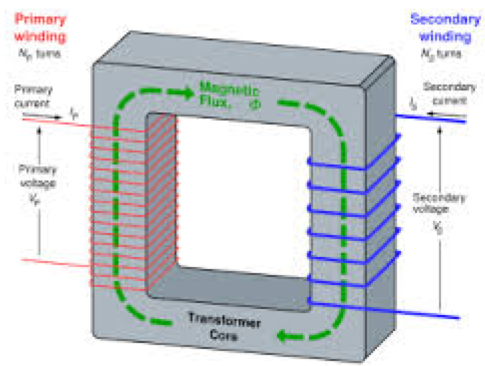
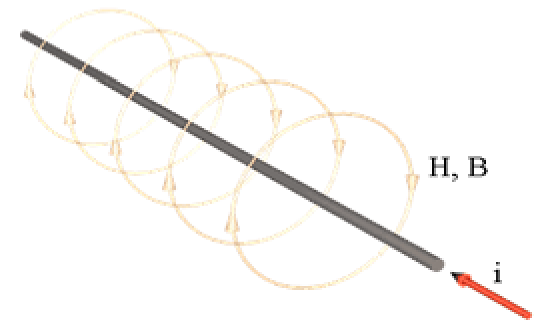
EE373-Electrical Machines

Topic 1: Magnetic Circuits





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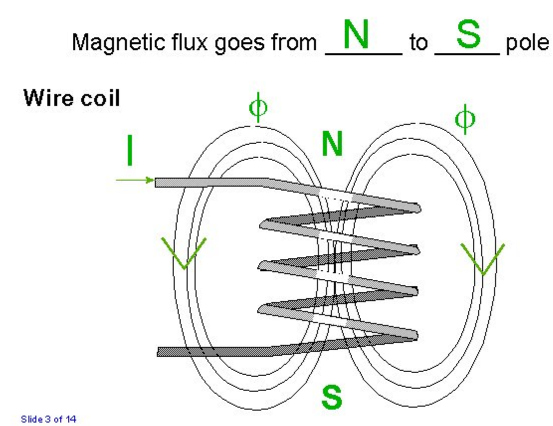


Topic 1: Magnetic Circuits

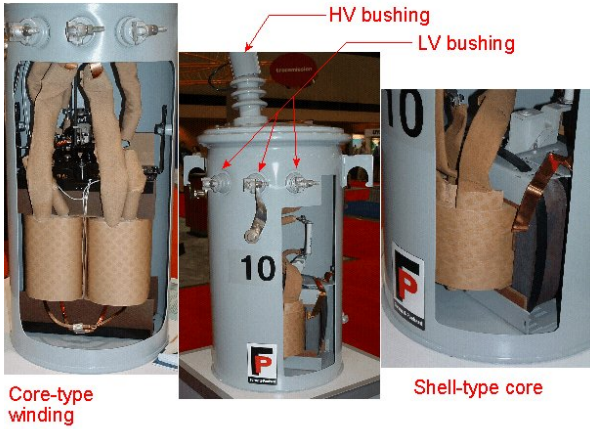
An **electric machine** is a devise that can convert electrical energy to mechanical or/and mechanical energy to electrical.

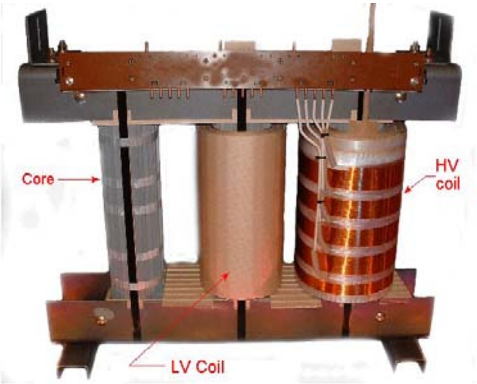
1.1. Generators convert mechanical energy from a prime mover to electrical energy through the action of the magnetic field.

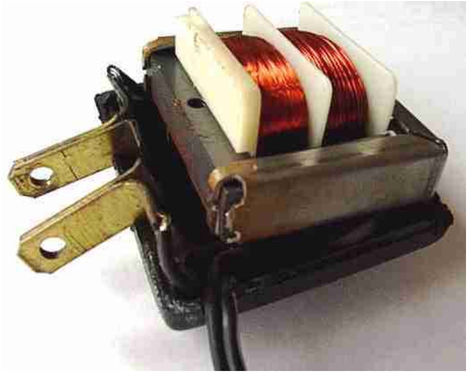
1.2. Motors convert electrical energy from a power source to mechanical energy through the action of the magnetic field.

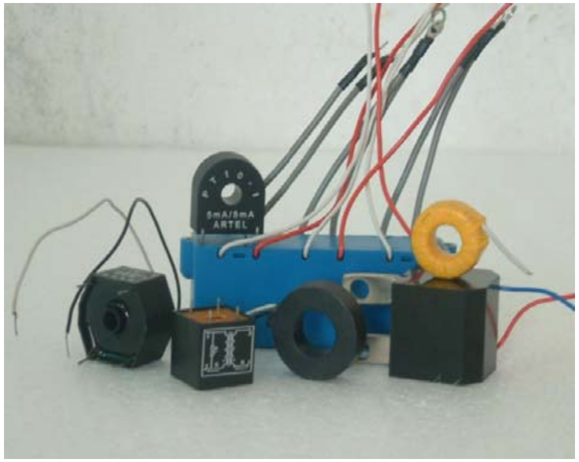
Topic 1: Magnetic Circuits

1.3. Transformers convert AC electrical energy at one voltage level to AC electrical energy at (an)other voltage level(s).









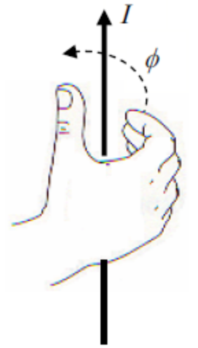
Basic principles underlying usage of magnetic field

1. A wire carrying a current produces a magnetic field around it.

2. A time-changing magnetic field induces a voltage in a coil of wire if it passes through that coil (*transformer action*).

3. A wire carrying a current in the presence of a magnetic field experiences a force induced on it (*motor action*).

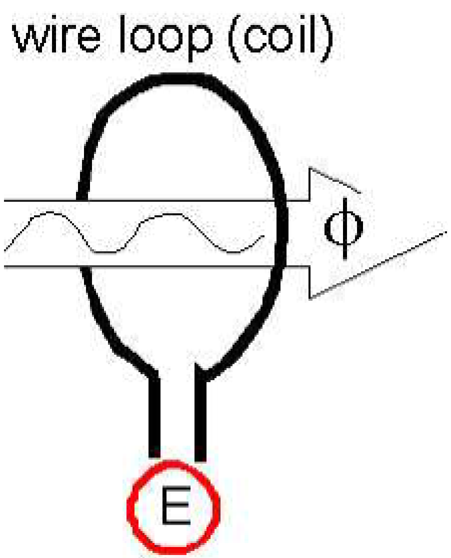
4. A wire moving in a presence of a magnetic field gets a voltage induced in it (*generator action*).

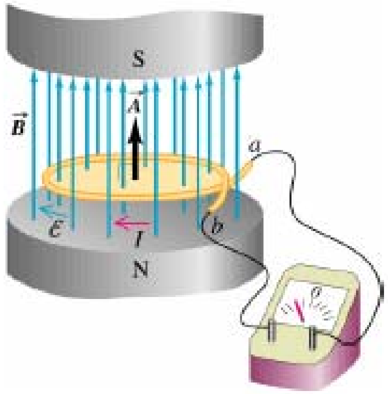


1. The Faradays law

If a flux passes through a turn of a coil of wire, a voltage will be induced in that turn that is directly proportional to the rate of change in the flux with respect to time:

$$e_{ind} = -\frac{d\phi}{dt}$$





Or, for a coil having N turns:

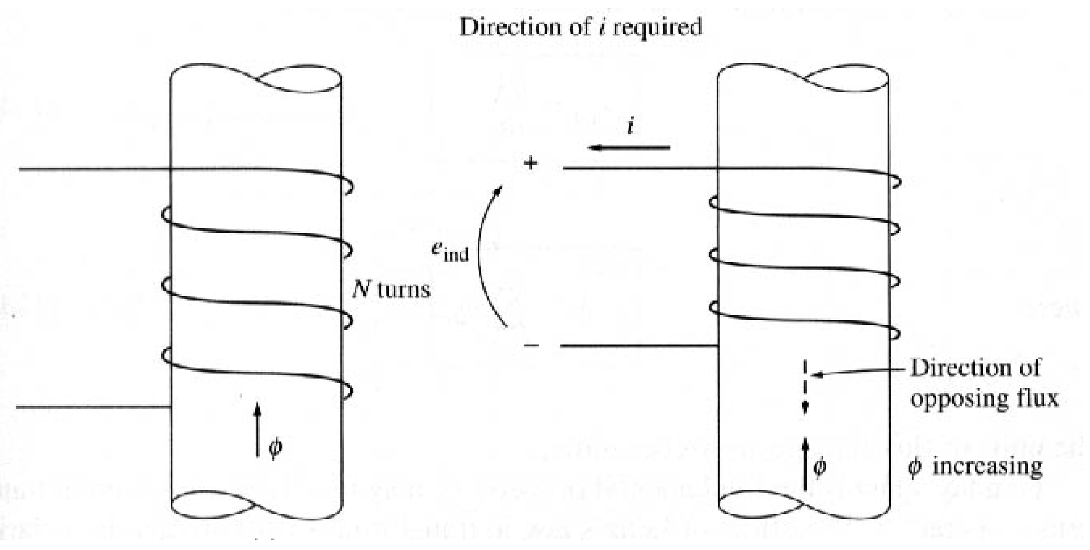
$$e_{ind} = -N\frac{d\phi}{dt}$$

e_{ind} – voltage induced in the coil
N – number of turns of wire in the coil
ϕ - magnetic flux passing through the coil

1. The Faradays law

The "minus" sign in the equation is a consequence of the Lentz's law stating that the direction of the voltage buildup in the coil is such that if the coil terminals were short circuited, it would produce a current that would cause a flux opposing the original flux change.

If the initial flux is increasing, the voltage buildup in the coil will tend to establish a flux that will oppose the increase. Therefore, a current will flow as indicated and the polarity of the induced voltage can be determined.



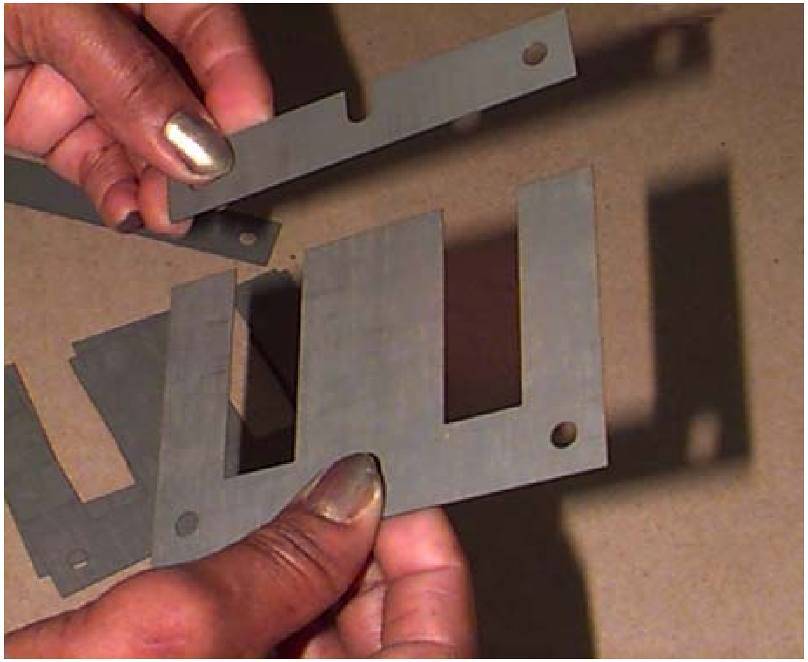
The minus sign is frequently omitted since the polarity is easy to figure out.

1. The Faradays law

A nature of eddy current losses:

Voltages are generated within a ferromagnetic core by a time-changing magnetic flux same way as they are induced in a wire. These voltages cause currents flowing in the resistive material (ferromagnetic core) called eddy currents. Therefore, energy is dissipated by these currents in the form of heat.

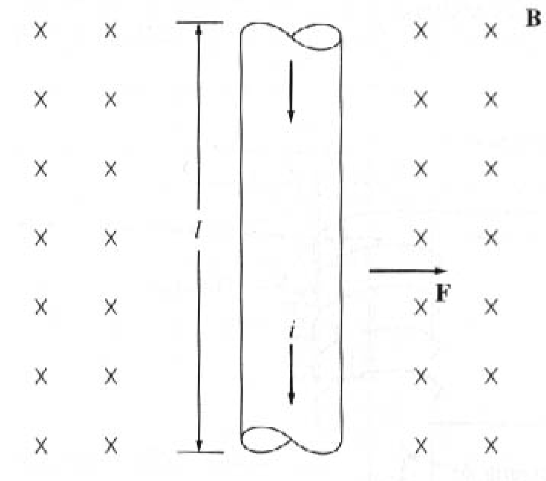
The amount of energy lost to eddy currents is proportional to the size of the paths they travel within the core. Therefore, ferromagnetic cores are frequently laminated: core consists of a set of tiny isolated strips. Eddy current losses are proportional to the square of the lamination thickness.

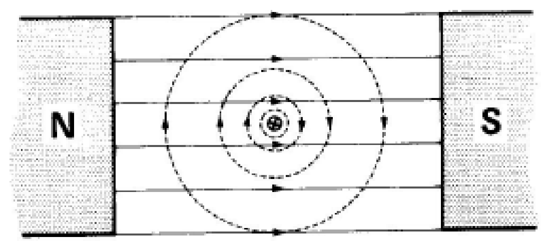


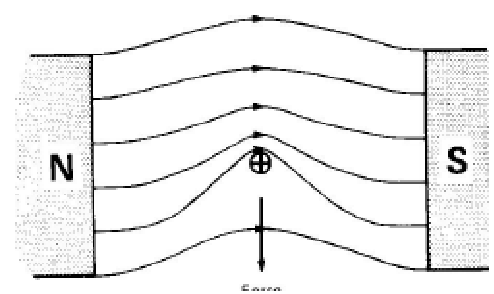
2. Production of induced force on a wire

A second major effect of a magnetic field is that it induces a force on a wire caring a current within the field.

$$F = I \times B$$







Where I is a vector of current, B is the magnetic flux density vector.

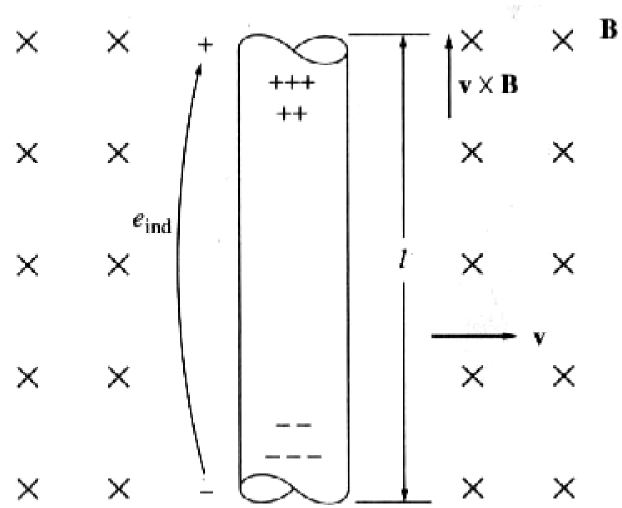
For a wire of length l caring a current i in a magnetic field with a flux density B that makes an angle θ to the wire, the magnitude of the force is:

$$F = ilB\sin\theta$$

This is a basis for a motor action.

3. Induced voltage on a conductor moving in a magnetic field

The third way in which a magnetic field interacts with its surrounding is by an induction of voltage in the wire with the proper orientation moving through a magnetic field.



$$e_{ind} = (v \times B)l$$

Where v is the velocity of the wire, l is its length in the magnetic field, B – the magnetic flux density

This is a basis for a generator action.

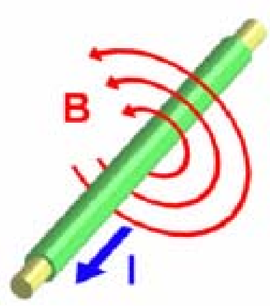
4. The magnetic field

4.1. Production of magnetic field

Magnetic field When a conductor carries current a magnetic field in the area around it is produced.

The direction of H (magnetic field intensity) can be determined by thumb rule.

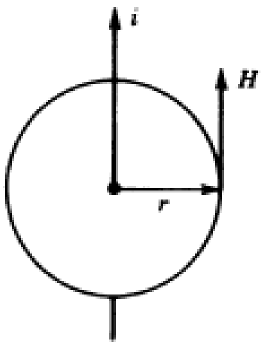
Ampere's circuit law The line integral of the magnetic field intensity H around a closed path is equal to the current enclosed.

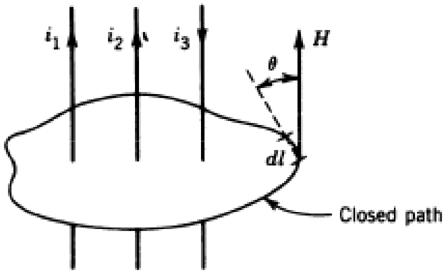


Consider a conductor carries a current *i*

The field intensity H at distance r

$$Hl = i \;\Rightarrow\; H = \frac{i}{2\pi r}$$





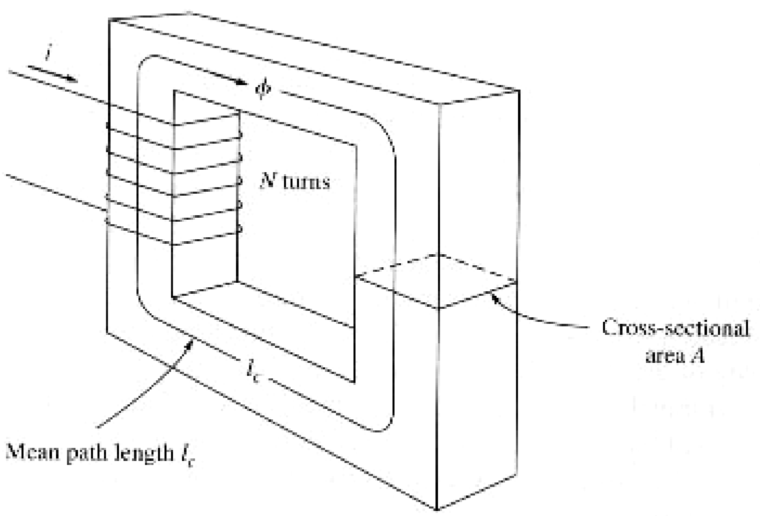
4. The magnetic field

The Ampere's law:

$$\phi = \oint H.dl = I_{net}$$

Where *H [A-turns/m]* is the intensity of the magnetic field produced by the current I_{net}

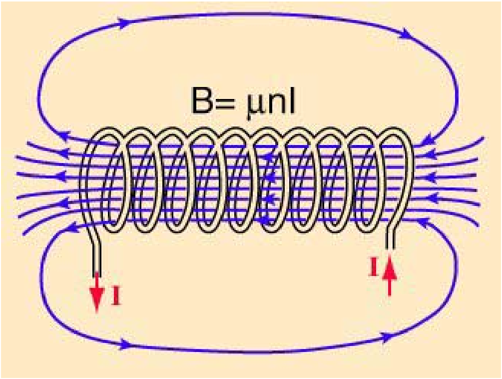
For the ferromagnetic cores, almost all the magnetic field produced by the current remains inside the core, therefore the integration path would be l_c and the current passes it *N* times.



$$I_{net} = Ni \Rightarrow H = \frac{Ni}{l_c}$$

4. The magnetic field

Magnetic flux density: $$B = \mu H = \frac{\mu N i}{l_c}$$

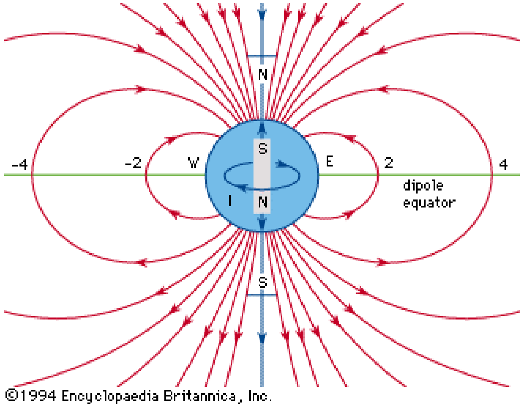


where $\mu = \mu_o \mu_r$ is the magnetic permeability of a material.

μ_r – the relative permeability

$\mu_o = 4\pi \times 10^{-7}$ *H/m – the* permeability of free space

The total flux in a given area: $$\phi = \int_A B.dA$$



If the magnetic flux density vector *B* is perpendicular to a plane of the area:

$$\Phi = BA = \frac{\mu N i A}{l_c}$$

4. The magnetic field

The <u>magnetic field intensity H</u> produce a <u>magnetic flux density B (Tesla)</u> at every point given by

$$B = \mu H \ \ wb/m^2 \ or \ Tesla$$

$$B = \mu_0 \mu_r H \ \ wb/m^2 \ or \ Tesla$$

Where:

μ= permeability of a material

μr = Relative permeability

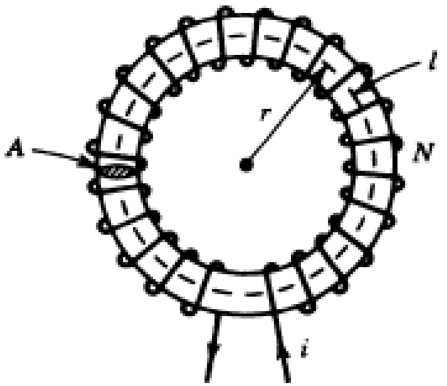
μ_o = permeability of free-space

Permeability For ferromagnetic materials, $\mu_r = 2000\text{-}6000$ **and for non-magnetic materials,** $\mu_r = 1$

4. The magnetic field

4.2. Magnetic circuits

Consider a toroid



$$\oint H \cdot dl = \sum i = Ni \qquad Hl = Ni$$

Magneto-Motive Force (mmf) F= Ni is the external force required to set up the magnetic flux lines within the magnetic material.

$$H = \frac{F}{l} = \frac{NI}{l}$$

$$B = \frac{\mu N I}{l}$$

Where:

F = Magneto-Motive Force (At)

H = Magnetizing Force (At/m)

N = number of turns (turns, t)

B=magnetic flux density (T)

L = is the mean length (m)

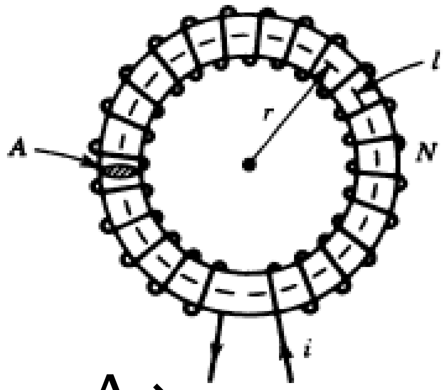
4.2. Magnetic circuits

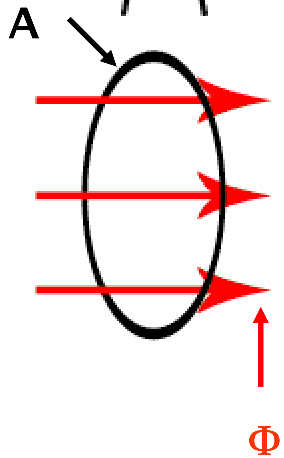
Magnetic Flux Φ (Webers) through a given surface is given (All the flux is confined and no leakage flux)

$$\Phi = \int B\, dA = BA \quad \Rightarrow \quad B = \frac{\Phi}{A}$$

$$\because H = \frac{NI}{l} \quad \Rightarrow \quad \Phi = \mu H A = \frac{\mu N I A}{l}$$

$$\Phi = \frac{NI}{l / \mu A} \quad \Rightarrow \quad \Phi = \frac{NI}{\Re}$$





Where:

Φ = is the flux (wb)

B = is the average flux density

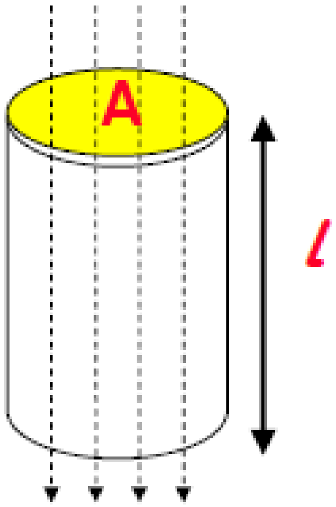
A = is cross-sectional area (m^2)

$\Re$ = is the reluctance

4.2. Magnetic circuits

Reluctance is a measure of the opposition the magnetic circuit offer to flux analogues to the resistance in an electric circuit

$$\Re = \frac{l}{\mu A}$$



Materials with high permeability such as ferromagnetic materials have small reluctance and will result in increased measure of flux through the core.

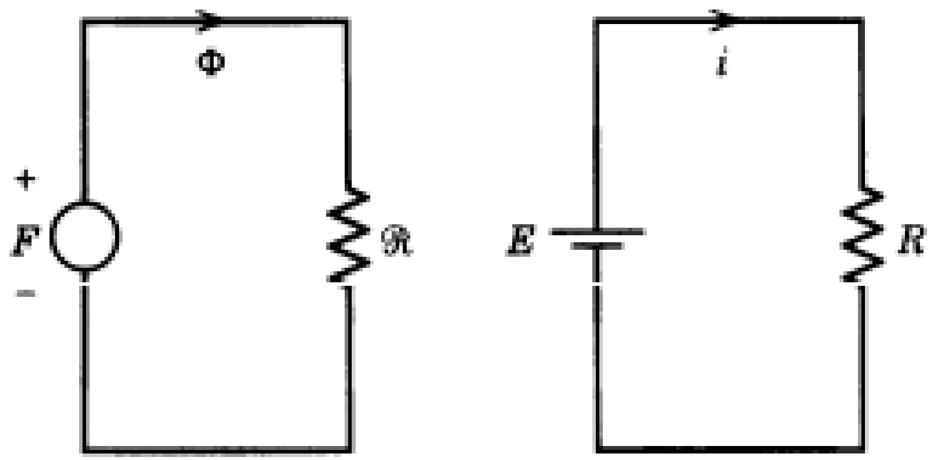
Where:

l = length of the circuit (m)

A = Cross-sectional area (m^2)

μ = permeability of the material

4.2. Magnetic circuits

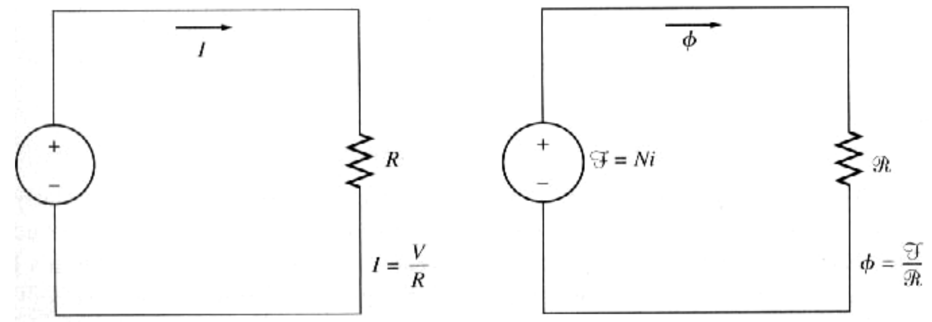


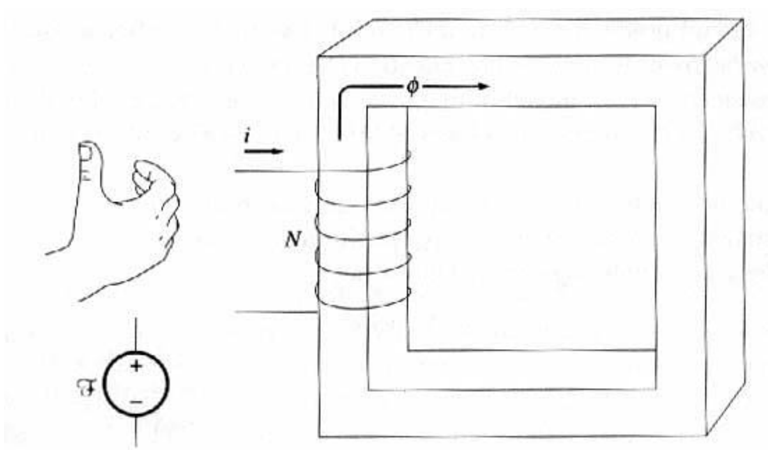
	Electric Circuit	Magnetic Circuit
Driving Force	**Emf (E)**	**Mmf (F)**
Produce	**Current i**	**Flux Φ**
Limited by	**Resistance R**	**Reluctance**

4.2. Magnetic circuits

Similarly to electric circuits, there are magnetic circuits …

Instead of electromotive force (voltage) magnetomotive force (mmf) is what drives magnetic circuits.





$$F = Ni$$

Direction of mmf is determined by RHR…

Like the Ohm's law, the Hopkinson's Law:

$$F = \phi R$$

F: mmf;

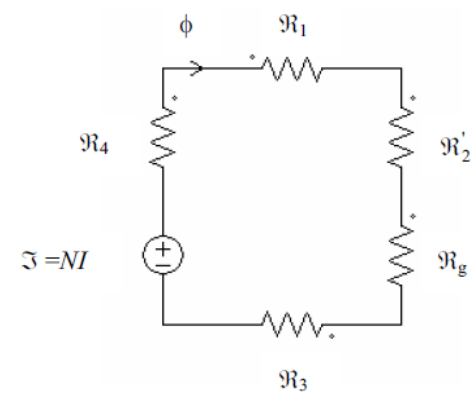
ϕ: magnetic flux

R: reluctance

4.2. Magnetic circuits

Permeance: $$P = \frac{1}{R}$$

Magnetic flux: $$\phi = F P = BA = \frac{\mu N i A}{l_c} = F \frac{\mu A}{l_c}$$



Therefore, the reluctance: $$R = \frac{l_c}{\mu A}$$

Serial connection: $$R_{eq} = R_1 + R_2 + \ldots + R_N$$

Parallel connection: $$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_N}$$

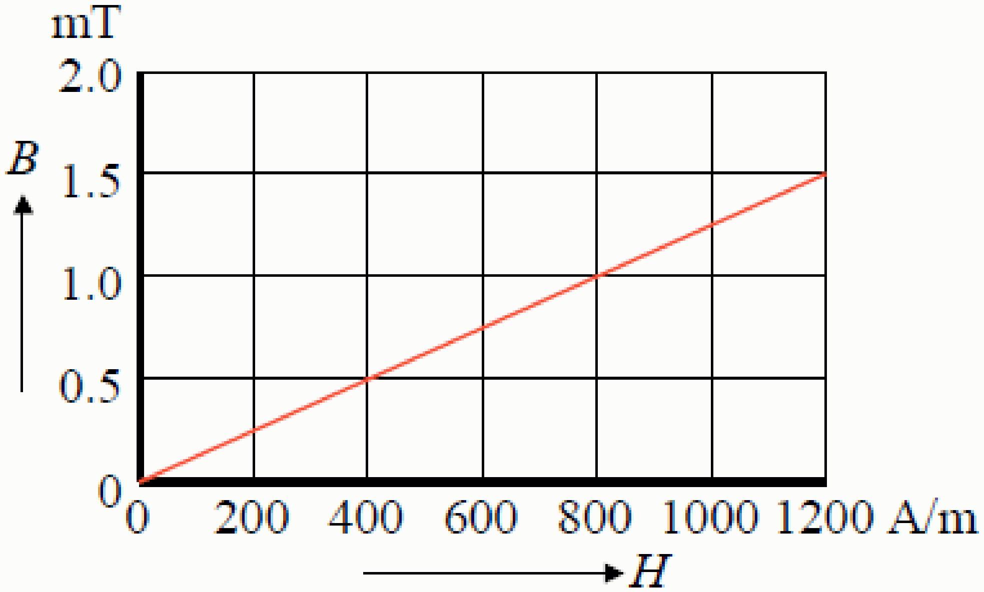
$$\frac{1}{\Re_{eq}} = \frac{1}{\Re_1} + \frac{1}{\Re_2} + \frac{1}{\Re_3}$$

Relationship between B-H in Free Space

- In free space, the magnetic flux density B is directly proportional to the magnetic field intensity H
- The constant of proportionality for free space is called the permeability constant, μ_0

$$B = \mu_0 H$$

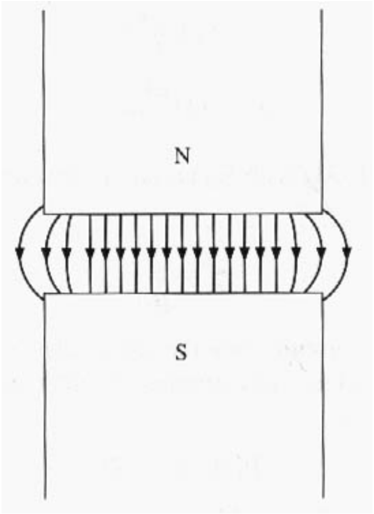
  – in the SI system, $\mu_0 = 4\pi\times10\text{-}7$ H/m [henry/meter]



4.2. Magnetic circuits

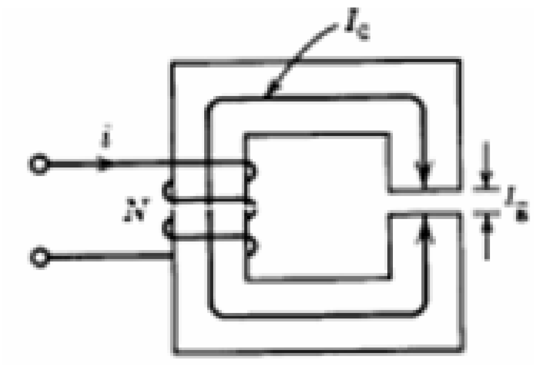
Calculations of magnetic flux are **always** approximations!

1. We assume that all flux is confined within the magnetic core but a leakage flux exists outside the core since permeability of air is non-zero!
2. A mean path length and cross-sectional area are assumed…
3. In ferromagnetic materials, the permeability varies with the flux.
4. In air gaps, the cross-sectional area is bigger due to the fringing effect.

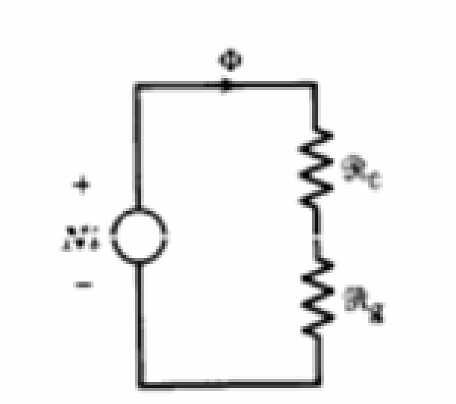


Core with Air-gap

Magnetic circuit with an air gap



$$\Re_c = \frac{l_c}{\mu_c A_c} \qquad \Re_g = \frac{l_g}{\mu_0 A_g}$$



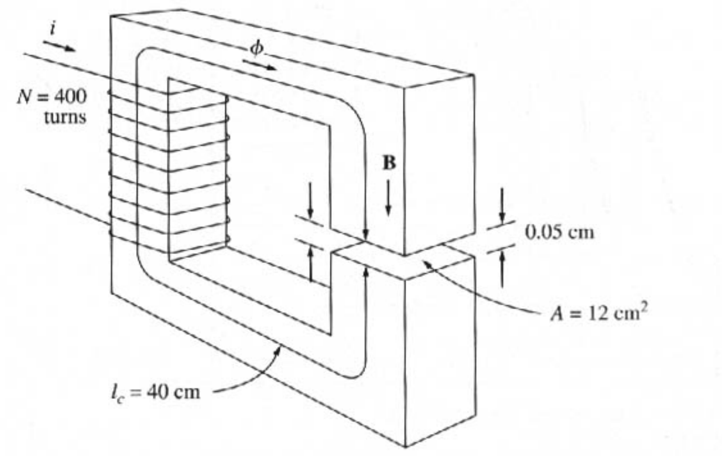
$$\Phi = \frac{Ni}{\Re_c + \Re_g}$$

$$Ni = H_c l_c + H_g l_g$$

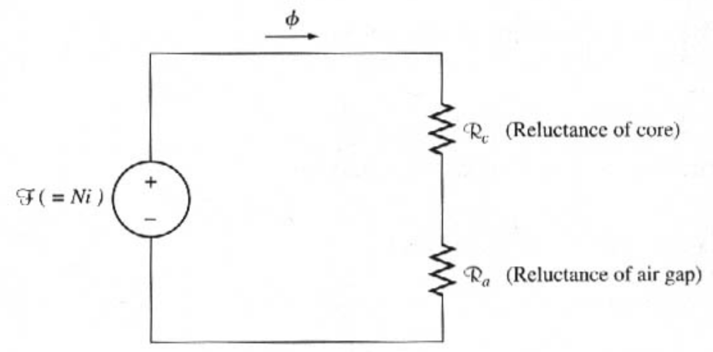
$$B_c = \frac{\Phi_c}{A_c} \qquad B_g = \frac{\Phi_g}{A_g}$$

4.2. Magnetic circuits

Example 1: A ferromagnetic core with a mean path length of 40 cm, an air gap of 0.05 cm, a cross-section 12 cm^2, and μ_r = 4000 has a coil of wire with 400 turns. Assume that fringing in the air gap increases the cross-sectional area of the gap by 5%, find (a) the total reluctance of the system (core and gap), (b) the current required to produce a flux density of 0.5 T in the gap.



The equivalent circuit



4.2. Magnetic circuits

(a) The reluctance of the core:

$$\mathcal{R}_c = \frac{l_c}{\mu A_c} = \frac{l_c}{\mu_0 \mu_r A_c} = \frac{0.4}{4000 \cdot 4\pi \cdot 10^{-7} \cdot 0.0012} = 66\,300\ A-turns/Wb$$

Since the effective area of the air gap is 1.05 x 12 = 12.6 cm^2, its reluctance:

$$\mathcal{R}_a = \frac{l_a}{\mu_0 A_a} = \frac{0.0005}{4\pi \cdot 10^{-7} \cdot 0.00126} = 316\,000\ A-turns/Wb$$

The total reluctance:

$$\mathcal{R}_{eq} = \mathcal{R}_c + \mathcal{R}_a = 66300 + 316000 = 382\,300\ \ A-turns/Wb$$

The air gap contribute most of the reluctance!

4.2. Magnetic circuits

(b) The mmf:

$$F = \phi R = Ni = BAR$$

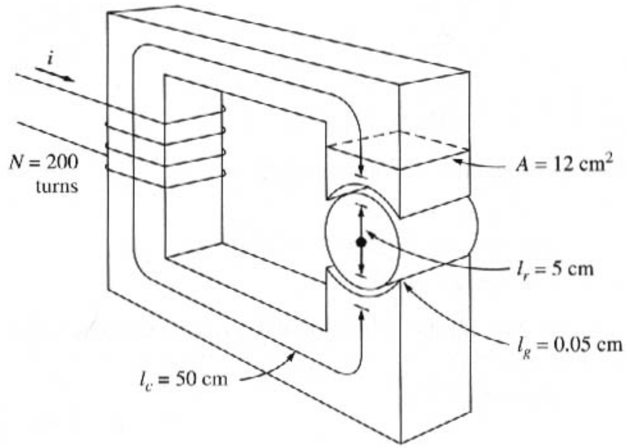
Therefore:

$$i = \frac{BAR}{N} = \frac{0.5 \cdot 0.00126 \cdot 383200}{400} = 0.602\ A$$

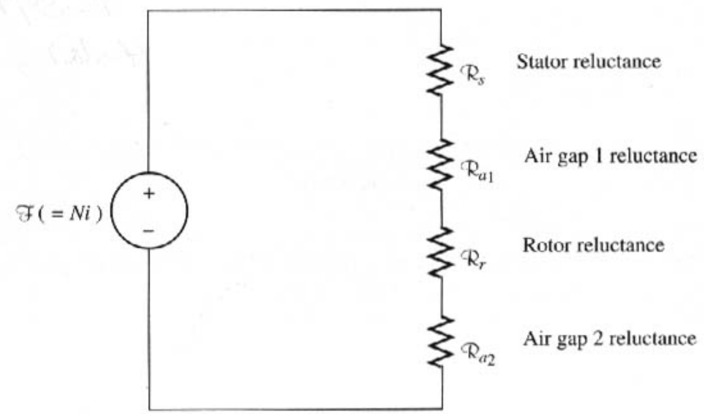
Since the air gap flux was required, the effective area of the gap was used.

4.2. Magnetic circuits

Example 2: In a simplified rotor and stator motor, the mean path length of the stator is 50 cm, its cross-sectional area is 12 cm^2, and μ_r = 2000. The mean path length of the rotor is 5 cm and its cross-sectional area is also 12 cm^2, and μ_r = 2000. Each air gap is 0.05 cm wide, and the cross-section of each gap (including fringing) is 14 cm^2. The coil has 200 turns of wire. If the current in the wire is 1A, what will the resulting flux density in the air gaps be?



The equivalent circuit



4.2. Magnetic circuits

The reluctance of the stator is:

$$\mathcal{R}_s = \frac{l_s}{\mu_r \mu_0 A_s} = \frac{0.5}{2000 \cdot 4\pi \cdot 10^{-7} \cdot 0.0012} = 166000 \;\; A-turns/Wb$$

The reluctance of the rotor is:

$$\mathcal{R}_r = \frac{l_r}{\mu_r \mu_0 A_r} = \frac{0.05}{2000 \cdot 4\pi \cdot 10^{-7} \cdot 0.0012} = 16600 \;\; A-turns/Wb$$

The reluctance of each gap is:

$$\mathcal{R}_a = \frac{l_a}{\mu_0 A_a} = \frac{0.0005}{4\pi \cdot 10^{-7} \cdot 0.0014} = 284000 \;\; A-turns/Wb$$

The total reluctance is:

$$\mathcal{R}_{eq} = \mathcal{R}_s + \mathcal{R}_{a1} + \mathcal{R}_r + \mathcal{R}_{a2} = 751000 \;\; A-turns/Wb$$

4.2. Magnetic circuits

The net mmf is: $F = Ni$

The magnetic flux in the core is:

$$\phi = \frac{F}{R} = \frac{Ni}{R}$$

Finally, the magnetic flux density in the gap is:

$$B = \frac{\phi}{A} = \frac{Ni}{\mathrm{R}\ A} = \frac{200 \cdot 1}{751000 \cdot 0.0014} = 0.19\,T$$

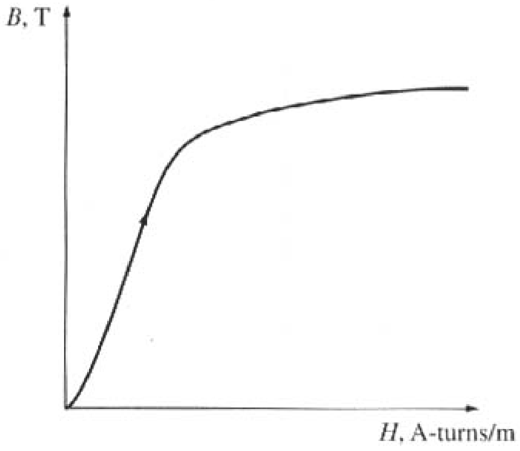
4. The magnetic field

4.3. Magnetic behavior of ferromagnetic materials

Magnetic permeability can be defined as: $$\mu = \frac{B}{H}$$

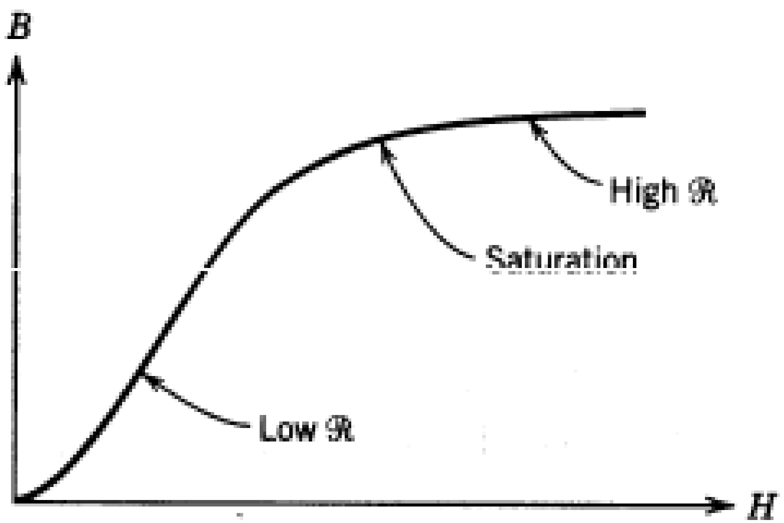
and was previously assumed as constant. However, for the ferromagnetic materials (for which permeability can be up to 6000 times the permeability of air), permeability is not a constant…

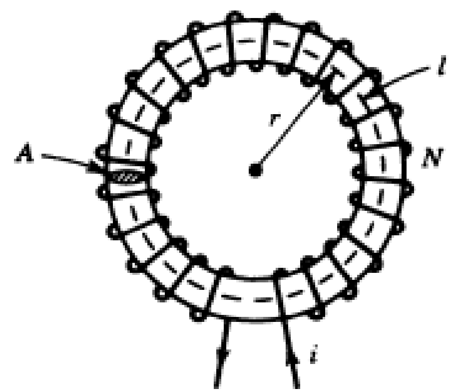
A saturation (magnetization) curve for a DC source



4.3. Magnetic behavior of ferromagnetic materials

Magnetization Curve If the magnetic field intensity H in the core increased (by increasing the current), the flux density increases almost linearly at low values of H, but at higher values of H the magnetic material shows the effect of Saturation i.e. the change of B is non-liner- The relation is as shown in the B-H curve





B-H Curve (Magnetization Curve)

4.3. Magnetic behavior of ferromagnetic materials

The magnetizing intensity is:

$$H = \frac{Ni}{l_c} = \frac{F}{l_c}$$

The magnetic flux density:

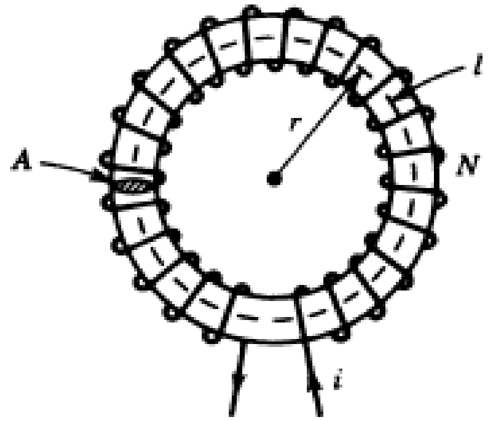
$$\phi = BA$$

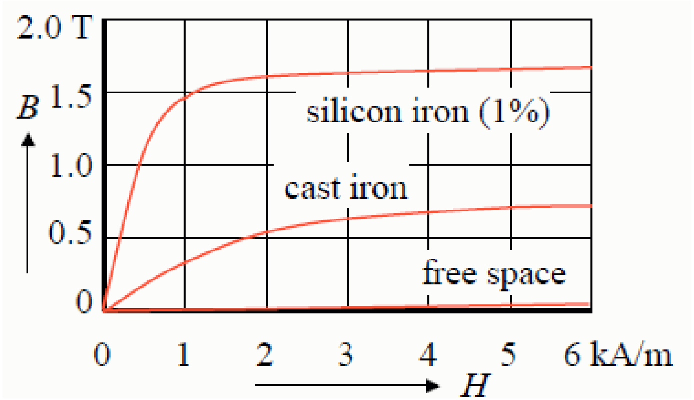
Therefore, the magnetizing intensity is directly proportional to mmf and the magnetic flux density is directly proportional to magnetic flux for any magnetic core.

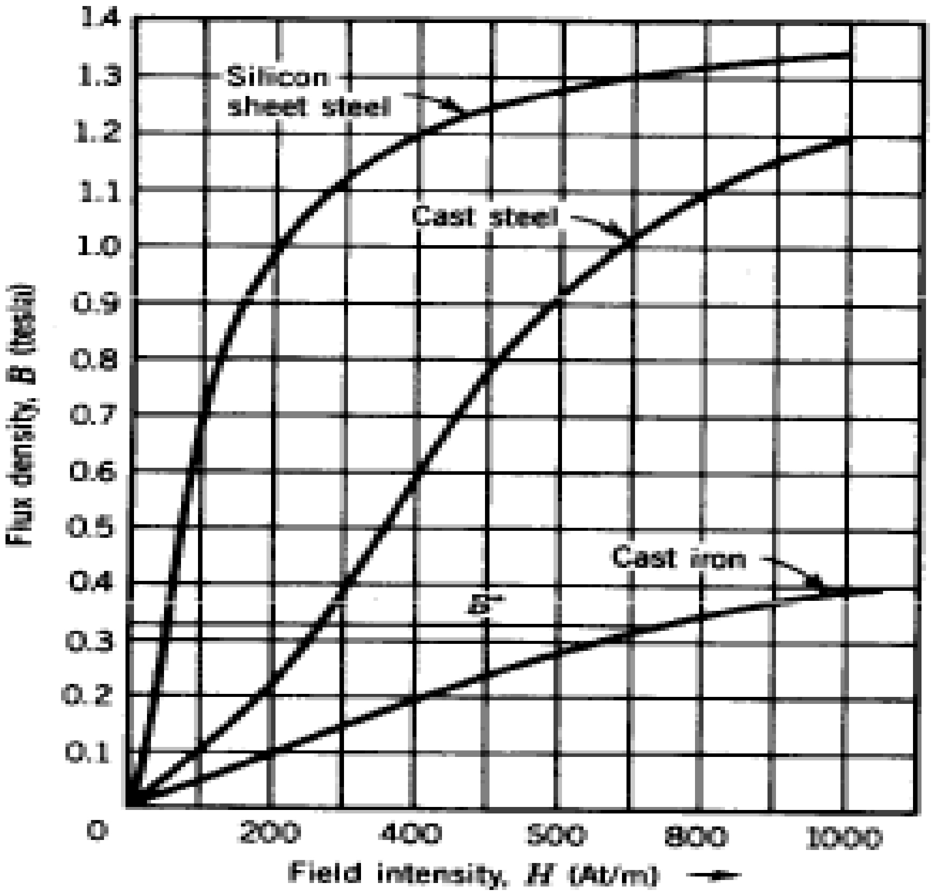
Ferromagnetic materials are essential since they allow to produce much more flux for the given mmf than when air is used.

4.3. Magnetic behavior of ferromagnetic materials

Magnetization curves of three different materials

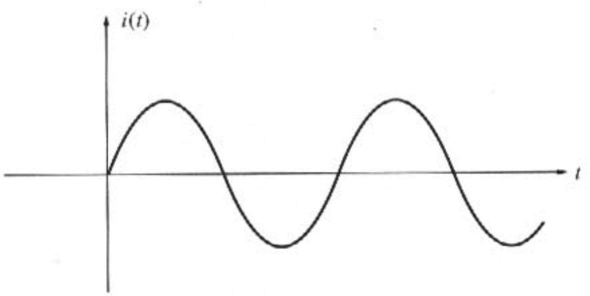




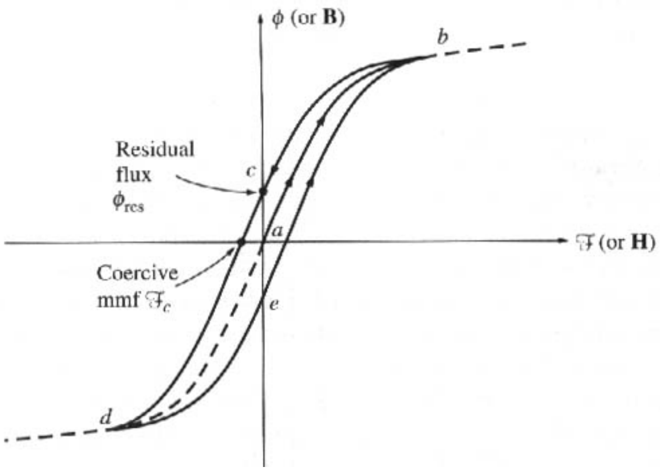


4. The magnetic field

4.4. Energy losses in a ferromagnetic core



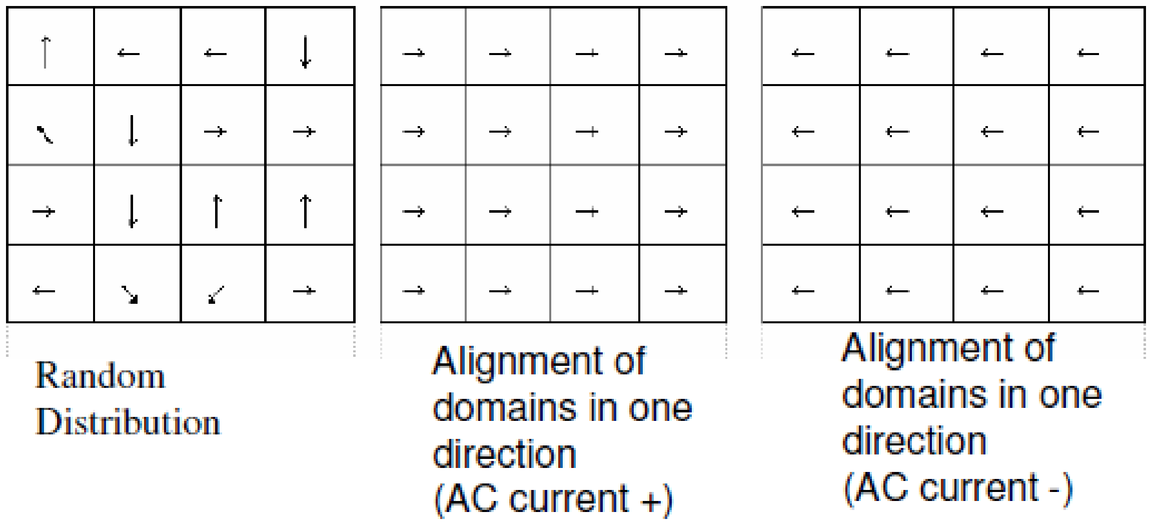
If instead of a DC, a sinusoidal current is applied to a magnetic core, a hysteresis loop will be observed…



If a large mmf is applied to a core and then removed, the flux in a core does not go to zero! A magnetic field (or flux), called the residual field (or flux), will be left in the material. To force the flux to zero, an amount of mmg (coercive mmf) is needed.

4.4. Energy losses in a ferromagnetic core

Ferromagnetic materials consist of small domains, within which magnetic moments of atoms are aligned. However, magnetic moments of domains are oriented randomly.



When an external magnetic field is applied, the domains pointing in the direction of that field grow since the atoms at their boundaries physically switch their orientation and align themselves in the direction of magnetic field. This increases magnetic flux in the material which, in turn, causes more atoms to change orientation. As the strength of the external field increases, more domains change orientation until almost all atoms and domains are aligned with the field. Further increase in mmf can cause only the same flux increase as it would be in a vacuum. This is a saturation.

4.4. Energy losses in a ferromagnetic core

When the external field is removed, the domains do not completely randomize again. Realigning the atoms would require energy! Initially, such energy was provided by the external field.

Atoms can be realigned by an external mmf in other direction, mechanical shock, or heating.

The hysteresis loss in the core is the energy required to reorient domains during each cycle of AC applied to the core.

Another type of energy losses is an eddy currents loss, which will be examined later.

Hysteresis and eddy current losses

When a magnetic material undergoes cyclic magnetization, two kind of power losses occur in it;

Hysteresis (P_h) and

Eddy current losses (P_e)

which known together as

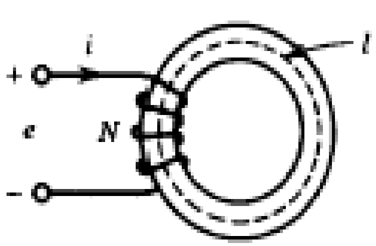
Core losses (P_c)

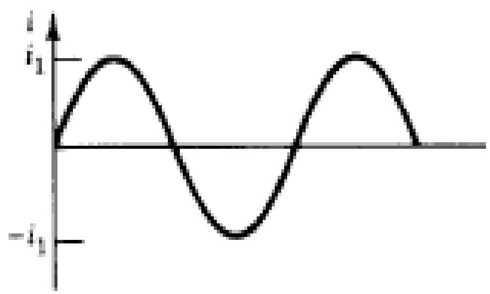
The loss appears as a heat in the core and it is important in determining, heating, temperature rise and efficiency.

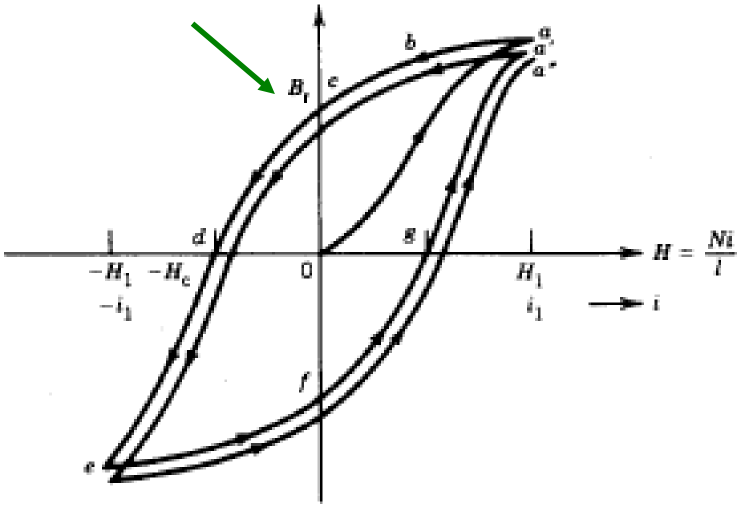
Hysteresis losses

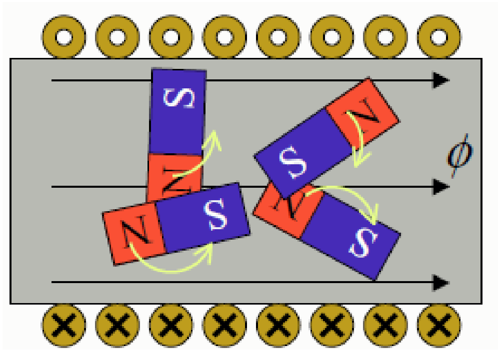
Consider the coil core assembly initially un-magnetized 0, If H increased by increasing i, results curve 0-a. If H decreased results *abc* curve. when $H = 0$ the core retained

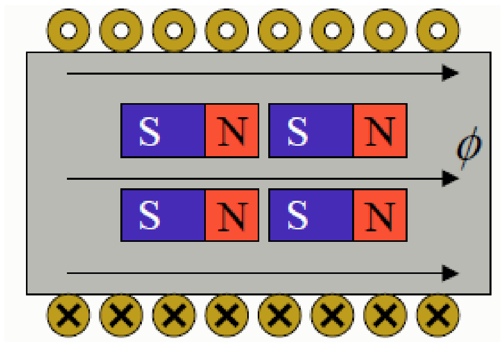
B_r (residual flux density)

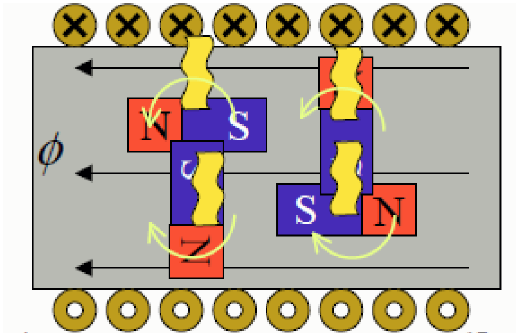










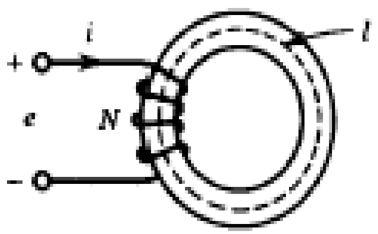


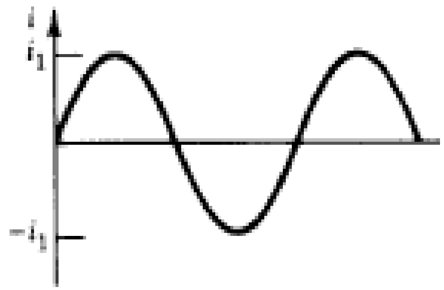
Hysteresis losses

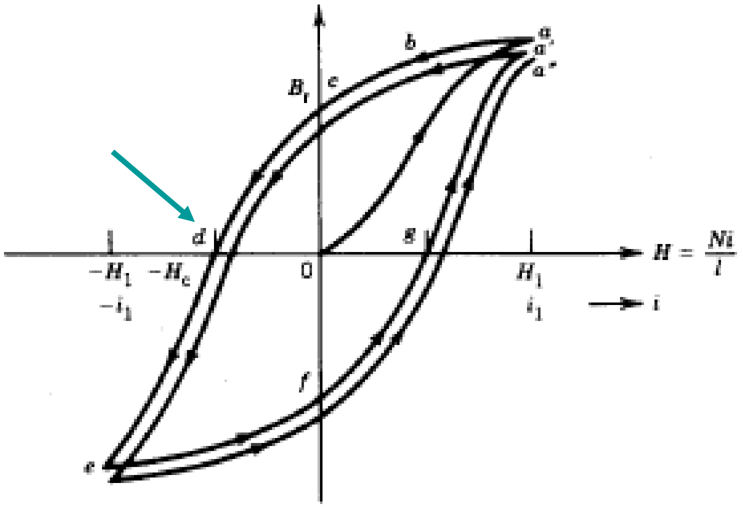
If H is reversed (by i) results

H_c (Coercivity or coercive force)

B_r is removed c d. If H is further increased B increases (in the reverse direction) e, if H decreased to 0, then to H_1 results the curve $efga'$, The loop does not close a' and another cycle a''



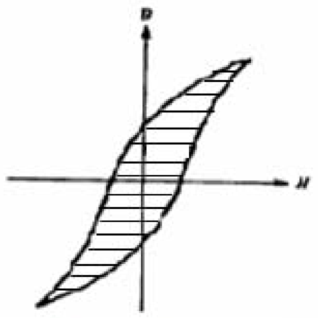




Hysteresis Loss

The energy transfer over one cycle

$$W = V_{core} \times Area\ of\ the\ B-H\ loop = V_{core} \times W_h$$



The hysteresis loss in volume V of material operated at f Hz

$$P_h = V_{core} \times W_h \times f \qquad W$$

An empirical formula based on experimental studies is often used to avoid the need of computing the loop area:

$$P_h = K_h \times B_{max}^n \times f \qquad W/m^3$$

Where:

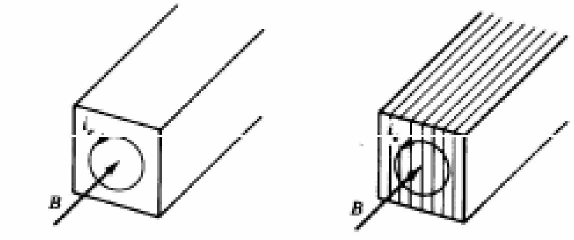
K_h is constant depends on core material and volume

B_{max} is the maximum flux density

n is constant vary from 1.5 to 2.5 depending on the material

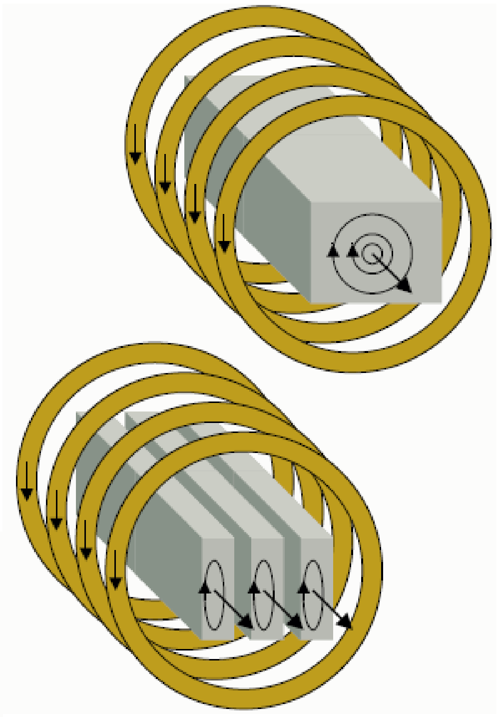
Eddy Current Loss

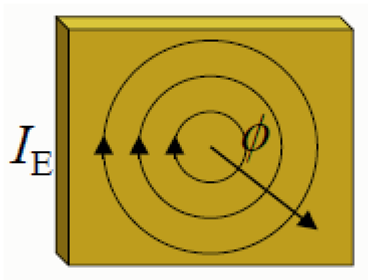
- When a magnetic flux density varies, a voltage is induced results circulating currents known as eddy currents in the core
- Since all magnetic materials have resistance, a power loss i_e^2R associated with them called eddy current loss.



$$i_e \alpha e = \frac{dB}{dt}$$

- Losses can be reduced by splitting the core into sections (lamination)–subdividing causes the losses to decrease progressively–varnish coatings insulate the laminates from current flows–silicon in the iron increases the resistance.





Eddy Current Loss

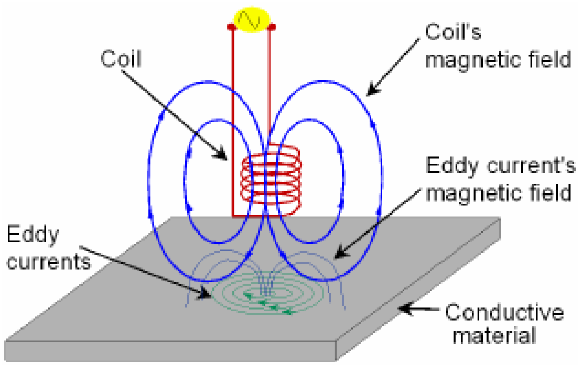
The eddy current loss can be reduced by:

Increasing the resistivity of the core material

Using a laminated core

The eddy current losses can be calculated

$$P_e = K_e \times B_{max}^2 \times f^2 \quad W/m^3$$



Where:

Ke is constant depends on core material and its lamination thicknesses

Core Loss

The core losses is the combination of hysteresis loss and the eddy current loss in the material.

$$P_c = P_h + P_e$$

In practice manufactures supply data used to estimate the core loss

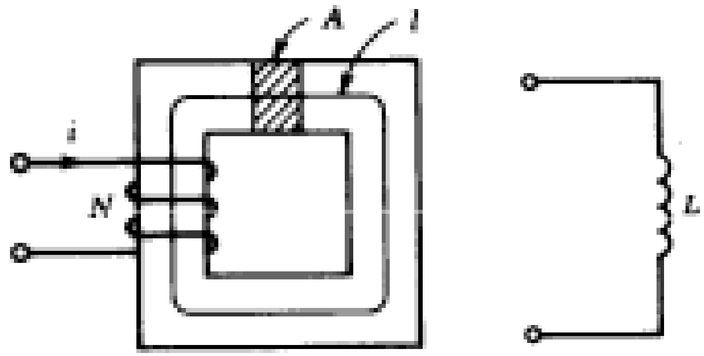
Inductance

A magnetic circuit excited by current i produces Φ

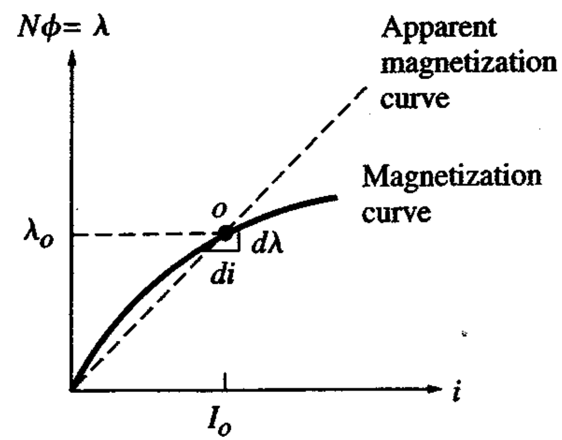
Each turn encloses or links the flux Φ

$$\lambda = N\Phi$$

Flux Linkage



Faraday's law $$e = N\frac{d\Phi}{dt}$$



Self inductance of the coil

$$L = \frac{\lambda}{i} \qquad L = \frac{N^2}{\Re}$$