***R5–4.**

A portion of an airplane fuselage can be approximated by the cross section shown. If the thickness of its 2014-T6-aluminum skin is 10 mm, determine the maximum wing torque **T** that can be applied if $\tau_{allow} = 4$ MPa. Also, in a 4-m-long section, determine the angle of twist.



SOLUTION

$$\tau_{avg} = \frac{T}{2tA_m}$$

$$4(10^6) = \frac{T}{2(0.01)[(\pi)(0.75)^2 + 2(1.5)]}$$

$$T = 381.37(10^3) = 381\ \text{kN}\cdot\text{m} \qquad \textbf{Ans.}$$

$$\phi = \frac{TL}{4A_m^2 G}\int \frac{ds}{t}$$

$$\phi = \frac{381.37(10^3)(4)}{4[(\pi(0.75)^2 + 2(1.5))^2 27(10^9)]}\left[\frac{4 + 2\pi(0.75)}{0.010}\right]$$

$$\phi = 0.542(10^{-3})\ \text{rad} = 0.0310° \qquad \textbf{Ans.}$$

Ans:
$T = 381\ \text{kN}\cdot\text{m}$, $\phi = 0.0310°$

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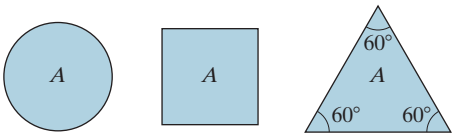
R5–5.

The material of which each of three shafts is made has a yield stress of τ_Y and a shear modulus of G. Determine which shaft geometry will resist the largest torque without yielding. What percentage of this torque can be carried by the other two shafts? Assume that each shaft is made from the same amount of material and that it has the same cross-sectional area A.



SOLUTION

For circular shaft:

$$A = \pi c^2; \qquad c = \left(\frac{A}{\pi}\right)^{\frac{1}{2}}$$

$$\tau_{\max} = \frac{T c}{J}, \qquad \tau_Y = = \frac{T c}{\frac{a}{2} c^4}$$

$$T = \frac{\pi c^3}{2}\tau_Y = \frac{\pi\left(\frac{A}{\pi}\right)^{\frac{3}{2}}}{2}\tau_Y$$

$$T_{cir} = 0.2821\, A^{\frac{1}{2}}\tau_Y$$

For the square shaft:

$$A = a^2; \qquad a = A^{\frac{1}{2}}$$

$$\tau_{\max} = \frac{4.81\, T}{a^3}; \qquad \tau_Y = \frac{4.81\, T}{A^{\frac{3}{2}}}$$

$$T = 0.2079\, A^{\frac{3}{2}}\tau_Y$$

For the triangular shaft:

$$A = \frac{1}{2}(a)(a \sin 60°); \qquad a = 1.5197 A^{\frac{1}{2}}$$

$$\tau_{\max} = \frac{20\, T}{a^3}; \qquad \tau_Y = \frac{20\, T}{(1.5197)^3 A^{\frac{3}{2}}}$$

$$T = 0.1755 A^{\frac{3}{2}}\tau_Y$$

The circular shaft will resist the largest torque. **Ans.**

For the square shaft:

$$\% = \frac{0.2079}{0.2821}(100\%) = 73.7\%$$ **Ans.**

For the triangular shaft:

$$\% = \frac{0.1755}{0.2821}(100\%) = 62.2\ \%$$ **Ans.**

Ans:
The circular shaft will resist the largest torque.
For the square shaft: 73.7%,
For the triangular shaft: 62.2%

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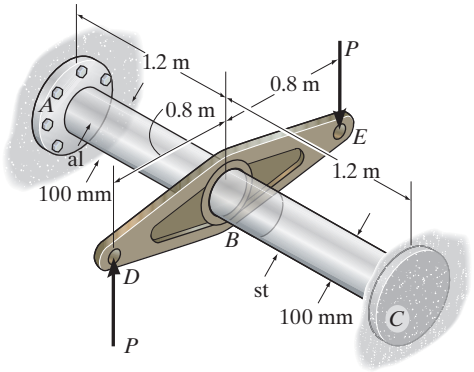
R5–6. Segments AB and BC of the assembly are made from 6061-T6 aluminum and A992 steel, respectively. If couple forces $P = 15$ kN are applied to the lever arm, determine the maximum shear stress developed in each segment. The assembly is fixed at A and C.



SOLUTION

Equilibrium: Referring to the free-body diagram of the assembly shown in Fig. *a*,

$$\Sigma M_x = 0; \qquad T_A + T_C - 15(1.6) = 0 \tag{1}$$

Compatibility Equation: It is required that

$$\phi_{B/A} = \phi_{B/C}$$

$$\frac{T_A L_{AB}}{JG_{al}} = \frac{T_C L_{BC}}{JG_{st}}$$

$$\frac{T_A L}{J[26(10^9)]} = \frac{T_C L}{J[75(10^9)]}$$

$$T_A = 0.3467\, T_C \tag{2}$$

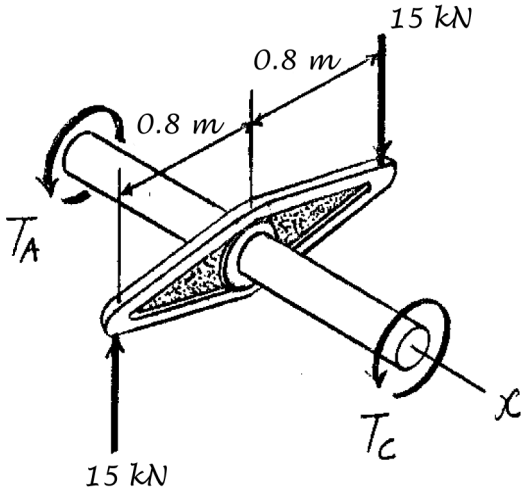
Solving Eqs. (1) and (2),

$$T_C = 17.82 \text{ kN} \cdot \text{m} \qquad T_A = 6.178 \text{ kN} \cdot \text{m}$$

Maximum Shear Stress:

$$(\tau_{\max})_{AB} = \frac{T_A c}{J} = \frac{[6.178(10^3)](0.05)}{\frac{\pi}{2}(0.05^4)} = 31.47(10^6) \text{ N/m}^2 = 31.5 \text{ MPa} \qquad \textbf{Ans.}$$

$$(\tau_{\max})_{BC} = \frac{T_C c}{J} = \frac{[17.82(10^3)](0.05)}{\frac{\pi}{2}(0.05^4)} = 90.77(10^6) \text{ N/m}^2 = 90.8 \text{ MPa} \qquad \textbf{Ans.}$$



(a)

Ans.
$(\tau_{\max})_{AB} = 31.5$ MPa
$(\tau_{\max})_{BC} = 90.8$ MPa

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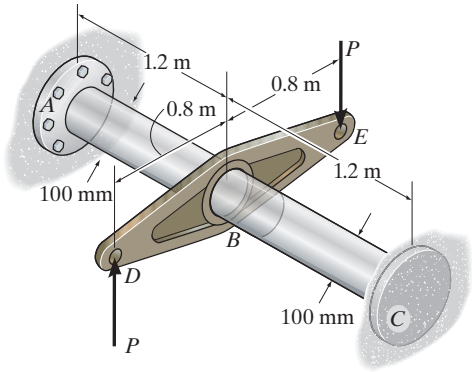
R5–7. Segments AB and BC of the assembly are made from 6061-T6 aluminum and A992 steel, respectively. If the allowable shear stress for the aluminum is $(\tau_{\text{allow}})_{al} = 90$ MPa and for the steel $(\tau_{\text{allow}})_{st} = 120$ MPa, determine the maximum allowable couple forces P that can be applied to the lever arm. The assembly is fixed at A and C.



SOLUTION

Equilibrium: Referring to the free-body diagram of the assembly shown in Fig. *a*,

$$\Sigma M_x = 0; \qquad T_A + T_C - P(1.6) = 0 \tag{1}$$

Compatibility Equation: It is required that

$$\phi_{B/A} = \phi_{B/C}$$

$$\frac{T_A L_{AB}}{JG_{\text{al}}} = \frac{T_C L_{BC}}{JG_{\text{st}}}$$

$$\frac{T_A L}{J[26(10^9)]} = \frac{T_C L}{J[75(10^9)]}$$

$$T_A = 0.3467\, T_C \tag{2}$$

Solving Eqs. (1) and (2),

$$T_C = 1.1881P \qquad T_A = 0.4119P$$

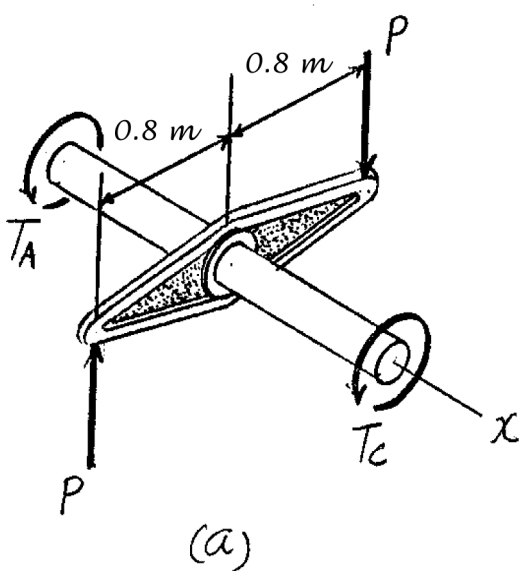
Allowable Shear Stress:

$$(\tau_{\text{allow}})_{al} = \frac{T_A c}{J}; \qquad 90(10^6) = \frac{(0.4119P)(0.05)}{\frac{\pi}{2}(0.05^4)}$$

$$P = 42.90(10^3)\ \text{N} = 42.9\ \text{kN}$$

$$(\tau_{\text{allow}})_{st} = \frac{T_C c}{J}; \qquad 120(10^6) = \frac{(1.1881P)(0.05)}{\frac{\pi}{2}(0.05^4)}$$

$$P = 19.83(10^3)\ \text{N} = 19.8\ \text{kN (controls)} \qquad \textbf{Ans.}$$



Ans:
$P = 19.8$ kN

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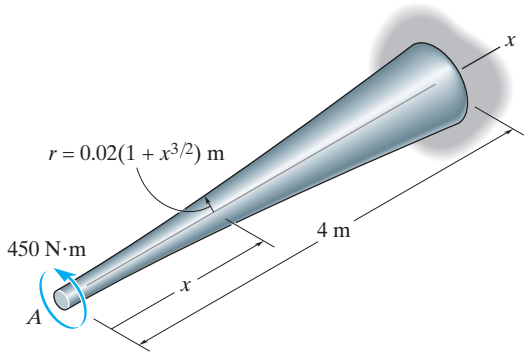
***R5–8.**

The tapered shaft is made from 2014-T6 aluminum alloy, and has a radius which can be described by the equation $r = 0.02(1 + x^{3/2})$ m, where x is in meters. Determine the angle of twist of its end A if it is subjected to a torque of $450\ \text{N} \cdot \text{m}$.



SOLUTION

$T = 450\ \text{N} \cdot \text{m}$

$$\phi_A = \int \frac{T dx}{JG} = \int_0^4 \frac{450\ dx}{\frac{\pi}{2}(0.02)^4(1 + x^{\frac{3}{2}})^4(27)(10^9)} = 0.066315 \int_0^4 \frac{dx}{(1 + x^{\frac{3}{2}})^4}$$

Evaluating the integral numerically, we have

$\phi_A = 0.066315\ [0.4179]$ rad

$= 0.0277$ rad $= 1.59°$ **Ans.**

Ans:
$\phi_A = 1.59°$

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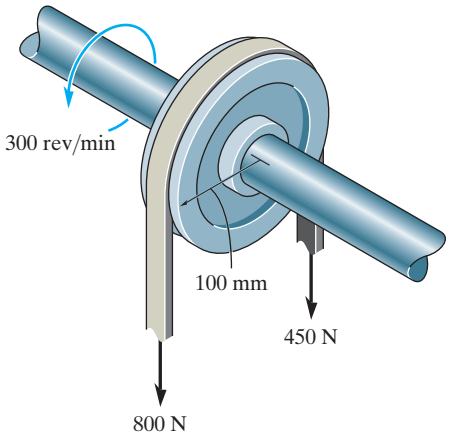
R5–9.

The 60-mm-diameter shaft rotates at 300 rev/min. This motion is caused by the unequal belt tensions on the pulley of 800 N and 450 N. Determine the power transmitted and the maximum shear stress developed in the shaft.



SOLUTION

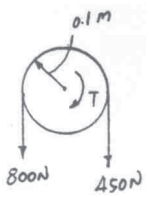
$$\omega = 300\frac{\text{rev}}{\text{min}}\left[\frac{2\pi \text{ rad}}{1 \text{ rev}}\right]\frac{1 \text{ min}}{60 \text{ s}} = 10\,\pi \text{ rad/s}$$

$$T + 450(0.1) - 800(0.1) = 0$$

$$T = 35.0 \text{ N} \cdot \text{m}$$

$$P = T\omega = 35.0(10\pi) = 1100 \text{ W} = 1.10 \text{ kW} \qquad \textbf{Ans.}$$

$$\tau_{\max} = \frac{Tc}{J} = \frac{35.0(0.03)}{\frac{\pi}{2}(0.03^4)} = 825 \text{ kPa} \qquad \textbf{Ans.}$$



Ans:
$P = 1.10$ kW,
$\tau_{\max} = 825$ kPa

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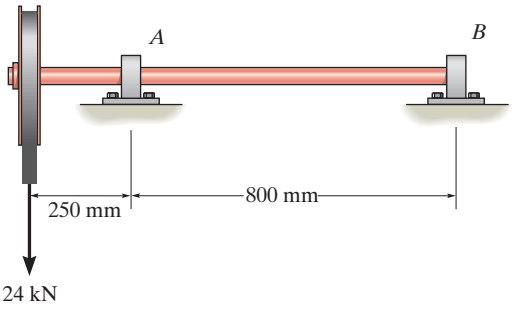
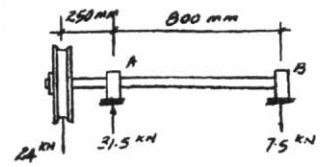
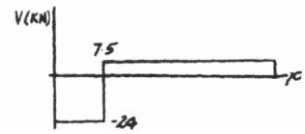
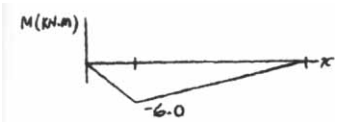
6–1. Draw the shear and moment diagrams for the shaft. The bearings at *A* and *B* exert only vertical reactions on the shaft.









These solutions represent a preliminary version of the Instructors' Solutions Manual (ISM). It is possible and even likely that at this preliminary stage of preparing the ISM there are some omissions and errors in the draft solutions. These will be corrected and this manual will be republished.

Ans:
$x = 0.25^{-}, V = -24, M = -6$

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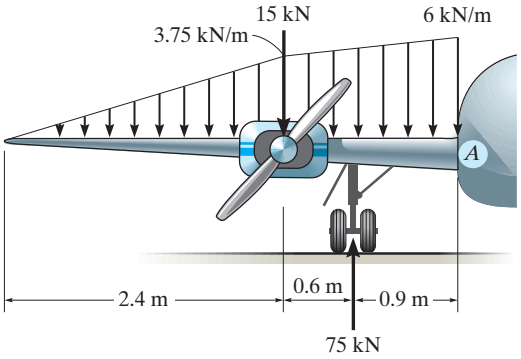
6–2. The dead-weight loading along the centerline of the airplane wing is shown. If the wing is fixed to the fuselage at A, determine the reactions at A, and then draw the shear and moment diagram for the wing.



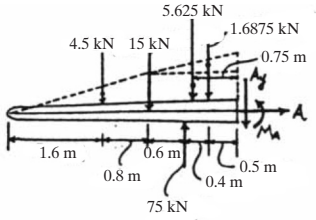
Support Reactions:

$$+\uparrow \Sigma F_y = 0; \qquad -4.5 - 15 + 75 - 5.625 - 1.6875 - A_y = 0$$

$$A_y = 48.1875 \text{ kN} = 48.2 \text{ kN}$$ **Ans.**

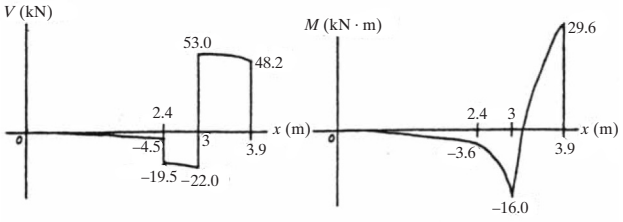
$$\circlearrowleft + \Sigma M_A = 0; \qquad 4.5(2.3) + 15(1.5) - 75(0.9) + 5.625(0.75) + 1.6875(0.5) + M_A = 0$$

$$M_A = 29.5875 \text{ kN} \cdot \text{m} = 29.6 \text{ kN} \cdot \text{m}$$ **Ans.**



$$\stackrel{+}{\rightarrow} \Sigma F_x = 0; \qquad A_x = 0$$ **Ans.**

Shear and Moment Diagram:



Ans:
$A_y = 48.2 \text{ kN}, A_x = 0$

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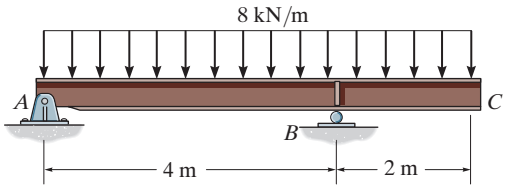
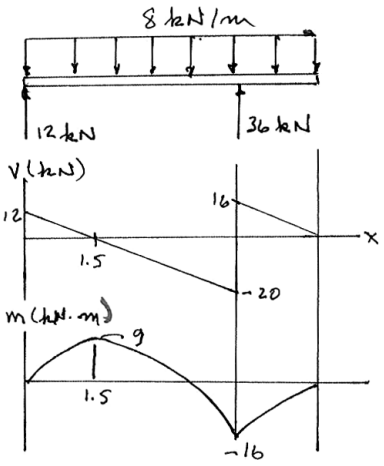
6–3. Draw the shear and moment diagrams for the overhang beam.





Ans:
$x = 1.5, V = 0, M = 9, x = 4^{-}$,
$V = -20, M = -16$

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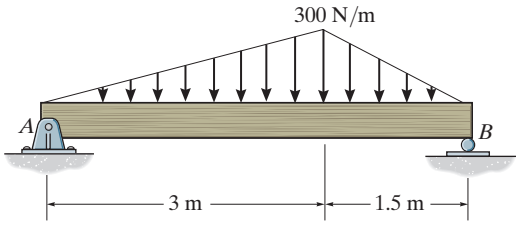
***6–4.**

Express the shear and moment in terms of x for $0 < x < 3$ m and $3\text{ m} < x < 4.5$ m, and then draw the shear and moment diagrams for the simply supported beam.



SOLUTION

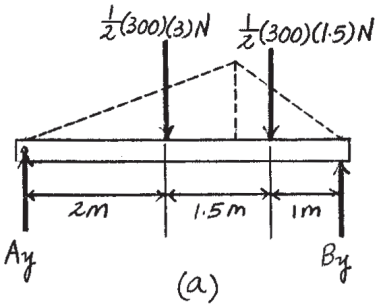
Support Reactions: Referring to the free-body diagram of the entire beam shown in Fig. *a*,

$$\zeta+\Sigma M_A = 0; \qquad B_y(4.5) - \frac{1}{2}(300)(3)(2) - \frac{1}{2}(300)(1.5)(3.5) = 0$$

$$B_y = 375\text{ N}$$

$$+\uparrow\Sigma F_y = 0; \qquad A_y + 375 - \frac{1}{2}(300)(3) - \frac{1}{2}(300)(1.5) = 0$$

$$A_y = 300\text{ N}$$



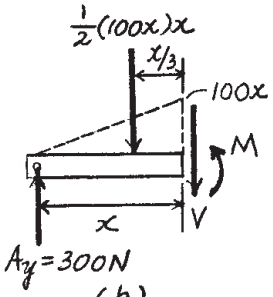
Shear and Moment Function: For $0 \le x < 3$ m, we refer to the free-body diagram of the beam segment shown in Fig. *b*.

$$+\uparrow\Sigma F_y = 0; \qquad 300 - \frac{1}{2}(100x)x - V = 0$$

$$V = \{300 - 50x^2\}\text{ N} \qquad \textbf{Ans.}$$

$$\zeta+\Sigma M = 0; \quad M + \frac{1}{2}(100x)x\left(\frac{x}{3}\right) - 300x = 0$$

$$M = \left\{300x - \frac{50}{3}x^3\right\}\text{N}\cdot\text{m} \qquad \textbf{Ans.}$$



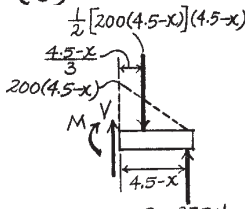
When $V = 0$, from the shear function,

$$0 = 300 - 50x^2 \qquad x = \sqrt{6}\text{ m}$$

Substituting this result into the moment equation,

$$M|_{x=\sqrt{6}\,\text{m}} = 489.90\text{ N}\cdot\text{m}$$

For $3\text{ m} < x \le 4.5$ m, we refer to the free-body diagram of the beam segment shown in Fig. *c*.

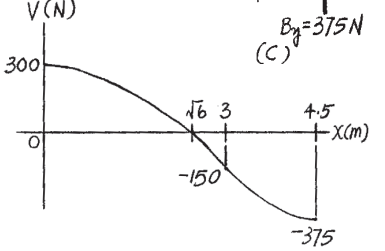


$$+\uparrow\Sigma F_y = 0; \qquad V + 375 - \frac{1}{2}[\,200(4.5 - x)\,](4.5 - x) = 0$$

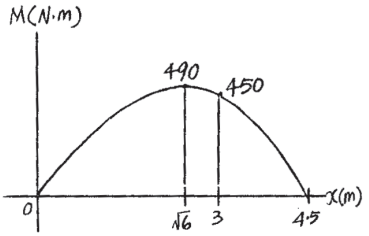
$$V = \left\{100(4.5 - x)^2 - 375\right\}\text{N} \qquad \textbf{Ans.}$$

$$\zeta+\Sigma M = 0; \quad 375(4.5 - x) - \frac{1}{2}[200(4.5 - x)](4.5 - x)\left(\frac{4.5 - x}{3}\right) - M = 0$$

$$M = \left\{375(4.5 - x) - \frac{100}{3}(4.5 - x)^3\right\}\text{N}\cdot\text{m} \qquad \textbf{Ans.}$$



Shear and Moment Diagrams: As shown in Figs. *d* and *e*.



Ans:
For $0 \le x < 3$ m, $V = \{300 - 50x^2\}$ N,
$M = \left\{300x - \frac{50}{3}x^3\right\}\text{N}\cdot\text{m}$,
For $3\text{ m} < x \le 4.5$ m, $V = \left\{100(4.5 - x)^2 - 375\right\}\text{N}$,
$M = \left\{375(4.5 - x) - \frac{100}{3}(4.5 - x)^3\right\}\text{N}\cdot\text{m}$

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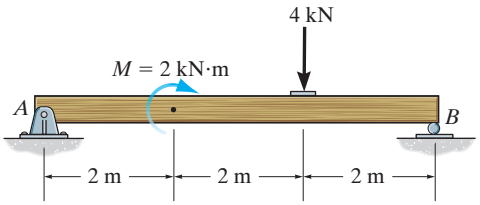
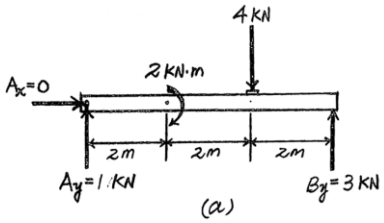
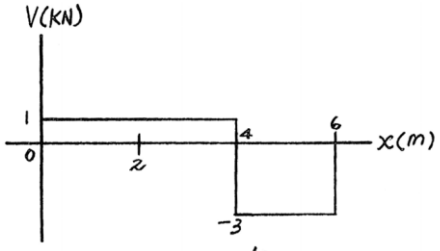
6–5. Draw the shear and moment diagrams for the simply supported beam.



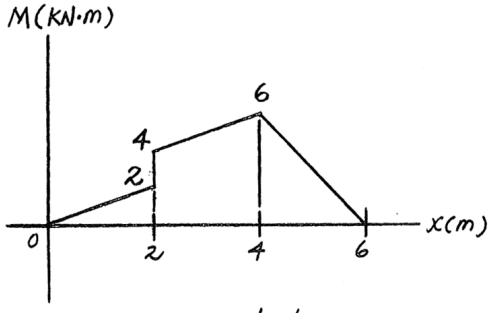


(a)



Shear diagram

(b)



moment diagram

(c)

Ans:
$x = 2^-, V = 1, M = 2, x = 4^-, V = 1, M = 6$

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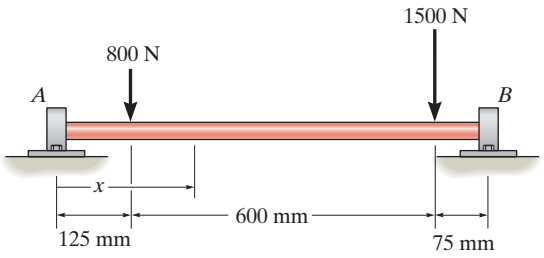
6–6.

Draw the shear and moment diagrams for the shaft. The bearings at A and B exert only vertical reactions on the shaft. Also, express the shear and moment in the shaft as a function of x within the region $125\text{ mm} < x < 725\text{ mm}$.



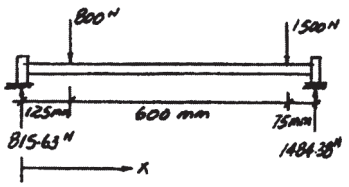
SOLUTION

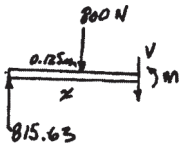
$+\uparrow \Sigma F_y = 0; \qquad 815.63 - 800 - V = 0$

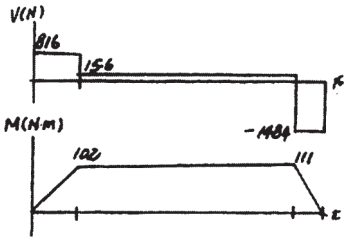
$$V = 15.6\text{ N}$$ **Ans.**

$\zeta + \Sigma M = 0; \qquad M + 800(x - 0.125) - 815.63\,x = 0$

$$M = (15.6x + 100)\text{ N}\cdot\text{m}$$ **Ans.**







Ans:
$V = 15.6\text{ N}$,
$M = (15.6x + 100)\text{ N}\cdot\text{m}$

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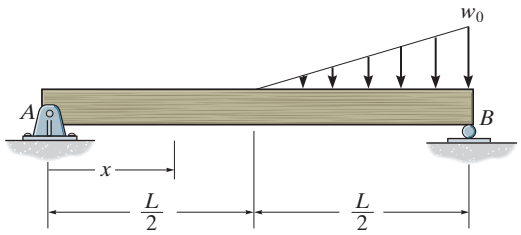
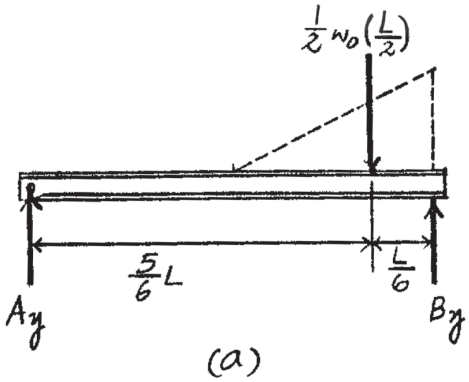
6–7.

Express the internal shear and moment in terms of x for $0 \le x < L/2$, and $L/2 < x < L$, and then draw the shear and moment diagrams.



SOLUTION

Support Reactions: Referring to the free-body diagram of the entire beam shown in Fig. *a*,



$$\zeta+\Sigma M_A = 0; \qquad B_y(L) - \frac{1}{2}w_0\left(\frac{L}{2}\right)\left(\frac{5}{6}L\right) = 0$$

$$B_y = \frac{5}{24}w_0 L$$

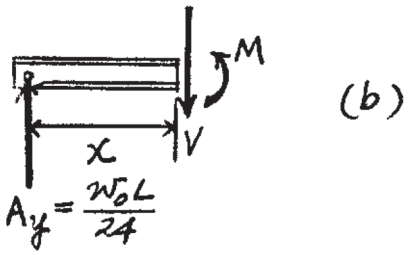
$$+\uparrow \Sigma F_y = 0; \qquad A_y + \frac{5}{24}w_0 L - \frac{1}{2}w_0\left(\frac{L}{2}\right) = 0$$

$$A_y = \frac{w_0 L}{24}$$

Shear and Moment Function: For $0 \le x < \frac{L}{2}$, we refer to the free-body diagram of the beam segment shown in Fig. *b*.

$$+\uparrow \Sigma F_y = 0; \qquad \frac{w_0 L}{24} - V = 0$$

$$V = \frac{w_0 L}{24} \qquad \textbf{Ans.}$$



$$\zeta+\Sigma M = 0; \qquad M - \frac{w_0 L}{24}x = 0$$

$$M = \frac{w_0 L}{24}x \qquad \textbf{Ans.}$$

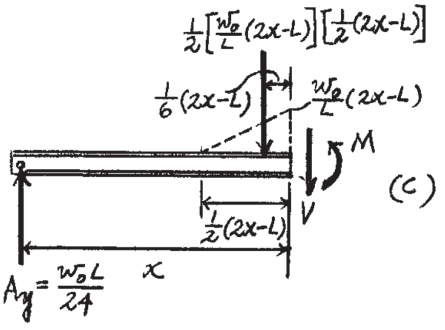
For $\frac{L}{2} < x \le L$, we refer to the free-body diagram of the beam segment shown in Fig. *c*.

$$+\uparrow \Sigma F_y = 0; \qquad \frac{w_0 L}{24} - \frac{1}{2}\left[\frac{w_0}{L}(2x - L)\right]\left[\frac{1}{2}(2x - L)\right] - V = 0$$

$$V = \frac{w_0}{24L}\left[L^2 - 6(2x - L)^2\right] \qquad \textbf{Ans.}$$

$$\zeta+\Sigma M = 0; \qquad M + \frac{1}{2}\left[\frac{w_0}{L}(2x - L)\right]\left[\frac{1}{2}(2x - L)\right]\left[\frac{1}{6}(2x - L)\right] - \frac{w_0}{24L}x = 0$$

$$M = \frac{w_0}{24L}\left[L^2 x - (2x - L)^3\right] \qquad \textbf{Ans.}$$



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6–7. Continued

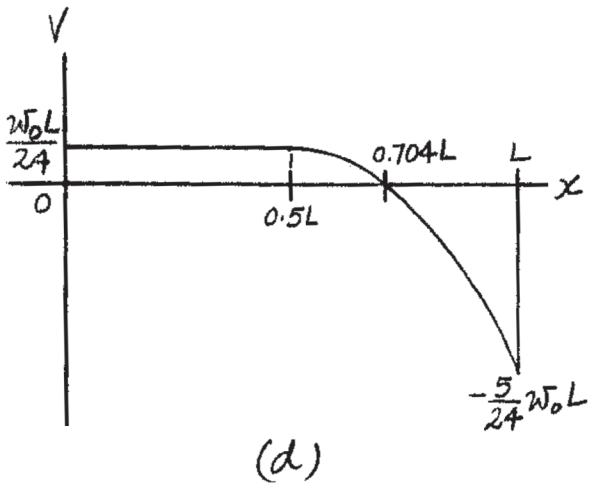
When $V = 0$, the shear function gives

$$0 = L^2 - 6(2x - L)^2 \qquad x = 0.7041L$$

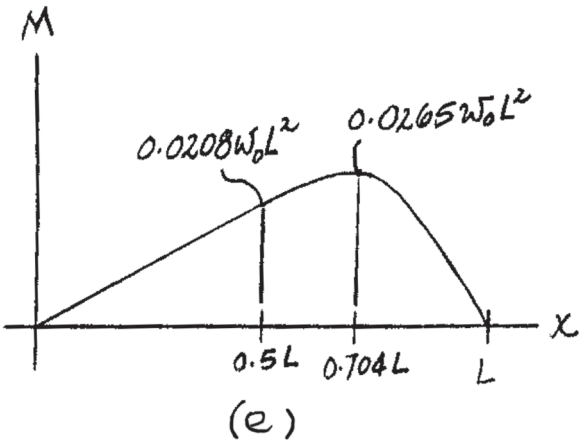
Substituting this result into the moment equation,

$$M|_{x=0.7041L} = 0.0265 w_0 L^2$$

Shear and Moment Diagrams: As shown in Figs. *d* and *e*.



(d)



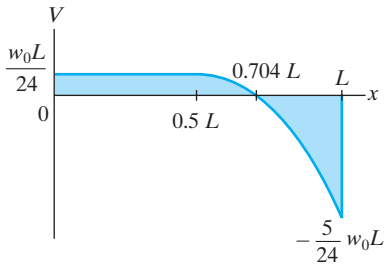
(e)

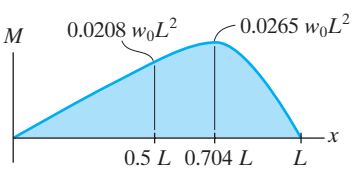
Ans:

For $0 \le x < \frac{L}{2}$: $V = \frac{w_0 L}{24}$, $M = \frac{w_0 L}{24} x$,

For $\frac{L}{2} < x \le L$: $V = \frac{w_0}{24L}\left[L^2 - 6(2x - L)^2\right]$,

$M = \frac{w_0}{24L}\left[L^2 x - (2x - L)^3\right]$





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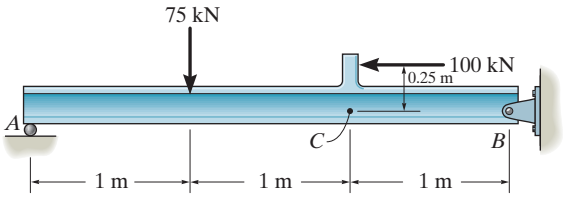
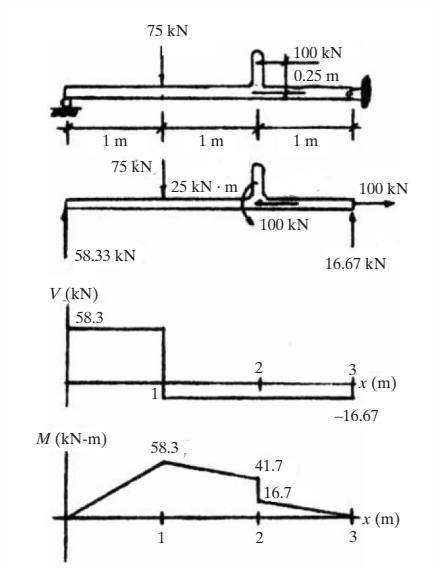
***6–8.** Draw the shear and moment diagrams for the beam. *Hint:* The 100-kN load must be replaced by equivalent loadings at point *C* on the axis of the beam.





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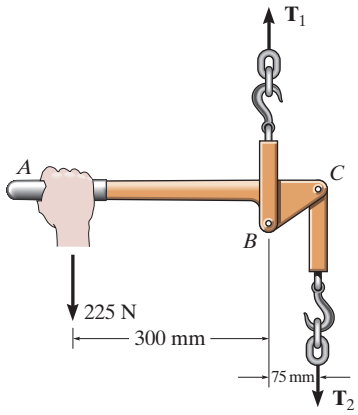
6–9. The load binder is used to support a load. If the force applied to the handle is 225 N, determine the tensions T_1 and T_2 in each end of the chain and then draw the shear and moment diagrams for the arm ABC.

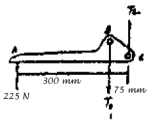


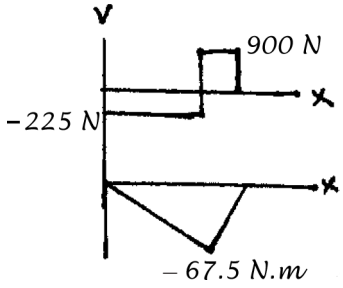
$\circlearrowleft + \Sigma M_C = 0;$ $\quad 225(375) - T_1 \quad (75) = 0$

$T_1 = 1125\ \text{N}$ **Ans.**

$+\downarrow \Sigma F_y = 0;$ $\quad 225 - 1125 + T_2 = 0$

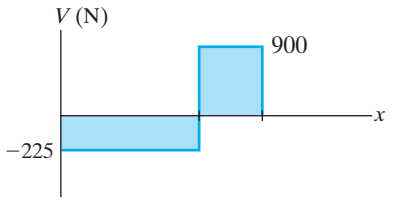
$T_2 = 900\ \text{N}$ **Ans.**

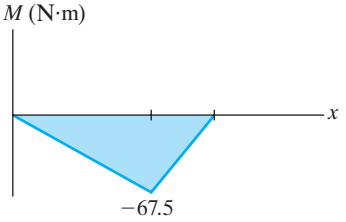




Ans:

$T_1 = 1125\ \text{N},\ T_2 = 900\ \text{N}$





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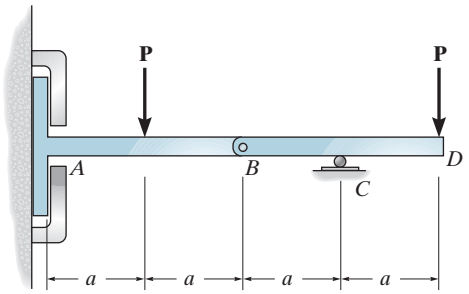
6–10. Draw the shear and moment diagrams for the compound beam. It is supported by a smooth plate at A which slides within the groove and so it cannot support a vertical force, although it can support a moment and axial load.



Support Reactions:

From the FBD of segment BD

$$\circlearrowleft +\Sigma M_C = 0; \quad B_y(a) - P(a) = 0 \quad B_y = P$$

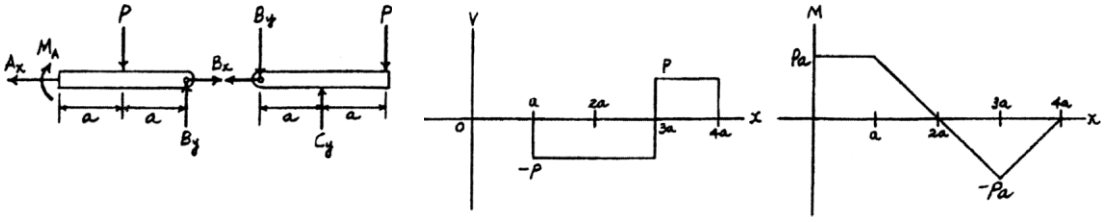
$$+\uparrow \Sigma F_y = 0; \quad C_y - P - P = 0 \quad C_y = 2P$$

$$\pm \rightarrow \Sigma F_x = 0; \quad B_x = 0$$

From the FBD of segment AB

$$\circlearrowleft +\Sigma M_A = 0; \quad P(2a) - P(a) - M_A = 0 \quad M_A = Pa$$

$$+\uparrow \Sigma F_y = 0; \quad P - P = 0 \text{ (equilibrium is statisfied!)}$$



Ans:
$x = 3a^{-}, V = -P, M = -Pa$

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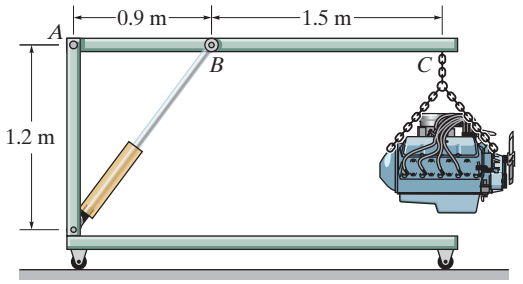
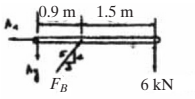
6–11. The engine crane is used to support the engine, which has a weightof 6 kN. Draw the shear and moment diagrams of the boom ABC when it is in the horizontal position shown.

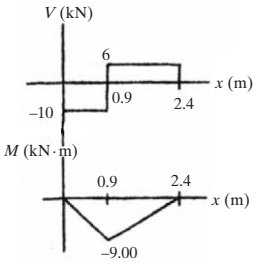


$\zeta + \Sigma M_A = 0; \quad \frac{4}{5} F_B(0.9) - 6(2.4) = 0 \quad F_B = 20 \text{ kN}$

$+\uparrow \Sigma F_y = 0; \quad -A_y + \frac{4}{5}(20) - 6 = 0; \quad A_y = 10 \text{ kN}$

$\pm \Sigma F_x = 0; \quad A_x - \frac{3}{5}(20) = 0; \quad A_x = 12 \text{ kN}$





Ans:

$x = 0.9^{-}\text{m}, V = -10 \text{ kN} \cdot x = 0.9^{+}\text{m}, V = 6 \text{ kN}$

$x = 0.9 \text{ m}, M = -9.00 \text{ kN} \cdot \text{m}$

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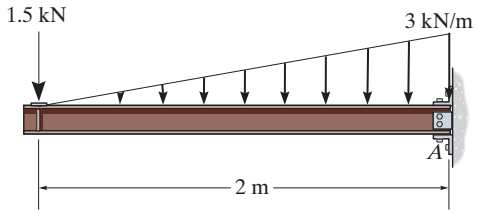
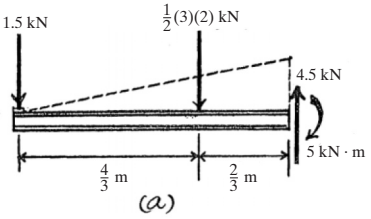
***6–12.** Draw the shear and moment diagrams for the cantilevered beam.



The free-body diagram of the beam's left segment sectioned through an arbitrary point shown in Fig. *b* will be used to write the shear and moment equations. The intensity of the triangular distributed load at the point of sectioning is

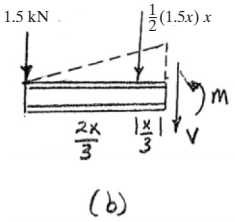
$$w = 3\left(\frac{x}{2}\right) = 1.5x$$



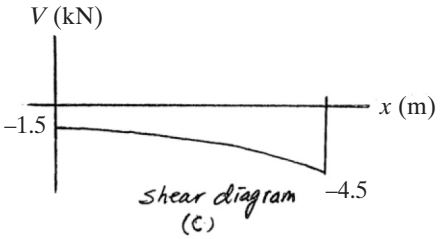
Referring to Fig. *b*,

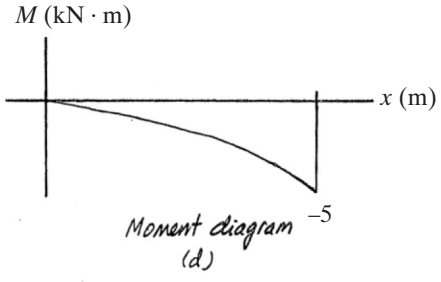
$$+\uparrow \Sigma F_y = 0; \qquad -1.5 - \frac{1}{2}(1.5x)(x) - V = 0 \qquad V = \{-1.5 - 0.75x^2\}\ \text{kN} \qquad (1)$$

$$\circlearrowleft + \Sigma M = 0; \qquad M + \frac{1}{2}(1.5x)(x)\left(\frac{x}{3}\right) + 1.5x = 0 \qquad M = \{-1.5x - 0.25x^3\}\ \text{kN}\cdot\text{m} \qquad (2)$$



The shear and moment diagrams shown in Figs. *c* and *d* are plotted using Eqs. (1) and (2), respectively.





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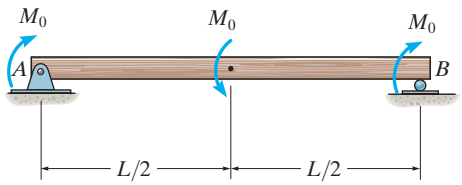
6–13.

Draw the shear and moment diagrams for the beam.



SOLUTION

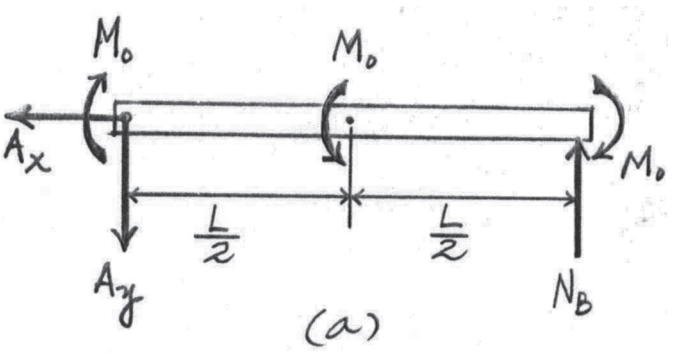
Support Reactions: Referring to the FBD of the beam, Fig. *a*,

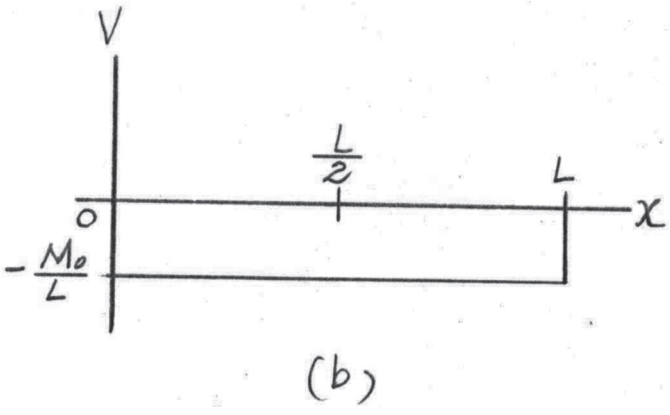
$$\zeta+\Sigma M_A = 0; \quad N_B(L) + M_0 - M_0 - M_0 = 0 \quad N_B = \frac{M_0}{L}$$

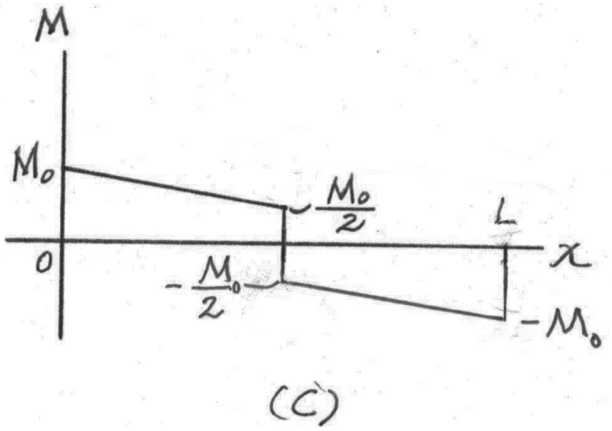
$$\zeta+\Sigma M_B = 0; \quad A_y(L) + M_0 - M_0 - M_0 = 0 \quad A_y = \frac{M_0}{L}$$

$$\overset{+}{\leftarrow} \Sigma F_x = 0; \quad A_x = 0.$$

Shear and Moment Diagram: Using the results of the support reaction, the shear and moment diagrams shown in Fig. *b* and *c* respectively can be plotted.







Ans:

$$V = -\frac{M_0}{L},$$

$$\text{For } 0 \le x < \frac{L}{2}, M = M_0 - \left(\frac{M_0}{L}\right)x,$$

$$\text{For } \frac{L}{2} < x \le L, M = -\left(\frac{M_0}{L}\right)x$$

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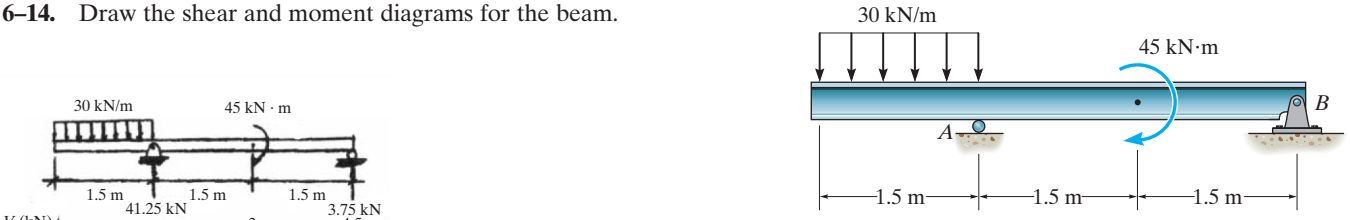
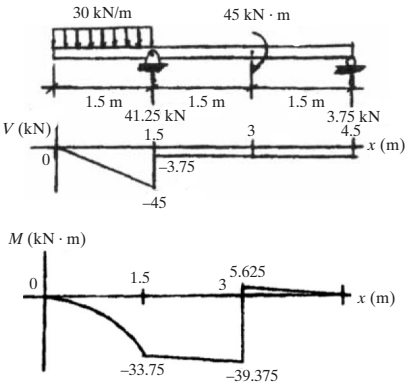
6–14. Draw the shear and moment diagrams for the beam.





Ans:

$x = 1.5^{-}$ m, $V = -45$ kN · $x = 1.5^{+}$m,
$V = -3.75$ kN
$x = 1.5$ m, $M = -33.75$ kN · m · $x = 3^{-}$m
$M = -39.375$ kN · m
$x = 3^{+}$m, $M = 5.625$ kN · m

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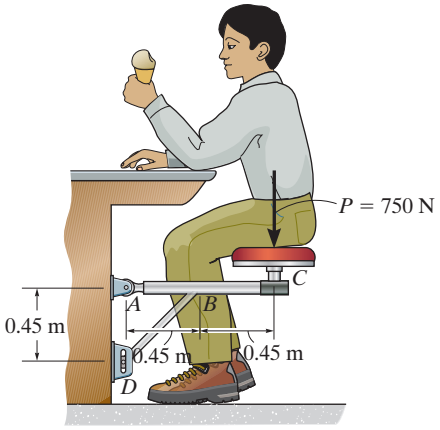
6–15. Members ABC and BD of the counter chair are rigidly connected at B and the smooth collar at D is allowed to move freely along the vertical slot. Draw the shear and moment diagrams for member ABC.



Equations of Equilibrium: Referring to the free-body diagram of the frame shown in Fig. *a*,

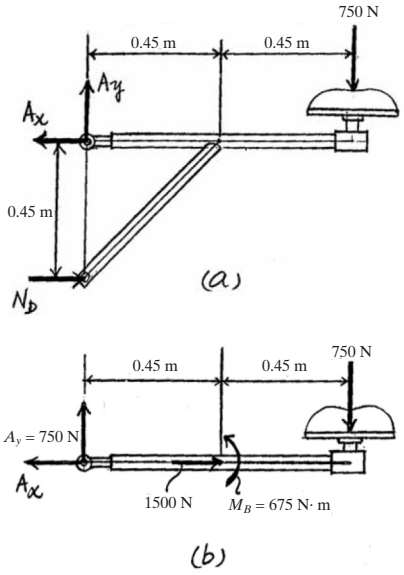
$+\uparrow \Sigma F_y = 0;$ $\quad A_y - 750 = 0$

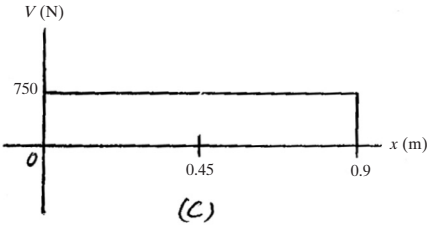
$A_y = 750\ \text{N}$

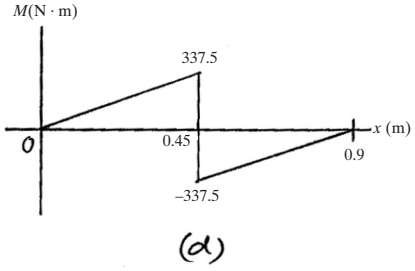
$\circlearrowleft + \Sigma M_A = 0;$ $\quad N_D(0.45) - 750(0.9) = 0$

$N_D = 1500\ \text{N}$

Shear and Moment Diagram: The couple moment acting on B due to N_D is $M_B = 1500(0.45) = 675\ \text{N} \cdot \text{m}$. The loading acting on member ABC is shown in Fig. *b* and the shear and moment diagrams are shown in Figs. *c* and *d*.







Ans:

$0 \le x \le 0.9\ \text{m},\ V = 750\ \text{N} \cdot x = 0.45^{-}\text{m},$
$M = -337.5\ \text{N} \cdot \text{m} \cdot x = 0.45^{+}\text{m},$
$M = 337.5\ \text{N} \cdot \text{m}$

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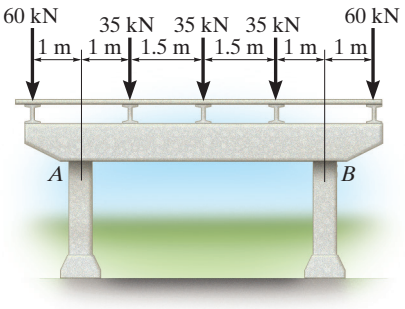
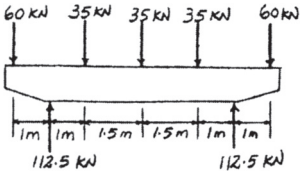
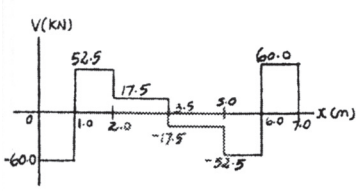
***6–16.**

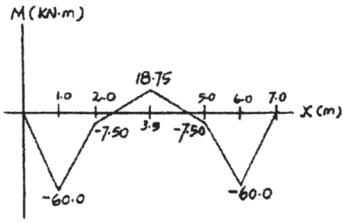
A reinforced concrete pier is used to support the stringers for a bridge deck. Draw the shear and moment diagrams for the pier. Assume the columns at *A* and *B* exert only vertical reactions on the pier.



SOLUTION







Ans:
N/A

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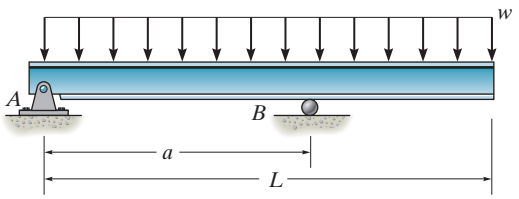
6–17. Determine the placement distance a of the roller support so that the largest absolute value of the moment is a minimum. Draw the shear and moment diagrams for this condition.



$$+\uparrow \Sigma F_y = 0; \qquad wL - \frac{wL^2}{2a} - wx = 0$$

$$x = L - \frac{L^2}{2a}$$

$$\circlearrowleft + \Sigma M = 0; \qquad M_{\max(+)} + wx\left(\frac{x}{2}\right) - \left(wL - \frac{wL^2}{2a}\right)x = 0$$

Substitute $x = L - \frac{L^2}{2a}$;

$$M_{\max(+)} = \left(wL - \frac{wL^2}{2a}\right)\left(L - \frac{L^2}{2a}\right) - \frac{w}{2}\left(L - \frac{L^2}{2a}\right)^2$$

$$= \frac{w}{2}\left(L - \frac{L^2}{2a}\right)^2$$

$$\Sigma M = 0; \qquad M_{\max(-)} - w(L-a)\frac{(L-a)}{2} = 0$$

$$M_{\max(-)} = \frac{w(L-a)^2}{2}$$

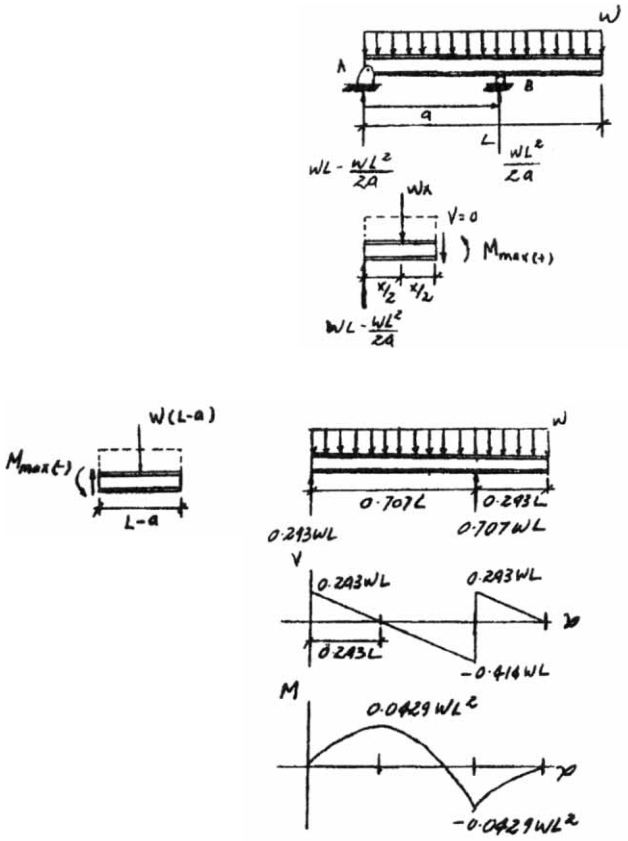
To get absolute minimum moment,

$$M_{\max(+)} = M_{\max(-)}$$

$$\frac{w}{2}(L - \frac{L^2}{2a})^2 = \frac{w}{2}(L-a)^2$$

$$L - \frac{L^2}{2a} = L - a$$

$$a = \frac{L}{\sqrt{2}},$$



Ans.

Ans:

$a = \dfrac{L}{\sqrt{2}}$

$x = 0, V = 0.243\, w\mathrm{L} \cdot x = 0.243\ \mathrm{L}, V = 0 \cdot$

$x = 0.707\ \mathrm{L}^{-}, V = -0.414\, w\mathrm{L}$

$x = 0.707\ \mathrm{L}^{+}, V = 0.293\, w\mathrm{L}$

$x = 0.243\ \mathrm{L}, M = 0.0429\, w\mathrm{L}^2 \cdot x = 0.707\ \mathrm{L},$

$M = -0.0429\, w\mathrm{L}^2$

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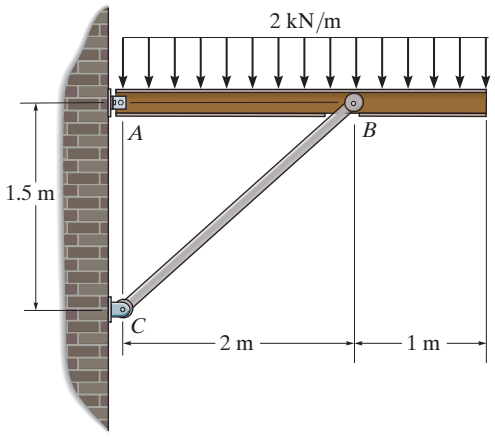
6–18. The beam is subjected to the uniform distributed load shown. Draw the shear and moment diagrams for the beam.



Equations of Equilibrium: Referring to the free-body diagram of the beam shown in Fig. *a*,

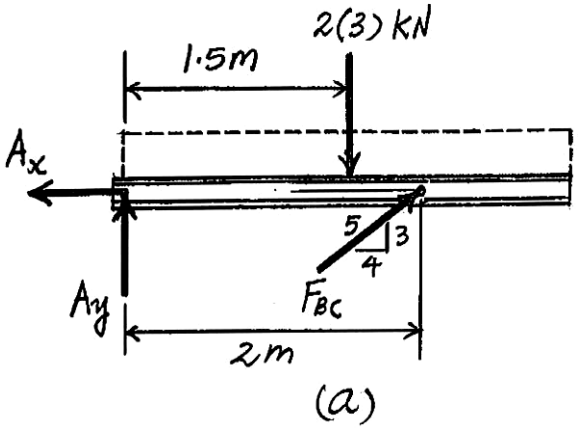
$$\zeta + \Sigma M_A = 0; \qquad F_{BC}\left(\frac{3}{5}\right)(2) - 2(3)(1.5) = 0$$

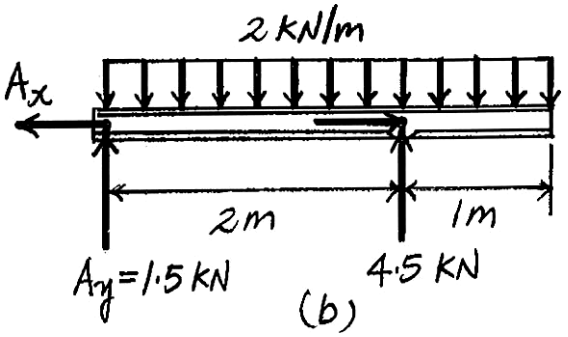
$$F_{BC} = 7.5 \text{ kN}$$

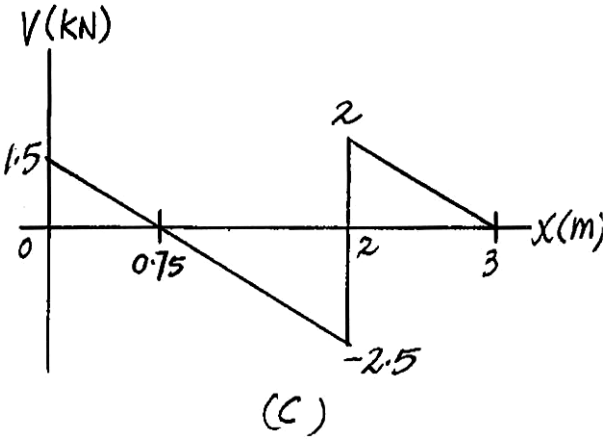
$$+\uparrow \Sigma F_y = 0; \qquad A_y + 7.5\left(\frac{3}{5}\right) - 2(3) = 0$$

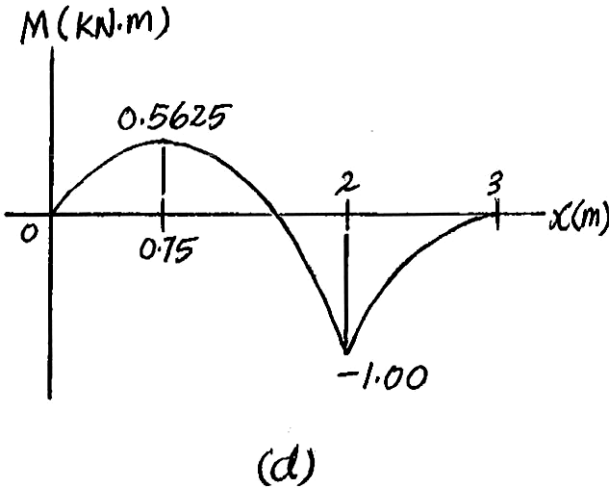
$$A_y = 1.5 \text{ kN}$$

Shear and Moment Diagram: The vertical component of $\mathbf{F}_{BC}$ is $\left(F_{BC}\right)_y = 7.5\left(\frac{3}{5}\right)$ $= 4.5$ kN. The shear and moment diagrams are shown in Figs. *c* and *d*.









Ans:
$x = 0.75, V = 0, M = 0.5625,$
$F_{BC} = 7.5$ kN, $A_y = 1.5$ kN

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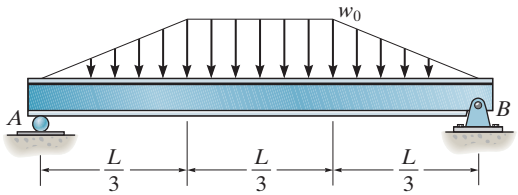
6–19.

Draw the shear and moment diagrams for the beam.



SOLUTION

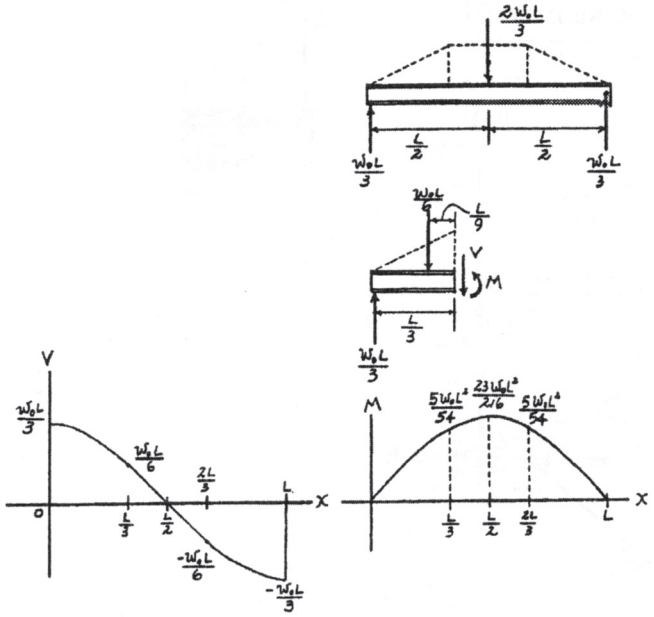
Support Reactions: As shown on FBD.

Shear and Moment Diagram: Shear and moment at $x = L/3$ can be determined using the method of sections.

$$+\uparrow \Sigma F_y = 0; \qquad \frac{w_0 L}{3} - \frac{w_0 L}{6} - V = 0 \qquad V = \frac{w_0 L}{6}$$

$$\circlearrowleft +\Sigma M_{NA} = 0; \qquad M + \frac{w_0 L}{6}\left(\frac{L}{9}\right) - \frac{w_0 L}{3}\left(\frac{L}{3}\right) = 0$$

$$M = \frac{5 w_0 L^2}{54}$$



Ans:

$$V_A = \frac{w_0 L}{3},$$

$$M_{\max} = \frac{23\, w_0 L^2}{216}$$

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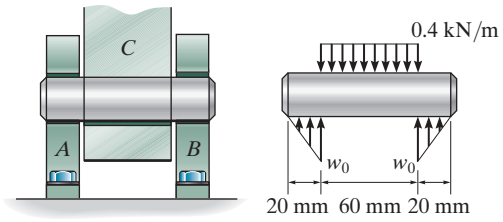
***6–20.**

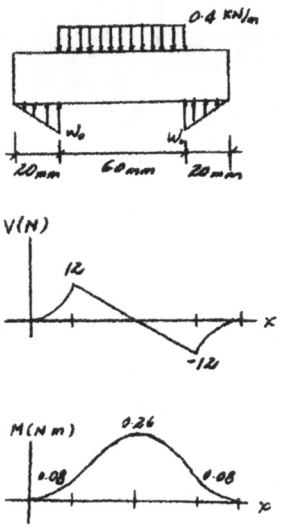
The smooth pin is supported by two leaves A and B and subjected to a compressive load of 0.4 kN/m caused by bar C. Determine the intensity of the distributed load w_0 of the leaves on the pin and draw the shear and moment diagram for the pin.



SOLUTION

$$+\uparrow \Sigma F_y = 0; \qquad 2(w_0)(20)\left(\frac{1}{2}\right) - 60(0.4) = 0$$

$$w_0 = 1.2\ \text{kN/m}$$ **Ans.**



Ans:
$w_0 = 1.2\ \text{kN/m}$

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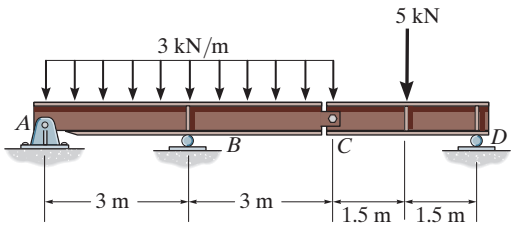
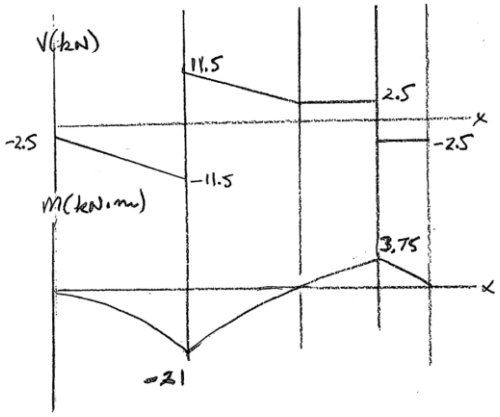
6–21. Draw the shear and moment diagrams for the compound beam.





Ans:
$x = 3^{-}, V = -11.5, M = -21$

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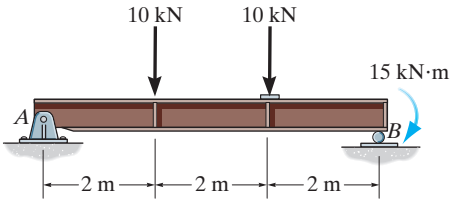
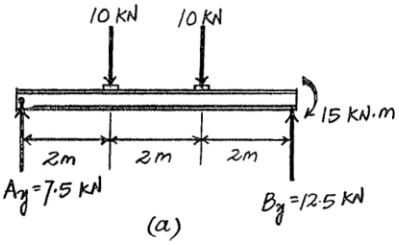
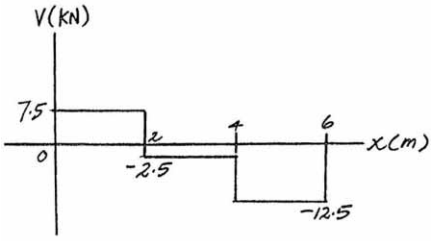
6–22. Draw the shear and moment diagrams for the simply supported beam.



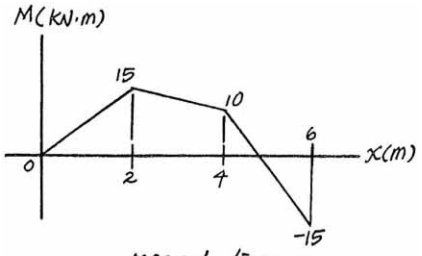


(a)



Shear diagram

(b)



Moment diagram

(c)

Ans:

$x = 0, V = 7.5\ \text{kN} \cdot x = 2^{-}\text{m}, V = 7.5\ \text{kN}$
$x = 2^{+}\text{m}, V = -2.5\ \text{kN}$
$x = 4^{+}\text{m}\ V = -12.5\ \text{kN}$
$x = 2\ \text{m}, M = 15\ \text{kN} \cdot \text{m} \cdot x = 4\ \text{m},$
$M = 10\ \text{kN} \cdot \text{m} \cdot x = 6\ \text{m}, \text{M} = -15\ \text{kN} \cdot \text{m}$

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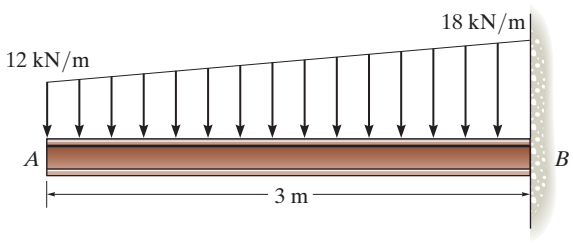
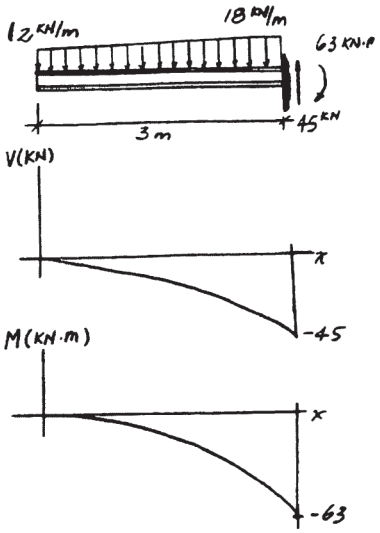
6–23.

Draw the shear and moment diagrams for the beam.



SOLUTION



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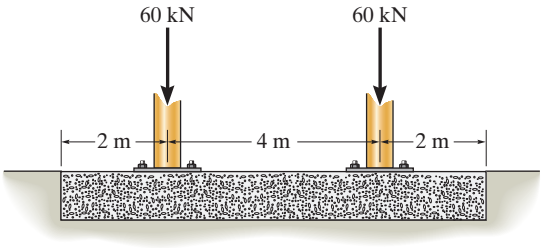
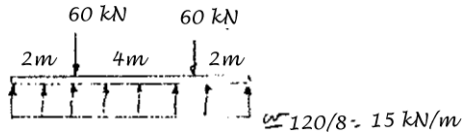
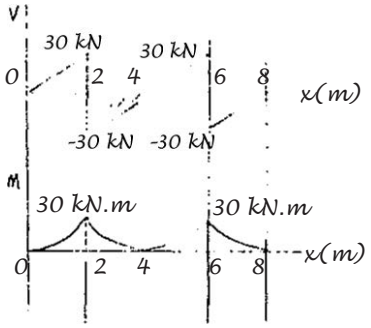
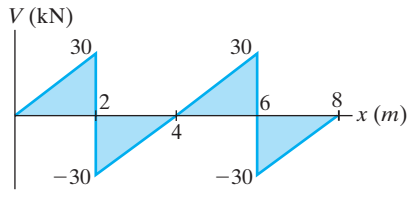
***6–24.** The footing supports the load transmitted by the two columns. Draw the shear and moment diagrams for the footing if the reaction of soil pressure on the footing is assumed to be uniform.

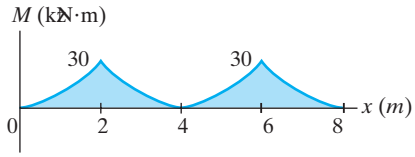






Ans:





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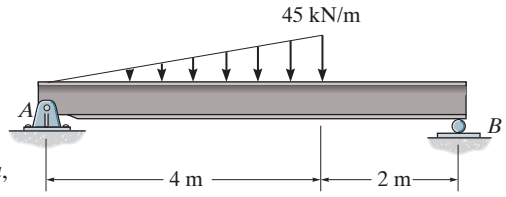
6–25. Draw the shear and moment diagrams for the overhanging beam.



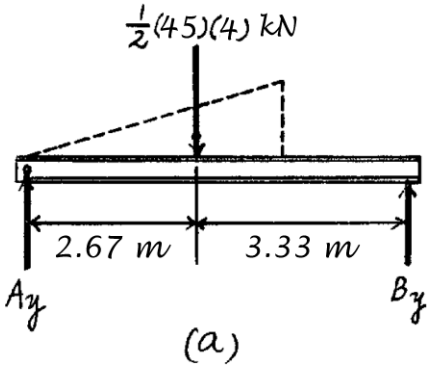
Support Reactions: Referring to the free-body diagram of the beam shown in Fig. *a*,

$$\zeta + \Sigma M_A = 0; \qquad B_y(6) - \frac{1}{2}(45)(4)(2.67) = 0$$

$$B_y = 40.0 \text{ kN}$$

$$+\uparrow \Sigma F_y = 0; \qquad A_y + 40.0 - \frac{1}{2}(45)(4) = 0$$

$$A_y = 50.0 \text{ kN}$$



(a)

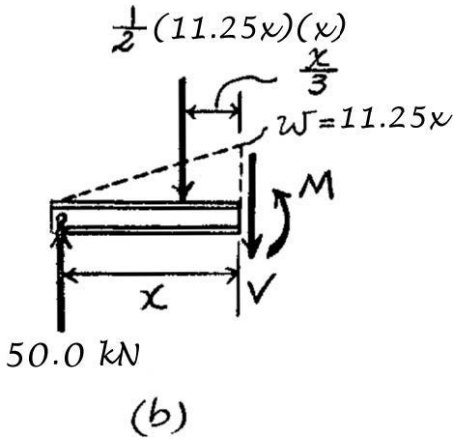
Shear and Moment Functions: For $0 \le x < 4$ m, we refer to the free-body diagram of the beam segment shown in Fig. *b*.

$$+\uparrow \Sigma F_y = 0; \qquad 50.0 - \frac{1}{2}(11.25x)(x) - V = 0$$

$$V = \{50.0 - 5.625x^2\} \text{ kN} \qquad \textbf{Ans.}$$

$$\zeta + \Sigma M = 0; \qquad M + \frac{1}{2}(11.25x)(x)\left(\frac{x}{3}\right) - 50.0x = 0$$

$$M = \{50.0x - 1.875x^3\} \text{ kN} \cdot \text{m} \qquad \textbf{Ans.}$$



(b)

When $V = 0$, from the shear function,

$$0 = 50.0 - 5.625x^2 \qquad x = 2.981 \text{ m}$$

Substituting this result into the moment function,

$$M|_{x=2.981\text{ m}} = 99.38 \text{ kN} \cdot \text{m}$$

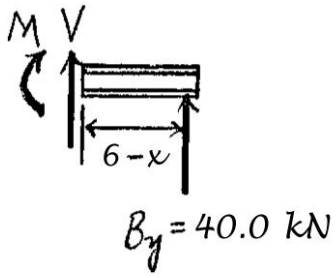
For $4 \text{ m} < x \le 6$ m, we refer to the free-body diagram of the beam segment shown in Fig. *c*.

$$+\uparrow \Sigma F_y = 0; \qquad V + 40.0 \text{ kN} = 0$$

$$V = -40.0 \text{ kN} \qquad \textbf{Ans.}$$

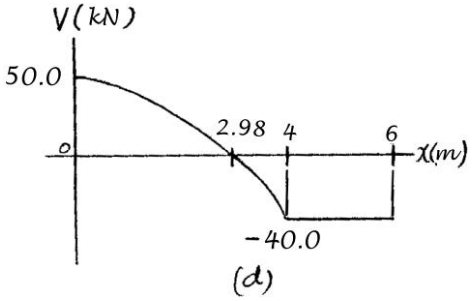
$$\zeta + \Sigma M = 0; \; 40.05(6 - x) - M = 0$$

$$M = \{40.0(6 - x)\} \text{ kN} \cdot \text{m} \qquad \textbf{Ans.}$$

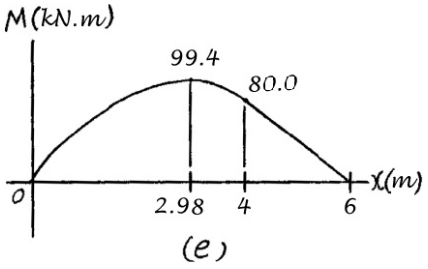


(c)

Shear and Moment Diagrams: As shown in Figs. *d* and *e*.



(d)



(e)

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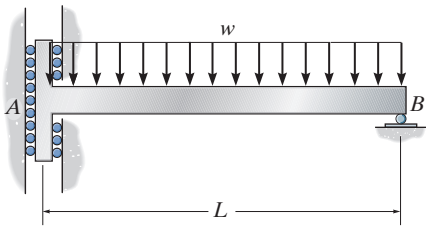
6–26.

The support at A allows the beam to slide freely along the vertical guide so that it cannot support a vertical force. Draw the shear and moment diagrams for the beam.



SOLUTION

Equations of Equilibrium: Referring to the free-body diagram of the beam shown in Fig. *a*,

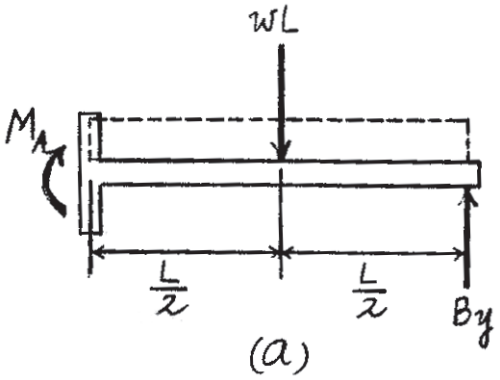
$$\circlearrowleft + \Sigma M_B = 0; \qquad wL\left(\frac{L}{2}\right) - M_A = 0$$

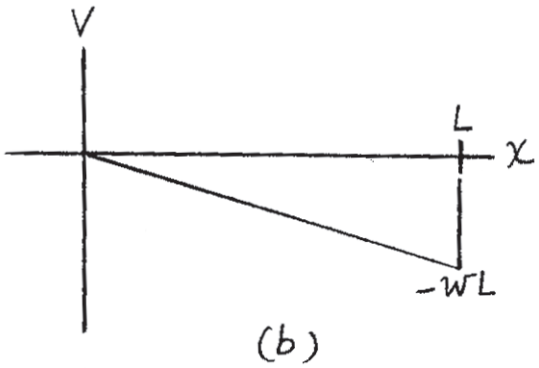
$$M_A = \frac{wL^2}{2}$$

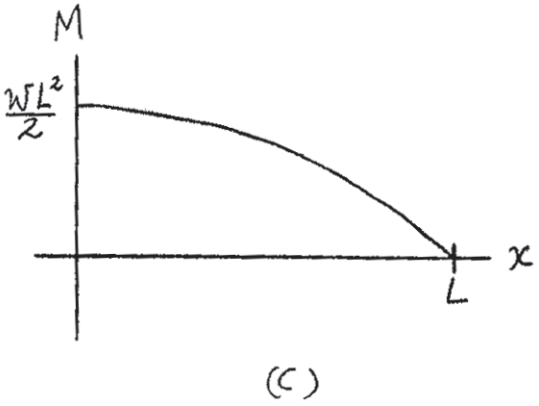
$$+\uparrow \Sigma F_y = 0; \qquad B_y - wL = 0$$

$$B_y = wL$$

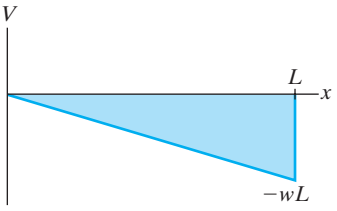
Shear and Moment Diagram: As shown in Figs. *b* and *c*.

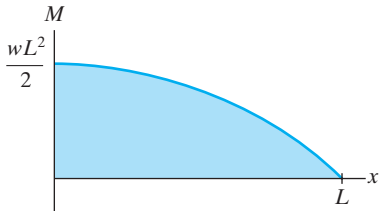






Ans:





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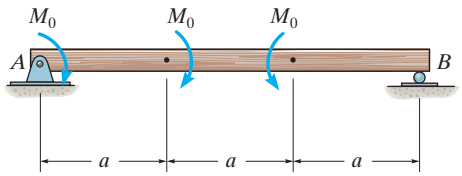
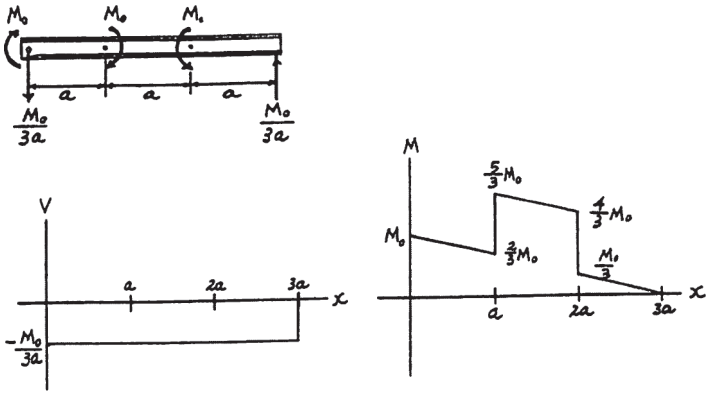
6–27.

Draw the shear and moment diagrams for the beam.



SOLUTION



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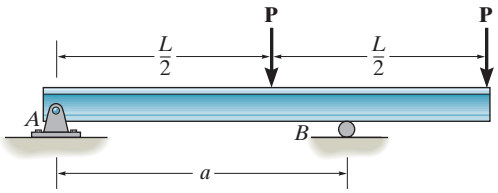
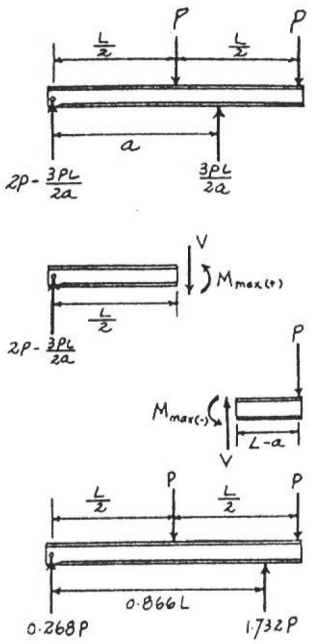
***6–28.**

Determine the placement distance a of the roller support so that the largest absolute value of the moment is a minimum. Draw the shear and moment diagrams for this condition.



SOLUTION

Support Reactions: As shown on FBD.



Absolute Minimum Moment: In order to get the absolute minimum moment, the maximum positive and maximum negative moment must be equal that is $M_{\max(+)} = M_{\max(-)}$.

For the positive moment:

$$\zeta+\Sigma M_{NA} = 0; \qquad M_{\max(+)} - \left(2P - \frac{3PL}{2a}\right)\left(\frac{L}{2}\right) = 0$$

$$M_{\max(+)} = PL - \frac{3PL^2}{4a}$$

For the negative moment:

$$\zeta+\Sigma M_{NA} = 0; \qquad M_{\max(-)} - P(L - a) = 0$$

$$M_{\max(-)} = P(L - a)$$

$$M_{\max(+)} = M_{\max(-)}$$

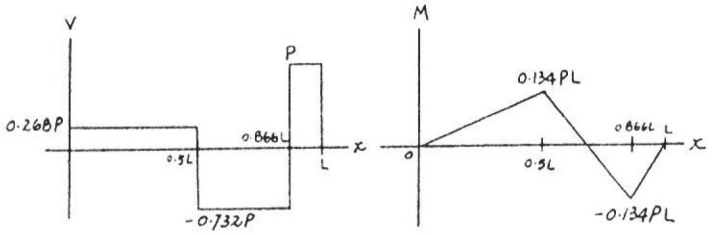
$$PL - \frac{3PL^2}{4a} = P(L - a)$$

$$4a\,L - 3L^2 = 4a\,L - 4a^2$$

$$a = \frac{\sqrt{3}}{2}L = 0.866L$$

Ans.

Shear and Moment Diagram:



Ans:
$a = 0.866L$

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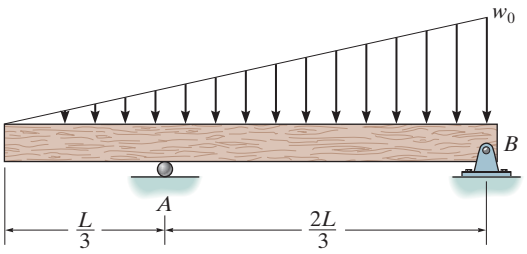
6–29.

Draw the shear and moment diagrams for the beam.



SOLUTION

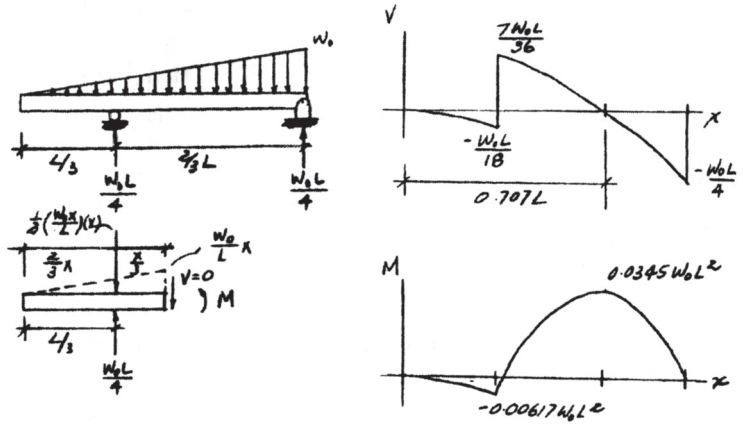
$$+\uparrow \Sigma F_y = 0; \qquad \frac{w_0 L}{4} - \frac{1}{2}\left(\frac{w_0 x}{L}\right)(x) = 0$$

$$x = 0.7071\, L$$

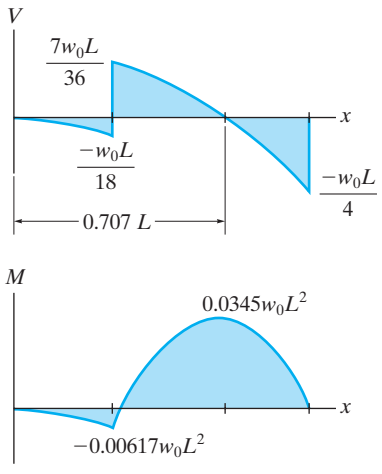
$$\zeta + \Sigma M_{NA} = 0; \qquad M + \frac{1}{2}\left(\frac{w_0 x}{L}\right)(x)\left(\frac{x}{3}\right) - \frac{w_0 L}{4}\left(x - \frac{L}{3}\right) = 0$$

Substitute $x = 0.7071L$,

$$M = 0.0345\, w_0 L^2$$



Ans:



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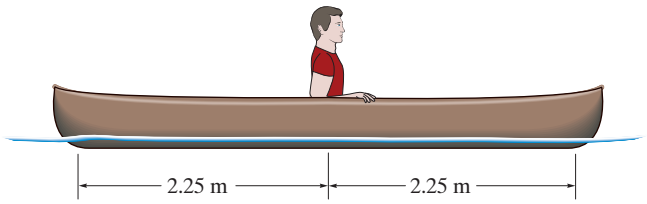
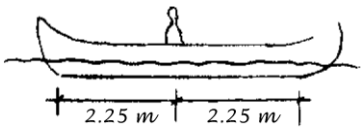
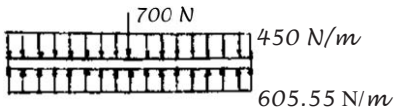
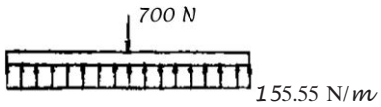
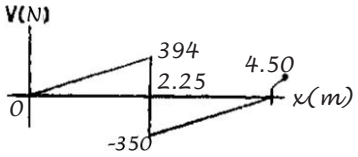
6–30. The 700-N man sits in the center of the boat, which has a uniform width and a weight per linear foot of 450 N/m. Determine the maximum bending moment exerted on the boat. Assume that the water exerts a uniform distributed load upward on the bottom of the boat.

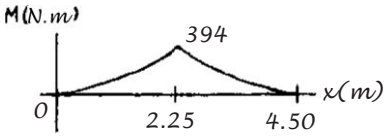






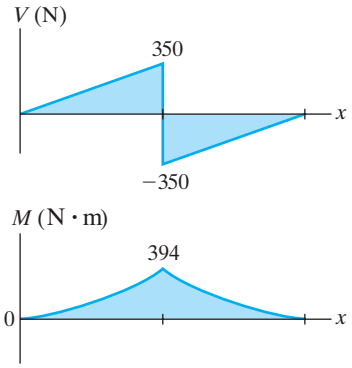






$M_{max} = 394 \text{ N} \cdot \text{m}$ **Ans.**

Ans:
$M_{max} = 394 \text{ N} \cdot \text{m}$



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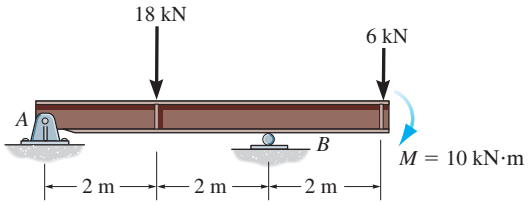
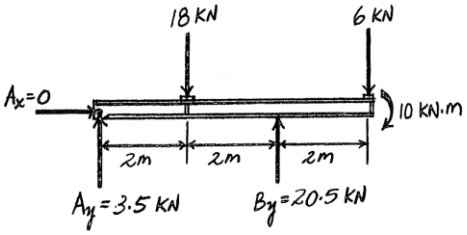
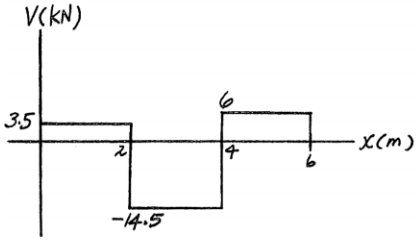
6–31. Draw the shear and moment diagrams for the overhang beam.



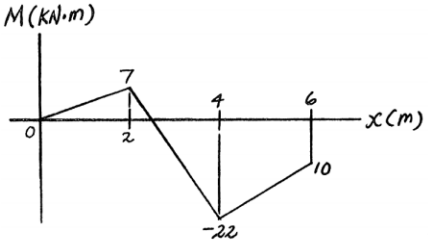


(a)



shear diagram

(b)



moment diagram

(c)

Ans:

$x = 0, V = 3.5\ \text{kN} \cdot x = 2^{+}\text{m}, V = -14.5\ \text{kN} \cdot$
$x = 4^{+}\text{m}, V = 6\ \text{kN}$
$x = 2\ \text{m}, M = 7\ \text{kN} \cdot \text{m}\, x = 4\ \text{m},$
$M = -22\ \text{kN} \cdot \text{m} \cdot x = 6\ \text{m}$
$M = 10\ \text{kN} \cdot \text{m}$

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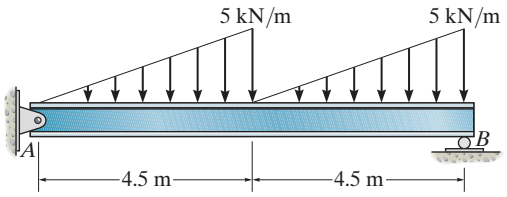
***6–32.** Draw the shear and moment diagrams for the beam.



From FBD(a)

$$+\uparrow \Sigma F_y = 0; \qquad 9.375 - 0.5556x^2 = 0 \qquad x = 4.108 \text{ m}$$

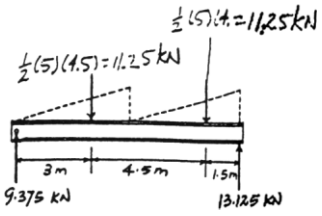
$$\circlearrowleft +\Sigma M_{NA} = 0; \qquad M + (0.5556)\left(4.108^2\right)\left(\frac{4.108}{3}\right) - 9.375(4.108) = 0$$

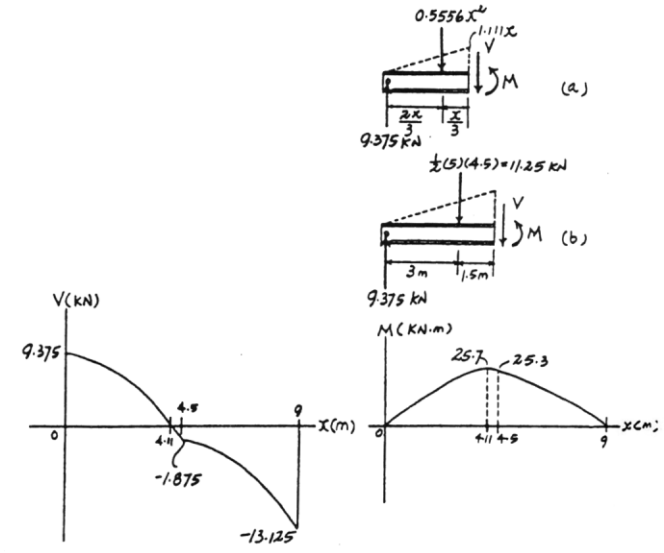
$$M = 25.67 \text{ kN} \cdot \text{m}$$

From FBD(b)

$$\circlearrowleft +\Sigma M_{NA} = 0; \qquad M + 11.25(1.5) - 9.375(4.5) = 0$$

$$M = 25.31 \text{ kN} \cdot \text{m}$$





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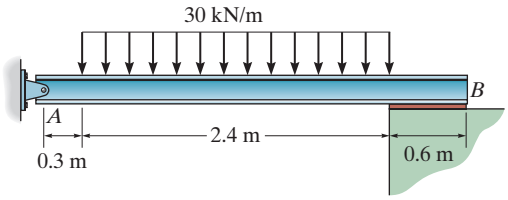
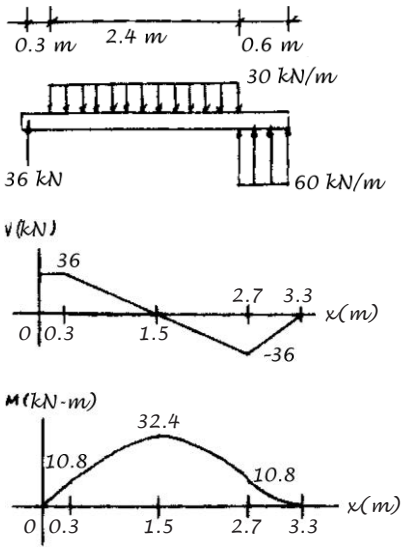
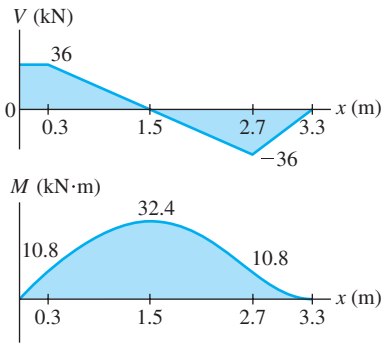
6–33. The beam is bolted or pinned at A and rests on a bearing pad at B that exerts a uniform distributed loading on the beam over its 0.6-m length. Draw the shear and moment diagrams for the beam if it supports a uniform loading of 30 kN/m.





Ans:



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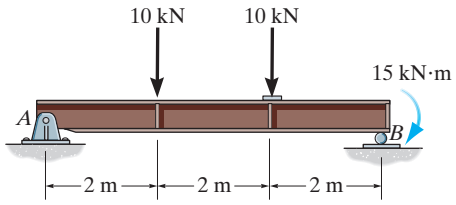
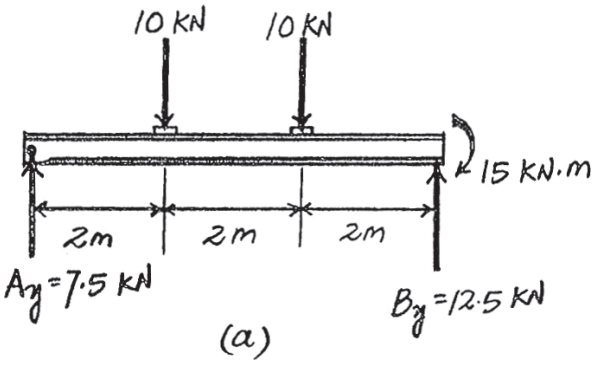
6–34.

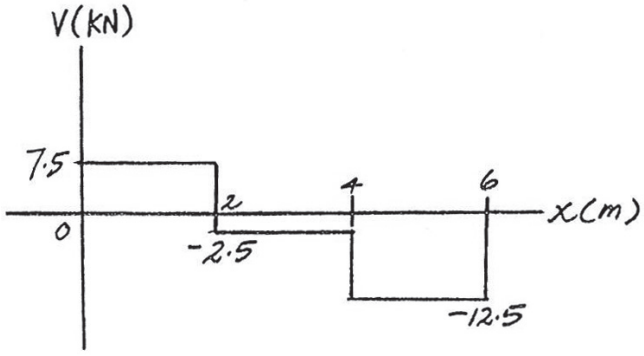
Draw the shear and moment diagrams for the simply supported beam.



SOLUTION

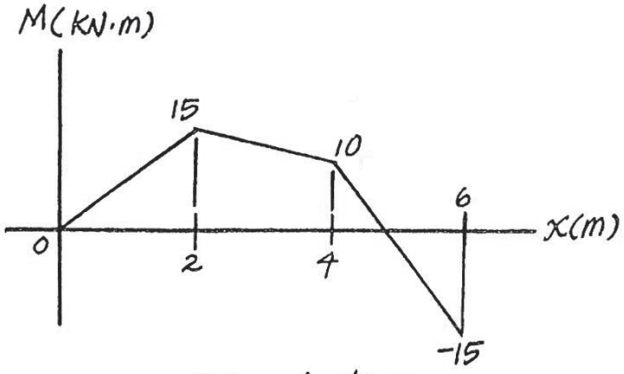


(a)



Shear diagram

(b)



Moment diagram

(c)

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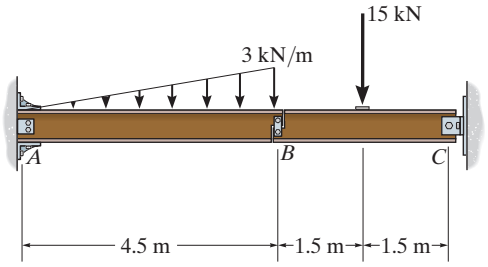
6–35.

A short link at B is used to connect beams AB and BC to form the compound beam. Draw the shear and moment diagrams for the beam if the supports at A and C are considered fixed and pinned, respectively.



SOLUTION

Support Reactions: Referring to the free-body diagram of segment BC shown in Fig. a,

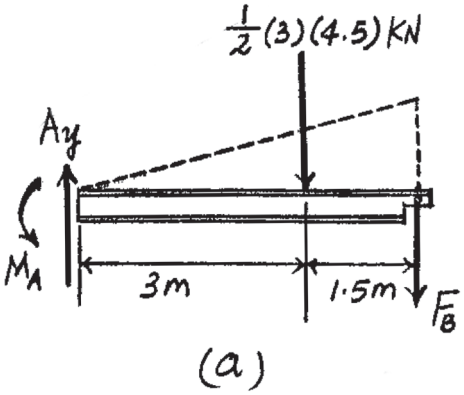
$$\zeta+\Sigma M_C = 0; \qquad 15(1.5) - F_B(3) = 0$$
$$F_B = 7.5\ \text{kN}$$

$$+\uparrow \Sigma F_y = 0; \qquad C_y + 7.5 - 15 = 0$$
$$C_y = 7.5\ \text{kN}$$

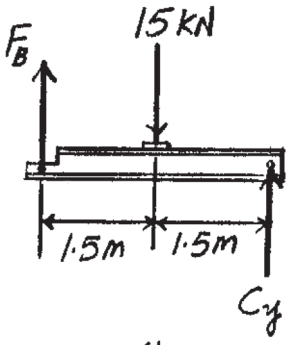
Using the result of $\mathbf{F}_B$ and referring to the free-body diagram of segment AB, Fig. b,

$$+\uparrow \Sigma F_y = 0; \qquad A_y - \frac{1}{2}(3)(4.5) - 7.5 = 0$$
$$A_y = 14.25\ \text{kN}$$

$$\zeta+\Sigma M_A = 0; \qquad M_A - \frac{1}{2}(3)(4.5)(3) - 7.5(4.5) = 0$$
$$M_A = 54\ \text{kN}\cdot\text{m}$$

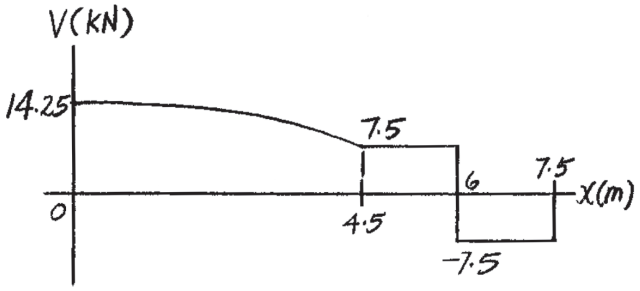


(a)

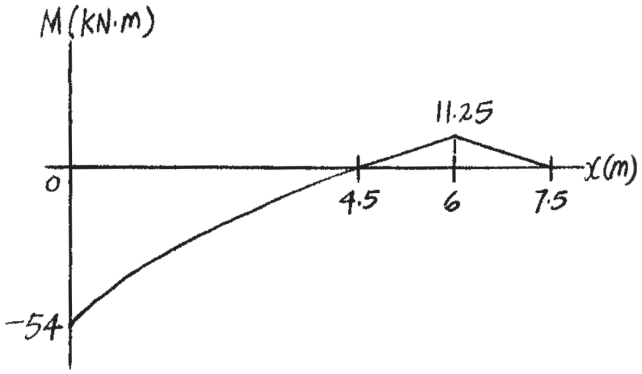


(b)

Shear and Moment Diagrams: As shown in Figs. c and d.

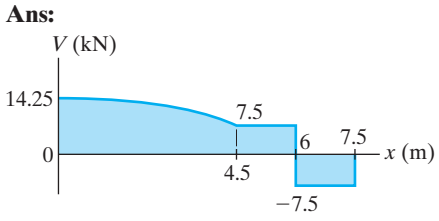


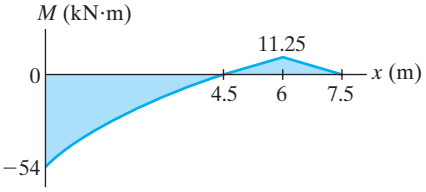
(c)



(d)

Ans:





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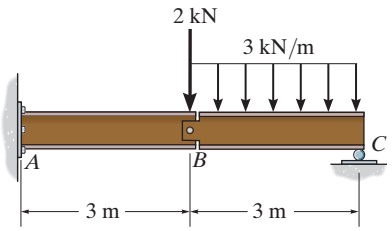
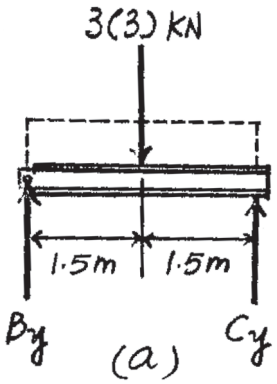
***6–36.**

The compound beam is fixed at A, pin connected at B, and supported by a roller at C. Draw the shear and moment diagrams for the beam.



SOLUTION



Support Reactions: Referring to the free-body diagram of segment BC shown in Fig. a,

$$\zeta+\Sigma M_B = 0; \qquad C_y(3) - 3(3)(1.5) = 0$$

$$C_y = 4.5 \text{ kN}$$

$$\uparrow \Sigma F_y = 0; \qquad B_y + 4.5 - 3(3) = 0$$

$$B_y = 4.5 \text{ kN}$$

Using the result of $\mathbf{B}_y$ and referring to the free-body diagram of segment AB, Fig. b,

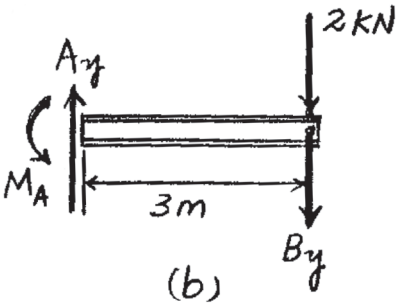
$$\uparrow \Sigma F_y = 0; \qquad A_y - 2 - 4.5 = 0$$

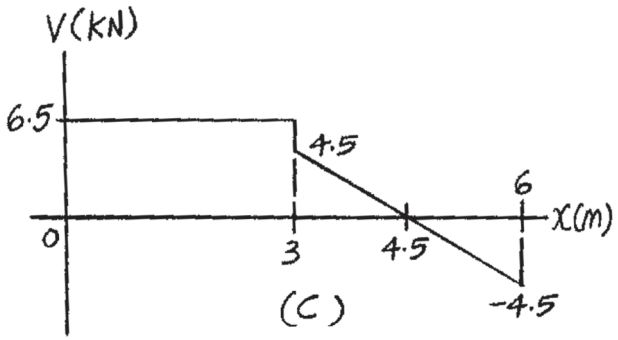
$$A_y = 6.5 \text{ kN}$$

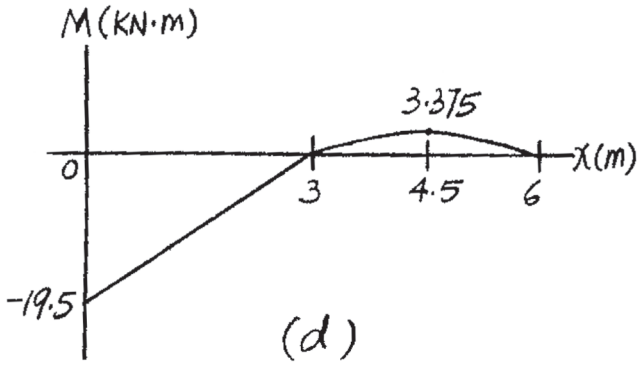
$$\zeta+\Sigma M_A = 0; \qquad M_A - 2(3) - 4.5(3) = 0$$

$$M_A = 19.5 \text{ kN}\cdot\text{m}$$

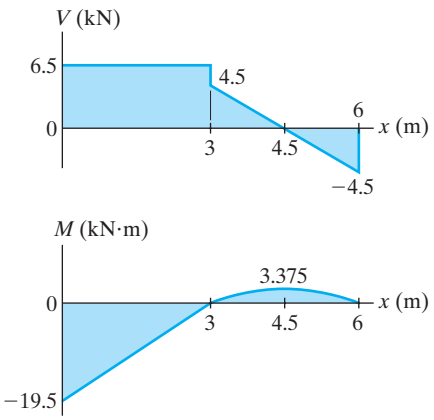
Shear and Moment Diagrams: As shown in Figs. c and d.







Ans:



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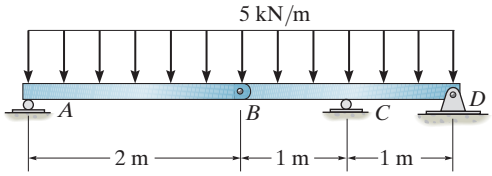
6–37.

Draw the shear and moment diagrams for the compound beam.



SOLUTION

Support Reactions:

From the FBD of segment AB

$\zeta+\Sigma M_A = 0; \quad B_y\,(2) - 10.0(1) = 0 \quad B_y = 5.00\ \text{kN}$

$+\uparrow \Sigma F_y = 0; \quad A_y - 10.0 + 5.00 = 0 \quad A_y = 5.00\ \text{kN}$

From the FBD of segment BD

$\zeta+\Sigma M_C = 0; \quad 5.00(1) + 10.0(0) - D_y\,(1) = 0$

$D_y = 5.00\ \text{kN}$

$+\uparrow \Sigma F_y = 0; \quad C_y - 5.00 - 5.00 - 10.0 = 0$

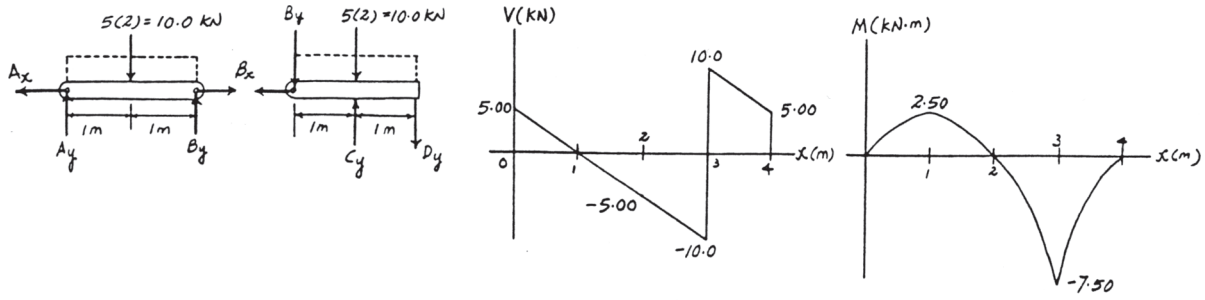
$C_y = 20.0\ \text{kN}$

$\pm \rightarrow \Sigma F_x = 0; \quad B_x = 0$

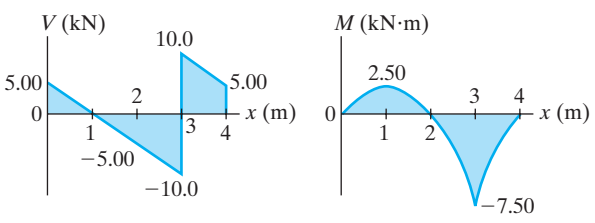
From the FBD of segment AB

$\pm \rightarrow \Sigma F_x = 0; \quad A_x = 0$

Shear and Moment Diagram:



Ans:



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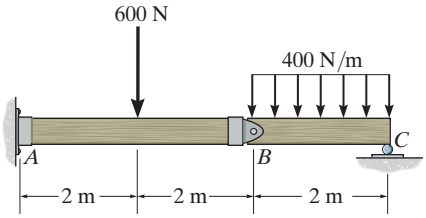
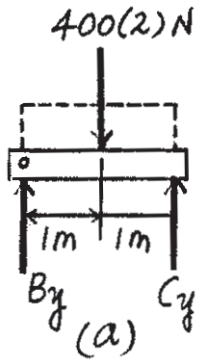
6–38.

The compound beam is fixed at A, pin connected at B, and supported by a roller at C. Draw the shear and moment diagrams for the beam.



SOLUTION



Support Reactions: Referring to the free-body diagram of segment BC shown in Fig. a,

$$\zeta+\Sigma M_B = 0; \qquad C_y(2) - 400(2)(1) = 0$$

$$C_y = 400 \text{ N}$$

$$+\uparrow \Sigma F_y = 0; \qquad B_y + 400 - 400(2) = 0$$

$$B_y = 400 \text{ N}$$

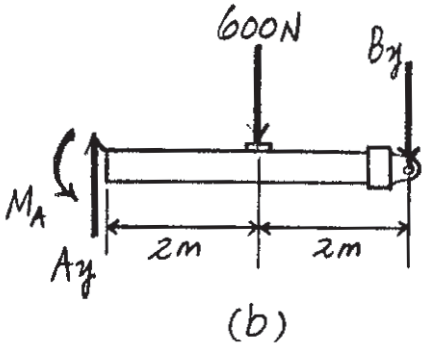
Using the result of $\mathbf{B}_y$ and referring to the free-body diagram of segment AB, Fig. b,

$$+\uparrow \Sigma F_y = 0; \qquad A_y - 600 - 400 = 0$$

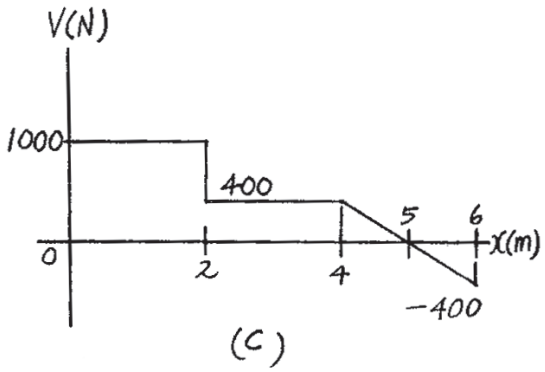
$$A_y = 1000 \text{ N}$$

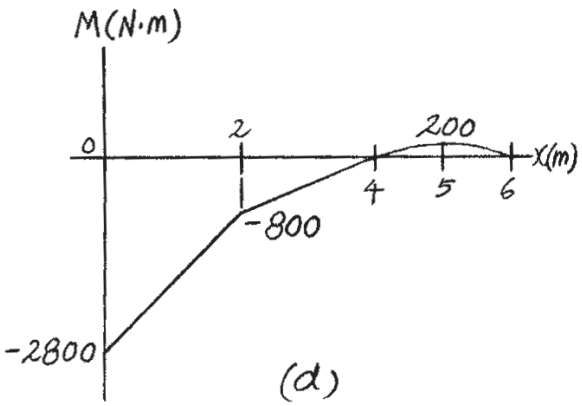
$$\zeta+\Sigma M_A = 0; \qquad M_A - 600(2) - 400(4) = 0$$

$$M_A = 2800 \text{ N}$$

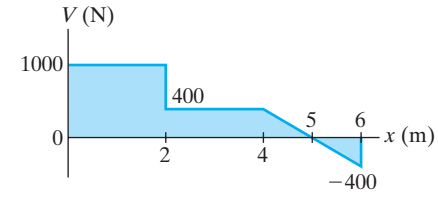


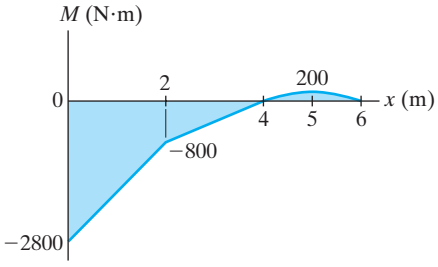
Shear and Moment Diagrams: As shown in Figs. c and d.





Ans:





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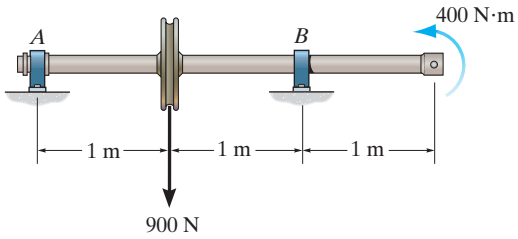
6–39.

The shaft is supported by a smooth thrust bearing at A and smooth journal bearing at B. Draw the shear and moment diagrams for the shaft.



SOLUTION

Equations of Equilibrium: Referring to the free-body diagram of the shaft shown in Fig. *a*,

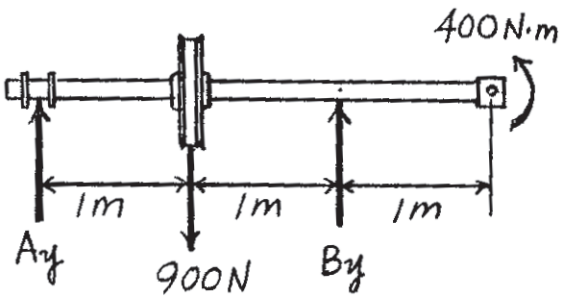
$$\zeta + \Sigma M_A = 0; \qquad B_y(2) + 400 - 900(1) = 0$$

$$B_y = 250\ \text{N}$$

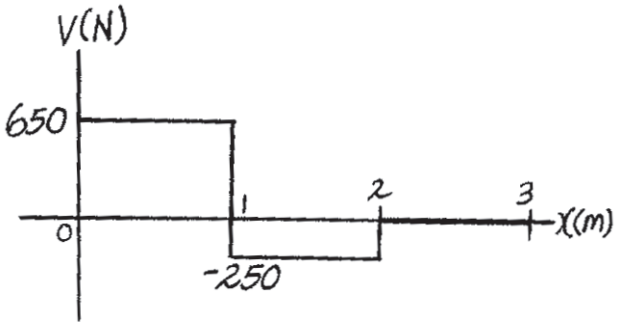
$$+\uparrow \Sigma F_y = 0; \qquad A_y + 250 - 900 = 0$$

$$A_y = 650\ \text{N}$$

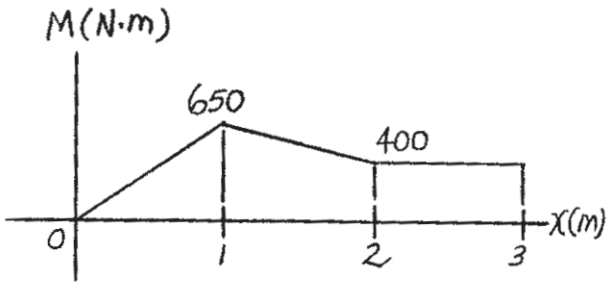
Shear and Moment Diagram: As shown in Figs. *b* and *c*.



(a)

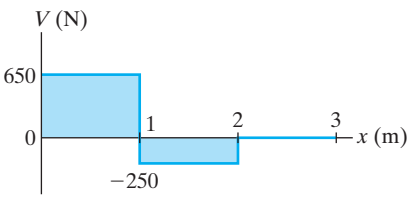


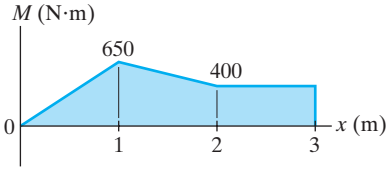
(b)



(c)

Ans:





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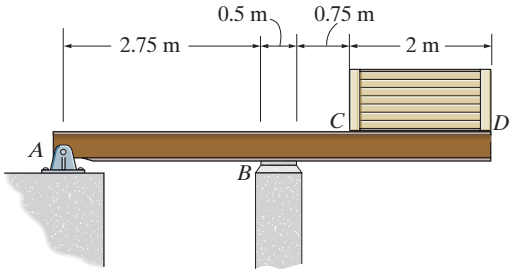
***6–40.**

The beam is used to support a uniform load along *CD* due to the 6-kN weight of the crate. Also, the reaction at the bearing support *B* can be assumed uniformly distributed along its width. Draw the shear and moment diagrams for the beam.



SOLUTION

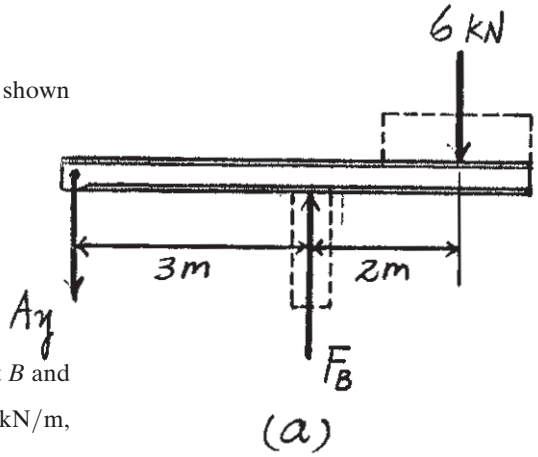
Equations of Equilibrium: Referring to the free-body diagram of the beam shown in Fig. *a*,

$\circlearrowleft + \Sigma M_A = 0;$ $\quad F_B(3) - 6(5) = 0$

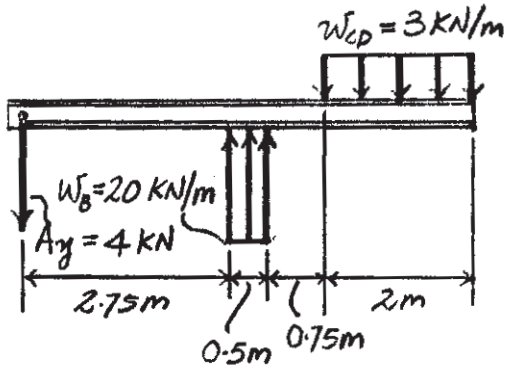
$F_B = 10\ \text{kN}$

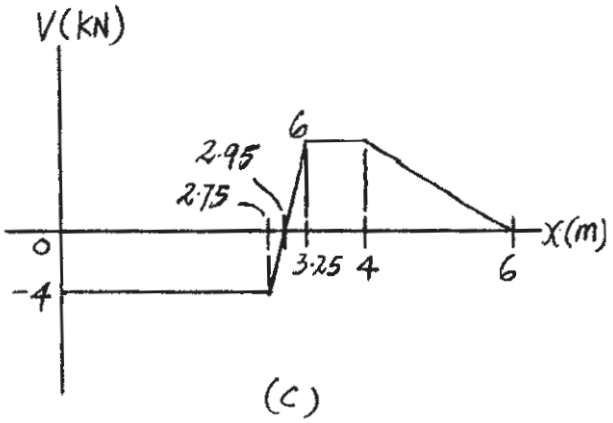
$+\uparrow \Sigma F_y = 0;$ $\quad 10 - 6 - A_y = 0$

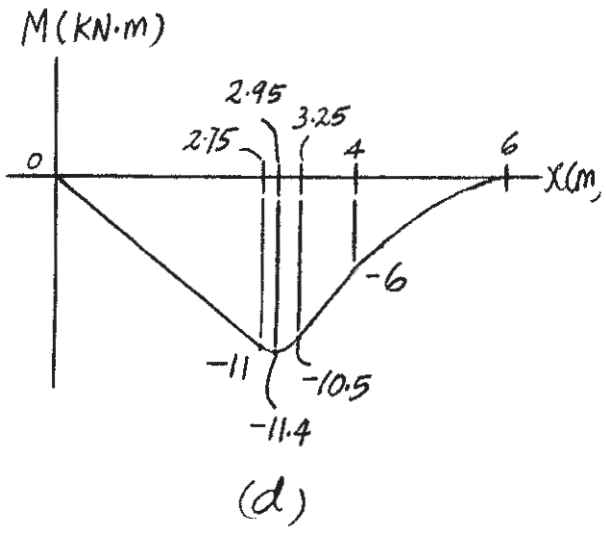
$A_y = 4\ \text{kN}$



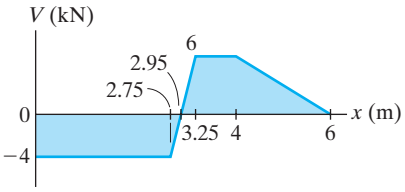
Shear and Moment Diagram: The intensity of the distributed load at support *B* and portion *CD* of the beam are $w_B = \frac{F_B}{0.5} = \frac{10}{0.5} = 20\ \text{kN/m}$ and $w_{CD} = \frac{6}{2} = 3\ \text{kN/m}$, Fig. *b*. The shear and moment diagrams are shown in Figs. *c* and *d*.

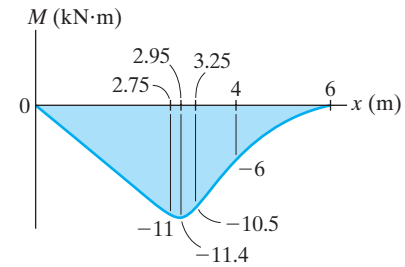






Ans:





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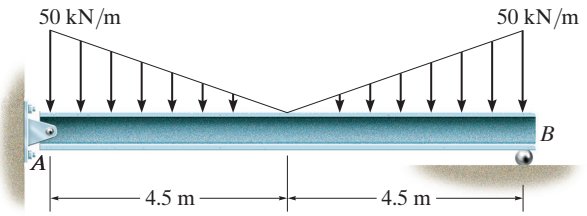
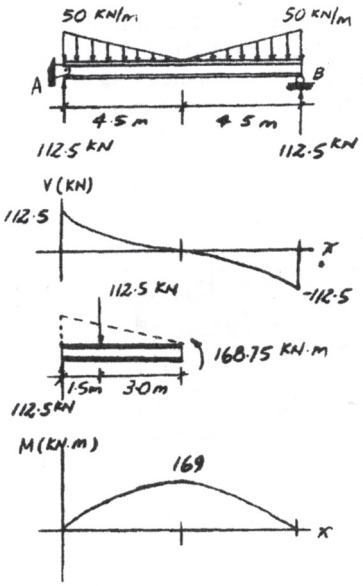
6–41.

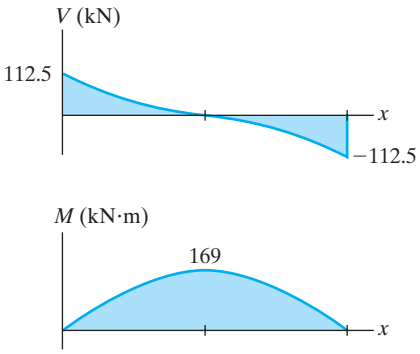
Draw the shear and moment diagrams for the beam.



SOLUTION



Ans:



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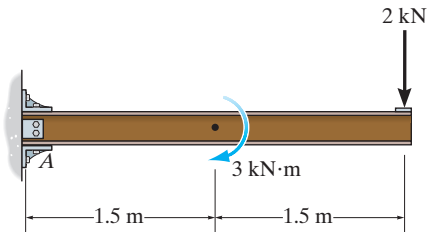
6–42.

Draw the shear and moment diagrams for the cantilever beam.



SOLUTION

Equations of Equilibrium: Referring to the free-body diagram of the beam shown in Fig. *a*,

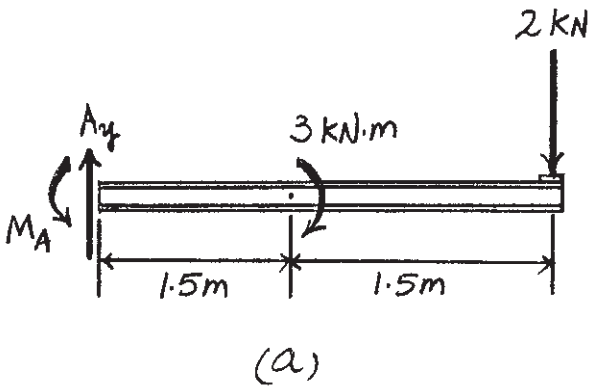
$+\uparrow \Sigma F_y = 0;$ $\quad A_y - 2 = 0$

$A_y = 2\ \text{kN}$

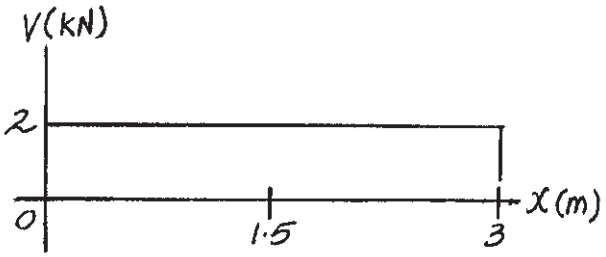
$\circlearrowleft + \Sigma M_A = 0;$ $\quad M_A - 3 - 2(3) = 0$

$M_A = 9\ \text{kN} \cdot \text{m}$

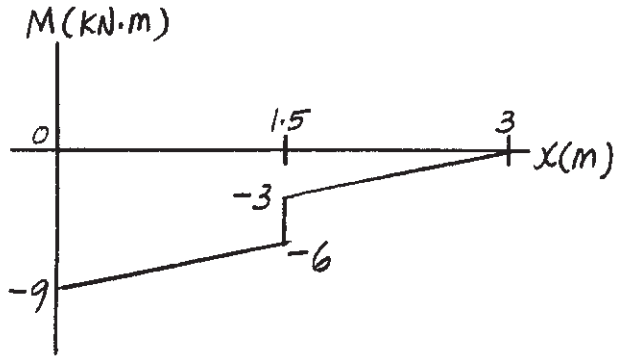
Shear and Moment Diagram: As shown in Figs. *b* and *c*.



(a)

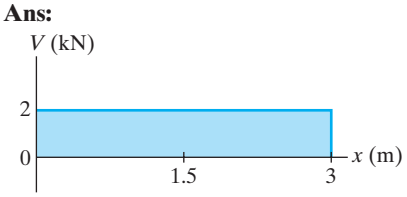


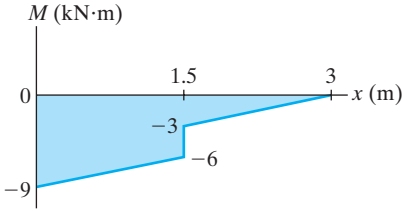
(b)



(c)

Ans:





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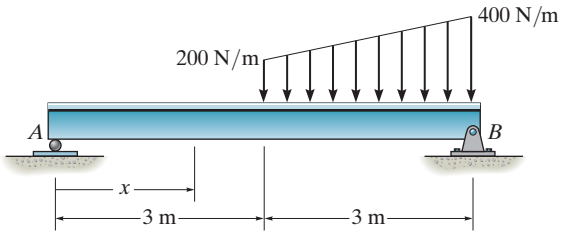
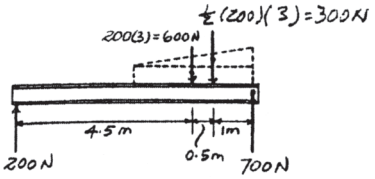
6–43.

Draw the shear and moment diagrams for the beam.



SOLUTION

Support Reactions: As shown on FBD.



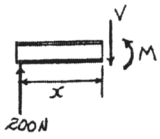
Shear and Moment Functions:

For $0 \le x < 3$ m:

$$+\uparrow \Sigma F_y = 0; \quad 200 - V = 0 \quad V = 200 \text{ N}$$ **Ans.**

$$\zeta + \Sigma M_{NA} = 0; \quad M - 200\,x = 0$$

$$M = \{200\,x\} \text{ N} \cdot \text{m}$$ **Ans.**



For $3 \text{ m} < x \le 6$ m:

$$+\uparrow \Sigma F_y = 0; \quad 200 - 200(x-3) - \frac{1}{2}\left[\frac{200}{3}(x-3)\right](x-3) - V = 0$$

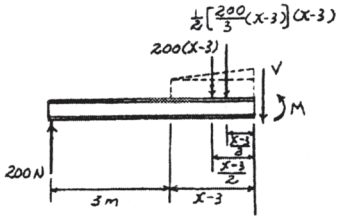
$$V = \left\{-\frac{100}{3}x^2 + 500\right\} \text{N}$$ **Ans.**

Set $V = 0, x = 3.873$ m

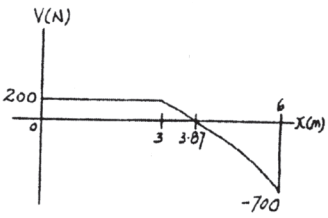
$$\zeta + \Sigma M_{NA} = 0; \quad M + \frac{1}{2}\left[\frac{200}{3}(x-3)\right](x-3)\left(\frac{x-3}{3}\right)$$

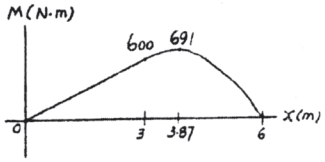
$$+ 200(x-3)\left(\frac{x-3}{2}\right) - 200x = 0$$

$$M = \left\{-\frac{100}{9}x^3 + 500x - 600\right\} \text{N} \cdot \text{m}$$ **Ans.**

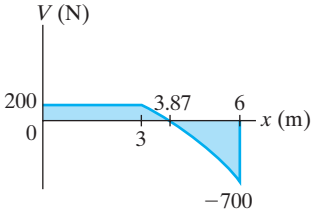


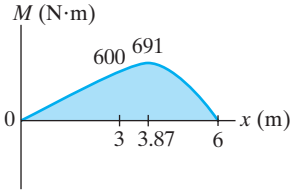
Substitute $x = 3.87$ m, $M = 691$ N · m





Ans:
For $0 \le x < 3$ m: $V = 200$ N, $M = \{200x\}$ N · m,
For $3 \text{ m} < x \le 6$ m: $V = \left\{-\frac{100}{3}x^2 + 500\right\}$ N,
$M = \left\{-\frac{100}{9}x^3 + 500x - 600\right\}$ N · m





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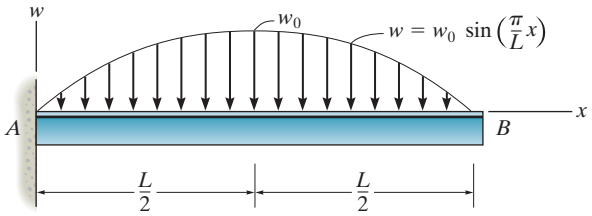
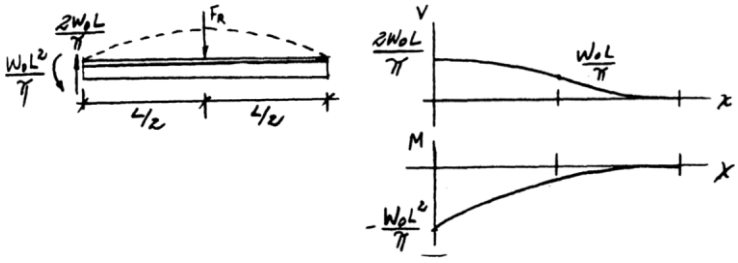
***6–44.** Draw the shear and moment diagrams for the beam.



$$F_R = \int_A dA = w_0 \int_0^L \sin\left(\frac{\pi}{L}x\right)\mathrm{d}x = \frac{2w_0 L}{\pi}$$



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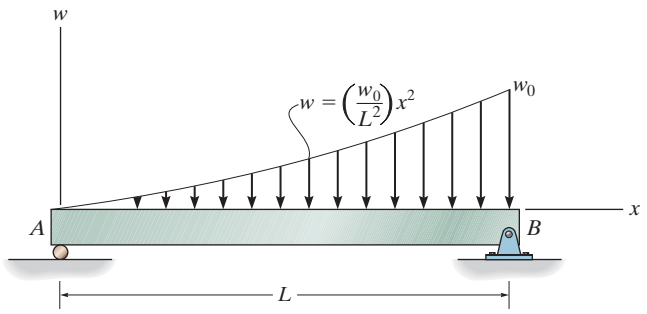
6–45. Draw the shear and moment diagrams for the beam.



$$F_R = \int_A dA = \int_0^L w dx = \frac{w_0}{L^2}\int_0^L x^2 dx = \frac{w_0 L}{3}$$

$$\overline{x} = \frac{\int_A x dA}{\int_A dA} = \frac{\frac{w_0}{L^2}\int_0^L x^3 dx}{\frac{w_0 L}{3}} = \frac{3L}{4}$$

$$+\uparrow \Sigma F_y = 0; \qquad \frac{w_0 L}{12} - \frac{w_0 x^3}{3L^2} = 0$$

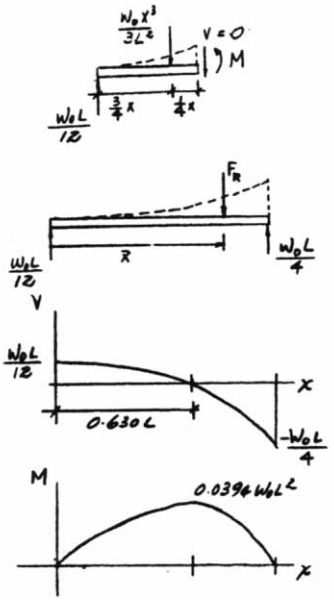
$$x = \left(\frac{1}{4}\right)^{1/3} L = 0.630\, L$$

$$\circlearrowleft + \Sigma M = 0; \qquad \frac{w_0 L}{12}(x) - \frac{w_0 x^3}{3L^2}\left(\frac{1}{4}x\right) - M = 0$$

$$M = \frac{w_0 L x}{12} - \frac{w_0 x^4}{12L^2}$$

Substitute $x = 0.630L$

$$M = 0.0394\, w_0 L^2$$



Ans:

$x = 0.630\, L, V = 0, M = 0.0394 w_0 L^2,$

$$M = \frac{w_0 L x}{12} - \frac{w_0 x^4}{12L^2}$$

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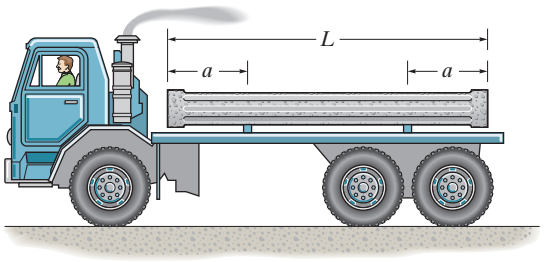
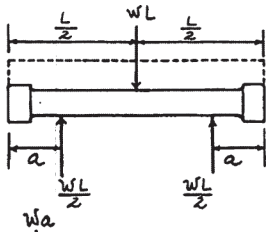
6–46.

The truck is to be used to transport the concrete column. If the column has a uniform weight of w (force/length), determine the equal placement a of the supports from the ends so that the absolute maximum bending moment in the column is as small as possible. Also, draw the shear and moment diagrams for the column.



SOLUTION



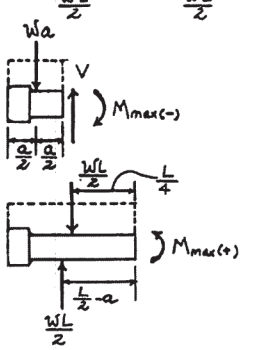
Support Reactions: As shown on FBD.

Absolute Minimum Moment: In order to get the absolute minimum moment, the maximum positive and maximum negative moment must be equal that is $M_{\max(+)} = M_{\min(-)}$.

For the positive moment:

$$\zeta+\Sigma M_{NA} = 0; \qquad M_{\max(+)} + \frac{wL}{2}\left(\frac{L}{4}\right) - \frac{wL}{2}\left(\frac{L}{2} - a\right) = 0$$

$$M_{\max(+)} = \frac{wL^2}{8} - \frac{waL}{2}$$



For the negative moment:

$$\zeta+\Sigma M_{NA} = 0; \qquad wa\left(\frac{a}{2}\right) - M_{\max(-)} = 0$$

$$M_{\max(-)} = \frac{wa^2}{2}$$

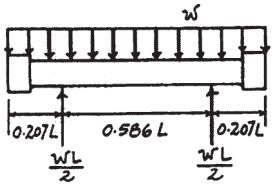
$$M_{\max(+)} = M_{\max(-)}$$

$$\frac{wL^2}{8} - \frac{wL}{2}a = \frac{wa^2}{2}$$

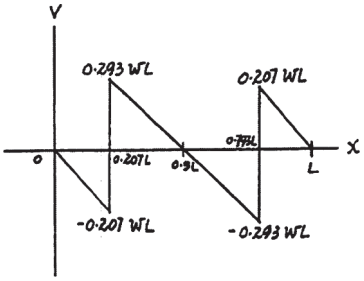
$$4a^2 + 4La - L^2 = 0$$

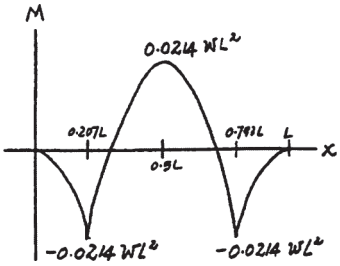
$$a = \frac{-4L \pm \sqrt{16L^2 - 4(4)(-L^2)}}{2(4)}$$

$$a = 0.207L$$ **Ans.**



Shear and Moment Diagram:





Ans:
$a = 0.207L$

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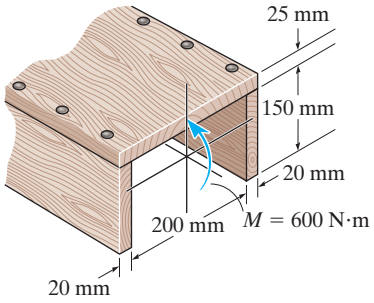
6–47.

The beam is made from three boards nailed together as shown. If the moment acting on the cross section is $M = 600\ \text{N}\cdot\text{m}$, determine the maximum bending stress in the beam. Sketch a three-dimensional view of the stress distribution and cover the cross section.



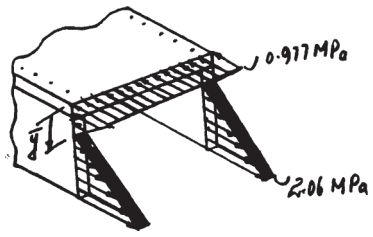
SOLUTION

$$\bar{y} = \frac{(0.0125)(0.24)(0.025) + 2(0.1)(0.15)(0.02)}{0.24(0.025) + 2(0.15)(0.02)} = 0.05625\ \text{m}$$

$$I = \frac{1}{12}(0.24)\left(0.025^3\right) + (0.24)(0.025)\left(0.04375^2\right)$$

$$+2\left(\frac{1}{12}\right)(0.02)\left(0.15^3\right) + 2(0.15)(0.02)\left(0.04375^2\right)$$

$$= 34.53125\left(10^{-6}\right)\text{m}^4$$



$$\sigma_{\max} = \sigma_B = \frac{Mc}{I}$$

$$= \frac{600(0.175 - 0.05625)}{34.53125\left(10^{-6}\right)}$$

$$= 2.06\ \text{MPa}$$ **Ans.**

$$\sigma_c = \frac{My}{I} = \frac{600(0.05625)}{34.53125\left(10^{-6}\right)} = 0.977\ \text{MPa}$$

Ans:
$\sigma_{\max} = 2.06\ \text{MPa}$

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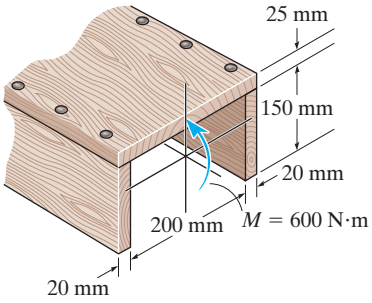
***6–48.**

The beam is made from three boards nailed together as shown. If the moment acting on the cross section is $M = 600\ \text{N}\cdot\text{m}$, determine the resultant force the bending stress produces on the top board.



SOLUTION

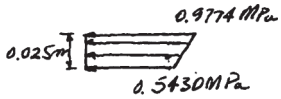
$$\bar{y} = \frac{(0.0125)(0.24)(0.025) + 2(0.15)(0.1)(0.02)}{0.24(0.025) + 2(0.15)(0.02)} = 0.05625\ \text{m}$$

$$I = \frac{1}{12}(0.24)\left(0.025^3\right) + (0.24)(0.025)\left(0.04375^2\right) + 2\left(\frac{1}{12}\right)(0.02)\left(0.15^3\right) + 2(0.15)(0.02)\left(0.04375^2\right)$$

$$= 34.53125\left(10^{-6}\right)\ \text{m}^4$$

$$\sigma_t = \frac{My}{I} = \frac{600(0.05625)}{34.53125\left(10^{-6}\right)} = 0.9774\ \text{MPa}$$

$$\sigma_b = \frac{My}{I} = \frac{600(0.05625 - 0.025)}{34.53125\left(10^{-6}\right)} = 0.5430\ \text{MPa}$$



$$F = \frac{1}{2}(0.025)(0.9774 + 0.5430)\left(10^6\right)(0.240) = 4.56\ \text{kN}$$ **Ans.**

Ans:
$F = 4.56\ \text{kN}$

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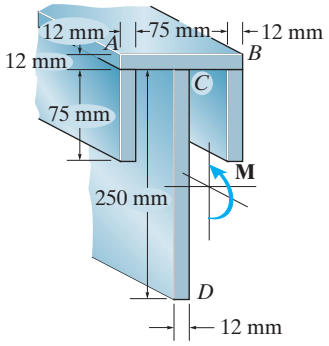
6–49. Determine the moment M that will produce a maximum stress of 70 MPa on the cross-section.



Section Properties:

$$\bar{y} = \frac{\Sigma \bar{y} A}{\Sigma A}$$

$$= \frac{0.006(0.099)(0.012) + 2[0.0495(0.012)(0.075)] + 0.137(0.012)(0.25)}{0.099(0.012) + 2(0.012)(0.075) + 0.012(0.25)} = 0.08471 \text{ m}$$

$$I_{NA} = \frac{1}{12}(0.099)(0.012^3) + (0.099)(0.012)(0.08471 - 0.006)^2$$

$$+ 2\left[\frac{1}{12}(0.012)(0.075^3) + (0.012)(0.075)(0.08471 - 0.0495)^2\right]$$

$$+ \frac{1}{12}(0.012)(0.25^3) + (0.012)(0.25)(0.137 - 0.08471)^2$$

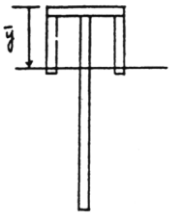
$$= 34.2773(10^{-6}) \text{ m}^4$$

Maximum Bending Stress: Applying the flexure formula

$$\sigma_{\max} = \frac{Mc}{I}$$

$$70(10^6) = \frac{M(0.262 - 0.08471)}{34.2773(10^{-6})}$$

$$M = 13.53(10^3) \text{ N} \cdot \text{m} = 13.5 \text{ kN} \cdot \text{m}$$ **Ans.**



Ans:

$M = 13.5 \text{ kN} \cdot \text{m}$

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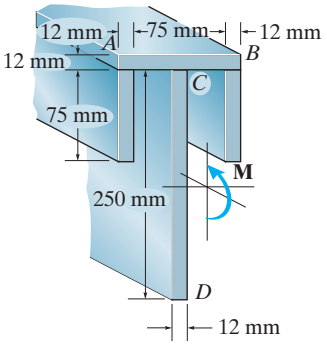
6–50. Determine the maximum tensile and compressive bending stress in the beam if it is subjected to a moment of $M = 6\ \text{kN} \cdot \text{m}$.



Section Properties:

$$\bar{y} = \frac{\Sigma \bar{y} A}{\Sigma A}$$

$$= \frac{0.006(0.099)(0.012) + 2[0.0495(0.012)(0.075)] + 0.137(0.012)(0.25)}{0.099(0.012) + 2(0.012)(0.075) + 0.012(0.25)} = 0.08471\ \text{m}$$

$$I_{NA} = \frac{1}{12}(0.099)(0.012^3) + (0.099)(0.012)(0.08471 - 0.006)^2$$

$$+ 2\left[\frac{1}{12}(0.012)(0.075^3) + (0.012)(0.075)(0.08471 - 0.0495)^2\right]$$

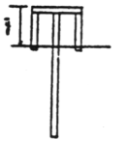
$$+ \frac{1}{12}(0.012)(0.25^3) + (0.012)(0.25)(0.137 - 0.08471)^2$$

$$= 34.2773(10^{-6})\ \text{m}^4$$

Maximum Bending Stress: Applying the flexure formula $\sigma_{\max} = \frac{Mc}{I}$

$$(\sigma_t)_{\max} = \frac{[6(10^3)](0.262 - 0.08471)}{34.2773(10^{-6})} = 31.03(10^6)\ \text{N/m}^2 = 31.0\ \text{MPa}$$ **Ans.**

$$(\sigma_c)_{\max} = \frac{[6(10^3)](0.08471)}{34.2773(10^{-6})} = 14.83(10^6)\ \text{N/m}^2 = 14.8\ \text{MPa}$$ **Ans.**



Ans:
$\bar{y} = 0.08471$ m,
$I_{NA} = 34.2773\ (10^{-6})\ \text{m}^4$,
$(\sigma_t)_{\max} = 31.0$ MPa, $(\sigma_c)_{\max} = 14.8$ MPa

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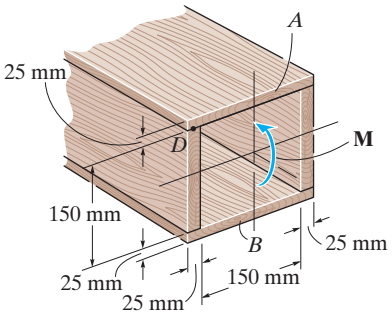
6–51. The beam is subjected to a moment **M**. Determine the percentage of this moment that is resisted by the stresses acting on both the top and bottom boards, A and B, of the beam.



Section Property:

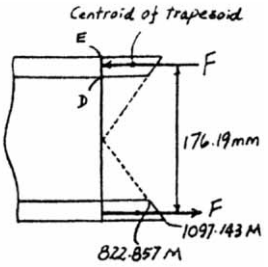
$$I = \frac{1}{12}(0.2)\left(0.2^3\right) - \frac{1}{12}(0.15)\left(0.15^3\right) = 91.14583\left(10^{-6}\right)\ \text{m}^4$$

Bending Stress: Applying the flexure formula

$$\sigma = \frac{My}{I}$$

$$\sigma_E = \frac{M(0.1)}{91.14583(10^{-6})} = 1097.143\ M$$

$$\sigma_D = \frac{M(0.075)}{91.14583(10^{-6})} = 822.857\ M$$



Resultant Force and Moment: For board A or B

$$F = 822.857M(0.025)(0.2) + \frac{1}{2}(1097.143M - 822.857M)(0.025)(0.2)$$

$$= 4.800\ M$$

$$M' = F(0.17619) = 4.80M(0.17619) = 0.8457\ M$$

$$\sigma_c\left(\frac{M'}{M}\right) = 0.8457(100\%) = 84.6\ \%$$ **Ans.**

Ans:
84.6%

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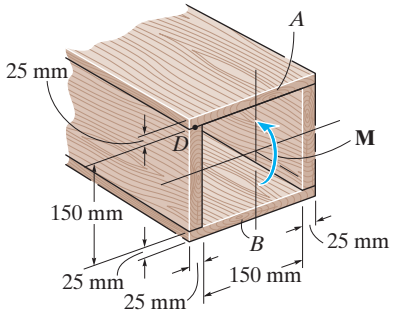
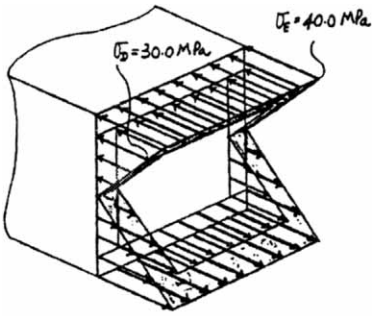
***6–52.** Determine the moment **M** that should be applied to the beam in order to create a compressive stress at point D of $\sigma_D = 30\,\text{MPa}$. Also sketch the stress distribution acting over the cross section and compute the maximum stress developed in the beam.



Section Property:

$$I = \frac{1}{12}(0.2)\left(0.2^3\right) - \frac{1}{12}(0.15)\left(0.15^3\right) = 91.14583\left(10^{-6}\right)\text{m}^4$$



Bending Stress: Applying the flexure formula

$$\sigma = \frac{My}{I}$$

$$30\left(10^6\right) = \frac{M(0.075)}{91.14583(10^{-6})}$$

$$M = 36458\,\text{N}\cdot\text{m} = 36.5\,\text{kN}\cdot\text{m} \qquad \textbf{Ans.}$$

$$\sigma_{\max} = \frac{Mc}{I} = \frac{36458(0.1)}{91.14583(10^{-6})} = 40.0\,\text{MPa} \qquad \textbf{Ans.}$$

Ans:

$M = 36.5\,\text{kN}\cdot\text{m},\ \sigma_{\max} = 40.0\,\text{MPa}$

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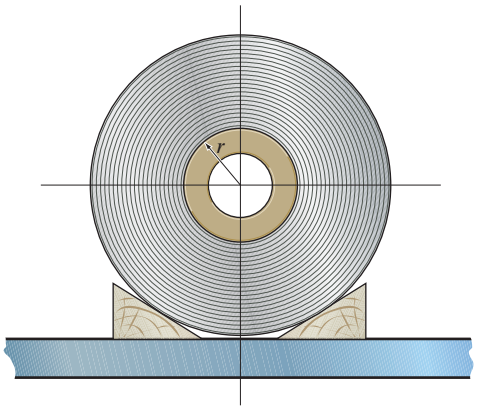
6–53.

An A-36 steel strip has an allowable bending stress of 165 MPa. If it is rolled up, determine the smallest radius r of the spool if the strip has a width of 10 mm and a thickness of 1.5 mm. Also, find the corresponding maximum internal moment developed in the strip.



SOLUTION

Bending Stress-Curvature Relation:

$$\sigma_{\text{allow}} = \frac{Ec}{\rho}; \qquad 165(10^6) = \frac{200(10^9)[\,0.75(10^{-3})\,]}{r}$$

$$r = 0.9091 \text{ m} = 909 \text{ mm} \qquad \textbf{Ans.}$$

Moment Curvature Relation:

$$\frac{1}{\rho} = \frac{M}{EI}; \quad \frac{1}{0.9091} = \frac{M}{200(10^9)\left[\frac{1}{12}(1)(0.0015^3)\right]}$$

$$M = 61.875 \text{ N} \cdot \text{m} = 61.9 \text{ N} \cdot \text{m} \qquad \textbf{Ans.}$$

Ans:
$r = 909$ mm, $M = 61.9 \text{ N} \cdot \text{m}$

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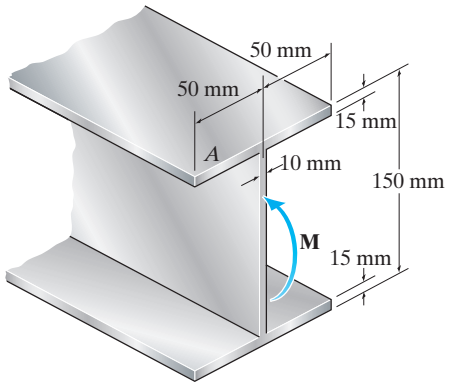
6–54. If the beam is subjected to an internal moment of $M = 30\ \text{kN}\cdot\text{m}$, determine the maximum bending stress in the beam. The beam is made from A992 steel. Sketch the bending stress distribution on the cross section.



Section Properties: The moment of inertia of the cross section about the neutral axis is

$$I = \frac{1}{12}(0.1)(0.15^3) - \frac{1}{12}(0.09)(0.12^3) = 15.165(10^{-6})\ \text{m}^4$$

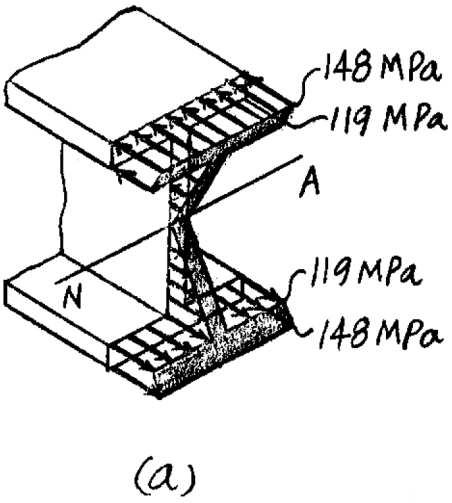
Maximum Bending Stress: The maximum bending stress occurs at the top and bottom surfaces of the beam since they are located at the furthest distance from the neutral axis. Thus, $c = 75\ \text{mm} = 0.075\ \text{m}$.

$$\sigma_{\max} = \frac{Mc}{I} = \frac{30(10^3)(0.075)}{15.165(10^{-6})} = 148\ \text{MPa}$$ **Ans.**

At $y = 60\ \text{mm} = 0.06\ \text{m}$,

$$\sigma|_{y=0.06\ \text{m}} = \frac{My}{I} = \frac{30(10^3)(0.06)}{15.165(10^{-6})} = 119\ \text{MPa}$$

The bending stress distribution across the cross section is shown in Fig. *a*.



(a)

Ans:
$\sigma_{\max} = 148\ \text{MPa}$

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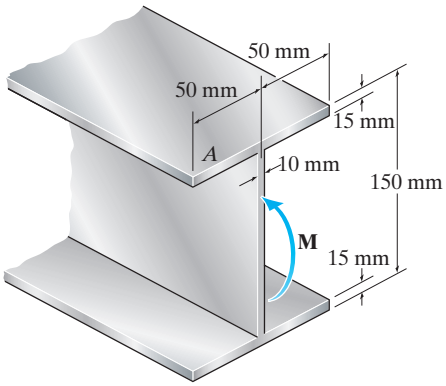
6–55. If the beam is subjected to an internal moment of $M = 30$ kN· m, determine the resultant force caused by the bending stress distribution acting on the top flange A.



Section Properties: The moment of inertia of the cross section about the neutral axis is

$$I = \frac{1}{12}(0.1)(0.15^3) - \frac{1}{12}(0.09)(0.12^3) = 15.165(10^{-6})\ \text{m}^4$$

Bending Stress: The distance from the neutral axis to the top and bottom surfaces of flange A is $y_t = 75$ mm $= 0.075$ m and $y_b = 60$ mm $= 0.06$ m.

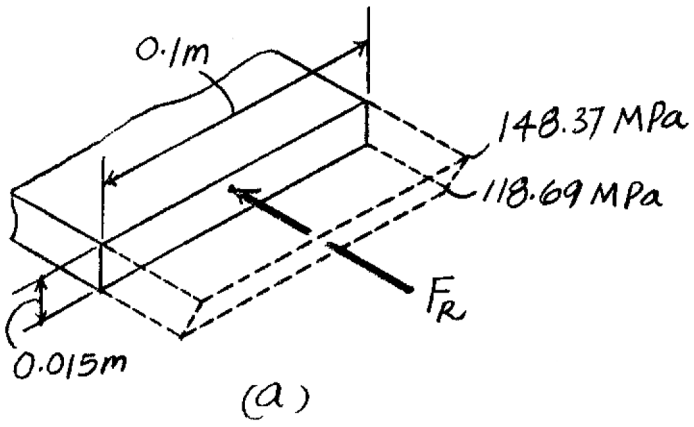
$$\sigma_t = \frac{My_t}{I} = \frac{30(10^3)(0.075)}{15.165(10^{-6})} = 148.37 = 148\ \text{MPa}$$

$$\sigma_b = \frac{My_b}{I} = \frac{30(10^3)(0.06)}{15.165(10^{-6})} = 118.69 = 119\ \text{MPa}$$

Resultant Force: The resultant force acting on flange A is equal to the volume of the trapezoidal stress block shown in Fig. a. Thus,

$$F_R = \frac{1}{2}(148.37 + 118.69)(10^6)(0.1)(0.015)$$

$$= 200\,296.74\ \text{N} = 200\ \text{kN}$$ **Ans.**



Ans:
$F_R = 200$ kN

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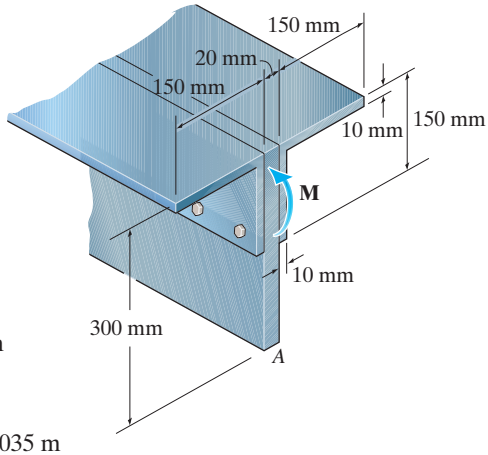
***6–56.**

If the built-up beam is subjected to an internal moment of $M = 75\,\text{kN}\cdot\text{m}$, determine the maximum tensile and compressive stress acting in the beam.



SOLUTION

Section Properties: The neutral axis passes through centroid C of the cross section as shown in Fig. a. The location of C is

$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{0.15(0.3)(0.02) + 2[\,0.225(0.15)(0.01)] + 2[\,0.295(0.01)(0.14)\,]}{0.3(0.02) + 2(0.15)(0.01) + 2(0.01)(0.14)} = 0.2035\ \text{m}$$

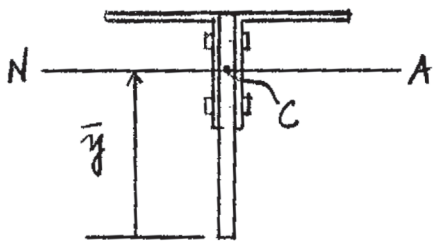
Thus, the moment of inertia of the cross section about the neutral axis is

$$\begin{aligned} I &= \Sigma \bar{I} + Ad^2 \\ &= \frac{1}{12}(0.02)(0.3^3) + 0.02(0.3)(0.2035 - 0.15)^2 \\ &\quad + 2\left[\frac{1}{12}(0.01)(0.15^3) + 0.01(0.15)(0.225 - 0.2035)^2\right] \\ &\quad + 2\left[\frac{1}{12}(0.14)(0.01^3) + 0.14(0.01)(0.295 - 0.2035)^2\right] \\ &= 92.6509(10^{-6})\ \text{m}^4 \end{aligned}$$

Maximum Bending Stress: The maximum compressive and tensile stress occurs at the top and bottom-most fiber of the cross section.

$$(\sigma_{\max})_c = \frac{My}{I} = \frac{75(10^3)(0.3 - 0.2035)}{92.6509(10^{-6})} = 78.1\ \text{MPa} \qquad \textbf{Ans.}$$

$$(\sigma_{\max})_t = \frac{Mc}{I} = \frac{75(10^3)(0.2035)}{92.6509(10^{-6})} = 165\ \text{MPa} \qquad \textbf{Ans.}$$



(a)

Ans:
$(\sigma_{\max})_c = 78.1\ \text{MPa}$, $(\sigma_{\max})_t = 165\ \text{MPa}$

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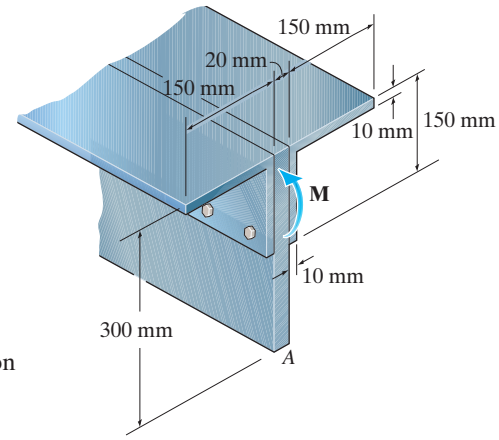
6–57.

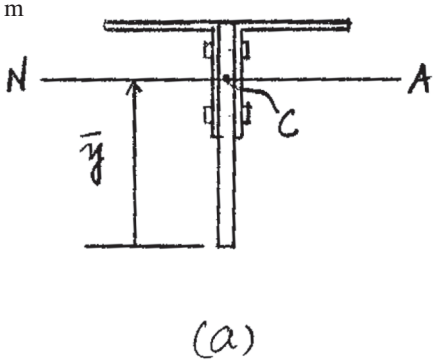
If the built-up beam is subjected to an internal moment of $M = 75\,\text{kN}\cdot\text{m}$, determine the amount of this internal moment resisted by plate A.



SOLUTION

Section Properties: The neutral axis passes through centroid C of the cross section as shown in Fig. *a*. The location of C is

$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{0.15(0.3)(0.02) + 2[0.225(0.15)(0.01)] + 2[0.295(0.01)(0.14)]}{0.3(0.02) + 2(0.15)(0.01) + 2(0.01)(0.14)} = 0.2035\ \text{m}$$



(a)

Thus, the moment of inertia of the cross section about the neutral axis is

$$I = \bar{I} + Ad^2$$

$$= \frac{1}{12}(0.02)(0.3^3) + 0.02(0.3)(0.2035 - 0.15)^2$$

$$+ 2\left[\frac{1}{12}(0.01)(0.15^3) + 0.01(0.15)(0.225 - 0.2035)^2\right]$$

$$+ 2\left[\frac{1}{12}(0.14)(0.01^3) + 0.14(0.01)(0.295 - 0.2035)^2\right]$$

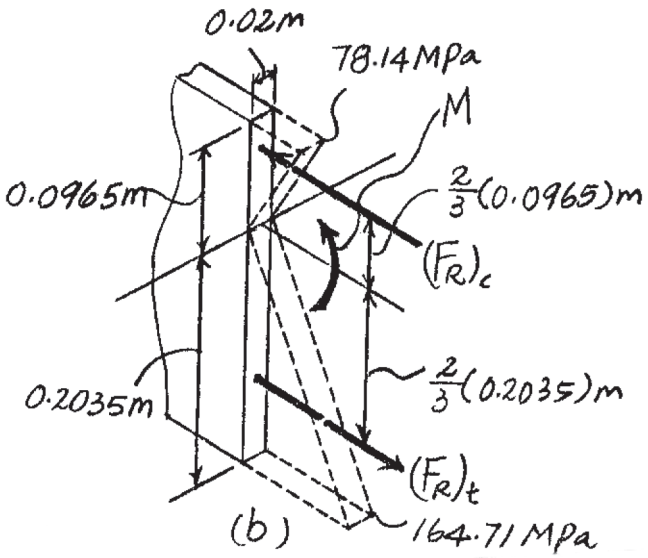
$$= 92.6509(10^{-6})\ \text{m}^4$$

Bending Stress: The distance from the neutral axis to the top and bottom of plate A is $y_t = 0.3 - 0.2035 = 0.0965$ m and $y_b = 0.2035$ m.

$$\sigma_t = \frac{My_t}{I} = \frac{75(10^3)(0.0965)}{92.6509(10^{-6})} = 78.14\ \text{MPa (C)}$$

$$\sigma_b = \frac{My_b}{I} = \frac{75(10^3)(0.2035)}{92.6509(10^{-6})} = 164.71\ \text{MPa (T)}$$

The bending stress distribution across the cross section of plate A is shown in Fig. *b*. The resultant forces of the tensile and compressive triangular stress blocks are



(b)

$$(F_R)_t = \frac{1}{2}(164.71)(10^6)(0.2035)(0.02) = 335\,144.46\ \text{N}$$

$$(F_R)_c = \frac{1}{2}(78.14)(10^6)(0.0965)(0.02) = 75\,421.50\ \text{N}$$

Thus, the amount of internal moment resisted by plate A is

$$M = 335144.46\left[\frac{2}{3}(0.2035)\right] + 75421.50\left[\frac{2}{3}(0.0965)\right]$$

$$= 50315.65\ \text{N}\cdot\text{m} = 50.3\ \text{kN}\cdot\text{m}$$ **Ans.**

Ans:
$M = 50.3\ \text{kN}\cdot\text{m}$

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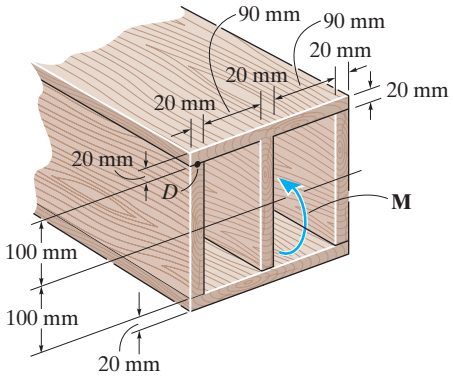
6–58.

The beam is subjected to a moment M. Determine the percentage of this moment that is resisted by the stresses acting on both the top and bottom boards of the beam.

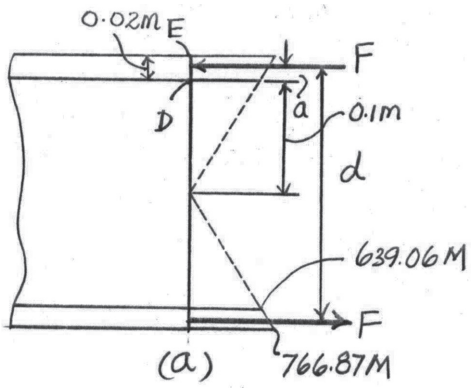


SOLUTION

Section Properties: The moment of inertia of the beam's cross-section about the neutral axis is

$$I = \frac{1}{12}(0.24)(0.24^3) - \frac{1}{12}(0.18)(0.2^3) = 0.15648(10^{-3})\ \text{m}^4$$

Bending Stress: Applying the flexure formula, $\sigma = \frac{My}{I}$, the bending stress on points D and E, Fig. a, is



$$\sigma_D = \frac{M(0.1)}{0.15648(10^{-3})} = 639.06M$$

$$\sigma_E = \frac{M(0.12)}{0.15648(10^{-3})} = 766.87M$$

Resultant Force and Moment: The resultant of the stress block acting on boards A and B, Fig. a, is

$$F = \frac{1}{2}(639.06M + 766.87M)(0.02)(0.24) = 3.3742M$$

The location of line of action of F is

$$a = \frac{1}{3}\left[\frac{2(766.87M) + 639.06M}{639.06M + 766.87M}\right](0.02) = 0.010303\ \text{m}$$

Thus, the moment arm of F is

$$a' = 2(0.1 + 0.010303) = 0.22061\ \text{m}$$

Then

$$M' = Fd = 3.3742M(0.22061) = 0.74438M$$

$$\%\left(\frac{M'}{M}\right) = \left(\frac{0.74438M}{M}\right)(100) = 74.4\% \qquad \textbf{Ans.}$$

Ans:
$\frac{M'}{M} = 74.4\%$

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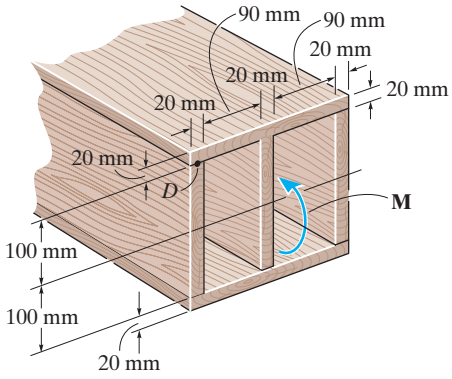
6–59.

Determine the moment M that should be applied to the beam in order to create a compressive stress at point D of $\sigma_D = 10$ MPa. Also sketch the stress distribution acting over the cross section and calculate the maximum stress developed in the beam.



SOLUTION

Section Properties: The moment of inertia of the beam's cross-section about the neutral axis is

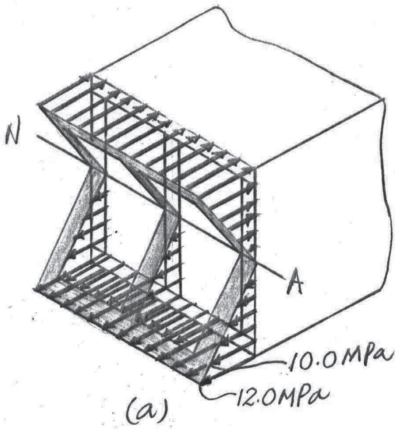
$$I = \frac{1}{12}(0.24)(0.24^3) - \frac{1}{12}(0.18)(0.2^3) = 0.15648(10^{-3})\,M^4$$

Bending Stress: Applying the flexure formula,

$$\sigma_D = \frac{My_D}{I}$$

$$10(10^6) = \frac{M(0.1)}{0.15648(10^{-3})}$$

$$M = 15.648(10^{-3})\ \text{N}\cdot\text{m} = 15.6\ \text{kN}\cdot\text{m} \qquad \textbf{Ans.}$$



The maximum bending stress is

$$\sigma_{\max} = \frac{MC}{I} = \frac{[15.648(10^3)](0.12)}{0.15648(10^{-3})} = 12.0(10^6)\ \text{Pa} = 12.0\ \text{MPa} \qquad \textbf{Ans.}$$

The sketch of Bending stress distribution on the beam's cross-section is shown in Fig. *a*.

Ans:
$M = 15.6\ \text{kN}\cdot\text{m}$,
$\sigma_{\max} = 12.0$ MPa

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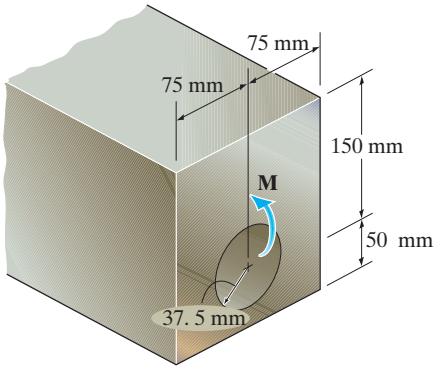
***6–60.** If the beam is subjected to an internal moment of of $M = 150$ kN · m, determine the maximum tensile and compressive bending stress in the beam.



Section Properties: The neutral axis passes through centroid C of the cross section as shown in Fig. *a*. The location of C is

$$\bar{y} = \frac{\Sigma \bar{y} A}{\Sigma A} = \frac{0.1(0.15)(0.2) - 0.05[\pi(0.0375^2)]}{0.15(0.2) - \pi(0.0375^2)} = 0.10863 \text{ m}$$

Thus, the moment of inertia of the cross section about the neutral axis is

$$I = \Sigma \bar{I} + Ad^2$$

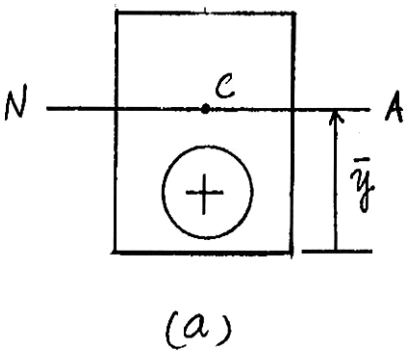
$$= \frac{1}{12}(0.15)(0.2^3) + 0.15(0.2)(0.10863 - 0.1)^2 - \left[\frac{\pi}{4}(0.0375^4) + \pi(0.0375^2)(0.10863 - 0.05)^2\right]$$

$$= 85.4948(10^{-6}) \text{ m}^4$$

Maximum Bending Stress: The maximum compressive and tensile bending stress occurs at the top and bottom edges of the cross section.

$$(\sigma_{\max})_T = \frac{MC}{I} = \frac{[150(10^3)](0.10863)}{85.4948(10^{-6})} = 190.60(10^6) \text{ N/m}^2 = 191 \text{ MPa} \quad \textbf{Ans.}$$

$$(\sigma_{\max})_C = \frac{MC}{I} = \frac{[150(10^3)](0.2 - 0.10863)}{85.4948(10^{-6})} = 160.30(10^6) \text{ N/m}^2 = 160 \text{ MPa} \quad \textbf{Ans.}$$



(a)

Ans:
$(\sigma_{\max})_T = 191$ MPa,
$(\sigma_{\max})_C = 160$ MPa

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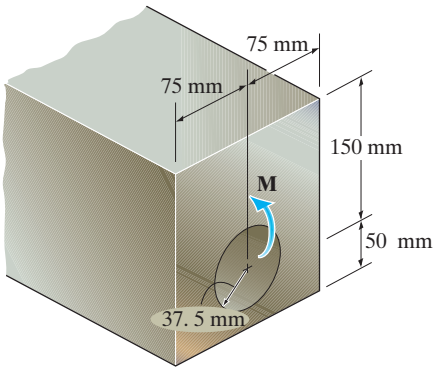
6–61. If the beam is made of material having an allowable tensile and compressive stress of $(\sigma_{allow})_t$ = 168 MPa and $(\sigma_{allow})_c$ = 154 MPa, respectively, determine the maximum allowable internal moment **M** that can be applied to the beam.



Section Properties: The neutral axis passes through centroid *C* of the cross section as shown in Fig. *a*. The location of *C* is

$$\bar{y} = \frac{\Sigma \bar{y} A}{\Sigma A} = \frac{0.1(0.15)(0.2) - 0.05[\pi(0.0375^2)]}{0.15(0.2) - \pi(0.0375^2)} = 0.10863 \text{ m}$$

Thus, the moment of inertia of the cross section about the neutral axis is

$$I = \Sigma \bar{I} + Ad^2$$

$$= \frac{1}{12}(0.15)(0.2^3) + 0.15(0.2)(0.10863 - 0.1)^2 - \left[\frac{\pi}{4}(0.0375^4) + \pi(0.0375^2)(0.10863 - 0.05)^2\right]$$

$$= 85.4948(10^{-6}) \text{ m}^4$$

Allowable Bending Stress: The maximum compressive and tensile bending stress occurs at the top and bottom edges of the cross section. For the top edge,

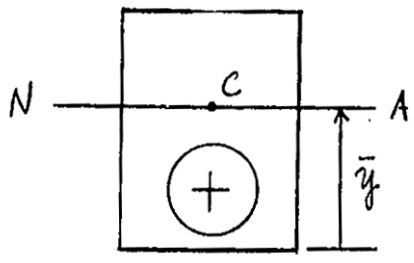
$$(\sigma_{allow})_c = \frac{My}{I}; \qquad 154(10^6) = \frac{M(0.2 - 0.10863)}{85.4948(10^{-6})}$$

$$M = 144.11(10^3) \text{ N} \cdot \text{m} = 144 \text{ kN} \cdot \text{m}$$

For the bottom edge,

$$(\sigma_{max})_t = \frac{Mc}{I}; \qquad 154(10^6) = \frac{M(0.10863)}{85.4948(10^{-6})}$$

$$M = 132.22(10^3) \text{ N} \cdot \text{m} = 132 \text{ kN} \cdot \text{m} \ (\textit{controls}!) \qquad \textbf{Ans.}$$



(a)

Ans:

$M = 132 \text{ kN} \cdot \text{m}$

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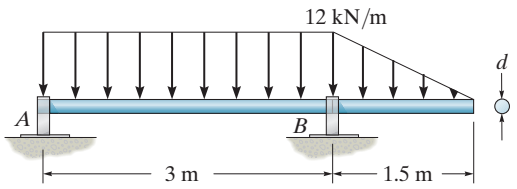
6–62.

The shaft is supported by smooth journal bearings at A and B that only exert vertical reactions on the shaft. If $d = 90$ mm, determine the absolute maximum bending stress in the beam, and sketch the stress distribution acting over the cross section.



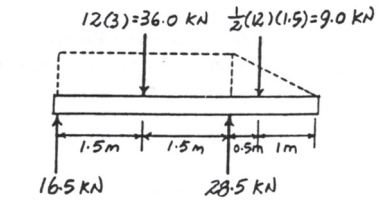
SOLUTION

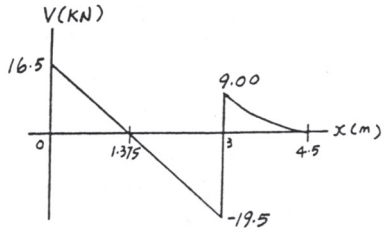
Absolute Maximum Bending Stress: The maximum moment is $M_{\max} = 11.34\ \text{kN} \cdot \text{m}$ as indicated on the moment diagram. Applying the flexure formula

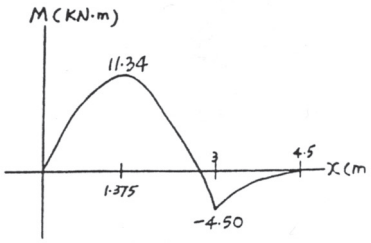
$$\sigma_{\max} = \frac{M_{\max}\, c}{I}$$

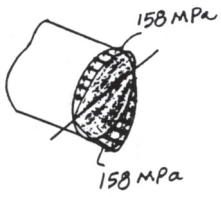
$$= \frac{11.34(10^3)(0.045)}{\frac{\pi}{4}(0.045^4)}$$

$$= 158\ \text{MPa} \qquad \textbf{Ans.}$$









Ans:
$\sigma_{\max} = 158\ \text{MPa}$

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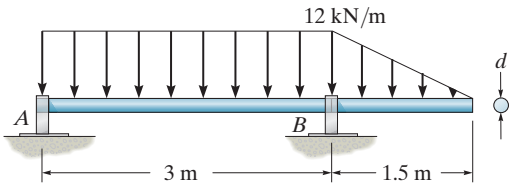
6–63.

The shaft is supported by smooth journal bearings at A and B that only exert vertical reactions on the shaft. Determine its smallest diameter d if the allowable bending stress is $\sigma_{\text{allow}} = 180$ MPa.



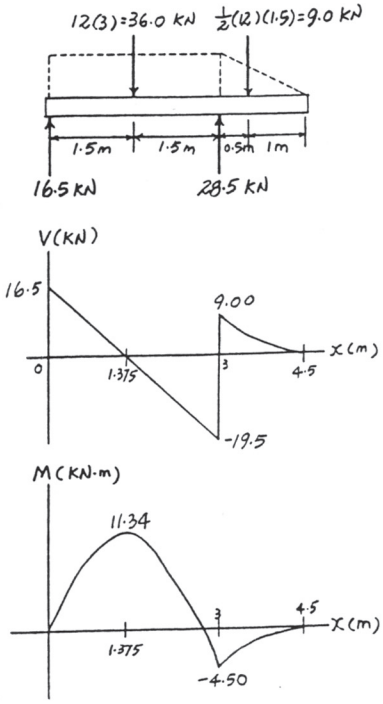
SOLUTION

Allowable Bending Stress: The maximum moment is $M_{\max} = 11.34\ \text{kN}\cdot\text{m}$ as indicated on the moment diagram. Applying the flexure formula

$$\sigma_{\max} = \sigma_{\text{allow}} = \frac{M_{\max}\, c}{I}$$

$$180(10^6) = \frac{11.34(10^3)\left(\frac{d}{2}\right)}{\frac{\pi}{4}\left(\frac{d}{2}\right)^4}$$

$$d = 0.08626\ \text{m} = 86.3\ \text{mm} \qquad \textbf{Ans.}$$



Ans:
$d = 86.3$ mm

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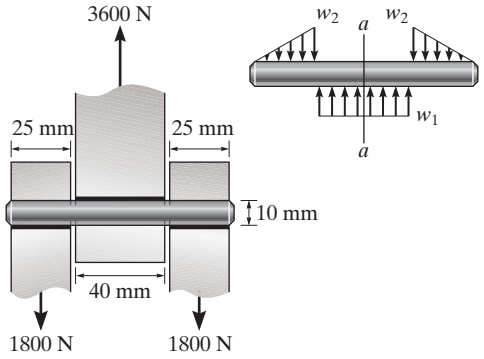
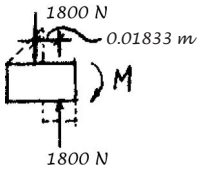
***6–64.** The pin is used to connect the three links together. Due to wear, the load is distributed over the top and bottom of the pin as shown on the free-body diagram. If the diameter of the pin is 10 mm, determine the maximum bending stress on the cross-sectional area at the center section *a–a*. For the solution it is first necessary to determine the load intensities w_1 and w_2.



$$\frac{1}{12}w_2(0.25) = 1800; \quad w_2 = 144(10^3)\ \text{N/m}$$

$$w_1(0.04) = 3600; \quad w_1 = 90(10^3)\ \text{N/m}$$



$$M = 1800(0.01833) = 33.0\ \text{N}\cdot\text{m}$$

$$I = \frac{1}{4}\pi(0.005^4) = 0.49087(10^{-9})\ \text{m}^4$$

$$\sigma_{\max} = \frac{Mc}{I} = \frac{33.0(0.005)}{0.49087(10^{-9})} = 336.14(10^6)\ \text{N/m}^2 = 336\ \text{MPa}$$

Ans.

Ans:
$\sigma_{\max} = 324.45$ MPa

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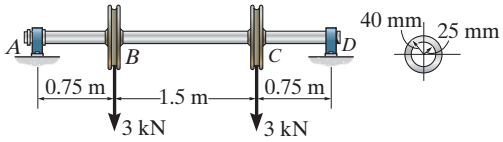
6–65.

The shaft is supported by a thrust bearing at A and journal bearing at D. If the shaft has the cross section shown, determine the absolute maximum bending stress in the shaft.



SOLUTION

Shear and Moment Diagrams: As shown in Fig. *a*.

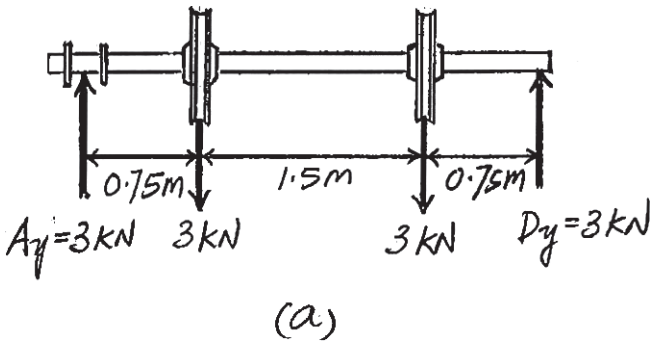
Maximum Moment: Due to symmetry, the maximum moment occurs in region BC of the shaft. Referring to the free-body diagram of the segment shown in Fig. *b*.

Section Properties: The moment of inertia of the cross section about the neutral axis is

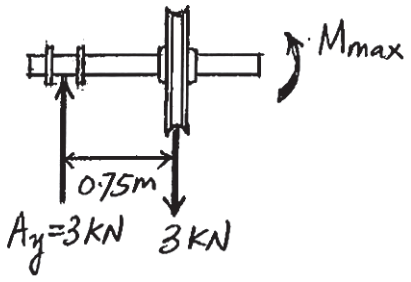
$$I = \frac{\pi}{4}(0.04^4 - 0.025^4) = 1.7038(10^{-6})\,\text{m}^4$$

Absolute Maximum Bending Stress:

$$\sigma_{\max} = \frac{M_{\max} c}{I} = \frac{2.25(10^3)(0.04)}{1.7038(10^{-6})} = 52.8\,\text{MPa} \qquad \textbf{Ans.}$$



(a)



(b)

Ans:
$\sigma_{\max} = 52.8\,\text{MPa}$

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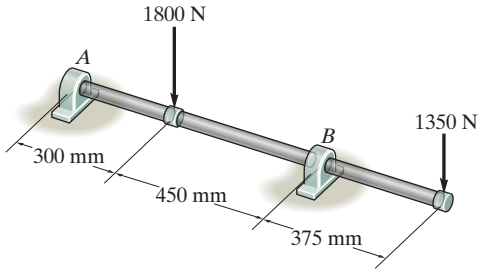
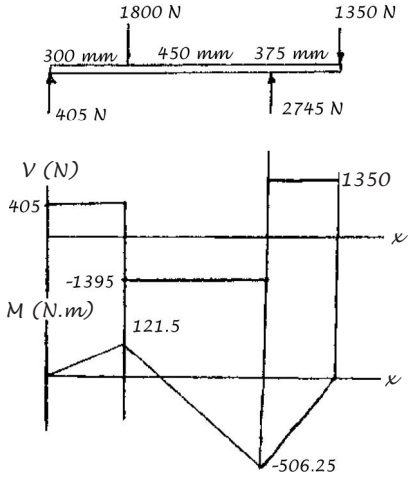
6–66. Determine the absolute maximum bending stress in the 40-mm-diameter shaft which is subjected to the concentrated forces. The sleeve bearings at A and B support only vertical forces.



$M_{\max} = 506.25\ \text{N}\cdot\text{m}$

$$\sigma = \frac{Mc}{I} = \frac{506.25(0.02)}{\frac{\pi}{4}(0.02^4)} = 80.57(10^6)\ \text{N/m}^2 = 80.6\ \text{MPa}$$ **Ans.**



Ans:
$\sigma_{\max} = 52.8\ \text{MPa}$

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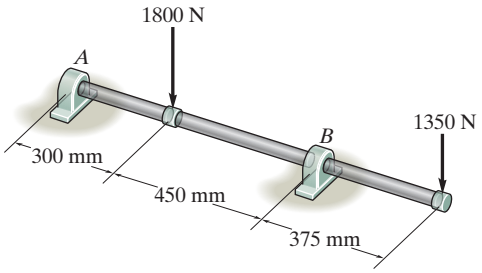
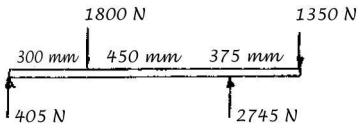
6–63. Determine the smallest allowable diameter of the shaft which is subjected to the concentrated forces. The sleeve bearings at A and B support only vertical forces, and the allowable bending stress is $\sigma_{\text{allow}} = 154$ MPa.



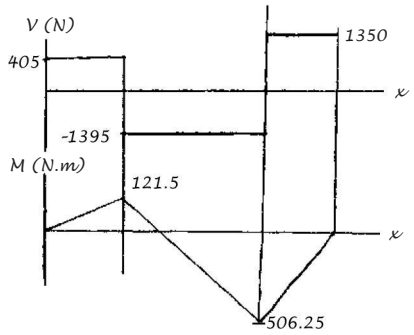


$M_{\max} = 506.25 \text{ N} \cdot \text{m}$

$$10.5(10^6) = \frac{(506.25)c}{\frac{\pi}{4}c^4}$$

$c = 0.01612$ m

$d = 2c = 2(0.01612) = 0.03223 \text{ m} = 32.2 \text{ mm}$ **Ans.**



Ans:
$d = 32.2$ mm

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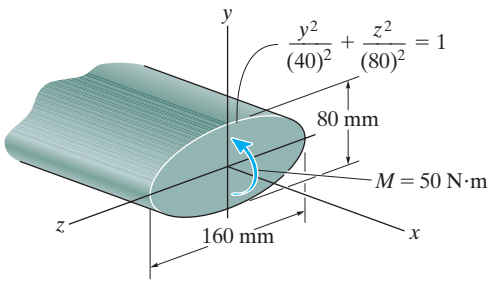
***6–68.**

A shaft is made of a polymer having an elliptical cross section. If it resists an internal moment of $M = 50\ \text{N}\cdot\text{m}$, determine the maximum bending stress in the material (a) using the flexure formula, where $I_z = \frac{1}{4}\pi(0.08\ \text{m})(0.04\ \text{m})^3$, (b) using integration. Sketch a three-dimensional view of the stress distribution acting over the cross-sectional area. Here $I_x = \frac{1}{4}\pi(0.08\ \text{m})(0.04\ \text{m})^3$.

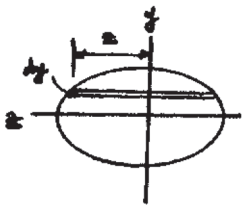


SOLUTION

(a)

$$I = \frac{1}{4}\pi\, ab^3 = \frac{1}{4}\pi(0.08)(0.04)^3 = 4.021238(10^{-6})\ \text{m}^4$$

$$\sigma_{\max} = \frac{Mc}{I} = \frac{50(0.04)}{4.021238(10^{-6})} = 497\ \text{kPa} \qquad \textbf{Ans.}$$



(b)

$$M = \frac{\sigma_{\max}}{c}\int_A y^2 dA$$

$$= \frac{\sigma_{\max}}{c}\int y^2 2z\,dy$$

$$z = \sqrt{0.0064 - 4y^2} = 2\sqrt{(0.04)^2 - y^2}$$

$$2\int_{-0.04}^{0.04} y^2 z\,dy = 4\int_{-0.04}^{0.04} y^2\sqrt{(0.04)^2 - y^2}\,dy$$

$$= 4\left[\frac{(0.04)^4}{8}\sin^{-1}\left(\frac{y}{0.04}\right) - \frac{1}{8}y\sqrt{(0.04)^2 - y^2}(0.04^2 - 2y^2)\right]\Bigg|_{-0.04}^{0.04}$$

$$= \frac{(0.04)^4}{2}\sin^{-1}\left(\frac{y}{0.04}\right)\Bigg|_{-0.04}^{0.04}$$

$$= 4.021238(10^{-6})\ \text{m}^4$$

$$\sigma_{\max} = \frac{50(0.04)}{4.021238(10^{-6})} = 497\ \text{kPa} \qquad \textbf{Ans.}$$

Ans:
(a) $\sigma_{\max} = 497\ \text{kPa}$,
(b) $\sigma_{\max} = 497\ \text{kPa}$

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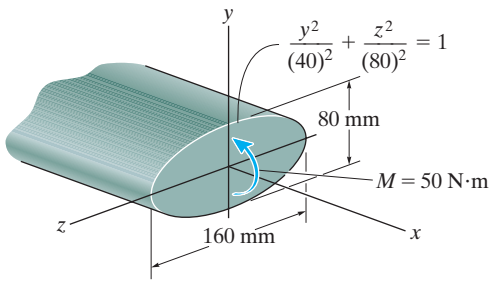
6–69.

Solve Prob. 6–65 if the moment $M = 50\,\text{N}\cdot\text{m}$ is applied about the y axis instead of the x axis. Here $I_y = \frac{1}{4}\pi\,(0.04\,\text{m})(0.08\,\text{m})^3$.

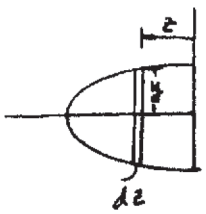


SOLUTION

(a)

$$I = \frac{1}{4}\pi\,ab^3 = \frac{1}{4}\pi(0.04)(0.08)^3 = 16.085(10^{-6})\,\text{m}^4$$

$$\sigma_{\max} = \frac{Mc}{I} = \frac{50(0.08)}{16.085(10^{-6})} = 249\,\text{kPa}$$ **Ans.**



(b)

$$M = \int_A z(\sigma\,dA) = \int_A z\left(\frac{\sigma_{\max}}{0.08}\right)(z)(2y)dz$$

$$50 = 2\left(\frac{\sigma_{\max}}{0.04}\right)\int_0^{0.08} z^2\left(1 - \frac{z^2}{(0.08)^2}\right)^{1/2}(0.04)dz$$

$$50 = 201.06(10^{-6})\sigma_{\max}$$

$$\sigma_{\max} = 249\,\text{kPa}$$ **Ans.**

Ans:
(a) $\sigma_{\max} = 249\,\text{kPa}$,
(b) $\sigma_{\max} = 249\,\text{kPa}$

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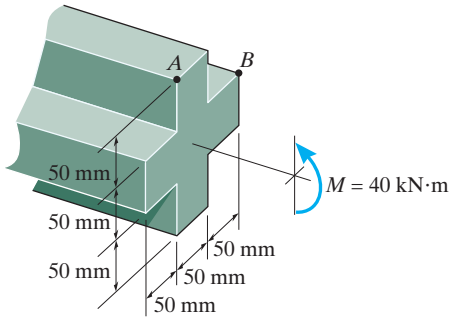
6–70.

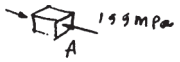
The beam is subjected to a moment of $M = 40\ \text{kN}\cdot\text{m}$. Determine the bending stress at points A and B. Sketch the results on a volume element acting at each of these points.



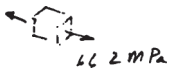
SOLUTION

$$I = \frac{1}{12}(0.150)(0.05)^3 + 2\left[\frac{1}{12}(0.05)(0.05)^3 + (0.05)(0.05)(0.05)^2\right] = 15.1042\left(10^{-6}\right)\text{m}^4$$

$$\sigma_A = \frac{Mc}{I} = \frac{40\left(10^3\right)(0.075)}{15.1042\left(10^{-6}\right)} = 199\ \text{MPa} \qquad \textbf{Ans.}$$



$$\sigma_B = \frac{My}{I} = \frac{40\left(10^3\right)(0.025)}{15.1042\left(10^{-6}\right)} = 66.2\ \text{MPa} \qquad \textbf{Ans.}$$



Ans:
$\sigma_A = 199$ MPa,
$\sigma_B = 66.2$ MPa

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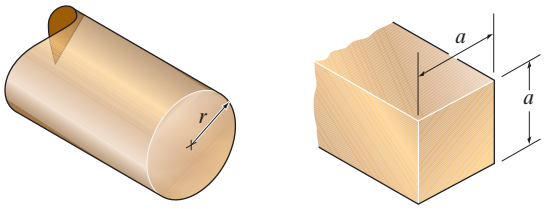
6–71. Determine the dimension a of a beam having a square cross section in terms of the radius r of a beam with a circular cross section if both beams are subjected to the same internal moment which results in the same maximum bending stress.



Section Properties: The moments of inertia of the square and circular cross sections about the neutral axis are

$$I_S = \frac{1}{12}a\left(a^3\right) = \frac{a^4}{12} \qquad I_C = \frac{1}{4}\pi r^4$$

Maximum Bending Stress: For the square cross section, $c = a/2$.

$$\left(\sigma_{\max}\right)_S = \frac{Mc}{I_S} = \frac{M(a/2)}{a^4/12} = \frac{6M}{a^3}$$

For the circular cross section, $c = r$.

$$\left(\sigma_{\max}\right)_c = \frac{Mc}{I_c} = \frac{Mr}{\frac{1}{4}\pi r^4} - \frac{4M}{\pi r^3}$$

It is required that

$$\left(\sigma_{\max}\right)_S = \left(\sigma_{\max}\right)_C$$

$$\frac{6M}{a^3} = \frac{4M}{\pi r^3}$$

$$a = 1.677r \qquad \textbf{Ans.}$$

Ans:
$a = 1.677r$

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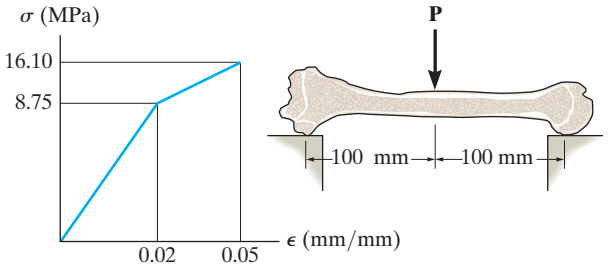
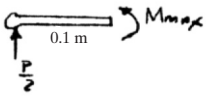
***6–72.** A portion of the femur can be modeled as a tube having an inner diameter of 9.5 mm and an outer diameter of 32 mm. Determine the maximum elastic static force P that can be applied to its center. Assume the bone to be roller supported at its ends. The $\sigma-\epsilon$ diagram for the bone mass is shown and is the same in tension as in compression.



$$I = \frac{\pi}{4}(0.016^4 - 0.00475^4) = 51.0720(10^{-9})\ \text{m}^4$$



$$M_{\max} = \frac{P}{2}(0.1) = 0.05P$$

Require $\sigma_{\max} = 8.75$ MPa

$$\sigma_{\max} = \frac{Mc}{I}$$

$$8.75(10^6) = \frac{(0.05P)(0.016)}{51.0720(10^{-9})}$$

$$P = 558.6\ \text{N} = 559\ \text{N}$$ **Ans.**

Ans:

$P = 559$ N

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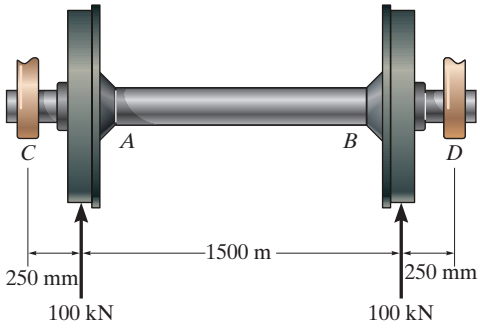
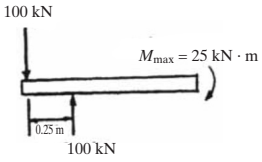
6–73. The axle of the freight car is subjected to wheel loadings of 100 kN. If it is supported by two journal bearings at C and D, determine the maximum bending stress developed at the center of the axle, where the diameter is 137.5 mm.



$$\sigma_{\max} = \frac{MC}{I} = \frac{[25(10^3)](0.06875)}{\frac{\pi}{4}(0.06875^4)} = 97.96(10^6)\ \text{N/m}^2 = 98\ \text{MPa}$$ **Ans.**



Ans:
$\sigma_{\max} = 98.0\ \text{MPa}$

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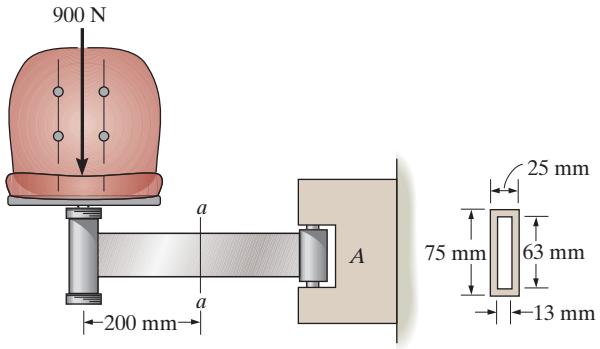
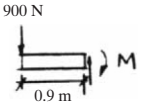
6–74. The chair is supported by an arm that is hinged so it rotates about the vertical axis at A. If the load on the chair is 900 N and the arm is a hollow tube section having the dimensions shown, determine the maximum bending stress at section a–a.



$\zeta + \Sigma M = 0; \qquad M - 900(0.9) = 0$

$M = 810 \text{ N} \cdot \text{m}$



$$I_x = \frac{1}{12}(0.025)(0.075^3) - \frac{1}{12}(0.013)(0.063^3) = 0.608022(10^{-6}) \text{ m}^4$$

$$\sigma_{\max} = \frac{MC}{I} = \frac{810(0.0375)}{0.608022(10^{-6})} = 49.96(10^{-6}) \text{ N/m}^2 = 50.0 \text{ MPa}$$ **Ans.**

Ans:
$\sigma_{\max} = 11.1$ MPa

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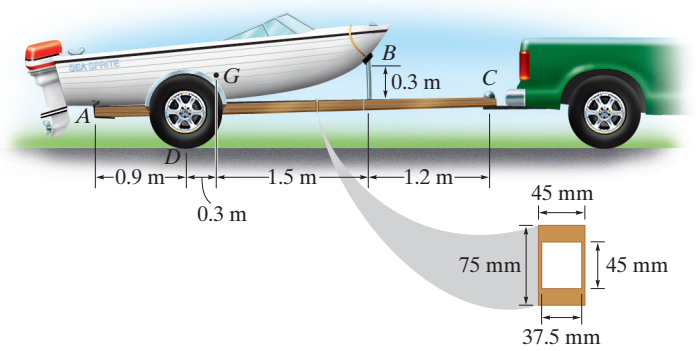
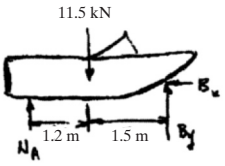
6–75. The boat has a weight of 11.5 kN and a center of gravity at *G*. If it rests on the trailer at the smooth contact *A* and can be considered pinned at *B*, determine the absolute maximum bending stress developed in the main strut of the trailer. Consider the strut to be a box-beam having the dimensions shown and pinned at *C*.



Boat:



$\stackrel{+}{\rightarrow}\Sigma F_x = 0; \quad B_x = 0$

$\circlearrowleft +\Sigma M_B = 0; \quad -N_A(2.7) + 11.5(1.5) = 0$

$N_A = 6.3889 \text{ kN}$

$+\uparrow \Sigma F_y = 0; \quad 6.3889 - 11.5 + B_y = 0$

$B_y = 5.1111 \text{ kN}$

Assembly:

$\circlearrowleft +\Sigma M_C = 0; \quad -N_D(3.0) + 11.5(2.7) = 0$

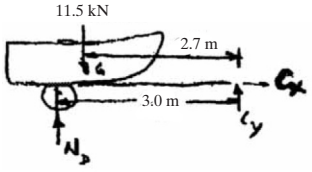
$N_D = 10.35 \text{ kN}$

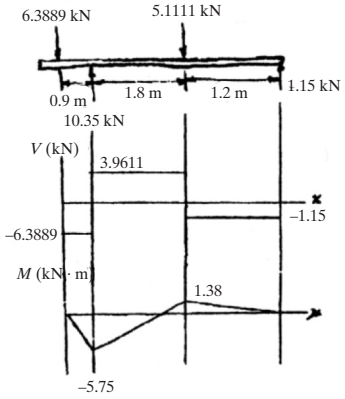
$+\uparrow \Sigma F_y = 0; \quad C_y + 10.35 - 11.5 = 0$

$C_y = 1.15 \text{ kN}$

$$I = \frac{1}{12}(0.045)(0.075^3) - \frac{1}{12}(0.0375)(0.045^3) = 1.2973(10^{-6})\ \text{m}^4$$

$$\sigma_{\max} = \frac{MC}{I} = \frac{[5.75(10^3)](0.0375)}{1.2973(10^{-6})} = 166.21(10^6)\ \text{N-m}^2 = 166\ \text{MPa}$$ **Ans.**





Ans:
$\sigma_{\max} = 166 \text{ MPa}$

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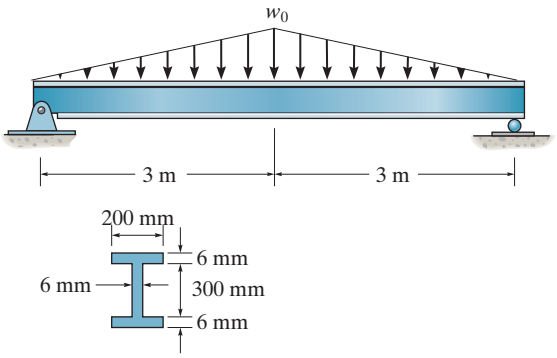
***6–76.** The steel beam has the cross-sectional area shown. Determine the largest intensity of the distributed load w_0 that it can support so that the maximum bending stress in the beam does not exceed $\sigma_{allow} = 160$ MPa.



Support Reactions. The FBD of the beam is shown in Fig. *a*.

The shear and moment diagrams are shown in Fig. *a* and *b*, respectively. As indicated on the moment diagram, $M_{max} = 3w_o$.

The moment of inertia of the cross-section about the neutral axis is

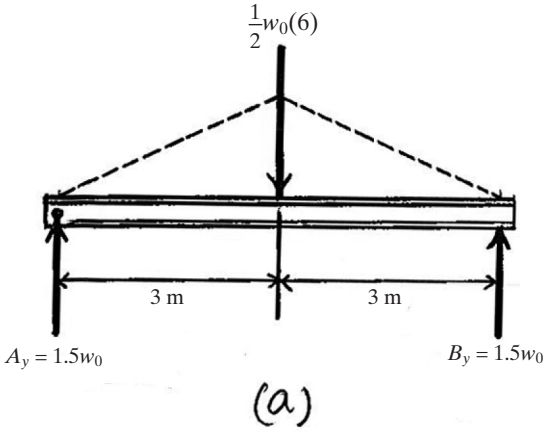
$$I = \frac{1}{12}(0.2)(0.312^3) - \frac{1}{12}(0.194)(0.3^3) = 69.6888(10^{-6})\ \text{m}^4$$

Hence, $\bar{y} = 0.156$ m. Thus,

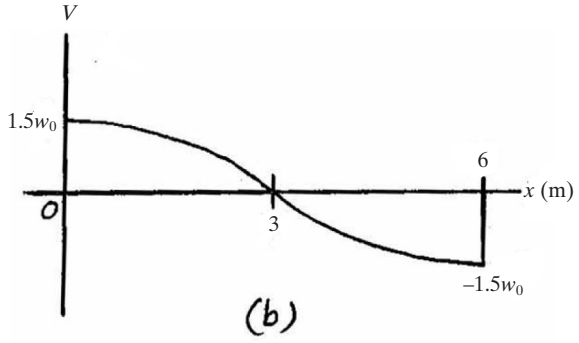
$$\sigma_{allow} = \frac{M_{max}\, c}{I};$$

$$160(10^6) = \frac{(3w_o)(0.156)}{69.6888(10^{-6})}$$

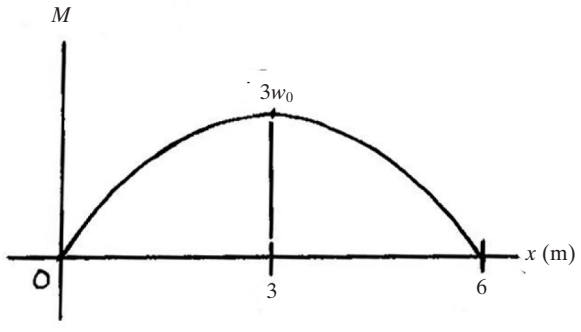
$$w_o = 23.83(10^3)\ \text{N/m} = 23.8\ \text{kN/m}$$ **Ans.**



(a)



(b)



(c)

Ans:

$w_o = 23.8$ kN/m

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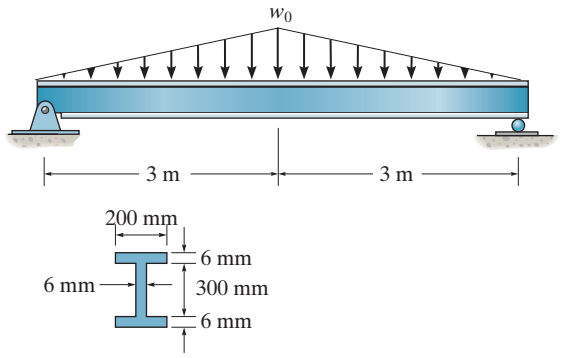
6–77. The steel beam has the cross-sectional area shown. If $w_0 = 30$ kN/m, determine the maximum bending stress in the beam.



The FBD of the beam is shown in Fig. *a*

The shear and moment diagrams are shown in Fig. *b* and *c*, respectively. As indicated on the moment diagram, $M_{max} = 90$ kN · m.

The moment of inertia of the I cross-section about the bending axis is

$$I = \frac{1}{12}(0.2)(0.312^3) - \frac{1}{12}(0.194)(0.3^3)$$

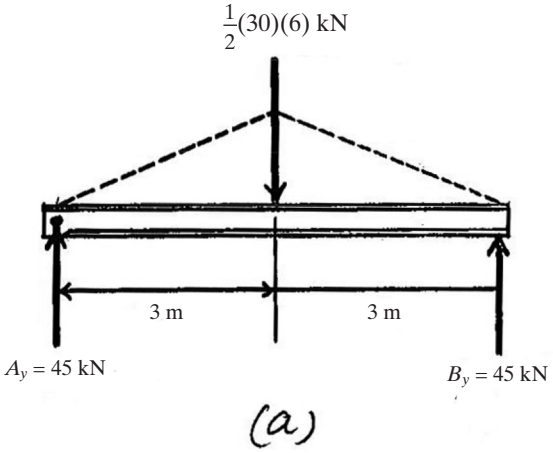
$$= 69.6888(10^{-6})\ \text{m}^4$$

Hence, $c = 0.156$ m. Thus,

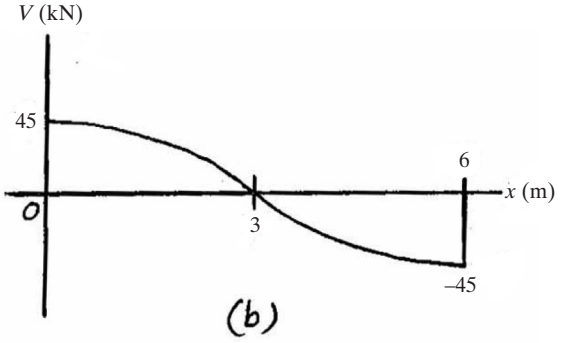
$$\sigma_{max} = \frac{M_{max}\, c}{I}$$

$$= \frac{[90(0.3^3)](0.156)}{69.6888(10^{-6})}$$

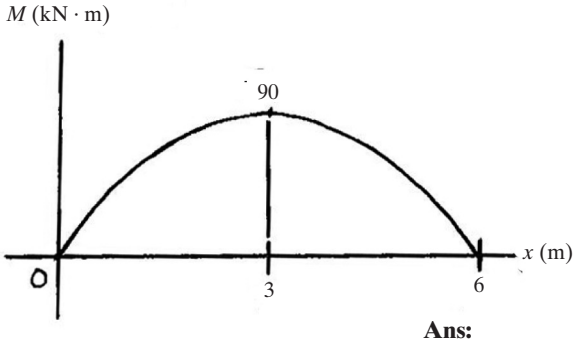
$$= 201.47(10^6)\ \text{N-m}^2 = 201\ \text{MPa}$$ **Ans.**



(a)



(b)



(c)

Ans:
$I = 69.6888\,(10^{-6})\ \text{m}^4$,
$\sigma_{max} = 201$ MPa

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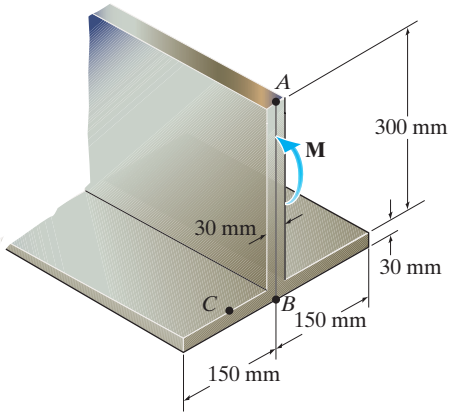
6–78.

If the beam is subjected to a moment of $M = 100\ \text{kN}\cdot\text{m}$, determine the bending stress at points A, B, and C. Sketch the bending stress distribution on the cross section.



SOLUTION

Section Properties: The neutral axis passes through centroid C of the cross section as shown in Fig. *a*. The location of C is

$$\bar{y} = \frac{\Sigma \tilde{y}A}{\Sigma A} = \frac{0.015(0.03)(0.3) + 0.18(0.3)(0.03)}{0.03(0.3) + 0.3(0.03)} = 0.0975\ \text{m}$$

Thus, the moment of inertia of the cross section about the neutral axis is

$$I = \frac{1}{12}(0.3)(0.03^3) + 0.3(0.03)(0.0975 - 0.015)^2 + \frac{1}{12}(0.03)(0.3^3)$$
$$+ 0.03(0.3)(0.18 - 0.0975)^2$$
$$= 0.1907(10^{-3})\ \text{m}^4$$

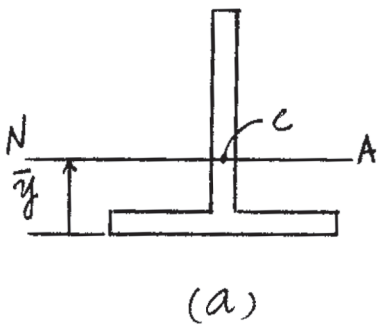
Bending Stress: The distance from the neutral axis to points A, B, and C is $y_A = 0.33 - 0.0975 = 0.2325$ m, $y_B = 0.0975$ m, and $y_C = 0.0975 - 0.03 = 0.0675$ m.

$$\sigma_A = \frac{My_A}{I} = \frac{100(10^3)(0.2325)}{0.1907(10^{-3})} = 122\ \text{MPa (C)} \qquad \textbf{Ans.}$$

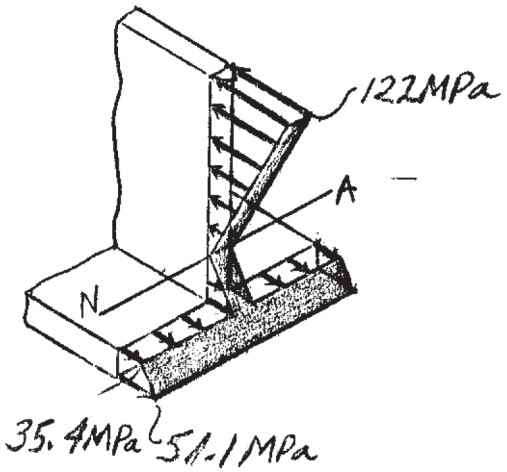
$$\sigma_B = \frac{My_B}{I} = \frac{100(10^3)(0.0975)}{0.1907(10^{-3})} = 51.1\ \text{MPa (T)} \qquad \textbf{Ans.}$$

$$\sigma_C = \frac{My_C}{I} = \frac{100(10^3)(0.0675)}{0.1907(10^{-3})} = 35.4\ \text{MPa (T)} \qquad \textbf{Ans.}$$

Using these results, the bending stress distribution across the cross section is shown in Fig. *b*.



(a)



(b)

Ans:
$\sigma_A = 122$ MPa (C),
$\sigma_B = 51.1$ MPa (T),
$\sigma_C = 35.4$ MPa (T)

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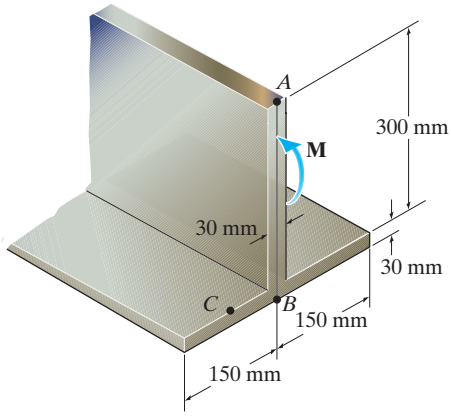
6–79.

If the beam is made of material having an allowable tensile and compressive stress of $(\sigma_{\text{allow}})_t = 125$ MPa and $(\sigma_{\text{allow}})_c = 150$ MPa, respectively, determine the maximum moment **M** that can be applied to the beam.



SOLUTION

Section Properties: The neutral axis passes through centroid C of the cross section as shown in Fig. a. The location of C is

$$\bar{y} = \frac{\Sigma \tilde{y}A}{\Sigma A} = \frac{0.015(0.03)(0.3) + 0.18(0.3)(0.03)}{0.03(0.3) + 0.3(0.03)} = 0.0975 \text{ m}$$

Thus, the moment of inertia of the cross section about the neutral axis is

$$I = \frac{1}{12}(0.3)(0.03^3) + 0.3(0.03)(0.0975 - 0.015)^2 + \frac{1}{12}(0.03)(0.3^3)$$
$$+ 0.03(0.3)(0.18 - 0.0975)^2$$
$$= 0.1907(10^{-3})\,\text{m}^4$$

Allowable Bending Stress: The maximum compressive and tensile stress occurs at the top and bottom-most fibers of the cross section. For the top-most fiber,

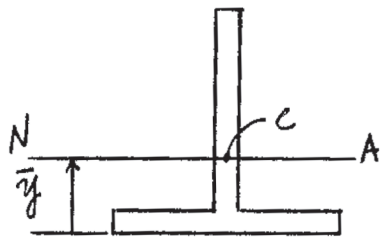
$$(\sigma_{\text{allow}})_c = \frac{Mc}{I} \qquad 150(10^6) = \frac{M(0.33 - 0.0975)}{0.1907(10^{-3})}$$

$$M = 123024.19\ \text{N}\cdot\text{m} = 123\ \text{kN}\cdot\text{m (controls)} \qquad \textbf{Ans.}$$

For the bottom-most fiber,

$$(\sigma_{\text{allow}})_t = \frac{My}{I} \qquad 125(10^6) = \frac{M(0.0975)}{0.1907(10^{-3})}$$

$$M = 244\,471.15\ \text{N}\cdot\text{m} = 244\ \text{kN}\cdot\text{m}$$



(a)

Ans:
$M = 123\ \text{kN}\cdot\text{m}$

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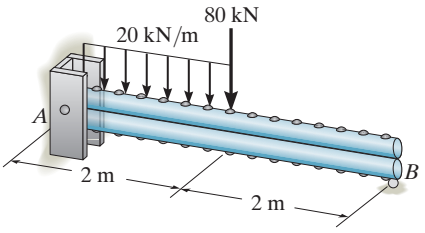
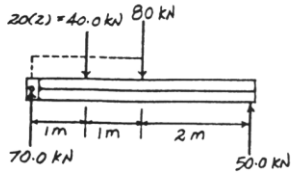
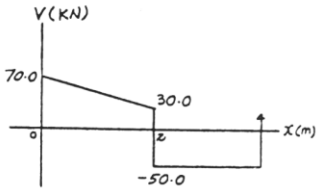
***6–80.** The two solid steel rods are bolted together along their length and support the loading shown. Assume the support at A is a pin and B is a roller. Determine the required diameter d of each of the rods if the allowable bending stress is $\sigma_{\text{allow}} = 130$ MPa.





Section Property:

$$I = 2\left[\frac{\pi}{4}\left(\frac{d}{2}\right)^4 + \frac{\pi}{4}d^2\left(\frac{d}{2}\right)^2\right] = \frac{5\pi}{32}d^4$$

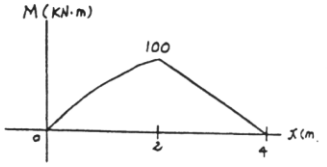


Allowable Bending Stress: The maximum moment is $M_{\max} = 100\ \text{kN}\cdot\text{m}$ as indicated on moment diagram. Applying the flexure formula

$$\sigma_{\max} = \sigma_{\text{allow}} = \frac{M_{\max}\,c}{I}$$

$$130\left(10^6\right) = \frac{100(10^3)(d)}{\frac{5\pi}{32}d^4}$$

$$d = 0.1162\ \text{m} = 116\ \text{mm}$$ **Ans.**



Ans:
$d = 116$ mm

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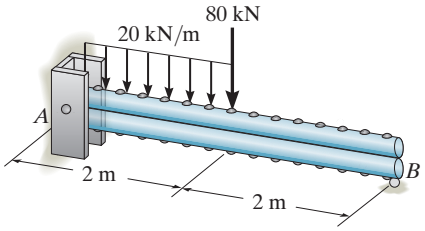
6–81. Solve Prob. 6–94 if the rods are rotated 90° so that both rods rest on the supports at A (pin) and B (roller).



Section Property:

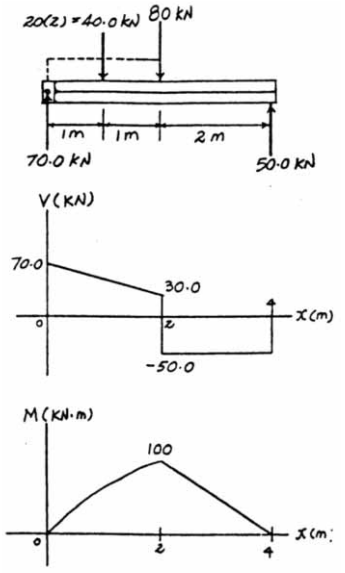
$$I = 2\left[\frac{\pi}{4}\left(\frac{d}{2}\right)^4\right] = \frac{\pi}{32}d^4$$

Allowable Bending Stress: The maximum moment is $M_{\max} = 100\ \text{kN}\cdot\text{m}$ as indicated on the moment diagram. Applying the flexure formula

$$\sigma_{\max} = \sigma_{\text{allow}} = \frac{M_{\max}\,c}{I}$$

$$130\left(10^6\right) = \frac{100(10^3)(d)}{\frac{\pi}{32}d^4}$$

$$d = 0.1986\ \text{m} = 199\ \text{mm}$$ **Ans.**



Ans:
$d = 199\ \text{mm}$

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6–82.

If the compound beam in Prob. 6–42 has a square cross section of side length a, determine the minimum value of a if the allowable bending stress is $\sigma_{\text{allow}} = 150$ MPa.

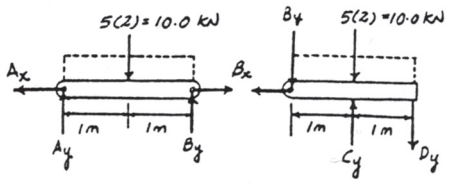
SOLUTION

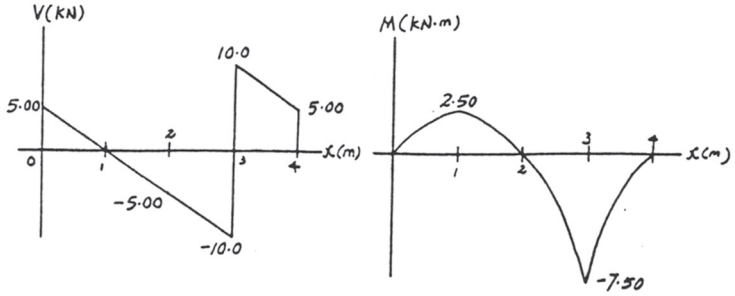
Allowable Bending Stress: The maximum moment is $M_{\max} = 7.50\,\text{kN}\cdot\text{m}$ as indicated on moment diagram. Applying the flexure formula

$$\sigma_{\max} = \sigma_{\text{allow}} = \frac{M_{\max}\, c}{I}$$

$$150\left(10^6\right) = \frac{7.50(10^3)\left(\frac{a}{2}\right)}{\frac{1}{12}a^4}$$

$$a = 0.06694\ \text{m} = 66.9\ \text{mm} \qquad \textbf{Ans.}$$





Ans:
$a = 66.9$ mm

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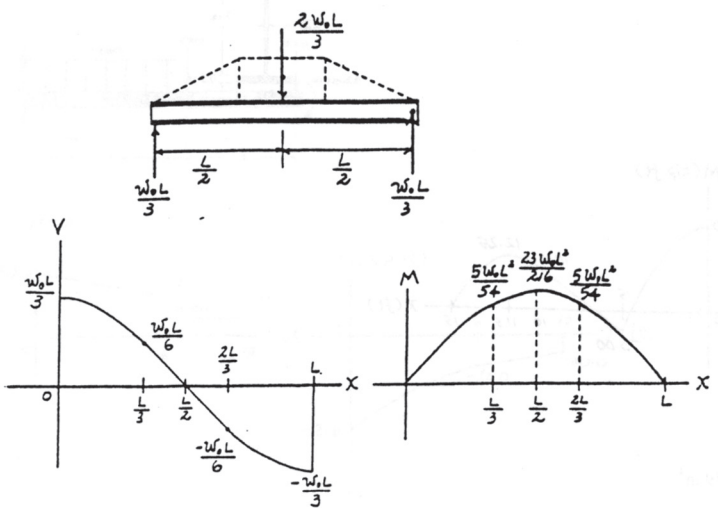
6–83.

If the beam in Prob. 6–28 has a rectangular cross section with a width b and a height h, determine the absolute maximum bending stress in the beam.

SOLUTION

Absolute Maximum Bending Stress: The maximum moment is $M_{\max} = \dfrac{23w_0 L^2}{216}$ as indicated on the moment diagram. Applying the flexure formula

$$\sigma_{\max} = \frac{M_{\max} c}{I} = \frac{\frac{23w_0 L^2}{216}\left(\frac{h}{2}\right)}{\frac{1}{12} bh^3} = \frac{23w_0 L^2}{36bh^2} \qquad \textbf{Ans.}$$



Ans:

$$\sigma_{\max} = \frac{23w_0 L^2}{36\, bh^2}$$

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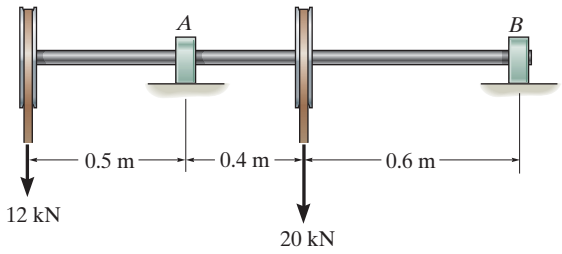
***6–84.**

Determine the absolute maximum bending stress in the 80-mm-diameter shaft which is subjected to the concentrated forces. There is a journal bearing at A and a thrust bearing at B.



SOLUTION

The FBD of the shaft is shown in Fig. *a*.

The shear and moment diagrams are shown in Fig. *b* and *c*, respectively. As indicated on the moment diagram, $|M_{max}| = 6\ \text{kN}\cdot\text{m}$.

The moment of inertia of the cross section about the neutral axis is

$$I = \frac{\pi}{4}(0.04^4) = 0.64(10^{-6})\pi\ \text{m}^4$$

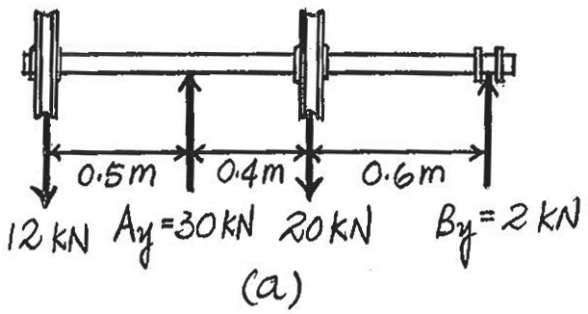
Here, $c = 0.04$ m. Thus

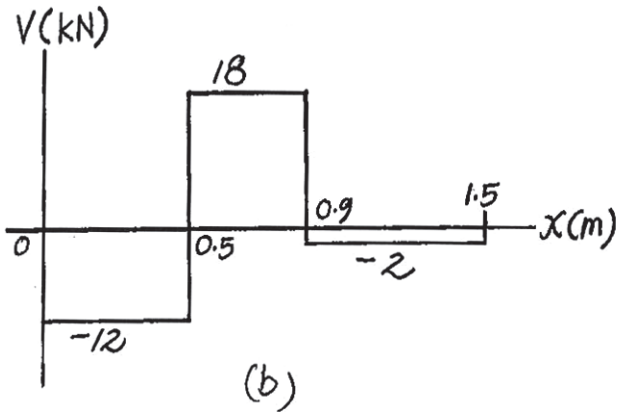
$$\sigma_{max} = \frac{M_{max}\,c}{I} = \frac{6(10^3)(0.04)}{0.64(10^{-6})\pi}$$

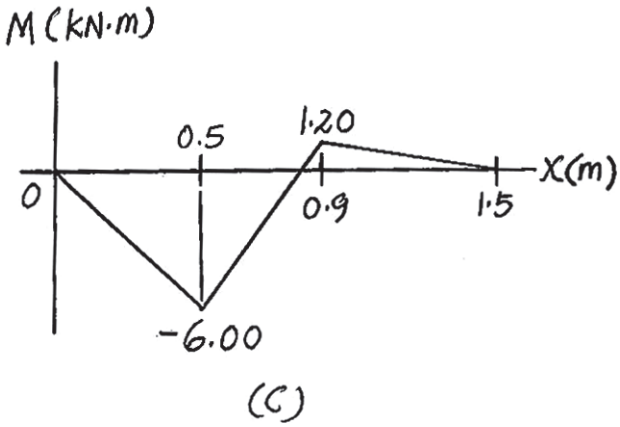
$$= 119.37(10^6)\ \text{Pa}$$

$$= 119\ \text{MPa}$$

Ans.







Ans:
$\sigma_{max} = 119$ MPa

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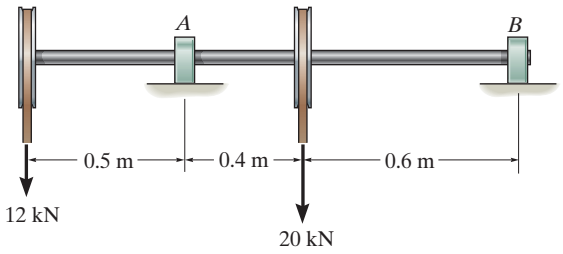
6–85.

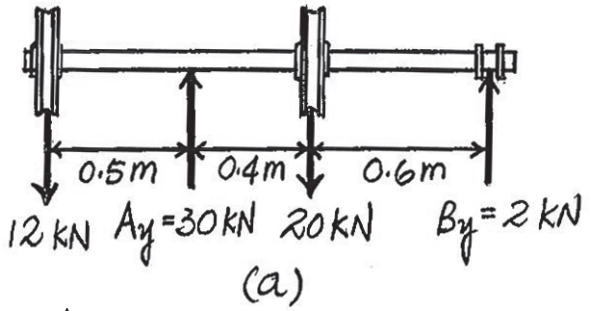
Determine, to the nearest millimeter, the smallest allowable diameter of the shaft which is subjected to the concentrated forces. There is a journal bearing at A and a thrust bearing at B. The allowable bending stress is $\sigma_{\text{allow}} = 150$ MPa.



SOLUTION

The FBD of the shaft is shown in Fig. *a*.

The shear and moment diagrams are shown in Fig. *b* and *c*, respectively. As indicated on the moment diagram, $\left| M_{\max} \right| = 6\ \text{kN} \cdot \text{m}$.



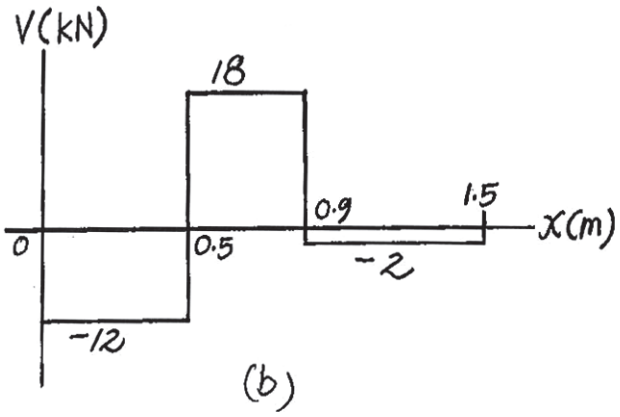
The moment of inertia of the cross section about the neutral axis is

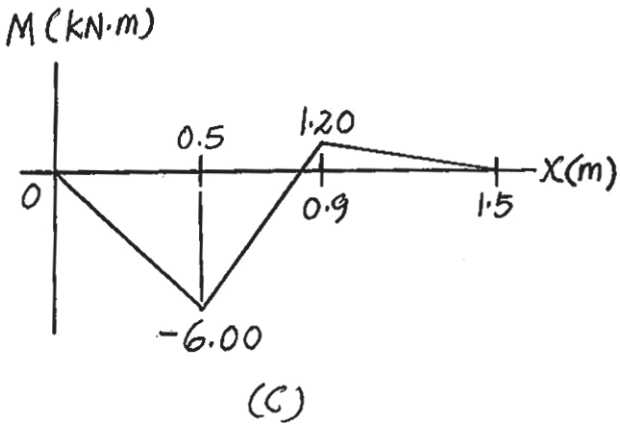
$$I = \frac{\pi}{4}\left(\frac{d}{2}\right)^4 = \frac{\pi d^4}{64}$$

Here, $c = d/2$. Thus

$$\sigma_{\text{allow}} = \frac{M_{\max}\, c}{I}; \qquad 150(10^6) = \frac{6(10^3)(^d/_2)}{\pi d^4/64}$$

$$d = 0.07413\ \text{m} = 74.13\ \text{mm} = 75\ \text{mm} \qquad \textbf{Ans.}$$





Ans:
$d = 75$ mm

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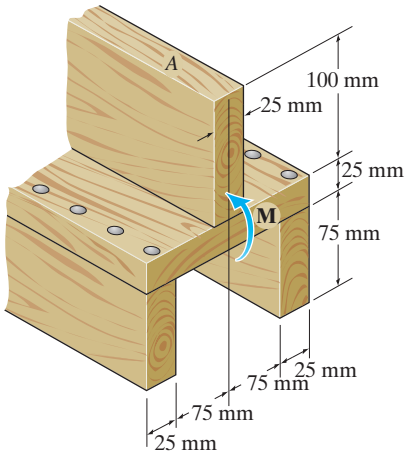
6–86. If the beam is subjected to an internal moment of $M = 3\,\text{kN}\cdot\text{m}$, determine the maximum tensile and compressive stress in the beam. Also, sketch the bending stress distribution on the cross section.



Section Properties: The neutral axis passes through centroid C of the cross section as shown in Fig. *a*. The location of C is

$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{2[0.0375(0.025)(0.075)] + 0.0875(0.2)(0.025) + 0.15(0.025)(0.1)}{2(0.025)(0.075) + 0.2(0.025) + 0.025(0.1)} = 0.08472\ \text{m}$$

Thus, the moment of inertia of the cross section about the neutral axis is

$$I = \Sigma \bar{I} + Ad^2$$
$$= 2\left[\frac{1}{12}(0.025)(0.075^3) + 0.025(0.075)(0.08472 - 0.0375)^2\right] + \frac{1}{12}(0.2)(0.025^3) + 0.2(0.025)(0.0875 - 0.08472)^2$$
$$+ \frac{1}{12}(0.025)(0.1^3) + 0.025(0.1)(0.15 - 0.08472)^2$$
$$= 23.1554(10^{-6})\ \text{m}^4$$

Maximum Bending Stress: The maximum compressive and tensile stress occurs at the top and bottom-most fibers of the cross section.

$$(\sigma_{\max})_c = \frac{Mc}{I} = \frac{[3(10^3)](0.2 - 0.08472)}{23.1554(10^{-6})} = 14.94(10^6)\ \text{N/m}^2 = 14.9\ \text{MPa}$$ **Ans.**

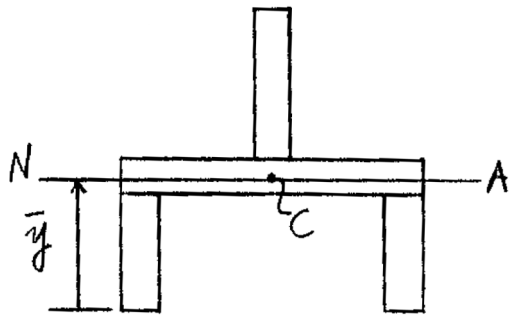
$$(\sigma_{\max})_t = \frac{My}{I} = \frac{[3(10^3)](0.08472)}{23.1554(10^{-6})} = 10.98(10^6)\ \text{N/m}^2 = 11.0\ \text{MPa}$$ **Ans.**

The bending stresses at $y = 0.01528$ m and $y = -0.009722$ m are

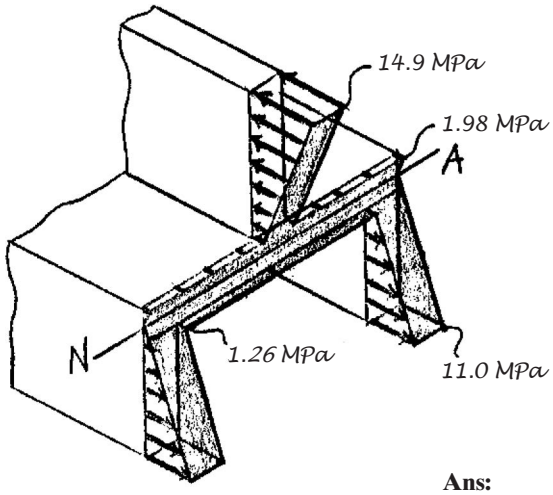
$$\sigma|_{y=0.1528\ \text{m}} = \frac{My}{I} = \frac{[3(10^3)](0.01528)}{23.1554(10^{-6})} = 1.979(10^6)\ \text{N/m}^2 = 1.98\ \text{MPa (C)}$$

$$\sigma|_{y=-0.009722\ \text{m}} = \frac{My}{I} = \frac{[3(10^3)](0.009722)}{23.1554(10^{-6})} = 1.260(10^6)\ \text{N/m}^2 = 1.26\ \text{MPa (T)}$$

The bending stress distribution across the cross section is shown in Fig. *b*.



(a)



(b)

Ans:
$(\sigma_{\max})_c = 14.9$ MPa,
$(\sigma_{\max})_t = 11.0$ MPa

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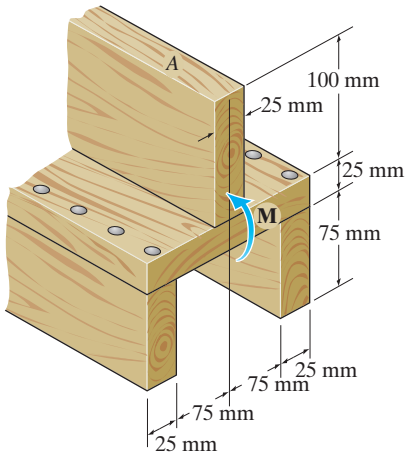
6–87. If the allowable tensile and compressive stress for the beam are $(\sigma_{allow})_t = 14$ MPa and $(\sigma_{allow})_c = 21$ MPa, respectively, determine the maximum allowable internal moment **M** that can be applied on the cross section.



Section Properties: The neutral axis passes through the centroid C of the cross section as shown in Fig. a. The location of C is given by

$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{2[0.0375(0.025)(0.075)] + 0.0875(0.2)(0.025) + 0.15(0.025)(0.1)}{2(0.025)(0.075) + 0.2(0.025) + 0.025(0.1)}$$

$$= 0.08472 \text{ m}$$

Thus, the moment of inertia of the cross section about the neutral axis is

$$I = \Sigma \bar{I} + Ad^2$$

$$= 2\left[\frac{1}{12}(0.025)(0.075^3) + 0.025(0.075)(0.08472 - 0.0375)^2\right] + \frac{1}{12}(0.2)(0.025^3) + 0.2(0.025)(0.0875 - 0.08472)^2$$

$$+ \frac{1}{12}(0.025)(0.1^3) + 0.025(0.1)(0.15 - 0.08472)^2$$

$$= 23.1554(10^{-6}) \text{ m}^4$$

Allowable Bending Stress: The maximum compressive and tensile stress occurs at the top and bottom-most fibers of the cross section.

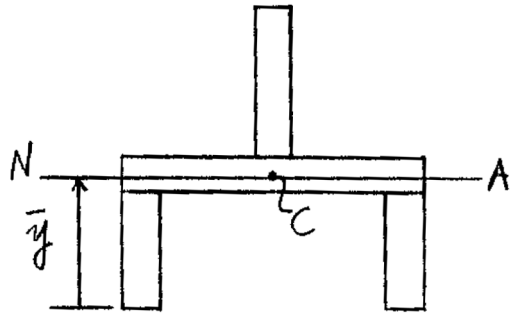
$$(\sigma_{allow})_c = \frac{Mc}{I}; \qquad 21(10^6) = \frac{M(0.2 - 0.08472)}{23.1554(10^{-6})}$$

$$M = 4.218(10^3) \text{ N} \cdot \text{m} = 4.22 \text{ kN} \cdot \text{m}$$

For the bottom-most fiber,

$$(\sigma_{allow})_t = \frac{My}{I}; \qquad 21(10^6) = \frac{M(0.08472)}{23.1554(10^{-6})}$$

$$M = 3.826(10^3) \text{ N} \cdot \text{m} = 3.83 \text{ kN} \cdot \text{m (Controls)} \qquad \textbf{Ans.}$$



(a)

Ans:

$M = 3.83 \text{ kN} \cdot \text{m}$

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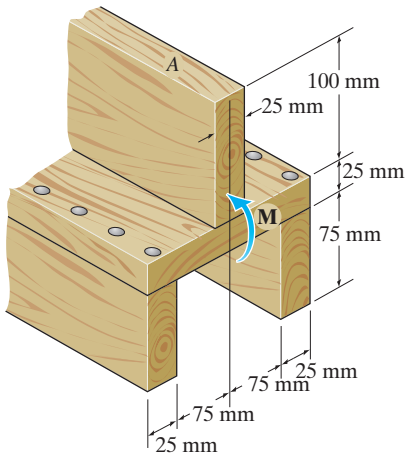
***6–88.** If the beam is subjected to an internal moment of $M = 3\ \text{kN}\cdot\text{m}$, determine the resultant force of the bending stress distribution acting on the top vertical board A.



Section Properties: The neutral axis passes through the centroid C of the cross section as shown in Fig. *a*. The location of C is given by

$$\bar{y} = \frac{\Sigma \tilde{y}A}{\Sigma A} = \frac{2[0.0375(0.025)(0.075)] + 0.0875(0.2)(0.025) + 0.15(0.025)(0.1)}{2(0.025)(0.075) + 0.2(0.025) + 0.025(0.1)}$$
$$= 0.08472\ \text{m}$$

Thus, the moment of inertia of the cross section about the neutral axis is

$$I = \Sigma \bar{I} + Ad^2$$
$$= 2\left[\frac{1}{12}(0.025)(0.075^3) + 0.025(0.075)(0.08472 - 0.0375)^2\right] + \frac{1}{12}(0.2)(0.025^3) + 0.2(0.025)(0.0875 - 0.08472)^2$$
$$+ \frac{1}{12}(0.025)(0.1^3) + 0.025(0.1)(0.15 - 0.08472)^2$$
$$= 23.1554(10^{-6})\ \text{m}^4$$

Bending Stress: The distance from the neutral axis to the top and bottom of board A is $y_t = 0.2 - 0.08472 = 0.11528$ m and $y_b = 0.1 - 0.08472 = 0.01528$ m. We have

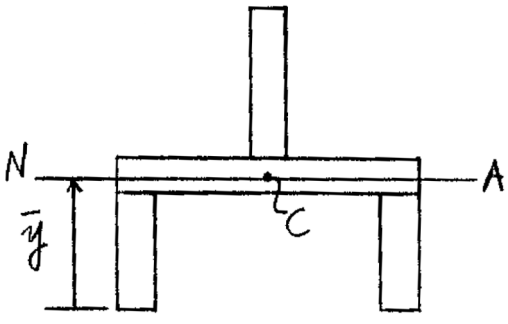
$$\sigma_t = \frac{My_t}{I} = \frac{[3(10^3)](0.11528)}{23.1554(10^{-6})} = 14.94(10^6)\ \text{N/m}^2 = 14.94\ \text{MPa}$$

$$\sigma_b = \frac{My_b}{I} = \frac{[3(10^3)](0.01528)}{23.1554(10^{-6})} = 1.979(10^6)\ \text{N/m}^2 = 1.979\ \text{MPa}$$

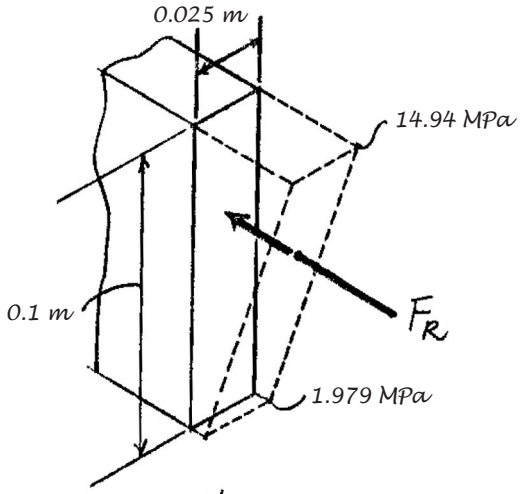
Resultant Force: The resultant force acting on board A is equal to the volume of the trapezoidal stress block shown in Fig. *b*. Thus,

$$F_R = \frac{1}{2}[(14.94 + 1.979)(10^6)](0.025)(0.1) = 21.14(10^3)\ \text{N} = 21.1\ \text{kN}$$

Ans.



(a)



(b)

Ans:
$F_R = 21.1\ \text{kN}$

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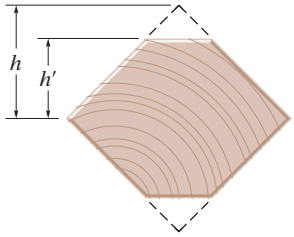
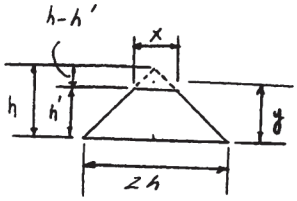
6–89.

A timber beam has a cross section which is originally square. If it is oriented as shown, determine the dimension h' so that it can resist the maximum moment possible. By what factor is this moment greater than that of the beam without its top or bottom flattened?



SOLUTION



$$\frac{x}{h-h'} = \frac{2h}{h}; \qquad x = 2(h-h')$$

$$y = h' + \frac{h-h'}{3} = \frac{2h'+h}{3}$$

$$I = 2\left\{\frac{1}{12}(2h)(h^3) - \left[\frac{1}{36}(2)(h-h')(h-h')^3 + \frac{1}{2}(2)(h-h')(h-h')\left(\frac{2h'+h}{3}\right)^2\right]\right\}$$

$$= \frac{1}{3}h^4 - \frac{1}{9}(h-h')^4 - \frac{2}{9}(h-h')^2(2h'+h)^2$$

$$= \frac{1}{3}h^4 - \frac{1}{9}(h-h')^2\left[3h^2 + 9h'^2 + 6hh'\right]$$

$$= \frac{1}{3}h^4 - \frac{1}{9}\left(3h^4 + 9h'^4 - 12hh'^3\right)$$

$$= \frac{4}{3}hh'^3 - h'^4$$

$$\sigma_{max} = \frac{Mc}{I}$$

$$M = \frac{I}{c}\sigma_{max} \qquad \textbf{(1)}$$

$$= \frac{\frac{4}{3}hh'^3 - h'^4}{h'}\sigma_{max} = \left(\frac{4}{3}hh'^2 - h'^3\right)\sigma_{max}$$

$$\frac{dM}{dh'} = \left(\frac{8}{3}hh' - 3h'^2\right)\sigma_{max}$$

In-order to have maximum moment,

$$\frac{dM}{dh'} = 0 = \frac{8}{3}hh' - 3h'^2$$

$$h' = \frac{8}{9}h \qquad \textbf{Ans.}$$

For the square beam,

$$I = \bar{I} + Ad^2$$

$$I = 2\left[\frac{1}{36}(2h)(h)^3 + \frac{1}{12}(2h)(h)\left(\frac{h}{3}\right)^2\right] = \frac{h^4}{3}$$

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6–89. Continued

From Eq. (1)

$$M = \frac{\frac{h^4}{3}}{h}\sigma_{\max} = \frac{h^3}{3}\sigma_{\max} = 0.3333h^3\,\sigma_{\max}$$

For the flattened beam:

$$I = \frac{4}{3}h\left(\frac{8}{9}h\right)^3 - \left(\frac{8}{9}h\right)^4 = 0.312147\,h^4$$

From Eq. (1)

$$M' = \frac{0.312147\,h^4}{\frac{8}{9}h}\sigma_{\max} = 0.35117\,h^3\,\sigma_{\max}$$

$$\text{Factor} = \frac{M'}{M'} = \frac{0.35117\,h^3\,\sigma_{\max}}{0.3333\,h^3\,\sigma_{\max}} = 1.05 \qquad \textbf{Ans.}$$

Ans:
$h' = \frac{8}{9}h,$
Factor $= 1.05$

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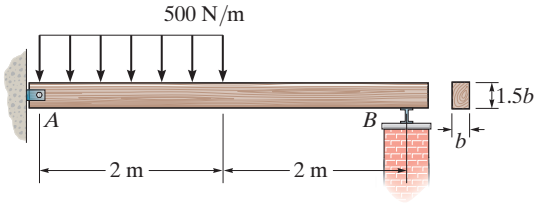
6–90. The wood beam has a rectangular cross section in the proportion shown. Determine its required dimension b if the allowable bending stress is $\sigma_{\text{allow}} = 10$ MPa.

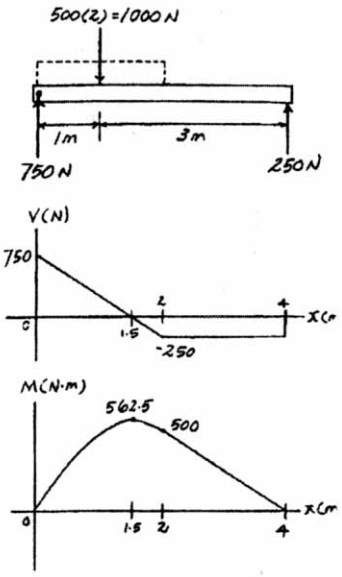


Allowable Bending Stress: The maximum moment is $M_{\max} = 562.5\ \text{N} \cdot \text{m}$ as indicated on the moment diagram. Applying the flexure formula

$$\sigma_{\max} = \sigma_{\text{allow}} = \frac{M_{\max}\, c}{I}$$

$$10\left(10^6\right) = \frac{562.5(0.75b)}{\frac{1}{12}(b)(1.5b)^3}$$

$$b = 0.05313\ \text{m} = 53.1\ \text{mm}$$ **Ans.**



Ans:
$b = 53.1$ mm

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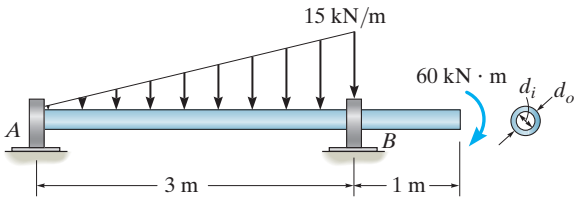
6–91. Determine the absolute maximum bending stress in the tubular shaft if $d_i = 160$ mm and $d_o = 200$ mm.



Section Property:

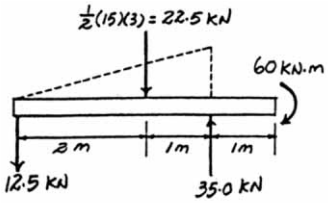
$$I = \frac{\pi}{4}\left(0.1^4 - 0.08^4\right) = 46.370\left(10^{-6}\right)\text{ m}^4$$

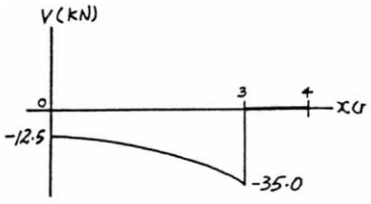
Absolute Maximum Bending Stress: The maximum moment is $M_{\max} = 60.0\text{ kN}\cdot\text{m}$ as indicated on the moment diagram. Applying the flexure formula

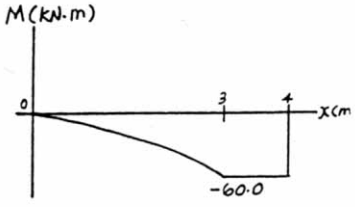
$$\sigma_{\max} = \frac{M_{\max} c}{I}$$

$$= \frac{60.0(10^3)(0.1)}{46.370(10^{-6})}$$

$$= 129\text{ MPa}$$ **Ans.**







Ans:
$\sigma_{\max} = 129\text{ MPa}$

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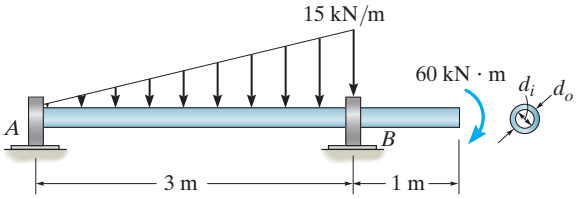
***6–92.** The tubular shaft is to have a cross section such that its inner diameter and outer diameter are related by $d_i = 0.8d_o$. Determine these required dimensions if the allowable bending stress is $\sigma_{\text{allow}} = 155$ MPa.



Section Property:

$$I = \frac{\pi}{4}\left[\left(\frac{d_o}{2}\right)^4 - \left(\frac{d_l}{2}\right)^4\right] = \frac{\pi}{4}\left[\frac{d_o{}^4}{16} - \left(\frac{0.8d_o}{2}\right)^4\right] = 0.009225\pi d_o^4$$

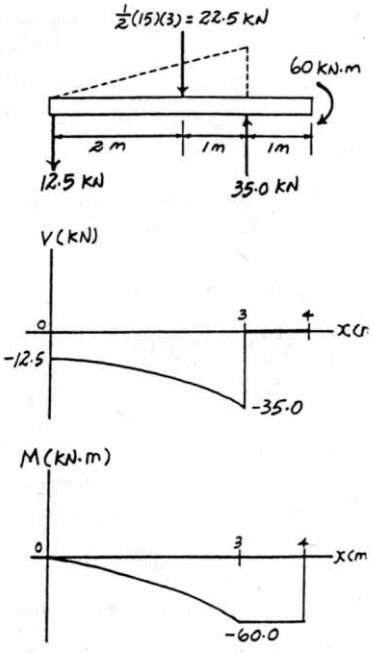
Allowable Bending Stress: The maximum moment is $M_{\max} = 60.0\ \text{kN}\cdot\text{m}$ as indicated on the moment diagram. Applying the flexure formula

$$\sigma_{\max} = \sigma_{\text{allow}} = \frac{M_{\max} c}{I}$$

$$155(10^6) = \frac{60.0(10^3)\left(\frac{d_o}{2}\right)}{0.009225\pi d_o^4}$$

$$d_o = 0.1883\ \text{m} = 188\ \text{mm} \qquad \textbf{Ans.}$$

Thus, $d_l = 0.8d_o = 151\ \text{mm}$ **Ans.**



Ans:

$d_o = 0188\ \text{mm}, d_l = 151\ \text{mm}$

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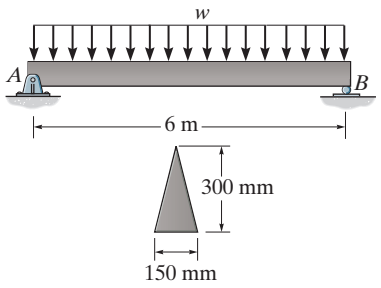
6–93.

If the intensity of the load $w = 15$ kN/m, determine the absolute maximum tensile and compressive stress in the beam.



SOLUTION

Support Reactions: Shown on the free-body diagram of the beam, Fig. *a*.

Maximum Moment: The maximum moment occurs when $V = 0$. Referring to the free-body diagram of the beam segment shown in Fig. *b*,

$+\uparrow \Sigma F_y = 0; \qquad 45 - 15x = 0 \qquad x = 3\text{ m}$

$\circlearrowleft + \Sigma M = 0; \qquad M_{\max} + 15(3)\left(\frac{3}{2}\right) - 45(3) = 0 \qquad M_{\max} = 67.5\text{ kN}\cdot\text{m}$

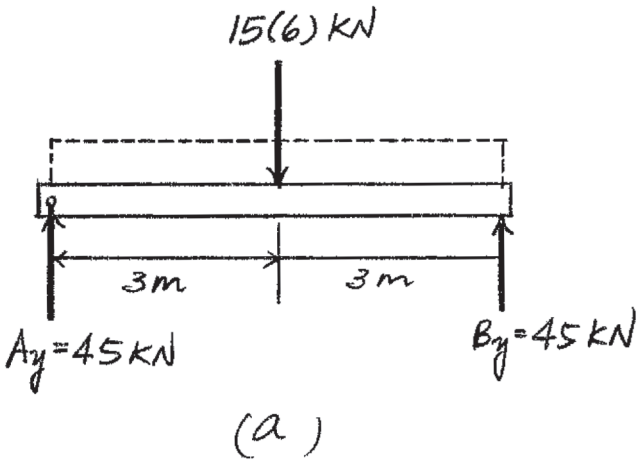
Section Properties: The moment of inertia of the cross section about the neutral axis is

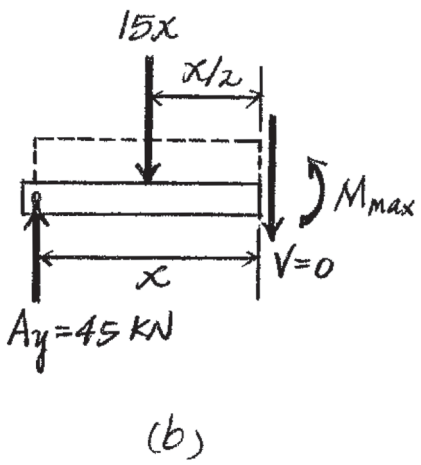
$$I = \frac{1}{36}(0.15)(0.3^3) = 0.1125(10^{-3})\text{ m}^4$$

Absolute Maximum Bending Stress: The maximum compressive and tensile stresses occur at the top and bottom-most fibers of the cross section.

$$(\sigma_{\max})_c = \frac{Mc}{I} = \frac{67.5(10^3)(0.2)}{0.1125(10^{-3})} = 120\text{ MPa (C)}$$ **Ans.**

$$(\sigma_{\max})_t = \frac{My}{I} = \frac{67.5(10^3)(0.1)}{0.1125(10^{-3})} = 60\text{ MPa (T)}$$ **Ans.**





Ans:
$(\sigma_{\max})_c = 120$ MPa (C),
$(\sigma_{\max})_t = 60$ MPa (T)

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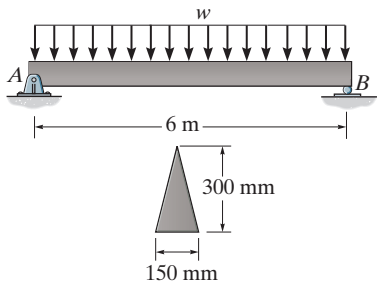
6–94.

If the allowable bending stress is $\sigma_{\text{allow}} = 150$ MPa, determine the maximum intensity w of the uniform distributed load.



SOLUTION

Support Reactions: As shown on the free-body diagram of the beam, Fig. *a*.

Maximum Moment: The maximum moment occurs when $V = 0$. Referring to the free-body diagram of the beam segment shown in Fig. *b*,

$+\uparrow \Sigma F_y = 0; \qquad 3w - wx = 0 \qquad x = 3\text{ m}$

$\circlearrowleft + \Sigma M = 0; \qquad M_{\max} + w(3)\left(\frac{3}{2}\right) - 3w(3) = 0 \qquad M_{\max} = \frac{9}{2}w$

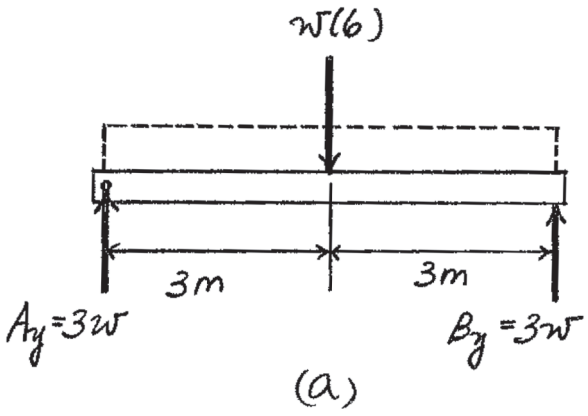
Section Properties: The moment of inertia of the cross section about the neutral axis is

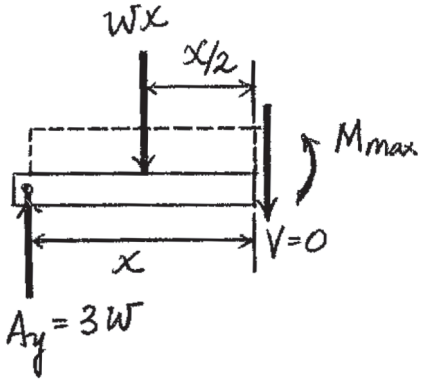
$$I = \frac{1}{36}(0.15)(0.3^3) = 0.1125(10^{-3})\,\text{m}^4$$

Absolute Maximum Bending Stress: Here, $c = \frac{2}{3}(0.3) = 0.2$ m.

$\sigma_{\text{allow}} = \frac{Mc}{I}; \qquad 150(10^6) = \frac{\frac{9}{2}w(0.2)}{0.1125(10^{-3})}$

$w = 18750\text{ N/m} = 18.75\text{ kN/m}$ **Ans.**





Ans:
$w = 18.75\text{ kN/m}$

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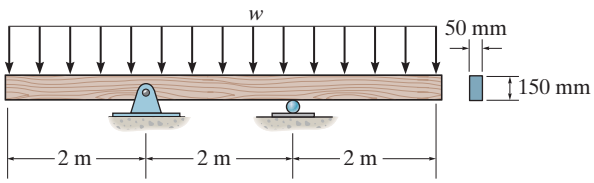
6–95.

The beam has a rectangular cross section as shown. Determine the largest intensity w of the uniform distributed load so that the bending stress in the beam does not exceed $\sigma_{max} = 10$ MPa.

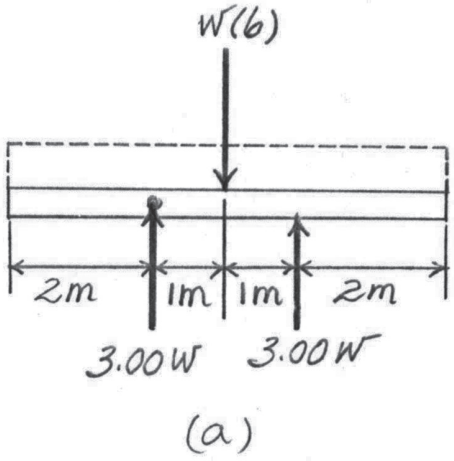


SOLUTION

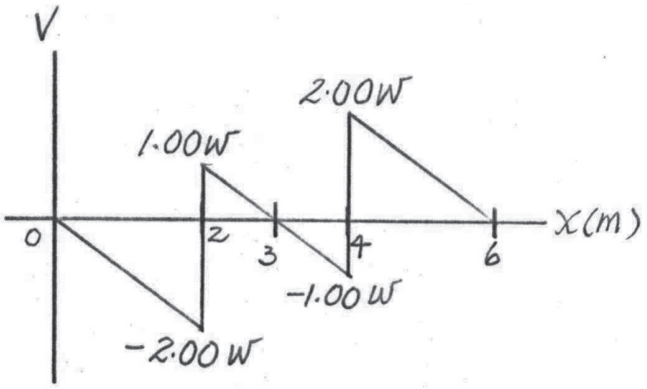
Absolute Maximum Bending Stress: The support reactions, shear diagram, and moment diagrams are shown in Figs. *a*, *b*, and *c*, respectively. From the moment diagram, the maximum moment is $M_{max} = 2.00\,w$, which occurs at the pin support and roller support. Applying the flexure formula,

$$\sigma_{\substack{\text{abs}\\ \text{max}}} = \frac{M_{max}C}{I}; \qquad 10(10^6) = \frac{(2.00\,w)(0.075)}{\frac{1}{12}(0.05)(0.15^3)}$$

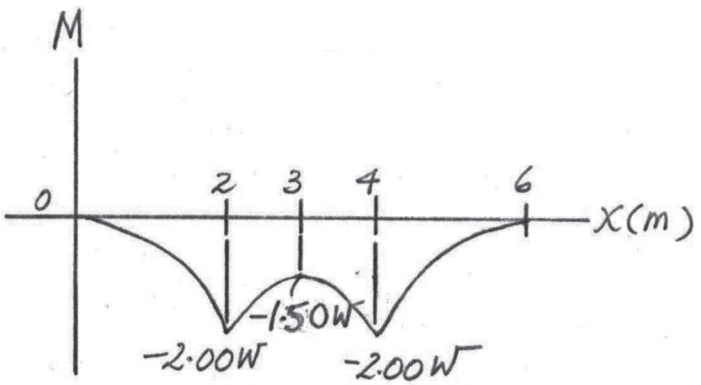
$$w = 937.5\ \text{N/m} \qquad \textbf{Ans.}$$



(a)



(b)



(c)

Ans:
$w = 937.5$ N/m

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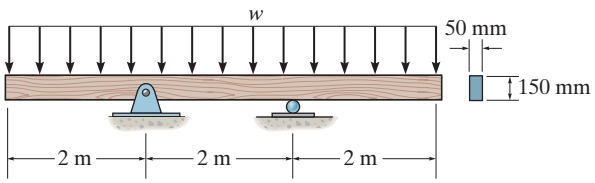
***6–96.**

The beam has the rectangular cross section shown. If $w = 1$ kN/m, determine the maximum bending stress in the beam. Sketch the stress distribution acting over the cross section.

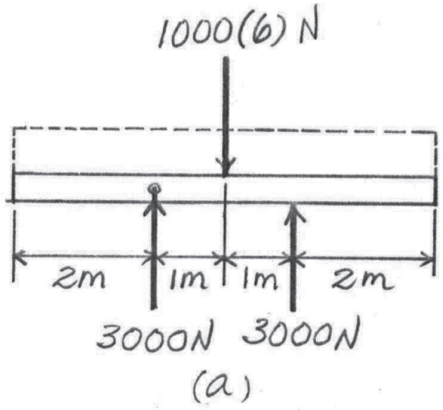


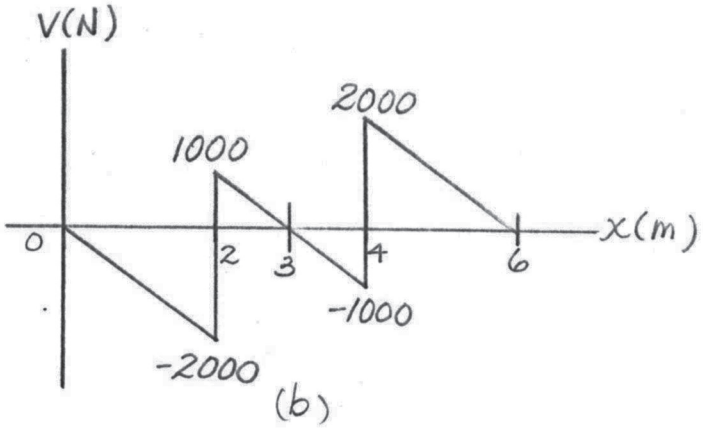
SOLUTION

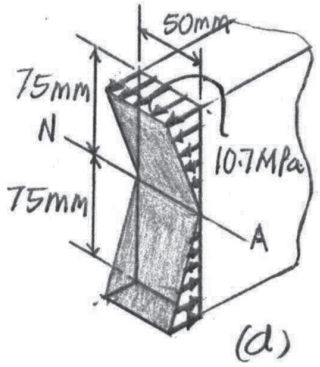
Absolute Maximum Bending Stress: The support reactions, shear diagram, and moment diagram are shown in Figs. *a*, *b,* and *c*, respectively. From the moment diagram, the maximum moment is $M_{\max} = 2000\ \text{N} \cdot \text{m}$, which occurs at the supports. Applying the flexure formula,

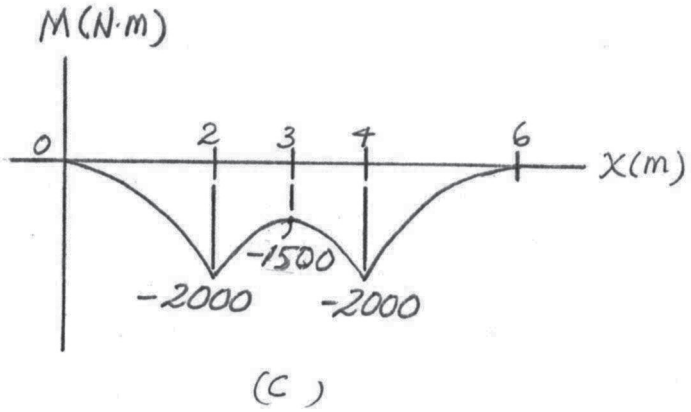
$$\sigma_{\substack{\text{abs}\\\max}} = \frac{M_{\max} C}{I} = \frac{2000(0.075)}{\frac{1}{12}(0.05)(0.15^3)} = 10.67(10^6)\ \text{MPa} = 10.7\ \text{MPa} \qquad \textbf{Ans.}$$

Using this result, the bending stress distribution on the beam's cross-section shown in Fig. *d* can be sketched.









Ans:
$\sigma_{\substack{\text{abs}\\\max}} = 10.7$ MPa

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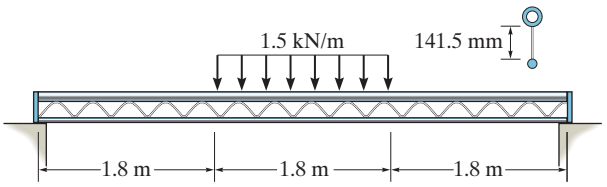
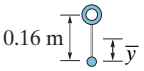
6–97. The simply supported truss is subjected to the central distributed load. Neglect the effect of the diagonal lacing and determine the absolute maximum bending stress in the truss. The top member is a pipe having an outer diameter of 25 mm and thickness of 5 mm, and the bottom member is a solid rod having a diameter of 12 mm.





$$\bar{y} = \frac{\Sigma \bar{y}A}{\Sigma A} = \frac{0 + 0.16[\pi(0.0125^2 - 0.0075^2)]}{\pi(0.006^2) + \pi(0.0125^2 - 0.0075^2)} = 0.11765 \text{ m}$$

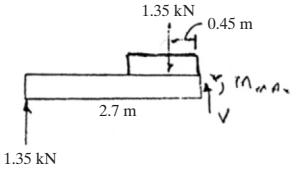
$$I = \frac{\pi}{4}(0.006^4) + \pi(0.006^2)(0.11765^2) + \frac{\pi}{4}(0.0125^4 - 0.0075^4)$$

$$+ \pi(0.0125^2 - 0.0075^2)(0.16 - 0.11765)^2 = 2.1466(10^{-6})\ \text{m}^4$$

$$M_{\max} = 1.35(2.7) - 1.35(0.45) = 3.0375\ \text{kN} \cdot \text{m}$$

$$\sigma_{\max} = \frac{MC}{I} = \frac{[3.0375(10^3)](0.11765 + 0.006)}{2.1466(10^{-6})}$$

$$= 174.96(10^6)\ \text{N-m}^2 = 175\ \text{MPa}$$ **Ans.**



Ans:
$(\sigma_{\max})_c = 120$ MPa (C), $(\sigma_{\max})_t = 60$ MPa (T)

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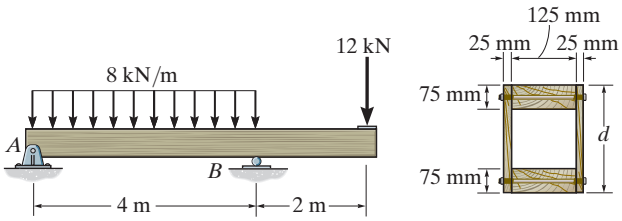
6–98.

If $d = 450$ mm, determine the absolute maximum bending stress in the overhanging beam.



SOLUTION

Support Reactions: Shown on the free-body diagram of the beam, Fig. *a*.

Maximum Moment: The shear and moment diagrams are shown in Figs. *b* and *c*. As indicated on the moment diagram, $M_{\max} = 24\,\text{kN}\cdot\text{m}$.

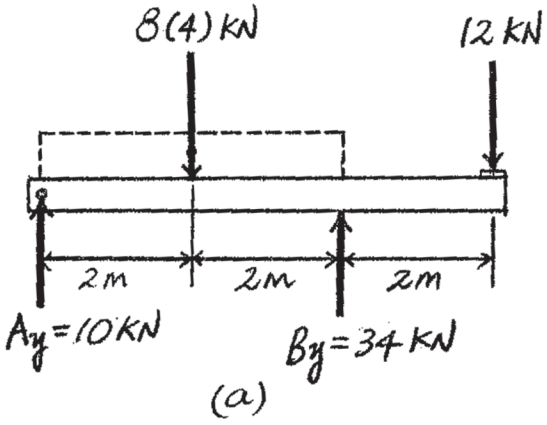
Section Properties: The moment of inertia of the cross section about the neutral axis is

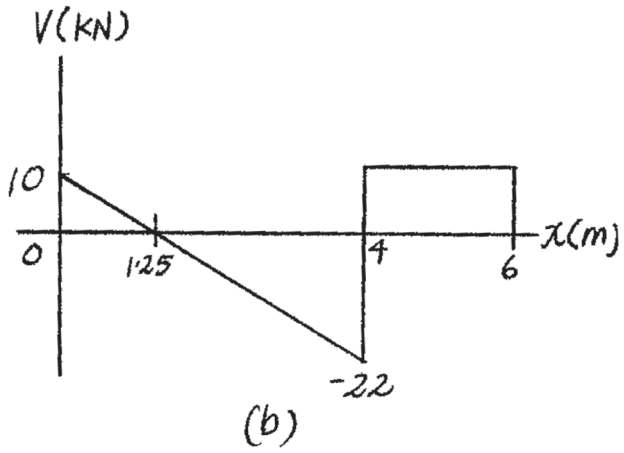
$$I = \frac{1}{12}(0.175)(0.45^3) - \frac{1}{12}(0.125)(0.3^3)$$

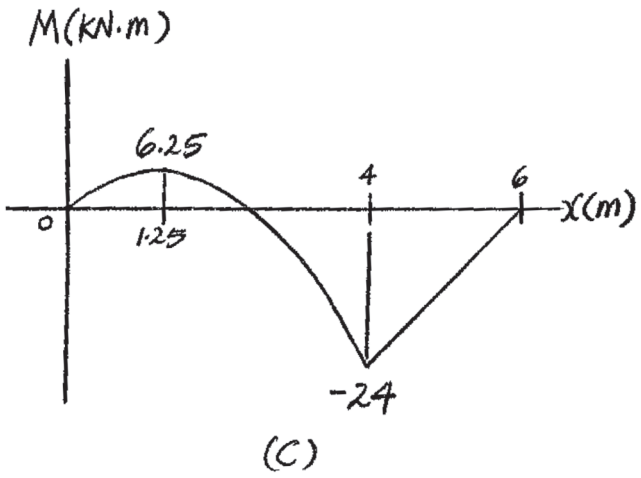
$$= 1.0477(10^{-3})\ \text{m}^4$$

Absolute Maximum Bending Stress: Here, $c = \dfrac{0.45}{2} = 0.225$ m.

$$\sigma_{\max} = \frac{M_{\max} c}{I} = \frac{24(10^3)(0.225)}{1.0477(10^{-3})} = 5.15\ \text{MPa}$$ **Ans.**







Ans:
$\sigma_{\max} = 5.15$ MPa

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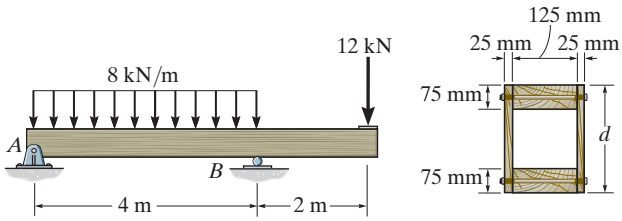
6–99.

If the allowable bending stress is $\sigma_{allow} = 6$ MPa, determine the minimum dimension d of the beam's cross-sectional area to the nearest mm.



SOLUTION

Support Reactions: Shown on the free-body diagram of the beam, Fig. *a*.

Maximum Moment: The shear and moment diagrams are shown in Figs. *b* and *c*. As indicated on the moment diagram, $M_{max} = 24\,\text{kN}\cdot\text{m}$.

Section Properties: The moment of inertia of the cross section about the neutral axis is

$$I = \frac{1}{12}(0.175)d^3 - \frac{1}{12}(0.125)(d - 0.15)^3$$

$$= 4.1667(10^{-3})d^3 + 4.6875(10^{-3})d^2 - 0.703125(10^{-3})d + 35.15625(10^{-6})$$

Absolute Maximum Bending Stress: Here, $c = \frac{d}{2}$.

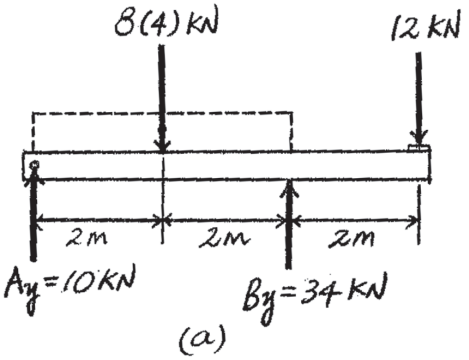
$$\sigma_{allow} = \frac{Mc}{I};$$

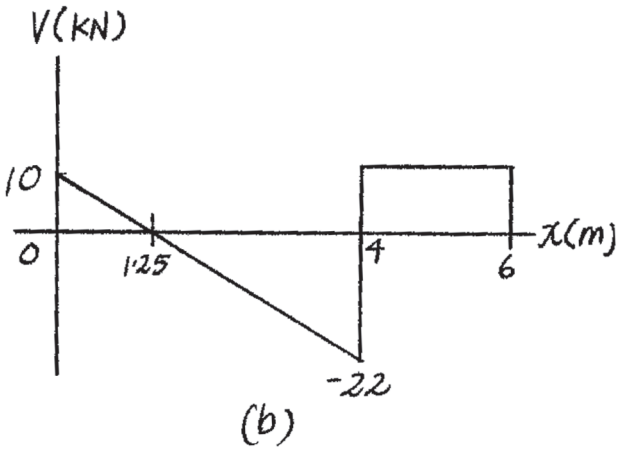
$$6(10^6) = \frac{24(10^3)\frac{d}{2}}{4.1667(10^{-3})d^3 + 4.6875(10^{-3})d^2 - 0.703125(10^{-3})d + 35.15625(10^{-6})}$$

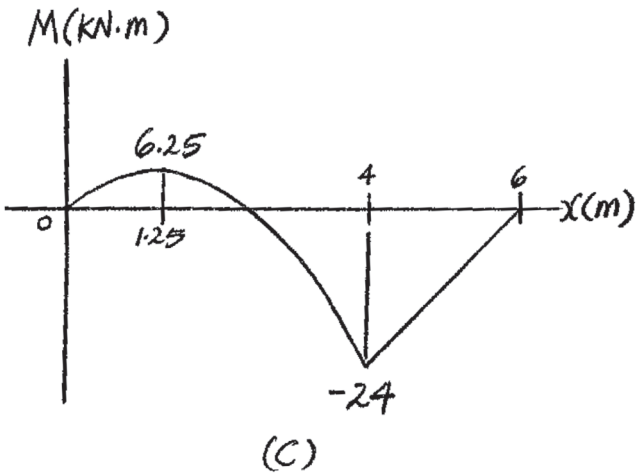
$$4.1667(10^{-3})d^3 + 4.6875(10^{-3})d^2 - 2.703125(10^{-3})d + 35.15625(10^{-6}) = 0$$

Solving,

$d = 0.4094$ m $= 410$ mm **Ans.**







Ans:
$d = 410$ mm

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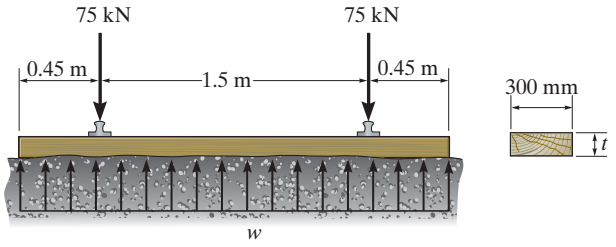
***6–100.** If the reaction of the ballast on the railway tie can be assumed uniformly distributed over its length as shown, determine the maximum bending stress developed in the tie. The tie has the rectangular cross section with thickness $t = 150$ mm.



Support Reactions: Referring to the free - body diagram of the tie shown in Fig. *a*, we have

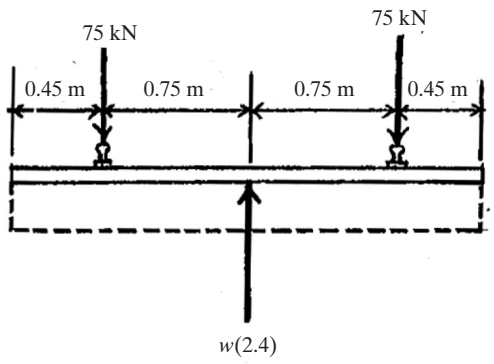
$+\uparrow \Sigma F_y = 0;$ $\quad w(2.4) - 2(75) = 0$

$w = 62.5$ kN/m

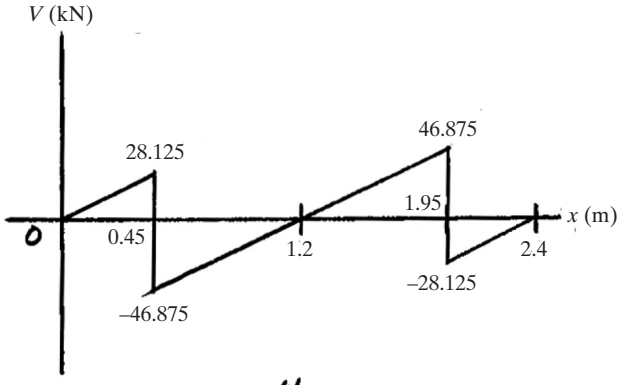
Maximum Moment: The shear and moment diagrams are shown in Figs. *b* and *c*. As indicated on the moment diagram, the maximum moment is $|M_{\max}| = 11.25$ kN · m

Absolute Maximum Bending Stress:

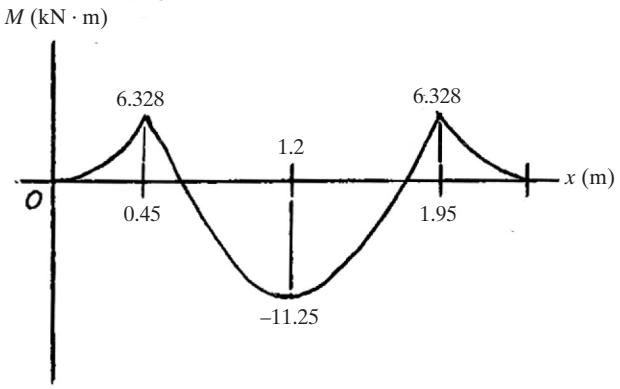
$$\sigma_{\max} = \frac{M_{\max} c}{I} = \frac{[11.25(10^3)](0.075)}{\frac{1}{12}(0.3)(0.15^3)} = 10.0(10^6)\ \text{N/m}^2 = 10.0\ \text{MPa}$$ **Ans.**



(a)



(b)



(c)

Ans:

$\sigma_{\max} = 10.0$ MPa

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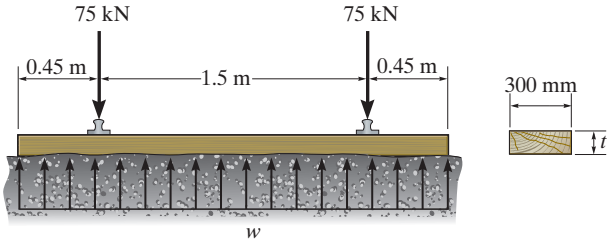
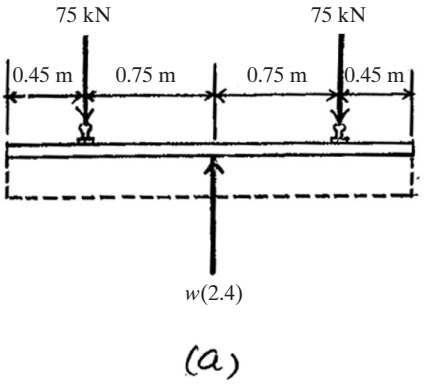
6–101. The reaction of the ballast on the railway tie can be assumed uniformly distributed over its length as shown. If the wood has an allowable bending stress of σ_{allow} = 10.5 MPa, determine the required minimum thickness t of the rectangular cross sectional area of the tie to the nearest multiples of 5 mm.



Support Reactions: Referring to the free-body diagram of the tie shown in Fig. *a*, we have

$$+\uparrow \Sigma F_y = 0; \qquad w = (2.4) - 2(75) = 0$$

$$w = 62.5 \text{ kN/m}$$



(a)

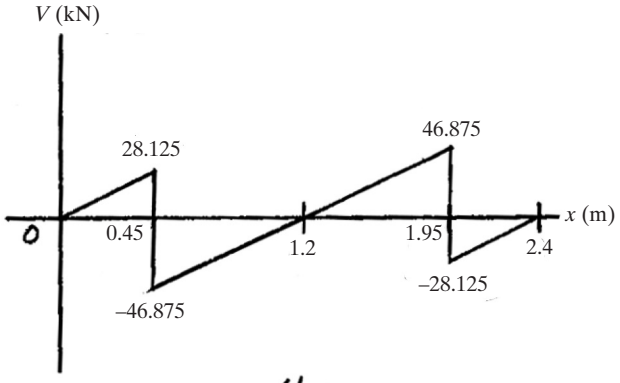
Maximum Moment: The shear and moment diagrams are shown in Figs. *b* and *c*. As indicated on the moment diagram, the maximum moment is $|M_{max}| = 11.25 \text{ kN} \cdot \text{m}$.

Absolute Maximum Bending Stress:

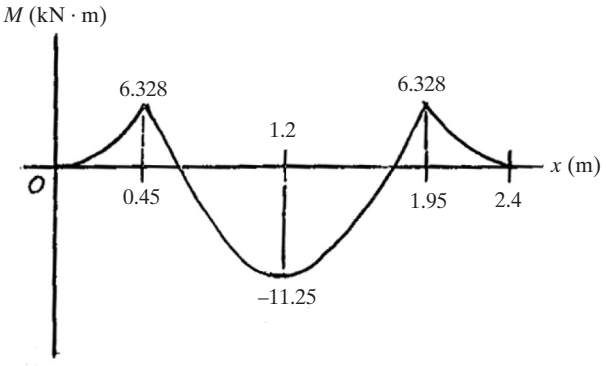
$$\sigma_{max} = \frac{Mc}{I}; \qquad 10.5(10^6) = \frac{[11.25(10^3)]\left(\frac{t}{2}\right)}{\frac{1}{12}(0.3)(t^3)}$$

$$t = 0.14638 \text{ m} = 146.38 \text{ mm}$$

Use t = 150 mm **Ans.**



(b)



(c)

Ans:
Use t = 150 mm

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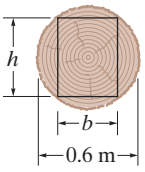
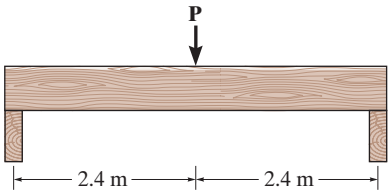
6–102. A log that is 0.6 m in diameter is to be cut into a rectangular section for use as a simply supported beam. If the allowable bending stress for the wood is $\sigma_{allow} = 56$ MPa, determine the required width b and height h of the beam that will support the largest load possible. What is this load?



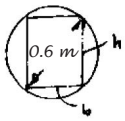


$$0.6^2 = b^2 + h^2$$

$$h^2 = 0.36 - b^2$$

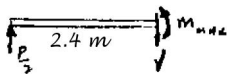
$$M_{max} = \frac{P}{2}(2.4) = 1.2P$$

$$\sigma_{allow} = \frac{Mc}{I} = \frac{M_{max}(\frac{h}{2})}{\frac{1}{12}(b)(h)^3}$$



$$\sigma_{allow} = \frac{6\,M_{max}}{bh^2}$$

$$56(10^6) = \frac{6(1.2P)}{b(0.36 - b^2)}$$



$$P = 7.7778(10^6)(0.36b - b^3)$$

Set $\frac{dP}{db} = 0$ gives

$$\frac{dP}{db} = 7.7778(10^6)(0.36 - b^2) = 0$$

$b = 0.3464$ m

Thus, from the above equations, **Ans.**

$b = 346{\cdot}41$ mm $= 346$ mm **Ans.**

$h = 0.4899$ m $= 490$ mm **Ans.**

$P = 646.63(10^3)$ N $= 647$ kN

Ans:
$b = 346$ mm, $h = 490$ mm, $P = 647$ kN

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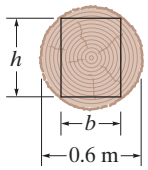
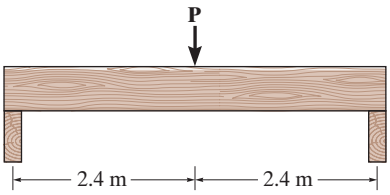
6–103. A log that is 0.6 m in diameter is to be cut into a rectangular section for use as a simply supported beam. the allowable bending stress for the wood is $\sigma_{allow} = 56$ MPa, determine the largest load P that can be supported if the width of the beam is $b = 200$ mm.



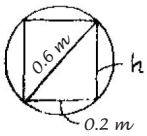


$0.6^2 = h^2 + 0.2^2$

$h = 0.5657$ m

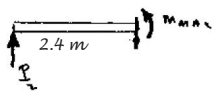
$M_{max} = \frac{P}{2}(2.4) = 1.2P$

$\sigma_{allow} = \frac{M_{max}c}{I}$



$56(10^6) = \frac{(1.2P)(0.5657/2)}{\frac{1}{12}(0.2)(0.5657^3)}$

$P = 497.78(10^3) = 498$ kN **Ans.**



Ans:
$P = 498$ kN

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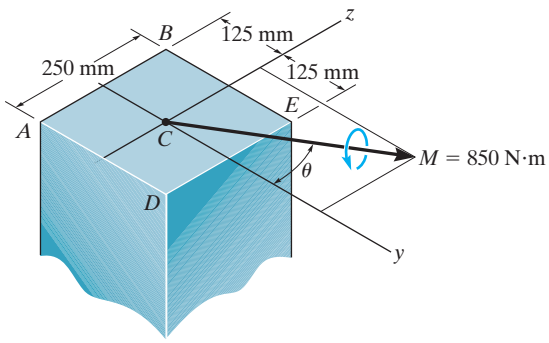
***6–104.**

The member has a square cross section and is subjected to the moment $M = 850\,\text{N}\cdot\text{m}$. Determine the stress at each corner and sketch the stress distribution. Set $\theta = 45°$.



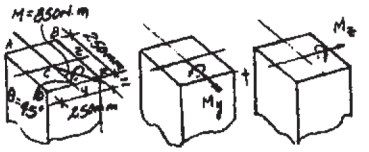
SOLUTION

$M_y = 850\cos 45° = 601.04\,\text{N}\cdot\text{m}$

$M_z = 850\sin 45° = 601.04\,\text{N}\cdot\text{m}$

$$I_z = I_y = \frac{1}{12}(0.25)(0.25)^3 = 0.3255208(10^{-3})\,\text{m}^4$$

$$\sigma = -\frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

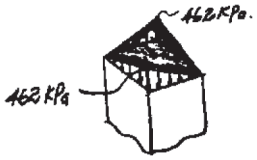


$$\sigma_A = -\frac{601.04(-0.125)}{0.3255208(10^{-3})} + \frac{601.04(-0.125)}{0.3255208(10^{-3})} = 0 \qquad \textbf{Ans.}$$

$$\sigma_B = -\frac{601.04(-0.125)}{0.3255208(10^{-3})} + \frac{601.04(0.125)}{0.3255208(10^{-3})} = 462\,\text{kPa} \qquad \textbf{Ans.}$$

$$\sigma_D = -\frac{601.04(0.125)}{0.3255208(10^{-3})} + \frac{601.04(-0.125)}{0.3255208(10^{-3})} = -462\,\text{kPa} \qquad \textbf{Ans.}$$

$$\sigma_E = -\frac{601.04(0.125)}{0.3255208(10^{-3})} + \frac{601.04(0.125)}{0.3255208(10^{-3})} = 0 \qquad \textbf{Ans.}$$



The negative sign indicates compressive stress.

Ans:
$\sigma_A = 0$,
$\sigma_B = 462\,\text{kPa}$,
$\sigma_D = -462\,\text{kPa}$,
$\sigma_E = 0$

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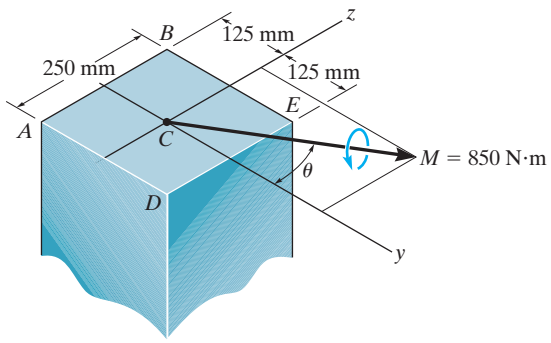
6–105.

The member has a square cross section and is subjected to the moment $M = 850\ \text{N}\cdot\text{m}$ as shown. Determine the stress at each corner and sketch the stress distribution. Set $\theta = 30°$.



SOLUTION

$M_y = 850 \cos 30° = 736.12\ \text{N}\cdot\text{m}$

$M_z = 850 \sin 30° = 425\ \text{N}\cdot\text{m}$

$$I_z = I_y = \frac{1}{12}(0.25)(0.25)^3 = 0.3255208(10^{-3})\ \text{m}^4$$

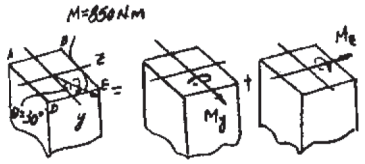
$$\sigma = -\frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

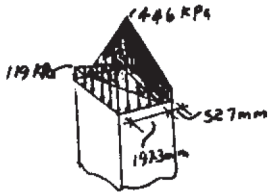
$$\sigma_A = -\frac{425(-0.125)}{0.3255208(10^{-3})} + \frac{736.12(-0.125)}{0.3255208(10^{-3})} = -119\ \text{kPa}$$ **Ans.**

$$\sigma_B = -\frac{425(-0.125)}{0.3255208(10^{-3})} + \frac{736.12(0.125)}{0.3255208(10^{-3})} = 446\ \text{kPa}$$ **Ans.**

$$\sigma_D = -\frac{425(0.125)}{0.3255208(10^{-3})} + \frac{736.12(-0.125)}{0.3255208(10^{-3})} = -446\ \text{kPa}$$ **Ans.**

$$\sigma_E = -\frac{425(0.125)}{0.3255208(10^{-3})} + \frac{736.12(0.125)}{0.3255208(10^{-3})} = 119\ \text{kPa}$$ **Ans.**





The negative signs indicate compressive stress.

Ans:
$\sigma_A = -119$ kPa, $\sigma_B = 446$ kPa, $\sigma_D = -446$ kPa,
$\sigma_E = 119$ kPa

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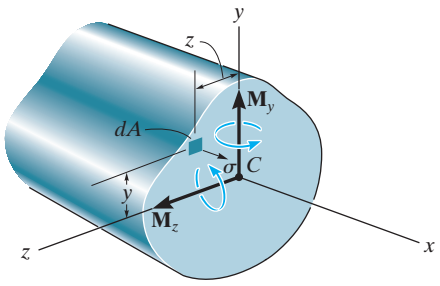
6–106.

Consider the general case of a prismatic beam subjected to bending-moment components $\mathbf{M}_y$ and $\mathbf{M}_z$ when the x, y, z axes pass through the centroid of the cross section. If the material is linear elastic, the normal stress in the beam is a linear function of position such that $\sigma = a + by + cz$. Using the equilibrium conditions $0 = \int_A \sigma\, dA$, $M_y = \int_A z\sigma\, dA$, $M_z = \int_A -y\sigma\, dA$, determine the constants a, b, and c, and show that the normal stress can be determined from the equation $\sigma = [-(M_z I_y + M_y I_{yz})y + (M_y I_z + M_z I_{yz})z]/(I_y I_z - I_{yz}{}^2)$, where the moments and products of inertia are defined in Appendix A.



SOLUTION

Equilibrium Condition: $\sigma_x = a + by + cz$

$$0 = \int_A \sigma_x\, dA$$

$$0 = \int_A (a + by + cz)\, dA$$

$$0 = a\int_A dA + b\int_A y\, dA + c\int_A z\, dA \qquad \textbf{(1)}$$

$$M_y = \int_A z\,\sigma_x\, dA$$

$$= \int_A z(a + by + cz)\, dA$$

$$= a\int_A z\, dA + b\int_A yz\, dA + c\int_A z^2\, dA \qquad \textbf{(2)}$$

$$M_z = \int_A -y\,\sigma_x\, dA$$

$$= \int_A -y(a + by + cz)\, dA$$

$$= -a\int_A y dA - b\int_A y^2\, dA - c\int_A yz\, dA \qquad \textbf{(3)}$$

Section Properties: The integrals are defined in Appendix A. Note that $\int_A y\, dA = \int_A z\, dA = 0$. Thus,

From Eq. (1) $\qquad Aa = 0$

From Eq. (2) $\qquad M_y = bI_{yz} + cI_y$

From Eq. (3) $\qquad M_z = -bI_z - cI_{yz}$

Solving for a, b, c:

$a = 0$ (Since $A \neq 0$) **Ans.**

$$b = -\left(\frac{M_z I_y + M_y I_{yz}}{I_y I_z - I_{yz}^2}\right) \qquad c = \frac{M_y I_z + M_z I_{yz}}{I_y I_z - I_{yz}^2} \qquad \textbf{Ans.}$$

Thus, $\sigma_x = -\left(\dfrac{M_z I_y + M_y I_{yz}}{I_y I_z - I_{yz}^2}\right)y + \left(\dfrac{M_y I_y + M_z I_{yz}}{I_y I_z - I_{yz}^2}\right)z$ **(Q.E.D.)**

Ans:

$$a = 0, b = -\left(\frac{M_z I_y + M_y I_{yz}}{I_y I_z - I_{yz}^2}\right), c = \frac{M_y I_z + M_z I_{yz}}{I_y I_z - I_{yz}^2}$$

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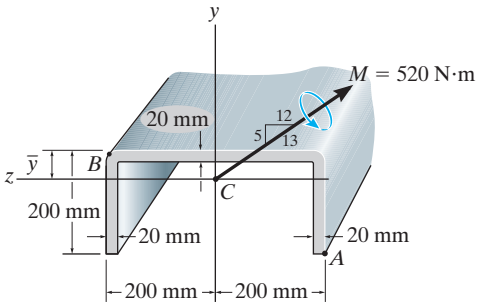
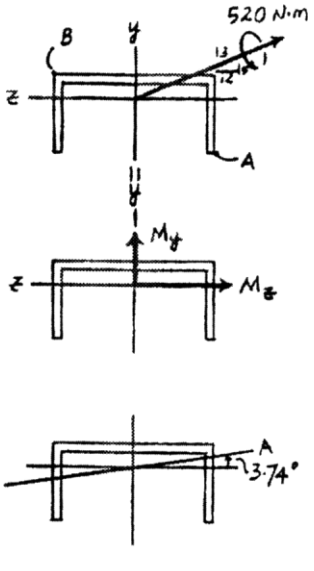
6–107. If the resultant internal moment acting on the cross section of the aluminum strut has a magnitude of $M = 520\ \text{N}\cdot\text{m}$ and is directed as shown, determine the bending stress at points A and B. The location $\bar{y}$ of the centroid C of the strut's cross-sectional area must be determined. Also, specify the orientation of the neutral axis.



Internal Moment Components:

$$M_z = -\frac{12}{13}(520) = -480\ \text{N}\cdot\text{m} \qquad M_y = \frac{5}{13}(520) = 200\ \text{N}\cdot\text{m}$$



Section Properties:

$$\bar{y} = \frac{\Sigma \bar{y} A}{\Sigma A} = \frac{0.01(0.4)(0.02) + 2[(0.110)(0.18)(0.02)]}{0.4(0.02) + 2(0.18)(0.02)}$$

$$= 0.057368\ \text{m} = 57.4\ \text{mm} \qquad \textbf{Ans.}$$

$$I_z = \frac{1}{12}(0.4)\left(0.02^3\right) + (0.4)(0.02)(0.057368 - 0.01)^2$$

$$+ \frac{1}{12}(0.04)\left(0.18^3\right) + 0.04(0.18)(0.110 - 0.057368)^2$$

$$= 57.6014\left(10^{-6}\right)\ \text{m}^4$$

$$I_y = \frac{1}{12}(0.2)\left(0.4^3\right) - \frac{1}{12}(0.18)\left(0.36^3\right) = 0.366827\left(10^{-3}\right)\ \text{m}^4$$

Maximum Bending Stress: Applying the flexure formula for biaxial at points A and B

$$\sigma = -\frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

$$\sigma_A = -\frac{-480(-0.142632)}{57.6014(10^{-6})} + \frac{200(-0.2)}{0.366827(10^{-3})}$$

$$= -1.298\ \text{MPa} = 1.30\ \text{MPa}\ (\mathbf{C}) \qquad \textbf{Ans.}$$

$$\sigma_B = -\frac{-480(0.057368)}{57.6014(10^{-6})} + \frac{200(0.2)}{0.366827(10^{-3})}$$

$$= 0.587\ \text{MPa}\ (\mathbf{T}) \qquad \textbf{Ans.}$$

Orientation of Neutral Axis:

$$\tan\alpha = \frac{I_z}{I_y}\tan\theta$$

$$\tan\alpha = \frac{57.6014(10^{-6})}{0.366827(10^{-3})}\tan(-22.62°)$$

$$\alpha = -3.74° \qquad \textbf{Ans.}$$

Ans:

$\bar{y} = 57.4$ mm, $\sigma_A = 1.30$ MPa (C),
$\sigma_B = 0.587$ MPa (T),
$\alpha = -3.74°$

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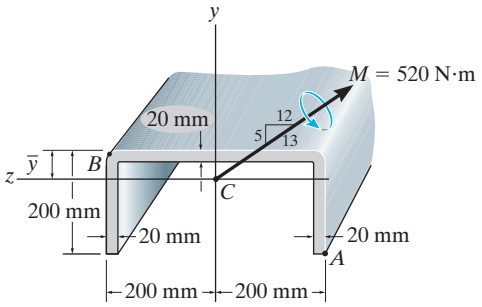
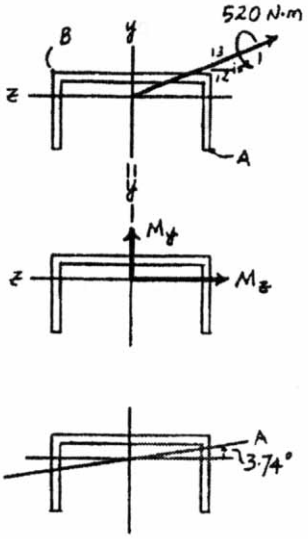
***6–108.** The resultant internal moment acting on the cross section of the aluminum strut has a magnitude of $M = 520\ \text{N}\cdot\text{m}$ and is directed as shown. Determine maximum bending stress in the strut. The location $\bar{y}$ of the centroid C of the strut's cross-sectional area must be determined. Also, specify the orientation of the neutral axis.



Internal Moment Components:

$$M_z = -\frac{12}{13}(520) = -480\ \text{N}\cdot\text{m} \qquad M_y = \frac{5}{13}(520) = 200\ \text{N}\cdot\text{m}$$



Section Properties:

$$\bar{y} = \frac{\Sigma \bar{y} A}{\Sigma A} = \frac{0.01(0.4)(0.02) + 2[(0.110)(0.18)(0.02)]}{0.4(0.02) + 2(0.18)(0.02)}$$

$$= 0.057368\ \text{m} = 57.4\ \text{mm} \qquad \textbf{Ans.}$$

$$I_z = \frac{1}{12}(0.4)\left(0.02^3\right) + (0.4)(0.02)(0.057368 - 0.01)^2$$

$$+ \frac{1}{12}(0.04)\left(0.18^3\right) + 0.04(0.18)(0.110 - 0.057368)^2$$

$$= 57.6014\left(10^{-6}\right)\text{m}^4$$

$$I_y = \frac{1}{12}(0.2)\left(0.4^3\right) - \frac{1}{12}(0.18)\left(0.36^3\right) = 0.366827\left(10^{-3}\right)\text{m}^4$$

Maximum Bending Stress: By inspection, the maximum bending stress can occur at either point A or B. Applying the flexure formula for biaxial bending at points A and B

$$\sigma = -\frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

$$\sigma_A = -\frac{-480(-0.142632)}{57.6014(10^{-6})} + \frac{200(-0.2)}{0.366827(10^{-3})}$$

$$= -1.298\ \text{MPa} = 1.30\ \text{MPa (C) (Max)} \qquad \textbf{Ans.}$$

$$\sigma_B = -\frac{-480(0.057368)}{57.6014(10^{-6})} + \frac{200(0.2)}{0.366827(10^{-3})}$$

$$= 0.587\ \text{MPa (T)}$$

Orientation of Neutral Axis:

$$\tan\alpha = \frac{I_z}{I_y}\tan\theta$$

$$\tan\alpha = \frac{57.6014(10^{-6})}{0.366827(10^{-3})}\tan(-22.62°)$$

$$\alpha = -3.74° \qquad \textbf{Ans.}$$

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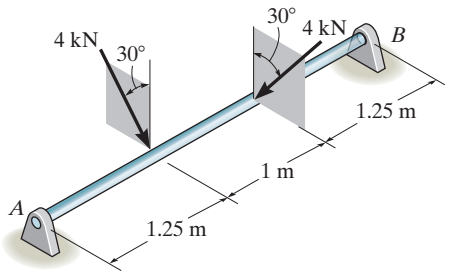
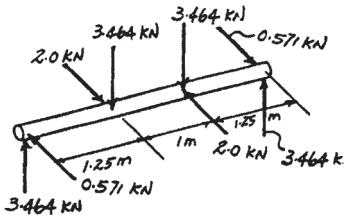
6–109.

The steel shaft is subjected to the two loads. If the journal bearings at A and B do not exert an axial force on the shaft, determine the required diameter of the shaft if the allowable bending stress is $\sigma_{\text{allow}} = 180$ MPa.

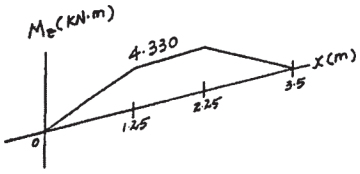


SOLUTION

Support Reactions: As shown on FBD.



Internal Moment Components: The shaft is subjected to two bending moment components M_y and M_z. The moment diagram for each component is drawn.

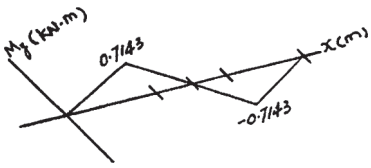


Allowable Bending Stress: Since all the axes through the circle's center for a circular shaft are principal axes, then the resultant moment $M = \sqrt{M_y^2 + M_z^2}$ can be used for the design. The maximum resultant moment is $M_{\max} = \sqrt{4.330^2 + 0.7143^2} = 4.389\ \text{kN} \cdot \text{m}$. Applying the flexure formula,

$$\sigma_{\max} = \sigma_{\text{allow}} = \frac{M_{\max} c}{I}$$

$$180(10^6) = \frac{4.389(10^3)\left(\frac{d}{2}\right)}{\frac{\pi}{4}\left(\frac{d}{2}\right)^4}$$

$$d = 0.06286\ \text{m} = 62.9\ \text{mm} \qquad \textbf{Ans.}$$



Ans:
$d = 62.9$ mm

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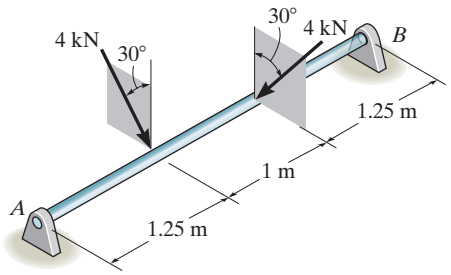
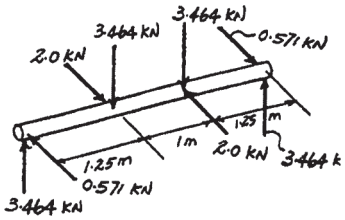
6–110.

The 65-mm-diameter steel shaft is subjected to the two loads. If the journal bearings at A and B do not exert an axial force on the shaft, determine the absolute maximum bending stress developed in the shaft.

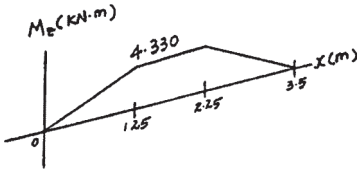


SOLUTION

Support Reactions: As shown on FBD.



Internal Moment Components: The shaft is subjected to two bending moment components M_y and M_z. The moment diagram for each component is drawn.

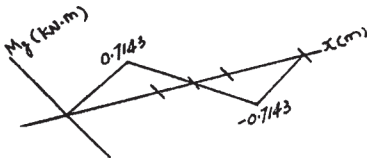


Maximum Bending Stress: Since all the axes through the circle's center for a circular shaft are principal axes, then the resultant moment $M = \sqrt{M_y^2 + M_z^2}$ can be used to determine the maximum bending stress. The maximum resultant moment is $M_{\max} = \sqrt{4.330^2 + 0.7143^2} = 4.389 \text{ kN} \cdot \text{m}$. Applying the flexure formula,

$$\sigma_{\max} = \frac{M_{\max} c}{I}$$

$$= \frac{4.389\left(10^3\right)(0.0325)}{\frac{\pi}{4}\left(0.0325^4\right)}$$

$$= 163 \text{ MPa} \qquad \textbf{Ans.}$$



Ans:
$\sigma_{\max} = 163$ MPa

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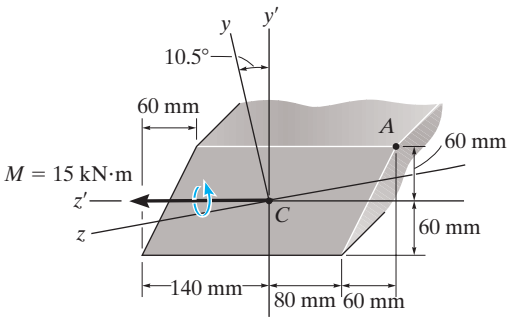
6–111.

For the section, $I_{z'} = 31.7(10^{-6})\ \text{m}^4$, $I_{y'} = 114(10^{-6})\ \text{m}^4$, $I_{y'z'} = -15.8(10^{-6})\ \text{m}^4$. Using the techniques outlined in Appendix A, the member's cross-sectional area has principal moments of inertia of $I_z = 28.8(10^{-6})\ \text{m}^4$ and $I_y = 117(10^{-6})\ \text{m}^4$, calculated about the principal axes of inertia y and z, respectively. If the section is subjected to the moment $M = 15\ \text{kN} \cdot \text{m}$, determine the stress at point A using Eq. 6–17.



SOLUTION

Internal Moment Components: The y and z components are

$$M_y = 15 \sin 10.5° = 2.7335\ \text{kN} \cdot \text{m} \qquad M_z = 15 \cos 10.5° = 14.7488\ \text{kN} \cdot \text{m}$$

Section Properties: Given that $I_y = 117(10^{-6})\ \text{m}^4$ and $I_z = 28.8(10^{-6})\ \text{m}^4$, the coordinates of point A with respect to the y and z axes are

$$y = 0.06 \cos 10.5° - 0.14 \sin 10.5° = 0.03348\ \text{m}$$

$$z = -(0.06 \sin 10.5° + 0.14 \cos 10.5°) = -0.14859\ \text{m}$$

Bending Stress: Applying the flexure formula for biaxial bending,

$$\sigma = -\frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

$$\sigma_A = -\frac{14.7488(10^3)(0.03348)}{28.8(10^{-6})} + \frac{2.7335(10^3)(-0.14869)}{117(10^{-6})}$$

$$= -20.62(10^6)\ \text{Pa}$$

$$= 20.6\ \text{MPa (C)} \qquad \textbf{Ans.}$$

Ans:
$\sigma_A = 20.6\ \text{MPa (C)}$

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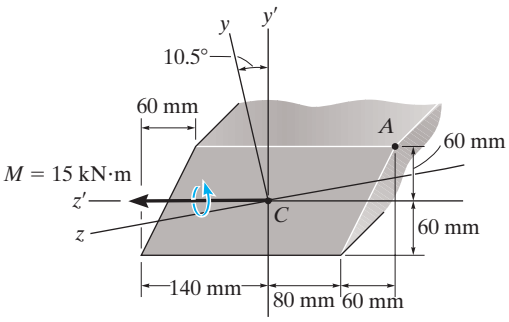
***6–112.**

Solve Prob. 6–111 using the equation developed in Prob. 6–106.



SOLUTION

Internal Moment Components: The y' and z' components are

$$M_{z'} = 15\ \text{kN}\cdot\text{m} \qquad M_{y'} = 0$$

Section Properties: Given that $I_{y'} = 114(10^{-6})\ \text{m}^4$, $I_{z'} = 31.7(10^{-6})\ \text{m}^4$ and $I_{y'z'} = -15.8(10^{-6})\ \text{m}^4$, the coordinates of point A with respect to the y' and z axes are

$$y' = 0.06\ \text{m} \qquad z' = -0.14\ \text{m}$$

Bending Stress: Using the formula,

$$\sigma = \frac{-(M_{z'}I_{y'} + M_{y'}I_{y'z'})y' + (M_{y'}I_{z'} + M_{z'}I_{y'z'})z'}{I_{y'}I_{z'} - I_{y'z'}^2}$$

$$\sigma_A = \frac{-\left[15(10^3)(114)(10^{-6}) + 0\right](0.06) + \left[0 + 15(10^3)(-15.8)(10^{-6})\right](-0.14)}{\left[114(10^{-6})\right]\left[31.7(10^{-6})\right] - \left[-15.8(10^{-6})\right]^2}$$

$$= -20.64(10^6)\ \text{Pa}$$

$$= 20.6\ \text{MPa (C)} \qquad \textbf{Ans.}$$

Ans:
$\sigma_A = 20.6\ \text{MPa (C)}$

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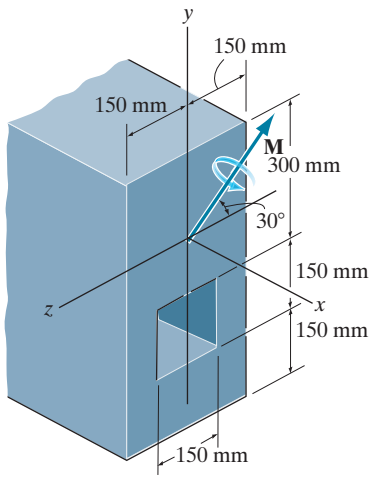
6–113. If the beam is subjected to the internal moment of $M = 1200$ kN · m, determine the maximum bending stress acting on the beam and the orientation of the neutral axis.



Internal Moment Components: The y component of M is positive since it is directed towards the positive sense of the y axis, whereas the z component of **M**, which is directed towards the negative sense of the z axis, is negative, Fig. *a*. Thus,

$$M_y = 1200 \sin 30° = 600 \text{ kN} \cdot \text{m}$$

$$M_z = -1200 \cos 30° = -1039.23 \text{ kN} \cdot \text{m}$$

Section Properties: The location of the centroid of the cross-section is given by

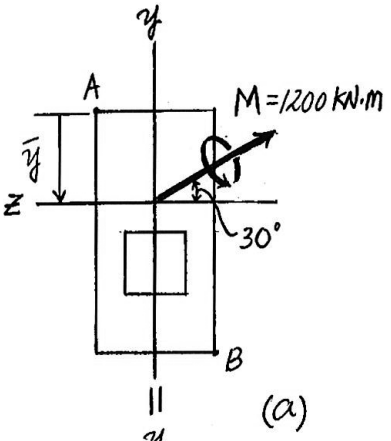
$$\bar{y} = \frac{\Sigma \bar{y} A}{\Sigma A} = \frac{0.3(0.6)(0.3) - 0.375(0.15)(0.15)}{0.6(0.3) - 0.15(0.15)} = 0.2893 \text{ m}$$

The moments of inertia of the cross section about the principal centroidal y and z axes are

$$I_y = \frac{1}{12}(0.6)\left(0.3^3\right) - \frac{1}{12}(0.15)\left(0.15^3\right) = 1.3078\left(10^{-3}\right) \text{m}^4$$

$$I_z = \frac{1}{12}(0.3)\left(0.6^3\right) + 0.3(0.6)(0.3 - 0.2893)^2 - \left[\frac{1}{12}(0.15)\left(0.15^3\right) + 0.15(0.15)(0.375 - 0.2893)^2\right]$$

$$= 5.2132\left(10^{-3}\right)\text{m}^4$$



(a)

Bending Stress: By inspection, the maximum bending stress occurs at either corner A or B.

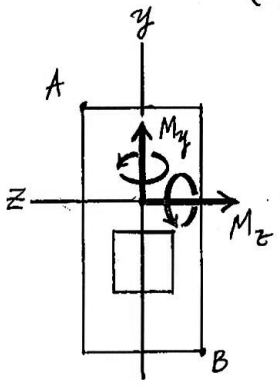
$$\sigma = -\frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

$$\sigma_A = -\frac{\left[-1039.23\left(10^3\right)\right](0.2893)}{5.2132\left(10^{-3}\right)} + \frac{600\left(10^3\right)(0.15)}{1.3078\left(10^{-3}\right)}$$

$$= 126 \text{ MPa (T)}$$

$$\sigma_B = -\frac{\left[-1039.23\left(10^3\right)\right](-0.3107)}{5.2132\left(10^{-3}\right)} + \frac{600\left(10^3\right)(-0.15)}{1.3078\left(10^{-3}\right)}$$

$$= -131 \text{ MPa} = 131 \text{ MPa (C)(Max.)} \quad \textbf{Ans.}$$

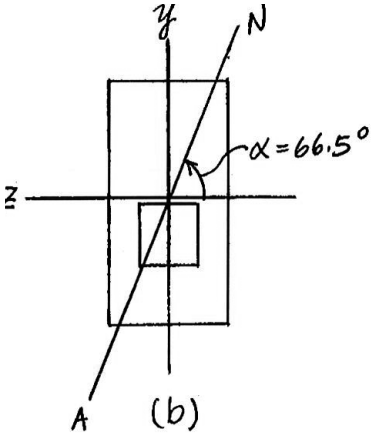


Orientation of Neutral Axis: Here, $\theta = -30°$.

$$\tan \alpha = \frac{I_z}{I_y} \tan \theta$$

$$\tan \alpha = \frac{5.2132\left(10^{-3}\right)}{1.3078\left(10^{-3}\right)} \tan(-30°)$$

$$\alpha = -66.5° \quad \textbf{Ans.}$$



(b)

The orientation of the neutral axis is shown in Fig. *b*.

Ans:
$\sigma_B = 131$ MPa (C), $\alpha = -66.5°$

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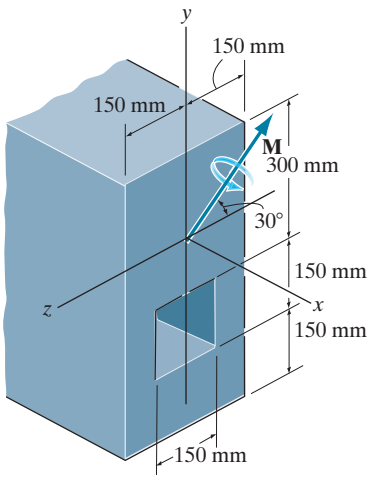
6–114. If the beam is made from a material having an allowable tensile and compressive stress of $(\sigma_{\text{allow}})_t = 125$ MPa and $(\sigma_{\text{allow}})_c = 150$ MPa, respectively, determine the maximum allowable internal moment **M** that can be applied to the beam.



Internal Moment Components: The y component of **M** is positive since it is directed towards the positive sense of the y axis, whereas the z component of **M**, which is directed towards the negative sense of the z axis, is negative, Fig. *a*. Thus,

$$M_y = M \sin 30° = 0.5M$$

$$M_z = -M \cos 30° = -0.8660M$$

Section Properties: The location of the centroid of the cross section is

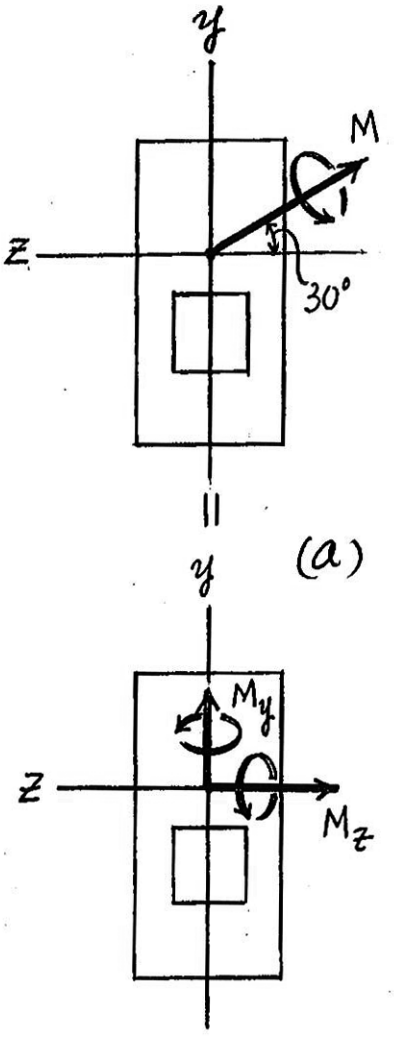
$$\bar{y} = \frac{\Sigma \bar{y} A}{\Sigma A} = \frac{0.3(0.6)(0.3) - 0.375(0.15)(0.15)}{0.6(0.3) - 0.15(0.15)} = 0.2893 \text{ m}$$

The moments of inertia of the cross section about the principal centroidal y and z axes are

$$I_y = \frac{1}{12}(0.6)\left(0.3^3\right) - \frac{1}{12}(0.15)\left(0.15^3\right) = 1.3078\left(10^{-3}\right) \text{m}^4$$

$$I_z = \frac{1}{12}(0.3)\left(0.6^3\right) + 0.3(0.6)(0.3 - 0.2893)^2 - \left[\frac{1}{12}(0.15)\left(0.15^3\right) + 0.15(0.15)(0.375 - 0.2893)^2\right]$$

$$= 5.2132\left(10^{-3}\right)\text{m}^4$$



Bending Stress: By inspection, the maximum bending stress can occur at either corner A or B. For corner A which is in tension,

$$\sigma_A = (\sigma_{\text{allow}})_t = -\frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y}$$

$$125\left(10^6\right) = -\frac{(-0.8660M)(0.2893)}{5.2132\left(10^{-3}\right)} + \frac{0.5M(0.15)}{1.3078\left(10^{-3}\right)}$$

$$M = 1185\,906.82 \text{ N} \cdot \text{m} = 1186 \text{ kN} \cdot \text{m} \text{ (controls)} \qquad \textbf{Ans.}$$

For corner B which is in compression,

$$\sigma_B = (\sigma_{\text{allow}})_c = -\frac{M_z y_B}{I_z} + \frac{M_y z_B}{I_y}$$

$$-150\left(10^6\right) = -\frac{(-0.8660M)(-0.3107)}{5.2132\left(10^{-3}\right)} + \frac{0.5M(-0.15)}{1.3078\left(10^{-3}\right)}$$

$$M = 1376\,597.12 \text{ N} \cdot \text{m} = 1377 \text{ kN} \cdot \text{m}$$

Ans:

$M = 1186 \text{ kN} \cdot \text{m}$

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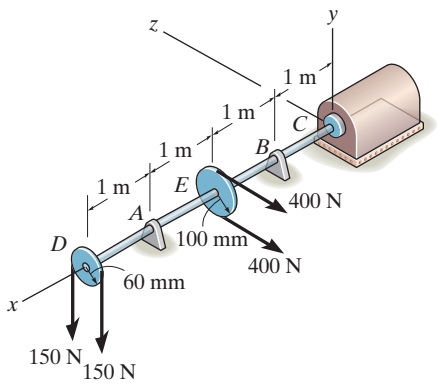
6–115.

The shaft is subjected to the vertical and horizontal loadings of two pulleys D and E as shown. It is supported on two journal bearings at A and B which offer no resistance to axial loading. Furthermore, the coupling to the motor at C can be assumed not to offer any support to the shaft. Determine the required diameter d of the shaft if the allowable bending stress is $\sigma_{allow} = 180$ MPa.



SOLUTION

Support Reactions: As shown on FBD.

Internal Moment Components: The shaft is subjected to two bending moment components M_y and M_z.

Allowable Bending Stress: Since all the axes through the circle's center for a circular shaft are principal axes, then the resultant moment $M = \sqrt{M_y^2 + M_z^2}$ can be used for the design. The maximum resultant moment is $M_{max} = \sqrt{400^2 + 150^2} = 427.2\ \text{N} \cdot \text{m}$. Applying the flexure formula

$$\sigma_{max} = \sigma_{allow} = \frac{M_{max} c}{I}$$

$$180(10^6) = \frac{427.2\left(\frac{d}{2}\right)}{\frac{\pi}{4}\left(\frac{d}{2}\right)^4}$$

$$d = 0.02891\ \text{m} = 28.9\ \text{mm} \qquad \textbf{Ans.}$$

Ans:
$d = 28.9$ mm

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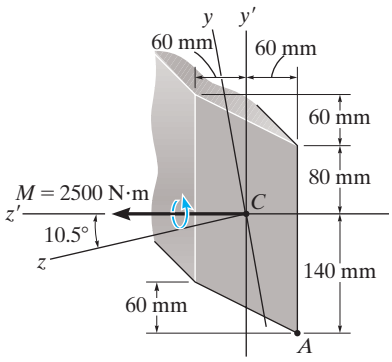
***6–116.**

For the section, $I_{y'} = 31.7(10^{-6})\,\text{m}^4$, $I_{z'} = 114(10^{-6})\,\text{m}^4$, $I_{y'z'} = 15.8(10^{-6})\,\text{m}^4$. Using the techniques outlined in Appendix A, the member's cross-sectional area has principal moments of inertia of $I_y = 28.8(10^{-6})\,\text{m}^4$ and $I_z = 117(10^{-6})\,\text{m}^4$, calculated about the principal axes of inertia y and z, respectively. If the section is subjected to a moment of $M = 2500\,\text{N}\cdot\text{m}$, determine the stress produced at point A, using Eq. 6–17.



SOLUTION

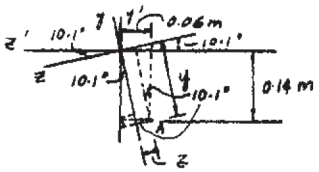
$I_z = 117(10^{-6})\,\text{m}^4 \qquad I_y = 28.8(10^{-6})\,\text{m}^4$

$M_y = 2500 \sin 10.5° = 455.59\,\text{N}\cdot\text{m}$

$M_z = 2500 \cos 10.5° = 2458.14\,\text{N}\cdot\text{m}$

$y = -0.06 \sin 10.5° - 0.14 \cos 10.5° = -0.14859\,\text{m}$

$z = 0.14 \sin 10.5° - 0.06 \cos 10.5° = -0.03348\,\text{m}$



$$\sigma_A = \frac{-M_z y}{I_z} + \frac{M_y z}{I_y}$$

$$= \frac{2458.14(-0.14859)}{117(10^{-6})} + \frac{455.59(-0.03348)}{28.8(10^{-6})} = 2.59\,\text{MPa (T)} \qquad \textbf{Ans.}$$

Ans:
$\sigma_A = 2.59\,\text{MPa (T)}$

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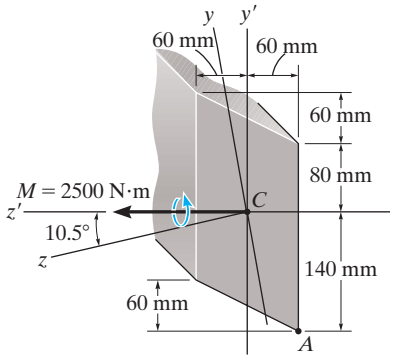
6–117.

Solve Prob. 6–116 using the equation developed in Prob. 6–106.



SOLUTION

$$\sigma_A = \frac{-(M_{z'}I_{y'} + M_{y'}I_{y'z'})y' + (M_{y'}I_{z'} + M_{z'}I_{y'z'})z'}{I_{y'}I_{z'} - I_{y'z'}^2}$$

$$= \frac{-[2500(31.7)(10^{-6}) + 0](-0.14) + [0 + 2500(15.8)(10^{-6})](-0.06)}{31.7(10^{-6})(114)(10^{-6}) - [(15.8)(10^{-6})]^2}$$

$$= 2.59 \text{ MPa (T)} \qquad \textbf{Ans.}$$

Ans:
$\sigma_A = 2.59$ MPa (T)

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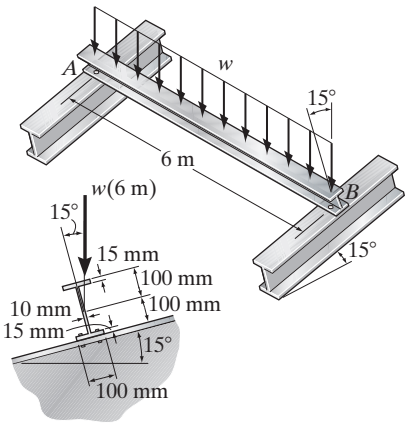
6–118.

If the applied distributed loading of $w = 4$ kN/m can be assumed to pass through the centroid of the beam's cross-sectional area, determine the absolute maximum bending stress in the joist and the orientation of the neutral axis. The beam can be considered simply supported at A and B.



SOLUTION

Internal Moment Components: The uniform distributed load **w** can be resolved into its y and z components as shown in Fig. a.

$$w_y = 4\cos 15° = 3.864 \text{ kN/m}$$

$$w_z = 4\sin 15° = 1.035 \text{ kN/m}$$

$\mathbf{w}_y$ and $\mathbf{w}_z$ produce internal moments in the beam about the z and y axes, respectively. For the simply supported beam subjected to the uniform distributed load, the maximum moment in the beam is $M_{\max} = \dfrac{wL^2}{8}$. Thus,

$$(M_z)_{\max} = \frac{w_y L^2}{8} = \frac{3.864(6^2)}{8} = 17.387 \text{ kN}\cdot\text{m}$$

$$(M_y)_{\max} = \frac{w_z L^2}{8} = \frac{1.035(6^2)}{8} = 4.659 \text{ kN}\cdot\text{m}$$

As shown in Fig. b, $(\mathbf{M}_z)_{\max}$ and $(\mathbf{M}_y)_{\max}$ are positive since they are directed towards the positive sense of their respective axes.

Section Properties: The moments of inertia of the cross section about the principal centroidal y and z axes are

$$I_y = 2\left[\frac{1}{12}(0.015)(0.1^3)\right] + \frac{1}{12}(0.17)(0.01^3) = 2.5142(10^{-6}) \text{ m}^4$$

$$I_z = \frac{1}{12}(0.1)(0.2^3) - \frac{1}{12}(0.09)(0.17^3) = 29.8192(10^{-6}) \text{ m}^4$$

Bending Stress: By inspection, the maximum bending stress occurs at points A and B.

$$\sigma = -\frac{(M_z)_{\max} y}{I_z} + \frac{(M_y)_{\max} z}{I_y}$$

$$\sigma_{\max} = \sigma_A = -\frac{17.387(10^3)(-0.1)}{29.8192\,(10^{-6})} + \frac{4.659(10^3)(0.05)}{2.5142(10^{-6})}$$

$$= 150.96 \text{ MPa} = 151 \text{ MPa (T)} \qquad \textbf{Ans.}$$

$$\sigma_{\max} = \sigma_B = -\frac{17.387(10^3)(0.1)}{29.8192\,(10^{-6})} + \frac{4.659(10^3)(-0.05)}{2.5142(10^{-6})}$$

$$= -150.96 \text{ MPa} = 151 \text{ MPa (C)} \qquad \textbf{Ans.}$$

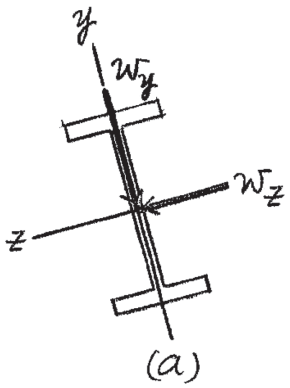
Orientation of Neutral Axis: Here, $\theta = \tan^{-1}\left[\dfrac{(M_y)_{\max}}{(M_z)_{\max}}\right] = \tan^{-1}\left(\dfrac{4.659}{17.387}\right) = 15°$.

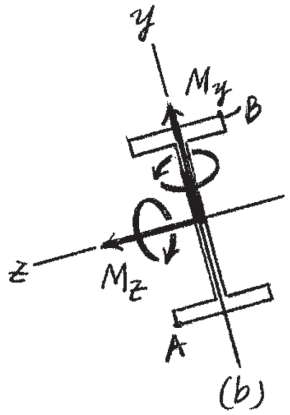
$$\tan\alpha = \frac{I_z}{I_y}\tan\theta$$

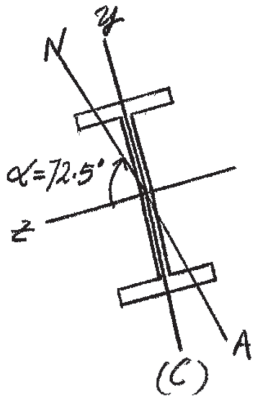
$$\tan\alpha = \frac{29.8192(10^{-6})}{2.5142(10^{-6})}\tan 15°$$

$$\alpha = 72.5° \qquad \textbf{Ans.}$$

The orientation of the neutral axis is shown in Fig. c.







Ans:
$\sigma_{\max} = 151$ MPa, $\alpha = 72.5°$

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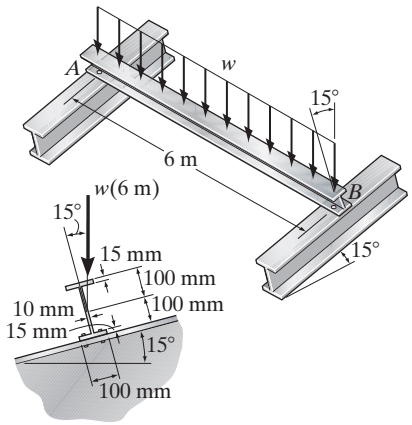
6–119.

Determine the maximum allowable intensity w of the uniform distributed load that can be applied to the beam. Assume w passes through the centroid of the beam's cross-sectional area, and the beam is simply supported at A and B. The allowable bending stress is $\sigma_{\text{allow}} = 165$ MPa.



SOLUTION

Internal Moment Components: The uniform distributed load **w** can be resolved into its y and z components as shown in Fig. a.

$$w_y = w\cos 15° = 0.9659w$$

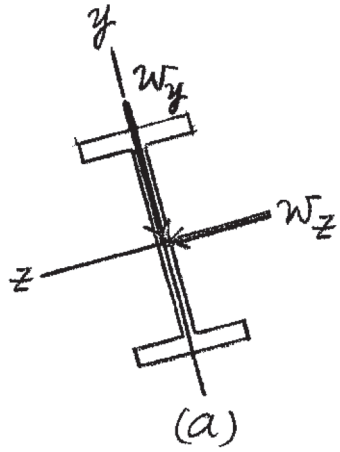
$$w_z = w\sin 15° = 0.2588w$$

$\mathbf{w}_y$ and $\mathbf{w}_z$ produce internal moments in the beam about the z and y axes, respectively. For the simply supported beam subjected to the a uniform distributed load, the maximum moment in the beam is $M_{\max} = \dfrac{wL^2}{8}$. Thus,

$$(M_z)_{\max} = \frac{w_y L^2}{8} = \frac{0.9659w(6^2)}{8} = 4.3476w$$

$$(M_y)_{\max} = \frac{w_z L^2}{8} = \frac{0.2588w(6^2)}{8} = 1.1647w$$

As shown in Fig. b, $(\mathbf{M}_z)_{\max}$ and $(\mathbf{M}_y)_{\max}$ are positive since they are directed towards the positive sense of their respective axes.



Section Properties: The moments of inertia of the cross section about the principal centroidal y and z axes are

$$I_y = 2\left[\frac{1}{12}(0.015)(0.1^3)\right] + \frac{1}{12}(0.17)(0.01^3) = 2.5142(10^{-6})\ \text{m}^4$$

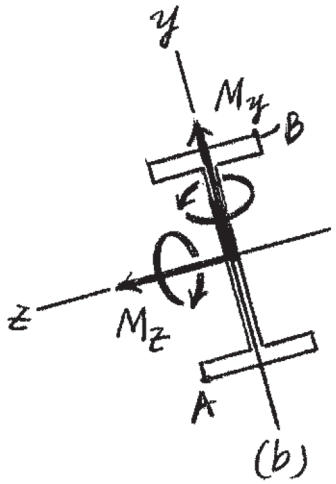
$$I_z = \frac{1}{12}(0.1)(0.2^3) - \frac{1}{12}(0.09)(0.17^3) = 29.8192(10^{-6})\ \text{m}^4$$

Bending Stress: By inspection, the maximum bending stress occurs at points A and B. We will consider point A.

$$\sigma_A = \sigma_{\text{allow}} = -\frac{(M_z)_{\max} y_A}{I_z} + \frac{(M_y)_{\max} z_A}{I_y}$$

$$165(10^6) = -\frac{4.3467w(-0.1)}{29.8192(10^{-6})} + \frac{1.1647w(0.05)}{2.5142(10^{-6})}$$

$$w = 4372.11\ \text{N/m} = 4.37\ \text{kN/m} \qquad \textbf{Ans.}$$



Ans:
$w = 4.37$ kN/m

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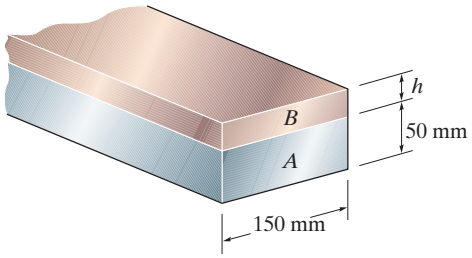
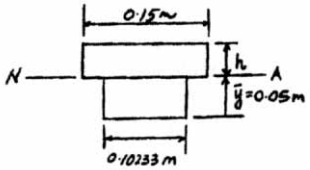
***6–120.** The composite beam is made of 6061-T6 aluminum (A) and C83400 red brass (B). Determine the dimension h of the brass strip so that the neutral axis of the beam is located at the seam of the two metals. What maximum moment will this beam support if the allowable bending stress for the aluminum is $(\sigma_{\text{allow}})_{\text{al}} = 128$ MPa and for the brass $(\sigma_{\text{allow}})_{\text{br}} = 35$ MPa?





Section Properties:

$$n = \frac{E_{\text{al}}}{E_{\text{br}}} = \frac{68.9(10^9)}{101(10^9)} = 0.68218$$

$$b_{\text{br}} = nb_{\text{al}} = 0.68218(0.15) = 0.10233\ \text{m}$$

$$\bar{y} = \frac{\Sigma \bar{y} A}{\Sigma A}$$

$$0.05 = \frac{0.025(0.10233)(0.05) + (0.05 + 0.5h)(0.15)h}{0.10233(0.05) + (0.15)h}$$

$$h = 0.04130\ \text{m} = 41.3\ \text{mm} \qquad \textbf{Ans.}$$

$$I_{NA} = \frac{1}{12}(0.10233)\left(0.05^3\right) + 0.10233(0.05)(0.05 - 0.025)^2$$

$$+ \frac{1}{12}(0.15)\left(0.04130^3\right) + 0.15(0.04130)(0.070649 - 0.05)^2$$

$$= 7.7851\left(10^{-6}\right)\ \text{m}^4$$

Allowable Bending Stress: Applying the flexure formula

Assume failure of red brass

$$(\sigma_{\text{allow}})_{\text{br}} = \frac{Mc}{I_{NA}}$$

$$35\left(10^6\right) = \frac{M(0.04130)}{7.7851(10^{-6})}$$

$$M = 6598\ \text{N} \cdot \text{m} = 6.60\ \text{kN} \cdot \text{m}\ (\textit{controls!}) \qquad \textbf{Ans.}$$

Assume failure of aluminium

$$(\sigma_{\text{allow}})_{\text{al}} = n\frac{Mc}{I_{NA}}$$

$$128\left(10^6\right) = 0.68218\left[\frac{M(0.05)}{7.7851(10^{-6})}\right]$$

$$M = 29215\ \text{N} \cdot \text{m} = 29.2\ \text{kN} \cdot \text{m}$$

Ans:
$h = 41.3$ mm,
$M = 6.60\ \text{kN} \cdot \text{m}$ (*controls!*)

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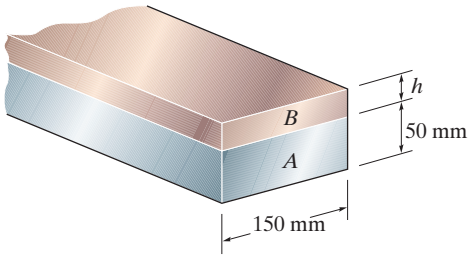
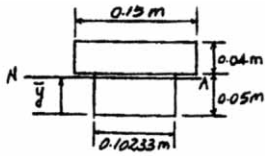
6–121. The composite beam is made of 6061-T6 aluminum (A) and C83400 red brass (B). If the height $h = 40$ mm, determine the maximum moment that can be applied to the beam if the allowable bending stress for the aluminum is $(\sigma_{allow})_{al} = 128$ MPa and for the brass $(\sigma_{allow})_{br} = 35$ MPa.





Section Properties: For transformed section.

$$n = \frac{E_{al}}{E_{br}} = \frac{68.9(10^9)}{101.0(10^9)} = 0.68218$$

$$b_{br} = nb_{al} = 0.68218(0.15) = 0.10233 \text{ m}$$

$$\bar{y} = \frac{\Sigma \bar{y} A}{\Sigma A}$$

$$= \frac{0.025(0.10233)(0.05) + (0.07)(0.15)(0.04)}{0.10233(0.05) + 0.15(0.04)}$$

$$= 0.049289 \text{ m}$$

$$I_{NA} = \frac{1}{12}(0.10233)\left(0.05^3\right) + 0.10233(0.05)(0.049289 - 0.025)^2$$

$$+ \frac{1}{12}(0.15)\left(0.04^3\right) + 0.15(0.04)(0.07 - 0.049289)^2$$

$$= 7.45799\left(10^{-6}\right) \text{ m}^4$$

Allowable Bending Stress: Applying the flexure formula

Assume failure of red brass

$$(\sigma_{allow})_{br} = \frac{Mc}{I_{NA}}$$

$$35\left(10^6\right) = \frac{M(0.09 - 0.049289)}{7.45799(10^{-6})}$$

$$M = 6412 \text{ N} \cdot \text{m} = 6.41 \text{ kN} \cdot \text{m} \text{ (}controls!\text{)}$$ **Ans.**

Assume failure of aluminium

$$(\sigma_{allow})_{al} = n\frac{Mc}{I_{NA}}$$

$$128\left(10^6\right) = 0.68218\left[\frac{M(0.049289)}{7.45799(10^{-6})}\right]$$

$$M = 28391 \text{ N} \cdot \text{m} = 28.4 \text{ kN} \cdot \text{m}$$

Ans:

$M = 6.41 \text{ kN} \cdot \text{m}$

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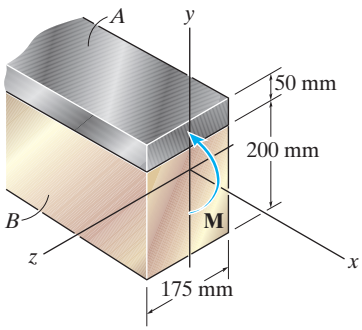
6–122.

The composite beam is made of steel (A) bonded to brass (B) and has the cross section shown. If it is subjected to a moment of $M = 6.5\ \text{kN}\cdot\text{m}$, determine the maximum bending stress in the brass and steel. Also, what is the stress in each material at the seam where they are bonded together? $E_{\text{br}} = 100\ \text{GPa}$, $E_{\text{st}} = 200\ \text{GPa}$.



SOLUTION

$$n = \frac{E_{\text{st}}}{E_{\text{br}}} = \frac{200(10^9)}{100(10^9)} = 2$$

$$\bar{y} = \frac{(350)(50)(25) + (175)(200)(150)}{350(50) + 175(200)} = 108.33\,\text{mm}$$

$$I = \frac{1}{12}(0.35)(0.05^3) + (0.35)(0.05)(0.08333^2) + \frac{1}{12}(0.175)(0.2^3) + (0.175)(0.2)(0.04167^2) = 0.3026042(10^{-3})\ \text{m}^4$$

Maximum stress in brass:

$$(\sigma_{\text{br}})_{\max} = \frac{Mc_1}{I} = \frac{6.5(10^3)(0.14167)}{0.3026042(10^{-3})} = 3.04\ \text{MPa} \qquad \textbf{Ans.}$$

Maximum stress in steel:

$$(\sigma_{\text{st}})_{\max} = \frac{nMc_2}{I} = \frac{(2)(6.5)(10^3)(0.10833)}{0.3026042(10^{-3})} = 4.65\ \text{MPa} \qquad \textbf{Ans.}$$

Stress at the junction:

$$\sigma_{\text{br}} = \frac{M\rho}{I} = \frac{6.5(10^3)(0.05833)}{0.3026042(10^{-3})} = 1.25\ \text{MPa} \qquad \textbf{Ans.}$$

$$\sigma_{\text{st}} = n\sigma_{\text{br}} = 2(1.25) = 2.51\ \text{MPa} \qquad \textbf{Ans.}$$

Ans:
$(\sigma_{\text{br}})_{\max} = 3.04\ \text{MPa}$,
$(\sigma_{\text{st}})_{\max} = 4.65\ \text{MPa}$,
$\sigma_{\text{br}} = 1.25\ \text{MPa}$,
$\sigma_{\text{st}} = 2.51\ \text{MPa}$

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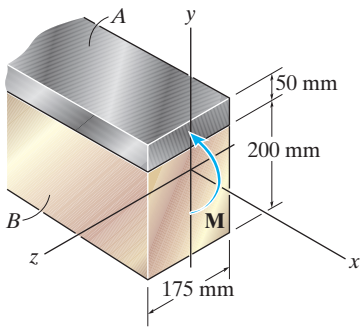
6–123.

The composite beam is made of steel (A) bonded to brass (B) and has the cross section shown. If the allowable bending stress for the steel is $(\sigma_{allow})_{st} = 180$ MPa, and for the brass $(\sigma_{allow})_{br} = 60$ MPa, determine the maximum moment M that can be applied to the beam. $E_{br} = 100$ GPa, $E_{st} = 200$ GPa.



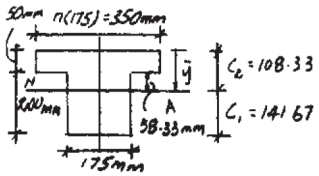
SOLUTION

$$n = \frac{E_{st}}{E_{br}} = \frac{200(10^9)}{100(10^9)} = 2$$

$$\bar{y} = \frac{(350)(50)(25) + (175)(200)(150)}{350(50) + 175(200)} = 108.33 \text{ mm}$$

$$I = \frac{1}{12}(0.35)(0.05^3) + (0.35)(0.05)(0.08333^2) + \frac{1}{12}(0.175)(0.2^3) +$$

$$(0.175)(0.2)(0.04167^2) = 0.3026042(10^{-3}) \text{ m}^4$$



$$(\sigma_{st})_{allow} = \frac{nMc_2}{I}; \qquad 180(10^6) = \frac{(2)\,M(0.10833)}{0.3026042(10^{-3})}$$

$$M = 251 \text{ kN} \cdot \text{m}$$

$$(\sigma_{br})_{allow} = \frac{Mc_1}{I}; \qquad 60(10^6) = \frac{M(0.14167)}{0.3026042(10^{-3})}$$

$$M = 128 \text{ kN} \cdot \text{m} \text{ (controls)} \qquad \textbf{Ans.}$$

Ans:
$M = 128 \text{ kN} \cdot \text{m}$

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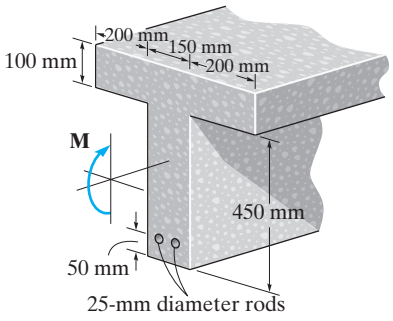
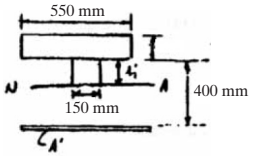
***6–124.** The reinforced concrete beam is made using two steel reinforcing rods. If the allowable tensile stress for for the steel is $(\sigma_{st})_{allow} = 280$ MPa and the allowable compressive stress for the concrete is $(\sigma_{conc})_{allow} = 21$ MPa, determine the maximum moment M that can be applied to the section. Assume the concrete cannot support a tensile stress. $E_{st} = 200$ GPa, $E_{conc} = 26.5$ GPa.



$$A_{st} = 2[\pi(0.0125^2)] = 0.3125\pi(10^{-3})\ \text{m}^2$$



$$A' = nA_{st} = \left(\frac{200}{26.5}\right)[0.3125\pi(10^{-3})] = 7.4094(10^{-3})\ \text{m}^2$$

$$\Sigma \bar{y}A = 0; \qquad (h' + 0.05)(0.55)(0.1) + \left(\frac{h'}{2}\right)(0.15)h' - 7.4094(10^{-3})(0.4 - h') = 0$$

$$0.075h'^2 + 0.062409h' - 0.21377(10^{-3}) = 0$$

Solving for the positive root:

$$h' = 0.0034112\ \text{m}$$

$$I = \frac{1}{12}(0.55)(0.1^3) + (0.55)(0.1)(0.0034112 + 0.05)^2$$

$$+ \frac{1}{12}(0.15)(0.0034112^3) + 0.15(0.0034112)(0.0034112/2)^2$$

$$+ 7.4094(10^{-3})(0.4 - 0.0034112)^2$$

$$= 1.36811(10^{-3})\ \text{m}^4$$

Assume concrete fails:

$$(\sigma_{con})_{allow} = \frac{My}{I}; \qquad 21(10^6) = \left(\frac{200}{26.5}\right)\left[\frac{M(0.4 - 0.0034112)}{1.36811(10^{-3})}\right]$$

$$M = 277.83(10^3)\ \text{N} \cdot \text{m} = 278\ \text{kN} \cdot \text{m}$$

Assume steel fails:

$$(\sigma_{st})_{allow} = n\left(\frac{My}{I}\right); \qquad 280(10^6) = \frac{M(0.4 + 0.0034112)}{1.36811(10^{-3})}$$

$$M = 127.98(10^3)\ \text{N} \cdot \text{m} = 128\ \text{kN} \cdot \text{m (Controls)} \qquad \textbf{Ans.}$$

Ans:
$M = 128\ \text{kN} \cdot \text{m}$ (Controls)

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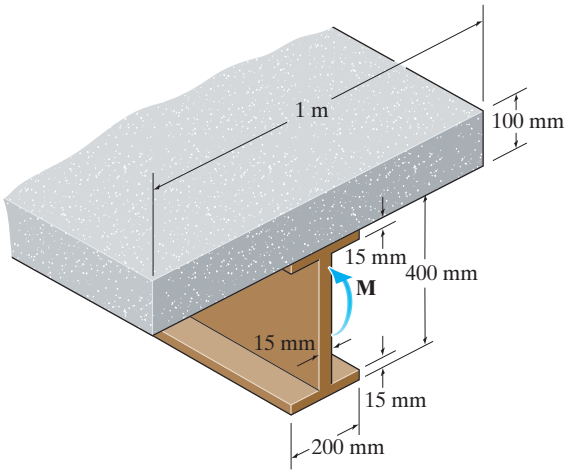
6–125. The low strength concrete floor slab is integrated with a wide-flange A-36 steel beam using shear studs (not shown) to form the composite beam. If the allowable bending stress for the concrete is $(\sigma_{allow})_{con} = 10$ MPa, and allowable bending stress for steel is $(\sigma_{allow})_{st} = 165$ MPa, determine the maximum allowable internal moment **M** that can be applied to the beam.



Section Properties: The beam cross section will be transformed into that of steel. Here, $n = \frac{E_{con}}{E_{st}} = \frac{22.1}{200} = 0.1105$. Thus, $b_{st} = nb_{con} = 0.1105(1) = 0.1105$ m. The location of the transformed section is

$$\bar{y} = \frac{\Sigma \bar{y} A}{\Sigma A}$$

$$= \frac{0.0075(0.015)(0.2) + 0.2(0.37)(0.015) + 0.3925(0.015)(0.2) + 0.45(0.1)(0.1105)}{0.015(0.2) + 0.37(0.015) + 0.015(0.2) + 0.1(0.1105)}$$

$$= 0.3222 \text{ m}$$

The moment of inertia of the transformed section about the neutral axis is

$$I = \Sigma \bar{I} + Ad^2 = \frac{1}{12}(0.2)\left(0.015^3\right) + 0.2(0.015)(0.3222 - 0.0075)^2 + \frac{1}{12}(0.015)\left(0.37^3\right) + 0.015(0.37)(0.3222 - 0.2)^2 + \frac{1}{12}(0.2)\left(0.015^3\right) + 0.2(0.015)(0.3925 - 0.3222)^2 + \frac{1}{12}(0.1105)\left(0.1^3\right) + 0.1105(0.1)(0.45 - 0.3222)^2$$

$$= 647.93\left(10^{-6}\right) \text{ m}^4$$

Bending Stress: Assuming failure of steel,

$$(\sigma_{allow})_{st} = \frac{Mc_{st}}{I}; \quad 165\left(10^6\right) = \frac{M(0.3222)}{647.93\left(10^{-6}\right)}$$

$$M = 331\,770.52 \text{ N} \cdot \text{m} = 332 \text{ kN} \cdot \text{m}$$

Assuming failure of concrete,

$$(\sigma_{allow})_{con} = n\frac{Mc_{con}}{I}; \qquad 10\left(10^6\right) = 0.1105\left[\frac{M(0.5 - 0.3222)}{647.93\left(10^{-6}\right)}\right]$$

$$M = 329\,849.77 \text{ N} \cdot \text{m} = 330 \text{ kN} \cdot \text{m} \text{ (controls)} \quad \textbf{Ans.}$$

Ans:

$M = 330 \text{ kN} \cdot \text{m}$

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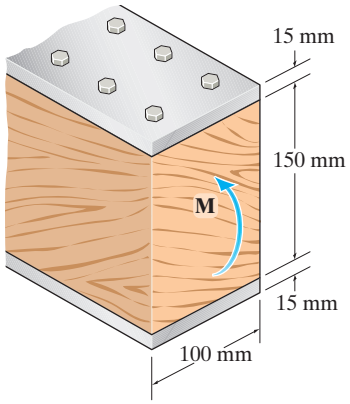
6–126.

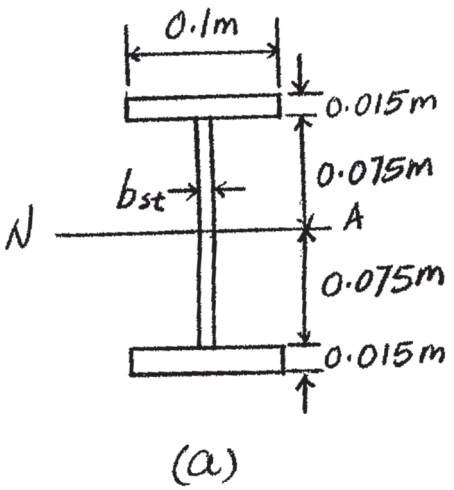
The wooden section of the beam is reinforced with two steel plates as shown. Determine the maximum moment M that the beam can support if the allowable stresses for the wood and steel are $(\sigma_{allow})_w = 6$ MPa, and $(\sigma_{allow})_{st} = 150$ MPa, respectively. Take $E_w = 10$ GPa and $E_{st} = 200$ GPa.



SOLUTION

Section Properties: The cross section will be transformed into that of steel as shown in Fig. *a*. Here, $n = \frac{E_w}{E_{st}} = \frac{10}{200} = 0.05$. Thus, $b_{st} = nb_w = 0.05(0.1) = 0.005$ m. The moment of inertia of the transformed section about the neutral axis is

$$I = \frac{1}{12}(0.1)(0.18^3) - \frac{1}{12}(0.095)(0.15^3) = 21.88125(10^{-6})\ \text{m}^4$$



Bending Stress: Assuming failure of the steel,

$$(\sigma_{allow})_{st} = \frac{Mc_{st}}{I}; \qquad 150(10^6) = \frac{M(0.09)}{21.88125(10^{-6})}$$

$$M = 36\,468.75\ \text{N}\cdot\text{m} = 36.5\ \text{kN}\cdot\text{m}$$

Assuming failure of wood,

$$(\sigma_{allow})_w = n\frac{Mc_w}{I}; \qquad 6(10^6) = 0.05\left[\frac{M(0.075)}{21.88125(10^{-6})}\right]$$

$$M = 35\,010\ \text{N}\cdot\text{m} = 35.0\ \text{kN}\cdot\text{m}\ \text{(controls)} \qquad \textbf{Ans.}$$

Ans:
$M = 35.0\ \text{kN}\cdot\text{m}$

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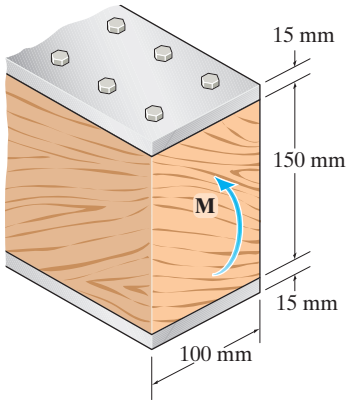
6–127.

The wooden section of the beam is reinforced with two steel plates as shown. If the beam is subjected to a moment of $M = 30\ \text{kN} \cdot \text{m}$, determine the maximum bending stresses in the steel and wood. Sketch the stress distribution over the cross section. Take $E_w = 10\ \text{GPa}$ and $E_{st} = 200\ \text{GPa}$.



SOLUTION

Section Properties: The cross section will be transformed into that of steel as shown in Fig. *a*. Here, $n = \dfrac{E_w}{E_{st}} = \dfrac{10}{200} = 0.05$. Thus, $b_{st} = nb_w = 0.05(0.1) = 0.005\,\text{m}$. The moment of inertia of the transformed section about the neutral axis is

$$I = \frac{1}{12}(0.1)(0.18^3) - \frac{1}{12}(0.095)(0.15^3) = 21.88125(10^{-6})\ \text{m}^4$$

Maximum Bending Stress: For the steel,

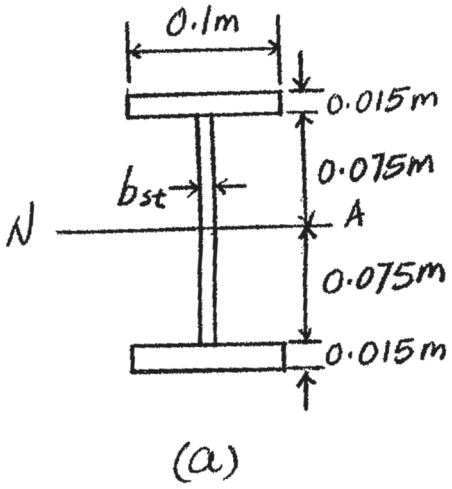
$$(\sigma_{max})_{st} = \frac{Mc_{st}}{I} = \frac{30(10^3)(0.09)}{21.88125(10^{-6})} = 123\ \text{MPa} \qquad \textbf{Ans.}$$

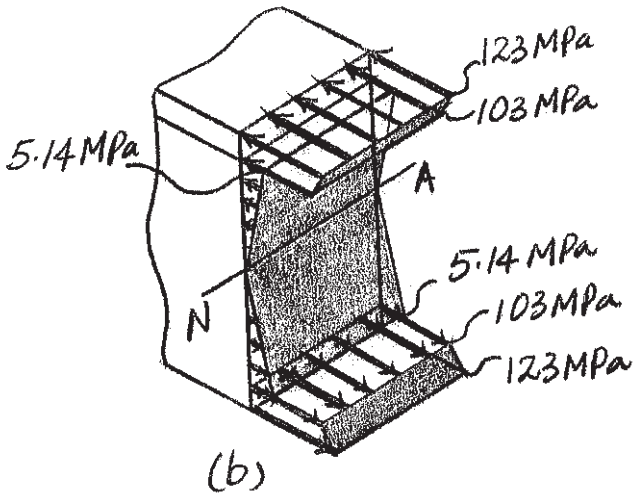
$$\sigma_{st}|_{y=0.075\ \text{m}} = \frac{My}{I} = \frac{30(10^3)(0.075)}{21.88125(10^{-6})} = 103\ \text{MPa}$$

For the wood,

$$(\sigma_{max})_w = n\frac{Mc_w}{I} = 0.05\left[\frac{30(10^3)(0.075)}{21.88125(10^{-6})}\right] = 5.14\ \text{MPa} \qquad \textbf{Ans.}$$

The bending stress distribution across the cross section is shown in Fig. *b*.





Ans:
$(\sigma_{max})_{st} = 123\ \text{MPa}$, $(\sigma_{max})_w = 5.14\ \text{MPa}$

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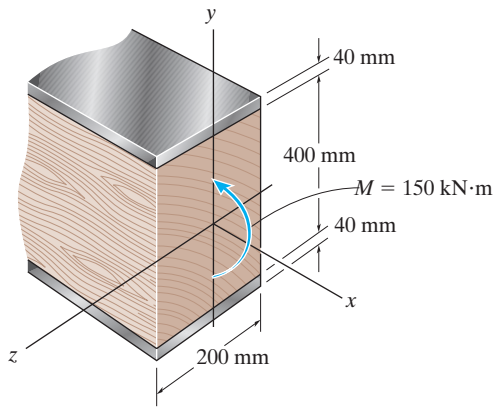
***6–128.**

A wood beam is reinforced with steel straps at its top and bottom as shown. Determine the maximum bending stress developed in the wood and steel if the beam is subjected to a moment of $M = 150\ \text{kN}\cdot\text{m}$. Sketch the stress distribution acting over the cross section. Take $E_w = 10$ GPa, $E_{st} = 200$ GPa.



SOLUTION

Section Properties: Here, $n = \dfrac{E_w}{E_{st}} = \dfrac{10\ \text{GPa}}{200\ \text{GPa}} = 0.05$. Then $b_{st} = nb_w = 0.05(0.2) = 0.01$ m. For the transformed section shown in Fig. *a*,

$$I_{NA} = \frac{1}{12}(0.2)(0.48^3) - \frac{1}{12}(0.19)(0.4^3) = 0.82987\,(10^{-3})\ \text{m}^4$$

Maximum Bending Stress: Applying the flexure formula,

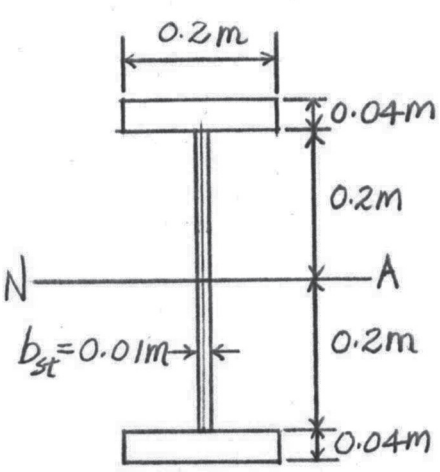
$$(\sigma_{st})_{max} = \frac{Mc_{st}}{I} = \frac{150(10^3)(0.24)}{0.82987(10^{-3})} = 43.38\,(10^6)\ \text{Pa} = 43.4\ \text{MPa} \qquad \textbf{Ans.}$$

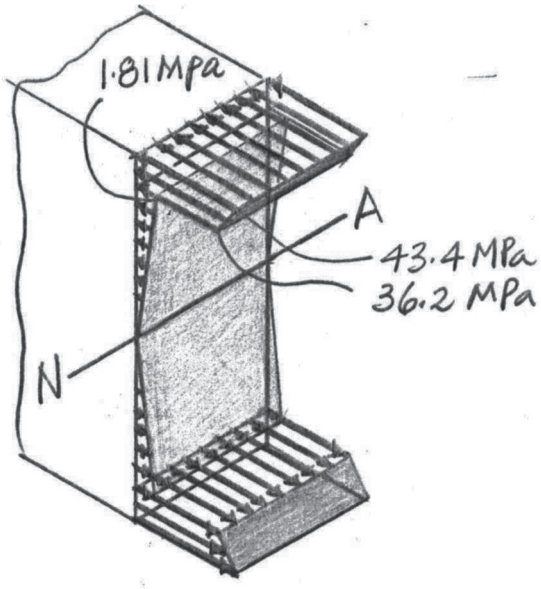
$$(\sigma_w)_{max} = n\frac{Mc_w}{I} = 0.05\left[\frac{150(10^3)(0.2)}{0.82987(10^{-3})}\right] = 1.8075(10^6)\ \text{Pa} = 1.81\ \text{MPa} \qquad \textbf{Ans.}$$

The bending stress on the steel at $y = 0.2$ m is

$$\sigma_{st} = \frac{My}{I} = \frac{150(10^3)(0.2)}{0.82987(10^{-3})} = 36.15\,(10^6)\ \text{Pa} = 36.2\ \text{MPa}$$

Using these results, the stress distribution on the beam's cross-section shown in Fig. *b* can be sketched.





Ans:
$(\sigma_{st})_{max} = 43.4$ MPa,
$(\sigma_w)_{max} = 1.81$ MPa

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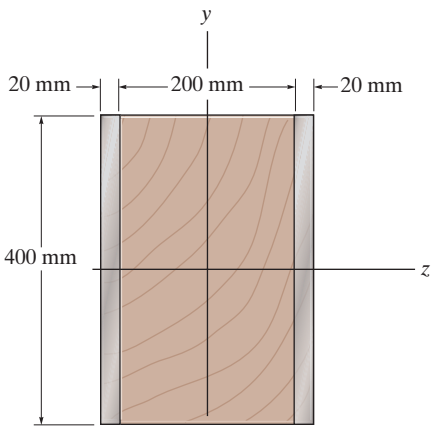
6–129.

The Douglas Fir beam is reinforced with A-992 steel straps at its sides. Determine the maximum stress in the wood and steel if the beam is subjected to a moment of $M_z = 80\ \text{kN}\cdot\text{m}$. Sketch the stress distribution acting over the cross section.



SOLUTION

Section Properties: Here, $n = \dfrac{E_w}{E_{st}} = \dfrac{13.1\ \text{GPa}}{200\ \text{GPa}} = 0.0655$. Then $b_{st} = nb_w = 0.0655(0.2) = 0.0131$ m. For the transformed section shown in Fig. *a*,

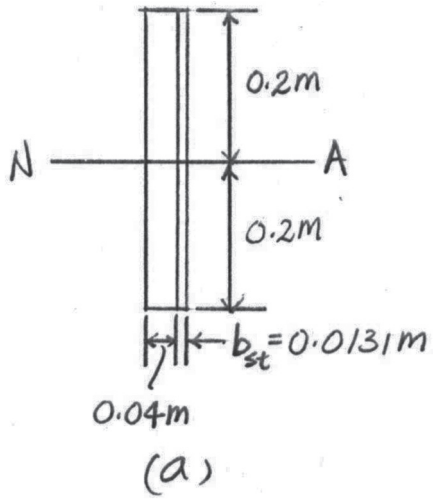
$$I_{NA} = \frac{1}{12}(0.04 + 0.0131)(0.4^3) = 0.2832\,(10^{-3})\ \text{m}^4$$

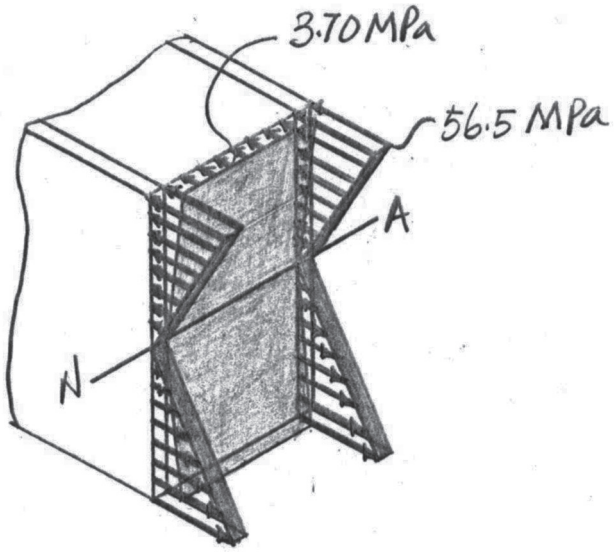
Maximum Bending Stress: Applying the flexure formula,

$$(\sigma_{st})_{max} = \frac{Mc}{I} = \frac{80(10^3)(0.2)}{0.2832(10^{-3})} = 56.497\,(10^6)\ \text{Pa} = 56.5\ \text{MPa} \qquad \textbf{Ans.}$$

$$(\sigma_w)_{max} = n\frac{Mc}{I} = 0.0655\left[\frac{80(10^3)(0.2)}{0.2832(10^{-3})}\right] = 3.7006(10^6)\ \text{Pa} = 3.70\ \text{MPa} \qquad \textbf{Ans.}$$

Using these results, the stress distribution on the beam's cross-section shown in Fig. *b* can be sketched.





Ans:
$(\sigma_{st})_{max} = 56.5$ MPa,
$(\sigma_w)_{max} = 3.70$ MPa

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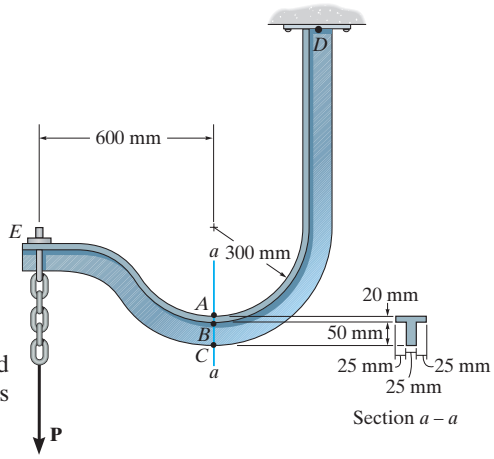
6–130.

If $P = 3$ kN, determine the bending stress at points A, B, and C of the cross section at section a–a. Using these results, sketch the stress distribution on section a–a.



SOLUTION

Internal Moment: The internal moment developed at section a–a can be determined by writing the moment equation of equilibrium about the neutral axis of the cross section at a–a. Using the free-body diagram shown in Fig. a,

$$\zeta + \Sigma M_{NA=0;} \qquad 3(0.6) - M_{a-a} = 0 \qquad M_{a-a} = 1.8\ \text{kN}\cdot\text{m}$$

Here, $\mathbf{M}_{a-a}$ is considered negative since it tends to reduce the curvature of the curved segment of the beam.

Section Properties: Referring to Fig. b, the location of the centroid of the cross section from the center of the beam's curvature is

$$\bar{r} = \frac{\Sigma \bar{r} A}{\Sigma A} = \frac{0.31(0.02)(0.075) + 0.345(0.05)(0.025)}{0.02(0.075) + 0.05(0.025)} = 0.325909\ \text{m}$$

The location of the neutral surface from the center of the beam's curvature can be determined from

$$R = \frac{A}{\Sigma \int_A \frac{dA}{r}}$$

where $A = 0.02(0.075) + 0.05(0.025) = 2.75(10^{-3})\ \text{m}^2$

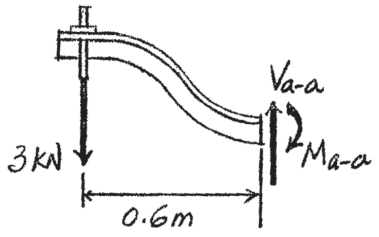
$$\Sigma \int_A \frac{dA}{r} = 0.075 \ln \frac{0.32}{0.3} + 0.025 \ln \frac{0.37}{0.32} = 8.46994(10^{-3})\ \text{m}$$

Thus,

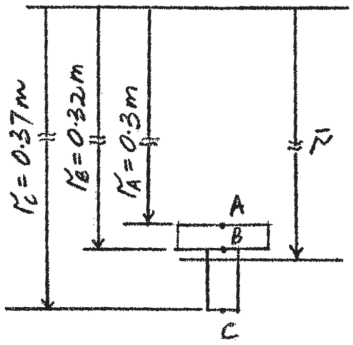
$$R = \frac{2.75(10^{-3})}{8.46994(10^{-3})} = 0.324678\ \text{m}$$

then

$$e = \bar{r} - R = 0.325909 - 0.324678 = 1.23144(10^{-3})\ \text{m}$$



(a)



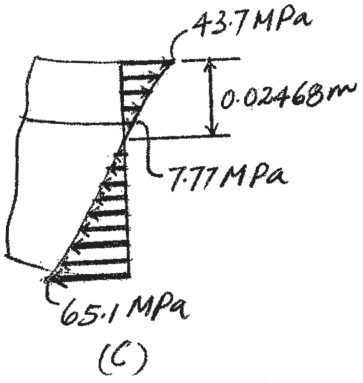
(b)

Normal Stress:

$$\sigma_A = \frac{M(R - r_A)}{Aer_A} = \frac{1.8(10^3)(0.324678 - 0.3)}{2.75(10^{-3})(1.23144)(10^{-3})(0.3)} = 43.7\ \text{MPa (T)} \qquad \textbf{Ans.}$$

$$\sigma_B = \frac{M(R - r_B)}{Aer_B} = \frac{1.8(10^3)(0.324678 - 0.32)}{2.75(10^{-3})(1.23144)(10^{-3})(0.32)} = 7.77\ \text{MPa (T)} \qquad \textbf{Ans.}$$

$$\sigma_C = \frac{M(R - r_C)}{Aer_C} = \frac{1.8(10^3)(0.324678 - 0.37)}{2.75(10^{-3})(1.23144)(10^{-3})(0.37)} = -65.1\ \text{MPa (C)} \qquad \textbf{Ans.}$$



(c)

Ans:
$\sigma_A = 43.7$ MPa (T), $\sigma_B = 7.77$ MPa (T),
$\sigma_C = -65.1$ MPa (C)

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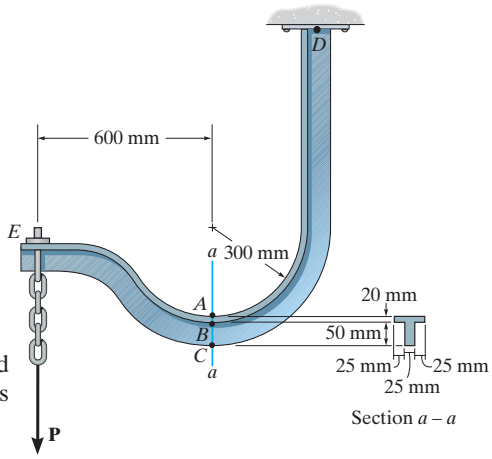
6–131.

If the maximum bending stress at section *a–a* is not allowed to exceed $\sigma_{allow} = 150$ MPa, determine the maximum allowable force **P** that can be applied to the end *E*.



SOLUTION

Internal Moment: The internal Moment developed at section *a–a* can be determined by writing the moment equation of equilibrium about the neutral axis of the cross section at *a–a*.

$$\zeta+\Sigma M_{NA} = 0; \qquad P(0.6) - M_{a-a} = 0 \qquad M_{a-a} = 0.6P$$

Here, $\mathbf{M}_{a-a}$ is considered positive since it tends to decrease the curvature of the curved segment of the beam.

Section Properties: Referring to Fig. *b*, the location of the centroid of the cross section from the center of the beam's curvature is

$$\bar{r} = \frac{\Sigma \tilde{r}A}{\Sigma A} = \frac{0.31(0.02)(0.075) + 0.345(0.05)(0.025)}{0.02(0.075) + 0.05(0.025)} = 0.325909 \text{ m}$$

The location of the neutral surface from the center of the beam's curvature can be determined from

$$R = \frac{A}{\Sigma \int_A \frac{dA}{r}}$$

where $A = 0.02(0.075) + 0.05(0.025) = 2.75(10^{-3})\ \text{m}^2$

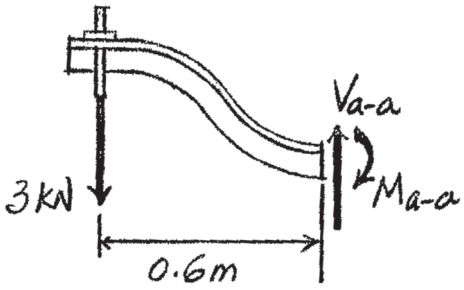
$$\Sigma \int_A \frac{dA}{r} = 0.075 \ln \frac{0.32}{0.3} + 0.025 \ln \frac{0.37}{0.32} = 8.46994(10^{-3}) \text{ m}$$

Thus,

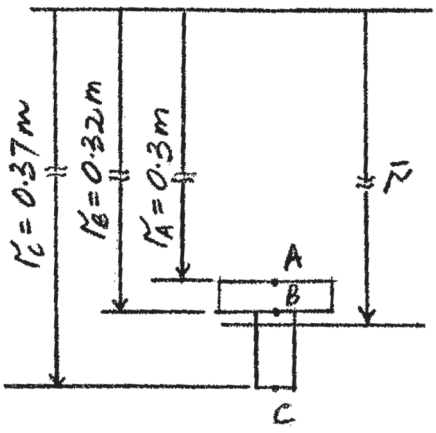
$$R = \frac{2.75(10^{-3})}{8.46994(10^{-3})} = 0.324678 \text{ m}$$

then

$$e = \bar{r} - R = 0.325909 - 0.324678 = 1.23144(10^{-3}) \text{ m}$$



(a)



(b)

Allowable Normal Stress: The maximum normal stress occurs at either points *A* or *C*. For point *A*, which is in tension,

$$\sigma_{allow} = \frac{M(R - r_A)}{Aer_A}; \qquad 150(10^6) = \frac{0.6P(0.324678 - 0.3)}{2.75(10^{-3})(1.23144)(10^{-3})(0.3)}$$

$$P = 10\,292.09 \text{ N} = 10.3 \text{ kN}$$

For point *C*, which is in compression,

$$\sigma_{allow} = \frac{M(R - r_C)}{Aer_C}; \qquad -150(10^6) = \frac{0.6P(0.324678 - 0.37)}{2.75(10^{-3})(1.23144)(10^{-3})(0.37)}$$

$$P = 6911.55 \text{ N} = 6.91 \text{ kN (controls)} \qquad \textbf{Ans.}$$

Ans:
$P = 6.91$ kN

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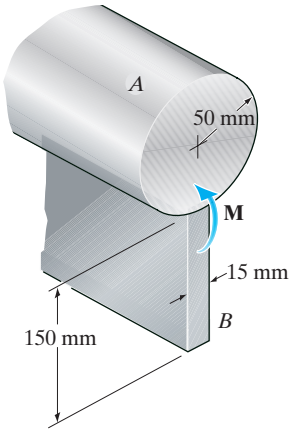
***6–132.**

If the beam is subjected to a moment of $M = 45\ \text{kN}\cdot\text{m}$, determine the maximum bending stress in the A-36 steel section A and the 2014-T6 aluminum alloy section B.



SOLUTION

Section Properties: The cross section will be transformed into that of steel as shown in Fig. a. Here, $n = \dfrac{E_{al}}{E_{st}} = \dfrac{73.1\left(10^9\right)}{200\left(10^9\right)} = 0.3655$. Thus, $b_{st} = nb_{al} = 0.3655(0.015) = 0.0054825$ m. The location of the transformed section is

$$\bar{y} = \frac{\Sigma \bar{y}A}{\Sigma A} = \frac{0.075(0.15)(0.0054825) + 0.2\left[\pi\left(0.05^2\right)\right]}{0.15(0.0054825) + \pi\left(0.05^2\right)}$$

$$= 0.1882\ \text{m}$$

The moment of inertia of the transformed section about the neutral axis is

$$I = \Sigma \bar{I} + Ad^2 = \frac{1}{12}(0.0054825)\left(0.15^3\right) + 0.0054825(0.15)(0.1882 - 0.075)^2$$

$$+ \frac{1}{4}\pi\left(0.05^4\right) + \pi\left(0.05^2\right)(0.2 - 0.1882)^2$$

$$= 18.08\left(10^{-6}\right)\ \text{m}^4$$

Maximum Bending Stress: For the steel,

$$(\sigma_{max})_{st} = \frac{Mc_{st}}{I} = \frac{45\left(10^3\right)(0.06185)}{18.08\left(10^{-6}\right)} = 154\ \text{MPa} \qquad \textbf{Ans.}$$

For the aluminum alloy,

$$(\sigma_{max})_{al} = n\frac{Mc_{al}}{I} = 0.3655\left[\frac{45\left(10^3\right)(0.1882)}{18.08\left(10^{-6}\right)}\right] = 171\ \text{MPa} \qquad \textbf{Ans.}$$

Ans:
$(\sigma_{max})_{st} = 154\ \text{MPa},\ (\sigma_{max})_{al} = 171\ \text{MPa}$

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6–133.

For the curved beam in Fig. 6–40*a*, show that when the radius of curvature approaches infinity, the curved-beam formula, Eq. 6–24, reduces to the flexure formula, Eq. 6–13.

SOLUTION

Normal Stress: Curved-beam formula

$$\sigma = \frac{M(R - r)}{Ar(\bar{r} - R)} \qquad \text{where } A' = \int_A \frac{dA}{r} \quad \text{and } R = \frac{A}{\int_A \frac{dA}{r}} = \frac{A}{A'}$$

$$\sigma = \frac{M(A - rA')}{Ar(\bar{r}A' - A)} \tag{1}$$

$$r = \bar{r} + y \tag{2}$$

$$\bar{r}A' = \bar{r}\int_A \frac{dA}{r} = \int_A \left(\frac{\bar{r}}{\bar{r} + y} - 1 + 1\right) dA$$

$$= \int_A \left(\frac{\bar{r} - \bar{r} - y}{\bar{r} + y} + 1\right) dA$$

$$= A - \int_A \frac{y}{\bar{r} + y}\, dA \tag{3}$$

Denominator of Eq. (1) becomes,

$$Ar(\bar{r}A' - A) = Ar\left(A - \int_A \frac{y}{\bar{r} + y}\, dA - A\right) = -Ar\int_A \frac{y}{\bar{r} + y}\, dA$$

Using Eq. (2),

$$Ar(\bar{r}A' - A) = -A\int_A \left(\frac{\bar{r}y}{\bar{r} + y} + y - y\right) dA - Ay\int_A \frac{y}{\bar{r} + y}\, dA$$

$$= A\int_A \frac{y^2}{\bar{r} + y}\, dA - A\int_A y\, dA - Ay\int_A \frac{y}{\bar{r} + y}\, dA$$

$$= \frac{A}{\bar{r}}\int_A \left(\frac{y^2}{1 + \frac{y}{\bar{r}}}\right) dA - A\int_A y\, dA - \frac{Ay}{\bar{r}}\int_A \left(\frac{y}{1 + \frac{y}{\bar{r}}}\right) dA$$

But, $\int_A y\, dA = 0, \qquad \text{as } \frac{y}{\bar{r}} \to 0$

Then, $Ar(\bar{r}A' - A) \to \frac{A}{\bar{r}}I$

Eq. (1) becomes $\sigma = \frac{M\bar{r}}{AI}(A - rA')$

Using Eq. (2), $\sigma = \frac{M\bar{r}}{AI}(A - \bar{r}A' - yA')$

Using Eq. (3),

$$\sigma = \frac{M\bar{r}}{AI}\left[A - \left(A - \int_A \frac{y}{\bar{r} + y}\, dA\right) - y\int_A \frac{dA}{\bar{r} + y}\right]$$

$$= \frac{M\bar{r}}{AI}\left[\int_A \frac{y}{\bar{r} + y}\, dA - y\int_A \frac{dA}{\bar{r} + y}\right]$$

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6–133. (Continued)

$$= \frac{M\bar{r}}{AI}\left[\int_A \left(\frac{\frac{y}{\bar{r}}}{1+\frac{y}{\bar{r}}}\right)dA - \frac{y}{\bar{r}}\int_A \left(\frac{dA}{1+\frac{y}{\bar{r}}}\right)\right]$$

As $\frac{y}{\bar{r}} \to 0$

$$\int_A \left(\frac{\frac{y}{\bar{r}}}{1+\frac{y}{\bar{r}}}\right)dA = 0 \quad \text{and} \quad \frac{y}{\bar{r}}\int_A \left(\frac{dA}{1+\frac{y}{\bar{r}}}\right) = \frac{y}{\bar{r}}\int_A dA = \frac{yA}{\bar{r}}$$

Therefore, $\sigma = \frac{M\bar{r}}{AI}\left(-\frac{yA}{\bar{r}}\right) = -\frac{My}{I}$ **(Q.E.D)**

Ans:
N/A

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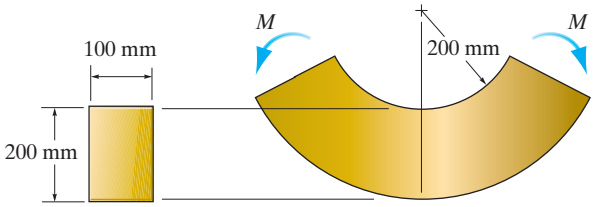
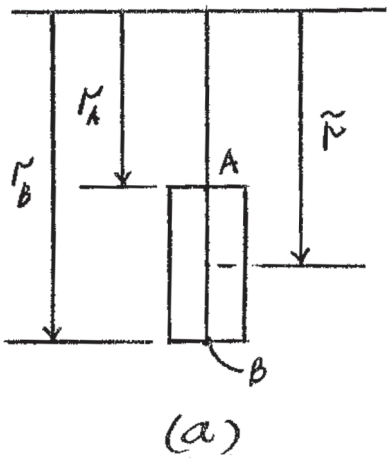
6–134.

The curved member is subjected to the moment of $M = 50\ \text{kN}\cdot\text{m}$. Determine the percentage error introduced in the calculation of maximum bending stress using the flexure formula for straight members.



SOLUTION



Straight Member: The maximum bending stress developed in the straight member

$$\sigma_{\max} = \frac{Mc}{I} = \frac{50(10^3)(0.1)}{\frac{1}{12}(0.1)(0.2^3)} = 75\ \text{MPa}$$

Curved Member: When $r = 0.2$ m, $\bar{r} = 0.3$ m, $r_A = 0.2$ m and $r_B = 0.4$ m, Fig. *a*. The location of the neutral surface from the center of curvature of the curve member is

$$R = \frac{A}{\int_A \frac{dA}{r}} = \frac{0.1(0.2)}{0.1\ln\frac{0.4}{0.2}} = 0.288539\ \text{m}$$

Then

$$e = \bar{r} - R = 0.011461\ \text{m}$$

Here, $M = 50\ \text{kN}\cdot\text{m}$. Since it tends to decrease the curvature of the curved member,

$$\sigma_B = \frac{M(R - r_B)}{Aer_B} = \frac{50(10^3)(0.288539 - 0.4)}{0.1(0.2)(0.011461)(0.4)}$$

$$= -60.78\ \text{MPa} = 60.78\ \text{MPa (C)}$$

$$\sigma_A = \frac{M(R - r_A)}{Aer_A} = \frac{50(10^3)(0.288539 - 0.2)}{0.1(0.2)(0.011461)(0.2)}$$

$$= 96.57\ \text{MPa (T) (Max.)}$$

Thus,

$$\%\ \text{of error} = \left(\frac{96.57 - 75}{96.57}\right)100 = 22.3\%$$ **Ans.**

Ans:
% of error = 22.3%

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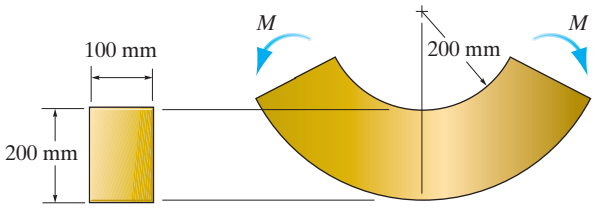
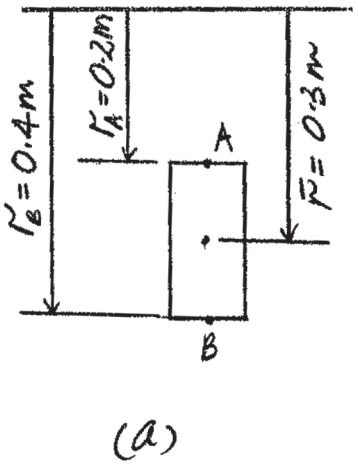
6–135.

The curved member is made from material having an allowable bending stress of $\sigma_{\text{allow}} = 100$ MPa. Determine the maximum allowable moment M that can be applied to the member.



SOLUTION



(a)

Internal Moment: M is negative since it tends to decrease the curvature of the curved member.

Section Properties: Referring to Fig. *a*, the location of the neutral surface from the center of curvature of the curve beam is

$$R = \frac{A}{\int_A \frac{dA}{r}} = \frac{0.1(0.2)}{0.1\ln\frac{0.4}{0.2}} = 0.288539\text{ m}$$

Then

$$e = \bar{r} - R = 0.3 - 0.288539 = 0.011461\text{ m}$$

Allowable Bending Stress: The maximum stress occurs at either point A or B. For point A, which is in tension,

$$\sigma_{\text{allow}} = \frac{M(R - r_A)}{Aer_A}; \qquad 100(10^6) = \frac{M(0.288539 - 0.2)}{0.1(0.2)(0.011461)(0.2)}$$

$$M = 51\,778.27\text{ N}\cdot\text{m} = 51.8\text{ kN}\cdot\text{m (controls)} \quad \textbf{Ans.}$$

For point B, which is in compression,

$$\sigma_{\text{allow}} = \frac{M(R - r_B)}{Aer_B}; \qquad -100(10^6) = \frac{M(0.288539 - 0.4)}{0.1(0.2)(0.011461)(0.4)}$$

$$M = 82\,260.10\text{ N}\cdot\text{m} = 82.3\text{ kN}\cdot\text{m}$$

Ans:
$M = 51.8\text{ kN}\cdot\text{m}$

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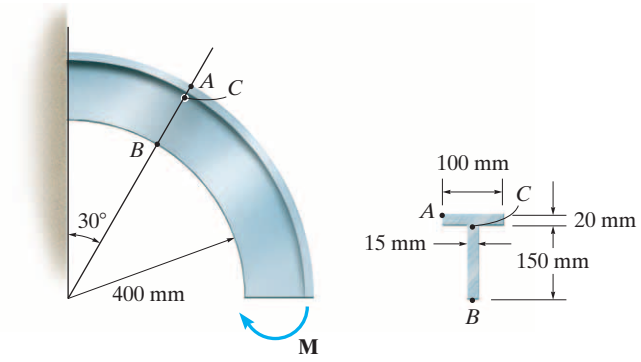
***6–136.** The curved beam is subjected to a bending moment of $M = 900\ \text{N}\cdot\text{m}$ as shown. Determine the stress at points A and B, and show the stress on a volume element located at each of these points.



Internal Moment: $M = -900\ \text{N}\cdot\text{m}$ is negative since it tends to decrease the beam's radius curvature.

Section Properties:

$$\Sigma A = 0.15(0.015) + 0.1(0.02) = 0.00425\ \text{m}^2$$

$$\Sigma \bar{r}A = 0.475(0.15)(0.015) + 0.56(0.1)(0.02) = 2.18875(10^{-3})\ \text{m}^3$$

$$\bar{r} = \frac{\Sigma \bar{r}A}{\Sigma A} = \frac{2.18875\ (10^{-3})}{0.00425} = 0.5150\ \text{m}$$

$$\Sigma \int_A \frac{dA}{r} = 0.015 \ln \frac{0.55}{0.4} + 0.1 \ln \frac{0.57}{0.55} = 8.348614(10^{-3})\ \text{m}$$

$$R = \frac{A}{\Sigma \int_A \frac{dA}{r}} = \frac{0.00425}{8.348614(10^{-3})} = 0.509067\ \text{m}$$

$$\bar{r} - R = 0.515 - 0.509067 = 5.933479(10^{-3})\ \text{m}$$

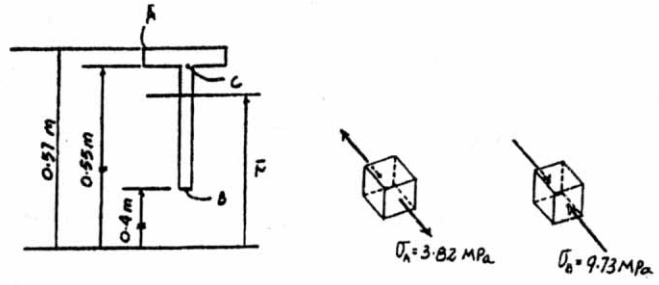
Normal Stress: Applying the curved-beam formula

$$\sigma_A = \frac{M(R - r_A)}{Ar_A(\bar{r} - R)} = \frac{-900(0.509067 - 0.57)}{0.00425(0.57)(5.933479)(10^{-3})}$$

$$= 3.82\ \text{MPa (T)} \qquad \textbf{Ans.}$$

$$\sigma_B = \frac{M(R - r_B)}{Ar_B(\bar{r} - R)} = \frac{-900(0.509067 - 0.4)}{0.00425(0.4)(5.933479)(10^{-3})}$$

$$= -9.73\ \text{MPa} = 9.73\ \text{MPa (C)} \qquad \textbf{Ans.}$$



Ans:

$\sigma_A = 1{,}60\ \text{KN_\&R'*}$

$\sigma_B = 9.73\ \text{MPa (C)}$

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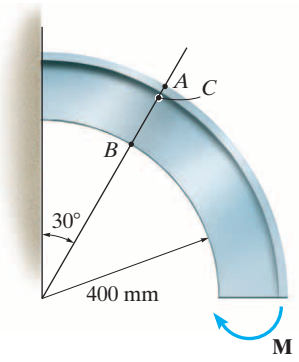
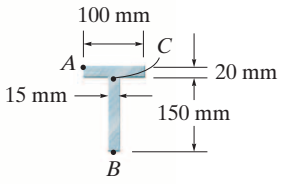
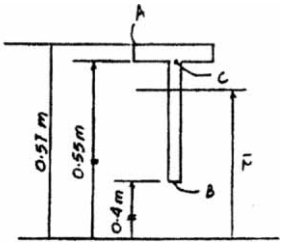
6–137. The curved beam is subjected to a bending moment of $M = 900\ \text{N}\cdot\text{m}$. Determine the stress at point C.





Internal Moment: $M = -900\ \text{N}\cdot\text{m}$ is negative since it tends to decrease the beam's radius of curvature.



Section Properties:

$$\Sigma A = 0.15(0.015) + 0.1(0.02) = 0.00425\ \text{m}^2$$

$$\Sigma \bar{r}A = 0.475(0.15)(0.015) + 0.56(0.1)(0.02) = 2.18875(10^{-3})\ \text{m}$$

$$\bar{r} = \frac{\Sigma \bar{r}A}{\Sigma A} = \frac{2.18875\ (10^{-3})}{0.00425} = 0.5150\ \text{m}$$

$$\Sigma \int_A \frac{dA}{r} = 0.015 \ln \frac{0.55}{0.4} + 0.1 \ln \frac{0.57}{0.55} = 8.348614(10^{-3})\ \text{m}$$

$$R = \frac{A}{\Sigma \int_A \frac{dA}{r}} = \frac{0.00425}{8.348614(10^{-3})} = 0.509067\ \text{m}$$

$$\bar{r} - R = 0.515 - 0.509067 = 5.933479(10^{-3})\ \text{m}$$

Normal Stress: Applying the curved-beam formula

$$\sigma_C = \frac{M(R - r_C)}{Ar_C(\bar{r} - R)} = \frac{-900(0.509067 - 0.55)}{0.00425(0.55)(5.933479)(10^{-3})}$$

$$= 2.66\ \text{MPa (T)} \qquad \textbf{Ans.}$$

Ans:
$\Sigma A = 0.00425\ \text{m}^2$, $\bar{r} = 0.5150\ \text{m}$,
$\Sigma \int_A \frac{dA}{r} = 8.348614(10^{-3})\ \text{m}$,
$\sigma_C = 2.66\ \text{MPa (T)}$
$(\sigma_{\max})_{\text{pvc}} = 12.3\ \text{MPa}$

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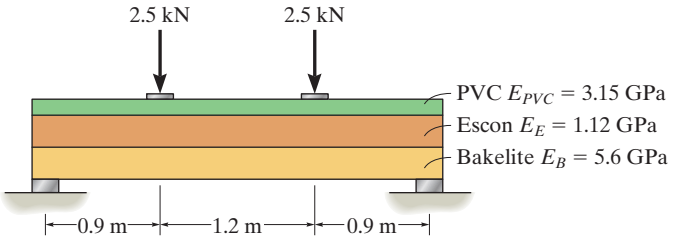
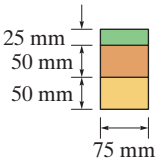
6–138. The beam is made from three types of plastic that are identified and have the moduli of elasticity shown in the figure. Determine the maximum bending stress in the PVC.



$$(b_{bk})_1 = n_1 b_{Es} = \left(\frac{1.12}{5.6}\right)(0.075) = 0.015 \text{ m}$$

$$(b_{bk})_2 = n_2 b_{pvc} = \left(\frac{3.15}{5.6}\right)(0.075) = 0.0421875 \text{ m}$$



$$\bar{y} = \frac{\Sigma \bar{y} A}{\Sigma A} = \frac{0.025(0.075)(0.05) + 0.075(0.015)(0.05) + 0.1125(0.0421875)(0.025)}{0.075(0.05) + 0.015(0.05) + 0.0421875(0.025)}$$

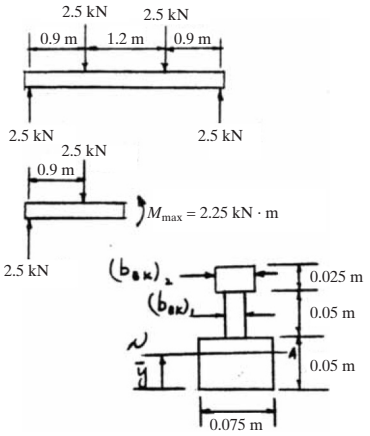
$$= 0.048365 \text{ m}$$

$$I = 2\left[\frac{1}{12}(0.075)(0.05^3) + 0.075(0.05)(0.048365 - 0.025)^2\right]$$

$$+ \frac{1}{12}(0.015)(0.05^3) + 0.015(0.05)(0.075 - 0.048365)^2$$

$$+ \frac{1}{12}(0.0421875)(0.025^3) + 0.0421875(0.025)(0.1125 - 0.048365)^2$$

$$= 7.90996(10^{-6}) \text{ m}^4$$



$$(\sigma_{max})_{pvc} = n_2 \frac{Mc}{I} = \left(\frac{3.15}{5.6}\right)\left\{\frac{[2.25(10^3)](0.1125 - 0.048365)}{7.90996(10^{-6})}\right\}$$

$$= 12.26(10^6) \text{ N-m}^2 = 12.3 \text{ MPa}$$ **Ans.**

Ans:
$(\sigma_{max})_{pvc} = 12.3$ MPa

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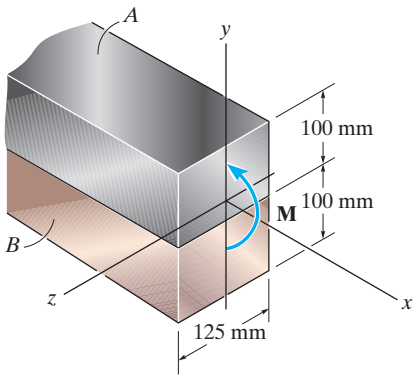
6–139.

The composite beam is made of A-36 steel (A) bonded to C83400 red brass (B) and has the cross section shown. If it is subjected to a moment of $M = 6.5$ kN · m, determine the maximum stress in the brass and steel. Also, what is the stress in each material at the seam where they are bonded together?



SOLUTION

Section Properties: For the transformed section.

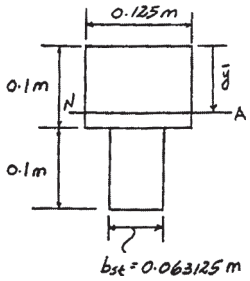
$$n = \frac{E_{br}}{E_{st}} = \frac{101(10^9)}{200(10^9)} = 0.505$$

$$b_{st} = nb_{br} = 0.505(0.125) = 0.063125 \text{ m}$$

$$\bar{y} = \frac{\Sigma \bar{y} A}{\Sigma A}$$

$$= \frac{0.05(0.125)(0.1) + 0.15(0.1)(0.063125)}{0.125(0.1) + 0.1(0.063125)}$$

$$= 0.08355 \text{ m}$$



$$I_{NA} = \frac{1}{12}(0.125)(0.1^3) + 0.125(0.1)(0.08355 - 0.05)^2$$

$$+ \frac{1}{12}(0.063125)(0.1^3) + 0.063125(0.1)(0.15 - 0.08355)^2$$

$$= 57.62060(10^{-6}) \text{ m}^4$$

Maximum Bending Stress: Applying the flexure formula

$$(\sigma_{max})_{st} = \frac{My}{I} = \frac{6.5(10^3)(0.08355)}{57.62060(10^{-6})}$$

$$= 9.42 \text{ MPa} \quad \textbf{Ans.}$$

$$(\sigma_{max})_{br} = n\frac{Mc}{I} = 0.505\left[\frac{6.5(10^3)(0.2 - 0.08355)}{57.62060(10^{-6})}\right]$$

$$= 6.63 \text{ MPa} \quad \textbf{Ans.}$$

Bending Stress: At seam

$$\sigma_{st} = \frac{My}{I}$$

$$= \frac{6.5(10^3)(0.1 - 0.08355)}{57.62060(10^{-6})}$$

$$= 1.855 \text{ MPa} = 1.86 \text{ MPa} \quad \textbf{Ans.}$$

$$\sigma_{br} = n\frac{My}{I} = 0.505(1.855) = 0.937 \text{ MPa} \quad \textbf{Ans.}$$

Ans:
$(\sigma_{max})_{st} = 9.42$ MPa,
$(\sigma_{max})_{br} = 6.63$ MPa,
$\sigma_{st} = 1.86$ MPa,
$\sigma_{br} = 0.937$ MPa

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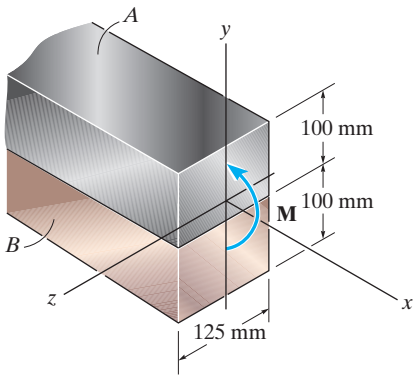
***6–140.**

The composite beam is made of A-36 steel (A) bonded to C83400 red brass (B) and has the cross section shown. If the allowable bending stress for the steel is $(\sigma_{allow})_{st} = 180$ MPa and for the brass $(\sigma_{allow})_{br} = 60$ MPa, determine the maximum moment M that can be applied to the beam.



SOLUTION

Section Properties: For the transformed section.

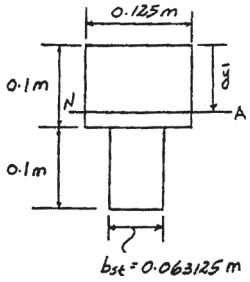
$$n = \frac{E_{br}}{E_{st}} = \frac{101(10^9)}{200(10^9)} = 0.505$$

$$b_{st} = nb_{br} = 0.505(0.125) = 0.063125 \text{ m}$$

$$\bar{y} = \frac{\Sigma \bar{y} A}{\Sigma A}$$

$$= \frac{0.05(0.125)(0.1) + 0.15(0.1)(0.063125)}{0.125(0.1) + 0.1(0.063125)}$$

$$= 0.08355 \text{ m}$$



$$I_{NA} = \frac{1}{12}(0.125)(0.1^3) + 0.125(0.1)(0.08355 - 0.05)^2$$

$$+ \frac{1}{12}(0.063125)(0.1^3) + 0.063125(0.1)(0.15 - 0.08355)^2$$

$$= 57.62060(10^{-6}) \text{ m}^4$$

Allowable Bending Stress: Applying the flexure formula

Assume failure of steel

$$(\sigma_{max})_{st} = (\sigma_{allow})_{st} = \frac{My}{I}$$

$$180(10^6) = \frac{M(0.08355)}{57.62060(10^{-6})}$$

$$M = 124130 \text{ N} \cdot \text{m} = 124 \text{ kN} \cdot \text{m}$$

Assume failure of brass

$$(\sigma_{max})_{br} = (\sigma_{allow})_{br} = n\frac{Mc}{I}$$

$$60(10^6) = 0.505\left[\frac{M(0.2 - 0.08355)}{57.62060(10^{-6})}\right]$$

$$M = 58792 \text{ N} \cdot \text{m}$$

$$= 58.8 \text{ kN} \cdot \text{m} \quad \textit{(Controls!)}$$ **Ans.**

Ans:
$M = 58.8 \text{ kN} \cdot \text{m}$

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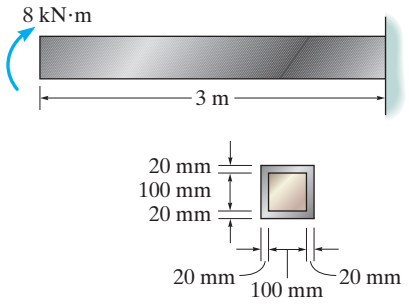
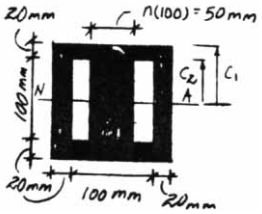
6–141. The member has a brass core bonded to a steel casing. If a couple moment of 8 kN # m is applied at its end, determine the maximum bending stress in the member. $E_{br} = 100$ GPa, $E_{st} = 200$ GPa.



$$n = \frac{E_{br}}{E_{st}} = \frac{100}{200} = 0.5$$



$$I = \frac{1}{12}(0.14)(0.14)^3 - \frac{1}{12}(0.05)(0.1)^3 = 27.84667(10^{-6})\text{m}^4$$

Maximum stress in steel:

$$(\sigma_{st})_{\max} = \frac{Mc_1}{I} = \frac{8(10^3)(0.07)}{27.84667(10^{-6})} = 20.1 \text{ MPa} \quad \text{(max)}$$ **Ans.**

Maximum stress in brass:

$$(\sigma_{br})_{\max} = \frac{nMc_2}{I} = \frac{0.5(8)(10^3)(0.05)}{27.84667(10^{-6})} = 7.18 \text{ MPa}$$

Ans:
$(\sigma_{st})_{\max} = 20.1$ MPa

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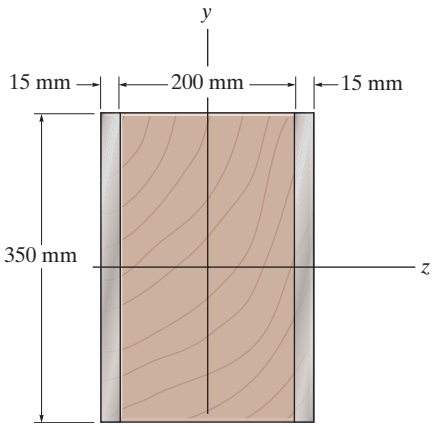
6–142.

The Douglas Fir beam is reinforced with A-36 steel straps at its sides. Determine the maximum stress in the wood and steel if the beam is subjected to a bending moment of $M_z = 4\ \text{kN} \cdot \text{m}$. Sketch the stress distribution acting over the cross section.



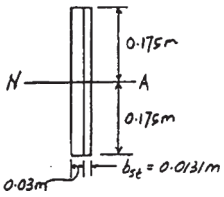
SOLUTION

Section Properties: For the transformed section,

$$n = \frac{E_w}{E_{st}} = \frac{13.1(10^9)}{200(10^9)} = 0.0655$$

$$b_{st} = nb_w = 0.0655(0.2) = 0.0131\ \text{m}$$

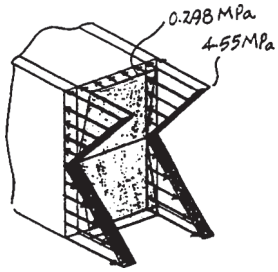
$$I_{NA} = \frac{1}{12}(0.03 + 0.0131)(0.35^3) = 153.99(10^{-6})\ \text{m}^4$$



Maximum Bending Stress: Applying the flexure formula

$$(\sigma_{max})_{st} = \frac{Mc}{I} = \frac{4(10^3)(0.175)}{153.99(10^{-6})} = 4.55\ \text{MPa}$$ **Ans.**

$$(\sigma_{max})_w = n\frac{Mc}{I} = 0.0655\left[\frac{4(10^3)(0.175)}{153.99(10^{-6})}\right] = 0.298\ \text{MPa}$$ **Ans.**



Ans:
$(\sigma_{max})_{st} = 4.55\ \text{MPa}$,
$(\sigma_{max})_w = 0.298\ \text{MPa}$

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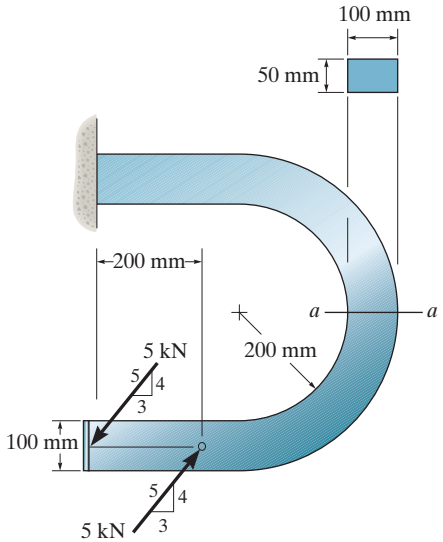
6–143.

The curved bar used on a machine has a rectangular cross section. If the bar is subjected to a couple as shown, determine the maximum tensile and compressive stress acting at section *a–a*. Sketch the stress distribution on the section in three dimensions.



SOLUTION

Internal Moment: Referring to the FBD of the lower segment of the curve beam sectioned through *a–a*, Fig. *a*,

$$\zeta+\Sigma M_o = 0; \qquad 5\left(\frac{4}{5}\right)(0.2) - M = 0 \qquad M = 0.8\ \text{kN}\cdot\text{m} = 800\ \text{N}\cdot\text{m}$$

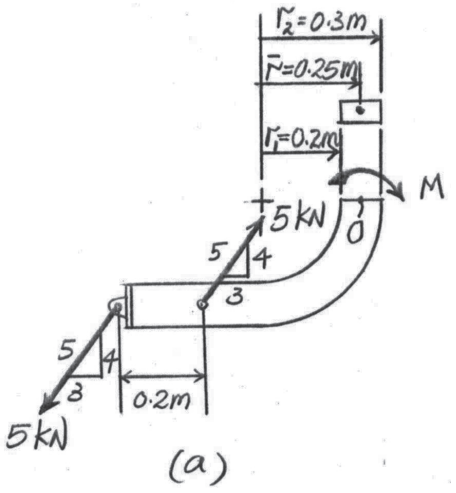
Section Properties: Referring to Fig. *a*,

$$\bar{r} = \frac{0.2 + 0.3}{2} = 0.25\ \text{m} \qquad A = (0.05)(0.1) = 0.005\ \text{m}^2$$

$$\int_A \frac{dA}{r} = b/\text{n}\,\frac{r_2}{r_1} = 0.05/\text{n}\left(\frac{0.3}{0.2}\right) = 0.020273255\ \text{m}$$

$$R = \frac{A}{\int_A \frac{dA}{r}} = \frac{0.005}{0.020273255} = 0.2466303\ \text{m}$$

$$\bar{r} - R = 0.25 - 0.2466303 = 0.003369654\ \text{m}$$



Normal Stress: Here $M = 800\ \text{N}\cdot\text{m}$ is positive, since it tends to increase the beam's radius of curvature. Applying the curved beam formula,

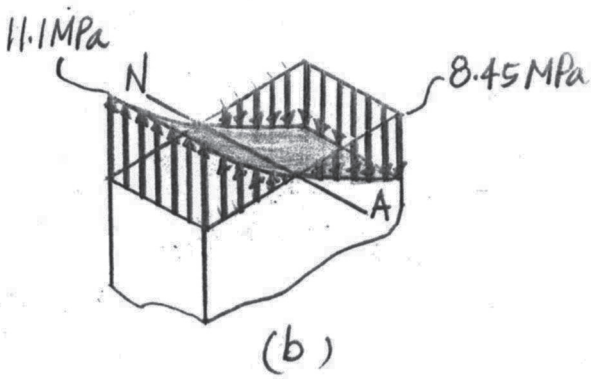
$$(\sigma_T)_{\max} = \frac{M(R - r_1)}{Ar_1(\bar{r} - R)}$$

$$= \frac{800(0.2466303 - 0.2)}{0.005(0.2)(0.003369654)} = 11.07(10^6)\ \text{Pa} = 11.1\ \text{MPa (T)} \qquad \textbf{Ans.}$$

$$(\sigma_C)_{\max} = \frac{M(R - r_2)}{Ar_2(\bar{r} - R)}$$

$$= \frac{800(0.2466303 - 0.3)}{0.005(0.3)(0.003369654)} = -8.447(10^6)\ \text{Pa} = 8.45\ \text{MPa (C)} \qquad \textbf{Ans.}$$

Using these results, the normal stress distribution on the beam's cross-section shown in Fig. *b* can be sketched.



Ans:
$(\sigma_T)_{\max} = 11.1\ \text{MPa (T)}$,
$(\sigma_C)_{\max} = 8.45\ \text{MPa (C)}$

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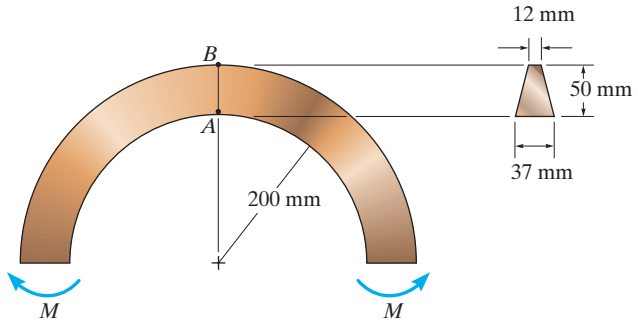
***6–144.** The curved member is symmetric and is subjected to a moment of $M = 900\ \text{N}\cdot\text{m}$. Determine the bending stress in the member at points A and B. Show the stress acting on volume elements located at these points.



$$A = 0.012(0.05) + \frac{1}{2}(0.025)(0.05) = 0.001225\ \text{m}^2$$

$$\bar{r} = \frac{\Sigma \bar{r} A}{\Sigma A} = \frac{0.225(0.012)(0.05) + 0.21667\left[\frac{1}{2}(0.025)(0.05)\right]}{0.001225} = 0.220748\ \text{m}$$

$$\int_A \frac{dA}{r} = 0.012 \ln\left(\frac{0.25}{0.2}\right) + \left[\frac{0.025(0.25)}{0.25 - 0.2}\right] \ln\left(\frac{0.25}{0.2}\right) - 0.025 = 0.005570667\ \text{m}$$

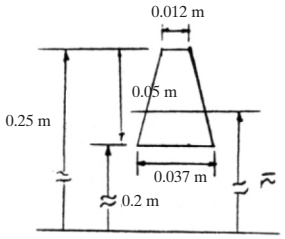
$$R = \frac{A}{\int_A \frac{dA}{r}} = \frac{0.001225}{0.005570667} = 0.2199018$$

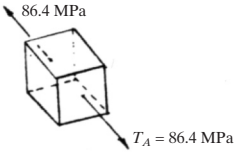
$$\bar{r} - R = 0.220748 - 0.2199018 = 0.846427(10^{-3})\ \text{m}$$

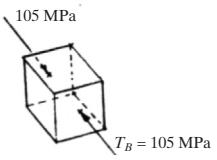
$$\sigma = \frac{M(R - r)}{Ar(\bar{r} - R)}$$

$$\sigma_A = \frac{900(0.2199018 - 0.2)}{0.001225(0.2)[0.846427(10^{-3})]} = 86.37(10^6)\ \text{N/m}^2 = 86.4\ \text{MPa (T)} \qquad \textbf{Ans.}$$

$$\sigma_B = \frac{900(0.2199018 - 0.25)}{0.001225(0.25)[0.846427(10^{-3})]} = -104.50(10^6)\ \text{N/m}^2 = 105\ \text{MPa (C)} \qquad \textbf{Ans.}$$







Ans:
$\sigma_A = 86.4\ \text{MPa (T)}, \sigma_B = 105\ \text{MPa (C)}$

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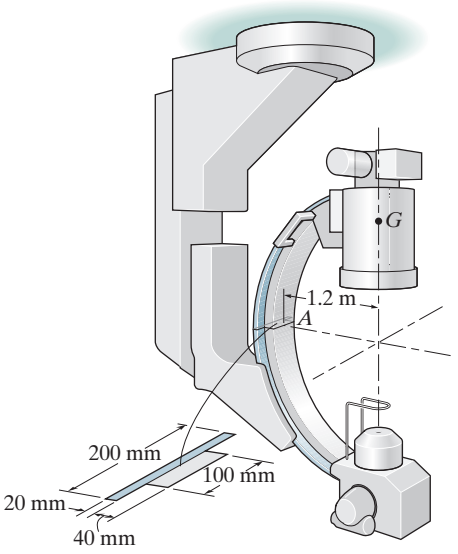
6–145. The ceiling-suspended C-arm is used to support the X-ray camera used in medical diagnoses. If the camera has a mass of 150 kg, with center of mass at G, determine the maximum bending stress at section A.



Section Properties:

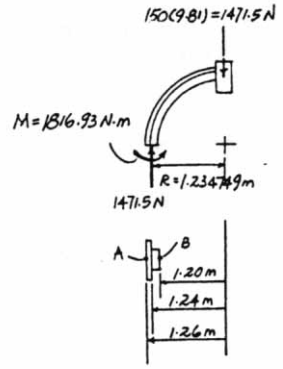
$$\bar{r} = \frac{\Sigma \bar{r} A}{\Sigma A} = \frac{1.22(0.1)(0.04) + 1.25(0.2)(0.02)}{0.1(0.04) + 0.2(0.02)} = 1.235 \text{ m}$$

$$\Sigma \int_A \frac{dA}{r} = 0.1 \ln \frac{1.24}{1.20} + 0.2 \ln \frac{1.26}{1.24} = 6.479051\left(10^{-3}\right) \text{m}$$

$$A = 0.1(0.04) + 0.2(0.02) = 0.008 \text{ m}^2$$

$$R = \frac{A}{\int_A \frac{dA}{r}} = \frac{0.008}{6.479051\ (10^{-3})} = 1.234749 \text{ m}$$

$$\bar{r} - R = 1.235 - 1.234749 = 0.251183\left(10^{-3}\right) \text{ m}$$



Internal Moment: The internal moment must be computed about the neutral axis as shown on FBD. $M = -1816.93 \text{ N} \cdot \text{m}$ is negative since it tends to decrease the beam's radius of curvature.

Maximum Normal Stress: Applying the curved-beam formula

$$\sigma_A = \frac{M(R - r_A)}{A r_A (\bar{r} - R)}$$

$$= \frac{-1816.93(1.234749 - 1.26)}{0.008(1.26)(0.251183)(10^{-3})}$$

$$= 18.1 \text{ MPa (T)}$$

$$\sigma_B = \frac{M(R - r_B)}{A r_B (\bar{r} - R)}$$

$$= \frac{-1816.93(1.234749 - 1.20)}{0.008(1.20)(0.251183)(10^{-3})}$$

$$= -26.2 \text{ MPa} = 26.2 \text{ MPa (C)} \quad (\textbf{Max}) \qquad \textbf{Ans.}$$

Ans:

$\bar{r} = 1.235$ m, $\Sigma \int_A \frac{dA}{r} = 6.479051(10^{-3})$ m,
$A = 0.008 \text{ m}^2$, $\sigma_B = 26.2$ MPa (C)

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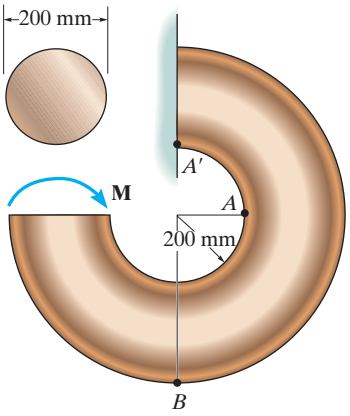
6–146

The member has a circular cross section. If it is subjected to a moment of $M = 5\ \text{kN}\cdot\text{m}$, determine the stress at points A and B. Is the stress at point A', which is located on the member near the wall, the same as that at A? Explain.



SOLUTION

Section Properties: Referring to Fig. *a*

$$\bar{r} = \frac{0.2 + 0.4}{2} = 0.3\ \text{m} \qquad A = \pi C^2 = \pi(0.1^2) = 0.01\pi\ \text{m}^2$$

$$\int_A \frac{dA}{r} = 2\pi\left(r - \sqrt{\bar{r}^2 - C^2}\right)$$

$$= 2\pi\left(0.3 - \sqrt{0.3^2 - 0.1^2}\right) = 0.1078024\ \text{m}$$

$$R = \frac{A}{\displaystyle\int_A \frac{dA}{r}} = \frac{0.012r}{0.1078024} = 0.291421\ \text{m}$$

$$\bar{r} - R = 0.3 - 0.291421 = 0.00857864\ \text{m}$$

Bending Stress: Here, $M = 5\ \text{kN}\cdot\text{m}$ is negative since it tends to decrease the beam's radius of curvature. Applying the curved beam formula,

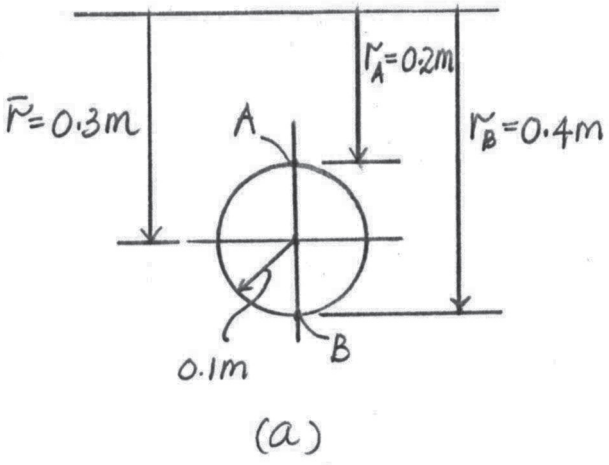
$$\sigma_A = \frac{M(R - r_A)}{Ar_A(\bar{r} - R)} = \frac{-5(10^3)(0.291421 - 0.2)}{(0.01\pi)(0.2)(0.00857864)} = -8.4805(10^6)\ \text{Pa} = 8.48\ \text{MPa (C)}$$

Ans.

$$\sigma_B = \frac{M(R - r_B)}{Ar_B(\bar{r} - R)} = \frac{-5(10^3)(0.291421 - 0.4)}{(0.01\pi)(0.4)(0.00857864)} = 5.0360(10^6)\ \text{Pa} = 5.04\ \text{MPa (T)}$$

Ans.

No! Point A' is at the fixed support where stress concentration occurs. The application of the curved beam formula is only valid for sections sufficiently removed from the support in accordance to Saint-Venant's principle.



(a)

Ans:
$\sigma_A = 8.48\ \text{MPa (C)}$,
$\sigma_B = 5.04\ \text{MPa (T)}$
No, it is not the same.

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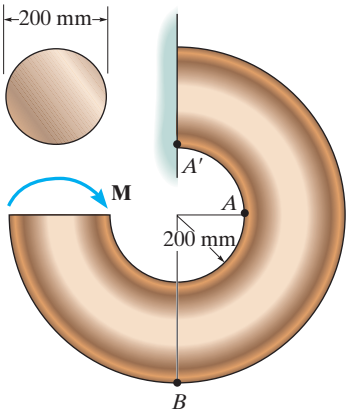
6–147.

The member has a circular cross section. If the allowable bending stress is $\sigma_{\text{allow}} = 100$ MPa, determine the maximum moment M that can be applied to the member.



SOLUTION

Section Properties: Referring to Fig. *a,*

$$\bar{r} = \frac{0.2 + 0.4}{2} = 0.3 \text{ m} \qquad A = \pi C^2 = \pi(0.1^2) = 0.01\pi \text{ m}^2$$

$$\int_A \frac{dA}{r} = 2\pi\left(r - \sqrt{\bar{r}^2 - C^2}\right)$$

$$= 2\pi\left(0.3 - \sqrt{0.3^2 - 0.1^2}\right) = 0.1078024 \text{ m}$$

$$R = \frac{A}{\int_A \frac{dA}{r}} = \frac{0.01\pi}{0.1078024} = 0.291421 \text{ m}$$

$$\bar{r} - R = 0.3 - 0.291421 = 0.00857864 \text{ m}$$

Bending Stress: Here, M is negative since it tends to decrease the beam's radius of curvature. Applying the curve beam's formula by assuming tension failure,

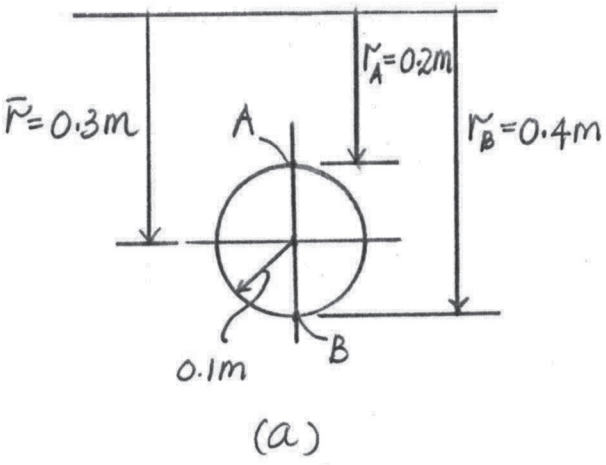
$$\sigma_B = \sigma_{\text{allow}} = \frac{M(R - r_B)}{Ar_B(\bar{r} - R)}; \quad 100(10^6) = \frac{-M(0.291421 - 0.4)}{(0.01\pi)(0.4)(0.00857864)}$$

$$M = 99.29(10^3) \text{ N} \cdot \text{m} = 99.3 \text{ kN} \cdot \text{m}$$

Assuming compression failure,

$$\sigma_A = \sigma_{\text{allow}} = \frac{M(R - r_A)}{Ar_A(\bar{r} - R)}; \quad -100(10^6) = \frac{-M(0.291421 - 0.2)}{(0.01\pi)(0.2)(0.00857864)}$$

$$M = 58.96(10^3) \text{ N} \cdot \text{m} = 59.0 \text{ kN} \cdot \text{m (control!)} \quad \textbf{Ans.}$$



(a)

Ans:
$M = 59.0 \text{ kN} \cdot \text{m}$

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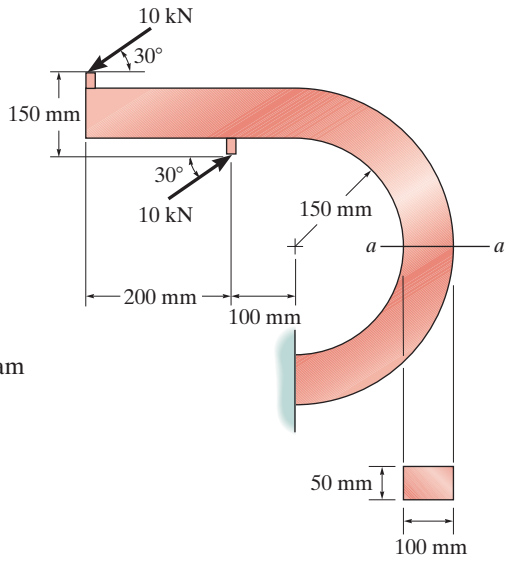
***6–148.**

The curved bar used on a machine has a rectangular cross section. If the bar is subjected to a couple as shown, determine the maximum tensile and compressive stresses acting at section *a–a*. Sketch the stress distribution on the section in three dimensions.



SOLUTION

Internal Moment: Referring to the FBD of the upper segment of the curved beam sectioned through *a–a*, Fig. *a*

$$\zeta+\Sigma M_0 = 0 \quad (10\sin 30°)(0.2) + 10\cos 30°(0.15) - M = 0 \quad M = 2.2990\ \text{kN}\cdot\text{m}$$

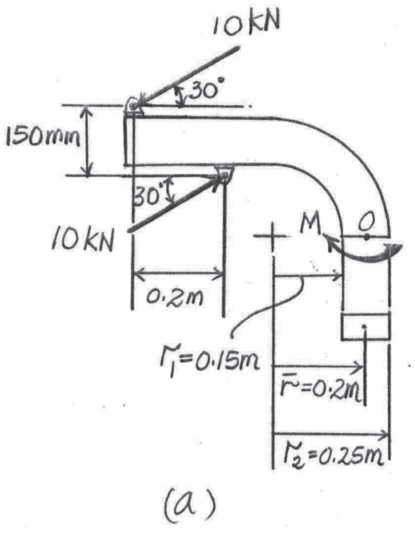
Section Properties: Referring to Fig. *a*

$$\bar{r} = \frac{0.15 + 0.25}{2} = 0.2\ \text{m} \qquad A = 0.05(0.1) = 0.005\ \text{m}^2$$

$$\int_A \frac{dA}{r} = b/n\,\frac{r_2}{r_1} = 0.05\ \text{m}\left(\frac{0.25}{0.15}\right) = 0.025541281\ \text{m}$$

$$R = \frac{A}{\rho_A \frac{dA}{r}} = \frac{0.005}{0.025541281} = 0.1957615\ \text{m}$$

$$\bar{r} - R = 0.2 - 0.1957615 = 0.00423848\ \text{m}$$



Normal Stress: Here, $M = -2.2990\ \text{kN}\cdot\text{m}$ is negative since it tends to decrease the beam's radius of curvature. Applying curved-beam formula,

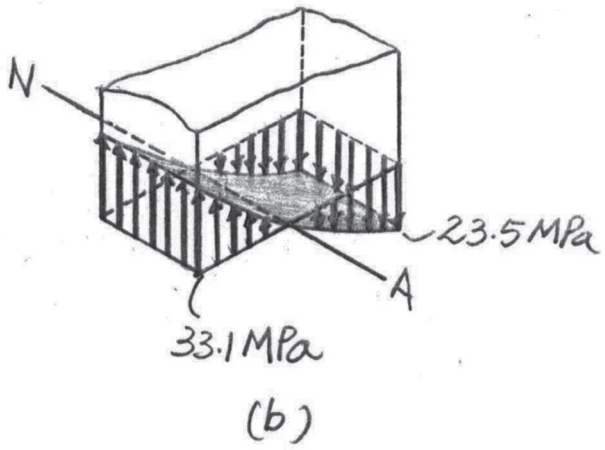
$$(\sigma_C)_{\max} = \frac{M(R - r_1)}{Ar_1(\bar{r} - R)} = \frac{-2.2990(10^3)(0.1957615 - 0.15)}{0.005(0.15)(0.00423848)} = -33.10(10^6)\ \text{Pa}$$

$$= -33.10\,(10^6)\ \text{Pa} = 33.1\ \text{MPa (C)} \qquad \textbf{Ans.}$$

$$(\sigma_T)_{\max} = \frac{M(R - r_2)}{Ar_2(\bar{r} - R)} = \frac{-2.2990(10^3)(0.1957615 - 0.25)}{0.005(0.25)(0.00423848)} =$$

$$= 23.54\,(10^6)\ \text{Pa} = 23.5\ \text{MPa (T)} \qquad \textbf{Ans.}$$

Using these results, the normal stress distribution on the beam's cross-section shown in Fig. *b* can be sketched.



Ans:
$(\sigma_C)_{\max} = 33.1\ \text{MPa (C)}$,
$(\sigma_T)_{\max} = 23.5\ \text{MPa (T)}$

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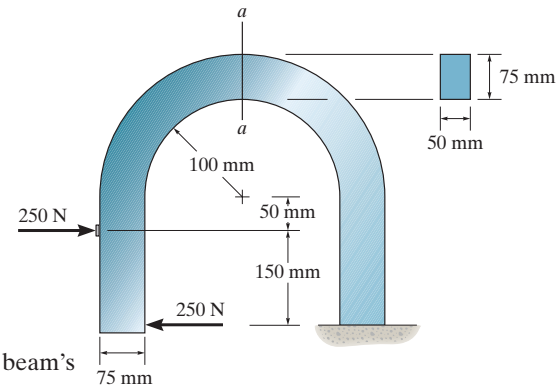
6–149.

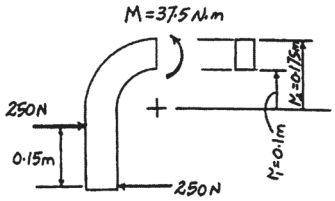
The curved bar used on a machine has a rectangular cross section. If the bar is subjected to a couple as shown, determine the maximum tensile and compressive stresses acting at section *a–a*. Sketch the stress distribution on the section in three dimensions.



SOLUTION

Internal Moment: $M = 37.5\ \text{N}\cdot\text{m}$ is positive since it tends to increase the beam's radius of curvature.

Section Properties:



$$\bar{r} = \frac{0.1 + 0.175}{2} = 0.1375\ \text{m}$$

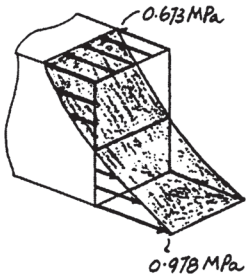
$$A = 0.075(0.05) = 0.00375\ \text{m}^2$$

$$\int_A \frac{dA}{r} = 0.05 \ln \frac{0.175}{0.1} = 0.027981\ \text{m}$$

$$R = \frac{A}{\int_A \frac{dA}{r}} = \frac{0.00375}{0.027981} = 0.134021\ \text{m}$$

$$\bar{r} - R = 0.1375 - 0.134021 = 3.479478\left(10^{-3}\right)\ \text{m}$$

Normal Stress: Applying the curved-beam formula



$$(\sigma_{\max})_t = \frac{M(R - r_1)}{Ar_1(\bar{r} - R)}$$

$$= \frac{37.5(0.134021 - 0.1)}{0.00375(0.1)(3.479478)\left(10^{-3}\right)}$$

$$= 0.978\ \text{MPa (T)} \qquad \textbf{Ans.}$$

$$(\sigma_{\max})_c = \frac{M(R - r_2)}{Ar_2(\bar{r} - R)}$$

$$= \frac{37.5(0.134021 - 0.175)}{0.00375(0.175)(3.479478)\left(10^{-3}\right)}$$

$$= -0.673\ \text{MPa} = 0.673\ \text{MPa (C)} \qquad \textbf{Ans.}$$

Ans:
$(\sigma_{\max})_t = 0.978$ MPa (T),
$(\sigma_{\max})_c = 0.673$ MPa (C)

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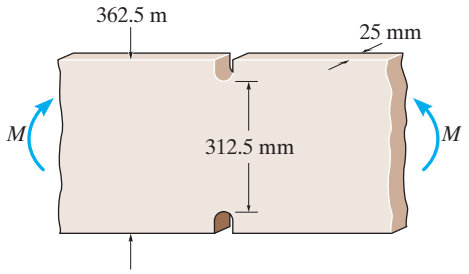
6–150. If the radius of each notch on the plate is $r = 12.5$ mm, determine the largest moment that can be applied. The allowable bending stress for the material is stress of $\sigma_{allow} = 125$ MPa.



$$b = \frac{362.5 - 312.5}{2} = 25 \text{ mm}$$

$$\frac{b}{r} = \frac{25}{12.5} = 2.0 \qquad \frac{r}{h} = \frac{12.5}{312.5} = 0.04$$

From Fig. 6-44:

$$K = 2.60$$

$$\sigma_{max} = K\frac{Mc}{I}$$

$$125(10^6) = 2.60\left[\frac{M(0.15625)}{\frac{1}{12}(0.025)(0.3125^3)}\right]$$

$$M = 19.56(10^3) \text{ N} \cdot \text{m} = 19.6 \text{ kN} \cdot \text{m}$$ **Ans.**

Ans:

$K = 2.60, M = 19.6 \text{ kN} \cdot \text{m}$

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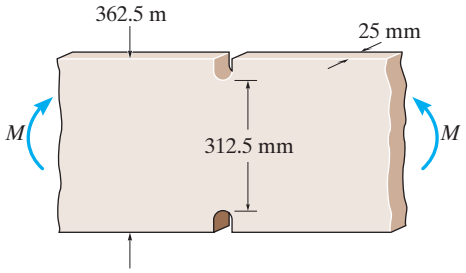
6–151. The symmetric notched plate is subjected to bending. If the radius of each notch is $r = 12.5$ mm and the applied moment is $M = 15\ \text{kN} \cdot \text{m}$, determine the maximum bending stress in the plate.



$$b = \frac{362.5 - 312.5}{2} = 25 \text{ mm}$$

$$\frac{b}{r} = \frac{25}{12.5} = 2.0 \qquad \frac{r}{h} = \frac{12.5}{312.5} = 0.04$$

From Fig. 6-44:

$$K = 2.60$$

$$\sigma_{\max} = K\frac{Mc}{I} = 2.60\left[\frac{[15(10^3)](0.15625)}{\frac{1}{12}(0.025)(0.3125^3)}\right]$$

$$= 36.864(10^6)\ \text{N-m}^2 = 36.9\ \text{MPa}$$ **Ans.**

Ans:

$\sigma_{\max} = 36.9$ MPa

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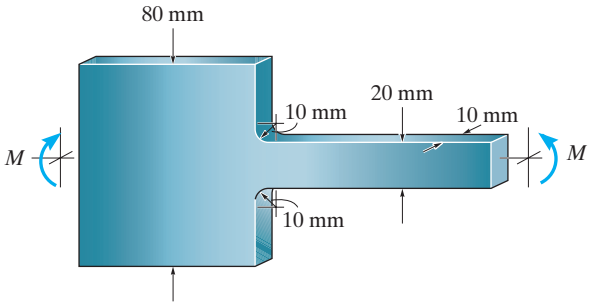
***6–152.**

The bar is subjected to a moment of $M = 100\ \text{N} \cdot \text{m}$. Determine the maximum bending stress in the bar and sketch, approximately, how the stress varies over the critical section.



SOLUTION

Stress Concentration Factor: Entering $\frac{w}{h} = \frac{80}{20} = 4.0$ and $\frac{r}{h} = \frac{10}{20} = 0.5$ into Fig. 6–43, we obtain $K = 1.25$.

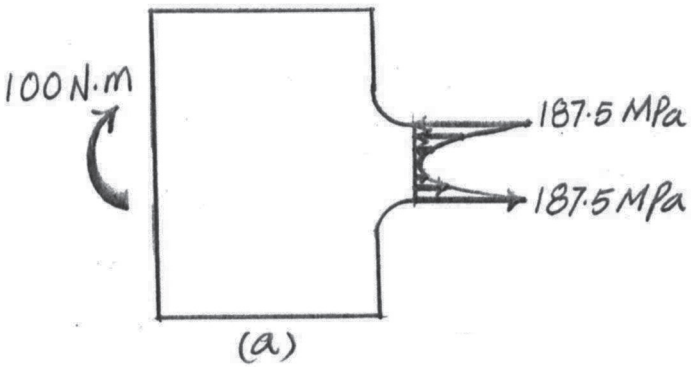
Maximum Bending Stress:

$$\sigma_{\max} = K\frac{Mc}{I}$$

$$= 1.25\left[\frac{100(0.01)}{\frac{1}{12}(0.01)(0.02^3)}\right]$$

$$= 187.5(10^6)\ \text{Pa} = 187.5\ \text{MPa} \qquad \textbf{Ans.}$$

The bending stress distribution across the critical section is shown in Fig. *a*.



Ans:
$\sigma_{\max} = 187.5\ \text{MPa}$

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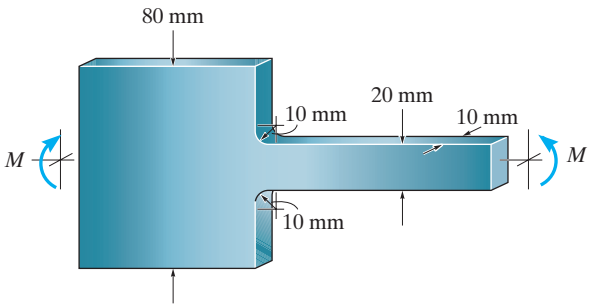
6–153.

The allowable bending stress for the bar is $\sigma_{allow} = 200$ MPa. Determine the maximum moment M that can be applied to the bar.



SOLUTION

Stress Concentration Factor: Entering $\frac{w}{h} = \frac{80}{20} = 4.0$ and $\frac{r}{h} = \frac{10}{20} = 0.5$ into Fig. 6–43, we obtain $K = 1.25$.

Maximum Bending Stress:

$$\sigma_{max} = \sigma_{allow} = K\frac{Mc}{I}$$

$$200(10^6) = 1.25\left[\frac{M(0.01)}{\frac{1}{12}(0.01)(0.02^3)}\right]$$

$$M = 106.67\ \text{N}\cdot\text{m}$$

$$= 107\ \text{N}\cdot\text{m} \qquad \textbf{Ans.}$$

Ans:
$M = 107\ \text{N}\cdot\text{m}$

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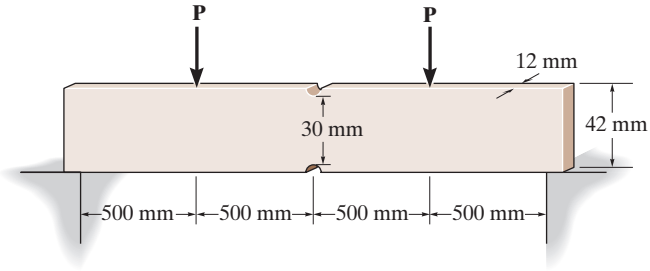
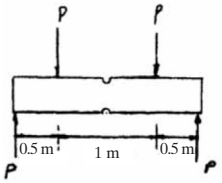
6–154. The simply supported notched bar is subjected to two forces **P**. Determine the largest magnitude of **P** that can be applied without causing the material to yield. The material is A-36 steel. Each notch has a radius of $r = 3$ mm.

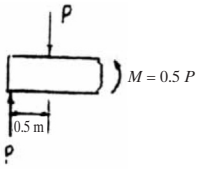


$$b = \frac{42 - 30}{2} = 6 \text{ mm}$$

$$\frac{b}{r} = \frac{6}{3} = 2; \qquad \frac{r}{h} = \frac{3}{30} = 0.1$$

From Fig. 6-44. $K = 1.92$





$$\sigma_Y = K\frac{Mc}{I}; \qquad 250(10^6) = 1.92\left[\frac{0.5P(0.015)}{\frac{1}{12}(0.012)(0.3^3)}\right]$$

$P = 468.75 \text{ N} = 469 \text{ N}$ **Ans.**

Ans:

$P = 469$ N

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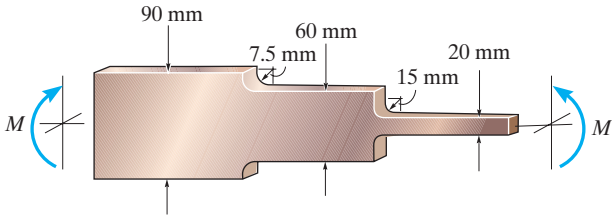
6–155.

The stepped bar has a thickness of 10 mm. Determine the maximum moment that can be applied to its ends if the allowable bending stress is $\sigma_{\text{allow}} = 150$ MPa.



SOLUTION

Stress Concentration Factor: For the smaller section, entering $\frac{w}{h} = \frac{60}{20} = 3.0$ and $\frac{r}{h} = \frac{15}{20} = 0.75$ into Fig. 6–43, we obtain $K = 1.15$. For the larger section, $\frac{w}{h} = \frac{90}{60} = 1.5$ and $\frac{r}{h} = \frac{7.5}{60} = 0.125$ gives $K = 1.65$.

Maximum Bending Stress: For the smaller section,

$$\sigma_{\max} = \sigma_{\text{allow}} = K\frac{Mc}{I}$$

$$150(10^6) = 1.15\left[\frac{M(0.01)}{\frac{1}{12}(0.01)(0.02^3)}\right]$$

$$M = 86.96\ \text{N}\cdot\text{m} = 87.0\ \text{N}\cdot\text{m}\ \text{(controls!)} \qquad \textbf{Ans.}$$

For the larger section,

$$\sigma_{\max} = \sigma_{\text{allow}} = K\frac{Mc}{I}$$

$$150(10^6) = 1.65\left[\frac{M(0.03)}{\frac{1}{12}(0.01)(0.06^3)}\right]$$

$$M = 545.45\ \text{N}\cdot\text{m}$$

Ans:
$M = 87.0\ \text{N}\cdot\text{m}$

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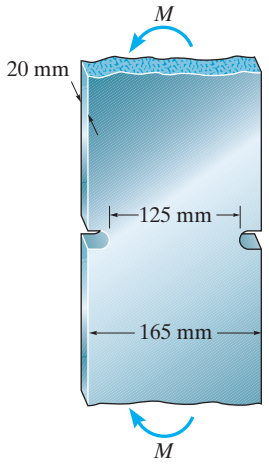
***6–156.**

If the radius of each notch on the plate is $r = 10$ mm, determine the largest moment M that can be applied. The allowable bending stress is $\sigma_{\text{allow}} = 180$ MPa.



SOLUTION

Stress Concentration Factor: From the graph in the text with $\frac{b}{r} = \frac{20}{10} = 2$ and $\frac{r}{h} = \frac{10}{125} = 0.08$, then $K = 2.1$.

Allowable Bending Stress:

$$\sigma_{\max} = \sigma_{\text{allow}} = K\frac{Mc}{I}$$

$$180(10^6) = 2.1\left[\frac{M(0.0625)}{\frac{1}{12}(0.02)(0.125^3)}\right]$$

$$M = 4464\ \text{N}\cdot\text{m} = 4.46\ \text{kN}\cdot\text{m} \qquad \textbf{Ans.}$$

Ans:
$M = 4.46\ \text{kN}\cdot\text{m}$

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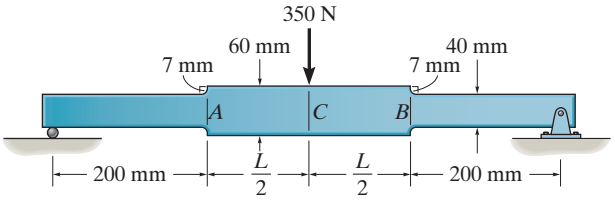
6–157. Determine the length L of the center portion of the bar so that the maximum bending stress at A, B, and C is the same. The bar has a thickness of 10 mm.



$$\frac{w}{h} = \frac{60}{40} = 1.5 \qquad \frac{r}{h} = \frac{7}{40} = 0.175$$

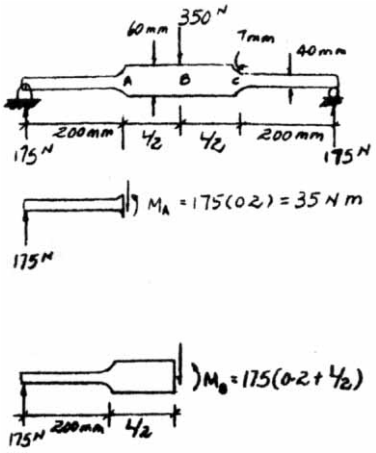
From Fig. 6-43, $K = 1.5$

$$(\sigma_A)_{\max} = K\frac{M_A c}{I} = 1.5\left[\frac{(35)(0.02)}{\frac{1}{12}(0.01)(0.04^3)}\right] = 19.6875 \text{ MPa}$$

$$(\sigma_B)_{\max} = (\sigma_A)_{\max} = \frac{M_B c}{I}$$

$$19.6875(10^6) = \frac{175(0.2 + \frac{L}{2})(0.03)}{\frac{1}{12}(0.01)(0.06^3)}$$

$$L = 0.95 \text{ m} = 950 \text{ mm}$$ **Ans.**



Ans:

$M = 87.0 \text{ kN} \cdot \text{m}$

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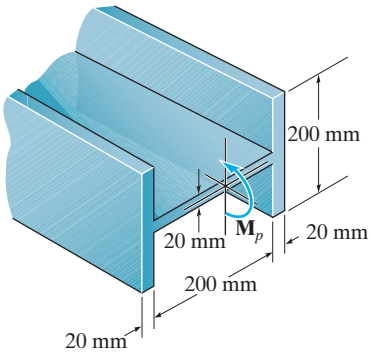
6–158. Determine the shape factor for the cross section of the H-beam.



$$I_x = \frac{1}{12}(0.2)(0.02^3) + 2\left(\frac{1}{12}\right)(0.02)(0.2^3) = 26.8(10^{-6})\text{m}^4$$

$$C_1 = T_1 = \sigma_Y(2)(0.09)(0.02) = 0.0036\sigma_y$$

$$C_2 = T_2 = \sigma_Y(0.01)(0.24) = 0.0024\sigma_y$$

$$M_p = 0.0036\sigma_Y(0.11) + 0.0024\sigma_Y(0.01) = 0.00042\sigma_Y$$

$$\sigma_Y = \frac{M_Y c}{I}$$

$$M_Y = \frac{\sigma_Y(26.8)(10^{-6})}{0.1} = 0.000268\sigma_Y$$

$$k = \frac{M_p}{M_Y} = \frac{0.00042\sigma_Y}{0.000268\sigma_Y} = 1.57$$ **Ans.**

Ans:
$I_x = 26.8\,(10^{-6})\,\text{m}^4$, $M_p = 0.00042\sigma_Y$,
$M_Y = 0.000268\sigma_Y$, $k = 1.57$

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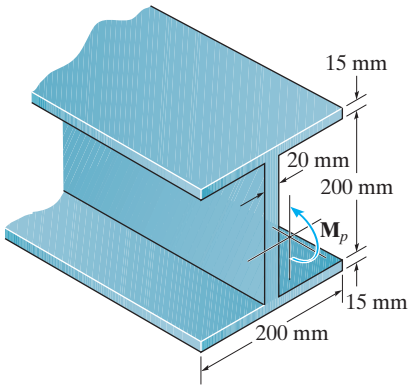
6–159.

Determine the shape factor for the wide-flange beam.



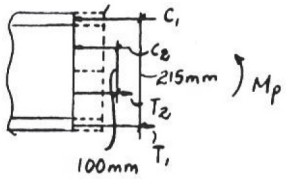
SOLUTION

$$I_x = \frac{1}{12}(0.2)(0.23)^3 - \frac{1}{12}(0.18)(0.2)^3 = 82.78333(10^{-6})\ \text{m}^4$$

$$C_1 = T_1 = \sigma_Y(0.2)(0.015) = 0.003\,\sigma_Y$$

$$C_2 = T_2 = \sigma_Y(0.1)(0.02) = 0.002\,\sigma_Y$$

$$M_p = 0.003\,\sigma_Y(0.215) + 0.002\,\sigma_Y(0.1) = 0.000845\,\sigma_Y$$



$$\sigma_Y = \frac{M_Y c}{I}$$

$$M_Y = \frac{\sigma_Y(82.78333)(10^{-6})}{0.115} = 0.000719855\,\sigma_Y$$

$$k = \frac{M_p}{M_Y} = \frac{0.000845\,\sigma_Y}{0.000719855\,\sigma_Y} = 1.17 \qquad \textbf{Ans.}$$

Ans:
$k = 1.17$

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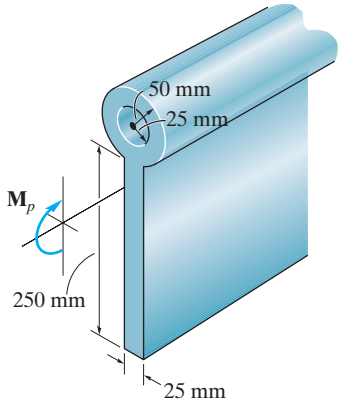
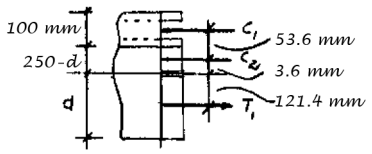
***6–160.** Determine the plastic moment $\mathbf{M}_p$ that can be supported by a beam having the cross section shown. $\sigma_Y = 210$ MPa.



$$\int \sigma \, dA = 0$$

$$C_1 + C_2 - T_1 = 0$$



$$[\pi(0.05^2 - 0.025^2)][210(10^6)] + [(0.25 - d)(0.025)][210(10^6)] - [d(0.025)][210(10^6)] = 0$$

$$d = 0.24281 \text{ m} < 0.25 \text{ m} \quad \text{(o.k.!)}$$

$$M_p = [\pi(0.05^2 - 0.025^2)][210(10^6)](0.0571903)$$

$$+ [0.0071903(0.025)][210(10^6)](0.0035951)$$

$$+ [0.242810(0.025)][210(10^6)](0.121405)$$

$$= 225.64(10^3) \text{ N} \cdot \text{m} = 226 \text{ kN} \cdot \text{m}$$ **Ans.**

Ans:
$M_p = 226 \text{ kN} \cdot \text{m}$.

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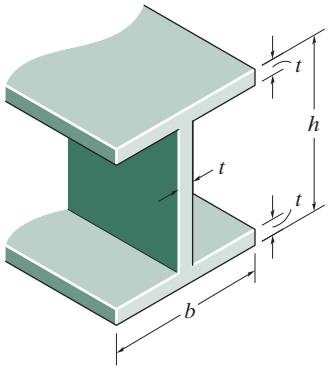
6–161.

The beam is made of an elastic perfectly plastic material for which $\sigma_Y = 250$ MPa. Determine the residual stress in the beam at its top and bottom after the plastic moment $\mathbf{M}_p$ is applied and then released.



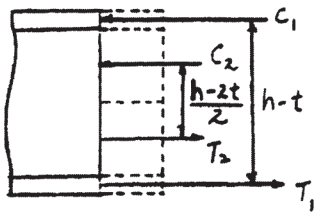
SOLUTION

Plastic analysis:

$$T_1 = C_1 = \sigma_Y bt; \qquad T_2 = C_2 = \sigma_Y\left(\frac{h-2t}{2}\right)t$$

$$M_p = \sigma_Y bt(h-t) + \sigma_Y\left(\frac{h-2t}{2}\right)(t)\left(\frac{h-2t}{2}\right)$$

$$= \sigma_Y\left[\, bt(h-t) + \frac{t}{4}(h-2t)^2 \,\right]$$



Elastic analysis:

$$I = \frac{1}{12}bh^3 - \frac{1}{12}(b-t)(h-2t)^3$$

$$= \frac{1}{12}\left[\, bh^3 - (b-t)(h-2t)^3 \,\right]$$

$$M_Y = \frac{\sigma_y I}{c} = \frac{\sigma_Y\left(\frac{1}{12}\right)[bh^3 - (b-t)(h-2t)^3]}{\frac{h}{2}}$$

$$= \frac{bh^3 - (b-t)(h-2t)^3}{6h}\sigma_Y$$

Shape factor:

$$k = \frac{M_P}{M_Y} = \frac{[bt(h-t) + \frac{t}{4}(h-2t)^2]\sigma_Y}{\frac{bh^3 - (b-t)(h-2t)^3}{6h}\sigma_Y}$$

$$= \frac{3h}{2}\left[\frac{4bt(h-t) + t(h-2t)^2}{bh^3 - (b-t)(h-2t)^3}\right]$$

Ans.

Ans:

$$k = \frac{3h}{2}\left[\frac{4bt(h-t) + t(h-2t)^2}{bh^3 - (b-t)(h-2t)^3}\right]$$

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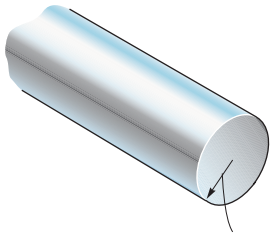
6–162.

The rod has a circular cross section. If it is made of an elastic perfectly plastic material, determine the shape factor.



SOLUTION

Maximum Elastic Moment: Applying the flexure formula with $\sigma = \sigma_Y$ and $I = \frac{\pi}{4} r^4 = \frac{\pi}{4}(0.1^4) = 25(10^{-6})\pi \text{ m}^4$,

$$\sigma = \frac{Mc}{I}; \qquad \sigma_Y = \frac{M_Y(0.1)}{25(10^{-6})\pi}$$

$$M_Y = 0.25\,(10^{-3})\pi\sigma_Y$$

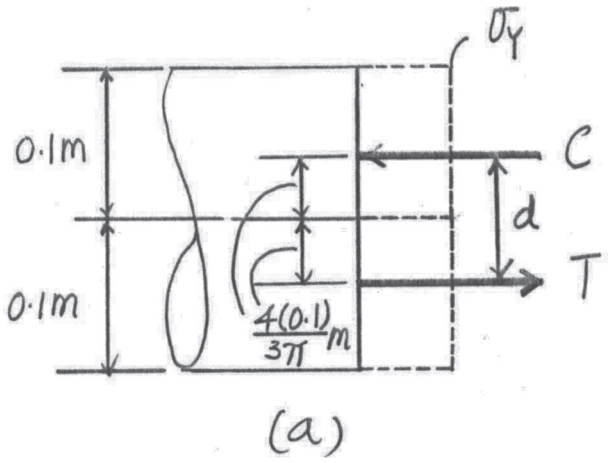
Plastic Moment: Referring to Fig. *a*,

$$C = T = \sigma_Y\left(\frac{\pi r^2}{2}\right) = \sigma_Y\left[\frac{\pi(0.1^2)}{2}\right] = 5(10^{-3})\pi\sigma_Y$$

$$d = 2\left(\frac{4r}{3\pi}\right) = 2\left[\frac{4(0.1)}{3\pi}\right] = \frac{0.26667}{\pi} = \text{m}$$

Thus,

$$M_p = Cd = (5(10^{-3})\pi\,\sigma_Y)\left(\frac{0.26667}{\pi}\right) = 1.3333(10^{-3})\sigma_Y$$



Shape Factor:

$$k = \frac{M_p}{M_Y} = \frac{1.3333(10^{-3})\sigma_Y}{0.25(10^{-3})\pi\sigma_Y} = 1.6976 = 1.70$$ **Ans.**

Ans:
$k = 1.70$

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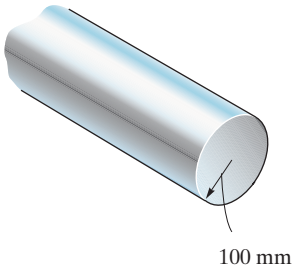
6–163.

The rod has a circular cross section. If it is made of an elastic perfectly plastic material where $\sigma_Y = 345$ MPa, determine the maximum elastic moment and plastic moment that can be applied to the cross section.



SOLUTION

Maximum Elastic Moment: Applying the flexure formula with $\sigma = \sigma_y = 345$ MPa and $I = \frac{\pi}{4} r^4 = \frac{\pi}{4}(0.1^4) = 25(10^{-6})\pi \text{ m}^4$,

$$\sigma = \frac{Mc}{I}; \qquad 345(10^6) = \frac{M_Y(0.1)}{25(10^{-6})\pi}$$

$$M_Y = 270.96(10^3) \text{ N} \cdot \text{m} = 271 \text{ kN} \cdot \text{m} \qquad \textbf{Ans.}$$

Plastic Moment: Referring to Fig. *a*,

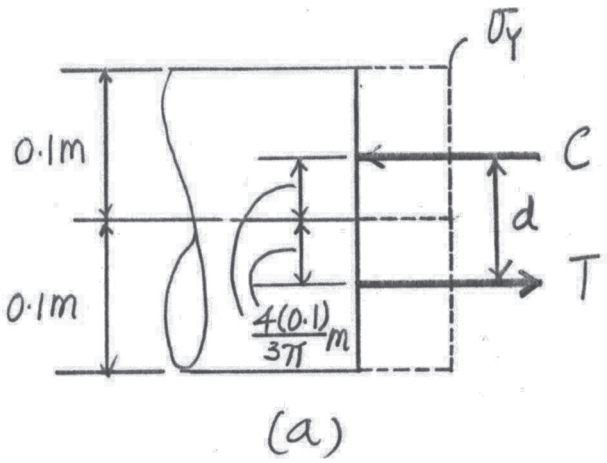
$$C = T = \sigma_y\left(\frac{\pi r^2}{2}\right) = 345(10^6)\left[\frac{\pi(0.1^2)}{2}\right] = 1.725(10^6)\pi \text{ N}$$

$$d = 2\left(\frac{4r}{3\pi}\right) = 2\left[\frac{4(0.1)}{3\pi}\right] = \frac{0.26667}{\pi} \text{ m}$$

Thus,

$$M_P = Cd = \left[1.725(10^6)\pi\right]\left[\frac{0.26667}{\pi}\right]$$

$$= 460(10^3) \text{ N} \cdot \text{m} = 460 \text{ kN} \cdot \text{m} \qquad \textbf{Ans.}$$



Ans:
$M_Y = 271 \text{ kN} \cdot \text{m}$,
$M_P = 460 \text{ kN} \cdot \text{m}$

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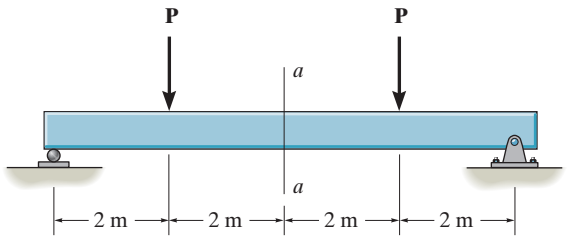
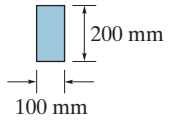
***6–164.**

The beam is made of an elastic perfectly plastic material for which $\sigma_Y = 200$ MPa. If the largest moment in the beam occurs within the center section *a–a*, determine the magnitude of each force **P** that causes this moment to be (a) the largest elastic moment and (b) the largest plastic moment.





SOLUTION

$M = 2P$ **(1)**

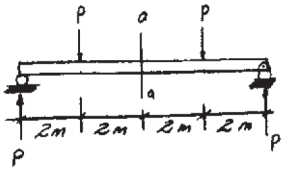
a) Elastic moment

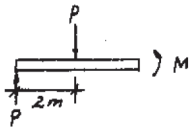
$$I = \frac{1}{12}(0.1)(0.2^3) = 66.667(10^{-6})\ \text{m}^4$$

$$\sigma_Y = \frac{M_Y c}{I}$$

$$M_Y = \frac{200(10^6)(66.667)(10^{-6})}{0.1}$$

$$= 133.33\ \text{kN} \cdot \text{m}$$





From Eq. (1)

$133.33 = 2P$

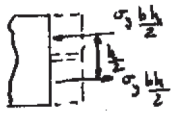
$P = 66.7$ kN **Ans.**

b) Plastic moment

$$M_p = \frac{b h^2}{4}\sigma_Y$$

$$= \frac{0.1(0.2^2)}{4}(200)(10^6)$$

$$= 200\ \text{kN} \cdot \text{m}$$



From Eq. (1)

$200 = 2P$

$P = 100$ kN **Ans.**

Ans:
Elastic: $P = 66.7$ kN,
Plastic: $P = 100$ kN

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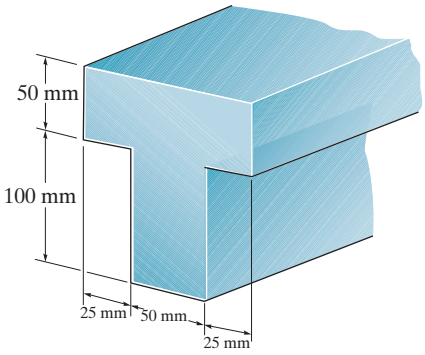
6–165. Determine the shape factor of the beam's cross section.



Referring to Fig. *a*, the location of centroid of the cross-section is

$$\bar{y} = \frac{\Sigma \bar{y} A}{\Sigma A} = \frac{0.125(0.1)(0.05) + 0.05(0.05)(0.1)}{0.1(0.05) + 0.05(0.1)} = 0.0875 \text{ m}$$

The moment of inertia of the cross-section about the neutral axis is

$$I = \frac{1}{12}(0.05)(0.1^3) + 0.05(0.1)(0.0875 - 0.05)^2$$

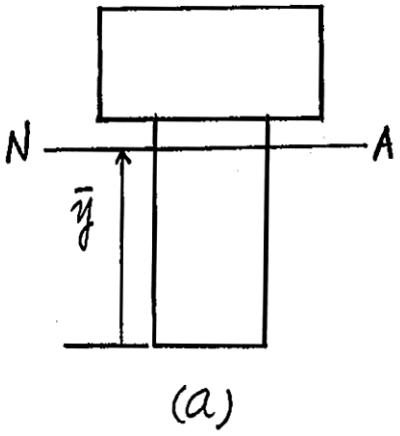
$$+ \frac{1}{12}(0.1)(0.05^3) + 0.1(0.05)(0.125 - 0.0875)^2$$

$$= 19.27083(10^{-6}) \text{ m}^4$$

Here $\sigma_{\max} = \sigma_Y$ and $c = \bar{y} = 0.0875$ m. Thus

$$\sigma_{\max} = \frac{Mc}{I}; \qquad \sigma_Y = \frac{M_Y(0.0875)}{19.27083(10^{-6})}$$

$$M_Y = 0.22024(10^{-3})\sigma_Y$$



(a)

Referring to the stress block shown in Fig. *b*,

$$\int_A \sigma dA = 0; \qquad T - C_1 - C_2 = 0$$

$$[d(0.05)]\sigma_Y - [(0.1 - d)(0.05)]\sigma_Y - [0.05(0.1)]\sigma_Y = 0$$

$$d = 0.1 \text{ m}$$

Since $d = 0.1$ m, $c_1 = 0$, Fig. c. Here

$$T = C = [0.05(0.1)]\sigma_Y = 0.005\sigma_Y$$

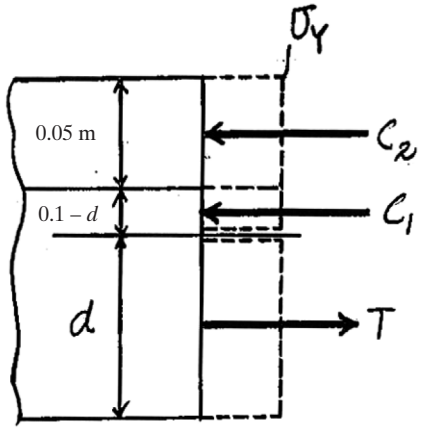
Thus,

$$M_p = T(0.075) = 0.005\sigma_Y(0.075) = 0.375(10^{-3})\sigma_Y$$

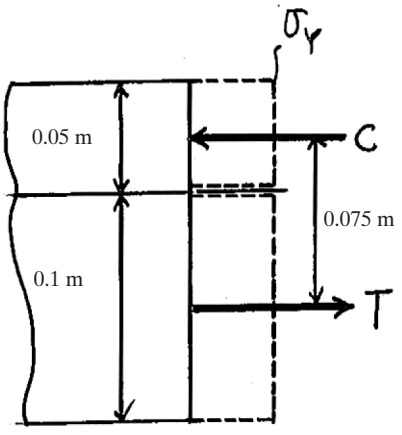
Thus,

$$k = \frac{M_p}{M_Y} = \frac{0.375(10^{-3})\sigma_Y}{0.22024(10^{-3})\sigma_Y} = 1.7027 = 1.70$$

Ans.



(b)



(c)

Ans:
$k = 1.70$

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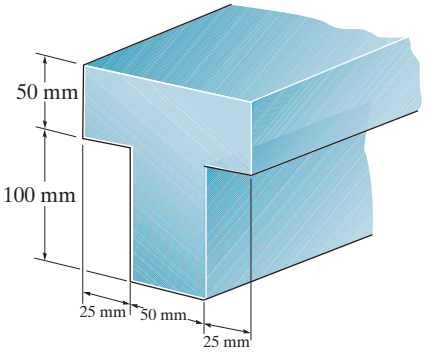
6–166. The beam is made of elastic-perfectly plastic material. Determine the maximum elastic moment and the plastic moment that can be applied to the cross section. Take $\sigma_Y = 250$ MPa.



Referring to Fig. *a*, the location of centroid of the cross-section is

$$\bar{y} = \frac{\Sigma \bar{y} A}{\Sigma A} = \frac{0.125(0.1)(0.05) + 0.05(0.05)(0.1)}{0.1(0.05) + 0.05(0.1)} = 0.0875 \text{ m}$$

The moment of inertia of the cross-section about the neutral axis is

$$I = \frac{1}{12}(0.05)(0.1^3) + 0.05(0.1)(0.0875 - 0.05)^2$$

$$+ \frac{1}{12}(0.1)(0.05^3) + 0.1(0.05)(0.125 - 0.0875)^2$$

$$= 19.27083(10^{-6}) \text{ m}^4$$

Here $\sigma_{\max} = \sigma_Y = 250$ MPa and $\not{c} = \bar{y} = 0.0875$ m. Then

$$\sigma_{\max} = \frac{Mc}{I}; \qquad 250(10^6) = \frac{M_Y(0.0875)}{19.27083(10^{-6})}$$

$$M_Y = 55.06(10^3) \text{ N} \cdot \text{m} = 55.1 \text{ kN} \cdot \text{m}$$ **Ans.**

Referring to the stress block shown in Fig. *b*,

$$\int_A \sigma dA = 0; \qquad T - C_1 - C_2 = 0$$

$$[d(0.05)][250(10^6)] - [(0.1 - d)(0.05)][250(10^6)] - [0.05(0.1)][250(10^6)] = 0$$

$$d = 0.1 \text{ m}$$

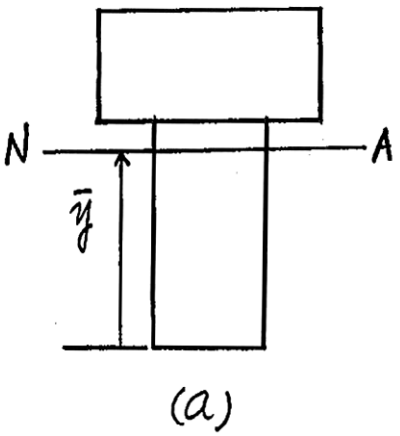
Since $d = 0.1$ m, $c_1 = 0$,

Here,

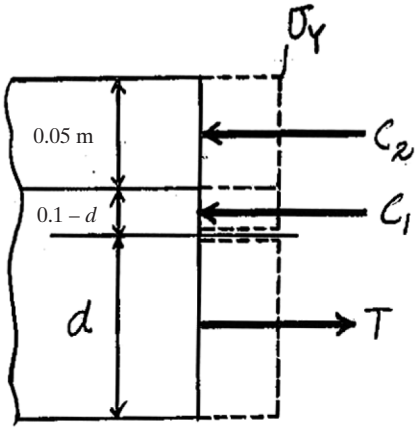
$$T = C = [0.05(0.1)][250(10^6)] = 1250(10^3) \text{ N} = 1250 \text{ kN}$$

Thus,

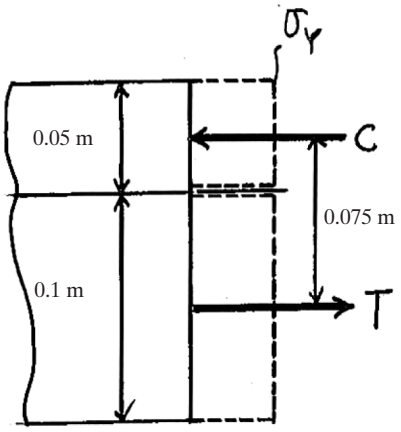
$$M_p = T(0.075) = (1250)(0.075) = 93.75 \text{ kN}$$ **Ans.**



(a)



(b)



(c)

Ans:

$M_Y = 55.1 \text{ kN} \cdot \text{m}$, $M_P = 93.75 \text{ kN} \cdot \text{m}$

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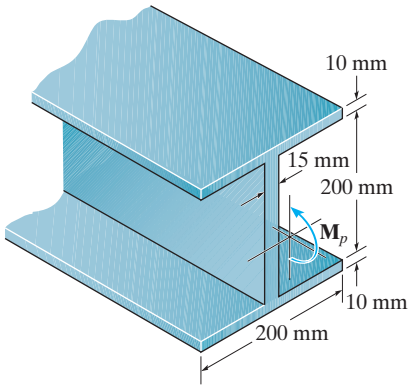
6–167.

Determine the shape factor for the beam.



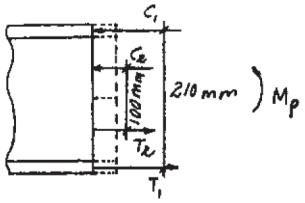
SOLUTION

$$I = \frac{1}{12}(0.2)(0.22)^3 - \frac{1}{12}(0.185)(0.2)^3 = 54.133(10^{-6})\ \text{m}^4$$

$$C_1 = \sigma_Y(0.01)(0.2) = (0.002)\,\sigma_Y$$

$$C_2 = \sigma_Y(0.1)(0.015) = (0.0015)\,\sigma_Y$$

$$M_p = 0.002\,\sigma_Y(0.21) + 0.0015\,\sigma_Y(0.1) = 0.0005\,\sigma_Y$$



$$\sigma_Y = \frac{M_Y c}{I}$$

$$M_Y = \frac{\sigma_Y(54.133)(10^{-6})}{0.11} = 0.000492\,\sigma_Y$$

$$k = \frac{M_p}{M_Y} = \frac{0.00057\,\sigma_Y}{0.000492\,\sigma_Y} = 1.16 \qquad \textbf{Ans.}$$

Ans:
$k = 1.16$

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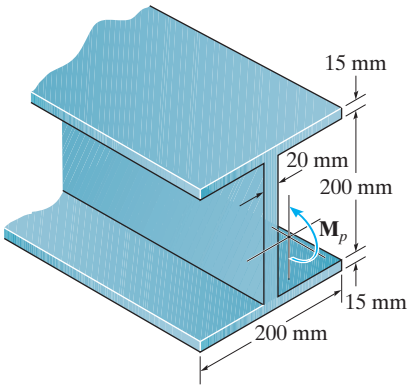
***6–168.**

The beam is made of an elastic perfectly plastic material for which $\sigma_Y = 250$ MPa. Determine the residual stress in the beam at its top and bottom after the plastic moment $\mathbf{M}_p$ is applied and then released.



SOLUTION

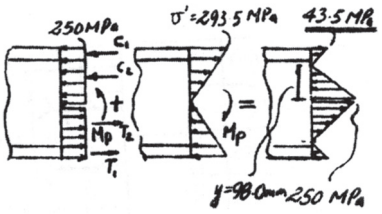
$$I_x = \frac{1}{12}(0.2)(0.23)^3 - \frac{1}{12}(0.18)(0.2)^3 = 82.78333(10^{-6})\ \text{m}^4$$

$$C_1 = T_1 = \sigma_Y(0.2)(0.015) = 0.003\,\sigma_Y$$

$$C_2 = T_2 = \sigma_Y(0.1)(0.02) = 0.002\,\sigma_Y$$

$$M_p = 0.003\,\sigma_Y(0.215) + 0.002\,\sigma_Y(0.1) = 0.000845\,\sigma_Y$$

$$= 0.000845(250)(10^6) = 211.25\ \text{kN}\cdot\text{m}$$



$$\sigma' = \frac{M_p c}{I} = \frac{211.25(10^3)(0.115)}{82.78333(10^{-6})} = 293.5\ \text{MPa}$$

$$\frac{y}{250} = \frac{0.115}{293.5}; \qquad y = 0.09796\ \text{m} = 98.0\ \text{mm}$$

$$\sigma_{\text{top}} = \sigma_{\text{bottom}} = 293.5 - 250 = 43.5\ \text{MPa}$$ **Ans.**

Ans:
$\sigma_{\text{top}} = \sigma_{\text{bottom}} = 43.5$ MPa

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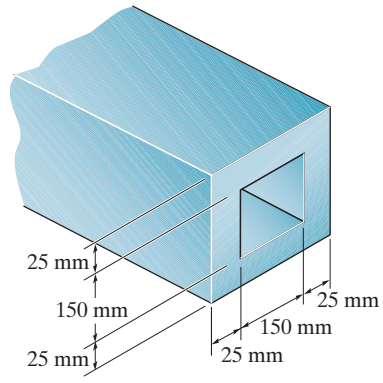
6–169.

The box beam is made of an elastic perfectly plastic material for which $\sigma_Y = 250$ MPa. Determine the residual stress in the top and bottom of the beam after the plastic moment $\mathbf{M}_p$ is applied and then released.



SOLUTION

Plastic Moment:

$$M_P = 250(10^6)\,(0.2)(0.025)(0.175) + 250(10^6)\,(0.075)(0.05)(0.075)$$

$$= 289062.5\ \text{N}\cdot\text{m}$$

Modulus of Rupture: The modulus of rupture σ_r can be determined using the flexure formula with the application of reverse, plastic moment $M_P = 289062.5\ \text{N}\cdot\text{m}$.

$$I = \frac{1}{12}(0.2)(0.2^3) - \frac{1}{12}(0.15)(0.15^3)$$

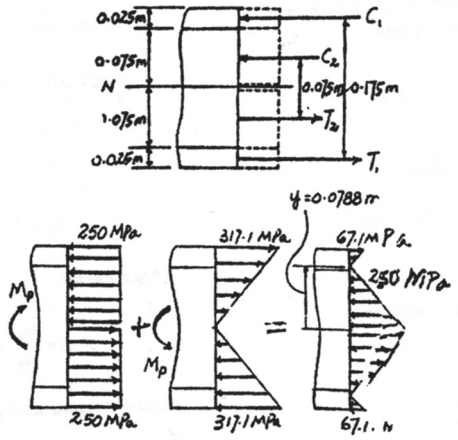
$$= 91.14583(10^{-6})\ \text{m}^4$$

$$\sigma_r = \frac{M_P\, c}{I} = \frac{289062.5\,(0.1)}{91.14583(10^{-6})} = 317.41\ \text{MPa}$$

Residual Bending Stress: As shown on the diagram.

$$\sigma'_{\text{top}} = \sigma'_{\text{bot}} = \sigma_r - \sigma_Y$$

$$= 317.14 - 250 = 67.1\ \text{MPa} \qquad \textbf{Ans.}$$



Ans:

$\sigma_{\text{top}} = \sigma_{\text{bottom}} = 67.1$ MPa

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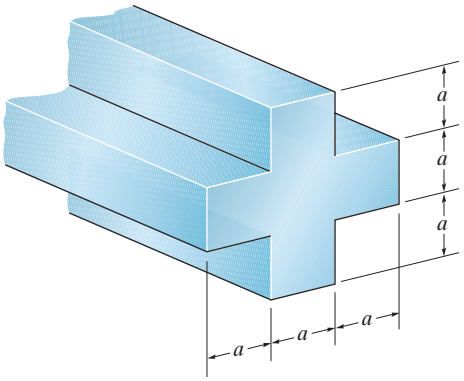
6–170.

Determine the shape factor of the cross section.



SOLUTION

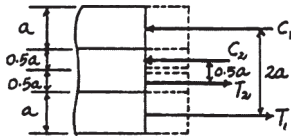
Maximum Elastic Moment: The moment of inertia about neutral axis must be determined first.

$$I_{NA} = \frac{1}{12}(a)(3a)^3 + \frac{1}{12}(2a)(a^3) = 2.41667\,a^4$$

Applying the flexure formula with $\sigma = \sigma_Y$, we have

$$\sigma_Y = \frac{M_Y c}{I}$$

$$M_Y = \frac{\sigma_Y I}{c} = \frac{\sigma_Y(2.41667a^4)}{1.5a} = 1.6111a^3\sigma_Y$$



Plastic Moment:

$$M_p = \sigma_Y(a)(a)(2a) + \sigma_Y(0.5a)(3a)(0.5a)$$

$$= 2.75a^3\sigma_Y$$

Shape Factor:

$$k = \frac{M_p}{M_Y} = \frac{2.75a^3\sigma_Y}{1.6111a^3\sigma_Y} = 1.71$$ **Ans.**

Ans:
$k = 1.71$

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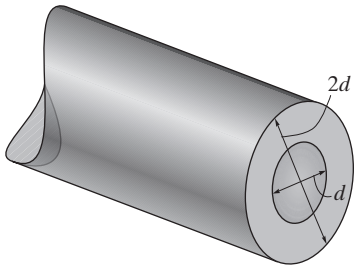
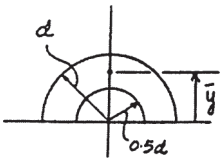
6–171.

Determine the shape factor for the member having the tubular cross section.



SOLUTION

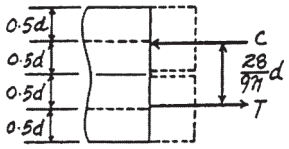
Maximum Elastic Moment: The moment of inertia about neutral axis must be determined first.



$$I_{NA} = \frac{\pi}{4}d^4 - \frac{\pi}{4}\left(\frac{d}{2}\right)^4 = \frac{15\pi}{64}d^4$$

Applying the flexure formula with $\sigma = \sigma_Y$, we have

$$\sigma_Y = \frac{M_Y\, c}{I}$$



$$M_Y = \frac{\sigma_Y\, I}{c} = \frac{\sigma_Y\left(\frac{15\pi}{64}d^4\right)}{d} = \frac{15\pi}{64}d^3\,\sigma_Y$$

Plastic Moment:

$$\bar{y} = \frac{\Sigma \bar{y}A}{\Sigma A} = \frac{\frac{4d}{3\pi}\left(\frac{\pi d^2}{2}\right) - \frac{4\left(\frac{d}{2}\right)}{3\pi}\left(\frac{\frac{\pi}{4}d^2}{2}\right)}{\frac{\pi d^2}{2} - \frac{\frac{\pi}{4}d^2}{2}} = \frac{14}{9\pi}d$$

$$M_P = \sigma_Y\left(\frac{\pi d^2}{2} - \frac{\frac{\pi}{4}d^2}{2}\right)\frac{28}{9\pi}d = \frac{7}{6}d^3\,\sigma_Y$$

Shape Factor:

$$k = \frac{M_P}{M_Y} = \frac{\frac{7}{6}d^3\,\sigma_Y}{\frac{15\pi}{64}d^3\,\sigma_Y} = 1.58 \qquad \textbf{Ans.}$$

Ans:
$k = 1.58$

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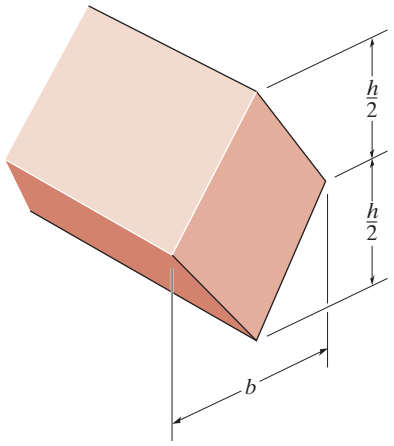
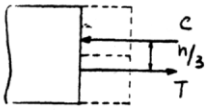
***6–172.** Determine the shape factor for the member.



Plastic analysis:

$$T = C = \frac{1}{2}(b)\left(\frac{h}{2}\right)\sigma_Y = \frac{b\,h}{4}\,\sigma_Y$$

$$M_P = \frac{b\,h}{4}\,\sigma_Y\left(\frac{h}{3}\right) = \frac{b\,h^2}{12}\,\sigma_Y$$



Elastic analysis:

$$I = 2\left[\frac{1}{12}(b)\left(\frac{h}{2}\right)^3\right] = \frac{b\,h^3}{48}$$

$$M_Y = \frac{\sigma_Y I}{c} = \frac{\sigma_Y\left(\frac{bh^3}{48}\right)}{\frac{h}{2}} = \frac{b\,h^2}{24}\,\sigma_Y$$

Shape factor:

$$k = \frac{M_p}{M_Y} = \frac{\frac{bh^2}{12}\,\sigma_Y}{\frac{bh^2}{24}\,\sigma_Y} = 2$$ **Ans.**

Ans:

$$k = \frac{M_p}{M_Y} = \frac{\frac{bh^2}{12}\,\sigma_Y}{\frac{bh^2}{24}\,\sigma_Y} = 2$$

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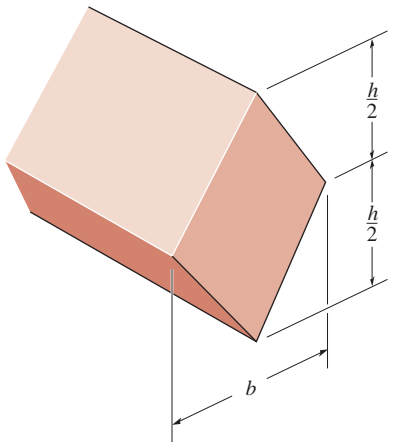
6–173. The member is made from an elastic-plastic material. Determine the maximum elastic moment and the plastic moment that can be applied to the cross section. Take $b = 100$ mm, $h = 150$ mm, $\sigma_Y = 250$ MPa.

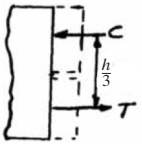


Elastic analysis:

$$I = 2\left[\frac{1}{12}(0.1)(0.075^3)\right] = 7.03125(10^{-6})\ \text{m}^4$$

$$M_Y = \frac{\sigma_Y I}{c} = \frac{[250(10^6)][7.03125(10^{-6})]}{0.075} = 23.4375(10^3)\ \text{N}\cdot\text{m} = 23.4\ \text{kN}\cdot\text{m}$$ **Ans.**

Plastic analysis:



$$T = C = \left[\frac{1}{2}(0.1)(0.075)\right][250(10^6)] = 937.5(10^3)\ \text{N}$$

$$M_p = T\left(\frac{h}{3}\right) = [937.5(10^3)]\left(\frac{0.15}{3}\right) = 46.875(10^3)\ \text{N}\cdot\text{m} = 46.9\ \text{kN}\cdot\text{m}$$ **Ans.**

Ans:

$M_Y = 23.4\ \text{kN}\cdot\text{m}$, $M_P = 46.9\ \text{kN}\cdot\text{m}$

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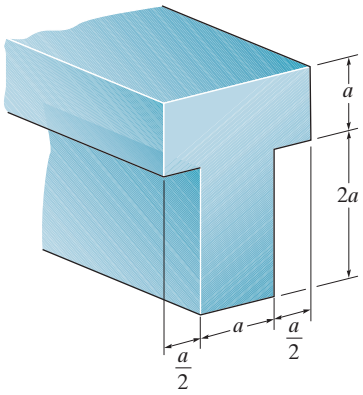
6–174.

Determine the shape factor of the cross section.



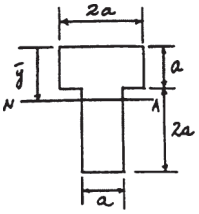
SOLUTION

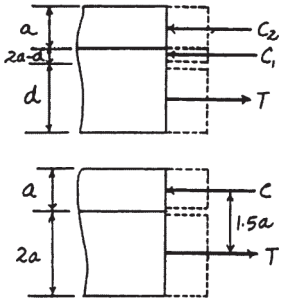
Maximum Elastic Moment: The centroid and the moment of inertia about neutral axis must be determined first.

$$\bar{y} = \frac{\Sigma \bar{y} A}{\Sigma A} = \frac{0.5a(a)\,(2a) + 2a(2a)\,(a)}{a(2a) + 2a(a)} = 1.25a$$

$$I_{NA} = \frac{1}{12}(2a)\,(a^3) + 2a(a)\,(1.25a - 0.5a)^2 + \frac{1}{12}(a)\,(2a)^3 + a(2a)\,(2a - 1.25a)^2$$

$$= 3.0833a^4$$





Applying the flexure formula with $\sigma = \sigma_Y$, we have

$$\sigma_Y = \frac{M_Y c}{I}$$

$$M_Y = \frac{\sigma_Y I}{c} = \frac{\sigma_Y (3.0833a^4)}{(3a - 1.25a)}$$

$$= 1.7619a^3\,\sigma_Y$$

Plastic Moment:

$$\int_A \sigma dA = 0; \qquad T - C_1 - C_2 = 0$$

$$\sigma_Y\,(d)\,(a) - \sigma_Y\,(2a - d)\,(a) - \sigma_Y\,(a)\,(2a) = 0$$

$$d = 2a$$

$$M_P = \sigma_Y\,(2a)\,(a)\,(1.5a) = 3.00a^3\,\sigma_Y$$

Shape Factor:

$$k = \frac{M_P}{M_Y} = \frac{3.00a^3\,\sigma_Y}{1.7619a^3\,\sigma_Y} = 1.70 \qquad \textbf{Ans.}$$

Ans:
$k = 1.70$

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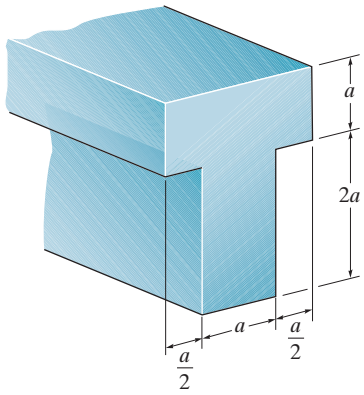
6–175.

The beam is made of elastic perfectly plastic material. Determine the maximum elastic moment and the plastic moment that can be applied to the cross section. Take $a = 50$ mm and $\sigma_Y = 230$ MPa.



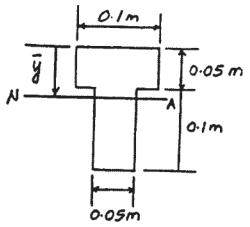
SOLUTION

Maximum Elastic Moment: The centroid and the moment of inertia about neutral axis must be determined first.

$$\bar{y} = \frac{\Sigma \bar{y} A}{\Sigma A} = \frac{0.025(0.05)\,(0.1) + 0.1\,(0.1)\,(0.05)}{0.05(0.1) + 0.1(0.05)} = 0.0625 \text{ m}$$

$$I_{NA} = \frac{1}{12}(0.1)\left(0.05^3\right) + 0.1(0.05)\,(0.0625 - 0.025)^2 + \frac{1}{12}(0.05)\left(0.1^3\right) + 0.05(0.1)\,(0.1 - 0.0625)^2$$

$$= 19.2709\left(10^{-6}\right) \text{ m}^4$$

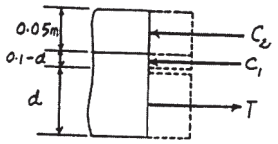


Applying the flexure formula with $\sigma = \sigma_Y$, we have

$$\sigma_Y = \frac{M_Y c}{I}$$

$$M_Y = \frac{\sigma_Y I}{c} = \frac{230\left(10^6\right)(19.2709)\left(10^{-6}\right)}{(0.15 - 0.0625)}$$

$$= 50654.8 \text{ N} \cdot \text{m} = 50.7 \text{ kN} \cdot \text{m} \qquad \textbf{Ans.}$$



Plastic Moment:

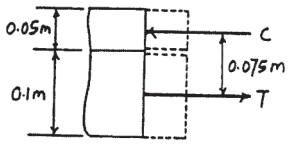
$$\int_A \sigma dA = 0; \qquad T - C_1 - C_2 = 0$$

$$\sigma_Y (d)\,(0.05) - \sigma_Y (0.1 - d)(0.05) - \sigma_Y (0.05)(0.1) = 0$$

$$d = 0.100 \text{ m}$$

$$M_P = 230\left(10^6\right)(0.100)\,(0.05)\,(0.075)$$

$$= 86250 \text{ N} \cdot \text{m} = 86.25 \text{ kN} \cdot \text{m} \qquad \textbf{Ans.}$$



Ans:
$M_Y = 50.7 \text{ kN} \cdot \text{m}$,
$M_P = 86.25 \text{ kN} \cdot \text{m}$

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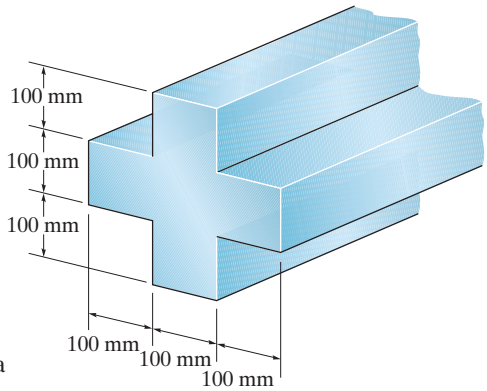
***6–176.**

The beam is made of elastic perfectly plastic material for which $\sigma_Y = 345$ MPa. Determine the maximum elastic moment and the plastic moment that can be applied to the cross section.



SOLUTION

Maximum Elastic Moment: Applying the flexure formula with $\sigma = \sigma_y = 345$ MPa and $I = \frac{1}{12}(0.1)(0.3^3) + \frac{1}{12}(0.2)(0.1^3) = 0.24167(10^{-3})\ \text{m}^4$,

$$\sigma = \frac{Mc}{I}; \qquad 345(10^6) = \frac{M_Y(0.15)}{0.24167(10^{-3})}$$

$$M_Y = 555.83(10^3)\ \text{N}\cdot\text{m} = 556\ \text{kN}\cdot\text{m} \qquad \textbf{Ans.}$$

Plastic Moment: Referring to Fig. *a*,

$$C_1 = T_1 = \sigma_y(0.1)(0.1) = 0.01\sigma_y = 0.01\left[345(10^6)\right] = 3.45(10^6)\ \text{N}$$

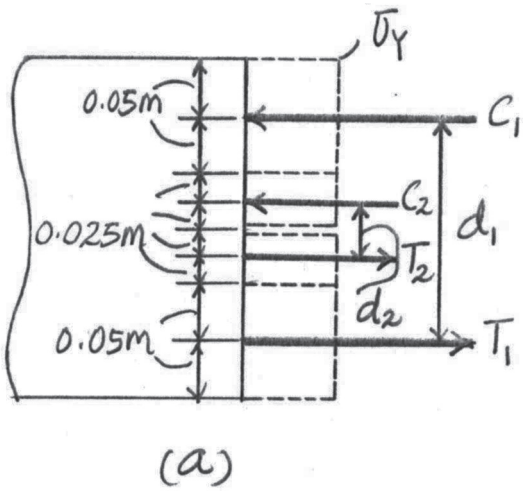
$$C_2 = T_2 = \sigma_y(0.3)(0.05) = 0.015\sigma_y = 0.015\left[345(10^6)\right] = 5.175(10^6)\ \text{N}$$

$$d_1 = 4(0.05) = 0.2\ \text{m} \qquad d_2 = 2(0.025) = 0.05\ \text{m}$$

Thus,

$$M_P = C_1 d_1 + C_2 d_2 = \left[3.45(10^6)\right](0.2) + \left[5.175(10^6)\right](0.05)$$

$$= 948.75(10^3)\ \text{N}\cdot\text{m} = 949\ \text{kN}\cdot\text{m} \qquad \textbf{Ans.}$$



(a)

Ans:
$M_Y = 556\ \text{kN}\cdot\text{m}$,
$M_P = 949\ \text{kN}\cdot\text{m}$

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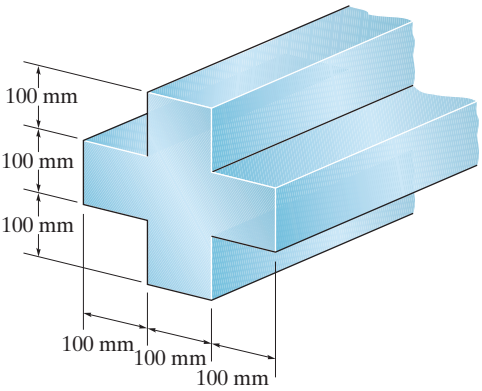
6–177.

Determine the shape factor of the cross section.



SOLUTION

Maximum Elastic Moment: Applying the flexure formula with $\sigma = \sigma_y$ and $I = \frac{1}{12}(0.1)(0.3^3) + \frac{1}{12}(0.2)(0.1^3) = 0.24167(10^{-3})\ \text{m}^4$,

$$\sigma = \frac{Mc}{I}; \qquad \sigma_y = \frac{M_y(0.15)}{0.24167(10^{-3})} \qquad M_y = 1.6111(10^{-3})\sigma_y$$

Plastic Moment: Referring to Fig. *a*,

$$C_1 = T_1 = \sigma_y(0.1)(0.1) = 0.01\,\sigma_y$$

$$C_2 = T_2 = \sigma_y(0.3)(0.05) = 0.015\,\sigma_y$$

$$d_1 = 4(0.05) = 0.2\ \text{m}$$

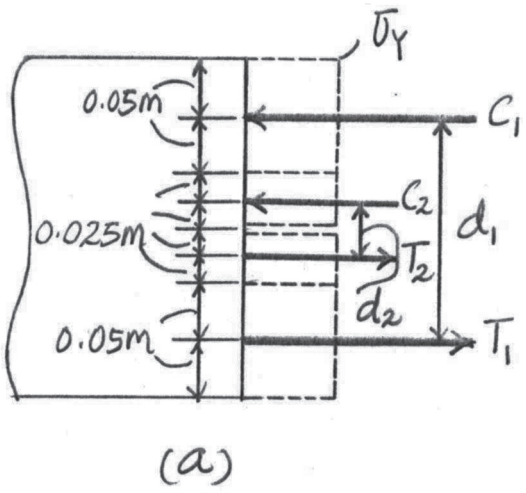
$$d_2 = 2(0.025) = 0.05\ \text{m}$$

Thus,

$$M_p = C_1 d_1 + C_2 d_2 = (0.01\,\sigma_y)(0.2) + (0.015\,\sigma_y)(0.05)$$
$$= 2.75\,(10^{-3})\sigma_y$$

Shape Factor:

$$k = \frac{M_p}{M_y} = \frac{2.75(10^{-3})\sigma_y}{1.6111(10^{-3})\sigma_y} = 1.7069 = 1.71 \qquad \textbf{Ans.}$$



Ans:
$k = 1.71$

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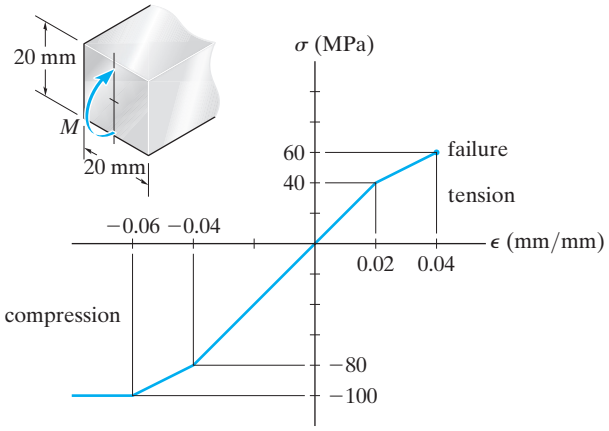
6–178.

The plexiglass bar has a stress–strain curve that can be approximated by the straight-line segments shown. Determine the largest moment M that can be applied to the bar before it fails.

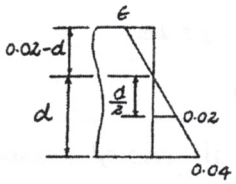


SOLUTION

Ultimate Moment:

$$\int_A \sigma\, dA = 0; \quad C - T_2 - T_1 = 0$$

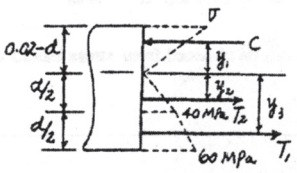
$$\sigma\left[\frac{1}{2}(0.02 - d)(0.02)\right] - 40(10^6)\left[\frac{1}{2}\left(\frac{d}{2}\right)(0.02)\right] - \frac{1}{2}(60 + 40)(10^6)\left[(0.02)\frac{d}{2}\right] = 0$$



Since $\frac{0.04}{d} = \frac{\epsilon}{0.02-d}$, then $0.02 - d = 25\epsilon d$.

And since $\frac{\sigma}{\epsilon} = \frac{40(10^6)}{0.02} = 2(10^9)$, then $\epsilon = \frac{\sigma}{2(10^9)}$.

So then $0.02 - d = \frac{23\sigma d}{2(10^9)} = 1.25(10^{-8})\sigma d$.



Substituting for $0.02 - d$, then solving for σ, yields $\sigma = 74.833$ MPa. Then $\epsilon = 0.037417$ mm/mm and $d = 0.010334$ m.

Therefore,

$$C = 74.833\,(10^6)\left[\frac{1}{2}(0.02 - 0.010334)(0.02)\right] = 7233.59\text{ N}$$

$$T_1 = \frac{1}{2}(60 + 40)\,(10^6)\left[(0.02)\left(\frac{0.010334}{2}\right)\right] = 5166.85\text{ N}$$

$$T_2 = 40(10^6)\left[\frac{1}{2}(0.02)\left(\frac{0.010334}{2}\right)\right] = 2066.74\text{ N}$$

$$y_1 = \frac{2}{3}(0.02 - 0.010334) = 0.0064442\text{ m}$$

$$y_2 = \frac{2}{3}\left(\frac{0.010334}{2}\right) = 0.0034445\text{ m}$$

$$y_3 = \frac{0.010334}{2} + \left[1 - \frac{1}{3}\left(\frac{2(40) + 60}{40 + 60}\right)\right]\left(\frac{0.010334}{2}\right) = 0.0079225\text{ m}$$

$$M = 7233.59(0.0064442) + 2066.74(0.0034445) + 5166.85(0.0079255)$$

$$= 94.7\text{ N}\cdot\text{m} \qquad \textbf{Ans.}$$

Ans:
$M = 94.7\text{ N}\cdot\text{m}$

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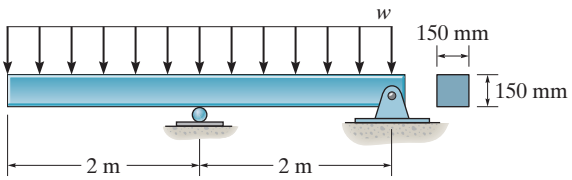
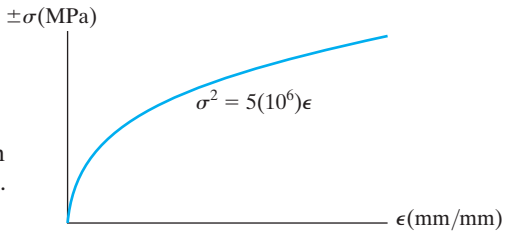
6–179.

The beam is made of phenolic, a structural plastic, that has the stress–strain curve shown. If a portion of the curve can be represented by the equation $\sigma = (5(10^6)\epsilon)^{1/2}$ MPa, determine the magnitude w of the distributed load that can be applied to the beam without causing the maximum strain in its fibers at the critical section to exceed $\epsilon_{max} = 0.005$ mm/mm.





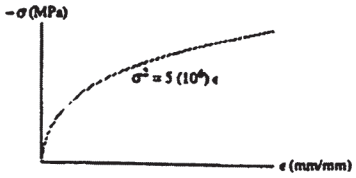
SOLUTION

Resultant Internal Forces: The resultant internal forces T and C can be evaluated from the volume of the stress block which is a paraboloid. When $\epsilon = 0.005$ mm/mm, then

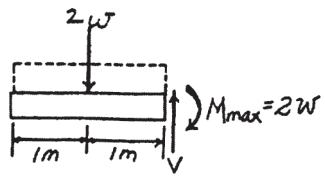
$$\sigma = \sqrt{5(10^6)(0.005)} = 158.11 \text{ MPa}$$

$$T = C = \frac{2}{3}\left[158.11(10^6)(0.075)\right](0.150) = 1.1859 \text{ MN}$$

$$d = 2\left[\frac{3}{5}(0.075)\right] = 0.090 \text{ m}$$



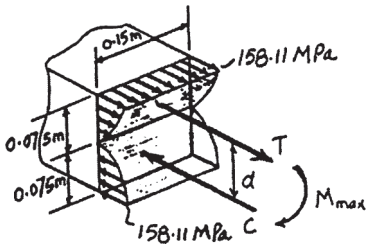
Maximum Internal Moment: The maximum internal moment $M = 2w$ occurs at the overhang support as shown on FBD.



$$M_{max} = Td$$

$$2w = 1.1859(10^6)(0.090)$$

$$w = 53363 \text{ N/m} = 53.4 \text{ kN/m} \quad \textbf{Ans.}$$



Ans:
$w = 53.4$ kN/m

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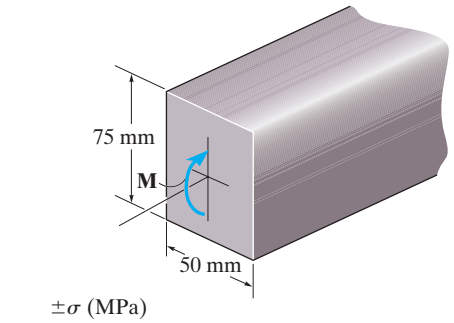
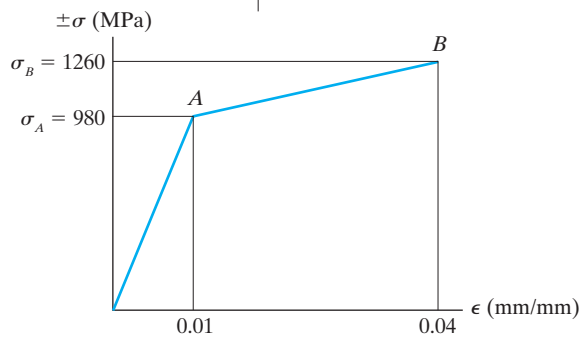
***6–180.** The stress–strain diagram for a titanium alloy can be approximated by the two straight lines. If a strut made of this material is subjected to bending, determine the moment resisted by the strut if the maximum stress reaches a value of (a) σ_A and (b) σ_B.





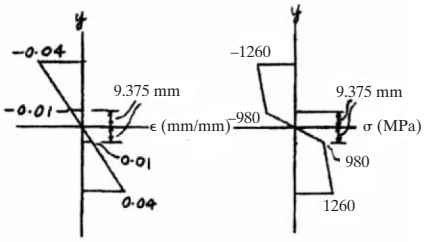
a) ***Maximum Elastic Moment :*** Since the stress is linearly related to strain up to point A, the flexure formula can be applied.

$$\sigma_A = \frac{Mc}{I}$$

$$M = \frac{\sigma_A I}{c}$$

$$= \frac{[980(10^6)]\left[\frac{1}{12}(0.05)(0.075^3)\right]}{0.0375}$$

$$= 45.9375(10^3)\ \text{N}\cdot\text{m} = 45.9\ \text{kN}\cdot\text{m} \qquad \textbf{Ans.}$$



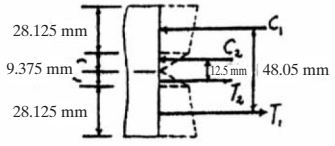
b) ***The Ultimate Moment :***

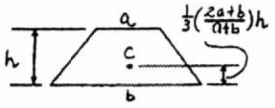
$$C_1 = T_1 = \frac{1}{2}[(1260 + 980)(10^6)](0.028125)(0.05) = 1575(10^3)\ \text{N}$$

$$C_2 = T_2 = \frac{1}{2}[980(10^6)](0.009375)(0.05) = 229.6875(10^3)\ \text{N}$$

$$M = [1575(10^3)](0.04805) + [229.6875(10^3)](0.0125)$$

$$= 78.54(10^3)\ \text{N}\cdot\text{m} = 78.5\ \text{kN}\cdot\text{m} \qquad \textbf{Ans.}$$





Note: The centroid of a trapezodial area was used in calculation of moment.

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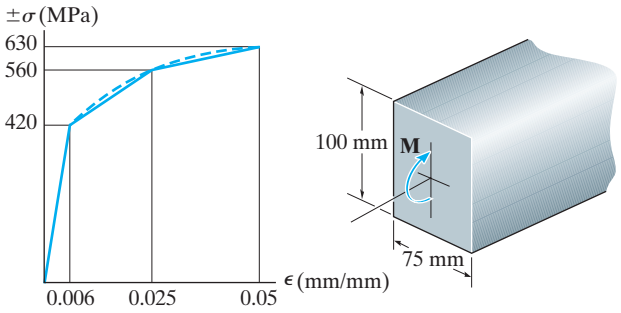
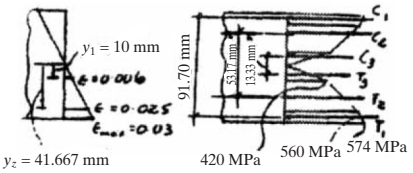
6–181. The bar is made of an aluminum alloy having a stress–strain diagram that can be approximated by the straight line segments shown. Assuming that this diagram is the same for both tension and compression, determine the moment the bar will support if the maximum strain at the top and bottom fibers of the beam is $\epsilon_{max} = 0.03$.



$$\frac{\sigma - 560}{0.03 - 0.025} = \frac{630 - 560}{0.05 - 0.025}; \qquad \sigma = 574 \text{ MPa}$$



$$C_1 = T_1 = \frac{1}{2}[(560 + 574)(10^6)](0.008333)(0.075) = 354.375(10^3) \text{ N}$$

$$C_2 = T_2 = \frac{1}{2}[(560 + 420)(10^6)](0.031667)(0.075) = 1163.75(10^3) \text{ N}$$

$$C_3 = T_3 = \frac{1}{2}[420(10^6)](0.01)(0.075) = 157.5(10^3) \text{ N}$$

$$M = [354.375(10^3)](0.09170) + [1163.75(10^3)](0.05317) + [157.5(10^3)](0.01333)$$

$$= 96.48(10^3) \text{ N} \cdot \text{m} = 96.5 \text{ kN} \cdot \text{m} \qquad \textbf{Ans.}$$

Note: The centroid of a trapezodial area was used in calculation of moment areas.

Ans:

$\sigma = 574$ MPa, $M = 96.5$ kN · m

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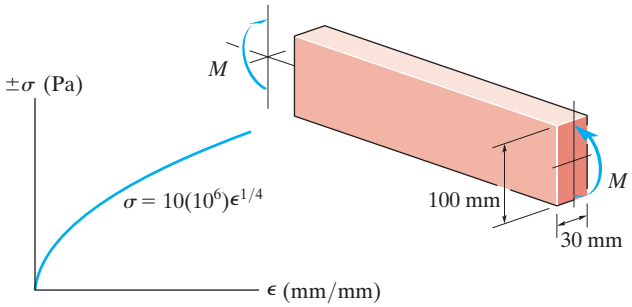
6–182.

A beam is made from polypropylene plastic and has a stress–strain diagram that can be approximated by the curve shown. If the beam is subjected to a maximum tensile and compressive strain of $\epsilon = 0.02$ mm/mm, determine the moment M.



SOLUTION

$\epsilon_{max} = 0.02$

$\sigma_{max} = 10\left(10^6\right)(0.02)^{1/4} = 3.761 \text{ MPa}$

$$\frac{0.02}{0.05} = \frac{\epsilon}{y}$$

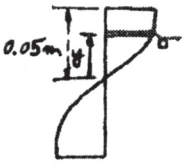
$$\epsilon = 0.4\,y$$

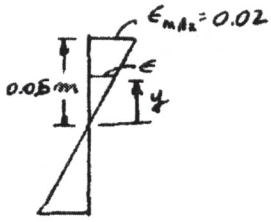
$$\sigma = 10\left(10^6\right)(0.4)^{1/4} y^{1/4}$$

$$M = \int_A y\,\sigma\,dA = 2\int_0^{0.05} y(7.9527)\left(10^6\right) y^{1/4}(0.03)dy$$

$$M = 0.47716\left(10^6\right)\int_0^{0.05} y^{5/4}dy = 0.47716\left(10^6\right)\left(\frac{4}{9}\right)(0.05)^{9/4}$$

$M = 251 \text{ N} \cdot \text{m}$ **Ans.**





Ans:
$M = 251 \text{ N} \cdot \text{m}$

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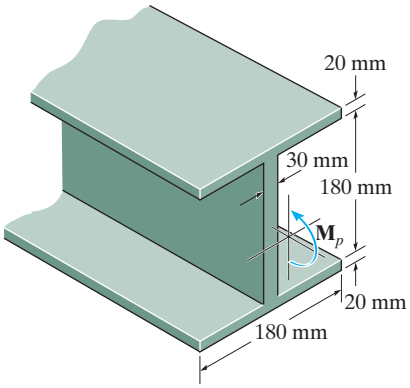
R6–1.

Determine the shape factor for the wide-flange beam.

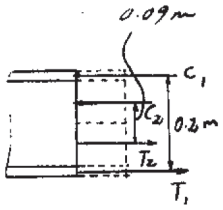


SOLUTION

$$I = \frac{1}{12}(0.18)(0.22^3) - \frac{1}{12}(0.15)(0.18^3)$$

$$= 86.82(10^{-6})\ \text{m}^4$$

Plastic moment:



$$M_p = \sigma_Y(0.18)(0.02)(0.2) + \sigma_Y(0.09)(0.03)(0.09)$$

$$= 0.963(10^{-3})\sigma_Y$$

Shape Factor:

$$M_Y = \frac{\sigma_Y I}{c} = \frac{\sigma_Y(86.82)(10^{-6})}{0.11} = 0.789273(10^{-3})\sigma_Y$$

$$k = \frac{M_p}{M_Y} = \frac{0.963(10^{-3})\sigma_Y}{0.789273(10^{-3})\sigma_Y} = 1.22$$ **Ans.**

Ans:
$k = 1.22$

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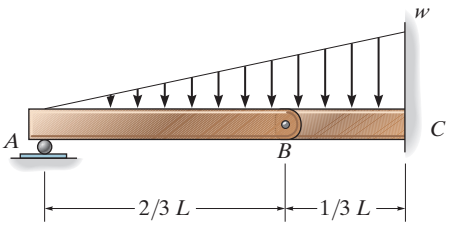
R6–2.

The compound beam consists of two segments that are pinned together at B. Draw the shear and moment diagrams if it supports the distributed loading shown.



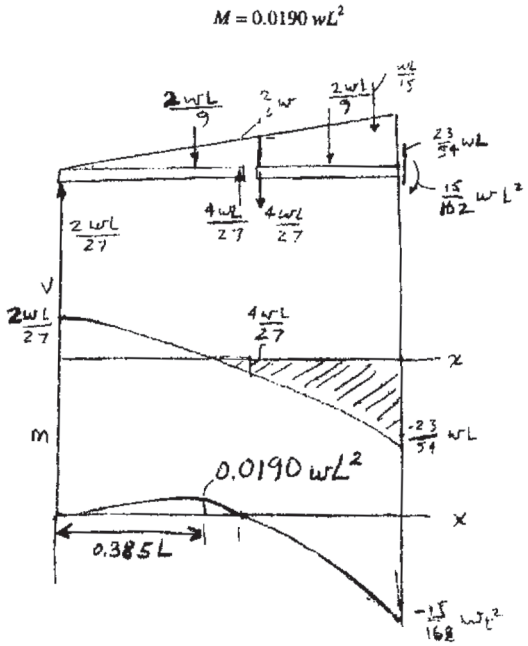
SOLUTION

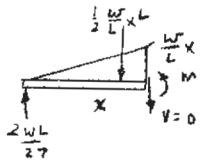
$$+\uparrow \Sigma F_y = 0; \qquad \frac{2wL}{27} - \frac{1}{2}\frac{w}{L}x^2 = 0$$

$$x = \sqrt{\frac{4}{27}}L = 0.385\,L$$

$$\circlearrowleft +\Sigma M = 0; \qquad M + \frac{1}{2}\frac{w}{L}(0.385L)^2\left(\frac{1}{3}\right)(0.385L) - \frac{2wL}{27}(0.385L) = 0$$

$$M = 0.0190\,wL^2$$





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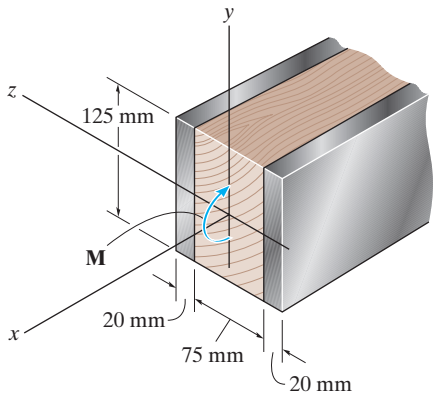
R6–3.

The composite beam consists of a wood core and two plates of steel. If the allowable bending stress for the wood is $(\sigma_{allow})_w = 20$ MPa, and for the steel $(\sigma_{allow})_{st} = 130$ MPa, determine the maximum moment that can be applied to the beam. $E_w = 11$ GPa, $E_{st} = 200$ GPa.



SOLUTION

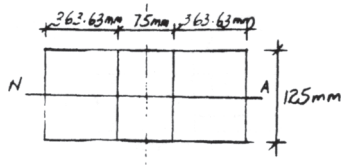
$$n = \frac{E_{st}}{E_w} = \frac{200(10^9)}{11(10^9)} = 18.182$$

$$I = \frac{1}{12}(0.80227)(0.125^3) = 0.130578(10^{-3})\text{m}^4$$

Failure of wood:

$$(\sigma_w)_{max} = \frac{Mc}{I}$$

$$20(10^6) = \frac{M(0.0625)}{0.130578(10^{-3})}; \qquad M = 41.8\ \text{kN}\cdot\text{m}$$



Failure of steel:

$$(\sigma_{st})_{max} = \frac{nMc}{I}$$

$$130(10^6) = \frac{18.182(M)(0.0625)}{0.130578(10^{-3})}$$

$$M = 14.9\ \text{kN}\cdot\text{m} \quad \text{(controls)}$$

Ans.

Ans:
$M = 14.9\ \text{kN}\cdot\text{m}$

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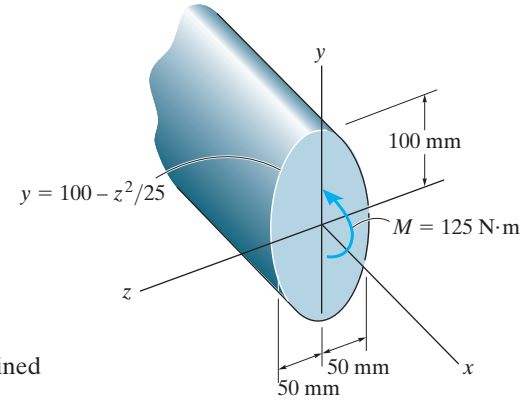
***R6–4.**

A shaft is made of a polymer having a parabolic upper and lower cross section. If it resists a moment of $M = 125\ \text{N}\cdot\text{m}$, determine the maximum bending stress in the material (a) using the flexure formula and (b) using integration. Sketch a three-dimensional view of the stress distribution acting over the cross-sectional area. *Hint:* The moment of inertia is determined using Eq. A–3 of Appendix A.



SOLUTION

Maximum Bending Stress: The moment of inertia about y axis must be determined first in order to use flexure formula

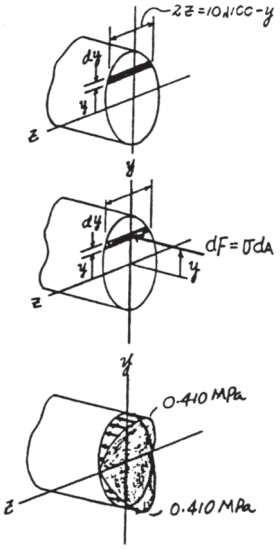
$$I = \int_A y^2\, dA$$

$$= 2\int_0^{100\text{ mm}} y^2\,(2z)\, dy$$

$$= 20\int_0^{100\text{ mm}} y^2\sqrt{100 - y}\, dy$$

$$= 20\left[-\frac{3}{2}y^2(100 - y)^{\frac{3}{2}} - \frac{8}{15}y\,(100 - y)^{\frac{5}{2}} - \frac{16}{105}(100 - y)^{\frac{7}{2}}\right]\Bigg|_0^{100}\text{mm}$$

$$= 30.4762\left(10^6\right)\text{mm}^4 = 30.4762\left(10^{-6}\right)\text{m}^4$$



Thus,

$$\sigma_{\max} = \frac{Mc}{I} = \frac{125(0.1)}{30.4762(10^{-6})} = 0.410\ \text{MPa}$$ **Ans.**

Maximum Bending Stress: Using integration

$$dM = 2[y(\sigma\, dA)] = 2\left\{y\left[\left(\frac{\sigma_{\max}}{100}\right)y\right](2z\, dy)\right\}$$

$$M = \frac{\sigma_{\max}}{5}\int_0^{100\text{ mm}} y^2\sqrt{100 - y}\, dy$$

$$125\left(10^3\right) = \frac{\sigma_{\max}}{5}\left[-\frac{3}{2}y^2(100 - y)^{\frac{3}{2}} - \frac{8}{15}y(100 - y)^{\frac{5}{2}} - \frac{16}{105}(100 - y)^{\frac{7}{2}}\right]\Bigg|_0^{100}\text{mm}$$

$$125\left(10^3\right) = \frac{\sigma_{\max}}{5}(1.5238)\left(10^6\right)$$

$$\sigma_{\max} = 0.410\ \text{N/mm}^2 = 0.410\ \text{MPa}$$ **Ans.**

Ans:
$\sigma_{\max} = 0.410\ \text{MPa}$

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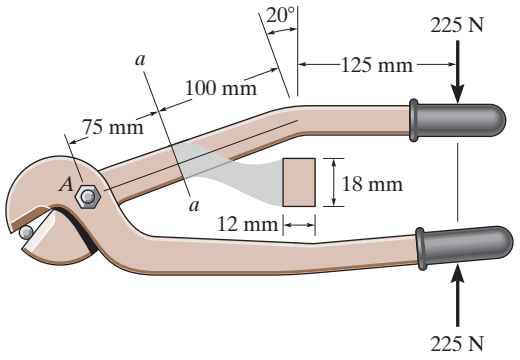
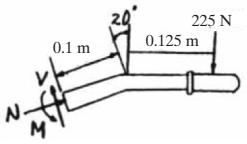
R6–5. Determine the maximum bending stress in the handle of the cable cutter at section *a–a*. A force of 225 N is applied to the handles. The cross-sectional area is shown in the figure.



$\zeta + \Sigma M = 0;$ $\quad M - 225(0.125 + 0.1\cos 20°) = 0$

$M = 49.27 \text{ N} \cdot \text{m}$

$$\sigma_{\max} = \frac{Mc}{I} = \frac{49.27(0.009)}{\frac{1}{12}(0.012)(0.018^3)} = 76.03(10^6) \text{ N/m}^2 = 76.0 \text{ MPa}$$ **Ans.**



Ans:
$\sigma_{\max} = 76.0 \text{ MPa}$

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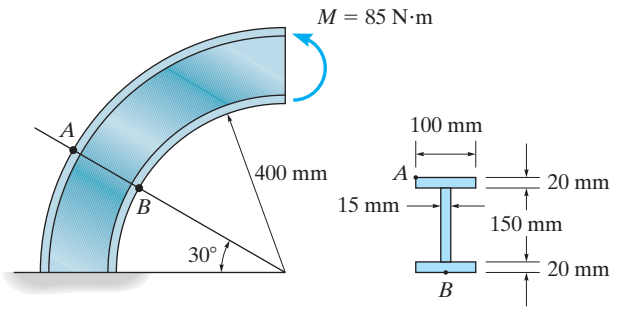
R6–6.

The curved beam is subjected to a bending moment of $M = 85\ \text{N}\cdot\text{m}$ as shown. Determine the stress at points A and B and show the stress on a volume element located at these points.



SOLUTION

$$\int_A \frac{dA}{r} = b \ln \frac{r_2}{r_1} = 0.1 \ln \frac{0.42}{0.40} + 0.015 \ln \frac{0.57}{0.42} + 0.1 \ln \frac{0.59}{0.57}$$

$$= 0.012908358\ \text{m}$$

$$A = 2(0.1)(0.02) + (0.15)(0.015) = 6.25(10^{-3})\ \text{m}^2$$

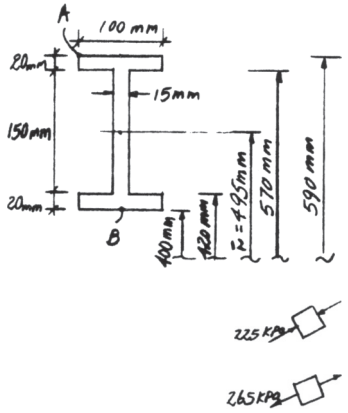
$$R = \frac{A}{\int_A \frac{dA}{r}} = \frac{6.25(10^{-3})}{0.012908358} = 0.484182418\ \text{m}$$

$$\bar{r} - R = 0.495 - 0.484182418 = 0.010817581\ \text{m}$$

$$\sigma_A = \frac{M(R - r_A)}{Ar_A(\bar{r} - R)} = \frac{85(0.484182418 - 0.59)}{6.25(10^{-3})(0.59)(0.010817581)} = -225.48\ \text{kPa}$$

$\sigma_A = 225\ \text{kPa (C)}$ **Ans.**

$$\sigma_B = \frac{M(R - r_B)}{Ar_B(\bar{r} - R)} = \frac{85(0.484182418 - 0.40)}{6.25(10^{-3})(0.40)(0.010817581)} = 265\ \text{kPa (T)}$$ **Ans.**



Ans:
$\sigma_A = 225\ \text{kPa (C)}, \sigma_B = 265\ \text{kPa (T)}$

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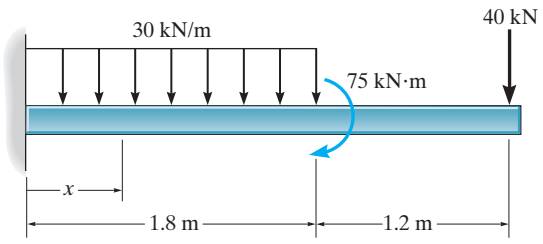
R6–7. Draw the shear and moment diagrams for the beam and determine the shear and moment in the beam as functions of x, where $0 \le x < 1.8$ m.

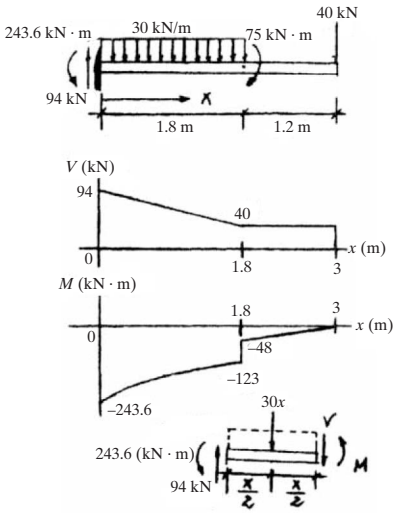


$+\uparrow \ \Sigma F_y = 0; \quad 94 - 30x - V = 0$

$V = \{94 - 30x\}$ kN **Ans.**

$\circlearrowleft + \Sigma M_{NA} = 0; \quad 94x - 243.6 - 30x\left(\frac{x}{2}\right) - M = 0$

$M = \{-15x^2 + 94x - 243.6\}$ kN·m **Ans.**



Ans:

$V = 94 - 30x$, $M = -15x^2 + 94x - 243.6$

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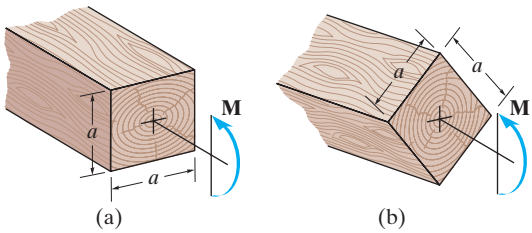
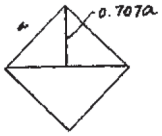
***R6–8.**

A wooden beam has a square cross section as shown. Determine which orientation of the beam provides the greatest strength at resisting the moment **M**. What is the difference in the resulting maximum stress in both cases?



SOLUTION

Case (a):



$$\sigma_{max} = \frac{Mc}{I} = \frac{M(a/2)}{\frac{1}{12}(a)^4} = \frac{6M}{a^3}$$

Case (b):

$$I = 2\left[\frac{1}{36}\left(\frac{2}{\sqrt{2}}a\right)\left(\frac{1}{\sqrt{2}}a\right)^3 + \frac{1}{2}\left(\frac{2}{\sqrt{2}}a\right)\left(\frac{1}{\sqrt{2}}a\right)\left[\left(\frac{1}{\sqrt{2}}a\right)\left(\frac{1}{3}\right)\right]^2\right] = 0.08333\,a^4$$

$$\sigma_{max} = \frac{Mc}{I} = \frac{M\left(\frac{1}{\sqrt{2}}a\right)}{0.08333\,a^4} = \frac{8.4853\,M}{a^3}$$

Case (a) provides higher strength, since the resulting maximum stress is less for a given M and a.

$$\Delta\sigma_{max} = \frac{8.4853\,M}{a^3} - \frac{6M}{a^3} = 2.49\left(\frac{M}{a^3}\right)$$ **Ans.**

Ans:
Case (a),
$\Delta\sigma_{max} = 2.49\left(\frac{M}{a^3}\right)$

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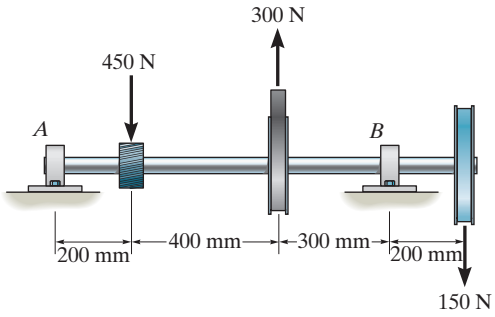
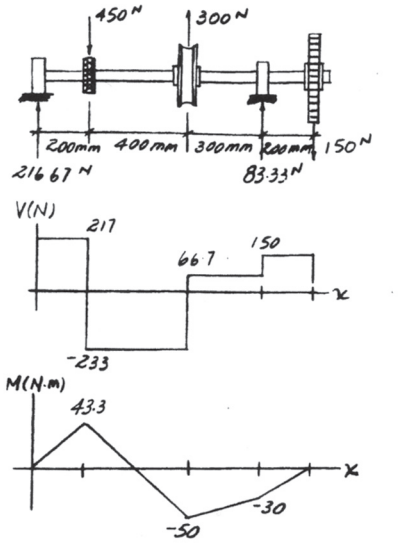
R6–9.

Draw the shear and moment diagrams for the shaft if it is subjected to the vertical loadings. The bearings at A and B exert only vertical reactions on the shaft.



SOLUTION



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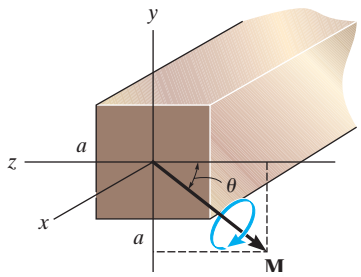
R6–10.

The strut has a square cross section a by a and is subjected to the bending moment **M** applied at an angle θ as shown. Determine the maximum bending stress in terms of a, M, and θ. What angle θ will give the largest bending stress in the strut? Specify the orientation of the neutral axis for this case.



SOLUTION

Internal Moment Components:

$$M_z = -M\cos\theta \qquad M_y = -M\sin\theta$$

Section Property:

$$I_y = I_z = \frac{1}{12}a^4$$

Maximum Bending Stress: By inspection, maximum bending stress occurs at A and B. Applying the flexure formula for biaxial bending at point A

$$\sigma_{\max} = -\frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

$$= -\frac{-M\cos\theta\left(\frac{a}{2}\right)}{\frac{1}{12}a^4} + \frac{-M\sin\theta\left(-\frac{a}{2}\right)}{\frac{1}{12}a^4}$$

$$= \frac{6M}{a^3}(\cos\theta + \sin\theta) \qquad \textbf{Ans.}$$

$$\frac{d\sigma}{d\theta} = \frac{6M}{a^3}(-\sin\theta + \cos\theta) = 0$$

$$\cos\theta - \sin\theta = 0$$

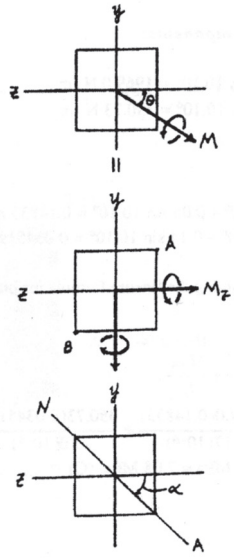
$$\theta = 45° \qquad \textbf{Ans.}$$

Orientation of Neutral Axis:

$$\tan\alpha = \frac{I_z}{I_y}\tan\theta$$

$$\tan\alpha = (1)\tan(45°)$$

$$\alpha = 45° \qquad \textbf{Ans.}$$



Ans:

$\sigma_{\max} = \frac{6M}{a^3}(\cos\theta + \sin\theta)$,

$\theta = 45°$, $\alpha = 45°$

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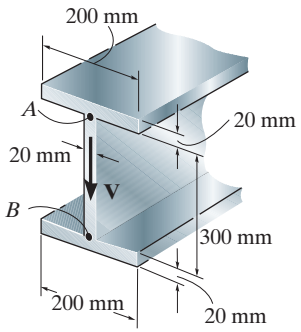
7–1.

If the wide-flange beam is subjected to a shear of $V = 20$ kN, determine the shear stress on the web at A. Indicate the shear-stress components on a volume element located at this point.



SOLUTION

The moment of inertia of the cross-section about the neutral axis is

$$I = \frac{1}{12}(0.2)(0.34^3) - \frac{1}{12}(0.18)(0.3^3) = 0.2501(10^{-3})\ \text{m}^4$$

From Fig. *a*,

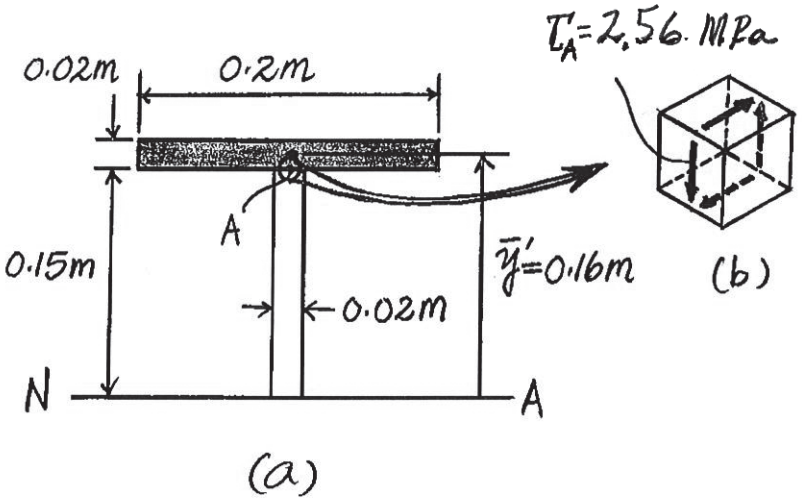
$$Q_A = \bar{y}'A' = 0.16\,(0.02)(0.2) = 0.64(10^{-3})\ \text{m}^3$$

Applying the shear formula,

$$\tau_A = \frac{VQ_A}{It} = \frac{20(10^3)[0.64(10^{-3})]}{0.2501(10^{-3})(0.02)}$$

$$= 2.559(10^6)\ \text{Pa} = 2.56\ \text{MPa} \qquad \textbf{Ans.}$$

The shear stress component at A is represented by the volume element shown in Fig. *b*.



These solutions represent a preliminary version of the Instructors' Solutions Manual (ISM). It is possible and even likely that at this preliminary stage of preparing the ISM there are some omissions and errors in the draft solutions. These will be corrected and this manual will be republished.

Ans:
$\tau_A = 2.56$ MPa

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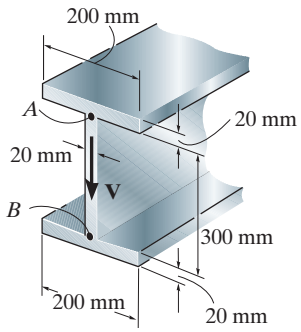
7–2.

If the wide-flange beam is subjected to a shear of $V = 20$ kN, determine the maximum shear stress in the beam.



SOLUTION

The moment of inertia of the cross-section about the neutral axis is

$$I = \frac{1}{12}(0.2)(0.34^3) - \frac{1}{12}(0.18)(0.3^3) = 0.2501(10^{-3})\ \text{m}^4$$

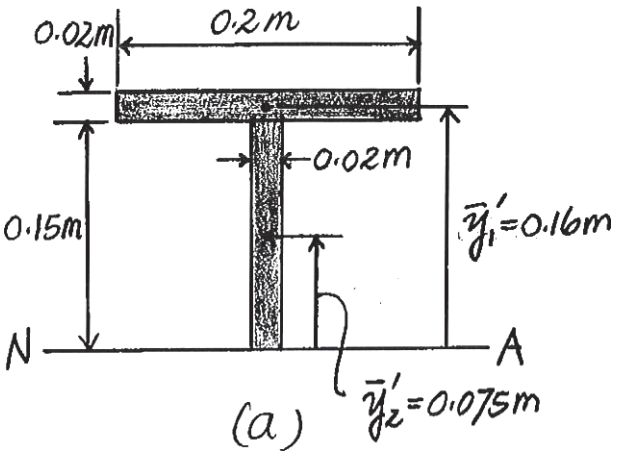
From Fig. *a*.

$$Q_{\max} = \Sigma \bar{y}'\text{A}' = 0.16\,(0.02)(0.2) + 0.075\,(0.15)(0.02) = 0.865(10^{-3})\ \text{m}^3$$

The maximum shear stress occurs at the points along neutral axis since Q is maximum and thickness t is the smallest.

$$\tau_{\max} = \frac{VQ_{\max}}{It} = \frac{20(10^3)\,[0.865(10^{-3})]}{0.2501(10^{-3})\,(0.02)}$$

$$= 3.459(10^6)\ \text{Pa} = 3.46\ \text{MPa} \qquad \textbf{Ans.}$$



Ans:
$\tau_{\max} = 3.46$ MPa

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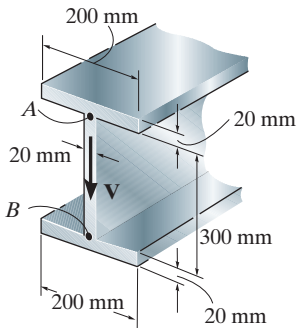
7–3.

If the wide-flange beam is subjected to a shear of $V = 20$ kN, determine the shear force resisted by the web of the beam.



SOLUTION

The moment of inertia of the cross-section about the neutral axis is

$$I = \frac{1}{12}(0.2)(0.34^3) - \frac{1}{12}(0.18)(0.3^3) = 0.2501(10^{-3})\ \text{m}^4$$

For $0 \le y < 0.15$ m, Fig. *a*, Q as a function of y is

$$Q = \Sigma \bar{y}' A' = 0.16\,(0.02)(0.2) + \frac{1}{2}(y + 0.15)(0.15 - y)(0.02)$$

$$= 0.865(10^{-3}) - 0.01y^2$$

For $0 \le y < 0.15$ m, $t = 0.02$ m. Thus,

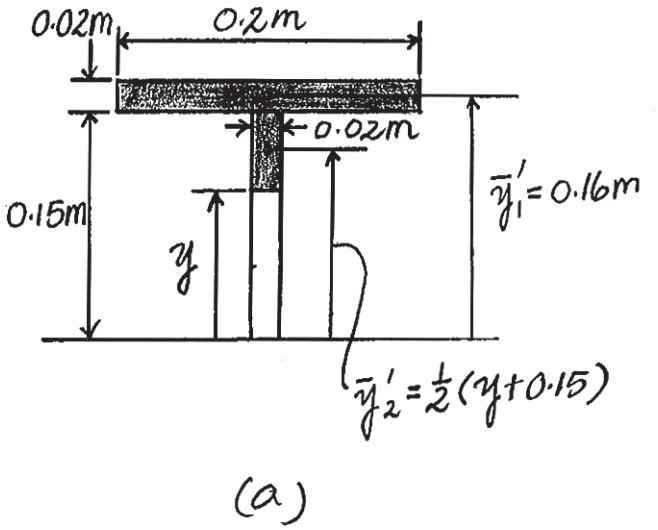
$$\tau = \frac{VQ}{It} = \frac{20(10^3)\left[0.865(10^{-3}) - 0.01y^2\right]}{0.2501(10^{-3})\,(0.02)}$$

$$= \left\{3.459(10^6) - 39.99(10^6)\,y^2\right\}\ \text{Pa}$$

The sheer force resisted by the web is

$$V_w = 2\int_0^{0.15\ \text{m}} \tau dA = 2\int_0^{0.15\ \text{m}} \left[3.459(10^6) - 39.99(10^6)\,y^2\right](0.02\,dy)$$

$$= 18.95\,(10^3)\ \text{N} = 19.0\ \text{kN} \qquad \textbf{Ans.}$$



(a)

Ans:
$V_w = 19.0$ kN

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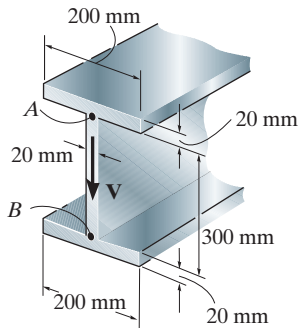
***7–4.**

If the beam is subjected to a shear of $V = 30$ kN, determine the web's shear stress at A and B. Indicate the shear-stress components on a volume element located at these points. Set $w = 200$ mm. Show that the neutral axis is located at $\bar{y} = 0.2433$ m from the bottom and $I = 0.5382(10^{-3})$ m^4.



SOLUTION

Section Properties: The location of the centroid measured from the bottom is

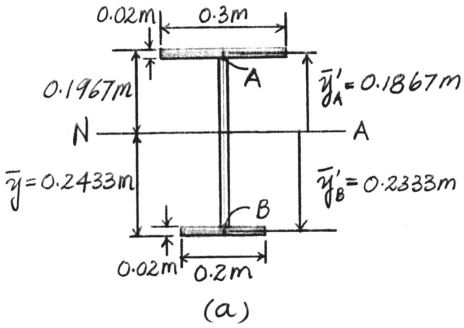
$$\bar{y} = \frac{(0.01)(0.2)(0.02) + 0.22(0.02)(0.4) + 0.430(0.3)(0.02)}{0.2(0.02) + 0.02(0.4) + 0.3(0.02)}$$

$$= 0.2433 \text{ m}$$

The moment of inertia of the cross-section about the neutral axis is

$$I = \frac{1}{12}(0.2)\left(0.02^3\right) + 0.2(0.02)(0.2433 - 0.01)^2 + \frac{1}{12}(0.02)\left(0.4^3\right) + 0.02(0.4)(0.2433 - 0.22)^2 + \frac{1}{12}(0.3)\left(0.02^3\right) + 0.3(0.02)(0.43 - 0.2433)^2$$

$$= 0.5382\left(10^{-3}\right) \text{ m}^4$$



Referring to Fig. *a,*

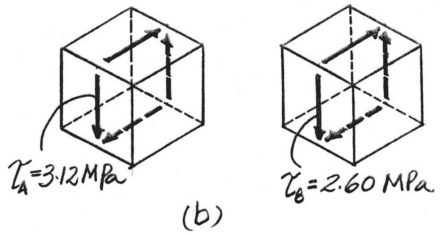
$$Q_A = \bar{y}'_A A'_A = 0.1867[0.3(0.02)] = 1.12\left(10^{-3}\right) \text{ m}^3$$

$$Q_B = \bar{y}'_B A'_B = 0.2333[0.2(0.02)] = 0.9333\left(10^{-3}\right) \text{ m}^3$$

Shear Stress: Applying the shear formula,

$$\tau_A = \frac{VQ_A}{It_A} = \frac{30\left(10^3\right)\left[1.12\left(10^{-3}\right)\right]}{0.5382\left(10^{-3}\right)(0.02)} = 3.122\left(10^6\right)\text{Pa} = 3.12 \text{ MPa} \quad \textbf{Ans.}$$

$$\tau_B = \frac{VQ_B}{It_B} = \frac{30\left(10^3\right)\left[0.9333\left(10^{-3}\right)\right]}{0.5382\left(10^{-3}\right)(0.02)} = 2.601\left(10^6\right)\text{Pa} = 2.60 \text{ MPa} \quad \textbf{Ans.}$$



These shear stresses on the volume element at points A and B are shown in Fig. *b*.

Ans:
$\tau_A = 3.12$ MPa,
$\tau_B = 2.60$ MPa

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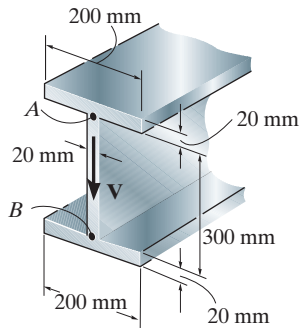
7–5.

If the wide-flange beam is subjected to a shear of $V = 30$ kN, determine the maximum shear stress in the beam. Set $w = 300$ mm.



SOLUTION

Section Properties: The moment of inertia of the cross section about the neutral axis is

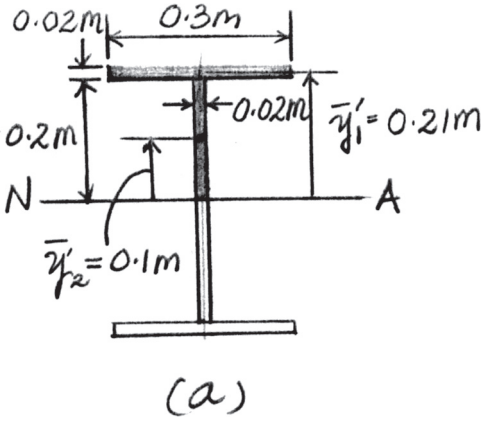
$$I = \frac{1}{12}(0.3)(0.44^3) - \frac{1}{12}(0.28)(0.4^3) = 0.63627(10^{-3})\ \text{m}^4$$

Maximum shear stress occurs at the neutral axis. Referring to Fig. *a*,

$$Q_{\max} = \Sigma \bar{y}' A' = 0.21[0.3(0.02)] + 0.1[0.2(0.02)]$$
$$= 1.66(10^{-3})\ \text{m}^3$$

Maximum Shear Stress: Applying the shear formula,

$$\tau_{\max} = \frac{VQ_{\max}}{It} = \frac{30(10^3)\left[1.66(10^{-3})\right]}{0.63627(10^{-3})(0.02)}$$
$$= 3.913(10^6)\text{Pa} = 3.91\ \text{MPa}$$ **Ans.**



(a)

Ans:
$\tau_{\max} = 3.91$ MPa

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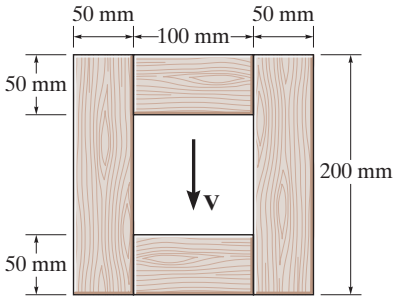
7–6.

The wood beam has an allowable shear stress of $\tau_{allow} = 7$ MPa. Determine the maximum shear force V that can be applied to the cross section.



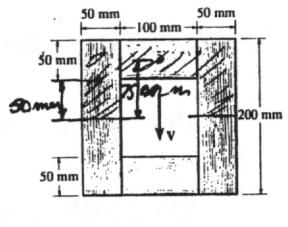
SOLUTION

$$I = \frac{1}{12}(0.2)(0.2)^3 - \frac{1}{12}(0.1)(0.1)^3 = 125(10^{-6})\ \text{m}^4$$

$$\tau_{allow} = \frac{VQ_{max}}{It}$$

$$7(10^6) = \frac{V[(0.075)(0.1)(0.05) + 2(0.05)(0.1)(0.05)]}{125(10^{-6})(0.1)}$$

$V = 100$ kN **Ans.**



Ans:
$V_{max} = 100$ kN

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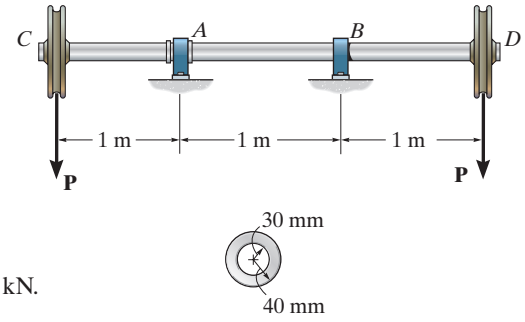
7–7.

The shaft is supported by a thrust bearing at A and a journal bearing at B. If $P = 20$ kN, determine the absolute maximum shear stress in the shaft.



SOLUTION

Support Reactions: As shown on the free-body diagram of the beam, Fig. *a*.

Maximum Shear: The shear diagram is shown in Fig. *b*. As indicated, $V_{max} = 20$ kN.

Section Properties: The moment of inertia of the hollow circular shaft about the neutral axis is

$$I = \frac{\pi}{4}(0.04^4 - 0.03^4) = 0.4375(10^{-6})\pi \text{ m}^4$$

Q_{max} can be computed by taking the first moment of the shaded area in Fig. *c* about the neutral axis.

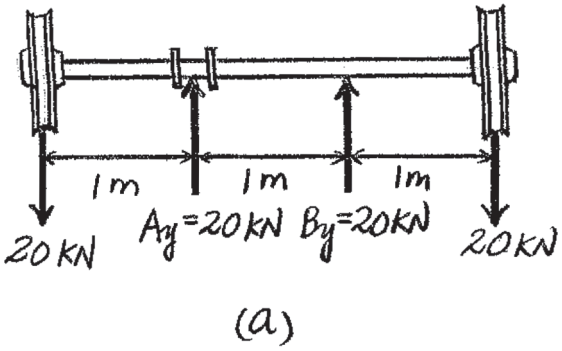
Here, $\bar{y}'_1 = \frac{4(0.04)}{3\pi} = \frac{4}{75\pi}$ m and $\bar{y}'_2 = \frac{4(0.03)}{3\pi} = \frac{1}{25\pi}$ m. Thus,

$$Q_{max} = \bar{y}'_1 A'_1 - \bar{y}'_2 A'_2$$

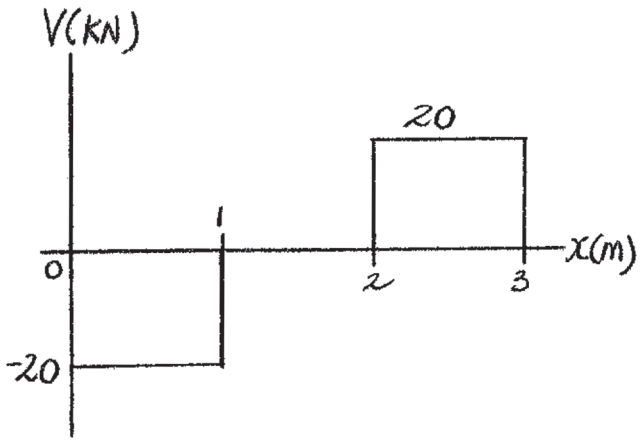
$$= \frac{4}{75\pi}\left[\frac{\pi}{2}(0.04^2)\right] - \frac{1}{25\pi}\left[\frac{\pi}{2}(0.03^2)\right] = 24.667(10^{-6}) \text{ m}^3$$

Shear Stress: The maximum shear stress occurs at points on the neutral axis since Q is maximum and the thickness $t = 2(0.04 - 0.03) = 0.02$ m is the smallest.

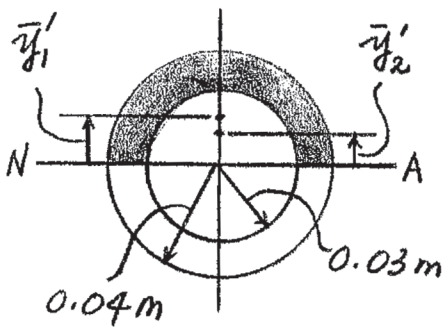
$$\tau_{max} = \frac{V_{max} Q_{max}}{It} = \frac{20(10^3)(24.667)(10^{-6})}{0.4375(10^{-6})\pi(0.02)} = 17.9 \text{ MPa}$$ **Ans.**



(a)



(b)



(c)

Ans:
$\tau_{max} = 17.9$ MPa

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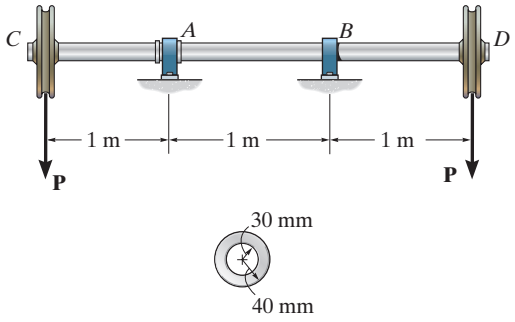
***7–8.**

The shaft is supported by a thrust bearing at A and a journal bearing at B. If the shaft is made from a material having an allowable shear stress of $\tau_{allow} = 75$ MPa, determine the maximum value for P.



SOLUTION

Support Reactions: As shown on the free-body diagram of the shaft, Fig. *a*.

Maximum Shear: The shear diagram is shown in Fig. *b*. As indicated, $V_{max} = P$.

Section Properties: The moment of inertia of the hollow circular shaft about the neutral axis is

$$I = \frac{\pi}{4}(0.04^4 - 0.03^4) = 0.4375(10^{-6})\pi \text{ m}^4$$

Q_{max} can be computed by taking the first moment of the shaded area in Fig. *c* about the neutral axis.

Here, $\bar{y}'_1 = \frac{4(0.04)}{3\pi} = \frac{4}{75\pi}$ m and $\bar{y}'_2 = \frac{4(0.03)}{3\pi} = \frac{1}{25\pi}$ m. Thus,

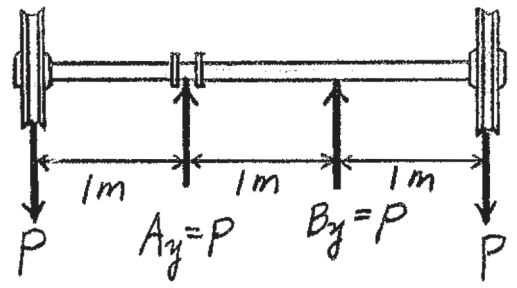
$$Q_{max} = \bar{y}'_1 A'_1 - \bar{y}'_2 A'_2$$

$$= \frac{4}{75\pi}\left[\frac{\pi}{2}(0.04^2)\right] - \frac{1}{25\pi}\left[\frac{\pi}{2}(0.03^2)\right] = 24.667(10^{-6}) \text{ m}^3$$

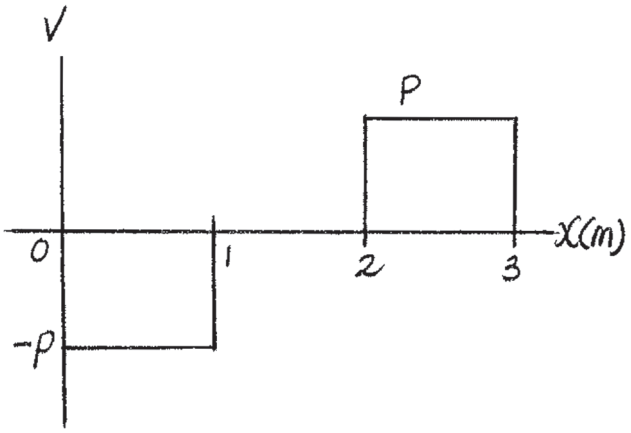
Shear Stress: The maximum shear stress occurs at points on the neutral axis since Q is maximum and the thickness $t = 2(0.04 - 0.03) = 0.02$ m.

$$\tau_{allow} = \frac{V_{max} Q_{max}}{It}; \qquad 75(10^6) = \frac{P(24.667)(10^{-6})}{0.4375(10^{-6})\pi(0.02)}$$

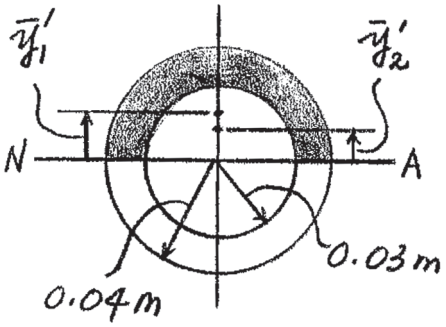
$$P = 83\,581.22 \text{ N} = 83.6 \text{ kN} \qquad \textbf{Ans.}$$



(a)



(b)



(c)

Ans:
$P = 83.6$ kN

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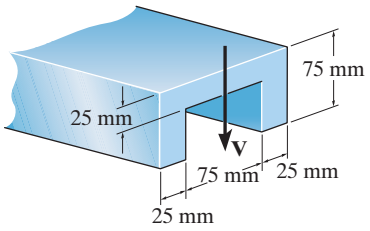
7–9. Determine the largest shear force V that the member can sustain if the allowable shear stress is $\tau_{allow} = 56$ MPa.

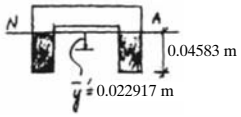


SOLUTION

$$\bar{y} = \frac{(0.0125)(0.125)(0.025) + 2[0.05(0.025)(0.05)]}{0.125(0.025) + 2(0.025)(0.05)} = 0.029167 \text{ m}$$

$$I = \frac{1}{12}(0.125)(0.025^3) + 0.125(0.025)(0.029167 - 0.0125)^2$$

$$+ 2\left[\frac{1}{12}(0.025)(0.05^3) + 0.025(0.05)(0.05 - 0.029167)^2\right] = 2.6367(10^{-6}) \text{ m}^4$$



$$Q_{max} = \Sigma \bar{y}' A' = 2[(0.022917)(0.025)(0.04583)] = 52.5174(10^{-6}) \text{ m}^3$$

$$\tau_{max} = \tau_{allow} = \frac{VQ_{max}}{It}$$

$$56(10^6) = \frac{V[52.5174(10^{-6})]}{[2.6367(10^{-6})][2(0.025)]}$$

$V = 140.58(10^3)$ N $= 141$ kN **Ans.**

Ans:
$V = 141$ kN

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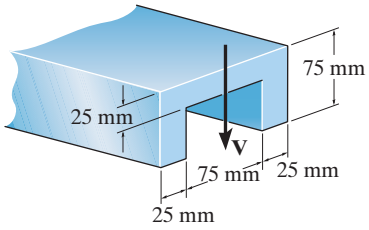
7–10. If the applied shear force $V = 90$ kN, determine the maximum shear stress in the member.



SOLUTION

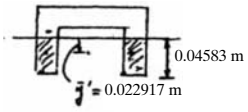
$$\bar{y} = \frac{(0.0125)(0.125)(0.025) + 2[0.05(0.025)(0.05)]}{0.125(0.025) + 2(0.025)(0.05)} = 0.029167 \text{ m}$$

$$I = \frac{1}{12}(0.125)(0.025^3) + 0.125(0.025)(0.029167 - 0.0125)^2$$

$$+ 2\left[\frac{1}{12}(0.025)(0.05^3) + 0.025(0.05)(0.05 - 0.029167)^2\right] = 2.6367(10^{-6}) \text{ m}^4$$

$$Q_{\max} = \Sigma \bar{y}' A' = 2[(0.022917)(0.025)(0.04583)] = 52.5174(10^{-6}) \text{ m}^3$$

$$\tau_{\max} = \frac{VQ_{\max}}{I\,t} = \frac{[90(10^3)][52.5174(10^{-6}) \text{ m}^3]}{[2.6367(10^{-6})][2(0.025)]} = 35.85(10^6) \text{ N/m}^3 = 35.9 \text{ MPa} \quad \textbf{Ans.}$$



Ans:
$\tau_{\max} = 35.9$ MPa

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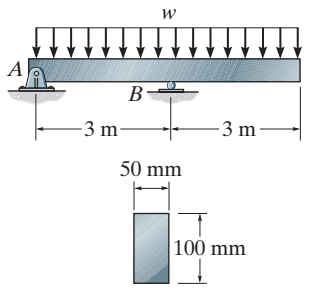
7–11.

The overhang beam is subjected to the uniform distributed load having an intensity of $w = 50$ kN/m. Determine the maximum shear stress in the beam.



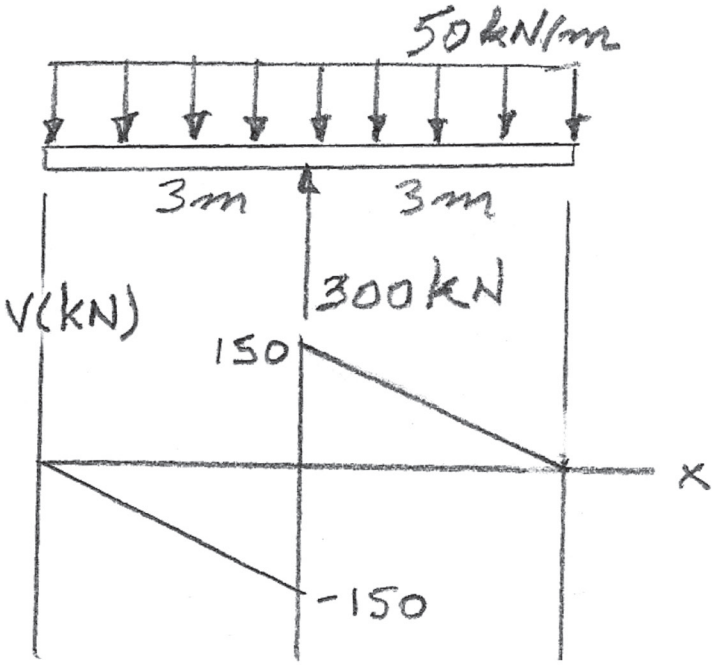
SOLUTION

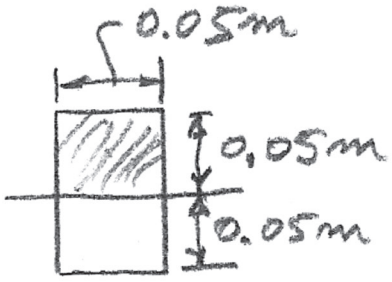
$$\tau_{max} = \frac{VQ}{It} = \frac{150(10^3)\text{ N}\,(0.025\text{ m})(0.05\text{ m})(0.05\text{ m})}{\frac{1}{12}(0.05\text{ m})(0.1\text{ m})^3(0.05\text{ m})}$$

$$\tau_{max} = 45.0\text{ MPa} \qquad \textbf{Ans.}$$

Because the cross section is a rectangle, then also,

$$\tau_{max} = 1.5\frac{V}{A} = 1.5\frac{150(10^3)\text{ N}}{(0.05\text{ m})(0.1\text{ m})} = 45.0\text{ MPa} \qquad \textbf{Ans.}$$





Ans:
$\tau_{max} = 45.0$ MPa

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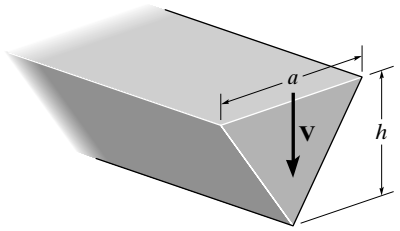
***7–12.** A member has a cross section in the form of an equilateral triangle. If it is subjected to a shear force **V**, determine the maximum average shear stress in the member using the shear formula. Should the shear formula actually be used to predict this value? Explain.



SOLUTION

$$I = \frac{1}{36}(a)(h)^3$$

$$\frac{y}{x} = \frac{h}{a/2}; \qquad y = \frac{2h}{a}x$$

$$Q = \int_{A'} y\, dA = 2\left[\left(\frac{1}{2}\right)(x)(y)\left(\frac{2}{3}h - \frac{2}{3}y\right)\right]$$

$$Q = \left(\frac{4h^2}{3a}\right)(x^2)\left(1 - \frac{2x}{a}\right)$$

$$t = 2x$$

$$\tau = \frac{VQ}{It} = \frac{V(4h^2/3a)(x^2)(1 - \frac{2x}{a})}{((1/36)(a)(h^3))(2x)}$$

$$\tau = \frac{24V(x - \frac{2}{a}x^2)}{a^2h}$$

$$\frac{d\tau}{dx} = \frac{24V}{a^2h^2}\left(1 - \frac{4}{a}x\right) = 0$$

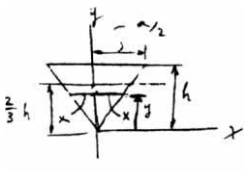
At $x = \frac{a}{4}$

$$y = \frac{2h}{a}\left(\frac{a}{4}\right) = \frac{h}{2}$$

$$\tau_{\max} = \frac{24V}{a^2h}\left(\frac{a}{4}\right)\left(1 - \frac{2}{a}\left(\frac{a}{4}\right)\right)$$

$$\tau_{\max} = \frac{3V}{ah}$$ **Ans.**

No, because the shear stress is not perpendicular to the boundary. See Sec. 7-3.



Ans.

$$\tau_{\max} = \frac{3V}{ah}$$

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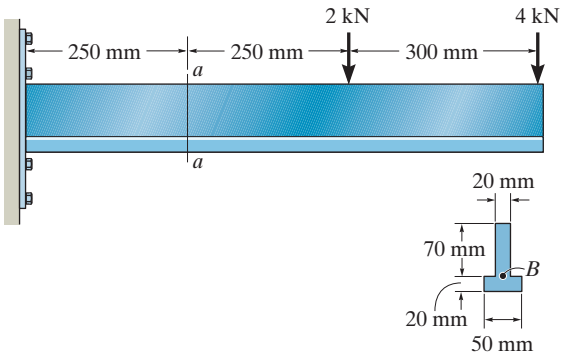
7–13.

Determine the shear stress at point B on the web of the cantilevered strut at section a–a.

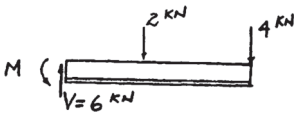


SOLUTION

$$\bar{y} = \frac{(0.01)(0.05)(0.02) + (0.055)(0.07)(0.02)}{(0.05)(0.02) + (0.07)(0.02)} = 0.03625 \text{ m}$$

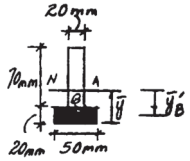
$$I = \frac{1}{12}(0.05)\left(0.02^3\right) + (0.05)(0.02)(0.03625 - 0.01)^2$$

$$+ \frac{1}{12}(0.02)\left(0.07^3\right) + (0.02)(0.07)(0.055 - 0.03625)^2 = 1.78625\left(10^{-6}\right) \text{m}^4$$



$$\bar{y}'_B = 0.03625 - 0.01 = 0.02625 \text{ m}$$

$$Q_B = (0.02)(0.05)(0.02625) = 26.25\left(10^{-6}\right) \text{m}^3$$



$$\tau_B = \frac{VQ_B}{It} = \frac{6\left(10^3\right)(26.25)\left(10^{-6}\right)}{1.78622\left(10^{-6}\right)(0.02)}$$

$$= 4.41 \text{ MPa}$$ **Ans.**

Ans:
$\tau_B = 4.41$ MPa

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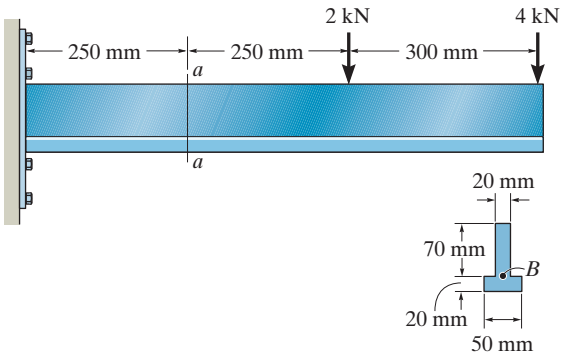
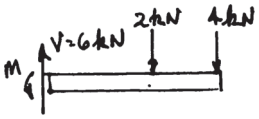
7–14.

Determine the maximum shear stress acting at section *a–a* of the cantilevered strut.



SOLUTION

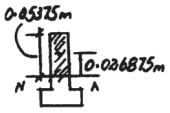
$$\bar{y} = \frac{(0.01)(0.05)(0.02) + (0.055)(0.07)(0.02)}{(0.05)(0.02) + (0.07)(0.02)} = 0.03625 \text{ m}$$



$$I = \frac{1}{12}(0.05)\left(0.02^3\right) + (0.05)(0.02)(0.03625 - 0.01)^2$$

$$+ \frac{1}{12}(0.02)\left(0.07^3\right) + (0.02)(0.07)(0.055 - 0.03625)^2 = 1.78625\left(10^{-6}\right) \text{m}^4$$

$$Q_{\max} = \bar{y}'A' = (0.026875)(0.05375)(0.02) = 28.8906\left(10^{-6}\right) \text{m}^3$$



$$\tau_{\max} = \frac{VQ_{\max}}{It} = \frac{6\left(10^3\right)(28.8906)\left(10^{-6}\right)}{1.78625\left(10^{-6}\right)(0.02)}$$

$$= 4.85 \text{ MPa}$$ **Ans.**

Ans:
$\tau_{\max} = 4.85$ MPa

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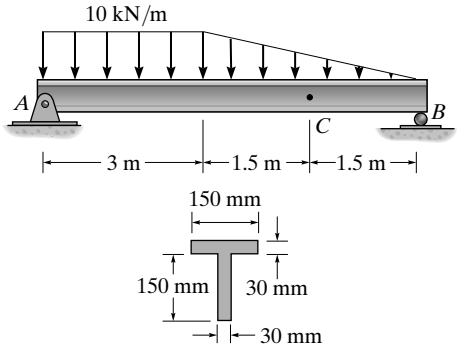
7–15. Determine the maximum shear stress in the T-beam at the critical section where the internal shear force is maximum.



The FBD of the beam is shown in Fig. *a*,

The shear diagram is shown in Fig. *b*. As indicated, $V_{max} = 27.5$ kN

The neutral axis passes through centroid *c* of the cross-section, Fig. *c*.

$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{0.075(0.15)(0.03) + 0.165(0.03)(0.15)}{0.15(0.03) + 0.03(0.15)}$$

$$= 0.12 \text{ m}$$

$$I = \frac{1}{12}(0.03)(0.15^3) + 0.03(0.15)(0.12 - 0.075)^2 + \frac{1}{12}(0.15)(0.03^3) + 0.15(0.03)(0.165 - 0.12)^2$$

$$= 27.0(10^{-6}) \text{ m}^4$$

From Fig. *d*,

$$Q_{max} = \bar{y}' A' = 0.06(0.12)(0.03)$$

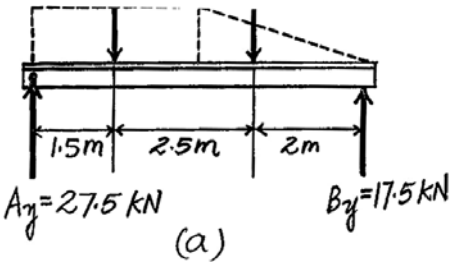
$$= 0.216(10^{-3}) \text{ m}^3$$

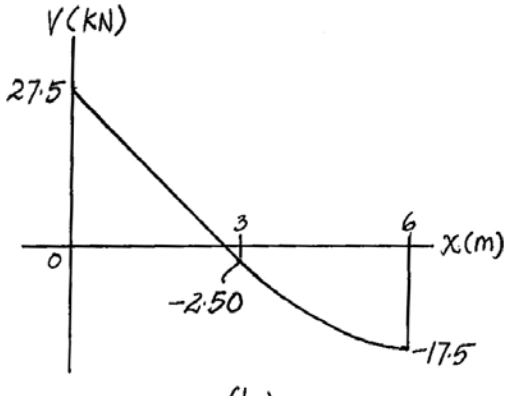
The maximum shear stress occurs at points on the neutral axis since *Q* is maximum and thickness $t = 0.03$ m is the smallest.

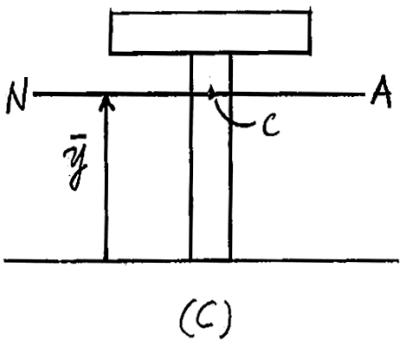
$$\tau_{max} = \frac{V_{max} Q_{max}}{I t} = \frac{27.5(10^3)\left[0.216(10^{-3})\right]}{27.0(10^{-6})(0.03)}$$

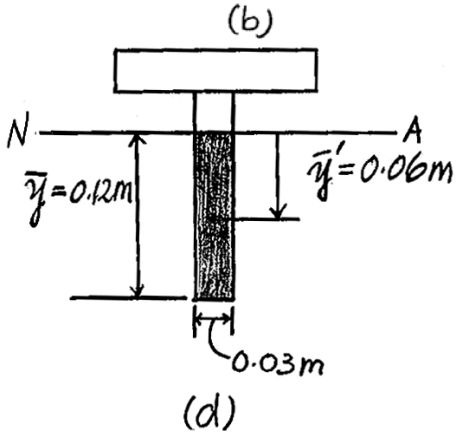
$$= 7.333(10^6) \text{ Pa}$$

$$= 7.33 \text{ MPa}$$ **Ans.**









Ans.
$\tau_{max} = 7.33$ MPa

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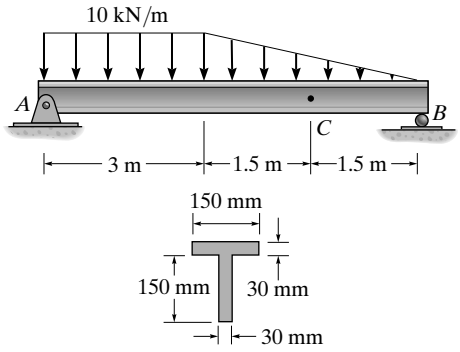
***7–16.** Determine the maximum shear stress in the T-beam at point *C*. Show the result on a volume element at this point.



SOLUTION

using the method of sections,

$$+\uparrow \Sigma F_y = 0; \qquad V_C + 17.5 - \frac{1}{2}(5)(1.5) = 0$$

$$V_C = -13.75 \text{ kN}$$

The neutral axis passes through centroid *C* of the cross-section,

$$\bar{y} = \frac{\Sigma \bar{y} A}{\Sigma A} = \frac{0.075\,(0.15)(0.03) + 0.165(0.03)(0.15)}{0.15(0.03) + 0.03(0.15)}$$

$$= 0.12 \text{ m}$$

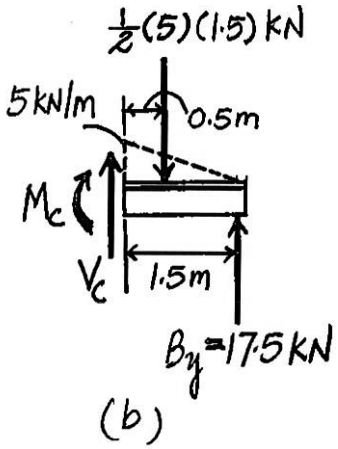
$$I = \frac{1}{12}(0.03)(0.15) + 0.03(0.15)(0.12 - 0.075)^2$$

$$+ \frac{1}{12}(0.15)(0.03^3) + 0.15(0.03)(0.165 - 0.12)^2$$

$$= 27.0\,(10^{-6})\text{ m}^4$$

$$Q_{\max} = \bar{y}' A' = 0.06\,(0.12)(0.03)$$

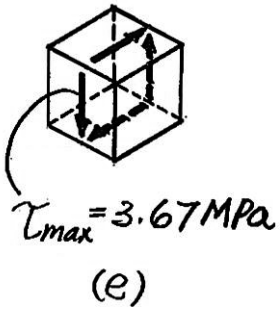
$$= 0.216\,(10^{-3})\text{ m}^3 \; 490$$



The maximum shear stress occurs at points on the neutral axis since *Q* is maximum and thickness $t = 0.03$ m is the smallest.

$$\tau_{\max} = \frac{V_C Q_{\max}}{It} = \frac{13.75(10^3)\left[0.216(10^{-3})\right]}{27.0(10^{-6})\,(0.03)}$$

$$= 3.667(10^6)\text{ Pa} = 3.67\text{ MPa} \qquad \textbf{Ans.}$$



Ans.

$\tau_{\max} = 3.67$ MPa

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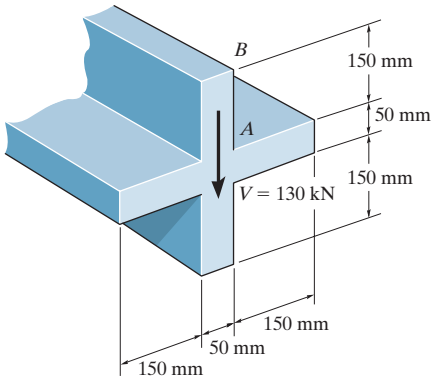
7–17. The strut is subjected to a vertical shear of $V = 130$ kN. Plot the intensity of the shear-stress distribution acting over the cross-sectional area, and compute the resultant shear force developed in the vertical segment AB.



SOLUTION

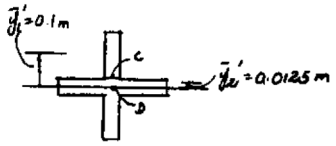
$$I = \frac{1}{12}(0.05)(0.35^3) + \frac{1}{12}(0.3)(0.05^3) = 0.18177083(10^{-3})\ \text{m}^4$$

$$Q_C = \bar{y}'A' = (0.1)(0.05)(0.15) = 0.75(10^{-3})\ \text{m}^3$$

$$Q_D = \Sigma \bar{y}'A' = (0.1)(0.05)(0.15) + (0.0125)(0.35)(0.025)$$

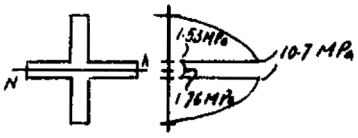
$$= 0.859375(10^{-3})\ \text{m}^3$$

$$\tau = \frac{VQ}{It}$$

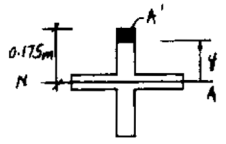


$$(\tau_C)_{t = 0.05\ \text{m}} = \frac{130(10^3)(0.75)(10^{-3})}{0.18177083(10^{-3})(0.05)} = 10.7\ \text{MPa}$$

$$(\tau_C)_{t = 0.35\ \text{m}} = \frac{130(10^3)(0.75)(10^{-3})}{0.18177083(10^{-3})(0.35)} = 1.53\ \text{MPa}$$



$$\tau_D = \frac{130(10^3)(0.859375)(10^{-3})}{0.18177083(10^{-3})(0.35)} = 1.76\ \text{MPa}$$



$$A' = (0.05)(0.175 - y)$$

$$\bar{y}' = y + \frac{(0.175 - y)}{2} = \frac{1}{2}(0.175 + y)$$

$$Q = \bar{y}'A' = 0.025(0.030625 - y^2)$$

$$\tau = \frac{VQ}{It}$$

$$= \frac{130(0.025)(0.030625 - y^2)}{0.18177083(10^{-3})(0.05)}$$

$$= 10951.3 - 357593.1\ y^2$$

$$V_{AB} = \int \tau\, dA \qquad dA = 0.05\, dy$$

$$= \int_{0.025}^{0.175} (10951.3 - 357593.1y^2)(0.05\, dy)$$

$$= \int_{0.025}^{0.175} (547.565 - 17879.66y^2)\, dy$$

$$= 50.3\ \text{kN}$$ **Ans.**

Ans:
$V_{AB} = 50.3$ kN

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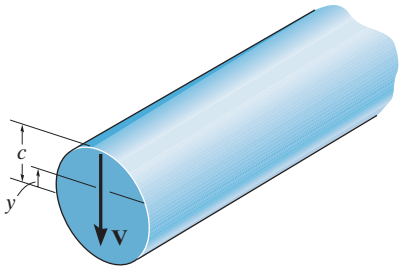
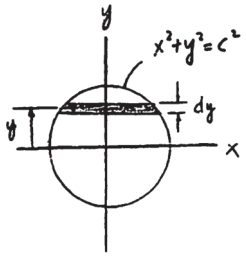
7–18.

Plot the shear-stress distribution over the cross section of a rod that has a radius c. By what factor is the maximum shear stress greater than the average shear stress acting over the cross section?



SOLUTION



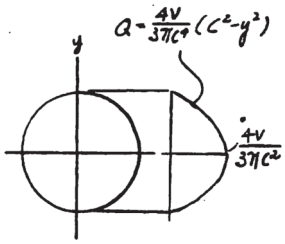
$$x = \sqrt{c^2 - y^2}; \qquad I = \frac{\pi}{4}c^4$$

$$t = 2x = 2\sqrt{c^2 - y^2}$$

$$dA = 2x\,dy = 2\sqrt{c^2 - y^2}\,dy$$

$$dQ = y\,dA = 2y\sqrt{c^2 - y^2}\,dy$$

$$Q = \int_y^c 2y\sqrt{c^2 - y^2}\,dy = -\frac{2}{3}(c^2 - y^2)^{\frac{3}{2}}\Big|_y^c = \frac{2}{3}(c^2 - y^2)^{\frac{3}{2}}$$



$$\tau = \frac{VQ}{It} = \frac{V\left[\frac{2}{3}(c^2 - y^2)^{\frac{3}{2}}\right]}{\left(\frac{\pi}{4}c^4\right)\left(2\sqrt{c^2 - y^2}\right)} = \frac{4V}{3\pi c^4}(c^2 - y^2)$$

The maximum shear stress occur when $y = 0$

$$\tau_{\max} = \frac{4V}{3\pi c^2}$$

$$\tau_{\text{avg}} = \frac{V}{A} = \frac{V}{\pi c^2}$$

$$\text{The factor} = \frac{\tau_{\max}}{\tau_{\text{avg}}} = \frac{\frac{4V}{3\pi c^2}}{\frac{V}{\pi c^2}} = \frac{4}{3}$$

Ans.

Ans:

$$\text{The factor} = \frac{4}{3}$$

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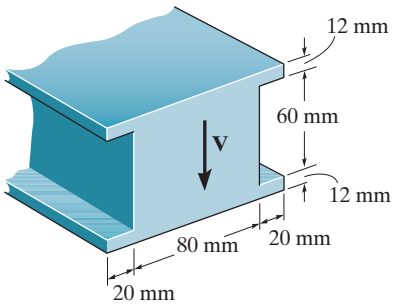
7–19.

Determine the maximum shear stress in the strut if it is subjected to a shear force of $V = 20$ kN.



SOLUTION

Section Properties:

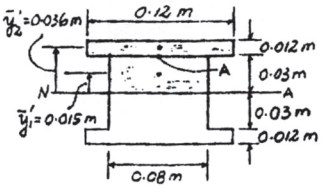
$$I_{NA} = \frac{1}{12}(0.12)(0.084^3) - \frac{1}{12}(0.04)(0.06^3)$$

$$= 5.20704\,(10^{-6})\ \text{m}^4$$

$$Q_{\max} = \Sigma \bar{y}' A'$$

$$= 0.015(0.08)(0.03) + 0.036(0.012)(0.12)$$

$$= 87.84(10^{-6})\ \text{m}^3$$



Maximum Shear Stress: Maximum shear stress occurs at the point where the neutral axis passes through the section.

Applying the shear formula

$$\tau_{\max} = \frac{VQ_{\max}}{It}$$

$$= \frac{20(10^3)(87.84)(10^{-6})}{5.20704(10^{-6})(0.08)}$$

$$= 4.\,22\ \text{MPa}$$ **Ans.**

Ans:
$\tau_{\max} = 4.22$ MPa

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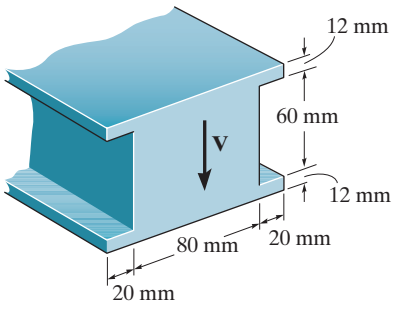
***7–20.**

Determine the maximum shear force V that the strut can support if the allowable shear stress for the material is $\tau_{allow} = 40$ MPa.



SOLUTION

Section Properties:

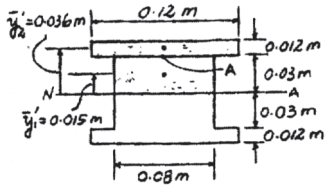
$$I_{NA} = \frac{1}{12}(0.12)(0.084^3) - \frac{1}{12}(0.04)(0.06^3)$$

$$= 5.20704(10^{-6})\ \text{m}^4$$

$$Q_{\max} = \Sigma \bar{y}' A'$$

$$= 0.015(0.08)(0.03) + 0.036(0.012)(0.12)$$

$$= 87.84(10^{-6})\ \text{m}^3$$



Allowable Shear Stress: Maximum shear stress occurs at the point where the neutral axis passes through the section.

Applying the shear formula

$$\tau_{\max} = \tau_{allow} = \frac{VQ_{\max}}{It}$$

$$40(10^6) = \frac{V(87.84)(10^{-6})}{5.20704(10^{-6})(0.08)}$$

$$V = 189\,692\ \text{N} = 190\ \text{kN} \qquad \textbf{Ans.}$$

Ans:
$V_{\max} = 190$ kN

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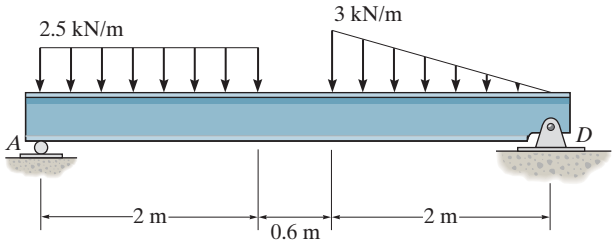
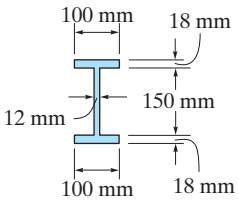
7–21. Determine the maximum shear stress acting in the fiberglass beam at the section where the internal shear force is maximum.

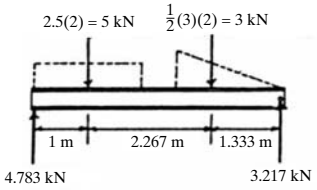




SOLUTION

Support Reactions: As shown on FBD.

Internal Shear Force: As shown on shear diagram, $V_{\max} = 4.783$ kN.



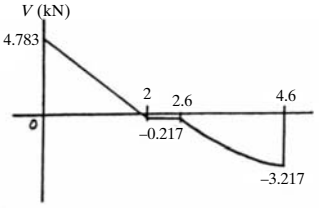
Section Properties:

$$I_{NA} = \frac{1}{12}(0.1)(0.186^3) - \frac{1}{12}(0.088)(0.15^3) = 28.8738(10^{-6})\ \text{m}^4$$

$$Q_{\max} = \Sigma y' \text{A}'$$

$$= 0.084(0.1)(0.018) + 0.0375(0.012)(0.075) = 0.18495(10^{-3})\ \text{m}^3$$

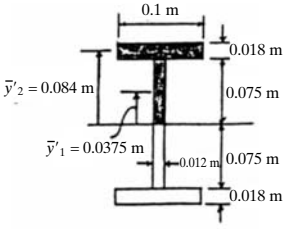
Maximum Shear Stress: Maximum shear stress occurs at the point where the neutral axis passes through the section.



Applying the shear formula

$$\tau_{\max} = \frac{VQ_{\max}}{It}$$

$$= \frac{[4.783(10^3)][0.18495(10^{-3})]}{[28.8738(10^{-6})](0.012)} = 2.553(10^6)\ \text{N/m}^3 = 2.55\ \text{MPa} \qquad \textbf{Ans.}$$



Ans.
$\tau_{\max} = 2.55$ MPa

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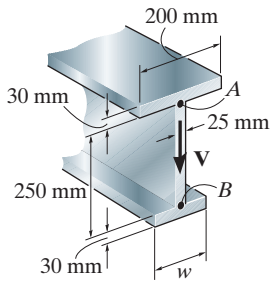
7–22.

If the beam is subjected to a shear of $V = 15$ kN, determine the web's shear stress at A and B. Indicate the shear-stress components on a volume element located at these points. Set $w = 125$ mm. Show that the neutral axis is located at $\bar{y} = 0.1747$ m from the bottom and $I_{NA} = 0.2182(10^{-3})$ m^4.



SOLUTION

$$\bar{y} = \frac{(0.015)(0.125)(0.03) + (0.155)(0.025)(0.25) + (0.295)(0.2)(0.03)}{0.125(0.03) + (0.025)(0.25) + (0.2)(0.03)} = 0.1747 \text{ m}$$

$$I = \frac{1}{12}(0.125)(0.03^3) + 0.125(0.03)(0.1747 - 0.015)^2$$

$$+ \frac{1}{12}(0.025)(0.25^3) + 0.25(0.025)(0.1747 - 0.155)^2$$

$$+ \frac{1}{12}(0.2)(0.03^3) + 0.2(0.03)(0.295 - 0.1747)^2 = 0.218182(10^{-3}) \text{ m}^4$$

$Q_A = \bar{y}A'_A = (0.310 - 0.015 - 0.1747)(0.2)(0.03) = 0.7219(10^{-3})$ m^3

$Q_B = \bar{y}A'_B = (0.1747 - 0.015)(0.125)(0.03) = 0.59883(10^{-3})$ m^3

$$\tau_A = \frac{VQ_A}{It} = \frac{15(10^3)(0.7219)(10^{-3})}{0.218182(10^{-3})0.025} = 1.99 \text{ MPa}$$ **Ans.**

$$\tau_B = \frac{VQ_B}{It} = \frac{15(10^3)(0.59883)(10^{-3})}{0.218182(10^{-3})0.025} = 1.65 \text{ MPa}$$ **Ans.**

Ans:
$\tau_A = 1.99$ MPa, $\tau_B = 1.65$ MPa

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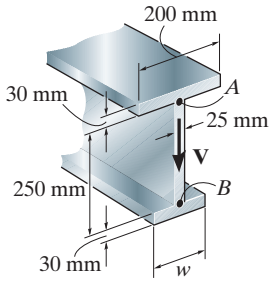
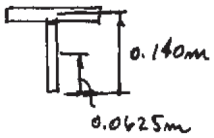
7–23.

If the wide-flange beam is subjected to a shear of $V = 30$ kN, determine the maximum shear stress in the beam. Set $w = 200$ mm.



SOLUTION

Section Properties:



$$I = \frac{1}{12}(0.2)(0.310)^3 - \frac{1}{12}(0.175)(0.250)^3 = 268.652(10)^{-6}\ \text{m}^4$$

$$Q_{\max} = \Sigma \bar{y}A = 0.0625(0.125)(0.025) + 0.140(0.2)(0.030) = 1.0353(10)^{-3}\ \text{m}^3$$

$$\tau_{\max} = \frac{VQ}{It} = \frac{30(10)^3\,(1.0353)(10)^{-3}}{268.652(10)^{-6}(0.025)} = 4.62\ \text{MPa} \qquad \textbf{Ans.}$$

Ans:
$\tau_{\max} = 4.62$ MPa

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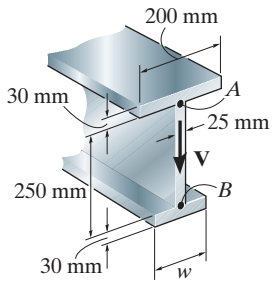
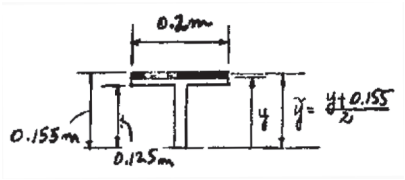
***7–24.**

If the wide-flange beam is subjected to a shear of $V = 30$ kN, determine the shear force resisted by the web of the beam. Set $w = 200$ mm.



SOLUTION



$$I = \frac{1}{12}(0.2)(0.310)^3 - \frac{1}{12}(0.175)(0.250)^3 = 268.652(10)^{-6}\ \text{m}^4$$

$$Q = \left(\frac{0.155 + y}{2}\right)(0.155 - y)(0.2) = 0.1(0.024025 - y^2)$$

$$\tau_f = \frac{30(10)^3(0.1)(0.024025 - y^2)}{268.652(10)^{-6}(0.2)}$$

$$V_f = \int \tau_f\, dA = 55.8343(10)^6 \int_{0.125}^{0.155} (0.024025 - y^2)(0.2\, dy)$$

$$= 11.1669(10)^6 \left[0.024025y - \frac{1}{3}y^3\right]_{0.125}^{0.155}$$

$$V_f = 1.457\ \text{kN}$$

$$V_w = 30 - 2(1.457) = 27.1\ \text{kN}$$ **Ans.**

Ans:
$V_w = 27.1$ kN

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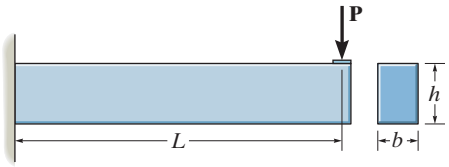
7–25.

Determine the length of the cantilevered beam so that the maximum bending stress in the beam is equivalent to the maximum shear stress.



SOLUTION

$V_{\max} = P$

$M_{\max} = PL$

$$\sigma_{\max} = \frac{Mc}{I} = \frac{PL(h/2)}{I} = \frac{PLh}{2I}$$

$$\tau_{\max} = \frac{VQ}{It} = \frac{P(h/2)(b)(h/4)}{Ib} = \frac{Ph^2}{8I}$$

Require,

$$\sigma_{\max} = \tau_{\max}$$

$$\frac{PLh}{2I} = \frac{Ph^2}{8I}$$

$$L = \frac{h}{4}$$ **Ans.**

Ans:

$$L = \frac{h}{4}$$

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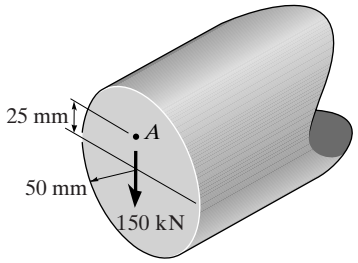
7–26. The steel rod is subjected to a shear of 150 kN. Determine the shear stress at point A. Show the result on a volume element at this point.



The moment of inertia of the circular cross-section about the neutral axis (x axis) is

$$I = \frac{\pi}{4} r^4 = \frac{\pi}{4} (50^4) = 1.5625\,\pi(10^6)\ \text{mm}^4$$

Q for the differential area shown in Fig. a is

$$dQ = y dA = y\,(2x dy) = 2xy\,dy$$

However, from the equation of the circle, $x = (2500 - y^2)^{\frac{1}{2}}$, Then

$$dQ = 2y(2500 - y^2)^{\frac{1}{2}}\,dy$$

Thus, Q for the area above y is

$$Q = \int_y^{50\ \text{mm}} 2y(2500 - y^2)^{\frac{1}{2}}\,dy$$

$$= -\frac{2}{3}\,(2500 - y^2)^{\frac{3}{2}}\Big|_y^{50\ \text{mm}} = \frac{2}{3}(2500 - y^2)^{\frac{3}{2}}$$

Here $t = 2x = 2\,(2500 - y^2)^{\frac{1}{2}}$. Thus

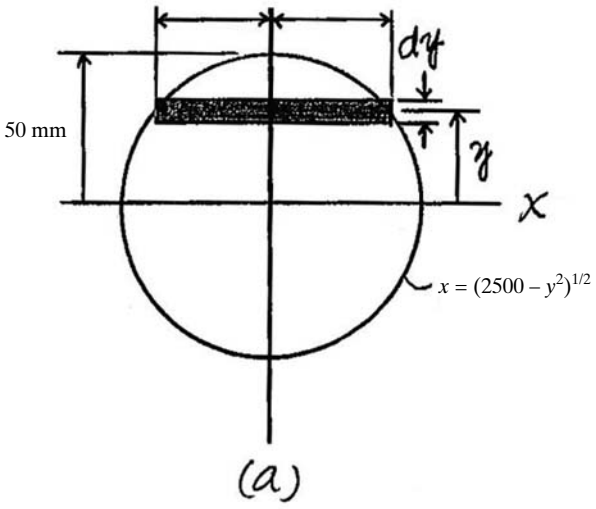
$$\tau = \frac{VQ}{It} = \frac{150(10^3)[\frac{2}{3}(2500 - y^2)^{\frac{3}{2}}]}{1.5625\pi(10^6)[2(2500 - y^2)^{\frac{1}{2}}]}$$

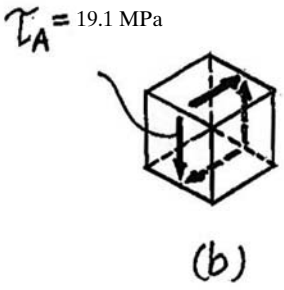
$$\tau = \frac{4}{125\pi}\,(2500 - y^2)\ \text{Ksi}$$

For point A, $y = 25$ mm. Thus

$$\tau_A = \frac{4}{125\pi}\,(2500 - 25^2) = 19.1\ \text{MPa}$$ **Ans.**

The state of shear stress at point A can be represented by the volume element shown in Fig. b.





Ans:
$P = 5.9988$ kN

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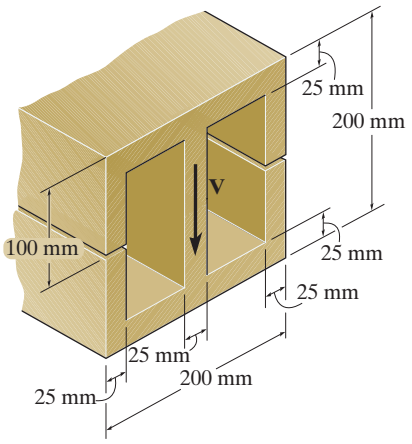
7–27.

The beam is slit longitudinally along both sides. If it is subjected to a shear of $V = 250$ kN, compare the maximum shear stress in the beam before and after the cuts were made.



SOLUTION

Section Properties: The moment of inertia of the cross section about the neutral axis is

$$I = \frac{1}{12}(0.2)(0.2^3) - \frac{1}{12}(0.125)(0.15^3) = 98.1771(10^{-6})\ \text{m}^4$$

$Q_{\max}$ is the first moment of the shaded area shown in Fig. *a* about the neutral axis. Thus,

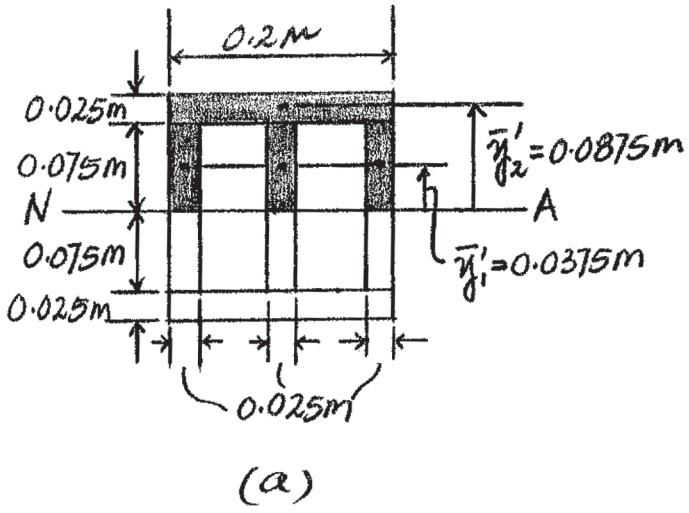
$$\begin{aligned} Q_{\max} &= 3\bar{y}'_1 A'_1 + \bar{y}'_2 A'_2 \\ &= 3\left[0.0375(0.075)(0.025)\right] + 0.0875(0.025)(0.2) \\ &= 0.6484375(10^{-3})\ \text{m}^3 \end{aligned}$$

Maximum Shear Stress: The maximum shear stress occurs at the points on the neutral axis since Q is maximum and t is minimum. Before the cross section is slit, $t = 3(0.025) = 0.075$ m.

$$\tau_{\max} = \frac{VQ_{\max}}{It} = \frac{250(10^3)(0.6484375)(10^{-3})}{98.1771(10^{-6})(0.075)} = 22.0\ \text{MPa} \qquad \textbf{Ans.}$$

After the cross section is slit, $t = 0.025$ m.

$$(\tau_{\max})_s = \frac{VQ_{\max}}{It} = \frac{250(10^3)(0.6484375)(10^{-3})}{98.1771(10^{-6})(0.025)} = 66.0\ \text{MPa} \qquad \textbf{Ans.}$$



(a)

Ans:
$\tau_{\max} = 22.0$ MPa, $(\tau_{\max})_s = 66.0$ MPa

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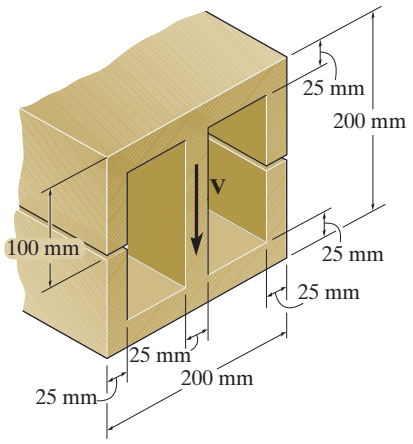
***7–28.**

The beam is to be cut longitudinally along both sides as shown. If it is made from a material having an allowable shear stress of $\tau_{allow} = 75$ MPa, determine the maximum allowable shear force **V** that can be applied before and after the cut is made.



SOLUTION

Section Properties: The moment of inertia of the cross section about the neutral axis is

$$I = \frac{1}{12}(0.2)(0.2^3) - \frac{1}{12}(0.125)(0.15^3) = 98.1771(10^{-6})\ \text{m}^4$$

Q_{max} is the first moment of the shaded area shown in Fig. *a* about the neutral axis. Thus,

$$\begin{aligned} Q_{max} &= 3\bar{y}'_1 A'_1 + \bar{y}'_2 A'_2 \\ &= 3(0.0375)(0.075)(0.025) + 0.0875(0.025)(0.2) \\ &= 0.6484375(10^{-3})\ \text{m}^3 \end{aligned}$$

Shear Stress: The maximum shear stress occurs at the points on the neutral axis since Q is maximum and thickness t is minimum. Before the cross section is slit, $t = 3(0.025) = 0.075$ m.

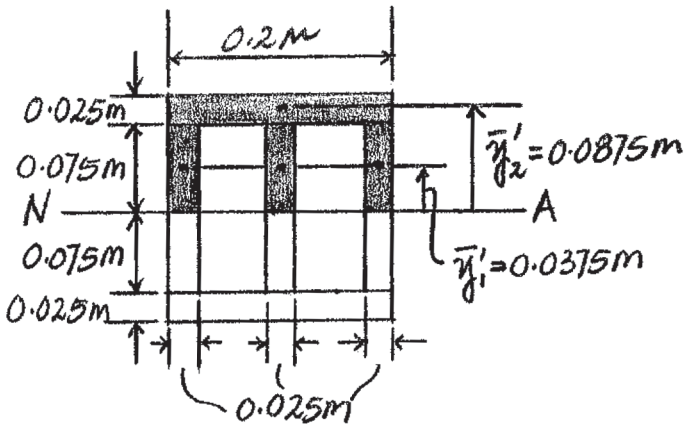
$$\tau_{allow} = \frac{VQ_{max}}{It}; \qquad 75(10^6) = \frac{V(0.6484375)(10^{-3})}{98.1771(10^{-6})(0.075)}$$

$$V = 851\,656.63\ \text{N} = 852\ \text{kN} \qquad \textbf{Ans.}$$

After the cross section is slit, $t = 0.025$ m.

$$\tau_{allow} = \frac{VQ_{max}}{It}; \qquad 75(10^6) = \frac{V_s(0.6484375)(10^{-3})}{98.1771(10^{-6})(0.025)}$$

$$V_s = 283\,885.54\ \text{N} = 284\ \text{kN} \qquad \textbf{Ans.}$$



(a)

Ans:
$V = 852$ kN,
$V_s = 284$ kN

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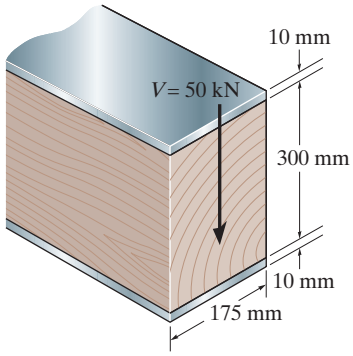
7–29.

The composite beam is constructed from wood and reinforced with a steel strap. Use the method of Sec. 6.6 and calculate the maximum shear stress in the beam when it is subjected to a shear of $V = 50$ kN. Take $E_{st} = 200$ GPa, $E_w = 15$ GPa.



SOLUTION

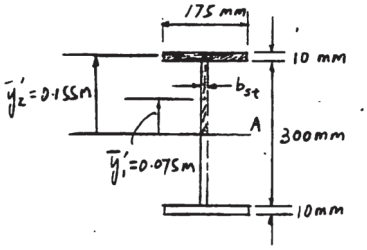
$$b_{st} = nb_w = \frac{15}{200}(0.175) = 0.013125 \text{ m}$$

$$I = \frac{1}{12}(0.175)(0.32^3) - \frac{1}{12}(0.175 - 0.013125)(0.3^3) = 0.113648(10^{-3}) \text{ m}^4$$

$$Q_{max} = \Sigma \bar{y}' A' = 0.075(0.013125)(0.15) + 0.155(0.175)(0.01) = 0.4189(10^{-3}) \text{ m}^3$$

$$\tau_{max} = n\frac{VQ_{max}}{It} = \left(\frac{15}{200}\right)\frac{50(10^3)(0.4189)(10^{-3})}{0.113648(10^{-3})(0.013125)}$$

$$= 1.05 \text{ MPa} \quad \textbf{Ans.}$$



Ans:
$\tau_{max} = 1.05$ MPa

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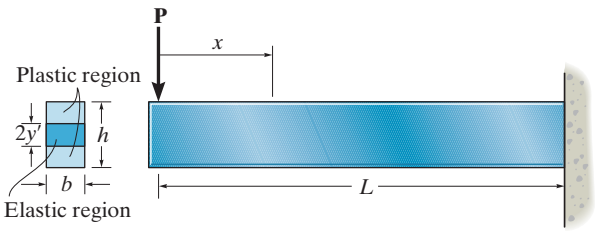
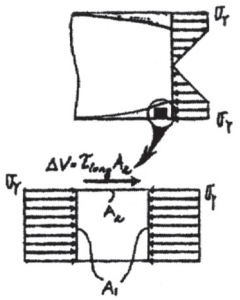
7–30.

The beam has a rectangular cross section and is subjected to a load P that is just large enough to develop a fully plastic moment $M_p = PL$ at the fixed support. If the material is elastic perfectly plastic, then at a distance $x < L$ the moment $M = Px$ creates a region of plastic yielding with an associated elastic core having a height $2y'$. This situation has been described by Eq. 6–30 and the moment **M** is distributed over the cross section as shown in Fig. 6–48*e*. Prove that the maximum shear stress in the beam is given by $\tau_{max} = \frac{3}{2}(P/A')$, where $A' = 2y'b$, the cross-sectional area of the elastic core.



SOLUTION

Force Equilibrium: The shaded area indicates the plastic zone. Isolate an element in the plastic zone and write the equation of equilibrium.



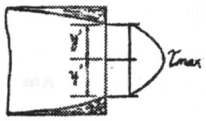
$$\pm \Sigma F_x = 0; \qquad \tau_{long} A_2 + \sigma_\gamma A_1 - \sigma_\gamma A_1 = 0$$

$$\tau_{long} = 0$$

This proves that the longitudinal shear stress. τ_{long}, is equal to zero. Hence the corresponding transverse stress, τ_{max}, is also equal to zero in the plastic zone. Therefore, the shear force $V = P$ is carried by the material only in the elastic zone.

Section Properties:

$$I_{NA} = \frac{1}{12}(b)(2y')^3 = \frac{2}{3}b\,y'^3$$



$$Q_{max} = \bar{y}'\,A' = \frac{y'}{2}(y')(b) = \frac{y'^2 b}{2}$$

Maximum Shear Stress: Applying the shear formula

$$\tau_{max} = \frac{VQ_{max}}{It} = \frac{V\left(\frac{y'^2 b}{2}\right)}{\left(\frac{2}{3}by'^3\right)(b)} = \frac{3P}{4by'}$$

However, $A' = 2by'$ hence

$$\tau_{max} = \frac{3P}{2A'}, \qquad \text{(Q.E.D.)}$$

Ans:
N/A

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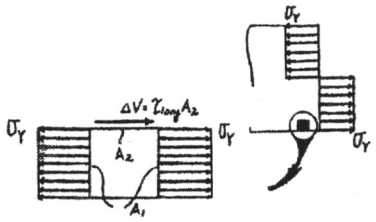
7–31.

The beam in Fig. 6–48*f* is subjected to a fully plastic moment $\mathbf{M}_p$. Prove that the longitudinal and transverse shear stresses in the beam are zero. *Hint:* Consider an element of the beam shown in Fig. 7–4*d*.

SOLUTION

Force Equilibrium: If a fully plastic moment acts on the cross section, then an element of the material taken from the top or bottom of the cross section is subjected to the loading shown. For equilibrium



$$\pm \Sigma F_x = 0; \qquad \sigma_\gamma A_1 + \tau_{\text{long}} A_2 - \sigma_\gamma A_1 = 0$$

$$\tau_{\text{long}} = 0$$

Thus no shear stress is developed on the longitudinal or transverse plane of the element. (*Q. E. D.*)

Ans:
N/A

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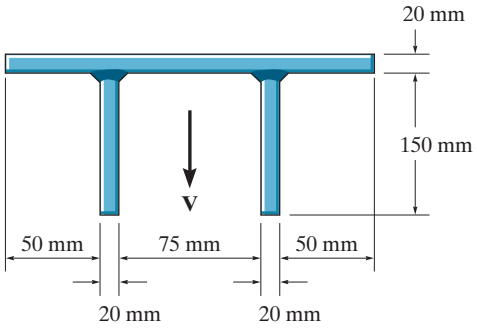
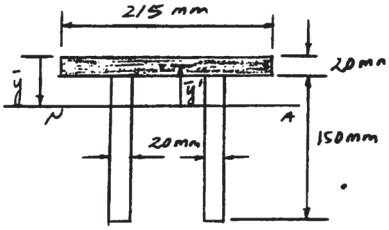
***7–32.**

The double T-beam is fabricated by welding the three plates together as shown. Determine the shear stress in the weld necessary to support a shear force of $V = 80$ kN.



SOLUTION

$$\bar{y} = \frac{\Sigma \bar{y} A}{\Sigma A} = \frac{0.01(0.215)(0.02) + 2[0.095(0.15)(0.02)]}{0.215(0.02) + 2(0.15)(0.02)} = 0.059515 \text{ m}$$



$$I = \frac{1}{12}(0.215)\left(0.02^3\right) + 0.215(0.02)(0.059515 - 0.01)^2$$

$$+ 2\left[\frac{1}{12}(0.02)\left(0.15^3\right) + 0.02(0.15)(0.095 - 0.059515)^2\right] = 29.4909\left(10^{-6}\right) \text{ m}^4$$

$$\bar{y}' = 0.059515 - 0.01 = 0.049515 \text{ m}$$

$$Q = \bar{y}' A' = 0.049515(0.215)(0.02) = 0.2129\left(10^{-3}\right) \text{ m}^3$$

Shear stress:

$$\tau = \frac{VQ}{It} = \frac{80\left(10^3\right)(0.2129)\left(10^{-3}\right)}{29.4909\left(10^{-6}\right)(2)(0.02)}$$

$$= 14.4 \text{ MPa} \qquad \textbf{Ans.}$$

Ans:
$\tau = 14.4$ MPa

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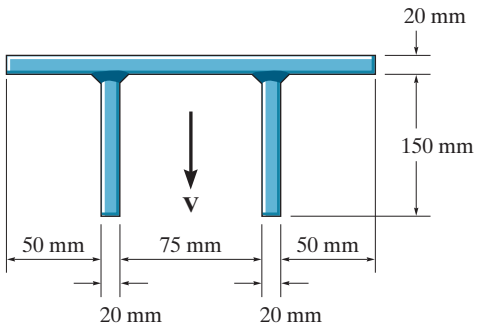
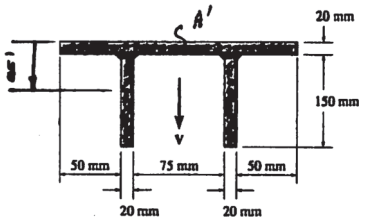
7–33.

The double T-beam is fabricated by welding the three plates together as shown. If the weld can resist a shear stress $\tau_{allow} = 90$ MPa, determine the maximum shear V that can be applied to the beam.



SOLUTION

$$\bar{y} = \frac{\Sigma \bar{y} A}{\Sigma A} = \frac{0.01(0.215)(0.02) + 2[0.095(0.15)(0.02)]}{0.215(0.02) + 2(0.15)(0.02)} = 0.059515 \text{ m}$$



$$I = \frac{1}{12}(0.215)(0.02^3) + 0.215(0.02)(0.059515 - 0.01)^2$$

$$+ 2\left[\frac{1}{12}(0.02)(0.15^3) + 0.02(0.15)(0.095 - 0.059515)^2\right] = 29.4909(10^{-6}) \text{ m}^4$$

$$\bar{y}' = 0.059515 - 0.01 = 0.049515 \text{ m}$$

$$Q = \bar{y}' A' = 0.049515(0.215)(0.02) = 0.2129(10^{-3}) \text{ m}^3$$

$$\tau = \frac{VQ}{It}$$

$$90(10^6) = \frac{V(0.2129)(10^{-3})}{29.491(10^{-6})(2)(0.02)}$$

$$V = 499 \text{ kN} \qquad \textbf{Ans.}$$

Ans:
$V = 499$ kN

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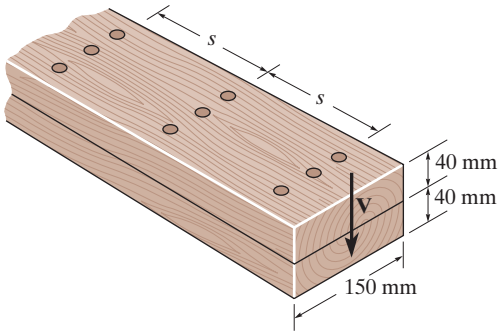
7–34. The beam is constructed from two boards fastened together with three rows of nails spaced $s = 50$ mm apart. If each nail can support a 2.25-kN shear force, determine the maximum shear force V that can be applied to the beam. The allowable shear stress for the wood is $\tau_{allow} = 2.1$ MPa.



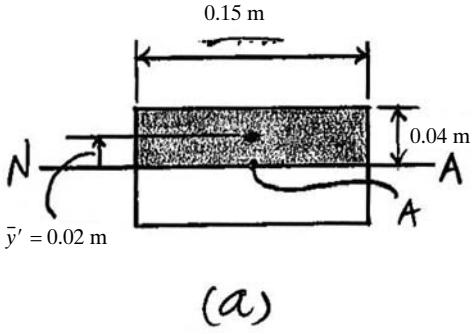
SOLUTION

The moment of inertia of the cross-section about the neutral axis is

$$I = \frac{1}{12}(0.15)(0.08^3) = 6.40(10^{-6})\ \text{m}^4$$

Refering to Fig. *a*,

$$Q_A = Q_{max} = \bar{y}'A' = 0.02(0.15)(0.04) = 0.12(10^{-3})\ \text{m}^3$$



The maximum shear stress occurs at the points on the neutral axis where Q is maximum and $t = 0.15$ m.

$$\tau_{allow} = \frac{VQ_{max}}{It}; \qquad 2.1(10^6) = \frac{V[0.12(10^{-3})]}{[6.40(10^{-6})](0.15)}$$

$$V = 16.8(10^3)\ \text{N} = 16.8\ \text{kN}$$

Shear Flow: Since there are three rows of nails,

$$q_{allow} = 3\left(\frac{F}{s}\right) = 3\left[\frac{2.25(10^3)}{0.05}\right] = 135(10^3)\ \text{N/m}$$

$$q_{allow} = \frac{VQ_A}{I}; \qquad 135(10^3) = \frac{V[0.12(10^{-3})]}{6.40(10^{-6})}$$

$$V = 7.20(10^3)\ \text{N} = 7.20\ \text{kN}$$ **Ans.**

Ans.
$V = 7.20$ kN

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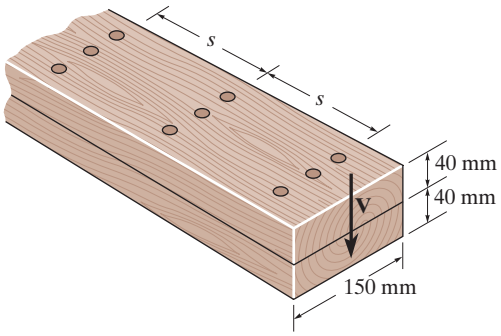
7–35. The beam is constructed from two boards fastened together with three rows of nails. If the allowable shear stress for the wood is $\tau_{allow} = 1$ MPa. determine the maximum shear force V that can be applied to the beam. Also, find the maximum spacing s of the nails if each nail can resist 3.25 kN in shear.



SOLUTION

The moment of inertia of the cross-section about the neutral axis is

$$I = \frac{1}{12}(0.15)(0.08^3) = 6.40(10^{-6})\ \text{m}^4$$

Refering to Fig. *a*,

$$Q_A = Q_{max} = \bar{y}'A' = 0.02(0.15)(0.04) = 0.12(10^{-3})\ \text{m}^3$$

The maximum shear stress occurs at the points on the neutral axis where Q is maximum and $t = 0.15$ m.

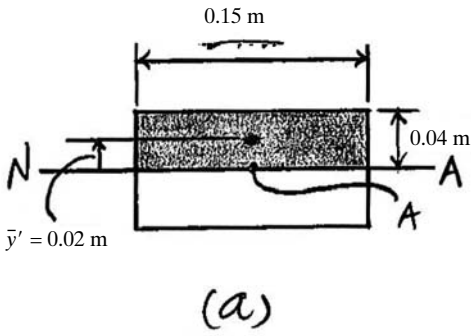
$$\tau_{allow} = \frac{VQ_{max}}{It}; \qquad 1(10^6) = \frac{V[0.12(10^{-3})]}{[6.40(10^{-6})](0.15)}$$

$$V = 8.00(10^3)\ \text{N} = 8.00\ \text{kN} \qquad \textbf{Ans.}$$

Since there are three rows of nails, $q_{allow} = 3\left[\frac{2.25(10^3)}{s}\right] = \left(\frac{9750}{s}\right)\ \text{N/m}$

$$q_{allow} = \frac{VQ_A}{I}; \qquad \frac{9750}{s} = \frac{[8(10^3)][0.12(10^{-3})]}{6.40(10^{-6})}$$

$$s = 0.0650\ \text{m} = 65.0\ \text{mm} \qquad \textbf{Ans.}$$



(a)

Ans.
$V = 8.00$ kN, $s = 65.0$ mm

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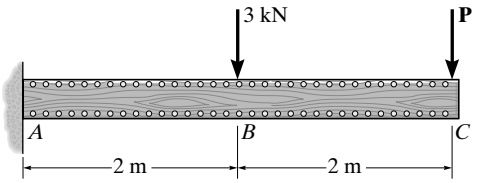
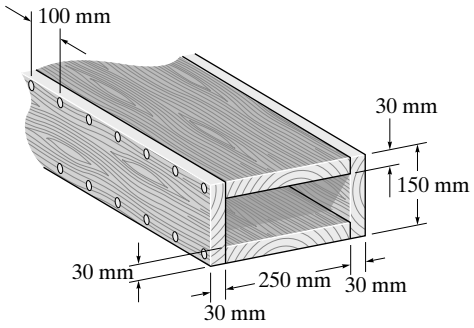
***7–36.** The beam is constructed from four boards which are nailed together. If the nails are on both sides of the beam and each can resist a shear of 3 kN, determine the maximum load P that can be applied to the end of the beam.





SOLUTION

Support Reactions: As shown on FBD.

Internal Shear Force: As shown on shear diagram, $V_{AB} = (P + 3)$ kN.

Section Properties:

$$I_{NA} = \frac{1}{12}(0.31)\left(0.15^3\right) - \frac{1}{12}(0.25)\left(0.09^3\right)$$
$$= 72.0\left(10^{-6}\right)\text{ m}^4$$

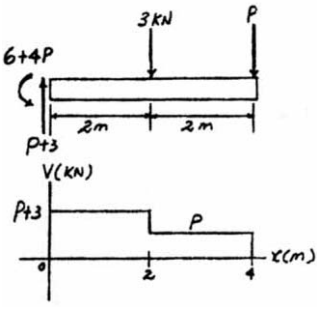
$$Q = \bar{y}'A' = 0.06(0.25)(0.03) = 0.450\left(10^{-3}\right)\text{ m}^3$$

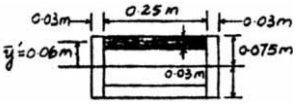
Shear Flow: There are two rows of nails. Hence the allowable shear flow is $q = \frac{3(2)}{0.1} = 60.0$ kN/m.

$$q = \frac{VQ}{I}$$

$$60.0\left(10^3\right) = \frac{(P + 3)(10^3)0.450(10^{-3})}{72.0(10^{-6})}$$

$$P = 6.60\text{ kN}$$ **Ans.**





Ans.
$P = 6.60$ kN

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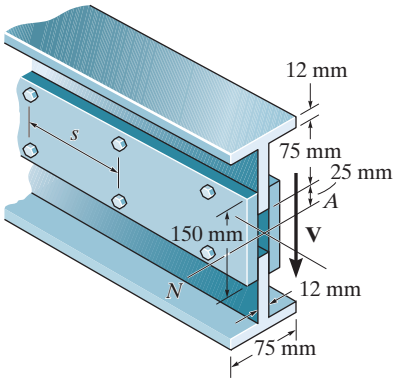
7–37. The beam is fabricated from two equivalent structural tees and two plates. Each plate has a height of 150 mm and a thickness of 12 mm. If a shear of $V = 250$ kN is applied to the cross section, determine the maximum spacing of the bolts. Each bolt can resist a shear force of 75 kN.

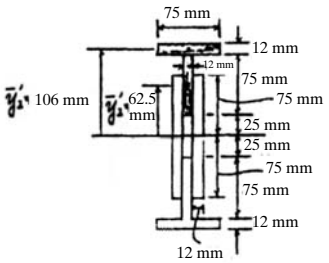


SOLUTION

Section Properties:

$$I_{NA} = \frac{1}{12}(0.075)(0.224^3) - \frac{1}{12}(0.063)(0.2^3)$$
$$-\frac{1}{12}(0.012)(0.05^3) + \frac{1}{12}(0.024)(0.15^3)$$
$$= 34.8714(10^{-6})\ \text{m}^4$$

$$Q = \Sigma \bar{y}' A' = 0.0625(0.012)(0.075) + 0.106(0.075)(0.012) = 0.15165(10^{-3})\ \text{m}^3$$



Shear Flow: Since there are two shear planes on the bolt, the allowable shear flow is $q = \frac{2(75)(10^3)}{s} = \frac{150(10^3)}{s}$.

$$q = \frac{VQ}{I}$$

$$\frac{150(10^3)}{s} = \frac{[250(10^3)][0.15165(10^{-3})]}{34.8714(10^{-6})}$$

$$s = 0.1380\ \text{m} = 138\ \text{mm}$$ **Ans.**

Ans.
$s = 138$ mm

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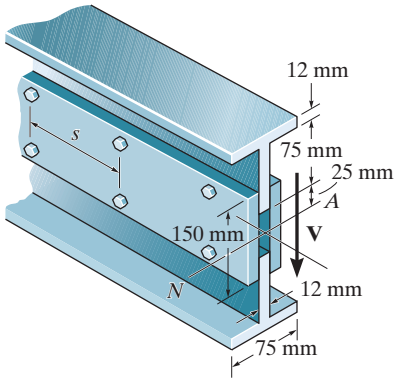
7–38. The beam is fabricated from two equivalent structural tees and two plates. Each plate has a height of 150 mm and a thickness of 12 mm. If the bolts are spaced at $s = 200$ mm, determine the maximum shear force V that can be applied to the cross section. Each bolt can resist a shear force of 75 kN.

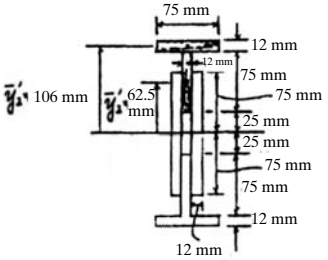


SOLUTION

Section Properties:

$$I_{NA} = \frac{1}{12}(0.075)(0.224^3) - \frac{1}{12}(0.063)(0.2^3)$$
$$-\frac{1}{12}(0.012)(0.05^3) + \frac{1}{12}(0.024)(0.15^3)$$
$$= 34.8714(10^{-6})\ \text{m}^4$$

$$Q = \Sigma \bar{y}' A' = 0.0625(0.012)(0.075) + 0.106(0.075)(0.012) = 0.15165(10^{-3})\ \text{m}^3$$



Shear Flow: Since there are two shear planes on the bolt, the allowable shear flow is

$$q = \frac{2(75)(10^3)}{0.2} = 750(10^3)\ \text{N/m}$$

$$q = \frac{VQ}{I}$$

$$750(10^3) = \frac{V[0.15165(10^{-3})]}{34.8714(10^{-6})}$$

$$V = 172.46(10^3)\ \text{N} = 172\ \text{kN}$$ **Ans.**

Ans.
$I_{NA} = 34.8714(10^{-6})\ \text{m}^4$, $Q = 0.15165\ (10^{-3})\ \text{m}^3$
$V = 172$ kN

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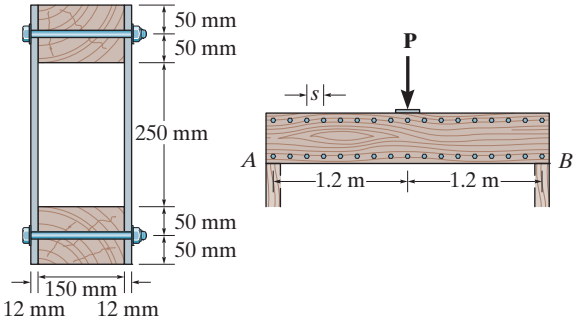
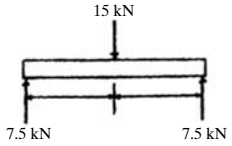
7–39. The double-web girder is constructed from two plywood sheets that are secured to wood members at its top and bottom. If each fastener can support 3 kN in single shear, determine the required spacing s of the fasteners needed to support the loading $P = 15$ kN. Assume A is pinned and B is a roller.

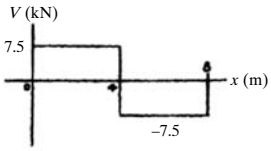


SOLUTION

Support Reactions: As shown on FBD.



Internal Shear Force: As shown on shear diagram, $V_{\max} = 7.5$ kN.

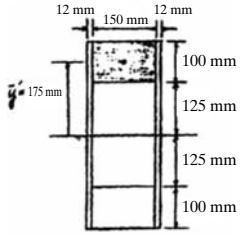


Section Properties:

$$I_{NA} = \frac{1}{12}(0.174)(0.45^3) - \frac{1}{12}(0.15)(0.25^3) = 1.126(10^{-3})\ \text{m}^4$$

$$Q = \bar{y}'A' = 0.175(0.15)(0.1) = 2.625(10^{-3})\ \text{m}^3$$

Shear Flow: Since there are two shear planes on the bolt, the allowable shear flow is $q = \dfrac{2(3)(10^3)}{s} = \dfrac{6000}{s}$.



$$q = \frac{VQ}{I}$$

$$\frac{6000}{s} = \frac{7.5(10^3)[2.625(10^{-3})]}{1.126(10^{-3})}$$

$$s = 0.3432\ \text{m} = 343\ \text{mm}$$ **Ans.**

Ans.
$s = 343$ mm

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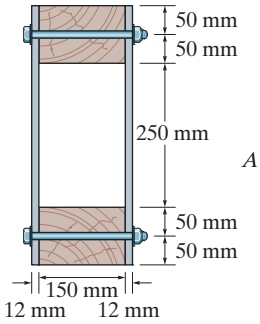
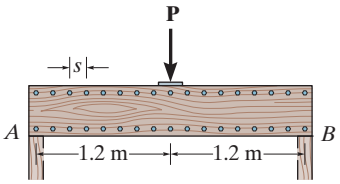
***7–40.** The double-web girder is constructed from two plywood sheets that are secured to wood members at its top and bottom. The allowable bending stress for the wood is $\sigma_{allow} = 56$ MPa and the allowable shear stress is $\tau_{allow} = 21$ MPa. If the fasteners are spaced $s = 150$ mm and each fastener can support 3 kN in single shear, determine the maximum load P that can be applied to the beam.

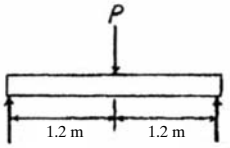




SOLUTION

Support Reactions: As shown on FBD.

Internal Shear Force and Moment: As shown on shear and moment diagram, $V_{max} = 0.500P$ and $M_{max} = 2.00P$.

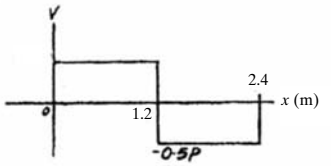


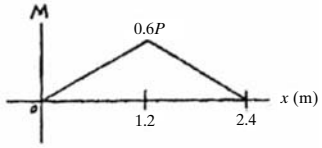
Section Properties:

$$I_{NA} = \frac{1}{12}(0.174)(0.45^3) - \frac{1}{12}(0.15)(0.25^3) = 1.126(10^{-3})\ \text{m}^4$$

$$Q = \bar{y}'_2 A' = 0.175(0.15)(0.1) = 2.625(10^{-3})\ \text{m}^3$$

$$Q_{max} = \Sigma \bar{y}' \text{A}' = 0.175(0.15)(0.1) + 0.1125(0.024)(0.225) = 3.2325(10^{-3})\ \text{m}^3$$



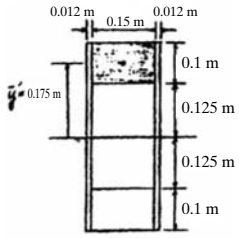


Shear Flow: Assume bolt failure. Since there are two shear planes on the bolt, the allowable shear flow is $q = \frac{2(3)(10^3)}{0.15} = 40(10^3)$ N/m

$$q = \frac{VQ}{I}$$

$$40(10^3) = \frac{0.5P[2.625(10^{-3})]}{1.126(10^{-3})}$$

$$P = 34.32(10^3)\ \text{N} = 34.3\ \text{kN} \quad \text{(Controls!)}$$ **Ans.**



Shear Stress: Assume failure due to shear stress.

$$\tau_{max} = \tau_{allow} = \frac{VQ_{max}}{It}$$

$$21(10^6) = \frac{0.5P[3.2325(10^{-3})]}{[1.126(10^{-3})](0.024)}$$

$$P = 351.12(10^3)\ \text{N} = 351\ \text{kN}$$

Bending Stress: Assume failure due to bending stress.

$$\sigma_{max} = \sigma_{allow} = \frac{Mc}{I}$$

$$56(10^6) = \frac{0.6P(0.225)}{1.126(10^{-3})}$$

$$P = 467.08(10^3)\ \text{N} = 467\ \text{kN}$$

Ans.
$P = 34.3$ kN

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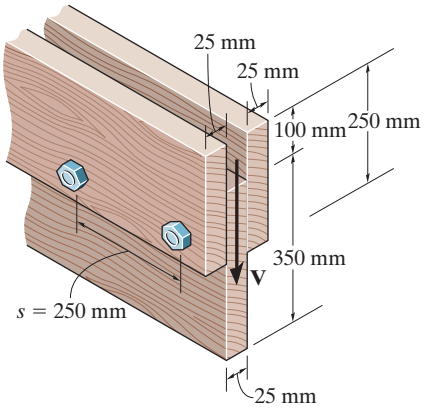
7–41.

A beam is constructed from three boards bolted together as shown. Determine the shear force in each bolt if the bolts are spaced $s = 250$ mm apart and the shear is $V = 35$ kN.



SOLUTION

$$\bar{y} = \frac{2\,(0.125)(0.25)(0.025) + 0.275\,(0.35)(0.025)}{2\,(0.25)(0.025) + 0.35\,(0.025)} = 0.18676\text{ m}$$

$$I = (2)\left(\frac{1}{12}\right)(0.025)(0.25^3) + 2\,(0.025)(0.25)(0.18676 - 0.125)^2$$

$$+ \frac{1}{12}(0.025)(0.35)^3 + (0.025)(0.35)(0.275 - 0.18676)^2$$

$$= 0.270236\,(10^{-3})\text{ m}^4$$

$$Q = \bar{y}'A' = 0.06176(0.025)(0.25) = 0.386(10^{-3})\text{ m}^3$$

$$q = \frac{VQ}{I} = \frac{35(0.386)(10^{-3})}{0.270236(10^{-3})} = 49.997\text{ kN/m}$$

$F = q(s) = 49.997\,(0.25) = 12.5$ kN **Ans.**

Ans:
$F = 12.5$ kN

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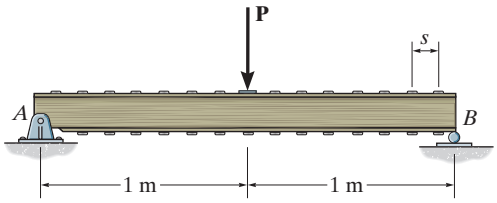
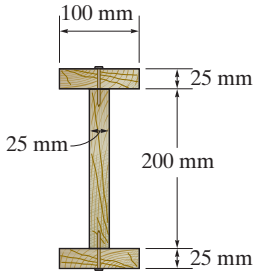
7–42.

The simply supported beam is built up from three boards by nailing them together as shown. The wood has an allowable shear stress of $\tau_{allow} = 1.5$ MPa, and an allowable bending stress of $\sigma_{allow} = 9$ MPa. The nails are spaced at $s = 75$ mm, and each has a shear strength of 1.5 kN. Determine the maximum allowable force **P** that can be applied to the beam.





SOLUTION

Support Reactions: As shown on the free-body diagram of the beam shown in Fig. *a*.

Maximum Shear and Moment: The shear diagram is shown in Fig. *b*. As indicated, $V_{max} = \frac{P}{2}$.

Section Properties: The moment of inertia of the cross section about the neutral axis is

$$I = \frac{1}{12}(0.1)(0.25^3) - \frac{1}{12}(0.075)(0.2^3)$$

$$= 80.2083(10^{-6})\ \text{m}^4$$

Referring to Fig. *d*,

$$Q_B = \bar{y}'_2 A'_2 = 0.1125(0.025)(0.1) = 0.28125(10^{-3})\ \text{m}^3$$

Shear Flow: Since there is only one row of nails, $q_{allow} = \frac{F}{s} = \frac{1.5(10^3)}{0.075} = 20(10^3)\ \text{N/m}$.

$$q_{allow} = \frac{V_{max} Q_B}{I}; \qquad 20(10^3) = \frac{\frac{p}{2}\left[0.28125(10^{-3})\right]}{80.2083(10^{-6})}$$

$$P = 11417.41\ \text{N} = 11.4\ \text{kN (controls)} \qquad \textbf{Ans.}$$

Bending,

$$\sigma_{max} = \frac{Mc}{I}$$

$$\sigma(10^6)\ \text{N/m}^2 = \frac{\left(\frac{P}{2}\right)(0.125\ \text{m})}{80.2083(10^{-6})\ \text{m}^4}$$

$$P = 11.550\ \text{N} = 11.55\ \text{kN}$$

Shear,

$$\tau_{max} = \frac{VQ}{It}$$

$$Q = (0.1125)(0.025)(0.1) + (0.05)(0.1)(0.025)$$

$$= 0.40625(10^{-3})\ \text{m}^3$$

$$1.5(10^6) = \frac{\left(\frac{P}{2}\right)(0.40625)(10^{-3})\ \text{m}^3}{80.2083(10^{-6})\ \text{m}^4(0.025\ \text{m})}$$

$$P = 14.808\ \text{N} = 14.8\ \text{kN}$$

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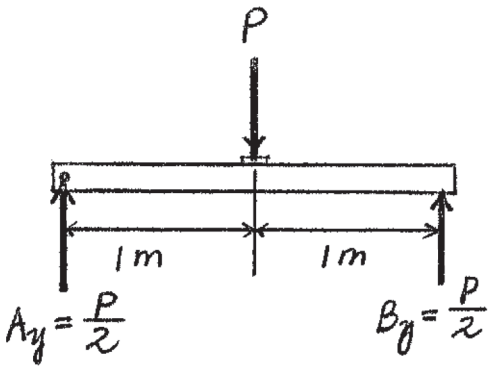
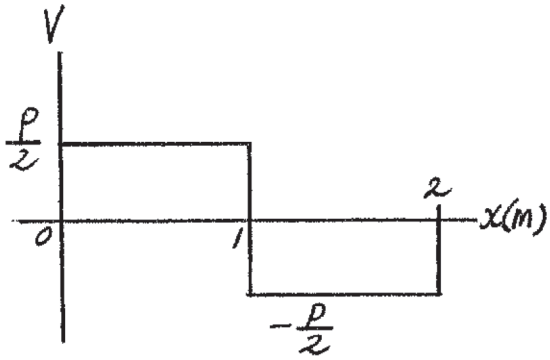
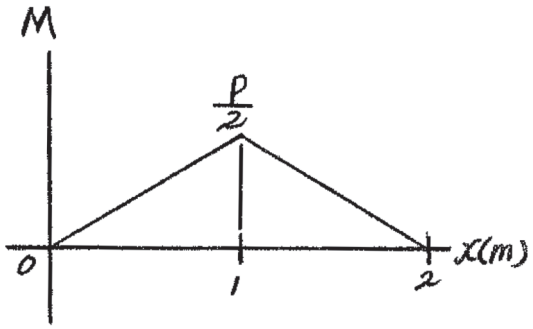
7–42. Continued



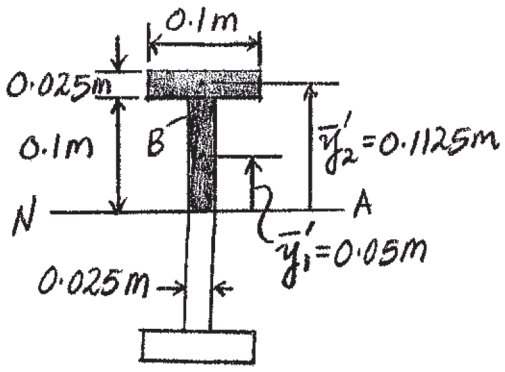
(a)



(b)



(c)



(d)

Ans:
$P = 11.4$ kN

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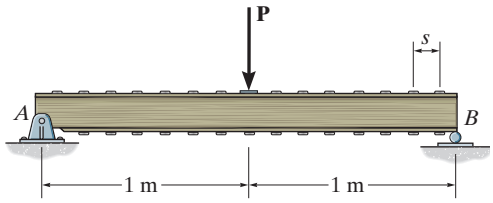
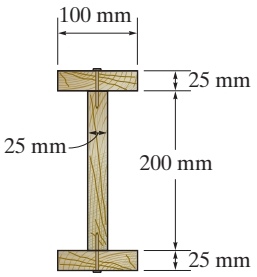
7–43.

The simply supported beam is built up from three boards by nailing them together as shown. If $P = 12$ kN, determine the maximum allowable spacing s of the nails to support that load, if each nail can resist a shear force of 1.5 kN.





SOLUTION

Support Reactions: As shown on the free-body diagram of the beam shown in Fig. *a*.

Maximum Shear and Moment: The shear diagram is shown in Fig. *b*. As indicated,

$$V_{\max} = \frac{P}{2} = \frac{12}{2} = 6 \text{ kN}.$$

Section Properties: The moment of inertia of the cross section about the neutral axis is

$$I = \frac{1}{12}(0.1)(0.25^3) - \frac{1}{12}(0.075)(0.2^3)$$
$$= 80.2083(10^{-6}) \text{ m}^4$$

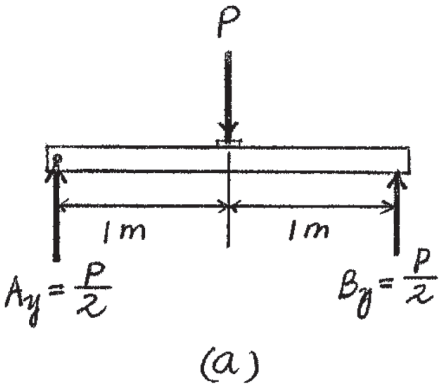
Referring to Fig. *d*,

$$Q_B = \bar{y}'_2 A'_2 = 0.1125(0.025)(0.1) = 0.28125(10^{-3}) \text{ m}^3$$

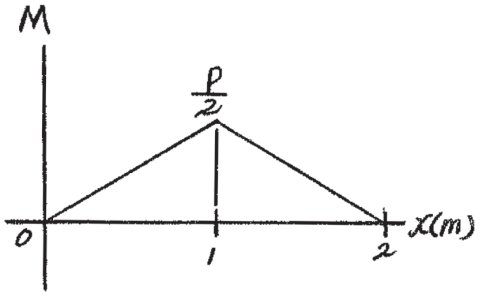
Shear Flow: Since there is only one row of nails, $q_{\text{allow}} = \frac{F}{s} = \frac{1.5(10^3 1)}{s}$.

$$q_{\text{allow}} = \frac{V_{\max} Q_B}{I}; \qquad \frac{1.5(10^3 1)}{s} = \frac{6000\left[0.28125(10^{-3})\right]}{80.2083(10^{-6} 1)}$$

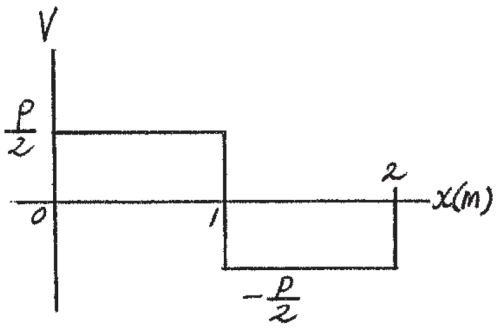
$$s = 0.07130 \text{ m} = 71.3 \text{ mm}$$ **Ans.**



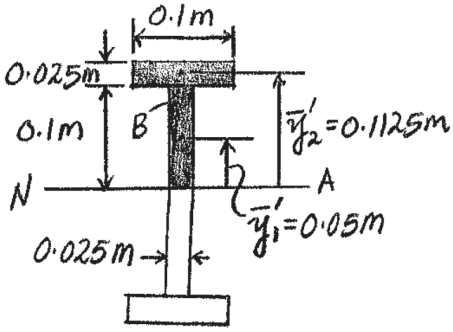
(a)



(c)



(b)



(d)

Ans:
$s = 71.3$ mm

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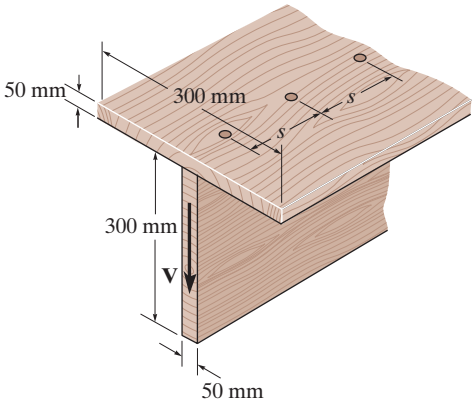
***7–44.** The T-beam is nailed together as shown. If the nails can each support a shear force of 4.5 kN, determine the maximum shear force V that the beam can support and the corresponding maximum nail spacing s to the nearest multiples of 5 mm. The allowable shear stress for the wood is $\tau_{allow} = 3$ MPa.



SOLUTION

The neutral axis passes through the centroid c of the cross-section as shown in Fig. a.

$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{0.325(0.3)(0.05) + 0.15(0.05)(0.3)}{0.3(0.05) + 0.05(0.3)} = 0.2375 \text{ m}$$

$$I = \frac{1}{12}(0.05)(0.3^3) + 0.05(0.3)(0.2375 - 0.15)^2 + \frac{1}{12}(0.3)(0.05^3) + 0.3(0.05)(0.325 - 0.2375)^2$$

$$= 0.3453125(10^{-3}) \text{ m}^4$$

Refering to Fig. a, Q_{max} and Q_A are

$$Q_{max} = \bar{y}_1' A_1' = 0.11875(0.05)(0.2375) = 1.41016(10^{-3}) \text{ m}^3$$

$$Q_A = \bar{y}_2' A_2' = 0.0875(0.3)(0.05) = 1.3125(10^{-3}) \text{ m}^3$$

The maximum shear stress occurs at the points on the neutral axis where Q is maximum and $t = 0.05$ m.

$$\tau_{allow} = \frac{VQ_{max}}{It}; \quad 3(10^6) = \frac{V[1.41016(10^{-3})]}{[0.3453125(10^{-3})](0.05)}$$

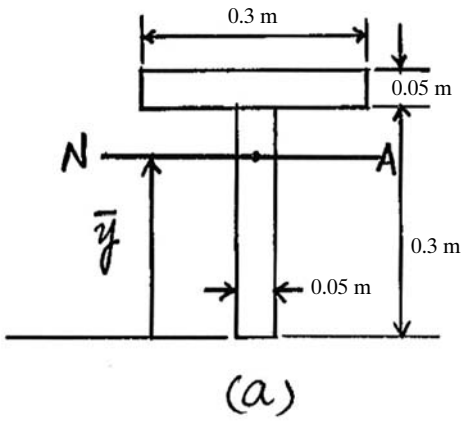
$$V = 36.73(10^3) \text{ N} = 36.7 \text{ kN} \quad \textbf{Ans.}$$

Here, $q_{allow} = \frac{F}{s} = \left[\frac{4.5(10^3)}{s}\right]$ N/m. Then

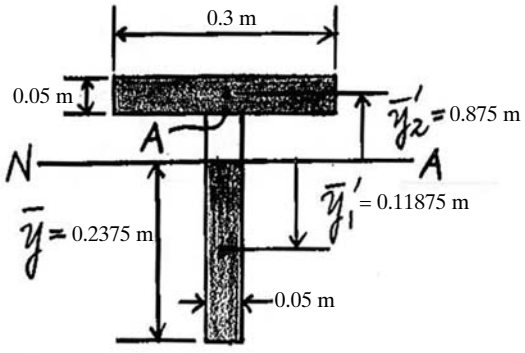
$$q_{allow} = \frac{VQ_A}{I}; \quad \left[\frac{4.5(10^3)}{s}\right] = \frac{[36.73(10^3)][1.3125(10^{-3})]}{0.3453125(10^{-3})}$$

$$s = 0.03223 \text{ m} = 32.23 \text{ mm} \quad \textbf{Ans.}$$

Use $s = 35$ mm



(a)



Ans.
$V = 36.7$ kN, $s = 32.23$ mm

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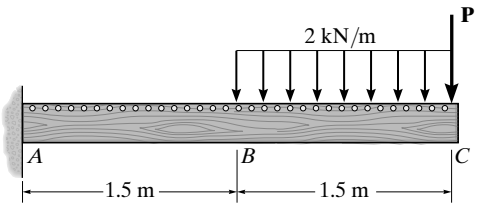
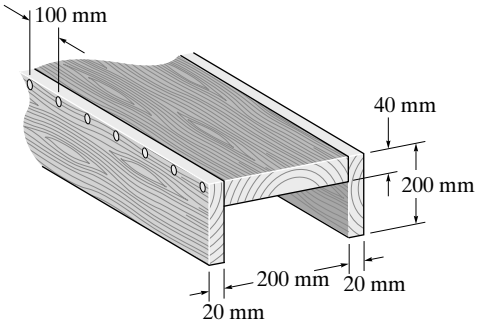
7–45. The nails are on both sides of the beam and each can resist a shear of 2 kN. In addition to the distributed loading, determine the maximum load P that can be applied to the end of the beam. The nails are spaced 100 mm apart and the allowable shear stress for the wood is $\tau_{allow} = 3$ MPa.



SOLUTION



The FBD is shown in Fig. *a*.

As indicated the shear diagram, Fig. *b*, the maximum shear occurs in region AB of Constant value, $V_{max} = (P + 3)$ kN.

The neutral axis passes through Centroid C of the cross-section as shown in Fig. *c*.

$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{0.18(0.04)(0.2) + 0.1(0.2)(0.04)}{0.04(0.2) + 0.2(0.04)} = 0.14 \text{ m}$$

$$I = \frac{1}{12}(0.04)(0.2^3) + 0.04(0.2)(0.14 - 0.1)^2 + \frac{1}{12}(0.2)(0.04^3) + 0.2(0.04)(0.18 - 0.14^2) = 53.333(10^{-6}) \text{ m}^4$$

Refering to Fig. *d*,

$$Q_{max} = \bar{y}_1' A_1' = 0.07(0.14)(0.04) = 0.392(10^{-3}) \text{ m}^3$$

$$Q_A = \bar{y}_2' A_2' = 0.04(0.04)(0.2) = 0.32(10^{-3}) \text{ m}^3$$

The maximum shear stress occurs at the points on Neutral axis where Q is maximum and $t = 0.04$ m.

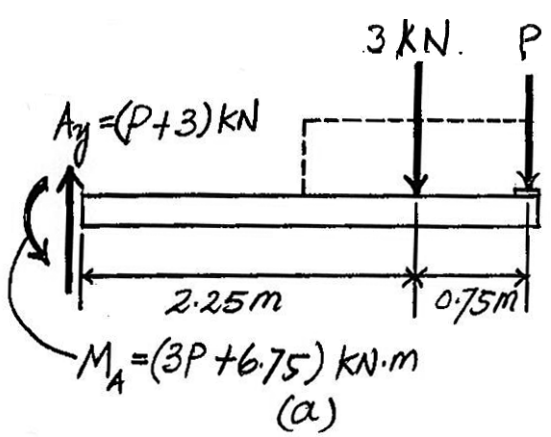
$$\tau_{allow} = \frac{V_{max} Q_{max}}{It}; \qquad 3(10^6) = \frac{(P + 3)(10^3)\left[0.392(10^{-3})\right]}{53.333(10^{-6})(0.04)}$$

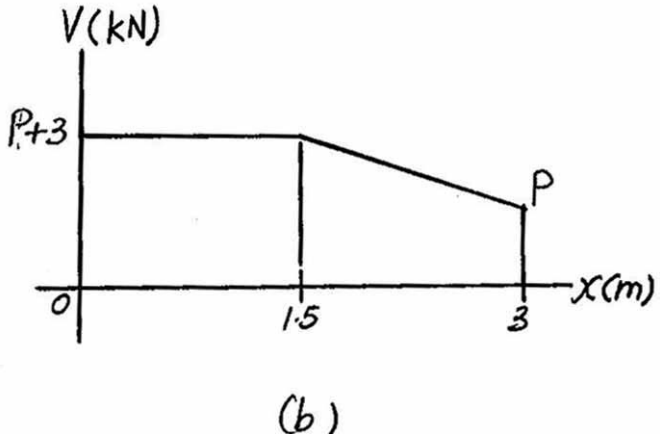
$$P = 13.33 \text{ kN}$$

Since there are two rows of nails $q_{allow} = 2\left(\frac{F}{s}\right) = 2\left[\frac{2(10^3)}{0.1}\right] = 40\,000$ N/m.

$$q_{allow} = \frac{V_{max} Q_A}{I}; \qquad 40\,000 = \frac{(P + 3)(10^3)\left[0.32(10^{-3})\right]}{53.333(10^{-6})}$$

$$P = 3.67 \text{ kN (Controls!)} \qquad \textbf{Ans.}$$





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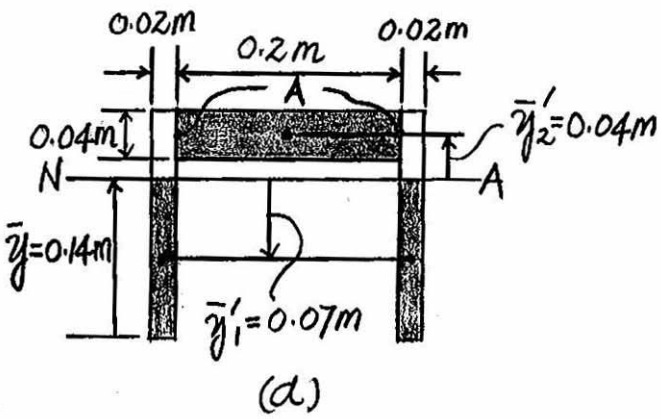
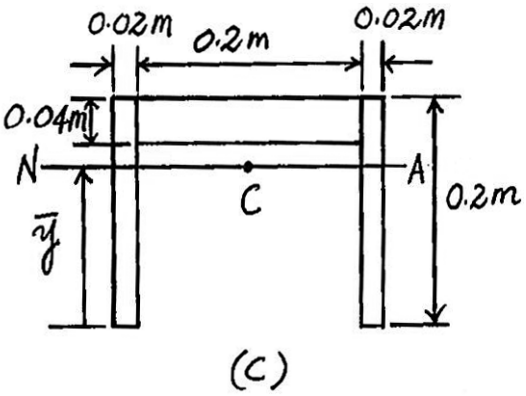
7–45. Continued



(a)



(c)

Ans.
$P = 3.67$ kN

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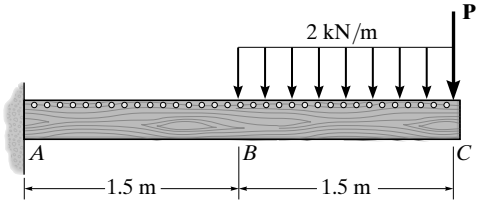
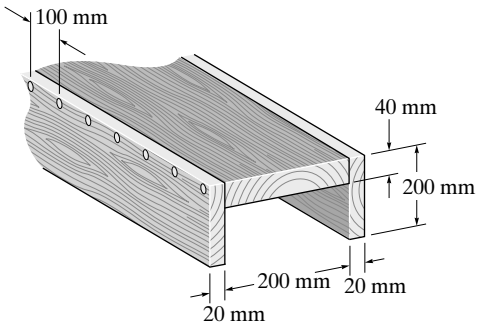
7–46. Determine the average shear stress developed in the nails within region AB of the beam. The nails are located on each side of the beam and are spaced 100 mm apart. Each nail has a diameter of 4 mm. Take $P = 2$ kN.



The FBD is shown in Fig. *a*.

As indicated in Fig. *b*, the internal shear force on the cross-section within region AB is constant that is $V_{AB} = 5$ kN.



The neutral axis passes through centroid C of the cross section as shown in Fig. *c*.

$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{0.18(0.04)(0.2) + 0.1(0.2)(0.04)}{0.04(0.2) + 0.2(0.04)}$$
$$= 0.14 \text{ m}$$

$$I = \frac{1}{12}(0.04)(0.2^3) + 0.04(0.2)(0.14 - 0.1)^2$$
$$+ \frac{1}{12}(0.2)(0.04^3) + 0.2(0.04)(0.18 - 0.14)^2$$
$$= 53.333(10^{-6}) \text{ m}^4$$

Q for the shaded area shown in Fig. *d* is

$$Q = \bar{y}' A' = 0.04(0.04)(0.2) = 0.32(10^{-3}) \text{ m}^3$$

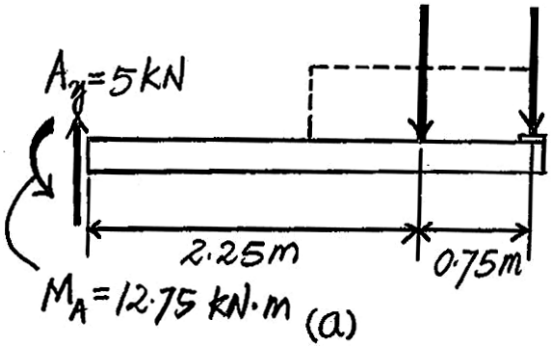
Since there are two rows of nail, $q = 2\left(\frac{F}{s}\right) = 2\left(\frac{F}{0.1}\right) = 20F$ N/m.

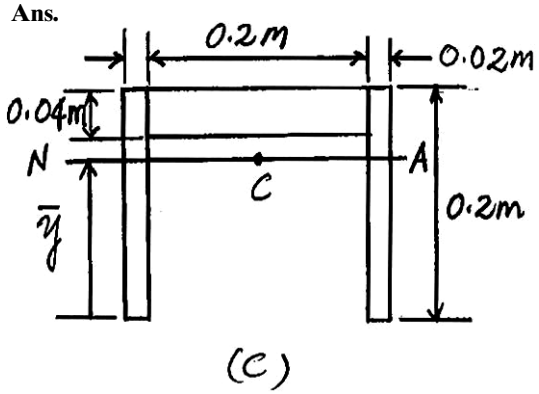
$$q = \frac{V_{AB} Q}{I}; \qquad 20F = \frac{5(10^3)\left[0.32(10^{-3})\right]}{53.333(10^{-6})}$$
$$F = 1500 \text{ N}$$

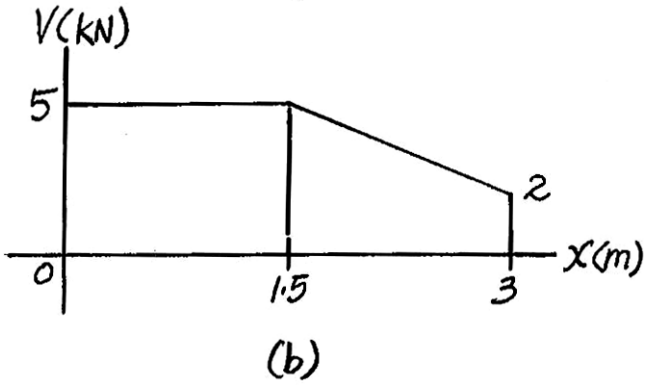
Thus, the average shear stress developed in each nail is

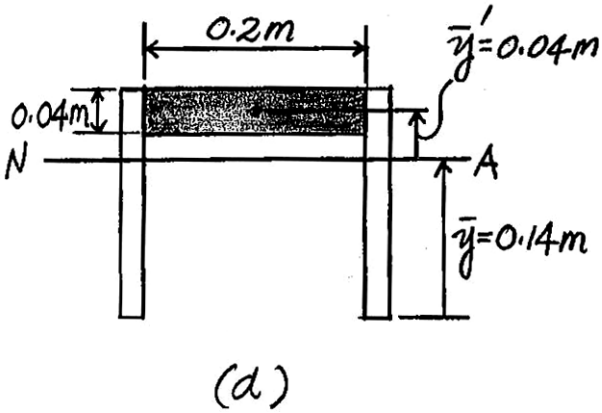
$$\left(\tau_{\text{nail}}\right)_{\text{avg}} = \frac{F}{A_{\text{nail}}} = \frac{1500}{\frac{\pi}{4}(0.004^2)} = 119.37(10^6)\text{Pa} = 119 \text{ MPa}$$

Ans.









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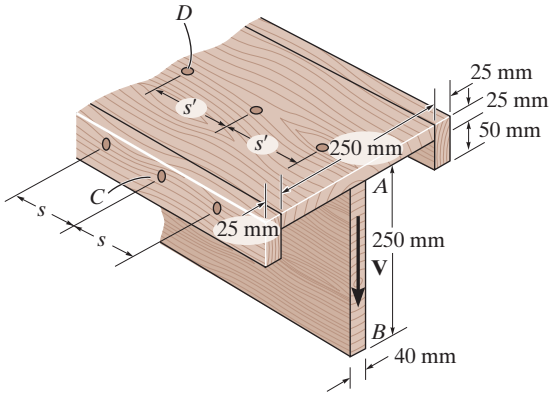
7–47. The beam is made from four boards nailed together as shown. If the nails can each support a shear force of 500 N, determine their required spacing s and s' if the beam is subjected to a shear of $V = 3.5$ kN.



SOLUTION

Section Properties:

$$\bar{y} = \frac{\Sigma \bar{y} A}{\Sigma A} = \frac{0.0125(0.25)(0.025) + 0.0375(0.05)(0.75) + 0.15(0.04)(0.25)}{0.25(0.025) + 0.05(0.075) + 0.04(0.25)}$$

$$= 0.0859375 \text{ m}$$

$$I_{NA} = \frac{1}{12}(0.25)(0.025^3) + 0.25(0.025)(0.0859375 - 0.0125)^2$$

$$+ \frac{1}{12}(0.05)(0.075^3) + 0.05(0.075)(0.0859375 - 0.0375)^2$$

$$+ \frac{1}{12}(0.04)(0.25^3) + 0.04(0.25)(0.15 - 0.0859375)^2$$

$$= 0.137712(10^{-3}) \text{ m}^4$$

$$Q_C = \bar{y}_1' A' = (0.0859375 - 0.0375)(0.25)(0.075) = 90.8203(10^{-6}) \text{ m}^3$$

$$Q_D = \bar{y}_2' A' = (0.0859375 - 0.0125)(0.25)(0.025) + [(0.0859375 - 0.0375)](0.25)(0.075) = 0.640625(10^{-3}) \text{ m}^3$$

Shear Flow: The allowable shear flow at points C and D is $q_C = \frac{500}{s}$ and $q_B = \frac{500}{s'}$, respectively.

$$q_C = \frac{VQ_C}{I}$$

$$\frac{500}{s} = \frac{[3.5(10^3)][90.8203(10^{-6})]}{0.137712(10^{-3})}$$

$$s = 0.2166 \text{ m} = 216 \text{ mm}$$ **Ans.**

$$q_D = \frac{VQ_D}{I}$$

$$\frac{500}{s'} = \frac{[3.5(10^3)][0.640625(10^{-3})]}{0.137712(10^{-3})}$$

$$s' = 0.03071 \text{ m} = 30 \text{ mm}$$ **Ans.**

Ans.
$s = 216$ mm, $s' = 30$ mm

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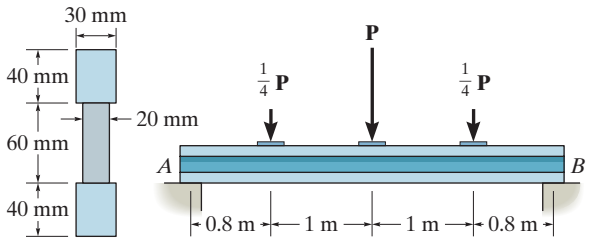
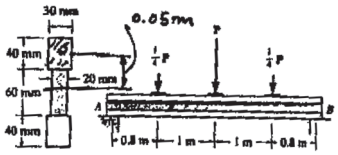
***7–48.**

The beam is made from three polystyrene strips that are glued together as shown. If the glue has a shear strength of 80 kPa, determine the maximum load P that can be applied without causing the glue to lose its bond.



SOLUTION

Maximum shear is at supports.



$$V_{max} = \frac{3P}{4}$$

$$I = \frac{1}{12}(0.02)(0.06)^3 + 2\left[\frac{1}{12}(0.03)(0.04)^3 + (0.03)(0.04)(0.05)^2\right] = 6.68(10^{-6})\ \text{m}^4$$

$$\tau = \frac{VQ}{It}; \qquad 80(10^3) = \frac{(3P/4)(0.05)(0.04)(0.03)}{6.68(10^{-6})(0.02)}$$

$$P = 238\ \text{N} \qquad \textbf{Ans.}$$

Ans:
$P = 238$ N

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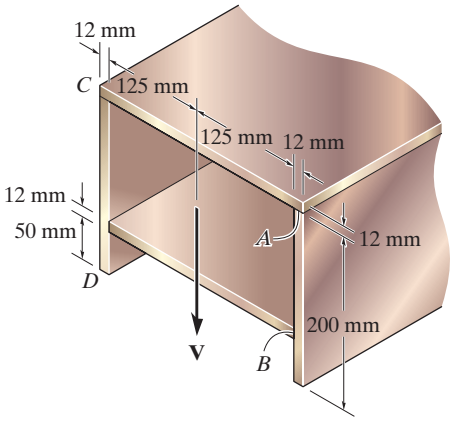
7–50. The beam is subjected to a shear force of $V = 25$ kN. Determine the shear flow at points A and B.

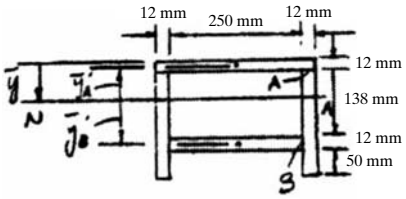


SOLUTION

$$\bar{y} = \frac{\Sigma \bar{y} A}{\Sigma A} = \frac{0.006(0.274)(0.012) + 2[0.112(0.012)(0.2)] + 0.156(0.25)(0.012)}{0.274(0.012) + 2(0.012)(0.2) + 0.25(0.012)}$$

$$= 0.092472 \text{ m}$$

$$I = \frac{1}{12}(0.274)(0.012^3) + 0.274(0.012)(0.092472 - 0.006)^2$$
$$+ 2\left[\frac{1}{12}(0.012)(0.2^3) + (0.012)(0.2)(0.112 - 0.092472)^2\right]$$
$$+ \frac{1}{12}(0.25)(0.012^3) + 0.25(0.012)(0.156 - 0.092472)^2$$
$$= 54.5990(10^{-6})\ \text{m}^4$$



$$\bar{y}'_A = 0.092472 - 0.006 = 0.086472 \text{ m}$$

$$\bar{y}'_B = 0.156 - 0.092472 = 0.063528 \text{ m}$$

$$Q_A = \bar{y}'_A A' = 0.086472(0.274)(0.012) = 0.28432(10^{-3})$$

$$Q_B = \bar{y}'_B A' = 0.063528(0.25)(0.012) = 0.19058(10^{-3})$$

$$q_A = \frac{1}{2}\left(\frac{VQ_A}{I}\right) = \frac{1}{2}\left\{\frac{[25(10^3)][0.28432(10^{-3})]}{54.5990(10^{-6})}\right\} = 65.09(10^3)\ \text{N/m}$$
$$= 65.1\ \text{kN/m}$$ **Ans.**

$$q_B = \frac{1}{2}\left(\frac{VQ_B}{I}\right) = \frac{1}{2}\left\{\frac{[25(10^3)][0.19058(10^{-3})]}{54.5990(10^{-6})}\right\} = 43.63(10^3)\ \text{N/m}$$
$$= 43.6\ \text{kN/m}$$ **Ans.**

Ans.
$q_A = 65.1$ kN/m, $q_B = 43.6$ kN/m

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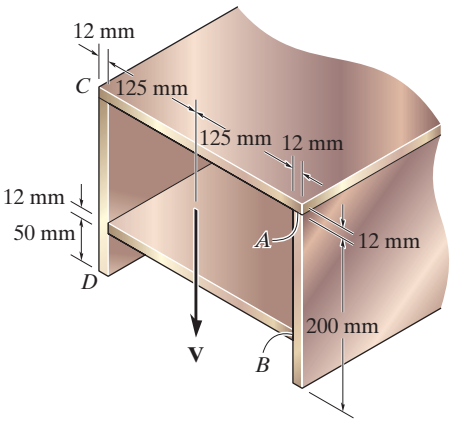
7–51. The beam is constructed from four plates and is subjected to a shear force of $V = 25$ kN. Determine the maximum shear flow in the cross section.



SOLUTION

$$\bar{y} = \frac{\Sigma \bar{y} A}{\Sigma A} = \frac{0.006(0.274)(0.012) + 2[0.112(0.012)(0.2)] + 0.156(0.25)(0.012)}{0.274(0.012) + 2(0.012)(0.2) + 0.25(0.012)}$$

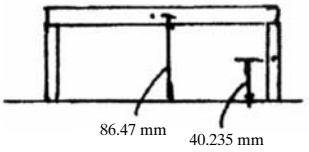
$$= 0.092472 \text{ m}$$

$$I = \frac{1}{12}(0.274)(0.012^3) + 0.274(0.012)(0.092472 - 0.006)^2$$
$$+ 2\left[\frac{1}{12}(0.012)(0.2^3) + (0.012)(0.2)(0.112 - 0.092472)^2\right]$$
$$+ \frac{1}{12}(0.25)(0.012^3) + 0.25(0.012)(0.156 - 0.092472)^2$$
$$= 54.5990(10^{-6}) \text{ m}^4$$

$$Q_{\max} = 0.086472(0.274)(0.012) + 0.040236(0.012)(0.086472) = 0.36203(10^{-3}) \text{ m}^3$$

$$q_{\max} = \frac{1}{2}\left(\frac{VQ_{\max}}{I}\right) = \frac{1}{2}\left\{\frac{[25(10^3)][0.36203(10^{-3})]}{54.5990(10^{-6})}\right\} = 82.88(10^3) \text{ N/m}$$

$$= 82.9 \text{ kN/m}$$ **Ans.**



Ans.
$\bar{y} = 0.092472$ m, $I = 54.5990(10^{-6})$ m^4,
$q_{\max} = 82.9$ kN/m

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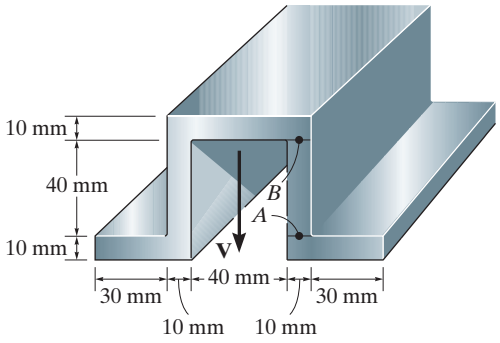
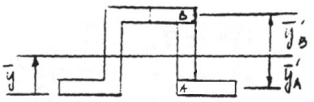
***7–52.**

The aluminum strut is 10 mm thick and has the cross section shown. If it is subjected to a shear of $V = 150$ N, determine the shear flow at points A and B.



SOLUTION

$$\bar{y} = \frac{2[0.005(0.03)(0.01)] + 2[0.03(0.06)(0.01)] + 0.055(0.04)(0.01)}{2(0.03)(0.01) + 2(0.06)(0.01) + 0.04(0.01)} = 0.027727 \text{ m}$$



$$I = 2\left[\frac{1}{12}(0.03)(0.01)^3 + 0.03(0.01)(0.027727 - 0.005)^2\right]$$

$$+ 2\left[\frac{1}{12}(0.01)(0.06)^3 + 0.01(0.06)(0.03 - 0.027727)^2\right]$$

$$+ \frac{1}{12}(0.04)(0.01)^3 + 0.04(0.01)(0.055 - 0.027727)^2 = 0.98197(10^{-6}) \text{ m}^4$$

$$\bar{y}_B' = 0.055 - 0.027727 = 0.027272 \text{ m}$$

$$\bar{y}_A' = 0.027727 - 0.005 = 0.022727 \text{ m}$$

$$Q_A = \bar{y}_A' A' = 0.022727(0.04)(0.01) = 9.0909(10^{-6}) \text{ m}^3$$

$$Q_B = \bar{y}_B' A' = 0.027272(0.03)(0.01) = 8.1818(10^{-6}) \text{ m}^3$$

$$q_A = \frac{VQ_A}{I} = \frac{150(9.0909)(10^{-6})}{0.98197(10^{-6})} = 1.39 \text{ kN/m} \qquad \textbf{Ans.}$$

$$q_B = \frac{VQ_B}{I} = \frac{150(8.1818)(10^{-6})}{0.98197(10^{-6})} = 1.25 \text{ kN/m} \qquad \textbf{Ans.}$$

Ans:
$q_A = 1.39$ kN/m, $q_B = 1.25$ kN/m

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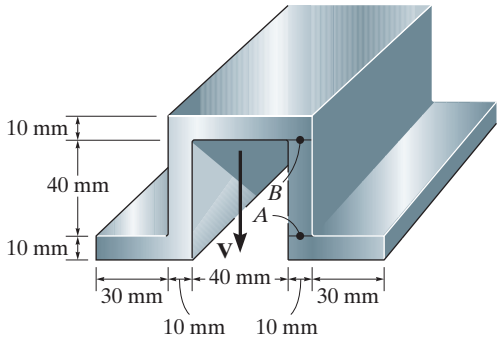
7–53.

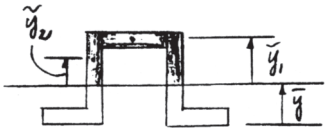
The aluminum strut is 10 mm thick and has the cross section shown. If it is subjected to a shear of $V = 150$ N, determine the maximum shear flow in the strut.



SOLUTION

$$\bar{y} = \frac{2[0.005(0.03)(0.01)] + 2[0.03(0.06)(0.01)] + 0.055(0.04)(0.01)}{2(0.03)(0.01) + 2(0.06)(0.01) + 0.04(0.01)}$$

$$= 0.027727 \text{ m}$$



$$I = 2\left[\frac{1}{12}(0.03)(0.01)^3 + 0.03(0.01)(0.027727 - 0.005)^2\right]$$

$$+ 2\left[\frac{1}{12}(0.01)(0.06)^3 + 0.01(0.06)(0.03 - 0.027727)^2\right]$$

$$+ \frac{1}{12}(0.04)(0.01)^3 + 0.04(0.01)(0.055 - 0.027727)^2$$

$$= 0.98197(10^{-6}) \text{ m}^4$$

$$Q_{\max} = (0.055 - 0.027727)(0.04)(0.01) + 2[(0.06 - 0.027727)(0.01)]\left(\frac{0.06 - 0.0277}{2}\right)$$

$$= 21.3(10^{-6}) \text{ m}^3$$

$$q_{\max} = \frac{1}{2}\left(\frac{VQ_{\max}}{I}\right) = \frac{1}{2}\left(\frac{150(21.3(10^{-6}))}{0.98197(10^{-6})}\right) = 1.63 \text{ kN/m}$$ **Ans.**

Ans:
$q_{\max} = 1.63$ kN/m

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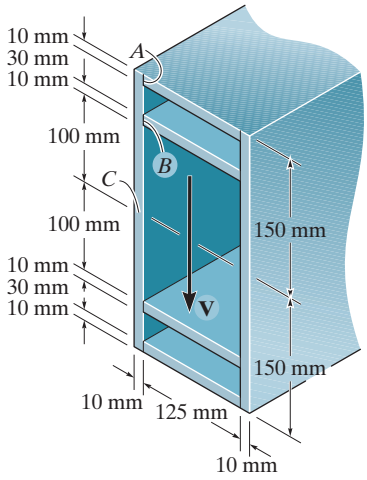
7–54.

A shear force of $V = 18$ kN is applied to the box girder. Determine the shear flow at points A and B.



SOLUTION

Section Properties:

$$I_{NA} = \frac{1}{12}(0.145)(0.3^3) - \frac{1}{12}(0.125)(0.28^3) + 2\left[\frac{1}{12}(0.125)(0.01^3) + 0.125(0.01)(0.105^2)\right]$$

$$= 125.17(10^{-6})\ \text{m}^4$$

$$Q_A = \bar{y}'_2 A' = 0.145(0.125)(0.01) = 0.18125(10^{-3})\ \text{m}^3$$

$$Q_B = \bar{y}'_1 A' = 0.105(0.125)(0.01) = 0.13125(10^{-3})\ \text{m}^3$$

Shear Flow:

$$q_A = \frac{1}{2}\left[\frac{VQ_A}{I}\right]$$

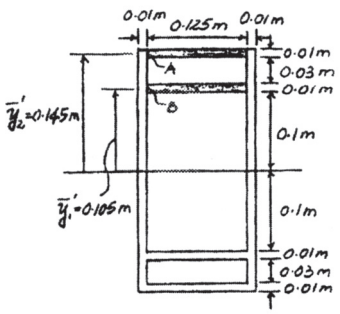
$$= \frac{1}{2}\left[\frac{18(10^3)(0.18125)(10^{-3})}{125.17(10^{-6})}\right]$$

$$= 13033\ \text{N/m} = 13.0\ \text{kN/m}$$ **Ans.**

$$q_B = \frac{1}{2}\left[\frac{VQ_B}{I}\right]$$

$$= \frac{1}{2}\left[\frac{18(10^3)(0.13125)(10^{-3})}{125.17(10^{-6})}\right]$$

$$= 9437\ \text{N/m} = 9.44\ \text{kN/m}$$ **Ans.**



Ans:
$q_A = 13.0$ kN/m, $q_B = 9.44$ kN/m

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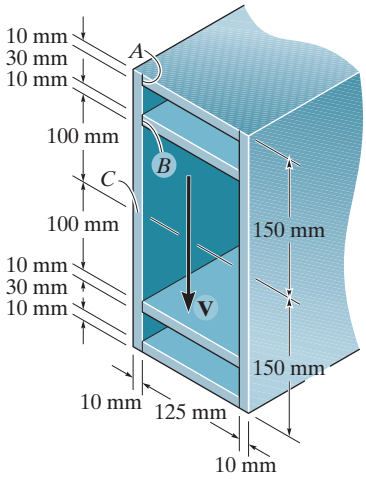
7–55.

A shear force of $V = 18$ kN is applied to the box girder. Determine the shear flow at point C.



SOLUTION

Section Properties:

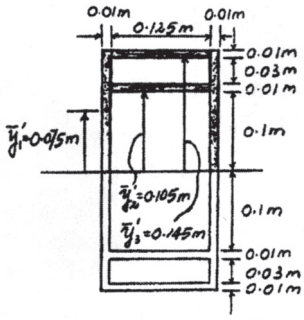
$$I_{NA} = \frac{1}{12}(0.145)(0.3^3) - \frac{1}{12}(0.125)(0.28^3) + 2\left[\frac{1}{12}(0.125)(0.01^3) + 0.125(0.01)(0.105^2)\right]$$

$$= 125.17(10^{-6})\ \text{m}^4$$

$$Q_C = \Sigma \bar{y}' A'$$

$$= 0.145(0.125)(0.01) + 0.105(0.125)(0.01) + 0.075(0.15)(0.02)$$

$$= 0.5375(10^{-3})\ \text{m}^3$$



Shear Flow:

$$q_C = \frac{1}{2}\left[\frac{VQ_C}{I}\right]$$

$$= \frac{1}{2}\left[\frac{18(10^3)(0.5375)(10^{-3})}{125.17(10^{-4})}\right]$$

$$= 38648\ \text{N/m} = 38.6\ \text{kN/m}$$ **Ans.**

Ans:
$q_C = 38.6$ kN/m

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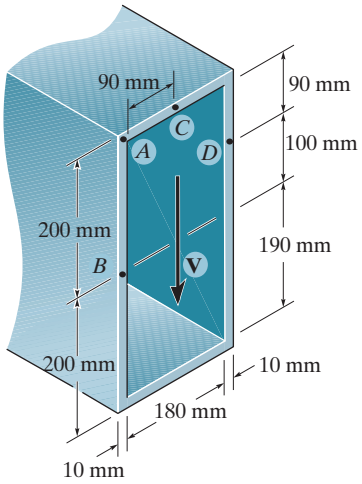
***7–56.**

A shear force of $V = 300$ kN is applied to the box girder. Determine the shear flow at points A and B.



SOLUTION

The moment of inertia of the cross section about the neutral axis is

$$I = \frac{1}{12}(0.2)(0.4^3) - \frac{1}{12}(0.18)(0.38^3) = 0.24359(10^{-3})\ \text{m}^4$$

Referring to Fig. a, Fig. b,

$$Q_A = \bar{y}'_1 A'_1 = 0.195(0.01)(0.19) = 0.3705(10^{-3})\ \text{m}^3$$

$$Q_B = 2\bar{y}'_2 A'_2 + \bar{y}'_3 A'_3 = 2[0.1(0.2)(0.01)] + 0.195(0.01)(0.18) = 0.751(10^{-3})\ \text{m}^3$$

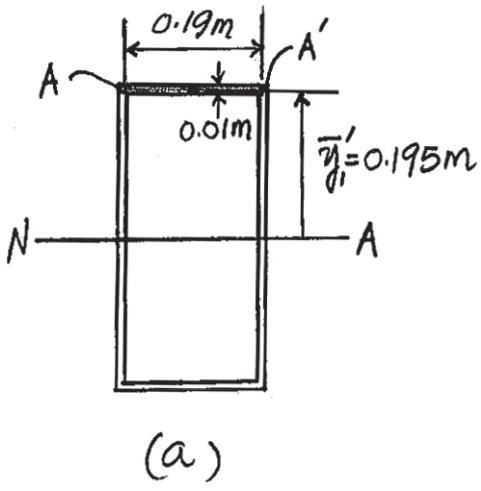
Due to symmetry, the shear flow at points A and A', Fig. a, and at points B and B', Fig. b, are the same. Thus

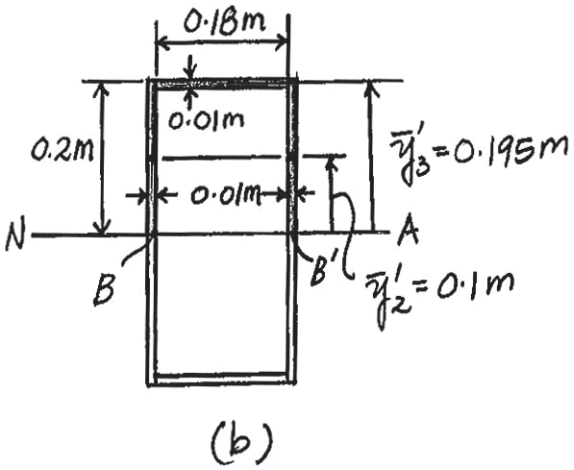
$$q_A = \frac{1}{2}\left(\frac{VQ_A}{I}\right) = \frac{1}{2}\left\{\frac{300(10^3)\left[0.3705(10^{-3})\right]}{0.24359(10^{-3})}\right\}$$

$$= 228.15(10^3)\ \text{N/m} = 228\ \text{kN/m} \qquad \textbf{Ans.}$$

$$q_B = \frac{1}{2}\left(\frac{VQ_B}{I}\right) = \frac{1}{2}\left\{\frac{300(10^3)\left[0.751(10^{-3})\right]}{0.24359(10^{-3})}\right\}$$

$$= 462.46(10^3)\ \text{N/m} = 462\ \text{kN/m} \qquad \textbf{Ans.}$$





Ans:
$q_A = 228$ kN/m, $q_B = 462$ kN/m

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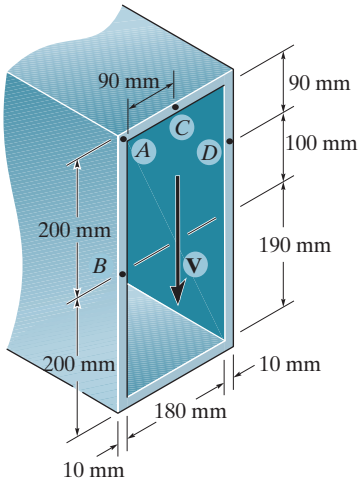
7–57.

A shear force of $V = 450$ kN is applied to the box girder. Determine the shear flow at points C and D.



SOLUTION

The moment of inertia of the cross section about the neutral axis is

$$I = \frac{1}{12}(0.2)(0.4^3) - \frac{1}{12}(0.18)(0.38^3) = 0.24359(10^{-3})\ \text{m}^4$$

Referring to Fig. *a*, due to symmetry $A'_C = 0$. Thus

$$Q_C = 0$$

Then referring to Fig. *b*,

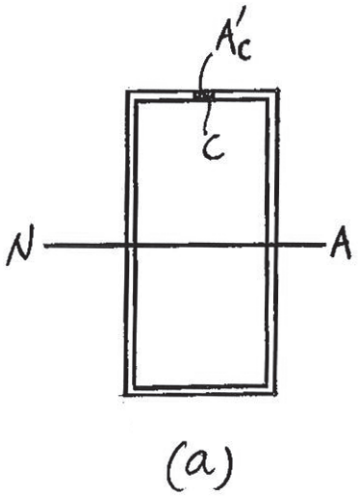
$$Q_D = \bar{y}'_1 A'_1 + \bar{y}'_2 A'_2 = 0.195(0.01)(0.09) + 0.15(0.1)(0.01)$$
$$= 0.3255(10^{-3})\ \text{m}^3$$

Thus,

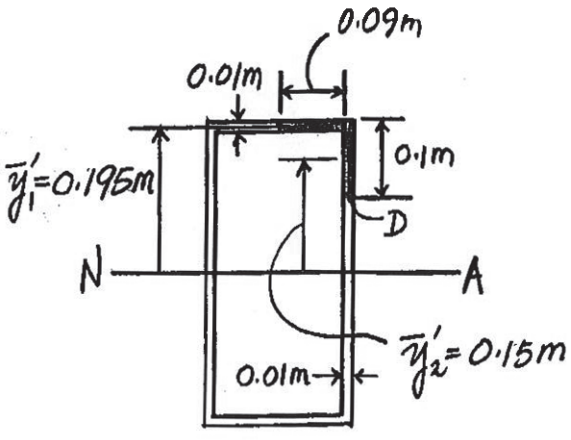
$$q_C = \frac{VQ_C}{I} = 0 \qquad \textbf{Ans.}$$

$$q_D = \frac{VQ_D}{I} = \frac{450(10^3)\left[0.3255(10^{-3})\right]}{0.24359(10^{-3})}$$

$$= 601.33(10^3)\ \text{N/m} = 601\ \text{kN/m} \qquad \textbf{Ans.}$$



(a)



(b)

Ans:
$q_C = 0, q_D = 601$ kN/m

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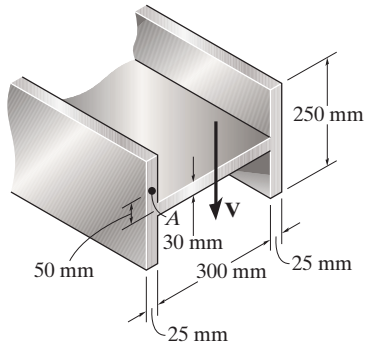
7–58.

The H-beam is subjected to a shear of $V = 80$ kN. Determine the shear flow at point A.

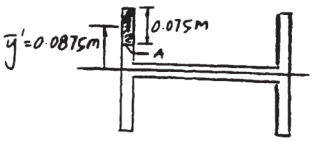


SOLUTION

$$I = 2\left[\frac{1}{12}(0.025)(0.25^3) + \frac{1}{12}(0.3)(0.03^3)\right] = 65.7792(10^{-6})\ \text{m}^4$$

$$Q_A = \bar{y}'A' = 0.0875(0.075)(0.025) = 0.1641(10^{-3})\ \text{m}^3$$

$$q_A = \frac{VQ_A}{I} = \frac{80(10^3)(0.1641)(10^{-3})}{65.7792(10^{-6})} = 200\ \text{kN/m}$$ **Ans.**



Ans:
$q_A = 200\ \text{kN/m}$

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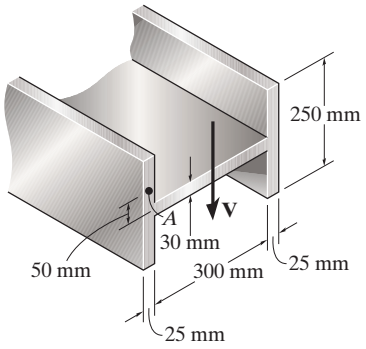
7–59.

The H-beam is subjected to a shear of $V = 80$ kN. Sketch the shear-stress distribution acting along one of its side segments. Indicate all peak values.

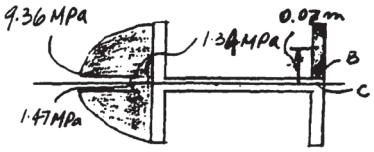


SOLUTION

$$I = 2\left[\frac{1}{12}(0.025)(0.25^3)\right] + \frac{1}{12}(0.3)(0.03^3) = 65.7792(10^{-6})\ \text{m}^4$$

$$Q_B = (0.070)(0.025)(0.110) = 0.1925(10^{-3})\ \text{m}^3$$

$$\tau_B = \frac{VQ}{It} = \frac{80(10^3)(0.1925)(10^{-3})}{65.7792(10^{-6})(0.025)} = 9.36\ \text{MPa}$$



$$\tau_B = \frac{VQ}{It} = \frac{80(10^3)\left[2(0.1925)(10^{-3})\right]}{65.7792(10^{-6})(0.35)} = 1.3378\ \text{MPa}$$

$$Q_{\max} = 2(0.07)(0.025)(0.110) + (0.0075)(0.35)(0.015) = 0.4244(10^{-3})\ \text{m}^3$$

$$\tau_C = \frac{VQ}{It} = \frac{80(10^3)(0.4244)(10^{-3})}{65.7792(10^{-6})(0.35)} = 1.47\ \text{MPa}$$

Ans:
$\tau_{\max} = 9.36$ MPa

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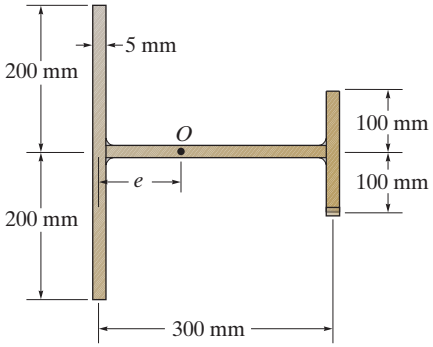
***7–60.**

The built-up beam is formed by welding together the thin plates of thickness 5 mm. Determine the location of the shear center O.



SOLUTION

Shear Center: Referring to Fig. *a* and summing moments about point A, we have

$$\circlearrowleft + \Sigma(M_R)_A = \Sigma M_A; \qquad -Pe = -(F_w)_1(0.3)$$

$$e = \frac{0.3(F_w)_1}{P} \tag{1}$$

Section Properties: The moment of inertia of the cross section about the axis of symmetry is

$$I = \frac{1}{12}(0.005)(0.4^3) + \frac{1}{12}(0.005)(0.2^3) = 30(10^{-6})\ \text{m}^4$$

Referring to Fig. b, $\bar{y}' = (0.1 - s) + \frac{s}{2} = (0.1 - 0.5s)$ m. Thus, Q as a function of s is

$$Q = \bar{y}'A' = (0.1 - 0.5s)(0.005s) = [0.5(10^{-3})s - 2.5(10^{-3})s^2]\ \text{m}^3$$

Shear Flow:

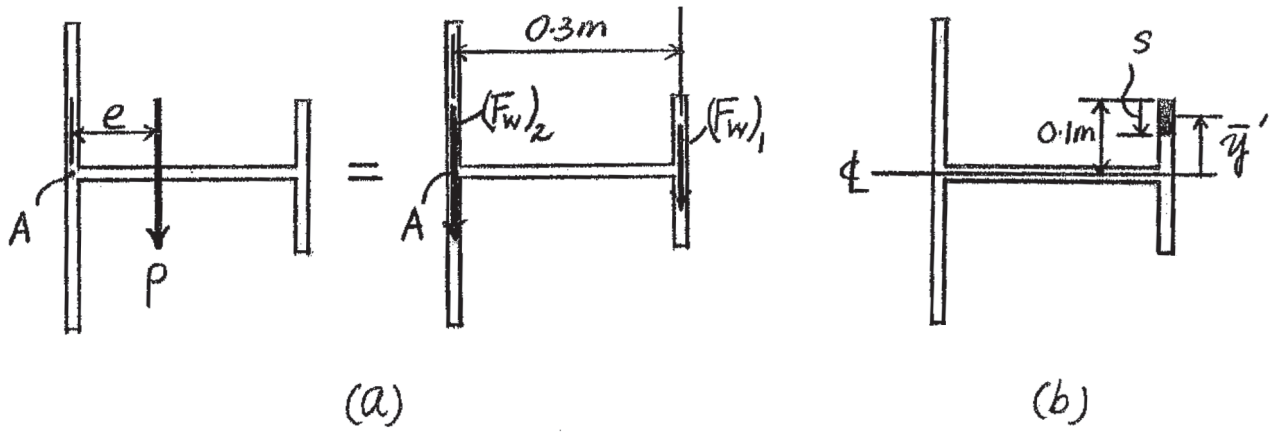
$$q = \frac{VQ}{I} = \frac{P[0.5(10^{-3})s - 2.5(10^{-3})s^2]}{30(10^{-6})} = P(16.6667s - 83.3333s^2)$$

Resultant Shear Force: The shear force resisted by the shorter web is

$$(F_w)_1 = 2\int_0^{0.1\text{ m}} q\,ds = 2\int_0^{0.1\text{ m}} P(16.6667s - 83.3333s^2)\,ds = 0.1111P$$

Substituting this result into Eq. (1),

$$e = 0.03333\ \text{m} = 33.3\ \text{m} \qquad \textbf{Ans.}$$



Ans:
$e = 33.3$ m

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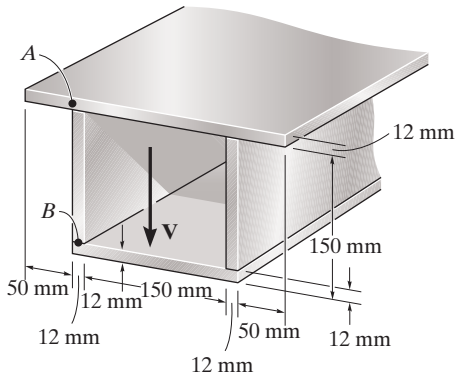
7–61. The assembly is subjected to a vertical shear of $V = 35$ kN. Determine the shear flow at points A and B and the maximum shear flow in the cross section.



SOLUTION

$$\bar{y} = \frac{\Sigma \bar{y} A}{\Sigma A} = \frac{0.006(0.274)(0.012) + 2[0.081(0.012)(0.138)] + 0.156(0.174)(0.012)}{0.274(0.012) + 2(0.012)(0.138) + 0.174(0.012)}$$
$$= 0.070641 \text{ m}$$

$$I = \frac{1}{12}(0.274)(0.012^3) + 0.274(0.012)(0.070641 - 0.006)^2$$
$$+ 2\left[\frac{1}{12}(0.012)(0.138^3) + (0.012)(0.138)(0.081 - 0.070641)^2\right]$$
$$+ \frac{1}{12}(0.174)(0.012^3) + 0.174(0.012)(0.156 - 0.070641)^2$$
$$= 34.6283(10^{-6}) \text{ m}^4$$

$$Q_A = \bar{y}_1' A_1' = 0.06464(0.05)(0.012) = 0.28432(10^{-3}) \text{ m}^3$$

$$Q_B = \bar{y}_2' A_2' = 0.08536(0.174)(0.012) = 0.17823(10^{-3}) \text{ m}^3$$

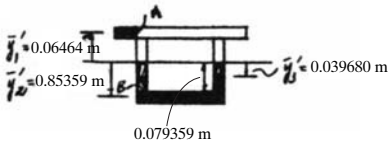
$$Q_{\max} = \Sigma \bar{y}' \mathrm{A}' = 0.08536(0.174)(0.012) + 2[0.039680(0.012)(0.079359)] = 0.25380(10^{-3}) \text{ m}^3$$

$$q = \frac{VQ}{I}$$

$$q_A = \frac{[35(10^3)][38.7845(10^{-6})]}{34.6283(10^{-6})} = 39.20(10^3) \text{ N/m} = 39.2 \text{ kN/m}$$ **Ans.**

$$q_B = \frac{1}{2}\left\{\frac{[35(10^3)][0.17823(10^{-3})]}{34.6283(10^{-6})}\right\} = 90.07(10^3) \text{ N/m} = 90.1 \text{ kN/m}$$ **Ans.**

$$q_{\max} = \frac{1}{2}\left\{\frac{[35(10^3)][0.25380(10^{-3})]}{34.6283(10^{-6})}\right\} = 128.26(10^3) \text{ N/m} = 128 \text{ kN/m}$$ **Ans.**



Ans.
$\bar{y} = 0.070641$ m, $I = 34.6283(10^{-6})$ m^4,
$q_A = 39.2$ kN/m, $q_B = 90.1$ kN/m,
$q_{\max} = 128$ kN/m

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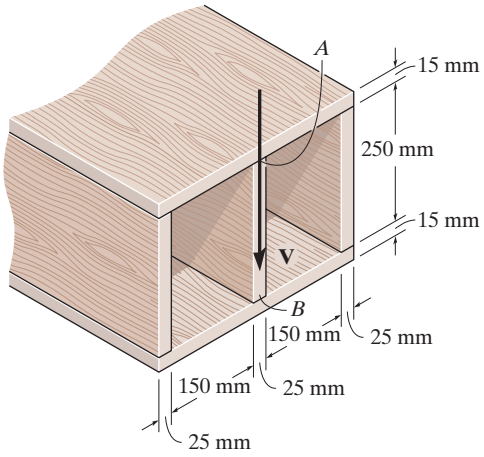
7–62.

The box girder is subjected to a shear of $V = 15$ kN. Determine the shear flow at point B and the maximum shear flow in the girder's web AB.



SOLUTION

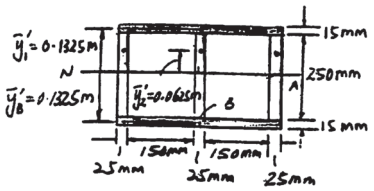
$$I = \frac{1}{12}(0.375)(0.28^3) - \frac{1}{12}(0.3)(0.25^3) = 0.295375(10^{-3})\ \text{m}^4$$

$$Q_B = \bar{y}'_B A' = 0.1325(0.375)(0.015) = 0.7453125(10^{-3})\ \text{m}^3$$

$$Q_{\max} = \Sigma \bar{y}'\text{A}' = 0.1325(0.375)(0.015) + 3[(0.0625)(0.125)(0.025)] = 1.33125(10^{-3})\ \text{m}^3$$

$$q_B = \frac{1}{3}\left[\frac{VQ_B}{I}\right] = \frac{1}{3}\left[\frac{15(10^3)(0.7453125)(10^{-3})}{0.295375(10^{-3})}\right] = 12.6\ \text{kN/m}$$ **Ans.**

$$q_{\max} = \frac{1}{3}\left[\frac{VQ_{\max}}{I}\right] = \frac{1}{3}\left[\frac{15(10^3)(1.33125)(10^{-3})}{0.295375(10^{-3})}\right] = 22.5\ \text{kN/m}$$ **Ans.**



Ans:
$q_B = 12.6$ kN/m, $q_{\max} = 22.5$ kN/m

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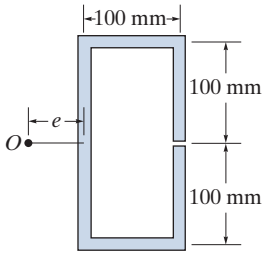
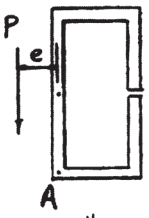
7–63.

Determine the location e of the shear center, point O, for the thin-walled member having a slit along its section.



SOLUTION



Suming moments about *A*,

$$Pe = 2V_1(100) + F(200) \qquad (1)$$

$$I = 2\left[\frac{1}{12}t(0.2^3)\right] + 2\left[(0.1)(t)(0.1^2)\right] = 3.3333(10^{-3})t\ \text{m}^4$$

$$Q_1 = \bar{y}_1' A' = \frac{y}{2}(y)t = 0.5y^2 t$$

$$Q_2 = \Sigma \bar{y}' A = 0.05(0.1)(t) + 0.1(x)(t) = 0.005t + 0.1xt$$

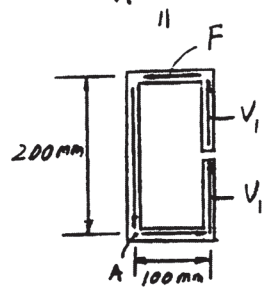
$$q_1 = \frac{VQ_1}{I} = \frac{P(0.5y^2 t)}{3.3333(10^{-3})t} = 150P\,y^2$$

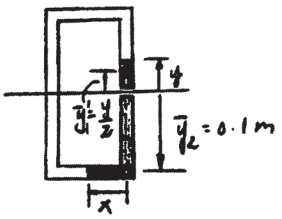
$$q_2 = \frac{VQ_2}{I} = \frac{P(0.005t + 0.1xt)}{3.3333(10^{-3})t} = 300P(0.005 + 0.1x)$$

$$V_1 = \int_0^{0.1} q_1\,dy = 150P\int_0^{0.1} y^2 dy = 150P\left[\frac{y^3}{3}\right]_0^{0.1} = 0.05P$$

$$F = \int_0^{0.1} q_2\,dx = 300P\int_0^{0.1} (0.005 + 0.1x)\,dx$$

$$= 300P\left[0.005x + \frac{0.1x^2}{2}\right]_0^{0.1} = 0.3P$$





From Eq. (1);

$$Pe = 2(0.05P)(100) + 0.3P(200)$$

$$e = 70\ \text{mm} \qquad \textbf{Ans.}$$

Ans:
$e = 70$ mm

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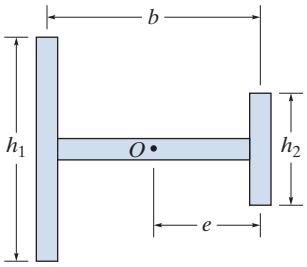
***7–64.**

Determine the location e of the shear center, point O, for the thin-walled member. The member segments have the same thickness t.



SOLUTION

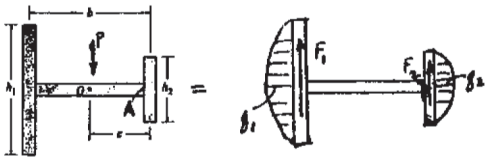
Summing moments about A,

$$eP = bF_1 \qquad (1)$$

$$I = \frac{1}{12}(t)(h_1)^3 + \frac{1}{12}(t)(h_2)^3 = \frac{1}{12}t\,(h_1^3 + h_2^3)$$

$$q_1 = \frac{P(h_1/2)(t)(h_1/4)}{I} = \frac{Ph_1^2 t}{8I}$$

$$F_1 = \frac{2}{3}q_1(h_1) = \frac{Ph_1^3 t}{12I}$$



From Eq. (1),

$$e = \frac{b}{P}\left(\frac{Ph_1^3 t}{12I}\right)$$

$$= \frac{h_1^3 b}{(h_1^3 + h_2^3)}$$

$$= \frac{b}{1 + (h_2/h_1)^3} \qquad \textbf{Ans.}$$

Ans:

$$e = \frac{b}{1 + (h_2/h_1)^3}$$

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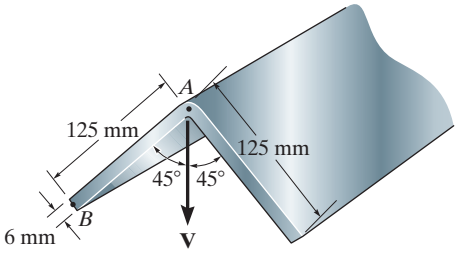
7–65. The angle is subjected to a shear of $V = 10$ kN. Sketch the distribution of shear flow along the leg AB. Indicate numerical values at all peaks.



SOLUTION

Section Properties:

$$b = \frac{0.006}{\sin 45°} = 0.00848528 \text{ m}$$

$$h = 0.125 \cos 45° = 0.08839 \text{ m}$$

$$I_{NA} = 2\left[\frac{1}{12}(0.00848528)(0.08839^3)\right] = 0.97656(10^{-6}) \text{ m}^4$$

$$Q = \bar{y}'A' = [0.25(0.08839) + 0.5y]\left(0.625 - \frac{y}{\sin 45°}\right)(0.006)$$

$$= 8.2864(10^{-6}) - 4.2426(10^{-3})y^2$$

Shear Flow:

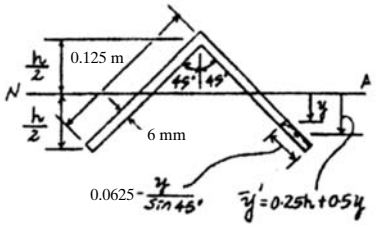
$$q = \frac{VQ}{I}$$

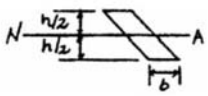
$$= \frac{[10(10^3)][8.2864(10^{-6}) - 4.2426(10^{-3})y^2]}{0.97656(10^{-6})}$$

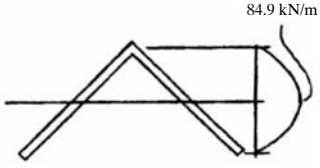
$$= [84.8528(10^3) - 43.4446(10^6)y^2] \text{ N/m}$$ **Ans.**

$$= [84.9 - 43.4y^2] \text{ kN/m}$$

At $y = 0$, $q = q_{\max} = 84.9$ kN/m **Ans.**







Ans.

At $y = 0, q = q_{\max} = 84.9$ kN/m

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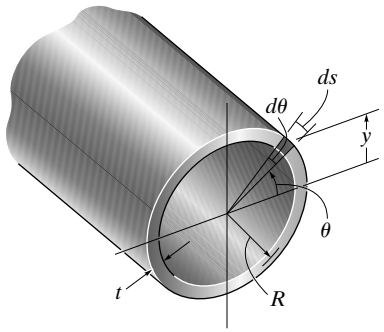
7–66. Determine the shear-stress variation over the cross section of the thin-walled tube as a function of elevation y and show that $\tau_{max} = 2V/A$, where $A = 2\pi rt$. *Hint:* Choose a differential area element $dA = Rt\, d\theta$. Using $dQ = y\, dA$, formulate Q for a circular section from θ to $(\pi - \theta)$ and show that $Q = 2R^2 t \cos\theta$, where $\cos\theta = \sqrt{R^2 - y^2}/R$.



SOLUTION

$dA = R\, t\, d\theta$

$dQ = y\, dA = yR\, t\, d\theta$

Here $y = R \sin\theta$

Therefore $dQ = R^2\, t \sin\theta\, d\theta$

$$Q = \int_{\theta}^{\pi-\theta} R^2\, t \sin\theta\, d\theta = R^2\, t(-\cos\theta)\Big|_{\theta}^{\pi-\theta}$$

$$= R^2 t\,[-\cos(\pi - \theta) - (-\cos\theta)] = 2R^2 t\cos\theta$$

$$dI = y^2\, dA = y^2\, R\, t\, d\theta = R^3\, t \sin^2\theta\, d\theta$$

$$I = \int_0^{2\pi} R^3\, t \sin^2\theta\, d\theta = R^3\, t \int_0^{2\pi} \frac{(1 - \cos 2\theta)}{2}\, d\theta$$

$$= \frac{R^3\, t}{2}\left[\theta - \frac{\sin 2\theta}{2}\right]\Big|_0^{2\pi} = \frac{R^3\, t}{2}[2\pi - 0] = \pi R^3\, t$$

$$\tau = \frac{VQ}{I\, t} = \frac{V(2R^2 t\cos\theta)}{\pi R^3\, t(2t)} = \frac{V\cos\theta}{\pi R\, t}$$

Here $\cos\theta = \dfrac{\sqrt{R^2 - y^2}}{R}$

$$\tau = \frac{V}{\pi R^2 t}\sqrt{R^2 - y^2}$$ **Ans.**

τ_{max} occurs at $y = 0$; therefore

$$\tau_{max} = \frac{V}{\pi R\, t}$$

$A = 2\pi Rt$; therefore

$$\tau_{max} = \frac{2V}{A}$$ **QED**

Ans.

$$\tau = \frac{V}{\pi R^2 t}\sqrt{R^2 - y^2}$$

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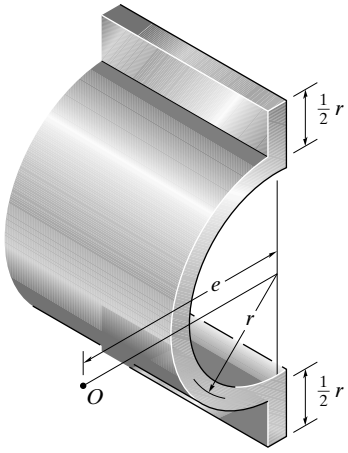
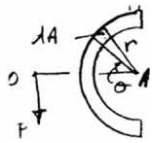
7–67. Determine the location e of the shear center, point O, for the beam having the cross section shown. The thickness is t.



SOLUTION

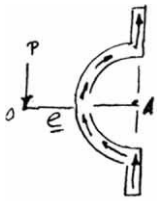
$$I = (2)\left[\frac{1}{12}(t)(r/2)^3 + (r/2)(t)\left(r + \frac{r}{4}\right)^2\right] + I_{\text{semi-circle}}$$

$$= 1.583333t\,r^3 + I_{\text{semi-circle}}$$



$$I_{\text{semi-circle}} = \int_{-\pi/2}^{\pi/2} (r\sin\theta)^2\, t\, r\, d\theta = t\, r^3 \int_{-\pi/2}^{\pi/2} \sin^2\theta\, d\theta$$

$$I_{\text{semi-circle}} = t\, r^3\left(\frac{\pi}{2}\right)$$



Thus,

$$I = 1.583333t\,r^3 + t\,r^3\left(\frac{\pi}{2}\right) = 3.15413t\,r^3$$

$$Q = \left(\frac{r}{2}\right)t\left(\frac{r}{4} + r\right) + \int_{\theta}^{\pi/2} r\sin\theta\,(t\,r\,d\theta)$$

$$Q = 0.625\,t\,r^2 + t\,r^2\cos\theta$$

$$q = \frac{VQ}{I} = \frac{P(0.625 + \cos\theta)t\,r^2}{3.15413\,t\,r^3}$$

Summing moments about A:

$$Pe = \int_{-\pi/2}^{\pi/2} (q\,r\,d\theta)r$$

$$Pe = \frac{Pr}{3.15413}\int_{-\pi/2}^{\pi/2} (0.625 + \cos\theta)d\theta$$

$$e = \frac{r\,(1.9634 + 2)}{3.15413}$$

$$e = 1.26\,r \qquad \textbf{Ans.}$$

Ans.
$e = 1.26\,r$

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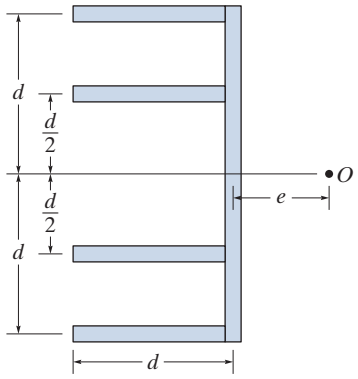
***7–68.**

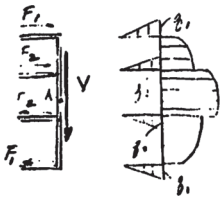
Determine the location e of the shear center, point O, for the thin-walled member. The member segments have the same thickness t.



SOLUTION

Summing moments about point A:

$$Pe = F_2 d + F_1(2d) \qquad \textbf{(1)}$$



$$I = 2[dt(d)^2] + 2[dt(d/2)^2] = \frac{1}{12}t(2d)^3 = \frac{19}{6}td^3$$

$$q_1 = \frac{P(dt)(d)}{\frac{19}{6}td^3} = \frac{6P}{19d}$$

$$F_1 = \frac{1}{2}\left(\frac{6P}{19d}\right)(d) = \frac{3}{19}P$$

$$q_2 = \frac{P(dt)(d/2)}{\frac{19}{6}td^3} = \frac{3P}{19d}$$

$$F_2 = \frac{1}{2}\left(\frac{3P}{19d}\right)d = \frac{1.5P}{19}$$

From Eq. (1):

$$Pe = 2d\left(\frac{3}{19}P\right) + d\left(\frac{1.5P}{19}\right)$$

$$e = \frac{7.5}{19}d = \frac{15}{38}d \qquad \textbf{Ans.}$$

Ans:
$e = \frac{15}{38}d$

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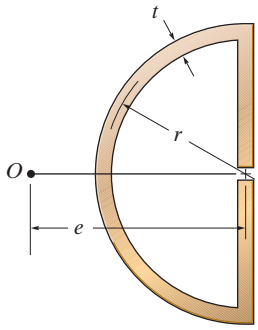
7–69.

A thin plate of thickness t is bent to form the beam having the cross section shown. Determine the location of the shear center O.



SOLUTION

Section Properties: For the arc segment, Fig. a, $y = r\cos\theta$ and the area of the differential element shown shaded is $dA = t\,ds = tr\,d\theta$. Then, the moment of inertia of the entire cross section about the axis of symmetry is

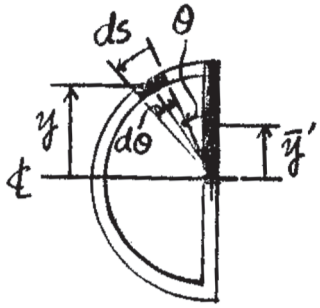
$$I = \frac{1}{12}(t)(2r)^3 + \int y^2\,dA$$

$$= \frac{2}{3}r^3t + \int_0^{\pi}(r\cos\theta)^2\,trd\theta$$

$$= \frac{2}{3}r^3t + \frac{r^3t}{2}\int_0^{\pi}(\cos 2\theta + 1)\,d\theta$$

$$= \frac{2}{3}r^3t + \frac{r^3t}{2}\left(\frac{1}{2}\sin 2\theta + \theta\right)\Big|_0^{\pi}$$

$$= \frac{r^3t}{6}(4 + 3\pi)$$



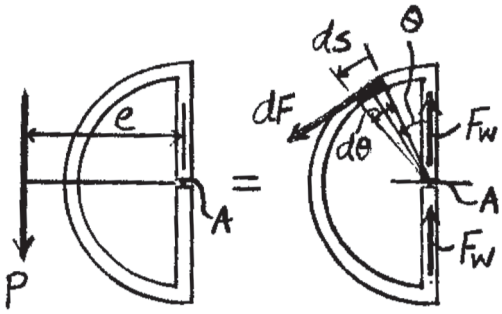
(a)

Referring to Fig. a, $\bar{y}' = \frac{r}{2}$. Thus, Q as a function of s is

$$Q = \bar{y}'A' + \int ydA = \frac{r}{2}(rt) + \int_0^{\theta} r\cos\theta(trd\theta)$$

$$= \frac{1}{2}r^2t + r^2t\int_0^{\theta}\cos\theta d\theta$$

$$= \frac{r^2t}{2}(1 + 2\sin\theta)$$



(b)

Shear Flow:

$$q = \frac{VQ}{I} = \frac{P\left[\frac{r^2t}{2}(1 + 2\sin\theta)\right]}{\frac{r^3t}{6}(4 + 3\pi)} = \frac{3P}{(4 + 3\pi)r}(1 + 2\sin\theta)$$

Resultant Shear Force: The shear force resisted by the arc segment is

$$F = \int qds = \int_0^{\pi} qrd\theta = \int_0^{\pi}\frac{3P}{(4 + 3\pi)r}(1 + 2\sin\theta)rd\theta$$

$$= \frac{3P}{4 + 3\pi}(\theta - 2\cos\theta)\Big|_0^{\pi}$$

$$= \frac{3P(\pi + 4)}{4 + 3\pi}$$

Shear Center: Referring to Fig. b and summing the moments about point A,

$$\zeta + \Sigma(M_R)_A = \Sigma M_A; \qquad Pe = r\int dF$$

$$Pe = r\left[\frac{3P(\pi + 4)}{4 + 3\pi}\right]$$

$$e = \left[\frac{3(\pi + 4)}{4 + 3\pi}\right]r \qquad \textbf{Ans.}$$

Ans:

$$e = \left[\frac{3(\pi + 4)}{4 + 3\pi}\right]r$$

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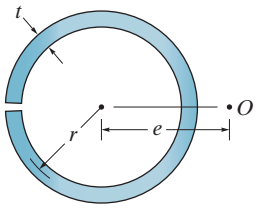
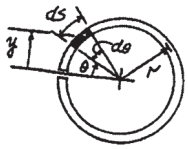
7–70.

Determine the location e of the shear center, point O, for the tube having a slit along its length.



SOLUTION



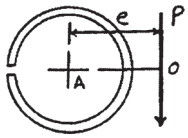
Section Properties:

$$dA = t\,ds = t\,r\,d\theta \qquad y = r\sin\theta$$

$$dI = y^2\,dA = r^2\sin^2\theta(t\,r\,d\theta) = r^3 t\sin^2\theta\,d\theta$$

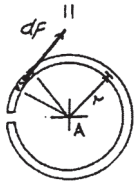
$$I = r^3 t\int_0^{2\pi}\sin^2\theta\,d\theta$$

$$= r^3 t\int_0^{2\pi}\left(\frac{1-\cos 2\theta}{2}\right)d\theta = \pi r^3 t$$



$$dQ = \bar{y}'A' = y\,dA = r\sin\theta(t\,r\,d\theta) = r^2 t\sin\theta\,d\theta$$

$$Q = r^2 t\int_0^{\theta}\sin\theta\,d\theta = r^2 t\,(1-\cos\theta)$$



Shear Flow Resultant:

$$q = \frac{VQ}{I} = \frac{P\,r^2 t(1-\cos\theta)}{\pi r^3 t} = \frac{P}{\pi r}(1-\cos\theta)$$

$$F = \int_0^{2\pi} q\,ds = \int_0^{2\pi}\frac{P}{\pi r}(1-\cos\theta)\,r\,d\theta$$

$$= \frac{P}{\pi}\int_0^{2\pi}(1-\cos\theta)\,d\theta$$

$$= 2P$$

Shear Center: Summing moments about point A.

$$Pe = Fr$$

$$Pe = 2Pr$$

$$e = 2r \qquad \textbf{Ans.}$$

Ans:
$e = 2r$

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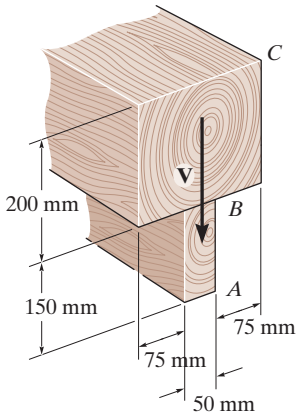
R7–1. Sketch the intensity of the shear-stress distribution acting over the beam's cross-sectional area, and determine the resultant shear force acting on the segment AB. The shear acting at the section is $V = 175$ kN. Show that $I_{NA} = 0.3048(10^{-3})$ m^4.



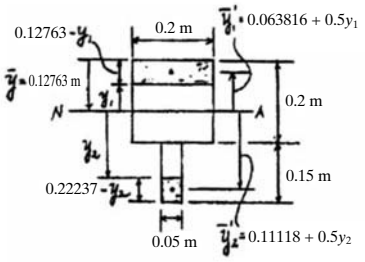
SOLUTION

Section Properties:

$$\bar{y} = \frac{\Sigma \bar{y} A}{\Sigma A} = \frac{0.1(0.2)(0.2) + 0.275(0.05)(0.15)]}{0.2(0.2) + 0.05(0.15)} = 0.12763 \text{ m}$$

$$I_{NA} = \frac{1}{12}(0.2)(0.2^3) + 0.2(0.2)(0.12763 - 0.1)^2 + \frac{1}{12}(0.05)(0.15^3) + (0.05)(0.15)(0.275 - 0.12763)^2$$

$$= 0.3408(10^{-3}) \text{ m}^4 \quad (Q.E.D)$$



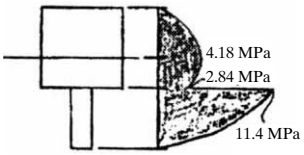
$$Q_1 = \bar{y}'_1 A' = (0.063816 + 0.5y_1)(0.2)(0.12763 - y_1)$$

$$= 1.62898(10^{-3}) - 0.1y_1^2$$

$$Q_2 = \bar{y}'_2 A' = (0.11118 + 0.5y_2)(0.05)(0.22237 - y_2)$$

$$= 1.23619(10^{-3}) - 0.25y_2^2$$

Shear Stress: Applying the shear formula $\tau = \frac{VQ}{It}$,



$$\tau_{CB} = \frac{VQ_1}{It} = \frac{[175(10^3)][1.62898(10^{-3}) - 0.1y_1^2]}{[0.3408(10^{-3})](0.2)}$$

$$= \left(4.18218 - 256.74y_1^2\right)(10^6)$$

At $y_1 = 0$, $\tau_{CB} = 4.18$ MPa

At $y_1 = -0.07237$ m $\tau_{CB} = 2.84$ MPa

$$\tau_{AB} = \frac{VQ_2}{It} = \frac{[175(10^3)][1.23619(10^{-3}) - 0.25y_2^2]}{[0.3408(10^{-3})](0.05)}$$

$$= \left(12.6950 - 256.74y_2^2\right)(10^6)$$

At $y_2 = 0.07237$ m $\tau_{AB} = 11.4$ MPa

Resultant Shear Force: For segment AB.

$$V_{AB} = \int \tau_{AB}\, dA$$

$$= \int_{0.07237\text{ m}}^{0.22237\text{ m}} \left[\left(12.6950 - 256.74y_2^2\right)(10^6)\right](0.05dy)$$

$$= 49.78(10^3) \text{ N} = 49.8 \text{ kN} \quad \textbf{Ans.}$$

Ans.
$V_{AB} = 49.78$ kN

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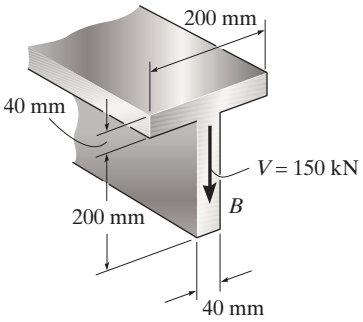
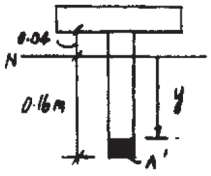
R7–2.

The T-beam is subjected to a shear of $V = 150$ kN. Determine the amount of this force that is supported by the web B.



SOLUTION

$$\bar{y} = \frac{(0.02)(0.2)(0.04) + (0.14)(0.2)(0.04)}{0.2(0.04) + 0.2(0.04)} = 0.08 \text{ m}$$



$$I = \frac{1}{12}(0.2)(0.04^3) + 0.2(0.04)(0.08 - 0.02)^2 + \frac{1}{12}(0.04)(0.2^3) + 0.2(0.04)(0.14 - 0.08)^2 = 85.3333(10^{-6}) \text{ m}^4$$

$$A' = 0.04(0.16 - y)$$

$$\bar{y}' = y + \frac{(0.16 - y)}{2} = \frac{(0.16 + y)}{2}$$

$$Q = \bar{y}'A' = 0.02(0.0256 - y^2)$$

$$\tau = \frac{VQ}{It} = \frac{150(10^3)(0.02)(0.0256 - y^2)}{85.3333(10^{-6})(0.04)} = 22.5(10^6) - 878.9(10^6)y^2$$

$$V = \int \tau \, dA, \qquad dA = 0.04 \, dy$$

$$V = \int_{-0.04}^{0.16} (22.5(10^6) - 878.9(10^6)y^2) \, 0.04 \, dy$$

$$= \int_{-0.04}^{0.16} (900(10^3) - 35.156(10^6)y^2) dy$$

$$= 131\,250 \text{ N} = 131 \text{ kN}$$ **Ans.**

Ans:
N/A

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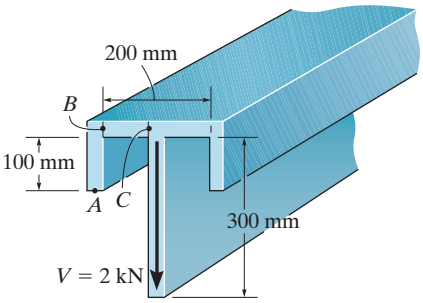
R7–3.

The member is subject to a shear force of $V = 2$ kN. Determine the shear flow at points A, B, and C. The thickness of each thin-walled segment is 15 mm.



SOLUTION

Section Properties:

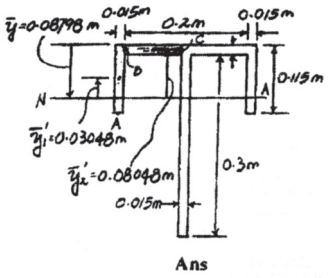
$$\bar{y} = \frac{\Sigma \bar{y} A}{\Sigma A}$$

$$= \frac{0.0075(0.2)(0.015) + 0.0575(0.115)(0.03) + 0.165(0.3)(0.015)}{0.2(0.015) + 0.115(0.03) + 0.3(0.015)}$$

$$= 0.08798 \text{ m}$$

$$I_{NA} = \frac{1}{12}(0.2)(0.015^3) + 0.2(0.015)(0.08798 - 0.0075)^2 + \frac{1}{12}(0.03)(0.115^3) + 0.03(0.115)(0.08798 - 0.0575)^2 + \frac{1}{12}(0.015)(0.3^3) + 0.015(0.3)(0.165 - 0.08798)^2$$

$$= 86.93913(10^{-6}) \text{ m}^4$$



$$Q_A = 0 \qquad \textbf{Ans.}$$

$$Q_B = \bar{y}_1' A' = 0.03048(0.115)(0.015) = 52.57705(10^{-6}) \text{ m}^{-3}$$

$$Q_C = \Sigma \overline{y1} A'$$

$$= 0.03048(0.115)(0.015) + 0.08048(0.0925)(0.015)$$

$$= 0.16424(10^{-3}) \text{ m}^3$$

Shear Flow:

$$q_A = \frac{VQ_A}{I} = 0$$

$$q_B = \frac{VQ_B}{I} = \frac{2(10^3)(52.57705)(10^{-6})}{86.93913(10^{-6})} = 1.21 \text{ kN/m} \qquad \textbf{Ans.}$$

$$q_C = \frac{VQ_C}{I} = \frac{2(10^3)(0.16424)(10^{-3})}{86.93913(10^{-6})} = 3.78 \text{ kN/m} \qquad \textbf{Ans.}$$

Ans:
$q_A = 0, q_B = 1.21$ kN/m, $q_C = 3.78$ kN/m

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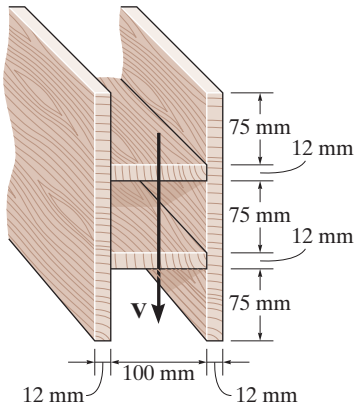
***R7–4.** The beam is constructed from four boards glued together at their seams. If the glue can withstand 15 kN/m, what is the maximum vertical shear V that the beam can support?



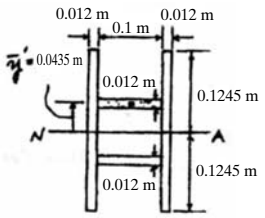
SOLUTION

Section Properties:

$$I_{NA} = 2\left[\frac{1}{12}(0.012)(0.249^3)\right] + 2\left[\frac{1}{12}(0.1)(0.012^3) + 0.1(0.012)(0.0435^2)\right]$$

$$= 35.4467(10^{-6})\ \text{m}^4$$

$$Q = \bar{y}'A' = 0.0435(0.1)(0.012) = 52.2(10^{-6})\ \text{m}^3$$



Shear Flow: There are two glue joints in this case, hence the allowable shear flow is $2[15(10^3)] = 30(10^3)$ N/m.

$$q = \frac{VQ}{I}$$

$$30(10^3) = \frac{V[52.2(10^{-6})]}{35.4467(10^{-6})}$$

$$V = 20.37(10^3)\ \text{N} = 20.4\ \text{kN}$$ **Ans.**

Ans.
$V = 20.4$ kN

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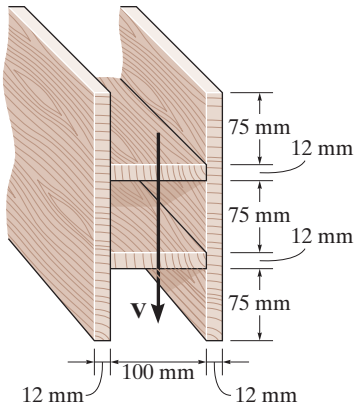
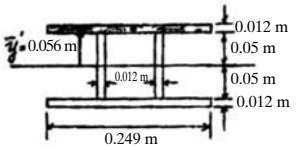
R7–5. Solve Prob. R7–4 if the beam is rotated 90° from the position shown.



SOLUTION

Section Properties:

$$I_{NA} = \frac{1}{12}(0.249)(0.124^3) - \frac{1}{12}(0.225)(0.1^3) = 20.8124(10^{-6})\ \text{m}^4$$



$$Q = \bar{y}'A' = 0.056(0.249)(0.012) = 1.167328(10^{-3})\ \text{m}^3$$

Shear Flow: There are two glue joints in this case, hence the allowable shear flow is $2[15(10^3)] = 30(10^3)$ N/m.

$$q = \frac{VQ}{I}$$

$$30(10^3) = \frac{V[1.167328(10^{-3})]}{20.8124(10^{-6})}$$

$$V = 3.731(10^3)\ \text{N} = 3.73\ \text{kN}$$ **Ans.**

Ans.
$V = 3.73$ kN

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8–1.

A spherical gas tank has an inner radius of $r = 1.5$ m. If it is subjected to an internal pressure of $p = 300$ kPa, determine its required thickness if the maximum normal stress is not to exceed 12 MPa.

SOLUTION

$$\sigma_{\text{allow}} = \frac{p\,r}{2\,t}; \qquad 12(10^6) = \frac{300(10^3)(1.5)}{2\,t}$$

$t = 0.0188\text{ m} = 18.8\text{ mm}$ **Ans.**

These solutions represent a preliminary version of the Instructors' Solutions Manual (ISM). It is possible and even likely that at this preliminary stage of preparing the ISM there are some omissions and errors in the draft solutions. These will be corrected and this manual will be republished.

Ans:
$t = 18.8$ mm

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8–2. A pressurized spherical tank is to be made of 12-mm-thick steel. If it is subjected to an internal pressure of $p = 1.4$ MPa, determine its outer radius if the maximum normal stress is not to exceed 105 MPa.

SOLUTION

$$\sigma_{\text{allow}} = \frac{p\,r}{2\,t}; \qquad 105(10^6) = \frac{[1.4(10^6)]r_i}{2(0.012)}$$

$r_i = 1.80$ m

$r_o = 1.80 + 0.012 = 1.812$ m **Ans.**

Ans.
$r_o = 1.812$ m

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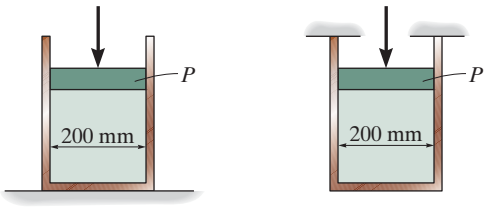
8–3. The thin-walled cylinder can be supported in one of two ways as shown. Determine the state of stress in the wall of the cylinder for both cases if the piston P causes the internal pressure to be 0.5 MPa. The wall has a thickness of 6 mm and the inner diameter of the cylinder is 200 mm.



(a) (b)

SOLUTION

Case (a):

$$\sigma_1 = \frac{pr}{t}; \qquad \sigma_1 = \frac{0.5(100)}{6} = 8.33 \text{ MPa} \qquad \textbf{Ans.}$$

$$\sigma_2 = 0 \qquad \textbf{Ans.}$$

Case (b):

$$\sigma_1 = \frac{pr}{t}; \qquad \sigma_1 = \frac{0.5(100)}{6} = 8.33 \text{ MPa} \qquad \textbf{Ans.}$$

$$\sigma_2 = \frac{pr}{2t}; \qquad \sigma_2 = \frac{0.5(100)}{2(6)} = 4.17 \text{ MPa} \qquad \textbf{Ans.}$$

Ans.
Case (a): $\sigma_1 = 8.33$ MPa; $\sigma_2 = 0$
Case (b): $\sigma_1 = 8.33$ MPa; $\sigma_2 = 4.17$ MPa

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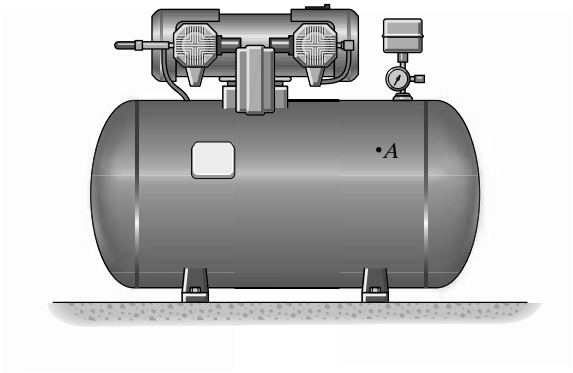
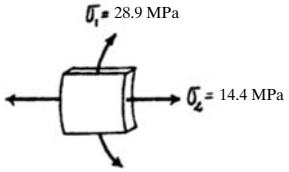
***8–4.** The tank of the air compressor is subjected to an internal pressure of 0.63 MPa. If the internal diameter of the tank is 550 mm, and the wall thickness is 6 mm, determine the stress components acting at point *A*. Draw a volume element of the material at this point, and show the results on the element.



SOLUTION

Hoop Stress for Cylindrical Vessels: Since $\frac{r}{t} = \frac{275}{6} = 45.8 > 10$, then *thin wall* analysis can be used. Applying Eq. 8–1



$$\sigma_1 = \frac{pr}{t} = \frac{0.63(275)}{6} = 28.875 \text{ MPa} = 28.9 \text{ MPa} \qquad \textbf{Ans.}$$

Longitudinal Stress for Cylindrical Vessels: Applying Eq. 8–2

$$\sigma_2 = \frac{pr}{2t} = \frac{0.63(275)}{2(6)} = 14.4375 \text{ MPa} = 14.4 \text{ MPa} \qquad \textbf{Ans.}$$

Ans.
$\sigma_1 = 28.9$ MPa, $\sigma_2 = 14.4$ MPa

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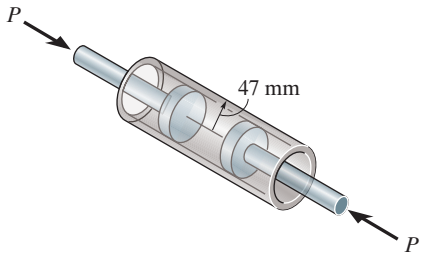
8–5.

Air pressure in the cylinder is increased by exerting forces $P = 2$ kN on the two pistons, each having a radius of 45 mm. If the cylinder has a wall thickness of 2 mm, determine the state of stress in the wall of the cylinder.



SOLUTION

$$p = \frac{P}{A} = \frac{2(10^3)}{\pi(0.045^2)} = 314\,380.13 \text{ Pa}$$

$$\sigma_1 = \frac{p\,r}{t} = \frac{314\,380.13(0.045)}{0.002} = 7.07 \text{ MPa} \qquad \textbf{Ans.}$$

$$\sigma_2 = 0 \qquad \textbf{Ans.}$$

The pressure P is supported by the surface of the pistons in the longitudinal direction.

Ans:
$\sigma_1 = 7.07$ MPa, $\sigma_2 = 0$

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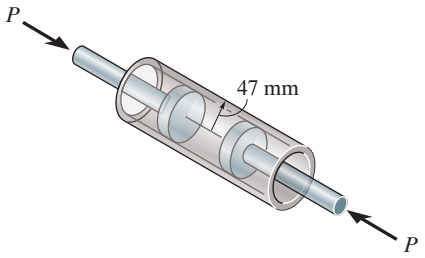
8–6.

Determine the maximum force P that can be exerted on each of the two pistons so that the circumferential stress in the cylinder does not exceed 3 MPa. Each piston has a radius of 45 mm and the cylinder has a wall thickness of 2 mm.



SOLUTION

$$\sigma = \frac{p\,r}{t}; \qquad 3(10^6) = \frac{p(0.045)}{0.002}$$

$$p = 133.3\ \text{kPa}$$

$$P = pA = 133.3\left(10^3\right)(\pi)(0.045)^2 = 848\ \text{N}$$ **Ans.**

Ans:
$P = 848$ N

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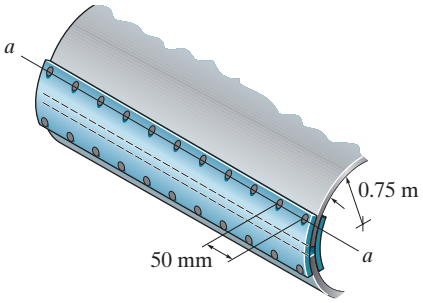
8–7.

A boiler is constructed of 8-mm-thick steel plates that are fastened together at their ends using a butt joint consisting of two 8-mm cover plates and rivets having a diameter of 10 mm and spaced 50 mm apart as shown. If the steam pressure in the boiler is 1.35 MPa, determine (a) the circumferential stress in the boiler's plate away from the seam, (b) the circumferential stress in the outer cover plate along the rivet line *a–a,* and (c) the shear stress in the rivets.



SOLUTION

a) $\sigma_1 = \dfrac{pr}{t} = \dfrac{1.35(10^6)(0.75)}{0.008} = 126.56(10^6) = 127\ \text{MPa}$ **Ans.**

b) $126.56\,(10^6)(0.05)(0.008) = \sigma_1'(2)(0.04)(0.008)$

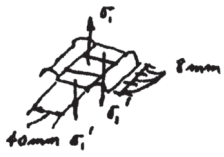
$\sigma_1' = 79.1\,\text{MPa}$ **Ans.**

c) From FBD(a)

$+\uparrow \Sigma F_y = 0; \qquad F_b - 79.1(10^6)[(0.008)(0.04)] = 0$

$F_b = 25.3\ \text{kN}$

$(\tau_{\text{avg}})_b = \dfrac{F_b}{A} = \dfrac{25312.5}{\frac{\pi}{4}(0.01)^2} = 322\ \text{MPa}$ **Ans.**



Ans:
(a) $\sigma_1 = 127$ MPa,
(b) $\sigma_1' = 79.1$ MPa,
(c) $(\tau_{avg})_b = 322$ MPa

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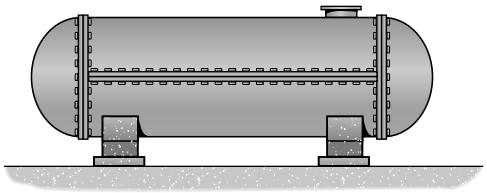


***8–8.** The gas storage tank is fabricated by bolting together two half cylindrical thin shells and two hemispherical shells as shown. If the tank is designed to withstand a pressure of 3 MPa, determine the required minimum thickness of the cylindrical and hemispherical shells and the minimum required number of longitudinal bolts per meter length at each side of the cylindrical shell. The tank and the 25 mm diameter bolts are made from material having an allowable normal stress of 150 MPa and 250 MPa, respectively. The tank has an inner diameter of 4 m.

Normal Stress: For the cylindrical portion of the tank, the hoop stress is twice as large as the longitudinal stress.

$$\sigma_{\text{allow}} = \frac{pr}{t}; \qquad 150\left(10^6\right) = \frac{3\left(10^6\right)(2)}{t_c}$$

$$t_c = 0.04\,\text{m} = 40\,\text{mm} \qquad \textbf{Ans.}$$

For the hemispherical cap,

$$\sigma_{\text{allow}} = \frac{pr}{t}; \qquad 150\left(10^6\right) = \frac{3\left(10^6\right)(2)}{2t_s}$$

$$t_s = 0.02\,\text{m} = 20\,\text{mm} \qquad \textbf{Ans.}$$

Since $\frac{r}{t} < 10$, thin-wall analysis is valid.

Referring to the free-body diagram of the per meter length of the cylindrical portion, Fig. *a,* where $P = pA = 3\left(10^6\right)[4(1)] = 12\left(10^6\right)\text{N}$, we have

$$+\uparrow \Sigma F_y = 0; \qquad 12\left(10^6\right) - n_c(P_b)_{\text{allow}} - n_c(P_b)_{\text{allow}} = 0$$

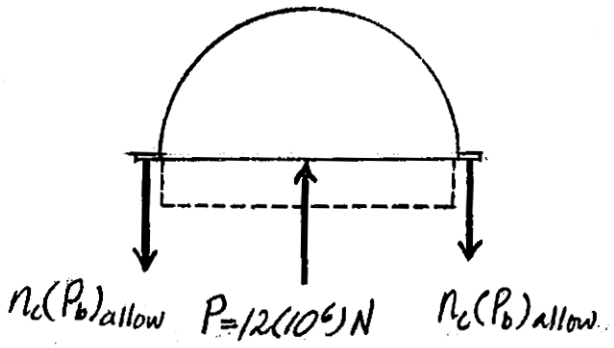
$$n_c = \frac{6\left(10^6\right)}{(P_b)_{\text{allow}}} \qquad \textbf{(1)}$$

The allowable tensile force for each bolt is

$$(P_b)_{\text{allow}} = \sigma_{\text{allow}} A_b = 250\left(10^6\right)\left[\frac{\pi}{4}\left(0.025^2\right)\right] = 122.72\left(10^3\right)\text{N}$$

Substituting this result into Eq. (1),

$$n_c = 48.89 = 49\ \text{bolts/meter} \qquad \textbf{Ans.}$$



(a)

Ans.
$t_c = 40\,\text{mm}$, $t_s = 20\,\text{mm}$,
$n_c = 49$ bolts/meter

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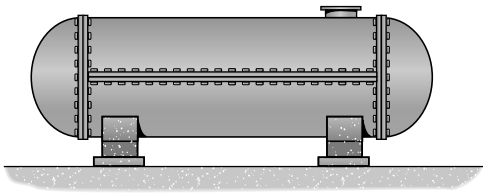
8–9. The gas storage tank is fabricated by bolting together two half cylindrical thin shells and two hemispherical shells as shown. If the tank is designed to withstand a pressure of 3 MPa, determine the required minimum thickness of the cylindrical and hemispherical shells and the minimum required number of bolts for each hemispherical cap. The tank and the 25 mm diameter bolts are made from material having an allowable normal stress of 150 MPa and 250 MPa, respectively. The tank has an inner diameter of 4 m.



Normal Stress: For the cylindrical portion of the tank, the hoop stress is twice as large as the longitudinal stress.

$$\sigma_{\text{allow}} = \frac{pr}{t}; \qquad 150\left(10^6\right) = \frac{3\left(10^6\right)(2)}{t_c}$$

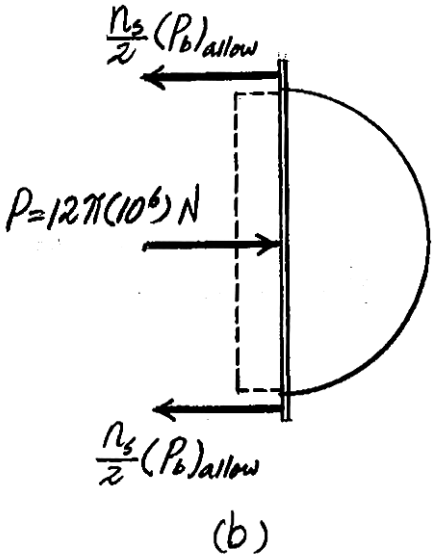
$$t_c = 0.04\text{ m} = 40\text{ mm} \qquad \textbf{Ans.}$$

For the hemispherical cap,

$$\sigma_{\text{allow}} = \frac{pr}{t}; \qquad 150\left(10^6\right) = \frac{3\left(10^6\right)(2)}{2t_s}$$

$$t_s = 0.02\text{ m} = 20\text{ mm} \qquad \textbf{Ans.}$$

Since $\frac{r}{t} < 10$, thin-wall analysis is valid.



The allowable tensile force for each bolt is

$$(P_b)_{\text{allow}} = \sigma_{\text{allow}} A_b = 250\left(10^6\right)\left[\frac{\pi}{4}\left(0.025^2\right)\right] = 122.72\left(10^3\right)\text{N}$$

Referring to the free-body diagram of the hemispherical cap, Fig. *b,* where $P = pA = 3\left(10^6\right)\left[\frac{\pi}{4}\left(4^2\right)\right] = 12\pi\left(10^6\right)\text{N}$,

$$\stackrel{+}{\rightarrow}\Sigma F_x = 0; \qquad 12\pi\left(10^6\right) - \frac{n_s}{2}(P_b)_{\text{allow}} - \frac{n_s}{2}(P_b)_{\text{allow}} = 0$$

$$n_s = \frac{12\pi\left(10^6\right)}{(P_b)_{\text{allow}}} \qquad \textbf{(1)}$$

Substituting this result into Eq. (1),

$$n_s = 307.2 = 308\text{ bolts} \qquad \textbf{Ans.}$$

Ans.
$t_c = 40$ mm, $t_s = 20$ mm,
$(P_b)_{\text{allow}} = 122.72(10^3)$ N, $n_s = 308$ bolts

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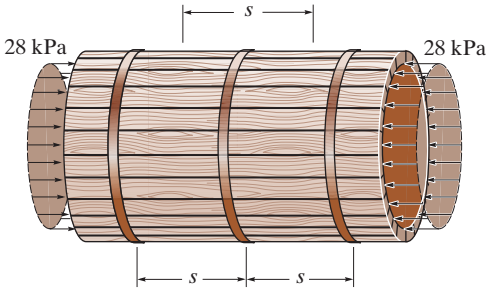
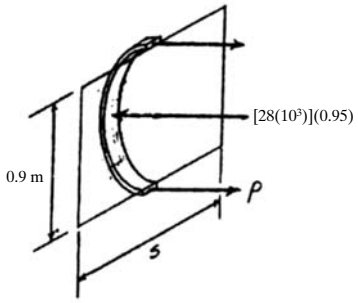
8–10. A wood pipe having an inner diameter of 0.9 m is bound together using steel hoops each having a cross-sectional area of 125 mm^2. If the allowable stress for the hoops is $\sigma_{allow} = 84$ MPa, determine their maximum spacing s along the section of pipe so that the pipe can resist an internal gauge pressure of 28 kPa. Assume each hoop supports the pressure loading acting along the length s of the pipe.



SOLUTION



Equilibrium for the steel Hoop: From the FBD

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 2P - [28(10^3)](0.93) = 0 \qquad P = 12.6(10^3)s$$

Hoop Stress for the Steel Hoop:

$$\sigma_1 = \sigma_{allow} = \frac{P}{A}$$

$$84(10^6) = \frac{12.6(10^3)s}{125(10^{-6})}$$

$$s = 833.33 \text{ mm} = 0.833 \text{ m}$$ **Ans.**

Ans.
$s = 0.833$ m

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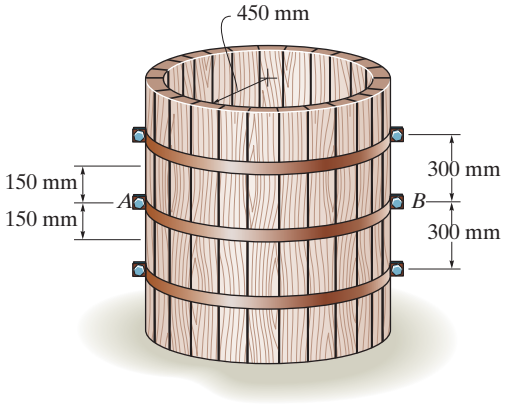
8–11. The staves or vertical members of the wooden tank are held together using semicircular hoops having a thickness of 12 mm and a width of 50 mm. Determine the normal stress in hoop AB if the tank is subjected to an internal gauge pressure of 14 kPa and this loading is transmitted directly to the hoops. Also, if 6-mm-diameter bolts are used to connect each hoop together, determine the tensile stress in each bolt at A and B. Assume hoop AB supports the pressure loading within a 300-mm length of the tank as shown.

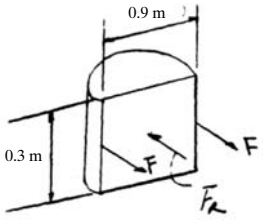


SOLUTION

$$F_R = [14(10^3)][(0.9)(0.3^3)] = 3.78(10^3)\ \text{N}$$

$$\Sigma F = 0; \quad 3.78(10^3) - 2F = 0; \quad F = 1.89(10^3)\ \text{N}$$

$$\sigma_h = \frac{F}{A_h} = \frac{1.89(10^3)}{0.012(0.05)} = 3.15(10^6)\ \text{N/m}^3 = 3.15\ \text{MPa}$$ **Ans.**



$$\sigma_b = \frac{F}{A_b} = \frac{1.89(10^3)}{\frac{\pi}{4}(0.006^2)} = 66.84(10^6)\ \text{N/m}^3 = 66.8\ \text{MPa}$$ **Ans.**

Ans.
$\sigma_h = 3.15$ MPa, $\sigma_b = 66.8$ MPa

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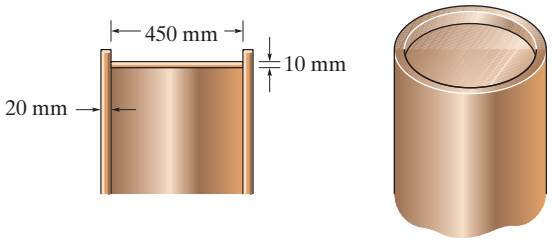
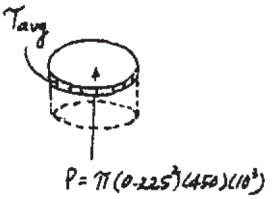
***8–12.**

A pressure-vessel head is fabricated by welding the circular plate to the end of the vessel as shown. If the vessel sustains an internal pressure of 450 kPa, determine the average shear stress in the weld and the state of stress in the wall of the vessel.



SOLUTION



$+\uparrow \Sigma F_y = 0; \quad \pi(0.225)^2 450(10^3) - \tau_{avg}(2\pi)(0.225)(0.01) = 0;$

$\tau_{avg} = 5.06 \text{ MPa}$ **Ans.**

$$\sigma_1 = \frac{p\,r}{t} = \frac{450(10^3)(0.225)}{0.02} = 5.06 \text{ MPa}$$ **Ans.**

$$\sigma_2 = \frac{p\,r}{2\,t} = \frac{450(10^3)(0.225)}{2(0.02)} = 2.53 \text{ MPa}$$ **Ans.**

Ans:
$\tau_{avg} = 5.06$ MPa,
$\sigma_1 = 5.06$ MPa,
$\sigma_2 = 2.53$ MPa

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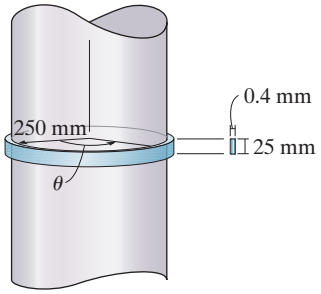
8–13. The 304 stainless steel band initially fits snugly around the smooth rigid cylinder. If the band is then subjected to a nonlinear temperature drop of $\Delta T = 12 \sin^2 \theta$ °C, where θ is in radians, determine the circumferential stress in the band.



SOLUTION

Compatibility: Since the band is fixed to a rigid cylinder (it does not deform under load), then

$$\delta_F - \delta_T = 0$$

$$\frac{P(2\pi r)}{AE} - \int_0^{2\pi} \alpha \Delta T r d\theta = 0$$

$$\frac{2\pi r}{E}\left(\frac{P}{A}\right) = 12\alpha r \int_0^{2\pi} \sin^2 \theta d\theta \qquad \text{however, } \frac{P}{A} = \sigma_c$$

$$\frac{2\pi}{E}\sigma_c = 6\alpha \int_0^{2\pi} (1 - \cos 2\theta) d\theta$$

$$\sigma_c = 6\alpha E$$

$$= 6[17(10^{-6})][193(10^9)] = 19.686(10^6)\ \text{N/m}^2 = 19.7\ \text{MPa} \qquad \textbf{Ans.}$$

Ans.
$\delta_F - \delta_T = 0, \sigma_c = 19.7$ MPa

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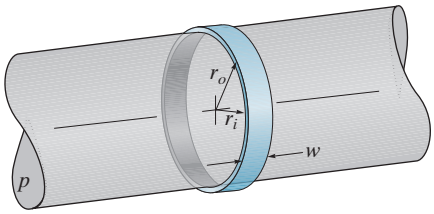
8–14.

The ring, having the dimensions shown, is placed over a flexible membrane which is pumped up with a pressure p. Determine the change in the inner radius of the ring after this pressure is applied. The modulus of elasticity for the ring is E.



SOLUTION

Equilibrium for the Ring: From the FBD

$$\stackrel{+}{\rightarrow} \Sigma F_x = 0;\ 2P - 2pr_i w = 0 \qquad P = pr_i w$$

Hoop Stress and Strain for the Ring:

$$\sigma_1 = \frac{P}{A} = \frac{pr_i w}{(r_o - r_i)w} = \frac{pr_i}{r_o - r_i}$$

Using Hooke's Law

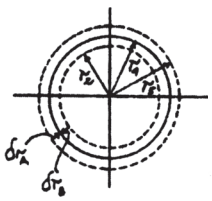
$$\epsilon_1 = \frac{\sigma_1}{E} = \frac{pr_i}{E(r_o - r_i)} \qquad \textbf{(1)}$$

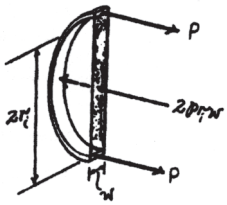
However, $\epsilon_1 = \dfrac{2\pi(r_i)_1 - 2\pi r_i}{2\pi r_i} = \dfrac{(r_i)_1 - r_i}{r_i} = \dfrac{\delta r_i}{r_i}$.

Then, from Eq. (1)

$$\frac{\delta r_i}{r_i} = \frac{pr_i}{E(r_o - r_i)}$$

$$\delta r_i = \frac{pr_i^2}{E(r_o - r_i)} \qquad \textbf{Ans.}$$





Ans:

$$\delta r_i = \frac{pr_i^2}{E(r_o - r_i)}$$

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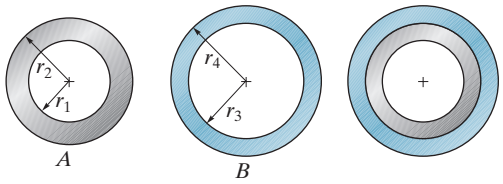
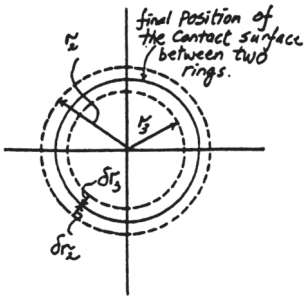
8–15.

The inner ring A has an inner radius r_1 and outer radius r_2. The outer ring B has an inner radius r_3 and an outer radius r_4, and $r_2 > r_3$. If the outer ring is heated and then fitted over the inner ring, determine the pressure between the two rings when ring B reaches the temperature of the inner ring. The material has a modulus of elasticity of E and a coefficient of thermal expansion of α.



SOLUTION



Equilibrium for the Ring: From the FBD

$$\stackrel{+}{\rightarrow} \Sigma F_x = 0; 2P - 2pr_i w = 0 \qquad P = pr_i w$$

Hoop Stress and Strain for the Ring:

$$\sigma_1 = \frac{P}{A} = \frac{pr_i w}{(r_o - r_i)w} = \frac{pr_i}{r_o - r_i}$$

Using Hooke's Law

$$\epsilon_1 = \frac{\sigma_1}{E} = \frac{pr_i}{E(r_o - r_i)} \qquad \textbf{(1)}$$

However, $\epsilon_1 = \dfrac{2\pi(r_i)_1 - 2\pi r_i}{2\pi r_i} = \dfrac{(r_i)_1 - r_i}{r_i} = \dfrac{\delta r_i}{r_i}$.

Then, from Eq. (1)

$$\frac{\delta r_i}{r_i} = \frac{pr_i}{E(r_o - r_i)}$$

$$\delta r_i = \frac{pr_i^2}{E(r_o - r_i)}$$

Compatibility: The pressure between the rings requires

$$\delta r_2 + \delta r_3 = r_2 - r_3 \qquad \textbf{(2)}$$

From the result obtained above

$$\delta r_2 = \frac{pr_2^2}{E(r_2 - r_1)} \qquad \delta r_3 = \frac{pr_3^2}{E(r_4 - r_3)}$$

Substitute into Eq. (2)

$$\frac{pr_2^2}{E(r_2 - r_1)} + \frac{pr_3^2}{E(r_4 - r_3)} = r_2 - r_3$$

$$p = \frac{E(r_2 - r_3)}{\dfrac{r_2^2}{r_2 - r_1} + \dfrac{r_3^2}{r_4 - r_3}} \qquad \textbf{Ans.}$$

Ans:

$$p = \frac{E(r_2 - r_3)}{\dfrac{r_2^2}{r_2 - r_1} + \dfrac{r_3^2}{r_4 - r_3}}$$

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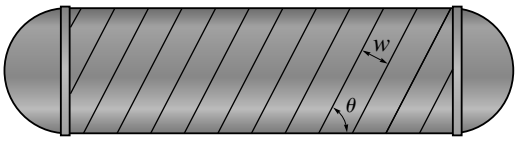
***8–16.** The cylindrical tank is fabricated by welding a strip of thin plate helically, making an angle θ with the longitudinal axis of the tank. If the strip has a width w and thickness t, and the gas within the tank of diameter d is pressured to p, show that the normal stress developed along the strip is given by $\sigma_\theta = (pd/8t)(3 - \cos 2\theta)$.



Normal Stress:

$$\sigma_h = \sigma_1 = \frac{pr}{t} = \frac{p(d/2)}{t} = \frac{pd}{2t}$$

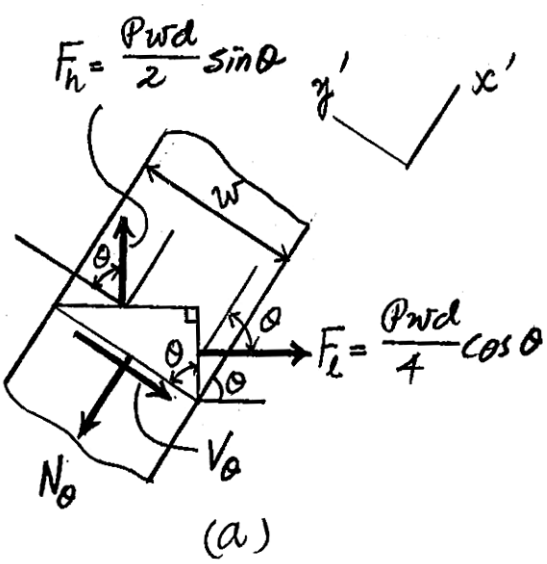
$$\sigma_l = \sigma_2 = \frac{pr}{2t} = \frac{p(d/2)}{2t} = \frac{pd}{4t}$$

Equilibrium: We will consider the triangular element cut from the strip shown in Fig. *a.* Here,

$A_h = (w \sin\theta)t$ and $A_l = (w \cos\theta)t$. Thus,

$$F_h = \sigma_h A_h = \frac{pd}{2t}(w \sin\theta)t = \frac{pwd}{2}\sin\theta \quad \text{and}$$

$$F_l = \sigma_l A_l = \frac{pd}{4t}(w \cos\theta)t = \frac{pwd}{4}\cos\theta.$$



Writing the force equation of equilibrium along the x' axis,

$$\Sigma F_{x'} = 0; \quad \left[\frac{pwd}{2}\sin\theta\right]\sin\theta + \left[\frac{pwd}{4}\cos\theta\right]\cos\theta - N_\theta = \theta$$

$$N_\theta = \frac{pwd}{4}\left(2\sin^2\theta + \cos^2\theta\right)$$

However, $\sin^2\theta + \cos^2\theta = 1$. This equation becomes

$$N_\theta = \frac{pwd}{4}\left(\sin^2\theta + 1\right)$$

Also, $\sin^2\theta = \frac{1}{2}(1 - \cos 2\theta)$, so that

$$N_\theta = \frac{pwd}{8}(3 - \cos 2\theta)$$

Since $A_\theta = wt$, then

$$\sigma_\theta = \frac{N_\theta}{A_\theta} = \frac{\frac{pwd}{8}(3 - \cos 2\theta)}{wt}$$

$$\sigma_\theta = \frac{pd}{8t}(3 - \cos 2\theta) \qquad \textbf{(Q.E.D.)}$$

Ans.

$$\sigma_\theta = \frac{pd}{8t}(3 - \cos 2\theta)$$

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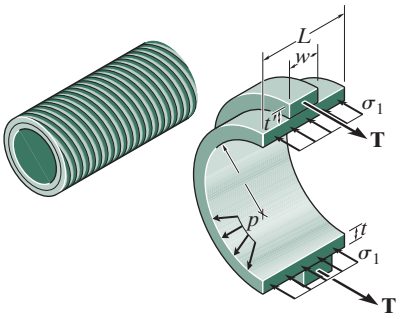
8–17.

In order to increase the strength of the pressure vessel, filament winding of the same material is wrapped around the circumference of the vessel as shown. If the pretension in the filament is T and the vessel is subjected to an internal pressure p, determine the hoop stresses in the filament and in the wall of the vessel. Use the free-body diagram shown, and assume the filament winding has a thickness t' and width w for a corresponding length L of the vessel.



SOLUTION

Normal Stress in the Wall and Filament Before the Internal Pressure is Applied: The entire length L of wall is subjected to pretension filament force T. Hence, from equilibrium, the normal stress in the wall at this state is

$$2T - (\sigma')_w (2Lt) = 0 \qquad (\sigma')_w = \frac{T}{Lt}$$

and for the filament the normal stress is

$$(\sigma')_{fil} = \frac{T}{wt'}$$

Normal Stress in the Wall and Filament After the Internal Pressure is Applied: In order to use $\sigma_1 = pr/t$, developed for a vessel of uniform thickness, we redistribute the filament's cross-section as if it were thinner and wider, to cover the vessel with no gaps. The modified filament has width L and thickness $t'w/L$, still with cross-sectional area wt' subjected to tension T. Then the stress in the filament becomes

$$\sigma_{fil} = \sigma + (\sigma')_{fil} = \frac{pr}{(t + t'w/L)} + \frac{T}{wt'} \qquad \textbf{Ans.}$$

And for the wall,

$$\sigma_w = \sigma - (\sigma')_w = \frac{pr}{(t + t'w/L)} - \frac{T}{Lt} \qquad \textbf{Ans.}$$

Check: $2wt'\sigma_{fil} + 2Lt\sigma_w = 2rLp$ OK

Ans:

$$\sigma_{fil} = \frac{pr}{t + t'w/L} + \frac{T}{wt'},$$

$$\sigma_w = \frac{pr}{t + t'w/L} - \frac{T}{Lt}$$

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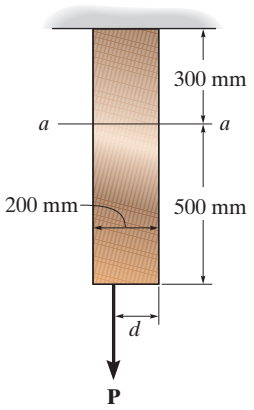
8–18.

Determine the shortest distance d to the edge of the plate at which the force **P** can be applied so that it produces no compressive stresses in the plate at section *a–a*. The plate has a thickness of 10 mm and **P** acts along the centerline of this thickness.



SOLUTION

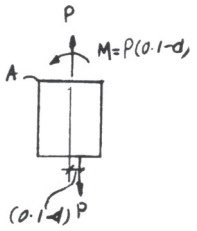
$$\sigma_A = 0 = \sigma_a - \sigma_b$$

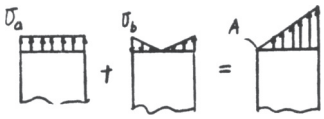
$$0 = \frac{P}{A} - \frac{M\,c}{I}$$

$$0 = \frac{P}{(0.2)(0.01)} - \frac{P(0.1 - d)(0.1)}{\frac{1}{12}(0.01)(0.2^3)}$$

$$P(-1000 + 15000\,d) = 0$$

$$d = 0.0667\text{ m} = 66.7\text{ mm}$$ **Ans.**





Ans:
$d = 66.7$ mm

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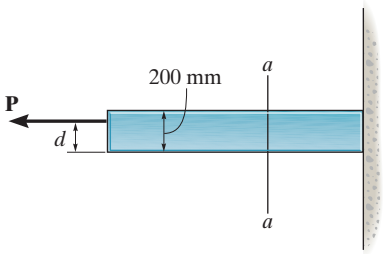
8–19.

Determine the maximum distance d to the edge of the plate at which the force **P** can be applied so that it produces no compressive stresses on the plate at section a–a. The plate has a thickness of 20 mm and **P** acts along the centerline of this thickness.

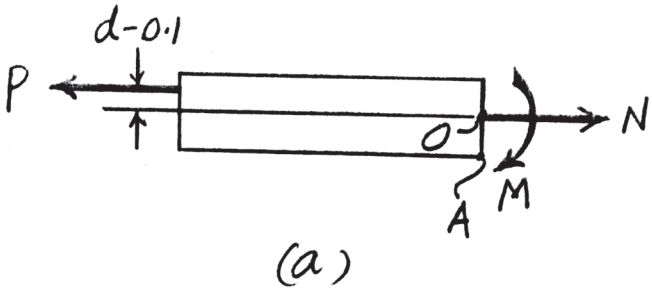


SOLUTION

Internal Loadings: Consider the equilibrium of the left segment of the plate sectioned through section a–a, Fig. a,

$$\stackrel{+}{\rightarrow}\Sigma F_x = 0; \qquad N - P = 0 \qquad N = P$$

$$\circlearrowleft +\Sigma M_o = 0; \qquad P(d - 0.1) - M = 0 \qquad M = P(d - 0.1)$$



Section Properties: For the rectangular cross section,

$$A = 0.2(0.02) = 0.004\ \text{m}^2$$

$$I = \frac{1}{12}(0.02)\left(0.2^3\right) = 13.3333\left(10^{-6}\right)\ \text{m}^4$$

Normal Stress: It is required that $\sigma_A = 0$. For the combined loadings,

$$\sigma_A = \frac{N}{A} - \frac{Mc}{I}$$

$$0 = \frac{P}{0.004} - \frac{P(d - 0.1)(0.1)}{13.3333\left(10^{-6}\right)}$$

$$d = 0.1333\ \text{m} = 133\ \text{mm} \qquad \textbf{Ans.}$$

Ans:
$d = 133$ mm

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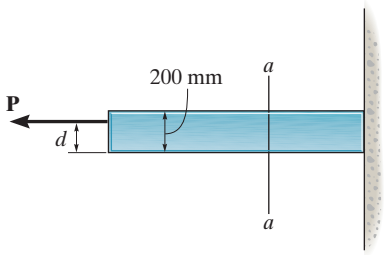
***8–20.**

The plate has a thickness of 20 mm and the force $P = 3$ kN acts along the centerline of this thickness such that $d = 150$ mm. Plot the distribution of normal stress acting along section *a–a*.



SOLUTION

Internal Loadings: Consider the equilibrium of the left segment of the plate sectioned through section *a–a*, Fig. *a*,

$$\stackrel{+}{\rightarrow} \Sigma F_x = 0; \qquad N - 3 = 0 \qquad N = 3.00 \text{ kN}$$

$$\circlearrowleft + \Sigma M_o = 0; \qquad 3(0.05) - M = 0 \qquad M = 0.150 \text{ kN} \cdot \text{m}$$

Section Properties: For the rectangular cross section,

$$A = 0.2(0.02) = 0.004 \text{ m}^2$$

$$I = \frac{1}{12}(0.02)(0.2^3) = 13.3333(10^{-6}) \text{ m}^4$$

Normal Stress: For the combined loadings, the normal stress at points A and B can be determined from

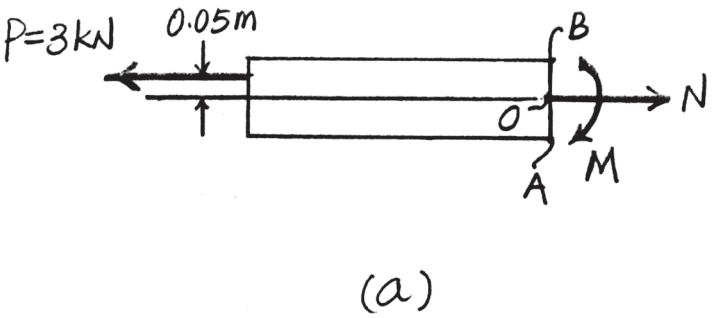
$$\sigma = \frac{N}{A} \pm \frac{Mc}{I} = \frac{3.00(10^3)}{0.004} \pm \frac{0.150(10^3)(0.1)}{13.3333(10^{-6})}$$

$$\sigma_A = 750(10^3) - 1.125(10^6) = -0.375(10^6) \text{ Pa} = 0.375 \text{ MPa (C)} \qquad \textbf{Ans.}$$

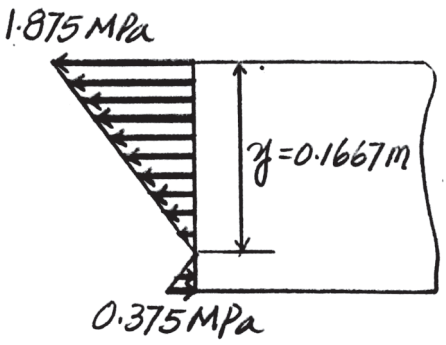
$$\sigma_B = 750(10^3) + 1.125(10^6) = 1.875(10^6) \text{ Pa} = 1.875 \text{ MPa (T)} \qquad \textbf{Ans.}$$

Using similar triangles, the location of the neutral axis can be determined

$$\frac{y}{1.875} = \frac{0.2 - y}{0.375}; \qquad y = 0.1667 \text{ m}$$



(a)



(b)

Ans:
$\sigma_A = 0.375$ MPa (C),
$\sigma_B = 1.875$ MPa (T)

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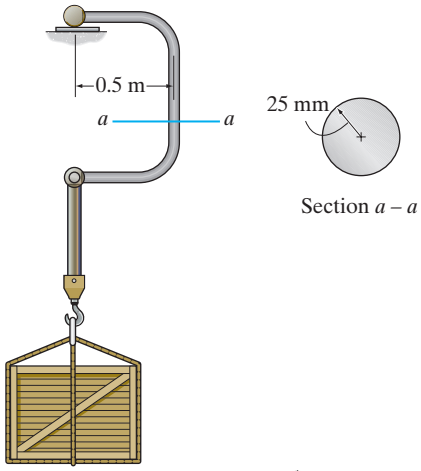
8–21. If the load has a weight of 2700 N, determine the maximum normal stress developed on the cross section of the supporting member at section *a–a*. Also, plot the normal stress distribution over the cross-section.



SOLUTION

Internal Loadings: Consider the equilibrium of the free-body diagram of the bottom cut segment shown in Fig. a.

$$\zeta + \uparrow \Sigma F_y = 0; \qquad N - 2700 = 0 \qquad N = 2700\ \text{N}$$

$$\zeta + \Sigma M_C = 0; \qquad 2700(0.5) - M = 0 \qquad M = 1350\ \text{N} \cdot \text{m}$$

Section Properties: The cross-sectional area and the moment of inertia about the centroidal axis of the member are

$$A = \pi(0.025^2) = 0.625(10^{-3})\pi\ \text{m}^2 \qquad I = \frac{\pi}{4}(0.025^4) = 0.30680(10^{-6})\ \text{m}^4$$

Normal Stress: The normal stress is the combination of axial and bending stress. Thus,

$$\sigma = \frac{N}{A} \pm \frac{Mc}{I}$$

By observation, the maximum normal stress occurs at point *B*, Fig. *b*. Thus,

$$\sigma_{\max} = \sigma_B = \frac{2700}{0.625(10^{-3})\pi} + \frac{1350(0.025)}{0.30680(10^{-6})}$$

$$= 111.38(10^6)\ \text{N/m}^2 = 111\ \text{MPa (T)} \qquad \textbf{Ans.}$$

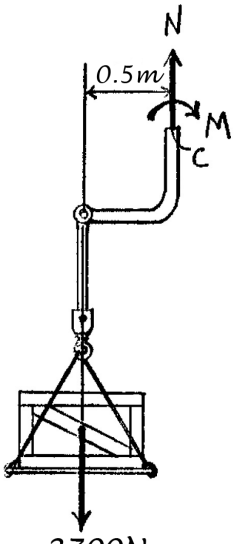
For Point *A*,

$$\sigma_A = \frac{2700}{0.625(10^{-3})\pi} - \frac{1350(0.025)}{0.30680(10^{-6})}$$

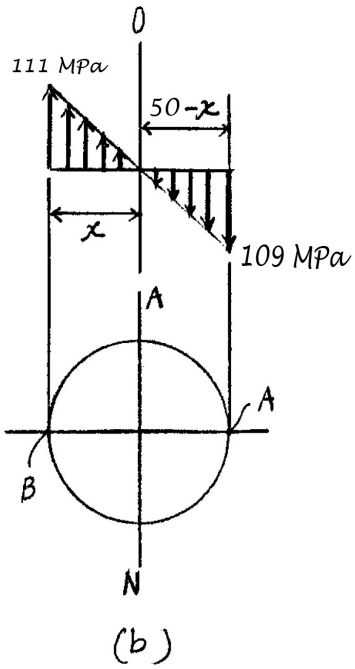
$$= -108.63(10^6)\ \text{N/m}^2 = 109\ \text{MPa (C)} \qquad \textbf{Ans.}$$

Using these results, the normal stress distribution over the cross section is shown in Fig. *b*. The location of the neutral axis can be determined from

$$\frac{50 - x}{108.63} = \frac{x}{111.38}; \qquad x = 25.3\ \text{mm}$$



(a)



(b)

Ans:

$\sigma_{\max} = 111\ \text{MPa (T)}$

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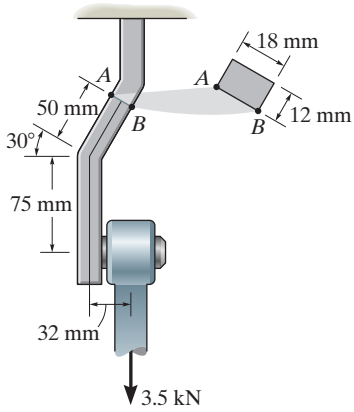
8–22. The bearing pin supports the load of 3.5 kN. Determine the stress components in the support member at point A. The support is 12 mm thick.



SOLUTION

$\Sigma F_x = 0; \quad N - 3.5\cos 30° = 0; \quad N = 3.0311 \text{ kN}$

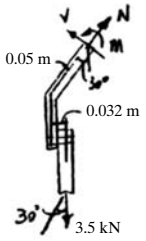
$\Sigma F_y = 0; \quad V - 3.5\sin 30° = 0; \quad V = 1.75 \text{ kN}$

$\zeta + \Sigma M = 0; \quad M - 3.5(0.032 - 0.05\sin 30°) = 0; \quad M = 0.0245 \text{ KN}\cdot\text{m} = 24.5 \text{ N}\cdot\text{m}$

$$\sigma_A = \frac{N}{A} - \frac{Mc}{I} = \frac{3.0311(10^3)}{0.012(0.018)} - \frac{24.5(0.009)}{\frac{1}{12}(0.012)(0.018^3)}$$

$\sigma_A = -23.78(10^6) \text{ N/m}^2 = 23.8 \text{ MPa (C)}$ **Ans.**

$\tau_A = 0 \quad$ (since $Q_A = 0$) **Ans.**



Ans.
$\sigma_A = 23.8 \text{ MPa(C)}, T_A = 0$

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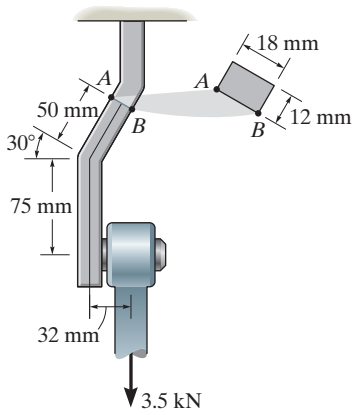
8–23. The bearing pin supports the load of 3.5 kN. Determine the stress components in the support member at point B. The support is 12 mm thick.



SOLUTION

$\Sigma F_x = 0; \quad N - 3.5\cos 30° = 0; \quad N = 3.0311 \text{ kN}$

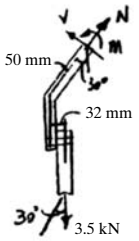
$\Sigma F_y = 0; \quad V - 3.5\sin 30° = 0; \quad V = 1.75 \text{ kN}$

$\zeta + \Sigma M = 0; \quad M - 3.5(0.032 - 0.05\sin 30°) = 0; \quad M = 0.0245 \text{ KN}\cdot\text{m} = 24.5 \text{ N}\cdot\text{m}$

$$\sigma_B = \frac{N}{A} + \frac{Mc}{I} = \frac{3.0311(10^3)}{0.012(0.018)} + \frac{24.5(0.009)}{\frac{1}{12}(0.012)(0.018^3)}$$

$\sigma_B = 51.84(10^6) \text{ N/m}^2 = 51.8 \text{ MPa (T)}$ **Ans.**

$\tau_B = 0 \quad$ (since $Q_B = 0$) **Ans.**



Ans.
$N = 3.031$ kN, $V = 1.75$ kN,
$M = 24.5 \text{ N}\cdot\text{m}$,
$\sigma_B = 51.8$ MPa (T), $\tau_B = 0$

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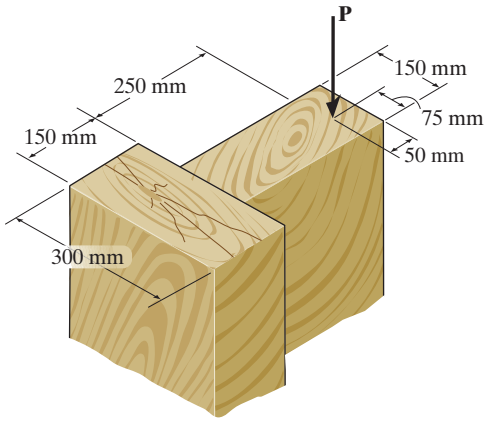
***8–24.**

The column is built up by gluing the two boards together. Determine the maximum normal stress on the cross section when the eccentric force of $P = 50$ kN is applied.



SOLUTION

Section Properties: The location of the centroid of the cross section, Fig. *a*, is

$$\bar{y} = \frac{\Sigma \bar{y} A}{\Sigma A} = \frac{0.075(0.15)(0.3) + 0.3(0.3)(0.15)}{0.15(0.3) + 0.3(0.15)} = 0.1875 \text{ m}$$

The cross-sectional area and the moment of inertia about the z axis of the cross section are

$$A = 0.15(0.3) + 0.3(0.15) = 0.09 \text{ m}^2$$

$$I_z = \frac{1}{12}(0.3)(0.15^3) + 0.3(0.15)(0.1875 - 0.075)^2 + \frac{1}{12}(0.15)(0.3^3) + 0.15(0.3)(0.3 - 0.1875)^2$$

$$= 1.5609(10^{-3}) \text{ m}^4$$

Equivalent Force System: Referring to Fig. *b*,

$+\uparrow \Sigma F_x = (F_R)_x;$ $\quad -50 = -F$ $\quad F = 50$ kN

$\Sigma M_z = (M_R)_z;$ $\quad -50(0.2125) = -M$ $\quad M = 10.625$ kN · m

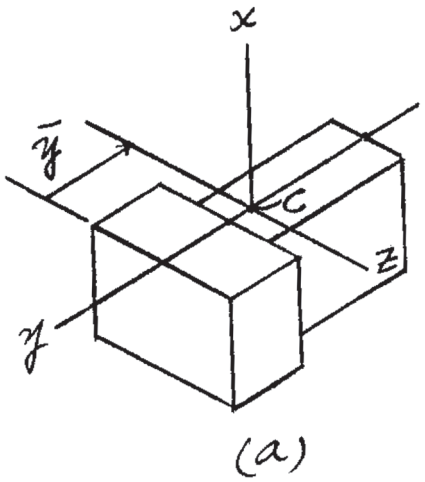
Normal Stress: The normal stress is a combination of axial and bending stress. Thus,

$$\sigma = \frac{N}{A} + \frac{My}{I}$$

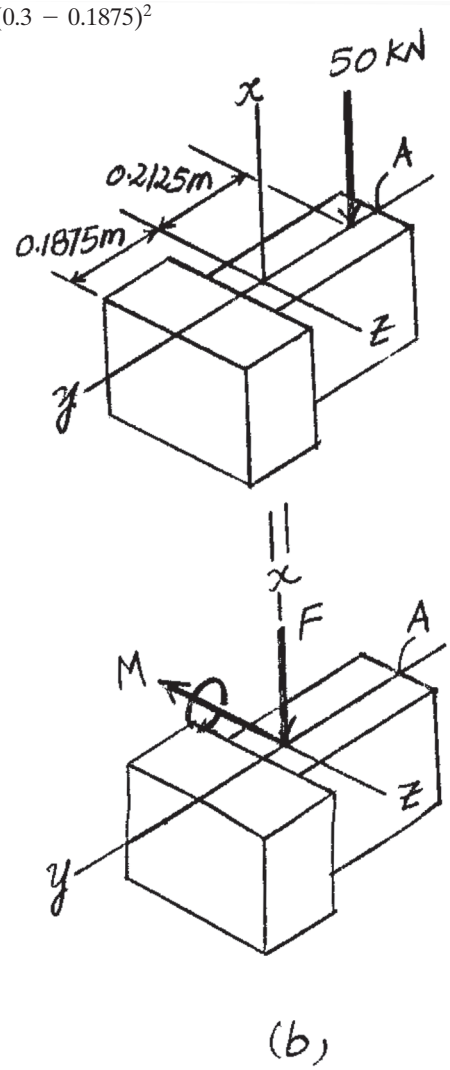
By inspection, the maximum normal stress occurs at points along the edge where $y = 0.45 - 0.1875 = 0.2625$ m such as point A. Thus,

$$\sigma_{\max} = \frac{-50(10^3)}{0.09} - \frac{10.625(10^3)(0.2625)}{1.5609(10^{-3})}$$

$$= -2.342 \text{ MPa} = 2.34 \text{ MPa (C)} \quad \textbf{Ans.}$$



(a)



(b)

Ans:
$\sigma_{\max} = 2.34$ MPa (C)

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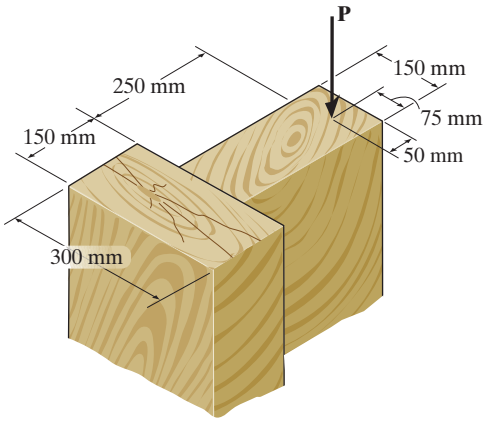
8–25.

The column is built up by gluing the two boards together. If the wood has an allowable normal stress of $\sigma_{allow} = 6$ MPa, determine the maximum allowable eccentric force **P** that can be applied to the column.



SOLUTION

Section Properties: The location of the centroid c of the cross section, Fig. a, is

$$\bar{y} = \frac{\Sigma \bar{y} A}{\Sigma A} = \frac{0.075(0.15)(0.3) + 0.3(0.3)(0.15)}{0.15(0.3) + 0.3(0.15)} = 0.1875 \text{ m}$$

The cross-sectional area and the moment of inertia about the z axis of the cross section are

$$A = 0.15(0.3) + 0.3(0.15) = 0.09 \text{ m}^2$$

$$I_z = \frac{1}{12}(0.3)(0.15^3) + 0.3(0.15)(0.1875 - 0.075)^2 + \frac{1}{12}(0.15)(0.3^3) + 0.15(0.3)(0.3 - 0.1875)^2$$

$$= 1.5609(10^{-3}) \text{ m}^4$$

Equivalent Force System: Referring to Fig. b,

$$+\uparrow \Sigma F_x = (F_R)_x; \qquad -P = -F \qquad F = P$$

$$\Sigma M_z = (M_R)_z; \qquad -P(0.2125) = -M \qquad M = 0.2125P$$

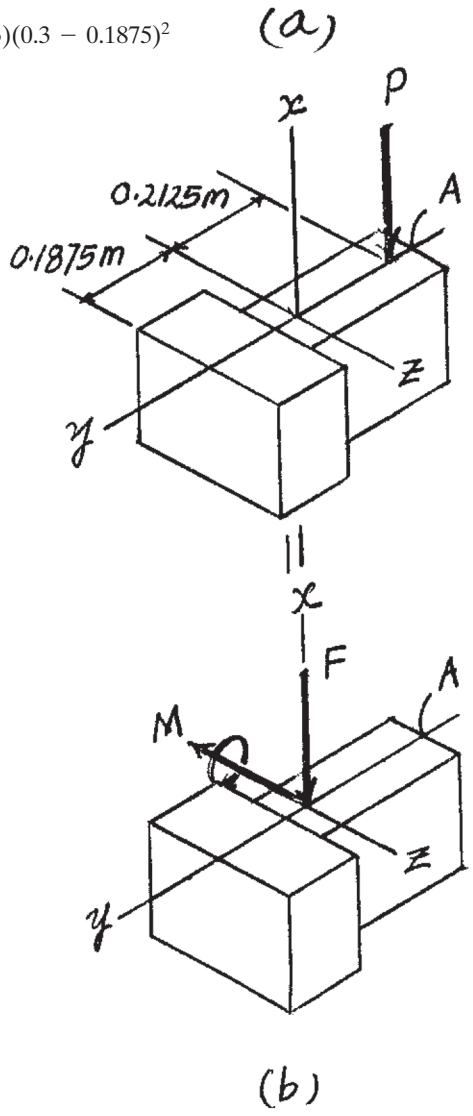
Normal Stress: The normal stress is a combination of axial and bending stress. Thus,

$$F = \frac{N}{A} + \frac{My}{I}$$

By inspection, the maximum normal stress, which is compression, occurs at points along the edge where $y = 0.45 - 0.1875 = 0.2625$ m such as point A. Thus,

$$-6(10^6) = \frac{-P}{0.09} - \frac{0.2125P(0.2625)}{1.5609(10^{-3})}$$

$$P = 128\,076.92 \text{ N} = 128 \text{ kN} \qquad \textbf{Ans.}$$



Ans:
$P_{max} = 128$ kN

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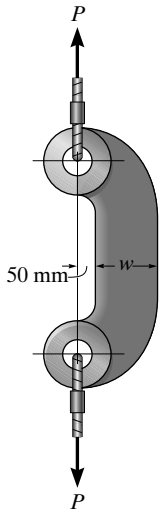
8–26. The offset link supports the loading of $P = 30$ kN. Determine its required width w if the allowable normal stress is $\sigma_{\text{allow}} = 73$ MPa. The link has a thickness of 40 mm.



SOLUTION

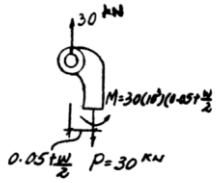
σ due to axial force:

$$\sigma_a = \frac{P}{A} = \frac{30(10^3)}{(w)(0.04)} = \frac{750(10^3)}{w}$$

σ due to bending:

$$\sigma_b = \frac{Mc}{I} = \frac{30(10^3)(0.05 + \frac{w}{2})(\frac{w}{2})}{\frac{1}{12}(0.04)(w)^3}$$

$$= \frac{4500\,(10^3)(0.05 + \frac{w}{2})}{w^2}$$



$$\sigma_{\max} = \sigma_{\text{allow}} = \sigma_a + \sigma_b$$

$$73(10^6) = \frac{750(10^3)}{w} + \frac{4500(10^3)(0.05 + \frac{w}{2})}{w^2}$$

$$73\,w^2 = 0.75\,w + 0.225 + 2.25\,w$$

$$73\,w^2 - 3\,w - 0.225 = 0$$

$$w = 0.0797 \text{ m} = 79.7 \text{ mm}$$ **Ans.**

Ans.
$w = 79.7$ mm

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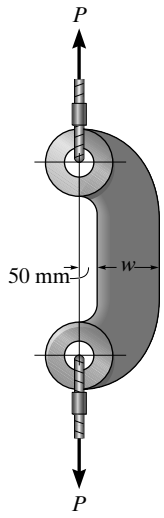
8–27. The offset link has a width of $w = 200$ mm and a thickness of 40 mm. If the allowable normal stress is $\sigma_{allow} = 75$ MPa, determine the maximum load P that can be applied to the cables.



SOLUTION

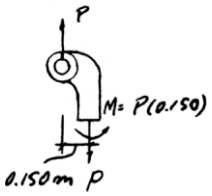
$$A = 0.2(0.04) = 0.008\ \text{m}^2$$

$$I = \frac{1}{12}(0.04)(0.2)^3 = 26.6667(10^{-6})\ \text{m}^4$$

$$\sigma = \frac{P}{A} + \frac{Mc}{I}$$

$$75(10^6) = \frac{P}{0.008} + \frac{0.150\,P(0.1)}{26.6667(10^{-6})}$$

$$P = 109\ \text{kN}$$ **Ans.**



Ans.

$P = 109$ kN

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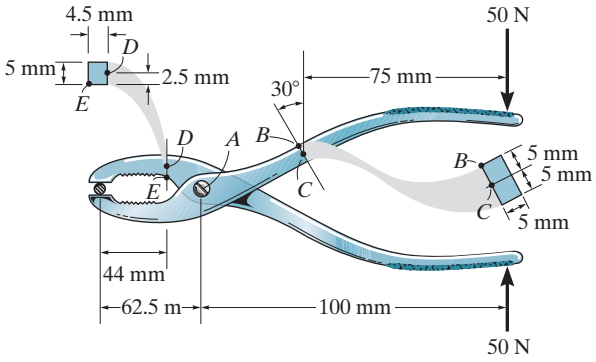
***8–28.** The pliers are made from two steel parts pinned together at A. If a smooth bolt is held in the jaws and a gripping force of 50 N is applied at the handles, determine the state of stress developed in the pliers at points B and C. Here the cross section is rectangular, having the dimensions shown in the figure.



SOLUTION

$^{+}\nearrow \Sigma F_x = 0; \quad N - 50\sin 30° = 0; \quad N = 25\ \text{N}$

$\nwarrow^{+} \Sigma F_y = 0; \quad V - 50\cos 30° = 0; \quad V = 43.30\ \text{N}$

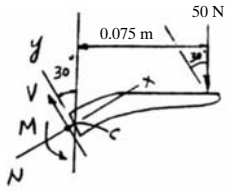
$\circlearrowleft + \Sigma M_C = 0; \quad M - 50(0.075) = 0; \quad M = 3.75\ \text{N}\cdot\text{m}$

$A = (0.005)(0.01) = 50(10^{-6})\ \text{m}^2$

$I = \frac{1}{12}(0.005)(0.01^3) = 0.41667(10^{-9})\ \text{m}^4$

$Q_B = 0$

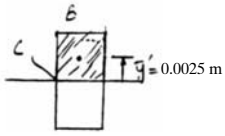
$Q_C = \bar{y}'A' = (0.0025)(0.005)(0.005) = 62.5(10^{-9})\ \text{m}^3$



Point B:

$$\sigma_B = \frac{N}{A} + \frac{My}{I} = \frac{-25}{50(10^{-6})} + \frac{3.75(0.005)}{0.41667(10^{-9})}$$

$$= 44.5(10^6)\ \text{N/m}^2 = 44.5\ \text{MPa (T)} \qquad \textbf{Ans.}$$



$$\tau_B = \frac{VQ}{It} = 0 \qquad \textbf{Ans.}$$

Point C:

$$\sigma_C = \frac{N}{A} + \frac{My}{I} = \frac{-25}{50(10^{-6})} + 0 = -0.500(10^6)\ \text{N/m}^2 = 0.500\ \text{MPa (C)} \qquad \textbf{Ans.}$$

Shear Stress :

$$\tau_C = \frac{VQ}{It} = \frac{43.30[62.5(10^{-9})]}{[0.41667(10^{-9}](0.005)} = 1.299(10^6)\ \text{N/m}^2 = 1.30\ \text{MPa} \qquad \textbf{Ans.}$$

Ans.
$\sigma_B = 44.5$ MPa, $\tau_B = 0$,
$\sigma_C = 0.50$ MPa, $\tau_C = 1.30$ MPa

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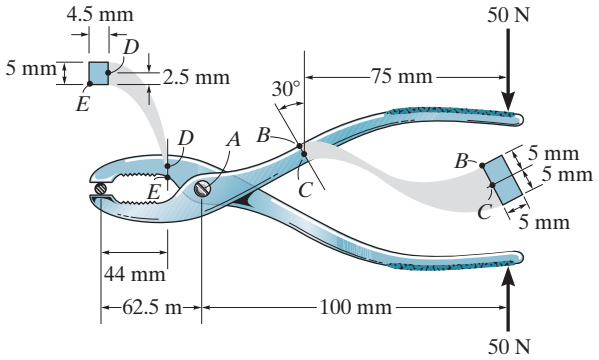
8–29. Solve Prob. 8–33 for points D and E.



SOLUTION

$\zeta + \Sigma M_A = 0; \quad -F(0.0625) + 50(0.1) = 0; \quad F = 80 \text{ N}$

Point D:

$$\sigma_D = 0 \qquad \textbf{Ans.}$$

$$\tau_D = \frac{VQ}{It} = \frac{80[(0.00125)(0.0045)(0.0025)]}{\left[\frac{1}{12}(0.0045)(0.005^3)\right](0.0045)}$$

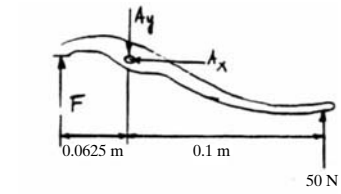
$$= 5.333(10^6) \text{ N/m}^2 = 5.33 \text{ MPa} \qquad \textbf{Ans.}$$

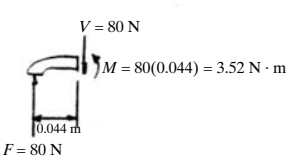
Point E:

$$\sigma_E = \frac{My}{I} = \frac{3.52(0.0025)}{\frac{1}{12}(0.0045)(0.005^3)}$$

$$= 187.73(10^6) \text{ N/m}^2 = 188 \text{ MPa} \qquad \textbf{Ans.}$$

$$\tau_E = 0 \qquad \textbf{Ans.}$$





Ans.
$\sigma_D = 0, \tau_D = 5.33 \text{ MPa}, \sigma_E = 188 \text{ MPa}, \tau_E = 0$

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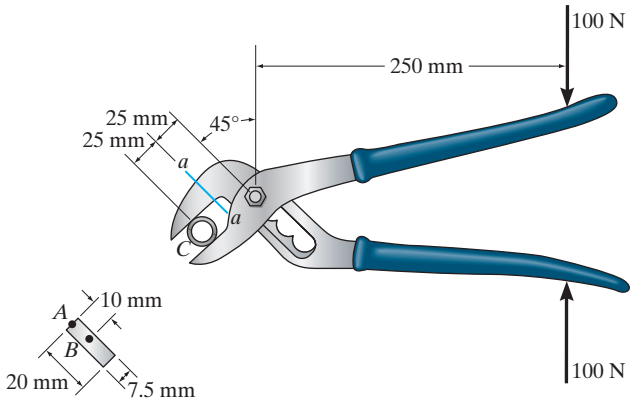
8–30.

The rib-joint pliers are used to grip the smooth pipe C. If the force of 100 N is applied to the handles, determine the state of stress at points A and B on the cross section of the jaw at section a–a. Indicate the results on an element at each point.



Section $a - a$

SOLUTION

Support Reactions: Referring to the free-body diagram of the handle shown in Fig. a,

$$\zeta+\Sigma M_D = 0; \qquad 100(0.25) - F_C(0.05) = 0 \qquad F_C = 500 \text{ N}$$

Internal Loadings: Consider the equilibrium of the free-body diagram of the segment shown in Fig. b.

$$\Sigma F_{y'} = 0; \qquad 500 - V = 0 \qquad V = 500 \text{ N}$$

$$\zeta+\Sigma M_C = 0; \qquad M - 500(0.025) = 0 \qquad M = 12.5 \text{ N} \cdot \text{m}$$

Section Properties: The moment of inertia of the cross section about the centroidal axis is

$$I = \frac{1}{12}(0.0075)(0.02^3) = 5(10^{-9}) \text{ m}^4$$

Referring to Fig. c, Q_A and Q_B are

$$Q_A = 0$$

$$Q_B = \bar{y}'A' = 0.005(0.01)(0.0075) = 0.375(10^{-6}) \text{ m}^3$$

Normal Stress: The normal stress is contributed by bending stress only. Thus

$$\sigma = \frac{My}{I}$$

For point A, $y = 0.01$ m. Then

$$\sigma_A = -\frac{12.5(0.01)}{5(10^{-9})} = -25 \text{ MPa} = 25 \text{ MPa (C)} \qquad \textbf{Ans.}$$

For point B, $y = 0$. Then

$$\sigma_B = 0 \qquad \textbf{Ans.}$$

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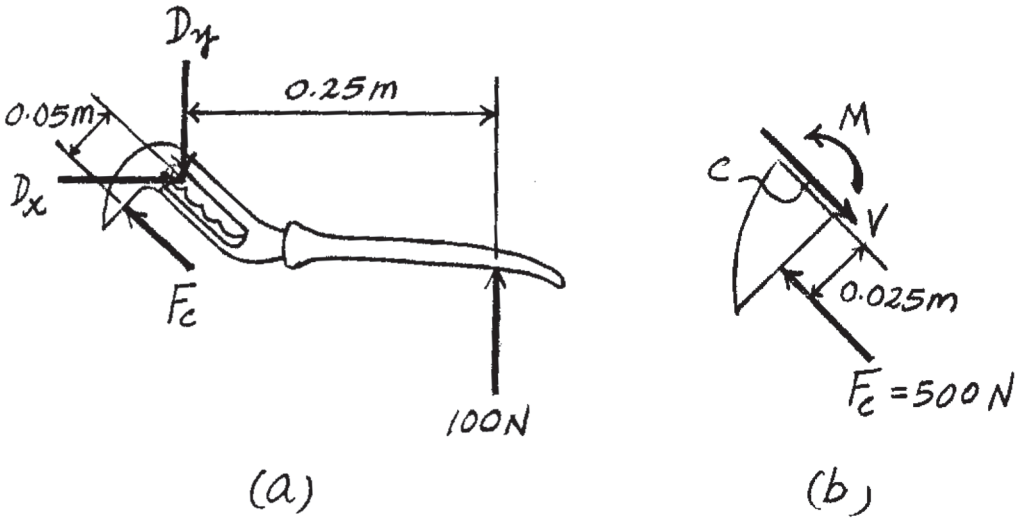
8–30. Continued

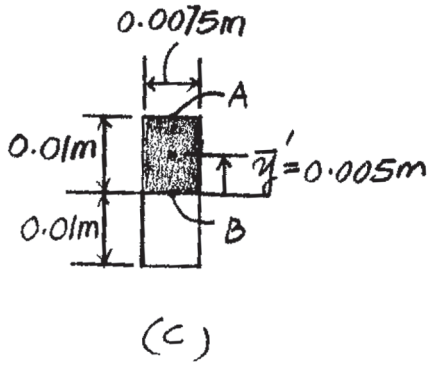
Shear Stress: The shear stress is contributed by transverse shear stress only. Thus,

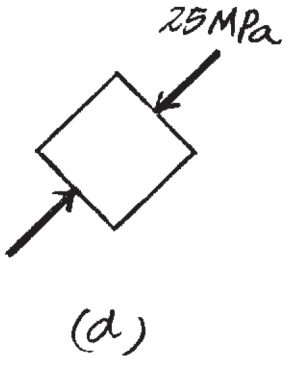
$$\tau_A = \frac{VQ_A}{It} = 0 \qquad \textbf{Ans.}$$

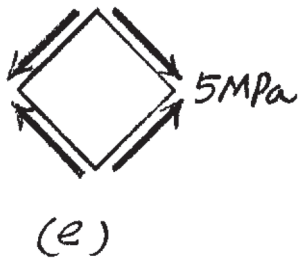
$$\tau_B = \frac{VQ_B}{It} = \frac{500[0.375(10^{-6})]}{5(10^{-9})(0.0075)} = 5\ \text{MPa} \qquad \textbf{Ans.}$$

The state of stress of points A and B are represented by the elements shown in Figs. *d* and *e* respectively.









Ans:
$\sigma_A = 25$ MPa (C), $\sigma_B = 0$,
$\tau_A = 0$, $\tau_B = 5$ MPa

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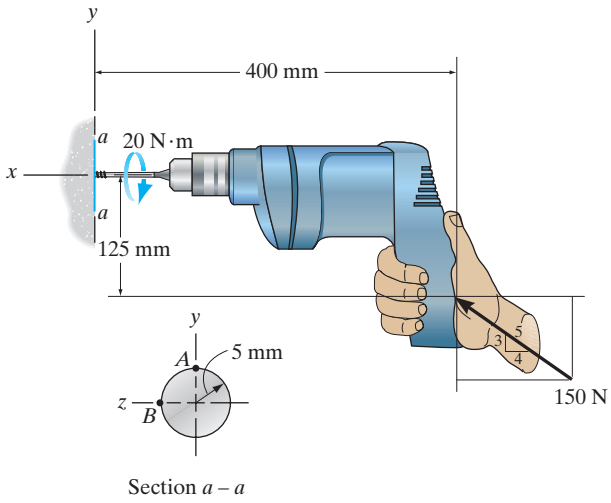
8–31.

The drill is jammed in the wall and is subjected to the torque and force shown. Determine the state of stress at point A on the cross section of the drill bit at section a–a.



SOLUTION

Internal Loadings: Consider the equilibrium of the free-body diagram of the drill's right cut segment, Fig. *a*.

$$\Sigma F_x = 0; \quad N - 150\left(\frac{4}{5}\right) = 0 \qquad N = 120\ \text{N}$$

$$\Sigma F_y = 0; \quad 150\left(\frac{3}{5}\right) - V_y = 0 \qquad V_y = 90\ \text{N}$$

$$\Sigma M_x = 0;\ 20 - T = 0 \qquad T = 20\ \text{N}\cdot\text{m}$$

$$\Sigma M_z = 0;\ -150\left(\frac{3}{5}\right)(0.4) + 150\left(\frac{4}{5}\right)(0.125) + M_z = 0$$

$$M_z = 21\ \text{N}\cdot\text{m}$$

Section Properties: The cross-sectional area, the moment of inertia about the z axis, and the polar moment of inertia of the drill's cross section are

$$A = \pi\left(0.005^2\right) = 25\pi\left(10^{-6}\right)\ \text{m}^2$$

$$I_z = \frac{\pi}{4}\left(0.005^4\right) = 0.15625\pi\left(10^{-9}\right)\ \text{m}^4$$

$$J = \frac{\pi}{2}\left(0.005^4\right) = 0.3125\pi\left(10^{-9}\right)\ \text{m}^4$$

Referring to Fig. *b*, Q_A is

$$Q_A = 0$$

Normal Stress: The normal stress is a combination of axial and bending stress. Thus,

$$\sigma = \frac{N}{A} - \frac{M_z y}{I_z}$$

For point A, $y = 0.005$ m. Then

$$\sigma_A = \frac{-120}{25\pi\left(10^{-6}\right)} - \frac{21(0.005)}{0.15625\pi\left(10^{-9}\right)} = -215.43\ \text{MPa} = 215\ \text{MPa (C)} \qquad \textbf{Ans.}$$

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8–31. Continued

Shear Stress: The transverse shear stress developed at point A is

$$\left[\left(\tau_{xy}\right)_V\right]_A = \frac{V_y Q_A}{I_z t} = 0$$

The torsional shear stress developed at point A is

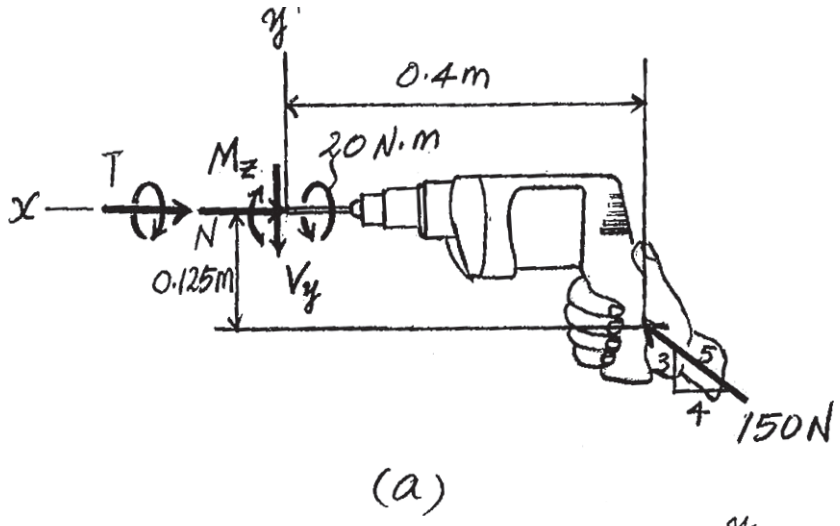
$$\left[\left(\tau_{xz}\right)_T\right]_A = \frac{Tc}{J} = \frac{20(0.005)}{0.3125\pi\left(10^{-9}\right)} = 101.86 \text{ MPa}$$

Thus,

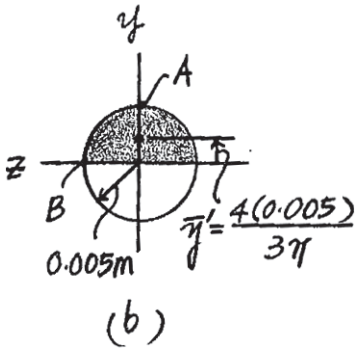
$$\left(\tau_{xy}\right)_A = 0 \qquad \textbf{Ans.}$$

$$\left(\tau_{xz}\right)_A = \left[\left(\tau_{xz}\right)_T\right]_A = 102 \text{ MPa} \qquad \textbf{Ans.}$$

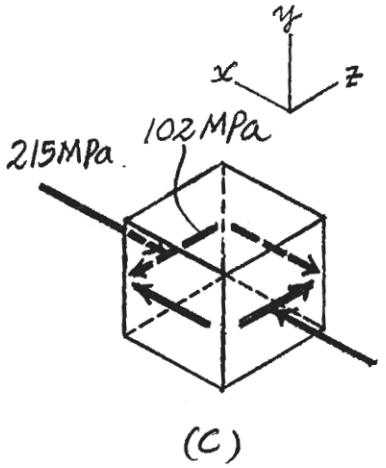
The state of stress at point A is represented on the element shown in Fig. *c*.



(a)



(b)



(C)

Ans:
$\sigma_A = 215$ MPa (C),
$\left(\tau_{xy}\right)_A = 0$,
$\left(\tau_{xz}\right)_A = 102$ MPa

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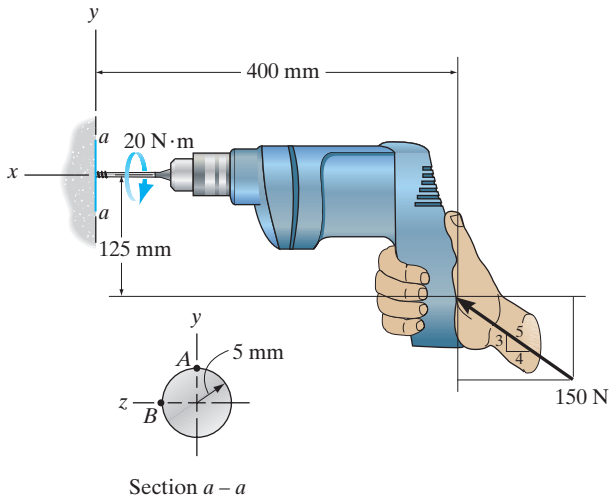
***8–32.**

The drill is jammed in the wall and is subjected to the torque and force shown. Determine the state of stress at point B on the cross section of the drill bit at section a–a.



SOLUTION

Internal Loadings: Consider the equilibrium of the free-body diagram of the drill's right cut segment, Fig. *a*.

$$\Sigma F_x = 0; \quad N - 150\left(\frac{4}{5}\right) = 0 \qquad N = 120 \text{ N}$$

$$\Sigma F_y = 0; \quad 150\left(\frac{3}{5}\right) - V_y = 0 \qquad V_y = 90 \text{ N}$$

$$\Sigma M_x = 0; \quad 20 - T = 0 \qquad T = 20 \text{ N} \cdot \text{m}$$

$$\Sigma M_z = 0; \; -150\left(\frac{3}{5}\right)(0.4) + 150\left(\frac{4}{5}\right)(0.125) + M_z = 0$$

$$M_z = 21 \text{ N} \cdot \text{m}$$

Section Properties: The cross-sectional area, the moment of inertia about the z axis, and the polar moment of inertia of the drill's cross section are

$$A = \pi\left(0.005^2\right) = 25\pi\left(10^{-6}\right) \text{m}^2$$

$$I_z = \frac{\pi}{2}\left(0.005^4\right) = 0.15625\pi\left(10^{-9}\right) \text{m}^4$$

$$J = \frac{\pi}{2}\left(0.005^4\right) = 0.3125\pi\left(10^{-9}\right) \text{m}^4$$

Referring to Fig. *b*, Q_B is

$$Q_B = \bar{y}'A' = \frac{4(0.005)}{3\pi}\left[\frac{\pi}{2}\left(0.005^2\right)\right] = 83.333\left(10^{-9}\right) \text{m}^3$$

Normal Stress: The normal stress is a combination of axial and bending stress. Thus,

$$\sigma = \frac{N}{A} - \frac{M_z y}{I_z}$$

For point B, $y = 0$. Then

$$\sigma_B = \frac{-120}{25\pi\left(10^{-6}\right)} - 0 = -1.528 \text{ MPa} = 1.53 \text{ MPa (C)} \qquad \textbf{Ans.}$$

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***8–32. Continued**

Shear Stress: The transverse shear stress developed at point B is

$$\left[\left(\tau_{xy}\right)_V\right]_B = \frac{V_y Q_B}{I_z t} = \frac{90\left[83.333\left(10^{-9}\right)\right]}{0.15625\pi\left(10^{-9}\right)(0.01)} = 1.528 \text{ MPa}$$

The torsional shear stress developed at point B is

$$\left[\left(\tau_{xy}\right)_T\right]_B = \frac{Tc}{J} = \frac{20(0.005)}{0.3125\pi\left(10^{-9}\right)} = 101.86 \text{ MPa}$$

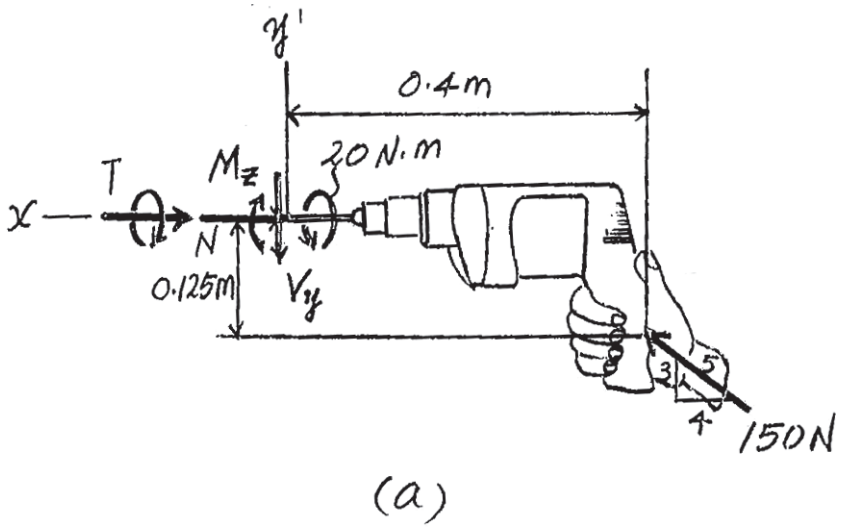
Thus,

$$\left(\tau_{xz}\right)_B = 0 \qquad \textbf{Ans.}$$

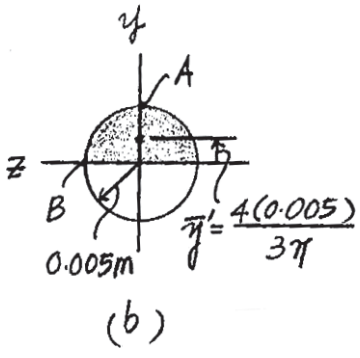
$$\left(\tau_{xy}\right)_B = \left[\left(\tau_{xy}\right)_T\right]_B - \left[\left(\tau_{xy}\right)_V\right]_B$$

$$= 101.86 - 1.528 = 100.33 \text{ MPa} = 100 \text{ MPa} \qquad \textbf{Ans.}$$

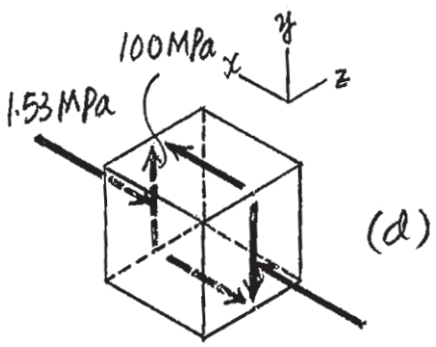
The state of stress at point B is represented on the element shown in Fig. *d*.



(a)



(b)



(d)

Ans:
$\sigma_B = 1.53$ MPa (C), $\left(\tau_{xz}\right)_B = 100$ MPa

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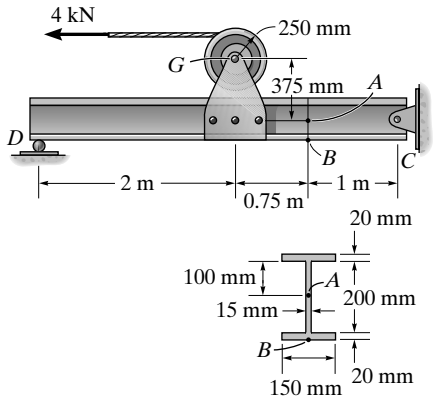
8–33. Determine the state of stress at point A when the beam is subjected to the cable force of 4 kN. Indicate the result as a differential volume element.



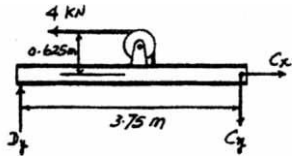
SOLUTION

Support Reactions:

$$\zeta + \Sigma M_D = 0; \quad 4(0.625) - C_y(3.75) = 0$$

$$C_y = 0.6667 \text{ kN}$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad C_x - 4 = 0 \quad C_x = 4.00 \text{ kN}$$

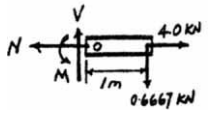


Internal Forces and Moment:

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad 4.00 - N = 0 \quad N = 4.00 \text{ kN}$$

$$+\uparrow \Sigma F_y = 0; \quad V - 0.6667 = 0 \quad V = 0.6667 \text{ kN}$$

$$\zeta + \Sigma M_o = 0; \quad M - 0.6667(1) = 0 \quad M = 0.6667 \text{ kN} \cdot \text{m}$$



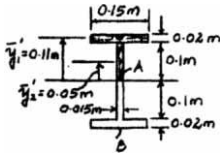
Section Properties:

$$A = 0.24(0.15) - 0.2(0.135) = 9.00\left(10^{-3}\right) \text{m}^2$$

$$I = \frac{1}{12}(0.15)\left(0.24^3\right) - \frac{1}{12}(0.135)\left(0.2^3\right) = 82.8\left(10^{-6}\right) \text{m}^4$$

$$Q_A = \Sigma \bar{y}' A' = 0.11(0.15)(0.02) + 0.05(0.1)(0.015)$$

$$= 0.405\left(10^{-3}\right) \text{m}^3$$

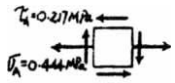


Normal Stress:

$$\sigma = \frac{N}{A} \pm \frac{My}{I}$$

$$\sigma_A = \frac{4.00(10^3)}{9.00(10^{-3})} + \frac{0.6667(10^3)(0)}{82.8(10^{-6})}$$

$$= 0.444 \text{ MPa (T)} \qquad \textbf{Ans.}$$



Shear Stress: Applying shear formula.

$$\tau_A = \frac{VQ_A}{It}$$

$$= \frac{0.6667(10^3)\left[0.405(10^{-3})\right]}{82.8(10^{-6})(0.015)} = 0.217 \text{ MPa} \qquad \textbf{Ans.}$$

Ans.

$\sigma_A = 0.444$ MPa (T), $\tau_A = 0.217$ MPa

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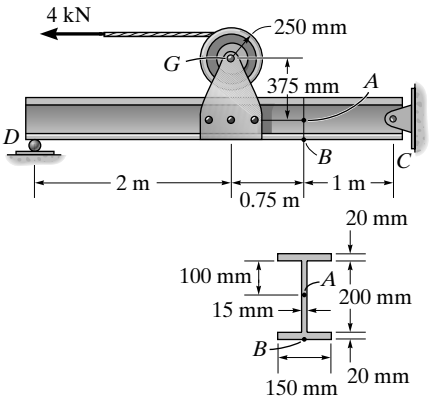
8–34. Determine the state of stress at point B when the beam is subjected to the cable force of 4 kN. Indicate the result as a differential volume element.



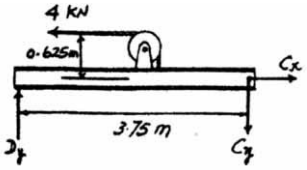
SOLUTION

Support Reactions:

$$\zeta + \Sigma M_D = 0; \qquad 4(0.625) - C_y(3.75) = 0$$

$$C_y = 0.6667 \text{ kN}$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad C_x - 4 = 0 \qquad C_x = 4.00 \text{ kN}$$



Internal Forces and Moment:

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 4.00 - N = 0 \qquad N = 4.00 \text{ kN}$$

$$+\uparrow \Sigma F_y = 0; \qquad V - 0.6667 = 0 \qquad V = 0.6667 \text{ kN}$$

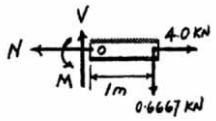
$$\zeta + \Sigma M_o = 0; \qquad M - 0.6667(1) = 0 \qquad M = 0.6667 \text{ kN} \cdot \text{m}$$

Section Properties:

$$A = 0.24(0.15) - 0.2(0.135) = 9.00\left(10^{-3}\right) \text{m}^2$$

$$I = \frac{1}{12}(0.15)\left(0.24^3\right) - \frac{1}{12}(0.135)\left(0.2^3\right) = 82.8\left(10^{-6}\right) \text{m}$$

$$Q_B = 0$$

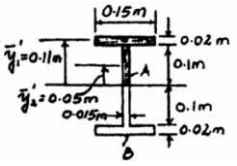


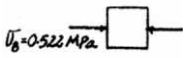
Normal Stress:

$$\sigma = \frac{N}{A} \pm \frac{My}{I}$$

$$\sigma_B = \frac{4.00(10^3)}{9.00(10^{-3})} - \frac{0.6667(10^3)(0.12)}{82.8(10^{-6})}$$

$$= -0.522 \text{ MPa} = 0.522 \text{ MPa (C)} \qquad \textbf{Ans.}$$





Shear Stress: Since $Q_B = 0$, then

$$\tau_B = 0 \qquad \textbf{Ans.}$$

Ans.
$A = 9.00\,(10^{-3})\text{ m}^2$, $I = 82.8\,(10^{-6})\text{ m}$,
$Q_B = 0$, $\sigma_B = 0.522$ MPa (C), $\tau_B = 0$

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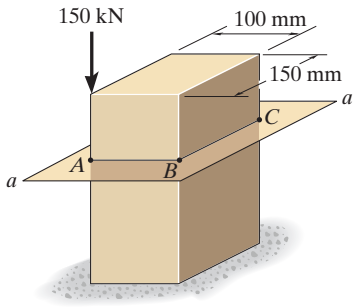
8–35.

The block is subjected to the eccentric load shown. Determine the normal stress developed at points A and B. Neglect the weight of the block.



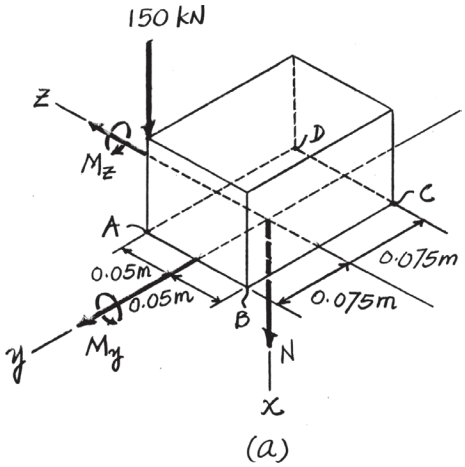
SOLUTION

Internal Loadings: Consider the equilibrium of the upper segment of the block sectioned through a–a, Fig. a.

$$\Sigma F_x = 0; \quad N + 150 = 0 \quad N = -150 \text{ kN}$$

$$\Sigma M_y = 0; \quad M_y + 150(0.05) = 0 \quad M_y = -7.50 \text{ kN} \cdot \text{m}$$

$$\Sigma M_z = 0; \quad M_z - 150(0.075) = 0 \quad M_z = 11.25 \text{ kN} \cdot \text{m}$$



Section Properties: For the rectangular cross section,

$$A = 0.1(0.15) = 0.015 \text{ m}^2$$

$$I_y = \frac{1}{12}(0.15)(0.1^3) = 12.5(10^{-6}) \text{ m}^4$$

$$I_z = \frac{1}{12}(0.1)(0.15^3) = 28.125(10^{-6}) \text{ m}^4$$

Normal Stresses: For the combined loadings, the normal stress can be determined from

$$\sigma = \frac{N}{A} - \frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

For point A, $y_A = 0.075$ m and $z_A = 0.05$ m

$$\sigma_A = \frac{-150(10^3)}{0.015} - \frac{11.25(10^3)(0.075)}{28.125(10^{-6})} + \frac{[-7.50(10^3)](0.05)}{12.5(10^{-6})}$$

$$= -70.0(10^6) \text{ Pa} = 70.0 \text{ MPa (C)} \quad \textbf{Ans.}$$

For point B, $y_B = 0.075$ m and $z_B = -0.05$ m

$$\sigma_B = \frac{-150(10^3)}{0.015} - \frac{11.25(10^3)(0.075)}{28.125(10^{-6})} + \frac{[-7.50(10^3)](-0.05)}{12.5(10^{-6})}$$

$$= -10.0(10^6) \text{ Pa} = 10.0 \text{ MPa (C)} \quad \textbf{Ans.}$$

Ans:
$\sigma_A = 70.0$ MPa (C),
$\sigma_B = 10.0$ MPa (C)

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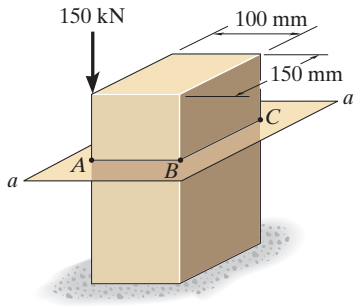
***8–36.**

The block is subjected to the eccentric load shown. Sketch the normal-stress distribution acting over the cross section at section *a–a*. Neglect the weight of the block.



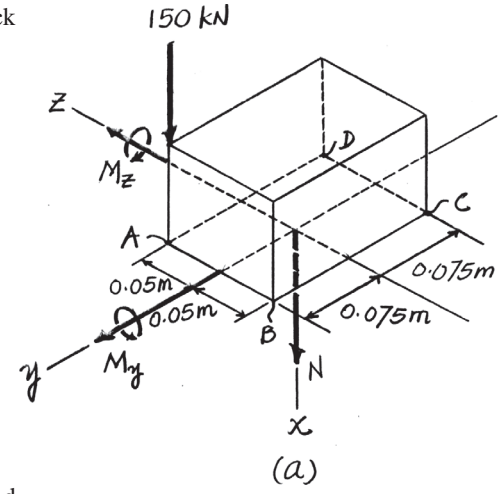
SOLUTION

Internal Loadings: Consider the equilibrium of the upper segment of the block sectioned through *a–a*, Fig. *a*.

$$\Sigma F_x = 0; \quad N + 150 = 0 \quad N = -150\ \text{kN}$$

$$\Sigma M_y = 0; \quad M_y + 150(0.05) = 0 \quad M_y = -7.50\ \text{kN}\cdot\text{m}$$

$$\Sigma M_z = 0; \quad M_z - 150(0.075) = 0 \quad M_z = 11.25\ \text{kN}\cdot\text{m}$$



Section Properties: For the rectangular cross section,

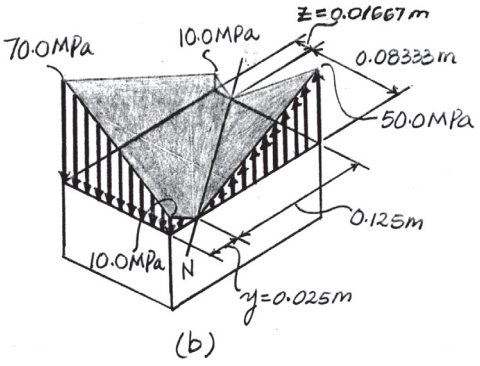
$$A = 0.1(0.15) = 0.015\ \text{m}^2$$

$$I_y = \frac{1}{12}(0.15)(0.1^3) = 12.5(10^{-6})\ \text{m}^4$$

$$I_z = \frac{1}{12}(0.1)(0.15^3) = 28.125(10^{-6})\ \text{m}^4$$

Normal Stress: For the combined loadings, the normal stress can be determined from

$$\sigma = \frac{N}{A} - \frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$



For point A, $y_A = 0.075$ m and $z_A = 0.05$ m

$$\sigma_A = \frac{-150(10^3)}{0.015} - \frac{11.25(10^3)(0.075)}{28.125(10^{-6})} + \frac{(-7.50)(10^3)(0.05)}{12.5(10^{-6})}$$

$$= -70.0(10^6)\ \text{Pa} = 70.0\ \text{MPa (C)} \qquad \textbf{Ans.}$$

For point B, $y_B = 0.075$ m and $z_B = -0.05$ m

$$\sigma_B = \frac{-150(10^3)}{0.015} - \frac{11.25(10^3)(0.075)}{28.125(10^{-6})} + \frac{(-7.50)(10^3)(-0.05)}{12.5(10^{-6})}$$

$$= -10.0(10^6)\ \text{Pa} = 10.0\ \text{MPa (C)} \qquad \textbf{Ans.}$$

For point C, $y_C = -0.075$ m and $z_C = -0.05$ m

$$\sigma_C = \frac{-150(10^3)}{0.015} - \frac{11.25(10^3)(-0.075)}{28.125(10^{-6})} + \frac{(-7.50)(10^3)(-0.05)}{12.5(10^{-6})}$$

$$= 50.0(10^6)\ \text{Pa} = 50.0\ \text{MPa (T)} \qquad \textbf{Ans.}$$

For point D, $y_D = -0.075$ m and $z_D = 0.05$ m

$$\sigma_D = \frac{-150(10^3)}{0.015} - \frac{11.25(10^3)(-0.075)}{28.125(10^{-6})} + \frac{(-7.50)(10^3)(0.05)}{12.5(10^{-6})}$$

$$= -10.0(10^6)\ \text{Pa} = 10.0\ \text{MPa (C)} \qquad \textbf{Ans.}$$

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***8–36. Continued**

The location of neutral axis can be found using similar triangles.

$$\frac{y}{10.0} = \frac{0.15 - y}{50.0}; \qquad y = 0.025 \text{ m}$$

$$\frac{z}{10.0} = \frac{0.1 - z}{50.0}; \qquad z = 0.01667 \text{ m}$$

Using these result, the normal stress distribution over the cross section shown in Fig. *b* can be sketched.

Ans:
$\sigma_A = 70.0$ MPa (C),
$\sigma_B = 10.0$ MPa (C),
$\sigma_C = 50.0$ MPa (T),
$\sigma_D = 10.0$ MPa (C)

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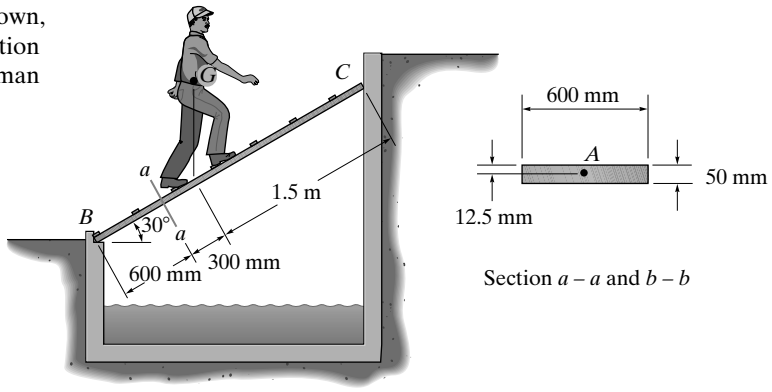
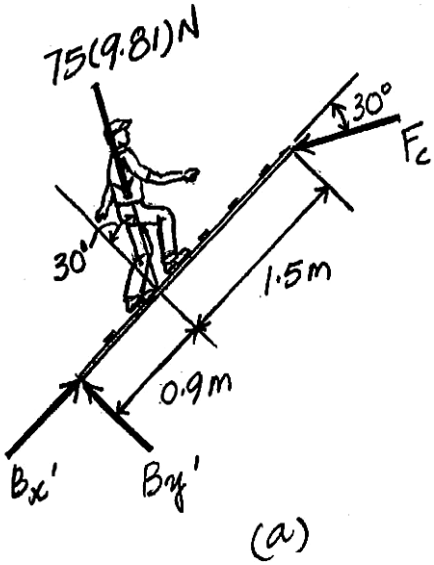
8–37. If the 75-kg man stands in the position shown, determine the state of stress at point A on the cross section of the plank at section a–a. The center of gravity of the man is at G. Assume that the contact point at C is smooth.



SOLUTION

Support Reactions: Referring to the free-body diagram of the entire plank, Fig. a,



$$\zeta + \Sigma M_B = 0; \qquad F_C \sin 30°(2.4) - 75(9.81)\cos 30°(0.9) = 0$$

$$F_C = 477.88 \text{ N}$$

$$\Sigma F_{x'} = 0; \quad B_{x'} - 75(9.81)\sin 30° - 477.88 \cos 30° = 0$$

$$B_{x'} = 781.73 \text{ N}$$

$$\Sigma F_{y'} = 0; \quad B_{y'} + 477.88 \sin 30° - 75(9.81)\cos 30° = 0$$

$$B_{y'} = 398.24 \text{ N}$$

Internal Loadings: Consider the equilibrium of the free-body diagram of the plank's lower segment, Fig. b,

$$\Sigma F_{x'} = 0; \quad 781.73 - N = 0 \qquad N = 781.73 \text{ N}$$

$$\Sigma F_{y'} = 0; \quad 398.24 - V = 0 \qquad V = 398.24 \text{ N}$$

$$\zeta + \Sigma M_O = 0; \qquad M - 398.24(0.6) = 0 \qquad M = 238.94 \text{ N} \cdot \text{m}$$

Section Properties: The cross-sectional area and the moment of inertia about the centroidal axis of the plank's cross section are

$$A = 0.6(0.05) = 0.03 \text{ m}^2$$

$$I = \frac{1}{12}(0.6)\left(0.05^3\right) = 6.25\left(10^{-6}\right)\text{m}^4$$

Referring to Fig. c, Q_A is

$$Q_A = \bar{y}'A' = 0.01875(0.0125)(0.6) = 0.140625\left(10^{-3}\right) \text{ m}^3$$

Normal Stress: The normal stress is the combination of axial and bending stress. Thus,

$$\sigma = \frac{N}{A} \pm \frac{My}{I}$$

For point A, $y = 0.0125$ m. Then

$$\sigma_A = \frac{-781.73}{0.03} - \frac{238.94(0.0125)}{6.25\left(10^{-6}\right)}$$

$$= -503.94 \text{ kPa} = 504 \text{ kPa (C)} \qquad \textbf{Ans.}$$

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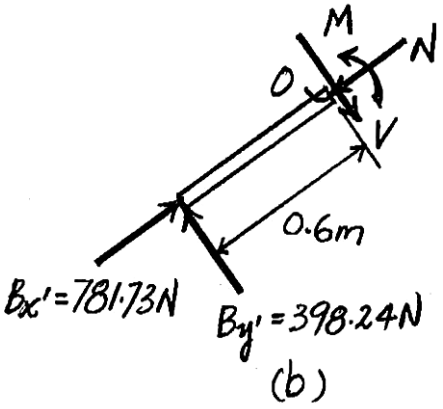
8–37. Continued

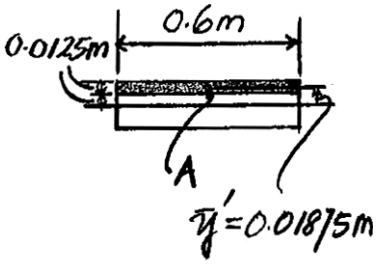
Shear Stress: The shear stress is contributed by transverse shear stress. Thus,

$$\tau_A = \frac{VQ_A}{It} = \frac{398.24\left[0.140625\left(10^{-3}\right)\right]}{6.25\left(10^{-6}\right)(0.6)} = 14.9 \text{ kPa}$$ **Ans.**

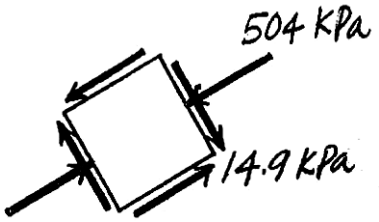
The state of stress at point A is represented on the element shown in Fig. *d*.



(b)



(c)



(d)

Ans.
$\sigma_A = 504$ kPa (C), $\tau_A = 14.9$ kPa

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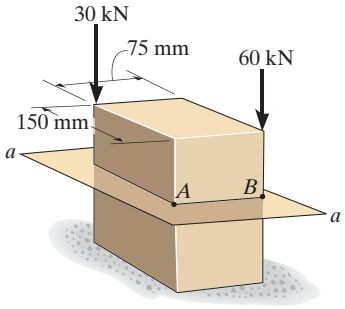
8–38. Determine the normal stress developed at points A and B. Neglect the weight of the block.



Referring to Fig. *a*,

$\Sigma F_x = (F_R)_x;$ $\quad -30 - 60 = F \quad F = -90$ kN

$\Sigma M_y = (M_R)_y;$ $\quad 30(0.0375) - 60(0.0375) = My \quad My = -1.125$ kN · m

$\Sigma M_z = (M_R)_z;$ $\quad 60(0.075) - 30(0.075) = Mz \quad Mz = 2.25$ kN · m

The cross-sectional area and moment of inertia about the y and z axes of the cross-section are

$$A = 0.075(0.15) = 0.01125 \text{ m}^2$$

$$I_y = \frac{1}{12}(0.15)(0.075^3) = 5.27344(10^{-6}) \text{ m}^4$$

$$I_z = \frac{1}{12}(0.75)(0.15^3) = 21.09375(10^{-6}) \text{ m}^4$$

The normal stress developed is the combination of axial and bending stress. Thus,

$$\sigma = \frac{F}{A} - \frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

For point A, $y = 0.075$ m and $z = -0.0375$ m

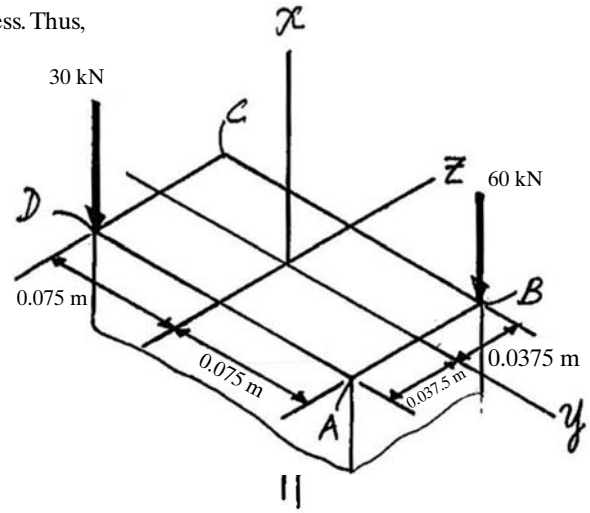
$$\sigma_A = \frac{-90(10^3)}{0.01125} - \frac{[2.25(10^3)](0.075)}{21.09375(10^{-6})} + \frac{[-1.125(10^3)](-0.0375)}{5.27344(10^{-6})}$$

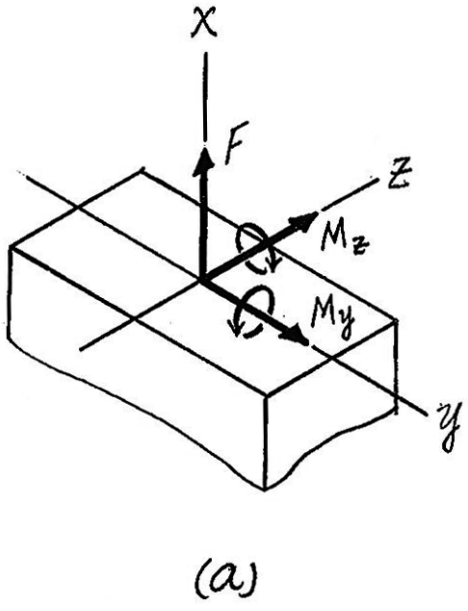
$$= -8.00(10^6) \text{ N/m}^2 = 8.00 \text{ MPa (C)}$$ **Ans.**

For point B, $y = 0.075$ m and $z = 0.0375$ m

$$\sigma_B = \frac{-90(10^3)}{0.01125} - \frac{[2.25(10^3)](0.075)}{21.09375(10^{-6})} + \frac{[-1.125(10^3)](0.0375)}{5.27344(10^{-6})}$$

$$= -24.00(10^6) \text{ N/m}^2 = 24.00 \text{ MPa (C)}$$ **Ans.**





(a)

Ans.
$\sigma_A = 8.00$ MPa (C), $\sigma_B = 240$ MPa (C)

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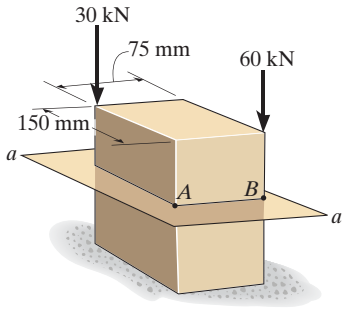
8–39. Sketch the normal stress distribution acting over the cross section at section *a–a*. Neglect the weight of the block.



SOLUTION

Referring to Fig. *a*,

$\Sigma F_x = (F_R)_x;\quad -30 - 60 = F \qquad F = -90 \text{ kN}$

$\Sigma M_y = (M_R)_y;\quad 30(0.0375) - 60(0.0375) = My \qquad My = -1.125 \text{ kN} \cdot \text{m}$

$\Sigma M_z = (M_R)_z;\quad 60(0.075) - 30(0.075) = Mz \qquad Mz = 2.25 \text{ kN} \cdot \text{m}$

The cross-sectional area and the moment of inertia about the y and z axes of the cross-section are

$$A = 0.075(0.15) = 0.01125 \text{ m}^2$$

$$I_y = \frac{1}{12}(0.15)(0.075^3) = 5.27344(10^{-6}) \text{ m}^4$$

$$I_z = \frac{1}{12}(0.75)(0.15^3) = 21.09375(10^{-6}) \text{ m}^4$$

The normal stress developed is the combination of axial and bending stress. Thus,

$$\sigma = \frac{F}{A} - \frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

For point A, $y = 0.075$ m and $z = -0.0375$ m

$$\sigma_A = \frac{-90(10^3)}{0.01125} - \frac{[2.25(10^3)](0.075)}{21.09375(10^{-6})} + \frac{[-1.125(10^3)](-0.0375)}{5.27344(10^{-6})}$$

$$= -8.00(10^6) \text{ N/m}^2 = 8.00 \text{ MPa (C)}$$

For point B, $y = 0.075$ m and $z = 0.0375$ m

$$\sigma_B = \frac{-90(10^3)}{0.01125} - \frac{[2.25(10^3)](0.075)}{21.09375(10^{-6})} + \frac{[-1.125(10^3)](0.0375)}{5.27344(10^{-6})}$$

$$= -24.00(10^6) \text{ N/m}^2 = 24.00 \text{ MPa (C)}$$

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8–39. Continued

For point C, $y = -0.075$ m and $z = 0.0375$ m

$$\sigma_C = \frac{-90(10^3)}{0.01125} - \frac{[2.25(10^3)](-0.075)}{21.09375(10^{-6})} + \frac{[-1.125(10^3)](0.0375)}{5.27344(10^{-6})}$$

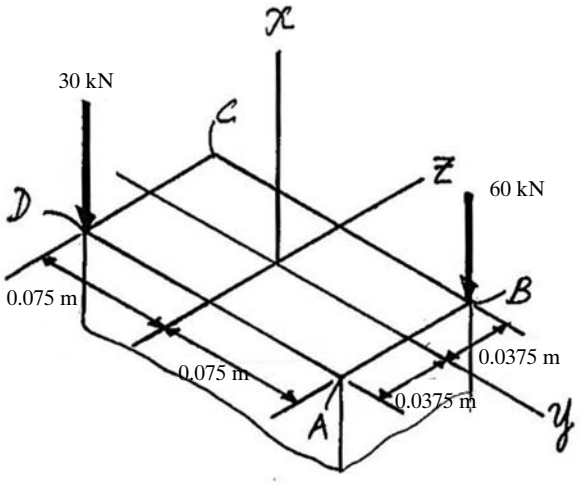
$$= -8.00(10^6)\ \text{N/m}^2 = 8.00\ \text{MPa (C)}$$

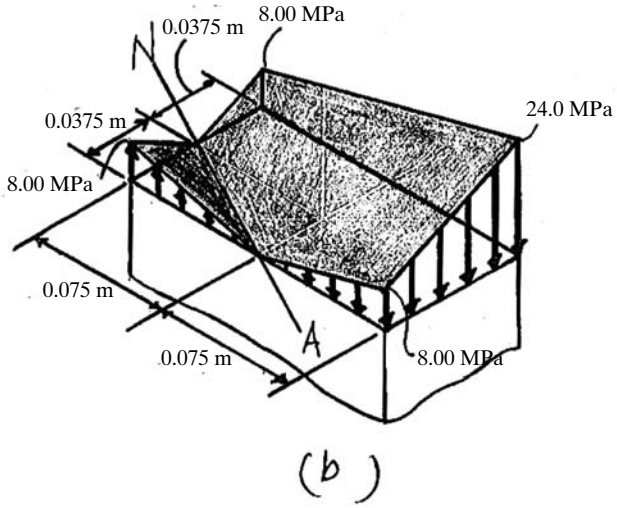
For point D, $y = -0.075$ m and $z = -0.0375$ m

$$\sigma_D = \frac{-90(10^3)}{0.01125} - \frac{[2.25(10^3)](-0.075)}{21.09375(10^{-6})} + \frac{[-1.125(10^3)](-0.0375)}{5.27344(10^{-6})}$$

$$= 8.00(10^6)\ \text{N/m}^2 = 8.00\ \text{MPa (T)}$$

The normal stress distribution over the cross-section is shown in Fig. *b*





Ans.
$A = 0.01125$ m, $I_y = 5.27344(10^{-6})$ m^4,
$I_z = 21.09375\ (10^{-6})$ m^4, $\sigma_C = 8.00$ MPa (C)

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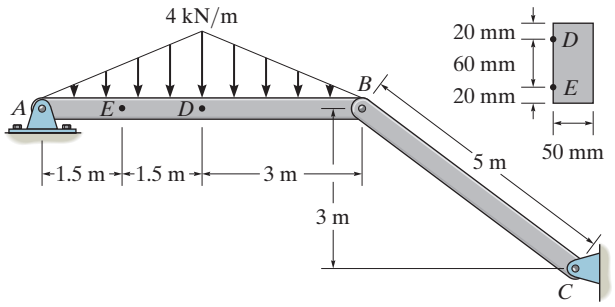
***8–40.**

The frame supports the distributed load shown. Determine the state of stress acting at point D. Show the results on a differential element at this point.

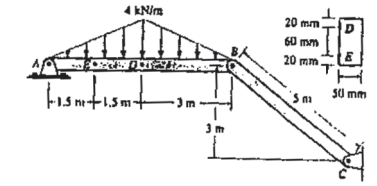


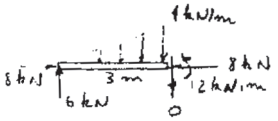
SOLUTION

$$\sigma_D = -\frac{P}{A} - \frac{My}{I} = -\frac{8(10^3)}{(0.1)(0.05)} - \frac{12(10^3)(0.03)}{\frac{1}{12}(0.05)(0.1)^3}$$

$\sigma_D = -88.0$ MPa **Ans.**

$\tau_D = 0$ **Ans.**





Ans:
$\sigma_D = -88.0$ MPa, $\tau_D = 0$

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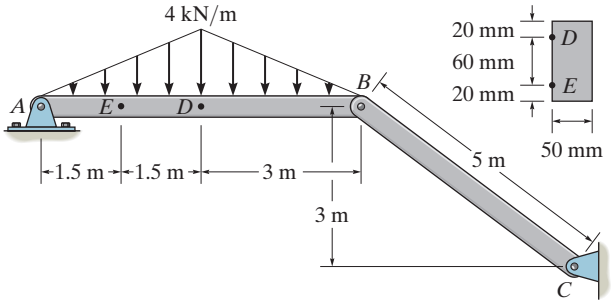
8–41.

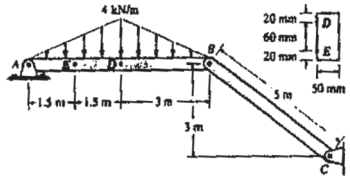
The frame supports the distributed load shown. Determine the state of stress acting at point E. Show the results on a differential element at this point.

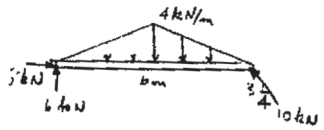


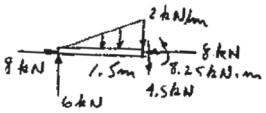
SOLUTION

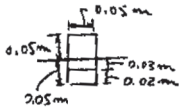
$$\sigma_E = -\frac{P}{A} - \frac{My}{I} = \frac{8(10^3)}{(0.1)(0.05)} + \frac{8.25(10^3)(0.03)}{\frac{1}{12}(0.05)(0.1)^3} = 57.8 \text{ MPa}$$ **Ans.**

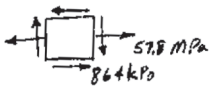
$$\tau_E = \frac{VQ}{It} = \frac{4.5(10^3)(0.04)(0.02)(0.05)}{\frac{1}{12}(0.05)(0.1)^3(0.05)} = 864 \text{ kPa}$$ **Ans.**











Ans:
$\sigma_E = 57.8$ MPa, $\tau_E = 864$ kPa

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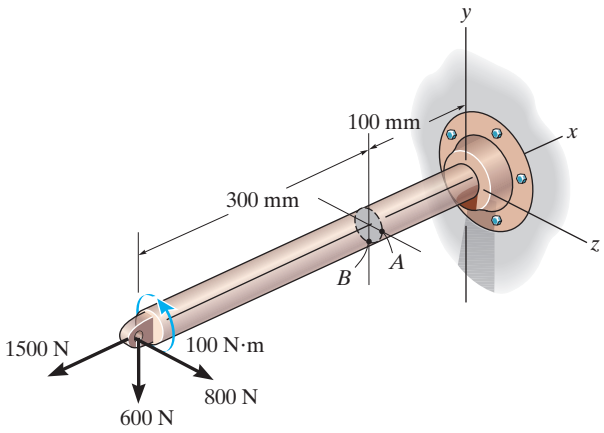
8–42.

The rod has a diameter of 40 mm. If it is subjected to the force system shown, determine the stress components that act at point A, and show the results on a volume element located at this point.



SOLUTION

Internal Loadings: Consider the equilibrium of the left segment of the rod being sectioned, Fig. *a*.

$$\Sigma F_x = 0; \quad N_x - 1500 = 0 \quad N_x = 1500\ \text{N}$$

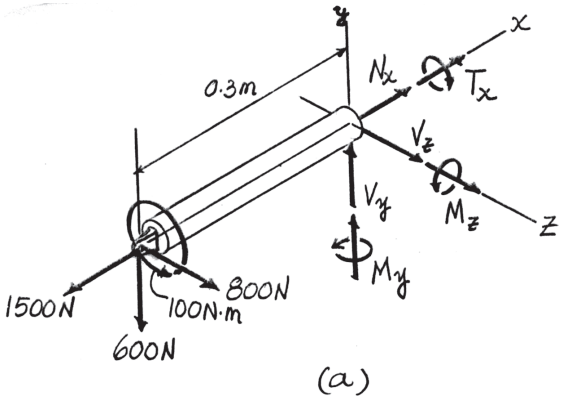
$$\Sigma F_y = 0; \quad V_y - 600 = 0 \quad V_y = 600\ \text{N}$$

$$\Sigma F_z = 0; \quad V_z + 800 = 0 \quad V_z = -800\ \text{N}$$

$$\Sigma M_x = 0; \quad T_x - 100 = 0 \quad T_x = 100\ \text{N}\cdot\text{m}$$

$$\Sigma M_y = 0; \quad M_y + 800(0.3) = 0 \quad M_y = -240\ \text{N}\cdot\text{m}$$

$$\Sigma M_z = 0; \quad M_z + 600(0.3) = 0 \quad M_z = -180\ \text{N}\cdot\text{m}$$



Section Properties: For the circular cross section, Fig. *b*,

$$A = \pi c^2 = \pi(0.02^2) = 0.400(10^{-3})\pi\ \text{m}^2$$

$$I_y = I_z = \frac{\pi}{4}c^4 = \frac{\pi}{4}(0.02^4) = 40.0(10^{-9})\pi\ \text{m}^4$$

$$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.02^4) = 80.0(10^{-9})\pi\ \text{m}^4$$

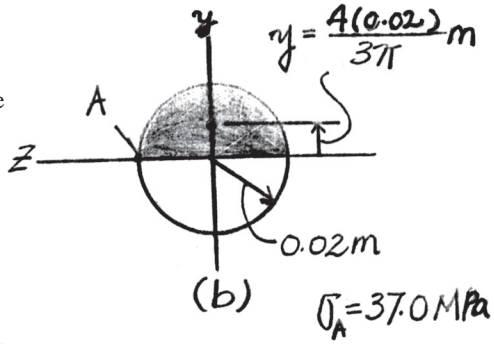
$$(Q_A)_z = 0 \qquad (Q_A)_y = \bar{y}'A' = \frac{4(0.02)}{3\pi}\left[\frac{\pi}{2}(0.02^2)\right] = 5.3333(10^{-6})\ \text{m}^3$$

Normal Stress: For the combined loadings, the normal stress at point A can be determined from

$$\sigma_x = \sigma_A = \frac{N_x}{A} - \frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y}$$

$$= \frac{1500}{0.400(10^{-3})\pi} - \frac{(-180)(0)}{40.0(10^{-9})\pi} + \frac{(-240)(0.02)}{40.0(10^{-9})\pi}$$

$$= -37.00(10^6)\ \text{Pa} = 37.0\ \text{MPa (C)} \qquad \textbf{Ans.}$$



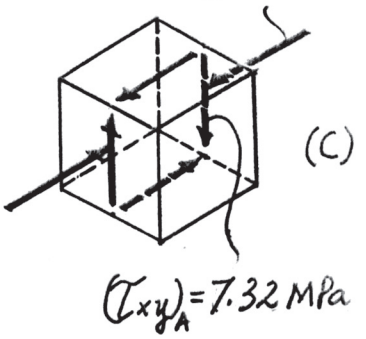
Shear Stress: The transverse shear stress in z and y directions and the torsional shear stress can be obtained using the shear formula $\tau_V = \frac{VQ}{It}$ and the torsion formula $\tau_T = \frac{T\rho}{J}$ respectively.

$$(\tau_{xy})_A = (\tau_V)_y - \tau_T$$

$$= \frac{600\left[5.3333(10^{-6})\right]}{40.0(10^{-9})\pi(0.04)} - \frac{100(0.02)}{80.0(10^{-9})\pi}$$

$$= -7.321(10^6)\ \text{Pa} = -7.32\ \text{MPa} \qquad \textbf{Ans.}$$

$$(\tau_{xz})_A = (\tau_V)_z = 0 \qquad \textbf{Ans.}$$



Using these results, the state of stress at point A can be represented by the differential volume element show in Fig. *c*.

Ans:
$\sigma_A = 37.0$ MPa (C),
$(\tau_{xy})_A = -7.32$ MPa,
$(\tau_{xz})_A = 0$

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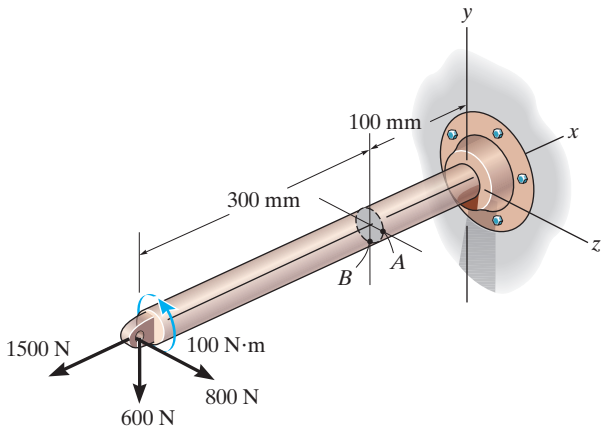
8–43.

Solve Prob. 8–40 for point B.



SOLUTION

Internal Loadings: Consider the equilibrium of the left segment of the rod being sectioned, Fig. *a*.

$$\Sigma F_x = 0; \quad N_x - 1500 = 0 \quad N_x = 1500\ \text{N}$$

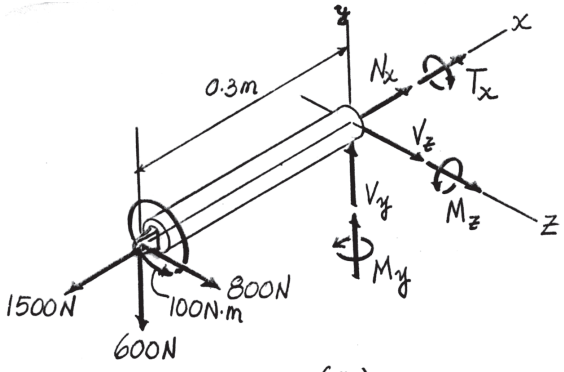
$$\Sigma F_y = 0; \quad V_y - 600 = 0 \quad V_y = 600\ \text{N}$$

$$\Sigma F_z = 0; \quad V_z + 800 = 0 \quad V_z = -800\ \text{N}$$

$$\Sigma M_x = 0; \quad T_x - 100 = 0 \quad T_x = 100\ \text{N}\cdot\text{m}$$

$$\Sigma M_y = 0; \quad M_y + 800(0.3) = 0 \quad M_y = -240\ \text{N}\cdot\text{m}$$

$$\Sigma M_z = 0; \quad M_z + 600(0.3) = 0 \quad M_z = -180\ \text{N}\cdot\text{m}$$



(a)

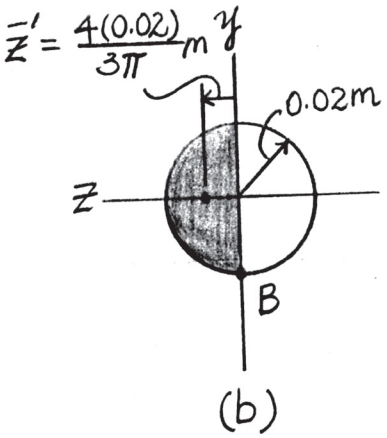
Section Properties: For the circular cross section,

$$A = \pi c^2 = \pi(0.02^2) = 0.400(10^{-3})\pi\ \text{m}^2$$

$$I_y = I_z = \frac{\pi}{4}c^4 = \frac{\pi}{4}(0.02^4) = 40.0(10^{-9})\pi\ \text{m}^4$$

$$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.02^4) = 80.0(10^{-9})\pi\ \text{m}^4$$

$$(Q_B)_z = \bar{z}'A' = \frac{4(0.02)}{3\pi}\left[\frac{\pi}{2}(0.02^2)\right] = 5.3333(10^{-6})\ \text{m}^3 \quad (Q_B)_y = 0$$



(b)

Normal Stress: For the combined loadings, the normal stress at point B can be determined from

$$\sigma_x = \sigma_B = \frac{N_x}{A} - \frac{M_z y_B}{I_z} + \frac{M_y z_B}{I_y}$$

$$= \frac{1500}{0.400(10^{-3})\pi} - \frac{(-180)(-0.02)}{40.0(10^{-9})\pi} + \frac{(-240)(0)}{40.0(10^{-9})\pi}$$

$$= -27.45(10^6) = 27.5\ \text{MPa (C)} \qquad \textbf{Ans.}$$

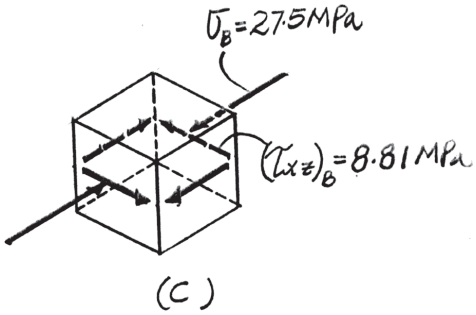
Shear Stress: The transverse shear stress in z and y directions and the torsional shear stress can be obtained using the shear formula $\tau_V = \frac{VQ}{It}$ and the torsion formula $\tau_T = \frac{T\rho}{J}$, respectively.

$$(\tau_{xz})_B = (\tau_V)_z - \tau_T$$

$$= \frac{-800\left[5.3333(10^{-6})\right]}{40.0(10^{-9})\pi(0.04)} - \frac{100(0.02)}{80.0(10^{-9})\pi}$$

$$= -8.807(10^6)\ \text{Pa} = -8.81\ \text{MPa} \qquad \textbf{Ans.}$$

$$(\tau_{xy})_B = (\tau_V)_y = 0 \qquad \textbf{Ans.}$$



(c)

Using these results, the state of stress at point B, can be represented by the differential volume element shown in Fig. *c*.

Ans:
$\sigma_B = 27.5$ MPa (C),
$(\tau_{xz})_B = -8.81$ MPa,
$(\tau_{xy})_B = 0$

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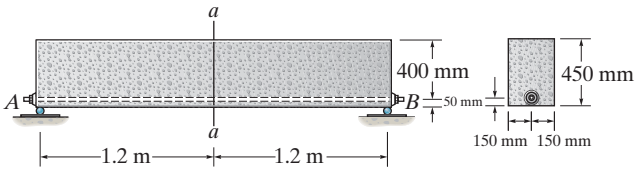
***8–44.** Since concrete can support little or no tension, this problem can be avoided by using wires or rods to *prestress* the concrete once it is formed. Consider the simply supported beam shown, which has a rectangular cross section of 450 mm by 300 mm. If concrete has a specific weight of 24 kN/m^3, determine the required tension in rod AB, which runs through the beam so that no tensile stress is developed in the concrete at its center section *a–a*. Neglect the size of the rod and any deflection of the beam.



SOLUTION

Support Reactions: As shown on FBD.

Internal Force and Moment:

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad T - N = 0 \qquad N = T$$

$$\circlearrowleft + \Sigma M_o = 0; \qquad M + T(0.175) - 3.888(10^3)(0.6) = 0$$

$$M = \{2.3328(10^3) - 0.175T\}\ \text{N}\cdot\text{m}$$

Section Properties:

$$A = 0.3(0.45) = 0.135\ \text{m}^2$$

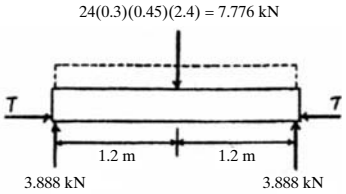
$$I = \frac{1}{12}0.3(0.45^3) = 2.278125(10^{-3})\ \text{m}^4$$

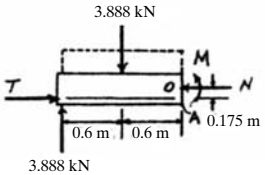
Normal Stress: Requires $\sigma_A = 0$

$$\sigma_A = 0 = \frac{N}{A} + \frac{Mc}{I}$$

$$0 = \frac{-T}{0.135} + \frac{[2.3328(10^3) - 0.175T](0.225)}{2.278125(10^{-3})}$$

$$T = 9.3312(10^3)\ \text{N} = 9.33\ \text{kN} \qquad \textbf{Ans.}$$





Ans.
$T = 9.33$ kN

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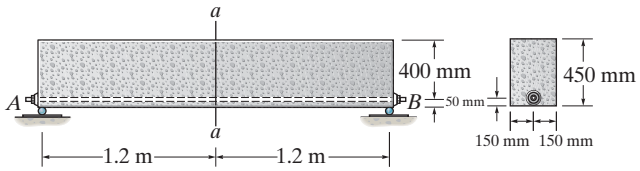
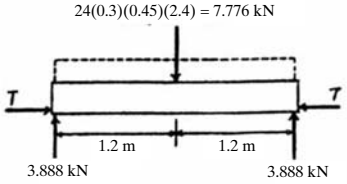
8–45. Solve Prob. 8–38 if the rod has a diameter of 12 mm. Use the transformed area method discussed in Sec 6.6. $E_{st} = 200$ GPa, $E_c = 25$ GPa.



SOLUTION

Support Reactions: As shown on FBD.

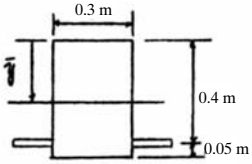
Section Properties:



$$n = \frac{E_{st}}{E_{\text{con}}} = \frac{200}{25} = 8.00$$

$$A_{\text{con}} = (n-a)A_{\text{st}} = (8.00-1)\left[\left(\frac{\pi}{4}\right)(0.012^2)\right] = 0.79168(10^{-3})\ \text{m}^2$$

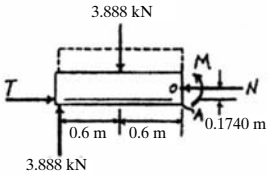
$$A = 0.3(0.45) + 0.79168(10^{-3}) = 0.135792\ \text{m}^2$$



$$\bar{y} = \frac{\Sigma \bar{y} A}{\Sigma A} = \frac{0.225(0.3)(0.45) + 0.4[0.79168(10^{-3})]}{0.135792} = 0.22602\ \text{m}$$

$$I = \frac{1}{12}0.3(0.45^3) + (0.3)(0.45)(0.22602 - 0.225)^2 + 0.79168(10^{-3})(0.4 - 0.22602)^2 = 2.30223(10^{-3})\ \text{m}^4$$

Internal Force and Moment:



$\overset{+}{\rightarrow} \Sigma F_x = 0;\quad T - N = 0 \quad N = T$

$\circlearrowleft + \Sigma M_o = 0;\quad M + T(0.1740) - [3.888(10^6)](0.6) = 0$

$$M = \{2.3328(10^3) - 0.1740T\}\ \text{N}\cdot\text{m}$$

Normal Stress: Requires $\sigma_A = 0$

$$\sigma_A = 0 = \frac{N}{A} + \frac{Mc}{I}$$

$$0 = \frac{-T}{0.135792} + \frac{[2.3328(10^3) - 0.1740T](0.45 - 0.22602)}{2.30223(10^{-3})}$$

$$T = 9.343(10^3)\ \text{N} = 9.34\ \text{kN}$$ **Ans.**

Ans.
$T = 9.34$ kN

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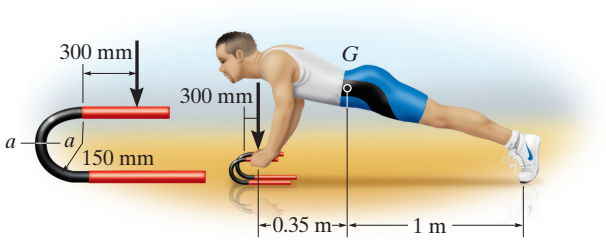
8–46.

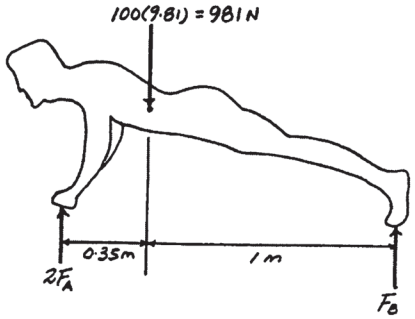
The man has a mass of 100 kg and center of mass at G. If he holds himself in the position shown, determine the maximum tensile and compressive stress developed in the curved bar at section a–a. He is supported uniformly by two bars, each having a diameter of 25 mm. Assume the floor is smooth. Use the curved-beam formula to calculate the bending stress.



SOLUTION

Equilibrium: For the man

$$\zeta+\Sigma M_B = 0; \quad 981(1) - 2F_A(1.35) = 0 \quad F_A = 363.33 \text{ N}$$



Section Properties:

$$\bar{r} = 0.15 + \frac{0.025}{2} = 0.1625 \text{ m}$$

$$\int_A \frac{dA}{r} = 2\pi(\bar{r} - \sqrt{\bar{r}^2 - c^2})$$

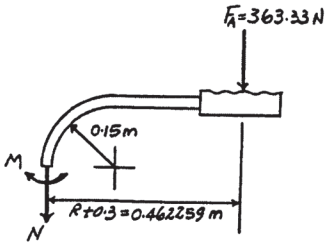
$$= 2\pi(0.1625 - \sqrt{0.1625^2 - 0.0125^2})$$

$$= 3.02524(10^{-3}) \text{ m}$$

$$A = \pi(0.0125^2) = 0.490874(10^{-3}) \text{ m}^2$$

$$R = \frac{A}{\int_A \frac{dA}{r}} = \frac{0.490874(10^{-3})}{3.02524(10^{-3})} = 0.162259 \text{ m}$$

$$\bar{r} - R = 0.1625 - 0.162259 = 0.240741(10^{-3}) \text{ m}$$



Internal Force and Moment: The internal moment must be computed about the neutral axis.

$$+\uparrow \Sigma F_y = 0; \quad -363.33 - N = 0 \quad N = -363.33 \text{ N}$$

$$\zeta+\Sigma M_o = 0; \quad -M - 363.33(0.462259) = 0 \quad M = -167.95 \text{ N}\cdot\text{m}$$

Normal Stress: Apply the curved-beam formula.
For tensile stress

$$(\sigma_t)_{\max} = \frac{N}{A} + \frac{M(R - r_2)}{Ar_2(\bar{r} - R)}$$

$$= \frac{-363.33}{0.490874(10^{-3})} + \frac{-167.95(0.162259 - 0.175)}{0.490874(10^{-3})(0.175)0.240741(10^{-3})}$$

$$= 103 \text{ MPa (T)} \qquad \textbf{Ans.}$$

For compressive stress,

$$(\sigma_c)_{\max} = \frac{N}{A} + \frac{M(R - r_1)}{Ar_1(\bar{r} - R)}$$

$$= \frac{-363.33}{0.490874(10^{-3})} + \frac{-167.95(0.162259 - 0.15)}{0.490874(10^{-3})(0.15)0.240741(10^{-3})}$$

$$= -117 \text{ MPa} = 117 \text{ MPa (C)} \qquad \textbf{Ans.}$$

Ans:
$(\sigma_t)_{\max} = 103$ MPa (T),
$(\sigma_c)_{\max} = 117$ MPa (C)

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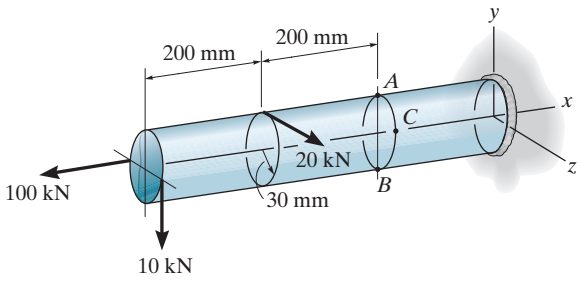
8–47.

The solid rod is subjected to the loading shown. Determine the state of stress at point A, and show the results on a differential volume element located at this point.



SOLUTION

Internal Loadings: Consider the equilibrium of the left segment of the rod being sectioned, Fig. *a*.

$$\Sigma F_x = 0; \quad N_x - 100 = 0 \quad N_x = 100\ \text{kN}$$

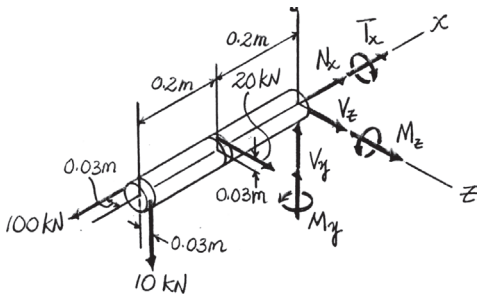
$$\Sigma F_y = 0; \quad V_y - 10 = 0 \quad V_y = 10\ \text{kN}$$

$$\Sigma F_z = 0; \quad V_z + 20 = 0 \quad V_z = -20\ \text{kN}$$

$$\Sigma M_x = 0; \quad T_x + 20(0.03) + 10(0.03) = 0 \quad T_x = -0.9\ \text{kN}\cdot\text{m}$$

$$\Sigma M_y = 0; \quad M_y + 20(0.2) + 100(0.03) = 0 \quad M_y = -7.00\ \text{kN}\cdot\text{m}$$

$$\Sigma M_z = 0; \quad M_z + 10(0.4) = 0 \quad M_z = -4.00\ \text{kN}\cdot\text{m}$$



Section Properties: For the circular cross section, Fig. *b*,

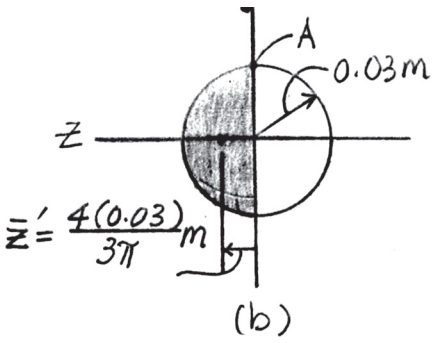
$$A = \pi c^2 = \pi(0.03^2) = 0.9(10^{-3})\pi\ \text{m}^2$$

$$I_y = I_z = \frac{\pi}{4}c^4 = \frac{\pi}{4}(0.03^4) = 0.2025(10^{-6})\pi\ \text{m}^4$$

$$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.03^4) = 0.405(10^{-6})\pi\ \text{m}^4$$

$$(Q_A)_z = \bar{z}'A' = \frac{4(0.03)}{3\pi}\left[\frac{\pi}{2}(0.03^2)\right] = 18.0(10^{-6})\ \text{m}^3$$

$$(Q_A)_y = 0$$

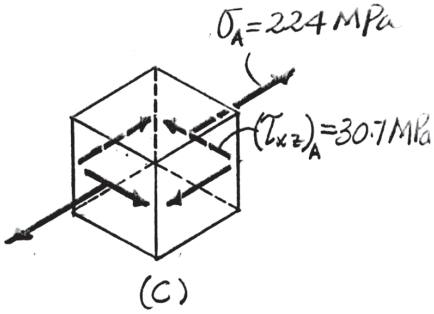


Normal Stress: For the combined loadings, the normal stress at point A can be determined from

$$\sigma_x = \sigma_A = \frac{N_x}{A} - \frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y}$$

$$= \frac{100(10^3)}{0.9(10^{-3})\pi} - \frac{[-4.00(10^3)](0.03)}{0.2025(10^{-6})\pi} + \frac{[-7.00(10^3)](0)}{0.2025(10^{-6})\pi}$$

$$= 224.00(10^6)\ \text{Pa} = 224\ \text{MPa (T)} \qquad \textbf{Ans.}$$



Shear Stress: The transverse shear stress in z and y directions and the torsional shear stress can be obtained using the shear formula $\tau_V = \frac{VQ}{It}$ and the torsion formula $\tau_T = \frac{T\rho}{J}$, respectively.

$$(\tau_{xz})_A = (\tau_V)_z + \tau_T$$

$$= \frac{-20(10^3)[18.0(10^{-6})]}{0.2025(10^{-6})\pi(0.06)} + \frac{-0.9(10^3)(0.03)}{0.405(10^{-6})\pi}$$

$$= -30.65(10^6)\ \text{Pa} = -30.7\ \text{MPa} \qquad \textbf{Ans.}$$

$$(\tau_{xy})_A = (\tau_V)_y = 0 \qquad \textbf{Ans.}$$

Using these results, the state of stress at point A can be represented by the differential volume element shown in Fig. *c*.

Ans:
$\sigma_A = 224$ MPa (T),
$(\tau_{xz})_A = -30.7$ MPa,
$(\tau_{xy})_A = 0$

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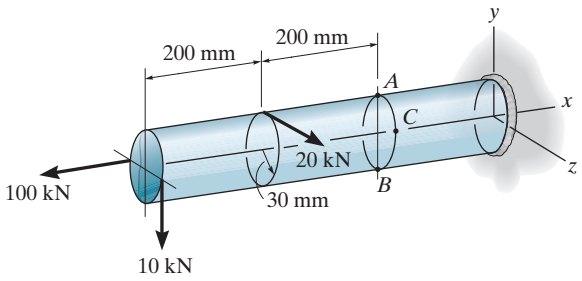
***8–48.**

The solid rod is subjected to the loading shown. Determine the state of stress at point B, and show the results on a differential volume element at this point.



SOLUTION

Internal Loadings: Consider the equilibrium of the left segment of the rod being sectioned, Fig. *a*.

$$\Sigma F_x = 0; \quad N_x - 100 = 0 \quad N_x = 100\ \text{kN}$$

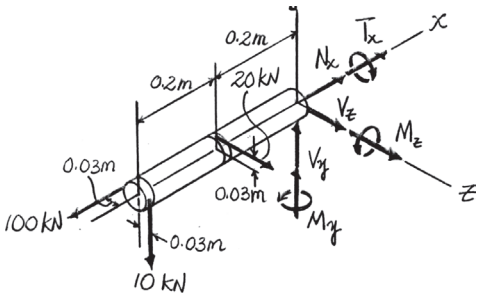
$$\Sigma F_y = 0; \quad V_y - 10 = 0 \quad V_y = 10\ \text{kN}$$

$$\Sigma F_z = 0; \quad V_z + 20 = 0 \quad V_z = -20\ \text{kN}$$

$$\Sigma M_x = 0; \quad T_x + 20(0.03) + 10(0.03) = 0 \quad T_x = -0.9\ \text{kN}\cdot\text{m}$$

$$\Sigma M_y = 0; \quad M_y + 20(0.2) + 100(0.03) = 0 \quad M_y = -7.00\ \text{kN}\cdot\text{m}$$

$$\Sigma M_z = 0; \quad M_z + 10(0.4) = 0 \quad M_z = -4.00\ \text{kN}\cdot\text{m}$$



(a)

Section Properties: For the circular cross section, Fig. *b*,

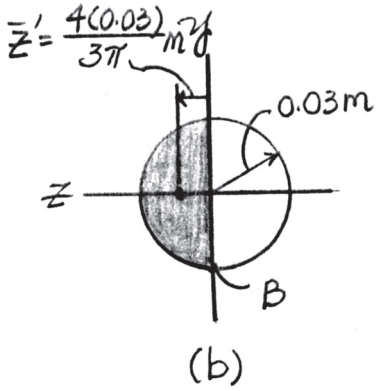
$$A = \pi c^2 = \pi(0.03^2) = 0.9(10^{-3})\pi\ \text{m}^2$$

$$I_y = I_z = \frac{\pi}{4}c^4 = \frac{\pi}{4}(0.03^4) = 0.2025(10^{-6})\pi\ \text{m}^4$$

$$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.03^4) = 0.405(10^{-6})\pi\ \text{m}^4$$

$$(Q_B)_z = \bar{z}'A' = \frac{4(0.03)}{3\pi}\left[\frac{\pi}{2}(0.03^2)\right] = 18.0(10^{-6})\ \text{m}^3$$

$$(Q_B)_y = 0$$



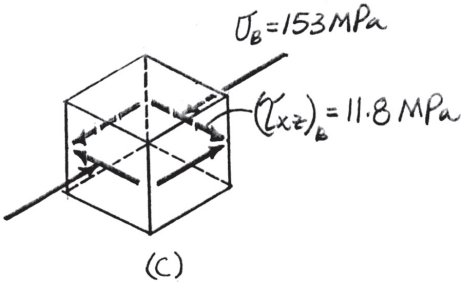
(b)

Normal Stress: For the combined loadings, the normal stress at point B can be determined from

$$\sigma_x = \sigma_B = \frac{N_x}{A} - \frac{M_z y_B}{I_z} + \frac{M_y z_B}{I_y}$$

$$= \frac{100(10^3)}{0.9(10^{-3})\pi} - \frac{[-4.00(10^3)](-0.03)}{0.2025(10^{-6})\pi} + \frac{[-7.00(10^3)](0)}{0.2025(10^{-6})\pi}$$

$$= -153.26(10^6)\ \text{Pa} = 153\ \text{MPa (C)} \qquad \textbf{Ans.}$$



(c)

Shear Stress: The transverse shear stress in z and y directions and the torsional shear stress can be determined using the shear formula $\tau_V = \frac{VQ}{It}$ and the torsion formula $\tau_T = \frac{T\rho}{J}$, respectively.

$$(\tau_{xz})_B = -\tau_T + (\tau_V)_z$$

$$= \frac{0.9(10^3)(0.03)}{0.405(10^{-6})\pi} + \frac{-20(10^3)[18.0(10^{-6})]}{0.2025(10^{-6})\pi(0.06)}$$

$$= 11.79(10^6)\ \text{Pa} = 11.8\ \text{MPa} \qquad \textbf{Ans.}$$

$$(\tau_{xy})_B = (\tau_V)_y = 0 \qquad \textbf{Ans.}$$

Using these results, the state of stress at point B can be represented by the differential volume element shown in Fig. *c*.

Ans:
$\sigma_B = 153$ MPa (C),
$(\tau_{xz})_B = 11.8$ MPa,
$(\tau_{xy})_B = 0$

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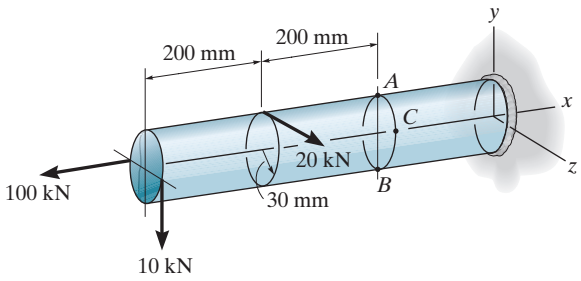
8–49.

The solid rod is subjected to the loading shown. Determine the state of stress at point C, and show the results on a differential volume element at this point.



SOLUTION

Internal Loadings: Consider the equilibrium of the left segment of the rod being sectioned, Fig. *a*.

$$\Sigma F_x = 0; \quad N_x - 100 = 0 \quad N_x = 100 \text{ kN}$$

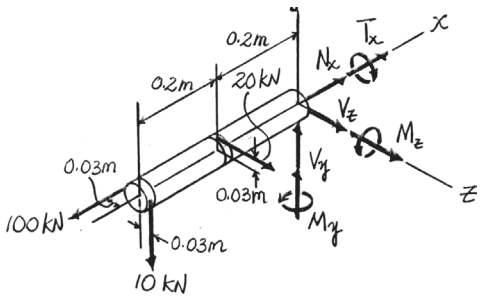
$$\Sigma F_y = 0; \quad V_y - 10 = 0 \quad V_y = 10 \text{ kN}$$

$$\Sigma F_z = 0; \quad V_z + 20 = 0 \quad V_z = -20 \text{ kN}$$

$$\Sigma M_x = 0; \quad T_x + 20(0.03) + 10(0.03) = 0 \quad T_x = -0.9 \text{ kN} \cdot \text{m}$$

$$\Sigma M_y = 0; \quad M_y + 20(0.2) + 100(0.03) = 0 \quad M_y = -7.00 \text{ kN} \cdot \text{m}$$

$$\Sigma M_z = 0; \quad M_z + 10(0.4) = 0 \quad M_z = -4.00 \text{ kN} \cdot \text{m}$$



Section Properties: For the circular cross section, Fig. *b*,

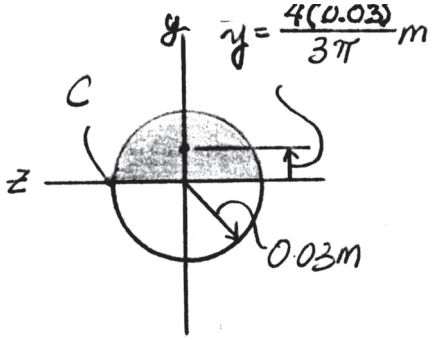
$$A = \pi c^2 = \pi(0.03^2) = 0.9(10^{-3})\pi \text{ m}^2$$

$$I_y = I_z = \frac{\pi}{4}c^4 = \frac{\pi}{4}(0.03^4) = 0.2025(10^{-6})\pi \text{ m}^4$$

$$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.03^4) = 0.405(10^{-6})\pi \text{ m}^4$$

$$(Q_C)_y = \bar{y}'A' = \left[\frac{4(0.03)}{3\pi}\right]\left[\frac{\pi}{2}(0.03^2)\right] = 18.0(10^{-6}) \text{ m}^3$$

$$(Q_C)_z = 0$$

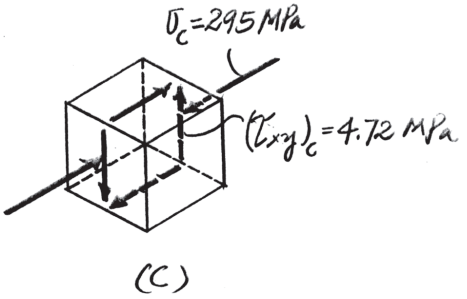


Normal Stress: For the combine loadings, the normal stress at point C can be determined from

$$\sigma_x = \sigma_C = \frac{N_x}{A} - \frac{M_z y_C}{I_z} + \frac{M_y z_C}{I_y}$$

$$= \frac{100(10^3)}{0.9(10^{-3})\pi} - \frac{[-4.00(10^3)](0)}{0.2025(10^{-6})\pi} + \frac{[-7.00(10^3)](0.03)}{0.2025(10^{-6})\pi}$$

$$= -294.73(10^6) \text{ Pa} = 295 \text{ MPa (C)} \qquad \textbf{Ans.}$$



Shear Stress: The transverse shear stress in z and y directions and the torsional shear stress can be determined using the shear formula $\tau_V = \frac{VQ}{It}$ and the torsion formula $\tau_T = \frac{T\rho}{J}$, respectively.

$$(\tau_{xy})_C = (\tau_V)_y - \tau_T$$

$$= \frac{10(10^3)[18.0(10^{-6})]}{0.2025(10^{-6})\pi(0.06)} - \frac{-0.9(10^3)(0.03)}{0.405(10^{-6})\pi}$$

$$= 25.94(10^6) \text{ Pa} = 25.9 \text{ MPa} \qquad \textbf{Ans.}$$

$$(\tau_{xz})_C = (\tau_V)_z = 0 \qquad \textbf{Ans.}$$

Using these results, the state of stress at point C can be represented by the differential volume element shown in Fig. *c*.

Ans:
$\sigma_C = 295$ MPa (C),
$(\tau_{xy})_C = 25.9$ MPa,
$(\tau_{xz})_C = 0$

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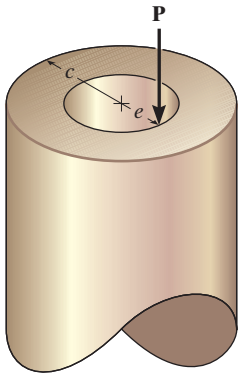
8–50.

The post has a circular cross section of radius c. Determine the maximum radius e at which the load **P** can be applied so that no part of the post experiences a tensile stress. Neglect the weight of the post.

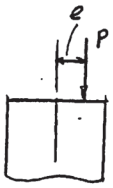


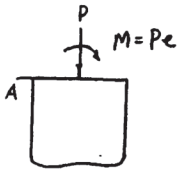
SOLUTION

Require $\sigma_A = 0$

$$\sigma_A = 0 = \frac{P}{A} + \frac{Mc}{I}; \qquad 0 = \frac{-P}{\pi c^2} + \frac{(Pe)c}{\frac{\pi}{4}c^4}$$

$$e = \frac{c}{4}$$ **Ans.**





Ans:
$e = \frac{c}{4}$

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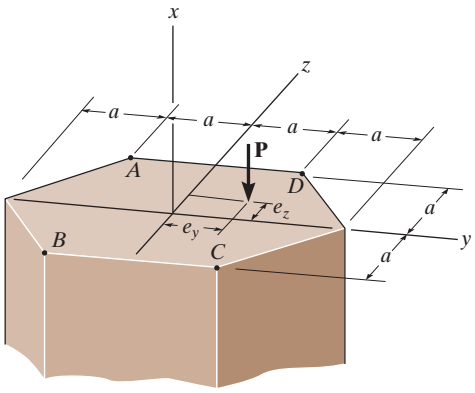
8–51.

The post having the dimensions shown is subjected to the load **P**. Specify the region to which this load can be applied without causing tensile stress at points A, B, C, and D.



SOLUTION

Equivalent Force System: As shown on FBD.

Section Properties:

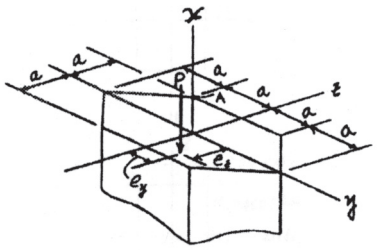
$$A = 2a(2a) + 2\left[\frac{1}{2}(2a)a\right] = 6a^2$$

$$I_z = \frac{1}{12}(2a)(2a)^3 + 2\left[\frac{1}{36}(2a)\,a^3 + \frac{1}{2}(2a)\,a\left(a + \frac{a}{3}\right)^2\right]$$

$$= 5a^4$$

$$I_y = \frac{1}{12}(2a)(2a)^3 + 2\left[\frac{1}{36}(2a)\,a^3 + \frac{1}{2}(2a)\,a\left(\frac{a}{3}\right)^2\right]$$

$$= \frac{5}{3}a^4$$

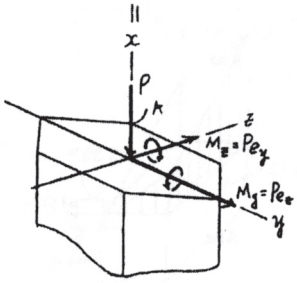


Normal Stress:

$$\sigma = \frac{N}{A} - \frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

$$= \frac{-P}{6a^2} - \frac{Pe_y y}{5a^4} - \frac{Pe_z z}{\frac{5}{3}a^4}$$

$$= \frac{P}{30a^4}\left(-5a^2 - 6e_y y - 18e_z z\right)$$

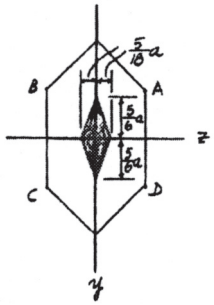


At point B where $y = -a$ and $z = -a$, we require $\sigma_B < 0$.

$$0 > \frac{P}{30a^4}\left[-5a^2 - 6(-a)\,e_y - 18(-a)\,e_z\right]$$

$$0 > -5a + 6e_y + 18e_z$$

$$6e_y + 18e_z < 5a \qquad \textbf{Ans.}$$



When $e_z = 0$, $\quad e_y < \frac{5}{6}a$

When $e_y = 0$, $\quad e_z < \frac{5}{18}a$

Repeat the same procedures for point A, C and D. The region where P can be applied without creating tensile stress at points A, B, C, and D is shown shaded in the diagram.

Ans:
$6e_y + 18e_z < 5a$

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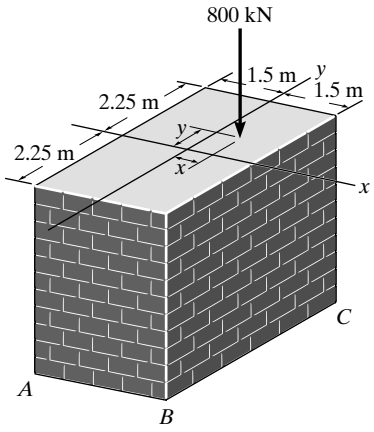
***8–52.** The masonry pier is subjected to the 800-kN load. Determine the equation of the line $y = f(x)$ along which the load can be placed without causing a tensile stress in the pier. Neglect the weight of the pier.



SOLUTION

$A = 3(4.5) = 13.5\ \text{m}^2$

$I_x = \frac{1}{12}(3)(4.5^3) = 22.78125\ \text{m}^4$

$I_y = \frac{1}{12}(4.5)(3^3) = 10.125\ \text{m}^4$

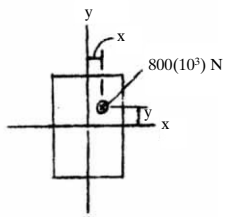
Normal Stress: Require $\sigma_A = 0$

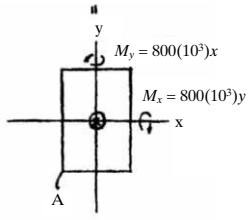
$$\sigma_A = \frac{P}{A} + \frac{M_x y}{I_x} + \frac{M_y x}{I_y}$$

$$0 = \frac{-800(10^3)}{13.5} + \frac{[800(10^3)y](2.25)}{22.78125} + \frac{[800(10^3)x](1.5)}{10.125}$$

$$0 = 79.01y + 118.52x - 59.26$$

$y = 0.75 - 1.5\,x$ **Ans.**





Ans.

$y = 0.75 - 1.5\,x$

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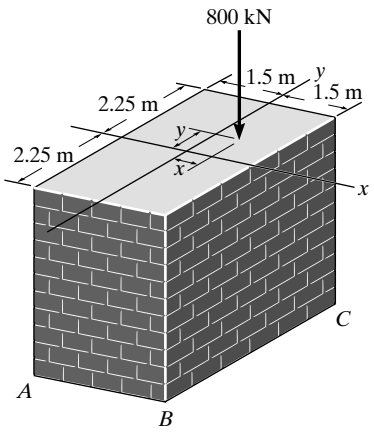
8–53. The masonry pier is subjected to the 800-kN load. If $x = 0.25$ m and $y = 0.5$ m, determine the normal stress at each corner A, B, C, D (not shown) and plot the stress distribution over the cross section. Neglect the weight of the pier.



$$A = 3(4.5) = 13.5\ \text{m}^2$$

$$I_x = \frac{1}{12}(3)(4.5^3) = 22.78125\ \text{m}^4$$

$$I_y = \frac{1}{12}(4.5)(3^3) = 10.125\ \text{m}^4$$

$$\sigma = \frac{P}{A} + \frac{M_x y}{I_x} + \frac{M_y x}{I_y}$$

$$\sigma_A = \frac{-800(10^3)}{13.5} + \frac{400(10^3)(2.25)}{22.78125} + \frac{200(10^3)(1.5)}{10.125}$$

$$= 9.88\ \text{kPa (T)} \qquad \textbf{Ans.}$$

$$\sigma_B = \frac{-800(10^3)}{13.5} + \frac{400(10^3)(2.25)}{22.78125} - \frac{200(10^3)(1.5)}{10.125}$$

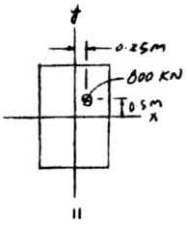
$$= -49.4\ \text{kPa} = 49.4\ \text{kPa (C)} \qquad \textbf{Ans.}$$

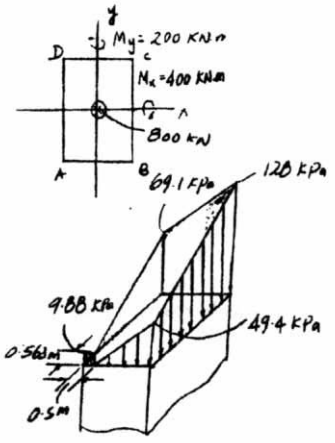
$$\sigma_C = \frac{-800(10^3)}{13.5} - \frac{400(10^3)(2.25)}{22.78125} + \frac{200(10^3)(1.5)}{10.125}$$

$$= -128\ \text{kPa} = 128\ \text{kPa (C)} \qquad \textbf{Ans.}$$

$$\sigma_D = \frac{-800(10^3)}{13.5} - \frac{400(10^3)(2.25)}{22.78125} + \frac{200(10^3)(1.5)}{10.125}$$

$$= -69.1\ \text{kPa} = 69.1\ \text{kPa (C)} \qquad \textbf{Ans.}$$





Ans.
$\sigma_A = 9.88$ kPa (T), $\sigma_B = 49.4$ kPa (C),
$\sigma_C = 128$ kPa (C), $\sigma_D = 69.1$ kPa (C)

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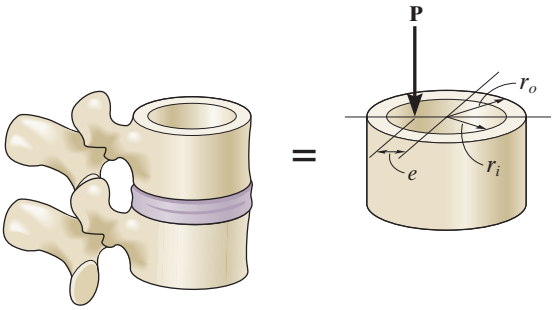
8–54.

The vertebra of the spinal column can support a maximum compressive stress of σ_{max}, before undergoing a compression fracture. Determine the smallest force P that can be applied to a vertebra, if we assume this load is applied at an eccentric distance e from the centerline of the bone, and the bone remains elastic. Model the vertebra as a hollow cylinder with an inner radius r_i and outer radius r_o.



SOLUTION

$$\sigma_{\max} = \frac{P}{A} + \frac{Per_o}{\frac{\pi}{4}(r_0^4 - r_i^4)}$$

$$\sigma_{\max} = P\left[\frac{1}{\pi(r_0^2 - r_i^2)} + \frac{4er_0}{\pi(r_0^4 - r_i^4)}\right]$$

$$\sigma_{\max} = \frac{P}{\pi(r_0^2 - r_i^2)}\left[1 + \frac{4er_0}{(r_0^2 + r_i^2)}\right]$$

$$\sigma_{\max} = \frac{P(r_0^2 + r_i^2 + 4er_0)}{\pi(r_0^2 - r_i^2)(r_0^2 + r_i^2)}$$

$$\sigma_{\max} = \frac{P(r_0^2 + r_i^2 + 4er_0)}{\pi(r_0^4 - r_i^4)}$$

$$P = \frac{\delta_{\max}\pi(r_0^4 - r_i^4)}{r_0^2 + r_i^2 + 4er_0}$$

Ans.

Ans:

$$P = \frac{\delta_{\max}\pi(r_0^4 - r_i^4)}{r_0^2 + r_i^2 + 4er_0}$$

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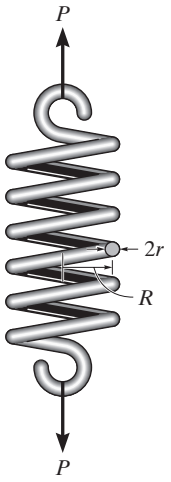
8–55.

The coiled spring is subjected to a force P. If we assume the shear stress caused by the shear force at any vertical section of the coil wire to be uniform, show that the maximum shear stress in the coil is $\tau_{max} = P/A + PRr/J$, where J is the polar moment of inertia of the coil wire and A is its cross-sectional area.

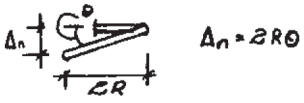


SOLUTION

$$\frac{Tc}{J} = \text{max on perimeter} = \frac{PRr}{J}$$

$$\tau_{max} = \frac{V}{A} + \frac{Tc}{J} = \frac{P}{A} + \frac{PRr}{J}$$

QED



Ans:
N/A

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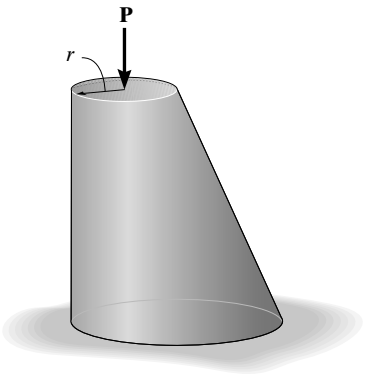
***8–56.** The support is subjected to the compressive load **P**. Determine the maximum and minimum normal stress acting in the material. All horizontal cross sections are circular.



Section Properties:

$$d' = 2r + x$$

$$A = \pi(r + 0.5x)^2$$

$$I = \frac{\pi}{4}(r + 0.5x)^4$$

Internal Force and Moment: As shown on FBD.

Normal Stress:

$$\sigma = \frac{N}{A} \pm \frac{Mc}{I}$$

$$= \frac{-P}{\pi(r + 0.5x)^2} \pm \frac{0.5Px(r + 0.5x)}{\frac{\pi}{4}(r + 0.5)^4}$$

$$= \frac{P}{\pi}\left[\frac{-1}{(r + 0.5x)^2} \pm \frac{2x}{(r + 0.5x)^3}\right]$$

$$\sigma_A = -\frac{P}{\pi}\left[\frac{1}{(r + 0.5x)^2} + \frac{2x}{(r + 0.5x)^3}\right]$$

$$= -\frac{P}{\pi}\left[\frac{r + 2.5x}{(r + 0.5x)^3}\right] \qquad \textbf{[1]}$$

$$\sigma_B = \frac{P}{\pi}\left[\frac{-1}{(r + 0.5x)^2} + \frac{2x}{(r + 0.5x)^3}\right]$$

$$= \frac{P}{\pi}\left[\frac{1.5x - r}{(r + 0.5x)^3}\right] \qquad \textbf{[2]}$$

In order to have maximum normal stress, $\frac{d\sigma_A}{dx} = 0$.

$$\frac{d\sigma_A}{dx} = -\frac{P}{\pi}\left[\frac{(r + 0.5x)^3(2.5) - (r + 2.5x)(3)(r + 0.5x)^2(0.5)}{(r + 0.5x)^6}\right] = 0$$

$$-\frac{P}{\pi(r + 0.5x)^4}(r - 2.5x) = 0$$

Since $\frac{P}{\pi(r + 0.5x)^4} \neq 0$, then

$$r - 2.5x = 0 \qquad x = 0.400r$$

Substituting the result into Eq. [1] yields

$$\sigma_{\max} = -\frac{\mathrm{P}}{\pi}\left[\frac{\mathrm{r} + 2.5(0.400\mathrm{r})}{[\mathrm{r} + 0.5(0.400\mathrm{r})]^3}\right]$$

$$= -\frac{0.368P}{r^2} = \frac{0.368P}{r^2}\,(\mathrm{C}) \qquad \textbf{Ans.}$$

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***8–56. Continued**

In order to have minimum normal stress, $\frac{d\sigma_B}{dx} = 0$.

$$\frac{d\sigma_B}{dx} = \frac{P}{\pi}\left[\frac{(r + 0.5x)^3(1.5) - (1.5x - r)(3)(r + 0.5x)^2(0.5)}{(r + 0.5x)^6}\right] = 0$$

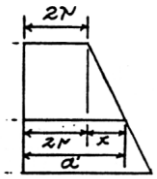
$$\frac{P}{\pi(r + 0.5x)^4}(3r - 1.5x) = 0$$

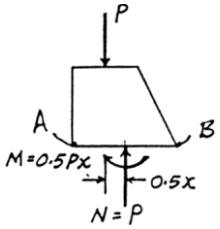
Since $\frac{P}{\pi(r + 0.5x)^4} \neq 0$, then

$$3r - 1.5x = 0 \qquad x = 2.00r$$

Substituting the result into Eq. [2] yields

$$\sigma_{\min} = \frac{P}{\pi}\left[\frac{1.5(2.00r) - r}{[r + 0.5(2.00r)]^3}\right] = \frac{0.0796P}{r^2}\ (\mathbf{T}) \qquad \textbf{Ans.}$$





Ans.

$$\sigma_{\max} = \frac{0.368P}{r^2}\ (\text{C}),\quad \sigma_{\min} = \frac{0.0796P}{r^2}\ (\text{T})$$

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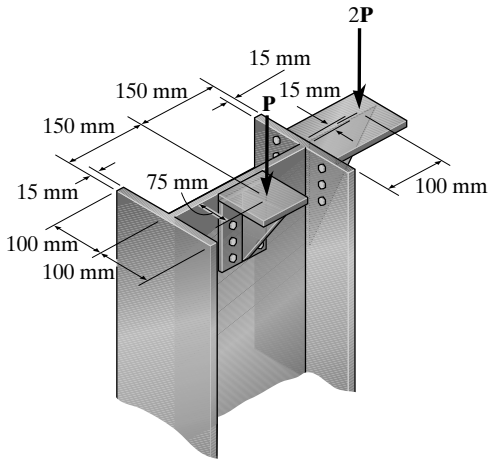
8–57. If $P = 60$ kN, determine the maximum normal stress developed on the cross section of the column.



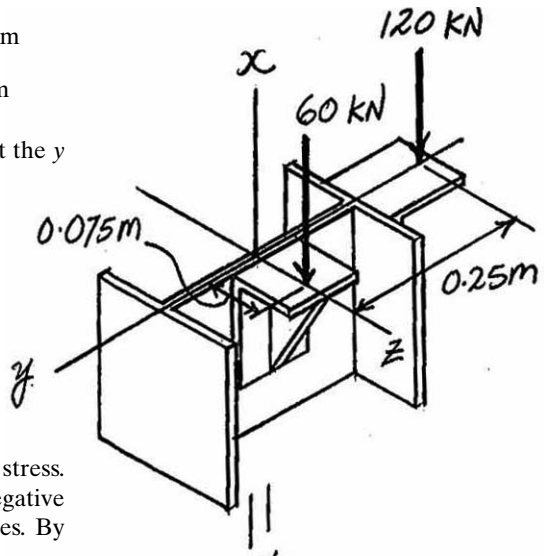
SOLUTION

Equivalent Force System: Referring to Fig. *a*,

$+\uparrow \Sigma F_x = (F_R)_x; \quad -60 - 120 = -F \quad F = 180\ \text{kN}$

$\Sigma M_y = (M_R)_y; \quad -60(0.075) = -M_y \quad M_y = 4.5\text{kN}\cdot\text{m}$

$\Sigma M_z = (M_R)_z; \quad -120(0.25) = -M_z \quad M_z = 30\text{kN}\cdot\text{m}$



Section Properties: The cross-sectional area and the moment of inertia about the y and z axes of the cross section are

$$A = 0.2(0.3) - 0.185(0.27) = 0.01005\ \text{m}^2$$

$$I_z = \frac{1}{12}(0.2)\left(0.3^3\right) - \frac{1}{12}(0.185)\left(0.27^3\right) = 0.14655\left(10^{-3}\right)\text{m}^4$$

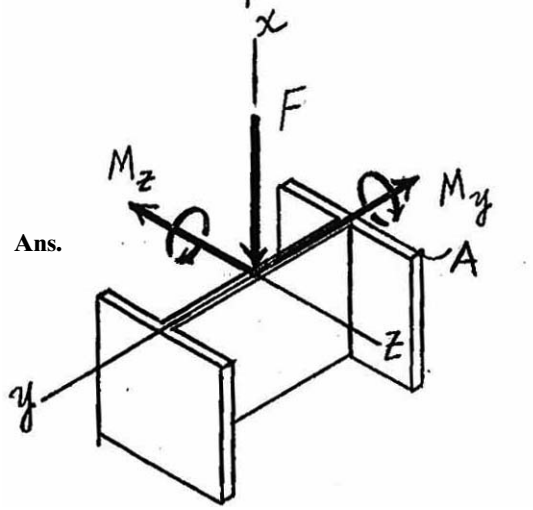
$$I_y = 2\left[\frac{1}{12}(0.015)\left(0.2^3\right)\right] + \frac{1}{12}(0.27)\left(0.015^3\right) = 20.0759\left(10^{-6}\right)\text{m}^4$$

Normal Stress: The normal stress is the combination of axial and bending stress. Here, **F** is negative since it is a compressive force. Also, $\mathbf{M}_y$ and $\mathbf{M}_z$ are negative since they are directed towards the negative sense of their respective axes. By inspection, point A is subjected to a maximum normal stress. Thus,

$$\sigma = \frac{N}{A} - \frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

$$\sigma_{\max} = \sigma_A = \frac{-180\left(10^3\right)}{0.01005} - \frac{\left[-30\left(10^3\right)\right](-0.15)}{0.14655\left(10^{-3}\right)} + \frac{\left[-4.5\left(10^3\right)\right](0.1)}{20.0759\left(10^{-6}\right)}$$

$$= -71.0\ \text{MPa} = 71.0\ \text{MPa(C)}$$ **Ans.**



Ans.

$\sigma_{\max} = 71.0\ \text{MPa (C)}$

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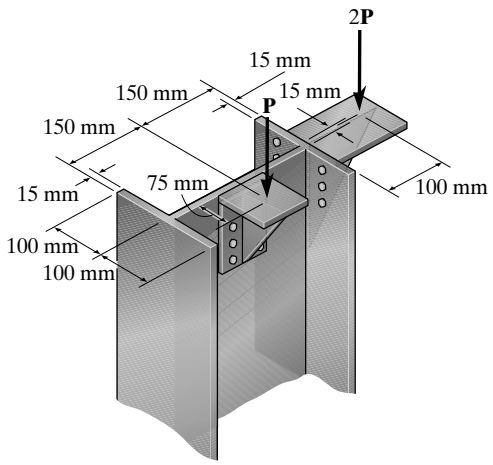
8–58. Determine the maximum allowable force **P**, if the column is made from material having an allowable normal stress of $\sigma_{\text{allow}} = 100$ MPa.



SOLUTION

Equivalent Force System: Referring to Fig. *a*,

$$+\uparrow \Sigma F_x = (F_R)_x; \qquad -P - 2P = -F$$

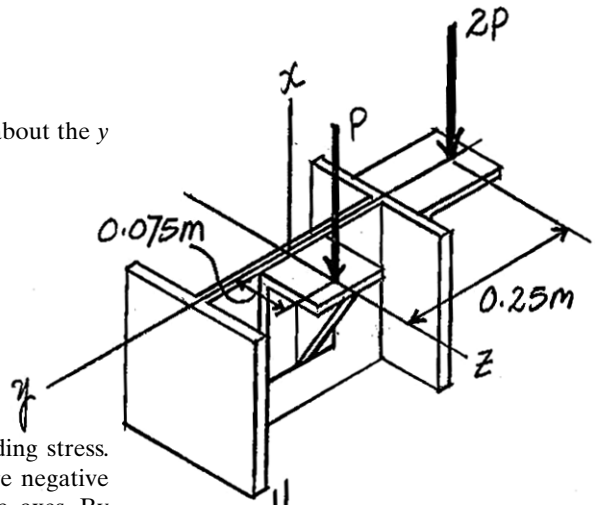
$$F = 3P$$

$$\Sigma M_y = (M_R)_y; \qquad -P(0.075) = -M_y$$

$$M_y = 0.075\,P$$

$$\Sigma M_z = (M_R)_z; \qquad -2P(0.25) = -M_z$$

$$M_z = 0.5P$$



Section Properties: The cross-sectional area and the moment of inertia about the y and z axes of the cross section are

$$A = 0.2(0.3) - 0.185(0.27) = 0.01005\ \text{m}^2$$

$$I_z = \frac{1}{12}(0.2)\left(0.3^3\right) - \frac{1}{12}(0.185)\left(0.27^3\right) = 0.14655\left(10^{-3}\right)\text{m}^4$$

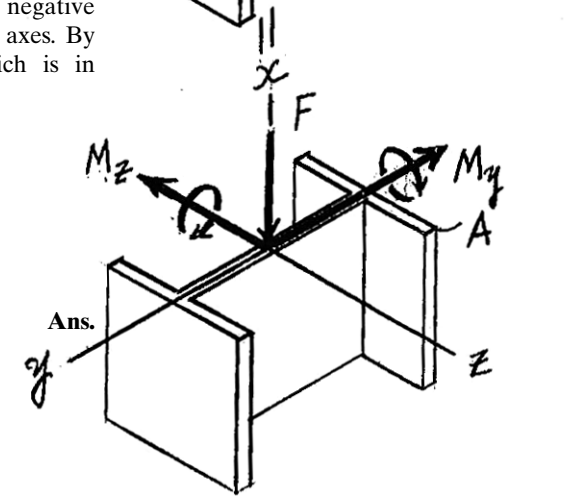
$$I_y = 2\left[\frac{1}{12}(0.15)\left(0.2^3\right)\right] + \frac{1}{12}(0.27)\left(0.015^3\right) = 20.0759\left(10^{-6}\right)\text{m}^4$$

Normal Stress: The normal stress is the combination of axial and bending stress. Here, **F** is negative since it is a compressive force. Also, $\mathbf{M}_y$ and $\mathbf{M}_z$ are negative since they are directed towards the negative sense of their respective axes. By inspection, point A is subjected to a maximum normal stress, which is in compression. Thus,

$$\sigma = \frac{N}{A} - \frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

$$-100\left(10^6\right) = -\frac{3P}{0.01005} - \frac{(-0.5P)(-0.15)}{0.14655\left(10^{-3}\right)} + \frac{-0.075P(0.1)}{20.0759\left(10^{-6}\right)}$$

$$P = 84470.40\ \text{N} = 84.5\ \text{kN}$$



Ans.

Ans.

$P = 84.5$ kN

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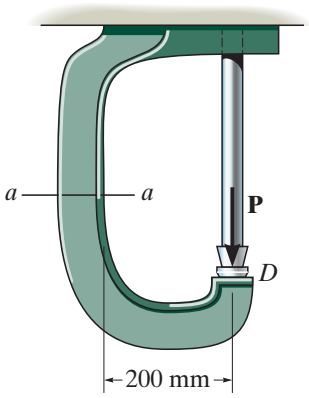
8–59.

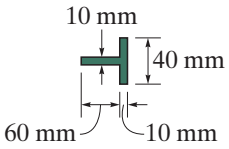
The C-frame is used in a riveting machine. If the force at the ram on the clamp at D is $P = 8$ kN, sketch the stress distribution acting over the section a–a.



SOLUTION

$$\bar{x} = \frac{\Sigma \bar{x} A}{\Sigma A} = \frac{(0.005)(0.04)(0.01) + 0.04(0.06)(0.01)}{0.04(0.01) + 0.06(0.01)} = 0.026 \text{ m}$$

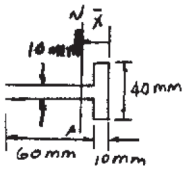
$$A = 0.04(0.01) + 0.06(0.01) = 0.001 \text{ m}^2$$



$$I = \frac{1}{12}(0.04)(0.01^3) + (0.04)(0.01)(0.026 - 0.005)^2 + \frac{1}{12}(0.01)(0.06^3) + 0.01(0.06)(0.040 - 0.026)^2 = 0.4773(10^{-6}) \text{ m}^4$$

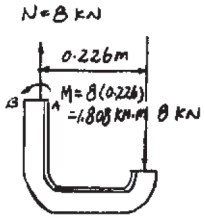
$$(\sigma_{\max})_t = \frac{P}{A} + \frac{Mx}{I} = \frac{8(10^3)}{0.001} + \frac{1.808(10^3)0.026}{0.4773(10^{-6})}$$

$$= 106.48 \text{ MPa} = 106 \text{ MPa} \quad \textbf{Ans.}$$

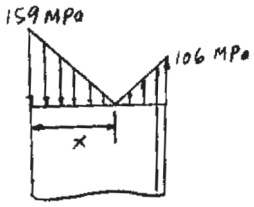


$$(\sigma_{\max})_c = \frac{P}{A} - \frac{Mc}{I} = \frac{8(10^3)}{0.001} - \frac{1.808(10^3)(0.070 - 0.026)}{0.4773(10^{-6})}$$

$$= -158.66 \text{ MPa} = -159 \text{ MPa} \quad \textbf{Ans.}$$



$$\frac{x}{158.66} = \frac{70 - x}{106.48}; \qquad x = 41.9 \text{ mm}$$



Ans:
$(\sigma_{\max})_t = 106$ MPa, $(\sigma_{\max})_c = -159$ MPa

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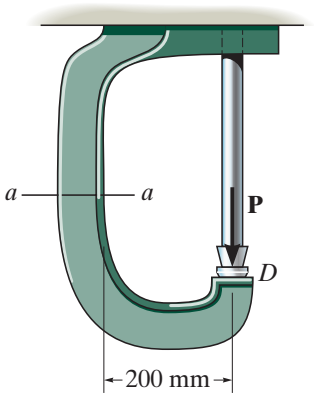
***8–60.**

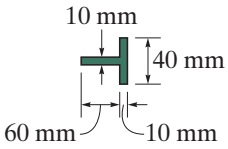
Determine the maximum ram force P that can be applied to the clamp at D if the allowable normal stress for the material is $\sigma_{allow} = 180$ MPa.



SOLUTION

$$\bar{x} = \frac{\Sigma \bar{x} A}{\Sigma A} = \frac{(0.005)(0.04)(0.01) + 0.04(0.06)(0.01)}{0.04(0.01) + 0.06(0.01)} = 0.026 \text{ m}$$

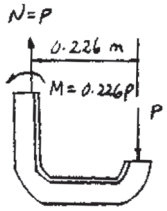
$$A = 0.04(0.01) + 0.06(0.01) = 0.001 \text{ m}^2$$



$$I = \frac{1}{12}(0.04)(0.01^3) + (0.04)(0.01)(0.026 - 0.005)^2 + \frac{1}{12}(0.01)(0.06^3) + 0.01(0.06)(0.040 - 0.026)^2 = 0.4773(10^{-6}) \text{ m}^4$$

$$\sigma = \frac{P}{A} \pm \frac{Mx}{I}$$

Assume tension failure,



$$180(10^6) = \frac{P}{0.001} + \frac{0.226P(0.026)}{0.4773(10^{-6})}$$

$$P = 13524 \text{ N} = 13.5 \text{ kN}$$

Assume compression failure,

$$-180(10^6) = \frac{P}{0.001} - \frac{0.226P(0.070 - 0.026)}{0.4773(10^{-6})}$$

$$P = 9076 \text{ N} = 9.08 \text{ kN (controls)} \qquad \textbf{Ans.}$$

Ans:
$P_{max} = 9.08$ kN

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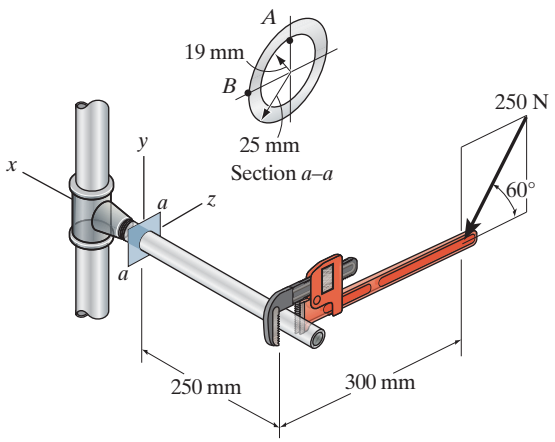
8–61. Determine the state of stress at point A on the cross section of the pipe at section a–a.



SOLUTION

Internal Loadings: Referring to the free - body diagram of the pipe's right segment, Fig. *a,*

$\Sigma F_y = 0;\quad V_y - 250\sin 60° = 0$ $\qquad V_y = 216.51\ \text{N}$

$\Sigma F_z = 0;\quad V_z - 250\cos 60° = 0$ $\qquad V_z = 125\ \text{N}$

$\Sigma M_x = 0;\quad T + (250\sin 60°)(0.300) = 0$ $\qquad T = -64.95\ \text{N}\cdot\text{m}$

$\Sigma M_y = 0;\quad M_y - (250\cos 60°)(0.250) = 0$ $\qquad M_y = 31.25\ \text{N}\cdot\text{m}$

$\Sigma M_z = 0;\quad M_z + (250\sin 60°)(0.250) = 0$ $\qquad M_z = -54.13\ \text{N}\cdot\text{m}$

Section Properties: The moment of inertia about the y and z axes and the polar moment of inertia of the pipe are

$$I_y = I_z = \frac{\pi}{4}(0.025^4 - 0.019^4) = 65.076(10^{-9})\pi\ \text{m}^4$$

$$J = \frac{\pi}{2}(0.025^4 - 0.019^4) = 0.130152(10^{-6})\pi\ \text{m}^4$$

Referring to Fig. *b,*

$$(Q_y)_A = 0$$

$$(Q_z)_A = \bar{y}_1' A_1' - \bar{y}_2' A_2' = \left[\frac{4(0.025)}{3\pi}\right]\left[\frac{\pi}{2}(0.025^2)\right] - \left[\frac{4(0.019)}{3\pi}\right]\left[\frac{\pi}{2}(0.019^2)\right] = 5.844(10^{-6})\ \text{m}^3$$

Normal Stress: The normal stress is contributed by bending stress only. Thus,

$$\sigma = -\frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

For point A, $y = 0.019$ m and $z = 0$. Then

$$\sigma_A = \frac{(-54.13)(0.019)}{65.076(10^{-9})\pi} + 0 = 5.030(10^6)\ \text{N/m}^2 = 5.03\ \text{MPa (T)} \qquad \textbf{Ans.}$$

Shear Stress: The torsional shear stress developed at point A is

$$\left[(\tau_{xz})_T\right]_A = \frac{T\rho_A}{J} = \frac{64.95(0.019)}{0.130152(10^{-6})\pi} = 3.0182(10^6)\ \text{N/m}^2 = 3.0182\ \text{MPa}$$

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8–61. Continued

The transverse shear stress developed at point A is

$$\left[\left(\tau_{xy}\right)_V\right]_A = 0$$

$$\left[\left(\tau_{xz}\right)_V\right]_A = \frac{V_z\left(Q_z\right)_A}{I_y t} = \frac{125[5.844(10^{-6})]}{[65.076(10^{-9})\pi][2(0.006)]}$$

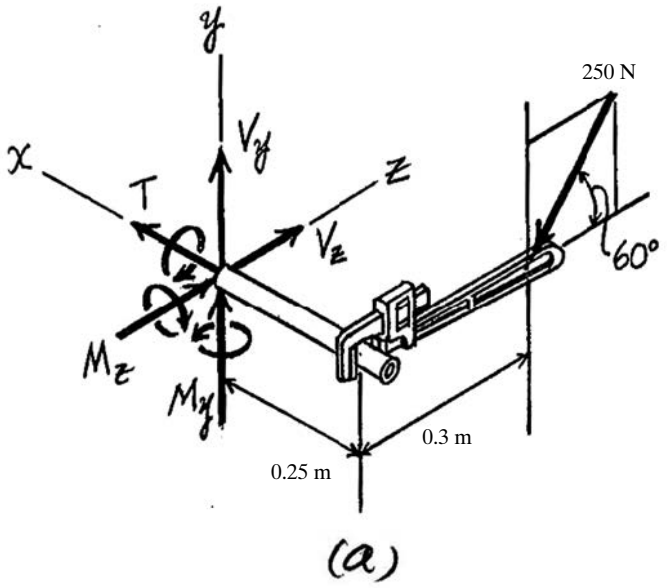
$$= 0.2978(10^6)\ \text{N/m}^2 = 0.2978\ \text{MPa}$$

Combining these two shear stress components,

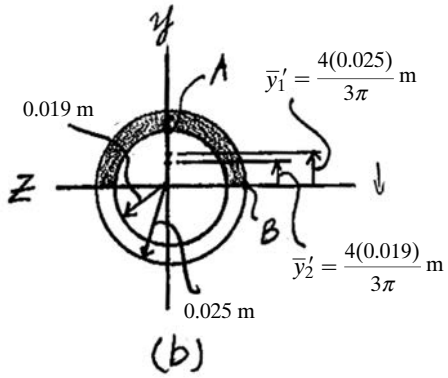
$$\left(\tau_{xy}\right)_A = 0 \qquad \textbf{Ans.}$$

$$\left(\tau_{xz}\right)_A = \left[\left(\tau_{xz}\right)_T\right]_A - \left[\left(\tau_{xz}\right)_V\right]_A$$

$$= 3.0182 - 0.2978 = 2.72\ \text{MPa} \qquad \textbf{Ans.}$$



(a)



(b)

Ans.
$T = -64.95\ \text{N}\cdot\text{m}$, $M_y = 31.25\ \text{N}\cdot\text{m}$,
$M_z = -54.13\ \text{N}\cdot\text{m}$, $I_y = I_z = 65.076\ (10^{-9})\ \pi\text{m}^4$
$J = 0.130152\ (10^{-6})\ \pi\text{m}^4$
$(Q_z)_A = 5.844\ (10^{-6})\ \text{m}^3$
$\sigma_A = 5.03\ \text{MPa (T)}$, $\tau_A = 2.72\ \text{MPa}$

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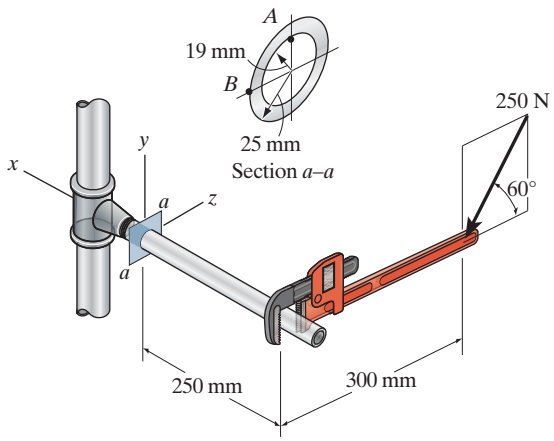
8–62. Determine the state of stress at point B on the cross section of the pipe at section a–a.



SOLUTION

Internal Loadings: Referring to the free - body diagram of the pipe's right segment, Fig. *a,*

$\Sigma F_y = 0;\quad V_y - 250 \sin 60° = 0$ $\qquad V_y = 216.5$ kN

$\Sigma F_z = 0;\quad V_z - 250 \cos 60° = 0$ $\qquad V_z = 125$ kN

$\Sigma M_x = 0;\quad T + 250 \sin 60°(0.300) = 0$ $\qquad T = -64.95$ N · m $\qquad$ lb · in

$\Sigma M_y = 0;\quad M_y - 250 \cos 60°(0.250) = 0$ $\qquad M_y = 31.25$ N · m

$\Sigma M_z = 0;\quad M_z + 250 \sin 60°(0.250) = 0$ $\qquad M_z = -54.13$ N · m $\qquad$ lb · in

Section Properties: The moment of inertia about the y and z axes and the polar moment of inertia of the pipe are

$$I_y = I_z = \frac{\pi}{4}(0.025^4 - 0.019^4) = 65.076(10^{-9})\pi \text{ m}^4$$

$$J = \frac{\pi}{2}(0.025^4 - 0.019^4) = 0.130152(10^{-6})\pi \text{ m}^4$$

Referring to Fig. *b,*

$$(Q_z)_B = 0$$

$$(Q_y)_B = \bar{y}_1' A_1' - \bar{y}_2' A_2' = \left[\frac{4(0.025)}{3\pi}\right]\left[\frac{\pi}{2}(0.025^2)\right] - \left[\frac{4(0.019)}{3\pi}\right]\left[\frac{\pi}{2}(0.019^2)\right] = 5.844(10^{-6}) \text{ m}^3$$

Normal Stress: The normal stress is contributed by bending stress only. Thus,

$$\sigma = -\frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

For point B, $y = 0$ and $z = -0.025$ m. Then

$$\sigma_B = -0 + \frac{31.25(-0.025)}{65.076(10^{-9})\pi} = -3.8214(10^6) \text{ N/m}^2 = 3.82 \text{ MPa (C)} \qquad \textbf{Ans.}$$

Shear Stress: The torsional shear stress developed at point B is

$$\left[(\tau_{xy})_T\right]_B = \frac{T\rho_C}{J} = \frac{64.95(0.025)}{0.130152(10^{-6})\pi} = 3.9713(10^6) \text{ N/m}^2 = 3.97 \text{ MPa}$$

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8–62. Continued

The transverse shear stress developed at point B is

$$\left[\left(\tau_{xz}\right)_V\right]_B = 0$$

$$\left[\left(\tau_{xy}\right)_V\right]_B = \frac{V_y\left(Q_y\right)_B}{I_z t} = \frac{216.51[5.844(10^{-6})]}{[65.076(10^{-9})\pi][2(0.006)]}$$

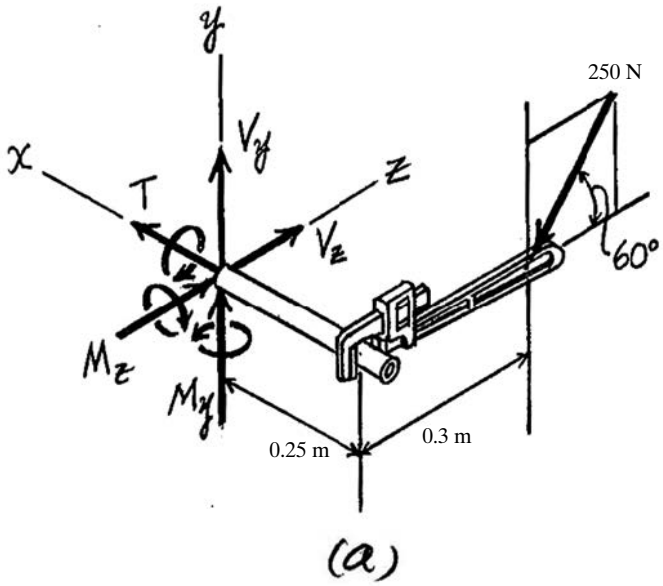
$$= 0.5157(10^6)\ \text{N/m}^2 = 0.5157\ \text{MPa}$$

Combining these two shear stress components,

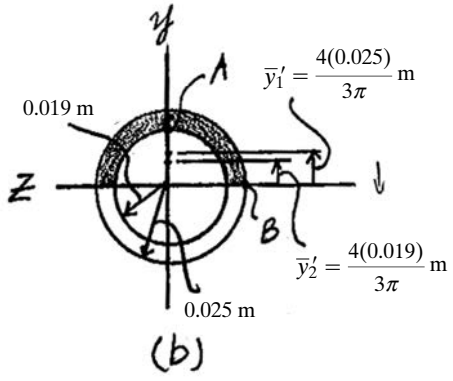
$$\left(\tau_{xy}\right)_B = \left[\left(\tau_{xy}\right)_T\right]_B - \left[\left(\tau_{xy}\right)_V\right]_B$$

$$= 3.9713 - 0.5157 = 3.4555\ \text{MPa} = 3.46\ \text{MPa} \qquad \textbf{Ans.}$$

$$\left(\tau_{xz}\right)_B = 0 \qquad \textbf{Ans.}$$



(a)



(b)

Ans.
$\sigma_B = 3.82\ \text{MPa (C)}, \tau_B = 3.46\ \text{MPa}$

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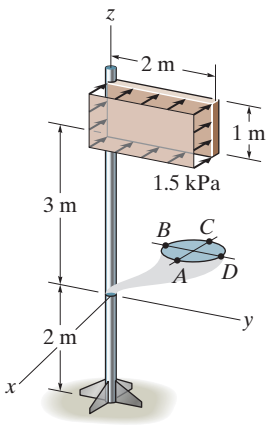
8–63.

The sign is subjected to the uniform wind loading. Determine the stress components at points A and B on the 100-mm-diameter supporting post. Show the results on a volume element located at each of these points.

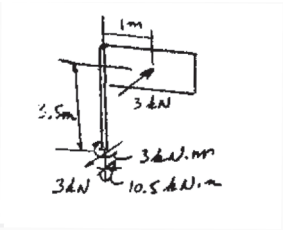


SOLUTION

Point A:

$$\sigma_A = \frac{Mc}{I} = \frac{10.5(10^3)(0.05)}{\frac{\pi}{4}(0.05)^4} = 107 \text{ MPa (T)} \qquad \textbf{Ans.}$$

$$\tau_A = \frac{Tc}{J} = \frac{3(10^3)(0.05)}{\frac{\pi}{4}(0.05)^4} = 15.279(10^6) = 15.3 \text{ MPa} \qquad \textbf{Ans.}$$

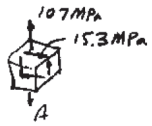


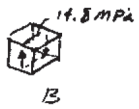
Point B:

$$\sigma_B = 0 \qquad \textbf{Ans.}$$

$$\tau_B = \frac{Tc}{J} - \frac{VQ}{It} = 15.279(10^6) - \frac{3000(4(0.05)/3\pi))(\frac{1}{2})(\pi)(0.05)^2}{\frac{\pi}{4}(0.05)^4(0.1)}$$

$$\tau_B = 14.8 \text{ MPa} \qquad \textbf{Ans.}$$





Ans:
$\sigma_A = 107$ MPa (T), $\tau_A = 15.3$ MPa,
$\sigma_B = 0$, $\tau_B = 14.8$ MPa

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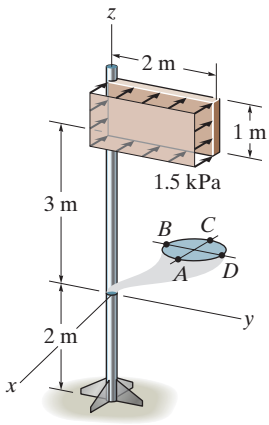
***8–64.**

The sign is subjected to the uniform wind loading. Determine the stress components at points *C* and *D* on the 100-mm-diameter supporting post. Show the results on a volume element located at each of these points.

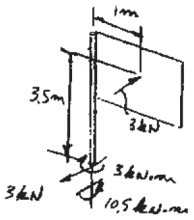


SOLUTION

Point *C*:

$$\sigma_C = \frac{Mc}{I} = \frac{10.5(10^3)(0.05)}{\frac{\pi}{4}(0.05)^4} = 107 \text{ MPa (C)} \qquad \textbf{Ans.}$$

$$\tau_C = \frac{T_C}{J} = \frac{3(10^3)(0.05)}{\frac{\pi}{2}(0.05)^4} = 15.279(10^6) = 15.3 \text{ MPa} \qquad \textbf{Ans.}$$

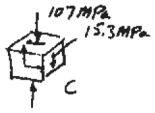


Point *D*:

$$\sigma_D = 0 \qquad \textbf{Ans.}$$

$$\tau_D = \frac{Tc}{J} + \frac{VQ}{It} = 15.279(10^6) + \frac{3(10^3)(4(0.05)/3\pi)(\frac{1}{2})(\pi)(0.05)^2}{\frac{\pi}{4}(0.05)^4(0.1)} = 15.8 \text{ MPa}$$

Ans.



Ans:
$\sigma_C = 107$ MPa (C), $\tau_C = 15.3$ MPa,
$\sigma_D = 0$, $\tau_D = 15.8$ MPa

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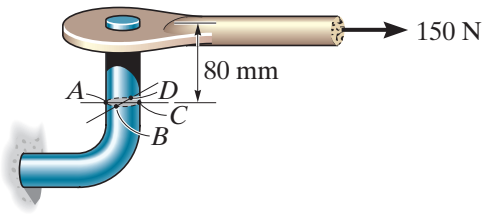
8–65.

The pin support is made from a steel rod and has a diameter of 20 mm. Determine the stress components at points A and B and represent the results on a volume element located at each of these points.



SOLUTION

$$I = \frac{1}{4}(\pi)(0.01^4) = 7.85398(10^{-9})\ \text{m}^4$$

$$Q_B = \bar{y}A' = \frac{4(0.01)}{3\pi}\left(\frac{1}{2}\right)(\pi)(0.01^2) = 0.66667(10^{-6})\ \text{m}^3$$

$$Q_A = 0$$

$$\sigma_A = \frac{Mc}{I} = \frac{12(0.01)}{7.85398(10^{-9})} = 15.3\ \text{MPa} \qquad \textbf{Ans.}$$

$$\tau_A = 0 \qquad \textbf{Ans.}$$

$$\sigma_B = 0 \qquad \textbf{Ans.}$$

$$\tau_B = \frac{VQ_B}{It} = \frac{150(0.6667)(10^{-6})}{7.85398(10^{-9})(0.02)} = 0.637\ \text{MPa} \qquad \textbf{Ans.}$$

Ans:
$\sigma_A = 15.3$ MPa, $\tau_A = 0$,
$\sigma_B = 0, \tau_B = 0.637$ MPa

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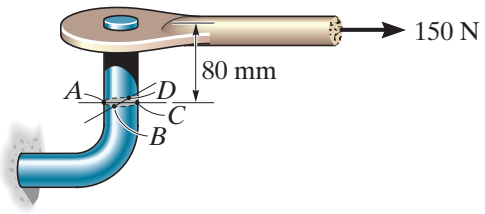
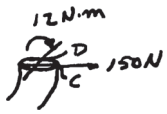
8–66.

Solve Prob. 8–65 for points C and D.



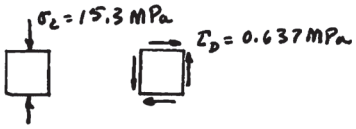
SOLUTION



$$I = \frac{1}{4}(\pi)(0.01^4) = 7.85398(10^{-9})\ \text{m}^4$$

$$Q_D = \bar{y}'A' = \frac{4(0.01)}{3\pi}\left(\frac{1}{2}\right)(\pi)(0.01^2) = 0.66667(10^{-6})\ \text{m}^3$$

$$Q_C = 0$$



$$\sigma_C = \frac{Mc}{I} = \frac{12(0.01)}{7.85398(10^{-9})} = 15.3\ \text{MPa}$$ **Ans.**

$$\tau_C = 0$$ **Ans.**

$$\sigma_D = 0$$ **Ans.**

$$\tau_D = \frac{VQ_D}{It} = \frac{150(0.6667)(10^{-6})}{7.8539(10^{-9})(0.02)} = 0.637\ \text{MPa}$$ **Ans.**

Ans:
$\sigma_C = 15.3$ MPa, $\tau_C = 0$,
$\sigma_D = 0, \tau_D = 0.637$ MPa

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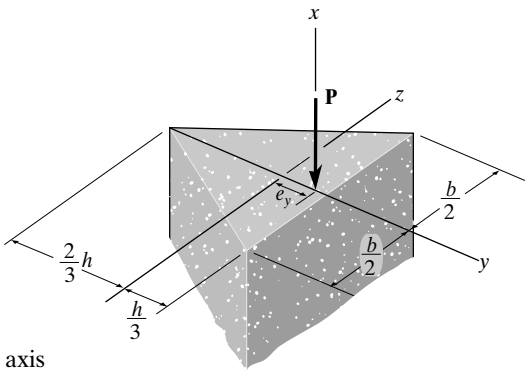
8–67. The eccentric force **P** is applied at a distance e_y from the centroid on the concrete support shown. Determine the range along the y axis where **P** can be applied on the cross section so that no tensile stress is developed in the material.



SOLUTION

Internal Loadings: As shown on the free - body diagram, Fig. *a.*

Section Properties: The cross-sectional area and moment of inertia about the z axis of the triangular concrete support are

$$A = \frac{1}{2}bh \qquad I_z = \frac{1}{36}bh^3$$

Normal Stress: The normal stress is the combination of axial and bending stress. Thus,

$$\sigma = \frac{N}{A} - \frac{M_z y}{I_z}$$

$$\sigma = \frac{-P}{\frac{1}{2}bh} - \frac{(Pe_y)y}{\frac{1}{36}bh^3}$$

$$\sigma = -\frac{2P}{bh^3}\left(h^2 + 18e_y y\right) \tag{1}$$

Here, it is required that $\sigma_A \le 0$ and $\sigma_B \le 0$. For point A, $y = \frac{h}{3}$, Then. Eq. (1) gives

$$0 \ge -\frac{2P}{bh^3}\left[h^2 + 18e_y\left(\frac{h}{3}\right)\right]$$

$$0 \le h^2 + 6he_y$$

$$e_y \ge -\frac{h}{6}$$

For Point B, $y = -\frac{2}{3}h$. Then. Eq. (1) gives

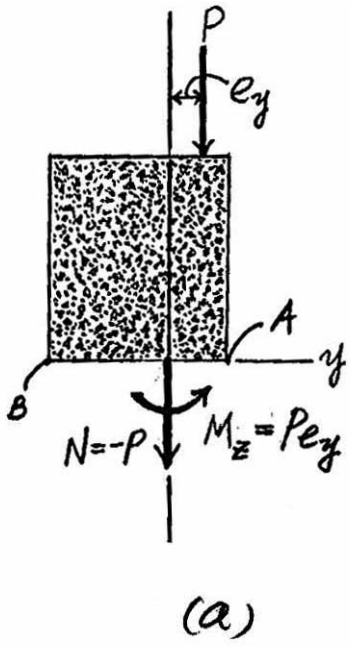
$$0 \ge -\frac{2P}{bh^3}\left[h^2 + 18e_y\left(-\frac{2}{3}h\right)\right]$$

$$0 \le h^2 - 12he_y$$

$$e_y \le \frac{h}{12}$$

Thus, in order that no tensile stress be developed in the concrete support, e_y must be in the range of

$$-\frac{h}{6} \le e_y \le \frac{h}{12}$$ **Ans.**



(a)

Ans.

$$\sigma = -\frac{2P}{bh^3}(h^2 + 18e_y y), -\frac{h}{6} \le e_y \le \frac{h}{12}$$

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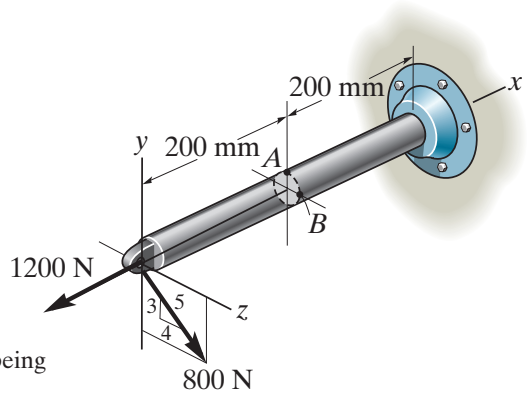
***8–68.**

The bar has a diameter of 40 mm. Determine the state of stress at point A and show the results on a differential volume element located at this point.



SOLUTION

Internal Loadings: Consider the equilibrium of the left segment of the rod being sectioned, Fig. *a.*

$$\Sigma F_x = 0; \quad N_x - 1200 = 0 \quad N_x = 1200\ \text{N}$$

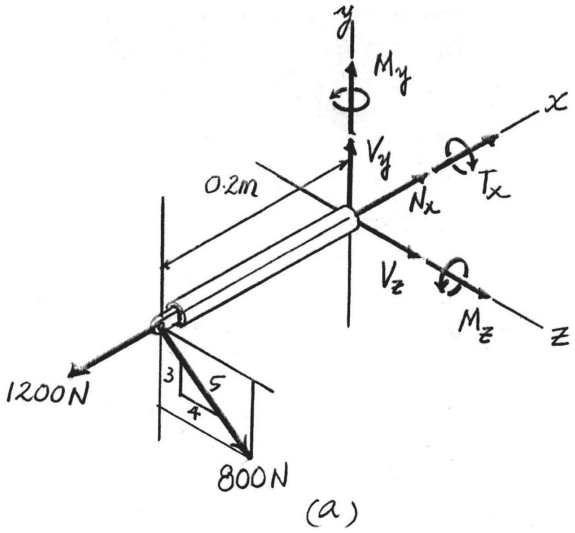
$$\Sigma F_y = 0; \quad V_y - 800\left(\frac{3}{5}\right) = 0 \quad V_y = 480\ \text{N}$$

$$\Sigma F_z = 0; \quad V_z + 800\left(\frac{4}{5}\right) = 0 \quad V_z = -640\ \text{N}$$

$$\Sigma M_x = 0; \quad T_x = 0$$

$$\Sigma M_y = 0; \quad M_y + 800\left(\frac{4}{5}\right)(0.2) = 0 \quad M_y = -128\ \text{N}\cdot\text{m}$$

$$\Sigma M_z = 0; \quad M_z + 800\left(\frac{3}{5}\right)(0.2) = 0 \quad M_z = -96\ \text{N}\cdot\text{m}$$



(a)

Section Properties: For the circular cross section, Fig. *b,*

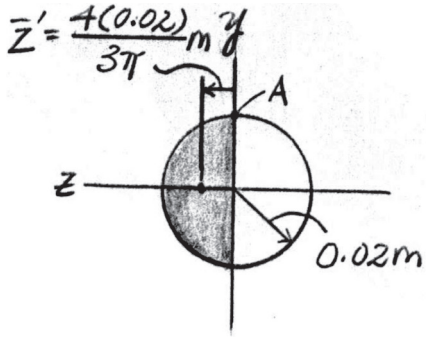
$$A = \pi c^2 = \pi(0.02^2) = 0.4(10^{-3})\pi\ \text{m}^2$$

$$I_y = I_z = \frac{\pi}{4}c^4 = \frac{\pi}{4}(0.02^4) = 40.0(10^{-9})\pi\ \text{m}^4$$

$$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.02^4) = 80.0(10^{-9})\pi\ \text{m}^4$$

$$(Q_A)_z = \bar{z}'A' = \frac{4(0.02)}{3\pi}\left[\frac{\pi}{2}(0.02^2)\right] = 5.3333(10^{-6})\ \text{m}^3$$

$$(Q_A)_y = 0$$



(b)

Normal Stress: For the combined loading, the normal stress at point A can be determined from

$$\sigma_x = \sigma_A = \frac{N_x}{A} - \frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y}$$

$$= \frac{1200}{0.4(10^{-3})\pi} - \frac{(-96)(0.02)}{40.0(10^{-9})\pi} + \frac{(-128)(0)}{40.0(10^{-9})\pi}$$

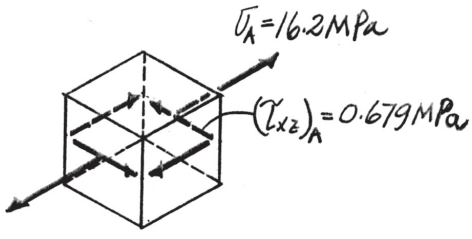
$$= 16.23(10^6)\ \text{Pa} = 16.2\ \text{MPa (T)} \qquad \textbf{Ans.}$$

Shear Stress: Since $T_x = 0$, the shear stress in the z and y directions is contributed by transverse shear stress only which can be obtained using the shear formula, $\tau_V = \frac{VQ}{It}$

$$(\tau_{xz})_A = (\tau_V)_z = \frac{-640\left[5.3333(10^{-6})\right]}{40.0(10^{-9})\pi(0.04)} = -0.6791(10^6)\ \text{Pa} = -0.679\ \text{MPa} \qquad \textbf{Ans.}$$

$$(\tau_{xy})_A = (\tau_V)_y = 0 \qquad \textbf{Ans.}$$

Using these results, the state of stress at point A can be represented by the volume element shown in Fig. *c.*



(c)

Ans:
$\sigma_A = 16.2$ MPa (T),
$(\tau_{xz})_A = -0.679$ MPa,
$(\tau_{xy})_A = 0$

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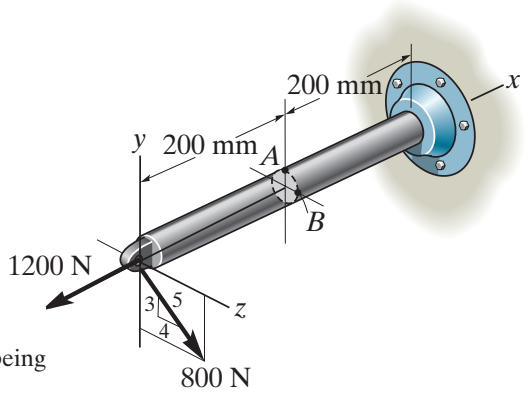
8–69.

Solve Prob. 8–68 for point *B*.



SOLUTION

Internal Loadings: Consider the equilibrium of the left segment of the rod being sectioned, Fig. *a*.

$$\Sigma F_x = 0; \quad N_x - 1200 = 0 \quad N_x = 1200\ \text{N}$$

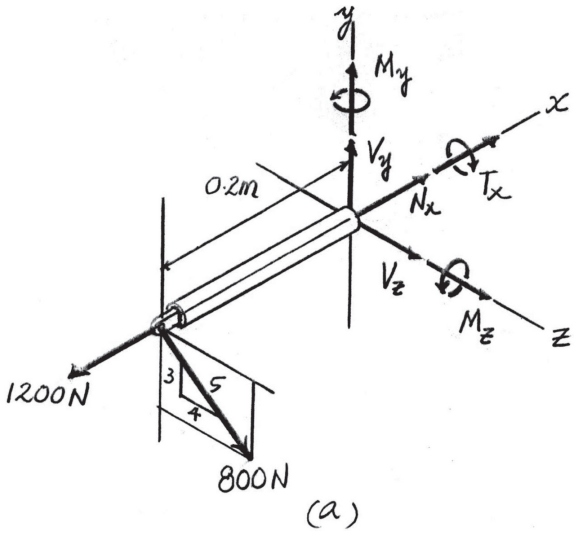
$$\Sigma F_y = 0; \quad V_y - 800\left(\frac{3}{5}\right) = 0 \quad V_y = 480\ \text{N}$$

$$\Sigma F_z = 0; \quad V_z + 800\left(\frac{4}{5}\right) = 0 \quad V_z = -640\ \text{N}$$

$$\Sigma M_x = 0; \quad T_x = 0$$

$$\Sigma M_y = 0; \quad M_y + 800\left(\frac{4}{5}\right)(0.2) = 0 \quad M_y = -128\ \text{N}\cdot\text{m}$$

$$\Sigma M_z = 0; \quad M_z + 800\left(\frac{3}{5}\right)(0.2) = 0 \quad M_z = -96\ \text{N}\cdot\text{m}$$



(a)

Section Properties: For the circular cross section, Fig. *b*,

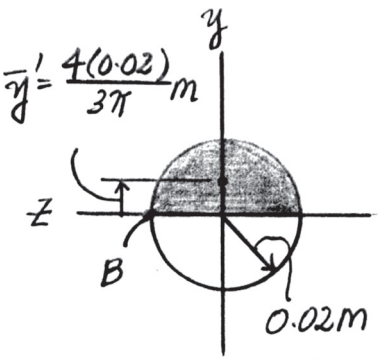
$$A = \pi c^2 = \pi(0.02^2) = 0.4(10^{-3})\pi\ \text{m}^2$$

$$I_y = I_z = \frac{\pi}{4}c^4 = \frac{\pi}{4}(0.02^4) = 40.0(10^{-9})\pi\ \text{m}^4$$

$$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.02^4) = 80.0(10^{-9})\pi\ \text{m}^4$$

$$(Q_B)_y = \bar{y}'A' = \frac{4(0.02)}{3\pi}\left[\frac{\pi}{2}(0.02^2)\right] = 5.3333(10^{-6})\ \text{m}^3$$

$$(Q_B)_z = 0$$



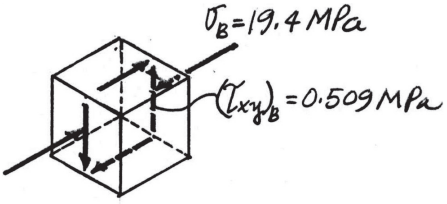
(b)

Normal Stress: For the combined loading, the normal stress at point *B* can be determined from

$$\sigma_x = \sigma_B = \frac{N_x}{A} - \frac{M_z y_B}{I_z} + \frac{M_y z_B}{I_y}$$

$$= \frac{1200}{0.4(10^{-3})\pi} - \frac{(-96)(0)}{40.0(10^{-9})\pi} + \frac{(-128)(0.02)}{40.0(10^{-9})\pi}$$

$$= -19.42(10^6)\ \text{Pa} = 19.4\ \text{MPa (C)} \qquad \textbf{Ans.}$$



(c)

Shear Stress: Since $T_x = 0$, the shear stress in z and y directions is contributed by transverse shear stress only, which can be obtained using the shear formula, $\tau_V = \frac{VQ}{It}$.

$$(\tau_{xy})_B = (\tau_V)_y = \frac{480\left[5.3333(10^{-6})\right]}{40.0(10^{-9})\pi(0.04)} = 0.5093(10^6)\ \text{Pa} = 0.509\ \text{MPa} \qquad \textbf{Ans.}$$

$$(\tau_{xz})_B = (\tau_V)_z = 0 \qquad \textbf{Ans.}$$

Using these results, the state of stress at point *B* can be represented by the volume element shown in Fig. *c*.

Ans:
$\sigma_B = 19.4$ MPa (C),
$(\tau_{xy})_B = 0.509$ MPa,
$(\tau_{xz})_B = 0$

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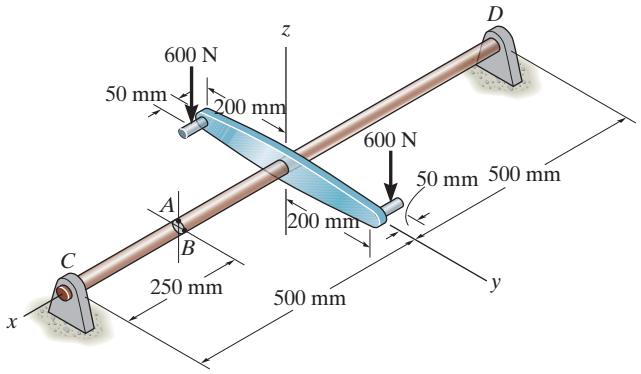
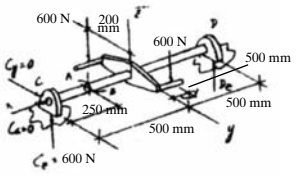
8-70. The 18 mm-diameter shaft is subjected to the loading shown. Determine the stress components at point A. Sketch the results on a volume element located at this point. The journal bearing at C can exert only force components $\mathbf{C}_y$ and $\mathbf{C}_z$ on the shaft, and the thrust bearing at D can exert force components $\mathbf{D}_x$, $\mathbf{D}_y$, and $\mathbf{D}_z$ on the shaft.



SOLUTION

$$A = \frac{\pi}{4}(0.018^2) = 81.0(10^{-6})\pi \text{ m}^2$$

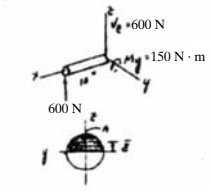
$$I = \frac{\pi}{4}(0.009^4) = 1.64025(10^{-9})\pi \text{ m}^4$$

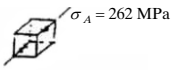


$$Q_A = 0$$

$$\tau_A = 0 \qquad \textbf{Ans.}$$

$$\sigma_A = -\frac{M_y c}{I} = -\frac{150(0.009)}{1.64025(10^{-9})\pi} = -261.98(10^6) \text{ N/m}^2 = 262 \text{ MPa (C)} \qquad \textbf{Ans.}$$





Ans.

$\tau_A = 0, \sigma_A = 262.0 \text{ MPa (C)}$

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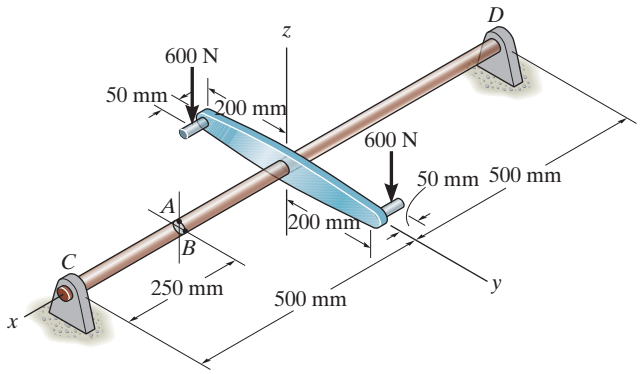
8–71. Solve Prob. 8–70 for the stress components at point *B*.



SOLUTION

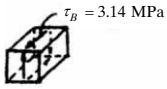
$$A = \frac{\pi}{4}(0.018^2) = 81.0(10^{-6})\pi \text{ m}^2$$

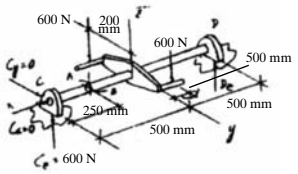
$$I = \frac{\pi}{4}(0.009^4) = 1.64025(10^{-9})\pi \text{ m}^4$$

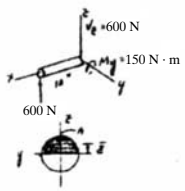
$$Q_B = y'A' = \left[\frac{4(0.009)}{3\pi}\right]\left[\frac{\pi}{2}(0.009^2)\right] = 0.486(10^{-6}) \text{ m}^3$$

$$\sigma_B = 0 \qquad \textbf{Ans.}$$

$$\tau_B = \frac{V_z Q_B}{I\,t} = \frac{600[0.486(10^{-6})]}{\left[1.64025(10^{-9})\pi\right](0.018)} = 3.144(10^{-6}) \text{ N/m}^2 = 3.14 \text{ MPa} \qquad \textbf{Ans.}$$







Ans.
$\sigma_B = 0, \tau_B = 3.14$ MPa

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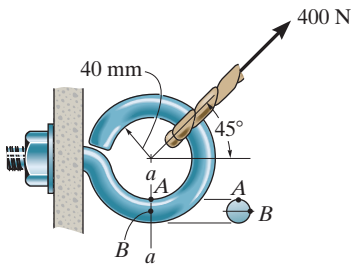
***8–72.** The hook is subjected to the force of 400 N. Determine the state of stress at point A at section a–a. The cross section is circular and has a diameter of 12 mm. Use the curved-beam formula to compute the bending stress.



The location of the neutral surface from the center of curvature of the hook, Fig. a, can be determined from

$$R = \frac{A}{\Sigma \int_A \frac{dA}{r}}$$

where $A = \pi(0.006^2) = 36.0(10^{-6})\pi \text{ m}^2$

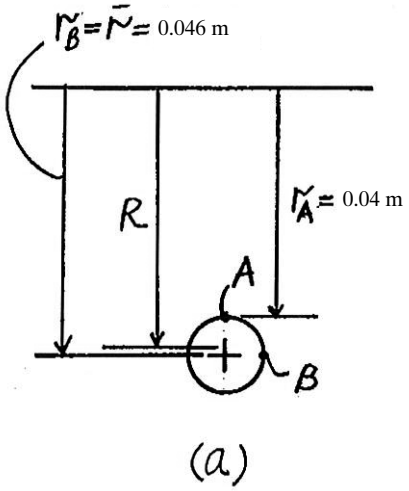
$$\Sigma \int_A \frac{dA}{r} = 2\pi\left(\bar{r} - \sqrt{\bar{r}^2 - c^2}\right) = 2\pi\left(0.046 - \sqrt{0.046^2 - 0.006^2}\right) = 2.4692(10^{-3}) \text{ m}$$

Thus,

$$R = \frac{36.0(10^{-6})\pi}{2.4692(10^{-3})} = 0.0458035 \text{ m}$$

Then

$$e = \bar{r} - R = 0.046 - 0.0458035 = 0.0001965 \text{ m}$$

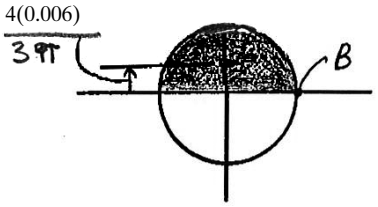


(a)

Referring to Fig. b, I and Q_A are

$$I = \frac{\pi}{4}(0.006^4) = 0.324(10^{-9})\pi \text{ m}^4$$

$$Q_A = 0$$



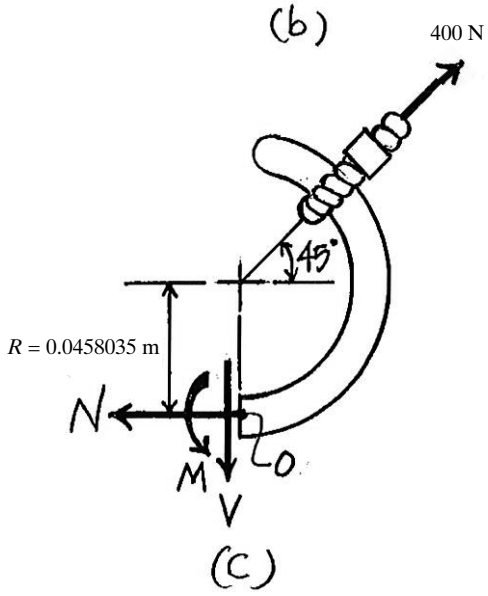
(b)

Consider the equilibrium of the FBD of the hook's cut segment, Fig. c,

$$\stackrel{+}{\leftarrow} \Sigma F_x = 0; \quad N - 400\cos 45° = 0 \qquad N = 282.84 \text{ N}$$

$$+\uparrow \Sigma F_y = 0; \quad 400\sin 45° - V = 0 \qquad V = 282.84 \text{ N}$$

$$\zeta + \Sigma M_o = 0; \quad M - (400\cos 45°)(0.0458035) = 0 \qquad M = 12.9552 \text{ N} \cdot \text{m}$$



(c)

The normal stress developed is the combination of axial and bending stress. Thus,

$$\sigma = \frac{N}{A} + \frac{M(R - r)}{Aer}$$

Here, $M = 12.9552 \text{ N} \cdot \text{m}$ since it tends to reduce the curvature of the hook. For point A, $r = 0.04$ m. Then

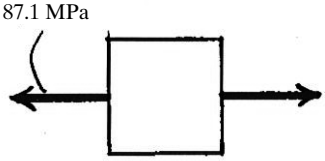
$$\sigma = \frac{282.84}{36.0(10^{-6})\pi} + \frac{12.9552(0.0458035 - 0.04)}{[36.0(10^{-6})\pi](0.0001965)(0.04)}$$

$$= 87.08(10^{-6}) \text{ N/m}^2 = 87.1 \text{ MPa (T)} \qquad \textbf{Ans.}$$

The shear stress in contributed by the transverse shear stress only. Thus

$$\tau = \frac{VQ_A}{It} = 0 \qquad \textbf{Ans.}$$

The state of strees of point A can be represented by the element shown in Fig. d.



(d)

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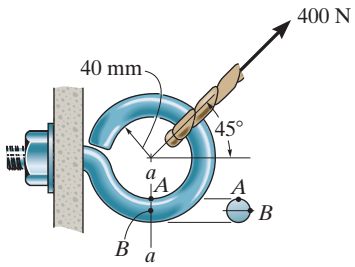
8–73. The hook is subjected to the force of 400 N. Determine the state of stress at point B at section a–a. The cross section has a diameter of 12 mm. Use the curved-beam formula to compute the bending stress.



The location of the neutral surface from the center of curvature of the hook, Fig. *a*, can be determined from

$$R = \frac{A}{\Sigma \int_A \frac{dA}{r}}$$

where $A = \pi(0.006^2) = 36.0(10^{-6})\pi \text{ m}^2$

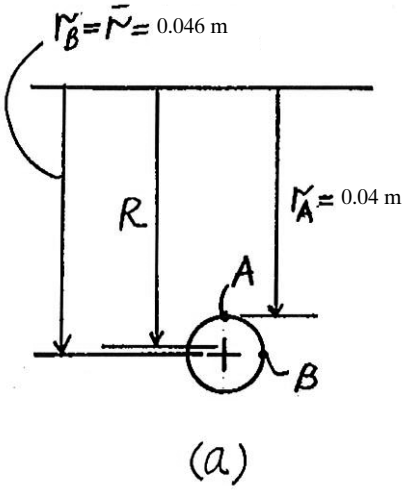
$$\Sigma \int_A \frac{dA}{r} = 2\pi\left(\bar{r} - \sqrt{\bar{r}^2 - c^2}\right) = 2\pi\left(0.046 - \sqrt{0.046^2 - 0.006^2}\right) = 2.4692(10^{-3}) \text{ m}$$

Thus,

$$R = \frac{36.0(10^{-6})\pi}{2.4692(10^{-3})} = 0.0458035 \text{ m}$$

Then

$$e = \bar{r} - R = 0.046 - 0.0458035 = 0.0001965 \text{ m}$$

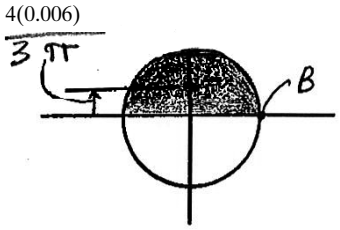


(a)

Referring to Fig. *b*, I and Q_A are

$$I = \frac{\pi}{4}(0.006^4) = 0.324(10^{-9})\pi \text{ m}^4$$

$$Q_B = \bar{y}'A' = \left[\frac{4(0.006)}{3\pi}\right]\left[\frac{\pi}{2}(0.006^2)\right] = 0.144(10^{-6}) \text{ m}^3$$



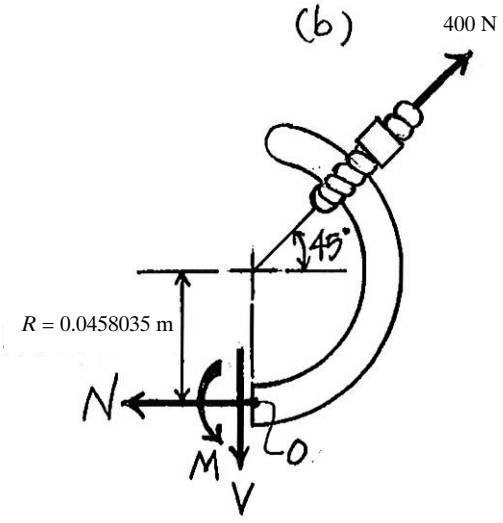
(b)

Consider the equilibrium of the FBD of the hook's cut segment, Fig. *c*,

$\stackrel{+}{\leftarrow} \Sigma F_x = 0;$ $\quad N - 400\cos 45° = 0$ $\quad N = 282.84$ N

$+\uparrow \Sigma F_y = 0;$ $\quad 400\sin 45° - V = 0$ $\quad V = 282.84$ N

$\circlearrowleft + \Sigma M_o = 0;$ $\quad M - (400\cos 45°)(0.0458035) = 0$ $\quad M = 12.9552 \text{ N} \cdot \text{m}$



(c)

The normal stress developed is the combination of axial and bending stress. Thus,

$$\sigma = \frac{N}{A} + \frac{M(R - r)}{Aer}$$

Here, $M = 12.9552 \text{ N} \cdot \text{m}$ since it tends to reduce the curvature of the hook. For point B, $r = 0.04.$ m. Then

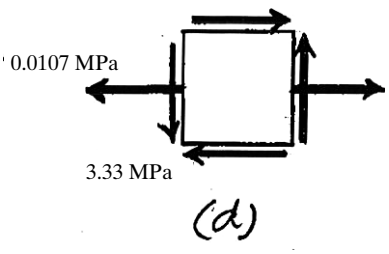
$$\sigma = \frac{282.84}{36.0(10^{-6})\pi} + \frac{12.9552(0.0458035 - 0.046)}{[36.0(10^{-6})\pi](0.0001965)(0.046)}$$

$$= 0.01068(10^6) \text{ N/m}^2 = 0.0107 \text{ MPa}$$ **Ans.**

The shear stress is contributed by the transverse shear stress only. Thus,

$$\tau = \frac{VQ_B}{It} = \frac{282.84[0.144(10^{-6})]}{[0.324(10^{-9})\pi](0.012)} = 3.3345(10^6) \text{ N/m}^2 = 3.33 \text{ MPa}$$ **Ans.**

The state of stress of point B can be represented by the element shown in Fig. *d*.



(d)

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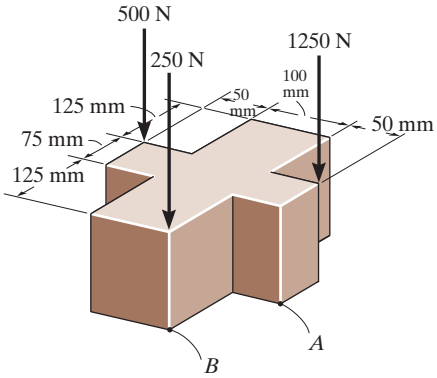
R8–1. The block is subjected to the three axial loads shown. Determine the normal stress developed at points *A* and *B*. Neglect the weight of the block.



SOLUTION

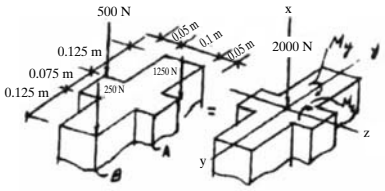
$$M_z = -1250(0.0375) - 500(0.0375) + 250(0.1625) = -25.0\ \text{N}\cdot\text{m}$$

$$M_y = -1250(0.1) - 250(0.05) + 500(0.1) = -87.5\ \text{N}\cdot\text{m}$$

$$I_z = \frac{1}{12}(0.1)(0.325^3) + \frac{1}{12}(0.1)(0.075^3) = 0.28958(10^{-3})\ \text{m}^4$$

$$I_y = \frac{1}{12}(0.75)(0.2^3) + \frac{1}{12}(0.25)(0.1^3) = 70.8333(10^{-6})\ \text{m}^4$$

$$A = 0.1(0.325) + 0.1(0.075) = 0.04\ \text{m}^2$$



$$\sigma = \frac{P}{A} - \frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

$$\sigma_A = \frac{-2000}{0.04} - \frac{(-25.0)(0.0375)}{0.28958(10^{-3})} + \frac{(-87.5)(0.1)}{70.8333(10^{-6})}$$

$$= -170.29(10^3)\ \text{N/m}^2 = 170\ \text{kPa (C)} \qquad \textbf{Ans.}$$

$$\sigma_B = \frac{-2000}{0.04} - \frac{(-25.0)(0.1625)}{0.28958(10^{-3})} + \frac{(-87.5)(0.1)}{70.8333(10^{-6})}$$

$$= 97.74(10^3)\ \text{N/m}^2 = 97.7\ \text{kPa (C)} \qquad \textbf{Ans.}$$

Ans.

$\sigma_A = 170$ kPa (C), $\sigma_B = 97.7$ kPa (C)

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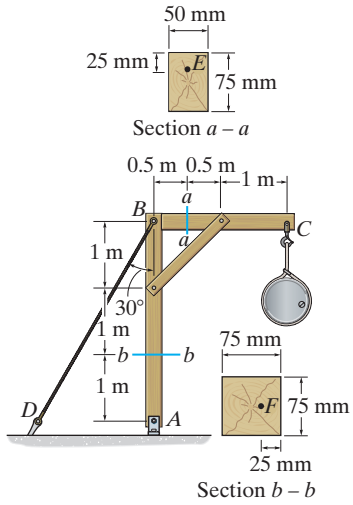
R8–2.

The 20-kg drum is suspended from the hook mounted on the wooden frame. Determine the state of stress at point E on the cross section of the frame at section a–a. Indicate the results on an element.



SOLUTION

Support Reactions: Referring to the free-body diagram of member BC shown in Fig. a,

$\circlearrowleft +\Sigma M_B = 0;\quad F\sin 45°(1) - 20(9.81)(2) = 0 \quad F = 554.94\text{ N}$

$\pm\rightarrow \Sigma F_x = 0;\quad 554.94\cos 45° - B_x = 0 \quad B_x = 392.4\text{ N}$

$+\uparrow \Sigma F_y = 0;\quad 554.94\sin 45° - 20(9.81) - B_y = 0 \quad B_y = 196.2\text{ N}$

Internal Loadings: Consider the equilibrium of the free-body diagram of the right segment shown in Fig. b.

$\pm\rightarrow \Sigma F_x = 0;\quad N - 392.4 = 0 \quad N = 392.4\text{ N}$

$+\uparrow \Sigma F_y = 0;\quad V - 196.2 = 0 \quad V = 196.2\text{ N}$

$\circlearrowleft +\Sigma M_C = 0;\quad 196.2(0.5) - M = 0 \quad M = 98.1\text{ N}\cdot\text{m}$

Section Properties: The cross-sectional area and the moment of inertia of the cross section are

$$A = 0.05(0.075) = 3.75\left(10^{-3}\right)\text{m}^2$$

$$I = \frac{1}{12}(0.05)\left(0.075^3\right) = 1.7578\left(10^{-6}\right)\text{m}^4$$

Referring to Fig. c, Q_E is

$$Q_E = \bar{y}'A' = 0.025(0.025)(0.05) = 31.25\left(10^{-6}\right)\text{m}^3$$

Normal Stress: The normal stress is the combination of axial and bending stress. Thus,

$$\sigma = \frac{N}{A} \pm \frac{My}{I}$$

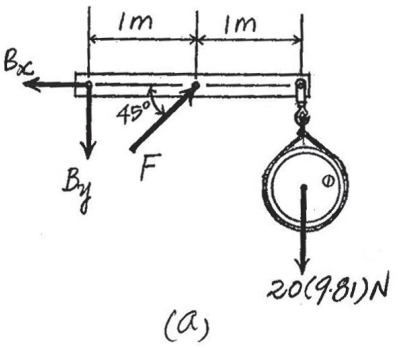
For point E, $y = 0.0375 - 0.025 = 0.0125\,\text{m}$. Then

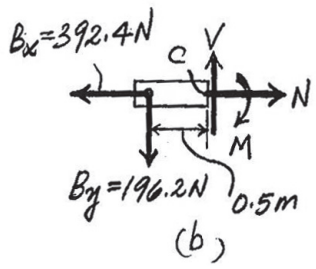
$$\sigma_E = \frac{392.4}{3.75\left(10^{-3}\right)} + \frac{98.1(0.0125)}{1.7578\left(10^{-6}\right)} = 802\text{ kPa} \qquad \textbf{Ans.}$$

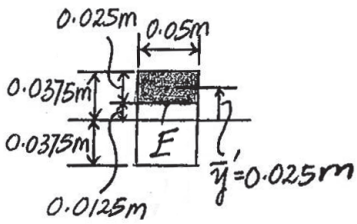
Shear Stress: The shear stress is contributed by transverse shear stress only. Thus,

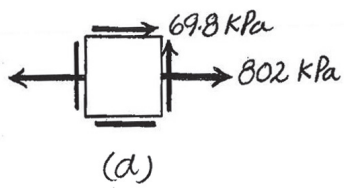
$$\tau_E = \frac{VQ_A}{It} = \frac{196.2\left[31.25\left(10^{-6}\right)\right]}{1.7578\left(10^{-6}\right)(0.05)} = 69.8\text{ kPa} \qquad \textbf{Ans.}$$

The state of stress at point E is represented on the element shown in Fig. d.









Ans:
$\sigma_E = 802\text{ kPa},\ \tau_E = 69.8\text{ kPa}$

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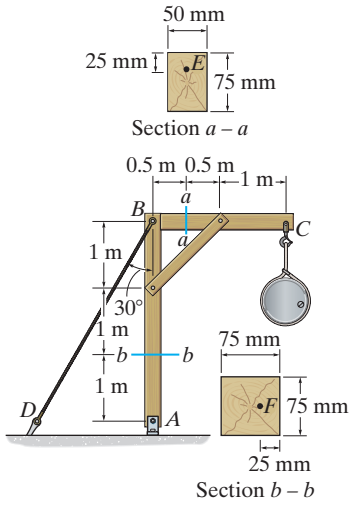
R8–3.

The 20-kg drum is suspended from the hook mounted on the wooden frame. Determine the state of stress at point F on the cross section of the frame at section b–b. Indicate the results on an element.



SOLUTION

Support Reactions: Referring to the free-body diagram of the entire frame shown in Fig. a,

$\zeta+\Sigma M_A = 0;\quad F_{BD}\sin 30°(3) - 20(9.81)(2) = 0 \quad F_{BD} = 261.6\ \text{N}$

$+\uparrow \Sigma F_y = 0;\quad A_y - 261.6\cos 30° - 20(9.81) = 0 \quad A_y = 422.75\ \text{N}$

$\pm\rightarrow \Sigma F_x = 0;\quad A_x - 261.6\sin 30° = 0 \quad A_x = 130.8\ \text{N}$

Internal Loadings: Consider the equilibrium of the free-body diagram of the lower cut segment, Fig. b.

$\pm\rightarrow \Sigma F_x = 0;\quad 130.8 - V = 0 \quad V = 130.8\ \text{N}$

$+\uparrow \Sigma F_y = 0;\quad 422.75 - N = 0 \quad N = 422.75\ \text{N}$

$\zeta+\Sigma M_C = 0;\quad 130.8(1) - M = 0 \quad M = 130.8\ \text{N}\cdot\text{m}$

Section Properties: The cross-sectional area and the moment of inertia about the centroidal axis of the cross section are

$$A = 0.075(0.075) = 5.625\left(10^{-3}\right)\text{m}^2$$

$$I = \frac{1}{12}(0.075)\left(0.075^3\right) = 2.6367\left(10^{-6}\right)\text{m}^4$$

Referring to Fig. c, Q_E is

$$Q_F = \bar{y}'A' = 0.025(0.025)(0.075) = 46.875\left(10^{-6}\right)\text{m}^3$$

Normal Stress: The normal stress is the combination of axial and bending stress. Thus,

$$\sigma = \frac{N}{A} \pm \frac{My}{I}$$

For point F, $y = 0.0375 - 0.025 = 0.0125$ m. Then

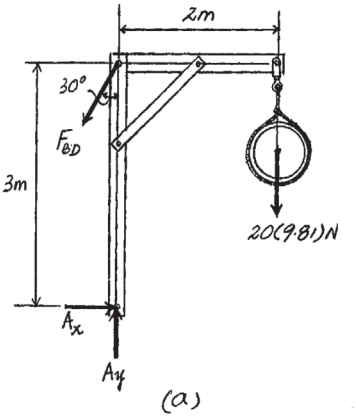
$$\sigma_F = \frac{-422.75}{5.625\left(10^{-3}\right)} - \frac{130.8(0.0125)}{2.6367\left(10^{-6}\right)}$$

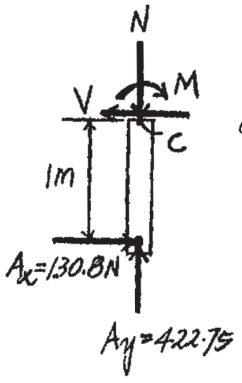
$$= -695.24\ \text{kPa} = 695\ \text{kPa (C)} \qquad \textbf{Ans.}$$

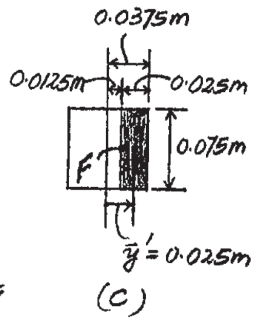
Shear Stress: The shear stress is contributed by transverse shear stress only. Thus,

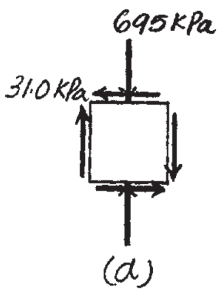
$$\tau_A = \frac{VQ_A}{It} = \frac{130.8\left[46.875\left(10^{-6}\right)\right]}{2.6367\left(10^{-6}\right)(0.075)} = 31.0\ \text{kPa} \qquad \textbf{Ans.}$$

The state of stress at point A is represented on the element shown in Fig. d.









Ans:
$\sigma_F = 695$ kPa (C),
$\tau_A = 31.0$ kPa

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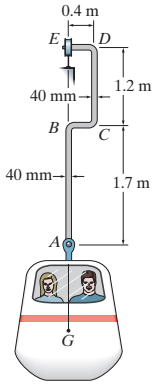
***R8–4.** The gondola and passengers have a weight of 7.5 kN and center of gravity at G. The suspender arm AE has a square cross-sectional area of 40 mm by 40 mm, and is pin connected at its ends A and E. Determine the largest tensile stress developed in regions AB and DC of the arm.

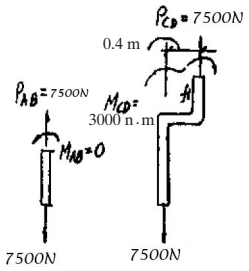


SOLUTION

Segment AB:

$$(\sigma_{\max})_{AB} = \frac{P_{AB}}{A} = \frac{7500}{0.04(0.04)} = 4.6875(10^6)\ \text{N/m}^2 = 4.69\ \text{MPa}$$ **Ans.**

Segment CD:



$$\sigma_a = \frac{P_{CD}}{A} = \frac{7500}{0.04(0.04)} = 4.6875(10^6)\ \text{N/m}^2 = 4.6875\ \text{MPa}$$

$$\sigma_b = \frac{Mc}{I} = \frac{3000(0.02)}{\frac{1}{12}(0.04)(0.04^3)} = 281.25(10^6)\ \text{N/m}^2 = 281.25\ \text{MPa}$$

$$(\sigma_{\max})_{CD} = \sigma_a + \sigma_b = 4.6875 + 281.25$$

$$= 285.9375\ \text{MPa} = 286\ \text{MPa}$$ **Ans.**

Ans:
$(\sigma_{\max})_{AB} = 4.69$ MPa, $(\sigma_{\max})_{CD} = 286$ MPa

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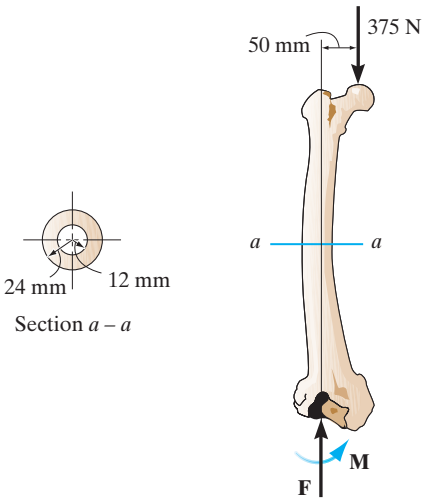
R8–5. If the cross section of the femur at section *a–a* can be approximated as a circular tube as shown, determine the maximum normal stress developed on the cross section at section *a–a* due to the load of 375 N.



SOLUTION

Internal Loadings: Considering the equilibrium for the free-body diagram of the femur's upper segment, Fig. *a*,

$+\uparrow \Sigma F_y = 0;$ $\quad N - 375 = 0$ $\quad N = 375 \text{ N}$

$\circlearrowleft + \Sigma M_O = 0;$ $\quad M - 375(0.05) = 0$ $\quad M = 18.75 \text{ N} \cdot \text{m}$

Section Properties: The cross-sectional area, the moment of inertia about the centroidal axis of the femur's cross section are

$$A = \pi(0.024^2 - 0.012^2) = 0.432(10^{-3})\pi \text{ m}^2$$

$$I = \frac{\pi}{4}(0.024^2 - 0.012^2) = 77.76(10^{-9})\pi \text{ m}^4$$

Normal Stress: The normal stress is a combination of axial and bending stress. Thus,

$$\sigma = \frac{N}{A} + \frac{My}{I}$$

By inspection, the maximum normal stress is in compression.

$$\sigma_{\max} = \frac{-375}{0.432(10^{-3})\pi} - \frac{18.75(0.024)}{77.76(10^{-9})\pi}$$

$$= -2.118(10^6) \text{ N/m}^2 = 2.12 \text{ MPa (C)} \qquad \textbf{Ans.}$$

Ans.
$\sigma_C = 93.7$ MPa (T), $\tau_C = 0$,
$\sigma_D = 187$ MPa (C), $\tau_D = 0$

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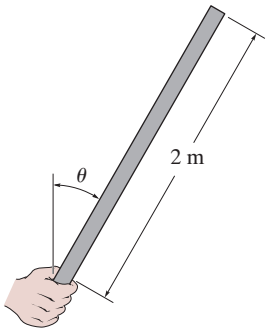
R8–6.

A bar having a square cross section of 30 mm by 30 mm is 2 m long and is held upward. If it has a mass of 5 kg/m, determine the largest angle θ, measured from the vertical, at which it can be supported before it is subjected to a tensile stress along its axis near the grip.



SOLUTION

$A = 0.03(0.03) = 0.9(10^{-3})\ \text{m}^2$

$I = \frac{1}{12}(0.03)(0.03^3) = 67.5(10^{-9})\ \text{m}^4$

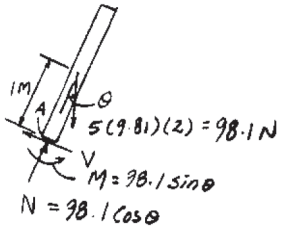
Require $\sigma_A = 0$

$\sigma_A = 0 = \frac{P}{A} + \frac{Mc}{I}$

$0 = \frac{-98.1\cos\theta}{0.9(10^{-3})} + \frac{98.1\sin\theta(0.015)}{67.5(10^{-9})}$

$0 = -1111.11\cos\theta + 222222.22\sin\theta$

$\tan\theta = 0.005; \qquad \theta = 0.286°$ **Ans.**



Ans:
$\theta = 0.286°$

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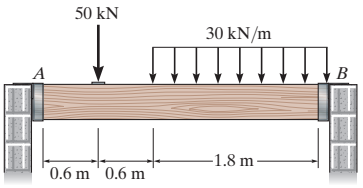
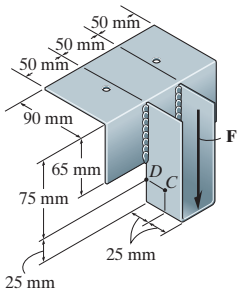
R8–7. The wall hanger has a thickness of 6 mm and is used to support the vertical reactions of the beam that is loaded as shown. If the load is transferred uniformly to each strap of the hanger, determine the state of stress at points *C* and *D* on the strap at *A*. Assume the vertical reaction **F** at this end acts in the center and on the edge of the bracket as shown.





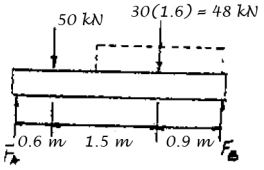
SOLUTION

$\zeta + \Sigma M_B = 0;$ $\quad 54(0.9) + 50(2.4) - F_A(3) = 0$

$F_A = 56.2$ kN

$$I = 2\left[\frac{1}{12}(0.006)(0.05^3)\right] = 0.125(10^{-6})\ \text{m}^4$$

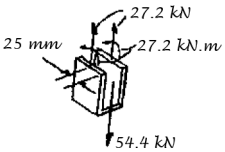
$$A = 2(0.006)(0.05) = 0.6(10^{-3})\ \text{m}^2$$



At point *C*,

$$\sigma_C = \frac{P}{A} = \frac{2[28.1(10^3)]}{0.6(10^{-3})} = 93.67(10^6)\ \text{N/m}^2 = 93.7\ \text{MPa (T)}$$ **Ans.**

$\tau_C = 0$ **Ans.**



At point *D*,

$$\sigma_D = \frac{P}{A} - \frac{Mc}{I} = \frac{2[28.1(10^3)]}{0.6(10^{-3})} - \frac{2[0.7025(10^3)](0.025)}{0.125(10^{-6})}$$

$$= -187.33(10^6)\ \text{N/m}^2 = 187\ \text{MPa (C)}$$ **Ans.**

$\tau_D = 0$ **Ans.**

Ans:
$\sigma_C = 93.7$ MPa (T), $\tau_C = 0$,
$\sigma_D = -187$ MPa (C), $\tau_D = 0$

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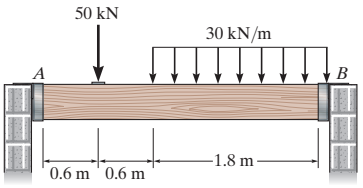
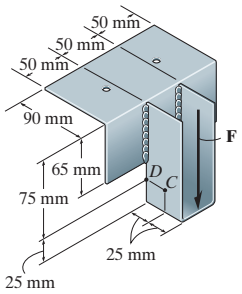
***R8–8.** The wall hanger has a thickness of 6 mm and is used to support the vertical reactions of the beam that is loaded as shown. If the load is transferred uniformly to each strap of the hanger, determine the state of stress at points *C* and *D* on the strap at *B*. Assume the vertical reaction **F** at this end acts in the center and on the edge of the bracket as shown.





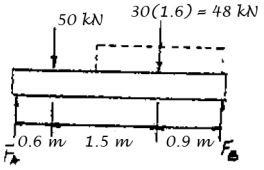
SOLUTION

$\zeta + \Sigma M_A = 0;$ $\quad -54(2.1) - 50(0.6) + F_B(3) = 0$

$F_B = 47.8$ kN

$A = 2(0.006)(0.05) = 0.6(10^{-3})\ \text{m}^2$

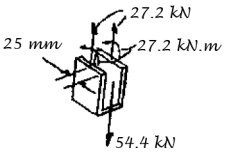
$I = 2\left[\frac{1}{12}(0.006)(0.05^3)\right] = 0.125(10^{-6})\ \text{m}^4$



At point *C*,

$$\sigma_C = \frac{P}{A} = \frac{2[23.9(10^3)]}{0.6(10^{-3})} = 79.67(10^6)\ \text{N/m}^2 = 79.7\ \text{MPa (T)}$$ **Ans.**

$\tau_C = 0$ **Ans.**



At point *D*,

$$\sigma_D = \frac{P}{A} - \frac{Mc}{I} = \frac{2[23.9(10^3)]}{0.6(10^{-3})} - \frac{2[0.5975(10^3)](0.025)}{0.125(10^{-6})}$$

$= -159.33(10^6)\ \text{N/m}^2 = 159\ \text{MPa (C)}$ **Ans.**

$\tau_D = 0$ **Ans.**

Ans:
$\sigma_C = 79.7$ MPa (T), $\tau_C = 0$,
$\sigma_D = 159$ MPa (C), $\tau_D = 0$

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9–1.

Prove that the sum of the normal stresses $\sigma_x + \sigma_y = \sigma_{x'} + \sigma_{y'}$ is constant. See Figs. 9–2*a* and 9–2*b*.

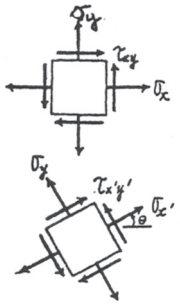
SOLUTION

Stress Transformation Equations: Applying Eqs. 9–1 and 9–3 of the text.

$$\sigma_{x'} + \sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$+ \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

$$\sigma_{x'} + \sigma_{y'} = \sigma_x + \sigma_y \qquad \textbf{(Q. E. D.)}$$



These solutions represent a preliminary version of the Instructors' Solutions Manual (ISM). It is possible and even likely that at this preliminary stage of preparing the ISM there are some omissions and errors in the draft solutions. These will be corrected and this manual will be republished.

Ans:
N/A

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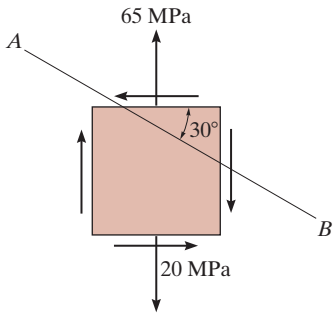
9–2.

Determine the stress components acting on the inclined plane AB. Solve the problem using the method of equilibrium described in Sec. 9.1.



SOLUTION

$\swarrow +\Sigma F_{x'} = 0;\quad \sigma_{x'}\Delta A + 20\Delta A \sin 30^\circ \cos 30^\circ + 20\Delta A \cos 30^\circ \cos 60^\circ$

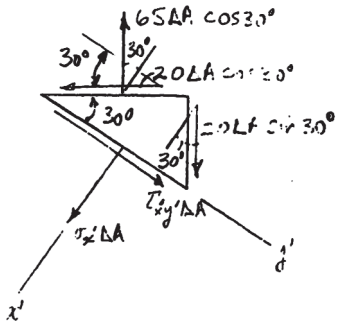
$$- 65\Delta A \cos 30^\circ \cos 30^\circ = 0$$

$$\sigma_{x'} = 31.4 \text{ MPa} \qquad \textbf{Ans.}$$

$\searrow +\Sigma F_{y'} = 0;\quad \tau_{x'y'}\Delta A + 20\Delta A \sin 30^\circ \sin 30^\circ - 20\Delta A \cos 30^\circ \sin 60^\circ$

$$- 65\Delta A \cos 30^\circ \sin 30^\circ = 0$$

$$\tau_{x'y'} = 38.1 \text{ MPa} \qquad \textbf{Ans.}$$



Ans:
$\sigma_{x'} = 31.4$ MPa,
$\tau_{x'y'} = 38.1$ MPa

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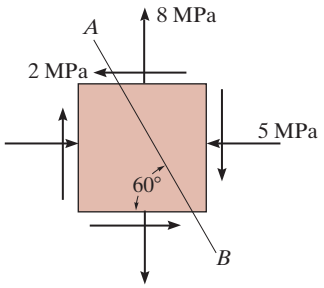
9–3. The state of stress at a point in a member is shown on the element. Determine the stress components acting on the inclined plane AB. Solve the problem using the method of equilibrium described in Sec. 9.1.



SOLUTION

Referring to Fig *a*, if we assume that the areas of the inclined plane AB is ΔA, then the area of the horizontal and vertical of the triangular element are $\Delta A\cos 60°$ and $\Delta A\sin 60°$ respectively. The forces act acting on these two faces indicated on the FBD of the triangular element, Fig. *b*.

$$+\nearrow \Sigma F_{x'} = 0; \quad \Delta F_{x'} + 2\Delta A\sin 60°\cos 60° + 5\Delta A\sin 60°\sin 60° + 2\Delta A\cos 60°\sin 60° - 8\Delta A\cos 60°\cos 60° = 0$$

$$\Delta F_{x'} = -3.482\,\Delta A$$

$$+\nwarrow \Sigma F_{y'} = 0; \quad \Delta F_{y'} + 2\Delta A\sin 60°\sin 60° - 5\Delta A\sin 60°\cos 60° - 8\Delta A\cos 60°\sin 60° - 2\Delta A\cos 60°\cos 60° = 0$$

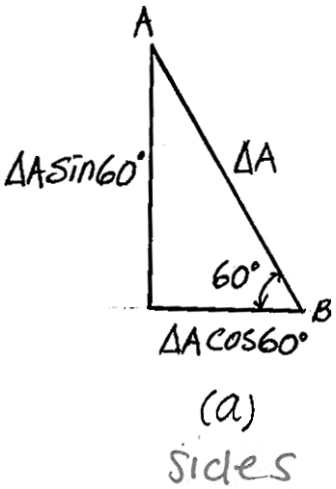
$$\Delta F_{y'} = 4.629\,\Delta A$$

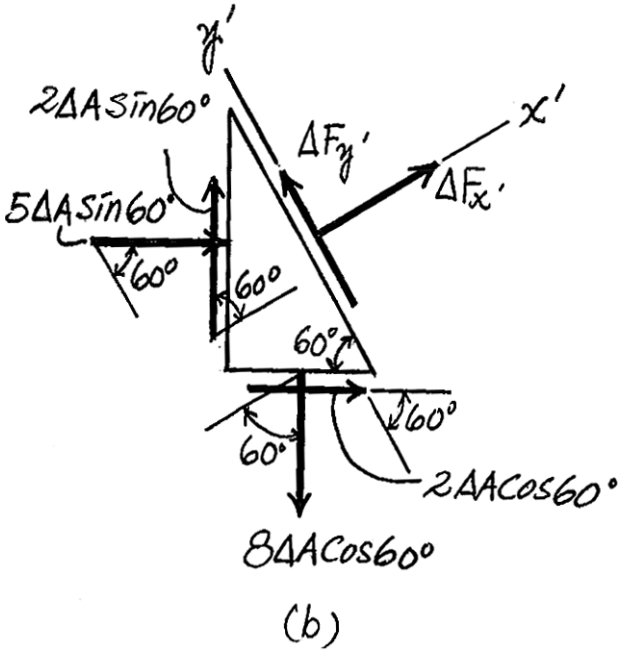
From the definition,

$$\sigma_{x'} = \lim_{\Delta A\to 0}\frac{\Delta F_{x'}}{\Delta A} = -3.48\,\text{MPa}$$ **Ans.**

$$\tau_{x'y'} = \lim_{\Delta A\to 0}\frac{\Delta F_{y'}}{\Delta A} = 4.63\,\text{MPa}$$ **Ans.**

The negative sign indicates that $\sigma_{x'}$, is a compressive stress.





Ans.

$\sigma_{x'} = -3.48$ MPa, $\tau_{x'y'} = 4.63$ MPa

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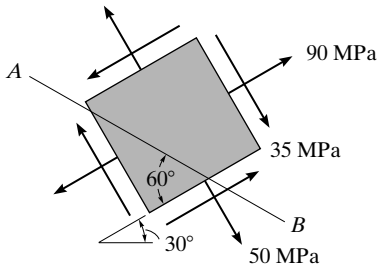
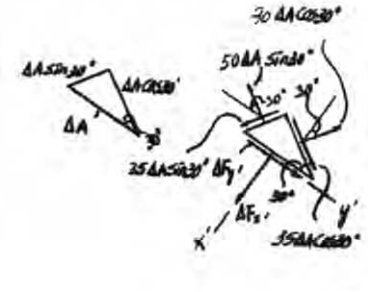
***9–4.** The state of stress at a point in a member is shown on the element. Determine the stress components acting on the inclined plane AB. Solve the problem using the method of equilibrium described in Sec. 9.1.



SOLUTION



$\nearrow+\Sigma F_{y'} = 0 \qquad \Delta F_{y'} - 50\Delta A \sin 30^\circ \cos 30^\circ - 35\Delta A \sin 30^\circ \cos 60^\circ +$

$$90\Delta A \cos 30^\circ \sin 30^\circ + 35\Delta A \cos 30^\circ \sin 60^\circ = 0$$

$$\Delta F_{y'} = -34.82\Delta A$$

$\nwarrow+\Sigma F_{x'} = 0 \qquad \Delta F_{x'} - 50\Delta A \sin 30^\circ \sin 30^\circ + 35\Delta A \sin 30^\circ \sin 60^\circ$

$$-90\Delta A \cos 30^\circ \cos 30^\circ + 35\Delta A \cos 30^\circ \cos 60^\circ = 0$$

$$\Delta F_{x'} = 49.69\ \Delta A$$

$\sigma_{x'} = \lim_{\Delta A \to 0} \dfrac{\Delta F_{x'}}{\Delta A} = 49.7$ MPa **Ans.**

$\tau_{x'y'} = \lim_{\Delta A \to 0} \dfrac{\Delta F_{y'}}{\Delta A} = -34.8$ MPa **Ans.**

The negative signs indicate that the sense of $\sigma_{x'}$, and $\tau_{x'y'}$ are opposite to the shown on FBD.

Ans.

$\sigma_{x'} = 49.7$ MPa, $\tau_{x'y'} = -34.8$ MPa

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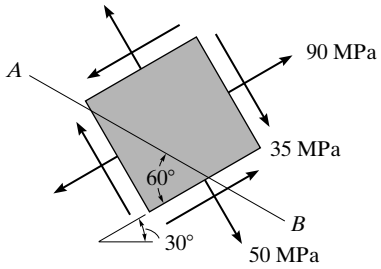
9–5. Solve Prob. 9–6 using the stress-transformation equations developed in Sec. 9.2. Show the result on a sketch.



SOLUTION

$\sigma_x = 90\ \text{MPa} \quad \sigma_y = 50\ \text{MPa} \quad \tau_{xy} = -35\ \text{MPa} \quad \theta = -150°$

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= \frac{90 + 50}{2} + \frac{90 - 50}{2}\cos(-300°) + (-35)\sin(-300°)$$

$$= 49.7\ \text{MPa} \qquad \textbf{Ans.}$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$= -\left(\frac{90 - 50}{2}\right)\sin(-300°) + (-35)\cos(-300°) = -34.8\ \text{MPa} \qquad \textbf{Ans.}$$

The negative sign indicates $\tau_{x'y'}$ acts in $-y'$ direction.

Ans.

$\sigma_{x'} = 49.7$ MPa, $\tau_{x'y'} = -34.8$ MPa

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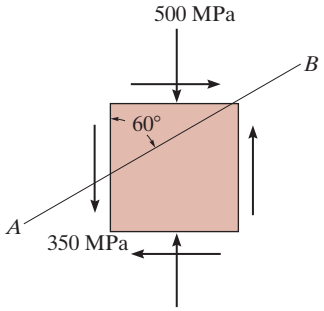
9–6. The state of stress at a point in a member is shown on the element. Determine the stress components acting on the inclined plane AB. Solve the problem using the method of equilibrium described in Sec. 9.1.



SOLUTION

Referring to Fig. *a*, if we assume that the area of the inclined plane AB is ΔA, then the areas of the horizontal and vertical surfaces of the triangular element are $\Delta A \sin 60°$ and $\Delta A \cos 60°$ respectively. The force acting on these two faces are indicated on the FBD of the triangular element, Fig. *b*

$$+\searrow \Sigma F_{x'} = 0; \quad \Delta F_{x'} + 500\,\Delta A \sin 60° \sin 60° + 350\Delta A \sin 60° \cos 60°$$

$$+350\Delta A \cos 60° \sin 60° = 0$$

$$\Delta F_{x'} = -678.11\,\Delta A$$

$$+\nearrow \Sigma F_{y'} = 0; \quad \Delta F_{y'} + 350\Delta A \sin 60° \sin 60° - 500\Delta A \sin 60° \cos 60°$$

$$-350\Delta A \cos 60° \cos 60° = 0$$

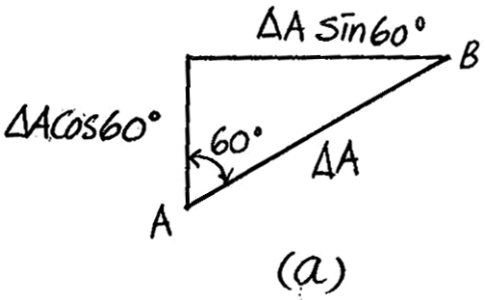
$$\Delta F_{y'} = 41.51\,\Delta A$$

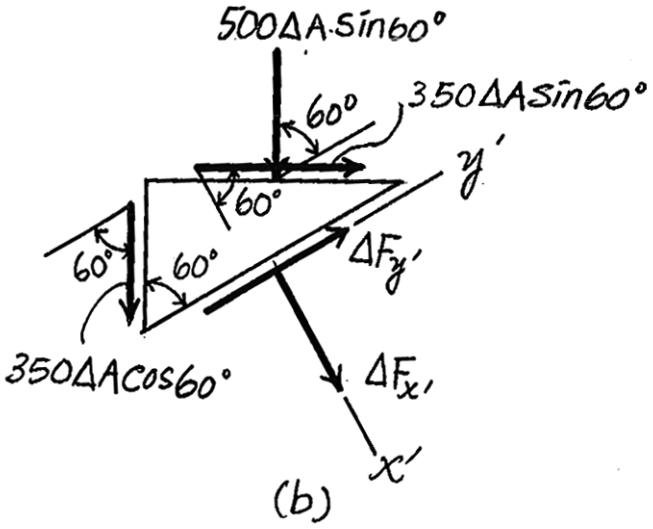
From the definition

$$\sigma_{x'} = \lim_{\Delta A \to 0} \frac{\Delta F_{x'}}{\Delta A} = -678 \text{ MPa} \qquad \textbf{Ans.}$$

$$\tau_{x'y'} = \lim_{\Delta A \to 0} \frac{\Delta F_{y'}}{\Delta A} = 41.5 \text{ MPa} \qquad \textbf{Ans.}$$

The negative sign indicates that $\sigma_{x'}$, is a compressive stress.





Ans.
$\sigma_{x'} = -678$ MPa, $\tau_{x'y'} = 41.5$ MPa

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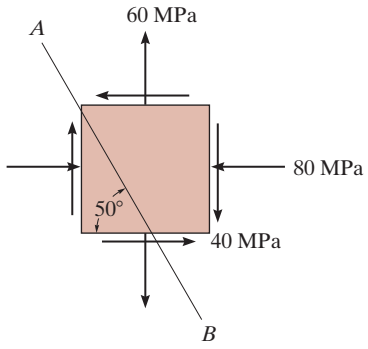
9–7.

Determine the stress components acting on the inclined plane AB. Solve the problem using the method of equilibrium described in Sec. 9.1.



SOLUTION

Force Equilibrium: The areas, thus the forces acting on the faces of the lower segment of the sectioned element are shown in Fig. *a*

$$\nwarrow + \Sigma F_{y'} = 0; \quad \Delta F_{y'} + (40\ \Delta A \sin 50°) \sin 50° - (80\ \Delta A \sin 50°) \cos 50° - (40\ \Delta A \cos 50°) \cos 50° - (60\ \Delta A \cos 50°) \sin 50° = 0$$

$$\Delta F_{y'} = 61.99\ \Delta A$$

$$+\nearrow \Sigma F_{x'} = 0; \quad \Delta F_{x'} + (40\ \Delta A \sin 50°) \cos 50° + (80\ \Delta A \sin 50°) \sin 50° + (40\ \Delta A \cos 50°) \sin 50° - (60\ \Delta A \cos 50°) \cos 50° = 0$$

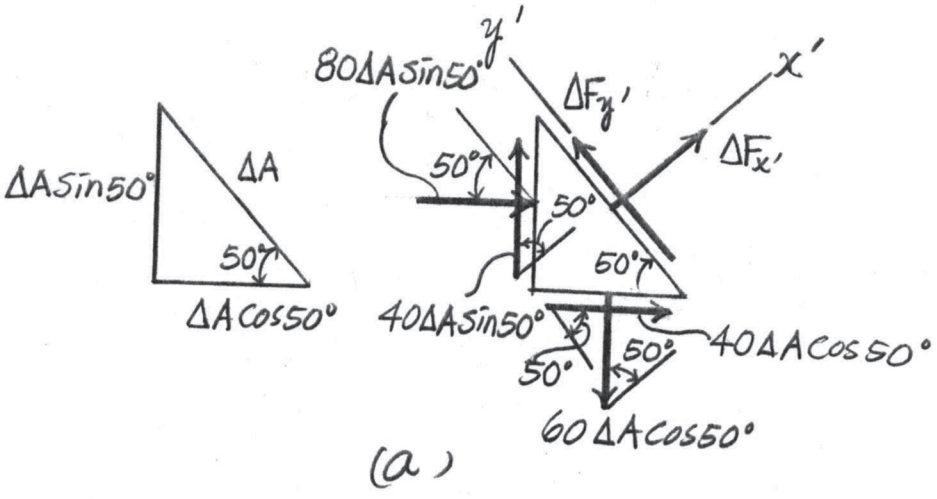
$$\Delta F_{x'} = -61.54\ \Delta A$$

Normal And Shear Stresses: For the inclined plane that has an area of ΔA,

$$\sigma_{x'} = \lim_{\Delta A \to 0} \frac{\Delta F_{x'}}{\Delta A} = -61.54\ \text{MPa} = -61.5\ \text{MPa} \qquad \textbf{Ans.}$$

$$\tau_{x'y'} = \lim_{\Delta A \to 0} \frac{\Delta F_{y'}}{\Delta A} = 61.99\ \text{MPa} = 62.0\ \text{MPa} \qquad \textbf{Ans.}$$

The negative sign indicates that $\sigma_{x'}$ is a compressive normal stress.



Ans:
$\sigma_{x'} = -61.5$ MPa,
$\tau_{x'y'} = 62.0$ MPa

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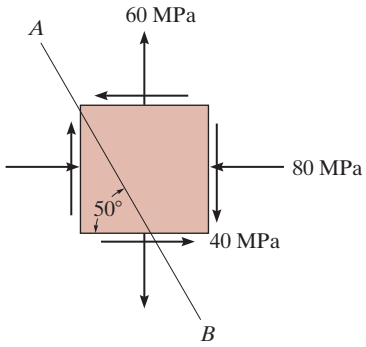
***9–8.**

Solve Prob. 9–7 using the stress transformation equations developed in Sec. 9.2.



SOLUTION

Normal And Shear Stress: In accordance with the established sign conventions,

$\theta = +40°$ (Fig. *a*) $\quad \sigma_x = -80$ MPa $\quad \sigma_y = 60$ MPa $\quad \tau_{xy} = -40$ MPa

Stress Transformation Equations:

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= \frac{-80 + 60}{2} + \frac{-80 - 60}{2}\cos 80° + (-40)\sin 80°$$

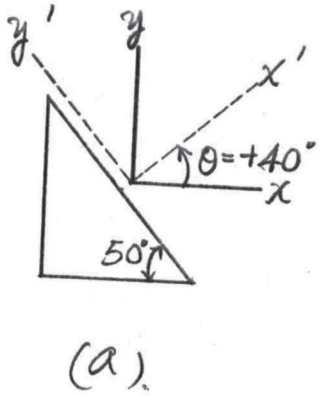
$$= -61.54 \text{ MPa} = -61.5 \text{ MPa} \qquad \textbf{Ans.}$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

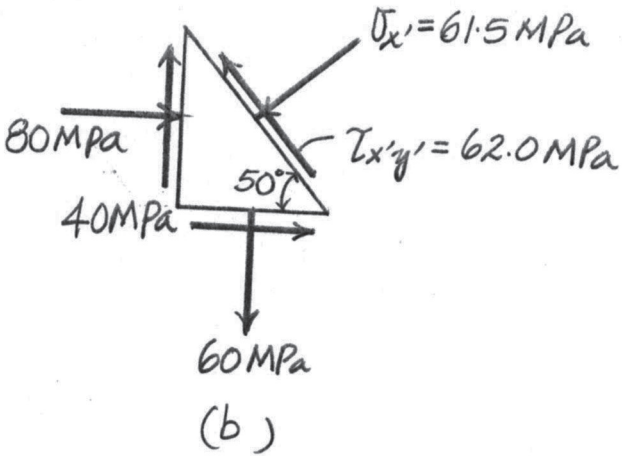
$$= -\left(\frac{-80 - 60}{2}\right)\sin 80° + (-40)\cos 80°$$

$$= 61.99 \text{ MPa} = 62.0 \text{ MPa} \qquad \textbf{Ans.}$$

These results are indicated on the sectioned element shown in Fig. *b*



(a)



(b)

Ans:
$\sigma_{x'} = -61.5$ MPa,
$\tau_{x'y'} = 62.0$ MPa

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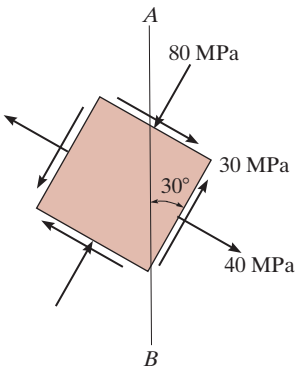
9–9.

Determine the stress components acting on the plane AB. Solve the problem using the method of equilibrium described in Sec. 9.1.



SOLUTION

Force Equilibrium: The areas, thus the forces acting on the faces of the upper segment of the sectioned element are shown in Fig. *a*

$$+\downarrow \Sigma F_{y'} = 0; \quad \Delta F_{y'} + (80\ \Delta A \sin 30°) \cos 30° + (30\ \Delta A \sin 30°) \sin 30° - (30\ \Delta A \cos 30°) \cos 30° + (40\ \Delta A \cos 30°) \sin 30° = 0$$

$$\Delta F_{y'} = -36.96\ \Delta A$$

$$\overset{+}{\leftarrow} \Sigma F_{x'} = 0; \quad \Delta F_{x'} - (30\ \Delta A \sin 30°) \cos 30° + (80\ \Delta A \sin 30°) \sin 30° - (30\ \Delta A \cos 30°) \sin 30° - (40\ \Delta A \cos 30°) \cos 30° = 0$$

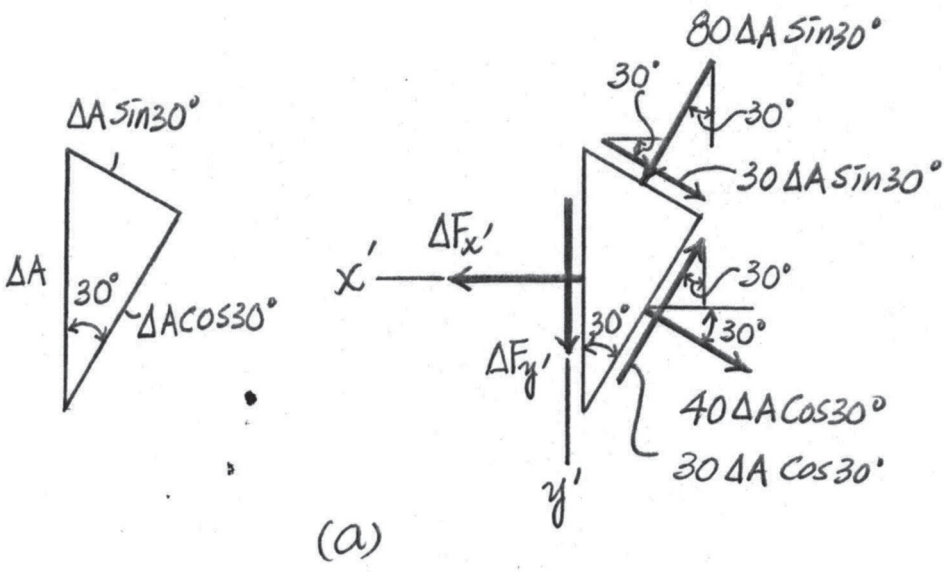
$$\Delta F_{x'} = 35.98\ \Delta A$$

Normal And Shear Stress: For the inclined plane that has an area of ΔA,

$$\sigma_{x'} = \lim_{\Delta A \to 0} = \frac{\Delta F_{x'}}{\Delta A} = 35.98 \text{ MPa} = 36.0 \text{ MPa} \qquad \textbf{Ans.}$$

$$\tau_{x'y'} = \lim_{\Delta A \to 0} = \frac{\Delta F_{y'}}{\Delta A} = -36.96 \text{ MPa} = -37.0 \text{ MPa} \qquad \textbf{Ans.}$$

The negative sign indicates that $\tau_{x'y'}$ acts in the sense opposite to that shown in the FBD.



Ans:
$\sigma_{x'} = 36.0$ MPa,
$\tau_{x'y'} = -37.0$ MPa

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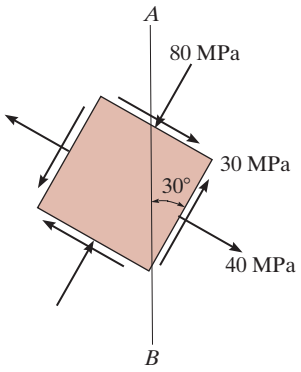
9–10.

Solve Prob. 9–9 using the stress transformation equation developed in Sec. 9.2.



SOLUTION

Normal And Shear Stress: In accordance with the established sign conventions,

$\theta = +120°$ (Fig. *a*) $\quad \sigma_x = -80$ MPa $\quad \sigma_y = 40$ MPa $\quad \tau_{xy} = -30$ MPa

Stress Transformation Equations:

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= \frac{-80 + 40}{2} + \left(\frac{-80 - 40}{2}\right)\cos 240° + (-30)\sin 240°$$

$$= 35.98 \text{ MPa} = 36.0 \text{ MPa} \qquad \textbf{Ans.}$$

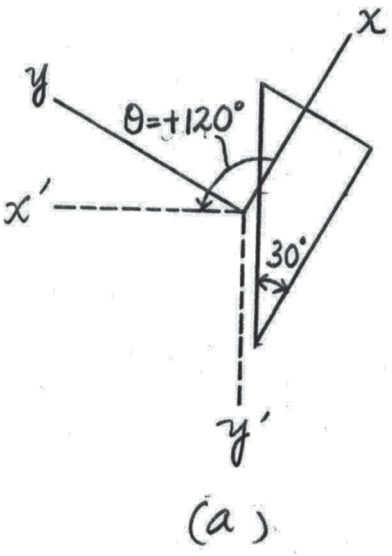
$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

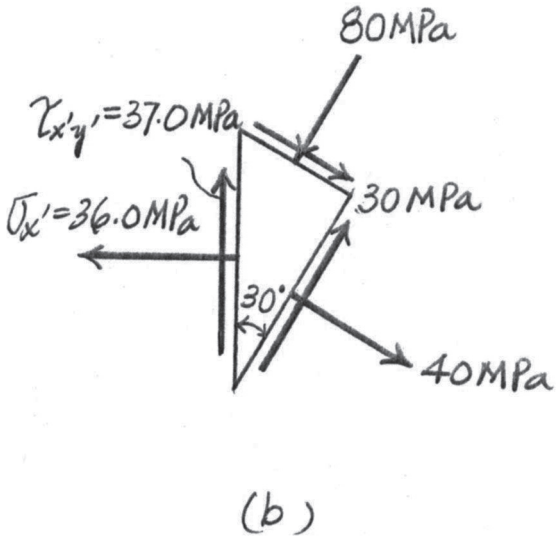
$$= -\left(\frac{-80 - 40}{2}\right)\sin 240° + (-30)\cos 240°$$

$$= -36.96 \text{ MPa} = -37.0 \text{ MPa} \qquad \textbf{Ans.}$$

The negative sign indicates that $\tau_{x'y'}$ acts in the negative y' direction.

These results are indicated on the sectioned element shown in Fig. *b*





Ans:
$\sigma_{x'} = 36.0$ MPa,
$\tau_{x'y'} = -37.0$ MPa

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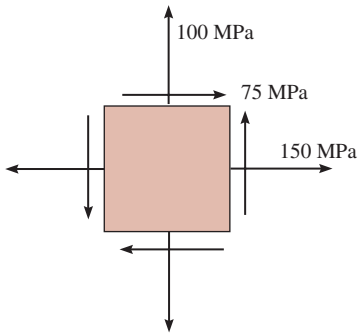
9–11.

Determine the equivalent state of stress on an element at the same point oriented 60° clockwise with respect to the element shown. Sketch the results on the element.



SOLUTION

Stress Transformation Equations:

$\theta = -60°$ (Fig. *a*) $\quad \sigma_x = 150\ \text{MPa} \quad \sigma_y = 100\ \text{MPa} \quad \tau_{xy} = 75\ \text{MPa}$

We obtain,

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= \frac{150 + 100}{2} + \frac{150 - 100}{2}\cos(-120°) + 75\sin(-120°)$$

$$= 47.5\ \text{MPa} \qquad \textbf{Ans.}$$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

$$= \frac{150 + 100}{2} - \frac{150 - 100}{2}\cos(-120°) - 75\sin(-120°)$$

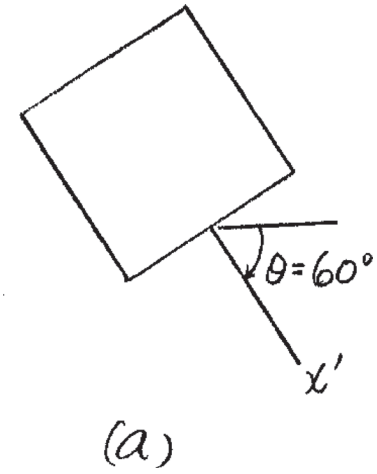
$$= 202\ \text{MPa} \qquad \textbf{Ans.}$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

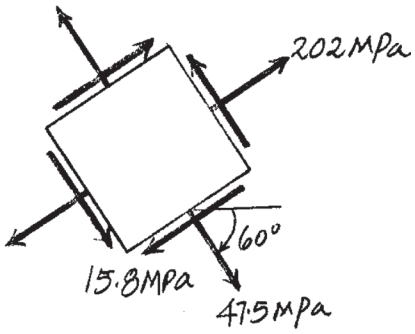
$$= -\frac{150 - 100}{2}\sin(-120°) + 75\cos(-120°)$$

$$= -15.8\ \text{MPa} \qquad \textbf{Ans.}$$

The negative sign indicates that $\tau_{x'y'}$ is directed towards the negative sense of the y' axis. These results are indicated on the element shown in Fig. *b*.



(a)



(b)

Ans:
$\sigma_{x'} = 47.5$ MPa, $\sigma_{y'} = 202$ MPa,
$\tau_{x'y'} = -15.8$ MPa

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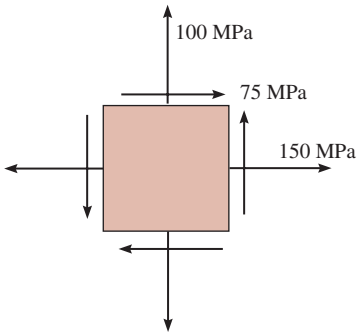
***9–12.**

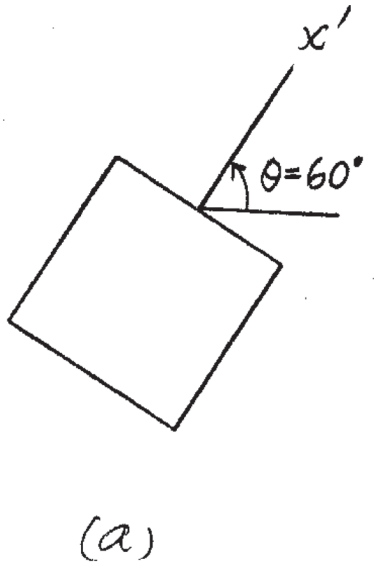
Determine the equivalent state of stress on an element at the same point oriented 60° counterclockwise with respect to the element shown. Sketch the results on the element.



SOLUTION

Stress Transformation Equations:

$\theta = +60°$ (Fig. *a*) $\quad \sigma_x = 150$ MPa $\quad \sigma_y = 100$ MPa $\quad \tau_{xy} = 75$ MPa



(a)

We obtain,

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= \frac{150 + 100}{2} + \frac{150 - 100}{2}\cos 120° + 75 \sin 120°$$

$$= 177 \text{ MPa} \qquad \textbf{Ans.}$$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

$$= \frac{150 + 100}{2} - \frac{150 - 100}{2}\cos 120° - 75 \sin 120°$$

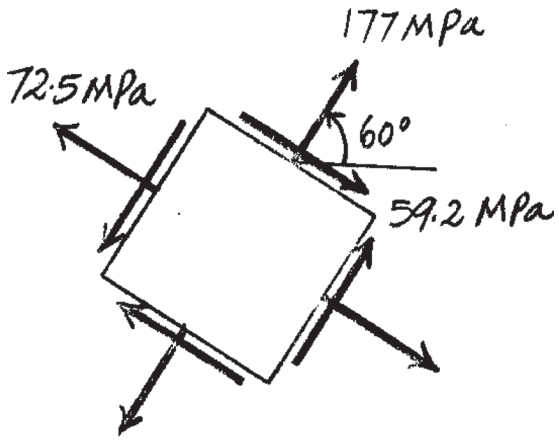
$$= 72.5 \text{ MPa} \qquad \textbf{Ans.}$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$= -\frac{150 - 100}{2}\sin 120° + 75 \cos 120°$$

$$= -59.2 \text{ MPa} \qquad \textbf{Ans.}$$

The negative sign indicates that $\tau_{x'y'}$ is directed towards the negative sense of the y' axis. These results are indicated on the element shown in Fig. *b*.



(b)

Ans:
$\sigma_{x'} = 177$ MPa, $\sigma_{y'} = 72.5$ MPa,
$\tau_{x'y'} = -59.2$ MPa

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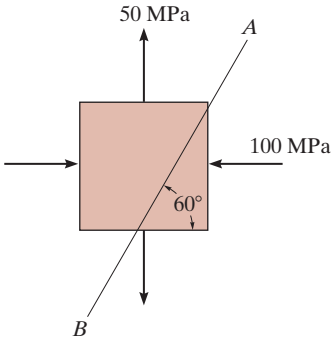
9–13.

Determine the stress components acting on the inclined plane AB. Solve the problem using the method of equilibrium described in Sec. 9.1.



SOLUTION

Force Equilibrium: The areas, thus the forces acting on the faces of the lower segment of the sectioned element, are shown in Fig. *a*.

$+\swarrow \Sigma F_{y'} = 0; \quad \Delta F_{y'} + (100\ \Delta A \sin 60°) \cos 60° + (50\ \Delta A \cos 60°) \sin 60° = 0$

$$\Delta F_{y'} = -64.95\ \Delta A$$

$\nwarrow + \Sigma F_{x'} = 0; \quad \Delta F_{x'} + (100\ \Delta A \sin 60°) \sin 60° - (50\ \Delta A \cos 60°) \cos 60° = 0$

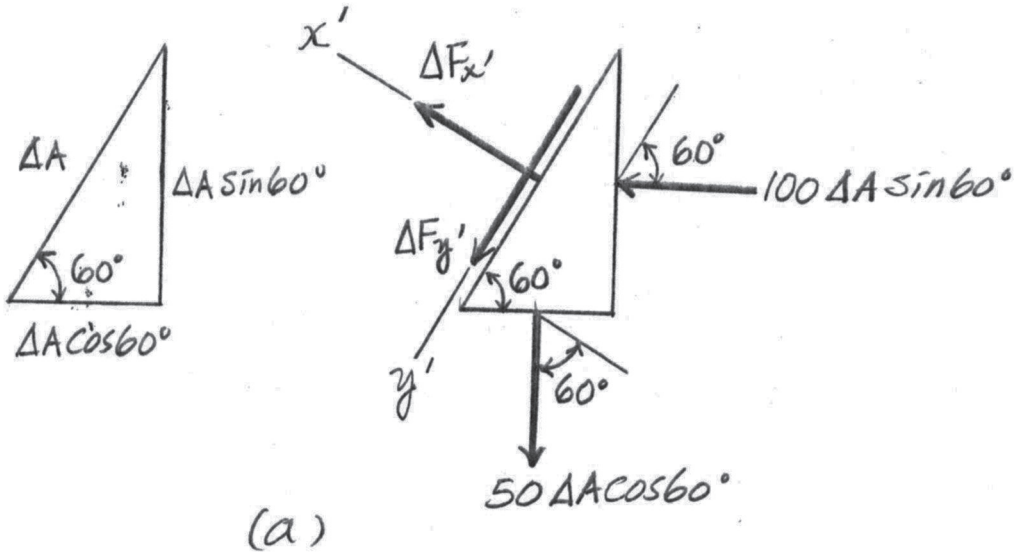
$$\Delta F_{x'} = -62.5\ \Delta A$$

Normal And Shear Stress: For the incline plane that has an area of ΔA,

$$\sigma_{x'} = \lim_{\Delta A \to 0} \frac{\Delta F_{x'}}{\Delta A} = -62.5\ \text{MPa} = -62.5\ \text{MPa} \qquad \textbf{Ans.}$$

$$\tau_{x'y'} = \lim_{\Delta A \to 0} \frac{\Delta F_{y'}}{\Delta A} = -64.95\ \text{MPa} = -65.0\ \text{MPa} \qquad \textbf{Ans.}$$

The negative signs indicate that $\sigma_{x'}$ is a compressive normal stress and $\tau_{x'y'}$ acts in the sense opposite to that shown in the FBD.



Ans:
$\sigma_{x'} = -62.5$ MPa,
$\tau_{x'y'} = -65.0$ MPa

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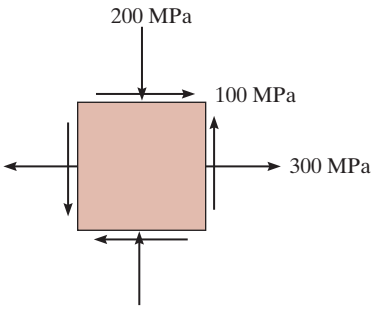
9–14.

Determine (a) the principal stresses and (b) the maximum in-plane shear stress and average normal stress at the point. Specify the orientation of the element in each case.



SOLUTION

Normal And Shear Stress: In accordance with the established sign conventions,

$\sigma_x = 300\ \text{MPa} \qquad \sigma_y = -200\ \text{MPa} \qquad \tau_{xy} = 100\ \text{MPa}$

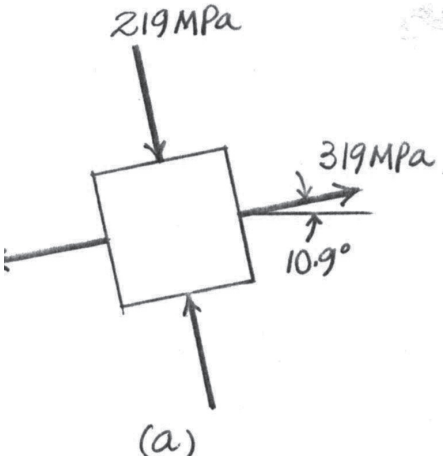
a) In-Plane Principal Stresses:

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + {\tau_{xy}}^2}$$

$$= \frac{300 + (-200)}{2} \pm \sqrt{\left[\frac{300 - (-200)}{2}\right]^2 + 100^2}$$

$$= 50 \pm 269.26$$

$\sigma_1 = 319.26\ \text{MPa} = 319\ \text{MPa} \qquad \sigma_2 = -219.26\ \text{MPa} = -219\ \text{MPa}$ **Ans.**



Orientation of Principal Plane:

$$\tan 2\theta_p = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2} = \frac{100}{[300 - (-200)]/2} = 0.4$$

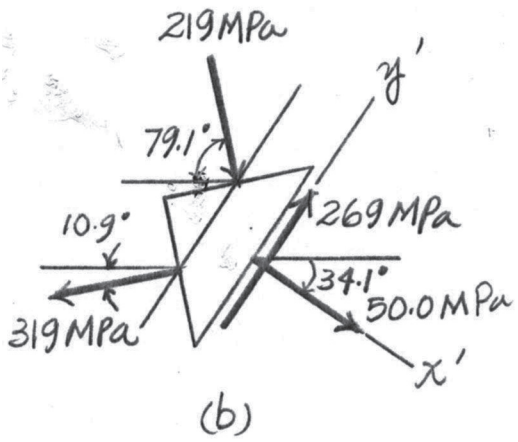
$$\theta_p = 10.90° \text{ and } -79.10°$$

Substitute the result of $\theta = 10.90°$

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= \frac{300 + (-200)}{2} + \frac{300 - (-200)}{2}\cos 21.80° + 100 \sin 21.80°$$

$$= 319.26\ \text{MPa} = \sigma_1$$



Hence,

$\theta_{p1} = 10.90° = 10.9° \qquad \theta_{p2} = -79.10° = -79.1°$ **Ans.**

Using these results, the state of in-plane principal stress can be represented by the differential element shown in Fig. *a*.

b) Maximum In-Plane Shear Stress:

$$\tau_{\substack{\max \\ \text{in-plane}}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + {\tau_{xy}}^2}$$

$$= \sqrt{\left[\frac{300 - (-200)}{2}\right]^2 + 100^2} = 269.26\ \text{MPa} = 269\ \text{MPa}$$ **Ans.**

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9–14. Continued

Orientation of the Plane for Maximum In-Plane Shear Stress:

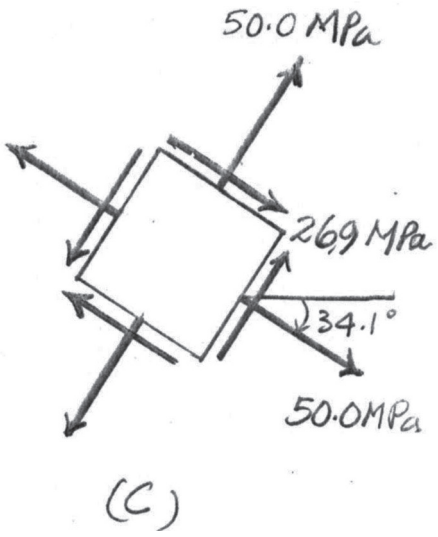
$$\tan 2\theta_s = \frac{-(\sigma_x - \sigma_y)/2}{\tau_{xy}} = \frac{-[300 - (-200)]/2}{100} = -2.5$$

$\theta_s = -34.10° = -34.1°$ and $55.90° = 55.9°$ **Ans.**

Average Normal Stress:

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{300 + (-200)}{2} = 50.0 \text{ MPa}$$ **Ans.**

By observing the segment of the element of principal stresses sectioned through the diagonal, Fig. *b*, equilibrium along y' axis requires that $\tau_{\text{max}}_{\text{in-plane}}$ to act in the direction shown. Thus, the state of maximum in-plane shear stress can be represented by the differential element shown in Fig. *c*.



Ans:
$\sigma_1 = 319$ MPa,
$\sigma_2 = -219$ MPa,
$\theta_{p1} = 10.9°$,
$\theta_{p2} = -79.1°$,
$\tau_{\text{max}}_{\text{in-plane}} = 269$ MPa,
$\theta_s = -34.1°$ and $55.9°$,
$\sigma_{avg} = 50.0$ MPa

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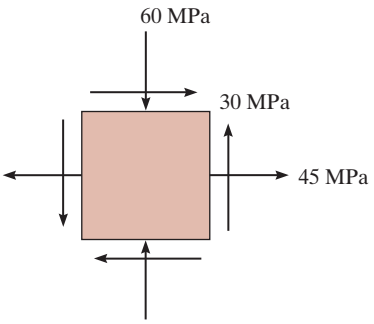
9–15.

The state of stress at a point is shown on the element. Determine (a) the principal stresses and (b) the maximum in-plane shear stress and average normal stress at the point. Specify the orientation of the element in each case.



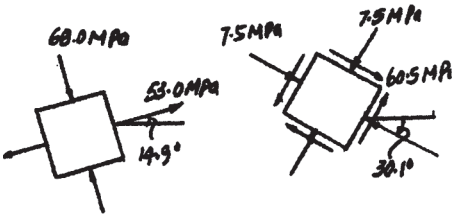
SOLUTION

$\sigma_x = 45$ MPa $\quad \sigma_y = -60$ MPa $\quad \tau_{xy} = 30$ MPa

a) $$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{45 - 60}{2} \pm \sqrt{\left(\frac{45 - (-60)}{2}\right)^2 + (30)^2}$$

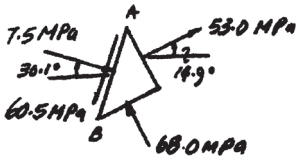
$\sigma_1 = 53.0$ MPa $\qquad \sigma_2 = -68.0$ MPa **Ans.**



Orientation of principal stress:

$$\tan 2\theta_p = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2} = \frac{30}{(45 - (-60))/2} = 0.5714$$

$\theta_p = 14.87, \quad -75.13$



Use Eq. 9–1 to determine the principal plane of σ_1 and σ_2:

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta, \qquad \text{where } \theta = 14.87°$$

$$= \frac{45 + (-60)}{2} + \frac{45 - (-60)}{2}\cos 29.74° + 30 \sin 29.74° = 53.0 \text{ MPa}$$

Therefore $\theta_{p1} = 14.9°$ and $\theta_{p2} = -75.1°$ **Ans.**

b)

$$\tau_{\substack{\max \\ \text{in-plane}}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{45 - (-60)}{2}\right)^2 + 30^2}$$

$= 60.5$ MPa **Ans.**

$$\sigma_{\text{avg}} = \frac{\sigma_x + \sigma_y}{2} = \frac{45 + (-60)}{2} = -7.50 \text{ MPa}$$ **Ans.**

Orientation of maximum in-plane shear stress:

$$\tan 2\theta_s = \frac{-(\sigma_x - \sigma_y)/2}{\tau_{xy}} = \frac{-(45 - (-60))/2}{30} = -1.75$$

$\theta_s = -30.1°$ and $\theta_s = 59.9°$ **Ans.**

By observation, in order to preserve equilibrium along AB, $\tau_{\max}$ has to act in the direction shown.

Ans:
$\sigma_1 = 53.0$ MPa,
$\sigma_2 = -68.0$ MPa,
$\theta_{p1} = 14.9°$,
$\theta_{p2} = -75.1°$,
$\tau_{\substack{\max \\ \text{in-plane}}} = 60.5$ MPa,
$\sigma_{\text{avg}} = -7.50$ MPa,
$\theta_s = -30.1°, \theta_s = 59.9°$

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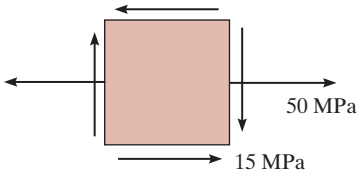
***9–16.**

Determine the equivalent state of stress on an element at the point which represents (a) the principal stresses and (b) the maximum in-plane shear stress and the associated average normal stress. Also, for each case, determine the corresponding orientation of the element with respect to the element shown and sketch the results on the element.



SOLUTION

Normal and Shear Stress:

$$\sigma_x = 50 \text{ MPa} \qquad \sigma_y = 0 \qquad \tau_{xy} = -15 \text{ MPa}$$

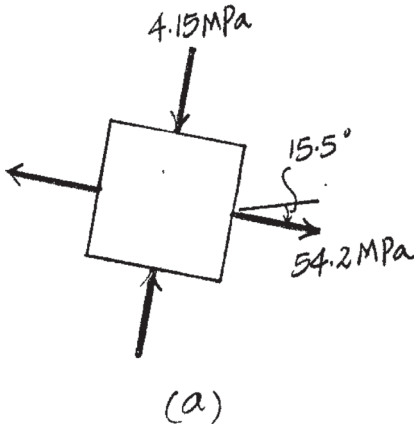
In-Plane Principal Stresses:

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + {\tau_{xy}}^2}$$

$$= \frac{50 + 0}{2} \pm \sqrt{\left(\frac{50 - 0}{2}\right)^2 + (-15)^2}$$

$$= 25 \pm \sqrt{850}$$

$$\sigma_1 = 54.2 \text{ MPa} \qquad \sigma_2 = -4.15 \text{ MPa} \qquad \textbf{Ans.}$$



Orientation of Principal Plane:

$$\tan 2\theta_p = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2} = \frac{-15}{(50 - 0)/2} = -0.6$$

$$\theta_p = -15.48° \text{ and } 74.52°$$

Substitute $\theta = -15.48°$ into

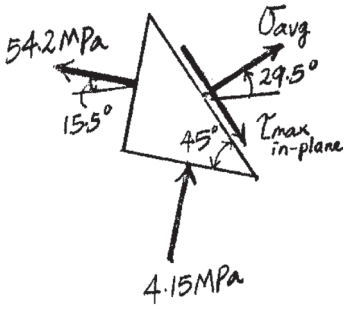
$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

$$= \frac{50 + 0}{2} + \frac{50 - 0}{2} \cos(-30.96°) + (-15) \sin(-30.96°)$$

$$= 54.2 \text{ MPa} = \sigma_1$$

Thus,

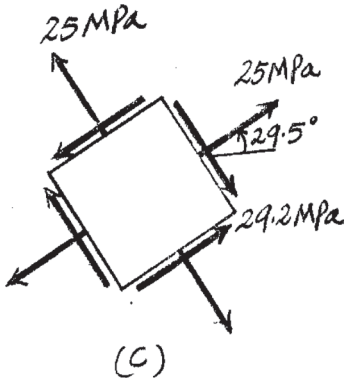
$$(\theta_p)_1 = -15.5° \text{ and } (\theta_p)_2 = 74.5° \qquad \textbf{Ans.}$$



The element that represents the state of principal stress is shown in Fig. *a*.

Maximum In-Plane Shear Stress:

$$\tau_{\substack{\max \\ \text{in-plane}}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + {\tau_{xy}}^2} = \sqrt{\left(\frac{50 - 0}{2}\right)^2 + (-15)^2} = 29.2 \text{ MPa} \qquad \textbf{Ans.}$$



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***9–16. Continued**

Orientation of the Plane of Maximum In-Plane Shear Stress:

$$\tan 2\theta_s = \frac{-(\sigma_x - \sigma_y)/2}{\tau_{xy}} = \frac{-(50 - 0)/2}{-15} = 1.667$$

$\theta_s = 29.5°$ and $120°$ **Ans.**

By inspection, $\tau_{\text{max}}_{\text{in-plane}}$ has to act in the same sense shown in Fig. *b* to maintain equilibrium.

Average Normal Stress:

$$\sigma_{\text{avg}} = \frac{\sigma_x + \sigma_y}{2} = \frac{50 + 0}{2} = 25 \text{ MPa}$$ **Ans.**

The element that represents the state of maximum in-plane shear stress is shown in Fig. *c*.

Ans:
$\sigma_1 = 54.2$ MPa,
$\sigma_2 = -4.15$ MPa,
$(\theta_p)_1 = -15.5°$,
$(\theta_p)_2 = 74.5°$,
$\tau_{\text{max}}_{\text{in-plane}} = 29.2$ MPa,
$\theta_s = 29.5°$ and $120°$,
$\sigma_{\text{avg}} = 25$ MPa

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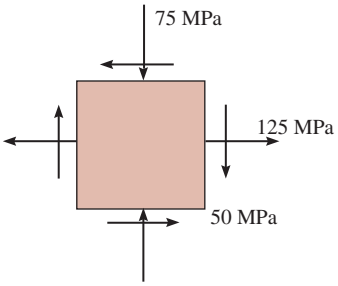
9–17.

Determine the equivalent state of stress on an element at the same point which represents (a) the principal stress, and (b) the maximum in-plane shear stress and the associated average normal stress. Also, for each case, determine the corresponding orientation of the element with respect to the element shown and sketch the results on the element.



SOLUTION

Normal and Shear Stress:

$\sigma_x = 125\ \text{MPa}$ $\sigma_y = -75\ \text{MPa}$ $\tau_{xy} = -50\ \text{MPa}$

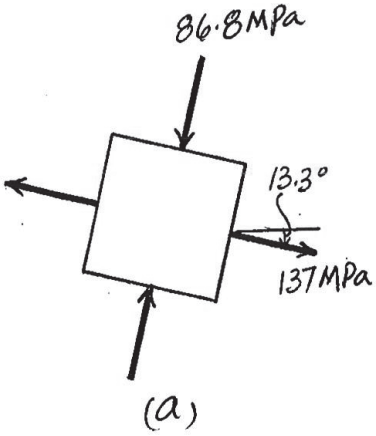
In - Plane Principal Stresses:

$$\sigma_{1,2} = \frac{\sigma_x - \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$$

$$= \frac{125 + (-75)}{2} \pm \sqrt{\left(\frac{125 - (-75)}{2}\right)^2 + (-50)^2}$$

$$= 25 \pm \sqrt{12500}$$

$\sigma_1 = 137\ \text{MPa}$ $\sigma_2 = -86.8\ \text{MPa}$ **Ans.**



Orientation of Principal Plane:

$$\tan 2\theta_P = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2} = \frac{-50}{(125-(-75))/2} = -0.5$$

$\theta_p = -13.28°$ and $76.72°$

Substitute $\theta = -13.28°$ into

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

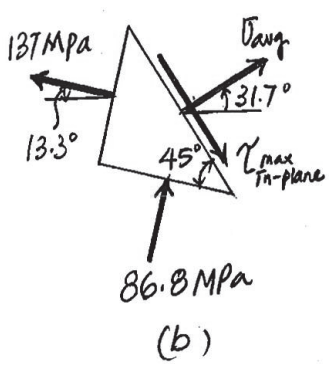
$$= \frac{125 + (-75)}{2} + \frac{125 - (-75)}{2}\cos(-26.57°) + (-50)\sin(-26.57°)$$

$= 137\ \text{MPa} = \sigma_1$

Thus,

$$(\theta_p)_1 = -13.3° \text{ and } (\theta_p)_2 = 76.7°$$ **Ans.**

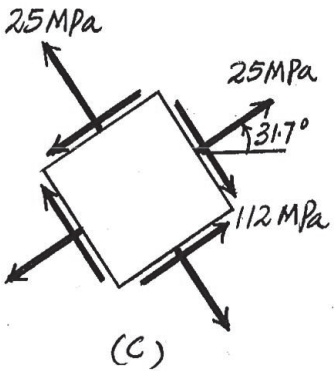
$125 - (-75)/(-50)$



The element that represents the state of principal stress is shown in Fig. *a*.

Maximum In - Plane Shear Stress:

$$\tau_{\substack{\max \\ \text{in-plane}}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2} = \sqrt{\left(\frac{125 - (-75)}{2}\right)^2 + 50^2} = 112\ \text{MPa}$$ **Ans.**



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9–17. Continued

Orientation of the Plane of Maximum In - Plane Shear Stress:

$$\tan 2\theta_s = -\frac{(\sigma_x - \sigma_y)/2}{\tau_{xy}} = -\frac{(125 - (-75))/2}{-50} = 2$$

$\theta_s = 31.7°$ and $122°$ **Ans.**

By inspection, $\tau_{\text{max}}_{\text{in-plane}}$ has to act in the same sense shown in Fig. *b* to maintain equilibrium.

Average Normal Stress:

$$\sigma_{\text{avg}} = \frac{\sigma_x + \sigma_y}{2} = \frac{125 + (-75)}{2} = 25 \text{ MPa}$$ **Ans.**

The element that represents the state of maximum in-plane shear stress is shown in Fig. *c*.

Ans:
$\sigma_1 = 137$ MPa, $\sigma_2 = -86.8$ MPa,
$\theta_{p1} = -13.3°$, $\theta_{p2} = 76.7°$, $\tau_{\text{max}}_{\text{in-plane}} = 112$ MPa,
$\theta_s = 31.7°$ and $122°$, $\sigma_{\text{avg}} = 25$ MPa

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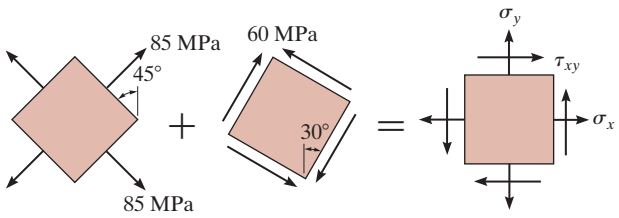
9–18.

A point on a thin plate is subjected to the two stress components. Determine the resultant state of stress represented on the element oriented as shown on the right.



SOLUTION

For element *a*:

$\sigma_x = \sigma_y = 85$ MPa $\quad \tau_{xy} = 0 \quad \theta = -45°$

$$(\sigma_{x'})_a = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

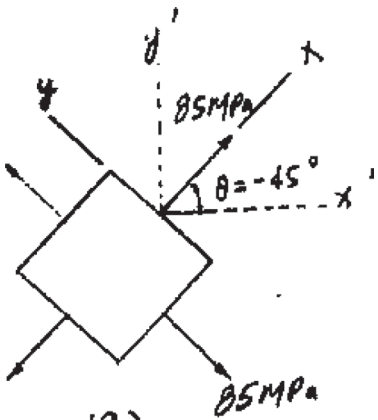
$$= \frac{85 + 85}{2} + \frac{85 - 85}{2}\cos(-90°) + 0 = 85 \text{ MPa}$$

$$(\sigma_{y'})_a = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

$$= \frac{85 + 85}{2} - \frac{85 - 85}{2}\cos(-90°) - 0 = 85 \text{ MPa}$$

$$(\tau_{x'y'})_a = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$= -\frac{85 - 85}{2}\sin(-90°) + 0 = 0$$



For element *b*:

$\sigma_x = \sigma_y = 0 \quad \tau_{xy} = 60$ MPa $\quad \theta = -60°$

$$(\sigma_{x'})_b = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

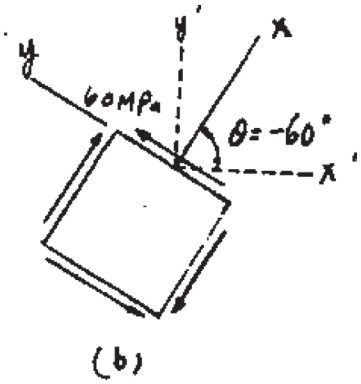
$$= 0 + 0 + 60\sin(-120°) = -51.96 \text{ MPa}$$

$$(\sigma_{y'})_b = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

$$= 0 - 0 - 60\sin(-120°) = 51.96 \text{ MPa}$$

$$(\tau_{x'y'})_b = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta - \tau_{xy}\cos 2\theta$$

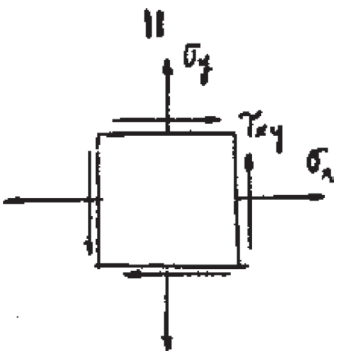
$$= -\frac{85 - 85}{2}\sin(-120°) + 60\cos(-120°) = -30 \text{ MPa}$$



$\sigma_x = (\sigma_{x'})_a + (\sigma_{x'})_b = 85 + (-51.96) = 33.0$ MPa **Ans.**

$\sigma_y = (\sigma_{y'})_a + (\sigma_{y'})_b = 85 + 51.96 = 137$ MPa **Ans.**

$\tau_{xy} = (\tau_{x'y'})_a + (\sigma_{x'y'})_b = 0 + (-30) = -30$ MPa **Ans.**



Ans:
$\sigma_x = 33.0$ MPa, $\sigma_y = 137$ MPa, $\tau_{xy} = -30$ MPa

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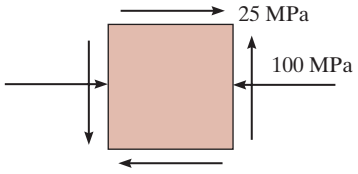
9–19.

Determine the equivalent state of stress on an element at the same point which represents (a) the principal stress, and (b) the maximum in-plane shear stress and the associated average normal stress. Also, for each case, determine the corresponding orientation of the element with respect to the element shown and sketch the results on the element.



SOLUTION

Normal and Shear Stress:

$$\sigma_x = -100\ \text{MPa} \qquad \sigma_y = 0 \qquad \tau_{xy} = 25\ \text{MPa}$$

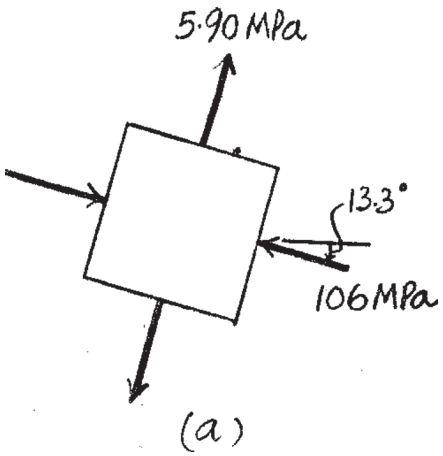
In-Plane Principal Stresses:

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{-100 + 0}{2} \pm \sqrt{\left(\frac{-100 - 0}{2}\right)^2 + 25^2}$$

$$= -50 \pm \sqrt{3125}$$

$$\sigma_1 = 5.90\ \text{MPa} \qquad \sigma_2 = -106\ \text{MPa} \qquad \textbf{Ans.}$$



Orientation of Principal Plane:

$$\tan 2\theta_p = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2} = \frac{25}{(-100 - 0)/2} = -0.5$$

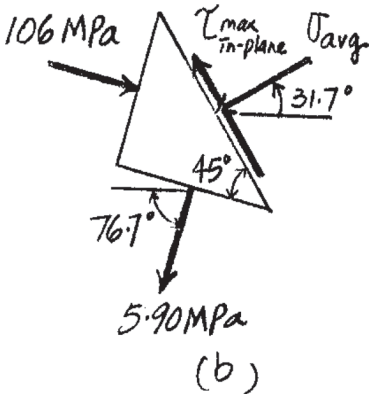
$$\theta_p = -13.28° \text{ and } 76.72°$$

Substitute $\theta = -13.28°$ into

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

$$= \frac{-100 + 0}{2} + \frac{-100 - 0}{2} \cos(-26.57°) + 25 \sin(-26.57°)$$

$$= -106\ \text{MPa} = \sigma_2$$



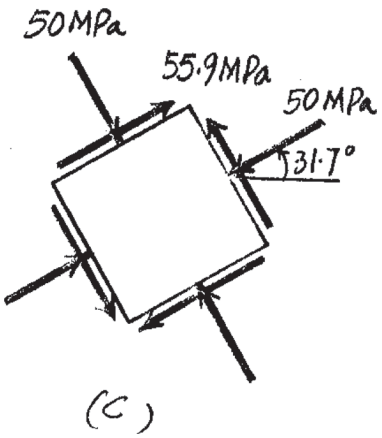
Thus,

$$(\theta_p)_1 = 76.7° \text{ and } (\theta_p)_2 = -13.3° \qquad \textbf{Ans.}$$

The element that represents the state of principal stress is shown in Fig. *a*.

Maximum In-Plane Shear Stress:

$$\tau_{\substack{\max \\ \text{in-plane}}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{-100 - 0}{2}\right)^2 + 25^2} = 55.9\ \text{MPa} \qquad \textbf{Ans.}$$



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9–19. Continued

Orientation of the Plane of Maximum In-Plane Shear Stress:

$$\tan 2\theta_s = -\frac{(\sigma_x - \sigma_y)/2}{\tau_{xy}} = -\frac{(-100 - 0)/2}{25} = 2$$

$\theta_s = 31.7°$ and $122°$ **Ans.**

By inspection, $\tau_{\text{max}}_{\text{in-plane}}$ has to act in the same sense shown in Fig. *b* to maintain equilibrium.

Average Normal Stress:

$$\sigma_{\text{avg}} = \frac{\sigma_x + \sigma_y}{2} = \frac{-100 + 0}{2} = -50 \text{ MPa}$$ **Ans.**

The element that represents the state of maximum in-plane shear stress is shown in Fig. *c*.

Ans:
$\sigma_1 = 5.90$ MPa, $\sigma_2 = -106$ MPa,
$\theta_{p1} = 76.7°$ and $\theta_{p2} = -13.3°$,
$\tau_{\text{max}}_{\text{in-plane}} = 55.9$ MPa, $\sigma_{\text{avg}} = -50$ MPa,
$\theta_s = 31.7°$ and $122°$

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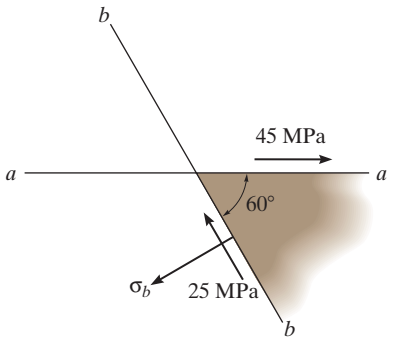
***9–20.**

The stress along two planes at a point is indicated. Determine the normal stresses on plane *b–b* and the principal stresses.

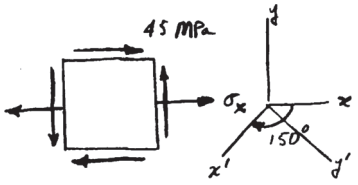


SOLUTION

$$\tau_b = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$-25 = -\frac{(\sigma_x - 0)}{2}\sin(-300°) + 45\cos(-300°)$$

$$\sigma_x = 109.70\ \text{MPa}$$



$$\sigma_b = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= \frac{109.70 + 0}{2} + \frac{109.70 - 0}{2}\cos(-300°) + 45\sin(-300°)$$

$$\sigma_b = 121\ \text{MPa}$$ **Ans.**

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + {\tau_{xy}}^2}$$

$$= \frac{109.70 + 0}{2} \pm \sqrt{\left(\frac{109.70 - 0}{2}\right)^2 + (45)^2}$$

$$\sigma_1 = 126\ \text{MPa}$$ **Ans.**

$$\sigma_2 = -16.1\ \text{MPa}$$ **Ans.**

Ans:
$\sigma_b = 121$ MPa,
$\sigma_1 = 126$ MPa,
$\sigma_2 = -16.1$ MPa

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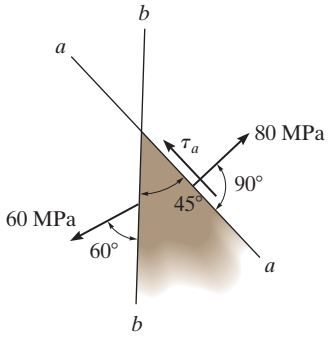
9–21. The stress acting on two planes at a point is indicated. Determine the shear stress on plane *a–a* and the principal stresses at the point.



SOLUTION

$$\sigma_x = 60 \sin 60° = 51.962 \text{ MPa}$$

$$\tau_{xy} = 60 \cos 60° = 30 \text{ MPa}$$

$$\sigma_a = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

$$80 = \frac{51.962 + \sigma_y}{2} + \frac{51.962 - \sigma_y}{2} \cos (90°) + 30 \sin (90°)$$

$$\sigma_y = 48.038 \text{ MPa}$$

$$\tau_a = -\left(\frac{\sigma_x - \sigma_y}{2}\right) \sin 2\theta + \tau_{xy} \cos \theta$$

$$= -\left(\frac{51.962 - 48.038}{2}\right) \sin (90°) + 30 \cos (90°)$$

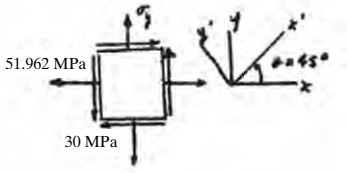
$$\tau_a = -1.96 \text{ MPa}$$ **Ans.**

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{51.962 + 48.038}{2} \pm \sqrt{\left(\frac{51.962 - 48.038}{2}\right)^2 + (30)^2}$$

$$\sigma_1 = 80.1 \text{ MPa}$$ **Ans.**

$$\sigma_2 = 19.9 \text{ MPa}$$ **Ans.**



Ans.

$\sigma_x = 61.962$ MPa, $\tau_{xy} = 30$ MPa

$\sigma_1 = 80.1$ MPa, $\sigma_2 = 19.9$ MPa

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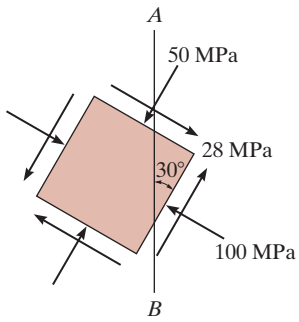
9–22.

The state of stress at a point in a member is shown on the element. Determine the stress components acting on the plane AB.



SOLUTION

Construction of the Circle: In accordance with the sign convention, $\sigma_x = -50$ MPa, $\sigma_y = -100$ MPa, and $\tau_{xy} = -28$ MPa. Hence,

$$\sigma_{\text{avg}} = \frac{\sigma_x + \sigma_y}{2} = \frac{-50 + (-100)}{2} = -75.0 \text{ MPa}$$

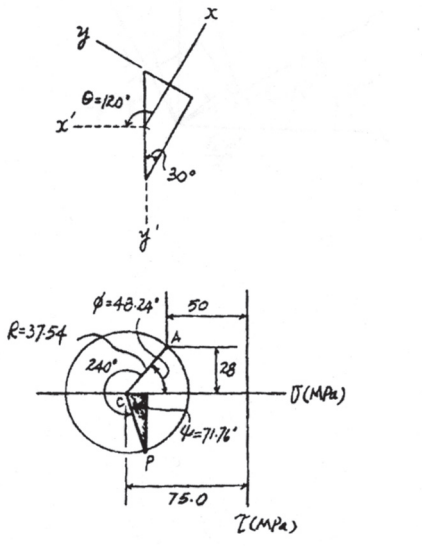
The coordinates for reference points A and C are $A(-50, -28)$ and $C(-75.0, 0)$.

The radius of the circle is $R = \sqrt{(75.0 - 50)^2 + 28^2} = 37.54$ MPa.

Stress on the Rotated Element: The normal and shear stress components $(\sigma_{x'}$ and $\tau_{x'y'})$ are represented by the coordinates of point P on the circle

$\sigma_{x'} = -75.0 + 37.54 \cos 71.76° = -63.3$ MPa **Ans.**

$\tau_{x'y'} = 37.54 \sin 71.76° = 35.7$ MPa **Ans.**



Ans:
$\sigma_{x'} = -63.3$ MPa, $\tau_{x'y'} = 35.7$ MPa

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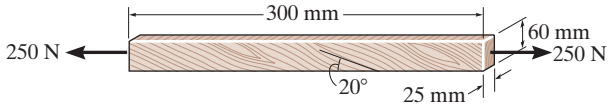
9–23.

The grains of wood in the board make an angle of 20° with the horizontal as shown. Determine the normal and shear stress that act perpendicular and parallel to the grains if the board is subjected to an axial load of 250 N.



SOLUTION

$$\sigma_x = \frac{P}{A} = \frac{250}{(0.06)(0.025)} = 166.67 \text{ kPa}$$

$$\sigma_y = 0 \qquad \tau_{xy} = 0$$

$$\theta = 70°$$

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= \frac{166.67 + 0}{2} + \frac{166.67 - 0}{2}\cos 140° + 0 = 19.5 \text{ kPa} \quad \textbf{Ans.}$$

$$\tau_{x'y'} = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$= -\left(\frac{166.67 - 0}{2}\right)\sin 140° + 0 = -53.6 \text{ kPa} \quad \textbf{Ans.}$$

Ans:
$\sigma_{x'} = 19.5$ kPa, $\tau_{x'y'} = -53.6$ kPa

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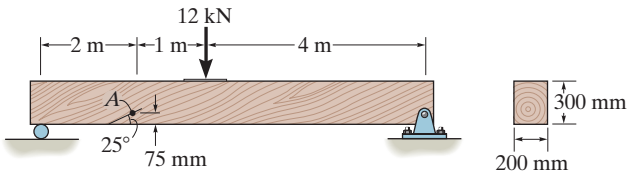
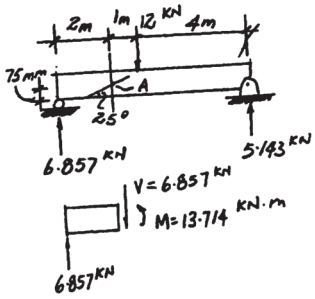
***9–24.**

The wood beam is subjected to a load of 12 kN. If grains of wood in the beam at point A make an angle of 25° with the horizontal as shown, determine the normal and shear stress that act perpendicular to the grains due to the loading.



SOLUTION

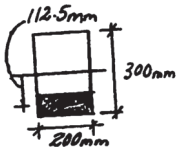


$$I = \frac{1}{12}(0.2)(0.3)^3 = 0.45(10^{-3})\ \text{m}^4$$

$$Q_A = \bar{y}A' = 0.1125(0.2)(0.075) = 1.6875(10^{-3})\ \text{m}^3$$

$$\sigma_A = \frac{My_A}{I} = \frac{13.714(10^3)(0.075)}{0.45(10^{-3})} = 2.2857\ \text{MPa (T)}$$

$$\tau_A = \frac{VQ_A}{It} = \frac{6.875(10^3)(1.6875)(10^{-3})}{0.45(10^{-3})(0.2)} = 0.1286\ \text{MPa}$$



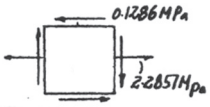
$\sigma_x = 2.2857\ \text{MPa} \qquad \sigma_y = 0 \qquad \tau_{xy} = -0.1286\ \text{MPa}$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + {\tau_{xy}}^2}$$

$$= \frac{2.2857 + 0}{2} \pm \sqrt{\left(\frac{2.2857 - 0}{2}\right)^2 + (-0.1286)^2}$$

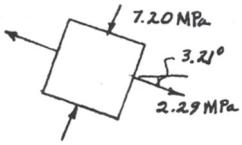
$\sigma_1 = 2.29\ \text{MPa}$ **Ans.**

$\sigma_2 = -7.20\ \text{kPa}$ **Ans.**



$$\tan 2\theta_p = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2} = \frac{-0.1286}{(2.2857 - 0)/2}$$

$\theta_p = -3.21°$ **Ans.**



Check direction of principal stress:

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= \frac{2.2857 + 0}{2} + \frac{2.2857 - 0}{2}\cos(-6.42°) - 0.1285\sin(-6.42)$$

$$= 2.29\ \text{MPa}$$

Ans:
$\sigma_1 = 2.29$ MPa,
$\sigma_2 = -7.20$ kPa,
$\theta_p = -3.21°$

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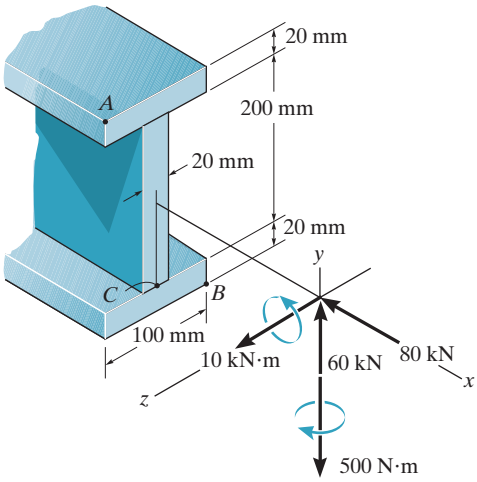
9–25.

The internal loadings at a section of the beam are shown. Determine the in-plane principal stresses at point A. Also compute the maximum in-plane shear stress at this point.



SOLUTION

Section Properties: For the wide flange section, Fig. *a*,

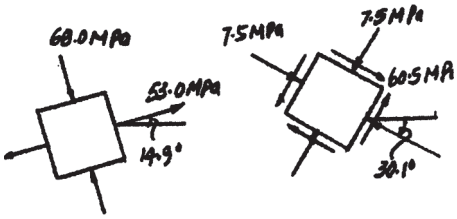
$$A = 0.1(0.24) - 0.08(0.2) = 0.008\ \text{m}^2$$

$$I_z = \frac{1}{12}(0.1)(0.24^3) - \frac{1}{12}(0.08)(0.2^3) = 61.8667(10^{-6})\ \text{m}^4$$

$$I_y = 2\left[\frac{1}{12}(0.02)(0.1^3)\right] + \frac{1}{12}(0.2)(0.02^3) = 3.4667(10^{-6})\ \text{m}^4$$

$$(Q_A)_y = 0$$

Internal Loadings: Here, $N_x = -80\ \text{kN}$, $V_y = 60\ \text{kN}$, $M_y = -0.5\ \text{kN}\cdot\text{m}$ and $M_z = 10\ \text{kN}\cdot\text{m}$.

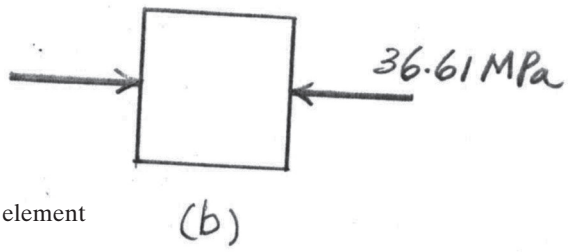


Normal Stress: For the combine loadings,

$$\sigma = \frac{N}{A} - \frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

$$\sigma_A = \frac{-80(10^3)}{0.008} - \frac{10(10^3)(0.12)}{61.8667(10^{-6})} + \frac{[-0.5(10^3)](0.05)}{3.4667(10^{-6})}$$

$$= -36.6080(10^6)\ \text{Pa} = 36.61\ \text{MPa (C)}$$



Shear Stress: Since $(Q_A)_y = 0$,

$$\tau_A = 0$$

Thus, the state of stress at point A can be represented by the differential element shown in Fig. *b*

In-Plane Principal Stresses: In accordance to the sign convention $\sigma_x = -36.61$ MPa, $\sigma_y = 0$ and $\tau_{xy} = 0$ for point A, Fig. *a*. Since no shear stress is acting on the element,

$$\sigma_1 = \sigma_y = 0 \qquad \textbf{Ans.}$$

$$\sigma_2 = \sigma_x = -36.61\ \text{MPa} = -36.6\ \text{MPa} \qquad \textbf{Ans.}$$

Maximum In-Plane Shear Stress:

$$\tau_{\substack{\max \\ \text{in-plane}}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{\left(\frac{-36.61 - 0}{2}\right)^2 + 0}$$

$$= 18.30\ \text{MPa} = 18.3\ \text{MPa} \qquad \textbf{Ans.}$$

Ans:
$\sigma_1 = 0$,
$\sigma_2 = -36.6$ MPa,
$\tau_{\substack{\max \\ \text{in-plane}}} = 18.3$ MPa

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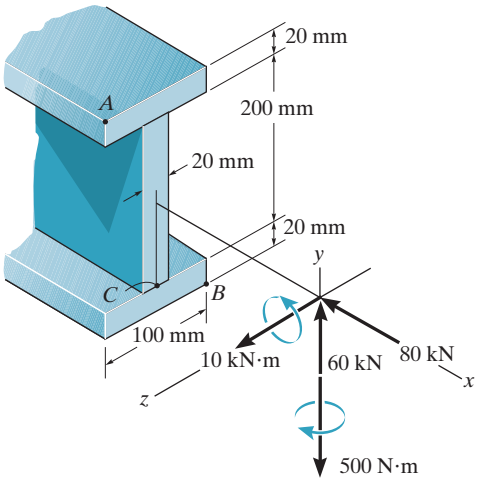
9–26.

Solve Prob. 9–25 for point B.



SOLUTION

Section Properties: For the wide flange section, Fig. *a*,

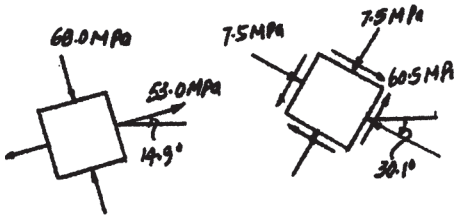
$$A = 0.1(0.24) - 0.08(0.2) = 0.008\ \text{m}^2$$

$$I_z = \frac{1}{12}(0.1)(0.24^3) - \frac{1}{12}(0.08)(0.2^3) = 61.8667(10^{-6})\ \text{m}^4$$

$$I_y = 2\left[\frac{1}{12}(0.02)(0.1^3)\right] + \frac{1}{12}(0.2)(0.02^3) = 3.4667(10^{-6})\ \text{m}^4$$

$$(Q_B)_y = 0$$

Internal Loadings: Here, $N_x = -80$ kN, $V_y = 60$ kN, $M_y = -0.5$ kN · m and $M_z = 10$ kN · m

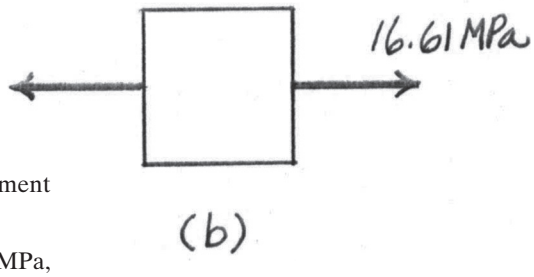


Normal Stress: For the combine loadings,

$$\sigma = \frac{N}{A} - \frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

$$\sigma_B = \frac{-80(10^3)}{0.008} - \frac{10(10^3)(-0.12)}{61.8667(10^{-6})} + \frac{[-0.5(10^3)](-0.05)}{3.4667(10^{-6})}$$

$$= 16.6081(10^6)\ \text{Pa} = 16.61\ \text{MPa (T)}$$



(b)

Shear Stress: Since $(Q_B)_y = 0$, $\tau_B = 0$.

Thus, the state of stress at point B can be represented by the differential element shown in Fig. *b*

In-Plane Principal Stresses: In accordance to the sign convention $\sigma_x = 16.61$ MPa, $\sigma_y = 0$ and $\tau_{xy} = 0$ for point B, Fig. *a*. Since no shear stress acting on the element,

$$\sigma_1 = \sigma_x = 16.61\ \text{MPa} = 16.6\ \text{MPa} \qquad \textbf{Ans.}$$

$$\sigma_2 = \sigma_y = 0 \qquad \textbf{Ans.}$$

Maximum In-Plane Shear Stress:

$$\tau_{\substack{\max \\ \text{in-plane}}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{\left(\frac{16.61 - 0}{2}\right)^2 + 0}$$

$$= 8.304\ \text{MPa} = 8.30\ \text{MPa} \qquad \textbf{Ans.}$$

Ans:
$\sigma_1 = 16.6$ MPa,
$\sigma_2 = 0$,
$\tau_{\substack{\max \\ \text{in-plane}}} = 8.30$ MPa

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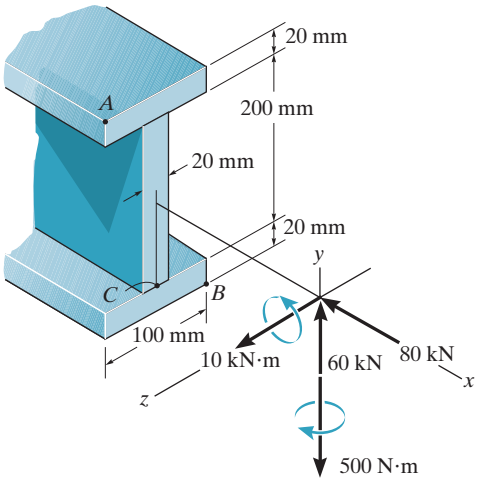
9–27.

Solve Prob. 9–25 for point *C*.



SOLUTION

Section Properties: For the wide-flange section, Fig. *a*

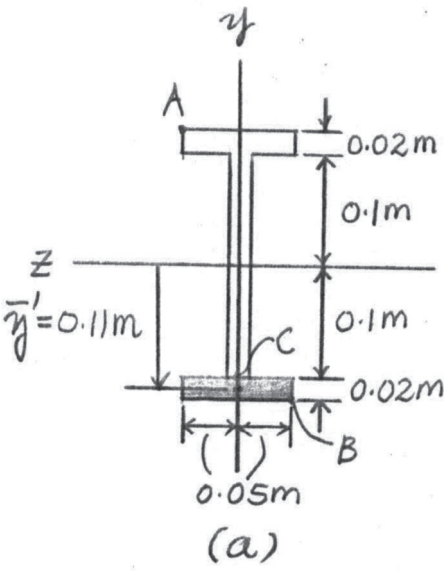
$$A = 0.1(0.24) - 0.08(0.2) = 0.008\ \text{m}^2$$

$$I_z = \frac{1}{12}(0.1)(0.24^3) - \frac{1}{12}(0.08)(0.2^3) = 61.8667(10^{-6})\ \text{m}^4$$

$$I_y = 2\left[\frac{1}{12}(0.02)(0.1^3)\right] + \frac{1}{12}(0.2)(0.02^3) = 3.4667(10^{-6})\ \text{m}^4$$

For the area shown shaded in Fig. *a*,

$$(Q_C)_y = \bar{y}'A' = 0.11[0.1(0.02)] = 0.22(10^{-3})\ \text{m}^3$$



Internal Loadings: Here, $N_x = -80\ \text{kN}$, $V_y = 60\ \text{kN}$, $M_y = -0.5\ \text{kN}\cdot\text{m}$ and $M_z = 10\ \text{kN}\cdot\text{m}$.

Normal Stress: For the combine loadings,

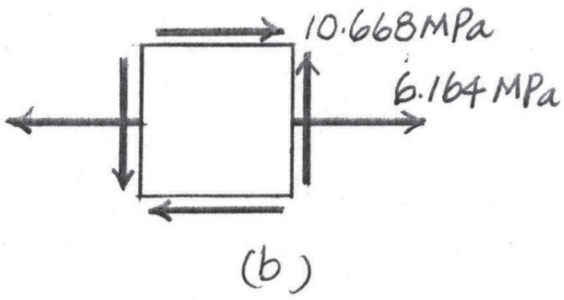
$$\sigma = \frac{N}{A} - \frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

$$\sigma_C = \frac{-80(10^3)}{0.008} - \frac{10(10^3)(-0.1)}{61.8667(10^{-6})} + \frac{[-0.5(10^3)](0)}{3.4667(10^{-6})}$$

$$= 6.164(10^6)\ \text{Pa} = 6.164\ \text{MPa (T)}$$

Shear Stress: Applying the shear formula,

$$\tau_c = \frac{V_y(Q_C)_y}{I_z t} = \frac{60(10^3)[0.22(10^{-3})]}{61.8667(10^{-6})(0.02)} = 10.668(10^6)\ \text{Pa} = 10.668\ \text{MPa}$$



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9–27. Continued

Thus, the state of stress at point C can be represented by the differential element shown in Fig. b.

In-Plane Principal Stress: In accordance to the sign convention, $\sigma_x = 6.164$ MPa, $\sigma_y = 0$ and $\tau_{xy} = 10.668$ MPa for point C, Fig. a.

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + {\tau_{xy}}^2}$$

$$= \frac{6.164 + 0}{2} \pm \sqrt{\left(\frac{6.164 - 0}{2}\right)^2 + 10.668^2}$$

$$= 3.082 \pm 11.104$$

$\sigma_1 = 14.19$ MPa $= 14.2$ MPa $\qquad \sigma_2 = -8.022$ MPa $= -8.02$ MPa **Ans.**

Maximum In-Plane Shear Stress:

$$\tau_{\substack{\max \\ \text{in-plane}}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + {\tau_{xy}}^2}$$

$$= \sqrt{\left(\frac{6.164 - 0}{2}\right)^2 + 10.668^2}$$

$= 11.104$ MPa $= 11.1$ MPa **Ans.**

Ans:
$\sigma_1 = 14.2$ MPa,
$\sigma_2 = -8.02$ MPa,
$\tau_{\substack{\max \\ \text{in-plane}}} = 11.1$ MPa

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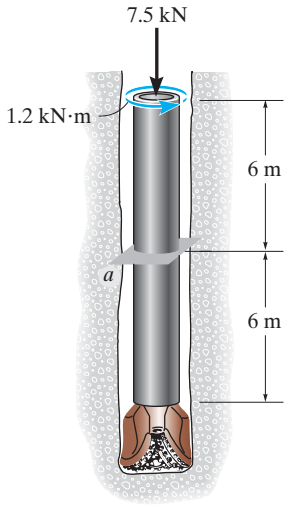
***9–28.** The drill pipe has an outer diameter of 75 mm, a wall thickness of 6 mm, and a weight of 0.8 kN/m. If it is subjected to a torque and axial load as shown, determine (a) the principal stress and (b) the maximum in-plane shear stress at a point on its surface at section *a*.



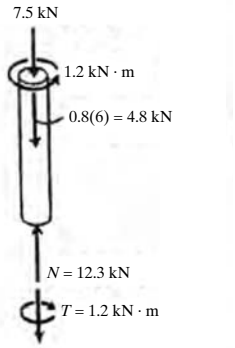
SOLUTION

Internal Forces and Torque: As shown on FBD(a).

Section Properties:

$$A = \frac{\pi}{4}(0.075^2 - 0.063^2) = 0.414(10^{-3})\pi \text{ m}^2$$

$$J = \frac{\pi}{2}(0.0375^4 - 0.0315^4) = 1.55977(10^{-6}) \text{ m}^4$$



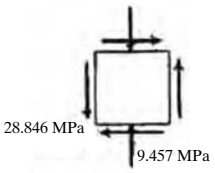
Normal Stress:

$$\sigma = \frac{N}{A} = \frac{-12.3(10^3)}{0.414(10^{-3})\pi} - 9.457(10^6) \text{ N/m}^2 = -9.457 \text{ MPa}$$

Shear Stress: Applying the torsion formula.

$$\tau = \frac{T\,c}{J} = \frac{[1.2(10^3)](0.0375)}{1.55977(10^{-6})} = 28.85(10^6) \text{ N/m}^2 = 28.85 \text{ MPa}$$

a) ***In - Plane Principal Stresses:*** $\sigma_x = 0, \sigma_y = -9.457$ MPa and $\tau_{xy} = 28.846$ MPa for any point on the shaft's surface. Applying Eq. 9-5.



$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{0 + (-9.457)}{2} \pm \sqrt{\left[\frac{0 - (-9.457)}{2}\right]^2 + (28.85^2)}$$

$$= -4.7285 \pm 29.2354$$

$\sigma_1 = 24.5$ MPa **Ans.**

$\sigma_2 = -34.0$ MPa **Ans.**

b) ***Maximum In - Plane Shear Stress:*** Applying Eq. 9-7

$$\tau_{\text{in-plane}}^{\max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{\left[\frac{0 - (-9.457)}{2}\right]^2 + (28.85^2)}$$

$= 29.2$ **Ans.**

Ans.

$\sigma_1 = 24.5$ MPa, $\sigma_2 = -34.0$ MPa,

$\tau_{\text{in-plane}}^{\max} = 29.2$ MPa

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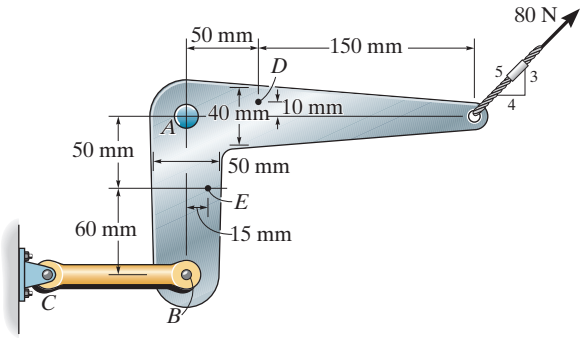
9–29.

The bell crank is pinned at A and supported by a short link BC. If it is subjected to the force of 80 N, determine the principal stresses at (a) point D and (b) point E. The crank is constructed from an aluminum plate having a thickness of 20 mm.



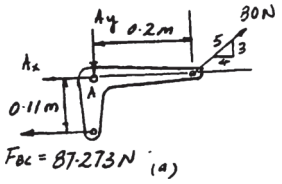
SOLUTION

Point *D*:

$$A = 0.04(0.02) = 0.8(10^{-3})\ \text{m}^2$$

$$I = \frac{1}{12}(0.02)(0.04^3) = 0.1067(10^{-6})\ \text{m}^4$$

$$Q_D = \bar{y}'A' = 0.015(0.02)(0.01) = 3(10^{-6})\ \text{m}^3$$

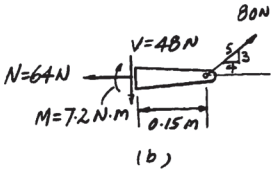


Normal stress:

$$\sigma_D = \frac{P}{A} + \frac{My}{I} = \frac{64}{0.8(10^{-3})} - \frac{7.2(0.01)}{0.1067(10^{-6})} = -0.595\ \text{MPa}$$

Shear stress:

$$\tau_D = \frac{VQ}{It} = \frac{48(3)(10^{-6})}{0.1067(10^{-6})(0.02)} = 0.0675\ \text{MPa}$$

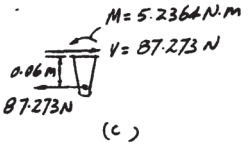


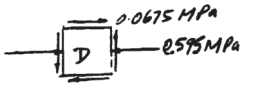
Principal stress: $\sigma_x = -0.595\ \text{MPa} \quad \sigma_y = 0 \quad \tau_{xy} = 0.0675\ \text{MPa}$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{-0.595 + 0}{2} \pm \sqrt{\left(\frac{-0.595 - 0}{2}\right)^2 + 0.0675^2}$$

$\sigma_1 = 7.56\ \text{kPa}$ **Ans.**
$\sigma_2 = -603\ \text{kPa}$ **Ans.**

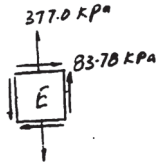




Point *E*:

$$I = \frac{1}{12}(0.02)(0.05^3) = 0.2083(10^{-6})\ \text{m}^4$$

$$Q_E = \bar{y}'A' = 0.02(0.01)(0.02) = 4.0(10^{-6})\ \text{m}^3$$



Normal stress:

$$\sigma_E = \frac{My}{I} = \frac{5.2364(0.015)}{0.2083(10^{-6})} = 377.0\ \text{kPa}$$

Shear stress:

$$\tau_E = \frac{VQ}{It} = \frac{87.273(4.0)(10^{-6})}{0.2083(10^{-6})(0.02)} = 83.78\ \text{kPa}$$

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9–29. Continued

Principal stress: $\sigma_x = 0 \quad \sigma_y = 377.0\ \text{kPa} \quad \tau_{xy} = 83.78\ \text{kPa}$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + {\tau_{xy}}^2}$$

$$= \frac{0 + 377.0}{2} \pm \sqrt{\left(\frac{0 - 377.0}{2}\right)^2 + 83.78^2}$$

$\sigma_1 = 395\ \text{kPa}$ **Ans.**

$\sigma_2 = -17.8\ \text{kPa}$ **Ans.**

Ans:
Point *D*:
$\sigma_1 = 7.56\ \text{kPa},$
$\sigma_2 = -603\ \text{kPa},$
Point *E*:
$\sigma_1 = 395\ \text{kPa},$
$\sigma_2 = -17.8\ \text{kPa}$

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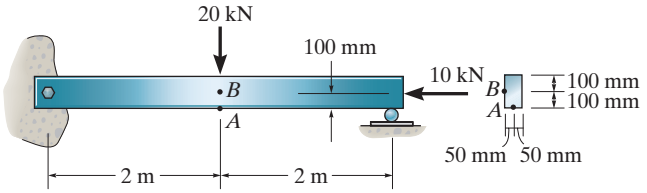
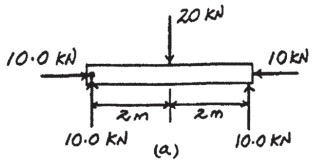
9–30.

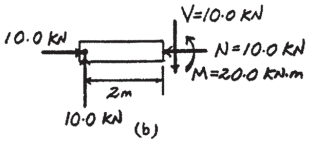
The beam has a rectangular cross section and is subjected to the loadings shown. Determine the principal stresses at point A and point B, which are located just to the left of the 20-kN load. Show the results on elements located at these points.



SOLUTION

Internal Forces and Moment: As shown on FBD(b).





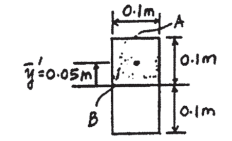
Section Properties:

$$A = 0.1(0.2) = 0.020\ \text{m}^2$$

$$I = \frac{1}{12}(0.1)\left(0.2^3\right) = 66.667\left(10^{-6}\right)\ \text{m}^4$$

$$Q_A = 0$$

$$Q_B = \bar{y}'A' = 0.05(0.1)(0.1) = 0.50\left(10^{-3}\right)\ \text{m}^3$$

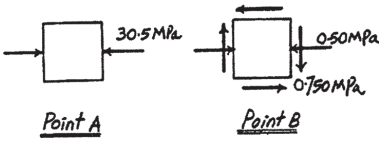


Normal Stresses:

$$\sigma = \frac{N}{A} \pm \frac{My}{I}$$

$$\sigma_A = \frac{-10.0\left(10^3\right)}{0.020} - \frac{20.0\left(10^3\right)(0.1)}{66.667\left(10^{-6}\right)} = -30.5\ \text{MPa}$$

$$\sigma_B = \frac{-10.0\left(10^3\right)}{0.020} - \frac{20.0\left(10^3\right)(0)}{66.667\left(10^{-6}\right)} = -0.500\ \text{MPa}$$



Shear Stress: Applying the shear formula $\tau = \frac{VQ}{It}$,

$$\tau_A = 0$$

$$\tau_B = \frac{10.0\left(10^3\right)\left[0.50\left(10^{-3}\right)\right]}{66.667\left(10^{-6}\right)(0.1)} = 0.750\ \text{MPa}$$

In-Plane Principal Stresses: $\sigma_x = -30.5$ MPa, $\sigma_y = 0$, and $\tau_{xy} = 0$ for point A. Since no shear stress acts on the element.

$$\sigma_1 = \sigma_y = 0 \qquad \textbf{Ans.}$$

$$\sigma_2 = \sigma_x = -30.5\ \text{MPa} \qquad \textbf{Ans.}$$

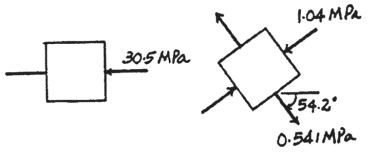
$\sigma_x = -0.500$ MPa, $\sigma_y = 0$, and $\tau_{xy} = -0.750$ MPa for point B. Applying Eq. 9–5,

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + {\tau_{xy}}^2}$$

$$= \frac{-0.500 + 0}{2} \pm \sqrt{\left(\frac{-0.500 - 0}{2}\right)^2 + (-0.750)^2}$$

$$= -0.250 \pm 0.7906$$

$$\sigma_1 = 0.541\ \text{MPa} \qquad \sigma_2 = -1.04\ \text{MPa} \qquad \textbf{Ans.}$$



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9–30. Continued

Orientation of Principal Plane: Applying Eq. 9–4 for point B.

$$\tan 2\theta_p = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2} = \frac{-0.750}{(-0.500 - 0)/2} = 3.000$$

$$\theta_P = 35.78^\circ \qquad \text{and} \qquad -54.22^\circ$$

Substituting the results into Eq. 9–1 with $\theta = 35.78^\circ$ yields

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$
$$= \frac{-0.500 + 0}{2} + \frac{-0.500 - 0}{2}\cos 71.56^\circ + (-0.750 \sin 71.56^\circ)$$
$$= -1.04 \text{ MPa} = \sigma_2$$

Hence.

$\theta_{p1} = -54.2^\circ \qquad \theta_{p2} = 35.8^\circ$ **Ans.**

Ans:
Point A,
$\sigma_1 = \sigma_y = 0$,
$\sigma_2 = \sigma_x = -30.5$ MPa,
Point B,
$\sigma_1 = 0.541$ MPa,
$\sigma_2 = -1.04$ MPa,
$\theta_{p1} = -54.2^\circ$,
$\theta_{p2} = 35.8^\circ$

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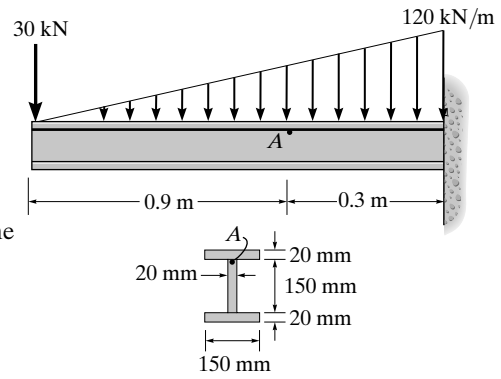
9–31. The wide-flange beam is subjected to the loading shown. Determine the principal stress in the beam at point A, which is located at the top of the web. Although it is not very accurate, use the shear formula to determine the shear stress. Show the result on an element located at this point.



Using the method of sections and consider the FBD of the left cut segment of the bean, Fig. *a*

$+\uparrow \Sigma F_y = 0; \quad V - \frac{1}{2}(90)(0.9) - 30 = 0 \quad V = 70.5 \text{ kN}$

$\circlearrowleft + \Sigma M_C = 0; \quad \frac{1}{2}(90)(0.9)(0.3) + 30(0.9) - M = 0 \quad M = 39.15 \text{ kN} \cdot \text{m}$

The moment of inertia of the cross - section about the bending axis is

$$I = \frac{1}{12}(0.15)(0.19^3) - \frac{1}{12}(0.13)(0.15^3) = 49.175(10^{-6}) \text{ m}^4$$

Referring to Fig. *b*,

$$Q_A = \bar{y}' A' = 0.085\,(0.02)(0.15) = 0.255\,(10^{-3}) \text{ m}^3$$

The normal stress developed is contributed by bending stress only. For point A, $y = 0.075$ m. Thus,

$$\sigma = \frac{My}{I} = \frac{39.15(10^3)(0.075)}{49.175(10^{-6})} = 59.71(10^6)\text{Pa} = 59.71 \text{ MPa (T)}$$

The shear stress is contributed by the transverse shear stress only. Thus

$$\tau = \frac{VQ_A}{It} = \frac{70.5(10^3)\left[0.255(10^{-3})\right]}{49.175(10^{-6})\,(0.02)} = 18.28(10^6)\text{Pa} = 18.28 \text{ MPa}$$

Here, $\sigma_x = 59.71$ MPa, $\sigma_y = 0$ and $\tau_{xy} = 18.28$ MPa.

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}}$$

$$= \frac{59.71 + 0}{2} \pm \sqrt{\left(\frac{59.71 - 0}{2}\right)^2 + 18.28^2}$$

$$= 29.86 \pm 35.01$$

$\sigma_1 = 64.9$ MPa $\quad \sigma_2 = -5.15$ MPa **Ans.**

$$\tan 2\theta_P = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2} = \frac{18.28}{(59.71 - 0)/2} = 0.6122$$

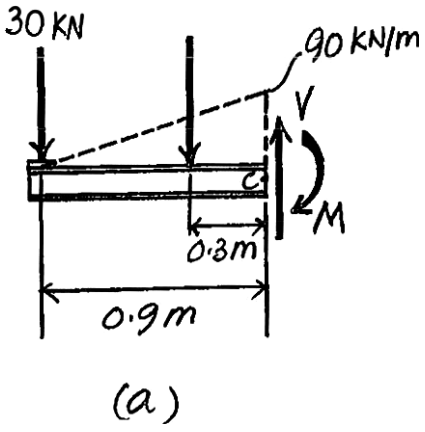
$\theta_P = 15.74°$ and $-74.26°$

Substitute $\theta = 15.74°$,

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= \frac{59.71 + 0}{2} + \frac{59.71 - 0}{2}\cos 31.48° + 18.28 \sin 31.48°$$

$$= 64.9 \text{ MPa} = \sigma_1$$



(a)

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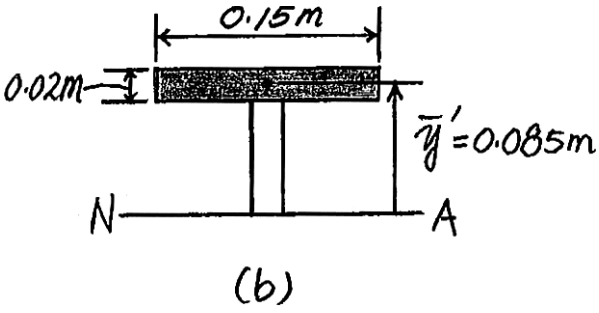
9–31. Continued

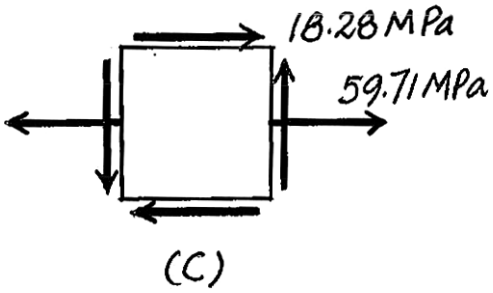
Thus,

$$(\theta_P)_1 = 15.7^\circ \qquad (\theta_P)_2 = -74.3^\circ \qquad \textbf{Ans.}$$

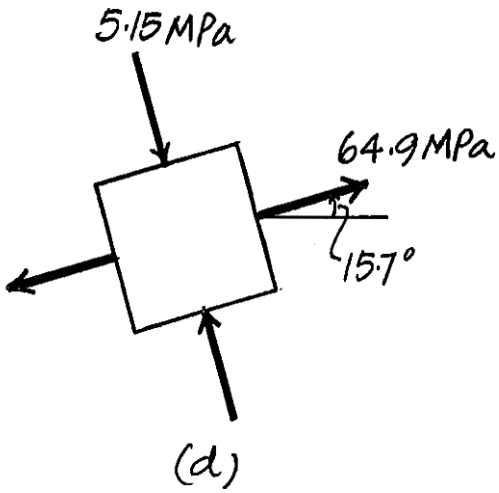
The state of principal stress can be represented by the element shown in Fig. *d*



(b)



(C)



(d)

Ans.

$V = 70.5$ kN, $M = 39.15$ kN · m,
$I = 49.175(10^{-6})$ m^4, $Q_A = 0.255(10^{-3})$ m^3,
$\sigma_1 = 64.9$ MPa, $\sigma_2 = -5.15$ MPa,
$(\theta_p)_1 = 15.7^\circ$, $(\theta_p)_2 = -74.3^\circ$

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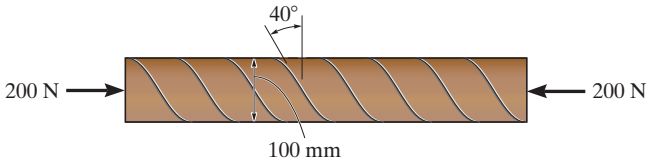
***9–32.**

A paper tube is formed by rolling a cardboard strip in a spiral and then gluing the edges together as shown. Determine the shear stress acting along the seam, which is at 50° from the horizontal, when the tube is subjected to an axial compressive force of 200 N. The paper is 2 mm thick and the tube has an outer diameter of 100 mm.



SOLUTION

Normal And Shear Stresses: The normal stress is caused by the axial force only. Thus

$$\sigma = \frac{N}{A} = \frac{-200}{\frac{\pi}{4}(0.1^2 - 0.096^2)} = -324.81(10^3)\text{ Pa} = 324.81\text{ kPa (C)}$$

Since there is no shear force on the cross-section,

$$\tau = 0$$

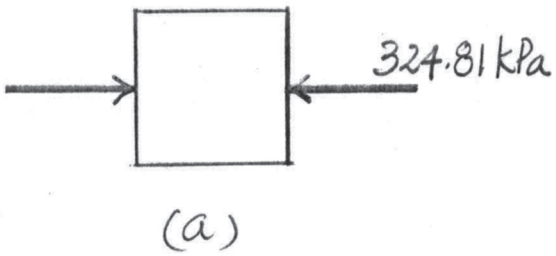
The state of stress of a point on the cross-section can be represented by the element shown in Fig. *a*

Stress Transformation Equations: With $\sigma_x = -324.81$ kPa, $\sigma_y = 0$, $\tau_{xy} = 0$ and $\theta = +40°$ (Fig. *b*),

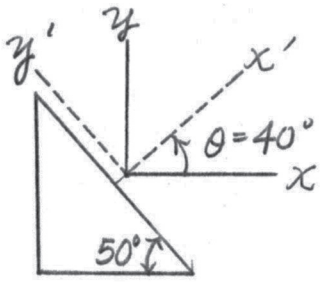
$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$= -\frac{(-324.81 - 0)}{2}\sin 80° + 0$$

$$= 159.94\text{ kPa} = 160\text{ kPa}$$ **Ans.**



(a)



(b)

Ans:
$\tau_{x'y'} = 160$ kPa

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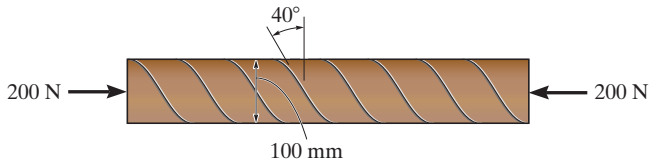
9–33.

Solve Prob. 9–31 for the normal stress acting perpendicular to the seam.



SOLUTION

Normal and Shear Stresses: The normal stress is caused by the axial force only. Thus

$$\sigma = \frac{N}{A} = \frac{-200}{\frac{\pi}{4}(0.1^2 - 0.096^2)} = -324.81(10^3)\text{ Pa} = 324.81\text{ kPa (C)}$$

Since there is no shear force on the cross-section,

$$\tau = 0$$

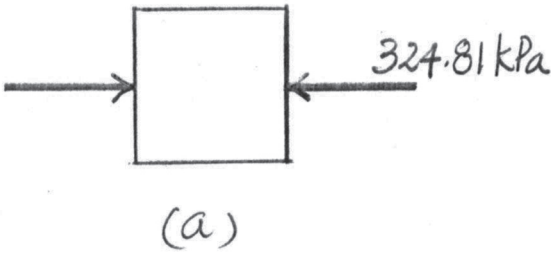
The state of stress of a point on the cross-section can be represented by the element shown in Fig. *a*.

Stress Transformation Equations: With $\sigma_x = -324.81$ kPa, $\sigma_y = 0$, $\tau_{xy} = 0$ and $\theta = +40°$ (Fig. *b*),

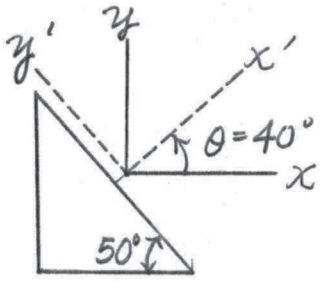
$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= \frac{-324.81 + 0}{2} + \frac{(-324.81 - 0)}{2}\cos 80° + 0$$

$$= -190.60\text{ kPa} = -191\text{ kPa}$$ **Ans.**



(a)



(b)

Ans:
$\sigma_{x'} = -191$ kPa

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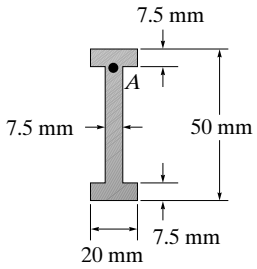
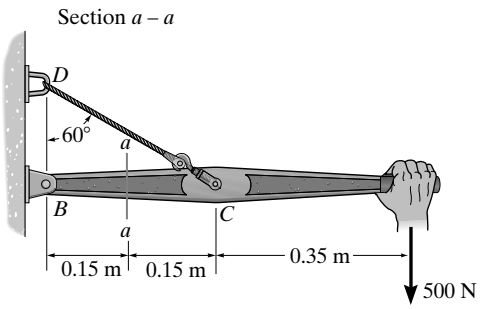
9–34. Determine the principal stress at point A on the cross section of the arm at section a–a. Specify the orientation of this state of stress and indicate the results on an element at the point.





Support Reactions: Referring to the free - body diagram of the entire arm shown in Fig. a,

$\Sigma M_B = 0;\ F_{CD}\sin 30°(0.3) - 500(0.65) = 0$ $\quad F_{CD} = 2166.67\,\text{N}$

$\stackrel{+}{\rightarrow}\Sigma F_x = 0;\quad B_x - 2166.67\cos 30° = 0$ $\quad B_x = 1876.39\,\text{N}$

$+\uparrow\Sigma F_y = 0;\quad 2166.67\sin 30° - 500 - B_y = 0$ $\quad B_y = 583.33\,\text{N}$

Internal Loadings: Consider the equilibrium of the free - body diagram of the arm's left segment, Fig. b.

$\stackrel{+}{\rightarrow}\Sigma F_x = 0;\quad 1876.39 - N = 0$ $\quad N = 1876.39\,\text{N}$

$+\uparrow\Sigma F_y = 0;\quad V - 583.33 = 0$ $\quad V = 583.33\,\text{N}$

$+\Sigma M_O = 0;\quad 583.33(0.15) - M = 0$ $\quad M = 87.5\,\text{N}\cdot\text{m}$

Section Properties: The cross - sectional area and the moment of inertia about the z axis of the arm's cross section are

$$A = 0.02(0.05) - 0.0125(0.035) = 0.5625\left(10^{-3}\right)\text{m}^2$$

$$I = \frac{1}{12}(0.02)\left(0.05^3\right) - \frac{1}{12}(0.0125)\left(0.035^3\right) = 0.16367\left(10^{-6}\right)\text{m}^4$$

Referring to Fig. b,

$$Q_A = \bar{y}'A' = 0.02125(0.0075)(0.02) = 3.1875\left(10^{-6}\right)\text{m}^3$$

Normal and Shear Stress: The normal stress is a combination of axial and bending stress. Thus,

$$\sigma_A = \frac{N}{A} + \frac{My_A}{I}$$

$$= \frac{-1876.39}{0.5625\left(10^{-3}\right)} + \frac{87.5(0.0175)}{0.16367\left(10^{-6}\right)} = 6.020\ \text{MPa}$$

The shear stress is caused by transverse shear stress.

$$\tau_A = \frac{VQ_A}{It} = \frac{583.33\left[3.1875\left(10^{-6}\right)\right]}{0.16367\left(10^{-6}\right)(0.0075)} = 1.515\ \text{MPa}$$

The share of stress at point A can be represented on the element shown in Fig. d.

In - Plane Principal Stress: $\sigma_x = 6.020$ MPa, $\sigma_y = 0$, and $\tau_{xy} = 1.515$ MPa. We have

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + {\tau_{xy}}^2}$$

$$= \frac{6.020 + 0}{2} \pm \sqrt{\left(\frac{6.020 - 0}{2}\right)^2 + 1.515^2}$$

$\sigma_1 = 6.38\ \text{MPa}$ $\quad \sigma_2 = -0.360\ \text{MPa}$ $\quad$ **Ans.**

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9–34. Continued

Orientation of the Principal Plane:

$$\tan 2\theta_P = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2} = \frac{1.515}{(6.020 - 0)/2} = 0.5032$$

$$\theta_p = 13.36^\circ \text{ and } 26.71^\circ$$

Substituting $\theta = 13.36^\circ$ into

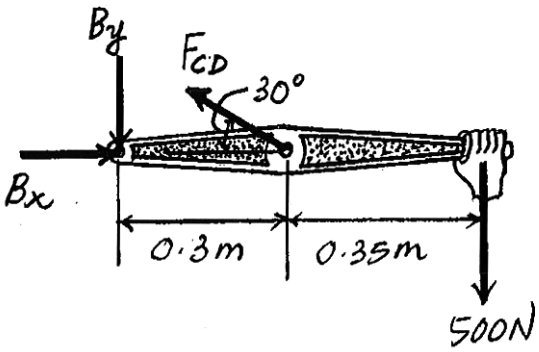
$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= \frac{6.020 - 0}{2} + \frac{6.020 + 0}{2}\cos 26.71^\circ + 1.515 \sin 26.71^\circ$$

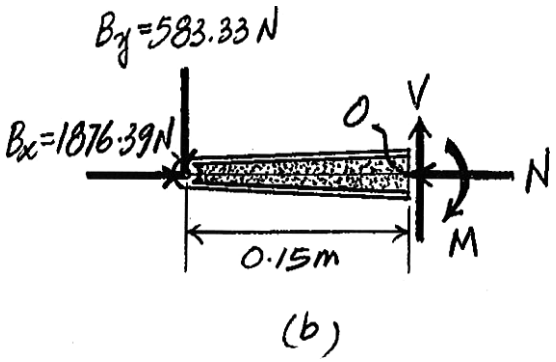
$$= 6.38 \text{ MPa} = \sigma_1$$

Thus, $(\theta_P)_1 = 13.4$ and $(\theta_P)_2 = 26.71^\circ$ **Ans.**

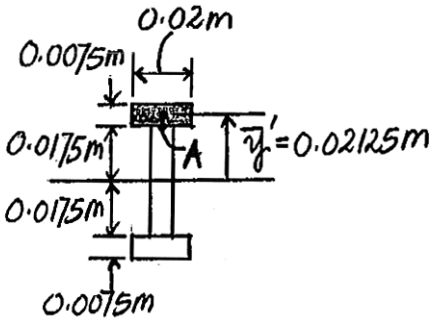
The state of principal stresses is represented by the element shown in Fig. *e*.



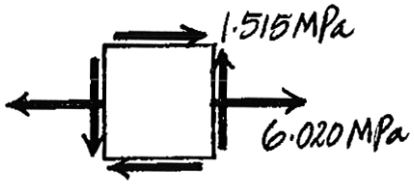
(a)



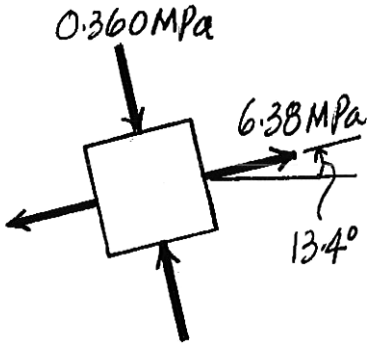
(b)



(c)



(d)



(e)

Ans.

$\sigma_1 = 6.38$ MPa, $\sigma_2 = -0.360$ MPa,
$(\theta_p)_1 = 13.4^\circ$, $(\theta_p)_2 = 103^\circ$

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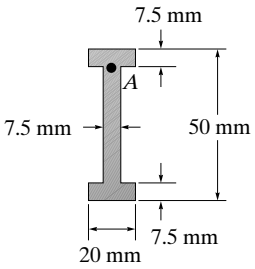
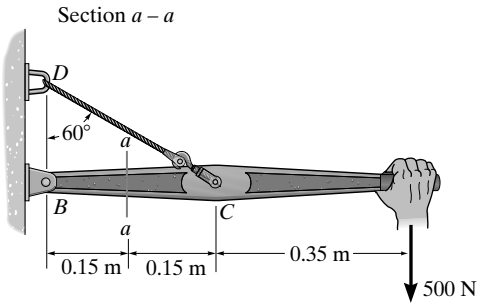
9–35. Determine the maximum in-plane shear stress developed at point A on the cross section of the arm at section a–a. Specify the orientation of this state of stress and indicate the results on an element at the point.



SOLUTION



Support Reactions: Referring to the free - body diagram of the entire arm shown in Fig. a,

$\Sigma M_B = 0;\ F_{CD}\sin 30°(0.3) - 500(0.65) = 0$ $\quad F_{CD} = 2166.67\,\text{N}$

$\stackrel{+}{\rightarrow}\Sigma F_x = 0;\quad B_x - 2166.67\cos 30° = 0$ $\quad B_x = 1876.39\,\text{N}$

$+\uparrow\Sigma F_y = 0;\quad 2166.67\sin 30° - 500 - B_y = 0$ $\quad B_y = 583.33\,\text{N}$

Internal Loadings: Considering the equilibrium of the free - body diagram of the arm's left cut segment, Fig. b,

$\stackrel{+}{\rightarrow}\Sigma F_x = 0;\quad 1876.39 - N = 0$ $\quad N = 1876.39\,\text{N}$

$+\uparrow\Sigma F_y = 0;\quad V - 583.33 = 0$ $\quad V = 583.33\,\text{N}$

$+\Sigma M_O = 0;\quad 583.33(0.15) - M = 0$ $\quad M = 87.5\,\text{N}\cdot\text{m}$

Section Properties: The cross - sectional area and the moment of inertia about the z axis of the arm's cross section are

$$A = 0.02(0.05) - 0.0125(0.035) = 0.5625\left(10^{-3}\right)\text{m}^2$$

$$I = \frac{1}{12}(0.02)\left(0.05^3\right) - \frac{1}{12}(0.0125)\left(0.035^3\right) = 0.16367\left(10^{-6}\right)\text{m}^4$$

Referring to Fig. b,

$$Q_A = \bar{y}'A' = 0.02125(0.0075)(0.02) = 3.1875\left(10^{-6}\right)\text{m}^3$$

Normal and Shear Stress: The normal stress is a combination of axial and bending stress. Thus,

$$\sigma_A = \frac{N}{A} + \frac{My_A}{I}$$

$$= \frac{-1876.39}{0.5625\left(10^{-3}\right)} + \frac{87.5(0.0175)}{0.16367\left(10^{-6}\right)} = 6.020\,\text{MPa}$$

The shear stress is contributed only by transverse shear stress.

$$\tau_A = \frac{VQ_A}{It} = \frac{583.33\left[3.1875\left(10^{-6}\right)\right]}{0.16367\left(10^{-6}\right)(0.0075)} = 1.515\,\text{MPa}$$

Maximum In - Plane Shear Stress: $\sigma_x = 6.020$ MPa, $\sigma_y = 0$, and $\tau_{xy} = 1.515$ MPa.

$$\tau_{\substack{\max\\ \text{in-plane}}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + {\tau_{xy}}^2} = \sqrt{\left(\frac{6.020 - 0}{2}\right)^2 + 1.515^2} = 3.37\,\text{MPa}$$ **Ans.**

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9–35. Continued

Orientation of the Plane of Maximum In - Plane Shear Stress:

$$\tan 2\theta_s = -\frac{(\sigma_x - \sigma_y)/2}{\tau_{xy}} = -\frac{(6.020 - 0)/2}{1.515} = -1.9871$$

$$\theta_s = -31.6° \text{ and } 58.4° \qquad \textbf{Ans.}$$

Substituting $\theta = -31.6°$ into

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$= -\frac{6.020 - 0}{2}\sin(-63.29°) + 1.515\cos(-63.29°)$$

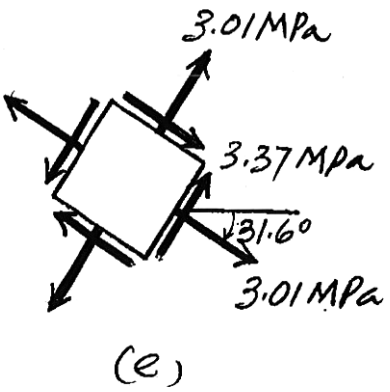
$$= 3.37 \text{ MPa} = \tau_{\substack{\max \\ \text{in-plane}}}$$

This indicates that $\tau_{\substack{\max \\ \text{in-plane}}}$ is directed in the positive sense of the y' axis on the face of the element defined by $\theta_s = -31.6°$.

Average Normal Stress:

$$\sigma_{\text{avg}} = \frac{\sigma_x + \sigma_y}{2} = \frac{6.020 + 0}{2} = 3.01 \text{ MPa} \qquad \textbf{Ans.}$$

The state of maximum in - plane shear stress is represented on the element shown in Fig. *e*.



(e)

Ans.

$\theta_s = -31.6°$ and $58.4°$, $\sigma_{\text{avg}} = 3.01$ MPa, $\tau_{\substack{\max \\ \text{in-plane}}} = 3.37$ MPa

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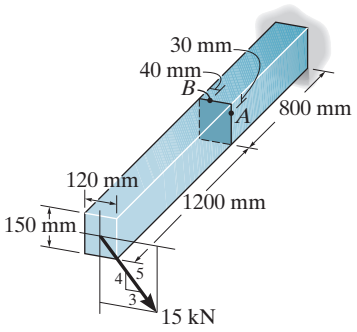
***9–36.**

Determine the principal stresses in the cantilevered beam at points A and B.



SOLUTION

Internal Forces and Moment: As shown on FBD.

Section Properties:

$$I_z = \frac{1}{12}(0.12)(0.15^3) = 33.75(10^{-6})\ \text{m}^4$$

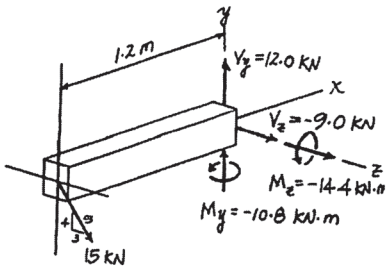
$$I_y = \frac{1}{12}(0.15)(0.12^3) = 21.6(10^{-6})\ \text{m}^4$$

$$(Q_A)_y = \bar{y}'A' = 0.06(0.03)(0.12) = 0.216(10^{-3})\ \text{m}^3$$

$$(Q_A)_z = 0$$

$$(Q_B)_z = \bar{z}'A' = 0.04(0.04)(0.15) = 0.240(10^{-3})\ \text{m}^3$$

$$(Q_B)_y = 0$$



Normal Stress:

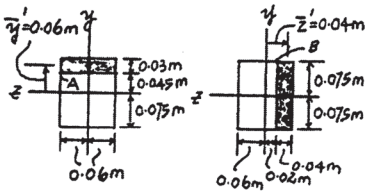
$$\sigma = -\frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

$$\sigma_A = -\frac{-14.4(10^3)(0.045)}{33.75(10^{-6})} + \frac{-10.8(10^3)(0.06)}{21.6(10^{-6})}$$

$$= -10.8\ \text{MPa}$$

$$\sigma_B = -\frac{-14.4(10^3)(0.075)}{33.75(10^{-6})} + \frac{-10.8(10^3)(-0.02)}{21.6(10^{-6})}$$

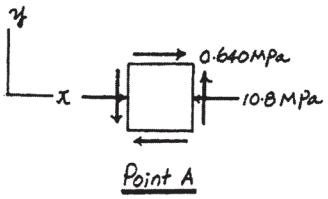
$$= 42.0\ \text{MPa}$$

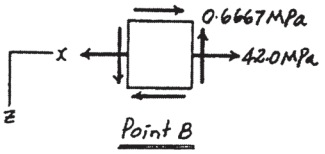


Shear Stress: Applying the shear formula

$$\tau_A = \frac{V_y(Q_A)_y}{I_z t} = \frac{12.0(10^3)\left[0.216(10^{-3})\right]}{33.75(10^{-6})(0.12)} = 0.640\ \text{MPa}$$

$$\tau_B = \frac{V_z(Q_B)_z}{I_y t} = \frac{-9.00(10^3)\left[0.240(10^{-3})\right]}{21.6(10^{-6})(0.15)} = -0.6667\ \text{MPa}$$





In-Plane Principal Stress: $\sigma_x = -10.8\ \text{MPa}$, $\sigma_y = 0$ and $\tau_{xy} = 0.640\ \text{MPa}$ for point A.

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{-10.8 + 0}{2} \pm \sqrt{\left(\frac{-10.8 - 0}{2}\right)^2 + 0.640^2}$$

$$= -5.40 \pm 5.4378$$

$$\sigma_1 = 37.8\ \text{kPa} \qquad \sigma_2 = -10.8\ \text{MPa} \qquad \textbf{Ans.}$$

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***9–36. Continued**

$\sigma_x = 42.0$ MPa, $\sigma_z = 0$, and $\tau_{xz} = 0.6667$ MPa for point *B*.

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + {\tau_{xy}}^2}$$

$$= \frac{42.0 + 0}{2} \pm \sqrt{\left(\frac{42.0 - 0}{2}\right)^2 + 0.6667^2}$$

$$= 21.0 \pm 21.0105$$

$\sigma_1 = 42.0$ MPa $\quad \sigma_2 = -10.6$ kPa **Ans.**

Ans:
Point *A*,
$\sigma_1 = 37.8$ kPa,
$\sigma_2 = -10.8$ MPa,
Point *B*,
$\sigma_1 = 42.0$ MPa,
$\sigma_2 = -10.6$ kPa

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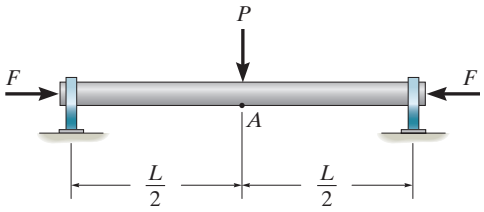
9–37.

The shaft has a diameter d and is subjected to the loadings shown. Determine the principal stresses and the maximum in-plane shear stress at point A. The bearings only support vertical reactions.



SOLUTION

Support Reactions: As shown on FBD(a).

Internal Forces and Moment: As shown on FBD(b).

Section Properties:

$$A = \frac{\pi}{4}d^2 \quad I = \frac{\pi}{4}\left(\frac{d}{2}\right)^4 = \frac{\pi}{64}d^4 \quad Q_A = 0$$

Normal Stress:

$$\sigma = \frac{N}{A} \pm \frac{Mc}{I}$$

$$= \frac{-F}{\frac{\pi}{4}d^2} \pm \frac{\frac{PL}{4}\left(\frac{d}{2}\right)}{\frac{\pi}{64}d^4}$$

$$\sigma_A = \frac{4}{\pi d^2}\left(\frac{2PL}{d} - F\right)$$

Shear Stress: Since $Q_A = 0, \tau_A = 0$

In-Plane Principal Stress: $\sigma_x = \frac{4}{\pi d^2}\left(\frac{2PL}{d} - F\right)$.

$\sigma_y = 0$ and $\tau_{xy} = 0$ for point A. Since no shear stress acts on the element,

$$\sigma_1 = \sigma_x = \frac{4}{\pi d^2}\left(\frac{2PL}{d} - F\right) \quad \textbf{Ans.}$$

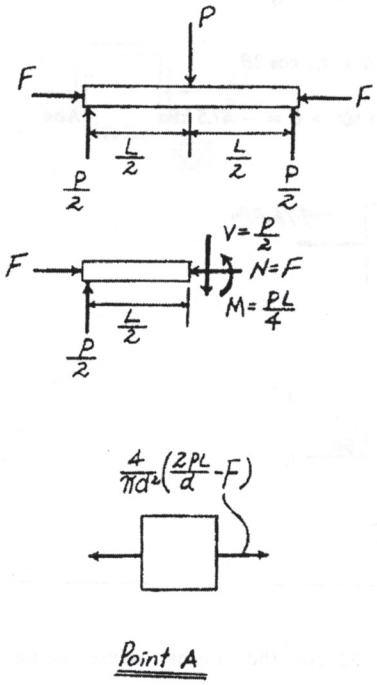
$$\sigma_2 = \sigma_y = 0 \quad \textbf{Ans.}$$

Maximum In-Plane Shear Stress: Applying Eq. 9–7 for point A,

$$\tau_{\substack{\max \\ \text{in-plane}}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$$

$$= \sqrt{\left(\frac{\frac{4}{\pi d^2}\left(\frac{2PL}{d} - F\right) - 0}{2}\right)^2 + 0}$$

$$= \frac{2}{\pi d^2}\left(\frac{2PL}{d} - F\right) \quad \textbf{Ans.}$$



Ans:

$$\sigma_1 = \frac{4}{\pi d^2}\left(\frac{2PL}{d} - F\right), \sigma_2 = 0,$$

$$\tau_{\substack{\max \\ \text{in-plane}}} = \frac{2}{\pi d^2}\left(\frac{2PL}{d} - F\right)$$

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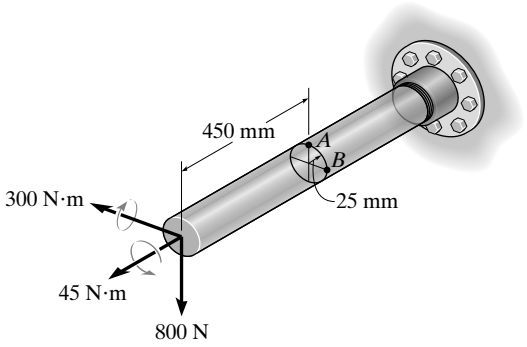
9–38. The solid shaft is subjected to a torque, bending moment, and shear force as shown. Determine the principal stresses acting at point A.

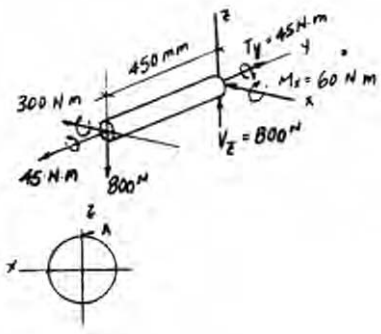


SOLUTION

$$I_x = I_y = \frac{\pi}{4}(0.025)^4 = 0.306796(10^{-6})\ \text{m}^4$$

$$J = \frac{\pi}{2}(0.025)^4 = 0.613592(10^{-6})\ \text{m}^4$$

$$Q_A = 0$$



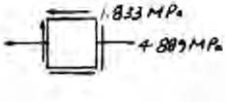
$$\sigma_A = \frac{M_x c}{I} = \frac{60(0.025)}{0.306796(10^{-6})} = 4.889\ \text{MPa}$$

$$\tau_A = \frac{T_y c}{J} = \frac{45(0.025)}{0.613592(10^{-6})} = 1.833\ \text{MPa}$$

$$\sigma_x = 4.889\ \text{MPa} \qquad \sigma_y = 0 \qquad \tau_{xy} = -1.833\ \text{MPa}$$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + {\tau_{xy}}^2}$$

$$= \frac{4.889 + 0}{2} \pm \sqrt{\left(\frac{4.889 - 0}{2}\right)^2 + (-1.833)^2}$$



$$\sigma_1 = 5.50\ \text{MPa}$$ **Ans.**

$$\sigma_2 = -0.611\ \text{MPa}$$ **Ans.**

Ans.

$\sigma_1 = 5.50\ \text{MPa}, \sigma_2 = -0.611\ \text{MPa}$

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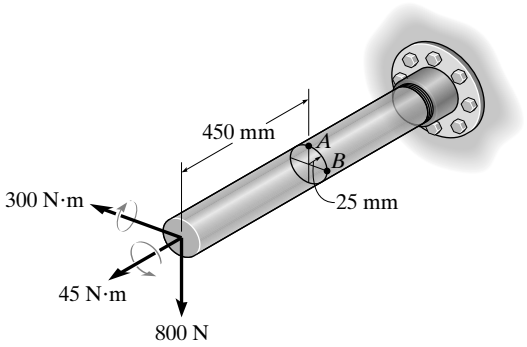
9–39. Solve Prob. 9–47 for point B.



SOLUTION

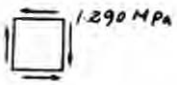
$$I_x = I_y = \frac{\pi}{4}(0.025)^4 = 0.306796(10^{-6})\ \text{m}^4$$

$$J = \frac{\pi}{2}(0.025)^4 = 0.613592(10^{-6})\ \text{m}^4$$

$$Q_B = \bar{y}A' = \frac{4(0.025)}{3\pi}\left(\frac{1}{2}\right)\pi\,(0.025^2) = 10.4167(10^{-6})\ \text{m}^3$$

$$\sigma_B = 0$$

$$\tau_B = \frac{V_z Q_B}{It} - \frac{T_y c}{J} = \frac{800(10.4167)(10^{-6})}{0.306796(10^{-6})(0.05)} - \frac{45(0.025)}{0.61359(10^{-6})} = -1.290\ \text{MPa}$$



$$\sigma_x = 0 \qquad \sigma_y = 0 \qquad \tau_{xy} = -1.290\ \text{MPa}$$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + {\tau_{xy}}^2}$$

$$= 0 \pm \sqrt{(0)^2 + (-1.290)^2}$$

$\sigma_1 = 1.29\ \text{MPa}$ **Ans.**

$\sigma_2 = -1.29\ \text{MPa}$ **Ans.**

Ans.

$\sigma_1 = 1.29\ \text{MPa}, \sigma_2 = -1.29\ \text{MPa}$

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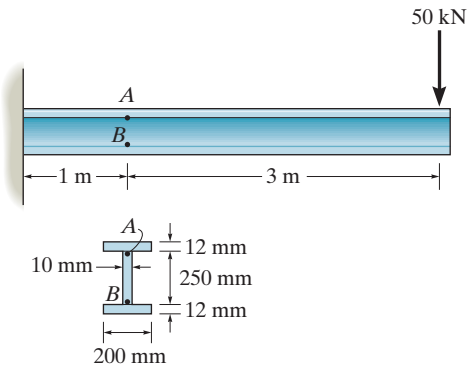
***9–40.**

The wide-flange beam is subjected to the 50-kN force. Determine the principal stresses in the beam at point A located on the *web* at the bottom of the upper flange. Although it is not very accurate, use the shear formula to calculate the shear stress.



SOLUTION

$$I = \frac{1}{12}(0.2)(0.274)^3 - \frac{1}{12}(0.19)(0.25)^3 = 95.451233(10^{-6})\ \text{m}^4$$

$$Q_A = (0.131)(0.012)(0.2) = 0.3144(10^{-3})\ \text{m}^3$$

$$\sigma_A = \frac{My}{I} = \frac{150(10^3)(0.125)}{95.451233(10^{-6})} = 196.43\ \text{MPa}$$

$$\tau_A = \frac{VQ_A}{It} = \frac{50(10^3)(0.3144)(10^{-3})}{95.451233(10^{-6})(0.01)} = 16.47\ \text{MPa}$$

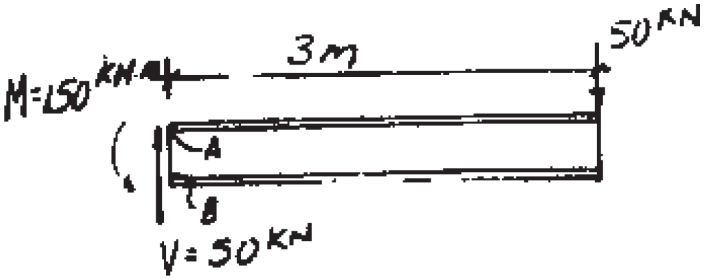
$$\sigma_x = 196.43\ \text{MPa} \qquad \sigma_y = 0 \qquad \tau_{xy} = -16.47\ \text{MPa}$$

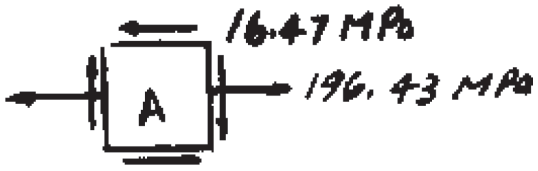
$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{196.43 + 0}{2} \pm \sqrt{\left(\frac{196.43 - 0}{2}\right)^2 + (-16.47)^2}$$

$\sigma_1 = 198\ \text{MPa}$ **Ans.**

$\sigma_2 = -1.37\ \text{MPa}$ **Ans.**





Ans:
$\sigma_1 = 198\ \text{MPa},$
$\sigma_2 = -1.37\ \text{MPa}$

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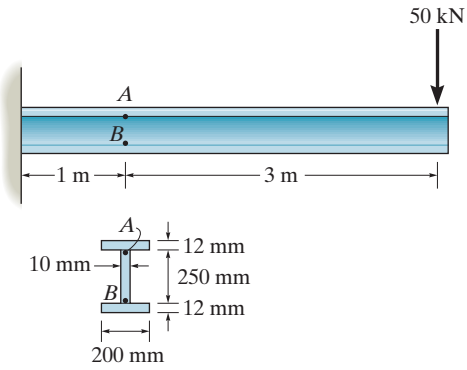
9–41.

Solve Prob. 9–40 for point B located on the *web* at the top of the bottom flange.



SOLUTION

$$I = \frac{1}{12}(0.2)(0.247)^3 - \frac{1}{12}(0.19)(0.25)^3 = 95.451233(10^{-6})\ \text{m}^4$$

$$Q_B = (0.131)(0.012)(0.2) = 0.3144(10^{-3})$$

$$\sigma_B = -\frac{My}{I} = -\frac{150(10^3)(0.125)}{95.451233(10^{-6})} = -196.43\ \text{MPa}$$

$$\tau_B = \frac{VQ_B}{It} = \frac{50(10^3)(0.3144)(10^{-3})}{95.451233(10^{-6})(0.01)} = 16.47\ \text{MPa}$$

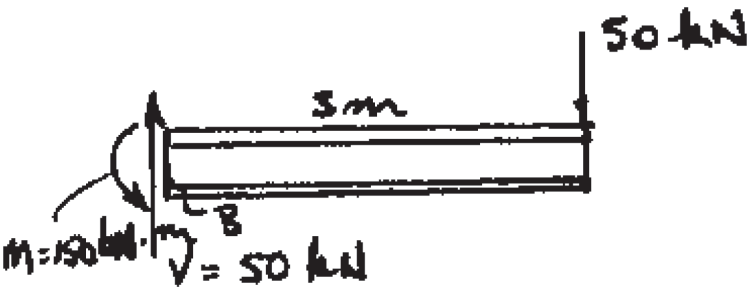
$\sigma_x = -196.43$ MPa $\quad \sigma_y = 0 \quad \tau_{xy} = -16.47$ MPa

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + {\tau_{xy}}^2}$$

$$= \frac{-196.43 + 0}{2} \pm \sqrt{\left(\frac{-196.43 - 0}{2}\right)^2 + (-16.47)^2}$$

$\sigma_1 = 1.37$ MPa **Ans.**

$\sigma_2 = -198$ MPa **Ans.**



Ans:
$\sigma_1 = 1.37$ MPa, $\sigma_2 = -198$ MPa

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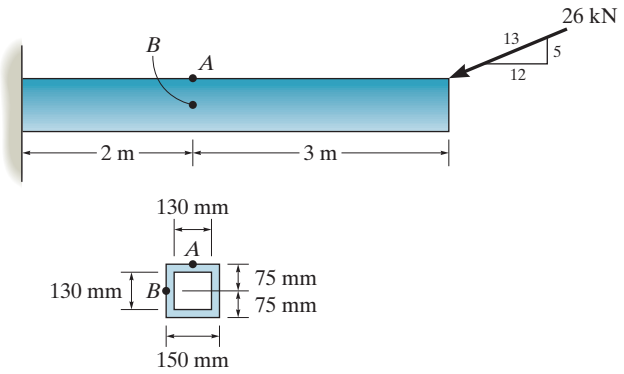
9–42.

The box beam is subjected to the 26-kN force that is applied at the center of its width, 75 mm from each side. Determine the principal stresses at point A and show the results in an element located at this point. Use the shear formula to calculate the shear stress.



SOLUTION

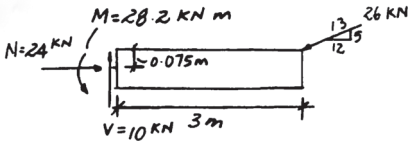
$$I = \frac{1}{12}(0.15)(0.15^3) - \frac{1}{12}(0.13)(0.13^3) = 18.38667(10^{-6})\ \text{m}^4$$

$$A = 0.15^2 - 0.13^2 = 5.6(10^{-3})\ \text{m}^2$$

$$Q_A = 0$$

$$\tau_A = 0$$

$$\sigma_A = -\frac{P}{A} + \frac{Mc}{I} = \frac{-24(10^3)}{5.6(10^{-3})} + \frac{28.2(10^3)(0.075)}{18.38667(10^{-6})} = 111\ \text{MPa}$$



$\sigma_1 = 111\ \text{MPa}$ **Ans.**

$\sigma_2 = 0$ **Ans.**

Ans:
$\sigma_1 = 111$ MPa,
$\sigma_2 = 0$

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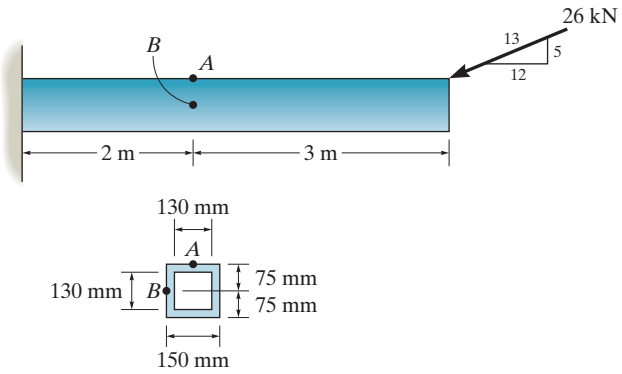
9–43.

Solve Prob. 9–42 for point B.



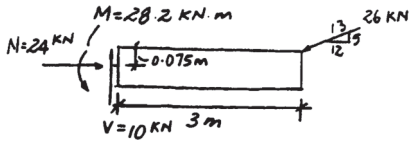
SOLUTION

$$I = \frac{1}{12}(0.15)(0.15^3) - \frac{1}{12}(0.13)(0.13^3) = 18.38667(10^{-6})\ \text{m}^4$$

$$A = 0.15^2 - 0.13^2 = 5.6(10^{-3})\ \text{m}^2$$

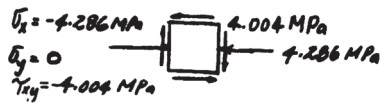
$$Q_B = (0.07)(0.15)(0.01) + 2(0.0325)(0.065)(0.01) = 0.14725(10^{-3})\ \text{m}^3$$

$$\sigma_B = -\frac{P}{A} = -\frac{24(10^3)}{5.6(10^{-3})} = -4.286\ \text{MPa}$$



$$\tau_B = \frac{VQ_B}{It} = \frac{10(10^3)(0.14725)(10^{-3})}{18.38667(10^{-6})(0.02)} = 4.004\ \text{MPa}$$

$$\sigma_x = -4.286\ \text{MPa} \qquad \sigma_y = 0 \qquad \tau_{xy} = -4.004\ \text{MPa}$$



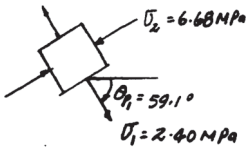
$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + {\tau_{xy}}^2}$$

$$= \frac{-4.286 + 0}{2} \pm \sqrt{\left(\frac{-4.286 - 0}{2}\right)^2 + (-4.004^2)}$$

$$\sigma_1 = 2.40\ \text{MPa} \qquad \sigma_2 = -6.68\ \text{MPa} \qquad \textbf{Ans.}$$

$$\tan 2\theta_P = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2} = \frac{-4.004}{(-4.286 - 0)/2}$$

$$\theta_P = 30.9° \quad \text{or} \quad -59.1°$$



Use Eq. 9–1,

$$\theta_{P1} = -59.1° \qquad \theta_{P2} = 30.9°$$

Ans:
$\sigma_1 = 2.40$ MPa,
$\sigma_2 = -6.68$ MPa

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***9–44.**

Solve Prob. 9–2 using Mohr's circle.

SOLUTION

$$\frac{\sigma_x + \sigma_y}{2} = \frac{0 + 65}{2} = 32.5 \text{ MPa}$$

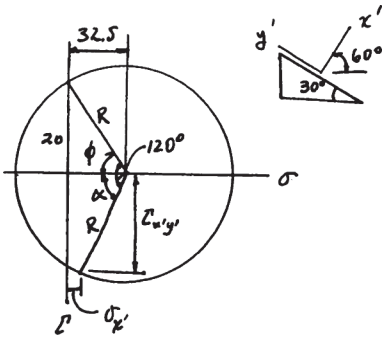
$$R = \sqrt{(32.5)^2 + (20)^2} = 38.1608$$

$$\phi = \tan^{-1}\frac{20}{32.5} = 31.6075°$$

$$\alpha = 120° - 31.6075° = 88.392°$$

$\sigma_{x'} = 32.5 - 38.1608 \cos 88.392° = 31.4$ MPa **Ans.**

$\tau_{x'y'} = 38.1608 \sin 88.392° = 38.1$ MPa **Ans.**



Ans:
$\sigma_{x'} = 31.4$ MPa,
$\tau_{x'y'} = 38.1$ MPa

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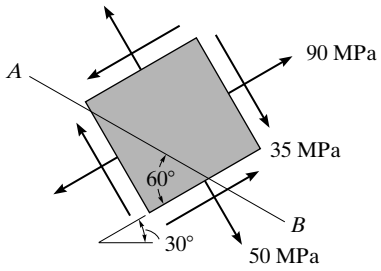
9–45. Solve Prob. 9–4 using Mohr's circle.



SOLUTION

$\sigma_x = 90$ MPa $\quad \sigma_y = 50$ MPa $\quad \tau_{xy} = -35$ MPa $\quad A(90, -35)$

$$\frac{\sigma_x + \sigma_y}{2} = \frac{90 + 50}{2} = 70$$

$$R = \sqrt{(90 - 70)^2 + (35)^2} = 40.311$$

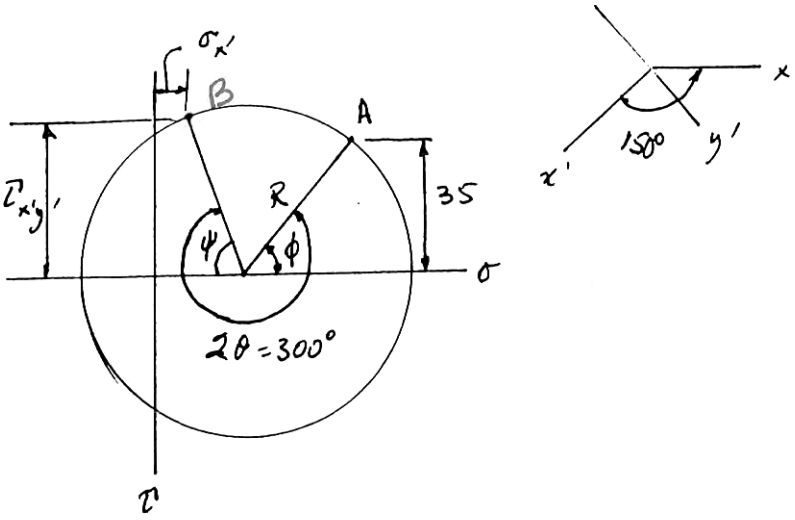
Coordinates of point B:

$$\phi = \tan^{-1}\left(\frac{35}{20}\right) = 60.255°$$

$$\psi = 300° - 180° - 60.255° = 59.745°$$

$\sigma_{x'} = 70 - 40.311 \cos 59.745° = 49.7$ MPa **Ans.**

$\tau_{x'} = -40.311 \sin 59.745° = -34.8$ MPa **Ans.**



Ans.

$\sigma_{x'} = -388$ psi, $\tau_{x'y'} = 455$ psi

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9–46.

SOLUTION

Construction of the Circle: In accordance to the sign convention, $\sigma_x = 0$, $\sigma_y = -500$ MPa and $\tau_{xy} = 350$ MPa. Hence

$$\sigma_{\text{avg}} = \frac{\sigma_x + \sigma_y}{2} = \frac{0 + (-500)}{2} = -250 \text{ MPa}$$

The coordinates for reference point A and center C of the circle are

$$A(0, 350) \qquad C(-250, 0)$$

The radius of the circle is

$$R = \sqrt{[0 - (-250)]^2 + 350^2} = 430.12 \text{ MPa}$$

Using the results, the circle shown in Fig. a can be constructed.

Stress on the Rotated Element: Plane AB can be represented by an element rotating clockwise through an angle $\theta = 60°$, Fig. b. The normal and shear stress components on plane AB $\left(\sigma_{x'} \text{ and } \tau_{x'y'}\right)$ are represented by coordinates of point P on the circle. From the geometry shown on the circle

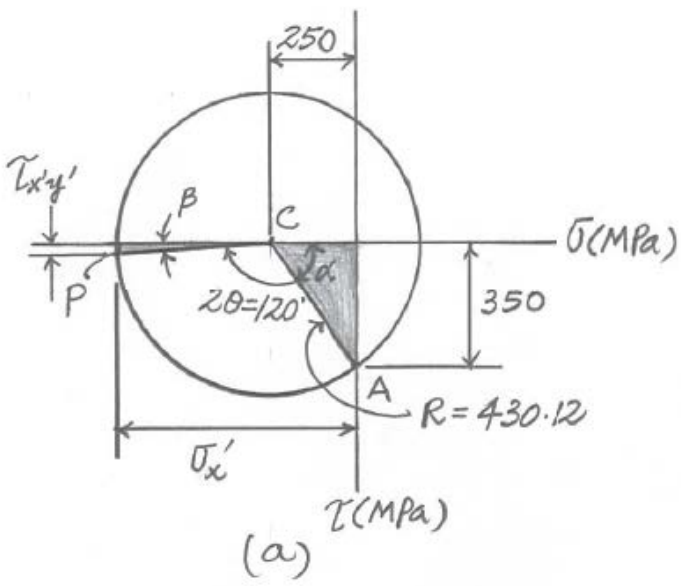
$$\alpha = \tan^{-1}\left(\frac{350}{250}\right) = 54.46°$$

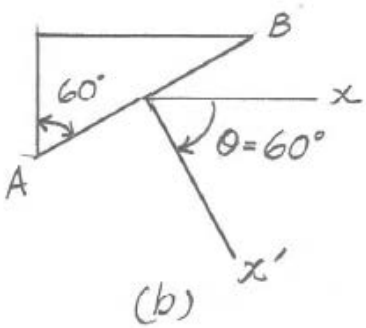
$$\beta = 180° - \alpha - 2\theta = 180° - 54.46° - 120° = 5.538°$$

Then

$$\sigma_{x'} = -250 - 430.12\cos 5.538° = -678 \text{ MPa}$$ **Ans**

$$\tau_{x'y'} = 430.12\sin 5.538° = 41.5 \text{ MPa}$$ **Ans**





Ans:

$\sigma_{x'} = -678$ MPa, $\tau_{x'y'} = 41.5$ MPa

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9–47.

Solve Prob. 9–11 using Mohr's circle.

SOLUTION

Construction of the Circle: $\theta = -60°$, $\sigma_x = 150$ MPa, $\sigma_y = 100$ MPa, $\tau_{xy} = 75$ MPa. Thus

$$\sigma_{\text{avg}} = \frac{\sigma_x + \sigma_y}{2} = \frac{150 + 100}{2} = 125 \text{ MPa}$$

The coordinates of the reference point A and center C of the circle are

$$A(150, 75) \qquad C(125, 0)$$

Thus, the radius of the circle is

$$R = CA = \sqrt{(150 - 125)^2 + (75)^2} = 79.06 \text{ MPa}$$

Normal and Shear Stress on Rotated Element: Here $\theta = 60°$ clockwise. By rotating the radial line CA clockwise $2\theta = 120°$, it coincides with the radial line OP and the coordinates of reference point $P(\sigma_{x'}, \tau_{x'y'})$ represent the normal and shear stresses on the face of the element defined by $\theta = -60°$. $\sigma_{y'}$ can be determined by calculating the coordinates of point Q. From the geometry of the circle, Fig. (a),

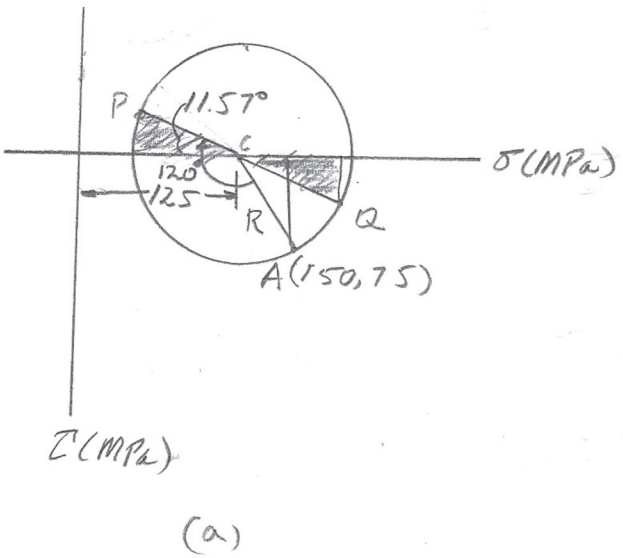
$$\sin \alpha = \frac{75}{79.06},\ \alpha = 71.57°,\ \beta = 120° + 71.57° - 180° = 11.57°$$

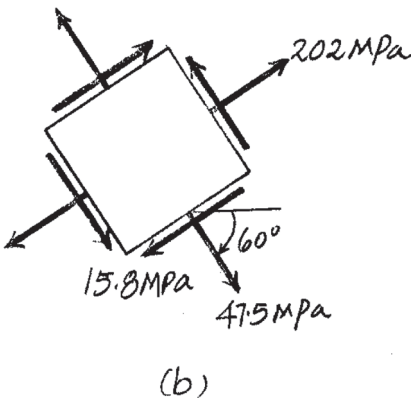
$\sigma_{x'} = 125 - 79.06 \cos 11.57° = 47.5$ MPa **Ans.**

$\tau_{x'y'} = -79.06 \sin 11.57° = -15.8$ MPa **Ans.**

$\sigma_{y'} = 125 + 79.06 \cos 11.57° = 202$ MPa **Ans.**

The results are shown in Fig. (b).





Ans:
$\sigma_{x'} = 47.5$ MPa, $\tau_{x'y'} = -15.8$ MPa,
$\sigma_{y'} = 202$ MPa

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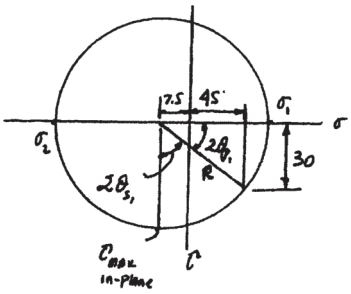
***9–48.**

Solve Prob. 9–15 using Mohr's circle.

SOLUTION



$$\frac{\sigma_x + \sigma_y}{2} = \frac{45 - 60}{2} = -7.5 \text{ MPa}$$

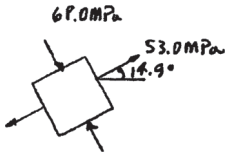
$$R = \sqrt{(45 + 7.5)^2 + (30)^2} = 60.467 \text{ MPa}$$

$\sigma_1 = 60.467 - 7.5 = 53.0$ MPa **Ans.**

$\sigma_2 = -60.467 - 7.5 = -68.0$ MPa **Ans.**

$$2\theta_{p1} = \tan^{-1}\frac{30}{(45 + 7.5)}$$

$\theta_{p1} = 14.9°$ counterclockwise **Ans.**

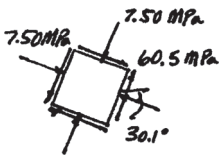


$\tau_{\text{max}}_{\text{in-plane}} = 60.5$ MPa **Ans.**

$\sigma_{avg} = -7.50$ MPa **Ans.**

$$2\theta_{s1} = 90° - \tan^{-1}\frac{30}{(45 + 7.5)}$$

$\theta_{s1} = 30.1°$ clockwise **Ans.**



Ans:
$\sigma_1 = 53.0$ MPa,
$\sigma_2 = -68.0$ MPa,
$\theta_{p1} = 14.9°$ counterclockwise,
$\tau_{\text{max}}_{\text{in-plane}} = 60.5$ MPa,
$\sigma_{avg} = -7.50$ MPa,
$\theta_{s1} = 30.1°$ clockwise

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9–49.

Solve Prob. 9–16 using Mohr's circle.

SOLUTION

Construction of Circle: $\sigma_x = 50$ MPa, $\sigma_y = 0$, $\tau_{xy} = -15$ MPa. Thus,

$$\sigma_{\text{avg}} = \frac{\sigma_x + \sigma_y}{2} = \frac{50 + 0}{2} = 25 \text{ MPa} \qquad \textbf{Ans.}$$

The coordinates of reference point A and center C of the circle are

$$A(50, -15) \qquad C(25, 0)$$

Thus, the radius of the circle is

$$R = CA = \sqrt{(50 - 25)^2 + (-15)^2} = \tau_{\substack{\max \\ \text{in-plane}}} = 29.15 \text{ MPa}$$

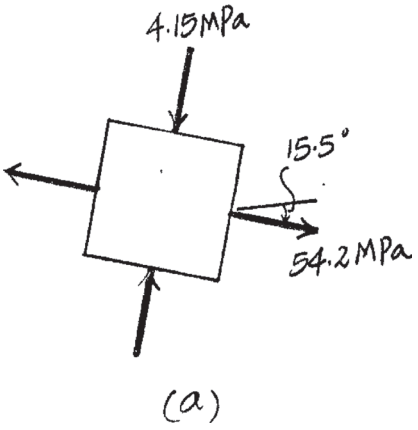
See Fig. (*a*).

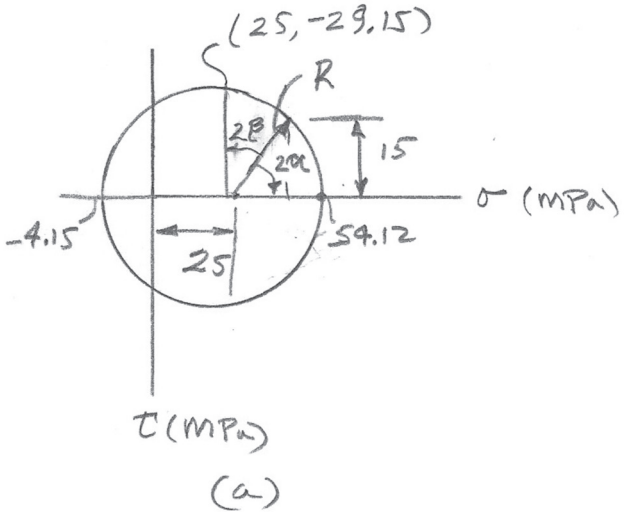
a) **Principal Stress:**

$$\sigma_1 = 54.2 \text{ MPa}, \qquad \sigma_2 = -4.15 \text{ MPa} \qquad \textbf{Ans.}$$

$$\sin 2\alpha = \frac{15}{29.15}, \qquad \alpha = 15.5° \qquad \textbf{Ans.}$$

See Fig. (*b*).





Ans:
$\sigma_1 = 54.2$ MPa, $\sigma_2 = -4.15$ MPa, $\theta_p = -15.5°$,
$\sigma_{\text{avg}} = 25$ MPa, $\tau_{\substack{\max \\ \text{in-plane}}} = 29.2$ MPa, $\theta_s = 29.5°$,
$\alpha = 15.5°$

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9–50.

Mohr's circle for the state of stress is shown in Fig. 9–17*a*. Show that finding the coordinates of point $P(\sigma_{x'}, \tau_{x'y'})$ on the circle gives the same value as the stress transformation Eqs. 9–1 and 9–2.

SOLUTION

$A(\sigma_x, \tau_{xy}) \qquad B(\sigma_y, -\tau_{xy}) \qquad C\left(\left(\frac{\sigma_x + \sigma_y}{2}\right), 0\right)$

$$R = \sqrt{\left[\sigma_x - \left(\frac{\sigma_x + \sigma_y}{2}\right)\right]^2 + \tau_{xy}^2} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_x' = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}\cos\theta' \qquad \textbf{(1)}$$

$$\theta' = 2\theta_P - 2\theta$$

$$\cos(2\theta_P - 2\theta) = \cos 2\theta_P \cos 2\theta + \sin 2\theta_p \sin 2\theta \qquad \textbf{(2)}$$

From the circle:

$$\cos 2\theta_P = \frac{\sigma_x - \frac{\sigma_x + \sigma_y}{2}}{\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}} \qquad \textbf{(3)}$$

$$\sin 2\theta_P = \frac{\tau_{xy}}{\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}} \qquad \textbf{(4)}$$

Substitute Eq. (2), (3) and into Eq. (1)

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta \qquad \textbf{QED}$$

$$\tau_{x'y'} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}\sin\theta' \qquad \textbf{(5)}$$

$$\sin\theta' = \sin(2\theta_P - 2\theta)$$

$$= \sin 2\theta_P \cos 2\theta - \sin 2\theta \cos 2\theta_P \qquad \textbf{(6)}$$

Substitute Eq. (3), (4), (6) into Eq. (5),

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta \qquad \textbf{QED}$$

Ans:
N/A

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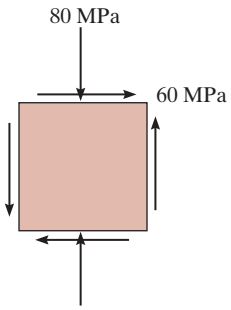
9–51.

Determine (a) the principal stresses and (b) the maximum in-plane shear stress and average normal stress. Specify the orientation of the element in each case.



SOLUTION

Construction of The Circle: In accordance to the sign convention, $\sigma_x = 0, \sigma_y = -80$ MPa and $\tau_{xy} = 60$ MPa. Hence,

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{0 + (-80)}{2} = -40.0 \text{ MPa}$$ **Ans.**

The coordinates for reference point A and center of circle C are

$A(0, 60)$ $C(-40.0, 0)$

Thus, the radius of the circle is

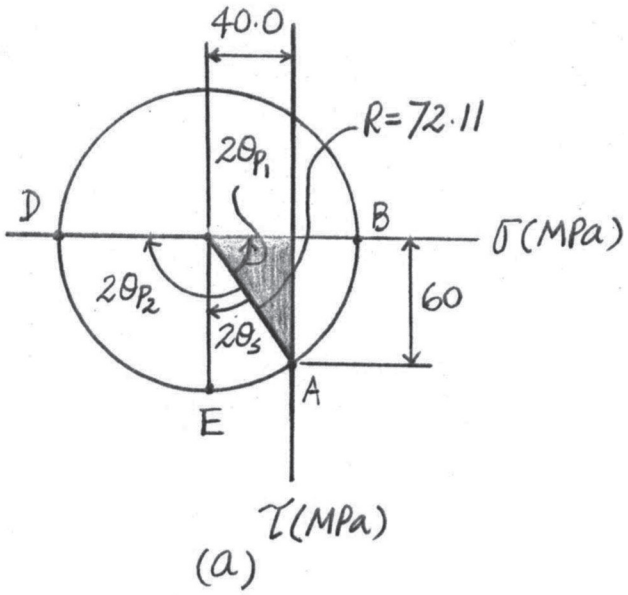
$$R = CA = \sqrt{[0 - (-40.0)]^2 + (60 - 0)^2} = 72.11 \text{ MPa}$$

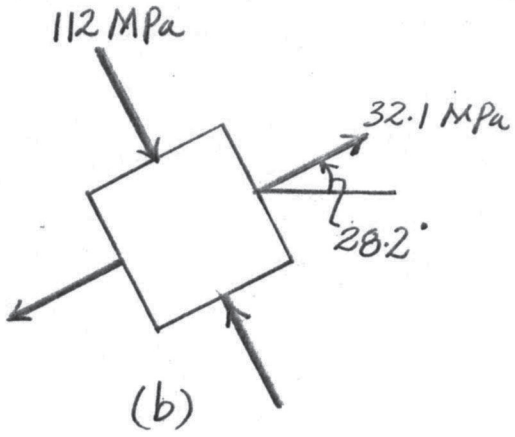
Using these results, the circle shown in Fig. *a* can be constructed.

a) In-plane Principal Stresses: The coordinates of points B and D on the circle represent σ_1 and σ_2, respectively.

$\sigma_1 = -40.0 + 72.11 = 32.11 \text{ MPa} = 32.1 \text{ MPa}$ **Ans.**

$\sigma_2 = -40.0 - 72.11 = -112.11 \text{ MPa} = -112 \text{ MPa}$ **Ans.**





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9–51. Continued

Orientation of Principal Plane: From the shaded triangle on the circle,

$$\tan 2\theta_{p1} = \frac{60}{40.0} = 1.5$$

$$2\theta_{p1} = 56.31°$$

$$\theta_{p1} = 28.15° = 28.2° \textbf{ (counterclockwise)} \qquad \textbf{Ans.}$$

Using these results the state of in-plane principal stresses can be represented by the differential element shown in Fig. *b*.

b) Maximum In-plane Shear Stress: Represented by the coordinates of point *E* on the circle.

$$\tau_{\substack{\max \\ \text{in-plane}}} = \text{R} = 72.11 \text{ MPa} = 72.1 \text{ MPa} \qquad \textbf{Ans.}$$

Orientation of The Plane For Maximum in-plane Shear Stress: From the circle,

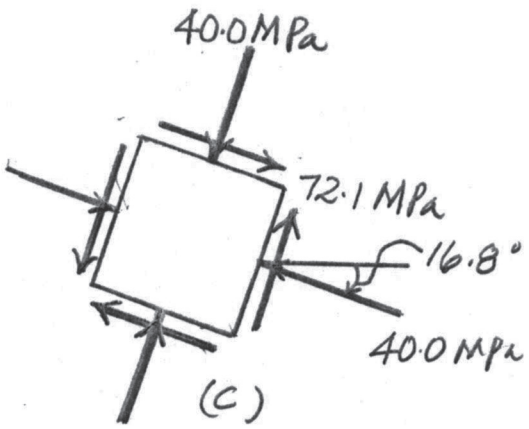
$$\tan 2\theta_s = \frac{40.0}{60} = 0.6667$$

$$2\theta_s = 33.69°$$

$$\theta_s = 16.845° = 16.8° \textbf{ (clockwise)} \qquad \textbf{Ans.}$$

Using these results, the state of maximum in-plane shear stress can be represented by the differential element shown in Fig. *c*.

$$\theta_s = -16.8°$$



Ans:
$\sigma_{\text{avg}} = -40.0$ MPa,
$\sigma_1 = 32.1$ MPa,
$\sigma_2 = -112$ MPa,
$\theta_{p1} = 28.2°$,
$\tau_{\substack{\max \\ \text{in-plane}}} = 72.1$ MPa,
$\theta_s = -16.8°$

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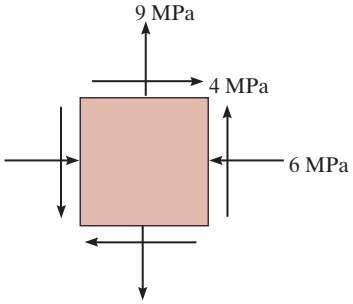
***9–52.** Determine the equivalent state of stress if an element is oriented 30° clockwise from the element shown. Show the result on the element.



In accordance to the established sign convention, $\sigma_x = -6$ MPa, $\sigma_y = 9$ MPa, and $\tau_{xy} = 4$ MPa. Thus,

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{-6 + 9}{2} = 1.50 \text{ MPa}$$

Then, the coordinates of reference point A and C are

$$A(-6, 4) \qquad C(1.5, 0)$$

The radius of the circle is

$$R = CA = \sqrt{(-6 - 1.5)^2 + 4^2} = 8.50 \text{ MPa}$$

Using these results, the circle shown in Fig. *a* can be constructed.

Referring to the geometry of the circle, Fig. *a*,

$$\alpha = \tan^{-1}\left(\frac{4}{6 + 1.5}\right) = 28.07° \qquad \beta = 60° - 28.07° = 31.93°$$

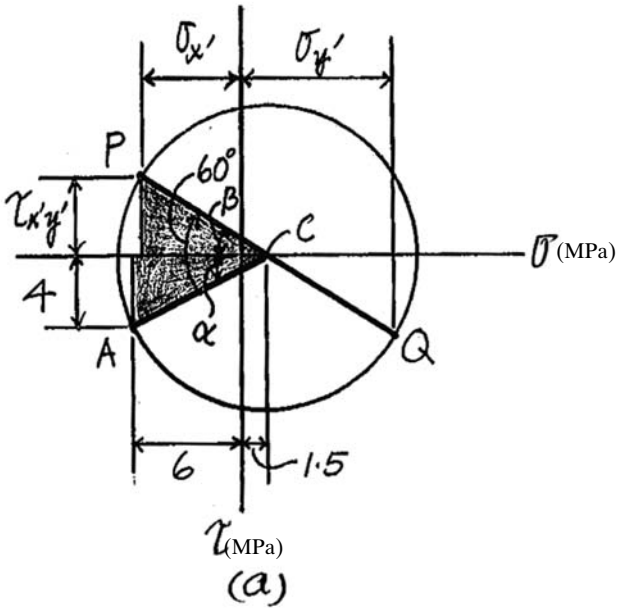
Then,

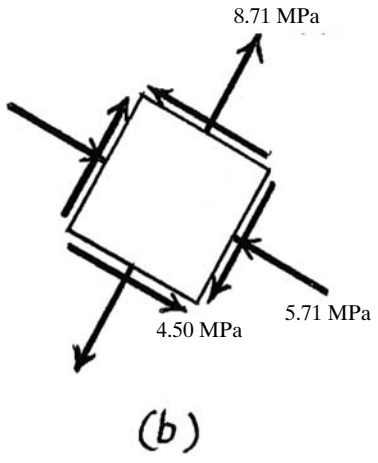
$$\sigma_{x'} = 1.5 - 8.50 \cos 31.93° = -5.71 \text{ MPa} \quad \textbf{Ans.}$$

$$\tau_{x'y'} = -8.5 \sin 31.95° = -4.50 \text{ MPa} \quad \textbf{Ans.}$$

$$\sigma_{y'} = 8.71 \text{ MPa} \quad \textbf{Ans.}$$

The results are shown in Fig. *b*.





Ans.
$\sigma_{x'} = -5.71$ MPa, $\tau_{x'y'} = -4.50$ MPa,
$\sigma_{y'} = 8.71$ MPa

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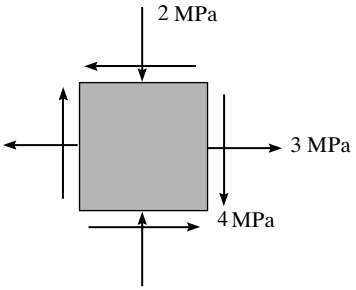
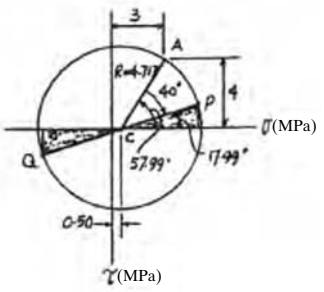
9–53. Determine the equivalent state of stress if an element is oriented 20° clockwise from the element shown.



SOLUTION

Construction of the Circle: In accordance with the sign convention, $\sigma_x = 3$ MPa, $\sigma_y = -2$ MPa, $\tau_{xy} = -4$ MPa. Hence,

$$\sigma_{\text{avg}} = \frac{\sigma_x + \sigma_y}{2} = \frac{3 + (-2)}{2} = 0.500 \text{ MPa}$$



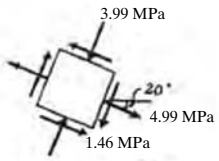
The coordinates for reference points A and C are

$$A(3, -4) \qquad C(0.500, 0)$$

The radius of the circle is

$$R = \sqrt{(3 - 0.500)^2 + 4^2} = 4.717 \text{ MPa}$$

Stress on the Rotated Element: The normal and shear stress components $(\sigma_{x'}$ and $\tau_{x'y'})$ are represented by the coordinate of point P on the circle, $\sigma_{y'}$, can be determined by calculating the coordinates of point Q on the circle.



$$\sigma_{x'} = 0.500 + 4.717 \cos 17.99° = 4.99 \text{ MPa} \qquad \textbf{Ans.}$$

$$\tau_{x'y'} = -4.717 \sin 17.99° = -1.46 \text{ MPa} \qquad \textbf{Ans.}$$

$$\sigma_{y'} = 0.500 - 4.717 \cos 17.99° = -3.99 \text{ MPa} \qquad \textbf{Ans.}$$

Ans.
$\sigma_{x'} = 4.99$ MPa, $\tau_{x'y'} = -1.46$ MPa,
$\sigma_{y'} = -3.99$ MPa

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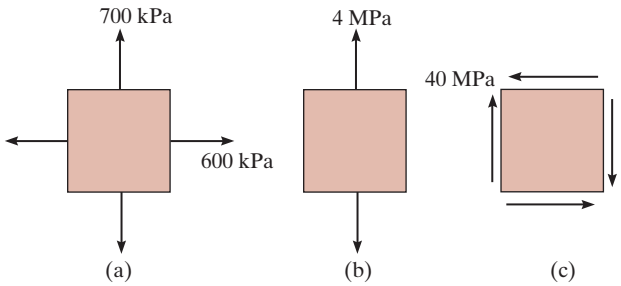
9–54. Draw Mohr's circle that describes each of the following states of stress.



SOLUTION

a) Here, $\sigma_x = 600$ KPa, $\sigma_y = 700$ KPa and $\tau_{xy} = 0$. Thus,

$$\sigma_{\text{avg}} = \frac{\sigma_x + \sigma_y}{2} = \frac{600 + 700}{2} = 650 \text{ KPa}$$

Thus, the coordinate of reference point A and center of circle are

$$A(600, 0) \qquad C(650, 0)$$

Then the radius of the circle is

$$R = CA = 650 - 600 = 50 \text{ KPa}$$

The Mohr's circle represents this state of stress is shown in Fig. *a*.

b) Here, $\sigma_x = 0, \sigma_y = 4$ MPa and $\tau_{xy} = 0$. Thus,

$$\sigma_{\text{avg}} = \frac{\sigma_x + \sigma_y}{2} = \frac{0 + 4}{2} = 2 \text{ MPa}$$

Thus, the coordinate of reference point A and center of circle are

$$A(0, 0) \qquad C(2, 0)$$

Then the radius of the circle is

$$R = CA = 2 - 0 = 2 \text{ MPa}$$

c) Here, $\sigma_x = \sigma_y = 0$ and $\tau_{xy} = -40$ MPa. Thus,

$$\sigma_{\text{avg}} = \frac{\sigma_x + \sigma_y}{2} = 0$$

Thus, the coordinate of reference point A and the center of circle are

$$A(0, -40) \qquad C(0, 0)$$

Then, the radius of the circle is

$$R = CA = 40 \text{ MPa}$$

The Mohr's circle represents this state of stress shown in Fig. *c*

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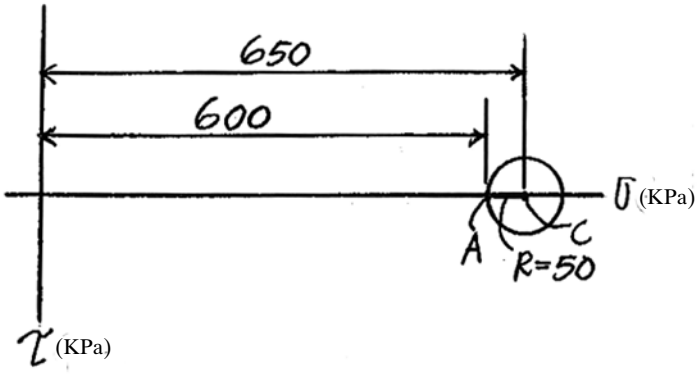
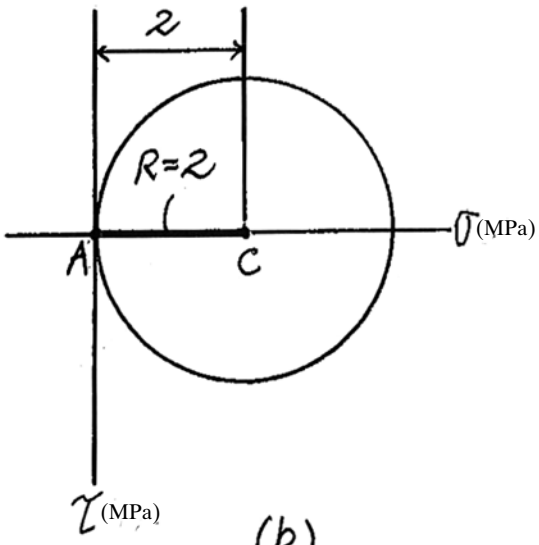
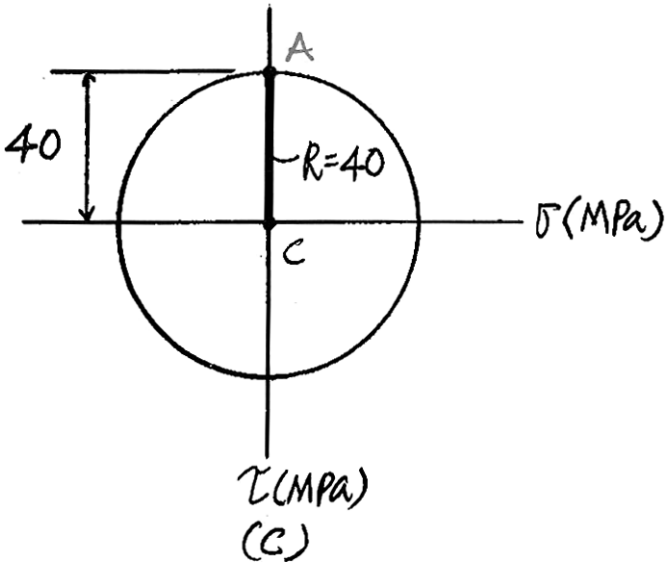
9–54. Continued



(a)



(b)



(c)

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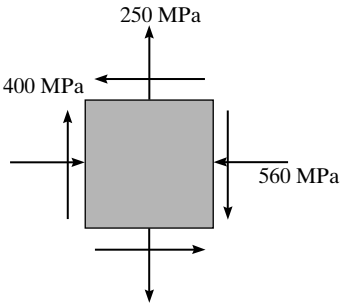
9–55. Determine the equivalent state of stress for an element oriented 60° counterclockwise from the element shown. Show the result on the element.



In accordance to the established sign convention, $\sigma_x = -560$ MPa, $\sigma_y = 250$ MPa and $\tau_{xy} = -400$ MPa. Thus,

$$\sigma_{\text{avg}} = \frac{\sigma_x + \sigma_y}{2} = \frac{-560 + 250}{2} = -155 \text{ MPa}$$

Then, the coordinate of reference points A and C are

$$A(-560, -400) \qquad C(-155, 0)$$

The radius of the circle is

$$R = CA = \sqrt{\left[-560 - (-155)\right]^2 + (-400)^2} = 569.23 \text{ MPa}$$

Using these results, the circle shown in Fig. *a* can be constructed.

Referring to the geometry of the circle, Fig. *a*

$$\alpha = \tan^{-1}\left(\frac{400}{560 - 155}\right) = 44.64° \qquad \beta = 120° - 44.64° = 75.36°$$

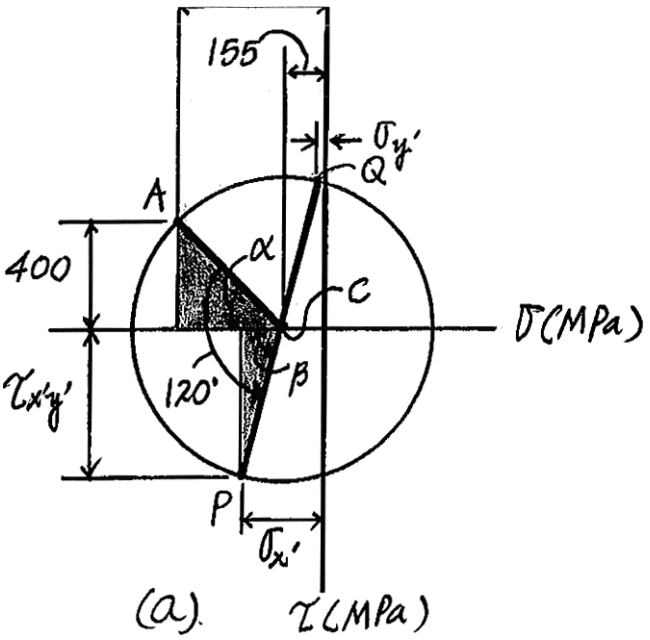
Then,

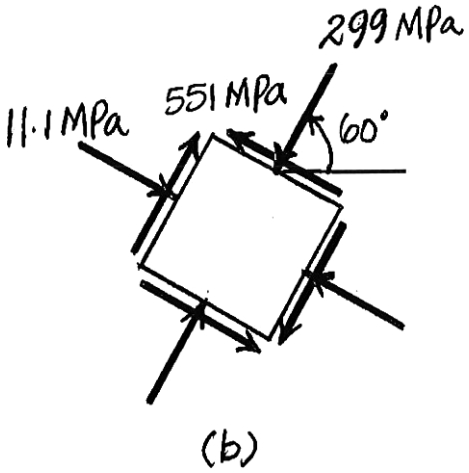
$$\sigma_{x'} = -155 - 569.23 \cos 75.36° = -299 \text{ MPa} \qquad \textbf{Ans.}$$

$$\tau_{x'y'} = 569.23 \sin 75.36° = 551 \text{ MPa} \qquad \textbf{Ans.}$$

$$\sigma_{y'} = -155 + 569.23 \cos 75.36° = -11.1 \text{ MPa} \qquad \textbf{Ans.}$$

The results are shown in Fig. *b*.





Ans.
$\sigma_{\text{avg}} = -155$ MPa, $R = 569.23$ MPa,
$\sigma_{x'} = -299$ MPa, $\tau_{x'y'} = 551$ MPa,
$\sigma_{y'} = -11.1$ MPa

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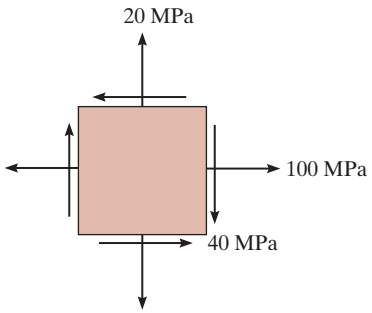
***9–56.**

Determine (a) the principal stresses and (b) the maximum in-plane shear stress and average normal stress. Specify the orientation of the element in each case.



SOLUTION

Construction of the Circle: In accordance to the sign convention, $\sigma_x = 100$ MPa, $\sigma_y = 20$ MPa, and $\tau_{xy} = -40$ MPa. Hence

$$\sigma_{\text{avg}} = \frac{\sigma_x + \sigma_y}{2} = \frac{100 + 20}{2} = 60.0 \text{ MPa} \qquad \textbf{Ans.}$$

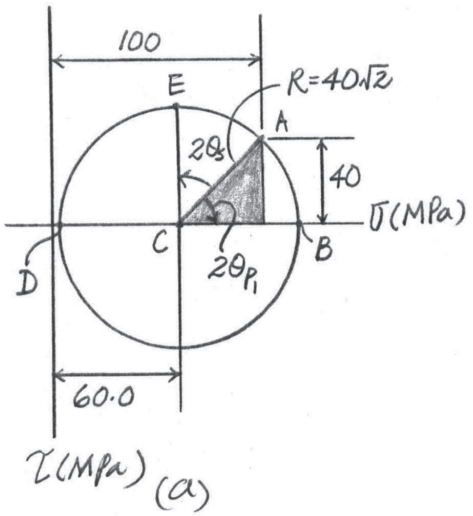
The coordinates for reference point A and center of circle C are

$$A(100, -40) \qquad C(60.0, 0)$$

Thus, the radius of the circle is

$$R = CA = \sqrt{(100 - 60.0)^2 + (-40 - 0)^2} = 40\sqrt{2} \text{ MPa}$$

Using these results, the circle shown in Fig. *a* can be constructed.



a) In-Plane Principal Stresses: The coordinates of points B and D on the circle represent σ_1 and σ_2 respectively.

$$\sigma_1 = 60.0 + 40\sqrt{2} = 116.57 \text{ MPa} = 117 \text{ MPa} \qquad \textbf{Ans.}$$

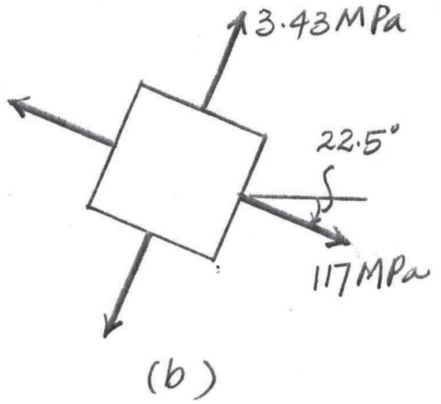
$$\sigma_2 = 60.0 - 40\sqrt{2} = 3.4315 \text{ MPa} = 3.43 \text{ MPa} \qquad \textbf{Ans.}$$

Orientation of Principal Plane: From the shaded triangle on the circle,

$$\tan 2\theta_{p1} = \frac{40}{100 - 60} = 1 \qquad 2\theta_{p1} = 45.0°$$

$$\theta_{p1} = 22.5° \textbf{ (Clockwise)} \qquad \textbf{Ans.}$$

Using these results, the state of in-plane principal stress can be represented by the differential element shown in Fig. *b*.



b) Maximum In-Plane Shear Stress: Represented by the coordinates of point E on the circle.

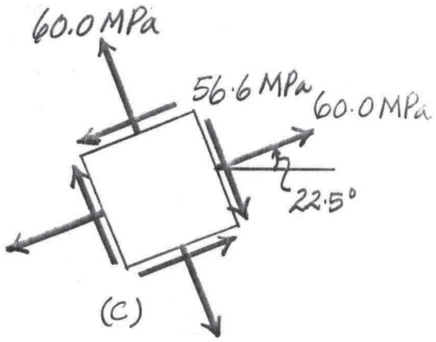
$$\tau_{\substack{\max \\ \text{in-plane}}} = -R = -40\sqrt{2} \text{ MPa} = -56.6 \text{ MPa} \qquad \textbf{Ans.}$$

Orientation of the Plane for Maximum In-Plane Shear Stress: From the circle,

$$\tan 2\theta_s = \frac{100 - 60.0}{40} = 1 \qquad 2\theta_s = 45.0°$$

$$\theta_s = 22.5° \textbf{ (Counterclockwise)} \qquad \textbf{Ans.}$$

Using these results, the state of maximum in-plane shear stress can be represented by the differential element shown in Fig. *c*.



Ans:
$\sigma_{\text{avg}} = 60.0$ MPa, $\sigma_1 = 117$ MPa, $\sigma_2 = 3.43$ MPa,
$\tau_{\substack{\max \\ \text{in-plane}}} = -56.6$ MPa, $\theta_s = 22.5°$,
$\theta_{p1} = 22.5°$ **(Clockwise)**

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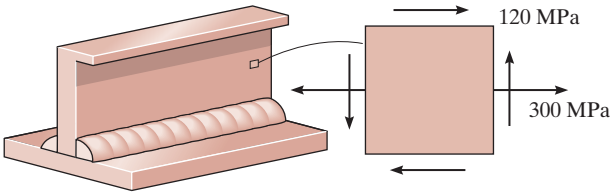
9–57. Determine the principal stress, the maximum in-plane shear stress, and average normal stress. Specify the orientation of the element in each case.



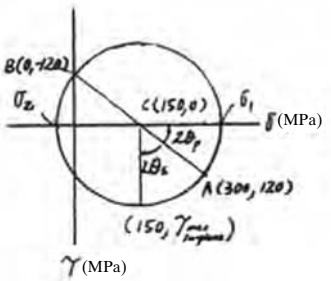
SOLUTION

$A(300, 120)$ $B(0, -120)$ $C(150, 0)$

$$R = \sqrt{(300 - 150)^2 + 120^2} = 192.094$$

$$\sigma_1 = 150 + 192.094 = 342 \text{ MPa}$$ **Ans.**

$$\sigma_2 = 150 - 192.094 = -42.1 \text{ MPa}$$ **Ans.**

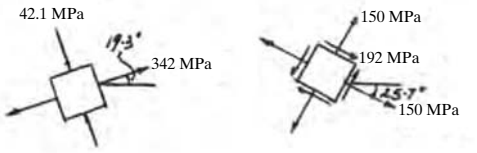


$$\tan 2\theta_P = \frac{120}{300 - 150} = 0.8$$

$$\theta_{P_1} = 19.3^\circ \quad (Counterclockwise)$$ **Ans.**

$$\sigma_{avg} = 150 \text{ MPa}$$ **Ans.**

$$\tau_{\substack{\max \\ \text{in-plane}}} = 192 \text{ MPa}$$ **Ans.**



$$\tan 2\theta_s = \frac{300 - 150}{120} = 1.25$$

$$\theta_s = 25.7^\circ \quad (Clockwise)$$

Ans.

$R = 192.094, \sigma_1 = 342$ MPa,
$\sigma_2 = -42.1$ MPa, $\theta_P = 19.3^\circ$ (*Counterclockwise*),
$\sigma_{avg} = 150$ MPa, $\tau_{\substack{\max \\ \text{in-plane}}} = 19.2$ MPa,
$\theta_s = 25.7^\circ$ (*Clockwise*)

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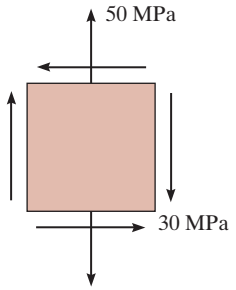
9–58.

Determine (a) the principal stresses and (b) the maximum in-plane shear stress and average normal stress. Specify the orientation of the element in each case.



SOLUTION

$A(0, -30)$ $B(50, 30)$ $C(25, 0)$

$R = CA = \sqrt{(25 - 0)^2 + 30^2} = 39.05$

$\sigma_1 = 25 + 39.05 = 64.1 \text{ MPa}$ **Ans.**

$\sigma_2 = 25 - 39.05 = -14.1 \text{ MPa}$ **Ans.**

$$\tan 2\theta_P = \frac{30}{25 - 0} = 1.2$$

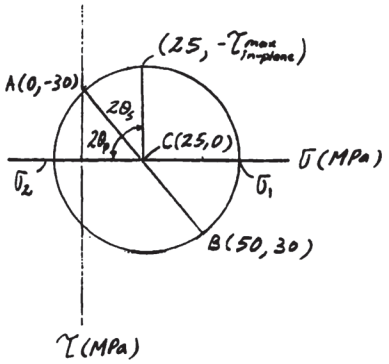
$\theta_P = 25.1°$ **Ans.**

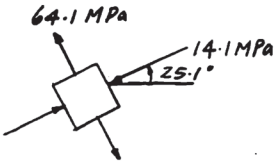
$\sigma_{\text{avg}} = 25.0 \text{ MPa}$ **Ans.**

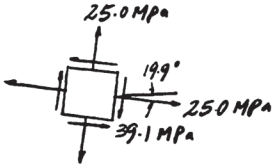
$\tau_{\substack{\max \\ \text{in-plane}}} = R = 39.1 \text{ MPa}$ **Ans.**

$$\tan 2\theta_s = \frac{25 - 0}{30} = 0.8333$$

$\theta_s = -19.9°$ **Ans.**







Ans:
$\sigma_1 = 64.1 \text{ MPa}$,
$\sigma_2 = -14.1 \text{ MPa}$,
$\theta_P = 25.1°$,
$\sigma_{\text{avg}} = 25.0 \text{ MPa}$,
$\tau_{\substack{\max \\ \text{in-plane}}} = 39.1 \text{ MPa}$,
$\theta_s = -19.9°$

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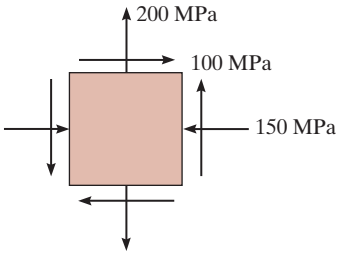
9–59.

Determine (a) the principal stresses and (b) the maximum in-plane shear stress and average normal stress. Specify the orientation of the element in each case.



SOLUTION

$A(-150, 100)$ $B(200, -100)$ $C(25, 0)$

$$R = CA = \sqrt{(150 + 25)^2 + 100^2} = 201.556$$

$$\tan 2\theta_P = \frac{100}{150 + 25} = 0.5714$$

$\theta_P = -14.9°$ **Ans.**

$\sigma_1 = 25 + 201.556 = 227$ MPa **Ans.**

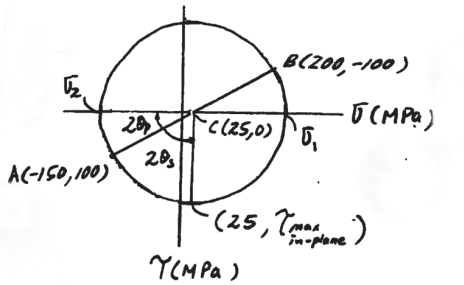
$\sigma_2 = 25 - 201.556 = -177$ MPa **Ans.**

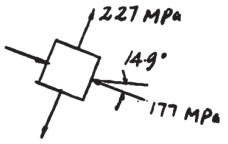
$\tau_{\substack{\max \\ \text{in-plane}}} = R = 202$ MPa **Ans.**

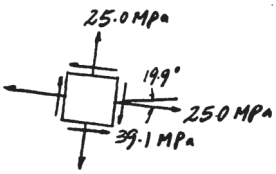
$\sigma_{avg} = 25$ MPa **Ans.**

$$\tan 2\theta_s = \frac{150 + 25}{100} = 1.75$$

$\theta_s = 30.1°$ **Ans.**







Ans:
$\theta_P = -14.9°$,
$\sigma_1 = 227$ MPa,
$\sigma_2 = -177$ MPa,
$\tau_{\substack{\max \\ \text{in-plane}}} = 202$ MPa,
$\sigma_{avg} = 25$ MPa,
$\theta_s = 30.1°$

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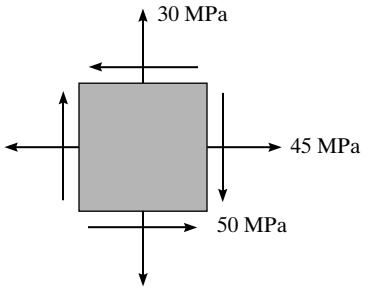
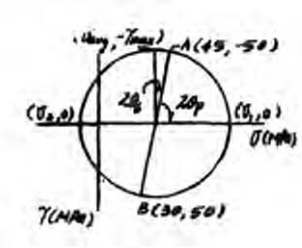
***9–60.** Determine the principal stress, the maximum in-plane shear stress, and average normal stress. Specify the orientation of the element in each case.



SOLUTION

$A(45, -50)$ $B(30, 50)$ $C(37.5, 0)$

$R = CA = CB = \sqrt{7.5^2 + 50^2} = 50.56$

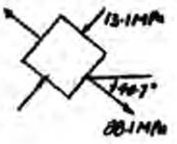


a)

$$\sigma_1 = 37.5 + 50.56 = 88.1 \text{ MPa}$$ **Ans.**

$$\sigma_2 = 37.5 - 50.56 = -13.1 \text{ MPa}$$ **Ans.**

$$\tan 2\theta_P = \frac{50}{7.5} \qquad 2\theta_P = 81.47^\circ \qquad \theta_P = 40.7^\circ \qquad (\textit{Clockwise})$$ **Ans.**



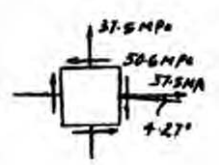
b)

$$\tau_{\substack{\max \\ \text{in-plane}}} = R = 50.6 \text{ MPa}$$ **Ans.**

$$\sigma_{\text{avg}} = 37.5 \text{ MPa}$$ **Ans.**

$$2\theta_s = 90 - 2\theta_P$$

$$\theta_s = 4.27^\circ \qquad (\textit{Counterclockwise})$$ **Ans.**



Ans.

$\sigma_1 = 88.1$ MPa, $\sigma_2 = -13.1$ MPa, $\theta_p = 40.7^\circ$

$\tau_{\substack{\max \\ \text{in-plane}}} = 50.6$ MPa, $\sigma_{\text{avg}} = 37.5$ MPa, $\theta_s = 4.27^\circ$

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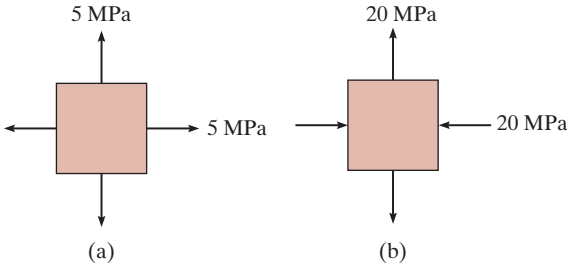
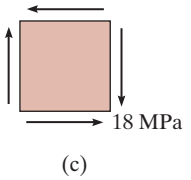
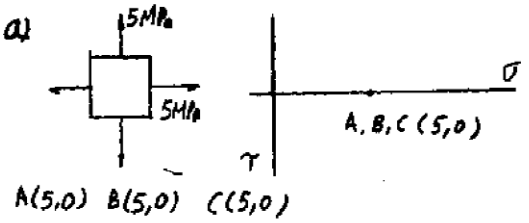
9–61. Draw Mohr's circle that describes each of the following states of stress.

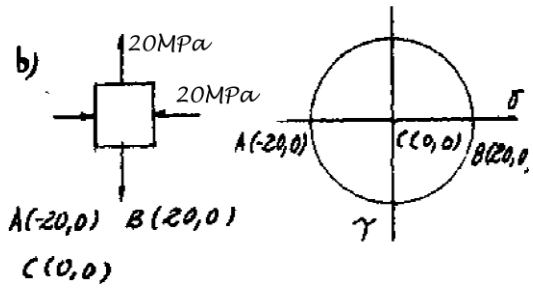




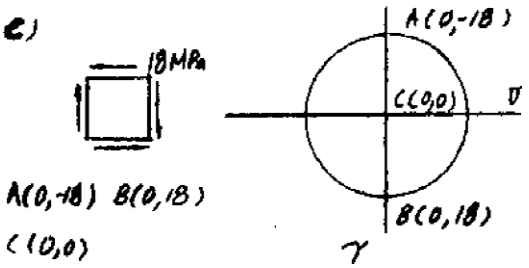
a)



b)



c)



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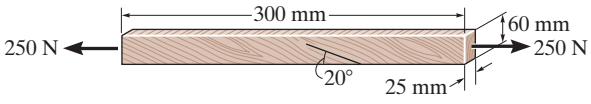
9–62.

The grains of wood in the board make an angle of 20° with the horizontal as shown. Determine the normal and shear stresses that act perpendicular and parallel to the grains if the board is subjected to an axial load of 250 N.



SOLUTION

$$\sigma_x = \frac{P}{A} = \frac{250}{(0.06)(0.025)} = 166.67 \text{ kPa}$$

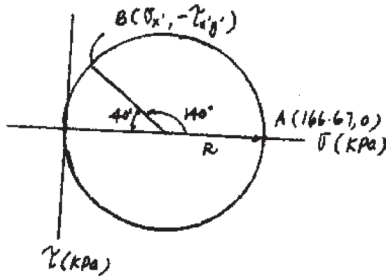
$R = 83.33$

Coordinates of point *B*:

$\sigma_{x'} = 83.33 - 83.33 \cos 40°$

$\sigma_{x'} = 19.5 \text{ kPa}$ **Ans.**

$\tau_{x'y'} = -83.33 \sin 40° = -53.6 \text{ kPa}$ **Ans.**



Ans:
$\sigma_{x'} = 19.5 \text{ kPa}, \tau_{x'y'} = -53.6 \text{ kPa}$

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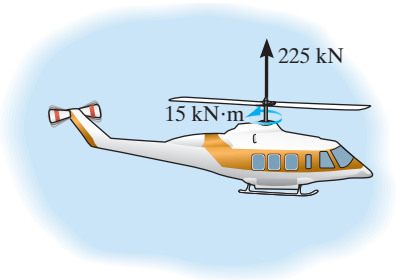
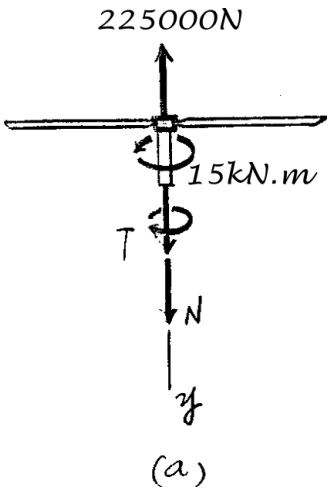
9–63. The rotor shaft of the helicopter is subjected to the tensile force and torque shown when the rotor blades provide the lifting force to suspend the helicopter at midair. If the shaft has a diameter of 150 mm, determine the principal stress and maximum in-plane shear stress at a point located on the surface of the shaft.



SOLUTION



Internal Loadings: Considering the equilibrium of the free-body diagram of the rotor shaft's upper segment, Fig. *a*,

$$\Sigma F_y = 0; \qquad N - 225 = 0 \qquad N = 225 \text{ kN}$$

$$\Sigma M_y = 0; \qquad T - 15 = 0 \qquad T = 15 \text{ kN.m}$$

Section Properties: The cross-sectional area and the polar moment of inertia of the rotor shaft's cross section are

$$A = \pi(0.075^2) = 5.625(10^{-3})\pi \text{ m}^2$$

$$J = \frac{\pi}{2}(0.075^4) = 49.7010(10^{-6}) \text{ m}^4$$

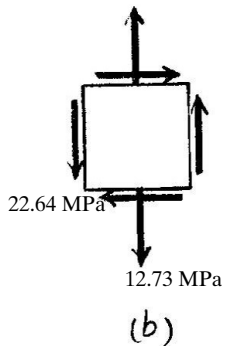
Normal and Shear Stress: The normal stress is contributed by axial stress only.

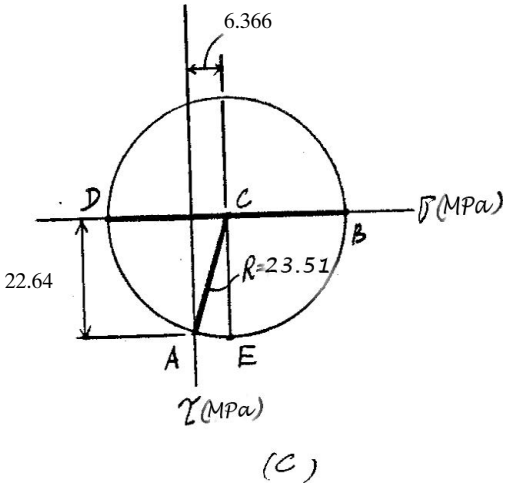
$$\sigma = \frac{N}{A} = \frac{225(10^3)}{5.625(10^{-3})\pi} = 12.73(10^6) \text{ N/m}^2 = 12.73 \text{ MPa}$$

The shear stress is contributed by the torsional shear stress only.

$$\tau = \frac{[15(10^3)](0.075)}{49.7010(10^{-6})} = 22.64(10^6) \text{ N/m}^2 = 22.64 \text{ MPa}$$

The state of stress at point *A* is represented by the element shown in Fig. *b*.





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9–63. Continued

Construction of the Circle: $\sigma_x = 0$, $\sigma_y = 12.73$ MPa, and $\tau_{xy} = 22.64$ MPa. Thus,

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{0 + 12.73}{2} = 6.366 \text{ MPa}$$

The coordinates of reference point A and the center C of the circle are

$$A(0, 22.64) \qquad C(6.366, 0)$$

Thus, the radius of the circle is

$$R = CA = \sqrt{(0 - 6.366)^2 + 22.64^2} = 23.51 \text{ MPa}$$

Using these results, the circle is shown in Fig. *c*.

In-Plane Principal Stress: The coordinates of reference points B and D represent σ_1 and σ_2, respectively.

$$\sigma_1 = 6.366 + 23.51 = 29.9 \text{ MPa} \qquad \textbf{Ans.}$$

$$\sigma_2 = 6.366 - 23.51 = -17.1 \text{ MPa} \qquad \textbf{Ans.}$$

Maximum In-Plane Shear Stress: The state of maximum shear stress is represented by the coordinates of point E, Fig. *a*.

$$\tau_{\substack{\max \\ \text{in-plane}}} = R = 23.5 \text{ MPa} \qquad \textbf{Ans.}$$

Ans.

$\tau_{\substack{\max \\ \text{in-plane}}} = 23.5$ MPa, $\sigma_1 = 29.9$ MPa, $\sigma_2 = -17.1$ MPa

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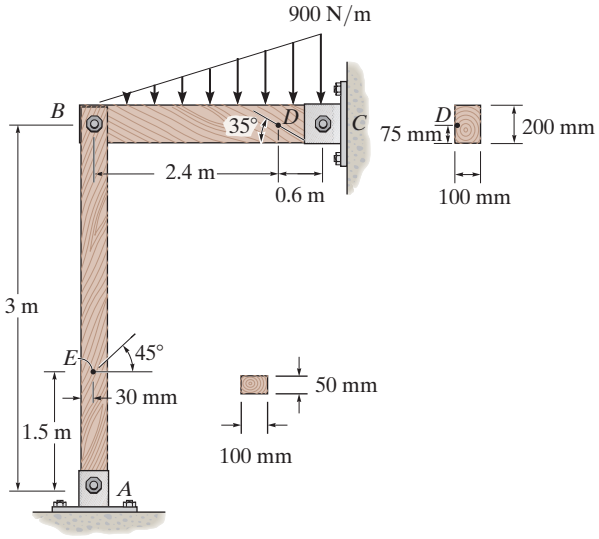
***9–64.**

The frame supports the triangular distributed load shown. Determine the normal and shear stresses at point D that act perpendicular and parallel, respectively, to the grains. The grains at this point make an angle of 35° with the horizontal as shown.



SOLUTION

Support Reactions and Internal Loadings: Referring to the FBD of the entire beam, Fig. *a*,

$$\zeta+\Sigma M_C = 0; \qquad \left[\frac{1}{2}(900)(3)\right](1) - B_y(3) = 0 \qquad B_y = 450\ \text{N}$$

Referring to the FBD of the left segment of the sectioned beam, Fig. *b*,

$$+\uparrow\Sigma F_y = 0; \qquad 450 - \frac{1}{2}(720)(2.4) - V_D = 0 \qquad V_D = -414\ \text{N}$$

$$\zeta+\Sigma M_D = 0; \qquad M_D + \left[\frac{1}{2}(720)(2.4)\right](0.8) - 450(2.4) = 0 \qquad M_D = 388.8\ \text{N}\cdot\text{m}$$

Section Properties: For the rectangular cross section, Fig. *c*,

$$I = \frac{1}{12}(0.1)\left(0.2^3\right) = 66.6667\left(10^{-6}\right)\ \text{m}^4$$

$$Q_D = \tilde{y}'A' = 0.0625[0.1(0.075)] = 0.46875\left(10^{-3}\right)\ \text{m}^3$$

Normal Stress: The normal stress is contributed by bending stress only. Applying the flexure formula,

$$\sigma_D = \frac{M_D y_D}{I} = \frac{388.8(0.025)}{66.6667\left(10^{-6}\right)} = 145.8\left(10^3\right)\ \text{Pa} = 145.8\ \text{kPa (T)}$$

Shear Stress: The shear stress is contributed by the transverse shear stress only. Applying the shear formula,

$$\tau_D = \frac{VQ}{It} = \frac{414\left[0.46875\left(10^{-3}\right)\right]}{66.6667\left(10^{-6}\right)(0.1)} = 29.11\left(10^3\right)\ \text{Pa} = 29.11\ \text{kPa}$$

Using these results, the state of stress at point D can be represented by the differential element shown in Fig. *d*.

Construction of The Circle: In accordance to the sign convention, $\sigma_x = 145.8$ kPa, $\sigma_y = 0$ $\tau_{xy} = 29.11$ kPa and $\theta = 55°$ (Counterclockwise, Fig. *e*). Hence,

$$\sigma_{\text{avg}} = \frac{\sigma_x + \sigma_y}{2} = \frac{145.8 + 0}{2} = 72.9\ \text{kPa}$$

The coordinates of reference point A and center of circle C are

$$A(145.8, 29.11) \qquad C(72.9, 0)$$

The radius of the circle is

$$R = CA = \sqrt{(145.8 - 72.9)^2 + (29.11 - 0)^2} = 78.497\ \text{kPa}$$

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***9–64. Continued**

Using these results, the circle shown in Fig. *f*, can be constructed.

Stress on the Inclined Plane: The normal stress and shear stress on the inclined plane are represented by the coordinates of point $P(\sigma_{x'}, \tau_{x'y'})$ on the circle which can be established by rotating radial line CA $2\theta = 110°$ counterclockwise to coincide with radial line CP, Fig. *f*. Here,

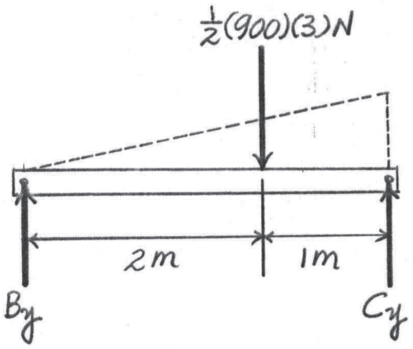
$$\alpha = \tan^{-1}\left(\frac{29.11}{145.8 - 72.9}\right) = 21.77°$$

$$\phi = 110° - 21.77° = 88.23°$$

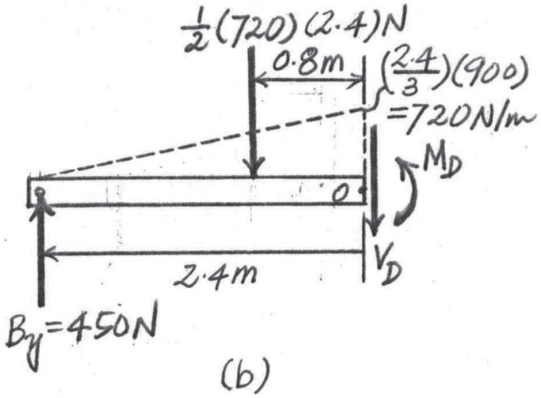
Then

$\sigma_{x'} = 72.9 + 78.497 \cos 88.23° = 75.32 \text{ kPa} = 75.3 \text{ kPa}$ **Ans.**

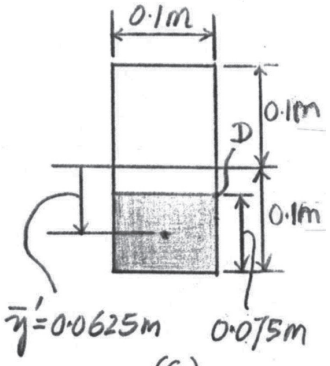
$\tau_{x'y'} = -78.497 \sin 88.23° = -78.46 \text{ kPa} = -78.5 \text{ kPa}$ **Ans.**



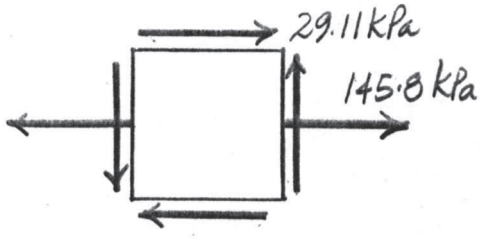
(a)



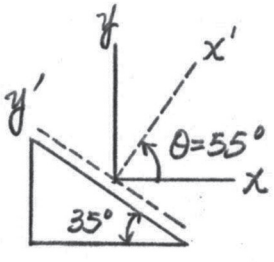
(b)



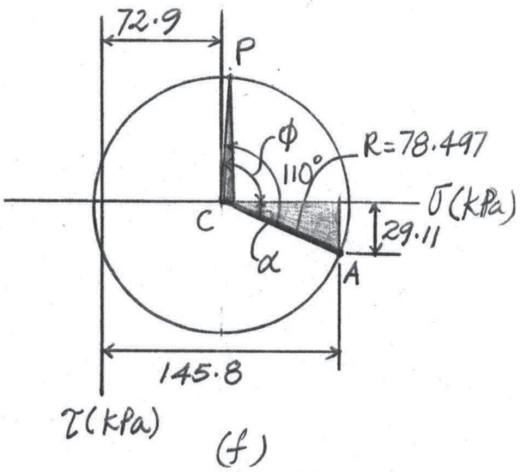
(c)



(d)



(e)



(f)

Ans:
$\sigma_{x'} = 75.3$ kPa,
$\tau_{x'y'} = -78.5$ kPa

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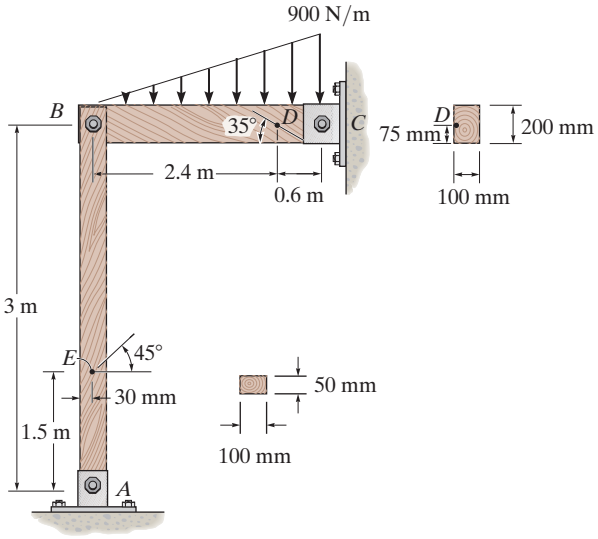
9–65.

The frame supports the triangular distributed load shown. Determine the normal and shear stresses at point E that act perpendicular and parallel, respectively, to the grains. The grains at this point make an angle of 45° with the horizontal as shown.



SOLUTION

Support Reactions and Internal Loadings: Referring to the FBD of the entire beam, Fig. *a*,

$$\circlearrowleft +\Sigma M_C = 0; \qquad \left[\frac{1}{2}(900)(3)\right](1) - B_y(3) = 0 \qquad B_y = 450\ \text{N}$$

Referring to the FBD of the upper segment of the sectioned column, Fig. *b*,

$$+\uparrow \Sigma F_y = 0; \qquad -450 - N_E = 0 \qquad N_E = -450\ \text{N}$$

Section Properties: For the rectangular cross-section,

$$A = 0.05(0.1) = 5.00\left(10^{-3}\right)\ \text{m}^2$$

Normal Stress: The normal stress is contributed by axial load only.

$$\sigma_E = \frac{N_E}{A} = \frac{-450}{5.00\left(10^{-3}\right)} = -90.0\ \text{kPa}$$

Using this result, the state of stress at point E can be represented by the differential element shown in Fig. *c*.

Construction of the Circle: In accordance to the sign convention, $\sigma_x = 0$, $\sigma_y = -90.0$ kPa, $\tau_{xy} = 0$ and $\theta = 135°$ (Counterclockwise, Fig. *d*). Hence,

$$\sigma_{\text{avg}} = \frac{\sigma_x + \sigma_y}{2} = \frac{0 + (-90.0)}{2} = -45.0\ \text{kPa}$$

The coordinates of reference point A and center of circle C are

$$A(0, 0) \qquad C(-45.0, 0)$$

The radius of the circle is

$$R = CA = 0 - (-45.0) = 45.0\ \text{kPa}$$

Using the results, the circle shown in Fig. *e* can be constructed.

Stress on the Inclined Plane: The normal stress and shear stress on the inclined plane are represented by the coordinates of point $P(\sigma_{x'}, \tau_{x'y'})$ on the circle which can be established by rotating the radial line CA $2\theta = 270°$ counterclockwise to coincide with radial line CP, Fig. *e*. Thus,

$$\sigma_{x'} = -45.0\ \text{kPa} \qquad \textbf{Ans.}$$

$$\tau_{x'y'} = 45.0\ \text{kPa} \qquad \textbf{Ans.}$$

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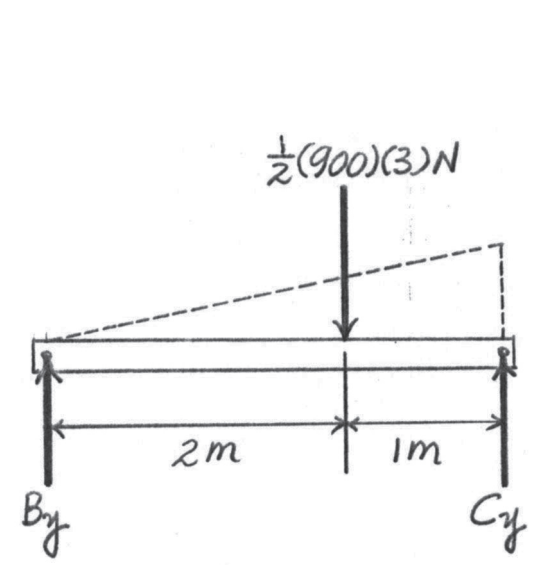
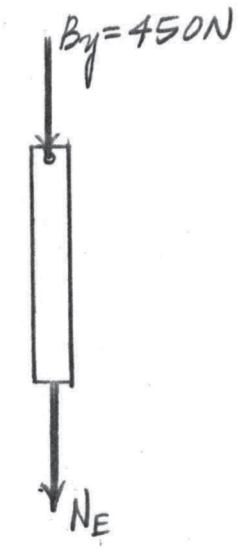
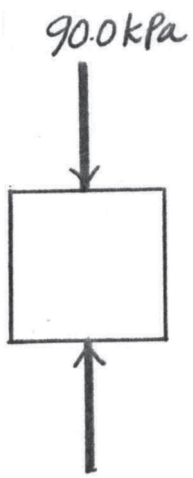
9–65. Continued



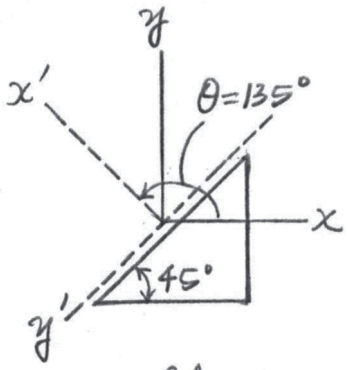
(a)



(b)



(c)



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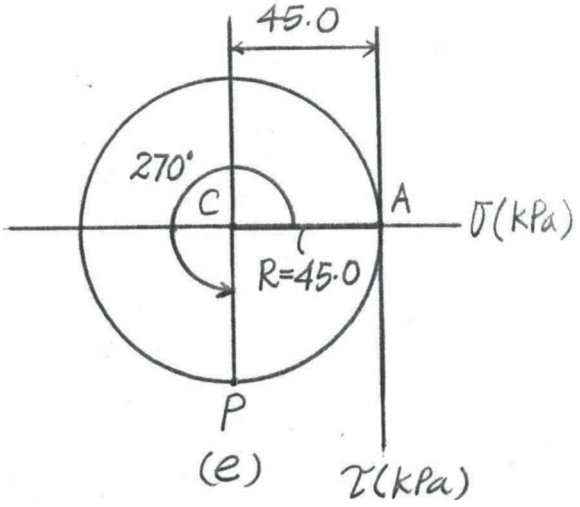

(e)

Ans:
$\sigma_{x'} = -45.0$ kPa,
$\tau_{x'y'} = 45.0$ kPa

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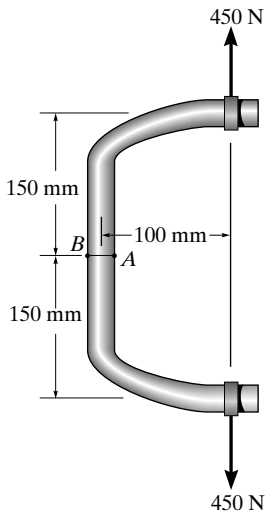
9–66. Determine the principal stresses and the maximum in-plane shear stress that are developed at point A. Show the results on an element located at this point. The rod has a diameter of 40 mm.



SOLUTION

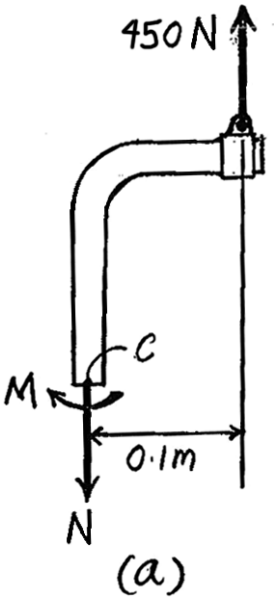
Using the method of sections and consider the FBD of the member's upper cut segment, Fig. *a*,

$+\uparrow \Sigma F_y = 0; \quad 450 - N = 0 \quad N = 450\ \text{N}$

$\circlearrowleft + \Sigma M_C = 0; \quad 450(0.1) - M = 0 \quad M = 45\ \text{N}\cdot\text{m}$

$$A = \pi(0.02^2) = 0.4(10^{-3})\pi\ \text{m}^2$$

$$I = \frac{\pi}{4}(0.02^4) = 40(10^{-9})\pi\ \text{m}^4$$



The normal stress is the combination of axial and bending stress. Thus,

$$\sigma = \frac{N}{A} + \frac{My}{I}$$

For point A, $y = C = 0.02$ m.

$$\sigma = \frac{450}{0.4(10^{-3})\pi} + \frac{45\ (0.02)}{40(10^{-9})\pi} = 7.520\ \text{MPa}$$

Since no transverse shear and torque is acting on the cross - section

$$\tau = 0$$

The state of stress at point A can be represented by the element shown in Fig. *b*.

In accordance to the established sign convention $\sigma_x = 0$, $\sigma_y = 7.520$ MPa and $\tau_{xy} = 0$. Thus

$$\sigma_{\text{avg}} = \frac{\sigma_x + \sigma_y}{2} = \frac{0 + 7.520}{2} = 3.760\ \text{MPa}$$

Then, the coordinates of reference point A and the center C of the circle are

$$A(0, 0) \qquad C(3.760, 0)$$

Thus, the radius of the circle is

$$R = CA = 3.760\ \text{MPa}$$

Using this results, the circle shown in Fig. *c* can be constructed. Since no shear stress acts on the element,

$$\sigma_1 = \sigma_y = 7.52\ \text{MPa} \qquad \sigma_2 = \sigma_x = 0 \qquad \textbf{Ans.}$$

The state of principal stresses can also be represented by the element shown in Fig. *b*.

The state of maximum in - plane shear stress is represented by point B on the circle, Fig. *c*. Thus.

$$\tau_{\substack{\text{max} \\ \text{in-plane}}} = R = 3.76\ \text{MPa} \qquad \textbf{Ans.}$$

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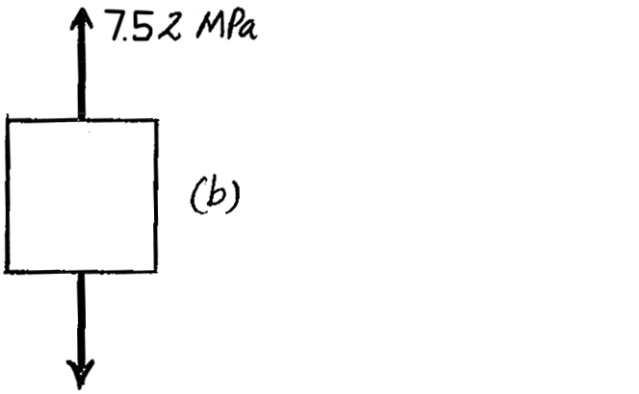
9–66. Continued

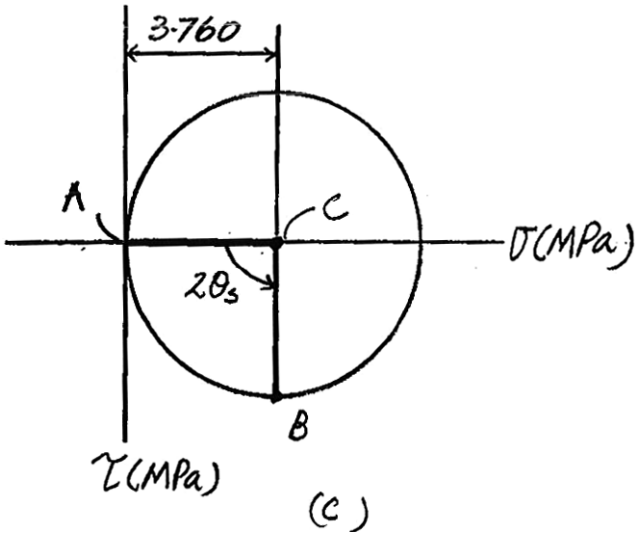
From the circle,

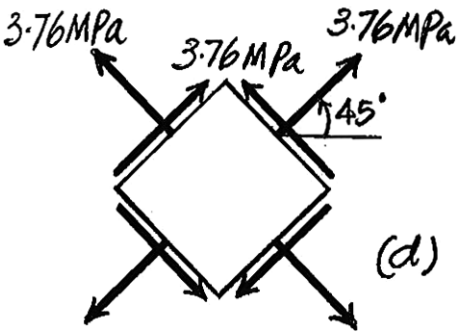
$$2\theta_s = 90°$$

$$\theta_s = 45° \quad (\textit{counter clockwise}) \qquad \textbf{Ans.}$$

The state of maximum In - Plane shear stress can be represented by the element shown in Fig. *d*.







Ans.

$\sigma_1 = 7.52$ MPa, $\sigma_2 = 0$,
$\tau_{\text{max}}_{\text{in-plane}} = 3.76$ MPa,
$\theta_s = 45°$ (*Counterclockwise*)

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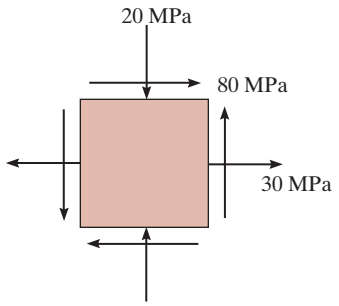
9–67.

Determine the principal stresses, the maximum in-plane shear stress, and average normal stress. Specify the orientation of the element in each case.



SOLUTION

In accordance to the established sign convention, $\sigma_x = 30$ MPa, $\sigma_y = -20$ MPa and $\tau_{xy} = 80$ MPa. Thus,

$$\sigma_{\text{avg}} = \frac{\sigma_x + \sigma_y}{2} = \frac{30 + (-20)}{2} = 5 \text{ MPa} \qquad \textbf{Ans.}$$

Then, the coordinates of reference point A and the center C of the circle is

$$A(30, 80) \qquad C(5, 0)$$

Thus, the radius of circle is given by

$$R = CA = \sqrt{(30 - 5)^2 + (80 - 0)^2} = 83.815 \text{ MPa}$$

Using these results, the circle shown in Fig. *a*, can be constructed.

The coordinates of points B and D represent σ_1 and σ_2 respectively. Thus

$$\sigma_1 = 5 + 83.815 = 88.8 \text{ MPa} \qquad \textbf{Ans.}$$

$$\sigma_2 = 5 - 83.815 = -78.8 \text{ MPa} \qquad \textbf{Ans.}$$

Referring to the geometry of the circle, Fig. *a*

$$\tan 2(\theta_P)_1 = \frac{80}{30 - 5} = 3.20$$

$$\theta_P = 36.3° \text{ (}\textit{Counterclockwise}\text{)} \qquad \textbf{Ans.}$$

The state of maximum in-plane shear stress is represented by the coordinate of point E. Thus

$$\tau_{\substack{\max \\ \text{in-plane}}} = R = 83.8 \text{ MPa} \qquad \textbf{Ans.}$$

From the geometry of the circle, Fig. *a*,

$$\tan 2\theta_s = \frac{30 - 5}{80} = 0.3125$$

$$\theta_s = 8.68° \quad \text{(}\textit{Clockwise}\text{)} \qquad \textbf{Ans.}$$

The state of maximum in-plane shear stress is represented by the element in Fig. *c*.

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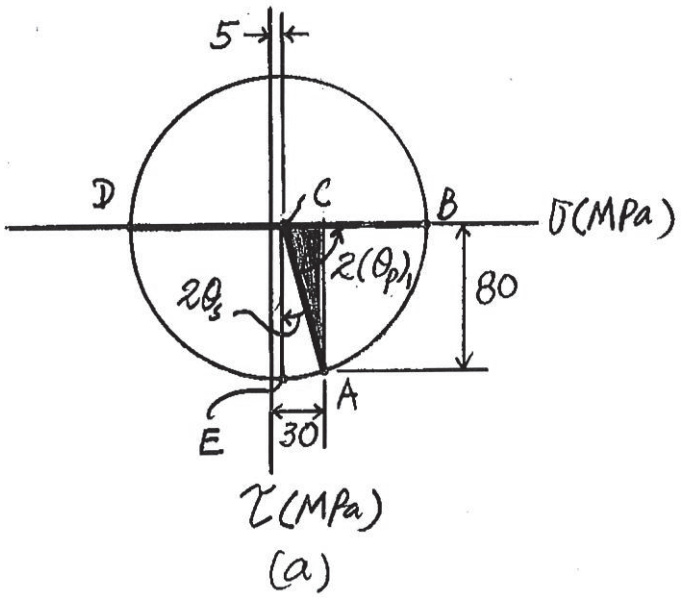
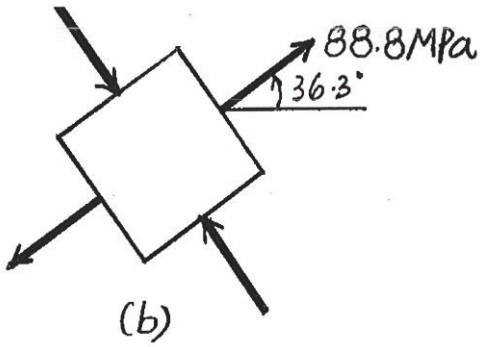
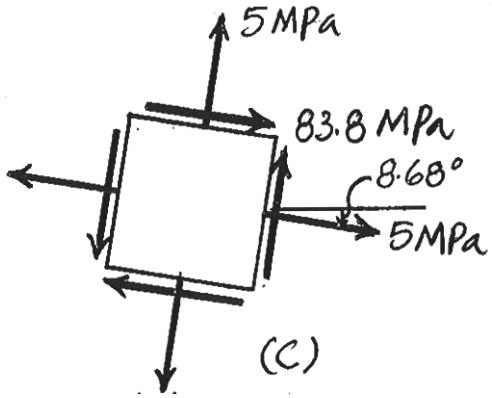
9–67. Continued







Ans:

$\sigma_{avg} = 5$ MPa,
$\sigma_1 = 88.8$ MPa,
$\sigma_2 = -78.8$ MPa,
$\theta_P = 36.3°$ (*Counterclockwise*),
$\tau_{\substack{\max \\ \text{in-plane}}} = 83.8$ MPa,
$\theta_s = 8.68°$ (*Clockwise*)

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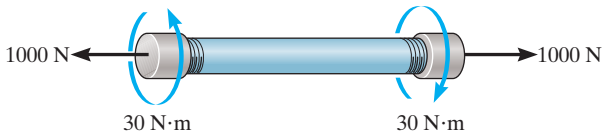
***9–68.** The thin-walled pipe has an inner diameter of 12 mm and a thickness of 0.6 mm. If it is subjected to an internal pressure of 3.5 MPa and the axial tension and torsional loadings shown, determine the principal stress at a point on the surface of the pipe.



SOLUTION

Section Properties:

$$A = \frac{\pi}{4}(0.0066^2 - 0.006^2) = 7.56(10^{-6})\pi \text{ m}^2$$

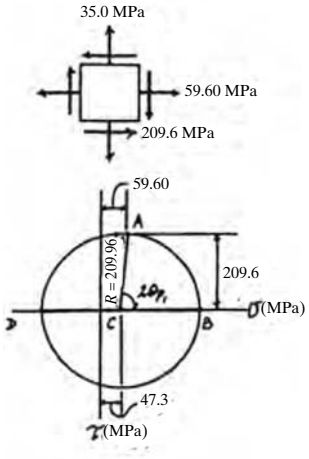
$$J = \frac{\pi}{2}(0.0066^4 - 0.006^4) = 0.94479(10^{-9}) \text{ m}^4$$

Normal Stress: Since $\frac{r}{t} = \frac{6}{0.6} = 10$, thin wall analysis is valid.

$$\sigma_{\text{long}} = \frac{N}{A} + \frac{pr}{2t} = \frac{1000}{7.56(10^{-6})\pi} + \frac{[3.5(10^6)](0.006)}{2(0.0006)}$$

$$= 59.60(10^6) \text{ N/m}^2 = 59.60 \text{ MPa}$$

$$\sigma_{\text{hoop}} = \frac{pr}{t} = \frac{[3.5(10^6)](0.006)}{0.0006} = 35.0(10^6) \text{ N/m}^2 = 35.0 \text{ MPa}$$



Shear Stress: Applying the torsion formula,

$$\tau = \frac{Tc}{J} = \frac{30(0.0066)}{0.94479(10^{-9})} = 209.57(10^6) \text{ N/m}^2 = 209.57 \text{ MPa}$$

Construction of the Circle: In accordance with the sign convention $\sigma_x = 59.60$ MPa, $\sigma_y = 35.0$ MPa, and $\tau_{xy} = -209.57$ MPa. Hence,

$$\sigma_{\text{avg}} = \frac{\sigma_x + \sigma_y}{2} = \frac{59.60 + 35.0}{2} = 47.30 \text{ MPa}$$

The coordinates for reference points A and C are

$$A(59.60 - 209.57) \qquad C(47.30, 0)$$

The radius of the circle is

$$R = \sqrt{(59.60 - 47.30)^2 + 209.57^2} = 209.93 \text{ MPa}$$

In - Plane Principal Stress: The coordinates of point B and D represent σ_1 and σ_2, respectively.

$$\sigma_1 = 47.30 + 209.93 = 257 \text{ MPa} \qquad \textbf{Ans.}$$

$$\sigma_2 = 47.30 - 209.93 = -163 \text{ MPa} \qquad \textbf{Ans.}$$

Ans.
$\sigma_1 = 257$ MPa, $\sigma_2 = -163$ MPa

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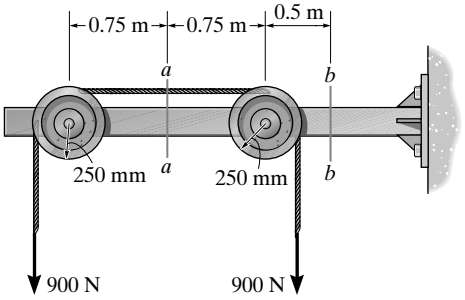
9–69. Determine the principal stress at point A on the cross section of the hanger at section a–a. Specify the orientation of this state of stress and indicate the result on an element at the point.



SOLUTION

Internal Loadings: Considering the equilibrium of the free - body diagram of the hanger's left cut segment, Fig. *a*,

$$\stackrel{+}{\rightarrow}\Sigma F_x = 0; \qquad 900 - N = 0 \qquad N = 900\,\text{N}$$

$$+\uparrow \Sigma F_y = 0; \qquad V - 900 = 0 \qquad V = 900\,\text{N}$$

$$\circlearrowleft + \Sigma M_O = 0; \qquad 900(1) - 900(0.25) - M = 0 \qquad M = 675\,\text{N}\cdot\text{m}$$

Section Properties: The cross - sectional area and the moment of inertia about the centroidal axis of the hanger's cross section are

$$A = 0.05(0.1) - 0.04(0.09) = 1.4\left(10^{-3}\right)\text{m}^2$$

$$I = \frac{1}{12}(0.05)\left(0.1^3\right) - \frac{1}{12}(0.04)\left(0.09^3\right) = 1.7367\left(10^{-6}\right)\text{m}^4$$

Referring to Fig. *b*,

$$Q_A = 2\bar{y}_1' A_1' + \bar{y}_2' A_2' = 2[0.0375(0.025)(0.005)] + 0.0475(0.005)(0.04)$$
$$= 18.875\left(10^{-6}\right)\text{m}^3$$

Normal and Shear Stress: The normal stress is a combination of axial and bending stresses. Thus,

$$\sigma_A = \frac{N}{A} + \frac{My_A}{I} = -\frac{900}{1.4\left(10^{-3}\right)} + \frac{675(0.025)}{1.7367\left(10^{-6}\right)} = 9.074\ \text{MPa}$$

The shear stress is caused by the transverse shear stress.

$$\tau_A = \frac{VQ_A}{It} = \frac{900\left[18.875\left(10^{-6}\right)\right]}{1.7367\left(10^{-6}\right)(0.01)} = 0.9782\ \text{MPa}$$

The state of stress at point A is represented by the element shown in Fig. *c*.

Construction of the Circle: $\sigma_x = 9.074$ MPa, $\sigma_y = 0$, and $\tau_{xy} = 0.9782$ MPa. Thus,

$$\sigma_{\text{avg}} = \frac{\sigma_x + \sigma_y}{2} = \frac{9.074 + 0}{2} = 4.537\ \text{MPa}$$

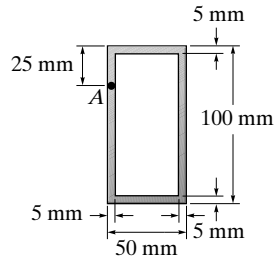
The coordinates of reference points A and the center C of the circle are

$$A(9.074, 0.9782) \qquad C(4.537, 0)$$

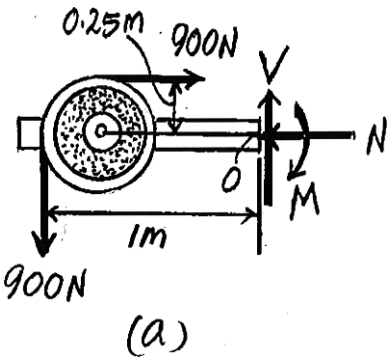
Thus, the radius of the circle is

$$R = CA = \sqrt{(9.074 - 4.537)^2 + 0.9782^2} = 4.641\ \text{MPa}$$

Using these results, the circle is shown in Fig. *d*.



Sections $a - a$ and $b - b$



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9–69. Continued

In - Plane Principal Stress: The coordinates of point B and D represent σ_1 and σ_2, respectively.

$$\sigma_1 = 4.537 + 4.641 = 9.18\ \text{MPa}$$ **Ans.**

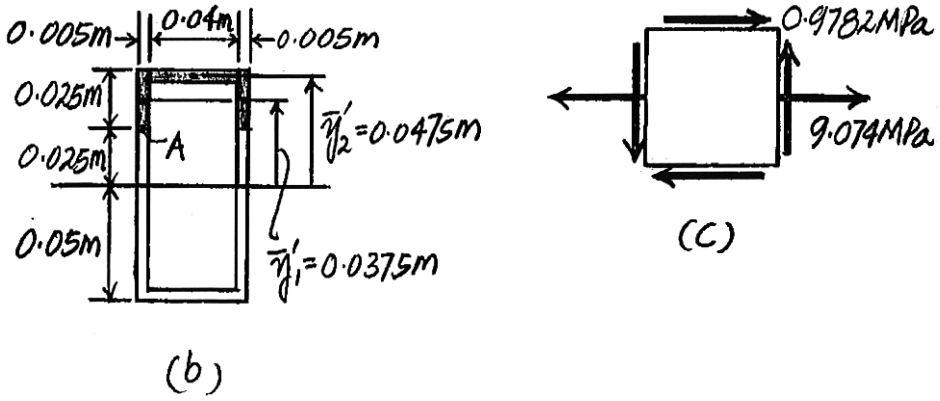
$$\sigma_2 = 4.537 - 4.641 = -0.104\ \text{MPa}$$ **Ans.**

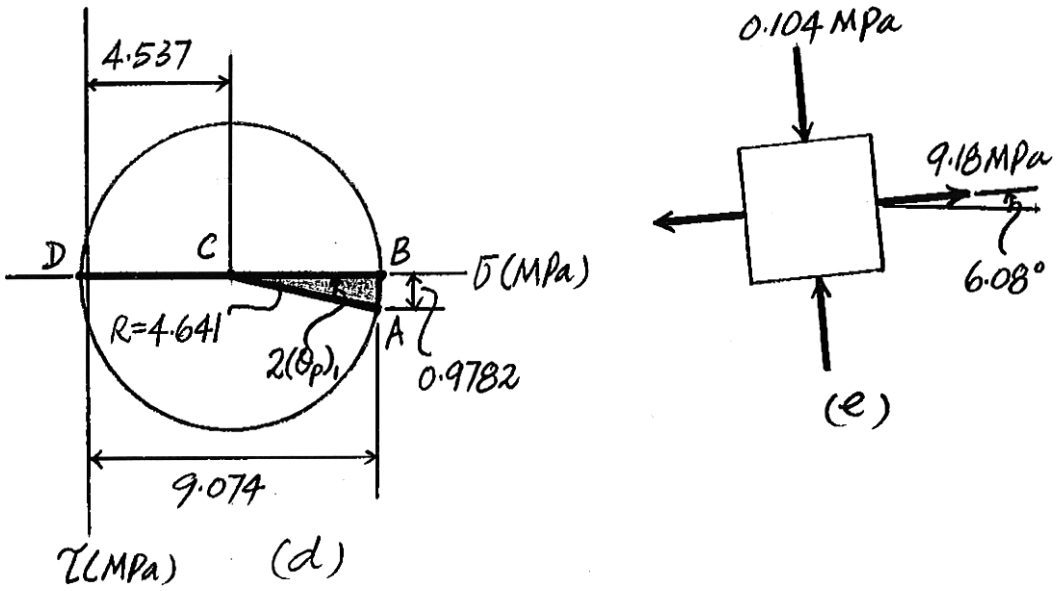
Orientaion of Principal Plane: Referring to the geometry of the circle, Fig. *d*,

$$\tan 2\left(\theta_P\right)_1 = \frac{0.9782}{9.074 - 4.537} = 0.2156$$

$$\left(\theta_P\right)_1 = 6.08°\ \text{(counterclockwise)}$$ **Ans.**

The state of principal stresses is represented on the element shown in Fig. *e*.





Ans.
$N = 900$ N, $V = 900$ N, $M = 675\,\text{N}\cdot\text{m}$,
$A = 1.4(10^{-3})\ \text{m}^2$, $I = 1.7367(10^{-6})\ \text{m}^4$,
$\sigma_1 = 9.18$ MPa, $\sigma_2 = -0.104$ MPa,
$(\theta_p)_1 = 6.08°$ (*Counterclockwise*)

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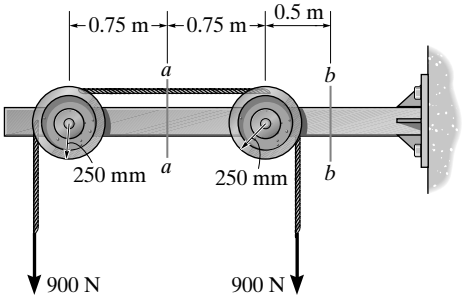
9–70. Determine the principal stress at point A on the cross section of the hanger at section b–b. Specify the orientation of the state of stress and indicate the results on an element at the point.



SOLUTION

Internal Loadings: Considering the equilibrium of the free - body diagram of the hanger's left cut segment, Fig. *a*,

$+\uparrow \Sigma F_y = 0; \qquad V - 900 - 900 = 0 \qquad V = 1800\,\text{N}$

$\zeta + \Sigma M_O = 0; \qquad 900(2.25) + 900(0.25) - M = 0 \qquad M = 2250\,\text{N}\cdot\text{m}$

Section Properties: The cross - sectional area and the moment of inertia about the centroidal axis of the hanger's cross section are

$$A = 0.05(0.1) - 0.04(0.09) = 1.4\left(10^{-3}\right)\text{m}^2$$

$$I = \frac{1}{12}(0.05)\left(0.1^3\right) - \frac{1}{12}(0.04)\left(0.09^3\right) = 1.7367\left(10^{-6}\right)\text{m}^4$$

Referring to Fig. *b*.

$$Q_A = 2\bar{y}_1' A_1' + \bar{y}_2' A_2' = 2[0.0375(0.025)(0.005)] + 0.0475(0.005)(0.04)$$
$$= 18.875\left(10^{-6}\right)\text{m}^3$$

Normal and Shear Stress: The normal stress is contributed by the bending stress only.

$$\sigma_A = \frac{My_A}{I} = \frac{2250(0.025)}{1.7367\left(10^{-6}\right)} = 32.39\ \text{MPa}$$

The shear stress is contributed by the transverse shear stress only.

$$\tau_A = \frac{VQ_A}{It} = \frac{1800\left[18.875\left(10^{-6}\right)\right]}{1.7367\left(10^{-6}\right)(0.01)} = 1.956\ \text{MPa}$$

The state stress at point A is represented by the element shown in Fig. *c*.

Construction of the Circle: $\sigma_x = 32.39$ MPa, $\sigma_y = 0$, and $\tau_{xy} = 1.956$ MPa. Thus,

$$\sigma_{\text{avg}} = \frac{\sigma_x + \sigma_y}{2} = \frac{32.39 + 0}{2} = 16.19\ \text{MPa}$$

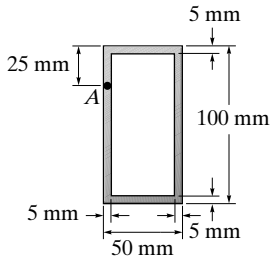
The coordinates of reference point A and the center C of the circle are

$A(32.39, 1.956) \qquad C(16.19, 0)$

Thus, the radius of the circle is

$$R = CA = \sqrt{(32.39 - 16.19)^2 + 1.956^2} = 16.313\ \text{MPa}$$

Using these results, the cricle is shown in Fig. *d*.



Sections $a - a$ and $b - b$

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9–70. Continued

In - Plane Principal Stresses: The coordinates of reference point B and D represent σ_1 and σ_2, respectively.

$$\sigma_1 = 16.19 + 16.313 = 32.5 \text{ MPa} \qquad \textbf{Ans.}$$

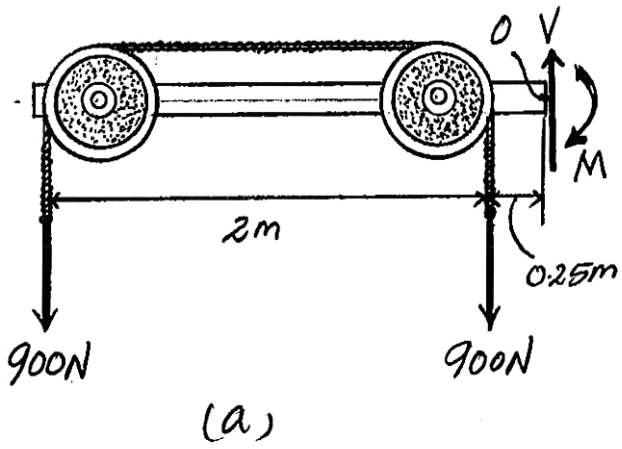
$$\sigma_2 = 16.19 - 16.313 = -0.118 \text{ MPa} \qquad \textbf{Ans.}$$

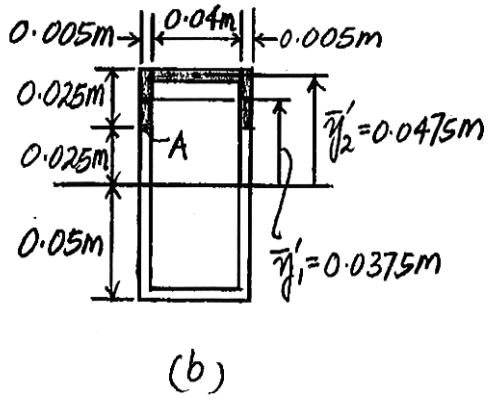
Orientaion of Principal Plane: Referring to the geometry of the circle, Fig. *d*,

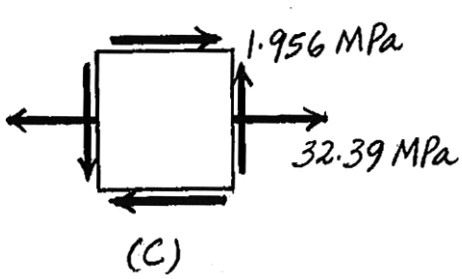
$$\tan 2(\theta_P)_1 = \frac{1.956}{32.39 - 16.19} = 0.1208$$

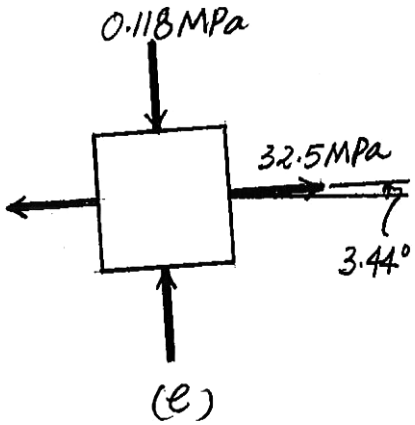
$$(\theta_P)_1 = 3.44° \quad \text{(counterclockwise)} \qquad \textbf{Ans.}$$

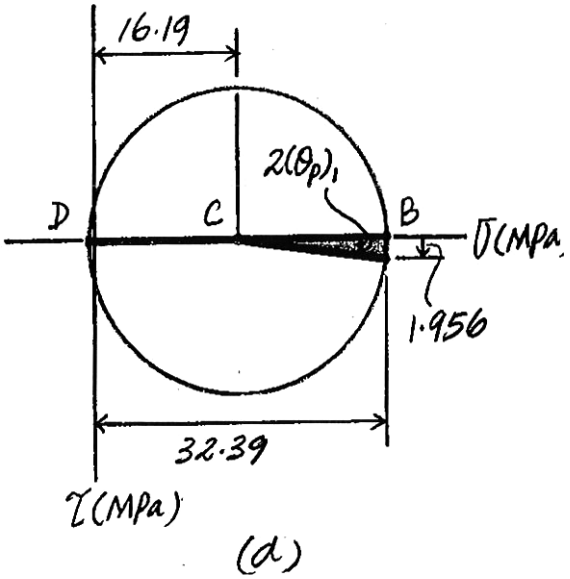
The state of principal stresses is represented on the element shown in Fig. *e*.











Ans.

$\sigma_1 = 32.5$ MPa, $\sigma_2 = -0.118$ MPa,
$(\theta_p)_1 = 3.44°$ (*Counterclockwise*)

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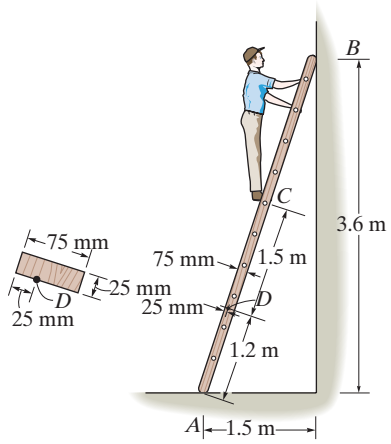
9–71. The ladder is supported on the rough surface at A and by a smooth wall at B. If a man weighing 675 N stands upright at C, determine the principal stresses in one of the legs at point D. Each leg is made from a 25 mm-thick board having a rectangular cross section. Assume that the total weight of the man is exerted vertically on the rung at C and is shared equally by each of the ladder's two legs. Neglect the weight of the ladder and the forces developed by the man's arms.



SOLUTION

$$A = 0.025(0.075) = 1.875(10^{-3})\ \text{m}^2$$

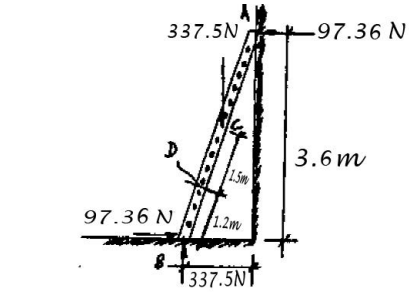
$$I = \frac{1}{12}0.025(0.075^3) = 0.87891(10^{-6})\ \text{m}^4$$

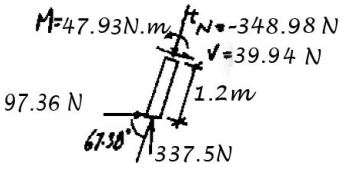
$$Q_D = y'A' = 0.025(0.025)(0.025) = 15.625(10^{-6})\ \text{m}^3$$

$$\sigma_D = \frac{-N}{A} - \frac{M_y}{I} = \frac{-348.98}{1.875(10^{-3})} - \frac{(47.93)(0.0125)}{0.87891(10^{-6})}$$

$$= -867.78(10^3)\ \text{N/m}^2 = -867.78\ \text{KPa}$$

$$\tau_D = \frac{VQ_D}{It} = \frac{39.94[15.625(10^{-6})]}{[0.87891(10^{-6})](0.025)} = 28.40(10^3)\ \text{N/m}^2 = 28.40\ \text{KPa}$$



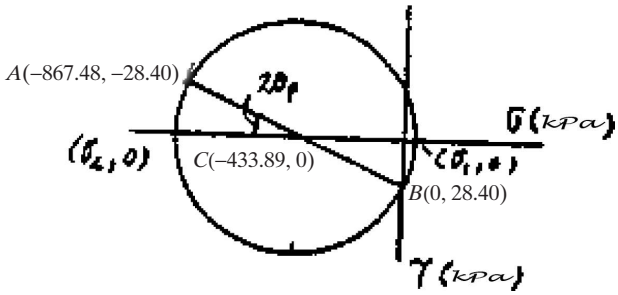


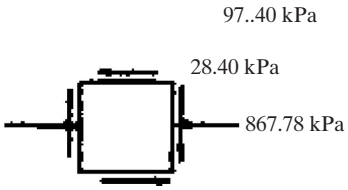
$A(-867.78, -28.40)$ $B(0, 28.40)$ $C(-433.89, 0)$

$$R = CA = \sqrt{(-867.78 - 433.89)^2 + 28.40^2} = 434.82\ \text{KPa}$$

$$\sigma_1 = -433.89 + 434.82 = 0.929\ \text{KPa}$$ **Ans.**

$$\sigma_2 = -433.89 - 434.82 = -869\ \text{KPa}$$ **Ans.**





Ans.

$\sigma_1 = 0.929$ kPa, $\sigma_2 = -869$ kPa

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***9–72.**

A spherical pressure vessel has an inner radius of 1.5 m and a wall thickness of 12 mm. Draw Mohr' Draw Mohr's circle for the state of stress at a point on the vessel and explain the significance of the result. The vessel is subjected to an 0.56 MPa.

SOLUTION

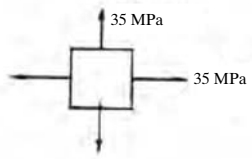
Normal Stress:

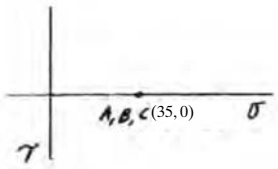
$$\sigma_1 = \sigma_2 = \frac{p\,r}{2\,t} = \frac{0.56(1.5)(1000)}{2(12)} = 35 \text{ MPa}$$

Mohr's circle:

$A(35, 0)$ $B(35, 0)$ $C(35, 0)$

Regardless of the orientation of the element, the shear stress is zero and the state of stress is represented by the same two normal stress components.





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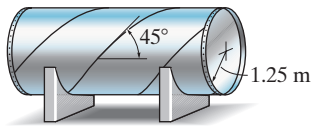
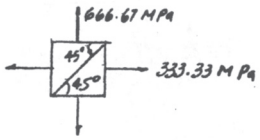
9–73.

The cylindrical pressure vessel has an inner radius of 1.25 m and a wall thickness of 15 mm. It is made from steel plates that are welded along the 45° seam. Determine the normal and shear stress components along this seam if the vessel is subjected to an internal pressure of 8 MPa.



SOLUTION



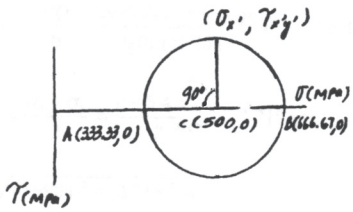
$$\sigma_x = \frac{pr}{2t} = \frac{8(1.25)}{2(0.015)} = 333.33 \text{ MPa}$$

$$\sigma_y = 2\sigma_x = 666.67 \text{ MPa}$$

$A(333.33, 0)$ $B(666.67, 0)$ $C(500, 0)$

$$\sigma_{x'} = \frac{333.33 + 666.67}{2} = 500 \text{ MPa}$$ **Ans.**

$$\tau_{x'y'} = -R = 500 - 666.67 = -167 \text{ MPa}$$ **Ans.**



Ans:
$\sigma_{x'} = 500$ MPa, $\tau_{x'y'} = -167$ MPa

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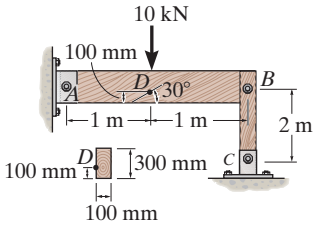
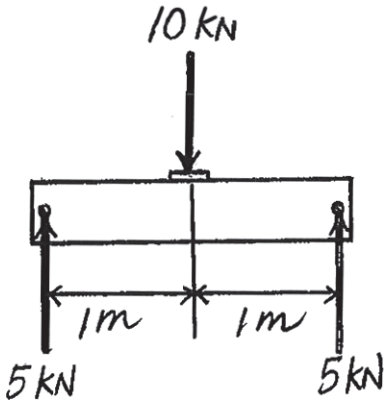
9–74.

Determine the normal and shear stresses at point D that act perpendicular and parallel, respectively, to the grains. The grains at this point make an angle of 30° with the horizontal as shown. Point D is located just to the left of the 10-kN force.



SOLUTION



Using the method of section and consider the FBD of the left cut segment, Fig. *a*

$+\uparrow \Sigma F_y = 0; \quad 5 - V = 0 \quad V = 5\ \text{kN}$

$\circlearrowleft + \Sigma M_C = 0; \quad M - 5(1) = 0 \quad M = 5\ \text{kN}\cdot\text{m}$

The moment of inertia of the rectangular cross section about the neutral axis is

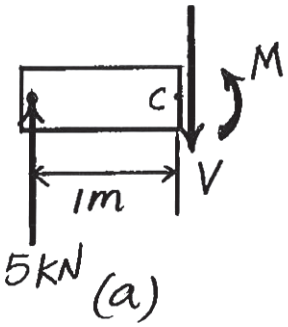
$$I = \frac{1}{12}(0.1)(0.3^3) = 0.225(10^{-3})\ \text{m}^4$$

Referring to Fig. *b*,

$$Q_D = \bar{y}'A' = 0.1(0.1)(0.1) = 0.001\ \text{m}^3$$

The normal stress developed is contributed by bending stress only. For point D, $y = 0.05$ m. Then

$$\sigma = \frac{My}{I} = \frac{5(10^3)(0.05)}{0.225(10^{-3})} = 1.111\ \text{MPa (T)}$$



The shear stress is contributed by the transverse shear stress only. Thus,

$$\tau = \frac{VQ_D}{It} = \frac{5(10^3)(0.001)}{0.225(10^{-3})(0.1)} = 0.2222\ \text{MPa}$$

The state of stress at point D can be represented by the element shown in Fig. *c*

In accordance with the established sign convention, $\sigma_x = 1.111$ MPa, $\sigma_y = 0$ and $\tau_{xy} = -0.2222$ MPa. Thus,

$$\sigma_{\text{avg}} = \frac{\sigma_x + \sigma_y}{2} = \frac{1.111 + 0}{2} = 0.5556\ \text{MPa}$$

Then, the coordinate of reference point A and the center C of the circle are

$$A(1.111, -0.2222) \qquad C(0.5556, 0)$$

Thus, the radius of the circle is given by

$$R = \sqrt{(1.111 - 0.5556)^2 + (-0.2222)^2} = 0.5984\ \text{MPa}$$

Using these results, the circle shown in Fig. *d* can be constructed.

Referring to the geometry of the circle, Fig. *d*,

$$\alpha = \tan^{-1}\left(\frac{0.2222}{1.111 - 0.5556}\right) = 21.80° \qquad \beta = 180° - (120° - 21.80°) = 81.80°$$

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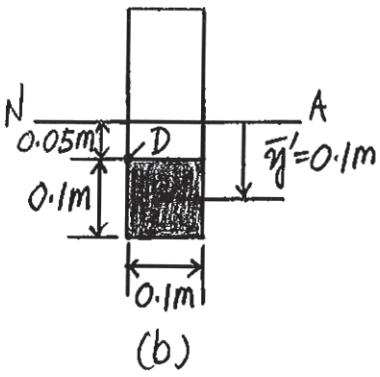
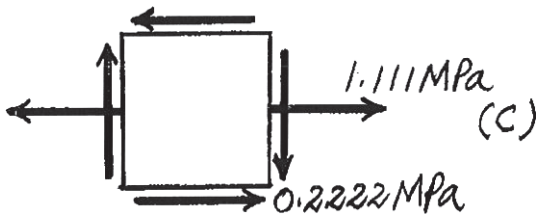
9–74. Continued

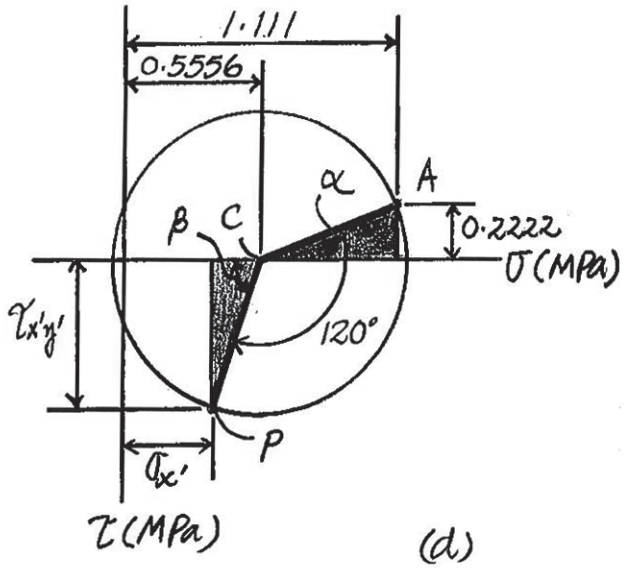
Then

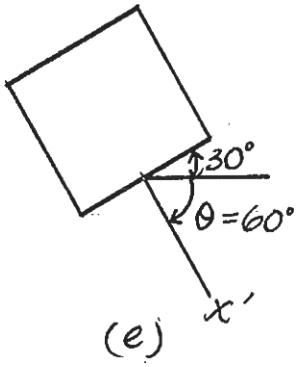
$\sigma_{x'} = 0.5556 - 0.5984 \cos 81.80° = 0.4702 \text{ MPa} = 470 \text{ kPa}$ **Ans.**

$\tau_{x'y'} = 0.5984 \sin 81.80° = 0.5922 \text{ MPa} = 592 \text{ kPa}$ **Ans.**









Ans:
$\sigma_{x'} = 470$ kPa,
$\tau_{x'y'} = 592$ kPa

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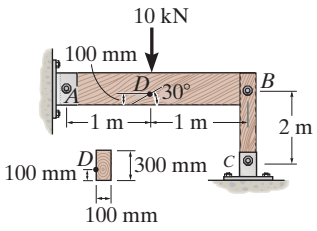
9–75.

Determine the principal stress at point D, which is located just to the left of the 10-kN force.



SOLUTION

Using the method of section and consider the FBD of the left cut segment, Fig. *a*,

$+\uparrow \Sigma F_y = 0; \quad 5 - V = 0 \quad V = 5\ \text{kN}$

$\circlearrowleft + \Sigma M_C = 0; \quad M - 5(1) = 0 \quad M = 5\ \text{kN}\cdot\text{m}$

$$I = \frac{1}{12}(0.1)(0.3^3) = 0.225(10^{-3})\ \text{m}^4$$

Referring to Fig. *b*,

$$Q_D = \bar{y}'A' = 0.1(0.1)(0.1) = 0.001\ \text{m}^3$$

The normal stress developed is contributed by bending stress only. For point D, $y = 0.05$ m

$$\sigma = \frac{My}{I} = \frac{5(10^3)(0.05)}{0.225(10^{-3})} = 1.111\ \text{MPa (T)}$$

The shear stress is contributed by the transverse shear stress only. Thus,

$$\tau = \frac{VQ_D}{It} = \frac{5(10^3)(0.001)}{0.225(10^{-3})(0.1)} = 0.2222\ \text{MPa}$$

The state of stress at point D can be represented by the element shown in Fig. *c*.

In accordance with the established sign convention, $\sigma_x = 1.111$ MPa, $\sigma_y = 0$, and $\tau_{xy} = -0.2222$ MPa. Thus,

$$\sigma_{\text{avg}} = \frac{\sigma_x + \sigma_y}{2} = \frac{1.111 + 0}{2} = 0.5556\ \text{MPa}$$

Then, the coordinate of reference point A and center C of the circle are

$$A(1.111, -0.2222) \qquad C(0.5556, 0)$$

Thus, the radius of the circle is

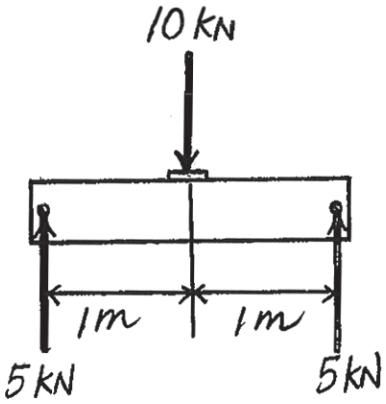
$$R = CA = \sqrt{(1.111 - 0.5556)^2 + (-0.2222)^2} = 0.5984\ \text{MPa}$$

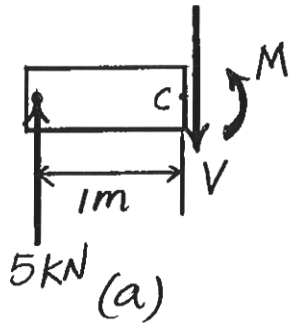
Using these results, the circle shown in Fig. *d* can be constructed.

In-Plane Principal Stresses: The coordinates of points B and D represent σ_1 and σ_2, respectively. Thus,

$$\sigma_1 = 0.5556 + 0.5984 = 1.15\ \text{MPa} \qquad \textbf{Ans.}$$

$$\sigma_2 = 0.5556 - 0.5984 = -0.0428\ \text{MPa} \qquad \textbf{Ans.}$$





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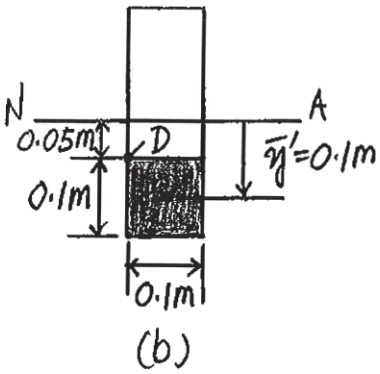
9–75. Continued

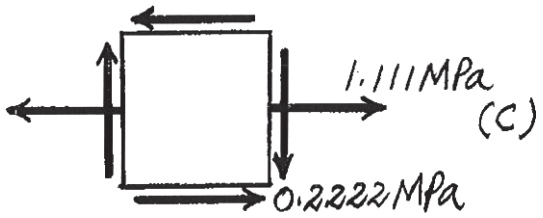
Referring to the geometry of the circle, Fig. *d*,

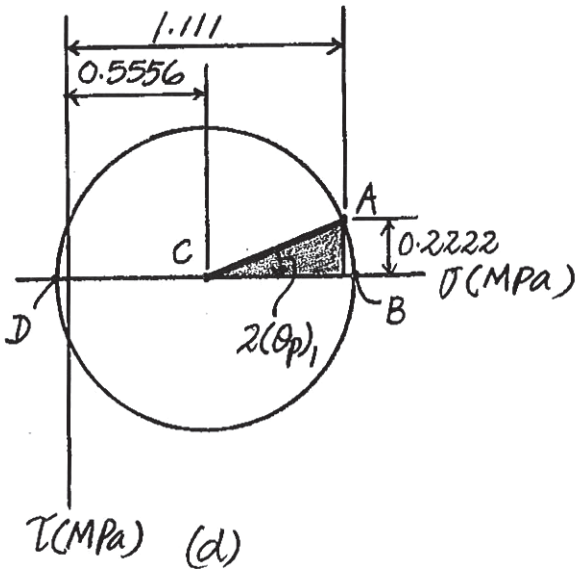
$$\tan (2\theta_P)_1 = \frac{0.2222}{1.111 - 0.5556} = 0.4$$

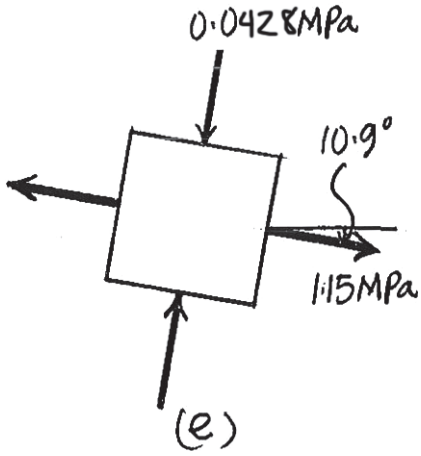
$$(\theta_P)_1 = 10.9^\circ \ (\textit{Clockwise}) \qquad \textbf{Ans.}$$

The state of principal stresses is represented by the element shown in Fig. *e*.









Ans:
$\sigma_1 = 1.15$ MPa, $\sigma_2 = -0.0428$ MPa

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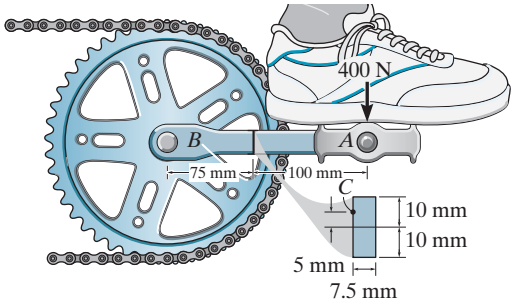
***9–76.** The pedal crank for a bicycle has the cross section shown. If it is fixed to the gear at B and does not rotate while subjected to a force of 400 N, determine the principal stress in the material on the cross section at point C.



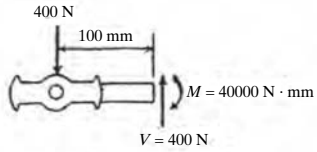
SOLUTION

Internal Forces and Moment: As shown on FBD

Section Properties:

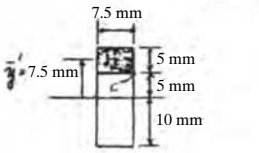
$$I = \frac{1}{12}(0.0075)(0.02^3) = 5(10^{-9})\ \text{m}^4$$

$$Q_C = \bar{y}'A' = (0.0075)(0.0075)(0.005) = 0.28125(10^{-6})\ \text{m}^3$$



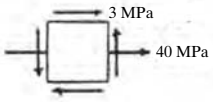
Normal Stress: Applying the flexure formula.

$$\sigma_C = -\frac{My}{I} = -\frac{(-40)(0.005)}{5(10^{-9})} = 40(10^6)\ \text{N/m}^2 = 40\ \text{MPa}$$



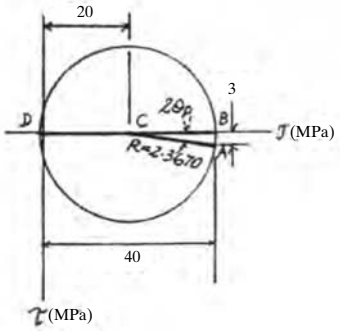
Shear Stress: Applying the shear formula.

$$\tau_C = \frac{VQ_C}{It} = \frac{400[0.28125(10^{-6})]}{[5(10^{-9})](0.0075)} = 3(10^6)\ \text{N/m}^2 = 3\ \text{MPa}$$



Construction of the Circle: In accordance with the sign convention, $\sigma_x = 40$ MPa, $\sigma_y = 0$, and $\tau_{xy} = 3$ MPa. Hence,

$$\sigma_{\text{avg}} = \frac{\sigma_x + \sigma_y}{2} = \frac{40 + 0}{2} = 20\ \text{MPa}$$



The coordinates for reference points A and C are

$$A(40, 3) \qquad C(20, 0)$$

The radius of the circle is

$$R = \sqrt{(40 - 20)^2 + 3^2} = 20.224\ \text{MPa}$$

In - Plane Principal Stress: The coordinates of point B and D represent σ_1 and σ_2, respectively.

$\sigma_1 = 20 + 20.224 = 40.2$ MPa **Ans.**

$\sigma_2 = 20 - 20.224 = -0.224$ MPa **Ans.**

Ans.
$\sigma_1 = 40.2$ MPa, $\sigma_2 = -0.224$ MPa

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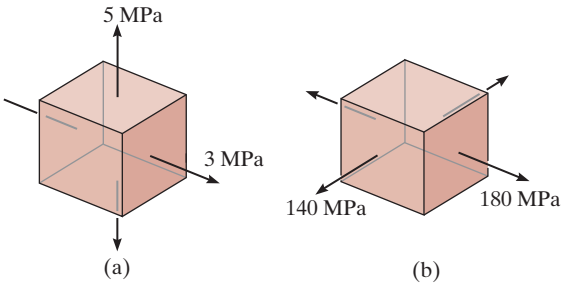
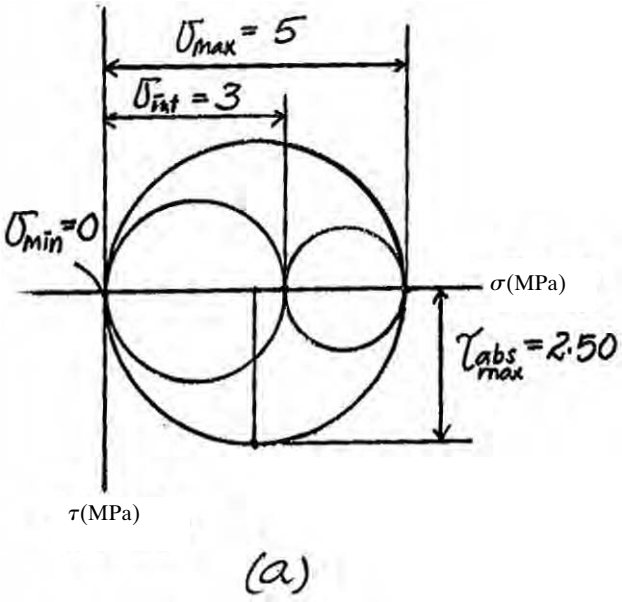
9–77. Draw the three Mohr's circles that describe each of the following states of stress.

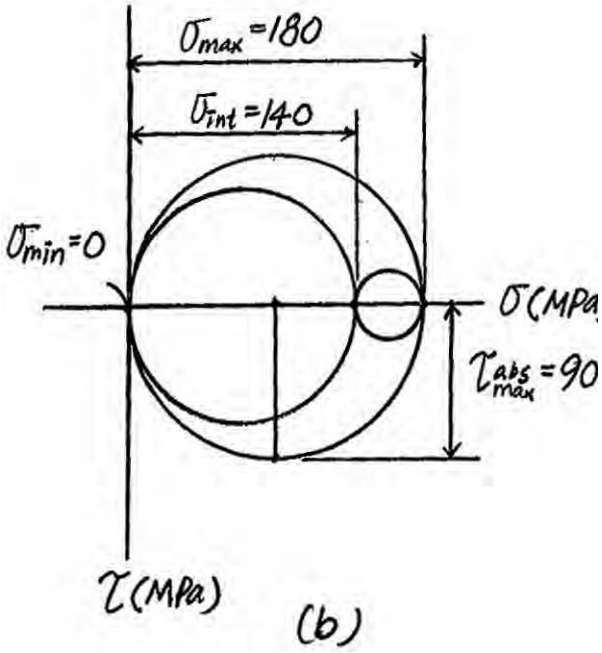


SOLUTION

(a) Here, $\sigma_{min} = 0$, $\sigma_{int} = 3$ MPa and $\sigma_{max} = 5$ MPa. The three Mohr's circle of this state of stress are shown in Fig. *a*

(b) Here, $\sigma_{min} = 0$, $\sigma_{int} = 140$ MPa and $\sigma_{max} = 180$ MPa. The three Mohr's circle of this state of stress are shown in Fig. *b*





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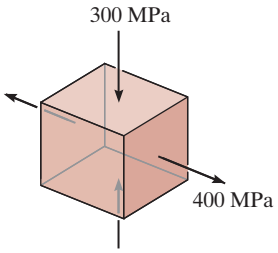
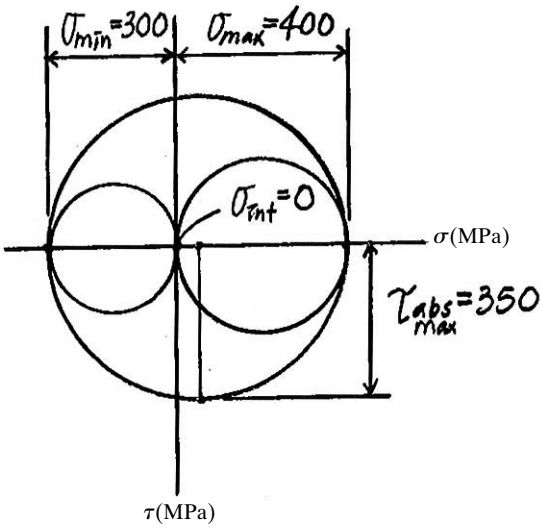
9–78. Draw the three Mohr's circles that describe the following state of stress.



SOLUTION

Here, $\sigma_{min} = -300$ MPa, $\sigma_{int} = 0$ and $\sigma_{max} = 400$ MPa. The three Mohr's circle for this state of stress is shown in Fig. *a*.



(a)

Ans:
$\sigma_{min} = -300$ MPa, $\sigma_{int} = 0$, $\sigma_{max} = 400$ MPa

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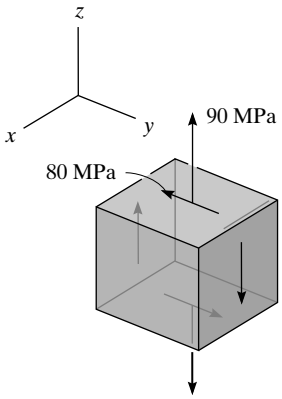
9–79. The stress at a point is shown on the element. Determine the principal stress and the absolute maximum shear stress.



SOLUTION

For $y - z$ plane:

$$A(0, -80) \qquad B(90, 80) \qquad C(45, 0)$$

$$R = \sqrt{45^2 + 80^2} = 91.79$$

$$\sigma_1 = 45 + 91.79 = 136.79 \text{ MPa}$$

$$\sigma_2 = 45 - 91.79 = -46.79 \text{ MPa}$$

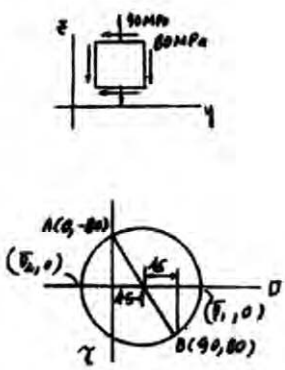
Thus,

$$\sigma_{\text{Int}} = 0 \qquad \textbf{Ans.}$$

$$\sigma_{\text{Max}} = 137 \text{ MPa} \qquad \textbf{Ans.}$$

$$\sigma_{\text{Min}} = -46.8 \text{ MPa} \qquad \textbf{Ans.}$$

$$\tau_{\substack{\text{abs} \\ \text{max}}} = \frac{\sigma_{\max} - \sigma_{\min}}{2} = \frac{136.79 - (-46.79)}{2} = 91.8 \text{ MPa} \qquad \textbf{Ans.}$$



Ans.
$\sigma_{\text{int}} = 0, \sigma_{\max} = 137 \text{ MPa},$
$\sigma_{\min} = -46.8 \text{ MPa},$
$\tau_{\substack{\text{abs} \\ \text{max}}} = 91.8 \text{ MPa}$

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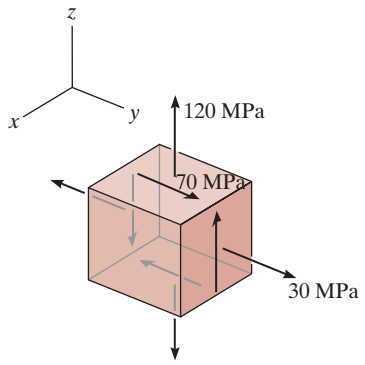
***9–80.** The stress at a point is shown on the element. Determine the principal stress and the absolute maximum shear stress.



SOLUTION

Mohr's circle for the element in y - z plane, Fig. *a*, will be drawn first. In accordance to the established sign convention, $\sigma_y = 30\,\text{MPa}, \sigma_z = 120\,\text{MPa}$ and $\tau_{yz} = 70\,\text{MPa}$. Thus

$$\sigma_{\text{avg}} = \frac{\sigma_y + \sigma_z}{2} = \frac{30 + 120}{2} = 75\ \text{MPa}$$

Thus the coordinates of reference point A and the center C of the circle are

$$A(30, 70) \qquad C(75, 0)$$

Thus, the radius of the circle is

$$R = CA = \sqrt{(75 - 30)^2 + 70^2} = 83.217\ \text{MPa}$$

Using these results, the circle shown in Fig. *b*.

The coordinates of point B and D represent the principal stresses

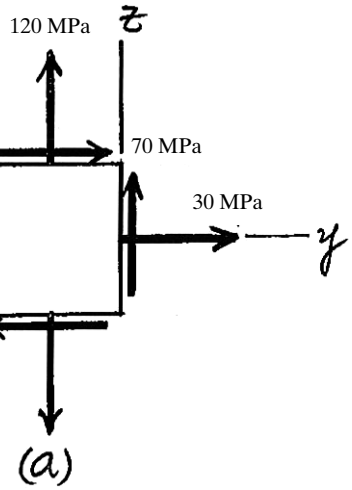
From the results,

$\sigma_{\text{max}} = 158.22\ \text{MPa} = 158\ \text{MPa}$ $\sigma_{\text{min}} = -8.22\ \text{MPa} = -8.22\ \text{MPa}$

$\sigma_{\text{int}} = 0\ \text{MPa}$

Using these results, the three Mohr's circle are shown in Fig. *c*,

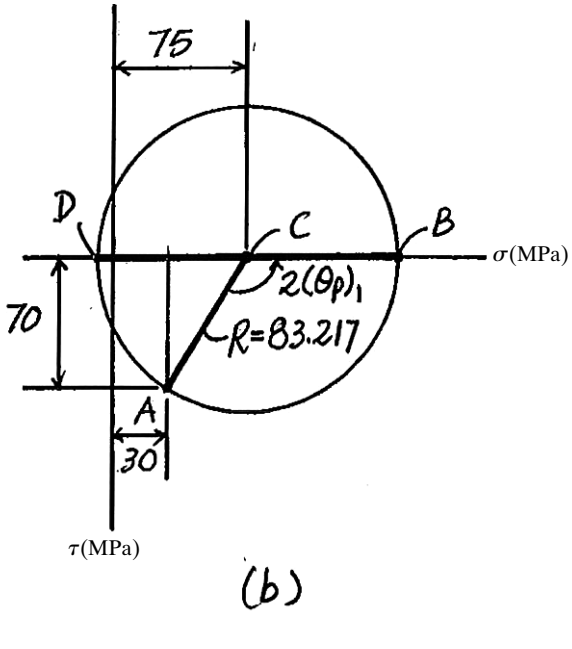
From the geometry of the three circles,

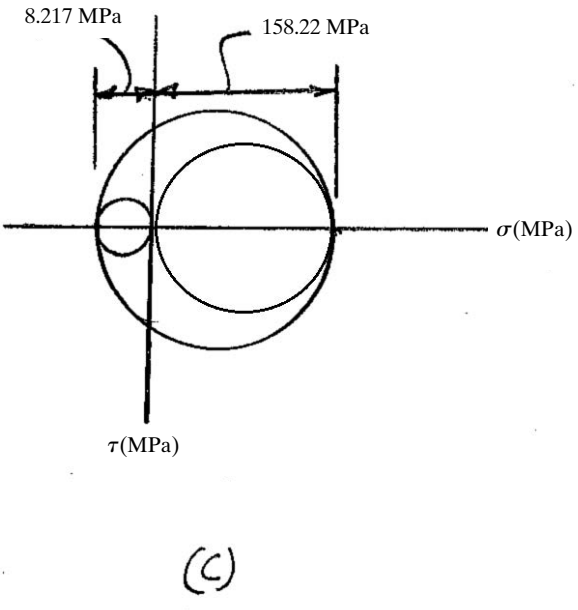


Ans.

$$\tau_{\text{abs}}_{\text{max}} = \frac{\sigma_{\text{max}} - \sigma_{\text{min}}}{2} = \frac{158.22 - (-8.217)}{2} = 83.2\ \text{MPa}$$

Ans.





Ans.

$\sigma_{\text{int}} = 0, \sigma_{\text{max}} = 158\ \text{MPa},$

$\sigma_{\text{min}} = -8.22\ \text{MPa},$

$\tau_{\text{abs}}_{\text{max}} = 83.2\ \text{MPa}$

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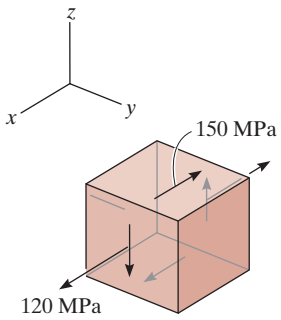
9–81.

Determine the principal stresses and the absolute maximum shear stress.



SOLUTION

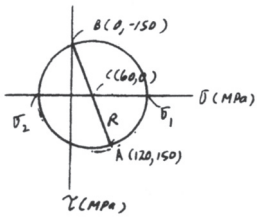
For x - z plane:

$$R = CA = \sqrt{(120 - 60)^2 + 150^2} = 161.55$$

$$\sigma_1 = 60 + 161.55 = 221.55 \text{ MPa}$$

$$\sigma_2 = 60 - 161.55 = -101.55 \text{ MPa}$$

$\sigma_1 = 222$ MPa $\quad \sigma_2 = -102$ MPa **Ans.**



$$\tau_{\substack{\text{abs}\\\text{max}}} = \frac{\sigma_{\max} - \sigma_{\min}}{2} = \frac{221.55 - (-101.55)}{2} = 162 \text{ MPa}$$ **Ans.**

Ans:
$\sigma_1 = 222$ MPa, $\sigma_2 = -102$ MPa,
$\tau_{\substack{\text{abs}\\\text{max}}} = 162$ MPa

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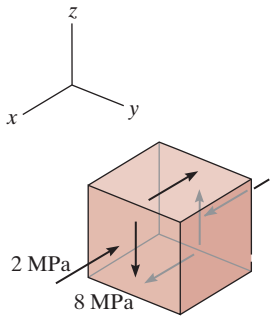
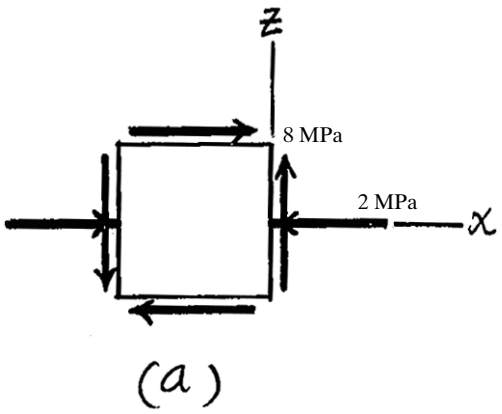
9–82. The stress at a point is shown on the element. Determine the principal stress and the absolute maximum shear stress.



SOLUTION

Mohr's circle for the element in x - z plane, Fig. a, will be drawn first. In accordance to the established sign convention, $\sigma_x = -2$ MPa, $\sigma_z = 0$ and $\tau_{xz} = 8$ MPa. Thus

$$\sigma_{avg} = \frac{\sigma_x + \sigma_z}{2} = \frac{-2 + 0}{2} = -1 \text{ MPa}$$



(a)

Thus, the coordinates of reference point A and the center C of the circle are

$$A(-2, 8) \qquad C(-1, 0)$$

Thus, the radius of the circle is

$$R = CA = \sqrt{[-2 - (-1)]^2 + 8^2} = \sqrt{65} \text{ MPa}$$

Using these results, the circle in shown in Fig. b,

The coordinates of points B and D represent σ_1 and σ_2, respectively.

$$\sigma_1 = -1 + \sqrt{65} = 7.062 \text{ MPa}$$

$$\sigma_2 = -1 - \sqrt{65} = -9.062 \text{ MPa}$$

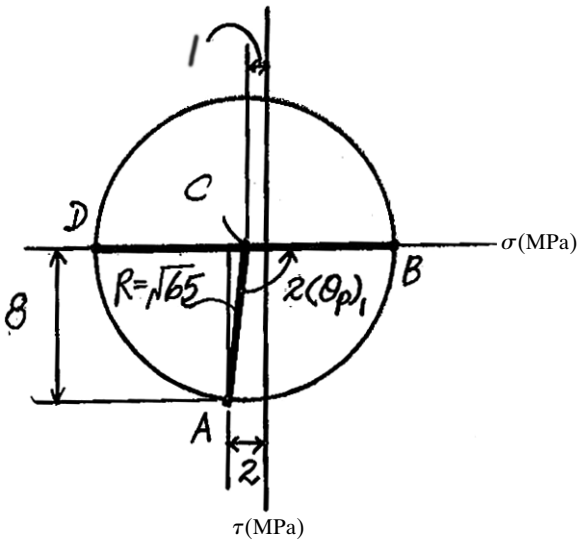
From the results obtained,

$$\sigma_{int} = 0 \text{ MPa} \quad \sigma_{max} = 7.06 \text{ MPa} \quad \sigma_{min} = -9.06 \text{ MPa}$$ **Ans.**

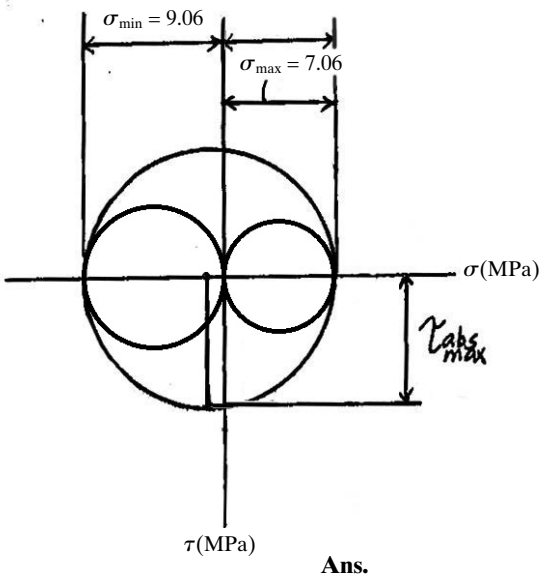
Using these results, the three Mohr's circles are shown in Fig, c.

From the geometry of the cricle,

$$\tau_{\substack{abs \\ max}} = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{7.062 - (-9.062)}{2} = 8.06 \text{ MPa}$$ **Ans.**



(b)



(c)

Ans.

$\sigma_{int} = 0$ MPa, $\sigma_{max} = 7.06$ MPa,
$\sigma_{min} = -9.06$ MPa, $\tau_{\substack{abs \\ max}} = 8.06$ MPa

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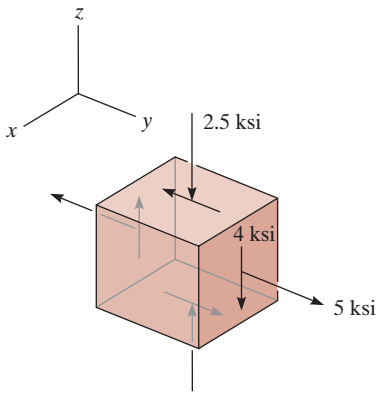
9–83.

Determine the principal stresses and the absolute maximum shear stress.



SOLUTION

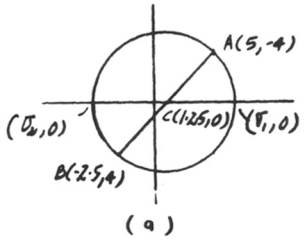
For *y* - *z* plane:

$$A(5, -4) \qquad B(-2.5, 4) \qquad C(1.25, 0)$$

$$R = \sqrt{3.75^2 + 4^2} = 5.483$$

$$\sigma_1 = 1.25 + 5.483 = 6.733 \text{ ksi}$$

$$\sigma_2 = 1.25 - 5.483 = -4.233 \text{ ksi}$$



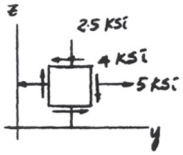
Thus,

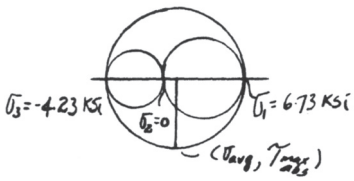
$$\sigma_1 = 6.73 \text{ ksi} \qquad \textbf{Ans.}$$

$$\sigma_2 = -4.23 \text{ ksi} \qquad \textbf{Ans.}$$

$$\sigma_{avg} = \frac{6.73 + (-4.23)}{2} = 1.25 \text{ ksi}$$

$$\tau_{\substack{abs \\ max}} = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{6.73 - (-4.23)}{2} = 5.48 \text{ ksi} \qquad \textbf{Ans.}$$





Ans:
$\sigma_1 = 6.73$ ksi, $\sigma_2 = -4.23$ ksi,
$\tau_{\substack{abs \\ max}} = 5.48$ ksi

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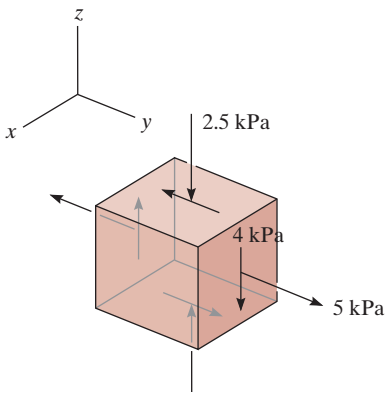
***9–84.** The state of stress at a point is shown on the element. Determine the principal stress and the absolute maximum shear stress.



SOLUTION

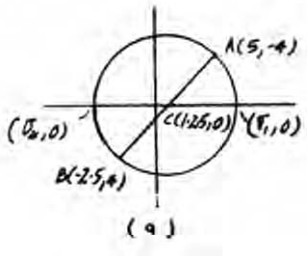
For y - z plane:

$$A(5, -4) \qquad B(-2.5, 4) \qquad C(1.25, 0)$$

$$R = \sqrt{3.75^2 + 4^2} = 5.483$$

$$\sigma_1 = 1.25 + 5.483 = 6.733 \text{ kPa}$$

$$\sigma_2 = 1.25 - 5.483 = -4.233 \text{ kPa}$$



Thus,

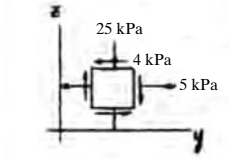
$$\sigma_{\text{Max}} = 6.73 \text{ kPa} \quad \textbf{Ans.}$$

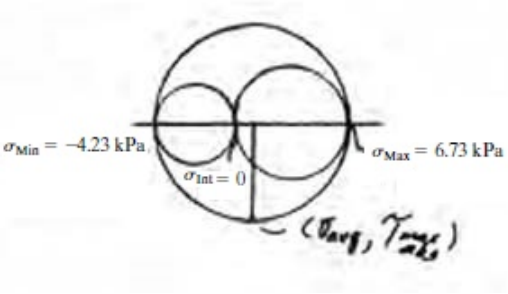
$$\sigma_{\text{Int}} = 0 \quad \textbf{Ans.}$$

$$\sigma_{\text{Min}} = -4.23 \text{ kPa} \quad \textbf{Ans.}$$

$$\sigma_{\text{avg}} = \frac{6.733 + (-4.233)}{2} = 1.25 \text{ kPa}$$

$$\tau_{\substack{\text{abs} \\ \text{max}}} = \frac{\sigma_{\text{max}} - \sigma_{\text{min}}}{2} = \frac{6.733 - (-4.233)}{2} = 5.48 \text{ kPa} \quad \textbf{Ans.}$$





Ans.

$\sigma_{\text{int}} = 0, \sigma_{\text{max}} = 6.73 \text{ MPa},$

$\sigma_{\text{min}} = -4.23 \text{ MPa},$

$\tau_{\substack{\text{abs} \\ \text{max}}} = 5.48 \text{ MPa}$

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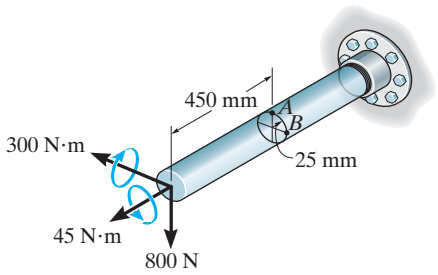
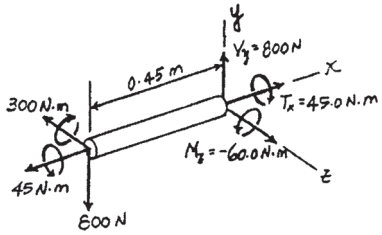
9–85.

The solid shaft is subjected to a torque, bending moment, and shear force. Determine the principal stresses at points A and B and the absolute maximum shear stress.



SOLUTION

Internal Forces and Moment: As shown on FBD.



Section Properties:

$$I_z = \frac{\pi}{4}\left(0.025^4\right) = 0.306796\left(10^{-6}\right)\text{m}^4$$

$$J = \frac{\pi}{2}\left(0.025^4\right) = 0.613592\left(10^{-6}\right)\text{m}^4$$

$$(Q_A)_v = 0$$

$$(Q_B)_y = \bar{y}'A'$$

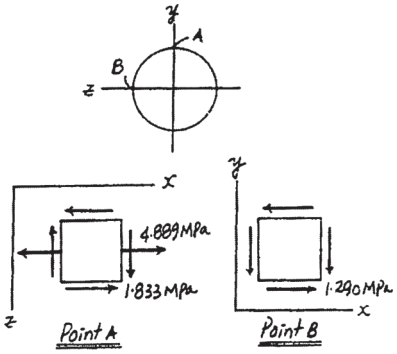
$$= \frac{4(0.025)}{3\pi}\left[\frac{1}{2}(\pi)\left(0.025^2\right)\right] = 10.417\left(10^{-6}\right)\text{m}^3$$

Normal Stress: Applying the flexure formula,

$$\sigma = -\frac{M_z y}{I_z}$$

$$\sigma_A = -\frac{-60.0(0.025)}{0.306796\left(10^{-6}\right)} = 4.889\text{ MPa}$$

$$\sigma_B = -\frac{-60.0(0)}{0.306796\left(10^{-6}\right)} = 0$$



Shear Stress: Applying the torsion formula for point A,

$$\tau_A = \frac{Tc}{J} = \frac{45.0(0.025)}{0.613592\left(10^{-6}\right)} = 1.833\text{ MPa}$$

The transverse shear stress in the y direction and the torsional shear stress can be obtained using shear formula and torsion formula.

$\tau_V = \dfrac{VQ}{It}$ and $\tau_{\text{twist}} = \dfrac{T\rho}{J}$, respectively.

$$\tau_B = (\tau_V)_y - \tau_{\text{twist}}$$

$$= \frac{800\left[10.417\left(10^{-6}\right)\right]}{0.306796\left(10^{-6}\right)(0.05)} - \frac{45.0(0.025)}{0.613592\left(10^{-6}\right)}$$

$$= -1.290\text{ MPa}$$

Construction of the Circle: $\sigma_x = 4.889$ MPa, $\sigma_z = 0$, and $\tau_{xz} = -1.833$ MPa for point A. Hence,

$$\sigma_{\text{avg}} = \frac{\sigma_x + \sigma_z}{2} = \frac{4.889 + 0}{2} = 2.445\text{ MPa}$$

The coordinates for reference points A and C are A (4.889, −1.833) and C(2.445, 0).

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9–85. Continued

The radius of the circle is

$$R = \sqrt{(4.889 - 2.445)^2 + 1.833^2} = 3.056 \text{ MPa}$$

$\sigma_x = \sigma_z = 0$ and $\tau_{xy} = -1.290$ MPa for point B. Hence,

$$\sigma_{avg} = \frac{\sigma_x + \sigma_z}{2} = 0$$

The coordinates for reference points A and C are $A(0, -1.290)$ and $C(0, 0)$

The radius of the circle is $R = 1.290$ MPa

In-Plane Principal Stresses: The coordinates of points B and D represent σ_1 and σ_2, respectively. For point A

$$\sigma_1 = 2.445 + 3.056 = 5.50 \text{ MPa}$$
$$\sigma_2 = 2.445 - 3.056 = -0.611 \text{ MPa}$$

For point B,

$$\sigma_1 = 0 + 1.290 = 1.29 \text{ MPa}$$
$$\sigma_2 = 0 - 1.290 = -1.290 \text{ MPa}$$

Three Mohr's Circles: From the results obtained above, the principal stresses for point A are

$\sigma_1 = 5.50$ MPa $\quad \sigma_2 = -0.611$ MPa **Ans.**

And for point B,

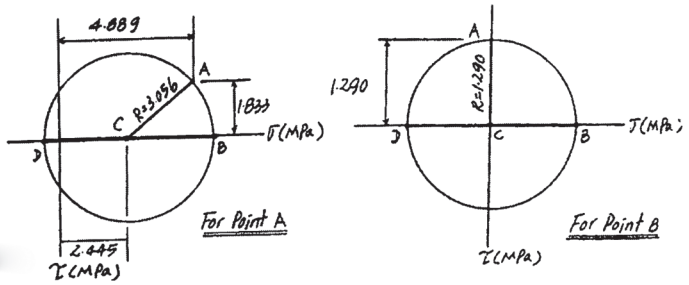
$\sigma_1 = 1.29$ MPa $\quad \sigma_2 = -1.29$ MPa **Ans.**

Absolute Maximum Shear Stress: From point A,

$$\tau_{\substack{\text{abs}\\\text{max}}} = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{5.50 - (-0.611)}{2} = 3.06 \text{ MPa}$$ **Ans.**

For point B,

$$\tau_{\substack{\text{abs}\\\text{max}}} = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{1.29 - (-1.29)}{2} = 1.29 \text{ MPa}$$ **Ans.**



Ans:
$\sigma_1 = 5.50$ MPa, $\sigma_2 = -0.611$ MPa,
$\sigma_1 = 1.29$ MPa, $\sigma_2 = -1.29$ MPa,
$\tau_{\substack{\text{abs}\\\text{max}}} = 3.06$ MPa,
$\tau_{\substack{\text{abs}\\\text{max}}} = 1.29$ MPa

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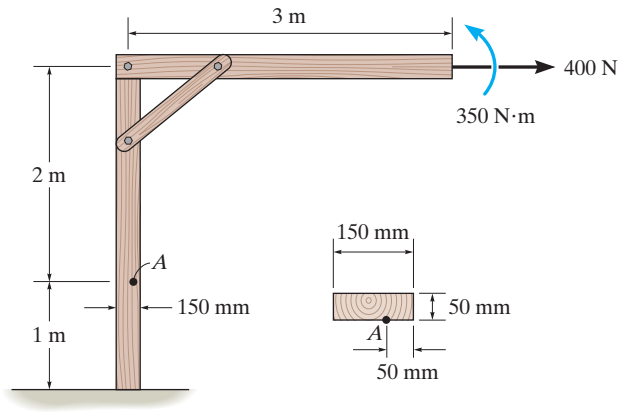
9–86.

The frame is subjected to a horizontal force and couple moment. Determine the principal stresses and the absolute maximum shear stress at point A. The cross-sectional area at this point is shown.



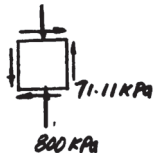
SOLUTION

$$I = \frac{1}{12}(0.05)(0.15^3) = 14.0625(10^{-6})\ \text{m}^4$$

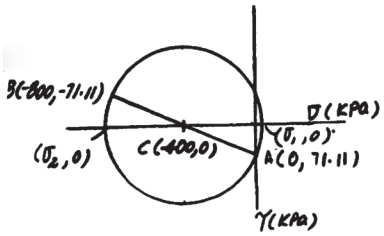
$$Q_A = 0.05(0.05)(0.05) = 0.125(10^{-3})\ \text{m}^3$$

$$\sigma_A = -\frac{Mx}{I} = -\frac{450(0.025)}{14.0625(10^{-6})} = -800\ \text{kPa}$$

$$\tau_A = \frac{VQ_A}{It} = \frac{400(0.125)(10^{-3})}{14.0625(10^{-6})(0.05)} = 71.11\ \text{kPa}$$



$A(0, 71.11)$ $B(-800, -71.11)$ $C(-400, 0)$

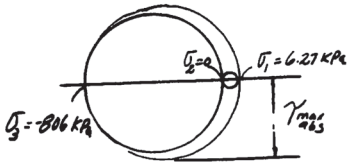


$$R = \sqrt{400^2 + 71.11^2} = 406.272$$
$$\sigma_1 = -400 + 406.272 = 6.72\ \text{kPa}$$
$$\sigma_2 = -400 - 406.272 = -806\ \text{kPa}$$

Thus,

$\sigma_1 = 6.27\ \text{kPa}$ **Ans.**
$\sigma_2 = -806\ \text{kPa}$ **Ans.**

$$\tau_{\substack{\text{abs}\\\text{max}}} = \frac{\sigma_{\max} - \sigma_{\min}}{2} = \frac{6.27 - (-806.27)}{2} = 406\ \text{kPa}$$ **Ans.**



Ans:
$\sigma_1 = 6.27\ \text{kPa}$,
$\sigma_2 = -806\ \text{kPa}$,
$\tau_{\substack{\text{abs}\\\text{max}}} = 406\ \text{kPa}$

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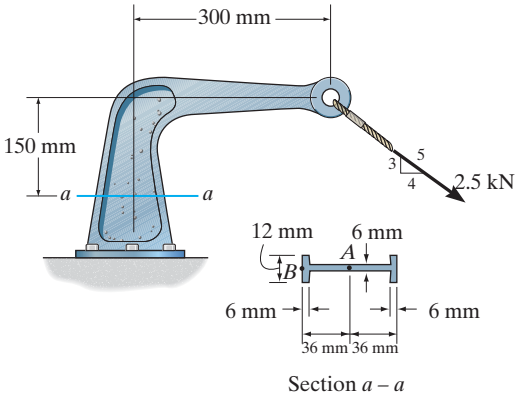
9–87. Determine the principal stress and absolute maximum shear stress developed at point B on the cross section of the bracket at section a–a.



Section $a-a$

Internal Loadings: Considering the equilibrium of the free - body diagram of the bracket's upper cut segment, Fig. *a*,

$$+\uparrow \Sigma F_y = 0; \qquad N - 2.5\left(\frac{3}{5}\right) = 0 \qquad N = 1.5 \text{ kN}$$

$$\stackrel{+}{\leftarrow} \Sigma F_x = 0; \qquad V - 2.5\left(\frac{4}{5}\right) = 0 \qquad V = 2.0 \text{ kN}$$

$$\Sigma M_O = 0; \; M - 2.5\left(\frac{3}{5}\right)(0.300) - 2.5\left(\frac{4}{5}\right)(0.150) = 0 \qquad M = 0.75 \text{ kN} \cdot \text{m}$$

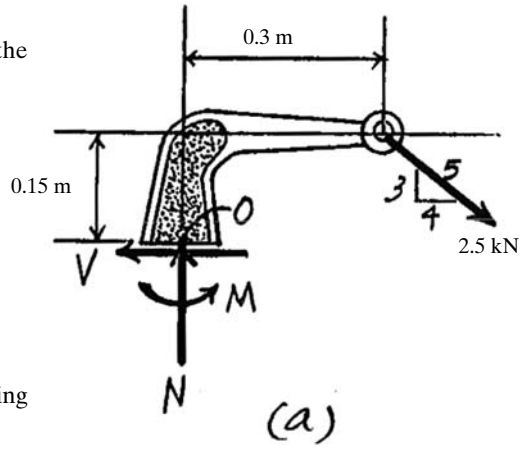
Section Properties: The cross - sectional area and the moment of inertia about the centroidal axis of the bracket's cross section are

$$A = 0.012(0.072) - 0.006(0.06) = 0.504(10^{-3}) \text{ m}^2$$

$$I = \frac{1}{12}(0.012)(0.072^3) - \frac{1}{12}(0.006)(0.06^3) = 0.265248(10^{-6}) \text{ m}^4$$

Referring to Fig. *b*,

$$Q_B = 0$$

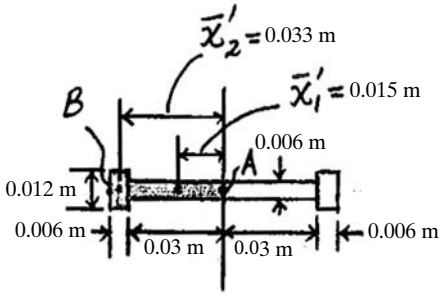


(a)

Normal and Shear Stress: The normal stress is a combination of axial and bending stress.

$$\sigma_B = \frac{N}{A} + \frac{Mx_B}{I} = \frac{-1.5(10^3)}{0.504(10^{-3})} + \frac{[0.75(10^3)](0.036)}{0.265248(10^{-6})}$$
$$= 98.82(10^6) \text{ N/m}^2 = 98.82 \text{ MPa}$$

Since $Q_B = 0$, $\tau_B = 0$. The state of stress at point B is represented on the element shown in Fig. *c*.



(b)

In - Plane Principal Stresses: Since no shear stress acts on the element,

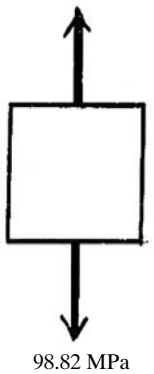
$$\sigma_1 = 98.8 \text{ MPa} \qquad \sigma_2 = 0$$

Three Mohr's Circles: Using these results,

$$\sigma_{\max} = 98.8 \text{ MPa} \qquad \sigma_{\text{int}} = \sigma_{\min} = 0 \qquad \textbf{Ans.}$$

Absolute Maximum Shear Stress:

$$\tau_{\substack{\text{abs} \\ \max}} = \frac{\sigma_{\max} - \sigma_{\min}}{2} = \frac{98.82 - 0}{2} = 49.4 \text{ MPa} \qquad \textbf{Ans.}$$



(c)

Ans.

$\sigma_{\max} = 98.8$ MPa, $\sigma_{\text{int}} = \sigma_{\min} = 0$

$\tau_{\substack{\text{abs} \\ \max}} = 49.4$ MPa

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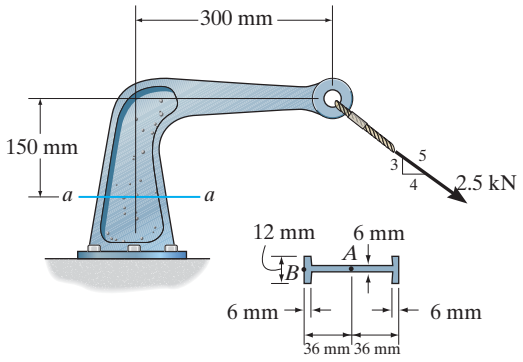
***9–88.** Determine the principal stress and absolute maximum shear stress developed at point A on the cross section of the bracket at section a–a.



SOLUTION

Internal Loadings: Considering the equilibrium of the free - body diagram of the bracket's upper cut segment, Fig. *a*,

$$+\uparrow \Sigma F_y = 0; \qquad N - 2.5\left(\frac{3}{5}\right) = 0 \qquad N = 1.5 \text{ kN}$$

$$\overset{+}{\leftarrow} \Sigma F_x = 0; \qquad V - 2.5\left(\frac{4}{5}\right) = 0 \qquad V = 2.0 \text{ kN}$$

$$\Sigma M_O = 0; \; M - 2.5\left(\frac{3}{5}\right)(0.300) - 2.5\left(\frac{4}{5}\right)(0.150) = 0 \qquad M = 0.75 \text{ kN} \cdot \text{m}$$

Section Properties: The cross - sectional area and the moment of inertia of the bracket's cross section are

$$A = 0.012(0.072) - 0.006(0.06) = 0.504(10^{-3}) \text{ m}^2$$

$$I = \frac{1}{12}(0.012)(0.072^3) - \frac{1}{12}(0.006)(0.06^3) = 0.265248(10^{-6}) \text{ m}^4$$

Referring to Fig. *b*.

$$Q_A = \bar{x}'_1 A'_1 + \bar{x}'_2 A'_2 = 0.015(0.006)(0.03) + 0.033(0.012)(0.006) = 5.076(10^{-6}) \text{ m}^3$$

Normal and Shear Stress: The normal stress is

$$\sigma_A = \frac{N}{A} = \frac{1.5(10^3)}{0.504(10^{-3})} = -2.976(10^6) \text{ N/m}^2 = -2.976 \text{ MPa}$$

The shear stress is contributed by the transverse shear stress.

$$\tau_A = \frac{VQ_A}{It} = \frac{[2.0(10^3)][5.076(10^{-6})]}{[0.265248(10^{-6})](0.006)} = 6.379(10^6) \text{ N/m}^2 = 6.379 \text{ MPa}$$

The state of stress at point A is represented by the element shown in Fig. *c*.

Construction of the Circle: $\sigma_x = 0, \sigma_y = -2.976$ MPa, and $\tau_{xy} = 6.379$ MPa. Thus,

$$\sigma_{\text{avg}} = \frac{\sigma_x + \sigma_y}{2} = \frac{0 + (-2.976)}{2} = -1.488 \text{ MPa}$$

The coordinates of reference point A and the center C of the circle are

$$A(0, -6.379) \qquad C(-1.488, 0)$$

Thus, the radius of the circle is

$$R = CA = \sqrt{[0 - (-1.488)]^2 + 6.379^2} = 6.5502 \text{ MPa}$$

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***9–88. Continued**

Using these results, the cricle is shown in Fig. *d*.

In - Plane Principal Stresses: The coordinates of reference point *B* and *D* represent σ_1 and σ_2, respectively.

$$\sigma_1 = -1.488 + 6.5502 = 5.062 \text{ MPa}$$

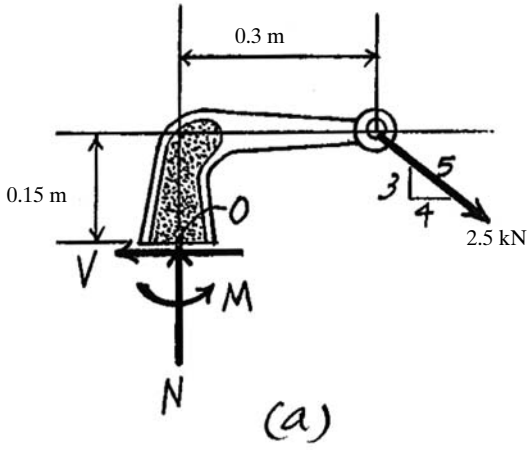
$$\sigma_2 = -1.488 - 6.5502 = -8.038 \text{ MPa}$$

Three Mohr's Circles: Using these results,

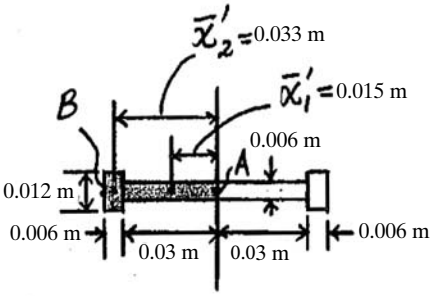
$$\sigma_{max} = 5.06 \text{ MPa} \quad \sigma_{int} = 0 \quad \sigma_{min} = -8.04 \text{ MPa}$$ **Ans.**

Absolute Maximum Shear Stress:

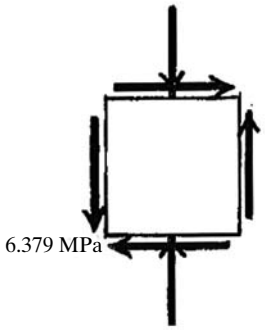
$$\tau_{\substack{abs \\ max}} = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{5.062 - (-8.038)}{2} = 6.55 \text{ MPa}$$ **Ans.**



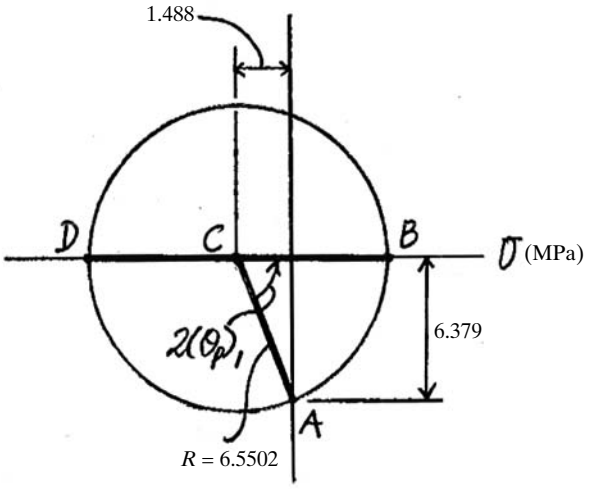
(a)



(b)



(c)



(d)

Ans.

$\sigma_{max} = 98.8$ MPa, $\sigma_{int} = \sigma_{min} = 0$,

$\tau_{\substack{abs \\ max}} = 49.4$ MPa

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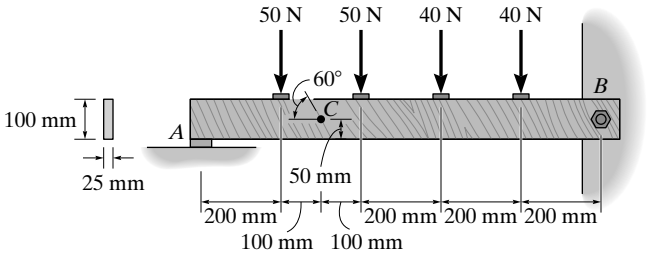
R9–1.

The wooden strut is subjected to the loading shown. Determine the principal stresses that act at point C and specify the orientation of the element at this point. The strut is supported by a bolt (pin) at B and smooth support at A.



SOLUTION

$Q_C = \bar{y}' A' = 0.025(0.05)(0.025) = 31.25(10^{-6})\ \text{m}^3$

$I = \frac{1}{12}(0.025)(0.1^3) = 2.0833(10^{-6})\ \text{m}^4$

Normal stress: $\sigma_C = 0$

Shear stress:

$$\tau = \frac{VQ_C}{It} = \frac{44(31.25)(10^{-6})}{2.0833(10^{-6})(0.025)} = 26.4\ \text{kPa}$$

Principal stress:

$\sigma_x = \sigma_y = 0; \quad \tau_{xy} = -26.4\ \text{kPa}$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2_{xy}}$$

$$= 0 \pm \sqrt{0 + (26.4)^2}$$

$\sigma_1 = 26.4\ \text{kPa} \quad ; \quad \sigma_2 = -26.4\ \text{kPa}$ **Ans.**

Orientation of principal stress:

$$\tan 2\theta_p = \frac{\tau_{xy}}{\frac{(\sigma_x - \sigma_y)}{2}} = -\infty$$

$\theta_p = +45°$ and $-45°$

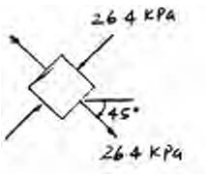
Use Eq. 9-1 to determine the principal plane of σ_1 and σ_2

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$\theta = \theta_p = -45°$

$\sigma_{x'} = 0 + 0 + (-26.4)\sin(-90°) = 26.4\ \text{kPa}$

Therefore, $\theta_{p_1} = -45°; \quad \theta_{p_2} = 45°$ **Ans.**



Ans.

$Q_C = 31.25(10^{-6})\ \text{m}^3, I = 2.0833(10^{-6})\ \text{m}^4,$
$\tau = 26.4\ \text{kPa}, \sigma_1 = 26.4\ \text{kPa}, \sigma_2 = -26.4\ \text{kPa},$
$\theta_{p_1} = -45°; \quad \theta_{p_2} = 45°$

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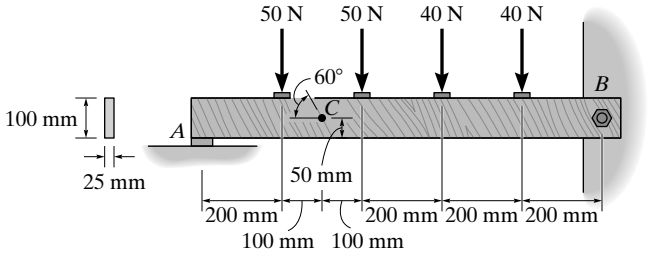
R9–2.

The wooden strut is subjected to the loading shown. If grains of wood in the strut at point C make an angle of 60° with the horizontal as shown, determine the normal and shear stresses that act perpendicular and parallel to the grains, respectively, due to the loading. The strut is supported by a bolt (pin) at B and smooth support at A.



SOLUTION

$$Q_C = y'A' = 0.025(0.05)(0.025) = 31.25(10^{-6})\ \text{m}^3$$

$$I = \frac{1}{12}(0.025)(0.1^3) = 2.0833(10^{-6})\ \text{m}^4$$

Normal stress: $\sigma_C = 0$

Shear stress:

$$\tau = \frac{VQ_C}{I\,t} = \frac{44(31.25)(10^{-6})}{2.0833(10^{-6})(0.025)} = 26.4\ \text{kPa}$$

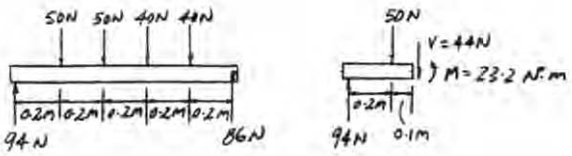
Stress transformation: $\sigma_x = \sigma_y = 0$; $\tau_{xy} = -26.4$ kPa; $\theta = 30°$

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= 0 + 0 + (-26.4)\sin 60° = -22.9\ \text{kPa}$$ **Ans.**

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$= -0 + (-26.4)\cos 60° = -13.2\ \text{kPa}$$ **Ans.**



Ans.

$\sigma_{x'} = -22.9$ kPa, $\tau_{x'y'} = -13.2$ kPa

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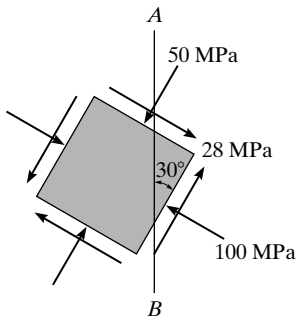
R9–3.

The state of stress at a point in a member is shown on the element. Determine the stress components acting on the plane AB.



SOLUTION

Construction of the Circle: In accordance with the sign convention, $\sigma_x = -50$ MPa, $\sigma_y = -100$ MPa, and $\tau_{xy} = -28$ MPa. Hence,

$$\sigma_{\text{avg}} = \frac{\sigma_x + \sigma_y}{2} = \frac{-50 + (-100)}{2} = -75.0 \text{ MPa}$$

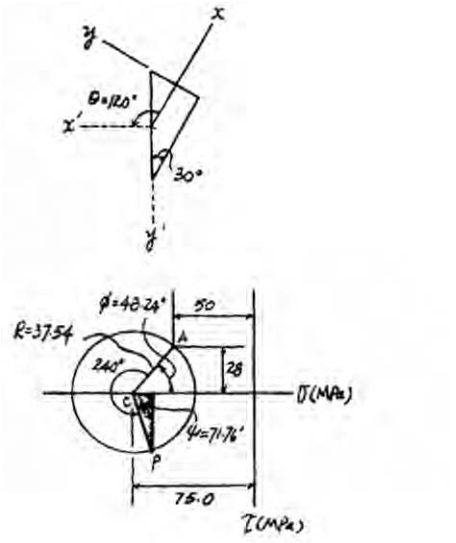
The coordinates for reference points A and C are $A(-50, -28)$ and $C(-75.0, 0)$.

The radius of the circle is $R = \sqrt{(75.0 - 50)^2 + 28^2} = 37.54$ MPa.

Stress on the Rotated Element: The normal and shear stress components $(\sigma_{x'}$ and $\tau_{x'y'})$ are represented by the coordinates of point P on the circle

$$\sigma_{x'} = -75.0 + 37.54 \cos 71.76° = -63.3 \text{ MPa} \quad \textbf{Ans.}$$

$$\tau_{x'y'} = 37.54 \sin 71.76° = 35.7 \text{ MPa} \quad \textbf{Ans.}$$



Ans.

$\sigma_{x'} = -63.3$ MPa, $\tau_{x'y'} = 35.7$ MPa

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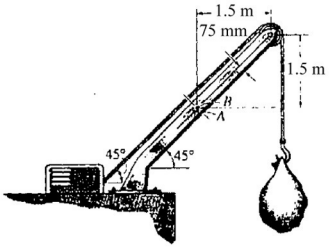
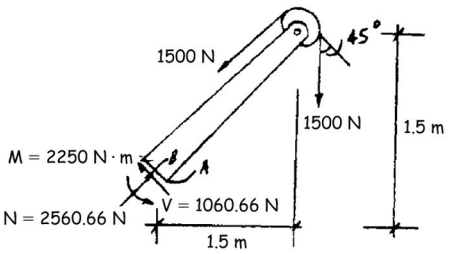
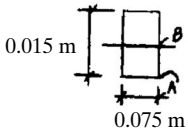
***R9–4.**

The crane is used to support the 1500 N load. Determine the principal stresses acting in the boom at points A and B. The cross section is rectangular and has a width of 150 mm and a thickness of 75 mm. Use Mohr's circle.







SOLUTION

$A = 0.075(0.15) = 0.01125 \text{ m}^2$

$I = \frac{1}{12}(0.075)(0.15^3) = 21.09375(10^{-6}) \text{ m}^4$

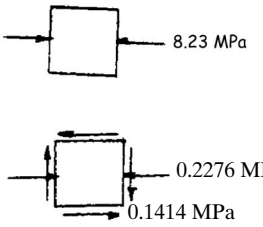
$Q_B = 0.0375(0.075)(0.075) = 0.2109375(10^{-3}) \text{ m}^3$

$Q_A = 0$

Point A:

$$\sigma_A = -\frac{P}{A} - \frac{My}{I} = \frac{-2560.66}{0.01125} - \frac{2250(0.075)}{21.09375(10^{-6})}$$

$$= -8.228(10^6) \text{ N/m}^2 = -8.23 \text{ MPa}$$



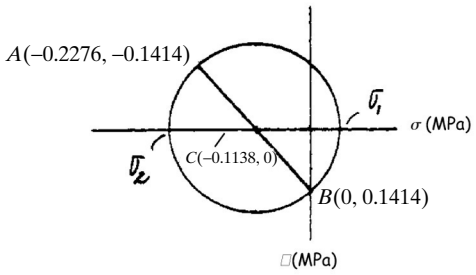
$\tau_A = 0$

$\sigma_1 = 0 \qquad \sigma_2 = -8.23 \text{ MPa}$ **Ans**

Point B:

$$\sigma_B = -\frac{P}{A} = \frac{-2560.66}{0.01125} = -0.2276(10^6) \text{ N/m}^2 = -0.2276 \text{ MPa}$$

$$\tau_B = \frac{VQ_B}{It} = \frac{1060.66[0.2109375(10^{-3})]}{[21.09375(10^{-6})](0.075)} = 0.1414(10^6) \text{ N/m}^2 = 0.1414 \text{ MPa}$$



$A(0.2276, -0.1414) \qquad B(0, 0.1414) \qquad C(-0.1138, 0)$

$R = \sqrt{0.1138^2 + 0.1414^2} = 0.1815$

$\sigma_1 = -0.1138 + 0.1815 = 0.06772 \text{ MPa} = 67.72 \text{ KPa}$ **Ans**

$\sigma_2 = -0.1138 - 0.1815 = -0.2953 \text{ MPa} = -295 \text{ KPa}$ **Ans**

Ans:
For point A : $\sigma_1 = 0$, $\sigma_2 = -8.23$ MPa,
For point B: $\sigma_1 = 67.7$ kPa, $\sigma_2 = -295$ kPa

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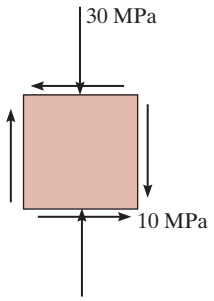
R9–5.

Determine the equivalent state of stress on an element at the same point which represents (a) the principal stresses, and (b) the maximum in-plane shear stress and the associated average normal stress. Also, for each case, determine the corresponding orientation of the element with respect to the element shown and sketch the results on the element.



SOLUTION

Normal and Shear Stress:

$$\sigma_x = 0 \qquad \sigma_y = -30 \text{ MPa} \qquad \tau_{xy} = -10 \text{ MPa}$$

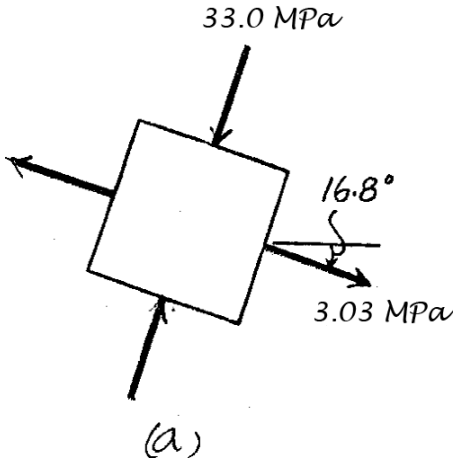
In-Plane Principal Stresses:

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{0 + (-30)}{2} \pm \sqrt{\left[\frac{0 - (-30)}{2}\right]^2 + (-10)^2}$$

$$= -15 \pm \sqrt{325}$$

$$\sigma_1 = 3.03 \text{ MPa} \qquad \sigma_2 = -33.0 \text{ MPa} \qquad \textbf{Ans.}$$



Orientation of Principal Plane:

$$\tan 2\theta_p = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2} = \frac{-10}{[0 - (-30)]/2} = -0.6667$$

$$\theta_p = -16.845° \text{ and } 73.155°$$

Substituting $\theta = -16.845°$ into

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

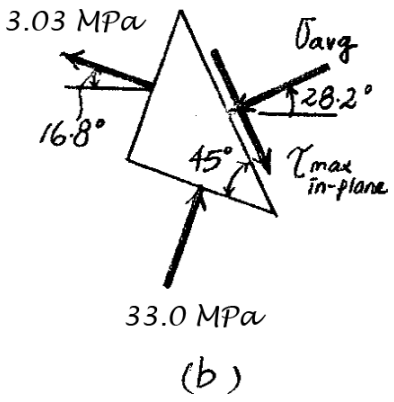
$$= \frac{0 + (-30)}{2} + \frac{0 - (-30)}{2} \cos(-33.69°) - 10 \sin(-33.69°)$$

$$= 3.03 \text{ MPa} = \sigma_1$$

Thus,

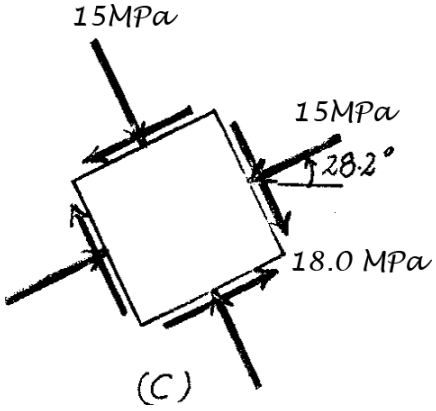
$$(\theta_p)_1 = -16.8° \text{ and } (\theta_p)_2 = 73.2° \qquad \textbf{Ans.}$$

The element that represents the state of principal stress is shown in Fig. *a*.



Maximum In-Plane Shear Stress:

$$\tau_{\substack{\max \\ \text{in-plane}}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left[\frac{0 - (-30)}{2}\right]^2 + (-10)^2} = 18.0 \text{ MPa} \qquad \textbf{Ans.}$$



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R9–5. Continued

Orientation of the Plane of Maximum In-Plane Shear Stress:

$$\tan 2\theta_s = -\frac{(\sigma_x - \sigma_y)/2}{\tau_{xy}} = -\frac{[0-(-30)]/2}{-10} = 1.5 \qquad \textbf{Ans.}$$

$$\theta_s = 28.2^\circ \text{ and } 118^\circ$$

By inspection, $\tau_{\substack{\max \\ \text{in-plane}}}$ has to act in the same sense shown in Fig. *b* to maintain equilibrium.

Average Normal Stress:

$$\sigma_{\text{avg}} = \frac{\sigma_x + \sigma_y}{2} = \frac{0 + (-30)}{2} = -15 \text{ MPa} \qquad \textbf{Ans.}$$

The element that represents the state of maximum in-plane shear stress is shown in Fig. *c*.

Ans:
$\sigma_1 = 3.03\,\text{MPa}, \sigma_2 = -33.0\text{ MPa},$
$\theta_{p1} = -16.8^\circ \text{ and } \theta_{p2} = 73.2^\circ,$
$\tau_{\substack{\max \\ \text{in-plane}}} = 18.0\,\text{MPa}, \sigma_{\text{avg}} = -15\,\text{MPa}, \quad \theta_s = 28.2^\circ$

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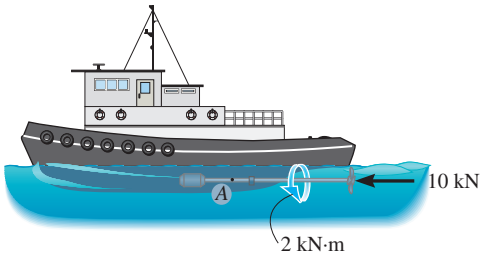
R9–6.

The propeller shaft of the tugboat is subjected to the compressive force and torque shown. If the shaft has an inner diameter of 100 mm and an outer diameter of 150 mm, determine the principal stresses at a point A located on the outer surface.



SOLUTION

Internal Loadings: Considering the equilibrium of the free-body diagram of the propeller shaft's right segment, Fig. *a*,

$\Sigma F_x = 0; \quad 10 - N = 0 \qquad N = 10\ \text{kN}$

$\Sigma M_x = 0; \quad T - 2 = 0 \qquad T = 2\ \text{kN}\cdot\text{m}$

Section Properties: The cross-sectional area and the polar moment of inertia of the propeller shaft's cross section are

$$A = \pi\left(0.075^2 - 0.05^2\right) = 3.125\pi\left(10^{-3}\right)\ \text{m}^2$$

$$J = \frac{\pi}{2}\left(0.075^4 - 0.05^4\right) = 12.6953125\pi\left(10^{-6}\right)\ \text{m}^4$$

Normal and Shear Stress: The normal stress is a contributed by axial stress only.

$$\sigma_A = \frac{N}{A} = -\frac{10\left(10^3\right)}{3.125\pi\left(10^{-3}\right)} = -1.019\ \text{MPa}$$

The shear stress is contributed by the torsional shear stress only.

$$\tau_A = \frac{Tc}{J} = \frac{2\left(10^3\right)(0.075)}{12.6953125\pi\left(10^{-6}\right)} = 3.761\ \text{MPa}$$

The state of stress at point A is represented by the element shown in Fig. *b*.

Construction of the Circle: $\sigma_x = -1.019$ MPa, $\sigma_y = 0$, and $\tau_{xy} = -3.761$ MPa. Thus,

$$\sigma_{\text{avg}} = \frac{\sigma_x + \sigma_y}{2} = \frac{-1.019 + 0}{2} = -0.5093\ \text{MPa}$$

The coordinates of reference point A and the center C of the circle are

$A(-1.019, -3.761) \qquad C(-0.5093, 0)$

Thus, the radius of the circle is

$$R = CA = \sqrt{[-1.019 - (-0.5093)]^2 + (-3.761)^2} = 3.795\ \text{MPa}$$

Using these results, the circle is shown in Fig. *c*.

In-Plane Principal Stress: The coordinates of reference points B and D represent σ_1 and σ_2, respectively.

$\sigma_1 = -0.5093 + 3.795 = 3.29\ \text{MPa}$ **Ans.**

$\sigma_2 = -0.5093 - 3.795 = -4.30\ \text{MPa}$ **Ans.**

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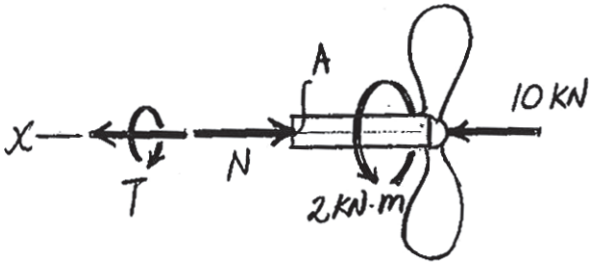
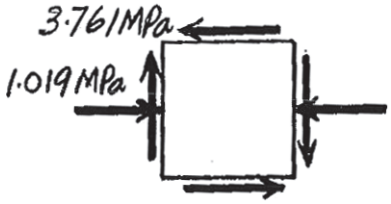
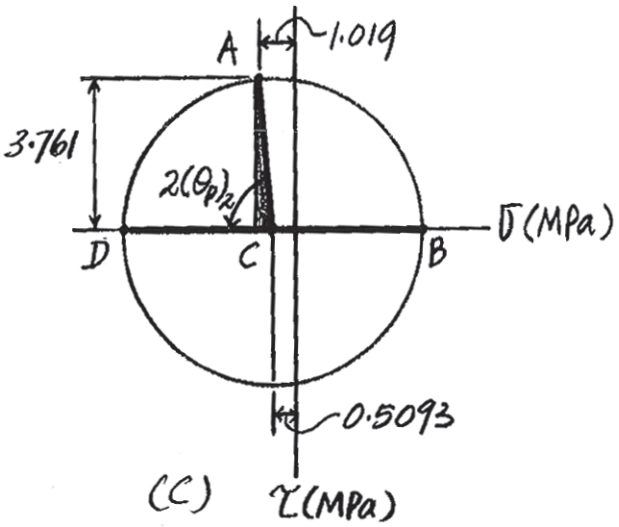
R9–6. Continued



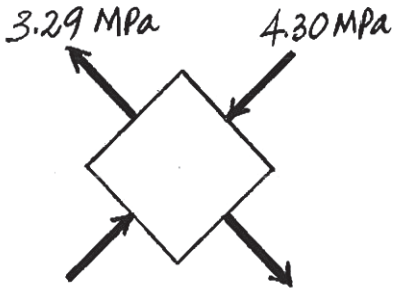
(a)



(b)



(c)



(d)

Ans:
$\sigma_1 = 3.29$ MPa,
$\sigma_2 = -4.30$ MPa

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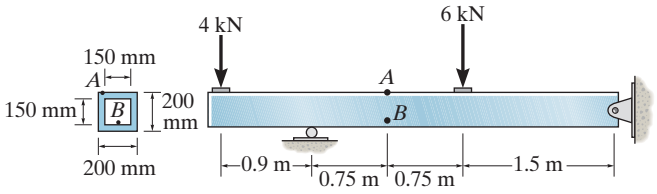
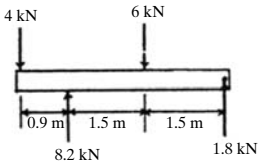
R9–7.

The box beam is subjected to the loading shown. Determine the principal stress in the beam at points A and B.



SOLUTION

Support Reactions: As shown on FBD(a).



Internal Forces and Moment: As shown on FBD(b).

Section Properties:

$$I = \frac{1}{12}(0.2)(0.2^3) - \frac{1}{12}(0.15)(0.15^3) = 91.1458(10^{-6})\ \text{m}^4$$

$$Q_A = Q_B = 0$$

Normal Stress: Applying the flexure formula.

$$\sigma = -\frac{My}{I}$$

$$\sigma_A = -\frac{[-0.45(10^3)](0.1)}{91.1458(10^{-6})} = 0.4937(10^6)\ \text{N/m}^2 = 0.494\ \text{MPa}$$

$$\sigma_B = -\frac{[-0.45(10^3)](-0.075)}{91.1458(10^{-6})} = -0.3703(10^6)\ \text{N/m}^2 = -0.370\ \text{MPa}$$

Shear Stress: Since $Q_A = Q_B = 0$, then $\tau_A = \tau_B = 0$.

In - Plane Principal Stress: $\sigma_x = 0.494$ MPa, $\sigma_y = 0$, and $\tau_{xy} = 0$. for point A. Since no shear stress acts on the element,

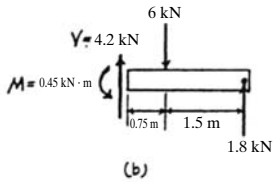
$$\sigma_1 = \sigma_x = 0.494\ \text{MPa} \qquad \textbf{Ans.}$$

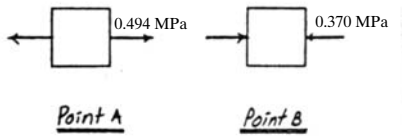
$$\sigma_2 = \sigma_y = 0 \qquad \textbf{Ans.}$$

$\sigma_x = -0.370$ MPa, $\sigma_y = 0$, and $\tau_{xy} = 0$ for point B. Since no shear stress acts on the element,

$$\sigma_1 = \sigma_y = 0 \qquad \textbf{Ans.}$$

$$\sigma_2 = \sigma_x = -0.370\ \text{MPa} \qquad \textbf{Ans.}$$





Ans.
$\sigma_1 = \sigma_x = 0.494$ MPa, $\sigma_2 = \sigma_y = 0$;
$\sigma_1 = \sigma_y = 0, \sigma_2 = \sigma_x = -0.370$ MPa

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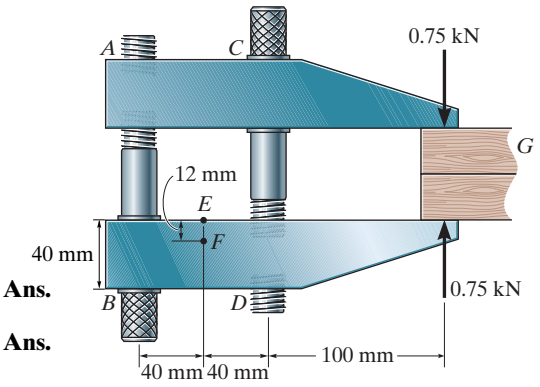
*** R9–8.**

The clamp exerts a force of 0.75 kN on the boards at *G*. Determine the axial force in each screw, *AB* and *CD*, and then compute the principal stresses at points *E* and *F*. Show the results on properly oriented elements located at these points. The section through *EF* is rectangular and is 25 mm wide, 40 mm deep.



Support Reactions: FBD(a).

$\zeta + \Sigma M_B = 0; \quad F_{CD}(0.080) - 0.75(180) = 0 \quad F_{CD} = 1.6875 \text{ kN}$ **Ans.**

$+\uparrow \Sigma F_y = 0; \quad 1.6875 - 0.75 - F_{AB} = 0 \quad F_{AB} = 0.9375 \text{ kN}$ **Ans.**

Internal Forces and Moment: As shown on FBD(b).

Section Properties:

$$I = \frac{1}{12}(0.025)(0.04^3) = 0.1333(10^{-6}) \text{ m}^4$$

$$Q_E = 0$$

$$Q_F = \overline{y}'A' = 0.014(0.025)(0.012) = 4.2(10^{-6}) \text{ m}^3$$

Normal Stress: Applying the flexure formula $\sigma = -\frac{My}{I}$,

$$\sigma_E = -\frac{(-37.5)(0.02)}{0.1333(10^{-6})} = 5.625(10^6) \text{ N/m}^2 = 5.625 \text{ MPa}$$

$$\sigma_F = -\frac{-37.5(10^3)(8)}{133333} - \frac{(-37.5)(0.008)}{0.1333(10^{-6})} = 2.25(10^6) \text{ N/m}^2 = 2.25 \text{ MPa}$$

Shear Stress: Applying the shear formula $\tau = \frac{VQ}{It}$,

$$\tau_E = \frac{[0.9375(10^{-3})](0)}{[0.1333(10^{-6})](0.025)} = 0$$

$$\tau_F = \frac{[0.9375(10^{-3})][4.2(10^{-6})]}{[0.1333(10^{-6})](0.025)}$$

$$= 1.18125(10^6) \text{ N/m}^2 = 1.18125 \text{ MPa}$$

In - Plane Principal Stress: $\sigma_x = 5.625$ MPa, $\sigma_y = 0$, and $\tau_{xy} = 0$. for point *E*. Since no shear stress acts upon the element.

$$\sigma_1 = \sigma_x = 5.625 \text{ MPa}$$ **Ans.**

$$\sigma_2 = \sigma_y = 0$$ **Ans.**

$\sigma_x = 2.25$ MPa, $\sigma_y = 0$, and $\tau_{xy} = 1.1825$ MPa for point *F*. Applying Eq. 9-5

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{2.25 + 0}{2} \pm \sqrt{\left(\frac{2.25 - 0}{2}\right)^2 + 1.18125^2}$$

$$= 1.125 \pm 1.63125$$

$\sigma_1 = 2.76$ MPa $\quad \sigma_2 = -0.506$ MPa **Ans.**

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***R9–8. Continued**

Orientation of Principal Plane: Applying Eq. 9-4 for point *F*,

$$\tan 2\theta_p = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2} = \frac{1.18125}{(2.25-0)/2} = 1.05$$

$$\theta_p = 23.20° \quad \text{and} \quad -66.80°$$

Substituting the results into Eq. 9-1 with $\theta = 23.20°$ yields

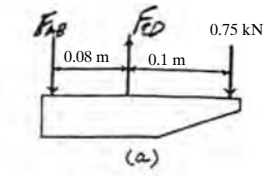
$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

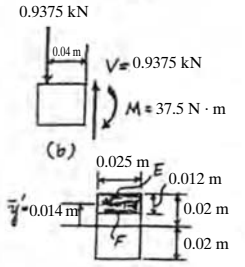
$$= \frac{2.25+0}{2} + \frac{2.25-0}{2}\cos 46.40° + 1.18125\sin 46.40°$$

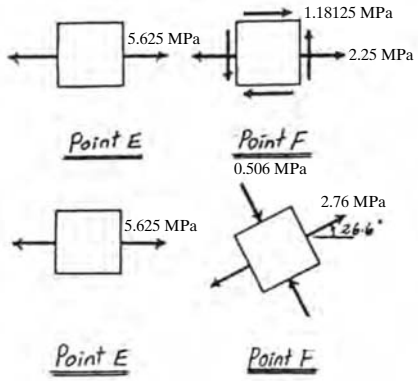
$$= 2.76 \text{ MPa} = \sigma_1$$

Hence,

$$\theta_{p1} = 23.2° \qquad \theta_{p2} = -66.8°$$ **Ans.**







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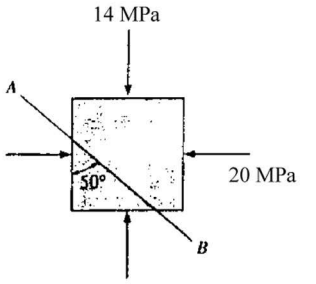
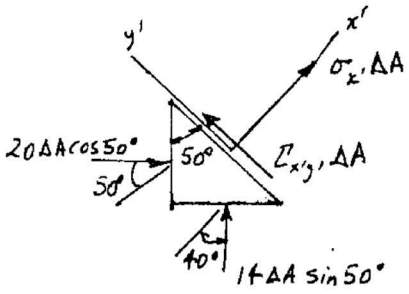
R9–9 The state of stress at a point in a member is shown on the element. Determine the stress components acting on the inclined plane AB. Solve the problem using the method of equilibrium described in Sec. 9.1.





$+\nearrow \Sigma F_{x'} = 0;$ $\quad \sigma_{x'}\Delta A + 14\,\Delta A \sin 50° \cos 40° + 20\,\Delta A \cos 50° \cos 50° = 0$

$\sigma_{x'} = -16.5$ MPa **Ans**

$+\nwarrow \Sigma F_{y'} = 0;$ $\quad \tau_{x'y'}\,\Delta A + 14\,\Delta A \sin 50° \sin 40° - 20\,\Delta A \cos 50° \sin 50° = 0$

$\tau_{xy} = 2.95$ MPa **Ans**

Ans:
$\sigma_{x'} = -16.5$ MPa, $\tau_{x'y'} = 2.95$ MPa

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10–1.

Prove that the sum of the normal strains in perpendicular directions is constant, i.e., $\epsilon_x + \epsilon_y = \epsilon_{x'} + \epsilon_{y'}$.

SOLUTION

$$\epsilon_{x'} = \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta \qquad \textbf{(1)}$$

$$\epsilon_{y'} = \frac{\epsilon_x + \epsilon_y}{2} - \frac{\epsilon_x - \epsilon_y}{2}\cos 2\theta - \frac{\gamma_{xy}}{2}\sin 2\theta \qquad \textbf{(2)}$$

Adding Eq. (1) and Eq. (2) yields:

$$\epsilon_{x'} + \epsilon_{y'} = \epsilon_x + \epsilon_y = \text{constant} \qquad \textbf{(Q.E.D.)}$$

These solutions represent a preliminary version of the Instructors' Solutions Manual (ISM). It is possible and even likely that at this preliminary stage of preparing the ISM there are some omissions and errors in the draft solutions. These will be corrected and this manual will be republished.

Ans:
N/A

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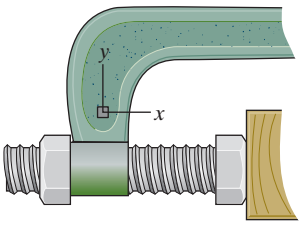
10–2.

The state of strain at the point on the arm has components of $\epsilon_x = 200\,(10^{-6})$, $\epsilon_y = -300\,(10^{-6})$, and $\gamma_{xy} = 400(10^{-6})$. Use the strain transformation equations to determine the equivalent in-plane strains on an element oriented at an angle of 30° counterclockwise from the original position. Sketch the deformed element due to these strains within the x–y plane.



SOLUTION

In accordance with the established sign convention,

$$\epsilon_x = 200(10^{-6}), \quad \epsilon_y = -300(10^{-6}) \quad \gamma_{xy} = 400(10^{-6}) \quad \theta = 30°$$

$$\epsilon_{x'} = \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$= \left[\frac{200 + (-300)}{2} + \frac{200 - (-300)}{2}\cos 60° + \frac{400}{2}\sin 60°\right](10^{-6})$$

$$= 248\,(10^{-6}) \qquad \textbf{Ans.}$$

$$\frac{\gamma_{x'y'}}{2} = -\left(\frac{\epsilon_x - \epsilon_y}{2}\right)\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta$$

$$\gamma_{x'y'} = \left\{-\left[200 - (-300)\right]\sin 60° + 400\cos 60°\right\}(10^{-6})$$

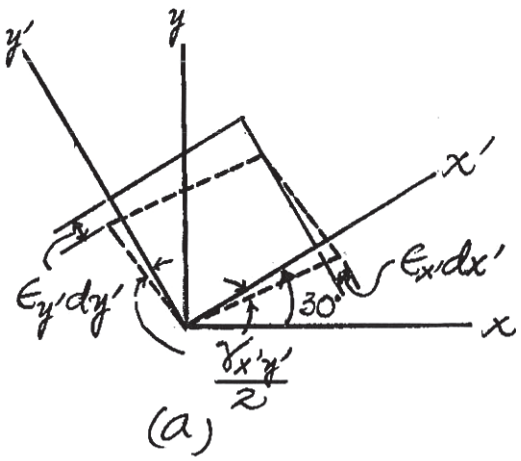
$$= -233(10^{-6}) \qquad \textbf{Ans.}$$

$$\epsilon_{y'} = \frac{\epsilon_x + \epsilon_y}{2} - \frac{\epsilon_x - \epsilon_y}{2}\cos 2\theta - \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$= \left[\frac{200 + (-300)}{2} - \frac{200 - (-300)}{2}\cos 60° - \frac{400}{2}\sin 60°\right](10^{-6})$$

$$= -348(10^{-6}) \qquad \textbf{Ans.}$$

The deformed element of this equivalent state of strain is shown in Fig. *a*.



Ans:
$\epsilon_{x'} = 248(10^{-6})$, $\gamma_{x'y'} = -233(10^{-6})$,
$\epsilon_{y'} = -348(10^{-6})$

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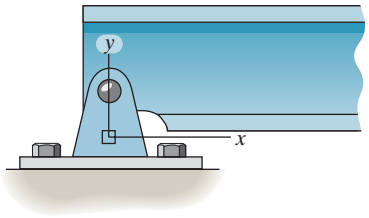
10–3.

The state of strain at the point on the pin leaf has components of $\epsilon_x = 200(10^{-6})$, $\epsilon_y = 180(10^{-6})$, and $\gamma_{xy} = -300(10^{-6})$. Use the strain transformation equations and determine the equivalent in-plane strains on an element oriented at an angle of $\theta = 60°$ counterclockwise from the original position. Sketch the deformed element due to these strains within the x–y plane.

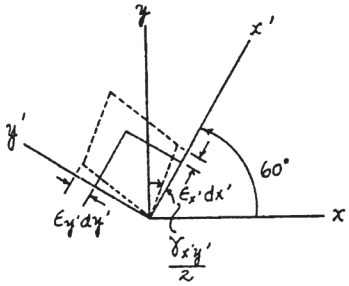


SOLUTION

Normal Strain and Shear Strain: In accordance with the sign convention,

$$\epsilon_x = 200(10^{-6}) \qquad \epsilon_y = 180(10^{-6}) \qquad \gamma_{xy} = -300(10^{-6})$$

$$\theta = +60°$$



Strain Transformation Equations: Applying Eqs. 10–5, 10–6, and 10–7,

$$\epsilon_{x'} = \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$= \left(\frac{200 + 180}{2} + \frac{200 - 180}{2}\cos 120° + \frac{-300}{2}\sin 120°\right)(10^{-6})$$

$$= 55.1(10^{-6}) \qquad \textbf{Ans.}$$

$$\frac{\gamma_{x'y'}}{2} = -\frac{\epsilon_x - \epsilon_y}{2}\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta$$

$$\gamma_{x'y'} = [-(200 - 180)\sin 120° + (-300)\cos 120°](10^{-6})$$

$$= 133(10^{-6}) \qquad \textbf{Ans.}$$

$$\epsilon_{y'} = \frac{\epsilon_x + \epsilon_y}{2} - \frac{\epsilon_x - \epsilon_y}{2}\cos 2\theta - \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$= \left(\frac{200 + 180}{2} - \frac{200 - 180}{2}\cos 120° - \frac{-300}{2}\sin 120°\right)(10^{-6})$$

$$= 325(10^{-6}) \qquad \textbf{Ans.}$$

Ans:
$\epsilon_{x'} = 55.1(10^{-6})$, $\gamma_{x'y'} = 133(10^{-6})$,
$\epsilon_{y'} = 325(10^{-6})$

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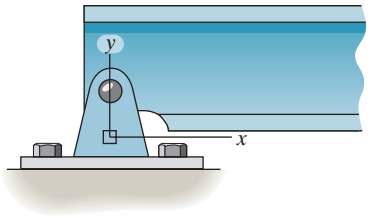
***10–4.**

Solve Prob. 10–3 for an element oriented $\theta = 30°$ clockwise.

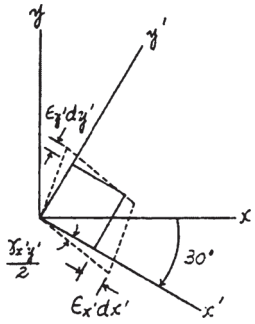


SOLUTION

Normal Strain and Shear Strain: In accordance with the sign convention,

$$\epsilon_x = 200(10^{-6}) \qquad \epsilon_y = 180(10^{-6}) \qquad \gamma_{xy} = -300(10^{-6})$$

$$\theta = -30°$$



Strain Transformation Equations: Applying Eqs. 10–5, 10–6, and 10–7,

$$\epsilon_{x'} = \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$= \left[\frac{200 + 180}{2} + \frac{200 - 180}{2}\cos(-60°) + \frac{-300}{2}\sin(-60°)\right](10^{-6})$$

$$= 325(10^{-6}) \qquad \textbf{Ans.}$$

$$\frac{\gamma_{x'y'}}{2} = -\frac{\epsilon_x - \epsilon_y}{2}\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta$$

$$\gamma_{x'y'} = [-(200 - 180)\sin(-60°) + (-300)\cos(-60°)](10^{-6})$$

$$= -133(10^{-6}) \qquad \textbf{Ans.}$$

$$\epsilon_{y'} = \frac{\epsilon_x + \epsilon_y}{2} - \frac{\epsilon_x - \epsilon_y}{2}\cos 2\theta - \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$= \left(\frac{200 + 180}{2} - \frac{200 - 180}{2}\cos(-60°) - \frac{-300}{2}\sin(-60°)\right)(10^{-6})$$

$$= 55.1(10^{-6}) \qquad \textbf{Ans.}$$

Ans:
$\epsilon_{x'} = 325(10^{-6})$, $\gamma_{x'y'} = -133(10^{-6})$,
$\epsilon_{y'} = 55.1(10^{-6})$

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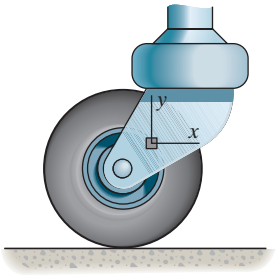
10–5.

The state of strain at the point on the leaf of the caster assembly has components of $\epsilon_x = -400(10^{-6})$, $\epsilon_y = 860(10^{-6})$, and $\gamma_{xy} = 375(10^{-6})$. Use the strain transformation equations to determine the equivalent in-plane strains on an element oriented at an angle of $\theta = 30°$ counterclockwise from the original position. Sketch the deformed element due to these strains within the x–y plane.

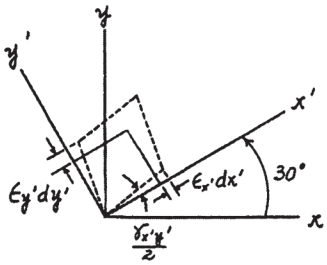


SOLUTION

Normal Strain and Shear Strain: In accordance with the sign convention,

$$\epsilon_x = -400(10^{-6}) \qquad \epsilon_y = 860(10^{-6}) \qquad \gamma_{xy} = 375(10^{-6})$$

$$\theta = +30°$$



Strain Transformation Equations: Applying Eqs. 10–5, 10–6, and 10–7,

$$\epsilon_{x'} = \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$= \left(\frac{-400 + 860}{2} + \frac{-400 - 860}{2}\cos 60° + \frac{375}{2}\sin 60°\right)(10^{-6})$$

$$= 77.4(10^{-6}) \qquad \textbf{Ans.}$$

$$\frac{\gamma_{x'y'}}{2} = -\frac{\epsilon_x - \epsilon_y}{2}\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta$$

$$\gamma_{x'y'} = [-(-400 - 860)\sin 60° + 375\cos 60°](10^{-6})$$

$$= 1279(10^{-6}) \qquad \textbf{Ans.}$$

$$\epsilon_{y'} = \frac{\epsilon_x + \epsilon_y}{2} - \frac{\epsilon_x - \epsilon_y}{2}\cos 2\theta - \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$= \left(\frac{-400 + 860}{2} - \frac{-400 - 860}{2}\cos 60° - \frac{375}{2}\sin 60°\right)(10^{-6})$$

$$= 383(10^{-6}) \qquad \textbf{Ans.}$$

Ans:
$\epsilon_{x'} = 77.4(10^{-6})$, $\gamma_{x'y'} = 1279(10^{-6})$,
$\epsilon_{y'} = 383(10^{-6})$

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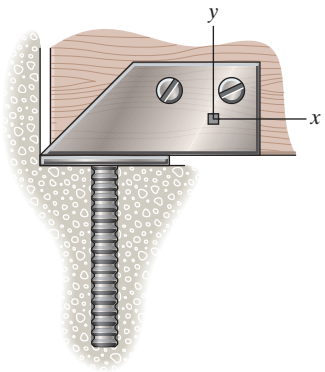
10–6.

The state of strain at a point on the bracket has components of $\epsilon_x = 150(10^{-6})$, $\epsilon_y = 200(10^{-6})$, $\gamma_{xy} = -700(10^{-6})$. Use the strain transformation equations and determine the equivalent in-plane strains on an element oriented at an angle of $\theta = 60°$ counterclockwise from the original position. Sketch the deformed element within the x–y plane due to these strains.



SOLUTION

$\epsilon_x = 150(10^{-6})$ $\epsilon_y = 200(10^{-6})$ $\gamma_{xy} = -700(10^{-6})$ $\theta = 60°$

$$\epsilon_{x'} = \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$= \left[\frac{150 + 200}{2} + \frac{150 - 200}{2}\cos 120° + \left(\frac{-700}{2}\right)\sin 120°\right]10^{-6}$$

$$= -116(10^{-6}) \quad \textbf{Ans.}$$

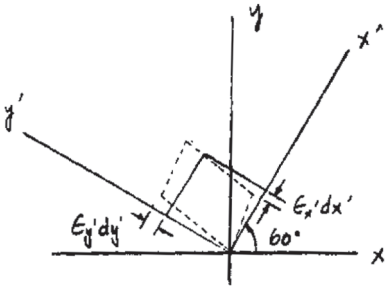
$$\epsilon_{y'} = \frac{\epsilon_x + \epsilon_y}{2} - \frac{\epsilon_x - \epsilon_y}{2}\cos 2\theta - \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$= \left[\frac{150 + 200}{2} - \frac{150 - 200}{2}\cos 120° - \left(\frac{-700}{2}\right)\sin 120°\right]10^{-6}$$

$$= 466(10^{-6}) \quad \textbf{Ans.}$$

$$\frac{\gamma_{x'y'}}{2} = -\frac{\epsilon_x - \epsilon_y}{2}\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta$$

$$\gamma_{x'y'} = 2\left[-\frac{150 - 200}{2}\sin 120° + \left(\frac{-700}{2}\right)\cos 120°\right]10^{-6} = 393(10^{-6}) \quad \textbf{Ans.}$$



Ans:
$\epsilon_{x'} = -116(10^{-6})$, $\epsilon_{y'} = 466(10^{-6})$,
$\gamma_{x'y'} = 393(10^{-6})$

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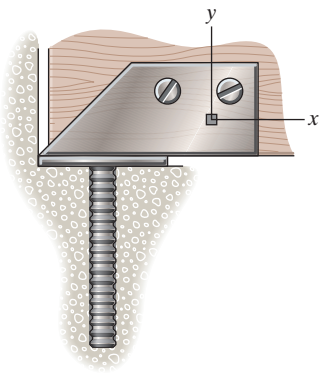
10–7.

Solve Prob. 10–6 for an element oriented $\theta = 30°$ clockwise.



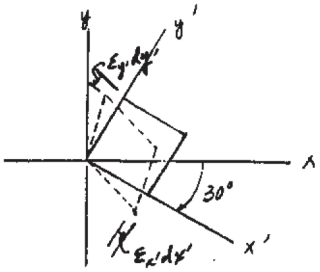
SOLUTION

$\epsilon_x = 150(10^{-6}) \quad \epsilon_y = 200(10^{-6}) \quad \gamma_{xy} = -700(10^{-6}) \quad \theta = -30°$

$$\epsilon_{x'} = \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$= \left[\frac{150 + 200}{2} + \frac{150 - 200}{2}\cos(-60°) + \left(\frac{-700}{2}\right)\sin(-60°)\right]10^{-6}$$

$$= 466(10^{-6}) \qquad \textbf{Ans.}$$



$$\epsilon_{y'} = \frac{\epsilon_x + \epsilon_y}{2} - \frac{\epsilon_x - \epsilon_y}{2}\cos 2\theta - \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$= \left[\frac{150 + 200}{2} - \frac{150 - 200}{2}\cos(-60°) - \left(\frac{-700}{2}\right)\sin(-60°)\right]10^{-6}$$

$$= -116(10^{-6}) \qquad \textbf{Ans.}$$

$$\frac{\gamma_{x'y'}}{2} = -\frac{\epsilon_x - \epsilon_y}{2}\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta$$

$$\gamma_{x'y'} = 2\left[-\frac{150 - 200}{2}\sin(-60°) + \frac{-700}{2}\cos(-60°)\right]10^{-6}$$

$$= -393(10^{-6}) \qquad \textbf{Ans.}$$

Ans:
$\epsilon_{x'} = 466(10^{-6})$, $\epsilon_{y'} = -116(10^{-6})$,
$\gamma_{x'y'} = -393(10^{-6})$

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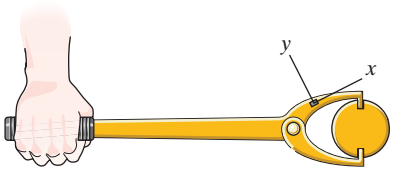
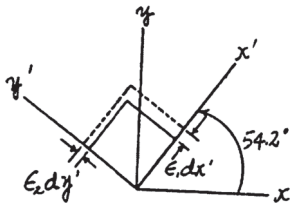
***10–8.**

The state of strain at the point on the spanner wrench has components of $\epsilon_x = 260(10^{-6})$, $\epsilon_y = 320(10^{-6})$, and $\gamma_{xy} = 180(10^{-6})$. Use the strain transformation equations to determine (a) the in-plane principal strains and (b) the maximum in-plane shear strain and average normal strain. In each case specify the orientation of the element and show how the strains deform the element within the x–y plane.



SOLUTION

a)



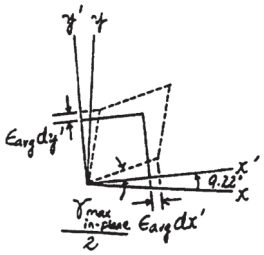
In-Plane Principal Strain: Applying Eq. 10–9,

$$\epsilon_{1,2} = \frac{\epsilon_x + \epsilon_y}{2} \pm \sqrt{\left(\frac{\epsilon_x - \epsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$= \left[\frac{260 + 320}{2} \pm \sqrt{\left(\frac{260 - 320}{2}\right)^2 + \left(\frac{180}{2}\right)^2}\right](10^{-6})$$

$$= 290 \pm 94.87$$

$$\epsilon_1 = 385(10^{-6}) \qquad \epsilon_2 = 195(10^{-6}) \qquad \textbf{Ans.}$$



Orientation of Principal Strain: Applying Eq. 10–8,

$$\tan 2\theta_p = \frac{\gamma_{xy}}{\epsilon_x - \epsilon_y} = \frac{180(10^{-6})}{(260 - 320)(10^{-6})} = -3.000$$

$$\theta_p = -35.78° \qquad \text{and} \qquad 54.22°$$

Use Eq. 10–5 to determine which principal strain deforms the element in the x' direction with $\theta = -35.78°$.

$$\epsilon_{x'} = \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$= \left[\frac{260 + 320}{2} + \frac{260 - 320}{2}\cos(-71.56°) + \frac{180}{2}\sin(-71.56°)\right](10^{-6})$$

$$= 195(10^{-6}) = \epsilon_2$$

Hence, $\theta_{p1} = 54.2°$ and $\theta_{p1} = -35.8°$ **Ans.**

b)

Maximum In-Plane Shear Strain: Applying Eq. 10–11,

$$\frac{\gamma_{\text{in-plane}}^{\max}}{2} = \sqrt{\left(\frac{\epsilon_x - \epsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$\gamma_{\text{in-plane}}^{\max} = 2\left[\sqrt{\left(\frac{260 - 320}{2}\right)^2 + \left(\frac{180}{2}\right)^2}\right](10^{-6})$$

$$= 190(10^{-6}) \qquad \textbf{Ans.}$$

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***10–8. Continued**

Orientation of Maximum In-Plane Shear Strain: Applying Eq. 10–10,

$$\tan 2\theta_s = -\frac{\epsilon_x - \epsilon_y}{\gamma_{xy}} = -\frac{260 - 320}{180} = 0.3333$$

$$\theta_s = 9.22° \quad \text{and} \quad -80.8° \qquad \textbf{Ans.}$$

Normal Strain and Shear Strain: In accordance with the sign convention,

$$\epsilon_x = 260(10^{-6}) \qquad \epsilon_y = 320(10^{-6}) \qquad \gamma_{xy} = 180(10^{-6})$$

The proper sign of $\gamma_{\substack{\max \\ \text{in-plane}}}$ can be determined by substituting $\theta = 9.22°$ into Eq. 10–6.

$$\frac{\gamma_{x'y'}}{2} = -\frac{\epsilon_x - \epsilon_y}{2}\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta$$

$$\gamma_{x'y'} = [-(260 - 320)\sin 18.44° + 180\cos 18.44°](10^{-6})$$

$$= 190(10^{-6})$$

Average Normal Strain: Applying Eq. 10–12,

$$\epsilon_{\text{avg}} = \frac{\epsilon_x + \epsilon_y}{2} = \left[\frac{260 + 320}{2}\right](10^{-6}) = 290(10^{-6}) \qquad \textbf{Ans.}$$

Ans:
$\epsilon_1 = 385(10^{-6}), \epsilon_2 = 195(10^{-6})$,
$\theta_{p1} = 54.2°, \theta_{p1} = -35.8°$,
$\gamma_{\substack{\max \\ \text{in-plane}}} = 190(10^{-6})$
$\theta_s = 9.22° \quad \text{and} \quad -80.8°$,
$\epsilon_{\text{avg}} = 290(10^{-6})$

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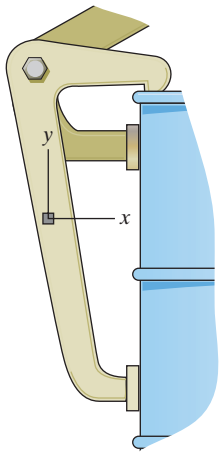
10–9.

The state of strain at the point on the member has components of $\epsilon_x = 180(10^{-6})$, $\epsilon_y = -120(10^{-6})$, and $\gamma_{xy} = -100(10^{-6})$. Use the strain transformation equations to determine (a) the in-plane principal strains and (b) the maximum in-plane shear strain and average normal strain. In each case specify the orientation of the element and show how the strains deform the element within the x–y plane.



SOLUTION

a) In accordance with the established sign convention, $\epsilon_x = 180(10^{-6})$, $\epsilon_y = -120(10^{-6})$ and $\gamma_{xy} = -100(10^{-6})$.

$$\epsilon_{1,2} = \frac{\epsilon_x + \epsilon_y}{2} \pm \sqrt{\left(\frac{\epsilon_x - \epsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$= \left\{\frac{180 + (-120)}{2} \pm \sqrt{\left[\frac{180 - (-120)}{2}\right]^2 + \left(\frac{-100}{2}\right)^2}\right\}(10^{-6})$$

$$= \left(30 \pm 158.11\right)(10^{-6})$$

$\epsilon_1 = 188(10^{-6})$ $\quad$ $\epsilon_2 = -128(10^{-6})$ **Ans.**

$$\tan 2\theta_P = \frac{\gamma_{xy}}{\epsilon_x - \epsilon_y} = \frac{-100(10^{-6})}{\left[180 - (-120)\right](10^{-6})} = -0.3333$$

$\theta_P = -9.217°$ $\quad$ and $\quad$ $80.78°$

Substitute $\theta = -9.217°$,

$$\epsilon_{x'} = \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$= \left[\frac{180 + (-120)}{2} + \frac{180 - (-120)}{2}\cos(-18.43°) + \frac{-100}{2}\sin(-18.43)\right](10^{-6})$$

$$= 188(10^{-6}) = \epsilon_1$$

Thus,

$(\theta_P)_1 = -9.22°$ $\quad$ $(\theta_P)_2 = 80.8°$ **Ans.**

The deformed element is shown in Fig (a).

b) $\epsilon_{avg} = \dfrac{180(10^{-6}) + (-120)(10^{-6})}{2} = 30(10^{-6})$ **Ans.**

$$\frac{\gamma_{\substack{\max\\ \text{in-plane}}}}{2} = \sqrt{\left(\frac{\epsilon_x - \epsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$\gamma_{\substack{\max\\ \text{in-plane}}} = \left\{2\sqrt{\left[\frac{180 - (-120)}{2}\right]^2 + \left(\frac{-100}{2}\right)^2}\right\}(10^{-6}) = 316\left(10^{-6}\right)$$ **Ans.**

$$\tan 2\theta_s = -\left(\frac{\epsilon_x - \epsilon_y}{\gamma_{xy}}\right) = -\left\{\frac{\left[180 - (-120)\right](10^{-6})}{-100(10^{-6})}\right\} = 3$$

$\theta_s = 35.78° = 35.8°$ $\quad$ and $\quad$ $-54.22° = -54.2°$ **Ans.**

Ans:
$\epsilon_1 = 188(10^{-6})$, $\epsilon_2 = -128(10^{-6})$,
$(\theta_P)_1 = -9.22°$, $(\theta_P)_2 = 80.8°$
$\epsilon_{avg} = 30(10^{-6})$
$\gamma_{\substack{\max\\ \text{in-plane}}} = 316\left(10^{-6}\right)$,
$\theta_s = 35.8°$ and $-54.2°$

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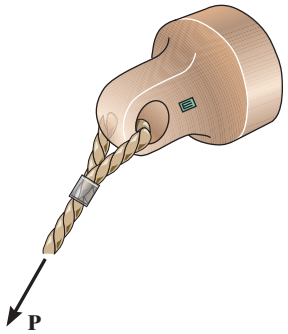
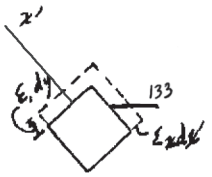
10–10.

The state of strain at the point on the support has components of $\epsilon_x = 350(10^{-6})$, $\epsilon_y = 400(10^{-6})$, $\gamma_{xy} = -675(10^{-6})$. Use the strain-transformation equations to determine (a) the in-plane principal strains and (b) the maximum in-plane shear strain and average normal strain. In each case specify the orientation of the element and show how the strains deform the element within the x–y plane.



SOLUTION

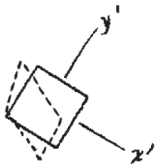
a)



$$\epsilon_{1,2} = \frac{\epsilon_x + \epsilon_y}{2} \pm \sqrt{\left(\frac{\epsilon_x - \epsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$= \frac{350 + 400}{2} \pm \sqrt{\left(\frac{350 - 400}{2}\right)^2 + \left(\frac{-675}{2}\right)^2}$$

$\epsilon_1 = 713\,(10^{-6})$ **Ans.** $\epsilon_2 = 36.6\,(10^{-6})$ **Ans.**



$$\tan 2\theta_p = \frac{\gamma_{xy}}{\epsilon_x - \epsilon_y} = \frac{-675}{(350 - 400)}$$

$\theta_{p1} = 133°$ **Ans.**

b)

$$\frac{(\gamma_{x'y'})_{\max}}{2} = \sqrt{\left(\frac{\epsilon_x - \epsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$\frac{(\gamma_{x'y'})_{\max}}{2} = \sqrt{\left(\frac{350 - 400}{2}\right)^2 + \left(\frac{-675}{2}\right)^2}$$

$(\gamma_{x'y'})_{\max} = 677(10^{-6})$ **Ans.**

$$\epsilon_{\text{avg}} = \frac{\epsilon_x + \epsilon_y}{2} = \frac{350 + 400}{2} = 375\,(10^{-6})$$ **Ans.**

$$\tan 2\theta_s = \frac{-(\epsilon_x - \epsilon_y)}{\gamma_{xy}} = \frac{350 - 400}{675}$$

$\theta_s = -2.12°$ **Ans.**

Ans:
(a) $\epsilon_1 = 713(10^{-6})$, $\epsilon_2 = 36.6(10^{-6})$, $\theta_{p1} = 133°$
(b) $\gamma_{\underset{\text{in-plane}}{\max}} = 677(10^{-6})$, $\epsilon_{\text{avg}} = 375(10^{-6})$,
$\theta_s = -2.12°$

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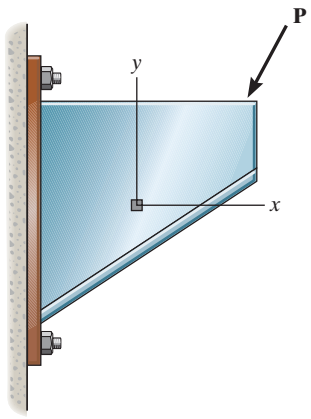
10–11.

Due to the load **P**, the state of strain at the point on the bracket has components of $\epsilon_x = 500(10^{-6})$, $\epsilon_y = 350(10^{-6})$, and $\gamma_{xy} = -430(10^{-6})$. Use the strain transformation equations to determine the equivalent in-plane strains on an element oriented at an angle of $\theta = 30°$ clockwise from the original position. Sketch the deformed element due to these strains within the x–y plane.



SOLUTION

Normal Strain and Shear Strain: In accordance with the sign convention,

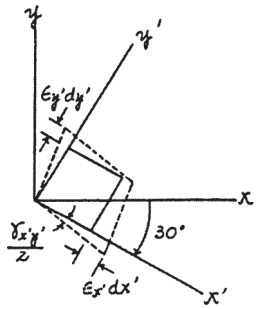
$$\epsilon_x = 500(10^{-6}) \qquad \epsilon_y = 350(10^{-6}) \qquad \gamma_{xy} = -430(10^{-6})$$
$$\theta = -30°$$

Strain Transformation Equations:

$$\epsilon_{x'} = \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$= \left[\frac{500 + 350}{2} + \frac{500 - 350}{2}\cos(-60°) + \frac{-430}{2}\sin(-60°)\right](10^{-6})$$

$$= 649(10^{-6}) \qquad \textbf{Ans.}$$



$$\frac{\gamma_{x'y'}}{2} = -\frac{\epsilon_x - \epsilon_y}{2}\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta$$

$$\gamma_{x'y'} = [-(500 - 350)\sin(-60°) + (-430)\cos(-60°)](10^{-6})$$

$$= -85.1(10^{-6}) \qquad \textbf{Ans.}$$

$$\epsilon_{y'} = \frac{\epsilon_x + \epsilon_y}{2} - \frac{\epsilon_x - \epsilon_y}{2}\cos 2\theta - \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$= \left(\frac{500 + 350}{2} - \frac{500 - 350}{2}\cos(-60°) - \frac{-430}{2}\sin(-60°)\right)(10^{-6})$$

$$= 201(10^{-6}) \qquad \textbf{Ans.}$$

Ans:
$\epsilon_{x'} = 649(10^{-6})$, $\gamma_{x'y'} = -85.1(10^{-6})$,
$\epsilon_{y'} = 201(10^{-6})$

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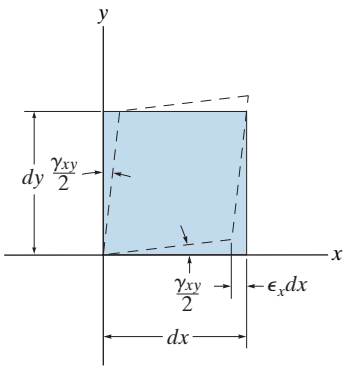
***10–12.**

The state of strain on an element has components $\epsilon_x = -400(10^{-6})$, $\epsilon_y = 0$, $\gamma_{xy} = 150(10^{-6})$. Determine the equivalent state of strain on an element at the same point oriented 30° clockwise with respect to the original element. Sketch the results on this element.



SOLUTION

Strain Transformation Equations:

$$\epsilon_x = -400(10^{-6}) \qquad \epsilon_y = 0 \qquad \gamma_{xy} = 150(10^{-6}) \qquad \theta = -30°$$

We obtain

$$\epsilon_{x'} = \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$= \left[\frac{-400 + 0}{2} + \frac{-400 - 0}{2}\cos(-60°) + \frac{150}{2}\sin(-60°)\right](10^{-6})$$

$$= -365(10^{-6}) \qquad \textbf{Ans.}$$

$$\frac{\gamma_{x'y'}}{2} = -\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta$$

$$\gamma_{x'y'} = [-(-400 - 0)\sin(-60°) + 150\cos(-60°)](10^{-6})$$

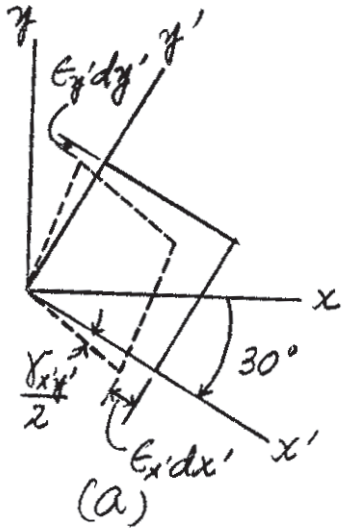
$$= -271(10^{-6}) \qquad \textbf{Ans.}$$

$$\epsilon_{y'} = \frac{\epsilon_x + \epsilon_y}{2} - \frac{\epsilon_x - \epsilon_y}{2}\cos 2\theta - \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$= \left[\frac{-400 + 0}{2} - \frac{-400 - 0}{2}\cos(-60°) - \frac{150}{2}\sin(-60°)\right](10^{-6})$$

$$= -35.0(10^{-6}) \qquad \textbf{Ans.}$$

The deformed element for this state of strain is shown in Fig. *a*.



Ans:

$\epsilon_{x'} = -365(10^{-6})$, $\gamma_{x'y'} = -271(10^{-6})$,

$\epsilon_{y'} = -35.0(10^{-6})$

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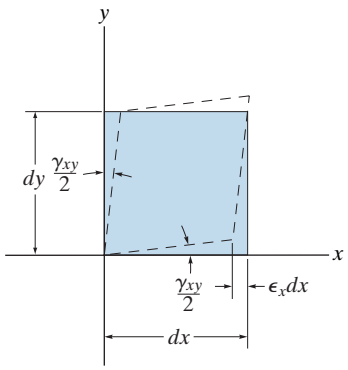
10–13.

The state of plane strain on the element is $\epsilon_x = -300(10^{-6})$, $\epsilon_y = 0$, and $\gamma_{xy} = 150(10^{-6})$. Determine the equivalent state of strain which represents (a) the principal strains, and (b) the maximum in-plane shear strain and the associated average normal strain. Specify the orientation of the corresponding elements for these states of strain with respect to the original element.



SOLUTION

In-Plane Principal Strains: $\epsilon_x = -300\left(10^{-6}\right)$, $\epsilon_y = 0$, and $\gamma_{xy} = 150\left(10^{-6}\right)$. We obtain

$$\epsilon_{1,2} = \frac{\epsilon_x + \epsilon_y}{2} \pm \sqrt{\left(\frac{\epsilon_x - \epsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$= \left[\frac{-300 + 0}{2} \pm \sqrt{\left(\frac{-300 - 0}{2}\right)^2 + \left(\frac{150}{2}\right)^2}\right]\left(10^{-6}\right)$$

$$= (-150 \pm 167.71)\left(10^{-6}\right)$$

$$\epsilon_1 = 17.7\left(10^{-6}\right) \qquad \epsilon_2 = -318\left(10^{-6}\right) \qquad \textbf{Ans.}$$

Orientation of Principal Strain:

$$\tan 2\theta_p = \frac{\gamma_{xy}}{\epsilon_x - \epsilon_y} = \frac{150\left(10^{-6}\right)}{(-300 - 0)\left(10^{-6}\right)} = -0.5$$

$\theta_P = -13.28°$ and $76.72°$

Substituting $\theta = -13.28°$ into Eq. 9–1,

$$\epsilon_{x'} = \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$= \left[\frac{-300 + 0}{2} + \frac{-300 - 0}{2}\cos(-26.57°) + \frac{150}{2}\sin(-26.57°)\right]\left(10^{-6}\right)$$

$$= -318\left(10^{-6}\right) = \epsilon_2$$

Thus,

$\left(\theta_P\right)_1 = 76.7°$ and $\left(\theta_P\right)_2 = -13.3°$ **Ans.**

The deformed element of this state of strain is shown in Fig. *a*.

Maximum In-Plane Shear Strain:

$$\frac{\gamma_{\substack{\max \\ \text{in-plane}}}}{2} = \sqrt{\left(\frac{\epsilon_x - \epsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$\gamma_{\substack{\max \\ \text{in-plane}}} = \left[2\sqrt{\left(\frac{-300 - 0}{2}\right)^2 + \left(\frac{150}{2}\right)^2}\right]\left(10^{-6}\right) = 335\left(10^{-6}\right) \qquad \textbf{Ans.}$$

Orientation of the Maximum In-Plane Shear Strain:

$$\tan 2\theta_s = -\left(\frac{\epsilon_x - \epsilon_y}{\gamma_{xy}}\right) = -\left[\frac{(-300 - 0)\left(10^{-6}\right)}{150\left(10^{-6}\right)}\right] = 2$$

$\theta_s = 31.7°$ and $122°$ **Ans.**

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10–13. Continued

The algebraic sign for $\gamma_{\substack{\max \\ \text{in-plane}}}$ when $\theta = \theta_s = 31.7°$ can be obtained using

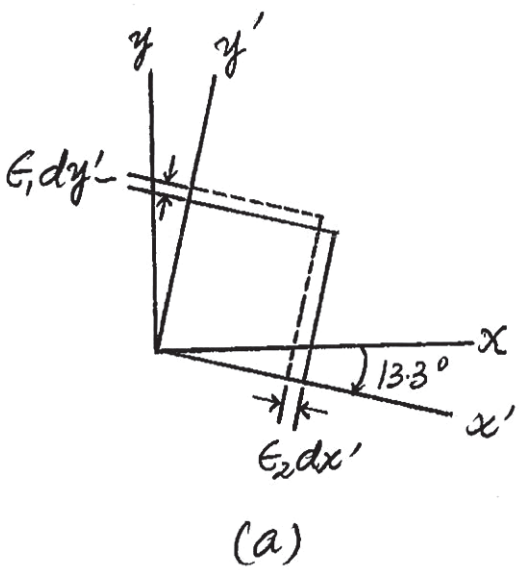
$$\frac{\gamma_{x'y'}}{2} = -\left(\frac{\epsilon_x - \epsilon_y}{2}\right)\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta$$

$$\gamma_{x'y'} = [-(-300 - 0)\sin 63.43° + 150\cos 63.43°]\left(10^{-6}\right)$$
$$= 335\left(10^{-6}\right)$$

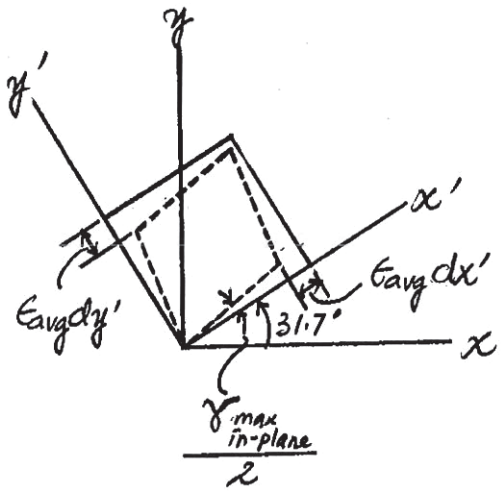
Average Normal Strain:

$$\epsilon_{\text{avg}} = \frac{\epsilon_x + \epsilon_y}{2} = \left(\frac{-300 + 0}{2}\right)\left(10^{-6}\right) = -150\left(10^{-6}\right)$$ **Ans.**

The deformed element for this state of strain is shown in Fig. *b*.



(a)



(b)

Ans:

$\epsilon_1 = 17.7(10^{-6})$, $\epsilon_2 = -318(10^{-6})$,
$\theta_{p1} = 76.7°$ and $\theta_{p2} = -13.3°$,
$\gamma_{\substack{\max \\ \text{in-plane}}} = 335(10^{-6})$, $\theta_s = 31.7°$ and $122°$,
$\epsilon_{\text{avg}} = -150(10^{-6})$

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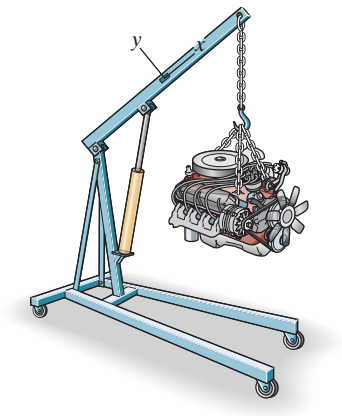
10–14.

The state of strain at the point on a boom of a shop crane has components of $\epsilon_x = 250(0^{-6})$, $\epsilon_y = 300(10^{-6})$, and $\gamma_{xy} = -180(10^{-6})$. Use the strain transformation equations to determine (a) the in-plane principal strains and (b) the maximum in-plane shear strain and average normal strain. In each case, specify the orientation of the element and show how the strains deform the element within the x–y plane.



SOLUTION

a)

In-Plane Principal Strain: Applying Eq. 10–9,

$$\epsilon_{1,2} = \frac{\epsilon_x + \epsilon_y}{2} \pm \sqrt{\left(\frac{\epsilon_x - \epsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$= \left[\frac{250 + 300}{2} \pm \sqrt{\left(\frac{250 - 300}{2}\right)^2 + \left(\frac{-180}{2}\right)^2}\right](10^{-6})$$

$$= 275 \pm 93.41$$

$$\epsilon_1 = 368(10^{-6}) \qquad \epsilon_2 = 182(10^{-6}) \qquad \textbf{Ans.}$$

Orientation of Principal Strain: Applying Eq. 10–8,

$$\tan 2\theta_P = \frac{\gamma_{xy}}{\epsilon_x - \epsilon_y} = \frac{-180(10^{-6})}{(250 - 300)(10^{-6})} = 3.600$$

$$\theta_P = 37.24° \quad \text{and} \quad -52.76°$$

Use Eq. 10–5 to determine which principal strain deforms the element in the x' direction with $\theta = 37.24°$.

$$\epsilon_{x'} = \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$= \left[\frac{250 + 300}{2} + \frac{250 - 300}{2}\cos 74.48° + \frac{-180}{2}\sin 74.48°\right](10^{-6})$$

$$= 182(10^{-6}) = \epsilon_2$$

Hence,

$$\theta_{p1} = -52.8° \quad \text{and} \quad \theta_{p2} = 37.2° \qquad \textbf{Ans.}$$

b)

Maximum In-Plane Shear Strain: Applying Eq. 10–11,

$$\frac{\gamma_{\substack{\max \\ \text{in-plane}}}}{2} = \sqrt{\left(\frac{\epsilon_x - \epsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$\gamma_{\substack{\max \\ \text{in-plane}}} = 2\left[\sqrt{\left(\frac{250 - 300}{2}\right)^2 + \left(\frac{-180}{2}\right)^2}\right](10^{-6})$$

$$= 187(10^{-6}) \qquad \textbf{Ans.}$$

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10–14. Continued

Orientation of the Maximum In-Plane Shear Strain: Applying Eq. 10–10,

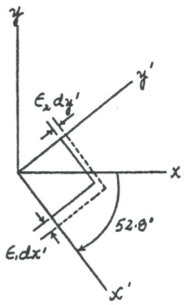
$$\tan 2\theta_s = -\frac{\epsilon_x - \epsilon_y}{\gamma_{xy}} = -\frac{250 - 300}{-180} = -0.2778$$

$$\theta_s = -7.76^\circ \quad \text{and} \quad 82.2^\circ \qquad \textbf{Ans.}$$

The proper sign of $\gamma_{\substack{\max \\ \text{in-plane}}}$ can be determined by substituting $\theta = -7.76^\circ$ into Eq. 10–6.

$$\frac{\gamma_{x'y'}}{2} = -\frac{\epsilon_x - \epsilon_y}{2}\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta$$

$$\gamma_{x'y'} = \{-[250 - 300]\sin(-15.52^\circ) + (-180)\cos(-15.52^\circ)\}\left(10^{-6}\right)$$
$$= -187\left(10^{-6}\right)$$

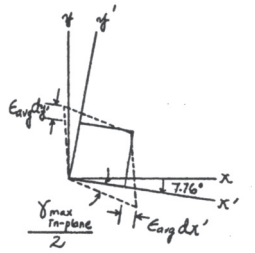


Normal Strain and Shear Strain: In accordance with the sign convention,

$$\epsilon_x = 250\left(10^{-6}\right) \quad \epsilon_y = 300\left(10^{-6}\right) \quad \gamma_{xy} = -180\left(10^{-6}\right)$$

Average Normal Strain: Applying Eq. 10–12,

$$\epsilon_{\text{avg}} = \frac{\epsilon_x + \epsilon_y}{2} = \left[\frac{250 + 300}{2}\right]\left(10^{-6}\right) = 275\left(10^{-6}\right) \qquad \textbf{Ans.}$$



Ans:
$\epsilon_1 = 368(10^{-6})$, $\epsilon_2 = 182(10^{-6})$,
$\theta_{p1} = -52.8^\circ$ and $\theta_{p2} = 37.2^\circ$,
$\gamma_{\substack{\max \\ \text{in-plane}}} = 187(10^{-6})$, $\theta_s = -7.76^\circ$ and 82.2°,
$\epsilon_{\text{avg}} = 275\,(10^{-6})$

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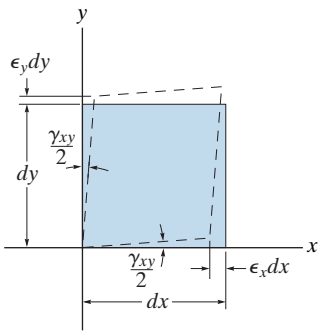
***10–16.**

The state of strain on the element has components $\epsilon_x = -300(10^{-6})$, $\epsilon_y = 100(10^{-6})$, $\gamma_{xy} = 150(10^{-6})$. Determine the equivalent state of strain, which represents (a) the principal strains, and (b) the maximum in-plane shear strain and the associated average normal strain. Specify the orientation of the corresponding elements for these states of strain with respect to the original element.



SOLUTION

In-Plane Principal Strains: $\epsilon_x = -300(10^{-6})$, $\epsilon_y = 100(10^{-6})$, and $\gamma_{xy} = 150(10^{-6})$. We obtain

$$\epsilon_{1,2} = \frac{\epsilon_x + \epsilon_y}{2} \pm \sqrt{\left(\frac{\epsilon_x - \epsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$= \left[\frac{-300 + 100}{2} \pm \sqrt{\left(\frac{-300 - 100}{2}\right)^2 + \left(\frac{150}{2}\right)^2}\right](10^{-6})$$

$$= (-100 \pm 213.60)(10^{-6})$$

$$\epsilon_1 = 114(10^{-6}) \qquad \epsilon_2 = -314(10^{-6}) \qquad \textbf{Ans.}$$

Orientation of Principal Strains:

$$\tan 2\theta_p = \frac{\gamma_{xy}}{\epsilon_x - \epsilon_y} = \frac{150(10^{-6})}{(-300 - 100)(10^{-6})} = -0.375$$

$$\theta_p = -10.28^\circ \text{ and } 79.72^\circ$$

Substituting $\theta = -10.28^\circ$ into

$$\epsilon_{x'} = \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$= \left[\frac{-300 + 100}{2} + \frac{-300 - 100}{2}\cos(-20.56^\circ) + \frac{150}{2}\sin(-20.56^\circ)\right](10^{-6})$$

$$= -314(10^{-6}) = \epsilon_2$$

Thus,

$$(\theta_p)_1 = 79.7^\circ \text{ and } (\theta_p)_2 = -10.3^\circ \qquad \textbf{Ans.}$$

The deformed element for the state of principal strain is shown in Fig. *a*.

Maximum In-Plane Shear Strain:

$$\frac{\gamma_{\substack{\max \\ \text{in-plane}}}}{2} = \sqrt{\left(\frac{\epsilon_x - \epsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$\gamma_{\substack{\max \\ \text{in-plane}}} = \left[2\sqrt{\left(\frac{-300 - 100}{2}\right)^2 + \left(\frac{150}{2}\right)^2}\right](10^{-6}) = 427(10^{-6}) \qquad \textbf{Ans.}$$

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***10–16. Continued**

Orientation of Maximum In-Plane Shear Strain:

$$\tan 2\theta_s = -\left(\frac{\epsilon_x - \epsilon_y}{\gamma_{xy}}\right) = -\left[\frac{(-300 - 100)(10^{-6})}{150(10^{-6})}\right](10^{-6}) = 2.6667$$

$$\theta_s = 34.7° \text{ and } 125° \qquad \textbf{Ans.}$$

The algebraic sign for $\gamma_{\substack{\max \\ \text{in-plane}}}$ when $\theta = \theta_s = 34.7°$ can be determined using

$$\frac{\gamma_{x'y'}}{2} = -\left(\frac{\epsilon_x - \epsilon_y}{2}\right)\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta$$

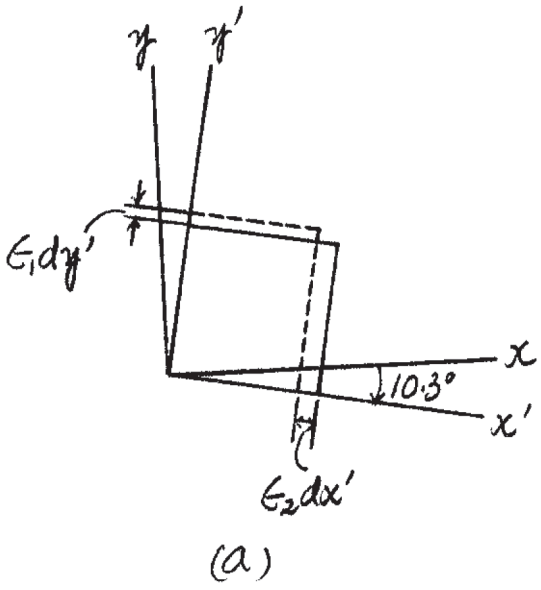
$$\gamma_{x'y'} = [-(-300 - 100)\sin 69.44° + 150\cos 69.44°](10^{-6})$$

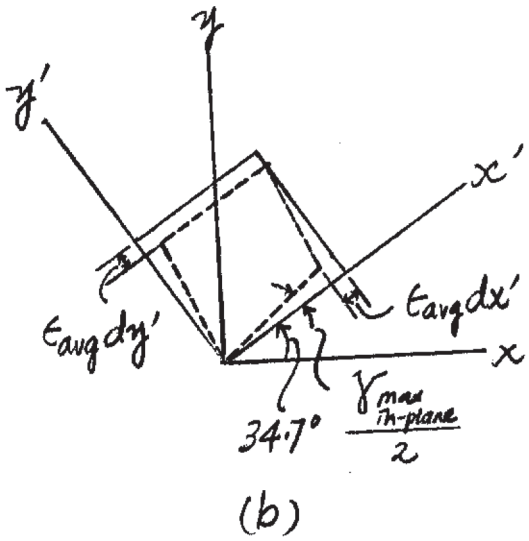
$$= 427(10^{-6})$$

Average Normal Strain:

$$\epsilon_{\text{avg}} = \frac{\epsilon_x + \epsilon_y}{2} = \left(\frac{-300 + 100}{2}\right)(10^{-6}) = -100(10^{-6}) \qquad \textbf{Ans.}$$

The deformed element for this state of maximum in-plane shear strain is shown in Fig. *b*.





Ans:
$\epsilon_1 = 114(10^{-6})$, $\epsilon_2 = -314(10^{-6})$,
$(\theta_p)_1 = 79.7°$ and $(\theta_p)_2 = -10.3°$,
$\gamma_{\substack{\max \\ \text{in-plane}}} = 427(10^{-6})$, $\theta_s = 34.7°$ and $125°$,
$\epsilon_{\text{avg}} = -100(10^{-6})$

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10–17.

Solve Prob. 10–3 using Mohr's circle.

SOLUTION

Construction of the Circle: In accordance with the sign convention, $\varepsilon_x = 200(10^{-6})$, $\varepsilon_y = 180(10^{-6})$, and $\frac{\gamma_{xy}}{2} = -150(10^{-6})$. Hence,

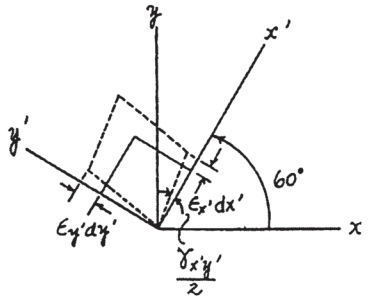
$$\varepsilon_{\text{avg}} = \frac{\varepsilon_x + \varepsilon_y}{2} = \left(\frac{200 + 180}{2}\right)(10^{-6}) = 190(10^{-6})$$

The coordinates for reference points A and C are

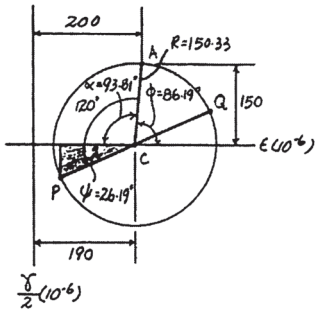
$$A(200, -150)(10^{-6}) \qquad C(190, 0)(10^{-6})$$

The radius of the circle is

$$R = \left(\sqrt{(200 - 190)^2 + 150^2}\right)(10^{-6}) = 150.33(10^{-6})$$



Strain on The Inclined Element: The normal and shear strain $\left(\varepsilon_{x'} \text{ and } \frac{\gamma_{x'y'}}{2}\right)$ are represented by coordinates of point P on the circle. $\varepsilon_{y'}$ can be determined by calculating the coordinates of point Q on the circle.



$$\varepsilon_{x'} = (190 - 150.33\cos 26.19°)(10^{-6}) = 55.1(10^{-6}) \qquad \textbf{Ans.}$$

$$\frac{\gamma_{x'y'}}{2} = (150.33\sin 26.19°)(10^{-6})$$

$$\gamma_{x'y'} = 133(10^{-6}) \qquad \textbf{Ans.}$$

$$\varepsilon_{y'} = (190 + 150.33\cos 26.19°)\,(10^{-6}) = 325(10^{-6}) \qquad \textbf{Ans.}$$

Ans:
$\epsilon_{x'} = 55.1(10^{-6})$, $\gamma_{x'y'} = 133(10^{-6})$,
$\epsilon_{y'} = 325(10^{-6})$

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10–18.

Solve Prob. 10–4 using Mohr's circle.

SOLUTION

Construction of the Circle: In accordance with the sign convention, $\varepsilon_x = 200(10^{-6})$, $\varepsilon_y = 180(10^{-6})$, and $\frac{\gamma_{xy}}{2} = -150(10^{-6})$. Hence,

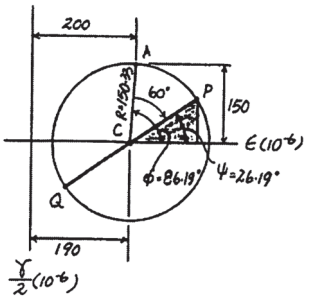
$$\varepsilon_{\text{avg}} = \frac{\varepsilon_x + \varepsilon_y}{2} = \left(\frac{200 + 180}{2}\right)(10^{-6}) = 190(10^{-6})$$

The coordinates for reference points A and C are

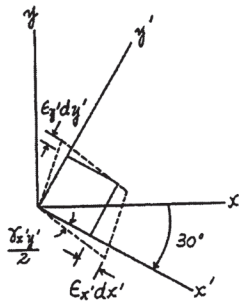
$$A(200, -150)(10^{-6}) \qquad C(190, 0)(10^{-6})$$

The radius of the circle is

$$R = \left(\sqrt{(200 - 190)^2 + 150^2}\right)(10^{-6}) = 150.33(10^{-6})$$



Strain on the Inclined Element: The normal and shear strain $\left(\varepsilon_{x'} \text{ and } \frac{\gamma_{x'y'}}{2}\right)$ are represented by coordinates of point P on the circle. $\varepsilon_{y'}$ can be determined by calculating the coordinates of point Q on the circle.



$$\varepsilon_{x'} = (190 + 150.33\cos 26.19°)(10^{-6}) = 325(10^{-6}) \qquad \textbf{Ans.}$$

$$\frac{\gamma_{x'y'}}{2} = -(150.33\sin 26.19°)(10^{-6})$$

$$\gamma_{x'y'} = -133(10^{-6}) \qquad \textbf{Ans.}$$

$$\varepsilon_{y'} = (190 - 150.33\cos 26.19°)(10^{-6}) = 55.1(10^{-6}) \qquad \textbf{Ans.}$$

Ans:
$\epsilon_{x'} = 325(10^{-6})$, $\gamma_{x'y'} = -133(10^{-6})$,
$\epsilon_{y'} = 55.1(10^{-6})$

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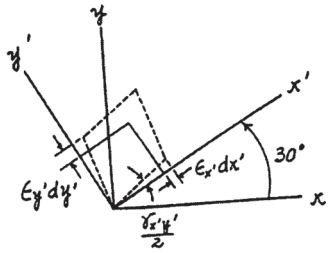
10–19.

Solve Prob. 10–5 using Mohr's circle.

SOLUTION



Construction of the Circle: In accordance with the sign convention $\varepsilon_x = -400(10^{-6})$, $\varepsilon_y = 860(10^{-6})$ and

$\frac{\gamma_{xy}}{2} = 187.5(10^{-6})$. Hence,

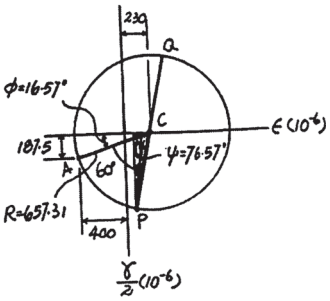
$$\varepsilon_{\text{avg}} = \frac{\varepsilon_x + \varepsilon_y}{2} = \left(\frac{-400 + 860}{2}\right)(10^{-6}) = 230(10^{-6})$$

The coordinates for reference points A and C are

$$A(-400, 187.5)(10^{-6}) \qquad C(230, 0)(10^{-6})$$

The radius of the circle is

$$R = \left(\sqrt{(400 + 230)^2 + 187.5^2}\right)(10^{-6}) = 657.31(10^{-6})$$



Strain on the Inclined Element: The normal and shear strain $\left(\varepsilon_{x'} \text{ and } \frac{\gamma_{x'y'}}{2}\right)$ are represented by the coordinates of point P on the circle. $\varepsilon_{y'}$ can be determined by calculating the coordinates of point Q on the circle.

$$\varepsilon_{x'} = (230 - 657.31\cos 76.57°)(10^{-6}) = 77.4(10^{-6}) \qquad \textbf{Ans.}$$

$$\frac{\gamma_{x'y'}}{2} = (657.31\sin 76.57°)(10^{-6})$$

$$\gamma_{x'y'} = 1279(10^{-6}) \qquad \textbf{Ans.}$$

$$\varepsilon_{y'} = (230 + 657.31\cos 76.57°)(10^{-6}) = 383(10^{-6}) \qquad \textbf{Ans.}$$

Ans:
$\epsilon_{x'} = 77.4(10^{-6})$, $\gamma_{x'y'} = 1279(10^{-6})$,
$\epsilon_{y'} = 383(10^{-6})$

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***10–20.**

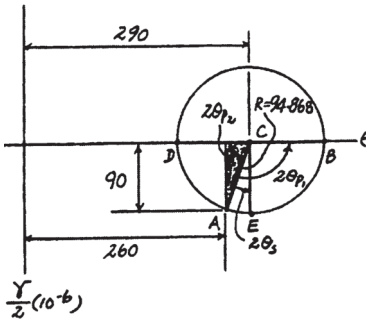
Solve Prob. 10–8 using Mohr's circle.

SOLUTION

Construction of the Circle: In accordance with the sign convention $\varepsilon_x = 260(10^{-6})$, $\varepsilon_y = 320(10^{-6})$, and

$\frac{\gamma_{xy}}{2} = 90(10^{-6})$. Hence,

$$\varepsilon_{\text{avg}} = \frac{\varepsilon_x + \varepsilon_y}{2} = \left(\frac{260 + 320}{2}\right)(10^{-6}) = 290(10^{-6}) \qquad \textbf{Ans.}$$



The coordinates for reference points A and C are

$A(260, 90)(10^{-6})$ $\quad$ $C(290, 0)(10^{-6})$

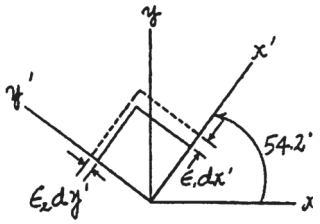
The radius of the circle is

$$R = \left(\sqrt{(290 - 260)^2 + 90^2}\right)(10^{-6}) = 94.868(10^{-6})$$

In-Plane Principal Strain: The coordinates of points B and D represent ε_1 and ε_2, respectively.

$\varepsilon_1 = (290 + 94.868)(10^{-6}) = 385(10^{-6})$ **Ans.**

$\varepsilon_2 = (290 - 94.868)(10^{-6}) = 195(10^{-6})$ **Ans.**

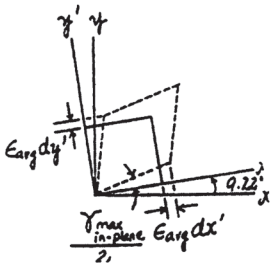


Orientation of Principal Strain: From the circle,

$$\tan 2\theta_{P_2} = \frac{90}{290 - 260} = 3.000 \qquad 2\theta_{P_2} = 71.57°$$

$$2\theta_{P_1} = 180° - 2\theta_{P_2}$$

$$\theta_{P_1} = \frac{180° - 71.57°}{2} = 54.2° \qquad \textbf{(Counterclockwise)} \qquad \textbf{Ans.}$$



Maximum In-Plane Shear Strain: Represented by the coordinates of point E on the circle.

$$\frac{\gamma_{\text{in-plane}}^{\max}}{2} = R = 94.868(10^{-6})$$

$$\gamma_{\text{in-plane}}^{\max} = 190(10^{-6}) \qquad \textbf{Ans.}$$

Orientation of Maximum In-Plane Shear Strain: From the circle,

$$\tan 2\theta_s = \frac{290 - 260}{90} = 0.3333$$

$\theta_s = 9.22°$ **(Counterclockwise)** **Ans.**

Ans:

$\varepsilon_{\text{avg}} = 290(10^{-6})$,
$\varepsilon_1 = 385(10^{-6})$,
$\varepsilon_2 = 195(10^{-6})$,
$\theta_{P_1} = 54.2°$ **(Counterclockwise)**,
$\gamma_{\text{in-plane}}^{\max} = 190(10^{-6})$,
$\theta_s = 9.22°$ **(Counterclockwise)**

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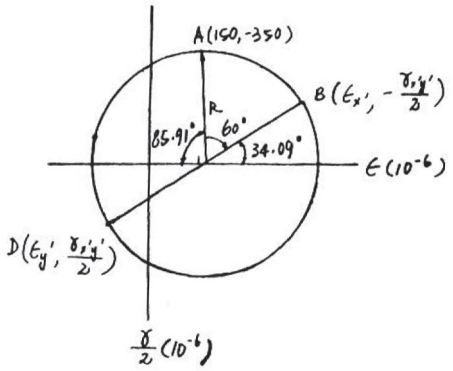
10–21.

Solve Prob. 10–7 using Mohr's circle.

SOLUTION

$\epsilon_x = 150(10^{-6})$ $\epsilon_y = 200(10^{-6})$ $\gamma_{xy} = -700(10^{-6})$ $\dfrac{\gamma_{xy}}{2} = -350(10^{-6})$



$\theta = -30°$ $2\theta = -60°$

$A(150, -350)$; $C(175, 0)$

$R = \sqrt{(175 - 150)^2 + (-350)^2} = 350.89$

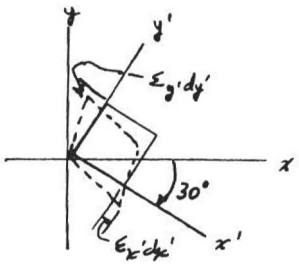
Coordinates of point B:

$\epsilon_{x'} = 350.89 \cos 34.09° + 175$

$= 466(10^{-6})$ **Ans.**

$\dfrac{\gamma_{x'y'}}{2} = -350.89 \sin 34.09°$

$\gamma_{x'y'} = -393(10^{-6})$ **Ans.**



Coordinates of point D:

$\epsilon_{y'} = 175 - 350.89 \cos 34.09°$

$= -116(10^{-6})$ **Ans.**

Ans:
$\epsilon_{x'} = 466(10^{-6})$, $\gamma_{x'y'} = -393(10^{-6})$,
$\epsilon_{y'} = -116(10^{-6})$

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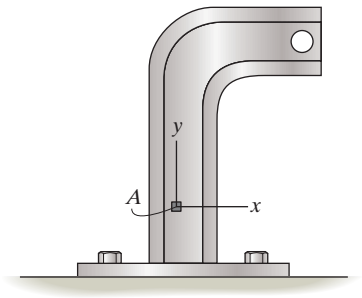
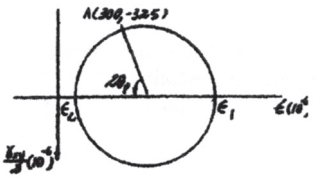
10–22.

The strain at point A on the bracket has components $\epsilon_x = 300(10^{-6})$, $\epsilon_y = 550(10^{-6})$, $\gamma_{xy} = -650(10^{-6})$, $\epsilon_z = 0$. Determine (a) the principal strains at A in the x–y plane, (b) the maximum shear strain in the x–y plane, and (c) the absolute maximum shear strain.



SOLUTION

$\epsilon_x = 300(10^{-6})$ $\epsilon_y = 550(10^{-6})$ $\gamma_{xy} = -650(10^{-6})$ $\dfrac{\gamma_{xy}}{2} = -325(10^{-6})$



$A(300, -325)10^{-6}$ $C(425, 0)10^{-6}$

$R = \left[\sqrt{(425 - 300)^2 + (-325)^2}\right]10^{-6} = 348.2(10^{-6})$

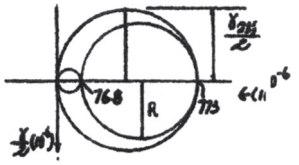
a)

$\epsilon_1 = (425 + 348.2)(10^{-6}) = 773(10^{-6})$ **Ans.**

$\epsilon_2 = (425 - 348.2)(10^{-6}) = 76.8(10^{-6})$ **Ans.**

b)

$\gamma_{\substack{\max \\ \text{in-plane}}} = 2R = 2(348.2)(10^{-6}) = 696(10^{-6})$ **Ans.**



c)

$\dfrac{\gamma_{\substack{\max \\ \text{in-plane}}}}{2} = \dfrac{773(10^{-6})}{2}$; $\gamma_{\substack{\text{abs} \\ \max}} = 773(10^{-6})$ **Ans.**

Ans:
(a) $\epsilon_1 = 773(10^{-6})$, $\epsilon_2 = 76.8(10^{-6})$,
(b) $\gamma_{\substack{\max \\ \text{in-plane}}} = 696(10^{-6})$,
(c) $\gamma_{\substack{\text{abs} \\ \max}} = 773(10^{-6})$

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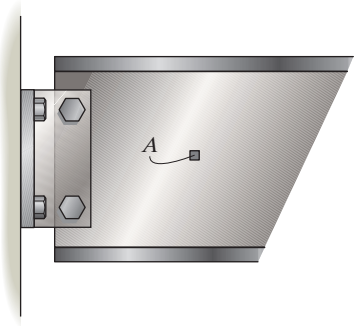
10–23.

The strain at point A on a beam has components $\epsilon_x = 450(10^{-6})$, $\epsilon_y = 825(10^{-6})$, $\gamma_{xy} = 275(10^{-6})$, $\epsilon_z = 0$. Determine (a) the principal strains at A, (b) the maximum shear strain in the x–y plane, and (c) the absolute maximum shear strain.

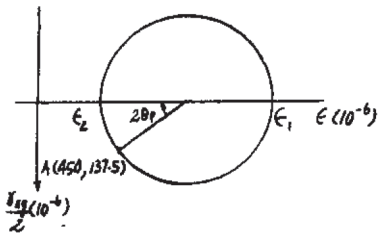


SOLUTION

$\epsilon_x = 450(10^{-6})$ $\epsilon_y = 825(10^{-6})$ $\gamma_{xy} = 275(10^{-6})$ $\dfrac{\gamma_{xy}}{2} = 137.5(10^{-6})$

$A(450, 137.5)10^{-6}$ $C(637.5, 0)10^{-6}$

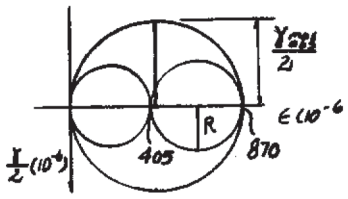
$R = [\sqrt{(637.5 - 450)^2 + 137.5^2}]10^{-6} = 232.51(10^{-6})$



a)

$\epsilon_1 = (637.5 + 232.51)(10^{-6}) = 870(10^{-6})$ **Ans.**

$\epsilon_2 = (637.5 - 232.51)(10^{-6}) = 405(10^{-6})$ **Ans.**



b)

$\gamma_{\text{in-plane}}^{\max} = 2R = 2(232.51)(10^{-6}) = 465(10^{-6})$ **Ans.**

c)

$\dfrac{\gamma_{\text{max}}^{\text{abs}}}{2} = \dfrac{870(10^{-6})}{2}$; $\gamma_{\text{max}}^{\text{abs}} = 870(10^{-6})$ **Ans.**

Ans:

(a) $\epsilon_1 = 870(10^{-6})$, $\epsilon_2 = 405(10^{-6})$,

(b) $\gamma_{\text{in-plane}}^{\max} = 465(10^{-6})$,

(c) $\gamma_{\text{max}}^{\text{abs}} = 870(10^{-6})$

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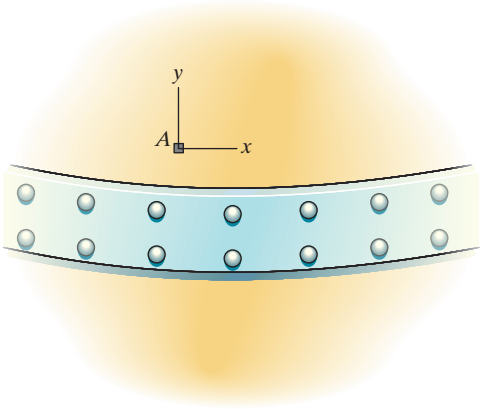
***10–24.**

The strain at point A on the pressure-vessel wall has components $\epsilon_x = 480(10^{-6})$, $\epsilon_y = 720(10^{-6})$, $\gamma_{xy} = 650(10^{-6})$. Determine (a) the principal strains at A, in the x–y plane, (b) the maximum shear strain in the x–y plane, and (c) the absolute maximum shear strain.



SOLUTION

$\epsilon_x = 480(10^{-6})$ $\epsilon_y = 720(10^{-6})$ $\gamma_{xy} = 650(10^{-6})$ $\dfrac{\gamma_{xy}}{2} = 325(10^{-6})$

$A(480, 325)10^{-6}$ $C(600, 0)10^{-6}$

$R = (\sqrt{(600 - 480)^2 + 325^2})10^{-6} = 346.44(10^{-6})$

a)

$\epsilon_1 = (600 + 346.44)10^{-6} = 946(10^{-6})$ **Ans.**

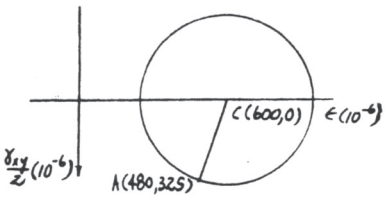
$\epsilon_2 = (600 - 346.44)10^{-6} = 254(10^{-6})$ **Ans.**

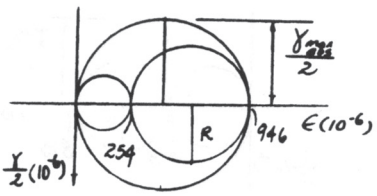
b)

$\gamma_{\substack{\max \\ \text{in-plane}}} = 2R = 2(346.44)10^{-6} = 693(10^{-6})$ **Ans.**

c)

$\dfrac{\gamma_{\substack{\text{abs} \\ \max}}}{2} = \dfrac{946(10^{-6})}{2}$; $\gamma_{\substack{\text{abs} \\ \max}} = 946(10^{-6})$ **Ans.**





Ans:
$\epsilon_1 = 946(10^{-6})$,
$\epsilon_2 = 254(10^{-6})$,
$\gamma_{\substack{\max \\ \text{in-plane}}} = 693(10^{-6})$,
$\gamma_{\substack{\text{abs} \\ \max}} = 946(10^{-6})$

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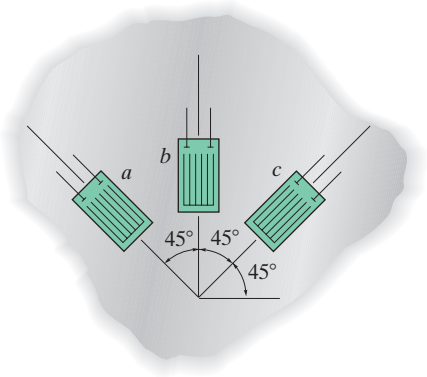
10–25.

The 45° strain rosette is mounted on the surface of a shell. The following readings are obtained for each gage: $\epsilon_a = -200(10^{-6})$, $\epsilon_b = 300(10^{-6})$, and $\epsilon_c = 250(10^{-6})$. Determine the in-plane principal strains.



SOLUTION

Strain Rosettes (45°): Applying the equation in the text with $\epsilon_a = -200(10^{-6})$, $\epsilon_b = 300(10^{-6})$, $\epsilon_c = 250(10^{-6})$, $\theta_a = 135°$, $\theta_b = 90°$ and $\theta_c = 45°$.

$$300(10^{-6}) = \epsilon_x \cos^2 90° + \epsilon_y \sin^2 90° + \gamma_{xy} \sin 90° \cos 90°$$

$$\epsilon_y = 300(10^{-6})$$

$$-200(10^{-6}) = \epsilon_x \cos^2 135° + 300(10^{-6}) \sin^2 135° + \gamma_{xy} \sin 135° \cos 135°$$

$$\epsilon_x - \gamma_{xy} = -700(10^{-6}) \quad \textbf{(1)}$$

$$250(10^{-6}) = \epsilon_x \cos^2 45° + 300(10^{-6}) \sin^2 45° + \gamma_{xy} \sin 45° \cos 45°$$

$$\epsilon_x + \gamma_{xy} = 200(10^{-6}) \quad \textbf{(2)}$$

Solving Eqs. (1) and (2)

$$\epsilon_x = -250(10^{-6}) \qquad \gamma_{xy} = 450(10^{-6})$$

Construction of The Circle: With $\epsilon_x = -250(10^{-6})$, $\epsilon_y = 300(10^{-6})$ and $\frac{\gamma_{xy}}{2} = 225(10^{-6})$,

$$\epsilon_{avg} = \frac{\epsilon_x + \epsilon_y}{2} = \left(\frac{-250 + 300}{2}\right)(10^{-6}) = 25.0(10^{-6})$$

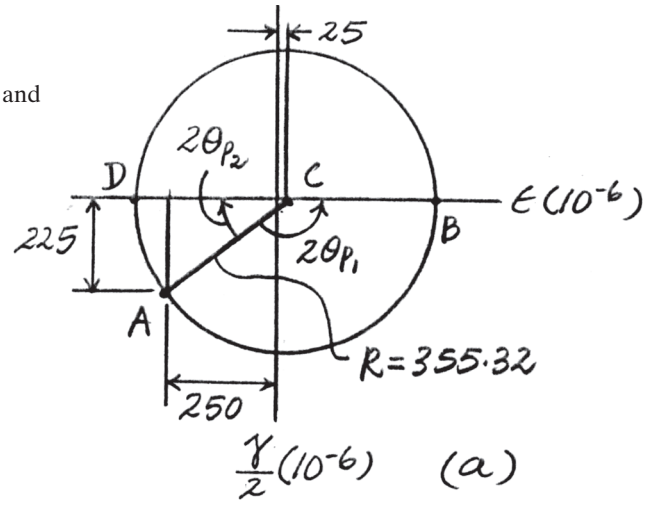
The coordinates of reference point A and center of circle C are

$$A(-250, 225)(10^{-6}) \qquad C(25.0, 0)(10^{-6})$$

The radius of the circle is

$$R = CA = \left[\sqrt{(-250 - 25)^2 + (225 - 0)^2}\right](10^{-6}) = 355.32(10^{-6})$$

Using these results, the circle shown in Fig. *a* can be constructed.



In-Plane Principal Strain: The coordinates of points B and D represent ϵ_1 and ϵ_2 respectively.

$$\epsilon_1 = (25.0 + 355.32)(10^{-6}) = 380.32(10^{-6}) = 380(10^{-6}) \quad \textbf{Ans.}$$

$$\epsilon_2 = (25.0 - 355.32)(10^{-6}) = -330.32(10^{-6}) = -330(10^{-6}) \quad \textbf{Ans.}$$

Ans:
$\epsilon_1 = 380(10^{-6})$,
$\epsilon_2 = -330(10^{-6})$

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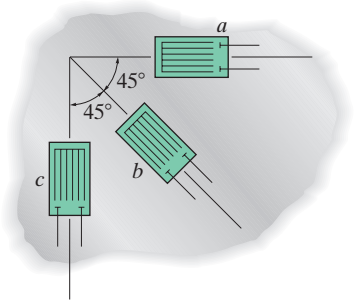
10–26.

The 45° strain rosette is mounted on the surface of a pressure vessel. The following readings are obtained for each gage: $\epsilon_a = 475(10^{-6})$, $\epsilon_b = 250(10^{-6})$, and $\epsilon_c = -360(10^{-6})$. Determine the in-plane principal strains.



SOLUTION

Strain Rosettes (45°): Applying the equations in the text with $\epsilon_a = 475(10^{-6})$, $\epsilon_b = 250(10^{-6})$, $\epsilon_c = -360(10^{-6})$, $\theta_a = 0°$, $\theta_b = -45°$, and $\theta_c = -90°$.

$$475(10^{-6}) = \epsilon_x \cos^2 0° + \epsilon_y \sin^2 0° + \gamma_{xy} \sin 0° \cos 0°$$

$$\epsilon_x = 475(10^{-6})$$

$$250(10^{-6}) = 475(10^{-6}) \cos^2(-45°) + \epsilon_y \sin^2(-45°) + \gamma_{xy} \sin(-45°) \cos(-45°)$$

$$250(10^{-6}) = 237.5(10^{-6}) + 0.5\,\epsilon_y - 0.5\,\gamma_{xy}$$

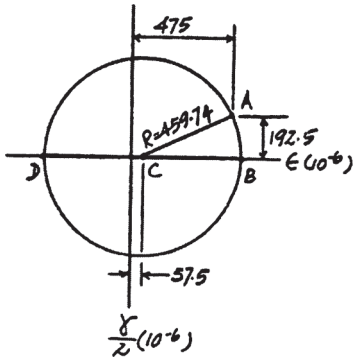
$$0.5\epsilon_y - 0.5\,\gamma_{xy} = 12.5(10^{-6}) \quad \textbf{(1)}$$

$$-360(10^{-6}) = 475(10^{-6}) \cos^2(-90°) + \epsilon_y \sin^2(-90°) + \gamma_{xy} \sin(-90°) \cos(-90°)$$

$$\epsilon_y = -360(10^{-6})$$

From Eq. (1), $\gamma_{xy} = -385(10^{-6})$

Therefore, $\epsilon_x = 475(10^{-6})$ $\epsilon_y = -360(10^{-6})$ $\gamma_{xy} = -385(10^{-6})$



Construction of the Circle: With $\frac{\gamma_{xy}}{2} = -192.5(10^{-6})$ and

$$\epsilon_{avg} = \frac{\epsilon_x + \epsilon_y}{2} = \left(\frac{475 + (-360)}{2}\right)(10^{-6}) = 57.5(10^{-6})$$

The coordinates for reference points A and C are

$$A(475, -192.5)(10^{-6}) \qquad C(57.5, 0)(10^{-6})$$

The radius of the circle is

$$R = \left(\sqrt{(475 - 57.5)^2 + 192.5^2}\right)(10^{-6}) = 459.74(10^{-6})$$

In-Plane Principal Strain: The coordinates of points B and D represent ϵ_1 and ϵ_2, respectively.

$$\epsilon_1 = (57.5 + 459.74)(10^{-6}) = 517(10^{-6}) \quad \textbf{Ans.}$$

$$\epsilon_2 = (57.5 - 459.74)(10^{-6}) = -402(10^{-6}) \quad \textbf{Ans.}$$

Ans:
$\epsilon_1 = 517(10^{-6}), \epsilon_2 = -402(10^{-6})$

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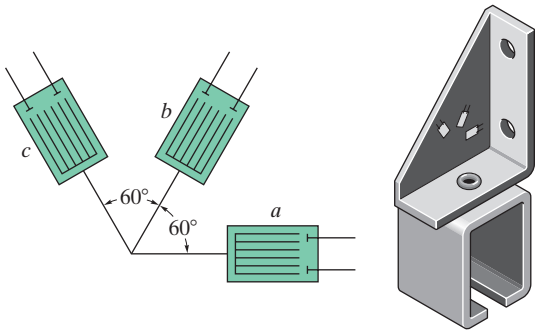
10–27.

The 60° strain rosette is mounted on the surface of the bracket. The following readings are obtained for each gage: $\epsilon_a = -780(10^{-6})$, $\epsilon_b = 400(10^{-6})$, and $\epsilon_c = 500(10^{-6})$. Determine (a) the principal strains and (b) the maximum in-plane shear strain and associated average normal strain. In each case show the deformed element due to these strains.



SOLUTION

Strain Rosettes (60°): Applying the equations in the text with $\epsilon_a = -780(10^{-6})$, $\epsilon_b = 400(10^{-6})$, $\epsilon_c = 500(10^{-6})$, $\theta_a = 0°$, $\theta_b = 60°$, and $\theta_c = 120°$,

$$\epsilon_x = \epsilon_a = -780(10^{-6})$$

$$\epsilon_y = \frac{1}{3}(2\epsilon_b + 2\epsilon_c - \epsilon_a)$$

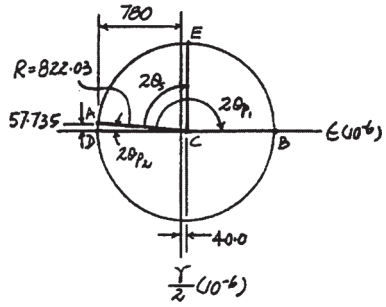
$$= \frac{1}{3}[2(400) + 2(500) - (-780)](10^{-6})$$

$$= 860(10^{-6})$$

$$\gamma_{xy} = \frac{2}{\sqrt{3}}(\epsilon_b - \epsilon_c)$$

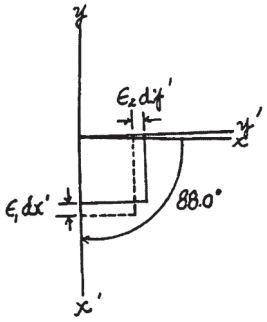
$$= \frac{2}{\sqrt{3}}(400 - 500)(10^{-6})$$

$$= -115.47(10^{-6})$$



Construction of the Circle: With $\epsilon_x = -780(10^{-6})$, $\epsilon_y = 860(10^{-6})$, and $\frac{\gamma_{xy}}{2} = -57.735(10^{-6})$.

$$\epsilon_{avg} = \frac{\epsilon_x + \epsilon_y}{2} = \left(\frac{-780 + 860}{2}\right)(10^{-6}) = 40.0(10^{-6})$$ **Ans.**



The coordinates for reference points A and C are

$$A(-780, -57.735)(10^{-6}) \qquad C(40.0, 0)(10^{-6})$$

The radius of the circle is

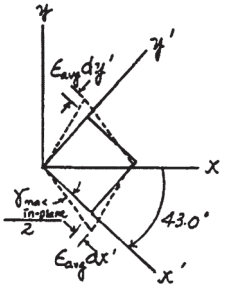
$$R = \left(\sqrt{(780 + 40.0)^2 + 57.735^2}\right)(10^{-6}) = 822.03(10^{-6})$$

a)

In-Plane Principal Strain: The coordinates of point B and D represent ϵ_1 and ϵ_2, respectively.

$$\epsilon_1 = (40.0 + 822.03)(10^{-6}) = 862(10^{-6})$$ **Ans.**

$$\epsilon_2 = (40.0 - 822.03)(10^{-6}) = -782(10^{-6})$$ **Ans.**



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10–27. Continued

Orientation of Principal Strain: From the circle,

$$\tan 2\theta_{p2} = \frac{57.735}{780 + 40} = 0.07041 \quad 2\theta_{p2} = 4.03°$$

$$2\theta_{p1} = 180° - 2\theta_{p2}$$

$$\theta_{p1} = \frac{180° - 4.03°}{2} = 88.0° \quad \textbf{(Clockwise)} \qquad \textbf{Ans.}$$

b)

Maximum In-Plane Shear Strain: Represented by the coordinates of point E on the circle.

$$\frac{\gamma_{\substack{\max \\ \text{in-plane}}}}{2} = -R = -822.03\left(10^{-6}\right)$$

$$\gamma_{\substack{\max \\ \text{in-plane}}} = -1644\left(10^{-6}\right) \qquad \textbf{Ans.}$$

Orientation of Maximum In-Plane Shear Strain: From the circle,

$$\tan 2\theta_s = \frac{780 + 40}{57.735} = 14.2028$$

$$\theta_s = 43.0° \quad \textbf{(Clockwise)} \qquad \textbf{Ans.}$$

Ans:
$\epsilon_{\text{avg}} = 40.0\left(10^{-6}\right)$,
$\epsilon_1 = 862\left(10^{-6}\right)$,
$\epsilon_2 = -782\left(10^{-6}\right)$,
$\theta_{p1} = 88.0°$ **(Clockwise)**,
$\gamma_{\substack{\max \\ \text{in-plane}}} = -1644\left(10^{-6}\right)$,
$\theta_s = 43.0°$ **(Clockwise)**

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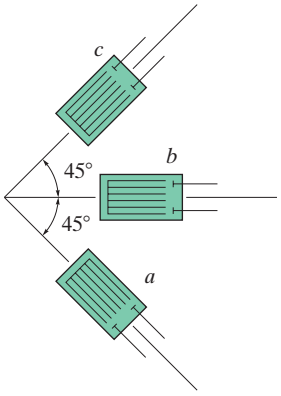
***10–28.**

The 45° strain rosette is mounted on a steel shaft. The following readings are obtained from each gage: $\epsilon_a = 800(10^{-6})$, $\epsilon_b = 520(10^{-6})$, $\epsilon_c = -450(10^{-6})$. Determine the in-plane principal strains.



SOLUTION

$\epsilon_a = 800(10^{-6})$ $\epsilon_b = 520(10^{-6})$ $\epsilon_c = -450(10^{-6})$

$\theta_a = -45°$ $\theta_b = 0°$ $\theta_c = 45°$

$$\epsilon_b = \epsilon_x \cos^2\theta_b + \epsilon_y \sin^2\theta_b + \gamma_{xy} \sin\theta_b \cos\theta_b$$

$$520(10^{-6}) = \epsilon_x \cos^2 0° + \epsilon_y \sin^2 0° + \gamma_{xy} \sin 0° \cos 0°$$

$$\epsilon_a = 520(10^{-6})$$

$$\epsilon_a = \epsilon_x \cos^2\theta_a + \epsilon_y \sin^2\theta_a + \gamma_{xy} \sin\theta_a \cos\theta_a$$

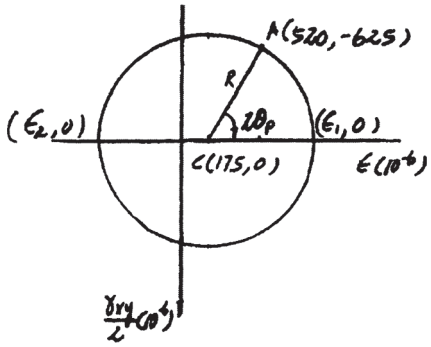
$$800(10^{-6}) = \epsilon_x \cos^2(-45°) + \epsilon_y \sin^2(-45°) + \gamma_{xy} \sin(-45°) \cos(-45°)$$

$$800(10^{-6}) = 0.5\epsilon_x + 0.5\epsilon_y - 0.5\gamma_{xy} \quad \textbf{(1)}$$

$$\epsilon_c = \epsilon_x \cos^2\theta_c + \epsilon_y \sin^2\theta_c + \gamma_{xy} \sin\theta_c \cos\theta_c$$

$$-450(10^{-6}) = \epsilon_x \cos^2 45° + \epsilon_y \sin^2 45° + \gamma_{xy} \sin 45° \cos 45°$$

$$-450(10^{-6}) = 0.5\epsilon_x + 0.5\epsilon_y + 0.5\gamma_{xy} \quad \textbf{(2)}$$



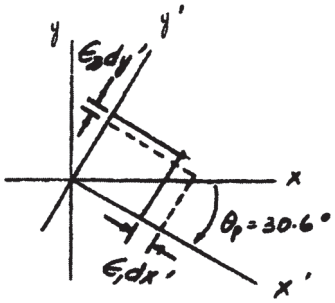
Subtract Eq. (2) from Eq. (1)

$$1250(10^{-6}) = -\gamma_{xy}$$

$$\gamma_{xy} = -1250(10^{-6})$$

$$\epsilon_y = -170(10^{-6})$$

$$\frac{\gamma_{xy}}{2} = -625(10^{-6})$$



$A(520, -625)10^{-6}$ $C(175, 0)10^{-6}$

$$R = \left[\sqrt{(520 - 175)^2 + 625^2}\right]10^{-6} = 713.90(10^{-6})$$

$$\epsilon_1 = (175 + 713.9)10^{-6} = 889(10^{-6}) \quad \textbf{Ans.}$$

$$\epsilon_2 = (175 - 713.9)10^{-6} = -539(10^{-6}) \quad \textbf{Ans.}$$

Ans:
$\epsilon_1 = 889(10^{-6})$,
$\epsilon_2 = -539(10^{-6})$

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10–30.

For the case of plane stress, show that Hooke's law can be written as

$$\sigma_x = \frac{E}{(1-\nu^2)}(\epsilon_x + \nu\epsilon_y), \quad \sigma_y = \frac{E}{(1-\nu^2)}(\epsilon_y + \nu\epsilon_x)$$

SOLUTION

Generalized Hooke's Law: For plane stress, $\sigma_z = 0$. Applying Eq. 10–18,

$$\epsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y)$$

$$\nu E\epsilon_x = (\sigma_x - \nu\sigma_y)\nu$$

$$\nu E\epsilon_x = \nu\sigma_x - \nu^2\sigma_y \qquad \textbf{(1)}$$

$$\epsilon_y = \frac{1}{E}(\sigma_y - \nu\sigma_x)$$

$$E\epsilon_y = -\nu\sigma_x + \sigma_y \qquad \textbf{(2)}$$

Adding Eq. (1) and Eq. (2) yields.

$$\nu E\epsilon_x + E\epsilon_y = \sigma_y - \nu^2\sigma_y$$

$$\sigma_y = \frac{E}{1-\nu^2}(\nu\epsilon_x + \epsilon_y) \qquad \textbf{\textit{(Q.E.D.)}}$$

Substituting σ_y into Eq. (2)

$$E\epsilon_y = -\nu\sigma_x + \frac{E}{1-\nu^2}(\nu\epsilon_x + \epsilon_y)$$

$$\sigma_x = \frac{E(\nu\epsilon_x + \epsilon_y)}{\nu(1-\nu^2)} - \frac{E\epsilon_y}{\nu}$$

$$= \frac{E\nu\epsilon_x + E\epsilon_y - E\epsilon_y + E\epsilon_y\nu^2}{\nu(1-\nu^2)}$$

$$= \frac{E}{1-\nu^2}(\epsilon_x + \nu\epsilon_y) \qquad \textbf{\textit{(Q.E.D.)}}$$

Ans:
N/A

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10–31.

Use Hooke's law, Eq. 10–18, to develop the strain tranformation equations, Eqs. 10–5 and 10–6, from the stress tranformation equations, Eqs. 9–1 and 9–2.

SOLUTION

Stress Transformation Equations:

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta \quad \textbf{(1)}$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta \quad \textbf{(2)}$$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta \quad \textbf{(3)}$$

Hooke's Law:

$$\epsilon_x = \frac{\sigma_x}{E} - \frac{\nu\,\sigma_y}{E} \quad \textbf{(4)}$$

$$\epsilon_y = \frac{-\nu\,\sigma_x}{E} + \frac{\sigma_y}{E} \quad \textbf{(5)}$$

$$\tau_{xy} = G\,\gamma_{xy} \quad \textbf{(6)}$$

$$G = \frac{E}{2(1+\nu)} \quad \textbf{(7)}$$

From Eqs. (4) and (5)

$$\epsilon_x + \epsilon_y = \frac{(1-\nu)(\sigma_x + \sigma_y)}{E} \quad \textbf{(8)}$$

$$\epsilon_x - \epsilon_y = \frac{(1+\nu)(\sigma_x - \sigma_y)}{E} \quad \textbf{(9)}$$

From Eqs. (6) and (7)

$$\tau_{xy} = \frac{E}{2(1+\nu)}\gamma_{xy} \quad \textbf{(10)}$$

From Eq. (4)

$$\epsilon_{x'} = \frac{\sigma_{x'}}{E} - \frac{\nu\,\sigma_{y'}}{E} \quad \textbf{(11)}$$

Substitute Eqs. (1) and (3) into Eq. (11)

$$\epsilon_{x'} = \frac{(1-\nu)(\sigma_x + \sigma_y)}{2\,E} + \frac{(1+\nu)(\sigma_x - \sigma_y)}{2\,E}\cos 2\theta + \frac{(1+\nu)\tau_{xy}\sin 2\theta}{E} \quad \textbf{(12)}$$

By using Eqs. (8), (9) and (10) and substitute into Eq. (12),

$$\epsilon_{x'} = \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta \quad \textbf{(Q.E.D.)}$$

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10–31. Continued

From Eq. (6).

$$\tau_{x'y'} = G\gamma_{x'y'} = \frac{E}{2(1 + \nu)}\gamma_{x'y'} \qquad \textbf{(13)}$$

Substitute Eqs. (13), (6) and (9) into Eq. (2),

$$\frac{E}{2(1 + \nu)}\gamma_{x'y'} = -\frac{E(\epsilon_x - \epsilon_y)}{2(1 + \nu)}\sin 2\theta + \frac{E}{2(1 + \nu)}\gamma_{xy}\cos 2\theta$$

$$\frac{\gamma_{x'y'}}{2} = -\frac{(\epsilon_x - \epsilon_y)}{2}\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta \qquad \textbf{\textit{(Q.E.D.)}}$$

Ans:
N/A

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***10–32.** A bar of copper alloy is loaded in a tension machine and it is determined that $\epsilon_x = 940(10^{-6})$ and $\sigma_x = 100$ MPa, $\sigma_y = 0$, $\sigma_z = 0$. Determine the modulus of elasticity, E_{cu}, and the dilatation, e_{cu}, of the copper. $\nu_{cu} = 0.35$.

SOLUTION

$$\varepsilon_x = \frac{1}{E}[\sigma_x - \nu(\sigma_y + \sigma_z)]$$

$$940(10^{-6}) = \frac{1}{E_{cu}}[100(10^6) - 0.35(0 + 0)]$$

$$E_{cu} = 106.38(10^9)\ \text{Pa} = 106\ \text{GPa}$$ **Ans.**

$$e_{cu} = \frac{1 - 2\nu}{E}(\sigma_x + \sigma_y + \sigma_z) = \frac{1 - 2(0.35)}{106.38(10^9)}[100(10^6) + 0 + 0] = 0.282(10^3)$$ **Ans.**

Ans.
$E_{cu} = 106$ GPa,
$e_{cu} = 0.282(10)^{-3}$

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10–33.

A rod has a radius of 10 mm. If it is subjected to an axial load of 15 N such that the axial strain in the rod is $\epsilon_x = 2.75(10^{-6})$, determine the modulus of elasticity E and the change in the rod's diameter. $\nu = 0.23$.

SOLUTION

$$\sigma_x = \frac{15}{\pi(0.01)^2} = 47.746\ \text{kPa}, \quad \sigma_y = 0, \quad \sigma_z = 0$$

$$\epsilon_x = \frac{1}{E}[\sigma_x - \nu(\sigma_y + \sigma_z)]$$

$$2.75(10^{-6}) = \frac{1}{E}[47.746(10^3) - 0.23(0 + 0)]$$

$$E = 17.4\ \text{GPa}$$ **Ans.**

$$\epsilon_y = \epsilon_z = -\nu\epsilon_x = -0.23(2.75)(10^{-6}) = -0.632(10^{-6})$$

$$\Delta d = 20(-0.632(10^{-6})) = -12.6(10^{-6})\ \text{mm}$$ **Ans.**

Ans:
$E = 17.4$ GPa, $\Delta d = -12.6(10^{-6})$ mm

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10–34. The principal strains at a point on the aluminum fuselage of a jet aircraft are $\epsilon_1 = 780(10^{-6})$ and $\epsilon_2 = 400(10^{-6})$. Determine the associated principal stresses at the point in the same plane. $E_{al} = 70$ GPa, $\nu_{al} = 0.33$. *Hint:* See Prob. 10–30.

SOLUTION

Plane stress, $\sigma_3 = 0$

See Prob 10-30,

$$\sigma_1 = \frac{E}{1 - \nu^2}(\varepsilon_1 + \nu\varepsilon_2)$$

$$= \frac{70(10^9)}{1-0.33^2}[780(10^{-6}) + 0.33(400)(10^{-6})] = 71.64(10^6)\text{ Pa} = 71.6\text{ MPa}$$ **Ans.**

$$\sigma_2 = \frac{E}{1 - \nu^2}(\varepsilon_2 + \nu\varepsilon_1)$$

$$= \frac{70(10^9)}{1-0.33^2}[400(10^{-6}) + 0.33(780)(10^{-6})] = 51.64(10^6)\text{ Pa} = 51.6\text{ MPa}$$ **Ans.**

Ans:

$\sigma_1 = 71.6$ MPa, $\sigma_2 = 51.6$ MPa

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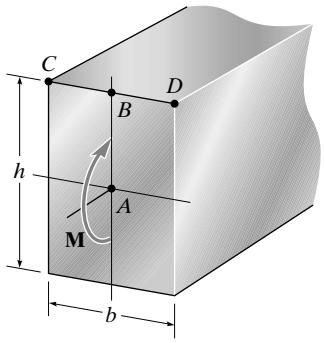
10–35. The cross section of the rectangular beam is subjected to the bending moment **M**. Determine an expression for the increase in length of lines AB and CD. The material has a modulus of elasticity E and Poisson's ratio is ν.



SOLUTION

For line AB,

$$\sigma_z = -\frac{My}{I} = \frac{My}{\frac{1}{12}b\,h^3} = -\frac{12My}{b\,h^3}$$

$$\varepsilon_y = -\frac{\nu\,\sigma_z}{E} = \frac{12\,\nu\,My}{E\,b\,h^3}$$

$$\Delta L_{AB} = \int_0^{\frac{h}{2}} \varepsilon_y\,dy = \frac{12\,\nu\,M}{E\,b\,h^3}\int_0^{\frac{h}{2}} y\,dy$$

$$= \frac{3\,\nu\,M}{2\,E\,b\,h}$$ **Ans.**

For line CD,

$$\sigma_z = -\frac{Mc}{I} = -\frac{M\frac{h}{2}}{\frac{1}{12}b\,h^3} = -\frac{6M}{bh^2}$$

$$\varepsilon_x = -\frac{\nu\,\sigma_z}{E} = \frac{6\,\nu\,M}{E\,b\,h^2}$$

$$\Delta L_{CD} = \varepsilon_x\,L_{CD} = \frac{6\,\nu\,M}{E\,b\,h^2}(b)$$

$$= \frac{6\,\nu\,M}{E\,h^2}$$ **Ans.**

Ans.

$$\sigma_z = -\frac{12\,My}{bh^3},\ \epsilon_y = \frac{12\,\nu My}{Ebh^3},$$
$$\Delta L_{AB} = \frac{3\,\nu M}{2Ebh},$$
$$\Delta L_{CD} = \frac{6\,\nu M}{Eh^2}$$

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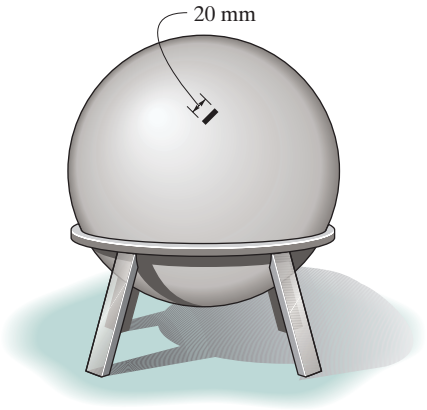
***10–36.**

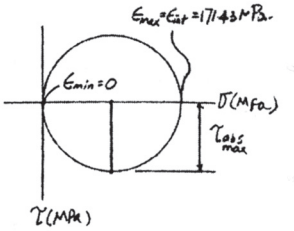
The spherical pressure vessel has an inner diameter of 2 m and a thickness of 10 mm. A strain gage having a length of 20 mm is attached to it, and it is observed to increase in length by 0.012 mm when the vessel is pressurized. Determine the pressure causing this deformation, and find the maximum in-plane shear stress, and the absolute maximum shear stress at a point on the outer surface of the vessel. The material is steel, for which $E_{st} = 200$ GPa and $\nu_{st} = 0.3$.



SOLUTION

Normal Stresses: Since $\frac{r}{t} = \frac{1000}{10} = 100 > 10$, the *thin wall* analysis is valid to determine the normal stress in the wall of the spherical vessel. This is a plane stress problem where $\sigma_{min} = 0$ since there is no load acting on the outer surface of the wall.

$$\sigma_{max} = \sigma_{lat} = \frac{pr}{2t} = \frac{p(1000)}{2(10)} = 50.0p \qquad \textbf{(1)}$$



Normal Strains: Applying the generalized Hooke's Law with

$$\epsilon_{max} = \epsilon_{lat} = \frac{0.012}{20} = 0.600\left(10^{-3}\right) \text{ mm/mm}$$

$$\epsilon_{max} = \frac{1}{E}\left[\sigma_{max} - \nu\left(\sigma_{lat} + \sigma_{min}\right)\right]$$

$$0.600\left(10^{-3}\right) = \frac{1}{200(10^9)}\left[50.0p - 0.3\left(50.0p + 0\right)\right]$$

$$p = 3.4286 \text{ MPa} = 3.43 \text{ MPa} \qquad \textbf{Ans.}$$

From Eq. (1) $\sigma_{max} = \sigma_{lat} = 50.0(3.4286) = 171.43$ MPa

Maximum In-Plane Shear (Sphere's Surface): Mohr's circle is simply a dot. As the result, the state of stress is the same, consisting of two normal stresses with zero shear stress regardless of the orientation of the element.

$$\tau_{\substack{\text{max} \\ \text{in-plane}}} = 0 \qquad \textbf{Ans.}$$

Absolute Maximum Shear Stress:

$$\tau_{\substack{\text{abs} \\ \text{max}}} = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{171.43 - 0}{2} = 85.7 \text{ MPa} \qquad \textbf{Ans.}$$

Ans:
$p = 3.43$ MPa
$\tau_{\substack{\text{max} \\ \text{in-plane}}} = 0$
$\tau_{\substack{\text{abs} \\ \text{max}}} = 85.7$ MPa

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10–37. Determine the bulk modulus for each of the following materials: (a) rubber, $E_r = 2.8$ MPa, $\nu_r = 0.48$, and (b) glass, $E_g = 56$ GPa, $\nu_g = 0.24$.

SOLUTION

a) For rubber:

$$K_r = \frac{E_r}{3\,(1 - 2\,\nu_r)} = \frac{2.8}{3[1 - 2(0.48)]} = 23.3 \text{ MPa}$$ **Ans.**

b) For glass:

$$K_g = \frac{E_g}{3\,(1 - 2\,\nu_g)} = \frac{56}{3[1 - 2(0.24)]} = 35.9 \text{ GPa}$$ **Ans.**

Ans.

(a) $K_r = 23.3$ MPa, (b) $K_g = 35.9$ GPa

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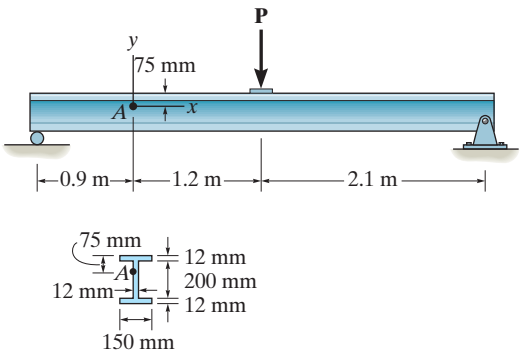
10–38. The strain in the x direction at point A on the steel beam is measured and found to be $\epsilon_x = -100(10^{-6})$. Determine the applied load P. What is the shear strain γ_{xy} at point A? $E_{st} = 200$ GPa, $\nu_{st} = 0.3$.



SOLUTION

$$I_x = \frac{1}{12}(0.15)(0.224^3) - \frac{1}{12}(0.138)(0.2^3) = 48.4928(10^{-6})\ \text{m}^4$$

$$Q_A = 0.106(0.15)(0.012) + 0.0685(0.012)(0.063) = 0.242586(10^{-3})\ \text{m}^3$$

$$\sigma = E\varepsilon_x = [200(10^9)][100(10^{-6})] = 20(10^6)\ \text{N/m}^2$$

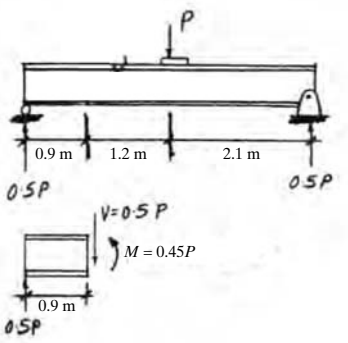
$$\sigma = \frac{My}{I}, \qquad 20(10^6) = \frac{(0.45P)(0.037)}{48.4928(10^{-6})}$$

$$P = 58.24(10^3)\ \text{N} = 58.2\ \text{kN}$$ **Ans.**

$$\tau_A = \frac{VQ}{It} = \frac{0.5[58.24(10^3)][0.242586(10^{-3})]}{[48.4928(10^{-6})](0.012)} = 12.14(10^6)\ \text{Pa}$$

$$G = \frac{E}{2(1+\nu)} = \frac{200}{2(1+0.3)} = 76.92\ \text{GPa}$$

$$\gamma_{xy} = \frac{\tau_{xy}}{G} = \frac{12.14(10^6)}{76.92(10^9)} = 0.1578(10^{-3})\ \text{rad} = 0.158(10^{-3})\ \text{rad}$$ **Ans.**



Ans.

$P = 58.2$ kN, $\gamma_{xy} = 0.158(10^{-3})$ rad

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10–39. The principal strains in a plane, measured experimentally at a point on the aluminum fuselage of a jet aircraft, are $\epsilon_1 = 630(10^{-6})$ and $\epsilon_2 = 350(10^{-6})$. If this is a case of plane stress, determine the associated principal stresses at the point in the same plane. $E_{al} = 70$ GPa and $\nu_{al} = 0.33$.

SOLUTION

Normal Stresses: For plane stress, $\sigma_3 = 0$.

Normal Strains: Applying the generalized Hooke's Law.

$$\varepsilon_1 = \frac{1}{E}\left[\sigma_1 - \nu\,(\sigma_2 + \sigma_3)\right]$$

$$630(10^{-6}) = \frac{1}{70(10^9)}[\sigma_1 - 0.33(\sigma_2 + 0)]$$

$$44.1(10^6) = \sigma_1 - 0.33\sigma_2 \qquad [1]$$

$$\varepsilon_2 = \frac{1}{E}\left[\sigma_2 - \nu\,(\sigma_1 + \sigma_3)\right]$$

$$350(10^{-6}) = \frac{1}{70(10^9)}[\sigma_2 - 0.33(\sigma_1 + 0)]$$

$$24.5(10^6) = \sigma_2 - 0.33\sigma_1 \qquad [2]$$

Solving Eqs.[1] and [2] yields:

$\sigma_1 = 58.56(10^6)$ Pa = 58.6 MPa $\qquad \sigma_2 = 43.83(10^6)$ Pa = 43.8 MPa **Ans.**

Ans:
$\sigma_1 = 58.6$ MPa, $\sigma_2 = 43.8$ MPa

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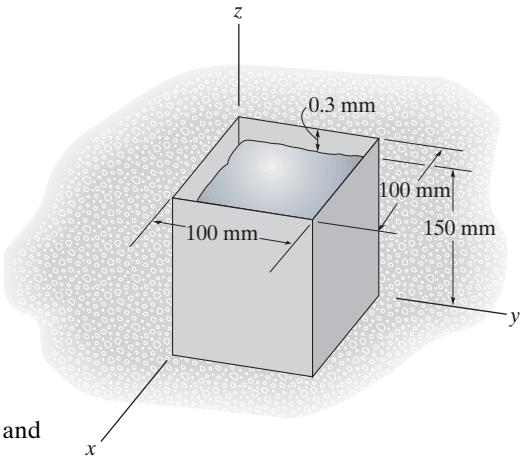
***10–40.** The smooth rigid-body cavity is filled with liquid 6061-T6 aluminum. When cooled it is 0.3 mm from the top of the cavity. If the top of the cavity is covered and the temperature is increased by 110°C, determine the stress components σ_x, σ_y, and σ_z in the aluminum. *Hint:* Use Eqs. 10–18 with an additional strain term of $\alpha\Delta T$ (Eq. 4–4).



SOLUTION

Normal Strains: Since the aluminum is confined at its sides by a rigid container and allowed to expand in the z direction, $\varepsilon_x = \varepsilon_y = 0$; whereas $\varepsilon_z = \dfrac{0.3}{150} = 0.002$. Applying the generalized Hooke's Law with the additional thermal strain,

$$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \nu(\sigma_y + \sigma_z)\right] + \alpha\Delta T$$

$$0 = \frac{1}{68.9(10^9)}[\sigma_x - 0.35(\sigma_y + \sigma_z)] + [24(10^{-6})](110)$$

$$0 = \sigma_x - 0.35\sigma_y - 0.35\sigma_z + 181.896(10^6) \qquad [1]$$

$$\varepsilon_y = \frac{1}{E}\left[\sigma_y - \nu(\sigma_x + \sigma_z) + \alpha\Delta T\right.$$

$$0 = \frac{1}{68.9(10^9)}[\sigma_y - 0.35(\sigma_x + \sigma_z)] + [24(10^{-6})](110)$$

$$0 = \sigma_y - 0.35\sigma_x - 0.35\sigma_z + 181.896(10^6) \qquad [2]$$

$$\varepsilon_z = \frac{1}{E}\left[\sigma_z - \nu\left(\sigma_x + \sigma_y\right)\right] + \alpha\Delta T$$

$$0.002 = \frac{1}{68.9(10^9)}[\sigma_z - 0.35(\sigma_x + \sigma_z)] + [24(10^{-6})](110)$$

$$0 = \sigma_z - 0.35\sigma_x - 0.35\sigma_z + 44.096(10^6) \qquad [3]$$

Solving Eqs.[1], [2] and [3] yields:

$$\sigma_x = \sigma_y = -487.23(10^6)\ \text{N/m}^2 = -487\ \text{MPa}$$ **Ans.**

$$\sigma_z = -385.16(10^6)\ \text{N/m}^2 = -385\ \text{MPa}$$ **Ans.**

Ans.
$\sigma_x = \sigma_y = -487$ MPa, $\sigma_z = -385$ MPa

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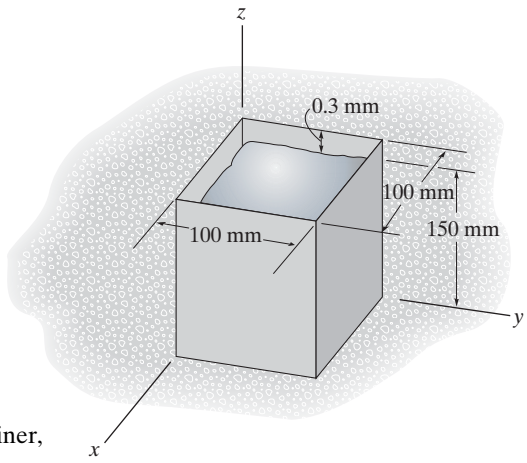
10–41. The smooth rigid-body cavity is filled with liquid 6061-T6 aluminum. When cooled it is 0.3 mm from the top of the cavity. If the top of the cavity is not covered and the temperature is increased by 110°C, determine the stress components ϵ_x, ϵ_y, and ϵ_z in the aluminum. *Hint:* Use Eqs. 10–18 with an additional strain term of $\alpha\Delta T$ (Eq. 4–4).



SOLUTION

Normal Strains: Since the aluminum is confined at its sides by a rigid container, then

$$\varepsilon_x = \varepsilon_y = 0 \qquad \textbf{Ans.}$$

and since it is not restrained in z *direction*, $\sigma_z = 0$. Applying the generalized Hooke's Law with the additional thermal strain,

$$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \nu\left(\sigma_y + \sigma_z\right)\right] + \alpha\Delta T$$

$$0 = \frac{1}{68.9(10^9)}[\sigma_x - 0.35(\sigma_y + \sigma_z)] + [24(10^{-6})](110)$$

$$0 = \sigma_x - 0.35\sigma_y - 0.35\sigma_z + 181.896(10^6) \qquad [1]$$

$$\varepsilon_y = \frac{1}{E}\left[\sigma_y - \nu(\sigma_x + \sigma_z)\right] + \alpha\Delta T$$

$$0 = \frac{1}{68.9(10^9)}[\sigma_y - 0.35(\sigma_x + \sigma_z)] + [24(10^{-6})](110)$$

$$0 = \sigma_y - 0.35\sigma_x - 0.35\sigma_z + 181.896(10^6) \qquad [2]$$

Solving Eqs. [1] and [2] yields:

$$\sigma_x = \sigma_y = -279.84(10^6)\ \text{N/m}^2$$

$$\varepsilon_z = \frac{1}{E}\left[\sigma_z - \nu\left(\sigma_x + \sigma_y\right)\right] + \alpha\Delta T$$

$$= \frac{1}{68.9(10^9)}\left\{0 - 0.35[-279.84(10^6) + (-279.84)(10^6)]\right\} + [24(10^{-6})](110)$$

$$= 5.483(10^{-3}) = 0.00548 \qquad \textbf{Ans.}$$

Ans.

$\epsilon_x = \epsilon_y = 0, \epsilon_z = 5.483(10^{-3})$

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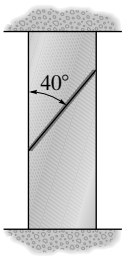
10–42. The block is fitted between the fixed supports. If the glued joint can resist a maximum shear stress of $\tau_{allow} = 14$ MPa, determine the temperature rise that will cause the joint to fail. Take $E = 70$ GPa, $\nu = 0.2$, and $\alpha = 11(10^{-6})/°C$.

Hint: Use Eq. 10–18 with an additional strain term of $\alpha\Delta T$ (Eq. 4–4).



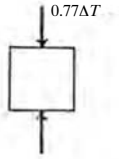
SOLUTION

Normal Strain: Since the block is confined along the *y direction* by the rigid frame, then $\varepsilon_y = 0$ and $\sigma_x = \sigma_z = 0$. Applying the generalized Hooke's Law with the additional thermal strain,

$$\varepsilon_y = \frac{1}{E}\left[\sigma_y - \nu(\sigma_x + \sigma_z)\right] + \alpha\Delta T$$

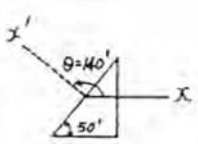
$$0 = \frac{1}{70(10^9)}[\sigma_y - 0.2(0+0)] + 11(10^{-6})(\Delta T)$$

$$\sigma_y = -770(10^3)\Delta T$$



Construction of the Circle: In accordance with the sign convention. $\sigma_x = 0$, $\sigma_y = -770(10^3)\Delta T$ and $\tau_{xy} = 0$. Hence.

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{0 + [-770(10^3)\Delta T]}{2} = -385(10^3)\Delta T$$

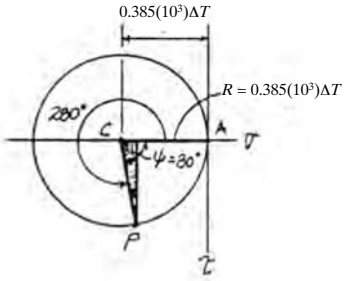


The coordinates for reference points A and C are $A(0, 0)$ and $C(-385(10^3)\Delta T, 0)$.

The radius of the circle is $R = \sqrt{\{0 - [-385(10^3)\Delta T]\}^2 + 0} = 385(10^3)\Delta T$

Stress on The inclined plane: The shear stress components $\tau_{x'y'}$, are represented by the coordinates of point P on the circle.

$$\tau_{x'y'} = 385(10^3)\Delta T \sin 80° = 379.15(10^3)\Delta T$$



Allowable Shear Stress:

$$\tau_{allow} = \tau_{x'y'}$$

$$14(10^6) = 379.15(10^3)\Delta T$$

$$\Delta T = 36.92° = 36.9°$$ **Ans.**

Ans.

$\Delta T = 36.92°C$

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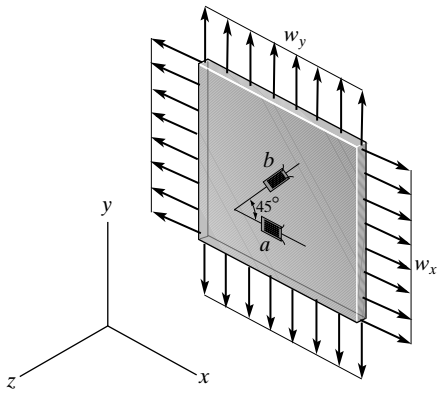
10–43. Two strain gauges a and b are attached to a plate made from a material having a modulus of elasticity of $E = 70$ GPa and Poisson's ratio $\nu = 0.35$. If the gauges give a reading of $\epsilon_a = 450(10^{-6})$ and $\epsilon_b = 100(10^{-6})$, determine the intensities of the uniform distributed load w_x and w_y acting on the plate. The thickness of the plate is 25 mm.



SOLUTION

Normal Strain: Since no shear force acts on the plane along the x and y axes, $\gamma_{xy} = 0$. With $\theta_a = 0$ and $\theta_b = 45°$, we have

$$\varepsilon_a = \varepsilon_x \cos^2 \theta_a + \varepsilon_y \sin^2 \theta_a + \gamma_{xy} \sin \theta_a \cos \theta_a$$

$$450\left(10^{-6}\right) = \varepsilon_x \cos^2 0° + \varepsilon_y \sin^2 0° + 0$$

$$\varepsilon_x = 450\left(10^{-6}\right)$$

$$\varepsilon_b = \varepsilon_x \cos^2 \theta_b + \varepsilon_y \sin^2 \theta_b + \gamma_{xy} \sin \theta_b \cos \theta_b$$

$$100\left(10^{-6}\right) = 450\left(10^{-6}\right) \cos^2 45° + \varepsilon_y \sin^2 45° + 0$$

$$\varepsilon_y = -250\left(10^{-6}\right)$$

Generalized Hooke's Law: This is a case of plane stress. Thus, $\sigma_z = 0$.

$$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \nu\left(\sigma_y + \sigma_z\right)\right]$$

$$450\left(10^{-6}\right) = \frac{1}{70\left(10^{9}\right)}\left[\sigma_y - 0.35\left(\sigma_y + 0\right)\right]$$

$$\sigma_x - 0.35\sigma_y = 31.5\left(10^{6}\right) \tag{1}$$

$$\varepsilon_y = \frac{1}{E}\left[\sigma_y - \nu\left(\sigma_x + \sigma_z\right)\right]$$

$$-250\left(10^{-6}\right) = \frac{1}{70\left(10^{9}\right)}\left[\sigma_y - 0.35\left(\sigma_y + 0\right)\right]$$

$$\sigma_y - 0.35\sigma_x = -17.5\left(10^{6}\right) \tag{2}$$

Solving Eqs. (1) and (2),

$$\sigma_y = -7.379\left(10^{6}\right)\text{N/m}^2 \qquad \sigma_x = 28.917\left(10^{6}\right)\text{N/m}^2$$

Then,

$$w_y = \sigma_y t = -7.379\left(10^{6}\right)(0.025) = -184 \text{ N/m} \quad \textbf{Ans.}$$

$$w_x = \sigma_x t = 28.917\left(10^{6}\right)(0.025) = 723 \text{ N/m} \quad \textbf{Ans.}$$

Ans.
$w_y = -184$ kN/m, $w_x = 723$ kN/m

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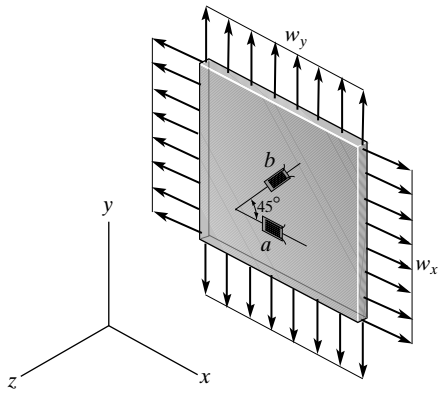
***10–44.** Two strain gauges a and b are attached to the surface of the plate which is subjected to the uniform distributed load $w_x = 700$ kN/m and $w_y = -175$ kN/m. If the gauges give a reading of $\varepsilon_a = 450(10^{-6})$ and $\varepsilon_b = 100(10^{-6})$, determine the modulus of elasticity E, shear modulus G, and Poisson's ratio v for the material.



Normal Stress and Strain: The normal stresses along the x, y, and z axes are

$$\sigma_x = \frac{700(10^3)}{0.025} = 28(10^6)\,\text{N/m}^2$$

$$\sigma_y = -\frac{175(10^3)}{0.025} = -7(10^6)\,\text{N/m}^2$$

$$\sigma_z = 0 \text{ (plane stress)}$$

Since no shear force acts on the plane along the x and y axes, $\gamma_{xy} = 0$. With $\theta_a = 0°$ and $\theta_b = 45°$, we have

$$\varepsilon_a = \varepsilon_x \cos^2\theta_a + \varepsilon_y \sin^2\theta_a + \gamma_{xy} \sin\theta_a \cos\theta_a$$

$$450(10^{-6}) = \varepsilon_x \cos^2 0° + \varepsilon_y \sin^2 0° + 0$$

$$\varepsilon_x = 450(10^{-6})$$

$$\varepsilon_b = \varepsilon_x \cos^2\theta_b + \varepsilon_y \sin^2\theta_b + \gamma_{xy} \sin\theta_b \cos\theta_b$$

$$100(10^{-6}) = 450(10^{-6}) \cos^2 45° + \varepsilon_y \sin^2 45° + 0$$

$$\varepsilon_y = -250(10^{-6})$$

Generalized Hooke's Law:

$$\varepsilon_x = \frac{1}{E}\left[\sigma_x - v(\sigma_y + \sigma_z)\right]$$

$$450(10^{-6}) = \frac{1}{E}\left[28(10^6) - v\left[-7(10^6) + 0\right]\right]$$

$$450(10^{-6})E - 7(10^6)v = 28(10^6) \qquad (1)$$

$$\varepsilon_y = \frac{1}{E}\left[\sigma_y - v(\sigma_x + \sigma_z)\right]$$

$$-250(10^{-6}) = \frac{1}{E}\left\{-7(10^6) - v\left[28(10^6) + 0\right]\right\}$$

$$250(10^{-6})E - 28(10^6)v = 7(10^6) \qquad (2)$$

Solving Eqs. (1) and (2),

$$E = 67.74(10^9)\,\text{N/m}^2 = 67.7\,\text{GPa} \qquad \textbf{Ans.}$$

$$v = 0.3548 = 0.355 \qquad \textbf{Ans.}$$

Using the above results,

$$G = \frac{E}{2(1+v)} = \frac{67.74(10^9)}{2(1 + 0.3548)}$$

$$= 25.0(10^9)\,\text{N/m}^2 = 25.0\,\text{GPa} \qquad \textbf{Ans.}$$

Ans.
$E = 67.7$ GPa, $v = 0.355$,
$G = 25.0$ GPa

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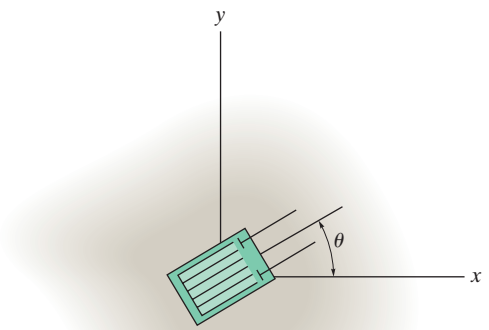
10–45.

A material is subjected to principal stresses σ_x and σ_y. Determine the orientation θ of the strain gage so that its reading of normal strain responds only to σ_y and not σ_x. The material constants are E and ν.



SOLUTION

$$\sigma_x = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

Since $\tau_{xy} = 0$,

$$\sigma_n = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta$$

$$\cos 2\theta = 2\cos^2\theta - 1$$

$$\sigma_n = \frac{\sigma_x}{2} + \frac{\sigma_y}{2} + (\sigma_x - \sigma_y)\cos^2\theta - \frac{\sigma_x}{2} + \frac{\sigma_y}{2}$$

$$= \sigma_y(1 - \cos^2\theta) + \sigma_x\cos^2\theta$$

$$= \sigma_x\cos^2\theta + \sigma_y\sin^2\theta$$

$$\sigma_{n+90^\circ} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta$$

$$= \frac{\sigma_x}{2} + \frac{\sigma_y}{2} - \left(\frac{\sigma_x - \sigma_y}{2}\right)(2\cos^2\theta - 1)$$

$$= \frac{\sigma_x}{2} + \frac{\sigma_y}{2} - (\sigma_x - \sigma_y)\cos^2\theta + \frac{\sigma_x}{2} - \frac{\sigma_y}{2}$$

$$= \sigma_x(1 - \cos^2\theta) + \sigma_y\cos^2\theta$$

$$= \sigma_x\sin^2\theta + \sigma_y\cos^2\theta$$

$$\epsilon_n = \frac{1}{E}(\sigma_n - \nu\,\sigma_{n+90^\circ})$$

$$= \frac{1}{E}(\sigma_x\cos^2\theta + \sigma_y\sin^2\theta - \nu\,\sigma_x\sin^2\theta - \nu\,\sigma_y\cos^2\theta)$$

If ϵ_n is to be independent of σ_x, then

$$\cos^2\theta - \nu\sin^2\theta = 0 \qquad \text{or} \qquad \tan^2\theta = 1/\nu$$

$$\theta = \tan^{-1}\left(\frac{1}{\sqrt{\nu}}\right)$$ **Ans.**

Ans:

$$\theta = \tan^{-1}\left(\frac{1}{\sqrt{\nu}}\right)$$

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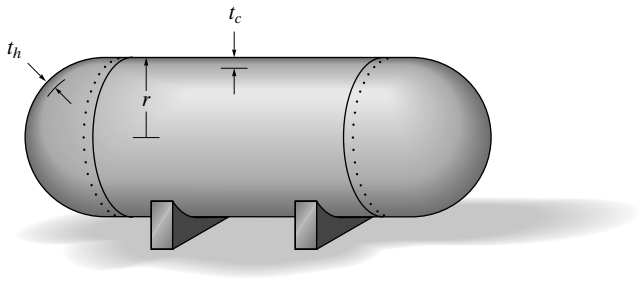
10–46. The cylindrical pressure vessel is fabricated using hemispherical end caps in order to reduce the bending stress that would occur if flat ends were used. The bending stresses at the seam where the caps are attached can be eliminated by proper choice of the thickness t_h and t_c of the caps and cylinder, respectively. This requires the radial expansion to be the same for both the hemispheres and cylinder. Show that this ratio is $t_c/t_h = (2 - \nu)/(1 - \nu)$. Assume that the vessel is made of the same material and both the cylinder and hemispheres have the same inner radius. If the cylinder is to have a thickness of 12 mm, what is the required of the hemispheres? Take $\nu = 0.3$.



SOLUTION

For cylindrical vessel:

$$\sigma_1 = \frac{p\,r}{t_c}; \qquad \sigma_2 = \frac{p\,r}{2\,t_c}$$

$$\varepsilon_1 = \frac{1}{E}\left[\sigma_1 - \nu\,(\sigma_2 + \sigma_3)\right] \qquad \sigma_3 = 0$$

$$= \frac{1}{E}\left(\frac{p\,r}{t_c} - \frac{\nu\,p\,r}{2\,t_c}\right) = \frac{p\,r}{E\,t_c}\left(1 - \frac{1}{2}\nu\right)$$

$$d\,r = \varepsilon_1\,r = \frac{p\,r2}{E\,t_c}\left(1 - \frac{1}{2}\nu\right) \qquad (1)$$

For hemispherical end caps:

$$\sigma_1 = \sigma_2 = \frac{p\,r}{2\,t_h}$$

$$\varepsilon_1 = \frac{1}{E}\left[\sigma_1 - \nu\,(\sigma_2 + \sigma_3)\right]; \qquad \sigma_3 = 0$$

$$= \frac{1}{E}\left(\frac{p\,r}{2\,t_h} - \frac{\nu\,p\,r}{2\,t_h}\right) = \frac{p\,r}{2\,E\,t_h}(1 - \nu)$$

$$d\,r = \varepsilon_1\,r = \frac{p\,r^2}{2\,E\,t_h}(1 - \nu) \qquad (2)$$

Equate Eqs. (1) and (2):

$$\frac{p\,r^2}{E\,t_c}\left(1 - \frac{1}{2}\nu\right) = \frac{p\,r^2}{2\,E\,t_h}(1 - \nu)$$

$$\frac{t_c}{t_h} = \frac{2\,(1 - \frac{1}{2}\nu)}{1 - \nu} = \frac{2 - \nu}{1 - \nu} \qquad \textbf{QED}$$

$$t_h = \frac{(1 - \nu)\,t_c}{2 - \nu} = \frac{(1 - 0.3)\,(12)}{2 - 0.3} = 4.94 \text{ mm} \qquad \textbf{Ans.}$$

Ans.
$t_h = 4.94$ mm

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10–47. A thin-walled cylindrical pressure vessel has an inner radius r, thickness t, and length L. If it is subjected to an internal pressure p, show that the increase in its inner radius is $dr = r\epsilon_1 = pr^2(1 - \frac{1}{2}\nu)/Et$ and the increase in its length is $\Delta L = pLr(\frac{1}{2} - \nu)/Et$. Using these results, show that the change in internal volume becomes $dV = \pi r^2(1 + \epsilon_1)^2(1 + \epsilon_2)L - \pi r^2 L$. Since ϵ_1 and ϵ_2 are small quantities, show further that the change in volume per unit volume, called *volumetric strain*, can be written as $dV/V = pr(2.5 - 2\nu)/Et$.

SOLUTION

Normal stress:

$$\sigma_1 = \frac{p\,r}{t}; \qquad \sigma_2 = \frac{p\,r}{2\,t}$$

Normal strain: Applying Hooke's law

$$\varepsilon_1 = \frac{1}{E}\left[\sigma_1 - \nu\left(\sigma_2 + \sigma_3\right)\right], \qquad \sigma_3 = 0$$

$$= \frac{1}{E}\left(\frac{p\,r}{t} - \frac{\nu\,p r}{2\,t}\right) = \frac{p\,r}{E\,t}\left(1 - \frac{1}{2}\,\nu\right)$$

$$d\,r = \varepsilon_t\,r = \frac{p\,r^2}{E\,t}\left(1 - \frac{1}{2}\,\nu\right) \qquad \textbf{QED}$$

$$\varepsilon_2 = \frac{1}{E}\left[\sigma_2 - \nu\left(\sigma_1 + \sigma_3\right)\right], \qquad \sigma_3 = 0$$

$$= \frac{1}{E}\left(\frac{p\,r}{2\,t} - \frac{\nu\,p r}{t}\right) = \frac{p\,r}{E\,t}\left(\frac{1}{2} - \nu\right)$$

$$\Delta L = \varepsilon_2\,L = \frac{p\,L\,r}{E\,t}\left(\frac{1}{2} - \nu\right) \qquad \textbf{QED}$$

$$V' = \pi(r + \varepsilon_1\,r)^2\,(L + \varepsilon_2 L)\,; \qquad V = \pi\,r^2\,L$$

$$dV = V' - V = \pi r^2\,(1 + \varepsilon_1)^2\,(1 + \varepsilon_2)L - \pi r^2\,L \qquad \textbf{QED}$$

$(1 + \varepsilon_1)^2 = 1 + 2\,\varepsilon_1$ neglect ε_1^2 term

$(1 + \varepsilon_1)^2\,(1 + \varepsilon_2) = (1 + 2\,\varepsilon_1)(1 + \varepsilon_2) = 1 + \varepsilon_2 + 2\,\varepsilon_1$ neglect $\varepsilon_1\varepsilon_2$ term

$$\frac{dV}{V} = 1 + \varepsilon_2 + 2\,\varepsilon_1 - 1 = \varepsilon_2 + 2\,\varepsilon_1$$

$$= \frac{p\,r}{E\,t}\left(\frac{1}{2} - \nu\right) + \frac{2\,p r}{E\,t}\left(1 - \frac{1}{2}\,\nu\right)$$

$$= \frac{p r}{E t}\,(2.5 - 2\,\nu) \qquad \textbf{QED}$$

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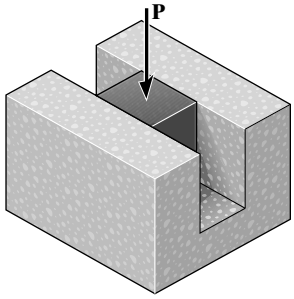
10–48. The rubber block is confined in the U-shape smooth rigid block. If the rubber has a modulus of elasticity E and Poisson's ratio ν, determine the effective modulus of elasticity of the rubber under the confined condition.



SOLUTION

Generalized Hooke's Law: Under this confined condition, $\varepsilon_x = 0$ and $\sigma_y = 0$. We have

$$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \nu\left(\sigma_y + \sigma_z\right)\right]$$

$$0 = \frac{1}{E}\left(\sigma_x - \nu\sigma_z\right)$$

$$\sigma_x = \nu\sigma_z \tag{1}$$

$$\varepsilon_z = \frac{1}{E}\left[\sigma_z - \nu\left(\sigma_x + \sigma_y\right)\right]$$

$$\varepsilon_z = \frac{1}{E}\left[\sigma_z - \nu(\sigma_x + 0)\right]$$

$$\varepsilon_z = \frac{1}{E}\left(\sigma_z - \nu\sigma_x\right) \tag{2}$$

Substituting Eq. (1) into Eq. (2),

$$\varepsilon_z = \frac{\sigma_z}{E}\left(1 - \nu^2\right)$$

The effective modulus of elasticity of the rubber block under the confined condition can be determined by considering the rubber block as unconfined but rather undergoing the same normal strain of ε_z when it is subjected to the same normal stress σ_z, Thus,

$$\sigma_z = E_{\text{eff}}\,\varepsilon_z$$

$$E_{\text{eff}} = \frac{\sigma_z}{\varepsilon_z} = \frac{\sigma_z}{\frac{\sigma_z}{E}\left(1 - \nu^2\right)} = \frac{E}{1 - \nu^2} \qquad \textbf{Ans.}$$

Ans.

$$E_{\text{eff}} = \frac{E}{1 - \nu^2}$$

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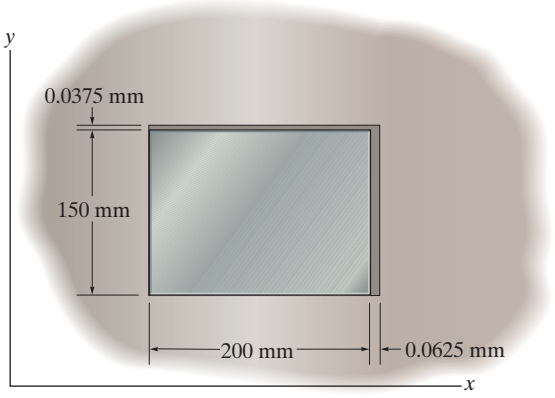
10–49. Initially, gaps between the A-36 steel plate and the rigid constraint are as shown. Determine the normal stresses σ_x and σ_y developed in the plate if the temperature is increased by $\Delta T = 55°\text{C}$. To solve, add the thermal strain $\alpha\Delta T$ to the equations for Hooke's Law.



SOLUTION

Generalized Hooke's Law: Since there are gaps between the sides of the plate and the rigid constraint, the plate is allowed to expand before it comes in contact with the constraint. Thus, $\varepsilon_x = \frac{\delta_x}{L_x} = \frac{0.0625}{200} = 0.3125\left(10^{-3}\right)$ and $\varepsilon_y = \frac{\delta_y}{L_y} = \frac{0.0375}{150} = 0.25\left(10^{-3}\right)$. However, the plate is allowed to have free expansion along the z direction. Thus, $\sigma_z = 0$. With the additional thermal strain term, we have

$$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \nu\left(\sigma_y + \sigma_z\right)\right] + \alpha\Delta T$$

$$0.3125(10^{-3}) = \frac{1}{200(10^9)}[\sigma_x - 0.32(\sigma_y + 0)] + [12(10^{-6})](55)$$

$$\sigma_x - 0.32\sigma_y = -69.5(10^6) \qquad (1)$$

$$\varepsilon_y = \frac{1}{E}\left[\sigma_y - \nu\left(\sigma_x + \sigma_z\right)\right] + \alpha\Delta T$$

$$0.25(10^{-3}) = \frac{1}{200(10^9)}[\sigma_y - 0.32(\sigma_x + 0)] + [12(10^{-6})](55)$$

$$\sigma_y - 0.32\sigma_x = -82(10^6) \qquad (2)$$

Solving Eqs. (1) and (2),

$$\sigma_x = -106.66(10^6)\ \text{N/m}^2 = 107\ \text{MPa (C)} \qquad \textbf{Ans.}$$

$$\sigma_y = -116.13(10^6)\ \text{N/m}^2 = 116\ \text{MPa (C)} \qquad \textbf{Ans.}$$

Since $\sigma_x < \sigma_Y$ and $\sigma_y < \sigma_Y$, the above results are valid.

Ans.
$\sigma_x = 107$ MPa (C), $\sigma_y = 116.1$ MPa (C)

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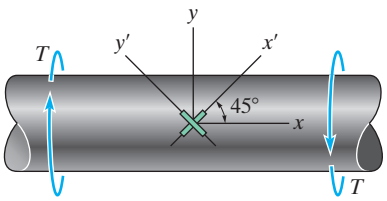
10–50.

The steel shaft has a radius of 15 mm. Determine the torque T in the shaft if the two strain gages, attached to the surface of the shaft, report strains of $\epsilon_{x'} = -80(10^{-6})$ and $\epsilon_{y'} = 80(10^{-6})$. Also, determine the strains acting in the x and y directions. $E_{st} = 200$ GPa, $\nu_{st} = 0.3$.



SOLUTION

$\epsilon_{x'} = -80(10^{-6})$ $\quad$ $\epsilon_{y'} = 80(10^{-6})$

Pure shear $\epsilon_x = \epsilon_y = 0$ **Ans.**

$\epsilon_{x'} = \epsilon_x \cos^2\theta + \epsilon_y \sin^2\theta + \gamma_{xy} \sin\theta \cos\theta$

$\theta = 45°$

$-80(10^{-6}) = 0 + 0 + \gamma_{xy} \sin 45° \cos 45°$

$\gamma_{xy} = -160(10^{-6})$ **Ans.**

Also, $\theta = 135°$

$80(10^{-6}) = 0 + 0 + \gamma \sin 135° \cos 135°$

$\gamma_{xy} = -160(10^{-6})$

$$G = \frac{E}{2(1+\nu)} = \frac{200(10^9)}{2(1+0.3)} = 76.923(10^9)$$

$\tau = G\gamma = 76.923(10^9)(160)(10^{-6}) = 12.308(10^6)$ Pa

$$T = \frac{\tau J}{c} = \frac{12.308(10^6)\left(\frac{\pi}{2}\right)(0.015)^4}{0.015} = 65.2\ \text{N}\cdot\text{m}$$ **Ans.**

Ans:
$\epsilon_x = \epsilon_y = 0$, $\gamma_{xy} = -160(10^{-6})$, $T = 65.2\ \text{N}\cdot\text{m}$

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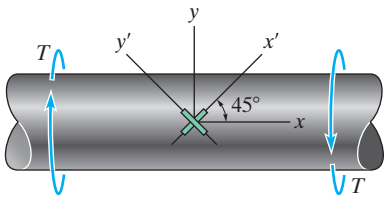
10–51.

The shaft has a radius of 15 mm and is made of L2 tool steel. Determine the strains in the x' and y' direction if a torque $T = 2\ \text{kN} \cdot \text{m}$ is applied to the shaft.



SOLUTION

$$\tau = \frac{Tc}{J} = \frac{2(10^3)(0.015)}{\frac{\pi}{2}(0.015^4)} = 377.26\ \text{MPa}$$

Stress-Strain Relationship:

$$\gamma_{xy} = \frac{\tau_{xy}}{G} = -\frac{377.26(10^6)}{75.0(10^9)} = -5.030(10^{-3})\ \text{rad}$$

This is a pure shear case, therefore,

$$\epsilon_x = \epsilon_y = 0$$

Applying Eq. 10–15,

$$\epsilon_{x'} = \epsilon_x \cos^2\theta_a + \epsilon_y \sin^2\theta_a + \gamma_{xy} \sin\theta_a \cos\theta_a$$

Here $\theta_a = 45°$

$$\epsilon_{x'} = 0 + 0 - 5.030(10^{-3}) \sin 45° \cos 45° = -2.52(10^{-3})$$

$$\epsilon_{x'} = -2.52(10^{-3}) \quad \textbf{Ans.}$$

$$\epsilon_{y'} = 2.52(10^{-3}) \quad \textbf{Ans.}$$

Ans:
$\varepsilon_{x'} = -2.52(10^{-3}),\ \varepsilon_{y'} = 2.52(10^{-3})$

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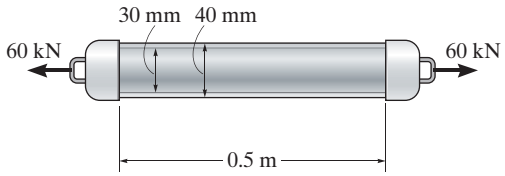
***10–52.**

The A-36 steel pipe is subjected to the axial loading of 60 kN. Determine the change in volume of the material after the load is applied.



SOLUTION

Normal Stress: The pipe is subjected to uniaxial load. Therefore,

$\sigma_v = \sigma_z = 0$ and $\sigma_x = \dfrac{N}{A}$.

Dilatation: Applying Eq. 10–23.

$$\frac{\delta V}{V} = \frac{1 - 2v}{E}(\sigma_x + \sigma_y + \sigma_z)$$

$$\frac{\delta V}{V} = \frac{1 - 2v}{E}\left(\frac{N}{A}\right)$$

$$\delta V = \frac{1 - 2v}{E}\left(\frac{N}{A}\right)V \qquad \text{However, } V = AL$$

$$\delta V = \left(\frac{1 - 2v}{E}\right)NL$$

$$= \left[\frac{1 - 2(0.32)}{200(10^9)}\right](60)(10^3)(0.5)$$

$$= 54.0\,(10^{-9})\ \text{m}^3 = 54.0\ \text{mm}^3 \qquad \textbf{Ans.}$$

Ans:
$\delta V = 54.0\ \text{mm}^3$

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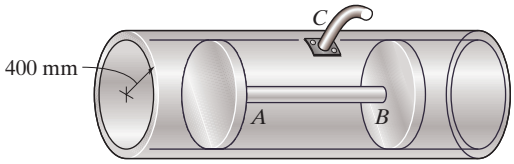
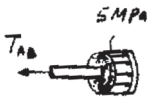
10–53.

Air is pumped into the steel thin-walled pressure vessel at C. If the ends of the vessel are closed using two pistons connected by a rod AB, determine the increase in the diameter of the pressure vessel when the internal gage pressure is 5 MPa. Also, what is the tensile stress in rod AB if it has a diameter of 100 mm? The inner radius of the vessel is 400 mm, and its thickness is 10 mm. $E_{st} = 200$ GPa and $\nu_{st} = 0.3$.



SOLUTION



Circumferential Stress:

$$\sigma = \frac{p\,r}{t} = \frac{5(400)}{10} = 200 \text{ MPa}$$

Note: longitudinal and radial stresses are zero.

Circumferential Strain:

$$\epsilon = \frac{\sigma}{E} = \frac{200(10^6)}{200(10^9)} = 1.0(10^{-3})$$

$$\Delta d = \epsilon\, d = 1.0(10^{-3})(800) = 0.800 \text{ mm}$$ **Ans.**

For rod AB:

$$\overset{+}{\leftarrow} \Sigma F_x = 0; \qquad T_{AB} - 5(10^6)\left(\frac{\pi}{4}\right)(0.8^2 - 0.1^2) = 0$$

$$T_{AB} = 2474 \text{ kN}$$

$$\sigma_{AB} = \frac{T_{AB}}{A_{AB}} = \frac{2474(10^3)}{\frac{\pi}{4}(0.1^2)} = 315 \text{ MPa}$$ **Ans.**

Ans:
$\Delta d = 0.800$ mm, $\sigma_{AB} = 315$ MPa

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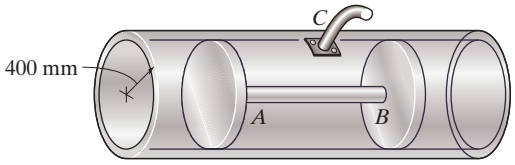
10–54.

Determine the increase in the diameter of the pressure vessel in Prob. 10–53 if the pistons are replaced by walls connected to the ends of the vessel.



SOLUTION

Principal Stress:

$$\sigma_1 = \frac{p\,r}{t} = \frac{5(400)}{10} = 200\ \text{MPa}; \qquad \sigma_3 = 0$$

$$\sigma_2 = \frac{1}{2}\,\sigma_1 = 100\ \text{MPa}$$

Circumferential Strain:

$$\epsilon_1 = \frac{1}{E}\left[\sigma_1 - \nu(\sigma_2 + \sigma_3)\right] = \frac{1}{200(10^9)}\left[200(10^6) - 0.3\{100(10^6) + 0\}\right]$$

$$= 0.85(10^{-3})$$

$$\Delta d = \epsilon_1\, d = 0.85(10^{-3})(800) = 0.680\ \text{mm}$$ **Ans.**

Ans:
$\Delta d = 0.680$ mm

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10–55.

A thin-walled spherical pressure vessel having an inner radius r and thickness t is subjected to an internal pressure p. Show that the increase in the volume within the vessel is $\Delta V = (2p\pi r^4/Et)(1 - \nu)$. Use a small-strain analysis.

SOLUTION

$$\sigma_1 = \sigma_2 = \frac{pr}{2t}$$

$$\sigma_3 = 0$$

$$\epsilon_1 = \epsilon_2 = \frac{1}{E}(\sigma_1 - \nu\sigma_2)$$

$$\epsilon_1 = \epsilon_2 = \frac{pr}{2tE}(1 - \nu)$$

$$\epsilon_3 = \frac{1}{E}(-\nu(\sigma_1 + \sigma_2))$$

$$\epsilon_3 = -\frac{\nu\, pr}{t\, E}$$

$$V = \frac{4\pi r^3}{3}$$

$$V + \Delta V = \frac{4\pi}{3}(r + \Delta r)^3 = \frac{4\pi r^3}{3}\left(1 + \frac{\Delta r}{r}\right)^3$$

where $\Delta V \ll V, \Delta r \ll r$

$$V + \Delta V = \frac{4\pi r^3}{3}\left(1 + 3\frac{\Delta r}{r}\right)$$

$$\epsilon_{\text{Vol}} = \frac{\Delta V}{V} = 3\left(\frac{\Delta r}{r}\right)$$

Since $\epsilon_1 = \epsilon_2 = \dfrac{2\pi(r + \Delta r) - 2\pi r}{2\pi r} = \dfrac{\Delta r}{r}$

$$\epsilon_{\text{Vol}} = 3\epsilon_1 = \frac{3pr}{2tE}(1 - \nu)$$

$$\Delta V = V\epsilon_{\text{Vol}} = \frac{2p\pi r^4}{Et}(1 - \nu) \qquad \textbf{(Q.E.D.)}$$

Ans:
N/A

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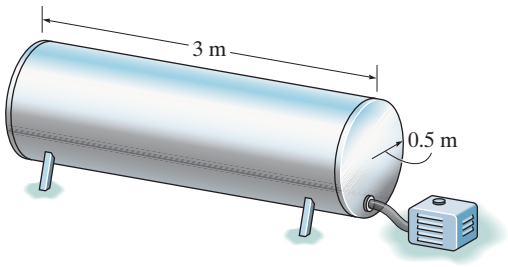
***10–56.**

The thin-walled cylindrical pressure vessel of inner radius r and thickness t is subjected to an internal pressure p. If the material constants are E and ν, determine the strains in the circumferential and longitudinal directions. Using these results, calculate the increase in both the diameter and the length of a steel pressure vessel filled with air and having an internal gage pressure of 15 MPa. The vessel is 3 m long, and has an inner radius of 0.5 m and a thickness of 10 mm. $E_{st} = 200$ GPa, $\nu_{st} = 0.3$.



SOLUTION

Normal Stress:

$$\sigma_1 = \frac{p\,r}{t} \qquad \sigma_2 = \frac{p\,r}{2\,t} \qquad \sigma_3 = 0$$

Normal Strain:

$$\epsilon_{cir} = \frac{1}{E}[\sigma_1 - \nu(\sigma_2 + \sigma_3)]$$

$$= \frac{1}{E}\left(\frac{p\,r}{t} - \frac{\nu\,p\,r}{2\,t}\right) = \frac{p\,r}{2\,E\,t}(2 - \nu) \qquad \textbf{Ans.}$$

$$\epsilon_{long} = \frac{1}{E}[\sigma_2 - \nu(\sigma_1 + \sigma_3)]$$

$$= \frac{1}{E}\left(\frac{p\,r}{2\,t} - \frac{\nu\,p\,r}{t}\right) = \frac{p\,r}{2\,E\,t}(1 - 2\nu) \qquad \textbf{Ans.}$$

Numerical Substitution:

$$\epsilon_{cir} = \frac{15(10^6)(0.5)}{2(200)(10^9)(0.01)}\,(2 - 0.3) = 3.1875\,(10^{-3})$$

$$\Delta d = \epsilon_{cir}\,d = 3.1875\,(10^{-3})(1000) = 3.19\text{ mm} \qquad \textbf{Ans.}$$

$$\epsilon_{long} = \frac{15(10^6)(0.5)}{2(200)(10^9)(0.01)}\,(1 - 2(0.3)) = 0.75(10^{-3})$$

$$\Delta L = \epsilon_{long}\,L = 0.75\,(10^{-3})(3000) = 2.25\text{ mm} \qquad \textbf{Ans.}$$

Ans:

$\epsilon_{cir} = \dfrac{p\,r}{2E\,t}(2 - \nu)$,

$\epsilon_{long} = \dfrac{p\,r}{2E\,t}(1 - 2\nu)$,

$\Delta d = 3.19$ mm,

$\Delta L = 2.25$ mm

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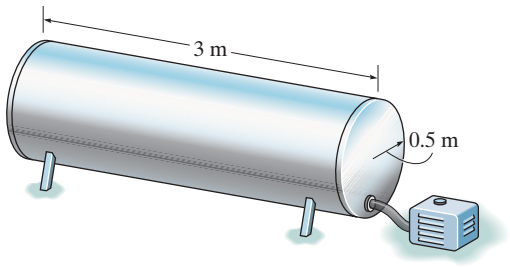
10–57.

Estimate the increase in volume of the pressure vessel in Prob. 10–56.



SOLUTION

By basic principles,

$$\Delta V = \pi(r + \Delta r)^2(L + \Delta L) - \pi r^2 L = \pi(r^2 + \Delta r^2 + 2r\,\Delta r)(L + \Delta L) - \pi r^2 L$$
$$= \pi(r^2 L + r^2\,\Delta L + \Delta r^2 L + \Delta r^2 \Delta L + 2r\,\Delta r\,L + 2r\,\Delta r\,\Delta L - r^2 L)$$
$$= \pi(r^2 \Delta L + \Delta r^2\,L + \Delta r^2 \Delta L + 2r\,\Delta r\,L + 2r\,\Delta r\,\Delta L)$$

Neglecting the second order terms,

$$\Delta V = \pi(r^2 \Delta L + 2r\,\Delta r\,L)$$

From Prob. 10–56,

$$\Delta L = 0.00225 \text{ m} \qquad \Delta r = \frac{\Delta d}{2} = 0.00159375 \text{ m}$$

$$\Delta V = \pi[(0.5^2)(0.00225) + 2(0.5)(0.00159375)(3)] = 0.0168 \text{ m}^3 \qquad \textbf{Ans.}$$

Ans:
$\Delta V = 0.0168 \text{ m}^3$

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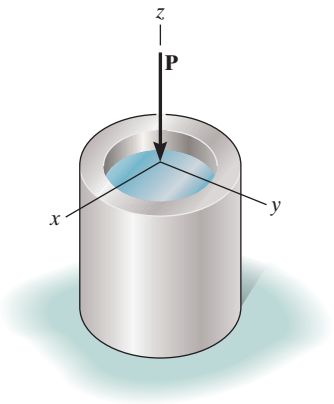
10–58.

A soft material is placed within the confines of a rigid cylinder which rests on a rigid support. Assuming that $\epsilon_x = 0$ and $\epsilon_y = 0$, determine the factor by which the stiffness of the material, or the apparent modulus of elasticity, will be increased when a load is applied, if $\nu = 0.3$ for the material.



SOLUTION

Normal Strain: Since the material is confined in a rigid cylinder. $\epsilon_x = \epsilon_y = 0$. Applying the generalized Hooke's Law,

$$\epsilon_x = \frac{1}{E}\left[\sigma_x - \nu(\sigma_y + \sigma_z)\right]$$

$$0 = \sigma_x - \nu(\sigma_y + \sigma_z) \qquad \textbf{(1)}$$

$$\epsilon_y = \frac{1}{E}\left[\sigma_y - \nu(\sigma_x + \sigma_z)\right]$$

$$0 = \sigma_y - \nu(\sigma_x + \sigma_z) \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields:

$$\sigma_x = \sigma_y = \frac{\nu}{1 - \nu}\sigma_z$$

Thus,

$$\epsilon_z = \frac{1}{E}\left[\sigma_z - \nu(\sigma_x + \sigma_y)\right]$$

$$= \frac{1}{E}\left[\sigma_z - \nu\left(\frac{\nu}{1 - \nu}\sigma_z + \frac{\nu}{1 - \nu}\sigma_z\right)\right]$$

$$= \frac{\sigma_z}{E}\left[1 - \frac{2\nu^2}{1 - \nu}\right]$$

$$= \frac{\sigma_z}{E}\left[\frac{1 - \nu - 2\nu^2}{1 - \nu}\right]$$

$$= \frac{\sigma_z}{E}\left[\frac{(1 + \nu)(1 - 2\nu)}{1 - \nu}\right]$$

Thus, when the material is not being confined and undergoes the same normal strain of ϵ_z, then the required modulus of elasticity is

$$E' = \frac{\sigma_z}{\epsilon_z} = \frac{1 - \nu}{(1 - 2\nu)(1 + \nu)}E$$

The increase factor is $k = \dfrac{E'}{E} = \dfrac{1 - \nu}{(1 - 2\nu)(1 + \nu)}$

$$= \frac{1 - 0.3}{[1 - 2(0.3)](1 + 0.3)}$$

$$= 1.35 \qquad \textbf{Ans.}$$

Ans:
$k = 1.35$

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10–59.

A material is subjected to plane stress. Express the distortion energy theory of failure in terms of σ_x, σ_y, and τ_{xy}.

SOLUTION

Maximum distortion energy theory:

$$(\sigma_1^2 - \sigma_1\,\sigma_2 + \sigma_2^2) = \sigma_Y^2 \qquad \textbf{(1)}$$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

Let $a = \dfrac{\sigma_x + \sigma_y}{2}$ and $b = \sqrt{\left(\dfrac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$$\sigma_1 = a + b; \qquad \sigma_2 = a - b$$

$$\sigma_1^2 = a^2 + b^2 + 2\,a\,b; \qquad \sigma_2^2 = a^2 + b^2 - 2\,a\,b$$

$$\sigma_1\sigma_2 = a^2 - b^2$$

From Eq. (1)

$$(a^2 + b^2 + 2\,a\,b - a^2 + b^2 + a^2 + b^2 - 2\,a\,b) = \sigma_Y^2$$

$$(a^2 + 3\,b^2) = \sigma_Y^2$$

$$\frac{(\sigma_x + \sigma_y)^2}{4} + 3\,\frac{(\sigma_x - \sigma_y)^2}{4} + 3\,\tau_{xy}^2 = \sigma_Y^2$$

$$\sigma_x^2 + \sigma_y^2 - \sigma_x\sigma_y + 3\,\tau_{xy}^2 = \sigma_Y^2 \qquad \textbf{Ans.}$$

Ans:
$\sigma_x^2 + \sigma_y^2 - \sigma_x\sigma_y + 3\tau_{xy}^2 = \sigma_y^2$

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***10–60.**

A material is subjected to plane stress. Express the maximum shear stress theory of failure in terms of σ_x, σ_y, and τ_{xy}. Assume that the principal stresses are of different algebraic signs.

SOLUTION

Maximum shear stress theory:

$$|\sigma_1 - \sigma_2| = \sigma_Y \qquad \textbf{(1)}$$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\left|\sigma_1 - \sigma_2\right| = 2\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

From Eq. (1)

$$4\left[\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2\right] = \sigma_Y^2$$

$$(\sigma_x - \sigma_y)^2 + 4\,\tau_{xy}^2 = \sigma_Y^2 \qquad \textbf{Ans.}$$

Ans:
$(\sigma_x - \sigma_y)^2 + 4\,\tau_{xy}^2 = \sigma_Y^2$

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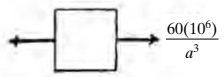
10–61. A bar with a square cross-sectional area is made of a material having a yield stress of $\sigma_Y = 840$ MPa. If the bar is subjected to a bending moment of 10 kN · m, determine the required size of the bar according to the maximum-distortion-energy theory. Use a factor of safety of 1.5 with respect to yielding.

SOLUTION

Normal and Shear Stress: Applying the flexure formula,

$$\sigma = \frac{Mc}{I} = \frac{[10(10^3)](a/2)}{\frac{1}{12}a^4} = \frac{60(10^3)}{a^3}$$

In-Plane Principal Stress: Since no shear stress acts on the element



$$\sigma_1 = \sigma_x = \frac{60(10^3)}{a^3} \qquad \sigma_2 = \sigma_y = 0$$

Maximum Distortion Energy Theory:

$$\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2 = \sigma_{\text{allow}}^2$$

$$\left[\frac{60(10^3)}{a^3}\right]^2 - 0 + 0 = \left[\frac{840(10^6)}{1.5}\right]^2$$

$a = 0.047495$ m $= 47.5$ mm **Ans.**

Ans.

$a = 47.5$ m

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10–62. Solve Prob. 10–64 using the maximum-shear-stress theory.

SOLUTION

Normal and Shear Stress: Applying the flexure formula,

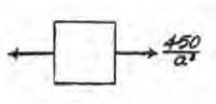
$$\sigma = \frac{Mc}{I} = \frac{[10(10^3)][(a/2)]}{\frac{1}{12}a^4} = \frac{60(10^3)}{a^3}$$

In-Plane Principal Stress: Since no shear stress acts on the element.

$$\sigma_1 = \sigma_x = \frac{60(10^3)}{a^3} \qquad \sigma_2 = \sigma_x = 0$$

Maximum Shear Stress Theory:

$$|\sigma_2| = 0 < \sigma_{\text{allow}} = \frac{840}{1.5} = 560 \text{ MPa} \qquad \textit{(O.K!)}$$



$$|\sigma_1| = \sigma_{\text{allow}}$$

$$\frac{60(10^3)}{a^3} = \frac{840(10^6)}{1.5}$$

$$a = 0.047495 \text{ m} = 47.5 \text{ mm} \qquad \textbf{Ans.}$$

Ans.

$$\sigma = \frac{60(10^6)}{a^3}, a = 47.50 \text{ mm}$$

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10–63. Derive an expression for an equivalent bending moment M_e that, if applied alone to a solid bar with a circular cross section, would cause the same energy of distortion as the combination of an applied bending moment M and torque T.

SOLUTION

Principal stresses:

$$\sigma_1 = \frac{M_e\, c}{I}; \qquad \sigma_2 = 0$$

$$u_d = \frac{1 + v}{3\,E}(\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2)$$

$$(u_d)_1 = \frac{1 + v}{3\,E}\left(\frac{M_e^2 c^2}{I^2}\right) \tag{1}$$

Principal stress:

$$\sigma_{1,2} = \frac{\sigma + 0}{2} \pm \sqrt{\left(\frac{\sigma - 0}{2}\right)^2 + \tau^3}$$

$$\sigma_1 = \frac{\sigma}{2} + \sqrt{\frac{\sigma^2}{4} + \tau^2}; \qquad \sigma_2 = \frac{\sigma}{2} - \sqrt{\frac{\sigma^2}{4} + \tau^2}$$

Distortion Energy:

Let $a = \frac{\sigma}{2}$ $b = \sqrt{\frac{\sigma^2}{4} + \tau^2}$

$$\sigma_1^2 = a^2 + b^2 + 2\,a\,b$$

$$\sigma_1\sigma_2 = a^2 - b^2$$

$$\sigma_2^2 = a^2 + b^2 - 2\,a\,b$$

$$\sigma_2^2 - \sigma_1\sigma_2 + \sigma_2^2 = 3\,b^2 + a^2$$

Apply $\sigma = \frac{M\,c}{I}; \qquad \tau = \frac{T\,c}{J}$

$$(u_d)_2 = \frac{1 + v}{3\,E}(3\,b^2 + a^2) = \frac{1 + v}{3\,E}\left(\frac{\sigma^2}{4} + \frac{3\sigma^2}{4} + 3\,\tau^2\right)$$

$$= \frac{1 + v}{3\,E}(\sigma^2 + 3\,\tau^2) = \frac{1 + v}{3\,E}\left(\frac{M^2 c^2}{I^2} + \frac{3\,T^2 c^2}{J^2}\right) \tag{2}$$

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10–63. Continued

Equating Eq. (1) and (2) yields:

$$\frac{(1+\nu)}{3E}\left(\frac{M_e c^2}{I^2}\right) = \frac{1+\nu}{3E}\left(\frac{M^2 c^2}{I^2} + \frac{3T^2 c^2}{J^2}\right)$$

$$\frac{M_e^2}{I^2} = \frac{M^1}{I^2} + \frac{3T^2}{J^2}$$

$$M_e^2 = M^1 + 3T^2\left(\frac{I}{J}\right)^2$$

For circular shaft

$$\frac{I}{J} = \frac{\frac{\pi}{4}c^4}{\frac{\pi}{2}c^4} = \frac{1}{2}$$

Hence, $M_e^2 = M^2 + 3T^2\left(\frac{1}{2}\right)^2$

$$M_e = \sqrt{M^2 + \frac{3}{4}T^2}$$ **Ans.**

Ans.

$$M_e = \sqrt{M^2 + \frac{3}{4}T^2}$$

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***10–64.** Derive an expression for an equivalent bending moment M_e that, if applied alone to a solid bar with a circular cross section, would cause the same maximum shear stress as the combination of an applied moment M and torque T. Assume that the principal stresses are of opposite algebraic signs.

SOLUTION

Bending and Torsion:

$$\sigma = \frac{M\,c}{I} = \frac{M\,c}{\frac{\pi}{4}c^4} = \frac{4\,M}{\pi\,c^3}; \qquad \tau = \frac{T\,c}{J} = \frac{T\,c}{\frac{\pi}{2}c^4} = \frac{2\,T}{\pi\,c^3}$$

The principal stresses:

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \frac{\frac{4\,M}{\pi c^3} + 0}{2} \pm \sqrt{\left(\frac{\frac{4\,M}{\pi\,c^3} - 0}{2}\right)^2 + \left(\frac{2T}{\pi\,c^3}\right)^2}$$

$$= \frac{2\,M}{\pi\,c^3} \pm \frac{2}{\pi\,c^3}\sqrt{M^2 + T^2}$$

$$\tau_{\substack{\text{abs}\\\text{max}}} = \sigma_1 - \sigma_2 = 2\left[\frac{2}{\pi\,c^3}\sqrt{M^2 + T^2}\right] \tag{1}$$

Pure bending:

$$\sigma_1 = \frac{M\,c}{I} = \frac{M_e\,c}{\frac{\pi}{4}c^4} = \frac{4\,M_e}{\pi\,c^3}; \qquad \sigma_2 = 0$$

$$\tau_{\substack{\text{abs}\\\text{max}}} = \sigma_1 - \sigma_2 = \frac{4\,M_e}{\pi\,c^3} \tag{2}$$

Equating Eq. (1) and (2) yields:

$$\frac{4}{\pi\,c^3}\sqrt{M^2 + T^2} = \frac{4\,M_e}{\pi\,c^3}$$

$$M_e = \sqrt{M^2 + T^2} \qquad \textbf{Ans.}$$

Ans.
$M_e = \sqrt{M^2 + T^2}$

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10–65.

Derive an expression for an equivalent torque T_e that, if applied alone to a solid bar with a circular cross section, would cause the same energy of distortion as the combination of an applied bending moment M and torque T.

SOLUTION

$$\tau = \frac{T_e\, c}{J}$$

Principal Stress:

$$\sigma_1 = \tau \quad \sigma_2 = -\tau$$

$$u_d = \frac{1 + \nu}{3\, E}(\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2)$$

$$(u_d)_1 = \frac{1 + \nu}{3\, E}(3\, \tau^2) = \frac{1 + \nu}{3\, E}\left(\frac{3\, T_e^2\, c^2}{J^2}\right)$$

Bending Moment and Torsion:

$$\sigma = \frac{M\, c}{I}; \qquad \tau = \frac{T\, c}{J}$$

Principal Stress:

$$\sigma_{1,\,2} = \frac{\sigma + 0}{2} \pm \sqrt{\left(\frac{\sigma - 0}{2}\right)^2 + \tau^2}$$

$$\sigma_1 = \frac{\sigma}{2} + \sqrt{\frac{\sigma^2}{4} + \tau^2}; \qquad \sigma_2 = \frac{\sigma}{2} - \sqrt{\frac{\sigma^2}{4} + \tau^2}$$

Let $a = \dfrac{\sigma}{2} \qquad b = \sqrt{\dfrac{\sigma^2}{4} + \tau^2}$

$$\sigma_1^2 = a^2 + b^2 + 2\, a\, b$$

$$\sigma_1\sigma_2 = a^2 - b^2$$

$$\sigma_2^2 = a^2 + b^2 - 2\, a\, b$$

$$\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2 = 3\, b^2 + a^2$$

$$u_d = \frac{1 + \nu}{3\, E}(\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2)$$

$$(u_d)_2 = \frac{1 + \nu}{3\, E}(3\, b^2 + a^2) = \frac{1 + \nu}{3\, E}\left(\frac{3\, \sigma^2}{4} + 3\tau^2 + \frac{\sigma^2}{4}\right)$$

$$= \frac{1 + \nu}{3\, E}(\sigma^2 + 3\, \tau^2) = \frac{c^2(1 + \nu)}{3\, E}\left(\frac{M^2}{I^2} + \frac{3\, T^2}{J^2}\right)$$

$$(u_d)_1 = (u_d)_2$$

$$\frac{c^2(1 + \nu)}{3\, E}\frac{3\, T_e^2}{J^2} = \frac{c^2(1 + \nu)}{3\, E}\left(\frac{M^2}{I^2} + \frac{3\, T^2}{J^2}\right)$$

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10–65. Continued

For circular shaft

$$\frac{J}{I} = \frac{\frac{\pi}{2}c^4}{\frac{\pi}{4}c^4} = 2$$

$$T_e = \sqrt{\frac{J^2}{I^2}\frac{M^2}{3} + T^2}$$

$$T_e = \sqrt{\frac{4}{3}M^2 + T^2}$$ **Ans.**

Ans:

$$T_e = \sqrt{\frac{4}{3}M^2 + T^2}$$

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10–66. An aluminum alloy 6061-T6 is to be used for a solid drive shaft such that it transmits 33 kW at 2400 rev/min. Using a factor of safety of 2 with respect to yielding, determine the smallest-diameter shaft that can be selected based on the maximum-shear-stress theory.

SOLUTION

$$\omega = \left(2400\,\frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi\text{ rad}}{\text{rev}}\right)\left(\frac{1\text{ min}}{60\text{s}}\right) = 80\,\pi\text{ rad/s}$$

$$T = \frac{P}{\omega} = \frac{33(10^3)}{80\pi} = \frac{3300}{(8\pi)}\,\text{N}\cdot\text{m}$$

Applying $\tau = \dfrac{T\,c}{J}$

$$\tau = \frac{\left(\frac{3300}{8\pi}\right)c}{\frac{\pi}{2}\,c^4} = \frac{6600}{8\pi^3 c^3}$$

The principal stresses:

$$\sigma_1 = \tau = \frac{6600}{8\pi^2 c^3}; \qquad \sigma_2 = -\tau = \frac{6600}{8\pi^2 c^3}$$

Maximum shear stress theory: Both principal stresses have opposite sign, hence,

$$\left|\sigma_1 - \sigma_2\right| = \frac{\sigma_Y}{\text{F.S.}}; \qquad 2\left(\frac{6600}{8\pi^2 c^3}\right) = \left|\frac{255(10^6)}{2}\right|$$

$c = 0.010945\text{ m} = 10.945\text{ mm}$

$d = 21.89\text{ mm}$ **Ans.**

Ans.

$\omega = 80\,\pi\text{ rad/s}$, $T = \dfrac{3300}{8\pi}\,\text{N}\cdot\text{m}$,

$d = 21.89\text{ mm}$

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10–67. Solve Prob. 10–61 using the maximum-distortion-energy theory.

SOLUTION

$$\omega = \left(2400\,\frac{\text{rev}}{\text{min}}\right)\left(\frac{2\text{p rad}}{\text{rev}}\right)\left(\frac{1\text{ min}}{60\text{s}}\right) = 80\,\pi\text{ rad/s}$$

$$T = \frac{P}{\omega} = \frac{33(10^3)}{80\pi} = \frac{3300}{(8\pi)}\,\text{N}\cdot\text{m}$$

Applying $\tau = \dfrac{T\,c}{J}$

$$\tau = \frac{\left(\frac{3300}{8\pi}\right)c}{\frac{\pi}{2}\,c^4} = \frac{6600}{8\pi^3 c^3}$$

The principal stresses:

$$\sigma_1 = \tau = \frac{6600}{8\pi^2 c^3}; \qquad \sigma_2 = -\tau = -\frac{6600}{8\pi^2 c^3}$$

The maximum distortion-energy theory:

$$\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2 = \left(\frac{\sigma_Y}{\text{F.S.}}\right)^2$$

$$3\left[\frac{6600}{8\pi^2 c^3}\right]^2 = \left(\frac{255(10^6)}{2}\right)^2$$

$c = 0.0104328$ m = 10.4328 mm

$d = 20.87$ mm **Ans.**

Ans.
$d = 20.87$ mm

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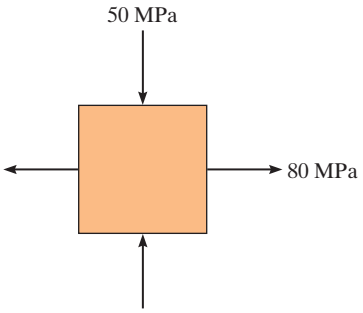
***10–68.**

If the material is machine steel having a yield stress of $\sigma_Y = 700$ MPa, determine the factor of safety with respect to yielding if the maximum shear stress theory is considered.



SOLUTION

$\sigma_{max} = 80$ MPa $\quad \sigma_{min} = -50$ MPa

$$\tau_{\substack{abs \\ max}} = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{80 - (-50)}{2} = 65 \text{ MPa}$$

$$\tau_{max} = \frac{\sigma_Y}{2} = \frac{700}{2} = 350 \text{ MPa}$$

$$\text{F.S.} = \frac{\tau_{max}}{\tau_{\substack{abs \\ max}}} = \frac{350}{65} = 5.38$$ **Ans.**

Ans:
F.S. = 5.38

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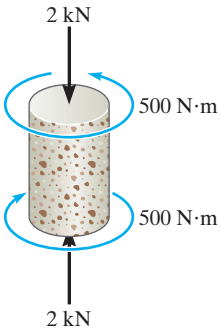
10–69.

The short concrete cylinder having a diameter of 50 mm is subjected to a torque of 500 N · m and an axial compressive force of 2 kN. Determine if it fails according to the maximum normal stress theory. The ultimate stress of the concrete is $\sigma_{ult} = 28$ MPa.



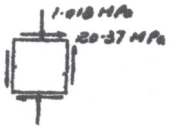
SOLUTION

$$A = \frac{\pi}{4}(0.05)^2 = 1.9635(10^{-3})\ \text{m}^2$$

$$J = \frac{\pi}{2}(0.025)^4 = 0.61359(10^{-4})\ \text{m}^4$$

$$\sigma = \frac{P}{A} = \frac{2(10^3)}{1.9635(10^{-3})} = 1.019\ \text{MPa}$$

$$\tau = \frac{Tc}{J} = \frac{500(0.025)}{0.61359(10^{-6})} = 20.372\ \text{MPa}$$



$$\sigma_x = 0 \qquad \sigma_y = -1.019\ \text{MPa} \qquad \tau_{xy} = 20.372\ \text{MPa}$$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + {\tau_{xy}}^2}$$

$$\sigma_{1,2} = \frac{0 - 1.018}{2} \pm \sqrt{\left(\frac{0 - (-1.019)}{2}\right)^2 + 20.372^2}$$

$$\sigma_1 = 19.87\ \text{MPa} \qquad \sigma_2 = -20.89\ \text{MPa}$$

Failure criteria:

$|\sigma_1| < \sigma_{alt} = 28$ MPa **OK**

$|\sigma_2| < \sigma_{alt} = 28$ MPa **OK**

No. **Ans.**

Ans:
No

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10–70. A bar with a circular cross-sectional area is made of SAE 1045 carbon steel having a yield stress of σ_Y = 1000 MPa. If the bar is subjected to a torque of 3.75 kN · m and a bending moment of 7 kN · m, determine the required diameter of the bar according to the maximum-distortion-energy theory. Use a factor of safety of 2 with respect to yielding.

SOLUTION

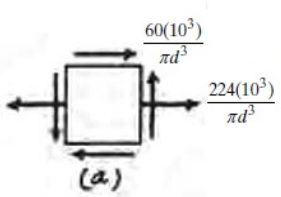
Normal and Shear Stresses: Applying the flexure and torsion formulas.

$$\sigma = \frac{Mc}{I} = \frac{[7(10^3)](d/2)}{\frac{\pi}{4}(d/2)^4} = \frac{224(10^3)}{\pi d^3}$$

$$\tau = \frac{Tc}{J} = \frac{[3.75(10^3)](d/2)}{\frac{\pi}{2}(-d/2)^4} = \frac{60(10^3)}{\pi d^3}$$

The critical state of stress is shown in Fig. (a) or (b), where

$$\sigma_x = \frac{224(10^3)}{\pi d^3} \qquad \sigma_y = 0 \qquad \tau_{xy} = \frac{60(10^3)}{\pi d^3}$$



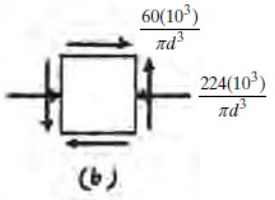
In - Plane Principal Stresses : Applying Eq. 9-5,

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \left[\frac{\frac{224}{\pi d^3} + 0}{2} \pm \sqrt{\left(\frac{\frac{224}{\pi d^3} - 0}{2}\right)^2 + \left(\frac{60}{\pi d^3}\right)^2}\right](10^3)$$

$$= \left(\frac{112}{\pi d^3} \pm \frac{127.06}{\pi d^3}\right)(10^3)$$

$$\sigma_1 = \left(\frac{239.06}{\pi d^3}\right)(10^3) \qquad \sigma_2 = \left(-\frac{15.06}{\pi d^3}\right)(10^3)$$



Maximum Distortion Energy Theory :

$$\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2 = \sigma_{\text{allow}}^2$$

$$\left(\frac{239.06}{\pi d^3}\right)^2 - \left(\frac{239.06}{\pi d^3}\right)\left(-\frac{15.06}{\pi d^3}\right) + \left(-\frac{15.06}{\pi d^3}\right)^2 (10^3)^2 = \left[\frac{1000(10^6)}{2}\right]^2$$

$d = 0.05397$ m = 54.0 mm **Ans.**

Ans.
d = 54.0 m

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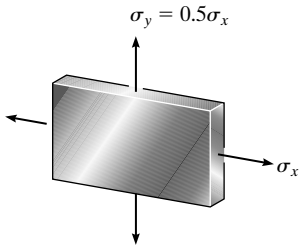
10–71. The plate is made of hard copper, which yields at $\sigma_Y = 735$ MPa. Using the maximum-shear-stress theory, determine the tensile stress σ_x that can be applied to the plate if a tensile stress $\sigma_y = 0.5\sigma_x$ is also applied.



SOLUTION

$\sigma_1 = \sigma_x \qquad \sigma_2 = \frac{1}{2}\sigma_x$

$|\sigma_1| = \sigma_Y$

$\sigma_x = 735$ MPa **Ans.**

Ans.
$\sigma_x = 735$ MPa

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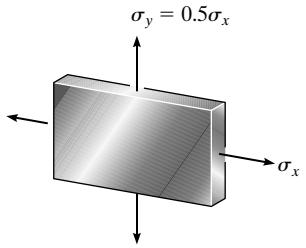
***10–72.** Solve Prob. 10–80 using the maximum-distortion-energy theory.



SOLUTION

$\sigma_1 = \sigma_x$

$\sigma_2 = \dfrac{\sigma_x}{2}$

$\sigma_1^2 - \sigma_1 \sigma_2 + \sigma_2^2 = \sigma_Y^2$

$\sigma_x^2 - \dfrac{\sigma_x^2}{2} + \dfrac{\sigma_x^2}{4} = 735^2$

$\sigma_x = 848.7 \text{ MPa} = 849 \text{ MPa}$ **Ans.**

Ans.

$\sigma_x = 849 \text{ MPa}$

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10–73. The state of stress acting at a critical point on the seat frame of an automobile during a crash is shown in the figure. Determine the smallest yield stress for a steel that can be selected for the member, based on the maximum-shear-stress theory.

SOLUTION

Normal and Shear Stress: In accordance with the sign convention.

$$\sigma_x = 560 \text{ MPa} \qquad \sigma_y = 0 \qquad \tau_{xy} = 175 \text{ MPa}$$

In - Plane Principal Stress: Applying Eq. 9-5.

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{560 + 0}{2} \pm \sqrt{\left(\frac{560 - 0}{2}\right)^2 + 175^2}$$

$$= 280 \pm 330.19$$

$$\sigma_1 = 610.19 \text{ MPa} \qquad \sigma_2 = -50.19 \text{ MPa}$$

Maximum Shear Stress Theory: σ_1 and σ_2 have opposite signs so

$$|\sigma_1 - \sigma_2| = \sigma_Y$$

$$|610.19 - (-50.19)| = \sigma_Y$$

$$\sigma_Y = 660.38 \text{ MPa} = 660 \text{ MPa}$$ **Ans.**

Ans.

$\sigma_Y = 660$ MPa

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10–74. Solve Prob. 10–82 using the maximum-distortion-energy theory.

SOLUTION

Normal and Shear Stress: In accordance with the sign convention.

$$\sigma_x = 560 \text{ MPa} \qquad \sigma_y = 0 \qquad \tau_{xy} = 175 \text{ MPa}$$

In - Plane Principal Stress: Applying Eq. 9-5.

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{560 + 0}{2} \pm \sqrt{\left(\frac{560 - 0}{2}\right)^2 + 175^2}$$

$$= 280 \pm 330.189$$

$$\sigma_1 = 610.19 \text{ MPa} \qquad \sigma_2 = -50.19 \text{ MPa}$$

Maximum Distortion Energy Theory:

$$\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2 = \sigma_Y^2$$

$$610.19^2 - (610.19)(-50.19) + (-50.19)^2 = \sigma_Y^2$$

$$\sigma_Y = 636.77 \text{ MPa} = 637 \text{ MPa}$$ **Ans.**

Ans.

$\sigma_Y = 637$ MPa

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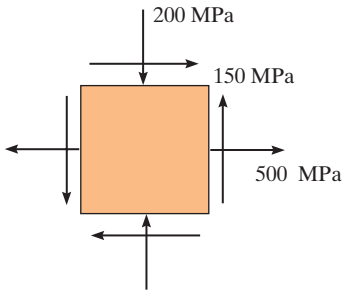
10–75.

The components of plane stress at a critical point on a thin steel shell are shown. Determine if failure (yielding) has occurred on the basis of the maximum distortion energy theory. The yield stress for the steel is $\sigma_Y = 700$ MPa.



SOLUTION

Normal And Shear Stresses: In accordance with the sign convention,

$$\sigma_x = 500 \text{ MPa} \qquad \sigma_y = -200 \text{ MPa} \qquad \tau_{xy} = 150 \text{ MPa}$$

In-Plane Principal Stresses:

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{500 + (-200)}{2} \pm \sqrt{\left[\frac{500 - (-200)}{2}\right]^2 + 150^2}$$

$$= 150 \pm 380.79$$

$$\sigma_1 = 530.79 \text{ MPa} \qquad \sigma_2 = -230.79 \text{ MPa}$$

Maximum Distortion Energy Theory: This theory gives

$$\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2$$

$$= 530.79^2 - 530.79(-230.79) + (-230.79)^2$$

$$= 457500 < \sigma_Y^2 = 490000$$

Based on the results obtained above, **the material will not yield according to the maximum distortion energy theory.** **Ans.**

Ans:
No

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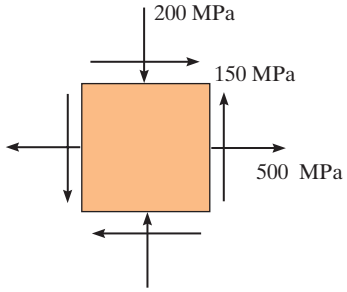
***10–76.**

Solve Prob. 10–75 using the maximum shear stress theory.



SOLUTION

Normal And Shear Stresses: In accordance with the sign convention,

$$\sigma_x = 500\ \text{MPa} \qquad \sigma_y = -200\ \text{MPa} \qquad \tau_{xy} = 150\ \text{MPa}$$

In-Plane Principal Stresses:

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + {\tau_{xy}}^2}$$

$$= \frac{500 + (-200)}{2} \pm \sqrt{\left[\frac{500 - (-200)}{2}\right]^2 + 150^2}$$

$$= 150 \pm 380.79$$

$$\sigma_1 = 530.79\ \text{MPa} \qquad \sigma_2 = -230.79\ \text{MPa}$$

Maximum Shear-Stress Theory: Here, σ_1 and σ_2 have opposite signs. So

$$|\sigma_1 - \sigma_2|$$

$$= 530.79 - (-230.79)$$

$$= 761.58\ \text{MPa} > \sigma_Y = 700\ \text{MPa}$$

Based on the results obtained above, **the material yield according to the maximum shear-stress theory.** **Ans.**

Ans:
Yes

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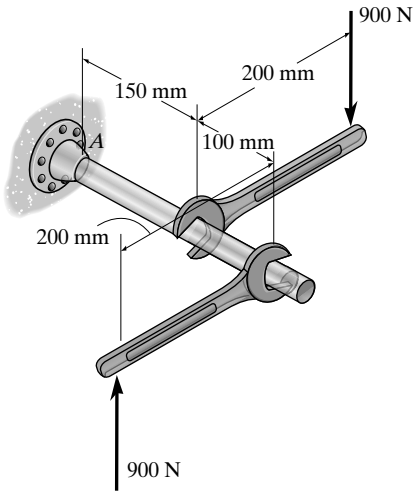
10–77. If the A-36 steel pipe has outer and inner diameters of 30 mm and 20 mm, respectively, determine the factor of safety against yielding of the material at point A according to the maximum-shear-stress theory.



SOLUTION

Internal Loadings. Considering the equilibrium of the free - body diagram of the post's right cut segment Fig. a,

$\Sigma F_y = 0;\quad V_y + 900 - 900 = 0 \qquad V_y = 0$

$\Sigma M_x = 0;\ T + 900(0.4) = 0 \qquad T = -360\,\mathrm{N}\cdot\mathrm{m}$

$\Sigma M_z = 0;\ M_z + 900(0.15) - 900(0.25) = 0\ M_z = 90\,\mathrm{N}\cdot\mathrm{m}$

Section Properties. The moment of inertia about the z axis and the polar moment of inertia of the pipe's cross section are

$$I_z = \frac{\pi}{4}\left(0.015^4 - 0.01^4\right) = 10.15625\pi\left(10^{-9}\right)\mathrm{m}^4$$

$$J = \frac{\pi}{2}\left(0.015^4 - 0.01^4\right) = 20.3125\pi\left(10^{-9}\right)\mathrm{m}^4$$

Normal Stress and Shear Stress. The normal stress is contributed by bending stress. Thus,

$$\sigma_Y = -\frac{My_A}{I_z} = -\frac{90(0.015)}{10.15625\pi\left(10^{-9}\right)} = -42.31\mathrm{MPa}$$

The shear stress is contributed by torsional shear stress.

$$\tau = \frac{Tc}{J} = \frac{360(0.015)}{20.3125\pi\left(10^{-9}\right)} = 84.62\ \mathrm{MPa}$$

The state of stress at point A is represented by the two - dimensional element shown in Fig. b.

In - Plane Principal Stress. $\sigma_x = -42.31$ MPa, $\sigma_z = 0$ and $\tau_{xz} = 84.62$ MPa. We have

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_z}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_z}{2}\right)^2 + {\tau_{xz}}^2}$$

$$= \frac{-42.31 + 0}{2} \pm \sqrt{\left(\frac{-42.31 - 0}{2}\right)^2 + 84.62^2}$$

$$= (-21.16 \pm 87.23)\ \mathrm{MPa}$$

$\sigma_1 = 66.07$ MPa $\qquad \sigma_2 = -108.38$ MPa

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10–77. Continued

Maximum Shear Stress Theory. σ_1 and σ_2 have opposite signs. This requires

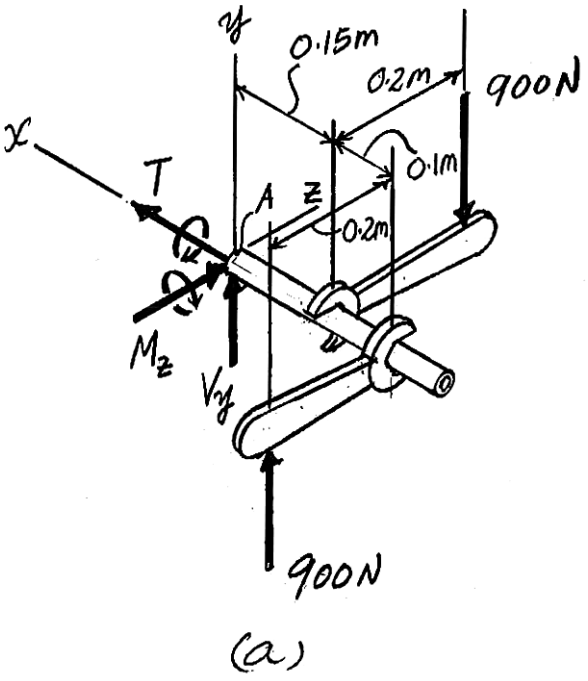
$$|\sigma_1 - \sigma_2| = \sigma_{\text{allow}}$$

$$66.07 - (-108.38) = \sigma_{\text{allow}}$$

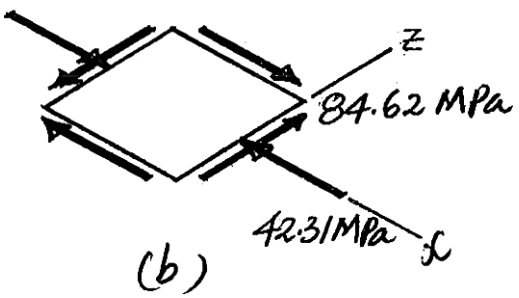
$$\sigma_{\text{allow}} = 174.45 \text{ MPa}$$

The factor of safety is

$$F.S. = \frac{\sigma_Y}{\sigma_{\text{allow}}} = \frac{250}{174.45} = 1.43$$ **Ans.**



(a)



(b)

Ans.
$F.S. = 1.43$

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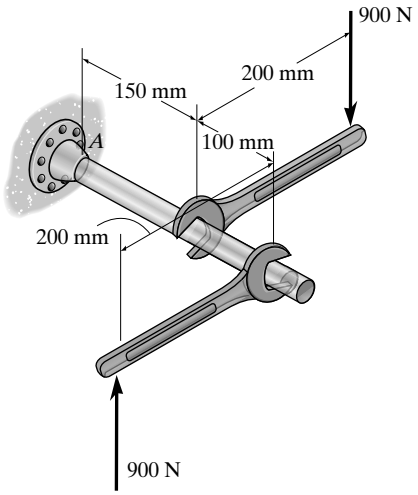
10–78. If the A-36 steel pipe has an outer and inner diameter of 30 mm and 20 mm, respectively, determine the factor of safety against yielding of the material at point A according to the maximum-distortion-energy theory.



SOLUTION

Internal Loadings: Considering the equilibrium of the free - body diagram of the pipe's right cut segment Fig. a,

$\Sigma F_y = 0;\quad V_y + 900 - 900 = 0 \qquad V_y = 0$

$\Sigma M_x = 0;\ T + 900(0.4) = 0 \qquad T = -360\,\mathrm{N}\cdot\mathrm{m}$

$\Sigma M_z = 0;\ M_z + 900(0.15) - 900(0.25) = 0\ M_z = 90\,\mathrm{N}\cdot\mathrm{m}$

Section Properties. The moment of inertia about the z axis and the polar moment of inertia of the pipe's cross section are

$$I_z = \frac{\pi}{4}\left(0.015^4 - 0.01^4\right) = 10.15625\pi\left(10^{-9}\right)\mathrm{m}^4$$

$$J = \frac{\pi}{2}\left(0.015^4 - 0.01^4\right) = 20.3125\pi\left(10^{-9}\right)\mathrm{m}^4$$

Normal Stress and Shear Stress. The normal stress is caused by bending stress. Thus,

$$\sigma_Y = -\frac{My_A}{I_z} = -\frac{90(0.015)}{10.15625\pi\left(10^{-9}\right)} = -42.31\mathrm{MPa}$$

The shear stress is caused by torsional stress.

$$\tau = \frac{Tc}{J} = \frac{360(0.015)}{20.3125\pi\left(10^{-9}\right)} = 84.62\ \mathrm{MPa}$$

The state of stress at point A is represented by the two -dimensional element shown in Fig. b.

In - Plane Principal Stress. $\sigma_x = -42.31$ MPa, $\sigma_z = 0$ and $\tau_{xz} = 84.62$ MPa. We have

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_z}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_z}{2}\right)^2 + {\tau_{xz}}^2}$$

$$= \frac{-42.31 + 0}{2} \pm \sqrt{\left(\frac{-42.31 - 0}{2}\right)^2 + 84.62^2}$$

$$= (-21.16 \pm 87.23)\ \mathrm{MPa}$$

$\sigma_1 = 66.07$ MPa $\qquad \sigma_2 = -108.38$ MPa

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10–78. Continued

Maximum Distortion Energy Theory.

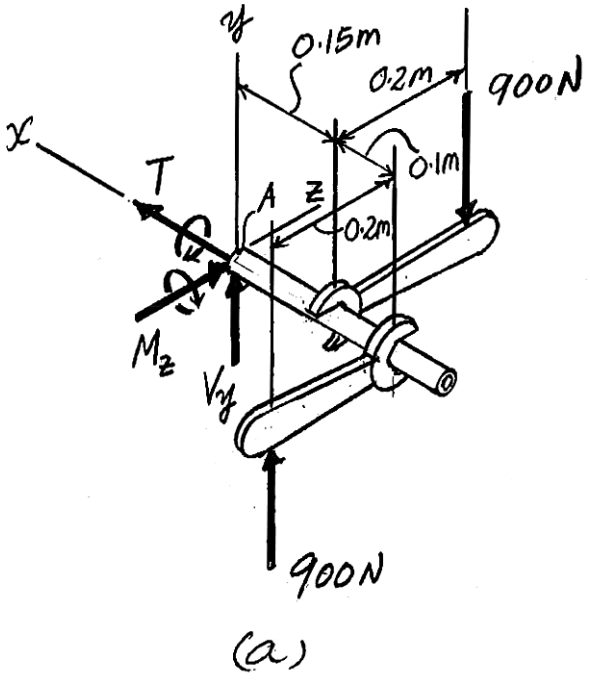
$$\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2 = \sigma_{\text{allow}}^2$$

$$66.07^2 - 66.07(-108.38) + (-108.38)^2 = \sigma_{\text{allow}}^2$$

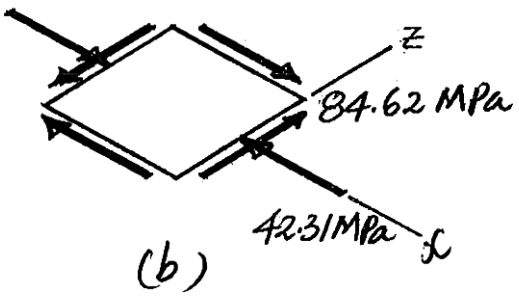
$$\sigma_{\text{allow}} = 152.55 \text{ MPa}$$

Thus, the factor of safety is

$$F.S. = \frac{\sigma_Y}{\sigma_{\text{allow}}} = \frac{250}{152.55} = 1.64$$ **Ans.**



(a)



(b)

Ans.

$F.S. = 1.64$

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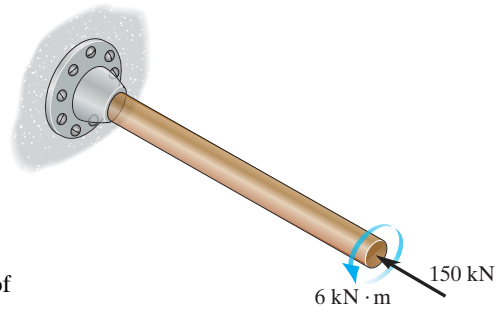
10–79. If the 50-mm diameter shaft is made from brittle material having an ultimate strength of $\sigma_{ult} = 350$ MPa for both tension and compression, determine if the shaft fails according to the maximum-normal-stress theory. Use a factor of safety of 1.5 against rupture.



SOLUTION

Normal Stress and Shear Stresses. The cross-sectional area and polar moment of inertia of the shaft's cross-section are

$$A = \pi(0.025^2) = 0.625(10^{-3})\pi \text{ m}^2$$

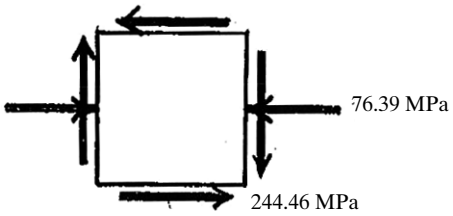
$$J = \frac{\pi}{2}(0.025^4) = 0.1953125(10^{-6})\pi \text{ m}^4$$

The normal stress is caused by axial stress.

$$\sigma = \frac{N}{A} = -\frac{150(10^3)}{0.625(10^{-3})\pi} = -76.39(10^6) \text{ N/m}^2 = -76.39 \text{ MPa}$$

The shear stress is contributed by torsional shear stress.

$$\tau = \frac{Tc}{J} = \frac{[6(10^3)](0.025)}{0.1953125(10^{-6})\pi} = 244.46(10^6) \text{ N/m}^2 = 244.46 \text{ MPa}$$



(a)

The state of stress at the points on the surface of the shaft is represented on the element shown in Fig. *a*.

In-Plane Principal Stress. $\sigma_x = -76.39$ MPa, $\sigma_y = 0$ and $\tau_{xy} = -244.46$ MPa. We have

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{-76.39 + 0}{2} \pm \sqrt{\left(\frac{-76.39 - 0}{2}\right)^2 + (-244.46)^2}$$

$$= -30.20 \pm 247.43$$

$\sigma_1 = 209.23$ MPa $\qquad \sigma_2 = -285.63$ MPa

Maximum Normal-Stress Theory.

$$\sigma_{\text{allow}} = \frac{\sigma_{\text{ult}}}{F.S.} = \frac{350}{1.5} = 233.3 \text{ MPa}$$

$|\sigma_1| = 209.23 < \sigma_{\text{allow}} = 233.33$ MPa (O.K.)

$|\sigma_2| = 285.63 > \sigma_{\text{allow}} = 233.33$ MPa (N.G.)

Based on these results, the material *fails* according to the maximum normal-stress theory.

Ans:
Yes

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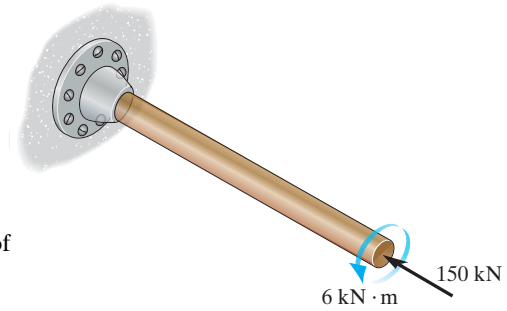
***10–80.** If the 50-mm diameter shaft is made from cast iron having tensile and compressive ultimate strengths of $(\sigma_{ult})_t$ = 350 MPa and $(\sigma_{ult})_c$ = 525 MPa, respectively, determine if the shaft fails in accordance with Mohr's failure criterion.



SOLUTION

Normal Stress and Shear Stresses. The cross-sectional area and polar moment of inertia of the shaft's cross-section are

$$A = \pi(0.025^2) = 0.625(10^{-3})\pi \text{ m}^2$$

$$J = \frac{\pi}{2}(0.025^4) = 0.1953125(10^{-6})\pi \text{ m}^4$$

The normal stress is caused by axial stress.

$$\sigma = \frac{N}{A} = -\frac{150(10^3)}{0.625(10^{-3})\pi} = -76.39(10^6) \text{ N/m}^2 = -76.39 \text{ MPa}$$

The shear stress is contributed by torsional shear stress.

$$\tau = \frac{Tc}{J} = \frac{[6(10^3)](0.025)}{0.1953125(10^{-6})\pi} = 244.46(10^6) \text{ N/m}^2 = 244.46 \text{ MPa}$$

The state of stress at the points on the surface of the shaft is represented on the element shown in Fig. *a*.

In-Plane Principal Stress. $\sigma_x = -76.39$ MPa, $\sigma_y = 0$ and $\tau_{xy} = -244.46$ MPa. We have

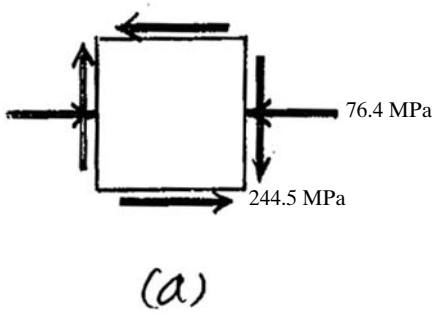
$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{-76.39 + 0}{2} \pm \sqrt{\left(\frac{-76.39 - 0}{2}\right)^2 + (-244.46)^2}$$

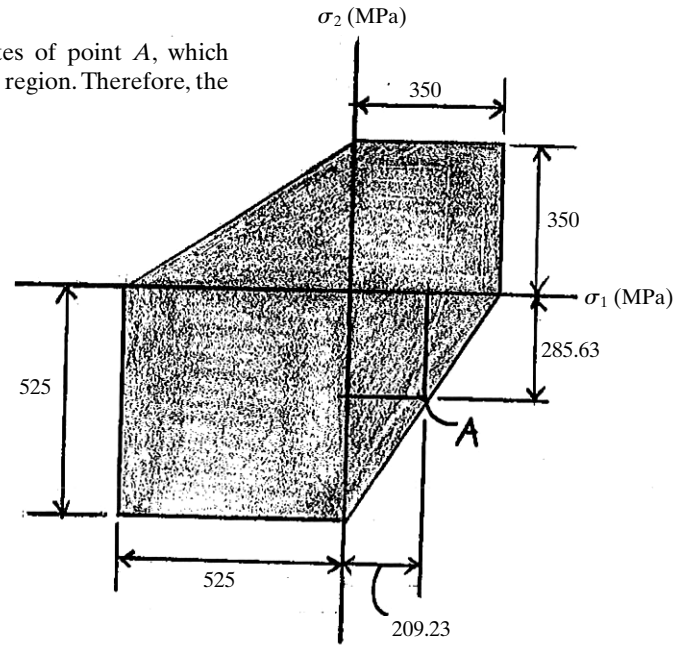
$$= -30.20 \pm 247.43$$

$$\sigma_1 = 209.23 \text{ MPa} \qquad \sigma_2 = -285.63 \text{ MPa}$$

Mohr's Failure Criteria. As shown in Fig. *b*, the coordinates of point *A*, which represent the principal stresses, are located inside the shaded region. Therefore, the material *does not fail* according to Mohr's failure criteria.



(a)



(b)

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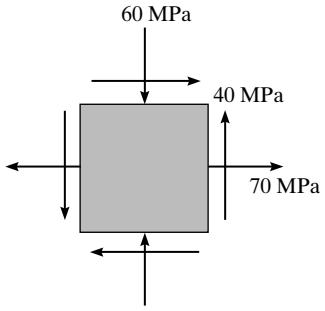
10–81. The components of plane stress at a critical point on an A-36 steel shell are shown. Determine if failure (yielding) has occurred on the basis of the maximum-shear-stress theory.



SOLUTION

In accordance to the established sign convention, $\sigma_x = 70$ MPa, $\sigma_y = -60$ MPa and $\tau_{xy} = 40$ MPa.

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$$

$$= \frac{70 + (-60)}{2} \pm \sqrt{\left[\frac{70 - (-60)}{2}\right]^2 + 40^2}$$

$$= 5 \pm \sqrt{5825}$$

$$\sigma_1 = 81.32 \text{ MPa} \qquad \sigma_2 = -71.32 \text{ MPa}$$

In this case, σ_1 and σ_2 have opposite sign. Thus,

$$|\sigma_1 - \sigma_2| = |81.32 - (-71.32)| = 152.64 \text{ MPa} < \sigma_Y = 250 \text{ MPa}$$

Based on this result, **the steel shell does not yield according to the maximum shear stress theory**.

Ans:
No

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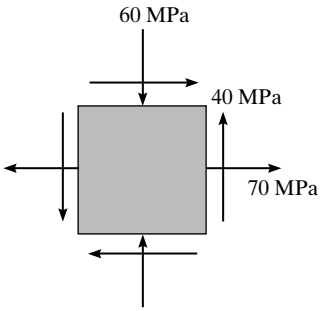
10–82. The components of plane stress at a critical point on an A-36 steel shell are shown. Determine if failure (yielding) has occurred on the basis of the maximum-distortion-energy theory.



SOLUTION

In accordance to the established sign convention, $\sigma_x = 70$ MPa, $\sigma_y = -60$ MPa and $\tau_{xy} = 40$ MPa.

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + {\tau_{xy}}^2}$$

$$= \frac{70 + (-60)}{2} \pm \sqrt{\left[\frac{70 - (-60)}{2}\right]^2 + 40^2}$$

$$= 5 \pm \sqrt{5825}$$

$$\sigma_1 = 81.32 \text{ MPa} \qquad \sigma_2 = -71.32 \text{ MPa}$$

$${\sigma_1}^2 - \sigma_1\sigma_2 + {\sigma_2}^2 = 81.32^2 - 81.32(-71.32) + (-71.32)^2 = 17{,}500 < {\sigma_Y}^2 = 62500$$

Based on this result, **the steel shell does not yield according to the maximum distortion energy theory**.

Ans:
No

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10–83. The yield stress for heat-treated beryllium copper is $\sigma_Y = 900$ MPa. If this material is subjected to plane stress and elastic failure occurs when one principal stress is 1000 MPa, what is the smallest magnitude of the other principal stress? Use the maximum-distortion-energy theory.

SOLUTION

Maximum Distortion Energy Theory : With $\sigma_1 = 1000$ MPa,

$$\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2 = \sigma_Y^2$$

$$1000^2 - 1000\sigma_2 + \sigma_2^2 = 900^2$$

$$\sigma_2^2 - 1000\sigma_2 + 190000 = 0$$

$$\sigma_2 = \frac{-(-1000) \pm \sqrt{(1000)^2 - 4(1)(190000)}}{2(1)}$$

$$= 500 \pm 244.95$$

Choose the smaller root, $\sigma_2 = 255.05$ MPa = 255 MPa **Ans.**

Ans:
$\sigma_2 = 255$ MPa

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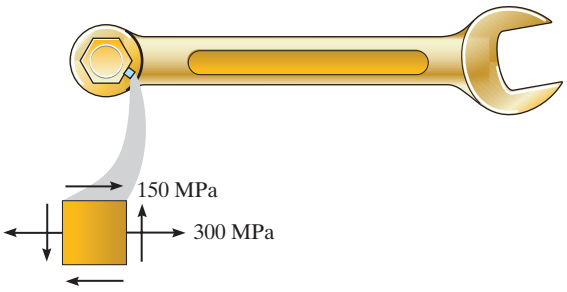
***10–84.**

The state of stress acting at a critical point on a wrench is shown. Determine the smallest yield stress for steel that might be selected for the part, based on the maximum distortion energy theory.



SOLUTION

Normal And Shear Stress: In accordance with the sign convention,

$$\sigma_x = 300\ \text{MPa} \qquad \sigma_y = 0 \qquad \tau_{xy} = 150\ \text{MPa}$$

In-Plane Principal Stress:

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + {\tau_{xy}}^2}$$

$$= \frac{300 + 0}{2} \pm \sqrt{\left(\frac{300 - 0}{2}\right)^2 + 150^2}$$

$$= 150 \pm 150\sqrt{2}$$

$$\sigma_1 = 362.13\ \text{MPa} \qquad \sigma_2 = -62.13\ \text{MPa}$$

Maximum Distortion Energy Theory: The theory gives

$${\sigma_1}^2 - \sigma_1\sigma_2 + {\sigma_2}^2 = {\sigma_Y}^2$$

$$362.13^2 - 362.13(-62.13) + (-62.13)^2 = {\sigma_Y}^2$$

$$\sigma_Y = 396.86\ \text{MPa} = 397\ \text{MPa} \qquad \textbf{Ans.}$$

Ans:
$\sigma_Y = 397$ MPa

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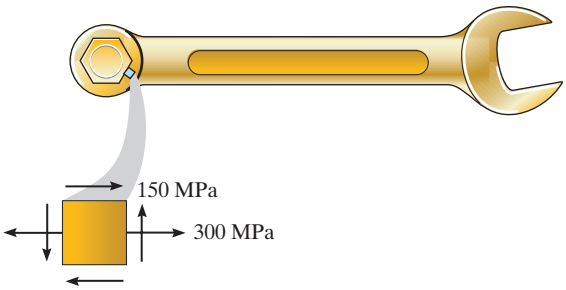
10–85.

The state of stress acting at a critical point on a wrench is shown in the figure. Determine the smallest yield stress for steel that might be selected for the part, based on the maximum shear stress theory.



SOLUTION

Normal And Shear Stresses: In accordance to the sign convention,

$$\sigma_x = 300 \text{ MPa} \qquad \sigma_y = 0 \qquad \tau_{xy} = 150 \text{ MPa}$$

In-Plane Principal Stress:

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + {\tau_{xy}}^2}$$

$$= \frac{300 + 0}{2} \pm \sqrt{\left(\frac{300 - 0}{2}\right)^2 + 150^2}$$

$$= 150 \pm 150\sqrt{2}$$

$$\sigma_1 = 362.13 \text{ MPa} \qquad \sigma_2 = -62.13 \text{ MPa}$$

Maximum Shear Stress Theory: Here, σ_1 and σ_2 have opposite signs. So

$$|\sigma_1 - \sigma_2| = \sigma_Y$$

$$|362.13 - (-62.13)| = \sigma_Y$$

$$\sigma_Y = 424.26 \text{ MPa} = 424 \text{ MPa}$$ **Ans.**

Ans:
$\sigma_Y = 424$ MPa

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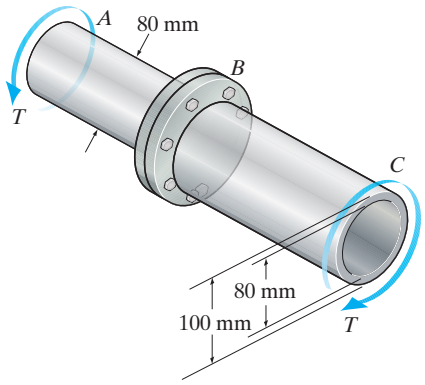
10–86.

The shaft consists of a solid segment AB and a hollow segment BC, which are rigidly joined by the coupling at B. If the shaft is made from A-36 steel, determine the maximum torque T that can be applied according to the maximum shear stress theory. Use a factor of safety of 1.5 against yielding.



SOLUTION

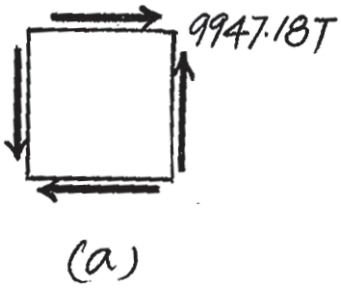
Shear Stress: This is a case of pure shear, and the shear stress is contributed by torsion. For the hollow segment, $J_h = \frac{\pi}{2}\left(0.05^4 - 0.04^4\right) = 1.845\pi\left(10^{-6}\right)\ \text{m}^4$. Thus,

$$(\tau_{\max})_h = \frac{Tc_h}{J_h} = \frac{T(0.05)}{1.845\pi\left(10^{-6}\right)} = 8626.28T$$

For the solid segment, $J_s = \frac{\pi}{2}\left(0.04^4\right) = 1.28\pi\left(10^{-6}\right)\ \text{m}^4$. Thus,

$$(\tau_{\max})_s = \frac{Tc_s}{J_s} = \frac{T(0.04)}{1.28\pi\left(10^{-6}\right)} = 9947.18T$$

By comparision, the points on the surface of the solid segment are critical and their state of stress is represented on the element shown in Fig. a.



In - Plane Principal Stress. $\sigma_x = \sigma_y = 0$ and $\tau_{xy} = 9947.18T$. We have

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{0 + 0}{2} \pm \sqrt{\left(\frac{0 - 0}{2}\right)^2 + (9947.18T)^2}$$

$$\sigma_1 = 9947.18T \qquad \sigma_2 = -9947.18T$$

Maximum Shear Stress Theory.

$$\sigma_{\text{allow}} = \frac{\sigma_Y}{F.S.} = \frac{250}{1.5} = 166.67\ \text{MPa}$$

Since σ_1 and σ_2 have opposite sings,

$$|\sigma_1 - \sigma_2| = \sigma_{\text{allow}}$$

$$9947.18T - (-9947.18T) = 166.67\left(10^6\right)$$

$$T = 8377.58\ \text{N}\cdot\text{m} = 8.38\ \text{kN}\cdot\text{m} \qquad \textbf{Ans.}$$

Ans:
$T = 8.38\ \text{kN}\cdot\text{m}$

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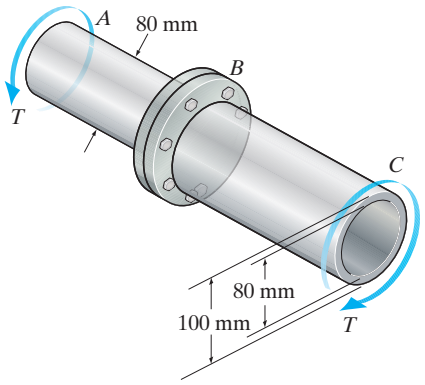
10–87.

The shaft consists of a solid segment AB and a hollow segment BC, which are rigidly joined by the coupling at B. If the shaft is made from A-36 steel, determine the maximum torque T that can be applied according to the maximum distortion energy theory. Use a factor of safety of 1.5 against yielding.



SOLUTION

Shear Stress. This is a case of pure shear, and the shear stress is contributed by torsion. For the hollow segment, $J_h = \frac{\pi}{2}\left(0.05^4 - 0.04^4\right) = 1.845\pi\left(10^{-6}\right)\ \text{m}^4$. Thus,

$$(\tau_{\max})_h = \frac{Tc_h}{J_h} = \frac{T(0.05)}{1.845\pi\left(10^{-6}\right)} = 8626.28T$$

For the solid segment, $J_s = \frac{\pi}{2}\left(0.04^4\right) = 1.28\pi\left(10^{-6}\right)\ \text{m}^4$. Thus,

$$(\tau_{\max})_s = \frac{Tc_s}{J_s} = \frac{T(0.04)}{1.28\pi\left(10^{-6}\right)} = 9947.18T$$

By comparision, the points on the surface of the solid segment are critical and their state of stress is represented on the element shown in Fig. *a*.

In - Plane Principal Stress. $\sigma_x = \sigma_y = 0$ and $\tau_{xy} = 9947.18T$. We have

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{0 + 0}{2} \pm \sqrt{\left(\frac{0 - 0}{2}\right)^2 + (9947.18T)^2}$$

$$\sigma_1 = 9947.18T \qquad \sigma_2 = -9947.18T$$

Maximum Distortion Energy Theory.

$$\sigma_{\text{allow}} = \frac{\sigma_Y}{F.S.} = \frac{250}{1.5} = 166.67\ \text{MPa}$$

Then,

$$\sigma_1{}^2 - \sigma_1\sigma_2 + \sigma_2{}^2 = \sigma_{\text{allow}}{}^2$$

$$(9947.18T)^2 - (9947.18T)(-9947.18T) + (-9947.18T)^2 = \left[166.67\left(10^6\right)\right]^2$$

$$T = 9673.60\ \text{N}\cdot\text{m} = 9.67\ \text{kN}\cdot\text{m} \qquad \textbf{Ans.}$$

Ans:

$T = 9.67\ \text{kN}\cdot\text{m}$

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***10–88.**

The principal stresses acting at a point on a thin-walled cylindrical pressure vessel are $\sigma_1 = pr/t$, $\sigma_2 = pr/2t$, and $\sigma_3 = 0$. If the yield stress is σ_Y, determine the maximum value of p based on (a) the maximum shear stress theory and (b) the maximum distortion energy theory.

SOLUTION

a) Maximum Shear Stress Theory: σ_1 and σ_2 have the same signs, then

$$|\sigma_2| = \sigma_Y \qquad \left|\frac{pr}{2t}\right| = \sigma_Y \qquad p = \frac{2t}{r}\sigma_Y$$

$$|\sigma_1| = \sigma_Y \qquad \left|\frac{pr}{t}\right| = \sigma_Y \qquad p = \frac{t}{r}\sigma_Y \text{ (Controls!)} \qquad \textbf{Ans.}$$

b) Maximum Distortion Energy Theory:

$$\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2 = \sigma_Y^2$$

$$\left(\frac{pr}{t}\right)^2 - \left(\frac{pr}{t}\right)\left(\frac{pr}{2t}\right) + \left(\frac{pr}{2t}\right)^2 = \sigma_Y^2$$

$$p = \frac{2t}{\sqrt{3}r}\sigma_Y \qquad \textbf{Ans.}$$

Ans:

(a) $p = \frac{t}{r}\sigma_y$,

(b) $p = \frac{2t}{\sqrt{3}r}\sigma_y$

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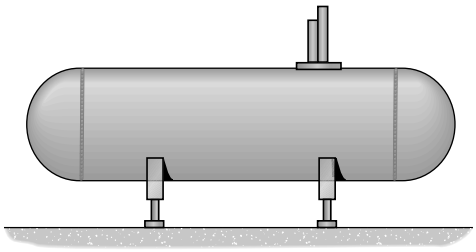
10–89. The gas tank has an inner diameter of 1.50 m and a wall thickness of 25 mm. If it is made from A-36 steel and the tank is pressured to 5 MPa, determine the factor of safety against yielding using (a) the maximum-shear-stress theory, and (b) the maximum-distortion-energy theory.



SOLUTION

(a) **Normal Stress.** Since $\frac{r}{t} = \frac{0.75}{0.025} = 30 > 10$, thin - wall analysis can be used. We have

$$\sigma_1 = \sigma_h = \frac{pr}{t} = \frac{5(0.75)}{0.025} = 150 \text{ MPa}$$

$$\sigma_2 = \sigma_{\text{long}} = \frac{pr}{2t} = \frac{5(0.75)}{2(0.025)} = 75 \text{ MPa}$$

Maximum Shear Stress Theory. σ_1 and σ_2 have the sign. Thus,

$$|\sigma_1| = \sigma_{\text{allow}}$$

$$\sigma_{\text{allow}} = 150 \text{ MPa}$$

The factor of safety is

$$F.S. = \frac{\sigma_Y}{\sigma_{\text{allow}}} = \frac{250}{150} = 1.67 \qquad \textbf{Ans.}$$

(b) **Maximum Distortion Energy Theory.**

$$\sigma_1{}^2 - \sigma_1\sigma_2 + \sigma_2{}^2 = \sigma_{\text{allow}}{}^2$$

$$150^2 - 150(75) + 75^2 = \sigma_{\text{allow}}{}^2$$

$$\sigma_{\text{allow}} = 129.90 \text{ MPa}$$

The factor of safety is

$$F.S. = \frac{\sigma_Y}{\sigma_{\text{allow}}} = \frac{250}{129.90} = 1.92 \qquad \textbf{Ans.}$$

Ans:

$F.S. = 1.67, F.S. = 1.92$

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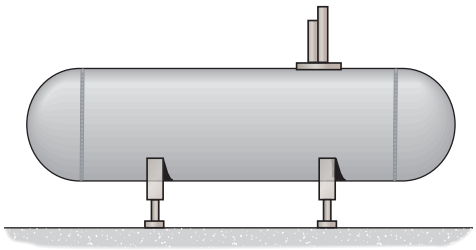
10–90.

The gas tank is made from A-36 steel and has an inner diameter of 1.50 m. If the tank is designed to withstand a pressure of 5 MPa, determine the required minimum wall thickness to the nearest millimeter using (a) the maximum shear stress theory, and (b) maximum distortion energy theory. Apply a factor of safety of 1.5 against yielding.



SOLUTION

(a) Normal Stress: Assuming that thin-wall analysis is valid, we have

$$\sigma_1 = \sigma_h = \frac{pr}{t} = \frac{5\left(10^6\right)(0.75)}{t} = \frac{3.75\left(10^6\right)}{t}$$

$$\sigma_2 = \sigma_{\text{long}} = \frac{pr}{2t} = \frac{5\left(10^6\right)(0.75)}{2t} = \frac{1.875\left(10^6\right)}{t}$$

Maximum Shear Stress Theory:

$$\sigma_{\text{allow}} = \frac{\sigma_Y}{FS.} = \frac{250\left(10^6\right)}{1.5} = 166.67\left(10^6\right)\text{Pa}$$

σ_1 and σ_2 have the same sign. Thus,

$$|\sigma_1| = \sigma_{\text{allow}}$$

$$\frac{3.75\left(10^6\right)}{t} = 166.67\left(10^6\right)$$

$$t = 0.0225 \text{ m} = 22.5 \text{ mm}$$ **Ans.**

Since $\frac{r}{t} = \frac{0.75}{0.0225} = 33.3 > 10$, thin-wall analysis is valid.

(b) Maximum Distortion Energy Theory:

$$\sigma_{\text{allow}} = \frac{\sigma_Y}{F.S.} = \frac{250\left(10^6\right)}{1.5} = 166.67\left(10^6\right)\text{Pa}$$

Thus,

$$\sigma_1{}^2 - \sigma_1\sigma_2 + \sigma_2{}^2 = \sigma_{\text{allow}}{}^2$$

$$\left[\frac{3.75\left(10^6\right)}{t}\right]^2 - \left[\frac{3.75\left(10^6\right)}{t}\right]\left[\frac{1.875\left(10^6\right)}{t}\right] + \left[\frac{1.875\left(10^6\right)}{t}\right]^2 = \left[166.67\left(10^6\right)\right]^2$$

$$\frac{3.2476\left(10^6\right)}{t} = 166.67\left(10^6\right)$$

$$t = 0.01949 \text{ m} = 19.5 \text{ mm}$$ **Ans.**

Since $\frac{r}{t} = \frac{0.75}{0.01949} = 38.5 > 10$, thin-wall analysis is valid.

Ans:
(a) $t = 22.5$ mm,
(b) $t = 19.5$ mm

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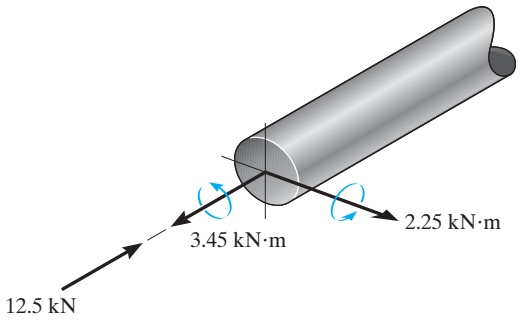
10–91. The internal loadings at a critical section along the steel drive shaft of a ship are calculated to be a torque of 3.45 KN · m, a bending moment of 2.25 kN · m, and an axial thrust of 12.5 kN. If the yield points for tension and shear are $\sigma_Y = 700$ MPa and $\sigma_Y = 350$ MPa, respectively, determine the required diameter of the shaft using the maximum-shear-stress theory.



SOLUTION

$$A = \pi c^2 \qquad I = \frac{\pi}{4}c^4 \qquad J = \frac{\pi}{2}c^4$$

$$\sigma_A = \frac{P}{A} + \frac{Mc}{I} = -\left[\frac{12.5(10^3)}{\pi c^2} + \frac{2.25(10^3)c}{\frac{\pi}{4}c^4}\right] = -\left[\frac{12.5(10^3)}{\pi c^2} + \frac{9(10^3)}{\frac{\pi}{4}c^3}\right]$$

$$\tau = \frac{Tc}{J} = \frac{3.45(10^3)c}{\frac{\pi}{2}c^4} = \frac{6.9(10^3)}{\pi c^3}$$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= -\left[\frac{12.5(10^3)c + 9(10^3)}{2\pi c^3}\right] \pm \sqrt{\left[\frac{12.5(10^3)c + 9(10^3)}{2\pi c^3}\right]^2 + \left[\frac{6.9(10^3)}{\pi c^3}\right]^2} \qquad (1)$$

Assume σ_1 and σ_2 have opposite signs:

$$|\sigma_1 - \sigma_2| = \sigma_Y$$

$$2\sqrt{\left[\frac{12.5(10^3)c + 9(10^3)}{2\pi c^3}\right]^2 + \left[\frac{6.9(10^3)}{\pi c^3}\right]^2} = 700(10^6)$$

$$490(10^9)\pi^2 c^6 - 156.25c^2 - 225c - 271.44 = 0$$

By trial and error:

$c = 0.019621$ m

Substitute c into Eq. (1):

$\sigma_1 = 155.20(10^6)$ N/m^2 = 155.20 MPa

$\sigma_2 = -544.81(10^6)$ N/m^2 = –544.81 MPa

σ_1 and σ_2 are of opposite signs **OK**

Therefore,

$d = 2(0.019621) = 0.039242$ m = 39.2 mm **Ans.**

Ans.
$d = 39.2$ mm

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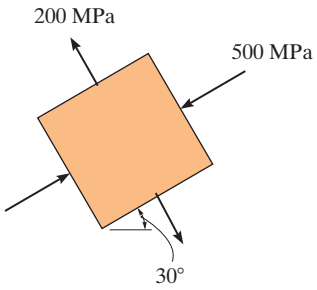
***10–92.**

If the material is machine steel having a yield stress of $\sigma_Y = 750$ MPa, determine the factor of safety with respect to yielding using the maximum distortion energy theory.



SOLUTION

Maximum Distortion Energy Theory: Here, $\sigma_1 = 200$ MPa and $\sigma_2 = -500$ MPa. The theory gives

$$\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2 = \sigma_{\text{allow}}^2$$

$$200^2 - (200)(-500) + (-500)^2 = \sigma_{\text{allow}}^2$$

$$\sigma_{\text{allow}} = 624.50 \text{ MPa}$$

The factor of safety is

$$\text{F.S.} = \frac{\sigma_Y}{\sigma_{\text{allow}}} = \frac{750}{624.50} = 1.20 \qquad \textbf{Ans.}$$

Ans:
F.S. = 1.20

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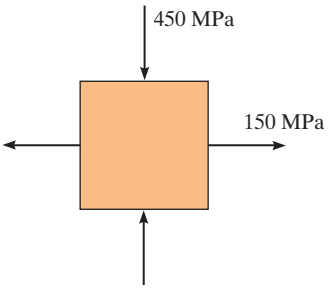
10–93.

If the material is machine steel having a yield stress of $\sigma_Y = 750$ MPa, determine the factor of safety with respect to yielding if the maximum shear stress theory is considered.



SOLUTION

Maximum Shear Stress Theory: Here, $\sigma_1 = 150$ MPa and $\sigma_2 = -450$ MPa since σ_1 and σ_2 have opposite signs, then

$$|\sigma_1 - \sigma_2| = \sigma_{\text{allow}}$$

$$|150 - (-450)| = \sigma_{\text{allow}}$$

$$\sigma_{\text{allow}} = 600 \text{ MPa}$$

The factor of safety is

$$\text{F.S.} = \frac{\sigma_Y}{\sigma_{\text{allow}}} = \frac{750}{600} = 1.25 \qquad \textbf{Ans.}$$

Ans:
F.S. = 1.25

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R10–1.

In the case of plane stress, where the in-plane principal strains are given by ϵ_1 and ϵ_2, show that the third principal strain can be obtained from

$$\epsilon_3 = \frac{-\nu(\epsilon_1 + \epsilon_2)}{(1 - \nu)}$$

where ν is Poisson's ratio for the material.

SOLUTION

Generalized Hooke's Law: In the case of plane stress, $\sigma_3 = 0$. Thus,

$$\epsilon_1 = \frac{1}{E}(\sigma_1 - \nu\sigma_2) \qquad \textbf{(1)}$$

$$\epsilon_2 = \frac{1}{E}(\sigma_2 - \nu\sigma_1) \qquad \textbf{(2)}$$

$$\epsilon_3 = -\frac{\nu}{E}(\sigma_1 + \sigma_2) \qquad \textbf{(3)}$$

Solving for σ_1 and σ_2 using Eqs. (1) and (2), we obtain

$$\sigma_1 = \frac{E(\epsilon_1 + \nu\epsilon_2)}{1 - \nu^2} \qquad \sigma_2 = \frac{E(\epsilon_2 + \nu\epsilon_1)}{1 - \nu^2}$$

Substituting these results into Eq. (3),

$$\epsilon_3 = -\frac{\nu}{E}\left[\frac{E(\epsilon_1 + \nu\epsilon_2)}{1 - \nu^2} + \frac{E(\epsilon_2 + \nu\epsilon_1)}{1 - \nu^2}\right]$$

$$\epsilon_3 = -\frac{\nu}{1 - \nu}\left[\frac{(\epsilon_1 + \epsilon_2) + \nu(\epsilon_1 + \epsilon_2)}{1 + \nu}\right]$$

$$\epsilon_3 = -\frac{\nu}{1 - \nu}\left[\frac{(\epsilon_1 + \epsilon_2)(1 + \nu)}{1 + \nu}\right]$$

$$\epsilon_3 = -\frac{\nu}{1 - \nu}(\epsilon_1 + \epsilon_2) \qquad \textbf{(Q.E.D.)} \qquad \textbf{Ans.}$$

Ans:
N/A

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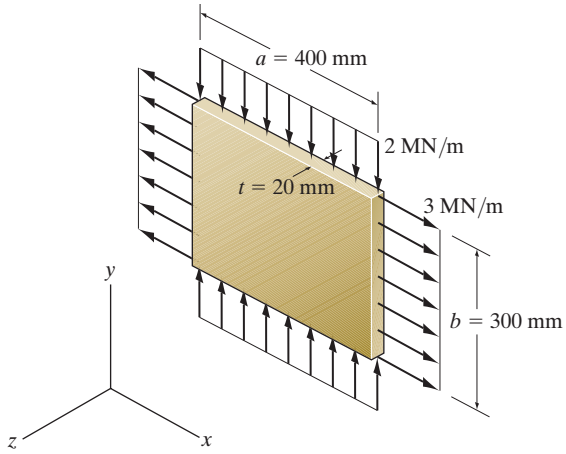
R10–2.

The plate is made of material having a modulus of elasticity $E = 200$ GPa and Poisson's ratio $\nu = \frac{1}{3}$. Determine the change in width a, height b, and thickness t when it is subjected to the uniform distributed loading shown.



SOLUTION

Normal Stress: The normal stresses along the x, y, and z axes are

$$\sigma_x = \frac{3(10^6)}{0.02} = 150 \text{ MPa}$$

$$\sigma_y = -\frac{2(10^6)}{0.02} = -100 \text{ MPa}$$

$$\sigma_z = 0$$

Generalized Hooke's Law:

$$\epsilon_x = \frac{1}{E}\left[\sigma_x - \nu(\sigma_y + \sigma_z)\right]$$

$$= \frac{1}{200(10^9)}\left\{150(10^6) - \frac{1}{3}\left[-100(10^6) + 0\right]\right\}$$

$$= 0.9167(10^{-3})$$

$$\epsilon_y = \frac{1}{E}\left[\sigma_y - \nu(\sigma_x + \sigma_z)\right]$$

$$= \frac{1}{200(10^9)}\left\{-100(10^6) - \frac{1}{3}\left[150(10^6) + 0\right]\right\}$$

$$= -0.75(10^{-3})$$

$$\epsilon_z = \frac{1}{E}\left[\sigma_z - \nu(\sigma_x + \sigma_y)\right]$$

$$= \frac{1}{200(10^9)}\left\{0 - \frac{1}{3}\left[150(10^6) + (-100)(10^6)\right]\right\}$$

$$= -83.33(10^{-6})$$

Thus, the changes in dimensions of the plate are

$$\delta_a = \epsilon_x a = 0.9167(10^{-3})(400) = 0.367 \text{ mm}$$ **Ans.**

$$\delta_b = \epsilon_y b = -0.75(10^{-3})(300) = -0.225 \text{ mm}$$ **Ans.**

$$\delta_t = \epsilon_z t = -83.33(10^{-6})(20) = -0.00167 \text{ mm}$$ **Ans.**

The negative signs indicate that b and t contract.

Ans:
$\delta_a = 0.367$ mm, $\delta_b = -0.255$ mm,
$\delta_t = -0.00167$ mm

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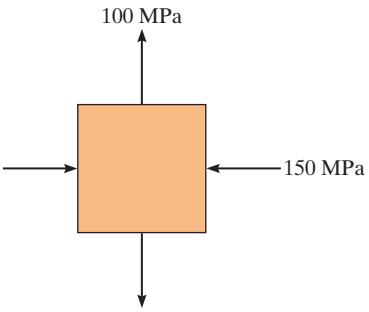
R10–3.

If the material is machine steel having a yield stress of $\sigma_Y = 500$ MPa, determine the factor of safety with respect to yielding if the maximum shear stress theory is considered.



SOLUTION

Here, the in plane principal stresses are

$$\sigma_1 = \sigma_y = 100 \text{ MPa} \quad \sigma_2 = \sigma_x = -150 \text{ MPa}$$

Since σ_1 and σ_2 have the same sign,

$$\text{F.S.} = \frac{\sigma_y}{|\sigma_1 - \sigma_2|} = \frac{500}{|100 - (-150)|} = 2 \qquad \textbf{Ans.}$$

Ans:
F.S. = 2

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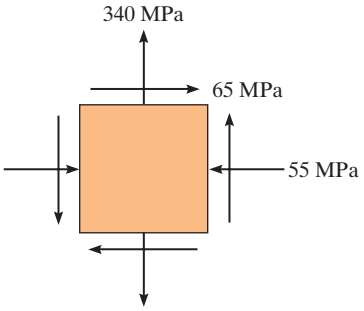
***R10–4.**

The components of plane stress at a critical point on a thin steel shell are shown. Determine if yielding has occurred on the basis of the maximum distortion energy theory. The yield stress for the steel is $\sigma_Y = 650$ MPa.



SOLUTION

$\sigma_x = -55$ MPa $\quad \sigma_y = 340$ MPa $\quad \tau_{xy} = 65$ MPa

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$$

$$= \frac{-55 + 340}{2} \pm \sqrt{\left(\frac{-55 - 340}{2}\right)^2 + 65^2}$$

$\sigma_1 = 350.42$ MPa $\quad \sigma_2 = -65.42$ MPa

$$(\sigma_1{}^2 - \sigma_1\sigma_2 + \sigma_2) = [350.42^2 - 350.42(-65.42) + (-65.42)^2]$$

$$= 150\,000 < \sigma_Y^2 = 422\,500 \qquad \textbf{OK}$$

No. **Ans.**

Ans:
No

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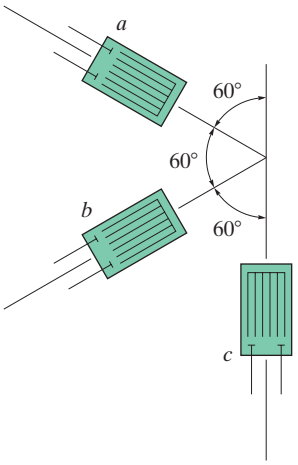
R10–5.

The 60° strain rosette is mounted on a beam. The following readings are obtained for each gage: $\epsilon_a = 600(10^{-6})$, $\epsilon_b = -700(10^{-6})$, and $\epsilon_c = 350(10^{-6})$. Determine (a) the in-plane principal strains and (b) the maximum in-plane shear strain and average normal strain. In each case show the deformed element due to these strains.



SOLUTION

Strain Rosettes (60°): Applying Eq. 10–15 with $\epsilon_x = 600\left(10^{-6}\right)$,

$\epsilon_b = -700\left(10^{-6}\right), \epsilon_c = 350\left(10^{-6}\right), \theta_a = 150°, \theta_b = -150°$ and $\theta_c = -90°$,

$$350\left(10^{-6}\right) = \epsilon_x \cos^2(-90°) + \epsilon_y \sin^2(-90°) + \gamma_{xy} \sin(-90°) \cos(-90°)$$

$$\epsilon_y = 350\left(10^{-6}\right)$$

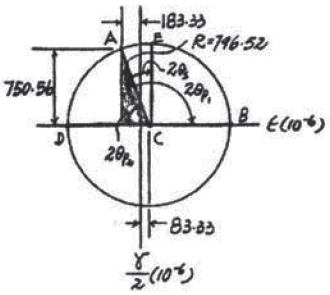
$$600\left(10^{-6}\right) = \epsilon_x \cos^2 150° + 350\left(10^{-6}\right) \sin^2 150° + \gamma_{xy} \sin 150° \cos 150°$$

$$512.5\left(10^{-6}\right) = 0.75\,\epsilon_x - 0.4330\,\gamma_{xy} \quad \textbf{(1)}$$

$$-700\left(10^{-6}\right) = \epsilon_x \cos^2(-150°) + 350\left(10^{-6}\right) \sin^2(-150°) + \gamma_{xy} \sin(-150°) \cos(-150°)$$

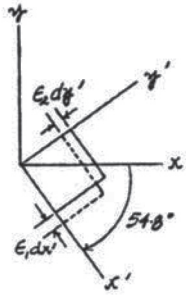
$$-787.5\left(10^{-6}\right) = 0.75\epsilon_x + 0.4330\,\gamma_{xy} \quad \textbf{(2)}$$

Solving Eq. (1) and (2) yields $\epsilon_x = -183.33\left(10^{-6}\right) \quad \gamma_{xy} = -1501.11\left(10^{-6}\right)$



Construction of the Circle: With $\epsilon_x = -183.33\left(10^{-6}\right)$, $\epsilon_y = 350\left(10^{-6}\right)$, and $\frac{\gamma_{xy}}{2} = -750.56\left(10^{-6}\right)$.

$$\epsilon_{avg} = \frac{\epsilon_x + \epsilon_y}{2} = \left(\frac{-183.33 + 350}{2}\right)\left(10^{-6}\right) = 83.3\left(10^{-6}\right) \quad \textbf{Ans.}$$



The coordinates for reference points A and C are

$A(-183.33, -750.56)\left(10^{-6}\right) \quad C(83.33, 0)\left(10^{-6}\right)$

The radius of the circle is

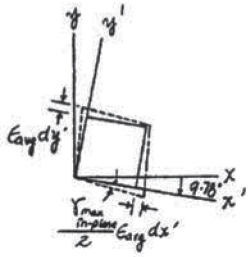
$$R = \left(\sqrt{(183.33 + 83.33)^2 + 750.56^2}\right)\left(10^{-6}\right) = 796.52\left(10^{-6}\right)$$

a)

In-Plane Principal Strain: The coordinates of points B and D represent ϵ_1 and ϵ_2, respectively.

$$\epsilon_1 = (83.33 + 796.52)\left(10^{-6}\right) = 880\left(10^{-6}\right) \quad \textbf{Ans.}$$

$$\epsilon_2 = (83.33 - 796.52)\left(10^{-6}\right) = -713\left(10^{-6}\right) \quad \textbf{Ans.}$$



Orientation of Principal Strain: From the circle,

$$\tan 2\theta_{P1} = \frac{750.56}{183.33 + 83.33} = 2.8145 \quad 2\theta_{P2} = 70.44°$$

$$2\theta_{P1} = 180° - 2\theta_{P2}$$

$$\theta_P = \frac{180° - 70.44°}{2} = 54.8° \quad \text{(Clockwise)} \quad \textbf{Ans.}$$

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R10–5. Continued

b)

Maximum In-Plane Shear Strain: Represented by the coordinates of point E on the circle.

$$\frac{\gamma_{\substack{\max \\ \text{in-plane}}}}{2} = -R = -796.52\left(10^{-6}\right)$$

$$\gamma_{\substack{\max \\ \text{in-plane}}} = -1593\left(10^{-6}\right) \quad \textbf{Ans.}$$

Orientation of Maximum In-Plane Shear Strain: From the circle.

$$\tan 2\theta_s = \frac{183.33 + 83.33}{750.56} = 0.3553$$

$$\theta_s = 9.78^\circ \text{ (Clockwise)} \quad \textbf{Ans.}$$

Ans:
$\epsilon_{\text{avg}} = 83.3(10^{-6})$, $\epsilon_1 = 880(10^{-6})$,
$\epsilon_2 = -713(10^{-6})$, $\theta_p = 54.8^\circ$ (clockwise),
$\gamma_{\substack{\max \\ \text{in-plane}}} = -1593(10^{-6})$,
$\theta_s = 9.78^\circ$ (clockwise)

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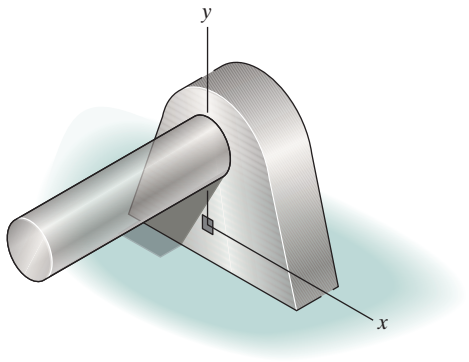
R10–6.

The state of strain at the point on the bracket has components $\epsilon_x = 350(10^{-6})$, $\epsilon_y = -860(10^{-6})$, $\gamma_{xy} = 250(10^{-6})$. Use the strain transformation equations to determine the equivalent in-plane strains on an element oriented at an angle of $\theta = 45°$ clockwise from the original position. Sketch the deformed element within the x–y plane due to these strains.



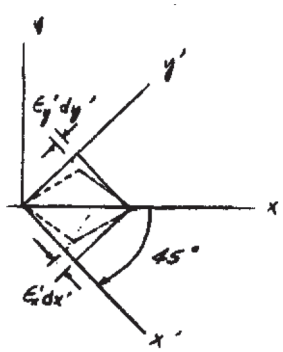
SOLUTION

$\epsilon_x = 350(10^{-6})$ $\epsilon_y = -860(10^{-6})$ $\gamma_{xy} = 250(10^{-6})$ $\theta = -45°$

$$\epsilon_{x'} = \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$= \left[\frac{350 - 860}{2} + \frac{350 - (-860)}{2}\cos(-90°) + \frac{250}{2}\sin(-90°)\right](10^{-6})$$

$$= -380(10^{-6}) \quad \textbf{Ans.}$$



$$\epsilon_{y'} = \frac{\epsilon_x + \epsilon_y}{2} - \frac{\epsilon_x - \epsilon_y}{2}\cos 2\theta - \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$= \left[\frac{350 - 860}{2} - \frac{350 - (-860)}{2}\cos(-90°) - \frac{250}{2}\sin(-90°)\right](10^{-6})$$

$$= -130(10^{-6}) \quad \textbf{Ans.}$$

$$\frac{\gamma_{x'y'}}{2} = -\frac{\epsilon_x - \epsilon_y}{2}\sin 2\theta + \frac{\gamma}{2}\cos 2\theta$$

$$\gamma_{x'y'} = 2\left[-\left(\frac{350 - (-860)}{2}\right)\sin(-90°) + \frac{250}{2}\cos(-90°)\right](10^{-6}) = 1.21(10^{-3}) \quad \textbf{Ans.}$$

Ans:
$\epsilon_{x'} = -380(10^{-6})$, $\epsilon_{y'} = -130(10^{-6})$,
$\gamma_{x'y'} = 1.21(10^{-3})$

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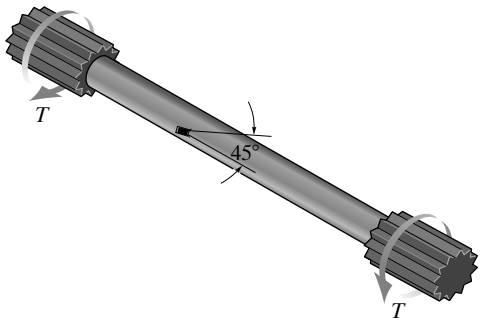
R10–7.

A strain gauge forms an angle of 45° with the axis of the 50-mm diameter shaft. If it gives a reading of $P = -200(10^{-6})$ when the torque **T** is applied to the shaft, determine the magnitude of **T**. The shaft is made from A-36 steel.

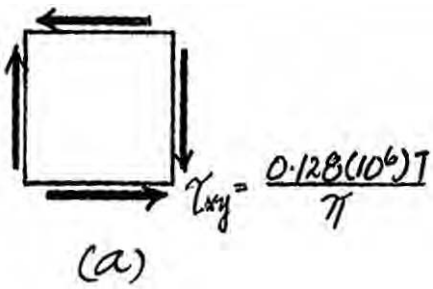


SOLUTION

Shear Stress. This is a case of pure shear, and the shear stress developed is contributed by torsional shear stress. Here, $J = \frac{\pi}{2}\left(0.025^4\right) = 0.1953125\pi\left(10^{-6}\right)\text{m}^4$. Then

$$\tau = \frac{Tc}{J} = \frac{T(0.025)}{0.1953125\pi\left(10^{-6}\right)} = \frac{0.128\left(10^6\right)T}{\pi}$$

The state of stress at points on the surface of the shaft can be represented by the element shown in Fig. *a*.



Shear Strain: For pure shear $\varepsilon_x = \varepsilon_y = 0$. We obtain,

$$\varepsilon_a = \varepsilon_x \cos^2\theta_a + \varepsilon_y \sin^2\theta_a + \gamma_{xy}\sin\theta_a \cos\theta_a$$

$$-200\left(10^{-6}\right) = 0 + 0 + \gamma_{xy}\sin 45° \cos 45°$$

$$\gamma_{xy} = -400\left(10^{-6}\right)$$

Shear Stress and Strain Relation: Applying Hooke's Law for shear,

$$\tau_{xy} = G\gamma_{xy}$$

$$-\frac{0.128\left(10^6\right)T}{\pi} = 75\left(10^9\right)\left[-400\left(10^{-6}\right)\right]$$

$$T = 736\ \text{N}\cdot\text{m} \qquad \textbf{Ans.}$$

Ans:

$T = 736\ \text{N}\cdot\text{m}$

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***R10–8.**

A differential element is subjected to plane strain that has the following components; $\epsilon_x = 950(10^{-6})$, $\epsilon_y = 420(10^{-6})$, $\gamma_{xy} = -325(10^{-6})$. Use the strain-transformation equations and determine (a) the principal strains and (b) the maximum in-plane shear strain and the associated average strain. In each case specify the orientation of the element and show how the strains deform the element.

SOLUTION

$$\epsilon_{1,2} = \frac{\epsilon_x + \epsilon_y}{2} \pm \sqrt{\left(\frac{\epsilon_x - \epsilon_y}{2}\right)^2 + {\gamma_{xy}}^2}$$

$$= \left[\frac{950 + 420}{2} \pm \sqrt{\left(\frac{950 - 420}{2}\right)^2 + \left(\frac{-325}{2}\right)^2}\right](10^{-6})$$

$$\epsilon_1 = 996(10^{-6}) \quad \textbf{Ans.}$$

$$\epsilon_2 = 374(10^{-6}) \quad \textbf{Ans.}$$

Orientation of ϵ_1 and ϵ_2:

$$\tan 2\theta_P = \frac{\gamma_{xy}}{\epsilon_x - \epsilon_y} = \frac{-325}{950 - 420}$$

$$\theta_P = -15.76°, 74.24°$$

Use Eq. 10–5 to determine the direction of ϵ_1 and ϵ_2.

$$\epsilon_{x'} = \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$\theta = \theta_P = -15.76°$$

$$\epsilon_{x'} = \left\{\frac{950 + 420}{2} + \frac{950 - 420}{2}\cos(-31.52°) + \frac{(-325)}{2}\sin(-31.52°)\right\}(10^{-6}) = 996(10^{-6})$$

$\theta_{P1} = -15.8°$ **Ans.**

$\theta_{P2} = 74.2°$ **Ans.**

b)

$$\frac{\gamma_{\text{in-plane}}^{\max}}{2} = \sqrt{\left(\frac{\epsilon_x - \epsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$\gamma_{\text{in-plane}}^{\max} = 2\left[\sqrt{\left(\frac{950 - 420}{2}\right)^2 + \left(\frac{-325}{2}\right)^2}\right](10^{-6}) = 622(10^{-6}) \quad \textbf{Ans.}$$

$$\epsilon_{\text{avg}} = \frac{\epsilon_x + \epsilon_y}{2} = \left(\frac{950 + 420}{2}\right)(10^{-6}) = 685(10^{-6}) \quad \textbf{Ans.}$$

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***R10–8. Continued**

Orientation of γ_{max}:

$$\tan 2\theta_P = \frac{-(\epsilon_x - \epsilon_y)}{\gamma_{xy}} = \frac{-(950 - 420)}{-325}$$

$\theta_P = 29.2°$ and $\theta_P = 119°$ **Ans.**

Use Eq. 10–6 to determine the sign of γ_{max} in-plane:

$$\frac{\gamma_{x'y'}}{2} = -\frac{\epsilon_x - \epsilon_y}{2} \sin 2\theta + \frac{\gamma_{xy}}{2} \cos 2\theta$$

$$\theta = \theta_s = 29.2°$$

$$\gamma_{x'y'} = 2\left[\frac{-(950 - 420)}{2} \sin (58.4°) + \frac{-325}{2} \cos (58.4°)\right](10^{-6})$$

$$\gamma_{xy} = -622(10^{-6})$$

Ans:
$\epsilon_1 = 996(10^{-6}), \epsilon_2 = 374(10^{-6}), \theta_{p1} = -15.8,$
$\theta_{p2} = 74.2, \gamma_{\text{max}}$ in-plane $= 622(10^{-6}),$
$\epsilon_{\text{avg}} = 685(10^{-6}), \theta_s = 29.2°$ and $119°$

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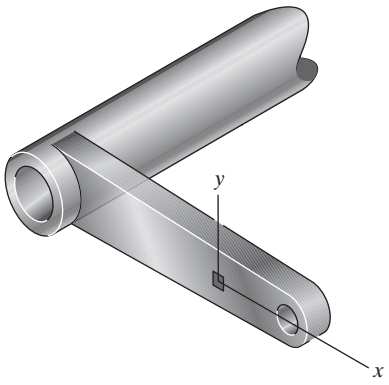
R10–9.

The state of strain at the point on the bracket has components $\epsilon_x = -130(10^{-6})$, $\epsilon_y = 280(10^{-6})$, $\gamma_{xy} = 75(10^{-6})$. Use the strain transformation equations to determine (a) the in-plane principal strains and (b) the maximum in-plane shear strain and average normal strain. In each case specify the orientation of the element and show how the strains deform the element within the x–y plane.



SOLUTION

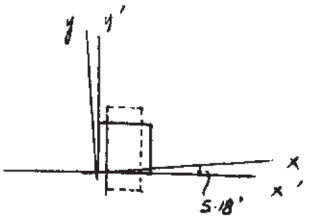
$\epsilon_x = -130(10^{-6})$ $\epsilon_y = 280(10^{-6})$ $\gamma_{xy} = 75(10^{-6})$

a)

$$\epsilon_{1,2} = \frac{\epsilon_x + \epsilon_y}{2} \pm \sqrt{\left(\frac{\epsilon_x - \epsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$= \left[\frac{-130 + 280}{2} \pm \sqrt{\left(\frac{-130 - 280}{2}\right)^2 + \left(\frac{75}{2}\right)^2}\right](10^{-6})$$

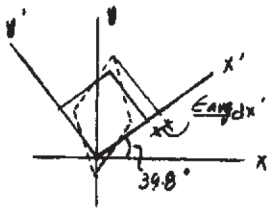
$\epsilon_1 = 283(10^{-6})$ **Ans.** $\epsilon_2 = -133(10^{-6})$ **Ans.**



Orientation of ϵ_1 and ϵ_2:

$$\tan 2\theta_p = \frac{\gamma_{xy}}{\epsilon_x - \epsilon_y} = \frac{75}{-130 - 280}$$

$\theta_p = -5.18°$ and $84.82°$



Use Eq. 10–5 to determine the direction of ϵ_1 and ϵ_2:

$$\epsilon_{x'} = \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$\theta = \theta_p = -5.18°$

$$\epsilon_{x'} = \left[\frac{-130 + 280}{2} + \frac{-130 - 280}{2}\cos(-10.37°) + \frac{75}{2}\sin(-10.37°)\right](10^{-6}) = -133(10^{-6})$$

Therefore $\theta_{p1} = 84.8°$ **Ans.** $\theta_{p2} = -5.18°$ **Ans.**

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R10–9. Continued

b)

$$\frac{\gamma_{\substack{\max\\ \text{in-plane}}}}{2} = \sqrt{\left(\frac{\epsilon_x - \epsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$\gamma_{\substack{\max\\ \text{in-plane}}} = 2\left[\sqrt{\left(\frac{-130 - 280}{2}\right)^2 + \left(\frac{75}{2}\right)^2}\right](10^{-6}) = 417(10^{-6}) \quad \textbf{Ans.}$$

$$\epsilon_{\text{avg}} = \frac{\epsilon_x + \epsilon_y}{2} = \left(\frac{-130 + 280}{2}\right)(10^{-6}) = 75.0(10^{-6}) \quad \textbf{Ans.}$$

Orientation of $\gamma_{\max}$:

$$\tan 2\theta_s = \frac{-(\epsilon_x - \epsilon_y)}{\gamma_{xy}} = \frac{-(-130 - 280)}{75}$$

$$\theta_s = 39.8° \text{ and } \theta_s = 130° \quad \textbf{Ans.}$$

Use Eq. 10–16 to determine the sign of $\gamma_{\substack{\max\\ \text{in-plane}}}$:

$$\frac{\gamma_{x'y'}}{2} = -\frac{\epsilon_x - \epsilon_y}{2}\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta; \qquad \theta = \theta_s = 39.8°$$

$$\gamma_{x'y'} = -(-130 - 280)\sin(79.6°) + (75)\cos(79.6°) = 417(10^{-6})$$

Ans:

$\epsilon_1 = 283(10^{-6})$, $\epsilon_2 = -133(10^{-6})$,
$\theta_{p1} = 84.8°$, $\theta_{p2} = -5.18°$, $\gamma_{\substack{\max\\ \text{in-plane}}} = 417(10^{-6})$,
$\epsilon_{\text{avg}} = 75.0(10^{-6})$, $\theta_s = 39.8°$ and $130°$

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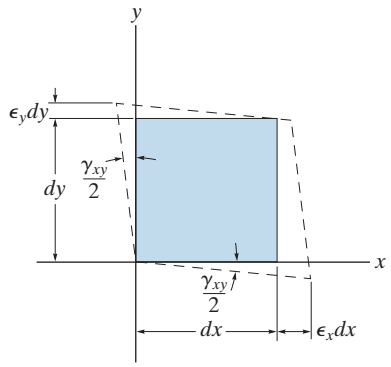
R10–10.

The state of plane strain on the element is $\epsilon_x = 400(10^{-6})$, $\epsilon_y = 200(10^{-6})$, and $\gamma_{xy} = -300(10^{-6})$. Determine the equivalent state of strain, which represents (a) the principal strains, and (b) the maximum in-plane shear strain and the associated average normal strain. Specify the orientation of the corresponding element at the point with respect to the original element. Sketch the results on the element.



SOLUTION

Construction of the Circle: $\epsilon_x = 400\left(10^{-6}\right)$, $\epsilon_y = 200\left(10^{-6}\right)$, and $\frac{\gamma_{xy}}{2} = -150\left(10^{-6}\right)$. Thus,

$$\epsilon_{\text{avg}} = \frac{\epsilon_x + \epsilon_y}{2} = \left(\frac{400 + 200}{2}\right)\left(10^{-6}\right) = 300\left(10^{-6}\right) \quad \textbf{Ans.}$$

The coordinates for reference points A and the center C of the circle are

$$A(400, -150)\left(10^{-6}\right) \qquad C(300, 0)\left(10^{-6}\right)$$

The radius of the circle is

$$R = CA = \sqrt{(400 - 300)^2 + (-150)^2} = 180.28\left(10^{-6}\right)$$

Using these results, the circle is shown in Fig. *a*.

In-Plane Principal Stresses: The coordinates of points B and D represent ϵ_1 and ϵ_2, respectively. Thus,

$$\epsilon_1 = (300 + 180.28)\left(10^{-6}\right) = 480\left(10^{-6}\right) \quad \textbf{Ans.}$$

$$\epsilon_2 = (300 - 180.28)\left(10^{-6}\right) = 120\left(10^{-6}\right) \quad \textbf{Ans.}$$

Orientation of Principal Plane: Referring to the geometry of the circle,

$$\tan 2\left(\theta_p\right)_1 = \frac{150}{400 - 300} = 1.5$$

$$\left(\theta_p\right)_1 = 28.2° \text{ (clockwise)} \quad \textbf{Ans.}$$

The deformed element for the state of principal strains is shown in Fig. *b*.

Maximum In-Plane Shear Stress: The coordinates of point E represent ϵ_{avg} and $\gamma_{\substack{\max \\ \text{in-plane}}}$. Thus

$$\frac{\gamma_{\substack{\max \\ \text{in-plane}}}}{2} = -R = -180.28\left(10^{-6}\right)$$

$$\gamma_{\substack{\max \\ \text{in-plane}}} = -361\left(10^{-6}\right) \quad \textbf{Ans.}$$

Orientation of the Plane of Maximum In-plane Shear Strain: Referring to the geometry of the circle,

$$\tan 2\theta_s = \frac{400 - 300}{150} = 0.6667$$

$$\theta_S = 16.8° \text{ (counterclockwise)} \quad \textbf{Ans.}$$

The deformed element for the state of maximum In-plane shear strain is shown in Fig. *c*.

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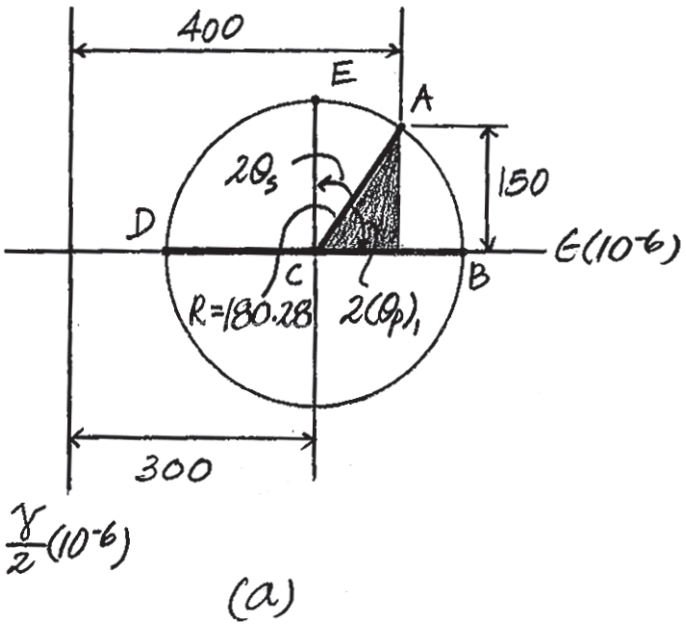
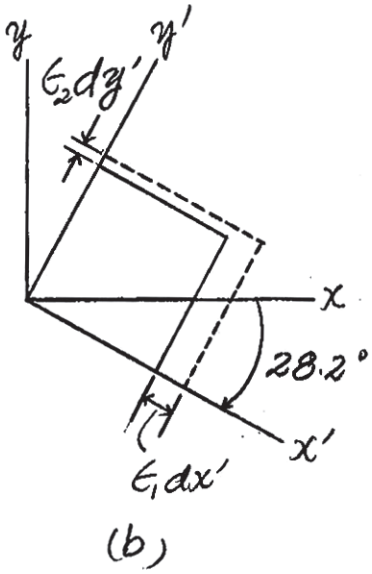
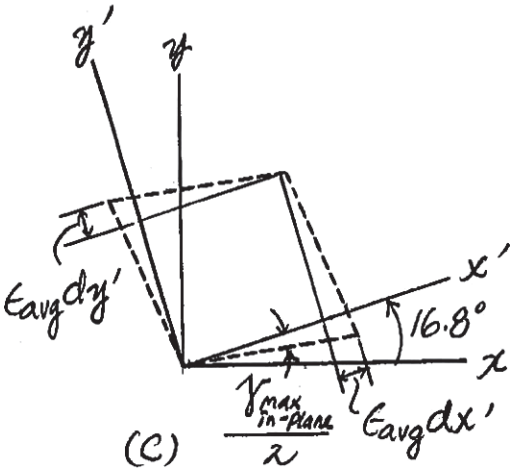
R10–10. Continued



(a)



(b)



(c)

Ans:
$\epsilon_1 = 480(10^{-6})$, $\epsilon_2 = 120(10^{-6})$,
$\theta_{p1} = 28.2°$ (clockwise),
$\gamma_{\text{in-plane}}^{\max} = -361(10^{-6})$,
$\theta_s = 16.8°$ (counterclockwise),
$\epsilon_{avg} = 300(10^{-6})$

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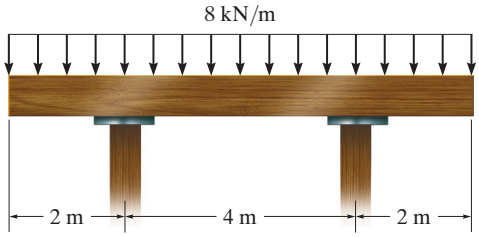
11–1.

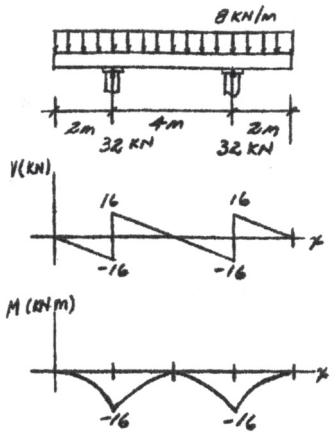
The beam is made of timber that has an allowable bending stress of $\sigma_{allow} = 6.5$ MPa and an allowable shear stress of $\tau_{allow} = 500$ kPa. Determine its dimensions if it is to be rectangular and have a height-to-width ratio of 1.25. Assume the beam rests on smooth supports.



SOLUTION

$$I_x = \frac{1}{12}(b)(1.25b)^3 = 0.16276b^4$$

$$Q_{max} = \bar{y}'A' = (0.3125b)(0.625b)(b) = 0.1953125b^3$$



Assume Bending Moment Controls:

$$M_{max} = 16 \text{ kN} \cdot \text{m}$$

$$\sigma_{allow} = \frac{M_{max}\, c}{I}$$

$$6.5(10^6) = \frac{16(10^3)(0.625b)}{0.16276b^4}$$

$$b = 0.21143 \text{ m} = 211 \text{ mm} \qquad \textbf{Ans.}$$

$$h = 1.25b = 264 \text{ mm} \qquad \textbf{Ans.}$$

Check Shear:

$$Q_{max} = 1.846159(10^{-3}) \text{ m}^3$$

$$I = 0.325248(10^{-3}) \text{ m}^4$$

$$\tau_{max} = \frac{VQ_{max}}{It} = \frac{16(10^3)(1.846159)(10^{-3})}{0.325248(10^{-3})(0.21143)} = 429 \text{ kPa} < 500 \text{ kPa}, \qquad \textbf{OK}$$

These solutions represent a preliminary version of the Instructors' Solutions Manual (ISM). It is possible and even likely that at this preliminary stage of preparing the ISM there are some omissions and errors in the draft solutions. These will be corrected and this manual will be republished.

Ans:
$b = 211$ mm, $h = 264$ mm

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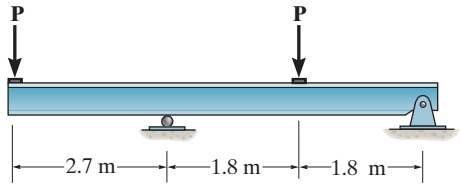
11–2. Select the lightest-weight W310 steel wide-flang beam from Appendix B that will safely support the loading shown, where $P = 30$ kN. The allowable bending stress is $\sigma_{allow} = 150$ MPa and the allowable shear stress is $\tau_{allow} = 84$ MPa.



SOLUTION

From the Moment Diagram, Fig. *a*, $M_{max} = 81$ kN · m.

$$S_{req'd} = \frac{M_{max}}{\sigma_{allow}}$$

$$= \frac{81(10^3)}{150(10^6)}$$

$$= 0.54(10^{-3})\ \text{m}^3 = 540(10^3)\ \text{mm}^3$$

Select W310 × 39 $[S_x = 547(10^3)\ \text{mm}^3, d = 310\ \text{mm}, t_w = 5.84\ \text{mm}]$

From the shear diagram, Fig. *a*, $V_{max} = 37.5$ kN. Provide the shear-stress check for W310 × 39.

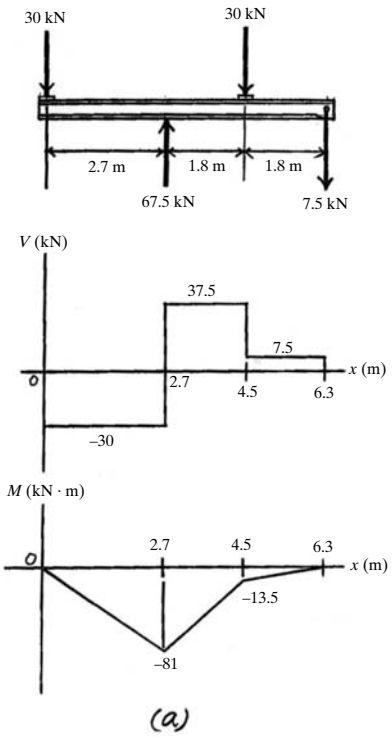
$$\tau_{max} = \frac{V_{max}}{t_w d}$$

$$= \frac{37.5(10^3)}{0.00584(0.310)} = 20.71(10^6)\ \text{N/m}^2$$

$$= 20.7\ \text{MPa} < \tau_{allow} = 84\ \text{MPa}\ (O.K.!)$$

Hence

Use W310 × 39 **Ans.**



(a)

Ans:

Use W310 × 39

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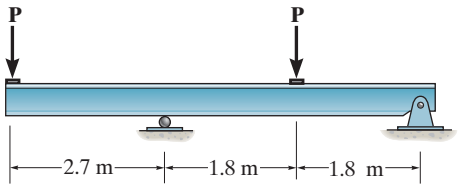
11–3. Select the lightest-weight W360 steel wide-flang beam from Appendix B that will safely support the loading shown, where $P = 60$ kN. The allowable bending stress is $\sigma_{allow} = 150$ MPa and the allowable shear stress is $\tau_{allow} = 84$ MPa.



SOLUTION

$$S_{req'd} = \frac{M_{max}}{\sigma_{allow}}$$

$$= \frac{162(10^3)}{150(10^6)}$$

$$= 1.08(10^{-3})\ m^3 = 1080(10^3)\ mm^3$$

Select W360 × 79 $[S_x = 1280(10^3)\ mm^3, d = 354\ mm, t_w = 9.4\ mm]$

From the shear diagram, Fig. *a*, $V_{max} = 75$ kN. Provide the shear-stress check for W360 × 79,

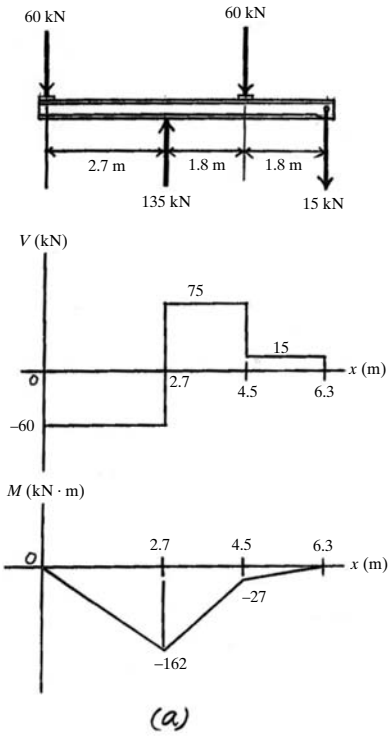
$$\tau_{max} = \frac{V_{max}}{t_w\, d}$$

$$= \frac{75(10^3)}{0.0094(0.354)} = 22.54(10^6)\ N/m^2$$

$$= 22.5\ MPa < \tau_{allow} = 84\ MPa\ (O.K!)$$

Hence,

Use W360 × 79 **Ans.**



Ans:

Use W360 × 79

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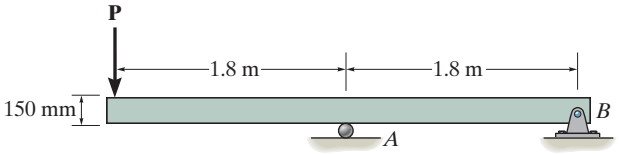
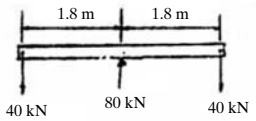
***11–4.** Determine the minimum width of the beam to the nearest multiples of 5 mm that will safely support the loading of $P = 40$ kN. The allowable bending stress is $\sigma_{allow} = 168$ MPa and the allowable shear stress is $\tau_{allow} = 100$ MPa.





SOLUTION

Beam design: Assume moment controls.

$$\sigma_{allow} = \frac{M\,c}{I}; \qquad 168(10^6) = \frac{[72.0(10^3)](0.075)}{\frac{1}{12}b(0.15^3)}$$

$b = 0.1143$ m $= 114.3$ mm

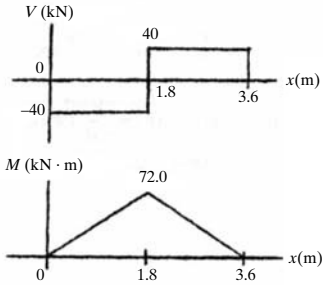
Use $b = 115$ mm

Check shear:

$$\tau_{max} = \frac{VQ}{I\,t} = \frac{[40(10^3)][0.0375(0.115)(0.075)]}{\left[\frac{1}{12}(0.115)(0.15^3)\right](0.115)}$$

$$= 3.478(10^6)\ \text{N/m}^2 = 3.48\ \text{MPa} < 100\ \text{MPa}\ \text{OK}$$

Ans.



Ans:

$b = 114.3$ mm

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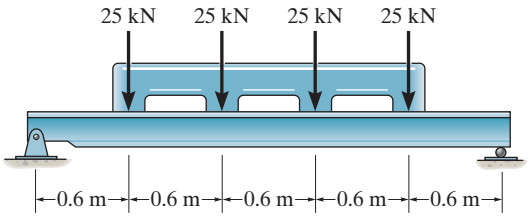
11–5. Select the lightest-weight steel wide-flange beam from Appendix B that will safely support the machine loading shown. The allowable bending stress is $\sigma_{allow} = 168$ MPa and the allowable shear stress is $\tau_{allow} = 100$ MPa.

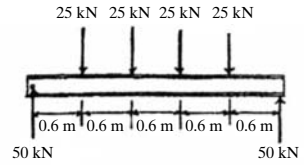


SOLUTION

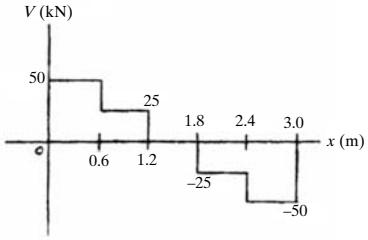
Bending Stress: From the moment diagram, $M_{max} = 45$ kN · m. Assume bending controls the design. Applying the flexure formula.

$$S_{req'd} = \frac{M_{max}}{\sigma_{allow}}$$

$$= \frac{45(10^3)}{168(10^6)} = 0.2679(10^{-3})\ \text{m}^3 = 267.9(10^3)\ \text{mm}^3$$



Select W310 × 24 ($S_x = 281(10^3)$ mm^3, $d = 305$ mm, $t_w = 5.59$ mm)

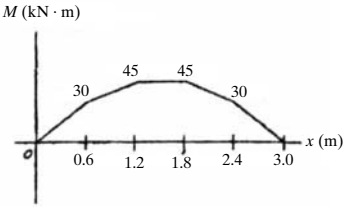


Shear Stress: Provide a shear stress check using $\tau = \frac{V}{t_w d}$ for the W310 × 24 wide-flange section. From the shear diagram, $V_{max} = 50$ kN

$$\tau_{max} = \frac{V_{max}}{t_w d}$$

$$= \frac{50(10^3)}{0.00559(0.305)} = 29.33(10^6)\ \text{N/m}^2$$

$$= 27.8\ \text{MPa} < \tau_{allow} = 100\ \text{MPa}\ (O.K!)$$



Hence,

Use W310 × 24 **Ans.**

Ans:
Use W310 × 24

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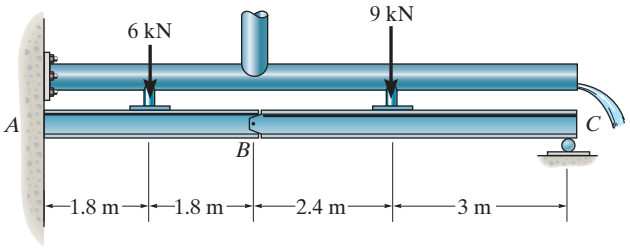
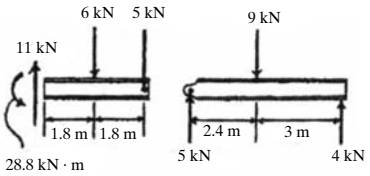
11–6. The compound beam is made from two sections, which are pinned together at B. Use Appendix B and select the lightest-weight wide-flange beam that would be safe for each section if the allowable bending stress is σ_{allow} = 168 MPa and the allowable shear stress is τ_{allow} = 100 MPa. The beam supports a pipe loading of 6 kN and 9 kN as shown.



SOLUTION

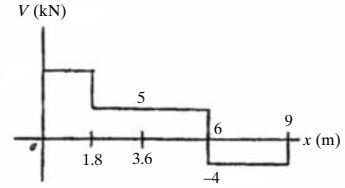
Bending Stress: From the moment diagram, $M_{max} = 28.8\ kN \cdot m$ for member AB. Assuming bending controls the design, applying the flexure formula.



$$S_{req'd} = \frac{M_{max}}{\sigma_{allow}}$$

$$= \frac{28.8(10^3)}{168(10^6)} = 0.1714(10^{-3})\ m^3 = 171.4(10^3)\ mm^3$$

Select W250 × 18 (S_x =179(10^3) mm^3, d = 251 mm, t_w = 4.83 mm)

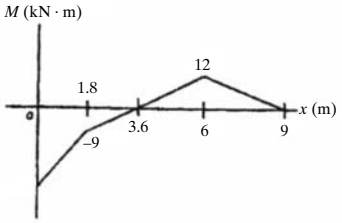


For member BC, $M_{max} = 12\ kN \cdot m$.

$$S_{req'd} = \frac{M_{max}}{\sigma_{allow}}$$

$$= \frac{12(10^3)}{168(10^6)} = 71.43(10^{-6})\ m^3 = 71.43(10^3)\ mm^3$$

Select W150 × 14 (S_x = 91.2(10^3) mm^3, d = 150 mm, t_w = 4.32 mm)



Shear Stress: Provide a shear stress check using $\tau = \frac{V}{t_w d}$ for the W250 × 18 wide-flange section for member AB. From the shear diagram, $V_{max} = 11$ kN.

$$\tau_{max} = \frac{V_{max}}{t_w d}$$

$$= \frac{11(10^3)}{0.00483(0.251)} = 9.073(10^6)\ N/m^2 = 9.07\ MPa$$

$$= 9.07\ MPa < \tau_{allow} = 100\ MPa\ (O.K!)$$

Use W250 × 18 **Ans.**

For member BC (W150 × 14), V_{max} = 5 kN

$$\tau_{max} = \frac{V_{max}}{t_w d}$$

$$= \frac{5(10^3)}{0.00432(0.15)} = 7.716(10^6)\ N/m^2 = 7.72\ MPa$$

$$= 7.72\ MPa < \tau_{allow} = 100\ MPa\ (O.K!)$$

Hence,

Use W150 × 14 **Ans.**

Ans:
Use W250 × 18, Use W150 × 14

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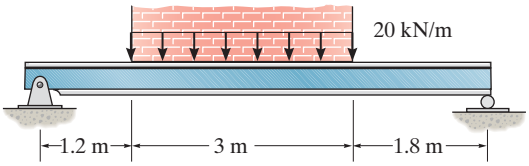
11–7. The brick wall exerts a uniform distributed load of 20 kN/m on the beam. If the allowable bending stress is $\sigma_{allow} = 154$ MPa and the allowable shear stress is $\tau_{allow} = 84$ MPa, select the lightest wide-flange section from Appendix B that will safely support the load.



SOLUTION

Bending Stress: From the moment diagram, $M_{max} = 66.825\ \text{kN} \cdot \text{m}$. Assuming bending controls the design and applying the flexure formula.

$$S_{req\ d} = \frac{M_{max}}{\sigma_{allow}}$$

$$= \frac{66.825(10^3)}{154(10^6)} = 0.4339(10^{-3})\ \text{m}^3 = 433.9(10^3)\ \text{mm}^3$$

Select W360 × 33 $(S_x = 475(10^3)\ \text{mm}^3, d = 349\ \text{mm}, t_w = 5.84\ \text{mm})$

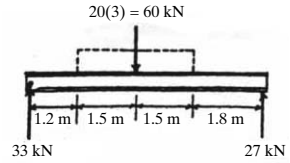
Shear Stress: Provide a shear stress check using $\tau = \frac{V}{t_w d}$ for the W360 × 33 wide-flange section. From the shear diagram, $V_{max} = 33$ kN.

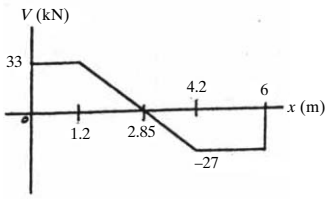
$$\tau_{max} = \frac{V_{max}}{t_w d}$$

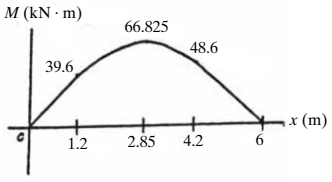
$$= \frac{33(10^3)}{0.00584(0.349)} = 16.19(10^6)\ \text{N/m}^2 = 16.2\ \text{MPa}$$

$$= 16.2\ \text{MPa} < \tau_{allow} = 84\ \text{MPa}\ (O.K!)$$

Hence, **Use** W360 × 33 **Ans.**







Ans:
Use W360 × 33

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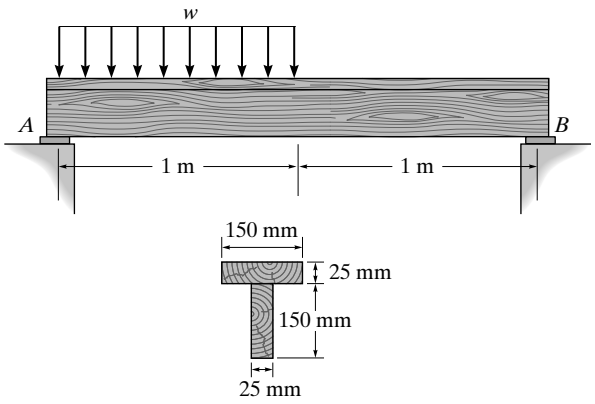
***11–8.** If the bearing pads at A and B support only vertical forces, determine the greatest magnitude of the uniform distributed loading w that can be applied to the beam. $\sigma_{allow} = 15$ MPa, $\tau_{allow} = 1.5$ MPa.



SOLUTION

The location of c, Fig. b, is

$$\bar{y} = \frac{\Sigma \bar{y} A}{\Sigma A} = \frac{0.1625(0.025)(0.15) + 0.075(0.15)(0.025)}{0.025(0.15) + 0.15(0.025)}$$

$$= 0.11875 \text{ m}$$

$$I = \frac{1}{12}(0.025)(0.15^3) + (0.025)(0.15)(0.04375)^2$$

$$+ \frac{1}{12}(0.15)(0.025^3) + 0.15(0.025)(0.04375)^2$$

$$= 21.5820(10^{-6}) \text{ m}^4$$

Referring to Fig. b,

$$Q_{max} = \bar{y}' A' = 0.059375\ (0.11875)(0.025)$$

$$= 0.17627(10^{-3}) \text{ m}^3$$

Referring to the moment diagram, $M_{max} = 0.28125\ w$. Applying the Flexure formula with $C = \bar{y} = 0.11875$ m,

$$\sigma_{allow} = \frac{M_{max}\, c}{I}; \qquad 15(10^6) = \frac{0.28125w(0.11875)}{21.5820(10^{-6})}$$

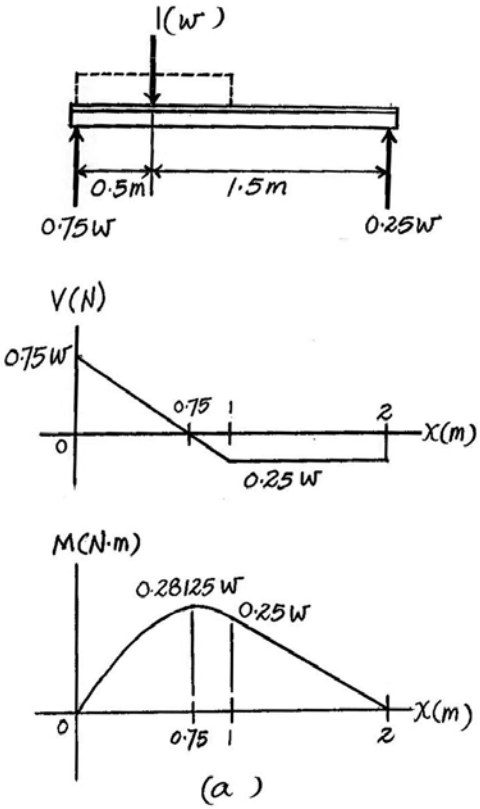
$$W = 9.693(10^3) \text{ N/m}$$

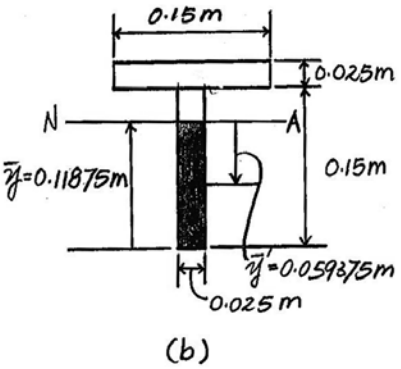
Referring to shear diagram, Fig. a, $V_{max} = 0.75\ w$.

$$\tau_{allow} = \frac{V_{allow} Q_{max}}{It}; \qquad 1.5(10^6) = \frac{0.75w\left[0.17627(10^{-3})\right]}{21.5820(10^{-6})(0.025)}$$

$$W = 6.122(10^3) \text{ N/m}$$

$$= 6.12 \text{ kN/m } (\textit{Control}!)$$





Ans.

Ans:

$W = 6.12$ kN/m ($\textit{Control}!$)

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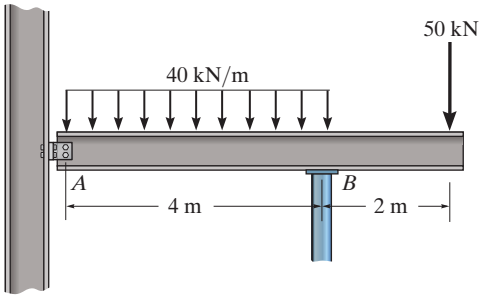
11–9.

Select the lightest W360 wide-flange beam from Appendix B that can safely support the loading. The beam has an allowable normal stress of $\sigma_{allow} = 150$ MPa and an allowable shear stress of $\tau_{allow} = 80$ MPa. Assume there is a pin at A and a roller support at B.



SOLUTION

Shear and Moment Diagram: As shown in Fig. *a*.

Bending Stress: Referring to the moment diagram, Fig. *a*, $M_{max} = 100\ \text{kN} \cdot \text{m}$. Applying the flexure formula,

$$S_{required} = \frac{M_{max}}{\sigma_{allow}} = \frac{100(10^3)}{150(10^6)}$$

$$= 0.6667(10^{-3})\ \text{m}^3 = 666.67(10^3)\ \text{mm}^3$$

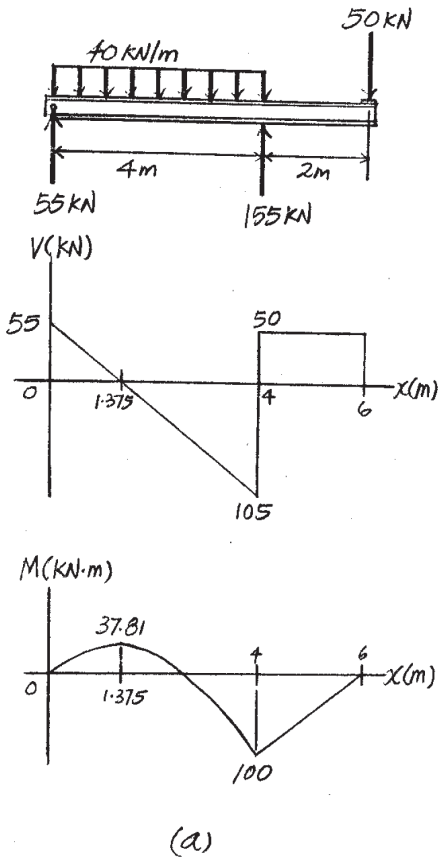
Select W360 × 45($S_x = 688(10^3)\ \text{mm}^3$, $d = 352$ mm and $t_w = 6.86$ mm)

Shear Stress: Referring to the shear diagram, Fig. *a*, $V_{max} = 105$ kN. We have

$$\tau_{max} = \frac{V_{max}}{t_w d} = \frac{105(10^3)}{6.86(10^{-3})(0.352)}$$

$$= 43.48\ \text{MPa} < \tau_{allow} = 80\ \text{MPa} \quad \textbf{(OK)}$$

Hence, use W360 × 45 **Ans.**



(a)

Ans:
Use W360 × 45

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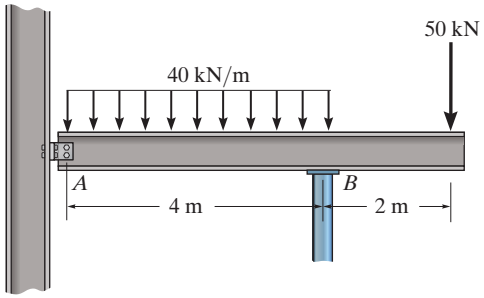
11–10.

Investigate if the W250 × 58 beam can safely support the loading. The beam has an allowable normal stress of $\sigma_{allow} = 150$ MPa and an allowable shear stress of $\tau_{allow} = 80$ MPa. Assume there is a pin at A and a roller support at B.



SOLUTION

Shear and Moment Diagram: As shown in Fig. *a*.

Bending Stress: Referring to the moment diagram, Fig. *a*, $M_{max} = 100\ \text{kN} \cdot \text{m}$. For a W250 × 58 section, $S_x = 693(10^3)\ \text{mm}^3 = 0.693(10^{-3})\ \text{m}^4$. Applying the flexure formula,

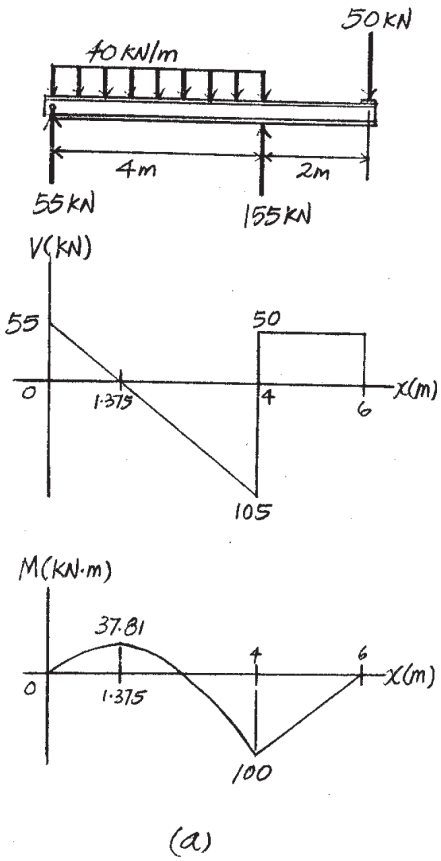
$$\sigma_{max} = \frac{M_{max}}{S_x} = \frac{100(10^3)}{0.693(10^{-3})} = 144.30\ \text{MPa} < \sigma_{allow} = 150\ \text{MPa} \qquad \textbf{(OK)}$$

Shear Stress: Referring to the shear diagram, Fig. *a*, $V_{max} = 105$ kN. For a W250 × 58 section, $d = 252$ mm and $t_w = 8.00$ mm. We have

$$\tau_{max} = \frac{V_{max}}{t_w d} = \frac{105(10^3)}{8.00(10^{-3})(0.252)}$$

$$= 52.08\ \text{MPa} < \tau_{allow} = 80\ \text{MPa} \qquad \textbf{(OK)}$$

The W250 × 58 can safely support the loading. **Ans.**



(a)

Ans:
Yes, it can

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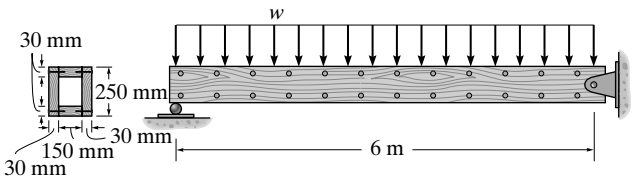
11–11. The box beam has an allowable bending stress of $\sigma_{allow} = 10$ MPa and an allowable shear stress of $\tau_{allow} = 775$ kPa. Determine the maximum intensity w of the distributed loading that it can safely support. Also, determine the maximum safe nail spacing for each third of the length of the beam. Each nail can resist a shear force of 200 N.



SOLUTION

Section Properties:

$$I = \frac{1}{12}(0.21)\left(0.25^3\right) - \frac{1}{12}(0.15)\left(0.19^3\right) = 0.1877\left(10^{-3}\right)\text{m}^4$$

$$Q_A = \bar{y}_1' A' = 0.11(0.03)(0.15) = 0.495\left(10^{-3}\right)\text{m}^3$$

$$Q_{max} = \Sigma \bar{y}' A' = 0.11(0.03)(0.15) + 0.0625(0.125)(0.06)$$

$$= 0.96375\left(10^{-3}\right)\text{m}^3$$

Bending Stress: From the moment diagram, $M_{max} = 4.50w$. Assume bending controls the design. Applying the flexure formula.

$$\sigma_{allow} = \frac{M_{max}\, c}{I}$$

$$10\left(10^6\right) = \frac{4.50w\,(0.125)}{0.1877\,(10^{-3})}$$

$$w = 3336.9\ \text{N/m}$$

Shear Stress: Provide a shear stress check using the shear formula. From the shear diagram, $V_{max} = 3.00w = 10.01$ kN.

$$\tau_{max} = \frac{V_{max}\, Q_{max}}{It}$$

$$= \frac{10.01(10^3)\left[0.96375(10^{-3})\right]}{0.1877(10^{-3})(0.06)}$$

$$= 857\ \text{kPa} > \tau_{allow} = 775\ \text{kPa}\ (\textit{No Good!})$$

Hence, shear stress controls.

$$\tau_{allow} = \frac{V_{max}\, Q_{max}}{It}$$

$$775\left(10^3\right) = \frac{3.00w\left[0.96375(10^{-3})\right]}{0.1877(10^{-3})(0.06)}$$

$$w = 3018.8\ \text{N/m} = 3.02\ \text{kN/m}$$ **Ans.**

Shear Flow: Since there are two rows of nails, the allowable shear flow is $q = \frac{2(200)}{s} = \frac{400}{s}$.

Ans:

$w = 3.02$ kN/m, $s = 16.7$ mm, $s = 50.2$ mm

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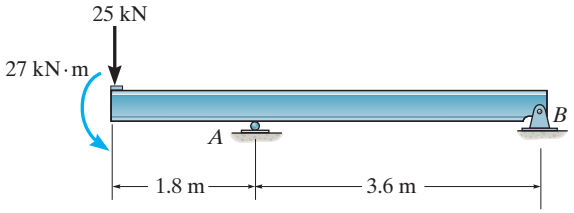
***11–12.** Select the lightest-weight steel wide-flange beam from Appendix B that will safely support the loading shown. The allowable bending stress is $\sigma_{allow} = 150$ MPa and the allowable shear stress is $\tau_{allow} = 84$ MPa.



SOLUTION

From the moment diagram, Fig. *a*, $M_{max} = 72$ kN ·m.

$$S_{req'd} = \frac{M_{max}}{\sigma_{allow}}$$

$$= \frac{72(10^3)}{150(10^6)} = 0.48(10^{-3})\ \text{m}^3 = 480(10^3)\ \text{mm}^3$$

$$= 480(10^3)\ \text{mm}^3$$

The choices are

W310 × 39 W360 × 39 W410 × 39

The section that has shortest depth is preferable, so

Select W310 × 39 [$S_x = 547(10^3)$ mm^3, $d = 310$ mm and $t_w = 5.84$ mm]

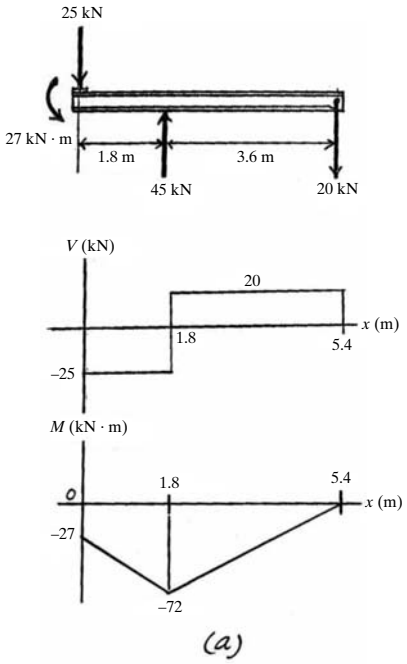
From the shear diagram, Fig. *a*, $V_{max} = 25$ kN. Provide the shear stress check for *W* 310 × 39,

$$\tau_{max} = \frac{V_{max}}{t_w d}$$

$$= \frac{25(10^3)}{0.00584(0.3101)} = 13.81(10^6)\ \text{N/m}^2 = 13.8\ \text{MPa}$$

$$= 13.8\ \text{MPa} < \tau_{allow} = 84\ \text{MPa}\ (O.K.!)$$

Use W310 × 39 **Ans.**



Ans:
Use W310 × 39

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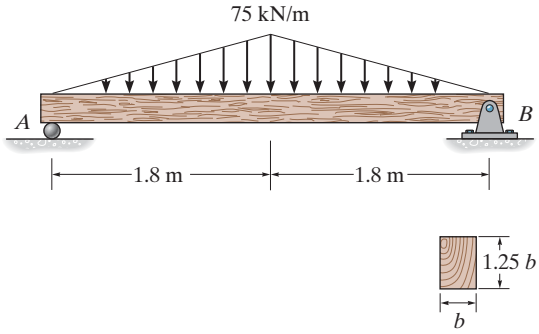
11–13. The simply supported beam is made of timber that has an allowable bending stress of $\sigma_{allow} = 6.72$ MPa and an allowable shear stress of $\tau_{allow} = 0.525$ MPa. Determine the dimension b if it is to be rectangular and have a height-to-width ratio of 1.25.



SOLUTION

$$I = \frac{1}{12}(b)(1.25b)^3 = 0.16276b^4$$

$$S_{req'd} = \frac{I}{c} = \frac{0.16276b^4}{0.625b} = 0.26042b^3$$

Assume bending moment controls:

$$M_{max} = 81 \text{ kN} \cdot \text{m}$$

$$\sigma_{allow} = \frac{M_{max}}{S_{req'd}}$$

$$6.72(10^6) = \frac{81(10^3)}{0.26042b^3}$$

$$b = 0.35904 \text{ m} = 359.04 \text{ mm}$$

Check shear:

$$\tau_{max} = \frac{1.5V}{A} = \frac{1.5[67.5(10^3)]}{(0.35904)[1.25(0.35904)]} = 0.6283(10^6) \text{ N/m}^2$$

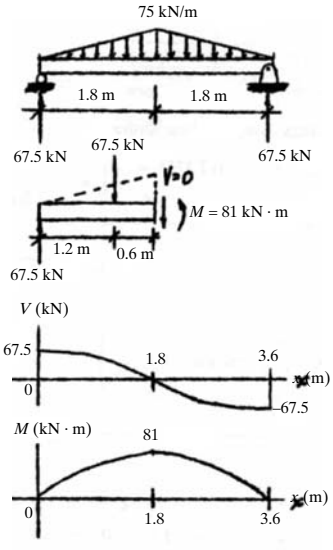
$$= 0.628 \text{ MPa} > 0.525 \text{ MPa (NO Good!)}$$

Shear controls:

$$\tau_{allow} = \frac{1.5V}{A}; \qquad 0.525(10^6) = \frac{1.5[67.5(10^3)]}{b(1.25b)}$$

$$b = 0.3928 \text{ m} = 393 \text{ mm}$$

Ans.



Ans:
$b = 393$ mm

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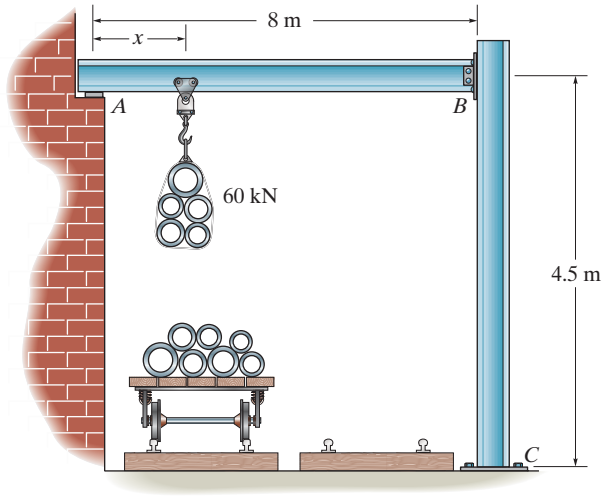
11–14. The beam is used in a railroad yard for loading and unloading cars. If the maximum anticipated hoist load is 60 kN, select the lightest-weight steel wide-flange section from Appendix B that will safely support the loading. The hoist travels along the bottom flange of the beam, $0.3\ \text{m} \le x \le 7.5\ \text{m}$, and has negligible size. Assume the beam is pinned to the column at B and roller supported at A. The allowable bending stress is $\sigma_{\text{allow}} = 168$ MPa and the allowable shear stress is $\tau_{\text{allow}} = 84$ MPa.



SOLUTION

Maximum moment occurs when load is in the center of beam.

$$M_{\max} = (30\ \text{kN})(4\ \text{m}) = 120\ \text{kN}\cdot\text{m}$$

$$S_{\text{req'd}} = \frac{M_{\max}}{\sigma_{\text{allow}}}$$

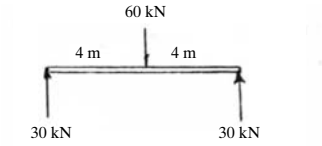
$$= \frac{120(10^3)}{168(10^6)} = 0.7143(10^{-3})\ \text{m}^3 = 714.3(10^3)\ \text{mm}^3$$

Select a W410 × 46 $\quad S_x = 774(10^3)\ \text{mm}^3,\ d = 403\ \text{mm},\ t_w = 6.99\ \text{mm}$

At $x = 0.3$ m, $V = 57.75$ kN

$$\tau = \frac{V}{A_{\text{web}}} = \frac{57.75(10^3)}{0.00699(0.403)} = 20.50(10^6)\ \text{N/m}^2 = 20.50\ \text{MPa} < 84\ \text{MPa}$$

Use W410 × 46 **Ans.**



Ans:
Use W410 × 46

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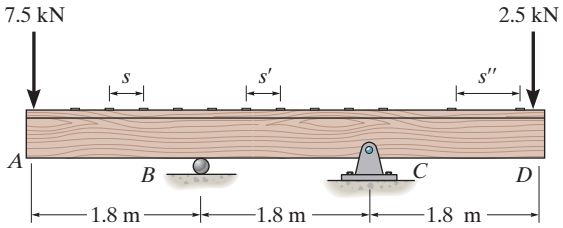
11–15. The beam is constructed from three boards as shown. If each nail can support a shear force of 1.5 kN, determine the maximum allowable spacing of the nails, s, s', s'', for regions AB, BC, and CD to the nearest 5 mm respectively. Also, if the allowable bending stress is σ_{allow} = 10 MPa and the allowable shear stress is τ_{allow} = 1 MPa, determine if it can safely support the load.



SOLUTION

The neutral axis passes through centroid c of the beam's cross-section. The location of c, Fig. b, is

$$\bar{y} = \frac{\Sigma \bar{y} A}{\Sigma A} = \frac{0.3(0.25)(0.1) + 2[0.125(0.05)(0.25)]}{0.25(0.1) + 2(0.05)(0.25)}$$

$$= 0.2125 \text{ m}$$

The moment of inertia of the beam's cross-section about the neutral axis is

$$I = \left[\frac{1}{12}(0.05)(0.25^3) + 0.05(0.25)(0.2125 - 0.125)^2\right]$$

$$+ \frac{1}{12}(0.25)(0.1^3) + 0.25(0.1)(0.3 - 0.2125)^2$$

$$= 0.53385(10^{-3}) \text{ m}^4$$

Referring to Fig. b,

$$Q_{max} = 2\bar{y}_2' A_2' = 2[(0.10625)(0.05)(0.2125)] = 2.2578(10^{-3}) \text{ m}^3$$

$$Q_A = \bar{y}_1' A_1' = 0.0875(0.25)(0.1) = 2.1875(10^{-3}) \text{ m}^3$$

Referring to the moment diagram, Fig. a, M_{max} = 13.5 kN · m. Applying flexure formula with c = $\bar{y}$ = 0.2125 mm,

$$\sigma_{max} = \frac{M_{max}\, c}{I}$$

$$= \frac{[13.5(10^3)](0.2125)}{0.53385(10^{-3})} = 5.374(10^6) \text{ N/m}^2$$

$$= 5.37 \text{ MPa} < \sigma_{allow} = 10 \text{ MPa } (O.K!)$$

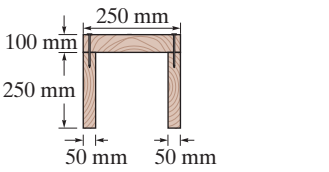
Referring to shear diagram, Fig. a, V_{max} = 7.5 kN.

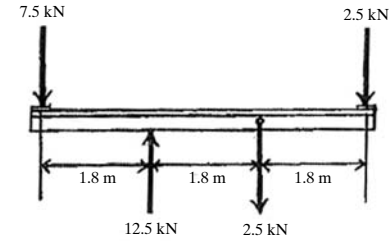
$$\tau_{max} = \frac{V_{max} Q_{max}}{It}$$

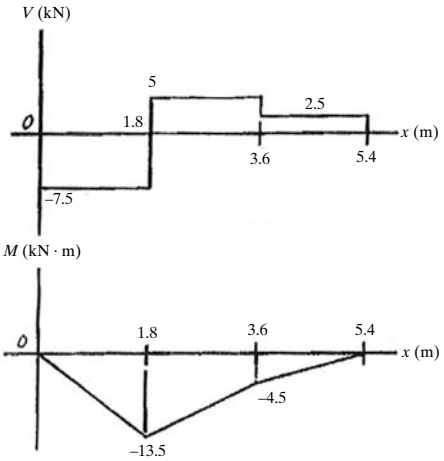
$$= \frac{[7.5(10^3)][2.2578(10^{-3})]}{[0.53385(10^{-3})](0.1)} = 0.3172(10^6) \text{ N/m}^2$$

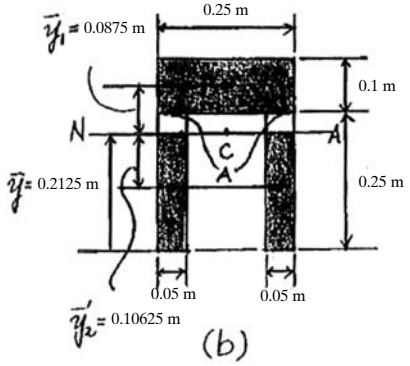
$$= 0.317 \text{ MPa} < \tau_{allow} = 1 \text{ MPa } (O.K!)$$

Yes, it can support the load. **Ans.**









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11–15. Continued

Since there are two rows of nails, the allowable shear flow is $q_{\text{allow}} = \frac{2F}{S} = \frac{2(1.5)(10^3)}{S} = \frac{3000}{S}$. For region AB, $V = 7.5$ kN. Thus

$$q_{\text{allow}} = \frac{VQ_A}{I}; \qquad \frac{3000}{S} = \frac{[7.5(10^3)][2.1875(10^{-3})]}{0.53385(10^{-3})} \qquad S = 0.09762 \text{ m} = 97.62 \text{ mm}$$

Use $S = 95$ mm **Ans.**

For region BC, $V = 5$ kN. Thus

$$q_{\text{allow}} = \frac{VQ_A}{I}; \qquad \frac{3000}{S'} = \frac{[5(10^3)][2.1875(10^{-3})]}{0.53385(10^{-3})} \qquad S' = 0.14643 \text{ m} = 146.43 \text{ mm}$$

Use $S' = 145$ mm **Ans.**

For region CD, $V = 2.5$ kN. Thus

$$q_{\text{allow}} = \frac{VQ_A}{I}; \qquad \frac{3000}{S''} = \frac{[2.5(10^3)][2.1875(10^{-3})]}{0.53385(10^{-3})} \qquad S'' = 0.29286 \text{ m} = 292.86 \text{ mm}$$

Use $S'' = 290$ mm **Ans.**

Ans:
Yes, it can support the load, Use $s = 95$ mm, Use $s' = 145$ mm, Use $s'' = 290$ mm

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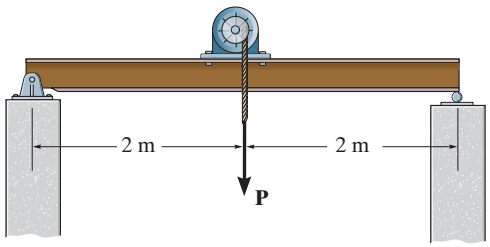
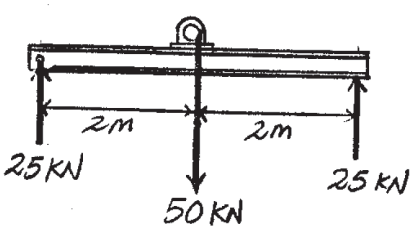
***11–16.**

If the cable is subjected to a maximum force of $P = 50$ kN, select the lightest W310 wide-flange beam that can safely support the load. The beam has an allowable normal stress of $\sigma_{allow} = 150$ MPa and an allowable shear stress of $\tau_{allow} = 85$ MPa.



SOLUTION



Shear and Moment Diagram: As shown in Fig. *a*.

Bending Stress: From the moment diagram, Fig. *a*, $M_{max} = 50\ \text{kN} \cdot \text{m}$. Applying the flexure formula,

$$S_{required} = \frac{M_{max}}{\sigma_{allow}} = \frac{50(10^3)}{150(10^6)}$$

$$= 0.3333(10^{-3})\ \text{m}^3 = 333.33(10^3)\ \text{mm}^3$$

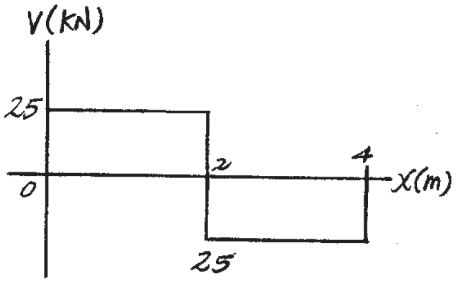
Select a W310 × 33[$S_x = 415(10^3)$ mm^3, $d = 313$ mm, and $t_w = 6.60$ mm]

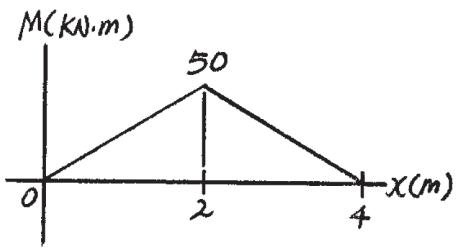
Shear Stress: From the shear diagram, Fig. *a*, $V_{max} = 25$ kN. We have

$$\tau_{max} = \frac{V_{max}}{t_w d} = \frac{25(10^3)}{6.60(10^{-3})(0.313)}$$

$$= 12.10\ \text{MPa} < \tau_{allow} = 85\ \text{MPa}$$ **OK**

Hence, Use W310 × 33. **Ans.**





(a)

Ans:
Use W310 × 33

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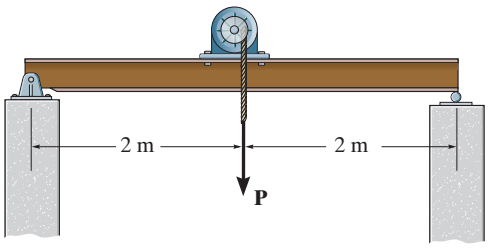
11–17.

If the W360 × 45 wide-flange beam has an allowable normal stress of $\sigma_{allow} = 150$ MPa and an allowable shear stress of $\tau_{allow} = 85$ MPa, determine the maximum cable force **P** that can safely be supported by the beam.



SOLUTION

Shear and Moment Diagram: As shown in Fig. *a*.

Bending Stress: From the moment diagram, Fig. *a*, $M_{max} = P$. For W360 × 45 section, $S_x = 688(10^3)\ mm^3 = 0.688(10^{-3})\ m^3$.

Applying the flexure formula,

$$M_{max} = S_x \sigma_{allow}$$

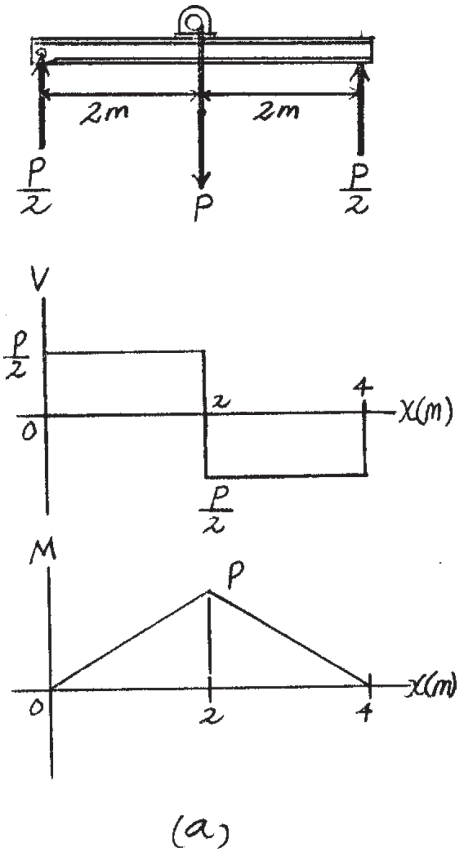
$$P = 0.688(10^{-3})[150(10^6)]$$

$$P = 103\,200\ N = 103\ kN \qquad \textbf{Ans.}$$

Shear Stress: From the shear diagram, Fig. *a*, $V_{max} = \frac{P}{2} = \frac{103\,200}{2} = 51\,600$ N. For W360 × 45 section, $d = 352$ mm and $t_w = 6.86$ mm. We have

$$\tau_{max} = \frac{V_{max}}{t_w d} = \frac{51\,600}{6.86(10^{-3})(0.352)}$$

$$= 21.37\ MPa < \tau_{allow} = 85\ MPa \qquad \textbf{(OK!)}$$



(a)

Ans:
$P = 103$ kN

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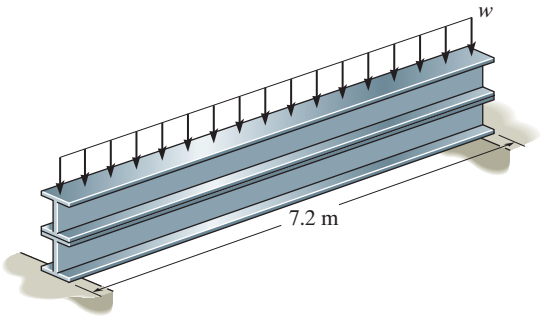
11–18. The simply supported beam is composed of two W310 × 33 sections built up as shown. Determine the maximum uniform loading w the beam will support if the allowable bending stress of σ_{allow} = 150 MPa and the allowable shear stress is τ_{allow} = 100 MPa.



SOLUTION

Section properties:

For W310 × 33 (d = 313 mm $I_x = 65(10^6)$ mm^4 t_w = 6.60 mm A = 4180 mm^2)

$$I = 2\left[65(10^6) + 4180\left(\frac{313}{2}\right)^2\right] = 334.76(10^6)\ \text{mm}^4$$

$$S = \frac{I}{c} = \frac{334.76(10^6)}{313} = 1.0695(10^6)\ \text{mm}^3 = 1.0695(10^{-3})\ \text{m}^3$$

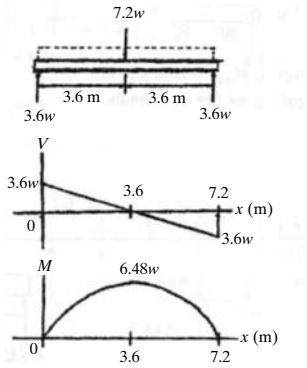
Maximum Loading: Assume moment controls.

$M = \sigma_{allow} S$;

$$6.48w = [150(10^6)][1.0695(10^{-3})]$$

$$w = 24.76(10^3)\ \text{N/m} = 24.8\ \text{kN/m}$$

Ans.



Check Shear: (Neglect area of flanges.)

$$\tau_{max} = \frac{V_{max}}{A_w} = \frac{3.6(24.76)(10^3)}{2(0.0066)(0.313)} = 21.57(10^6)\ \text{N/m}^2$$

$$= 21.6\ \text{MPa} < \tau_{allow} = 100\ \text{MPa}\ (O.K.!)$$

Ans:

w = 24.8 kN/m

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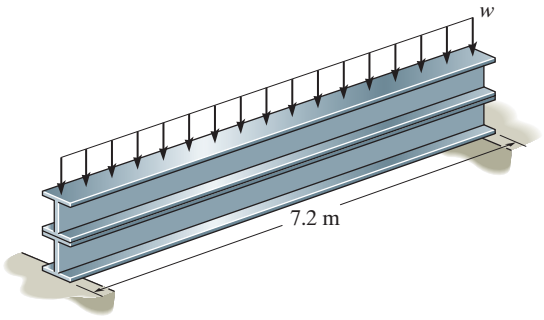
11–19. The simply supported beam is composed of two W310 × 33 sections built up as shown. Determine if the beam will safely support a loading of $w = 30$ kN/m. The allowable bending stress is $\sigma_{allow} = 150$ MPa and the allowable shear stress is $\tau_{allow} = 100$ MPa.



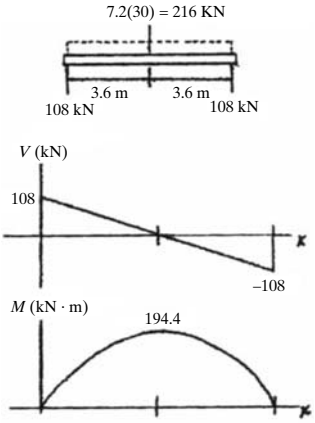
SOLUTION

Section properties:

For W310 × 33 ($d = 313$ mm $I_x = 65(10^6)$ mm^4 $t_w = 6.60$ mm $A = 4180$ mm^2)

$$I = 2\left[65(10^6) + 4180\left(\frac{313}{2}\right)^2\right] = 334.76(10^6)\ \text{mm}^4$$

$$S = \frac{I}{c} = \frac{334.76(10^6)}{313} = 1.0695(10^6)\ \text{mm}^3 = 1.0695(10^{-3})\ \text{m}^3$$



Bending stress:

$$\sigma_{max} = \frac{M_{max}}{S} = \frac{194.4(10^3)}{1.0695(10^{-3})} = 181.77(10^6)\ \text{N/m}^2$$

$$= 181.8\ \text{MPa} > \sigma_{allow} = 150\ \text{MPa}$$

No, the beam fails due to bending stress criteria. **Ans.**

Check shear: (Neglect area of flanges.)

$$\tau_{max} = \frac{V_{max}}{A_w} = \frac{108(10^3)}{2(0.0066)(0.313)} = 26.14(10^6)\ \text{N/m}^2$$

$$= 26.1\ \text{MPa} < \tau_{allow} = 100\ \text{MPa}\ \ \text{OK}$$

Ans:
The beam fails.

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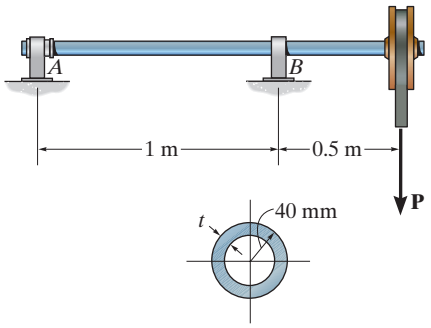
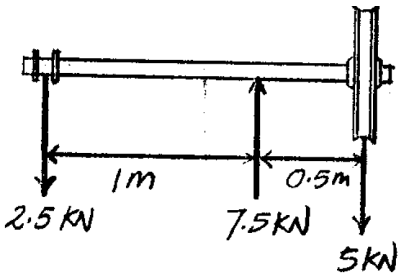
***11–20.** The shaft is supported by a smooth thrust bearing at A and a smooth journal bearing at B. If $P = 5$ kN and the shaft is made from steel having an allowable normal stress of $\sigma_{allow} = 150$ MPa and an allowable shear stress of $\tau_{allow} = 85$ MPa, determine the required minimum wall thickness t of the shaft to the nearest millimeter to safely support the load.



SOLUTION

Shear and Moment Diagram: As shown in Fig. *a*,

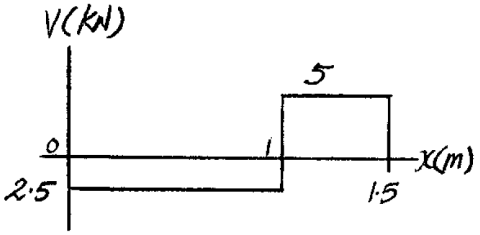


Bending Stress: From the moment diagram, Fig. *a*, $M_{max} = 2.5\ \text{kN}\cdot\text{m}$. The moment of inertia of the shaft about the bending axis is $I = \frac{\pi}{4}(0.04^4 - r_i{}^4)$. Applying the flexure formula,

$$\sigma_{allow} = \frac{M_{max}\,c}{I}$$

$$150(10^6) = \frac{2.5(10^3)(0.04)}{\frac{\pi}{4}(0.04^4 - r_i{}^4)}$$

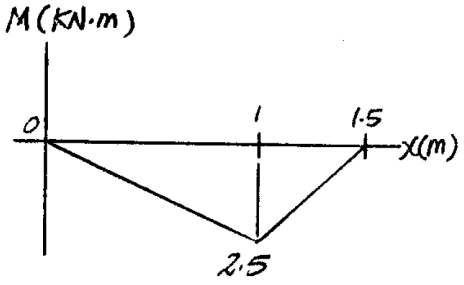
$$r_i = 0.03617\ \text{m} = 36.17\ \text{mm}$$



Thus,

$$t = r_0 - r_i = 40 - 36.17 = 3.83\ \text{mm}$$

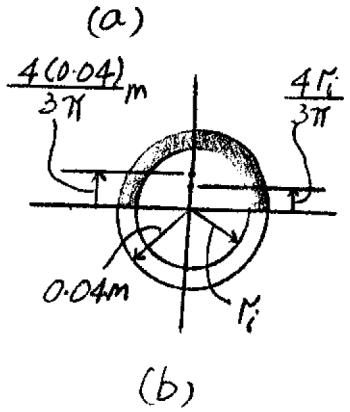
Use $t = 4$ mm **Ans.**



Shear Stress: Using this result, $r_i = 0.04 - 0.004 = 0.036$ m. Then $Q_{max} = \frac{4(0.04)}{3\pi}\left[\frac{\pi}{2}(0.04^2)\right] - \frac{4(0.036)}{3\pi}\left[\frac{\pi}{2}(0.036^2)\right] = 11.5627(10^{-6})\ \text{m}^3$, Fig. *b*, and $I = \frac{\pi}{4}(0.04^4 - 0.036^4) = 0.69145(10^{-6})\ \text{m}^4$. Referring to the shear diagram, Fig. *a*, $V_{max} = 5$ kN.

$$\tau_{max} = \frac{V_{max}Q_{max}}{It} = \frac{5(10^3)[11.5627(10^{-6})]}{0.69145(10^{-6})(0.008)}$$

$$= 10.45\ \text{MPa} < \tau_{allow} = 85\ \text{MPa (OK)}$$



Ans:
Use $t = 4$ mm

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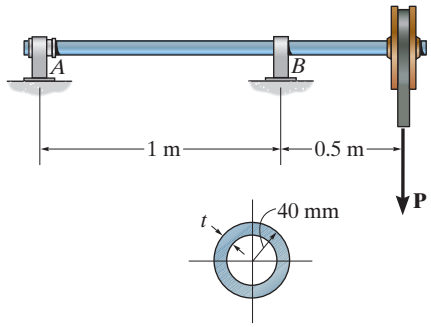
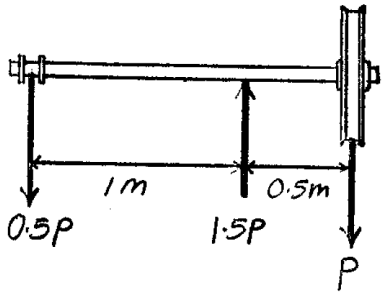
11–21. The shaft is supported by a smooth thrust bearing at A and a smooth journal bearing at B. If the shaft is made from steel having an allowable normal stress of $\sigma_{allow} = 150$ MPa and allowable shear stress of $\tau_{allow} = 85$ MPa, determine the maximum allowable force **P** that can be applied to the shaft. The thickness of the shaft's wall is $t = 5$ mm.



SOLUTION

Shear and Moment Diagram: As shown in Fig. *a*,

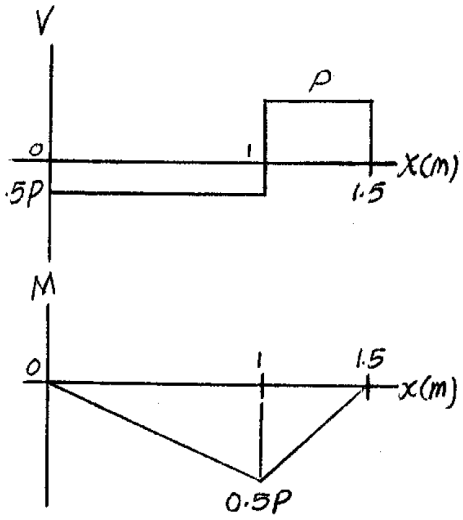


Bending Stress: The moment of inertia of the shaft about the bending axis is $I = \frac{\pi}{4}(0.04^4 - 0.035^4) = 0.8320(10^{-6})\ \text{m}^4$. Referring to the moment diagram, Fig. *a*, $M_{max} = 0.5P$. Applying the flexure formula,

$$\sigma_{allow} = \frac{M_{max}\, c}{I}$$

$$150(10^6) = \frac{0.5P(0.04)}{0.8320(10^{-6})}$$

$$P = 6240.23\text{N} = 6.24\ \text{kN}$$ **Ans.**



Shear Stress:

$$Q_{max} = \frac{4(0.04)}{3\pi}\left[\frac{\pi}{2}(0.04^2)\right] - \frac{4(0.035)}{3\pi}\left[\frac{\pi}{2}(0.035^2)\right]$$

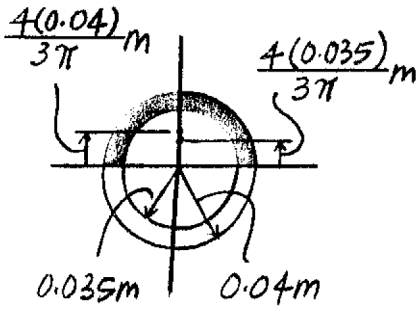
$$= 14.0833(10^{-6})\ \text{m}^3$$

Referring to the shear diagram, Fig. *a*, $V_{max} = P = 6240.23$ N. Applying the shear formula,

$$\tau_{max} = \frac{V_{max} Q_{max}}{I t} = \frac{6240.23[14.0833(10^{-6})]}{0.8320(10^{-6})(0.01)}$$

$$= 10.56\ \text{MPa} < \tau_{allow} = 85\ \text{MPa (OK!)}$$

(a)



(b)

Ans:
$P = 6.24$ kN

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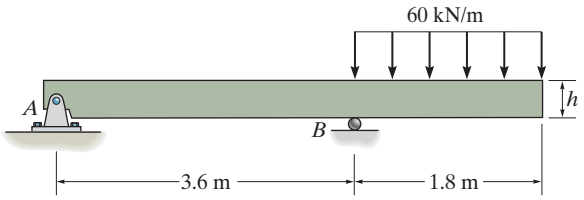
11–22. Determine the minimum depth h of the beam to the nearest multiples of 5 mm that will safely support the loading shown. The allowable bending stress is $\sigma_{allow} = 147$ MPa and the allowable shear stress is $\tau_{allow} = 70$ MPa. The beam has a uniform thickness of 75 mm.



SOLUTION

The section modulus of the rectangular cross-section is

$$S = \frac{I}{C} = \frac{\frac{1}{12}(0.075)(h^3)}{h/2} = 0.0125h^2$$

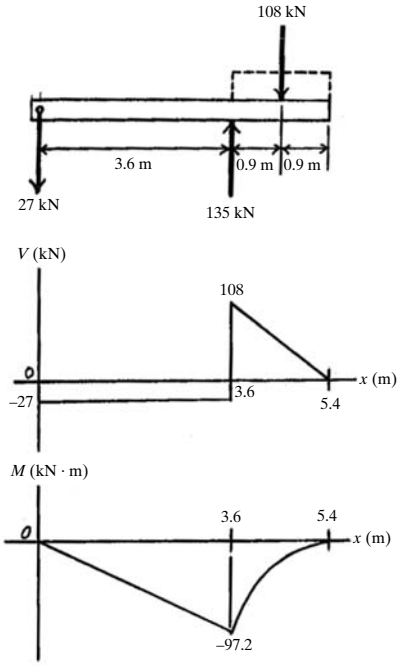
From the moment diagram, $M_{max} = 97.2$ kN · m.

$$S_{req'd} = \frac{M_{max}}{\sigma_{allow}}$$

$$0.0125h^2 = \frac{97.2(10^3)}{147(10^6)}$$

$$h = 0.229995 \text{ m} = 230 \text{ mm}$$

Use $h = 230$ mm **Ans.**



(a)

From the shear diagram, Fig. a, $V_{max} = 108$ kN. Referring to Fig. b,

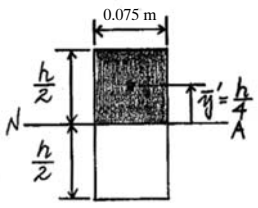
$Q_{max} = \bar{y}'A' = \left(\frac{0.23}{4}\right)(0.075)\left(\frac{0.23}{2}\right) = 0.4959375(10^{-3})$ m^3 and

$I = \frac{1}{12}(0.075)(0.23^3) = 76.04375(10^{-6})$ m^4. Provide the shear stress check by providing shear formula,

$$\tau_{max} = \frac{V_{max} Q_{max}}{It}$$

$$= \frac{[108(10^3)][0.4959375(10^{-3})]}{[76.04375(10^{-6})](0.075)}$$

$$= 9.391(10^6) \text{ N/m}^2 = 9.39 \text{ MPa} < \tau_{allow} = 70 \text{ MPa } (O.K!)$$



(b)

Ans:
Use $h = 230$ mm

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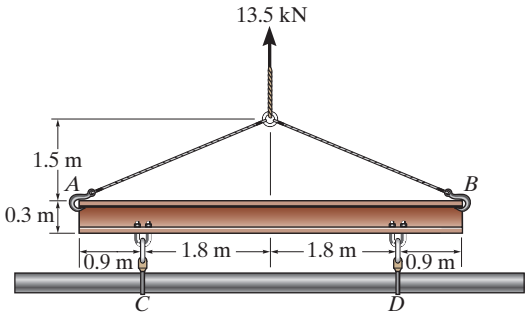
11–23. The spreader beam AB is used to slowly lift the 13.5 kN pipe that is centrally located on the straps at C and D. If the beam is a W310 × 67, determine if it can safely support the load. The allowable bending stress is $\sigma_{allow} = 154$ MPa and the allowable shear stress is $\tau_{allow} = 84$ MPa.



SOLUTION

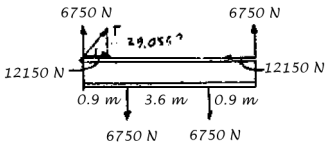
$$F_h = \frac{6750}{\tan 29.055°} = 12150 \text{ N}$$

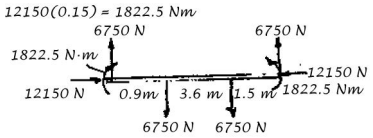
For W310 × 67 $[S = 948(10^3)\ \text{mm}^3,\ d = 306\ \text{mm},\ t_w = 8.51\ \text{mm}]$

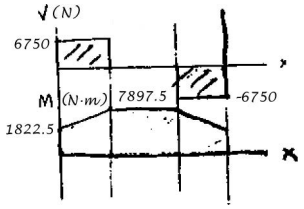
$$\sigma_{max} = \frac{M_{max}}{S};\quad \sigma_{max} = \frac{7897.5}{0.948(10^{-3})} = 8.331(10^6)\ \text{N/m}^2 = 8.33\ \text{MPa} < 154\ \text{MPa}\quad \text{OK}$$

$$\tau_{max} = \frac{V_{max}}{A_{web}};\quad \tau = \frac{6750}{(0.00851)(0.306)} = 2.592(10^6)\ \text{N/m}^2 = 2.59\ \text{MPa} < 84\ \text{MPa}\quad \text{OK}$$

Yes. **Ans.**







Ans:
Yes.

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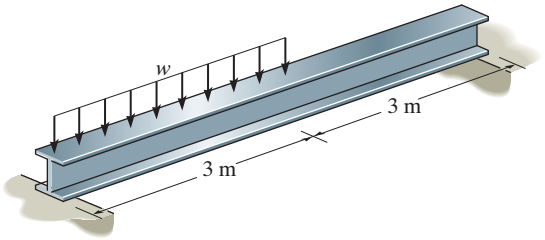
***11–24.** Determine the maximum uniform loading w the W310 × 21 beam will support if the allowable bending stress is $\sigma_{allow} = 150$ MPa and the allowable shear stress is $\tau_{allow} = 84$ MPa.



SOLUTION

From the moment diagram, Fig. *a*, $M_{max} = 2.53125\,w$. For W310 × 21, $S_x = 244(10^3)$ mm^3, $d = 303$ mm and $t_w = 5.08$ mm.

$$\sigma_{allow} = \frac{M_{max}}{S}$$

$$150(10^6) = \frac{2.53125w}{0.244(10^{-3})}$$

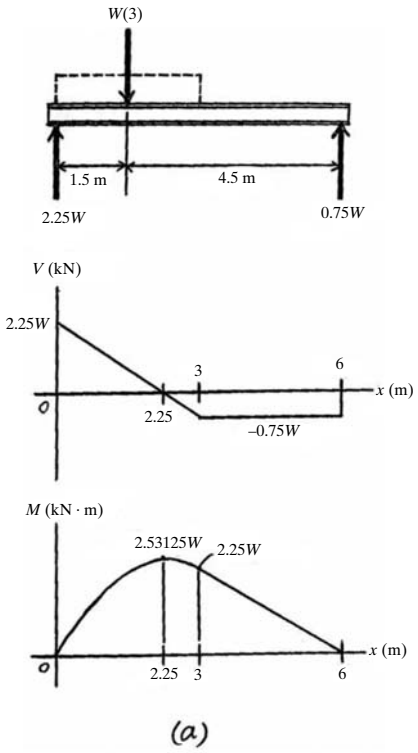
$$w = 14.46(10^3) \text{ N/m} = 14.5 \text{ kN/m}$$ **Ans.**

From the shear diagram, Fig. *a*, $V_{max} = 2.25(14.46) = 32.53$ kN. Provide a shear stress check on W310 × 21,

$$\tau_{max} = \frac{V_{max}}{t_w\, d}$$

$$= \frac{32.53(10^3)}{0.00508(0.303)} = 21.14(10^6) \text{ N/m}^2$$

$$= 21.1 \text{ MPa} < \tau_{allow} = 84 \text{ MPa } (O.K)$$



(a)

Ans:
$w = 14.5$ kN/m

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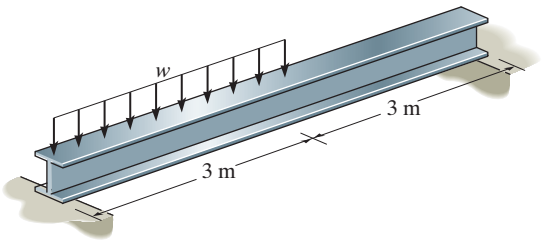
11–25. Determine if the W360 × 33 beam will safely support a loading of $w = 25$ kN/m. The allowable bending stress is $\sigma_{allow} = 150$ MPa and the allowable shear stress is $\tau_{allow} = 84$ MPa.



SOLUTION

For W360 × 33 $S_x = 475(10^3)$ mm^3 $d = 349$ mm and $t_w = 5.84$ mm. From the moment diagram, Fig. *a*, $M_{max} = 63.28125$ kN · m.

$$\sigma_{max} = \frac{M_{max}}{S}$$

$$= \frac{63.28125(10^3)}{0.475(10^{-3})} = 133.22(10^6)\ \text{N/m}^2$$

$$= 133\ \text{MPa} < \sigma_{allow} = 150\ \text{MPa}\ (O.K!)$$

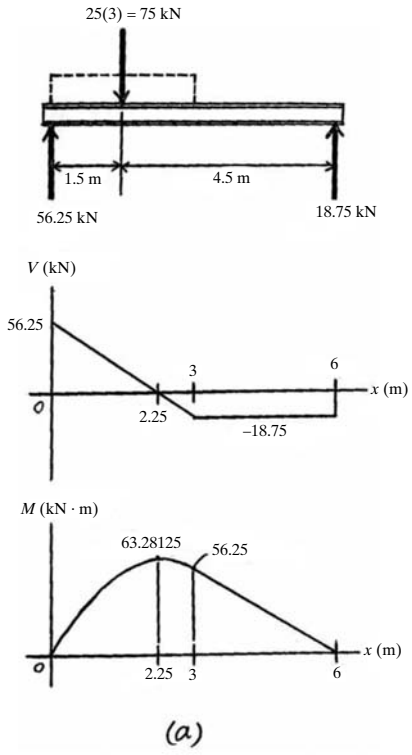
From the shear diagram, Fig. *a*, $V_{max} = 56.25$ kN.

$$\tau_{max} = \frac{V_{max}}{t_w\, d}$$

$$= \frac{56.25(10^3)}{0.00584(0.349)} = 27.60(10^6)\ \text{N/m}^2$$

$$= 27.6\ \text{MPa} < \tau_{allow} = 84\ \text{MPa}\ (\text{O.K!})$$

Based on the investigated results, we conclude that W360 × 33 can safely support the loading.



Ans:
Yes

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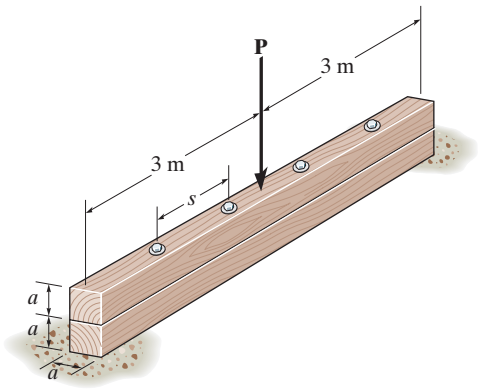
11–26. The simply supported beam supports a load of $P = 16$ kN. Determine the smallest dimension a of each timber if the allowable bending stress for the wood is $\sigma_{allow} = 30$ MPa and the allowable shear stress is $\tau_{allow} = 800$ kPa. Also, if each bolt can sustain a shear of 2.5 kN, determine the spacing s of the bolts at the calculated dimension a.

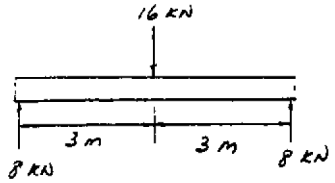


SOLUTION

Section Properties:

$$I = \frac{1}{12}(a)(2\,a)^3 = 0.66667\,a^4$$

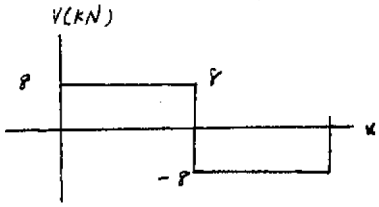
$$Q_{max} = \bar{y}'A' = \frac{a}{2}(a)(a) = 0.5\,a^3$$

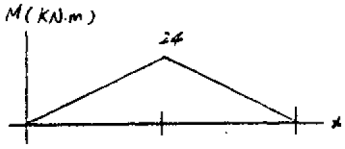


Assume bending controls.

$$\sigma_{allow} = \frac{M_{max}c}{I}; \qquad 30(10^6) = \frac{24(10^3)a}{0.66667\,a^4}$$

$$a = 0.106266\text{ m} = 106\text{ mm} \qquad \textbf{Ans.}$$





Check shear:

$$\tau_{max} = \frac{VQ}{It} = \frac{8(10^3)(0.106266/2)(0.106266)^2}{0.66667(0.106266^4)(0.106266)} = 531\text{ kPa} < \tau_{allow} = 800\text{ kPa} \quad \text{OK}$$

Bolt spacing:

$$q = \frac{VQ}{I} = \frac{8(10^3)(0.106266/2)(0.106266^2)}{0.66667(0.106266^4)} = 56462.16\text{ N/m}$$

$$s = \frac{2.5(10^3)}{56462.16} = 0.04427\text{ m} = 44.3\text{ mm} \qquad \textbf{Ans.}$$

Ans:
$a = 106$ mm, $s = 44.3$ mm

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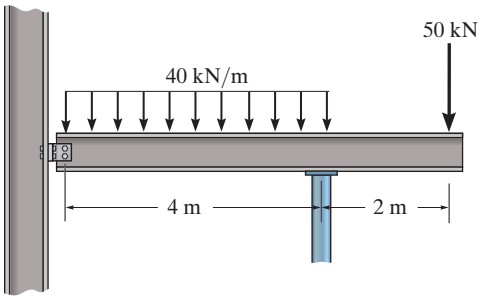
11–27. Select the lightest W360 shape section from Appendix B that can safely support the loading acting on the overhanging beam. The beam is made from steel having an allowable normal stress of $\sigma_{allow} = 150$ MPa and an allowable shear stress of $\tau_{allow} = 80$ MPa.



SOLUTION

Shear and Moment Diagram: As shown in Fig. *a*.

Bending Stress: Referring to the moment diagram, Fig. *a*, $M_{max} = 100\ \text{kN}\cdot\text{m}$. Applying the flexure formula,

$$S_{required} = \frac{M_{max}}{\sigma_{allow}} = \frac{100(10^3)}{150(10^6)}$$

$$= 0.6667(10^{-3})\ \text{m}^3 = 666.67(10^3)\ \text{mm}^3$$

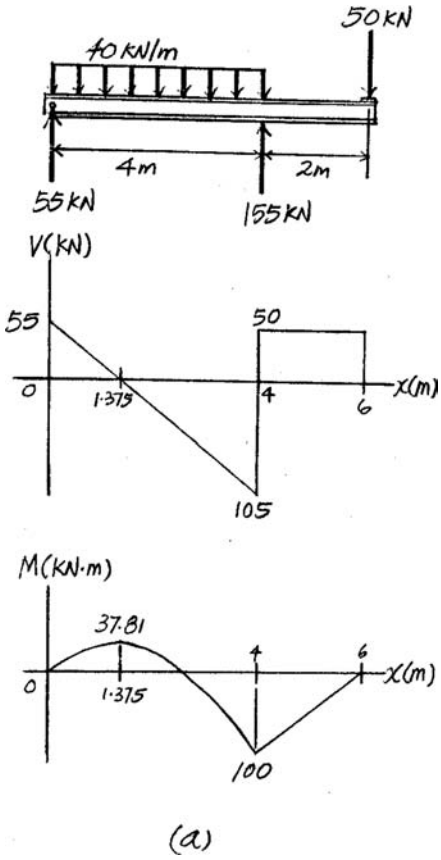
Select W360 × 45($S_x = 688(10^3)\ \text{mm}^3$, $d = 352$ mm and $t_w = 6.86$ mm)

Shear Stress: Referring to the shear diagram, Fig. *a*, $V_{max} = 105$ kN. We have

$$\tau_{max} = \frac{V_{max}}{t_w d} = \frac{105(10^3)}{6.86(10^{-3})(0.352)}$$

$$= 43.48\ \text{MPa} < \tau_{allow} = 80\ \text{MPa}\ \ (\text{OK})$$

Hence, use W360 × 45 **Ans.**



(a)

Ans:
Use W360 × 45

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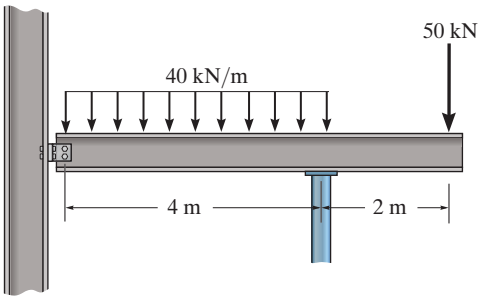
***11–28.** Investigate if a W250 × 58 shape section can safely support the loading acting on the overhanging beam. The beam is made from steel having an allowable normal stress of $\sigma_{allow} = 150$ MPa and an allowable shear stress of $\tau_{allow} = 80$ MPa.



SOLUTION

Shear and Moment Diagram: As shown in Fig. *a*,

Bending Stress: Referring to the moment diagram, Fig. *a*, $M_{max} = 100$ kN · m. For a W250 × 58 section, $S_x = 693(10^3)$ mm^3 $= 0.693(10^{-3})$ m^4. Applying the flexure formula,

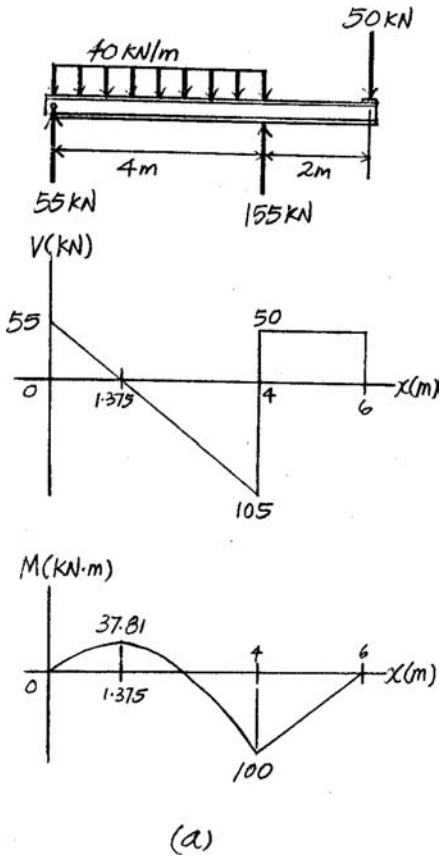
$$\sigma_{max} = \frac{M_{max}}{S_x} = \frac{100(10^3)}{0.693(10^{-3})} = 144.30 \text{ MPa} < \sigma_{allow} = 150 \text{ MPa} \quad \text{(OK)}$$

Shear Stress: Referring to the shear diagram, Fig. *a*, $V_{max} = 105$ kN. For a W250 × 58 section, $d = 252$ mm and $t_w = 8.00$ mm. We have

$$\tau_{max} = \frac{V_{max}}{t_w d} = \frac{105(10^3)}{8.00(10^{-3})(0.252)}$$

$$= 52.08 \text{ MPa} < \tau_{allow} = 80 \text{ MPa} \quad \text{(OK)}$$

The W250 × 58 can safely support the loading. **Ans.**



(a)

Ans:
Yes.

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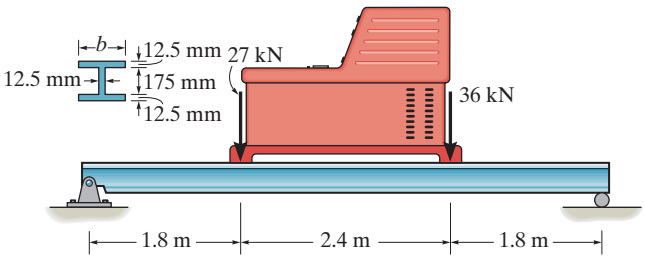
11–29. The beam is to be used to support the machine, which exerts the forces of 27 kN and 36 kN as shown. If the maximum bending stress is not to exceed $\sigma_{allow} = 154$ MPa, determine the required width b of the flanges.



SOLUTION

Section Properties:

$$I = \frac{1}{12}(b)(0.2^3) - \frac{1}{12}(b - 0.0125)(0.175^3)$$

$$= 0.220052(10^{-3})b + 5.58268(10^{-6})$$

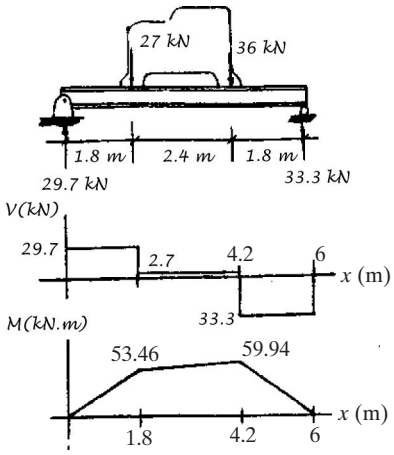
$$S = \frac{I}{c} = \frac{0.220052(10^{-3})b + 5.58268(10^{-6})}{0.1}$$

$$= 2.20052(10^{-3})b + 55.8268(10^{-6})$$

$$S_{req'd} = \frac{M_{max}}{\sigma_{allow}}$$

$$2.20052(10^{-3})b + 55.8268(10^{-6}) = \frac{59.94(10^3)}{154(10^6)}$$

$b = 0.15151$ m $= 152$ mm **Ans.**



Ans:
$b = 151.5$ mm.

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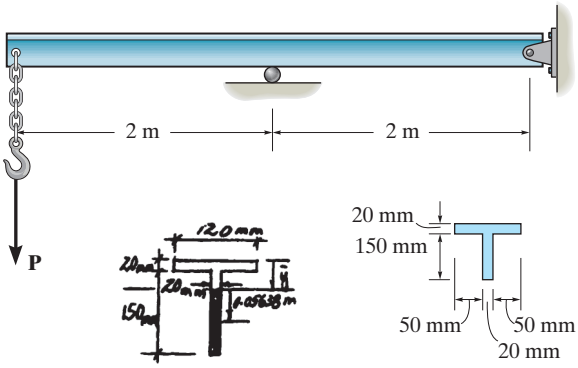
11–30.

The steel beam has an allowable bending stress $\sigma_{allow} = 140$ MPa and an allowable shear stress of $\tau_{allow} = 90$ MPa. Determine the maximum load that can safely be supported.



SOLUTION

Section Properties:

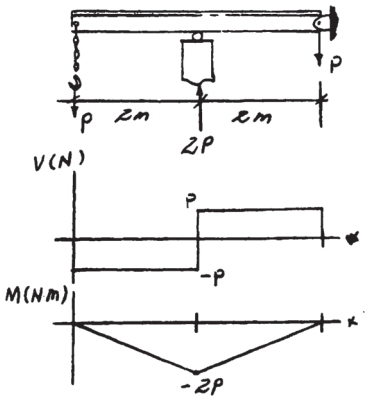
$$\bar{y} = \frac{(10)(120)(20) + (95)(150)(20)}{120(20) + 150(20)} = 57.22 \text{ mm}$$

$$Q_{max} = \bar{y}'A' = (0.05638)(0.02)(0.170 - 0.05722) = 0.127168(10^{-3}) \text{ m}^3$$

$$I = \frac{1}{12}(0.12)(0.02^3) + 0.12(0.02)(0.05722 - 0.01)^2 +$$

$$\frac{1}{12}(0.02)(0.15^3) + 0.15(0.02)(0.095 - 0.05722)^2 = 15.3383(10^{-6}) \text{ m}^4$$

$$S = \frac{I}{c} = \frac{15.3383(10^{-6})}{(0.170 - 0.05722)} = 0.136005(10^{-3}) \text{ m}^3$$



For Moment:

$$M = \sigma_{allow} S$$

$$2P = 140(10^6)(0.136005)(10^{-3})$$

$$P = 9520 \text{ N} = 9.52 \text{ kN} \quad \textbf{(Controls)} \qquad \textbf{Ans.}$$

For Shear:

$$V \overset{?}{=} \tau_{allow}\left(\frac{It}{Q_{max}}\right)$$

$$P = 90(10^6)\left(\frac{15.3383(10^{-6})(0.02)}{0.127168(10^{-3})}\right) = 217106 = 217 \text{ kN}$$

Ans:
$P = 9.52$ kN

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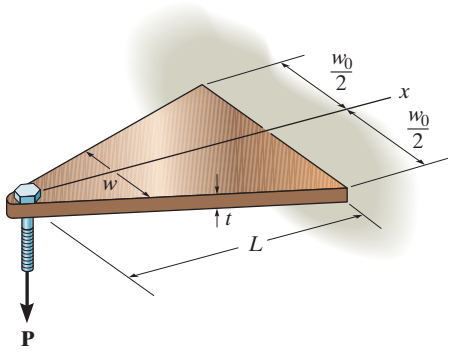
11–31.

Determine the variation in the width w as a function of x for the cantilevered beam that supports a concentrated force **P** at its end so that it has a maximum bending stress σ_{allow} throughout its length. The beam has a constant thickness t.

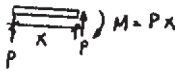


SOLUTION

Section Properties:

$$I = \frac{1}{12}(w)(t^3) \qquad S = \frac{I}{c} = \frac{\frac{1}{12}(w)(t^3)}{t/2} = \frac{wt^2}{6}$$

$$\sigma_{\text{allow}} = \frac{M}{S} = \frac{Px}{wt^2/6} \qquad \textbf{(1)}$$



At $x = L$,

$$\sigma_{\text{allow}} = \frac{PL}{w_0 t^2/6} \qquad \textbf{(2)}$$

Equate Eqs. (1) and (2),

$$\frac{Px}{wt^2/6} = \frac{PL}{w_0 t^2/6}$$

$$w = \frac{w_0}{L}x \qquad \textbf{Ans.}$$

Ans:

$$w = \frac{w_0}{L}x$$

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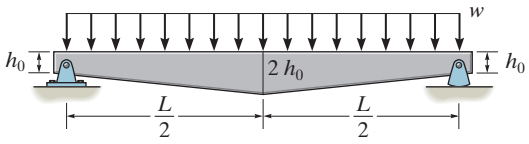
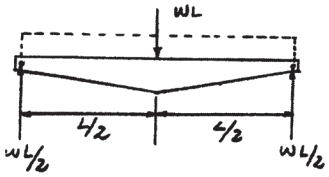
***11–32.**

The tapered beam supports a uniform distributed load w. If it is made from a plate and has a constant width b, determine the absolute maximum bending stress in the beam.



SOLUTION

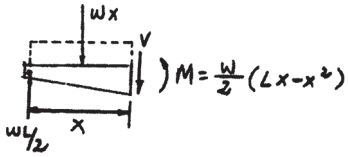


Section Properties:

$$\frac{h - h_0}{x} = \frac{h_0}{\frac{L}{2}}; \qquad h = h_0\left(\frac{2}{L}x + 1\right)$$

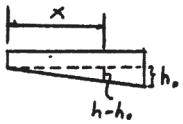
$$I = \frac{1}{12}bh_0^3\left(\frac{2}{L}x + 1\right)^3$$

$$S = \frac{I}{c} = \frac{\frac{1}{12}bh_0^3\left(\frac{2}{L}x + 1\right)^3}{\frac{h_0}{2}\left(\frac{2}{L}x + 1\right)} = \frac{1}{6}bh_0^2\left(\frac{2}{L}x + 1\right)^2$$



Bending Stress:

$$\sigma = \frac{M}{S} = \frac{\frac{w}{2}(Lx - x^2)}{\frac{1}{6}bh_0^2\left(\frac{2}{L}x + 1\right)^2} = \frac{3w}{bh_0^2}\left[\frac{Lx - x^2}{\left(\frac{2}{L}x + 1\right)^2}\right] \qquad \textbf{(1)}$$



$$\frac{d\sigma}{dx} = \frac{3w}{bh_0^2}\left[\frac{\left(\frac{2}{L}x + 1\right)^2(L - 2x) - (Lx - x^2)(2)\left(\frac{2}{L}x + 1\right)\left(\frac{2}{L}\right)}{\left(\frac{2}{L}x + 1\right)^4}\right] = 0$$

$$\left(\frac{2}{L}x + 1\right)(L - 2x) - \frac{4}{L}(Lx - x^2) = 0; \qquad x = \frac{L}{4}$$

Hence, from Eq. (1),

$$\sigma_{\max} = \frac{3w}{bh_0^2}\left[\frac{L\left(\frac{L}{4}\right) - \left(\frac{L}{4}\right)^2}{\left(\frac{2}{L}\left(\frac{L}{4}\right) + 1\right)^2}\right] = \frac{wL^2}{4bh_0^2} \qquad \textbf{Ans.}$$

Ans:

$$\sigma_{\max} = \frac{WL^2}{4bh_0^2}$$

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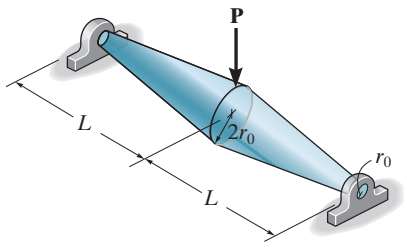
11–33.

The tapered beam supports the concentrated force **P** at its center. Determine the absolute maximum bending stress in the beam. The reactions at the supports are vertical.



SOLUTION

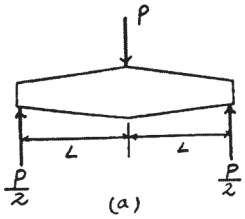
Moment Function: As shown on FBD(b).

Section Properties:

$$\frac{r - r_0}{x} = \frac{r_0}{L} \qquad r = \frac{r_0}{L}(x + L)$$

$$I = \frac{\pi}{4}\left[\frac{r_0}{L}(x + L)\right]^4 = \frac{\pi r_0^4}{4L^4}(x + L)^4$$

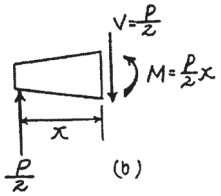
$$S = \frac{I}{c} = \frac{\frac{\pi r_0^4}{4L^4}(x + L)^4}{\frac{r_0}{L}(x + L)} = \frac{\pi r_0^3}{4L^3}(x + L)^3$$



Bending Stress: Applying the flexure formula,

$$\sigma = \frac{M}{S} = \frac{\frac{P}{2}x}{\frac{\pi r_0^3}{4L^3}(x + L)^3} = \frac{2PL^3x}{\pi r_0^3(x + L)^3} \qquad \textbf{(1)}$$

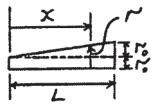
In order to have the absolute maximum bending stress, $\frac{d\sigma}{dx} = 0$.



$$\frac{d\sigma}{dx} = \frac{2PL^3}{\pi r_0^3}\left[\frac{(x + L)^3(1) - x(3)(x + L)^2(1)}{(x + L)^6}\right] = 0$$

$$x = \frac{L}{2}$$

Substituting $x = \frac{L}{2}$ into Eq. (1) yields



$$\sigma_{\max} = \frac{8PL}{27\pi r_0^3} \qquad \textbf{Ans.}$$

Ans:

$$\sigma_{\max} = \frac{8PL}{27\pi r_0^3}$$

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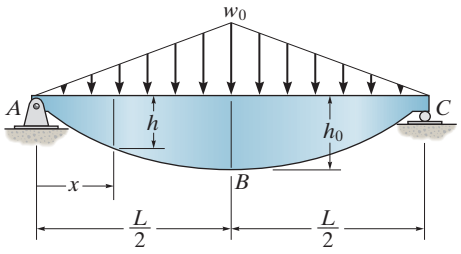
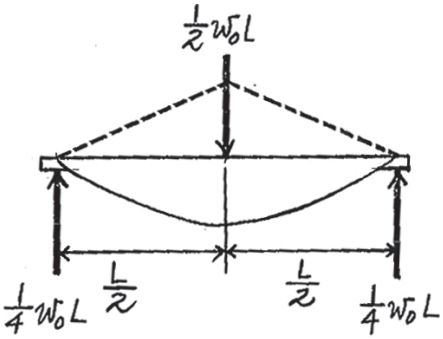
11–34.

The beam is made from a plate that has a constant thickness b. If it is simply supported and carries the distributed loading shown, determine the variation of its depth h as a function of x so that it maintains a constant maximum bending stress σ_{allow} throughout its length.



SOLUTION

Support Reactions: As shown on the free-body diagram of the entire beam, Fig. *a*.



(a)

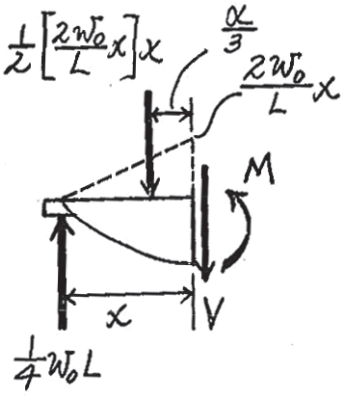
Moment Function: The distributed load as a function of x is

$$\frac{w}{x} = \frac{w_0}{L/2} \qquad w = \frac{2w_0}{L}x$$

The free-body diagram of the beam's left cut segment is shown in Fig. *b*. Considering the moment equilibrium of this free-body diagram,

$$\zeta + \Sigma M_O = 0; \qquad M + \frac{1}{2}\left[\frac{2w_0}{L}x\right]x\left(\frac{x}{3}\right) - \frac{1}{4}w_0Lx = 0$$

$$M = \frac{w_0}{12L}\left(3L^2x - 4x^3\right)$$



(b)

Section Properties: At position x, the height of the beam's cross section is h. Thus

$$I = \frac{1}{12}bh^3$$

Then

$$S = \frac{I}{c} = \frac{\frac{1}{12}bh^3}{h/2} = \frac{1}{6}bh^2$$

Bending Stress: The maximum bending stress σ_{max} as a function of x can be obtained by applying the flexure formula.

$$\sigma_{max} = \frac{M}{S} = \frac{\frac{w_0}{12L}\left(3L^2x - 4x^3\right)}{\frac{1}{6}bh^2} = \frac{w_0}{2bh^2L}\left(3L^2x - 4x^3\right), \qquad \textbf{(1)}$$

At $x = \frac{L}{2}$, $h = h_0$. From Eq. (1),

$$\sigma_{max} = \frac{w_0L^2}{2bh_0{}^2} \qquad \textbf{(2)}$$

Equating Eqs. (1) and (2),

$$\frac{w_0}{2bh^2L}\left(3L^2x - 4x^3\right) = \frac{w_0L^2}{2bh_0{}^2}$$

$$h = \frac{h_0}{L^{3/2}}\left(3L^2x - 4x^3\right)^{1/2} \qquad \textbf{Ans.}$$

Ans:

$$h = \frac{h_0}{L^{3/2}}(3L^2x - 4x^3)^{1/2}$$

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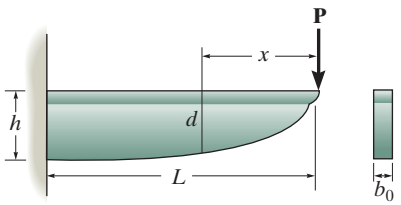
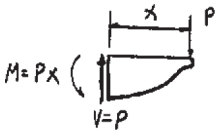
11–35.

Determine the variation in the depth d of a cantilevered beam that supports a concentrated force **P** at its end so that it has a constant maximum bending stress σ_{allow} throughout its length. The beam has a constant width b_0.



SOLUTION



Section Properties:

$$I = \frac{1}{12} b_0 d^3; \qquad S = \frac{I}{c} = \frac{\frac{1}{12} b_0 d^3}{\frac{d}{2}} = \frac{b_0 d^2}{6}$$

Maximum Bending Stress:

$$\sigma_{\text{allow}} = \frac{M}{S} = \frac{Px}{b_0 \frac{d^2}{6}} = \frac{6Px}{b_0 d^2} \qquad \textbf{(1)}$$

At $x = L, \quad d = h$

$$\sigma_{\text{allow}} = \frac{6PL}{b_0 h^2} \qquad \textbf{(2)}$$

Equating Eqs. (1) and (2),

$$\frac{6Px}{b_0 d^2} = \frac{6PL}{b_0 h^2}$$

$$d = h\sqrt{\frac{x}{L}} \qquad \textbf{Ans.}$$

Ans:

$$d = h\sqrt{\frac{x}{L}}$$

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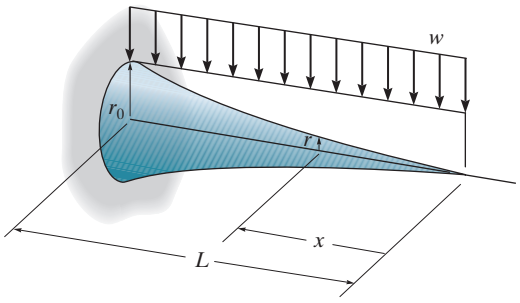
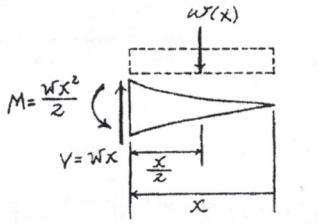
***11–36.**

Determine the variation of the radius r of the cantilevered beam that supports the uniform distributed load so that it has a constant maximum bending stress σ_{max} throughout its length.



SOLUTION

Moment Function: As shown on FBD.



Section Properties:

$$I = \frac{\pi}{4} r^4 \qquad S = \frac{I}{c} = \frac{\frac{\pi}{4} r^4}{r} = \frac{\pi}{4} r^3$$

Bending Stress: Applying the flexure formula,

$$\sigma_{max} = \frac{M}{S} = \frac{\frac{wx^2}{2}}{\frac{\pi}{4} r^3}$$

$$\sigma_{max} = \frac{2wx^2}{\pi r^3} \qquad \textbf{(1)}$$

At $x = L$, $r = r_0$. From Eq. (1),

$$\sigma_{max} = \frac{2wL^2}{\pi r_0^3} \qquad \textbf{(2)}$$

Equating Eq. (1) and (2) yields

$$r^3 = \frac{r_0^3}{L^2} x^2 \qquad \textbf{Ans.}$$

Ans:

$$r^3 = \frac{r_0^3}{L^2} x^2$$

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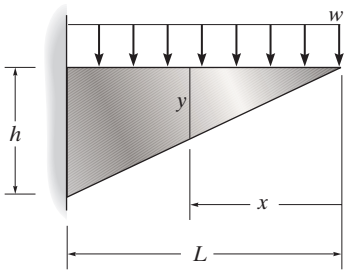
11–37.

The tapered beam supports a uniform distributed load w. If it is made from a plate that has a constant width b_0, determine the absolute maximum bending stress in the beam.



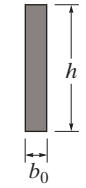
SOLUTION

Section Properties:

$$\frac{y}{h} = \frac{x}{L}; \qquad y = \frac{h}{L}x$$

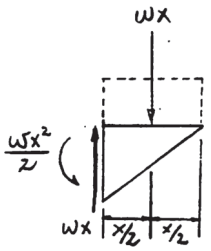
$$I = \frac{1}{12}(b_0)\left(\frac{h}{L}x\right)^3 = \frac{b_0 h^3}{12L^3}x^3$$

$$S = \frac{I}{c} = \frac{\frac{b_0 h^3}{12L^3}x^3}{\frac{h}{2L}x} = \frac{b_0 h^2}{6L^2}x^2$$

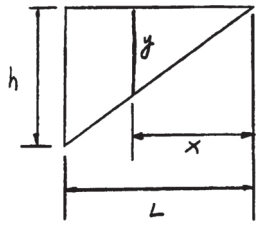


Bending Stress:

$$\sigma_{\max} = \frac{M}{S} = \frac{\frac{w}{2}x^2}{\frac{b_0 h^2}{6L^2}x^2} = \frac{3wL^2}{b_0 h^2}$$ **Ans.**



The bending stress is independent of x. Therefore, the stress is constant throughout the span.



Ans:

$$\sigma_{\max} = \frac{3wL^2}{b_0 h^2}$$

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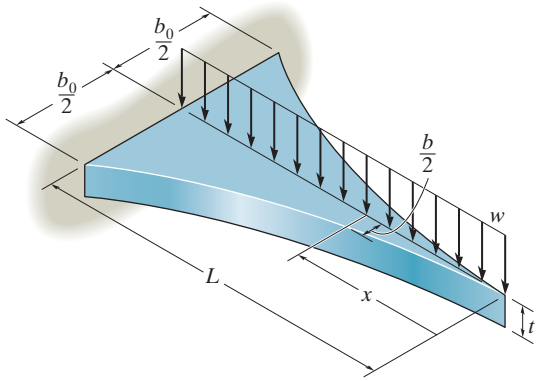
11–38.

Determine the variation in the width b as a function of x for the cantilevered beam that supports a uniform distributed load along its centerline so that it has the same maximum bending stress σ_{allow} throughout its length. The beam has a constant depth t.



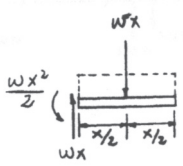
SOLUTION

Section Properties:

$$I = \frac{1}{12} b\, t^3 \qquad S = \frac{I}{c} = \frac{\frac{1}{12} b\, t^3}{\frac{t}{2}} = \frac{t^2}{6} b$$

Bending Stress:

$$\sigma_{\text{allow}} = \frac{M}{S} = \frac{\frac{wx^2}{2}}{\frac{t^2}{6} b} = \frac{3wx^2}{t^2 b} \qquad \textbf{(1)}$$



At $x = L, b = b_0$

$$\sigma_{\text{allow}} = \frac{3wL^2}{t^2 b_0} \qquad \textbf{(2)}$$

Equating Eqs. (1) and (2) yields:

$$\frac{3wx^2}{t^2 b} = \frac{3wL^2}{t^2 b_0}$$

$$b = \frac{b_0}{L^2} x^2 \qquad \textbf{Ans.}$$

Ans:

$$b = \frac{b_0}{L^2} x^2$$

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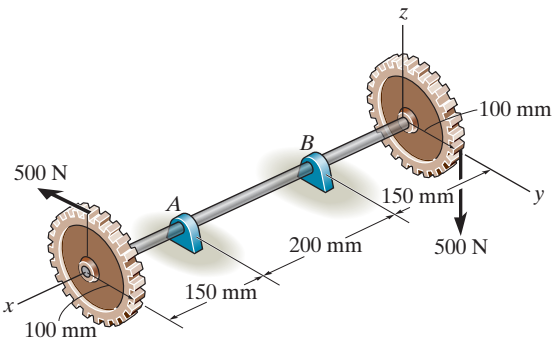
11–39.

The tubular shaft has an inner diameter of 15 mm. Determine to the nearest millimeter its minimum outer diameter if it is subjected to the gear loading. The bearings at A and B exert force components only in the y and z directions on the shaft. Use an allowable shear stress of $\tau_{allow} = 70$ MPa, and base the design on the maximum shear stress theory of failure.



SOLUTION

$$I = \frac{\pi}{4}(c^4 - 0.0075^4) \text{ and } J = \frac{\pi}{2}(c^4 - 0.0075^4)$$

$$\tau_{allow} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\tau_{allow} = \sqrt{\left(\frac{Mc}{2I}\right)^2 + \left(\frac{Tc}{J}\right)^2}$$

$$\tau_{allow}^2 = \frac{M^2c^2}{4I^2} + \frac{T^2c^2}{J^2}$$

$$\tau_{allow}^2\left(\frac{c^4 - 0.0075^4}{c}\right)^2 = \frac{4M^2}{\pi^2} + \frac{4T^2}{\pi^2}$$

$$\frac{c^4 - 0.0075^4}{c} = \frac{2}{\pi\,\tau_{allow}}\sqrt{M^2 + T^2}$$

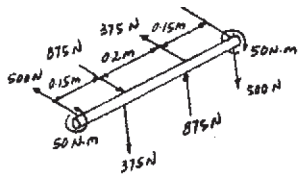
$$\frac{c^4 - 0.0075^4}{c} = \frac{2}{\pi(70)(10^6)}\sqrt{75^2 + 50^2}$$

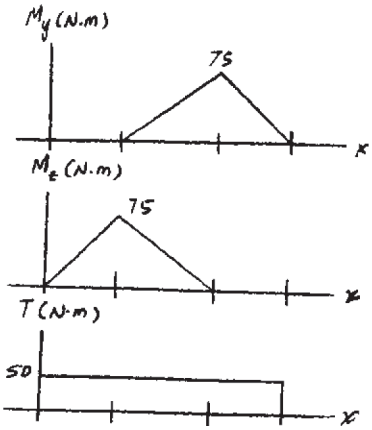
$$c^4 - 0.0075^4 = 0.8198(10^{-6})c$$

Solving, $c = 0.0103976$ m

$d = 2c = 0.0207952$ m $= 20.8$ mm

Use $d = 21$ mm





Ans.

Ans:
Use $d = 21$ mm

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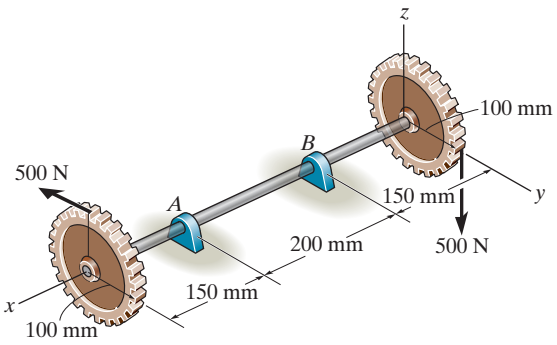
***11–40.**

Determine to the nearest millimeter the minimum diameter of the solid shaft if it is subjected to the gear loading. The bearings at A and B exert force components only in the y and z directions on the shaft. Base the design on the maximum distortion energy theory of failure with $\sigma_{allow} = 150$ MPa.



SOLUTION

$$\sigma_{1,2} = \frac{\sigma_x}{2} \pm \sqrt{\frac{\sigma_x^2}{4} + \tau_{xy}^2}$$

Let $a = \dfrac{\sigma_x}{2}, b = \sqrt{\dfrac{\sigma_x^2}{4} + \tau_{xy}^2}$

$$\sigma_1 = a + b, \quad \sigma_2 = a - b$$

Require

$$\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2 = \sigma_{allow}^2$$

$$a^2 + 2ab + b^2 - [a^2 - b^2] + a^2 - 2ab + b^2 = \sigma_{allow}$$

$$a^2 + 3b^2 = \sigma_{allow}^2$$

$$\frac{\sigma_x^2}{4} + 3\left(\frac{\sigma_x^2}{4} + \tau_{xy}^2\right) = \sigma_{allow}^2$$

$$\sigma_x^2 + 3\tau_{xy}^2 = \sigma_{allow}^2$$

$$\left(\frac{Mc}{\frac{\pi}{4}c^4}\right)^2 + 3\left(\frac{Tc}{\frac{\pi}{2}c^4}\right)^2 = \sigma_{allow}^2$$

$$\frac{1}{c^6}\left[\left(\frac{4M}{\pi}\right)^2 + 3\left(\frac{2T}{\pi}\right)^2\right] = \sigma_{allow}^2$$

$$c^6 = \frac{16}{\sigma_{allow}^2\pi^2}M^2 + \frac{12T^2}{\sigma_{allow}^2\pi^2}$$

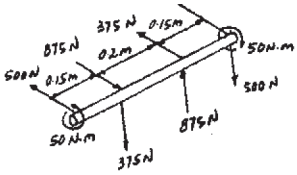
$$c = \left(\frac{4}{\sigma_{allow}^2\pi^2}\left(4M^2 + 3T^2\right)\right)^{\frac{1}{6}}$$

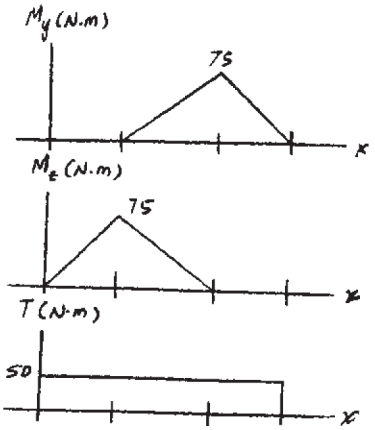
$$= \left[\frac{4}{(150(10^6))^2(\pi)^2}(4(75)^2 + 3(50)^2)\right]^{\frac{1}{6}} \approx 0.009025 \text{ m}$$

$$d = 2c = 0.0181 \text{ m}$$

Use $d = 19$ mm.

Ans.





Ans:
Use $d = 19$ mm

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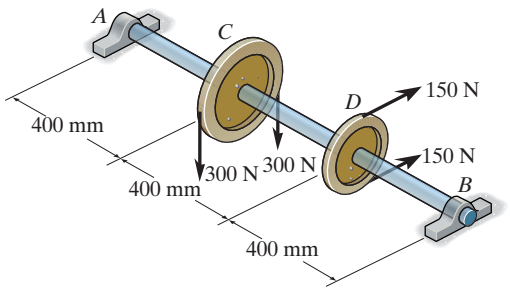
11–41.

The 50-mm-diameter shaft is supported by journal bearings at A and B. If the pulleys C and D are subjected to the loadings shown, determine the absolute maximum bending stress in the shaft.



SOLUTION

Internal Moment Components: The shaft is subjected to two bending moment components $\mathbf{M}_z$ and $\mathbf{M}_y$.

Bending Stress: Since all the axes through the centroid of the circular cross section of the shaft are principal axes, then the resultant moment $M = \sqrt{M_y{}^2 + M_z{}^2}$ can be used to determine the maximum bending stress. The maximum bending moment occurs at C ($x = 0.4$ m). Then, $M_{\max} = \sqrt{40^2 + 160^2} = 164.92\ \text{N} \cdot \text{m}$.

$$\sigma_{\max} = \frac{M_{\max}\, c}{I}$$

$$= \frac{164.92(0.025)}{\frac{\pi}{4}(0.025^4)}$$

$$= 13.439\ \text{MPa} = 13.4\ \text{MPa} \qquad \textbf{Ans.}$$

Ans:
$\sigma_{\max} = 13.4$ MPa

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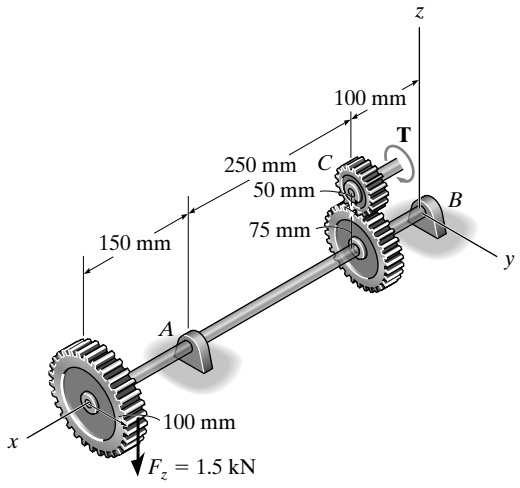
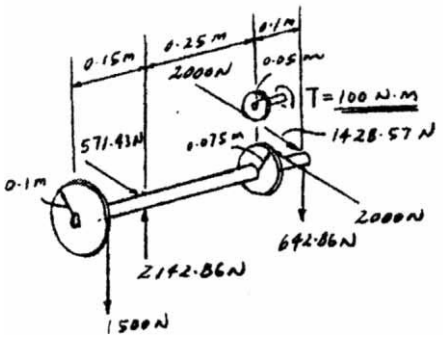
11–42. The end gear connected to the shaft is subjected to the loading shown. If the bearings at A and B exert only y and z components of force on the shaft, determine the equilibrium torque T at gear C and then determine the smallest diameter of the shaft to the nearest millimeter that will support the loading. Use the maximum-shear-stress theory of failure with $\tau_{allow} = 60$ MPa.



SOLUTION

From the free - body diagrams:

$T = 100\ \text{N} \cdot \text{m}$ **Ans.**



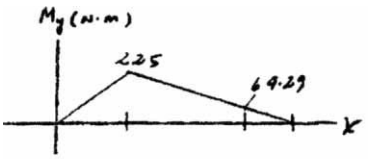
Critical section is at support A.

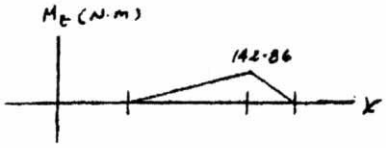
$$c = \left[\frac{2}{\pi\,\tau_{allow}}\sqrt{M^2 + T^2}\right]^{\frac{1}{3}} = \left[\frac{2}{\pi(60)(10^6)}\sqrt{225^2 + 150^2}\right]^{\frac{1}{3}}$$

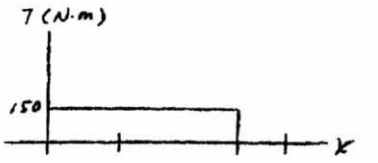
$$= 0.01421\ \text{m}$$

$d = 2c = 0.0284\ \text{m} = 28.4\ \text{mm}$

Use $d = 29$ mm **Ans.**







Ans:

$T = 100\ \text{N} \cdot \text{m}$, Use $d = 29$ mm

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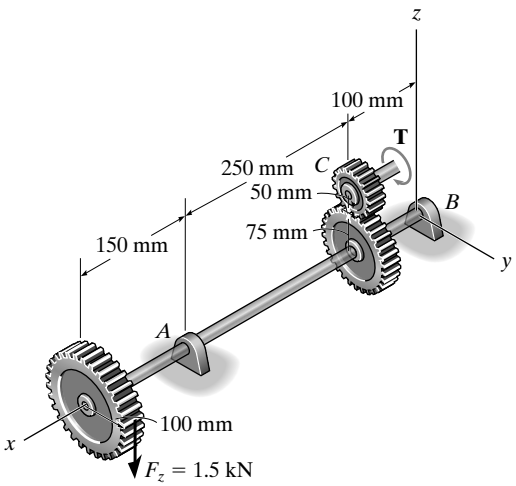
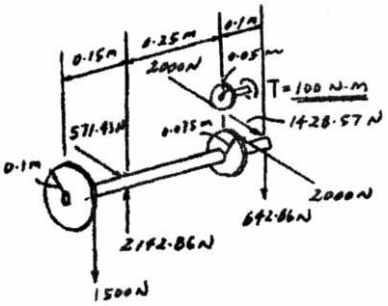
11–43. The end gear connected to the shaft is subjected to the loading shown. If the bearings at A and B exert only y and z components of force on the shaft, determine the equilibrium torque T at gear C and then determine the smallest diameter of the shaft to the nearest millimeter that will support the loading. Use the maximum-distortion-energy theory of failure with $\sigma_{\text{allow}} = 80$ MPa.



SOLUTION

From the free-body diagrams:

$T = 100\ \text{N} \cdot \text{m}$ **Ans.**



Critical section is at support A.

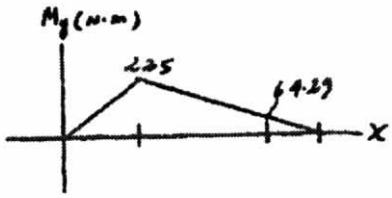
$$\sigma_{1,2} = \frac{\sigma_x}{2} \pm \sqrt{\frac{\sigma_x^2}{4} + \tau_{xy}^2}$$

Let $a = \dfrac{\sigma_x}{2}, b = \sqrt{\dfrac{\sigma_x^2}{4} + \tau_{xy}^2}$

$\sigma_1 = a + b, \sigma_2 = a - b$

Require,

$$\sigma_1^2 - \sigma_1 \sigma_2 + \sigma_2^2 = \sigma_{\text{allow}}^2 a^2 + 2ab + b^2 - [a^2 - b^2] + a^2 - 2ab + b^2 = \sigma_{\text{allow}}^2$$

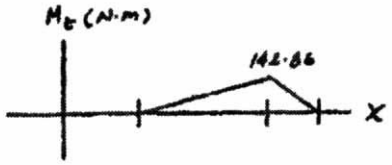


$$a^2 + 3b^2 = \sigma_{\text{allow}}^2$$

$$\frac{\sigma_x^2}{4} + 3\left(\frac{\sigma_x^2}{4} + \tau_{xy}^2\right) = \sigma_{\text{allow}}^2$$

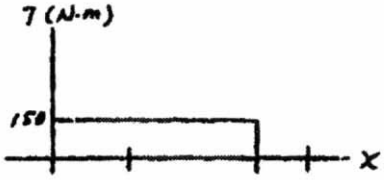
$$\sigma_x^2 + 3\tau_{xy}^2 = \sigma_{\text{allow}}^2$$

$$\left(\frac{Mt}{\frac{\pi}{4}c^4}\right)^2 + 3\left(\frac{Tc}{\frac{\pi}{2}c^4}\right)^2 = \sigma_{\text{allow}}^2$$



$$\frac{1}{c^4}\left[\left(\frac{4M}{\pi}\right)^2 + 3\left(\frac{2T}{\pi}\right)^2\right] = \sigma_{\text{allow}}^2$$

$$c^4 = \frac{16}{\sigma_{\text{allow}}^2 \pi^2} M^2 + \frac{12T^2}{\sigma_{\text{allow}}^2 \pi^2}$$



$$c = \left(\frac{4}{\sigma_{\text{allow}}^2 \pi^2}(4M^2 + 3T^2)\right)^{\frac{1}{2}}$$

$$= \left[\frac{4}{(80(10^6))^2(\pi)^2}(4(225)^2 + 3(150)^2)\right]^{\frac{1}{2}}$$

$$= 0.01605\ \text{m}$$

$d = 2c = 0.0321\ \text{m} = 32.1\ \text{mm}$

Use $d = 33$ mm **Ans.**

Ans:

$T = 100\ \text{N} \cdot \text{m}$, Use $d = 33$ mm

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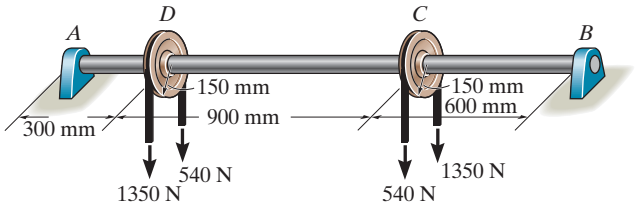
***11–44.** The two pulleys attached to the shaft are loaded as shown. If the bearings at A and B exert only vertical forces on the shaft, determine the required diameter of the shaft to the nearest mm using the maximum-distortion energy theory. $\sigma_{allow} = 469$ MPa.



SOLUTION

Section just to the left of point C is the most critical. Both states of stress will yield the same result.

$$\sigma_{a,b} = \frac{\sigma}{2} \pm \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2}$$

Let $\frac{\sigma}{2} = A$ and $\sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2} = B$

$$\sigma_a^2 = (A + B)^2 \qquad \sigma_b^2 = (A - B)^2$$

$$\sigma_a \sigma_b = (A + B)(A - B)$$

$$\sigma_a^2 - \sigma_a \sigma_b + \sigma_b^2 = A^2 + B^2 + 2AB - A^2 + B^2 + A^2 + B^2 - 2AB$$

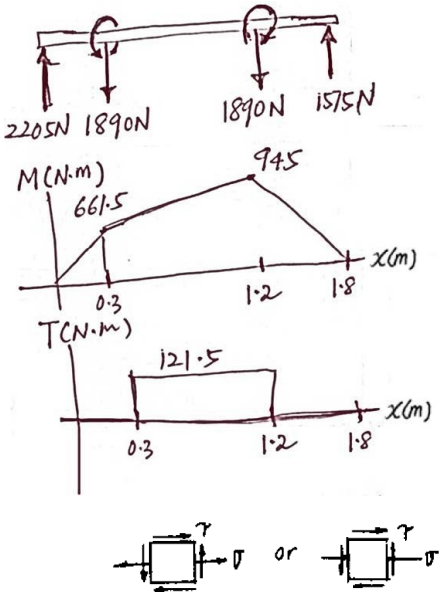
$$= A^2 + 3B^2$$

$$= \frac{\sigma^2}{4} + 3\left(\frac{\sigma^2}{4} + \tau^2\right)$$

$$= \sigma^2 + 3\tau^2$$

$$\sigma_a^2 - \sigma_a \sigma_b + \sigma_b^2 = \sigma_{allow}^2$$

$$\sigma^2 + 3\tau^2 = \sigma_{allow}^2 \qquad (1)$$



$$\sigma = \frac{Mc}{I} = \frac{Mc}{\frac{\pi}{4}c^4} = \frac{4M}{\pi c^3}$$

$$\tau = \frac{Tc}{J} = \frac{Tc}{\frac{\pi}{2}c^4} = \frac{2T}{\pi c^3}$$

From Eq (1)

$$\frac{16M^2}{\pi^2 c^6} + \frac{12T^2}{\pi^2 c^6} = \sigma_{allow}^2$$

$$c = \left(\frac{16M^2 + 12T^2}{\pi^2 \sigma_{allow}^2}\right)^{1/6} = \left\{\frac{16(945^2) + 12(121.5^2)}{\pi^2[469(10^6)]^2}\right\}^{1/6}$$

$c = 0.013718$ m

$d = 2c = 2(0.013718) = 0.027435$ m $= 27.44$ mm

Use $d = 28$ mm **Ans.**

Ans:
Use $d = 28$ mm

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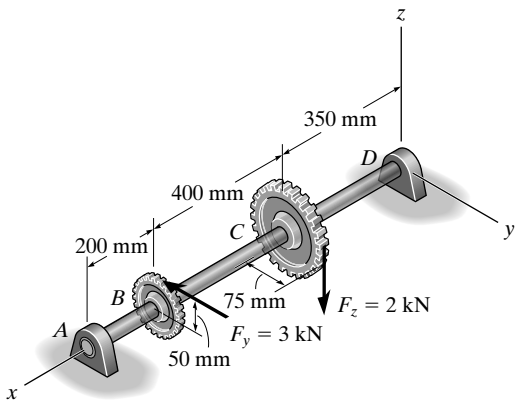
11–45. The bearings at A and D exert only y and z components of force on the shaft. If $\tau_{allow} = 60$ MPa, determine to the nearest millimeter the smallest-diameter shaft that will support the loading. Use the maximum-shear-stress theory of failure.



SOLUTION

Critical moment is at point B:

$$M = \sqrt{(473.7)^2 + (147.4)^2} = 496.1\ \text{N} \cdot \text{m}$$

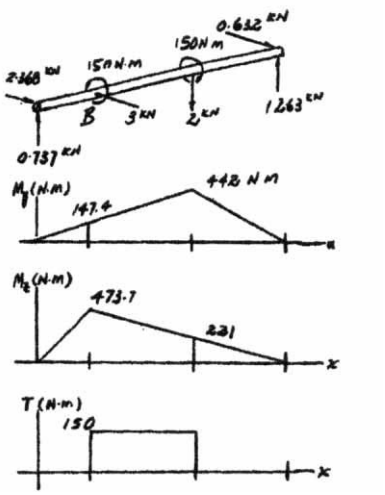
$$T = 150\ \text{N} \cdot \text{m}$$

$$c = \left(\frac{2}{\pi\, \tau_{allow}} \sqrt{M^2 + T^2}\right)^{1/3} = \left(\frac{2}{\pi(60)(10^6)} \sqrt{496.1^2 + 150^2}\right)^{1/3} = 0.0176\ \text{m}$$

$$c = 0.0176\ \text{m} = 17.6\ \text{mm}$$

$$d = 2c = 35.3\ \text{mm}$$

Use $d = 36$ mm **Ans.**



Ans:

Use $d = 36$ mm

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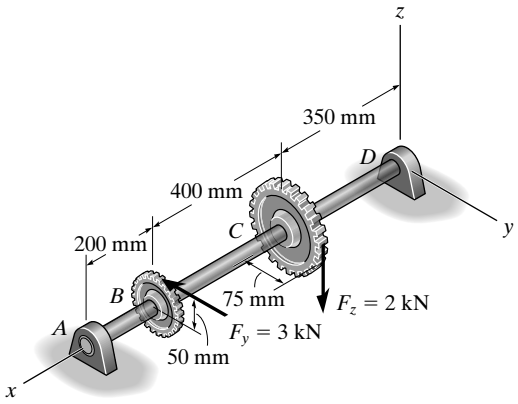
11–46. The bearings at A and D exert only y and z components of force on the shaft. If $\tau_{allow} = 60$ MPa, determine to the nearest millimeter the smallest-diameter shaft that will support the loading. Use the maximum-distortion-energy theory of failure. $\sigma_{allow} = 130$ MPa.



SOLUTION

The critical moment is at B.

$$M = \sqrt{(473.7)^2 + (147.4)^2} = 496.1\ \text{N}\cdot\text{m}$$

$$T = 150\ \text{N}\cdot\text{m}$$

Since,

$$\sigma_{a,b} = \frac{\sigma}{2} \pm \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2}$$

Let $\frac{\sigma}{2} = A$ and $\sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2} = B$

$$\sigma_a^2 = (A + B)^2 \qquad \sigma_b^2 = (A - B)^2$$

$$\sigma_a \sigma_b = (A + B)(A - B)$$

$$\sigma_a^2 - \sigma_a \sigma_b + \sigma_b^2 = A^2 + B^2 + 2AB - A^2 + B^2 + A^2 + B^2 - 2AB$$

$$= A^2 + 3B^2$$

$$= \frac{\sigma^2}{4} + 3\left(\frac{\sigma^2}{4} + \tau^2\right)$$

$$= \sigma^2 + 3\tau^2$$

$$\sigma_a^2 - \sigma_a\sigma_b + \sigma_b^2 = \sigma_{allow}^2$$

$$\sigma^2 + 3\tau^2 = \sigma_{allow}^2 \qquad \textbf{(1)}$$

$$\sigma = \frac{Mc}{I} = \frac{Mc}{\frac{\pi}{4}c^4} = \frac{4M}{\pi c^3}$$

$$\tau = \frac{Tc}{J} = \frac{Tc}{\frac{\pi}{2}c^4} = \frac{2T}{\pi c^3}$$

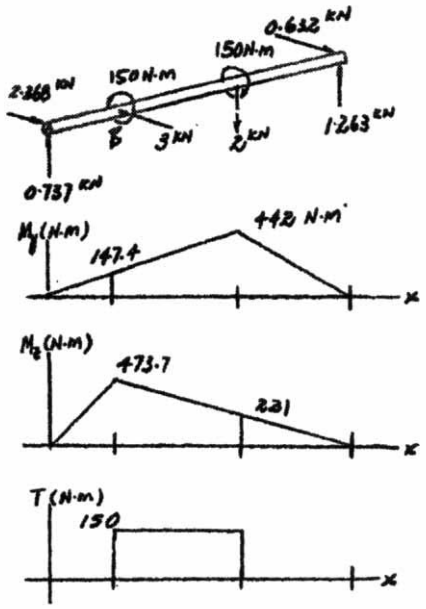
From Eq (1)

$$\frac{16M^2}{\pi^2 c^4} + \frac{12T^2}{\pi^2 c^4} = \sigma_{allow}^2$$

$$c = \left(\frac{16M^2 + 12T^2}{\pi^2 \sigma_{allow}^2}\right)^{1/6}$$

$$= \left(\frac{16(496.1)^2 + 12(150)^2}{\pi^2((130)(10^4))^2}\right)^{1/4} = 0.01712\ \text{m}$$

$$d = 2c = 34.3\ \text{mm} \qquad \textbf{Ans.}$$



Ans:

Use $d = 34.3$ mm

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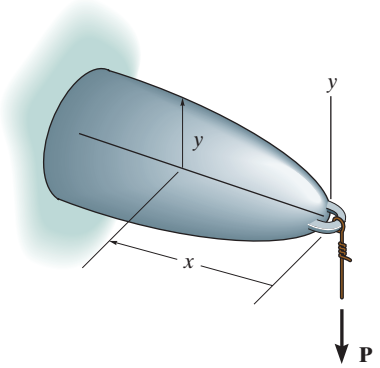
R11–1.

The cantilevered beam has a circular cross section. If it supports a force **P** at its end, determine its radius y as a function of x so that it is subjected to a constant maximum bending stress σ_{allow} throughout its length.



SOLUTION

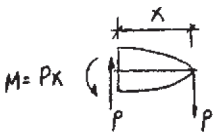
Section Properties:

$$I = \frac{\pi}{4} y^4$$

$$S = \frac{I}{c} = \frac{\frac{\pi}{4}y^4}{y} = \frac{\pi}{4} y^3$$

$$\sigma_{\text{allow}} = \frac{M}{S} = \frac{Px}{\frac{\pi}{4}y^3}$$

$$y = \left[\frac{4P}{\pi\, \sigma_{\text{allow}}} x\right]^{\frac{1}{3}}$$



Ans.

Ans:

$$y = \left[\frac{4P}{\pi\, \sigma_{\text{allow}}} x\right]^{1/3}$$

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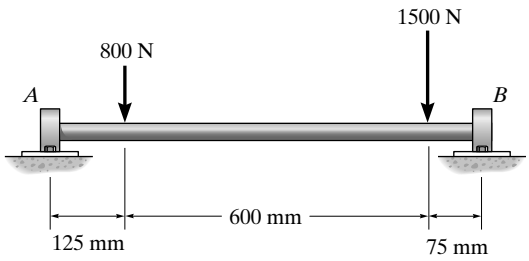
R11–2. Draw the shear and moment diagrams for the shaft, and then determine its required diameter to the nearest millimeter if $\sigma_{allow} = 140$ MPa and $\tau_{allow} = 80$ MPa. The bearings at A and B exert only vertical reactions on the shaft.



SOLUTION

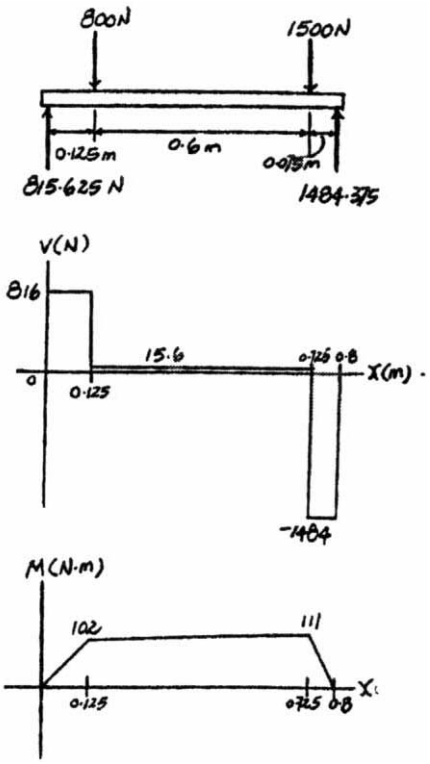
Bending Stress: From the moment diagram, $M_{max} = 111\ \text{N}\cdot\text{m}$. Assume bending controls the design. Applying the flexure formula.

$$\sigma_{allow} = \frac{M_{max}\, c}{I}$$

$$140\left(10^6\right) = \frac{111\left(\frac{d}{2}\right)}{\frac{\pi}{4}\left(\frac{d}{2}\right)^4}$$

$$d = 0.02008\ \text{m} = 20.1\ \text{mm}$$

Use $d = 21$ mm **Ans.**



Shear Stress: Provide a shear stress check using the shear formula with

$$I = \frac{\pi}{4}\left(0.0105^4\right) = 9.5466\left(10^{-9}\right)\ \text{m}^4$$

$$Q_{max} = \frac{4(0.0105)}{3\pi}\left[\frac{1}{2}(\pi)(0.0105)^2\right] = 0.77175\left(10^{-6}\right)\ \text{m}^3$$

From the shear diagram, $V_{max} = 1484$ N.

$$\tau_{max} = \frac{V_{max}\, Q_{max}}{It}$$

$$= \frac{1484\left[0.77175(10^{-6})\right]}{9.5466(10^{-9})(0.021)}$$

$$= 5.71\ \text{MPa} < \tau_{allow} = 80\ \text{MPa}\ (O.K!)$$

Ans:
Use $d = 21$ mm

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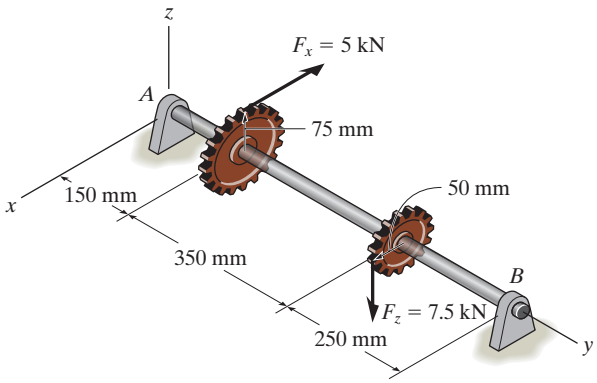
R11–3.

The journal bearings at A and B exert only x and z components of force on the shaft. Determine the shaft's diameter to the nearest millimeter so that it can resist the loadings without exceeding an allowable shear stress of $\tau_{\text{allow}} = 80$ MPa. Use the maximum shear stress theory of failure.



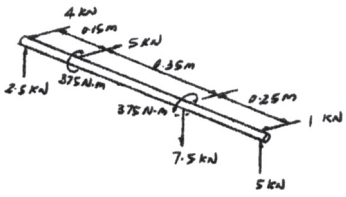
SOLUTION

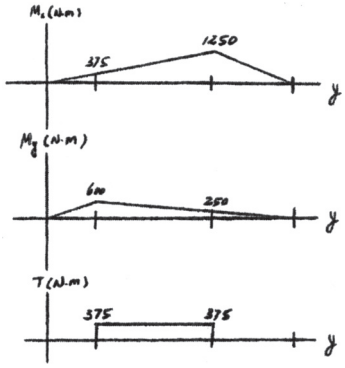
Maximum resultant moment $M = \sqrt{1250^2 + 250^2} = 1274.75\ \text{N} \cdot \text{m}$

$$c = \left[\frac{2}{\pi\,\tau_{\text{allow}}}\sqrt{M^2 + T^2}\right]^{\frac{1}{3}} = \left[\frac{2}{\pi(80)(10^6)}\sqrt{1274.75^2 + 375^2}\right]^{\frac{1}{3}} = 0.0219\ \text{m}$$

$d = 2c = 0.0439\ \text{m} = 43.9\ \text{mm}$

Use $d = 44$ mm. **Ans.**





Ans:
Use $d = 44$ mm

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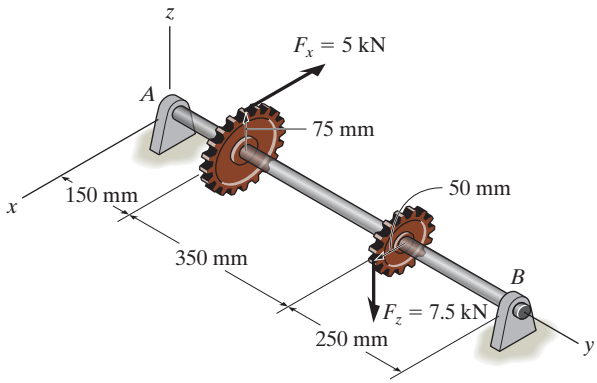
***R11–4.**

The journal bearings at A and B exert only x and z components of force on the shaft. Determine the shaft's diameter to the nearest millimeter so that it can resist the loadings. Use the maximum distortion energy theory of failure with $\sigma_{allow} = 200$ MPa.



SOLUTION

Maximum resultant moment $M = \sqrt{1250^2 + 250^2} = 1274.75\ \text{N}\cdot\text{m}$

$$\sigma_{1,2} = \frac{\sigma_x}{2} \pm \sqrt{\frac{\sigma_x^2}{4} + \tau_{xy}^2}$$

Let $a = \frac{\sigma_x}{2}, b = \sqrt{\frac{\sigma_x^2}{4} + \tau_{xy}^2}$

$$\sigma_1 = a + b, \qquad \sigma_2 = a - b$$

Require,

$$\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2 = \sigma_{allow}^2$$

$$a^2 + 2ab + b^2 - [a^2 - b^2] + a^2 - 2ab + b^2 = \sigma_{allow}^2$$

$$a^2 + 3b^2 = \sigma_{allow}^2$$

$$\frac{\sigma_x^2}{4} + 3\left(\frac{\sigma_x^2}{4} + \tau_{xy}^2\right) = \sigma_{allow}^2$$

$$\sigma_x^2 + 3\tau_{xy}^2 = \sigma_{allow}^2$$

$$\left(\frac{Mc}{\frac{\pi}{4}c^4}\right)^2 + 3\left(\frac{Tc}{\frac{\pi}{2}c^4}\right)^2 = \sigma_{allow}^2$$

$$\frac{1}{c^6}\left[\left(\frac{4M}{\pi}\right)^2 + 3\left(\frac{2T}{\pi}\right)^2\right] = \sigma_{allow}^2$$

$$c^6 = \frac{16}{\sigma_{allow}^2\,\pi^2}M^2 + \frac{12T^2}{\sigma_{allow}^2\,\pi^2}$$

$$c = \left[\frac{4}{\sigma_{allow}^2\,\pi^2}(4M^2 + 3\,T^2)\right]^{\frac{1}{6}}$$

$$= \left[\frac{4}{(200(10^6))^2(\pi)^2}(4(1274.75)^2 + 3(375)^2)\right]^{\frac{1}{6}}$$

$$= 0.0203\ \text{m} = 20.3\ \text{mm}$$

$d = 40.6$ mm

Use

$d = 41$ mm **Ans.**

Ans:
Use $d = 41$ mm

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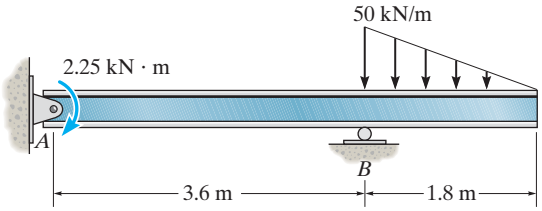
R11–5. Draw the shear and moment diagrams for the beam. Then select the lightest-weight steel wide-flange beam from Appendix B that will safely support the loading. Take $\sigma_{allow} = 150$ MPa and $\tau_{allow} = 84$ MPa.



SOLUTION

Bending Stress: From the moment diagram, $M_{max} = 27$ kN · m. Assume bending controls the design. Applying the flexure formula.

$$S_{req'd} = \frac{M_{max}}{\sigma_{allow}}$$

$$= \frac{27(10^3)}{150(10^6)} = 0.18(10^{-3})\ \text{m}^3 = 180(10^3)\ \text{mm}^3$$

Select W310 × 21 ($S_x = 244(10^3)$ mm^3, $d = 303$ mm and $t_w = 5.08$ mm)

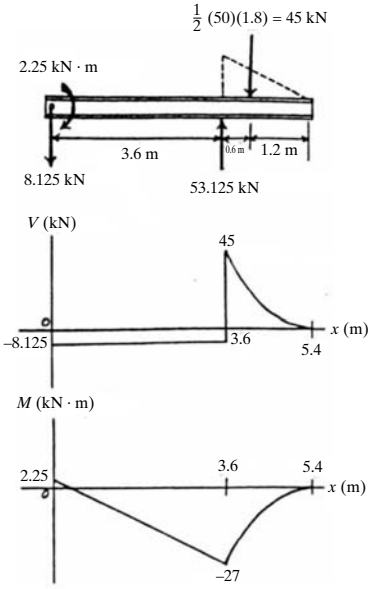
Shear Stress: Provide a shear stress check using $\tau = \frac{V}{t_w d}$ for the W310 × 21 wide-flange section. From the shear diagram, $V_{max} = 45$ kN

$$\tau_{max} = \frac{V_{max}}{t_w d}$$

$$= \frac{45(10^3)}{0.00508(0.303)} = 29.24(10^6)\ \text{N/m}^2$$

$$= 29.2\ \text{MPa} < \tau_{allow} = 84\ \text{MPa}\ (O.K!)$$

Hence, **Use** W310 × 21 **Ans.**



Ans:
Use W310 × 21

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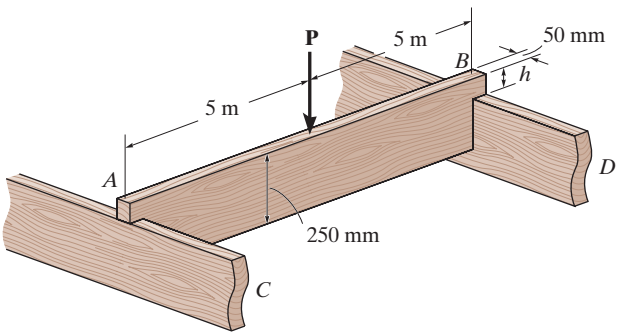
R11–6. The simply supported joist is used in the construction of a floor for a building. In order to keep the floor low with respect to the sill beams C and D, the ends of the joists are notched as shown. If the allowable shear stress for the wood is $\tau_{allow} = 2.5$ MPa and the allowable bending stress is $\sigma_{allow} = 10.5$ MPa determine the height h that will cause the beam to reach both allowable stresses at the same time. Also, what load P causes this to happen? Neglect the stress concentration at the notch.



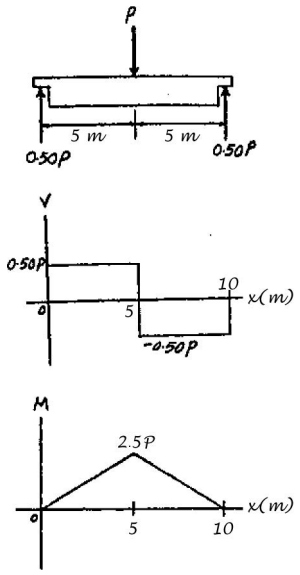
SOLUTION

Bending Stress: From the moment diagram, $M_{max} = 2.5P$. Applying the flexure formula.

$$\sigma_{allow} = \frac{M_{max}c}{I}$$

$$10.5(10^6) = \frac{(2.5P)(0.125)}{\frac{1}{12}(0.05)(0.25^3)}$$

$$P = 2187.5\text{ N} = 2.19\text{ kN}$$ **Ans.**



Shear Stress: From the shear diagram, $V_{max} = 0.500P = 1093.75$ N. The notch is the critical section. Using the shear formula for a rectangular section,

$$\tau_{allow} = \frac{3V_{max}}{2A}$$

$$2.5(10^6) = \frac{3(1093.75)}{2[0.05(h)]}$$

$$h = 0.013125\text{ m} = 13.1\text{ mm}$$ **Ans.**

Ans:

$P = 2.19$ kN, $h = 13.1$ mm

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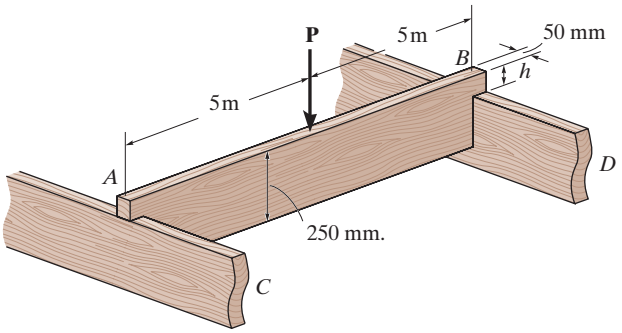
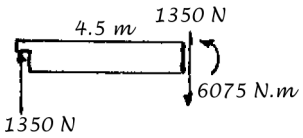
R11–7. The simply supported joist is used in the construction of a floor for a building. In order to keep the floor low with respect to the sill beams C and D, the ends of the joists are notched as shown. If the allowable shear stress for the wood is $\tau_{allow} = 2.5$ MPa and the allowable bending stress is $\sigma_{allow} = 12$ MPa, determine the smallest height h so that the beam will support a load of $P = 2$ kN. Also, will the entire joist safely support the load? Neglect the stress concentration at the notch.



SOLUTION

The reaction at the support is $\frac{2(10^3)}{2} = 1000$ N



$$\tau_{allow} = \frac{1.5V}{A}; \qquad 2.5(10^6) = \frac{1.5(1000)}{0.05h}$$

$h = 0.012$ m $= 12.0$ mm **Ans.**

$$\sigma_{max} = \frac{M_{max}c}{I} = \frac{5000(0.125)}{\frac{1}{12}(0.05)(0.25^3)}$$

$$= 9.60(10^6)\ \text{N/m}^2 = 9.60\ \text{MPa} < 9.60\ \text{MPa} \quad \text{OK}$$

Yes, the joist will safely support the load. **Ans.**

Ans:
$h = 16.2$ mm. Yes, the joist will support the load.

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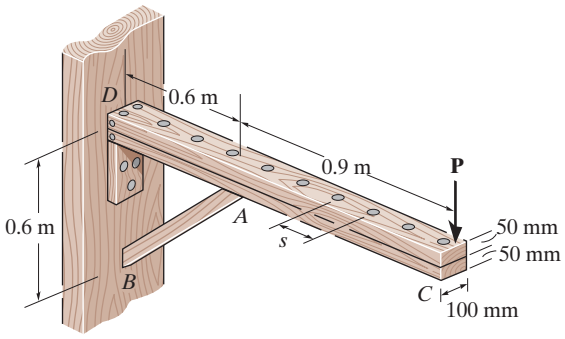
***R11–8** The overhang beam is constructed using two 50-mm by 100-mm pieces of wood braced as shown. If the allowable bending stress of $\sigma_{allow} = 4.2$ MPa, determine the largest load P that can be applied. Also, determine the associated maximum spacing of nails, s, along the beam section AC if each nail can resist a shear force of 4 kN. Assume the beam is pin-connected at A, B, and D. Neglect the axial force developed in the beam along DA.



SOLUTION

$M_A = M_{max} = 0.9P$

Section properties:

$$I = \frac{1}{12}(0.1)(0.1^3) = 8.3333(10^{-6})\ \text{m}^4$$

$$S = \frac{I}{c} = \frac{8.3333(10^{-6})}{0.05} = 0.16667(10^{-3})\ \text{m}^3$$

$$M_{max} = \sigma_{allow} S$$

$$0.9P = [4.2(10^{-6})][0.16667(10^{-3})]$$

$P = 777.78\ \text{N} = 778\ \text{N}$ **Ans.**

Nail Spacing:

$V = P = 777.78\ \text{N}$

$$Q = 0.025(0.1)(0.05) = 0.125(10^{-3})\ \text{m}^3$$

$$q_{allow} = \frac{4(10^3)}{S}$$

Then

$$q_{allow} = \frac{VQ}{I}; \quad \frac{4(10^3)}{S} = \frac{777.78[0.125(10^{-3})]}{8.3333(10^{-6})}$$

$S = 0.34286\ \text{m} = 343\ \text{mm}$

Use $S = 340$ mm **Ans.**

Ans.

Use $S = 340$ mm

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12–1. An A-36 steel strap havingathickness of 10 mm and a width of 20 mm is bent into a circular arc of radius $\rho = 10$ m. Determine the maximum bending stress in the strap.

SOLUTION

Moment-Curvature Relationship:

$$\frac{1}{\rho} = \frac{M}{EI} \qquad \text{however,} \qquad M = \frac{I}{c}\sigma$$

$$\frac{1}{\rho} = \frac{\frac{1}{c}\sigma}{EI}$$

$$\sigma = \frac{c}{\rho}E = \left(\frac{0.005}{10}\right)\left[200\left(10^9\right)\right] = 100 \text{ MPa}$$

These solutions represent a preliminary version of the Instructors' Solutions Manual (ISM). It is possible and even likely that at this preliminary stage of preparing the ISM there are some omissions and errors in the draft solutions. These will be corrected and this manual will be republished.

Ans:

$\sigma = 100$ MPa

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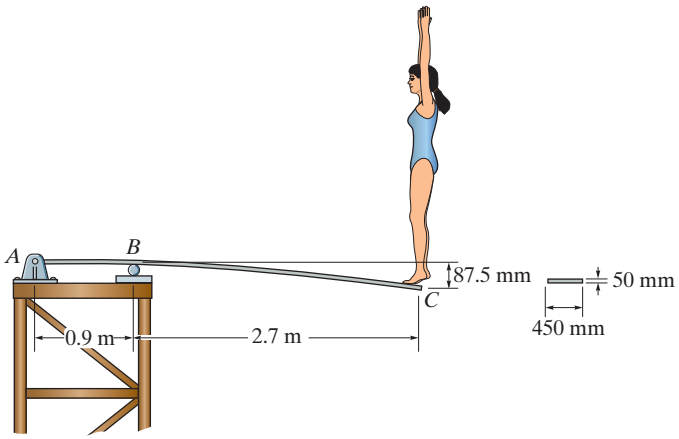
12–2. When the diver stands at end C of the diving board, it deflects downward 87.5 mm. Determine the mass of the diver. The board is made of material having a modulus of elasticity of $E = 10$ GPa.



SOLUTION

Support Reactions and Elastic Curve. As shown in Fig. *a*.

Moment Functions. Referring to the free-body diagrams of the diving board's cut segments, Fig. *b*, $M(x_1)$ is

$$\zeta + \Sigma M_O = 0; \qquad M(x_1) + 3Wx_1 = 0 \qquad M(x_1) = -3Wx_1$$

and $M(x_2)$ is

$$\zeta + \Sigma M_O = 0; \qquad -M(x_2) - Wx_2 = 0 \qquad M(x_2) = -Wx_2$$

Equations of Slope and Elastic Curve.

$$EI\frac{d^2v}{dx^2} = M(x)$$

For coordinate x_1,

$$EI\frac{d^2v_1}{dx_1^2} = -3Wx_1$$

$$EI\frac{dv_1}{dx_1} = -\frac{3}{2}Wx_1^2 + C_1 \tag{1}$$

$$EIv_1 = -\frac{1}{2}Wx_1^3 + C_1x_1 + C_2 \tag{2}$$

For coordinate x_2

$$EI\frac{d^2v_2}{dx_2^2} = -Wx_2$$

$$EI\frac{dv_2}{dx_2} = -\frac{1}{2}Wx_2^2 + C_3 \tag{3}$$

$$EIv_2 = -\frac{1}{6}Wx_2^3 + C_3x_2 + C_4 \tag{4}$$

Boundary Conditions. At $x_1 = 0, v_1 = 0$. Then, Eq. (2) gives

$$EI(0) = -\frac{1}{2}W(0^3) + C_1(0) + C_2 \qquad C_2 = 0$$

At $x_1 = 0.9$ m, $v_1 = 0$. Then Eq.(2) gives

$$EI(0) = -\frac{1}{2}W(0.9^3) + C_1(0.9) + 0 \qquad C_1 = 0.405W$$

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12-2. Continued

At $x_2 = 2.7$ m, $v_2 = 0$. Then Eq.(4) gives

$$EI(0) = -\frac{1}{6}W(2.7^3) + C_3(2.7) + C_4$$

$$2.7C_3 + C_4 = 3.2805W \qquad (5)$$

Continuity Conditions. At $x_1 = 0.9$ m and $x_2 = 2.7$ m, $\frac{dv_1}{dx_1} = -\frac{dv_2}{dx_2}$. Thus Eqs. (1) and (3) give

$$-\frac{3}{2}W(0.9^2) + 0.405W = -\left[-\frac{1}{2}W(2.7^2) + C_3\right] \qquad C_3 = 4.455W$$

Substituting the value of C_3 into Eq. (5),

$$C_4 = -8.748W$$

Substituting the values of C_3 and C_4 into Eq. (4),

$$v_2 = \frac{1}{EI}\left(-\frac{1}{6}Wx_2{}^3 + 4.455Wx_2 - 8.748W\right)$$

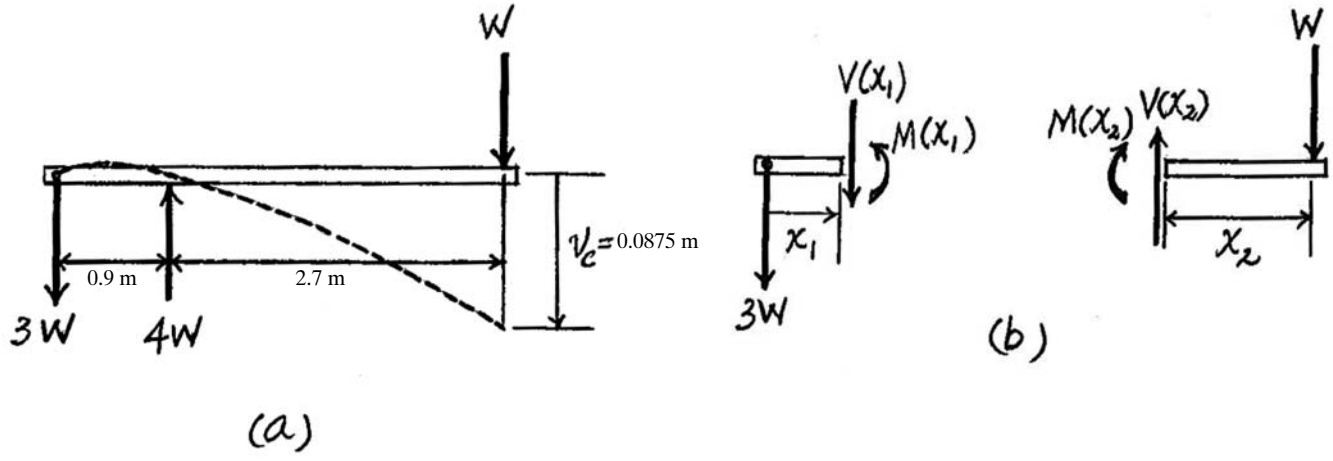
At $x_2 = 0$, $v_2 = -0.0875$ m. Then,

$$-0.0875 = \frac{-8.748W}{[10(10^9)]\left[\frac{1}{12}(0.45)(0.05^3)\right]}$$

$W = 468.86$ N

Then

$$m = \frac{W}{g} = \frac{468.86}{9.81} = 47.8 \text{ kg}$$ **Ans.**



Ans:
$W = 47.8$ kg

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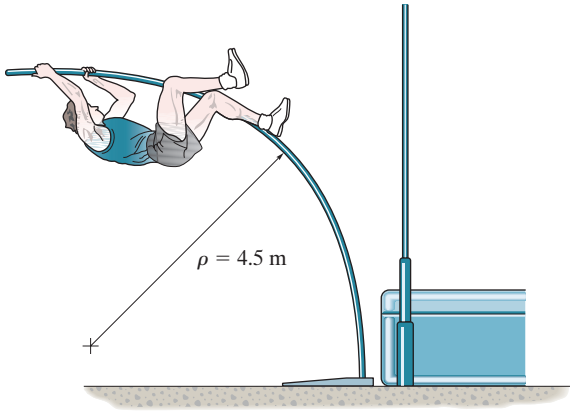
12–3.

A picture is taken of a man performing a pole vault, and the minimum radius of curvature of the pole is estimated by measurement to be 4.5 m. If the pole is 40 mm in diameter and it is made of a glass-reinforced plastic for which $E_g = 131$ GPa, determine the maximum bending stress in the pole.



SOLUTION

Moment-Curvature Relationship:

$$\frac{1}{\rho} = \frac{M}{EI} \qquad \text{however,} \qquad M = \frac{I}{c}\sigma$$

$$\frac{1}{\rho} = \frac{\frac{I}{c}\sigma}{EI}$$

$$\sigma = \frac{c}{\rho}E = \left(\frac{0.02}{4.5}\right)\left[131\left(10^9\right)\right] = 582 \text{ MPa} \qquad \textbf{Ans.}$$

Ans:
$\sigma = 582$ MPa

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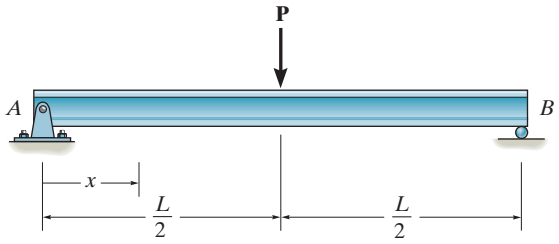
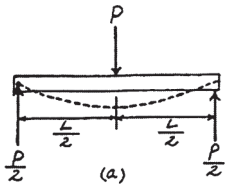
***12–4.**

Determine the equation of the elastic curve for the beam using the x coordinate that is valid for $0 \le x < L/2$. Specify the slope at A and the beam's maximum deflection. EI is constant.

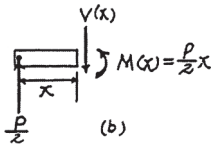


SOLUTION

Support Reactions and Elastic Curve: As shown on FBD(a).



Moment Function: As shown on FBD(b).



Slope and Elastic Curve:

$$EI\frac{d^2v}{dx^2} = M(x)$$

$$EI\frac{d^2v}{dx^2} = \frac{P}{2}x$$

$$EI\frac{dv}{dx} = \frac{P}{4}x^2 + C_1 \qquad \textbf{(1)}$$

$$EI\,v = \frac{P}{12}x^3 + C_1x + C_2 \qquad \textbf{(2)}$$

Boundary Conditions: Due to symmetry, $\frac{dv}{dx} = 0$ at $x = \frac{L}{2}$.

Also, $v = 0$ at $x = 0$.

From Eq. (1) $\quad 0 = \frac{P}{4}\left(\frac{L}{2}\right)^2 + C_1 \qquad C_1 = -\frac{PL^2}{16}$

From Eq. (2) $\quad 0 = 0 + 0 + C_2 \qquad C_2 = 0$

The Slope: Substitute the value of C_1 into Eq. (1),

$$\frac{dv}{dx} = \frac{P}{16EI}(4x^2 - L^2)$$

$$\theta_A = \left.\frac{dv}{dx}\right|_{x=0} = -\frac{PL^2}{16EI} \qquad \textbf{Ans.}$$

The negative sign indicates clockwise rotation.

The Elastic Curve: Substitute the values of C_1 and C_2 into Eq. (2),

$$v = \frac{Px}{48EI}(4x^2 - 3L^2) \qquad \textbf{Ans.}$$

$v_{\max}$ occurs at $x = \frac{L}{2}$,

$$v_{\max} = -\frac{PL^3}{48EI} \qquad \textbf{Ans.}$$

The negative sign indicates downward displacement.

Ans:

$$\theta_A = -\frac{PL^2}{16EI},$$

$$v = \frac{Px}{48EI}(4x^2 - 3L^2),$$

$$v_{\max} = -\frac{PL^3}{48EI}$$

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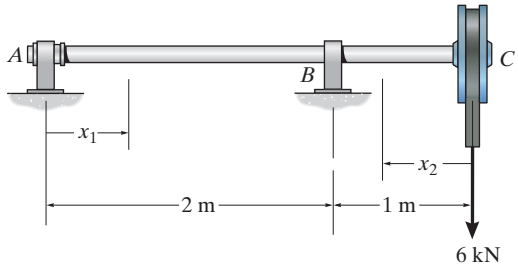
12–5.

Determine the deflection of end C of the 100-mm-diameter solid circular shaft. Take $E = 200$ GPa.



SOLUTION

Support Reactions and Elastic Curve. As shown in Fig. *a*.

Moment Functions. Referring to the free-body diagrams of the shaft's cut segments, Fig. *b*, $M(x_1)$ is

$$\zeta + \Sigma M_O = 0; \qquad M(x_1) + 3x_1 = 0 \qquad M(x_1) = -3x_1\ \text{kN}\cdot\text{m}$$

and $M(x_2)$ is

$$\zeta + \Sigma M_O = 0; \qquad -M(x_2) - 6x_2 = 0 \qquad M(x_2) = -6x_2\ \text{kN}\cdot\text{m}$$

Equations of Slope and Elastic Curve.

$$EI\frac{d^2v}{dx^2} = M(x)$$

For coordinate x_1,

$$EI\frac{d^2v_1}{dx_1^2} = -3x_1$$

$$EI\frac{dv_1}{dx_1} = -\frac{3}{2}x_1^2 + C_1 \qquad \textbf{(1)}$$

$$EIv_1 = -\frac{1}{2}x_1^3 + C_1x_1 + C_2 \qquad \textbf{(2)}$$

For coordinate x_2,

$$EI\frac{d^2v_2}{dx_2^2} = -6x_2$$

$$EI\frac{dv_2}{dx_2} = -3x_2^2 + C_3 \qquad \textbf{(3)}$$

$$EIv_2 = -x_2^3 + C_3x_2 + C_4 \qquad \textbf{(4)}$$

Boundary Conditions. At $x_1 = 0$, $v_1 = 0$. Then, Eq. (2) gives

$$EI(0) = -\frac{1}{2}(0^3) + C_1(0) + C_2 \qquad C_2 = 0$$

At $x_1 = 2$ m, $v_1 = 0$. Then, Eq. (2) gives

$$EI(0) = -\frac{1}{2}(2^3) + C_1(2) + 0 \qquad C_1 = 2\ \text{kN}\cdot\text{m}^2$$

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12–5. Continued

At $x_2 = 1$ m, $v_2 = 0$. Then, Eq. (4) gives

$$EI(0) = -(1^3) + C_3(1) + C_4$$

$$C_3 + C_4 = 1 \qquad \textbf{(5)}$$

Continuity Conditions. At $x_1 = 2$ m and $x_2 = 1$, $\frac{dv_1}{dx_1} = -\frac{dv_2}{dx_2}$. Thus, Eqs. (1) and (3) give

$$-\frac{3}{2}(2^2) + 2 = -[-3(1^2) + C_3] \qquad C_3 = 7\ \text{kN}\cdot\text{m}^2$$

Substituting the values of C_3 into Eq. (5),

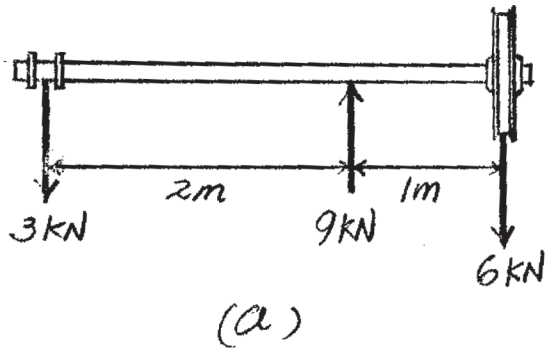
$$C_4 = -6\ \text{kN}\cdot\text{m}^3$$

Substituting the values of C_3 and C_4 into Eq. (4),

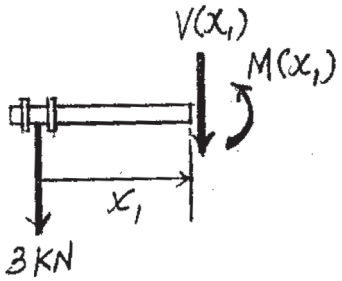
$$v_2 = \frac{1}{EI}(-x_2^3 + 7x_2 - 6)$$

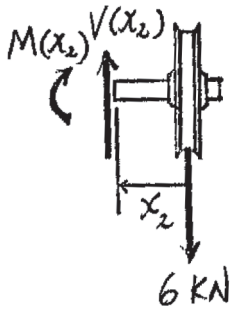
$$v_C = v_2|_{x_2=0} = -\frac{6\ \text{kN}\cdot\text{m}^3}{EI}$$

$$= -\frac{6(10^3)}{200(10^9)\left[\frac{\pi}{4}(0.05^4)\right]} = -0.006112\ \text{m} = -6.11\ \text{mm} \qquad \textbf{Ans.}$$



(a)





(b)

Ans:
$v_C = -6.11$ mm

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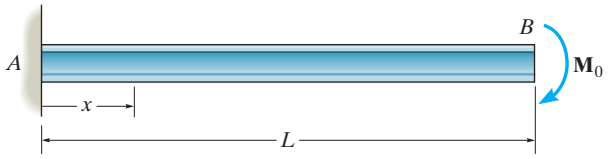
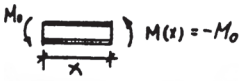
12–6.

Determine the elastic curve for the cantilevered beam, which is subjected to the couple moment M_0. Also calculate the maximum slope and maximum deflection of the beam. *EI* is constant.



SOLUTION



Elastic Curve and Slope:

$$EI\frac{d^2v}{dx^2} = M(x)$$

$$EI\frac{d^2v}{dx^2} = -M_0$$

$$EI\frac{dv}{dx} = -M_0x + C_1 \qquad \textbf{(1)}$$

$$EIv = \frac{-M_0x^2}{2} + C_1x + C_2 \qquad \textbf{(2)}$$

Boundary Conditions:

$$\frac{dv}{dx} = 0 \quad \text{at} \quad x = 0$$

From Eq. (1), $C_1 = 0$

$$v = 0 \quad \text{at} \quad x = 0$$

From Eq. (2), $C_2 = 0$

$$\frac{dv}{dx} = -\frac{M_0x}{EI}$$

$$\theta_{\max} = \left.\frac{dv}{dx}\right|_{x=L} = -\frac{M_0L}{EI} \qquad \textbf{Ans.}$$

The negative sign indicates clockwise rotation.

$$v = -\frac{M_0x^2}{2EI} \qquad \textbf{Ans.}$$

$$v_{\max} = \left.v\right|_{x=L} = -\frac{M_0L^2}{2EI} \qquad \textbf{Ans.}$$

Negative sign indicates downward displacement.

Ans:

$$\theta_{\max} = -\frac{M_0L}{EI},$$

$$v = -\frac{M_0x^2}{2EI},$$

$$v_{\max} = -\frac{M_0L^2}{2EI}$$

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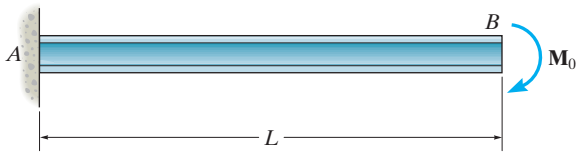
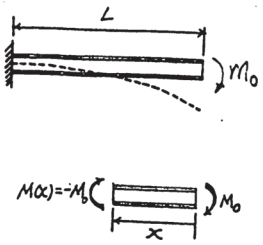
12–7.

The A-36 steel beam has a depth of 10 in. and is subjected to a constant moment $\mathbf{M}_0$, which causes the stress at the outer fibers to become $\sigma_Y = 36$ ksi. Determine the radius of curvature of the beam and the beam's maximum slope and deflection.



SOLUTION



Moment-Curvature Relationship:

$$\frac{1}{\rho} = \frac{M_0}{EI} \qquad \text{however,} \qquad M_0 = \frac{I}{c}\sigma_Y$$

$$\frac{1}{\rho} = \frac{\frac{I}{c}\sigma_Y}{EI}$$

$$\rho = \frac{Ec}{\sigma_Y} = \frac{29.0\,(10^3)(5)}{36} = 4027.78 \text{ in.} = 336 \text{ ft} \qquad \textbf{Ans.}$$

Elastic Curve: As shown.

Moment Function: As shown on FBD.

Slope and Elastic Curve:

$$EI\frac{d^2v}{dx^2} = M(x)$$

$$EI\frac{d^2v}{dx^2} = -M_0$$

$$EI\frac{dv}{dx} = -M_0x + C_1 \qquad \textbf{(1)}$$

$$EI\,v = -\frac{M_0}{2}x^2 + C_1x + C_2 \qquad \textbf{(2)}$$

Boundary Conditions: $\dfrac{dv}{dx} = 0$ at $x = L$, and $v = 0$ at $x = L$.

From Eq. (1), $\quad 0 = -M_0L + C_1 \qquad C_1 = M_0L$

From Eq. (2), $\quad 0 = -\dfrac{M_0}{2}(L^2) + M_0L^2 + C_2 \qquad C_2 = -\dfrac{M_0L^2}{2}$

The Slope: Substitute the value of C_1 into Eq. (1),

$$\frac{dv}{dx} = \frac{M_0}{EI}(-x + L)$$

The maximum slope occurs at $x = 0$.

$$\theta_{\max} = \left.\frac{dv}{dx}\right|_{x=0} = \frac{M_0L}{EI}\curvearrowright \qquad \textbf{Ans.}$$

The Elastic Curve: Substitute the values of C_1 and C_2 into Eq. (2),

$$v = \frac{M_0}{2EI}(-x^2 + 2Lx - L^2)$$

The maximum displacement occurs at $x = 0$.

$$v_{\max} = -\frac{M_0L^2}{2EI} \qquad \textbf{Ans.}$$

The negative sign indicates downward displacement.

Ans:
$\rho = 336$ ft,
$\theta_{\max} = \dfrac{M_0L}{EI}\curvearrowright$,
$v_{\max} = -\dfrac{M_0L^2}{2EI}$

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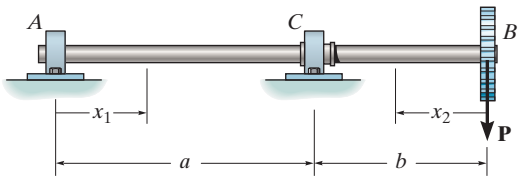
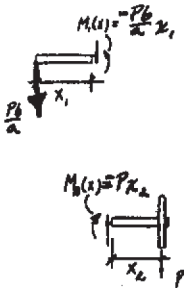
***12–8.**

Determine the equations of the elastic curve using the coordinates x_1 and x_2. EI is constant.



SOLUTION



Elastic Curve and Slope:

$$EI\frac{d^2v}{dx^2} = M(x)$$

For $M_1(x) = -\frac{Pb}{a}x_1$

$$EI\frac{d^2v_1}{dx_1^2} = -\frac{Pb}{a}x_1$$

$$EI\frac{dv_1}{dx_1} = -\frac{Pb}{2a}x_1^2 + C_1 \qquad (1)$$

$$EIv_1 = -\frac{Pb}{6a}x_1^3 + C_1x_1 + C_2 \qquad (2)$$

For $M_2(x) = -Px_2$

$$EI\frac{d^2v_2}{dx_2^2} = -Px_2$$

$$EI\frac{dv_2}{dx_2} = \frac{-Px_2^2}{2} + C_3 \qquad (3)$$

$$EIv_2 = \frac{-Px_2^3}{6} + C_3x_2 + C_4 \qquad (4)$$

Boundary Conditions:

$v_1 = 0$ at $x = 0$

From Eq. (2), $C_2 = 0$

$v_1 = 0$ at $x_1 = a$

From Eq. (2),

$$0 = -\frac{Pb}{6a}a^3 + C_1a$$

$$C_1 = \frac{Pab}{6}$$

$v_2 = 0$ at $x_2 = b$

From Eq. (4),

$$0 = \frac{Pb^3}{6} + C_3b + C_4$$

$$C_3b + C_4 = \frac{Pb^3}{6} \qquad (5)$$

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***12–8. Continued**

Continuity Conditions:

$$\frac{dv_1}{dx_1} = \frac{-dv_2}{dx_2} \quad \text{at} \quad x_1 = a \quad x_2 = b$$

From Eqs. (1) and (3)

$$-\frac{Pb}{2a}(a^2) + \frac{Pab}{6} = \frac{Pb^2}{2} - C_3$$

$$C_3 = \frac{Pab}{3} + \frac{Pb^2}{2}$$

Substitute C_3 into Eq. (5)

$$C_4 = \frac{Pb^3}{3} - \frac{Pab^2}{3}$$

$$v_1 = \frac{-Pb}{6aEI}(x_1^3 - a^2x_1) \qquad \textbf{Ans.}$$

$$v_2 = \frac{P}{6EI}[-x_2^3 + b(2a + 3b)x_2 - 2b^2(a + b)] \qquad \textbf{Ans.}$$

Ans:

$$v_1 = \frac{-Pb}{6aEI}(x_1^3 - a^2x_1),$$

$$v_2 = \frac{P}{6EI}[-x_2^3 + b(2a + 3b)x_2 - 2b^2(a + b)]$$

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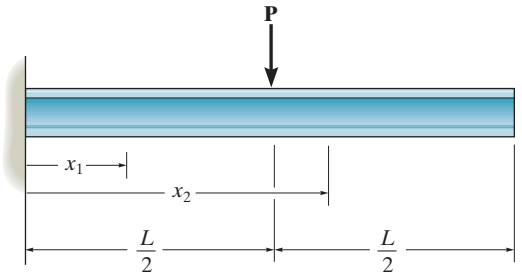
12–9.

Determine the equations of the elastic curve for the beam using the x_1 and x_2 coordinates. EI is constant.



SOLUTION

Referring to the FBDs of the beam's cut segments shown in Fig. *b* and *c*,

$$\zeta + \Sigma M_O = 0; \qquad M(x_1) + \frac{PL}{2} - Px_1 = 0 \qquad M(x_1) = Px_1 - \frac{PL}{2}$$

And

$$\zeta + \Sigma M_O = 0; \qquad M(x_2) = 0$$

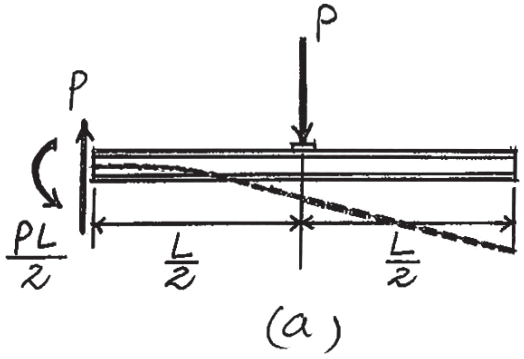
$$EI\frac{d^2v}{dx^2} = M(x)$$

For coordinate x_1,

$$EI\frac{d^2v_1}{dx_1^2} = Px_1 - \frac{PL}{2}$$

$$EI\frac{dv_1}{dx_1} = \frac{P}{2}x_1^{\,2} - \frac{PL}{2}x_1 + C_1 \qquad \textbf{(1)}$$

$$EI\,v_1 = \frac{P}{6}x_1^{\,3} - \frac{PL}{4}x_1^{\,2} + C_1x_1 + C_2 \qquad \textbf{(2)}$$



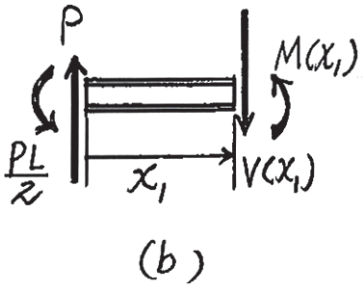
(a)

For coordinate x_2,

$$EI\frac{d^2v_2}{dx_2^2} = 0$$

$$EI\frac{dv_2}{dx_2} = C_3 \qquad \textbf{(3)}$$

$$EI\,v_2 = C_3x_2 = C_4 \qquad \textbf{(4)}$$



(b)

At $x_1 = 0, \frac{dv_1}{dx_1} = 0$. Then, Eq. (1) gives

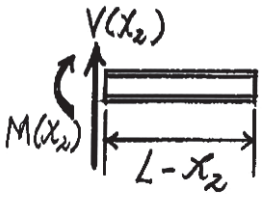
$$EI(0) = \frac{P}{2}(0^2) - \frac{PL}{2}(0) + C_1 \qquad C_1 = 0$$

At $x_1 = 0, v_1 = 0$. Then, Eq(2) gives

$$EI(0) = \frac{P}{6}(0^3) - \frac{PL}{4}(0^2) + 0 + C_2 \qquad C_2 = 0$$

At $x_1 = x_2 = \frac{L}{2}, \frac{dv_1}{dx_1} = \frac{dv_2}{dx_2}$. Thus, Eqs.(1) and (3) gives

$$\frac{P}{2}\left(\frac{L}{2}\right)^2 - \frac{PL}{2}\left(\frac{L}{2}\right) = C_3 \qquad C_3 = -\frac{PL^2}{8}$$



(c)

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12–9. Continued

Also, at $x_1 = x_2 = \frac{L}{2}$, $v_1 = v_2$. Thus, Eqs, (2) and (4) gives

$$\frac{P}{6}\left(\frac{L}{2}\right)^3 - \frac{PL}{4}\left(\frac{L}{2}\right)^2 = \left(-\frac{PL^2}{8}\right)\left(\frac{L}{2}\right) + C_4 \qquad C_4 = \frac{PL^3}{48}$$

Substitute the values of C_1 and C_2 into Eq. (2) and C_3 and C_4 into Eq (4),

$$v_1 = \frac{P}{12EI}\left(2x_1^3 - 3Lx_1^2\right) \qquad \textbf{Ans.}$$

$$v_2 = \frac{PL^2}{48EI}(-6x_2 + L) \qquad \textbf{Ans.}$$

Ans:

$$v_1 = \frac{P}{12EI}(2x_1^3 - 3Lx_1^2),$$

$$v_2 = \frac{PL^2}{48EI}(-6x_2 + L)$$

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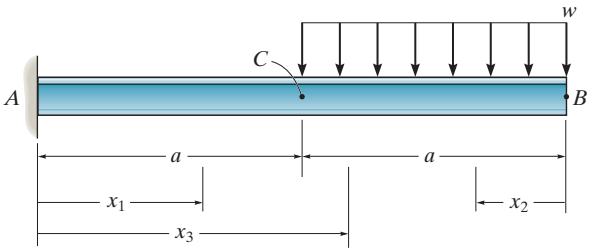
12–10.

Determine the equations of the elastic curve using the coordinates x_1 and x_2. What is the slope at C and displacement at B? EI is constant.



SOLUTION

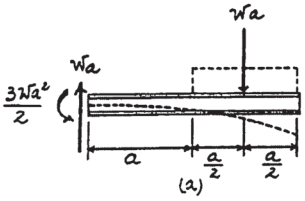
Support Reactions and Elastic Curve: As shown on FBD(a).

Moment Function: As shown on FBD(b) and (c).

Slope and Elastic Curve:

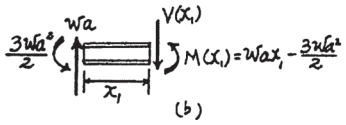
$$EI\frac{d^2v}{dx^2} = M(x)$$

For $M(x_1) = w\,ax_1 = -\frac{3wa^2}{2}$,



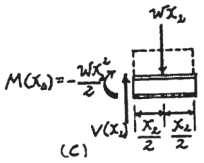
$$EI\frac{d^2v_1}{dx_1^2} = wax_1 - \frac{3wa^2}{2}$$

$$EI\frac{dv_1}{dx_1} = \frac{wa}{2}x_1^2 - \frac{3wa^2}{2}x_1 + C_1 \qquad \textbf{(1)}$$



$$EI\,v_1 = \frac{wa}{6}x_1^3 - \frac{3wa^2}{4}x_1^2 + C_1x_1 + C_2 \qquad \textbf{(2)}$$

For $M(x_2) = -\frac{w}{2}x_2^2$,



$$EI\frac{d^2v_2}{dx_2^2} = -\frac{w}{2}x_2^2$$

$$EI\frac{dv_2}{dx_2} = -\frac{w}{6}x_2^3 + C_3 \qquad \textbf{(3)}$$

$$EI\,v_2 = -\frac{w}{24}x_2^4 + C_3x_2 + C_4 \qquad \textbf{(4)}$$

Boundary Conditions:

$\frac{dv_1}{dx_1} = 0$ at $x_1 = 0$. From Eq. (1), $C_1 = 0$

$v_1 = 0$ at $x_1 = 0$. From Eq. (2), $C_2 = 0$

Continuity Conditions:

At $x_1 = a$ and $x_2 = a$, $\frac{dv_1}{dx_1} = -\frac{dv_2}{dx_2}$. From Eqs. (1) and (3),

$$\frac{wa^3}{2} - \frac{3wa^3}{2} = -\left(-\frac{wa^3}{6} + C_3\right) \qquad C_3 = \frac{7wa^3}{6}$$

At $x_1 = a$ and $x_2 = a$, $v_1 = v_2$. From Eqs. (2) and (4),

$$\frac{wa^4}{6} - \frac{3wa^4}{4} = -\frac{wa^4}{24} + \frac{5wa^4}{6} + C_4 \qquad C_4 = -\frac{11wa^4}{8}$$

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12–10. Continued

The Slope: Substituting into Eq. (1),

$$\frac{dv_1}{dx_1} = \frac{wax_1}{2EI}(x_1 - 3a)$$

$$\theta_C = \left.\frac{dv_1}{dx_1}\right|_{x_1=a} = -\frac{wa^3}{EI} \qquad \textbf{Ans.}$$

The Elastic Curve: Substituting the values of C_1, C_2, C_3, and C_4 into Eqs. (2) and (4), respectively,

$$v_1 = \frac{wax_1}{12EI}\left(2x_1^2 - 9ax_1\right) \qquad \textbf{Ans.}$$

$$v_2 = \frac{w}{24EI}\left(-x_2^4 + 28a^3x_2 - 41a^4\right) \qquad \textbf{Ans.}$$

$$v_B = v_2|_{x_2=0} = -\frac{41wa^4}{24EI} \qquad \textbf{Ans.}$$

Ans:

$$\theta_C = \left.\frac{dv_1}{dx_1}\right|_{x_1=a} = -\frac{wa^3}{EI},$$

$$v_1 = \frac{wax_1}{12EI}\left(2x_1^2 - 9ax_1\right),$$

$$v_2 = \frac{w}{24EI}\left(-x_2^4 + 28a^3x_2 - 41a^4\right),$$

$$v_B = v_2|_{x_2=0} = -\frac{41\,wa^4}{24\,EI}$$

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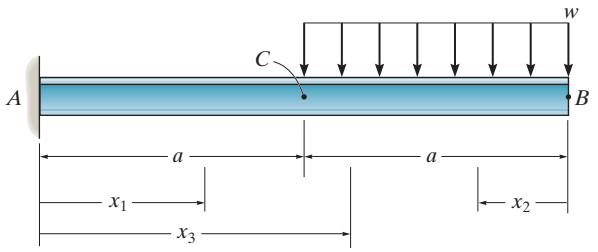
12–11.

Determine the equations of the elastic curve using the coordinates x_1 and x_3. What is the slope at B and deflection at C? EI is constant.



SOLUTION

Support Reactions and Elastic Curve: As shown on FBD(a).

Moment Function: As shown on FBD(b) and (c).

Slope and Elastic Curve:

$$EI\frac{d^2v}{dx^2} = M(x)$$

For $M(x_1) = wax_1 - \frac{3wa^2}{2}$,

$$EI\frac{d^2v_1}{dx_1^2} = wax_1 - \frac{3wa^2}{2}$$

$$EI\frac{dv_1}{dx_1} = \frac{wa}{2}x_1^2 - \frac{3wa^2}{2}x_1 + C_1 \qquad \textbf{(1)}$$

$$EI\,v_1 = \frac{wa}{6}x_1^3 - \frac{3wa^2}{4}x_1^2 + C_1x_1 + C_2 \qquad \textbf{(2)}$$

For $M(x_3) = 2wax_3 - \frac{w}{2}x_3^2 - 2wa^2$,

$$EI\frac{d^2v_3}{dx_3^2} = 2wax_3 - \frac{w}{2}x_3^2 - 2wa^2$$

$$EI\frac{dv_3}{dx_3} = wax_3^2 - \frac{w}{6}x_3^3 - 2wa^2x_3 + C_3 \qquad \textbf{(3)}$$

$$EI\,v_3 = \frac{wa}{3}x_3^3 - \frac{w}{24}x_3^4 - wa^2x_3^2 + C_3x_3 + C_4 \qquad \textbf{(4)}$$

Boundary Conditions:

$\frac{dv_1}{dx_1} = 0$ at $x_1 = 0$. From Eq. (1), $C_1 = 0$

$v_1 = 0$ at $x_1 = 0$. From Eq. (2), $C_2 = 0$

Continuity Conditions:

At $x_1 = a$ and $x_3 = a$, $\frac{dv_1}{dx_1} = \frac{dv_3}{dx_3}$. From Eqs. (1) and (3),

$$\frac{wa^3}{2} - \frac{3wa^3}{2} = wa^3 - \frac{wa^3}{6} - 2wa^3 + C_3 \qquad C_3 = \frac{wa^3}{6}$$

At $x_1 = a$ and $x_3 = a$, $v_1 = v_3$. From Eqs. (2) and (4),

$$\frac{wa^4}{6} - \frac{3wa^4}{4} = \frac{wa^4}{3} - \frac{wa^4}{24} - wa^4 + \frac{wa^4}{6} + C_4 \qquad C_4 = -\frac{wa^4}{24}$$

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12–11. Continued

The Slope: Substituting the value of C_1 into Eq. (1),

$$\frac{dv_3}{dx_3} = \frac{w}{6EI}(6ax_3^2 - x_3^3 - 12a^2x_3 + a^3)$$

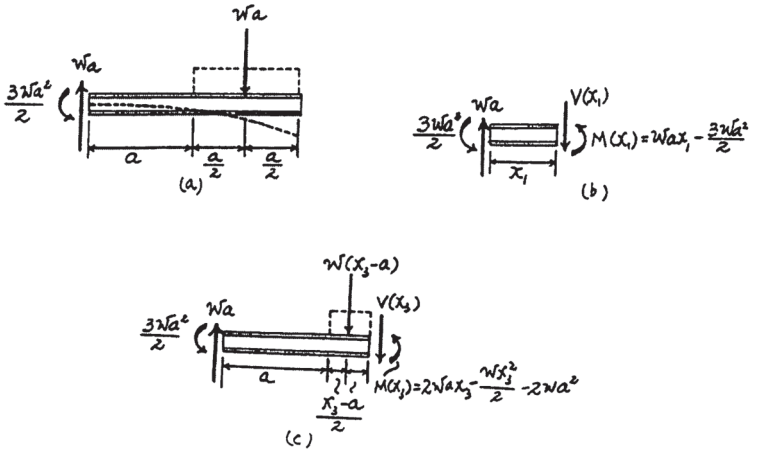
$$\theta_B = \left.\frac{dv_3}{dx_3}\right|_{x_3=2a} = -\frac{7wa^3}{6EI} \qquad \textbf{Ans.}$$

The Elastic Curve: Substituting the values of C_1, C_2, C_3, and C_4 into Eqs. (2) and (4), respectively,

$$v_1 = \frac{wax_1}{12EI}(2x_1^2 - 9ax_1) \qquad \textbf{Ans.}$$

$$v_C = v_1|_{x_1=a} = -\frac{7\,wa^4}{12EI} \qquad \textbf{Ans.}$$

$$v_3 = \frac{w}{24EI}(-x_3^4 + 8ax_3^3 - 24a^2x_3^2 + 4a^3x_3 - a^4) \qquad \textbf{Ans.}$$



Ans:

$$\theta_B = -\frac{7wa^3}{6EI},$$

$$v_1 = \frac{wax_1}{12EI}(2x_1^2 - 9ax_1),$$

$$v_C = v_1|_{x_1=a} = -\frac{7\,wa^4}{12EI},$$

$$v_3 = \frac{w}{24EI}(-x_3^4 + 8ax_3^3 - 24a^2x_3^2 + 4a^3x_3 - a^4)$$

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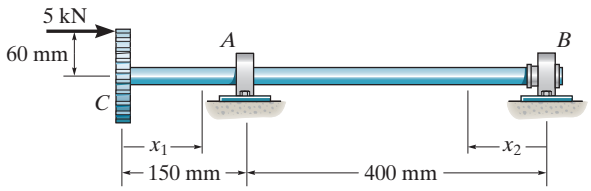
***12–12.**

Draw the bending-moment diagram for the shaft and then, from this diagram, sketch the deflection or elastic curve for the shaft's centerline. Determine the equations of the elastic curve using the coordinates x_1 and x_2. EI is constant.



SOLUTION

Elastic Curve: As shown.

Moment Function: As shown on FBD(b) and (c).

Slope and Elastic Curve:

$$EI\frac{d^2v}{dx^2} = M(x)$$

For $M(x_1) = 300\ \text{N}\cdot\text{m}$,

$$EI\frac{d^2v_1}{dx_1^2} = 300$$

$$EI\frac{dv_1}{dx_1} = 300x_1 + C_1 \qquad \textbf{(1)}$$

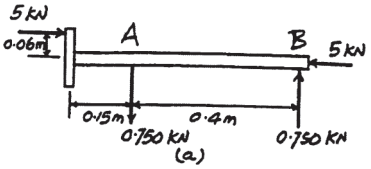
$$EI\,v_1 = 150x_1^2 + C_1x_1 + C_2 \qquad \textbf{(2)}$$

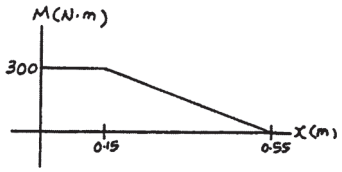
For $M(x_2) = 750x_2$,

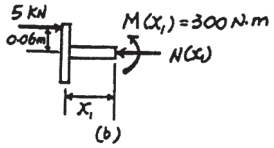
$$EI\frac{d^2v_2}{dx_2^2} = 750x_2$$

$$EI\frac{dv_2}{dx_2} = 375x_2^2 + C_3 \qquad \textbf{(3)}$$

$$EI\,v_2 = 125x_2^3 + C_3x_2 + C_4 \qquad \textbf{(4)}$$







Boundary Conditions:

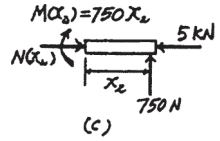
$v_1 = 0$ at $x_1 = 0.15$ m. From Eq. (2),

$$0 = 150(0.15^2) + C_1(0.15) + C_2 \qquad \textbf{(5)}$$

$v_2 = 0$ at $x_2 = 0$. From Eq. (4), $C_4 = 0$

$v_2 = 0$ at $x_2 = 0.4$ m. From Eq. (4),

$$0 = 125(0.4^3) + C_3(0.4) \qquad C_3 = -20.0$$



Continuity Condition:

At $x_1 = 0.15$ m and $x_2 = 0.4$ m, $\dfrac{dv_1}{dx_1} = -\dfrac{dv_2}{dx_2}$. From Eqs. (1) and (3),

$$300(0.15) + C_1 = -\left[375(0.4^2) - 20\right] \qquad C_1 = -85.0$$

From Eq. (5), $C_2 = 9.375$

The Elastic Curve: Substitute the values of C_1, C_2, C_3, and C_4 into Eqs. (2) and (4), respectively.

$$v_1 = \frac{1}{EI}(150x_1^2 - 85.0x_1 + 9.375)\ \text{N}\cdot\text{m}^3 \qquad \textbf{Ans.}$$

$$v_2 = \frac{1}{EI}(125x_2^3 - 20.0x_2)\ \text{N}\cdot\text{m}^3 \qquad \textbf{Ans.}$$

Ans:

$$v_1 = \frac{1}{EI}(150x_1^2 - 85.0x_1 + 9.375)\ \text{N}\cdot\text{m}^3,$$

$$v_2 = \frac{1}{EI}(125x_2^3 - 20.0x_2)\ \text{N}\cdot\text{m}^3$$

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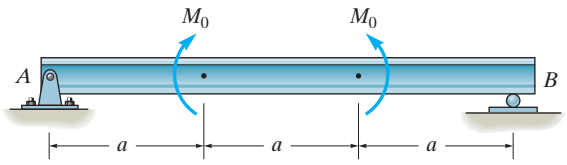
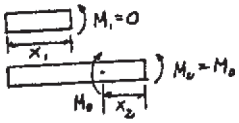
12–13.

Determine the maximum deflection of the beam and the slope at A. EI is constant.



SOLUTION



$M_1 = 0$

$$EI\frac{d^2v_1}{dx_1^2} = 0; \qquad EI\frac{dv_1}{dx_1} = C_1$$

$$EIv_1 = C_1x_1 + C_2$$

$$\text{At } x_1 = 0, \quad v_1 = 0; \quad C_2 = 0$$

$$M_2 = M_0; \qquad EI\frac{d^2v_1}{dx_2^2} = M_0$$

$$EI\frac{dv_2}{dx_2} = M_0x_2 + C_2$$

$$EI\,v_2 = \frac{1}{2}M_0x_2^2 + C_3x_2 + C_4$$

$$\text{At } x_2 = \frac{a}{2}, \quad \frac{dv_2}{dx_2} = 0; \qquad C_1 = \frac{-M_0a}{2}$$

$$\text{At } x_1 = a, \quad x_2 = 0, \quad v_1 = v_2, \quad \frac{dv_1}{dx_1} = \frac{dv_2}{dx_2}$$

$$C_1a = C_a$$

$$C_1 = \frac{-M_0a}{2}, \qquad C_a = \frac{-M_0a^2}{2}$$

$$\text{At } x_1 = 0,$$

$$EI\frac{dv_1}{dx_1} = -\frac{M_0a}{2}$$

$$\boldsymbol{\theta}_A = -\frac{M_0a}{2EI}$$ **Ans.**

$$\text{At } x_2 = \frac{a}{2},$$

$$EI\,v_{\max} = \frac{1}{2}M_0\left(\frac{a^2}{4}\right) - \frac{M_0a}{2}\left(\frac{a}{2}\right) - \frac{M_0a^2}{2}$$

$$v_{\max} = -\frac{5M_0a^2}{8EI}$$ **Ans.**

Ans:

$$\theta_A = -\frac{M_0a}{2EI},\ v_{\max} = -\frac{5M_0a^2}{8EI}$$

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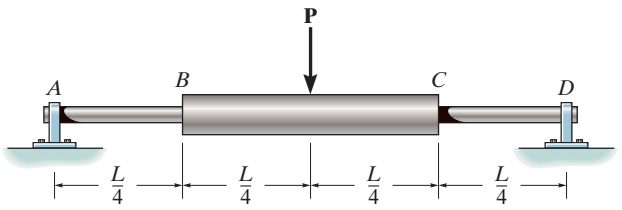
12–14.

The simply supported shaft has a moment of inertia of $2I$ for region BC and a moment of inertia I for regions AB and CD. Determine the maximum deflection of the shaft due to the load **P**.



SOLUTION

$$M_1(x) = \frac{P}{2}x_1$$

$$M_2(x) = \frac{P}{2}x_2$$

Elastic Curve and Slope:

$$EI\frac{d^2v}{dx^2} = M(x)$$

$$EI\frac{d^2v_1}{dx_1{}^2} = \frac{P}{2}x_1$$

$$EI\frac{dv_1}{dx_1} = \frac{Px_1^2}{4} + C_1 \qquad \textbf{(1)}$$

$$EIv_1 = \frac{Px_1^3}{12} + C_1x_1 + C_2 \qquad \textbf{(2)}$$

$$2EI\frac{d^2v_2}{dx_2{}^2} = \frac{P}{2}x_2$$

$$2EI\frac{dv_2}{dx_1} = \frac{Px_2^2}{4} + C_3 \qquad \textbf{(3)}$$

$$2EIv_2 = \frac{Px_2^3}{12} + C_3x_2 + C_4 \qquad \textbf{(4)}$$

Boundary Conditions:

$v_1 = 0$ at $x_1 = 0$

From Eq. (2), $C_2 = 0$

$$\frac{dv_2}{dx_2} = 0 \text{ at } x_2 = \frac{L}{2}$$

From Eq. (3),

$$0 = \frac{PL^2}{16} + C_3$$

$$C_3 = -\frac{PL^2}{16}$$

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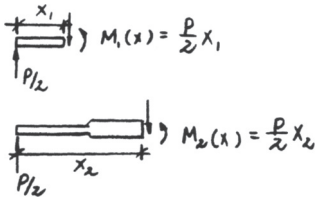
12–14. Continued

Continuity Conditions:

$$\frac{dv_1}{dx_1} = \frac{dv_2}{dx_2} \text{ at } x_1 = x_2 = \frac{L}{4}$$



From Eqs. (1) and (3),

$$\frac{PL^2}{64} + C_1 = \frac{PL^2}{128} - \frac{1}{2}\left(\frac{PL^2}{16}\right)$$

$$C_1 = \frac{-5PL^2}{128}$$

$$v_1 = v_2 \text{ at } x_1 = x_2 = \frac{L}{4}$$

From Eqs. (2) and (4)

$$\frac{PL^3}{768} - \frac{5PL^2}{128}\left(\frac{L}{4}\right) = \frac{PL^3}{1536} - \frac{1}{2}\left(\frac{PL^2}{16}\right)\left(\frac{L}{4}\right) + \frac{1}{2}C_4$$

$$C_4 = \frac{-PL^3}{384}$$

$$v_2 = \frac{P}{768EI}\left(32x_2^3 - 24L^2 x_2 - L^3\right)$$

$$v_{\max} = v_2\Big|_{x_2 = \frac{L}{2}} = \frac{-3PL^3}{256EI}$$ **Ans.**

Ans:

$$v_{\max} = -\frac{3PL^3}{256EI}$$

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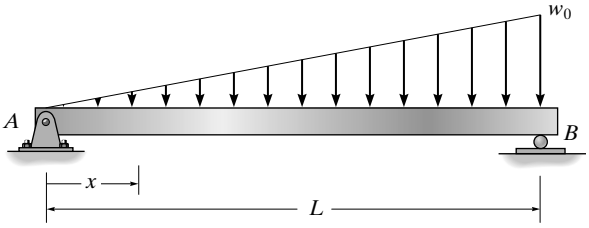
12–15. The beam is subjected to the linearly varying distributed load. Determine the maximum deflection of the beam. *EI* is constant.

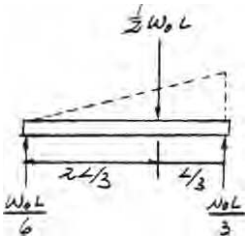


SOLUTION

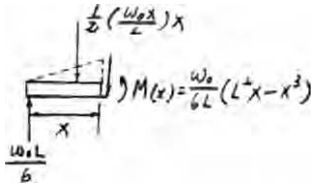
$$EI\frac{d^2v}{dx^2} = M(x)$$

$$EI\frac{d^2v}{dx^2} = \frac{w_0}{6L}\left(L^2x - x^3\right)$$

$$EI\frac{dv}{dx} = \frac{w_0}{6L}\left(\frac{L^2x^2}{2} - \frac{x^4}{4}\right) + C_1 \qquad (1)$$



$$EI\,v = \frac{w_0}{6L}\left(\frac{L^2x^3}{6} - \frac{x^5}{20}\right) + C_1x + C_2 \qquad (2)$$



Boundary conditions:

$v = 0$ at $x = 0$.

From Eq. (2), $C_2 = 0$

$v = 0$ at $x = L$.

From Eq. (2),

$$0 = \frac{w_0}{6L}\left(\frac{L^2}{6} - \frac{L^5}{20}\right) + C_1L; \qquad C_1 = -\frac{7w_0L^3}{360}$$

$$\frac{dv}{dx} = \frac{w_0}{6EIL}\left(\frac{L^2x^2}{2} - \frac{x^4}{4} - \frac{7L^4}{60}\right)$$

$$\frac{dv}{dx} = 0 = \left(\frac{L^2x^2}{2} - \frac{x^4}{4} - \frac{7L^4}{60}\right)$$

$$15x^4 - 30L^2x^2 + 7L^4 = 0; \qquad x = 0.5193L$$

$$v = \frac{w_0x}{360EIL}\left(10L^2x^2 - 3x^4 - 7L^4\right)$$

Substitute $x = 0.5193L$ into v,

$$v_{\max} = -\frac{0.00652w_0L^4}{EI}$$ **Ans.**

Ans:

$$v_{\max} = -\frac{0.00652w_0L^4}{EI}$$

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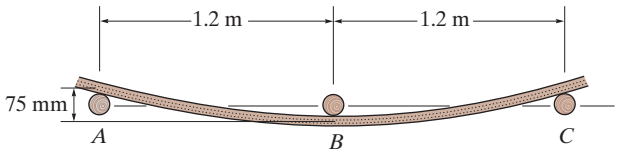
***12–16.** The fence board weaves between the three smooth fixed posts. If the posts remain along the same line, determine the maximum bending stress in the board. The board has a width of 150 mm and a thickness of 12 mm. $E = 12$ GPa Assume the displacement of each end of the board relative to its center is 75 mm.



Support Reactions and Elastic Curve: As shown on FBD(a).

Moment Function: As shown on FBD(b).

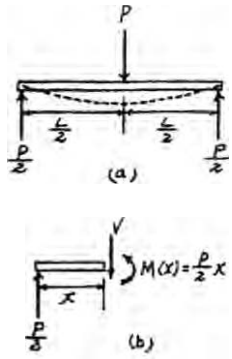
Slope and Elastic Curve:

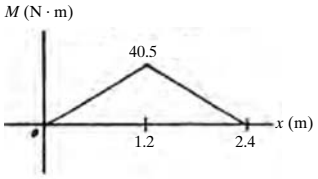
$$EI\frac{d^2v}{dx^2} = M(x)$$

$$EI\frac{d^2v}{dx^2} = \frac{P}{2}x$$

$$EI\frac{dv}{dx} = \frac{P}{4}x^2 + C_1 \qquad [1]$$

$$EI\,v = \frac{P}{12}x^3 + C_1x + C_2 \qquad [2]$$





Boundary Conditions: Due to symmetry, $\frac{dv}{dx} = 0$ at $x = \frac{L}{2}$.

Also, $v = 0$ at $x = 0$.

From Eq. [1] $0 = \frac{P}{4}\left(\frac{L}{2}\right)^2 + C_1 \qquad C_1 = -\frac{PL^2}{16}$

From Eq. [2] $0 = 0 + 0 + C_2 \qquad C_2 = 0$

The Elastic Curve: Substitute the values of C_1 and C_2 into Eq. [2],

$$v = \frac{Px}{48EI}\left(4x^2 - 3L^2\right) \qquad [1]$$

Require $x = 1.2$, $v = -0.075$ m. From Eq. [1],

$$-0.075 = \frac{P(1.2)}{48[12(10^9)]\left[\frac{1}{12}(0.150)(0.012^3)\right]}[4(1.2^2) - 3(2.4^2)]$$

$$P = 67.5 \text{ N}$$

Maximum Bending Stress: From the moment diagram, the maximum moment is $M_{max} = 40.5$ N · m. Applying the flexure formula,

$$\sigma_{max} = \frac{Mc}{I} = \frac{40.5(0.006)}{\frac{1}{12}(0.15)(0.012^3)} = 11.25(10^6) \text{ N/m}^2 = 11.25 \text{ MPa} \qquad \textbf{Ans.}$$

Ans:
$\sigma_{max} = 11.25$ MPa

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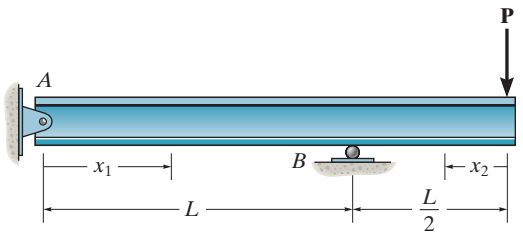
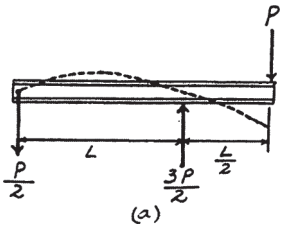
12–17.

Determine the equations of the elastic curve for the beam using the x_1 and x_2 coordinates. Specify the beam's maximum deflection. EI is constant.

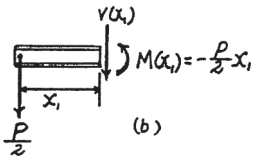


SOLUTION

Support Reactions and Elastic Curve: As shown on FBD(a).



Moment Function: As shown on FBD(b) and (c).



Slope and Elastic Curve:

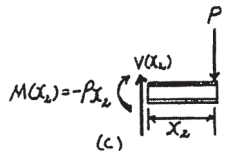
$$EI\frac{d^2v}{dx^2} = M(x)$$

For $M(x_1) = -\frac{P}{2}x_1$,

$$EI\frac{d^2v_1}{dx_1^2} = -\frac{P}{2}x_1$$

$$EI\frac{dv_1}{dx_1} = -\frac{P}{4}x_1^2 + C_1 \qquad \textbf{(1)}$$

$$EI\,v_1 = -\frac{P}{12}x_1^3 + C_1x_1 + C_2 \qquad \textbf{(2)}$$



For $M(x_2) = -Px_2$,

$$EI\frac{d^2v_2}{dx_2^2} = -Px_2$$

$$EI\frac{dv_2}{dx_2} = -\frac{P}{2}x_2^2 + C_3 \qquad \textbf{(3)}$$

$$EI\,v_2 = -\frac{P}{6}x_2^3 + C_3x_2 + C_4 \qquad \textbf{(4)}$$

Boundary Conditions:

$v_1 = 0$ at $x_1 = 0$. From Eq. (2), $C_2 = 0$

$v_1 = 0$ at $x_1 = L$. From Eq. (2),

$$0 = -\frac{PL^3}{12} + C_1L \qquad C_1 = \frac{PL^2}{12}$$

$v_2 = 0$ at $x_2 = \frac{L}{2}$. From Eq. (4),

$$0 = -\frac{PL^3}{48} + \frac{L}{2}C_3 + C_4 \qquad \textbf{(5)}$$

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12–17. Continued

Continuity Conditions:

At $x_1 = L$ and $x_2 = \frac{L}{2}$, $\quad \frac{dv_1}{dx_1} = -\frac{dv_2}{dx_2}$. From Eqs. (1) and (3),

$$-\frac{PL^2}{4} + \frac{PL^2}{12} = -\left(-\frac{PL^2}{8} + C_3\right) \qquad C_3 = \frac{7PL^2}{24}$$

From Eq. (5), $\qquad C_4 = -\frac{PL^3}{8}$

The Slope: Substitute the value of C_1 into Eq. (1),

$$\frac{dv_1}{dx_1} = \frac{P}{12EI}\left(L^2 - 3x_1^2\right)$$

$$\frac{dv_1}{dx_1} = 0 = \frac{P}{12EI}\left(L^2 - 3x_1^2\right) \qquad x_1 = \frac{L}{\sqrt{3}}$$

The Elastic Curve: Substitute the values of C_1, C_2, C_3, and C_4 into Eqs. (2) and (4), respectively,

$$v_1 = \frac{Px_1}{12EI}\left(-x_1^2 + L^2\right) \qquad \textbf{Ans.}$$

$$v_D = v_1\big|_{x_1 = \frac{L}{\sqrt{3}}} = \frac{P\left(\frac{L}{\sqrt{3}}\right)}{12EI}\left(-\frac{L^2}{3} + L^2\right) = \frac{0.0321PL^3}{EI}$$

$$v_2 = \frac{P}{24EI}\left(-4x_2^3 + 7L^2x_2 - 3L^3\right) \qquad \textbf{Ans.}$$

$$v_C = v_2\big|_{x_2 = 0} = -\frac{PL^3}{8EI}$$

Hence, $\qquad v_{\max} = v_C = \frac{PL^3}{8EI}$ **Ans.**

Ans:

$$v_1 = \frac{Px_1}{12EI}\left(-x_1^2 + L^2\right),$$

$$v_2 = \frac{P}{24EI}\left(-4x_2^3 + 7L^2x_2 - 3L^3\right),$$

$$v_{\max} = \frac{PL^3}{8EI}$$

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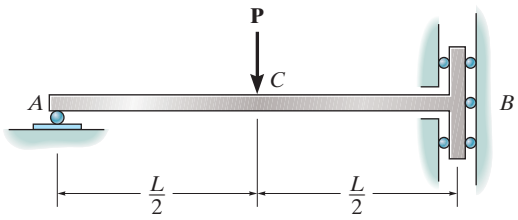
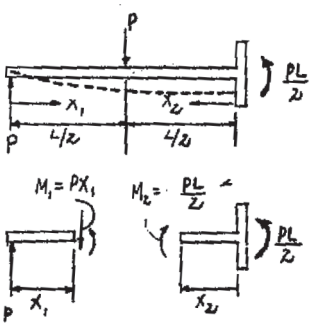
12–18.

The bar is supported by a roller constraint at B, which allows vertical displacement but resists axial load and moment. If the bar is subjected to the loading shown, determine the slope at A and the deflection at C. EI is constant.



SOLUTION



$$EI\frac{d^2v_1}{dx_1^2} = M_1 = Px_1$$

$$EI\,v_1 = \frac{Px_1^2}{6} + C_1$$

$$EI\,v_1 = \frac{Px_1^2}{6} + C_1x_1 + C_2$$

$$EI\frac{d^2v_2}{dx_2} = M_2 = \frac{PL}{2}$$

$$EI\frac{dv_2}{dx_2} = \frac{PL}{2}x_2 + C_3$$

$$EI\,v_2 = \frac{PL}{4}x_2^2 + C_3x_2 + C_4$$

Boundary Conditions:

$$\text{At } x_1 = 0, v_1 = 0$$

$$0 = 0 + 0 + C_2; \quad C_2 = 0$$

$$\text{At } x_2 = 0, \quad \frac{dv_2}{dx_2} = 0$$

$$0 + C_3 = 0; \quad C_3 = 0$$

$$\text{At } x_1 = \frac{L}{2}, \quad x_2 = \frac{L}{2}, v_1 = v_2, \frac{dv_1}{dx_1} = -\frac{dv_2}{dx_2}$$

$$\frac{P\left(\frac{1}{2}\right)^2}{6} + C_1\left(\frac{L}{2}\right) = \frac{PL\left(\frac{1}{2}\right)^2}{4} + C_4$$

$$\frac{P\left(\frac{1}{2}\right)^2}{2} + C_1 = -\frac{PL\left(\frac{1}{2}\right)}{2}; \quad C_1 = -\frac{3}{8}PL^2$$

$$C_4 = -\frac{11}{48}PL^3$$

$$\text{At } x_1 = 0$$

$$\frac{dv_1}{dx_1} = \theta_A = -\frac{3}{8}\frac{PL^2}{EI}$$ **Ans.**

$$\text{At } x_1 = \frac{L}{2}$$

$$v_C = \frac{P\left(\frac{1}{2}\right)^3}{6EI} - \left(\frac{3}{8EI}PL^3\right)\left(\frac{L}{2}\right) + 0$$

$$v_C = -\frac{PL^3}{6EI}$$ **Ans.**

Ans:

$$\theta_A = -\frac{3PL^2}{8EI}, v_C = -\frac{PL^3}{6EI}$$

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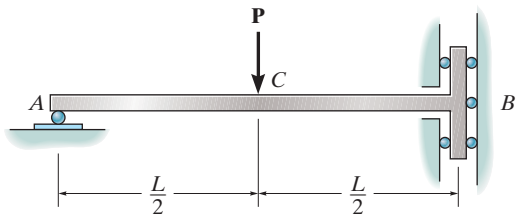
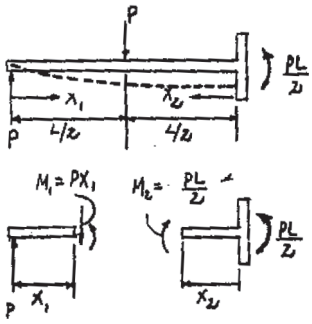
12–19.

Determine the deflection at B of the bar in Prob. 12–18.



SOLUTION



$$EI\frac{d^2v_1}{dx_1^2} = M_1 = Px_1$$

$$EI\frac{dv_1}{dx_1} = \frac{Px_1^2}{2} + C_1$$

$$EI\,v_1 = \frac{Px_1^2}{6} + C_1x_1 + C_2$$

$$EI\frac{d^2v_2}{dx_2} = M_2 = \frac{PL}{2}$$

$$EI\frac{dv_2}{dx_2} = \frac{PL}{2}x_2 + C_2$$

$$EI\,v_2 = \frac{PL}{4}x_2^2 + C_3x_2 + C_4$$

Boundary Conditions:

At $x_1 = 0, v_1 = 0$

$0 = 0 + 0 + C_2; \quad C_2 = 0$

At $x_2 = 0, \quad \frac{dv_2}{dx_2} = 0$

$0 + C_3 = 0; \quad C_3 = 0$

At $x_1 = \frac{L}{2}, \quad x_2 = \frac{L}{2}, v_1 = v_2, \frac{dv_1}{dx_1} = -\frac{dv_2}{dx_2}$

$$\frac{P\left(\frac{1}{2}\right)^2}{6} + C_1\left(\frac{L}{2}\right) = \frac{PL\left(\frac{1}{2}\right)^2}{4} + C_4$$

$$\frac{P\left(\frac{1}{2}\right)^2}{2} + C_1 = -\frac{PL\left(\frac{1}{2}\right)}{2}; \quad C_1 = -\frac{3}{8}PL^2$$

$$C_4 = -\frac{11}{48}PL^3$$

At $x_2 = 0$.

$$v_2 = v_B = -\frac{11PL^3}{48EI}$$ **Ans.**

Ans:

$$v_B = -\frac{11PL^3}{48EI}$$

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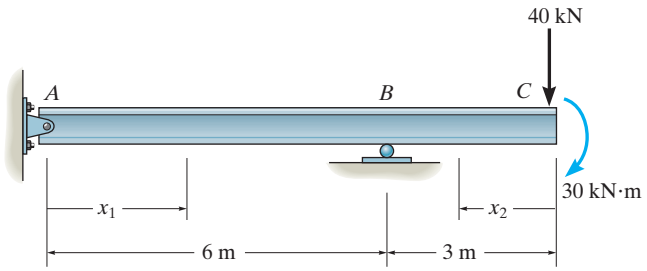
***12–20.** Determine the equations of the elastic curve using the x_1 and x_2 coordinates, and specify the slope at A and the deflection at C. EI is constant.



SOLUTION

Referring to the FBDs of the beam's cut segments shown in Fig. *b*, and *c*,

$$\zeta + \Sigma M_o = 0; \qquad M(x_1) + 25x_1 = 0 \qquad M(x_1) = (-25x_1)\ \text{kN} \cdot \text{m}$$

And

$$\zeta + \Sigma M_o = 0; \qquad -M(x_2) - 40x_2 - 30 = 0 \qquad M(x_2) = (-40x_2 - 30)\ \text{kN} \cdot \text{m}$$

$$EI\frac{d^2v}{dx^2} = M(x)$$

For coordinate x_1,

$$EI\frac{d^2v_1}{dx_1^2} = (-25x_1)\ \text{kN} \cdot \text{m}$$

$$EI\frac{dv_1}{dx_1} = \left(-\frac{25}{2}x_1^2 + C_1\right)\text{kN} \cdot \text{m}^2 \qquad (1)$$

$$EI\,v_1 = \left(-\frac{25}{6}x_1^3 + C_1x_1 + C_2\right)\text{kN} \cdot \text{m}^3 \qquad (2)$$

For coordinate x_2,

$$EI\frac{d^2v_2}{dx_2^2} = (-40x_2 - 30)\ \text{kN} \cdot \text{m}$$

$$EI\frac{dv_2}{dx_2} = (-20x_2^2 - 30x_2 + C_3)\ \text{kN} \cdot \text{m}^2 \qquad (3)$$

$$EI\,v_2 = \left(-\frac{20}{3}x_2^3 - 15\,x_2^2 + C_3x_2 + C_4\right)\text{kN} \cdot \text{m}^3 \qquad (4)$$

At $x_1 = 0, v_1 = 0$. Then, Eq (2) gives

$$EI(0) = -\frac{25}{6}(0^3) + C_1(0) + C_2 \qquad C_2 = 0$$

Also, at $x_1 = 6\ \text{m}$, $v_1 = 0$. Then, Eq (2) gives

$$EI(0) = -\frac{25}{6}(6^3) + C_1(6) + 0 \qquad C_1 = 150\ \text{kN} \cdot \text{m}^2$$

Also, at $x_2 = 3\ \text{m}$, $v_2 = 0$. Then, Eq. (4) gives

$$EI(0) = -\frac{20}{3}(3^3) - 15(3^2) + C_3(3) + C_4$$

$$3C_3 + C_4 = 315 \qquad (5)$$

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***12–20. Continued**

At $x_1 = 6$ m and $x_2 = 3$ m, $\frac{dv_1}{dx_1} = -\frac{dv_2}{dx_2}$. Then Eq. (1) and (3) gives

$$-\frac{25}{2}(6^2) + 150 = -[-20(3^2) - 30(3) + C_3]$$

$$C_3 = 570 \text{ kN} \cdot \text{m}^2$$

Substitute the value of C_3 into Eq (5),

$$C_4 = -1395 \text{ kN} \cdot \text{m}^3$$

Substitute the value of C_1 into Eq. (1),

$$\frac{dv_2}{dx_1} = \frac{1}{EI}\left(-\frac{25}{2}x_1^2 + 150\right) \text{kN} \cdot \text{m}^2$$

At A, $x_1 = 0$. Thus,

$$\theta_A = \left.\frac{dv_1}{dx_1}\right|_{x_1=0} = \frac{150 \text{ kN} \cdot \text{m}^2}{EI} \quad \measuredangle \theta_A \qquad \textbf{Ans.}$$

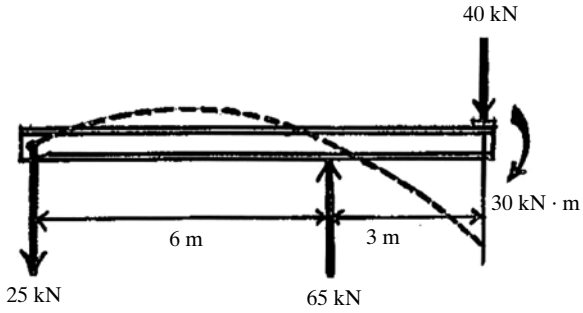
Substitute the values of C_1 and C_2 into Eq. (2) and C_3 and C_4 into Eq (4),

$$v_1 = \frac{1}{EI}\left(-\frac{25}{6}x_1^3 + 150x_1\right) \text{kN} \cdot \text{m}^3 \qquad \textbf{Ans.}$$

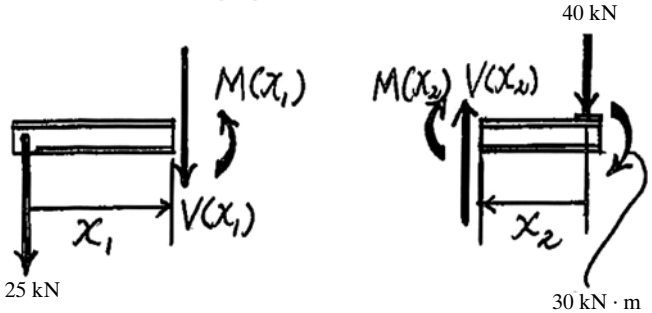
$$v_2 = \frac{1}{EI}\left(-\frac{20}{3}x_2^3 - 15x_2^2 + 570x_2 - 1395\right) \text{kN} \cdot \text{m}^3 \qquad \textbf{Ans.}$$

At C, $x_2 = 0$. Thus

$$v_C = v_2\big|_{x_2=0} = -\frac{1395 \text{ kN} \cdot \text{m}^3}{EI} = \frac{1395 \text{ kN} \cdot \text{m}^3}{EI} \downarrow \qquad \textbf{Ans.}$$



(a)



(b) (c)

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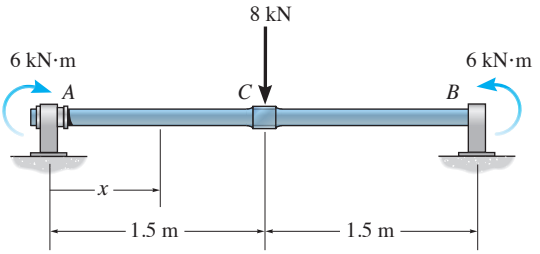
12–21.

Determine the maximum deflection of the solid circular shaft. The shaft is made of steel having $E = 200$ GPa. It has a diameter of 100 mm.



SOLUTION

Support Reactions and Elastic Curve. As shown in Fig. *a*.

Moment Function. Referring to the free-body diagram of the beam's cut segment, Fig. *b*,

$$\zeta+\Sigma M_O = 0; \qquad M(x) - 4x - 6 = 0 \qquad M(x) = (4x + 6)\ \text{kN}\cdot\text{m}$$

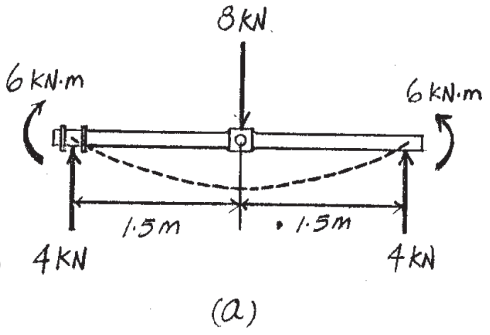
Equations of Slope and Elastic Curve.

$$EI\frac{d^2v}{dx^2} = M(x)$$

$$EI\frac{d^2v}{dx^2} = 4x + 6$$

$$EI\frac{dv}{dx} = 2x^2 + 6x + C_1 \qquad \textbf{(1)}$$

$$EIv = \frac{2}{3}x^3 + 3x^2 + C_1x + C_2 \qquad \textbf{(2)}$$

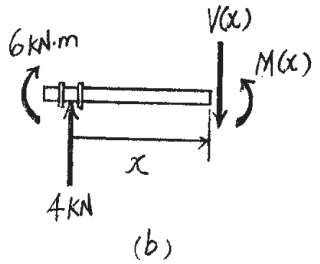


Boundary Conditions. Due to symmetry, $\frac{dv}{dx} = 0$ at $x = 1.5$ m. Then Eq. (1) gives

$$EI(0) = 2(1.5^2) + 6(1.5) + C_1 \qquad C_1 = -13.5\ \text{kN}\cdot\text{m}^2$$

Also, at $x = 0$, $v = 0$. Then Eq. (2) gives

$$EI(0) = \frac{2}{3}(0^3) + 3(0^2) + C_1(0) + C_2 \qquad C_2 = 0$$



Substituting the values of C_1 and C_2 into Eq. (2),

$$v = \frac{1}{EI}\left(\frac{2}{3}x^3 + 3x^2 - 13.5x\right)$$

$v_{\max}$ occurs at $x = 1.5$ m, where $\frac{dv}{dx} = 0$. Thus,

$$v_{\max} = v|_{x=1.5\ \text{m}} = \frac{1}{EI}\left[\frac{2}{3}(1.5^3) + 3(1.5^2) - 13.5(1.5)\right]$$

$$= -\frac{11.25\ \text{kN}\cdot\text{m}^3}{EI}$$

$$= -\frac{11.25(10^3)}{200(10^9)\left[\frac{\pi}{4}(0.05^4)\right]}$$

$$= -0.01146\ \text{m} = -11.5\ \text{mm} \qquad \textbf{Ans.}$$

Ans:
$v_{\max} = -11.5$ mm

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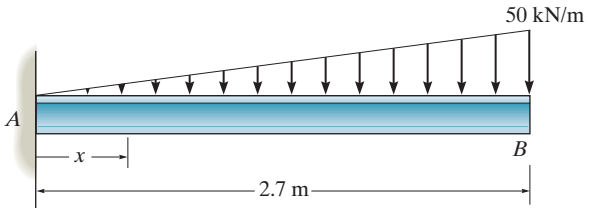
12–22. Determine the elastic curve for the cantilevered W360 × 45 beam using the x coordinate. Specify the maximum slope and maximum deflection. $E = 200$ GPa.



Referring to the FBD of the beam's cut segment shown in Fig. *b*,

$$\zeta + \Sigma M_o = 0; \quad M(x) + 121.5 + \tfrac{1}{2}\left(\tfrac{50}{2.7}x\right)(x)\left(\tfrac{x}{3}\right) - 67.5x = 0$$

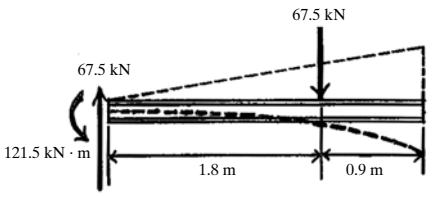
$$M(x) = (67.5x - 3.0864x^3 - 121.5)\ \text{kN}\cdot\text{m}$$

$$EI\frac{d^2v}{dx^2} = M(x)$$

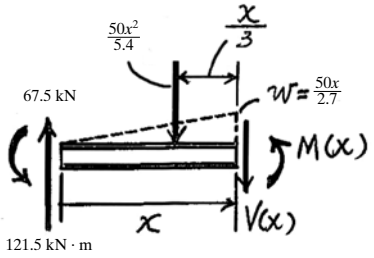
$$EI\frac{d^2v}{dx^2} = (67.5x - 3.0864x^3 - 121.5)\ \text{kN}\cdot\text{m}$$

$$EI\frac{dv}{dx} = (33.75x^2 - 0.7716x^4 - 121.5x + C_1)\ \text{kN}\cdot\text{m}^2 \qquad (1)$$

$$EI\,v = (11.25x^3 - 0.15432x^5 - 60.75x^2 + C_1x + C_2)\ \text{kN}\cdot\text{m}^3 \qquad (2)$$



(a)



(b)

At $x = 0, \frac{dv}{dx} = 0$. Then, Eq (1) gives

$$EI(0) = 33.75(0^2) - 0.7716(0^4) - 121.5(0) + C_1 \qquad C_1 = 0$$

Also, at $x = 0, v = 0$. Then Eq. (2) gives

$$EI(0) = 11.25(0^3) - 0.15432(0^5) - 60.75(0^2) + 0 + C_2 \qquad C_2 = 0$$

Substitute the value of C_1 into Eq (1) gives.

$$\frac{dv}{dx} = \frac{1}{EI}(33.75x^2 - 0.7716x^4 - 121.5x)\ \text{kN}\cdot\text{m}^2$$

The Maximum Slope occurs at $x = 2.7$ m. Thus,

$$\theta_{\max} = \left.\frac{dv}{dx}\right|_{x=2.7\text{m}} = -\frac{123.02\ \text{kN}\cdot\text{m}^2}{EI}$$

$$= \frac{123.02\ \text{kN}\cdot\text{m}^2}{EI}$$

For W360 × 45, $I = 121(10^6)\ \text{mm}^4 = 121(10^{-6})\ \text{m}^4$. Thus

$$\theta_{\max} = \frac{123.02(10^3)\ \text{N}\cdot\text{m}^2}{[200(10^9)\ \text{N/m}^2][121(10^{-6})\ \text{m}^4]} = 0.00508\ \text{rad} \quad \curvearrowright\ \theta_{\max} \qquad \textbf{Ans.}$$

Substitute the values of C_1 and C_2 into Eq (2),

$$v = \frac{1}{EI}(11.25x^3 - 0.15432x^5 - 60.75x^2)\ \text{kN}\cdot\text{m}^3$$

The maximum deflection occurs at $x = 2.7$ m. Thus,

$$v_{\max} = v\big|_{x=2.7\text{m}} = -\frac{243.58\ \text{kN}\cdot\text{m}^3}{EI}$$

$$= \frac{243.58\ \text{kN}\cdot\text{m}^3}{EI}\downarrow$$

$$= \frac{243.58(10^3)\ \text{N}\cdot\text{m}^3}{[200(10^9)\ \text{N/m}^2][121(10^{-6})\ \text{m}^4]}$$

$$= 0.01007\ \text{m} = 10.1\ \text{mm}\downarrow$$

Ans.

Ans:
$\theta_{\max} = -0.00508$

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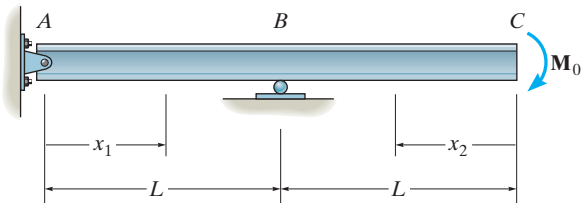
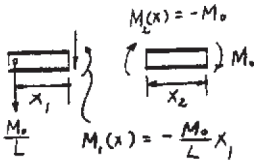
12–23.

Determine the equations of the elastic curve using the coordinates x_1 and x_2. What is the deflection and slope at C? EI is constant.



SOLUTION



$$EI\frac{d^2v}{dx^2} = M(x)$$

For $M_1(x_1) = -\frac{M_0}{L}x_1$

$$EI\frac{d^2v_1}{dx_1^2} = -\frac{M_0}{L}x_1$$

$$EI\frac{dv_1}{dx_1} = -\frac{M_0}{2L}x_1^2 + C_1 \qquad \textbf{(1)}$$

$$EIv_1 = -\frac{M_0}{6L}x_1^3 + C_1x_1 + C_2 \qquad \textbf{(2)}$$

For $M_2(x) = -M_0$; $\qquad EI\frac{d^2v_2}{dx_2^2} = -M_0$

$$EI\frac{dv_2}{dx_2} = -M_0x_2 + C_3 \qquad \textbf{(3)}$$

$$EIv_2 = -\frac{M_0}{2}x_2^2 + C_3x_2 + C_4 \qquad \textbf{(4)}$$

Boundary Conditions:

At $x_1 = 0, v_1 = 0$

From Eq. (2),

$0 = 0 + 0 + C_2;\ C_2 = 0$

At $x_1 = x_2 = L, v_1 = v_2 = 0$

From Eq. (2),

$$0 = -\frac{M_0L^2}{6} + C_1L; \qquad C_1 = \frac{M_0L}{6}$$

From Eq. (4),

$$0 = -\frac{M_0L^2}{2} + C_3L + C_4 \qquad \textbf{(5)}$$

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12–23. Continued

Continuity Condition:

$$\text{At } x_1 = x_2 = L, \frac{dv_1}{dx_1} = -\frac{dv_2}{dx_2}$$

From Eqs. (1) and (3),

$$-\frac{M_0 L}{2} + \frac{M_0 L}{6} = -(-M_0 L + C_3); \qquad C_3 = \frac{4M_0 L}{3}$$

Substituting C_3 into Eq. (5) yields,

$$C_4 = -\frac{5M_0 L^2}{6}$$

The slope:

$$\frac{dv_2}{dx_2} = \frac{1}{EI}\left[-M_0 x_2 + \frac{4M_0 L}{3}\right]$$

$$\theta_C = \left.\frac{dv_2}{dx_2}\right|_{x_2=0} = \frac{4M_0 L}{3EI} \circlearrowright \qquad \textbf{Ans.}$$

The elastic Curve:

$$v_1 = \frac{M_0}{6EIL}(-x_1^3 + L^2 x_1) \qquad \textbf{Ans.}$$

$$v_2 = \frac{M_0}{6EIL}(-3Lx_2^2 + 8L^2 x_2 - 5L^3) \qquad \textbf{Ans.}$$

$$v_C = v_2|_{x_2=0} = -\frac{5M_0 L^2}{6EI} \qquad \textbf{Ans.}$$

The negative sign indicates downward deflection.

Ans:

$$\theta_C = \frac{4M_0 L}{3EI} \circlearrowright, v_1 = \frac{M_0}{6EIL}(-x_1^3 + L^2 x_1),$$

$$v_2 = \frac{M_0}{6EIL}(-3Lx_2^2 + 8L^2 x_2 - 5L^3),$$

$$v_C = -\frac{5M_0 L^2}{6EI}$$

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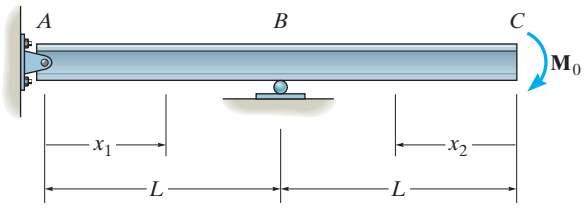
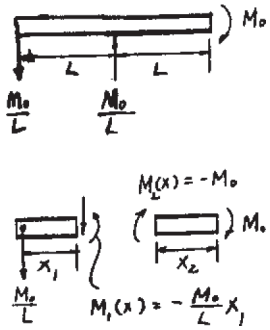
***12–24.**

Determine the equations of the elastic curve using the coordinates x_1 and x_2. What is the slope at A? EI is constant.



SOLUTION



$$EI\frac{d^2v}{dx^2} = M(x)$$

For $M_1(x_1) = -\frac{M_0}{L}x_1$

$$EI\frac{d^2v_1}{dx_1^2} = -\frac{M_0}{L}x_1$$

$$EI\frac{dv_1}{dx_1} = -\frac{M_0}{2L}x_1^2 + C_1 \qquad \textbf{(1)}$$

$$EIv_1 = -\frac{M_0}{6L}x_1^3 + C_1x_1 + C_2 \qquad \textbf{(2)}$$

For $M_2(x) = -M_0$; $\quad EI\frac{d^2v_2}{dx_2^2} = -M_0$

$$EI\frac{dv_2}{dx_2} = -M_0x_2 + C_3 \qquad \textbf{(3)}$$

$$EIv_2 = -\frac{M_0}{2}x_2^2 + C_3x_2 + C_4 \qquad \textbf{(4)}$$

Boundary Conditions:

At $x_1 = 0, v_1 = 0$

From Eq. (2),

$0 = 0 + 0 + C_2; \quad C_2 = 0$

At $x_1 = x_2 = L, v_1 = v_2 = 0$

From Eq. (2),

$$0 = -\frac{M_0L^2}{6} + C_1L; \quad C_1 = \frac{M_0L}{6}$$

From Eq. (4),

$$0 = -\frac{M_0L^2}{2} + C_3L + C_4 \qquad \textbf{(5)}$$

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***12–24. Continued**

Continuity Condition:

At $x_1 = x_2 = L$, $\quad \dfrac{dv_1}{dx_1} = -\dfrac{dv_2}{dx_2}$

From Eqs. (1) and (3),

$$-\frac{M_0 L}{2} + \frac{M_0 L}{6} = -(-M_0 L + C_3); \qquad C_3 = \frac{4M_0 L}{3}$$

Substiuting C_3 into Eq. (5) yields,

$$C_4 = -\frac{5M_0 L^2}{6}$$

The Elastic Curve:

$$v_1 = \frac{M_0}{6EIL}(-x_1^3 + L^2 x_1) \qquad \textbf{Ans.}$$

$$v_2 = \frac{M_0}{6EIL}(-3Lx_2^2 + 8L^2 x_2 - 5L^3) \qquad \textbf{Ans.}$$

From Eq. (1),

$$EI\frac{dv_1}{dx_1} = 0 + C_1 = \frac{M_0 L}{6}$$

$$\theta_A = \left.\frac{dv_1}{dx_1}\right|_{x_1 = 0} = \frac{M_0 L}{6EI} \qquad \textbf{Ans.}$$

Ans:

$$v_1 = \frac{M_0}{6EIL}(-x_1^3 + L^2 x_1),$$

$$v_2 = \frac{M_0}{6EIL}(-3Lx_2^2 + 8L^2 x_2 - 5L^3),$$

$$\theta_A = \frac{M_0 L}{6EI}$$

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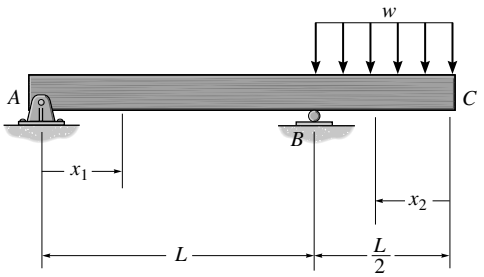
12–25. Determine the elastic curve in terms of the and coordinates and the deflection of end C of the overhang beam. EI is constant.



Support Reactions and Elastic Curve. As shown in Fig. *a*.

Moment Functions. Referring to the free-body diagrams of the beam's cut segments, Fig. *b*, $M(x_1)$ is

$$\zeta + \Sigma M_O = 0; \qquad M(x_1) + \frac{wL}{8}x_1 = 0 \qquad M(x_1) = -\frac{wL}{8}x_1$$

and $M(x_2)$ is

$$\zeta + \Sigma M_O = 0; \qquad -M(x_2) - wx_2\left(\frac{x_2}{2}\right) = 0 \qquad M(x_2) = -\frac{w}{2}{x_2}^2$$

Equations of Slope and Elastic Curve.

$$EI\frac{d^2v}{dx^2} = M(x)$$

For coordinate x_1,

$$EI\frac{d^2v_1}{d{x_1}^2} = -\frac{wL}{8}x_1$$

$$EI\frac{dv_1}{dx_1} = -\frac{wL}{16}{x_1}^2 + C_1 \tag{1}$$

$$EIv_1 = -\frac{wL}{48}{x_1}^3 + C_1x_1 + C_2 \tag{2}$$

For coordinate x_2,

$$EI\frac{d^2v_2}{d{x_2}^2} = -\frac{w}{2}{x_2}^2$$

$$EI\frac{dv_2}{dx_2} = -\frac{w}{6}{x_2}^3 + C_3 \tag{3}$$

$$EIv_2 = -\frac{w}{24}{x_2}^4 + C_3x_2 + C_4 \tag{4}$$

Boundary Conditions. At $x_1 = 0, v_1 = 0$. Then, Eq. (2) gives

$$EI(0) = -\frac{wL}{48}\left(0^3\right) + C_1(0) + C_2 \qquad C_2 = 0$$

At $x_1 = L, v_1 = 0$. Then, Eq. (2) gives

$$EI(0) = -\frac{wL}{48}\left(L^3\right) + C_1L + 0 \qquad C_1 = \frac{wL^3}{48}$$

At $x_2 = \frac{L}{2}, v_2 = 0$. Then, Eq. (4) gives

$$EI(0) = -\frac{w}{24}\left(\frac{L}{2}\right)^4 + C_3\left(\frac{L}{2}\right) + C_4$$

$$\frac{L}{2}C_3 + C_4 = \frac{wL^4}{384} \tag{5}$$

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12–25. Continued

Continuity Conditions. At $x_1 = L$ Land $x_2 = \frac{L}{2}, \frac{dv_1}{dx_1} = -\frac{dv_2}{dx_2}$. Thus, Eqs. (1) and (3) give

$$-\frac{wL}{16}\left(L^2\right) + \frac{wL^3}{48} = -\left[-\frac{w}{6}\left(\frac{L}{2}\right)^3 + C_3\right] \qquad C_3 = \frac{wL^3}{16}$$

Substituting the value of C_3 into Eq. (5),

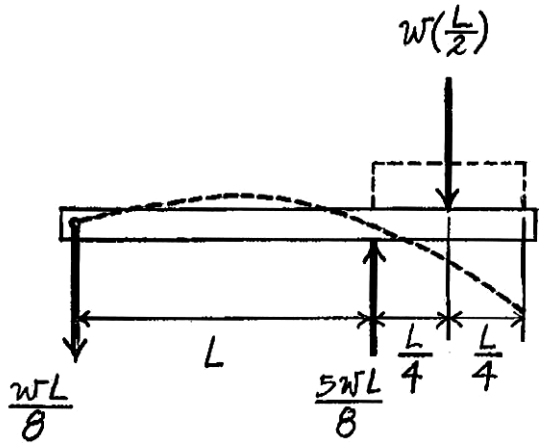
$$C_4 = -\frac{11wL^4}{384}$$

Substituting the values of C_3 and C_4 into Eq. (4),

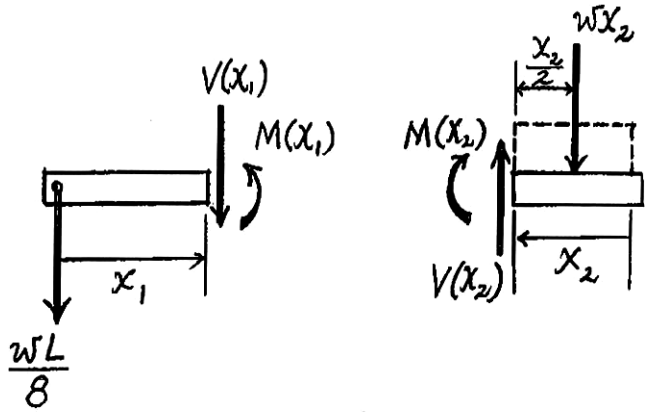
$$v_2 = \frac{w}{384EI}\left(-16{x_2}^4 + 24L^3x_2 - 11L^4\right)$$

At $C, x_2 = 0$. Thus,

$$v_C = v_2\big|_{x_2=0} = -\frac{11wL^4}{384EI} = \frac{11wL^4}{384EI} \downarrow$$ **Ans.**



(a)



(b)

Ans:

$$v_C = \frac{11wL^4}{384EI} \downarrow$$

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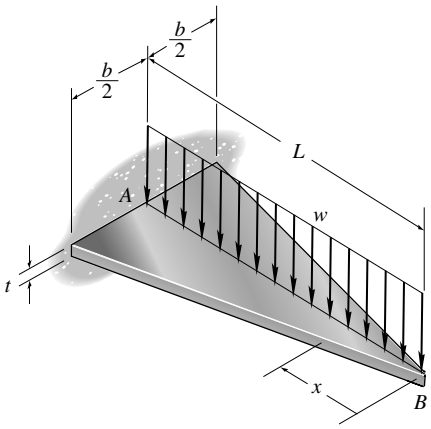
12–26. Determine the slope at end B and the maximum deflection of the cantilevered triangular plate of constant thickness t. The plate is made of material having a modulus of elasticity E.



Section Properties. Referring to the geometry shown in Fig. *a*,

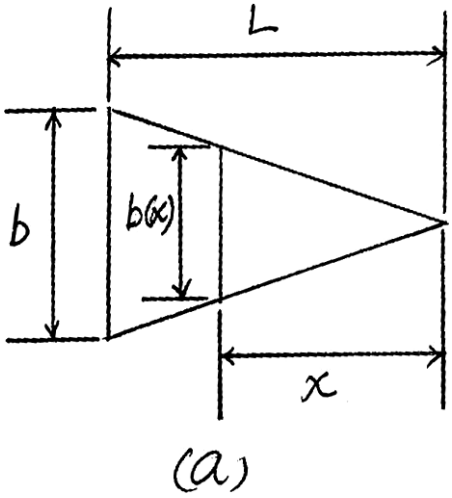
$$\frac{b(x)}{x} = \frac{b}{L}; \qquad b(x) = \frac{b}{L}x$$

Thus, the moment of the plate as a function of x is

$$I(x) = \frac{1}{12}\big[b(x)\big]t^3 = \frac{bt^3}{12L}x$$

Moment Functions. Referring to the free-body diagram of the plate's cut segments, Fig. b,

$$+\Sigma M_O = 0; \qquad -M(x) - w(x)\left(\frac{x}{2}\right) = 0 \qquad M(x) = -\frac{w}{2}x^2$$



(a)

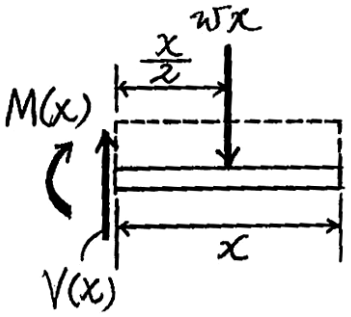
Equations of Slope and Elastic Curve.

$$E\frac{d^2v}{dx^2} = \frac{M(x)}{I(x)}$$

$$E\frac{d^2v}{dx^2} = \frac{-\frac{w}{2}x^2}{\frac{bt^3}{12L}x} = -\frac{6wL}{bt^3}x$$

$$E\frac{dv}{dx} = -\frac{3wL}{bt^3}x^2 + C_1 \tag{1}$$

$$Ev = -\frac{wL}{bt^3}x^3 + C_1x + C_2 \tag{2}$$



(b)

Boundary Conditions. At $x = L, \frac{dv}{dx} = 0$. Then Eq. (1) gives

$$E(0) = -\frac{3wL}{bt^3}\big(L^2\big) + C_1 \qquad C_1 = \frac{3wL^3}{bt^3}$$

At $x = L, v = 0$. Then Eq. (2) gives

$$E(0) = -\frac{wL}{bt^3}\big(L^3\big) + C_1(L) + C_2 \qquad C_2 = -\frac{2wL^4}{bt^3}$$

Substituting the value of C_1 into Eq. (1),

$$\frac{dv}{dx} = \frac{3wL}{Ebt^3}\left(-x^2 + L^2\right)$$

At $B, x = 0$. Thus,

$$\theta_B = \left.\frac{dv}{dx}\right|_{x=0} = \frac{3wL^3}{Ebt^3}$$

Substituting the values of C_1 and C_2 into Eq. (2),

$$v = \frac{wL}{Ebt^3}\left(-x^3 + 3L^2x - 2L^3\right)$$

$v_{\max}$ occurs at $x = 0$. Thus,

$$v_{\max} = v\big|_{x=0} = -\frac{2wL^4}{Ebt^3} = \frac{2wL^4}{Ebt^3} \quad \downarrow$$

Ans.

Ans:

$$v_{\max} = \frac{2wL^4}{Ebt^3}\downarrow$$

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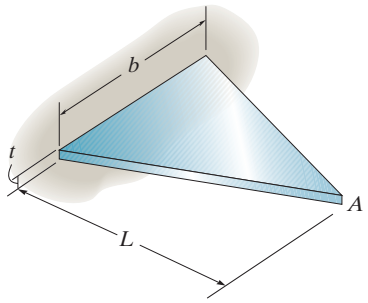
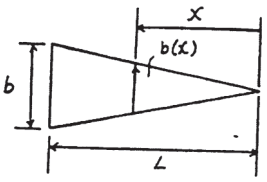
12–27.

The beam is made of a material having a specific weight γ. Determine the displacement and slope at its end A due to its weight. The modulus of elasticity for the material is E.



SOLUTION

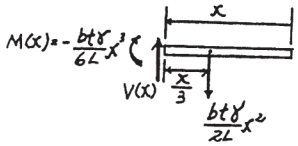
Section Properties:



$$b(x) = \frac{b}{L}x \qquad V(x) = \frac{1}{2}\left(\frac{b}{L}x\right)(x)(t) = \frac{bt}{2L}x^2$$

$$I(x) = \frac{1}{12}\left(\frac{b}{L}x\right)t^3 = \frac{bt^3}{12L}x$$

Moment Function: As shown on FBD.



Slope and Elastic Curve:

$$E\frac{d^2v}{dx^2} = \frac{M(x)}{I(x)}$$

$$E\frac{d^2v}{dx^2} = -\frac{\frac{bt\gamma}{6L}x^3}{\frac{bt^3}{12L}x} = -\frac{2\gamma}{t^2}x^2$$

$$E\frac{dv}{dx} = -\frac{2\gamma}{3t^2}x^3 + C_1 \qquad \textbf{(1)}$$

$$Ev = -\frac{\gamma}{6t^2}x^4 + C_1x + C_2 \qquad \textbf{(2)}$$

Boundary Conditions: $\frac{dv}{dx} = 0$ at $x = L$ and $v = 0$ at $x = L$.

From Eq. (1), $\quad 0 = -\frac{2\gamma}{3t^2}(L^3) + C_1 \qquad C_1 = \frac{2\gamma L^3}{3t^2}$

From Eq. (2), $\quad 0 = -\frac{\gamma}{6t^2}(L^4) + \left(\frac{2\gamma L^3}{3t^2}\right)(L) + C_2$

$$C_2 = -\frac{\gamma L^4}{2^2}$$

The Slope: Substituting the value of C_1 into Eq. (1),

$$\frac{dv}{dx} = \frac{2\gamma}{3t^2E}(-x^3 + L^3)$$

$$\theta_A = \left.\frac{dv}{dx}\right|_{x=0} = \frac{2\gamma L^3}{3t^2E} \qquad \textbf{Ans.}$$

The Elastic Curve: Substituting the value of C_1 and C_2 into Eq. (2),

$$v = \frac{\gamma}{6t^2E}(-x^4 + 4L^3x - 3L^4)$$

$$v_A = v|_{x=0} = -\frac{\gamma L^4}{2t^2E} \qquad \textbf{Ans.}$$

The negative sign indicates downward displacement.

Ans:

$\theta_A = \frac{2\gamma L^3}{3t^2E}$,

$v_A = -\frac{\gamma L^4}{2t^2E}$

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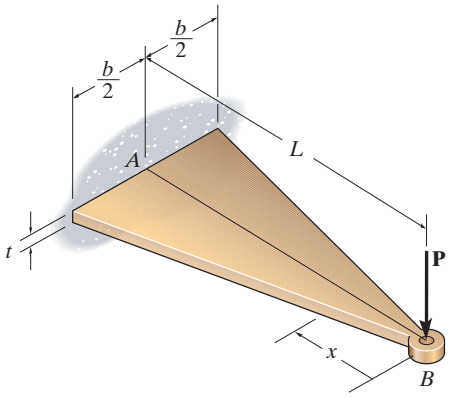
***12–28.**

Determine the slope at end B and the maximum deflection of the cantilever triangular plate of constant thickness t. The plate is made of material having a modulus of elasticity of E.



SOLUTION

Section Properties: Referring to the geometry shown in Fig. *a*,

$$\frac{b(x)}{x} = \frac{b}{L}; \qquad b(x) = \frac{b}{L}x$$

Thus, the moment of the plate as a function of x is

$$I(x) = \frac{1}{12}[b(x)]t^3 = \frac{bt^3}{12L}x$$

Moment Functions. Referring to the free–body diagram of the plate's cut segment, Fig. *b*,

$$\zeta + \Sigma M_O = 0; \qquad -M(x) - Px = 0 \qquad M(x) = -Px$$

Equations of Slope and Elastic Curve.

$$E\frac{d^2v}{dx^2} = \frac{M(x)}{I(x)}$$

$$E\frac{d^2v}{dx^2} = \frac{-Px}{\frac{bt^3}{12L}x} = -\frac{12PL}{bt^3}$$

$$E\frac{dv}{dx} = -\frac{12PL}{bt^3}x + C_1 \qquad \textbf{(1)}$$

$$Ev = -\frac{6PL}{bt^3}x^2 + C_1x + C_2 \qquad \textbf{(2)}$$

Boundary Conditions. At $x = L, \frac{dv}{dx} = 0$. Then Eq. (1) gives

$$E(0) = -\frac{12PL}{bt^3}(L) + C_1 \qquad C_1 = \frac{12PL^2}{bt^3}$$

At $x = L, v = 0$. Then Eq. (2) gives

$$E(0) = -\frac{6PL}{bt^3}(L^2) + C_1(L) + C_2 \qquad C_2 = -\frac{6PL^3}{bt^3}$$

Substituting the value of C_1 into Eq. (1),

$$\frac{dv}{dx} = \frac{12PL}{bt^3E}(-x + L)$$

At $B, x = 0$. Thus,

$$\theta_B = \left.\frac{dv}{dx}\right|_{x=0} = \frac{12PL^2}{bt^3E} \qquad \textbf{Ans.}$$

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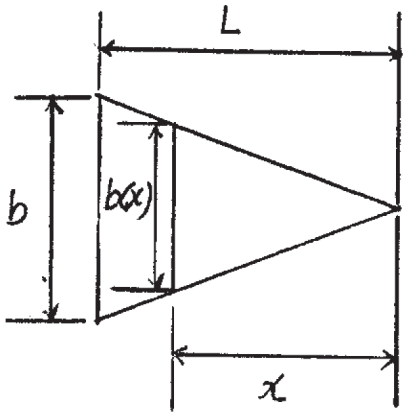
***12–28. Continued**

Substituting the values of C_1 and C_2 into Eq. (2),

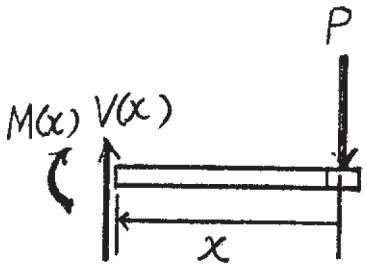
$$v = \frac{6PL}{Ebt^3}(-x^2 + 12Lx - L^2)$$

v_{max} occurs at $x = 0$. Thus,

$$v_{max} = v|_{x=0} = --\frac{6PL^3}{Ebt^3} = \frac{6PL^3}{Ebt^3} \downarrow$$ **Ans.**



(a)



(b)

Ans:

$\theta_B = \frac{12PL^2}{bt^3E}$,

$v_{max} = \frac{6PL^3}{Ebt^3} \downarrow$

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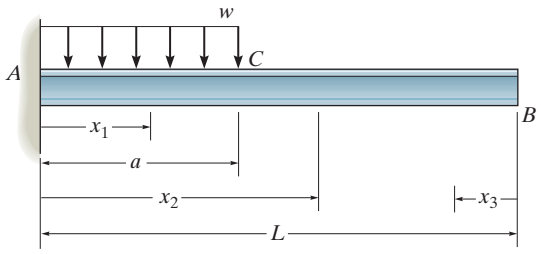
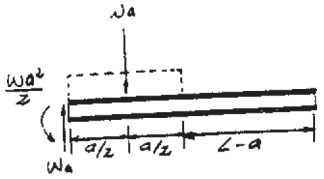
12–29.

Determine the equation of the elastic curve using the coordinates x_1 and x_2. What is the slope and deflection at B? EI is constant.



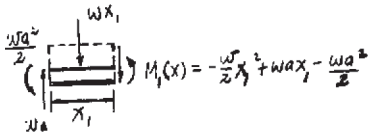
SOLUTION



$$EI\frac{d^2v}{dx^2} = M(x)$$

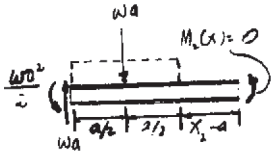
For $M_1(x) = -\frac{w}{2}x_1^2 + wax_1 - \frac{wa^2}{2}$

$$EI\frac{d^2v_1}{dx_1^2} = -\frac{w}{2}x_1^2 + wax_1 - \frac{wa^2}{2}$$



$$EI\frac{dv_1}{dx_1} = -\frac{w}{6}x_1^3 + \frac{wa}{2}x_1^2 - \frac{wa^2}{2}x_1 + C_1 \qquad \textbf{(1)}$$

$$EI\,v_1 = -\frac{w}{24}x_1^4 + \frac{wa}{6}x_1^3 - \frac{wa^2}{4}x_1^2 + C_1x_1 + C_2 \qquad \textbf{(2)}$$



For $M_2(x) = 0$; $\quad EI\frac{d^2v_2}{dx_2^2} = 0$

$$EI\frac{dv_2}{dx_2} = C_3 \qquad \textbf{(3)}$$

$$EI\,v_2 = C_3x_2 + C_4 \qquad \textbf{(4)}$$

Boundary Conditions:

At $x_1 = 0$, $\quad \frac{dv_1}{dx_1} = 0$

From Eq. (1), $\quad C_1 = 0$

At $x_1 = 0$, $v_1 = 0$

From Eq. (2); $\quad C_2 = 0$

Continuity Conditions:

At $x_1 = a$, $\quad x_2 = a$; $\quad \frac{dv_1}{dx_1} = \frac{dv_2}{dx_2}$

From Eqs. (1) and (3),

$$-\frac{wa^3}{6} + \frac{wa^3}{2} - \frac{wa^3}{2} = C_3; \qquad C_3 = -\frac{wa^3}{6}$$

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12–29. Continued

From Eqs. (2) and (4),

At $x_1 = a$, $x_2 = a$ $v_1 = v_2$

$$-\frac{wa^4}{24} + \frac{wa^4}{6} - \frac{wa^4}{4} = -\frac{wa^4}{6} + C_4; \qquad C_4 = \frac{wa^4}{24}$$

The slope, from Eq. (3),

$$\theta_B = \frac{dv_1}{dx_2} = -\frac{wa^3}{6EI}$$ **Ans.**

The Elastic Curve:

$$v_1 = \frac{w}{24EI}(-x_1^4 + 4ax_1^3 - 6a^2x_1^2)$$ **Ans.**

$$v_2 = \frac{wa^3}{24EI}(-4x_2 + a)$$ **Ans.**

$$v_B = v_2\Big|_{x_2=L} = \frac{wa^3}{24EI}(-4L + a)$$ **Ans.**

Ans:

$$\theta_B = -\frac{wa^3}{6EI},$$

$$v_1 = \frac{w}{24EI}(-x_1^4 + 4ax_1^3 - 6a^2x_1^2),$$

$$v_2 = \frac{wa^3}{24EI}(-4x_2 + a),$$

$$v_B = \frac{wa^3}{24EI}(-4L + a)$$

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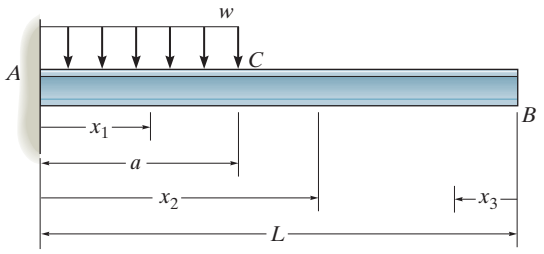
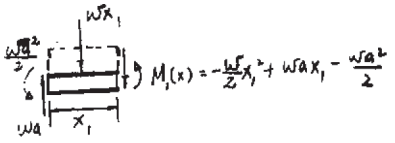
12–30.

Determine the equations of the elastic curve using the coordinates x_1 and x_3. What is the slope and deflection at point B? EI is constant.



SOLUTION



$$EI\frac{d^2v}{dx^2} = M(x)$$

For $M_1(x) = -\frac{w}{2}x_1^2 + wax_1 - \frac{wa^2}{2}$

$$EI\frac{d^2v_1}{dx_1^2} = -\frac{w}{2}x_1^2 + wax_1 - \frac{wa^2}{2}$$

$$EI\frac{dv_1}{dx_1} = -\frac{w}{6}x_1^3 + \frac{wa}{2}x_1^2 - \frac{wa^2}{2}x_1 + C_1 \qquad \textbf{(1)}$$

$$EI\,v_1 = -\frac{w}{24}x_1^4 + \frac{wa}{6}x_1^3 - \frac{wa^2}{4}x_1^2 + C_1x_1 + C_2 \qquad \textbf{(2)}$$

For $M_3(x) = 0$; $\qquad EI\frac{d^2v_3}{dx_3^2} = 0 \qquad \textbf{(3)}$

$$EI\frac{dv_3}{dx_3} = C_3 \qquad \textbf{(4)}$$

$$EI\,v_3 = C_3x_3 + C_4$$

Boundary Conditions:

At $x_1 = 0, \frac{dv_1}{dx_1} = 0$

From Eq. (1),

$0 = -0 + 0 - 0 + C_1; \quad C_1 = 0$

At $x_1 = 0, \quad v_1 = 0$

From Eq. (2),

$0 = -0 - 0 - 0 + 0 + C_2; \quad C_2 = 0$

Continuity Conditions:

At $x_1 = a, \qquad x_3 = L - a; \qquad \frac{dv_1}{dx_1} = -\frac{dv_3}{dx_3}$

$$-\frac{wa^3}{6} + \frac{wa^3}{2} - \frac{wa^3}{2} = -C_3; \quad C_3 = +\frac{wa^3}{6}$$

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12–30. Continued

At $x_1 = a, x_3 = L - a \quad v_1 = v_3$

$$-\frac{wa^4}{24} + \frac{wa^4}{6} - \frac{wa^4}{4} = \frac{wa^3}{6}(L - a) + C_4; \quad C_4 = \frac{wa^4}{24} - \frac{wa^3 L}{6}$$

The Slope:

$$\frac{dv_1}{dx_3} = \frac{wa^3}{6EI}$$

$$\theta_B = \left.\frac{dv_1}{dx_3}\right|_{x_3=0} = \frac{wa^3}{6EI}\circlearrowright$$ **Ans.**

The Elastic Curve:

$$v_1 = \frac{wx_1^2}{24EI}(-x_1^2 + 4ax_1 - 6a^2)$$ **Ans.**

$$v_2 = \frac{wa^3}{24EI}(4x_3 + a - 4L)$$ **Ans.**

$$v_B = v_3\Big|_{x_3=a} = \frac{wa^3}{24EI}(a - 4L)$$ **Ans.**

Ans:

$\theta_B = -\frac{wa^3}{6EI}$, $v_1 = \frac{wx_1^2}{24EI}(-x_1^2 + 4ax_1 - 6a^2)$,

$v_2 = \frac{wa^3}{24EI}(4x_3 + a - 4L)$, $v_B = \frac{wa^3}{24EI}(a - 4L)$

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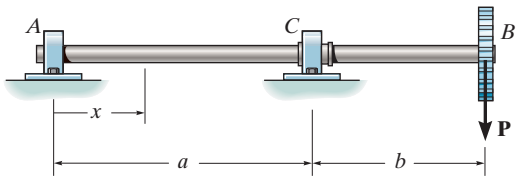
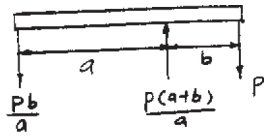
12–31.

The shaft is supported at A by a journal bearing and at C by a thrust bearing. Determine the equation of the elastic curve. EI is constant.



SOLUTION



$$M = -\frac{Pb}{a}\langle x - 0\rangle - \left(-\frac{P(a+b)}{a}\langle x - a\rangle\right) = -\frac{Pb}{a}x + \frac{P(a+b)}{a}\langle x - a\rangle$$

$$EI\frac{d^2v}{dx^2} = M$$

$$EI\frac{d^2v}{dx^2} = -\frac{Pb}{a}x + \frac{P(a+b)}{a}\langle x - a\rangle$$

$$EI\frac{dv}{dx} = -\frac{Pb}{2a}x^2 + \frac{P(a+b)}{2a}\langle x - a\rangle^2 + C_1 \qquad \textbf{(1)}$$

$$EIv = -\frac{Pb}{6a}x^3 + \frac{P(a+b)}{6a}\langle x - a\rangle^3 + C_1x + C_2 \qquad \textbf{(2)}$$

Boundary Conditions:

At $x = 0, v = 0$

From Eq. (2)

$0 = -0 + 0 + 0 + C_2; \quad C_2 = 0$

At $x = a, v = 0$

From Eq. (2)

$$0 = -\frac{Pb}{6a}(a^3) + 0 + C_1a + 0; \quad C_1 = \frac{Pab}{6}$$

From Eq. (2),

$$v = \frac{1}{EI}\left[-\frac{Pb}{6a}x^3 + \frac{P(a+b)}{6a}\langle x - a\rangle^3 + \frac{Pab}{6}x\right] \qquad \textbf{Ans.}$$

Ans:

$$v = \frac{1}{EI}\left[-\frac{Pb}{6a}x^3 + \frac{P(a+b)}{6a}\langle x - a\rangle^3 + \frac{Pab}{6}x\right]$$

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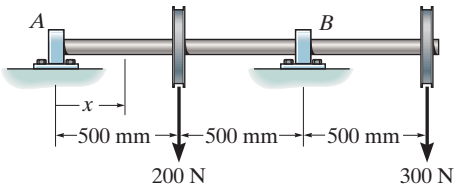
***12–32.**

The shaft supports the two pulley loads shown. Determine the equation of the elastic curve. The bearings at A and B exert only vertical reactions on the shaft. EI is constant.



SOLUTION

$$M = -50\langle x-0\rangle - 200\langle x-0.5\rangle + 550\langle x-1.0\rangle$$

$$M = -50x - 200\langle x-0.5\rangle + 550\langle x-1.0\rangle$$

Elastic curve and slope:

$$EI\frac{d^2v}{dx^2} = M$$

$$EI\frac{d^2v}{dx^2} = -50x - 200\langle x-0.5\rangle + 550\langle x-1.0\rangle$$

$$EI\frac{dv}{dx} = -25x^2 - 100\langle x-0.5\rangle^2 + 275\langle x-1.0\rangle^2 + C_1$$

$$EIv = -8.333x^3 - 33.333\langle x-0.5\rangle^3 + 91.67\langle x-1.0\rangle^3 + C_1x + C_2 \qquad (1)$$

Boundary conditions:

$$v = 0 \text{ at } x = 0$$

From Eq. (1):

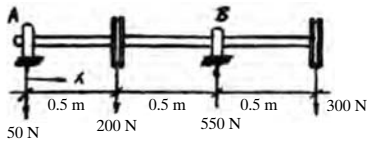
$$C_2 = 0$$

$$v = 0 \text{ at } x = 1.0\text{ m}$$

$$0 = -8.333 - 4.167 + 0 + C_1$$

$$C_1 = 12.5$$

$$v = \frac{1}{EI}\left[-8.33x^3 - 33.3\langle x-0.5\rangle^3 + 91.7\langle x-1.0\rangle^3 + 12.5x\right]\text{N}\cdot\text{m}^3 \qquad \textbf{Ans.}$$



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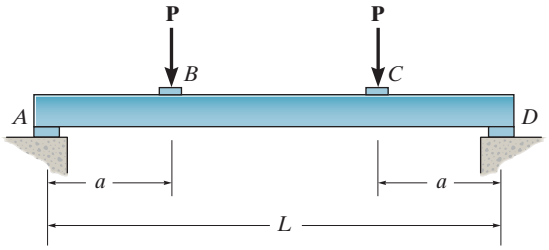
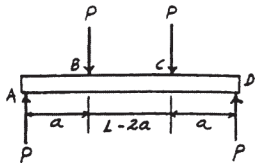
12–33.

The beam is made of a ceramic material. If it is subjected to the elastic loading shown, and the moment of inertia is I and the beam has a measured maximum deflection Δ at its center, determine the modulus of elasticity, E. The supports at A and D exert only vertical reactions on the beam.



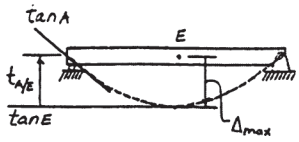
SOLUTION



Moment-Area Theorems: Due to symmetry, the slope at midspan (point E) is zero. Hence the maximum displacement is

$$\Delta_{\max} = t_{A/E} = \left(\frac{Pa}{EI}\right)\left(\frac{L-2a}{2}\right)\left(a + \frac{L-2a}{4}\right) + \frac{1}{2}\left(\frac{Pa}{EI}\right)(a)\left(\frac{2}{3}a\right)$$

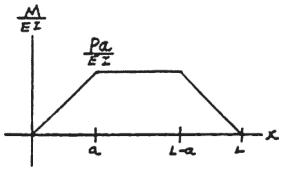
$$= \frac{Pa}{24EI}(3L^2 - 4a^2)$$



Require, $\Delta_{\max} = \Delta$, then

$$\Delta = \frac{Pa}{24EI}(3L^2 - 4a^2)$$

$$E = \frac{Pa}{24\Delta I}(3L^2 - 4a^2) \qquad \textbf{Ans.}$$



Ans:

$$E = \frac{Pa}{24\Delta I}(3L^2 - 4a^2)$$

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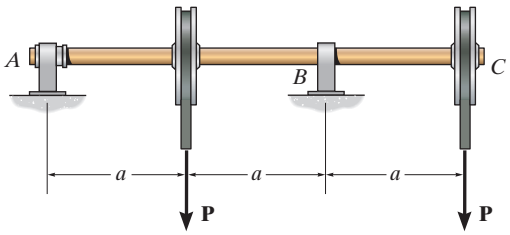
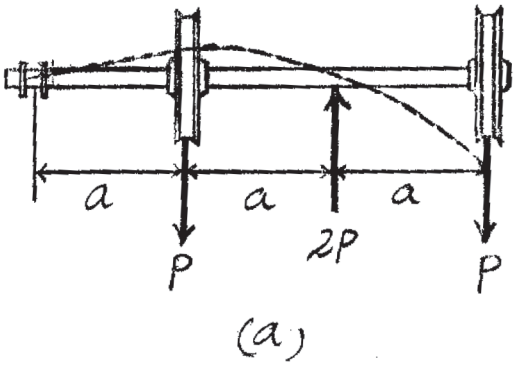
12–34.

Determine the equation of the elastic curve, the maximum deflection in region AB, and the deflection of end C. EI is constant.



SOLUTION



Support Reactions and Elastic Curve: As shown in Fig. *a*.

Moment Function:

$$M = -P\langle x - a\rangle - (-2P)\langle x - 2a\rangle$$
$$= -P\langle x - a\rangle + 2P\langle x - 2a\rangle$$

Equations of Slope and Elastic Curve:

$$EI\frac{d^2v}{dx^2} = M$$

$$EI\frac{d^2v}{dx^2} = -P\langle x - a\rangle + 2P\langle x - 2a\rangle$$

$$EI\frac{dv}{dx} = \frac{-P}{2}\langle x - a\rangle^2 + P\langle x - 2a\rangle^2 + C_1 \qquad \textbf{(1)}$$

$$EIv = \frac{-P}{6}\langle x - a\rangle^3 + \frac{P}{3}\langle x - 2a\rangle^3 + C_1x + C_2 \qquad \textbf{(2)}$$

Boundary Conditions: At $x = 0, v = 0$. Then Eq. (2) gives

$$EI(0) = -0 + 0 + C_1(0) + C_2 \qquad C_2 = 0$$

At $x = 2a, v = 0$. Then Eq. (2) gives

$$EI(0) = -\frac{P}{6}(2a - a)^3 + \frac{P}{3}(2a - 2a)^3 + C_1(2a) + 0 \qquad C_1 = \frac{Pa^2}{12}$$

Substituting the value of C_1 into Eq. (1),

$$\frac{dv}{dx} = \frac{P}{12EI}[-6\langle x - a\rangle^2 + 12\langle x - 2a\rangle^2 + a^2]$$

Assuming that $\frac{dv}{dx} = 0$ occurs in the region $a < x < 2a$,

$$-6(x - a)^2 + 0 + a^2 = 0 \qquad x = 1.4082a \qquad \textbf{(O.K.)}$$

Substituting the values of C_1 and C_2 into Eq. (2),

$$v = \frac{P}{12EI}[-2\langle x - a\rangle^3 + 4\langle x - 2a\rangle^3 + a^2x] \qquad \textbf{Ans.}$$

$(v_{\max})_{AB}$ occurs at $x = 1.4082a$, where $\frac{dv}{dx} = 0$. Thus,

$$(v_{\max})_{AB} = v|_{x=1.4082a} = \frac{P}{12EI}[-2\langle 1.4082a - a\rangle^3 + 0 + a^2(1.4082a)]$$
$$= \frac{0.106Pa^3}{EI} \qquad \textbf{Ans.}$$

At $C, x = 3a$. Thus,

$$v_C = v|_{x=3a} = \frac{P}{12EI}[-2\langle 3a - a\rangle^3 + 4(3a - 2a)^3 + a^2(3a)]$$
$$= -\frac{3Pa^3}{4EI} \qquad \textbf{Ans.}$$

Ans:

$$v = \frac{P}{12EI}[-2\langle x - a\rangle^3 + 4\langle x - 2a\rangle^3 + a^2x],$$
$$(v_{\max})_{AB} = \frac{0.106Pa^3}{EI}, v_C = -\frac{3Pa^3}{4EI}$$

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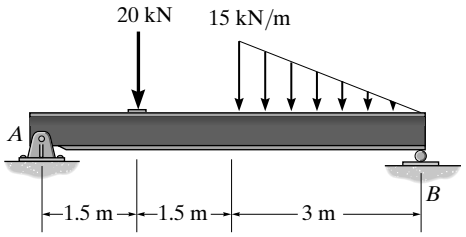
12–35. Determine the maximum deflection of the simply supported beam. $E = 200$ GPa and $I = 65.0(10^6)$ mm^4.



Support Reactions and Elastic Curve. As shown in Fig. *a*.

Moment Function. From Fig. *b*, we obtain

$$M = -(-22.5)\langle x - 0\rangle - 20\langle x - 1.5\rangle - \frac{15}{2}\langle x - 3\rangle^2 - \left(-\frac{5}{6}\right)\langle x - 3\rangle^3$$

$$= 22.5x - 20\langle x - 1.5\rangle - 7.5\langle x - 3\rangle^2 + \frac{5}{6}\langle x - 3\rangle^3$$

Equations of Slope and Elastic Curve.

$$EI\frac{d^2v}{dx^2} = M$$

$$EI\frac{d^2v}{dx^2} = 22.5x - 20\langle x - 1.5\rangle - 7.5\langle x - 3\rangle^2 + \frac{5}{6}\langle x - 3\rangle^3$$

$$EI\frac{dv}{dx} = 11.25x^2 - 10\langle x - 1.5\rangle^2 - 2.5\langle x - 3\rangle^3 + \frac{5}{24}\langle x - 3\rangle^4 + C_1 \qquad (1)$$

$$EIv = 3.75x^3 - \frac{10}{3}\langle x - 1.5\rangle^3 - 0.625\langle x - 3\rangle^4 + \frac{1}{24}\langle x - 3\rangle^5 + C_1x + C_2 \qquad (2)$$

Boundary Conditions. At $x = 0, v = 0$. Then, Eq. (2) gives

$$0 = 0 - 0 - 0 + 0 + C_1(0) + C_2 \qquad C_2 = 0$$

At $x = 6$ m, $v = 0$. Then Eq. (2) gives

$$0 = 3.75\left(6^3\right) - \frac{10}{3}\langle 6 - 1.5\rangle^3 - 0.625\langle 6 - 3\rangle^4 + \frac{1}{24}\langle 6 - 3\rangle^5 + C_1(6) + C_2$$

$$C_1 = -77.625\,\text{kN}\cdot\text{m}^2$$

Substituting the value of C_1 into Eq. (1),

$$\frac{dv}{dx} = \frac{1}{EI}\left[11.25x^2 - 10\langle x - 1.5\rangle^2 - 2.5\langle x - 3\rangle^3 + \frac{5}{24}\langle x - 3\rangle^4 - 77.625\right]$$

Assuming that $\frac{dv}{dx} = 0$ occurs in the region 1.5 m $< x <$ 3 m, then

$$\frac{dv}{dx} = 0 = \frac{1}{EI}\left[11.25x^2 - 10\langle x - 1.5\rangle^2 - 0 + 0 - 77.625\right]$$

Solving for the root 1.5 m $< x <$ 3 m,

$$x = 2.970\text{ m O.K.}$$

Substituting the values of C_1 and C_2 into Eq. (2),

$$v = \frac{1}{EI}\left[3.75x^3 - \frac{10}{3}\langle x - 1.5\rangle^3 - 0.625\langle x - 3\rangle^4 + \frac{1}{24}\langle x - 3\rangle^5 - 77.625x\right]$$ **Ans.**

$v_{\max}$ occurs at $x = 2.970$ m, where $\frac{dv}{dx} = 0$. Thus,

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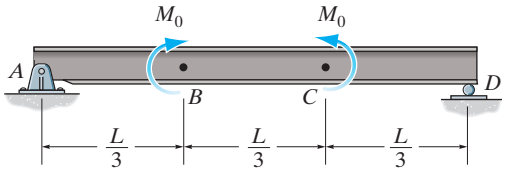
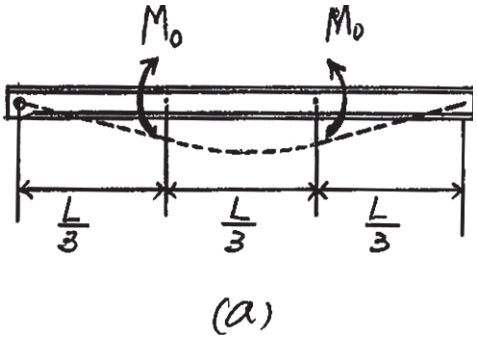
***12–36.**

Determine the equation of the elastic curve, the slope at A, and the deflection at B. EI is constant.



SOLUTION



(a)

Support Reactions and Elastic Curve: As shown in Fig. *a*.

Moment Function:

$$M = -(-M_0)\left\langle x - \frac{L}{3}\right\rangle^0 - M_0\left\langle x - \frac{2}{3}L\right\rangle^0$$

$$= M_0\left\langle x - \frac{L}{3}\right\rangle^0 - M_0\left\langle x - \frac{2}{3}L\right\rangle^0$$

Equations of Slope and Elastic Curve:

$$EI\frac{d^2v}{dx^2} = M$$

$$EI\frac{d^2v}{dx^2} = M_0\left\langle x - \frac{L}{3}\right\rangle^0 - M_0\left\langle x - \frac{2}{3}L\right\rangle^0$$

$$EI\frac{dv}{dx} = M_0\left\langle x - \frac{L}{3}\right\rangle - M_0\left\langle x - \frac{2}{3}L\right\rangle + C_1 \qquad \textbf{(1)}$$

$$EIv\frac{M_0}{2}\left\langle x - \frac{L}{3}\right\rangle^2 - \frac{M_0}{2}\left\langle x - \frac{2}{3}L\right\rangle^2 + C_1x + C_2 \qquad \textbf{(2)}$$

Boundary Conditions: Due to symmetry, $\frac{dv}{dx} = 0$ at $x = \frac{L}{2}$. Then Eq. (1) gives

$$EI(0) = M_0\left(\frac{L}{2} - \frac{L}{3}\right) - 0 + C_1 \qquad C_1 = -\frac{M_0L}{6}$$

At $x = 0, v = 0$. Then, Eq. (2) gives

$$EI(0) = 0 - 0 + C_1(0) + C_2 \qquad C_2 = 0$$

Substituting the value of C_1 into Eq. (1),

$$\frac{dv}{dx} = \frac{M_0}{6EI}\left[6\left\langle x - \frac{L}{3}\right\rangle - 6\left\langle x - \frac{2}{3}L\right\rangle - L\right]$$

At $A, x = 0$. Thus,

$$\theta_A = \left.\frac{dv}{dx}\right|_{x=0} = \frac{M_0}{6EI}[6(0) - 6(0) - L] = -\frac{M_0L}{6EI} \qquad \textbf{Ans.}$$

Substituting the values of C_1 and C_2 into Eq. (2),

$$v = \frac{M_0}{6EI}\left[3\left\langle x - \frac{L}{3}\right\rangle^2 - 3\left\langle x - \frac{2}{3}L\right\rangle^2 - Lx\right] \qquad \textbf{Ans.}$$

At $B, x = \frac{L}{3}$. Thus,

$$v_B = v|_{x=\frac{L}{3}} = \frac{M_0}{6EI}\left[3(0) - 3(0) - L\left(\frac{L}{3}\right)\right]$$

$$= -\frac{M_0L^2}{18EI} \qquad \textbf{Ans.}$$

Ans:

$$\theta_A = -\frac{M_0L}{6EI},$$

$$v = \frac{M_0}{6EI}\left[3\left\langle x - \frac{L}{3}\right\rangle^2 - 3\left\langle x - \frac{2}{3}L\right\rangle^2 - Lx\right],$$

$$v_B = -\frac{M_0L^2}{18EI}$$

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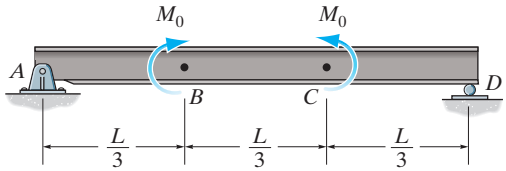
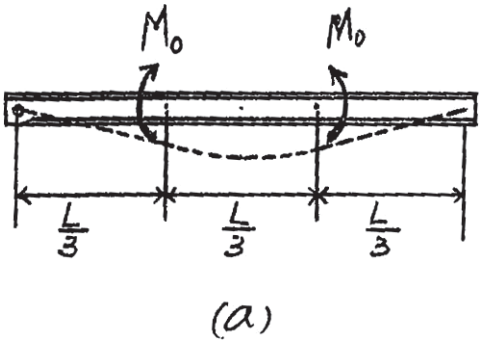
12–37.

Determine the equation of the elastic curve and the maximum deflection of the simply supported beam. *EI* is constant.



SOLUTION

Support Reactions and Elastic Curve: As shown in Fig. *a*.



Moment Function:

$$M = -(-M_0)\left\langle x - \frac{L}{3}\right\rangle^0 - M_0\left\langle x - \frac{2}{3}L\right\rangle^0$$

$$= M_0\left\langle x - \frac{L}{3}\right\rangle^0 - M_0\left\langle x - \frac{2}{3}L\right\rangle^0$$

Equations of Slope and Elastic Curve:

$$EI\frac{d^2v}{dx^2} = M$$

$$EI\frac{d^2v}{dx^2} = M_0\left\langle x - \frac{L}{3}\right\rangle^0 - M_0\left\langle x - \frac{2}{3}L\right\rangle^0$$

$$EI\frac{dv}{dx} = M_0\left\langle x - \frac{L}{3}\right\rangle - M_0\left\langle x - \frac{2}{3}L\right\rangle + C_1 \qquad \textbf{(1)}$$

$$EIv\frac{M_0}{2}\left\langle x - \frac{L}{3}\right\rangle^2 - \frac{M_0}{2}\left\langle x - \frac{2}{3}L\right\rangle^2 + C_1x + C_2 \qquad \textbf{(2)}$$

Boundary Conditions: Due to symmetry, $\frac{dv}{dx} = 0$ at $x = \frac{L}{2}$. Then Eq. (1) gives

$$EI(0) = M_0\left(\frac{L}{2} - \frac{L}{3}\right) - 0 + C_1 \qquad C_1 = -\frac{M_0L}{6}$$

At $x = 0, v = 0$. Then, Eq. (2) gives

$$EI(0) = 0 - 0 + C_1(0) + C_2 \qquad C_2 = 0$$

Substituting the values of C_1 and C_2 into Eq. (2),

$$v = \frac{M_0}{6EI}\left[3\left\langle x - \frac{L}{3}\right\rangle^2 - 3\left\langle x - \frac{2}{3}L\right\rangle^2 - Lx\right] \qquad \textbf{Ans.}$$

$v_{\max}$ occurs at $x = \frac{L}{2}$, where $\frac{dv}{dx} = 0$. Then,

$$v_{\max} = v|_{x=\frac{L}{2}} = \frac{M_0}{6EI}\left[3\left(\frac{L}{2} - \frac{L}{3}\right)^2 - 0 - L\left(\frac{L}{2}\right)\right]$$

$$= -\frac{5M_0L^2}{72EI} \qquad \textbf{Ans.}$$

Ans:

$$v = \frac{M_0}{6EI}\left[3\left\langle x - \frac{L}{3}\right\rangle^2 - 3\left\langle x - \frac{2}{3}L\right\rangle^2 - Lx\right],$$

$$v_{\max} = -\frac{5M_0L^2}{72EI}$$

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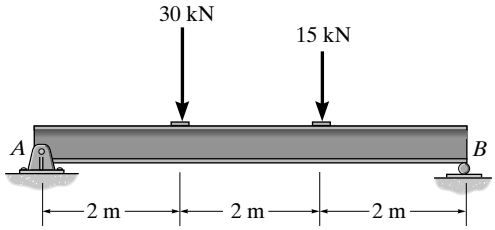
12–38. Determine the maximum deflection of the simply supported beam. $E = 200$ GPa and $I = 65.0(10^6)$ mm^4.



Support Reactions and Elastic Curve. As shown in Fig. *a*.

Moment Function. From Fig. *a*, we obtain

$$M = -(-25)\langle x - 0\rangle - 30\langle x - 2\rangle - 15\langle x - 4\rangle$$

$$= 25x - 30\langle x - 2\rangle - 15\langle x - 4\rangle$$

Equations of Slope and Elastic Curve.

$$EI\frac{d^2v}{dx^2} = M$$

$$EI\frac{d^2v}{dx^2} = 25x - 30\langle x - 2\rangle - 15\langle x - 4\rangle$$

$$EI\frac{dv}{dx} = 12.5x^2 - 15\langle x - 2\rangle^2 - 7.5\langle x - 4\rangle^2 + C_1 \qquad (1)$$

$$EIv = 4.1667x^3 - 5\langle x - 2\rangle^3 - 2.5\langle x - 4\rangle^3 + C_1x + C_2 \qquad (2)$$

Boundary Conditions. At $x = 0, v = 0$. Then, Eq. (2) gives

$$0 = 0 - 0 - 0 + C_1(0) + C_2 \qquad C_2 = 0$$

At $x = 6$ m, $v = 0$. Then Eq. (2) gives

$$0 = 4.1667\left(6^3\right) - 5\langle 6 - 2\rangle^3 - 2.5\langle 6 - 4\rangle^3 + C_1(6) + C_2$$

$$C_1 = -93.333\ \text{kN}\cdot\text{m}^3$$

Substituting the value of C_1 into Eq. (1),

$$\frac{dv}{dx} = \frac{1}{EI}\left[12.5x^2 - 15\langle x - 2\rangle^2 - 7.5\langle x - 4\rangle^2 - 93.333\right]$$

Assuming that $\frac{dv}{dx} = 0$ occurs in the region $2\text{ m} < x < 4$ m. Then

$$\frac{dv}{dx} = 0 = \frac{1}{EI}\left[12.5x^2 - 15\langle x - 2\rangle^2 - 93.333\right]$$

$$12.5x^2 - 15(x - 2)^2 - 93.333 = 0$$

$$2.5x^2 - 60x + 153.333 = 0$$

Solving for the root $2\text{ m} < x < 4$ m,

$$x = 2.9079\text{ m}\quad \text{O.K.}$$

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12–38. Continued

Substituting the values of C_1 and C_2 into Eq. (2),

$$v = \frac{1}{EI}\left[4.1667x^3 - 5\langle x-2\rangle^3 - 2.5\langle x-4\rangle^3 - 93.333x\right] \quad \textbf{Ans.}$$

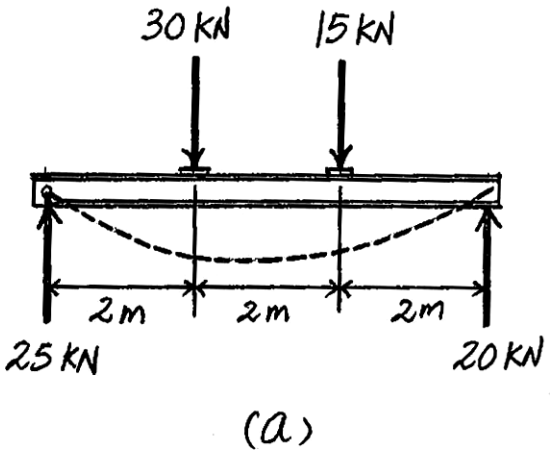
$v_{\max}$ occurs at $x = 2.9079$ m, where $\frac{dv}{dx} = 0$. Thus,

$$v_{\max} = v|_{x=2.9079\text{ ft}}$$

$$= \frac{1}{EI}\left[4.1667\left(2.9079^3\right) - 5\langle 2.9079-2\rangle^3 - 0 - 93.333(2.9079)\right]$$

$$= -\frac{172.69\text{kN}\cdot\text{m}^3}{EI} = -\frac{172.69\left(10^3\right)}{200\left(10^9\right)\left[65.0\left(10^{-6}\right)\right]}$$

$$= -0.01328\text{ m} = 13.3\text{ mm}\downarrow \quad \textbf{Ans.}$$



(a)

Ans:

$$v = \frac{1}{EI}[4.1667x^3 - 5(x-2)^3 - 2.5(x-4)^3 - 93.333x],\ v_{\max} = 13.3\text{ mm}\downarrow$$

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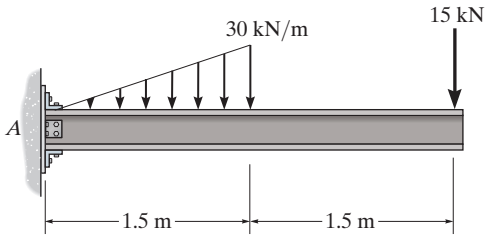
12–39.

Determine the maximum deflection of the cantilevered beam. Take $E = 200$ GPa and $I = 65.0(10^6)$ mm^4.



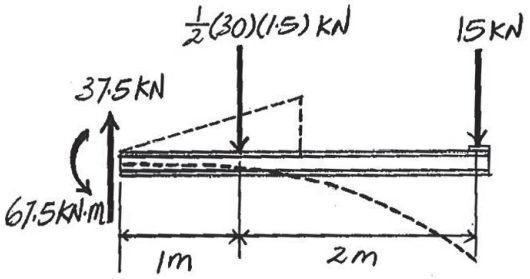
SOLUTION

Support Reactions and Elastic Curve: As shown in Fig. *a*.

Moment Function: From Fig. *b*, we obtain

$$M = -(-37.5)\langle x - 0\rangle - 67.5\langle x - 0\rangle^0 - \frac{20}{6}\langle x - 0\rangle^3 - \left(-\frac{20}{6}\right)\langle x - 1.5\rangle^3 - \left(-\frac{30}{2}\right)\langle x - 1.5\rangle^2$$

$$= 37.5x - 67.5 - \frac{10}{3}x^3 + \frac{10}{3}\langle x - 1.5\rangle^3 + 15\langle x - 1.5\rangle^2$$



(a)

Equations of Slope and Elastic Curve:

$$EI\frac{d^2v}{dx^2} = M$$

$$EI\frac{d^2v}{dx^2} = 37.5x - 67.5 - \frac{10}{3}x^3 + \frac{10}{3}\langle x - 1.5\rangle^3 + 15\langle x - 1.5\rangle^2$$

$$EI\frac{dv}{dx} = 18.75x^2 - 67.5x - \frac{5}{6}x^4 + \frac{5}{6}\langle x - 1.5\rangle^4 + 5\langle x - 1.5\rangle^3 + C_1 \quad \textbf{(1)}$$

$$EIv = 6.25x^3 - 33.75x^2 - \frac{1}{6}x^5 + \frac{1}{6}\langle x - 1.5\rangle^5 + \frac{5}{4}\langle x - 1.5\rangle^4 + C_1x + C_2 \quad \textbf{(2)}$$

Boundary Conditions: At $x = 0, \frac{dv}{dx} = 0$. Then Eq. (1) gives

$$0 = 0 - 0 - 0 + 0 + 0 + C_1 \qquad C_1 = 0$$

At $x = 0, v = 0$. Then Eq. (2) gives

$$0 = 0 - 0 - 0 + 0 + 0 + 0 + C_2 \qquad C_2 = 0$$

Substituting the values of C_1 and C_2 into Eq. (2),

$$v = \frac{1}{EI}\left[6.25x^3 - 33.75x^2 - \frac{1}{6}x^5 + \frac{1}{6}\langle x - 1.5\rangle^5 + \frac{5}{4}\langle x - 1.5\rangle^4\right]$$

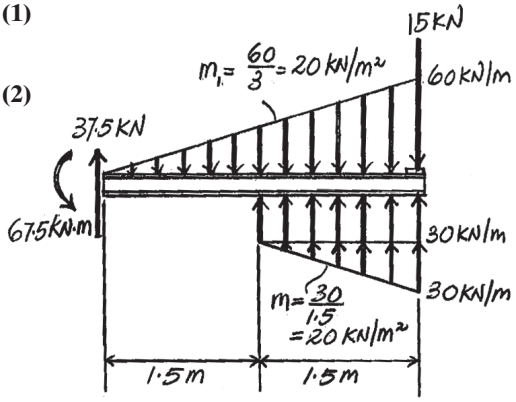
v_{max} occurs at $x = 3$ m. Thus

$$v_{max} = v|_{x=3\text{ m}}$$

$$= \frac{1}{EI}\left[6.25(3^3) - 33.75(3^2) - \frac{1}{6}(3^5) + \frac{1}{6}(3 - 1.5)^5 + \frac{5}{4}(3 - 1.5)^4\right]$$

$$= -\frac{167.91\text{ kN}\cdot\text{m}^3}{EI} = -\frac{167.91(10^3)}{200(10^9)\left[65.0(10^{-6})\right]}$$

$$= -0.01292\text{ m} = -12.9\text{ mm} \qquad \textbf{Ans.}$$



(b)

Ans:
$v_{max} = -12.9$ mm

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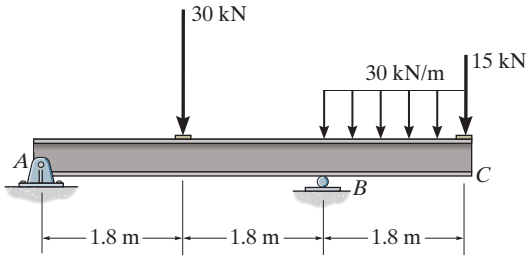
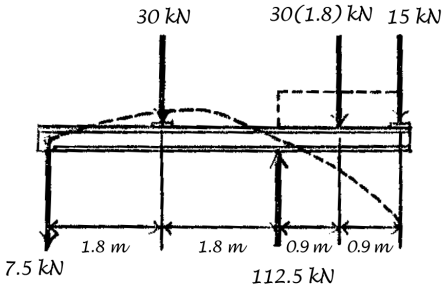
***12–40.** Determine the slope at A and the deflection of end C of the overhang beam. $E = 200$ GPa and $I = 84.9(10^{-6})\ \text{m}^4$.



SOLUTION

Support Reactions and Elastic Curve. As shown in Fig. *a*.



(a)

Moment Function. From Fig. *a*, we obtain

$$M = -6\langle x-0\rangle - 30\langle x-1.8\rangle + 105\langle x-3.6\rangle - \frac{30}{2}\langle x-3.6\rangle^2$$

$$= -6x - 30\langle x-1.8\rangle + 105\langle x-3.6\rangle - 15\langle x-3.6\rangle^2$$

Equations of Slope and Elastic Curve.

$$EI\frac{d^2v}{dx_2} = M$$

$$EI\frac{dv_2}{dx^2} = -6x - 30\langle x-1.8\rangle + 105\langle x-3.6\rangle - 15\langle x-3.6\rangle^2$$

$$EI\frac{dv}{dx} = -3x^2 - 15\langle x-1.8\rangle^2 + 52.5\langle x-3.6\rangle^2 - 5\langle x-3.6\rangle^3 + C_1 \qquad (1)$$

$$EIv = -x^3 - 5\langle x-1.8\rangle^3 + 17.5\langle x-3.6\rangle^3 - 1.25\langle x-3.6\rangle^4 + C_1x + C_2 \qquad (2)$$

Boundary Conditions. At $x = 0, v = 0$. Then Eq. (2) gives

$$0 = -0 - 0 + 0 - 0 + C_1(0) + C_2 \qquad C_2 = 0$$

At $x = 3.6$ m, $v = 0$. Then Eq. (2) gives

$$0 = -3.6^3 - 5\langle 3.6-1.8\rangle^3 + 0 - 0 + C_1(3.6) + 0 \qquad C_1 = 21.06\ \text{kN}\cdot\text{m}^2$$

Substituting the value of C_1 into Eq. (1),

$$\frac{dv}{dx} = \frac{1}{EI}\left[-3x^2 - 15\langle x-1.8\rangle^2 + 52.5\langle x-3.6\rangle^2 - 5\langle x-3.6\rangle^3 + 21.06\right]\ \text{kN}\cdot\text{m}^2$$

At $A, x = 0$. Thus,

$$\theta_A = \left.\frac{dv}{dx}\right|_{x=0} = \frac{1}{EI}[-0-0+0-0+21.06]$$

$$= \frac{21.06\ \text{kN}\cdot\text{m}^2}{EI} = \frac{21.06(10^3)\ \text{N}\cdot\text{m}^2}{[200(10^9)\ \text{N/m}^2][84.9(10^{-6})\ \text{m}^4]} = 0.00124\ \text{rad}\ \measuredangle\theta_A \qquad \textbf{Ans.}$$

Substituting the values of C_1 and C_2 into Eq. (2),

$$v = \frac{1}{EI}\left[-x^3 - 5\langle x-1.8\rangle^3 + 17.5\langle x-3.6\rangle^3 - 1.25\langle x-3.6\rangle^4 + 21.06x\right]\ \text{kN}\cdot\text{m}^3 \qquad \textbf{Ans.}$$

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12–40. Continued

At C, $x = 5.4$ m. Thus,

$$vC = v|_{x=5.4\text{ m}} = \frac{1}{EI}\left[-5.4x^3 - 5(5.4-1.8)^3 + 17.5(5.4-3.6)^3 - 1.25(5.4-3.6)^4 + 21.06(5.4)\right]\ \text{kN}\cdot\text{m}^3$$

$$= \frac{188.08\ \text{kN}\cdot\text{m}^3}{EI} = \frac{188.08(10^3)\ \text{N}\cdot\text{m}^3}{[200(10^9)\ \text{N/m}^2][84.9(10^{-6})\ \text{m}^4]}$$

$$= 0.1108\ \text{m} = 11.1\ \text{mm}\ \downarrow$$ **Ans.**

Ans:

$vC = 11.1$ mm $\downarrow$

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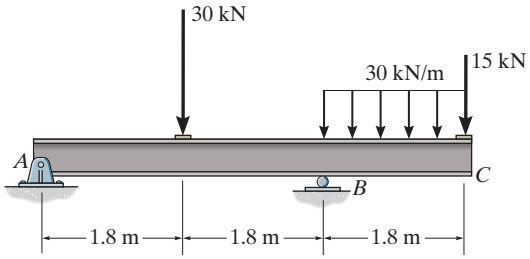
12–41. Determine the maximum deflection in region AB of the overhang beam. $E = 200$ GPa and $I = 84.9(10^{-6})\ \text{m}^4$.



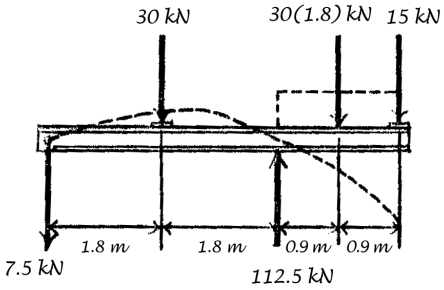
SOLUTION

Support Reactions and Elastic Curve. As shown in Fig. *a*.

Moment Function. From Fig. *a*, we obtain

$$M = -6\langle x-0\rangle - 30\langle x-1.8\rangle + 105\langle x-3.6\rangle - \frac{30}{2}\langle x-3.6\rangle^2$$

$$= -6x - 30\langle x-1.8\rangle + 105\langle x-3.6\rangle - 15\langle x-3.6\rangle^2$$



(a)

Equations of Slope and Elastic Curve.

$$EI\frac{d^2v}{dx_2} = M$$

$$EI\frac{d^2v}{dx^2} = -6x - 30\langle x-1.8\rangle + 105\langle x-3.6\rangle - 15\langle x-3.6\rangle^2$$

$$EI\frac{dv}{dx} = -3x^2 - 15\langle x-1.8\rangle^2 + 52.5\langle x-3.6\rangle^2 - 5\langle x-3.6\rangle^3 + C_1 \qquad (1)$$

$$EIv = -x^3 - 5\langle x-1.8\rangle^3 + 17.5\langle x-3.6\rangle^3 - 5\langle x-3.6\rangle^4 + C_1x + C_2 \qquad (2)$$

Boundary Conditions. At $x = 0, v = 0$. Then Eq. (2) gives

$$0 = -0 - 0 + 0 - 0 + C_1(0) + C_2 \qquad C_2 = 0$$

At $x = 3.6$ m, $v = 0$. Then Eq. (2) gives

$$0 = -3.6^3 - 5\langle 3.6-1.8\rangle^3 + 0 - 0 + C_1(3.6) + 0 \qquad C_1 = 21.06\ \text{kN}\cdot\text{m}^2$$

Substituting the value of C_1 into Eq. (1),

$$\frac{dv}{dx} = \frac{1}{EI}\left[-3x^2 - 15\langle x-1.8\rangle^2 + 52.5\langle x-3.6\rangle^2 - 5\langle x-3.6\rangle^3 + 21.06\right]\ \text{kN}\cdot\text{m}^2$$

Assuming that $\frac{dv}{dx} = 0$ occurs in the region $1.8\ \text{m} < x < 3.6\ \text{m}$, then

$$\frac{dv}{dx} = 0 = \frac{1}{EI}\left[-3x^2 - 15(x-1.8)^2 + 0 - 0 + 21.06\right]$$

$$= -3x^2 - 15(x-1.8)^2 + 21.06 = 0$$

$$18x^2 - 54x + 27.54 = 0$$

Solving for the root $1.8\ \text{m} < x < 3.6\ \text{m}$,

$$x = 2.3485$$

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12–41. Continued

Substituting the values of C_1 and C_2 into Eq. (2),

$$v = \frac{1}{EI}\left[-x^3 - 5\langle x - 1.8\rangle^3 + 17.5\langle x - 3.6\rangle^3 - 1.25\langle x - 3.6\rangle^4 + 21.06x\right] \text{kN}\cdot\text{m}^3$$

$$(v_{\max})_{AB} = v|_{x=2.3485\text{ m}} = \frac{1}{EI}\left[-2.3485^3 - 5(2.3485 - 1.8)^3 + 0 - 0 + 21.06(2.3485)\right] \text{kN}\cdot\text{m}^3$$

$$= \frac{35.68\text{ kN}\cdot\text{m}^3}{EI} = \frac{35.68(10^3)\text{ N}\cdot\text{m}^3}{[200(10^9)\text{ N/m}^2][84.9(10^{-6})\text{ m}^4]}$$

$$= 0.002101\text{ m} = 2.10\text{ mm}\ \uparrow$$ **Ans.**

Ans:
$(v_{\max})_{AB} = 3.128\text{ mm}\uparrow$

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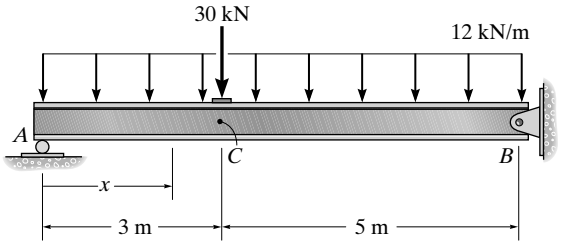
12–42. The beam is subjected to the load shown. Determine the slopes at A and B and the displacement at C. EI is constant.



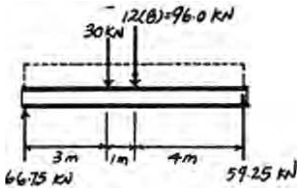
The negative sign indicates downward displacement.

Support Reactions and Elastic Curve: As shown on FBD.

Moment Function: Using the discontinuity function,

$$M = 66.75 < x - 0 > -6 < x - 0 >^2 - 30 < x - 3 >$$

$$= 66.75x - 6x^2 - 30 < x - 3 >$$



Slope and Elastic Curve:

$$EI\frac{d^2v}{dx^2} = M$$

$$EI\frac{d^2v}{dx^2} = 66.75x - 6x^2 - 30 < x - 3 >$$

$$EI\frac{dv}{dx} = 33.375x^2 - 2x^3 - 15 < x - 3 >^2 + C_1 \qquad [1]$$

$$EI\,v = 11.125x^3 - 0.5x^4 - 5 < x - 3 >^3 + C_1x + C_2 \qquad [2]$$

Boundary Conditions:

$v = 0$ at $x = 0$. From Eq.[2], $C_2 = 0$

$v = 0$ at $x = 8$ m. From Eq.[2],

$$0 = 11.125\left(8^3\right) - 0.5\left(8^4\right) - 5(8 - 3)^3 + C_1(8)$$

$$C_1 = -377.875$$

The Slope: Substituting the value of C_1 into Eq.[1],

$$\frac{dv}{dx} = \frac{1}{EI}\left\{33.375x^2 - 2x^3 - 15 < x - 3 >^2 - 377.875\right\}\ \text{kN}\cdot\text{m}^2$$

$$\theta_A = \left.\frac{dv}{dx}\right|_{x=0} = \frac{1}{EI}\{0 - 0 - 0 - 377.875\} = -\frac{378\ \text{kN}\cdot\text{m}^2}{EI} \qquad \textbf{Ans.}$$

$$\theta_B = \left.\frac{dv}{dx}\right|_{x=8\,\text{m}}$$

$$= \frac{1}{EI}\left\{33.375\left(8^2\right) - 2\left(8^3\right) - 15(8 - 3)^2 - 377.875\right\}$$

$$= \frac{359\ \text{kN}\cdot\text{m}^2}{EI} \qquad \textbf{Ans.}$$

The Elastic Curve: Substituting the values of C_1 and C_2 into Eq.[2],

$$v = \frac{1}{EI}\left\{11.125x^3 - 0.5x^4 - 5 < x - 3 >^3 - 377.875\,x\right\}\ \text{kN}\cdot\text{m}^3$$

$$v_C = v|_{x=3\,\text{m}} = \frac{1}{EI}\left\{11.125\left(3^3\right) - 0.5\left(3^4\right) - 0 - 377.875(3)\right\}$$

$$= -\frac{874\ \text{kN}\cdot\text{m}^3}{EI} \qquad \textbf{Ans.}$$

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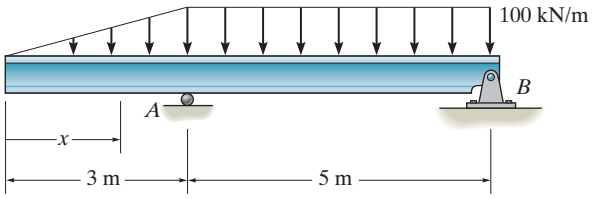
12–43. The beam is subjected to the load shown. Determine the equation of the elastic curve. *EI* is constant.



SOLUTION

$$M = \frac{1}{6}\left(-\frac{100}{3}\right)\langle x-0\rangle^3 + 430\langle x-3\rangle + \frac{1}{6}\left(-\frac{100}{3}\right)\langle x-3\rangle^3$$

$$M = \left(\frac{-50}{9}\right)x^3 + 430 < x-3 > + \left(\frac{50}{9}\right) < x-3 >^3$$

Elastic curve and slope:

$$EI\frac{d^2v}{dx^2} = M = \left(-\frac{50}{9}\right)x^3 + 430 < x-3 > + \left(\frac{50}{9}\right) < x-3 >^3$$

$$EI\frac{dv}{dx} = -\left(\frac{25}{18}\right)x^4 + 215 < x-3 >^2 + \left(\frac{25}{18}\right) < x-3 >^4 + C_1$$

$$EIv = -\left(\frac{5}{18}\right)x^5 + \left(\frac{215}{3}\right) < x-3 >^3 + \left(\frac{5}{18}\right) < x-3 >^5 + C_1x + C_2 \qquad (1)$$

Boundary conditions:

$v = 0$ at $x = 3$ m

From Eq.(1)

$$0 = -67.5 + 0 + 0 + 3C_1 + C_2$$

$$3C_1 + C_2 = 67.5 \qquad (2)$$

$v = 0$ at $x = 8$ m

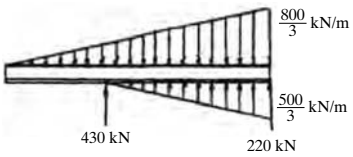
$$0 = -\left(\frac{81920}{9}\right) + \left(\frac{26875}{3}\right) + \left(\frac{15625}{18}\right) + 8C_1 + C_2$$

$$48C_1 + 6C_2 = -4345 \qquad (3)$$

Solving Eqs. (2) and (3) yields,

$C_1 = -158.33\ \text{kN}\cdot\text{m}^2 \qquad C_2 = 542.5\ \text{kN}\cdot\text{m}^3$

$$v = \frac{1}{EI}\left[-0.278x^5 + 71.7\langle x-3\rangle^3 + 0.278\langle x-3\rangle^5 - 158x + 542.5\right]\ \text{kN}\cdot\text{m}^3 \qquad \textbf{Ans.}$$



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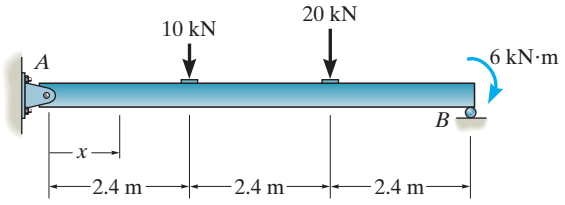
***12–44.** The beam is subjected to the loads shown. Determine the equation of the elastic curve. *EI* is constant.



SOLUTION

$M = 12.5\langle x-0\rangle - 10\langle x-2.4\rangle - 20\langle x-4.8\rangle$

$M = 12.5x - 10\langle x-2.4\rangle - 20\langle x-4.8\rangle$

Elastic curve and slope:

$$EI\frac{d^2v}{dx^2} = M = 12.5x - 10\langle x-2.4\rangle - 20\langle x-4.8\rangle$$

$$EI\frac{dv}{dx} = 6.25x^2 - 5\langle x-2.4\rangle^2 - 10\langle x-4.8\rangle^2 + C_1$$

$$EIv = 2.0833x^3 - 1.6667\langle x-2.4\rangle^3 - 3.3333\langle x-4.8\rangle^3 + C_1x + C_2 \qquad (1)$$

Boundary conditions:

$v = 0$ at $x = 0$

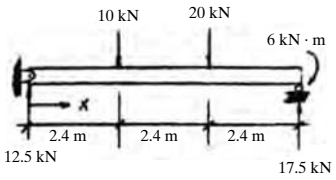
From Eq. (1), $C_2 = 0$

$v = 0$ at $x = 7.2$ m

$0 = 777.60 - 184.32 - 46.06 + 7.2C_1$

$C_1 = -76.00$

$$v = \frac{1}{EI}\left[2.08x^3 - 1.67\langle x-2.4\rangle^3 - 3.33\langle x-4.8\rangle^3 - 76.0x\right] \text{kN}\cdot\text{m}^3 \qquad \textbf{Ans.}$$



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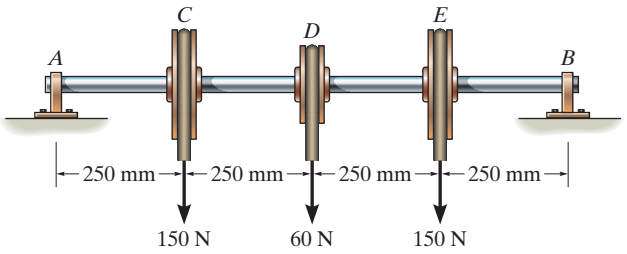
12–45.

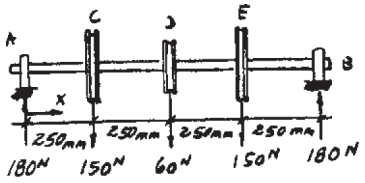
Determine the deflection at each of the pulleys *C, D*, and *E*. The shaft is made of steel and has a diameter of 30 mm. $E_{st} = 200$ GPa.



SOLUTION

$$M = -(-180)\langle x - 0\rangle - 150\langle x - 0.25\rangle - 60\langle x - 0.5\rangle - 150\langle x - 0.75\rangle$$

$$M = 180x - 150\langle x - 0.25\rangle - 60\langle x - 0.5\rangle - 150\langle x - 0.75\rangle$$



Elastic Curve and Slope:

$$EI\frac{d^2v}{dx^2} = M = 180x - 150\langle x - 0.25\rangle - 60\langle x - 0.5\rangle - 150\langle x - 0.75\rangle$$

$$EI\frac{dv}{dx} = 90x^2 - 75\langle x - 0.25\rangle^2 - 30\langle x - 0.50\rangle^2 - 75\langle x - 0.75\rangle^2 + C_1 \quad \textbf{(1)}$$

$$EIv = 30x^3 - 25\langle x - 0.25\rangle^3 - 10\langle x - 0.50\rangle^3 - 25\langle x - 0.75\rangle^3 + C_1x + C_2 \quad \textbf{(2)}$$

Boundary Conditions:

$v = 0$ at $x = 0$

From Eq. (2),

$C_2 = 0$

$v = 0$ at $x = 1.0$ m

$0 = 30 - 10.55 - 1.25 - 0.39 + C_1$

$C_1 = -17.8125$

$$v = \frac{1}{EI}[30x^3 - 25\langle x - 0.25\rangle^3 - 10\langle x - 0.5\rangle^3 - 25\langle x - 0.75\rangle^3 - 17.8125x]$$

$$v_C = v\Big|_{x=0.25\text{ m}} = \frac{-3.984}{EI} = \frac{-3.984}{200(10^9)\frac{\pi}{4}(0.015)^4}$$

$$= -0.000501\text{ m} = -0.501\text{ mm} \qquad \textbf{Ans.}$$

$$v_D = v\Big|_{x=0.5\text{ m}} = \frac{-5.547}{200(10^9)\frac{\pi}{4}(0.015)^4} = -0.000698\text{ m} = -0.698\text{ mm} \qquad \textbf{Ans.}$$

$$v_E = v\Big|_{x=0.75\text{ m}} = \frac{-3.984}{EI} = -0.501\text{ mm} \quad \text{(symmetry check !)} \qquad \textbf{Ans.}$$

The negative signs indicate downward displacement.

Ans:
$v_C = -0.501$ mm, $v_D = -0.698$ mm,
$v_E = -0.501$ mm

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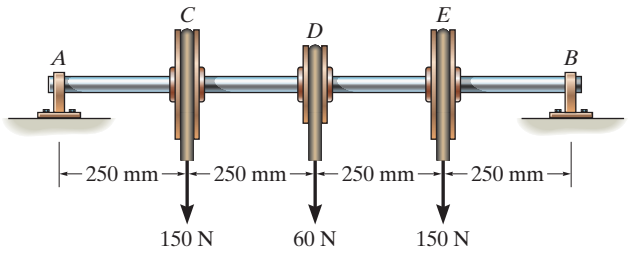
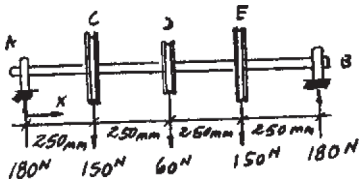
12–46.

Determine the slope of the shaft at A and B. The shaft is made of steel and has a diameter of 30 mm. The bearings only exert vertical reactions on the shaft. $E_{st} = 200$ GPa.



SOLUTION



$$M = -(-180)\langle x - 0\rangle - 150\langle x - 0.25\rangle - 60\langle x - 0.5\rangle - 150\langle x - 0.75\rangle$$

$$M = 180x - 150\langle x - 0.25\rangle - 60\langle x - 0.5\rangle - 150\langle x - 0.75\rangle$$

Elastic Curve and Slope:

$$EI\frac{d^2v}{dx^2} = M = 180x - 150\langle x - 0.25\rangle - 60\langle x - 0.5\rangle - 150\langle x - 0.75\rangle$$

$$EI\frac{dv}{dx} = 90x^2 - 75\langle x - 0.25\rangle^2 - 30\langle x - 0.50\rangle^2 - 75\langle x - 0.75\rangle^2 + C_1 \quad \textbf{(1)}$$

$$EIv = 30x^3 - 25\langle x - 0.25\rangle^3 - 10\langle x - 0.50\rangle^3 - 25\langle x - 0.75\rangle^3 + C_1x + x_2 \quad \textbf{(2)}$$

Boundary Conditions:

$v = 0$ at $x = 0$

From Eq. (2)

$C_2 = 0$

$v = 0$ at $x = 1.0$ m

$0 = 30 - 10.55 - 1.25 - 0.39 + C_1$

$C_1 = -17.8125$

$$\frac{dv}{dx} = \frac{1}{EI}[90x^2 - 75\langle x - 0.25\rangle^2 - 30\langle x - 0.5\rangle^2 - 75\langle x - 0.75\rangle^2 - 17.8125] \quad \textbf{(3)}$$

$$\theta_A = \left.\frac{dv}{dx}\right|_{x=0} = \frac{-17.8125}{EI} = \frac{-17.8125}{200(10^9)\frac{\pi}{4}(0.015)^4} = -0.00224 \text{ rad} = -0.128° \quad \textbf{Ans.}$$

The negative sign indicates clockwise rotation.

$$\theta_B = \left.\frac{dv}{dx}\right|_{x=1\text{ m}} = \frac{17.8125}{EI} = 0.128° \quad \textbf{Ans.}$$

The positive result indicates counterclockwise rotation.

Ans:
$\theta_A = -0.128°, \theta_B = 0.128°$

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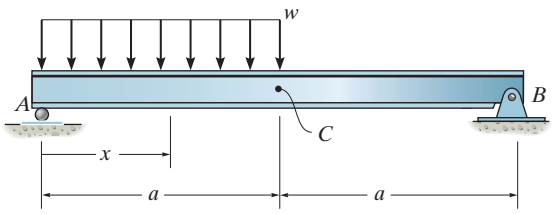
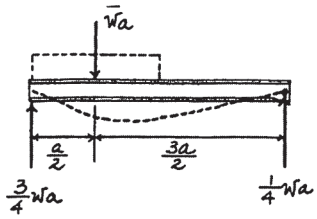
12–47.

Determine the equation of the elastic curve. Specify the slopes at A and B. EI is constant.



SOLUTION

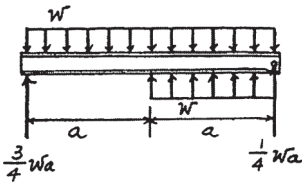


Support Reactions and Elastic Curve: As shown on FBD.

Moment Function: Using the discontinuity function,

$$M = \frac{3}{4}wa\langle x - 0\rangle - \frac{w}{2}\langle x - 0\rangle^2 - \left(-\frac{w}{2}\right)\langle x - a\rangle^2$$

$$= \frac{3wa}{4}x - \frac{w}{2}x^2 + \frac{w}{2}\langle x - a\rangle^2$$



Slope and Elastic Curve:

$$EI\frac{d^2v}{dx^2} = M$$

$$EI\frac{d^2v}{dx^2} = \frac{3wa}{4}x - \frac{w}{2}x^2 + \frac{w}{2}\langle x - a\rangle^2$$

$$EI\frac{dv}{dx} = \frac{3wa}{8}x^2 - \frac{w}{6}x^3 + \frac{w}{6}\langle x - a\rangle^3 + C_1 \qquad \textbf{(1)}$$

$$EI\,v = \frac{wa}{8}x^3 - \frac{w}{24}x^4 + \frac{w}{24}\langle x - a\rangle^4 + C_1x + C_2 \qquad \textbf{(2)}$$

Boundary Conditions:

$v = 0$ at $x = 0$. From Eq. (2), $C_2 = 0$

$v = 0$ at $x = 2a$. From Eq. (2),

$$0 = \frac{wa}{8}(2a)^3 - \frac{w}{24}(2a)^4 + \frac{w}{24}(2a - a)^4 + C_1(2a)$$

$$C_1 = -\frac{3wa^3}{16}$$

The Slope: Substituting the value C_1 into Eq. (1),

$$\frac{dv}{dx} = \frac{w}{48EI}\left\{18ax^2 - 8x^3 + 8\langle x - a\rangle^3 - 9a^3\right\}$$

$$\theta_A = \left.\frac{dv}{dx}\right|_{x=0} = \frac{w}{48EI}\left\{0 - 0 + 0 - 9a^3\right\} = -\frac{3wa^3}{16EI} \qquad \textbf{Ans.}$$

$$\theta_B = \left.\frac{dv}{dx}\right|_{x=2a} = \frac{w}{48EI}\left\{18a(2a)^2 - 8(2a)^3 + 8(2a - a)^3 - 9a^3\right\}$$

$$= \frac{7wa^3}{48EI} \qquad \textbf{Ans.}$$

The Elastic Curve: Substituting the value of C_1 and C_2 into Eq. (2),

$$v = \frac{w}{48EI}\left[6ax^3 - 2x^4 + 2\langle x - a\rangle^4 - 9a^3x\right] \qquad \textbf{Ans.}$$

Ans:

$$\theta_A = -\frac{3wa^3}{16EI}$$

$$\theta_B = \frac{7wa^3}{48EI}$$

$$v = \frac{w}{48EI}\left[6ax^3 - 2x^4 + 2\langle x - a\rangle^4 - 9a^3x\right]$$

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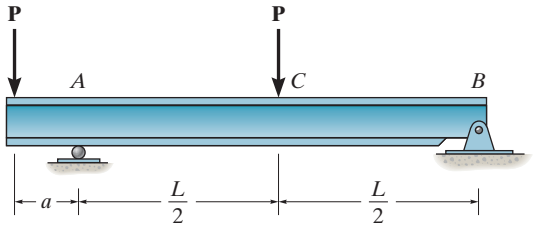
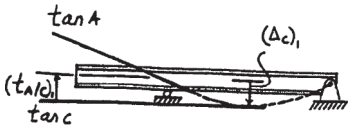
***12–48.**

Determine the value of a so that the displacement at C is equal to zero. EI is constant.



SOLUTION



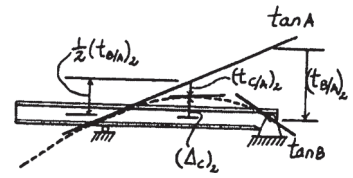
Moment-Area Theorems:

$$(\Delta_C)_1 = (t_{A/C})_1 = \frac{1}{2}\left(\frac{PL}{4EI}\right)\left(\frac{L}{2}\right)\left(\frac{L}{3}\right) = \frac{PL^3}{48EI}$$

$$(t_{B/A})_2 = \frac{1}{2}\left(-\frac{Pa}{EI}\right)(L)\left(\frac{2}{3}L\right) = -\frac{PaL^2}{3EI}$$

$$(t_{C/A})_2 = \left(-\frac{Pa}{2EI}\right)\left(\frac{L}{2}\right)\left(\frac{L}{4}\right) + \frac{1}{2}\left(-\frac{Pa}{2EI}\right)\left(\frac{L}{2}\right)\left(\frac{L}{3}\right) = -\frac{5PaL^2}{48EI}$$

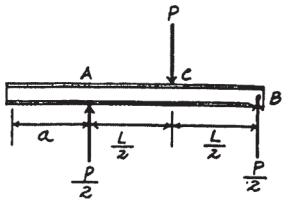
$$(\Delta_C)_2 = \frac{1}{2}|(t_{B/A})_2| - |(t_{C/A})_2| = \frac{1}{2}\left(\frac{PaL^2}{3EI}\right) - \frac{5PaL^2}{48EI} = \frac{PaL^2}{16EI}$$

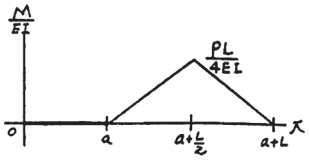


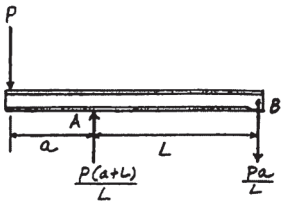
Require, $\Delta_C = 0 = (\Delta_C)_1 - (\Delta_C)_2$

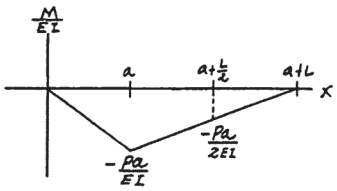
$$0 = \frac{PL^3}{48EI} - \frac{PaL^2}{16EI}$$

$$a = \frac{L}{3}$$ **Ans.**









Ans:

$$a = \frac{L}{3}$$

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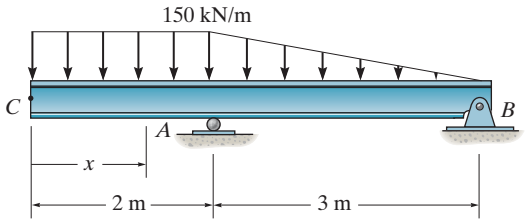
12–49. Determine the displacement at C and the slope at A of the beam.

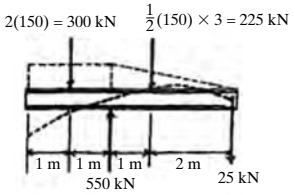


Support Reactions and Elastic Curve: As shown on FBD.

Moment Function: Using the discontinuity function,

$$M = -\frac{1}{2}(150)\langle x-0\rangle^2 - \frac{1}{6}\left(-\frac{150}{3}\right)\langle x-2\rangle^3 - (-550)\langle x-2\rangle$$

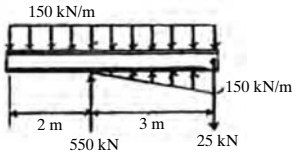
$$= -75x^2 + \frac{25}{3}\langle x-2\rangle^3 + 550\langle x-2\rangle$$



Slope and Elastic Curve:

$$EI\frac{d^2v}{dx^2} = M$$

$$EI\frac{d^2v}{dx^2} = -75x^2 + \left(\frac{25}{3}\right)\langle x-2\rangle^3 + 550\langle x-2\rangle$$



$$EI\frac{dv}{dx} = -25x^3 + \left(\frac{25}{12}\right)\langle x-2\rangle^4 + 275\langle x-2\rangle^2 + C_1 \qquad [1]$$

$$EIv = -6.25x^4 + \left(\frac{5}{12}\right)\langle x-2\rangle^5 + \left(\frac{275}{3}\right)\langle x-2\rangle^3 + C_1x + C_2 \qquad [2]$$

Boundary Conditions:

$v = 0$ at $x = 2$ m. From Eq.[2],

$$0 = -6.25(2^4) + 0 + 0 + C_1(2) + C_2$$

$$100 = 2C_1 + C_2 \qquad [3]$$

$v = 0$ at $x = 5$ m. From Eq.[2],

$$0 = -6.25(5)^4 + \left(\frac{5}{12}\right)(5-2)^5 + \left(\frac{275}{3}\right)(5-2)^3 + C_1(5) + C_2$$

$$1330 = 5C_1 + C_2 \qquad [4]$$

Solving Eqs. [3] and [4] yields,

$$C_1 = 410\ \text{kN}\cdot\text{m}^2 \qquad C_2 = -720\ \text{kN}\cdot\text{m}^3$$

The Slope: Substitute the value of C_1 into Eq.[1],

$$\frac{dv}{dx} = \frac{1}{EI}\left\{-25x^3 + \left(\frac{25}{12}\right)\langle x-2\rangle^4 + 275\langle x-2\rangle^2 + 410\right\}\ \text{kN}\cdot\text{m}^2$$

$$\theta_A = \left.\frac{dv}{dx}\right|_{x=2\,\text{m}} = \frac{1}{EI}\{-25(2^3) + 0 + 0 + 410\} = \frac{210\ \text{kN}\cdot\text{m}^3}{EI} \quad \measuredangle\,\theta_A \quad \textbf{Ans.}$$

The Elastic Curve: Substitute the values of C_1 and C_2 into Eq. [2],

$$v = \frac{1}{EI}\left\{-6.25x^4 + \left(\frac{5}{12}\right)\langle x-2\rangle^5 + \left(\frac{275}{3}\right)\langle x-2\rangle^3 + 410x - 720\right\}\ \text{kN}\cdot\text{m}^3$$

$$v_C = v|_{x=0} = \frac{1}{EI}\{-0 + 0 + 0 + 0 - 720\}\ \text{kN}\cdot\text{m}^3 = \frac{720\ \text{kN}\cdot\text{m}^3}{EI} \quad \downarrow \quad \textbf{Ans.}$$

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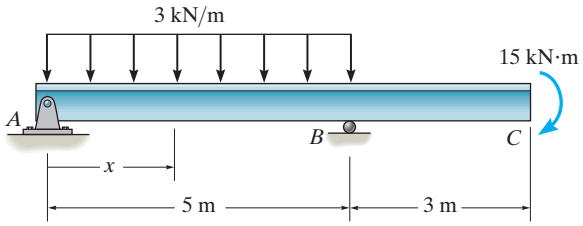
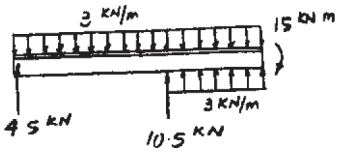
12–50.

Determine the equations of the slope and elastic curve. *EI* is constant.



SOLUTION



$$M = -(-4.5)\langle x - 0\rangle - \frac{3}{2}\langle x - 0\rangle^2 - (-10.5)\langle x - 5\rangle - \left(\frac{-3}{2}\right)\langle x - 5\rangle^2$$

$$M = 4.5x - 1.5x^2 + 10.5\langle x - 5\rangle + 1.5\langle x - 5\rangle^2$$

Elastic Curve and Slope:

$$EI\frac{d^2v}{dx^2} = M = 4.5x - 1.5x^2 + 10.5\langle x - 5\rangle + 1.5\langle x - 5\rangle^2$$

$$EI\frac{dv}{dx} = 2.25x^2 - 0.5x^3 + 5.25\langle x - 5\rangle^2 + 0.5\langle x - 5\rangle^3 + C_1 \quad \textbf{(1)}$$

$$EIv = 0.75x^3 - 0.125x^4 + 1.75\langle x - 5\rangle^3 + 0.125\langle x - 5\rangle^4 + C_1x + C_2 \quad \textbf{(2)}$$

Boundary Conditions:

$$v = 0 \quad \text{at} \quad x = 0$$

From Eq. (2), $C_2 = 0$

$$v = 0 \quad \text{at} \quad x = 5$$

$$0 = 93.75 - 78.125 + 5C_1$$

$$C_1 = -3.125$$

$$\frac{dv}{dx} = \frac{1}{EI}[2.25x^2 - 0.5x^3 + 5.25\langle x - 5\rangle^2 + 0.5\langle x - 5\rangle^3 - 3.125]\ \text{kN}\cdot\text{m}^2 \quad \textbf{Ans.}$$

$$v = \frac{1}{EI}[0.75x^3 - 0.125x^4 + 1.75\langle x - 5\rangle^3 + 0.125\langle x - 5\rangle^4 - 3.125x]\ \text{kN}\cdot\text{m}^3 \quad \textbf{Ans.}$$

Ans:

$$\frac{dv}{dx} = \frac{1}{EI}[2.25x^2 - 0.5x^3 + 5.25\langle x - 5\rangle^2 + 0.5\langle x - 5\rangle^3 - 3.125]\ \text{kN}\cdot\text{m}^2,$$

$$v = \frac{1}{EI}[0.75x^3 - 0.125x^4 + 1.75\langle x - 5\rangle^3 + 0.125\langle x - 5\rangle^4 - 3.125x]\ \text{kN}\cdot\text{m}^3$$

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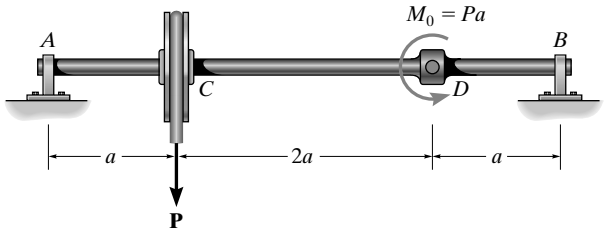
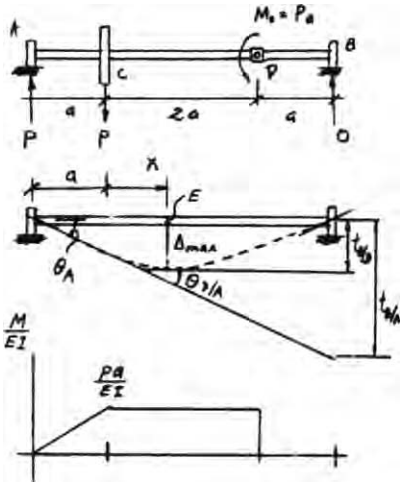
12–51. If the bearings at A and B exert only vertical reactions on the shaft, determine the slope at A and the maximum deflection.



SOLUTION

$$t_{B/A} = \frac{1}{2}\left(\frac{Pa}{EI}\right)(a)\left(3a + \frac{a}{3}\right) + \left(\frac{Pa}{EI}\right)(2a)(a + a) = \frac{17Pa^3}{3EI}$$

$$\theta_A = \frac{|t_{B/A}|}{4a} = \frac{17Pa^2}{12EI} \qquad \textbf{Ans.}$$



Assume $\Delta_{\max}$ is at point E located at $0 < x < 2a$

$$\theta_{E/A} = \frac{1}{2}\left(\frac{Pa}{EI}\right)(a) + \left(\frac{Pa}{EI}\right)(x) = \frac{Pa^2}{2EI} + \frac{Pax}{EI}$$

$$\theta_E = 0 = \theta_{E/A} + \theta_A$$

$$0 = \frac{Pa^2}{2EI} + \frac{Pax}{EI} + \left(\frac{-17Pa^2}{12EI}\right)$$

$$x = \frac{11}{12}a$$

$$\Delta_{\max} = |t_{B/E}| = \left(\frac{Pa}{EI}\right)\left(2a - \frac{11}{12}a\right)\left[\frac{(2a - \frac{11}{12}a)}{2} + a\right] = \frac{481Pa^3}{288EI} \qquad \textbf{Ans.}$$

Ans:

$$\theta_A = \frac{17\,Pa^2}{12EI},\ \Delta_{\max} = \frac{481\,Pa^3}{288EI}$$

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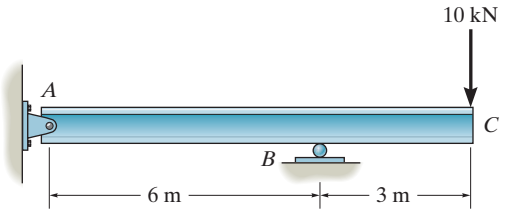
***12–52.**

Determine the slope and deflection at *C*. *EI* is constant.



SOLUTION

Referring to Fig. *b*,

$$|\theta_{C/A}| = \frac{1}{2}\left(\frac{30}{EI}\right)(9) = \frac{135\ \text{kN}\cdot\text{m}^2}{EI}$$

$$|t_{B/A}| = \frac{6}{3}\left[\frac{1}{2}\left(\frac{30}{EI}\right)(6)\right] = \frac{180\ \text{kN}\cdot\text{m}^3}{EI}$$

$$|t_{C/A}| = \left(\frac{6}{3}+3\right)\left[\frac{1}{2}\left(\frac{30}{EI}\right)(6)\right] + \left[\frac{2}{3}(3)\right]\left[\frac{1}{2}\left(\frac{30}{EI}\right)(3)\right]$$

$$= \frac{540\ \text{kN}\cdot\text{m}^3}{EI}$$

From the geometry shown in Fig. *b*,

$$\theta_A = \frac{|t_{B/A}|}{6} = \frac{180/EI}{6} = \frac{30\ \text{kN}\cdot\text{m}^2}{EI}$$

Here,

$$+\circlearrowright \theta_C = \theta_A + \theta_{C/A}$$

$$\theta_C = -\frac{30}{EI} + \frac{135}{EI}$$

$$\theta_C = \frac{105\ \text{kN}\cdot\text{m}^2}{EI}$$

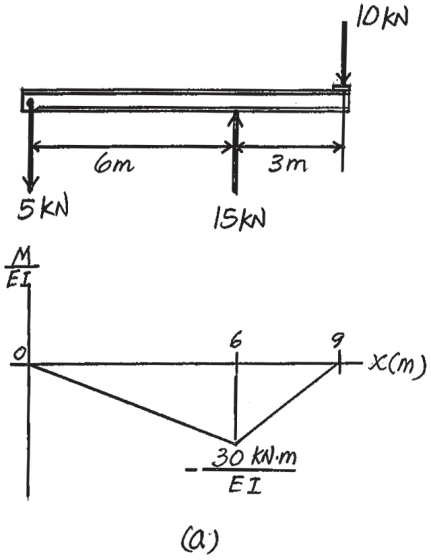
Ans.

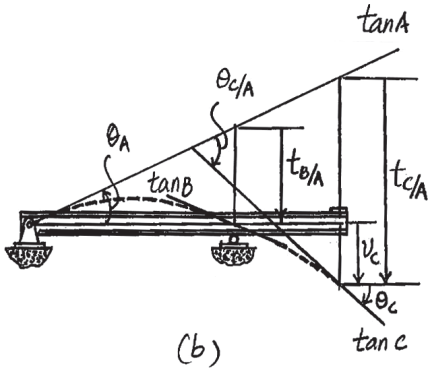
$$v_C = \left|t_{C/A}\right| - \left|t_{B/A}\right|\left(\frac{9}{6}\right)$$

$$= \frac{540}{EI} - \frac{180}{EI}\left(\frac{9}{6}\right)$$

$$= \frac{270\ \text{kN}\cdot\text{m}^3}{EI}\ \downarrow$$

Ans.





Ans:

$$\theta_C = \frac{105\ \text{kN}\cdot\text{m}^2}{EI},$$

$$v_C = \frac{270\ \text{kN}\cdot\text{m}^3}{EI}\ \downarrow$$

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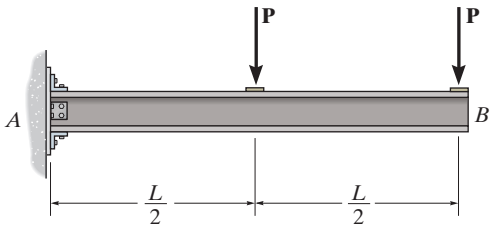
12–53.

Determine the deflection of end B of the cantilever beam. EI is constant.



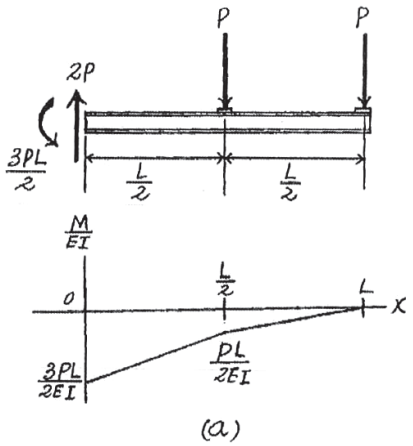
SOLUTION

Support Reactions and $\frac{M}{EI}$ Diagram: As shown in Fig. *a*.

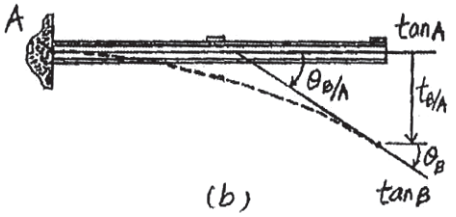
Moment Area Theorem: Since A is a fixed support, $\theta_A = 0$. Referring to the geometry of the elastic curve, Fig. *b*,

$$v_B = |t_{B/A}| = \left(\frac{3L}{4}\right)\left(\frac{PL}{2EI}\right)\left(\frac{L}{2}\right) + \frac{5L}{6}\left[\frac{1}{2}\left(\frac{PL}{EI}\right)\left(\frac{L}{2}\right)\right] + \frac{L}{3}\left[\frac{1}{2}\left(\frac{PL}{2EI}\right)\left(\frac{L}{2}\right)\right]$$

$$= \frac{7PL^3}{16EI} \quad \downarrow \qquad \textbf{Ans.}$$



(a)



(b)

Ans:

$$v_B = \frac{7PL^3}{16EI}\downarrow$$

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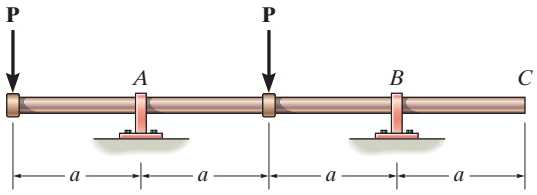
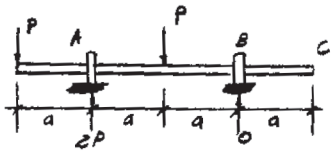
12–54.

Determine the slope at B and the deflection at C. EI is constant.



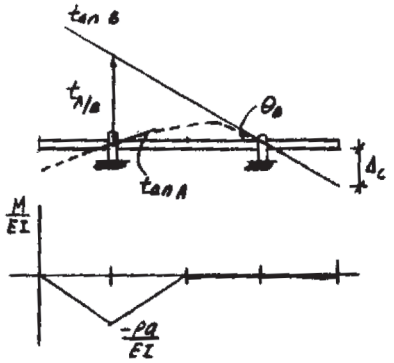
SOLUTION



$$t_{A/B} = \frac{1}{2}\left(-\frac{Pa}{EI}\right)(a)\left(\frac{a}{3}\right) = \frac{Pa^3}{6EI}$$

$$\theta_B = -\frac{|t_{A/B}|}{2a} = -\frac{Pa^3/6EI}{2a} = -\frac{Pa^2}{12EI}$$ **Ans.**

$$v_C = \theta_B a = \frac{Pa^2}{12EI}(a) = \frac{Pa^3}{12EI}$$ **Ans.**



Ans:

$$\theta_B = -\frac{Pa^2}{12EI}, v_C = \frac{Pa^3}{12EI}$$

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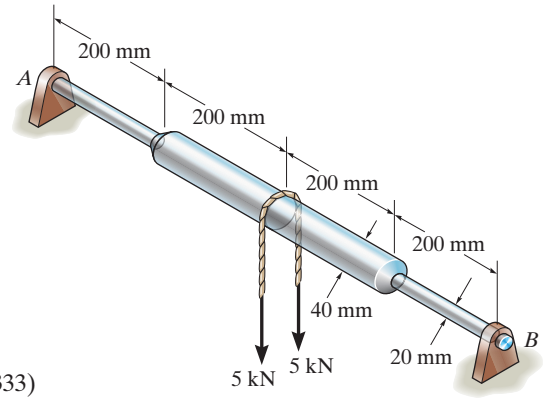
12–55.

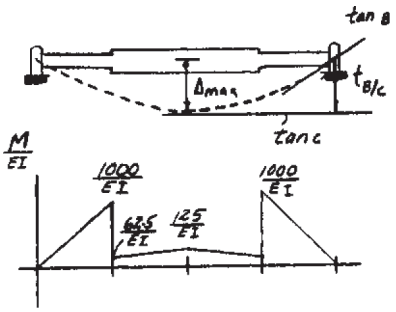
The composite simply supported steel shaft is subjected to a force of 10 kN at its center. Determine its maximum deflection. $E_{st} = 200$ GPa.



SOLUTION

$$v_{\max} = |t_{B/C}| = \frac{62.5}{EI}(0.2)(0.3) + \frac{1}{2}\left(\frac{62.5}{EI}\right)(0.2)(0.3333) + \frac{1}{2}\left(\frac{1000}{EI}\right)(0.2)(0.1333)$$

$$= \frac{19.167}{EI} = \frac{19.167}{200(10^9)(7.8540)(10^{-9})} = 0.0122\text{ m} = 12.2\text{ mm} \qquad \textbf{Ans.}$$



Ans:
$v_{\max} = 12.2$ mm

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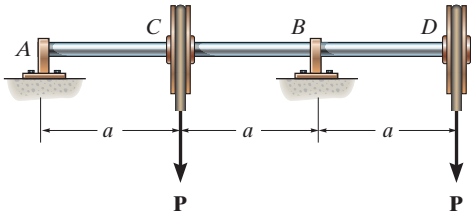
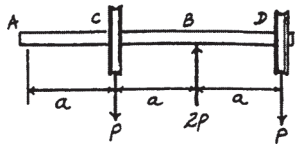
***12–56.**

Determine the slope of the shaft at A and the displacement at D. EI is constant.

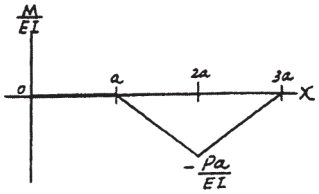


SOLUTION

Support Reactions and Elastic Curve: As shown.



M/EI Diagram: As shown.



Moment-Area Theorems:

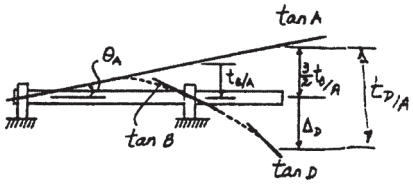
$$t_{B/A} = \frac{1}{2}\left(-\frac{Pa}{EI}\right)(a)\left(\frac{a}{3}\right) = -\frac{Pa^3}{6EI}$$

$$t_{D/A} = \frac{1}{2}\left(-\frac{Pa}{EI}\right)(a)\left(a + \frac{a}{3}\right) + \frac{1}{2}\left(-\frac{Pa}{EI}\right)(a)\left(\frac{2}{3}a\right) = -\frac{Pa^3}{EI}$$

The slope at A is

$$\theta_A = \frac{|t_{B/A}|}{L} = \frac{\frac{Pa^3}{6EI}}{2a} = \frac{Pa^2}{12EI}$$ **Ans.**

The displacement at D is



$$v_D = |t_{D/A}| - \left|\frac{3}{2}t_{B/A}\right|$$

$$= \frac{Pa^3}{EI} - \frac{3}{2}\left(\frac{Pa^3}{6EI}\right)$$

$$= \frac{3Pa^3}{4EI}$$ **Ans.**

Ans:

$$\theta_A = \frac{Pa^2}{12EI},$$

$$v_D = \frac{3Pa^3}{4EI}$$

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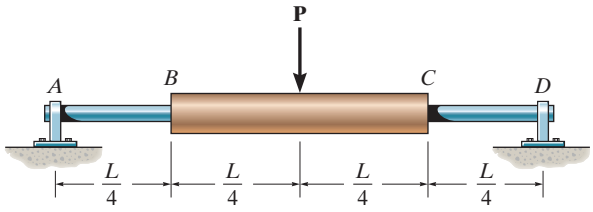
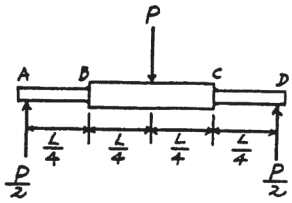
12–57.

The simply supported shaft has a moment of inertia of $2I$ for region BC and a moment of inertia I for regions AB and CD. Determine the maximum deflection of the shaft due to the load **P**. The modulus of elasticity is E.



SOLUTION



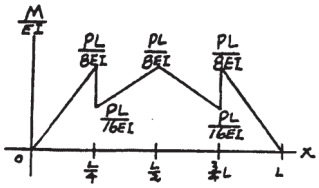
Support Reactions and Elastic Curve: As shown.

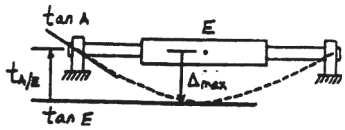
M/EI Diagram: As shown.

Moment-Area Theorems: Due to symmetry, the slope at midspan (point E) is zero. Hence,

$$v_{\max} = t_{A/E} = \frac{1}{2}\left(\frac{PL}{16EI}\right)\left(\frac{L}{4}\right)\left(\frac{L}{4}+\frac{L}{6}\right) + \left(\frac{PL}{16EI}\right)\left(\frac{L}{4}\right)\left(\frac{L}{4}+\frac{L}{8}\right) + \frac{1}{2}\left(\frac{PL}{8EI}\right)\left(\frac{L}{4}\right)\left(\frac{L}{6}\right)$$

$$= \frac{3PL^3}{256EI} \downarrow \qquad \textbf{Ans.}$$





Ans:

$$v_{\max} = \frac{3PL^3}{256EI} \downarrow$$

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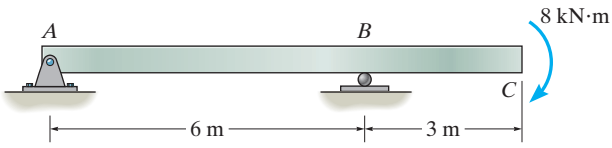
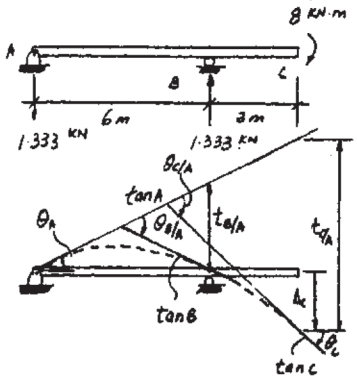
12–58.

Determine the deflection at C and the slope of the beam at A, B, and C. EI is constant.



SOLUTION



$$t_{B/A} = \frac{1}{2}\left(\frac{-8}{EI}\right)(6)(2) = \frac{-48}{EI}$$

$$t_{C/A} = \frac{1}{2}\left(\frac{-8}{EI}\right)(6)(3 + 2) + \left(\frac{-8}{EI}\right)(3)(1.5) = \frac{-156}{EI}$$

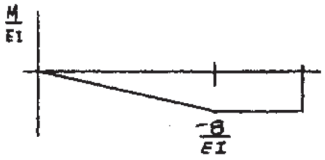
$$v_C = |t_{C/A}| - \frac{9}{6}|t_{B/A}| = \frac{156}{EI} - \frac{9(48)}{6(EI)} = \frac{84}{EI}$$ **Ans.**

$$\theta_A = \frac{|t_{B/A}|}{6} = \frac{8}{EI}$$ **Ans.**

$$\theta_{B/A} = \frac{1}{2}\left(\frac{-8}{EI}\right)(6) = \frac{-24}{EI}$$

$$\theta_B = \theta_{B/A} + \theta_A$$

$$\theta_B = -\frac{24}{EI} + \frac{8}{EI} = -\frac{16}{EI}$$ **Ans.**



$$\theta_{C/A} = \frac{1}{2}\left(\frac{-8}{EI}\right)(6) + \left(\frac{-8}{EI}\right)(3) = \frac{-48}{EI}$$

$$\theta_C = \theta_{C/A} + \theta_A$$

$$\theta_C = -\frac{48}{EI} + \frac{8}{EI} = -\frac{40}{EI}$$ **Ans.**

Ans:

$$v_C = -\frac{84}{EI}, \theta_A = \frac{8}{EI}, \theta_B = -\frac{16}{EI}, \theta_C = -\frac{40}{EI}$$

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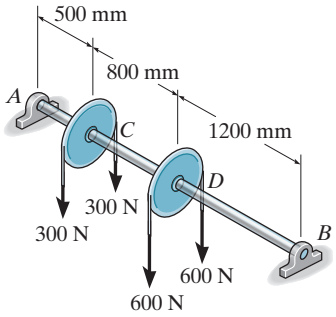
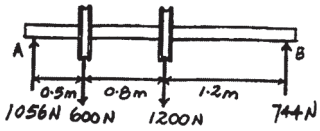
12–59.

Determine the maximum deflection of the 50-mm-diameter A-36 steel shaft.



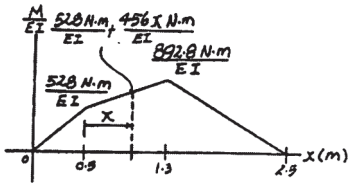
SOLUTION

Moment-Area Theorems:



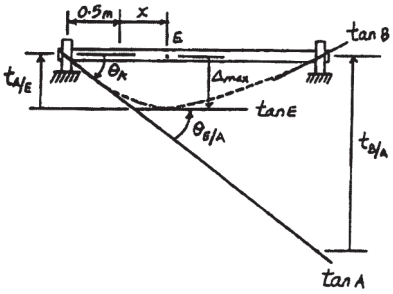
$$t_{B/A} = \frac{1}{2}\left(\frac{892.8}{EI}\right)(1.2)(0.8) + \frac{1}{2}\left(\frac{364.8}{EI}\right)(0.8)(1.4667) + \left(\frac{528}{EI}\right)(0.8)(1.6) + \frac{1}{2}\left(\frac{528}{EI}\right)(0.5)(2.1667)$$

$$= \frac{1604.4\ \text{N}\cdot\text{m}^3}{EI}$$



$$\theta_A = \frac{|t_{B/A}|}{L} = \frac{\dfrac{1604.4\ \text{N}\cdot\text{m}^3}{EI}}{2.5\ \text{m}} = \frac{641.76\ \text{N}\cdot\text{m}^2}{EI}$$

The maximum displacement occurs at point E, where $\theta_E = 0$.



$$\theta_{E/A} = \frac{1}{2}\left(\frac{528}{EI}\right)(0.5) + \left(\frac{528}{EI}\right)x + \frac{1}{2}\left(\frac{456}{EI}x\right)x$$

$$= \frac{1}{EI}(228x^2 + 528x + 132)$$

$$\theta_E = \theta_A + \theta_{E/A}$$

$$0 = -\frac{641.76}{EI} + \frac{1}{EI}(228x^2 + 528x + 132)$$

$$x = 0.7333\ \text{m} < 0.8\ \text{m} \qquad \textbf{(O.K!)}$$

The maximum displacement is,

$$v_{\max} = |t_{A/E}| = \frac{1}{2}\left(\frac{528}{EI}\right)(0.5)(0.3333) + \left(\frac{528}{EI}\right)(0.7333)(0.8666) + \frac{1}{2}\left(\frac{456}{EI}\right)(0.7333^2)(0.9888)$$

$$= \frac{500.76\ \text{N}\cdot\text{m}^3}{EI}$$

$$= \frac{500.76}{200(10^9)\left(\frac{\pi}{4}\right)(0.025^4)}$$

$$= 0.008161\ \text{m} = 8.16\ \text{mm}\downarrow \qquad \textbf{Ans.}$$

Ans:
$v_{\max} = 8.16\ \text{mm}\downarrow$

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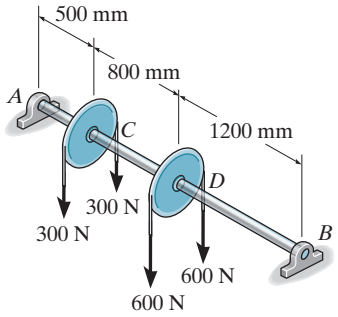
***12–60.**

Determine the slope of the 50-mm-diameter A-36 steel shaft at the journal bearings at A and B. The bearings exert only vertical reactions on the shaft.

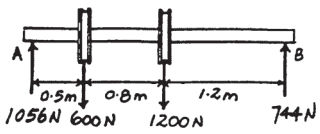


SOLUTION

Support Reactions and Elastic Curve: As shown.

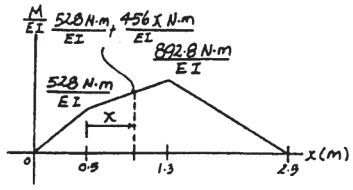
M/EI Diagram: As shown.

Moment-Area Theorems:



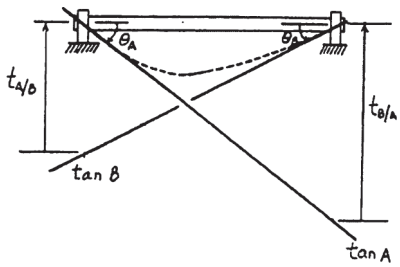
$$t_{B/A} = \frac{1}{2}\left(\frac{892.8}{EI}\right)(1.2)(0.8) + \frac{1}{2}\left(\frac{364.8}{EI}\right)(0.8)(1.4667) + \left(\frac{528}{EI}\right)(0.8)(1.6) + \frac{1}{2}\left(\frac{528}{EI}\right)(0.5)(2.1667)$$

$$= \frac{1604.4\ \text{N}\cdot\text{m}^3}{EI}$$



$$t_{A/B} = \frac{1}{2}\left(\frac{892.8}{EI}\right)(1.2)(1.7) + \frac{1}{2}\left(\frac{364.8}{EI}\right)(0.8)(1.0333) + \left(\frac{528}{EI}\right)(0.8)(0.9) + \frac{1}{2}\left(\frac{528}{EI}\right)(0.5)(0.3333)$$

$$= \frac{1485.6\ \text{N}\cdot\text{m}^3}{EI}$$



The slopes at A and B are,

$$\theta_A = \frac{|t_{B/A}|}{L} = \frac{\frac{1604.4\ \text{N}\cdot\text{m}^3}{EI}}{2.5\ \text{m}}$$

$$= \frac{641.76\ \text{N}\cdot\text{m}^2}{EI}$$

$$= \frac{641.76}{200(10^9)\left(\frac{\pi}{4}\right)(0.025^4)} = 0.0105\ \text{rad}$$ **Ans.**

$$\theta_B = \frac{|t_{A/B}|}{L} = \frac{\frac{1485.6\ \text{N}\cdot\text{m}^3}{EI}}{2.5\ \text{m}}$$

$$= \frac{594.24\ \text{N}\cdot\text{m}^2}{EI}$$

$$= \frac{594.24}{200(10^9)\left(\frac{\pi}{4}\right)(0.025^4)} = 0.00968\ \text{rad}$$ **Ans.**

Ans:
$\theta_A = 0.0105$ rad

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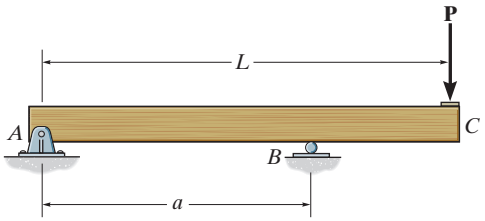
12–61.

Determine the position a of the roller support B in terms of L so that the deflection at end C is the same as the maximum deflection of region AB of the overhang beam. EI is constant.



SOLUTION

Support Reactions and $\frac{M}{EI}$ Diagram. As shown in Fig. *a*.

Moment Area Theorem. Referring to Fig. *b*,

$$|t_{B/A}| = \frac{a}{3}\left[\frac{1}{2}\left(\frac{P(L-a)}{EI}\right)(a)\right] = \frac{Pa^2(L-a)}{6EI}$$

$$|t_{C/A}| = \left(L - \frac{2}{3}a\right)\left[\frac{1}{2}\left(\frac{P(L-a)}{EI}\right)(a)\right] + \frac{2(L-a)}{3}\left[\frac{1}{2}\left(\frac{P(L-a)}{EI}\right)(L-a)\right]$$

$$= \frac{P(L-a)\left(2L^2 - aL\right)}{6EI}$$

From the geometry shown in Fig. *b*,

$$v_C = |t_{C/A}| - \frac{|t_{B/A}|}{a}L$$

$$= \frac{PL(L-a)\left(2L-a\right)}{6EI} - \frac{Pa^2(L-a)}{6EI}\left(\frac{L}{a}\right)$$

$$= \frac{PL(L-a)^2}{3EI}$$

$$\theta_A = \frac{|t_{B/A}|}{a} = \frac{\frac{Pa^2(L-a)}{6EI}}{a} = \frac{Pa(L-a)}{6EI}$$

The maximum deflection in region AB occurs at point D, where the slope of the elastic curve is zero ($\theta_D = 0$).

Thus,

$$|\theta_{D/A}| = \theta_A$$

$$\frac{1}{2}\left[\frac{P(L-a)}{EIa}x\right](x) = \frac{Pa(L-a)}{6EI}$$

$$x = \frac{\sqrt{3}}{3}a$$

Also,

$$v_D = |t_{4/D}| = \left(\frac{2\sqrt{3}}{9}a\right)\left[\frac{1}{2}\left[\frac{P(L-a)}{EIa}\left(\frac{\sqrt{3}}{3}a\right)\right]\right]\left(\frac{\sqrt{3}}{3}a\right) = \frac{\sqrt{3}Pa^2(L-a)}{27EI}$$

It is required that

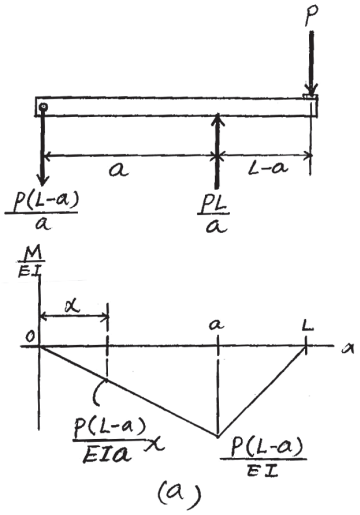
$$v_C = \Delta_D$$

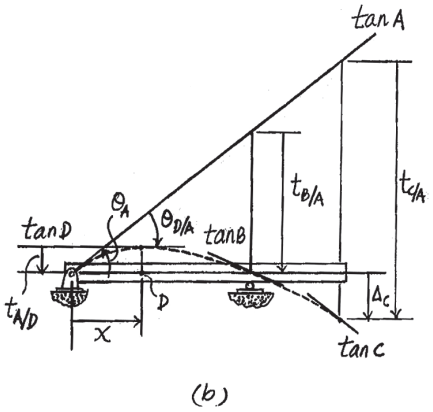
$$\frac{PL(L-a)^2}{3EI} = \frac{\sqrt{3}Pa^2(L-a)}{27EI}$$

$$\frac{\sqrt{3}}{9}a^2 + La - L^2 = 0$$

Solving for the positive root,

$$a = 0.858L \qquad \textbf{Ans.}$$





Ans:
$a = 0.858\,L$

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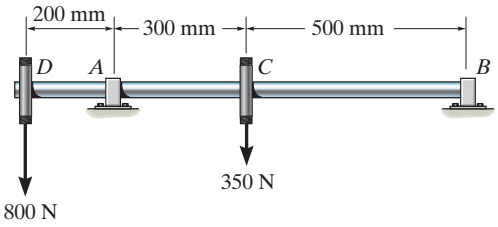
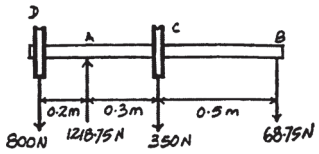
12–62.

Determine the slope of the 20-mm-diameter A-36 steel shaft at the journal bearings A and B.



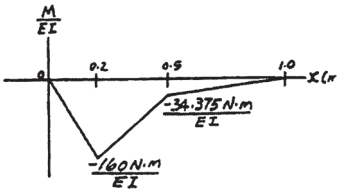
SOLUTION



Moment-Area Theorems:

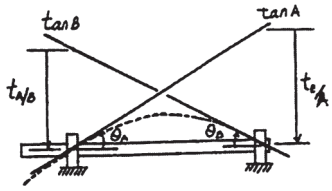
$$t_{B/A} = \frac{1}{2}\left(-\frac{34.375}{EI}\right)(0.5)(0.3333) + \frac{1}{2}\left(-\frac{125.625}{EI}\right)(0.3)(0.7) + \left(-\frac{34.375}{EI}\right)(0.3)(0.65)$$

$$= -\frac{22.75833\ \text{N}\cdot\text{m}^3}{EI}$$



$$t_{A/B} = \frac{1}{2}\left(-\frac{34.375}{EI}\right)(0.5)(0.4667) + \frac{1}{2}\left(-\frac{125.625}{EI}\right)(0.3)(0.1) + \left(-\frac{34.375}{EI}\right)(0.3)(0.15)$$

$$= -\frac{7.44167\ \text{N}\cdot\text{m}^3}{EI}$$



The slopes at A and B are,

$$\theta_A = \frac{|t_{B/A}|}{L} = \frac{\frac{22.75833\ \text{N}\cdot\text{m}^3}{EI}}{0.8\ \text{m}}$$

$$= \frac{28.448\ \text{N}\cdot\text{m}^2}{EI}$$

$$= \frac{28.448}{200(10^9)\left(\frac{\pi}{4}\right)(0.01^4)} = 0.0181\ \text{rad}$$ **Ans.**

$$\theta_B = \frac{|t_{A/B}|}{L} = \frac{\frac{7.44167\ \text{N}\cdot\text{m}^3}{EI}}{0.8\ \text{m}}$$

$$= \frac{9.302\ \text{N}\cdot\text{m}^2}{EI}$$

$$= \frac{9.302}{200(10^9)\left(\frac{\pi}{4}\right)(0.01^4)} = 0.00592\ \text{rad}$$ **Ans.**

Ans:
$\theta_A = 0.0181$ rad,
$\theta_B = 0.00592$ rad

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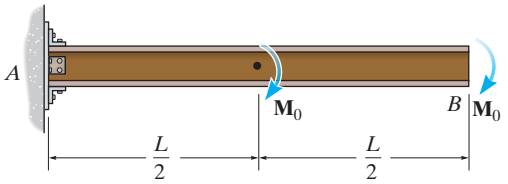
12–63.

Determine the slope and the deflection of end B of the cantilever beam. EI is constant.



SOLUTION

Support Reactions and $\frac{M}{EI}$ Diagram. As shown in Fig. *a*.

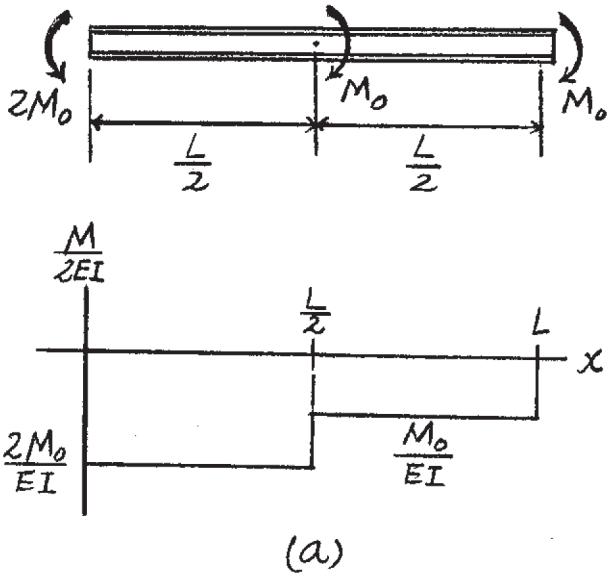
Moment Area Theorem: Since A is a fixed support, $\theta_A = 0$. Referring to the geometry of the elastic curve, Fig. *b*,

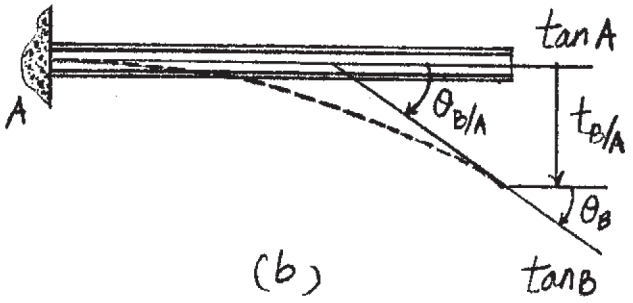
$$\theta_B = \theta_{B/A} = -\frac{2M_O}{EI}\left(\frac{L}{2}\right) + \left(-\frac{M_O}{EI}\right)\left(\frac{L}{2}\right)$$

$$= -\frac{3M_O L}{2EI}$$ **Ans.**

$$v_B = |t_{B/A}| = \frac{3L}{2}\left[\frac{2M_O}{EI}\left(\frac{L}{2}\right)\right] + \frac{L}{4}\left[\frac{M_O}{EI}\left(\frac{L}{2}\right)\right]$$

$$= \frac{7M_O L^2}{8EI}\downarrow$$ **Ans.**





Ans:

$$\theta_B = -\frac{3M_0 L}{2EI}, v_B = \frac{7M_0 L^2}{8EI}\downarrow$$

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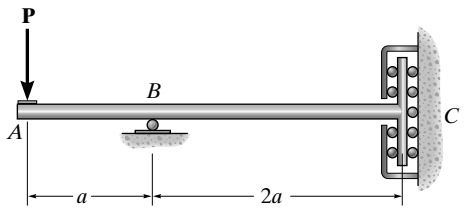
***12–64.** The bar is supported by the roller constraint at C, which allows vertical displacement but resists axial load and moment. If the bar is subjected to the loading shown, determine the slope and displacement at A. EI is constant.

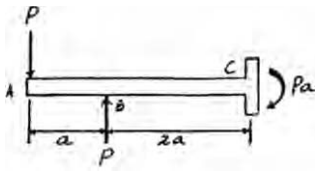


SOLUTION

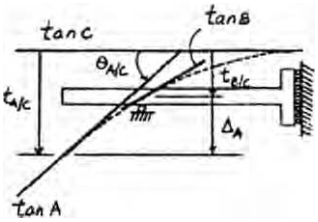
Support Reactions and Elastic Curve: As shown.

M/EI Diagram: As shown.

Moment-Area Theorems:



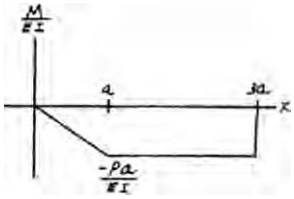
$$\theta_{A/C} = \left(-\frac{Pa}{EI}\right)(2a) + \frac{1}{2}\left(-\frac{Pa}{EI}\right)(a) = -\frac{5Pa^2}{2EI}$$



$$t_{B/C} = \left(-\frac{Pa}{EI}\right)(2a)(a) = \frac{2Pa^3}{EI}$$

$$t_{A/C} = \left(-\frac{Pa}{EI}\right)(2a)(2a) + \frac{1}{2}\left(-\frac{Pa}{EI}\right)(a)\left(\frac{2}{3}a\right) = -\frac{13Pa^3}{3EI}$$

Due to the moment constraint, the slope at support C is zero. Hence, the slope at A is



$$\theta_A = |\theta_{A/C}| = \frac{5Pa^2}{2EI}$$ **Ans.**

and the displacement at A is

$$\Delta_A = |t_{A/C}| - |t_{B/C}|$$

$$= \frac{13Pa^3}{3EI} - \frac{2Pa^3}{EI} = \frac{7Pa^3}{3EI} \quad \downarrow$$ **Ans.**

Ans:

$$\theta_A = |\theta_{A/C}| = \frac{5Pa^2}{2EI}\ ;\ \Delta_A = \frac{7Pa^3}{3EI} \quad \downarrow$$

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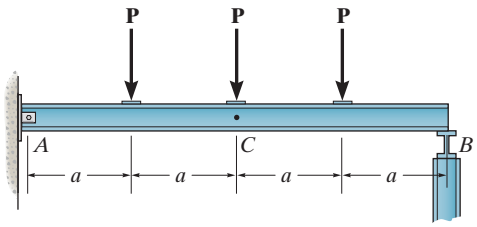
12–65.

Determine the slope at A and the displacement at C. Assume the support at A is a pin and B is a roller. EI is constant.



SOLUTION

Support Reactions and Elastic Curve: As shown.

M/EI Diagram: As shown.

Moment - Area Theorems: Due to symmetry, the slope at midspan (point C) is zero. Hence the slope at A is

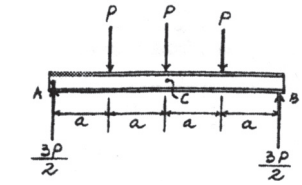
$$|\theta_A| = \theta_{A/C} = \frac{1}{2}\left(\frac{3Pa}{2EI}\right)(a) + \left(\frac{3Pa}{2EI}\right)(a) + \frac{1}{2}\left(\frac{Pa}{2EI}\right)(a)$$

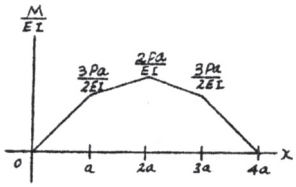
$$\theta_A = -\frac{5Pa^2}{2EI} \qquad \textbf{Ans.}$$

The displacement at C is

$$v_C = t_{A/C} = \frac{1}{2}\left(\frac{3Pa}{2EI}\right)(a)\left(\frac{2a}{3}\right) + \left(\frac{3Pa}{2EI}\right)\left(a + \frac{a}{2}\right) + \frac{1}{2}\left(\frac{Pa}{2EI}\right)(a)\left(a + \frac{2a}{3}\right)$$

$$= \frac{19Pa^3}{6EI}\ \downarrow \qquad \textbf{Ans.}$$





Ans:

$$\theta_A = -\frac{5Pa^2}{2EI}, v_C = \frac{19Pa^3}{6EI}\downarrow$$

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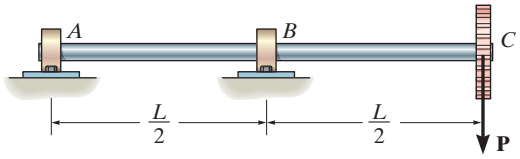
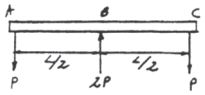
12–66.

Determine the deflection at C and the slopes at the bearings A and B. EI is constant.

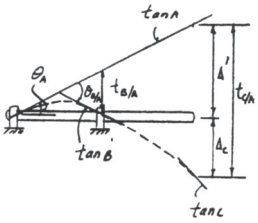


SOLUTION



$$t_{B/A} = \frac{1}{2}\left(\frac{-PL}{2EI}\right)\left(\frac{L}{2}\right)\left(\frac{L}{6}\right) = \frac{-PL^3}{48EI}$$

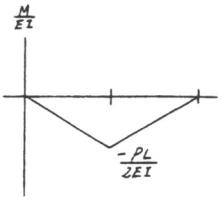
$$t_{C/A} = \frac{1}{2}\left(\frac{-PL}{2EI}\right)(L)\left(\frac{L}{2}\right) = \frac{-PL^3}{8EI}$$



$$|v_C| = |t_{C/A}| - \left(\frac{L}{\frac{L}{2}}\right)|t_{B/A}|$$

$$v_C = -\frac{PL^3}{8EI} + 2\left(\frac{PL^3}{48EI}\right) = -\frac{PL^3}{12EI}$$ **Ans.**

$$\theta_A = \frac{|t_{B/A}|}{\frac{L}{2}} = \frac{\frac{PL^3}{48\,EI}}{\frac{L}{2}} = \frac{PL^2}{24EI}$$ **Ans.**



$$\theta_{B/A} = \frac{1}{2}\left(\frac{-PL}{2EI}\right)\left(\frac{L}{2}\right) = \frac{-PL^2}{8EI}$$

$$\theta_B = \theta_{B/A} + \theta_A$$

$$\theta_B = -\frac{PL^2}{8EI} + \frac{PL^2}{24EI} = -\frac{PL^2}{12EI}$$ **Ans.**

Ans:

$$v_C = -\frac{PL^3}{12EI},\ \theta_A = \frac{PL^2}{24EI},\ \theta_B = -\frac{PL^2}{12EI}$$

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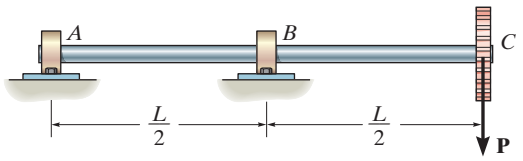
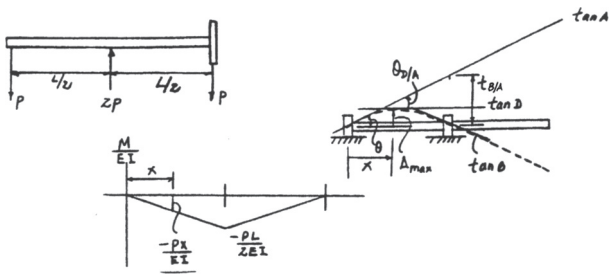
12–67.

Determine the maximum deflection within region AB. EI is constant.



SOLUTION



$$\theta_{D/A} = \frac{t_{B/A}}{\left(\frac{L}{2}\right)}$$

$$\frac{1}{2}\left(\frac{Px}{EI}\right)x = \frac{\frac{1}{2}\left(\frac{L}{2}\right)\left(\frac{PL}{2EI}\right)\left(\frac{1}{3}\right)\left(\frac{L}{2}\right)}{\left(\frac{L}{2}\right)}; \quad x = 0.288675\,L$$

$$v_{\max} = \frac{1}{2}\left(\frac{P(0.288675\,L)}{EI}\right)(0.288675\,L)\left(\frac{2}{3}\right)(0.288675\,L)$$

$$v_{\max} = \frac{0.00802PL^3}{EI}$$ **Ans.**

Ans:

$$v_{\max} = \frac{0.00802PL^3}{EI}$$

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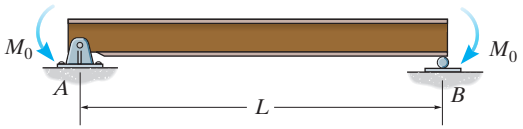
***12–68.**

Determine the slope at A and the maximum deflection of the simply supported beam. EI is constant.



SOLUTION

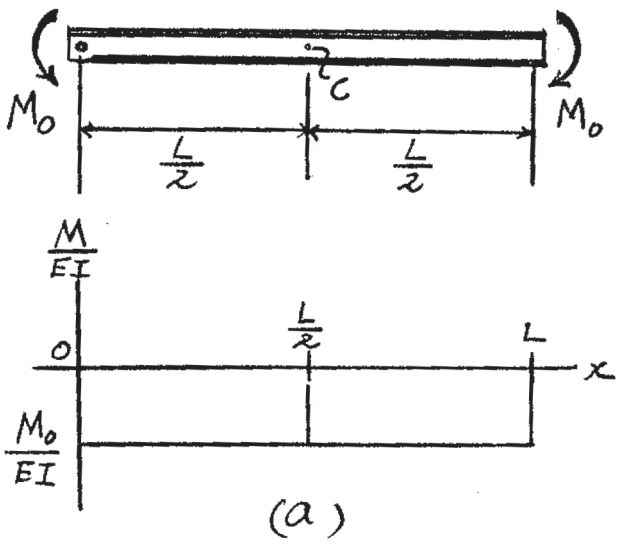
Support Reactions and $\frac{M}{EI}$ Diagram. As shown in Fig. *a*.

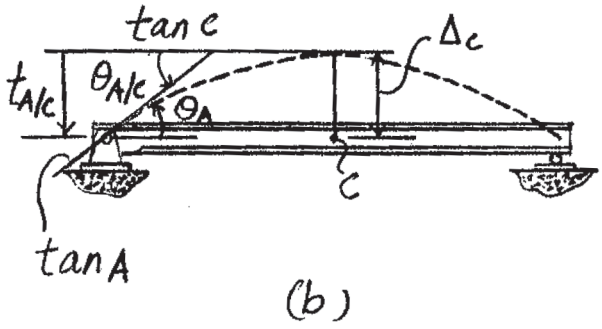
Moment Area Theorem. Due to symmetry, the slope at the midspan of the beam, i.e., point C is zero ($\theta_C = 0$). Thus, the maximum deflection of the beam occurs here. Referring to the geometry of the elastic curve, Fig. *b*,

$$\theta_A = |\theta_{A/C}| = \frac{M_O}{EI}\left(\frac{L}{2}\right) = \frac{M_O L}{2EI} \qquad \textbf{Ans.}$$

$$v_{\max} = v_C = |t_{A/C}| = \frac{L}{4}\left[\frac{M_O}{EI}\left(\frac{L}{2}\right)\right]$$

$$= \frac{M_O L^2}{8EI} \uparrow \qquad \textbf{Ans.}$$





Ans:

$$\theta_A = \frac{M_O L}{2EI},$$

$$v_{\max} = \frac{M_O L^2}{8EI} \uparrow$$

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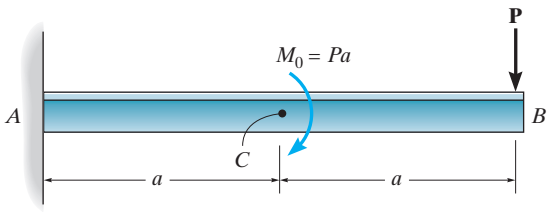
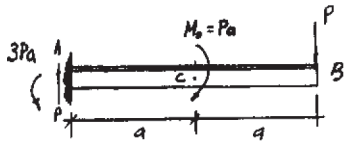
12–69.

Determine the slope at C and the deflection at B. EI is constant.

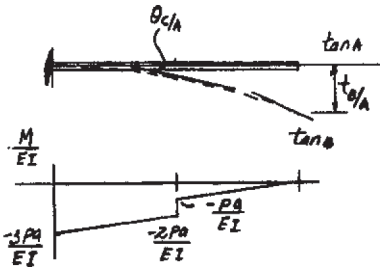


SOLUTION



$$\theta_{C/A} = \left(-\frac{2Pa}{EI}\right)(a) + \frac{1}{2}\left(-\frac{Pa}{EI}\right)(a)$$

$$= -\frac{5Pa^2}{2EI} \qquad \textbf{Ans.}$$



$$\theta_C = \theta_{C/A}$$

$$\curvearrowright + \theta_C = +\frac{5Pa^2}{2EI} \qquad \measuredangle$$

$$v_B = |t_{B/A}| = \frac{1}{2}\left(-\frac{Pa}{EI}\right)(a)\left(\frac{2a}{3}\right) + \frac{1}{2}\left(-\frac{Pa}{EI}\right)(a)\left(a + \frac{2a}{3}\right) + \left(-\frac{2Pa}{EI}\right)(a)\left(a + \frac{a}{2}\right)$$

$$= \frac{25Pa^3}{6EI}\downarrow \qquad \textbf{Ans.}$$

Ans:

$$\theta_C = -\frac{5Pa^2}{2EI},\ v_B = \frac{25Pa^3}{6EI}\downarrow$$

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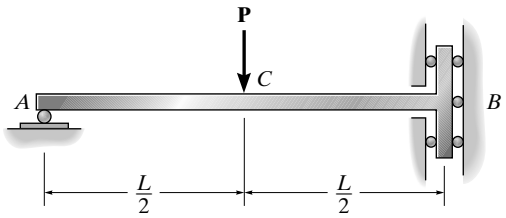
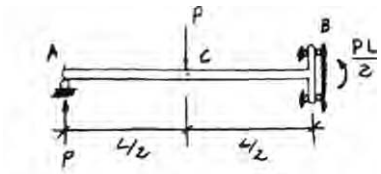
12–70. The bar is supported by a roller constraint at B, which allows vertical displacement but resists axial load and moment. If the bar is subjected to the loading shown, determine the slope at A and the deflection at C. EI is constant.



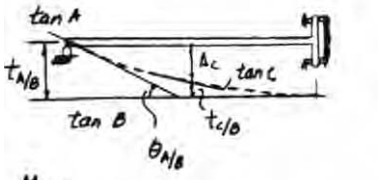
SOLUTION



$$\theta_{A/B} = \frac{1}{2}\left(\frac{PL}{2EI}\right)\left(\frac{L}{2}\right) + \frac{PL}{2EI}\left(\frac{L}{2}\right) = \frac{3PL^2}{8EI}$$

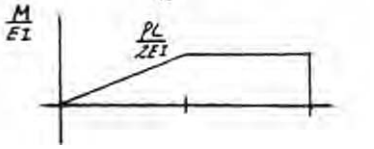
$$\theta_A = \theta_{A/B}$$

$$\theta_A = \frac{3PL^2}{8EI} \quad \textbf{Ans.}$$



$$t_{A/B} = \frac{1}{2}\left(\frac{PL}{2EI}\right)\left(\frac{L}{2}\right)\left(\frac{L}{3}\right) + \frac{PL}{2EI}\left(\frac{L}{2}\right)\left(\frac{L}{2} + \frac{L}{4}\right) = \frac{11PL^3}{48EI}$$

$$t_{C/B} = \frac{PL}{2EI}\left(\frac{L}{2}\right)\left(\frac{L}{4}\right) = \frac{PL^3}{16EI}$$



$$\Delta_C = t_{A/B} - t_{C/B} = \frac{11PL^3}{48EI} - \frac{PL^3}{16EI} = \frac{PL^3}{6EI} \quad \textbf{Ans.}$$

Ans:

$$\theta_A = \frac{3PL^2}{8EI},\ \Delta_C = \frac{PL^3}{6EI}$$

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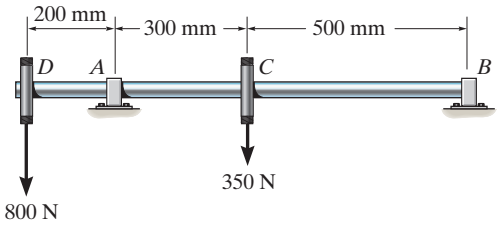
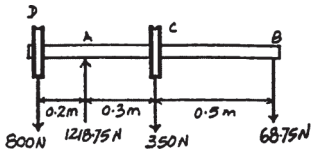
12–71.

Determine the displacement of the 20-mm-diameter A-36 steel shaft at D.

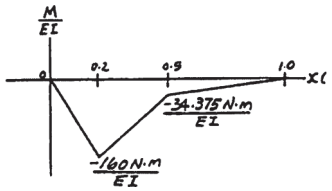


SOLUTION



Support Reactions and Elastic Curve: As shown.

M/EI Diagram: As shown.



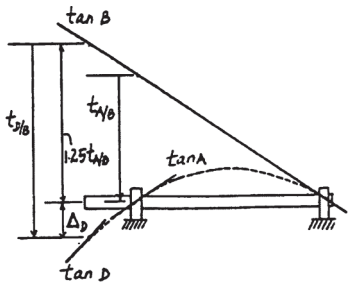
Moment-Area Theorems:

$$t_{D/B} = \frac{1}{2}\left(-\frac{34.375}{EI}\right)(0.5)(0.6667) + \frac{1}{2}\left(-\frac{125.625}{EI}\right)(0.3)(0.3) + \left(-\frac{34.375}{EI}\right)(0.3)(0.35) + \frac{1}{2}\left(-\frac{160}{EI}\right)(0.2)(0.1333)$$

$$= -\frac{17.125\ \text{N}\cdot\text{m}^3}{EI}$$

$$t_{A/B} = \frac{1}{2}\left(-\frac{34.375}{EI}\right)(0.5)(0.4667) + \frac{1}{2}\left(-\frac{125.625}{EI}\right)(0.3)(0.1) + \left(-\frac{34.375}{EI}\right)(0.3)(0.15)$$

$$= -\frac{7.44167\ \text{N}\cdot\text{m}^3}{EI}$$



The displacement at D is,

$$v_D = |t_{D/B}| - |1.25t_{A/B}|$$

$$= \frac{17.125}{EI} - 1.25\left(\frac{7.44167}{EI}\right)$$

$$= \frac{7.823\ \text{N}\cdot\text{m}^3}{EI}$$

$$= \frac{7.823}{200(10^9)\left(\frac{\pi}{4}\right)(0.01^4)}$$

$$= 0.00498\ \text{m} = 4.98\ \text{mm}\downarrow$$ **Ans.**

Ans:
$v_D = 4.98\ \text{mm}\downarrow$

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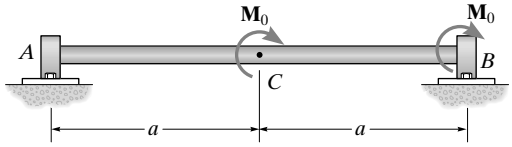
***12–72.** The shaft is subjected to the loading shown. If the bearings at A and B only exert vertical reactions on the shaft, determine the slope at A and the displacement at C. EI is constant.



M/EI Diagram: As shown.

Moment-Area Theorems:

$$t_{B/A} = \frac{1}{2}\left(-\frac{M_0}{EI}\right)(a)\left(\frac{a}{3}\right) + \frac{1}{2}\left(-\frac{M_0}{EI}\right)(a)\left(a + \frac{a}{3}\right)$$

$$= -\frac{5M_0 a^2}{6EI}$$

$$t_{C/A} = \frac{1}{2}\left(-\frac{M_0}{EI}\right)(a)\left(\frac{a}{3}\right) = -\frac{M_0 a^2}{6EI}$$

The slope at A is

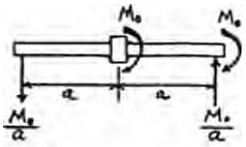
$$\theta_A = \frac{|t_{B/A}|}{L} = \frac{\frac{5M_0 a^2}{6EI}}{2a} = \frac{5M_0 a}{12EI}$$ **Ans.**

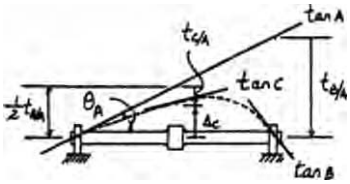
The displacement at C is,

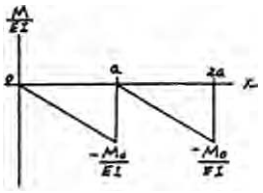
$$\Delta_C = \left|\frac{1}{2}t_{B/A}\right| - |t_{C/A}|$$

$$= \frac{1}{2}\left(\frac{5M_0 a^2}{6EI}\right) - \frac{M_0 a^2}{6EI}$$

$$= \frac{M_0 a^2}{4EI} \uparrow$$ **Ans.**







Ans:

$$\theta_A = \frac{5M_0 a}{12EI} \; ; \; \Delta_C = \frac{M_0 a^2}{4EI} \uparrow$$

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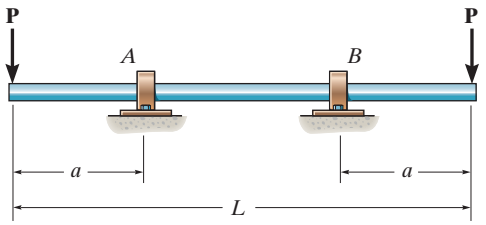
12–73.

At what distance a should the journal bearing supports at A and B be placed so that the deflection at the center of the shaft is equal to the deflection at its ends? EI is constant.

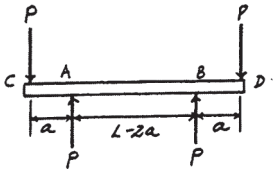


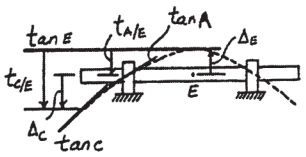
SOLUTION

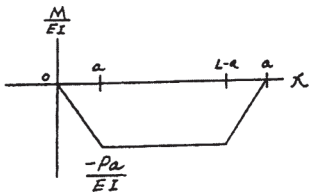
Support Reactions and Elastic Curve: As shown.

M/EI Diagram: As shown.

Moment-Area Theorems: Due to symmetry, the slope at midspan (point E) is zero.







$$v_E = |t_{A/E}| = \left(-\frac{Pa}{EI}\right)\left(\frac{L-2a}{2}\right)\left(\frac{L-2a}{4}\right) = \frac{Pa}{8EI}(L-2a)^2$$

$$t_{C/E} = \left(-\frac{Pa}{EI}\right)\left(\frac{L-2a}{2}\right)\left(a+\frac{L-2a}{4}\right) + \frac{1}{2}\left(-\frac{Pa}{EI}\right)(a)\left(\frac{2}{3}a\right)$$

$$= -\frac{Pa}{24EI}(3L^2-4a^2)$$

$$v_C = |t_{C/E}| - |t_{A/E}|$$

$$= \frac{Pa}{24EI}(3L^2-4a^2) - \frac{Pa}{8EI}(L-2a)^2$$

$$= \frac{Pa^2}{6EI}(3L-4a)$$

Require $v_E = v_C$, then,

$$\frac{Pa}{8EI}(L-2a)^2 = \frac{Pa^2}{6EI}(3L-4a)$$

$$28a^2 - 24aL + 3L^2 = 0$$

$$a = 0.152L \qquad \textbf{Ans.}$$

Ans:
$a = 0.152L$

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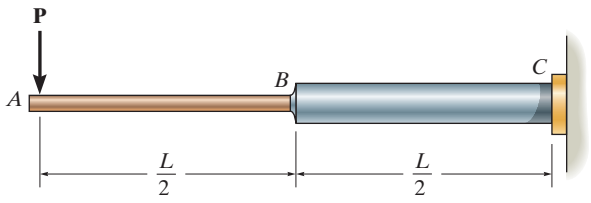
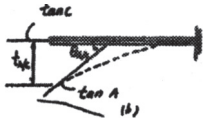
12–74.

The rod is constructed from two shafts for which the moment of inertia of AB is I and for BC it is $2I$. Determine the maximum slope and deflection of the rod due to the loading. The modulus of elasticity is E.



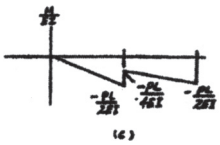
SOLUTION

$$\theta_{A/C} = \frac{1}{2}\left(\frac{-PL}{2EI}\right)\left(\frac{L}{2}\right) + \frac{1}{2}\left(\frac{-PL}{4EI}\right)\left(\frac{L}{2}\right) + \left(\frac{-PL}{4EI}\right)\left(\frac{L}{2}\right) = \frac{-5PL^2}{16EI} = \frac{5PL^2}{16EI}$$



$$\theta_A = \theta_{A/C} + \theta_C$$

$$\theta_{\max} = \theta_A = \frac{5PL^2}{16EI} + 0 = \frac{5PL^2}{16EI} \qquad \textbf{Ans.}$$



$$v_{\max} = v_A = |t_{A/C}|$$

$$= \left|\frac{1}{2}\left(\frac{-PL}{2\,EI}\right)\left(\frac{L}{2}\right)\left(\frac{L}{3}\right) + \frac{1}{2}\left(\frac{-PL}{4EI}\right)\left(\frac{L}{2}\right)\left(\frac{L}{2} + \frac{L}{3}\right) + \left(\frac{-PL}{4EI}\right)\left(\frac{L}{2}\right)\left(\frac{L}{2} + \frac{L}{4}\right)\right|$$

$$= \frac{3PL^3}{16EI}\downarrow \qquad \textbf{Ans.}$$

Ans:

$$\theta_{\max} = \frac{5PL^2}{16EI},\ v_{\max} = \frac{3PL^3}{16EI}\downarrow$$

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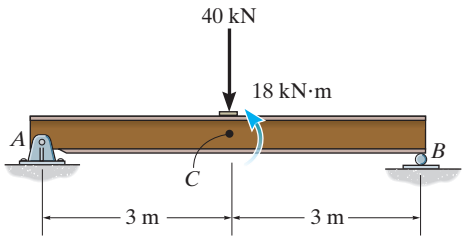
12–75.

Determine the slope at B and the deflection at C of the beam. $E = 200$ GPa and $I = 65.0(10^6)$ mm^4.



SOLUTION

Support Reactions and $\frac{M}{EI}$ Diagram: As shown in Fig. *a*.

Moment Area Theorem: Referring to Fig. *b*,

$$|t_{C/B}| = \left[\frac{1}{3}(3)\right]\left[\frac{1}{2}\left(\frac{51}{EI}\right)(3)\right]$$

$$= \frac{76.5\ \text{kN}\cdot\text{m}^3}{EI}$$

$$|t_{A/B}| = \left[\frac{1}{3}(3) + 3\right]\left[\frac{1}{2}\left(\frac{51}{EI}\right)(3)\right] + \left[\frac{2}{3}(3)\right]\left[\frac{1}{2}\left(\frac{69}{EI}\right)(3)\right]$$

$$= \frac{513\ \text{kN}\cdot\text{m}^3}{EI}$$

From the geometry of the elastic curve, Fig. *b*,

$$\theta_B = \frac{|t_{A/B}|}{L} = \frac{513/EI}{6} = \frac{85.5\ \text{kN}\cdot\text{m}^2}{EI}$$

$$= \frac{85.5(10^3)}{200(10^9)[65.0(10^{-6})]} = 0.00658\ \text{rad}$$ **Ans.**

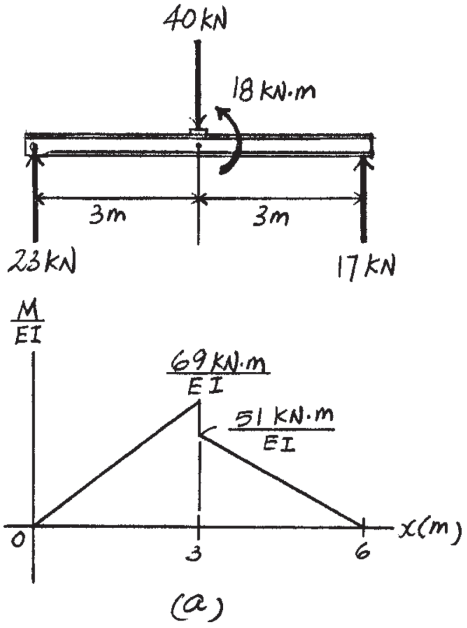
and

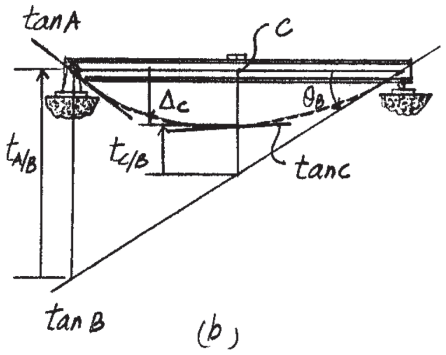
$$v_C = |t_{A/B}|\left(\frac{L_{BC}}{L}\right) - |t_{C/B}|$$

$$= \frac{513}{EI}\left(\frac{3}{6}\right) - \frac{76.5}{EI}$$

$$= \frac{180\ \text{kN}\cdot\text{m}^3}{EI} = \frac{180(10^3)}{200(10^9)[65.0(10^{-6})]}$$

$$= 0.0138\ \text{m} = 13.8\ \text{mm}\ \downarrow$$ **Ans.**





Ans:
$\theta_B = 0.00658$ rad, $v_C = 13.8$ mm$\downarrow$

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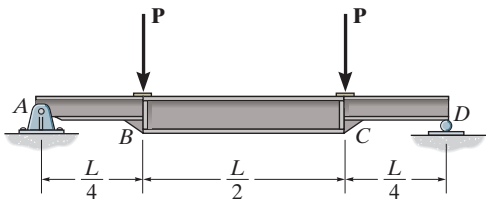
***12–76.**

Determine the slope at point A and the maximum deflection of the simply supported beam. The beam is made of material having a modulus of elasticity E. The moment of inertia of segments AB and CD of the beam is I, and the moment of inertia of segment BC is $2I$.



SOLUTION

Support Reactions and $\frac{M}{EI}$ Diagram: As shown in Fig. *a*.

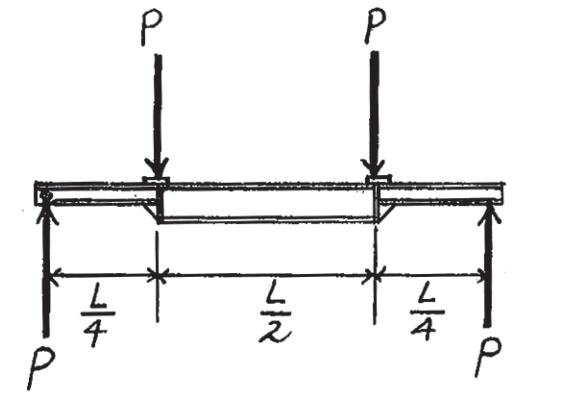
Moment Area Theorem: Due to symmetry, the slope at the midspan of the beam, i.e., point E, is zero ($\theta_E = 0$). Thus the maximum deflection occurs here. Referring to the geometry of the elastic curve, Fig. *b*,

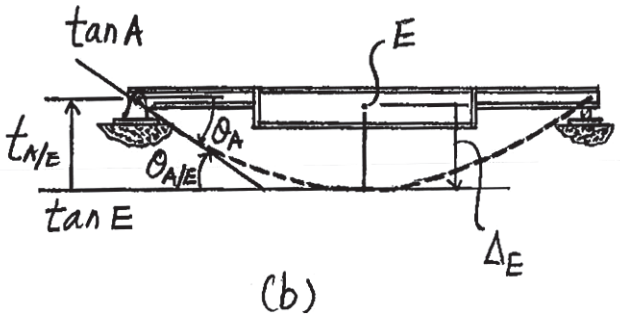
$$|\theta_A| = |\theta_{A/E}| = \frac{1}{2}\left(\frac{PL}{4EI}\right)\left(\frac{L}{4}\right) + \frac{PL}{8EI}\left(\frac{L}{4}\right)$$

$$\theta_A = -\frac{PL^2}{16EI} \qquad \textbf{Ans.}$$

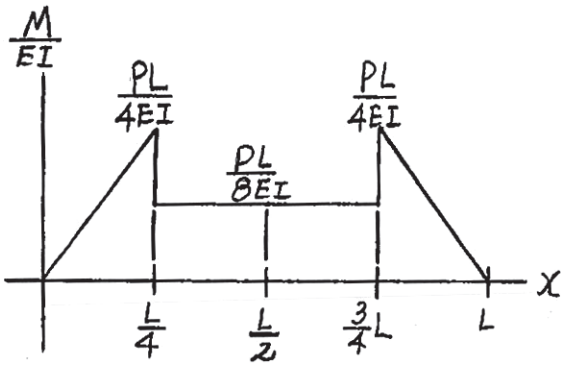
$$v_{\max} = v_E = |t_{A/E}| = \frac{3}{8}L\left[\frac{PL}{8EI}\left(\frac{L}{4}\right)\right] + \frac{L}{6}\left[\frac{1}{2}\left(\frac{PL}{4EI}\right)\left(\frac{L}{4}\right)\right]$$

$$= \frac{13PL^3}{768EI} \downarrow \qquad \textbf{Ans.}$$





(b)



(a)

Ans:

$\theta_A = -\frac{PL^2}{16EI}$,

$v_{\max} = \frac{13PL^3}{768EI} \downarrow$

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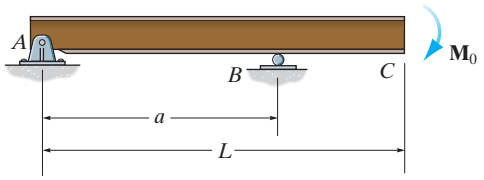
12–77.

Determine the position a of roller support B in terms of L so that deflection at end C is the same as the maximum deflection in region AB of the overhang beam. EI is constant.



SOLUTION

Support Reactions and $\frac{M}{EI}$ Diagram: As shown in Fig. *a*.

Moment Area Theorem: Referring to Fig. *b*,

$$|t_{B/A}| = \frac{a}{3}\left[\frac{1}{2}\left(\frac{M_O}{EI}\right)(a)\right] = \frac{M_O a^2}{6EI}$$

$$|t_{C/A}| = \left(L - \frac{2}{3}a\right)\left[\frac{1}{2}\left(\frac{M_O}{EI}\right)(a)\right] + \left(\frac{L-a}{2}\right)\left[\frac{M_O}{EI}(L-a)\right]$$

$$= \frac{M_O}{6EI}\left(a^2 + 3L^2 - 3La\right)$$

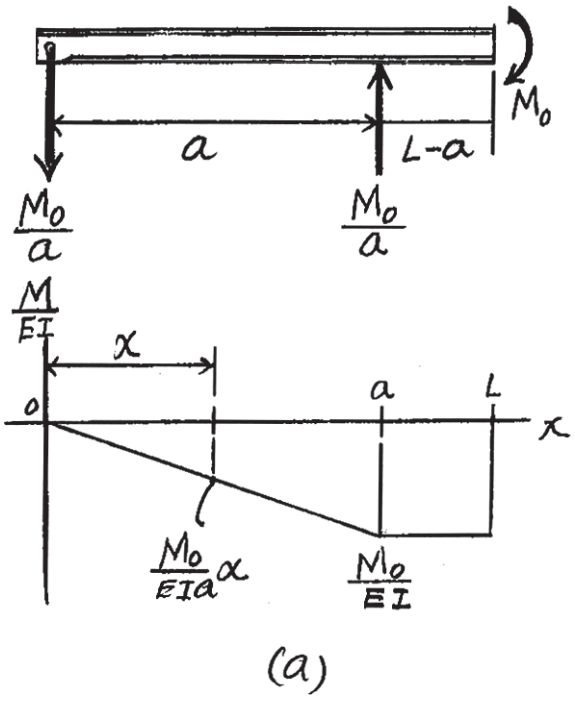
From the geometry shown in Fig. *b*,

$$v_C = |t_{C/A}| - \frac{|t_{B/A}|}{a}L$$

$$= \frac{M_O}{6EI}\left(a^2 + 3L^2 - 3La\right) - \frac{M_O a^2}{6EI}\left(\frac{L}{a}\right)$$

$$= \frac{M_O}{6EI}\left(a^2 + 3L^2 - 4La\right)$$

$$\theta_A = \frac{|t_{B/A}|}{a} = \frac{\frac{M_O a^2}{6EI}}{a} = \frac{M_O a}{6EI}$$



(a)

The maximum deflection in region AB occurs at point D, where the slope of the elastic curve is zero ($\theta_D = 0$).

Thus,

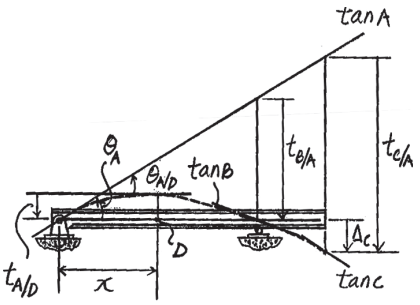
$$|\theta_{D/A}| = \theta_A$$

$$\frac{1}{2}\left(\frac{M_O}{EIa}\right)(x)^2 = \frac{M_O a}{6EI}$$

$$x = \frac{\sqrt{3}}{3}a$$

Also,

$$v_D = |t_{A/D}| = \frac{2}{3}\left(\frac{\sqrt{3}}{3}a\right)\left[\frac{1}{2}\left(\frac{M_O}{EIa}\right)\left(\frac{\sqrt{3}}{3}a\right)\right]\left(\frac{\sqrt{3}}{3}a\right) = \frac{\sqrt{3}M_O a^2}{27EI}$$



(b)

It is required that

$$v_C = v_D$$

$$\frac{M_O}{6EI}\left(a^2 + 3L^2 - 4La\right) = \frac{\sqrt{3}M_O a^2}{27EI}$$

$$0.6151a^2 - 4La + 3L^2 = 0$$

Solving for the root $< L$,

$$a = 0.865\,L$$ **Ans.**

Ans:
$a = 0.865\,L$

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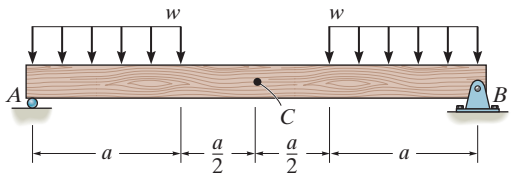
12–78.

Determine the slope at B and deflection at C. EI is constant.



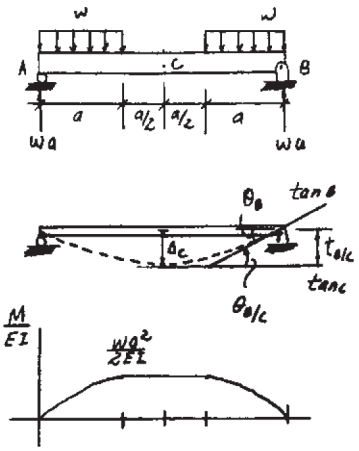
SOLUTION

$$\theta_{B/C} = \frac{wa^2}{2EI}\left(\frac{a}{2}\right) + \frac{2}{3}\left(\frac{wa^2}{2EI}\right)(a) = \frac{7wa^3}{12EI}$$

$$\theta_B = \theta_{B/C} = \frac{7wa^3}{12EI}$$ **Ans.**

$$v_C = t_{B/C} = \frac{wa^2}{2EI}\left(\frac{a}{2}\right)\left(a + \frac{a}{4}\right) + \frac{2}{3}\left(\frac{wa^2}{2EI}\right)(a)\left(\frac{5}{8}a\right)$$

$$= \frac{25wa^4}{48EI}\downarrow$$ **Ans.**



Ans:

$$\theta_B = \frac{7wa^3}{12EI},\ v_C = \frac{25wa^4}{48EI}\downarrow$$

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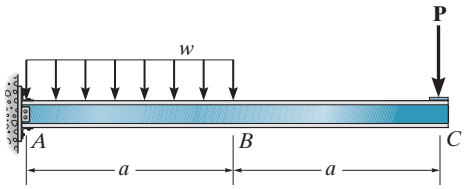
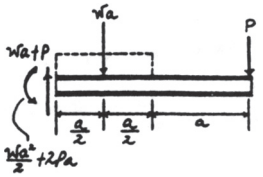
12–79.

Determine the slope and displacement at *C*. *EI* is constant.



SOLUTION

Support Reactions and Elastic Curve: As shown.

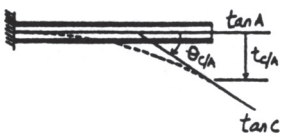


M/EI Diagrams: The *M/EI* diagrams for the uniform distributed load and concentrated load are drawn separately as shown.

Moment-Area Theorems: The slope at support *A* is zero. The slope at *C* is

$$\theta_C = |\theta_{C/A}| = \frac{1}{2}\left(-\frac{2Pa}{EI}\right)(2a) + \frac{1}{3}\left(-\frac{wa^2}{2EI}\right)(a)$$

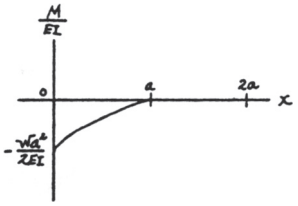
$$= \frac{a^2}{6EI}(12P + wa) \curvearrowright \quad \textbf{Ans.}$$

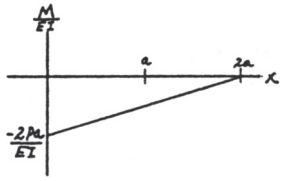


The displacement at *C* is

$$v_C = |t_{C/A}| = \frac{1}{2}\left(-\frac{2Pa}{EI}\right)(2a)\left(\frac{4}{3}a\right) + \frac{1}{3}\left(-\frac{wa^2}{2EI}\right)(a)\left(a + \frac{3}{4}a\right)$$

$$= \frac{a^3}{24EI}(64P + 7wa) \quad \downarrow \quad \textbf{Ans.}$$





Ans:

$$\theta_C = -\frac{a^2}{6EI}(12P + wa),$$

$$v_C = \frac{a^3}{24EI}(64P + 7wa)\downarrow$$

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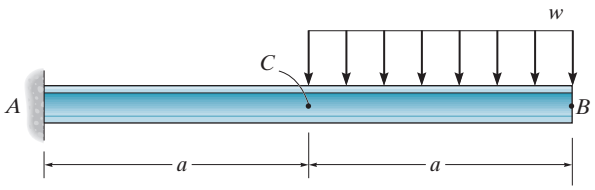
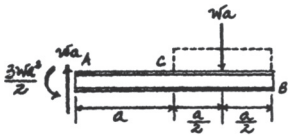
***12–80.**

Determine the slope at C and deflection at B. EI is constant.



SOLUTION

Support Reactions and Elastic Curve: As shown.

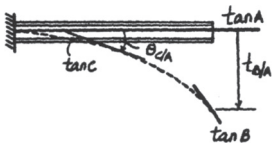


M/EI Diagram: As shown.

Moment-Area Theorems: The slope at support A is zero. The slope at C is

$$|\theta_C| = |\theta_{C/A}| = \frac{1}{2}\left(-\frac{wa^2}{EI}\right)(a) + \left(-\frac{wa^2}{2EI}\right)(a)$$

$$\theta_A = -\frac{wa^3}{EI} \qquad \textbf{Ans.}$$

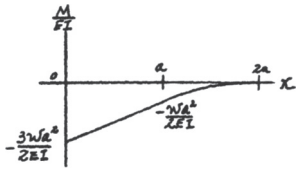


The displacement at B is

$$v_B = |t_{B/A}|$$

$$= \frac{1}{2}\left(-\frac{wa^2}{EI}\right)(a)\left(a + \frac{2}{3}a\right) + \left(-\frac{wa^2}{2EI}\right)(a)\left(a + \frac{a}{2}\right) + \frac{1}{3}\left(-\frac{wa^2}{2EI}\right)(a)\left(\frac{3}{4}a\right)$$

$$= \frac{41wa^4}{24EI} \downarrow \qquad \textbf{Ans.}$$



Ans:

$$\theta_A = -\frac{wa^3}{EI},$$

$$v_B = \frac{41wa^4}{24EI} \downarrow$$

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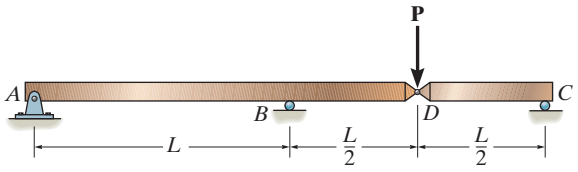
12–81.

The two bars are pin connected at D. Determine the slope at A and the deflection at D. EI is constant.



SOLUTION

$$t_{B/A} = \frac{1}{2}\left(\frac{-PL}{2EI}\right)(L)\left(\frac{L}{3}\right) = \frac{-PL^3}{12EI}$$

$$\theta_A = \frac{|t_{B/A}|}{L} = \frac{PL^2}{12EI}$$ **Ans.**

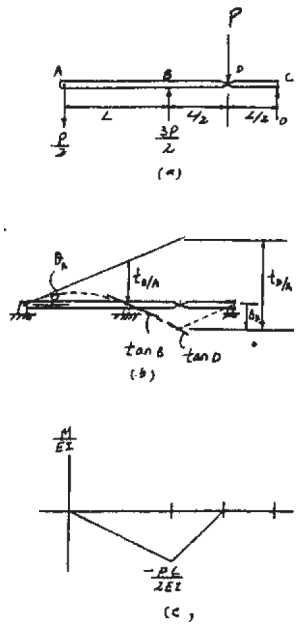
The Deflection:

$$t_{D/A} = \frac{1}{2}\left(\frac{-PL}{2EI}\right)(L)\left(\frac{L}{2} + \frac{L}{3}\right) + \frac{1}{2}\left(\frac{-PL}{2EI}\right)\left(\frac{L}{2}\right)\left(\frac{L}{3}\right)$$

$$= -\frac{PL^3}{4EI}$$

$$v_D = |t_{D/A}| - \left(\frac{\frac{3}{2}L}{L}\right)|t_{B/A}|$$

$$= \frac{PL^3}{4EI} - \frac{3}{2}\left(\frac{PL^3}{12EI}\right) = \frac{PL^3}{8EI}\downarrow$$ **Ans.**



Ans:

$$\theta_A = \frac{PL^2}{12EI}, v_D = \frac{PL^3}{8EI}\downarrow$$

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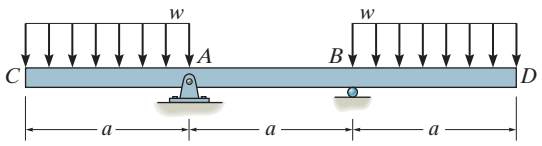
12–82.

Determine the maximum deflection of the beam. *EI* is constant.



SOLUTION

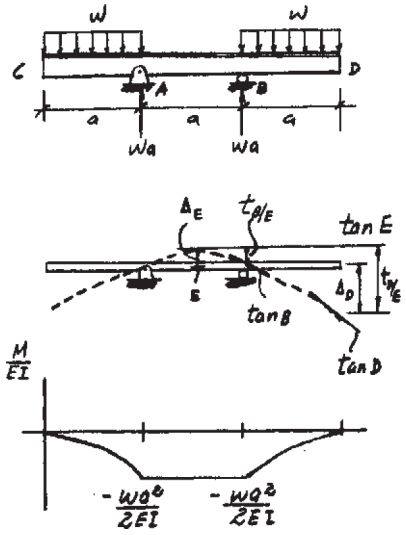
$$t_{B/E} = \left(\frac{-wa^2}{2EI}\right)\left(\frac{a}{2}\right)\left(\frac{a}{4}\right) = \frac{-wa^4}{16EI}$$

$$v_E = |t_{B/E}| = \frac{wa^4}{16EI}\uparrow$$

$$t_{D/E} = \left(\frac{-wa^2}{2EI}\right)\left(\frac{a}{2}\right)\left(a + \frac{a}{4}\right) + \frac{1}{3}\left(\frac{-wa^2}{2EI}\right)(a)\left(\frac{3a}{4}\right) = -\frac{7wa^4}{16EI}$$

$$v_D = |t_{D/E}| - |t_{B/E}| = \frac{7wa^4}{16EI} - \frac{wa^4}{16EI} = \frac{3wa^4}{8EI}$$

$$v_{\max} = v_D = \frac{3wa^4}{8EI}$$ **Ans.**



Ans:

$$v_{\max} = \frac{3wa^4}{8EI}$$

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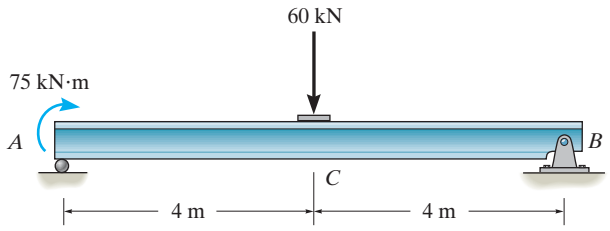
12–83. The W310 × 67 imply supported beam is made of A-36 steel and is subjected to the loading shown. Determine the deflection at its center *C*.



SOLUTION

$$(\Delta_C)_1 = \frac{PL^3}{48EI} = \frac{60(4^3)}{48EI} = \frac{80}{EI}\ \text{kN}\cdot\text{m}^3$$

$$\Delta_2(x) = \frac{Mx}{6EIL}(L^2 - x^2)$$

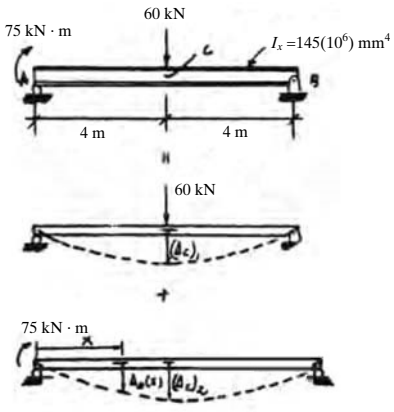
At point C, $x = \frac{L}{2}$

$$(\Delta_C)_2 = \frac{M\left(\frac{L}{2}\right)}{6EIL}\left[L^2 - \left(\frac{L}{2}\right)^2\right]$$

$$= \frac{ML^2}{16EI} = \frac{75(8)^2}{16EI} = \frac{300}{EI}\ \text{kN}\cdot\text{m}^3\ \downarrow$$

$$\Delta_C = (\Delta_C)_1 + (\Delta_C)_2 = \frac{80}{EI} + \frac{300}{EI} = \frac{380}{EI}\ \text{kN}\cdot\text{m}^3$$

$$= \frac{380(10^3)\ \text{N}\cdot\text{m}^3}{[200(10^9)\ \text{N/m}^2][145(10^{-6})\ \text{m}^4]} = 0.01310\ \text{m} = 13.1\ \text{mm}\ \downarrow$$ **Ans.**



Ans:

$\Delta_C = 13.1\ \text{mm}\ \downarrow$

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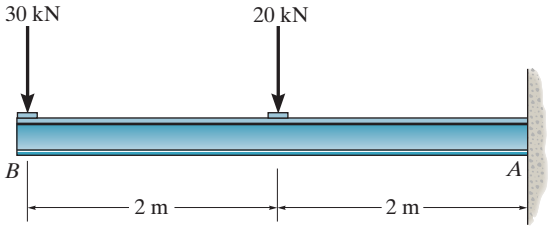
***12–84.** The W250 × 22 cantilevered beam is made of A-36 steel and is subjected to the loading shown. Determine the displacement at B and the slope at B.



SOLUTION

Using the table in appendix, the required slopes and deflections for each load case are computed as follow:

$$(\Delta_B)_1 = \frac{5PL^3}{48EI} = \frac{5(20)(4^3)}{48EI} = \frac{133.33\ \text{kN}\cdot\text{m}^3}{EI} \downarrow$$

$$(\theta_B)_1 = \frac{PL^2}{8EI} = \frac{20(4^2)}{8EI} = \frac{40\ \text{kN}\cdot\text{m}^2}{EI}$$

$$(\Delta_B)_2 = \frac{PL^3}{3EI} = \frac{30(4^3)}{3EI} = \frac{640\ \text{kN}\cdot\text{m}^3}{EI} \downarrow$$

$$(\theta_B)_2 = \frac{PL^2}{2EI} = \frac{30(4^2)}{2EI} = \frac{240\ \text{kN}\cdot\text{m}^2}{EI}$$

Then the slope and deflection at B are

$$\theta_B = (\theta_B)_1 + (\theta_B)_2$$

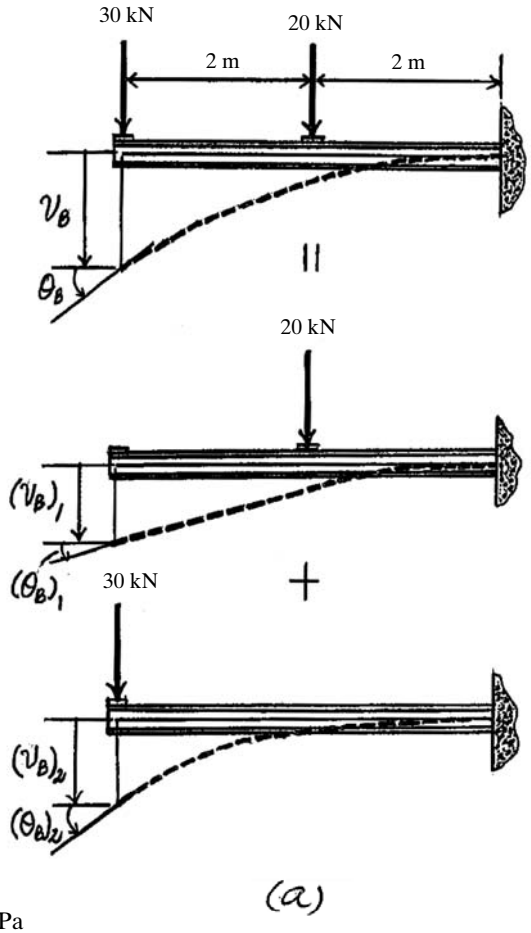
$$= \frac{40}{EI} + \frac{240}{EI}$$

$$= \frac{280\ \text{kN}\cdot\text{m}^2}{EI}$$

$$\Delta_B = (\Delta_B)_1 + (\Delta_B)_2$$

$$= \frac{133.33}{EI} + \frac{640}{EI}$$

$$= \frac{773.33\ \text{kN}\cdot\text{m}^3}{EI}$$



For A36 steel W250 × 22, $I = 28.8(10^6)\ \text{mm}^4 = 28.8(10^{-6})\ \text{m}^4$ and $E = 200\ \text{GPa}$

$$\theta_B = \frac{280(10^3)\ \text{N}\cdot\text{m}^2}{[200(10^9)\ \text{N/m}^2][28.8(10^{-6})\ \text{m}^4]} = 0.0486\ \text{rad}$$ **Ans.**

$$\Delta_B = \frac{773.33(10^3)\ \text{N}\cdot\text{m}^3}{[200(10^9)\ \text{N/m}^2][28.8(10^{-6})\ \text{m}^4]} = 0.1343\ \text{m} = 134\ \text{mm}$$ **Ans.**

Ans:

$\theta_B = 0.0486$ rad; $\Delta_B = 134$ mm

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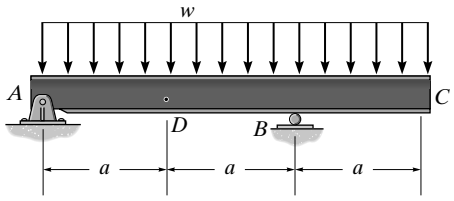
12–85. Determine the slope and deflection at end C of the overhang beam. EI is constant.



SOLUTION

Elastic Curves. The uniform distributed load on the beam is equivalent to the sum of the seperate loadings shown in Fig.*a*. The elastic curve for each seperate loading is shown Fig. *a*.

Method of Superposition. Using the table in the appendix, the required slopes and deflections are

$$(\theta_C)_1 = (\theta_B)_1 = \frac{wL^3}{24EI} = \frac{w(2a)^3}{24EI} = \frac{wa^3}{3EI}$$

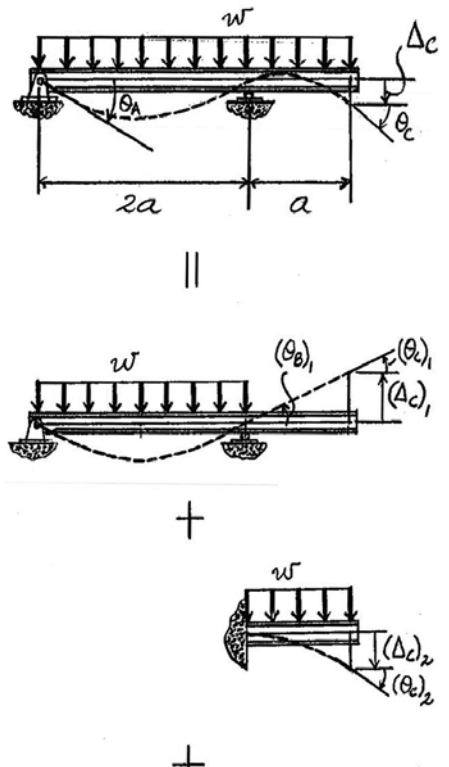
$$(\Delta_C)_1 = (\theta_B)_1(a) = \frac{wa^3}{3EI}(a) = \frac{wa^4}{3EI} \quad \uparrow$$

$$(\theta_C)_2 = \frac{wL^3}{6EI} = \frac{wa^3}{6EI}$$

$$(\Delta_C)_2 = \frac{wL^4}{8EI} = \frac{wa^4}{8EI} \quad \downarrow$$

$$(\theta_C)_3 = (\theta_B)_3 = \frac{M_O L}{3EI} = \frac{\left(\frac{wa^2}{2}\right)(2a)}{3EI} = \frac{wa^3}{3EI}$$

$$(\Delta_C)_3 = (\theta_B)_3\,(a) = \frac{wa^3}{3EI}(a) = \frac{wa^4}{3EI} \quad \downarrow$$



Then the slope and deflection of C are

$$\theta_C = (\theta_C)_1 + (\theta_C)_2 + (\theta_C)_3$$

$$= -\frac{wa^3}{3EI} + \frac{wa^3}{6EI} + \frac{wa^3}{3EI}$$

$$= \frac{wa^3}{6EI}$$ **Ans.**

$$\Delta_C = (\Delta_C)_1 + (\Delta_C)_2 + (\Delta_C)_3$$

$$= -\frac{wa^4}{3EI} + \frac{wa^4}{8EI} + \frac{wa^4}{3EI}$$

$$= \frac{wa^4}{8EI} \downarrow$$ **Ans.**

Ans:

$$\theta_C = \frac{wa^3}{6EI},\ \Delta_C = \frac{wa^4}{8EI} \downarrow$$

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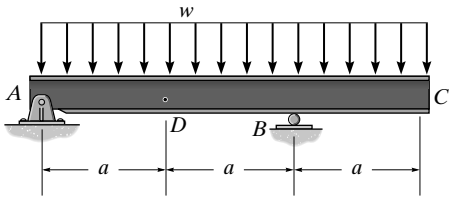
12–86. Determine the slope at A and the deflection at point D of the overhang beam. EI is constant.



SOLUTION

Elastic Curves. The uniform distributed load on the deformation of span AB is equivalent to the sum of the seperate loadings shown in Fig. a. The elastic curve for each seperate loading is shown in Fig. a.

Method of Superposition. Using the table in the appendix, the required slope and deflections are

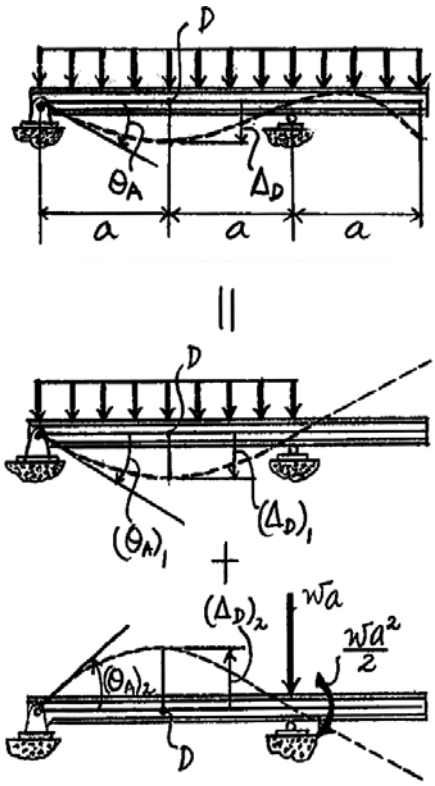
$$(\theta_A)_1 = \frac{wL^3}{24EI} = \frac{w(2a)^3}{24EI} = \frac{wa^3}{3EI}$$

$$(\Delta_D)_1 = \frac{5wL^4}{384EI} = \frac{5w(2a)^4}{384EI} = \frac{5wa^4}{24EI} \quad \downarrow$$

$$(\theta_A)_2 = \frac{M_O L}{6EI} = \frac{\frac{wa^2}{2}(2a)}{6EI} = \frac{wa^3}{6EI}$$

$$(\Delta_D)_2 = \frac{M_O x}{6EIL}\left(L^2 - x^2\right) = \frac{\left(\frac{wa^2}{2}\right)(a)}{6EI(2a)}\left[(2a)^2 - a^2\right]$$

$$= \frac{wa^4}{8EI} \quad \uparrow$$



(a)

Then the slope and deflection of point D are

$$\theta_A = (\theta_A)_1 + (\theta_A)_2$$

$$= \frac{wa^3}{3EI} - \frac{wa^3}{6EI} = \frac{wa^3}{6EI}$$ **Ans.**

$$\Delta_D = (\Delta_D)_1 + (\Delta_D)_2$$

$$= \frac{5wa^4}{24EI} - \frac{wa^4}{8EI} = \frac{wa^4}{12EI}\downarrow$$ **Ans.**

Ans:

$$\theta_A = \frac{wa^3}{6EI}, \Delta_D = \frac{wa^4}{12EI}\downarrow$$

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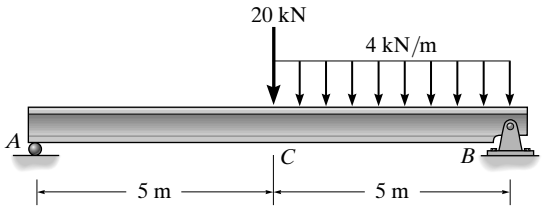
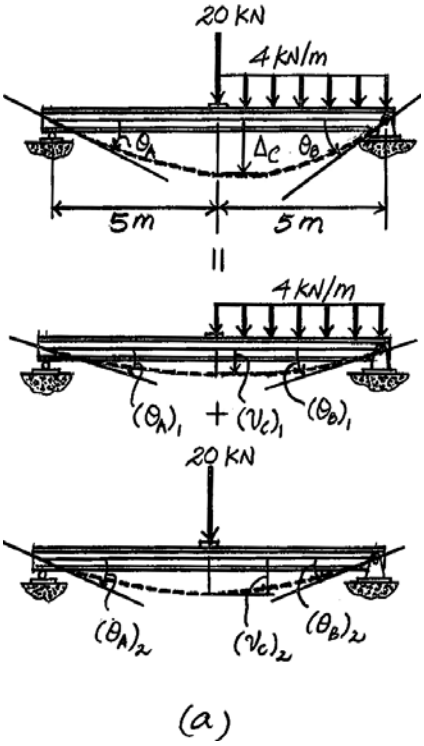
12–87. The simply supported beam is made of A-36 steel and is subjected to the loading shown. Determine the deflection at its center C. $I = 0.1457(10^{-3})\ \text{m}^4$.



SOLUTION

Using the table in appendix, the required deflections for each load case are computed as follow:



$$(v_C)_1 = \frac{5wL^4}{768EI} = \frac{5(4)\left(10^4\right)}{768\ EI}$$

$$= \frac{260.42\ \text{kN}\cdot\text{m}^3}{EI} \quad \downarrow$$

$$(v_C)_2 = \frac{PL^3}{48EI} = \frac{20N\left(10^3\right)}{48EI} = \frac{416.67\ \text{kN}\cdot\text{m}^3}{EI} \quad \downarrow$$

Then the deflection of point C is

$$v_C = (v_C)_1 + (v_C)_2$$

$$= \frac{260.42}{EI} + \frac{416.67}{EI}$$

$$= \frac{677.08\ \text{kN}\cdot\text{m}^3}{EI} \quad \downarrow$$

$$= 0.1457\left(10^{-3}\right)\text{m}^4$$

and $E = 200GPa$

$$\Delta_C = \frac{677.08\left(10^3\right)}{200\left(10^9\right)\left[0.1457\left(10^{-3}\right)\right]} = 0.0232\ \text{m} = 23.2\ \text{m} \downarrow$$ **Ans.**

Ans:

$\Delta_C = 23.2\ \text{m} \downarrow$

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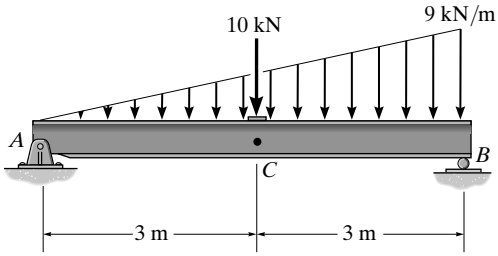
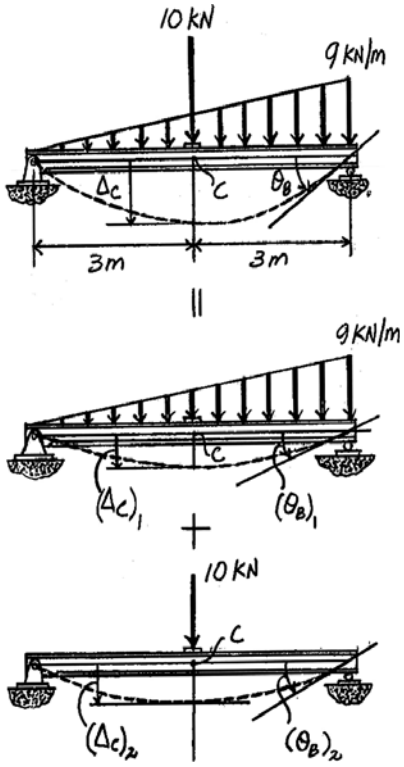
***12–88.** Determine the slope at B and the deflection at point C of the simply supported beam. $E = 200$ GPa and $I = 45.5(10^6)$ mm^4.



SOLUTION

Elastic Curves. The loading system on the beam is equivalent to the sum of the seperate loadings shown in Fig. *a*. The elastic curves for each loading are shown in Fig. *a*.



Method of Superposition. Using the table in the appendix, the required slope and deflections are

$$(\theta_B)_1 = \frac{w_O L^3}{45EI} = \frac{9(6^3)}{45EI} = \frac{43.2\,\text{kN}\cdot\text{m}^2}{EI}$$

$$(\Delta_C)_1 = \frac{w_O x}{360EIL}\left(3x^4 - 10L^2x^2 + 7L^4\right)$$

$$= \frac{9(3)}{360EI(6)}\left[3(3^4) - 10(6^2)(3^2) + 7(6^4)\right]$$

$$= \frac{75.9375\,\text{kN}\cdot\text{m}^3}{EI} \quad \downarrow$$

$$(\theta_B)_2 = \frac{PL^2}{16EI} = \frac{10(6^2)}{16EI} = \frac{22.5\,\text{kN}\cdot\text{m}^2}{EI}$$

$$(\Delta_C)_2 = \frac{PL^3}{48EI} = \frac{10(6^3)}{48EI} = \frac{45\,\text{kN}\cdot\text{m}^3}{EI} \quad \downarrow$$

Then the slope at B and deflection at C are

$$\theta_B = (\theta_B)_1 + (\theta_B)_2$$

$$= \frac{43.2}{EI} + \frac{22.5}{EI} = \frac{65.7\,\text{kN}\cdot\text{m}^2}{EI} = \frac{65.7(10^3)}{200(10^9)[45.5(10^{-6})]} = 0.00722\ \text{rad} \quad \textbf{Ans.}$$

$$\Delta_C = (\Delta_C)_1 + (\Delta_C)_2$$

$$= \frac{75.9375}{EI} + \frac{45}{EI} = \frac{120.9375\,\text{kN}\cdot\text{m}^3}{EI} = \frac{120.9375(10^3)}{200(10^9)[45.5(10^{-6})]}$$

$$= 0.01329\ \text{m} = 13.3\ \text{mm} \downarrow \quad \textbf{Ans.}$$

Ans:

$\theta_B = 0.00722$ rad;
$\Delta_C = 13.3$ mm $\downarrow$

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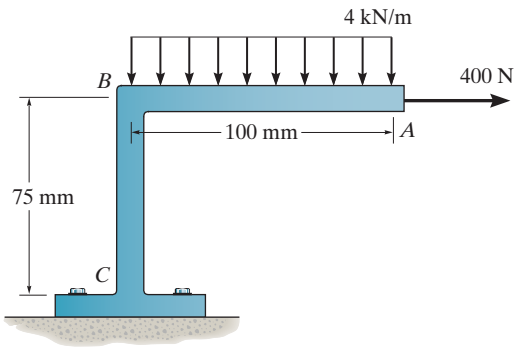
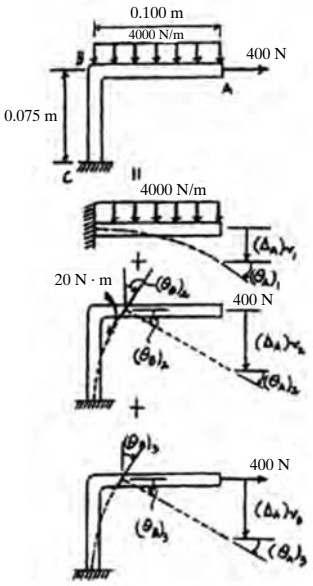
12–89. Determine the vertical deflection and slope at the end A of the bracket. Assume that the bracket is fixed supported at its base, and neglect the axial deformation of segment AB. EI is constant.



SOLUTION

Elastic Curve: The elastic curves for the concentrated load, uniform distibuted load, and couple moment are drawn separately as shown.



Method of Superposition: Using the table in Appendix C, the required slopes and displacements are

$$(\theta_A)_1 = \frac{wL_{AB}^3}{6EI} = \frac{4(10^3)(0.100)^3}{6EI} = \frac{0.6667\ \text{N}\cdot\text{m}^2}{EI}$$

$$(\theta_A)_2 = (\theta_B)_2 = \frac{M_0\, L_{BC}}{EI} = \frac{20(0.075)}{EI} = \frac{1.5\ \text{N}\cdot\text{m}^2}{EI}$$

$$(\theta_A)_3 = (\theta_B)_3 = \frac{PL_{BC}^2}{2EI} = \frac{400(0.075)^2}{2EI} = \frac{1.125\ \text{N}\cdot\text{m}^2}{EI}$$

$$(\Delta_A)_{v_1} = \frac{wL_{AB}^4}{8EI} = \frac{4(0.1)^4(10^3)}{8EI} = \frac{0.05\ \text{N}\cdot\text{m}^3}{EI}\ \downarrow$$

$$(\Delta_A)_{v_2} = (\theta_B)_2\,(L_{AB}) = \frac{1.5}{EI}(0.1) = \frac{0.15\ \text{N}\cdot\text{m}^3}{EI}\ \downarrow$$

$$(\Delta_A)_{v_3} = (\theta_B)_3\,(L_{AB}) = \frac{1.125}{EI}(0.1) = \frac{0.1125\ \text{N}\cdot\text{m}^3}{EI}\ \downarrow$$

The slope at A is

$$\theta_A = (\theta_A)_1 + (\theta_A)_2 + (\theta_A)_3$$
$$= \frac{0.6667}{EI} + \frac{1.5}{EI} + \frac{1.125}{EI}$$
$$= \frac{3.29\ \text{N}\cdot\text{m}^2}{EI}$$

Ans.

The vertical displacement at A is

$$(\Delta_A)_v = (\Delta_A)_{v_1} + (\Delta_A)_{v_2}\,(\Delta_A)_{v_3}$$
$$= \frac{0.05}{EI} + \frac{0.15}{EI} + \frac{0.1125}{EI}$$
$$= \frac{0.3125\ \text{N}\cdot\text{m}^3}{EI}\ \downarrow$$

Ans.

Ans:

$$\theta_A = \frac{3.29\ \text{N}\cdot\text{m}^2}{EI},\ (\Delta_A)_v = \frac{0.3125\ \text{N}\cdot\text{m}^3}{EI}\ \downarrow$$

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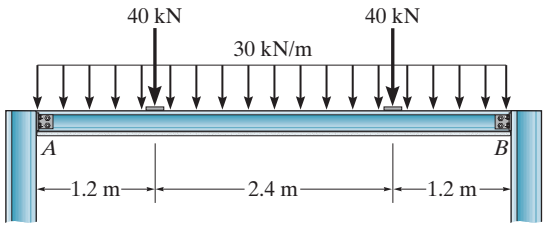
12–90. The simply supported beam carries a uniform load of 30 kN/m. Code restrictions, due to a plaster ceiling, require the maximum deflection not to exceed 1/360 of the span length. Select the lightest-weight A-36 steel wide-flange beam from Appendix B that will satisfy this requirement and safely support the load. The allowable bending stress is σ_{allow} = 168 MPa and the allowable shear stress is τ_{allow} = 100 MPa. Assume A is a pin and B a roller support.



$M_{max} = 134.4 \text{ kN} \cdot \text{m}$

Strength criterion:

$$\sigma_{allow} = \frac{M}{S_{req'd}}; \quad 168(10^6) = \frac{134.4(10^3)}{S_{req'd}}$$

$$S_{req'd} = 0.8(10^{-3}) \text{ m}^3 = 800(10^3) \text{ mm}^3$$

Choose W460 × 52 $S = 942(10^3) \text{ mm}^3$ t_w = 7.62 mm, d = 450 mm, I = $212(10^6) \text{ mm}^4$

$$\tau_{allow} = \frac{V}{A_{web}}$$

$$100 \geq \frac{112(10^3)}{(0.00749)(0.403)} = 37.1 \text{ MPa} \quad \text{O.K.}$$

Deflection criterion:

Maximum is at center.

$$v_{max} = \frac{5wL^4}{384EI} + 2\left[\frac{Pbx}{6EIL}(L^2 - b^2 - x^2)\right]$$

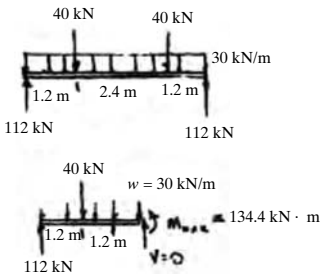
$$= \frac{5(30)(4.8)^4}{384EI} + 2\left\{\left[\frac{40(1.2)(2.4)}{6EI(4.8)}\right](4.8^2 - 1.2^2 - 2.4^2)\right\}$$

$$= \frac{334.08 \text{ kN} \cdot \text{m}^3}{EI} = \frac{334.08(10^3) \text{ N} \cdot \text{m}^3}{[200(10^9) \text{ N/m}^2][212(10^{-6}) \text{ m}^4]}$$

$$= 0.007879 \text{ m} = 7.88 \text{ mm} < \frac{1}{360}[4.8(10^3)]$$

$$= 13.3 \text{ mm} \quad \text{(O.K.)}$$

Use W460 × 52 **Ans.**



Ans:

Use W460 × 52

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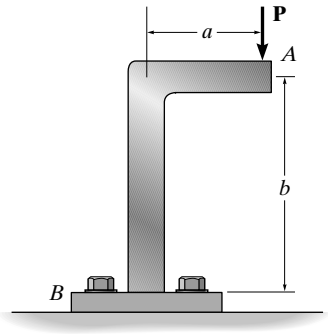
12–91. Determine the vertical deflection at the end A of the bracket. Assume that the bracket is fixed supported at its base B and neglect axial deflection. EI is constant.



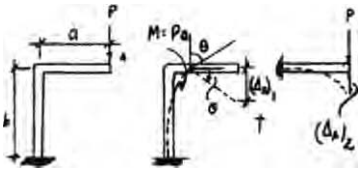
SOLUTION

$$\theta = \frac{ML}{EI} = \frac{Pab}{EI}$$

$$(\Delta_A)_1 = \theta(a) = \frac{Pa^2b}{EI}$$

$$(\Delta_A)_2 = \frac{PL^3}{3EI} = \frac{Pa^3}{3EI}$$

$$\Delta_A = (\Delta_A)_1 + (\Delta_A)_2 = \frac{Pa^2b}{EI} + \frac{Pa^3}{3EI} = \frac{Pa^2\,(3b + a)}{3EI}$$ **Ans.**



Ans:

$$\Delta_A = \frac{Pa^2\,(3b + a)}{3EI}$$

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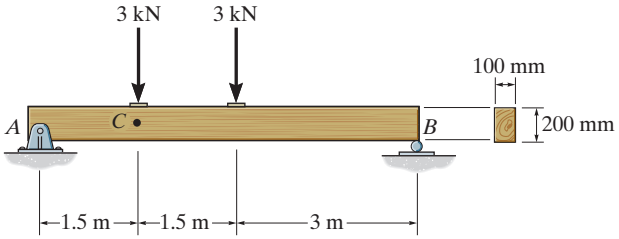
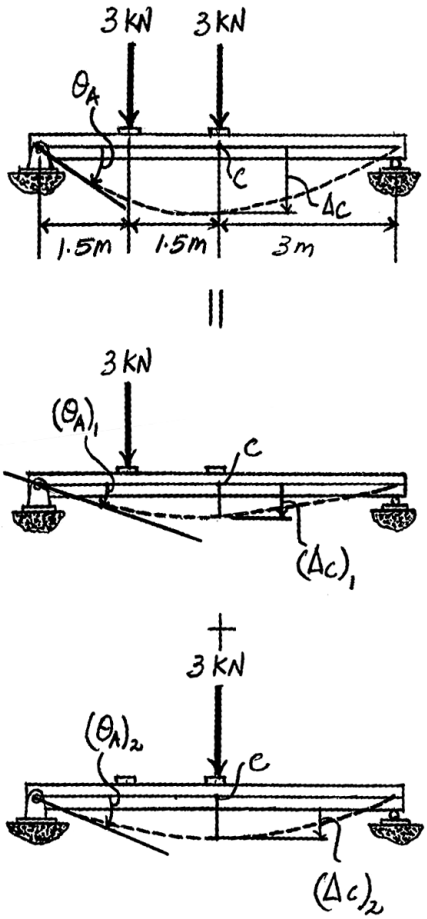
***12–92.** Determine the slope at A and the deflection at point C of the simply supported beam. The modulus of elasticity of the wood is $E = 10$ GPa.



SOLUTION

Elastic Curves. The two concentrated forces **P** are applied separately on the beam and the resulting elastic curves are shown in Fig. *a*.



Method of Superposition. Using the table in the appendix, the required slopes and deflections are

$$(\theta_A)_1 = \frac{Pab(L + b)}{6EIL} = \frac{3(1.5)(4.5)(6 + 4.5)}{6EI(6)} = \frac{5.90625\ \text{kN}\cdot\text{m}^2}{EI}$$

$$(\Delta_C)_1 = \frac{Pbx}{6EIL}\left(L^2 - b^2 - x^2\right) = \frac{3(4.5)(1.5)}{6EI(6)}\left(6^2 - 4.5^2 - 1.5^2\right)$$

$$= \frac{7.594\ \text{kN}\cdot\text{m}^3}{EI} \quad \downarrow$$

$$(\theta_A)_2 = \frac{PL^2}{16EL} = \frac{3\left(6^2\right)}{16EI} = \frac{6.75\,\text{kN}\cdot\text{m}^2}{EI}$$

$$(\Delta_C)_2 = \frac{Px}{48EI}\left(3L^2 - 4x^2\right) = \frac{3(1.5)}{48EI}\left(3(6)^2 - 4(1.5)^2\right) = \frac{9.281}{EI}$$

Then the slope at A and deflection at C are

$$\theta_A = (\theta_A)_1 + (\theta_A)_2$$

$$= \frac{5.90625}{EI} + \frac{6.75}{EI}$$

$$= \frac{12.65625\ \text{kN}\cdot\text{m}^2}{EI} = \frac{12.6525\left(10^3\right)}{10\left(10^9\right)\left[\frac{1}{12}(0.1)\left(0.2^3\right)\right]} = 0.0190\ \text{rad} \quad \searrow \qquad \textbf{Ans.}$$

and

$$\Delta_C = (\Delta_C)_1 + (\Delta_C)_2$$

$$= \frac{7.594}{EI} + \frac{9.281}{EI} = \frac{16.88\left(10^3\right)}{10\left(10^9\right)\left[\frac{1}{12}(0.1)\left(0.2^3\right)\right]} = 0.0253\ \text{m} = 25.3\ \text{mm} \qquad \textbf{Ans.}$$

Ans:

$\theta_A = 0.0190$ rad; $\Delta_C = 25.3$ mm

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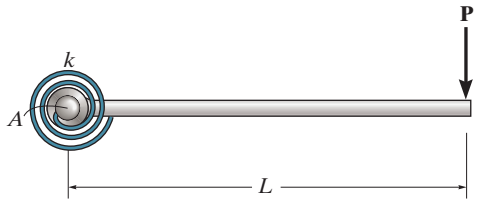
12–93.

The rod is pinned at its end A and attached to a torsional spring having a stiffness k, which measures the torque per radian of rotation of the spring. If a force **P** is always applied perpendicular to the end of the rod, determine the displacement of the force. EI is constant.



SOLUTION

In order to maintain equilibrium, the rod has to rotate through an angle θ.

$$\zeta+\Sigma M_A = 0; \quad k\theta - PL = 0; \quad \theta = \frac{PL}{k}$$

Hence,

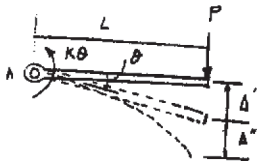
$$v' = L\theta = L\left(\frac{PL}{k}\right) = \frac{PL^2}{k}$$

Elastic deformation:

$$v'' = \frac{PL^3}{3EI}$$

Therefore,

$$v = v' + v'' = \frac{PL^2}{k} + \frac{PL^3}{3EI} = PL^2\left(\frac{1}{k} + \frac{L}{3EI}\right)$$ **Ans.**



Ans:

$$v = PL^2\left(\frac{1}{k} + \frac{L}{3EI}\right)$$

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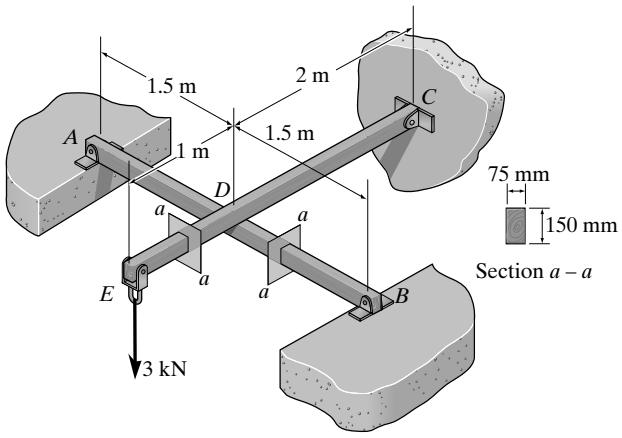
12–94. Determine the deflection at end E of beam CDE. The beams are made of wood having a modulus of elasticity of $E = 10$ GPa.



SOLUTION

Method of Superposition. Referring to the table in the appendix, the deflection of point D is

$$\Delta_D = \frac{PL^3}{48EI} = \frac{4.5\left(3^3\right)}{48EI} = \frac{2.53125\ \text{kN}\cdot\text{m}^3}{EI} \quad \downarrow$$

Subsequently,

$$(\Delta_E)_1 = \Delta_D\left(\frac{3}{2}\right) = \frac{2.53125}{EI}\left(\frac{3}{2}\right) = \frac{3.796875\ \text{kN}\cdot\text{m}^3}{EI} \downarrow$$

Also,

$$(\Delta_E)_2 = \frac{PL^3}{3EI} = \frac{3\left(1^3\right)}{3EI} = \frac{1\ \text{kN}\cdot\text{m}^3}{EI} \downarrow$$

$$(\theta_D)_3 = \frac{M_O L}{3EI} = \frac{3(2)}{3EI} = \frac{2\ \text{kN}\cdot\text{m}^2}{EI}$$

$$(\Delta_E)_3 = (\theta_D)_3 L = \frac{2}{EI}(1) = \frac{2\ \text{kN}\cdot\text{m}^3}{EI} \downarrow$$

Thus, the deflection of end E is

$$\Delta_E = (\Delta_E)_1 + (\Delta_E)_2 + (\Delta_E)_3$$

$$= \frac{3.796875}{EI} + \frac{1}{EI} + \frac{2}{EI} = \frac{6.796875\text{kN}\cdot\text{m}^3}{EI} = \frac{6.796875\left(10^3\right)}{10\left(10^9\right)\left[\frac{1}{12}(0.075)\left(0.15^3\right)\right]}$$

$$= 0.03222\ \text{m} = 32.2\ \text{mm} \downarrow \qquad \textbf{Ans.}$$

Ans:

$\Delta_E = 32.2$ mm $\downarrow$

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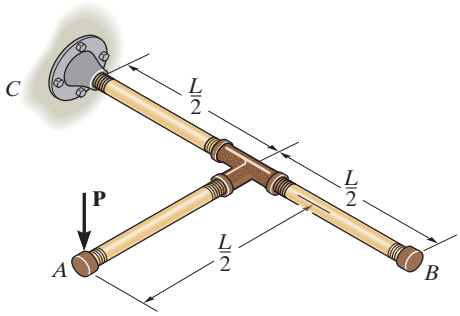
12–95.

The pipe assembly consists of three equal-sized pipes with flexibility stiffness *EI* and torsional stiffness *GJ*. Determine the vertical deflection at *A*.



SOLUTION

$$v_D = \frac{P\left(\frac{L}{2}\right)^3}{3EI} = \frac{PL^3}{24EI}$$

$$(v_A)_1 = \frac{P\left(\frac{L}{2}\right)^3}{3EI} = \frac{PL^3}{24EI}$$

$$\theta = \frac{TL}{GJ} = \frac{(PL/2)\left(\frac{L}{2}\right)}{GJ} = \frac{PL^2}{4GJ}$$

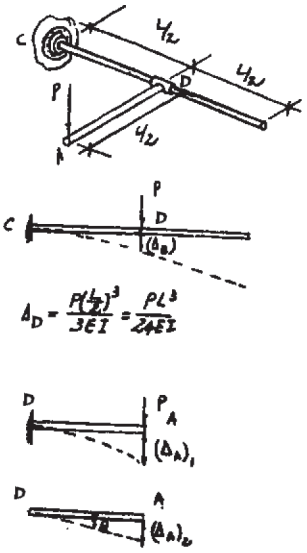
$$(v_A)_2 = \theta\left(\frac{L}{2}\right) = \frac{PL^3}{8GJ}$$

$$v_A = v_D + (v_A)_1 + (v_A)_2$$

$$= \frac{PL^3}{24EI} + \frac{PL^3}{24EI} + \frac{PL^3}{8GJ}$$

$$= PL^3\left(\frac{1}{12EI} + \frac{1}{8GJ}\right)\downarrow$$

Ans.



Ans:

$$v_A = PL^3\left(\frac{1}{12EI} + \frac{1}{8GJ}\right)\downarrow$$

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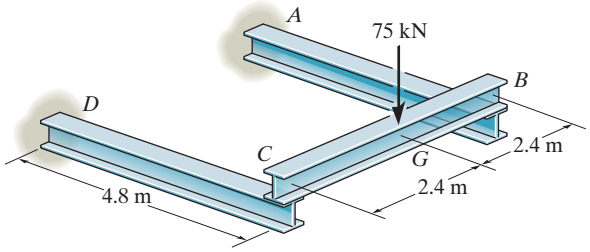
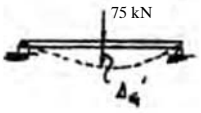
***12–96.** The framework consists of two A-36 steel cantilevered beams *CD* and *BA* and a simply supported beam *CB*. If each beam is made of steel and has a moment of inertia about its principal axis of $I_x = 46(10^6)\ \text{mm}^4$, determine the deflection at the center *G* of beam *CB*.



SOLUTION



$$\Delta_C = \frac{PL^3}{3EI} = \frac{37.5(4.8^3)}{3EI} = \frac{1382.4\ \text{kN}\cdot\text{m}^3}{EI} \quad \downarrow$$

$$\Delta_G' = \frac{PL^3}{48EI} = \frac{75(4.8^3)}{48EI} = \frac{172.8\ \text{kN}\cdot\text{m}^3}{EI} \quad \downarrow$$

$$\Delta_G = \Delta_C + \Delta_G'$$

$$= \frac{1382.4}{EI} + \frac{172.8}{EI} = \frac{1555.2\ \text{kN}\cdot\text{m}^3}{EI}$$

$$= \frac{1555.2(10^3)\ \text{N}\cdot\text{m}^3}{[200(10^9)\ \text{N/m}^2][46(10^{-6})\ \text{m}^4]} = 0.1690\ \text{m} = 169\ \text{mm} \quad \downarrow$$

Ans.

Ans:

$\Delta_G = 169\ \text{mm} \downarrow$

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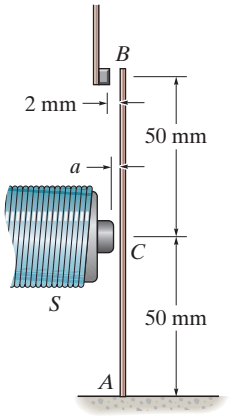
12–97.

The relay switch consists of a thin metal strip or armature AB that is made of red brass C83400 and is attracted to the solenoid S by a magnetic field. Determine the smallest force F required to attract the armature at C in order that contact is made at the free end B. Also, what should the distance a be for this to occur? The armature is fixed at A and has a moment of inertia of $I = 0.18(10^{-12})\ \text{m}^4$.



SOLUTION

Elastic Curve: As shown.

Method of Superposition: Using the table in Appendix C, the required slopes and displacements are

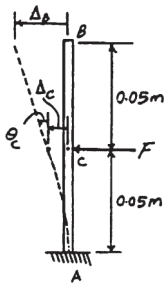
$$\theta_C = \frac{PL_{AC}^2}{2EI} = \frac{F(0.05^2)}{2EI} = \frac{0.00125F\,\text{m}^2}{EI}$$

$$\Delta_C = \frac{PL_{AC}^3}{3EI} = \frac{F(0.05^3)}{3EI} = \frac{41.667(10^{-6})F\,\text{m}^3}{EI} \qquad \textbf{(1)}$$

$$\Delta_B = \Delta_C + \theta_C L_{CB}$$

$$= \frac{41.667(10^{-6})F}{EI} + \frac{0.00125(10^{-6})F}{EI}(0.05)$$

$$= \frac{104.167(10^{-6})F\,\text{m}^3}{EI} \qquad \textbf{(2)}$$



Required the displacement $\Delta_B = 0.002$ m. From Eq. (2),

$$0.002 = \frac{104.167(10^{-6})F}{101(10^9)(0.18)(10^{-12})}$$

$$F = 0.349056\ \text{N} = 0.349\ \text{N} \qquad \textbf{Ans.}$$

From Eq. (1),

$$a = \Delta_C = \frac{41.667(10^{-6})(0.349056)}{101(10^9)(0.18)(10^{-12})}$$

$$= 0.800(10^{-3})\ \text{m} = 0.800\ \text{mm} \qquad \textbf{Ans.}$$

Ans:
$F = 0.349$ N,
$a = 0.800$ mm

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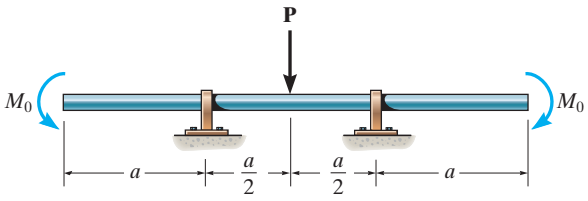
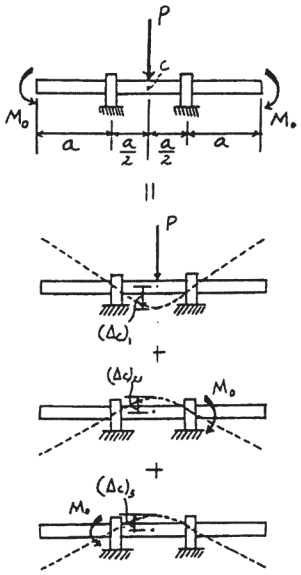
12–98.

Determine the moment M_0 in terms of the load P and dimension a so that the deflection at the center of the shaft is zero. EI is constant.



SOLUTION



Elastic Curve: The elastic curves for the concentrated load and couple moment are drawn separately as shown.

Method of Superposition: Using the table in Appendix C, the required slope and displacement are

$$(\Delta_C)_1 = \frac{Pa^3}{48EI}\downarrow$$

$$(\Delta_C)_2 = (\Delta_C)_3 = \frac{M_0 x}{6EIL}\left(x^2 - 3Lx + 2L^2\right)$$

$$= \frac{M_0\left(\frac{a}{2}\right)}{6EIa}\left[\left(\frac{a}{2}\right)^2 - 3(a)\left(\frac{a}{2}\right) + 2a^2\right]$$

$$= \frac{M_0 a^2}{16EI}\uparrow$$

Require the displacement at C to equal zero.

$$(+\uparrow) \qquad \Delta_C = 0 = (\Delta_C)_1 + (\Delta_C)_2 + (\Delta_C)_3$$

$$0 = -\frac{Pa^3}{48EI} + \frac{M_0 a^2}{16EI} + \frac{M_0 a^2}{16EI}$$

$$M_0 = \frac{Pa}{6} \qquad \textbf{Ans.}$$

Ans:

$$M_0 = \frac{Pa}{6}$$

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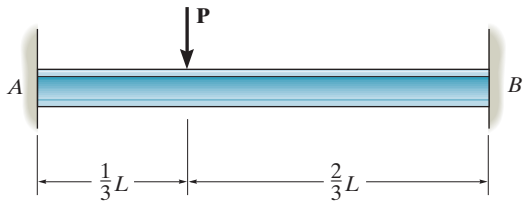
12–99.

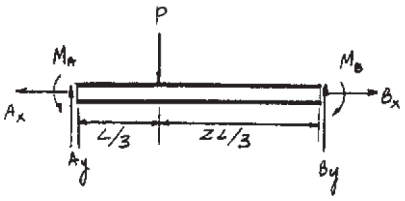
Determine the reactions at the supports A and B, then draw the shear and moment diagram. EI is constant. Neglect the effect of axial load.



SOLUTION

$$\circlearrowleft + \Sigma M_A = 0; \qquad M_A + B_y L - P\left(\frac{L}{3}\right) - M_B = 0 \tag{1}$$

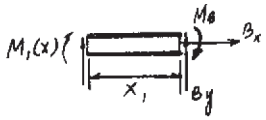
$$+ \uparrow \Sigma F_y = 0; \qquad A_y + B_y - P = 0 \tag{2}$$

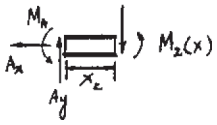


Moment Functions:

$$M_1(x_1) = B_y x_1 - M_B$$

$$M_2(x_2) = A_y x_2 - M_A$$





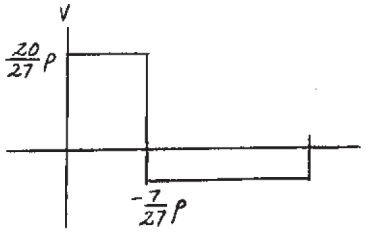
Slope and Elastic Curve:

$$EI\frac{d^2v}{dx^2} = M(x)$$

For $M_1(x) = B_y x_1 - M_B$; $\qquad EI\frac{d^2v_1}{dx_1^2} = B_y x_1 - M_B$

$$EI\frac{dv_2}{dx_2} = \frac{B_y x_1^2}{2} - M_B x_1 + C_1 \tag{3}$$

$$EIv_1 = \frac{B_y x_1^3}{6} - \frac{M_B x_1^2}{2} + C_1 x + C_2 \tag{4}$$

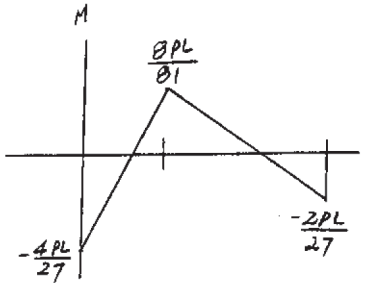


For $M_2(x) = A_y x_2 - M_A$

$$EI\frac{d^2v_2}{dx_2^2} = A_y x_2 - M_A$$

$$EI\frac{dv_2}{dx_2^2} = \frac{A_y x_2^2}{2} - M_A x_2 + C_3 \tag{5}$$

$$EIv_2 = \frac{A_y x_2^3}{6} - \frac{M_A x_2^3}{2} + C_3 x_2 + C_4 \tag{6}$$



Boundary Conditions:

At $x_1 = 0$, $\quad \frac{dv_1}{dx_1} = 0$

From Eq. (3),

$0 = 0 - 0 + C_1; \quad C_1 = 0$

At $x_1 = 0, \quad v_1 = 0$

From Eq. (4),

$0 = 0 - 0 + 0 + C_2; \quad C_2 = 0$

Similarly, from Eqs. (5) and (6), $C_3 = C_4 = 0$.

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12–99. Continued

At $x_1 = \frac{2}{3}L, x_2 = \frac{1}{3}L, v_1 = v_2$ and $\frac{dv_1}{dx_1} = -\frac{dv_2}{dx_2}$.

From Eqs. (4) and (6),

$$\frac{B_y}{6}\left(\frac{2}{3}L\right)^3 - \frac{M_B}{2}\left(\frac{2}{3}L\right)^2 = \frac{A_y}{6}\left(\frac{1}{3}L\right)^3 - \frac{M_A}{2}\left(\frac{1}{3}L\right)^2$$

$$8B_yL - 36M_B = A_yL - 9M_A \qquad \textbf{(7)}$$

From Eqs. (3) and (5),

$$\frac{B_y}{2}\left(\frac{2}{3}L\right)^2 - M_B\left(\frac{2}{3}L\right) = -\frac{A_y}{2}\left(\frac{1}{3}L\right)^2 + M_A\left(\frac{1}{3}L\right)$$

$$4B_yL - 12M_B = -A_yL + 6M_A \qquad \textbf{(8)}$$

Solving Eqs. (1), (2), (7) and (8) simultaneously,

$$A_y = \frac{20}{27}P \qquad \textbf{Ans.}$$

$$M_A = \frac{4}{27}PL \qquad \textbf{Ans.}$$

$$B_y = \frac{7}{27}P \qquad \textbf{Ans.}$$

$$M_B = \frac{2}{27}PL \qquad \textbf{Ans.}$$

Ans:
$A_x = B_x = 0,$
$A_y = \frac{20}{27}P,$
$M_A = \frac{4}{27}PL,$
$B_y = \frac{7}{27}P,$
$M_B = \frac{2}{27}PL$

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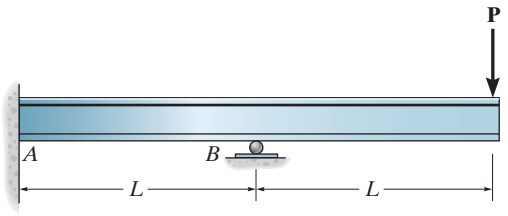
***12–100.**

Determine the reactions at the supports, then draw the shear and moment diagram. *EI* is constant.



SOLUTION

Support Reactions: FBD(a).

$$\stackrel{+}{\rightarrow}\Sigma F_x = 0; \qquad A_x = 0 \qquad \textbf{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \qquad B_y - A_y - P = 0 \qquad \textbf{(1)}$$

$$\circlearrowleft + \Sigma M_B = 0; \qquad A_y L - M_A - PL = 0 \qquad \textbf{(2)}$$

Moment Functions: FBD(b) and (c).

$$M(x_1) = -Px_1$$

$$M(x_2) = M_A - A_y x_2$$

Slope and Elastic Curve:

$$EI\frac{d^2v}{dx^2} = M(x)$$

For $M(x_1) = -Px_1$,

$$EI\frac{d^2v_1}{dx_1^2} = -Px_1$$

$$EI\frac{dv_1}{dx_1} = -\frac{P}{2}x_1^2 + C_1 \qquad \textbf{(3)}$$

$$EI\ v_1 = -\frac{P}{6}x_1^3 + C_1x_1 + C_2 \qquad \textbf{(4)}$$

For $M(x_2) = M_A - A_y x_2$,

$$EI\frac{d^2v_2}{dx_2^2} = M_A - A_y x_2$$

$$EI\frac{dv_2}{dx_2} = M_A x_2 - \frac{A_y}{2}x_2^2 + C_3 \qquad \textbf{(5)}$$

$$EI\ v_2 = \frac{M_A}{2}x_2^2 - \frac{A_y}{6}x_2^3 + C_3x_2 + C_4 \qquad \textbf{(6)}$$

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***12–100. Continued**

Boundary Conditions:

$v_2 = 0$ at $x_2 = 0$. From Eq. (6), $C_4 = 0$

$\frac{dv_2}{dx_2} = 0$ at $x_2 = 0$. From Eq. (5), $C_3 = 0$

$v_2 = 0$ at $x_2 = L$. From Eq. (6),

$$0 = \frac{M_A L^2}{2} - \frac{A_y L^3}{6} \qquad \textbf{(7)}$$

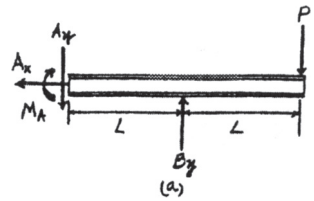
Solving Eqs. (2) and (7) yields,

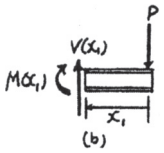
$$M_A = \frac{PL}{2} \qquad A_y = \frac{3P}{2} \qquad \textbf{Ans.}$$

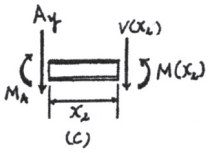
Substituting the value of A_y into Eqs. (1),

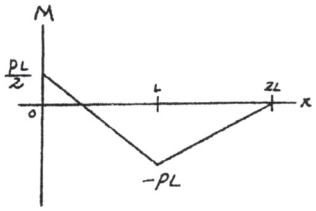
$$B_y = \frac{5P}{2} \qquad \textbf{Ans.}$$

Note: The other boundary and continuity condition can be used to determine the constants C_1 and C_2 which are not needed here.









Ans:

$M_A = \frac{PL}{2}$,

$A_y = \frac{3P}{2}$,

$B_y = \frac{5P}{2}$

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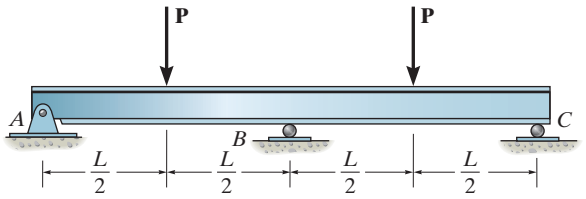
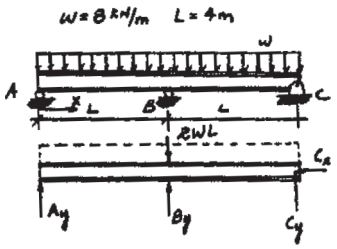
12–101.

Determine the reactions at the supports A, B, and C, then draw the shear and moment diagrams. EI is constant.



SOLUTION

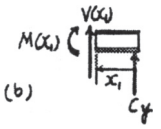


Support Reactions: FBD(a).

$$\stackrel{+}{\rightarrow}\Sigma F_x = 0; \qquad A_x = 0 \qquad \textbf{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \qquad A_y + B_y + C_y - 2P = 0 \qquad \textbf{(1)}$$

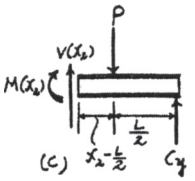
$$\circlearrowleft +\Sigma M_A = 0; \qquad B_y L + C_y(2L) - P\left(\frac{L}{2}\right) - P\left(\frac{3L}{2}\right) = 0 \qquad \textbf{(2)}$$



Moment Function: FBD(b) and (c).

$$M(x_1) = C_y x_1$$

$$M(x_2) = C_y x_2 - P x_2 + \frac{PL}{2}$$



Slope and Elastic Curve:

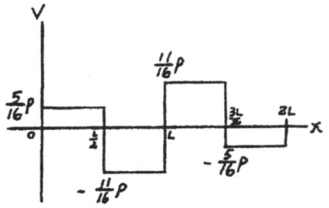
$$EI\frac{d^2v}{dx^2} = M(x)$$

For $M(x_1) = C_y x_1$,

$$EI\frac{d^2v_1}{dx_1^2} = C_y x_1$$

$$EI\frac{dv_1}{dx_1} = \frac{C_y}{2}x_1^2 + C_1 \qquad \textbf{(3)}$$

$$EI\,v_1 = \frac{C_y}{6}x_1^3 + C_1 x_1 + C_2 \qquad \textbf{(4)}$$

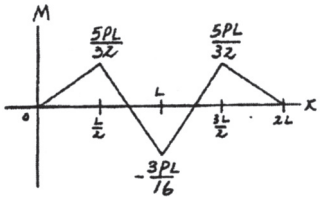


For $M(x_2) = C_y x_2 - P x_2 + \frac{PL}{2}$,

$$EI\frac{d^2v_2}{dx_2^2} = C_y x_2 - P x_2 + \frac{PL}{2}$$

$$EI\frac{dv_2}{dx_2} = \frac{C_y}{2}x_2^2 - \frac{P}{2}x_2^2 + \frac{PL}{2}x_2 + C_3 \qquad \textbf{(5)}$$

$$EI\,v_2 = \frac{C_y}{6}x_2^3 - \frac{P}{6}x_2^3 + \frac{PL}{4}x_2^2 + C_3 x_2 + C_4 \qquad \textbf{(6)}$$



Boundary Conditions:

$v_1 = 0$ at $x_1 = 0$. From Eq. (4), $C_2 = 0$

Due to symmetry, $\frac{dv_2}{dx_2} = 0$ at $x_2 = L$. From Eq. (5),

$$0 = \frac{C_y L^2}{2} - \frac{PL^2}{2} + \frac{PL^2}{2} + C_3 \qquad C_3 = -\frac{C_y L^2}{2}$$

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12–101. Continued

$v_2 = 0$ at $x_2 = L$. From Eq. (6),

$$0 = \frac{C_y L^3}{6} - \frac{PL^3}{6} + \frac{PL^3}{4} + \left(-\frac{C_y L^2}{2}\right)L + C_4$$

$$C_4 = \frac{C_y L^3}{3} - \frac{PL^3}{12}$$

Continuity Conditions:

At $x_1 = x_2 = \frac{L}{2}, \frac{dv_1}{dx_1} = \frac{dv_2}{dx_2}$. From Eqs. (3) and (5),

$$\frac{C_y}{2}\left(\frac{L}{2}\right)^2 + C_1 = \frac{C_y}{2}\left(\frac{L}{2}\right)^2 - \frac{P}{2}\left(\frac{L}{2}\right)^2 + \frac{PL}{2}\left(\frac{L}{2}\right) - \frac{C_y L^2}{2}$$

$$C_1 = \frac{PL^2}{8} - \frac{C_y L^2}{2}$$

At $x_1 = x_2 = \frac{L}{2}, v_1 = v_2$. From Eqs. (4) and (6),

$$\frac{C_y}{6}\left(\frac{L}{2}\right)^3 + \left(\frac{PL^2}{8} - \frac{C_y L^2}{2}\right)\left(\frac{L}{2}\right)$$

$$= \frac{C_y}{6}\left(\frac{L}{2}\right)^3 - \frac{P}{6}\left(\frac{L}{2}\right)^3 + \frac{PL}{4}\left(\frac{L}{2}\right)^2 + \left(-\frac{C_y L^2}{2}\right)\left(\frac{L}{2}\right) + \frac{C_y L^3}{3} - \frac{PL^3}{12}$$

$$C_y = \frac{5}{16}P \qquad \textbf{Ans.}$$

Substituting C_y into Eqs. (1) and (2),

$$B_y = \frac{11}{8}P \qquad A_y = \frac{5}{16}P \qquad \textbf{Ans.}$$

Ans:

$A_x = 0, C_y = \frac{5}{16}P, B_y = \frac{11}{8}P, A_y = \frac{5}{16}P$

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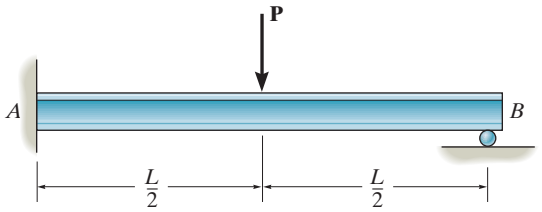
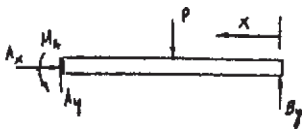
12–102.

Determine the reactions at the supports A and B, then draw the shear and moment diagrams. EI is constant.



SOLUTION



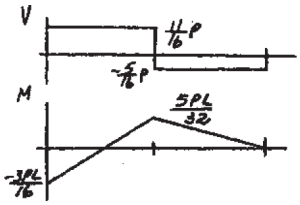
$\pm\rightarrow \Sigma F_x = 0; \quad A_x = 0$ **Ans.**

$+\uparrow \Sigma F_y = 0 \quad A_y + B_y - P = 0$

$$A_y = P - B_y \qquad (1)$$

$\circlearrowleft +\Sigma M_A = 0 \quad M_A + B_y(L) - P(L/2) = 0$

$$M_A = \frac{PL}{2} - B_y L \qquad (2)$$



Bending Moment $M(x)$:

$$M(x) = -(-B_y)\langle x - 0\rangle - P\left\langle x - \frac{L}{2}\right\rangle = B_y x - P\left\langle x - \frac{L}{2}\right\rangle$$

Elastic Curve and Slope:

$$EI\frac{d^2v}{dx^2} = M(x)$$

$$EI\frac{d^2v}{dx^2} = B_y x - P\left\langle x - \frac{L}{2}\right\rangle$$

$$EI\frac{dv}{dx} = \frac{B_y x^2}{2} - \frac{P}{2}\left\langle x - \frac{L}{2}\right\rangle^2 + C_1 \qquad (3)$$

$$EIv = \frac{B_y x^3}{6} - \frac{P}{6}\left\langle x - \frac{L}{2}\right\rangle^3 + C_1 x + C_3 \qquad (4)$$

Boundary Conditions:

$v = 0 \quad$ at $\quad x = 0$

From Eq. (4)

$C_1 = 0$

$v = 0 \quad$ at $\quad x = L$

From Eq. (4)

$$0 = \frac{B_y L^3}{6} - \frac{PL^3}{48} + C_1 L \qquad (5)$$

$$\frac{dv}{dx} = 0 \quad \text{at} \quad x = L$$

From Eq. (3),

$$0 = \frac{B_y L^2}{2} - \frac{PL^2}{8} + C_1 \qquad (6)$$

Solving Eqs. (5) and (6) yields;

$$B_y = \frac{5}{16}P \quad \textbf{Ans.} \qquad C_1 = \frac{-PL^3}{32}$$

Substitute: $B_y = \frac{5}{16}P$ into Eqs. (1) and (2),

$$A_y = \frac{11}{16}P \quad \textbf{Ans.} \qquad M_A = \frac{3PL}{16} \quad \textbf{Ans.}$$

Ans:

$$B_y = \frac{5}{16}P, A_y = \frac{11}{16}P, M_A = \frac{3PL}{16}$$

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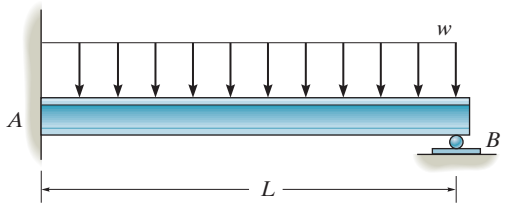
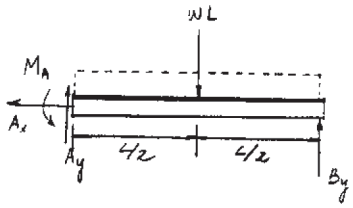
12–103.

Determine the reactions at the supports A and B, then draw the shear and moment diagrams. EI is constant.



SOLUTION



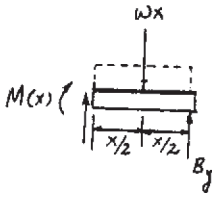
$\leftarrow \Sigma F_a = 0; \quad A_x = 0$ **Ans.**

$+\uparrow \Sigma F_y = 0; \quad A_y + B_y - wL = 0$ **(1)**

$\circlearrowleft + \Sigma M_A = 0; \quad M_A + B_y L - wL\left(\frac{L}{2}\right) = 0$ **(2)**

$\circlearrowleft + \Sigma M_{NA} = 0; \quad B_y(x) - wx\left(\frac{x}{2}\right) - M(x) = 0$

$$M(x) + B_y x - \frac{wx^2}{2}$$

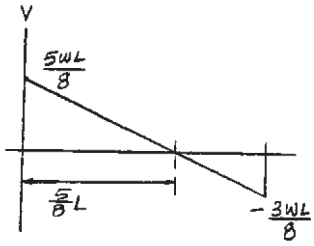


$$EI\frac{d^2v}{dx^2} = M(x)$$

$$EI\frac{d^2v}{dx^2} = B_y x - \frac{wx^2}{2}$$

$$EI\frac{dv}{dx} = \frac{B_y x^2}{2} - \frac{wx^3}{6} + C_1 \quad \textbf{(3)}$$

$$EIv = \frac{B_y x^3}{6} - \frac{wx^4}{24} + C_1 x + C_2 \quad \textbf{(4)}$$



Boundary Conditions:

At $x = 0, \quad v = 0$

From Eq. (4),

$0 = 0 - 0 + 0 + C_2; \quad C_2 = 0$

At $x = L, \quad \frac{dv}{dx} = 0$

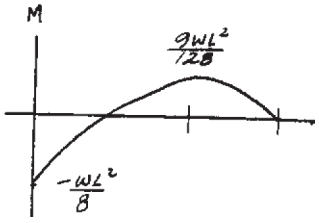
From Eq. (3),

$$0 = \frac{B_y L^2}{2} - \frac{wL^3}{6} + C_1 \quad \textbf{(5)}$$

At $x = L, \quad v = 0$

From Eq. (4),

$$0 = \frac{B_y L^3}{6} - \frac{wL^4}{24} + C_1 L \quad \textbf{(6)}$$



Solving Eqs. (5) and (6) yields:

$$B_y = \frac{3wL}{8} \quad \textbf{Ans.}$$

$$C_1 = -\frac{wL^3}{48}$$

Substituting B_y into Eqs. (1) and (2) yields:

$$A_y = \frac{5wL}{8} \quad \textbf{Ans.}$$

$$M_A = \frac{wL^2}{8} \quad \textbf{Ans.}$$

Ans:

$A_x = 0, B_y = \frac{3wL}{8},$

$A_y = \frac{5wL}{8}, M_A = \frac{wL^2}{8}$

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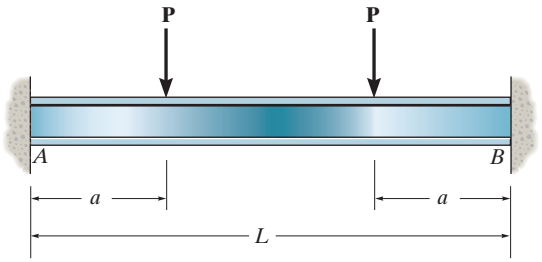
***12–104.**

Determine the moment reactions at the supports A and B. EI is constant.

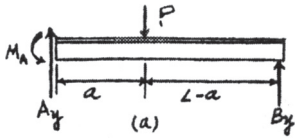


SOLUTION

Support Reactions: FBD(a).

$$\zeta+\Sigma M_B = 0; \quad Pa + P(L - a) + M_A - A_y L - M_B = 0$$

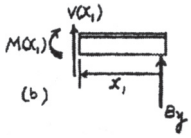
$$PL + M_A - A_y L - M_B = 0 \tag{1}$$



Moment Functions: FBD(b) and (c).

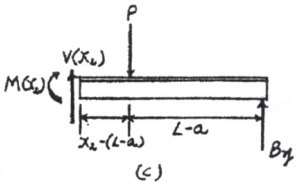
$$M(x_1) = A_y x_1 - M_A$$

$$M(x_2) = A_y x_2 - Px_2 + Pa - M_A$$



Slope and Elastic Curve:

$$EI\frac{d^2v}{dx^2} = M(x)$$

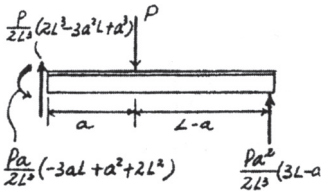


For $M(x_1) = A_y x_1 - M_A$,

$$EI\frac{d^2v_1}{dx_1^2} = A_y x_1 - M_A$$

$$EI\frac{dv_1}{dx_1} = \frac{A_y}{2}x_1^2 - M_A x_1 + C_1 \tag{2}$$

$$EI\,v_1 = \frac{A_y}{6}x_1^3 - \frac{M_A}{2}x_1^2 + C_1x_1 + C_2 \tag{3}$$

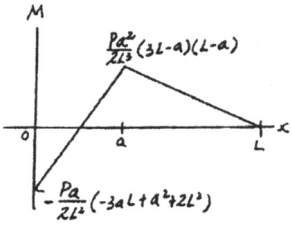


For $M(x_2) = A_y x_2 - Px_2 + Pa - Ma$,

$$EI\frac{d^2v}{dx_2^2} = A_y x_2 - Px_2 + Pa - M_A$$

$$EI\frac{dv_2}{dx_2} = \frac{A_y}{2}x_2^2 - \frac{P}{2}x_2^2 + Pax_2 - M_A x_2 + C_3 \tag{4}$$

$$EI\,v_2 = \frac{A_y}{6}x_2^3 - \frac{P}{6}x_2^3 + \frac{Pa}{2}x_2^2 - \frac{M_A}{2}x_2^2 + C_3x_2 + C_4 \tag{5}$$



Boundary Conditions:

$\frac{dv_1}{dx_1} = 0$ at $x_1 = 0$. From Eq. (2), $C_1 = 0$

$v_1 = 0$ at $x_1 = 0$. From Eq. (3), $C_2 = 0$

Due to symmetry, $\frac{dv_2}{dx_2} = 0$ at $x_2 = \frac{L}{2}$. From Eq. (4),

$$0 = \frac{A_y}{2}\left(\frac{L}{2}\right)^2 - \frac{P}{2}\left(\frac{L}{2}\right)^2 + Pa\left(\frac{L}{2}\right) - M_A\left(\frac{L}{2}\right) + C_3$$

$$C_3 = -\frac{A_y L^2}{8} + \frac{PL^2}{8} - \frac{PaL}{2} + \frac{M_A L}{2}$$

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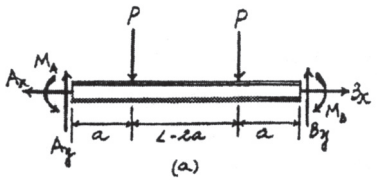
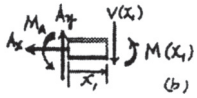
***12–104. Continued**



Due to symmetry, $\frac{dv_1}{dx_1} = -\frac{dv_2}{dx_2}$ at $x_1 = a$ and $x_2 = L - a$. From Eqs. (2) and (4),

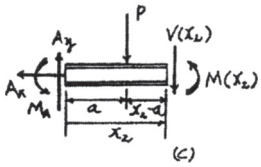
$$\frac{A_y a^2}{2} - M_A a = -\frac{A_y}{2}(L-a)^2 + \frac{P}{2}(L-a)^2 - Pa(L-a) + M_A(L-a) + \frac{A_y L^2}{8} - \frac{PL^2}{8} + \frac{PaL}{2} - \frac{M_A L}{2}$$

$$-A_y a^2 - \frac{3A_y L^2}{8} + A_y aL + \frac{3PL^2}{8} - \frac{3PaL}{2} + \frac{3Pa^2}{2} + \frac{M_A L}{2} = 0 \qquad \textbf{(6)}$$



Continuity Conditions:

At $x_1 = x_2 = a$, $\frac{dv_1}{dx_1} = \frac{dv_2}{dx_2}$. From Eqs. (2) and (4),



$$\frac{A_y a^2}{2} - M_A a = \frac{A_y a^2}{2} - \frac{Pa^2}{2} + Pa^2 - M_A a - \frac{A_y L^2}{8} + \frac{PL^2}{8} - \frac{PaL}{2} + \frac{M_A L}{2}$$

$$\frac{Pa^2}{2} - \frac{A_y L^2}{8} + \frac{PL^2}{8} - \frac{PaL}{2} + \frac{M_A L}{2} = 0 \qquad \textbf{(7)}$$

Solving Eqs. (6) and (7) yields,

$$M_A = \frac{Pa}{L}(L-a) \qquad \textbf{Ans.}$$

$$A_y = P$$

Substitute the value of M_A and A_y obtained into Eqs. (1),

$$M_B = \frac{Pa}{L}(L-a) \qquad \textbf{Ans.}$$

Ans:

$$M_A = \frac{Pa}{L}(L-a),$$

$$M_B = \frac{Pa}{L}(L-a)$$

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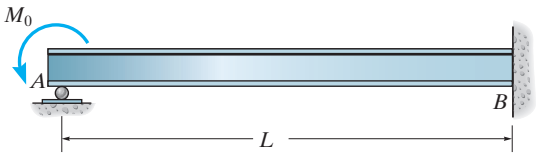
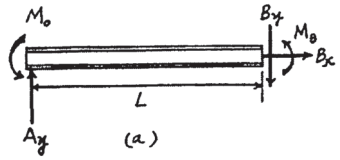
12–105.

Determine the reactions at the supports A and B, then draw the moment diagram. EI is constant.



SOLUTION

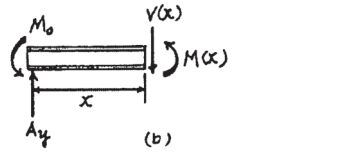


Support Reactions: FBD(a).

$$\pm\rightarrow \Sigma F_x = 0; \quad A_x = 0 \qquad \textbf{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \quad A_y - B_y = 0 \qquad \textbf{(1)}$$

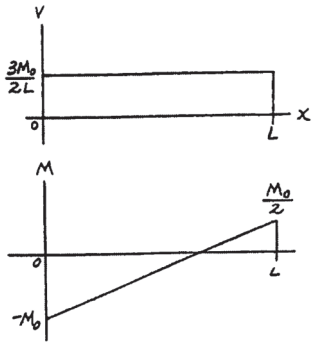
$$\circlearrowleft +\Sigma M_B = 0; \quad M_0 - A_y L + M_B = 0 \qquad \textbf{(2)}$$



Moment Function: FBD(b)

$$\circlearrowleft +\Sigma M_{NA} = 0; \quad M(x) + M_0 - A_y x = 0$$

$$M(x) = A_y x - M_0$$



Slope and Elastic Curve:

$$EI\frac{d^2v}{dx^2} = M(x)$$

$$EI\frac{d^2v}{dx^2} = A_y x - M_0$$

$$EI\frac{dv}{dx} = \frac{A_y}{2}x^2 - M_0 x + C_1 \qquad \textbf{(3)}$$

$$EIv = \frac{A_y}{6}x^3 - \frac{M_0}{2}x^2 + C_1 x + C_2 \qquad \textbf{(4)}$$

Boundary Conditions:

At $x = 0, v = 0$. From Eq. (4), $C_2 = 0$

At $x = L, \frac{dv}{dx} = 0$. From Eq. (3),

$$0 = \frac{A_y L^2}{2} - M_0 L + C_1 \qquad \textbf{(5)}$$

At $x = L, v = 0$. From Eq. (4),

$$0 = \frac{A_y L^3}{6} - \frac{M_0 L^2}{2} + C_1 L \qquad \textbf{(6)}$$

Solving Eqs. (5) and (6) yields,

$$A_y = \frac{3M_0}{2L} \qquad \textbf{Ans.}$$

$$C_1 = \frac{M_0 L}{4}$$

Substituting A_y, into Eqs. (1) and (2) yields:

$$B_y = \frac{3M_0}{2L} \qquad M_B = \frac{M_0}{2} \qquad \textbf{Ans.}$$

Ans:
$A_x = 0,$
$A_y = \frac{3M_0}{2L},$
$B_y = \frac{3M_0}{2L},$
$M_B = \frac{M_0}{2}$

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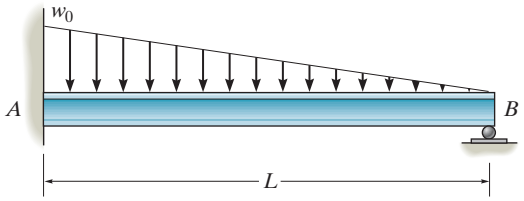
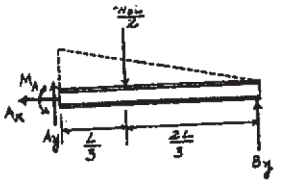
12–106.

Determine the reactions at the support A and B. EI is constant.



SOLUTION

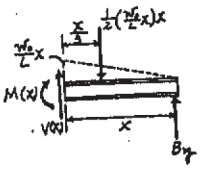


Support Reactions: FBD(a).

$$\stackrel{+}{\rightarrow}\Sigma F_x = 0; \qquad A_x = 0 \qquad \textbf{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \qquad A_y + B_y - \frac{w_0 L}{2} = 0 \qquad \textbf{(1)}$$

$$\circlearrowleft + \Sigma M_A = 0; \qquad B_y L + M_A - \frac{w_0 L}{2}\left(\frac{L}{3}\right) = 0 \qquad \textbf{(2)}$$



Moment Function: FBD(b).

$$\circlearrowleft + \Sigma M_{NA} = 0; \qquad -M(x) - \frac{1}{2}\left(\frac{w_0}{L}x\right)x\left(\frac{x}{3}\right) + B_y x = 0$$

$$M(x) = B_y x - \frac{w_0}{6L}x^3$$

Slope and Elastic Curve:

$$EI\frac{d^2v}{dx^2} = M(x)$$

$$EI\frac{d^2v}{dx^2} = B_y x - \frac{w_0}{6L}x^3$$

$$EI\frac{dv}{dx} = \frac{B_y}{2}x^2 - \frac{w_0}{24L}x^4 + C_1 \qquad \textbf{(3)}$$

$$EIv = \frac{B_y}{6}x^3 - \frac{w_0}{120L}x^5 + C_1 x + C_2 \qquad \textbf{(4)}$$

Boundary Conditions:

At $x = 0$, $v = 0$. From Eq. (4), $C_2 = 0$

At $x = L$, $\frac{dv}{dx} = 0$. From Eq. (3),

$$0 = \frac{B_y L^2}{2} - \frac{w_0 L^3}{24} + C_1$$

$$C_1 = \frac{B_y L^2}{2} + \frac{w_0 L^3}{24}$$

At $x = L$, $v = 0$. From Eq. (4),

$$0 = \frac{B_y L^3}{6} - \frac{w_0 L^4}{120} + \left(-\frac{B_y L^2}{2} + \frac{w_0 L^3}{24}\right)L$$

$$B_y = \frac{w_0 L}{10} \qquad \textbf{Ans.}$$

Substituting B_y into Eqs. (1) and (2) yields,

$$A_y = \frac{2w_0 L}{5} \qquad M_A = \frac{w_0 L^2}{15} \qquad \textbf{Ans.}$$

Ans:

$$A_x = 0, B_y = \frac{w_0 L}{10}, A_y = \frac{2w_0 L}{5}, M_A = \frac{w_0 L^2}{15}$$

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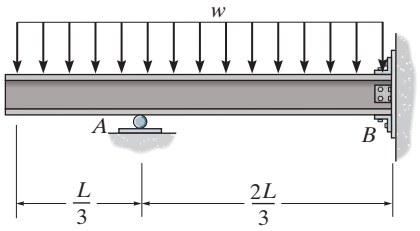
12–107.

Determine the reactions at roller support A and fixed support B.



SOLUTION

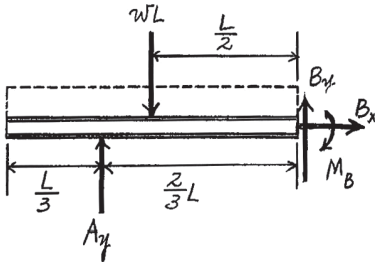
Equations of Equilibrium: Referring to the free-body diagram of the entire beam, Fig. *a*,

$$\pm\rightarrow \Sigma F_x = 0; \qquad B_x = 0 \qquad \textbf{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \qquad A_y + B_y - wL = 0 \qquad \textbf{(1)}$$

$$\circlearrowleft + \Sigma M_B = 0; \qquad wL\left(\frac{L}{2}\right) - A_y\left(\frac{2}{3}L\right) - M_B = 0$$

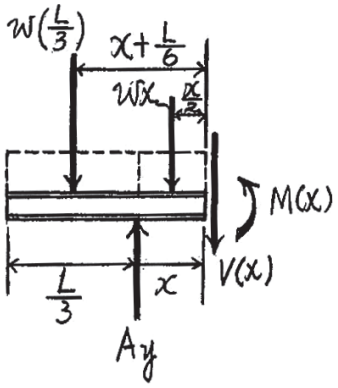
$$M_B = \frac{wL^2}{2} - \frac{2}{3}A_yL \qquad \textbf{(2)}$$



Moment Functions: Referring to the free-body diagram of the beam's segment, Fig. *b*,

$$\circlearrowleft + \Sigma M_O = 0; \qquad M(x) + wx\left(\frac{x}{2}\right) + w\left(\frac{L}{3}\right)\left(x + \frac{L}{6}\right) - A_yx = 0$$

$$M(x) = A_yx - \frac{w}{2}x^2 - \frac{wL}{3}x - \frac{wL^2}{18}$$



Equations of Slope and Elastic Curves:

$$EI\frac{d^2v}{dx^2} = M(x)$$

$$EI\frac{d^2v}{dx^2} = A_yx - \frac{w}{2}x^2 - \frac{wL}{3}x - \frac{wL^2}{18}$$

$$EI\frac{dv}{dx} = \frac{A_y}{2}x^2 - \frac{w}{6}x^3 - \frac{wL}{6}x^2 - \frac{wL^2}{18}x + C_1 \qquad \textbf{(3)}$$

$$EIv = \frac{A_y}{6}x^3 - \frac{w}{24}x^4 - \frac{wL}{18}x^3 - \frac{wL^2}{36}x^2 + C_1x + C_2 \qquad \textbf{(4)}$$

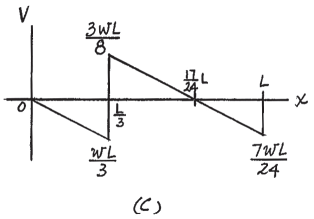
(b)

Boundary Conditions: At $x = 0, v = 0$. Then Eq. (4) gives

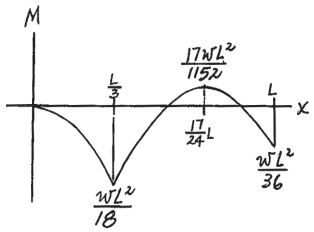
$0 = 0 - 0 - 0 - 0 + 0 + C_2 \quad C_2 = 0$ At $x = \frac{2}{3}L, \frac{dv}{dx} = 0$. Then Eq. (3) gives

$$0 = \frac{A_y}{2}\left(\frac{2}{3}L\right)^2 - \frac{w}{6}\left(\frac{2}{3}L\right)^3 - \frac{wL}{6}\left(\frac{2}{3}L\right)^2 - \frac{wL^2}{18}\left(\frac{2}{3}L\right) + C_1$$

$$C_1 = \frac{13wL^3}{81} - \frac{2A_yL^2}{9} \qquad \textbf{(5)}$$



(c)



(d)

At $x = \frac{2}{3}L, v = 0$. Then Eq. (4) gives

$$0 = \frac{A_y}{6}\left(\frac{2}{3}L\right)^3 - \frac{w}{24}\left(\frac{2}{3}L\right)^4 - \frac{wL}{18}\left(\frac{2}{3}L\right)^3 - \frac{wL^2}{36}\left(\frac{2}{3}L\right)^2 + C_1\left(\frac{2}{3}L\right)$$

$$C_1 = \frac{wL^3}{18} - \frac{2A_yL^2}{27} \qquad \textbf{(6)}$$

Solving Eqs. (5) and (6),

$A_y = \frac{17wL}{24}$ **Ans.** $C_1 = \frac{wL}{324}$ Substituting the result of A_y into Eqs. (1) and (2),

$$B_y = \frac{7wL}{24} \qquad M_B = \frac{wL^2}{36} \qquad \textbf{Ans.}$$

The shear and moment diagrams are shown in Figs. *c* and *d*, respectively.

Ans:
$B_x = 0$,
$A_y = \frac{17wL}{24}$,
$B_y = \frac{7wL}{24}$,
$M_B = \frac{wL^2}{36}$

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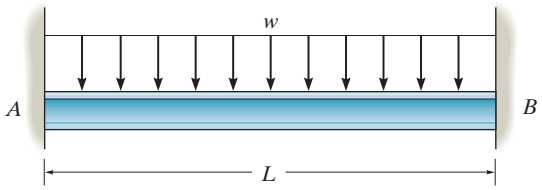
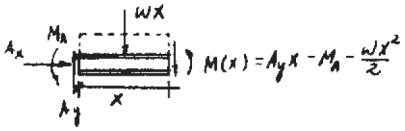
***12–108.**

Determine the moment reactions at the supports A and B, then draw the shear and moment diagrams. Solve by expressing the internal moment in the beam in terms of A_y and M_A. EI is constant.



SOLUTION



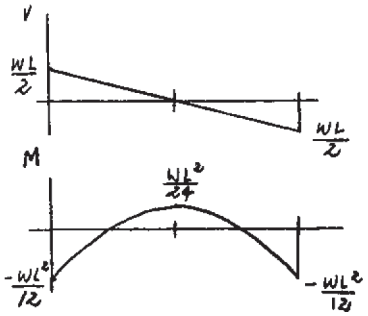
$$M(x) = A_y x - M_A - \frac{wx^2}{2}$$

Elastic Curve and Slope:

$$EI\frac{d^2v}{dx^2} = M(x) = A_y x - M_A - \frac{wx^2}{2}$$

$$EI\frac{dv}{dx} = \frac{A_y x^2}{2} - M_A x - \frac{wx^3}{6} + C_1 \qquad \textbf{(1)}$$

$$EIv = \frac{A_y x^3}{6} - \frac{M_A x^2}{2} - \frac{wx^4}{24} + C_1 x + C_2 \qquad \textbf{(2)}$$



Boundary Conditions:

$$\frac{dv}{dx} = 0 \quad \text{at} \quad x = 0$$

From Eq. (1)

$$C_1 = 0$$

$$v = 0 \quad \text{at} \quad x = 0$$

From Eq. (2)

$$C_2 = 0$$

$$\frac{dv}{dx} = 0 \quad \text{at} \quad x = L$$

From Eq. (1)

$$0 = \frac{A_y L^2}{2} - M_A L - \frac{wL^3}{6} \qquad \textbf{(3)}$$

$$v = 0 \quad \text{at} \quad x = L$$

From Eq. (2)

$$0 = \frac{A_y L^3}{6} - \frac{M_A L^2}{2} - \frac{wL^4}{24} \qquad \textbf{(4)}$$

Solving Eqs. (3) and (4) yields:

$$A_y = \frac{wL}{2}$$

$$M_A = \frac{wL^2}{12} \qquad \textbf{Ans.}$$

Due to symmetry:

$$M_B = \frac{wL^2}{12} \qquad \textbf{Ans.}$$

Ans:

$$M_A = \frac{wL^2}{12},$$

$$M_B = \frac{wL^2}{12}$$

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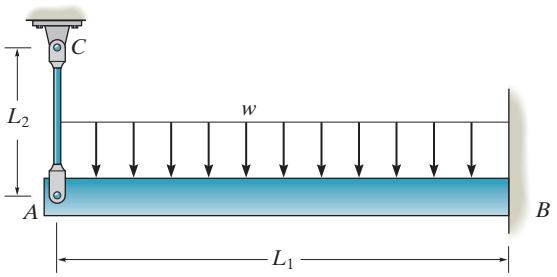
12–109.

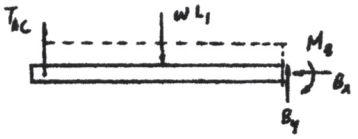
The beam has a constant E_1I_1 and is supported by the fixed wall at B and the rod AC. If the rod has a cross-sectional area A_2 and the material has a modulus of elasticity E_2, determine the force in the rod.



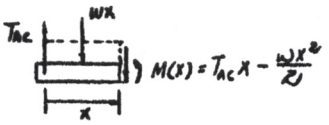
SOLUTION

$+\uparrow \Sigma F_y = 0 \qquad T_{AC} + B_y - wL_1 = 0$

$\circlearrowleft + \ \Sigma M_B = 0 \qquad T_{AC}(L_1) + M_B - \dfrac{wL_1^{\,2}}{2} = 0$ **(1)**



$$M_B = \frac{wL_1^{\,2}}{2} - T_{AC}L_1 \qquad \textbf{(2)}$$



Bending Moment $M(x)$:

$$M(x) = T_{AC}x - \frac{wx^2}{2}$$

Elastic Curve and Slope:

$$EI\frac{d^2v}{dx^2} = M(x) = T_{AC}x - \frac{wx^2}{2}$$

$$EI\frac{dv}{dx} = \frac{T_{AC}x^2}{2} - \frac{wx^3}{6} + C_1 \qquad \textbf{(3)}$$

$$EIv = \frac{T_{AC}x^3}{6} - \frac{wx^4}{24} + C_1x + C_2 \qquad \textbf{(4)}$$

Boundary Conditions:

$$v = \frac{T_{AC}L_2}{A_2E_2} \qquad x = 0$$

From Eq. (4)

$$-E_1I_1\left(\frac{T_{AC}L_2}{A_2E_2}\right) = 0 - 0 + 0 + C_2$$

$$C_2 = \left(\frac{-E_1I_1L_2}{A_2E_2}\right)T_{AC}$$

$v = 0 \qquad$ at $\qquad x = L_1$

From Eq. (4)

$$0 = \frac{T_{AC}L_1^{\,3}}{6} - \frac{wL_1^{\,4}}{24} + C_1L_1 - \frac{E_1I_1L_2}{A_2E_2}T_{AC} \qquad \textbf{(5)}$$

$$\frac{dv}{dx} = 0 \qquad \text{at} \qquad x = L_1$$

From Eq. (3)

$$0 = \frac{T_{AC}L_1^{\,2}}{2} - \frac{wL_1^{\,3}}{6} + C_1 \qquad \textbf{(6)}$$

Solving Eqs. (5) and (6) yields:

$$T_{AC} = \frac{3A_2E_2wL_1^{\,4}}{8\left(A_2E_2L_1^{\,3} + 3E_1I_1L_2\right)} \qquad \textbf{Ans.}$$

Ans:

$$T_{AC} = \frac{3A_2E_2wL_1^4}{8(A_2E_2L_1^3 + 3E_1I_1L_2)}$$

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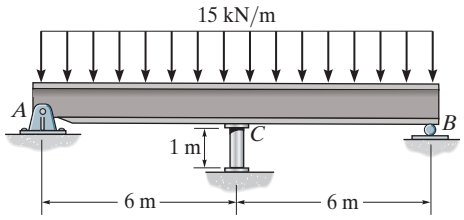
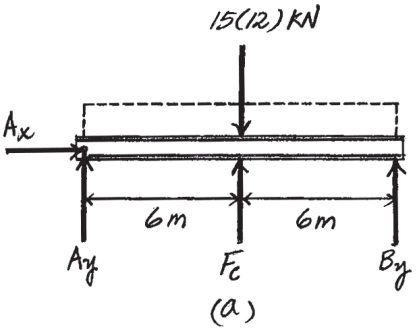
12–110.

The beam is supported by a pin at A, a roller at B, and a post having a diameter of 50 mm at C. Determine the support reactions at A, B, and C. The post and the beam are made of the same material having a modulus of elasticity $E = 200$ GPa, and the beam has a constant moment of inertia $I = 255(10^6)$ mm^4.



SOLUTION



Equations of Equilibrium: Referring to the free-body diagram of the entire beam, Fig. *a*,

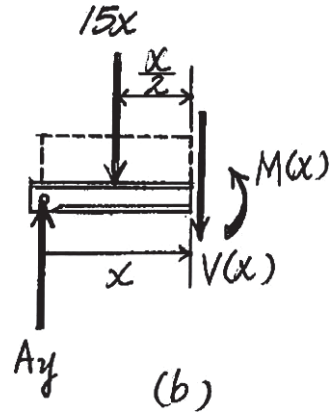
$$\stackrel{+}{\rightarrow}\Sigma F_x = 0; \qquad A_x = 0 \qquad \textbf{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \qquad A_y + B_y + F_C - 15(12) = 0 \qquad \textbf{(1)}$$

$$\circlearrowleft +\Sigma M_B = 0; \qquad 15(12)(6) - F_C(6) - A_y(12) = 0$$

$$2A_y + F_C = 180 \qquad \textbf{(2)}$$

Moment Functions: Referring to the free-body diagram of the beam's segment, Fig. *b*,



$$\circlearrowleft +\Sigma M_O = 0; \qquad M(x) + 15x\left(\frac{x}{2}\right) - A_y x = 0$$

$$M(x) = A_y x - 7.5x^2$$

Equations of Slope and Elastic Curves:

$$EI\frac{d^2v}{dx^2} = M(x)$$

$$EI\frac{d^2v}{dx^2} = A_y x - 7.5x^2$$

$$EI\frac{dv}{dx} = \frac{A_y}{2}x^2 - 2.5x^3 + C_1 \qquad \textbf{(3)}$$

$$EIv = \frac{A_y}{6}x^3 - 0.625x^4 + C_1 x + C_2 \qquad \textbf{(4)}$$

Boundary Conditions: At $x = 0$, $v = 0$. Then Eq. (4) gives

$$0 = 0 - 0 + 0 + C_2 \qquad C_2 = 0$$

At $x = 6$ m, $v = -\Delta_C = -\dfrac{F_C L_C}{A_C E} = -\dfrac{F_C(1)}{\frac{\pi}{4}(0.05^2)E} = -\dfrac{1600F_C}{\pi E}$. Then Eq. (4) gives

$$E\left[255\left(10^{-6}\right)\right]\left(-\frac{1600F_C}{\pi E}\right) = \frac{A_y}{6}\left(6^3\right) - 0.625\left(6^4\right) + C_1(6)$$

$$C_1 = 135 - 6A_y - 0.02165F_C$$

Due to symmetry, $\dfrac{dv}{dx} = 0$ at $x = 6$ m. Then Eq. (3) gives

$$0 = \frac{A_y}{2}\left(6^2\right) - 2.5\left(6^3\right) + 135 - 6A_y - 0.02165F_C$$

$$12A_y - 0.02165F_C = 405 \qquad \textbf{(5)}$$

Solving Eqs. (2) and (5),

$$F_C = 112.096 \text{ kN} = 112 \text{ kN} \qquad A_y = 33.95 \text{ kN} = 34.0 \text{ kN} \qquad \textbf{Ans.}$$

Substituting these results into Eq. (1),

$$B_y = 33.95 \text{ kN} = 34.0 \text{ kN} \qquad \textbf{Ans.}$$

Ans:
$A_x = 0$, $F_C = 112$ kN, $A_y = 34.0$ kN, $B_y = 34.0$ kN

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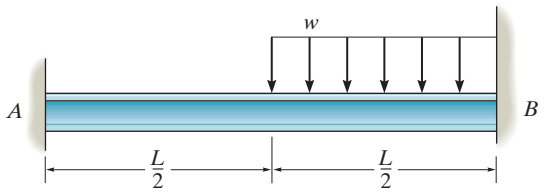
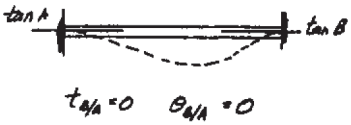
12–111.

Determine the moment reactions at the supports A and B. EI is constant.

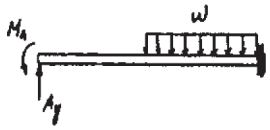


SOLUTION



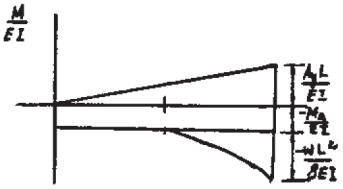
$$\theta_{B/A} = 0 = \frac{1}{2}\left(\frac{A_y L}{EI}\right)(L) + \left(\frac{-M_A}{EI}\right)(L) + \frac{1}{3}\left(\frac{-wL^2}{8EI}\right)\left(\frac{L}{2}\right)$$

$$0 = \frac{A_y L}{2} - M_A - \frac{wL^2}{48} \qquad (2)$$



$$t_{B/A} = 0 = \frac{1}{2}\left(\frac{A_y L}{EI}\right)(L)\left(\frac{L}{3}\right) + \left(\frac{-M_A}{EI}\right)(L)\left(\frac{L}{2}\right) + \frac{1}{3}\left(\frac{-wL^2}{8EI}\right)\left(\frac{L}{2}\right)\left(\frac{L}{8}\right)$$

$$0 = \frac{A_y L}{6} - \frac{M_A}{2} - \frac{wL^2}{384} \qquad (3)$$



Solving Eqs. (2) and (3) yields:

$$A_y = \frac{3wL}{32}$$

$$M_A = \frac{5wL^2}{192} \qquad \textbf{Ans.}$$

$$\circlearrowleft + \Sigma M_B = 0; \quad M_B + \frac{3wL}{32}(L) - \frac{5wL^2}{192} - \frac{wL}{2}\left(\frac{L}{4}\right) = 0$$

$$M_B = \frac{11wL^2}{192} \qquad \textbf{Ans.}$$

Ans:

$$M_A = \frac{5wL^2}{192}, M_B = \frac{11wL^2}{192}$$

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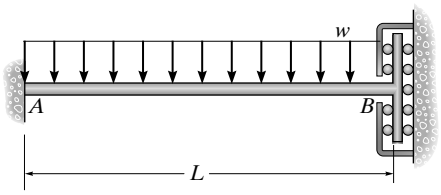
***12–112.** The rod is fixed at A, and the connection at B consists of a roller constraint which allows vertical displacement but resists axial load and moment. Determine the moment reactions at these supports. EI is constant.



SOLUTION

Support Reaction: FBD(a).

$$\zeta + \Sigma M_A = 0; \qquad M_B + M_A - wL\left(\frac{L}{2}\right) = 0 \qquad [1]$$

Elastic Curve: As shown.

M/EI Diagrams: M/EI diagrams for M_B and the uniform distributed load acting on a cantilever beam are shown.

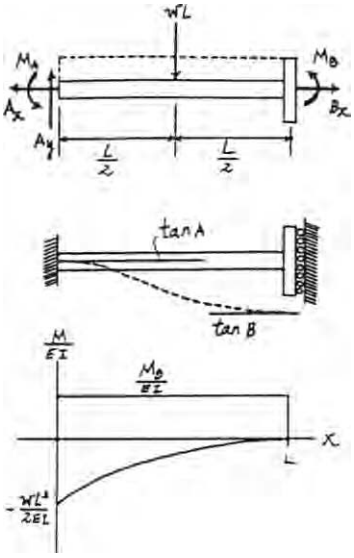
Moment-Area Theorems: Since both tangents at A and B are horizontal (parallel), $\theta_{B/A} = 0$.

$$\theta_{B/A} = 0 = \left(\frac{M_B}{EI}\right)(L) + \frac{1}{3}\left(-\frac{wL^2}{2EI}\right)(L)$$

$$M_B = \frac{wL^2}{6} \qquad \textbf{Ans.}$$

Substituting M_B into Eq.[1],

$$M_A = \frac{wL^2}{3} \qquad \textbf{Ans.}$$



Ans:

$$M_A = \frac{wL^2}{3};\ M_B = \frac{wL^2}{6}$$

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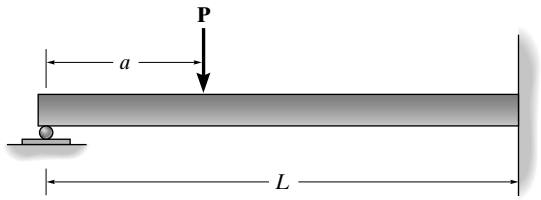
12–113. Determine the value of a for which the maximum positive moment has the same magnitude as the maximum negative moment. EI is constant.



SOLUTION

$$(t_{A/B})_1 = \frac{1}{2}\left(\frac{-P(L-a)}{EI}\right)(L-a)\left(a + \frac{2(L-a)}{3}\right) = \frac{-P(L-a)^2(2L+a)}{6EI}$$

$$(t_{A/B})_2 = \frac{1}{2}\left(\frac{A_y L}{EI}\right)(L)\left(\frac{2L}{3}\right) = \frac{A_y L^3}{3EI}$$

$$t_{A/B} = 0 = (t_{A/B})_1 + (t_{A/B})_2$$

$$0 = \frac{-P(L-a)^2(2L+a)}{6EI} + \frac{A_y L^3}{3EI}$$

$$A_y = \frac{P(L-a)^2(2L+a)}{2L^3}$$

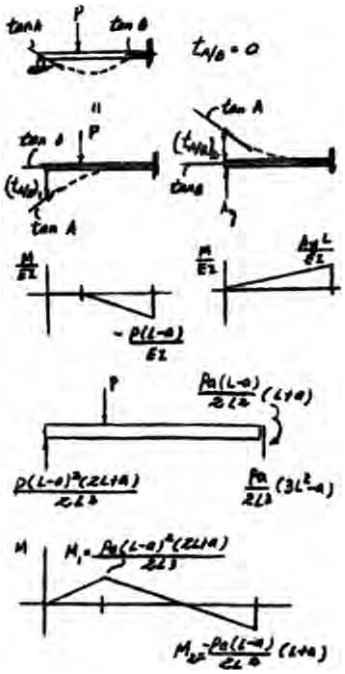
Require:

$$|M_1| = |M_2|$$

$$\frac{Pa(L-a)^2(2L+a)}{2L^3} = \frac{Pa(L-a)(L+a)}{2L^2}$$

$$a^2 + 2La - L^2 = 0$$

$$a = 0.414L \qquad \textbf{Ans.}$$



Ans:

$a = 0.414L$

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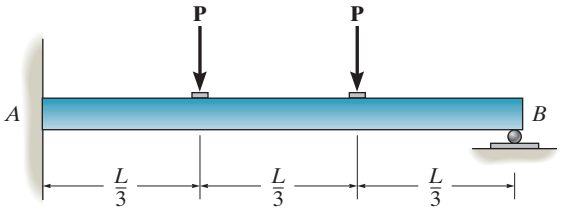
12–114.

Determine the reactions at the supports A and B, then draw the shear and moment diagrams. EI is constant.



SOLUTION

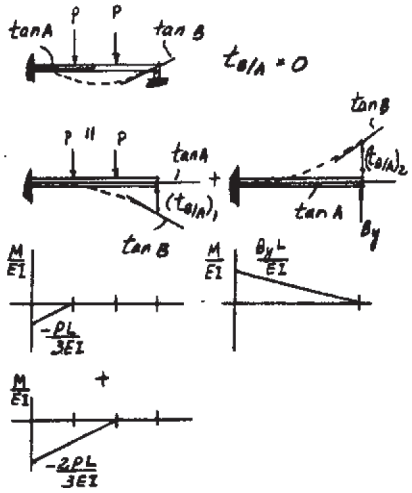
$$(t_{B/A})_1 = \frac{1}{2}\left(\frac{-PL}{3EI}\right)\left(\frac{L}{3}\right)\left(\frac{2L}{3}+\frac{2L}{9}\right) + \frac{1}{2}\left(\frac{-2PL}{3EI}\right)\left(\frac{2L}{3}\right)\left(\frac{L}{3}+\frac{4L}{9}\right) = -\frac{2PL^3}{9EI}$$

$$(t_{B/A})_2 = \frac{1}{2}\left(\frac{B_y L}{EI}\right)(L)\left(\frac{2L}{3}\right) = \frac{B_y L^3}{3EI}$$

$$t_{B/A} = 0 = (t_{B/A})_1 + (t_{B/A})_2$$

$$0 = -\frac{2PL^3}{9EI} + \frac{B_y L^3}{3EI}$$

$$B_y = \frac{2}{3}P \qquad \textbf{Ans.}$$

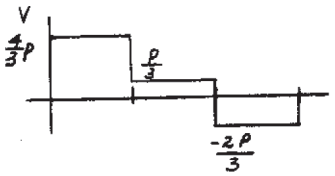


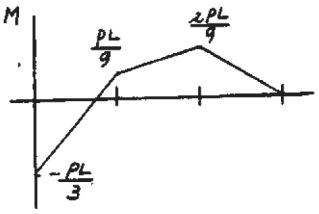
From the free-body diagram,

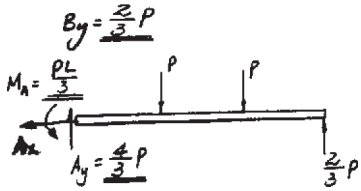
$$M_A = \frac{PL}{3} \qquad \textbf{Ans.}$$

$$A_y = \frac{4}{3}P \qquad \textbf{Ans.}$$

$$A_x = 0 \qquad \textbf{Ans.}$$







Ans:

$$B_y = \frac{2}{3}P,\ M_A = \frac{PL}{3},\ A_y = \frac{4}{3}P,\ A_x = 0$$

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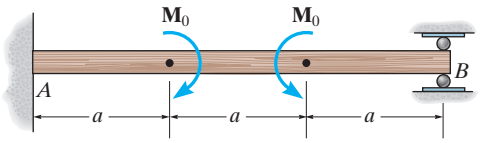
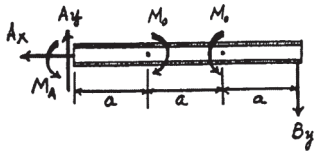
12–115.

Determine the reactions at the supports. *EI* is constant.



SOLUTION

Support Reaction: FBD(a).



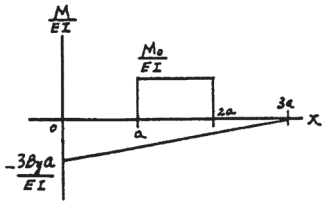
$$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad A_x = 0 \qquad \textbf{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \qquad -B_y + A_y = 0 \qquad \textbf{(1)}$$

$$\circlearrowleft +\Sigma M_A = 0; \qquad -B_y(3a) + M_A = 0 \qquad \textbf{(2)}$$

Elastic Curve: As shown.

***M/EI* Diagrams:** M/EI diagrams for B_y and M_0 acting on a cantilever beam are drawn.



Moment - Area Theorems: From the elastic curve, $t_{B/A} = 0$.

$$t_{B/A} = 0 = \frac{1}{2}\left(-\frac{3B_y a}{EI}\right)(3a)\left(\frac{2}{3}\right)(3a) + \left(\frac{M_0}{EI}\right)(a)\left(a + \frac{a}{2}\right)$$

$$B_y = \frac{M_0}{6a} \qquad \textbf{Ans.}$$

Substituting B_y into Eqs. (1) and (2) yields,

$$A_y = \frac{M_0}{6a} \qquad M_A = \frac{M_0}{2} \qquad \textbf{Ans.}$$

Ans:
$A_x = 0,$
$B_y = \frac{M_0}{6a},$
$A_y = \frac{M_0}{6a},$
$M_A = \frac{M_0}{2}$

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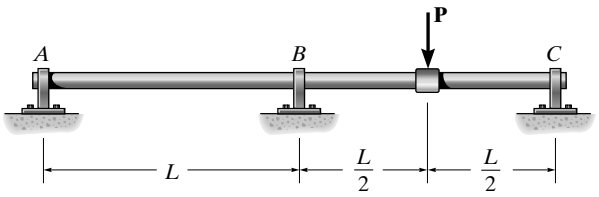
***12–116.** Determine the reactions at the supports, then draw the shear and moment diagrams. EI is constant. Support B is a thrust bearing.



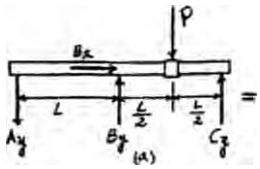
SOLUTION

Support Reactions: FBD(a).

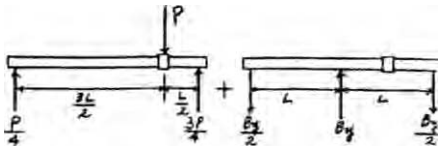
$$\stackrel{+}{\rightarrow}\Sigma F_x = 0; \qquad B_x = 0 \qquad \textbf{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \qquad -A_y + B_y + C_y - P = 0 \qquad [1]$$

$$\text{a}+\Sigma M_A = 0; \qquad B_y(L) + C_y(2L) - P\left(\frac{3L}{2}\right) = 0 \qquad [2]$$

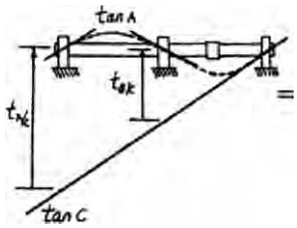


Elastic Curve: As shown.



M/EI Diagrams: *M/EI* diagrams for P and B_y acting on a simply supported beam are drawn separately.

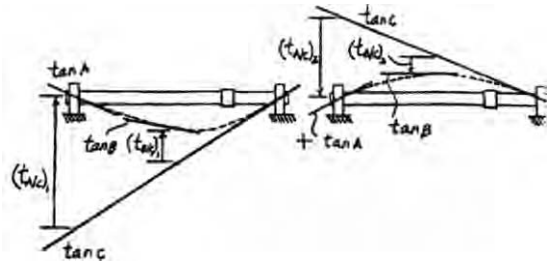
Moment-Area Theorems:



$$(t_{A/C})_1 = \frac{1}{2}\left(\frac{3PL}{8EI}\right)\left(\frac{3L}{2}\right)\left(\frac{2}{3}\right)\left(\frac{3L}{2}\right) + \frac{1}{2}\left(\frac{3PL}{8EI}\right)\left(\frac{L}{2}\right)\left(\frac{3L}{2} + \frac{L}{6}\right)$$

$$= \frac{7PL^3}{16EI}$$

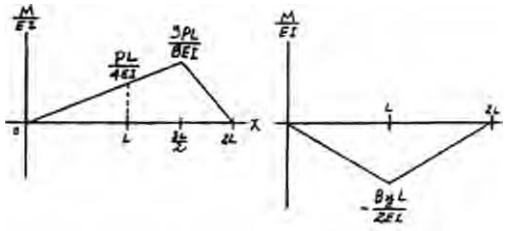
$$(t_{A/C})_2 = \frac{1}{2}\left(-\frac{B_y L}{2EI}\right)(2L)(L) = -\frac{B_y L^3}{2EI}$$



$$(t_{B/C})_1 = \frac{1}{2}\left(\frac{PL}{8EI}\right)\left(\frac{L}{2}\right)\left(\frac{2}{3}\right)\left(\frac{L}{2}\right) + \left(\frac{PL}{4EI}\right)\left(\frac{L}{2}\right)\left(\frac{L}{4}\right) + \frac{1}{2}\left(\frac{3PL}{8EI}\right)\left(\frac{L}{2}\right)\left(\frac{L}{2} + \frac{L}{6}\right)$$

$$= \frac{5PL^3}{48EI}$$

$$(t_{B/C})_2 = \frac{1}{2}\left(-\frac{B_y L}{2EI}\right)(L)\left(\frac{L}{3}\right) = -\frac{B_y L^3}{12EI}$$

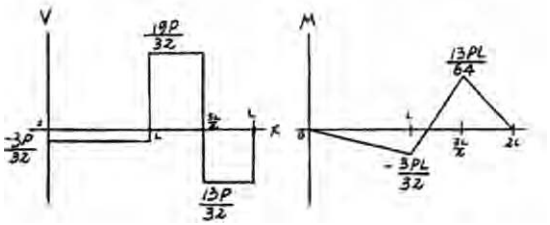


$$t_{A/C} = (t_{A/C})_1 + (t_{A/C})_2 = \frac{7PL^3}{16EI} - \frac{B_y L^3}{2EI}$$

$$t_{B/C} = (t_{B/C})_1 + (t_{B/C})_2 = \frac{5PL^3}{48EI} - \frac{B_y L^3}{12EI}$$

From the elastic curve,

$$t_{A/C} = 2t_{B/C}$$



$$\frac{7PL^3}{16EI} - \frac{B_y L^3}{2EI} = 2\left(\frac{5PL^3}{48EI} - \frac{B_y L^3}{12EI}\right)$$

$$B_y = \frac{11P}{16}$$

Ans.

Substituting B_y into Eqs. [1] and [2] yields,

$$C_y = \frac{13P}{32} \qquad A_y = \frac{3P}{32}$$

Ans.

Ans:

$$A_y = \frac{3P}{32};\ B_y = \frac{11P}{16};\ C_y = \frac{13P}{32}$$

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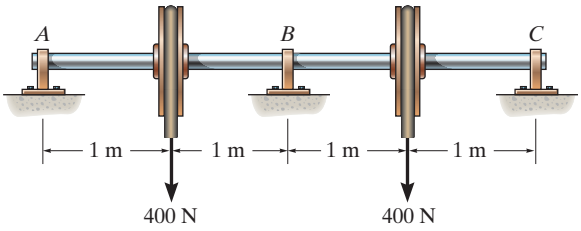
12–117.

Determine the reactions at the journal bearing supports A, B, and C of the shaft, then draw the shear and moment diagrams. EI is constant.

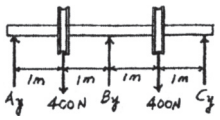


SOLUTION

Support Reactions: FBD(a).

$$+\uparrow \Sigma F_y = 0; \qquad A_y + B_y + C_y - 800 = 0 \qquad \textbf{(1)}$$

$$\circlearrowleft + \Sigma M_A = 0; \qquad B_y\,(2) + C_y\,(4) - 400(1) - 400(3) = 0 \qquad \textbf{(2)}$$

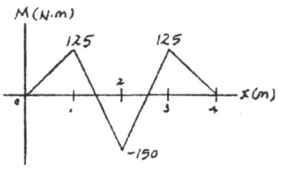


Method of Superposition: Using the table in Appendix C, the required displacements are

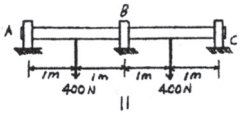
$$v_B' = \frac{Pbx}{6EIL}\left(L^2 - b^2 - x^2\right)$$

$$= \frac{400(1)(2)}{6EI(4)}\left(4^2 - 1^2 - 2^2\right)$$

$$= \frac{366.67\ \text{N}\cdot\text{m}^3}{EI} \quad \downarrow$$



$$v_B'' = \frac{PL^3}{48EI} = \frac{B_y\left(4^3\right)}{48EI} = \frac{1.3333B_y\,\text{m}^3}{EI} \quad \uparrow$$

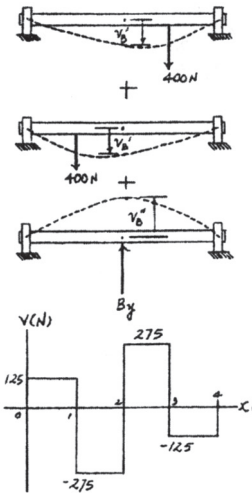


The compatibility condition requires

$$(+\downarrow) \qquad 0 = 2v_B' + v_B''$$

$$0 = 2\left(\frac{366.67}{EI}\right) + \left(-\frac{1.3333B_y}{EI}\right)$$

$$B_y = 550\ \text{N} \qquad \textbf{Ans.}$$



Substituting B_y into Eqs. (1) and (2) yields,

$$A_y = 125\ \text{N} \qquad C_y = 125\ \text{N} \qquad \textbf{Ans.}$$

Ans:
$B_y = 550$ N, $A_y = 125$ N, $C_y = 125$ N

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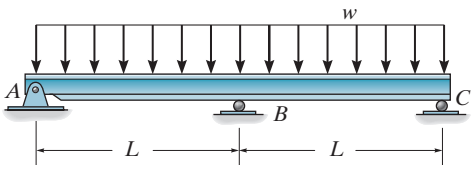
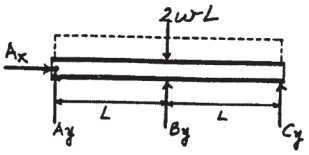
12–118.

Determine the reactions at the supports, then draw the shear and moment diagrams. *EI* is constant.



SOLUTION

Support Reactions: FBD(a).

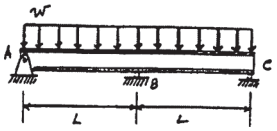


$$\stackrel{+}{\rightarrow}\Sigma F_x = 0; \qquad A_x = 0 \qquad \textbf{Ans.}$$

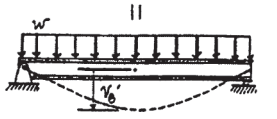
$$+\uparrow \Sigma F_y = 0; \qquad A_y + B_y + C_y - 2wL = 0 \qquad \textbf{(1)}$$

$$\circlearrowleft +\Sigma M_A = 0; \qquad B_y(L) + C_y(2L) - (2wL)(L) = 0 \qquad \textbf{(2)}$$

Method of Superposition: Using the table in Appendix C, the required displacements are

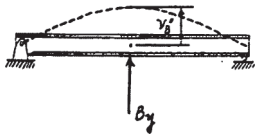


$$v_B' = \frac{5wL_{AC}^4}{384EI} = \frac{5w(2L)^4}{384EI} = \frac{5wL^4}{24EI} \downarrow$$



$$v_B'' = \frac{PL_{AC}^3}{48EI} = \frac{B_y(2L)^3}{48EI} = \frac{B_yL^3}{6EI} \uparrow$$

The compatibility condition requires

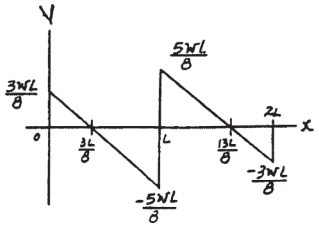


$$(+\downarrow) \qquad 0 = v_B' + v_B''$$

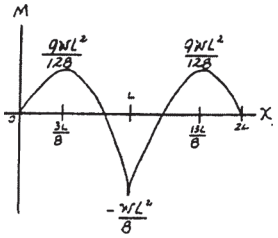
$$0 = \frac{5wL^4}{24EI} + \left(-\frac{B_yL^3}{6EI}\right)$$

$$B_y = \frac{5wL}{4} \qquad \textbf{Ans.}$$

Substituting the value of B_y into Eqs. (1) and (2) yields,



$$C_y = A_y = \frac{3wL}{8} \qquad \textbf{Ans.}$$



Ans:
$A_x = 0,$
$B_y = \frac{5wL}{4},$
$C_y = \frac{3wL}{8}$

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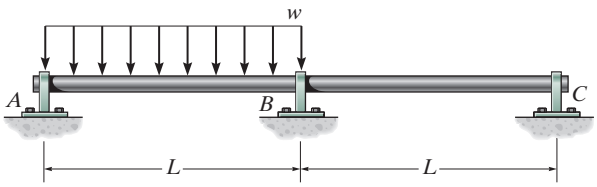
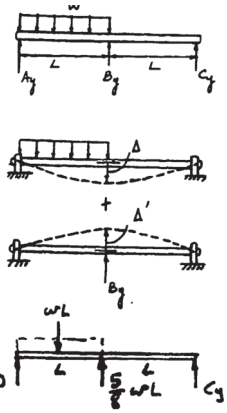
12–119.

Determine the reactions at the supports, then draw the shear and moment diagrams. *EI* is constant.



SOLUTION



$$\Delta = \frac{5w(2L)^4}{768EI} = \frac{5wL^4}{48EI} \downarrow$$

$$\Delta' = \frac{B_y(2L)^4}{48EI} = \frac{B_yL^3}{6EI} \uparrow$$

Require:

$$(+\downarrow) \qquad 0 = \Delta - \Delta'$$

$$0 = \frac{5wL^4}{48EI} - \frac{B_yL^3}{6EI}$$

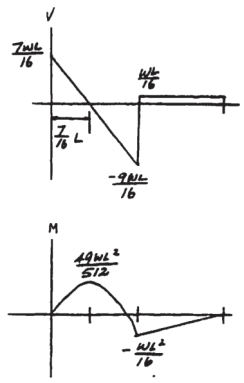
$$B_y = \frac{5}{8}wL \uparrow$$ **Ans.**

$$\zeta + \Sigma M_A = 0; \qquad wL\left(\frac{L}{2}\right) - \frac{5}{8}wL(L) - C_y(2L) = 0$$

$$C_y = -\frac{wL}{16} = \frac{wL}{16} \downarrow$$ **Ans.**

$$+\uparrow \Sigma F_y = 0; \qquad -wL - \frac{wL}{16} + \frac{5}{8}wL + A_y = 0$$

$$A_y = \frac{7}{16}wL \uparrow$$ **Ans.**



Ans:

$$B_y = \frac{5}{8}wL \uparrow,$$

$$C_y = \frac{wL}{16} \downarrow,$$

$$A_y = \frac{7}{16}wL \uparrow$$

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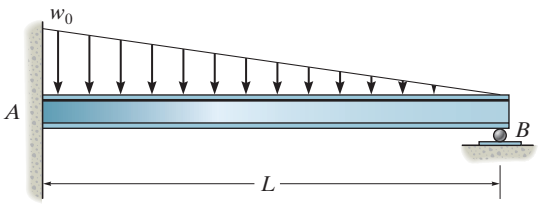
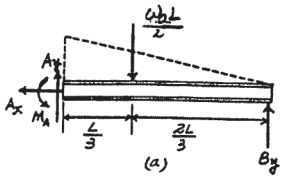
***12–120.**

Determine the reactions at the supports A and B. EI is constant.



SOLUTION

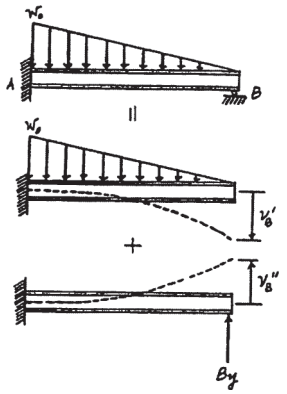


Support Reactions: FBD(a).

$$\stackrel{+}{\rightarrow}\Sigma F_x = 0; \qquad A_x = 0 \qquad \textbf{Ans.}$$

$$+\uparrow\Sigma F_y = 0; \qquad A_y + B_y - \frac{w_0 L}{2} = 0 \qquad \textbf{(1)}$$

$$\circlearrowleft+\Sigma M_A = 0; \qquad B_y L + M_A - \frac{w_0 L}{2}\left(\frac{L}{3}\right) = 0 \qquad \textbf{(2)}$$



Method of Superposition: Using the table in Appendix C, the required displacements are

$$v_B' = \frac{w_0 L^4}{30EI}\downarrow \qquad v_B'' = \frac{B_y L^3}{3EI}\uparrow$$

The compatibility condition requires

$$(+\downarrow) \qquad 0 = v_B' + v_B''$$

$$0 = \frac{w_0 L^4}{30EI} + \left(-\frac{B_y L^3}{3EI}\right)$$

$$B_y = \frac{w_0 L}{10} \qquad \textbf{Ans.}$$

Substituting B_y into Eqs. (1) and (2) yields,

$$A_y = \frac{2w_0 L}{5} \qquad M_A = \frac{w_0 L^2}{15} \qquad \textbf{Ans.}$$

Ans:
$A_x = 0,$
$B_y = \frac{w_0 L}{10},$
$A_y = \frac{2w_0 L}{5},$
$M_A = \frac{w_0 L^2}{15}$

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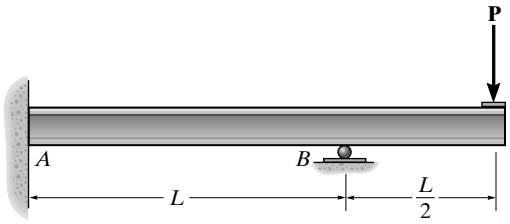
12–121. Determine the reactions at the supports A and B. EI is constant.



SOLUTION

Referring to the FBD of the beam, Fig. *a*

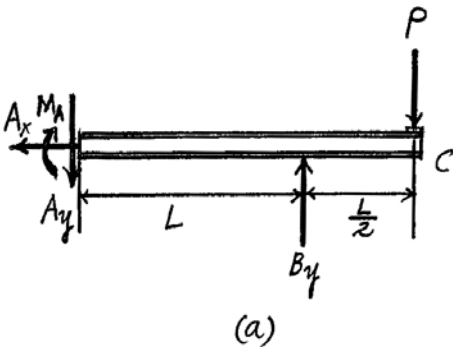
$$\stackrel{+}{\rightarrow}\Sigma F_x = 0; \qquad A_x = 0 \qquad \textbf{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \qquad B_y - P - A_y = 0$$

$$A_y = B_y - P \qquad (1)$$

$$\text{a}+\Sigma M_A = 0; \; -M_A + B_y L - P\left(\frac{3}{2}L\right) = 0$$

$$M_A = B_y L - \frac{3}{2}PL \qquad (2)$$



(a)

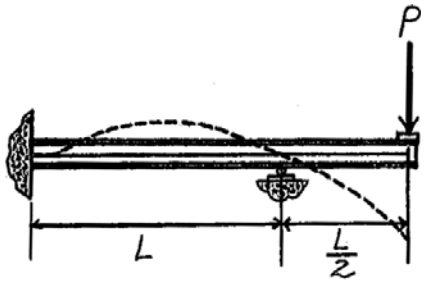
Referring to Fig. *b* and the table in appendix, the necessary deflections are computed as follow:

$$v_P = \frac{Px^2}{6EI}(3L_{AC} - x)$$

$$= \frac{P(L^2)}{6EI}\left[3\left(\frac{3}{2}L\right) - L\right]$$

$$= \frac{7PL^3}{12EI} \; \downarrow$$

$$v_{B_y} = \frac{PL_{AB}^3}{3EI} = \frac{B_y L^3}{3EI} \; \uparrow$$



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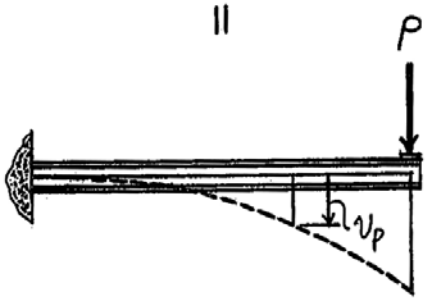

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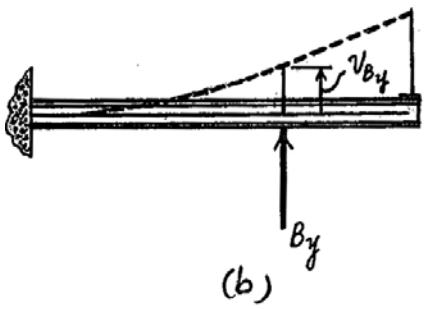

(b)

The compatibility condition at support B requires that

$$(+\downarrow) \qquad 0 = v_P + v_{B_y}$$

$$0 = \frac{7PL^3}{12EI} + \left(\frac{-B_y L^3}{3EI}\right)$$

$$B_y = \frac{7P}{4} \qquad \textbf{Ans.}$$

Substitute this result into Eq (1) and (2)

$$A_y = \frac{3P}{4} \qquad M_A = \frac{PL}{4} \qquad \textbf{Ans.}$$

Ans:

$$B_y = \frac{7P}{4}, A_y = \frac{3P}{4}, M_A = \frac{PL}{4}$$

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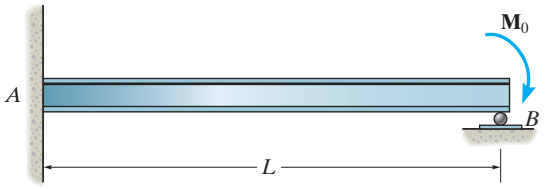
12–122.

Determine the reactions at the supports A and B. EI is constant.



SOLUTION

Referring to the FBD of the beam, Fig. *a*

$$\stackrel{+}{\rightarrow}\Sigma F_x = 0; \qquad A_x = 0 \qquad \textbf{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \qquad B_y - P - A_y = 0$$

$$A_y = B_y - P \qquad \textbf{(1)}$$

$$\circlearrowleft +\Sigma M_A = 0; \; -M_A + B_y L - P\left(\frac{3}{2}L\right) = 0$$

$$M_A = B_y L - \frac{3}{2}PL \qquad \textbf{(2)}$$

Referring to Fig. *b* and the table in appendix, the necessary deflections are computed as follow:

$$v_P = \frac{Px^2}{6EI}(3L_{AC} - x)$$

$$= \frac{P(L^2)}{6EI}\left[3\left(\frac{3}{2}L\right) - L\right]$$

$$= \frac{7PL^3}{12EI}\downarrow$$

$$v_{B_y} = \frac{PL_{AB}^3}{3EI} = \frac{B_y L^3}{3EI}\uparrow$$

The compatibility condition at support B requires that

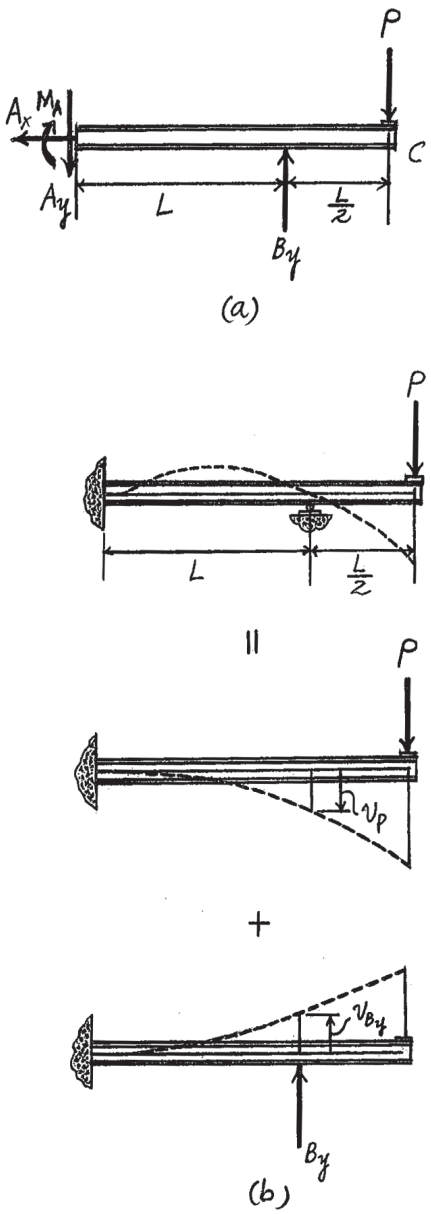
$$(+\downarrow) \qquad 0 = v_P + v_{B_y}$$

$$0 = \frac{7PL^3}{12EI} + \left(\frac{-B_y L^3}{3EI}\right)$$

$$B_y = \frac{7P}{4} \qquad \textbf{Ans.}$$

Substitute this result into Eqs. (1) and (2)

$$A_y = \frac{3P}{4} \qquad M_A = \frac{PL}{4} \qquad \textbf{Ans.}$$



Ans:
$A_x = 0,$
$B_y = \frac{7P}{4},$
$A_y = \frac{3P}{4},$
$M_A = \frac{PL}{4}$

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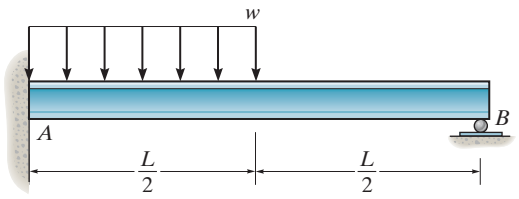
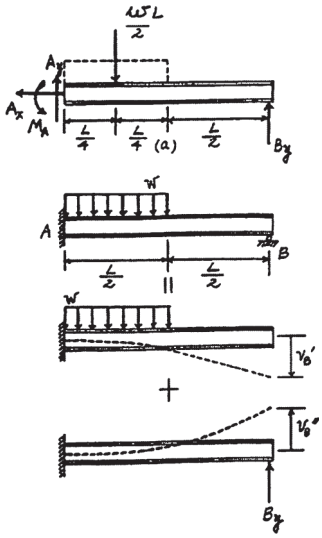
12–123.

Determine the reactions at the supports A and B. EI is constant.



SOLUTION



Support Reactions: FBD(a).

$$\stackrel{+}{\rightarrow}\Sigma F_x = 0; \qquad A_x = 0 \qquad \textbf{Ans.}$$

$$+\uparrow\Sigma F_y = 0; \qquad A_y + B_y - \frac{wL}{2} = 0 \qquad \textbf{(1)}$$

$$\circlearrowleft+\Sigma M_A = 0; \qquad B_y(L) + M_A - \left(\frac{wL}{2}\right)\left(\frac{L}{4}\right) = 0 \qquad \textbf{(2)}$$

Method of Superposition: Using the table in Appendix C, the required displacements are

$$v_B' = \frac{7wL^4}{384EI}\downarrow \qquad v_B'' = \frac{PL^3}{3EI} = \frac{B_yL^3}{3EI}\uparrow$$

The compatibility condition requires

$$(+\downarrow) \qquad 0 = v_B' + v_B''$$

$$0 = \frac{7wL^4}{384EI} + \left(-\frac{B_yL^3}{3EI}\right)$$

$$B_y = \frac{7wL}{128} \qquad \textbf{Ans.}$$

Substituting B_y into Eqs. (1) and (2) yields

$$A_y = \frac{57wL}{128} \qquad M_A = \frac{9wL^2}{128} \qquad \textbf{Ans.}$$

Ans:
$A_x = 0,$
$B_y = \frac{7wL}{128},$
$A_y = \frac{57wL}{128},$
$M_A = \frac{9wL^2}{128}$

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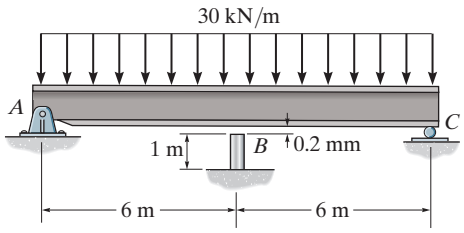
***12–124.**

Before the uniform distributed load is applied to the beam, there is a small gap of 0.2 mm between the beam and the post at B. Determine the support reactions at A, B, and C. The post at B has a diameter of 40 mm, and the moment of inertia of the beam is $I = 875(10^6)\ \text{mm}^4$. The post and the beam are made of material having a modulus of elasticity of $E = 200$ GPa.



SOLUTION

Equations of Equilibrium: Referring to the free-body diagram of the beam, Fig. *a*,

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad A_x = 0 \qquad \textbf{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \qquad A_y + F_B + C_y - 30(12) = 0 \qquad \textbf{(1)}$$

$$\circlearrowleft + \Sigma M_A = 0; \qquad F_B(6) + C_y(12) - 30(12)(6) = 0 \qquad \textbf{(2)}$$

Method of Superposition: Referring to Fig. *b* and the table in the appendix, the necessary deflections are

$$(v_B)_1 = \frac{5wL^4}{384EI} = \frac{5(30)\left(12^4\right)}{384EI} = \frac{8100\ \text{kN}\cdot\text{m}^3}{EI} \downarrow$$

$$(v_B)_2 = \frac{PL^3}{48EI} = \frac{F_B\left(12^3\right)}{48EI} = \frac{36F_B}{EI} \uparrow$$

The deflection of point B is

$$v_B = 0.2\left(10^{-3}\right) + \frac{F_B L_B}{AE} = 0.2\left(10^{-3}\right) + \frac{F_B(1)}{AE} \downarrow$$

The compatibility condition at support B requires

$$(+\downarrow) \qquad v_B = (v_B)_1 + (v_B)_2$$

$$0.2\left(10^{-3}\right) + \frac{F_B(1)}{AE} = \frac{8100}{EI} + \left(-\frac{36F_B}{EI}\right)$$

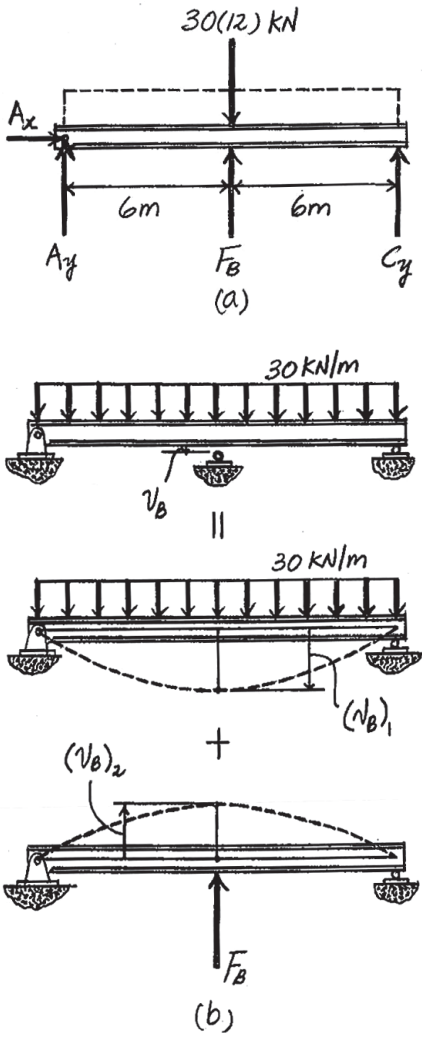
$$0.2\left(10^{-3}\right)E + \frac{F_B}{A} = \frac{8100}{I} - \frac{36F_B}{I}$$

$$\frac{F_B}{\frac{\pi}{4}\left(0.04^2\right)} + \frac{36F_B}{875\left(10^{-6}\right)} = \frac{8100}{875\left(10^{-6}\right)} - \frac{0.2\left(10^{-3}\right)\left[200\left(10^9\right)\right]}{1000}$$

$$F_B = 219.78\ \text{kN} = 220\ \text{kN} \qquad \textbf{Ans.}$$

Substituting the result of F_B into Eqs. (1) and (2),

$$A_y = C_y = 70.11\ \text{kN} = 70.1\ \text{kN} \qquad \textbf{Ans.}$$



Ans:
$A_x = 0$,
$F_B = 220$ kN,
$A_y = C_y = 70.1$ kN

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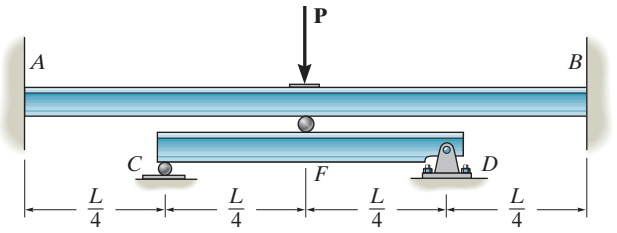
12–125.

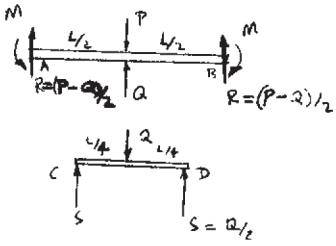
The fixed supported beam AB is strengthened using the simply supported beam CD and the roller at F which is set in place just before application of the load **P**. Determine the reactions at the supports if EI is constant.



SOLUTION

$\delta_F =$ Deflection of top beam at F

$\delta'_F =$ Deflection of bottom beam at F

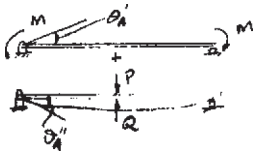


$$\delta_F = \delta'_F$$

$$(+\downarrow)\ \frac{(P-Q)(L^3)}{48EI} - \frac{2M\left(\frac{L}{2}\right)}{6\,EIL}\left[L^2 - \left(\frac{L}{2}\right)^2\right] = \frac{Q\left(\frac{L}{2}\right)^3}{48EI}$$

$$\frac{(P-Q)L}{48} - \frac{1}{6}M\frac{3}{4} = \frac{QL}{48(8)}$$

$$8PL - 48M = 9QL \qquad \textbf{(1)}$$



$$\theta_A = \theta'_A + \theta''_A = 0$$

$$\curvearrowright +\quad -\frac{ML}{6EI} - \frac{ML}{3EI} + \frac{(P-Q)L^2}{16EI} = 0$$

$$8M = (P-Q)L \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2):

$$M = QL/16$$

$$Q = 2P/3$$

$$S = P/3$$

$$R = P/6$$

$$M = PL/24$$

Thus,

$$M_A = M_B = \frac{1}{24}PL \qquad \textbf{Ans.}$$

$$A_y = B_y = \frac{1}{6}P \qquad \textbf{Ans.}$$

$$C_y = D_y = \frac{1}{3}P \qquad \textbf{Ans.}$$

$$D_x = 0 \qquad \textbf{Ans.}$$

Ans:

$M_A = M_B = \frac{1}{24}PL,\ A_y = B_y = \frac{1}{6}P,$

$C_y = D_y = \frac{1}{3}P,\ D_x = 0$

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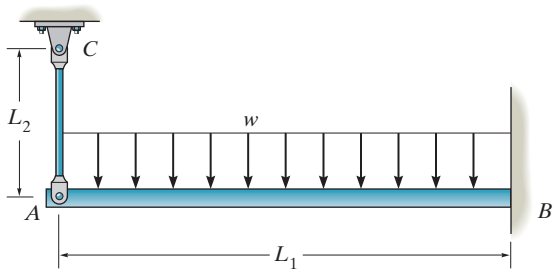
12–126.

The beam has a constant E_1I_1 and is supported by the fixed wall at B and the rod AC. If the rod has a cross-sectional area A_2 and the material has a modulus of elasticity E_2, determine the force in the rod.



SOLUTION

$$(\Delta_A)' = \frac{wL_1^4}{8E_1I_1}; \qquad \Delta_A = \frac{T_{AC}L_2}{A_2E_2}$$

$$\delta_A = \frac{T_{AC}L_1^3}{3E_1I_1}$$

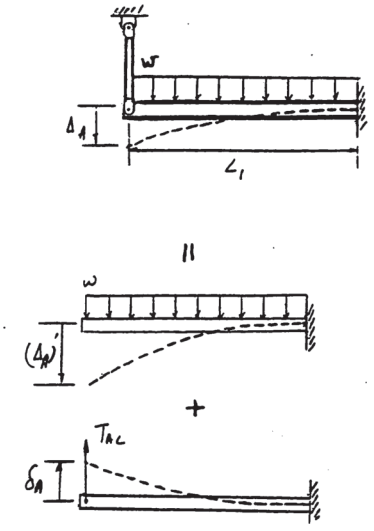
By Superposition:

$$(+\downarrow) \qquad \Delta_A = (\Delta_A)' - \delta_A$$

$$\frac{T_{AC}L_2}{A_2E_2} = \frac{wL_1^4}{8E_1I_1} - \frac{T_{AC}L_1^3}{3E_1I_1}$$

$$T_{AC}\left(\frac{L_2}{A_2E_2} + \frac{L_1^3}{3E_1I_1}\right) = \frac{wL_1^4}{8E_1I_1}$$

$$T_{AC} = \frac{3wA_2E_2L_1^4}{8(3E_1I_1L_2 + A_2E_2L_1^3)}$$



Ans.

Ans:

$$T_{AC} = \frac{3wA_2E_2L_1^4}{8(3E_1I_1L_2 + A_2E_2L_1^3)}$$

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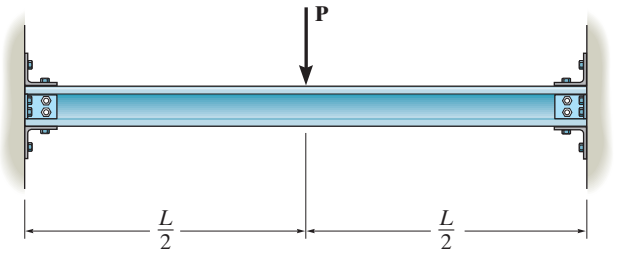
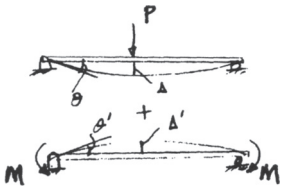
12–127.

The beam is supported by the bolted supports at its ends. When loaded these supports initially do not provide an actual fixed connection, but instead allow a slight rotation α before becoming fixed after the load is fully applied. Determine the moment at the supports and the maximum deflection of the beam.



SOLUTION



$$\theta - \theta' = \alpha$$

$$\frac{PL^2}{16EI} - \frac{ML}{3EI} - \frac{ML}{6EI} = \alpha$$

$$ML = \left(\frac{PL^2}{16EI} - \alpha\right)(2EI)$$

$$M = \frac{PL}{8} - \frac{2EI}{L}\alpha \qquad \textbf{Ans.}$$

$$\Delta_{\max} = \Delta - \Delta' = \frac{PL^3}{48EI} - 2\left[\frac{M(\frac{L}{2})}{6EIL}\left[L^2 - (L/2)^2\right]\right]$$

$$\Delta_{\max} = \frac{PL^3}{48EI} - \frac{L^2}{8EI}\left(\frac{PL}{8} - \frac{2EI\alpha}{L}\right)$$

$$\Delta_{\max} = \frac{PL^3}{192EI} + \frac{\alpha L}{4} \qquad \textbf{Ans.}$$

Ans:

$$M = \frac{PL}{8} - \frac{2EI}{L}\alpha,\ \Delta_{\max} = \frac{PL^3}{192EI} + \frac{\alpha L}{4}$$

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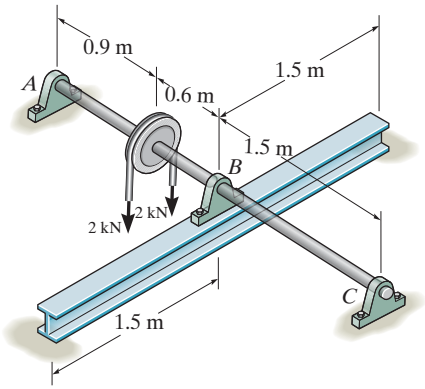
***12–128.** The 25-mm-diameter A-36 steel shaft is supported by unyielding bearings at A and C. The bearing at B rests on a simply supported steel wide-flange beam having a moment of inertia of $I = 195(10^6)\ \text{mm}^4$. If the belt loads on the pulley are 2 kN each, determine the vertical reactions at A, B, and C.



For the shaft:

$$(\Delta_b)_1 = \left[\frac{4(10^3)(0.9)(1.5)}{6EI_s(3)}\right](3^2 - 0.9^2 - 1.5^2) = \frac{1782}{EI_s}\ \downarrow$$

$$(\Delta_b)_2 = \frac{B_y(3^3)}{48EI_s} = \frac{0.5625B_y}{EI_s}\ \uparrow$$

For the beam:

$$\Delta_b = \frac{B_y(3^3)}{48EI_b} = \frac{0.5625B_y}{EI_b}$$

Compatibility condition:

$$+\downarrow \Delta_b = (\Delta_b)_1 - (\Delta_b)_2$$

$$\frac{0.5625B_y}{EI_b} = \frac{1782}{EI_s} - \frac{0.5625B_y}{EI_s}$$

$$I_s = \frac{\pi}{4}(0.0125^4) = 19.175(10^{-9})\ \text{m}^4$$

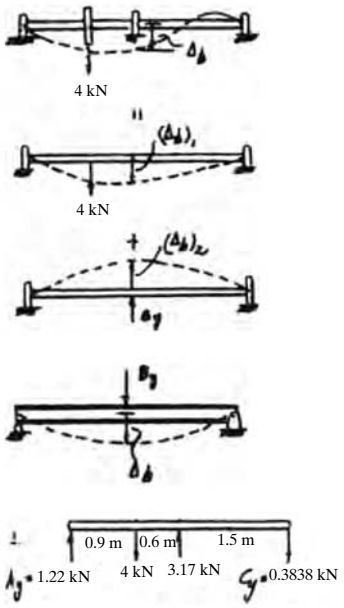
$$\frac{0.5625B_y}{E[195(10^{-6})]} = \frac{1782}{E[19.175(10^{-9})]} - \frac{0.5625B_y}{E[19.175(10^{-9})]}$$

$$B_y = 3167.69\ \text{N} = 3.17\ \text{kN}$$ **Ans.**

Form the free-body digram,

$$A_y = 1.22\ \text{kN}$$ **Ans.**

$$C_y = 0.384\ \text{kN}$$ **Ans.**



Ans:

$B_y = 3.17\ \text{kN};\ A_y = 1.22\ \text{kN};\ C_y = 0.384\ \text{kN}$

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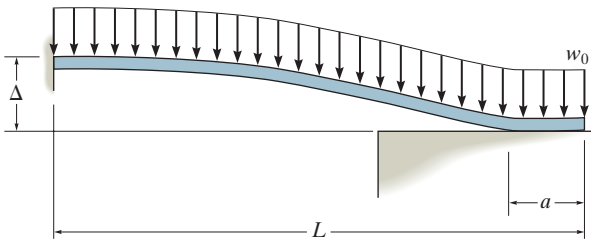
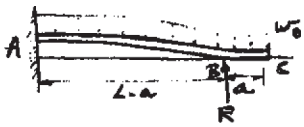
12–129.

The beam is made from a soft linear elastic material having a constant EI. If it is originally a distance Δ from the surface of its end support, determine the length a that rests on this support when it is subjected to the uniform load w_0, which is great enough to cause this to happen.



SOLUTION



The curvature of the beam in region BC is zero, therefore there is no bending moment in the region BC. The reaction R is at B where it touches the support. The slope is zero at this point and the deflection is Δ where

$$\Delta = \frac{w_0(L-a)^4}{8EI} - \frac{R(L-a)^3}{3EI}$$

$$\theta_x = 0 = \frac{w_0(L-a)^3}{6EI} - \frac{R(L-a)^2}{2EI}$$

Thus,

$$R = \frac{w_0(L-a)}{3}$$

$$\Delta = \frac{w_0(L-a)^4}{(72EI)}$$

$$L - a = \left(\frac{72\Delta EI}{w_0}\right)^{\frac{1}{4}}$$

$$a = L - \left(\frac{72\Delta EI}{w_0}\right)^{\frac{1}{4}}$$ **Ans.**

Ans:

$$a = L - \left(\frac{72\Delta EI}{w_0}\right)^{1/4}$$

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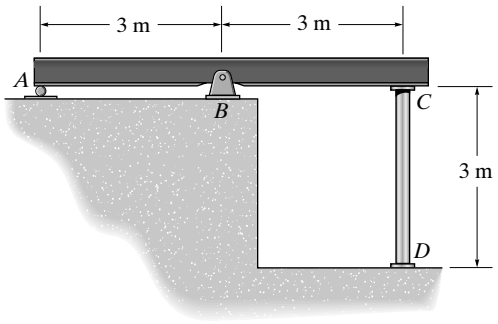
12–130. If the temperature of the 75-mm-diameter post *CD* is increased by 60°C, determine the force developed in the post. The post and the beam are made of A-36 steel, and the moment of inertia of the beam is $I = 255(10^6)\ \text{mm}^4$.



SOLUTION

Method of Superposition. Referring to Fig. *a* and the table in the Appendix, the necessary deflections are

$$(v_C)_1 = \frac{PL_{BC}{}^3}{3EI} = \frac{F_{CD}\left(3^3\right)}{3EI} = \frac{9F_{CD}}{EI} \uparrow$$

$$(v_C)_2 = (\theta_B)_2 L_{BC} = \frac{M_O L_{AB}}{3EI}(L_{BC}) = \frac{3F_{CD}(3)}{3EI}(3) = \frac{9F_{CD}}{EI} \uparrow$$

The compatibility condition at end *C* requires

$$\left(+\uparrow\right) \qquad v_C = (v_C)_1 + (v_C)_2$$

$$= \frac{9F_{CD}}{EI} + \frac{9F_{CD}}{EI} = \frac{18F_{CD}}{EI} \uparrow$$

Referring to Fig. *b*, the compatibility condition of post *CD* requires that

$$\delta_{F_{CD}} + v_C = \delta_T \qquad (1)$$

$$\delta F_{CD} = \frac{F_{CD} L_{CD}}{AE} = \frac{F_{CD}(3)}{AE}$$

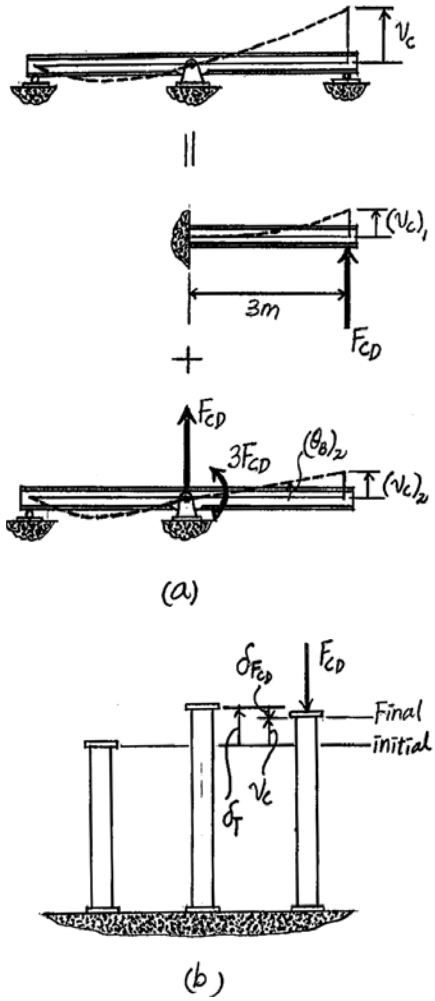
$$\delta_T = \alpha \Delta T L = 12\left(10^{-6}\right)(60)(3) = 2.16\left(10^{-3}\right)\text{m}$$

Thus, Eq. (1) becomes

$$\frac{3F_{CD}}{AE} + \frac{18F_{CD}}{EI} = 2.16\left(10^{-3}\right)$$

$$\frac{3F_{CD}}{\frac{\pi}{4}\left(0.075^2\right)} + \frac{18F_{CD}}{255\left(10^{-6}\right)} = 2.16\left(10^{-3}\right)\left[200\left(10^9\right)\right]$$

$$F_{CD} = 6061.69\text{N} = 6.06\ \text{kN}$$ **Ans.**



Ans:

$F_{CD} = 6.06\ \text{kN}$

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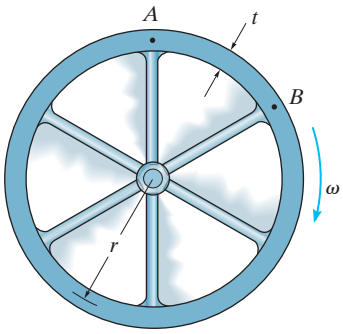
12–131.

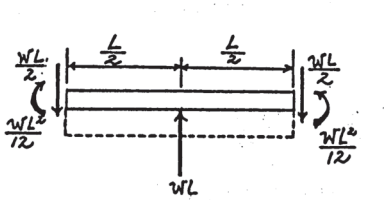
The rim on the flywheel has a thickness t, width b, and specific weight γ. If the flywheel is rotating at a constant rate of ω, determine the maximum moment developed in the rim. Assume that the spokes do not deform. *Hint*: Due to symmetry of the loading, the slope of the rim at each spoke is zero. Consider the radius to be sufficiently large so that the segment AB can be considered as a straight beam fixed at both ends and loaded with a uniform centrifugal force per unit length. Show that this force is $w = bt\gamma\omega^2 r/g$.



SOLUTION

Centrifugal Force: The centrifugal force acting on a unit length of the rim rotating at a constant rate of ω is

$$w = m\omega^2 r = bt\left(\frac{\gamma}{g}\right)\omega^2 r = \frac{bt\gamma\omega^2 r}{g} \qquad \textbf{(Q.E.D)}$$

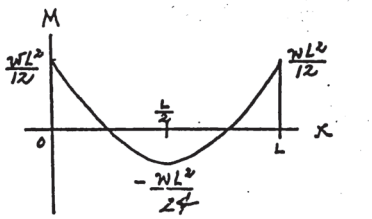


Elastic Curve: Member AB of the rim is modeled as a straight beam with both of its ends fixed and subjected to a uniform centrifugal force w.

Method of Superposition: Using the table in Appendix C, the required displacements are

$$\theta_B' = \frac{wL^3}{6EI} \qquad \theta_B'' = \frac{M_B L}{EI} \qquad \theta_B''' = \frac{B_y L^2}{2EI}$$

$$v_B' = \frac{wL^4}{8EI}\uparrow \qquad v_B'' = \frac{M_B L^2}{2EI}\uparrow \qquad v_B''' = \frac{B_y L^3}{3EI}\downarrow$$



Compatibility requires

$$0 = \theta_B' + \theta_B'' + \theta_B'''$$

$$0 = \frac{wL^3}{6EI} + \frac{M_B L}{EI} + \left(-\frac{B_y L^2}{2EI}\right)$$

$$0 = wL^2 + 6M_B - 3B_y L \qquad \textbf{(1)}$$

$(+\uparrow)$ $$0 = v_B' + v_B'' + v_B'''$$

$$0 = \frac{wL^4}{8EI} + \frac{M_B L^2}{2EI} + \left(-\frac{B_y L^3}{3EI}\right)$$

$$0 = 3wL^2 + 12M_B - 8B_y L \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields

$$B_y = \frac{wL}{2} \qquad M_B = \frac{wL^2}{12}$$

Due to symmetry, $A_y = \frac{wL}{2}$ $\qquad M_A = \frac{wL^2}{12}$

Maximum Moment: From the moment diagram, the maximum moment occurs at the two fixed end supports. With $w = \frac{bt\gamma\omega^2 r}{g}$ and $L = r\theta = \frac{\pi r}{3}$,

$$M_{\max} = \frac{wL^2}{12} = \frac{\frac{bt\gamma\omega^2 r}{g}\left(\frac{\pi r}{3}\right)^2}{12} = \frac{\pi^2 bt\gamma\omega^2 r^3}{108g} \qquad \textbf{Ans.}$$

Ans:

$$M_{\max} = \frac{\pi^2 bt\gamma\omega^2 r^3}{108g}$$

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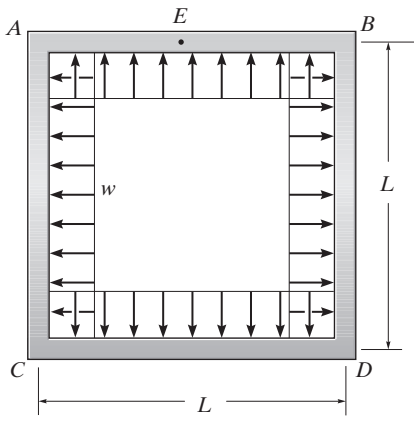
***12–132.**

The box frame is subjected to a uniform distributed loading w along each of its sides. Determine the moment developed in each corner. Neglect the deflection due to axial load. EI is constant.



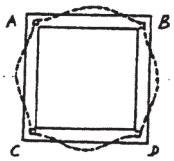
SOLUTION

Elastic Curve: In order to maintain the right angle and zero slope (due to symmetrical loading) at the four corner joints, the box frame deforms into the shape shown when it is subjected to the internal uniform distributed load. Therefore, member AB of the frame can be modeled as a beam with both ends fixed.

Method of Superposition: Using the table in Appendix C, the required displacements are

$$\theta_B' = \frac{wL^3}{6EI} \qquad \theta_B'' = \frac{M_B L}{EI} \qquad \theta_B''' = \frac{B_y L^2}{2EI}$$

$$v_B' = \frac{wL^4}{8EI}\uparrow \qquad v_B'' = \frac{M_B L^2}{2EI}\uparrow \qquad v_B''' = \frac{B_y L^3}{3EI}\downarrow$$



Compatibility conditions require

$$0 = \theta_B' + \theta_B'' + \theta_B'''$$

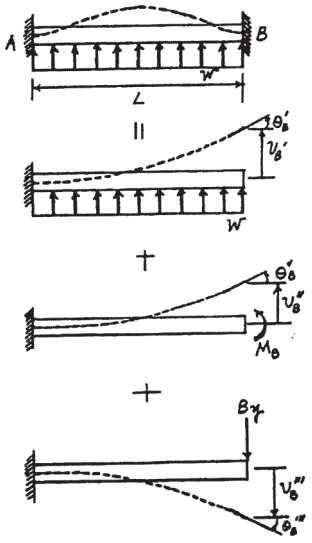
$$0 = \frac{wL^3}{6EI} + \frac{M_B L}{EI} + \left(-\frac{B_y L^2}{2EI}\right)$$

$$0 = wL^2 + 6M_B - 3B_y L \qquad \textbf{(1)}$$

$$(+\uparrow) \qquad 0 = v_B' + v_B'' + v_B'''$$

$$0 = \frac{wL^4}{8EI} + \frac{M_B L^2}{2EI} + \left(-\frac{B_y L^3}{3EI}\right)$$

$$0 = 3wL^2 + 12M_B - 8B_y L \qquad \textbf{(2)}$$



Solving Eqs. (1) and (2) yields

$$B_y = \frac{wL}{2}$$

$$M_B = \frac{wL^2}{12} \qquad \textbf{Ans.}$$

Ans:

$$M_B = \frac{wL^2}{12}$$

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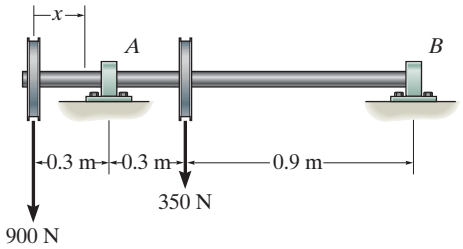
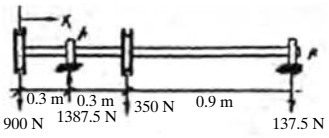
R12–1. The shaft supports the two pulley loads shown. Using discontinuity functions, determine the equation of the elastic curve. The bearings at A and B exert only vertical reactions on the shaft. EI is constant.



SOLUTION

$$M = -900\langle x - 0\rangle - (-1387.5)\langle x - 0.3\rangle - 350\langle x - 0.6\rangle$$

$$M = -900x + 1387.5\langle x - 0.3\rangle - 350\langle x - 0.6\rangle$$



Elastic curve and slope:

$$EI\frac{d^2v}{dx^2} = M = -900x + 1387.5\langle x - 0.3\rangle - 350\langle x - 0.6\rangle$$

$$EI\frac{dv}{dx} = -450x^2 + 693.75\langle x - 0.3\rangle^2 - 175\langle x - 0.6\rangle^2 + C_1$$

$$EIv = -150x^3 + 231.25\langle x - 0.3\rangle^3 - 58.333\langle x - 0.6\rangle^3 + C_1x + C_2 \qquad (1)$$

Boundary conditions:

$v = 0$ at $x = 0.3$ m

From Eq. (1)

$$0 = -4.05 + 0.3C_1 + C_2$$

$$0.3C_1 + C_2 = 4.05 \qquad (2)$$

$v = 0$ at $x = 1.5$ m

From Eq.(1)

$$0 = -506.25 + 399.6 - 42.525 + 1.5C_1 + C_2$$

$$1.5C_1 + C_2 = 149.175 \qquad (3)$$

Solving Eqs. (2) and (3) yields:

$$C_1 = 120.94 \qquad C_2 = -32.23$$

$$v = \frac{1}{EI}[-150x^3 + 231\langle x - 0.3\rangle^3 - 58.3\langle x - 0.6\rangle^3 + 121x - 32.2]\ \text{N}\cdot\text{m}^3$$ **Ans.**

Ans:

$$v = \frac{1}{EI}[-150x^3 + 231\langle x - 0.3\rangle^3 - 58.3\langle x - 0.6\rangle^3 + 121x - 32.2]\ \text{N}\cdot\text{m}^3$$

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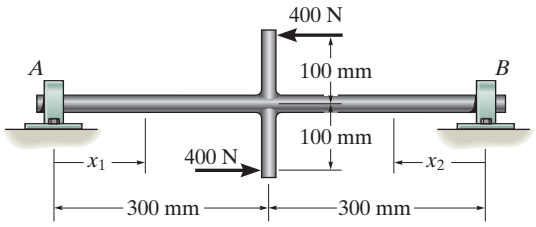
R12–2. The shaft is supported by a journal bearing at A, which exerts only vertical reactions on the shaft, and by a thrust bearing at B, which exerts both horizontal and vertical reactions on the shaft. Draw the bending-moment diagram for the shaft and then, from this diagram, sketch the deflection or elastic curve for the shaft's centerline. Determine the equations of the elastic curve using the coordinates x_1 and x_2. EI is constant.



For $M_1(x) = 133.33x_1$

$$EI\frac{d^2v_1}{dx_1^2} = 133.33x_1$$

$$EI\frac{dv_1}{dx_1} = 66.67x_1^2 + C_1 \qquad (1)$$

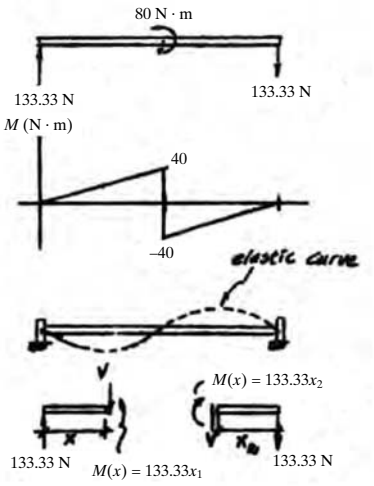
$$EIv_1 = 22.22x_1^3 + C_1x_1 + C_2 \qquad (2)$$

For $M_2(x) = -133.33x_2$

$$EI\frac{d^2v_2}{dx_2^2} = -133.33x_2$$

$$EI\frac{dv_2}{dx_2} = -66.67x_2^2 + C_3 \qquad (3)$$

$$EIv_2 = -22.22x_2^3 + C_3x_2 + C_4 \qquad (4)$$



Boundary conditions:

$v_1 = 0$ at $x_1 = 0$

From Eq.(2)

$C_2 = 0$

$v_2 = 0$ at $x_2 = 0$

$C_4 = 0$

Continuity conditions:

$$\frac{dv_1}{dx_1} = -\frac{dv_2}{dx_2} \quad \text{at} \quad x_1 = x_2 = 0.300\ \text{m}$$

From Eqs. (1) and (3)

$$6 + C_1 = -(-6 + C_3)$$

$$C_1 = -C_3 \qquad (5)$$

$v_1 = v_2$ at $x_1 = x_2 = 0.300$ m

$$0.6 + 0.3C_1 = -0.6 + 0.3C_3$$

$$C_3 - C_1 = 4 \qquad (6)$$

Solving Eqs. (5) and (6) yields:

$C_3 = 2\ \text{N}\cdot\text{m}^2 \quad C_1 = -2\ \text{N}\cdot\text{m}^2$

$$v_1 = \frac{1}{EI}(22.2x_1^3 - 2x_1)\text{N}\cdot\text{m}^3 \qquad \textbf{Ans.}$$

$$v_2 = \frac{1}{EI}(-22.2x_2^3 + 2x_2)\ \text{N}\cdot\text{m}^3 \qquad \textbf{Ans.}$$

Ans:

$$v_1 = \frac{1}{EI}(22.2x_1^3 - 2x_1)\text{N}\cdot\text{m}^3,$$

$$v_2 = \frac{1}{EI}(-22.2x_2^3 + 2x_2)\ \text{N}\cdot\text{m}^3$$

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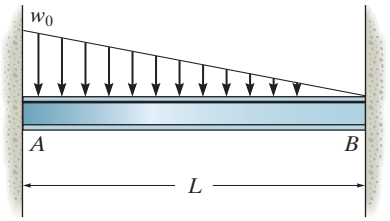
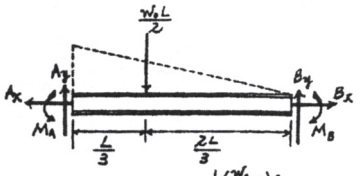
R12–3.

Determine the moment reactions at the supports A and B. Use the method of integration. EI is constant.



SOLUTION

Support Reactions: FBD(a).

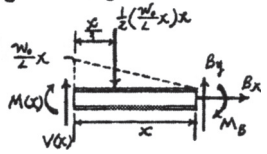


(a)

$$+\uparrow \Sigma F_y = 0; \qquad A_y + B_y - \frac{w_0 L}{2} = 0 \tag{1}$$

$$\zeta + \Sigma M_A = 0; \qquad B_y L + M_A - M_B - \frac{w_0 L}{2}\left(\frac{L}{3}\right) = 0 \tag{2}$$

Moment Function: FBD(b).



(b)

$$\zeta + \Sigma M_{\text{NA}} = 0; \qquad -M(x) - \frac{1}{2}\left(\frac{w_0}{L}x\right)x\left(\frac{x}{3}\right) - M_B + B_y x = 0$$

$$M(x) = B_y x - \frac{w_0}{6L}x^3 - M_B$$

Slope and Elastic Curve:

$$EI\frac{d^2v}{dx^2} = M(x)$$

$$EI\frac{d^2v}{dx^2} = B_y x - \frac{w_0}{6L}x^3 - M_B$$

$$EI\frac{dv}{dx} = \frac{B_y}{2}x^2 - \frac{w_0}{24L}x^4 - M_B x + C_1 \tag{3}$$

$$EI\,v = \frac{B_y}{6}x^3 - \frac{w_0}{120L}x^5 - \frac{M_B}{2}x^2 + C_1 x + C_2 \tag{4}$$

Boundary Conditions:

At $x = 0, \dfrac{dv}{dx} = 0$ From Eq. (3), $C_1 = 0$

At $x = 0, v = 0.$ From Eq. (4), $C_2 = 0$

At $x = L, \dfrac{dv}{dx} = 0.$ From Eq. (3),

$$0 = \frac{B_y L^2}{2} - \frac{w_0 L^3}{24} - M_B L$$

$$0 = 12B_y L - w_0 L^2 - 24M_B \tag{5}$$

At $x = L, v = 0.$ From Eq. (4),

$$0 = \frac{B_y L^3}{6} - \frac{w_0 L^4}{120} - \frac{M_B L^2}{2}$$

$$0 = 20B_y L - w_0 L^2 - 60M_B \tag{6}$$

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R12–3. Continued

Solving Eqs. (5) and (6) yields,

$$M_B = \frac{w_0 L^2}{30}$$ **Ans.**

$$B_y = \frac{3w_0 L}{20}$$

Substituting B_y and M_B into Eqs. (1) and (2) yields,

$$M_A = \frac{w_0 L^2}{20}$$ **Ans.**

$$A_y = \frac{7w_0 L}{20}$$

Ans:
$M_B = \frac{w_0 L^2}{30}, M_A = \frac{w_0 L^2}{20}$

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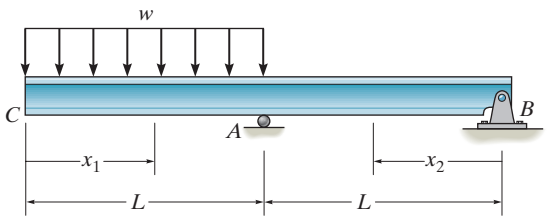
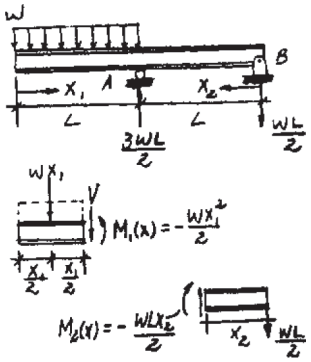
***R12–4.**

Determine the equations of the elastic curve for the beam using the x_1 and x_2 coordinates. Specify the slope at A and the maximum deflection. Use the method of integration. EI is constant.



SOLUTION



Elastic Curve and Slope:

$$EI\frac{d^2v}{dx^2} = M(x)$$

For $M_1(x) = \dfrac{-wx_1^1}{2}$

$$EI\frac{d^2v_1}{dx_1^2} = \frac{-wx_1^2}{2}$$

$$EI\frac{dv_1}{dx_1} = \frac{-wx_1^3}{6} + C_1 \qquad \textbf{(1)}$$

$$EIv_1 = \frac{-wx_1^4}{24} + C_1x_1 + C_2 \qquad \textbf{(2)}$$

For $M_2(x) = \dfrac{-wLx_2}{2}$

$$EI\frac{d^2v_2}{dx_2^2} = \frac{-wLx_2}{2}$$

$$EI\frac{dv_2}{dx_2} = \frac{-wLx_3^2}{4} + C_3 \qquad \textbf{(3)}$$

$$EIv_2 = \frac{-wLx_2^3}{12} + C_3x_2 + C_4 \qquad \textbf{(4)}$$

Boundary Conditions:

$v_2 = 0$ at $x_2 = 0$

From Eq. (4):

$C_4 = 0$

$v_2 = 0$ at $x_2 = L$

From Eq. (4):

$$0 = \frac{-wL^4}{12} + C_3L$$

$$C_3 = \frac{wL^3}{12}$$

$v_1 = 0$ at $x_1 = L$

From Eq. (2):

$$0 = -\frac{wL^4}{24} + C_1L + C_2 \qquad \textbf{(5)}$$

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***R12–4. Continued**

Continuity Conditions:

$$\frac{dv_1}{dx_1} = \frac{dv_2}{-dx_2} \qquad \text{at} \qquad x_1 = x_2 = L$$

From Eqs. (1) and (3)

$$-\frac{wL^3}{6} + C_1 = -\left(-\frac{wL^3}{4} + \frac{wL^3}{12}\right)$$

$$C_1 = \frac{wL^3}{3}$$

Substitute C_1 into Eq. (5)

$$C_2 = -\frac{7wL^4}{24}$$

$$\frac{dv_1}{dx_1} = \frac{w}{6EI}(2L^3 - x_1^3)$$

$$\frac{dv_2}{dx_2} = \frac{w}{12EI}(L^3 - 3Lx_2^2) \qquad \textbf{(6)}$$

$$\theta_A = \left.\frac{dv_1}{dx_1}\right|_{x_1 = L} = -\left.\frac{dv_2}{dx_2}\right|_{x_2 = L} = \frac{wL^3}{6EI} \qquad \textbf{Ans.}$$

$$v_1 = \frac{w}{24EI}(-x_1^4 + BL^3x_1 - 7L^4) \qquad \textbf{Ans.}$$

$$(v_1)_{\max} = \frac{-7wL^4}{24EI} \quad (x_1 = 0)$$

The negative sign indicates downward displacement.

$$v_2 = \frac{wL}{12EI}(L^2x_2 - x_2^3) \qquad \textbf{(7)} \qquad \textbf{Ans.}$$

$(v_2)_{\max}$ occurs when $\dfrac{dv_2}{dx_2} = 0$

From Eq. (6)

$$L^3 - 3Lx_2^2 = 0$$

$$x_2 = \frac{L}{\sqrt{3}}$$

Substitute x_2 into Eq. (7),

$$(v_2)_{\max} = \frac{wL^4}{18\sqrt{3}EI} \qquad \textbf{(8)}$$

$$v_{\max} = (v_1)_{\max} = -\frac{7wL^4}{24EI} \qquad \textbf{(9)} \qquad \textbf{Ans.}$$

Ans:

$$\theta_A = \frac{wL^3}{6EI},$$

$$v_1 = \frac{w}{24EI}(-x_1^4 + BL^3x_1 - 7L^4),$$

$$v_2 = \frac{wL}{12EI}(L^2x_2 - x_2^3),$$

$$v_{\max} = -\frac{7wL^4}{24EI}$$

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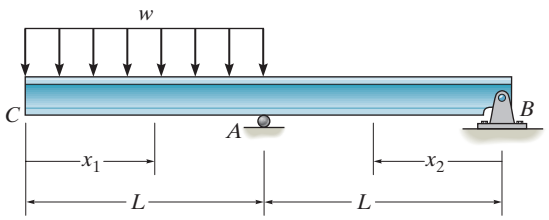
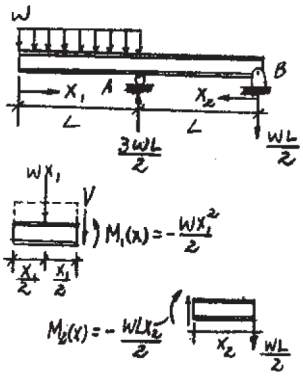
R12–5.

Determine the maximum deflection between the supports A and B. Use the method of integration. EI is constant.



SOLUTION



Elastic Curve and Slope:

$$EI\frac{d^2v}{dx^2} = M(x)$$

For $M_1(x) = \dfrac{-wx_1^2}{2}$

$$EI\frac{d^2v_1}{dx_1^2} = \frac{-wx_1^2}{2}$$

$$EI\frac{dv_1}{dx_1} = \frac{-wx_1^3}{6} + C_1 \qquad \textbf{(1)}$$

$$EIv_1 = \frac{-wx_1^4}{24} + C_1x_1 + C_2 \qquad \textbf{(2)}$$

For $M_2(x) = \dfrac{-wLx_2}{2}$

$$EI\frac{d^2v_2}{dx_2^2} = \frac{-wLx_2}{2}$$

$$EI\frac{dv_2}{dx_2} = \frac{-wLx_2^2}{4} + C_3 \qquad \textbf{(3)}$$

$$EIv_2 = \frac{-wLx_2^3}{12} + C_3x_2 + C_4 \qquad \textbf{(4)}$$

Boundary Conditions:

$v_2 = 0$ at $x_2 = 0$

From Eq. (4):

$C_4 = 0$

$v_2 = 0$ at $x_2 = L$

From Eq. (4):

$$0 = \frac{-wL^4}{12} + C_3L$$

$$C_3 = \frac{wL^3}{12}$$

$v_1 = 0$ at $x_1 = L$

From Eq. (2):

$$0 = -\frac{wL^4}{24} + C_1L + C_2 \qquad \textbf{(5)}$$

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R12–5. Continued

Continuity Conditions:

$$\frac{dv_1}{dx_1} = \frac{dv_2}{-dx_2} \qquad \text{at} \qquad x_1 = x_2 = L$$

From Eqs. (1) and (3)

$$-\frac{wL^3}{6} + C_1 = -\left(-\frac{wL^3}{4} + \frac{wL^3}{12}\right)$$

$$C_1 = \frac{wL^3}{3}$$

Substitute C_1 into Eq. (5)

$$C_2 = -\frac{7wL^4}{24}$$

$$\frac{dv_1}{dx_1} = \frac{w}{6EI}(2L^3 - x_1^3)$$

$$\frac{dv_2}{dx_2} = \frac{w}{12EI}(L^3 - 3Lx_2^2) \qquad \textbf{(6)}$$

$$\theta_A = \left.\frac{dv_1}{dx_1}\right|_{x_1 = L} = -\left.\frac{dv_2}{dx_2}\right|_{x_2 = L} = \frac{wL^3}{6EI}$$

$$v_1 = \frac{w}{24EI}(-x_1^4 + 8L^3x_1 - 7L^4)$$

$$(v_1)_{\max} = \frac{-7wL^4}{24EI} \qquad (x_1 = 0)$$

The negative sign indicates downward displacement.

$$v_2 = \frac{wL}{12EI}(L^2x_2 - x_2^3) \qquad \textbf{(7)}$$

$(v_2)_{\max}$ occurs when $\dfrac{dv_2}{dx_2} = 0$

From Eq. (6)

$$L^3 - 3Lx_2^2 = 0$$

$$x_2 = \frac{L}{\sqrt{3}}$$

Substitute x_2 into Eq. (7),

$$(v_2)_{\max} = \frac{wL^4}{18\sqrt{3}EI} \qquad \textbf{Ans.}$$

Ans:

$$(v_2)_{\max} = \frac{wL^4}{18\sqrt{3}EI}$$

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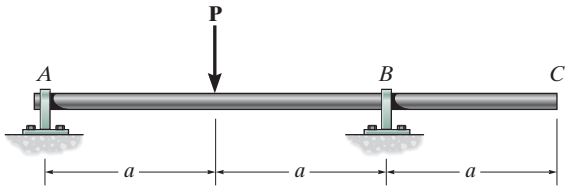
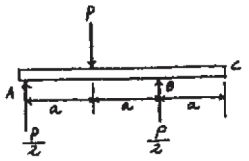
R12–6.

Determine the slope at B and the deflection at C. Use the moment-area theorems. EI is constant.



SOLUTION

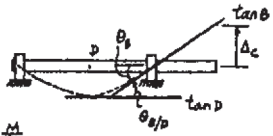


Support Reaction and Elastic Curve: As shown.

***M*/*EI* Diagram:** As shown.

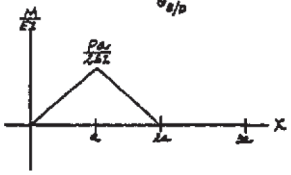
Moment-Area Theorems:

$$\theta_{B/D} = \frac{1}{2}\left(\frac{Pa}{2EI}\right)(a) = \frac{Pa^2}{4EI}$$



Due to symmetry, the slope at point D is zero. Hence, the slope at B is

$$\theta_B = |\theta_{B/D}| = \frac{Pa^2}{4EI}$$ **Ans.**



The displacement at C is

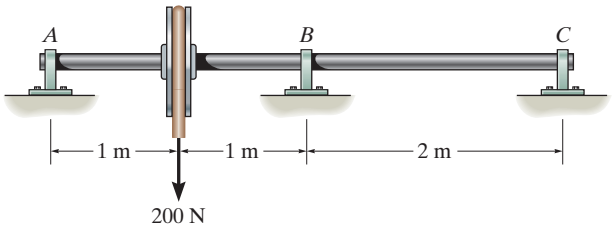
$$\Delta_C = \theta_B L_{BC} = \frac{Pa^2}{4EI}(a) = \frac{Pa^3}{4EI}\uparrow$$ **Ans.**

Ans:

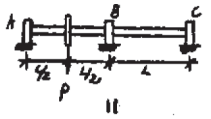
$$\theta_B = \frac{Pa^2}{4EI},\ \Delta_C = \frac{Pa^3}{4EI}\uparrow$$

R12–7.

Determine the reactions, then draw the shear and moment diagrams. Use the moment-area theorems. *EI* is constant.

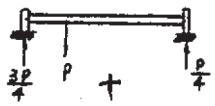


SOLUTION



$$(t_{B/A})_1 = \frac{1}{2}\left(\frac{3PL}{8EI}\right)\left(\frac{L}{2}\right)\left(\frac{L}{2}+\frac{L}{6}\right) + \frac{1}{2}\left(\frac{PL}{8EI}\right)\left(\frac{L}{2}\right)\left(\frac{L}{3}\right) + \frac{PL}{4EI}\left(\frac{L}{2}\right)\left(\frac{L}{4}\right) = \frac{5PL^3}{48EI}$$

$$(t_{C/A})_1 = \frac{1}{2}\left(\frac{3PL}{8EI}\right)\left(\frac{L}{2}\right)\left(\frac{3L}{2}+\frac{L}{6}\right) + \frac{1}{2}\left(\frac{3PL}{8EI}\right)\left(\frac{3L}{2}\right)(L) = \frac{7PL^3}{16EI}$$

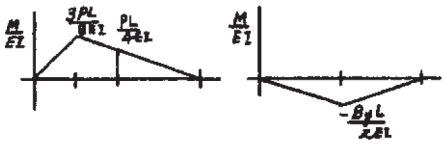


$$(t_{B/A})_2 = \frac{1}{2}\left(\frac{-B_yL}{2EI}\right)(L)\left(\frac{L}{3}\right) = \frac{-B_yL^3}{12EI}$$

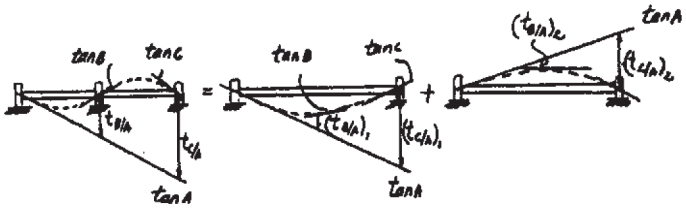
$$(t_{C/A})_2 = \frac{1}{2}\left(\frac{-B_yL}{2EI}\right)(2L)(L) = \frac{-B_yL^3}{2EI}$$

$$2t_{B/A} = t_{C/A}$$

$$2[(t_{B/A})_1 + (t_{B/A})_2] = (t_{C/A})_1 + (t_{C/A})_2$$



$$2\left[\frac{5PL^3}{48EI} + \left(\frac{-B_yL^3}{12EI}\right)\right] = \frac{7PL^3}{16EI} + \left(\frac{-B_yL^3}{2EI}\right)$$



$$B_y = \frac{11}{16}P$$

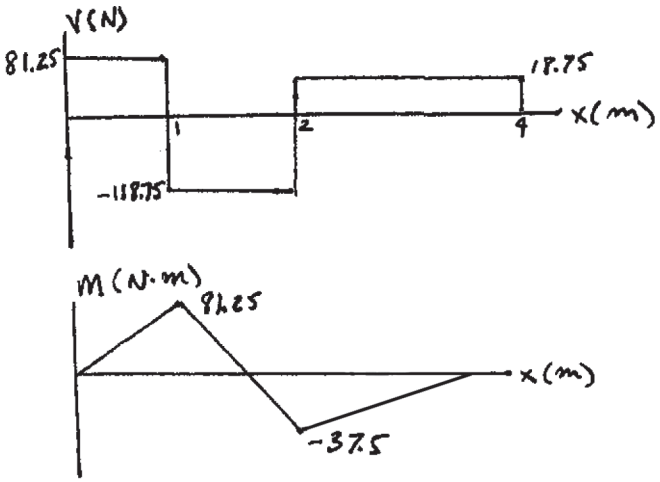
Thus,

$$B_y = \frac{11}{16}(200) = 138\ \text{N} \uparrow$$ **Ans.**

As shown on the free-body diagram

$A_y = 81.3\ \text{N} \uparrow$ **Ans.**

$C_y = 18.8\ \text{N} \downarrow$ **Ans.**



Ans:
$B_y = 138\ \text{N}\uparrow$, $A_y = 81.3\ \text{N}\uparrow$, $C_y = 18.8\ \text{N}\downarrow$

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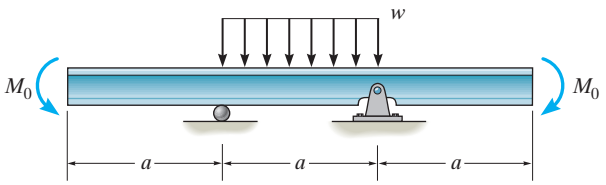
***R12–8.**

Using the method of superposition, determine the magnitude of $\mathbf{M}_0$ in terms of the distributed load w and dimension a so that the deflection at the center of the beam is zero. EI is constant.



SOLUTION

$$(\Delta_C)_1 = \frac{5wa^4}{384EI}\downarrow$$

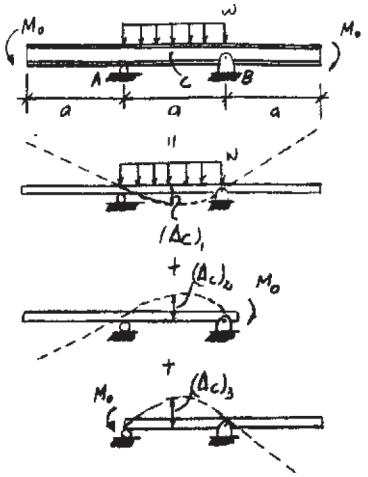
$$(\Delta_C)_2 = (\Delta_C)_3 = \frac{M_0a^2}{16EI}\uparrow$$

$$\Delta_C = 0 = (\Delta_C)_1 + (\Delta_C)_2 + (\Delta_C)_3$$

$$+\uparrow \qquad 0 = \frac{-5wa^4}{384EI} + \frac{M_0a^2}{8EI}$$

$$M_0 = \frac{5wa^2}{48}$$

Ans.



Ans:

$$M_0 = \frac{5wa^2}{48}$$

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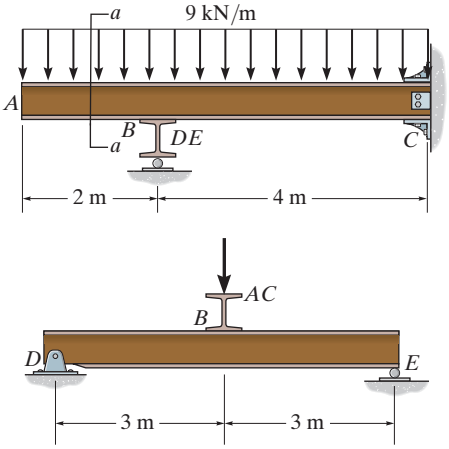
***12–9.** Beam ABC is supported by beam DBE and fixed at C. Determine the reactions at B and C. The beams are made of the same material having a modulus of elasticity $E = 200$ GPa, and the moment of inertia of both beams is $I = 25.0(10^6)\ \text{mm}^4$.



Section $a-a$

SOLUTION

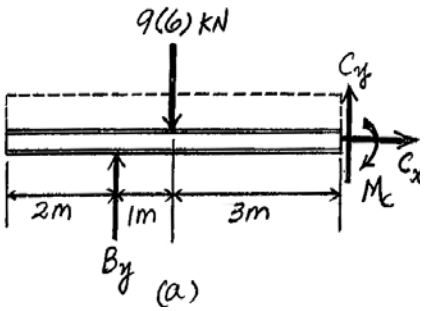
Equation of Equilibrium. Referring to the free-body diagram of the beam, Fig. *a*,

$\stackrel{+}{\rightarrow}\Sigma F_x = 0; \qquad C_x = 0$ **Ans.**

$+\uparrow \Sigma F_y = 0; \qquad B_y + C_y - 9(6) = 0 \qquad (1)$

$\text{a}+\Sigma M_C = 0; \qquad 9(6)(3) - B_y(4) - M_C = 0$

$M_C = 162 - 4B_y \qquad (2)$

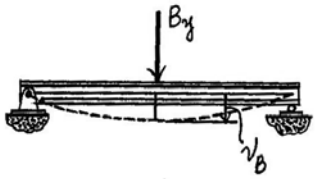


Method of superposition: Referring to Fig. *b* and the table in the appendix, the deflections are

$$v_B = \frac{PL_{DE}^3}{48EI} = \frac{B_y(6^3)}{48EI} = \frac{4.5B_y}{EI} \downarrow$$

$$(v_B)_1 = \frac{wx^2}{24EI}\left(x^2 - 4Lx + 6L^2\right) = \frac{9(4^2)}{24EI}\left[4^2 - 4(6)(4) + 6(6^2)\right]$$

$$= \frac{816\ \text{kN}\cdot\text{m}^3}{EI} \downarrow$$



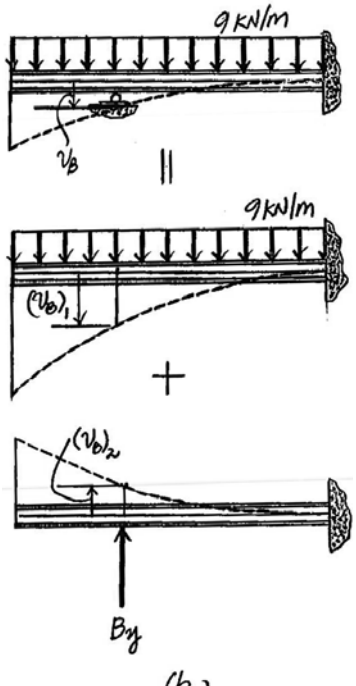
$$(v_B)_2 = \frac{PL_{BC}^3}{3EI} = \frac{B_y(4^3)}{3EI} = \frac{21.3333B_y}{EI} \uparrow$$

The compatibility condition at support B requires that

$(+\downarrow) \qquad v_B = (v_B)_1 + (v_B)_2$

$$\frac{4.5B_y}{EI} = \frac{816}{EI} + \left(-\frac{21.3333B_y}{EI}\right)$$

$B_y = 31.59\ \text{kN} = 31.6\ \text{kN}$ **Ans.**



Substituting the result of B_y into Eqs. (1) and (2),

$M_C = 35.65\ \text{kN}\cdot\text{m} = 35.7\ \text{kN}\cdot\text{m}$ **Ans.**

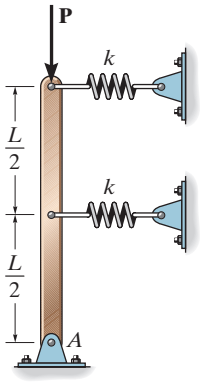
$C_y = 22.41\ \text{kN} = 22.4\ \text{kN}$ **Ans.**

Ans:

$C_x = 0$, $B_y = 31.6$ kN, $M_C = 35.7\ \text{kN}\cdot\text{m}$,
$C_y = 22.4$ kN

13–1.

Determine the critical buckling load for the column. The material can be assumed rigid.



SOLUTION

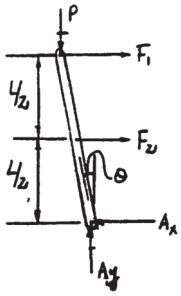
$F_1 = k(L\,\theta); \qquad F_2 = k\left(\frac{L}{2}\theta\right)$

$\zeta+\Sigma M_A = 0; \qquad P(\theta)(L) - (F_1 L) - F_2\left(\frac{L}{2}\right) = 0$

$P(\theta)(L) - kL^2\,\theta - k\left(\frac{L}{2}\right)^2\theta = 0$

Require:

$P_{cr} = kL + \frac{kL}{4} = \frac{5kL}{4}$ **Ans.**



These solutions represent a preliminary version of the Instructors' Solutions Manual (ISM). It is possible and even likely that at this preliminary stage of preparing the ISM there are some omissions and errors in the draft solutions. These will be corrected and this manual will be republished.

Ans:
$P_{cr} = \frac{5KL}{4}$

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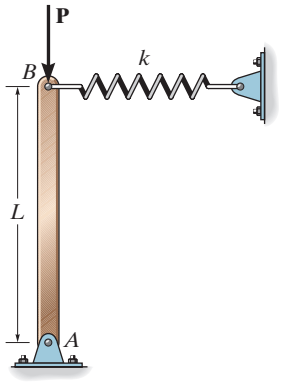
13–2.

The column consists of a rigid member that is pinned at its bottom and attached to a spring at its top. If the spring is unstretched when the column is in the vertical position, determine the critical load that can be placed on the column.



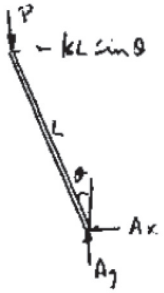
SOLUTION

$\zeta+\Sigma M_A = 0; \qquad PL\sin\theta - (kL\sin\theta)(L\cos\theta) = 0$

$$P = kL\cos\theta$$

Since θ is small $\qquad \cos\theta = 1$

$$P_{cr} = kL \qquad \textbf{Ans.}$$



Ans:
$P_{cr} = kL$

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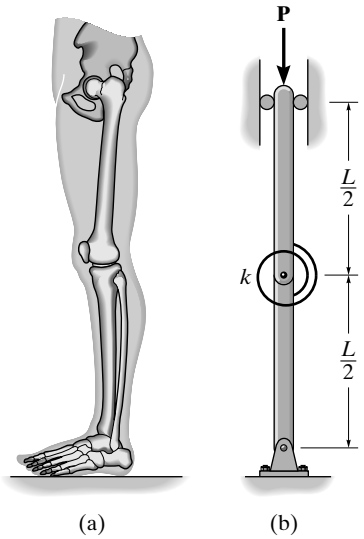
13–3. The leg in (a) acts as a column and can be modeled (b) by the two pin-connected members that are attached to a torsional spring having a stiffness k (torque/rad). Determine the critical buckling load. Assume the bone material is rigid.



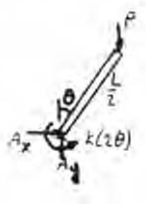
(a) (b)

SOLUTION

$$\zeta + \Sigma M_A = 0; \qquad -P(\theta)\left(\frac{L}{2}\right) + 2\,k\,\theta = 0$$

Require:

$$P_{\text{cr}} = \frac{4\,k}{L} \qquad \textbf{Ans.}$$



Ans:

$$P_{\text{cr}} = \frac{4\,k}{L}$$

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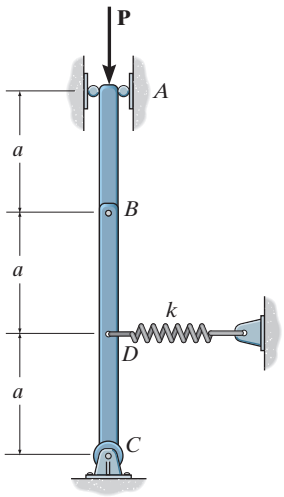
***13–4.**

Rigid bars AB and BC are pin connected at B. If the spring at D has a stiffness k, determine the critical load P_{cr} that can be applied to the bars.



SOLUTION

Equilibrium: The disturbing force F can be related to P by considering the equilibrium of joint A and then the equilibrium of member BC.

Joint A (Fig. b)

$$+\uparrow \Sigma F_y = 0; \qquad F_{AB}\cos\phi - P = 0 \qquad F_{AB} = \frac{P}{\cos\phi}$$

Member BC (Fig. c)

$$\Sigma M_C = 0;\ F(a\cos\theta) - \frac{P}{\cos\phi}\cos\phi\,(2a\sin\theta) - \frac{P}{\cos\phi}\sin\phi(2a\cos\theta) = 0$$

$$F = 2P(\tan\theta + \tan\phi)$$

Since θ and ϕ are small, $\tan\theta \cong \theta$ and $\tan\phi \cong \phi$. Thus,

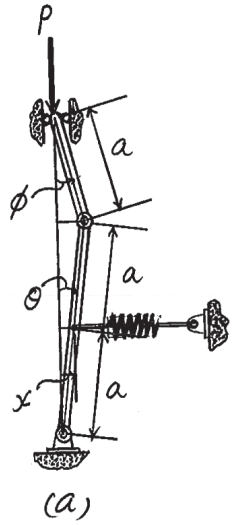
$$F = 2P(\theta + \phi) \qquad \textbf{(1)}$$

Also, from the geometry shown in Fig. a,

$$2a\theta = a\phi \qquad \phi = 2\theta$$

Thus Eq. (1) becomes

$$F = 2P(\theta + 2\theta) = 6P\theta$$



(a)

Spring Force: The restoring spring force F_{sp} can be determined using the spring formula, $F_{sp} = kx$, where $x = a\theta$, Fig. a. Thus,

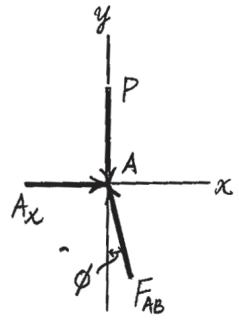
$$F_{sp} = kx = ka\theta$$

Critical Buckling Load: When the mechanism is on the verge of buckling the disturbing force F must be equal to the restoring spring force F_{sp}.

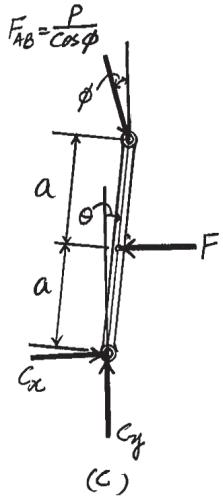
$$F = F_{sp}$$

$$6P_{cr}\theta = ka\theta$$

$$P_{cr} = \frac{ka}{6} \qquad \textbf{Ans.}$$



(b)



(c)

Ans:

$$P_{cr} = \frac{ka}{6}$$

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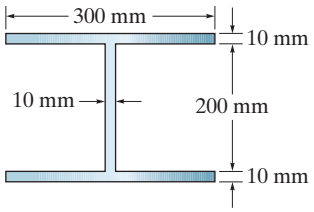
13–5.

A 2014-T6 aluminum alloy column has a length of 6 m and is fixed at one end and pinned at the other. If the cross-sectional area has the dimensions shown, determine the critical load. $\sigma_Y = 250$ MPa.



SOLUTION

Section Properties: For the cross-section shown

$$A = 0.3(0.22) - 0.29(0.2) = 8.00\left(10^{-3}\right)\ \text{m}^2$$

$$I_x = \frac{1}{12}(0.3)\left(0.22^3\right) - \frac{1}{12}(0.29)\left(0.2^3\right) = 72.8667\left(10^{-6}\right)\ \text{m}^4$$

$$I_y = 2\left[\frac{1}{12}(0.01)\left(0.3^3\right)\right] + \frac{1}{12}(0.2)\left(0.01^3\right) = 45.0167\left(10^{-6}\right)\ \text{m}^4 \text{ (controls)}$$

Critical Buckling Load: $K = 0.7$ for the column with one end fixed and the other end pinned. For 2014-T6 aluminium alloy, $E = 73.1$ GPa. Applying Euler's formula,

$$P_{\text{cr}} = \frac{\pi^2 EI}{(KL)^2}$$

$$= \frac{\pi^2\left[73.1\left(10^9\right)\right]\left[45.0167\left(10^{-6}\right)\right]}{[0.7(6)]^2}$$

$$= 1.8412\left(10^6\right)\ \text{N} = 1.84\ \text{MN} \qquad \textbf{Ans.}$$

Critical Stress: Euler's formula valid only if $\sigma_{\text{cr}} < \sigma_Y$. For 2014-T6 aluminium alloy, $\sigma_Y = 414$ MPa.

$$\sigma_{\text{cr}} = \frac{P_{\text{cr}}}{A} = \frac{1.8412\left(10^6\right)}{8.00\left(10^{-3}\right)} = 230.15\ \text{MPa} < \sigma_Y. \qquad \textbf{(O.K!)}$$

Ans:
$P_{\text{cr}} = 1.84$ MN

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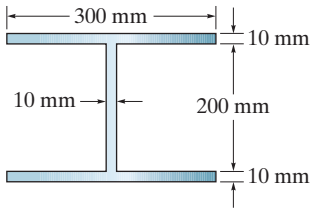
13–6.

Solve Prob. 13–5 if the column is pinned at its top and bottom.



SOLUTION

Section Properties: For the cross-section shown

$$A = 0.3(0.22) - 0.29(0.2) = 8.00\left(10^{-3}\right)\ \text{m}^2$$

$$I_x = \frac{1}{12}(0.3)\left(0.32^3\right) - \frac{1}{12}(0.29)\left(0.2^3\right) = 72.8667\left(10^{-6}\right)\ \text{m}^4$$

$$I_y = 2\left[\frac{1}{12}(0.01)\left(0.3^3\right)\right] + \frac{1}{12}(0.2)\left(0.01^3\right) = 45.0167\left(10^{-6}\right)\ \text{m}^4 \qquad \text{(controls!)}$$

Critical Buckling Load: $K = 1.0$ for the column pinned at both ends. For 2014-T6 aluminium alloy, $E = 73.1$ GPa. Applying Euler's formula,

$$P_{\text{cr}} = \frac{\pi^2 EI}{(KL)^2}$$

$$= \frac{\pi^2\left[73.1\left(10^9\right)\right]\left[45.0167\left(10^{-6}\right)\right]}{[1.0(6)]^2}$$

$$= 902.17\left(10^3\right)\ \text{N} = 902\ \text{kN} \qquad \textbf{Ans.}$$

Critical Stress: Euler's formula valid only if $\sigma_{\text{cr}} < \sigma_Y$. For 2014-T6 aluminium alloy, $\sigma_Y = 414$ MPa.

$$\sigma_{\text{cr}} = \frac{P_{\text{cr}}}{A} = \frac{902.17\left(10^3\right)}{8.00\left(10^{-3}\right)} = 112.77\ \text{MPa} < \sigma_Y \qquad \textbf{(O.K!)}$$

Ans:
$P_{\text{cr}} = 902\ \text{kN}$

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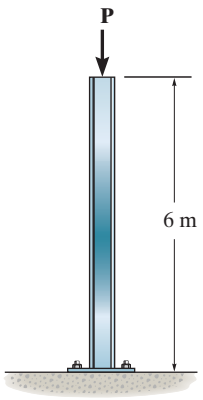
13–7. The W360 × 57 column is made of A-36 steel and is fixed supported at its base. If it is subjected to an axial load of $P = 75$ kN, determine the factor of safety with respect to buckling.



SOLUTION

From the table in appendix, the cross-sectional area and moment of inertia about weak axis (y-axis) for W360 × 57 are

$$A = 7200\ \text{mm}^2 = 7.2(10^{-3})\ \text{m}^2 \qquad I_y = 11.1(10^6)\ \text{mm}^4 = 11.1(10^{-6})\ \text{m}^4$$

The column is fixed at its base and free at top, $k = 2$. Here, the column will buckle about the weak axis (y axis). For A36 steel, $E = 200$ GPa and $\sigma_Y = 250$ MPa. Applying Euler's formula,

$$P_{cr} = \frac{\pi^2 E I_y}{(KL)^2} = \frac{\pi^2[200(10^9)][11.1(10^{-6})]}{[2(6)]^2} = 152.16(10^3)\ \text{N} = 152.16\ \text{kN}$$

Thus, the factor of safety with respect to buckling is

$$F.S = \frac{P_{cr}}{P} = \frac{152.16}{75} = 2.03 \qquad \textbf{Ans.}$$

The Euler's formula is valid only if $\sigma_{cr} < \sigma_Y$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{152.16(10^3)}{7.2(10^{-3})} = 21.13(10^6)\ \text{N-m}^2 = 21.1\ \text{MPa} < \sigma_Y$$
$$= 250\ \text{MPa} \qquad \text{O.K.}$$

Ans:
F.S. = 2.03

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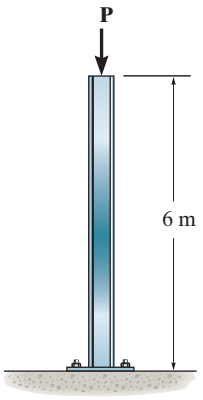
***13–8.** The W360 × 57 column is made of A-36 steel. Determine the critical load if its bottom end is fixed supported and its top is free to move about the strong axis and is pinned about the weak axis.



SOLUTION

From the table in appendix, the cross-sectional area and moment of inertia about weak axis (y-axis) for W360 × 57 are

$$A = 7200\ \text{mm}^2 = 7.2(10^{-3})\ \text{m}^2$$

$$I_x = 160(10^6)\ \text{mm}^4 = 160(10^{-6})\ \text{m}^4 \qquad I_y = 11.1(10^6)\ \text{mm}^4 = 11.1(10^{-6})\ \text{m}^4$$

The column is fixed at its base and free at top about strong axis. Thus, $k_x = 2$. For A36 steel, $E = 200$ GPa and $\sigma_\gamma = 250$ MPa.

$$P_{cr} = \frac{\pi^2 EI_x}{(K_x L_x)^2} = \frac{\pi^2[200(10^9)][160(10^{-6})]}{[2(6)]^2} = 2.193(10^6)\ \text{N} = 2.193\ \text{MN}$$

The column is fixed at its base and pinned at top about weak axis. Thus, $k_y = 0.7$.

$$P_{cr} = \frac{\pi^2 EI_y}{(K_y L_y)^2} = \frac{\pi^2[200(10^9)][11.1(10^{-6})]}{[0.7(6)]^2}$$

$$= 1.242(10^6)\ \text{N} = 1.24\ \text{MN} \quad (\textit{Control}) \qquad \textbf{Ans.}$$

The Euler's formula is valid only if $\sigma_{cr} < \sigma_Y$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{1.242(10^6)}{7.2(10^{-3})} = 172.51(10^6)\ \text{N/m}^2 = 172.51\ \text{MPa} < \sigma_Y$$

$$= 250\ \text{MPa} \qquad \text{O.K.}$$

Ans:
$P_{cr} = 1.24$ MN (*Control*)

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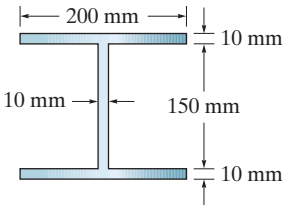
13–9.

A steel column has a length of 9 m and is fixed at both ends. If the cross-sectional area has the dimensions shown, determine the critical load. $E_{st} = 200$ GPa, $\sigma_Y = 250$ MPa.



SOLUTION

Section Properties:

$$A = 0.2(0.17) - 0.19(0.15) = 5.50(10^{-3})\ \text{m}^2$$

$$I_x = \frac{1}{12}(0.2)(0.17^3) - \frac{1}{12}(0.19)(0.15^3) = 28.44583(10^{-6})\ \text{m}^4$$

$$I_y = 2\left[\frac{1}{12}(0.01)(0.2^3)\right] + \frac{1}{12}(0.15)(0.01^3) = 13.34583(10^{-6})\ \text{m}^4 \quad \text{(Controls!)}$$

Critical Buckling Load: $K = 0.5$ for fixed support ends column.
Applying *Euler's* formula,

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

$$= \frac{\pi^2 (200)(10^9)(13.34583)(10^{-6})}{[0.5(9)]^2}$$

$$= 1300919\ \text{N} = 1.30\ \text{MN} \qquad \textbf{Ans.}$$

Critical Stress: *Euler's* formula is only valid if $\sigma_{cr} < \sigma_Y$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{1300919}{5.50(10^{-3})} = 236.53\ \text{MPa} < \sigma_Y = 250\ \text{MPa} \qquad \textbf{(O.K!)}$$

Ans:
$P_{cr} = 1.30$ MN

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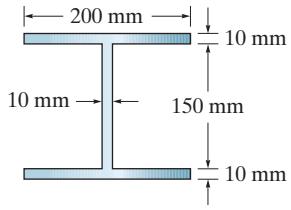
13–10.

A steel column has a length of 9 m and is pinned at its top and bottom. If the cross-sectional area has the dimensions shown, determine the critical load. $E_{st} = 200$ GPa, $\sigma_Y = 250$ MPa.



SOLUTION

Section Properties:

$$A = 0.2(0.17) - 0.19(0.15) = 5.50(10^{-3})\ \text{m}^2$$

$$I_x = \frac{1}{12}(0.2)(0.17^3) - \frac{1}{12}(0.19)(0.15^3) = 28.44583(10^{-6})\ \text{m}^4$$

$$I_y = 2\left[\frac{1}{12}(0.01)(0.2^3)\right] + \frac{1}{12}(0.15)(0.01^3) = 13.34583(10^{-6})\ \text{m}^4 \text{ (Controls!)}$$

Critical Buckling Load: $K = 1$ for pin supported ends column.
Applying *Euler's* formula,

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

$$= \frac{\pi^2 (200)(10^9)(13.34583)(10^{-6})}{[1(9)]^2}$$

$$= 325229.87\ \text{N} = 325\ \text{kN} \qquad \textbf{Ans.}$$

Critical Stress: *Euler's* formula is only valid if $\sigma_{cr} < \sigma_Y$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{325229.87}{5.50(10^{-3})} = 59.13\ \text{MPa} < \sigma_Y = 250\ \text{MPa} \qquad \textbf{(O.K!)}$$

Ans:
$P_{cr} = 325$ kN

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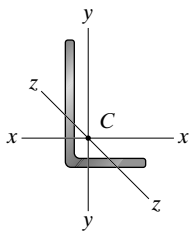
13–11. The A-36 steel angle has a cross-sectional area of $A = 1550\ \text{mm}^2$ and a radius of gyration about the x axis of $r_x = 31.5$ mm and about the y axis of $r_y = 21.975$ mm. The smallest radius of gyration occurs about the z axis and is $r_z = 16.1$ mm. If the angle is to be used as a pin-connected 3-m long column, determine the largest axial load that can be applied through its centroid C without causing it to buckle.



SOLUTION

The least radius of gyration:

$r_z = 16.1\ \text{mm} = 0.0161\ \text{m}$ controls.

$$\sigma_{\text{cr}} = \frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2}; \qquad K = 1.0$$

$$= \frac{\pi^2[200(10^9)]}{\left[\frac{1.0(3)}{0.0161}\right]^2} = 56.85(10^6)\ \text{N-m}^2 = 56.85\ \text{MPa} < \sigma_Y = 250\ \text{MPa} \qquad \text{O.K.}$$

$$P_{cr} = \sigma_{cr} A = [56.85(10^6)][1.55(10^{-3})] = 88.12(10^3)\ \text{N} = 88.1\ \text{kN} \qquad \textbf{Ans.}$$

Ans:
$P_{cr} = 88.1$ kN

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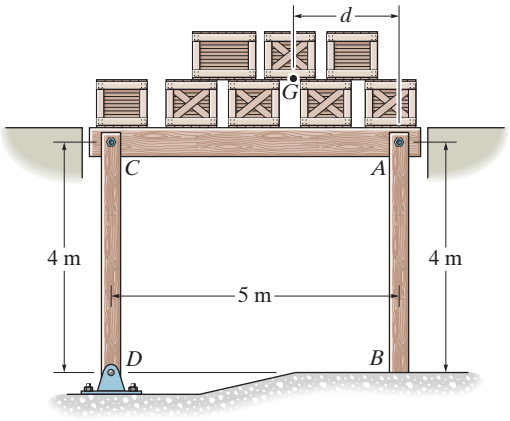
***13–12.**

The deck is supported by the two 40-mm-square columns. Column AB is pinned at A and fixed at B, whereas CD is pinned at C and D. If the deck is prevented from sidesway, determine the greatest weight of the load that can be applied without causing the deck to collapse. The center of gravity of the load is located at $d = 2$ m. Both columns are made from Douglas Fir.



SOLUTION

$$\circlearrowleft + \Sigma M_C = 0; \qquad F_{AB}(5) - W(3) = 0$$

$$F_{AB} = 0.6\,W$$

$$+\uparrow \Sigma F_y = 0; \qquad F_{CD} + 0.6\,W - W = 0$$

$$F_{CD} = 0.4\,W$$

Column CD:

$$P_{\text{cr}} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2(13.1)\left(10^9\right)\left(\frac{1}{12}\right)(0.04)^4}{(1(4))^2} = 0.4\,W$$

$$W = 4.31 \text{ kN}$$

Column AB:

$$P_{\text{cr}} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2(13.1)\left(10^9\right)\left(\frac{1}{12}\right)(0.04)^4}{(0.7(4))^2} = 0.6\,W$$

$$W = 5.86 \text{ kN}$$

Thus,

$$W = 4.31 \text{ kN}$$ **Ans.**

Ans:
$W = 4.31$ kN

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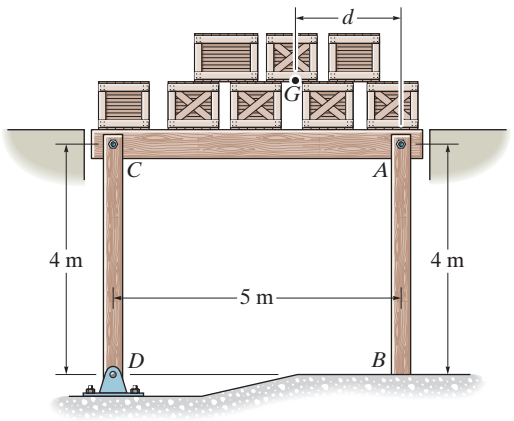
13–13.

The deck is supported by the two 40-mm-square columns. Column AB is pinned at A and fixed at B, whereas CD is pinned at C and D. If the deck is prevented from sidesway, determine the position d of the center of gravity of the load and the load's greatest magnitude without causing the deck to collapse. Both columns are made from Douglas Fir.



SOLUTION

Column CD:

$$P_{\text{cr}} = \frac{\pi^2 EI}{(KL)^2}$$

$$F_{CD} = \frac{\pi^2(13.1)(10^9)\left(\frac{1}{12}\right)(0.04)^4}{(1.0(4))^2} = 1.7239(10^3)\ \text{N}$$

Column AB:

$$P_{\text{cr}} = \frac{\pi^2 EI}{(KL)^2}$$

$$F_{AB} = \frac{\pi^2(13.1)(10^9)\left(\frac{1}{12}\right)(0.04)^4}{(0.7(4))^2} = 3.5181(10^3)\ \text{N}$$

Thus,

$$+\uparrow \Sigma F_y = 0; \qquad 1.7239(10^3) + 3.5181(10^3) - W = 0$$

$$W = 5.2420(10^3)\ \text{N} = 5.24\ \text{kN} \qquad \textbf{Ans.}$$

$$\circlearrowleft + \Sigma M_A = 0; \qquad 5.2420(10^3)(d) - 1.7239(10^3)(5) = 0$$

$$d = 1.64\ \text{m} \qquad \textbf{Ans.}$$

Ans:
$W = 5.24$ kN,
$d = 1.64$ m

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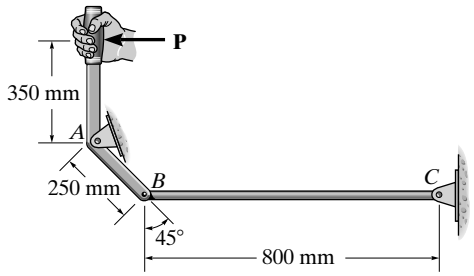
13–14. Determine the maximum force P that can be applied to the handle so that the A-36 steel control rod BC does not buckle. The rod has a diameter of 25 mm.



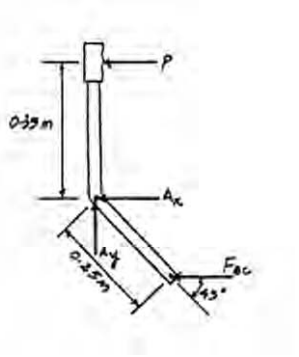
SOLUTION

Support Reactions:

$$\zeta + \Sigma M_A = 0; \qquad P(0.35) - F_{BC}\sin 45°(0.25) = 0$$

$$F_{BC} = 1.9799P$$

Section Properties:



$$A = \frac{\pi}{4}\left(0.025^2\right) = 0.15625\left(10^{-3}\right)\pi \text{ m}^2$$

$$I = \frac{\pi}{4}\left(0.0125^4\right) = 19.17476\left(10^{-9}\right) \text{m}^4$$

Critical Buckling Load: $K = 1$ for a column with both ends pinned. Appyling *Euler's* formula,

$$P_{\text{cr}} = F_{BC} = \frac{\pi^2 EI}{(KL_{BC})^2}$$

$$1.9799P = \frac{\pi^2(200)(10^9)\left[19.17476(10^{-9})\right]}{[1(0.8)]^2}$$

$$P = 29\,870 \text{ N} = 29.9 \text{ kN} \qquad \textbf{Ans.}$$

Critical Stress: *Euler's* formula is only valid if $\sigma_{\text{cr}} < \sigma_\gamma$.

$$\sigma_{\text{cr}} = \frac{P_{\text{cr}}}{A} = \frac{1.9799(29\,870)}{0.15625(10^{-3})\pi} = 120.5 \text{ MPa} < \sigma_\gamma = 250 \text{ MPa} \qquad \text{O.K.}$$

Ans:
$P = 29.9$ kN

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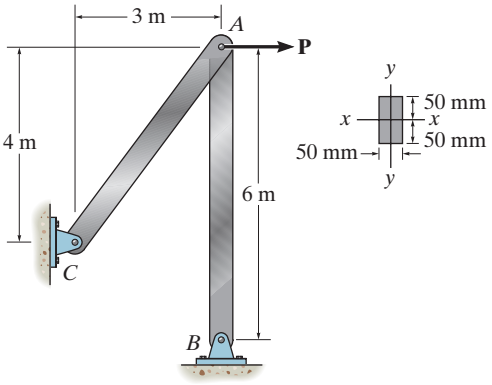
13–15.

Determine the maximum load P the frame can support without buckling member AB. Assume that AB is made of steel and is pinned at its ends for y–y axis buckling and fixed at its ends for x–x axis buckling. $E_{st} = 200$ GPa, $\sigma_Y = 360$ MPa.



SOLUTION

$$\stackrel{+}{\rightarrow}\Sigma F_x = 0; \qquad -F_{AC}\left(\frac{3}{5}\right) + P = 0$$

$$F_{AC} = \frac{5}{3}P$$

$$+\uparrow \Sigma F_y = 0; \qquad F_{AB} - \frac{5}{3}P\left(\frac{4}{5}\right) = 0$$

$$F_{AB} = \frac{4}{3}P$$

$$I_y = \frac{1}{12}(0.10)(0.05)^3 = 1.04167\left(10^{-6}\right)\text{m}^4$$

$$I_x = \frac{1}{12}(0.05)(0.10)^3 = 4.16667\left(10^{-6}\right)\text{m}^4$$

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

x–x axis buckling:

$$P_{cr} = \frac{\pi^2(200)\left(10^9\right)(4.16667)\left(10^{-6}\right)}{(0.5(6))^2} = 914\text{ kN}$$

y–y axis buckling:

$$P_{cr} = \frac{\pi^2(200)\left(10^9\right)(1.04167)\left(10^{-6}\right)}{(1(6))^2} = 57.12\text{ kN}$$

y–y axis buckling controls

$$\frac{4}{3}P = 57.12$$

$$P = 42.8\text{ kN} \qquad \textbf{Ans.}$$

Check:

$$\sigma_{cr} = \frac{P}{A} = \frac{57.12\left(10^3\right)}{(0.1)(0.05)} = 11.4\text{ MPa} < \sigma_Y \qquad \textbf{OK}$$

Ans:
$P = 42.8$ kN

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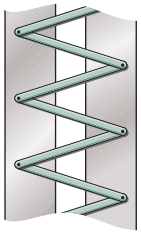
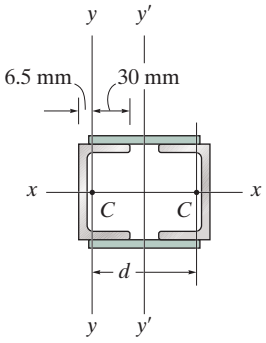
***13–16.** The two steel channels are to be laced together to form a 9-m long bridge column assumed to be pin connected at its ends. Each channel has a cross-sectional area of $A = 1950\ \text{mm}^2$ and moments of inertia $I_x = 21.60(10^6)\ \text{mm}^4$, $I_y = 0.15(10^6)\ \text{mm}^4$. The centroid C of its area is located in the figure. Determine the proper distance d between the centroids of the channels so that buckling occurs about the x–x and y'–y' axes due to the same load. What is the value of this critical load? Neglect the effect of the lacing. $E_{\text{st}} = 200\ \text{GPa}$, $\sigma_Y = 350\ \text{MPa}$.





SOLUTION

$$I_x = 2[21.60(10^{-6})\ \text{m}^4] = 43.2(10^{-6})\ \text{m}^4$$

$$I_y = 2[0.15(10^{-6}) + 1.95(10^{-3})(d/2)^2] = 0.3(10^{-6}) + 0.975(10^{-3})d^2$$

In order for the column to buckle about $x - x$ and $y - y$ at the same time, I_y must be equal to I_x

$$I_y = I_x$$

$$0.3(10^{-6}) + 0.975(10^{-3})d^2 = 43.2(10^{-6})$$

$$d = 0.20976\ \text{m} = 210\ \text{mm}$$ **Ans.**

Check:

$$d > 2(30) = 60\ \text{mm}$$ O.K.

$$P_{\text{cr}} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2[200(10^9)][43.2(10^{-6})]}{[1(9)]^2}$$

$$= 1.053(10^6)\ \text{N} = 1.05\ \text{MN}$$ **Ans.**

Check stress:

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{1.053(10^6)}{2[1.95(10^{-3})]} = 269.94(10^6)\ \text{N/m}^2 = 270\ \text{MPa} < \sigma_Y$$

Therefore, Euler's formula is valid.

Ans:
$P_{cr} = 1.05\ \text{MN}$

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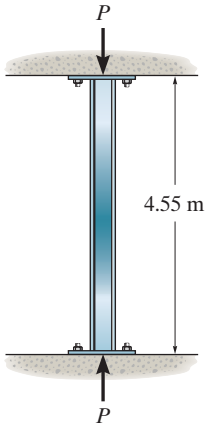
13–17. The W250 × 67 is made of A992 steel and is used as a column that has length of 4.55 m. If its ends are assumed pin supported, and it is subjected to an axial load of 500 kN, determine the factor of safety with respect to buckling.



SOLUTION

Critical Buckling Load:

$I_y = 22.2(10^6)\ \text{mm}^4 = 22.2(10^{-6})\ \text{m}^4$ for a W250 × 67 wide flange section and $K = 1$ for pin supported ends column. Applying *Euler's* formula,

$$P_{\text{cr}} = \frac{\pi^2 EI}{(KL)^2}$$

$$= \frac{\pi^2[200(10^9)][22.2(10^{-6})]}{[1(4.55)]^2}$$

$$= 2116.70(10^3)\ \text{N} = 2116.70\ \text{kN}$$

Critical Stress: *Euler's* formula is only valid if $\sigma_{\text{cr}} < \sigma_Y$.
$A = 8560\ \text{mm}^2 = 8.56(10^{-3})\ \text{m}^2$ for the W250 × 67 wide-flange section.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{2116.70(10^3)}{8.56(10^{-3})} = 247.28(10^6)\ \text{N/m}^2$$

$$= 247.28\ \text{MPa} < \sigma_Y = 345\ \text{MPa} \qquad \text{O.K.}$$

Factor of Safety:

$$\text{F.S.} = \frac{P_{cr}}{A} = \frac{2116.70}{500} = 4.23 \qquad \textbf{Ans.}$$

Ans:
F.S. = 4.23

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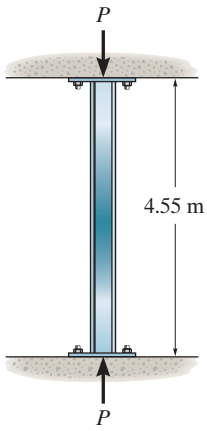
13–18. The W250 × 67 is made of A-36 steel and is used as a column that has length of 4.55 m. If the ends of the column are fixed supported, can the column support the critical load without yielding?



SOLUTION

Critical Buckling Load:

$I_y = 22.2(10^6)\ \text{mm}^4 = 22.2(10^{-6})\ \text{m}^4$ for a W250 × 67 wide flange section and $K = 0.5$ for fixed ends support column. Applying *Euler's* formula,

$$P_{\text{cr}} = \frac{\pi^2 EI}{(KL)^2}$$

$$= \frac{\pi^2[200(10^9)][22.2(10^{-6})]}{[0.5(4.55)]^2}$$

$$= 8.4668(10^6)\ \text{N} = 8.47\ \text{MN}$$

Critical Stress: *Euler's* formula is only valid if $\sigma_{\text{cr}} < \sigma_Y$.
$A = 8560\ \text{mm}^2 = 8.56(10^{-3})\ \text{m}^2$ for the W250 × 67 wide-flange section.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{8.4668(10^6)}{8.56(10^{-3})} = 989.11(10^6)\ \text{N/m}^2$$

$$= 989\ \text{MPa} < \sigma_Y = 345\ \text{MPa} \quad (No!) \qquad \textbf{Ans.}$$

The column will yield before the axial force achieves the critical load P_{cr} and so *Euler's* formula is not valid.

Ans:
$\sigma_{cr} = 345\ \text{MPa}\ (No!)$

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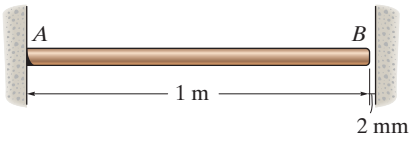
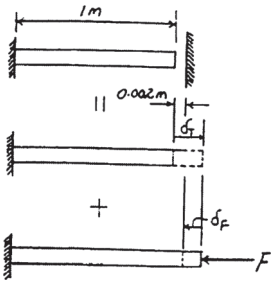
13–19.

The 50-mm-diameter C86100 bronze rod is fixed supported at A and has a gap of 2 mm from the wall at B. Determine the increase in temperature ΔT that will cause the rod to buckle. Assume that the contact at B acts as a pin.



SOLUTION



Section Properties:

$$A = \frac{\pi}{4}(0.05^2) = 0.625\pi(10^{-3})\ \text{m}^2$$

$$I = \frac{\pi}{4}(0.025^4) = 97.65625\pi(10^{-9})\ \text{m}^4$$

Compatibility Condition: This requires

$$(\pm_{\rightarrow}) \qquad 0.002 = \delta_T + \delta_F$$

$$0.002 = 17(10^{-6})(\Delta T)(1) - \frac{F(1)}{0.625\pi(10^{-3})(103)(10^9)}$$

$$F = 3438.08\Delta T - 404480.05$$

Critical Buckling Load: $K = 0.7$ for a column with one end fixed and the other end pinned. Applying *Euler's* formula,

$$P_{\text{cr}} = F = \frac{\pi^2 EI}{(KL)^2}$$

$$3438.08\Delta T - 404\,480.05 = \frac{\pi^2\,(103)(10^9)\left[97.65625\pi(10^{-9})\right]}{[0.7(1)]^2}$$

$$\Delta T = 302.78°\text{C} = 303°\text{C} \qquad \textbf{Ans.}$$

Critical Stress: *Euler's* formula is only valid if $\sigma_{\text{cr}} < \sigma_Y$.

$$P_{\text{cr}} = 3438.08(302.78) - 404\,480.05 = 636\,488.86\ \text{N}$$

$$\sigma_{\text{cr}} = \frac{P_{\text{cr}}}{A} = \frac{636\,488.86}{0.625\pi(10^{-3})} = 324.2\ \text{MPa} < \sigma_Y = 345\ \text{MPa} \qquad \textbf{(O.K!)}$$

Ans:
$\Delta T = 303°\text{C}$

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***13–20.**

An A992 steel W200 × 46 column of length 9 m is fixed at one end and free at its other end. Determine the allowable axial load the column can support if F.S. = 2 against buckling.

SOLUTION

Section Properties: From the table listed in the appendix, the cross-sectional area and moment of inertia about the y axis for a W200 × 46 are

$$A = 5890\ \text{mm}^2 = 5.89(10^{-3})\ \text{m}^2$$

$$I_y = 15.3(10^6)\ \text{mm}^4 = 15.3(10^{-6})\ \text{m}^4$$

Critical Buckling Load: The column will buckle about the weak (y) axis. Applying Euler's formula,

$$P_{\text{cr}} = \frac{\pi^2 E I_y}{(KL)^2} = \frac{\pi^2[200(10^9)][15.3(10^{-6})]}{[2(9)]^2} = 93.21\ \text{kN}$$

Thus, the allowable centric load is

$$P_{\text{allow}} = \frac{P_{\text{cr}}}{\text{F.S.}} = \frac{93.21}{2} = 46.61\ \text{kN} = 46.6\ \text{kN} \qquad \textbf{Ans.}$$

Critical Stress: Euler's formula is valid only if $\sigma_{\text{cr}} < \sigma_Y$.

$$\sigma_{\text{cr}} = \frac{P_{\text{cr}}}{A} = \frac{93.21(10^3)}{5.89(10^{-3})} = 15.83\ \text{MPa} < \sigma_Y = 345\ \text{MPa} \qquad \textbf{(O.K.)}$$

Ans:
$P_{\text{allow}} = 46.6\ \text{kN}$

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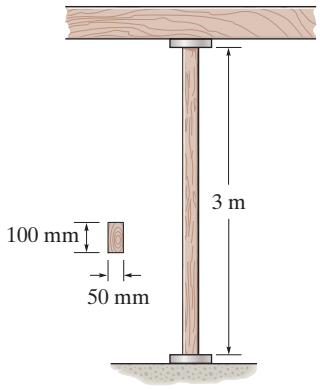
13–21. The 3-m wooden rectangular column has the dimension shown. Determine the critical load if the ends are assumed to be pin connected. $E_w = 12$ GPa, $\sigma_Y = 35$ MPa.



SOLUTION

Section Properties:

$$A = 0.1(0.05) = 0.005\ \text{m}^2$$

$$I_x = \frac{1}{12}(0.05)(0.1^3) = 4.1667(10^{-6})\ \text{m}^4$$

$$I_y = \frac{1}{12}(0.1)(0.05^3) = 1.04167(10^{-6})\ \text{m}^4 \quad (\textit{Controls!})$$

Critical Buckling Load: $K = 1$ for pin supported ends column. Applying *Euler's* formula,.

$$P_{\text{cr}} = \frac{\pi^2 EI}{(KL)^2}$$

$$= \frac{\pi^2[12(10^9)][1.04167(10^{-6})]}{[1(3)]^2}$$

$$= 13.71(10^3)\ \text{N} = 13.7\ \text{kN} \qquad \textbf{Ans.}$$

Critical Stress: *Euler's* formula is only valid if $\sigma_{\text{cr}} < \sigma_Y$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{13.71(10^3)}{0.005} = 2.742(10^6)\ \text{N/m}^2 = 2.74\ \text{MPa} < \sigma_Y = 35\ \text{MPa} \qquad \text{O.K.}$$

Ans:
$A = 5000\ \text{mm}^2$, $I_x = 4.1667\ (10^6)\ \text{mm}^4$,
$I_y = 1.041667\ (10^6)\ \text{mm}^4$, $P_{\text{cr}} = 13.7\ \text{kN}$

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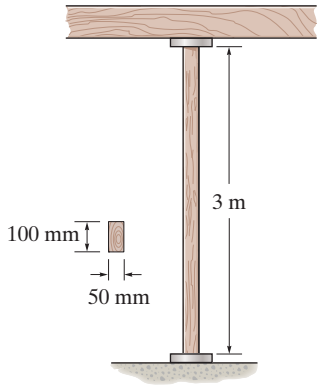
13–22. The 3-m column has the dimensions shown. Determine the critical load if the bottom is fixed and the top is pinned. $E_w = 12$ GPa, $\sigma_Y = 35$ MPa.



SOLUTION

Section Properties:

$$A = 0.1(0.05) = 0.005 \text{ m}^2$$

$$I_x = \frac{1}{12}(0.05)(0.1^3) = 4.1667(10^{-6}) \text{ m}^4$$

$$I_y = \frac{1}{12}(0.1)(0.05^3) = 1.04167(10^{-6}) \text{ m}^4 \quad (\textit{Controls!})$$

Critical Buckling Load: $K = 0.7$ for column with one end fixed and the other end pinned. Applying *Euler's* formula.

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

$$= \frac{\pi^2[12(10^9)][1.04167(10^{-6})]}{[0.7(3)]^2}$$

$$= 27.98(10^3) \text{ N} = 28.0 \text{ kN} \qquad \textbf{Ans.}$$

Critical Stress: *Euler's* formula is only valid if $\sigma_{cr} < \sigma_Y$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{27.98(10^3)}{0.005} = 5.595(10^6) \text{ N/m}^2 = 5.60 \text{ MPa} < \sigma_Y = 35 \text{ MPa} \qquad \text{O.K.}$$

Ans:
$P_{cr} = 28.0$ kN

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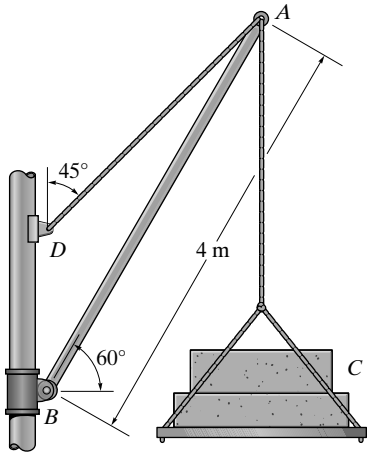
13–23. If load C has a mass of 500 kg, determine the required minimum diameter of the solid L2-steel rod AB to the nearest mm so that it will not buckle. Use F.S. = 2 against buckling.



SOLUTION

Equilibriun. The compressive force developed in rod AB can be determined by analyzing the equilibrium of joint A, Fig. a.

$$\Sigma F_{y'} = 0; \quad F_{AB} \sin 15° - 500(9.81) \cos 45° = 0 \qquad F_{AB} = 13\,400.71 \text{ N}$$

Section Properties. The cross-sectional area and moment of inertia of the solid rod are

$$A = \frac{\pi}{4} d^2 \qquad I = \frac{\pi}{4}\left(\frac{d}{2}\right)^4 = \frac{\pi}{64} d^4$$

Critical Buckling Load. Since the rod is pinned at both of its ends, $K = 1$. Here, $P_{cr} = F_{AB}$ (F.S.) = 13400.71(2) = 26801.42 N. Applying Euler's formula,

$$P_{cr} = \frac{\pi^2 E I_y}{(KL)^2}$$

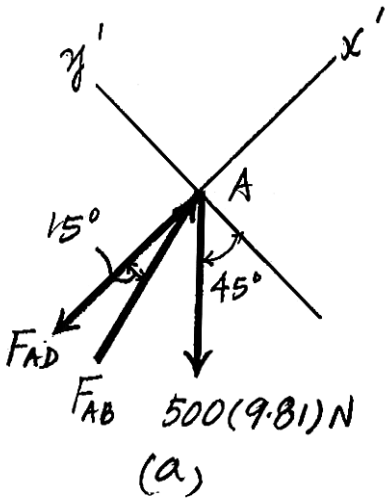
$$26801.42 = \frac{\pi^2\left[200\left(10^9\right)\right]\left[\frac{\pi}{64} d^4\right]}{[1(4)]^2}$$

$$d = 0.04587 \text{ m} = 45.87 \text{ mm}$$

Use $d = 46$ mm **Ans.**

Critical Stress. Euler's formula is valid only if $\sigma_{cr} < \sigma_Y$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{26801.42}{\frac{\pi}{4}\left(0.046^2\right)} = 16.13 \text{ MPa} < \sigma_Y = 703 \text{ MPa} \qquad \text{O.K.}$$



Ans:
Use $d = 46$ mm

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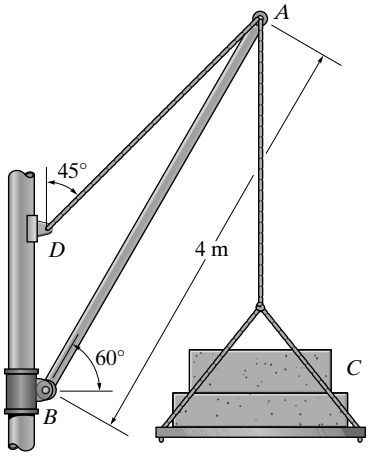
***13–24.** If the diameter of the solid L2-steel rod AB is 50 mm, determine the maximum mass C that the rod can support without buckling. Use F.S. = 2 against buckling.



SOLUTION

Equilibrium. The compressive force developed in rod AB can be determined by analyzing the equilibrium of joint A, Fig. a.

$$\Sigma F_{y'} = 0; \quad F_{AB} \sin 15° - m(9.81) \cos 45° = 0 \qquad F_{AB} = 26.8014m$$

Section Properties. The cross-sectional area and moment of inertia of the rod are

$$A = \frac{\pi}{4}\left(0.05^2\right) = 0.625\left(10^{-3}\right)\pi \text{m}^2$$

$$I = \frac{\pi}{4}\left(0.025^4\right) = 97.65625\left(10^{-9}\right)\pi \text{m}^4$$

Critical Buckling Load. Since the rod is pinned at both of its ends, $K = 1$. Here, $P_{cr} = F_{AB}\,(\text{F.S.}) = 26.8014m(2) = 53.6028m$. Applying Euler's formula,

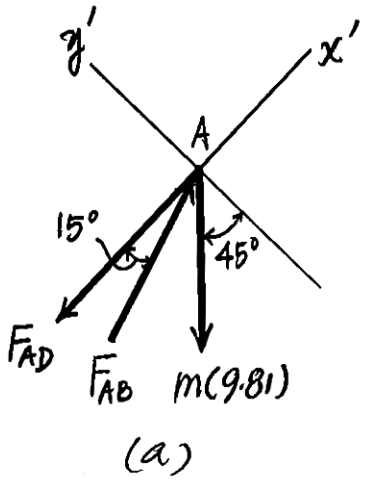
$$P_{cr} = \frac{\pi^2 E I_y}{(KL)^2}$$

$$53.6028m = \frac{\pi^2\left[200\left(10^9\right)\right]\left[97.65625\left(10^{-9}\right)\pi\right]}{[1(4)]^2}$$

$$m = 706.11 \text{ kg} = 7.06 \text{ kg} \qquad \textbf{Ans.}$$

Critical Stress. Euler's formula is valid only if $\sigma_{cr} < \sigma_Y$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{53.6028(706.11)}{\pi\, 0.625\left(10^{-3}\right)} = 19.28 \text{ MPa} < \sigma_Y = 703 \text{ MPa} \qquad \text{O.K.}$$



Ans:
$m = 7.06$ kg

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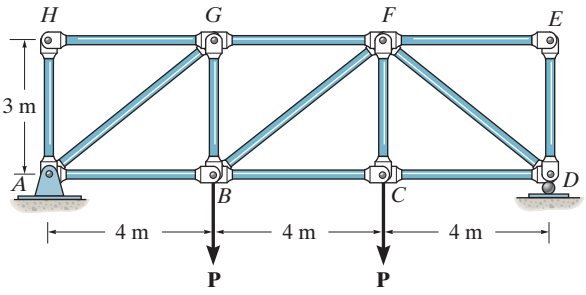
13–25. The members of the truss are assumed to be pin connected. If member *GF* is an A-36 steel rod having a diameter of 50 mm, determine the greatest magnitude of load **P** that can be supported by the truss without causing this member to buckle.



SOLUTION

Support Reactions: As shown on FBD(a).

Member Forces: Use the method of sections [FBD(b)].

$$+\Sigma M_B = 0; \qquad F_{GF}(3) - P(4) = 0 \qquad F_{GF} = 1.3333P \text{ (C)}$$

Section Properties:

$$A = \frac{\pi}{4}(0.05^2) = 0.625(10^{-3})\pi \text{ m}^2$$

$$I = \frac{\pi}{4}(0.025^4) = 97.65625(10^{-9})\pi \text{ m}^4$$

Critical Buckling Load: $K = 1$ for a column with both ends pinned. Applying *Euler's* formula,

$$P_{\text{cr}} = F_{GF} = \frac{\pi^2 EI}{(KL_{GF})^2}$$

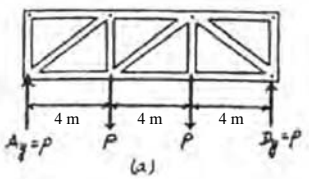
$$1.3333P = \frac{\pi^2[200(10^9)][97.65625(10^{-9})\pi]}{[1(4)]^2}$$

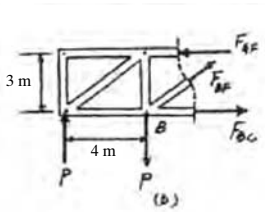
$$P = 28.39(10^3) \text{ N} = 28.4 \text{ kN} \qquad \textbf{Ans.}$$

Critical Stress: *Euler's* formula is only valid if $\sigma_{\text{cr}} < \sigma_Y$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{1.3333[28.39(10^3)]}{(0.625)(10^{-3})\pi} = 19.28(10^6) \text{ N/m}^2$$

$$= 19.3 \text{ MPa} < \sigma_Y = 250 \text{ MPa} \qquad \text{O.K.}$$





Ans:
$P = 28.4$ kN

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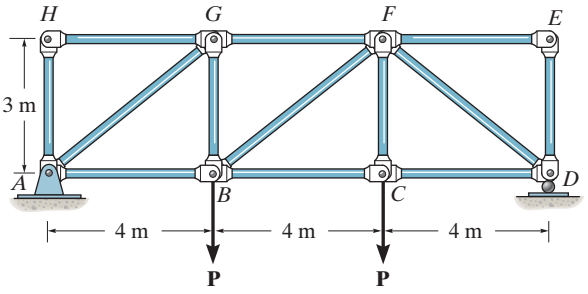
13–26. The members of the truss are assumed to be pin connected. If member AG is an A-36 steel rod having a diameter of 50 mm, determine the greatest magnitude of load **P** that can be supported by the truss without causing this member to buckle.



SOLUTION

Support Reactions: As shown on FBD(a).

Member Forces: Use the method of joints [FBD(b)].

$$+\uparrow \Sigma F_y = 0; \qquad P - \frac{3}{5} F_{AG} = 0 \qquad F_{AG} = 1.6667P \text{ (C)}$$

Section Properties:

$$L_{AG} = \sqrt{4^2 + 3^2} = 5 \text{ m}$$

$$A = \frac{\pi}{4}(0.05^2) = 0.625(10^{-3})\pi \text{ m}^2$$

$$I = \frac{\pi}{4}(0.025^4) = 97.65625(10^{-9})\pi \text{ m}^4$$

Critical Buckling Load: $K = 1$ for a column with both ends pinned. Applying *Euler's* formula,

$$P_{\text{cr}} = F_{GF} = \frac{\pi^2 EI}{(KL_{GF})^2}$$

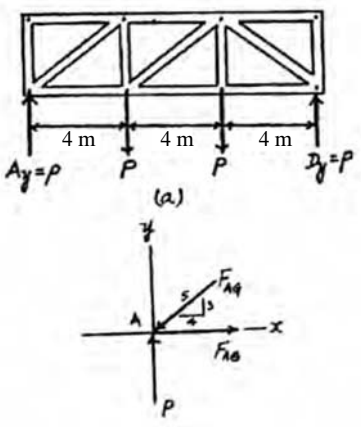
$$1.6667P = \frac{\pi^2[200(10^9)][97.65625(10^{-9})\pi]}{[1(5)]^2}$$

$$P = 14.53(10^3) \text{ N} = 14.5 \text{ kN} \qquad \textbf{Ans.}$$

Critical Stress: *Euler's* formula is only valid if $\sigma_{\text{cr}} = \sigma_Y$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{1.6667[14.53(10^3)]}{(0.625)(10^{-3})\pi} = 12.34(10^6) \text{ N/m}^2$$

$$= 12.3 \text{ MPa} < \sigma_Y = 250 \text{ MPa} \qquad \text{O.K.}$$



Ans:
$P = 14.5$ kN

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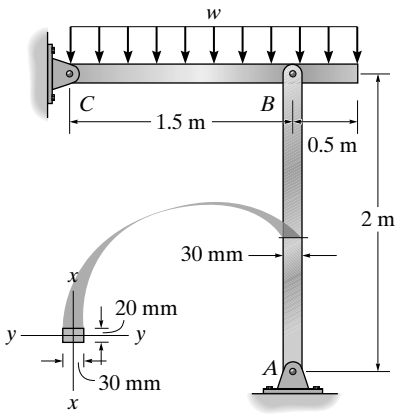
13–27. Determine the maximum allowable intensity w of the distributed load that can be applied to member BC without causing member AB to buckle. Assume that AB is made of steel and is pinned at its ends for x–x axis buckling and fixed at its ends for y–y axis buckling. Use a factor of safety with respect to buckling of 3. $E_{st} = 200$ GPa, $\sigma_Y = 360$ MPa.



SOLUTION

Moment of inertia:

$$I_x = \frac{1}{12}(0.02)(0.03^3) = 45.0(10^{-9})\text{m}^4$$

$$I_y = \frac{1}{12}(0.03)(0.02^3) = 20(10^{-9})\text{ m}^4$$

$x-x$ axis:

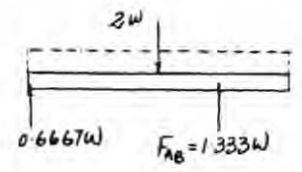
$$P_{cr} = F_{AB}\,(\text{F.S.}) = 1.333w(3) = 4.0\,w$$

$$K = 1.0, \qquad L = 2\text{m}$$

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

$$4.0w = \frac{\pi^2(200)(10^9)(45.0)(10^{-9})}{[(1.0)(2)]^2}$$

$$w = 5552\text{ N/m} = 5.55\text{ kN/m} \qquad \text{(controls)} \qquad \textbf{Ans.}$$



$y-y$ axis

$$K = 0.5, \qquad L = 2\text{m}$$

$$4.0w = \frac{\pi^2(200)(10^9)(20)(10^{-9})}{[(0.5)(2)]^2}$$

$$w = 9870\text{ N/m} = 9.87\text{ kN/m}$$

Check:

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{4(5552)}{(0.02)(0.03)} = 37.0\text{ MPa} < \sigma_\gamma \qquad \text{O.K.}$$

Ans:
$w = 5.55$ kN/m

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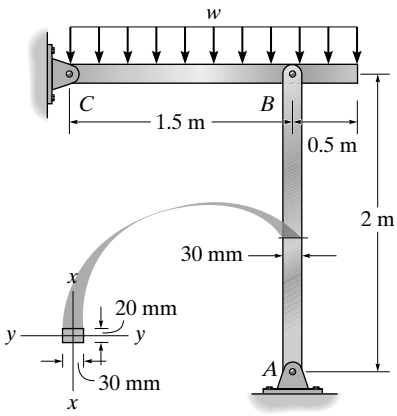
***13–28.** Determine if the frame can support a load of $w = 6$ kN/m if the factor of safety with respect to buckling of member AB is 3. Assume that AB is made of steel and is pinned at its ends for x–x axis buckling and fixed at its ends for y–y axis buckling. $E_{st} = 200$ GPa, $\sigma_Y = 360$ MPa.



SOLUTION

Check $x-x$ axis buckling:

$$I_x = \frac{1}{12}(0.02)(0.03)^3 = 45.0(10^{-9})\ \text{m}^4$$

$$K = 1.0 \qquad L = 2\ \text{m}$$

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2(200)(10^9)(45.0)(10^{-9})}{((1.0)(2))^2}$$

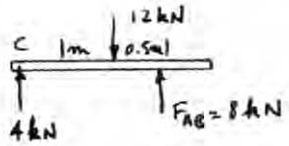
$$P_{cr} = 22.2\ \text{kN}$$

$$\zeta + \Sigma M_C = 0; \qquad F_{AB}(1.5) - 6(2)(1) = 0$$

$$F_{AB} = 8\ \text{kN}$$

$$P_{req'd} = 8(3) = 24\ \text{kN} > 22.2\ \text{kN}$$

No, AB will fail. **Ans.**



Ans:
No, AB will fail.

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13–29.

A 6061-T6 aluminum alloy solid circular rod of length 4 m is pinned at both of its ends. If it is subjected to an axial load of 15 kN and F.S. = 2 against buckling, determine the minimum required diameter of the rod to the nearest mm.

SOLUTION

Section Properties: The cross-sectional area and moment of inertia of the solid rod are

$$A = \frac{\pi}{4}d^2 \qquad I = \frac{\pi}{4}\left(\frac{d}{2}\right)^4 = \frac{\pi}{64}d^4$$

Critical Buckling Load: The critical buckling load is

$$P_{\text{cr}} = P_{\text{allow}}(\text{F.S.}) = 15(2) = 30\ \text{kN}$$

Applying Euler's formula,

$$P_{\text{cr}} = \frac{\pi^2 EI_y}{(KL)^2}$$

$$30(10^3) = \frac{\pi^2\left[68.9(10^9)\right]\left[\frac{\pi}{64}d^4\right]}{[1(4)]^2}$$

$$d = 0.06158\ \text{m} = 61.58\ \text{mm}$$

Use $d = 62$ mm **Ans.**

Critical Stress: Euler's formula is valid only if $\sigma_{\text{cr}} < \sigma_Y$.

$$\sigma_{\text{cr}} = \frac{P_{\text{cr}}}{A} = \frac{30(10^3)}{\frac{\pi}{4}(0.062^2)} = 9.94\ \text{MPa} < \sigma_Y = 255\ \text{MPa}$$ **(O.K.)**

Ans:
Use $d = 62$ mm

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13–30.

A 6061-T6 aluminum alloy solid circular rod of length 4 m is pinned at one end while fixed at the other end. If it is subjected to an axial load of 15 kN and F.S. = 2 against buckling, determine the minimum required diameter of the rod to the nearest mm.

SOLUTION

Section Properties: The cross-sectional area and moment of inertia of the solid rod are

$$A = \frac{\pi}{4}d^2 \qquad I = \frac{\pi}{4}\left(\frac{d}{2}\right)^4 = \frac{\pi}{64}d^4$$

Critical Buckling Load: The critical buckling load is

$$P_{\text{cr}} = P_{\text{allow}}(\text{F.S.}) = 15(2) = 30\ \text{kN}$$

Applying Euler's formula,

$$P_{\text{cr}} = \frac{\pi^2 EI_y}{(KL)^2}$$

$$30(10^3) = \frac{\pi^2\left[68.9(10^9)\right]\left[\frac{\pi}{64}d^4\right]}{[0.7(4)]^2}$$

$$d = 0.05152\ \text{m} = 51.52\ \text{mm}$$

Use $d = 52$ mm **Ans.**

Critical Stress: Euler's formula is valid only if $\sigma_{\text{cr}} < \sigma_Y$.

$$\sigma_{\text{cr}} = \frac{P_{\text{cr}}}{A} = \frac{30(10^3)}{\frac{\pi}{4}(0.052^2)} = 14.13\ \text{MPa} < \sigma_Y = 255\ \text{MPa} \qquad \text{(O.K.)}$$

Ans:
Use $d = 52$ mm

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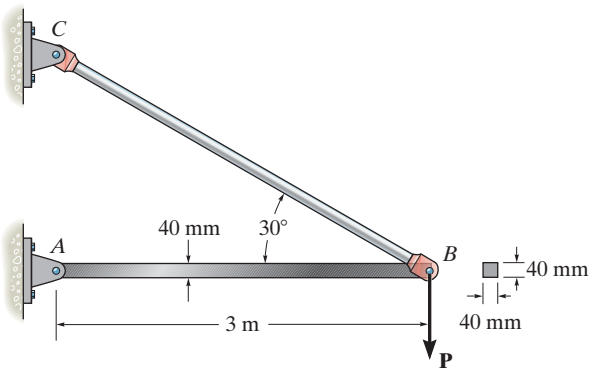
13–31. The A-36 steel bar AB has a square cross section. If it is pin connected at its ends, determine the maximum allowable load P that can be applied to the frame. Use a factor of safety with respect to buckling of 2.



SOLUTION

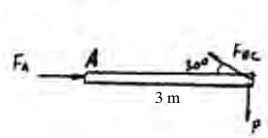
$$\zeta + \Sigma M_A = 0; \qquad F_{BC} \sin 30°(3) - P(3) = 0$$

$$F_{BC} = 2\,P$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad F_A - 2P\cos 30° = 0$$

$$F_A = 1.732\,P$$

Buckling load:



$$P_{\text{cr}} = F_A(\text{F.S.}) = 1.732\,P(2) = 3.464\,P$$

$$L = 3\text{ m}$$

$$I_y = \frac{1}{12}(0.04)(0.04^3) = 0.21333(10^{-6})\text{ m}^4$$

$$P_{\text{cr}} = \frac{\pi^2 EI}{(KL)^2}$$

$$0.464P = \frac{\pi^2[200(10^9)][0.21333(10^{-6})]}{[1(3)]^2}$$

$$P = 13.51(10^3)\text{ N} = 13.5\text{ kN}$$ **Ans.**

$$P_{\text{cr}} = F_A(\text{F.S.}) = 1.732(13.51)(2)\text{ N} = 46.79\text{ kN}$$

Check:

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{46.79(10^3)}{(0.04)(0.04)} = 29.24(10^6)\text{ N/m}^2 = 29.24\text{ MPa} < \sigma_Y$$ O.K.

Ans:
$P = 13.5$ kN

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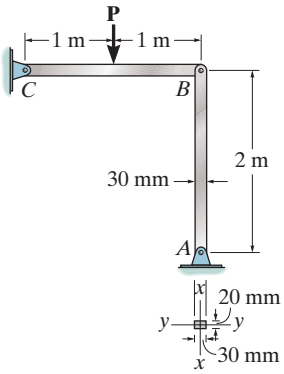
***13–32.**

Determine the maximum allowable load P that can be applied to member BC without causing member AB to buckle. Assume that AB is made of steel and is pinned at its ends for x–x axis buckling and fixed at its ends for y–y axis buckling. Use a factor of safety with respect to buckling of F.S. = 3. $E_{st} = 200$ GPa, $\sigma_Y = 360$ MPa.



SOLUTION

Support Reactions:

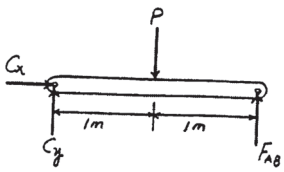
$$\zeta + \Sigma M_C = 0; \qquad F_{AB}(2) - P(1) = 0 \qquad F_{AB} = 0.500P$$

Section Properties:

$$A = 0.02(0.03) = 0.600(10^{-3})\ \text{m}^2$$

$$I_x = \frac{1}{12}(0.02)(0.03^3) = 45.0(10^{-9})\ \text{m}^4$$

$$I_y = \frac{1}{12}(0.03)(0.02^2) = 20.0(10^{-9})\ \text{m}^4$$



Critical Buckling Load: With respect to x–x axis, $K = 1$ (column with both ends pinned). Applying *Euler's* formula,

$$P_{cr} = 3F_{AB} = \frac{\pi^2 EI}{(KL)^2}$$

$$3(0.500P) = \frac{\pi^2(200)(10^9)\left[45.0(10^{-9})\right]}{[1(2)]^2}$$

$$P = 14804.4\ \text{N} = 14.8\ \text{kN (Controls!)} \qquad \textbf{Ans.}$$

With respect to y–y axis, $K = 0.5$ (column with both ends fixed).

$$P_{cr} = 3F_{AB} = \frac{\pi^2 EI}{(KL)^2}$$

$$3(0.500P) = \frac{\pi^2(200)(10^9)\left[20.0(10^{-9})\right]}{[0.5(2)]^2}$$

$$P = 26\,318.9\ \text{N}$$

Critical Stress: *Euler's* formula is only valid if $\sigma_{cr} < \sigma_Y$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{3(0.5)(14\,804.4)}{0.600(10^{-3})} = 37.01\ \text{MPa} < \sigma_Y = 360\ \text{MPa (O.K!)}$$

Ans:
$P = 14.8$ kN

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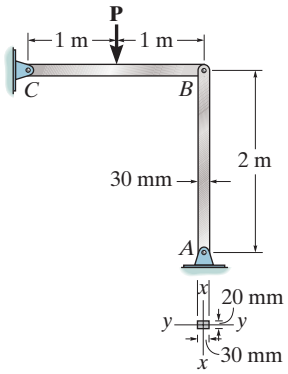
13–33.

Determine if the frame can support a load of $P = 20$ kN if the factor of safety with respect to buckling of member AB is F.S. = 3. Assume that AB is made of steel and is pinned at its ends for x–x axis buckling and fixed at its ends for y–y axis buckling. $E_{st} = 200$ GPa, $\sigma_Y = 360$ MPa.



SOLUTION

Support Reactions:

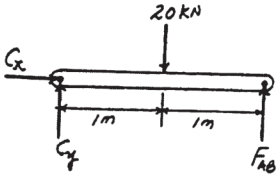
$$\zeta + \Sigma M_C = 0; \qquad F_{AB}(2) - 20(1) = 0 \qquad F_{AB} = 10.0\ \text{kN}$$

Section Properties:

$$A = 0.02(0.03) = 0.600(10^{-3})\ \text{m}^2$$

$$I_x = \frac{1}{12}(0.02)(0.03^3) = 45.0(10^{-9})\ \text{m}^4$$

$$I_y = \frac{1}{12}(0.03)(0.02^2) = 20.0(10^{-9})\ \text{m}^4$$



Critical Buckling Load: With respect to x–x axis, $K = 1$ (column with both ends pinned). Applying *Euler's* formula,

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

$$= \frac{\pi^2(200)(10^9)\left[45.0(10^{-9})\right]}{[1(2)]^2}$$

$$= 22\,206.61 = 22.207\ \text{kN (Controls!)}$$

With respect to y–y axis, $K = 0.5$ (column with both ends fixed).

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

$$= \frac{\pi^2(200)(10^9)\left[20.0(10^{-9})\right]}{[0.5(2)]^2}$$

$$= 39\,478.42\ \text{N}$$

Critical Stress: *Euler's* formula is only valid if $\sigma_{cr} < \sigma_Y$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{32\,127.6}{5.00(10^{-3})} = 6.426\ \text{MPa} < \sigma_Y = 360\ \text{MPa} \qquad \textbf{(O.K!)}$$

Factor of Safety: The required factor of safety is 3.

$$\text{F.S} = \frac{P_{cr}}{F_{AB}} = \frac{22.207}{10.0} = 2.22 < 3\ \text{(No Good!)}$$

Hence, the frame cannot support the load with the required F.S. **Ans.**

Ans:
No

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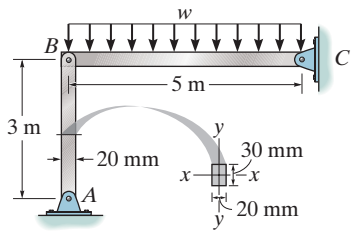
13–34.

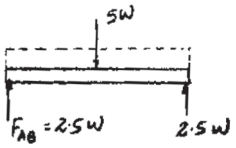
The steel bar AB has a rectangular cross section. If it is pin connected at its ends, determine the maximum allowable intensity w of the distributed load that can be applied to BC without causing AB to buckle. Use a factor of safety with respect to buckling of 1.5. $E_{st} = 200$ GPa, $\sigma_Y = 360$ MPa.



SOLUTION

Buckling Load:

$$P_{cr} = F_{AB}(\text{F.S.}) = 2.5\,w(1.5) = 3.75\,w$$



$$I = \frac{1}{12}(0.03)(0.02)^3 = 20\,(10^{-9})\ \text{m}^4$$

$$K = 1.0$$

$$P_{cr} = \frac{\pi^2 E I}{(KL)^2}$$

$$3.75\,w = \frac{\pi^2(200)(10^9)(20)(10^{-9})}{[(1.0)(3)]^2}$$

$$w = 1170\ \text{N/m} = 1.17\ \text{kN/m} \qquad \textbf{Ans.}$$

$$P_{cr} = 4.39\ \text{kN}$$

Check:

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{4.39\,(10^3)}{0.02\,(0.03)} = 7.31\ \text{MPa} < \sigma_Y \qquad \textbf{OK}$$

Ans:
$w = 1.17$ kN/m

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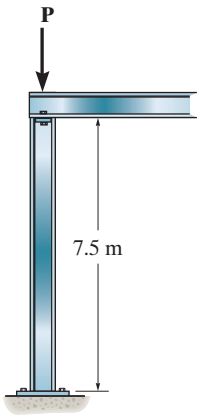
13–35. The W360 × 45 is used as a structural A-36 steel column that can be assumed pinned at both of its ends. Determine the largest axial force P that can be applied without causing it to buckle.



SOLUTION

From the table in appendix, the cross-sectional area and the moment of inertia about weak axis (y-axis) for W360 × 45 are

$$A = 5710\ \text{mm}^2 = 5.71(10^{-3})\ \text{m}^2 \qquad I_y = 8.16(10^6)\ \text{mm}^4 = 8.16(10^{-6})\ \text{m}^4$$

Critical Buckling Load: Since the column is pinned at its base and top, $K = 1$. For A36 steel, $E = 200$ GPa and $\sigma_Y = 250$ MPa. Here, the buckling occurs about the weak axis (y-axis).

$$P = P_{cr} = \frac{\pi^2 E I_y}{(KL)^2} = \frac{\pi^2[200(10^9)][8.16(10^{-6})]}{[1(7.5)]^2}$$

$$= 286.35(10^3)\ \text{N} = 286\ \text{kN} \qquad \textbf{Ans.}$$

Euler's formula is valid only if $\sigma_{cr} < \sigma_Y$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{286.35(10^3)}{5.71(10^{-3})} = 50.14(10^6)\ \text{N/m}^2 = 50.1\ \text{MPa} < \sigma_Y = 250\ \text{MPa} \qquad \text{O.K.}$$

Ans:
$A = 5710\ \text{mm}^2$, $I_y = 8.16\ (10^6)\ \text{mm}^4$, $P = 286\ \text{kN}$

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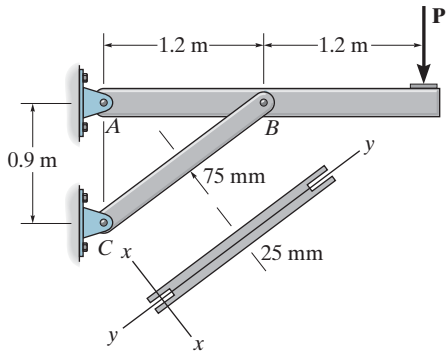
***13–36.** The beam supports the load of $P = 30$ kN. As a result, the A-36 steel member BC is subjected to a compressive load. Due to the forked ends on the member, consider the supports at B and C to act as pins for x–x axis buckling and as fixed supports for y–y axis buckling. Determine the factor of safety with respect to buckling about each of these axes.

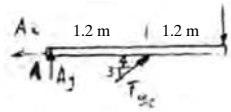


SOLUTION

$$\zeta + \Sigma M_A = 0; \qquad F_{BC}\left(\frac{3}{5}\right)(1.2) - 30(2.4) = 0$$

$$F_{BC} = 100 \text{ kN}$$

$x-x$ axis buckling:



$$P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2[200(10^9)]\left[\frac{1}{12}(0.025)(0.075^3)\right]}{[1.0(1.5)]^2} = 771.06(10^3) \text{ N} = 771.06 \text{ kN}$$

$$\text{F.S.} = \frac{771.06}{100} = 7.71 \qquad \textbf{Ans.}$$

$y-y$ axis buckling:

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2[200(10^9)]\left[\frac{1}{12}(0.075)(0.025^3)\right]}{[0.5(1.5)]^2} = 342.69(10^3) \text{ N} = 342.69 \text{ kN}$$

$$\text{F.S.} = \frac{342.69}{100} = 3.43 \qquad \textbf{Ans.}$$

Ans:
x-x axis buckling: F.S. = 7.71,
y-y axis buckling: F.S. = 3.43

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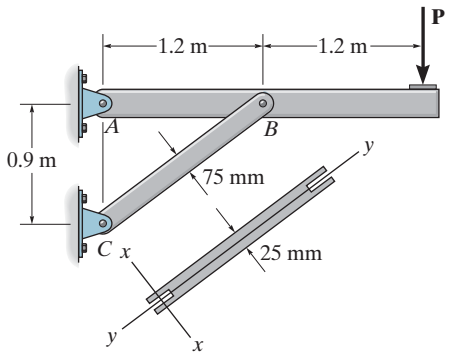
13–37. Determine the greatest load P the frame will support without causing the A-36 steel member BC to buckle. Due to the forked ends on the member, consider the supports at B and C to act as pins for x–x axis buckling and as fixed supports for y–y axis buckling.

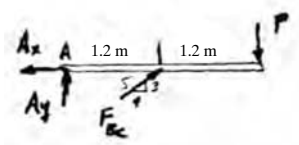


SOLUTION

$$\zeta + \Sigma M_A = 0; \quad F_{BC}\left(\frac{3}{5}\right)(1.2) - P(2.4) = 0$$

$$F_{BC} = \left(\frac{10}{3}\right)P$$

$x-x$ axis buckling:



$$P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2[200(10^9)]\left[\frac{1}{12}(0.025)(0.075^3)\right]}{[1.0(1.5)]^2} = 771.06(10^3)\text{ N} = 771.06\text{ kN}$$

$y-y$ axis buckling:

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2[200(10^9)]\left[\frac{1}{12}(0.075)(0.025^3)\right]}{[0.5(1.5)]^2} = 342.69(10^3)\text{ N} = 342.69\text{ kN}$$

Thus,

$$\left(\frac{10}{3}\right)P = 342.69$$

$P = 102.81\text{ kN} = 103\text{ kN}$ **Ans.**

Ans:
$P = 103\text{ kN}$

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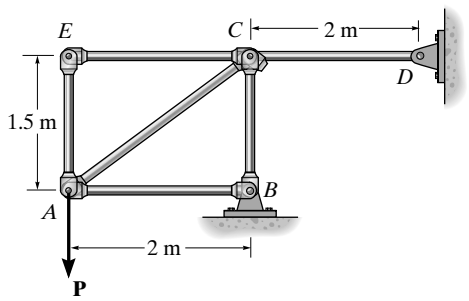
13–38. The members of the truss are assumed to be pin connected. If member AB is an A-36 steel rod of 40 mm diameter, determine the maximum force P that can be supported by the truss without causing the member to buckle.



SOLUTION

By inspecting the equilibrium of joint E, $F_{AB} = 0$. Then, the compressive force developed in member AB can be determined by analysing the equilibrium of joint A, Fig. *a*.

$$+\uparrow \Sigma F_y = 0; \qquad F_{AC}\left(\frac{3}{5}\right) - P = 0 \qquad F_{AC} = \frac{5}{3}P \text{ (T)}$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad \frac{5}{3}P\left(\frac{4}{5}\right) - F_{AB} = 0 \qquad F_{AB} = \frac{4}{3}P(c)$$

$$A = \pi(0.02^2) = 0.4(10^{-3})\pi \text{ m}^2 \qquad I = \frac{\pi}{4}(0.02^4) = 40(10^{-9})\,\pi \text{ m}^4$$

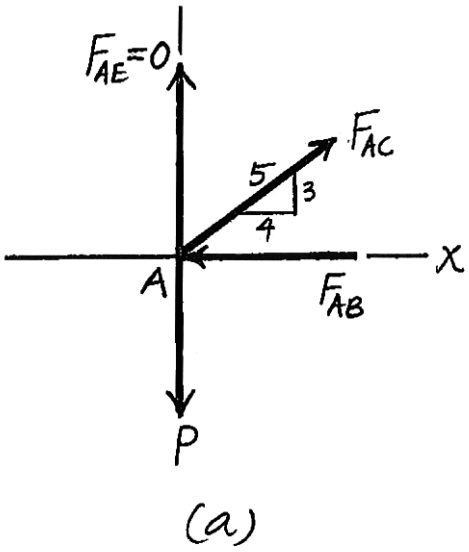
Since both ends of member AB are pinned, $K = 1$. For A36 steel, $E = 200$ GPa and $\sigma_\gamma = 250$ MPa.

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}; \qquad \frac{4}{3}P = \frac{\pi^2\left[200(10^9)\right]\left[40(10^{-9})\pi\right]}{\left[1(2)\right]^2}$$

$$P = 46.51(10^3) \text{ N} = 46.5 \text{ kN} \qquad \textbf{Ans.}$$

The Euler's formula is valid only if $\sigma_{cr} < \sigma_\gamma$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{\frac{4}{3}(46.51)(10^3)}{0.4(10^{-3})\pi} = 49.35(10^6) \text{ Pa} = 49.35 \text{ MPa} < \sigma_\gamma = 250 \text{ MPa O.K.}$$



(a)

Ans:
$P = 46.5$ kN

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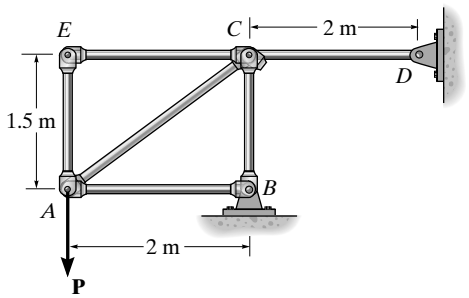
13–39. The members of the truss are assumed to be pin connected. If member *CB* is an A-36 steel rod of 40 mm diameter, determine the maximum load *P* that can be supported by the truss without causing the member to buckle.



SOLUTION

Section the truss through *a–a*, the FBD of the left cut segment is shown in Fig. *a*. The compressive force developed in member *CB* can be obtained directly by writing the force equation of equilibrium along *y* axis.

$$+\uparrow \Sigma F_y = 0; \qquad F_{CB} - P = 0 \qquad F_{CB} = P \text{ (C)}$$

$$A = \pi(0.02^2) = 0.4(10^{-3})\pi \text{ m}^2 \qquad I = \frac{\pi}{4}(0.02^4) = 40(10^{-9})\pi \text{ m}^4$$

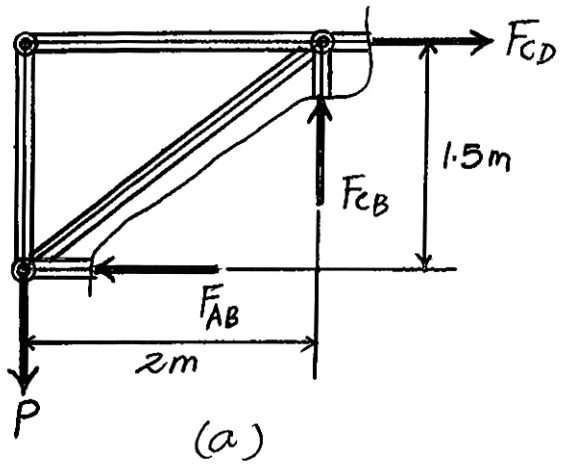
Since both ends of member *CB* are pinned, $K = 1$. For A36 steel, $E = 200$ GPa and $\sigma_\gamma = 250$ MPa.

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}; \qquad P = \frac{\pi^2\left[200(10^9)\right]\left[40(10^{-9})\pi\right]}{\left[1(1.5)\right]^2}$$

$$= 110.24(10^3) \text{ N} = 110 \text{ kN} \qquad \textbf{Ans.}$$

The Euler's formula is valid only if $\sigma_{cr} < \sigma_\gamma$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{110.24(10^3)}{0.4(10^{-3})\pi} = 87.73(10^6) \text{ Pa} = 87.73 \text{ MPa} < \sigma_\gamma = 250 \text{ MPa} \qquad \text{O.K.}$$



Ans:
$P = 110$ kN

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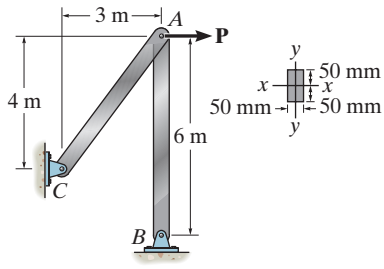
***13–40.**

The steel bar AB of the frame is assumed to be pin connected at its ends for y–y axis buckling. If $P = 18$ kN, determine the factor of safety with respect to buckling about the y–y axis. $E_{st} = 200$ GPa, $\sigma_Y = 360$ MPa.



SOLUTION

$$I_y = \frac{1}{12}(0.10)(0.05^3) = 1.04167\,(10^{-6})\ \text{m}^4$$

Joint *A*:

$$\stackrel{+}{\leftarrow} \Sigma F_x = 0; \quad \frac{3}{5}F_{AC} - 18 = 0$$

$$F_{AC} = 30\ \text{kN}$$

$$+\uparrow \Sigma F_y = 0; \quad F_{AB} - \frac{4}{5}(30) = 0$$

$$F_{AB} = 24\ \text{kN}$$

$$P_{\text{cr}} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2(200)(10^9)(1.04167)(10^{-6})}{[(1.0)(6)]^2} = 57116\ \text{N} = 57.12\ \text{kN}$$

$$\text{F.S.} = \frac{P_{\text{cr}}}{F_{AB}} = \frac{57.12}{24} = 2.38 \qquad \textbf{Ans.}$$

Check:

$$\sigma_{\text{cr}} = \frac{P_{\text{cr}}}{A} = \frac{57.12\,(10^3)}{0.1\,(0.05)} = 11.4\ \text{MPa} < \sigma_Y \qquad \textbf{OK}$$

Ans:
F.S. = 2.38

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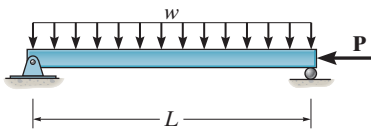
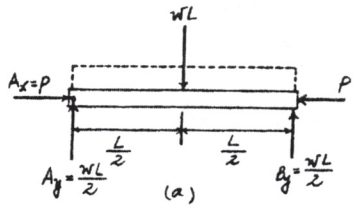
13–41.

The ideal column has a weight w (force/length) and is subjected to the axial load **P**. Determine the maximum moment in the column at midspan. EI is constant. *Hint:* Establish the differential equation for deflection, Eq. 13–1, with the origin at the midspan. The general solution is $v = C_1 \sin kx + C_2 \cos kx + (w/(2P))x^2 - (wL/(2P))x - (wEI/P^2)$ where $k^2 = P/EI$.



SOLUTION

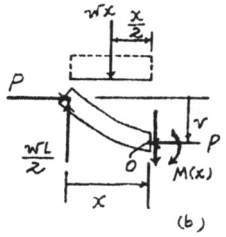
Moment Functions: FBD(b).



$$\zeta+\Sigma M_o = 0; \qquad wx\left(\frac{x}{2}\right) - M(x) - \left(\frac{wL}{2}\right)x - Pv = 0$$

$$M(x) = \frac{w}{2}\left(x^2 - Lx\right) - Pv \qquad \textbf{(1)}$$

Differential Equation of The Elastic Curve:



$$EI\frac{d^2v}{dx^2} = M(x)$$

$$EI\frac{d^2v}{dx^2} = \frac{w}{2}\left(x^2 - Lx\right) - Pv$$

$$\frac{d^2v}{dx^2} + \frac{P}{EI}v = \frac{w}{2EI}\left(x^2 - Lx\right)$$

The solution of the above differential equation is of the form

$$v = C_1 \sin\left(\sqrt{\frac{P}{EI}}x\right) + C_2 \cos\left(\sqrt{\frac{P}{EI}}x\right) + \frac{w}{2P}x^2 - \frac{wL}{2P}x - \frac{wEI}{P^2} \qquad \textbf{(2)}$$

and

$$\frac{dv}{dx} = C_1\sqrt{\frac{P}{EI}}\cos\left(\sqrt{\frac{P}{EI}}x\right) - C_2\sqrt{\frac{P}{EI}}\sin\left(\sqrt{\frac{P}{EI}}x\right) + \frac{w}{P}x - \frac{wL}{2P} \qquad \textbf{(3)}$$

The integration constants can be determined from the boundary conditions.

Boundary Condition:

At $x = 0$, $v = 0$. From Eq. (2),

$$0 = C_2 - \frac{wEI}{P^2} \qquad C_2 = \frac{wEI}{P^2}$$

At $x = \frac{L}{2}$, $\frac{dv}{dx} = 0$. From Eq. (3),

$$0 = C_1\sqrt{\frac{P}{EI}}\cos\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right) - \frac{wEI}{P^2}\sqrt{\frac{P}{EI}}\sin\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right) + \frac{w}{P}\left(\frac{L}{2}\right) - \frac{wL}{2P}$$

$$C_1 = \frac{wEI}{P^2}\tan\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right)$$

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13–41. Continued

SOLUTION

Elastic Curve:

$$v = \frac{w}{P}\left[\frac{EI}{P}\tan\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right)\sin\left(\sqrt{\frac{P}{EI}}x\right) + \frac{EI}{P}\cos\left(\sqrt{\frac{P}{EI}}x\right) + \frac{x^2}{2} - \frac{L}{2}x - \frac{EI}{P}\right]$$

However, $v = v_{\max}$ at $x = \frac{L}{2}$. Then,

$$v_{\max} = \frac{w}{P}\left[\frac{EI}{P}\tan\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right)\sin\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right) + \frac{EI}{P}\cos\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right) - \frac{L^2}{8} - \frac{EI}{P}\right]$$

$$= \frac{wEI}{P^2}\left[\sec\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right) - \frac{PL^2}{8EI} - 1\right]$$

Maximum Moment: The maximum moment occurs at $x = \frac{L}{2}$. From Eq. (1),

$$M_{\max} = \frac{w}{2}\left[\frac{L^2}{4} - L\left(\frac{L}{2}\right)\right] - Pv_{\max}$$

$$= -\frac{wL^2}{8} - P\left\{\frac{wEI}{P^2}\left[\sec\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right) - \frac{PL^2}{8EI} - 1\right]\right\}$$

$$= -\frac{wEI}{P}\left[\sec\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right) - 1\right] \qquad \textbf{Ans.}$$

Ans:

$$M_{\max} = -\frac{wEI}{P}\left[\sec\left(\frac{L}{2}\sqrt{\frac{P}{EI}}\right) - 1\right]$$

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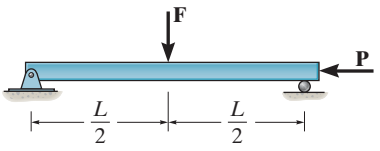
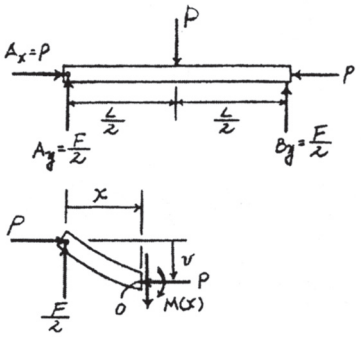
13–42.

The ideal column is subjected to the force **F** at its midpoint and the axial load **P**. Determine the maximum moment in the column at midspan. *EI* is constant. *Hint:* Establish the differential equation for deflection, Eq. 13–1. The general solution is $v = C_1 \sin kx + C_2 \cos kx - c^2x/k^2$, where $c^2 = F/2EI, k^2 = P/EI$.



SOLUTION

Moment Functions: FBD(b).



$$\zeta + \Sigma M_o = 0; \qquad M(x) + \frac{F}{2}x + P(v) = 0$$

$$M(x) = -\frac{F}{2}x - Pv \tag{1}$$

Differential Equation of The Elastic Curve:

$$EI\frac{d^2v}{dx^2} = M(x)$$

$$EI\frac{d^2v}{dx^2} = -\frac{F}{2}x - Pv$$

$$\frac{d^2v}{dx^2} + \frac{P}{EI}v = -\frac{F}{2EI}x$$

The solution of the above differential equation is of the form

$$v = C_1 \sin\left(\sqrt{\frac{P}{EI}}x\right) + C_2 \cos\left(\sqrt{\frac{P}{EI}}\,x\right) - \frac{F}{2P}x \tag{2}$$

and

$$\frac{dv}{dx} = C_1\sqrt{\frac{P}{EI}}\cos\left(\sqrt{\frac{P}{EI}}x\right) - C_2\sqrt{\frac{P}{EI}}\sin\left(\sqrt{\frac{P}{EI}}x\right) - \frac{F}{2P} \tag{3}$$

The integration constants can be determined from the boundary conditions.

Boundary Conditions:

At $x = 0, v = 0$. From Eq. (2), $C_2 = 0$

At $x = \frac{L}{2}, \frac{dv}{dx} = 0$. From Eq. (3),

$$0 = C_1\sqrt{\frac{P}{EI}}\cos\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right) - \frac{F}{2P}$$

$$C_1 = \frac{F}{2P}\sqrt{\frac{EI}{P}}\sec\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right)$$

Elastic Curve:

$$v = \frac{F}{2P}\sqrt{\frac{EI}{P}}\sec\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right)\sin\left(\sqrt{\frac{P}{EI}}x\right) - \frac{F}{2P}x$$

$$= \frac{F}{2P}\left[\sqrt{\frac{EI}{P}}\sec\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right)\sin\left(\sqrt{\frac{P}{EI}}x\right) - x\right]$$

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13–42. Continued

SOLUTION

However, $v = v_{\max}$ at $x = \frac{L}{2}$. Then,

$$v_{\max} = \frac{F}{2P}\left[\sqrt{\frac{EI}{P}}\sec\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right)\sin\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right) - \frac{L}{2}\right]$$

$$= \frac{F}{2P}\left[\sqrt{\frac{EI}{P}}\tan\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right) - \frac{L}{2}\right]$$

Maximum Moment: The maximum moment occurs at $x = \frac{L}{2}$. From Eq. (1),

$$M_{\max} = -\frac{F}{2}\left(\frac{L}{2}\right) - Pv_{\max}$$

$$= -\frac{FL}{4} - P\left\{\frac{F}{2P}\left[\sqrt{\frac{EI}{P}}\tan\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right) - \frac{L}{2}\right]\right\}$$

$$= -\frac{F}{2}\sqrt{\frac{EI}{P}}\tan\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right) \qquad \textbf{Ans.}$$

Ans:

$$M_{\max} = -\frac{F}{2}\sqrt{\frac{EI}{P}}\tan\left(\frac{L}{2}\sqrt{\frac{P}{EI}}\right)$$

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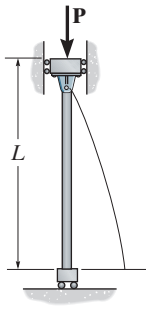
13–43.

The column with constant EI has the end constraints shown. Determine the critical load for the column.



SOLUTION

Moment Function. Referring to the free-body diagram of the upper part of the deflected column, Fig. *a*,

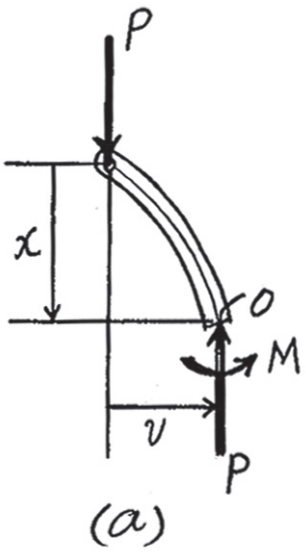
$$\zeta + \Sigma M_O = 0; \qquad M + Pv = 0 \qquad M = -Pv$$

Differential Equation of the Elastic Curve.

$$EI\frac{d^2v}{dx^2} = M$$

$$EI\frac{d^2v}{dx^2} = -Pv$$

$$\frac{d^2v}{dx^2} + \frac{P}{EI}v = 0$$



The solution is in the form of

$$v = C_1 \sin\left(\sqrt{\frac{P}{EI}}x\right) + C_2 \cos\left(\sqrt{\frac{P}{EI}}x\right) \qquad \textbf{(1)}$$

$$\frac{dv}{dx} = C_1\sqrt{\frac{P}{EI}}\cos\left(\sqrt{\frac{P}{EI}}x\right) - C_2\sqrt{\frac{P}{EI}}\sin\left(\sqrt{\frac{P}{EI}}x\right) \qquad \textbf{(2)}$$

Boundary Conditions. At $x = 0$, $v = 0$. Then Eq. (1) gives

$$0 = 0 + C_2 \qquad C_2 = 0$$

At $x = L$, $\frac{dv}{dx} = 0$. Then Eq. (2) gives

$$0 = C_1\sqrt{\frac{P}{EI}}\cos\left(\sqrt{\frac{P}{EI}}L\right)$$

$C_1 = 0$ is the trivial solution, where $v = 0$. This means that the column will remain straight and buckling will not occur regardless of the load P. Another possible solution is

$$\cos\left(\sqrt{\frac{P}{EI}}L\right) = 0$$

$$\sqrt{\frac{P}{EI}}L = \frac{n\pi}{2} \qquad n = 1, 3, 5$$

The smallest critical load occurs when $n = 1$, then

$$\sqrt{\frac{P_{cr}}{EI}}L = \frac{\pi}{2}$$

$$P_{cr} = \frac{\pi^2 EI}{4L^2} \qquad \textbf{Ans.}$$

Ans:

$$P_{cr} = \frac{\pi^2 EI}{4L^2}$$

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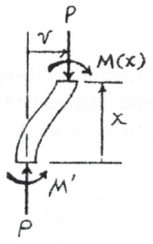
***13–44.**

Consider an ideal column as in Fig. 13–10*c*, having both ends fixed. Show that the critical load on the column is $P_{cr} = 4\pi^2 EI/L^2$. *Hint*: Due to the vertical deflection of the top of the column, a constant moment $\mathbf{M}'$ will be developed at the supports. Show that $d^2v/dx^2 + (P/EI)v = M'/EI$. The solution is of the form $v = C_1 \sin(\sqrt{P/EI}x) + C_2 \cos(\sqrt{P/EI}x) + M'/P$.

SOLUTION

Moment Functions:



$M(x) = M' - Pv$

Differential Equation of The Elastic Curve:

$$EI\frac{d^2v}{dx^2} = M(x)$$

$$EI\frac{d^2v}{dx^2} = M' - Pv$$

$$\frac{d^2v}{dx^2} + \frac{P}{EI}v = \frac{M'}{EI} \qquad \textbf{(Q.E.D.)}$$

The solution of the above differential equation is of the form

$$v = C_1 \sin\left(\sqrt{\frac{P}{EI}}x\right) + C_2 \cos\left(\sqrt{\frac{P}{EI}}x\right) + \frac{M'}{P} \qquad \textbf{(1)}$$

and

$$\frac{dv}{dx} = C_1\sqrt{\frac{P}{EI}}\cos\left(\sqrt{\frac{P}{EI}}x\right) - C_2\sqrt{\frac{P}{EI}}\sin\left(\sqrt{\frac{P}{EI}}x\right) \qquad \textbf{(2)}$$

The integration constants can be determined from the boundary conditions.

Boundary Conditions:

At $x = 0, v = 0$. From Eq. (1), $C_2 = -\dfrac{M'}{P}$

At $x = 0, \dfrac{dv}{dx} = 0$. From Eq. (2), $C_1 = 0$

Elastic Curve:

$$v = \frac{M'}{P}\left[1 - \cos\left(\sqrt{\frac{P}{EI}}x\right)\right]$$

and

$$\frac{dv}{dx} = \frac{M'}{P}\sqrt{\frac{P}{EI}}\sin\left(\sqrt{\frac{P}{EI}}x\right)$$

However, due to symmetry $\dfrac{dv}{dx} = 0$ at $x = \dfrac{L}{2}$. Then,

$$\sin\left[\sqrt{\frac{P}{EI}}\left(\frac{L}{2}\right)\right] = 0 \quad \text{or} \quad \sqrt{\frac{P}{EI}}\left(\frac{L}{2}\right) = n\pi \quad \text{where } n = 1, 2, 3,...$$

The smallest critical load occurs when $n = 1$.

$$P_{cr} = \frac{4\pi^2 EI}{L^2} \qquad \textbf{(Q.E.D.)}$$

Ans:
N/A

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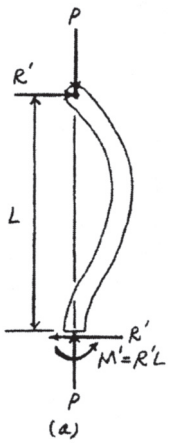
13–45.

Consider an ideal column as in Fig. 13–10*d*, having one end fixed and the other pinned. Show that the critical load on the column is $P_{cr} = 20.19EI/L^2$. *Hint*: Due to the vertical deflection at the top of the column, a constant moment $\mathbf{M}'$ will be developed at the fixed support and horizontal reactive forces $\mathbf{R}'$ will be developed at both supports. Show that $d^2v/dx^2 + (P/EI)v = (R'/EI)(L - x)$. The solution is of the form $v = C_1 \sin(\sqrt{P/EI}x) + C_2 \cos(\sqrt{P/EI}x) + (R'/P)(L - x)$. After application of the boundary conditions show that $\tan(\sqrt{P/EI}L) = \sqrt{P/EI}\,L$. Solve numerically for the smallest nonzero root.

SOLUTION

Equilibrium. FBD(a).



Moment Functions: FBD(b).

$M(x) = R'(L - x) - Pv$

Differential Equation of The Elastic Curve:

$$EI\frac{d^2v}{dx^2} = M(x)$$

$$EI\frac{d^2v}{dx^2} = R'(L - x) - Pv$$

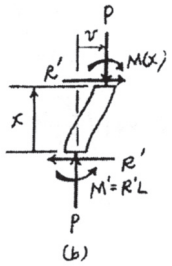
$$\frac{d^2v}{dx^2} + \frac{P}{EI}v = \frac{R'}{EI}(L - x) \qquad \textbf{(Q.E.D.)}$$

The solution of the above differential equation is of the form

$$v = C_1 \sin\left(\sqrt{\frac{P}{EI}}x\right) + C_2 \cos\left(\sqrt{\frac{P}{EI}}x\right) + \frac{R'}{P}(L - x) \qquad \textbf{(1)}$$

and

$$\frac{dv}{dx} = C_1\sqrt{\frac{P}{EI}}\cos\left(\sqrt{\frac{P}{EI}}x\right) - C_2\sqrt{\frac{P}{EI}}\sin\left(\sqrt{\frac{P}{EI}}x\right) - \frac{R'}{P} \qquad \textbf{(2)}$$



The integration constants can be determined from the boundary conditions.

Boundary Conditions:

At $x = 0$, $v = 0$. From Eq. (1), $C_2 = -\dfrac{R'L}{P}$

At $x = 0$, $\dfrac{dv}{dx} = 0$. From Eq. (2), $C_1 = \dfrac{R'}{P}\sqrt{\dfrac{EI}{P}}$

Elastic Curve:

$$v = \frac{R'}{P}\sqrt{\frac{EI}{P}}\sin\left(\sqrt{\frac{P}{EI}}x\right) - \frac{R'L}{P}\cos\left(\sqrt{\frac{P}{EI}}x\right) + \frac{R'}{P}(L - x)$$

$$= \frac{R'}{P}\left[\sqrt{\frac{EI}{P}}\sin\left(\sqrt{\frac{P}{EI}}x\right) - L\cos\left(\sqrt{\frac{P}{EI}}x\right) + (L - x)\right]$$

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13–45. Continued

SOLUTION

However, $v = 0$ at $x = L$. Then,

$$0 = \sqrt{\frac{EI}{P}}\sin\left(\sqrt{\frac{P}{EI}}L\right) - L\cos\left(\sqrt{\frac{P}{EI}}L\right)$$

$$\tan\left(\sqrt{\frac{P}{EI}}L\right) = \sqrt{\frac{P}{EI}}L \qquad \textbf{(Q.E.D.)}$$

By trial and error and choosing the smallest root, we have

$$\sqrt{\frac{P}{EI}}L = 4.49341$$

Then,

$$P_{\text{cr}} = \frac{20.19EI}{L^2} \qquad \textbf{(Q.E.D.)}$$

Ans:
N/A

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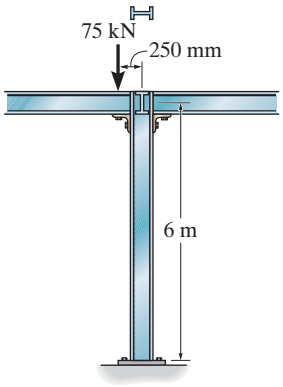
13–46. The W360 × 39 structural A-36 steel member is used as 6-m-long column that is assumed to be fixed at its top and fixed at its bottom. If the 75-kN load is applied at an eccentric distance of 250 mm, determine the maximum stress in the column



SOLUTION

Section Properties for W360 × 39

$A = 4960\ \text{mm}^2 = 4.96(10^{-3})\ \text{m}^2$

$d = 353\ \text{mm} = 0.353\ \text{m}$

$r_x = 143\ \text{mm} = 0.143\ \text{m}$

Yielding about $x-x$ axis:

$$\sigma_{\max} = \frac{P}{A}\left[1 + \frac{e\,c}{r^2}\sec\left(\frac{KL}{2\,r}\sqrt{\frac{P}{E\,A}}\right)\right]; \qquad K = 0.5$$

$$\frac{P}{A} = \frac{75(10^3)}{4.96(10^{-3})} = 15.12(10^6)\ \text{N/m}^2 = 15.12\ \text{MPa};$$

$$\frac{ec}{r^2} = \frac{0.25\left(\frac{0.353}{2}\right)}{0.143^2} = 2.157807$$

$$\frac{KL}{2\pi}\sqrt{\frac{P}{EA}} = \frac{0.5(6)}{2(0.143)}\sqrt{\frac{75(10^3)}{[200(10^9)][4.96(10^{-3})]}}$$

$\sigma_{\max} = 15.12[1 + 2.157807\sec(0.091207)]$

$= 47.9\ \text{MPa} < \sigma_Y = 250\ \text{MPa}$ O.K.

Ans.

Ans:
$\sigma_{\max} = 47.9\ \text{MPa}$

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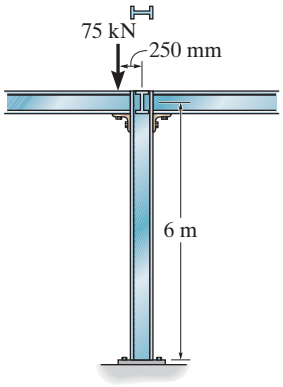
13-47. The W360 × 39 structural A-36 steel member is used as a column that is assumed to be fixed at its top and pinned at its bottom. If the 75-kN load is applied at an eccentric distance of 250 mm, determine the maximum stress in the column



SOLUTION

Section Properties for W360 × 39

$A = 4960 \text{ mm}^2 = 4.96(10^{-3}) \text{ m}^2$

$d = 353 \text{ mm} = 0.353 \text{ m}$

$r_x = 143 \text{ mm} = 0.143 \text{ m}$

Yielding about $x-x$ axis:

$$\sigma_{\max} = \frac{P}{A}\left[1 + \frac{e\,c}{r^2}\sec\left(\frac{KL}{2\,r}\sqrt{\frac{P}{E\,A}}\right)\right]; \qquad K = 0.7$$

$$\frac{P}{A} = \frac{75(10^3)}{4.96(10^{-3})} = 15.12(10^6) \text{ N-m}^2 = 15.12 \text{ MPa};$$

$$\frac{ec}{r^2} = \frac{0.25\left(\frac{0.353}{2}\right)}{0.143^2} = 2.157807$$

$$\frac{KL}{2\pi}\sqrt{\frac{P}{EA}} = \frac{0.7(6)}{2(0.143)}\sqrt{\frac{75(10^3)}{[200(10^9)][4.96(10^{-3})]}} = 0.127690$$

$\sigma_{\max} = 15.12[1 + 2.157807 \sec (0.127690)]$

$= 48.0 \text{ MPa} < \sigma_Y = 250 \text{ MPa}$ O.K.

Ans.

Ans:
$\sigma_{\max} = 48.0 \text{ MPa}$

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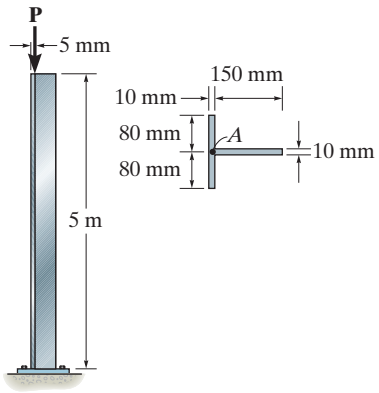
***13–48.**

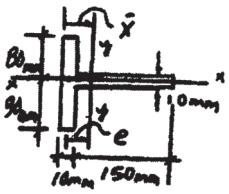
The aluminum column is fixed at the bottom and free at the top. Determine the maximum force P that can be applied at A without causing it to buckle or yield. Use a factor of safety of 3 with respect to buckling and yielding. $E_{al} = 70$ GPa, $\sigma_Y = 95$ MPa.



SOLUTION

$$\bar{x} = \frac{(0.005)(0.16)(0.01) + (0.085)(0.15)(0.01)}{0.16(0.01) + 0.15(0.01)} = 0.04371 \text{ m}$$

$$I_y = \frac{1}{12}(0.16)(0.01)^3 + (0.16)(0.01)(0.04371 - 0.005)^2 + \frac{1}{12}(0.01)(0.15)^3$$
$$+ (0.15)(0.01)(0.085 - 0.04371)^2 = 7.7807(10^{-6})\text{ m}^4$$



$$I_x = \frac{1}{12}(0.01)(0.16^3) + \frac{1}{12}(0.15)(0.01^3) = 3.42583(10^{-6})\text{ m}^4$$

$$A = (0.16)(0.01) + (0.15)(0.01) = 3.1(10^{-3})\text{ m}^2$$

$$r_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{7.7807(10^{-6})}{3.1(10^{-3})}} = 0.0501 \text{ m}$$

Buckling about *x–x* axis:

$$P = P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2(70)(10^9)(3.42583)(10^{-6})}{[(2.0)(5)]^2} = 23668 \text{ N}$$

$$P_{allow} = \frac{P_{cr}}{3} = 7.89 \text{ kN} \qquad \textbf{(controls)} \qquad \textbf{Ans.}$$

Check: $\sigma_{cr} = \frac{P_{cr}}{A} = \frac{23668 \text{ N}}{3.1(10^{-3})} = 7.63 \text{ MPa} < \sigma_Y$ **OK**

Yielding about *y–y* axis:

$$\sigma_{max} = \frac{P}{A}\left(1 + \frac{ec}{r^2}\sec\left(\frac{KL}{2r}\sqrt{\frac{P}{EA}}\right)\right)$$

$$\frac{ec}{r^2} = \frac{(0.03871)(0.04371)}{0.0501^2} = 0.6741$$

$$\left(\frac{KL}{2r}\right)\sqrt{\frac{P}{EA}} = \frac{2.0(5)}{2(0.0501)}\sqrt{\frac{P}{70(10^8)(3.1)(10^{-3})}} = 6.7749(10^{-3})\sqrt{P}$$

$$95(10^6)(3.1)(10^{-3}) = P[1 + 0.6741 \sec(6.7749(10^{-3})\sqrt{P}]$$

By trial and error:

$$P = 45.61 \text{ kN} \qquad P_{allow} = \frac{45.61}{3} = 15.2 \text{ kN}$$

Ans:
$P_{allow} = 7.89$ kN

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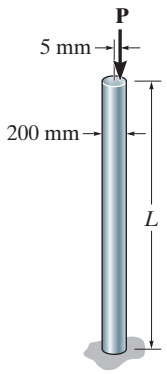
13–49.

The aluminum rod is fixed at its base and free at its top. If the eccentric load $P = 200$ kN is applied, determine the greatest allowable length L of the rod so that it does not buckle or yield. $E_{al} = 72$ GPa, $\sigma_Y = 410$ MPa.



SOLUTION

Section Properties:

$$A = \pi(0.1^2) = 0.031416\ \text{m}^2 \qquad I = \frac{\pi}{4}(0.1^4) = 78.54(10^{-6})\ \text{m}^4$$

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{78.54(10^{-6})}{0.031416}} = 0.05\ \text{m}$$

Yielding:

$$\sigma_{\max} = \frac{P}{A}\left[1 + \frac{ec}{r^2}\sec\left(\frac{KL}{2r}\sqrt{\frac{P}{EA}}\right)\right]$$

$$\frac{P}{A} = \frac{200(10^3)}{0.031416} = 6.3662(10^4)\ \text{Pa}$$

$$\frac{ec}{r^2} = \frac{0.005(0.1)}{(0.05)^2} = 0.2$$

$$\left(\frac{KL}{2r}\right)\sqrt{\frac{P}{EA}} = \frac{2.0(L)}{2(0.05)}\sqrt{\frac{200(10^3)}{72(10^9)(0.031416)}} = 0.188063L$$

$$410(10^4) = 6.3662(10^6)[1 + 0.2\sec(0.188063\ \text{L})]$$

$$L = 8.34\ \text{m} \qquad \text{(controls)} \qquad \textbf{Ans.}$$

Buckling about *x–x* axis:

$$\frac{P}{A} = 6.36\ \text{MPa} < \sigma_Y \qquad \text{Euler formula is valid.}$$

$$P_{\text{cr}} = \frac{\pi^2 EI}{(KL)^2}$$

$$200(10^3) = \frac{\pi^2(72)(10^9)(78.54)(10^{-4})}{[(2.0)(L)]^2}$$

$$L = 8.34\ \text{m}$$

Ans:
$L = 8.34$ m

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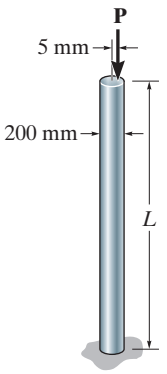
13–50.

The aluminum rod is fixed at its base and free and at its top. If the length of the rod is $L = 2$ m, determine the greatest allowable load P that can be applied so that the rod does not buckle or yield. Also, determine the largest sideway deflection of the rod due to the loading. $E_{al} = 72$ GPa, $\sigma_Y = 410$ MPa.



SOLUTION

Section Properties:

$$A = \pi(0.1^2) = 0.031416\ \text{m}^2 \qquad I = \frac{\pi}{4}(0.1^4) = 78.54(10^{-6})\ \text{m}^4$$

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{78.54(10^{-6})}{0.031416}} = 0.05\ \text{m}$$

Yielding:

$$\sigma_{\max} = \frac{P}{A}\left[1 + \frac{ec}{r^2}\sec\left(\frac{KL}{2r}\sqrt{\frac{P}{EA}}\right)\right]$$

$$\frac{ec}{r^2} = \frac{(0.005)(0.1)}{0.05^2} = 0.2$$

$$\left(\frac{KL}{2r}\right)\sqrt{\frac{P}{EA}} = \frac{2(2)}{2(0.05)}\sqrt{\frac{P}{72(10^9)(0.031416)}} = 0.8410\,(10^{-3})\sqrt{P}$$

$$410(10^4)(0.031416) = P[(1 + 0.2\sec(0.8410(10^{-3})\sqrt{P}\,)]$$

By trial and Error:

$P = 3.20$ MN (controls) **Ans.**

Buckling:

$$P = P_{\text{cr}} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2(72)(10^9)(78.54)(10^{-6})}{[(2.0)(2)]^2} = 3488\ \text{kN}$$

Check: $\sigma_{\text{cr}} = \dfrac{P_{\text{cr}}}{A} = \dfrac{3488(10^3)}{0.031416} = 111\ \text{MPa} < \sigma_Y$ OK

Maximum deflection:

$$v_{\max} = e\left[\sec\left(\sqrt{\frac{P}{EI}}\frac{KL}{2}\right) - 1\right]$$

$$\sqrt{\frac{P}{EI}}\frac{KL}{2} = \sqrt{\frac{3.20(10^6)}{72(10^9)(78.54)(10^{-6})}}\left(\frac{2.0(2)}{2}\right) = 1.5045$$

$v_{\max} = 5[\sec(1.5045) - 1] = 70.5$ mm **Ans.**

Ans:
$P = 3.20$ MN,
$v_{\max} = 70.5$ mm

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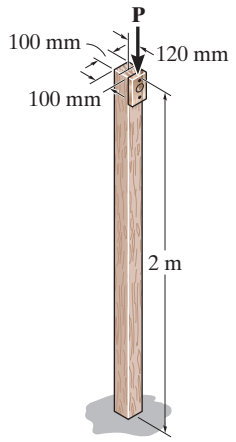
13–51.

The wood column is fixed at its base and free at its top. Determine the load P that can be applied to the edge of the column without causing the column to fail either by buckling or by yielding. $E_w = 12$ GPa, $\sigma_Y = 55$ MPa.



SOLUTION

Section Properties:

$$A = 0.1(0.1) = 0.01\ \text{m}^2 \quad I = \frac{1}{12}(0.1)(0.1)^3 = 8.333(10^{-6})\ \text{m}^4$$

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{8.333(10^{-6})}{0.01}} = 0.02887\ \text{m}$$

Buckling:

$$P_{\text{cr}} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2(12)(10^9)(8.333)(10^{-6})}{[2.0(2)]^2} = 61.7\ \text{kN}$$

Check: $\sigma_{\text{cr}} = \dfrac{P_{\text{cr}}}{A} = \dfrac{61.7(10^3)}{0.01} = 6.17\ \text{MPa} < \sigma_Y$ **OK**

Yielding:

$$\sigma_{\max} = \frac{P}{A}\left[1 + \frac{ec}{r^2}\sec\left(\frac{KL}{2r}\sqrt{\frac{P}{EA}}\right)\right]$$

$$\frac{ec}{r^2} = \frac{0.12(0.05)}{(0.02887)^2} = 7.20$$

$$\frac{KL}{2r}\sqrt{\frac{P}{EA}} = \frac{2.0(2)}{2(0.02887)}\sqrt{\frac{P}{12(10^9)(0.01)}} = 0.006324\sqrt{P}$$

$$55(10^6)(0.01) = P[1 + 7.20\sec(0.006324\sqrt{P})]$$

By trial and error:

$P = 31400\ \text{N} = 31.4\ \text{kN}$ controls **Ans.**

Ans:
$P = 31.4$ kN

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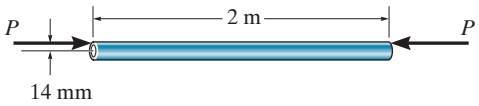
***13–52.**

The tube is made of copper and has an outer diameter of 35 mm and a wall thickness of 7 mm. Determine the eccentric load P that it can support without failure. The tube is pin supported at its ends. $E_{cu} = 120$ GPa, $\sigma_Y = 750$ MPa.



SOLUTION

Section Properties:

$$A = \frac{\pi}{4}(0.035^2 - 0.021^2) = 0.61575(10^{-3})\ \text{m}^2$$

$$I = \frac{\pi}{4}(0.0175^4 - 0.0105^4) = 64.1152(10^{-9})\ \text{m}^4$$

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{64.1152(10^{-9})}{0.61575(10^{-3})}} = 0.010204\ \text{m}$$

For a column pinned at both ends, $K = 1$. Then $KL = 1(2) = 2$ m.

Buckling: Applying *Euler's* formula,

$$P_{\max} = P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2(120)(10^9)\left[64.1152(10^{-9})\right]}{2^2} = 18983.7\ \text{N} = 18.98\ \text{kN}$$

Critical Stress: *Euler's* formula is only valid if $\sigma_{cr} < \sigma_\gamma$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{18983.7}{0.61575(10^{-3})} = 30.83\ \text{MPa} < \sigma_\gamma = 750\ \text{MPa} \qquad \text{O.K.}$$

Yielding: Applying the secant formula,

$$\sigma_{\max} = \frac{P_{\max}}{A}\left[1 + \frac{ec}{r^2}\sec\left(\frac{(KL)}{2r}\sqrt{\frac{P_{\max}}{EA}}\right)\right]$$

$$750\left(10^6\right) = \frac{P_{\max}}{0.61575(10^{-3})}\left[1 + \frac{0.014(0.0175)}{0.010204^2}\sec\left(\frac{2}{2(0.010204)}\sqrt{\frac{P_{\max}}{120(10^9)[0.61575(10^{-3})]}}\right)\right]$$

$$750\left(10^6\right) = \frac{P_{\max}}{0.61575(10^{-3})}\left(1 + 2.35294\sec 0.0114006\sqrt{P_{\max}}\right)$$

Solving by trial and error,

$$P_{\max} = 16\,884\ \text{N} = 16.9\ \text{kN} \quad (\textit{Controls!}) \qquad \textbf{Ans.}$$

Ans:
$P_{\max} = 16.9$ kN

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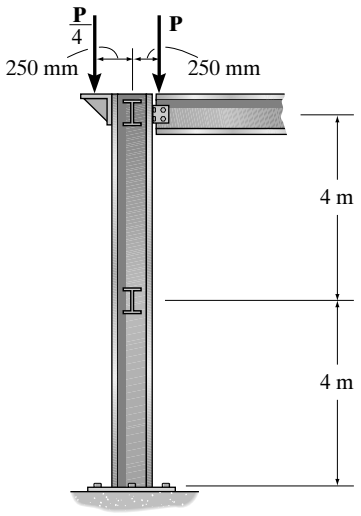
13–53. The W250 × 45 A-36-steel column is pinned at its top and fixed at its base. Also, the column is braced along its weak axis at mid-height. If $P = 250$ kN, investigate whether the column is adequate to support this loading. Use F.S. = 2 against buckling and F.S. = 1.5 against yielding.



SOLUTION

Section Properties. From the table listed in the appendix, the necessary section properties for a W250 × 45 are

$A = 5700\ \text{mm}^2 = 5.70\left(10^{-3}\right)\text{m}^2$ $\qquad r_x = 112\ \text{mm} = 0.112\ \text{m}$

$I_y = 7.03\left(10^6\right)\text{mm}^4 = 7.03\left(10^{-6}\right)\text{m}^4$ $\qquad c = \dfrac{d}{2} = \dfrac{266}{2} = 133\ \text{mm} = 0.133\ \text{m}$

The eccentricity of the equivalent force $P' = 250 + \dfrac{250}{4} = 312.5$ kN is

$$e = \frac{250(0.25) - \dfrac{250}{4}(0.25)}{250 + \dfrac{250}{4}} = 0.15\ \text{m}$$

Buckling About the Weak Axis. The column is braced along the weak axis at midheight and the support provided by the bracing can be considered as a pin. The top portion of the column is critical is since the top is pinned so $K_y = 1$ and $L = 4$ m Applying Euler's formula,

$$P_{cr} = \frac{\pi^2 E I_y}{(KL)_y{}^2} = \frac{\pi^2\left[200\left(10^9\right)\right]\left[7.03\left(10^{-6}\right)\right]}{[1(4)]^2} = 867.29\,\text{kN}$$

Euler's equation is valid only if $\sigma_{cr} < \sigma_Y$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{867.29\left(10^3\right)}{5.70\left(10^{-3}\right)} = 152.16\ \text{MPa} < \sigma_Y = 250\ \text{MPa} \qquad \text{O.K.}$$

Then,

$$P'_{\text{allow}} = \frac{P_{cr}}{\text{F.S.}} = \frac{867.29}{2} = 433.65\ \text{kN}$$

Since $P'_{\text{allow}} > P'$, the column *does not buckle*.

Yielding About Strong Axis. Since the column is fixed at its base and pinned at its top, $K_x = 0.7$ and $L = 8$ m. Applying the secant formula with $P'_{\max} = P'(\text{F.S.}) = 312.5(1.5) = 468.75$ kN

$$\sigma_{\max} = \frac{P'_{\max}}{A}\left[1 + \frac{ec}{r_x{}^2}\sec\left[\frac{(KL)_x}{2r_x}\sqrt{\frac{P'_{\max}}{EA}}\right]\right]$$

$$= \frac{468.75\left(10^3\right)}{5.70\left(10^{-3}\right)}\left[1 + \frac{0.15(0.133)}{0.112^2}\sec\left[\frac{0.7(8)}{2(0.112)}\sqrt{\frac{468.75\left(10^3\right)}{200\left(10^9\right)\left[5.70\left(10^{-3}\right)\right]}}\right]\right]$$

$$= 231.84\ \text{MPa}$$

Since $\sigma_{\max} < \sigma_Y = 250$ MPa, the column *does not yield*.

Ans:
The column is adequate.

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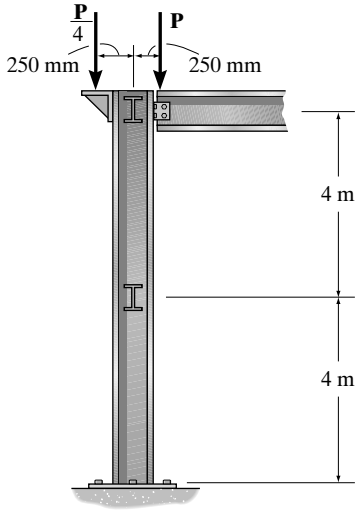
*13–54. The W250 × 45 A-36-steel column is pinned at its top and fixed at its base. Also, the column is braced along its weak axis at mid-height. Determine the allowable force P that the column can support without causing it either to buckle or yield. Use F.S. = 2 against buckling and F.S. = 1.5 against yielding.



Section Properties. From the table listed in the appendix, the necessary section properties for a W250 × 45 are

$A = 5700\ \text{mm}^2 = 5.70\left(10^{-3}\right)\ \text{m}^2$ $\quad r_x = 112\ \text{mm} = 0.112\ \text{m}$

$I_y = 7.03\left(10^6\right)\text{mm}^4 = 7.03\left(10^{-6}\right)\text{m}^4$ $\quad c = \dfrac{d}{2} = \dfrac{266}{2} = 133\ \text{mm} = 0.133\ \text{m}$

The eccentricity of the equivalent force $P' = P + \dfrac{P}{4} = 1.25P$ is

$$e = \frac{P(0.25) - \dfrac{P}{4}(0.25)}{P + \dfrac{P}{4}} = 0.15\ \text{m}$$

Buckling About the Weak Axis. The column is braced along the weak axis at midheight and the support provided by the bracing can be considered as a pin. The top portion of the column is critical is since the top is pinned so $K_y = 1$ and $L = 4$ m. Applying Euler's formula,

$$P_{cr} = \frac{\pi^2 EI_y}{(KL)_y{}^2} = \frac{\pi^2\left[200\left(10^9\right)\right]\left[7.03\left(10^{-6}\right)\right]}{[1(4)]^2} = 867.29\ \text{kN}$$

Euler's equation is valid only if $\sigma_{cr} < \sigma_Y$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{867.29\left(10^3\right)}{5.70\left(10^{-3}\right)} = 152.16\ \text{MPa} < \sigma_Y = 250\ \text{MPa} \qquad \text{O.K.}$$

Then,

$$P'_{\text{allow}} = \frac{P_{cr}}{\text{F.S.}}$$
$$1.25P_{\text{allow}} = \frac{867.29}{2}$$
$$P_{\text{allow}} = 346.92\ \text{kN}$$

Yielding About Strong Axis. Since the column is fixed at its base and pinned at its top, $K_x = 0.7$ and $L = 8$ m. Applying the secant formula,

$$\sigma_{\max} = \frac{P'_{\max}}{A}\left[1 + \frac{ec}{r_x{}^2}\sec\left[\frac{(KL)_x}{2r_x}\sqrt{\frac{P'_{\max}}{EA}}\right]\right]$$

$$250\left(10^6\right) = \frac{1.25P_{\max}}{5.70\left(10^{-3}\right)}\left[1 + \frac{0.15(0.133)}{0.112^2}\sec\left[\frac{0.7(8)}{2(0.112)}\sqrt{\frac{1.25P_{\max}}{200\left(10^9\right)\left[5.70\left(10^{-3}\right)\right]}}\right]\right]$$

$$250\left(10^6\right) = \frac{1.25P_{\max}}{5.70\left(10^{-3}\right)}\left(1 + 1.5904\sec\left(0.00082783\right)\sqrt{P_{\max}}\right)$$

Solving by trial and error,

$P_{\max} = 401.75\ \text{kN}$

Then,

$$P_{\text{allow}} = \frac{401.75}{1.5} = 267.83\ \text{kN} = 268\ \text{kN (controls)} \qquad \textbf{Ans.}$$

Ans:
$e = 0.15$ m, $P_{\text{allow}} = 346.92$ kN,
$P_{\text{allow}} = 268$ kN

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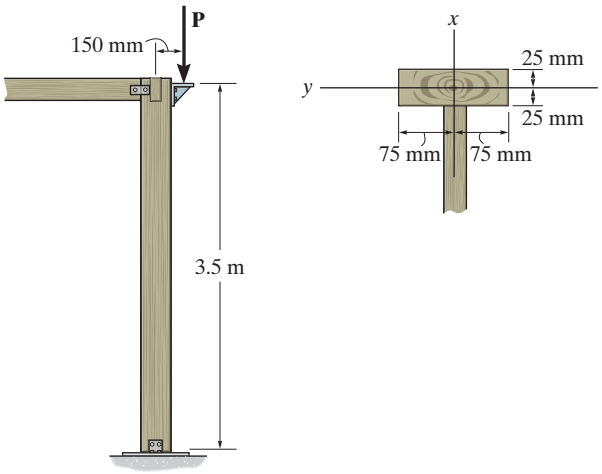
13–55.

The wood column is pinned at its base and top. If the eccentric force $P = 10$ kN is applied to the column, investigate whether the column is adequate to support this loading without buckling or yielding. Take $E = 10$ GPa and $\sigma_Y = 15$ MPa.



SOLUTION

Section Properties:

$$A = 0.05(0.15) = 7.5\left(10^{-3}\right)\text{m}^2$$

$$I_x = \frac{1}{12}(0.05)\left(0.15^3\right) = 14.0625\left(10^{-6}\right)\text{m}^4$$

$$r_x = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{14.0625\left(10^{-6}\right)}{7.5\left(10^{-3}\right)}} = 0.04330\text{ m}$$

$$I_y = \frac{1}{12}(0.15)\left(0.05^3\right) = 1.5625\left(10^{-6}\right)\text{m}^4$$

$e = 0.15$ m $\qquad c = 0.075$ m

For a column that is pinned at both ends, $K = 1$. Then,

$$(KL)_x = (KL)_y = 1(3.5) = 3.5\text{ m}$$

Buckling About the Weak Axis: Applying Euler's formula,

$$P_{\text{cr}} = \frac{\pi^2 EI_y}{(KL)_y{}^2} = \frac{\pi^2\left[10\left(10^9\right)\right]\left[1.5625\left(10^{-6}\right)\right]}{3.5^2} = 12.59\text{ kN}$$

Euler's formula is valid if $\sigma_{\text{cr}} < \sigma_Y$.

$$\sigma_{\text{cr}} = \frac{P_{\text{cr}}}{A} = \frac{12.59\left(10^3\right)}{7.5\left(10^{-3}\right)} = 1.68\text{ MPa} < \sigma_Y = 15\text{ MPa}$$ **O.K.**

Since $P_{\text{cr}} > P = 10$ kN, the column *will not buckle*.

Yielding About Strong Axis: Applying the secant formula,

$$\sigma_{\max} = \frac{P}{A}\left[1 + \frac{ec}{r_x{}^2}\sec\left[\frac{(KL)_x}{2r_x}\sqrt{\frac{P}{EA}}\right]\right]$$

$$= \frac{10\left(10^3\right)}{7.5\left(10^{-3}\right)}\left[1 + \frac{0.15(0.075)}{0.04330^2}\sec\left[\frac{3.5}{2(0.04330)}\sqrt{\frac{10\left(10^3\right)}{10\left(10^9\right)\left[7.5\left(10^{-3}\right)\right]}}\right]\right]$$

$$= 10.29\text{ MPa}$$

Since $\sigma_{\max} < \sigma_Y = 15$ MPa, the column *will not yield.* **Ans.**

Ans:
Yes

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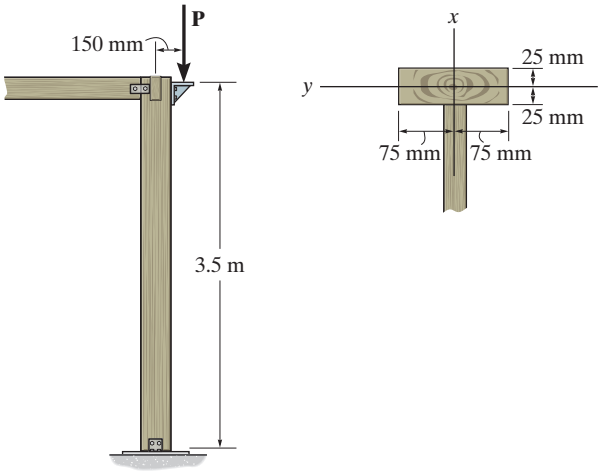
***13–56.**

The wood column is pinned at its base and top. Determine the maximum eccentric force P the column can support without causing it to either buckle or yield. Take $E = 10$ GPa and $\sigma_Y = 15$ MPa.



SOLUTION

Section Properties:

$$A = 0.05(0.15) = 7.5\left(10^{-3}\right)\text{m}^2$$

$$I_x = \frac{1}{12}(0.05)\left(0.15^3\right) = 14.0625\left(10^{-6}\right)\text{m}^4$$

$$r_x = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{14.0625\left(10^{-6}\right)}{7.5\left(10^{-3}\right)}} = 0.04330\,\text{m}$$

$$I_y = \frac{1}{12}(0.15)\left(0.05^3\right) = 1.5625\left(10^{-6}\right)\text{m}^4$$

$e = 0.15$ m $\qquad c = 0.075$ m

For a column that is pinned at both ends, $K = 1$. Then,

$$(KL)_x = (KL)_y = 1(3.5) = 3.5\text{ m}$$

Buckling About the Weak Axis: Applying Euler's formula,

$$P_{\text{cr}} = \frac{\pi^2 EI_y}{(KL)_y{}^2} = \frac{\pi^2\left[10\left(10^9\right)\right]\left[1.5625\left(10^{-6}\right)\right]}{3.5^2} = 12.59\text{ kN} = 12.6\text{ kN} \qquad \textbf{Ans.}$$

Euler's formula is valid if $\sigma_{\text{cr}} < \sigma_Y$.

$$\sigma_{\text{cr}} = \frac{P_{\text{cr}}}{A} = \frac{12.59\left(10^3\right)}{7.5\left(10^{-3}\right)} = 1.68\text{ MPa} < \sigma_Y = 15\text{ MPa} \qquad \textbf{O.K.}$$

Yielding About Strong Axis: Applying the secant formula with $P = P_{\text{cr}} = 12.59$ kN,

$$\sigma_{\max} = \frac{P}{A}\left[\left[1 + \frac{ec}{r_x{}^2}\sec\left[\frac{(KL)_x}{2r_x}\sqrt{\frac{P}{EA}}\right]\right]\right]$$

$$= \frac{12.59\left(10^3\right)}{7.5\left(10^{-3}\right)}\left[1 + \frac{0.15(0.075)}{0.04330^2}\sec\left[\frac{3.5}{2(0.04330)}\sqrt{\frac{12.59\left(10^3\right)}{10\left(10^9\right)\left[7.5\left(10^{-3}\right)\right]}}\right]\right]$$

$$= 13.31\text{ MPa} < \sigma_Y = 15\text{ MPa} \qquad \textbf{O.K.}$$

Ans:
$P_{\text{cr}} = 12.6$ kN

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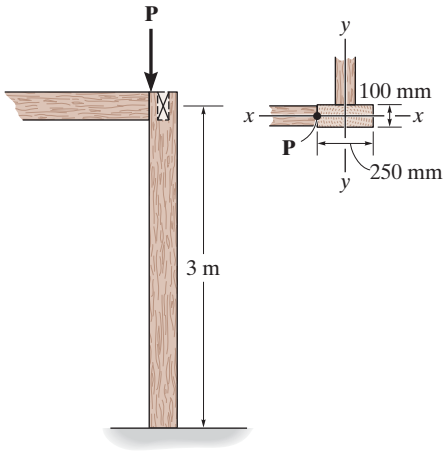
13–57. The wood column is fixed at its base and can be assumed pin connected at its top. Determine the maximum eccentric load P that can be applied without causing the column to buckle or yield. $E_w = 12$ GPa, $\sigma_Y = 56$ MPa.



SOLUTION

Section Properties:

$$A = 0.25(0.1) = 0.025 \text{ m}^2$$

$$I_y = \frac{1}{12}(0.1)(0.25^3) = 0.13021(10^{-3}) \text{ m}^4$$

$$I_x = \frac{1}{12}(0.25)(0.1^3) = 20.8333(10^{-6}) \text{ m}^4$$

$$r_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{0.13021(10^{-3})}{0.025}} = 0.072169 \text{ m}$$

Buckling about $x-x$ axis:

$$P = P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2[12(10^9)][20.8333(10^{-6})]}{[0.7(3)]^2} = 559.50(10^3) \text{ N} = 560 \text{ kN}$$

Check: $\sigma_{cr} = \frac{P_{cr}}{A} = \frac{559.50(10^3)}{0.025} = 22.38(10^6) \text{ N-m}^2 = 22.4 \text{ MPa} < \sigma_Y$ O.K.

Yielding about $y-y$ axis:

$$\sigma_{\max} = \frac{P}{A}\left[1 + \frac{ec}{r^2}\sec\left(\frac{KL}{2r}\right)\sqrt{\frac{P}{EA}}\right]$$

$$\frac{ec}{r^2} = \frac{0.125(0.125)}{0.072169^2} = 3.00$$

$$\left(\frac{KL}{2r}\right)\sqrt{\frac{P}{EA}} = \frac{0.7(3)}{2(0.072169)}\sqrt{\frac{P}{[12(10^9)](0.125)}} = 0.8400(10^{-3})\sqrt{P}$$

$$56(10^6) = \frac{P}{0.025}\left\{1 + 3.00\sec[0.8400(10^{-3})\sqrt{P}]\right\}$$

$$1.40(10^6) = P\left\{1 + 3.00\sec[0.8400(10^{-3})\sqrt{P}]\right\}$$

By trial and error:

$P = 320.08(10^3)$ N = 320 kN (controls) **Ans.**

Ans:
$P = 320$ kN

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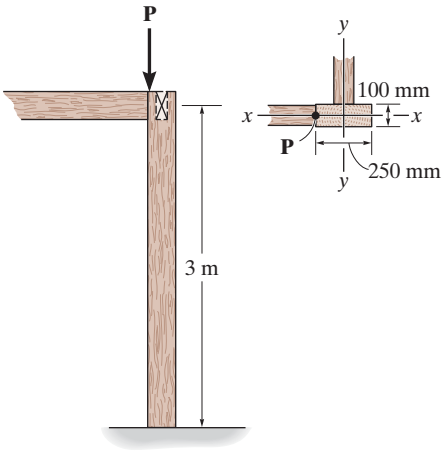
13–58. The wood column is fixed at its base and can be assumed fixed connected at its top. Determine the maximum eccentric load P that can be applied without causing the column to buckle or yield. E_w = 12 GPa, σ_Y = 56 MPa.



SOLUTION

Section Properties:

$$A = 0.25(0.1) = 0.025\ \text{m}^2$$

$$I_y = \frac{1}{12}(0.1)(0.25^3) = 0.13021(10^{-3})\ \text{m}^4$$

$$I_x = \frac{1}{12}(0.25)(0.1^3) = 20.8333(10^{-6})\ \text{m}^4$$

$$\pi_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{0.13021(10^{-3})}{0.025}} = 0.072169\ \text{m}$$

Buckling about $x-x$ axis:

$$P = P_{c\pi} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2[12(10^9)][20.8333(10^{-6})]}{[0.5(3)]^2} = 1.097(10^6)\ \text{N} = 1.10\ \text{MN}$$

Check: $\sigma_{c\pi} = \frac{P_{c\pi}}{A} = \frac{1.097(10^6)}{0.025} = 43.86(10^6)\ \text{N-m}^2 = 43.9\ \text{MPa} < \sigma_Y (= 56\ \text{MPa})$ O.K.

Yielding about $y-y$ axis:

$$\sigma_{\max} = \frac{P}{A}\left[1 + \frac{ec}{\pi^2}\sec\left(\frac{KL}{2\pi}\right)\sqrt{\frac{P}{EA}}\right]$$

$$\frac{ec}{\pi^2} = \frac{0.125(0.125)}{0.072169^2} = 3.00$$

$$\left(\frac{KL}{2\pi}\right)\sqrt{\frac{P}{EA}} = \frac{0.5(3)}{2(0.072169)}\sqrt{\frac{P}{[12(10^9)](0.025)}} = 0.600(10^{-3})\sqrt{P}$$

$$56(10^6) = \frac{P}{0.025}\left\{1 + 3.00\sec[0.600(10^{-3})\sqrt{P}]\right\}$$

$$1.4(10^6) = P\left\{1 + 3.00\sec[0.600(10^{-3})\sqrt{P}]\right\}$$

By trial and error:

$P = 334.13(10^3)\ \text{N} = 334\ \text{kN}$ (controls) **Ans.**

Ans:
$P = 334$ kN

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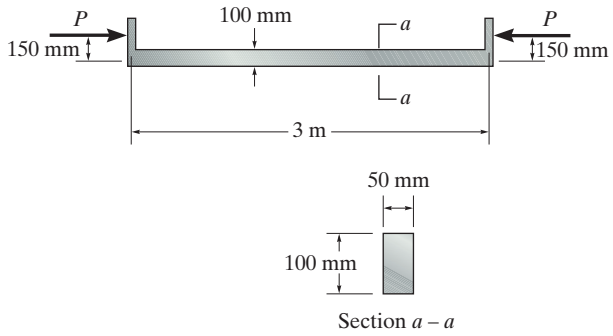
13–59.

Determine the maximum eccentric load P the 2014-T6-aluminum-alloy strut can support without causing it either to buckle or yield. The ends of the strut are pin connected.



SOLUTION

Section Properties: The necessary section properties are

$$A = 0.05(0.1) = 5\left(10^{-3}\right)\text{m}^2$$

$$I_y = \frac{1}{12}(0.1)\left(0.05^3\right) = 1.04167\left(10^{-6}\right)\text{ m}^4$$

$$r_x = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{4.1667\left(10^{-6}\right)}{5\left(10^{-3}\right)}} = 0.02887\text{ m}$$

For a column that is pinned at both of its ends $K = 1$. Thus,

$$(KL)_x = (KL)_y = 1(3) = 3\text{ m}$$

Buckling About the Weak Axis: Applying Euler's formula,

$$P_{\text{cr}} = \frac{\pi^2 EI_y}{(KL)_y{}^2} = \frac{\pi^2\left[73.1\left(10^9\right)\right]\left[1.04167\left(10^{-6}\right)\right]}{3^2} = 83.50\text{ kN} = 83.5\text{ kN} \qquad \textbf{Ans.}$$

Critical Stress: Euler's formula is valid only if $\sigma_{\text{cr}} < \sigma_Y$.

$$\sigma_{\text{cr}} = \frac{P_{\text{cr}}}{A} = \frac{83.50\left(10^3\right)}{5\left(10^{-3}\right)} = 16.70\text{ MPa} < \sigma_Y = 414\text{ MPa} \qquad \textbf{O.K.}$$

Yielding About Strong Axis: Applying the secant formula,

$$\sigma_{\max} = \frac{P}{A}\left[1 + \frac{ec}{r_x{}^2}\sec\left[\frac{(KL)_x}{2r_x}\sqrt{\frac{P}{EA}}\right]\right]$$

$$= \frac{83.50\left(10^3\right)}{5\left(10^{-3}\right)}\left[1 + \frac{0.15(0.05)}{0.02887^2}\sec\left[\frac{3}{2(0.02887)}\sqrt{\frac{83.50\left(10^3\right)}{73.1\left(10^9\right)\left[5\left(10^{-3}\right)\right]}}\right]\right]$$

$$= 229.27\text{ MPa} < \sigma_Y = 414\text{ MPa} \qquad \textbf{O.K.}$$

Ans:
$P_{\text{cr}} = 83.5\text{ kN}$

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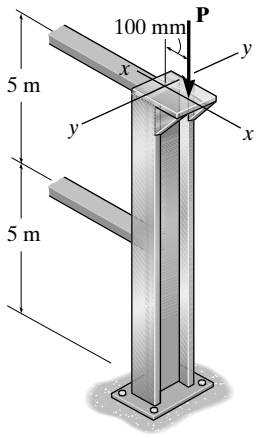
*13–60. The W200 × 22 A-36-steel column is fixed at its base. Its top is constrained to rotate about the *y–y* axis and free to move along the *y–y* axis. Also, the column is braced along the *x–x* axis at its mid-height. Determine the allowable eccentric force *P* that can be applied without causing the column either to buckle or yield. Use F.S. = 2 against buckling and F.S. = 1.5 against yielding.



Section Properties. From the table listed in the appendix, the necessary section properties for a W200 × 22 are

$A = 2860\,\text{mm}^2 = 2.86\left(10^{-3}\right)\text{m}^2$ $\quad r_y = 22.3\,\text{mm} = 0.0223\,\text{m}$

$I_x = 20.0\left(10^6\right)\text{mm}^4 = 20.0\left(10^{-6}\right)\text{m}^4$ $\quad c = \dfrac{b_f}{2} = \dfrac{102}{2} = 51\,\text{mm} = 0.051\,\text{m}$

$e = 0.1\text{m}$

Buckling About the Strong Axis. Since the column is fixed at the base and free at the top, $K_x = 2$. Applying Euler's formula,

$$P_{\text{cr}} = \frac{\pi^2 EI_x}{(KL)_x{}^2} = \frac{\pi^2\left[200\left(10^9\right)\right]\left[20.0\left(10^{-6}\right)\right]}{[2(10)]^2} = 98.70\text{kN}$$

Euler's formula is valid if $\sigma_{cr} < \sigma_Y$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{98.70\left(10^3\right)}{2.86\left(10^{-3}\right)} = 34.51\,\text{MPa} < \sigma_Y = 250\text{MPa} \qquad \text{O.K.}$$

Then,

$$P_{\text{allow}} = \frac{P_{cr}}{\text{F.S.}} = \frac{98.70}{2} = 49.35\,\text{kN}$$

Yielding About Weak Axis. Since the support provided by the bracing can be considered a pin connection, the upper portion of the column is pinned at both of its ends. Then $K_y = 1$ and $L = 5$ m. Applying the secant formula,

$$\sigma_{\max} = \frac{P_{\max}}{A}\left[1 + \frac{ec}{r_y{}^2}\sec\left[\frac{(KL)_y}{2r_y}\sqrt{\frac{P_{\max}}{EA}}\right]\right]$$

$$250\left(10^6\right) = \frac{P_{\max}}{2.86\left(10^{-3}\right)}\left[1 + \frac{0.1(0.051)}{0.0223^2}\sec\left[\frac{1(5)}{2(0.0223)}\sqrt{\frac{P_{\max}}{200\left(10^9\right)\left[2.86\left(10^{-3}\right)\right]}}\right]\right]$$

$$250\left(10^6\right) = \frac{P_{\max}}{2.86\left(10^{-3}\right)}\left[1 + 10.2556\sec 4.6875\left(10^{-3}\right)\sqrt{P_{\max}}\right]$$

Solving by trial and error,

$P_{\max} = 39.376\,\text{kN}$

Then,

$$P_{\text{allow}} = \frac{P_{\max}}{1.5} = \frac{39.376}{1.5} = 26.3\,\text{kN (controls)} \qquad \textbf{Ans.}$$

Ans:

$P_{\text{allow}} = 26.3\,\text{kN (controls)}$

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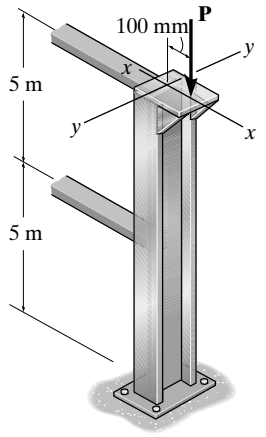
13–61. The W200 × 22 A-36-steel column is fixed at its base. Its top is constrained to rotate about the *y*–*y* axis and free to move along the *y*–*y* axis. Also, the column is braced along the *x*–*x* axis at its mid-height. If $P = 25$ kN, determine the maximum normal stress developed in the column.



SOLUTION

Section Properties. From the table listed in the appendix, necessary section properties for a W200 × 22 are

$A = 2860\ \text{mm}^2 = 2.86\left(10^{-3}\right)\ \text{m}^2$ $\quad r_y = 22.3\ \text{mm} = 0.0223\ \text{m}$

$I_x = 20.0\left(10^6\right)\ \text{mm}^4 = 20.0\left(10^{-6}\right)\text{m}^4$ $\quad c = \dfrac{b_f}{2} = \dfrac{102}{2} = 51\ \text{mm} = 0.051\ \text{m}$

$e = 0.1\text{m}$

Buckling About the Strong Axis. Since the column is fixed at the base and free at the top, $K_x = 2$. Applying Euler's formula,

$$P_{cr} = \frac{\pi^2 EI_x}{(KL)_x{}^2} = \frac{\pi^2\left[200\left(10^9\right)\right]\left[20.0\left(10^{-6}\right)\right]}{[2(10)]^2} = 98.70\text{kN}$$

Euler's formula is valid only if $\sigma_{cr} < \sigma_Y$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{98.70\left(10^3\right)}{2.86\left(10^{-3}\right)} = 34.51\ \text{MPa} < \sigma_Y = 250\ \text{MPa} \qquad \text{O.K.}$$

Since $P = 25\,\text{kN} < P_{cr}$, the column does not buckle.

Yielding About Weak Axis. Since the support provided by the bracing can be considered a pin connection, the upper portion of the column is pinned at both of its ends. Then $K_y = 1$ and $L = 5$ m. Applying the secant formula,

$$\sigma_{\max} = \frac{P}{A}\left[1 + \frac{ec}{r_y{}^2}\sec\left[\frac{(KL)}{2r_y}\sqrt{\frac{P}{EA}}\right]\right]$$

$$= \frac{2.5\left(10^3\right)}{2.86\left(10^{-3}\right)}\left[1 + \frac{0.1(0.051)}{0.0223^2}\sec\left[\frac{1(5)}{2(0.0223)}\sqrt{\frac{25\left(10^3\right)}{200\left(10^9\right)\left[2.86\left(10^{-3}\right)\right]}}\right]\right]$$

$= 130.26\ \text{MPa} = 130\ \text{MPa}$ **Ans.**

Since $\sigma_{\max} < \sigma_Y = 250$ MPa, the column does not yield.

Ans:
$\sigma_{\max} = 130$ MPa

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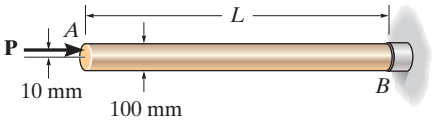
13–62.

The brass rod is fixed at one end and free at the other end. If the eccentric load $P = 200$ kN is applied, determine the greatest allowable length L of the rod so that it does not buckle or yield. $E_{br} = 101$ GPa, $\sigma_Y = 69$ MPa.



SOLUTION

Section Properties:

$$A = \frac{\pi}{4}(0.1^2) = 2.50(10^{-3})\pi \text{ m}^2$$

$$I = \frac{\pi}{4}(0.05^4) = 1.5625(10^{-6})\pi \text{ m}^4$$

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{1.5625(10^{-6})\pi}{2.50(10^{-3})\pi}} = 0.025 \text{ m}$$

For a column fixed at one end and free at the other end, $K = 2$. Then $KL = 2(L) = 2L$.

Buckling: Applying Euler's formula,

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

$$200(10^3) = \frac{\pi^2(101)(10^9)\left[1.5625(10^{-6})\pi\right]}{(2L)^2}$$

$$L = 2.473 \text{ m}$$

Critical Stress: *Euler's* formula is only valid if $\sigma_{cr} < \sigma_Y$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{200(10^3)}{2.50(10^{-3})\pi} = 25.46 \text{ MPa} < \sigma_Y = 69 \text{ MPa} \qquad \textbf{(O.K!)}$$

Yielding: Applying the secant formula,

$$\sigma_Y = \frac{P}{A}\left[1 + \frac{ec}{r^2}\sec\left(\frac{(KL)}{2r}\sqrt{\frac{P}{EA}}\right)\right]$$

$$69(10^6) = \frac{200(10^3)}{2.50(10^{-3})\pi}\left[1 + \frac{0.01(0.05)}{0.025^2}\sec\left(\frac{2L}{2(0.025)}\sqrt{\frac{200(10^3)}{101(10^9)\left[2.50(10^{-3})\pi\right]}}\right)\right]$$

$$69 = \frac{80}{\pi}(1 + 0.800\sec 0.635140L)$$

Solving by trial and error,

$$L = 1.7065 \text{ m} = 1.71 \text{ m} \qquad \textbf{(Controls!)} \qquad \textbf{Ans.}$$

Ans:
$L = 1.71$ m

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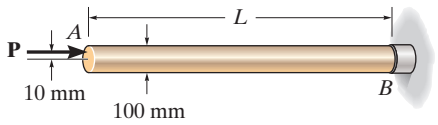
13–63.

The brass rod is fixed at one end and free at the other end. If the length of the rod is $L = 2$ m, determine the greatest allowable load P that can be applied so that the rod does not buckle or yield. Also, determine the largest sidesway deflection of the rod dueto the loading. $E_{br} = 101$ GPa, $\sigma_Y = 69$ MPa.



SOLUTION

Section Properties:

$$A = \frac{\pi}{4}(0.1^2) = 2.50(10^{-3})\pi \text{ m}^2$$

$$I = \frac{\pi}{4}(0.05^4) = 1.5625(10^{-6})\pi \text{ m}^4$$

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{1.5625(10^{-6})\pi}{2.50(10^{-3})\pi}} = 0.025 \text{ m}$$

For a column fixed at one end and free at the other end, $K = 2$. Then $KL = 2(2) = 4$ m.

Buckling: Applying *Euler's* formula,

$$P = P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

$$= \frac{\pi^2(101)(10^9)\left[1.5625(10^{-6})\pi\right]}{4^2}$$

$$= 305823.6 \text{ N} = 305.8 \text{ kN}$$

Critical Stress: *Euler's* formula is only valid if $\sigma_{cr} < \sigma_Y$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{305823.6}{2.50(10^{-3})\pi} = 38.94 \text{ MPa} < \sigma_Y = 69 \text{ MPa} \quad \textbf{(O.K!)}$$

Yielding: Applying the secant formula,

$$\sigma_Y = \frac{P}{A}\left[1 + \frac{ec}{r^2}\sec\left(\frac{(KL)}{2r}\sqrt{\frac{P}{EA}}\right)\right]$$

$$69(10^6) = \frac{P}{2.50(10^{-3})\pi}\left[1 + \frac{0.01(0.05)}{0.025^2}\sec\left(\frac{4}{2(0.025)}\sqrt{\frac{P}{101(10^9)\left[2.50(10^{-3})\pi\right]}}\right)\right]$$

$$69(10^6) = \frac{400P}{\pi}\left[1 + 0.800\sec 2.84043(10^{-3})\sqrt{P}\right]$$

Solving by trial and error,

$$P = 173700 \text{ N} = 174 \text{ kN} \quad \textbf{(Controls!)} \quad \textbf{Ans.}$$

Maximum Displacement:

$$v_{max} = e\left[\sec\left(\sqrt{\frac{P}{EI}}\frac{KL}{2}\right) - 1\right]$$

$$= 0.01\left[\sec\left(\sqrt{\frac{173700}{101(10^9)\left[1.5625(10^{-6})\pi\right]}}\left(\frac{4}{2}\right)\right) - 1\right]$$

$$= 0.01650 \text{ m} = 16.5 \text{ mm} \quad \textbf{Ans.}$$

Ans:
$P = 174$ kN
$v_{max} = 16.5$ mm

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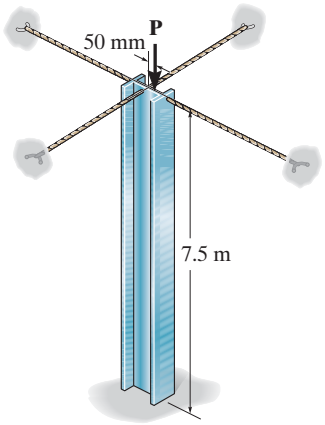
***13–64.** Determine the load P required to cause the steel W310 × 74 structural A-36 steel column to fail either by buckling or by yielding. The column is fixed at its bottom and the cables at its top act as a pin to hold it.



SOLUTION

Section Properties: For a wide flange section W310 × 74,

$$A = 9480\ \text{mm}^2 = 9.48(10^{-3})\ \text{m}^2$$

$$r_x = 132\ \text{mm} = 0.132\ \text{mm}$$

$$I_y = 23.4(10^6)\ \text{mm}^4 = 23.4(10^{-6})\ \text{m}^4$$

$$d = 310\ \text{mm} = 0.310\ \text{mm}$$

For a column fixed at one end and pinned at the other end, $K = 0.7$.

$$(KL)_y = (KL)_x = 0.7(7.5) = 5.25\ \text{m}$$

Buckling About y–y Axis: Applying *Euler's* formula,

$$P = P_{cr} = \frac{\pi^2 EI}{(KL)_y^2}$$

$$= \frac{\pi^2[200(10^9)][23.4(10^{-6})]}{5.25^2}$$

$$= 1.6758(10^6)\ \text{N} = 1.68\ \text{MN}$$

Critical Stress: *Euler's* formula is only valid if $\sigma_{cr} < \sigma_Y$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{1.6758(10^6)}{9.48(10^{-3})} = 176.77(10^6)\ \text{N/m}^2$$

$$= 177\ \text{MPa} < \sigma_Y (= 250\ \text{MPa})$$ O. K.

Yielding About x–x Axis: Applying the secant formula,

$$\sigma_{\max} = \frac{P}{A}\left\{1 + \frac{ec}{r_x^2}\sec\left[\frac{(KL)_x}{2r_x}\sqrt{\frac{P}{EA}}\right]\right\}$$

$$250(10^6) = \frac{P}{9.48(10^{-3})}\left\{1 + \frac{0.05\left(\frac{0.310}{2}\right)}{0.132^2}\sec\left[\frac{5.25}{2(0.132)}\sqrt{\frac{P}{200(10^9)(9.48)(10^{-3})}}\right]\right\}$$

$$2.37(10^6) = P\left\{1 + 0.444789\sec[0.45671(10^{-3})\sqrt{P}]\right\}$$

Solving by trial and error,

$$P_{\max} = 1.5511(10^6)\ \text{N} = 1.55\ \text{MN}\ (\textit{Controls}!)$$ **Ans.**

Ans:

$P_{\max} = 1.55$ MN (*Controls*!)

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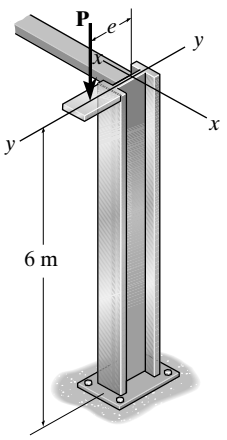
13–65. The W250 × 28 A-36-steel column is fixed at its base. Its top is constrained to rotate about the *y*–*y* axis and free to move along the *y*–*y* axis. If $e = 350$ mm, determine the allowable eccentric force P that can be applied without causing the column either to buckle or yield. Use F.S. = 2 against buckling and F.S. = 1.5 against yielding.



SOLUTION

Section Properties. From the table listed in the appendix, necessary section properties for a W250 × 28 are

$A = 3620\ \text{mm}^2 = 3.62\left(10^{-3}\right)\text{m}^2$ $\qquad r_x = 105\ \text{mm} = 0.105\ \text{m}$

$I_y = 1.78\left(10^6\right)\text{mm}^4 = 1.78\left(10^{-6}\right)\text{m}^4$ $\qquad c = \dfrac{d}{2} = \dfrac{260}{2} = 130\ \text{mm} = 0.13\ \text{m}$

$e = 0.35\ \text{m}$

Buckling About the Strong Axis. Since the column is fixed at the base and pinned at the top, $K_x = 0.7$. Applying Euler's formula,

$$P_{cr} = \frac{\pi^2 E I_y}{(KL)_y{}^2} = \frac{\pi^2\left[200\left(10^9\right)\right]\left[1.78\left(10^{-6}\right)\right]}{[0.7(6)]^2} = 199.18\ \text{kN}$$

Euler's formula is valid only if $\sigma_{cr} < \sigma_Y$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{199.18\left(10^3\right)}{3.62\left(10^{-3}\right)} = 55.02\ \text{MPa} < \sigma_Y = 250\ \text{MPa} \qquad \text{O.K.}$$

Thus,

$$P_{\text{allow}} = \frac{P_{cr}}{\text{F.S.}} = \frac{199.18}{2} = 99.59\,\text{kN}$$

Yielding About Strong Axis. Since the column is fixed at its base and free at its top, $K_x = 2$. Applying the secant formula,

$$\sigma_{\max} = \frac{P_{\max}}{A}\left[1 + \frac{ec}{r_x{}^2}\sec\left[\frac{(KL)_x}{2r_x}\sqrt{\frac{P_{\max}}{EA}}\right]\right]$$

$$250\left(10^6\right) = \frac{P_{\max}}{3.62\left(10^{-3}\right)}\left[1 + \frac{0.35(0.13)}{0.105^2}\sec\left[\frac{2(6)}{2(0.105)}\sqrt{\frac{P_{\max}}{200\left(10^9\right)\left[3.62\left(10^{-3}\right)\right]}}\right]\right]$$

$$250\left(10^6\right) = \frac{P_{\max}}{3.62\left(10^{-3}\right)}\left(1 + 4.1270\sec\left(0.0021237\right)\sqrt{P_{\max}}\right)$$

Solving by trial and error,

$P_{\max} = 133.45\ \text{kN}$

Then,

$$P_{\text{allow}} = \frac{P_{\max}}{1.5} = \frac{133.45}{1.5} = 88.97\ \text{kN} = 89.0\ \text{kN (controls)} \qquad \textbf{Ans.}$$

Ans:
Strong axis yielding controls.
$P_{\text{allow}} = 89.0\ \text{kN}$

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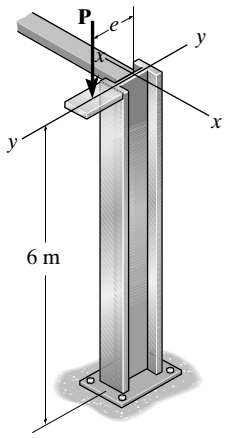
13–66. The W250 × 28 A-36-steel column is fixed at its base. Its top is constrained to rotate about the *y*–*y* axis and free to move along the *y*–*y* axis. Determine the force **P** and its eccentricity *e* so that the column will yield and buckle simultaneously.



SOLUTION

Section Properties. From the table listed in the appendix, the necessary section properties for a W250 × 28 are

$A = 3620\,\text{mm}^2 = 3.62\left(10^{-3}\right)\text{m}^2$ $\qquad r_x = 105\,\text{mm} = 0.105\,\text{m}$

$I_y = 1.78\left(10^6\right)\text{mm}^4 = 1.78\left(10^{-6}\right)\text{m}^4$ $\qquad c = \dfrac{d}{2} = \dfrac{260}{2} = 130\,\text{mm} = 0.13\,\text{m}$

Buckling About the Weak Axis. Since the column is fixed at the base and pinned at its top, $K_x = 0.7$. Applying Euler's formula,

$$P_{cr} = \frac{\pi^2 EI_y}{(KL)_y{}^2} = \frac{\pi^2\left[200\left(10^9\right)\right]\left[1.78\left(10^{-6}\right)\right]}{[0.7(6)]^2} = 199.18\,\text{kN} = 199\,\text{kN} \qquad \textbf{Ans.}$$

Euler's formula is valid only if $\sigma_{cr} < \sigma_Y$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{199.18\left(10^3\right)}{3.62\left(10^{-3}\right)} = 55.02\,\text{MPa} < \sigma_Y = 250\,\text{MPa} \qquad \text{O.K.}$$

Yielding About Strong Axis. Since the column is fixed at its base and free at its top, $K_x = 2$. Applying the secant formula with $P = P_{cr} = 199.18\,\text{kN}$,

$$\sigma_{\max} = \frac{P}{A}\left[1 + \frac{ec}{r_x{}^2}\sec\left[\frac{(KL)_x}{2r_x}\sqrt{\frac{P}{EA}}\right]\right]$$

$$250\left(10^6\right) = \frac{199.18\left(10^3\right)}{3.62\left(10^{-3}\right)}\left[1 + \frac{e(0.13)}{0.105^2}\sec\left[\frac{2(6)}{2(0.105)}\sqrt{\frac{199.18\left(10^3\right)}{200\left(10^9\right)\left[3.62\left(10^{-3}\right)\right]}}\right]\right]$$

$e = 0.1753\,\text{m} = 175\,\text{mm}$ $\qquad$ **Ans.**

Ans:
$P_{\text{cr}} = 199\,\text{kN}, e = 175\,\text{mm}$

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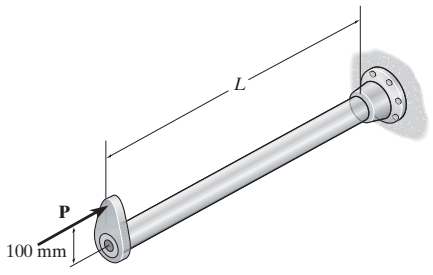
13–67.

The 6061-T6 aluminum alloy solid shaft is fixed at one end but free at the other end. If the shaft has a diameter of 100 mm, determine its maximum allowable length L if it is subjected to the eccentric force $P = 80$ kN.



SOLUTION

Section Properties.

$$A = \pi(0.05^2) = 2.5(10^{-3})\pi\,\text{m}^2$$

$$I = \frac{\pi}{4}(0.05^4) = 1.5625(10^{-6})\pi\,\text{m}^4$$

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{1.5625(10^{-6})\pi}{2.5(10^{-3})\pi}} = 0.025\ \text{m}$$

$e = 0.1$ m $\qquad c = 0.05$ m

For a column that is fixed at one end and free at the other, $K = 2$. Thus,

$$KL = 2L$$

Buckling. The critical buckling load is $P_{cr} = 80$ kN. Applying Euler's equation,

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

$$80(10^3) = \frac{\pi^2[68.9(10^9)][1.5625(10^{-6})\pi]}{(2L)^2}$$

$$L = 3.230\ \text{m}$$

Euler's equation is valid only if $\sigma_{cr} < \sigma_Y$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{80(10^3)}{2.5(10^{-3})\pi} = 10.19\ \text{MPa} < \sigma_Y = 255\ \text{MPa} \qquad \text{(O.K.)}$$

Yielding. Applying the secant formula,

$$\sigma_{max} = \frac{P}{A}\left[1 + \frac{ec}{r^2}\sec\left[\frac{KL}{2r}\sqrt{\frac{P}{EA}}\right]\right]$$

$$255(10^6) = \frac{80(10^3)}{2.5(10^{-3})\pi}\left[1 + \frac{0.1(0.05)}{0.025^2}\sec\left[\frac{2L}{2(0.025)}\sqrt{\frac{80(10^3)}{68.9(10^9)[2.5(10^{-3})\pi]}}\right]\right]$$

$$\sec 0.4864L = 3.0043$$

$$L = 2.532\ \text{m} = 2.53\ \text{m (controls)} \qquad \textbf{Ans.}$$

Ans:
$L = 2.53$ m

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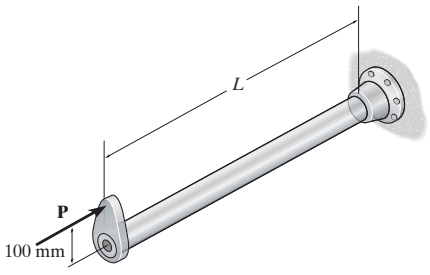
***13–68**

The 6061-T6 aluminum alloy solid shaft is fixed at one end but free at the other end. If the length is $L = 3$ m, determine its minimum required diameter if it is subjected to the eccentric force $P = 60$ kN.



SOLUTION

Section Properties.

$$A = \frac{\pi}{4}d^2 \qquad I = \frac{\pi}{4}\left(\frac{d}{2}\right)^4 = \frac{\pi}{64}d^4$$

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{\frac{\pi}{64}d^4}{\frac{\pi}{4}d^2}} = \frac{d}{4}$$

$$e = 0.1\text{ m} \qquad c = \frac{d}{2}$$

For a column that is fixed at one end and free at the other, $K = 2$. Thus,

$$KL = 2(3) = 6\text{ m}$$

Buckling. The critical buckling load is $P_{cr} = 60$ kN. Applying Euler's equation,

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

$$60(10^3) = \frac{\pi^2\left[68.9(10^9)\right]\left(\frac{\pi}{64}d^4\right)}{6^2}$$

$$d = 0.08969\text{ m} = 89.7\text{ mm}$$

Yielding. Applying the secant formula,

$$\sigma_{max} = \frac{P}{A}\left[1 + \frac{ec}{r^2}\sec\left[\frac{KL}{2r}\sqrt{\frac{P}{EA}}\right]\right]$$

$$255(10^6) = \frac{60(10^3)}{\frac{\pi}{4}d^2}\left[1 + \frac{0.1\left(\frac{d}{2}\right)}{\left(\frac{d}{4}\right)^2}\sec\left[\frac{6}{2\left(\frac{d}{4}\right)}\sqrt{\frac{60(10^3)}{68.9(10^9)\left(\frac{\pi}{4}d^2\right)}}\right]\right]$$

$$255(10^6) = \frac{240(10^3)}{\pi d^2}\left[1 + \frac{0.8}{d}\sec\left(\frac{0.012636}{d^2}\right)\right]$$

Solving by trial and error,

$$d = 0.09831\text{ m} = 98.3\text{ mm (controls)}$$ **Ans.**

Ans:
$d = 98.3$ mm

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13–69. A column of intermediate length buckles when the compressive stress is 280 MPa. If the slenderness ratio is 60, determine the tangent modulus.

SOLUTION

$$\sigma_{\text{cr}} = \frac{\pi^2 E_t}{\left(\frac{KL}{r}\right)^2}; \qquad \left(\frac{KL}{r}\right) = 60$$

$$280(10^6) = \frac{\pi^2 E_t}{60^2}$$

$$E_t = 102.13(10^9)\ \text{N/m}^2 = 102\ \text{GPa}$$ **Ans.**

Ans:
$E_t = 102.13$ GPa

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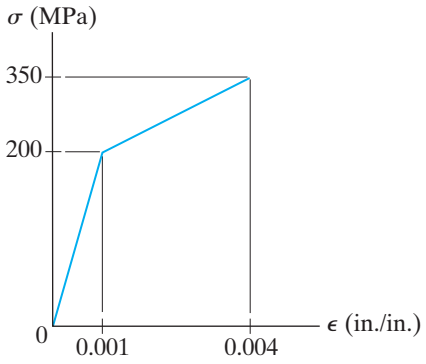
13–70.

The stress–strain diagram for the material of a column can be approximated as shown. Plot P/A vs. KL/r for the column.



SOLUTION

Tangent Moduli: From the stress-strain diagram,

$$(E_t)_1 = \frac{200\left(10^6\right)}{0.001} = 200\,\text{GPa} \qquad 0 \le \sigma < 200\ \text{MPa}$$

$$(E_t)_2 = \frac{(350 - 200)\left(10^6\right)}{0.004 - 0.001} = 50\,\text{GPa} \qquad 200\ \text{MPa} < \sigma \le 350\ \text{MPa}$$

Critical Stress: Applying Engesser's equation,

$$\sigma_{\text{cr}} = \frac{P}{A} = \frac{\pi^2 E_t}{\left(\frac{KL}{r}\right)^2} \qquad (1)$$

If $E_t = (E_t)_1 = 200\,\text{GPa}$, Eq. (1) becomes

$$\frac{P}{A} = \frac{\pi^2\left[200\left(10^9\right)\right]}{\left(\frac{KL}{r}\right)^2} = \frac{1.974\left(10^6\right)}{\left(\frac{KL}{r}\right)^2}\,\text{MPa}$$

When $\sigma_{\text{cr}} = \frac{P}{A} = \sigma_Y = 200\ \text{MPa}$, this equation becomes

$$200\left(10^6\right) = \frac{\pi^2\left[200\left(10^9\right)\right]}{\left(\frac{KL}{r}\right)^2}$$

$$\frac{KL}{r} = 99.346 = 99.3$$

If $E_t = (E_t)_2 = 50\ \text{GPa}$, Eq. (1) becomes

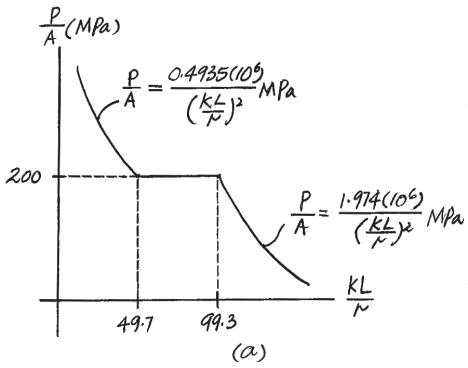
$$\frac{P}{A} = \frac{\pi^2\left[50\left(10^9\right)\right]}{\left(\frac{KL}{r}\right)^2} = \frac{0.4935\left(10^6\right)}{\left(\frac{KL}{r}\right)^2}\,\text{MPa}$$

when $\sigma_{\text{cr}} = \frac{P}{A} = \sigma_Y = 200\ \text{MPa}$, this equation gives

$$200\left(10^6\right) = \frac{\pi^2\left[50\left(10^9\right)\right]}{\left(\frac{KL}{r}\right)^2}$$

$$\frac{KL}{r} = 49.67 = 49.7$$

Using these results, the graphs of $\frac{P}{A}$ vs. $\frac{KL}{r}$ as shown in Fig. *a* can be plotted.



Ans:
N/A

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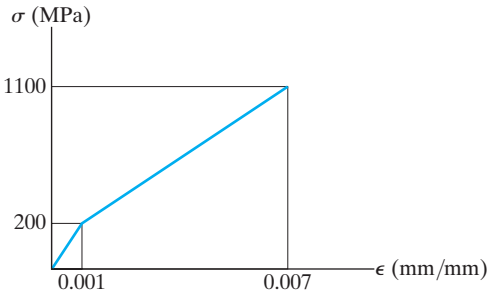
13–71.

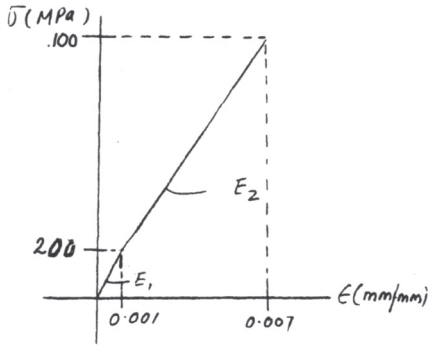
The stress–strain diagram for a material can be approximated by the two line segments shown. If a bar having a diameter of 80 mm and a length of 1.5 m is made from this material, determine the critical load provided the ends are pinned. Assume that the load acts through the axis of the bar. Use Engesser's equation.



SOLUTION

$$E_1 = \frac{200\,(10^6)}{0.001} = 200\text{ GPa}$$

$$E_2 = \frac{1100\,(10^6) - 200\,(10^6)}{0.007 - 0.001} = 150\text{ GPa}$$



Section properties:

$$I = \frac{\pi}{4}c^4;\quad A = \pi c^2$$

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{\frac{\pi}{4}c^4}{\pi c^2}} = \frac{c}{2} = \frac{0.04}{2} = 0.02\text{ m}$$

Engesser's equation:

$$\frac{KL}{r} = \frac{1.0(1.5)}{0.02} = 75$$

$$\sigma_{cr} = \frac{\pi^2 E_t}{\left(\frac{KL}{r}\right)^2} = \frac{\pi^2 E_t}{(75)^2} = 1.7546(10^{-3})\,E_t$$

Assume $E_t = E_1 = 200$ GPa

$$\sigma_{cr} = 1.7546\,(10^{-3})(200)(10^9) = 351\text{ MPa} > 200\text{ MPa}$$

Therefore, inelastic buckling occurs:

Assume $E_t = E_2 = 150$ GPa

$$\sigma_{cr} = 1.7546\,(10^{-3})(150)(10^9) = 263.2\text{ MPa}$$

$200\text{ MPa} < \sigma_{cr} < 1100\text{ MPa}$ **O.K.**

Critical load:

$$P_{cr} = \sigma_{cr} A = 263.2\,(10^6)(\pi)(0.04^2) = 1323\text{ kN}$$ **Ans.**

Ans:
$P_{cr} = 1323$ kN

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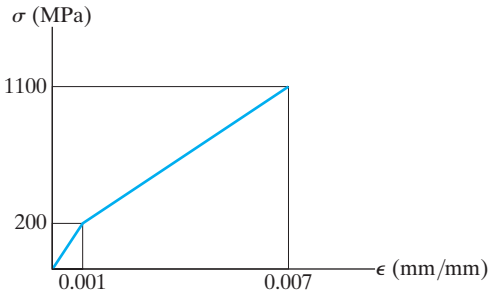
***13–72.**

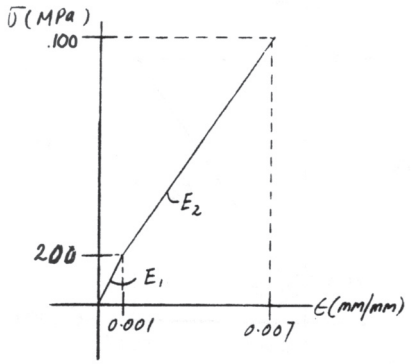
The stress–strain diagram for a material can be approximated by the two line segments shown. If a bar having a diameter of 80 mm and a length of 1.5 m is made from this material, determine the critical load provided the ends are fixed. Assume that the load acts through the axis of the bar. Use Engesser's equation.



SOLUTION

$$E_1 = \frac{200\,(10^6)}{0.001} = 200\ \text{GPa}$$

$$E_2 = \frac{1100\,(10^6) - 200\,(10^6)}{0.007 - 0.001} = 150\ \text{GPa}$$



Section properties:

$$I = \frac{\pi}{4}c^4; \quad A = \pi c^2$$

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{\frac{\pi}{4}c^4}{\pi c^2}} = \frac{c}{2} = \frac{0.04}{2} = 0.02\ \text{m}$$

Engesser's equation:

$$\frac{KL}{r} = \frac{0.5\,(1.5)}{0.02} = 37.5$$

$$\sigma_{cr} = \frac{\pi^2 E_t}{\left(\frac{KL}{r}\right)^2} = \frac{\pi^2 E_t}{(37.5)^2} = 7.018385(10^{-3})\,E_t$$

Assume $E_t = E_1 = 200$ GPa

$$\sigma_{cr} = 7.018385\,(10^{-3})(200)(10^9) = 1403.7\ \text{MPa} > 200\ \text{MPa}$$ NG

Assume $E_t = E_2 = 150$ GPa

$$\sigma_{cr} = 7.018385\,(10^{-3})(150)(10^9) = 1052.8\ \text{MPa}$$

$$200\ \text{MPa} < \sigma_{cr} < 1100\ \text{MPa}$$ **O.K.**

Critical load:

$$P_{cr} = \sigma_{cr} A = 1052.8\,(10^6)(\pi)(0.04^2) = 5292\ \text{kN}$$ **Ans.**

Ans:
$P_{cr} = 5292$ kN

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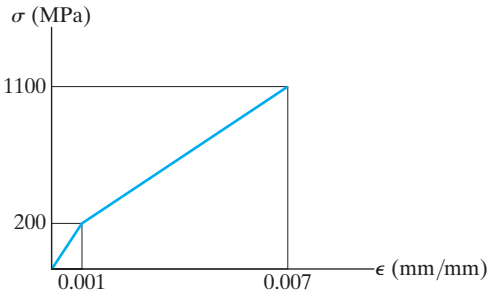
13–73.

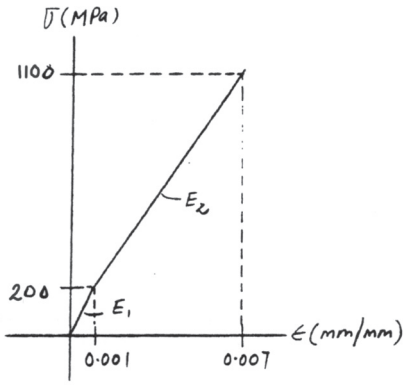
The stress–strain diagram for a material can be approximated by the two line segments shown. If a bar having a diameter of 80 mm and length of 1.5 m is made from this material, determine the critical load provided one end is pinned and the other is fixed. Assume that the load acts through the axis of the bar. Use Engesser's equation.



SOLUTION

$$E_1 = \frac{200\,(10^6)}{0.001} = 200\text{ GPa}$$

$$E_2 = \frac{1100\,(10^6) - 200\,(10^6)}{0.007 - 0.001} = 150\text{ GPa}$$



Section properties:

$$I = \frac{\pi}{4}c^4;\quad A = \pi c^2$$

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{\frac{\pi}{4}c^4}{\pi c^2}} = \frac{c}{2} = \frac{0.04}{2} = 0.02\text{ m}$$

Engesser's equation:

$$\frac{KL}{r} = \frac{0.7\,(1.5)}{0.02} = 52.5$$

$$\sigma_{cr} = \frac{\pi^2 E_t}{\left(\frac{KL}{r}\right)^2} = \frac{\pi^2 E_t}{(52.5)^2} = 3.58081\,(10^{-3})\,E_t$$

Assume $E_t = E_1 = 200$ GPa

$\sigma_{cr} = 3.58081\,(10^{-3})(200)(10^9) = 716.2\text{ MPa} > 200\text{ MPa}$ NG

Assume $E_t = E_2 = 150$ GPa

$\sigma_{cr} = 3.58081\,(10^{-3})(150)(10^9) = 537.1\text{ MPa}$

$200\text{ MPa} < \sigma_{cr} < 1100\text{ MPa}$ **O.K.**

Critical load:

$P_{cr} = \sigma_{cr} A = 537.1\,(10^6)(\pi)(0.04^2) = 2700\text{ kN}$ **Ans.**

Ans:
$P_{cr} = 2700$ kN

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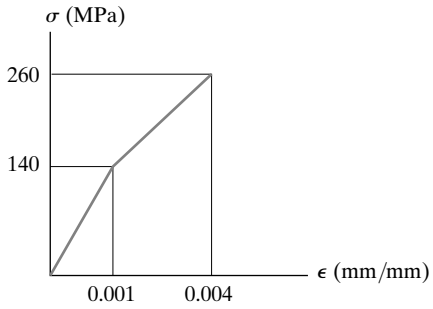
13–74. Construct the buckling curve, P/A versus L/r, for a column that has a bilinear stress–strain curve in compression as shown. The column is pinned at its ends.



SOLUTION

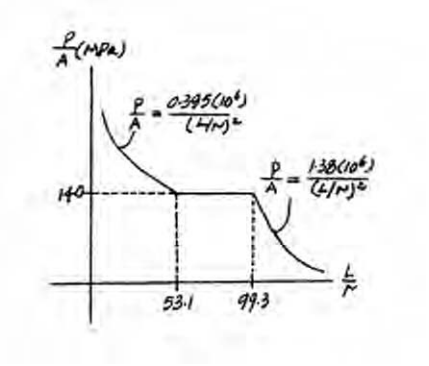
Tangent modulus: From the stress–strain diagram,

$$(E_t)_1 = \frac{140(10^6)}{0.001} = 140 \text{ GPa}$$

$$(E_t)_2 = \frac{(260 - 140)(10^6)}{0.004 - 0.001} = 40 \text{ GPa}$$

Critical Stress: Applying *Engesser's equation*,

$$\sigma_{cr}\frac{P}{A} = \frac{\pi^2 E_t}{\left(\frac{L}{r}\right)^2} \qquad [1]$$



Substituting $(E_t)_1 = 140$ GPa into Eq. [1], we have

$$\frac{P}{A} = \frac{\pi^2\left[140(10^9)\right]}{\left(\frac{L}{r}\right)^2}$$

$$\frac{P}{A} = \frac{1.38(10^6)}{\left(\frac{L}{r}\right)^2} \text{ MPa}$$

When $\frac{P}{A} = 140$ MPa, $\frac{L}{r} = 99.3$

Substitute $(E_t)_2 = 40$ GPa into Eq. [1], we have

$$\frac{P}{A} = \frac{\pi^2\left[40(10^9)\right]}{\left(\frac{L}{r}\right)^2}$$

$$\frac{P}{A} = \frac{0.395(10^6)}{\left(\frac{L}{r}\right)^2} \text{ MPa}$$

When $\frac{P}{A} = 140$ MPa, $\frac{L}{r} = 53.1$

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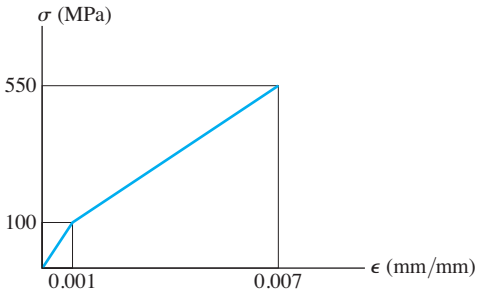
13–75.

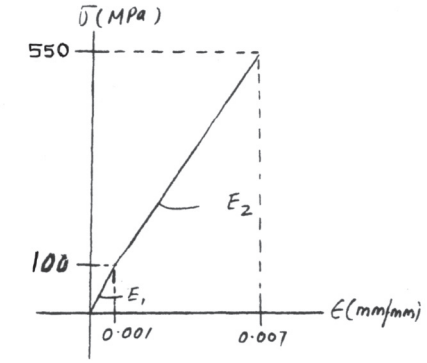
The stress–strain diagram of the material can be approximated by the two line segments. If a bar having a diameter of 80 mm and a length of 1.5 m is made from this material, determine the critical load provided the ends are pinned. Assume that the load acts through the axis of the bar. Use Engesser's equation.



SOLUTION

$$E_1 = \frac{100(10^6)}{0.001} = 100 \text{ GPa}$$

$$E_2 = \frac{550(10^6) - 100(10^6)}{0.007 - 0.001} = 75 \text{ GPa}$$



Section Properties:

$$I = \frac{\pi}{4}c^4; \qquad A = \pi c^2$$

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{\frac{\pi}{4}c^4}{\pi c^2}} = \frac{c}{2} = \frac{0.04}{2} = 0.02 \text{ m}$$

Engesser's Equation:

$$\frac{KL}{r} = \frac{1.0(1.5)}{0.02} = 75$$

$$\sigma_{cr} = \frac{\pi^2 E_r}{(\frac{KL}{r})^2} = \frac{\pi^2 E_r}{(75)^2} = 1.7546(10^{-3})\, E_t$$

Assume $E_t = E_1 = 100$ GPa

$\sigma_{cr} = 1.7546\,(10^{-3})(100)(10^9) = 175$ MPa > 100 MPa

Therefore, inelastic buckling occurs:

Assume $E_t = E_2 = 75$ GPa

$\sigma_{cr} = 1.7546\,(10^{-3})\,(75)(10^9) = 131.6$ MPa

100 MPa $< \sigma_{cr} <$ 550 MPa OK

Critical Load:

$P_{cr} = \sigma_{cr} A = 131.6(10^6)(\pi)(0.04^2) = 661$ kN **Ans.**

Ans:
$P_{cr} = 661$ kN

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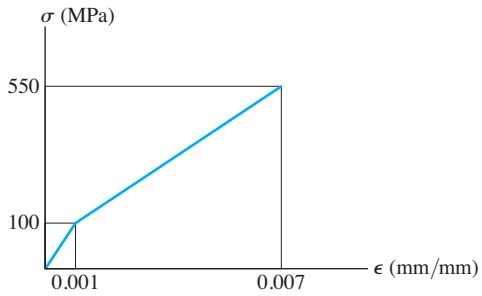
***13–76.**

The stress–strain diagram of the material can be approximated by the two line segments. If a bar having a diameter of 80 mm and a length of 1.5 m is made from this material, determine the critical load provided the ends are fixed. Assume that the load acts through the axis of the bar. Use Engesser's equation.



SOLUTION

$$E_1 = \frac{100\,(10^6)}{0.001} = 100\text{ GPa}$$

$$E_2 = \frac{550\,(10^6) - 100\,(10^6)}{0.007 - 0.001} = 75\text{ GPa}$$

Section Properties:

$$I = \frac{\pi}{4}c^4; \qquad A = \pi c^2$$

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{\frac{\pi}{4}c^4}{\pi c^2}} = \frac{c}{2} = \frac{0.04}{2} = 0.02\text{ m}$$

Engesser's Equation:

$$\frac{KL}{r} = \frac{0.5\,(1.5)}{0.02} = 37.5$$

$$\sigma_{cr} = \frac{\pi^2 E_r}{(\frac{KL}{r})^2} = \frac{\pi^2 E_r}{(37.5)^2} = 7.018385\,(10^{-3})\,E_t$$

Assume $E_t = E_1 = 100$ GPa

$$\sigma_{cr} = 7.018385\,(10^{-3})(100)(10^9) = 701.8\text{ MPa} > 100\text{ MPa} \qquad \text{NG}$$

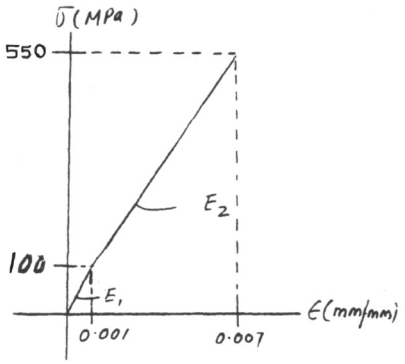
Assume $E_t = E_2 = 75$ GPa

$$\sigma_{cr} = 7.018385\,(10^{-3})(75)(10^9) = 526.4\text{ MPa}$$

$$100\text{ MPa} < \sigma_{cr} < 550\text{ MPa} \qquad \textbf{OK}$$

Critical Load:

$$P_{cr} = \sigma_{cr}A = 526.4\,(10^6)(\pi)(0.04^2) = 2.65(10^3)\text{ kN} \qquad \textbf{Ans.}$$



Ans:
$P_{cr} = 2.65(10^3)$ kN

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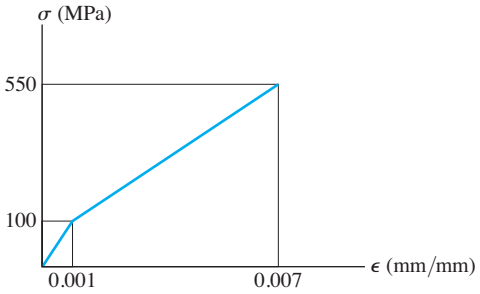
13–77.

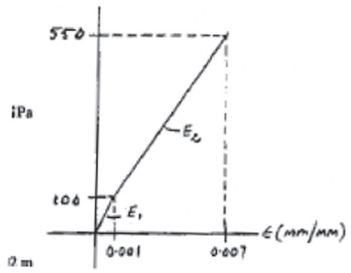
The stress–strain diagram of the material can be approximated by the two line segments. If a bar having a diameter of 80 mm and a length of 1.5 m is made from this material, determine the critical load provided one end is pinned and the other is fixed. Assume that the load acts through the axis of the bar. Use Engesser's equation.



SOLUTION

$$E_1 = \frac{100\,(10^6)}{0.001} = 100\text{ GPa}$$

$$E_2 = \frac{550\,(10^6) - 100\,(10^6)}{0.007 - 0.001} = 75\text{ GPa}$$



Section Properties:

$$I = \frac{\pi}{4}c^4; \qquad A = \pi c^2$$

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{\frac{\pi}{4}c^4}{\pi c^2}} = \frac{c}{2} = \frac{0.04}{2} = 0.02\text{ m}$$

Engesser's Equation:

$$\frac{KL}{r} = \frac{0.7\,(1.5)}{0.02} = 52.5$$

$$\sigma_{cr} = \frac{\pi^2 E_r}{(\frac{KL}{r})^2} = \frac{\pi^2 E_r}{(52.5)^2} = 3.58081\,(10^{-3})\,E_t$$

Assume $E_t = E_1 = 100$ GPa

$\sigma_{cr} = 3.58081\,(10^{-3})(100)(10^9) = 358.1\text{ MPa} > 100\text{ MPa}$ NG

Assume $E_t = E_2 = 75$ GPa

$\sigma_{cr} = 3.58081\,(10^{-3})(75)(10^9) = 268.6\text{ MPa}$

$100\text{ MPa} < \sigma_{cr} < 550\text{ MPa}$ OK

Critical Load:

$P_{cr} = \sigma_{cr} A = 268.6\,(10^6)(\pi)(0.04^2) = 1.35(10^3)\text{ kN}$ **Ans.**

Ans:
$P_{cr} = 1.35(10^3)\text{ kN}$

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13–78. Determine the largest length of a structural A-36 steel rod if it is fixed supported and subjected to an axial load of 100 kN. The rod has a diameter of 50 mm. Use the AISC equations.

SOLUTION

Section Properties:

$$A = \pi\left(0.025^2\right) = 0.625\left(10^{-3}\right)\pi \text{ m}^2$$

$$I = \frac{\pi}{4}\left(0.025^4\right) = 97.65625\left(10^{-9}\right)\pi \text{ m}^4$$

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{97.65625(10^{-9})\pi}{0.625(10^{-3})\pi}} = 0.0125 \text{ m}$$

Slenderness Ratio: For a column fixed at both ends, $K = 0.5$. Thus,

$$\frac{KL}{r} = \frac{0.5L}{0.0125} = 40.0L$$

AISC Column Formula: Assume a *long* column.

$$\sigma_{\text{allow}} = \frac{12\pi^2 E}{23\left(\frac{KL}{r}\right)^2}$$

$$\frac{100(10^3)}{0.625(10^{-3})\pi} = \frac{12\pi^2\left[200(10^9)\right]}{23(40.0L)^3}$$

$$L = 3.555 \text{ m}$$

Here, $\frac{KL}{r} = 40.0(3.555) = 142.2$ and for A–36 steel, $\left(\frac{KL}{r}\right)_e = \sqrt{\frac{2\pi^2 E}{\sigma_\gamma}}$ $= \sqrt{\frac{2\pi^2[200(10^9)]}{250(10^6)}} = 125.7$. Since $\left(\frac{KL}{r}\right)_e \le \frac{KL}{r} \le 200$, the assumption is correct.

Thus,

$$L = 3.56 \text{ m}$$ **Ans.**

Ans:
$L = 3.56$ m

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13–79. Check if a W250 × 58 column can safely support an axial force of $P = 1150$ kN. The column is 6 m long and is pinned at both ends and braced against its weak axis at mid-height. It is made of steel having $E = 200$ GPa and $\sigma_Y = 350$ MPa. use the AISC column design formulas.

SOLUTION

Section Properties. From the table listed in the appendix, the necessary section properties for a W250 × 58 are

$$A = 7400 \text{ mm}^2 = 7.4(10^{-3}) \text{ m}^2$$

$$r_x = 109 \text{ mm} = 0.109 \text{ m}$$

$$r_y = 50.4 \text{ mm} = 0.0504 \text{ m}$$

Slenderness Ratio. For a column pinned at both of its ends, $K = 1$. Here, $L_x = 6$ m and $L_y = 3$ m. Thus,

$$\left(\frac{KL}{r}\right)_x = \frac{1(6)}{0.109} = 55.05$$

$$\left(\frac{KL}{r}\right)_y = \frac{1(3)}{0.0504} = 59.52 \text{ (controls)}$$

AISC Column Formulas. For A-36 steel $\left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}}$

$= \sqrt{\frac{2\pi^2[200(10^9)]}{350(10^6)}} = 106.21.$ Since $\left(\frac{KL}{r}\right)_y < \left(\frac{KL}{r}\right)_c$, the column is an intermediate column.

$$\sigma_{\text{allow}} = \frac{\left[1 - \frac{(KL/r)^2}{2(KL/r)_c{}^2}\right]\sigma_Y}{\frac{5}{3} + \frac{3(KL/r)}{8(KL/r)_c} - \frac{(KL/r)^3}{8(KL/r)_c{}^3}}$$

$$= \frac{\left[1 - \frac{59.62^2}{2(106.21^2)}\right](350)}{\frac{5}{3} + \frac{3(59.52)}{8(106.21)} - \frac{(59.52^3)}{8(106.21^3)}}$$

$$= 159.06 \text{ MPa}$$

Thus, the allowable force is

$$P_{\text{allow}} = \sigma_{\text{allow}} A = [159.06(10^6)][7.4(10^{-3})] = 1177.04(10^3) \text{ N}$$

$$= 1177 \text{ kN} > P = 1150 \text{ kN} \qquad \text{O.K.}$$

Thus, a W250 × 58 column *is adequate.*

Ans:
Yes

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***13–80.** A W200 × 36 A-36-steel column of 9-m length is pinned at both ends and braced against its weak axis at mid-height. Determine the allowable axial force P that can be safely supported by the column. Use the AISC column design formulas.

SOLUTION

Section Properties. From the table listed in the appendix, the necessary section properties for a W200 × 36 are

$$A = 4570 \text{ mm}^2 = 4.57(10^{-3}) \text{ m}^2$$

$$r_x = 86.8 \text{ mm} = 0.0868 \text{ m} \qquad r_y = 40.9 \text{ mm} = 0.0409 \text{ m}$$

Slenderness Ratio. For a column pinned at both of its ends, $K = 1$. Here, $L_x = 9$ m and $L_y = 4.5$ m. Thus,

$$\left(\frac{KL}{r}\right)_x = \frac{1(9)}{0.0868} = 103.69$$

$$\left(\frac{KL}{r}\right)_y = \frac{1(4.5)}{0.0409} = 110.02 \text{ (controls)}$$

AISC Column Formulas. For A-36 steel $\left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}}$ $= \sqrt{\frac{2\pi^2[200(10^9)]}{250(10^6)}} = 125.66$. Since $\left(\frac{KL}{r}\right)_y < \left(\frac{KL}{r}\right)_c$, the column is an intermediate column.

$$\sigma_{\text{allow}} = \frac{\left[1 - \frac{(KL/r)^2}{2(KL/r)_c{}^2}\right]\sigma_Y}{\frac{5}{3} + \frac{3(KL/r)}{8(KL/r)_c} - \frac{(KL/r)^3}{8(KL/r)_c{}^3}}$$

$$= \frac{\left[1 - \frac{110.02^2}{2(125.66^2)}\right](250)}{\frac{5}{3} + \frac{3(110.02)}{8(125.66)} - \frac{(110.02^3)}{8(125.66^3)}}$$

$$= 80.67 \text{ MPa}$$

Thus, the allowable force is

$$P_{\text{allow}} = \sigma_{\text{allow}} A = [80.67(10^6)][4.57(10^{-3})] = 368.68(10^3) \text{ N} = 369 \text{ kN} \qquad \textbf{Ans.}$$

Ans:

$P_{\text{allow}} = 369$ kN

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13–81. Using the AISC equations, select from Appendix B the lightest-weight structural A-36 steel column that is 9 m long and supports an axial load of 1000 kN. The ends are fixed.

SOLUTION

Try W250×80

$A = 10200 \text{ mm}^2 = 0.0102 \text{ m}^2 \qquad r_y = 65.0 \text{ mm} = 0.065 \text{ m}$

$$\left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2[200(10^9)]}{250(10^6)}} = 125.66$$

$$\frac{KL}{r_y} = \frac{0.5(9)}{0.065} = 69.23$$

$\left(\frac{KL}{r_y}\right) < \left(\frac{KL}{r}\right)_c$ intermediate column.

$$\sigma_{\text{allow}} = \frac{\left\{1 - \frac{1}{2}\left[\frac{\frac{KL}{r}}{\left(\frac{KL}{r}\right)_c}\right]^2\right\}\sigma_Y}{\left\{\frac{5}{3} + \frac{3}{8}\left[\frac{\frac{KL}{r}}{\left(\frac{KL}{r}\right)_c}\right] - \frac{1}{8}\left[\frac{\frac{KL}{r}}{\left(\frac{KL}{r}\right)_c}\right]^3\right\}}$$

$$= \frac{\{1 - \frac{1}{2}[\frac{69.23}{125.66}]^2\}250}{\{\frac{5}{3} + \frac{3}{8}[\frac{69.23}{125.66}] - \frac{1}{8}[\frac{69.23}{125.66}]^3\}} = 114.48 \text{ MPa}$$

$$P_{\text{allow}} = \sigma_{\text{allow}} A = [114.48(10^6)](0.0102) = 1167.71(10^3) \text{ N}$$

$= 1168 \text{ kN} > P = 1000 \text{ kN}$ O.K.

Use W250 × 80 **Ans.**

Ans:
Use W250 × 80

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13–82. Using the AISC equations, select from Appendix B the lightest-weight structural A-36 steel column that is 7.2 m long and supports an axial load of 450 kN. The ends are fixed.

SOLUTION

Section Properties: Try a W200 × 36 wide flange section,

$$A = 4570 \text{ mm}^2 = 4.57(10^{-3}) \text{ m}^2 \qquad r_y = 40.9 \text{ mm} = 0.0409 \text{ m}$$

Slenderness Ratio: For a column fixed at both ends, $K = 0.5$. Thus,

$$\left(\frac{KL}{r}\right)_y = \frac{0.5(7.2)}{0.0409} = 88.02$$

AISC Column Formula: For A–36 steel, $\left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}}$ $= \sqrt{\frac{2\pi^2[200(10^9)]}{250(10^6)}} = 125.66$. Since $\frac{KL}{r} < \left(\frac{KL}{r}\right)_c$, the column is an *intermediate* column. Applying Eq. 13–23,

$$\sigma_{\text{allow}} = \frac{\left[1 - \frac{(KL/r)^2}{2(KL/r)_c^2}\right]\sigma_Y}{\frac{5}{3} + \frac{3(KL/r)}{8(KL/r)_c} - \frac{(KL/r)^3}{8(KL/r)_c^3}}$$

$$= \frac{\left[1 - \frac{(88.02^2)}{2(125.66^2)}\right](250)}{\frac{5}{3} + \frac{3(88.02)}{8(125.66)} - \frac{(88.02^3)}{8(125.66^3)}}$$

$$= 100.02 \text{ MPa}$$

The allowable load is

$$P_{\text{allow}} = \sigma_{\text{allow}} A$$

$$= [100.02(10^6)][4.57(10^{-3})] = 457.09(10^3) \text{ N}$$

$$= 457 \text{ kN} > P = 450 \text{ kN} \qquad \text{O.K.}$$

Thus, **Use** W200 × 36 **Ans.**

Ans:
Use W200 × 36

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13–83. Determine the largest length of a W250 × 67 A992 structural steel column if it is pin supported and subjected to an axial load of 1450 kN. $E_{st} = 200$ GPa, $\sigma_Y = 350$ MPa. Use the AISC equations.

SOLUTION

Section Properties: For W200 × 46 wide flange section,

$$A = 8560 \text{ mm}^2 = 8.56(10^{-3}) \text{ m}^2 \qquad r_y = 50.9 \text{ mm} = 0.0509 \text{ m}$$

Slenderness Ratio: For a column pinned at both ends, $K = 1$. Thus,

$$\left(\frac{KL}{r}\right)_y = \frac{1(L)}{0.0509} = 19.6464L$$

AISC Column Formula: Assume a *long* column,

$$\sigma_{\text{allow}} = \frac{12\pi^2 E}{23\left(\frac{KL}{r}\right)^2}$$

$$\frac{1450(10^3)}{8.56(10^{-3})} = \sqrt{\frac{12\pi^2[200(10^9)]}{23(19.6464L)^2}}$$

$$L = 3.9688 \text{ m}$$

Here, $\frac{KL}{r} = 19.6464(3.9688) = 77.97$ and for A992 steel, $\left(\frac{KL}{r}\right)_c = \frac{2\pi^2 E}{\sigma_Y}$

$= \sqrt{\frac{2\pi^2[200(10^9)]}{345(10^6)}} = 106.97$. Since $\frac{KL}{r} < \left(\frac{KL}{r}\right)_c$, the assumption is not correct.

Thus, the column is an *intermediate* column.

Applying Eq. 13–23,

$$\sigma_{\text{allow}} = \frac{\left[1 - \frac{KL/r)^2}{(2(KL/r)_c^2}\right]\sigma_Y}{\frac{5}{3} + \frac{3(KL/r)}{8(KL/r)_c} - \frac{(KL/r)^3}{8(KL/r)_c^3}}$$

$$\frac{1450(10^3)}{8.56(10^{-3})} = \frac{\left[1 - \frac{(19.6464L)^2}{2(106.97^2)}\right][345(10^6)]}{\frac{5}{3} + \frac{3(19.6464L)}{8(106.97)} - \frac{(19.6464L)^3}{8(106.97^3)}}$$

$$-3.80208(10^{-4})L^3 + 0.016865L^2 + 0.0338157L - 0.181679 = 0$$

Solving by trial and error,

$$L = 2.4790 \text{ m} = 2.48 \text{ m} \qquad \textbf{Ans.}$$

Ans:
$L = 2.48$ m

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***13–84.** Determine the largest length of a W250 × 18 structural A-36 steel section if it is pin supported and is subjected to an axial load of 140 kN. Use the AISC equations.

SOLUTION

For a W250 × 18, $A = 2280\ \text{mm}^2 = 2.28(10^{-3})\ \text{m}^2$

$r_y = 20.1\ \text{mm} = 0.0201\ \text{m}$

$$\sigma = \frac{P}{A} = \frac{140(10^3)}{2.28(10^{-3})} = 61.40(10^6)\ \text{N/m}^2 = 61.40\ \text{MPa}$$

Assume a long column:

$$\sigma_{\text{allow}} = \frac{12\pi^2 E}{23(KL/r)^2}$$

$$\left(\frac{KL}{r}\right) = \sqrt{\frac{12\pi^2 E}{23\sigma_{\text{allow}}}} = \sqrt{\frac{12\pi^2[200(10^9)]}{23[61.40(10^6)]}} = 129.51$$

$$\left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2[200(10^9)]}{250(10^6)}} = 125.66, \frac{KL}{r} > \left(\frac{KL}{r}\right)_c$$

Long column.

$$\frac{KL}{r} = 129.51$$

$$L = 129.51\left(\frac{r}{K}\right) = 129.51\left(\frac{0.0201}{1}\right) = 2.603\ \text{m} = 2.60\ \text{m} \quad \textbf{Ans.}$$

Ans:
$L = 2.60$ m

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13–85. Using the AISC equations, select from Appendix B the lightest-weight structural A-36 steel column that is 4.2 m long and supports an axial load of 200 kN. The ends are pinned.

SOLUTION

Try, W150 × 22

$A = 2860 \text{ mm}^2 = 2.86(10^{-3}) \text{ m}^2 \qquad r_y = 36.8 \text{ mm} = 0.0368 \text{ m}$

$$\left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2[200(10^9)]}{345(10^6)}} = 106.97$$

$$\left(\frac{KL}{r_y}\right) = \frac{1.0(4.2)}{0.0368} = 114.13, \quad \left(\frac{KL}{r_y}\right) > \left(\frac{KL}{r}\right)_c$$

Long column

$$\sigma_{\text{allow}} = \frac{12\,\pi^2 E}{23(KL/r)^2} = \frac{12\pi^2[200(10^9)]}{23(114.13^2)} = 79.06(10^6) \text{ N-m}^2 = 79.06 \text{ MPa}$$

$$P_{\text{allow}} = \sigma_{\text{allow}} A$$

$$= [79.06(10^6)][2.86(10^{-3})] = 226 \text{ kN} > 200 \text{ kN} \qquad \text{O.K.}$$

Use W150 × 22 **Ans.**

Ans:
Use W150 × 22

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13–86. Using the AISC equations, select from Appendix B the lightest-weight structural A-36 steel column that is 3.6 m long and supports an axial load of 200 kN. The ends are fixed.

SOLUTION

Tπy W150×14

$A = 1730\ \text{mm}^2 = 1.73(10^{-3})\ \text{m}^2 \qquad r_y = 23.0\ \text{mm} = 0.023\ \text{m}$

$$\left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2[200(10^9)]}{345(10^6)}} = 106.97$$

$$\frac{KL}{r_y} = \frac{0.5(3.6)}{0.023} = 78.26$$

$$\frac{KL}{r_y} < \left(\frac{KL}{r}\right)_c$$

Intermediate column

$$\sigma_{\text{allow}} = \frac{\left[1 - \frac{1}{2}\left(\frac{KL/r}{(KL/r)_c}\right)^2\right]\sigma_Y}{\left[\frac{5}{3} + \frac{3}{8}\left(\frac{KL/r}{(KL/r)_c}\right) - \frac{1}{8}\left(\frac{KL/r}{(KL/r)_c}\right)^3\right]} = \frac{\left[1 - \frac{1}{2}\left(\frac{78.26}{106.97}\right)^2\right](345\ \text{MPa})}{\left[\frac{5}{3} + \frac{3}{8}\left(\frac{78.26}{106.97}\right) - \frac{1}{8}\left(\frac{78.26}{106.97}\right)^3\right]} = 133.54\ \text{MPa}$$

$$\begin{aligned} P_{\text{allow}} &= \sigma_{\text{allow}} A \\ &= [133.54(10^6)][1.73(10^{-3})] \\ &= 231.03(10^3)\ \text{N} \\ &= 231\ \text{kN} > 200\ \text{kN} \end{aligned}$$

O.K.

Use W150 × 14 **Ans.**

Ans:
Use W150 × 14

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13–87. Check if a W250 × 67 column can safely support an axial force of P = 1000 kN. The column is 4.5 m long and is pinned at both of its ends. It is made of steel having E = 200 GPa and σ_Y = 350 MPa. Use the AISC column design formulas.

SOLUTION

Section Properties. Try W250 × 67. From the table listed in the appendix, the necessary section properties are

$$A = 8560 \text{ mm}^2 = 8.56(10^{-3}) \text{ m}^2 \qquad r_y = 50.9 \text{ mm} = 0.0509 \text{ m}$$

Slenderness Ratio. For a column pinned at both of its ends, $K = 1$. Thus,

$$\left(\frac{KL}{r}\right)_y = \frac{1(4.5)}{0.0509} = 88.41$$

AISC Column Formulas. Here, $\left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2[200(10^9)]}{350(10^6)}} = 106.21.$

Since $\left(\frac{KL}{r}\right)_y < \left(\frac{KL}{r}\right)_c$, the column is an intermediate column.

$$\sigma_{\text{allow}} = \frac{\left[1 - \frac{(KL/r)^2}{2(KL/r)_c{}^2}\right]\sigma_Y}{\frac{5}{3} + \frac{3(KL/r)}{8(KL/r)_c} - \frac{(KL/r)^3}{8(KL/r)_c{}^3}}$$

$$= \frac{\left[1 - \frac{88.41^2}{2(106.21^2)}\right](350)}{\frac{5}{3} + \frac{3(88.41)}{8(106.21)} - \frac{(88.41^3)}{8(106.21^3)}}$$

$$= 199.96 \text{ MPa}$$

Thus, the allowable force is

$$P_{\text{allow}} = \sigma_{\text{allow}} A = [119.96(10^6)][8.56(10^{-3})] = 1026.88(10^3) \text{ N}$$

$$= 1027 \text{ kN} > 1000 \text{ kN} \qquad \text{O.K.}$$

Thus, use

W250 × 67 **Ans.**

Ans:
Use W250 × 67

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***13–88.** A 1.5-m-long rod is used in a machine to transmit an axial compressive load of 15 kN. Determine its smallest diameter if it is pin connected at its ends and is made of a 2014-T6 aluminum alloy.

SOLUTION

Section properties:

$$A = \frac{\pi}{4} d^2; \qquad I = \frac{\pi}{4}\left(\frac{d}{2}\right)^4 = \frac{\pi d^4}{64}$$

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{\frac{\pi d^4}{64}}{\frac{\pi}{4} d^2}} = \frac{d}{4}$$

$$\sigma_{\text{allow}} = \frac{P}{A} = \frac{15(10^3)}{\frac{\pi}{4} d^2} = \frac{60(10^3)}{\pi d^2}$$

Assume long column:

$$\frac{KL}{r} = \frac{1.0(1.5)}{d/4} = \frac{6}{d}$$

$$\sigma_{\text{allow}} = \frac{372550 \text{ MPa}}{(KL/r)^2}; \quad \frac{60(10^3)}{\pi d^2} = \frac{372550(10^6)}{(6/d)^2}$$

$d = 0.03686 \text{ m} = 36.9 \text{ mm}$ **Ans.**

$\frac{KL}{r} = \frac{1.0(1.5)}{d/4} = \frac{6}{0.03686} = 162.79 > 55$ (Assumption is correct!) O.K.

Ans:
$d = 36.9$ mm

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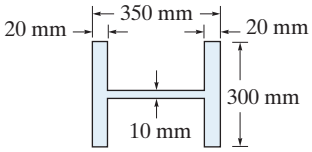
13–89.

Using the AISC equations, check if a column having the cross section shown can support an axial force of 1500 kN. The column has a length of 4 m, is made from A992 steel, and its ends are pinned.



SOLUTION

Section Properties:

$$A = 0.3(0.35) - 0.29(0.31) = 0.0151\ \text{m}^2$$

$$I_y = \frac{1}{12}(0.04)\left(0.3^3\right) + \frac{1}{12}(0.31)\left(0.01^3\right) = 90.025833\left(10^{-6}\right)\ \text{m}^4$$

$$r_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{90.02583(10^{-6})}{0.0151}} = 0.077214\ \text{m}$$

Slenderness Ratio: For a column pinned at both ends, $K = 1$. Thus,

$$\left(\frac{KL}{r}\right)_y = \frac{1(4)}{0.077214} = 51.80$$

AISC Column Formula: For A992 steel, $\left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_\gamma}}$ $= \sqrt{\frac{2\pi^2[200(10^9)]}{345(10^6)}} = 107$. Since $\frac{KL}{r} < \left(\frac{KL}{r}\right)_c$, the column is an *intermediate* column. Applying Eq. 13–23,

$$\sigma_{\text{allow}} = \frac{\left[1 - \frac{(KL/r)^2}{2(KL/r)_c^2}\right]\sigma_Y}{\frac{5}{3} + \frac{3(KL/r)}{8(KL/r)_c} - \frac{(KL/r)^3}{8(KL/r)_c^3}}$$

$$= \frac{\left[1 - \frac{(51.80^2)}{2(107^2)}\right](345)(10^6)}{\frac{5}{3} + \frac{3(51.80)}{8(107)} - \frac{(51.80^3)}{8(107^3)}}$$

$$= 166.1\ \text{MPa}$$

The allowable load is

$$P_{\text{allow}} = \sigma_{\text{allow}} A$$

$$= 166.1\left(10^6\right)(0.0151)$$

$$= 2508\ \text{kN} > P = 1500\ \text{kN} \qquad \textbf{O.K.}$$

Thus, the column is adequate. **Ans.**

Ans:
Yes

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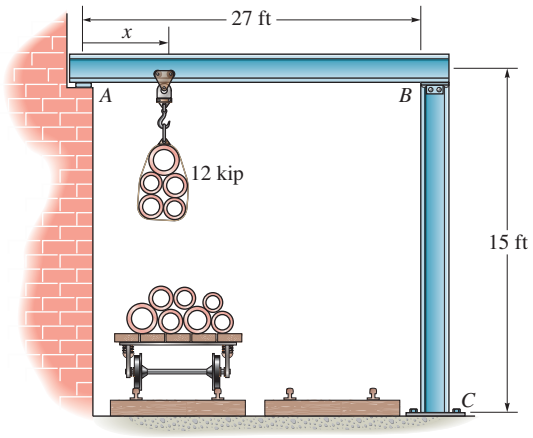
13–90.

The beam and column arrangement is used in a railroad yard for loading and unloading cars. If the maximum anticipated hoist load is 12 kip, determine if the W8 × 31 wide-flange A-36 steel column is adequate for supporting the load. The hoist travels along the bottom flange of the beam, 1 ft ≤ x ≤ 25 ft, and has negligible size. Assume the beam is pinned to the column at B and pin supported at A. The column is also pinned at C and it is braced so it will not buckle out of the plane of the loading.



SOLUTION

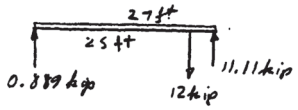
For W8 × 31, $r_x = 3.47$ in., $A = 9.13$ in^2

Maximum axial load occurs when $x = 25$ ft.

$$\frac{KL}{r} = \frac{(1.0)(15)(12)}{3.47} = 51.87$$

$$\left(\frac{KL}{r}\right)c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2(29)(10^3)}{36}} = 126.1$$

Here $0 < 51.87 < 126.1$



Intermediate Column:

$$\sigma_{\text{allow}} = \frac{\left[1 - \frac{1}{2}\left(\frac{KL/r}{(KL/r)c}\right)^2\right]\sigma_Y}{\left[\frac{5}{3} + \frac{3}{8}\left(\frac{KL/r}{(KL/r)c}\right) - \frac{1}{8}\left(\frac{KL/r}{(KL/r)c}\right)^3\right]}$$

$$= \frac{\left(1 - \frac{1}{2}(51.87/126.1)^2\right)36}{\left\{\frac{5}{3} + \left[\left(\frac{3}{8}\right)(51.87/126.1)\right] - \left[\frac{1}{8}(51.87/126.1)^3\right]\right\}} = 18.2 \text{ ksi}$$

$$\sigma = \frac{P}{A} = \frac{11.11}{9.13} = 1.22 \text{ ksi} < 18.2 \text{ ksi} \qquad \textbf{OK}$$

Yes, the column is adequate. **Ans.**

Ans:
Yes

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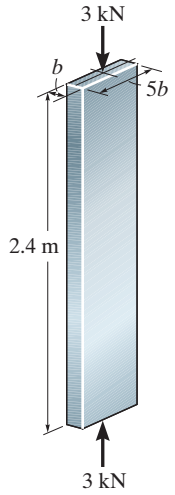
13–91. The bar is made of a 2014-T6 aluminum alloy. Determine its smallest thickness b if its width is $5b$. Assume that it is pin connected at its ends.



SOLUTION

Section Properties:

$$A = b(5b) = 5b^2$$

$$I_y = \frac{1}{12}(5b)\left(b^3\right) = \frac{5}{12}b^4$$

$$r_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{\frac{5}{12}b^4}{5b^2}} = \frac{\sqrt{3}}{6}b$$

Slenderness Ratio: For a column pinned at both ends, $K = 1$. Thus,

$$\left(\frac{KL}{\pi}\right)_y = \frac{1(2.4)}{\sqrt{\frac{3}{6}}b} = \frac{8.3138}{b}$$

Aluminum (2014 - T6 alloy) Column Formulas: Assume a *long* column and apply Eq. 13–26.

$$\sigma_{\text{allow}} = \frac{372550\ \text{MPa}}{(KL/\pi)^2}$$

$$\frac{3(10^3)}{5b^2} = \frac{372550(10^6)}{(8.3138/b)^2}$$

$$b = 0.01827\ \text{m} = 18.3\ \text{mm}$$

Here, $\frac{KL}{\pi} = \frac{8.3138}{0.01827} = 455.15$. Since $\frac{KL}{r} > 55$, the assumption is correct. Thus,

$$b = 18.3\ \text{mm}$$ **Ans.**

Ans:
$b = 18.3$ mm

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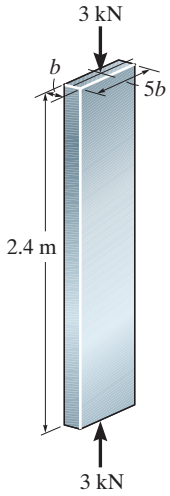
***13–92.** The bar is made of a 2014-T6 aluminum alloy. Determine its smallest thickness b if its width is $5b$. Assume that it is fixed connected at its ends.



SOLUTION

Section Properties:

$$A = b(5b) = 5b^2$$

$$I_y = \frac{1}{12}(5b)\left(b^3\right) = \frac{5}{12}b^4$$

$$r_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{\frac{5}{12}b^4}{5b^2}} = \frac{\sqrt{3}}{6}b$$

Slenderness Ratio: For a column fixed at both ends, $K = 0.5$. Thus,

$$\left(\frac{KL}{r}\right)_y = \frac{0.5(2.4)}{\sqrt{\frac{3}{6}}b} = \frac{4.1569}{b}$$

Aluminum (2014 - T6 alloy) Column Formulas: Assume a *long* column and apply Eq. 13–26.

$$\sigma_{\text{allow}} = \frac{372550\ \text{MPa}}{(KL/r)^2}$$

$$\frac{3(10^3)}{5b^2} = \frac{372550(10^6)}{(4.1569/b)^2}$$

$$b = 0.01292\ \text{m} = 12.9\ \text{mm}$$

Here, $\frac{KL}{r} = \frac{4.1569}{0.01292} = 321.84$. Since $\frac{KL}{r} > 55$, the assumption is correct.

Thus,

$$b = 12.9\ \text{mm}$$ **Ans.**

Ans:

$b = 12.9$ mm

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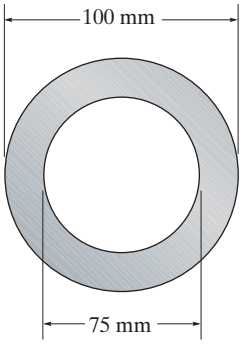
13–93. The 2014-T6 aluminum hollow section has the cross section shown. If the column is 3 m long and is fixed at both ends, determine the allowable axial force P that can be safely supported by the column.



SOLUTION

Section Properties.

$$A = \pi(0.05^2 - 0.0375^2) = 3.4361(10^{-3})\ \text{m}^2$$

$$I = \frac{\pi}{4}(0.05^4 - 0.0375^4) = 3.3556(10^{-6})\ \text{m}^4$$

$$\pi = \sqrt{\frac{I}{A}} = \sqrt{\frac{3.3556(10^{-6})}{3.4361(10^{-3})}} = 0.03125\ \text{m}$$

Slenderness Ratio. For a column fixed at both of its ends, $K = 0.5$. Thus,

$$\frac{KL}{\pi} = \frac{0.5(3)}{0.03125} = 48$$

2014-T6 Aluminum Alloy Column Formulas. Since $12 < \frac{KL}{r} < 55$, the column can be classified as an intermediate column.

$$\sigma_{\text{allow}} = \left[212 - 1.59\left(\frac{KL}{\pi}\right)\right]\ \text{MPa}$$
$$= [212 - 1.59(48)]\ \text{MPa}$$
$$= 135.68\ \text{MPa}$$

Thus, the allowable load is

$$P_{\text{allow}} = \sigma_{\text{allow}} A = [135.68(10^6)][3.4361(10^{-3})]$$
$$= 466.21(10^3)\ \text{N} = 466\ \text{kN}$$ **Ans.**

Ans:
$P_{\text{allow}} = 466\ \text{kN}$

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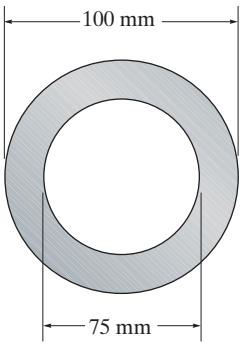
***13–94.** The 2014-T6 aluminum hollow section has the cross section shown. If the column is fixed at its base and pinned at its top, and is subjected to the axial force $P = 500$ kN, determine the maximum length of the column for it to safely support the load.



SOLUTION

Section Properties.

$$A = \pi(0.05^2 - 0.0375^2) = 3.4361(10^{-3})\ \text{m}^2$$

$$I = \frac{\pi}{4}(0.05^4 - 0.0375^4) = 3.3556(10^{-6})\ \text{m}^4$$

$$\pi = \sqrt{\frac{I}{A}} = \sqrt{\frac{3.3556(10^{-6})}{3.4361(10^{-3})}} = 0.03125\ \text{m}$$

Slenderness Ratio. For a column fixed at its base and pinned at its top, $K = 0.7$. Thus,

$$\frac{KL}{\pi} = \frac{0.7L}{0.03125} = 22.4L$$

2014-T6 Aluminum Alloy Column Formulas. Assuming an intermediate column,

$$\sigma_{\text{allow}} = \left[212 - 1.59\left(\frac{KL}{\pi}\right)\right]\ \text{MPa}$$

$$\frac{500(10^3)}{3.4361(10^{-3})} = [212 - 1.59(22.4L)](10^6)$$

$$L = 1.8668\ \text{m} = 1.87\ \text{m} \qquad \textbf{Ans.}$$

Hence, $\frac{KL}{\pi} = 22.4(1.8668) = 41.8$. Since $12 < \frac{KL}{\pi} < 55$, the assumptions of an intermediate column is correct.

Ans:
$L = 1.87$ m

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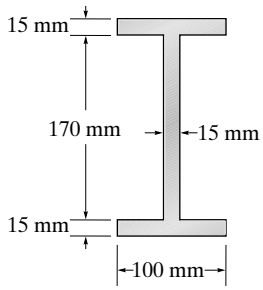
13–95. The 2014-T6 aluminum column of 3-m length has the cross section shown. If the column is pinned at both ends and braced against the weak axis at its mid-height, determine the allowable axial force P that can be safely supported by the column.



SOLUTION

Section Properties.

$$A = 0.1(0.2) - 0.085(0.17) = 5.55\left(10^{-3}\right)\text{m}^2$$

$$I_x = \frac{1}{12}(0.1)\left(0.2^3\right) - \frac{1}{12}(0.085)\left(0.17^3\right) = 31.86625\left(10^{-6}\right)\text{m}^4$$

$$I_y = 2\left[\frac{1}{12}(0.015)\left(0.1^3\right)\right] + \frac{1}{12}(0.17)\left(0.015^3\right) = 2.5478\left(10^{-6}\right)\text{m}^4$$

$$r_x = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{31.86625\left(10^{-6}\right)}{5.55\left(10^{-3}\right)}} = 0.07577$$

$$r_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{2.5478\left(10^{-6}\right)}{5.55\left(10^{-3}\right)}} = 0.02143\text{ m}$$

Slenderness Ratio. For a column pinned at both of its ends, $K = 1$. Here, $L_x = 3$ m and $L_y = 1.5$ m. Thus,

$$\left(\frac{KL}{r}\right)_x = \frac{(1)(3)}{0.07577} = 39.592$$

$$\left(\frac{KL}{r}\right)_y = \frac{(1)(1.5)}{0.02143} = 70.009\text{ (controls)}$$

2014-T6 Alumimum Alloy Column Formulas. Since $\left(\frac{KL}{r}\right)_y > 55$, the column can be classified a long column,

$$\sigma_{\text{allow}} = \left[\frac{372550}{\left(\frac{KL}{r}\right)^2}\right]\text{MPa}$$

$$= \left[\frac{372550}{70.009^2}\right]\text{MPa}$$

$$= 76.011\text{ MPa}$$

Thus, the allowed force is

$$P_{\text{allow}} = \sigma_{\text{allow}}A = [76.011(10^6)]\,[5.55(10^{-3})] = 421.86(10^3)\text{ N} = 422\text{ kN}$$ **Ans.**

Ans:
$A = 5.55\ (10^{-3})\ \text{m}^2$, $I_x = 31.86625\ (10^{-6})\ \text{m}^4$,
$I_y = 2.5478\ (10^{-6})\ \text{m}^4$, $P_{\text{allow}} = 422\text{ kN}$

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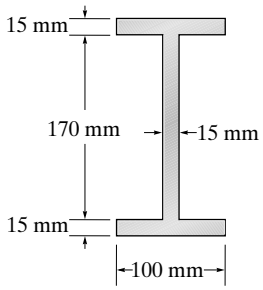
***13–96.** The 2014-T6 aluminum column has the cross section shown. If the column is pinned at both ends and subjected to an axial force $P = 100$ kN, determine the maximum length the column can have to safely support the loading.



SOLUTION

Section Properties.

$$A = 0.1(0.2) - 0.085(0.17) = 5.55\left(10^{-3}\right)\text{m}^2$$

$$I_y = 2\left[\frac{1}{12}(0.015)\left(0.1^3\right)\right] + \frac{1}{12}(0.17)\left(0.015^3\right) = 2.5478\left(10^{-6}\right)\text{m}^4$$

$$r_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{2.5478\left(10^{-6}\right)}{5.55\left(10^{-3}\right)}} = 0.02143\text{ m}$$

Slenderness Ratio. For a column pinned at both of its ends, $K = 1$. Then,

$$\left(\frac{KL}{r}\right)_y = \frac{1(L)}{0.02143} = 46.6727L$$

2014-T6 Alumimum Alloy Column Formulas. Assuming a long column,

$$\sigma_{\text{allow}} = \left[\frac{372550}{\left(\frac{KL}{r}\right)^2}\right]\text{MPa}$$

$$\frac{100\left(10^3\right)}{5.55\left(10^{-3}\right)} = \left[\frac{372550}{(46.6727L)^2}\right]\left(10^6\right)\text{Pa}$$

$$L = 3.0809\text{ m} = 3.08\text{ m} \qquad \textbf{Ans.}$$

Since $\left(\frac{KL}{r}\right)_y = 46.6727(3.0809) = 143.79 > 55$, the assumption is correct.

Ans:
$L = 3.08$ m

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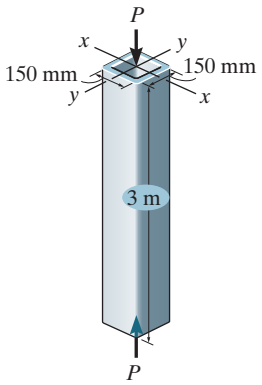
13–97. The tube is 6 mm thick, is made of a 2014-T6 aluminum alloy, and is fixed at its bottom and pinned at its top. Determine the largest axial load that it can support.



SOLUTION

Section Properties:

$$A = 0.15(0.15) - 0.138(0.138) = 3.456(10^{-3})\ \text{m}^2$$

$$I = \frac{1}{12}(0.15)(0.15^3) - \frac{1}{12}(0.138)(0.138^3) = 11.9647(10^{-6})\ \text{m}^4$$

$$\pi = \sqrt{\frac{I}{A}} = \sqrt{\frac{11.9647(10^{-6})}{3.456(10^{-3})}} = 0.058839\ \text{m}$$

Slenderness Ratio: For a column fixed at one end and pinned at the other end, $K = 0.7$. Thus,

$$\frac{KL}{\pi} = \frac{0.7(3)}{0.058839} = 35.69$$

Aluminium (2014 –∑ T6 alloy) Column Formulas: Since $12 < \frac{KL}{r} < 55$, the column is classified as an *intermediate* column. Applying Eq. 13–25,

$$\sigma_{\text{allow}} = \left[212 - 1.59\left(\frac{KL}{\pi}\right)\right]\ \text{MPa}$$

$$= [212 - 1.59(35.69)]$$

$$= 155.25\ \text{MPa}$$

The allowable load is

$$P_{\text{allow}} = \sigma_{\text{allow}} A = [155.25(10^6)][3.456(10^{-3})] = 536.55(10^3)\ \text{N} = 537\ \text{kN}$$ **Ans.**

Ans:
$P_{\text{allow}} = 537\ \text{kN}$

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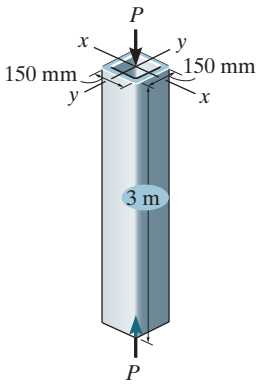
***13–98.** The tube is 6 mm thick, is made of a 2014-T6 aluminum alloy, and is fixed connected at its ends. Determine the largest axial load that it can support.



SOLUTION

Section Properties:

$$A = 0.15(0.15) - 0.138(0.138) = 3.456(10^{-3})\ \text{m}^2$$

$$I = \frac{1}{12}(0.15)(0.15^3) - \frac{1}{12}(0.138)(0.138^3) = 11.9647(10^{-6})\ \text{m}^4$$

$$\pi = \sqrt{\frac{I}{A}} = \sqrt{\frac{11.9647(10^{-6})}{3.456(10^{-3})}} = 0.058839\ \text{m}$$

Slenderness Ratio: For column fixed at both ends, $K = 0.5$. Thus,

$$\frac{KL}{\pi} = \frac{0.5(3)}{0.058839} = 25.49$$

Aluminium (2014 – T6 alloy) Column Formulas: Since $12 < \frac{KL}{r} < 55$, the column is classified as an *intermediate* column. Applying Eq. 13–25,

$$\sigma_{\text{allow}} = \left[212 - 1.59\left(\frac{KL}{\pi}\right)\right]\ \text{MPa}$$

$$= [212 - 1.59(25.49)]$$

$$= 171.46\ \text{MPa}$$

The allowable load is

$$P_{\text{allow}} = \sigma_{\text{allow}} A = [171.46(10^6)][3.456(10^{-3})] = 592.58(10^3)\ \text{N} = 593\ \text{kN}$$ **Ans.**

Ans:
$P_{\text{allow}} = 593\ \text{kN}$

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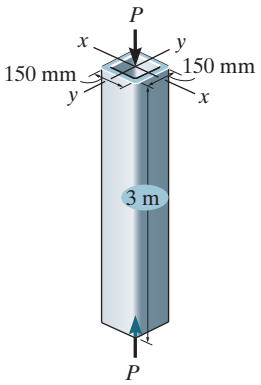
13–99. The tube is 6 mm thick, is made of a 2014-T6 aluminum alloy and is pin connected at its ends. Determine the largest axial load it can support.



SOLUTION

Section Properties:

$$A = 0.15(0.15) - 0.138(0.138) = 3.456(10^{-3})\ \text{m}^2$$

$$I = \frac{1}{12}(0.15)(0.15^3) - \frac{1}{12}(0.138)(0.138^3) = 11.9647(10^{-6})\ \text{m}^4$$

$$\pi = \sqrt{\frac{I}{A}} = \sqrt{\frac{11.9647(10^{-6})}{3.456(10^{-3})}} = 0.058839\ \text{m}$$

Slenderness Ratio: For a column pinned as both ends, $K = 1$. Thus,

$$\frac{KL}{\pi} = \frac{1(3)}{0.058839} = 50.99$$

Aluminum (2014 – T6 alloy) Column Formulas: Since $12 < \frac{KL}{r} < 55$, the column is classified as an *intermediate* column. Applying Eq. 13–25,

$$\sigma_{\text{allow}} = \left[212 - 1.59\left(\frac{KL}{\pi}\right)\right]\ \text{MPa}$$

$$= [212 - 1.59(50.99)]$$

$$= 130.93\ \text{MPa}$$

The allowable load is

$$P_{\text{allow}} = \sigma_{\text{allow}} A = [130.93(10^6)][3.456(10^{-3})] = 452.49(10^3)\ \text{N} = 452\ \text{kN} \qquad \textbf{Ans.}$$

Ans:
$P_{\text{allow}} = 452\ \text{kN}$

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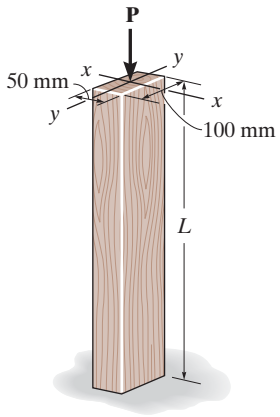
*__13–100.__ The column is made of wood. It is fixed at its bottom and free at its top. Use the NFPA formulas to determine its greatest allowable length if it supports an axial load of $P = 10$ KN.



SOLUTION

Slenderness Ratio: For a column fixed at one end and free at the other end, $K = 2$. Thus,

$$\frac{KL}{d} = \frac{2(L)}{0.05} = 40L$$

NFPA Timber Column Formulas: Assume a *long* column. Apply Eq. 13–29,

$$\sigma_{\text{allow}} = \frac{3725}{(KL/d)^2}\ \text{MPa}$$

$$\frac{10(10^3)}{0.05(0.1)} = \frac{3725(10^6)}{(40L)^2}$$

$$L = 1.079\ \text{m}$$

Here, $\frac{KL}{d} = 40(1.079) = 43.16$. Since $26 < \frac{KL}{d} < 50$, the assumption is correct. Thus,

$$L = 1.08\ \text{m}$$ **Ans.**

Ans:

$L = 1.08$ m

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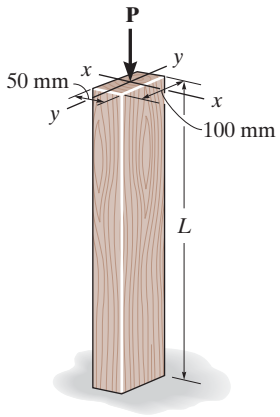
13–101. The column is made of wood. It is fixed at its bottom and free at its top. Use the NFPA formulas to determine the largest allowable axial load P that it can support if it has a length $L = 1.2$ m.



SOLUTION

Slenderness Ratio: For a column fixed at one end and free at the other end, $K = 2$. Thus,

$$\frac{KL}{d} = \frac{2(1.2)}{0.05} = 48.0$$

NFPA Timber Column Formulas: Since $26 < \frac{KL}{d} < 50$, it is a *long* column. Apply Eq. 13–29,

$$\sigma_{\text{allow}} = \frac{3725}{(KL/d)^2}\ \text{MPa}$$

$$= \frac{3725}{48.0^2}\ \text{MPa}$$

$$= 1.6168\ \text{MPa}$$

The allowable axial force is

$$P_{\text{allow}} = \sigma_{\text{allow}} A = [1.6168(10^6)][0.05(0.1)] = 8.084(10^3)\ \text{N} = 8.08\ \text{kN} \qquad \textbf{Ans.}$$

Ans:
$P_{\text{allow}} = 8.08$ kN

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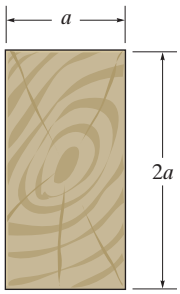
13–102.

A rectangular wooden column has the cross section shown. If $a = 3$ in. and the column is subjected to an axial force of $P = 15$ kip, determine the maximum length the column can have to safely support the load. The column is pinned at its top and fixed at its base.



SOLUTION

Slenderness Ratio: For a column fixed at its base and pinned at its top, $K = 0.7$. Then,

$$\frac{KL}{d} = \frac{0.7L}{3} = 0.2333L$$

NFPA Timber Column Formula: Assuming an intermediate column,

$$\sigma_{\text{allow}} = 1.20\left[1 - \frac{1}{3}\left(\frac{KL/d}{26.0}\right)^2\right] \text{ ksi}$$

$$\frac{15}{3(6)} = 1.20\left[1 - \frac{1}{3}\left(\frac{0.2333L}{26.0}\right)^2\right]$$

$$L = 106.68 \text{ in.} = 8.89 \text{ ft} \qquad \textbf{Ans.}$$

Here, $\frac{KL}{d} = 0.2333(106.68) = 24.89$. Since $11 < \frac{KL}{d} < 26$, the assumption is correct.

Ans:
$L = 8.89$ ft

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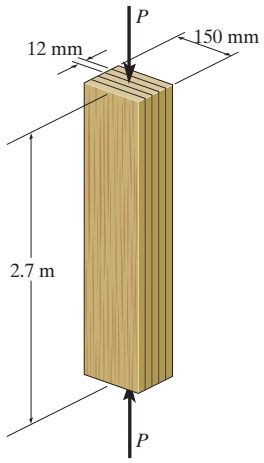
13–103. The wooden column shown is formed by gluing together the 150 mm × 12 mm boards. If the column is pinned at both ends and is subjected to an axial load P = 100 kN, determine the required number of boards needed to form the column in order to safely support the loading.



SOLUTION

Slenderness Ratio. For a column pinned at both of its ends, $K = 1$. If the number of the boards required is n and assuming that $n(0.012) < 0.15$ m. Then, $d = n(0.012)$. Thus,

$$\frac{KL}{d} = \frac{1(2.7)}{n(0.012)} = \frac{225}{n}$$

NFPA Timber Column Formula. Assuming an intermediate column,

$$\sigma_{\text{allow}} = 8.28\left[1 - \frac{1}{3}\left(\frac{KL/d}{26.0}\right)^2\right] \text{MPa}$$

$$\frac{100(10^3)}{[n(0.012)](0.15)} = 8.28\left[1 - \frac{1}{3}\left(\frac{1.8/d}{26.0}\right)^2\right](10^6)$$

$$n^2 - 6.70961n - 24.963 = 0$$

Solving for the positive root,

$$n = 9.373$$

Use $n = 10$ **Ans.**

Here, $\frac{KL}{d} = \frac{225}{10} = 22.5$. Since $n(0.012) = 10(0.012) = 0.12$ m < 0.15 m and $11 < \frac{KL}{d} < 26$, the assumptions made are correct.

Ans:
Use $n = 10$

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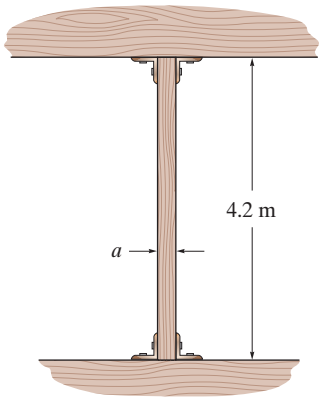
***13–104.** The timber column has a square cross section and is assumed to be pin connected at its top and bottom. If it supports an axial load of 250 kN, determine its smallest side dimension a to the nearest multiples of 15 mm. Use the NFPA formulas.



SOLUTION

Section properties:

$$A = a^2 \qquad \sigma_{\text{allow}} = \sigma = \frac{P}{A} = \frac{250(10^3)}{a^2}$$

Assume long column:

$$\sigma_{\text{allow}} = \frac{3725\ \text{MPa}}{(KL/d)^2}$$

$$\frac{250(10^3)}{a^2} = \frac{3725(10^6)}{[1.0(4.2)/a]^2}$$

$$a = 0.1855\ \text{m}$$

$$\frac{KL}{d} = \frac{1.0(4.2)}{0.1855} = 22.6, \quad \frac{KL}{d} < 26$$ Assumption NG

Assume intermediate column:

$$\sigma_{\text{allow}} = 8.28\left[1 - \frac{1}{3}\left(\frac{KL/d}{26.0}\right)^2\right]\ \text{MPa}$$

$$\frac{250(10^3)}{a^2} = 8.28\left\{1 - \frac{1}{3}\left[\frac{1.0(4.2)/a}{26.0}\right]^2\right\}(10^6)$$

$$a = 0.19721\ \text{m} = 197.21\ \text{mm}$$

$$\frac{KL}{d} == \frac{1(4.2)}{0.19721} = 21.3, \ 11 < \frac{KL}{d} < 26$$ Assumption O.K.

Use $a = 200$ mm **Ans.**

Ans:
Use $a = 200$ mm

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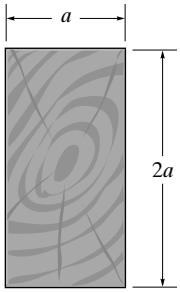
13–105. A rectangular wooden column has the cross section shown. If the column is 1.8 m long and subjected to an axial force of $P = 75$ kN, determine the required minimum dimension a of its cross-sectional area to the nearest multiples of 5 mm so that the column can safely support the loading. The column is pinned at both ends.



SOLUTION

Slenderness Ratio. For a column pinned at both of its ends, $K = 1$. Then,

$$\frac{KL}{d} = \frac{1(1.8)}{a} = \frac{1.8}{a}$$

NFPA Timber Column Formula. Assuming an intermediate column,

$$\sigma_{\text{allow}} = 8.28\left[1 - \frac{1}{3}\left(\frac{KL/d}{26.0}\right)^2\right] \text{MPa}$$

$$\frac{75(10^3)}{2a(\text{a})} = 8.28\left[1 - \frac{1}{3}\left(\frac{1.8/d}{26.0}\right)^2\right](10^6)$$

$$a = 0.07827 \text{ m} = 78.27 \text{ mm}$$

Use $a = 80$ mm **Ans.**

$\frac{KL}{d} = \frac{1.8}{0.08} = 22.5$. Since $11 < \frac{KL}{d} < 26$, the assumption is correct.

Ans:
Use $a = 80$ mm

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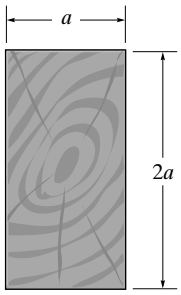
13–106. A rectangular wooden column has the cross section shown. If $a = 75$ mm and the column is 3.6 m long, determine the allowable axial force P that can be safely supported by the column if it is pinned at its top and fixed at its base.



SOLUTION

Slenderness Ratio. For a column fixed at its base and pinned at its top $K = 0.7$. Then,

$$\frac{KL}{d} = \frac{0.7(3.6)}{0.075} = 33.6$$

NFPA Timer Column Formula. Since $26 < \frac{KL}{d} < 50$, the column can be classified as a long column.

$$\sigma_{\text{allow}} = \frac{3725\text{ MPa}}{(KL/d)^2} = \frac{3725\text{ MPa}}{33.6^2} = 3.2995\text{ MPa}$$

The allowable force is

$$P_{\text{allow}} = \sigma_{\text{allow}} A = [3.2995(10^6)][(0.075)(0.015)] = 37.12(10^3)\text{ N} = 37.1\text{ N} \qquad \textbf{Ans.}$$

Ans:
$P_{\text{allow}} = 37.1$ kN

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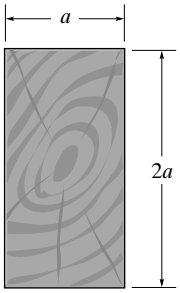
13–107. A rectangular wooden column has the cross section shown. If $a = 75$ mm and the column is subjected to an axial force of $P = 75$ kN, determine the maximum length the column can have to safely support the load. The column is pinned at its top and fixed at its base.



SOLUTION

Slenderness Ratio. For a column fixed at its base and pinned at its top, $K = 0.7$. Then,

$$\frac{KL}{d} = \frac{0.7L}{0.075} = 9.3333L$$

NFPA Timber Column Formula. Assuming an intermediate column,

$$\sigma_{\text{allow}} = 8.28\left[1 - \frac{1}{3}\left(\frac{KL/d}{26.0}\right)^2\right] \text{MPa}$$

$$\frac{75(10^3)}{0.075(0.15)} = 8.28\left[1 - \frac{1}{3}\left(\frac{9.3333L}{26.0}\right)^2\right](10^6)$$

$$L = 2.1298 \text{ m} = 2.13 \text{ m} \qquad \textbf{Ans.}$$

Here, $\frac{KL}{d} = 9.3333(2.1298) = 19.88$. Since $11 < \frac{KL}{d} < 26$, the assumption is correct.

Ans:
$L = 2.13$ m

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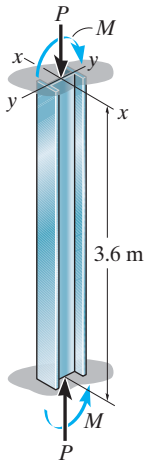
***13–108.** The W360 × 33 structural A-36 steel column fixed at its top and bottom. If a horizontal load (not shown) causes it to support end moments of $M = 15\ \text{kN} \cdot \text{m}$, determine the maximum allowable axial force P that can be applied. Bending is about the x–x axis. Use the AISC equations of Sec. 13.6 and Eq. 13–30.



SOLUTION

Section properties for W360 × 33:

$A = 4190\ \text{mm}^2 = 4.19(10^{-3})\ \text{m}^2$ $\quad d = 349\ \text{mm} = 0.349\ \text{m}$

$I_x = 82.9(10^6)\ \text{mm}^4 = 82.9(10^{-6})\ \text{m}^4$ $\quad r_y = 26.4\ \text{mm} = 0.0264\ \text{m}$

Allowable stress method:

$$\frac{KL}{r_y} = \frac{0.5(3.6)}{0.0264} = 68.182$$

$$\left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2[200(10^9)]}{250(10^6)}} = 125.66, \quad \frac{KL}{r_y} < \left(\frac{KL}{r}\right)_c$$

Hence,

$$(\sigma_a)_{\text{allow}} = \frac{\left[1 - \frac{1}{2}\left(\frac{\left(\frac{KL}{r}\right)^2}{\left(\frac{KL}{r}\right)_c^2}\right)\right]\sigma_Y}{\left[\frac{5}{3} + \frac{3}{8}\frac{\frac{KL}{r}}{\left(\frac{KL}{r}\right)_c} - \frac{1}{8}\frac{\left(\frac{KL}{r}\right)^3}{\left(\frac{KL}{r}\right)_c^3}\right]} = \frac{[1 - \frac{1}{2}(\frac{68.182}{125.66})^2]250}{[\frac{5}{3} + \frac{3}{8}(\frac{68.182}{125.66}) - \frac{1}{8}(\frac{68.182}{125.66})^3]} = 115.23\ \text{MPa}$$

$$\sigma_{\max} = (\sigma_a)_{\text{allow}} = \frac{P}{A} + \frac{M_y c}{I_x}$$

$$115.23(10^6) = \frac{P}{4.19(10^{-3})} + \frac{[15(10^3)](0.349/2)}{82.9(10^{-6})}$$

$P = 350.53(10^3)\ \text{N} = 351\ \text{kN}$ **Ans.**

Ans:
$P = 351\ \text{kN}$

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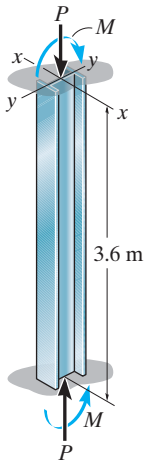
13–109. The W360 × 33 structural A-36 steel column is fixed at its top and bottom. If a horizontal load (not shown) causes it to support end moments of $M = 70$ kN · m, determine the maximum allowable axial force P that can be applied. Bending is about the x–x axis. Use the interaction formula with $(\sigma_b)_{\text{allow}} = 168$ MPa.



SOLUTION

Section Properties for W360 × 33:

$A = 4190 \text{ mm}^2 = 4.19(10^{-3}) \text{ m}^2$ $\quad d = 349 \text{ mm} = 0.349 \text{ m}$

$I_x = 82.9(10^6) \text{ mm}^4 = 82.9(10^{-6}) \text{ m}^4$ $\quad r_y = 26.4 \text{ mm} = 0.0264 \text{ m}$

Interaction method:

$$\frac{KL}{r_y} = \frac{0.5(3.6)(10^3)}{26.4} = 68.182$$

$$\left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2[200(10^9)]}{25(10^6)}} = 125.66, \quad \frac{KL}{r_y} < \left(\frac{KL}{r}\right)_c$$

Hence,

$$(\sigma_a)_{\text{allow}} = \frac{\left[1 - \frac{1}{2}\left(\frac{\left(\frac{KL}{r}\right)^2}{\left(\frac{KL}{r}\right)_c^2}\right)\right]\sigma_Y}{\left[\frac{5}{3} + \frac{3}{8}\frac{\frac{KL}{r}}{\left(\frac{KL}{r}\right)_c} - \frac{1}{8}\frac{\left(\frac{KL}{r}\right)^3}{\left(\frac{KL}{r}\right)_c^3}\right]} = \frac{[1 - \frac{1}{2}(\frac{68.182}{125.66})^2]250}{[\frac{5}{3} + \frac{3}{8}(\frac{68.182}{125.66}) - \frac{1}{8}(\frac{68.182}{125.66})^3]} = 115.2 \text{ MPa}$$

$$\sigma_a = \frac{P}{A} = \frac{P}{4.19(10^{-3})} = 238.66P$$

$$\sigma_b = \frac{M_x c}{I_x} = \frac{[70(10^3)](0.349/2)}{82.9(10^{-6})} = 147.35(10^6) \text{ N/m}^2$$

$$\frac{\sigma_a}{(\sigma_a)_{\text{allow}}} + \frac{\sigma_b}{(\sigma_b)_{\text{allow}}} = 1.0$$

$$\frac{238.66P}{115.23(10^6)} + \frac{147.35(10^6)}{168(10^6)} = 1.0$$

$P = 59.36(10^3) \text{ N} = 59.4 \text{ kN}$ **Ans.**

Since $\frac{\sigma_a}{\sigma_{\text{allow}}} = \frac{238.66[59.36(10^3)]}{115.23(10^6)} = 0.12 < 0.15$

Then the interaction formula is valid.

Ans:
$P = 59.4$ kN

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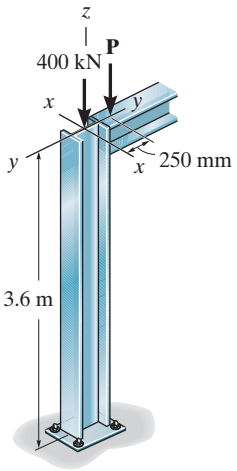
13–110. The W360 × 79 structural A-36 steel column supports an axial load of 400 kN in addition to an eccentric load *P*. Determine the maximum allowable value of *P* based on the AISC equations of Sec. 13.6 and Eq. 13–30. Assume the column is fixed at its base, and at its top it is free to sway in the *x*–*z* plane while it is pinned in the *y*–*z* plane.



SOLUTION

Section Properties: For a W360 × 79 wide flange section.

$A = 10100 \text{ mm}^2 = 0.0101 \text{ m}^2$

$d = 354 \text{ mm} = 0.354 \text{ m}$

$I_x = 227(10^6) \text{ mm}^4 = 227(10^{-6}) \text{ m}^4$

$r_x = 150 \text{ mm} = 0.15 \text{ m}$

$r_x = 48.9 \text{ mm} = 0.0489 \text{ m}$

Slenderness Ratio: By observation, the largest slenderness ratio is about $y-y$ axis. For a column fixed at one end and free at the other end, $K = 2$. Thus,

$$\left(\frac{KL}{r}\right)_y = \frac{2(3.6)}{0.0489} = 147.24$$

Allowable Stress: The allowable stress can be determined using *AISC Column Formulas*. For A–36 steel, $\left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2[200(10^9)]}{250(10^6)}} = 125.66$. Since $\left(\frac{KL}{r}\right)_c \le \frac{KL}{r} \le 200$, the column is a *long* column. Applying Eq. 13–21,

$$\sigma_{\text{allow}} = \frac{12\pi^2 E}{23(KL/r)^2}$$

$$= \frac{12\pi^2[200(10^9)]}{23(147.24^2)}$$

$$= 47.50(10^6) \text{ N/m}^2$$

Maximum Stress: Bending is about $x-x$ axis. Applying we have

$$\sigma_{\max} = \sigma_{\text{allow}} = \frac{P}{A} + \frac{Mc}{I}$$

$$47.50(10^6) = \frac{P + 400(10^3)}{0.0101} + \frac{[P(0.25)](0.354/2)}{227(10^{-6})}$$

$$P = 26.88(10^3) \text{ N} = 26.9 \text{ kN}$$ **Ans.**

Ans:
$P = 26.9$ kN

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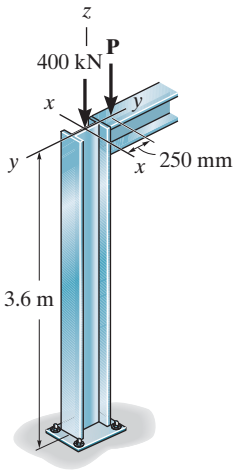
13–111. The W310 × 67 structural A-36 steel column supports an axial load of 400 kN in addition to an eccentric loads of $P = 30$ kN. Determine if the column fails based on the AISC equations of Sec. 13.6 and Eq. 13–30. Assume that the column is fixed at its base, and at its top it is free to sway in the x–z plane while it is pinned in the y–z plane.



SOLUTION

Section Properties: For a W310 × 67 wide flange section,

$A = 8530 \text{ mm}^2 = 8.53(10^{-3}) \text{ m}^2$

$d = 306 \text{ mm} = 0.306 \text{ m}$

$I_x = 145(10^6) \text{ mm}^4 = 145(10^{-6}) \text{ m}^4$

$r_x = 130 \text{ mm} = 0.130 \text{ m}$

$r_y = 49.3 \text{ mm} = 0.0483 \text{ m}$

Slenderness Ratio: By observation, the largest slenderness ratio is about $y-y$ axis. For a column fixed at one end and free at the other end, $K = 2$. Thus,

$$\left(\frac{KL}{r}\right)_y = \frac{2(3.6)}{0.0483} = 146.04$$

Allowable Stress: The allowable stress can be determined using *AISC Column Formulas*. For A–36 steel, $\left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2[200(10^9)]}{250(10^6)}} = 125.66$. Since $\left(\frac{KL}{r}\right)_c \le \frac{KL}{r} \le 200$, the column is a *long* column. Applying Eq. 13–21,

$$\sigma_{\text{allow}} = \frac{12\pi^2 E}{23(KL/r)^2}$$

$$= \frac{12\pi^2[200(10^9)]}{23(146.04^2)}$$

$$= 48.28(10^6) \text{ N/m}^2 = 48.28 \text{ MPa}$$

Maximum Stress: Bending is about $x-x$ axis. Applying Eq. 1 we have

$$\sigma_{\max} = \frac{P}{A} + \frac{Mc}{I}$$

$$= \frac{P + 430(10^3)}{8.53(10^{-3})} + \frac{[30(10^3)](0.306/2)}{145(10^{-6})}$$

$$= 82.07(10^6) \text{ N/m}^2 = 82.07 \text{ MPa}$$

Since $\sigma_{\max} > \sigma_{\text{allow}}$, the column is not adequate.

Ans:
The column is not adequate.

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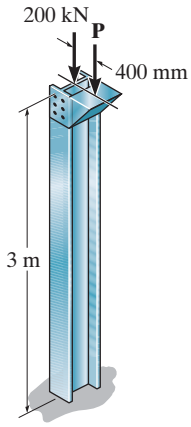
***13–112.** The W360 × 57 structural A-36 steel column is fixed at its bottom and free at its top. Determine the greatest eccentric load P that can be applied using Eq. 13–30 and the AISC equations of Sec. 13.6.



SOLUTION

Section properties for W360 × 57:

$A = 7200\ \text{mm}^2 = 7.2(10^{-3})\ \text{m}^2$ $\quad r_y = 39.3\ \text{mm} = 0.0393\ \text{m}$

$d = 358\ \text{mm} = 0.358\ \text{m}$ $\quad b_f = 172\ \text{mm} = 0.172\ \text{m}$

$I_x = 11.1(10^6)\ \text{mm}^4 = 11.1(10^{-6})\ \text{m}^4$

Allowable stress method:

$$\frac{KL}{r_y} = \frac{2(3)}{0.0393} = 152.67$$

$$\left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2[200(10^9)]}{250(10^6)}} = 125.66,\ \left(\frac{KL}{r}\right)_c < \frac{KL}{r_y} < 200$$

$$(\sigma_a)_{\text{allow}} = \frac{12\pi^2 E}{23(KL/r)^2} = \frac{12\pi^2[200(10^9)]}{23(152.67^2)} = 44.18(10^6)\ \text{N/m}^2$$

$$\sigma_{\max} = (\sigma_a)_{\text{allow}} = \frac{P}{A} + \frac{M_y c}{I_y}$$

$$44.18(10^6) = \frac{P + 200(10^3)}{7.2(10^{-3})} + \frac{[P(0.4)](0.172/2)}{11.1(10^{-6})}$$

$P = 5.067(10^3)\ \text{N} = 5.07\ \text{kN}$ **Ans.**

Ans:
$P = 5.07$ kN

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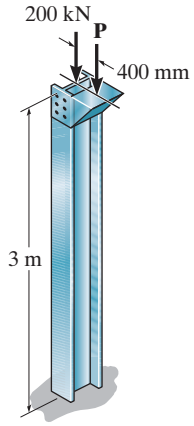
13–113. The W250 × 67 structural A-36 steel column is fixed at its bottom and free at its top. If it is subjected to a load of $P = 10$ kN, determine if it is safe based on the AISC equations of Sec. 13.6 and Eq. 13–30.



SOLUTION

Section Properties for W250 × 67:

$A = 8560 \text{ mm}^2 = 8.56(10^{-3}) \text{ m}^2$ $\quad d = 257 \text{ mm} = 0.257 \text{ m}$

$I_y = 22.2(10^6) \text{ mm}^4 = 22.2(10^{-6}) \text{ m}^4$ $\quad r_y = 50.9 \text{ mm} = 0.0509 \text{ m}$

$b_f = 204 \text{ mm} = 0.204 \text{ m}$

Allowable stress method:

$$\frac{KL}{r_y} = \frac{2(3)}{0.0509} = 117.88$$

$$\left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2[200(10^9)]}{250(10^6)}} = 125.66$$

$$\frac{KL}{r} < \left(\frac{KL}{r}\right)_c$$

$$(\sigma_a)_{\text{allow}} \geq \frac{\left[1 - \frac{1(KL/r)^2}{2(KL/r_c)^2}\right]\sigma_Y}{\frac{5}{3} + \frac{3}{8}\left(\frac{KL/r}{KL/r_c}\right) - \frac{1(KL/r)^3}{8(KL/r_c)^3}} = \frac{[1 - \frac{1}{2}(\frac{117.88}{125.66})^2](250)}{\frac{5}{3} + \frac{3}{8}[\frac{117.88}{125.66}] - \frac{1}{8}[\frac{117.88}{125.66}]^3} = 73.10 \text{ MPa}$$

$$(\sigma_a)_{\text{allow}} = \frac{P}{A} + \frac{M_y c}{I_y}$$

$$\sigma_{\max} = \frac{210(10^3)}{8.56(10^{-3})} + \frac{[10(10^3)](0.4)(0.204/2)}{22.2(10^{-3})}$$

$= 42.91(10^6) \text{ N/m}^2 = 42.91 \text{ MPa}$ O.K.

Since $\sigma_{\text{allow}} > \sigma_{\max}$ the column is safe.

Yes. **Ans.**

Ans:
Yes

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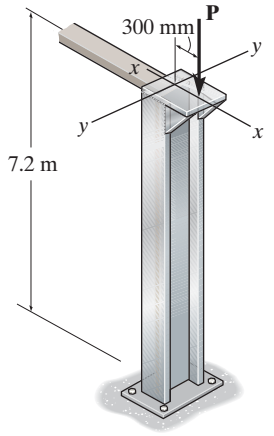
13–114. The A-36-steel W250 × 67 column is fixed at its base. Its top is constrained to move along the *x*–*x* axis but free to rotate about and move along the *y*–*y* axis. Determine the maximum eccentric force *P* that can be safely supported by the column using an interaction formula. The allowable bending stress is $(\sigma_b)_{\text{allow}} = 100$ MPa.



SOLUTION

Section Properties. From the table listed in the appendix, the section properties for a W250 × 67 are

$A = 8560 \text{ mm}^2 = 8.56(10^{-3}) \text{ m}^2$ $\quad r_x = 110 \text{ mm} = 0.11 \text{ m}$

$I_y = 22.2(10^6) \text{ mm}^4 = 22.2(10^{-6}) \text{ m}^4$ $\quad r_y = 50.9 \text{ mm} = 0.0509 \text{ m}$

$b_f = 204 \text{ mm} = 0.204 \text{ m}$

Slenderness Ratio. Here, $L_x = 7.2$ m and for a column fixed at its base and free at its top, $K_x = 2$. Thus,

$$\left(\frac{KL}{r}\right)_x = \frac{2(7.2)}{0.11} = 130.91 \text{ (controls)}$$

Since the column is fixed at its base and pinned at its top, $K_y = 0.7$ and $L_y = 7.2$ m. Then,

$$\left(\frac{KL}{r}\right)_y = \frac{0.7(7.2)}{0.0509} = 99.02$$

Allowable Stress. The allowable stress will be determined using the AISC column formulas. For A-36 steel, $\left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2[200(10^9)]}{250(10^6)}} = 125.66$. Since $\left(\frac{KL}{r}\right)_c < \left(\frac{KL}{r}\right)_x < 200$, the column is classified as a long column.

$$\sigma_{\text{allow}} = \frac{12\pi^2 E}{23(KL/r)^2}$$

$$= \frac{12\pi^2[200(10^9)]}{23(130.91^2)} = 60.10(10^6) \text{ N/m}^2$$

Interaction Formula. Bending is about the weak axis. Here, $M = P(0.3 \text{ m})$ and $c = \frac{b_f}{2} = \frac{0.204}{2} = 0.102$ m

$$\frac{P/A}{(\sigma_a)_{\text{allow}}} + \frac{Mc/Ar^2}{(\sigma_b)_{\text{allow}}} = 1$$

$$\frac{P/8.56(10^{-3})}{60.10(10^6)} + \frac{[P(0.3)](0.102)/[8.56(10^{-3})](0.0509^2)}{100(10^6)}$$

$P = 63.53(10^3) \text{ N} = 63.5 \text{ kN}$ **Ans.**

$$\frac{\sigma_a}{(\sigma_a)_{\text{allow}}} = \frac{63.53(10^3)/8.56(10^{-3})}{60.10(10^6)} = 0.123 < 0.15$$ O.K.

Ans:
$P = 63.5$ kN

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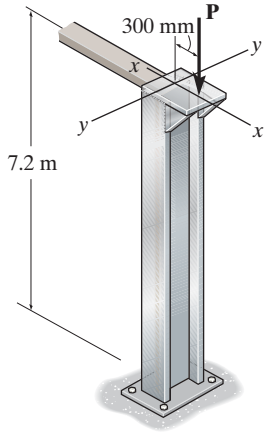
13–115. The A-36-steel W310 × 74 column is fixed at its base. Its top is constrained to move along the *x*–*x* axis but free to rotate about and move along the *y*–*y* axis. If the eccentric force $P = 75$ kN is applied to the column, investigate if the column is adequate to support the loading. Use the allowable stress method.



SOLUTION

Section Properties. From the table listed in the appendix, the section properties for a W310 × 74 are

$A = 9480 \text{ mm}^2 = 9.48(10^{-3}) \text{ m}^2$ $\quad r_x = 132 \text{ mm} = 0.132 \text{ m}$

$I_y = 23.4(10^6) \text{ mm}^4 = 22.3(10^{-6}) \text{ m}^4$ $\quad r_y = 49.7 \text{ mm} = 0.0497 \text{ m}$

$b_f = 205 \text{ mm} = 0.205 \text{ m}$

Slenderness Ratio. Here, $L_x = 7.2$ m and for a column fixed at its base and pinned at its top, $K_x = 2$. Thus,

$$\left(\frac{KL}{r}\right)_x = \frac{2(7.2)}{0.132} = 109.09 \text{ (controls)}$$

Since the column is fixed at its base and pinned at its top, $K_y = 0.7$ and $L_y = 7.2$ m. Then,

$$\left(\frac{KL}{r}\right)_y = \frac{0.7(7.2)}{0.0497} = 101.41$$

Allowable Stress. The allowable stress will be determined using the AISC column formulas. For A-36 steel, $\left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2[200(10^9)]}{250(10^6)}} = 125.66$. Since $\left(\frac{KL}{r}\right)_x < \left(\frac{KL}{r}\right)_c$, the column can be classified as an intermediate column.

$$\sigma_{\text{allow}} = \frac{\left[1 - \frac{(KL/r)^2}{2(KL/r)_C{}^2}\right]\sigma_Y}{\frac{5}{3} + \frac{3(KL/r)}{8(KL/r)_C} - \frac{(KL/r)^3}{8(KL/r)_C{}^3}}$$

$$= \frac{\left[1 - \left(\frac{1}{2}\right)\left(\frac{109.09}{125.66}\right)^2\right](250)}{\frac{5}{3} + \left(\frac{3}{8}\right)\left(\frac{109.09}{125.66}\right) - \left(\frac{1}{8}\right)\left(\frac{109.09}{125.66}\right)^3}$$

$$= 81.55 \text{ MPa}$$

Maximum Stress. Bending is about the weak axis. Since, $M = 75(0.3) = 22.5 \text{ kN} \cdot \text{m}$ and $c = \frac{b_f}{2} = \frac{0.205}{2} = 0.1025$ m

$$\sigma_{\max} = \frac{P}{A} + \frac{Mc}{I} = \frac{75(10^3)}{9.48(10^{-3})} + \frac{[22.5(10^3)](0.1025)}{23.4(10^{-6})}$$

$$= 106.47(10^6) \text{ N/m}^2 = 106.47 \text{ MPa}$$

Since $\sigma_{\max} > \sigma_{\text{allow}}$, the W310 × 74 column *is inadequate* according to the allowable stress method.

? Iq8
The column is not adequate.

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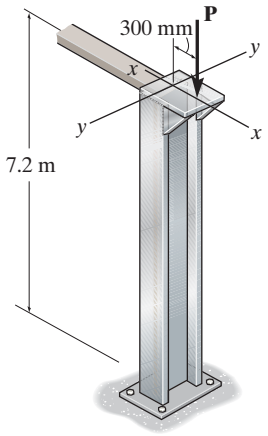
***13–116.** The A-36-steel W250 × 67 column is fixed at its base. Its top is constrained to move along the *x*–*x* axis but free to rotate about and move along the *y*–*y* axis. Determine the maximum eccentric force *P* that can be safely supported by the column using the allowable stress method.



SOLUTION

Section Properties. From the table listed in the appendix, the section properties for a W250 × 67 are

$A = 8560 \text{ mm}^2 = 8.56(10^{-3}) \text{ m}^2$ $\quad r_x = 110 \text{ mm} = 0.11 \text{ m}$

$I_y = 22.2(10^6) \text{ mm}^4 = 22.2(10^{-6}) \text{ m}^4$ $\quad r_y = 50.9 \text{ mm} = 0.0509 \text{ m}$

$b_f = 204 \text{ mm} = 0.204 \text{ m}$

Slenderness Ratio. Here, $L_x = 7.2$ m and for a column fixed at its base and free at its top, $K_x = 2$. Thus,

$$\left(\frac{KL}{r}\right)_x = \frac{2(7.2)}{0.11} = 130.91 \text{ (controls)}$$

Since the column is fixed at its base and pinned at its top, $K_y = 0.7$ and $L_y = 7.2$ m. Then,

$$\left(\frac{KL}{r}\right)_y = \frac{0.7(7.2)}{0.0509} = 99.02$$

Allowable Stress. The allowable stress will be determined using the AISC column formulas. For A-36 steel,

$\left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2[200(10^9)]}{250(10^6)}} = 125.66$. Since $\left(\frac{KL}{r}\right)_c < \left(\frac{KL}{r}\right)_x < 200$,

the column is classified as a long column.

$$\sigma_{\text{allow}} = \frac{12\pi^2 E}{23(KL/r)^2}$$

$$= \frac{12\pi^2[200(10^9)]}{23(130.91^2)} = 60.10(10^6) \text{ N/m}^2 = 60.10 \text{ MPa}$$

Maximum Stress. Bending is about the weak axis. Since $M = P(0.3 \text{ m})$ and $c = \frac{b_f}{2} = \frac{0.204}{2} = 0.102$ m

$$\sigma_{\text{allow}} = \frac{P}{A} + \frac{Mc}{I}$$

$$60.10(10^6) = \frac{P}{8.56(10^{-3})} + \frac{[P(0.3)](0.102)}{22.2(10^{-6})}$$

$P = 40.19(10^3) \text{ N} = 40.2 \text{ kN}$ **Ans.**

Ans:
$P = 40.2$ kN

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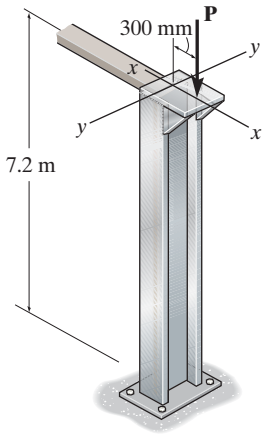
13–117. The A-36-steel W310 × 74 column is fixed at its base. Its top is constrained to move along the *x*–*x* axis but free to rotate about and move along the *y*–*y* axis. If the eccentric force $P = 65$ kN is applied to the column, investigate if the column is adequate to support the loading. Use the interaction formula. The allowable bending stress is $(\sigma_b)_{\text{allow}} = 100$ MPa



SOLUTION

Section Properties. From the table listed in the appendix, the section properties for a W310 × 74 are

$A = 9480 \text{ mm}^2 = 9.48(10^{-3}) \text{ m}^2$ $\quad r_x = 132 \text{ mm} = 0.132 \text{ m}$

$I_y = 23.4(10^6) \text{ mm}^4 = 22.3(10^{-6}) \text{ m}^4$ $\quad r_y = 49.7 \text{ mm} = 0.0497 \text{ m}$

$b_f = 205 \text{ mm} = 0.205 \text{ m}$

Slenderness Ratio. Here, $L_x = 7.2$ m and for a column fixed at its base and pinned at its top, $K_x = 2$. Thus,

$$\left(\frac{KL}{r}\right)_x = \frac{2(7.2)}{0.132} = 109.09 \text{ (controls)}$$

Since the column is fixed at its base and pinned at its top, $K_y = 0.7$ and $L_y = 7.2$ m. Thus,

$$\left(\frac{KL}{r}\right)_y = \frac{0.7(7.2)}{0.0497} = 101.41$$

Allowable Stress. The allowable stress will be determined using the AISC column formulas. For A-36 steel, $\left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2[200(10^9)]}{250(10^6)}} = 125.66$. Since $\left(\frac{KL}{r}\right)_x < \left(\frac{KL}{r}\right)_c$, the column can be classified as an intermediate column.

$$\sigma_{\text{allow}} = \frac{\left[1 - \frac{(KL/r)^2}{2(KL/r)_C{}^2}\right]\sigma_Y}{\frac{5}{3} + \frac{3(KL/r)}{8(KL/r)_C} - \frac{(KL/r)^3}{8(KL/r)_C{}^3}}$$

$$= \frac{\left[1 - \left(\frac{1}{2}\right)\left(\frac{109.09}{125.66}\right)^2\right](250)}{\frac{5}{3} + \left(\frac{3}{8}\right)\left(\frac{109.09}{125.66}\right) - \left(\frac{1}{8}\right)\left(\frac{109.09}{125.66}\right)^3}$$

$$= 81.55 \text{ MPa}$$

Interaction Formula. Bending is about the weak axis. Here $M = 65(0.3) = 19.5 \text{ kN} \cdot \text{m}$ and $c = \frac{b_f}{2} = \frac{0.205}{2} = 0.1025$ m

$$\frac{P/A}{(\sigma_a)_{\text{allow}}} + \frac{Mc/Ar^2}{(\sigma_b)_{\text{allow}}} = \frac{65(10^3)/9.48(10^{-3})}{81.55(10^6)} + \frac{[19.5(10^3)](0.1025)/[9.48(10^{-3})](0.0497^2)}{100(10^6)}$$

$$= 0.9376 < 1$$

$$\frac{\sigma_a}{(\sigma_a)_{\text{allow}}} = \frac{65(10^3)/9.48(10^{-3})}{81.55(10^6)} = 0.0841 < 0.15 \quad \text{O.K.}$$

Thus, a W310 × 74 column *is adequate* according to the interaction formula.

Ans:
The column is adequate.

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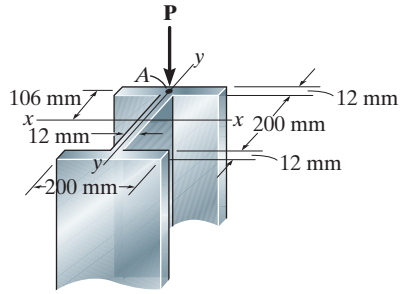
13–118. A 4.8-m-long column is made of aluminum alloy 2014-T6. If it is fixed at its top and bottom, and a compressive load **P** is applied at point A, determine the maximum allowable magnitude of **P** using the equations of Sec. 13.6 and Eq. 13–30.



SOLUTION

Section properties:

$$A = 0.012(0.2) + 2[0.2(0.012)] = 7.2(10^{-3})\ \text{m}^2$$

$$I_x = \frac{1}{12}(0.2)(0.224^3) - \frac{1}{12}(0.188)(0.2^3) = 61.9904(10^{-6})\ \text{m}^4$$

$$I_y = \frac{1}{12}(0.2)(0.012^3) + 2\left[\frac{1}{12}(0.012)(0.2^3)\right] = 16.0288(10^{-6})\ \text{m}^4$$

$$\pi_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{16.0288(10^{-6})}{7.2(10^{-3})}} = 0.04718\ \text{m}$$

Allowable stress method:

$$\frac{KL}{\pi_y} = \frac{0.5(4.8)}{0.04718} = 50.87,\ 12 < \frac{KL}{\pi_y} < 55$$

$$\sigma_{\text{allow}} = \left[212 - 1.59\left(\frac{KL}{\pi}\right)\right]\ \text{MPa}$$

$$= [212 - 1.59(50.87)] = 131.12\ \text{MPa}$$

$$\sigma_{\max} = \sigma_{\text{allow}} = \frac{P}{A} + \frac{M_x c}{I_x}$$

$$131.12(10^6) = \frac{P}{7.2(10^{-3})} + \frac{[P(0.106)](0.112)}{61.9904(10^{-6})}$$

$$P = 396.86(10^3)\ \text{N} = 397\ \text{kN}$$ **Ans.**

Ans:
$P = 397$ kN

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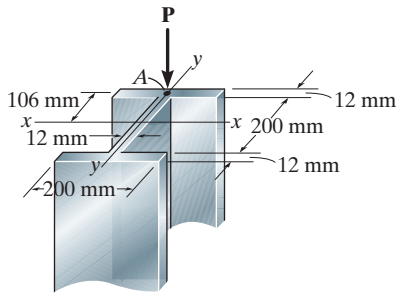
13–119. A 4.8-m-long column is made of aluminum alloy 2014-T6. If it is fixed at its top and bottom, and a compressive load **P** is applied at point A, determine the maximum allowable magnitude of **P** using the equations of Sec. 13.6 and the interaction formula with $(\sigma_b)_{\text{allow}} = 140$ MPa.



SOLUTION

Section properties:

$$A = 0.012(0.2) + 2[0.2(0.012)] = 7.2(10^{-3})\ \text{m}^2$$

$$I_x = \frac{1}{12}(0.2)(0.224^3) - \frac{1}{12}(0.188)(0.2^3) = 61.9904(10^{-6})\ \text{m}^4$$

$$I_y = \frac{1}{12}(0.2)(0.012^3) + 2\left[\frac{1}{12}(0.012)(0.2^3)\right] = 16.0288(10^{-6})\ \text{m}^4$$

$$r_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{16.0288(10^{-6})}{7.2(10^{-3})}} = 0.04718\ \text{m}$$

Allowable stress method:

$$\frac{KL}{r_y} = \frac{0.5(4.8)}{0.04718} = 50.87,\ 12 < \frac{KL}{r_y} < 55$$

$$\sigma_{\text{allow}} = \left[212 - 1.59\left(\frac{KL}{r}\right)\right]\ \text{MPa}$$

$$= [212 - 1.59(50.87)] = 131.12\ \text{MPa}$$

$$\sigma_a = \frac{P}{A} = \frac{P}{7.2(10^{-3})} = 138.89P$$

$$\sigma_b = \frac{Mc}{I_x} = \frac{[P(0.106)](0.112)}{61.9904(10^{-6})} = 191.51P$$

$$\frac{\sigma_a}{(\sigma_a)_{\text{allow}}} + \frac{\sigma_b}{(\sigma_b)_{\text{allow}}} = 1.0$$

$$\frac{138.89P}{131.12(10^6)} + \frac{191.51(10^6)}{140(10^6)} = 1.0$$

$P = 412.00(10^3)\ \text{N} = 412\ \text{kN}$ **Ans.**

Ans:
$P = 412$ kN

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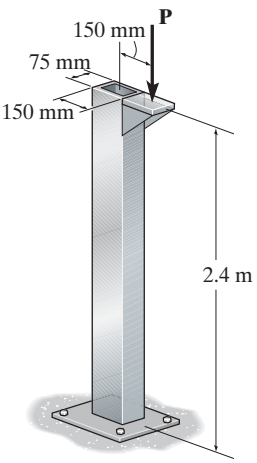
***13–120.** The 2014-T6 hollow column is fixed at its base and free at its top. Determine the maximum eccentric force P that can be safely supported by the column. Use the allowable stress method. The thickness of the wall for the section is $t = 12$ mm.



SOLUTION

Section Properties.

$$A = 0.15(0.075) - 0.126(0.051) = 4.824(10^{-3})\ \text{m}^2$$

$$I_x = \frac{1}{12}(0.075)(0.15^3) - \frac{1}{12}(0.051)(0.126^3) = 12.592152(10^{-6})\ \text{m}^4$$

$$I_y = \frac{1}{12}(0.15)(0.075^3) - \frac{1}{12}(0.126)(0.051^3) = 3.880602(10^{-6})\ \text{m}^4$$

$$r_x = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{12.592152(10^{-6})}{4.824(10^{-3})}} = 0.05109\ \text{m}$$

$$r_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{3.880602(10^{-6})}{4.824(10^{-3})}} = 0.02836\ \text{m}$$

Slenderness Ratio. For a column fixed at its base and free at its top, $K = 2$. Thus,

$$\left(\frac{KL}{r}\right)_y = \frac{2(2.4)}{0.02836} = 169.24$$

Allowable Stress. Since $\left(\frac{KL}{r}\right)_y > 55$, the column can be classified as a long column.

$$\sigma_{\text{allow}} = \frac{372550\ \text{MPa}}{(KL/r)^2} = \frac{372550\ \text{MPa}}{169.24^2} = 13.01\ \text{MPa}$$

Maximum Stress. Bending occurs about the strong axis so that $M = P(0.150\ \text{m})$ and $c = \frac{0.15}{2} = 0.075\ \text{m}$

$$\sigma_{\text{allow}} = \frac{P}{A} + \frac{Mc}{I}$$

$$13.01(10^6) = \frac{P}{4.824(10^{-3})} + \frac{[P(0.15)](0.075)}{12.592152(10^{-6})}$$

$$P = 11.82(10^3)\ \text{N} = 11.8\ \text{kN}$$ **Ans.**

Ans:
$P = 11.8$ kN

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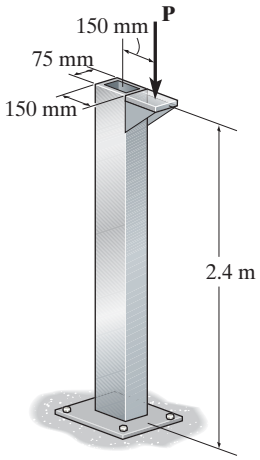
13–121. The 2014-T6 hollow column is fixed at its base and free at its top. Determine the maximum eccentric force P that can be safely supported by the column. Use the interaction formula. The allowable bending stress is $(\sigma_b)_{\text{allow}} = 200$ MPa. The thickness of the wall for the section is $t = 12$ mm.



SOLUTION

Section Properties.

$$A = 0.15(0.075) - 0.126(0.051) = 4.824(10^{-3})\ \text{m}^2$$

$$I_x = \frac{1}{12}(0.075)(0.15^3) - \frac{1}{12}(0.051)(0.126^3) = 12.592152(10^{-6})\ \text{m}^4$$

$$I_y = \frac{1}{12}(0.15)(0.075^3) - \frac{1}{12}(0.126)(0.051^3) = 3.880602(10^{-6})\ \text{m}^4$$

$$\pi_x = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{12.592152(10^{-6})}{4.824(10^{-3})}} = 0.05109\ \text{m}$$

$$\pi_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{3.880602(10^{-6})}{4.824(10^{-3})}} = 0.02836\ \text{m}$$

Slenderness Ratio. For a column fixed at its base and free at its top, $K = 2$. Thus,

$$\left(\frac{KL}{\pi}\right)_y = \frac{2(2.4)}{0.02836} = 169.24$$

Allowable Stress. Since $\left(\frac{KL}{r}\right)_y > 55$, the column can be classified as a long column.

$$\sigma_{\text{allow}} = \frac{372550\ \text{MPa}}{(KL/\pi)^2} = \frac{372550\ \text{MPa}}{169.24^2} = 13.01\ \text{MPa}$$

Maximum Stress. Bending occurs about the strong axis so that $M = P(0.150\ \text{m})$ and $c = \frac{0.15}{2} = 0.075\ \text{m}$

$$\frac{P/A}{(\sigma_a)_{\text{allow}}} + \frac{Mc/Ar^2}{(\sigma_b)_{\text{allow}}} = 1$$

$$\frac{P/4.824(10^{-3})}{13.01(10^6)} + \frac{P(0.15)(0.075)/[4.824(10^{-3})](0.05109^2)}{200(10^6)} = 1$$

$$P = 49.01(10^3)\ \text{N} = 49.0\ \text{kN} \qquad \textbf{Ans.}$$

Ans:
$P = 49.0$ kN

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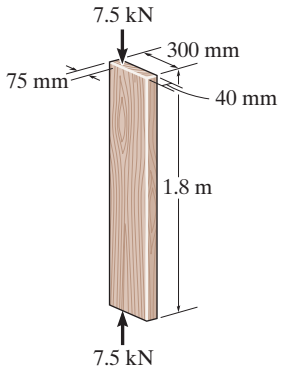
13–122. Determine if the column can support the eccentric compressive load of 7.5 kN. Assume that the ends are pin connected. Use the NFPA equations in Sec. 13.6 and Eq. 13–30.



SOLUTION

$A = 0.3(0.04) = 0.012\ \text{m}^2 \qquad I_x = \frac{1}{12}(0.04)(0.3^3) = 90.0(10^{-6})\ \text{m}^4$

$d = 0.04\ \text{m}$

$$\frac{KL}{d} = \frac{1.0(1.8)}{0.04} = 45$$

$$26 < \frac{KL}{d} < 50$$

$$(\sigma_a)_{\text{allow}} = \frac{3725\ \text{MPa}}{(KL/d)^2} = \frac{3725\ \text{MPa}}{45^2} = 1.8395\ \text{MPa}$$

$$\sigma_{\max} = \frac{P}{A} + \frac{M_x c}{I_x}$$

$$= \frac{7.5(10^3)}{0.012} + \frac{[7.5(10^3)](0.075)(0.15)}{90.0(10^{-6})}$$

$$= 1.5625(10^6)\ \text{N/m}^2 = 1.5625\ \text{MPa}$$

$(\sigma_a)_{\text{allow}} > \sigma_{\max}$

The column is adequate.

Yes. **Ans.**

Ans:
Yes.

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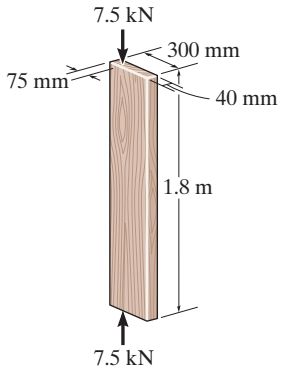
13–123. Determine if the column can support the eccentric compressive load of 7.5 kN. Assume that the bottom is fixed and the top is pinned. Use the NFPA equations in Sec. 13.6 and Eq. 13–30.



SOLUTION

$A = 0.3(0.04) = 0.012\ \text{m}^2 \qquad I_x = \frac{1}{12}(0.04)(0.3^3) = 90.0(10^{-6})\ \text{m}^4$

$d = 0.04\ \text{m}$

$$\frac{KL}{d} = \frac{0.7(1.8)}{0.04} = 31.5$$

$$26 < \frac{KL}{d} < 50$$

$$(\sigma_a)_{\text{allow}} = \frac{3725\ \text{MPa}}{(KL/d)^2} = \frac{3725\ \text{MPa}}{31.5^2} = 3.754\ \text{MPa}$$

$$\sigma_{\max} = \frac{P}{A} + \frac{M_x c}{I_x} = \frac{7.5(10^3)}{0.012} + \frac{[7.5(10^3)](0.075)(0.15)}{90.0(10^{-6})}$$

$$= 1.5625(10^6)\ \text{N/m}^2 = 1.5625\ \text{MPa}$$

$(\sigma_a)_{\text{allow}} > \sigma_{\max}$

The column is adequate.

Yes. **Ans.**

Ans:
Yes

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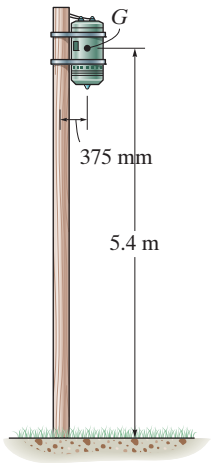
***13–124.** The 250-mm-diameter utility pole supports the transformer that has a weight of 3 kN and center of gravity at *G*. If the pole is fixed to the ground and free at its top, determine if it is adequate according to the NFPA equations of Sec. 13.6 and Eq. 13–30.



SOLUTION

$$\frac{KL}{d} = \frac{2(5.4)}{0.25} = 43.2$$

$$26 < \frac{KL}{d} < 50$$

Use Eq. 13–29,

$$\sigma_{\text{allow}} = \frac{3725\ \text{MPa}}{(KL/d)^2} = \frac{3725\ \text{MPa}}{43.2^2} = 1.9960\ \text{MPa} = 1.99\ \text{MPa}$$

$$\sigma_{\max} = \frac{P}{A} + \frac{Mc}{I}$$

$$\sigma_{\max} = \frac{3(10^3)}{\pi(0.125^2)} + \frac{[3(10^3)(0.375)](0.125)}{\frac{\pi}{4}(0.125^4)}$$

$$\sigma_{\max} = 0.7945(10^6)\ \text{N/m}^2 = 0.7945\ \text{MPa} < 1.9960\ \text{MPa}$$ O.K.

Yes. **Ans.**

Ans:
Yes

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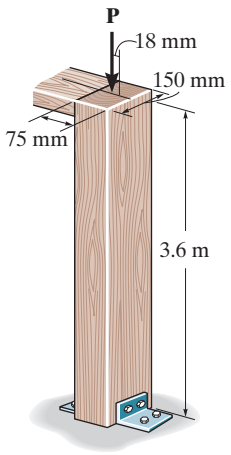
13–125. Using the NFPA equations of Sec. 13.6 and Eq. 13–30, determine the maximum allowable eccentric load P that can be applied to the wood column. Assume that the column is pinned at both its top and bottom.



SOLUTION

Section Properties:

$$A = 0.15(0.075) = 0.01125\ \text{m}^2$$

$$I_y = \frac{1}{12}(0.15)(0.075^3) = 5.27344(10^{-6})\ \text{m}^4$$

Slenderness Ratio: For a column pinned at both ends, $K = 1.0$. Thus,

$$\left(\frac{KL}{d}\right)_y = \frac{1.0(3.6)}{0.075} = 48.0$$

Allowable Stress: The allowable stress can be determined using *NFPA timber column formulas.* Since $26 < \frac{KL}{d} < 50$, it is a *long* column. Applying Eq. 13–29,

$$\sigma_{\text{allow}} = \frac{3725}{(KL/d)^2}\text{MPa}$$

$$= \frac{3725\ \text{MPa}}{48.0^2} = 1.6168\ \text{MPa}$$

Maximum Stress: Bending is about $y-y$ axis. Applying Eq. 13–30, we have

$$\sigma_{\max} = \sigma_{\text{allow}} = \frac{P}{A} + \frac{Mc}{I}$$

$$1.6168(10^6) = \frac{P}{0.01125} + \frac{[P(0.018)](0.0375)}{5.27344(10^{-6})}$$

$$P = 7.454(10^3)\ \text{N} = 7.45\ \text{kN} \qquad \textbf{Ans.}$$

Ans:
$P = 7.45$ kN

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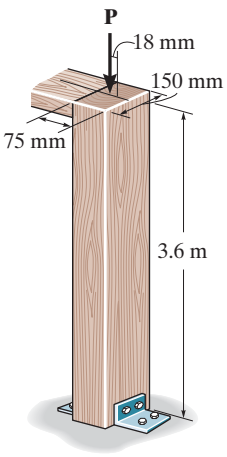
13–126. Using the NFPA equations of Sec. 13.6 and Eq. 13–30, determine the maximum allowable eccentric load P that can be applied to the wood column. Assume that the column is pinned at the top and fixed at the bottom.



SOLUTION

Section Properties:

$$A = 0.15(0.075) = 0.01125\ \text{m}^2$$

$$I_y = \frac{1}{12}(0.15)(0.075^3) = 5.27344(10^{-6})\ \text{m}^4$$

Slenderness Ratio: For a column pinned at one end and fixed at the other end, $K = 0.7$. Thus,

$$\left(\frac{KL}{d}\right)_y = \frac{0.7(3.6)}{0.075} = 33.6$$

Allowable Stress: The allowable stress can be determined using *NFPA timber column formulas*. Since $26 < \frac{KL}{d} < 50$, it is a *long* column. Applying Eq. 13–29,

$$\sigma_{\text{allow}} = \frac{3725}{(KL/d)^2}\text{MPa}$$

$$= \frac{3725\ \text{MPa}}{33.6^2} = 3.2995\ \text{MPa}$$

Maximum Stress: Bending is about $y-y$ axis. Applying Eq. 13–30, we have

$$\sigma_{\max} = \sigma_{\text{allow}} = \frac{P}{A} + \frac{Mc}{I}$$

$$3.2995(10^6) = \frac{P}{0.01125} + \frac{[P(0.018)](0.0375)}{5.27344(10^{-6})}$$

$$P = 15.21(10^3)\ \text{N} = 15.2\ \text{kN}$$ **Ans.**

Ans:
$P = 15.2$ kN

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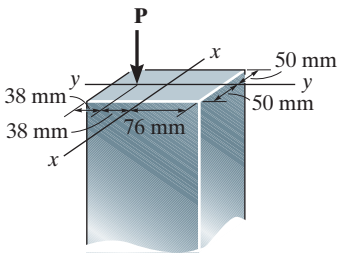
13–127. A 3-m-long bar is made of aluminum alloy 2014-T6. If it is fixed at its bottom and pinned at the top, determine the maximum allowable eccentric load **P** that can be applied using the formulas in Sec. 13.6 and Eq. 13–30.



SOLUTION

Section Properties:

$$A = 0.152(0.1) = 0.0152 \text{ m}^2$$

$$I_x = \frac{1}{12}(0.1)(0.152^3) = 29.2651(10^{-6}) \text{ m}^4$$

$$I_y = \frac{1}{12}(0.152)(0.1^3) = 12.6667(10^{-6}) \text{ m}^4$$

$$\pi_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{12.6667(10^{-6})}{0.0152}} = 0.02887 \text{ m}$$

Slenderness Ratio: The largest slenderness ratio is about $y-y$ axis. For a column pinned at one end fixed at the other end, $K = 0.7$. Thus,

$$\left(\frac{KL}{\pi}\right)_y = \frac{0.7(3)}{0.02887} = 72.74$$

Allowable Stress: The allowable stress can be determined using aluminum (2014 –T6 alloy) column formulas. Since $\frac{KL}{r} > 55$, the column is classified as a *long* column. Applying Eq. 13–26,

$$\sigma_{\text{allow}} = \left[\frac{372550}{(KL/\pi)^2}\right] \text{MPa}$$

$$= \frac{372550}{72.74^2} \text{MPa}$$

$$= 70.40 \text{ MPa}$$

Maximum Stress: Bending is about $x-x$ axis. Applying Eq. 13–30, we have

$$\sigma_{\max} = \sigma_{\text{allow}} = \frac{P}{A} + \frac{Mc}{I}$$

$$70.40(10^6) = \frac{P}{0.0152} + \frac{[P(0.038)](0.076)}{29.2651(10^{-6})}$$

$$P = 428.02(10^3) \text{ N} = 428 \text{ kN}$$ **Ans.**

Ans:
$P = 428$ kN

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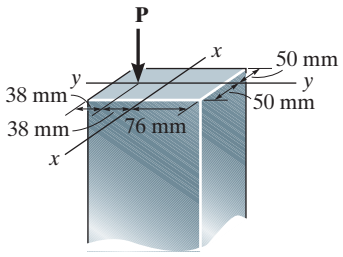
***13–128.** The 3-m-long bar is made of aluminum alloy 2014-T6. If it is fixed at its bottom and pinned at the top, determine the maximum allowable eccentric load **P** that can be applied using the equations of Sec. 13.6 and the interaction formula with $(\sigma_b)_{\text{allow}} = 126$ MPa.



SOLUTION

Section Properties:

$$A = 0.152(0.1) = 0.0152 \text{ m}^2$$

$$I_x = \frac{1}{12}(0.1)(0.152^3) = 29.2651(10^{-6}) \text{ m}^4$$

$$I_y = \frac{1}{12}(0.152)(0.1^3) = 12.6667(10^{-6}) \text{ m}^4$$

$$r_x = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{29.2651(10^{-6})}{0.0152}} = 0.04388 \text{ m}$$

$$r_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{12.6667(10^{-6})}{0.0152}} = 0.02887 \text{ m}$$

Slenderness Ratio: The largest slenderness radio is about $y-y$ axis. For a column pinned at one end and fixed at the other end, $K = 0.7$. Thus

$$\left(\frac{KL}{r}\right)_y = \frac{0.7(3)}{0.02887} = 72.74$$

Allowable Stress: The allowable stress can be determined using *aluminum (2014 –T6 alloy) column formulas.* Since $\frac{KL}{r} > 55$, the column is classified as a *long* column. Applying Eq. 13–26,

$$\sigma_{\text{allow}} = \left[\frac{372550}{(KL/r)^2}\right] \text{MPa}$$

$$= \frac{372550}{72.74^2} \text{MPa}$$

$$= 70.40 \text{ MPa}$$

Interaction Formula: Bending is about $x-x$ axis. Applying Eq. 13–31, we have

$$\frac{P/A}{(\sigma_a)_{\text{allow}}} + \frac{Mc/Ar^2}{(\sigma_b)_{\text{allow}}} = 1$$

$$\frac{P/0.0152}{70.40(10^6)} + \frac{[P(0.038)](0.076)/0.0152(0.04388^2)}{126(10^6)} = 1$$

$$P = 582.16(10^3) \text{ N} = 582 \text{ kN} \qquad \textbf{Ans.}$$

Ans:
$P = 582$ kN

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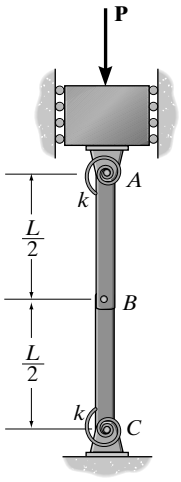
R13–1. If the torsional springs attached to ends A and C of the rigid members AB and BC have a stiffness k, determine the critical load P_{cr}.



SOLUTION

Equilibrium. When the system is given a slight lateral disturbance, the configuration shown in Fig. *a* is formed. The couple moment M can be related to P by considering the equilibrium of members AB and BC.

Member AB

$$+\uparrow \Sigma F_y = 0; \qquad B_y - P = 0 \tag{1}$$

$$\circlearrowleft +\Sigma M_A = 0; \qquad B_y\left(\frac{L}{2}\sin\theta\right) + B_x\left(\frac{L}{2}\cos\theta\right) - M = 0 \tag{2}$$

Member BC

$$\circlearrowleft +\Sigma M_C = 0; \; -B_y\left(\frac{L}{2}\sin\theta\right) + B_x\left(\frac{L}{2}\cos\theta\right) + M = 0 \tag{3}$$

Solving Eqs. (1), (2), and (3), we obtain

$$B_x = 0 \qquad B_y = \frac{2M}{L\sin\theta} \qquad M = \frac{PL}{2}\sin\theta$$

Since θ is very small, the small angle analysis gives $\sin\theta \cong \theta$. Thus,

$$M = \frac{PL}{2}\theta \tag{4}$$

Torslonal Spring Moment. The restoring couple moment M_{sp} can be determined using the torsional spring formula, $M = k\theta$. Thus,

$$M_{sp} = k\theta$$

Critical Buckling Load. When the mechanism is on the verge of bucklling M must equal M_{sp}.

$$M = M_{sp}$$

$$\frac{P_{cr}\,L}{2}\theta = k\theta$$

$$P_{cr} = \frac{2k}{L} \qquad \textbf{Ans.}$$

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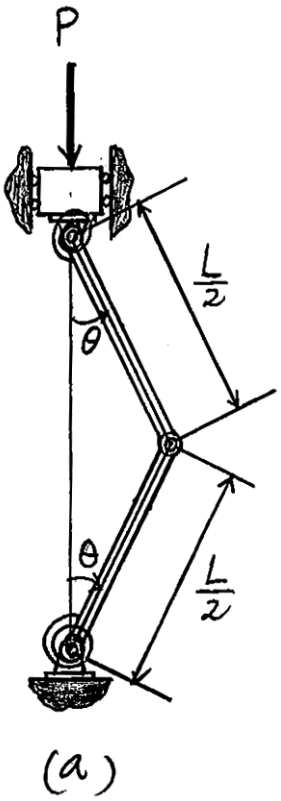
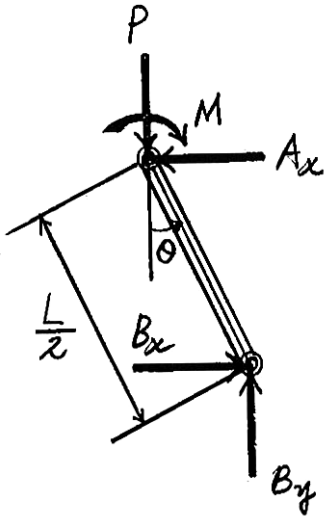
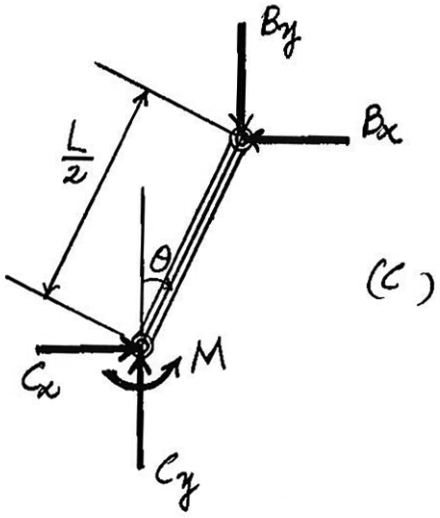
R13–1. Continued



(a)



(b)



(c)

Ans:

$B_x = 0, B_y = \dfrac{2M}{L \sin\theta}, M = \dfrac{PL}{2} \sin\theta,$

$M = \dfrac{PL}{2}\theta, P_{\text{cr}} = \dfrac{2k}{L}$

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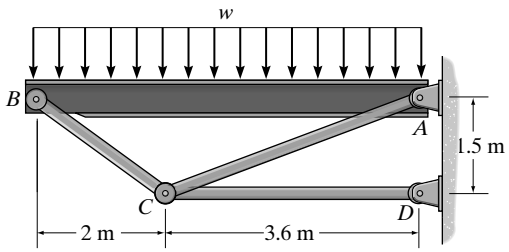
R13–2. Determine the maximum intensity w of the uniform distributed load that can be applied on the beam without causing the compressive members of the supporting truss to buckle. The members of the truss are made from A-36-steel rods having a 60-mm diameter. Use F.S. = 2 against buckling.



SOLUTION

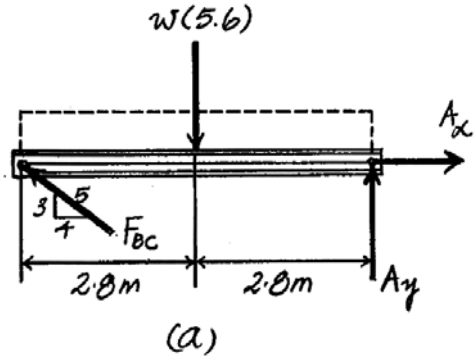
Equilibrium. The force developed in member BC can be determined by considering the equilibrium of the free-body diagram of the beam AB, Fig. *a*.

$$\zeta + \Sigma M_A = 0; \qquad w(5.6)(2.8) - F_{BC}\left(\frac{3}{5}\right)(5.6) = 0 \quad F_{BC} = 4.6667w$$

The Force developed in member CD can be obtained by analyzing the equilibrium of joint C, Fig. *b*,

$$+\uparrow \Sigma F_y = 0; \qquad F_{AC}\left(\frac{5}{13}\right) - 4.6667w\left(\frac{3}{5}\right) = 0 \qquad F_{AC} = 7.28w \text{ (T)}$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 4.6667w\left(\frac{4}{5}\right) + 7.28\left(\frac{12}{13}\right)w - F_{CD} = 0 \qquad F_{CD} = 10.4533w \text{ (C)}$$



Section Properties. The cross-sectional area and moment of inertia of the solid circular rod CD are

$$A = \pi\left(0.03^2\right) = 0.9\left(10^{-3}\right)\pi \text{ m}^2 \qquad I = \frac{\pi}{4}\left(0.03^4\right) = 0.2025\left(10^{-6}\right)\pi \text{ m}^4$$

Critical Buckling Load. Since both ends of member CD are pinned, $K = 1$. The critical buckling load is

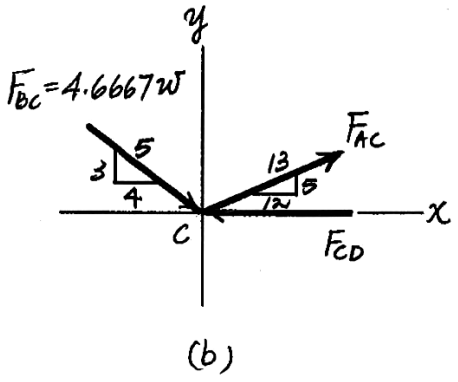
$$P_{cr} = F_{CD}\,(\text{F.S.}) = 10.4533w(2) = 20.9067w$$

Applying Euler's formula,

$$P_{\text{cr}} = \frac{\pi^2 EI}{(KL)^2}$$

$$20.9067w = \frac{\pi^2\left[200\left(10^9\right)\right]\left[0.2025\left(10^{-6}\right)\pi\right]}{[1(3.6)]^2}$$

$$w = 4634.63 \text{ N/m} = 4.63 \text{ kN/m} \qquad \textbf{Ans.}$$



Critical Stress: Euler's formula is valid only if $\sigma_{\text{cr}} < \sigma_Y$.

$$\sigma_{\text{cr}} = \frac{P_{\text{cr}}}{A} = \frac{20.907(4634.63)}{0.9\left(10^{-3}\right)\pi} = 34.27 \text{ MPa} < \sigma_Y = 250 \text{ MPa} \qquad \text{O.K.}$$

Ans:
$w = 4.63$ kN/m

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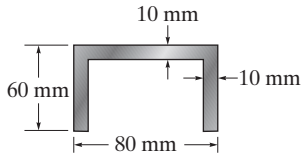
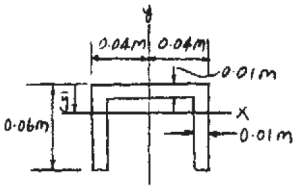
R13–3.

A steel column has a length of 5 m and is free at one end and fixed at the other end. If the cross-sectional area has the dimensions shown, determine the critical load. $E_{st} = 200$ GPa, $\sigma_Y = 360$ MPa.



SOLUTION

Section Properties:



$$A = 0.06\,(0.01) + 2\,(0.06)(0.01) = 1.80(10^{-3})\ \text{m}^2$$

$$\bar{y} = \frac{\Sigma \bar{y} A}{\Sigma A} = \frac{0.005\,(0.06)(0.01) + 2[0.03\,(0.06)(0.01)]}{0.06\,(0.01) + 2\,(0.06)(0.01)} = 0.02167\ \text{m}$$

$$I_x = \frac{1}{12}(0.06)(0.01)^3 + 0.06\,(0.01)(0.02167 - 0.005)^2$$

$$+ \left[\frac{1}{12}(0.01)(0.06)^3 + 0.01\,(0.06)(0.03 - 0.02167)^2\right] = 0.615\,(10^{-6})\ \text{m}^4 \quad \text{(controls)}$$

$$I_y = \frac{1}{12}(0.06)(0.08)^3 - \frac{1}{12}(0.05)(0.06)^3 = 1.66\,(10^{-6})\ \text{m}^4$$

Critical Load:

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}; \quad K = 2.0$$

$$= \frac{\pi^2\,(200)(10^9)(0.615)(10^{-6})}{[2.0\,(5)]^2}$$

$$= 12140\ \text{N} = 12.1\ \text{kN} \qquad \textbf{Ans.}$$

Check Stress:

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{12140}{1.80\,(10^{-3})} = 6.74\ \text{MPa} < \sigma_Y = 360\ \text{MPa}$$

Hence, Euler's equation is still valid.

Ans:
$P_{cr} = 12.1$ kN

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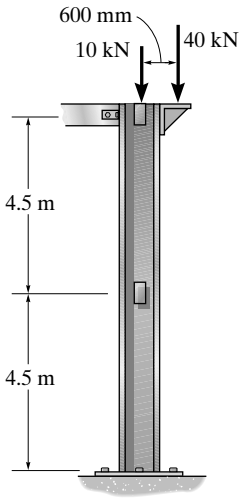
***R13–4.** The A-36-steel column can be considered pinned at its top and fixed at its base. Also, it is braced at its mid-height along the weak axis. Investigate whether a W250 × 45 section can safely support the loading shown. Use the interaction formula. The allowable bending stress is $(\sigma_b)_{\text{allow}} = 100$ MPa.



SOLUTION

Section Properties. From the table listed in the appendix, the necessary section properties for a W250 × 45 are

$A = 5700\ \text{mm}^2 = 5.70\left(10^{-3}\right)\text{m}^2 \qquad d = 266\ \text{mm} = 0.266\ \text{m}$

$I_x = 71.1\left(10^6\right)\text{mm}^4 = 71.1\left(10^{-6}\right)\text{m}^4 \qquad r_x = 112\ \text{mm} = 0.112\ \text{m} \qquad r_y = 35.1\ \text{mm} = 0.0351\ \text{mm}$

Slenderness Ratio. Here, $L_x = 9$ m and for a column fixed at its base and pinned at its top, $K_x = 0.7$. Thus,

$$\left(\frac{KL}{r}\right)_x = \frac{0.7(9)}{0.112} = 56.25$$

Since the bracing provides support equivalent to a pin, $K_y = 1$ and $L_y = 4..5$ m. Then,

$$\left(\frac{KL}{r}\right)_y = \frac{1(4.5)}{0.0351} = 128.21\ \text{(controls)}$$

Allowable Axial Stress. For A–36 steel, $\left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}}$ $= \sqrt{\frac{2\pi^2\left[200\left(10^9\right)\right]}{250\left(10^6\right)}} = 125.66$. Since $\left(\frac{KL}{r}\right)_c < \left(\frac{KL}{r}\right)_y < 200$, the column can be classified as a long column.

$$\sigma_{\text{allow}} = \frac{12\pi^2 E}{23(KL/r)^2} = \frac{12\pi^2\left[200\left(10^9\right)\right]}{23(128.21)^2} = 62.657\ \text{MPa}$$

Interaction Formula. Bending is about the strong axis. Here, $P = 10 + 40 = 50$ kN, $M = 40(0.6) = 24\ \text{kN}\cdot\text{m}$ and $c = \frac{d}{2} = \frac{0.266}{2} = 0.133$ m,

$$\frac{P/A}{(\sigma_a)_{\text{allow}}} + \frac{Mc/Ar^2}{(\sigma_b)_{\text{allow}}} = \frac{50\left(10^3\right)\Big/5.70\left(10^{-3}\right)}{62.657\left(10^6\right)} + \frac{24\left(10^3\right)(0.133)\Big/\left[5.70\left(10^{-3}\right)\left(0.112^2\right)\right]}{100\left(10^6\right)}$$

$$= 0.5864 < 1 \qquad \text{O.K.}$$

$$\frac{\sigma_a}{(\sigma_a)_{\text{allow}}} = \frac{50\left(10^3\right)\Big/5.7\left(10^{-3}\right)}{62.657\left(10^6\right)} = 0.140 < 0.15 \qquad \text{O.K.}$$

Thus, a W250 × 45 column is *adequate* according to the interaction formula.

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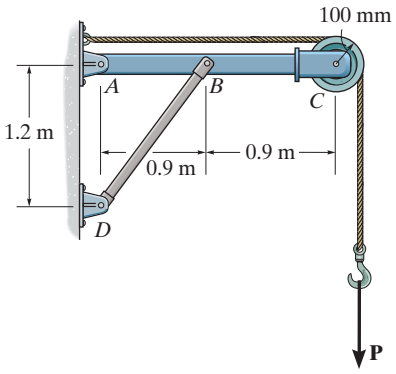
R13–5. If the A-36 steel solid circular rod *BD* has a diameter of 50 mm, determine the allowable maximum force **P** that can be supported by the frame without causing the rod to buckle. Use F.S. = 2 against buckling.



SOLUTION

Equilibrium. The compressive force developed in *BD* can be determined by considering the equilibrium of the free-body diagram of member *ABC*, Fig. *a*.

$$\zeta + \Sigma M_A = 0; \qquad F_{BD}\left(\frac{4}{5}\right)(0.9) - P(1.9) + P(0.1) = 0 \qquad F_{BD} = 2.5P$$

Section Properties. The cross-sectional area and moment of inertia of *BD* are

$$A = \pi(0.025^2) = 0.625(10^{-3})\pi \text{ m}^2 \qquad I = \frac{\pi}{4}(0.025^4) = 97.65625(10^{-9})\pi \text{ m}^4$$

Critical Buckling Load. Since *BD* is pinned at both of its ends, $K = 1$. The critical buckling load is

$$P_{\text{cr}} = F_{BD}(\text{F.S.}) = 2.5P(2) = 5P$$

The length of *BD* is $L = \sqrt{0.9^2 + 1.2^2} = 1.5$ m. Applying Euler's for,mula

$$P_{\text{cr}} = \frac{\pi^2 EI}{(KL)^2}$$

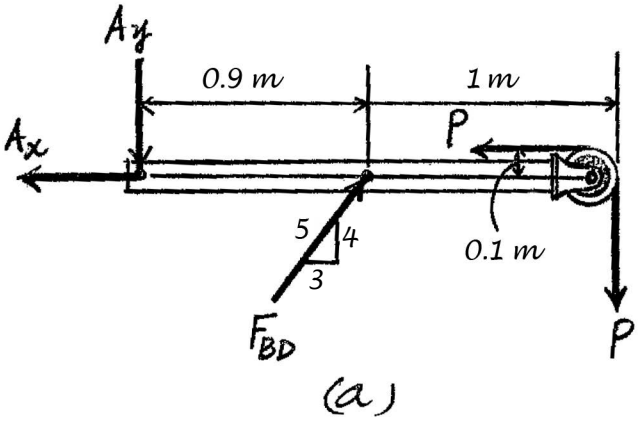
$$5P = \frac{\pi^2[200(10^9)][97.65625(10^{-9})\pi]}{[1.0(1.5)]^2}$$

$$P = 53.83(10^3) \text{ N} = 53.8 \text{ kN} \qquad \textbf{Ans.}$$

Critical Stress. Euler's formula is valid only if $\sigma_{\text{cr}} < \sigma_Y$.

$$\sigma_{\text{cr}} = \frac{P_{\text{cr}}}{A} = \frac{5[53.83(10^3)]}{0.625(10^{-3})\pi} = 137.08(10^6) \text{ N/m}^2$$

$$= 137.08 \text{ MPa} < \sigma_Y = 250 \text{ MPa} \qquad \text{(O.K.)}$$



Ans:
$P = 53.8$ kN

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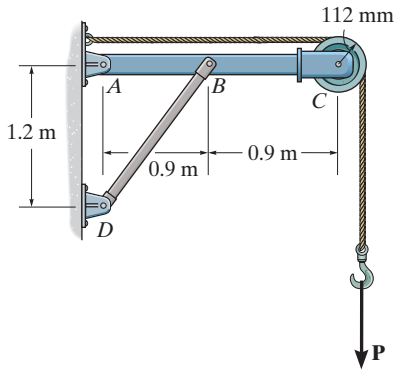
R13–6. If $P = 75$ kN, determine the required minimum diameter of the A992 steel solid circular rod BD to the nearest mm. Use F.S. = 2 against buckling.



SOLUTION

Equilibrium. The compressive force developed in BD can be determined by considering the equilibrium of the free-body diagram of member ABC, Fig. a,

$$\zeta + \Sigma M_A = 0; \quad F_{BD}\left(\frac{4}{5}\right)(0.9) - 75(1.9) + 75(0.1) = 0 \qquad F_{BC} = 187.5 \text{ kN}$$

Section Properties. The cross-sectional area and moment of inertia of BD are

$$A = \frac{\pi}{4}d^2 \qquad I = \frac{\pi}{4}\left(\frac{d}{2}\right)^4 = \frac{\pi}{64}d^4$$

Critical Buckling Load. Since BD is pinned at both of its ends, $K = 1$. The critical buckling load is

$$P_{\text{cr}} = F_{BD}(\text{F.S.}) = 187.5(2) = 375 \text{ kN}$$

The length of BD is $L = \sqrt{0.9^2 + 1.2^2} = 2.5$ m. Applying Euler's formula,

$$P_{\text{cr}} = \frac{\pi^2 EI}{(KL)^2}$$

$$375(10^3) = \frac{\pi^2[200(10^9)]\left(\frac{\pi}{64}d^4\right)}{[1.0(1.5)]^2}$$

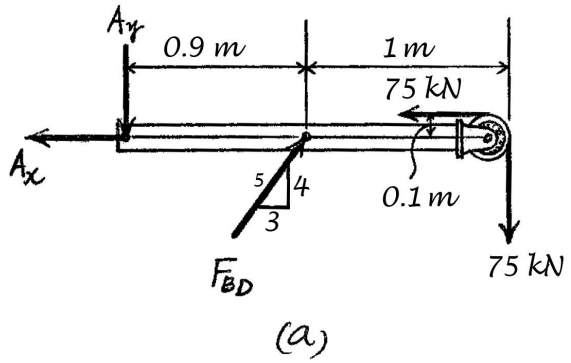
$$d = 0.05432 \text{ m} = 54.32 \text{ mm}$$

Use $d = 55$ mm **Ans.**

Critical Stress. Euler's formula is valid only if $\sigma_{cr} < \sigma_Y$.

$$\sigma_{\text{cr}} = \frac{P_{\text{cr}}}{A} = \frac{375(10^3)}{\frac{\pi}{4}(0.055^2)} = 157.84(10^6) \text{ N/m}^2$$

$$= 157.85 \text{ MPa} < \sigma_Y = 345 \text{ MPa} \qquad \text{(O.K.)}$$



(a)

Ans:

Use $d = 55$ mm

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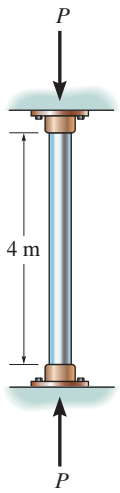
R13–7.

The steel pipe is fixed supported at its ends. If it is 4 m long and has an outer diameter of 50 mm, determine its required thickness so that it can support an axial load of $P = 100$ kN without buckling. $E_{st} = 200$ GPa, $\sigma_Y = 250$ MPa.



SOLUTION

$$I = \frac{\pi}{4}(0.025^4 - r_i{}^4)$$

Critical Load:

$$P_{cr} = \frac{\pi^2 EI}{(K\,L)^2}; \qquad K = 0.5$$

$$100(10^3) = \frac{\pi^2(200)(10^9)[\frac{\pi}{4}(0.025^4 - r_i{}^4)]}{[0.5(4)]^2}$$

$$r_i = 0.01908\text{ m} = 19.1\text{ mm}$$

$$t = 25\text{ mm} - 19.1\text{ mm} = 5.92\text{ mm}$$ **Ans.**

Check Stress:

$$\sigma = \frac{P_{cr}}{A} = \frac{100(10^3)}{\pi(0.025^2 - 0.0191^2)} = 122\text{ MPa} < \sigma_Y = 345\text{ MPa}$$ **(OK)**

Ans:
$t = 5.92$ mm

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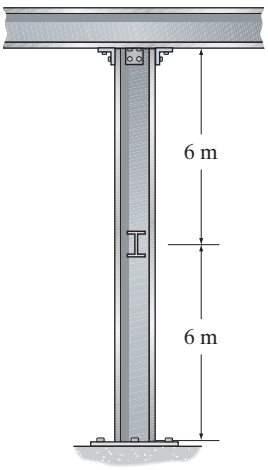
***R13–8.**

The W200 × 46 wide-flange A992-steel column can be considered pinned at its top and fixed at its base. Also, the column is braced at its mid-height against weak axis buckling. Determine the maximum axial load the column can support without causing it to buckle.



SOLUTION

Section Properties: From the table listed in the appendix, the section properties for a W200 × 46 are

$A = 5890\ \text{mm}^2 = 5.89\left(10^{-3}\right)\ \text{m}^2 \qquad I_x = 45.5\left(10^{6}\right)\ \text{mm}^4 = 45.5\left(10^{-6}\right)\ \text{m}^4$

$I_y = 15.3\left(10^{6}\right)\ \text{mm}^4 = 15.3\left(10^{-6}\right)\ \text{m}^4$

Critical Buckling Load: For buckling about the strong axis, $K_x = 0.7$ and $L_x = 12$ m. Since the column is fixed at its base and pinned at its top,

$$P_{\text{cr}} = \frac{\pi^2 EI_x}{(KL)_x{}^2} = \frac{\pi^2\left[200\left(10^{9}\right)\right]\left[45.5\left(10^{-6}\right)\right]}{[0.7(12)]^2} = 1.273\left(10^{6}\right)\text{N} = 1.27\ \text{MN}$$

For buckling about the weak axis, $K_y = 1$ and $L_y = 6$ m since the bracing provides a support equivalent to a pin. Applying Euler's formula,

$$P_{\text{cr}} = \frac{\pi^2 EI_y}{(KL)_y{}^2} = \frac{\pi^2\left[200\left(10^{9}\right)\right]\left[15.3\left(10^{-6}\right)\right]}{[1(6)]^2} = 838.92\ \text{kN} = 839\ \text{kN (controls)}$$

Ans.

Critical Stress: Euler's formula is valid only if $\sigma_{\text{cr}} < \sigma_Y$.

$$\sigma_{\text{cr}} = \frac{P_{\text{cr}}}{A} = \frac{838.92\left(10^{3}\right)}{5.89\left(10^{-3}\right)} = 142.43\ \text{MPa} < \sigma_Y = 345\ \text{MPa} \qquad \textbf{(O.K.)}$$

Ans:
$P_{\text{cr}} = 839$ kN

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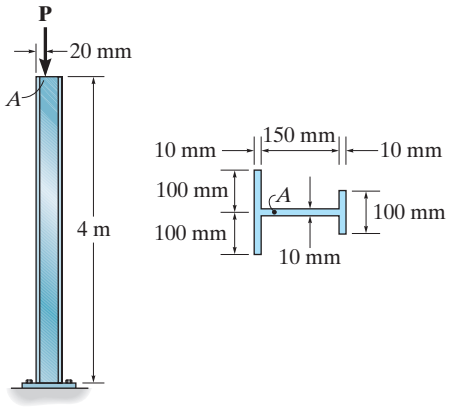
R13–9.

The wide-flange A992 steel column has the cross section shown. If it is fixed at the bottom and free at the top, determine the maximum force P that can be applied at A without causing it to buckle or yield. Use a factor of safety of 3 with respect to buckling and yielding.



SOLUTION

Section Properties:

$$\Sigma A = 0.2(0.01) + 0.15\,(0.01) + 0.1(0.01) = 4.5(10^{-3})\ \text{m}^2$$

$$\bar{x} = \frac{\Sigma \tilde{x} A}{\Sigma A} = \frac{0.005\,(0.2)(0.01) + 0.085\,(0.15)(0.01) + 0.165\,(0.1)(0.01)}{4.5(10^{-3})}$$

$$= 0.06722\ \text{m}$$

$$I_y = \frac{1}{12}(0.2)(0.01^3) + 0.2\,(0.01)(0.06722 - 0.005)^2$$

$$+ \frac{1}{12}(0.01)(0.15^3) + 0.01\,(0.15)(0.085 - 0.06722)^2$$

$$+ \frac{1}{12}(0.1)(0.01^3) + 0.1\,(0.01)(0.165 - 0.06722)^2$$

$$= 20.615278\,(10^{-6})\ \text{m}^4$$

$$I_x = \frac{1}{12}(0.01)(0.2^3) + \frac{1}{12}(0.15)(0.01^3) + \frac{1}{12}(0.01)(0.1^3)$$

$$= 7.5125\,(10^{-6})\ \text{m}^4$$

$$r_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{20.615278(10^{-6})}{4.5\,(10^{-3})}} = 0.0676844$$

Buckling about *x–x* axis:

$$P_{\text{cr}} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2(200)(10^9)(7.5125)(10^{-6})}{[2.0(4)]^2}$$

$$= 231.70\ \text{kN} \qquad \text{(controls)}$$

$$\sigma_{\text{cr}} = \frac{P_{\text{cr}}}{A} = \frac{231.7\,(10^3)}{4.5\,(10^{-3})} = 51.5\ \text{MPa} < \sigma_\gamma = 345\ \text{MPa}$$

Yielding about *y–y* axis:

$$\sigma_{\max} = \frac{P}{A}\left[1 + \frac{e\,c}{r^2}\sec\left(\frac{KL}{2r}\sqrt{\frac{P}{EA}}\right)\right]; \qquad e = 0.06722 - 0.02 = 0.04722\ \text{m}$$

$$\frac{e\,c}{r^2} = \frac{0.04722\,(0.06722)}{0.0676844^2} = 0.692919$$

$$\frac{KL}{2r}\sqrt{\frac{P}{EA}} = \frac{2.0\,(4)}{2(0.0676844)}\sqrt{\frac{P}{200\,(10^9)(4.5)(10^{-3})}} = 1.96992\,(10^{-3})\sqrt{P}$$

$$345(10^6)(4.5)(10^{-3}) = P[1 + 0.692919\sec(1.96992\,(10^{-3})\sqrt{P})]$$

By trial and error:

$$P = 434.342\ \text{kN}$$

Hence,

$$P_{\text{allow}} = \frac{231.70}{3} = 77.2\ \text{kN} \qquad \textbf{Ans.}$$

Ans:
$P_{\text{allow}} = 77.2$ kN

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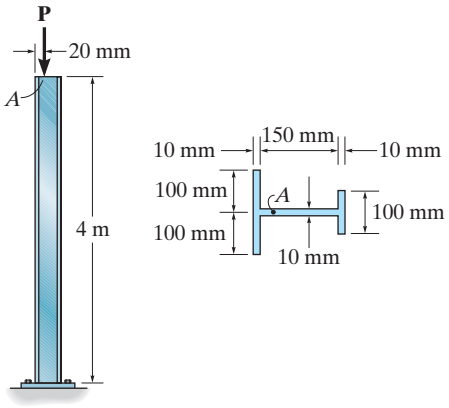
R13–10.

The wide-flange A992 steel column has the cross section shown. If it is fixed at the bottom and free at the top, determine if the column will buckle or yield when the load $P = 10$ kN is applied at A. Use a factor of safety of 3 with respect to buckling and yielding.



SOLUTION

Section Properties:

$$\Sigma A = 0.2\,(0.01) + 0.15\,(0.01) + 0.1\,(0.01) = 4.5\,(10^{-3})\ \text{m}^2$$

$$\bar{x} = \frac{\Sigma \tilde{x} A}{\Sigma A} = \frac{0.005\,(0.2)(0.01) + 0.085\,(0.15)(0.01) + 0.165\,(0.1)(0.01)}{4.5\,(10^{-3})} = 0.06722\ \text{m}$$

$$I_y = \frac{1}{12}(0.2)(0.01^3) + 0.2\,(0.01)(0.06722 - 0.005)^2$$

$$+ \frac{1}{12}(0.01)(0.15^3) + 0.01\,(0.15)(0.085 - 0.06722)^2$$

$$+ \frac{1}{12}(0.1)(0.01^3) + 0.1\,(0.01)(0.165 - 0.06722)^2 = 20.615278\,(10^{-6})\ \text{m}^4$$

$$I_x = \frac{1}{12}(0.01)(0.2^3) + \frac{1}{12}(0.15)(0.01^3) + \frac{1}{12}(0.01)(0.1^3) = 7.5125\,(10^{-6})\ \text{m}^4$$

$$r_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{20.615278\,(10^{-6})}{4.5\,(10^{-3})}} = 0.0676844\ \text{m}$$

Buckling about *x–x* axis:

$$P_{\text{cr}} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2(200)(10^9)(7.5125)(10^{-6})}{[2.0(4)]^2} = 231.70\ \text{kN}$$

$$\sigma_{\text{cr}} = \frac{P_{\text{cr}}}{A} = \frac{231.7\,(10^3)}{4.5\,(10^{-3})} = 51.5\ \text{MPa} < \sigma_\gamma = 345\ \text{MPa} \qquad \textbf{(O.K.)}$$

$$P_{\text{allow}} = \frac{P_{\text{cr}}}{\text{FS}} = \frac{231.7}{3} = 77.2\ \text{kN} > P = 10\ \text{kN}$$

Hence the column does not buckle.

Yielding about *y–y* axis:

$$\sigma_{\max} = \frac{P}{A}\left[1 + \frac{e\,c}{r^2}\sec\left(\frac{KL}{2r}\sqrt{\frac{P}{EA}}\right)\right] \qquad e = 0.06722 - 0.02 = 0.04722\ \text{m}$$

$$P = \frac{10}{3} = 3.333\ \text{kN}$$

$$\frac{P}{A} = \frac{3.333\,(10^3)}{4.5\,(10^{-3})} = 0.7407\ \text{MPa}$$

$$\frac{e\,c}{r^2} = \frac{0.04722\,(0.06722)}{(0.067844)^2} = 0.692919$$

$$\frac{K\,L}{2\,r}\sqrt{\frac{P}{E\,A}} = \frac{2.0\,(4)}{2(0.0676844)}\sqrt{\frac{3.333\,(10^3)}{200\,(10^9)(4.5)(10^{-3})}} = 0.113734$$

$$\sigma_{\max} = 0.7407\,[1 + 0.692919\sec(0.113734)] = 1.26\ \text{MPa} < \sigma_\gamma = 345\ \text{MPa}$$

Hence the column does not yield!

No. **Ans.**

Ans:
It does not buckle or yield

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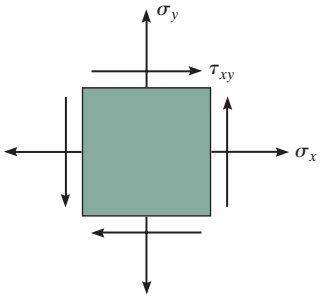
14–1.

A material is subjected to a general state of plane stress. Express the strain energy density in terms of the elastic constants E, G, and ν and the stress components σ_x, σ_y, and τ_{xy}.



SOLUTION

Strain Energy Due to Normal Stresses: We will consider the application of normal stresses on the element in two successive stages. For the first stage, we apply only σ_x on the element. Since σ_x is a constant,

$$(U_i)_1 = \int_\nu \frac{\sigma_x^2}{2E} dV = \frac{\sigma_x^2 V}{2E}$$

When σ_y is applied in the second stage, the normal strain ϵ_x will be strained by $\epsilon_x' = -\nu\epsilon_y = -\frac{v\sigma_y}{E}$. Therefore, the strain energy for the second stage is

$$(U_i)_2 = \int_\nu \left(\frac{\sigma_y^2}{2E} + \sigma_x \epsilon_x'\right) dV$$

$$= \int_\nu \left[\frac{\sigma_y^2}{2E} + \sigma_x\left(-\frac{\nu\sigma_y}{E}\right)\right] dV$$

Since σ_x and σ_y are constants,

$$(U_i)_2 = \frac{V}{2E}(\sigma_y^2 - 2\nu\sigma_x\sigma_y)$$

Strain Energy Due to Shear Stresses: The application of τ_{xy} does not strain the element in a normal direction. Thus, from Eq. 14–11, we have

$$(U_i)_3 = \int_\nu \frac{\tau_{xy}^2}{2G} dV = \frac{\tau_{xy}^2 V}{2G}$$

The total strain energy is

$$U_i = (U_i)_1 + (U_i)_2 + (U_i)_3$$

$$= \frac{\sigma_x^2 V}{2E} + \frac{V}{2E}(\sigma_y^2 - 2\nu\sigma_x\sigma_y) + \frac{\tau_{xy}^2 V}{2G}$$

$$= \frac{V}{2E}(\sigma_x^2 + \sigma_y^2 - 2\nu\sigma_x\sigma_y) + \frac{\tau_{xy}^2 V}{2G}$$

and the strain energy density is

$$\frac{U_i}{V} = \frac{1}{2E}(\sigma_x^2 + \sigma_y^2 - 2\nu\sigma_x\sigma_y) + \frac{\tau_{xy}^2}{2G}$$ **Ans.**

These solutions represent a preliminary version of the Instructors' Solutions Manual (ISM). It is possible and even likely that at this preliminary stage of preparing the ISM there are some omissions and errors in the draft solutions. These will be corrected and this manual will be republished.

Ans:

$$\frac{U_i}{V} = \frac{1}{2E}(\sigma_x^2 + \sigma_y^2 - 2\nu\sigma_x\sigma_y) + \frac{\tau_{xy}^2}{2G}$$

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14–2.

The strain-energy density for plane stress must be the same whether the state of stress is represented by σ_x, σ_y, and τ_{xy}, or by the principal stresses σ_1 and σ_2. This being the case, equate the strain–energy expressions for each of these two cases and show that $G = E/[2(1 + \nu)]$.

SOLUTION

$$U = \int_v \left[\frac{1}{2E}(\sigma_x^2 + \sigma_y^2) - \frac{\nu}{E}\sigma_x\sigma_y + \frac{1}{2G}\tau_{xy}^2 \right] dV$$

$$U = \int_v \left[\frac{1}{2E}(\sigma_1^2 + \sigma_2^2) - \frac{\nu}{E}\sigma_1\sigma_2 \right] dV$$

Equating the above two equations yields.

$$\frac{1}{2E}(\sigma_x^2 + \sigma_y^2) - \frac{\nu}{E}\sigma_x\sigma_y + \frac{1}{2G}\tau_{xy}^2 = \frac{1}{2E}(\sigma_1^2 + \sigma_2^2) - \frac{\nu}{E}\sigma_1\sigma_2 \qquad \textbf{(1)}$$

However, $\sigma_{1,2} = \dfrac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\dfrac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

Thus, $\left(\sigma_1^2 + \sigma_2^2\right) = \sigma_x^2 + \sigma_y^2 + 2\tau_{xy}^2$

and also $\sigma_1\sigma_2 = \sigma_x\sigma_y - \tau_{xy}^2$

Substitute into Eq. (1)

$$\frac{1}{2E}\left(\sigma_x^2 + \sigma_y^2\right) - \frac{\nu}{E}\sigma_x\sigma_y + \frac{1}{2G}\tau_{xy}^2 = \frac{1}{2E}(\sigma_x^2 + \sigma_y^2 + 2\tau_{xy}^2) - \frac{\nu}{E}\sigma_x\sigma_y + \frac{\nu}{E}\tau_{xy}^2$$

$$\frac{1}{2G}\tau_{xy}^2 = \frac{\tau_{xy}^2}{E} + \frac{\nu}{E}\tau_{xy}^2$$

$$\frac{1}{2G} = \frac{1}{E} + \frac{\nu}{E}$$

$$\frac{1}{2G} = \frac{1}{E}(1 + \nu)$$

$$G = \frac{E}{2(1 + \nu)} \qquad \textbf{QED}$$

Ans:
N/A

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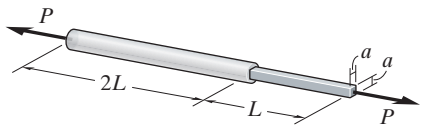
14–3.

The A-36 steel bar consists of two segments, one of circular cross section of radius r, and one of square cross section. If the bar is subjected to the axial loading of P, determine the dimensions a of the square segment so that the strain energy within the square segment is the same as in the circular segment.



SOLUTION

Axial Strain Energy: Applying Eq. 14–16 to the circular segment gives

$$(U_i)_c = \frac{N^2 L_c}{2AE} = \frac{P^2(2L)}{2(\pi r^2)E} = \frac{P^2 L}{\pi r^2 E}$$

Applying Eq. 14–16 to the square segment gives

$$(U_i)_s = \frac{N^2 L_s}{2AE} = \frac{P^2 L}{2(a^2)E} = \frac{P^2 L}{2a^2 E}$$

Require

$$(U_i)_c = (U_i)_s$$

$$\frac{P^2 L}{\pi r^2 E} = \frac{P^2 L}{2a^2 E}$$

$$a = \sqrt{\frac{\pi}{2}}\, r \qquad \textbf{Ans.}$$

Ans:

$$a = \sqrt{\frac{\pi}{2}}\, r$$

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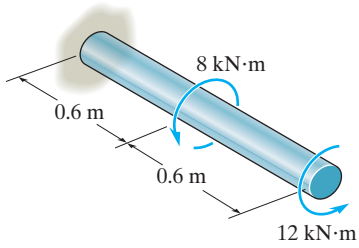
***14–4.**

Determine the torsional strain energy in the A992 steel shaft. The shaft has a radius of 50 mm.



SOLUTION

Internal Torque: As shown on the FBD in Fig. *a* and *b*.

Torsional Strain Energy: With polar moment of inertia $J = \frac{\pi}{2}(0.05^4) = 3.125(10^{-6})\pi \text{ m}^4$ and $G = 75$ GPa for A992 steel,

$$U_i = \Sigma \frac{T^2 L}{2GJ}$$

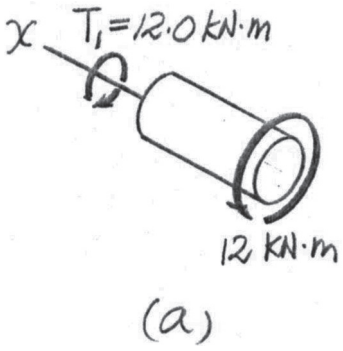
$$= \frac{1}{2GJ}\left\{\left[12.0(10^3)\right]^2(0.6) + \left[20.0(10^3)\right]^2(0.6)\right\}$$

$$= \frac{0.1632(10^9)\ \text{N}^2 \cdot \text{m}^3}{GJ}$$

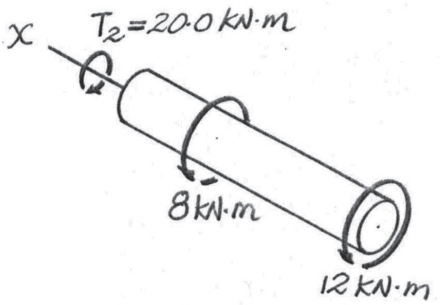
$$= \frac{0.1632(10^9)}{75(10^9)\left[3.125(10^{-6})\pi\right]}$$

$$= 221.65 \text{ J}$$

$$= 222 \text{ J}$$ **Ans.**



(a)



(b)

Ans:
$U_i = 222$ J

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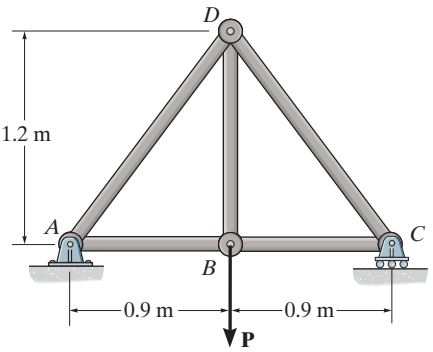
14–5. If $P = 50$ kN, determine the total strain energy stored in the truss. Each member has a diameter of 50 mm and is made of A992 steel.



SOLUTION

Normal Forces. The normal forces developed in each member of the truss can be determined using the method of joints.

Joint *B* (Fig. *a*)

$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad F_{BC} - F_{AB} = 0 \quad (1)$

$+\uparrow \Sigma F_y = 0; \quad F_{BD} - 50 = 0 \quad F_{BD} = 50 \text{ kN (T) (max)}$

Joint *D* (Fig. *b*)

$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad F_{AD} = \left(\frac{3}{5}\right) - F_{CD}\left(\frac{3}{5}\right) = 0 \quad F_{AD} = F_{CD} = F$

$+\uparrow \Sigma F_y = 0; \quad 2\left[F\left(\frac{4}{5}\right)\right] - 50 = 0 \quad F_{AD} = F_{CD} = F = 31.25 \text{ kN (C)}$

Joint *C* (Fig. *c*)

$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad 31.25\left(\frac{3}{5}\right) - F_{BC} = 0 \quad F_{BC} = 18.75 \text{ kN (T)}$

Using the result of $\mathbf{F}_{BC}$, Eq. (1) gives

$F_{AB} = 18.75 \text{ kN (T)}$

Axial Strain Energy. $A = \frac{\pi}{4}(0.05^2) = 0.625(10^{-3})\pi \text{m}^2$ and $L_{CD} = \sqrt{0.9^2 + 1.2^2} = 1.5$ m

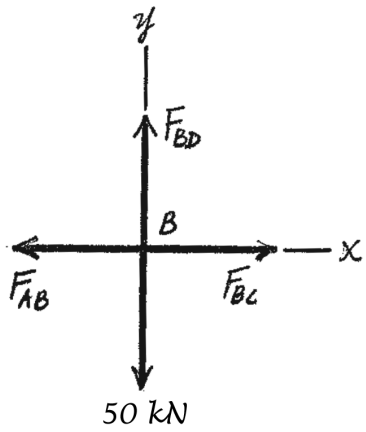
$$(U_i)_a = \Sigma \frac{N^2 L}{2AE}$$

$$= \frac{1}{2[0.625(10^{-3})\pi][200(10^9)]}\left\{[50(10^3)]^2(1.2) + 2[31.25(10^3)]^2(1.5) + 2[18.75(10^3)]^2(0.9)\right\}$$

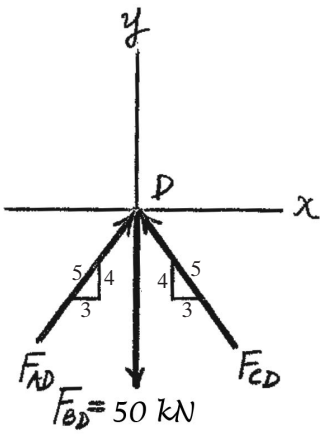
$$= 8.356 \text{ N} \cdot \text{m} = 8.36 \text{ J} \quad \textbf{Ans.}$$

This result is only valid if $\sigma < \sigma_Y$. We only need to check member BD since it is subjected to the greatest normal force

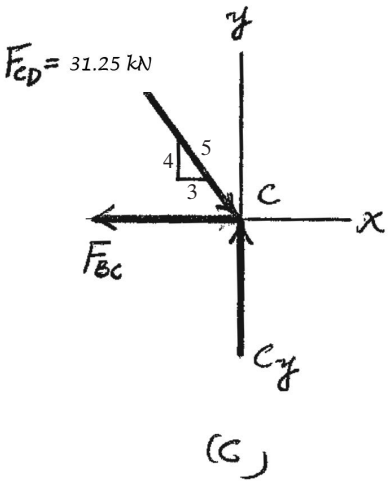
$$\sigma_{BD} = \frac{F_{BD}}{A} = \frac{50(10^3)}{0.625(10^{-3})\pi} = 25.46(10^6) \text{ N/m}^2 = 25.46 \text{ MPa} < \sigma_Y = 345 \text{ MPa} \quad \text{(O.K.)}$$



(a)



(b)



(c)

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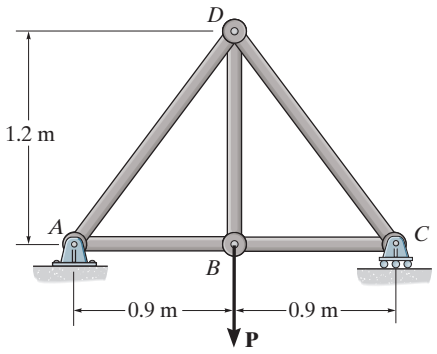
14–6. Determine the maximum force **P** and the corresponding maximum total strain energy that can be stored in the truss without causing any of the members to have permanent deformation. Each member of the truss has a diameter of 50 mm and is made of A-36 steel.



SOLUTION

Normal Forces. The normal force developed in each member of the truss can be determined using the method of joints.

Joint *B* (Fig. *a*)

$\overset{+}{\rightarrow}\Sigma F_x = 0; \quad F_{BC} - F_{AB} = 0 \quad (1)$

$+\uparrow \Sigma F_y = 0; \quad F_{BD} - P = 0 \quad F_{BD} = P(\text{T})$

Joint *D* (Fig. *b*)

$\overset{+}{\rightarrow}\Sigma F_x = 0; \quad F_{AD} = \left(\frac{3}{5}\right) - F_{CD}\left(\frac{3}{5}\right) = 0 \quad F_{AD} = F_{CD} = F$

$+\uparrow \Sigma F_y = 0; \quad 2\left[F\left(\frac{4}{5}\right)\right] - P = 0 \quad F_{AD} = F_{CD} = F = 0.625P\ (\text{C})$

Joint *C* (Fig. *c*)

$\overset{+}{\rightarrow}\Sigma F_x = 0; \quad 0.625\left(\frac{3}{5}\right) - F_{BC} = 0 \quad F_{BC} = 0.375P\ (\text{T})$

Using the result of $\mathbf{F}_{BC}$, Eq. (1) gives

$$F_{AB} = 0.375P\ (\text{T})$$

Axial Strain Energy. $A = \frac{\pi}{4}(0.05^2) = 0.625(10^{-3})\pi \text{m}^2$ and $L_{CD} = \sqrt{0.9^2 + 1.2^2} = 1.5$ m

Member *BD* is critical since it is subjected to greatest normal force. Thus,

$$\sigma_Y = \frac{F_{BD}}{A}$$

$$250(10^6) = \frac{P}{0.625(10^{-3})\pi}$$

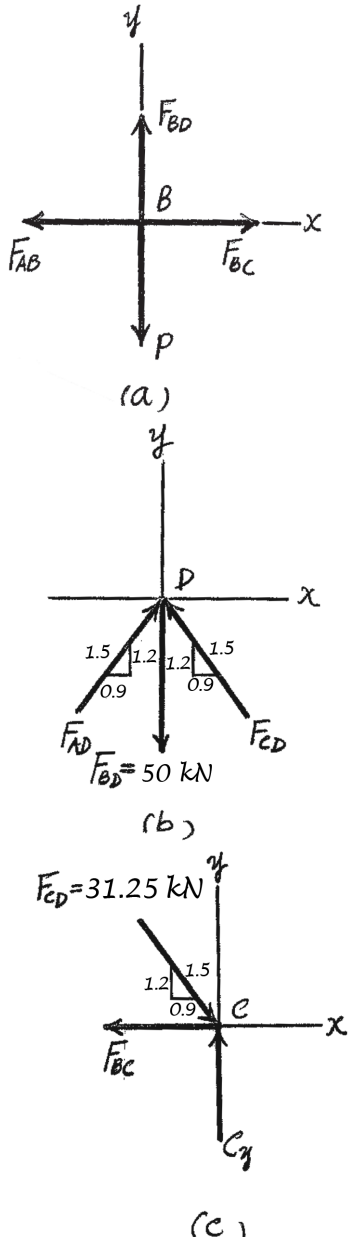
$$P = 490.87(10^3)\ \text{N} = 491\ \text{kN} \qquad \textbf{Ans.}$$

Using the result of *P*,

$F_{BD} = 490.87(10^3)$ N $\quad F_{AD} = F_{CD} = 306.80(10^3)$ N $\quad F_{BC} = F_{AB} = 184.08(10^3)$ N

$$(U_i)_a = \Sigma\frac{N^2L}{2AE} = \frac{1}{2[0.625(10^{-3})\pi][200(10^9)]}\left\{[490.87(10^3)]^2(1.2) + 2[306.80(10^3)]^2(1.5) + 2[184.08(10^3)]^2(0.9)\right\}$$

$$= 805.34\ \text{N}\cdot\text{m} = 805\ \text{J} \qquad \textbf{Ans.}$$



Ans:
$P = 491$ kN, $U_i = 0.805$ J

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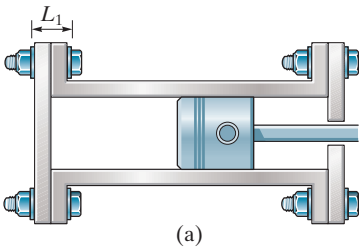
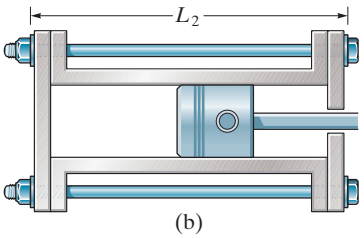
14–7.

Using bolts of the same material and cross-sectional area, two possible attachments for a cylinder head are shown. Compare the strain energy developed in each case, and then explain which design is better for resisting an axial shock or impact load.



(a)



(b)

SOLUTION

Case (a)

$$U_a = \frac{N^2 L_1}{2AE}$$ **Ans.**

Case (b)

$$U_b = \frac{N^2 L_2}{2AE}$$ **Ans.**

Since $U_b > U_a$, i.e., $L_2 > L_1$, the design for case (b) is better able to absorb energy.

Case (b) **Ans.**

Ans:

$U_a = \frac{N^2 L_1}{2AE}$, $U_b = \frac{N^2 L_2}{2AE}$

Since $U_b > U_a$, i.e., $L_2 > L_1$, the design for case (b) is better able to absorb energy.

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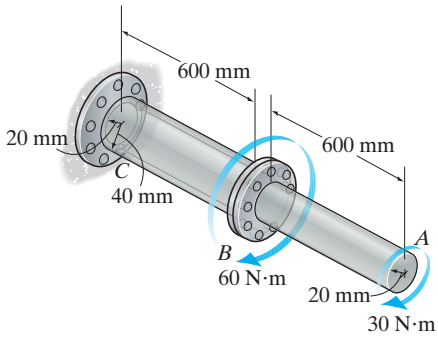
***14–8.**

The shaft assembly is fixed at C. The hollow segment BC has an inner radius of 20 mm and outer radius of 40 mm, while the solid segment AB has a radius of 20 mm. Determine the torsional strain energy stored in the shaft. The shaft is made of 2014-T6 aluminum alloy. The coupling at B is rigid.



SOLUTION

Internal Torque: Referring to the free-body diagram of segment AB, Fig. a,

$$\Sigma M_x = 0; \qquad T_{AB} + 30 = 0 \qquad T_{AB} = -30\,\text{N}\cdot\text{m}$$

Referring to the free-body diagram of segment BC, Fig. b,

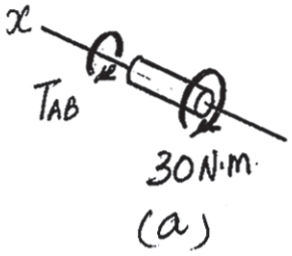
$$\Sigma M_x = 0; \qquad T_{BC} + 30 + 60 = 0 \qquad T_{AB} = -90\,\text{N}\cdot\text{m}$$

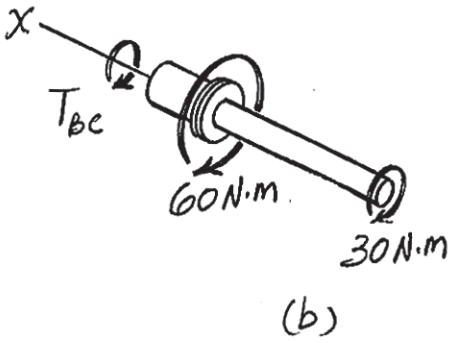
Torsional Strain Energy: Here, $J_{AB} = \frac{\pi}{2}\left(0.02^4\right) = 80\left(10^{-9}\right)\pi\ \text{m}^4$ and $J_{BC} = \frac{\pi}{2}\left(0.04^4 - 0.02^4\right) = 1200\left(10^{-9}\right)\pi\ \text{m}^4$,

$$(U_i)_t = \Sigma\frac{T^2L}{2GJ} = \frac{T_{AB}{}^2L_{AB}}{2GJ_{AB}} + \frac{T_{BC}{}^2L_{BC}}{2GJ_{BC}}$$

$$= \frac{(-30)^2(0.6)}{2\left[27\left(10^9\right)\right]\left[80\left(10^{-9}\right)\pi\right]} + \frac{(-90)^2(0.6)}{2\left[27\left(10^9\right)\right]\left[1200\left(10^{-9}\right)\pi\right]}$$

$$= 0.06379\ \text{J} = 0.0638\ \text{J} \qquad \textbf{Ans.}$$





Ans:
$U_i = 0.0638\ \text{J}$

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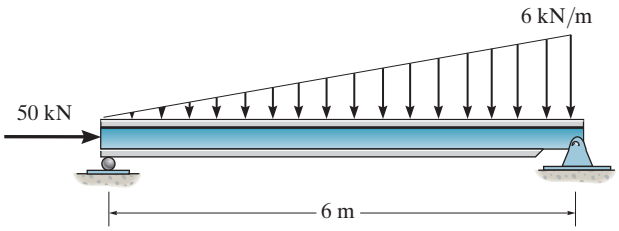
14–9.

Determine the total axial and bending strain energy in the A992 steel beam. $A = 2850\ \text{mm}^2$, $I = 28.9(10^6)\ \text{mm}^4$.



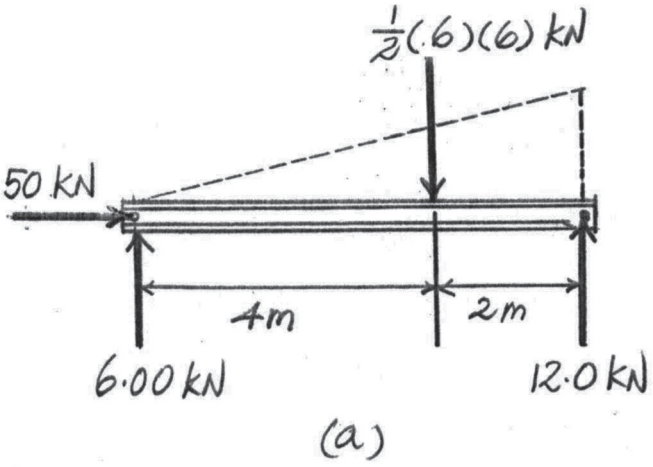
SOLUTION

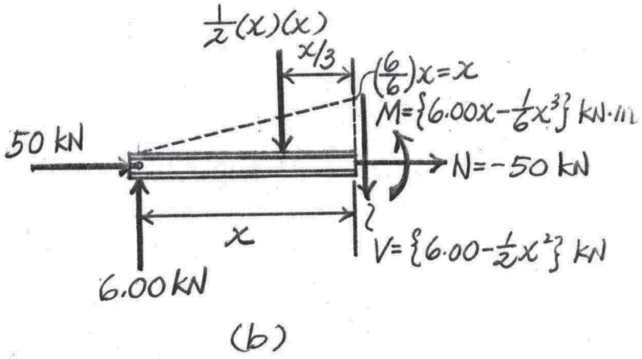
Support Reactions and Internal Loadings: The support reactions and the necessary moment function are shown on the FBD in Figs. *a* and *b*, respectively.

Strain Energy: For the axial load and bending,

$$\begin{aligned} U_i &= (U_a)_i + (U_b)_i \\ &= \frac{N^2 L}{2EA} + \int_0^L \frac{M^2 dx}{2EI} \\ &= \frac{\left[-50(10^3)\right]^2(6)}{2EA} + \int_0^{6\,\text{m}} \frac{\left[\left(6.00x - \frac{1}{6}x^3\right)(10^3)\right]^2 dx}{2EI} \\ &= \frac{7.50(10^9)\ \text{N}^2 \cdot \text{m}}{EA} + \frac{0.29623(10^9)\ \text{N}^2 \cdot \text{m}^3}{EI} \\ &= \frac{7.50(10^9)}{200(10^9)\left[2.85(10^{-3})\right]} + \frac{0.29623(10^9)}{200(10^9)\left[28.9(10^{-6})\right]} \\ &= 64.41\ \text{J} = 64.4\ \text{J} \end{aligned}$$

Ans.





Ans:
$U_i = 64.4\ \text{J}$

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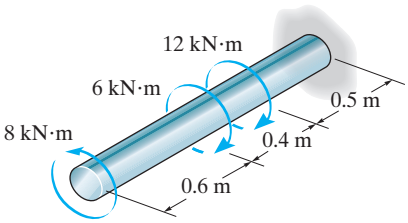
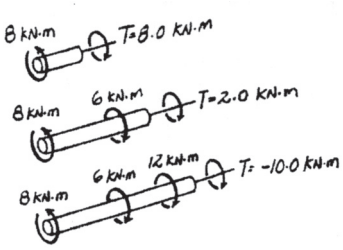
14–10.

Determine the torsional strain energy in the A-36 steel shaft. The shaft has a radius of 40 mm.



SOLUTION



Internal Torsional Moment: As shown on FBD.

Torsional Strain Energy: With polar moment of inertia $J = \frac{\pi}{2}\left(0.04^4\right) = 1.28\left(10^{-6}\right)\pi\ \text{m}^4$. Applying Eq. 14–22 gives

$$U_i = \sum \frac{T^2 L}{2GJ}$$

$$= \frac{1}{2GJ}\left[8000^2\,(0.6) + 2000^2\,(0.4) + \left(-10000^2\right)(0.5)\right]$$

$$= \frac{45.0(10^6)\ \text{N}^2\cdot\text{m}^3}{GJ}$$

$$= \frac{45.0(10^6)}{75(10^9)[1.28(10^{-6})\,\pi]}$$

$$= 149\ \text{J}$$ **Ans.**

Ans:
$U_i = 149$ J

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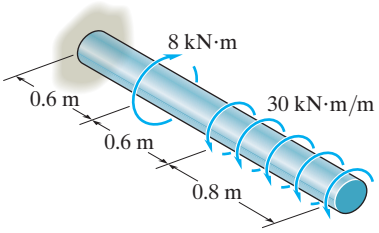
14–11.

Determine the torsional strain energy in the A992 steel shaft. The shaft has a radius of 40 mm.



SOLUTION

Internal Torque: As shown on the FBD in Fig. *a*, *b* and *c*.

Torsional Strain Energy: With polar moment of inertia $J = \frac{\pi}{2}(0.04^4) = 1.28(10^{-6})\pi\ \text{m}^4$ and $G = 75$ GPa for A992 steel,

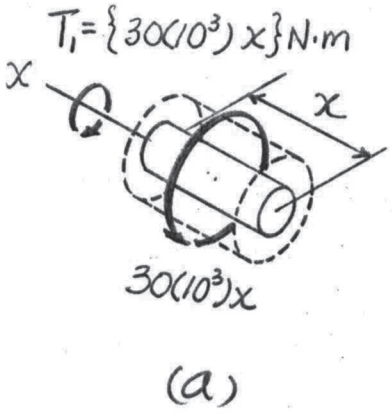
$$U_i = \int_0^L \frac{T^2 dx}{2GJ}$$

$$= \frac{1}{2GJ}\left\{\int_0^{0.8\,\text{m}} [30(10^3)x]^2 dx + [24.0(10^3)]^2(0.6) + [16.0(10^3)]^2(0.6)\right\}$$

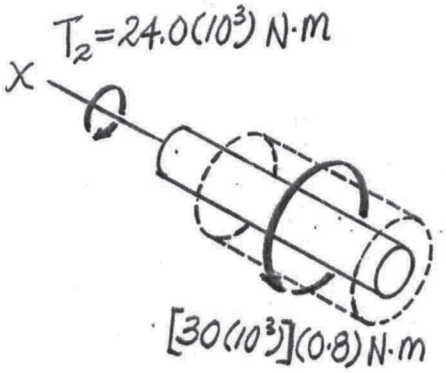
$$= \frac{0.3264(10^9)\ \text{N}^2\cdot\text{m}^3}{GJ}$$

$$= \frac{0.3264(10^9)}{75(10^9)[1.28(10^{-6})\pi]}$$

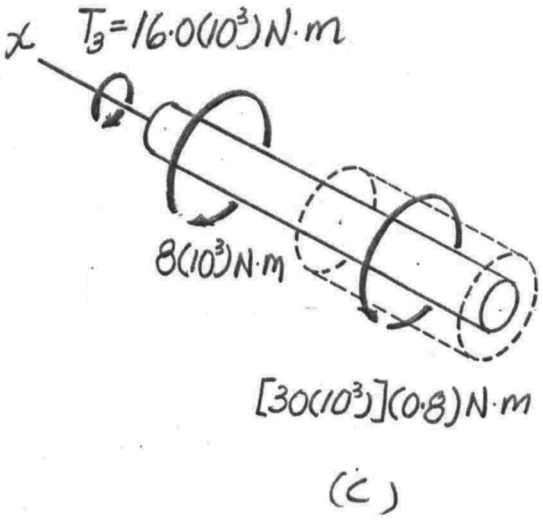
$$= 1082.25\ \text{J} = 1.08\ \text{kJ}$$ **Ans.**



(a)



(b)



(c)

Ans:
$U_i = 1.08$ kJ

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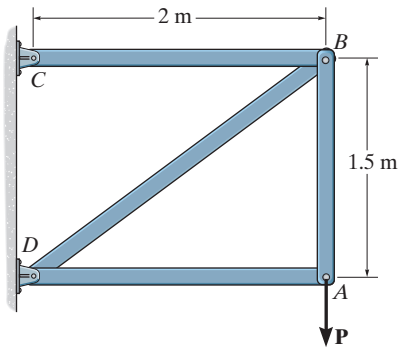
***14–12.**

If $P = 60$ kN, determine the total strain energy stored in the truss. Each member has a cross-sectional area of $2.5(10^3)$ mm^2 and is made of A-36 steel.



SOLUTION

Normal Forces: The normal force developed in each member of the truss can be determined using the method of joints.

Joint *A* (Fig. *a*)

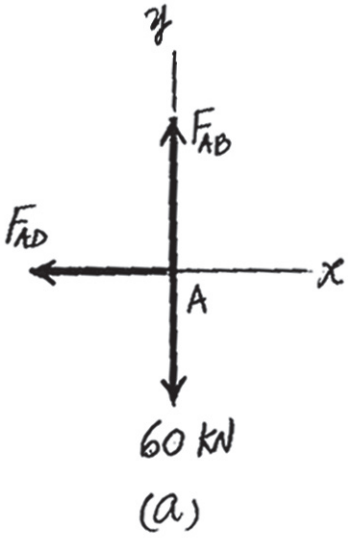
$\pm\rightarrow \Sigma F_x = 0;$ $\quad F_{AD} = 0$

$+\uparrow \Sigma F_y = 0;$ $\quad F_{AB} - 60 = 0$ $\quad F_{AB} = 60$ kN (T)

Joint *B* (Fig. *b*)

$+\uparrow \Sigma F_y = 0;$ $\quad F_{BD}\left(\frac{3}{5}\right) - 60 = 0$ $\quad F_{BD} = 100$ kN (C)

$\pm\rightarrow \Sigma F_x = 0;$ $\quad 100\left(\frac{4}{5}\right) - F_{BC} = 0$ $\quad F_{BC} = 80$ kN (T)



(a)

Axial Strain Energy: $A = 2.5(10^3)\text{ mm}^2 = 2.5(10^{-3})\text{ m}^2$ and $L_{BD} = \sqrt{2^2 + 1.5^2} = 2.5$ m

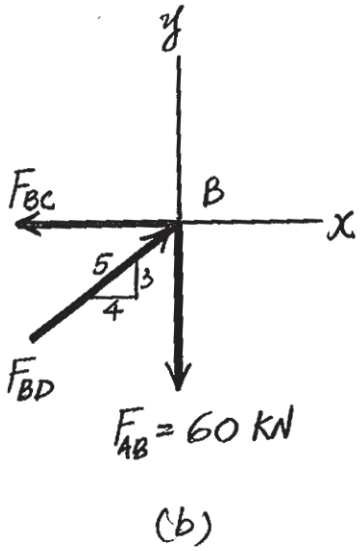
$$(U_i)_a = \Sigma \frac{N^2 L}{2AE}$$

$$= \frac{1}{2[2.5(10^{-3})][200(10^9)]}\Big[[60(10^3)]^2(1.5) + [100(10^3)]^2(2.5) + [80(10^3)]^2(2)\Big]$$

$$= 43.2\text{ J}$$ **Ans.**

This result is only valid if $\sigma < \sigma_Y$. We only need to check member BD since it is subjected to the greatest normal force

$$\sigma_{BD} = \frac{F_{BD}}{A} = \frac{100(10^3)}{2.5(10^{-3})} = 40\text{ MPa} < \sigma_Y = 250\text{ MPa}$$ **O.K.**



(b)

Ans:
$U_i = 43.2$ J

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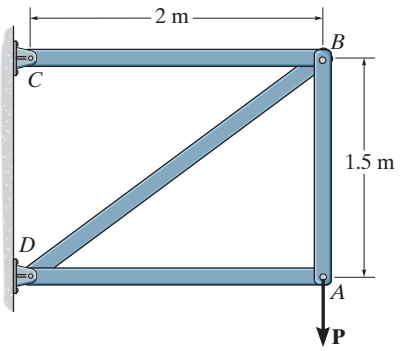
14–13.

Determine the maximum force **P** and the corresponding maximum total strain energy stored in the truss without causing any of the members to have permanent deformation. Each member has the cross-sectional area of $2.5(10^3)\ \text{mm}^2$ and is made of A-36 steel.



SOLUTION

Normal Forces: The normal force developed in each member of the truss can be determined using the method of joints.

Joint *A* (Fig. *a*)

$$\pm\rightarrow \Sigma F_x = 0; \qquad F_{AD} = 0$$

$$+\uparrow \Sigma F_y = 0; \qquad F_{AB} - P = 0 \qquad F_{AB} = P\,(T)$$

Joint *B* (Fig. *b*)

$$+\uparrow \Sigma F_y = 0; \qquad F_{BD}\left(\frac{3}{5}\right) - P = 0 \qquad F_{BD} = 1.6667P\ (\text{C})$$

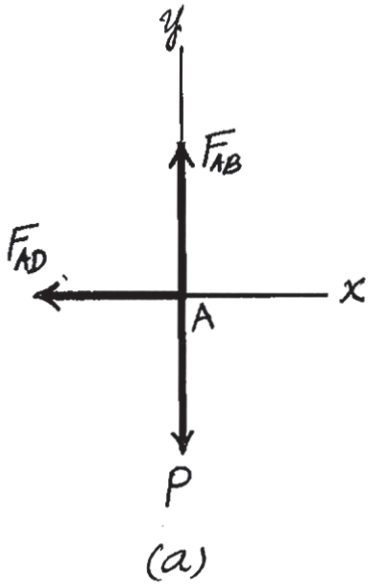
$$\pm\rightarrow \Sigma F_x = 0; \qquad 1.6667P\left(\frac{4}{5}\right) - F_{BC} = 0 \qquad F_{BC} = 1.3333P(\text{T})$$

Axial Strain Energy: $A = 2.5\left(10^3\right)\text{mm}^2 = 2.5\left(10^{-3}\right)\text{m}^2$. Member BD is critical since it is subjected to the greatest force. Thus,

$$\sigma_Y = \frac{F_{BD}}{A}$$

$$250\left(10^6\right) = \frac{1.6667P}{2.5\left(10^{-3}\right)}$$

$$P = 375\,\text{kN} \qquad \textbf{Ans.}$$



Using the result of P

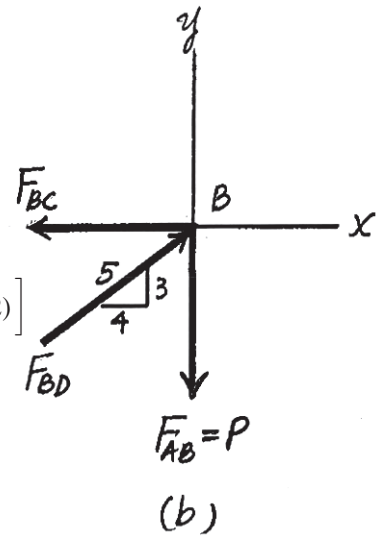
$$F_{AB} = 375\,\text{kN} \qquad F_{BD} = 625\,\text{kN} \qquad F_{BC} = 500\,\text{kN}$$

Here, $L_{BD} = \sqrt{1.5^2 + 2^2} = 2.5\ \text{m}$.

$$(U_i)_a = \Sigma \frac{N^2 L}{2AE}$$

$$= \frac{1}{2\left[2.5\left(10^{-3}\right)\right]\left[200\left(10^9\right)\right]}\left[\left[375\left(10^3\right)\right]^2(1.5) + \left[625\left(10^3\right)\right]^2(2.5) + \left[500\left(10^3\right)\right]^2(2)\right]$$

$$= 1687.5\,\text{J} = 1.69\,\text{kJ} \qquad \textbf{Ans.}$$



Ans:
$P = 375\ \text{kN}$, $U_i = 1.69\ \text{kJ}$

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14–14.

Consider the thin-walled tube of Fig. 5–26. Use the formula for shear stress, $\tau_{avg} = T/2tA_m$, Eq. 5–18, and the general equation of shear strain energy, Eq. 14–11, to show that the twist of the tube is given by Eq. 5–20. *Hint*: Equate the work done by the torque T to the strain energy in the tube, determined from integrating the strain energy for a differential element, Fig. 14–4, over the volume of material.

SOLUTION

$$U_i = \int_v \frac{\tau^2\, dV}{2\, G} \quad \text{but } \tau = \frac{T}{2\, t A_m}$$

Thus,

$$U_i = \int_v \frac{T^2}{8\, t^2\, A_m^2 G} dV$$

$$= \frac{T^2}{8\, A_m^2\, G} \int_v \frac{dV}{t^2} = \frac{T^2}{8\, A_m^2\, G} \int_A \frac{dV}{t^2} \int_0^L dx = \frac{T^2 L}{8\, A_m^2 G} \int_A \frac{dA}{t^2}$$

However, $dA = t\, ds$. Thus,

$$U_i = \frac{T^2 L}{8\, A_m^2 G} \int \frac{ds}{t}$$

$$U_e = \frac{1}{2} T\phi$$

$$U_e = U_i$$

$$\frac{1}{2}\, T\, \phi = \frac{T^2 L}{8\, A_m^2 G} \int \frac{ds}{t}$$

$$\phi = \frac{T\, L}{4\, A_m^2 G} \int \frac{ds}{t} \qquad \textbf{QED}$$

Ans:
N/A

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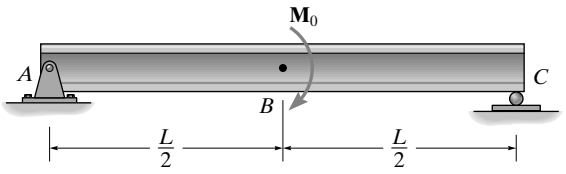
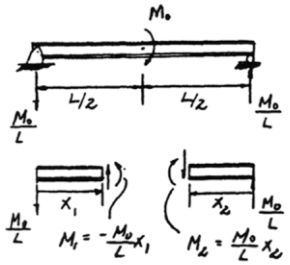
14–15. Determine the bending strain-energy in the beam due to the loading shown. *EI* is constant.



SOLUTION



$$U_i = \int_0^L \frac{M^2 dx}{2EI}$$

$$= \frac{1}{2EI}\left[\int_0^{L/2}\left(\frac{-M_0}{L}x_1\right)^2 dx_1 + \int_0^{L/2}\left(\frac{M_0}{L}x_2\right)^2 dx_2\right]$$

$$= \frac{M_0^2 L}{24EI} \qquad \textbf{Ans.}$$

Note: Strain energy is always positive regardless of the sign of the moment function.

Ans:

$$U_i = \frac{M_0^2 L}{24EI}$$

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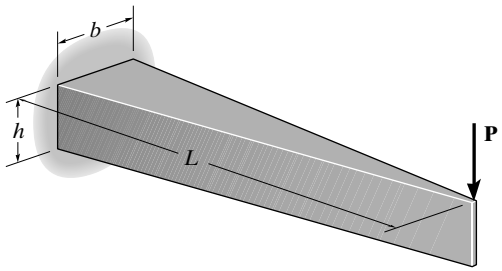
***14–16.** The beam shown is tapered along its width. If a force **P** is applied to its end, determine the strain energy in the beam and compare this result with that of a beam that has a constant rectangular cross section of width b and height h.



SOLUTION

Moment of Inertia: For the beam with the uniform section,

$$I = \frac{bh^3}{12} = I_0$$

For the beam with the tapered section,

$$I = \frac{1}{12}\left(\frac{b}{L}x\right)\left(h^3\right) = \frac{bh^3}{12L}x = \frac{I_0}{L}x$$

Internal Moment Function: As shown on FBD.

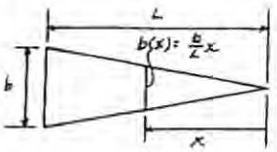
Bending Strain Energy: For the beam with the tapered section, applying Eq. 14–17 gives

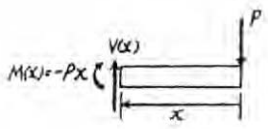
$$\begin{aligned} U_I &= \int_0^L \frac{M^2\,dx}{2EI} \\ &= \frac{1}{2E}\int_0^L \frac{(-Px)^2}{\frac{I_0}{L}x}dx \\ &= \frac{P^2L}{2EI_0}\int_0^L x dx \\ &= \frac{P^2L^3}{4EI_0} = \frac{3P^2L^3}{bh^3E} \end{aligned}$$ **Ans.**

For the beam with the uniform section,

$$\begin{aligned} U_i &= \int_0^L \frac{M^2 dx}{2EI} \\ &= \frac{1}{2EI_0}\int_0^L (-Px)^2\,dx \\ &= \frac{P^3L^3}{6EI_0} \end{aligned}$$

The strain energy in the capered beam is 1.5 times as great as that in the beam having a uniform cross section. **Ans.**





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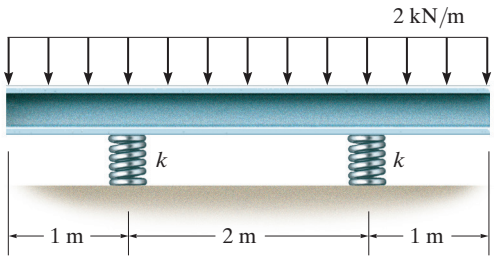
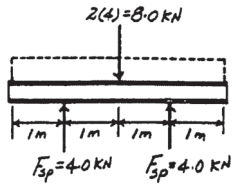
14–17.

The steel beam is supported on two springs, each having a stiffness of $k = 8$ MN/m. Determine the strain energy in each of the springs and the bending strain energy in the beam. $E_{st} = 200$ GPa, $I = 5(10^6)$ mm^4.

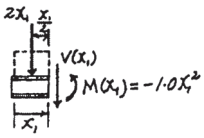


SOLUTION

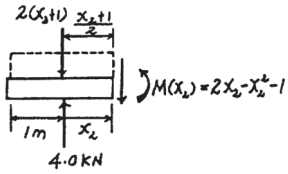
Spring Strain Energy: The spring deforms $\delta_{sp} = \dfrac{F_{sp}}{k} = \dfrac{4.00(10^3)}{8(10^6)} = 0.500(10^{-3})$ m under the applied load.



$$(U_i)_{sp} = \frac{1}{2}k\delta_{sp}^2$$
$$= \frac{1}{2}\left[8(10^6)\right]\left[0.500(10^{-})\right]^2$$
$$= 1.00 \text{ J} \qquad \textbf{Ans.}$$



Bending Strain Energy: Applying Eq. 14–17 gives



$$(U_i)_b = \int_0^L \frac{M^2 dx}{2EI}$$
$$= \frac{1}{2EI}\left[2\int_0^{1\text{m}} (-1.00x_1^2)^2\, dx_1 + \int_0^{2\text{m}} (2x_2 - x_2^2 - 1)^2\, dx_2\right]$$
$$= \frac{0.400 \text{ kN}^2 \cdot \text{m}^3}{EI}$$
$$= \frac{0.400(10^6)}{200(10^9)\left[5(10^{-6})\right]}$$
$$= 0.400 \text{ J} \qquad \textbf{Ans.}$$

Ans:
$(U_i)_{sp} = 1.00$ J
$(U_i)_b = 0.400$ J

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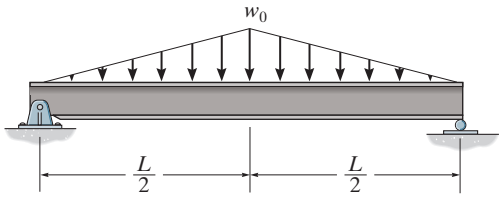
14–18.

Determine the bending strain energy in the simply supported beam. *EI* is constant.



SOLUTION

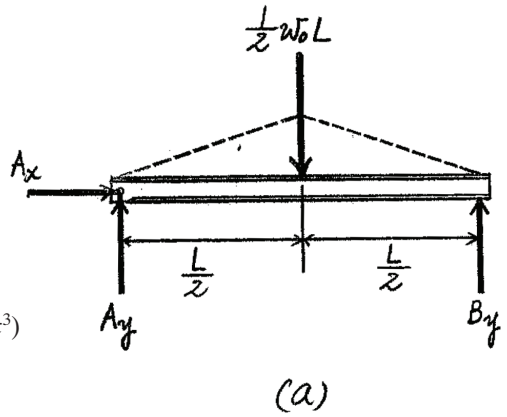
Support Reactions: Referring to the FBD of the entire beam, Fig. *a*,

$$\stackrel{+}{\rightarrow}\Sigma F_x = 0; \qquad A_x = 0$$

$$\circlearrowleft+\Sigma M_B = 0; \qquad \frac{1}{2}w_0 L\left(\frac{L}{2}\right) - A_y(L) = 0 \qquad A_y = \frac{w_0 L}{4}$$

Internal Moment: Referring to the FBD of the beam's left cut segment, Fig. *b*,

$$\circlearrowleft+\Sigma M_0 = 0; \quad M + \left[\frac{1}{2}\left(\frac{2w_0}{L}x\right)x\right]\left(\frac{x}{3}\right) - \frac{w_0 L}{4}x = 0 \quad M = \frac{w_0}{12L}(3L^2x - 4x^3)$$



(a)

Bending Strain Energy:

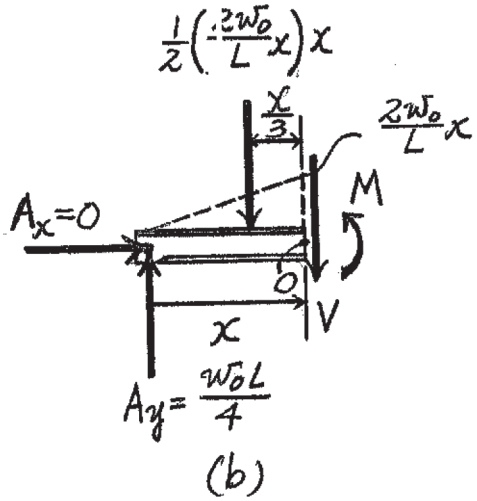
$$(U_i)_b = \Sigma\int_0^L \frac{M^2 dx}{2EI} = 2\int_0^{L/2} \frac{\left[\frac{w_0}{12L}(3L^2x - 4x^3)\right]^2 dx}{2EI}$$

$$= \frac{{w_0}^2}{144\,EIL^2}\int_0^{L/2}(9L^4x^2 + 16x^6 - 24L^2x^4)\,dx.$$

$$= \frac{{w_0}^2}{144\,EIL^2}\left(3L^4x^3 + \frac{16}{7}x^7 - \frac{24}{5}L^2x^5\right)\Big|_0^{L/2}$$

$$= \frac{17{w_0}^2L^5}{10\,080\,EI}$$

Ans.



(b)

Ans:

$$U_i = \frac{17w_0^2L^5}{10\,080\,EI}$$

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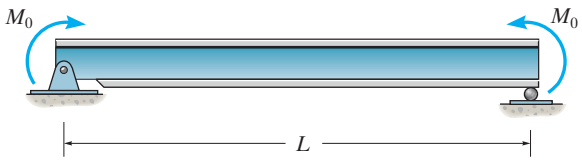
14–19.

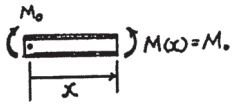
Determine the bending strain energy in the beam. EI is constant.



SOLUTION

Support Reactions: As shown on FBD(a).

Internal Moment Function: As shown on FBD(b).



Bending Strain Energy:

$$U_i = \int_0^L \frac{M^2 dx}{2EI}$$

$$= \frac{M_0}{2EI}\int_0^L dx$$

$$= \frac{M_0 L}{2EI}$$ **Ans.**

Ans:

$$U_i = \frac{M_0 L}{2EI}$$

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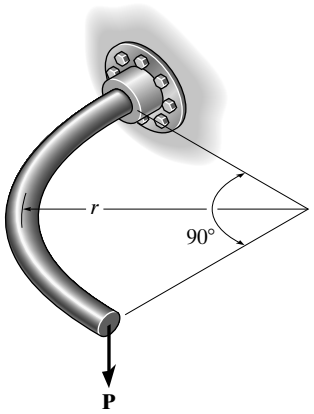
***14–20.** Determine the strain energy in the *horizontal* curved bar due to torsion. There is a *vertical* force **P** acting at its end. *JG* is constant.



SOLUTION

$T = Pr(1 - \cos\theta)$

Strain energy:

$$U_i = \int_0^L \frac{T^2\,ds}{2JG}$$

However,

$$s = r\theta; \qquad ds = r\,d\theta$$

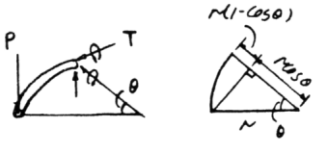
$$U_i = \int_0^\theta \frac{T^2 r\,d\theta}{2JG} = \frac{r}{2JG}\int_0^{\pi/2} [Pr(1 - \cos\theta)]^2 d\theta$$

$$= \frac{P^2 r^3}{2JG}\int_0^{\pi/2} (1 - \cos\theta)^2\,d\theta$$

$$= \frac{P^2 r^3}{2JG}\int_0^{\pi/2} (1 + \cos^2\theta - 2\cos\theta) d\theta$$

$$= \frac{P^2 r^3}{2JG}\int_0^{\pi/2} \left(1 + \frac{\cos 2\theta + 1}{2} - 2\cos\theta\right) d\theta$$

$$= \frac{P^2 r^3}{JG}\left(\frac{3\pi}{8} - 1\right)$$ **Ans.**



Ans:

$$U_i = \frac{P^2 r^3}{JG}\left(\frac{3\pi}{8} - 1\right)$$

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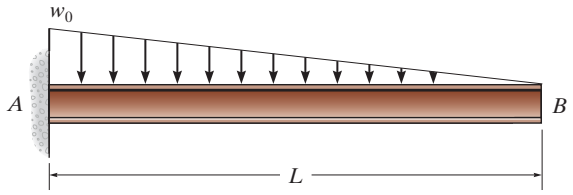
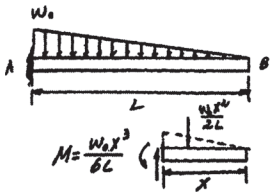
14–21.

Determine the bending strain energy in the beam. *EI* is constant.



SOLUTION

$$U_i = \int_0^L \frac{M^2 dx}{2EI} = \frac{1}{2EI}\int_0^L \left(\frac{w_0 x^3}{6L}\right)^2 dx = \frac{w_0^2 L^5}{504\,EI}$$ **Ans.**



Ans:
$U_i = \frac{w_0^2 L^5}{504\,EI}$

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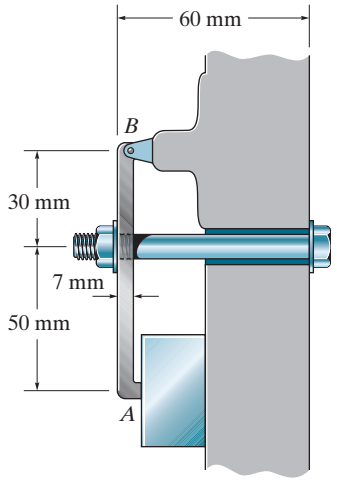
14–22.

The bolt has a diameter of 10 mm, and the arm AB has a rectangular cross section that is 12 mm wide by 7 mm thick. Determine the strain energy in the arm due to bending and in the bolt due to axial force. The bolt is tightened so that it has a tension of 500 N. Both members are made of A-36 steel. Neglect the hole in the arm.



SOLUTION

Axial Strain Energy: Applying Eq. 14–16 gives

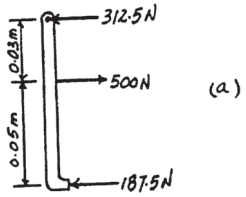
$$(U_i)_b = \frac{N^2 L}{2AE}$$

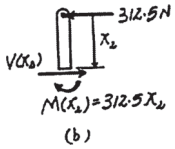
$$= \frac{500^2(0.06)}{2AE}$$

$$= \frac{7500\ \text{N}^2 \cdot \text{m}}{AE}$$

$$= \frac{7500}{\frac{\pi}{4}\left(0.01^2\right)\left[200\left(10^9\right)\right]}$$

$$= 0.477\left(10^{-3}\right)\ \text{J} \qquad \textbf{Ans.}$$





Bending Strain Energy: Applying Eq. 14–17 gives

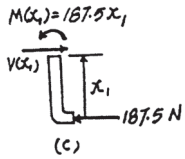
$$(U_i)_l = \int_0^L \frac{M^2 dx}{2EI}$$

$$= \frac{1}{2EI}\left[\int_0^{0.05\ \text{m}} (187.5x_1)^2 dx_1 + \int_0^{0.03\ \text{m}} (312.5x_2)^2 dx_2\right]$$

$$= \frac{1.171875\ \text{N}^2 \cdot \text{m}^3}{EI}$$

$$= \frac{1.171875}{200\left(10^9\right)\left[\frac{1}{12}(0.012)\left(0.007^3\right)\right]}$$

$$= 0.0171\ \text{J} \qquad \textbf{Ans.}$$



Ans:
$(U_i)_b = 0.477\left(10^{-3}\right)\ \text{J}$
$(U_i)_l = 0.0171\ \text{J}$

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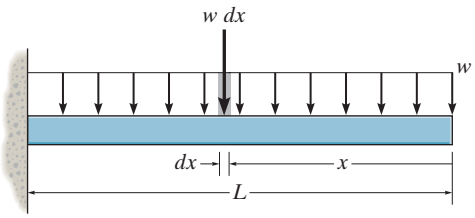
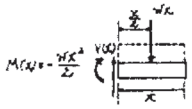
14–23.

Determine the bending strain energy in the cantilevered beam. Solve the problem two ways. (*a*) Apply Eq. 14–17. (*b*) The load $w\,dx$ acting on a segment dx of the beam is displaced a distance y, where $y = w(-x^4 + 4L^3x - 3L^4)/(24EI)$, the equation of the elastic curve. Hence the internal strain energy in the differential segment dx of the beam is equal to the external work, i.e., $dU_i = \frac{1}{2}(w\,dx)(-y)$. Integrate this equation to obtain the total strain energy in the beam. EI is constant.



SOLUTION

Internal Moment Function: As shown on FBD.



Bending Strain Energy: a) Applying Eq. 14–17 gives

$$U_i = \int_0^L \frac{M^2 dx}{2EI}$$

$$= \frac{1}{2EI}\left[\int_0^L \left[-\frac{w}{2}x^2\right]^2 dx\right]$$

$$= \frac{w^2}{8EI}\left[\int_0^L x^4 dx\right]$$

$$= \frac{w^2 L^5}{40EI} \qquad \textbf{Ans.}$$

b) Integrating $dU_i = \frac{1}{2}(wdx)(-y)$

$$dU_i = \frac{1}{2}(wdx)\left[-\frac{w}{24EI}\left(-x^4 + 4L^3x - 3L^4\right)\right]$$

$$dU_i = \frac{w^2}{48EI}\left(x^4 - 4L^3x + 3L^4\right)dx$$

$$U_i = \frac{w^2}{48EI}\int_0^L \left(x^4 - 4L^3x + 3L^4\right)dx$$

$$= \frac{w^2 L^5}{40EI} \qquad \textbf{Ans.}$$

Ans:

$$U_i = \frac{w^2 L^5}{40\,EI}$$

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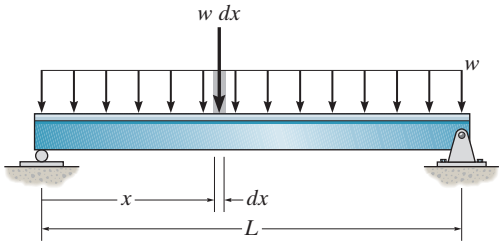
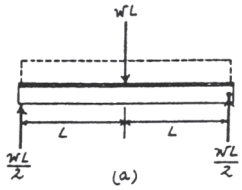
***14–24.**

Determine the bending strain energy in the simply supported beam. Solve the problem two ways. (a) Apply Eq. 14–17. (b) The load $w\,dx$ acting on the segment dx of the beam is displaced a distance y, where $y = w(-x^4 + 2Lx^3 - L^3x)/(24EI)$, the equation of the elastic curve. Hence the internal strain energy in the differential segment dx of the beam is equal to the external work, i.e., $dU_i = \frac{1}{2}(w\,dx)(-y)$. Integrate this equation to obtain the total strain energy in the beam. EI is constant.

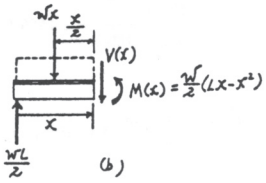


SOLUTION

Support Reactions: As shown on FBD(a).



Internal Moment Function: As shown on FBD(b).



Bending Strain Energy: a) Applying Eq. 14–17 gives

$$U_i = \int_0^L \frac{M^2 dx}{2EI}$$

$$= \frac{1}{2EI}\left[\int_0^L \left[\frac{w}{2}(Lx - x^2)\right]^2 dx\right]$$

$$= \frac{w^2}{8EI}\left[\int_0^L (L^2x^2 + x^4 - 2Lx^3)\,dx\right]$$

$$= \frac{w^2L^5}{240EI}$$ **Ans.**

b) Integrating $dU_i = \frac{1}{2}(wdx)(-y)$

$$dU_i = \frac{1}{2}(wdx)\left[-\frac{w}{24EI}(-x^4 + 2Lx^3 - L^3x)\right]$$

$$dU_i = \frac{w^2}{48EI}(x^4 - 2Lx^3 + L^3x)\,dx$$

$$U_i = \frac{w^2}{48EI}\int_0^L (x^4 - 2Lx^3 + L^3x)\,dx$$

$$= \frac{w^2L^5}{240EI}$$ **Ans.**

Ans:

$$U_i = \frac{w^2L^5}{240EI}$$

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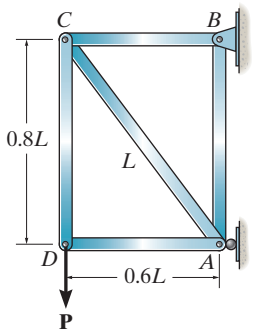
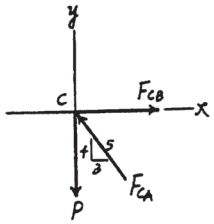
14–25.

Determine the vertical displacement of joint D. AE is constant.



SOLUTION

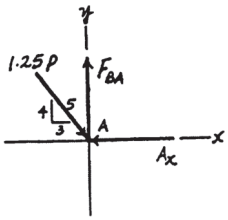
Member Forces: By inspection of joint D, member AD is a zero-force member and $F_{CD} = P(T)$. Applying the method of joints at C, we have



$$+\uparrow \Sigma F_y = 0; \qquad \frac{4}{5}F_{CA} - P = 0 \qquad F_{CA} = 1.25P \text{ (C)}$$

$$\stackrel{+}{\rightarrow} \Sigma F_x = 0; \qquad F_{CB} - \frac{3}{5}(1.25P) = 0 \qquad F_{CB} = 0.750P \text{ (T)}$$

At joint A



$$+\uparrow \Sigma F_y = 0; \qquad F_{AB} - \frac{4}{5}(1.25P) = 0 \qquad F_{BA} = 1.00P \text{ (T)}$$

Axial Strain Energy: Applying Eq. 14–16, we have

$$U_i = \sum \frac{N^2 L}{2AE}$$

$$= \frac{1}{2AE}\left[P^2(0.8L) + (-1.25P)^2(L) + (0.750P)^2(0.6L) + (1.00P)^2(0.8L)\right]$$

$$= \frac{1.750P^2L}{AE}$$

External Work: The external work done by force P is

$$U_e = \frac{1}{2}(P)(\Delta_D)_v$$

Conservation of Energy:

$$U_e = U_i$$

$$\frac{1}{2}(P)(\Delta_D)_v = \frac{1.750P^2L}{AE}$$

$$(\Delta_D)_v = \frac{3.50PL}{AE} \qquad \textbf{Ans.}$$

Ans:

$$(\Delta_D)_v = \frac{3.50PL}{AE}$$

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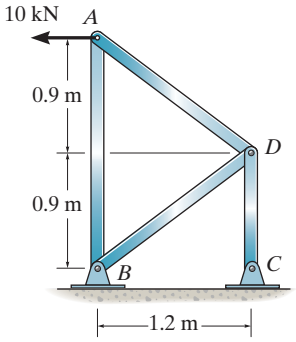
14–26. Determine the horizontal displacement of joint A. Each bar is made of A-36 steel and has a cross-sectional area of 950 mm^2.

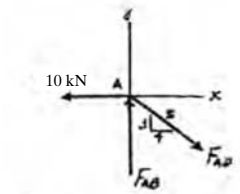


SOLUTION

Member Forces: Applying the method of joints to joint at A, we have

$$\stackrel{+}{\rightarrow}\Sigma F_x = 0; \quad \frac{4}{5}F_{AD} - 10 = 0 \qquad F_{AD} = 12.5\ \text{kN (T)}$$

$$+\uparrow \Sigma F_y = 0; \quad F_{AB} - \frac{3}{5}(12.5) = 0 \qquad F_{AB} = 7.5\ \text{kN (C)}$$

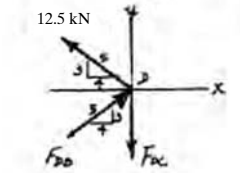


At joint D

$$\stackrel{+}{\rightarrow}\Sigma F_x = 0; \quad \frac{4}{5}F_{DB} - \frac{4}{5}(12.5) = 0 \qquad F_{DB} = 12.5\ \text{kN (C)}$$

$$+\uparrow \Sigma F_y = 0; \quad \frac{3}{5}(12.5) + \frac{3}{5}(12.5) - F_{DC} = 0$$

$$F_{DC} = 15\ \text{kN (T)}$$



Axial Strain Energy: Applying Eq. 14–16, we have

$$U_i = \sum \frac{N^2 L}{2AE}$$

$$= \frac{1}{2AE}[(12.5)^2(1.5) + (-7.5)^2(1.8) + (-12.5)^2(1.5) + (15)^2(0.9)]$$

$$= \frac{386.25\ \text{kN}^2 \cdot \text{m}}{AE}$$

$$= \frac{386.25(10^3)^2\ \text{N}^2 \cdot \text{m}}{[0.95(10^{-3})\ \text{m}^2][200(10^9)\ \text{N/m}^2]} = 2.0329\ \text{N} \cdot \text{m}$$

External Work: The external work done by 10-kN force is

$$U_e = \frac{1}{2}(10)(10^3)(\Delta_A)_h = 5(10^3)(\Delta_A)_h$$

Conservation of Energy:

$$U_e = U_i$$

$$(\Delta_A)_h = \frac{2.0329}{5(10^3)}$$

$$= 0.4066(10^{-3})\ \text{m} = 0.407\ \text{mm} \qquad \textbf{Ans.}$$

Ans:

$$(\Delta_D)_v = \frac{3.50PL}{AE}$$

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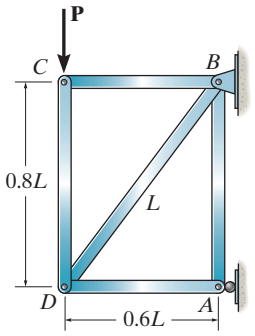
14–27.

Determine the vertical displacement of joint C. AE is constant.

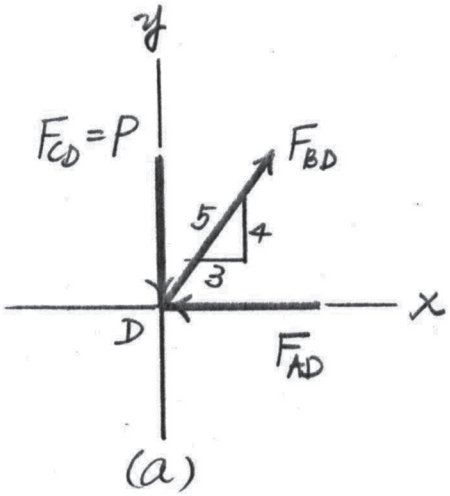


SOLUTION

Member Forces: By inspecting joint C, we notice that member BC is a zero-force member and $F_{CD} = P$ (C). Subsequently, consider the equilibrium of joint D, Fig. a.

$$+\uparrow \Sigma F_y = 0; \qquad F_{BD}\left(\frac{4}{5}\right) - P = 0 \qquad F_{BD} = \frac{5}{4}P \text{ (T)}$$

$$\stackrel{+}{\rightarrow} \Sigma F_x = 0; \qquad \frac{5}{4}P\left(\frac{3}{5}\right) - F_{AD} = 0 \qquad F_{AD} = \frac{3}{4}P \text{ (C)}$$



Axial Strain Energy:

$$(U_i)_a = \sum \frac{N^2 L}{2AE}$$
$$= \frac{1}{2AE}\left[P^2\left(\frac{4}{5}L\right) + \left(\frac{5}{4}P\right)^2(L) + \left(\frac{3}{4}P\right)^2\left(\frac{3}{5}L\right)\right]$$
$$= \frac{27P^2L}{20AE}$$

External Work: The external work done by force **P** is

$$U_e = \frac{1}{2}P\,(\Delta_C)_v$$

Using the concept of conservation of energy,

$$U_e = (U_i)_a$$
$$\frac{1}{2}P\,(\Delta_C)_v = \frac{27P^2L}{20AE}$$
$$(\Delta_C)_v = \frac{27PL}{10AE} \qquad \textbf{Ans.}$$

Ans:

$$(\Delta_C)_v = \frac{27PL}{10AE}$$

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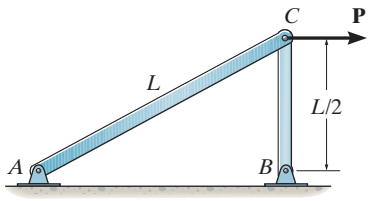
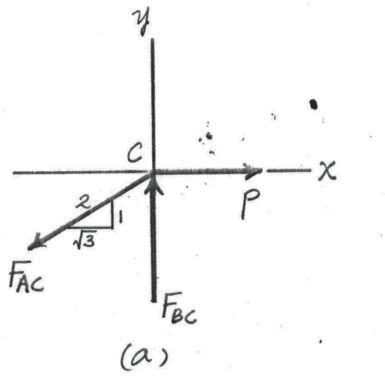
***14–28.**

Determine the horizontal displacement of joint C. AE is constant.



SOLUTION



Member Forces: Consider the equilibrium of joint C, Fig. a.

$$\stackrel{+}{\rightarrow}\Sigma F_x = 0; \qquad P - F_{AC}\left(\frac{\sqrt{3}}{2}\right) = 0 \qquad F_{AC} = \frac{2}{\sqrt{3}}P\,(\mathrm{T})$$

$$+\uparrow \Sigma F_y = 0; \qquad F_{BC} - \left(\frac{2}{\sqrt{3}}P\right)\left(\frac{1}{2}\right) = 0 \qquad F_{BC} = \frac{1}{\sqrt{3}}P\,(\mathrm{C})$$

Axial Strain Energy:

$$(U_i)_a = \sum \frac{N^2 L}{2AE}$$

$$= \frac{1}{2AE}\left[\left(\frac{2}{\sqrt{3}}P\right)^2 (L) + \left(\frac{1}{\sqrt{3}}P\right)^2 \left(\frac{L}{2}\right)\right]$$

$$= \frac{3P^2 L}{4AE}$$

External Work: The external work done by force **P** is

$$U_e = \frac{1}{2}P\,(\Delta_C)_h$$

Using the concept of conservation of energy,

$$U_e = (U_i)_a$$

$$\frac{1}{2}P\,(\Delta_C)_h = \frac{3P^2 L}{4AE}$$

$$(\Delta_C)_h = \frac{3PL}{2AE} \qquad \textbf{Ans.}$$

Ans:

$$(\Delta_C)_h = \frac{3PL}{2AE}$$

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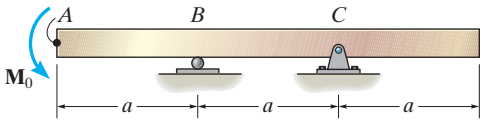
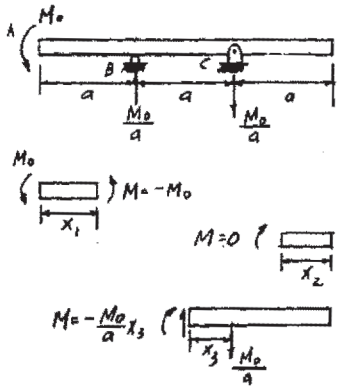
14–29.

Determine the slope at point A of the beam. EI is constant.



SOLUTION



$$U_i = \int_0^L \frac{M^2 dx}{2EI} = \frac{1}{2EI}\left[\int_0^a (-M_0)^2 dx_1 + \int_0^a (0) dx_2 + \int_0^b \left(-\frac{M_0}{a}x_3\right)^2 dx_3\right]$$

$$= \frac{2M_0^2 a}{3EI}$$

$$U_e = \frac{1}{2}M\theta = \frac{1}{2}M_0\,\theta_A$$

Conservation of Energy:

$$U_e = U_i$$

$$\frac{1}{2}M_0\,\theta_A = \frac{2M_0^2 a}{3EI}$$

$$\theta_A = \frac{4M_0\,a}{3EI}$$ **Ans.**

Ans:

$$\theta_A = \frac{4M_0\,a}{3EI}$$

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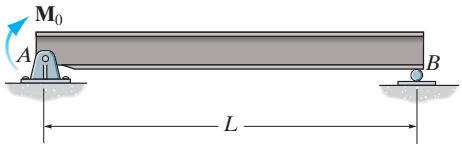
14–30.

Determine the slope of the beam at the pin support A. Consider only bending strain energy. EI is constant.



SOLUTION

Support Reactions: Referring to the FBD of the entire beam, Fig. *a*,

$$\zeta+ \Sigma M_A = 0; \qquad B_y L - M_0 = 0 \qquad B_y = \frac{M_0}{L}$$

Internal Moment: Referring to the FBD of the beam's right cut segment, Fig. *b*,

$$\zeta+ \Sigma M_B = 0; \qquad \frac{M_0}{L}x - M = 0 \qquad M = \frac{M_0}{L}x$$

Bending Strain Energy:

$$(U_i)_b = \int_0^L \frac{M^2 dx}{2EI} = \int_0^L \frac{\left(\frac{M_0}{L}x\right)^2 dx}{2EI} = \frac{{M_0}^2}{2EIL^2}\int_0^L x^2 dx = \frac{{M_0}^2}{2EIL^2}\left(\frac{x^3}{3}\right)\Big|_0^L$$

$$= \frac{{M_0}^2 L}{6EI}$$

External Work: The external work done by $\mathbf{M}_0$ is

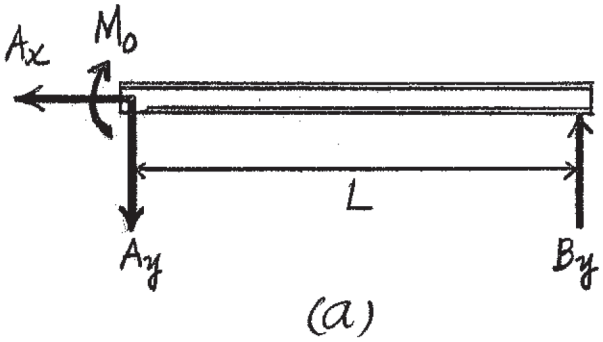
$$U_e = \frac{1}{2}M_0\theta_A$$

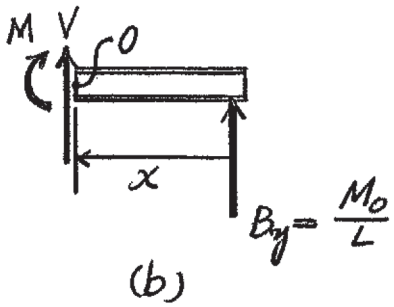
Conservation of Energy:

$$U_e = (U_i)_b$$

$$\frac{1}{2}M_0\theta_A = \frac{M_0^2 L}{6EI}$$

$$\theta_A = \frac{M_0 L}{3EI}\circlearrowright$$ **Ans.**





Ans:

$$\theta_A = -\frac{M_0 L}{3EI}$$

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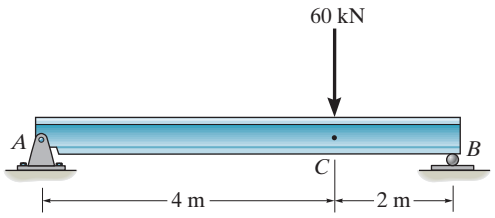
14–31.

Determine the vertical displacement of point C of the A992 steel beam. $I = 80(10^6)\ \text{mm}^4$.



SOLUTION

Support Reactions and Internal Loadings: The support reactions and the necessary moment functions are shown on the FBD in Figs. *a* and *b*, respectively,

Bending Strain Energy:

$$(U_i)_b = \int_0^L \frac{M^2 dx}{2EI}$$

$$= \frac{1}{2EI}\left[\int_0^{4\,\text{m}} (20.0x_1)^2\, dx_1 + \int_0^{2\,\text{m}} (40.0\, x_2^2)\, dx_2\right]$$

$$= \frac{1}{2EI}\left[\left(\frac{400}{3}x_1^3\right)\Big|_0^{4\,\text{m}} + \left(\frac{1600}{3}x_2^3\right)\Big|_0^{2\,\text{m}}\right]$$

$$= \frac{6400\ \text{kN}^2\cdot\text{m}^3}{EI}$$

For A992 steel, $E = 200$ GPa. Then

$$(U_i)_b = \frac{6400\,(1000^2)}{200(10^9)\,[\,80(10^{-6})\,]} = 400\ \text{J}$$

External Work: The external work done by 60 kN force is

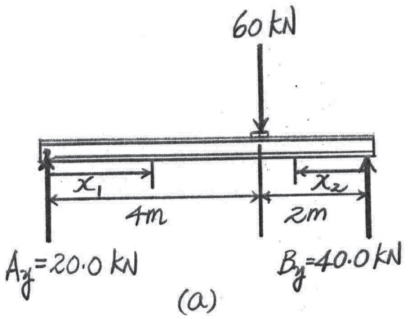
$$U_e = \frac{1}{2}P\,(\Delta_C)_v = \frac{1}{2}\,[\,60\,(10^3)\,]\,(\Delta_C)_v = 30(10^3)(\Delta_C)_v$$

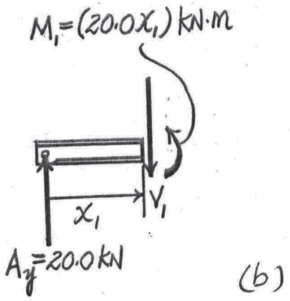
Using the concept of conservation of energy,

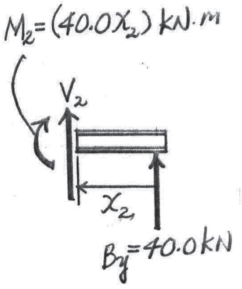
$$U_e = (U_i)_b$$

$$30(10^3)\,(\Delta_C)_v = 400$$

$$(\Delta_C)_v = 0.013333\ \text{m} = 13.3\ \text{mm}$$ **Ans.**







Ans:
$(\Delta_C)_v = 13.3\ \text{mm}$

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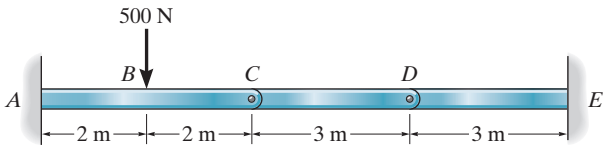
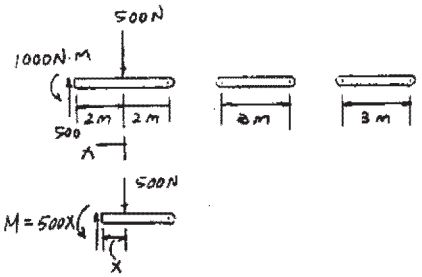
***14–32.**

The A992 steel bars are pin connected at C and D. If they each have the same rectangular cross section, with a height of 200 mm and a width of 100 mm, determine the vertical displacement at B. Neglect the axial load in the bars.



SOLUTION



Internal Strain Energy:

$$U_i = \int_0^L \frac{M^2\,dx}{2\,EI} = \frac{1}{2\,EI}\int_0^{2\,\text{m}} [500\,x]^2\,dx = \frac{0.3333\,(10^6)}{EI}$$

External Work:

$$U_e = \frac{1}{2}P\,\Delta_B = \frac{1}{2}(500)\,\Delta_B = 250\,\Delta_B$$

Conservation of Energy:

$$U_e = U_i$$

$$250\,\Delta_B = \frac{0.3333\,(10^6)}{EI}$$

$$\Delta_B = \frac{1333.33}{EI} = \frac{1333.33}{200\,(10^9)\left(\frac{1}{12}\right)(0.1)(0.2^3)}$$

$$= 0.1\,(10^{-3})\,\text{m} = 0.100\,\text{mm}$$ **Ans.**

Ans:
$\Delta_B = 0.100\,\text{mm}$

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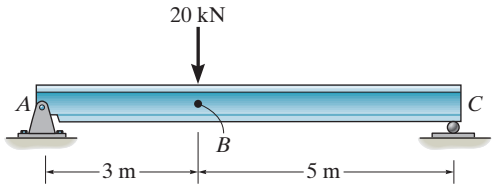
14–33.

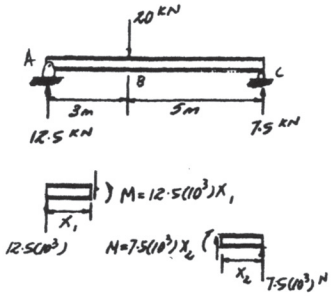
Determine the vertical displacement of point B on the A992 steel beam. $I = 80(10^6)\ \text{mm}^4$.



SOLUTION

$$U_i = \int_0^L \frac{M^2 dx}{2EI} = \frac{1}{2EI}\left[\int_0^3 [(12.5)(10^3)(x_1)]^2 dx_1 + \int_0^3 [(7.5)(10^3)(x_2)]^2 dx_2\right] = \frac{1.875(10^9)}{EI}$$

$$U_e = \frac{1}{2}P\Delta = \frac{1}{2}(20)(10^3)\Delta_B = 10(10^3)\Delta_B$$



Conservation of Energy:

$$U_e = U_i$$

$$10(10^3)\Delta_B = \frac{1.875(10^9)}{EI}$$

$$\Delta_B = \frac{187500}{EI} = \frac{187500}{200(10^9)(80)(10^{-6})} = 0.0117\ \text{m} = 11.7\ \text{mm}$$ **Ans.**

Ans:
$\Delta_B = 11.7\ \text{mm}$

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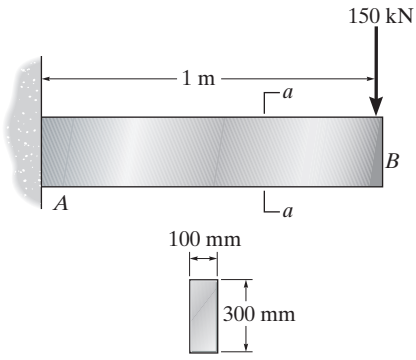
14–34.

Determine the vertical displacement of end B of the cantilevered 6061-T6 aluminum alloy rectangular beam. Consider both shearing and bending strain energy.



Section $a-a$

SOLUTION

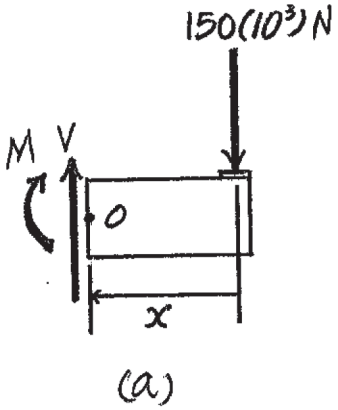
Internal Loadings: Referring to the FBD of beam's right cut segment, Fig. *a*,

$$+\uparrow \Sigma F_y = 0; \qquad V - 150(10^3) = 0 \qquad V = 150(10^3)\ \text{N}$$

$$\circlearrowleft + \Sigma M_0 = 0; \qquad -M - 150(10^3)\,x = 0 \qquad M = -150(10^3)x$$

Shearing Strain Energy: For the rectangular beam, the form factor is $f_s = \frac{6}{5}$.

$$(U_i)_v = \int_0^L \frac{f_s V^2 dx}{2GA} = \int_0^{1\text{ m}} \frac{\frac{6}{5}[150(10^3)]^2 dx}{2[26(10^9)][0.1(0.3)]} = 17.31\ \text{J}$$



Bending Strain Energy: $I = \frac{1}{12}(0.1)(0.3^3) = 0.225(10^{-3})\ \text{m}^4$. We obtain

$$(U_i)_b = \int_0^L \frac{M^2 dx}{2EI} = \int_0^{1\text{ m}} \frac{[-150(10^3)x]^2 dx}{2[68.9(10^9)][0.225(10^{-3})]}$$

$$= 725.689 \int_0^{1\text{ m}} x^2\, dx$$

$$= 725.689 \left(\frac{x^3}{3}\right)\Big|_0^{1\text{ m}}$$

$$= 241.90\ \text{J}$$

Thus, the strain energy stored in the beam is

$$U_i = (U_i)_v + (U_i)_b$$

$$= 17.31 + 241.90$$

$$= 259.20\ \text{J}$$

External Work: The work done by the external force $P = 150$ kN is

$$U_e = \frac{1}{2}P\Delta = \frac{1}{2}[150\ (10^3)]\ \Delta_B = 75(10^3)\ \Delta_B$$

Conservation of Energy:

$$U_e = U_i$$

$$75(10^3)\ \Delta_B = 259.20$$

$$\Delta_B = 3.456\ (10^{-3})\ \text{m} = 3.46\ \text{mm} \qquad \textbf{Ans.}$$

Ans:
$\Delta_B = 3.46$ mm

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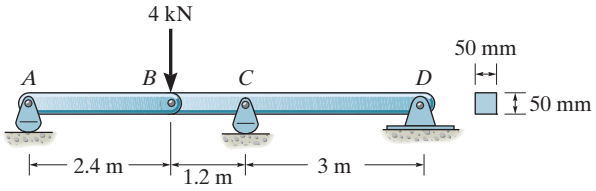
14–35. The A-36 steel bars are pin connected at B. If each has a square cross section, determine the vertical displacement at B.



SOLUTION

Support Reactions: As shown on FBD(a).

Moment Functions: As shown on FBD(b) and (c).

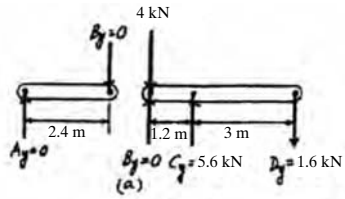
Bending Strain Energy: Applying 14–17, we have

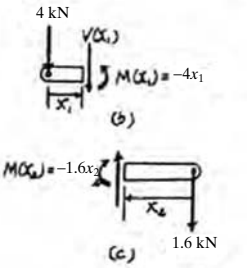
$$U_i = \int_0^L \frac{M^2 dx}{2EI}$$

$$= \frac{1}{2EI}\left[\int_0^{1.2\text{ m}} (-4x_1)^2\, dx_1 + \int_0^{3\text{ m}} (-1.6x_2)^2\, dx_2\right]$$

$$= \frac{16.128\text{ kN}^2 \cdot \text{m}^3}{EI}$$

$$= \frac{16.128(10^3\text{ N})^2 \cdot \text{m}^3}{[200(10^9)\text{ N/m}^2]\left[\frac{1}{12}(0.05\text{ m})(0.05\text{ m})^3\right]} = 154.829\text{ J}$$





External Work: The external work done by 4 kN force is

$$U_e = \frac{1}{2}[4(10^3)](\Delta_B) = 2000\Delta_B$$

Conservation of Energy:

$$U_e = U_i$$

$$2000\Delta_B = 154.829$$

$$\Delta_B = 0.07741\text{ m} = 77.41\text{ mm}$$ **Ans.**

Ans:
$\Delta_B = 77.4$ mm

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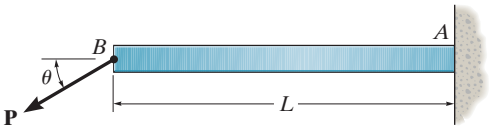
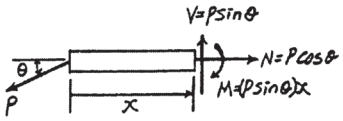
***14–36.**

The cantilevered beam has a rectangular cross-sectional area A, a moment of inertia I, and a modulus of elasticity E. If a load **P** acts at point B as shown, determine the displacement at B in the direction of **P**, accounting for bending, axial force, and shear.



SOLUTION



Strain Energy: Applying Eq. 14–15, 14–17 and 14–19, we have

$$U_i = \int_0^L \frac{N^2 dx}{2AE} + \int_0^L \frac{M^2 dx}{2EI} + \int_0^L \frac{f_s V^2 dx}{2GA}$$

However, $f_s = \frac{6}{5}$ for a rectangular section.

$$U_i = \int_0^L \frac{(P\cos\theta)^2\, dx}{2AE} + \int_0^L \frac{[(P\sin\theta)\, x]^2\, dx}{2EI} + \frac{6}{5}\int_0^L \frac{(P\sin\theta)^2\, dx}{2GA}$$

$$= \frac{P^2 L}{30}\left(\frac{15\cos^2\theta}{AE} + \frac{5L^2\sin^2\theta}{EI} + \frac{18\sin^2\theta}{GA}\right)$$

External Work: The external work done by force P is

$$U_e = \frac{1}{2}(P)(\Delta_B)$$

Conservation of Energy:

$$U_e = U_i$$

$$\frac{1}{2}(P)(\Delta_B) = \frac{P^2 L}{30}\left(\frac{15\cos^2\theta}{AE} + \frac{5L^2\sin^2\theta}{EI} + \frac{18\sin^2\theta}{GA}\right)$$

$$\Delta_B = \frac{PL}{15}\left(\frac{15\cos^2\theta}{AE} + \frac{5L^2\sin^2\theta}{EI} + \frac{18\sin^2\theta}{GA}\right) \quad \textbf{Ans.}$$

Ans:

$$\Delta_B = \frac{PL}{15}\left(\frac{15\cos^2\theta}{AE} + \frac{5L^2\sin^2\theta}{EI} + \frac{18\sin^2\theta}{GA}\right)$$

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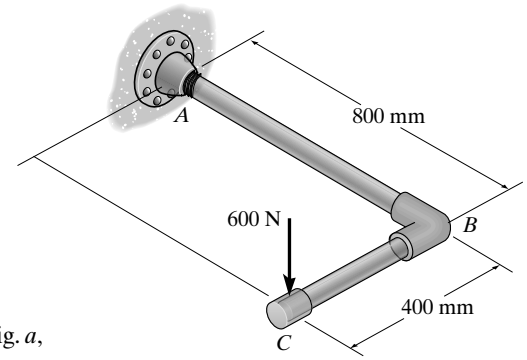
14–37. The pipe assembly is fixed at A. Determine the vertical displacement of end C of the assembly. The pipe has an inner diameter of 40 mm and outer diameter of 60 mm and is made of A-36 steel. Neglect the shearing strain energy.



SOLUTION

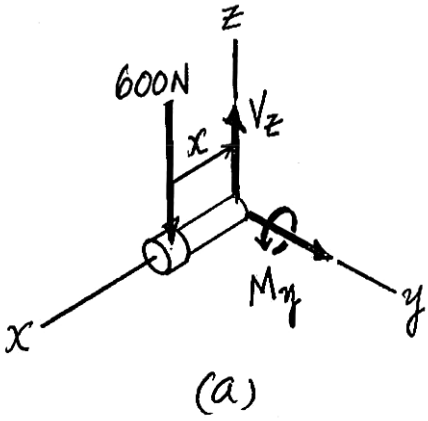
Internal Loading: Referring to the free-body diagram of the cut segment BC, Fig. a,

$$\Sigma M_y = 0;\ M_y + 600x = 0 \qquad M_y = -600x$$

Referring to the free-body diagram of the cut segment AB, Fig. b,

$$\Sigma M_x = 0;\ M_x - 600y = 0 \qquad M_x = 600y$$

$$\Sigma M_y = 0;\ 600(0.4) - T_y = 0 \qquad T_y = 240\,\text{N}\cdot\text{m}$$



(a)

Torsional Strain Energy. $J = \frac{\pi}{2}\left(0.03^4 - 0.02^4\right) = 0.325\left(10^{-6}\right)\pi\,\text{m}^4$. We obtain

$$(U_i)_t = \int_0^L \frac{T^2 dx}{2GJ} = \int_0^{0.8\,\text{m}} \frac{240^2 dx}{2\left[75\left(10^9\right)\right]\left[0.325\left(10^{-6}\right)\pi\right]} = 0.3009\ \text{J}$$

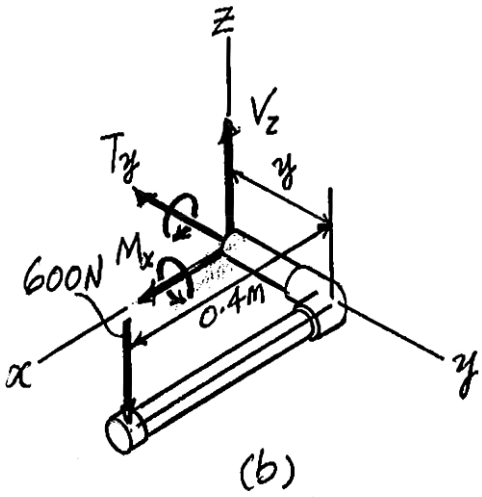
Bending Strain Energy. $I = \frac{\pi}{4}\left(0.03^4 - 0.02^4\right) = 0.1625\left(10^{-6}\right)\pi\,\text{m}^4$. We obtain

$$(U_i)_b = \int_0^L \frac{M^2 dx}{2EI} = \frac{1}{2EI}\left[\int_0^{0.4\,\text{m}} (-600x)^2\,dx + \int_0^{0.8\,\text{m}} (600y)^2\,dy\right]$$

$$= \frac{1}{2EI}\left[120\left(10^3\right)x^3\Big|_0^{0.4\,\text{m}} + 120\left(10^3\right)y^3\Big|_0^{0.8\,\text{m}}\right]$$

$$= \frac{34\,560\ \text{N}^2\cdot\text{m}^3}{EI}$$

$$= \frac{34\,560}{200\left(10^9\right)\left[0.1625\left(10^{-6}\right)\pi\right]} = 0.3385\ \text{J}$$



(b)

Thus, the strain energy stored in the pipe is

$$U_i = (U_i)_t + (U_i)_b$$
$$= 0.3009 + 0.3385$$
$$= 0.6394\ \text{J}$$

External Work. The work done by the external force $P = 600$ N is

$$U_e = \frac{1}{2}P\Delta = \frac{1}{2}(600)\Delta_C = 300\Delta_C$$

Conservation of Energy.

$$U_e = U_t$$
$$300\Delta_C = 0.6394$$
$$\Delta_C = 2.1312\left(10^{-3}\right) = 2.13\ \text{mm} \qquad \textbf{Ans.}$$

Ans:
$\Delta_C = 2.13$ mm

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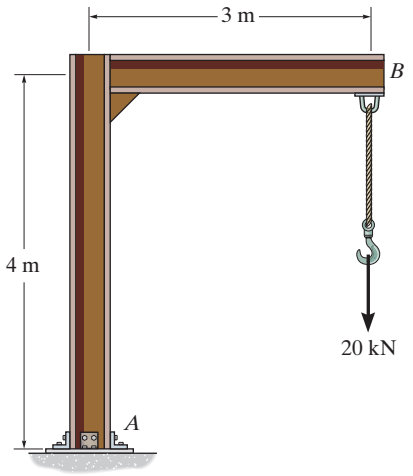
14–38.

Determine the vertical displacement of end B of the frame. Consider only bending strain energy. The frame is made using two A-36 steel W460 × 68 wide-flange sections.



SOLUTION

Internal Loading: Using the coordinates x_1 and x_2, the free-body diagrams of the frame's segments in Figs. *a* and *b* are drawn. For coordinate x_1,

$$\zeta+\Sigma M_O = 0; \qquad -M_1 - 20\left(10^3\right)x_1 = 0 \qquad M_1 = -20\left(10^3\right)x_1$$

For coordinate x_2,

$$\zeta+\Sigma M_O = 0; \qquad M_2 - 20\left(10^3\right)(3) = 0 \qquad M_2 = 60\left(10^3\right)\text{N}\cdot\text{m}$$

Bending Strain Energy:

$$(U_b)_i = \int_0^L \frac{M^2 dx}{2EI} = \frac{1}{2EI}\left[\int_0^{3\text{ m}}\left[-20\left(10^3\right)x_1\right]^2 dx_1 + \int_0^{4\text{ m}}\left[60\left(10^3\right)\right]^2 dx_2\right]$$

$$= \frac{1}{2EI}\left[\left(\frac{400\left(10^6\right)}{3}{x_1}^3\right)\Bigg|_0^{3\text{ m}} + 3.6\left(10^9\right)x\Big|_0^{4\text{ m}}\right]$$

$$= \frac{9\left(10^9\right)\text{N}^2\cdot\text{m}^3}{EI}$$

For a W460 × 68, $I = 297\left(10^6\right)\text{mm}^4 = 297\left(10^{-6}\right)\text{m}^4$. Then

$$(U_b)_i = \frac{9\left(10^9\right)}{200\left(10^9\right)(297)\left(10^{-6}\right)} = 151.52\text{ J}$$

External Work: The work done by the external force $P = 20$ kN is

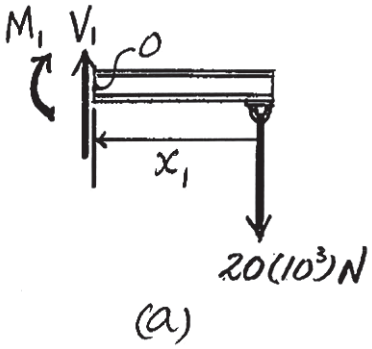
$$U_e = \frac{1}{2}P\Delta = \frac{1}{2}\left[20\left(10^3\right)\right]\Delta_B = 10\left(10^3\right)\Delta_B$$

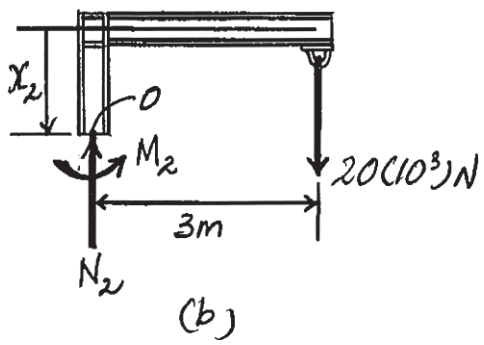
Conservation of Energy:

$$U_e = U_i$$

$$10\left(10^3\right)\Delta_B = 151.52$$

$$\Delta_B = 0.01515\text{ m} = 15.2\text{ mm} \qquad \textbf{Ans.}$$





Ans:
$\Delta_B = 15.2$ mm

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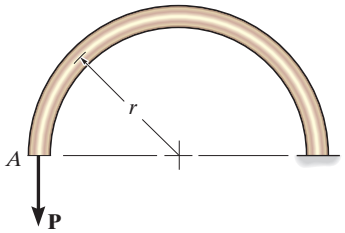
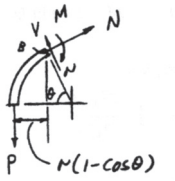
14–39.

The rod has a circular cross section with a moment of inertia I. If a vertical force **P** is applied at A, determine the vertical displacement at this point. Only consider the strain energy due to bending. The modulus of elasticity is E.



SOLUTION



Moment Function:

$$\zeta+\ \Sigma M_B = 0; \qquad P[r(1-\cos\theta)] - M = 0; \qquad M = Pr(1-\cos\theta)$$

Bending Strain Energy:

$$U_i = \int_0^S \frac{M^2\,ds}{2EI} \qquad ds = r\,d\theta$$

$$= \int_0^\theta \frac{M^2 r\,d\theta}{2EI} = \frac{r}{2EI}\int_0^\pi [P\,r(1-\cos\theta)]^2 d\theta$$

$$= \frac{P^2r^3}{2EI}\int_0^\pi (1+\cos^2\theta - 2\cos\theta)\,d\theta$$

$$= \frac{P^2r^3}{2EI}\int_0^\pi \left(1+\frac{1}{2}+\frac{\cos 2\theta}{2}-2\cos\theta\right)d\theta$$

$$= \frac{P^2r^3}{2EI}\int_0^\pi \left(\frac{3}{2}+\frac{\cos 2\theta}{2}-2\cos\theta\right)d\theta = \frac{P^2r^3}{2EI}\left(\frac{3}{2}\pi\right) = \frac{3\pi P^2r^3}{4EI}$$

Conservation of Energy:

$$U_e = U_i; \qquad \frac{1}{2}P\,\Delta_A = \frac{3\pi P^2r^3}{4EI}$$

$$\Delta_A = \frac{3\pi Pr^3}{2EI} \qquad \textbf{Ans.}$$

Ans:

$$\Delta_A = \frac{3\pi Pr^3}{2EI}$$

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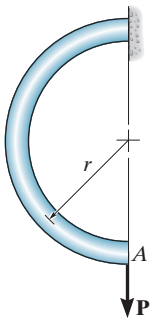
***14–40.**

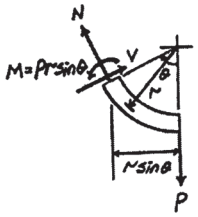
The rod has a circular cross section with a moment of inertia I. If a vertical force **P** is applied at A, determine the vertical displacement at this point. Only consider the strain energy due to bending. The modulus of elastcity is E.



SOLUTION

Bending Strain Energy: Applying 14–17 with $ds = rd\theta$, we have

$$\begin{aligned} U_i &= \int_0^s \frac{M^2 ds}{2EI} \\ &= \frac{1}{2EI}\int_0^\pi (Pr\sin\theta)^2\, rd\theta \\ &= \frac{P^2r^3}{2EI}\int_0^\pi \sin^2\theta d\theta \\ &= \frac{P^2r^3}{4EI}\int_0^\pi (1-\cos 2\theta)\, d\theta \\ &= \frac{\pi P^2 r^3}{4EI} \end{aligned}$$



External Work: The external work done by force P is

$$U_e = \frac{1}{2}(P)(\Delta_A)$$

Conservation of Energy:

$$U_e = U_i$$

$$\frac{1}{2}(P)(\Delta_A) = \frac{\pi P^2 r^3}{4EI}$$

$$\Delta_A = \frac{\pi P r^3}{2EI} \qquad \textbf{Ans.}$$

Ans:

$$\Delta_A = \frac{\pi P r^3}{2EI}$$

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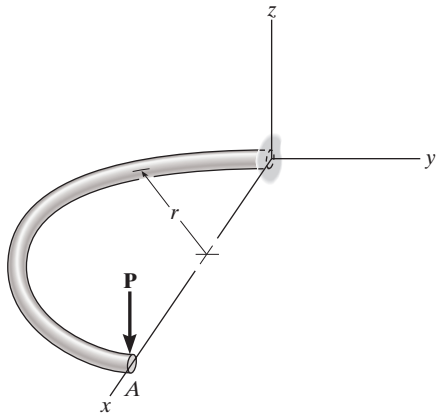
14–41.

The rod has a circular cross section with a polar moment of inertia J and moment of inertia I. If a vertical force **P** is applied at A, determine the vertical displcement at this point. Consider the strain energy due to bending and torsion. The material constants are E and G.



SOLUTION

$T = Pr(1 - \cos\theta); \quad M = Pr\sin\theta$

Torsion Strain Energy:

$$U_i = \int_0^s \frac{T^2 ds}{2GJ} = \int_0^\theta \frac{T^2 r d\theta}{2GJ}$$

$$= \frac{r}{2GJ}\int_0^\pi [Pr(1 - \cos\theta)]^2 d\theta$$

$$= \frac{P^2 r^3}{2GJ}\int_0^\pi (1 + \cos^2\theta - 2\cos\theta) d\theta$$

$$= \frac{P^2 r^3}{2GJ}\int_0^\pi \left(1 + \frac{\cos 2\theta + 1}{2} - 2\cos\theta\right) d\theta$$

$$= \frac{3P^2 r^3 \pi}{4GJ}$$

$T = Pr(1-\cos\theta)$
$M = Pr\sin\theta$

Bending Strain Energy:

$$U_i = \int_0^s \frac{M^2 ds}{2EI}$$

$$= \int_0^\theta \frac{M^2 r\, d\theta}{2EI} = \frac{r}{2EI}\int_0^\pi [Pr\sin\theta]^2 d\theta$$

$$= \frac{P^2 r^3}{2EI}\int_0^\pi \left(\frac{1 - \cos 2\theta}{2}\right) d\theta = \frac{P^2 r^3 \pi}{4EI}$$

Conservation of Energy:

$$U_e = U_i$$

$$\frac{1}{2}P\Delta_A = \frac{3P^2 r^3 \pi}{4GJ} + \frac{P^2 r^3 \pi}{4EI}$$

$$\Delta_A = \frac{Pr^3 \pi}{2}\left(\frac{3}{GJ} + \frac{1}{EI}\right) \qquad \textbf{Ans.}$$

Ans:

$$\Delta_A = \frac{Pr^3 \pi}{2}\left(\frac{3}{GJ} + \frac{1}{EI}\right)$$

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14–42.

A bar is 4 m long and has a diameter of 30 mm. Determine the total amount of elastic energy that it can absorb from an impact loading if (a) it is made of steel for which $E_{st} = 200$ GPa, $\sigma_Y = 800$ MPa, and (b) it is made from an aluminum alloy for which $E_{al} = 70$ GPa, $\sigma_Y = 405$ MPa.

SOLUTION

a) $\epsilon_Y = \dfrac{\sigma_Y}{E} = \dfrac{800(10^6)}{200(10^9)} = 4(10^{-3})\ \text{m/m}$

$$u_r = \frac{1}{2}(\sigma_Y)(\epsilon_Y) = \frac{1}{2}(800)(10^6)(\text{N/m}^2)(4)(10^{-3})\ \text{m/m} = 1.6\ \text{MJ/m}^3$$

$$V = \frac{\pi}{4}(0.03)^2(4) = 0.9(10^{-3})\pi\ \text{m}^2$$

$$U_i = 1.6(10^6)(0.9)(10^{-3})\pi = 4.52\ \text{kJ}$$ **Ans.**

b)

$$\epsilon_Y = \frac{\sigma_Y}{E} = \frac{405(10^6)}{70(10^9)} = 5.786(10^{-3})\ \text{m/m}$$

$$U_r = \frac{1}{2}(\sigma_Y)(\epsilon_Y) = \frac{1}{2}(405)(10^6)(\text{N/m}^2)(5.786)(10^{-3})\ \text{m/m} = 1.172\ \text{MJ/m}^3$$

$$V = \frac{\pi}{4}(0.03)^2(4) = 0.9(10^{-3})\pi\ \text{m}^3$$

$$U_i = 1.172(10^6)(0.9)(10^{-3})\pi = 3.31\ \text{kJ}$$ **Ans.**

Ans:
(a) $U_i = 4.52$ kJ
(b) $U_i = 3.31$ kJ

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14–43.

Determine the diameter of a red brass C83400 bar that is 8 ft long if it is to be used to absorb 800 ft · lb of energy in tension from an impact loading. No yielding occurs.

SOLUTION

Elastic Strain Energy: The yielding axial force is $P_Y = \sigma_Y A$.

$$U_i = \frac{N^2 L}{2AE} = \frac{(\sigma_Y A)^2 L}{2AE} = \frac{\sigma_Y^2 AL}{2E}$$

Substituting, we have

$$U_i = \frac{\sigma_Y^2 AL}{2E}$$

$$0.8(12) = \frac{11.4^2\left[\frac{\pi}{4}(d^2)\right](8)(12)}{2[14.6(10^3)]}$$

$$d = 5.35 \text{ in.} \qquad \textbf{Ans.}$$

Ans:
$d = 5.35$ in.

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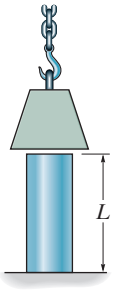
***14–44.**

Determine the speed v of the 50-Mg mass when it is just over the top of the steel post, if after impact, the maximun stress developed in the post is 550 MPa. The post has a length of $L = 1$ m and a cross-sectional area of 0.01 m^2. $E_{st} = 200$ GPa, $\sigma_Y = 600$ MPa.



SOLUTION

The Maximum Stress:

$$\sigma_{max} = \frac{P_{max}}{A}$$

$$550\,(10^6) = \frac{P_{max}}{0.01}; \qquad P_{max} = 5500\text{ kN}$$

$$\Delta_{max} = \frac{P_{max}}{k} \qquad \text{Here} \quad k = \frac{AE}{L} = \frac{0.01(200)\,(10^9)}{1} = 2\,(10^9)\text{ N/m}$$

$$= \frac{5500\,(10^3)}{2\,(10^9)} = 2.75\,(10^{-3})\text{ m}$$

Conservation of Energy:

$$U_e = U_i$$

$$\frac{1}{2}mv^2 + W\,\Delta_{max} = \frac{1}{2}k\,\Delta_{max}^2$$

$$\frac{1}{2}(50)\,(10^3)\,(v^2) + 50\,(10^3)\,(9.81)\,[2.75\,(10^{-3})] = \frac{1}{2}(2)\,(10^9)\,[2.75\,(10^{-3})]^2$$

$$v = 0.499\text{ m/s} \qquad \textbf{Ans.}$$

Ans:
$v = 0.499$ m/s

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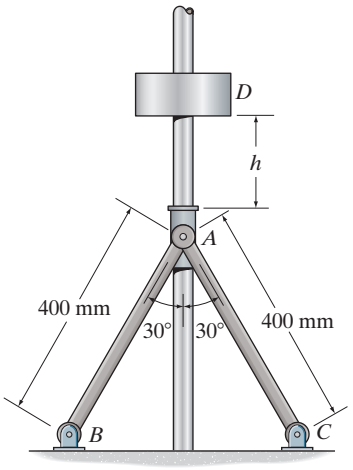
14–45. Rods AB and AC have a diameter of 20 mm and are made of 6061-T6 aluminum alloy. They are connected to the rigid collar which slides freely along the vertical guide rod. Determine the maximum height h from which the 50-kg block D can be dropped without causing yielding in the rods when the block strikes the collar.

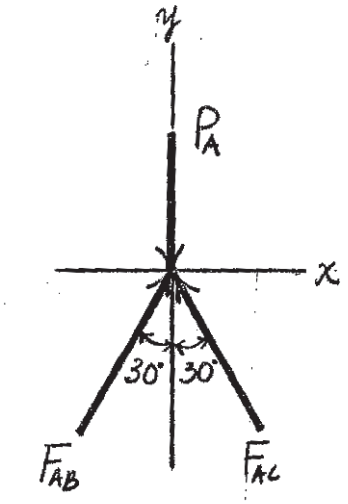


SOLUTION

Equilibrium. Referring to the free-body diagram of joint A, Fig. a

$$\stackrel{+}{\rightarrow}\Sigma Fx = 0; \quad F_{AB}\sin 30^\circ - F_{AC}\sin 30^\circ = 0 \qquad F_{AB} = F_{AC} = F$$

$$+\uparrow \Sigma F_y = 0; \quad 2F\cos 30^\circ - P_A = 0 \qquad P_A = 1.732F \qquad (1)$$



Compatibility Equation. From the geometry shown in Fig. b

$$\delta_F = \Delta_A \cos 30^\circ$$

$$\frac{F(0.4)}{\frac{\pi}{4}(0.02^2)[68.9(10^9)]} = \Delta_A \cos 30^\circ$$

$$\Delta_A = 21.3383(10^{-9})F$$

Thus, the equivalent spring constant for the system can be determined from

$$P_A = k\Delta_A$$

$$1.732F = k[21.3383(10^{-9})F]$$

$$k = 81.171(10^6)\ \text{N/m}$$

Maximum Stress. The maximum force that can be developed in members AB and AC is

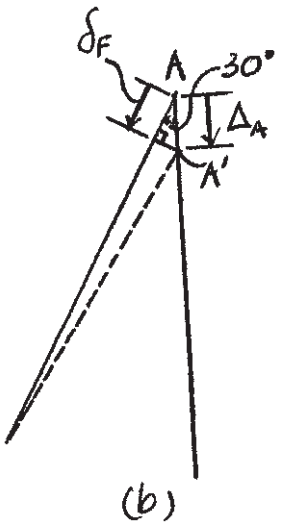
$$F_{\max} = \sigma_Y A = 255(10^6)\left[\frac{\pi}{4}(0.02^2)\right] = 80.11(10^3)\ \text{N}$$

From Eq. (1),

$$(P_A)_{\max} = 1.732F_{\max} = 1.732[80.11(10^3)] = 138.76(10^3)\ \text{N}$$

Then,

$$(\Delta_A)_{\max} = \frac{(P_A)_{\max}}{k} = \frac{138.76(10^3)}{81.171(10^6)} = 1.7094(10^{-3})\ \text{m}$$



Conservation of Energy.

$$mg[h + (\Delta_A)_{\max}] = \frac{1}{2}k(\Delta_A)_{\max}^2$$

$$50(9.81)[h + 1.7094(10^{-3})] = \frac{1}{2}[81.171(10^6)][1.7094(10^{-3})]^2$$

$$h = 0.240\ \text{m} \qquad \textbf{Ans.}$$

Ans:

$h = 0.240$ m

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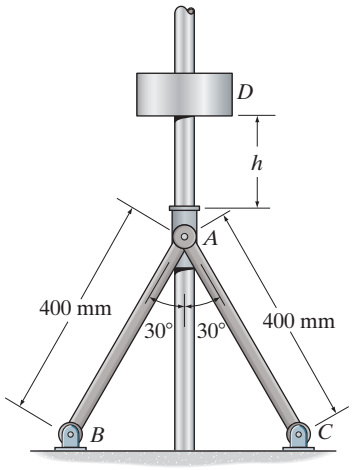
14–46. Rods AB and AC have a diameter of 20 mm and are made of 6061-T6 aluminum alloy. They are connected to the rigid collar A which slides freely along the vertical guide rod. If the 50-kg block D is dropped from height $h = 200$ mm above the collar, determine the maximum normal stress developed in the rods.



Equilibrium. Referring to the free-body diagram of joint A, Fig. a

$$\stackrel{+}{\rightarrow}\Sigma F_x = 0; \qquad F_{AB}\sin 30° - F_{AC}\sin 30° = 0 \qquad F_{AB} = F_{AC} = F$$

$$+\uparrow \Sigma F_y = 0; \qquad 2F\cos 30° - P_A = 0 \qquad P_A = 1.732F \qquad (1)$$

Compatibility Equation. From the geometry shown in Fig. b

$$\delta_F = \Delta_A \cos 30°$$

$$\frac{F(0.4)}{\frac{\pi}{4}(0.02^2)[68.9(10^9)]} = \Delta_A \cos 30°$$

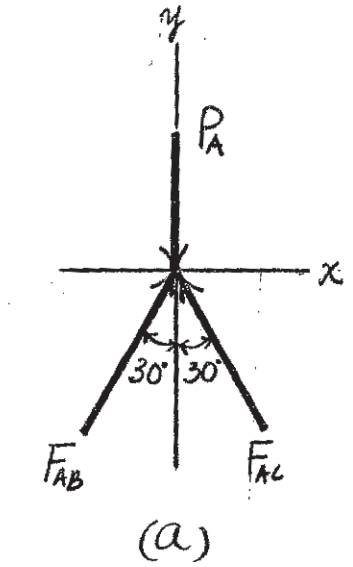
$$\Delta_A = 21.3383(10^{-9})F$$

Thus, the equivalent spring constant for the system can be determined from

$$P_A = k\Delta_A$$

$$1.732F = k[21.3383(10^{-9})F]$$

$$k = 81.171(10^6)\,\text{N/m}$$



(a)

Conservation of Energy.

$$mg[h + (\Delta_A)_{\max}] = \frac{1}{2}k(\Delta_A)_{\max}^2$$

$$50(9.81)[0.2 + (\Delta_A)_{\max}] = \frac{1}{2}[81.171(10^6)](\Delta_A)_{\max}^2$$

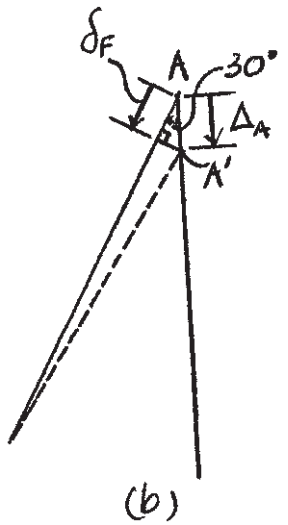
$$40.5855(10^6)(\Delta_A)_{\max}^2 - 490.5(\Delta_A)_{\max} - 98.1 = 0$$

Solving for the positive root,

$$(\Delta_A)_{\max} = 1.5608(10^{-3})\,\text{m}$$

Then,

$$(P_A)_{\max} = k(\Delta_A)_{\max} = 81.171(10^6)[1.5608(10^{-3})] = 126.69(10^3)\,\text{N}$$



(b)

Maximum Stress. From Eq. (1),

$$(P_A)_{\max} = 1.732F_{\max}$$

$$F_{\max} = 73.143(10^3)\,\text{N}$$

Thus, the maximum normal stress developed in members AB and AC is

$$(\sigma_{\max})_{AB} = (\sigma_{\max})_{AC} = \frac{F_{\max}}{A} = \frac{73.143(10^3)}{\frac{\pi}{4}(0.02^2)} = 232.82\,\text{MPa} = 233\,\text{MPa} \qquad \textbf{Ans.}$$

Since $(\sigma_{\max})_{AB} = (\sigma_{\max})_{AC} < \sigma_Y = 255$ MPa, this result is valid.

Ans:
$(\sigma_{\max})_{AB} = (\sigma_{\max})_{AC} = 233$ MPa

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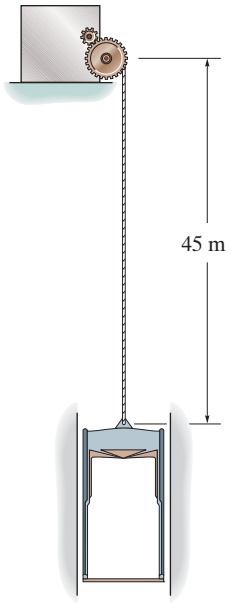
14–47. A steel cable having a diameter of 16 mm wraps over a drum and is used to lower an elevator having a mass of 400 kg. The elevator is 45 m below the drum and is descending at the constant rate of 0.6 m/s when the drum suddenly stops. Determine the maximum stress developed in the cable when this occurs. E_{st} = 200 GPa, when σ_Y = 350 MPa.



SOLUTION

$$k = \frac{AE}{L} = \frac{\left[\frac{\pi}{4}(0.016^2)\right][200(10^9)]}{45} = 893.61(10^3)\ \text{N/m}$$

$$U_e = U_i$$

$$\frac{1}{2}mv^2 + mg\,\Delta_{max} = \frac{1}{2}k\Delta_{max}^2$$

$$\frac{1}{2}(400)(0.6^2) + 400(9.81)\Delta_{max} = \frac{1}{2}[893.61(10^3)]\Delta_{max}^2$$

$$446.80(10^3)\Delta_{max}^2 - 3924\Delta_{max} - 72 = 0$$

$$\Delta_{max} = 0.017823\ \text{m}$$

$$P_{max} = k\Delta_{max} = [893.61(10^3)](0.017823) = 15.927(10^3)\ \text{N}$$

$$\sigma_{max} = \frac{P_{max}}{A} = \frac{15.927(10^3)}{\frac{\pi}{4}(0.016^2)} = 79.22(10^6)\ \text{N/m}^2$$

$$= 79.22\ \text{MPa} < \sigma_Y \quad \text{O.K.}$$

Ans.

Ans:
$\sigma_{max} = 79.2$ MPa

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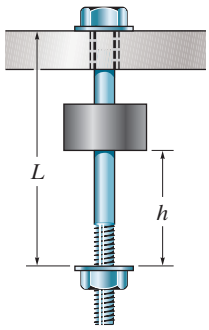
***14–48.**

The A-36 steel bolt is required to absorb the energy of a 2-kg mass that falls $h = 30$ mm. If the bolt has a diameter of 4 mm, determine its required length L so the stress in the bolt does not exceed 150 MPa.



SOLUTION

Maximum Stress: With $\Delta_{st} = \dfrac{WL}{AE} = \dfrac{2(9.81)(L)}{\frac{\pi}{4}(0.004^2)\left[200(10^9)\right]}$

$= 7.80655\,(10^{-6})L$ and $\sigma_{st} = \dfrac{W}{A} = \dfrac{2(9.81)}{\frac{\pi}{4}(0.004^2)} = 1.56131$ MPa, we have

$$\sigma_{max} = n\sigma_{st} \quad \text{where } n = 1 + \sqrt{1 + 2\left(\frac{h}{\Delta_{st}}\right)}$$

$$150(10^6) = \left[1 + \sqrt{1 + 2\left(\frac{0.03}{7.80655(10^{-6})L}\right)}\right]\left[1.56131(10^6)\right]$$

$$L = 0.8504 \text{ m} = 850 \text{ mm} \qquad \textbf{Ans.}$$

Ans:
$L = 850$ mm

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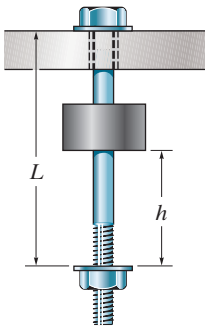
14–49.

The A-36 steel bolt is required to absorb the energy of a 2-kg mass that falls $h = 30$ mm. If the bolt has a diameter of 4 mm and a length of $L = 200$ mm, determine if the stress in the bolt will exceed 175 MPa.



SOLUTION

Maximum Stress: With

$$\Delta_{st} = \frac{WL}{AE} = \frac{2(9.81)(0.2)}{\frac{\pi}{4}(0.004^2)\left[200(10^9)\right]} = 1.56131(10^{-6})\ \text{m}$$

$$\sigma_{st} = \frac{W}{A} = \frac{2(9.81)}{\frac{\pi}{4}(0.004^2)} = 1.56131\ \text{MPa}$$

Applying Eq. 14–34, we have

$$n = 1 + \sqrt{1 + 2\left(\frac{h}{\Delta_{st}}\right)} = 1 + \sqrt{1 + 2\left(\frac{0.03}{1.56131(10^{-6})}\right)} = 197.04$$

Thus,

$$\sigma_{max} = n\sigma_{st} = 197.04(1.56131) = 307.6\ \text{MPa}$$

Yes, σ_{max} exceeded 175 MPa. **Ans.**

Ans:
Yes

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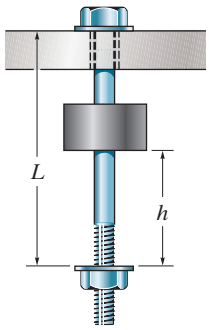
14–50.

The A-36 steel bolt is required to absorb the energy of a 2-kg mass that falls along the 4-mm-diameter bolt shank that is 150 mm long. Determine the maximum height h of release so the stress in the bolt does not exceed 150 MPa.



SOLUTION

Maximum Stress: With $\Delta_{st} = \dfrac{WL}{AE} = \dfrac{2(9.81)(0.15)}{\frac{\pi}{4}(0.004^2)\left[200(10^9)\right]}$

$= 1.17098(10^{-6})$ m and $\sigma_{st} = \dfrac{W}{A} = \dfrac{2(9.81)}{\frac{\pi}{4}(0.004^2)} = 1.56131$ MPa,

we have

$$\sigma_{max} = n\sigma_{st} \quad \text{where } n = 1 + \sqrt{1 + 2\left(\frac{h}{\Delta_{st}}\right)}$$

$$150(10^6) = \left[1 + \sqrt{1 + 2\left(\frac{h}{1.17098(10^{-6})}\right)}\right]\left[1.56131(10^6)\right]$$

$$h = 5.292(10^{-3})\text{ m} = 5.29\text{ mm} \qquad \textbf{Ans.}$$

Ans:
$h = 5.29$ mm

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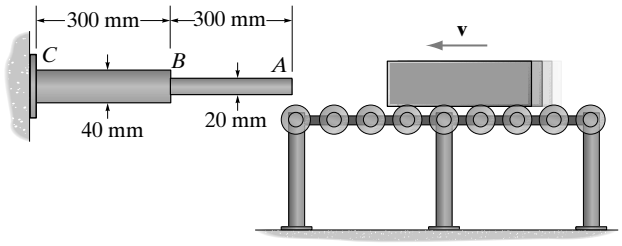
14–51. The 5-kg block is traveling with the speed of $v = 4$ m/s just before it strikes the 6061-T6 aluminum stepped cylinder. Determine the maximum normal stress developed in the cylinder.



SOLUTION

Equilibrium. The equivalent spring constant for segments AB and BC are

$$k_{AB} = \frac{A_{AB}E}{L_{AB}} = \frac{\frac{\pi}{4}\left(0.02^2\right)\left[68.9\left(10^9\right)\right]}{0.3} = 72.152\left(10^6\right)\text{N/m}$$

$$k_{BC} = \frac{A_{BC}E}{L_{BC}} = \frac{\frac{\pi}{4}\left(0.04^2\right)\left[68.9\left(10^9\right)\right]}{0.3} = 288.608\left(10^6\right)\text{N/m}$$

Equilibrium requires

$$F_{AB} = F_{BC}$$

$$k_{AB}\,\Delta_{AB} = k_{BC}\Delta_{BC}$$

$$72.152\left(10^6\right)\Delta_{AB} = 288.608\left(10^6\right)\Delta_{BC}$$

$$\Delta_{BC} = \frac{1}{4}\,\Delta_{AB} \qquad \textbf{(1)}$$

Conservation of Energy.

$$U_e = U_i$$

$$\frac{1}{2}mv^2 = \frac{1}{2}\,k_{AB}{\Delta_{AB}}^2 + \frac{1}{2}\,k_{BC}{\Delta_{BC}}^2 \qquad \textbf{(2)}$$

Substituting Eq. (1) into Eq. (2),

$$\frac{1}{2}\,mv^2 = \frac{1}{2}\,k_{AB}{\Delta_{AB}}^2 + \frac{1}{2}\,k_{BC}\left(\frac{1}{4}\Delta_{AB}\right)^2$$

$$\frac{1}{2}\,mv^2 = \frac{1}{2}\,k_{AB}{\Delta_{AB}}^2 + \frac{1}{32}\,k_{BC}{\Delta_{AB}}^2$$

$$\frac{1}{2}(5)\left(4^2\right) = \frac{1}{2}\left[72.152\left(10^6\right)\right]{\Delta_{AB}}^2 + \frac{1}{32}\left[288.608\left(10^6\right)\right]{\Delta_{AB}}^2$$

$$\Delta_{AB} = 0.9418\left(10^{-3}\right)\text{ m}$$

Maximum Stress. The force developed in segment AB is $F_{AB} = k_{AB}\Delta_{AB} = 72.152\left(10^6\right)\left[0.9418\left(10^{-3}\right)\right] = 67.954\left(10^3\right)\text{N}$.

Thus,

$$\sigma_{\max} = \sigma_{AB} = \frac{F_{AB}}{A_{AB}} = \frac{67.954\left(10^3\right)}{\frac{\pi}{4}\left(0.02^2\right)} = 216.30\text{ MPa} = 216\text{ MPa} \qquad \textbf{Ans.}$$

Since $\sigma_{\max} < \sigma_Y = 255$ MPa, this result is valid.

Ans:
$\sigma_{\max} = 216$ MPa

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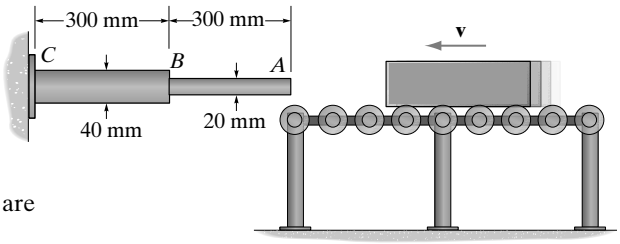
***14–52.** Determine the maximum speed v of the 5-kg block without causing the 6061-T6 aluminum stepped cylinder to yield after it is struck by the block.



SOLUTION

Equilibrium. The equivalent spring constant for segments AB and BC are

$$k_{AB} = \frac{A_{AB}E}{L_{AB}} = \frac{\frac{\pi}{4}\left(0.02^2\right)\left[68.9\left(10^9\right)\right]}{0.3} = 72.152\left(10^6\right)\text{N/m}$$

$$k_{BC} = \frac{A_{BC}E}{L_{BC}} = \frac{\frac{\pi}{4}\left(0.04^2\right)\left[68.9\left(10^9\right)\right]}{0.3} = 288.608\left(10^6\right)\text{N/m}$$

Equilibrium requires

$$F_{AB} = F_{BC}$$

$$k_{AB}\Delta_{AB} = k_{BC}\Delta_{BC}$$

$$72.152\left(10^6\right)\Delta_{AB} = 288.608\left(10^6\right)\Delta_{BC}$$

$$\Delta_{BC} = \frac{1}{4}\Delta_{AB} \qquad \textbf{(1)}$$

Conservation of Energy.

$$U_e = U_i$$

$$\frac{1}{2}mv^2 = \frac{1}{2}k_{AB}{\Delta_{AB}}^2 + \frac{1}{2}k_{BC}{\Delta_{BC}}^2 \qquad \textbf{(2)}$$

Substituting Eq. (1) into Eq. (2),

$$\frac{1}{2}mv^2 = \frac{1}{2}k_{AB}{\Delta_{AB}}^2 + \frac{1}{2}k_{BC}\left(\frac{1}{4}\Delta_{AB}\right)^2$$

$$\frac{1}{2}mv^2 = \frac{1}{2}k_{AB}{\Delta_{AB}}^2 + \frac{1}{32}k_{BC}{\Delta_{AB}}^2$$

$$\frac{1}{2}(5)v^2 = \frac{1}{2}\left[72.152\left(10^6\right)\right]{\Delta_{AB}}^2 + \frac{1}{32}\left[288.608\left(10^6\right)\right]{\Delta_{AB}}^2$$

$$\Delta_{AB} = 0.23545\left(10^{-3}\right)v$$

Maximum Stress. The force developed in segment AB is $F_{AB} = k_{AB}\Delta_{AB} = 72.152\left(10^6\right)\left[0.23545\left(10^{-3}\right)v\right] = 16988.46v$.

Thus,

$$\sigma_{\max} = \sigma_{AB} = \frac{F_{AB}}{A_{AB}}$$

$$255\left(10^6\right) = \frac{16988.46v}{\frac{\pi}{4}\left(0.02^2\right)}$$

$$v = 4.716\text{ m/s} = 4.72\text{ m/s} \qquad \textbf{Ans.}$$

Ans.

$v = 4.72$ m/s

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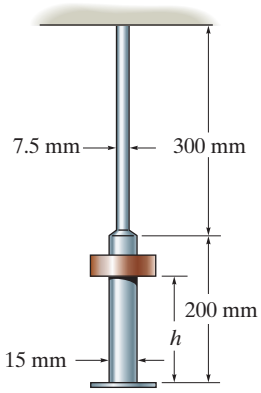
14–53.

The composite aluminum 2014-T6 bar is made from two segments having diameters of 7.5 mm and 15 mm. Determine the maximum axial stress developed in the bar if the 10-kg collar is dropped from a height of $h = 100$ mm.



SOLUTION

$$\Delta_{st} = \Sigma\frac{WL}{AE} = \frac{10(9.81)(0.3)}{\frac{\pi}{4}(0.0075)^2(73.1)(10^9)} + \frac{10(9.81)(0.2)}{\frac{\pi}{4}(0.015)^2(73.1)(10^9)}$$

$$= 10.63181147\,(10^{-6})\text{ m}$$

$$n = \left[1 + \sqrt{1 + 2\left(\frac{h}{\Delta_{st}}\right)}\right] = \left[1 + \sqrt{1 + 2\left(\frac{0.1}{10.63181147(10^{-6})}\right)}\right] = 138.16$$

$$\sigma_{max} = n\,\sigma_{st} \quad \text{Here } \sigma_{st} = \frac{W}{A} = \frac{10\,(9.81)}{\frac{\pi}{4}(0.0075^2)} = 2.22053\text{ MPa}$$

$$\sigma_{max} = 138.16\,(2.22053)$$

$$= 307\text{ MPa} < \sigma_Y = 414\text{ MPa} \quad \textbf{OK} \qquad \textbf{Ans.}$$

Ans:
$\sigma_{max} = 307$ MPa

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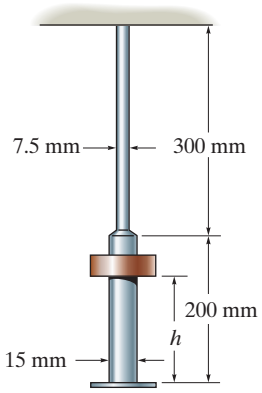
14–54.

The composite aluminum 2014-T6 bar is made from two segments having diameters of 7.5 mm and 15 mm. Determine the maximum height h from which the 10-kg collar should be dropped so that it produces a maximum axial stress in the bar of $\sigma_{max} = 300$ MPa.



SOLUTION

$$\Delta_{st} = \Sigma\frac{WL}{AE} = \frac{10(9.81)(0.3)}{\frac{\pi}{4}(0.0075)^2(73.1)(10^9)} + \frac{10(9.81)(0.2)}{\frac{\pi}{4}(0.015)^2(73.1)(10^9)}$$

$$= 10.63181147\,(10^{-6})\text{ m}$$

$$n = \left[1 + \sqrt{1 + 2\left(\frac{h}{\Delta_{st}}\right)}\right] = \left[1 + \sqrt{1 + 2\left(\frac{h}{10.63181147(10^{-6})}\right)}\right]$$

$$= \left[1 + \sqrt{1 + 188114.7\,h}\right]$$

$$\sigma_{max} = n\,\sigma_{st} \quad \text{Here } \sigma_{st} = \frac{W}{A} = \frac{10\,(9.81)}{\frac{\pi}{4}(0.0075^2)} = 2.22053\text{ MPa}$$

$$300\,(10^6) = \left[1 + \sqrt{1 + 188114.7\,h}\right](2220530)$$

$h = 0.09559\text{ m} = 95.6\text{ mm}$ **Ans.**

Ans:
$h = 95.6$ mm

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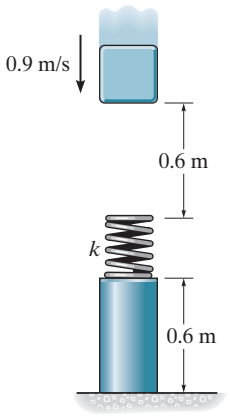
14–55. The 25-kg block is failing at 0.9 m/s at the instant it is 0.6 m above the spring and post assembly. Determine the maximum stress in the post if the spring has a stiffness of $k = 40$ MN/m. The post has a diameter of 75 mm and a modulus of elasticity of $E = 48$ GPa. Assume the material will not yield.



SOLUTION

Equilibrium: This requires $F_{sp} = F_P$. Hence

$$k_{sp}\Delta_{sp} = k_P\Delta_P \qquad \text{and} \qquad \Delta_{sp} = \frac{k_p}{k_{sp}}\Delta_P \qquad \textbf{[1]}$$

Conservation of Energy: The equivalent spring constant for the post is

$$k_p = \frac{AE}{L} = \frac{\left[\frac{\pi}{4}(0.075^2)\right][48(10^9)]}{0.6} = 353.43(10^6)\ \text{N/m}$$

$$U_e = U_i$$

$$\frac{1}{2}mv^2 + W(h + \Delta_{\max}) = \frac{1}{2}k_P\Delta_P^2 + \frac{1}{2}k_{sp}\Delta_{sp}^2 \qquad \textbf{[2]}$$

However, $\Delta_{\max} = \Delta_P + \Delta_{sp}$. Then, Eq. [2] becomes

$$\frac{1}{2}mv^2 + W\left(h + \Delta_P + \Delta_{sp}\right) = \frac{1}{2}k_P\Delta_P^2 + \frac{1}{2}k_{sp}\Delta_{sp}^2 \qquad \textbf{[3]}$$

Substituting Eq. [1] into [3] yields

$$\frac{1}{2}mv^2 + W\left(h + \Delta_P + \frac{k_p}{k_{sp}}\Delta_P\right) = \frac{1}{2}k_p\Delta_P^2 + \frac{1}{2}\left(\frac{k_P^2}{k_{sp}}\Delta_P^2\right)$$

$$\frac{1}{2}(25)(0.9^2) + [25(9.81)]\left[0.6 + \Delta_p + \frac{353.43(10^6)}{40(10^6)}\Delta_p\right]$$

$$= \frac{1}{2}[353.43(10^6)]\Delta_p^2 + \frac{1}{2}\left\{\frac{[353.43(10^6)]^2}{40(10^6)}\right\}\Delta_p^2$$

$$1.73812(10^9)\Delta_p^2 - 2412.21\Delta_p - 157.275 = 0$$

Solving for positive root, we have

$$\Delta_P = 0.301503(10^{-3})\ \text{m}$$

Maximum Stress: The maximum axial force for the post is $P_{\max} = k_p\Delta_p$ $= [353.43(10^6)][0.301503(10^{-3})] = 106.56(10^3)$

$$\sigma_{\max} = \frac{P_{\max}}{A} = \frac{106.56(10^3)}{\frac{\pi}{4}(0.075^2)} = 24.12(10^6)\ \text{N/m}^2 = 24.1\ \text{MPa} \qquad \textbf{Ans.}$$

Ans:

$\sigma_{\max} = 24.1$ MPa

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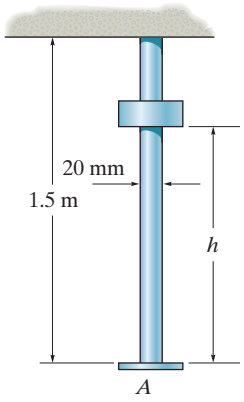
***14–56.**

The collar has a mass of 5 kg and falls down the titanium Ti-6A1-4V bar. If the bar has a diameter of 20 mm, determine the maximum stress developed in the bar if the weight is (a) dropped from a height of $h = 1$ m, (b) released from a height $h \approx 0$, and (c) placed slowly on the flange at A.



SOLUTION

Maximum Stress: With $\Delta_{st} = \dfrac{WL}{AE} = \dfrac{5(9.81)(1.5)}{\frac{\pi}{4}(0.02^2)\left[120(10^9)\right]} = 1.9516(10^{-6})$ m

and $\sigma_{st} = \dfrac{W}{A} = \dfrac{5(9.81)}{\frac{\pi}{4}(0.02^2)} = 0.156131$ MPa and Applying Eq. 14–34, we have

a)

$$n = 1 + \sqrt{1 + 2\left(\frac{h}{\Delta_{st}}\right)} = 1 + \sqrt{1 + 2\left(\frac{1}{1.9516(10^{-6})}\right)} = 1013.31$$

Thus,

$$\sigma_{max} = n\sigma_{st} = 1013.31(0.156131) = 158 \text{ MPa} \qquad \textbf{Ans.}$$

b)

$$n = 1 + \sqrt{1 + 2\left(\frac{h}{\Delta_{st}}\right)} = 1 + \sqrt{1 + 2\left(\frac{0}{1.9516(10^{-6})}\right)} = 2$$

Thus,

$$\sigma_{max} = n\sigma_{st} = 2(0.156131) = 0.312 \text{ MPa} \qquad \textbf{Ans.}$$

c)

$$\sigma_{max} = \sigma_{st} = 0.156 \text{ MPa} \qquad \textbf{Ans.}$$

Since all of the $\sigma_{max} < \sigma_Y = 924$ MPa, the above analysis is valid.

Ans:
a) $\sigma_{max} = 158$ MPa,
b) $\sigma_{max} = 0.312$ MPa,
c) $\sigma_{max} = 0.156$ MPa

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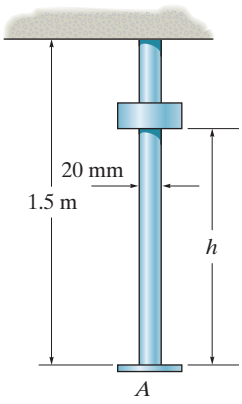
14–57.

The collar has a mass of 5 kg and falls down the titanium Ti-6A1-4V bar. If the bar has a diameter of 20 mm, determine if the weight can be released from rest at any point along the bar and not permanently damage the bar after striking the flange at A.



SOLUTION

Maximum Stress: With $\Delta_{st} = \dfrac{WL}{AE} = \dfrac{5(9.81)(1.5)}{\frac{\pi}{4}(0.02^2)\left[120(10^9)\right]} = 1.9516(10^{-6})$ m, $\sigma_{st} = \dfrac{W}{A} = \dfrac{5(9.81)}{\frac{\pi}{4}(0.02^2)} = 0.156131$ MPa and $h = h_{max} = 1.5$ m. Applying Eq. 14–34, we have

$$n = 1 + \sqrt{1 + 2\left(\frac{h}{\Delta_{st}}\right)} = 1 + \sqrt{1 + 2\left(\frac{1.5}{1.9516(10^{-6})}\right)} = 1240.83$$

Thus,

$$\sigma_{max} = n\sigma_{st} = 1240.83(0.156131) = 193.7 \text{ MPa}$$

Since $\sigma_{max} < \sigma_Y = 924$ MPa, the weight can be released from rest at any position along the bar without causing permanent damage to the bar. **Ans.**

Ans:
Yes, from any position

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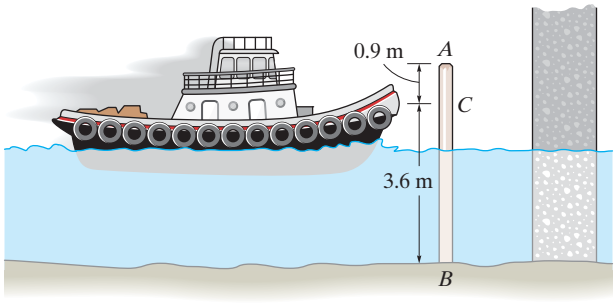
14–58. The tugboat has a mass of 60 tonnes and is traveling forward at 0.6 m/s when it strikes the 300-mm-diameter fender post AB used to protect a bridge pier. If the post is made from treated white spruce and is assumed fixed at the river bed, determine the maximum horizontal distance the top of the post will move due to the impact. Assume the tugboat is rigid and neglect the effect of the water.



SOLUTION

From Appendix C:

$$P_{\max} = \frac{3EI(\Delta_C)_{\max}}{(L_{BC})^3}$$

Conservation of energy:

$$\frac{1}{2}mv^2 = \frac{1}{2}P_{\max}(\Delta_C)_{\max}$$

$$\frac{1}{2}mv^2 = \frac{1}{2}\left[\frac{3EI(\Delta_C)^2_{\max}}{(L_{BC})^3}\right]$$

$$(\Delta_C)_{\max} = \sqrt{\frac{mv^2L^3_{BC}}{3EI}}$$

$$(\Delta_C)_{\max} = \sqrt{\frac{60(10^3)(0.6)^2(3.6)^3}{3(9.65)(10^9)(\frac{\pi}{64})(0.3)^4}} = 0.29589\text{ m} = 295.89\text{ mm}.$$

$$P_{\max} = \left[\frac{3(9.65)(10^9)\left(\frac{\pi}{64}\right)(0.3)^4}{(3.6)^3}\right](0.29589) = 73.00(10^3)\text{ N}$$

$$\theta_C = \frac{P_{\max}L^2_{BC}}{2EI} = \frac{73.00(10^3)(3.6)^2}{2(9.65)(10^9)\left(\frac{\pi}{64}\right)(0.3)^4} = 0.12329\ \pi\text{ad}$$

$$(\Delta_A)_{\max} = (\Delta_C)_{\max} + \theta_C(L_{CA})$$

$$(\Delta_A)_{\max} = 295.89 + 0.12329(0.9)(10^3) = 406.74\text{ mm} = 407\text{ mm}$$ **Ans.**

Ans:

$(\Delta_A)_{\max} = 407\text{ mm}$

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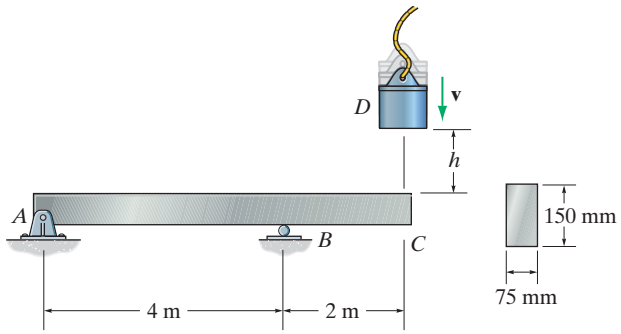
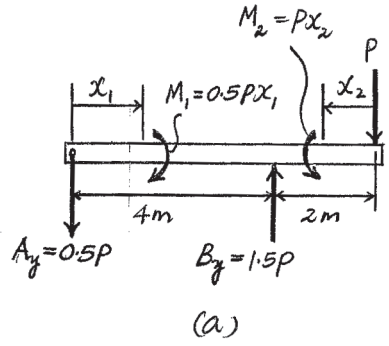
14–59.

The overhang beam is made of 2014-T6 aluminum. If the 75-kg block has a speed of $v = 3$ m/s at $h = 0.75$ m, determine the maximum bending stress in the beam.



SOLUTION

Equilibrium: The support reactions and the moment function for regions AB and BC of the beam under static conditions are indicated on the free-body diagram of the beam, Fig. *a*.



$$U_e = U_i$$

$$\frac{1}{2}P\Delta_{st} = \Sigma\int_0^L \frac{M^2 dx}{2EI}$$

$$\frac{1}{2}P\Delta_{st} = \frac{1}{2EI}\left[\int_0^{4\text{ m}} (0.5Px_1)^2 dx_1 + \int_0^{2\text{ m}} (Px_2)^2 dx_2\right]$$

$$\frac{1}{2}P\Delta_{st} = \frac{1}{2EI}\left[\left(\frac{0.25}{3}P^2x_1^3\right)\Big|_0^{4\text{ m}} + \frac{P^2}{3}x_2^3\Big|_0^{2\text{ m}}\right]$$

$$\Delta_{st} = \frac{8P}{EI}$$

$I = \frac{1}{12}(0.075)(0.15^3) = 21.09375(10^{-6})\text{ m}^4$ and $E = E_{al} = 73.1$ GPa. Then, the equivalent spring constant can be determined from

$$P = k\Delta_{st}$$

$$P = k\left(\frac{8P}{EI}\right)$$

$$k = \frac{EI}{8} = \frac{73.1(10^9)[21.09375(10^{-6})]}{8}$$

$$= 192.74(10^3)\text{ N/m}$$

Conservation of Energy:

$$U_e = U_i$$

$$\frac{1}{2}mv^2 + mg(h + \Delta_{max}) = \frac{1}{2}k\Delta_{max}^2$$

$$\frac{1}{2}(75)(3^2) + 75(9.81)(0.75 + \Delta_{max}) = \frac{1}{2}\left[192.74(10^3)\right]\Delta_{max}^2$$

$$96372.07\Delta_{max}^2 - 735.75\Delta_{max} - 889.3125 = 0$$

$$\Delta_{max} = 0.09996\text{ m}$$

Maximum Bending Stress: The maximum force on the beam is $P_{max} = k\Delta_{max} = 192.74(10^3)[0.09996] = 19.266(10^3)$ N. The maximum moment occurs at support B. Thus, $M_{max} = P_{max}(2) = 19.266(10^3)(2) = 38.531(10^3)\text{ N}\cdot\text{m}$.

Applying the flexure formula,

$$\sigma_{max} = \frac{M_{max}c}{I} = \frac{38.531(10^3)(0.15/2)}{21.09375(10^{-6})} = 137\text{ MPa}$$ **Ans.**

Since $\sigma_{max} < \sigma_Y = 414$ MPa, this result is valid.

Ans:
$\sigma_{max} = 137$ MPa

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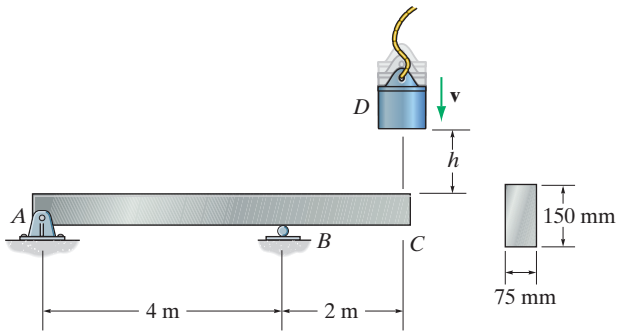
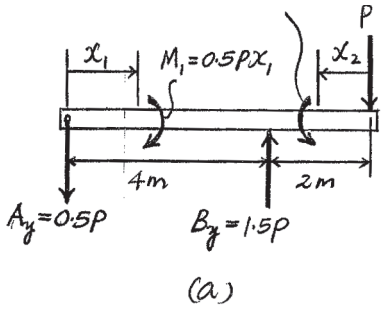
***14–60.**

The overhang beam is made of 2014-T6 aluminum. Determine the maximum height h from which the 100-kg block can be dropped from rest ($v = 0$), without causing the beam to yield.



SOLUTION

Equilibrium: The support reactions and the moment function for regions AB and BC of the beam under static conditions are indicated on the free-body diagram of the beam, Fig. *a*.



$$U_e = U_i$$

$$\frac{1}{2}P\Delta_{st} = \Sigma \int_0^L \frac{M^2 dx}{2EI}$$

$$\frac{1}{2}P\Delta_{st} = \frac{1}{2EI}\left[\int_0^{4\,m} (0.5Px_1)^2 dx_1 + \int_0^{2\,m} (Px_2)^2 dx_2\right]$$

$$\frac{1}{2}P\Delta_{st} = \frac{1}{2EI}\left[\left(\frac{0.25}{3}P^2x_1^3\right)\Big|_0^{4\,m} + \frac{P^2}{3}x_2^3\Big|_0^{2\,m}\right]$$

$$\Delta_{st} = \frac{8P}{EI}$$

$I = \frac{1}{12}(0.075)(0.15^3) = 21.09375(10^{-6})\ m^4$, $P = 100(9.81) = 981N$, and $E = E_{al} = 73.1$ GPa. Then,

$$\Delta_{st} = \frac{8(981)}{73.1(10^9)[21.09375(10^{-6})]} = 5.0896(10^{-3})\ m$$

Maximum Bending Stress: The maximum force on the beam is $P_{max} = nP = 981n$. The maximum moment occurs at support B. Thus, $M_{max} = P_{max}(2) = (981n)(2) = 1962n$. Applying the flexure formula,

$$\sigma_{max} = \frac{M_{max}c}{I}$$

$$414(10^6) = \frac{1962n(0.15/2)}{21.09375(10^{-6})}$$

$$n = 59.35$$

Impact Factor:

$$n = 1 + \sqrt{1 + 2\left(\frac{h}{\Delta_{st}}\right)}$$

$$59.35 = 1 + \sqrt{1 + 2\left[\frac{h}{5.0896(10^{-3})}\right]}$$

$$h = 8.66\ m \qquad \textbf{Ans.}$$

Ans:
$h = 8.66$ m

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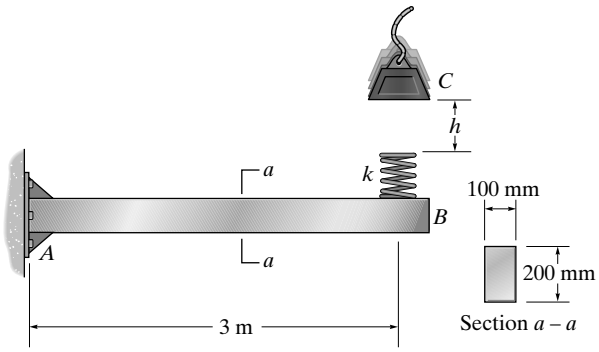
14–61. Block C of mass 50 kg is dropped from height $h = 0.9$ m onto the spring of stiffness $k = 150$ kN/m mounted on the end B of the 6061-T6 aluminum cantilever beam. Determine the maximum bending stress developed in the beam.



SOLUTION

Conservation of Energy. From the table listed in the appendix, the displacement of end B under static conditions is $\Delta_{st} = \frac{PL^3}{3EI}$. Thus, the equivalent spring constant for the beam is $k_b = \frac{3EI}{L^3}$, where $I = \frac{1}{12}(0.1)\left(0.2^3\right) = 66.6667\left(10^{-6}\right)\text{m}^4$, $L = 3$ m, and $E = E_{al} = 68.9$ GPa. Thus,

$$k_b = \frac{3EI}{L^3} = \frac{3\left[68.9\left(10^9\right)\right]\left[66.6667\left(10^{-6}\right)\right]}{3^3} = 510.37\left(10^3\right)\text{N/m}$$

Equilibrium requires,

$$F_{sp} = P$$

$$k_{sp}\Delta_{sp} = k_b\Delta_b$$

$$150\left(10^3\right)\Delta_{sp} = 510.37\left(10^3\right)\Delta_b$$

$$\Delta_{sp} = 3.4025\Delta_b \qquad \textbf{(1)}$$

We have,

$$U_e = U_i$$

$$mg\left(h + \Delta_{sp} + \Delta_b\right) = \frac{1}{2}k_b\,\Delta_b{}^2 + \frac{1}{2}k_{sp}\,\Delta_{sp}{}^2$$

Substituting Eq. (1) into this equation,

$$50(9.81)(0.9 + 3.4025\Delta_b + \Delta_b) = \frac{1}{2}\left[510.37\left(10^3\right)\right]\Delta_b{}^2 + \frac{1}{2}\left[150\left(10^3\right)\right](3.4025\Delta_b)^2$$

$$1123444.90\Delta_b{}^2 - 2159.41\Delta_b - 441.45 = 0$$

Solving for the positive root,

$$\Delta_b = 0.020807 \text{ m}$$

Maximum Stress. The maximum force on the beam is $P_{\max} = k_b\Delta_b = 510.37\left(10^3\right)(0.020807) = 10.619\left(10^3\right)$ N. The maximum moment occurs at fixed support A, where $M_{\max} = P_{\max}L = 10.619\left(10^3\right)(3) = 31.858\left(10^3\right)\text{N}\cdot\text{m}$. Applying the flexure formula with $c = \frac{0.2}{2} = 0.1$ m,

$$\sigma_{\max} = \frac{M_{\max}\,c}{I} = \frac{31.858\left(10^3\right)(0.1)}{66.6667\left(10^{-6}\right)} = 47.79 \text{ MPa} = 47.8 \text{ MPa} \qquad \textbf{Ans.}$$

Since $\sigma_{\max} < \sigma_Y = 255$ MPa, this result is valid.

Ans:

$\sigma_{\max} = 47.8$ MPa

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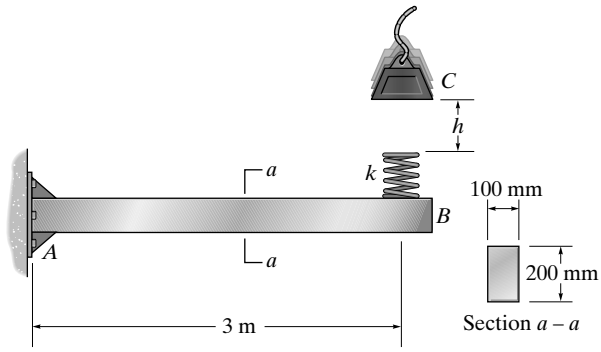
14–62. Determine the maximum height h from which 200-kg block C can be dropped without causing the 6061-T6 aluminum cantilever beam to yield. The spring mounted on the end B of the beam has a stiffness of $k = 150$ kN/m.



SOLUTION

Maximum Stress. From the table listed in the appendix, the displacement of end B under static conditions is $\Delta_{st} = \frac{PL^3}{3EI}$. Thus, the equivalent spring constant for the beam is $k_b = \frac{3EI}{L^3}$, where $I = \frac{1}{12}(0.1)\left(0.2^3\right) = 66.6667\left(10^{-6}\right)\text{m}^4$, $L = 3$ m, and $E = E_{al} = 68.9$ GPa. Thus,

$$k_b = \frac{3EI}{L^3} = \frac{3\left[68.9\left(10^9\right)\right]\left[66.6667\left(10^{-6}\right)\right]}{3^3} = 510.37\left(10^3\right)\text{N/m}$$

The maximum force on the beam is $P_{\max} = k_b\Delta_b = 510.37\left(10^3\right)\Delta_b$. The maximum moment occurs at the fixed support A, where $M_{\max} = P_{\max}L = 510.37\left(10^3\right)\Delta_b(3) = 1.5311\left(10^6\right)\Delta_b$. Applying the flexure formula with $\sigma_{\max} = \sigma_Y = 255$ MPa and $c = \frac{0.2}{2} = 0.1$ m,

$$\sigma_{\max} = \sigma_Y = \frac{M_{\max}\,c}{I}$$

$$255\left(10^6\right) = \frac{1.5311\left(10^6\right)\Delta_b(0.1)}{66.6667\left(10^{-6}\right)}$$

$$\Delta_b = 0.11103\text{ m}$$

Equilibrium requires,

$$F_{sp} = P$$

$$k_{sp}\Delta_{sp} = k_b\Delta_b$$

$$150\left(10^3\right)\Delta_{sp} = 510.37\left(10^3\right)(0.11103)$$

$$\Delta_{sp} = 0.37778\text{ m}$$

Conservation of Energy.

$$U_e = U_i$$

$$mg\left(h + \Delta_{sp} + \Delta_b\right) = \frac{1}{2}k_b\,\Delta_b{}^2 + \frac{1}{2}k_{sp}\,\Delta_{sp}{}^2$$

$$200(9.81)(h + 0.37778 + 0.11103) = \frac{1}{2}\left[510.37\left(10^3\right)\right](0.11103)^2 + \frac{1}{2}\left[150\left(10^3\right)\right](0.37778)^2$$

$h = 6.57$ m **Ans.**

Ans:

$h = 6.57$ m

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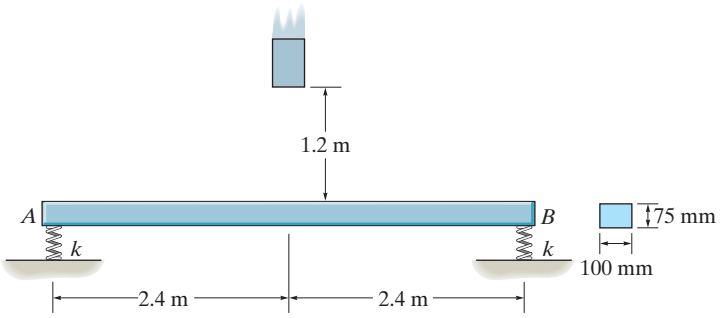
14–63. The weight of 90 kg is dropped from a height of 1.2 m from the top of the A-36 steel beam. Determine the maximum deflection and maximum stress in the beam if the supporting springs at A and B each have a stiffness of $k = 100$ kN/m. The beam is 75 mm thick and 100 mm wide.

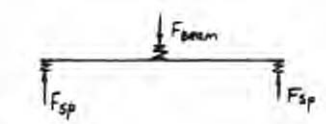


SOLUTION

From Appendix C:

$$\Delta_{\text{beam}} = \frac{PL^3}{48EI}$$

$$k_{\text{beam}} = \frac{48EI}{L^3} = \frac{48[200(10^9)]\left[\frac{1}{12}(0.1)(0.075^3)\right]}{4.8^3} = 305.18(10^3)\ \text{N/m}$$



From equilibrium (equivalent system):

$$2F_{\text{sp}} = F_{\text{beam}}$$

$$2k_{\text{sp}}\Delta_{\text{sp}} = k_{\text{beam}}\Delta_{\text{beam}}$$

$$\Delta_{\text{sp}} = \frac{305.18(10^3)}{2[100(10^3)]}\Delta_{\text{beam}}$$

$$\Delta_{\text{sp}} = 1.5259\,\Delta_{\text{beam}} \qquad \textbf{(1)}$$

Conservation of energy:

$$U_e = U_i$$

$$W(h + \Delta_{\text{sp}} + \Delta_{\text{beam}}) = \frac{1}{2}k_{\text{beam}}\Delta_{\text{beam}}^2 + 2\left(\frac{1}{2}\right)k_{\text{sp}}\Delta_{\text{sp}}^2$$

From Eq. (1):

$$90(9.81)[1.2 + 1.5259\Delta_{\text{beam}} + \Delta_{\text{beam}}] = \frac{1}{2}[305.18(10^3)]\Delta_{\text{beam}}^2 + [100(10^3)](1.5259\Delta_{\text{beam}})^2$$

$$385.4185(10^3)\Delta_{\text{beam}}^2 - 2230.0985\Delta_{\text{beam}} - 1059.48 = 0$$

$$\Delta_{\text{beam}} = 0.05540\ \text{m} = 55.40\ \text{mm}$$

From Eq. (1):

$$\Delta_{\text{sp}} = 84.54\ \text{mm}$$

$$\Delta_{\text{max}} = \Delta_{\text{sp}} + \Delta_{\text{beam}}$$

$$= 84.54 + 55.40 = 139.94\ \text{mm} = 140\ \text{mm} \qquad \textbf{Ans.}$$

$$F_{\text{beam}} = k_{\text{beam}}\Delta_{\text{beam}}$$

$$= [305.18(10^3)](0.05540) = 16.91(10^3)$$

$$M_{\text{max}} = \frac{F_{\text{beam}}L}{4} = \frac{[16.91(10^3)](4.8)}{4} = 20.29(10^3)\ \text{N}\cdot\text{m}$$

$$\sigma_{\text{max}} = \frac{M_{\text{max}}c}{I} = \frac{[20.29(10^3)](0.0375)}{\frac{1}{12}(0.1)(0.075^3)}$$

$$= 216.42(10^6)\ \text{N/m}^2 = 216\ \text{MPa} < \sigma_Y \quad \text{O.K.} \qquad \textbf{Ans.}$$

Ans:

$\sigma_{\text{max}} = 216\ \text{MPa}$

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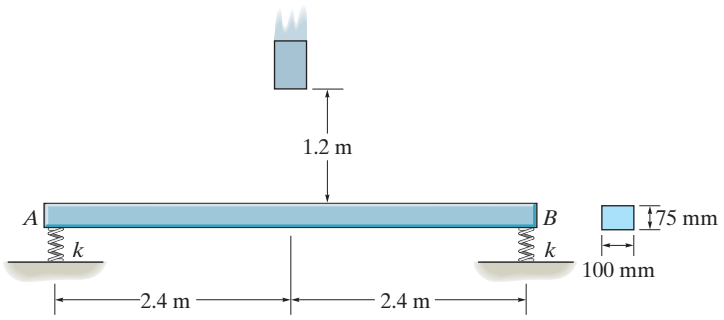
***14–64.** The weight of 90 kg is dropped from a height of 1.2 m from the top of the A-36 steel beam. Determine the load factor n if the supporting springs at A and B each have a stiffness of $k = 60$ kN/m. The beam is 75 mm thick and 100 mm wide.

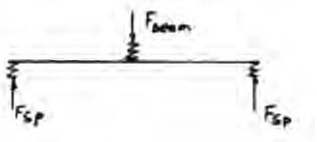


SOLUTION

From Appendix C:

$$\Delta_{\text{beam}} = \frac{PL^3}{48EI}$$

$$k_{\text{beam}} = \frac{48EI}{L^3} = \frac{48[200(10^9)]\left[\frac{1}{12}(0.1)(0.075^3)\right]}{4.8^3} = 305.18(10^3)\ \text{N/m}$$



From equilibrium (equivalent system):

$$2F_{\text{sp}} = F_{\text{beam}}$$

$$2k_{\text{sp}}\Delta_{\text{sp}} = k_{\text{beam}}\Delta_{\text{beam}}$$

$$\Delta_{\text{sp}} = \frac{305.18(10^3)}{2[60(10^3)]}\Delta_{\text{beam}}$$

$$\Delta_{\text{sp}} = 2.5431\ \Delta_{\text{beam}} \qquad \textbf{(1)}$$

Conservation of energy:

$$U_e = U_i$$

$$W(h + \Delta_{\text{beam}} + \Delta_{\text{sp}}) = \frac{1}{2}k_{\text{beam}}\Delta_{\text{beam}}^2 + 2\left(\frac{1}{2}\right)k_{\text{sp}}\Delta_{\text{sp}}^2$$

From Eq. (1):

$$90(9.81)[1.2 + \Delta_{\text{beam}} + 2.5431\Delta_{\text{beam}}] = \frac{1}{2}[305.18(10^3)]\Delta_{\text{beam}}^2 + [60(10^3)](2.5431\Delta_{\text{beam}})^2$$

$$540.6359(10^3)\Delta_{\text{beam}}^2 - 3128.23\Delta_{\text{beam}} - 1059.48 = 0$$

$$\Delta_{\text{beam}} = 0.047256\ \text{m}$$

$$F_{\text{beam}} = k_{\text{beam}}\Delta_{\text{beam}}$$

$$= [305.18(10^3)][0.047256] = 14.42(10^3)\ \text{N}$$

$$n = \frac{14.42(10^3)}{90(9.81)} = 16.33 = 16.3 \qquad \textbf{Ans.}$$

$$\sigma_{\max} = n(\sigma_{\text{st}})_{\max} = n\left(\frac{Mc}{I}\right)$$

$$M = \frac{90(9.81)(4.8)}{4} = 1059.48\ \text{N}\cdot\text{m}$$

$$\sigma_{\max} = 16.33\left[\frac{1059.48(0.0375)}{\frac{1}{12}(0.1)(0.075^3)}\right]$$

$$= 184.59(10^6)\ \text{N/m}^2 = 184.59\ \text{MPa} < \sigma_Y \quad \text{O.K.}$$

Ans:

$\sigma_{\max} = 184.59$ MPa

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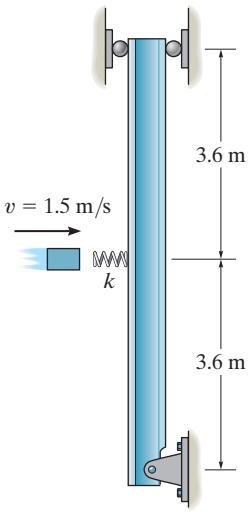
14–65. The simply supported W250 × 22 structural A-36 steel beam lies in the horizontal plane and acts as a shock absorber for the 250-kg block which is traveling toward it at 1.5 m/s. Determine the maximum deflection of the beam and the maximum stress in the beam during the impact. The spring has a stiffness of $k = 200$ kN/m.



For W250 × 22: $I = 28.8(10^6)\ \text{mm}^4 = 28.8(10^{-6})\ \text{m}^4$ $d = 254\ \text{mm} = 0.254\ \text{m}$

From Appendix C:

$$\Delta_{\text{beam}} = \frac{PL^3}{48EI}$$

$$k_{\text{beam}} = \frac{48EI}{L^3} = \frac{48[200(10^9)][28.8(10^{-6})]}{7.2^3} = 740.74(10^3)\ \text{N/m}$$

Equilibrium (equivalent system):

$$F_{\text{sp}} = F_{\text{beam}}$$

$$k_{\text{sp}}\Delta_{\text{sp}} = k_{\text{beam}}\Delta_{\text{beam}}$$

$$\Delta_{\text{sp}} = \frac{740.74(10^3)}{200(10^3)}\Delta_{\text{beam}}$$

$$\Delta_{\text{sp}} = 3.7037\,\Delta_{\text{beam}} \qquad \textbf{(1)}$$

Conservation of energy:

$$U_e = U_i$$

$$\frac{1}{2}mv^2 = \frac{1}{2}k_{\text{beam}}\,\Delta_{\text{beam}}^2 + \frac{1}{2}k_{\text{sp}}\,\Delta_{\text{sp}}^2$$

From Eq. (1):

$$\frac{1}{2}(250)(1.5^2) = \frac{1}{2}[740.74(10^3)]\Delta_{\text{beam}}^2 + \frac{1}{2}[200(10^3)](3.7037\Delta_{\text{beam}})^2$$

$$1.74211(10^6)\Delta_{\text{beam}}^2 = 281.25$$

$$\Delta_{\text{beam}} = 0.01271\ \text{m} = 12.7\ \text{mm} \qquad \textbf{Ans.}$$

$$F_{\text{beam}} = k_{\text{beam}}\,\Delta_{\text{beam}}$$

$$= [740.74(10^3)][0.01271] = 9.4118(10^3)\ \text{N} = 9.41\ \text{kN}$$

$$M_{\max} = \frac{[9.4118(10^3)](7.2)}{4} = 16.941(10^3)\ \text{N}\cdot\text{m}$$

$$\sigma_{\max} = \frac{M_{\max}c}{I} = \frac{[16.941(10^3)](0.25412)}{28.8(10^{-6})}$$

$$= 74.71(10^6)\ \text{N/m}^2 = 74.7\ \text{MPa} < \sigma_Y \quad \text{O.K.} \qquad \textbf{Ans.}$$

Ans:

$\Delta_{\text{beam}} = 12.7$ mm, $\sigma_{\max} = 74.7$ MPa

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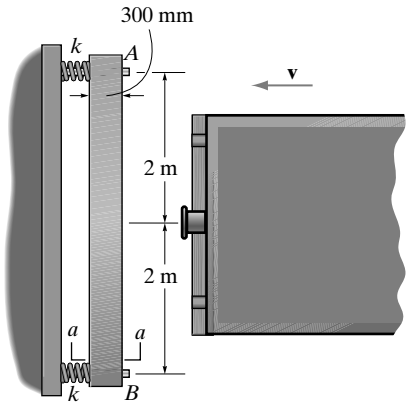
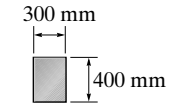
14–66. The 2014-T6 aluminum bar AB can slide freely along the guides mounted on the rigid crash barrier. If the railcar of mass 10 Mg is traveling with a speed of $v = 1.5$ m/s, determine the maximum bending stress developed in the bar. The springs at A and B have a stiffness of $k = 15$ MN/m.





Section $a-a$

SOLUTION

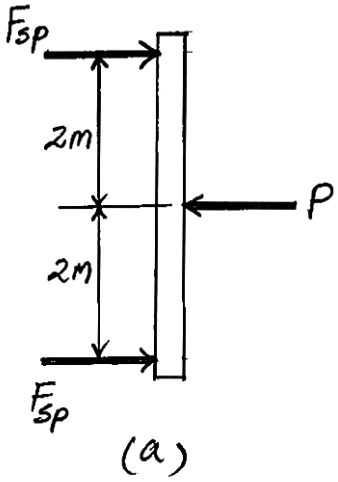
Equilibrium. Referring to the free-body diagram of the bar for static conditions, Fig. *a*,

$$\stackrel{+}{\rightarrow} \Sigma F_x = 0; \qquad 2F_{sp} - P = 0 \qquad F_{sp} = \frac{P}{2} \qquad \textbf{(1)}$$

Referring to the table listed in the appendix, the displacement of the bar at the position where **P** is applied under static conditions is $\Delta_{st} = \frac{PL^3}{48EI}$. Thus, the equivalent spring constant for the bar is $k_b = \frac{48EI}{L^3}$, where $I = \frac{1}{12}(0.4)\left(0.3^3\right) = 0.9\left(10^{-3}\right)\text{m}^4$, $L = 4$ m, and $E = E_{al} = 73.1$ GPa. Thus,

$$k_b = \frac{48\left[73.1\left(10^9\right)\right]\left[0.9\left(10^{-3}\right)\right]}{4^3} = 49.3425\left(10^6\right)\text{ N/m}$$

Using Eq. (1)



$$F_{sp} = \frac{P}{2}$$

$$k_{sp}\Delta_{sp} = \frac{1}{2}k_b\Delta_b$$

$$\Delta_{sp} = \frac{1}{2}\left(\frac{k_b}{k_{sp}}\right)\Delta_b = \frac{1}{2}\left[\frac{49.3425\left(10^6\right)}{15\left(10^6\right)}\right]\Delta_b = 1.64475\Delta_b \qquad (2)$$

Conservation of Energy.

$$\frac{1}{2}mv^2 = \frac{1}{2}k_b{\Delta_b}^2 + 2\left[\frac{1}{2}k_{sp}{\Delta_{sp}}^2\right]$$

Substituting Eq. (2) into this equation,

$$\frac{1}{2}mv^2 = \frac{1}{2}k_b{\Delta_b}^2 + k_{sp}\left(1.64475\Delta_b\right)^2$$

$$\frac{1}{2}mv^2 = \frac{1}{2}k_b{\Delta_b}^2 + 2.7052k_{sp}{\Delta_b}^2$$

$$\frac{1}{2}\left[10\left(10^3\right)\right]\left(1.5^2\right) = \frac{1}{2}\left[49.3425\left(10^6\right)\right]{\Delta_b}^2 + 2.7052\left[15\left(10^6\right)\right]{\Delta_b}^2$$

$$\Delta_b = 0.01313\text{ m}$$

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14–66. Continued

SOLUTION

Maximum Stress. The maximum force on the bar is $(P_b)_{\max} = k_b\,\Delta_b = 49.3425\left(10^6\right)(0.01313) = 647.90\left(10^3\right)$ N. The maximum moment occurs at the midspan of the bar, where $M_{\max} = \dfrac{(P_b)_{\max}\,L}{4} = \dfrac{647.90\left(10^3\right)(4)}{4}$ $= 647.90\left(10^3\right)$ N · m. Applying the flexure formula,

$$\sigma_{\max} = \frac{M_{\max}\,c}{I} = \frac{647.90\left(10^3\right)(0.15)}{0.9\left(10^{-3}\right)} = 107.98 \text{ MPa} = 108 \text{ MPa} \qquad \textbf{Ans.}$$

Since $\sigma_{\max} < \sigma_Y = 414$ MPa, this result is valid.

Ans:
$\sigma_{\max} = 108$ MPa

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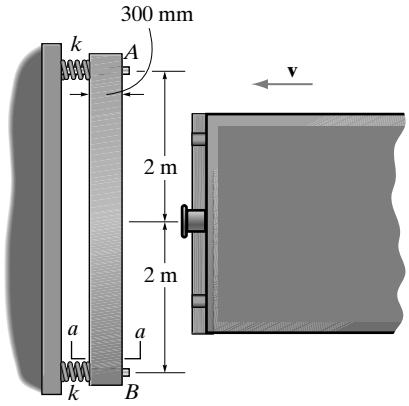
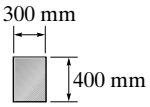
14–67. The 2014-T6 aluminum bar AB can slide freely along the guides mounted on the rigid crash barrier. Determine the maximum speed v the 10-Mg railcar without causing the bar to yield when it is struck by the railcar. The springs at A and B have a stiffness of $k = 15$ MN/m.





Section $a-a$

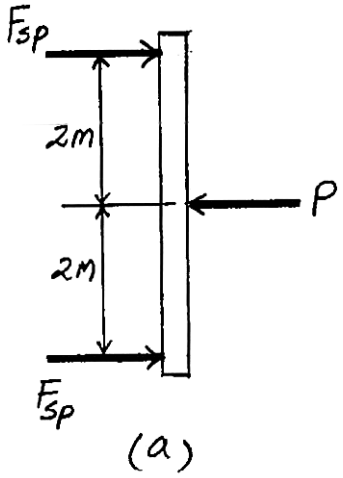
SOLUTION

Equilibrium. Referring to the free-body diagram of the bar for static conditions, Fig. *a*,

$$\stackrel{+}{\rightarrow}\Sigma F_x = 0; \qquad 2F_{sp} - P = 0 \qquad F_{sp} = \frac{P}{2} \qquad \textbf{(1)}$$

Referring to the table listed in the appendix, the displacement of the bar at the position where **P** is applied under static conditions is $\Delta_{st} = \frac{PL^3}{48EI}$. Thus, the equivalent spring constant for the bar is $k_b = \frac{48EI}{L^3}$, where $I = \frac{1}{12}(0.4)\left(0.3^3\right) = 0.9\left(10^{-3}\right)\text{m}^4$, $L = 4$ m, and $E = E_{al} = 73.1$ GPa. Thus,

$$k_b = \frac{48\left[73.1\left(10^9\right)\right]\left[0.9\left(10^{-3}\right)\right]}{4^3} = 49.3425\left(10^6\right)\text{N/m}$$



Using Eq. (1)

$$F_{sp} = \frac{P}{2}$$

$$k_{sp}\Delta_{sp} = \frac{1}{2}k_b\Delta_b$$

$$\Delta_{sp} = \frac{1}{2}\left(\frac{k_b}{k_{sp}}\right)\Delta_b = \frac{1}{2}\left[\frac{49.3425\left(10^6\right)}{15\left(10^6\right)}\right]\Delta_b = 1.64475\Delta_b \qquad \textbf{(2)}$$

Maximum Stress. The maximum force on the bar is $(P_b)_{\max} = k_b\,\Delta_b = 49.3425\left(10^6\right)\Delta_b$. The maximum moment occurs at the midspan of the bar, where $M_{\max} = \frac{(P_b)_{\max}L}{4} = \frac{49.3425\left(10^6\right)\Delta_b(4)}{4} = 49.3425\left(10^6\right)\Delta_b$. Applying the flexure formula with $\sigma_{\max} = \sigma_Y = 414$ MPa,

$$\sigma_{\max} = \frac{M_{\max}c}{I}$$

$$414\left(10^6\right) = \frac{49.3425\left(10^6\right)\Delta_b\,(0.15)}{0.9\left(10^{-3}\right)}$$

$$\Delta_b = 0.050342\text{ m}$$

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14–67. Continued

SOLUTION

Substituting this result into Eq. (2),

$$\Delta_{sp} = 0.0828 \text{ m}$$

Conservation of Energy.

$$\frac{1}{2} mv^2 = \frac{1}{2} k_b {\Delta_b}^2 + 2\left[\frac{1}{2} k_{sp} {\Delta_{sp}}^2\right]$$

$$\frac{1}{2}\left[10\left(10^3\right)\right]v^2 = \frac{1}{2}\left[49.3425\left(10^6\right)\right]\left(0.050342^2\right) + 2\left[\frac{1}{2}\left[15\left(10^6\right)\right]\left(0.0828^2\right)\right]$$

$v = 5.75 \text{ m/s}$ **Ans.**

Ans:

$k_b = 49.3425\ (10^6)$ N/m, $\Delta_b = 0.050342$ m,
$v = 5.75$ m/s

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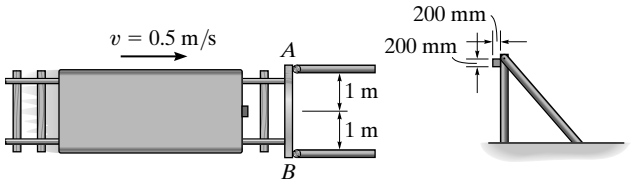
*__14–68.__ The steel beam AB acts to stop the oncoming railroad car, which has a mass of 10 Mg and is coasting towards it at $v = 0.5$ m/s. Determine the maximum stress developed in the beam if it is struck at its center by the car. The beam is simply supported and only horizontal forces occur at A and B. Assume that the railroad car and the supporting framework for the beam remains rigid. Also, compute the maximum deflection of the beam. $E_{st} = 200$ GPa, $\sigma_Y = 250$ MPa.



SOLUTION

From Appendix C:

$$\Delta_{st} = \frac{PL^3}{48EI} = \frac{10(10^3)(9.81)(2^3)}{48(200)(10^4)(\frac{1}{12})(0.2)(0.2^3)} = 0.613125(10^{-3})\text{ m}$$

$$k = \frac{W}{\Delta_{st}} = \frac{10(10^3)(9.81)}{0.613125(10^{-3})} = 160(10^6)\text{ N/m}$$

$$\Delta_{\max} = \sqrt{\frac{\Delta_{st} v^2}{g}} = \sqrt{\frac{0.613125(10^{-3})(0.5^2)}{9.81}} = 3.953(10^{-3})\text{ m} = 3.95\text{ mm} \qquad \textbf{Ans.}$$

$$W' = k\Delta_{\max} = 160(10^6)(3.953)(10^{-3}) = 632455.53\text{ N}$$

$$M' = \frac{w'L}{4} = \frac{632455.53(2)}{4} = 316228\text{ N}\cdot\text{m}$$

$$\sigma_{\max} = \frac{M'c}{I} = \frac{316228(0.1)}{\frac{1}{12}(0.2)(0.2^3)} = 237\text{ MPa} < \sigma_\gamma \qquad \text{O.K.} \qquad \textbf{Ans.}$$

Ans:

$\Delta_{\max} = 3.95$ mm, $\sigma_{\max} = 237$ MPa

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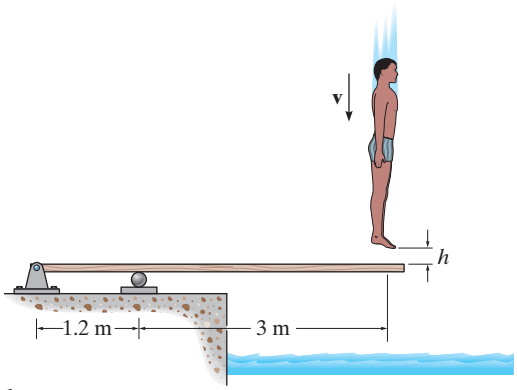
14–69. The diver weighs 750 N and, while holding himself rigid, strikes the end of a wooden diving board ($h = 0$) with a downward velocity of 1.2 m/s. Determine the maximum bending stress developed in the board. The board has a thickness of 40 mm and width of 450 mm. $E_w = 12.6$ GPa, $\sigma_Y = 56$ MPa.



SOLUTION

Static Displacement: The static displacement at the end of the diving board can be determined using the conservation of energy.

$$\frac{1}{2}P\Delta = \int_0^L \frac{M^2\,dx}{2EI}$$

$$\frac{1}{2}(750)\Delta_{st} = \frac{1}{2EI}\left[\int_0^{1.2\text{ m}} (-1875x_1)^2\,dx_1 + \int_0^{3\text{ m}} (-750x_2)^2\,dx_2\right]$$

$$\Delta_{st} = \frac{9450\text{ N}\cdot\text{m}^3}{EI}$$

$$= \frac{9450}{[12.6(10^9)]\left[\frac{1}{12}(0.45)(0.04^3)\right]}$$

$$= 0.3125\text{ m}$$

Conservation of Energy: The equivalent spring constant for the board is $k = \frac{W}{\Delta_{st}} = \frac{750}{0.3125} = 2.40(10^3)$ N/m,

$$U_e = U_i$$

$$\frac{1}{2}mv^2 + W\Delta_{max} = \frac{1}{2}k\Delta_{max}^2$$

$$\frac{1}{2}\left(\frac{750}{9.81}\right)(1.2^2) + 750\Delta_{max} = \frac{1}{2}[2.40(10^3)]\Delta_{max}^2$$

$$1.20(10^3)\Delta_{max}^2 - 750\Delta_{max} - 55.05 = 0$$

Solving for the positive root, we have

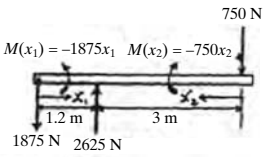
$$\Delta_{max} = 0.69135\text{ m}$$

Maximum Stress: The maximum force on to the beam is $P_{max} = k\Delta_{max}$ $[2.40(10^3)](0.69135) = 1.6592(10^3)$ N. The maximum moment occurs at the middle support support $M_{max} = [1.6592(10^3)](3) = 4.9777(10^3)\text{ N}\cdot\text{m}$

$$\sigma_{max} = \frac{M_{max}c}{I} = \frac{4.9777(10^3)(0.02)}{\frac{1}{12}(0.45)(0.04^3)}$$

$$= 41.48(10^6)\text{ N/m}^2 = 41.5\text{ MPa} < \sigma_Y \quad \text{O.K.} \qquad \textbf{Ans.}$$

Note: The result will be somewhat inaccurate since the static displacement is so large.



Ans:

$\sigma_{max} = 41.5$ MPa

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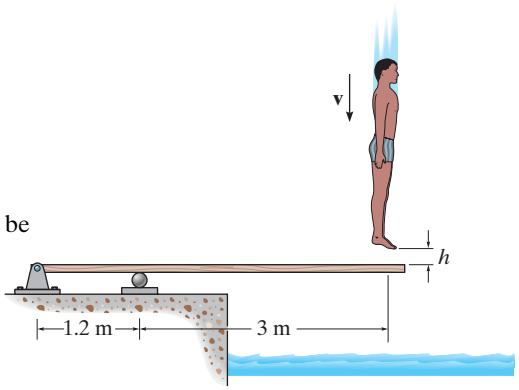
14–70. The diver weighs 750 N and, while holding himself rigid, strikes the end of the wooden diving board. Determine the maximum height h from which he can jump onto the board so that the maximum bending stress in the wood does not exceed 42 MPa. The board has a thickness of 40 mm and width of 450 mm. $E_w = 12.6$ GPa.



Static Displacement: The static displacement at the end of the diving board can be determined using the conservation of energy.

$$\frac{1}{2}P\Delta = \int_0^L \frac{M^2\,dx}{2EI}$$

$$\frac{1}{2}(750)\Delta_{st} = \frac{1}{2EI}\left[\int_0^{1.2\text{ m}} (-1875x_1)^2\,dx_1 + \int_0^{3\text{ m}} (-750x_2)^2\,dx_2\right]$$

$$\Delta_{st} = \frac{9450\text{ N}\cdot\text{m}^3}{EI}$$

$$= \frac{9450}{[12.6(10^9)]\left[\frac{1}{12}(0.45)(0.04^3)\right]}$$

$$= 0.3125\text{ m}$$

Maximum Stress: The maximum force on the beam is P_{max}. The maximum moment occurs at the middle support $M_{max} = P_{max}(3) = 3P_{max}$.

$$\sigma_{max} = \frac{M_{max}\,c}{I}$$

$$12.6(10^9) = \frac{(3P_{max})(0.02)}{\frac{1}{12}(0.45)(0.04^3)}$$

$$P_{max} = 1680\text{ N}$$

Conservation of Energy: The equivalent spring constant for the board is $k = \frac{W}{\Delta_{st}} = \frac{750}{0.3125} = 2.40(10^3)$ N/m. The maximum displacement at the end of the board is $\Delta_{max} = \frac{P_{max}}{k} = \frac{1680}{2.40(10^3)} = 0.700$ m.

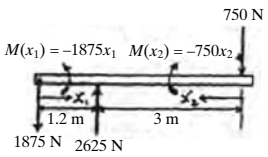
$$U_e = U_i$$

$$W(h + \Delta_{max}) = \frac{1}{2}k\Delta_{max}^2$$

$$750(h + 0.700) = \frac{1}{2}[2.40(10^3)](0.700^2)$$

$$h = 0.0840\text{ m} = 84.0\text{ mm}.$$ **Ans.**

Note: The result will be somewhat at inaccurate since the static displacement is so large.



Ans:
$h = 84.0$ mm

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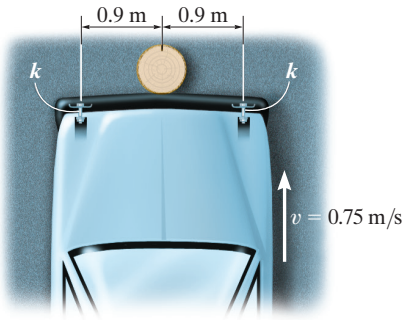
14–71.

The car bumper is made of polycarbonate-polybutylene terephthalate. If $E = 2.0$ GPa, determine the maximum deflection and maximum stress in the bumper if it strikes the rigid post when the car is coasting at $v = 0.75$ m/s. The car has a mass of 1.80 Mg, and the bumper can be considered simply supported on two spring supports connected to the rigid frame of the car. For the bumper take $I = 300(10^6)$ mm^4, $c = 75$ mm, $\sigma_Y = 30$ MPa and $k = 1.5$ MN/m.



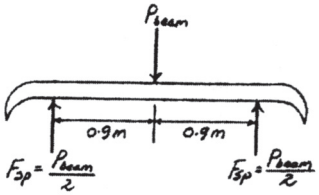
SOLUTION

Equilibrium: This requires $F_{sp} = \frac{P_{beam}}{2}$. Then

$$k_{sp}\,\Delta_{sp} = \frac{k\Delta_{beam}}{2} \qquad \text{or} \qquad \Delta_{sp} = \frac{k}{2k_{sp}}\,\Delta_{beam} \tag{1}$$

Conservation of Energy: The equivalent spring constant for the beam can be determined using the deflection table listed in the Appendix C.

$$k = \frac{48EI}{L^3} = \frac{48\left[2(10^9)\right]\left[300(10^{-6})\right]}{1.8^3} = 4\,938\,271.6\ \text{N/m}$$



Thus,

$$U_e = U_i$$

$$\frac{1}{2}mv^2 = \frac{1}{2}k\Delta_{beam}^2 + 2\left(\frac{1}{2}k_{sp}\,\Delta_{sp}^2\right) \tag{2}$$

Substitute Eq. (1) into (2) yields

$$\frac{1}{2}mv^2 = \frac{1}{2}k\Delta_{beam}^2 + \frac{k^2}{4k_{sp}}\Delta_{beam}^2$$

$$\frac{1}{2}(1800)\left(0.75^2\right) = \frac{1}{2}(493\,8271.6)\,\Delta_{beam}^2 + \frac{(4\,93\,8271.6)^2}{4[1.5(10^6)]}\Delta_{beam}^2$$

$$\Delta_{beam} = 8.8025\left(10^{-3}\right)\ \text{m}$$

Maximum Displacement: From Eq. (1) $\Delta_{sp} = \frac{4\,938\,271.6}{2[1.5(10^6)]}\left[8.8025\left(10^{-3}\right)\right] = 0.014490$ m.

$$\Delta_{max} = \Delta_{sp} + \Delta_{beam}$$
$$= 0.014490 + 8.8025\left(10^{-3}\right)$$
$$= 0.02329\ \text{m} = 23.3\ \text{mm} \qquad \textbf{Ans.}$$

Maximum Stress: The maximum force on the beam is $P_{beam} = k\Delta_{beam} = 4\,938\,271.6\left[8.8025\left(10^{-3}\right)\right] = 43\,469.3$ N. The maximum moment occurs at mid-span. $M_{max} = \frac{P_{beam}\,L}{4} = \frac{43\,469.3(1.8)}{4} = 19\,561.2\ \text{N}\cdot\text{m}$.

$$\sigma_{max} = \frac{M_{max}\,c}{I} = \frac{19\,561.2(0.075)}{300(10^{-6})} = 4.89\ \text{MPa} \qquad \textbf{Ans.}$$

Since $\sigma_{max} < \sigma_Y = 30$ MPa, the above analysis is valid.

Ans:
$\Delta_{max} = 23.3$ mm, $\sigma_{max} = 4.89$ MPa

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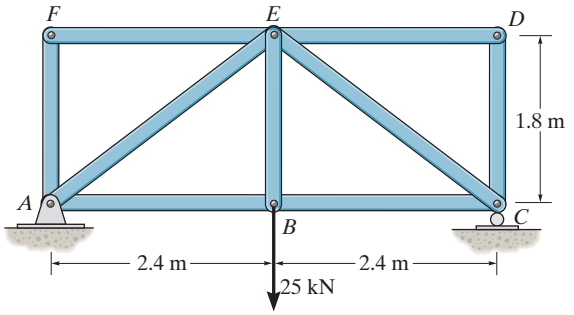
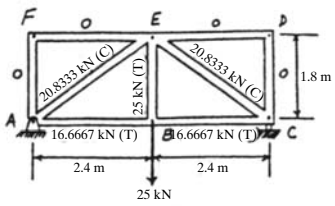
***14-72.** Determine thevertical displacement of point E. Each A-36 steel member has a cross-sectional area of 2800 mm^2.



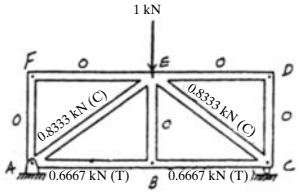
SOLUTION

Virtual-Work Equation: Applying Eq. 14–39, we have

Member	n (kN)	N (kN)	L (m)	nNL
AB	0.6667	16.6667	2.4	26.667
BC	0.6667	16.6667	2.4	26.667
CD	0	0	1.8	0
DE	0	0	2.4	0
EF	0	0	2.4	0
AF	0	0	1.8	0
AE	–0.8333	–20.8333	3.0	52.083
CE	–0.8333	–20.8333	3.0	52.083
BE	0	25.00	1.8	0
				$\Sigma 157.5\,\text{kN}^2\cdot\text{m}$



(a)



(c)

$$1\cdot\Delta = \sum\frac{nNL}{AE}$$

$$1\ \text{kN}\cdot(\Delta_E)_v = \frac{157.5\ \text{kN}\cdot\text{m}^2}{AE}$$

$$(\Delta_E)_v = \frac{157.5(10^3)}{[2.80(10^{-3})][200(10^9)]} = 0.28125(10^{-3})\ \text{m} = 0.281\ \text{mm}\ \downarrow \quad \textbf{Ans.}$$

Ans:
$(\Delta_E)_v = 0.281\ \text{mm}\ \downarrow$

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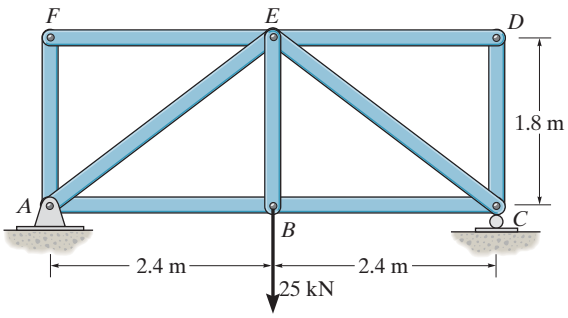
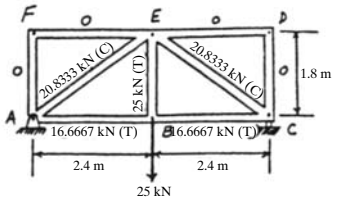
14–73. Determine the vertical displacement of point B. Each A-36 steel member has a cross-sectional area of 2800 mm^2.



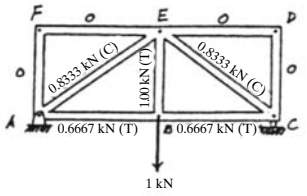
SOLUTION

Virtual-Work Equation: Applying Eq. 14–39, we have

Member	n (kN)	N (kN)	L (m)	nNL
AB	0.6667	16.6667	2.4	26.667
BC	0.6667	16.6667	2.4	26.667
CD	0	0	1.8	0
DE	0	0	2.4	0
EF	0	0	2.4	0
AF	0	0	1.8	0
AE	–0.8333	–20.8333	3.0	52.083
CE	–0.8333	–20.8333	3.0	52.083
BE	1.00	25.00	1.8	45.00
				$\Sigma 202.5\,\text{kN}^2\cdot\text{m}$



(a)



(b)

$$1\cdot\Delta = \sum \frac{nNL}{AE}$$

$$1\ \text{kN}\cdot(\Delta_B)_v = \frac{202.5\ \text{kN}\cdot\text{m}^2}{AE}$$

$$(\Delta_B)_v = \frac{202.5(10^3)}{[2.80(10^{-3})][200(10^9)]} = 0.3616(10^{-3})\ \text{m} = 0.362\ \text{mm}\ \downarrow$$ **Ans.**

Ans:
$(\Delta_B)_v = 0.362$ mm

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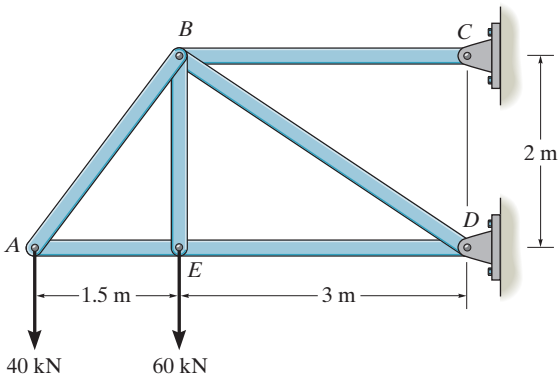
14–74.

Determine the vertical displacement of joint A. Each A992 steel member has a cross-sectional area of 400 mm^2.



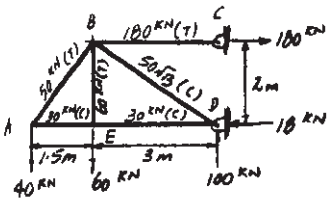
SOLUTION

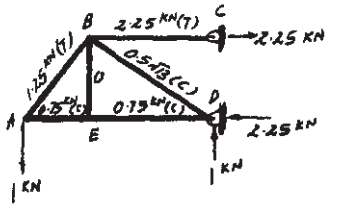
Member	n	N	L	nNL
AB	1.25	50	2.5	156.25
AE	−0.75	−30	1.5	33.75
BC	2.25	180	3.0	1215.00
BD	$-0.5\sqrt{13}$	$-50\sqrt{13}$	$\sqrt{13}$	1171.80
BE	0	60	2.0	0
DE	−0.75	−30	3.0	67.5

$$\Sigma = 2644.30$$

$$1 \cdot \Delta_{A_v} = \Sigma \frac{nNL}{AE}$$

$$(\Delta_A)_v = \frac{2644.30(10^3)}{400(10^{-6})(200)(10^9)} = 0.0331 \text{ m} = 33.1 \text{ mm}\downarrow$$





Ans.

Ans:
$(\Delta_A)_v = 33.1 \text{ mm}\downarrow$

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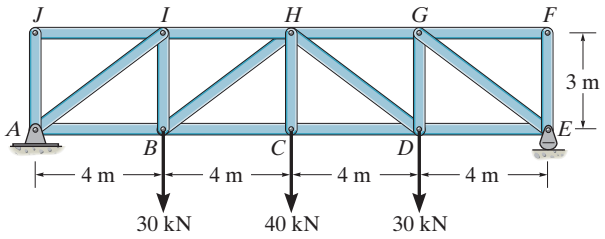
***14–75.** Determine the vertical displacement of joint H. Each A-36 steel member has a cross-sectional area of 2800 mm^2.

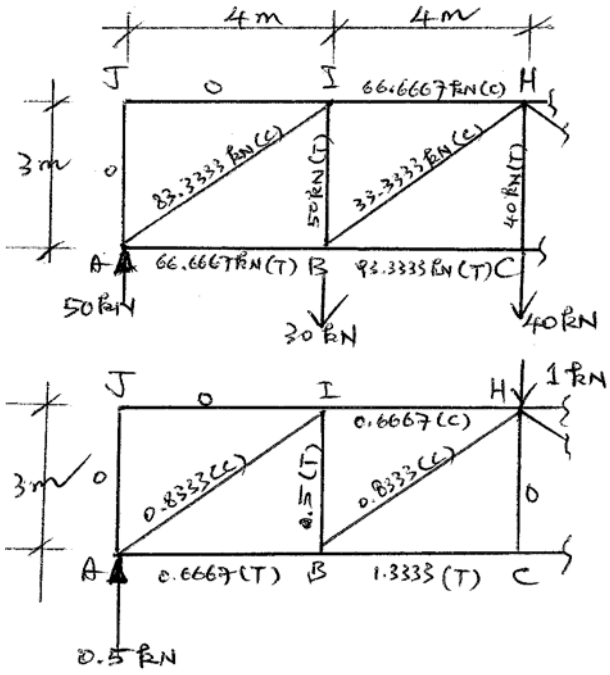


SOLUTION

$$1 \cdot \Delta_{N_v} = \Sigma \frac{nNL}{AE}$$

$$\Delta_{H_v} = \frac{2828.89(10^3)}{[2.80(10^{-3})][200(10^9)]} = 5.052(10^{-3})\ \text{m} = 5.05\ \text{mm} \downarrow$$

Ans.



Ans:

$\Delta_{H_v} = 5.05$ mm

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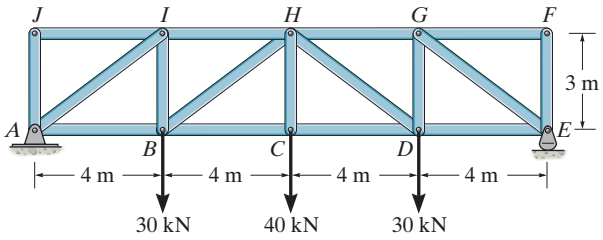
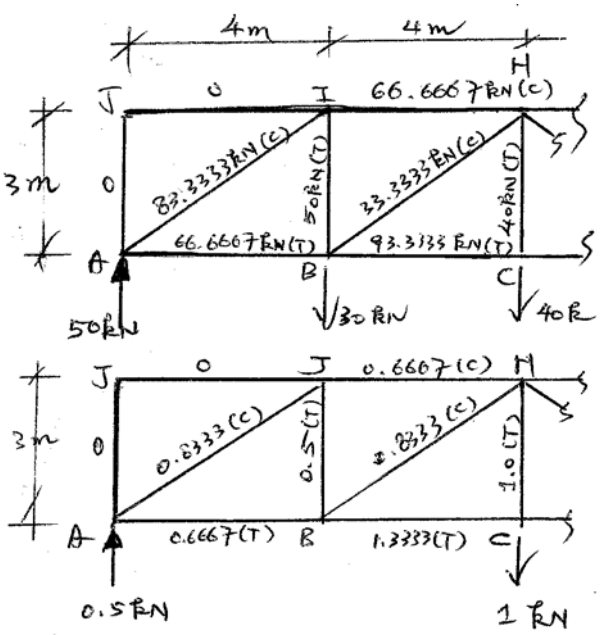
***14–76.** Determine the vertical displacement of joint *C*. Each A-36 steel member has a cross-sectional area of 2800 mm^2.



SOLUTION

$$1 \cdot \Delta_{C_v} = \Sigma \frac{nNL}{AE}$$

$$\Delta_{C_v} = \frac{2948.89(10^3)}{[2.80(10^{-3})][200(10^9)]} = 5.266(10^{-3})\ \text{m} = 5.27\ \text{mm} \downarrow \quad \textbf{Ans.}$$



Ans:

$\Delta_{C_v} = 5.27$ mm

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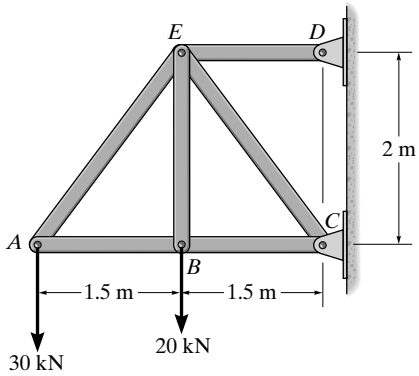
14–77. Determine the vertical displacement of point B. Each A-36 steel member has a cross-sectional area of 400 mm^2.



SOLUTION

Virtual-Work Equation:

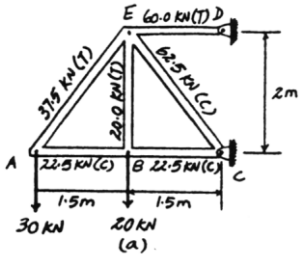
Member	*n*	*N*	*L*	*nNL*
AB	0	$-22.5(10^3)$	1.5	0
BC	0	$-22.5(10^3)$	1.5	0
AE	0	$37.5(10^3)$	2.5	0
CE	–1.25	$-62.5(10^3)$	2.5	$195.3125(10^3)$
BE	1.00	$22.0(10^3)$	2	$40.0(10^3)$
DE	0.750	$60.0(10^3)$	1.5	$67.5(10^3)$
				$\Sigma\ 302.8125(10^3)\ \text{N}^2\cdot\text{m}$

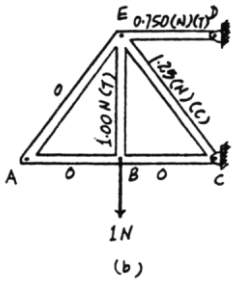
$$1\cdot\Delta = \sum \frac{nNL}{AE}$$

$$1\ \text{N}\cdot(\Delta_B)_v = \frac{302.8125(10^3)\ \text{N}^2\cdot\text{m}}{AE}$$

$$(\Delta_B)_v = \frac{302.8125(10^3)}{0.400(10^{-3})[200(10^9)]}$$

$$= 3.785(10^{-3})\ \text{m} = 3.79\ \text{mm}\downarrow$$ **Ans.**





Ans:

$(\Delta_B)_v = 3.79\ \text{mm}\downarrow$

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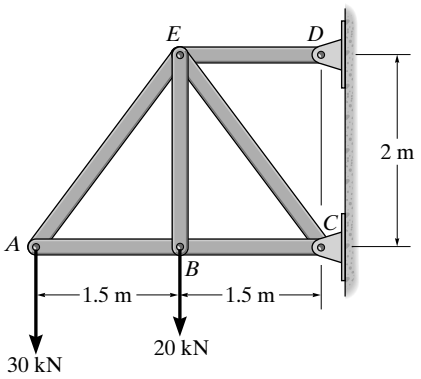
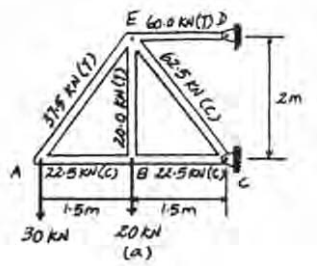
14–78. Determine the vertical displacement of point A. Each A-36 steel member has a cross-sectional area of 400 mm^2.



SOLUTION

Virtual-Work Equation:

Member	*n*	*N*	*L*	*nNL*
AB	–0.750	$-22.5(10^3)$	1.5	$25.3125(10^3)$
BC	–0.750	$-22.5(10^3)$	1.5	$25.3125(10^3)$
AE	1.25	$37.5(10^3)$	2.5	$117.1875(10^3)$
CE	–1.25	$-62.5(10^3)$	2.5	$195.3125(10^3)$
BE	0	$22.0(10^3)$	2	0
DE	1.50	$60.0(10^3)$	1.5	$135.00(10^3)$
				$\Sigma\ 498.125(10^3)\ \mathrm{N^2 \cdot m}$



$$1 \cdot \Delta = \sum \frac{nNL}{AE}$$

$$1\ \mathrm{N} \cdot (\Delta_A)_v = \frac{498.125(10^3)\ \mathrm{N^2 \cdot m}}{AE}$$

$$(\Delta_A)_v = \frac{498.125(10^3)}{0.400(10^{-3})[200(10^9)]}$$

$$= 6.227(10^{-3})\ \mathrm{m} = 6.23\ \mathrm{mm} \downarrow$$ **Ans.**

Ans:

$$1\ \mathrm{N} \cdot (\Delta_A)_v = \frac{498.125(10^3)\ \mathrm{N^2 \cdot m}}{AE},$$

$(\Delta_A)_v = 6.23\ \mathrm{mm} \downarrow$

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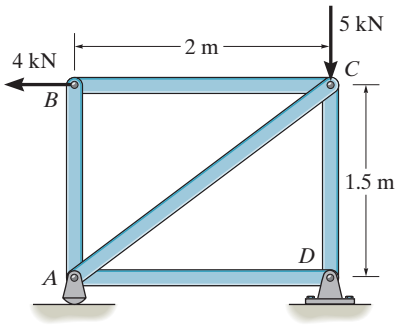
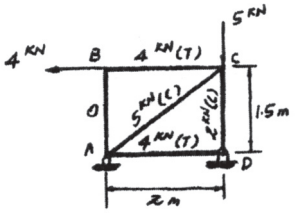
14–79.

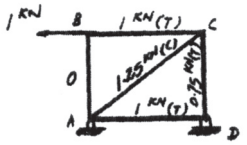
Determine the horizontal displacement of joint B of the truss. Each A992 steel member has a cross-sectional area of 400 mm^2.



SOLUTION

Member	n	N	L	nNL
AB	0	0	1.5	0
AC	–1.25	–5.00	2.5	15.625
AD	1.00	4.00	2.0	8.000
BC	1.00	4.00	2.0	8.000
CD	0.75	–2.00	1.5	–2.25
				$\Sigma = 29.375$





$$1 \cdot (\Delta_B)_h = \Sigma \frac{nNL}{AE}$$

$$(\Delta_B)_h = \frac{29.375(10^3)}{400(10^{-6})(200)(10^9)} = 0.3672(10^{-3})\ \text{m} = 0.367\ \text{mm} \leftarrow \qquad \textbf{Ans.}$$

Ans:
$(\Delta_B)_h = 0.367\ \text{mm} \leftarrow$

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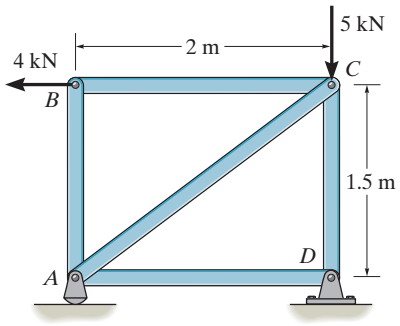
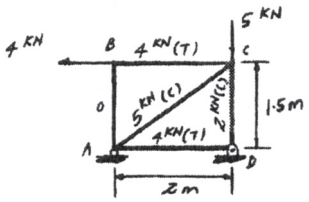
***14–80.**

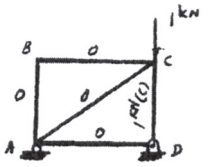
Determine the vertical displacement of joint C of the truss. Each A992 steel member has a cross-sectional area of 400 mm^2.



SOLUTION

Member	n	N	L	nNL
AB	0	0	1.5	0
AC	0	–5.00	2.5	0
AD	0	4.00	2.0	0
BC	0	4.00	2.0	0
CD	–1.00	–2.00	1.5	3.00
				$\Sigma = 3.00$





$$1 \cdot (\Delta_C)_v = \Sigma \frac{nNL}{AE}$$

$$(\Delta_C)_v = \frac{3.00\,(10^3)}{400(10^{-6})(200)(10^9)} = 37.5(10^{-6})\text{ m} = 0.0375\text{ mm}\downarrow \qquad \textbf{Ans.}$$

Ans:
$(\Delta_C)_v = 0.0375\text{ mm}\downarrow$

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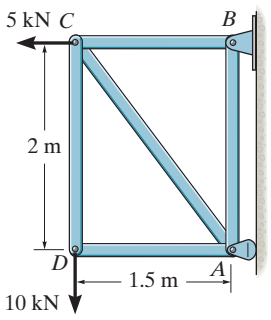
14–81.

Determine the horizontal displacement of joint C. Each A-36 steel member has a cross-sectional area of 400 mm^2.



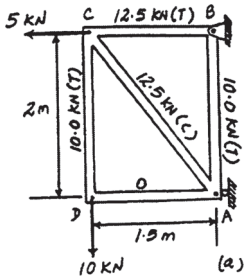
SOLUTION

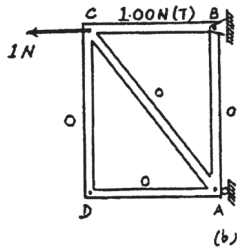
Member Real Forces *N*: As shown on Figure (a).

Member Virtual Forces *n*: As shown on Figure (b).

Virtual-Work Equation: Applying Eq. 14–39, we have

Member	n	N	L	nNL
AB	0	$10.0(10^3)$	2	0
BC	1.00	$12.5(10^3)$	1.5	$18.75(10^3)$
CD	0	$10.0(10^3)$	2	0
AD	0	0	1.5	0
AC	0	$-12.5(10^3)$	2.5	0
				$\Sigma 18.7(10^3)\ \text{N}^2\cdot\text{m}$





$$1\cdot\Delta = \Sigma\frac{nNL}{AE}$$

$$1\ \text{N}\cdot(\Delta_C)_h = \frac{18.75(10^3)\ \text{N}^2\cdot\text{m}}{AE}$$

$$(\Delta_C)_h = \frac{18.75(10^3)}{0.400(10^{-3})\left[200(10^9)\right]}$$

$$= 0.2344(10^{-3})\ \text{m} = 0.234\ \text{mm} \quad \leftarrow$$ **Ans.**

Ans:
$(\Delta_C)_h = 0.234\ \text{mm} \quad \leftarrow$

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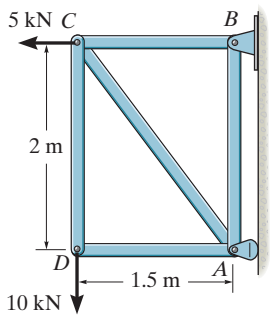
14–82.

Determine the vertical displacement of joint D. Each A-36 steel member has a cross-sectional area of 400 mm^2.



SOLUTION

Member Real Forces *N*: As shown on Figure (a).

Member Virtual Forces *n*: As shown on Figure (b).

Virtual-Work Equation: Applying Eq. 14–39, we have

Member	n	N	L	nNL
AB	1.00	$10.0(10^3)$	2	$20.0(10^3)$
BC	0.750	$12.5(10^3)$	1.5	$14.0625(10^3)$
CD	1.00	$10.0(10^3)$	2	$20.0(10^3)$
AD	0	0	1.5	0
AC	–1.25	$-12.5(10^3)$	2.5	$39.0625(10^3)$
				$\Sigma 93.125(10^3)\ \text{N}^2\cdot\text{m}$

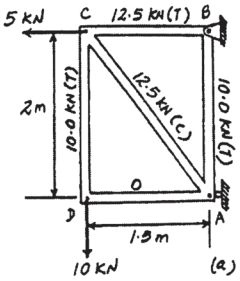
$$1\cdot\Delta = \Sigma\frac{nNL}{AE}$$

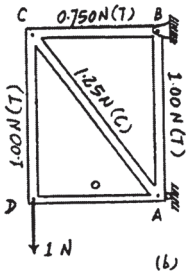
$$1\ \text{N}\cdot(\Delta_D)_v = \frac{93.125(10^3)\ \text{N}^2\cdot\text{m}}{AE}$$

$$(\Delta_D)_v = \frac{93.125(10^3)}{0.400(10^{-3})\left[200(10^9)\right]}$$

$$= 1.164(10^{-3})\ \text{m} = 1.16\ \text{mm} \quad \downarrow$$

Ans.





Ans:
$(\Delta_D)_v = 1.16\ \text{mm} \quad \downarrow$

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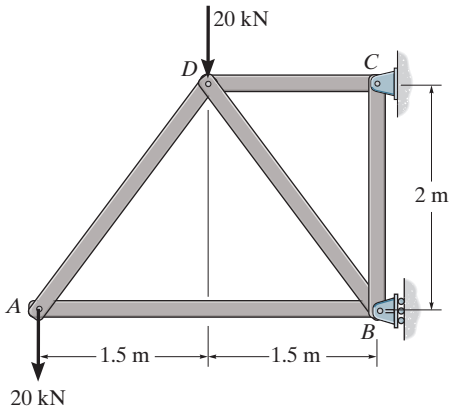
14–83.

Determine the vertical displacement of joint A. The truss is made from A992 steel rods having a diameter of 30 mm.



SOLUTION

Members Real Force *N*: As indicated in Fig. *a*.

Members Virtual Force *n*: As indicated in Fig. *b*.

Virtual Work Equation: Since $\sigma_{\max} = \dfrac{F_{BD}}{A} = \dfrac{50(10^3)}{\frac{\pi}{4}(0.03^2)} = 70.74\ \text{MPa} < \sigma_Y = 345\ \text{MPa}$,

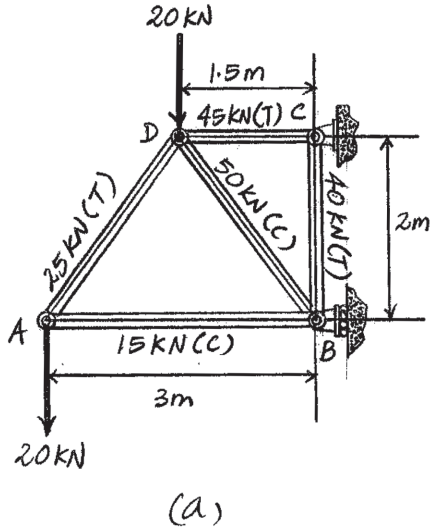
Member	n (N)	N (N)	L(m)	nNL(N$^2\cdot$m)
AB	-0.75	$-15(10^3)$	3	$33.75(10^3)$
AD	1.25	$25(10^3)$	2.5	$78.125(10^3)$
BC	1	$40(10^3)$	2	$80(10^3)$
BD	-1.25	$-50(10^3)$	2.5	$156.25(10^3)$
CD	1.5	$45(10^3)$	1.5	$101.25(10^3)$
				$\Sigma\ 449.375(10^3)$

Then

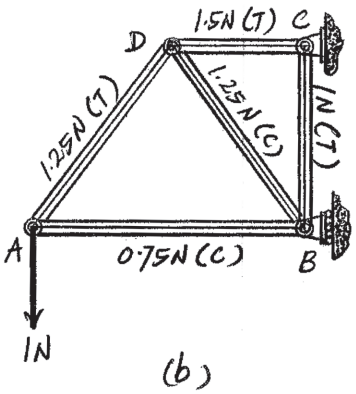
$$1\cdot\Delta = \Sigma\frac{nNL}{AE}$$

$$1\ \text{N}\cdot(\Delta_A)_v = \frac{449.375(10^3)}{\frac{\pi}{4}(0.03^2)[200(10^9)]}$$

$$(\Delta_A)_v = 3.179(10^{-3})\ \text{m} = 3.18\ \text{mm}\ \downarrow$$ **Ans.**



(a)



(b)

Ans:
$(\Delta_A)_v = 3.18\ \text{mm}\ \downarrow$

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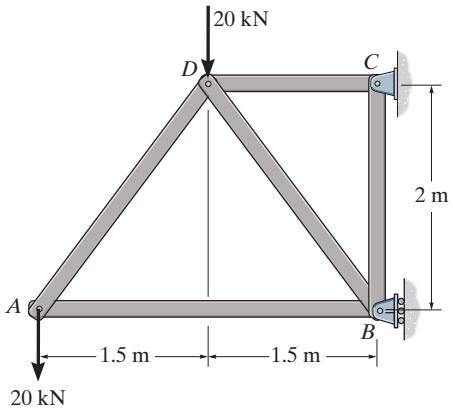
***14–84.**

Determine the vertical displacement of joint D. The truss is made from A992 steel rods having a diameter of 30 mm.



SOLUTION

Members Real Force *N*: As indicated in Fig. *a*.

Members Virtual Force *n*: As indicated in Fig. *b*.

Virtual Work Equation: Since $\sigma_{\max} = \dfrac{F_{BD}}{A} = \dfrac{50(10^3)}{\frac{\pi}{4}(0.03^2)} = 70.74\ \text{MPa} < \sigma_Y = 345\ \text{MPa}$,

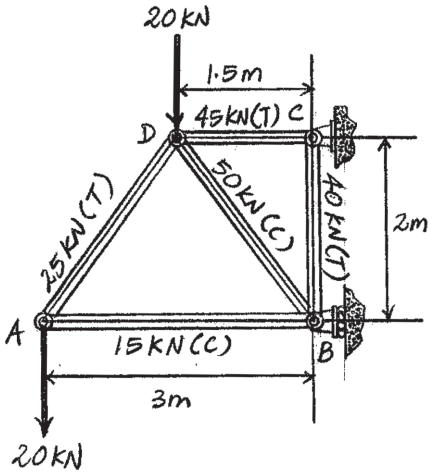
Member	n (N)	N (N)	L(m)	nNL(N$^2\cdot$m)
AB	0	$-15(10^3)$	3	0
AD	0	$25(10^3)$	2.5	0
BC	1	$40(10^3)$	2	$80(10^3)$
BD	-1.25	$-50(10^3)$	2.5	$156.25(10^3)$
CD	0.75	$45(10^3)$	1.5	$50.625(10^3)$
				$\Sigma\ 286.875(10^3)$

Then

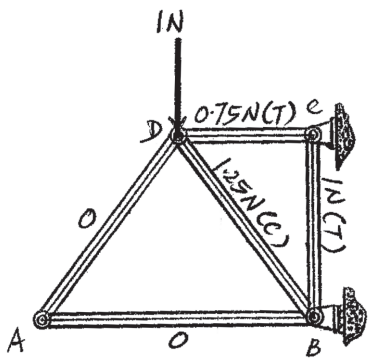
$$1\cdot\Delta = \Sigma\frac{nNL}{AE}$$

$$1\ \text{N}\cdot(\Delta_D)_v = \frac{286.875(10^3)}{\frac{\pi}{4}(0.03^2)[200(10^9)]}$$

$$(\Delta_D)_v = 2.029(10^{-3})\ \text{m} = 2.03\ \text{mm}\downarrow$$ **Ans.**



(a)



(b)

Ans:
$(\Delta_D)_v = 2.03\ \text{mm}\downarrow$

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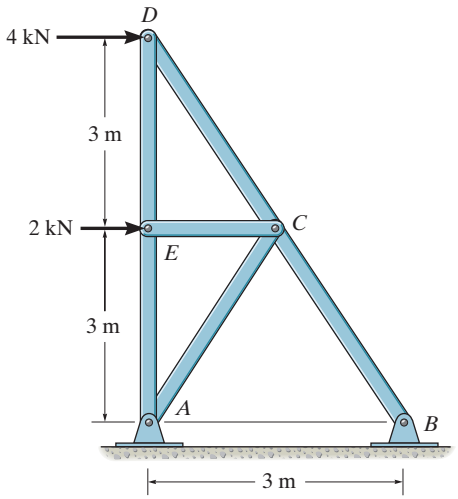
14–85.

Determine the horizontal displacement of joint D. Each A-36 steel member has a cross-sectional area of 300 mm^2.



SOLUTION

Virtual-Work Equation: Applying Eq. 14–39, we have

Member	n	N	L	nNL
AE	2.00	$800(10^3)$	3	$48.0(10^3)$
ED	2.00	$800(10^3)$	3	$48.0(10^3)$
CD	-2.236	$-8.944(10^3)$	3.354	$67.082(10^3)$
BC	-2.236	$-11.180(10^3)$	3.354	$83.853(10^3)$
CE	0	$-2.00(10^3)$	1.5	0
AC	0	$2.236(10^3)$	3.354	0

$$\Sigma 246.935(10^3)\ \text{N}^2\cdot\text{m}$$

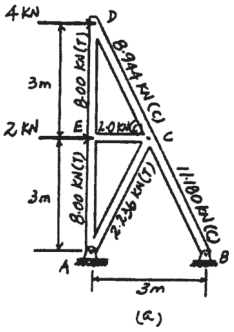
$$1\cdot\Delta = \Sigma\frac{nNL}{AE}$$

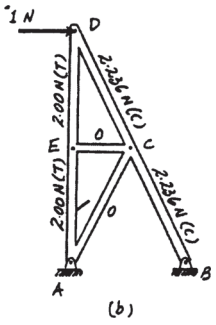
$$1\ \text{N}\cdot(\Delta_D)_h = \frac{246.935(10^3)\ \text{N}^2\cdot\text{m}}{AE}$$

$$(\Delta_D)_h = \frac{246.935(10^3)}{0.300(10^{-3})\left[200(10^9)\right]}$$

$$= 4.116(10^{-3})\ \text{m} = 4.12\ \text{mm} \quad \rightarrow$$

Ans.





Ans:
$(\Delta_D)_h = 4.12\ \text{mm} \quad \rightarrow$

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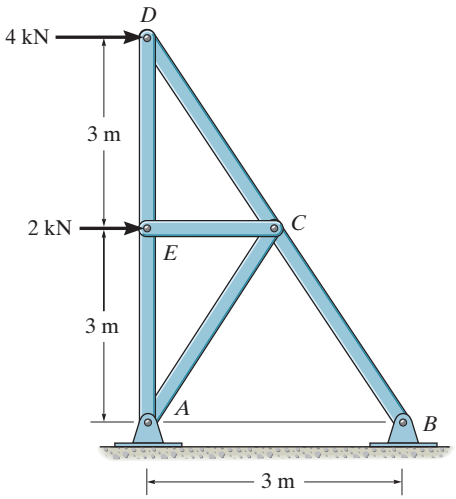
14–86.

Determine the horizontal displacement of joint E. Each A-36 steel member has a cross-sectional area of 300 mm^2.



SOLUTION

Virtual-Work Equation: Applying Eq. 14–39, we have

Member	n	N	L	nNL
AE	0	$800(10^3)$	3	0
ED	0	$800(10^3)$	3	0
CD	0	$-8.944(10^3)$	3.354	0
BC	-1.118	$-11.180(10^3)$	3.354	$41.926(10^3)$
CE	-1.00	$-2.00(10^3)$	1.5	$300(10^3)$
AC	1.118	$2.236(10^3)$	3.354	$8.385(10^3)$

$$\Sigma 53.312(10^3)\ \text{N}^2\cdot\text{m}$$

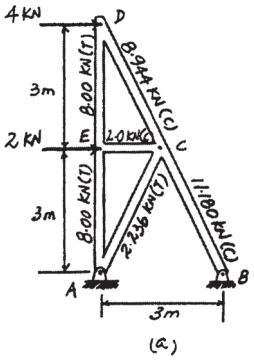
$$1\cdot\Delta = \Sigma\frac{nNL}{AE}$$

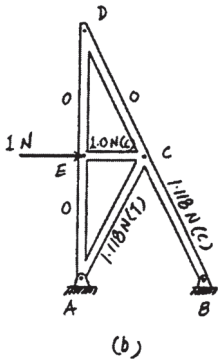
$$1\ \text{N}\cdot(\Delta_E)_h = \frac{53.312(10^3)\ \text{N}^2\cdot\text{m}}{AE}$$

$$(\Delta_E)_h = \frac{53.312(10^3)}{0.300(10^{-3})\left[200(10^9)\right]}$$

$$= 0.8885(10^{-3})\ \text{m} = 0.889\ \text{mm} \quad \rightarrow$$

Ans.





Ans:
$(\Delta_E)_h = 0.889\ \text{mm} \quad \rightarrow$

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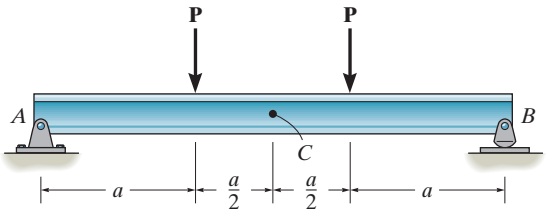
14–87.

Determine the displacement at point *C*. *EI* is constant.

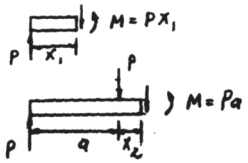


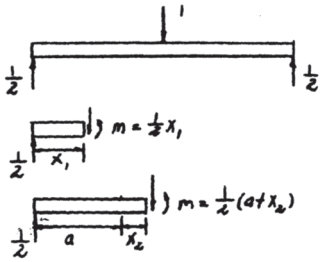
SOLUTION

$$1 \cdot \Delta_C = \int_0^L \frac{mM}{EI}\,dx$$

$$\Delta_C = 2\left(\frac{1}{EI}\right)\left[\int_0^a \left(\frac{1}{2}x_1\right)(Px_1)dx_1 + \int_0^{a/2} \frac{1}{2}(a + x_2)(Pa)dx_2\right]$$

$$= \frac{23Pa^3}{24EI}$$ **Ans.**





Ans:

$$\Delta_C = \frac{23Pa^3}{24EI}$$

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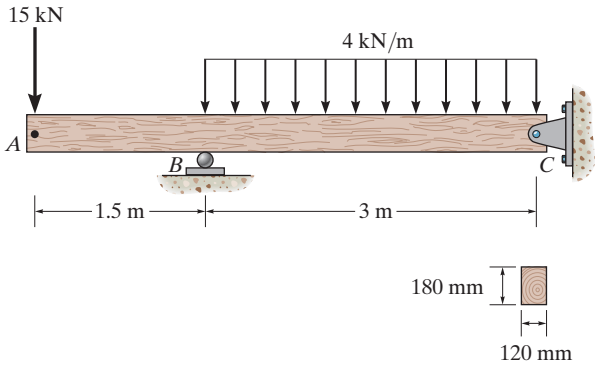
***14–88.**

The beam is made of southern pine for which $E_p = 13$ GPa. Determine the displacement at A.

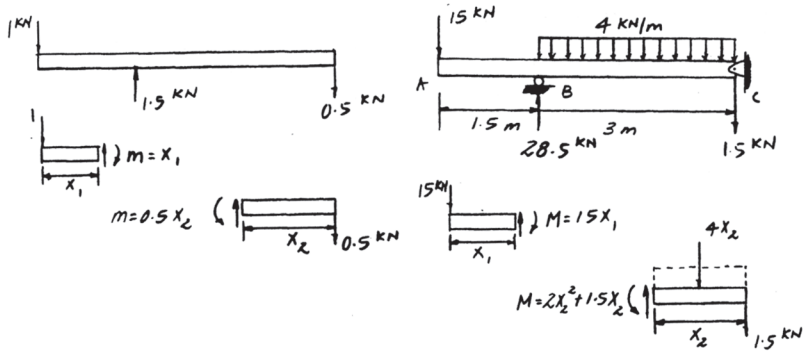


SOLUTION

$$1 \cdot \Delta_A = \int_0^L \frac{mM}{EI}$$

$$\Delta_A = \frac{1}{EI}\left[\int_0^{1.5} (x_1)(15x_1)dx_1 + \int_0^3 (0.5x_2)(2x_2^2 + 1.5x_2)dx_2\right]$$

$$= \frac{43.875\ \text{kN}\cdot\text{m}^3}{EI} = \frac{43.875(10^3)}{13(10^9)(\frac{1}{12})(0.12)(0.18)^3} = 0.0579\ \text{m} = 57.9\ \text{mm} \quad \textbf{Ans.}$$



Ans:
$\Delta_A = 57.9$ mm

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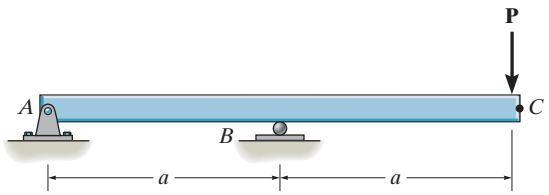
14–89.

Determine the displacement at point *C*. *EI* is constant.



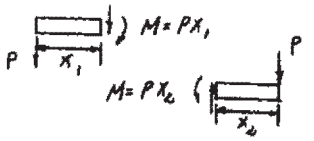
SOLUTION

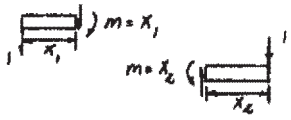
$$1 \cdot \Delta_C = \int_0^L \frac{mM}{EI}\,dx$$

$$\Delta_C = \frac{1}{EI}\left[\int_0^a (x_1)(Px_1)dx_1 + \int_0^a (x_2)(Px_2)dx_2\right]$$

$$= \frac{2Pa^3}{3EI}$$

Ans.





Ans:

$$\Delta_C = \frac{2Pa^3}{3EI}$$

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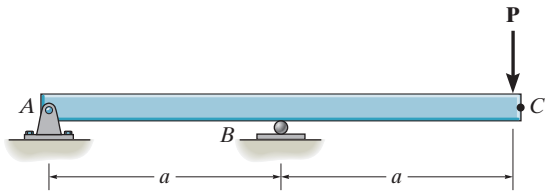
14–90.

Determine the slope at point *C*. *EI* is constant.

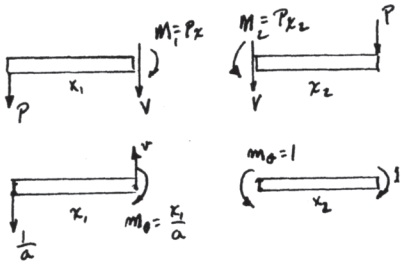


SOLUTION

$$1 \cdot \theta_C = \int_0^L \frac{m_\theta M dx}{EI}$$

$$\theta_C = \int_0^a \frac{\left(\frac{x_1}{a}\right) P x_1 dx_1}{EI} + \int_0^a \frac{(1) P x_2 dx_2}{EI}$$

$$= \frac{Pa^2}{3EI} + \frac{Pa^2}{2EI} = \frac{5Pa^2}{6EI} \circlearrowright$$ **Ans.**



Ans:

$$\theta_C = -\frac{5Pa^2}{6EI}$$

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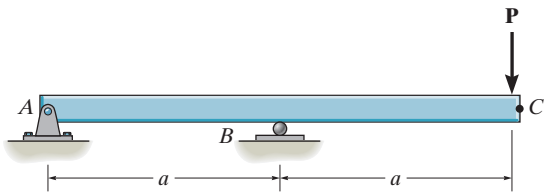
14–91.

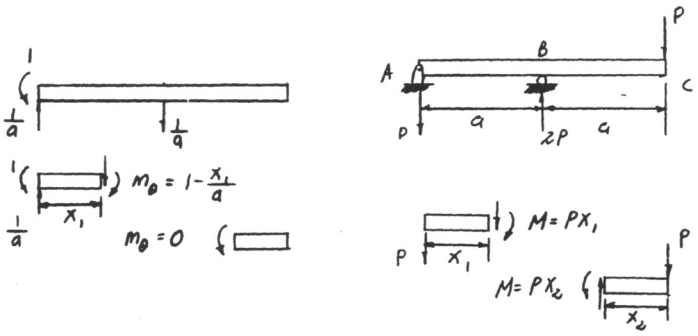
Determine the slope at point A. EI is constant.



SOLUTION

$$1 \cdot \theta_A = \int_0^L \frac{m_\theta M}{EI}\,dx$$

$$\theta_A = \frac{1}{EI}\left[\int_0^a \left(1 - \frac{x_1}{a}\right)(Px_1)\,dx_1 + \int_0^a (0)\,(Px_2)\,dx_2\right] = \frac{Pa^2}{6EI} \qquad \textbf{Ans.}$$



Ans:

$$\theta_A = \frac{Pa^2}{6EI}$$

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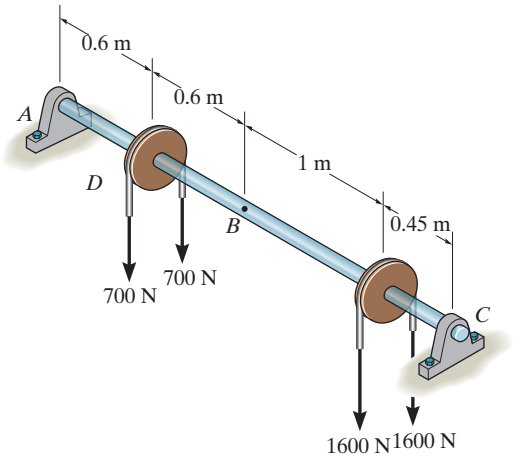
***14–92.** Determine the displacement at B of the 30-mm-diameter A-36 steel shaft.



SOLUTION

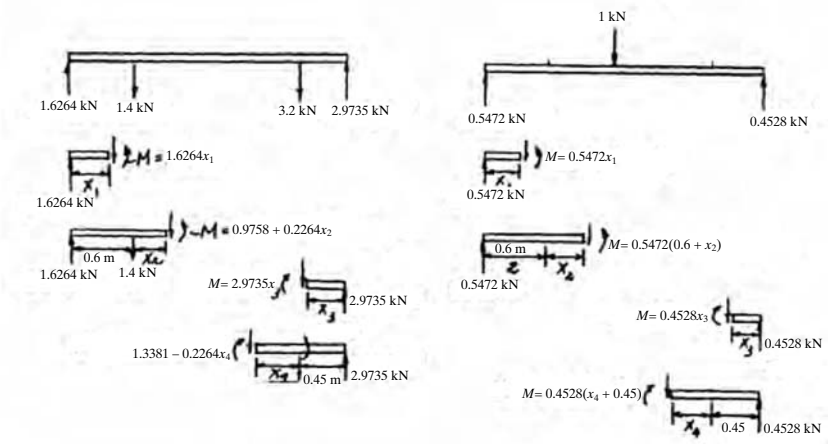
$$1 \cdot \Delta_B = \int_0^L \frac{mM}{EI}\,dx$$

$$\Delta_B = \frac{1}{EI}\left[\int_0^{0.6\text{m}} (0.5472x_1)(1.6264x_1)\,dx_1 + \int_0^{0.6\text{m}} (0.5472)(0.6 + x_2)(0.9758 + 0.2264x_2)\,dx_2 + \int_0^{0.45\text{m}} (0.4528x_3)(2.9731x_3)\,dx_3 + \int_0^{1\text{m}} 0.4528(x_4 + 0.45)(1.3381 - 0.2264x_4)\,dx_4\right]$$

$$= \frac{0.93401\ \text{kN}\cdot\text{m}^3}{EI} = \frac{0.93401(10^3)}{[200(10^9)]\left[\frac{\pi}{4}(0.015^4)\right]}$$

$$= 0.11745\ \text{m} = 117\ \text{mm}\downarrow$$

Ans.



Ans:

$\Delta_B = 117\ \text{mm}\downarrow$

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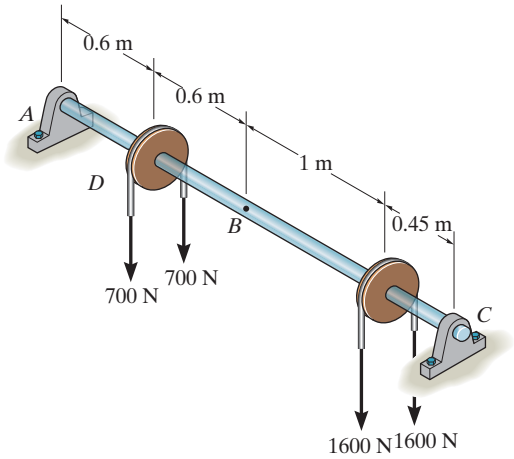
14–93. Determine the slope of the 30-mm-diameter A-36 steel shaft at the bearing support A.



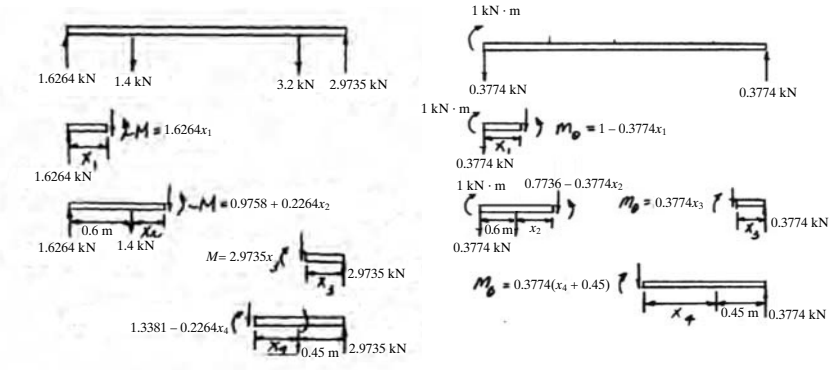
SOLUTION

$$1 \cdot \theta_A = \int_0^L \frac{m_\theta M}{EI}\,dx$$

$$\theta_A = \frac{1}{EI}\left[\int_0^{0.6\text{m}} (1 - 0.3774x_1)(1.6264x_1)\,dx_1 + \int_0^{0.6\text{m}} (0.7736 - 0.3774x_2)(0.9758 + 0.2264x_2)\,dx_2 + \int_0^{0.45\text{m}} 0.3774x_3(2.9735x_3)\,dx_3 + \int_0^{1\text{m}} 0.3774(x_4 + 0.45)(1.3381 - 0.2264x_4)\,dx_4\right]$$

$$= \frac{1.12668\ \text{kN}\cdot\text{m}^2}{EI} = \frac{1.12668(10^3)}{[200(10^9)]\left[\frac{\pi}{4}(0.015^4)\right]}$$

$$= 0.141682\ \text{rad} = 8.12°\ \text{(clockwise)}$$ **Ans.**



Ans:

$\theta_A = 8.12°$ (clockwise)

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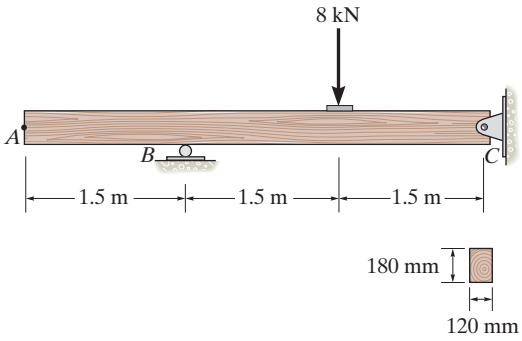
14–94.

The beam is made of Douglas fir. Determine the slope at C.



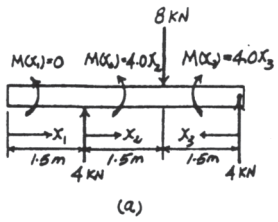
SOLUTION

Virtual Work Equation: For the slope at point C,

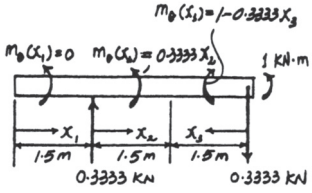
$$1 \cdot \theta = \int_0^L \frac{m_\theta M}{EI}\, dx$$

$$1\ \text{kN} \cdot \text{m} \cdot \theta_C = 0 + \frac{1}{EI}\int_0^{1.5\ \text{m}} (0.3333x_2)(4.00x_2)\, dx_2 + \frac{1}{EI}\int_0^{1.5\ \text{m}} (1 - 0.3333x_3)(4.00x_3)dx_3$$

$$\theta_C = \frac{4.50\ \text{kN} \cdot \text{m}^3}{EI} = \frac{4.50(1000)}{13.1(10^9)\left[\frac{1}{12}(0.12)(0.18^3)\right]} = 5.89\left(10^{-3}\right)\ \text{rad} = 0.337°$$ **Ans.**



(a)



(b)

Ans:
$\theta_C = 0.337°$

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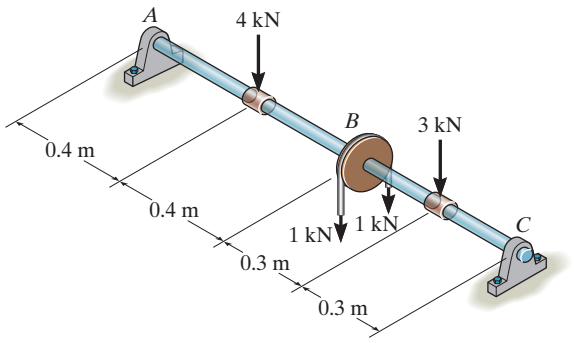
14–95.

Determine the displacement at pulley B. The A992 steel shaft has a diameter of 30 mm.



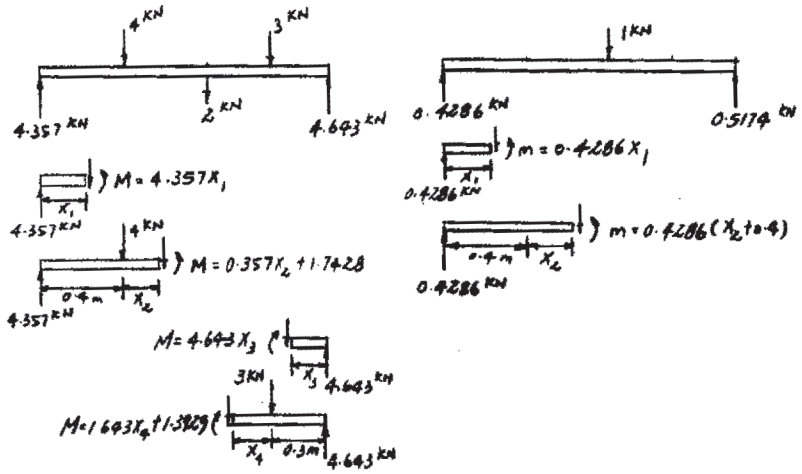
SOLUTION

$$1 \cdot \Delta_B = \int_0^L \frac{mM}{EI}\,dx$$

$$\Delta_B = \frac{1}{EI}\left[\int_0^{0.4} (0.4286x_1)(4.357x_1)\,dx_1 + \int_0^{0.4} 0.4286(x_2 + 0.4)(0.357x_2 + 1.7428)\,dx_2 \right.$$

$$\left. \int_0^{0.3} (0.5714x_3)(4.643x_3)\,dx_3 + \int_0^{0.3} 0.5714(x_4 + 0.3)(1.643x_4 + 1.3929)\,dx_4\right]$$

$$= \frac{0.37972\ \text{kN}\cdot\text{m}^3}{EI} = \frac{0.37972(10^3)}{200(10^9)\left(\frac{\pi}{4}\right)(0.015^4)} = 0.0478\ \text{m} = 47.8\ \text{mm}\ \downarrow \quad \textbf{Ans.}$$



Ans:
$\Delta_B = 47.8\ \text{mm}\ \downarrow$

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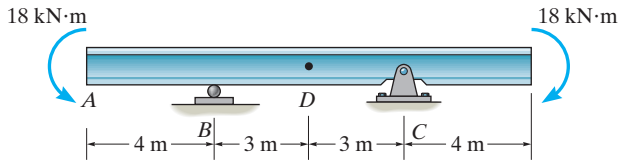
***14–96.**

The A992 steel beam has a moment of inertia of $I = 125(10^6)\ \text{mm}^4$. Determine the displacement at point D.

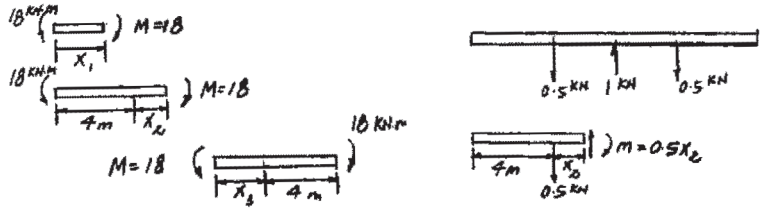


SOLUTION

$$1 \cdot \Delta_D = \int_0^L \frac{mM}{EI}\,dx$$

$$\Delta_D = (2)\frac{1}{EI}\left[\int_0^3 (0.5x_2)(18)\,dx_2\right] = \frac{81\ \text{kN}\cdot\text{m}^3}{EI} = \frac{81(10^3)}{200\,(10^9)(125)(10^{-6})}$$

$$= 3.24\,(10^{-3}) = 3.24\ \text{mm}\ \downarrow \qquad \textbf{Ans.}$$



Ans:
$\Delta_D = 3.24\ \text{mm}\ \downarrow$

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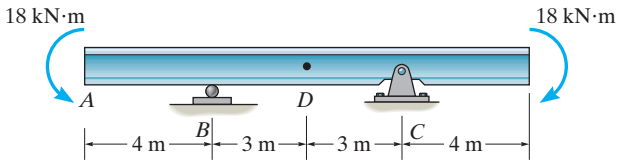
14–97.

The A992 steel beam has a moment of inertia of $I = 125(10^6)\ \text{mm}^4$. Determine the slope at A.

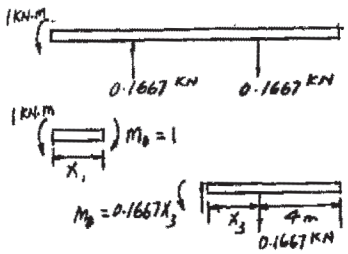


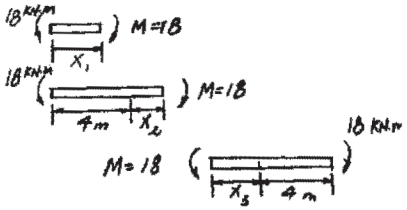
SOLUTION

$$1 \cdot \theta_A = \int_0^L \frac{m_\theta M}{EI}\,dx$$

$$\theta_A = \frac{1}{EI}\left[\int_0^4 (1)(18)\,(dx_1) + \int_0^6 (0.1667x_3)(18)\,dx_3\right] = \frac{126\ \text{kN}\cdot\text{m}^2}{EI}$$

$$= \frac{126\,(10^3)}{200\,(10^9)\,(125)\,(10^{-6})} = 5.04\,(10^{-3})\ \text{rad} = 0.289^\circ \qquad \textbf{Ans.}$$





Ans:
$\theta_A = 0.289^\circ$

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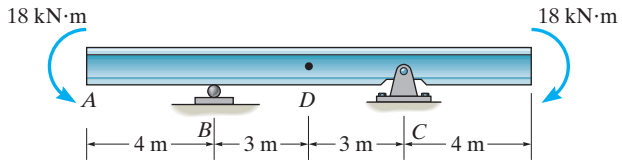
14–98.

The A992 structural steel beam has a moment of inertia of $I = 125(10^6)\ \text{mm}^4$. Determine the slope at B.

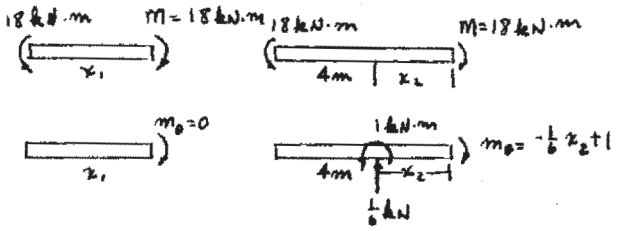


SOLUTION

$$1 \cdot \theta_B = \int_0^L \frac{m_\theta M}{EI}\,dx$$

$$\theta_B = 0 + \frac{1}{EI}\int_0^6 \frac{\left(-\frac{1}{6}x_2 + 1\right)(18)dx}{EI}$$

$$= \frac{54}{EI} = \frac{54\,(10^3)}{200\,(10^9)\,(125\,(10^{-6}))} = 0.00216\ \text{rad} = 0.124^\circ \qquad \textbf{Ans.}$$



Ans:
$\theta_B = 0.124^\circ$

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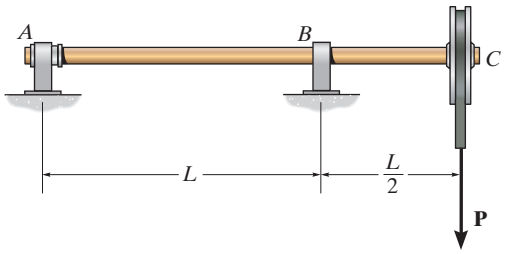
14–99.

Determine the displacement at point C of the shaft. EI is constant.



SOLUTION

Real Moment Function *M*: As indicated in Fig. *a*.

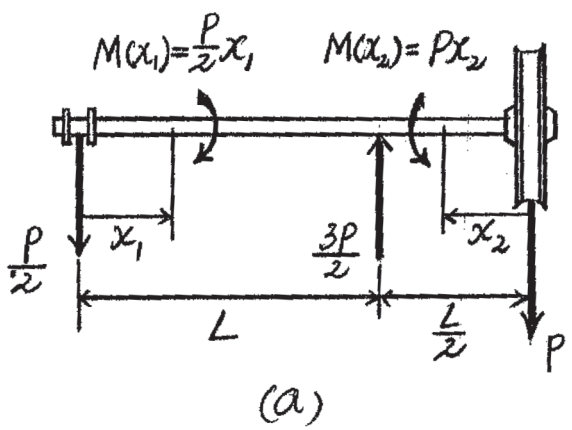
Virtual Moment Function *m*: As indicated in Fig. *b*.

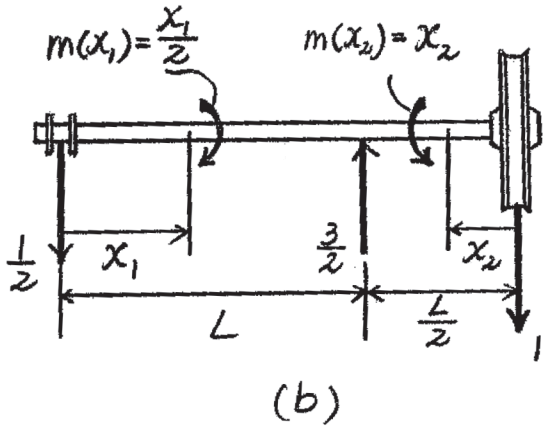
Virtual Work Equation:

$$1 \cdot \Delta = \int_0^L \frac{mM}{EI}\,dx$$

$$1 \cdot \Delta_C = \frac{1}{EI}\left[\int_0^L \left(\frac{x_1}{2}\right)\left(\frac{P}{2}x_1\right)dx_1 + \int_0^{L/2} x_2(Px_2)\,dx_2\right]$$

$$\Delta_C = \frac{PL^3}{8EI}\ \downarrow \qquad \textbf{Ans.}$$





Ans:

$$\Delta_C = \frac{PL^3}{8EI}\downarrow$$

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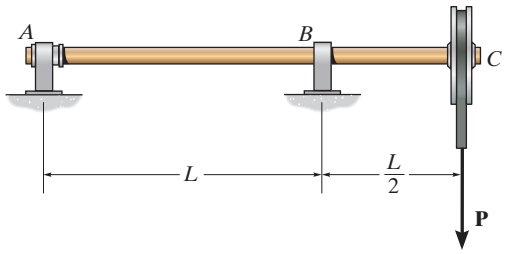
***14–100.**

Determine the slope at A of the shaft. EI is constant.



SOLUTION

Real Moment Function *M*: As indicated in Fig. *a*.

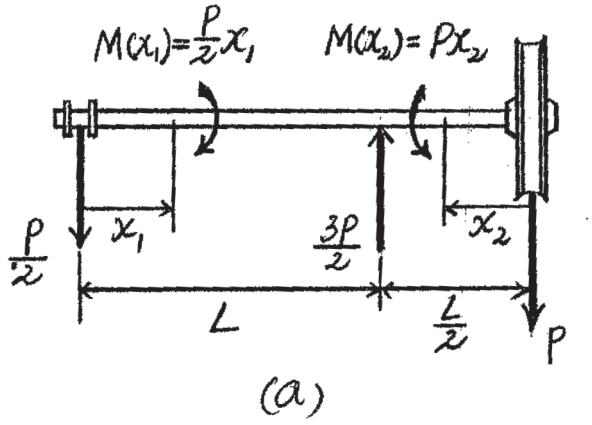
Virtual Moment Function *M*: As indicated in Fig. *b*.

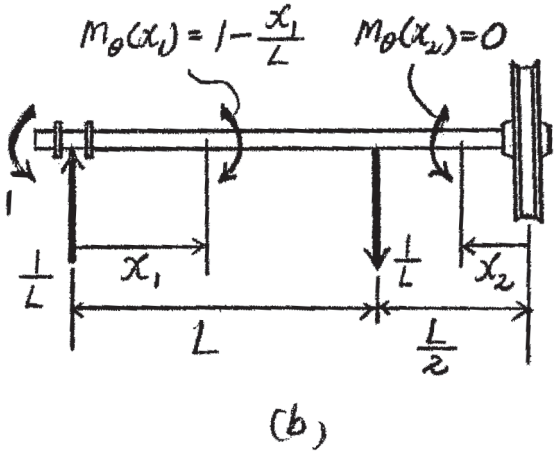
Virtual Work Equation:

$$1 \cdot \theta = \int_0^L \frac{m_\theta M}{EI}\, dx$$

$$1 \cdot \theta_A = \frac{1}{EI}\left[\int_0^L \left(1 - \frac{x_1}{L}\right)\left(\frac{P}{2}x_1\right) dx_1 + \int_0^{L/2} (0)\,(Px_2)\, dx_2\right]$$

$$\theta_A = \frac{PL^2}{12EI} \qquad \textbf{Ans.}$$





Ans:

$$\theta_A = \frac{PL^2}{12EI}$$

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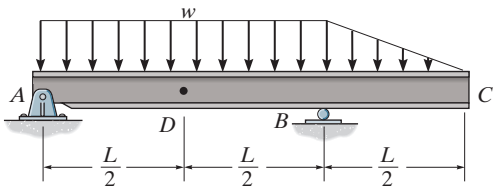
14–101.

Determine the slope of end C of the overhang beam. EI is constant.



SOLUTION

Real Moment Function M: As indicated in Fig. *a*.

Virtual Moment Function m_θ: As indicated in Fig. *b*.

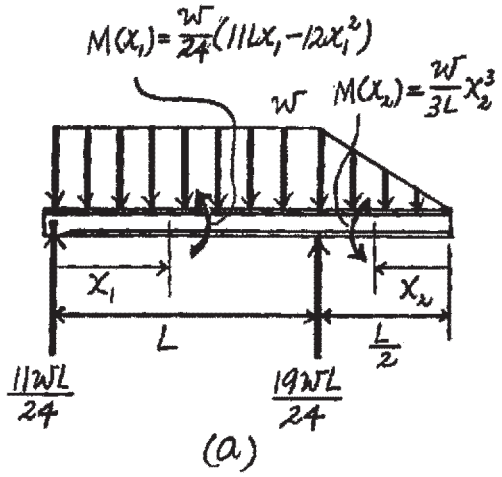
Virtual Work Equation:

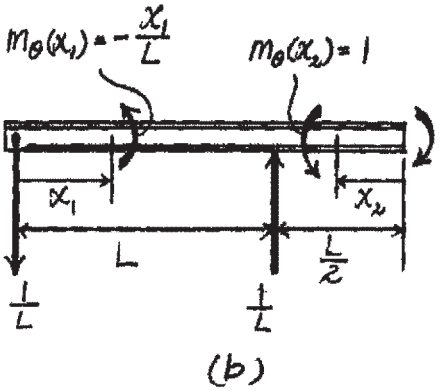
$$1 \cdot \theta = \int_0^L \frac{m_\theta M}{EI}\, dx$$

$$1 \cdot \theta_C = \frac{1}{EI}\left[\int_0^L \left(-\frac{x_1}{L}\right)\left[\frac{w}{24}\left(11Lx_1 - 12{x_1}^2\right)\right]dx_1 + \int_0^{L/2} (1)\left(\frac{w}{3L}{x_2}^3\right)dx_2\right]$$

$$\theta_C = \frac{1}{EI}\left[\frac{w}{24L}\int_0^L \left(12{x_1}^3 - 11L{x_1}^2\right)dx_1 + \frac{w}{3L}\int_0^{L/2} {x_2}^3\, dx_2\right]$$

$$\theta_C = -\frac{13wL^3}{576EI} = \frac{13wL^3}{576EI}\circlearrowleft$$ **Ans.**





Ans:

$$\theta_C = -\frac{13wL^3}{576\,EI}$$

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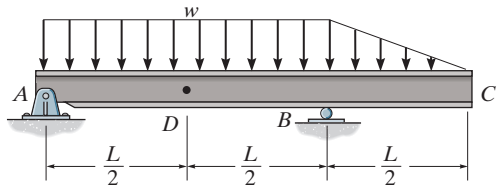
14–102.

Determine the displacement of point D of the overhang beam. EI is constant.



SOLUTION

Real Moment Function *M*: As indicated in Fig. *a*.

Virtual Moment Function *m*: As indicated in Fig. *b*.

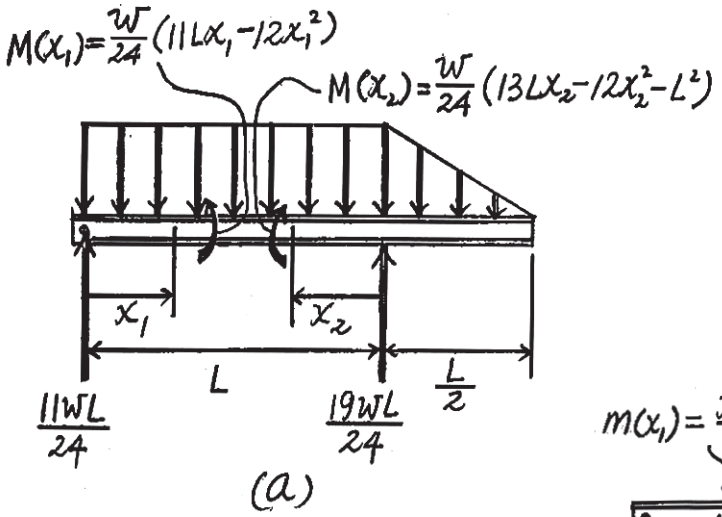
Virtual Work Equation:

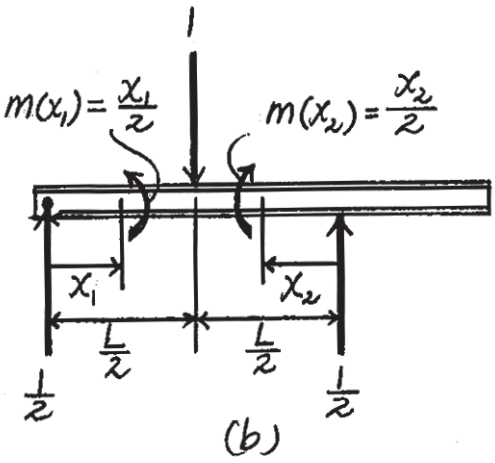
$$1 \cdot \Delta = \int_0^L \frac{mM}{EI}\,dx$$

$$1 \cdot \Delta_D = \frac{1}{EI}\left[\int_0^{L/2}\left(\frac{x_1}{2}\right)\left[\frac{w}{24}\left(11Lx_1 - 12{x_1}^2\right)\right]dx_1 + \int_0^{L/2}\left(\frac{x_2}{2}\right)\left[\frac{w}{24}\left(13Lx_2 - 12{x_2}^2 - L^2\right)\right]dx_2\right]$$

$$\Delta_D = \frac{w}{48EI}\left[\int_0^{L/2}\left(11Lx_1{}^2 - 12x_1{}^3\right)dx_1 + \int_0^{L/2}\left(13Lx_2{}^2 - 12x_2{}^3 - L^2x_2\right)dx_2\right]$$

$$\Delta_D = \frac{wL^4}{96EI}\downarrow$$ **Ans.**





Ans:

$$\Delta_D = \frac{wL^4}{96\,EI}\downarrow$$

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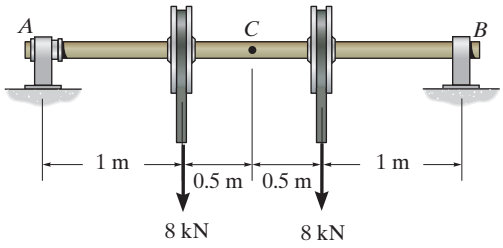
14–103.

Determine the slope at A of the 2014-T6 aluminum shaft having a diameter of 100 mm.



SOLUTION

Real Moment Function *M*: As indicated in Fig. *a*.

Virtual Moment Function *m*: As indicated in Fig. *b*.

Virtual Work Equation:

$$1 \cdot \theta = \int_0^L \frac{m_\theta M}{EI} dx$$

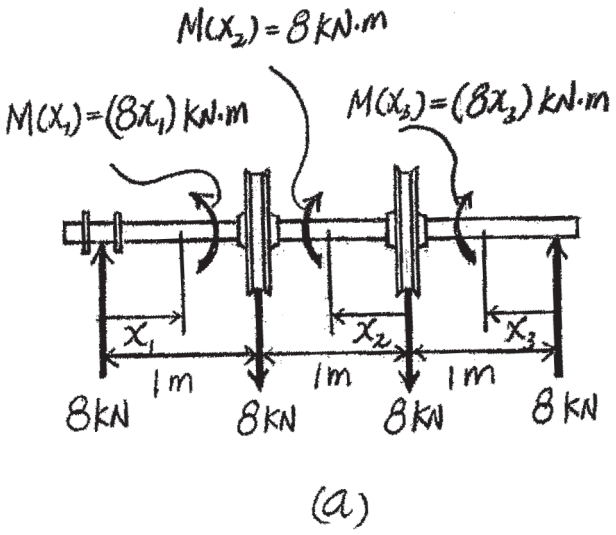
$$1\ \text{kN} \cdot \text{m} \cdot \theta_A = \frac{1}{EI}\left[\int_0^{1\,\text{m}} (1 - 0.3333x_1)(8x_1)\,dx_1 + \int_0^{1\,\text{m}} [0.3333(x_2+1)]\,8dx_2 + \int_0^{1\,\text{m}} (0.3333x_3)(8x_3)\,dx_3\right]$$

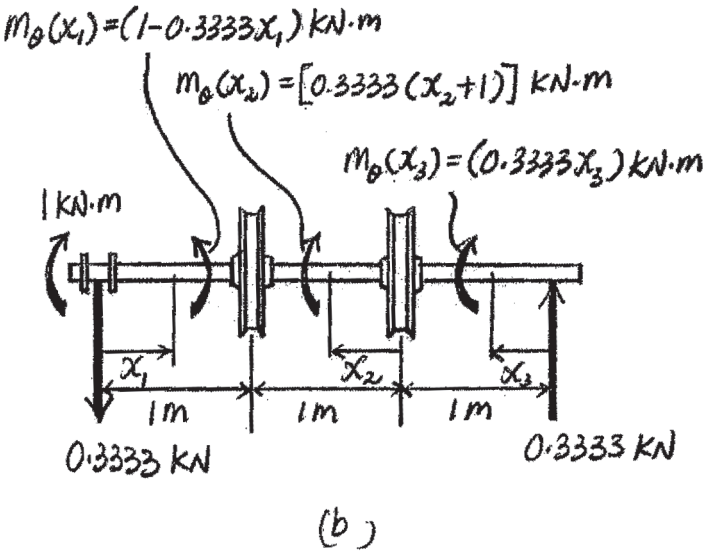
$$\theta_A = \frac{1}{EI}\left[8\int_0^{1\,\text{m}} (1 - 0.3333x_1)\,dx_1 + 2.6667\int_0^{1\,\text{m}} (x_2 + 1)\,dx_2 + 2.6667\int_0^{1\,\text{m}} x_3^2\,dx_3\right]$$

$$= \frac{8\ \text{kN} \cdot \text{m}^2}{EI}$$

$$= \frac{8(10^3)}{73.1(10^9)\left[\frac{\pi}{4}(0.05^4)\right]}$$

$$= 0.02229\ \text{rad} = 1.28^\circ \circlearrowright$$ **Ans.**





Ans:
$\theta_A = -1.28^\circ$

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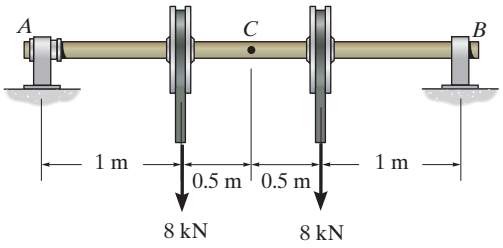
***14–104.**

Determine the displacement at point C of the 2014-T6 aluminum shaft having a diameter of 100 mm.



SOLUTION

Real Moment Function *M*: As indicated in Fig. *a*.

Virtual Moment Function *m*: As indicated in Fig. *b*.

Virtual Work Equation:

$$1 \cdot \Delta = \int_0^L \frac{mM}{EI} dx$$

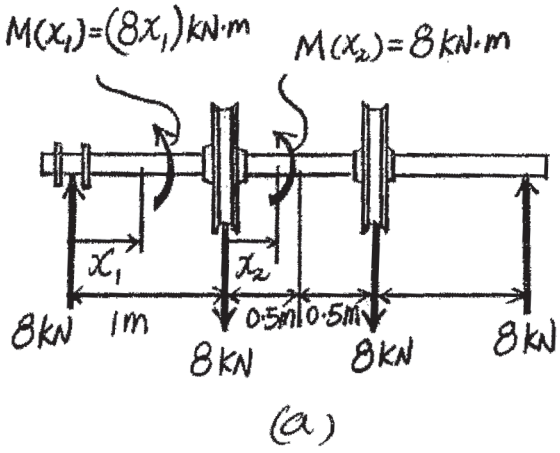
$$1 \cdot \Delta_C = 2\left[\frac{1}{EI}\left[\int_0^{1\text{ m}} (0.5x_1)(8x_1)dx_1 + \int_0^{0.5\text{ m}} [0.5(x_2 + 1)](8)dx_2\right]\right]$$

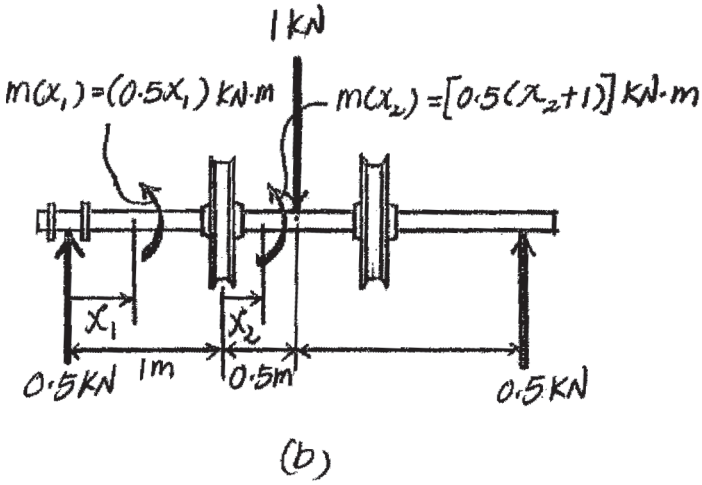
$$\Delta_C = \frac{2}{EI}\left[\int_0^{1\text{ m}} 4x_1^2 dx_1 + \int_0^{0.5\text{ m}} 4(x_2 + 1)dx_2\right]$$

$$= \frac{7.6667\text{ kN} \cdot \text{m}^3}{EI}$$

$$= \frac{7.6667(10^3)}{73.1(10^9)\left[\frac{\pi}{4}(0.05^4)\right]}$$

$$= 0.02137\text{ m} = 21.4\text{ mm} \downarrow$$ **Ans.**





Ans:
$\Delta_C = 21.4\text{ mm} \downarrow$

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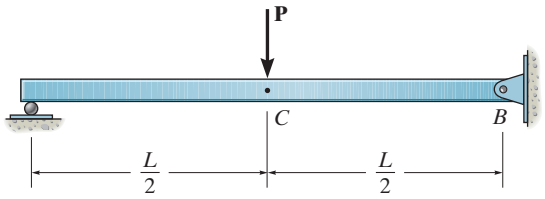
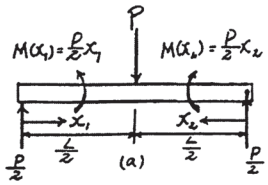
14–105.

Determine the displacement at point C and the slope at B. EI is constant.

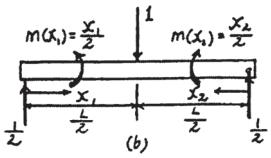


SOLUTION



Real Moment Function $M(x)$: As shown on Figure (a).

Virtual Moment Functions $m(x)$ and $m_\theta(x)$: As shown on Figure (b) and (c).

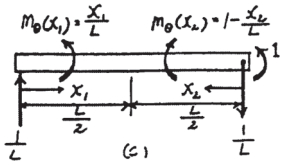


Virtual Work Equation: For the displacement at point C, apply Eq. 14–42.

$$1 \cdot \Delta = \int_0^L \frac{mM}{EI}\,dx$$

$$1 \cdot \Delta_C = 2\left[\frac{1}{EI}\int_0^{\frac{L}{2}}\left(\frac{x_1}{2}\right)\left(\frac{P}{2}x_1\right)dx_1\right]$$

$$\Delta_C = \frac{PL^3}{48EI}\downarrow \qquad \textbf{Ans.}$$



For the slope at B, apply Eq. 14–43.

$$1 \cdot \theta = \int_0^L \frac{m_\theta M}{EI}\,dx$$

$$1 \cdot \theta_B = \frac{1}{EI}\left[\int_0^{\frac{L}{2}}\left(\frac{x_1}{L}\right)\left(\frac{P}{2}x_1\right)dx_1 + \int_0^{\frac{L}{2}}\left(1 - \frac{x_2}{L}\right)\left(\frac{P}{2}x_2\right)dx_2\right]$$

$$\theta_B = \frac{PL^2}{16EI} \qquad \textbf{Ans.}$$

Ans:

$$\Delta_C = \frac{PL^3}{48EI}\downarrow$$

$$\theta_B = \frac{PL^2}{16EI}$$

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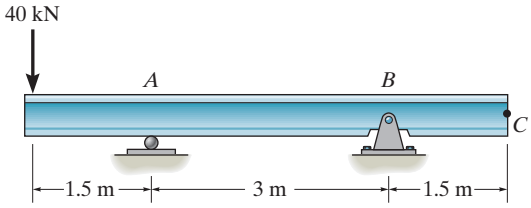
14–106. Determine the displacement of point C of the beam made from A992 steel and having a moment of inertia of $I = 22.3(10^6)\ \text{mm}^4$.



SOLUTION

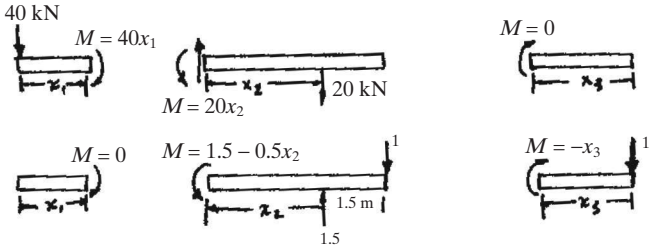
$$1 \cdot \Delta_C = \int_0^L \frac{mM}{EI}\,dx$$

$$\Delta_C = \frac{1}{EI}\left[0 + \int_0^{3\text{m}} (1.5 - 0.5x_2)(20x_2)\,dx_2 + 0\right]$$

$$= \frac{45\ \text{kN}\cdot\text{m}^2}{EI} = \frac{45(10^3)}{[200(10^9)][22.3(10^{-6})]}$$

$$= 0.01009\ \text{m} = 10.1\ \text{mm}\ \downarrow$$

Ans.



Ans:
$\Delta_C = 10.1\ \text{mm}$

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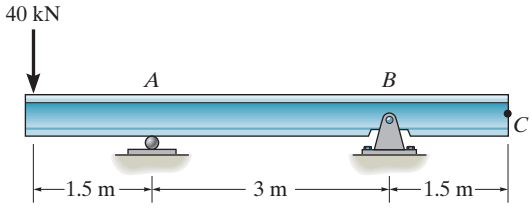
14–107. Determine the slope at B of the beam made from A992 steel and having a moment of inertia of $I = 22.3(10^6)$ mm^4.



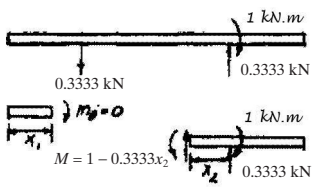
SOLUTION

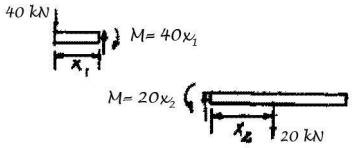
$$1 \cdot \theta_B = \int_0^L \frac{m_\theta M}{EI} dx$$

$$\theta_B = \frac{1}{EI}\left[\int_0^{1.5\text{m}} (0)(40x_1)dx_1 + \int_0^{3\text{m}} (1 - 0.3333x_2)(20x_2)dx_2 + 0\right]$$

$$= \frac{30\ \text{kN}\cdot\text{m}^2}{EI} = \frac{30(10^3)}{[200(10^9)][22.3(10^{-6})]}$$

$$= 6.7265(10^{-3})\ \text{rad} = 0.385° \quad \curvearrowright \theta_B$$ **Ans.**





Ans:
$\theta_B = 0.385° \quad \curvearrowright \theta_B$

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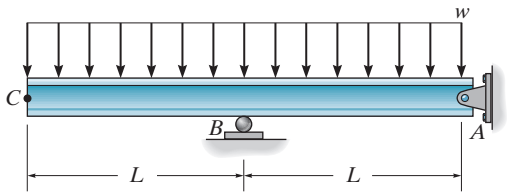
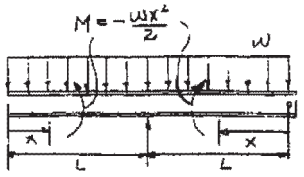
***14–108.**

Determine the slope at A. EI is constant.



SOLUTION

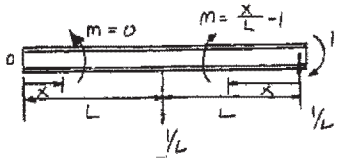


$$\theta_A = \int_0^L \frac{m_\theta M}{EI}\,dx$$

$$= 0 + \int_0^L \frac{\left(\frac{x}{L} - 1\right)\left(\frac{-wx^2}{2}\right)}{EI}\,dx$$

$$= \frac{-\frac{wL^4}{8L} + \frac{wL^3}{6}}{EI} = -\frac{w\,L^3}{24\,EI}$$

Ans.



Ans:

$$\theta_A = -\frac{wL^3}{24\,EI}$$

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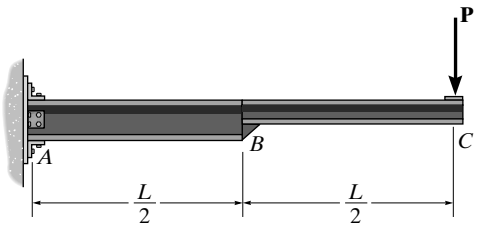
14–109. Determine the slope and displacement of end C of the cantilevered beam. The beam is made of a material having a modulus of elasticity of E. The moments of inertia for segments AB and BC of the beam are $2I$ and I, respectively.



Real Moment Function M. As indicated in Fig. *a*.

Virtual Moment Functions m_θ **and** M. As indicated in Figs. *b* and *c*.

Virtual Work Equation. For the slope at C,

$$1 \cdot \theta = \int_0^L \frac{m_\theta M}{EI}\, dx$$

$$1 \cdot \theta_C = \frac{1}{EI}\int_0^{L/2} 1(Px_1)dx_1 + \frac{1}{2\,EI}\int_0^{L/2} 1\left[P\left(x_2 + \frac{L}{2}\right)\right]dx_2$$

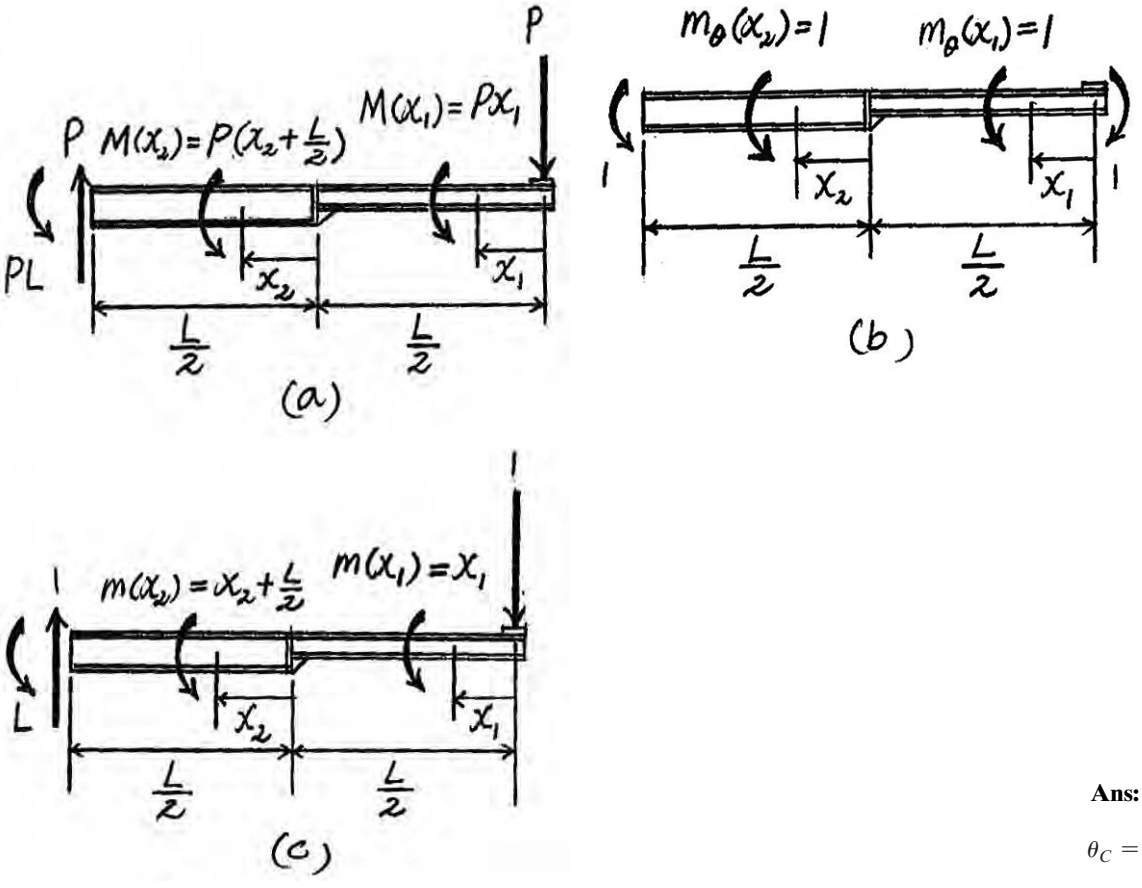
$$\theta_C = \frac{5PL^2}{16\,EI} \qquad \textbf{Ans.}$$

For the displacement at C,

$$1 \cdot \Delta = \int_0^L \frac{mM}{EI}\, dx$$

$$1 \cdot \Delta_C = \frac{1}{EI}\int_0^{L/2} x_1(Px_1)dx_1 + \frac{1}{2EI}\int_0^{L/2} \left(x_2 + \frac{L}{2}\right)\left[P\left(x_2 + \frac{L}{2}\right)\right]dx_2$$

$$\Delta_C = \frac{3PL}{16EI}\ \downarrow \qquad \textbf{Ans.}$$



Ans:

$$\theta_C = \frac{5PL^2}{16EI},\ \Delta_C = \frac{3PL}{16EI}\downarrow$$

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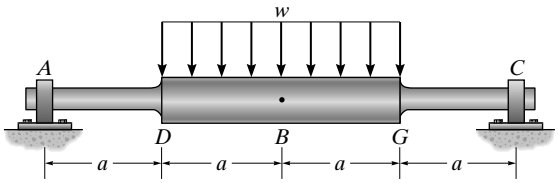
14–110. Determine the displacement at point B. The moment of inertia of the center portion DG of the shaft is $2I$, whereas the end segments AD and GC have a moment of inertia I. The modulus of elasticity for the material is E.



SOLUTION

Real Moment Function M(x): As shown on Fig. a.

Virtual Moment Functions m(x): As shown on Fig. b.

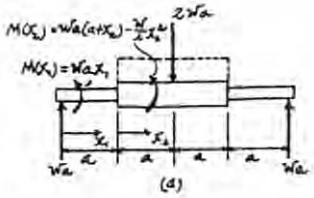
Virtual Work Equation: For the slope at point B, apply Eq. 14–42.

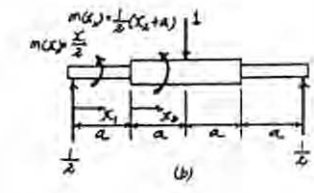
$$1 \cdot \Delta = \int_0^L \frac{mM}{EI}\,dx$$

$$1 \cdot \Delta_B = 2\left[\frac{1}{EI}\int_0^a \left(\frac{x_1}{2}\right)(wax_1)dx_1\right]$$

$$+ 2\left[\frac{1}{2EI}\int_0^a \frac{1}{2}(x_2 + a)\left[wa(a + x_2) - \frac{w}{2}x_2^2\right]dx_2\right]$$

$$\Delta_B = \frac{65wa^4}{48EI} \quad \downarrow$$ **Ans.**





Ans:

$M(x_2) = wa(a + x_2) - \frac{w}{2}x_2^2, M(x_1) = wax_1,$

$m(x_2) = \frac{1}{2}(x_2 + a), m(x_1) = \frac{x_1}{2}, \Delta_B = \frac{65wa^4}{48EI} \downarrow$

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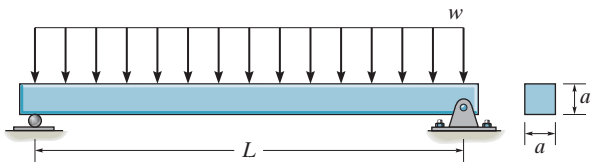
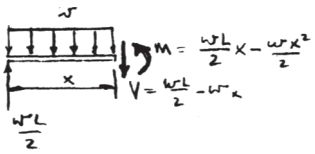
14–111.

Determine the maximum deflection of the beam caused only by bending, and caused by bending and shear. Take $E = 3G$.



SOLUTION



For bending and shear,

$$1 \cdot \Delta = \int_0^L \frac{mM}{EI}\,dx + \int_0^L \frac{f_s v V}{GA}\,dx$$

$$\Delta = 2\int_0^{L/2} \frac{\left(\frac{1}{2}x\right)\left(\frac{wL}{2}x - w\frac{x^2}{2}\right)dx}{EI} + 2\int_0^{L/2} \frac{\left(\frac{6}{5}\right)\left(\frac{1}{2}\right)\left(\frac{wL}{2} - wx\right)dx}{GA}$$

$$= \frac{1}{EI}\left(\frac{wL}{6}x^3 - \frac{wx^4}{8}\right)\Bigg|_0^{L/2} + \frac{\left(\frac{6}{5}\right)}{GA}\left(\frac{wL}{2}x - \frac{wx^2}{2}\right)\Bigg|_0^{L/2}$$

$$= \frac{5wL^4}{384EI} + \frac{3wL^2}{20\,GA}$$

$$\Delta = \frac{5wL^4}{384(3G)\left(\frac{1}{12}\right)a^4} + \frac{3wL^2}{20(G)a^2}$$

$$= \frac{20wL^4}{384Ga^4} + \frac{3wL^2}{20Ga^2}$$

$$= \left(\frac{w}{G}\right)\left(\frac{L}{a}\right)^2\left[\left(\frac{5}{96}\right)\left(\frac{L}{a}\right)^2 + \frac{3}{20}\right] \qquad \textbf{Ans.}$$

For bending only,

$$\Delta = \frac{5w}{96G}\left(\frac{L}{a}\right)^4 \qquad \textbf{Ans.}$$

Ans:

$$\Delta_{\text{tot}} = \left(\frac{w}{G}\right)\left(\frac{L}{a}\right)^2\left[\left(\frac{5}{96}\right)\left(\frac{L}{a}\right)^2 + \frac{3}{20}\right],$$

$$\Delta_b = \frac{5w}{96G}\left(\frac{L}{a}\right)^4$$

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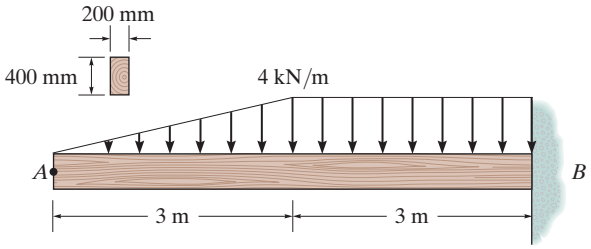
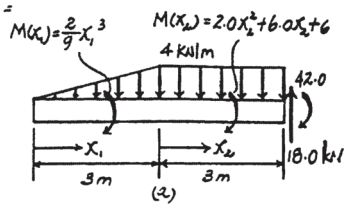
***14–112.**

The beam is made of oak, for which $E_o = 11$ GPa. Determine the slope and displacement at point A.



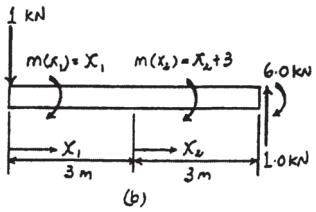
SOLUTION



Virtual Work Equation: For the displacement at point A, apply Eq. 14–42.

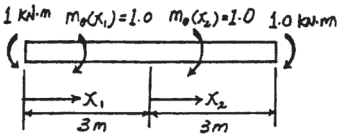
$$1 \cdot \Delta = \int_0^L \frac{mM}{EI}\,dx$$

$$1\ \text{kN} \cdot \Delta_A = \frac{1}{EI}\int_0^{3\ \text{m}} x_1\left(\frac{2}{9}x_1^3\right)dx_1 + \frac{1}{EI}\int_0^{3\ \text{m}} (x_2 + 3)\left(2.00x_2^2 + 6.00x_2 + 6.00\right)dx_2$$



$$\Delta_A = \frac{321.3\ \text{kN} \cdot \text{m}^3}{EI} = \frac{321.3\ (10^3)}{11(10^9)\left[\frac{1}{12}(0.2)(0.4^3)\right]}$$

$$= 0.02738\ \text{m} = 27.4\ \text{m}\ \downarrow \qquad \textbf{Ans.}$$



For the slope at A, apply Eq. 14–43.

$$1 \cdot \theta = \int_0^L \frac{m_\theta M}{EI}\,dx$$

$$1\ \text{kN} \cdot \text{m} \cdot \theta_A = \frac{1}{EI}\int_0^{3\ \text{m}} 1.00\left(\frac{2}{9}x_1^3\right)dx_1 + \int_0^{3\ \text{m}} 1.00\left(2.00x_2^2 + 6.00x_2 + 6.00\right)dx_2$$

$$\theta_A = \frac{67.5\ \text{kN} \cdot \text{m}^2}{EI} = \frac{67.5\ (1000)}{11(10^9)\left[\frac{1}{12}(0.2)(0.4^3)\right]}$$

$$= 5.75\ (10^{-3})\ \text{rad} \qquad \textbf{Ans.}$$

Ans:
$\Delta_A = 27.4\ \text{m}\ \downarrow$,
$\theta_A = 5.75(10^{-3})\ \text{rad}$

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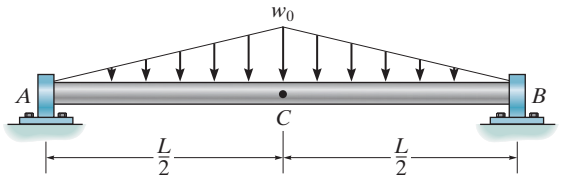
14–113.

Determine the slope of the shaft at the bearing support A. EI is constant.

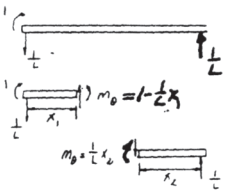


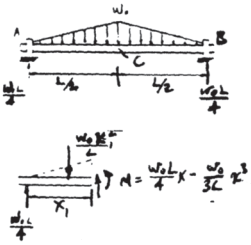
SOLUTION

$$1 \cdot \theta_A = \int_0^L \frac{m_\theta M}{EI} dx$$

$$\theta_A = \frac{1}{EI}\left[\int_0^{\frac{L}{2}} \left(1 - \frac{1}{L}x_1\right)\left(\frac{w_0 L}{4}x_1 - \frac{w_0}{3L}x_1^3\right)dx_1 + \int_0^{\frac{L}{2}} \left(\frac{1}{L}x_2\right)\left(\frac{w_0 L}{4}x_2 - \frac{w_0}{3L}x_2^3\right)dx_2\right]$$

$$= \frac{5w_0 L^3}{192EI} \curvearrowright \qquad \textbf{Ans.}$$





Ans:

$$\theta_A = -\frac{5w_0 L^3}{192\,EI}$$

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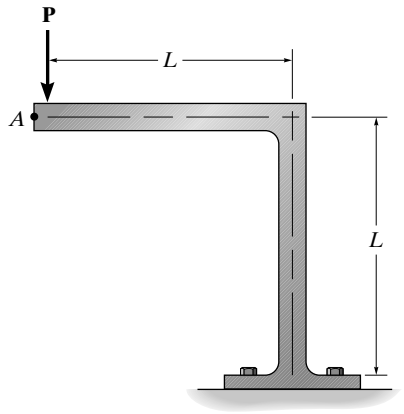
14–114. Determine the vertical displacement of point A on the angle bracket due to the concentrated force **P**. The bracket is fixed connected to its support. EI is constant. Consider only the effect of bending.

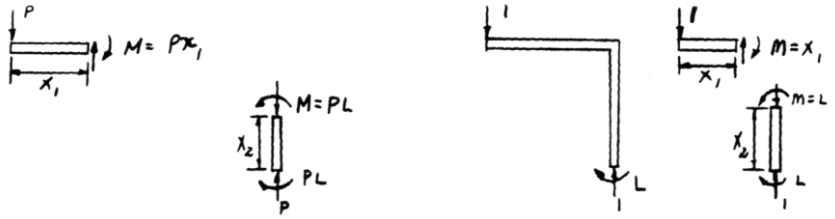


SOLUTION

$$1 \cdot \Delta_{A_v} = \int_0^L \frac{mM}{EI}\,dx$$

$$\Delta_{A_v} = \frac{1}{EI}\left[\int_0^L (x_1)(Px_1)dx_1 + \int_0^L (1L)(PL)dx_2\right]$$

$$= \frac{4PL^3}{3EI} \qquad \textbf{Ans.}$$



Ans:

$$\Delta_{A_v} = \frac{4PL^3}{3EI}$$

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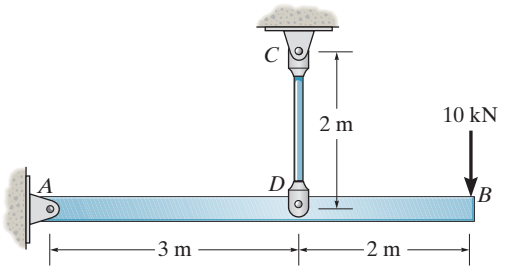
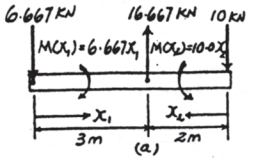
14–115.

Beam AB has a square cross section of 100 mm by 100 mm. Bar CD has a diameter of 10 mm. If both members are made of A992 steel, determine the vertical displacement of point B due to the loading of 10 kN.

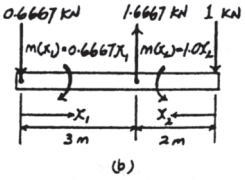


SOLUTION

Real Moment Function $M(x)$: As shown in Figure *a*.



Virtual Moment Functions $m(x)$: As shown in Figure *b*.



Virtual Work Equation: For the displacement at point B,

$$1 \cdot \Delta = \int_0^L \frac{mM}{EI}\,dx + \frac{nNL}{AE}$$

$$1\ \text{kN} \cdot \Delta_B = \frac{1}{EI}\int_0^{3\ \text{m}} (0.6667x_1)(6.667x_1)dx_1 + \frac{1}{EI}\int_0^{2\ \text{m}} (1.00x_2)(10.0x_2)dx_2 + \frac{1.667(16.667)(2)}{AE}$$

$$\Delta_B = \frac{66.667\ \text{kN} \cdot \text{m}^3}{EI} + \frac{55.556\ \text{kN} \cdot \text{m}}{AE}$$

$$= \frac{66.667(1000)}{200(10^9)\left[\frac{1}{12}(0.1)\left(0.1^3\right)\right]} + \frac{55.556(1000)}{\left[\frac{\pi}{4}\left(0.01^2\right)\right]\left[200\left(10^9\right)\right]}$$

$$= 0.04354\ \text{m} = 43.5\ \text{mm} \quad \downarrow$$

Ans.

Ans:
$\Delta_B = 43.5\ \text{mm} \downarrow$

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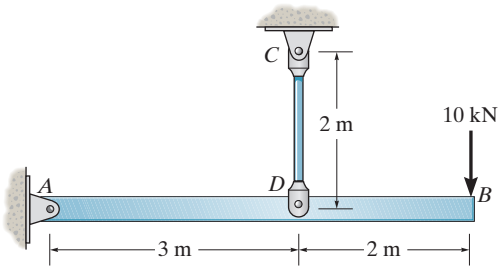
***14–116.**

Beam AB has a square cross section of 100 mm by 100 mm. Bar CD has a diameter of 10 mm. If both members are made of A992 steel, determine the slope at A due to the loading of 10 kN.



SOLUTION

Real Moment Function $M(x)$: As shown in Figure *a*.

Virtual Moment Functions $m_\theta(x)$: As shown in Figure *b*.

Virtual Work Equation: For the slope at point A,

$$1 \cdot \theta = \int_0^L \frac{m_\theta M}{EI}\,dx + \frac{nNL}{AE}$$

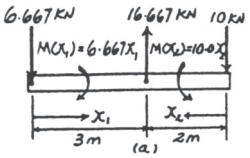
$$1\ \text{kN} \cdot \text{m} \cdot \theta_A = \frac{1}{EI}\int_0^{3\ \text{m}} (1 - 0.3333x_1)(6.667x_1)dx_1 + \frac{1}{EI}\int_0^{2\ \text{m}} 0(10.0x_2)dx_2 + \frac{(-0.3333)(16.667)(2)}{AE}$$

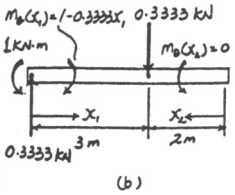
$$\theta_A = \frac{10.0\ \text{kN} \cdot \text{m}^2}{EI} - \frac{11.111\ \text{kN}}{AE}$$

$$= \frac{10.0(1000)}{200\left(10^9\right)\left[\frac{1}{12}(0.1)\left(0.1^3\right)\right]} - \frac{11.111(1000)}{\left[\frac{\pi}{4}\left(0.01^2\right)\right]\left[200\left(10^9\right)\right]}$$

$$= 0.00529\ \text{rad} = 0.303^\circ \curvearrowright$$

Ans.





Ans:
$\theta_A = -0.303^\circ$

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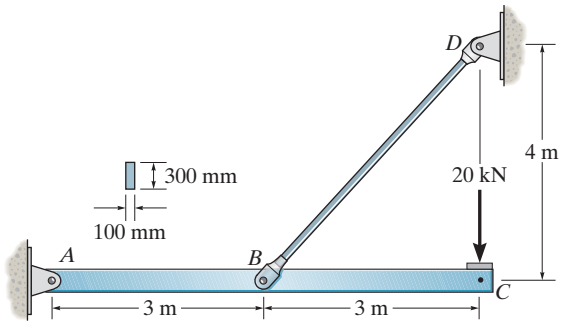
14–117.

Bar *ABC* has a rectangular cross section of 300 mm by 100 mm. Attached rod *DB* has a diameter of 20 mm. If both members are made of A-36 steel, determine the vertical displacement of point *C* due to the loading. Consider only the effect of bending in A*BC* and axial force in *DB*.



SOLUTION

Real Moment Function $M(x)$: As shown in Figure *a*.

Virtual Moment Functions $m(x)$: As shown in Figure *b*.

Virtual Work Equation: For the displacement at point *C*,

$$1 \cdot \Delta = \int_0^L \frac{mM}{EI}\,dx + \frac{nNL}{AE}$$

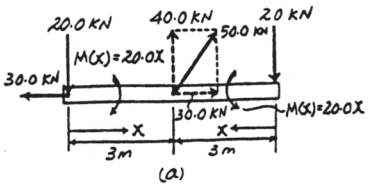
$$1\ \text{kN} \cdot \Delta_C = 2\left[\frac{1}{EI}\int_0^{3\ \text{m}} (1.00x)(20.0x)\,dx\right] + \frac{2.50(50.0)\,(5)}{AE}$$

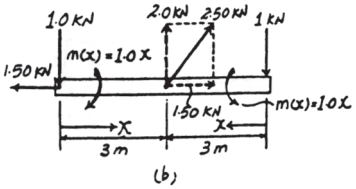
$$\Delta_C = \frac{360\ \text{kN} \cdot \text{m}^3}{EI} + \frac{625\ \text{kN} \cdot \text{m}}{AE}$$

$$= \frac{360(1000)}{200\left(10^9\right)\left[\frac{1}{12}(0.1)\left(0.3^3\right)\right]} + \frac{625(1000)}{\left[\frac{\pi}{4}\left(0.02^2\right)\right]\left[200\left(10^9\right)\right]}$$

$$= 0.017947\ \text{m} = 17.9\ \text{mm}\ \downarrow$$

Ans.





Ans:
$\Delta_C = 17.9\ \text{mm}\downarrow$

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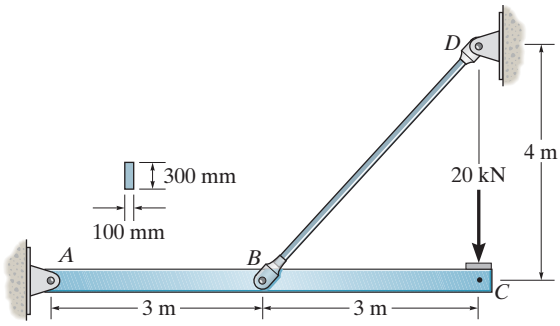
14–118.

Bar ABC has a rectangular cross section of 300 mm by 100 mm. Attached rod DB has a diameter of 20 mm. If both members are made of A-36 steel, determine the slope at A due to the loading. Consider only the effect of bending in ABC and axial force in DB.



SOLUTION

Real Moment Function $M(x)$: As shown in Figure *a*.

Virtual Moment Functions $m_\theta(x)$: As shown in Figure *b*.

Virtual Work Equation: For the slope at point A,

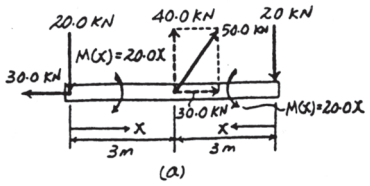
$$1 \cdot \theta = \int_0^L \frac{m_\theta M}{EI}\,dx + \frac{nNL}{AE}$$

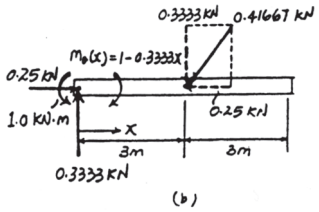
$$1\ \text{kN} \cdot \text{m} \cdot \theta_A = \frac{1}{EI}\int_0^{3\ \text{m}} (1 - 0.3333x)(20.0x)dx + \frac{(-0.41667)(50.0)(5)}{AE}$$

$$\theta_A = \frac{30.0\ \text{kN} \cdot \text{m}^2}{EI} - \frac{104.167\ \text{kN}}{AE}$$

$$= \frac{30.0(1000)}{200\left(10^9\right)\left[\frac{1}{12}(0.1)\left(0.3^3\right)\right]} - \frac{104.167(1000)}{\left[\frac{\pi}{4}\left(0.02^2\right)\right]\left[200\left(10^9\right)\right]}$$

$$= -0.991\left(10^{-3}\right)\ \text{rad} = 0.0568°\circlearrowright$$ **Ans.**





Ans:
$\theta_A = -0.0568°$

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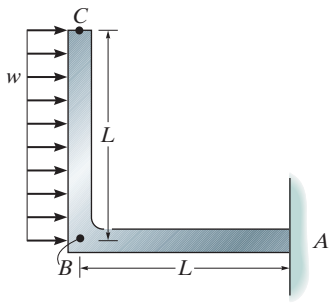
14–119.

The L-shaped frame is made from two segments, each of length L and flexural stiffness EI. If it is subjected to the uniform distributed load, determine the horizontal displacement of point C.

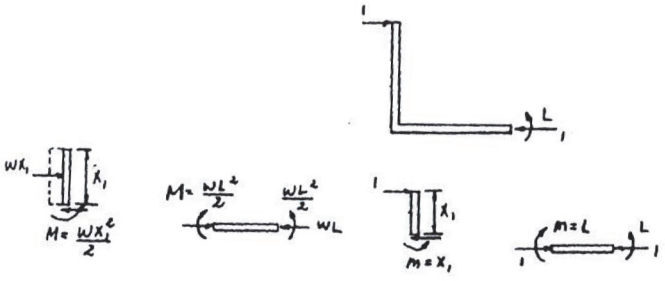


SOLUTION

$$1 \cdot \Delta_C = \int_0^L \frac{mM}{EI}\,dx$$

$$\Delta_C = \frac{1}{EI}\left[\int_0^L (1x_1)\left(\frac{wx_1^2}{2}\right)dx_1 + \int_0^L (1L)\left(\frac{wL^2}{2}\right)dx_2\right]$$

$$= \frac{5wL^4}{8EI} \rightarrow$$ **Ans.**



Ans:

$$(\Delta_C)_h = \frac{5wL^4}{8EI} \rightarrow$$

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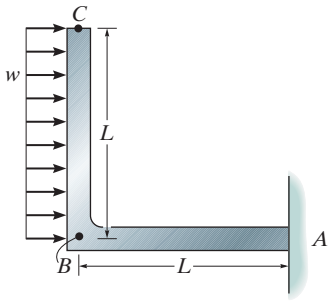
***14–120.**

The L-shaped frame is made from two segments, each of length L and flexural stiffness EI. If it is subjected to the uniform distributed load, determine the vertical displacement of point B.

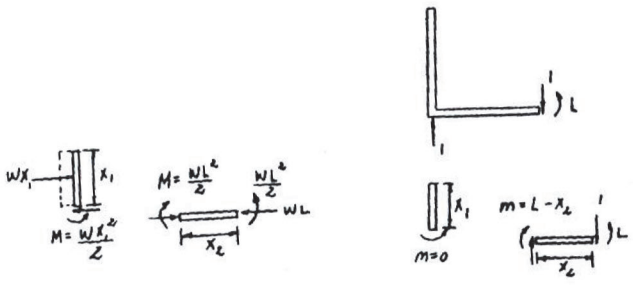


SOLUTION

$$1 \cdot \Delta_B = \int_0^L \frac{mM}{EI}\,dx$$

$$\Delta_B = \frac{1}{EI}\left[\int_0^L (0)\left(\frac{wx_1^2}{2}\right)dx_1 + \int_0^L (L - x_2)\left(\frac{wL^2}{2}\right)dx_2\right]$$

$$= \frac{wL^4}{4EI}\uparrow$$ **Ans.**



Ans:

$$(\Delta_B)_v = \frac{wL^4}{4EI}\uparrow$$

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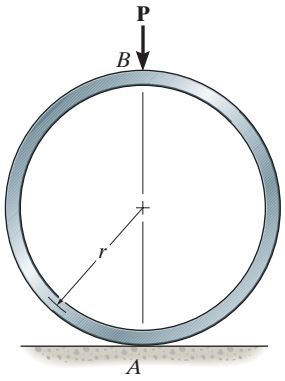
14–121.

Determine the vertical displacement of the ring at point B. EI is constant.



SOLUTION

Model: The ring can be modeled as a half ring as shown in Figure (a).

Real Moment Function $M(x)$: As shown on Figure (a).

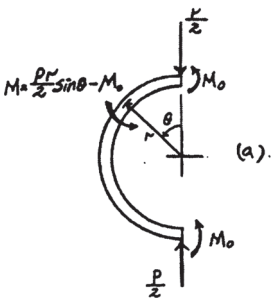
Virtual Moment Functions $m(x)$ and $m_\theta(x)$: As shown on Figure (b) and (c).

Virtual Work Equation: Due to symmetry, the slope at B remains horizontal, i.e., equal to zero. Applying Eq. 14–43, we have

$$1 \cdot \theta = \int_0^L \frac{m_\theta M}{EI}\, ds \qquad \text{Where } ds = r d\theta$$

$$1 \cdot \theta_B = 0 = \frac{1}{EI}\int_0^\pi 1.00\left(\frac{Pr}{2}\sin\theta - M_0\right) r d\theta$$

$$M_0 = \frac{Pr}{\pi}$$



For the vertical displacement at B, apply Eq. 14–42.

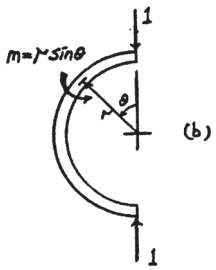
$$1 \cdot \Delta = \int_0^L \frac{mM}{EI}\, ds$$

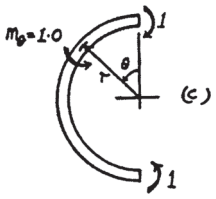
$$1 \cdot (\Delta_B)_v = \frac{1}{EI}\int_0^\pi (r\sin\theta)\left(\frac{Pr}{2}\sin\theta - \frac{Pr}{\pi}\right) r d\theta$$

$$= \frac{Pr^3}{2\pi EI}\int_0^\pi \left(\pi \sin^2\theta - 2\sin\theta\right) d\theta$$

$$= \frac{Pr^3}{4\pi EI}\int_0^\pi [\pi(1 - \cos 2\theta) - 4\sin\theta]\, d\theta$$

$$(\Delta_B)_v = \frac{Pr^3}{4\pi EI}\left(\pi^2 - 8\right)\downarrow \qquad \textbf{Ans.}$$





Ans:

$$(\Delta_B)_v = \frac{Pr^3}{4\pi EI}\left(\pi^2 - 8\right)\downarrow$$

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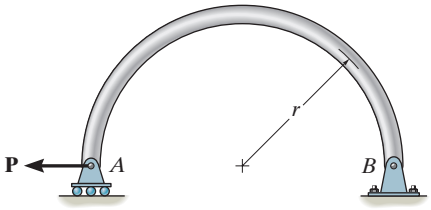
14–122.

Determine the horizontal displacement at the roller at A due to the loading. EI is constant.



SOLUTION

$$\Sigma F_x = 0; \qquad N - P\sin\theta = 0$$
$$N = P\sin\theta$$

$$\Sigma M_o = 0; \qquad M - P\sin\theta\, r = 0$$
$$M = Pr\sin\theta$$

$$\Sigma F_x = 0; \qquad n - 1\sin\theta = 0$$
$$n = 1\sin\theta$$

$$\zeta+\Sigma M_o = 0; \qquad m - (1\sin\theta)\, r = 0$$
$$m = r\sin\theta$$

$$1\cdot(\Delta_A)_h = \int_0^L \frac{mM}{EI}\,dx = \frac{Pr^2}{EI}\int_0^\pi \sin^2\theta\,(r d\theta) = \frac{Pr^3}{EI}\left(\frac{1}{2}\theta - \frac{1}{4}\sin 2\theta\right)\Big|_0^\pi$$

$$(\Delta_A)_h = \frac{\pi Pr^3}{2EI}\leftarrow$$ **Ans.**

Ans:

$$(\Delta_A)_h = \frac{\pi Pr^3}{2EI}\leftarrow$$

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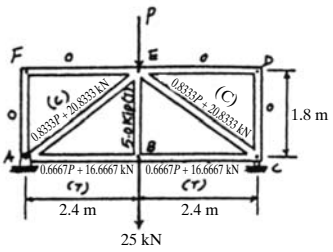
14–123. Solve Prob. 14–72 using Castigliano's theorem.



SOLUTION

Member Forces N: Member forces due to external force **P** and external applied forces are shown on the figure.

Castigliano's Second Theorem:

Member	N	$\frac{\partial N}{\partial P}$	$N(P=0)$	L	$N\left(\frac{\partial N}{\partial P}\right)L$
AB	$0.6667P+3.333$	0.6667	16.6667	2.4	26.667
BC	$0.6667P+3.333$	0.6667	16.6667	2.4	26.667
CD	0	0	0	1.8	0
DE	0	0	0	2.4	0
EF	0	0	0	2.4	0
AF	0	0	0	1.8	0
AE	$-(0.8333P + 4.167)$	-0.8333	-20.8333	3.0	52.083
CE	$-(0.8333P + 4.167)$	-0.8333	-20.8333	3.0	52.083
BE	5.0	0	25.00	1.8	0
					$\Sigma 157.5\,\text{kN}\cdot\text{m}$

$$\Delta = \sum N\left(\frac{\partial N}{\partial P}\right)\frac{L}{AE}$$

$$(\Delta_E)_v = \frac{157.5\ \text{kN}\cdot\text{m}}{AE}$$

$$= \frac{157.5(10^3)}{[2.80(10^{-3})][200(10^9)]} = 0.28125(10^{-3})\ \text{m} = 0.281\ \text{mm}\ \downarrow \qquad \textbf{Ans.}$$

Ans:

$(\Delta_E)_v = 0.281$ mm

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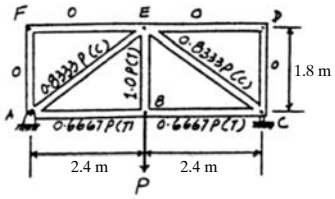
***14–124.** Solve Prob. 14–73 using Castigliano's theorem.

SOLUTION



Member Forces N: Member forces due to external force **P** and external applied forces are shown on the figure.

Castigliano's Second Theorem:

Member	N	$\frac{\partial N}{\partial P}$	$N(P = 25\text{ kN})$	L	$N\left(\frac{\partial N}{\partial P}\right)L$
AB	$0.6667P$	0.6667	16.6667	2.4	26.667
BC	$0.6667P$	0.6667	16.6667	2.4	26.667
CD	0	0	0	1.8	0
DE	0	0	0	2.4	0
EF	0	0	0	2.4	0
AF	0	0	0	1.8	0
AE	$-0.8333P$	−0.8333	−20.8333	3.0	52.083
CE	$-0.8333P$	−0.8333	−20.8333	3.0	52.083
BE	$1.00P$	1.00	25.00	1.8	45.0
					$\Sigma 202.5\text{ kN}\cdot\text{m}$

$$\Delta = \sum N\left(\frac{\partial N}{\partial P}\right)\frac{L}{AE}$$

$$(\Delta_B)_v = \frac{202.5\text{ kN}\cdot\text{m}}{AE}$$

$$= \frac{202.5(10^3)}{[2.80(10^{-3})][200(10^9)]} = 0.3616(10^{-3})\text{ m} = 0.362\text{ mm}\downarrow \qquad \textbf{Ans.}$$

Ans:

$(\Delta_B)_v = 0.362$ mm

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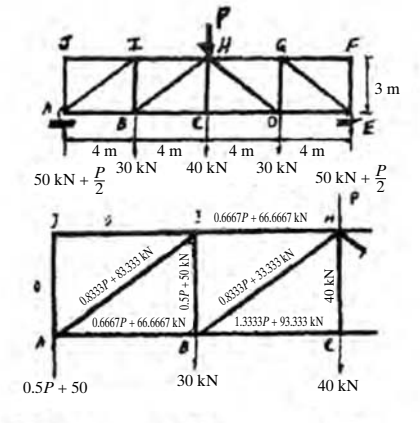
14–125. Solve Prob. 14–75 using Castigliano's theorem.

SOLUTION

$$\Delta_{Hv} = \Sigma N\left(\frac{\partial N}{\partial P}\right)\frac{L}{AE} = \frac{2828.89\ \text{kN}\cdot\text{m}}{AE} = \frac{2828.89(10^3)}{[2.80(10^{-3})][200(10^9)]}$$

$$= 5.052(10^{-3})\ \text{m} = 5.05\ \text{mm} \qquad \textbf{Ans.}$$



Ans:

$\Delta_{H_v} = 5.05\ \text{mm}$

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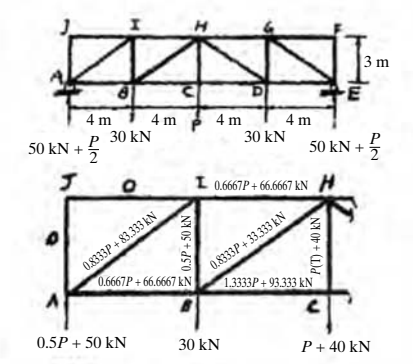
14–126. Solve Prob. 14–76 using Castigliano's theorem.

SOLUTION

$$\Delta_{C_v} = \Sigma N\left(\frac{\partial N}{\partial P}\right)\frac{L}{AE} = \frac{2948.89\ \text{kN}\cdot\text{m}}{AE} = \frac{2848.89(10^3)}{[2.80(10^{-3})][200(10^9)]}$$

$$= 5.266(10^{-3})\ \text{m} = 5.27\ \text{mm}\downarrow \qquad \textbf{Ans.}$$



Ans:

$\Delta_{C_v} = 5.27\ \text{mm}$

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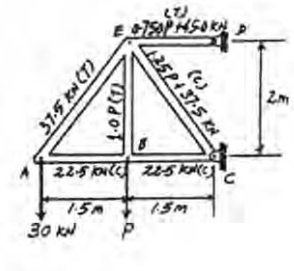
14–127. Solve Prob. 14–77 using Castigliano's theorem.



SOLUTION

Member Forces N: Member forces due to external force **P** and external applied forces are shown on the figure.

Castigliano's Second Theorem:

Member	N	$\frac{\partial N}{\partial P}$	$N(P = 20\text{ kN})$	L	$N\left(\frac{\partial N}{\partial P}\right)L$
AB	-22.5	0	-22.5	1.5	0
BC	-22.5	0	-22.5	1.5	0
AE	37.5	0	37.5	2.5	0
CE	$-(1.25P + 37.5)$	-1.25	-62.5	2.5	195.3125
BE	$1.00P$	1.00	20.0	2	40.0
DE	$0.750P + 45$	0.750	60.0	1.5	67.50
					$\sum 302.8125\text{ kN}\cdot\text{m}$

$$\Delta = \sum N\left(\frac{\partial N}{\partial P}\right)\frac{L}{AE}$$

$$(\Delta_B)_v = \frac{302.8125\text{ kN}\cdot\text{m}}{AE}$$

$$= \frac{302.8125\left(10^3\right)}{0.400\left(10^{-3}\right)\left[200\left(10^9\right)\right]}$$

$$= 3.785\left(10^{-3}\right)\text{ m} = 3.79\text{ mm}\ \downarrow$$ **Ans.**

Ans:

$(\Delta_B)_h = 0.699\,(10^{-3})$ in. $\rightarrow$

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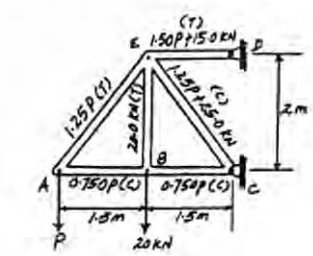
***14–128.** Solve Prob. 14–78 using Castigliano's theorem.



SOLUTION

Member Forces N: Member forces due to external force **P** and external applied forces are shown on the figure.

Castigliano's Second Theorem:

Member	N	$\frac{\partial N}{\partial P}$	$N(P = 30\text{ kN})$	L	$N\left(\frac{\partial N}{\partial P}\right)L$
AB	$-0.750P$	-0.750	-22.5	1.5	25.3125
BC	$-0.750P$	-0.750	-22.5	1.5	25.3125
AE	$1.25P$	1.25	37.5	2.5	117.1875
CE	$-(1.25P + 25.0)$	-1.25	-62.5	2.5	195.3125
BE	20.0	0	20.0	2	0
DE	$1.50P$+15.0	1.50	60.0	1.5	135.00
					$\sum 498.125\text{ kN}\cdot\text{m}$

$$\Delta = \sum N\left(\frac{\partial N}{\partial P}\right)\frac{L}{AE}$$

$$(\Delta_A)_v = \frac{498.125\text{ kN}\cdot\text{m}}{AE}$$

$$= \frac{498.125\left(10^3\right)}{0.400\left(10^{-3}\right)\left[200\left(10^9\right)\right]}$$

$$= 6.227\left(10^{-3}\right)\text{ m} = 6.23\text{ mm}\downarrow$$ **Ans.**

Ans:
$(\Delta_A)_v = 6.23$ mm

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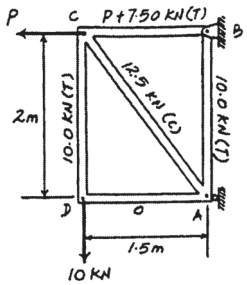
14–129.

Solve Prob. 14–81 using Castigliano's theorem.

SOLUTION



Member Forces *N*: Member forces due to external force **P** and external applied forces are shown on the figure.

Castigliano's Second Theorem: Applying Eq. 14–48, we have

Member	N	$\frac{\partial N}{\partial P}$	$N(P = 5\ \text{kN})$	L	$N\left(\frac{\partial N}{\partial P}\right)L$
AB	10.0	0	10.0	2	0
BC	$1.00P + 7.50$	1.00	12.5	1.5	18.75
CD	10.0	0	10.0	2	0
AD	0	0	0	1.5	0
AC	-12.5	0	-12.5	2.5	0
					$\Sigma\ 18.75\ \text{kN}\cdot\text{m}$

$$\Delta = \sum N\left(\frac{\partial N}{\partial P}\right)\frac{L}{AE}$$

$$(\Delta_C)_h = \frac{18.75\ \text{kN}\cdot\text{m}}{AE}$$

$$= \frac{18.75(10^3)}{0.400(10^{-3})[200(10^9)]}$$

$$= 0.2344(10^{-3})\ \text{m} = 0.234\ \text{mm} \leftarrow$$ **Ans.**

Ans:
$(\Delta_C)_h = 0.234\ \text{mm} \leftarrow$

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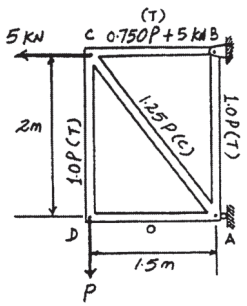
14–130.

Solve Prob. 14–82 using Castigliano's theorem.

SOLUTION



Member Forces *N*: Member forces due to external force **P** and external applied forces are shown on the figure.

Castigliano's Second Theorem: Applying Eq. 14–48, we have

Member	N	$\frac{\partial N}{\partial P}$	$N(P = 10\ \text{kN})$	L	$N\left(\frac{\partial N}{\partial P}\right)L$
AB	$1.00P$	1.00	10.0	2	20.00
BC	$0.750P + 5.00$	0.750	12.5	1.5	14.0625
CD	$1.00P$	1.00	10.0	2	20.00
AD	0	0	0	1.5	0
AC	$-1.25P$	−1.25	−12.5	2.5	39.0625
					Σ 93.125 kN · m

$$\Delta = \sum N\left(\frac{\partial N}{\partial P}\right)\frac{L}{AE}$$

$$(\Delta_D)_v = \frac{93.125\ \text{kN}\cdot\text{m}}{AE}$$

$$= \frac{93.125(10^3)}{0.400(10^{-3})[200(10^9)]}$$

$$= 1.164(10^{-3})\ \text{m} = 1.16\ \text{mm}\downarrow \qquad \textbf{Ans.}$$

Ans:
$(\Delta_D)_v = 1.16\ \text{mm}\downarrow$

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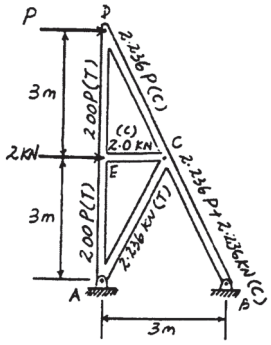
14–131.

Solve Prob. 14–85 using Castigliano's theorem.

SOLUTION



Member Forces *N*: Member forces due to external force **P** and external applied forces are shown on the figure.

Castigliano's Second Theorem: Applying Eq. 14–48, we have

Member	N	$\frac{\partial N}{\partial P}$	$N(P = 4\text{ kN})$	L	$N\left(\frac{\partial N}{\partial P}\right)L$
AE	$2.00P$	2.00	8.00	3	48.00
ED	$2.00P$	2.00	8.00	3	48.00
CD	$-2.236P$	−2.236	−8.944	3.354	67.082
BC	$-(2.236P + 2.236)$	−2.236	−11.180	3.354	83.853
CE	−2.00	0	−2.00	1.5	0
AC	2.236	0	2.236	3.354	0
					Σ 246.935 kN · m

$$\Delta = \sum N\left(\frac{\partial N}{\partial P}\right)\frac{L}{AE}$$

$$(\Delta_D)_h = \frac{246.935\text{ kN}\cdot\text{m}}{AE}$$

$$= \frac{246.935(10^3)}{0.300(10^{-3})[200(10^9)]}$$

$$= 4.116(10^{-3})\text{ m} = 4.12\text{ mm} \rightarrow$$ **Ans.**

Ans:
$(\Delta_D)_h = 4.12\text{ mm} \rightarrow$

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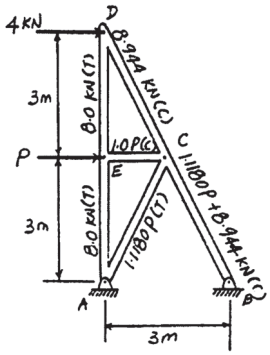
***14–132.**

Solve Prob. 14–86 using Castigliano's theorem.

SOLUTION



Member Forces *N*: Member forces due to external force **P** and external applied forces are shown on the figure.

Castigliano's Second Theorem: Applying Eq. 14–48, we have

Member	N	$\frac{\partial N}{\partial P}$	$N(P = 2\text{ kN})$	L	$N\left(\frac{\partial N}{\partial P}\right)L$
AE	8.00	0	8.00	3	0
ED	8.00	0	8.00	3	0
CD	-8.944	0	-8.944	3.354	0
BC	$-(1.118P + 8.944)$	-1.118	-11.180	3.354	41.926
CE	$-1.00P$	-1.00	-2.00	1.5	3.00
AC	$1.118P$	1.118	2.236	3.354	8.385
					Σ 53.312 kN · m

$$\Delta = \sum N\left(\frac{\partial N}{\partial P}\right)\frac{L}{AE}$$

$$(\Delta_E)_h = \frac{53.312\text{ kN}\cdot\text{m}}{AE}$$

$$= \frac{53.312(10^3)}{0.300(10^{-3})[200(10^9)]}$$

$$= 0.8885(10^{-3})\text{ m} = 0.889\text{ mm} \rightarrow$$ **Ans.**

Ans:
$(\Delta_E)_h = 0.889\text{ mm} \rightarrow$

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14–133.

Solve Prob. 14–90 using Castigliano's theorem.

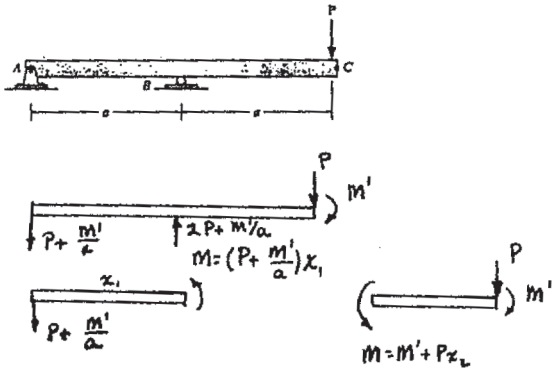
SOLUTION

Set $M' = 0$

$$\theta_C = \int_0^L M\left(\frac{\partial M}{\partial M'}\right)\frac{dx}{EI}$$

$$= \int_0^a \frac{(Px_1)(\frac{1}{a}x_1)dx_1}{EI} + \int_0^a \frac{(Px_2)(1)dx_2}{EI}$$

$$= \frac{Pa^2}{3EI} + \frac{Pa^2}{2EI} = \frac{5Pa^2}{6EI}$$ **Ans.**



Ans:

$$\theta_C = -\frac{5Pa^2}{6EI}$$

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14–134.

Solve Prob. 14–91 using Castigliano's theorem.

SOLUTION

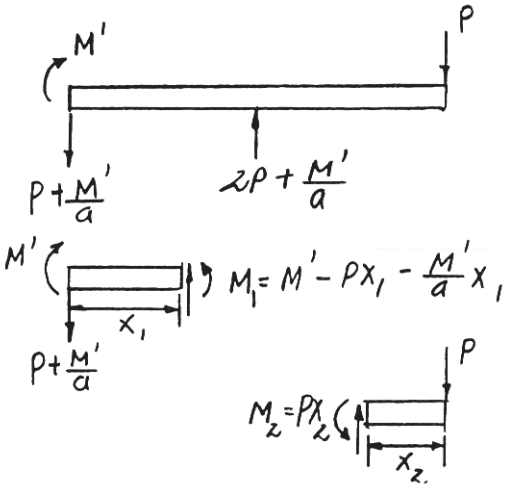
$\frac{\partial M_1}{\partial M'} = 1 - \frac{x_1}{a} \quad \frac{\partial M_2}{\partial M'} = 0$

Set $M' = 0$

$M_1 = -Px_1 \qquad M_2 = Px_2$

$$\theta_A = \int_0^L M\left(\frac{\partial M}{\partial M'}\right)\frac{dx}{EI} = \frac{1}{EI}\left[\int_0^a (-Px_1)\left(1 - \frac{x_1}{a}\right)dx_1 + \int_0^a (Px_2)(0)dx_2\right] = \frac{-Pa^2}{6EI}$$

$$= \frac{Pa^2}{6EI}$$ **Ans.**



Ans:

$\theta_A = \frac{Pa^2}{6EI}$

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14–135. Solve Prob. 14–106 using Castigliano's theorem.

SOLUTION

$$\frac{\partial M_1}{\partial p} = 0 \quad \frac{\partial M_2}{\partial p} = 1.5 - 0.5x_2 \quad \frac{\partial M_3}{\partial p} = x_3$$

Set $p = 0$

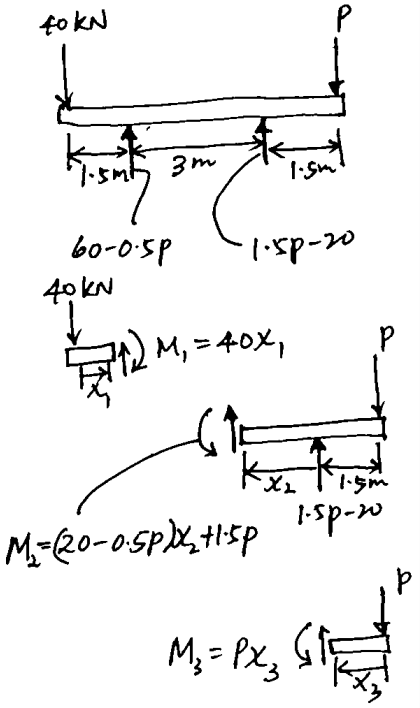
$$M_1 = 40x_1 \quad M_2 = 20x_2 \quad M_3 = 0$$

$$\Delta_C = \int_0^L M\left(\frac{\partial M}{\partial p}\right)\frac{dx}{EI}$$

$$= \frac{1}{EI}\left[\int_0^{1.5\text{m}} 40x_1(0)dx_1 + \int_0^{3\text{m}} 20x_2(1.5 - 0.5x_2)dx_2 + \int_0^{3\text{m}} (0)(x_3)dx_3\right]$$

$$= \frac{45\ \text{kN}\cdot\text{m}^3}{EI} = \frac{45(10^3)}{[200(10^9)][22.3(10^{-6})]}$$

$$= 0.01009\ \text{m} = 10.1\ \text{mm}\ \downarrow$$ **Ans.**



Ans:
$\Delta_C = 10.1$ mm

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***14–136.** Solve Prob. 14–107 using Castigliano's theorem.

SOLUTION

$$\frac{\partial M_1}{\partial M'} = 0 \quad \frac{\partial M_2}{\partial M'} = 1 - 0.3333x_2 \quad \frac{\partial M_3}{\partial M'} = 0$$

Set $M' = 0$

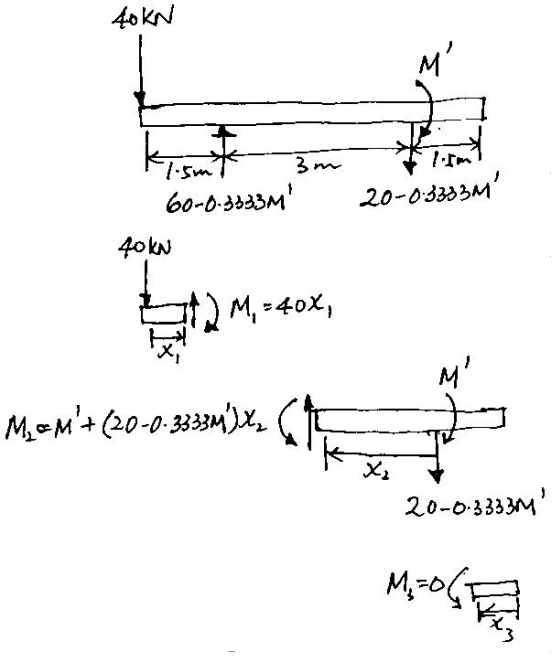
$$M_1 = 40x_1 \quad M_2 = 20x_2 \quad M_3 = 0$$

$$\theta_B = \int_0^L M\left(\frac{\partial M}{\partial M'}\right)\frac{dx}{EI}$$

$$= \frac{1}{EI}\left[\int_0^{1.5\text{m}} 40x_1(0)dx_1 + \int_0^{3\text{m}} 20x_2(1 - 0.3333x_2)dx_2 + 0\right]$$

$$= \frac{30\ \text{kN}\cdot\text{m}^2}{EI} = \frac{30(10^3)}{[200(10^9)][22.3(10^{-6})]}$$

$$= 6.7265(10^{-3})\ \text{rad} = 0.385^\circ \quad \curvearrowright \theta_B$$ **Ans.**



Ans:
$\theta_B = 0.385^\circ$

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14–137.

Solve Prob. 14–95 using Castigliano's theorem.

SOLUTION

$$\frac{\partial M_1}{\partial P} = 0.4286x_1 \qquad \frac{\partial M_2}{\partial P} = 0.4286x_2 + 0.17144$$

$$\frac{\partial M_3}{\partial P} = 0.5714x_3 \qquad \frac{\partial M_4}{\partial P} = 0.5714x_4 + 0.17144$$

Set $P = 2$ kN

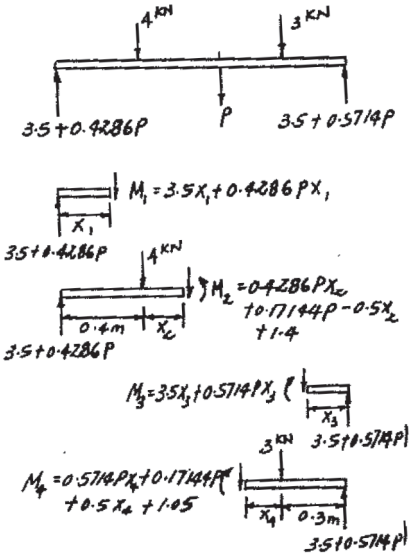
$$M_1 = 4.3572x_1 \qquad M_2 = 0.3572x_2 + 1.7429$$

$$M_3 = 4.6428x_3 \qquad M_4 = 1.6428x_4 + 1.3929$$

$$\Delta_B = \int_0^L M\left(\frac{\partial M}{\partial P}\right)\frac{dx}{EI}$$

$$= \frac{1}{EI}\left[\int_0^{0.4}(4.3572x_1)(0.4286x_1)\,dx_1 + \int_0^{0.4}(0.3572x_2 + 1.7429)(0.4286x_2 + 0.17144)dx_2 + \int_0^{0.3}(4.6428x_3)(0.5714x_3)dx_3 + \int_0^{0.3}(1.6428x_4 + 1.3929)(0.5714x_4 + 0.17144)dx_4\right]$$

$$= \frac{0.37944\ \text{kN}\cdot\text{m}^3}{EI} = \frac{0.37944(10^3)}{200(10^9)\frac{\pi}{4}(0.015)^4} = 0.0478\ \text{m} = 47.8\ \text{mm} \qquad \textbf{Ans.}$$



Ans:
$\Delta_B = 47.8$ mm

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14–138.

Solve Prob. 14–96 using Castigliano's theorem.

SOLUTION

$\frac{\partial M_1}{\partial P} = 0 \qquad \frac{\partial M_2}{\partial P} = -0.5x_2$

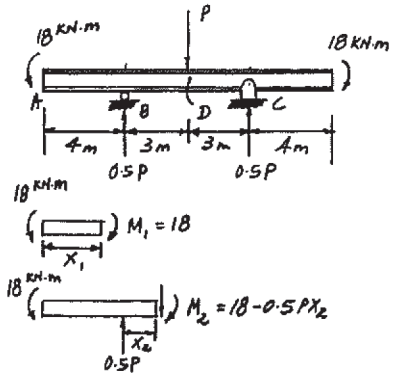
Set $P = 0$

$M_1 = 18 \qquad M_2 = 18$

$$\Delta_D = \int_0^L M\left(\frac{\partial M}{\partial P}\right)\frac{dx}{EI}$$

$$= (2)\frac{1}{EI}\left[\int_0^4 (18)(0)dx_1 + \int_0^3 (18)(-0.5x_2)dx_2\right]$$

$$= \frac{81\ \text{kN}\cdot\text{m}^3}{EI} = \frac{81(10)^3}{200(10^9)(125)(10^{-6})} = 3.24(10^{-3})\ \text{m} = 3.24\ \text{mm} \qquad \textbf{Ans.}$$



Ans:
$\Delta_D = 3.24\ \text{mm}$

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14–139.

Solve Prob. 14–97 using Castigliano's theorem.

SOLUTION

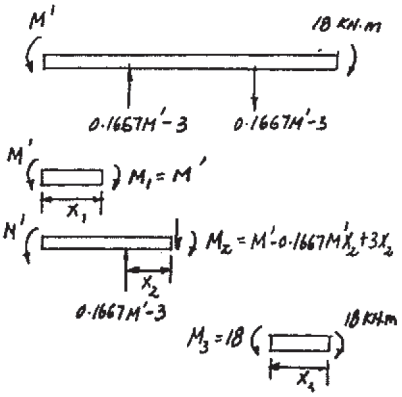
$$\frac{\partial M_1}{\partial M'} = 1 \qquad \frac{\partial M_2}{\partial M'} = 1 - 0.1667x_2 \qquad \frac{\partial M_3}{\partial M'} = 0$$

Set $M' = 18\ \text{kN}\cdot\text{m}$

$$M_1 = 18\ \text{kN}\cdot\text{m} \qquad M_2 = 18\ \text{kN}\cdot\text{m} \qquad M_3 = 18\ \text{kN}\cdot\text{m}$$

$$\theta_A = \int_0^L M\left(\frac{\partial M}{\partial M'}\right)\frac{dx}{EI} = \frac{1}{EI}\left[\int_0^4 (18)(1)dx_1 + \int_0^6 18(1 - 0.1667x_2)dx_2 + \int_0^4 (18)(0)dx_3\right]$$

$$= \frac{126\ \text{kN}\cdot\text{m}^2}{EI} = \frac{126(10^3)}{200(10^9)(125)(10^{-6})} = 5.04(10^{-3})\ \text{rad} = 0.289^\circ \qquad \textbf{Ans.}$$



Ans:
$\theta_A = 0.289^\circ$

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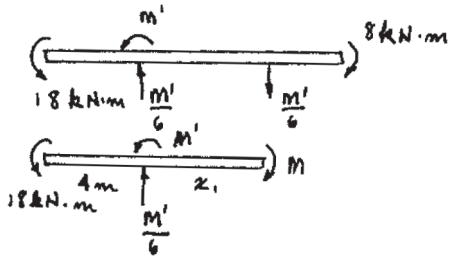
***14–140.**

Solve Prob. 14–98 using Castigliano's theorem.

SOLUTION

$$\theta_B = \int_0^L M\left(\frac{\partial M}{\partial M'}\right)\frac{dx}{EI} = \int_0^6 \frac{(-18)\left(-\frac{1}{6}x\right)dx(10^3)}{EI}$$

$$= \frac{18(6^2)(10^3)}{6(2)(200)(10^9)(125)(10^{-6})} = 0.00216 \text{ rad} = 0.124^\circ \qquad \textbf{Ans.}$$



Ans:
$\theta_B = 0.124^\circ$

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14–141.

Solve Prob. 14–108 using Castigliano's theorem.

SOLUTION

M' does not influence the moment within the overhang.

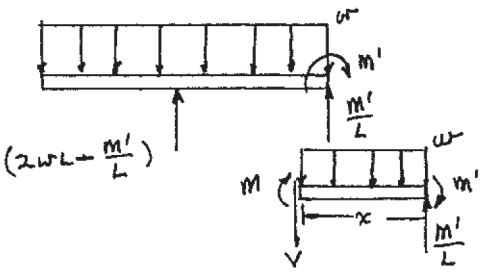
$$M = \frac{M'}{L}x - M' - \frac{wx^2}{2}$$

$$\frac{\partial M}{\partial M'} = \frac{x}{L} - 1$$

Setting $M' = 0$,

$$\theta_A = \int_0^L M\left(\frac{\partial M}{\partial M'}\right)\frac{dx}{EI} = \frac{1}{EI}\int_0^L\left(-\frac{wx^2}{2}\right)\left(\frac{x}{L} - 1\right)dx = \frac{-w}{2EI}\left[\frac{L^3}{4} - \frac{L^3}{3}\right]$$

$$= \frac{wL^3}{24EI}$$ **Ans.**



Ans:

$$\theta_A = \frac{wL^3}{24EI}$$

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14–142.

Solve Prob. 14–119 using Castigliano's theorem.

SOLUTION

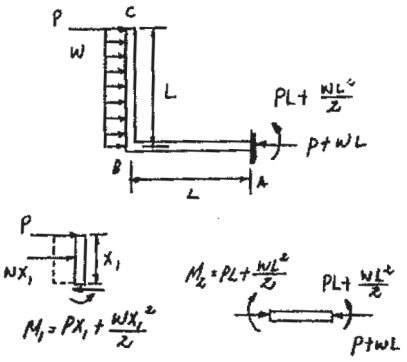
$$M_1 = Px_1 + \frac{wx_1^2}{2}$$

$$\frac{\partial M_1}{\partial P} = x_1 \qquad \frac{\partial M_2}{\partial P} = L$$

Setting $P = 0$

$$M_1 = \frac{wx_1^2}{2} \qquad M_2 = \frac{wL^2}{2}$$

$$\Delta_C = \int_0^L M\left(\frac{\partial M}{\partial P}\right)\frac{dx}{EI} = \frac{1}{EI}\left[\int_0^L \frac{wx_1^2}{2}(x_1)dx_1 + \int_0^L \frac{wL^2}{2}Ldx_2\right] = \frac{5wL^4}{8EI} \qquad \textbf{Ans.}$$



Ans:

$$\Delta_C = \frac{5wL^4}{8EI}$$

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14–143.

Solve Prob. 14–120 using Castigliano's theorem.

SOLUTION

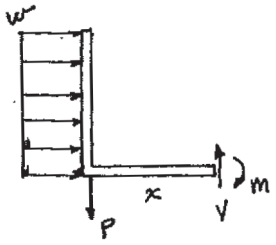
P does not influence moment within segment.

$$M = Px = \frac{wL^2}{2}$$

$$\frac{\partial M}{\partial P} = x$$

Set $P = 0$

$$\Delta_B = \int_0^L M\left(\frac{\partial M}{\partial P}\right)\frac{dx}{EI} = \int_0^L \left(-\frac{wL^2}{2}\right)(x)\frac{dx}{EI} = \frac{wL^4}{4EI}$$ **Ans.**



Ans:

$$\Delta_B = \frac{wL^4}{4EI}$$

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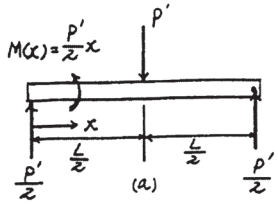
***14–144.**

Solve Prob. 14–105 using Castigliano's theorem.

SOLUTION



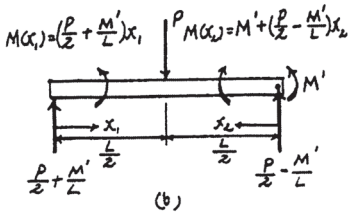
Internal Moment Function *M(x)*: The internal moment function in terms of the load $\mathbf{P}'$ and couple of moment $\mathbf{M}'$ and externally applied load are shown on figures (*a*) and (*b*), respectively.

Castigliano's Second Theorem: The displacement at *C* can be determined using Eq. 14–49 with $\frac{\partial M(x)}{\partial P'} = \frac{x}{2}$ and set $P' = P$.

$$\Delta = \int_0^L M\left(\frac{\partial M}{\partial P'}\right)\frac{dx}{EI}$$

$$\Delta_C = 2\left[\frac{1}{EI}\int_0^{\frac{L}{2}}\left(\frac{P}{2}x\right)\left(\frac{x}{2}\right)dx\right]$$

$$= \frac{PL^3}{48EI}\downarrow \qquad \textbf{Ans.}$$



To determine the slope at *B*, we apply Eq. 14–50 with $\frac{\partial M(x_1)}{\partial M'} = \frac{x_1}{L}$, $\frac{\partial M(x_2)}{\partial M'} = 1 - \frac{x_2}{L}$ and setting $M' = 0$.

$$\theta = \int_0^L M\left(\frac{\partial M}{\partial M'}\right)\frac{dx}{EI}$$

$$\theta_B = \frac{1}{EI}\int_0^{\frac{L}{2}}\left(\frac{P}{2}x_1\right)\left(\frac{x_1}{L}\right)dx_1 + \frac{1}{EI}\int_0^{\frac{L}{2}}\left(\frac{P}{2}x_2\right)\left(1 - \frac{x_2}{L}\right)dx_2$$

$$= \frac{PL^2}{16EI} \qquad \textbf{Ans.}$$

Ans:

$$\Delta_C = \frac{PL^3}{48EI}\downarrow$$

$$\theta_B = \frac{PL^2}{16EI}$$

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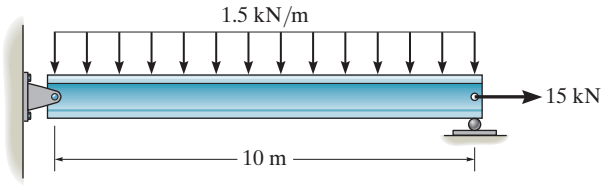
R14–1.

Determine the total axial and bending strain energy in the A992 steel beam. $A = 2300\ \text{mm}^2$, $I = 9.5(10^6)\ \text{mm}^4$.



SOLUTION

Axial Load:

$$(U_e)_i = \int_0^L \frac{N^2 dx}{2EA} = \frac{N^2 L}{2EA}$$

$$(U_e)_i = \frac{((15)(10^3))^2(10)}{2(200)(10^9)(2.3)(10^{-3})} = 2.4456\ \text{J}$$

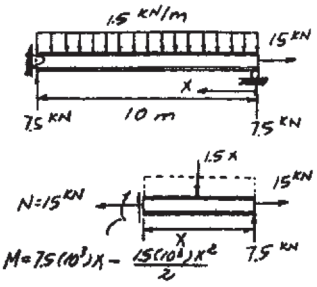
Bending:

$$(U_b)_i = \int_0^L \frac{M^2 dx}{2EI} = \frac{1}{2EI}\int_0^{10} [(7.5)(10^3)x - 0.75(10^3)x^2]^2 dx$$

$$= \frac{1}{2EI}\int_0^{10} [56.25(10^6)x^2 + 562.5(10^3)x^4 - 11.25(10^6)x^3]\, dx$$

$$(U_b)_i = \frac{0.9375(10^9)}{200(10^9)(9.5)(10^{-6})} = 493.4210\ \text{J}$$

$$U_i = (U_a)_i + (U_b)_i = 2.4456 + 493.4210 = 496\ \text{J}$$ **Ans.**



Ans:
$U_i = 496\ \text{J}$

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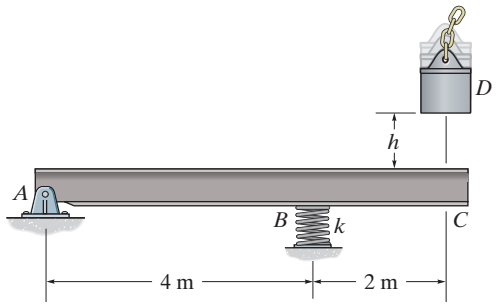
R14–2.

The 200-kg block D is dropped from a height $h = 1$ m onto end C of the A992 steel W200 × 36 overhang beam. If the spring at B has a stiffness $k = 200$ kN/m, determine the maximum bending stress developed in the beam.



SOLUTION

Equilibrium: The support reactions and the moment functions for regions AB and BC of the beam under static conditions are indicated on the free-body diagram of the beam, Fig. *a*.

$$U_e = U_i$$

$$\frac{1}{2}P\Delta_{st} = \Sigma \int_0^L \frac{M^2 dx}{2EI}$$

$$\frac{1}{2}P\Delta_{st} = \frac{1}{2EI}\left[\int_0^{4\,m}\left(\frac{P}{2}x_2\right)^2 dx + \int_0^{2\,m}(Px_1)^2\,dx\right]$$

$$\Delta_{st} = \frac{8P}{EI}$$

Here, $I = 34.4\left(10^6\right)\text{ mm}^4 = 34.4\left(10^{-6}\right)\text{ m}^4$ (see the appendix) and $E = E_{st} = 200$ GPa. Then, the equivalent spring constant can be determined from

$$P = k_b\,\Delta_{st}$$

$$P = k_b\left(\frac{8P}{EI}\right)$$

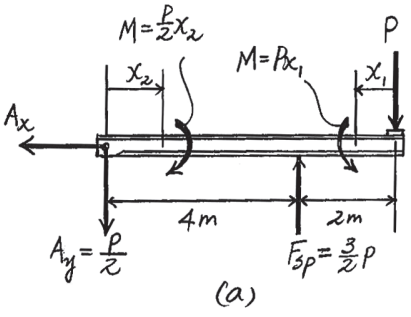
$$k_b = \frac{EI}{8} = \frac{200\left(10^9\right)\left[34.4\left(10^{-6}\right)\right]}{8} = 860\left(10^3\right)\text{ N/m}$$

From the free-body diagram,

$$F_{sp} = \frac{3}{2}P$$

$$k_{sp}\Delta_{sp} = \frac{3}{2}\left(k_b\Delta_b\right)$$

$$\Delta_{sp} = \frac{3}{2}\left(\frac{k_b}{k_{sp}}\right)\Delta_b = \frac{3}{2}\left(\frac{860\left(10^3\right)}{200\left(10^3\right)}\right)\Delta_b = 6.45\,\Delta_b \qquad (1)$$



Conservation of Energy:

$$mg\left(h + \Delta_b + \frac{3}{2}\Delta_{sp}\right) = \frac{1}{2}k_{sp}\Delta_{sp}^2 + \frac{1}{2}k_b\Delta_b^2$$

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R14–2. Continued

SOLUTION

Substiuting Eq. (1) into this equation,

$$200(9.81)\left[1 + \Delta_b + \frac{3}{2}(6.45\Delta_b)\right] = \frac{1}{2}\left[200\left(10^3\right)\right](6.45\Delta_b)^2 + \frac{1}{2}\left[860\left(10^3\right)\right]{\Delta_b}^2$$

$$4590.25\left(10^3\right){\Delta_b}^2 - 20944.35\Delta_b - 1962 = 0$$

Solving for the positive root

$$\Delta_b = 0.02308 \text{ m}$$

Maximum Stress: The maximum force on the beam is $P_{\max} = k_b\Delta_b = 860\left(10^3\right)(0.02308) = 19.85\left(10^3\right)$ N. The maximum moment occurs at the supporting spring, where $M_{\max} = P_{\max}L = 19.85\left(10^3\right)(2) = 39.70\left(10^3\right)$ N · m. Applying the flexure formula with $c = \frac{d}{2} = \frac{0.201}{2} = 0.1005$ m,

$$\sigma_{\max} = \frac{M_{\max}\,c}{I} = \frac{39.70\left(10^3\right)(0.1005)}{34.4\left(10^{-6}\right)} = 115.98 \text{ MPa} = 116 \text{ MPa} \qquad \textbf{Ans.}$$

Since $\sigma_{\max} < \sigma_Y = 345$ MPa, this result is valid.

Ans:
$\sigma_{\max} = 116$ MPa

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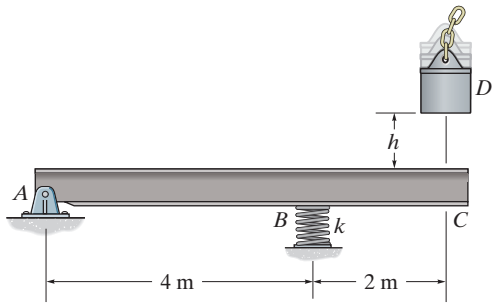
R14–3.

Determine the maximum height h from which the 200-kg block D can be dropped without causing the A992 steel W200 × 36 overhang beam to yield. The spring at B has a stiffness $k = 200$ kN/m.



SOLUTION

Equilibrium: The support reactions and the moment functions for regions AB and BC of the beam under static conditions are indicated on the free-body diagram of the beam, Fig. *a*.

$$U_e = U_i$$

$$\frac{1}{2}P\Delta_{st} = \Sigma\int_0^L \frac{M^2 dx}{2EI}$$

$$\frac{1}{2}P\Delta_{st} = \frac{1}{2EI}\left[\int_0^{4\text{ m}}\left(\frac{P}{2}x_2\right)^2 dx + \int_0^{2\text{ m}}(Px_1)^2\,dx\right]$$

$$\Delta_{st} = \frac{8P}{EI}$$

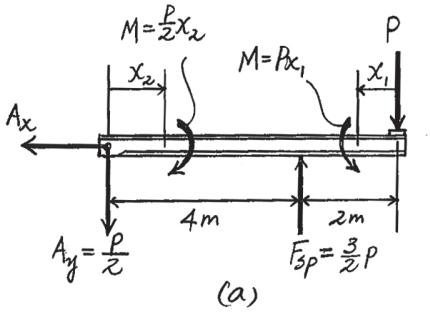
Here, $I = 34.4\left(10^6\right)\text{ mm}^4 = 34.4\left(10^{-6}\right)\text{ m}^4$ (see the appendix) and $E = E_{st} = 200$ GPa. Then, the equivalent spring constant can be determined from

$$P = k_b\,\Delta_{st}$$

$$P = k_b\left(\frac{8P}{EI}\right)$$

$$k_b = \frac{EI}{8} = \frac{200\left(10^9\right)\left[34.4\left(10^{-6}\right)\right]}{8} = 860\left(10^3\right)\text{ N/m}$$

From the free-body diagram,



$$F_{sp} = \frac{3}{2}P$$

$$k_{sp}\Delta_{sp} = \frac{3}{2}\left(k_b\Delta_b\right)$$

$$\Delta_{sp} = \frac{3}{2}\left(\frac{k_b}{k_{sp}}\right)\Delta_b = \frac{3}{2}\left(\frac{860\left(10^3\right)}{200\left(10^3\right)}\right)\Delta_b = 6.45\Delta_b \qquad \textbf{(1)}$$

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R14–3. Continued

SOLUTION

Maximum Stress: The maximum force on the beam is $P_{\max} = k_b\Delta_b = 860\left(10^3\right)\Delta_b$. The maximum moment occurs at the supporting spring, where $M_{\max} = P_{\max}L = 860\left(10^3\right)\Delta_b(2) = 1720\left(10^3\right)\Delta_b$. Applying the flexure formula with $c = \frac{d}{2} = \frac{0.201}{2} = 0.1005$ m,

$$\sigma_{\max} = \frac{M_{\max}c}{I}$$

$$345\left(10^6\right) = \frac{1720\left(10^3\right)\Delta_b(0.1005)}{34.4\left(10^{-6}\right)}$$

$$\Delta_b = 0.06866 \text{ m}$$

Substituting this result into Eq. (1),

$$\Delta_{sp} = 0.44284 \text{ m}$$

Conservation of Energy:

$$mg\left(h + \Delta_b + \frac{3}{2}\Delta_{sp}\right) = \frac{1}{2}k_{sp}\Delta_{sp}^2 + \frac{1}{2}k_b\Delta_b^2$$

$$200(9.81)\left[h + 0.06866 + \frac{3}{2}(0.44284)\right] = \frac{1}{2}\left[200\left(10^3\right)\right](0.44284)^2 + \frac{1}{2}\left[860\left(10^3\right)\right](0.06866)^2$$

$$h = 10.3 \text{ m} \quad \textbf{Ans.}$$

Ans:
$h = 10.3$ m

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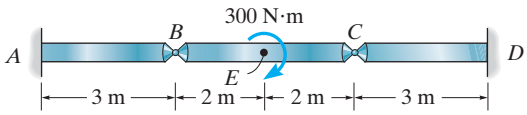
***R14–4.**

The A992 steel bars are pin connected at B and C. If they each have a diameter of 30 mm, determine the slope at E.



SOLUTION

$$U_i = \int_0^L \frac{M^2 dx}{2EI} = (2)\frac{1}{2EI}\int_0^3 (75x_1)^2 dx_1 + (2)\frac{1}{2EI}\int_0^2 (-75x_2)^2 dx_2 = \frac{65625}{EI}$$

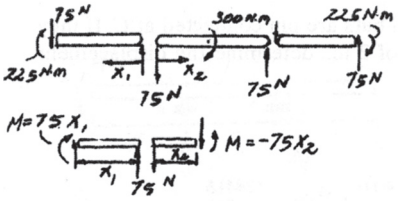
$$U_e = \frac{1}{2}(M')\theta = \frac{1}{2}(300)\theta_E = 150\theta_E$$

Conservation of Energy:

$$U_e = U_i$$

$$150\theta_E = \frac{65625}{EI}$$

$$\theta_E = \frac{473.5}{EI} = \frac{473.5}{(200)(10^9)(\frac{\pi}{4})(0.015^4)} = 0.0550 \text{ rad} = 3.15°\circlearrowleft$$ **Ans.**



Ans:
$\theta_E = -3.15°$

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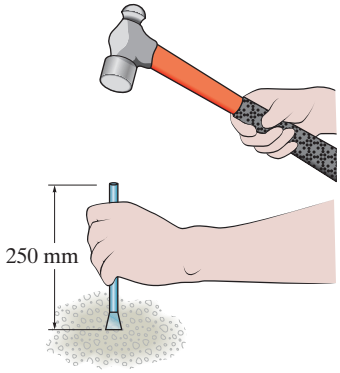
R14–5.

The steel chisel has a diameter of 12 mm and a length of 250 mm. It is struck by a hammer of mass 1.5 kg, and at the instant of impact it is moving at 3.6 m/s. Determine the maximum compressive stress in the chisel, assuming that 80% of the impacting energy goes into the chisel. $E_{st} = 200$ GPa, $\sigma_Y = 700$ MPa.



SOLUTION

$$k = \frac{AE}{L} = \frac{\left[\frac{\pi}{4}(0.012^2)\right][200(10^9)]}{0.25} = 90.4779(10^6)\ \text{N/m}$$

$$0.8U_e = U_i$$

$$0.8\left[\frac{1}{2}(1.5)(3.6^2) + 1.5(9.81)\Delta_{max}\right] = \frac{1}{2}[90.4779(10^6)]\Delta_{max}^2$$

$$45.2389(10^6)\Delta_{max}^2 - 11.772\Delta_{max} - 7.776 = 0$$

$$\Delta_{max} = 0.41472(10^{-3})\ \text{m}$$

$$P_{max} = k\Delta_{max} = [90.4779(10^6)][0.41472(10^{-3})] = 37.5233(10^3)\ \text{N}$$

$$\sigma_{max} = \frac{P_{max}}{A} = \frac{37.5233(10^3)}{\frac{\pi}{4}(0.012^2)} = 331.78(10^6)\ \text{N/m}^2 = 332\ \text{MPa} < \sigma_Y \quad \text{O.K.} \qquad \textbf{Ans.}$$

Ans:
$\sigma_{max} = 332$ MPa

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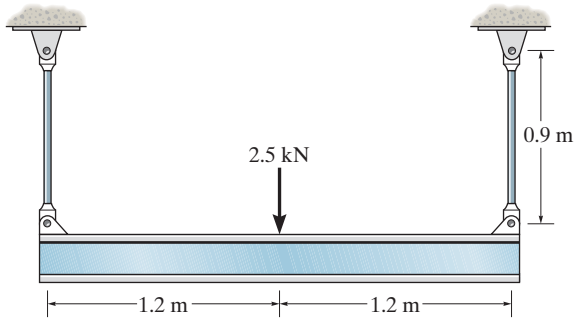
R14–6.

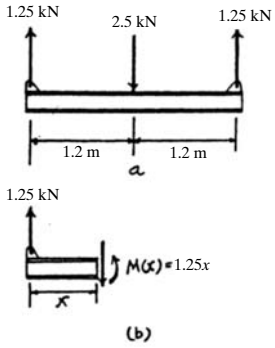
Determine the total strain energy in the A-36 steel assembly. Consider the axial strain energy in the two 12-mm-diameter rods and the bending strain energy in the beam for which $I = 17.0(10^6)\ \text{mm}^4$.



SOLUTION

Support Reactions: As shown FBD(a).

Internal Moment Function: As shown on FBD(b).



Total Strain Energy:

$$(U_i)_T = \int_0^L \frac{M^2\,dx}{2EI} + \frac{N^2L}{2AE}$$

$$= 2\left[\frac{1}{2EI}\int_0^{1.2\text{ m}} (1.25x)^2 dx\right] + 2\left[\frac{1.25^2(0.9)}{2AE}\right]$$

$$= \frac{0.9\ \text{kN}^2\cdot\text{m}^3}{EI} + \frac{1.40625\ \text{kN}^2\cdot\text{m}^3}{AE}$$

$$= \frac{0.9\ \text{kN}^2\cdot\text{m}^3}{[200(10^9)\ \text{N/m}^2][17.0(10^{-6})\ \text{m}^4]} + \frac{1.40625(10^3\ \text{N})^2\cdot\text{m}^3}{\left[\frac{\pi}{4}(0.012\ \text{m})^2\right][200(10^9)\ \text{N/m}^2]}$$

$$= 0.3269\ \text{N}\cdot\text{m} = 0.327\ \text{J}$$ **Ans.**

Ans:

$(U_i)_T = 0.327\ \text{J}$

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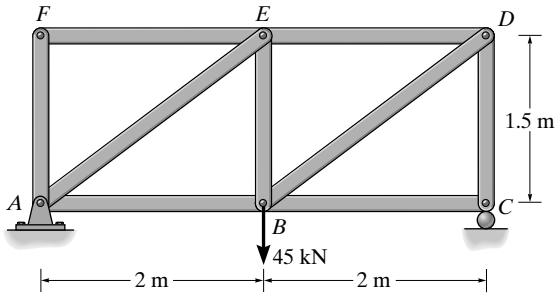
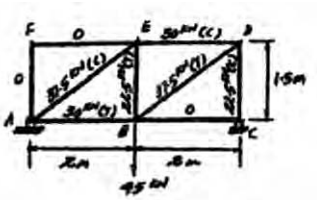
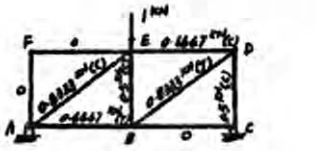
R14–7. Determine the vertical displacement of joint E. For each member $A = 400\ \text{mm}^2$, $E = 200\ \text{GPa}$. Use the method of virtual work.



SOLUTION

Member	n	N	L	nNL
AF	0	0	1.5	0
AE	−0.8333	−37.5	2.5	78.125
AB	0.6667	30.0	2.0	40.00
EF	0	0	2.0	0
EB	−0.50	22.5	1.5	−16.875
ED	−0.6667	−30.0	2.0	40.00
BC	0	0	2.0	0
BD	0.8333	37.5	2.5	78.125
CD	−0.5	−22.5	1.5	16.875
				$\Sigma = 236.25$





$$1 \cdot \Delta_{B_v} = \Sigma \frac{nNL}{AE}$$

$$\Delta_{B_v} = \frac{236.25\left(10^3\right)}{400\left(10^{-6}\right)(200)\left(10^9\right)} = 2.95\left(10^{-3}\right) = 2.95\ \text{mm}$$ **Ans.**

Ans:

$\Delta_{B_v} = 2.95\ \text{mm}$

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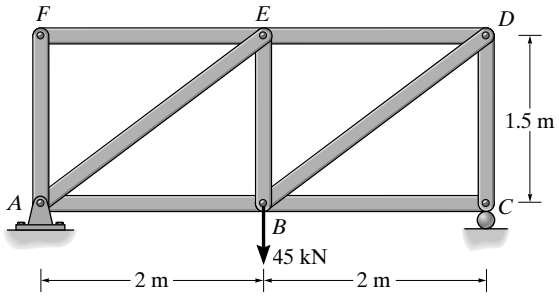
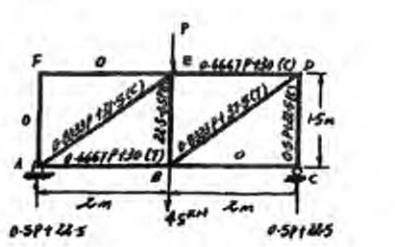
*__R14–8.__ Solve Prob. 14–152 using Castigliano's theorem.



SOLUTION

Member	N	$\partial N/\partial P$	$N(P = 45)$	L	$N(\partial N/\partial P)L$
AF	0	0	0	1.5	0
AE	$-(0.8333P + 37.5)$	-0.8333	-37.5	2.5	78.125
AB	$0.6667P + 30$	0.6667	30.0	2.0	40.00
BE	$22.5–0.5P$	-0.5	22.5	1.5	-16.875
BD	$0.8333P + 37.5$	0.8333	37.5	2.5	78.125
BC	0	0	0	2.0	0
CD	$-(0.5P + 22.5)$	-0.5	-22.5	1.5	16.875
DE	$-(0.6667P + 30)$	-0.6667	-30.0	2.0	40.00
EF	0	0	0	2.0	0
					$\Sigma = 236.25$



$$\Delta_{B_v} = \Sigma N \frac{\partial N}{\partial P}\frac{L}{AE} = \frac{236.25}{AE}$$

$$= \frac{236.25\left(10^3\right)}{400\left(10^{-6}\right)(200)\left(10^9\right)} = 2.95\left(10^{-3}\right)\text{m} = 2.95\text{ mm}$$ **Ans.**

Ans:

$\Delta_{B_v} = 2.95$ mm

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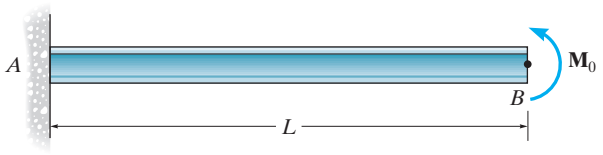
R14–9.

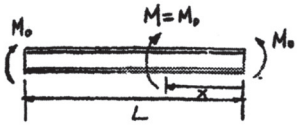
The cantilevered beam is subjected to a couple moment $\mathbf{M}_0$ applied at its end. Determine the slope of the beam at B. EI is constant. Use the method of virtual work.

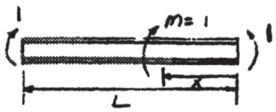


SOLUTION

$$\theta_B = \int_0^L \frac{m_\theta M}{EI}\,dx = \int_0^L \frac{(1)\,M_0}{EI}\,dx$$

$$= \frac{M_0 L}{EI}$$ **Ans.**





Ans:

$$\theta_B = \frac{M_0 L}{EI}$$

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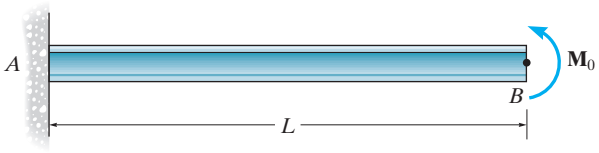
R14–10.

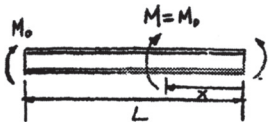
Solve Prob. R14–9 using Castigliano's theorem.



SOLUTION

$$\theta_B = \int_0^L m\left(\frac{dm}{dm'}\right)\frac{dy}{EI} = \int_0^L \frac{M_0(1)}{EI}\,dx$$

$$= \frac{M_0 L}{EI}$$ **Ans.**



Ans:

$$\theta_B = \frac{M_0 L}{EI}$$

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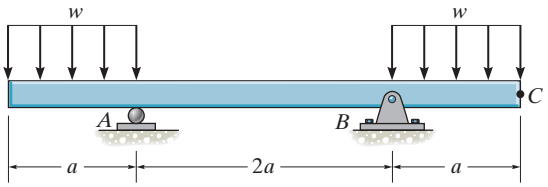
R14–11.

Determine the slope and displacement at point C. EI is constant.

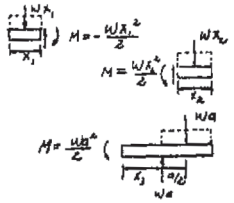


SOLUTION

$$\theta_C = \int_0^L \frac{m_\theta M}{EI}dx$$

$$= \frac{1}{EI}\left[\int_0^a (0)\left(\frac{wx_1^2}{2}\right)dx_1 + \int_0^a (1)\left(\frac{wx_2^2}{2}\right)dx_2 + \int_0^{2a}\left(1 - \frac{x_3}{2a}\right)\left(\frac{wa^2}{2}\right)dx_3\right]$$

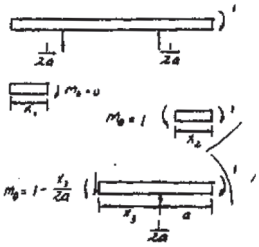
$$= -\frac{2\,w\,a^3}{3\,E\,I}\,\curvearrowright \quad \textbf{Ans.}$$

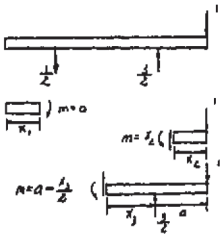


$$\Delta_C = \int_0^L \frac{m\,M}{EI}dx$$

$$= \frac{1}{EI}\left[\int_0^a (0)\left(\frac{wx_1^2}{2}\right)dx_1 + \int_0^a (x_2)\left(\frac{wx_2^2}{2}\right)dx_2 + \int_0^{2a}\left(a - \frac{x_3}{2}\right)\left(\frac{wa^2}{2}\right)dx_3\right]$$

$$= \frac{5\,w\,a^4}{8\,E\,I}\downarrow \quad \textbf{Ans.}$$





Ans:

$$\theta_C = -\frac{2wa^3}{3EI}$$

$$\Delta_C = \frac{5wa^4}{8EI}\downarrow$$

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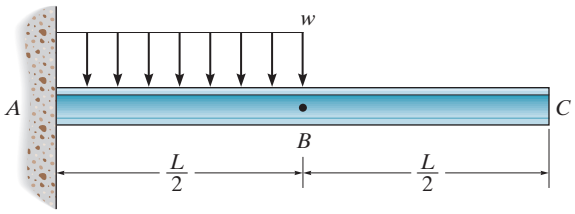
***R14–12.**

Determine the displacement at B. EI is constant.

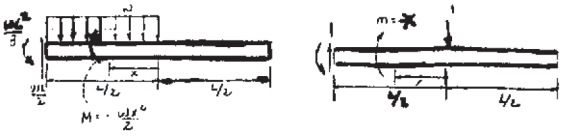


SOLUTION

$$1 \cdot \Delta_B = \int_0^L \frac{m M}{EI} dx$$

$$\Delta_B = \int_0^{\frac{L}{2}} \frac{(-1x)\left(\frac{-wx^2}{2}\right)}{EI} dx = \frac{w\left(\frac{L}{2}\right)^4}{8EI}$$

$$= \frac{wL^4}{128\,EI} \downarrow$$ **Ans.**



Ans:

$$\Delta_B = \frac{wL^4}{128\,EI} \downarrow$$