

Units

Standard units

القوالب بالقوانين الهندسية يتم دائماً باستخدام الوحدات الأساسية وليس الوحدات الفرعية.

عزم قصير I m^4 و Pa و القوة N و الحجم m^3 و المساحة m^2 و الطول m

Kilo $\equiv 10^3$
Mega $\equiv 10^6$
Gega $\equiv 10^9$

المعطى بالسؤال

القوالب بالقوانين الهندسية

الطول $L = 85 \text{ cm}$

$85 \times 10^{-2} \text{ m}$

$L = 43.7 \text{ mm}$

$43.7 \times 10^{-3} \text{ m}$

المساحة $A = 430 \text{ cm}^2$

$430 \times 10^{-4} \text{ m}^2$

$A = 145.7 \text{ mm}^2$

$145.7 \times 10^{-6} \text{ m}^2$

القوة $P = 45 \text{ kN}$

$45 \times 10^3 \text{ N}$

$F = 2.6 \text{ MN}$

$2.6 \times 10^6 \text{ N}$

الضغط $\sigma = 413 \text{ kPa}$

$413 \times 10^3 \text{ Pa}$

$\tau = 41 \text{ MPa}$

$41 \times 10^6 \text{ Pa}$

$E = 200 \text{ GPa}$

$200 \times 10^9 \text{ Pa}$

عزم قصير $I = 342 \text{ cm}^4$

$342 \times 10^{-8} \text{ m}^4$

$I = 4392 \text{ mm}^4$

$4392 \times 10^{-12} \text{ m}^4$

Dr. Hashem Alkhalaf
Unit, Stress, & Strength (1)

Chapter one

"Stress and Strain"

Pa
الوحدة بالضغط
(N/m²)

$$\text{Stress} = \frac{\text{Force}}{\text{Area}}$$

الضغط = القوة / المساحة

ليس له وحدة

$$\text{Strain} = \frac{\text{Elongation}}{\text{original length}}$$

الانفعال = الاستطالة / الطول الأصلي



الانفعال
Strain

Normal
Stress

Shear
Stress

Bearing
Stress

Normal
Strain

Shear
Strain

$$\sigma = \frac{N}{A}$$

$$\tau = \frac{V}{A}$$

$$\sigma_b = \frac{N}{td}$$

$$\epsilon = \frac{\Delta L}{L_0} = \frac{\delta}{L_0}$$

$\gamma \equiv$ مقدار الانفعال عند الزاوية القائمة



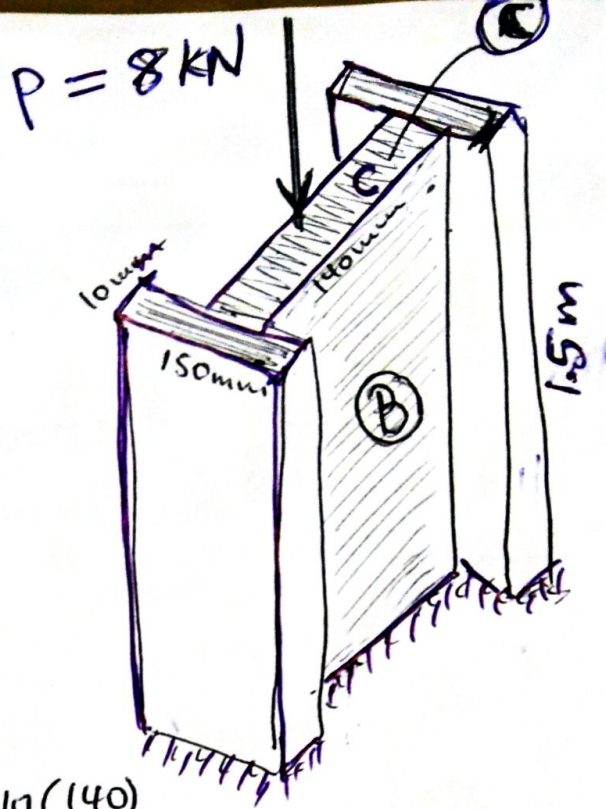
ملحوظة * غائب :

* Normal stress $\Rightarrow$ في الجبال والقضبان
Cables and Rods.

* Shear Stress $\Rightarrow$ في المسامير Bolts

* Bearing Stress $\Rightarrow$ في المسامير Bolts

Find the Normal and Shear stresses at (C) and (B) ?



جواب

الحل

$$A = (10)(150) + (10)(150) + 10(140)$$

$$A = 4400 \text{ mm}^2$$

$$\begin{aligned} * \sigma_c &= \frac{8 \times 10^3}{4400 \times 10^{-6}} = 1818181.8 \text{ Pa} \\ &= 1.82 \times 10^6 \text{ Pa} \\ &= 1.82 \text{ MPa} \end{aligned}$$

الجواب
ENG

$$* \tau_c = 0$$

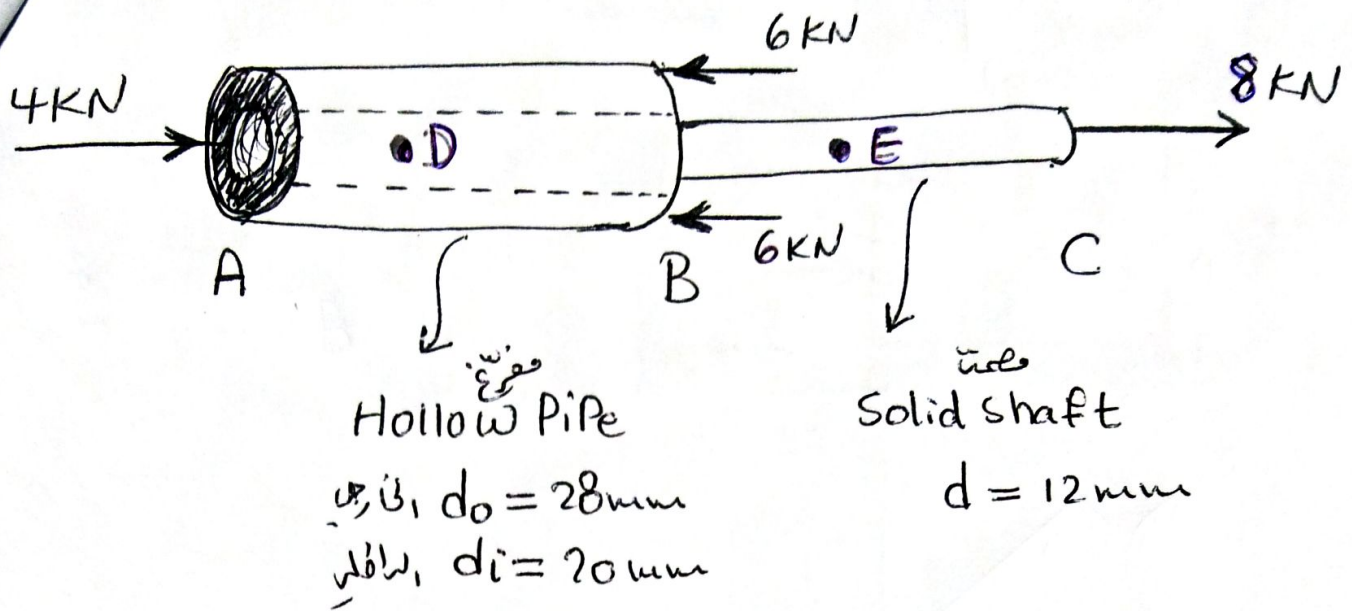
$$* \sigma_b = 0$$

الحل

$$A = (1.5)(140 \times 10^{-3})$$

$$* A = 0.21 \text{ m}^2$$

$$\begin{aligned} * \tau_b &= \frac{8 \times 10^3}{0.21} \\ &= 38.1 \text{ kPa} \end{aligned}$$



Find The Normal stress at points D and E?

المساحة الدائرية

$A_{\text{solid}} = \frac{\pi}{4} d^2 = \frac{\pi}{4} (12)^2 = 113.1 \text{ mm}^2$

المساحة الفارغة

$A_{\text{Hollow}} = \frac{\pi}{4} (d_o^2 - d_i^2) = \frac{\pi}{4} (28^2 - 20^2) = 301.6 \text{ mm}^2$

المساحة الفارغة

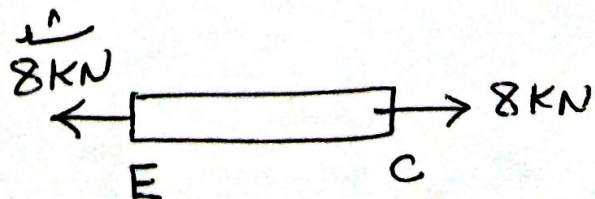
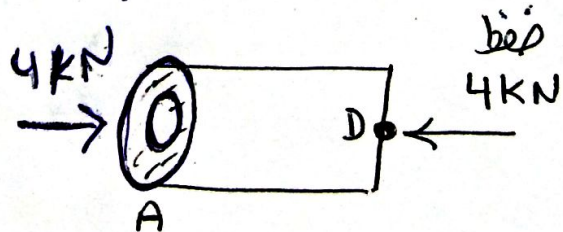
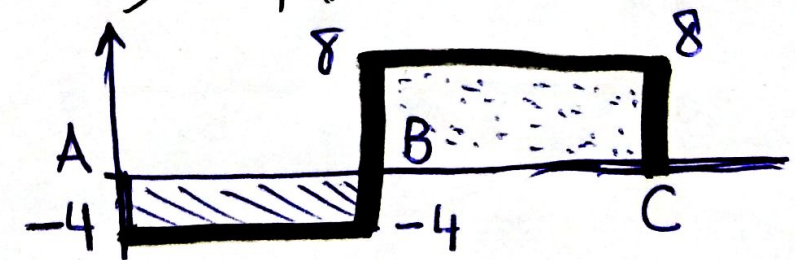
$A_{\text{Hollow}} = \frac{\pi}{4} (d_o^2 - d_i^2)$

Deep Comp.

$\sigma_D = \frac{4 \times 10^3}{301.6 \times 10^{-6}} = 13.3 \text{ MPa}$

Tension

$\sigma_E = \frac{8 \times 10^3}{113.1 \times 10^{-6}} = 70.7 \text{ MPa}$

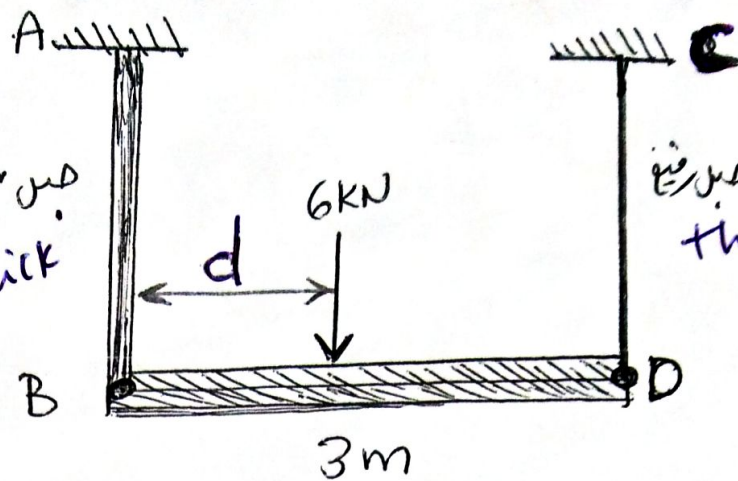


$$A_{AB} = 12 \text{ mm}^2$$

$$A_{CD} = 8 \text{ mm}^2$$

مِسْ كَمْسِ
Thick

مِسْ كَمْسِ
thin



"الوزن"
أولاً: عِطَانِ لِقَوَّةِ 6kN (عِطَانِ تَقْلِيلِ الْوِزْنِ)

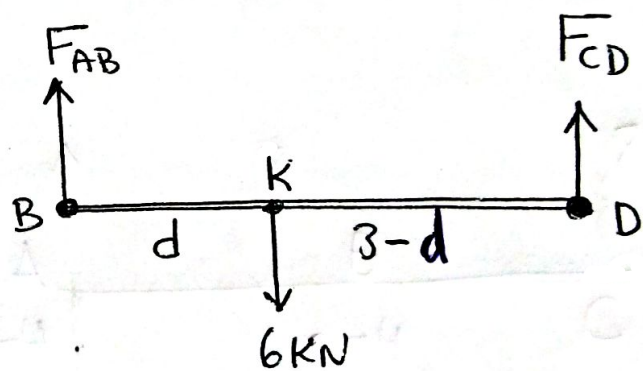
هَتِي لِيَصْبَحَ الْبُحْدَارُ فِي كِلِ الْاَوَّلِ كَمْسِ سَاوِي

Stress

الْبُحْدَارُ فِي كِلِ الْاَوَّلِ كَمْسِ سَاوِي

$$\Rightarrow (\sigma_{AB} = \sigma_{CD})$$

الكل



$$\sum M_K = 0$$

$$\Rightarrow F_{CD}(3-d) - F_{AB}(d) = 0$$

$$F_{AB} = \frac{3-d}{d} F_{CD}$$

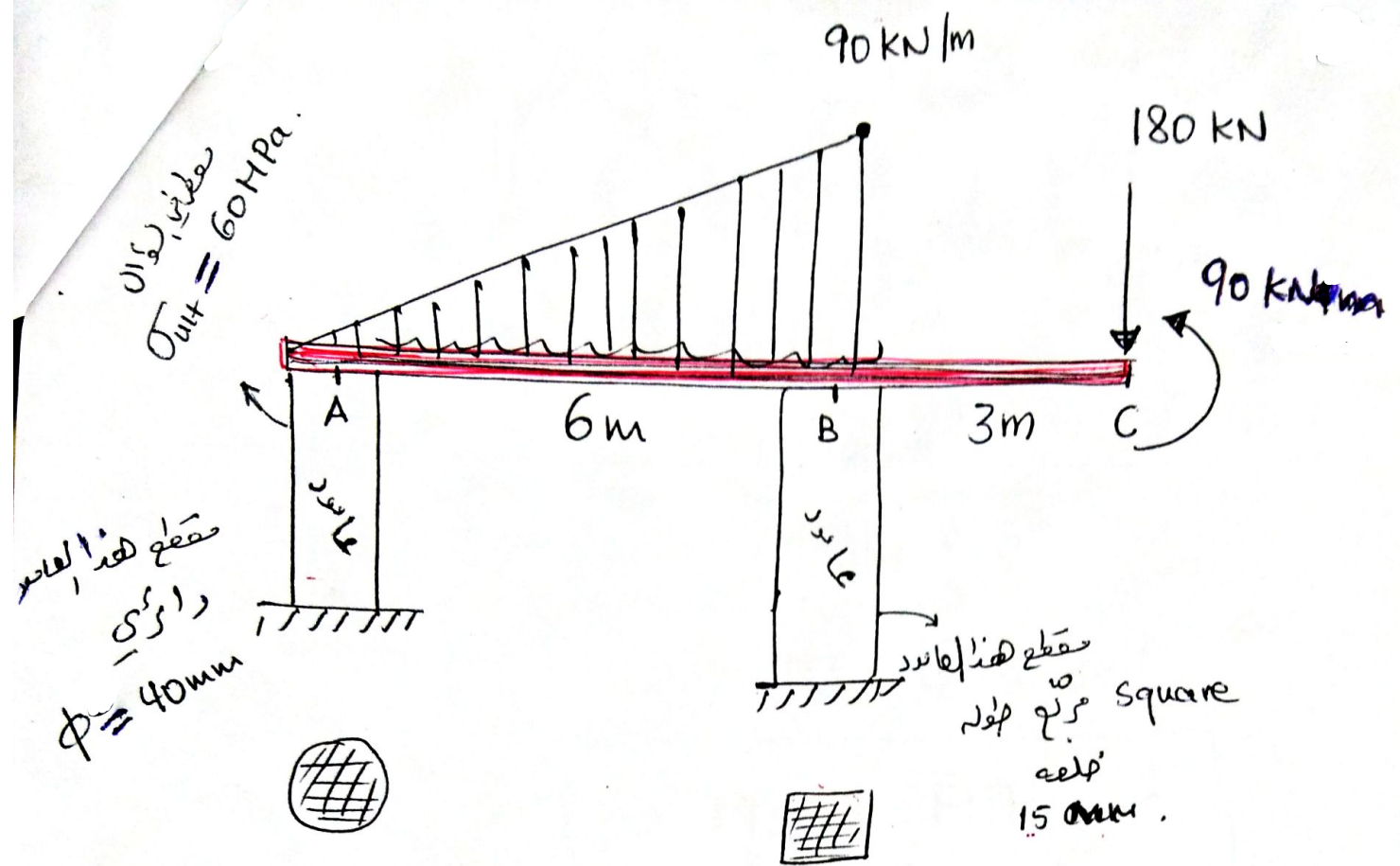
الكل

$$\sigma_{AB} = \sigma_{CD}$$

$$\frac{F_{AB}}{A_{AB}} = \frac{F_{CD}}{A_{CD}}$$

$$\Rightarrow \frac{\frac{3-d}{d} F_{CD}}{12 \times 10^{-6}} = \frac{F_{CD}}{8 \times 10^{-6}}$$

$$\Rightarrow \frac{3-d}{d} = 1.5 \Rightarrow 3-d = 1.5d \Rightarrow d = 1.20 \text{ m}$$

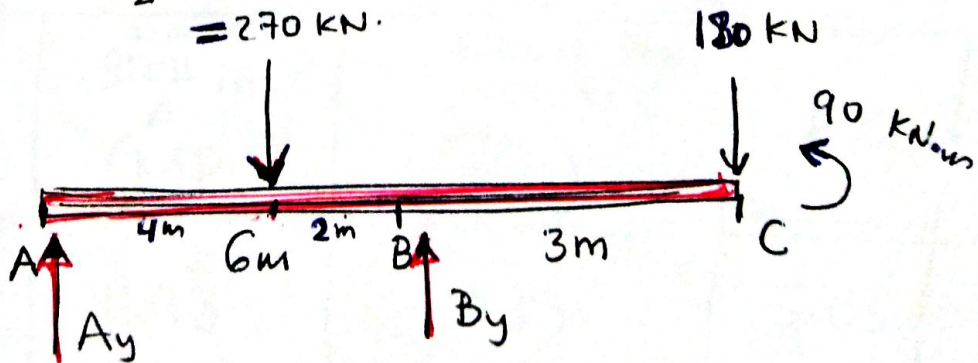


- 1) Find The Normal stress in Column A ?
- " in Column B ?

- 3) Find Factor of Safety? (F.S.) in A?
- $\frac{1}{2}(6)(90) = 270 \text{ kN}$

الوزن

سقف هذا العنصر



$$\sum M_A = 0 \rightarrow 6B_y - 270(4) - 180(9) + 90 = 0$$

$$B_y = 435 \text{ kN}$$

القوة في العمود B

$$\sum F_y = 0 \rightarrow A_y - 270 - 180 + 435 = 0 \rightarrow A_y = 15 \text{ kN}$$

القوة في العمود A

$$\sigma_{\text{allow}} = \frac{\text{Force}}{\text{Area}} = \frac{15 \times 10^3}{\frac{\pi}{4} (40)^2 \times 10^{-6}}$$

$$\sigma_A = 11.93 \text{ MPa}$$

استرس
ENG
و ب. ب. ب.

$$[2] \quad \sigma_{\text{allow}} = \frac{\text{Force}}{\text{Area}} = \frac{435 \times 10^3}{(15)(15) \times 10^{-6}}$$

$$= 1.933 \text{ GPa} \checkmark$$

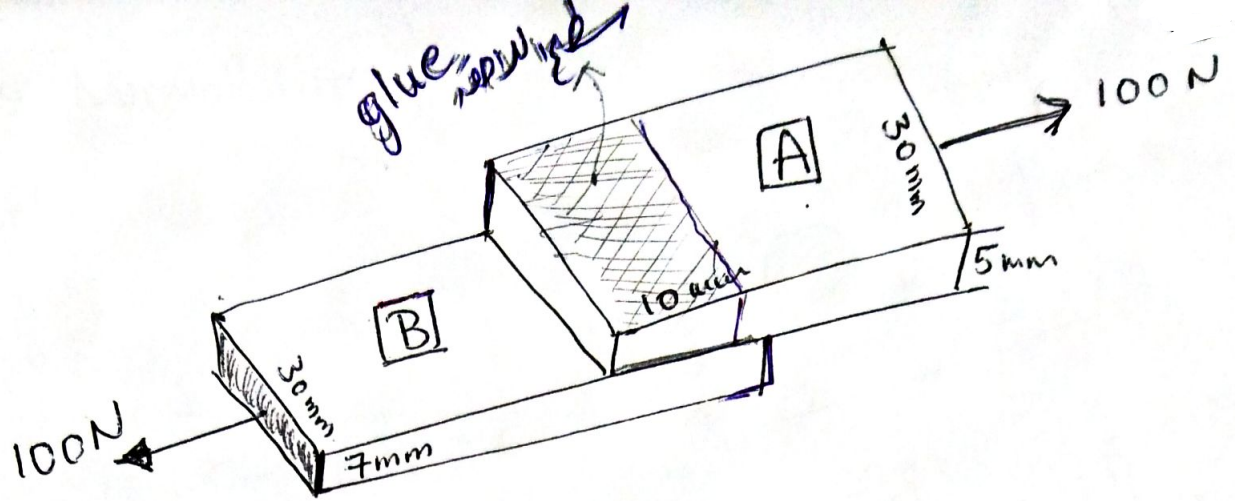
ENG.

$$= 1933.3 \text{ MPa} \checkmark$$

$$= 1933333333 \text{ Pa} \checkmark$$

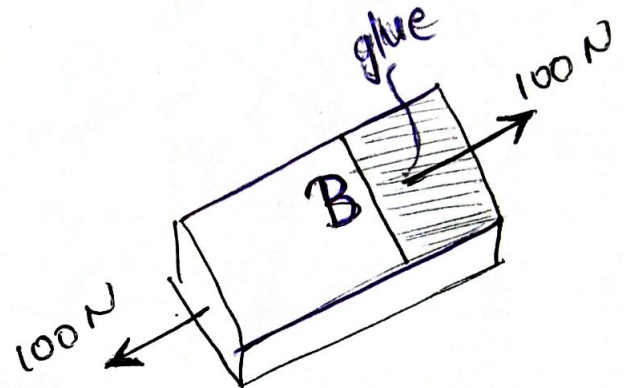
$$[3] \quad F.S. = \frac{\sigma_{\text{ult}}}{\sigma_{\text{all}}} = \frac{60 \times 10^6}{11.93 \times 10^6} = \underline{\underline{5.03}}$$

Force in A (kN)	Force in B (N)	Normal Stress in A (MPa)	Normal Stress in B (GPa)	F.S.
15	435,000	11.93	1.933	5.03



$$\sigma_B = \frac{100}{(7 \times 30) \times 10^{-6}} = \checkmark$$

$$\sigma_A = \frac{100}{(30 \times 5) \times 10^{-6}} = \checkmark$$



$$\tau_{\text{glue}} = \frac{100}{(10 \times 30) \times 10^{-6}} = \checkmark$$

Shear stress on the surface of glue?

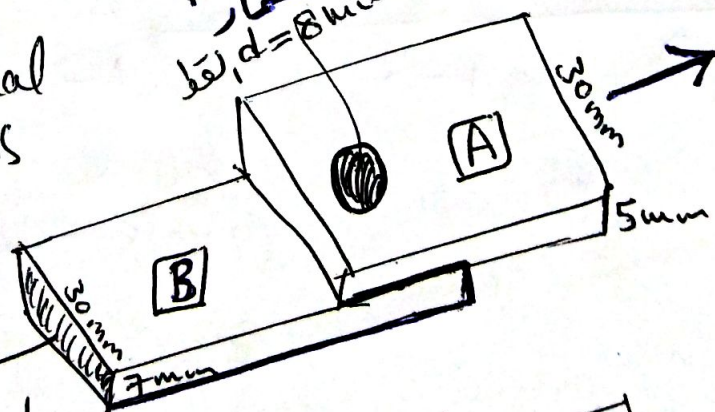


$$\sigma_B = \frac{100}{7 \times 30 \times 10^{-6}} \quad \left. \begin{array}{l} \sigma_A = \frac{100}{5 \times 30 \times 10^{-6}} \end{array} \right\} \text{Normal Stress}$$

Single shear

$$\tau_{\text{bolt}} = \frac{100}{\frac{\pi}{4} d^2} = \frac{100}{\frac{\pi}{4} (8)^2 \times 10^{-6}}$$

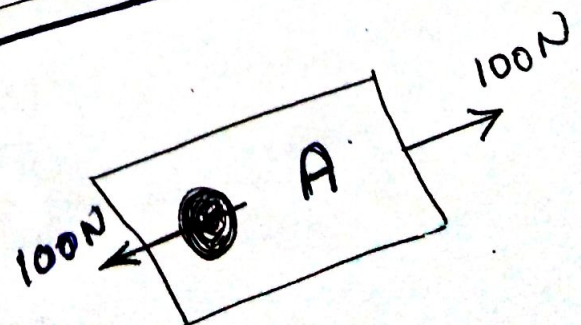
Bolt
dia = 8mm



Bearing

$$(\sigma_b)_A = \frac{100}{td} = \frac{100}{(5)(8) \times 10^{-6}}$$

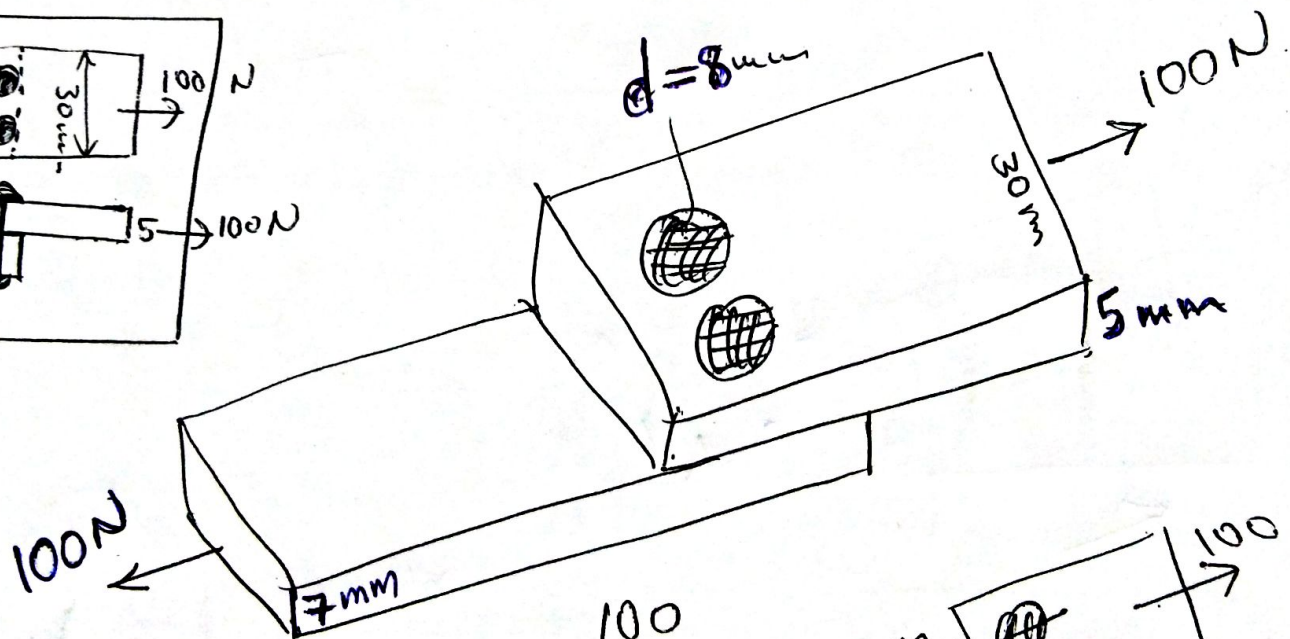
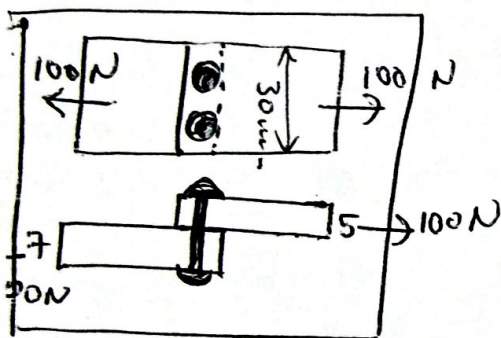
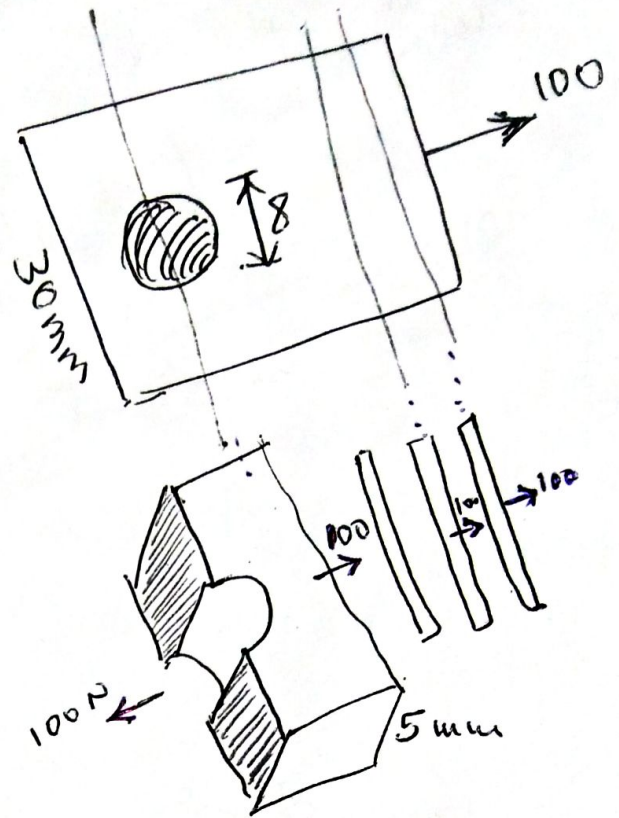
$$(\sigma_b)_B = \frac{100}{td} = \frac{100}{(7)(8) \times 10^{-6}}$$



max. Normal Stress

$$\sigma_{max} = \frac{100}{(30-8)(5) \times 10^{-6}}$$

$\downarrow$
 d



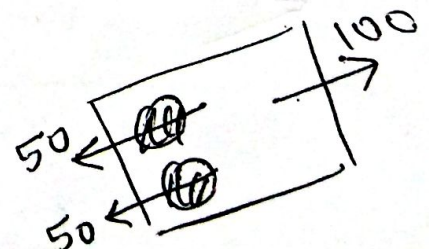
$$\sigma = \frac{100}{(30)(5)}$$

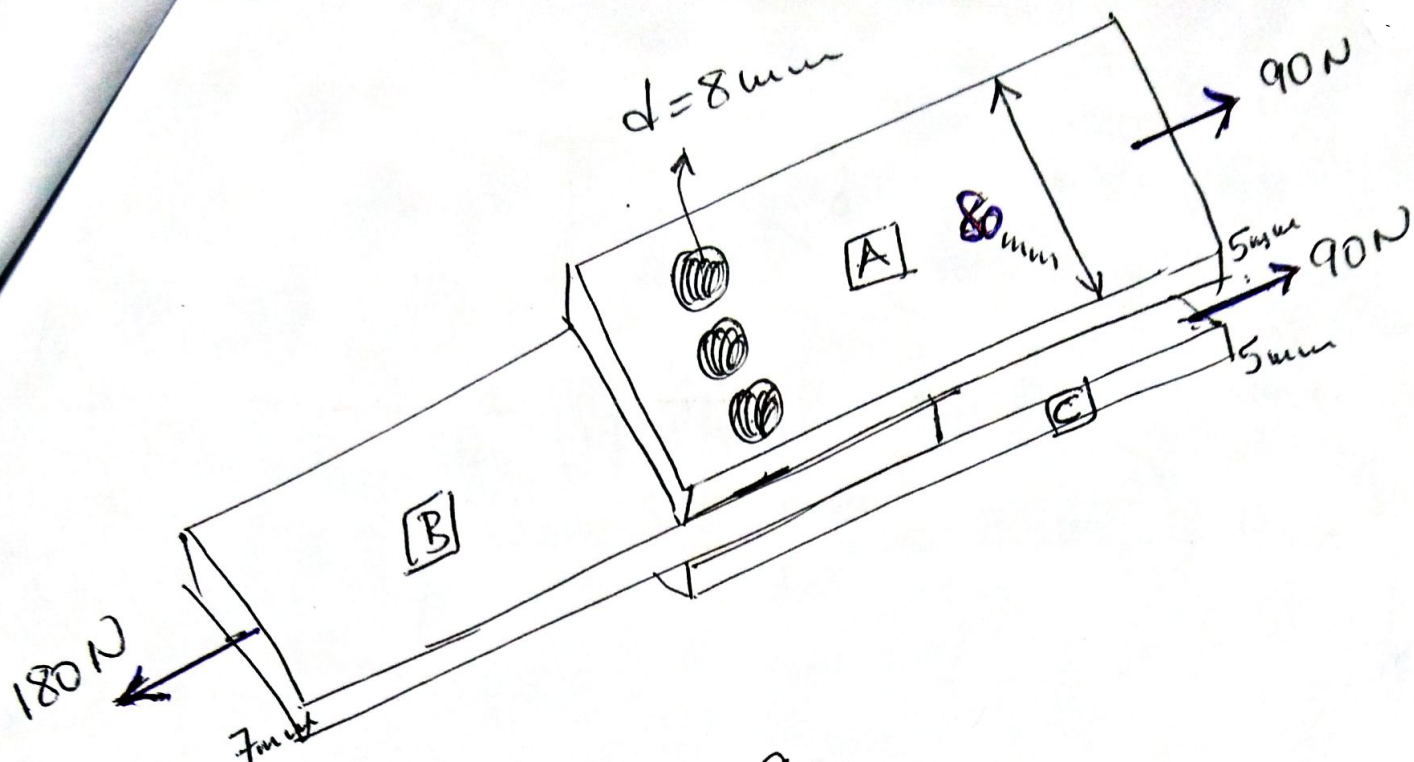
لا بد أن يكون
⇒ Double Shear

$$\tau = \frac{50}{\frac{\pi}{4}(8)^2}$$

$$\sigma_b = \frac{50}{td} = \frac{50}{(5)(8)}$$

$$\sigma_{max} = \frac{100}{(30-16)(5)} = \checkmark$$

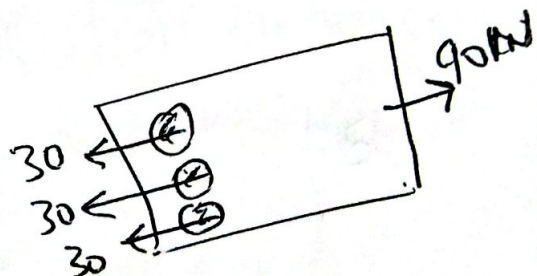




$$* \sigma_A = \frac{90}{(80)(5)}$$

$$* \sigma_B = \frac{180}{(80)(7)}$$

$$* \tau_{\text{Bolt}} = \frac{30}{\frac{\pi}{4}(8)^2}$$



Bearing Stress

$$* (\sigma_b)_A = \frac{30}{td} = \frac{30}{(5)(8)}$$

$$* (\sigma_b)_B = \frac{60}{td} = \frac{60}{(7)(8)}$$

$$* (\sigma_{\text{max}})_A = \frac{90}{(80 - 24)(5)}$$

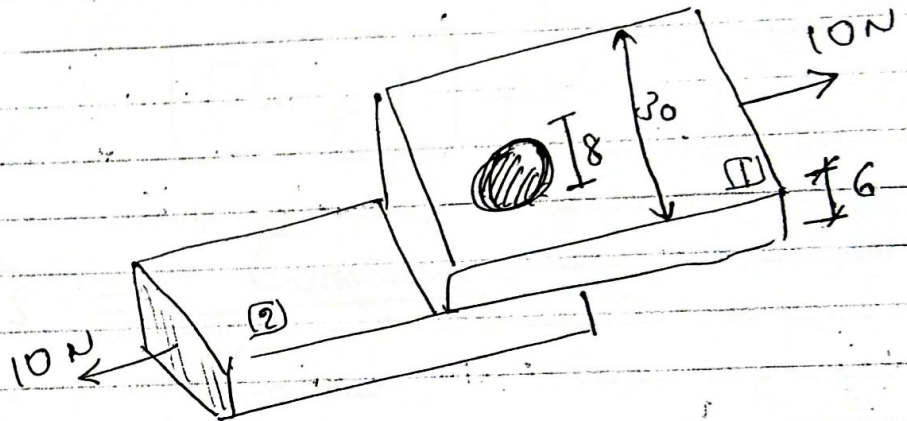
$$* (\sigma_{\text{max}})_B = \frac{180}{(80 - 24)(7)}$$

* Bearing stress (Crushing stress)

$$\sigma_B = \frac{F}{\text{Projected Area}}$$

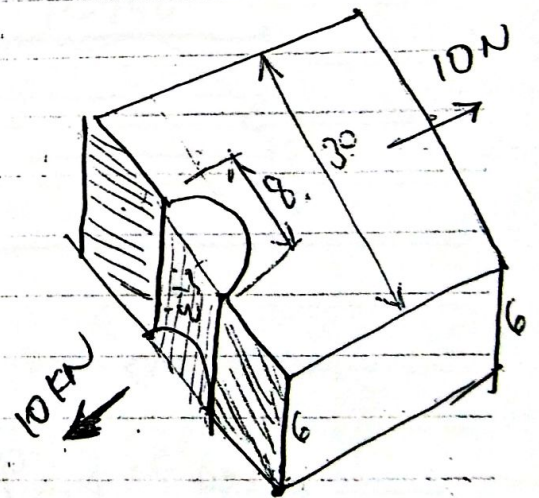
$$* \sigma_1 = \frac{10}{6(30)}$$

$$* \tau = \frac{10}{\frac{\pi}{4} d^2}$$



$$* (\sigma_1)_{\max} = \frac{10}{(30-8) 6}$$

الضغط، الشد، الانحناء



* Bearing stress between the bolt and the plate

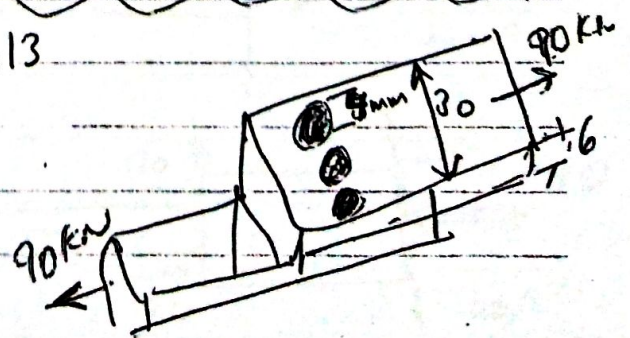
$$\sigma_B = \frac{10}{td} = \frac{10}{6(8)} = \checkmark$$

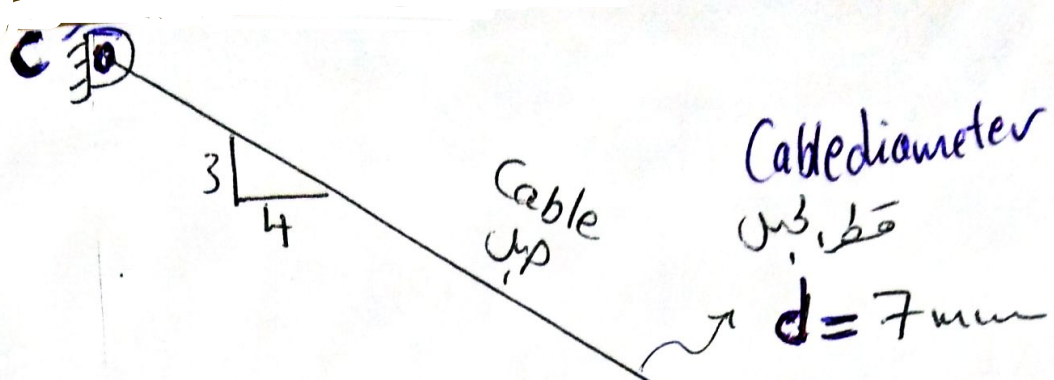
$$* \sigma_B = \frac{90}{6(30)}$$

$$* \tau = \frac{90/3}{\frac{\pi}{4} d^2}$$

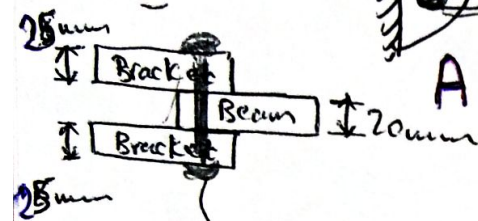
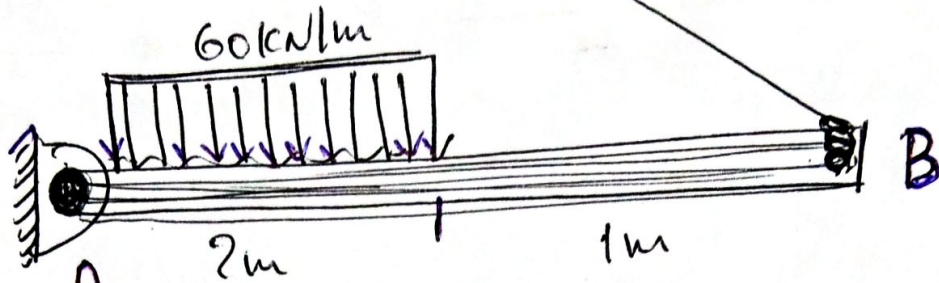
$$* \sigma_{\max} = \frac{90}{6(30-12)}$$

$$\sigma_B = \frac{30}{6(4)}$$





Double Shear
Pin, 2mm, 2mm



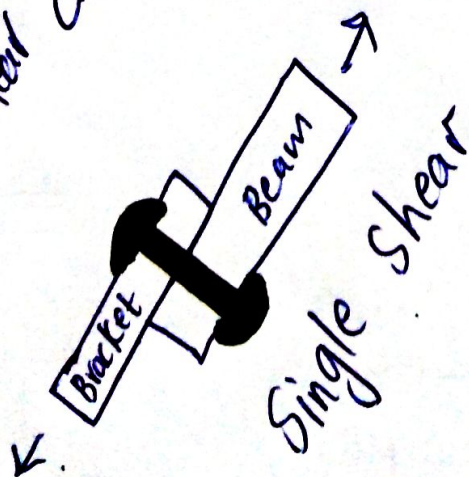
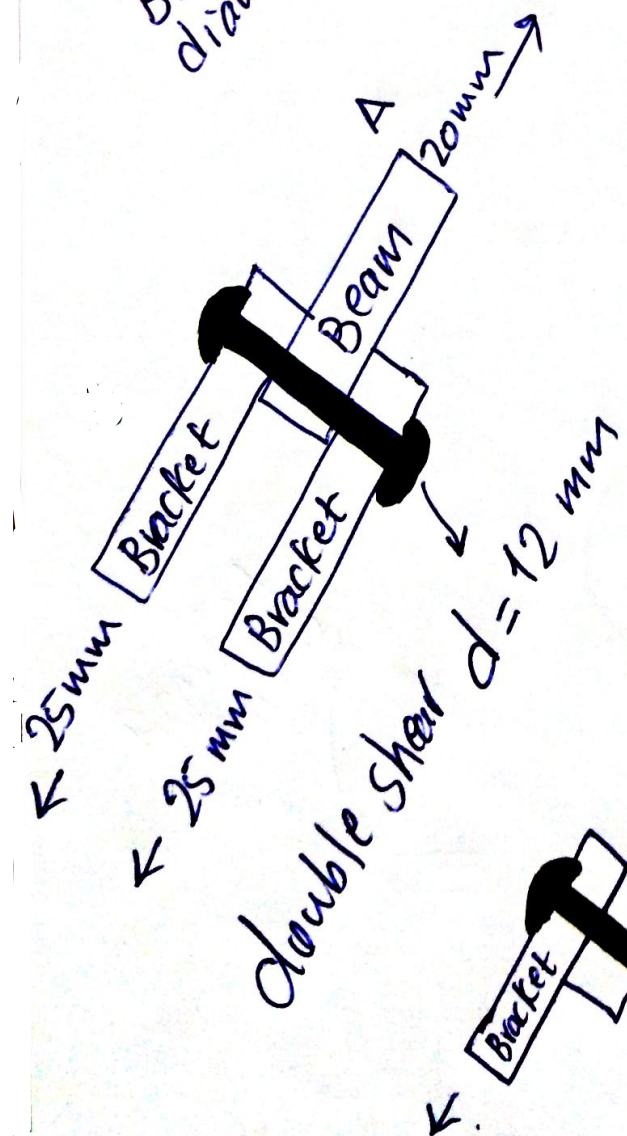
Bolt diameter $d = 12 \text{ mm}$

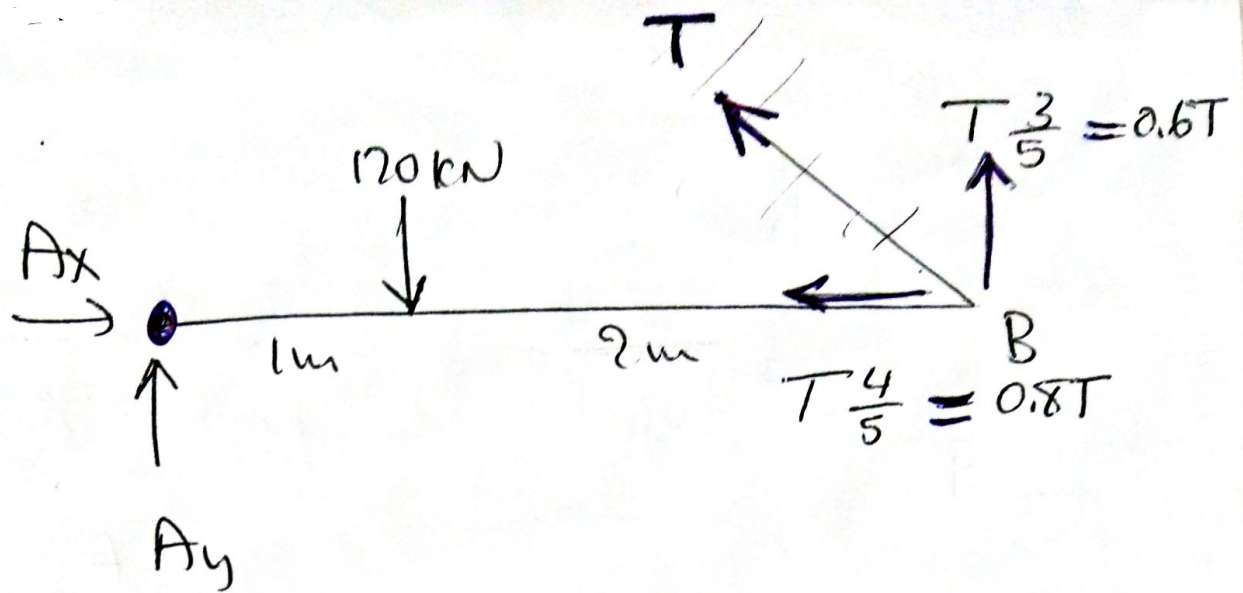
Find ① Normal Stress in cable?

② Shear Stress in Pin A?

③ Bearing stress between Pin A and Bracket?

④ Bearing stress between Pin A and Beam?





$$* \sum M_A = 0 \Rightarrow (0.6T)(3) - 120(1) = 0$$

القوة في الكابل $(T = 66.7 \text{ kN})$

$$* \sum F_x = 0 \Rightarrow A_x - 0.8T = 0$$

$$A_x = 0.8(66.7)$$

$$A_x = 53.36 \text{ kN}$$

$$+ \sum F_y = 0 \Rightarrow A_y - 120 + 0.6T = 0$$

$$A_y = 120 - 0.6(66.7)$$

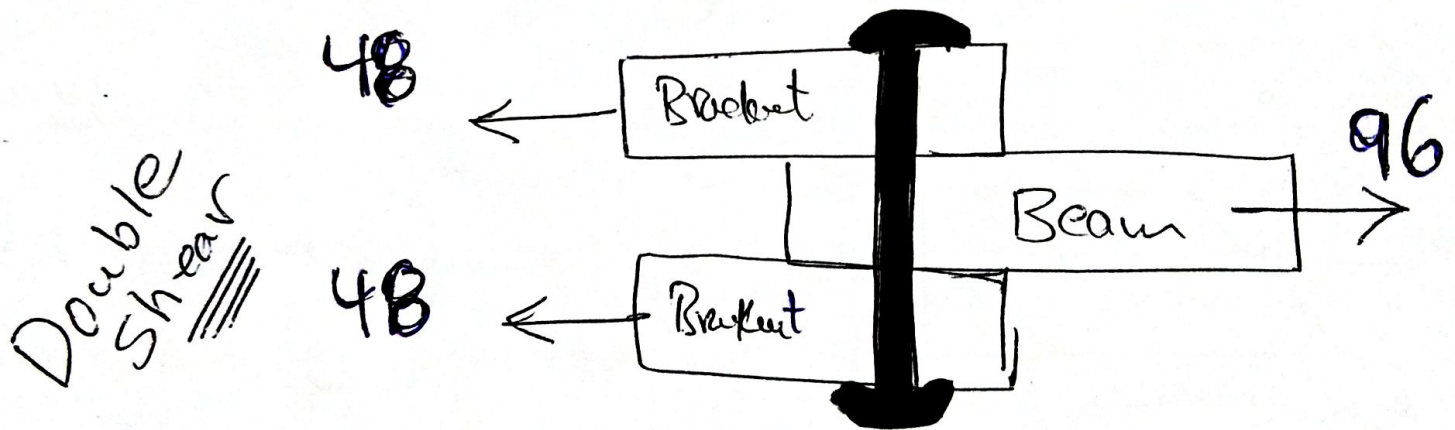
$$A_y = 80 \text{ kN}$$

A - resultant force, Resultant at A

$$\Rightarrow A = \sqrt{A_x^2 + A_y^2}$$

$$A = \sqrt{53.36^2 + 80^2}$$

$$A = 96 \text{ kN}$$

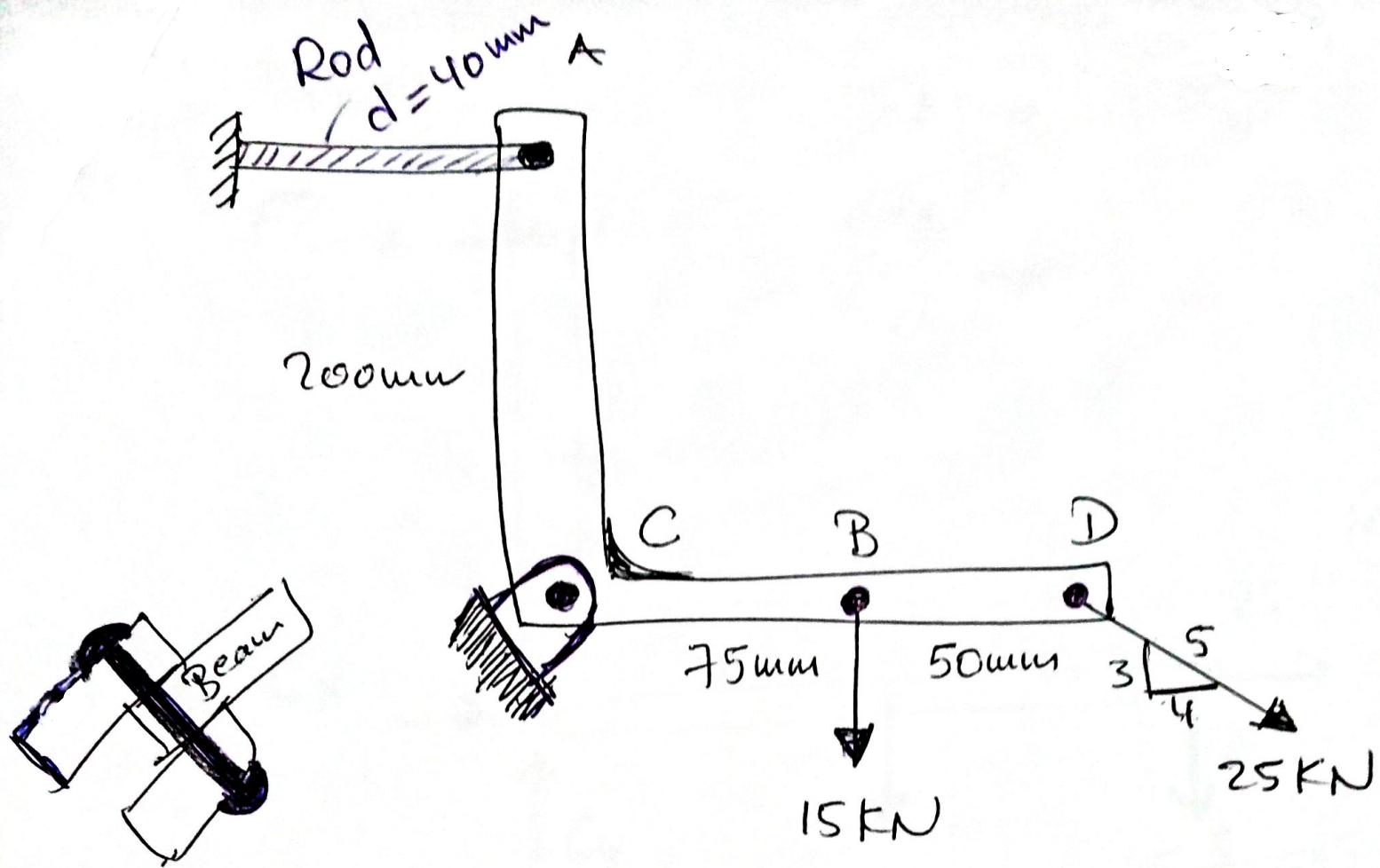


$$\textcircled{1} \quad \sigma = \frac{66.7 \times 10^3}{\frac{\pi}{4}(7)^2 \times 10^{-6}} = 1.733 \text{ GPa}$$

$$\textcircled{2} \quad \tau = \frac{48 \times 10^3}{\frac{\pi}{4}(12)^2 \times 10^{-6}} = 424.4 \text{ MPa}$$

$$\textcircled{3} \quad (\sigma_b)_{\text{Bracket}} = \frac{48 \times 10^3}{\frac{(25)(12)}{t} \times 10^{-6}} = 160 \text{ MPa}$$

$$\textcircled{4} \quad (\sigma_b)_{\text{Beam}} = \frac{96 \times 10^3}{(20)(12) \times 10^{-6}} = 400 \text{ MPa}$$



Double
Shear

$$\tau_{\text{allow}} = 55 \text{ MPa}$$

** Find the pin diameter ?
C = 50 mm, 75 mm, 50 mm

** Find the tensile Normal Stress in the Rod at A ?

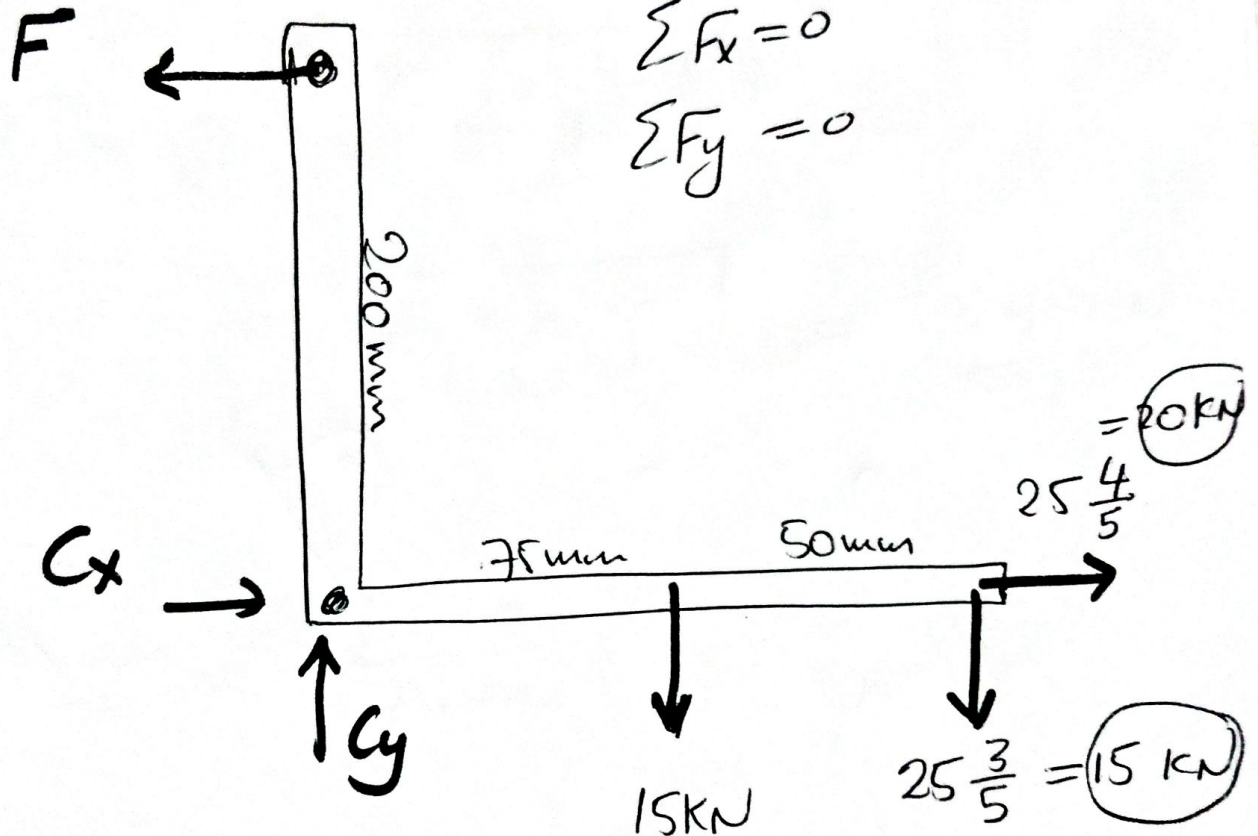
Q31

Statics

$$\sum M = 0$$

$$\sum F_x = 0$$

$$\sum F_y = 0$$



$$\sum M_c = 0 \Rightarrow 200F - 75(15) - 15(125) = 0$$

$$F = 15 \text{ kN}$$

$$\sum F_x = 0 \Rightarrow C_x + 20 - 15 = 0$$

$$C_x = -5 \text{ kN}$$

$$\sum F_y = 0 \Rightarrow C_y - 15 - 15 = 0$$

$$C_y = 30 \text{ kN}$$

Resultant $C = \sqrt{C_x^2 + C_y^2}$ - in steel also

$$C = \sqrt{C_x^2 + C_y^2}$$

$$C = \sqrt{(-5)^2 + (30)^2} = \underline{\underline{30.4 \text{ kN}}}$$

* $\tau = \frac{\text{Force}}{\text{Area}}$

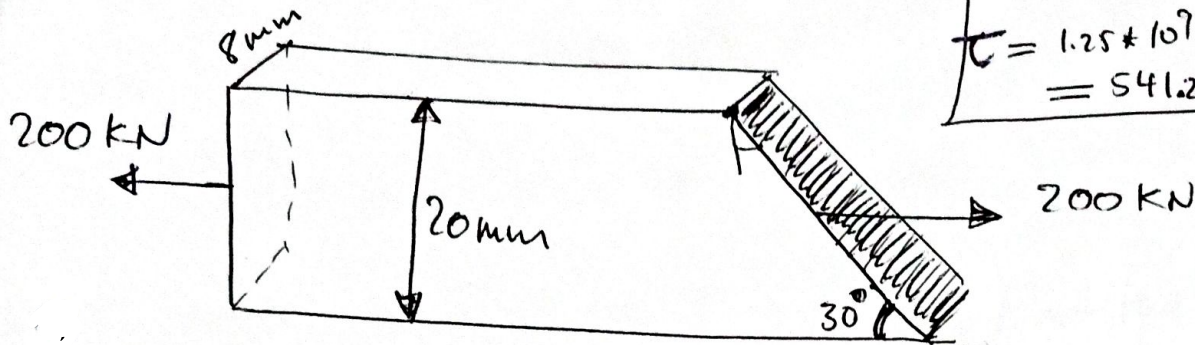
$55 \times 10^6 = \frac{\frac{30.4}{2} \times 10^3}{\frac{\pi}{4} d^2}$ → Double shear

① $d = 18.8 \text{ mm}$

② $\sigma_A = \frac{F}{A} \Rightarrow \frac{15 \times 10^3}{\frac{\pi}{4} (40 \times 10^{-3})^2} = \dots \checkmark$

Inclined Surface (Stress)

ايجاد الاجابة على السؤالين



$$\sigma_x = \frac{200 \times 10^3}{(20)(8) \times 10^{-6}} = 1.25 \text{ GPa}$$

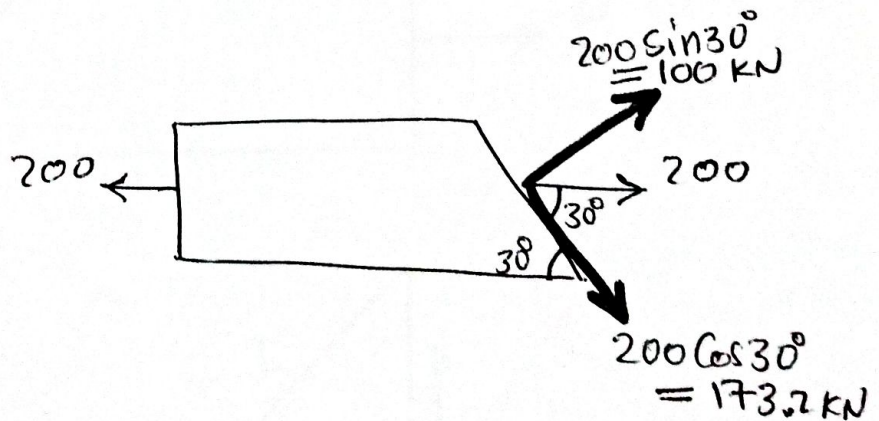
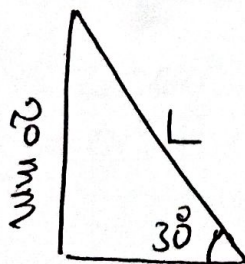
$$\theta = 60^\circ$$

$$\sigma = 1.25 \times 10^9 \cos^2 60^\circ = 312.5 \text{ MPa}$$

$$\tau = 1.25 \times 10^9 \cos 60^\circ \sin 60^\circ = 541.25 \text{ MPa}$$

* Find the Normal and shear stress on the inclined surface?

الحل



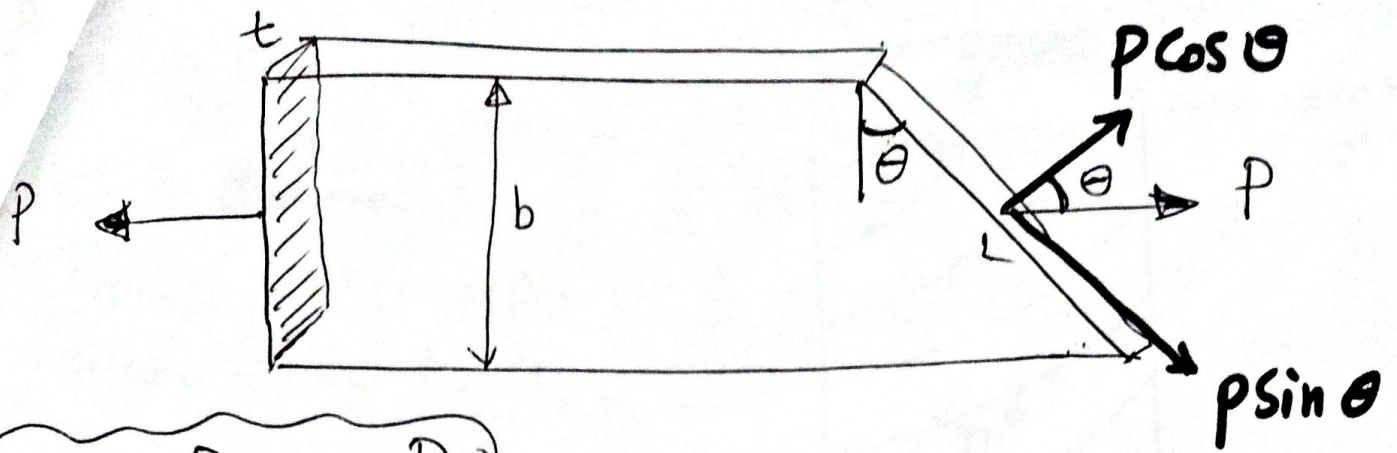
$$\sin 30^\circ = \frac{20}{L} \Rightarrow L = \frac{20}{\sin 30^\circ} \Rightarrow L = 40 \text{ mm}$$

مساحة السطح

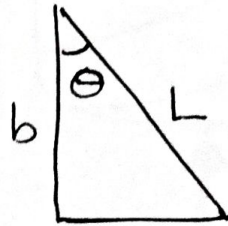
$$A = (40)(8) = 320 \text{ mm}^2$$

$$\sigma = \frac{100 \times 10^3}{320 \times 10^{-6}} = 312.5 \text{ MPa}$$

$$\tau = \frac{173.2 \times 10^3}{320 \times 10^{-6}} = 541.25 \text{ MPa}$$



$$\sigma_x = \frac{P}{bt} = \frac{P}{A}$$



$$\cos \theta = \frac{b}{L}$$

$$L = \frac{b}{\cos \theta}$$

$$A' = \text{Area} = \frac{b}{\cos \theta} t = \frac{bt}{\cos \theta}$$

$$A' = \frac{A}{\cos \theta}$$

$$* \sigma = \frac{P \cos \theta}{\frac{A}{\cos \theta}}$$

$$\Rightarrow \sigma = \frac{P}{A} \cos^2 \theta$$

$$\sigma = \sigma_x \cos^2 \theta$$

کاسٹونہ

$$* \tau = \frac{P \sin \theta}{\frac{A}{\cos \theta}}$$

$$\Rightarrow \tau = \frac{P}{A} \cos \theta \sin \theta$$

$$\tau = \sigma_x \cos \theta \sin \theta$$

کاسٹونہ

$$d_o = 12 \text{ mm}$$

$$t = 2 \text{ mm}$$

$$\sigma_{\text{allow}} = 150 \text{ MPa}$$

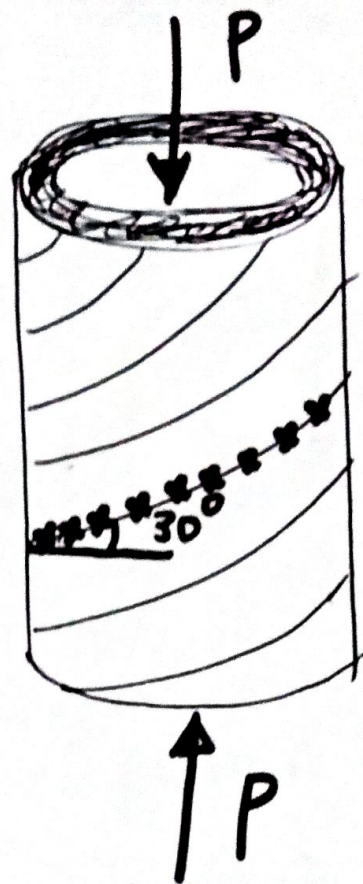
$$\tau_{\text{allow}} = 160 \text{ MPa}$$

Find P_{max} ?
 زاویه 30 درجه
عن الأفق

$$\theta = 30^\circ$$

مثال

$$\sigma_x = \frac{\text{Force}}{\text{Area}}$$



$$d_i = d_o - 2t$$

$$d_i = 12 - 4 = 8 \text{ mm}$$

$$\begin{aligned} \text{Area} &= \frac{\pi}{4} (d_o^2 - d_i^2) \\ &= \frac{\pi}{4} ((12)^2 - (8)^2) = 62.8 \text{ mm}^2 \end{aligned}$$

$$\sigma_x = \frac{1P}{62.8 \times 10^{-6}} = 15.92 \times 10^3 P$$

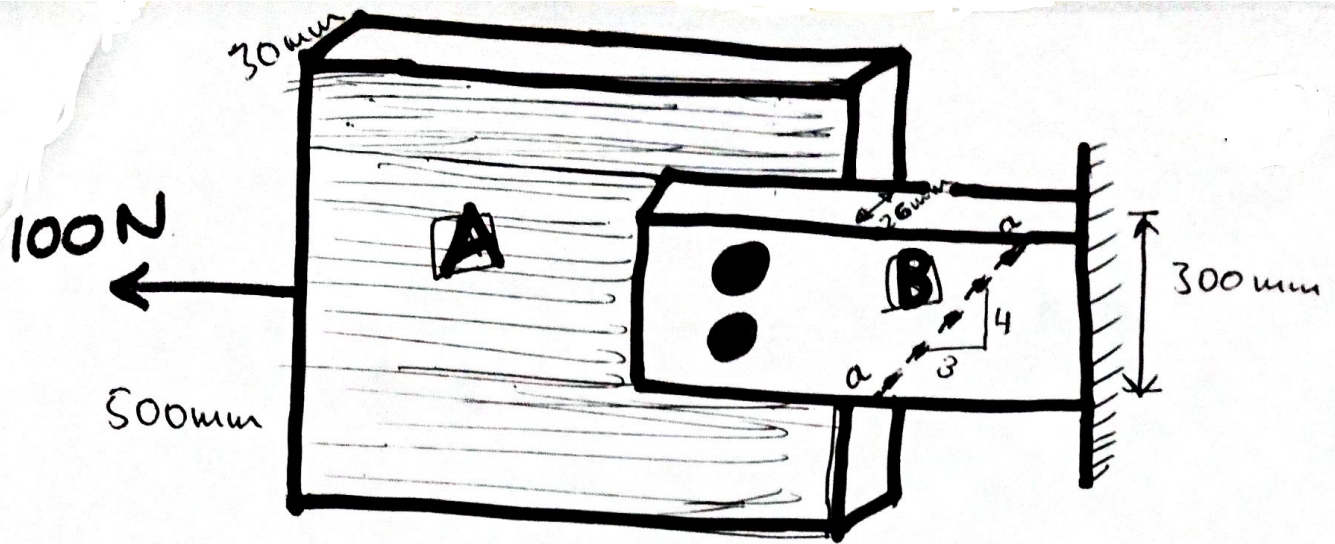
$$\sigma = \sigma_x \cos^2 \theta$$

$$150 \times 10^6 = (15.92 \times 10^3 P) \cos^2 30^\circ \Rightarrow P = 12.45 \text{ kN}$$

$$\tau = \sigma_x \cos \theta \sin \theta$$

$$160 \times 10^6 = 15.92 \times 10^3 P \cos 30^\circ \sin 30^\circ \Rightarrow P = 23.2 \text{ kN}$$

select $\Rightarrow P = 12.45 \text{ kN}$



$$d = 20 \text{ mm}$$

- 1) Find the Normal stress on plate A
- 2) " " " " " " B
- 3) Find the shear stress on the Bolt (pin)
- 4) Find the Bearing stress between pin & A
- 5) Find the Bearing " " " pin & B
- 6) Find the Normal and Shear stress on the inclined surface (a-a)

$$\sigma_A = \frac{\text{Force}}{\text{Area}}$$

$$= \frac{100 \text{ N}}{(300)(30) \times 10^{-6}} = 6.67 \text{ kPa}$$

$$(2) \quad \sigma_B = \frac{100}{(300)(26) \times 10^{-6}} = 12.8 \text{ kPa}$$

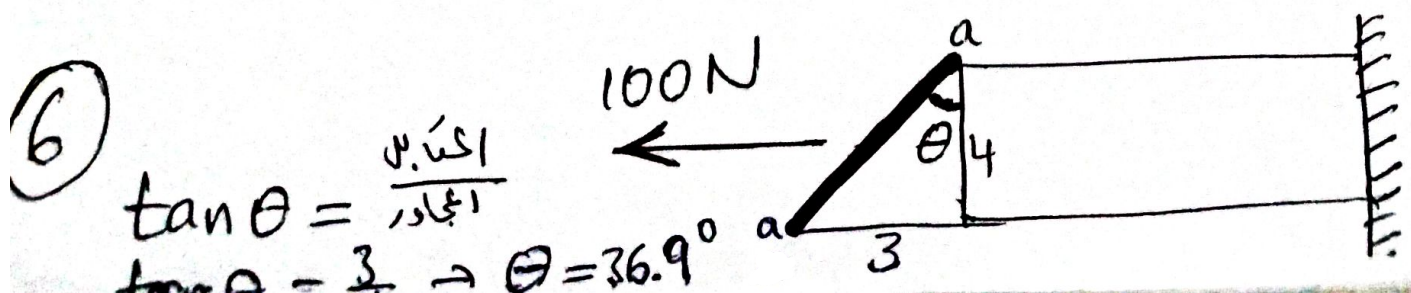
$$(3) \quad \tau = \frac{50}{\frac{\pi}{4}(20)^2 \times 10^{-6}} = 159.2 \text{ kPa.}$$

$$(4) \quad \text{Bearing } (\sigma_b)_A = \frac{50}{t d} = \frac{50}{(30)(20) \times 10^{-6}}$$

$$= 83.3 \text{ kPa.}$$

$$(5) \quad \text{Bearing } (\sigma_b)_B = \frac{50}{t d} = \frac{50}{(26)(20) \times 10^{-6}}$$

$$= 96.7 \text{ kPa}$$



$$\sigma_x = 12.8 \text{ kPa}$$

→ نفس المبدأ لو عدد
لحسب رشم ©

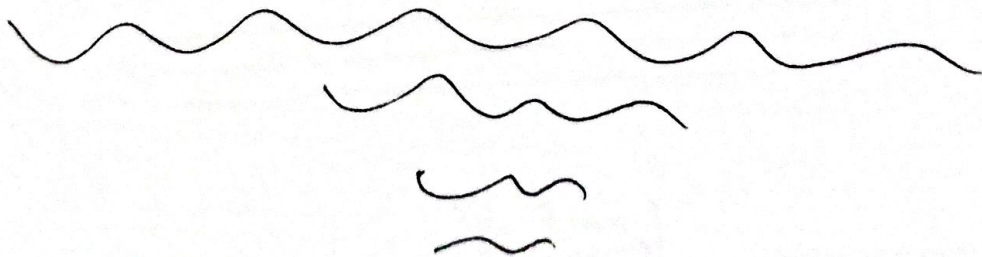
$$\sigma = \sigma_x \cos^2 \theta$$

$$\sigma = (12.8 \times 10^3) \cos^2 36.9^\circ = \underline{\underline{8.2 \text{ kPa}}}$$

$$* \tau = \sigma_x \sin \theta \cos \theta$$

$$= (12.8 \times 10^3) \sin(36.9^\circ) \cos(36.9^\circ)$$

$$= \underline{\underline{6.15 \text{ kPa}}}$$



Factor of Safety

معامل الأمان

$$F.S. = \frac{\sigma_{ult}}{\sigma_{allow}}$$

$$F.S. = \frac{T_{ult.}}{T_{allow}}$$

أكثر مما يمكن
أن يتحمله المعدن

الجهد المسموح
باعتباره في الأمان

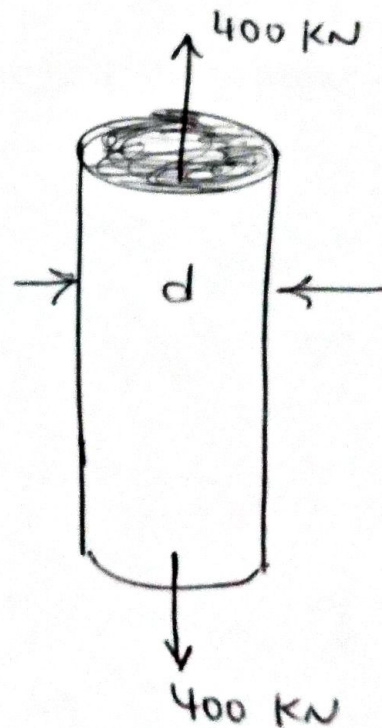
(الجهد المستخدم في عملية التصميم)
Design.

ex1

$$\sigma_{ult.} = 570 \text{ MPa}$$

$$F.S. = 3.5$$

Find the rod diameter?
أوجد قطر المعدن؟



الحل

$$\sigma_{allow} = \frac{570}{3.5} = 163 \text{ MPa}$$

$$\sigma_{allow} = \frac{\text{Force}}{\text{Area}} \Rightarrow 163 \times 10^3 = \frac{400 \times 10^3}{\frac{\pi}{4} d^2}$$

$$\Rightarrow d = 0.0559 \text{ m}$$

$$\Rightarrow d = 55.9 \text{ mm}$$

$$\Rightarrow d = 56 \text{ mm} \rightarrow \text{Round up.}$$

تقريباً لأعلى

Factor of safety : معام الأمان (F.S.)

$$F.S. = \frac{\sigma_{\text{Ultimate (fail)}}}{\sigma_{\text{Design (allow)}}}$$

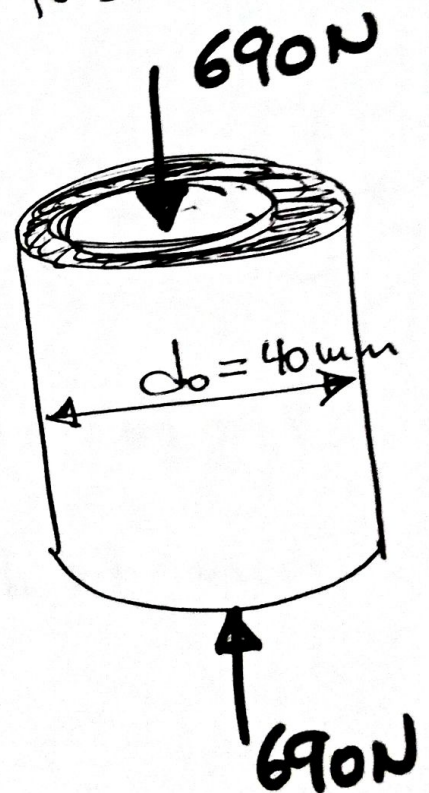
مقام تحمل الفشل → $\sigma_{\text{Ultimate (fail)}}$
 المقام الإجهاد المسموح به في التصميم (في ظل الأمان) → $\sigma_{\text{Design (allow)}}$

ex) Find the pipe thickness ^{أوجد سمك الأنبوب}

Assuming the ultimate stress equal

120 MPa

And the F.S. = 2.5



الحل

$$\left. \begin{array}{l} \sigma_u = 120 \text{ MPa} \\ F.S. = 2.5 \end{array} \right\} \Rightarrow$$

$$\sigma_{\text{allow}} = \frac{120}{2.5}$$

$$\sigma_{\text{allow}} = 48 \text{ MPa}$$

$$\sigma = \frac{\text{Force}}{\text{Area}} = \frac{F}{A}$$

$$\Rightarrow F = 690 \text{ N}$$

$$\Rightarrow A = \frac{\pi}{4} (d_o^2 - d_i^2)$$

$$* 48 * 10^6 = \frac{690}{A}$$

$$\Rightarrow A = 14.375 * 10^{-6} \text{ m}^2$$

جواب

$$14.375 * 10^{-6} = \frac{\pi}{4} [(40)^2 - d_i^2] * 10^{-6}$$

$$\Rightarrow d_i^2 = 1581.69718$$

بقدره، $d_i = 39.77 \text{ mm}$

$$\text{thickness} = \frac{d_o - d_i}{2}$$

$$t = \frac{40 - 39.77}{2}$$

$$t = 0.115 \text{ mm}$$

انتهى

** Factor of safety (F.S.)

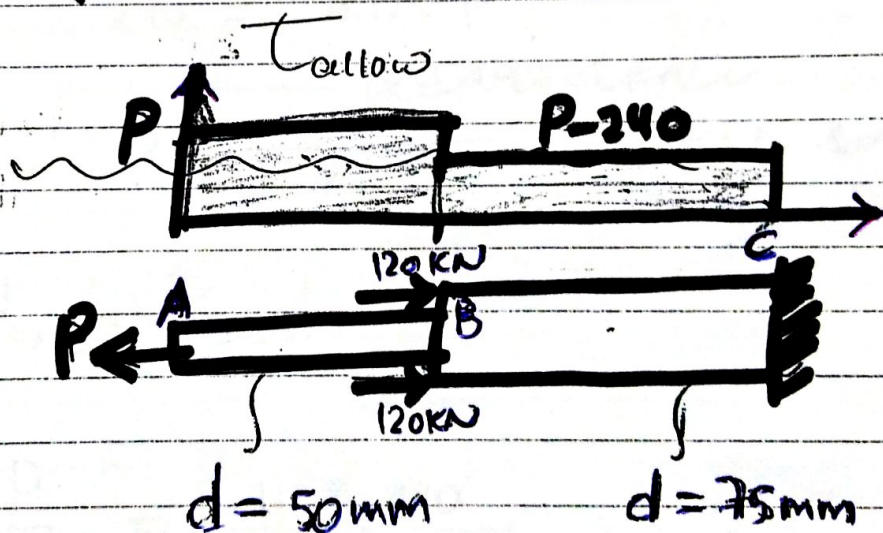
عوامل الأمان

$$** F.S. = \frac{\sigma_{ultimate}}{\sigma_{allow (Design)}} = \frac{\text{الضغط الأقصى المسموح به}}{\text{الضغط التصميمي المسموح به}}$$

$$F.S. \geq 1$$

$$** F.S. = \frac{\sigma_{ultimate}}{\sigma_{allow}}$$

Ex



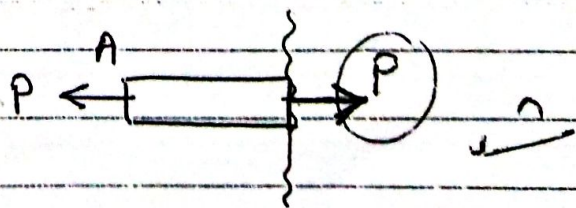
double.

أوجد فيه P التي تجعل الضغط في AB يساوي ضغط BC في BC

وحي

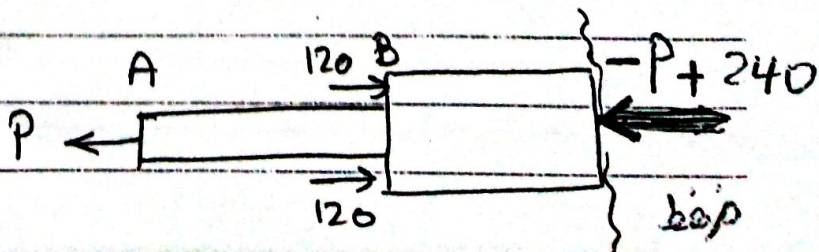
* AB

$$\sigma_{AB} = \frac{P}{\frac{\pi}{4} (50)^2}$$



* BC

$$\sigma_{BC} = \frac{-P + 240}{\frac{\pi}{4} (75)^2}$$



وأيضاً،

$$\sigma_{AB} = 2 \sigma_{BC}$$

$$\Rightarrow \frac{P \times 10^3}{\frac{\pi}{4} (50)^2 \times 10^{-6}} = 2 \frac{-(P+240) \times 10^3}{\frac{\pi}{4} (75)^2 \times 10^{-6}}$$

$$\Rightarrow \frac{P}{(50)^2} = 2 \frac{-(P+240)}{(75)^2}$$

⇓

$$\frac{(75)^2}{2(50)^2} P = -P + 240$$

أو 1.125 P = -P + 240

$$1.125 P = -P + 240$$

$$240 = P + 1.125 P$$

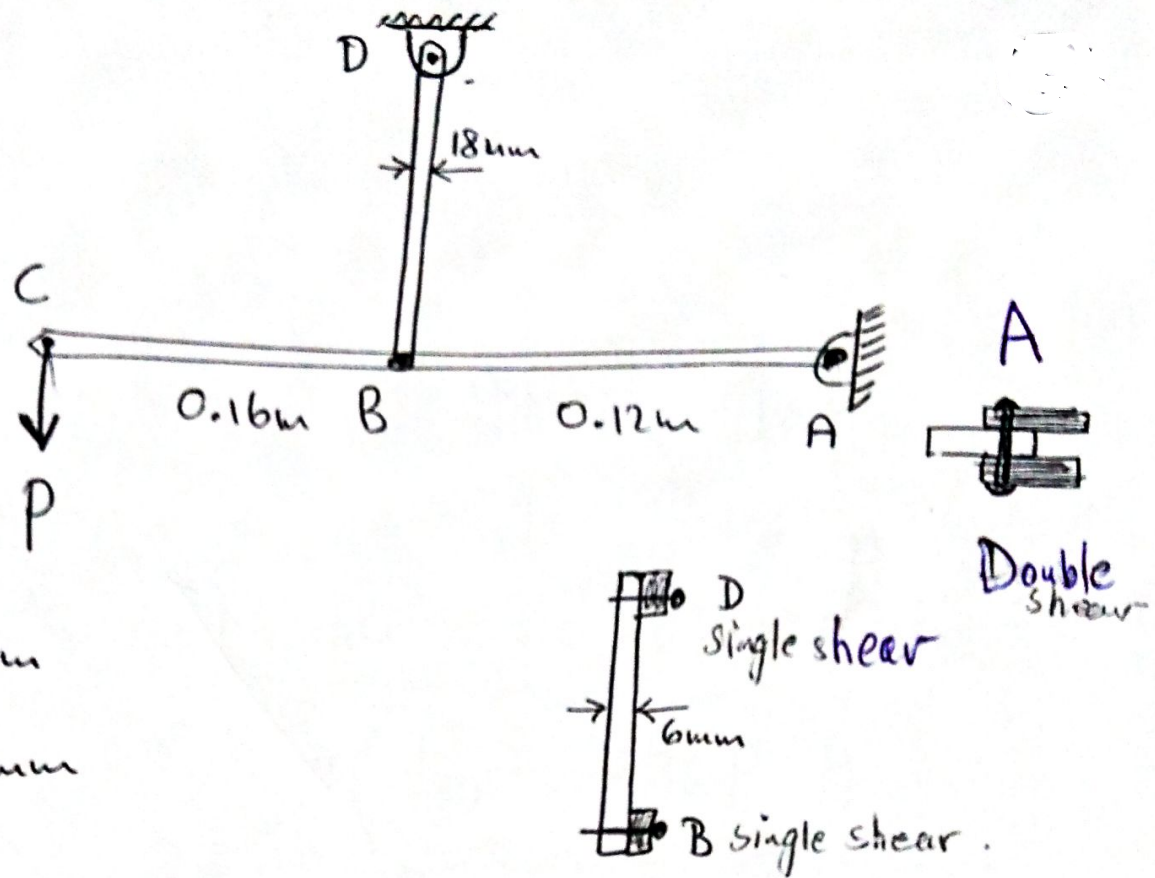
$$240 = 2.125 P$$

$$P = \frac{240}{2.125} = 112.9 \text{ KN} \quad \checkmark$$

أو بدلالة القوة (الضغط) في (القسم) AB

$$\sigma_{AB} = \frac{P}{A} = \frac{112.9 \times 10^3}{\frac{\pi}{4} (50)^2 \times 10^{-6}} = 57.5 \text{ MPa} \quad \checkmark$$

و



B قطر 10mm
D قطر 14mm
A قطر 6mm

$$\ast \tau_{ult} = 150 \text{ MPa}$$

$$\ast \sigma_{ult} = 400 \text{ MPa}$$

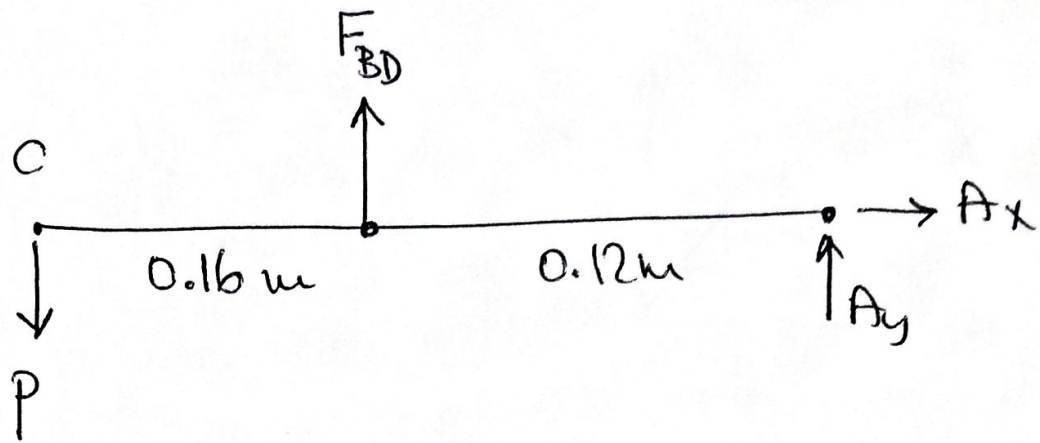
$$\ast F.S. = 3$$

Find the max. load P ?

و

$$\tau_{allow} = \frac{400}{3} = 133.3 \text{ MPa}$$

$$\tau_{allow} = \frac{150}{3} = 50 \text{ MPa}$$



$$* \sum M_A = 0 \Rightarrow 0.28P - 0.12F_{BD} = 0$$

$$\Rightarrow F_{BD} = 2.33P$$

$$* \sum F_x = 0 \Rightarrow A_x = 0$$

$$* \sum F_y = 0 \Rightarrow A_y + 2.33P - P = 0$$

$$A_y = -1.33P$$

Resultant.
* also
A_{res}

$$A = \sqrt{A_x^2 + A_y^2} \Rightarrow A = 1.33P$$

$$\leftarrow \text{Pin A} \Rightarrow \tau = \frac{\text{Force}/2}{\text{Area}} \Rightarrow 50 \times 10^6 = \frac{1.33P/2}{\frac{\pi}{4}(6)^2 \times 10^{-6}} \Rightarrow P = 2126 \text{ N}$$

$$* \text{Pin B} \Rightarrow \tau = \frac{\text{Force}}{\text{Area}} \Rightarrow 50 \times 10^6 = \frac{2.33P}{\frac{\pi}{4}(10)^2 \times 10^{-6}} \Rightarrow P = 1686 \text{ N}$$

$$* \text{Pin D} \Rightarrow \tau = \frac{\text{Force}}{\text{Area}} \Rightarrow 50 \times 10^6 = \frac{2.33P}{\frac{\pi}{4}(14)^2 \times 10^{-6}} \Rightarrow P = 3304 \text{ N}$$

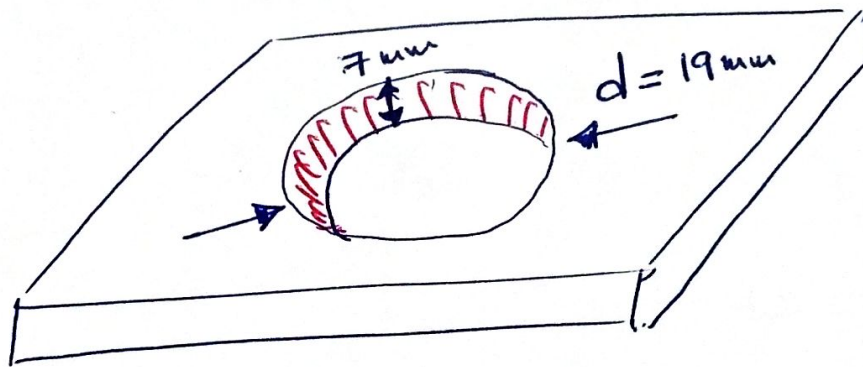
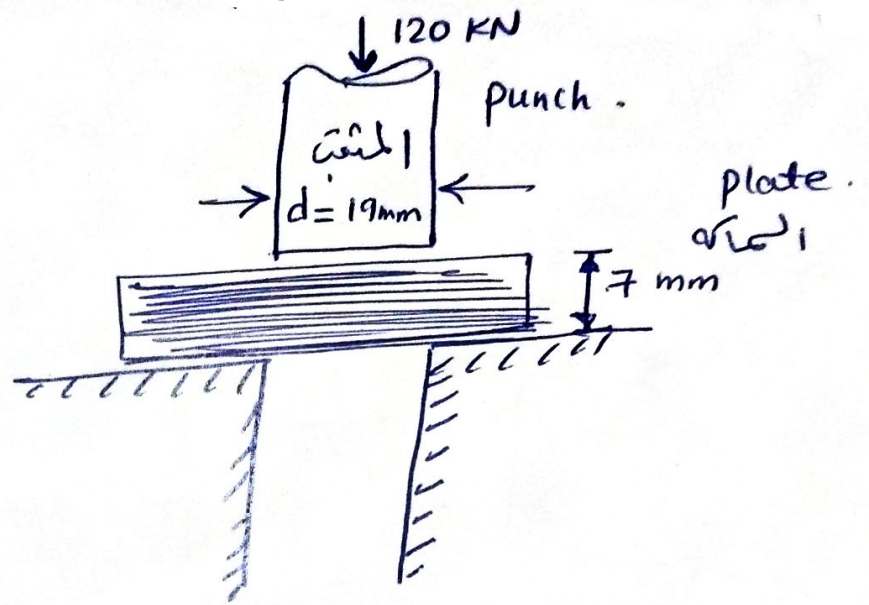
$$* \text{Rod BD} \Rightarrow \sigma = \frac{\text{Force}}{\text{Area}} \Rightarrow 133.3 \times 10^6 = \frac{2.33P}{(18)(6) \times 10^{-6}} \Rightarrow P = 6179 \text{ N}$$

~~***~~ The max load $P = 1686 \text{ N}$ ←

د. هشام الندي

Ex)

Find The Shear Stress (τ) in the plate .



الحل

$$\tau = \frac{\text{Force}}{\text{Area}}$$

$$\tau = \frac{120 \times 10^3}{\pi(19)(7) \times 10^{-6}}$$

$$\tau = 287 \text{ MPa .}$$

ENG .

مساحة سطح لدافتي
من الحفرة (لثقب)

هو

$$\text{Area} = (\pi d)(h)$$

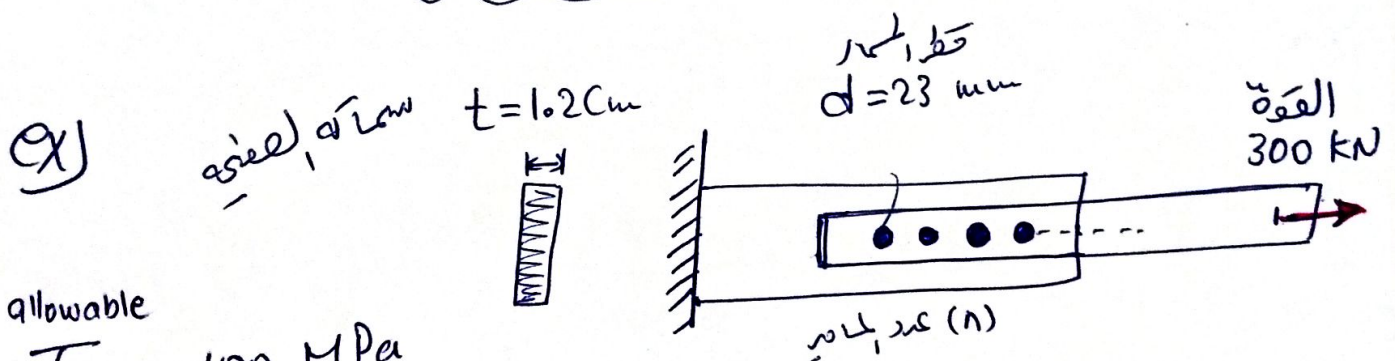
↓ ↓
قطر الدائرة السمك

Normal stress $\rightarrow \sigma = \frac{F}{A}$

* Shear stress $\rightarrow \tau = \frac{F}{A} = \frac{F}{\frac{\pi}{4} d^2}$

* Bearing stress $\rightarrow \sigma_b = \frac{F}{A} = \frac{F}{t d}$

القوة
المعبر لها



allowable $\tau_{all} = 100 \text{ MPa}$

$\tau_b = 280 \text{ MPa}$
Bearing.

أوجد عدد البراغي اللازمة
لثبات هذه القطعة دون
أن تنكسر؟؟

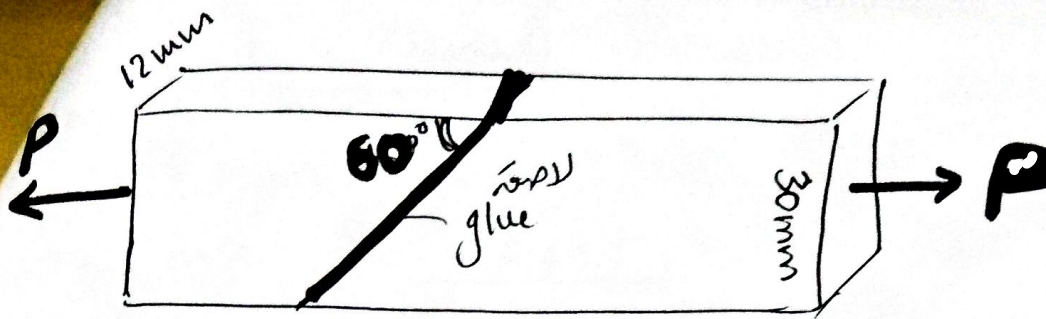
البن

$$\tau = \frac{F}{\left(\frac{\pi}{4} d^2\right) n} \Rightarrow 100 \times 10^6 = \frac{300 \times 10^3}{\frac{\pi}{4} (23)^2 n \times 10^6}$$

$\Rightarrow \boxed{n = 3.6}$ أربع براغي

$$\tau_b = \frac{F}{(t d) n} \Rightarrow 280 \times 10^6 = \frac{300 \times 10^3}{(1.2 \times 10^{-2}) (23 \times 10^{-3}) n}$$

$\Rightarrow \boxed{n = 3.88}$ أربع براغي



$$\sigma_{\text{ultimate}} = 860 \text{ MPa}$$

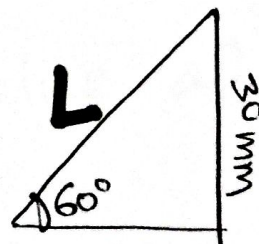
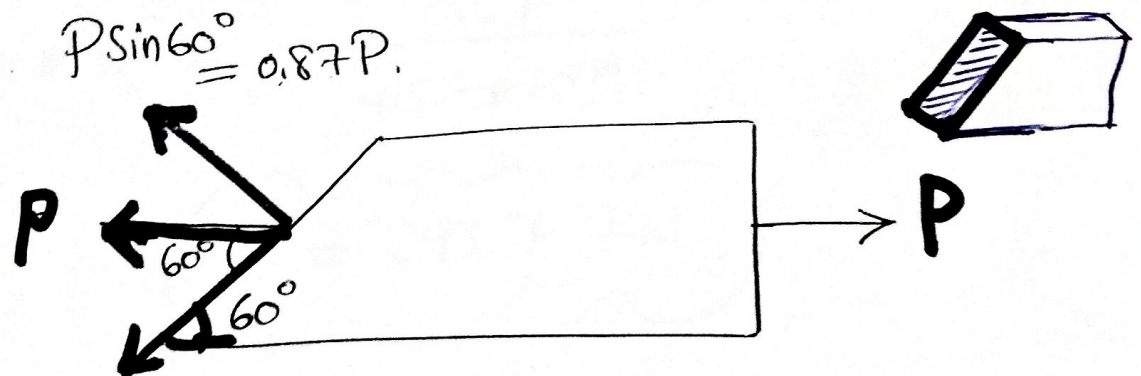
$$F.S. = 3$$

$$\tau_{\text{ultimate}} = 1060 \text{ MPa}$$

use $F.S. = 3$

$\Rightarrow$ Find the maximum Force P ?

Q31



$$* \quad L = \frac{30}{\sin 60^\circ} = 34.64 \text{ mm}$$

ساحة المثلث $*$

$$A = (34.64)(12) = 415.7 \text{ mm}^2$$

$$* \quad \sigma_{\text{allow}} = \frac{860}{3} = 286.7 \text{ MPa}$$

$$* \quad \tau_{\text{allow}} = \frac{1060}{3} = 353.3 \text{ MPa}$$

$$\sigma_{allow} = \frac{\text{Force (Normal)}}{\text{Area}}$$

$$286.7 \times 10^6 = \frac{0.87 P}{415.7 \times 10^6}$$

$$\Rightarrow P = 137 \text{ KN}$$

$$* \quad \tau_{allow} = \frac{\text{Force (Shear)}}{\text{Area}}$$

$$353.3 \times 10^6 = \frac{0.5 P}{415.7 \times 10^6}$$

$$P = 293.7 \text{ KN}$$

$\therefore$ the maximum value of force P is

$$P_{max} = 137 \text{ KN}$$

والتي لقوة
الانقل (الامن)

Safe

Strain الانفعال

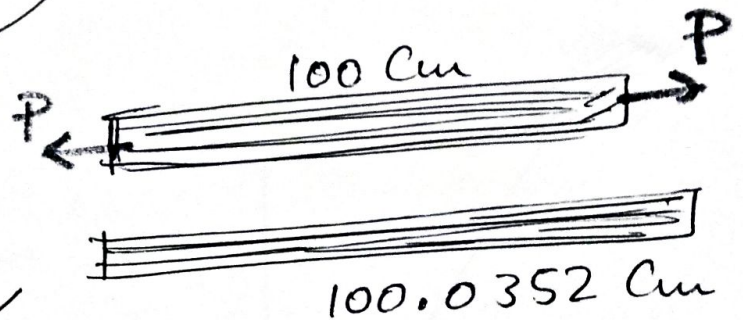
1) Normal strain ϵ (ϵ)

$$\epsilon = \frac{\Delta L}{L_0}$$

$$\epsilon = \frac{100.0352 - 100}{100}$$

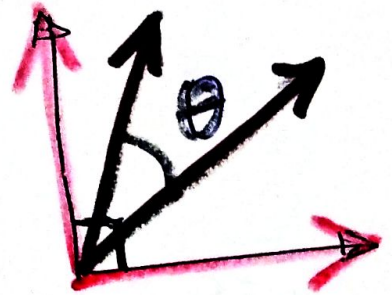
$$\epsilon = 3.25 \times 10^{-4}$$

new orig
⇒ unitless



2) Shear strain γ (γ)

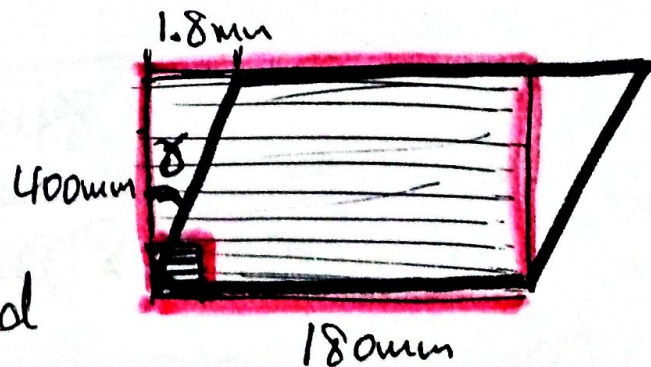
$$\gamma = \frac{\pi}{2} - \theta$$



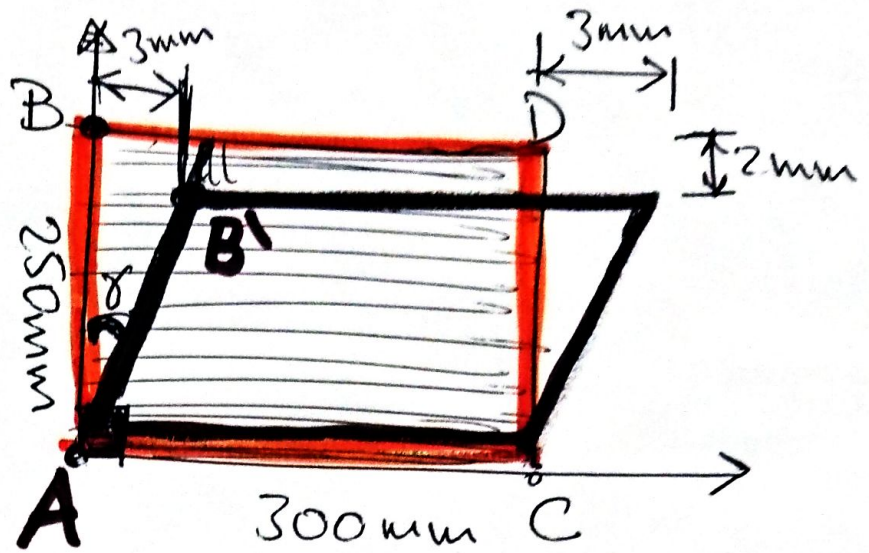
وفقاً، لثقلية، زواوية، (بالراديان)
Radian

$$\gamma = \frac{1.651}{400}$$

$$\gamma = \frac{1.8}{400} = 4.5 \times 10^{-3} \text{ rad}$$



- Find
- 1) E_{AB} ?
 - 2) γ ?

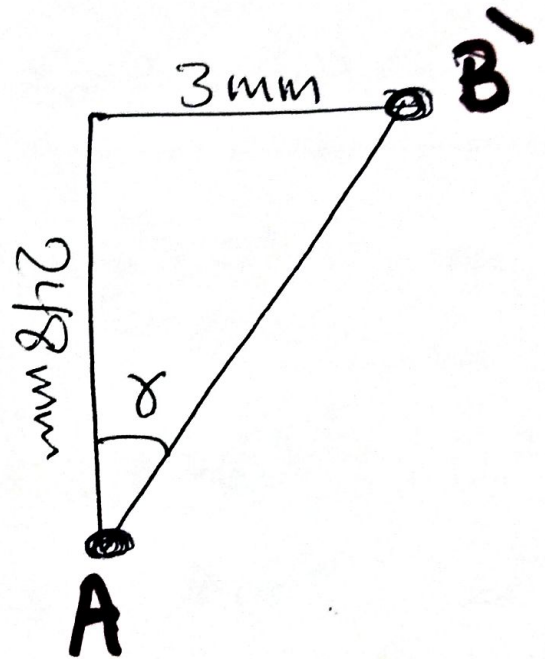


الحل

① $AB = 250 \text{ mm}$

$$AB' = \sqrt{(248)^2 + (3)^2}$$

$$\Rightarrow AB' = 248.0181 \text{ mm}$$



* $E_{AB} = \frac{\Delta L}{L_0}$

$$= \frac{248.0181 - 250}{250} = -7.93 \times 10^{-3}$$

② $\gamma = \frac{\Delta L}{L_0} = \frac{3}{248}$

$$= 0.0121 \text{ rad.}$$



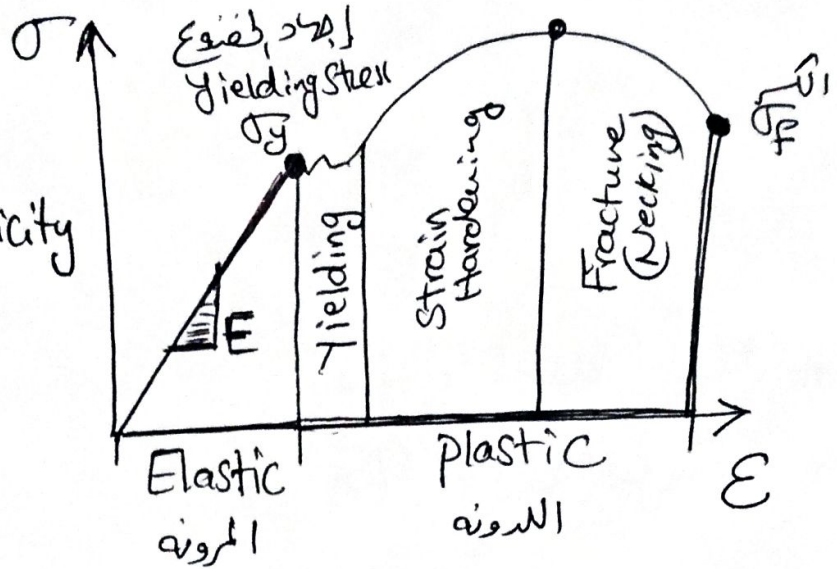
(Stress strain Diagram)

الضغط الأقصى (الحد)
ultimate stress
 σ_u

معامل مرونة
 $E \equiv$ Modulus of Elasticity

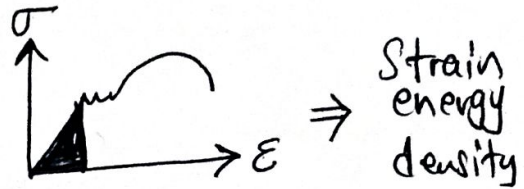
$$\sigma_{all} = E \epsilon_{long.}$$

Hook's law



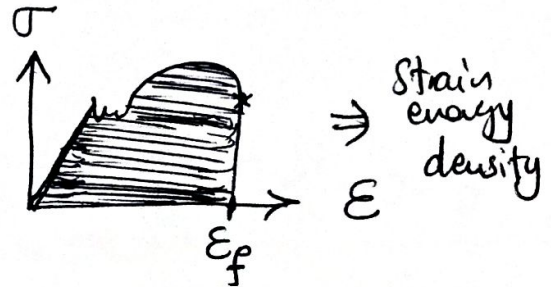
$$U_r = \frac{1}{2} \epsilon \sigma$$

Modulus of resilience
المرونة في التشوه المرن



$$U_t = \int_0^{\epsilon_f} \sigma d\epsilon$$

Modulus of Toughness
المرونة في التشوه المرن

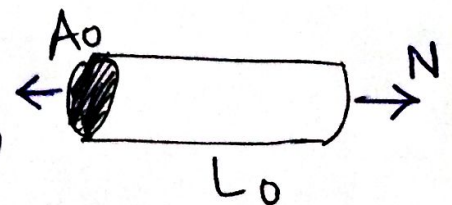


* Strain energy in elastic region = $(U_r)(\text{Volume})$
(Joule)

* Strain energy until fracture = $(U_t)(\text{Volume})$
(Joule)

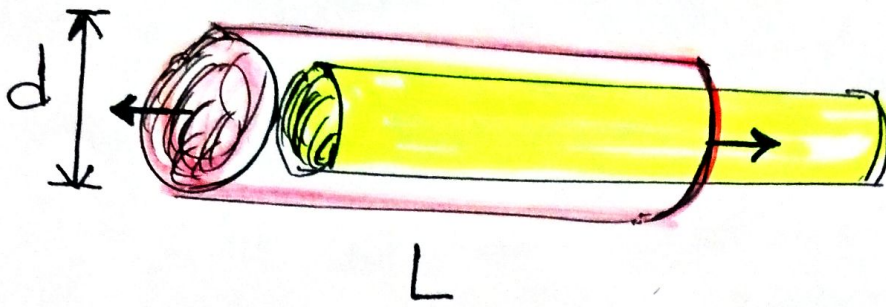
المرونة في التشوه المرن
Strain energy
 $U = \frac{1}{2} N \Delta L$

Volume = $A_0 L_0$
OR $U_r = \frac{U}{V}$



Poisson's Ratio (ν) معامل بویزون

Poisson's Ratio (ν)



* ازداد طول بمقدار ΔL
* ونقص قطر بمقدار Δd

الانفعال
الطولي

$$\epsilon_{\text{long}} = \frac{\Delta L}{L}$$

الانفعال
الجانبي

$$\epsilon_{\text{lat}} = \frac{\Delta d}{d}$$

$$\nu = \left| \frac{\epsilon_{\text{lat}}}{\epsilon_{\text{long}}} \right|$$

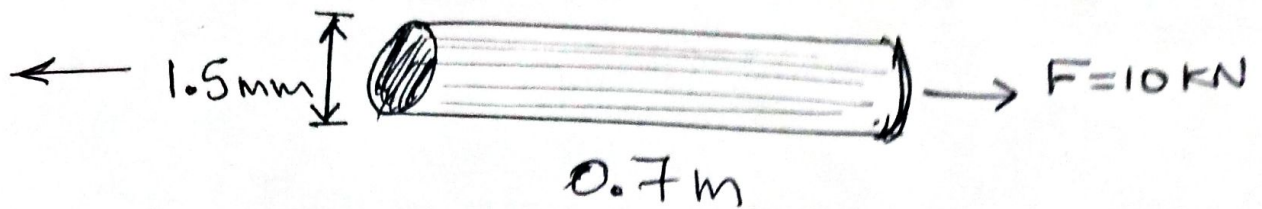
$$0 < \nu < 0.5$$

Hooke's
law

$$\sigma_{\text{all}} = E \epsilon_{\text{long}}$$

$$F.S. = \frac{\sigma_{\text{ult}}}{\sigma_{\text{all}}}$$

ex)



$$F = 10 \text{ kN} \Rightarrow d = 1.4892 \text{ mm}$$

لأنه أن يعطى

$$E = 200 \text{ GPa}$$

Find ① poisson's Ratio (ν)

Find ② The increase in its length
الزيادة في طول القيد

$$\text{ب1} \quad * \quad \epsilon_{\text{lat}} = \frac{\Delta d}{d} = \frac{1.5 - 1.4892}{1.5}$$

$$* \quad \epsilon_{\text{lat}} = 7.2 \times 10^{-3}$$

$$* \quad \sigma = \frac{F}{A} = \frac{10 \times 10^3}{\frac{\pi}{4} (1.5)^2 \times 10^{-6}} = 5.658 \text{ GPa}$$

$$* \quad \sigma = E \epsilon_{\text{long}} \Rightarrow \epsilon_{\text{long}} = \frac{5.658 \times 10^9}{200 \times 10^9}$$
$$\epsilon_{\text{long}} = 0.02829$$

$$① \quad \nu = \left| \frac{\epsilon_{lat}}{\epsilon_{long}} \right|$$

$$\nu = \frac{7.2 \times 10^{-3}}{0.02829}$$

$$\nu = 0.255$$

$$② \quad \epsilon_{long} = \frac{\Delta L}{L}$$

$$0.02829 = \frac{\Delta L}{0.7}$$

$$\Delta L = 0.0198 \text{ m}$$

$$\Delta L = 19.8 \text{ mm}$$

مقدار الزيادة في
طول القضيب
المطوي
(Rod)

$$\text{Strain } (\epsilon)_{\text{long}} \equiv \frac{\text{التغير في الطول}}{\text{الطول الأصلي}} = \frac{\Delta L}{L}$$

$$\begin{aligned} & \text{(Dilatation) Volumetric strain } \epsilon_v \equiv \frac{\text{التغير في الحجم}}{\text{الحجم الأصلي}} = \frac{\Delta V}{V} \end{aligned}$$

$$\Rightarrow \epsilon_v = \frac{\Delta V}{V}$$

$$\Rightarrow \epsilon_v = \epsilon_{\text{long}} [1 - 2\nu]$$

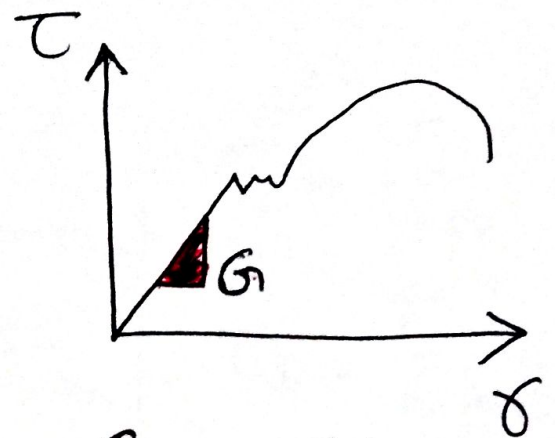
$$\sigma = E \epsilon$$

Hooke's law

$$\tau = G \gamma$$

$$G = \frac{E}{2(1+\nu)}$$

↓
modulus of rigidity

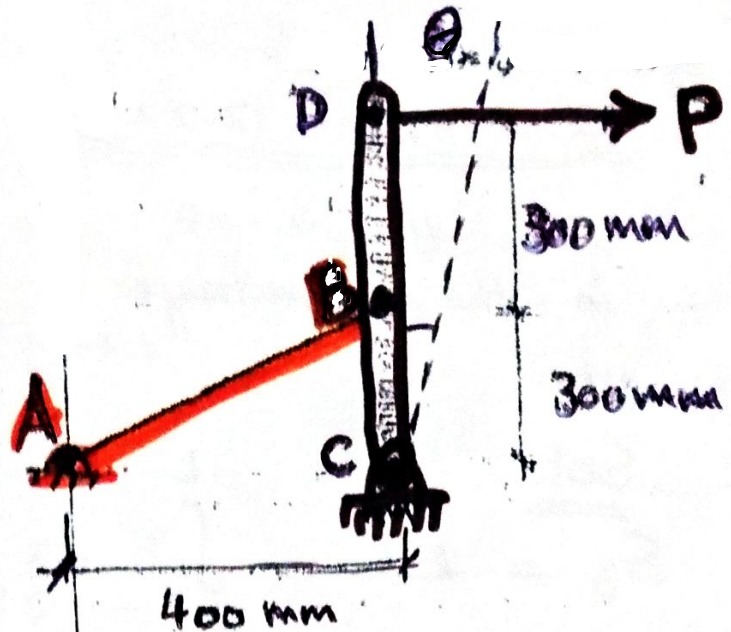


$G \equiv$ modulus of rigidity
 $\gamma \equiv$ shear strain

Ex.

Given $\epsilon_{AB} = 0.0035$

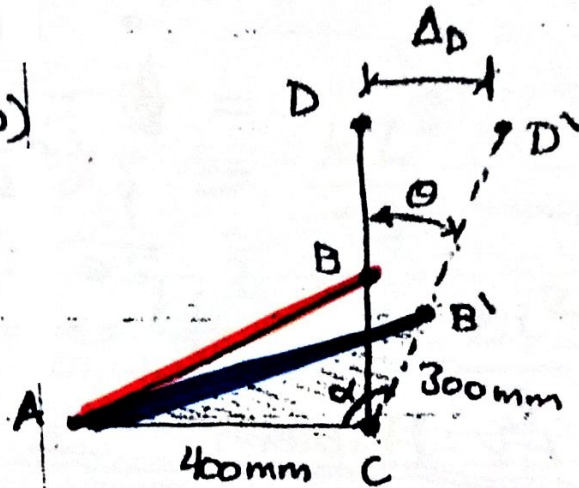
Find the displacement of point D?



Sol.

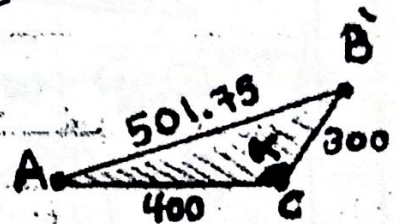
$$AB = \sqrt{300^2 + 400^2} = 500 \text{ mm}$$

$$\begin{aligned} AB' &= AB + \epsilon_{AB} AB \\ &= 500 + 0.0035(500) \\ &= 501.75 \text{ mm} \end{aligned}$$



$$(501.75)^2 = (300)^2 + (400)^2 - 2(300)(400) \cos \alpha$$

$$\Rightarrow \alpha = 90.4185^\circ$$



$$\therefore \theta = 90.4185^\circ - 90^\circ = 0.4185^\circ$$

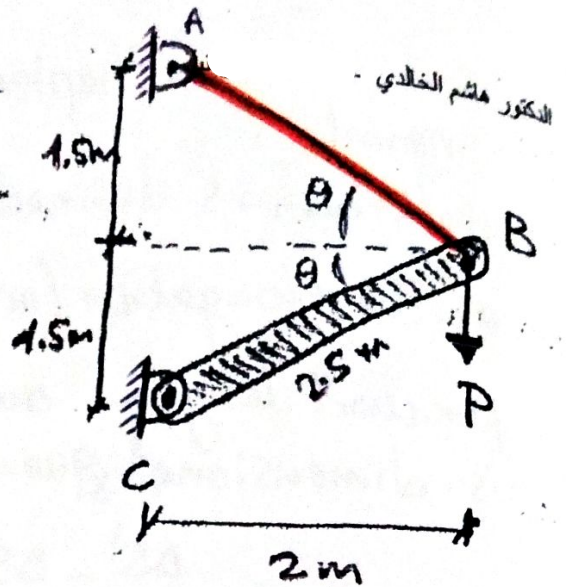
$$= \frac{\pi}{180^\circ} (0.4185^\circ) \text{ rad}$$

$$\therefore \Delta_D = 600 \theta$$

$$= 600 \left(\frac{\pi}{180^\circ} \right) (0.4185^\circ)$$

$$= 4.38 \text{ mm}$$

Ex. given $P = 15 \text{ kN}$
 $A_{\text{cable}} = 120 \text{ mm}^2$
 $A_{\text{CB}} = 250 \text{ mm}^2$



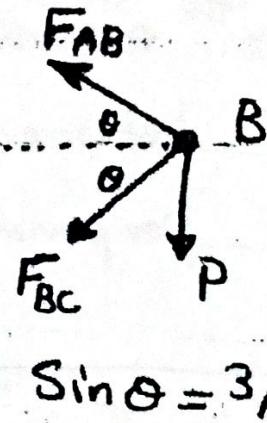
- (a) find σ_{AB} and σ_{BC} ?
 (b) if the cable elongates $1.3 \text{ mm} \rightarrow$ find ϵ_{AB} ?
 (c) if the bar shortens $0.62 \text{ mm} \rightarrow$ find ϵ_{BC} ?

Sol.

Statics

$$\sum F_x = 0 \rightarrow F_{AB} = -F_{BC}$$

$$\sum F_y = 0 \rightarrow F_{AB} \sin \theta - F_{BC} \sin \theta - P = 0$$



$$\sin \theta = 3/5$$

Solve the two equations:

* $F_{BC} = 12.5 \text{ kN}$ Compression

* $F_{AB} = 12.5 \text{ kN}$ Tension

✓ (a)

$$\sigma_{AB} = \frac{F_{AB}}{A_{\text{cable}}} = \frac{12.5 \text{ kN}}{120 \text{ mm}^2} = 104 \text{ MPa (Tens)}$$

and $\sigma_{BC} = \frac{F_{BC}}{A_{BC}} = \frac{12.5 \text{ kN}}{250 \text{ mm}^2} = 50 \text{ MPa (Comp)}$

✓ (b)

$$\epsilon_{AB} = \frac{\delta_{AB}}{L_{AB}} = \frac{1.3 \times 10^{-3}}{2.5} = 520 \times 10^{-6}$$

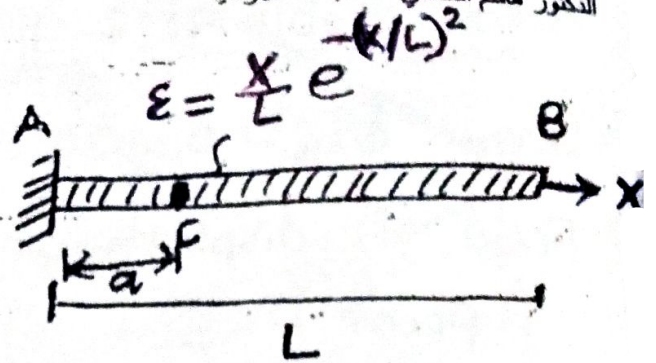
✓ (c)

$$\epsilon_{BC} = \frac{\delta_{BC}}{L_{BC}} = \frac{0.62 \times 10^{-3}}{2.5} = 248 \times 10^{-6}$$

$$\epsilon = \frac{x}{L} e^{-\left(\frac{x}{L}\right)^2}$$

Ex.

- 1) Find the increase in the beam length?
- 2) Find δ_c ?

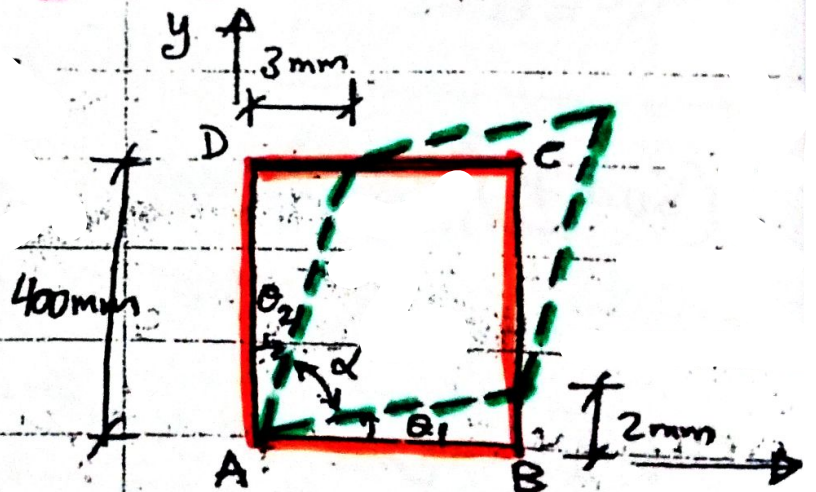


Sol.

$$\begin{aligned} \delta_B = \Delta L &= \int_0^L \epsilon \, dx \\ &= \frac{1}{L} \int_0^L x e^{-\left(\frac{x}{L}\right)^2} \, dx \\ &= -L \left[\frac{e^{-\left(\frac{x}{L}\right)^2}}{2} \right]_0^L = \frac{L}{2} [1 - (1/e)] \\ &= \frac{L}{2e} [e - 1] \end{aligned}$$

Ex.

Find the average shear strain γ_{xy} ?



Sol.

$$\theta_1 = \tan \theta_1 = \frac{2}{300} = 0.006667 \text{ rad}$$

$$\theta_2 = \tan \theta_2 = \frac{3}{400} = 0.0075 \text{ rad}$$

$$\begin{aligned} \gamma_{xy} &= \frac{\pi}{2} - \alpha = \frac{\pi}{2} - \left(\frac{\pi}{2} - (\theta_1 + \theta_2) \right) \\ &= \theta_1 + \theta_2 = 0.0142 \text{ rad} \end{aligned}$$

$$\sigma_{all} = \frac{P}{A} \rightarrow \text{Normal}$$

$$\tau = \frac{P}{A} \rightarrow \text{parallel}$$

Shear Stress

$$\epsilon_{long} = \frac{\Delta L}{L}$$

"axial" longitudinal strain الانفعال الطولي

$$\epsilon_{lat} = \frac{\Delta d}{d}$$

"lateral" strain الانفعال العرضي الجانبي

$$\nu = \frac{\epsilon_{lat}}{\epsilon_{long}}$$

Poisson's Ratio معامل بوزون

$$\sigma_{all} = E \epsilon_{long}$$

$$\epsilon_v = \frac{\Delta V}{V}$$

$\epsilon_v \equiv$ Dilatation
"Volumetric strain"
الانفعال الحجمي

$$\epsilon_v = \epsilon_{long} [1 - 2\nu]$$

$$V = AL$$

$$G = \frac{E}{2(1+\nu)}$$

$E \equiv$ modulus of elasticity معامل المرونة
 $G \equiv$ modulus of rigidity معامل الصلابة

$$U_r = \frac{1}{2} \sigma_{all} \epsilon_{long}$$

$U_r \equiv$ modulus of resilience معامل المروعة

$$U_r = \frac{U}{V}$$

$U \equiv$ strain energy طاقة الانفعال

$$U = \frac{1}{2} P \Delta L$$

$$A = \frac{\pi}{4} d^2 \quad \text{المساحة}$$

$$A = \frac{\pi}{4} (d_o^2 - d_i^2) \quad \text{المساحة المتوسطة}$$

Factor of safety معامل الأمان

$$F.S. = \frac{\sigma_{ult}}{\sigma_{all}} \rightarrow \text{allowable}$$

ultimate stress

قطر الحبل
 $d = 25 \text{ mm}$
 $E = 200 \text{ GPa}$

$\nu = 0.36$

$\sigma_{ult} = 715 \text{ MPa}$

١) أوجد إجهاد لفة في الحبل (F)

٢) أوجد إجهاد الشد في الحبل (σ)

٣) أوجد الانفعال في الحبل (ε)

٤) أوجد طاقة الانفعال في الحبل (U) Strain energy

٥) أوجد الانقسان في قطر الحبل (Δd)

٦) أوجد معامل المرونة (G)

٧) أوجد معامل الأمان في الحبل (F.S.)

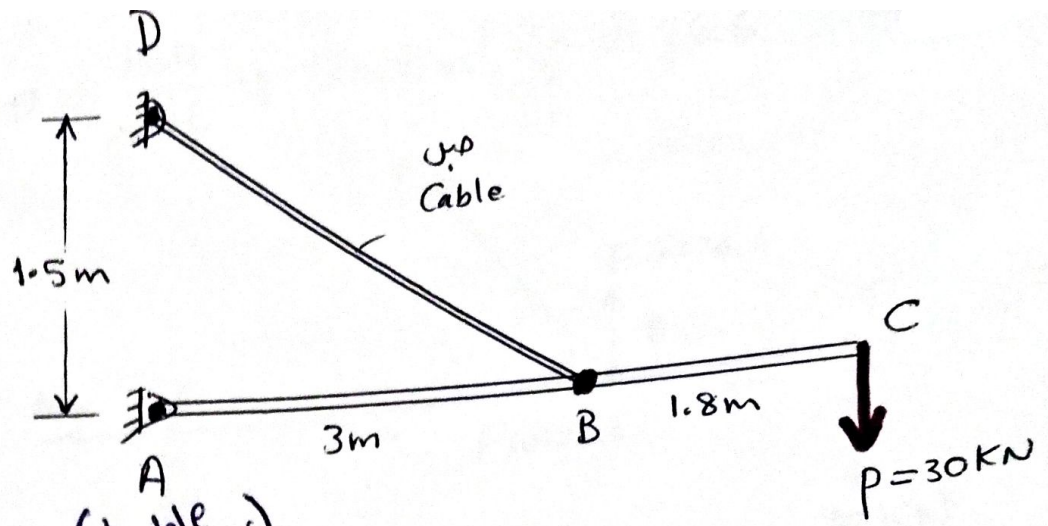
٨) أوجد Dilatation (ε_v)

٩) أوجد التغير في حجم الحبل (ΔV)

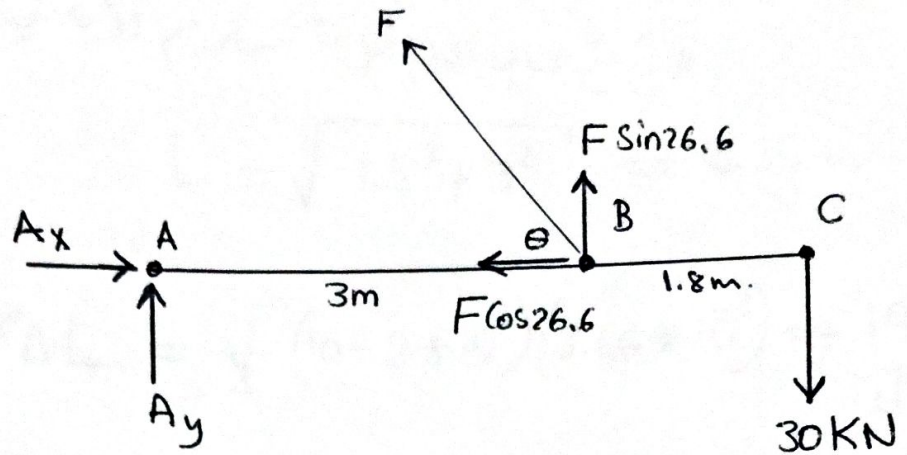
١٠) إذا علمت أن قطر الحبل عند

النقطة A يساوي 12 mm

١١) أوجد إجهاد لفة في الحبل عند النقطة A (T)
 عند جانبي الحبل A
 (Double shear)



$$\tan \theta = \frac{\text{المقابل}}{\text{المجاور}} \Rightarrow \theta = \tan^{-1} \frac{1.5}{3} = 26.6^\circ$$



$$\sum M_A = 0 \Rightarrow [F \sin 26.6][3] - [30][4.8] = 0$$

$$\textcircled{1} \Rightarrow F = 107.2 \text{ kN} \quad \text{القوة في الجيب}$$

$$* \sum F_x = 0 \Rightarrow A_x - 107.2 \cos 26.6^\circ = 0$$

$$A_x = 96 \text{ kN}$$

$$* \sum F_y = 0 \Rightarrow A_y - 30 + 107.2 \sin 26.6^\circ = 0$$

$$A_y = -18 \text{ kN}$$

$$* \text{المحصلة } A \Rightarrow A = \sqrt{A_x^2 + A_y^2} \Rightarrow \sqrt{96^2 + 18^2}$$

$$\text{القوة عند A} \Rightarrow A = 97.7 \text{ kN}$$

مساحة مقطع الجيب

$$* \text{Area} = \frac{\pi}{4} (25)^2 = 491 \text{ mm}^2$$

$$\textcircled{2} * \sigma_{all} = \frac{F}{\text{Area}} = \frac{107.2 \times 10^3}{491 \times 10^{-6}} = 218.3 \text{ MPa}$$

$$* \sigma_{all} = E \epsilon_{long} \Rightarrow \textcircled{3} \epsilon_{long} = \frac{218.3 \times 10^6}{200 \times 10^9} = 1.092 \times 10^{-3}$$

$$\epsilon_{\text{long}} = \frac{\Delta L}{L} \Rightarrow \Delta L = (1.092 \times 10^{-3})(3.35) = 3.66 \times 10^{-3} \text{ m}$$

مقدار تغییر طول میل با سازه نام نظریه میخاکورس

$$L = \sqrt{1.5^2 + 3^2} = 3.35 \text{ m}$$

$$\textcircled{4} \quad * \quad U = \frac{1}{2} F \Delta L = \frac{1}{2} (107.2 \times 10^3)(3.66 \times 10^{-3}) = 196.2 \text{ Joule}$$

$$* \quad \nu = \frac{\epsilon_{\text{lat}}}{\epsilon_{\text{long}}} \Rightarrow \epsilon_{\text{lat}} = (0.36)(1.092 \times 10^{-3}) = 3.93 \times 10^{-4}$$

$$* \quad \epsilon_{\text{lat}} = \frac{\Delta d}{d} \Rightarrow \textcircled{5} \quad \Delta d = (3.93 \times 10^{-4})(25 \times 10^{-3}) = 9.83 \times 10^{-6} \text{ m}$$

$$* \quad \textcircled{6} \quad G = \frac{E}{2(1+\nu)} = \frac{200 \times 10^9}{2(1+0.36)} = 73.5 \text{ GPa}$$

$$\textcircled{7} \quad * \quad F.S. = \frac{\sigma_{\text{ult}}}{\sigma_{\text{all}}} = \frac{715 \times 10^6}{218.3 \times 10^6} = 3.28$$

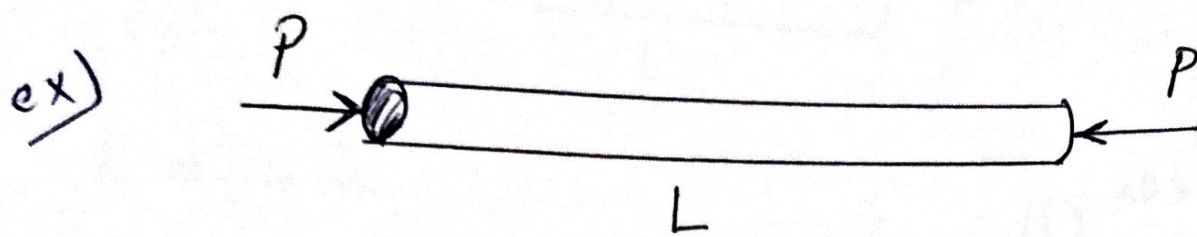
$$\textcircled{8} \quad * \quad \epsilon_v = \epsilon_{\text{long}} [1 - 2\nu] = (1.092 \times 10^{-3}) [1 - 2(0.36)] = 5.33 \times 10^{-4}$$

$$* \quad V = A L = (491 \times 10^{-6})(3.35) = 1.64 \times 10^{-3} \text{ m}^3$$

$$* \quad \epsilon_v = \frac{\Delta V}{V} \Rightarrow \textcircled{9} \quad \Delta V = (1.64 \times 10^{-3})(5.33 \times 10^{-4}) = 8.77 \times 10^{-7} \text{ m}^3$$

$$\textcircled{10} \quad * \quad \text{Double shear : } \tau = \frac{\text{Force}/2}{\text{Area}} = \frac{\frac{(97.7)}{2} \times 10^3}{\frac{\pi}{4} (12)^2 \times 10^{-6}}$$

$$\Rightarrow \tau = 431.9 \text{ MPa} .$$



$$L = 400 \text{ mm}$$

$$d_o = 60 \text{ mm}$$

$$d_i = 50 \text{ mm}$$

$$\epsilon_{\text{long}} = 650 \times 10^{-6}$$

$$\sigma = 50 \text{ MPa}$$

? ΔL أوجد ①

? P أوجد ②

? U أوجد ③

? E أوجد ④

الحل

$$\epsilon_{\text{long}} = \frac{\Delta L}{L} \Rightarrow \Delta L = (650 \times 10^{-6})(400 \times 10^{-3})$$

$$\text{①} \Rightarrow \Delta L = 2.6 \times 10^{-4} \text{ m}$$

$$\sigma = \frac{P}{A} \Rightarrow P = \left[\frac{\pi}{4} (60^2 - 50^2) \times 10^{-6} \right] [50 \times 10^6]$$

$$\text{②} \Rightarrow P = 43.2 \text{ kN}$$

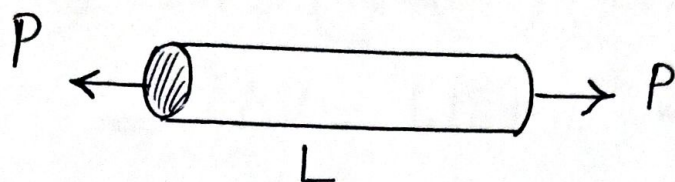
$$U = \frac{1}{2} P \Delta L = \frac{1}{2} (43.2 \times 10^3) (2.6 \times 10^{-4})$$

$$\text{③} \Rightarrow U = 5.62 \text{ joule}$$

$$\sigma = E \epsilon_{\text{long}} \Rightarrow E = \frac{50 \times 10^6}{650 \times 10^{-6}}$$

$$\text{④} \Rightarrow E = 76.9 \text{ GPa.}$$

ex)



$$L = 1.6 \text{ m}$$

$$d = 35 \text{ mm}$$

$$E = 75 \text{ GPa}$$

$$\nu = 0.33$$

$$\Delta L = 3.6 \text{ mm}$$

$$\cdot \epsilon_{\text{lat}} \text{ ①}$$

$$\cdot \Delta d \text{ ②}$$

$$\cdot P \text{ ③}$$

$$\cdot \Delta V \text{ ④}$$

$$\cdot G \text{ ⑤}$$

$$\cdot U \text{ ⑥}$$

$$\cdot U_r \text{ ⑦}$$

* if the ultimate load equals 540 kN ⑧

Find F.S. ?

$$\underline{\underline{\text{Sol}}}$$

$$\cdot \epsilon_{\text{long}} = \frac{\Delta L}{L} \Rightarrow$$

$$\epsilon_{\text{long}} = \frac{3.6 \times 10^{-3}}{1.6} = 2.25 \times 10^{-3}$$

$$\text{①} \cdot \epsilon_{\text{lat}} = \nu \epsilon_{\text{long}} = (0.33)(2.25 \times 10^{-3}) = 7.5 \times 10^{-4}$$

$$\cdot \epsilon_{\text{lat}} = \frac{\Delta d}{d} \Rightarrow \text{②} \Delta d = 2.625 \times 10^{-5} \text{ m}$$

$$\cdot \sigma = \frac{P}{A} \Rightarrow \sigma_{\text{all}} = E \epsilon_{\text{long}} = (75 \times 10^9)(2.25 \times 10^{-3})$$

$$\sigma = 169 \text{ MPa}$$

$$\cdot P = (169 \times 10^6) \frac{\pi}{4} (35)^2 \times 10^{-6}$$

$$\text{③} P = 162.4 \text{ kN}$$

$$\cdot \epsilon_v = \epsilon_{\text{long}} [1 - 2\nu] = 7.65 \times 10^{-4}$$

$$\cdot V = AL = \frac{\pi}{4} (35)^2 \times 10^{-6} [1.6] = 1.54 \times 10^{-3} \text{ m}^3$$

$$\epsilon_v = \frac{\Delta V}{V} \Rightarrow \Delta V = (1.54 \times 10^3) (7.65 \times 10^{-4})$$

$$\Rightarrow \textcircled{4} \Delta V = 1.18 \times 10^{-6} \text{ m}^3$$

$$* \textcircled{5} G = \frac{E}{2(1+\nu)} = \frac{75 \times 10^9}{2(1+0.33)} = 28.2 \text{ GPa}$$

$$* \textcircled{6} U = \frac{1}{2} P \Delta L = \frac{1}{2} (162.4 \times 10^3) (3.6 \times 10^{-3})$$

$$\Rightarrow U = 292.3 \text{ Joule. (J)}$$

$$\textcircled{7} u_r = \frac{U}{V} = \frac{292.3}{1.54 \times 10^3} = 190 \text{ KJ/m}^3$$



$$\textcircled{8} P_{ult} = 540 \text{ kN}$$

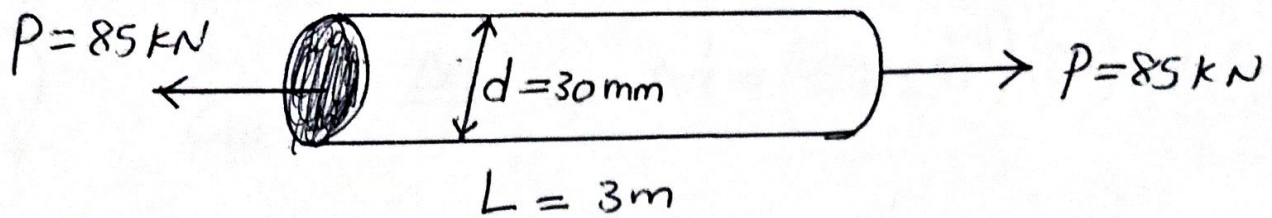
$$\Rightarrow \sigma_{ult} = \frac{P_{ult}}{\text{Area}} = \frac{540 \times 10^3}{\frac{\pi}{4} (35)^2 \times 10^{-6}}$$

$$\Rightarrow \sigma_{ult} = 561.3 \text{ MPa.}$$

$$* F.S. = \frac{\sigma_{ult}}{\sigma_{all}}$$

$$F.S. = \frac{561.3 \times 10^6}{169 \times 10^6}$$

$$\Rightarrow F.S. = 3.32$$



$$E = 70 \text{ GPa}$$

$$\nu = \frac{1}{3} = 0.333$$

- Find
- ① Elongation ΔL ? مقدار الاستطالة
 - ② decrease in diameter Δd ? النقص في القطر
 - ③ the increase in volume ΔV ? الزيادة في الحجم
 - ④ the modulus of rigidity G ? معامل الصلابة
 - ⑤ the Dilatation ϵ_v ? الانفعال الحجمي
 - ⑥ the volumetric strain ϵ_v ? الانفعال الحجمي
 - ⑦ the strain energy U ? طاقة الانفعال

الحل

$$* A = \frac{\pi}{4} (d^2) = \frac{\pi}{4} (30)^2 = 707 \text{ mm}^2$$

$$* V = AL = [(707) \times 10^{-6}] [3] = 2.12 \times 10^{-3} \text{ m}^3$$

$$* \sigma = \frac{P}{A} = \frac{85 \times 10^3}{707 \times 10^{-6}} = 120 \text{ MPa}$$

$$* \sigma = E \epsilon_{\text{long}} \Rightarrow \epsilon_{\text{long}} = \frac{120 \times 10^6}{70 \times 10^9} = 1.714 \times 10^{-3}$$

$$\textcircled{1} * \epsilon_{\text{long}} = \frac{\Delta L}{L} \Rightarrow \Delta L = (1.714 \times 10^{-3})(3) = 5.14 \times 10^{-3} = 5.14 \text{ mm}$$

$$\star \nu = \frac{\epsilon_{lat}}{\epsilon_{long}} \Rightarrow \epsilon_{lat} = (0.333)(1.714 \times 10^{-3}) = 5.71 \times 10^{-4}$$

$$\textcircled{2} \star \epsilon_{lat} = \frac{\Delta d}{d} \Rightarrow \Delta d = (5.71 \times 10^{-4})(30) = 0.01714 \text{ mm}$$

$$\textcircled{5} + \textcircled{6} \star \epsilon_v = \epsilon_{long}[1 - 2\nu] = 1.714 \times 10^{-3}[1 - 2(0.333)] = 5.71 \times 10^{-4}$$

$$\textcircled{3} \star \epsilon_v = \frac{\Delta V}{V} \Rightarrow \Delta V = [5.71 \times 10^{-4}][2.12 \times 10^{-3}] = 1.21 \times 10^{-6} \text{ m}^3$$

$$\textcircled{4} \star G = \frac{E}{2(1+\nu)} = \frac{70 \times 10^9}{2[1+0.333]} = 26.3 \text{ GPa}$$

$$\textcircled{7} \star U = \frac{1}{2} P \Delta L = \frac{1}{2} (85 \times 10^3)(5.14 \times 10^{-3}) = 218 \text{ joule}$$

* • * * • * * • * * • *

Ex

Find the Normal Stress (σ)
in the Leg AC?

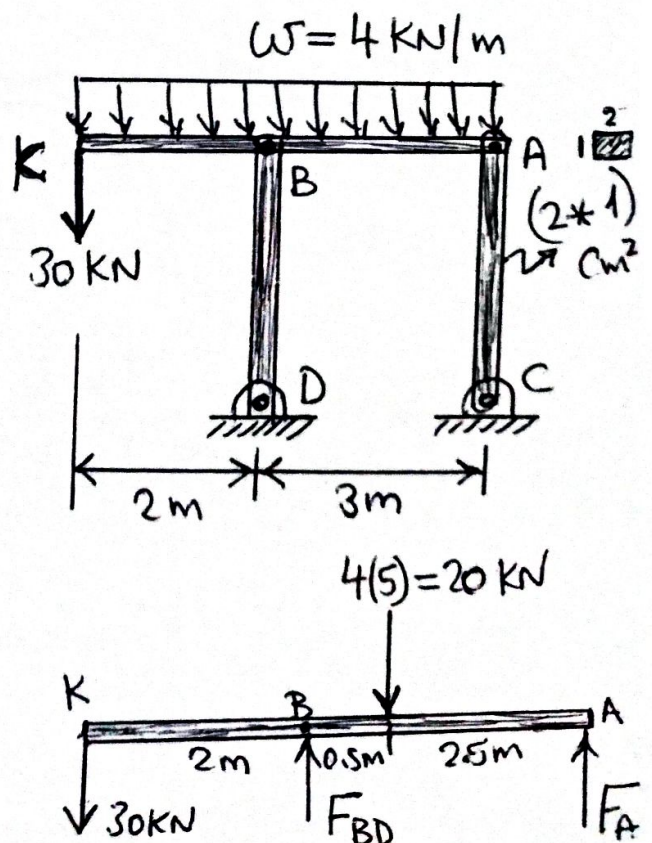
$$\sum M_B = 0$$

$$\Rightarrow 3F_{AC} - 20(0.5) + 30(2) = 0$$

$$\Rightarrow F_{AC} = -16.7 \text{ kN}$$

$$\star \sigma = \frac{F_{AC}}{A_{AC}} = \frac{16.7 \times 10^3}{(2)(1) \times 10^{-4}}$$

$$\Rightarrow \sigma = 83.5 \text{ MPa}$$



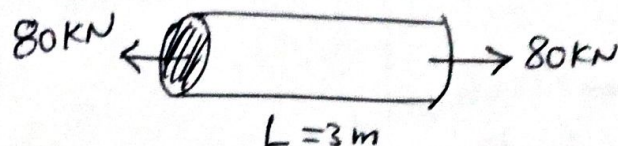
EX

$$L = 3 \text{ m}$$

$$d = 40 \text{ mm}$$

$$P = 80 \text{ kN}$$

$$E = 200 \text{ GPa}$$



Find ① Strain energy U ?

② Modulus of resilience U_r ?

~~Ex~~ *
$$\sigma = \frac{P}{A} = \frac{80 \times 10^3}{\frac{\pi}{4}(40)^2 \times 10^{-6}} = 63.7 \text{ MPa}$$

*
$$\sigma = E \epsilon \Rightarrow \epsilon = \frac{63.7 \times 10^6}{200 \times 10^9} = 3.18 \times 10^{-4}$$

*
$$\epsilon = \frac{\Delta L}{L} \Rightarrow \Delta L = (3.18 \times 10^{-4})(3) = 9.55 \times 10^{-4} \text{ m}$$

① *
$$U = \frac{1}{2} P \Delta L = \frac{1}{2} (80 \times 10^3) (9.55 \times 10^{-4})$$

$$U = 38.2 \text{ joule.}$$

② *
$$U_r = \frac{U}{V} = \frac{38.2}{\left[\frac{\pi}{4}(40)^2 \times 10^{-6}\right] [3]} = 10132 \text{ Joule/m}^3$$

~~Ex~~ *
$$U_r = \frac{1}{2} \sigma \epsilon = \frac{1}{2} (63.7 \times 10^6) (3.18 \times 10^{-4})$$

$$= 10132 \text{ joule /m}^3$$

Ex.

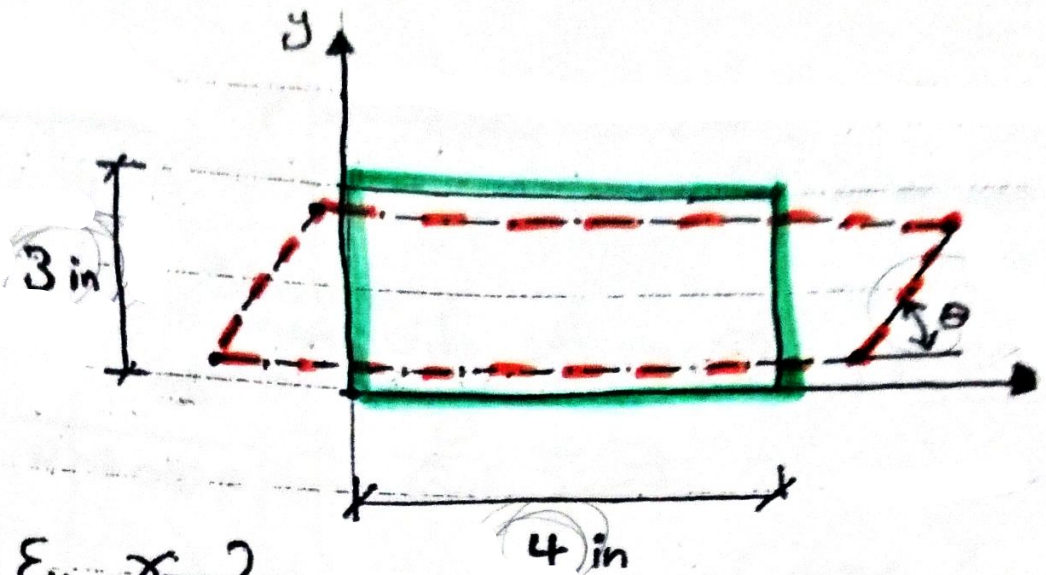
rubber block

$$\Delta x = 0.03 \text{ in}$$

$$\theta = 89.3^\circ$$

$$\nu = 0.5$$

Determine ϵ_x , ϵ_y , γ_{xy} ?



Sol.

$$\epsilon_x = \frac{\Delta x}{x} = \frac{0.03}{4} = 0.0075 \text{ in/in}$$

$$\epsilon_y = -\nu \epsilon_x = -0.5(0.0075)$$

$$= -0.00375 \text{ in/in}$$

$$\gamma_{xy} = \frac{\pi}{2} - \theta$$

$$= \frac{\pi}{2} - 89.3^\circ \left(\frac{\pi}{180^\circ} \right) = 0.0122 \text{ rad}$$



Ex.

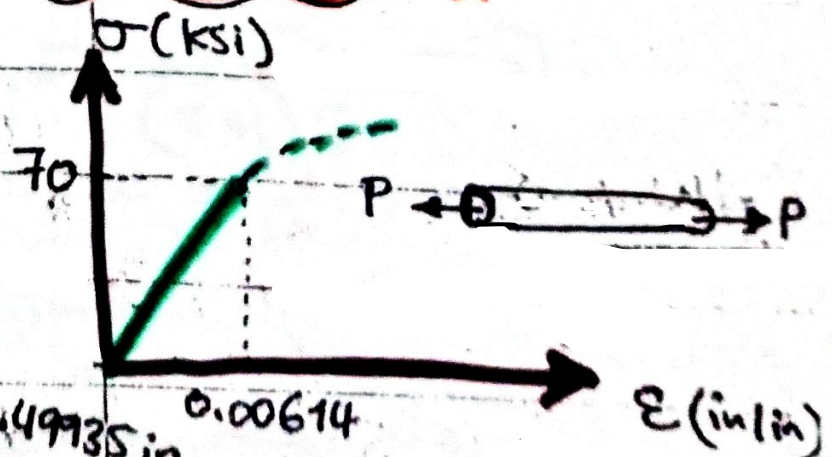
Given

$$L_o = 2 \text{ in}$$

$$d_o = 0.5 \text{ in}$$

When $P = 9 \text{ Kip} \rightarrow d' = 0.49935 \text{ in}$

* Find the shear modulus of rigidity G ?



Sol.

From the diagram:

$$E = \frac{\sigma}{\epsilon} = \frac{70}{0.00614} = 11400.65 \text{ ksi}$$

$$\text{After loading} \Rightarrow \sigma = \frac{9}{\frac{\pi}{4}(0.5)^2} = 45.84 \text{ ksi}$$

$$\epsilon_{\text{long}} = \frac{\sigma}{E} = \frac{45.84}{11400.65} = 0.0040208$$

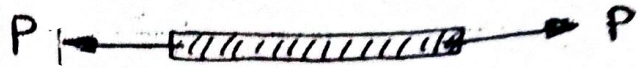
$$\epsilon_{\text{lat.}} = \frac{d' - d_0}{d_0} = \frac{0.49935 - 0.5}{0.5} = -0.0013$$

$$\nu = - \frac{\epsilon_{\text{lat.}}}{\epsilon_{\text{long}}} = - \frac{-0.0013}{0.0040208} = 0.32332$$

$$\therefore G = \frac{E}{2(1+\nu)} = \frac{11400.65}{2(1+0.32332)}$$

$$= 4.31 \times 10^3 \text{ ksi}$$


Example:



$$L_0 = 50 \text{ mm}$$

$$d_0 = 4 \text{ mm}$$

$$P_{\max} = 10 \text{ kN}$$

$$L_f = 80 \text{ mm}$$

$$d_f = 3 \text{ mm}$$

$$E = 200 \text{ GPa}$$

Calculate :

1. the Ultimate tensile stress UTS
2. Percent elongation ?
3. Percent reduction of area ?
4. Poisson's ratio ν ?
5. shear modulus of rigidity G ?

Sol.

$$1. \text{ UTS} = \frac{P_{\max}}{A_0} = \frac{10,000}{\frac{\pi}{4}(4)^2} = 796 \text{ MPa}$$

$$2. \text{ Elongation} = \frac{80 - 50}{50} \times 100 = 60\%$$

$$\begin{aligned} 3. \text{ Reduction of area} &= \frac{A_0 - A_f}{A_0} \times 100 \\ &= \frac{12.57 - 7.07}{12.57} \times 100 \\ &= 43.8\% \end{aligned}$$

$$4. \text{ } E_{\text{long}} = \frac{\Delta L}{L_0} = \frac{80 - 50}{50} = \frac{30}{50} = 0.6$$

$$\epsilon_{\text{lat}} = \frac{\Delta d}{d_0} = \frac{3 - 4}{4} = -\frac{1}{4} = -0.25$$

$$\therefore \nu = -\frac{\epsilon_{\text{lat}}}{E_{\text{long}}} = \frac{0.25}{0.6} = 0.416$$

$$6. \text{ } G = \frac{E}{2(1+\nu)} = \frac{200}{2(1+0.416)} = 70.6 \text{ GPa}$$

Mechanical Properties of Materials

Tension and Compression Test

is very important to determine the Mechanical Properties of the material, and Also the relationship between the average normal stress and average normal strain in many engineering materials.

** Tension Test :-

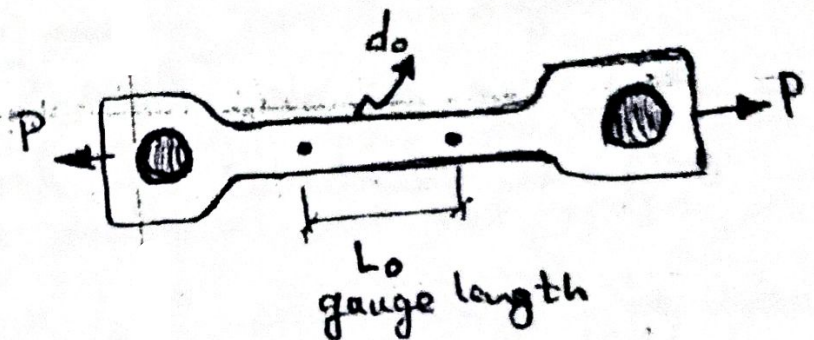
$$\delta = L - L_0$$

is the elongation
in the specimen

and is measured by :

- ① mechanical measurement by using the Caliper.
- ② Optical device called an extensometer.
- ③ electrical resistance strain gauge, which can be calibrated to read values of normal strain directly.

⇒ Change of the wire resistance is calibrated to give the strain



Stress-Strain diagram:

Conventional
Stress-Strain
diagram

True Stress-
Strain
diagram

* Conventional Stress-Strain diagram:-

→ nominal or engineering stress $\sigma = \frac{P}{A_0}$
and " " " " Strain $\epsilon = \frac{\delta}{L_0}$

- Here we divide "Always" by the original quantities of Area and Length (A_0) & (L_0)

- Steel is a Common used material, so its stress-strain diagram can be divided into four regions as:

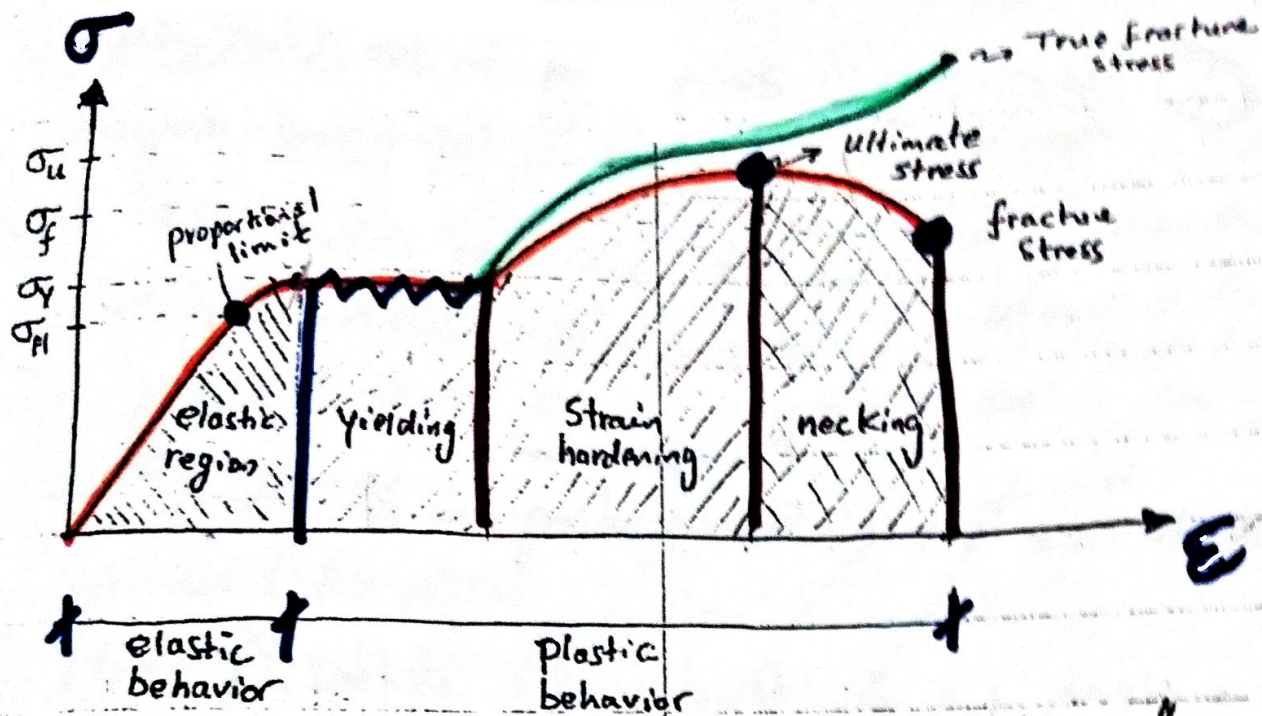
① Elastic region:

* the material is linear elastic (straight line)

* proportional limit is the upper stress limit of this line, i.e. σ_{PL}

* Elastic limit is the end of the small curvature after the proportional limit. Until

this point the material will return back to its original shape if the load is removed



"Conventional and True Stress-Strain diagram"

- ② Yielding :
- Plastic deformation starts
 - the material will not return back to its original shape when unloading
 - the upper yield point occurs first then the lower one does
 - the specimen will continue to elongate without any increase in load

- ③ Strain Hardening :
- occurs after yielding
 - reaches until the Ultimate Stress σ_u
 - decrease in the cross-sectional area of the specimen in a uniform way

- ④ Necking - decreasing in the cross-sectional area in a localized region.
- the specimen breaks at the fracture stress σ_f .

* * True Stress-Strain diagram :-

- Here we divide over the actual (current) Cross-Sectional area A and Current Specimen Length L .
- Here the material sustains increasing stress until failure σ_f and the necking region is always decreasing until fracture.

Ductile and Brittle Materials :-

* Ductile Material: which can be subjected to large strains before rupture. i.e (Steel)

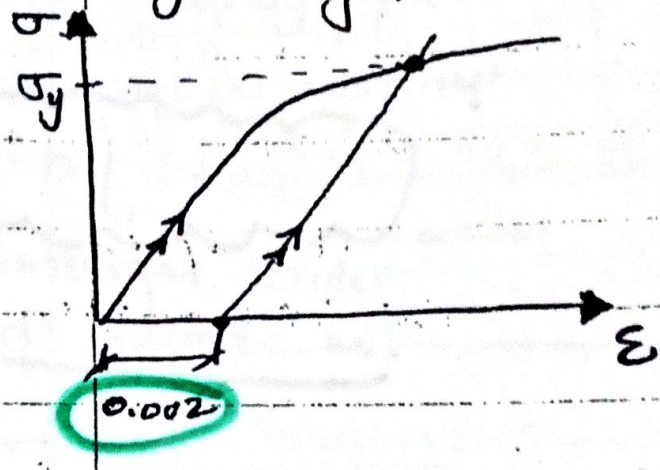
* Ductility

① Percent elongation at failure $= \frac{L_f - L_0}{L_0} (100\%)$

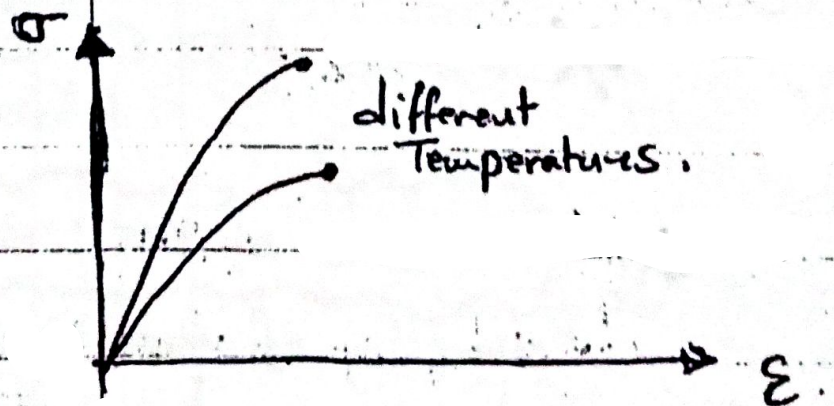
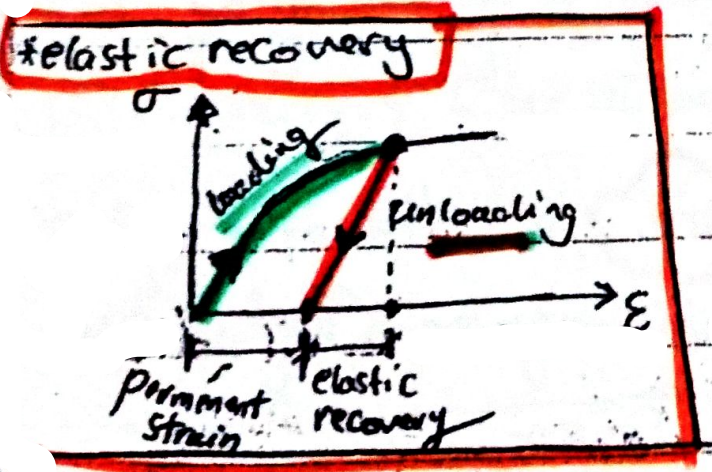
② Percent reduction in area at failure $= \frac{A_0 - A_f}{A_0} (100\%)$

** Some ductile materials (as Aluminum), the constant yielding will not appear OR is not clear just after the elastic region, therefore we use the "offset method" to determine the yielding stress and the yielding point on the graph.

⇒ We take offset strain of about $0.2\% = 0.002$



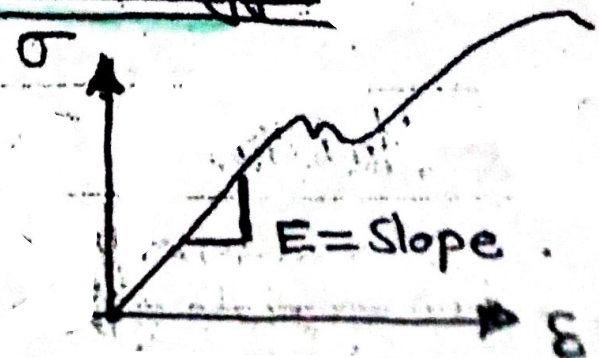
* Brittle material : that exhibit little or no yielding before failure, i.e. (glass)



Hook's Law and Strain Energy:

$$\sigma = E \epsilon$$

E = modulus of Elasticity
OR Young's Modulus.



**** Strain Energy** is the amount of energy stored in the material due to the applied strain.

**** the strain-energy density or the energy per unit volume u can be written as:**

$$u = \int \sigma d\epsilon$$

which represents the area under the stress-strain diagram.

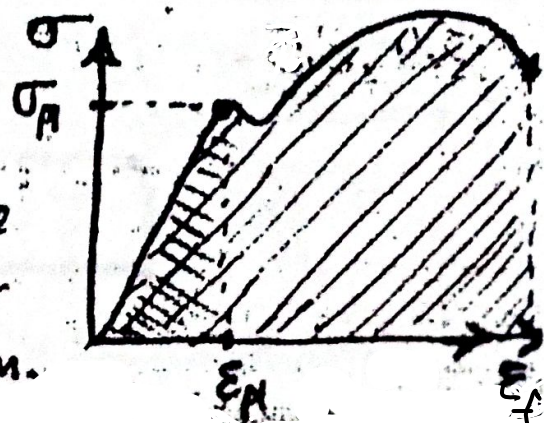
***** The strain energy per unit volume in the elastic region is given as:

$$* \quad u = \frac{1}{2} \sigma \epsilon = \frac{1}{2} \sigma \frac{\sigma}{E} = \frac{1}{2} \frac{\sigma^2}{E}$$

***** Modulus of resilience is the strain energy density of the elastic region (until the proportional limit), i.e.:-

$$* \quad U_r = \frac{1}{2} \sigma_p \epsilon_p = \frac{1}{2} \frac{\sigma_p^2}{E}$$

**** Modulus of Toughness** is the strain energy density under the whole stress-strain diagram until fracture (U_t).



Ex.

Given:

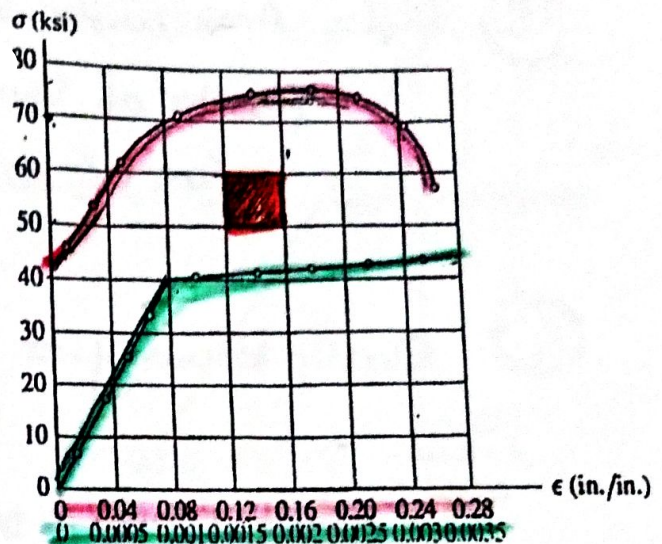
$$d_0 = 0.5 \text{ in}$$

$$L_0 = 2 \text{ in}$$

Find:

- ① modulus of elasticity E ?
- ② load which causes yielding?
- ③ ultimate load?
- ④ modulus of resilience U_r ?
- ⑤ Modulus of Toughness U_t ?
- ⑥ if the material is loaded until 70 ksi find:-

- (a) the amount of elastic recovery when unloaded?
- (b) the increase in the gauge length when unloaded?



Probs. 3-5/6/7

Solution:-

$$\textcircled{1} E = \text{slope} = \frac{40 - 0}{0.001 - 0} = 40 \times 10^3 \text{ ksi}$$

$$\textcircled{2} \sigma_y = 40 \text{ ksi}$$

$$\begin{aligned} \therefore P_y &= \sigma_y A \\ &= \sigma_y \frac{\pi}{4} d_0^2 \\ &= 7.85 \text{ kip} \end{aligned}$$

$$\textcircled{3} \sigma_u = 76.25 \text{ ksi}$$

$$\therefore P_u = \sigma_u A = \sigma_u \frac{\pi}{4} d_0^2 = 15 \text{ kip}$$

$$\textcircled{4} U_r = \text{Area under elastic curve} = \frac{1}{2} (40 \times 10^3) (0.001) = 20 \text{ Ib/in}^2$$

⑤ $U_t = \text{Area under Stress-Strain Curve}$
 $= (\text{number of squares}) (\text{area of one square})$
 $= (45) (10 \times 0.04) = 18 \text{ kip/in}^2$

⑥ Elastic Recovery $= \frac{\sigma}{E} = \frac{70}{40 \times 10^3} = 0.00175 \text{ in/in}$

$\therefore$ The amount of elastic recovery $= (0.00175) (2) = 0.0035 \text{ in}$

⑦ Permanent Set $= 0.08 - 0.00175 = 0.07825 \text{ in/in}$

Thus,

Permanent elongation $= (0.07825) (2) = 0.1565 \text{ in}$

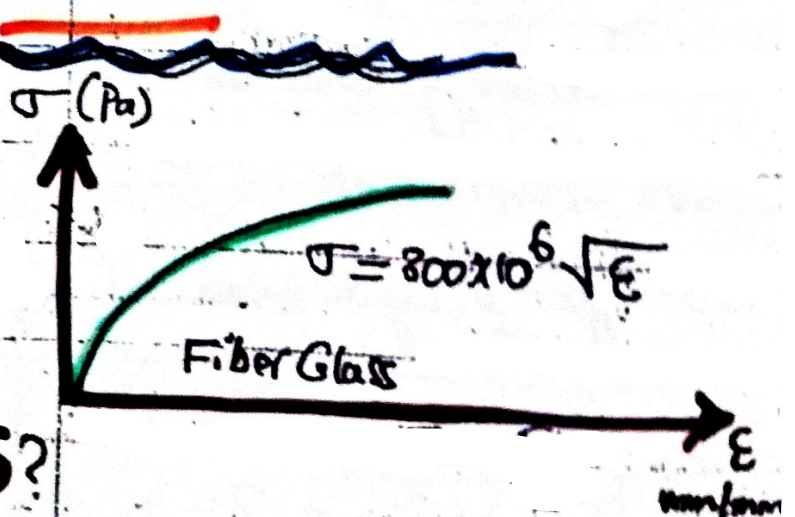
* Ex. Problem 3-12

Given $d_o = 50 \text{ mm}$

$L_o = 2 \text{ m}$

axial load $P = 60 \text{ kN}$

→ Find the elongation δ ?



Sol. $\sigma = \frac{P}{A} = \frac{60 \times 10^3}{\pi (0.025)^2} = 30.558 \text{ MPa}$

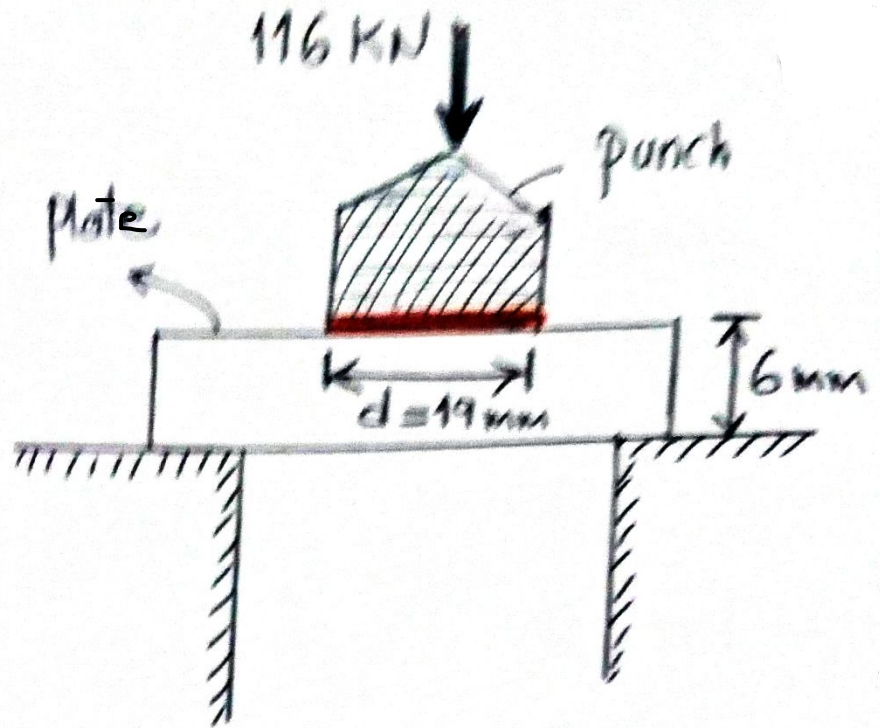
$\sigma = 800 \times 10^6 \sqrt{\epsilon} = 30.558 \times 10^6 \Rightarrow \epsilon = 0.010375$

$\therefore \delta = L_o \epsilon = (2) (0.010375) = 20.8 \text{ mm}$

Ex 1

Find

① the average Compressive stress in the punch? (σ)



② the average Shear stress (τ) in the plate?



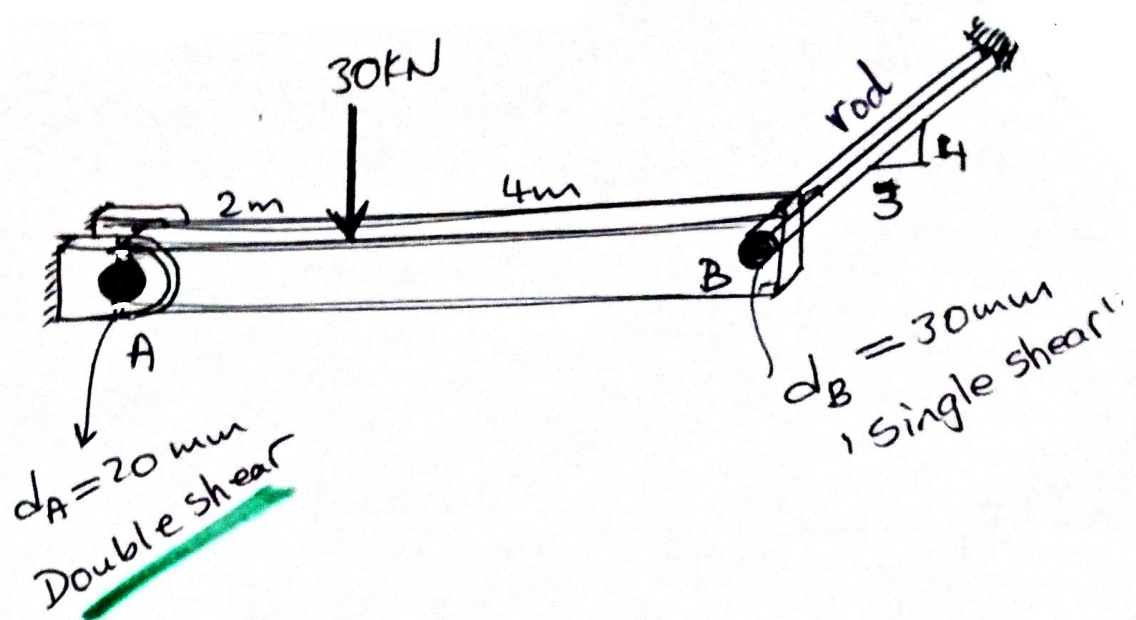
① $\sigma = \frac{\text{Force}}{\text{Area}} = \frac{116 \times 10^3}{\frac{\pi}{4}(19)^2 \times 10^{-6}} = 409 \text{ MPa}$

② $\tau = \frac{\text{Force}}{\text{Area}} = \frac{116 \times 10^3}{\pi(19)(6) \times 10^{-6}} = 324 \text{ MPa}$

$A = (\pi d)(t)$
 $A = \pi d t$

* • * • * • *

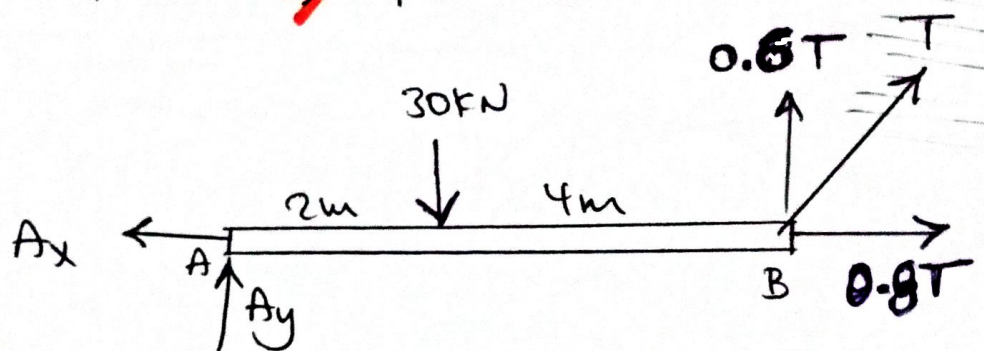
ex)



Find the average shear stress in the

- 1) Pin A
- 2) Pin B

Sol



$$\begin{aligned} * \sum M_A = 0 &\Rightarrow 0.6T(6) - 30(2) = 0 \\ &\Rightarrow T = 16.7 \text{ kN} \end{aligned}$$

$$\begin{aligned} * \sum F_y = 0 &\Rightarrow A_y - 30 + 0.6(16.7) = 0 \\ &A_y = 19.9 \text{ kN} \end{aligned}$$

$$\begin{aligned} * \sum F_x = 0 &\Rightarrow -A_x + 0.8(16.7) = 0 \\ &A_x = 13.4 \text{ kN} \end{aligned}$$

Ans

$$A = \sqrt{A_x^2 + A_y^2} \Rightarrow A = 24 \text{ kN}$$

$$\tau_A = \frac{\overset{\text{Double shear}}{\frac{24}{2} \times 10^3}}{\frac{\pi}{4} (20)^2 \times 10^{-6}} = 38.2 \text{ MPa.}$$

$$\tau_B = \frac{\overset{\text{single}}{16.7 \times 10^3}}{\frac{\pi}{4} (30)^2 \times 10^{-6}} = \text{--- MPa.}$$

* Find the diameter of pin A

assuming $\tau_{ult} = 100 \text{ MPa}$

and the factor of safety $F.S. = 2.5$?

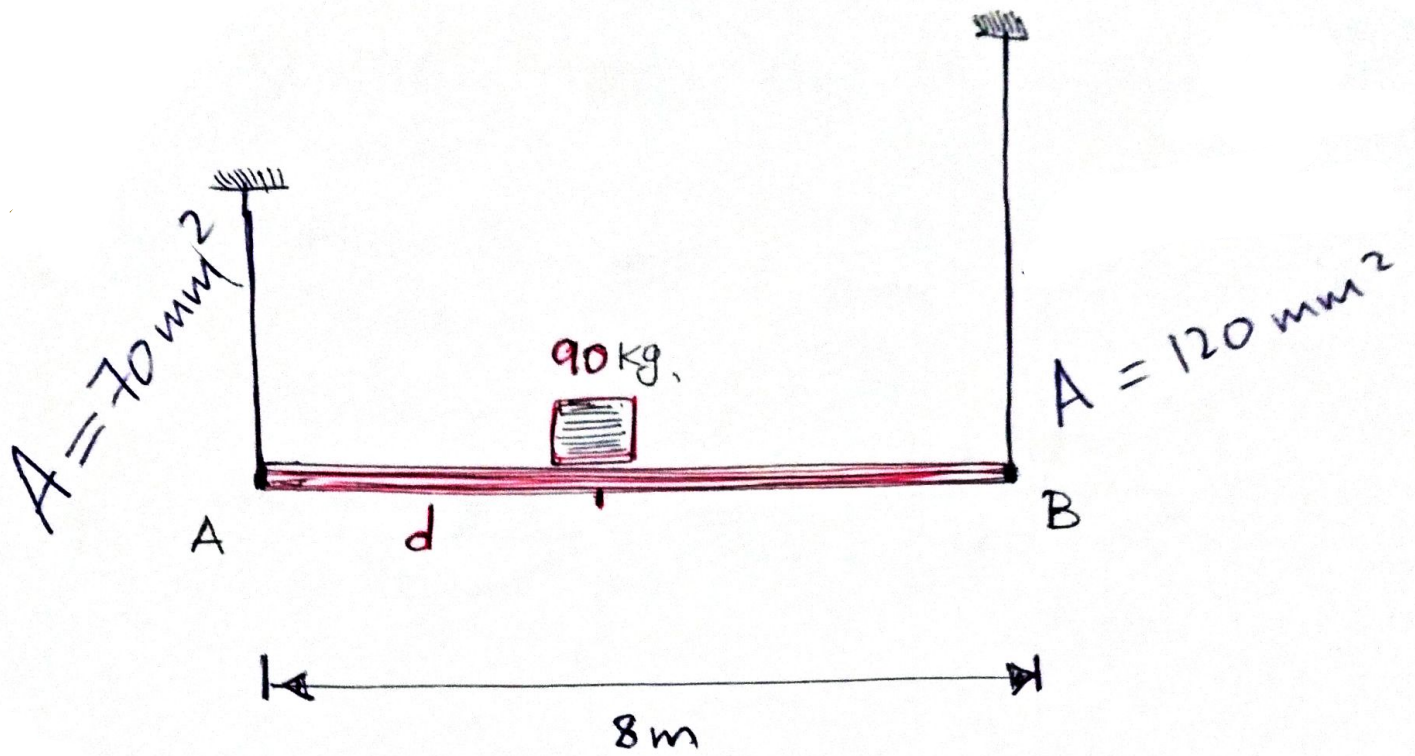
$$\Rightarrow \tau_{all} = \frac{\tau_{ult}}{F.S.} = \frac{100}{2.5} = \underline{40} \text{ MPa}$$

$$\Rightarrow \tau_{all} = \frac{\frac{24}{2} \times 10^3}{\frac{\pi}{4} (d)^2 \times 10^{-6}} = 40 \times 10^6 \Rightarrow$$

Solve
for d

$$\Rightarrow d = \text{--- mm}$$

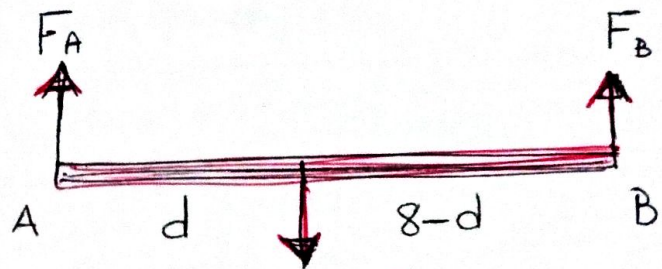




أوجد المسافة d بحيث أن الأوزان في الطرفين الأولى يساوي الأوزان في الطرفين الثاني
equal

الحل

مساوية



$$\sum M_A = 0$$

$$\Rightarrow 8F_B - 883d = 0$$

$$\Rightarrow F_B = 110.4d \quad \text{--- ①}$$

$$90(9.81) = 883 \text{ N}$$

الوزن
المعلقة

$$\sum F_y = 0 \Rightarrow F_A + 110.4d - 883 = 0$$

$$\Rightarrow F_A = 883 - 110.4d \quad \text{--- ②}$$

$$\text{Stress(A)} = \text{Stress (B)}$$

$$\Rightarrow \frac{\text{Force}}{\text{Area}} = \frac{\text{Force}}{\text{Area}}$$

$$\Rightarrow \frac{883 - 110.4 d}{70 \times 10^{-6}} = \frac{110.4 d}{120 \times 10^{-6}}$$

Solve

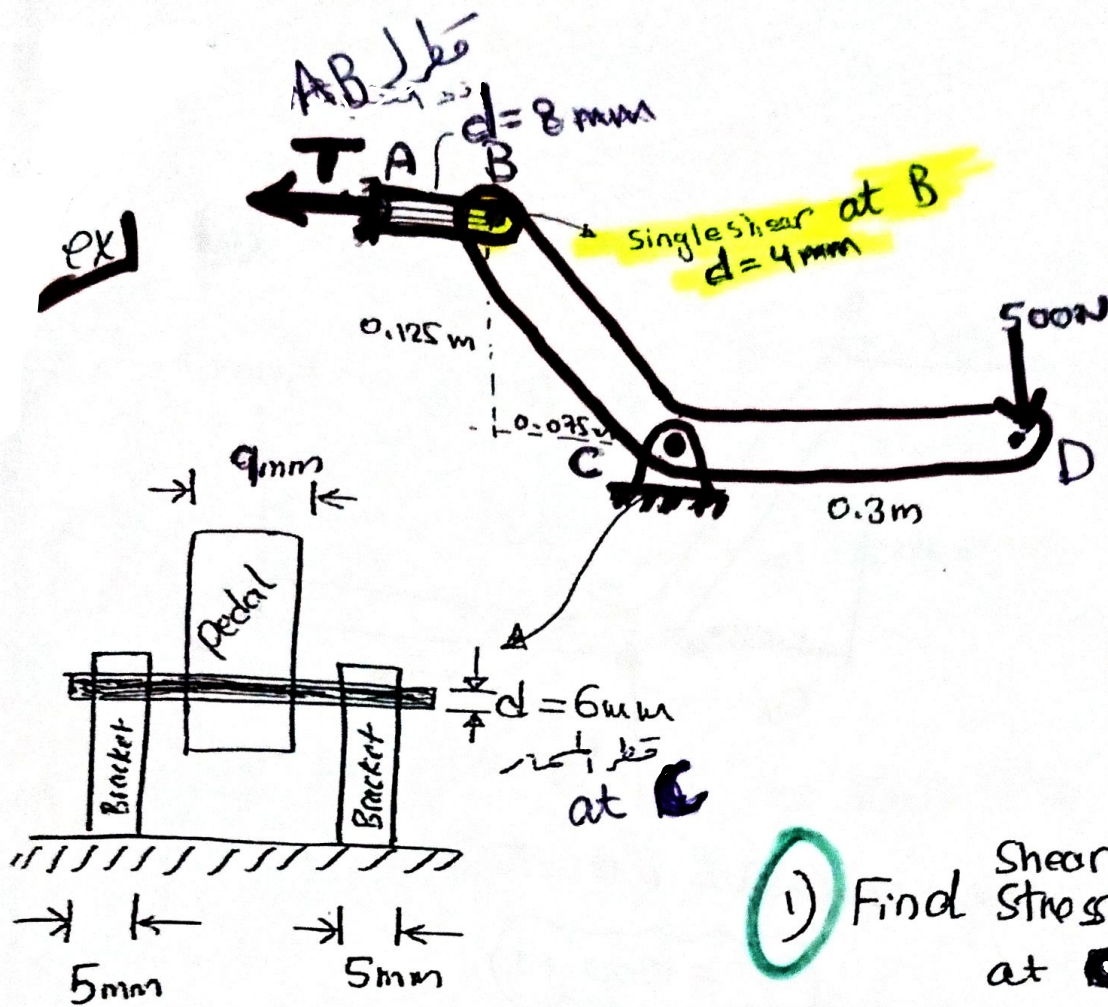
$$d = 5.1$$

$$(883 - 110.4 d) = \frac{(110.4)(70)}{120} d$$

$$883 - 110.4 d = 64.4 d$$

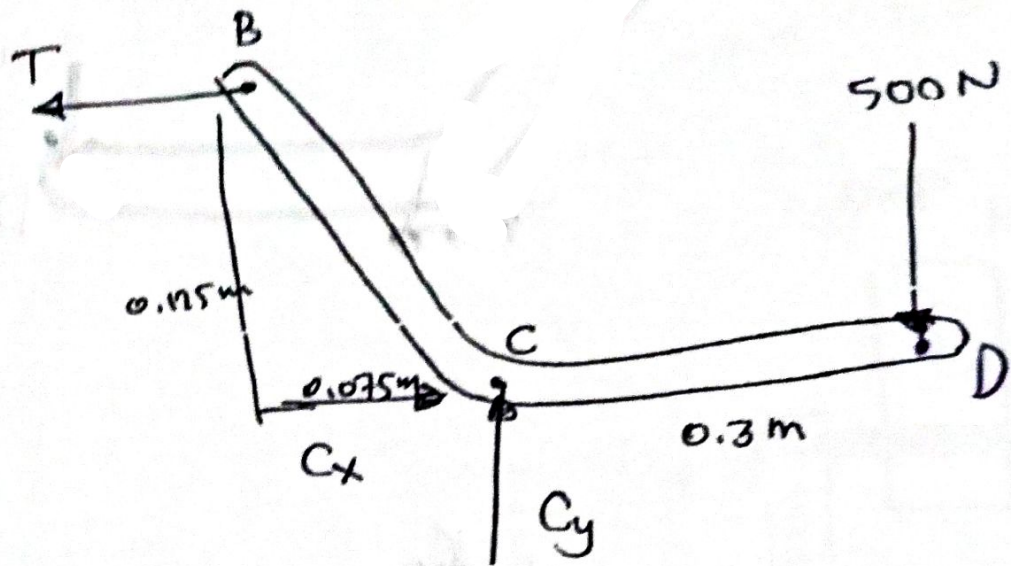
$$\rightarrow 174.8 d = 883$$

$$\rightarrow d = 5.1 \text{ m}$$



- 1) Find Shear Stress in Pin at C?
- 2) Bearing stress in Pedal? at C
- 3) Bearing stress in Bracket at C
- 4) Stress in rod AB
- 5) Shear stress of the pin at B (Single shear)

6) Factor of Safety at pin B if $\tau_{ult} = 420 \text{ MPa}$



$$* \sum M_C = 0 \Rightarrow 500(0.3) - 0.125 T = 0$$

$$T = 1200 \text{ N}$$

$$* \sum F_x = 0 \Rightarrow C_x - 1200 = 0 \Rightarrow C_x = 1200 \text{ N}$$

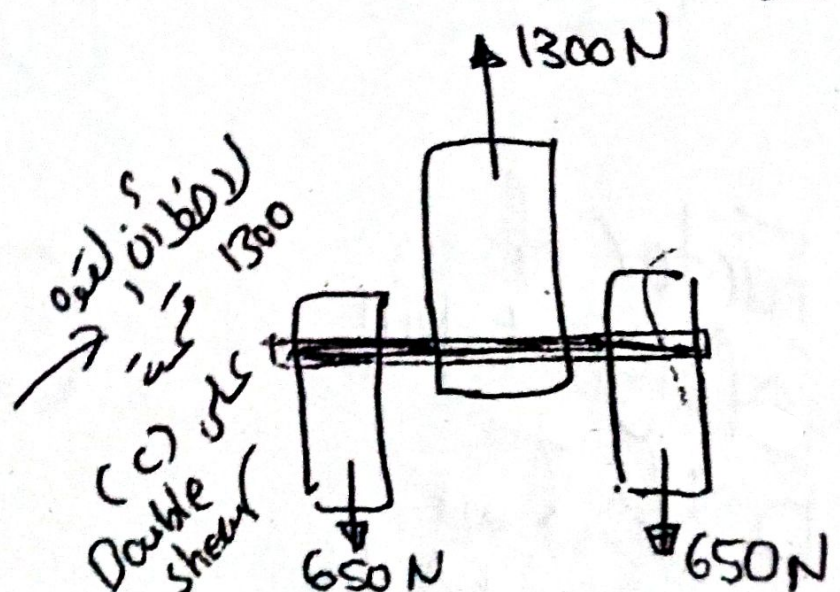
$$* \sum F_y = 0 \Rightarrow C_y - 500 = 0 \Rightarrow C_y = 500 \text{ N}$$

$$* \text{المحصلة } C = \sqrt{C_x^2 + C_y^2} = \sqrt{(1200)^2 + (500)^2} = 1300 \text{ N}$$

* ① Double Shear

$$\tau = \frac{650}{\frac{\pi}{4}(6)^2 \times 10^{-6}}$$

$$\tau = 23 \text{ MPa}$$



$$\textcircled{2} \quad \sigma_B = \frac{1300}{(6)(9) \times 10^{-6}} = 24.1 \text{ MPa}$$

$$\textcircled{3} \quad \sigma_B = \frac{650}{(6)(5) \times 10^{-6}} = 21.7 \text{ MPa}$$

$$\textcircled{4} \quad \sigma = \frac{1200}{\frac{\pi}{4}(8)^2 \times 10^{-6}} = 23.9 \text{ MPa}$$

$$\textcircled{5} \quad \tau_B = \frac{1200}{\frac{\pi}{4}(4)^2 \times 10^{-6}} = 95.5 \text{ MPa.}$$

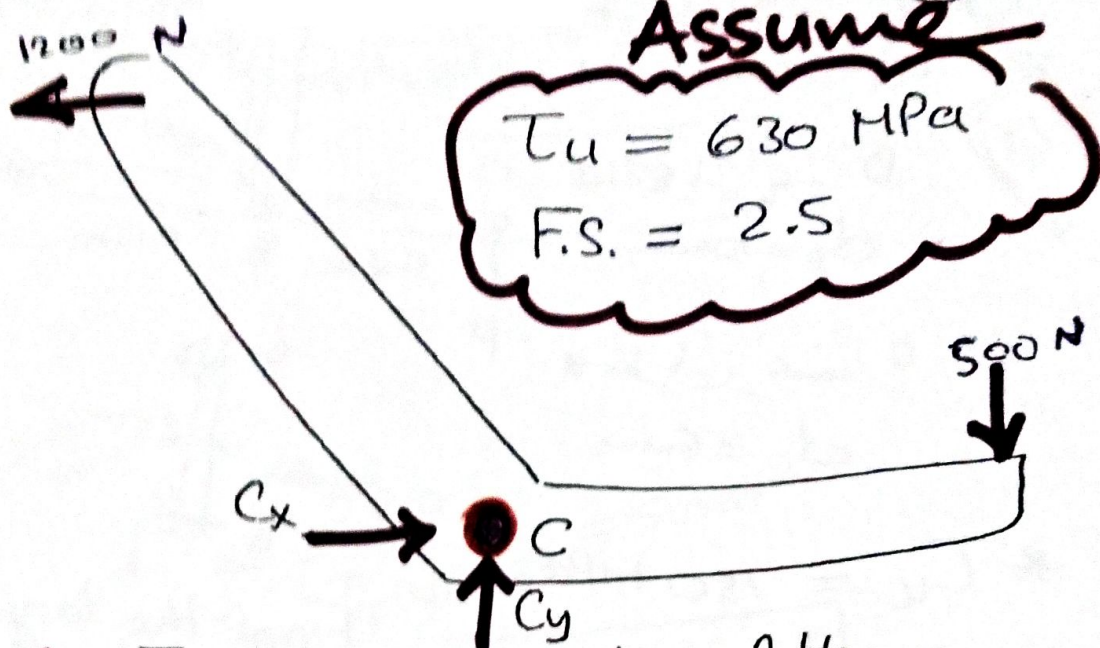
لا تخطئ عدم الخلط بين
القوة على (c)

الزني
(single shear)

$$\textcircled{6} \quad \begin{aligned} T_{ult} &= 420 \text{ MPa} \\ T_{all} &= 95.5 \text{ MPa} \end{aligned}$$

$$\Rightarrow F.S. = \frac{T_{ult}}{T_{all}}$$

$$F.S. = \frac{420}{95.5} = \dots \checkmark$$



⇒ Find the diameter of the pin at C?

Remind ⇒ $C = \sqrt{C_x^2 + C_y^2} = 1300 \text{ N}$

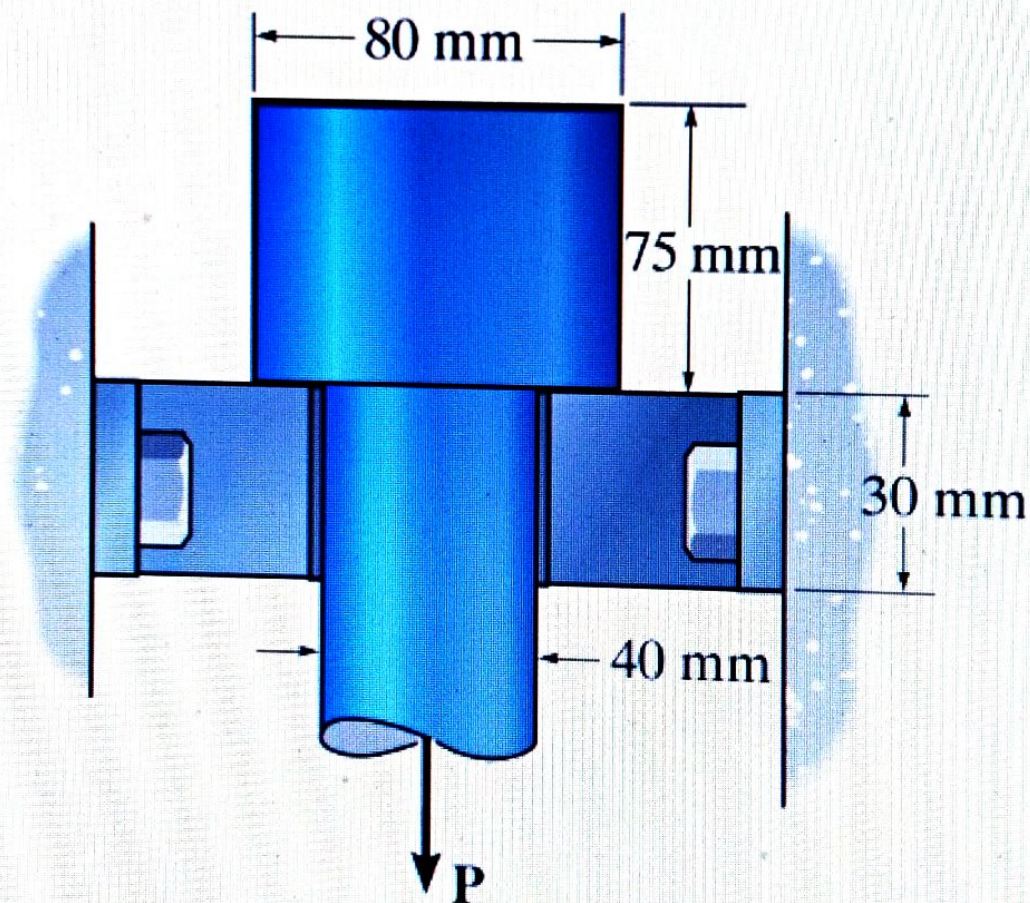
$\tau = \frac{F}{A}$ Double Shear $\tau_{\text{allow}} = \frac{630}{2.5}$

$252 \times 10^6 = \frac{\frac{1300}{2}}{\frac{\pi}{4} d^2}$

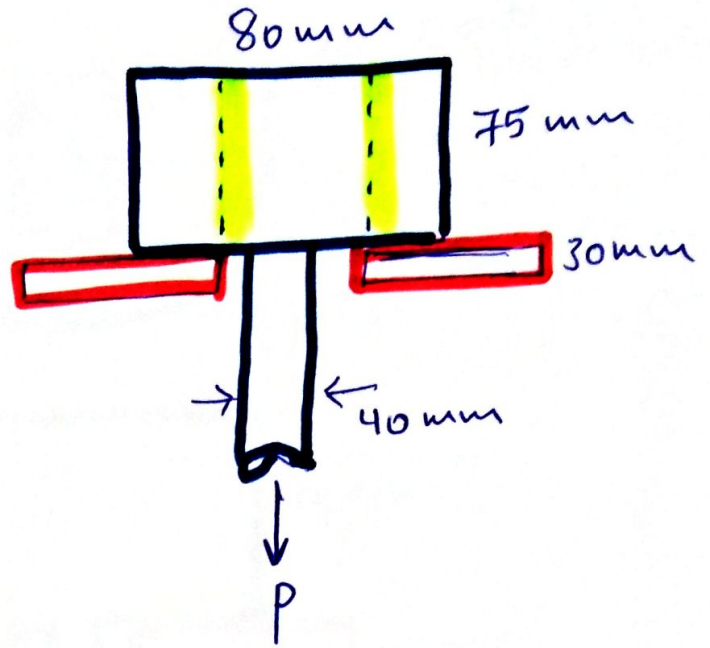
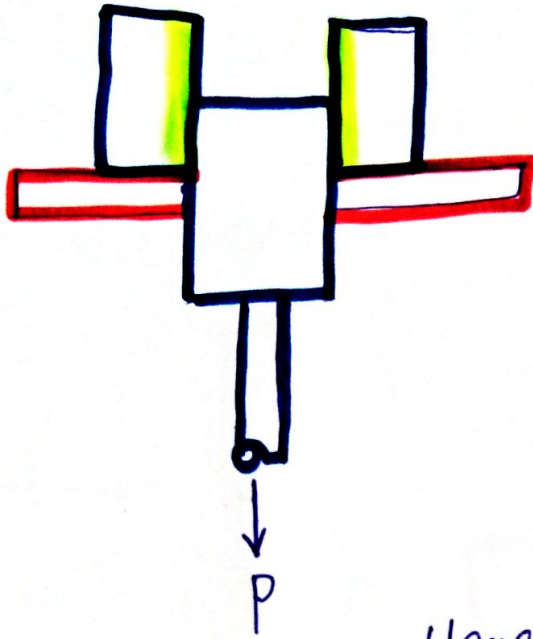
$\tau_{\text{allow}} = 252 \text{ MPa}$

⇒ Find $d = \dots$ ✓

If the bolt head and the supporting bracket are made of the same material having a failure shear stress of $\tau_{\text{fail}} = 120 \text{ MPa}$, determine the maximum allowable force **P** that can be applied to the bolt so that it does not pull through the plate. Apply a factor of safety of F.S. = 2.5 against shear failure.



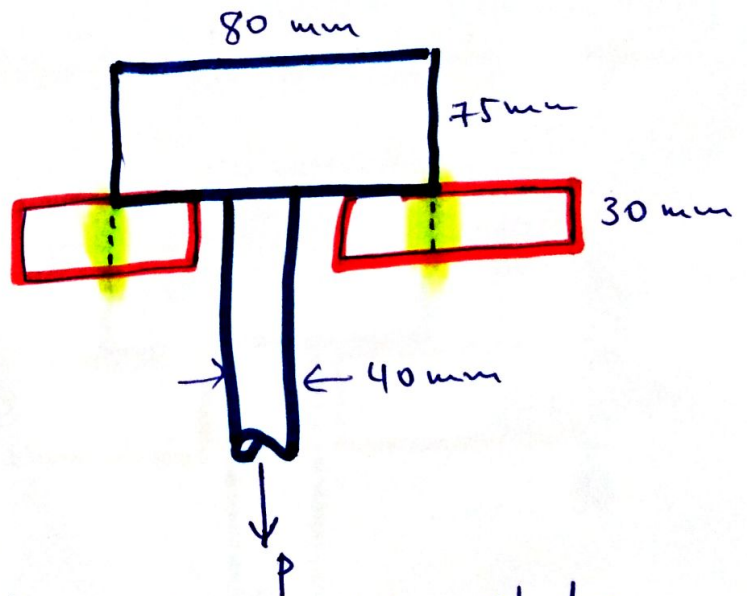
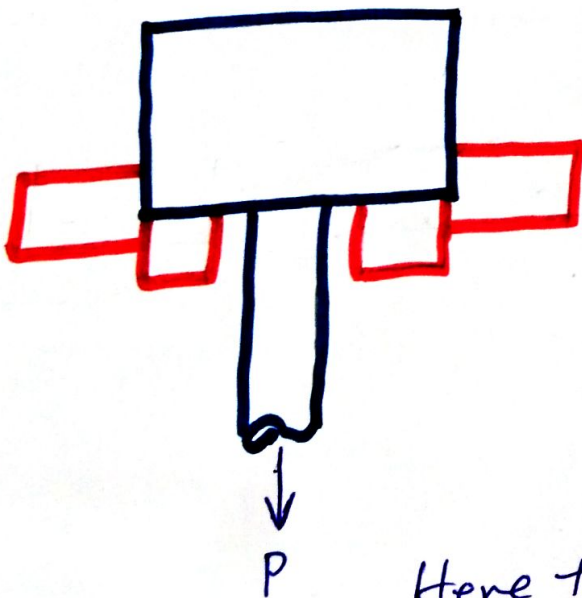
Bolt & plate



Here the shear is in the Bolt

$$\Rightarrow \text{Area} = \pi(40)(75), \text{ mm}^2$$

$$\Rightarrow \tau = \frac{P}{A} \quad \checkmark$$



Here the shear is in the plate

$$\Rightarrow \text{Area} = \pi(80)(30), \text{ mm}^2$$

$$\Rightarrow \tau = \frac{P}{A}$$

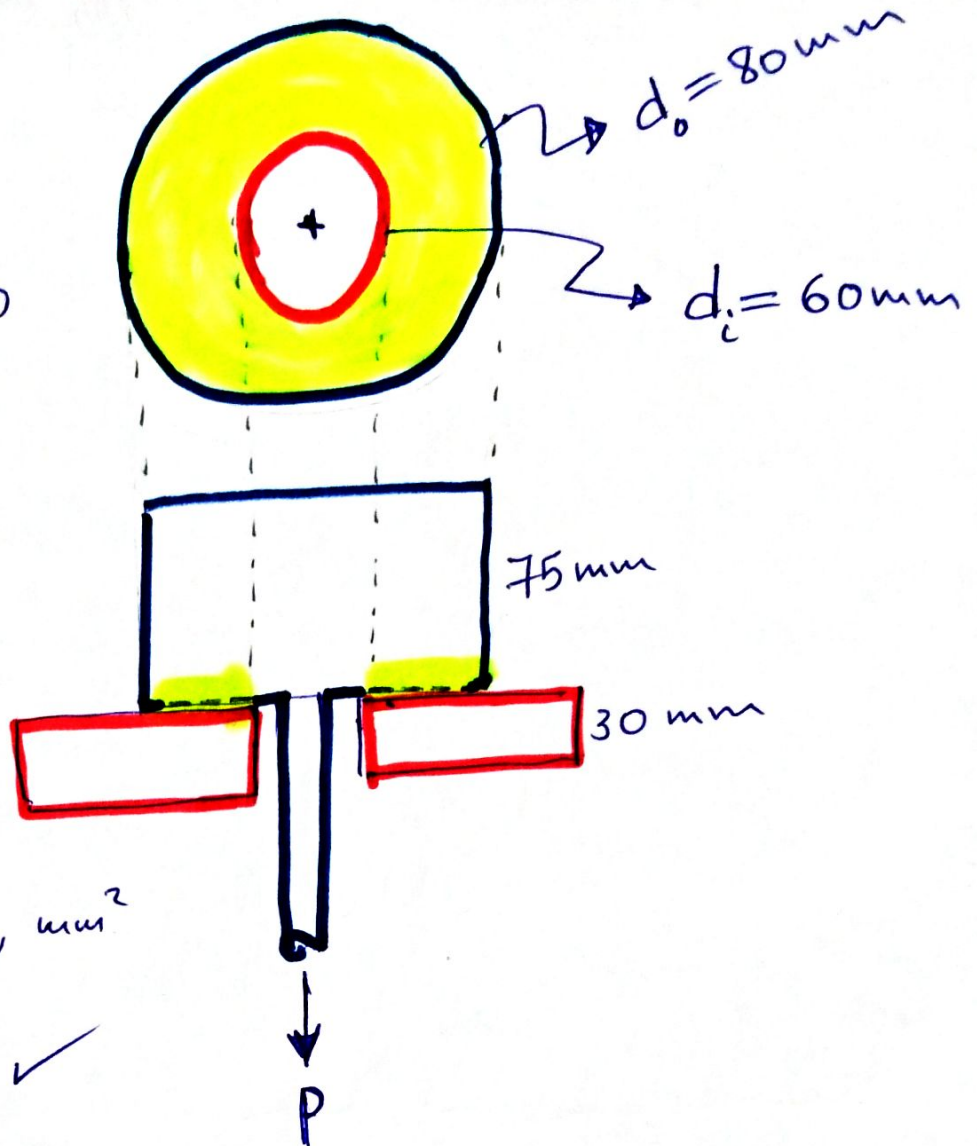
the Bearing
Stress between
the Bolt and
the plate ?

$$\Rightarrow \text{Area} = A$$

$$= \frac{\pi}{4} (d_o^2 - d_i^2)$$

$$= \frac{\pi}{4} (80^2 - 60^2), \text{ mm}^2$$

$$\Rightarrow \sigma_{\text{Bearing}} = \frac{P}{A} \checkmark$$



the shear stress
in the Bolt
at point C ?

$$\text{Area} = \frac{\pi}{4} (18)^2, \text{ mm}^2$$

$$\text{Force} = P/2$$

$$\Rightarrow \tau = \frac{P/2}{A}$$

