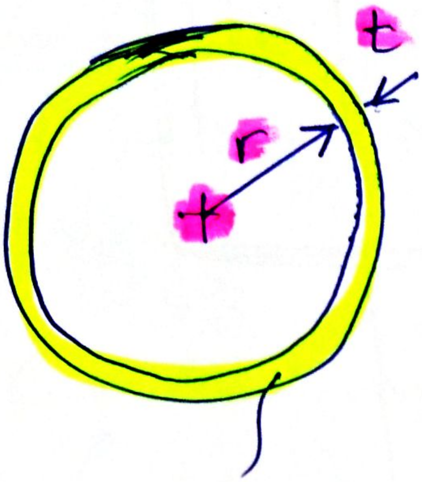
8

Applications of Plane Stress (Pressure Vessels, Beams, and Combined Loadings)

CHAPTER OVERVIEW

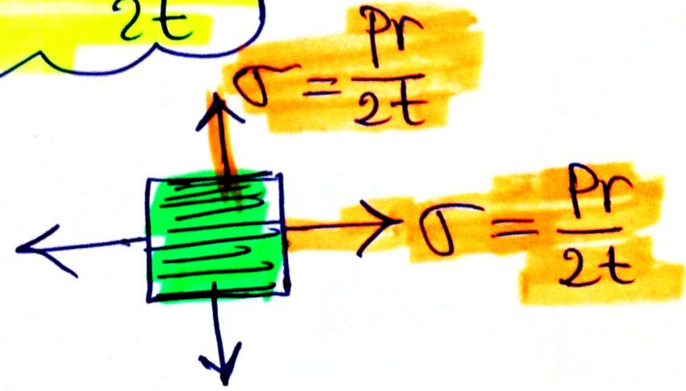
Chapter 8 deals with a number of applications of plane stress, a topic discussed in detail in Sections 7.2 through 7.5 of the previous chapter. Plane stress is a common stress condition that exists in all ordinary structures, including buildings, machines, vehicles, and aircraft. First, thin-wall shell theory is presented describing the behavior of spherical (Section 8.2) and cylindrical (Section 8.3) pressure vessels under internal pressure and having walls whose thickness t is small compared with radius r of the cross section (i.e., $r/t > 10$). We will determine the stresses and strains in the walls of these structures due to the internal pressures from the compressed gases or liquids. Only positive internal pressure (not the effects of external loads, reactions, the weight of the contents, and the weight of the structure) is considered. Linear-elastic behavior is assumed, and the formulas for membrane stresses in spherical tanks and hoop and axial stresses in cylindrical tanks are only valid in regions of the tank away from stress concentrations caused by openings and support brackets or legs. Next, the variation in principal stresses and maximum shear stresses in beams is investigated (Section 8.4), building upon the discussions of stresses in beams in Chapter 5. The variation in these stress quantities across the beam can be displayed using either stress trajectories or stress contours. Stress trajectories give the directions of the principal stresses, while stress contours connect points of equal principal stress at points throughout the beam. Finally, stresses at points of interest in structures under combined loadings (axial, shear, torsion, bending, and possibly internal pressure) are assessed (Section 8.5). Our objective is to determine the maximum normal and shear stresses at various points in these structures. Linear-elastic behavior is assumed so that superposition can be used to combine normal and shear stresses due to various loadings, all of which contribute to the state of plane stress at that point.

* Spherical Pressure vessels



* Thin vessel $\frac{r}{t} \geq 10$
* Thick vessel $\frac{r}{t} < 10$
* $\boxed{\sigma = \frac{pr}{2t}}$

Area
$A = (2\pi r)t$

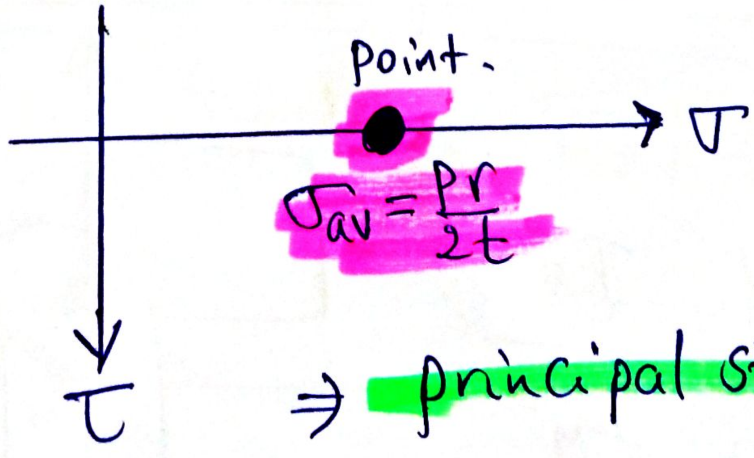


* from Mohr's Circle : $\sigma_x = \sigma_y = \frac{pr}{2t}$

$\tau_{xy} = 0$

$\Rightarrow \sigma_{av} = \frac{pr}{2t}$

$\Rightarrow$ Radius $R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 0$



$\Rightarrow$ principal stresses :

$\sigma_1 = \sigma_2 = \frac{pr}{2t}$

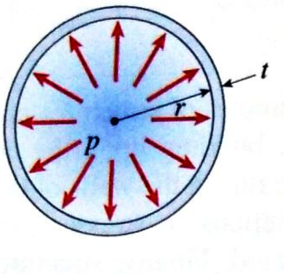


FIG. 8-2 Cross section of spherical pressure vessel showing inner radius r, wall thickness t, and internal pressure p

Because of the symmetry of the vessel and its loading (Fig. 8-3b), the tensile stress σ is uniform around the circumference. Furthermore, since the wall is thin, we can assume with good accuracy that the stress is uniformly distributed across the thickness t. The accuracy of this approximation increases as the shell becomes thinner and decreases as it becomes thicker.

The resultant of the tensile stresses σ in the wall is a horizontal force equal to the stress σ times the area over which it acts, or

$$\sigma(2\pi r_m t)$$

where t is the thickness of the wall and r_m is its mean radius:

$$r_m = r + \frac{t}{2} \tag{b}$$

Thus, equilibrium of forces in the horizontal direction (Fig. 8-3b) gives

$$\sum F_{\text{horiz}} = 0 \qquad \sigma(2\pi r_m t) - p(\pi r^2) = 0 \tag{c}$$

from which we obtain the *tensile stresses* in the wall of the vessel:

$$\sigma = \frac{pr^2}{2r_m t} \tag{d}$$

Since our analysis is valid only for thin shells, we can disregard the small difference between the two radii appearing in Eq. (d) and replace r by r_m or replace r_m by r. While either choice is satisfactory for this approximate analysis, it turns out that the stresses are closer to the theoretically exact stresses if we use the inner radius r instead of the mean radius r_m. Therefore, we will adopt the following formula for calculating the **tensile stresses in the wall of a spherical shell:**

$$\sigma = \frac{pr}{2t} \tag{8-1}$$

As is evident from the symmetry of a spherical shell, we obtain the same equation for the tensile stresses when we cut a plane through the center of the sphere in any direction whatsoever. Thus, we reach the following conclusion: *The wall of a pressurized spherical vessel is subjected to uniform tensile stresses σ in all directions*. This stress condition is represented in Fig. 8-3c by the small stress element with stresses σ acting in mutually perpendicular directions.

Stresses that act tangentially to the curved surface of a shell, such as the stresses σ shown in Fig. 8-3c, are known as **membrane stresses.** The name arises from the fact that these are the only stresses that exist in true membranes, such as soap films.

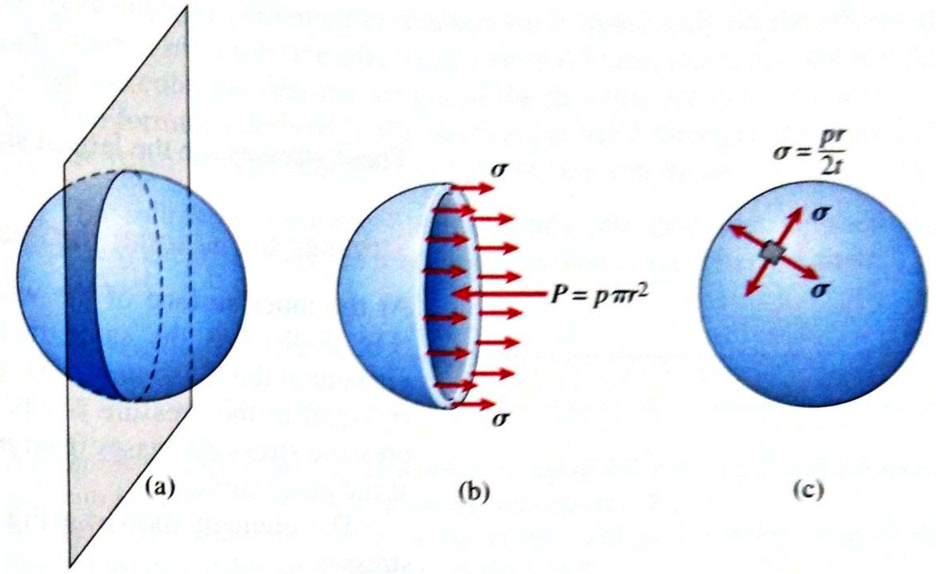


FIG. 8-3 Tensile stresses σ in the wall of a spherical pressure vessel

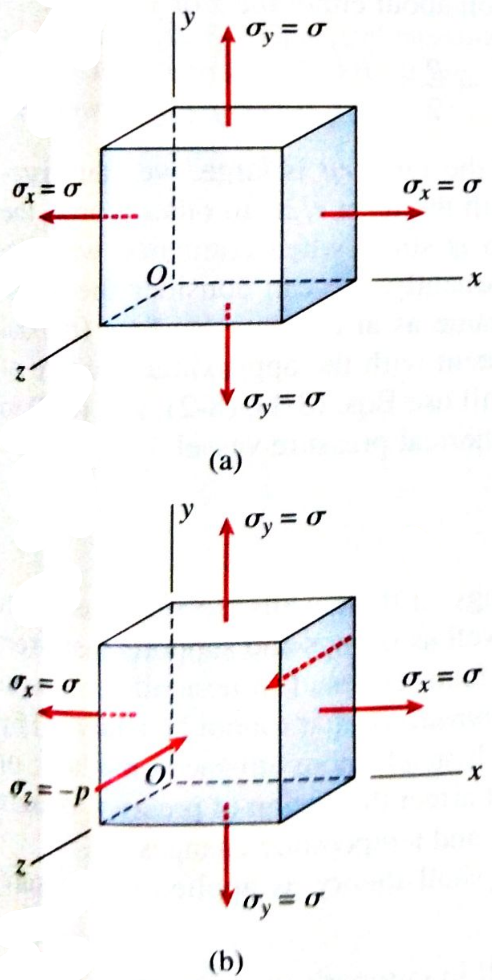


FIG. 8-4 Stresses in a spherical pressure vessel at (a) the outer surface and (b) the inner surface

Stresses at the Outer Surface

The outer surface of a spherical pressure vessel is usually free of any loads. Therefore, the element shown in Fig. 8-3c is in *biaxial stress*. To aid in analyzing the stresses acting on this element, we show it again in Fig. 8-4a, where a set of coordinate axes is oriented parallel to the sides of the element. The x and y axes are tangential to the surface of the sphere, and the z axis is perpendicular to the surface. Thus, the normal stresses σ_x and σ_y are the same as the membrane stresses σ, and the normal stress σ_z is zero. No shear stresses act on the sides of this element.

If we analyze the element of Fig. 8-4a by using the transformation equations for plane stress (see Fig. 7-1 and Eqs. 7-4a and 7-4b of Section 7.2), we find

$$\sigma_{x_1} = \sigma \quad \text{and} \quad \tau_{x_1 y_1} = 0$$

as expected. In other words, when we consider elements obtained by rotating the axes about the z axis, the normal stresses remain constant and there are no shear stresses. *Every plane is a principal plane and every direction is a principal direction.* Thus, the **principal stresses** for the element are

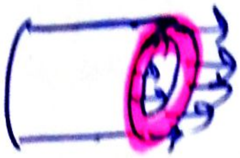
$$\sigma_1 = \sigma_2 = \frac{pr}{2t} \qquad \sigma_3 = 0 \tag{8-2a,b}$$

The stresses σ_1 and σ_2 lie in the xy plane and the stress σ_3 acts in the z direction.

To obtain the **maximum shear stresses**, we must consider out-of-plane rotations, that is, rotations about the x and y axes (because all in-plane shear stresses are zero). Elements oriented by making 45° rotations about the x and y axes have maximum shear stresses equal to $\sigma/2$ and normal stresses equal to $\sigma/2$. Therefore,

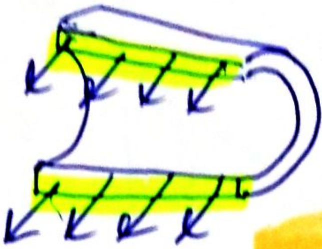
* Cylindrical Pressure vessels

* longitudinal (axial) stresses



$$\sigma_2 = \frac{Pr}{2t}$$

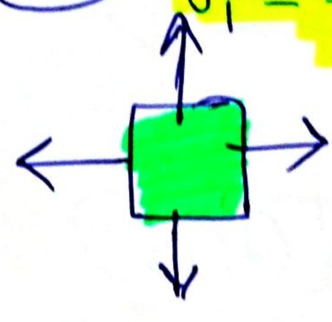
* Circumferential (Hoop) stresses



$$\sigma_1 = \frac{Pr}{t}$$

Note that : $\sigma_1 = 2\sigma_2$

$$\sigma_1 = \frac{Pr}{t}$$



$$\sigma_2 = \frac{Pr}{2t}$$

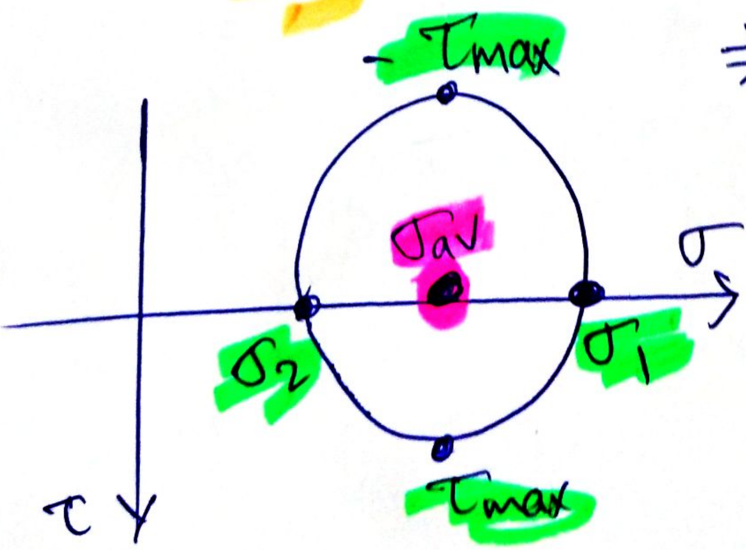
* Mohr's Circle:

$\sigma_x = \frac{Pr}{2t}$, $\tau_{xy} = 0$

$\sigma_y = \frac{Pr}{t}$

$\Rightarrow \sigma_{av} = \frac{3Pr}{4t}$

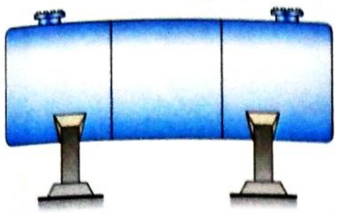
$\Rightarrow R = \frac{Pr}{2t}$



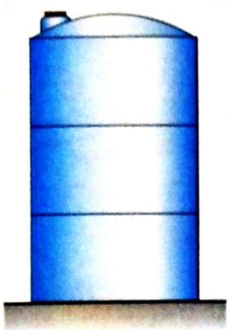
$$\sigma_1 = \frac{5Pr}{4t}$$

$$\sigma_2 = \frac{Pr}{4t}$$

$$\tau_{max} = \frac{Pr}{2t}$$



(a)



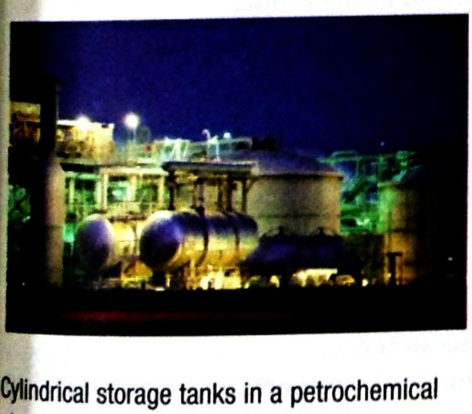
(b)

FIG. 8-6 Cylindrical pressure vessels with circular cross sections

Cylindrical pressure vessels with a circular cross section (Fig. 8-6) are found in industrial settings (compressed air tanks and rocket motors), in homes (fire extinguishers and spray cans), and in the countryside (propane tanks and grain silos). Pressurized pipes, such as water-supply pipes and penstocks, are also classified as cylindrical pressure vessels.

We begin our analysis of cylindrical vessels by determining the normal stresses in a *thin-walled circular tank AB* subjected to internal pressure (Fig. 8-7a). A *stress element* with its faces parallel and perpendicular to the axis of the tank is shown on the wall of the tank. The normal stresses σ_1 and σ_2 acting on the side faces of this element are the membrane stresses in the wall. No shear stresses act on these faces because of the symmetry of the vessel and its loading. Therefore, the stresses σ_1 and σ_2 are principal stresses.

Because of their directions, the stress σ_1 is called the **circumferential stress** or the **hoop stress**, and the stress σ_2 is called the **longitudinal stress** or the **axial stress**. Each of these stresses can be calculated from equilibrium by using appropriate free-body diagrams.



Cylindrical storage tanks in a petrochemical plant

Circumferential Stress

To determine the circumferential stress σ_1, we make two cuts (*mn* and *pq*) perpendicular to the longitudinal axis and distance b apart (Fig. 8-7a). Then we make a third cut in a vertical plane through the longitudinal axis of the tank, resulting in the free body shown in Fig. 8-7b. This free body consists not only of the half-circular piece of the tank but also of the fluid contained within the cuts. Acting on the longitudinal cut (plane *mpqn*) are the circumferential stresses σ_1 and the internal pressure p.

Stresses and pressures also act on the left-hand and right-hand faces of the free body. However, these stresses and pressures are not shown in the figure because they do not enter the equation of equilibrium that we will use. As in our analysis of a spherical vessel, we will disregard the weight of the tank and its contents.

The circumferential stresses σ_1 acting in the wall of the vessel have a resultant equal to $\sigma_1(2bt)$, where t is the thickness of the wall. Also, the resultant force P_1 of the internal pressure is equal to $2pbr$, where r is the inner radius of the cylinder. Hence, we have the following equation of equilibrium:

$$\sigma_1(2bt) - 2pbr = 0$$

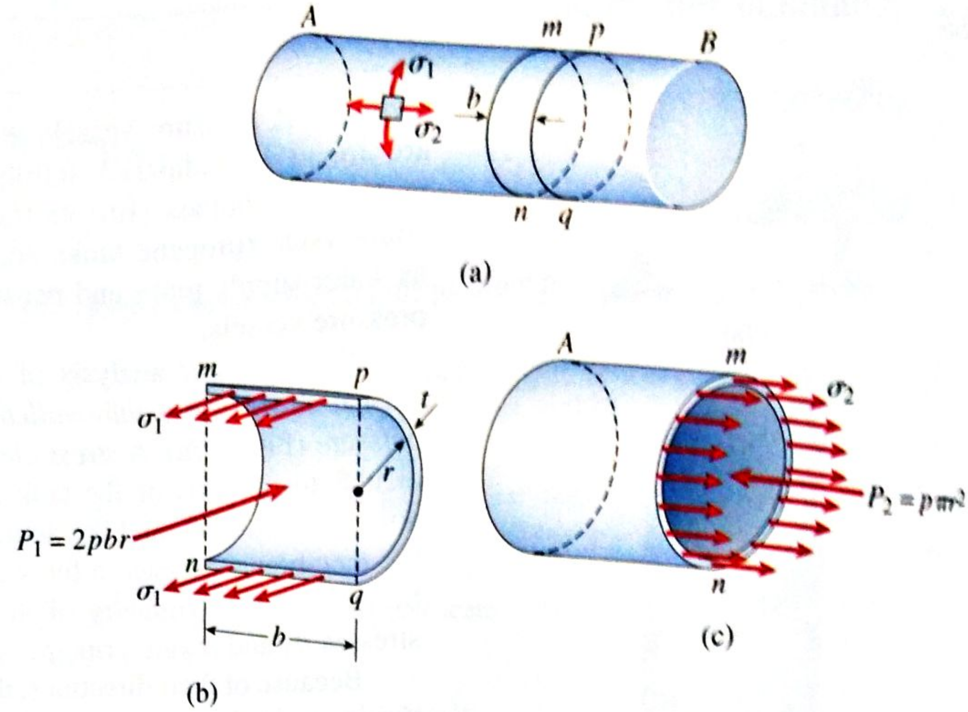


FIG. 8-7 Stresses in a circular cylindrical pressure vessel

From this equation we obtain the following formula for the *circumferential stress in a pressurized cylinder*:

$$\sigma_1 = \frac{pr}{t} \tag{8-5}$$

This stress is uniformly distributed over the thickness of the wall, provided the thickness is small compared to the radius.

Longitudinal Stress

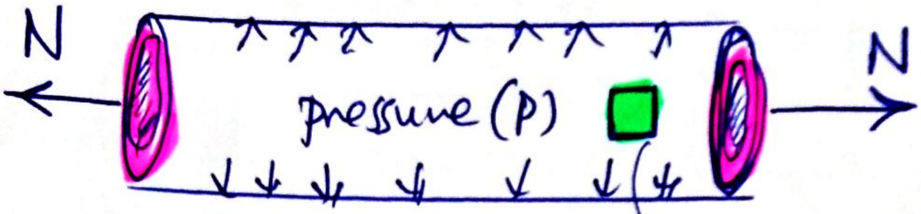
The longitudinal stress σ_2 is obtained from the equilibrium of a free body of the part of the vessel to the left of cross section *mn* (Fig. 8-7c). Again, the free body includes not only part of the tank but also its contents. The stresses σ_2 act longitudinally and have a resultant force equal to $\sigma_2(2\pi rt)$. Note that we are using the inner radius of the shell in place of the mean radius, as explained in Section 8.2.

The resultant force P_2 of the internal pressure is a force equal to $p\pi r^2$. Thus, the equation of equilibrium for the free body is

$$\sigma_2(2\pi rt) - p\pi r^2 = 0$$

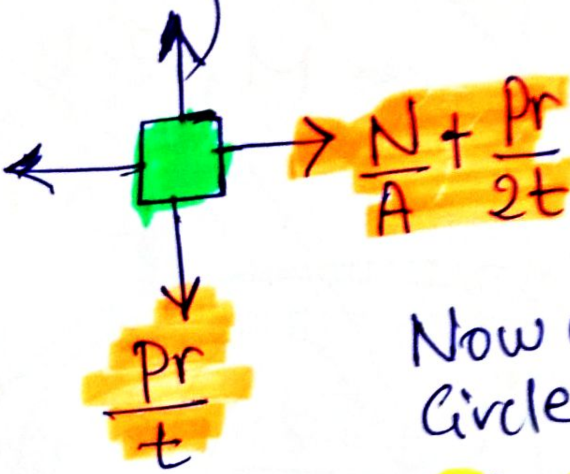
Solving this equation for σ_2, we obtain the following formula for the *longitudinal stress* in a cylindrical pressure vessel:

$$\sigma_2 = \frac{pr}{2t} \tag{8-6}$$

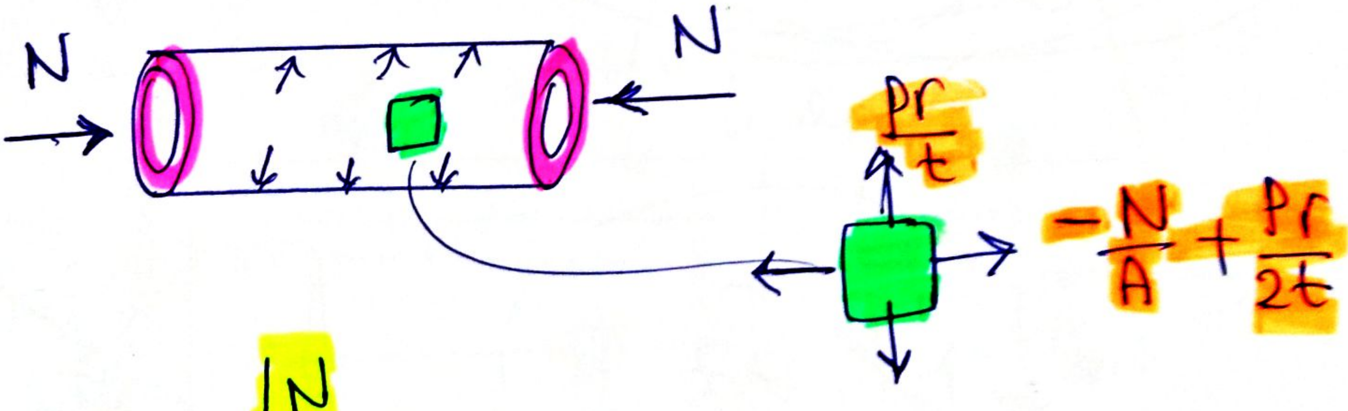


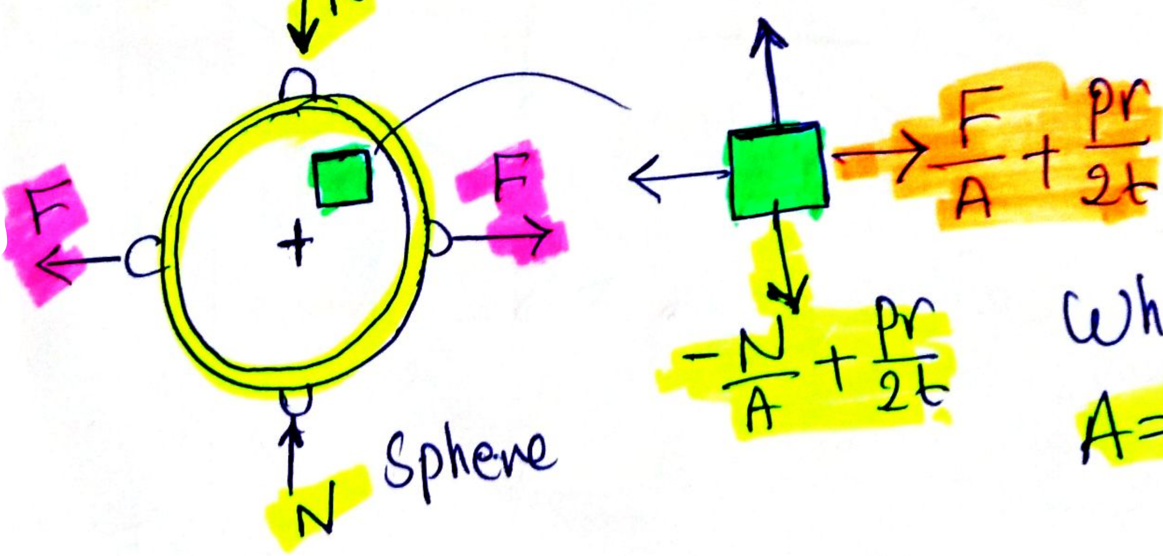
Area

$$A = (2\pi r)(t)$$



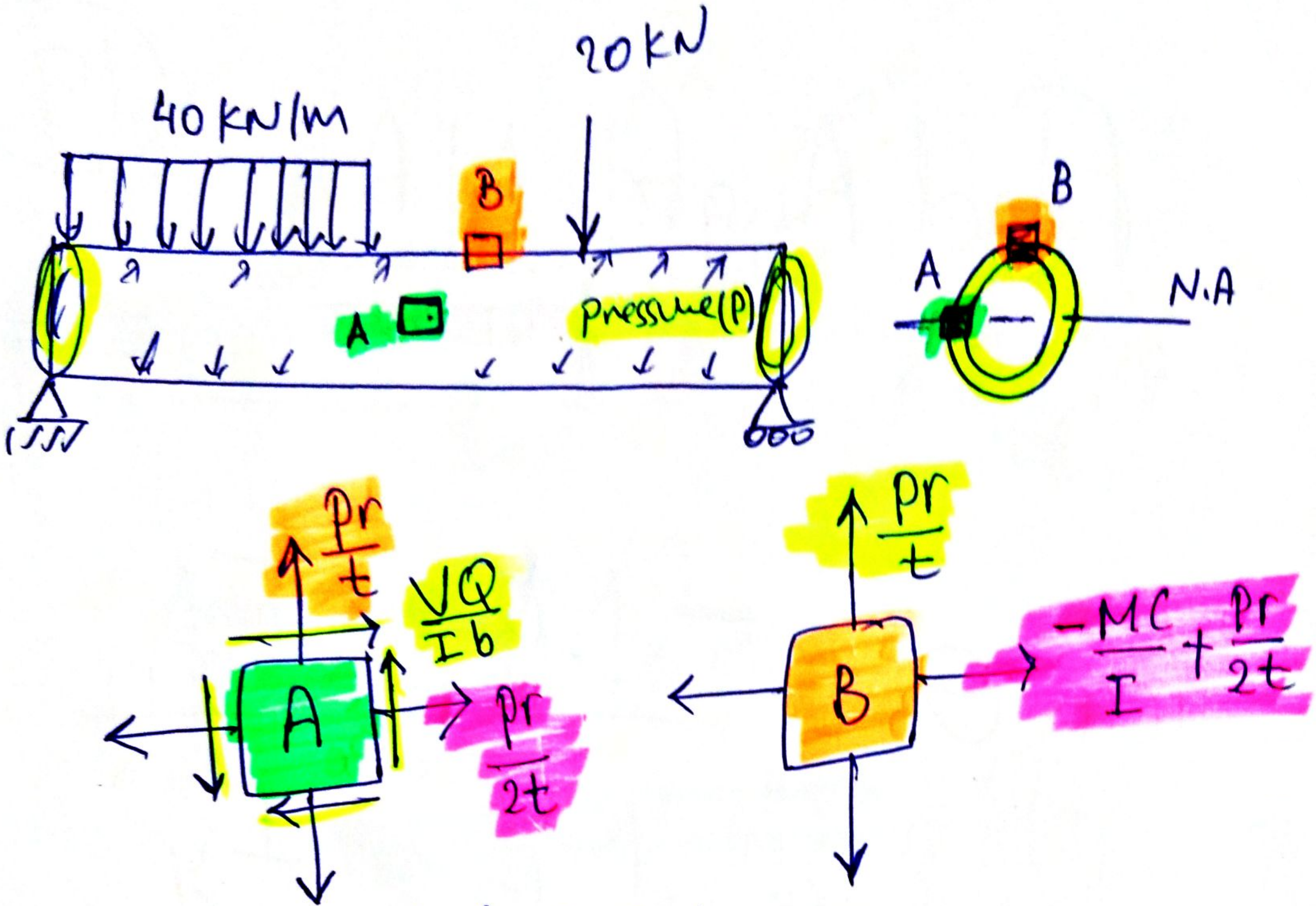
Now use Mohr's Circle to find σ_1, σ_2 and τ_{max}.



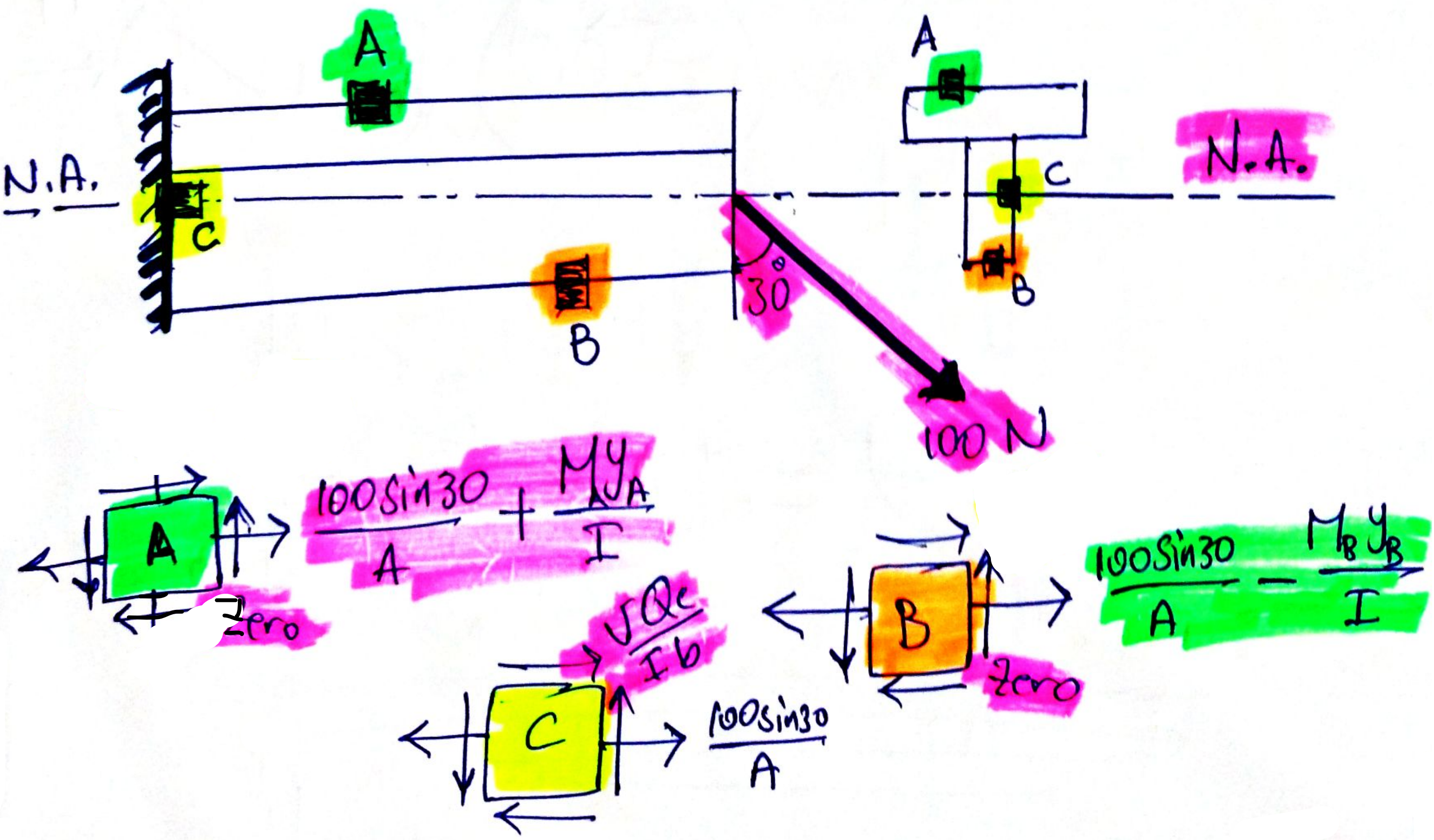


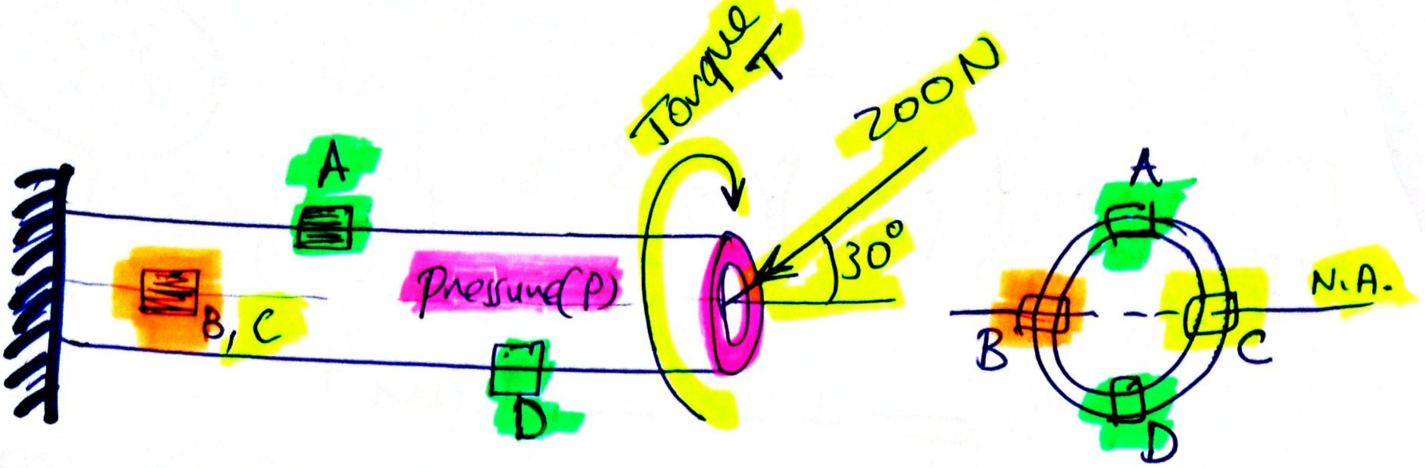
Where:

$$A = (2\pi r)(t)$$



Now use Mohr's Circle to find σ_1, σ_2, and τ_{max}.





$$\text{Area: } A = (2\pi r)(t)$$

$$r_i = r$$

$$r_o = r + t$$

$$I = \frac{\pi}{4}\left(r_o^4 - r_i^4\right)$$

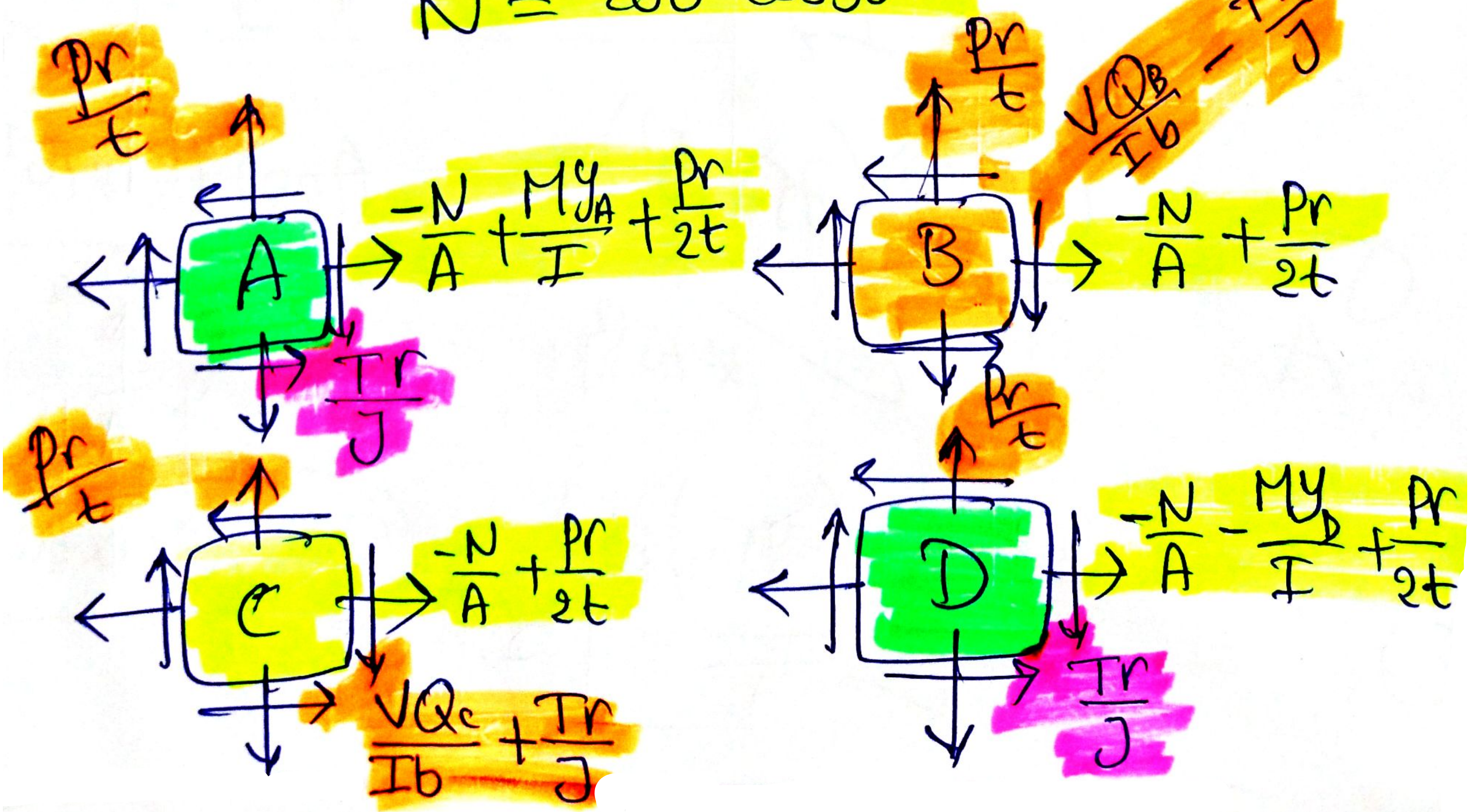
$$J = \frac{\pi}{2}\left(r_o^4 - r_i^4\right)$$

$$Q_B = Q_C = \frac{2}{3}\left(r_o^3 - r_i^3\right)$$

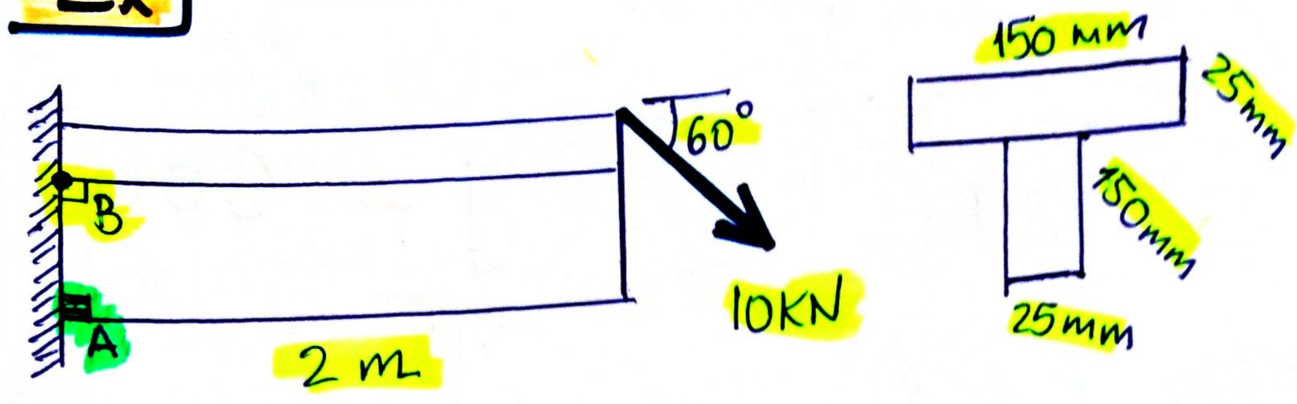
$$Q_A = Q_D = 0$$

$$V = 200 \sin 30^\circ$$

$$N = 200 \cos 30^\circ$$



Ex Cantilever Beam:



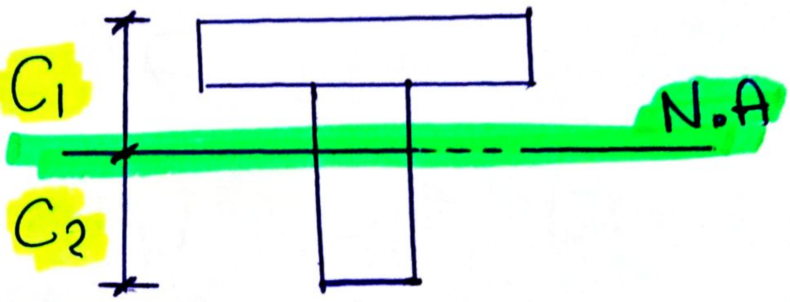
1) Determine the State of Stress at A and B?
2) Find principal and Max shear stresses at A and B?
3) Draw the Mohr's Circle at A and B?

$C_1 = 56.25$ mm

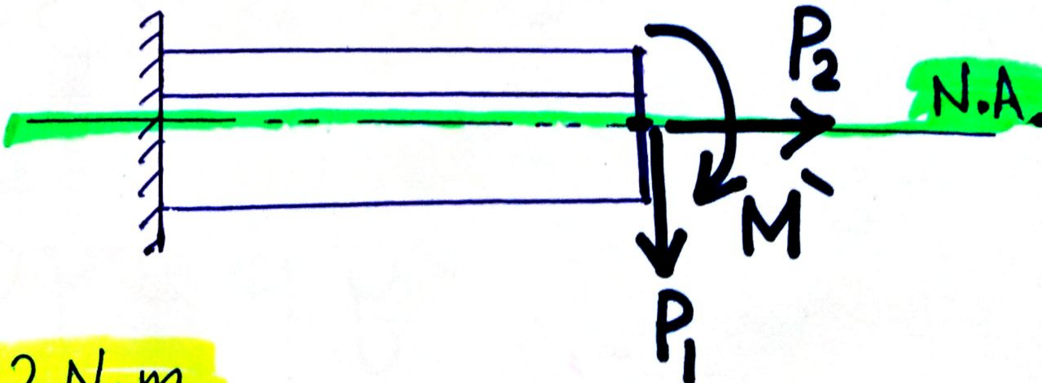
$C_2 = 118.75$ mm

$I_{N.A.} = 21.58 \times 10^6$ mm^4

$A = 7.5 \times 10^{-3}$ m^2



* $P_1 = 10 \sin 60°$
  $= 8.66$ KN



* $P_2 = 10 \cos 60°$
  $= 5$ KN

* $M' = P_2 C_1 = 281.2$ N.m

* $N = P_2 = 5$ KN
* $V = P_1 = 8.66$ KN
* $M = M' + P_1 L = 17.6$ KN.m

} Section at points A and B.

Point A

* $\sigma = \dfrac{N}{A} - \dfrac{MC_2}{I}$

$= -96.17$ MPa

* $\tau = 0$ (Since $Q = 0$)

$\sigma_x = -96.17$ MPa , $\sigma_y = 0$, $\tau_{xy} = 0$

$\sigma_{av} = \dfrac{\sigma_x + \sigma_y}{2} = -48.1$ MPa

$R = \sqrt{\left(\dfrac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 48.1$ MPa

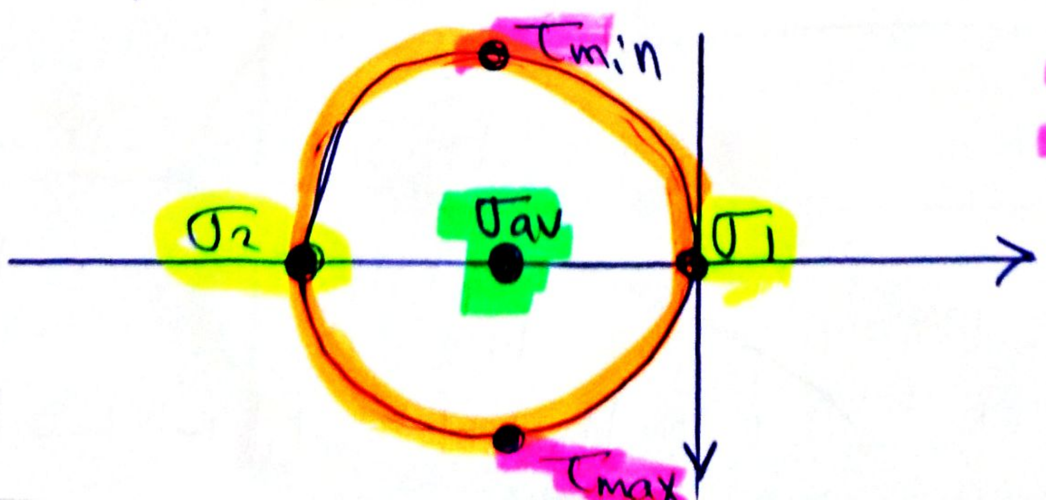
$\sigma_1 = \sigma_{av} + R = 0$

$\sigma_2 = \sigma_{av} - R = -96.17$ MPa

$\tau_{max} = \pm R = \pm 48.1$ MPa

$\tan 2\theta_p = \dfrac{2\tau_{xy}}{\sigma_x - \sigma_y} \Rightarrow \theta_p = 0$

$\theta_s = \theta_p \pm 45° \Rightarrow \theta_s = \pm 45°$



Mohr's Circle

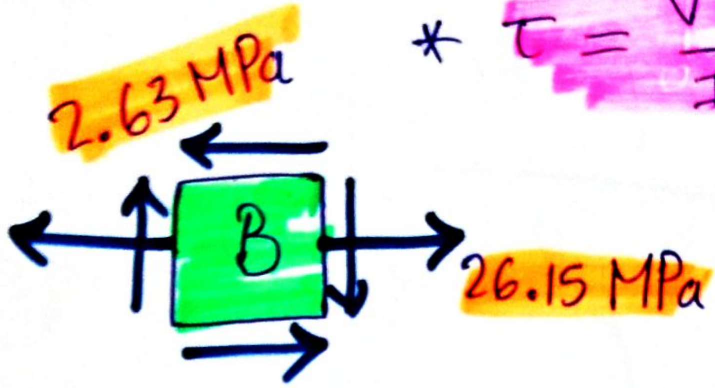
Point B

$c_1 = 25$

* $\sigma = \dfrac{MC}{I} + \dfrac{N}{A}$

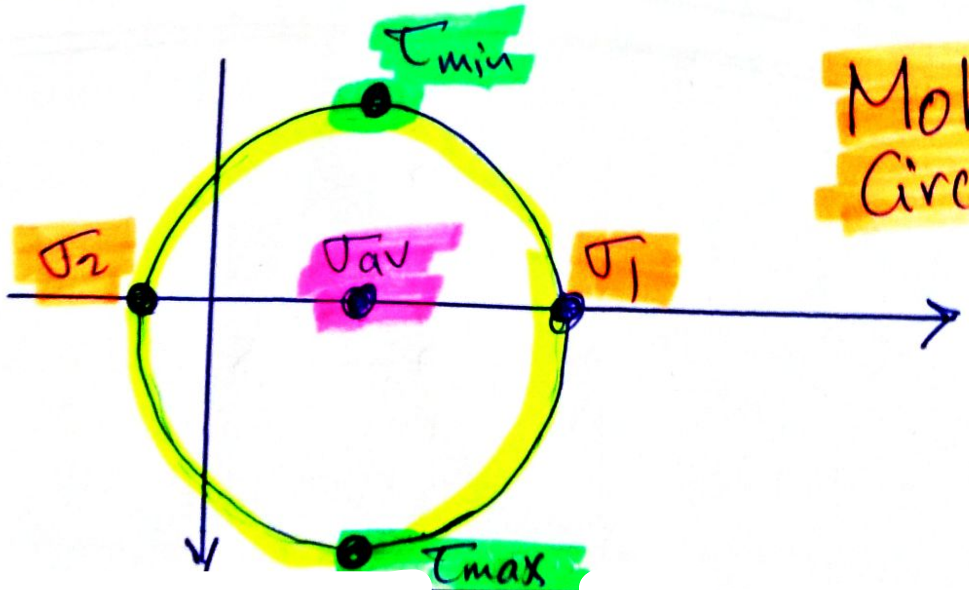
$= 26.15$ MPa

* $\tau = \dfrac{VQ_B}{Ib} = 2.63$ MPa



$\sigma_x = 26.15$ MPa , $\sigma_y = 0$, $\tau_{xy} = -2.63$ MPa

* $\sigma_{av} = 13.1$ MPa.
* $R = 13.34$ MPa.
* $\sigma_1 = \sigma_{av} + R = 26.44$ MPa.
* $\sigma_2 = \sigma_{av} - R = -0.24$ MPa.
* $\tau_{\substack{max \\ min}} = \pm R = \pm 13.34$ MPa.
* $\tan 2\theta_p = \dfrac{2\tau_{xy}}{\sigma_x - \sigma_y} \Rightarrow \theta_p = -11.4^\circ$
* $\theta_s = \theta_p \mp 45^\circ \Rightarrow \theta_s = 33.6^\circ$ OR $\theta_s = -56.4^\circ$

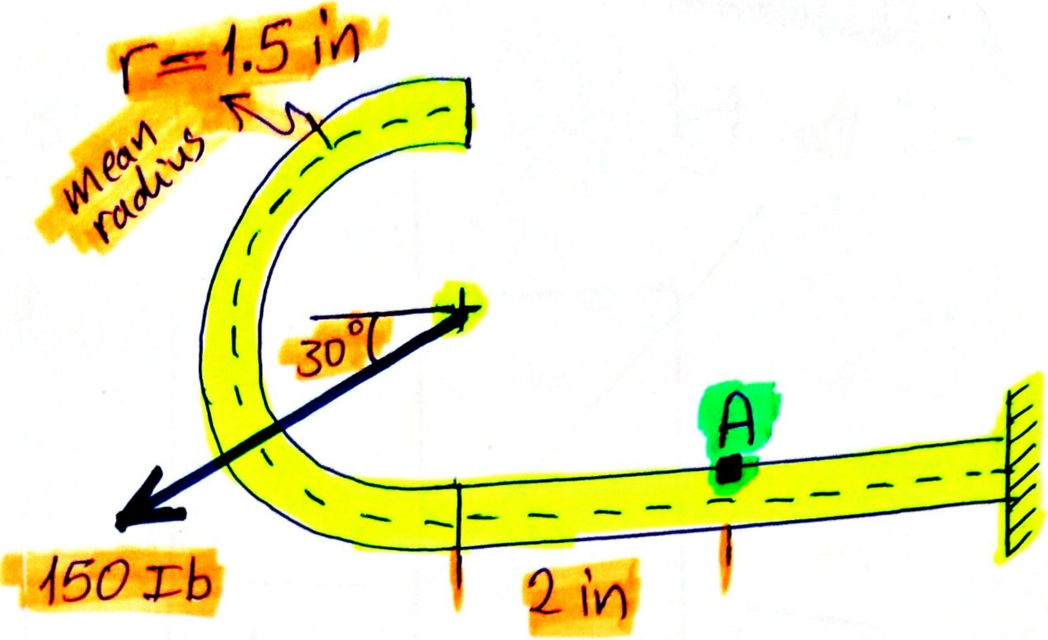


Mohr's Circle

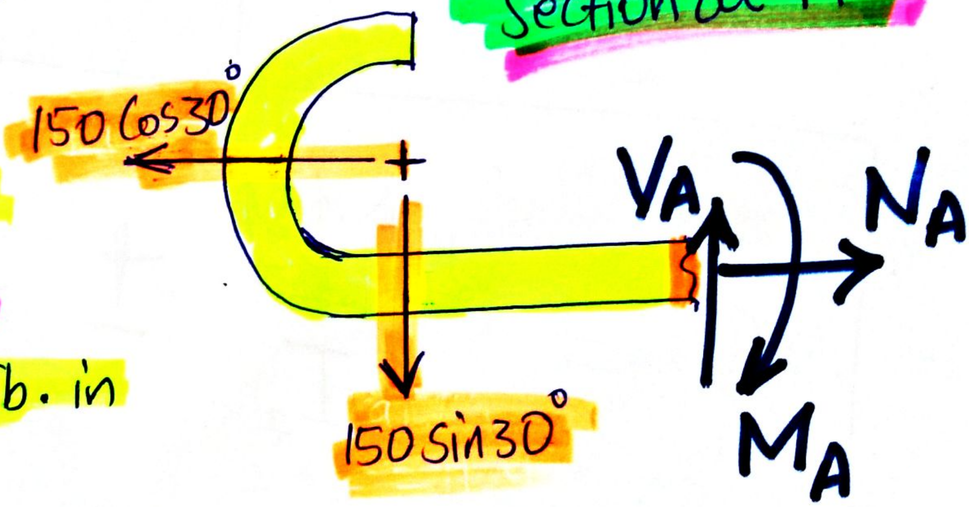
Ex

Hook of diameter $d = \frac{1}{2}$ in.

Find the State of Stress at point A and the principal and max. shear stresses?



Section at A



* $N_A = 129.9$ Ib
* $V_A = 75$ Ib
* $M_A = 344.9$ Ib. in

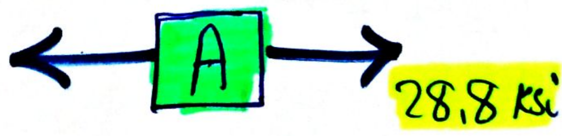
* $$\sigma_A = \frac{N_A}{A} + \frac{M_A C}{I}$$
$$= 28.8 \text{ ksi}$$

* $\tau_A = 0$ (since $Q_A = 0$)

where: $C = \frac{d}{2} = \frac{1/2}{2} = \frac{1}{4}$ in

and, $A = \frac{\pi}{4} d^2$

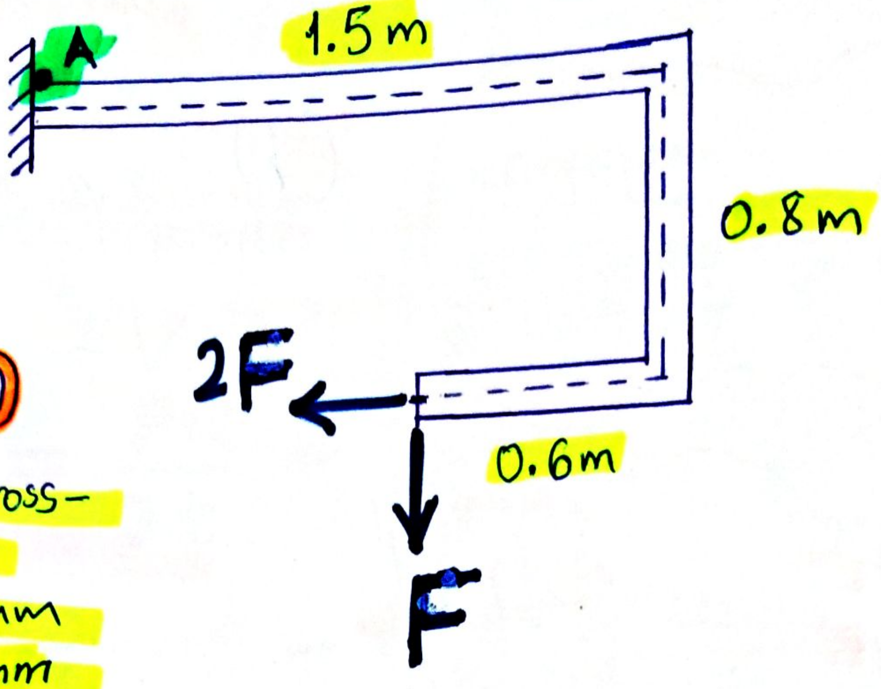
$I = \frac{\pi}{64} d^2$



* $\sigma_{av} = 14.4$ ksi
* $R = 14.4$ ksi
* $\sigma_1 = 28.8$ ksi
* $\sigma_2 = 0$
* $\tau_{\substack{max \\ min}} = \pm 14.4$ ksi

Ex

Find the Force **F**?



Hollow Cross-Section

$d_o = 40$ mm

$t = 2$ mm

* longitudinal strain at point A is $\varepsilon_x = 1.189 \times 10^{-3}$, m/m

* Assume $E = 200$ GPa and $\nu = 0.35$.

* Internal pressure $P = 800$ KPa

$d_i = d_o - 2t$

$d_i = 40 - 4 = 36$ mm

$I = \frac{\pi}{64}(d_o^4 - d_i^4)$

$= 4.322 \times 10^{-8}$ m^4

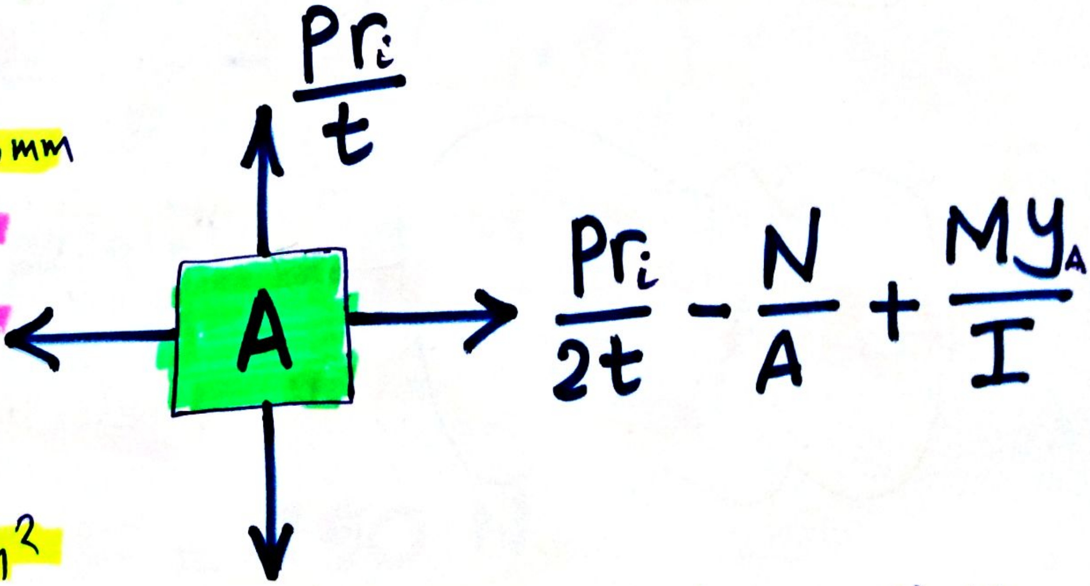
$A = \frac{\pi}{4}(d_o^2 - d_i^2)$

$= 2.388 \times 10^{-4}$ m^2

OR approximetly

$A = (2\pi r_i)(t)$

$= 2.262 \times 10^{-4}$ m^2



* There is No shear since point A is on the top

$(Q_A = 0)$

* $N = 2F$

* $M = 2F(0.8) + F(1.5 - 0.6)$

$\Rightarrow M = 0.7\,F$

* $y_A = \frac{d_o}{2} = r_o = 20\ mm$

* $$\sigma_x = \frac{Pr_i}{2t} - \frac{N}{A} + \frac{M\,y_A}{I}$$

$$= 4\times10^6 - 8375.2\,F + 323924.2\,F$$

$$= 315549\,F + 4\times10^6 \,,\ (N/m^2)$$

* $$\sigma_y = \frac{Pr_i}{t} = 8\times10^6 \,,\ (N/m^2)$$

* From Hooke's law :-

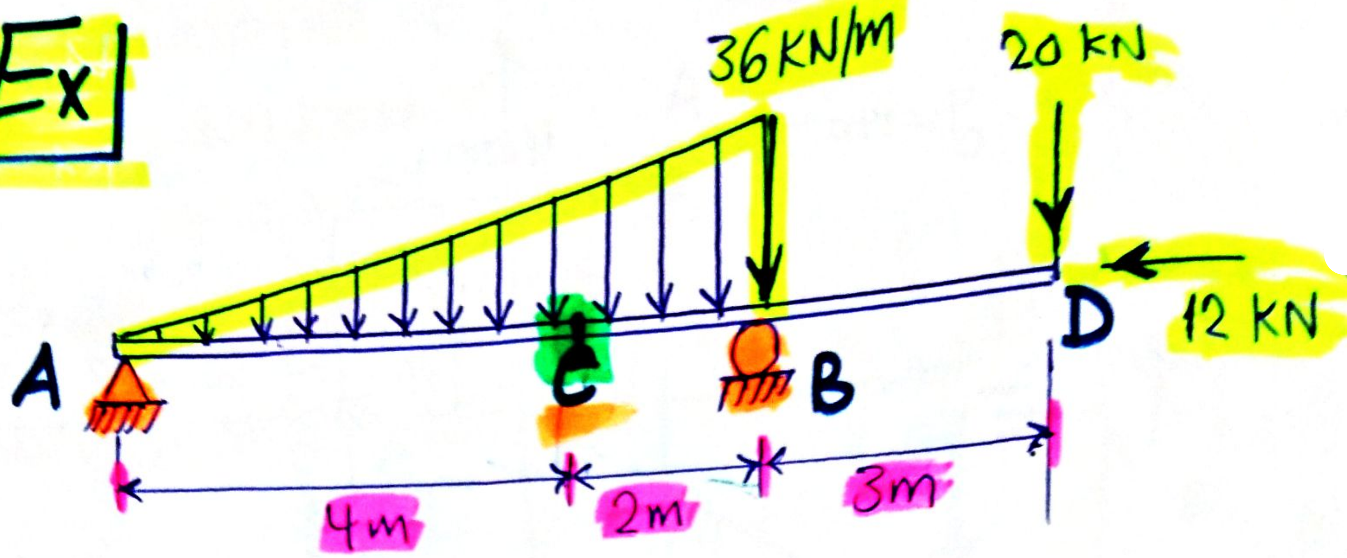
$$\varepsilon_x = \frac{\sigma_x}{E} - \frac{\nu}{E}\left(\sigma_y + \overset{0}{\cancel{\sigma_z}}\right)$$

$$(1.1893\times10^{-3})(200\times10^{9}) = 315549\,F + 4\times10^6 - 0.35\,(8\times10^6)$$

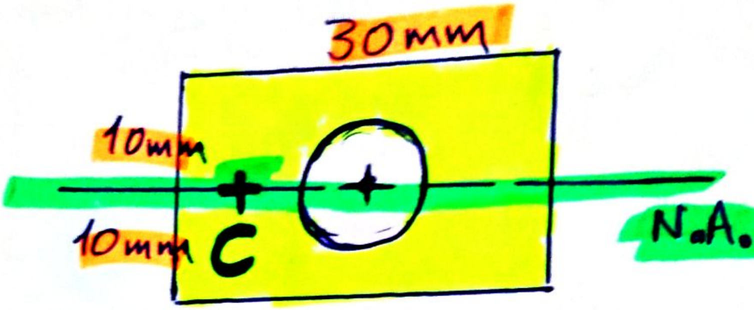
$\Rightarrow$ Solve for force F :

$\Rightarrow$ $F = 750\ N$

Ex



Find the principal Stresses and the maximum Shear Stress at point **C** ?



$r = 7$ mm

Cross-Section

* $I = \frac{1}{12} bh^3 - \frac{\pi}{4} r^4$

$I = \frac{1}{12}(0.03)(0.02)^3 - \frac{\pi}{4}(0.007)^4$

$I = 1.811 \times 10^{-8}\ m^4$

* $Q_c = (0.03)(0.01)\left(\frac{0.01}{2}\right) - \frac{2}{3}(0.007)^3$

$Q_c = 1.271 \times 10^{-6}\ m^3$

* Thickness of Cross Section at point C is :

$b = 30 - 2(7) = 16$ mm

* The Cross Sectional Area $A = (0.03)(0.02) - \pi(0.007)^2$

$\rightarrow A = 4.46 \times 10^{-4}\ m^2$

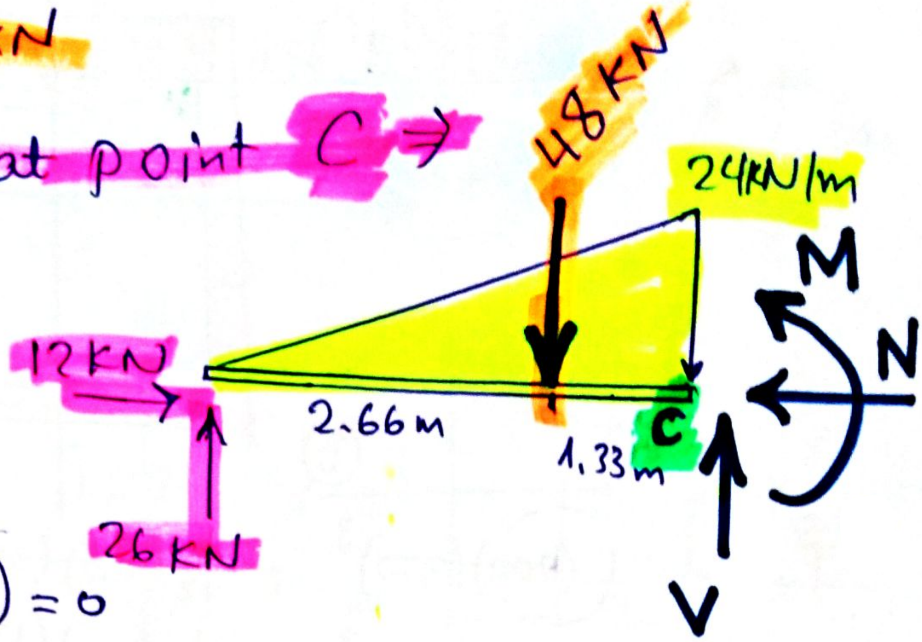
* Find Reactions at Support A & B :

$\Rightarrow A_x = 12\,kN$

$\Rightarrow A_y = 26\,kN$

$\Rightarrow B_y = 102\,kN$

* Now, Section at point C $\Rightarrow$

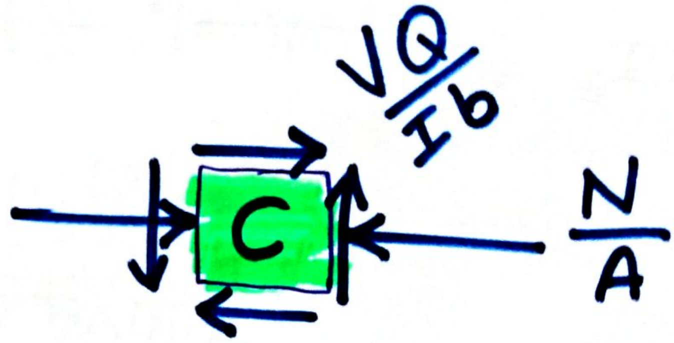


* $N = 12\,kN$
* $V = 48 - 26$

$\Rightarrow V = 22\,kN$

* $M + 48(1.33) - 26(4) = 0$

$\Rightarrow M = 40\,kN.m$



* $\sigma_x = \dfrac{-N}{A}$

$\sigma_x = -26.9\ MPa$

* $\tau_{xy} = \dfrac{VQ}{Ib} = 96.5\ MPa$

From Mohr's Circle :-

* $\sigma_{av} = \dfrac{\sigma_x + \sigma_y}{2} = -13.45\ MPa.$
* $R = \sqrt{\left(\dfrac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 97.43\ MPa.$
* $\sigma_1 = \sigma_{av} + R = 84\ MPa$
* $\sigma_2 = \sigma_{av} - R = -110.9\ MPa$

$\tau_{max} = R$

$\tau_{max} = 97.43\ MPa$

The spherical gas tank is fabricated by bolting together two hemispherical thin shells of thickness 30 mm. If the gas contained in the tank is under a gauge pressure of 2 MPa, determine the normal stress developed in the wall of the tank and in each of the bolts. The tank has an inner diameter of 8 m and is sealed with 900 bolts each 25 mm in diameter.

Normal Stress: Since $\frac{r}{t} = \frac{4}{0.03} = 133.33 > 10$, thin-wall analysis is valid. For the spherical tank's wall,

$$\sigma = \frac{pr}{2t} = \frac{2(4)}{2(0.03)} = 133 \text{ MPa} \qquad \textbf{Ans.}$$

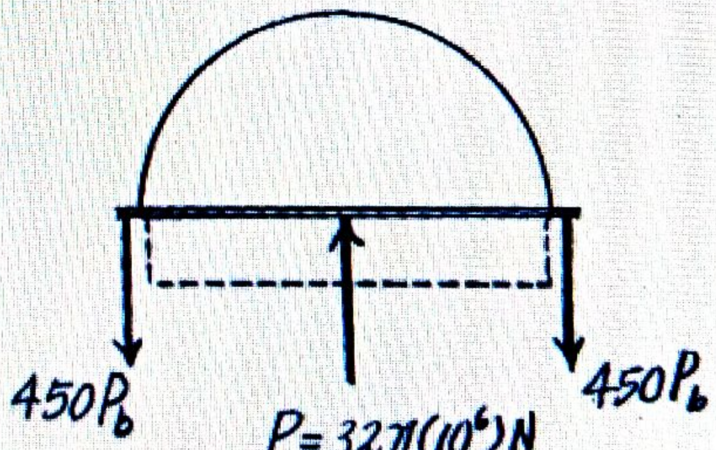
Referring to the free-body diagram shown in Fig. *a*, $P = pA = 2(10^6)\left[\frac{\pi}{4}(8^2)\right] = 32\pi(10^6)$ N. Thus,

$$+\uparrow \Sigma F_y = 0; \qquad 32\pi(10^6) - 450P_b - 450P_b = 0$$

$$P_b = 35.56(10^3)\pi \text{ N}$$

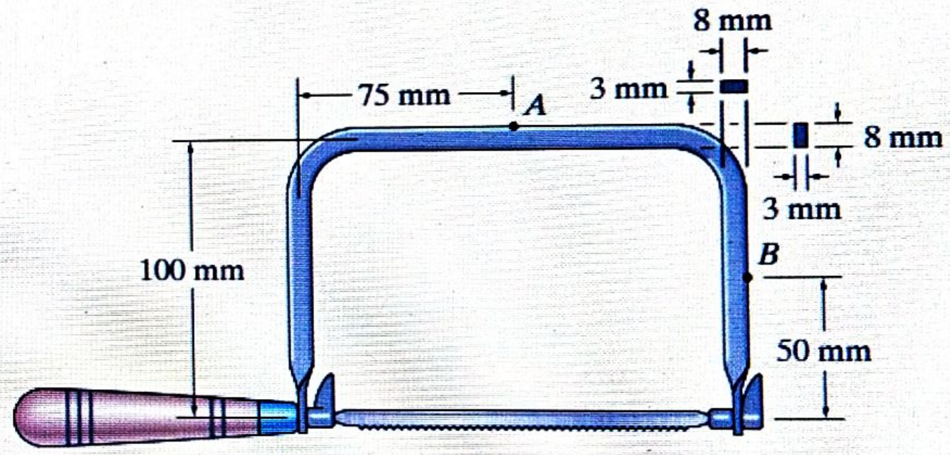
The normal stress developed in each bolt is then

$$\sigma_b = \frac{P_b}{A_b} = \frac{35.56(10^3)\pi}{\frac{\pi}{4}(0.025^2)} = 228 \text{ MPa} \qquad \textbf{Ans.}$$

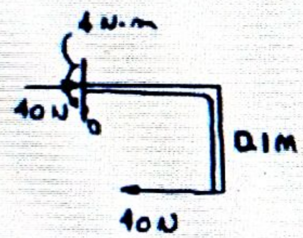


(a)

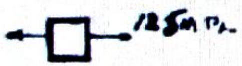
The coping saw has an adjustable blade that is tightened with a tension of 40 N. Determine the state of stress in the frame at points A and B.

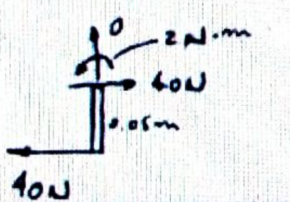


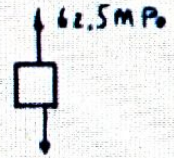
$$\sigma_A = -\frac{P}{A} + \frac{Mc}{I} = -\frac{40}{(0.008)(0.003)} + \frac{4(0.004)}{\frac{1}{12}(0.003)(0.008)^3} = 123 \text{ MPa} \qquad \textbf{Ans.}$$



$$\sigma_B = \frac{Mc}{I} = \frac{2(0.004)}{\frac{1}{12}(0.003)(0.008)^3} = 62.5 \text{ MPa} \qquad \textbf{Ans.}$$







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The offset link supports the loading of $P = 30$ kN. Determine its required width w if the allowable normal stress is $\sigma_{allow} = 73$ MPa. The link has a thickness of 40 mm.

σ due to axial force:

$$\sigma_a = \frac{P}{A} = \frac{30(10^3)}{(w)(0.04)} = \frac{750(10^3)}{w}$$

σ due to bending:

$$\sigma_b = \frac{Mc}{I} = \frac{30(10^3)(0.05 + \frac{w}{2})(\frac{w}{2})}{\frac{1}{12}(0.04)(w)^3}$$

$$= \frac{4500\,(10^3)(0.05 + \frac{w}{2})}{w^2}$$

My

$\frac{w}{2}$

I

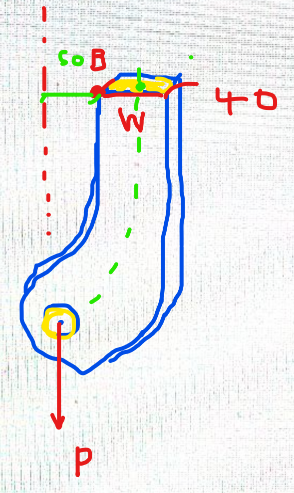
$$\sigma_{max} = \sigma_{allow} = \sigma_a + \sigma_b$$

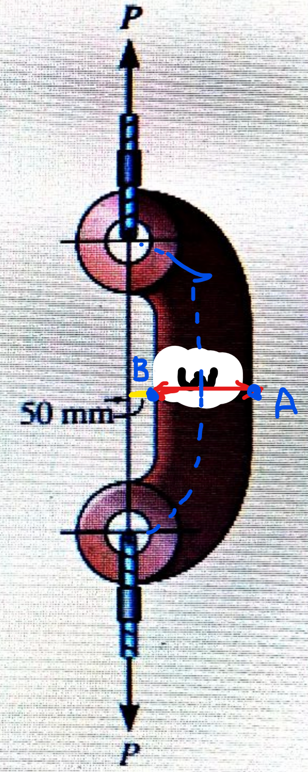
$$73(10^6) = \frac{750(10^3)}{w} + \frac{4500(10^3)(0.05 + \frac{w}{2})}{w^2}$$

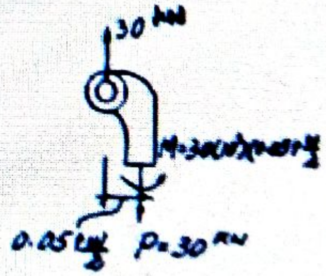
$$73\,w^2 = 0.75\,w + 0.225 + 2.25\,w$$

$$73\,w^2 - 3\,w - 0.225 = 0$$

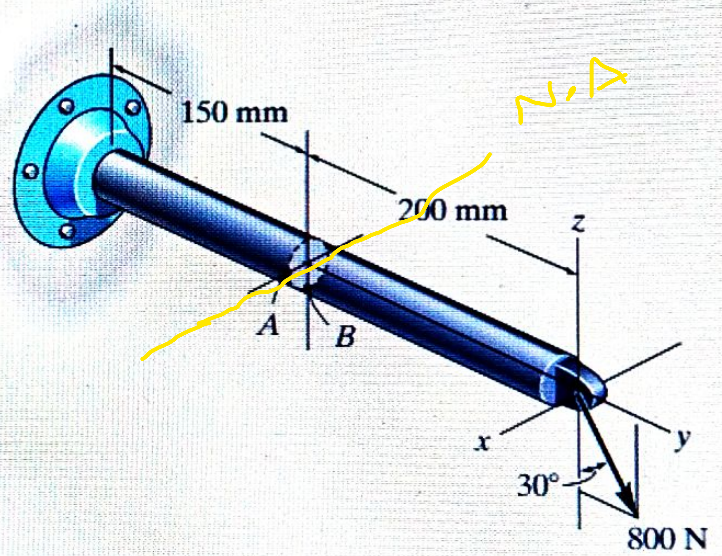
$$w = 0.0797 \text{ m} = 79.7 \text{ mm}$$ **Ans.**







The bar has a diameter of 40 mm. If it is subjected to a force of 800 N as shown, determine the stress components that act at point A and show the results on a volume element located at this point.



$$I = \frac{1}{4}\pi r^4 = \frac{1}{4}(\pi)(0.02^4) = 0.1256637\,(10^{-6})\,\mathrm{m}^4$$

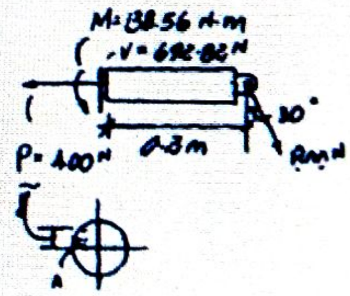
$$A = \pi r^2 = \pi(0.02^2) = 1.256637\,(10^{-3})\,\mathrm{m}^2$$

$$Q_A = \bar{y}'A' = \left(\frac{4\,(0.02)}{3\pi}\right)\left(\frac{\pi\,(0.02)^2}{2}\right) = 5.3333\,(10^{-6})\,\mathrm{m}^3$$

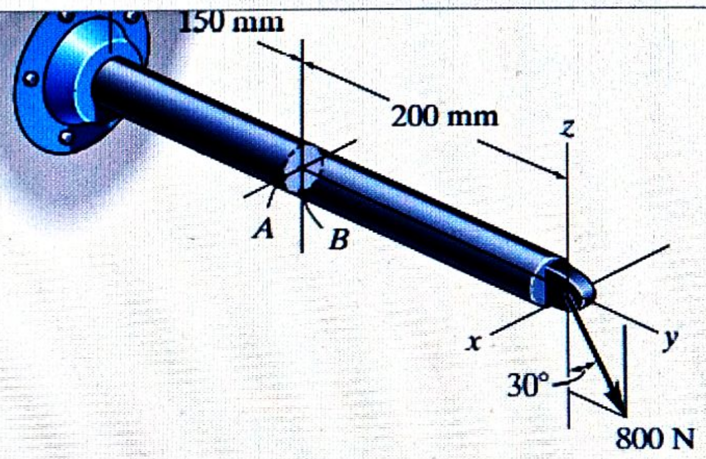
$$\sigma_A = \frac{P}{A} + \frac{Mz}{I}$$

A

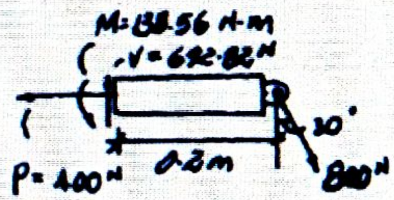
$$= \frac{400}{1.256637\,(10^{-3})} + 0 = 0.318\ \mathrm{MPa}$$ **Ans.**



$$\tau_A = \frac{VQ_A}{I\,t} = \frac{692.82\,(5.3333)\,(10^{-6})}{0.1256637\,(10^{-6})(0.04)} = 0.735\ \mathrm{MPa}$$ **Ans.**



$$I = \frac{1}{4}\pi r^4 = \frac{1}{4}(\pi)(0.02^4) = 0.1256637\,(10^{-6})\,\mathrm{m}^4$$



$$A = \pi r^2 = \pi(0.02^2) = 1.256637\,(10^{-3})\,\mathrm{m}^2$$

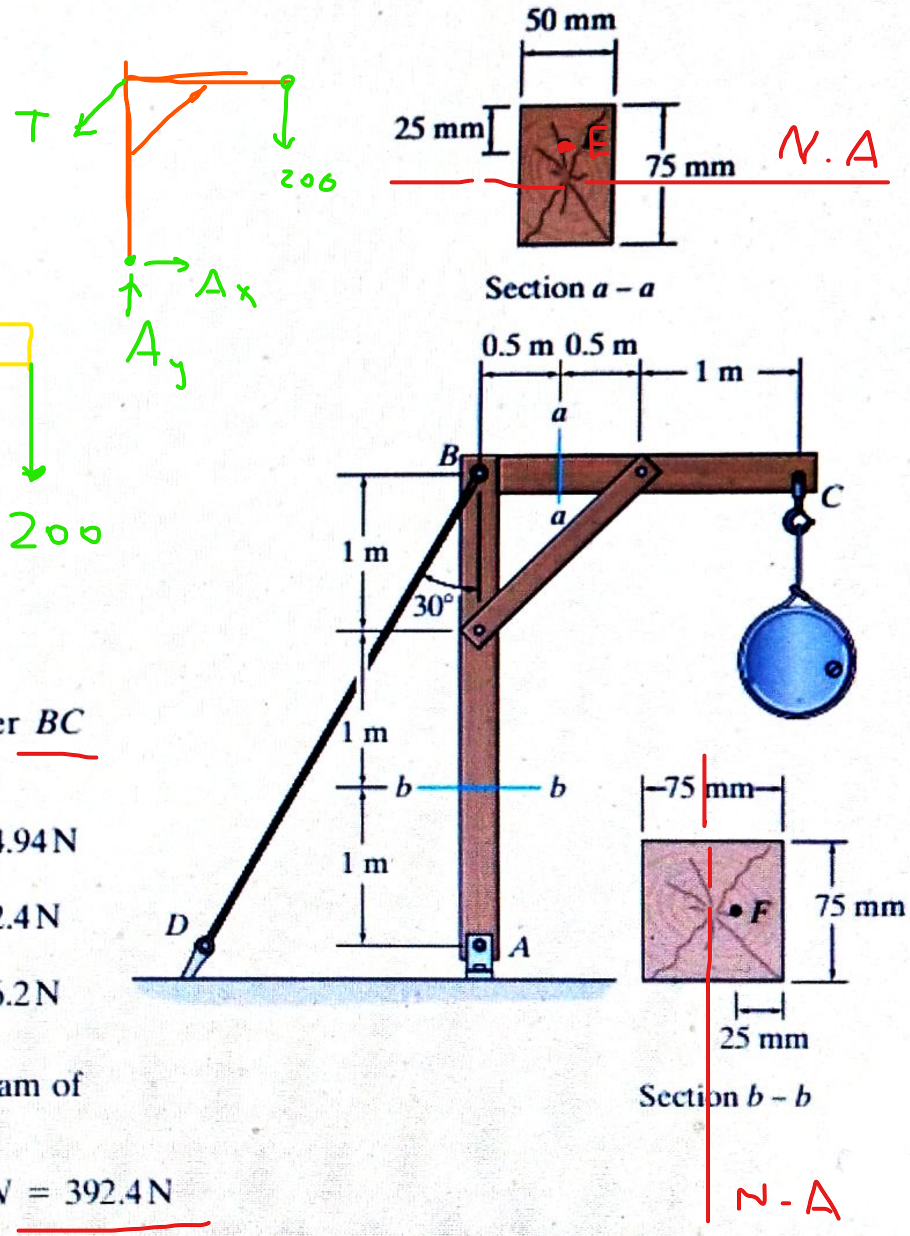
$\boxed{B}$

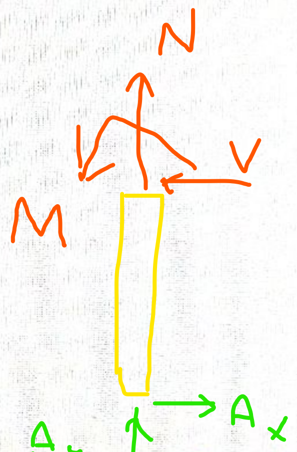
$$Q_B = 0$$

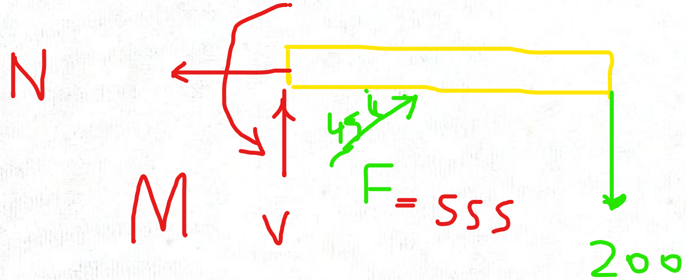
$$\sigma_B = \frac{P}{A} - \frac{Mc}{I} = \frac{400}{1.256637\,(10^{-3})} - \frac{138.56\,(0.02)}{0.1256637\,(10^{-6})} = -21.7\ \mathrm{MPa}$$ **Ans.**

$$\tau_B = 0$$ **Ans.**

The 20-kg drum is suspended from the hook mounted on the wooden frame. Determine the state of stress at point E on the cross section of the frame at section a–a. Indicate the results on an element.







Support Reactions: Referring to the free-body diagram of member BC shown in Fig. a,

$$\zeta + \Sigma M_B = 0; \quad F \sin 45°(1) - 20(9.81)(2) = 0 \qquad F = 554.94\text{ N}$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad 554.94 \cos 45° - B_x = 0 \qquad B_x = 392.4\text{ N}$$

$$+\uparrow \Sigma F_y = 0; \quad 554.94 \sin 45° - 20(9.81) - B_y = 0 \qquad B_y = 196.2\text{ N}$$

Internal Loadings: Consider the equilibrium of the free - body diagram of the right segment shown in Fig. b.

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad N - 392.4 = 0 \qquad N = 392.4\text{ N}$$

$\xrightarrow{+} \Sigma F_x = 0;$ $\quad N - 392.4 = 0$ $\quad N = 392.4\text{N}$

$+\uparrow \Sigma F_y = 0;$ $\quad V - 196.2 = 0$ $\quad V = 196.2\text{N}$

$\circlearrowleft + \Sigma M_C = 0;$ $\quad 196.2(0.5) - M = 0$ $\quad M = 98.1\,\text{N}\cdot\text{m}$

Section Properties: The cross -sectional area and the moment of inertia of the cross section are

$$A = 0.05(0.075) = 3.75\left(10^{-3}\right)\text{m}^2$$

$$I = \frac{1}{12}(0.05)\left(0.075^3\right) = 1.7578\left(10^{-6}\right)\text{m}^4$$

Referring to Fig. *c*, Q_E is

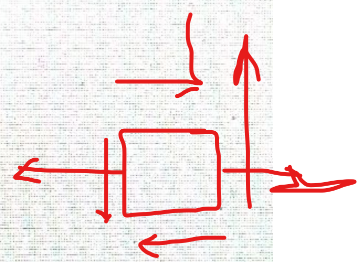
$$Q_E = \bar{y}'A' = 0.025(0.025)(0.05) = 3.125\left(10^{-6}\right)\text{m}^3$$

Normal Stress: The normal stress is the combination of axial and bending stress. Thus,

$$\sigma = \frac{N}{A} \pm \frac{My}{I}$$

For point A, $y = 0.0375 - 0.025 = 0.0125\,\text{m}$. Then

$$\sigma_E = \frac{392.4}{3.75\left(10^{-3}\right)} + \frac{98.1(0.0125)}{1.7578\left(10^{-6}\right)} = 802\text{ kPa} \qquad \textbf{Ans.}$$



802

Shear Stress: The shear stress is contributed by transverse shear stress only. Thus,

$$\tau_E = \frac{VQ_A}{It} = \frac{196.2\left[31.25\left(10^{-6}\right)\right]}{1.7578\left(10^{-6}\right)(0.05)} = 69.8\text{ kPa} \qquad \textbf{Ans.}$$

The state of stress at point E is represented on the element shown in Fig. *d*.

$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad A_x - 261.6 \sin 30^\circ = 0 \qquad A_x = 130.8\,\text{N}$

Internal Loadings: Consider the equilibrium of the free - body diagram of the lower cut segment, Fig. *b*,

$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 130.8 - V = 0 \qquad V = 130.8\,\text{N}$

$+\uparrow \Sigma F_y = 0; \qquad 422.75 - N = 0 \qquad N = 422.75\,\text{N}$

$\circlearrowleft + \Sigma M_C = 0; \qquad 130.8(1) - M = 0 \qquad M = 130.8\,\text{N}\cdot\text{m}$

Section Properties: The cross -sectional area and the moment of inertia about the centroidal axis of the cross section are

$$A = 0.075(0.075) = 5.625\left(10^{-3}\right)\text{m}^2$$

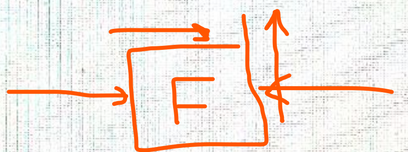
$$I = \frac{1}{12}(0.075)\left(0.075^3\right) = 2.6367\left(10^{-6}\right)\text{m}^4$$

Referring to Fig. c, Q_E is

$$Q_F = \bar{y}'A' = 0.025(0.025)(0.075) = 46.875\left(10^{-6}\right)\text{m}^3$$

Normal Stress: The normal stress is the combination of axial and bending stress. Thus,

$$\sigma = \frac{N}{A} \pm \frac{My}{I}$$



For point F, $y = 0.0375 - 0.025 = 0.0125$ m. Then

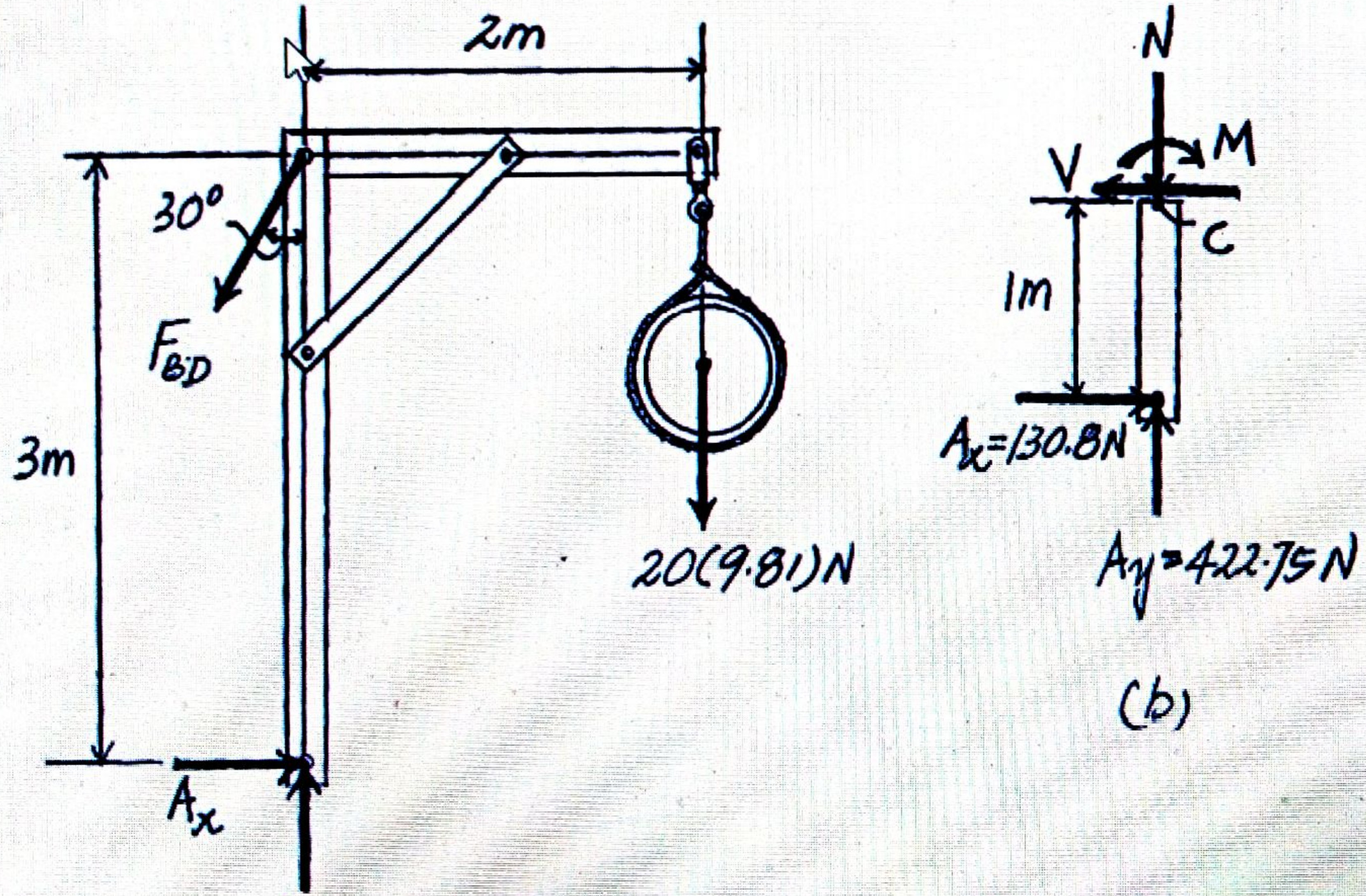
$$\sigma_F = \frac{-422.75}{5.625\left(10^{-3}\right)} - \frac{130.8(0.0125)}{2.6367\left(10^{-6}\right)}$$

$$= -695.24\,\text{kPa} = 695\,\text{kPa (C)} \qquad \textbf{Ans.}$$

Shear Stress: The shear stress is contributed by transverse shear stress only. Thus,

$$\tau_A = \frac{VQ_A}{It} = \frac{130.8\left[46.875(10^{-6})\right]}{2.6367(10^{-6})(0.075)} = 31.0\text{ kPa} \qquad \textbf{Ans.}$$

The state of stress at point *A* is represented on the element shown in Fig. *d*.



(b)