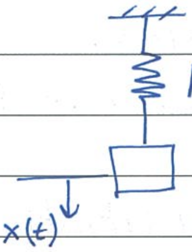
Classification of vibration :-

Vibration :- Any motion that repeats itself after an interval of time.

1] Free vibration :-



$F = kx$

$k = \frac{F}{x}$ (N/m)

For example: pendullum, spring.

"No External force acts on the system"

2] Forced vibration :-

"If system is subjected to an external force"; for example: diesel engines.

3] Damped Vibration:-

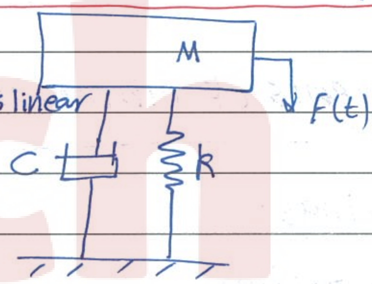
"if any energy is lost durding vibration"

في اي سؤال يتم تحويل السؤال
الى اربع عناصر :
mass, spring, damper, (motor, comp.) external force

4] un-damped Vibration:-

"if no energy energy is lost infriction ...."

5] linear vibration :-

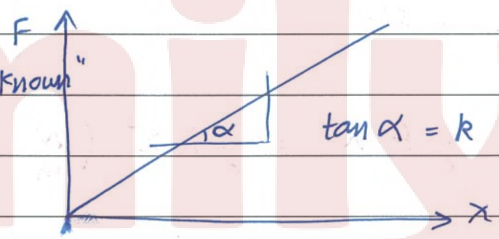


if all elemants behave linearly, so resultant vibration is linear

6] non-Linear Vibration:-

at least one element non-linear so resultant vabration non-linear.

spring :-



$\tan\alpha = k$

7] Deterministic vibration:-

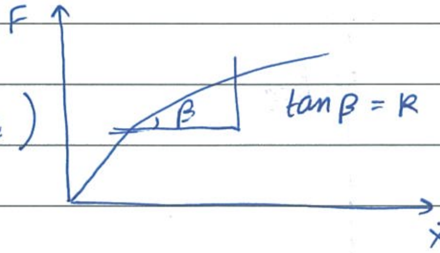
"if the value or magnitude of the (الإثارة، قوة) excitation is known"

8] un-deterministic Vibration:-

{Random Vib}.

"if external Force is unknown."

damper:-

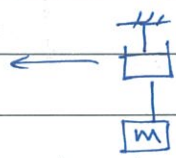


$\tan\beta = k$

* Degree of freedom :-

D.o.F = (# of masses)(# of ~~indep. variable~~ directions of ~~of each mass~~ motion of the masses.)

$F = c\dot{x}$



Note :-

- Discrete system :-

system with a finite number of degrees of freedom.

continous system :-

system with infinite number of degrees of freedom.

* Modeling in vibration :-

1. Understand the problem.
2. Mathmatical Model.
3. Solution.
4. Interpretation.

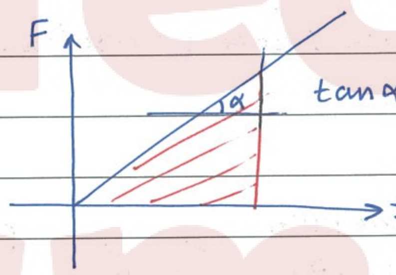
* Elements of vibration :-

1. springs :-

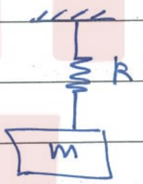
linear springs

$F = kx$

$\frac{dF}{dx} = const.$

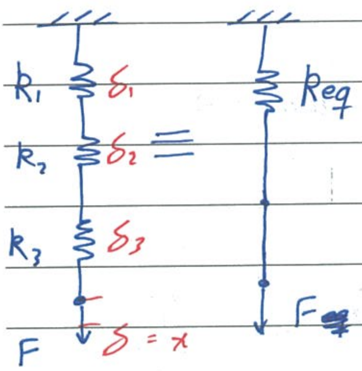


$\tan\alpha = k$



$\underset{P.E}{U} = \frac{1}{2} F x = \frac{1}{2} k x^2 \left(\frac{N}{m} \cdot m^2 = N.m = J \right)$ ⇐ massless spring.

spring in series

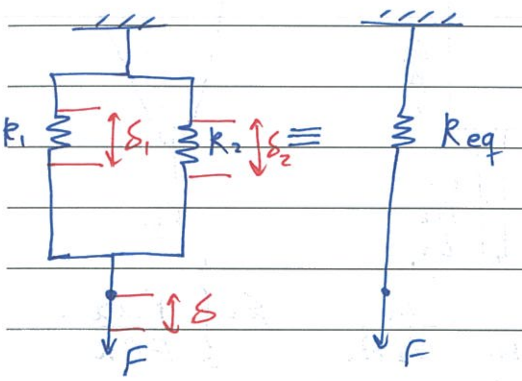


$\delta_1 + \delta_2 + \delta_3 = \delta$ ---- (1)

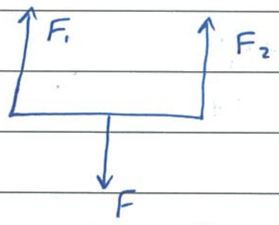
$F_1 = F_2 = F_3 = F$ ----- (2)

$$\boxed{\frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3}}$$

spring in parallel :-



$\delta_1 = \delta_2 = \delta$ ---- ①

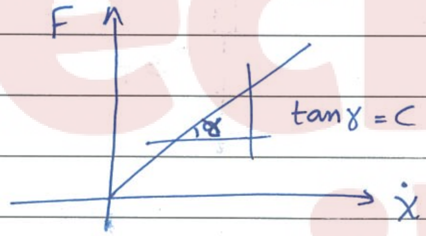


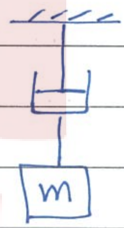
$F = F_1 + F_2$ --- ②

$k_{eq}\, x = k_1 \delta_1 + k_2 \delta_2$ [But $x = \delta = \delta_1 = \delta_2$]

$$\boxed{k_{eq} = k_1 + k_2}$$

2. Dash-pots [dampers] :-





damper in series :-

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

damper in parallel :-

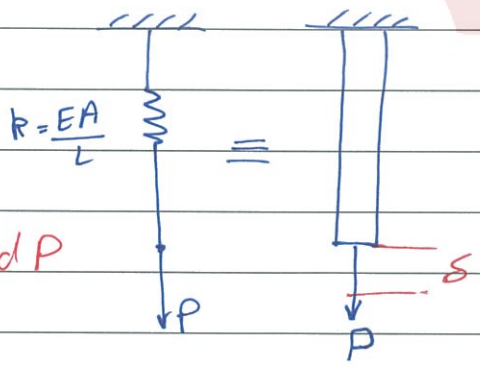
$$C_{eq} = C_1 + C_2 + \dots$$

Example :-

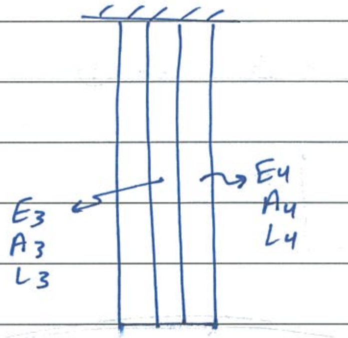
$$\delta = \frac{PL}{EA}$$

$$P = \frac{EA}{L}\,\delta$$

$\to k_{rod}$: under uniaxial load P

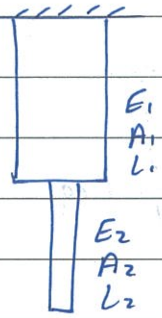


①



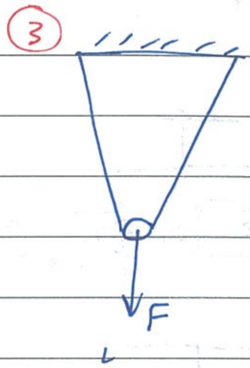
$$R_{eq} = \frac{E_3 A_3}{L_3} + \frac{E_4 A_4}{L_4}$$

②



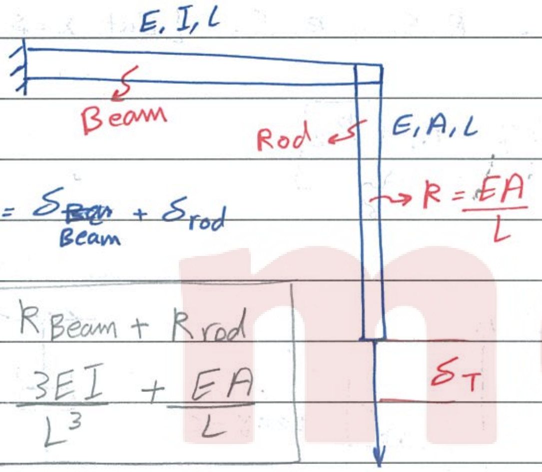
$$\frac{1}{R_{eq}} = \frac{L_1}{E_1 A_1} + \frac{L_2}{E_2 A_2}$$

③



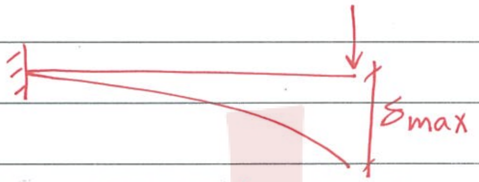
$$\delta = \int_0^L \frac{P\,dx}{EAx}.$$

④



$$\delta_T = \delta_{Beam} + \delta_{rod}$$

$$\boxed{\begin{aligned} R_T &= R_{Beam} + R_{rod} \\ R_T &= \frac{3EI}{L^3} + \frac{EA}{L} \end{aligned}}$$

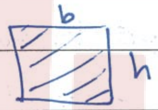


$$\delta_{max} = \frac{PL^3}{3EI}$$

$$R_{beam} = \frac{3EI}{L^3}$$

I: moment of inertia

$$I = \frac{1}{12} b h^3$$



$$\phi = \frac{TL}{JG}$$

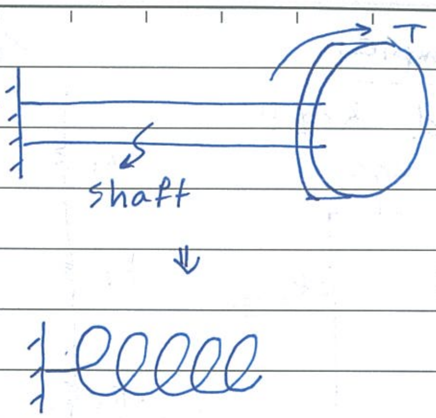
G: shear Muduls of Elasticity

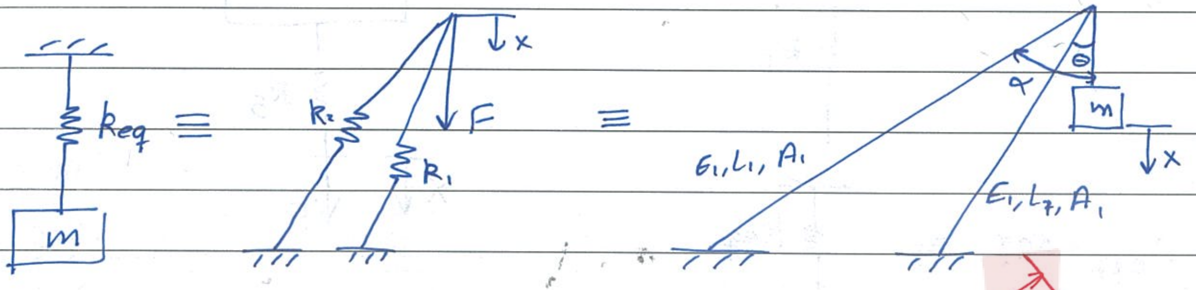
J: polar moment of inertia

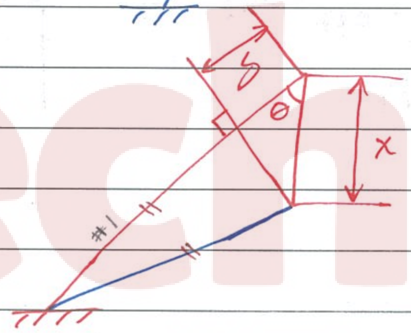
L: length

$$T = \frac{JG}{L}\,\phi$$

k_t: torsinal stiffness







$$\frac{\delta}{x} = \cos\phi$$

$$\delta = x\cos\phi \quad \text{----} \quad (1)$$

$$U_{eq} = \Sigma U_{(spring)}$$

$$\frac{1}{2} k_e x^2 = \frac{1}{2} k \delta^2 \Rightarrow k_{eq} = k\cos^2\theta$$ → use when spring is not Parallel & series

For two spring (k_1, k_2)

$$\cancel{\tfrac{1}{2}} k_{eq} x^2 = \cancel{\tfrac{1}{2}} k_1 \delta_1^2 + \cancel{\tfrac{1}{2}} k_2 \delta_2^2$$

$$k_{eq} \cancel{x^2} = k_1 \cancel{x^2} \cos^2\theta + k_2 \cancel{x^2} \cos^2\alpha$$

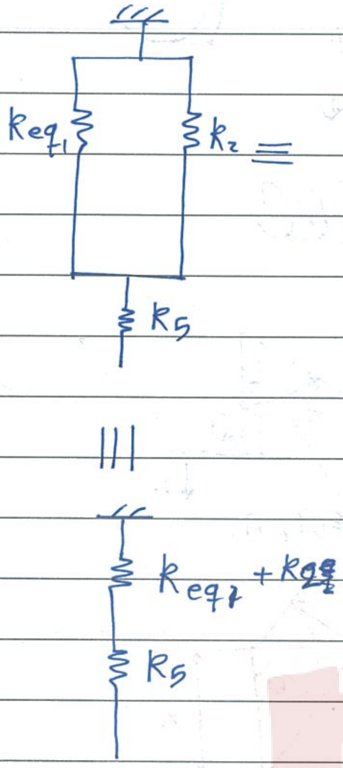
$$k_{eq} = \underbrace{k_1}_{\frac{A_1E_1}{L_1}} \cos^2\theta + \underbrace{k_2}_{\frac{A_2E_2}{L_2}} \cos^2\alpha$$ For vertical motion

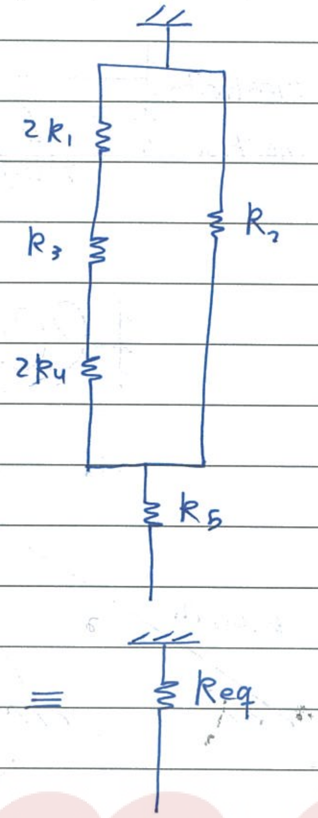
$$k_{eq} = k_1 \sin^2\theta + k_2 \sin^2\alpha$$ For horizantal motion

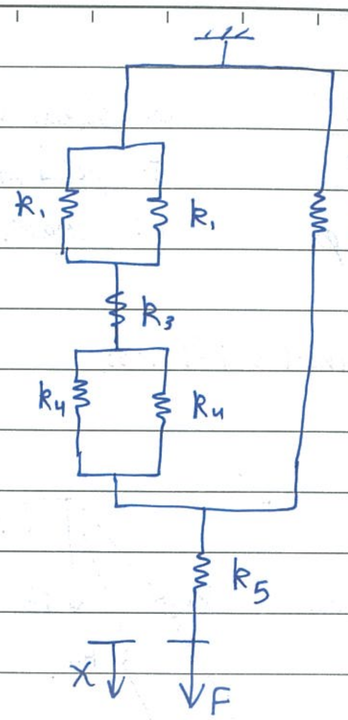
H.W = 1.7 ✓ 1.18
1.9 1.25
1.16 1.47

Example:-

find Keq :-



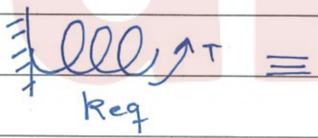


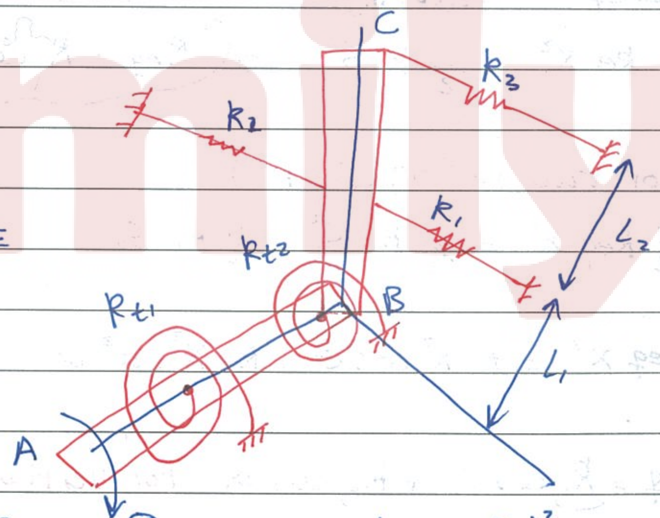


$$\frac{1}{k_{eq_1}} = \frac{1}{2k_1} + \frac{1}{k_3} + \frac{1}{2k_4}$$

$$\boxed{\frac{1}{k_{eq}} = \frac{1}{k_5} + \frac{1}{k_2 + k_{eq_1}}}$$

Example:-





sol.

$$U_{eq} = \sum_{n=1}^{5} U_{(\text{all spring})}$$

$$\frac{1}{2} k_{t\,eq}\theta^2 = \frac{1}{2} k_{t1}\theta^2 + \frac{1}{2} k_{t2}\theta^2 + \frac{1}{2} k_1 (l_1\theta)^2 + \frac{1}{2} k_2 (l_2\theta)^2 + \frac{1}{2} k_3 ((l_1+l_2)\theta)^2$$

* عندما يكون اكثر من نوع من spring

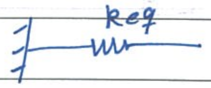
$$k_{t\,eq} = k_{t1} + k_{t2} + k_1 \ell_1^2 + k_2 \ell_1^2 + k_3 (\ell_1 + \ell_2)^2$$

نستخدم قانون P.E.

Note:-

لوطلب Req عند النقطة C بكون Req

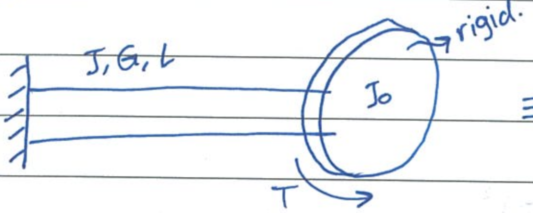
. Linear



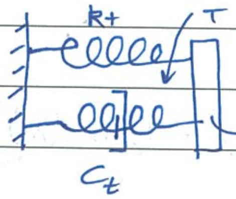
?!

$$\underset{\text{small assump.}}{\tan\theta} = \frac{\delta_1}{L_1} = \frac{\delta_2}{L_2}$$

$$\approx \theta = \frac{\delta_1}{L_1} = \frac{\delta_2}{L_2}$$



≡



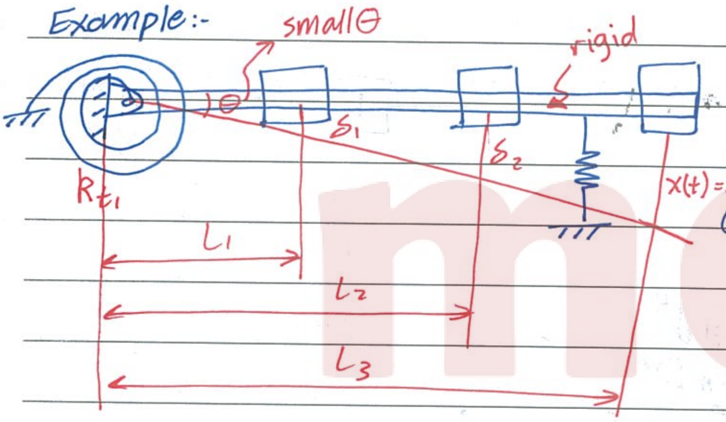
J_o : Mass moment of inertia $(kg \cdot m^2)$

$$T = J_o \alpha$$

J_o → geometric property.

α → $\ddot{\theta}(t)$

Example:-

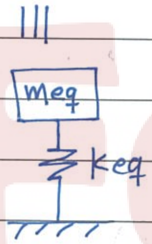


① Find Meq at C ?

② Find Jeq ?

* يتم حساب Meq عادة عند max. displacement

* لحساب Meq نستخدم قانون K.E .



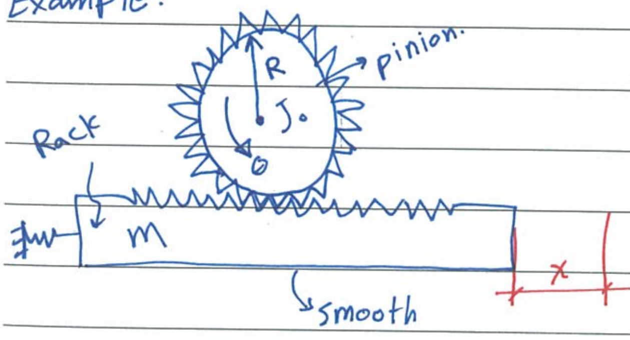
$$KE_{eq\ mass} = \sum KE_{(all\ mass)}$$

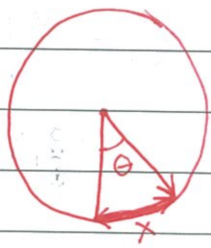
$$\frac{1}{2} M_{eq}\Big|_{@C} (\dot{x})^2 = \frac{1}{2} m_1 (\dot{\delta}_1)^2 + \frac{1}{2} m_2 (\dot{\delta}_2)^2 + \frac{1}{2} m_3 (\dot{\delta}_3)^2$$

$$\tan\theta \approx \theta = \frac{x}{L_3} = \frac{\delta_1}{L_1} = \frac{\delta_2}{L_2}$$

$$M_{eq} = M_3 + M_1 \left(\frac{L_3}{L_1}\right)^2 + M_2 \left(\frac{L_3}{L_2}\right)^2$$

Example:





$$\boxed{x = R\theta}$$

① Find Meq.

$$\tfrac{1}{2} M_{eq} (\dot{x})^2 = \tfrac{1}{2} m (\dot{x})^2 + \tfrac{1}{2} J_o \underbrace{(\dot{\theta})^2}_{\left(\frac{\dot{x}}{R}\right)^2}$$

$$M_{eq} = m + J_o \left(\frac{1}{R}\right)^2$$

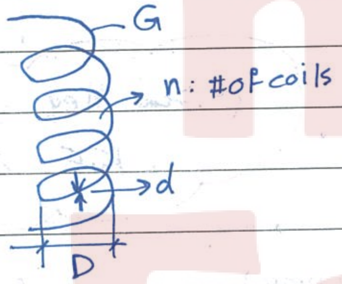
② Find Jeq.

$$\tfrac{1}{2} J_{eq} (\dot{\theta})^2 = \tfrac{1}{2} J_o (\dot{\theta})^2 + \tfrac{1}{2} m \underbrace{(\dot{x})^2}_{(R\theta)^2}$$

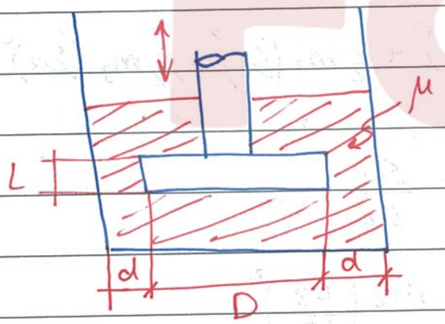
$$J_{eq} = J_o + mR^2$$

* dash pot (damper) :-

$F = c\dot{x}$ $\qquad C\left(\frac{N.s}{m}\right)$



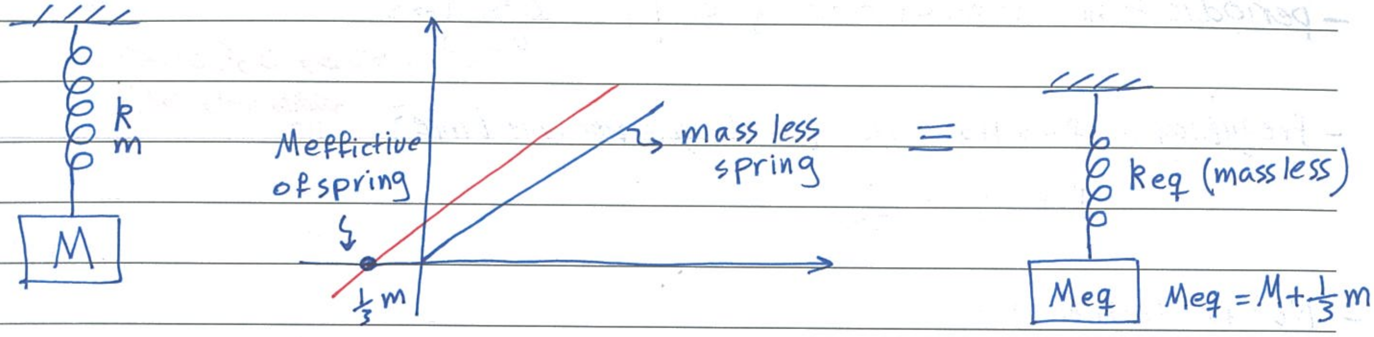
$$K_{spring} = \frac{G d^4}{8 n D^3}$$



$$C = \mu \frac{3\pi D^3 l}{4 d^3}\left(1 + \frac{2d}{D}\right)$$

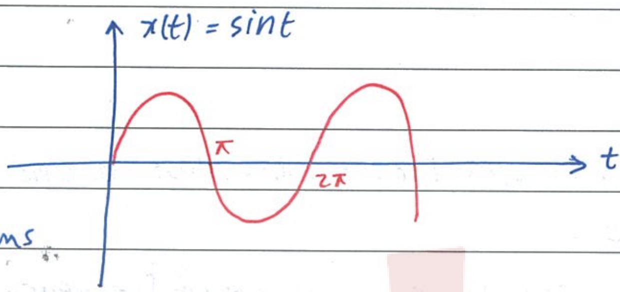
$$C = \frac{\mu A}{h}$$

NOTE :- [very important]



Harmonic : → Motion
→ Force

$\boxed{1}$ $x(t) = A\sin(\omega t)$

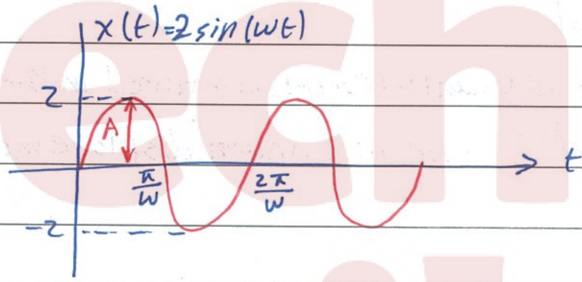


∴ Harmonic motion is repeated and $\ddot{x}(t)$ can be expressed in terms of $(x(t))$

$x(t) = A\sin(\omega t)$

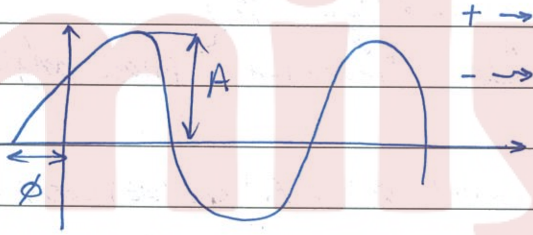
$\dot{x}(t) = A\omega\cos(\omega t)$

$\ddot{x}(t) = -A\omega^2\sin(\omega t) = -\omega^2 x(t)$



Harmonic Force

$x(t) = A\sin(\omega t \pm \phi)$ → phase angle

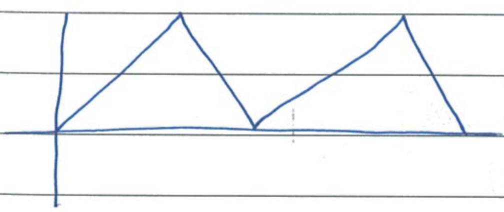


+ → shift to left.
− → shift to right.

$\boxed{2}$ $x(t) = A\cos(\omega t)$

$\boxed{3}$ $x(t) = Ae^{i\omega t}$

Euler Formulla :-
$e^{i\theta} = \cos\theta + i\sin\theta$



→ its periodic but not harmonic.
→ Needs fourier series.

- periodic time : time of one cycle $\left[\tau = \frac{2\pi}{\omega}\right] \rightarrow s$

- يجب ان يكون بوحدة rad/s وليس ~~Hz~~ Hz

- frequency :- $f \rightarrow Hz \equiv 1/s$ {# of cycle per unit time}

$$f = \frac{1}{\tau} = \frac{\omega}{2\pi}$$

- Natural frequency :-

The frequency with which it oscillates without external forces.

- Force frequency :-

- Beats:

when 2 H.M with frequencies close to one another.

- Octave :-

when the max. value of a range of frequency is twice its min. value [75 - 150 Hz]

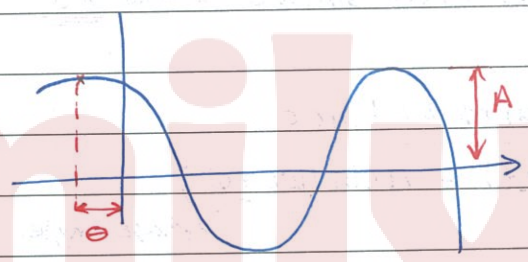
$2\,min = max$

- Decibel

The various quantities encountered in the field of vibration & sound (dis, velocity ..)

- Amplitude :- The maximum displacement of a vibrating Body.

$$\underset{\text{Amplitude}}{x(t) = A} \cos(\underset{\text{frequency}}{\omega} t + \underset{\text{phase angle}}{\phi})$$



** $A\cos(\omega t + \phi) = A\left[\cos(\omega t)\cos\phi - \sin(\omega t)\sin\phi\right]$

$$= C_1 \cos(\omega t) + C_2 \sin(\omega t)$$

such that :-

$C_1 = A\cos\phi \Rightarrow \tan\phi = C_2/C_1$

$C_2 = -A\sin\phi \qquad A^2 = C_1^2 + C_2^2$

$A\sin(\omega t + \phi) = A\left[\cos(\omega t)\sin\phi + \sin(\omega t)\cos\phi\right]$

$$= C_1 \cos(\omega t) + C_2 \sin(\omega t)$$

such that :-

$C_1 = A\sin\phi$

$C_2 = A\cos\phi$

OR $A\sin(\omega t+\phi) = A\cos(\omega t+(\phi-\frac{\pi}{2}))$

Back to :-

Fourier series.

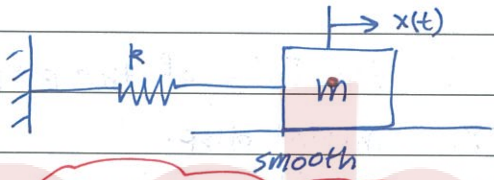
complex forier series.

Chapter 2

- Free vibration of single degree of freedom system :- → Undamped / → Damped.

Undamped system :-

systems ⟹ S.D.O.F



P.E + K.E = const.

$X = X(t)$

dep. variable ↲ ↳ indep. variable

$\frac{1}{2}kX^2 + \frac{1}{2}m\dot{X}^2 = const.$

$\frac{1}{2}k\,2X\dot{X} + \frac{1}{2}m\,2\dot{X}\ddot{X} = 0$

By dividing $[\dot{X}]$:

$\boxed{kX + m\ddot{X} = 0}$ Eq. o.M : 2^{nd}-order homog. linear ODE with const coeff.

This equation solve by char. eq :-

$mR^2 + k = 0$

$R^2 = \frac{-k}{m} \Rightarrow R = \pm\sqrt{\frac{k}{m}}\,i$

$X(t) = C_1\cos\sqrt{\frac{k}{m}}\,t + C_2\sin\sqrt{\frac{k}{m}}\,t$ where :- $\boxed{\omega = \sqrt{\frac{k}{m}}}$ ← netural frequency.

$= C_1\cos(\omega t) + C_2\sin(\omega t)$

$= A\cos(\omega t - \phi_1)$ ✓ where :- $A = \sqrt{C_1^2 + C_2^2}$

$= A\sin(\omega t + \phi_2)$ $\phi_1 = \tan^{-1}\left(\frac{C_2}{C_1}\right)$

Summary:-

For single DOF of Free vibration, undamed

EoM: $m\ddot{x} + kx = 0$ ---- *

solution: $x(t) = A\cos(\omega t - \phi)$ ---- **

$\omega^2 = \frac{k}{m}$ ⇝ Natural frequency.

OR as: $x(t) = C_1\cos\omega t + C_2\sin\omega t$ --- ***

where $A^2 = C_1^2 + C_2^2$, $\phi = \tan^{-1}\left(\frac{C_2}{C_1}\right)$

Assume that the I.C's of the proplem are given as :-

$x(0) = X_0$ initial disp.

$\dot{x}(0) = \dot{X}_0$ initial velocity.

from eq. ***

$\boxed{X_0 = C_1}$

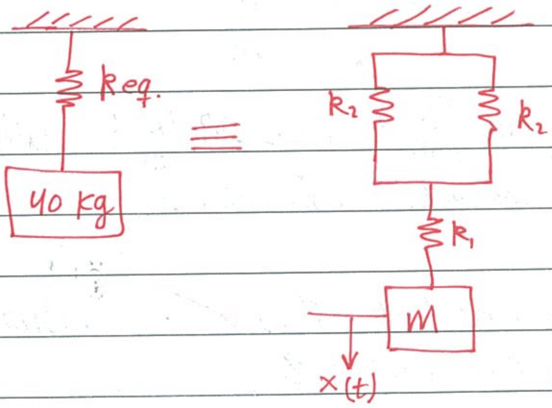
$\dot{x}(t) = -C_1\omega\sin\omega t + C_2\omega\cos(\omega t)$

$\dot{X}_0 = C_2\omega \rightarrow \boxed{C_2 = \frac{\dot{X}_0}{\omega}}$

Exact solution:-

$$x(t) = \sqrt{(X_0)^2 + \left(\frac{\dot{X}_0}{\omega}\right)^2}\ \cos\left(\omega t - \tan^{-1}\left(\frac{\dot{X}_0}{\omega X_0}\right)\right)$$

Example:-



$k_1 = 1000\ N/m$

$k_2 = 2000\ N/m$

$m = 40\ kg$

Given:-

$x(0) = 2\ cm$

$\dot{x}(0) = 0$

$k_{eq.1} = 2000 + 2000 = 4000$

$\frac{1}{k_{eq}} = \frac{1}{1000} + \frac{1}{4000}$

$k_{eq} = 800\ N/m$

Exact solution

$$m\ddot{x} + k_{eq}x = 0$$

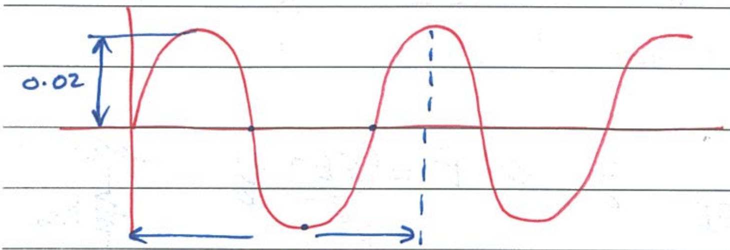
$$x(t) = \sqrt{(0.02)^2 + (0)^2}\ \cos(4.472t)$$

$$\omega = \sqrt{\frac{800}{40}} = \sqrt{20}$$

$$x(t) = 0.02\cos(4.472t)$$

$$\omega = 4.472\ \text{rad/s}$$

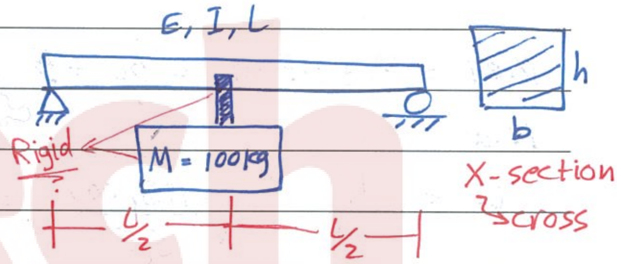
$$\phi = 0$$

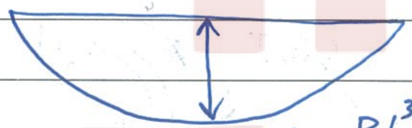


$$\tau = \frac{2\pi}{4.472} = 1.405\,s$$

Example #2 :-

Find the netural frequency of the beam at the middle point?

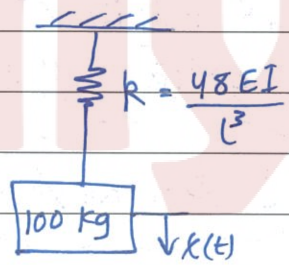




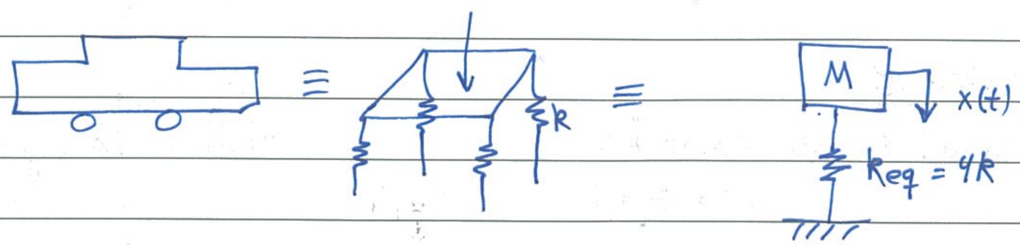
from the Table $\Leftarrow$ $y(L/2) = \dfrac{PL^3}{48EI}$ P: Force , y: deflection

$$P = \frac{48EI}{L^3}\,y \quad ; \quad k = \frac{48EI}{L^3}$$

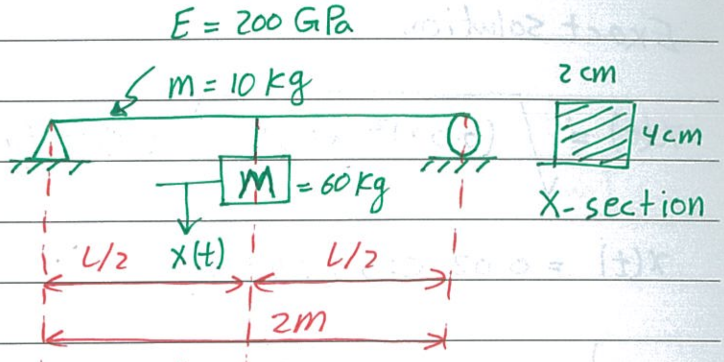
$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{48EI}{L^3(100)}}$$



Example #3 :-



Example :-



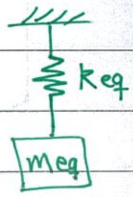
From Tabels :-

$a = 1m \;,\; L = 2m \;,\; b = L - a = 1m$

$$y(x) = \frac{P b X (L^2 - x^2 - b^2)}{6EIL}$$

X :- distance where deflection takes place (here = 1)

$$y = \frac{P(1)(1)(4-1-1)}{6(2)EI} = \frac{2P}{12EI} = \frac{P}{6EI} = y \Rightarrow P = \underbrace{6EI}_{k} y$$



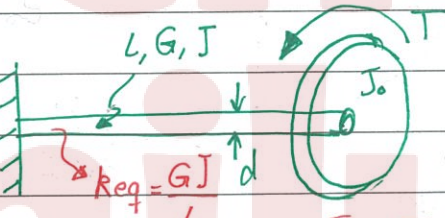
$$K_{eq} = 6 * 200 \times 10^9 * \frac{1}{12}(0.02)(0.04)^3 = 128{,}000 \text{ N/m}$$

$$M_{eq} = M + 0.5m = 60 + 5 = 65 \text{ kg}$$

Now,

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{128{,}000}{65}} = 44.38 \text{ rad/s}$$

Example :-



$, J = \frac{\pi}{32} d^4$

P.E + K.E = const.

$$\frac{1}{2} J_o \dot{\phi}^2 + \frac{1}{2} k_t \phi^2 = \text{const}$$

$$\cancel{\tfrac{1}{2}} J_o \cancel{2\dot{\phi}}(\ddot{\phi}) + \cancel{\tfrac{1}{2}} k_t (\cancel{2}\phi)(\cancel{\dot{\phi}}) = 0$$

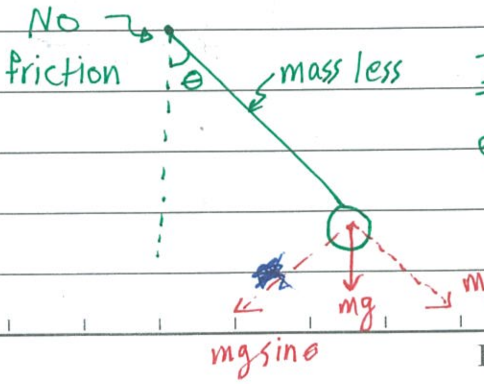
$$J_o \ddot{\phi} + k_t \phi = 0$$ equation of motiom.

$\phi(t) = \phi \cos(\omega t - \psi)$; $\phi(0) = 3^\circ \left(\frac{\pi}{180}\right) \{rad\}$ $\dot{\phi}(0) = 2$ rad/s

Example :-

small $\theta \Rightarrow \sin\theta = \theta$

$\cos\theta = 1$



I.C's:-

$\theta(0) = 3$

$\dot{\theta}(0) = 0$

E.O.M :- $\circlearrowleft \Sigma M_o = J_o \ddot{\theta}$

$$J_o \ddot{\theta} = -mgL\sin\theta$$

$$J_o \ddot{\theta} + mgL\sin\theta = 0$$

mass

moment of inertia $m\ell^2$

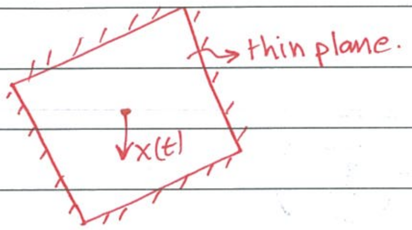
θ

$$\ddot{\theta} + \frac{g}{L}\theta = 0 \qquad \omega^2 = \frac{g}{L} \rightsquigarrow \boxed{\omega = \sqrt{\frac{g}{L}}}$$

$$\theta(t) = \phi\cos(\omega t - \phi) \xrightarrow[\text{I.C's}]{\text{from}} \phi = \frac{3\pi}{180}$$

$$\psi = 0$$

$$\therefore \theta(t) = \frac{3\pi}{180}\cos\left(\sqrt{\frac{g}{L}}\,t\right)$$

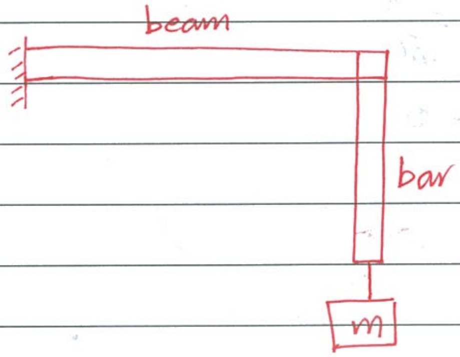


$$y_{center} = \frac{\alpha P a^2}{E t^3} \Rightarrow P = \frac{E t^3}{\alpha a^2} y_{center}$$

const. (α)

$\frac{E t^3}{\alpha a^2}$ → Req

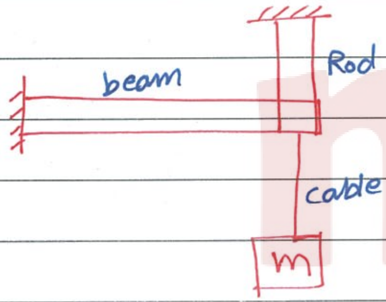
$$m\ddot{x} + kx = 0$$



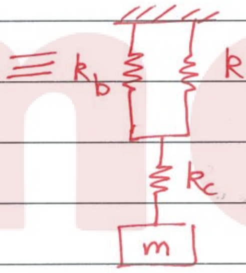
$\equiv$

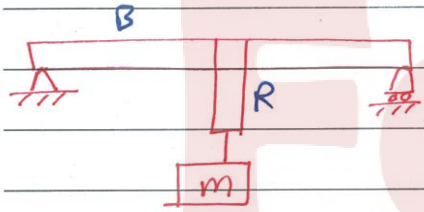
$$k_{beam} = \frac{3EI}{L^3}$$

$$k_{bar} = \frac{EA}{L}$$



$\equiv$





$\equiv$

$$k_B = \frac{48EI}{L^3}$$

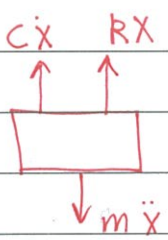
$$k_R = \frac{EA}{L}$$

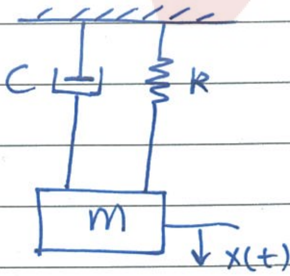
* Free damped vibration:-

Eq. o. M :-

$m\ddot{x} + c\dot{x} + kx = 0$ → 2^{nd}-order linear, Homo. DE with const. coeff.

F.B.D





char. eq :-

$$mR^2 + cR + k = 0$$

$$R_{1,2} = \frac{-c \pm \sqrt{c^2 - 4mk}}{2m}$$

→ critical vibration $\sqrt{c^2 - 4mk} = 0$

Case 1 :-

Assume that $c^2 = 4mk$ → Critical Damping. $[\xi = 1]$

$R_{1,2} = \frac{-c}{2m}$ → Repeated roots

solution :-

$$x(t) = C_1 e^{\frac{-c}{2m}t} + C_2 t e^{\frac{-c}{2m}t}$$

if given I.C is :-

$x(0) = x_0 \quad \rightarrow C_1 = X_0$

$\dot{x}(0) = \dot{x}_0$

$$\dot{x}(t) = C_1\left(\frac{-c}{2m}\right)e^{-c/2m\, t} + C_2 e^{\frac{-c}{2m}t} + C_2\left(\frac{-c}{2m}\right)t e^{\frac{-c}{2m}t}$$

$$\dot{x}(0) = X_0\left(\frac{-c}{2m}\right) + C_2 \rightarrow C_2 = \dot{x}_0 + \frac{C X_0}{2m}$$

$$x(t) = X_0 e^{\frac{-c}{2m}t} + \left(\dot{x}_0 + \frac{C X_0}{2m}\right)t e^{-c/2m\, t}$$

$w = \sqrt{\frac{k}{m}}$

$w^2 = \frac{k}{m} \rightarrow m = \frac{k}{w^2}$

$$\boxed{\therefore\ x(t) = X_0 e^{-wt} + (\dot{X}_0 + wX_0)t e^{-wt}}$$

** Definition:-

ξ is defined as damping Ratio (unit less)

$\xi = \frac{C}{C_{critical}}$,

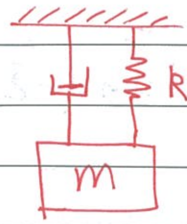
where :- $C_c = \sqrt{4mk} = \sqrt{4m(mw^2)}$

$$\boxed{C_c = 2mw}$$

Free vibration of viscous damping [summary] :-

$m\ddot{x} + c\dot{x} + kx = 0 \quad ---- (*)$

→ $R_{1,2} = \frac{-c \pm \sqrt{c^2 - 4mk}}{2m}$



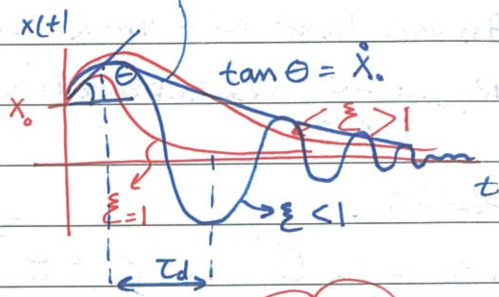
Def :- $C_c^2 = 4mk = 4m^2 w_n^2 \rightarrow \boxed{C_c = 2m w_n}$

Def :- $\xi = \frac{c}{C_c} = \frac{c}{2m w_n}$

any number $\leftarrow n > \xi > 0$

$\xi \neq$ negative

Def :- $w_n = \sqrt{\frac{k}{m}}$



case ① :- $\xi = 1$

$$x(t) = x_0 e^{-w_n t} + (\dot{x}_0 + w_n x_0) t e^{-w_n t}$$

case ② :- over damping $\{\xi > 1\}$

$$R_{1,2} = \frac{-c \pm \sqrt{c^2 - c_c^2}}{2m}$$

$$= \frac{-c \pm c_c\sqrt{\xi^2 - 1}}{2m} = -w_n \xi \pm w_n \sqrt{\xi^2 - 1}$$

$$= w_n \left(-\xi \pm \sqrt{\xi^2 - 1}\right)$$

$\xi > 1$

$c^2 > 4mk$

$c^2 > c_c^2$

$$\therefore x(t) = c_1 e^{R_1 t} + c_2 e^{R_2 t}$$

$$= c_1 e^{w_n(-\xi + \sqrt{\xi^2 - 1})t} + c_2 e^{w_n(-\xi - \sqrt{\xi^2 - 1})t}$$

If the IC's :-

$x(0) = x_0 \longrightarrow c_1 + c_2 = x_0$

$\dot{x}(0) = \dot{x}_0 \longrightarrow c_1 w_n \left(-\xi + \sqrt{\xi^2 - 1}\right) + c_2 \left(-\xi - \sqrt{\xi^2 - 1}\right) = \dot{x}_0$

solve for c_1 & c_2 :-

$$c_1 = \frac{x_0 w_n \left(\xi + \sqrt{\xi^2 - 1}\right) + \dot{x}_0}{2 w_n \sqrt{\xi^2 - 1}} \quad ; \quad c_2 = \frac{-x_0 w_n \left(\xi - \sqrt{\xi^2 - 1}\right) - \dot{x}_0}{2 w_n \sqrt{\xi^2 - 1}}$$

*** case ③ :-

Underdamping $[0 < \xi < 1]$

$$R_{1,2} = w_n \left(-\xi \pm \sqrt{\xi^2 - 1}\right)$$ → negative quantity.

$$= w_n \left(-\xi \pm \sqrt{1 - \xi^2}\, i\right)$$

positive quantity ; $i = \sqrt{-1}$

$$x(t) = e^{-\xi w_n t}\left(c_1 \cos\left(w_n \sqrt{1 - \xi^2}\, t\right) + c_2 \sin\left(w_n \sqrt{1 - \xi^2}\, t\right)\right)$$

w_d : damping frequancy.

$$\tau_d = \frac{2\pi}{w_d}$$

$$x(t) = e^{-\xi \omega_n t}\left[A\cos(\omega_d t - \phi)\right]$$

$$A = \sqrt{C_1^2 + C_2^2}$$

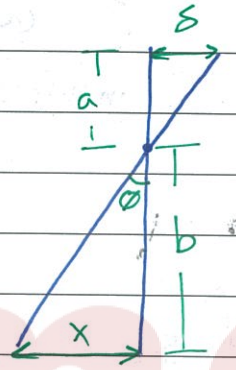
$$\phi = \tan^{-1}\left(\frac{C_2}{C_1}\right)$$

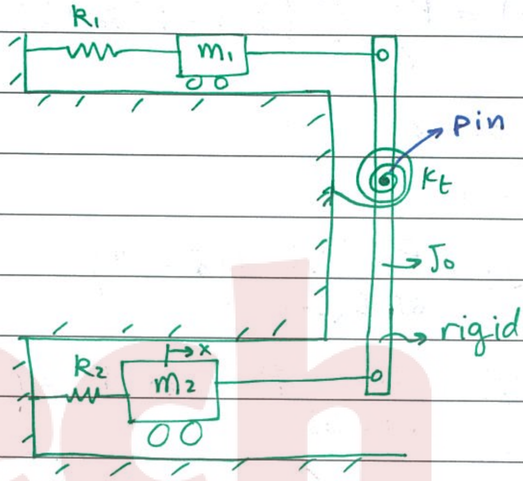
$$C_1 = X_0$$

$$\dot{X}_0 = -\xi \omega_n \overset{X_0}{C_1} + C_2 \omega_d \Rightarrow C_2 = \frac{\dot{X}_0 + \xi X_0 \omega_n}{\omega_d}$$

Example

$$\phi = \frac{\delta_1}{a} = \frac{x}{b}$$





$$\frac{1}{2} m_{eq} (\dot{x})^2 = \frac{1}{2} m_2 (\dot{x})^2 + \frac{1}{2} J_0 \underbrace{\dot{\theta}^2}_{\left(\frac{\dot{x}}{a}\right)^2} + \frac{1}{2} m_1 \underbrace{(\dot{\delta}_1)^2}_{\left(\frac{a}{b}\dot{x}\right)^2}$$

$$m_{eq} = m_2 + J_0\left(\frac{1}{a}\right)^2 + m_1\left(\frac{a}{b}\right)^2$$

$$\frac{1}{2} k_{eq} X^2 = \frac{1}{2} k_2 x^2 + \frac{1}{2} k_t \theta^2 + \frac{1}{2} k_1 \delta_1^2$$

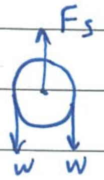
$\boxed{2.13}$

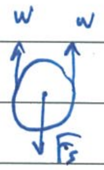
$$x = 2\delta_2 + 2\delta_1$$

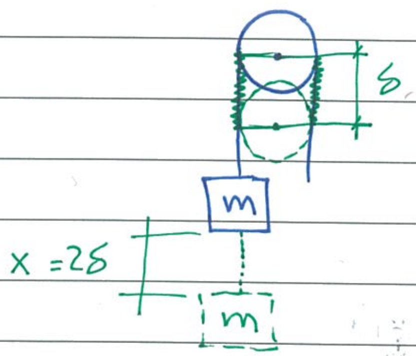
$$F = kX$$

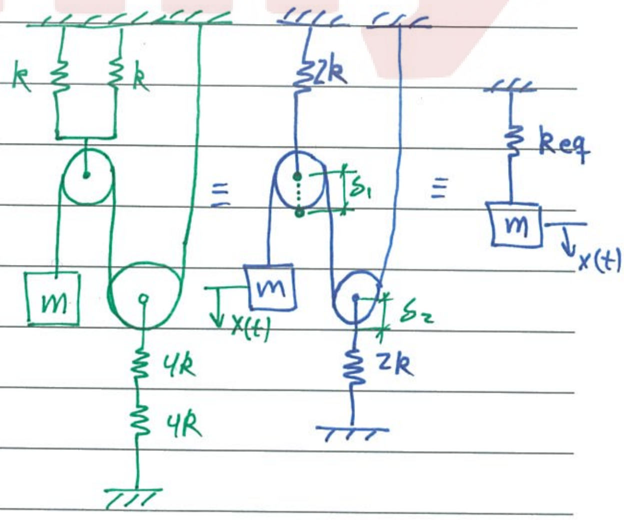
$$x = \frac{F}{k}$$

$\boxed{F_s = 2W}$







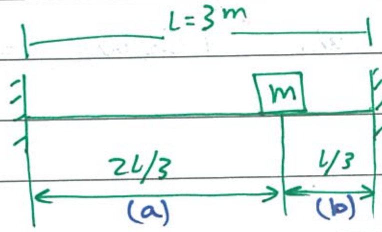


$$X = 2\delta_2 + 2\delta_1$$

$$\frac{\cancel{w}}{K_{eq}} = 2\left(\frac{2\cancel{w}}{2k}\right) + \left(\frac{2\cancel{w}}{2k}\right)2 = \frac{2}{k} + \frac{2}{k} = \frac{4}{k}$$

$$\boxed{K_{eq} = \frac{k}{4}}$$

Example:-



$$y(x) = \frac{P b^2 x^2}{6EIL^3}\left(3aL - x(3a+b)\right)$$

$x = a = 2$

in this example : $X = a = \frac{2L}{3}$

$b = 1$

$$y(x) = \frac{P(1)^2(4)}{6EI(27)}(18 - 14)$$

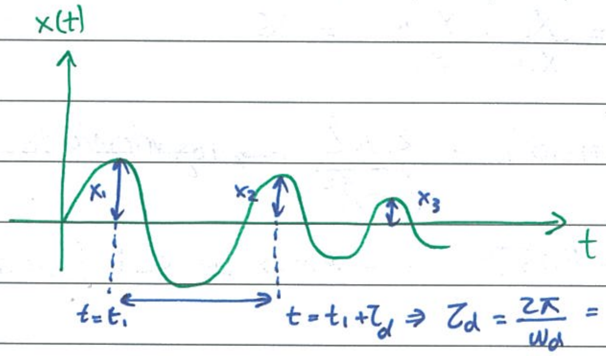
$$y(x) = \frac{16}{162EI}P \Rightarrow P = \underbrace{\frac{162EI}{16}}_{K} y$$

* Logarithmic Decrement :-

Recall that :-

$$x(t) = e^{-\xi \omega t}\left[A\cos(\omega_d t - \phi)\right]$$

$$\frac{x_1}{x_2} = \frac{e^{-\xi\omega_n t_1}\cancel{(A\cos(\omega_d t - \phi))}}{e^{-\xi\omega(t_1+\tau_d)}\cancel{(A\cos(\omega_d(t_1+\tau_d) - \phi))}}$$



$t = t_1 + \tau_d \Rightarrow \tau_d = \frac{2\pi}{\omega_d} = \frac{2\pi}{\omega_n\sqrt{1-\xi^2}}$

$$= \frac{1}{e^{-\xi\omega_n\tau_d}} = e^{\xi\omega_n\tau_d}$$

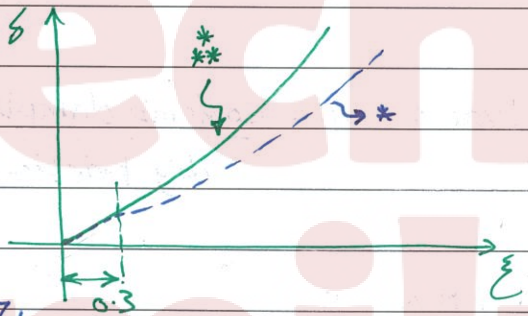
$$\delta = \ln\left(\frac{x_1}{x_2}\right)$$

$$\delta = \xi\omega_n\tau_d$$

$$\delta = \xi\,\frac{\omega_n(2\pi)}{\omega_n\sqrt{1-\xi^2}} = \frac{2\pi\xi}{\sqrt{1-\xi^2}} \quad ---- \; (**)$$

For small ξ (**) becomes

$$\delta \simeq 2\pi\xi \quad ---- \; (*)$$



$\frac{x_1}{x_m}$; $x \geq 2$

$$\frac{x_1}{x_m} = \frac{x_1}{x_2}\,\frac{x_2}{x_3}\;\cdots\;\frac{x_{m-1}}{x_m} = e^{m\xi\omega_n\tau_d}$$

$$\ln\left(\frac{x_1}{x_m}\right) = m\,\xi\,\omega_n\tau_d$$

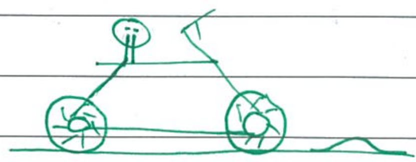
$$\delta = \frac{1}{m}\ln\left(\frac{x_1}{x_m}\right) = \xi\,\omega_n\tau_d$$

Ex.

$\tau_d = 2s$

$\frac{x_1}{x_{1.5}} = \frac{4}{1}$

$m = 200\ kg$



$$\frac{X_1}{X_2} = \frac{X_1}{X_{.5}} \cdot \frac{X_{1.5}}{X_2} = 16$$

$$\delta = \ln(16) = \frac{2\pi\xi}{\sqrt{1-\xi^2}}$$ → logaritmic increament

$$\xi = 0.4037$$ → must be < 1 [because it under damping]

$$\tau_d = 2 = \frac{2\pi}{\omega_d} \Rightarrow \omega_d =$$

$$\omega_d = \omega_n\sqrt{1-\xi^2} \Rightarrow \omega_n$$

$$\omega_n = \sqrt{\frac{k}{m}} \Rightarrow k =$$ ✓

$$\xi = \frac{C}{C_c} \Rightarrow C = \underbrace{C_c}_{2m\omega_n}\xi$$

Free Vibration with coulomb damping

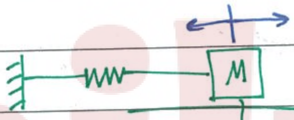
dry friction

$\mu = 0$

EOM: $m\ddot{x} + kx = 0$

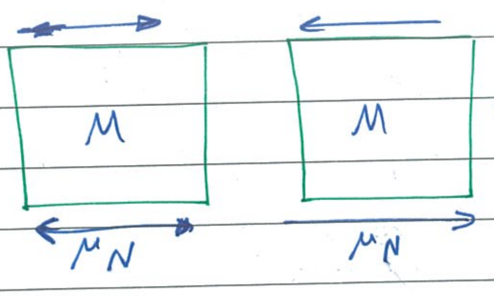
$\mu \neq 0$

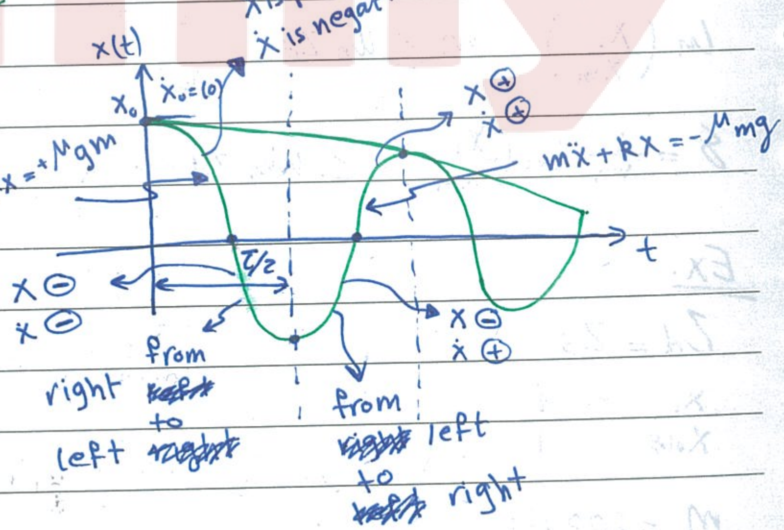
EOM: $m\ddot{x} + kx = \pm\mu N$ → Normal force $N = mg$



from left to right

from right to left





$m\ddot{x} + kx = \mu mg \rightsquigarrow$ Non homo. DE

$x(t) = x_p(t) + x_h(t) \rightsquigarrow x_h(t)$:- known $\rightsquigarrow x_h(t) = A\cos(\omega_n t - \phi)$

$$\left.\begin{aligned} x_p(t) &= A \\ \dot{x}_p &= 0 \\ \ddot{x}_p &= 0 \end{aligned}\right\} \Rightarrow kA = \mu mg \rightarrow A = \frac{\mu mg}{k}$$

$$x(t) = x_p(t) + x_h(t)$$
$$= C_1\cos(\omega_n t) + C_2\sin(\omega_n t) + \frac{\mu mg}{k}$$

I.C's :-

$x(0) = x_0 \Rightarrow C_1 = x_0 - \frac{\mu mg}{k}$

$\dot{x}(0) = 0 \Rightarrow C_2\omega_n = 0 \Rightarrow C_2 = 0$

$$\boxed{x(t) = \left(x_0 - \frac{\mu mg}{k}\right)\cos(\omega_n t) + \frac{\mu mg}{k}} \rightsquigarrow \text{valid for } 0 \le t \le \tau/2 \quad (*)$$

$\rightarrow$ for $\frac{\tau}{2} \le t \le \tau$

$m\ddot{x} + kx = -\mu mg \leftarrow$ Non Homo. DE

$\tau = \frac{2\pi}{\omega_n}$

$$x(t) = C_3\cos(\omega_n t) + C_4\sin(\omega_n t) - \frac{\mu mg}{k} \quad \text{-----} \; (**)$$

$x\left(\frac{\tau}{2}\right) \overset{(*)}{=} \frac{2\mu mg}{k} \quad C_3 = x_0 - \frac{3\mu mg}{k} \rightsquigarrow$ I.C's :- نأخذها من نهاية الفترة الاولى

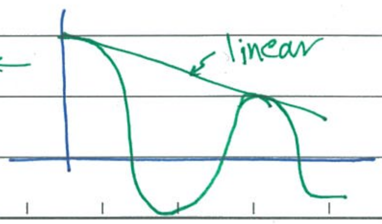
$\dot{x}\left(\tau/2\right) = 0 \Leftarrow$ find C_3 and C_4

$-C_4\omega_n = 0 \Rightarrow C_4 = 0$

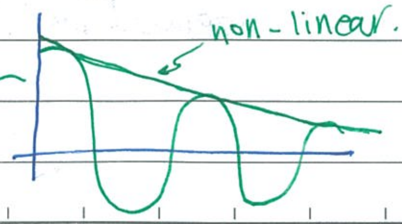
$$\boxed{x(t) = \left(x_0 - \frac{3\mu mg}{k}\right)\cos(\omega_n t) - \frac{\mu mg}{k}} \rightsquigarrow \text{valid for } \frac{\tau}{2} \le t \le \tau$$

← في viscous كان Wd بينما في coulomb ، يصبح Wn

coulomb ←



viscous ←



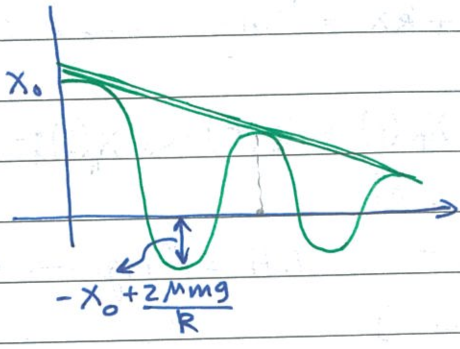
$\boxed{w_d < w_n}$

viscous: continous motion ←

columb: stop at $\frac{\mu N}{k}$ ←

$$x\left(\frac{\tau}{2}\right) = \left(-X_0 + \frac{2\mu m g}{k}\right)$$

$$x(\tau) = \left(X_0 - \frac{4\mu m g}{k}\right)$$



Xp:

$$x(t) = C_1 \cos(w_n t) + C_2 \sin(w_n t) - \frac{\mu m g}{k}$$

$$x(0) = X_0$$

$$C + \frac{\mu m g}{k} = X_0$$

Summary:-

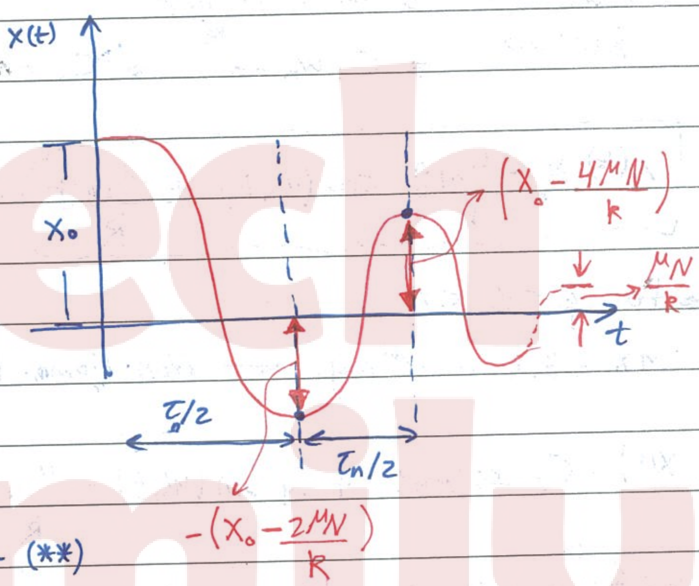
$$0 \le t \le \frac{\tau_n}{2}$$

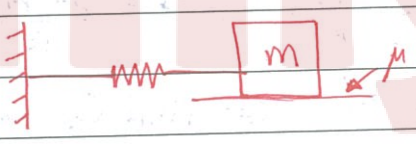
$$m\ddot{x} + kx = \mu N$$

$$x(t) = \left(X_0 - \frac{\mu N}{k}\right)\cos(w_n t) + \frac{\mu N}{k} \quad --- (*)$$

$$\frac{\tau_n}{2} \le t \le \tau_n$$

$$x(t) = \left(X_0 - \frac{3\mu N}{k}\right)\cos(w_n t) - \frac{\mu N}{k} \quad ---- (**)$$





# of half cycles

$$r \ge \frac{X_0 - \frac{\mu N}{k}}{\frac{2\mu N}{k}}$$

max. amplitude

→ min. Amplitude

→ reduction

viscous problem after 97

$$X_m = X_{(m-1)} - \frac{4\mu N}{k}$$

in Rotational:-

$$J\ddot{\theta} + k_t\theta = \pm T$$

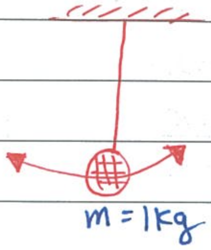
→ Now, (μN) is replaced by T.

$$\theta = \theta_0 -$$

Example 2-14
2-15

problem 97 :-

$f = \frac{1}{\tau} = 0.5\ Hz$, Find damping constant (C) ??



$f_{fluid} = \frac{1}{\tau_d} = 0.45\ HZ$

$\omega_n = \sqrt{\frac{g}{\ell}} \rightsquigarrow t = 2\pi / \omega_n$

$\omega_n = \frac{2\pi}{\tau} = 2\pi\left(\frac{1}{2}\right) = \pi$

$L = \frac{g}{\omega_n^2}$

$\omega_d = \frac{2\pi}{\tau_d} \rightarrow \tau_d = \frac{2\pi}{\omega_d} \Rightarrow \omega_d = 0.9\pi$

$\omega_d = \omega_n \sqrt{1-\xi^2} \rightsquigarrow \xi = 0.43$

$\rightsquigarrow C = \xi C_c = \xi (2 m \omega_n)$

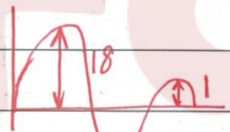
$\underset{m \times L^2}{J}\ \ddot{\theta} + C\dot{\theta} + mgL\theta = 0$

$C_c = \sqrt{4 \underset{m \times L^2}{J}\, m g L} =$

Now ; $\xi = \frac{c}{C_c} \Rightarrow C = \xi C_c$

problem 98 :-

$\frac{x_i}{x_{i+1}} = \frac{18}{1}$



$\delta = \ln\left(\frac{x_1}{x_2}\right) = \frac{2\pi\xi}{\sqrt{1-\xi^2}}$

$\ln(18) = \frac{2\pi\xi}{\sqrt{1-\xi^2}} \rightsquigarrow \xi = 0.418$

case ①:-

$\xi = 0.836$

$\delta =$ $\rightsquigarrow$ assume $x_2 = 1$, x_1

$\delta = 14365$

case ②

$\xi = 0.209$

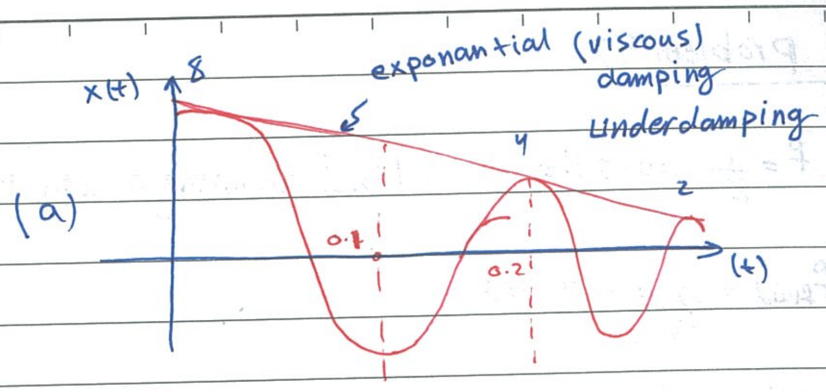
$x_2 = 1$, $x_1 =$

$\delta \approx 4$

problem 102 :-

$W = 500N$

$m = \frac{500}{9.81} = \quad kg$

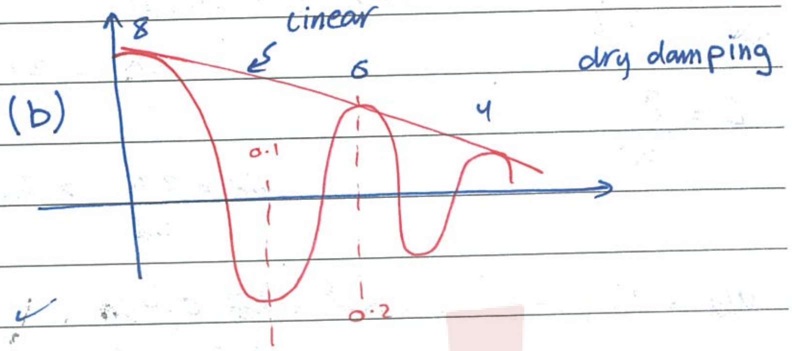


sol.

$\tau_n = 0.2 = \frac{2\pi}{\omega_n} \rightarrow$ from(b)

$\omega_n =$

$0.2 = \frac{2\pi}{\omega_d} \rightarrow \omega_d =$ from (a)



$\delta = \ln\left(\frac{x_1}{x_2}\right)$

$= \ln(2) = \frac{2\pi\xi}{\sqrt{1-\xi^2}} \rightsquigarrow \xi =$

$\omega_d = \omega_n\sqrt{1-\xi^2} \rightsquigarrow \omega_n =$

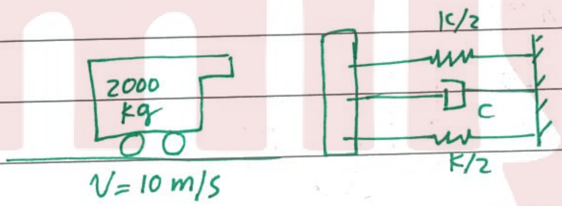
$\omega_n = \sqrt{\frac{k}{m}} \rightsquigarrow k =$

$2 = \frac{4\mu N}{k}$

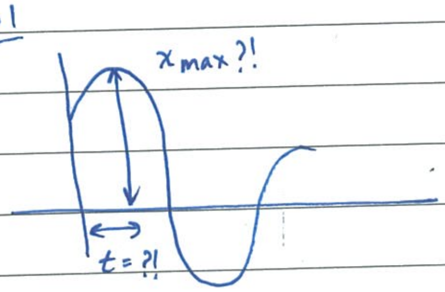
Example 2:104

$k = 80$ N/mm , find x_{max}?!?

$C = 20$ N.s/mm $\quad t$?!?



sol



$k = 80 \times 10^3$ N/m

$C = 20 \times 10^3$ N.s/m

$k_{eq} = \frac{k}{2} + \frac{k}{2} = 80 \times 10^3$ N/m

$C_{eq} = C = 20 \times 10^3$ N.s/m

$\xi = \frac{C}{C_c} = \frac{C}{2m\omega_n}$, $\omega_n = \sqrt{\frac{80\times10^3}{2000}} = \sqrt{40} = 6.325 \frac{rad}{s}$

$$\xi = \frac{20 \times 10^3}{4000\sqrt{40}} = 0.79 \rightsquigarrow \therefore \text{Under damping}$$

$$x(t) = e^{-\xi w_n t}\left[C_1 \cos(w_d t) + C_2 \sin(w_d t)\right]$$

$$w_d = w_n\sqrt{1-\xi^2} =$$

$$x(0) = 0 \rightsquigarrow C_1 = 0$$

$$\dot{x}(0) = 10$$

$$\dot{x}(t) = -\xi w_n e^{-\xi w_n t}\left[C_1 \cos(w_d t) + C_2 \sin(w_d t)\right] + e^{-\xi w_n t} w_d\left[-C_1 \sin(w_d t) + C_2 \cos(w_d t)\right]$$

$$C_2 = \frac{10}{w_d}$$

$$\therefore x(t) = \frac{10}{w_d} e^{-w_n t \xi} \sin(w_d t)$$

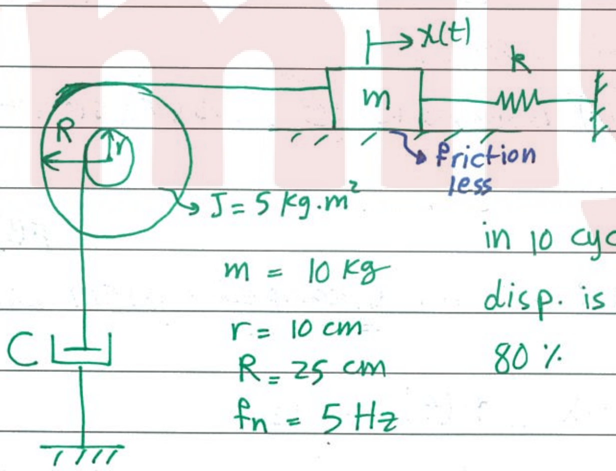
$$\dot{x}(t) = \frac{10(-w_n\xi)}{w_d} e^{-\xi w_n t} \sin(w_d t) + \frac{10}{\cancel{w_d}} \cancel{w_d} e^{-\xi w_n t} \cos(w_d t) = 0$$

$$\cancel{10}\, \cancel{e^{-\xi w_n t}} \cos(w_d t) = \frac{\cancel{10}\, w_n \xi}{w_d} \cancel{e^{-\xi w_n t}} \sin(w_d t)$$

$$\tan(w_d t) = \frac{w_d}{w_n \xi} \rightsquigarrow \frac{\cancel{w_d}}{\cancel{w_d}}\, \boxed{t = \tan^{-1}\left(\frac{w_d}{w_n \xi}\right) \cdot \frac{1}{w_d}}$$

Problem 119 :- 106, man.

Find k, C ?!?



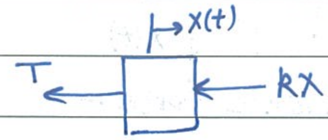
$m = 10$ kg

$r = 10$ cm

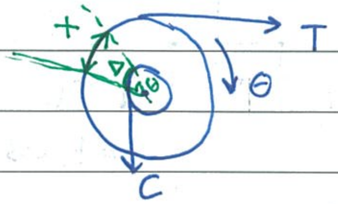
$R = 25$ cm

$f_n = 5$ Hz

in 10 cycles the disp. is reduced by 80 %



$$m\ddot{x} = -kx - T \text{ ------ } ①$$



$$\Delta = r\theta \quad \dots ②$$

$$x = R\theta$$

$$\theta = \frac{\delta}{r} = \frac{x}{R}$$

$$J\ddot{\theta} = -C\dot{\Delta}r + RT \text{ ---- } ③$$

from eq. 1

$$-m\ddot{x} - k(x)$$

$$J\left(\frac{\ddot{X}}{R}\right) + Cr\left(\frac{\dot{X}r}{R}\right) + Rm\ddot{X} + RkX = 0$$

$$\left(\frac{J}{R} + Rm\right)\ddot{X} + \left(\frac{Cr^2}{R}\right)\dot{X} + (kR)X = 0 \qquad R, r \rightsquigarrow \text{in meter.}$$

$$\frac{1}{\tau_n} = f_n$$

$$\tau_n = \frac{2\pi}{\omega_n} \rightsquigarrow \omega_n = \checkmark$$

$$\omega_n = \sqrt{\frac{kR}{\frac{J}{R} + Rm}}$$

$$\delta = \frac{1}{N}\ln\left(\frac{X_i}{X_{i+N}}\right) \Rightarrow \delta = \frac{\cancel{(10)}}{}\overset{0.1}{} \ln\left(\frac{1}{0.2}\right) = \frac{2\pi\xi}{\sqrt{1-\xi^2}} \Rightarrow \xi = \checkmark \quad \underline{0.0256}$$

$$\xi = \frac{C_{eq}}{C_c} = \frac{C_{eq}}{2\omega_n k_{eq}}$$

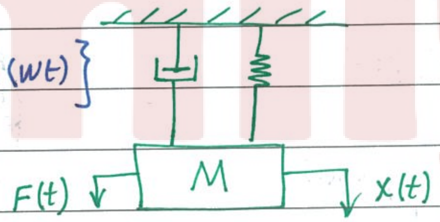
Chapter 3

* Forced Vibration

I) Harmonic Force. $\qquad$ w: is given

II) Harmonic distance

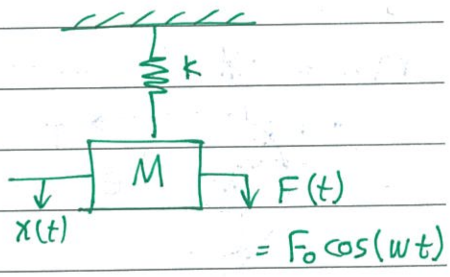
when $F(t)$ is harmonic $\{e^{i\omega t}, \cos(\omega t), \sin(\omega t)\}$



⇒ Responce of an undamped system under harmonic force :-

$$m\ddot{x} + kx = F(t) \quad \text{—— ①}$$

$$m\ddot{x} + kx = F_0\cos(\omega t) \quad \text{.... non-homo } 2^{nd} \text{ order ODE}$$



steady state :- $X_p(t)$

$$X_h(t) = C_1\cos(\omega_n t) + C_2\sin(\omega_n t)$$

I.C'S : C_1, C_2

$$X_p(t) = A\cos\omega t + B\sin\omega t$$
$$X_p'(t) = -\omega A\sin(\omega t) + \omega B\cos(\omega t)$$
$$X_p''(t) = -\omega^2 A\cos(\omega t) - \omega^2 B\sin(\omega t)$$

put in ①

$$m\left(-\omega^2 A\cos(\omega t) - \omega^2 B\sin(\omega t)\right) + K\left(A\cos(\omega t) + B\sin(\omega t)\right) = F_0\cos(\omega t)$$

$$-m\omega^2 A + KA = F_0 \rightsquigarrow A = \frac{F_0}{K - m\omega^2}$$

$$-m\omega^2 B + KB = 0 \rightsquigarrow B = 0$$

$$\Rightarrow A = \frac{F_0}{K\left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right]}$$

$$\Rightarrow X(t) = \underbrace{C_1\cos(\omega t) + C_2\sin(\omega t)}_{x_h(t)} + \underbrace{\left[\frac{F_0}{K\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]}\right]}_{X_p(t)}$$

$$\left.\begin{aligned} x(0) &= X_0 \\ \dot{x}(0) &= \dot{X}_0 \end{aligned}\right\} \quad C_1 = X_0 - \frac{F_0}{K\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]}$$

$$C_2 = \frac{\dot{X}_0}{\omega_n}$$

$$F = K\delta$$

$$\delta = \frac{F}{K} \rightsquigarrow \delta_{st.} = \frac{F_0}{K} \Rightarrow \text{static Amplitude.}$$

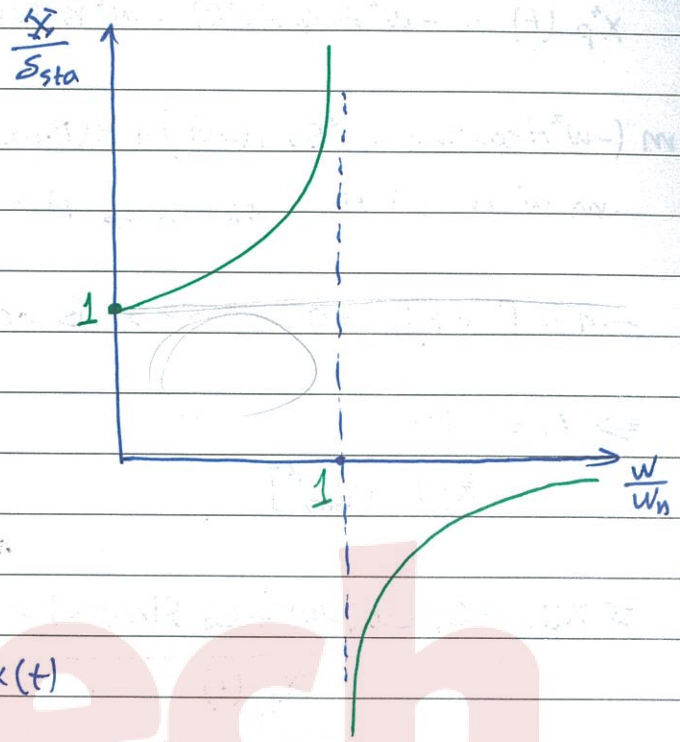
$$X = \frac{\delta_{st}}{\underbrace{1 - \left(\frac{\omega}{\omega_n}\right)^2}_{\text{dynamic Amplitude}}}$$

$$x(t) = (X_0 - X)\cos(\omega_n t) + \frac{\dot{X}_0}{\omega}\sin(\omega_n t) + X\cos(\omega t)$$

Steady state responce of the system :-

$x(t) = \underbrace{X}_{}\cos(wt)$

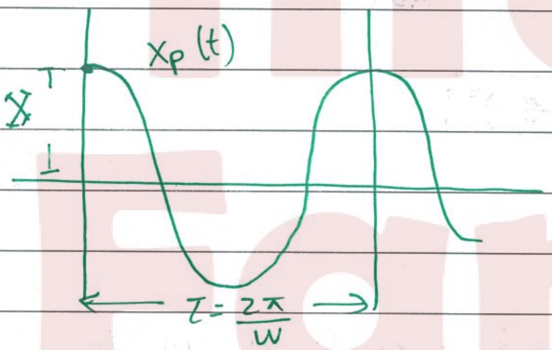
Dynamic amplitude for steady state solution



⇒ We have 3 Case's :-

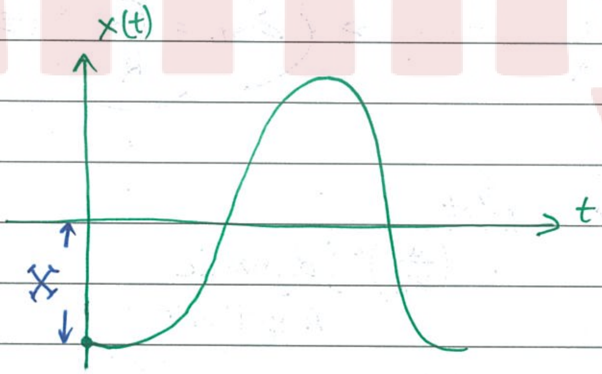
* Case ① :-

$\frac{w}{w_n} < 1$ ⇝ The same $x(t)$



Case ② :-

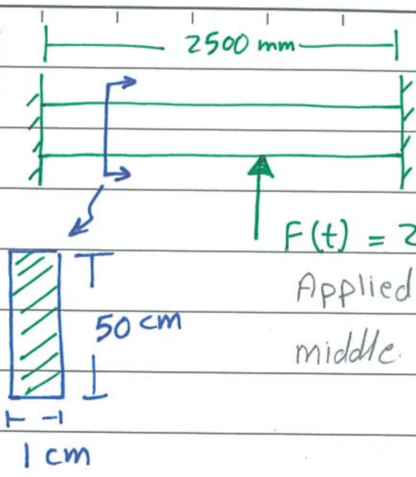
$\frac{w}{w_n} > 1$



Case ③ :-

$\frac{w}{w_n} = 1$ ⇝ Resonance.

$*$ Example :- {Find stedy response}



$F(t) = 220\cos(62.832t)$ Applied @ the middle.

$E = 200\ GPa$

$$k_{eq} = \frac{192\,EI}{L^3} = 256\ MN/m$$

$$m\ddot{x} + kx = F(t)$$

$$x_p = X\cos wt \qquad w_n = \sqrt{\frac{k_{eq}}{m}}$$

$$X = \frac{F_o}{k_{eq}\left(1 - \left(\frac{w}{w_n}\right)^2\right)}$$

from $F(t)$

$F_o = 220$

$w = 62.832$

⇒ Damped Vibration under harmonic Force :-

$$m\ddot{x} + c\dot{x} + kx = F(t) \qquad F(t) = F_o\cos(wt)$$

$$x_p(t) = X\cos(wt - \phi)$$

$$\dot{x}_p(t) = -Xw\sin(wt - \phi)$$

$$\ddot{x}_p(t) = -Xw^2\cos(wt - \phi)$$

$$-mXw^2\cos(wt - \phi) - cXw\sin(wt - \phi) + kX\cos(wt - \phi) = F_o\cos(wt)$$

$$[\cos wt\cos\phi + \sin wt\sin\phi][-mXw^2 + kX] + [\cos wt\sin\phi + \sin wt\cos\phi]$$

$$[-cXw] = F_o\cos(wt)$$

In general, the total response of a damped system subjected to harmonic force :-

$$x(t) = X_0 e^{-\xi w_n t} \cos(w_d t - \phi_0) + X \cos(wt - \phi)$$

where :- $0 < \xi < 1$

- $\xi = \dfrac{C}{C_c}$, $X_0 = \sqrt{(x_0 - X\cos\phi)^2 + \dfrac{1}{w_d^2}\left(\xi w_n x_0 + \dot{x}_0 - \xi w_n X\cos\phi - wX\sin\phi\right)^2}$

- $w_d = w_n\sqrt{1-\xi^2}$

$$\phi_0 = \tan^{-1}\left(\frac{\xi w_n x_0 + \dot{x}_0 - \xi w_n X\cos\phi - wX\sin\phi}{w_d(x_0 - X\cos\phi)}\right)$$

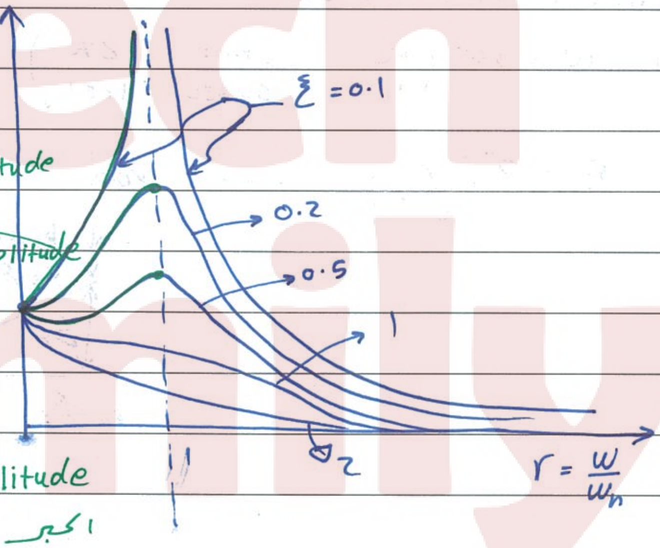
$$\phi = \tan^{-1}\left(\frac{2\xi r}{1-r^2}\right) \quad ; \quad r = \frac{w}{w_n}$$

$$X = \frac{F_0}{k\sqrt{(1-r^2)^2 + (2\xi r)^2}}$$

$$\delta_{st} = \frac{F_0}{k}$$

$$\frac{X}{\delta_{st}} = M = \frac{1}{\sqrt{(1-r^2)^2 + (2\xi r)^2}}$$

$M = \dfrac{X}{\delta_{st}}$



Dyn. Amplitude
اكبر من
static Amplitude
if $r = \dfrac{w}{w_n} < 1$

But if $r > 1$
static Amplitude
اكبر من
Dynamic Amplitude

① if $\xi = 0 \rightarrow M = \infty$

② اي زيادة في ال Damping
بتقلل ال (M)

* ولزيادة ال Damping نحتاج
لتكلفة اعلى.

99% من انظمة الفايبريشن تعمل
على $r > 1$.

③ if $r = 0$, $M = 1$

④ Reduction @ resonance is largest.

* عندما يكون هناك اهتزاز عالي وضجيج نقوم ~~بتغيير~~ بتغيير
w للجهاز او تغيير w_n وذلك بزيادة ال mass

من الكتاب

Example :-

$m = 10\ kg$

$C = 20\ N.s/m$

$k = 4000\ N/m$

$X_0 = 0.01\ m$

$\dot{X}_0 = 0$

a) $X_0 = 0.010012$

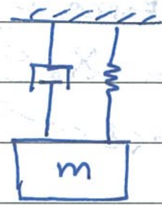
$\phi_0 = -2.865984$

b) $X = 0.0498$

$\phi = 3.814^\circ$

$X_0 = 0.0398$

$\phi_0 = 5.5867^\circ$



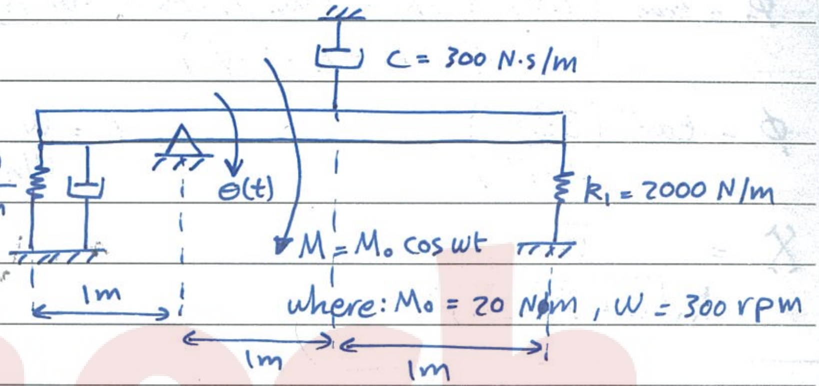
find total response if ⓐ $F = 0$ & ⓑ $F = 100\cos(10t)$

Example:-

I.C's :-

$\phi_0 = 2^\circ$

$\dot{\phi}_0 = 0$



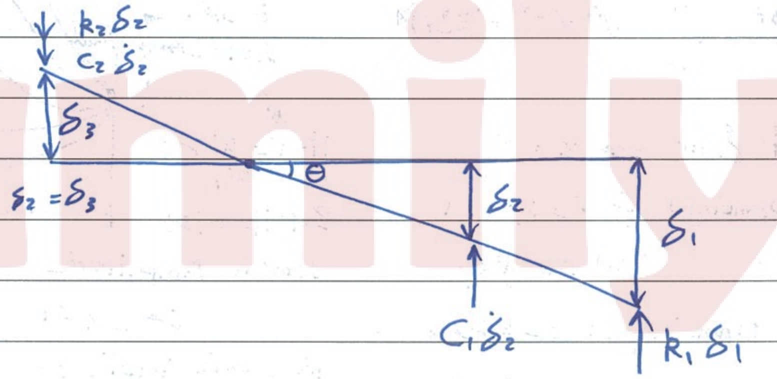
where: $M_0 = 20\ N.m$, $W = 300\ rpm$

find total response?

Sol: $w = 300\left(\frac{rev}{min} \cdot \frac{1\ min}{60\ sec} \cdot \frac{2\pi}{1\ rev}\right) = \dots\ rad/s$

* اول خطوة هي تحويل الوحدات ان وجد.

$\phi_0 = 2^\circ\left(\frac{\pi}{180^\circ}\right) = \dots\ rad$



* Responce of a damped system :-

under $F = F_0 e^{-iwt}$

recall Euler formulla

$e^{-i\theta} = \cos\theta + i\sin\theta$, $i = \sqrt{-1}$

E.O.M:- $m\ddot{X} + C\dot{X} + kX = F_0 e^{-iwt}$

we assume that the steady state solution is in the form:

$$x_p(t) = X e^{-i\omega t} \quad ; \quad X = \frac{F_0}{\sqrt{(k - m\omega^2)^2 + (c\omega)^2}}$$

~~where~~

$$x_p(t) = X_0 e^{i(\omega t - \phi)} \begin{cases} \to x_p(t) \text{ real} \\ \to x_p(t) \text{ imm} \end{cases}$$

where :-

$$X_0 = \frac{F_0}{k\sqrt{(1-r^2)^2 + (2\xi r)^2}} \quad , \quad \phi = \tan^{-1}\left(\frac{2\xi r}{1-r^2}\right)$$

$$m\ddot{x} + c\dot{x} + kx = F_0\left(\cos(\omega t) + i\sin(\omega t)\right)$$

→ case ①:-

if $F = F_0 \cos(\omega t)$

$x_p(t) \Rightarrow$ real part $= X_0 \cos(\omega t - \phi)$

→ case ②:-

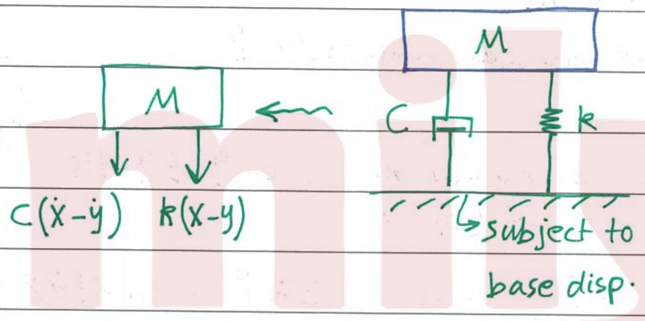
if $F = F_0 \sin(\omega t)$

$x_p(t) \Rightarrow$ (imma. part) $= X_0 \sin(\omega t - \phi)$

* Responce of a dammped system under Base-displacements :-

disp $y(t) = Y \sin(\omega t)$

let $z = x - y$



E.O.M :-

$$m\ddot{x} + c(\dot{x} - \dot{y}) + k(x - y) = 0 \quad \text{----} \quad (*)$$

$$m(\ddot{z} + \ddot{y}) + c\dot{z} + kz = 0$$

$$\to m\ddot{x} + c\dot{x} + kx = c\dot{y} + ky$$

$$\to m\ddot{x} + c\dot{x} + kx = cY\omega\cos(\omega t) + kY\sin(\omega t)$$

$$m\ddot{x} + c\dot{x} + kx = \underbrace{A\sin\phi_0}_{cY\omega}\cos(\omega t) + \underbrace{A\cos\phi_0}_{Yk}\sin(\omega t)$$

$$= A\sin(\omega t - \phi_0)$$

where :

$$A = \sqrt{(cYw)^2 + (kY)^2}$$

$$\phi_0 = \tan^{-1}\left(\frac{-k\cancel{Y}}{c\cancel{Y}w}\right)$$

$$X_p(t) = X \sin(wt - \phi_0 - \phi)$$

$$X = \sqrt{\frac{(cYw)^2 + (kY)^2}{(k - mw^2)^2 + (cw)^2}}$$

$$\phi_0 = \tan^{-1}\left(\frac{k}{cw}\right) \quad ; \quad \phi = \tan^{-1}\left(\frac{cw}{k - mw^2}\right)$$

Example :-

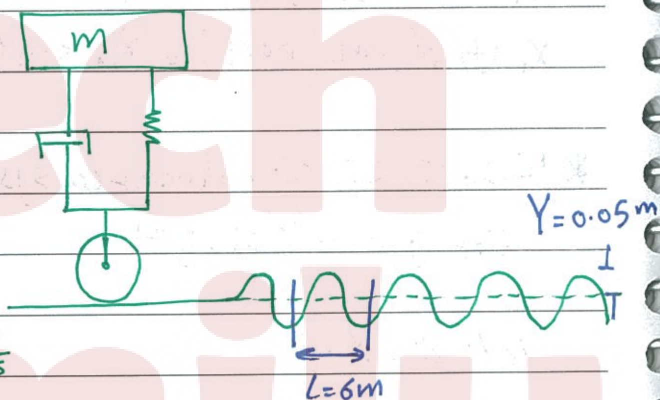
$\xi = 0.5$

$K = 20{,}000\ N/m$

$m = 1200\ kg$

$V = 20\ km/h$

Find the maximam amplitude ?



$$f = \frac{V}{L} = \frac{20 \times 10^3\ m}{6\ (60)^2\ s} = 0.9259\ Hz$$ $\frac{1}{s}$

$$\tau = \frac{1}{f} = \frac{2\pi}{w} \rightsquigarrow w = 2\pi f = 5.8178\ s$$

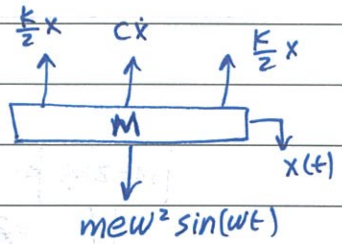
$$C_c = 2mw = \sqrt{4mk} \rightsquigarrow C_c = ✓$$

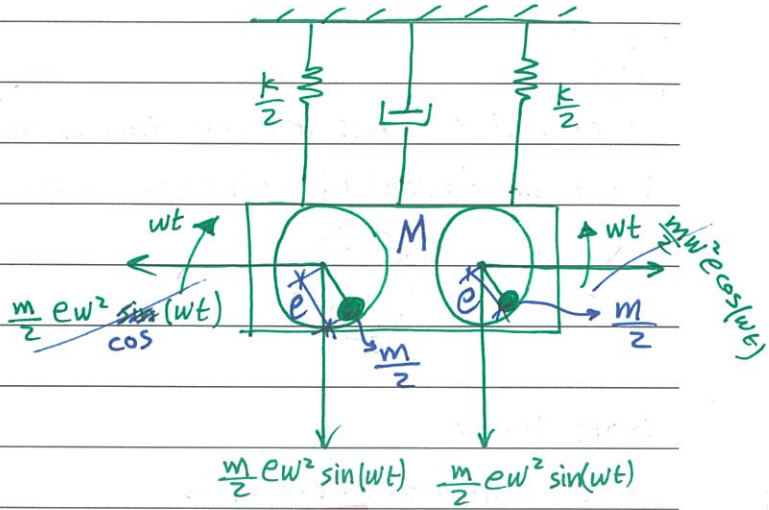
$$\xi = \frac{C}{C_c} \rightsquigarrow C = ✓$$

$$X = \sqrt{\frac{(cYw)^2 + (kY)^2}{(k - mw^2)^2 + (cw)^2}} =$$

Response of damped system under Rotating unbalance :-

Sol.





E.o.M :-

$$M\ddot{x} + c\dot{x} + kx = \underbrace{mew^2}_{F_0} \sin(wt)$$

$$x_p(t) = X \sin(wt - \phi)$$

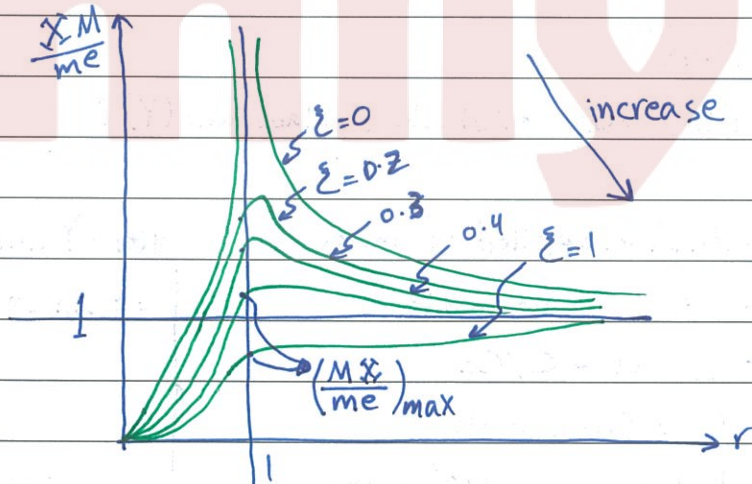
$$X = \frac{F_0}{k\sqrt{(1-r^2)^2 + (2\xi r)^2}} = \frac{mew^2}{k\sqrt{(1-r^2)^2+(2\xi r)^2}} \quad , \; w_n = \sqrt{\frac{k}{M}}$$

$$= \frac{mew^2}{M w_n^2 \sqrt{(1-r^2)^2+(2\xi r)^2}} \quad , \; r = \frac{w}{w_n}$$

$$= \frac{mer^2}{M\sqrt{(1-r^2)^2+(2\xi r)^2}}$$

$$\phi = \tan^{-1}\left(\frac{2\xi r}{1-r^2}\right)$$

$$\frac{XM}{me} = \frac{r^2}{\sqrt{(1-r^2)^2+(2\xi r)^2}}$$



max. point → تكون على يسار

ال resonanse

$$\frac{d}{dr}\left(\frac{XM}{me}\right) = 0$$

$$\left(\frac{MX}{me}\right)_{max} = \frac{1}{2\xi\sqrt{1-\xi^2}}$$

$$F_T = M\ddot{x}_p(t)$$

Force Transmitted To the Base

$$= mew^2 \sqrt{\frac{1+(2\xi r)^2}{(1-r^2)^2+(2\xi r)^2}}$$

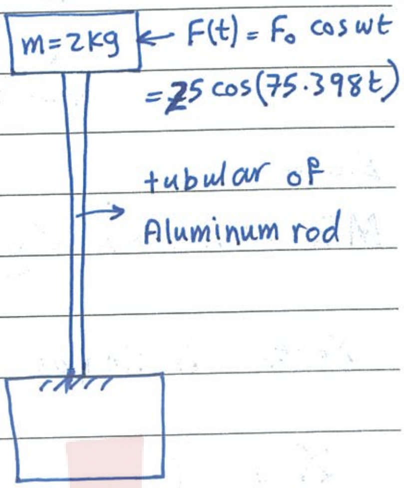
Figure 3-16

For $r<\sqrt{2}$ & ξ small $\rightarrow F_t$ small . اذا قلت X زادت F_t والعكس صحيح

For $r<\sqrt{2}$ & ξ hight $\rightarrow F_t$ hight

$\boxed{3.37}$

Determine cross section dimension if max. Amplitude vibration = 0.005 m



Note that :-

* force applied to the mass
* force is harmonic
* $\xi = 0$
* $E \rightarrow$ from table
* $k_{eq} = \dfrac{3EI}{L^3}$, $I = \dfrac{\pi}{64}(d_o^4 - d_i^4)$, assum $d_i = 0.01 \rightarrow d_o$ ✓

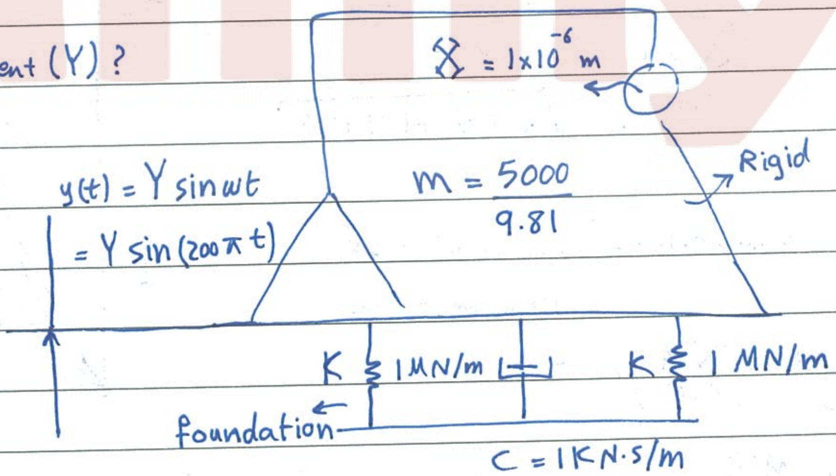
اذا طلعت ال thikness قليلة جداً نرفع di

عادةً بتكون thikness = $\square \times 10^{-2}$

$$X = \frac{F_o}{k(1-r^2)} = 0.005$$

$\boxed{3-45}$

Find max. axibtable displacement (Y) ?



* displac. applied to mass
* " is harmonic
* $\xi \neq 0 = C/C_c$.
* $E \rightarrow$ from table
* $K_{eq} = 2K = 2\times10^6$ N/m

$$\xi = \frac{C}{C_c} = \frac{1\times10^3}{2mW_n} =$$

$$W_n = \sqrt{\frac{2\times10^6}{\frac{5000}{9.81}}} = 62.6418$$

$r = \frac{w}{w_n} = \frac{200\pi}{\ } = 10$

$\bar{X} = \sqrt{\frac{(cYw)^2 + (kY)^2}{(k - mw^2)^2 + (cw)^2}}$ → كتبناها بهذا الشكل لأن (disp.) Force مُطبقة على (base) القاعدة.

$\therefore Y = 3.13 \times 10^{-8}\ m$

الفكرة

3.58, 59 :-

$m = 10\ kg$

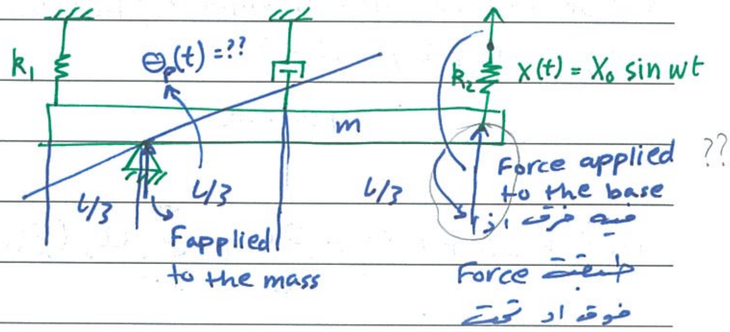
$X_0 = 1\ cm$

$w = 10\ rad/s$

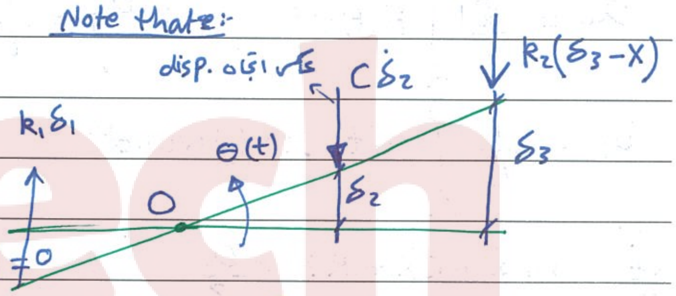
$L = 1\ m$

$k_1 = 1000\ N/m = k_2$

$C = 300\ N.s/m$



Note that:-



$\Sigma M_o = J_o \ddot{\theta}$

$J\ddot{\theta} + k_1 \frac{L}{3}\delta_1 + C\frac{L}{3}\dot{\delta}_2 + \frac{2}{3}k_2(\delta_3 - x) = 0$

$\theta = \frac{\delta_3}{2L/3} = \frac{\delta_2}{L/3} = \frac{\delta_1}{L/3}$

$J\ddot{\theta} + \left(k_1\left(\frac{L}{3}\right)^2 + k_2\left(\frac{2L}{3}\right)^2\right)\theta + C\left(\frac{L}{3}\right)^2\dot{\theta} = \frac{2L}{3}k_2 X_0 \sin wt$

← F_0 لأنه Force اعتبره applied to mass

$\bar{X} = \frac{\frac{2}{3}L k_2 X_0}{k\sqrt{(1-r^2)^2 + (2\xi r)^2}}$

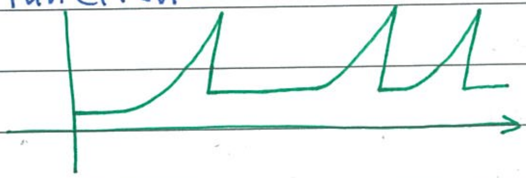
$\theta_p = \bar{X} \sin(wt)$

Chapter 4

Vibration Under General Forcing conditions

* For regular (not harmonic) periodic function:-

$m\ddot{x} + c\dot{x} + kx = \underbrace{F(t)}_{\text{periodic}}$



$$m\ddot{x} + c\dot{x} + kx = \frac{a_0}{2} + \sum_{J=1}^{\infty} a_J\,(J\omega t) + \sum b_J \sin(J\omega t)$$

$$a_0 = \frac{1}{L}\int_{-L}^{L} F(t)\,dt \quad ; \quad a_J = \frac{1}{L}\int_{-L}^{L} F(t)\cos(J\omega t)\,dt \qquad \text{since } L = \frac{\tau}{2}$$

$$b_J = \frac{1}{L}\int_{-L}^{L} F(t)\sin(J\omega t)\,dt$$

I $\quad m\ddot{x} + c\dot{x} + kx = \frac{a_0}{2}$

II $\quad m\ddot{x} + c\dot{x} + kx = a_J \cos(J\omega t)\ , \ j = 1, 2, \ldots$

III $\quad m\ddot{x} + c\dot{x} + kx = b_J \sin(J\omega t)\ , \ j = 1, 2, \ldots$

for I :-

$$X_p(t) = \frac{a_0}{2k}$$

for II & III

$F_0 = a_j \qquad\qquad \omega = j\omega$

$$X_p(t) = \frac{a_J}{k\sqrt{(1-(Jr)^2)^2 + (2jr\xi)^2}} \cos(J\omega t - \phi)$$

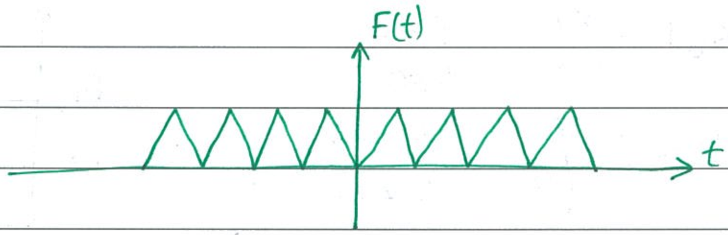
$$\phi = \tan^{-1}\left(\frac{2Jr\xi}{1-(Jr)^2}\right)$$

$$X_p(t) = \frac{b_J}{k\sqrt{\quad = \quad}} \sin(J\omega t - \phi)$$

$$X_p(t) = \frac{a_0}{2k} + \sum_{J=1}^{\infty} \frac{a_J}{k\sqrt{(1-(Jr)^2)^2 + (2\xi jr)^2}} \cos(J\omega t - \phi) + \sum_{J=1}^{\infty} \frac{b_J}{k\sqrt{\quad = \quad}} \sin(J\omega t - \phi)$$

if $F(t)$ = is even function :-

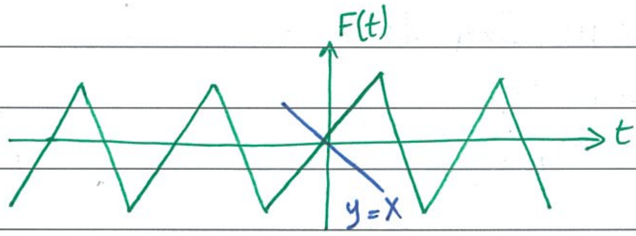
$b_j = 0$



if $F(t)$ is odd function

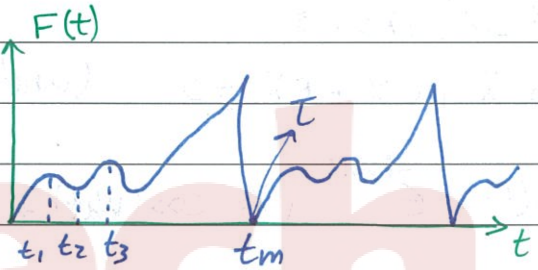
$a_j = 0$

$a_0 = 0$



* For irregular periodic functions:-

t	t_0	t_1	t_2	----	t_m
F	F_0	F_1	F_2	----	F_m



$$a_0 = \frac{2}{M} \sum_{i=1}^{M} F_i$$

$$a_j = \frac{2}{M} \sum_{i=1}^{M} F_i \cos(j \omega t_i)$$

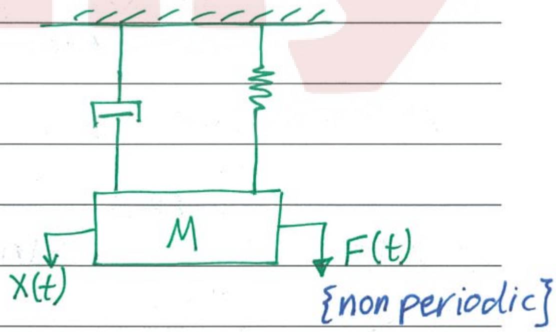
$$W = \frac{2\pi}{\tau}$$

$$b_j = \frac{2}{M} \sum_{i=1}^{M} F_i \sin(j M t_i)$$

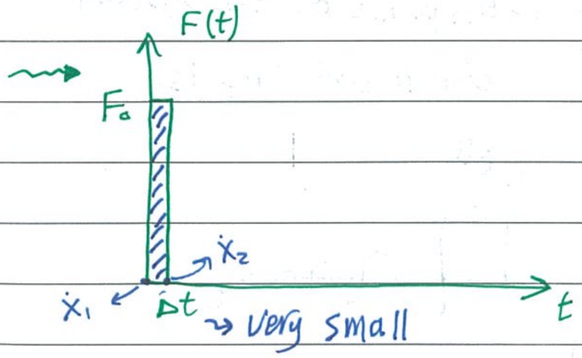
* Responce under non-periodic loading condition :-

(I) convolution Integral :-

* Impulse force :-



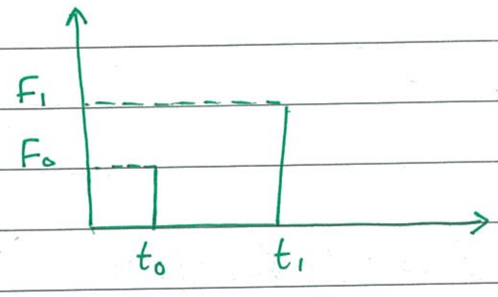
$$\text{impulse} = F_0 \Delta t = m\dot{x}_2 - m\dot{x}_1$$



⇒ Dirac delta function

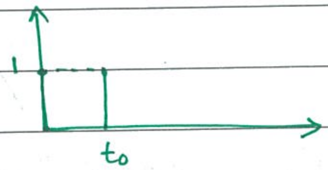
$$\delta(t) = \begin{cases} 1 & t = 0 \\ 0 & t \neq 0 \end{cases}$$

$$\delta(t-t_0) = \begin{cases} 1, & t = t_0 \\ 0, & t \neq t_0 \end{cases}$$



⇝ unit Impuls :-

$$f = \underbrace{1}_{[F_0 = 1]}\,\delta(t)$$



* Recall that :-

$$m\ddot{X} + C\dot{X} + kX = 0 \qquad \text{(under damping)}$$

$$X(t) = e^{-\xi w_n t}\left(X_0 \cos(w_d t) + \frac{\dot{X}_0 + \xi w_n X_0}{w_d} \sin(w_d t)\right)$$

$$w_d = w_n\sqrt{1-\xi^2}$$

$$\xi = C/C_c = \frac{C}{2m w_n}$$

$$w_n = \sqrt{\frac{k}{m}}$$

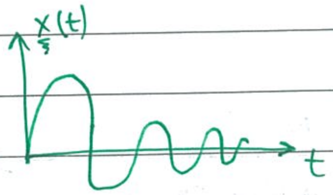
$$\text{Impuls} = 1 = f$$
$$= m\left(\dot{X}_2\Big|_{t=0^+}\right) - \cancel{m\left(\dot{X}_1\Big|_{t=0^-}\right)}$$

$\dot{X}_2$ as initial cond. ⇝ $\therefore\ \dot{X}_2 = \boxed{\dot{X}_0 = \frac{1}{m}}$

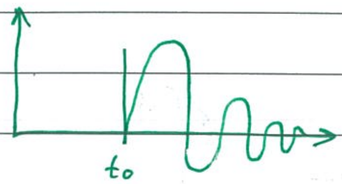
$$X(t) = \frac{e^{-\xi w_n t}}{m w_d}\sin(w_d t) = g(t)$$

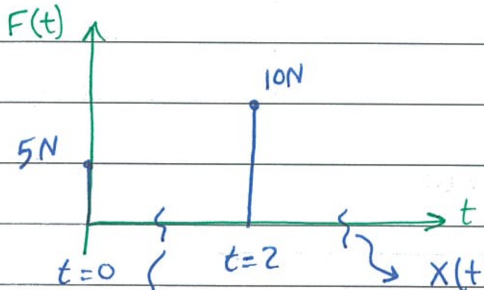
Responce of the system due to unit Impuls.

$$X(t) = \underbrace{F_0\, g(t)}_{\text{Impulse} = F_0\,\delta(t)}$$



$$X(t) = F_0\, g(t-t_0)$$
↳ $\text{Impuls} = F_0\,\delta(t-t_0)$

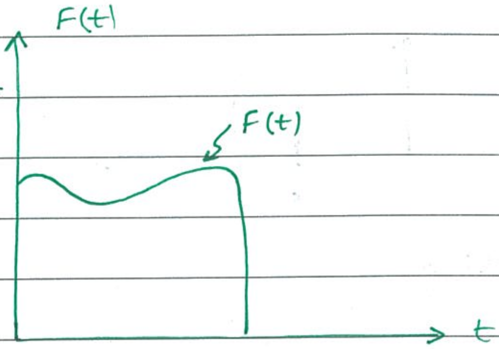




$x(t) = F_0\, g(t) + F_0\, g(t - t_0)$

$x(t) = F_0\, g(t)$

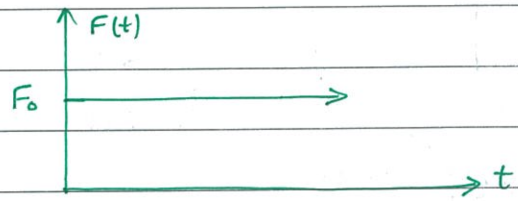
$$x(t) = \frac{1}{m\omega_d}\int_0^t F(\tau)\, e^{-\xi\omega_n(t-\tau)} \sin(\omega_d(t-\tau))\, d\tau \quad \text{--*}$$



Case 1 :-

Eq (*) :-

$$x(t) = \frac{F_o}{\omega_d m}\int_0^t e^{-\xi\omega_n(t-\tau)} \sin(\omega_d(t-\tau))\, d\tau$$

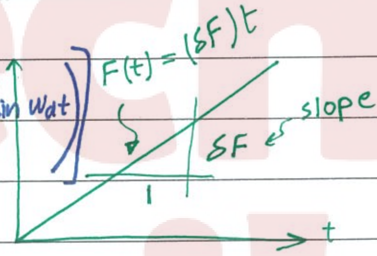


$$x(t) = \frac{F_o}{k}\left[1 - \frac{1}{\sqrt{1-\xi^2}}\, e^{-\xi\omega_n t} \cos(\omega_d t - \phi)\right] \text{ ---- ①}$$

$$\phi = \tan^{-1}\left(\frac{\xi}{\sqrt{1-\xi^2}}\right)$$

Case 2 :-

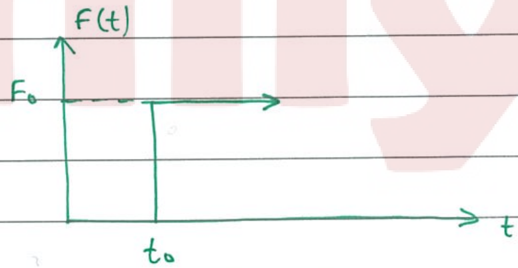
$$)\text{---}\ x(t) = \frac{\delta F}{k}\left[t - \frac{2\xi}{\omega_n} + e^{-\xi\omega_n t}\left(\frac{2\xi}{\omega_n}\cos\omega_d t - \left(\frac{\omega_d^2 - \xi^2\omega_n^2}{\omega_n^2\, \omega_d}\right)\sin\omega_d t\right)\right]$$



⇒ if the system is Undamped, $[\xi = 0,\ \omega_d = \omega_n]$ for all cases.

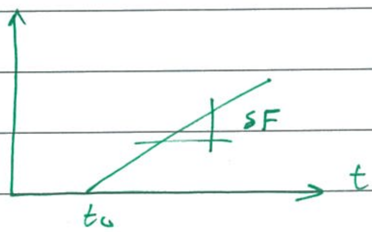
Case 3 :-

t is replaced by $(t - t_o)$ in Eq ①

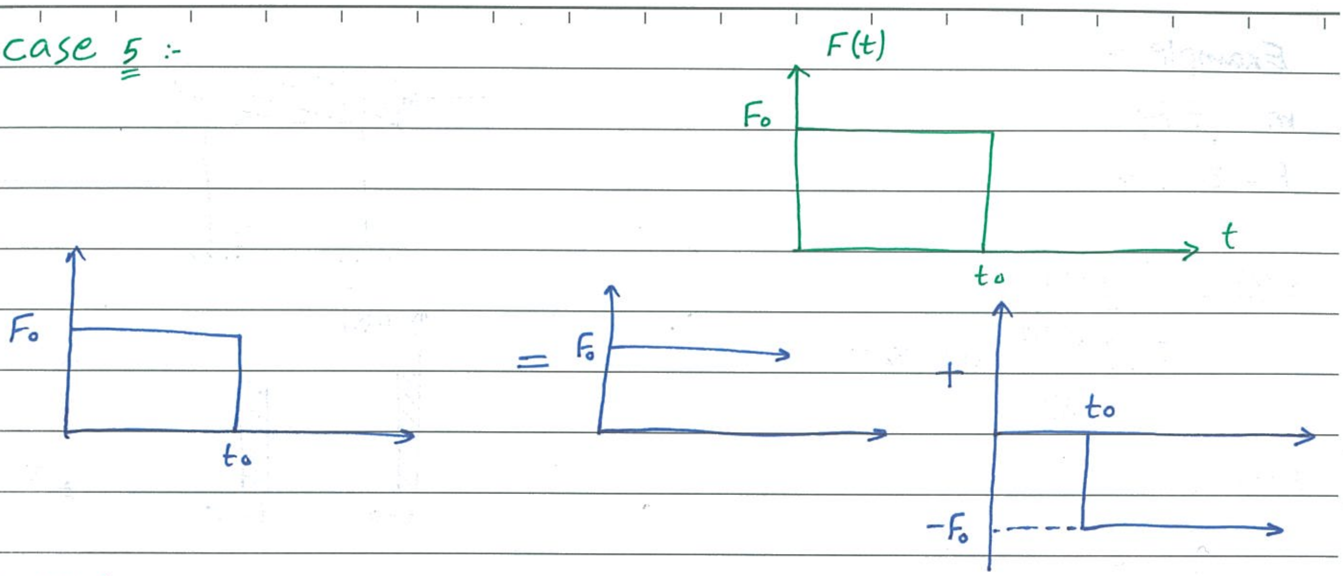


Case 4

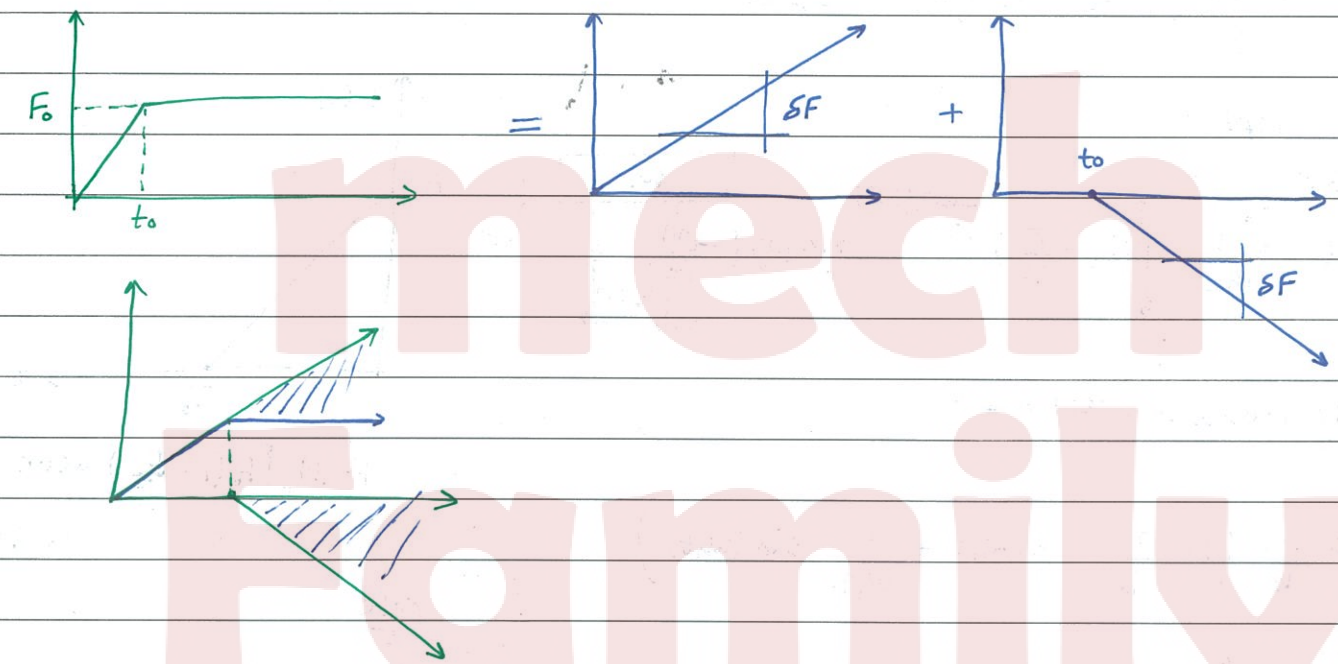
t is replaced by $(t - t_o)$ in Eq ②



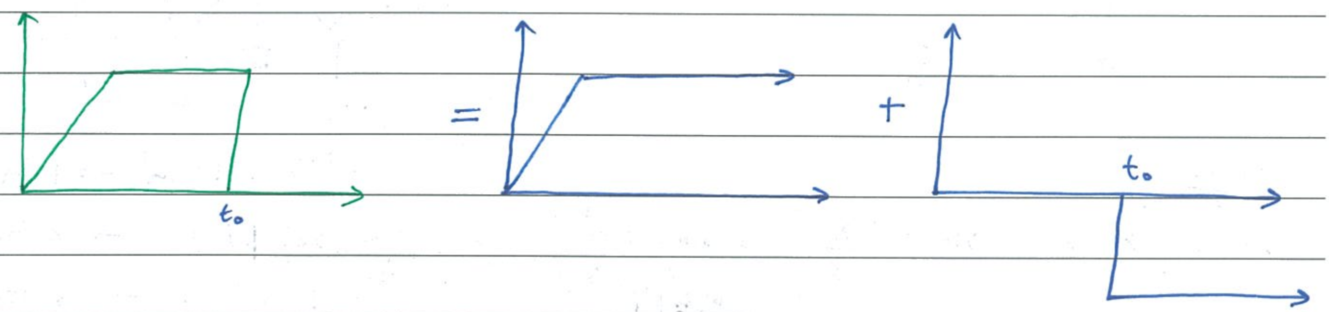
case 5 :-



case 6 :-



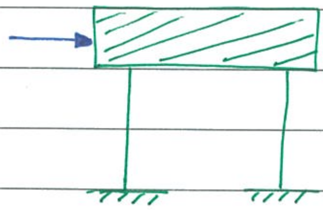
case 7 :-



Example :-

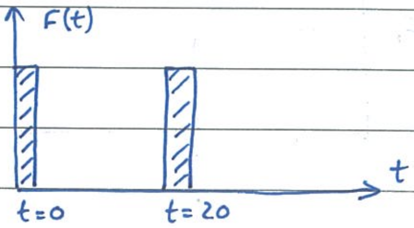
$m = 5\ kg$

$F = 20\ N\text{-}s$



$x(t) = F\, g(t)$

$= F \dfrac{e^{-\xi w_n t}}{m w_d} \sin(w_d t)$



Extra

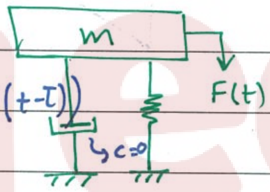
$$x(t) = \begin{cases} F g(t) & 0 < t < 20 \\ F g(t-20) + F g(t) & t > 20 \end{cases}$$

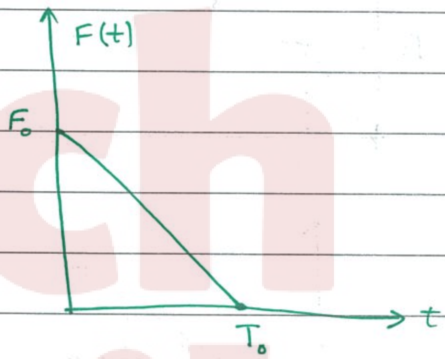
Example:

$0 < t < T_0$

$x(t) = \dfrac{1}{m w_d} \displaystyle\int_0^t F_0 \left(1 - \dfrac{\tau}{T_0}\right) e^{-w_n \xi (t-\tau)} \sin(w_d (t-\tau))$

⋮





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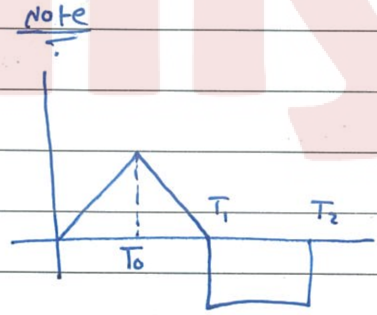
$$F(t) = \begin{cases} F_0 \left(1 - \dfrac{t}{T_0}\right) & 0 < t < T_0 \\ 0 & T_0 < t \end{cases}$$

$= \dfrac{F_0}{k} \left(1 - \dfrac{t}{T_0} - \cos w_n t + \dfrac{1}{w_n T_0} \sin(w_n t)\right)$

Note



$T_0 < t$

$x(t) = \displaystyle\int_0^{T_0} + \int_{T_0}^{t}$

Note

$\sin t \displaystyle\int_0^{t_0} x^2 \cdot dx = \sin t \left(\dfrac{x^3}{3}\right)_0^{t_0} = \underbrace{\sin t}_{\text{لا نعوض فيها}} \left(\dfrac{t_0^3}{3}\right)$

First interval → 1. integral

second " → 2 int

third " → 3 int

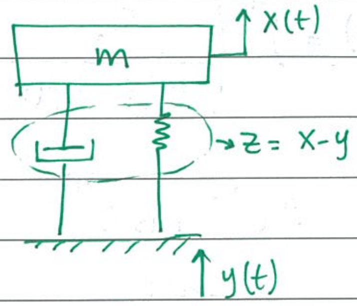
chapter 3

$e^{it} \rightarrow$ periodic

$e^{t} \rightarrow$ non periodic

type of Force :-
1- Harmonic
2- periodic, not Harmonic
3- non periodic

$$m\ddot{x} + c(\dot{x}-\dot{y}) + k(x-y) = 0$$



$$m\ddot{z} + c\dot{z} + kz = -m\ddot{y}$$

if non-periodic

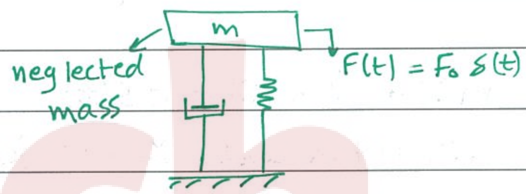
$$\Rightarrow z(t) = \frac{-1}{w_d}\int_0^t \ddot{y}\, e^{-\xi w_n(t-\tau)} \sin\left(w_d(t-\tau)\right) . d\tau$$

اذا كان فيه disp. على base
نشتق y مرتين ثم نجد
z(t) ثم نجمع y(t), z(t) لإيجاد x(t)

Laplace Transform :-

$$m\ddot{x} + c\dot{x} + kx = F(t)$$

m



4-47-57x
4-58-80

$$x(t) = \mathcal{L}^{-1}\left(\frac{F_o}{m(s-s_1)(s-s_2)}\right) \rightsquigarrow \text{Partial fraction } \frac{A}{s-s_1} + \frac{B}{s-s_2}$$

Find A and B

$$x(t) = A e^{s_1 t} + B e^{s_2 t}$$

$$= \frac{F_o}{m} e^{-\xi w_n t} \sin(w_d t)$$ ← similar to the results of conv. integral.

conv. integral :-

$m\ddot{x} + c\dot{x} + kx = 0$

But with initial cond. :- $\dot{X}_o = \frac{F_o}{m}$, $X_o = 0$

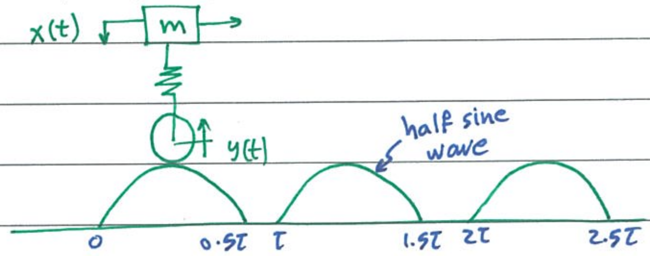
$m\ddot{X} + C\dot{X} + kX = F_o \delta(t)$

with initial cond. $X_o = 0$, $\dot{X}_o = 0$

For :- $m\ddot{x} + c\dot{x} + kX = F_o$

$\mathcal{L}\,[m\ddot{x} + c\dot{x} + kX] = \mathcal{L}\,[F_o]$

$\boxed{4-12}$



$m = 750$ kg

$k = 5\times10^6$ N/m

E.O.M :-

$$w = \frac{2\pi}{\tau}$$

$$m\ddot{x} + kx = k\,y(t)$$

$$y(t) = \begin{cases} \sin(wt) & , 0 < t < \tau/2 \\ 0 & , \frac{\tau}{2} < t < \tau \end{cases}$$

sin (wt)

$t \neq 0 \rightarrow$

Fourier Series

$$y(t) = \frac{a_0}{2} + \sum_{J=1}^{\infty} a_J \cos(wJt) + \sum_{J=1}^{\infty} b_J \sin(wJt)$$

But y(t) is even $\rightsquigarrow \therefore b_j = 0$

$$a_0 = \frac{1}{\tau}\int_0^{\tau} y(t)\,dt$$

$$= \frac{1}{\tau}\int_0^{\tau/2} + \cancel{\int_{\tau/2}^{\tau}}$$

$$= \frac{1}{w\tau} - \cos(wt)\Big|_0^{\tau/2} = \frac{-1}{w\tau}\cos\left(\frac{\tau w}{2}\right) + \frac{1}{w\tau} = \frac{2}{w\tau} = \frac{1}{\pi}$$

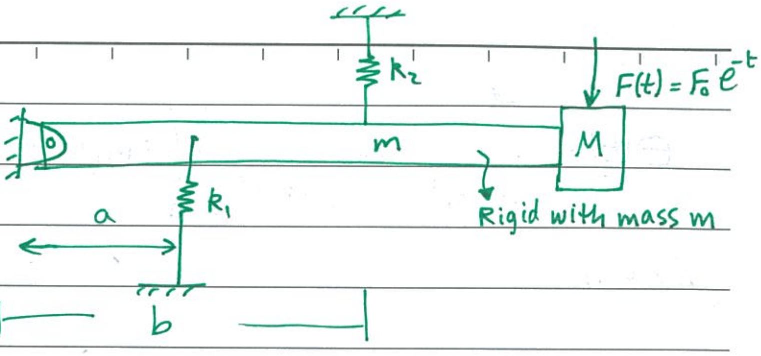
$\frac{2\pi}{\tau}$

$\therefore \cos(\pi) = -1$

$$m\ddot{x} + kx = k\,y(t) \rightarrow \frac{k a_0}{2} + \sum_{j=1}^{\infty} k a_j \cos(wt)$$

$$x(t) = \frac{a_0}{2} + \sum_{J=1}^{\infty} \frac{k a_j}{k(1-(rJ)^2)} \cos(wjt)$$

$\boxed{4.22}$



$J_{eq}\ddot{\theta} + K_{eq}\theta = F(t)$

$M_{eq}L^2 = L^2\left(M + \frac{m}{3}\right)$

$\Sigma M_o = J\ddot{\theta}$

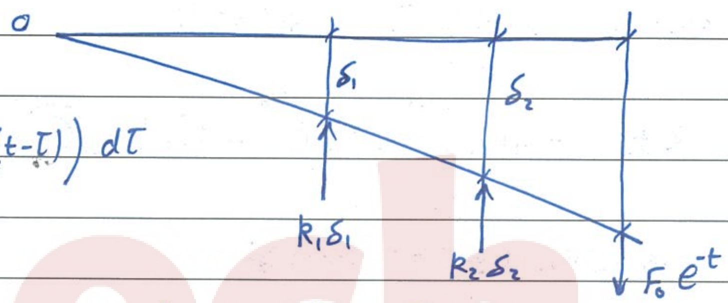
$k_1 = k_2 = 5000\ N/m$

$L = 1m\ ,\ a = 0.25m\ ,\ b = 0.5m$

$M = 50\ kg\ ,\ m = 10\ kg\ ,\ F_0 = 500\ N$

$J\ddot{\theta} + k_2 \underbrace{\delta_2}_{b\times\theta} b + k_1 \underbrace{\delta_1}_{a\cdot\theta} a = F_0 e^{-t} L$

$J\ddot{\theta} + (k_2 b^2 + k_1 a^2)\theta = F_0 L e^{-t}$



$\theta(t) = \frac{1}{m w_d}\int_0^t F(\tau)\, e^{-\xi w_n (t-\tau)} \sin\left(w_d(t-\tau)\right) d\tau$

But No Damping , $\xi = 0,\ w_d = w_n$

$\tan\theta = \frac{\delta_2}{b} = \frac{\delta_1}{a} \approx \theta$

$\theta(t) = \frac{1}{m w_n}\int_0^t F_0 L e^{-\tau} \sin\left(w_n(t-\tau)\right) \cdot d\tau$

$= \int \underbrace{e^{-\tau}}_{dW} \underbrace{\sin\left(w_n(t-\tau)\right)}_{u} \cdot d\tau \qquad du = -w_n \cos\left(w_n(t-\tau)\right) d\tau$

$dW = e^{-\tau} \cdot d\tau \rightarrow V = -e^{-\tau}$

$= -e^{-\tau} \sin\left(w_n(t-\tau)\right) - \int w_n \underbrace{e^{-\tau}}_{dW} \overbrace{\cos\left(w_n(t-\tau)\right)}^{u} \cdot d\tau \rightarrow du = +w_n \sin\left(w_n(t-\tau)\right)$

$V = -e^{-\tau}$

$I = -e^{-\tau} \sin\left(w_n(t-\tau)\right) + w_n\left[e^{-\tau}\cos\left(w_n(t-\tau)\right) + \underbrace{\int -e^{-\tau} \sin\left(w_n(t-\tau)\right) \cdot w_n \cdot d\tau}_{I}\right.$

$I\left(1 + w_n^2\right) = e^{-\tau}\left(\sin\left(w_n(t-\tau)\right) + w_n \cos\left(w_n(t-\tau)\right)\right)$

$I = \frac{e^{-\tau}}{1 + w_n^2}\left(\sin\left(w_n(t-\tau)\right) + w_n \cos\left(w_n(t-\tau)\right)\right)\Bigg|_0^t$

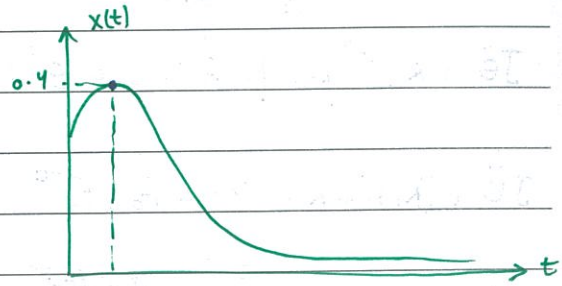
$$\Theta(t) = \frac{e^{-\frac{t}{w_n}}}{1+w_n^2} - \left(\frac{1}{1+w_n^2}\right)\left(\sin(w_n t) + w_n \cos(w_n t)\right) \quad \#$$

Example:

$M = 500\ kg$, $K = 70{,}000\ N/m$, $0.4\ m = X_{max}$, $\xi = 1$, $\dot{X}(0) = 0$

find: $C_c = ??$

initial velocity 0.1 from it's initial position ?



Because $\xi = 1$

case ② $x(t) = C_1 e^{-w_n t} + C_2 t e^{-w_n t}$

$$\dot{x}(t) = -C_1 w_n e^{-w_n t} + C_2 e^{-w_n t} - C_2 t w_n e^{-w_n t}$$

$C_1 = X_0$

$C_2 = \dot{X}_0 + W_n X_0$

~~$-C_1 W_n + C_2 = 0$ ---- ①~~

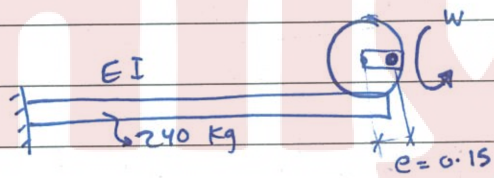
$\dot{x}(t) = 0$

$$-C_1 W_n \cancel{e^{-w_n t}} + C_2\left(\cancel{e^{-w_n t}} - t w_n \cancel{e^{-w_n t}}\right) = 0$$

$$t = \frac{-C_1 W_n + C_2}{C_2 W_n}$$

⋮

3-64:



$$M_{eq}\ddot{X} + KX + C\dot{X} = m e w^2 \sin(wt)$$

$$M_{eq} = 20 + 0.23(240) \qquad \frac{3EI}{L^3} \qquad \frac{C}{M_{eq}} = 2\xi W_n$$

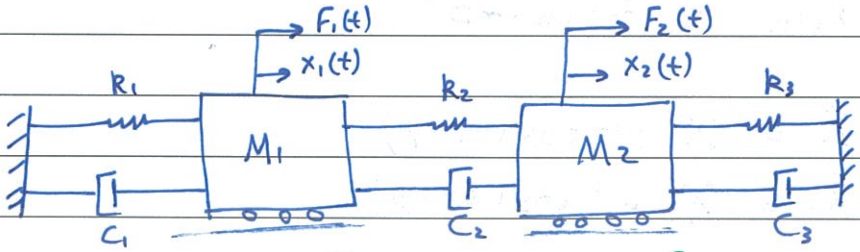
Chapter 5

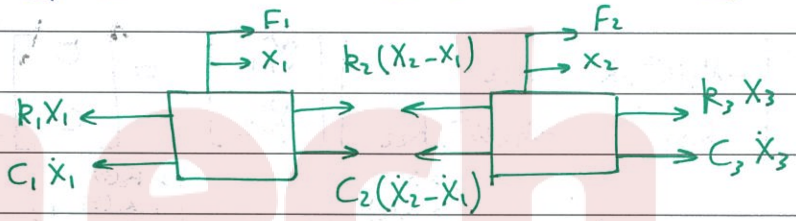
* Tow Degree of freedom system :-

2 Degree of freedom

\# of Diff equations ≡ # of natural frequancy ≡ # of ~~free~~ degree of freedom

㊀ Translational motion



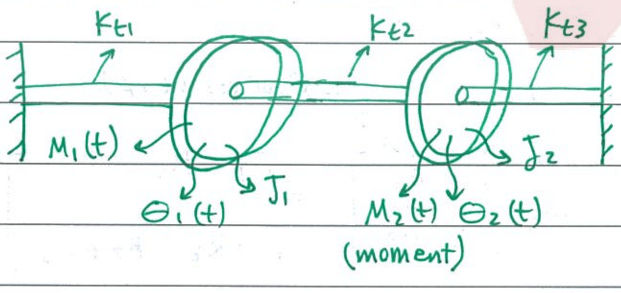


Equations of motion :-

$$M\vec{\ddot{X}} + C\vec{\dot{X}} + K\vec{X} = \vec{F}$$

$$\begin{bmatrix} M_1 & 0 \\ 0 & M_2 \end{bmatrix}\begin{bmatrix} \ddot{X}_1 \\ \ddot{X}_2 \end{bmatrix} + \begin{bmatrix} C_1+C_2 & -C_2 \\ -C_2 & C_2+C_3 \end{bmatrix}\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \end{bmatrix} + \begin{bmatrix} k_1+k_2 & -k_2 \\ -k_2 & k_2+k_3 \end{bmatrix}\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} F_1(t) \\ F_2(t) \end{bmatrix}$$

* Rotational motion :-



Equation of motion :-

$$\begin{bmatrix} J_1 & 0 \\ 0 & J_2 \end{bmatrix}\begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix} + \begin{bmatrix} k_{t1}+k_{t2} & -k_{t2} \\ -k_{t2} & k_2+k_3 \end{bmatrix}\begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} M_1(t) \\ M_2(t) \end{bmatrix}$$

if there is a damper we add the C part from the first equation

* Free Undamped 2D of freedom system :-

$$\begin{bmatrix} M_1 & 0 \\ 0 & M_2 \end{bmatrix} \begin{bmatrix} \ddot{X}_1 \\ \ddot{X}_2 \end{bmatrix} + \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

we assume :-

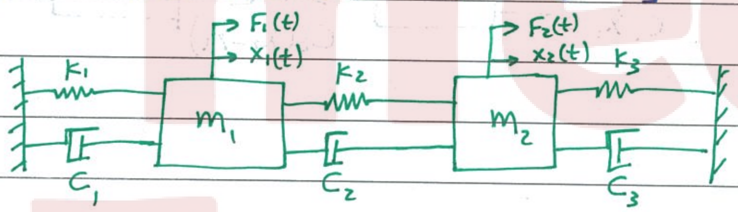
$x_1(t) = \mathcal{X}_1 \cos(wt + \phi)$

$x_2(t) = \mathcal{X}_2 \cos(wt + \phi)$

$\ddot{X}_1 = -\mathcal{X}_1 W^2 \cos(wt + \phi)$

$\ddot{X}_2 = -\mathcal{X}_2 W^2 \cos(wt - \phi)$

$$\begin{bmatrix} -M_1 W^2 + k_1 & k_{12} \\ k_{21} & -M_2 W^2 + k_{22} \end{bmatrix} \begin{bmatrix} \mathcal{X}_1 \\ \mathcal{X}_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$



$$* ** \rightarrow \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{bmatrix} \ddot{X}_1 \\ \ddot{X}_2 \end{bmatrix} + \begin{bmatrix} C_1 + C_2 & -C_2 \\ -C_2 & C_2 + C_3 \end{bmatrix} \begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \end{bmatrix} + \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 + k_3 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} F_1 \\ F_2 \end{bmatrix}$$

* Free Undamped two-degree of freedom system :-

$C_1 = C_2 = C_3 = 0$

$F_1 = F_2 = 0$

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{bmatrix} \overset{-\mathcal{X}_1 w^2 \cos(wt+\phi)}{\ddot{X}_1} \\ \ddot{X}_2 \end{bmatrix} + \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 + k_3 \end{bmatrix} \begin{bmatrix} \overset{\mathcal{X}_1 \cos(wt+\phi)}{X_1} \\ X_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\left.\begin{aligned} x_1(t) &= \mathcal{X}_1 \cos(wt + \phi) \\ x_2(t) &= \mathcal{X}_2 \cos(wt + \phi) \end{aligned}\right\} \leftarrow \text{assume a solution in this form.}$$

$$\begin{bmatrix} -w^2 m_1 + (k_1+k_2) & -k_2 \\ -k_2 & -w^2 m_2 + (k_2+k_3) \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \rightsquigarrow \text{homog. system.}$$

To have ~~a~~ non-trivial $\begin{pmatrix} X_1 \neq 0 \\ X_2 \neq 0 \end{pmatrix}$ we set :-

$$\begin{vmatrix} -m_1 w^2 + k_1 + k_2 & -k_2 \\ -k_2 & -m_2 w^2 + k_2 + k_3 \end{vmatrix} = 0$$

$$(-m_1 w^2 + k_1 + k_2)(-m_2 w^2 + k_2 + k_3) - k_2^2 = 0$$

$$\underbrace{m_1 m_2}_{a} \underbrace{w^4}_{z^2} - \Big(\underbrace{+m_2(k_1+k_2) + m_1(k_2+k_3)}_{b}\Big) \underbrace{w^2}_{z} + \Big(\underbrace{(k_1+k_2)(k_2+k_3) - k_2^2}_{c}\Big) = 0 \rightsquigarrow \text{char. equation}$$

take solved for the natural frequencies :-

let $w^2 = z$

$$z_{1,2} = \frac{-b \mp \sqrt{b^2 - 4ac}}{2a}$$

$\therefore w_{1,2} = \sqrt{z_{1,2}}$; $w_{1,2}$: Natural frequencies.

for $w_1 \rightarrow \begin{bmatrix} X_1^{(1)} \\ X_2^{(1)} \end{bmatrix} = \begin{bmatrix} X_1^{(1)} \\ r_1 X_2^{(1)} \end{bmatrix}$ $\qquad r_1 = \dfrac{-m_1 w_1^2 + (k_1+k_2)}{k_2} = \dfrac{k_2}{-m_2 w_1^2 + (k_2+k_3)}$

where:-

for $w_2 \rightarrow \begin{bmatrix} X_1^{(2)} \\ X_2^{(2)} \end{bmatrix} = \begin{bmatrix} X_1^{(2)} \\ r_2 X_2^{(2)} \end{bmatrix}$ $\qquad r_2 = \dfrac{-m_1 w_2^2 + (k_1+k_2)}{k_2} = \dfrac{k_2}{-m_2 w_2^2 + (k_2+k_3)}$

Thus, then solution(s) becomes :-

$X^{(1)}(t) = X_1^{(1)} \cos(w_1 t + \phi_1) + r_1 X_1^{(1)} \cos(w_1 t + \phi_1)$ @ w_1 first Mody.

$X^{(2)}(t) = X_1^{(2)} \cos(w_2 t + \phi_2) + r_2 X_1^{(2)} \cos(w_2 t + \phi_2)$ @ w_2 2^{nd} Mody.

$$X_1(t) = X_1^{(1)} \cos(\omega_1 t + \phi_1) + X_1^{(2)} \cos(\omega_2 t + \phi_2)$$

$$X_2(t) = r_1 X_1^{(1)} \cos(\omega_1 t + \phi_1) + r_2 X_1^{(2)} \cos(\omega_2 t + \phi_2)$$