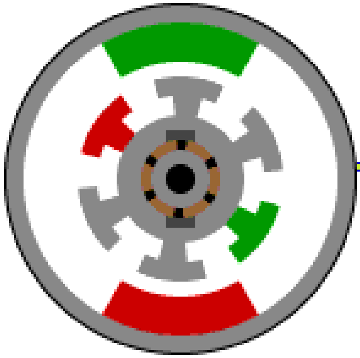
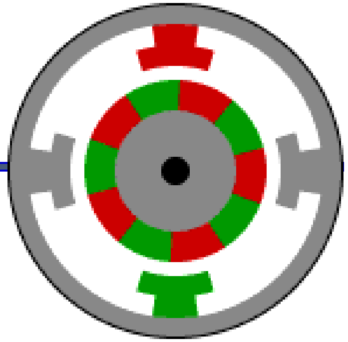
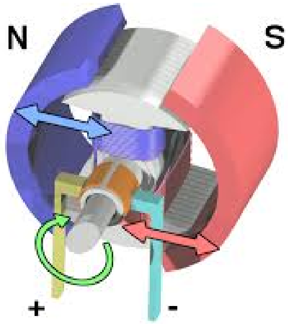
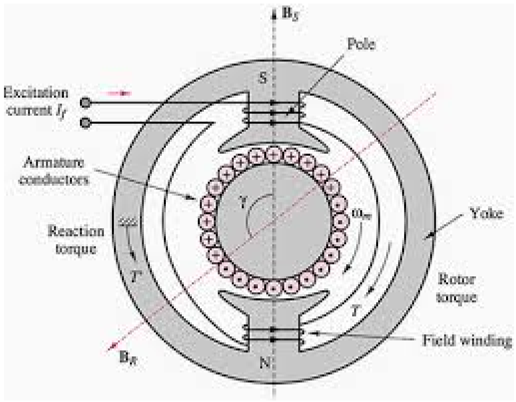
EE373-Electrical Machines

Topic 3: DC Machines

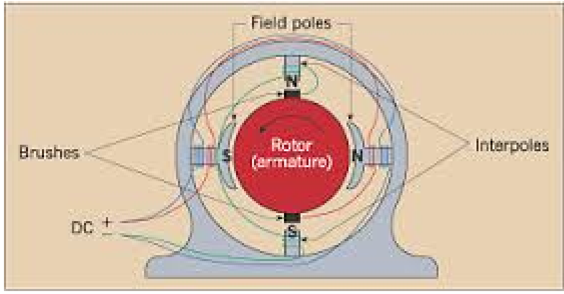








2500 HP, 1200 RPM DC motor for repair



DC Machines

LEARNING GOALS

Introduction
- Application of DC Machine
- Advantages & Disadvantages of DC Machine

Construction of DC Machine
- Field System
- Armature
- Commutator
- Brush

Principle of Operation
- Faraday's Law
- Armature Voltage & Developed Torque

Classification of DC Machine
- Permanent Magnet
- Self-Excited
- Separately-Excited

DC Machine Representation

Magnetization Curve (Saturation)

DC Motor Equations

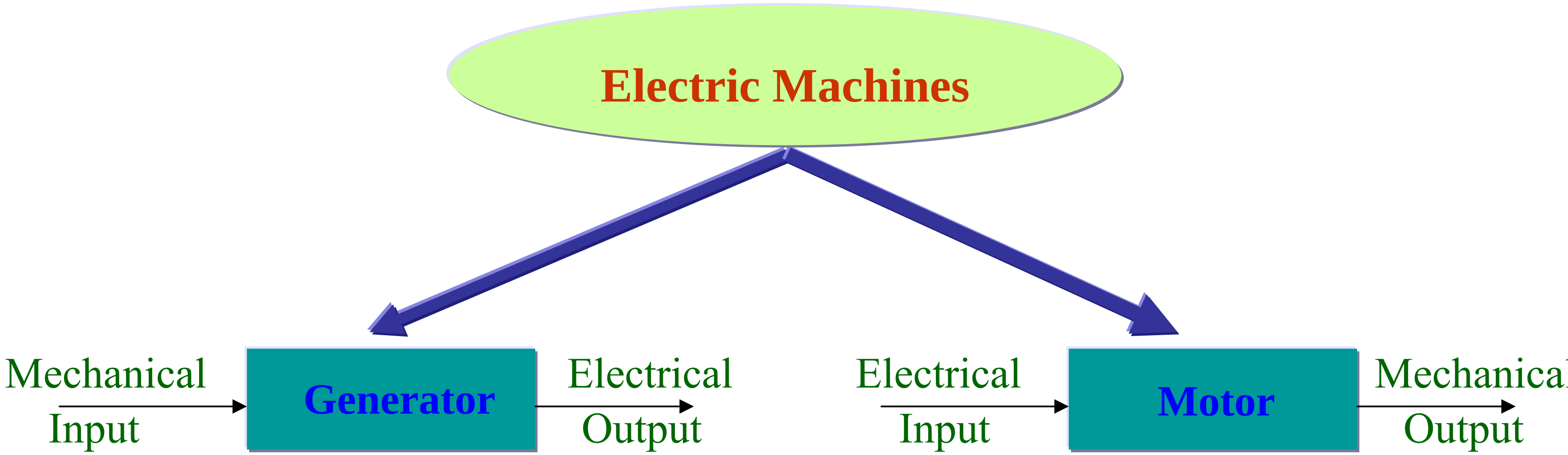
Power Flow & Efficiency

Torque-Speed Characteristics

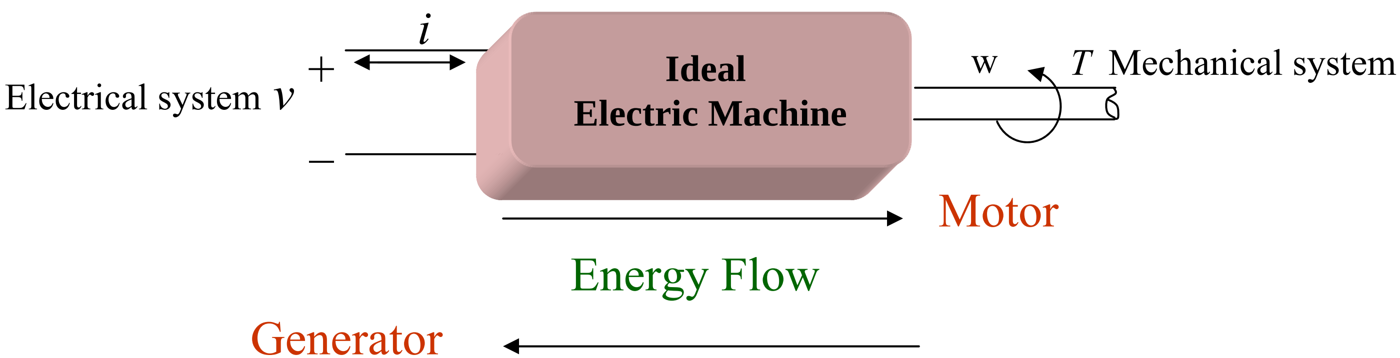
Starting of DC Machine

DC motor, field structure, and armature assembly. (Courtesy Reliance Electric Co.)

Introduction



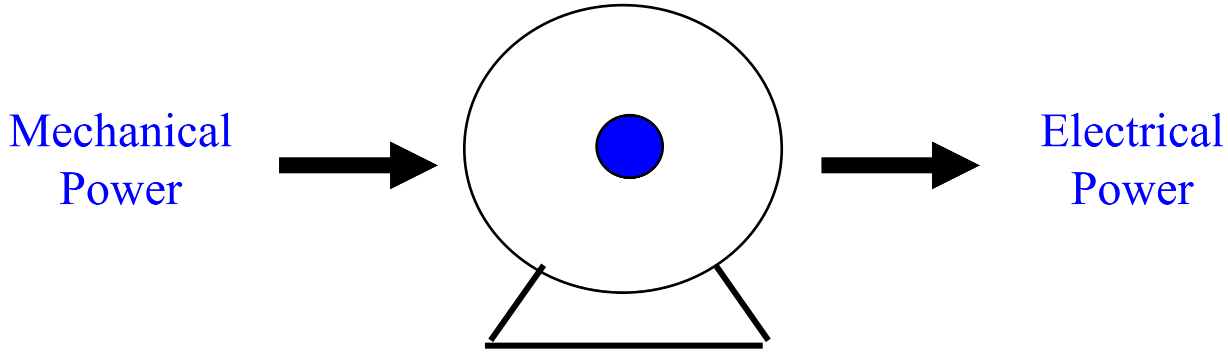
Electromechanical Energy Conversion



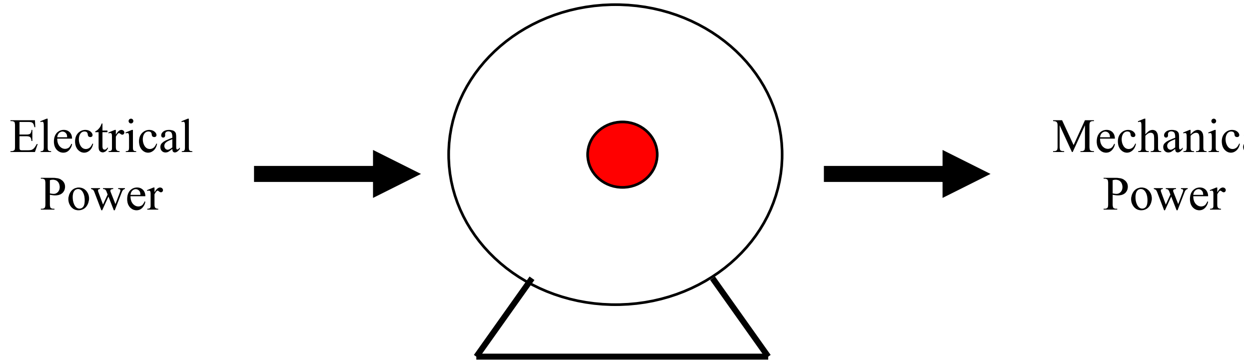
$$v\, i \approx T\, \omega$$

Introduction

- **Generator :** **The electromechanical energy conversion is from mechanical to electrical**



- **Motor:** **The electromechanical energy conversion is from electrical to mechanical**



Application of DC Machines

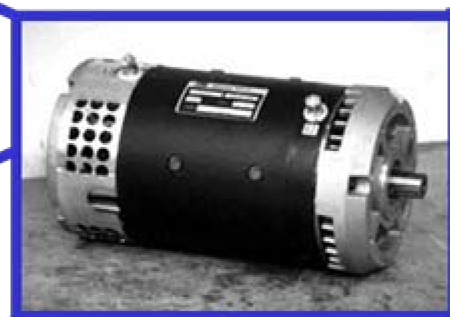
- The DC machine can operate as either a motor or a generator. At present, its use as a generator is limited because of the widespread use of ac power and power electronics converters.

- Large DC motors are used in machine tools, printing presses, fans, pumps, cranes, paper mill, traction, textile mills and so forth.

- Small DC machines (fractional horsepower rating) are used primarily as control devices such as tachogenerators for speed sensing and servomotors for position and tracking.

Application of DC Machines

DC Motor



Paper Mills

Steel Mills

Robots

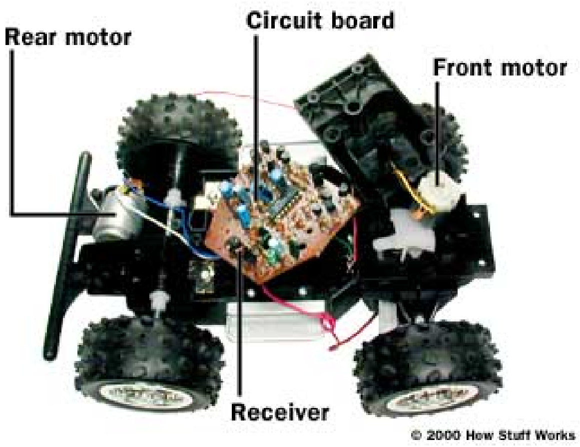
Mining

Machine Tools

Petrochemical

DC Motor Applications

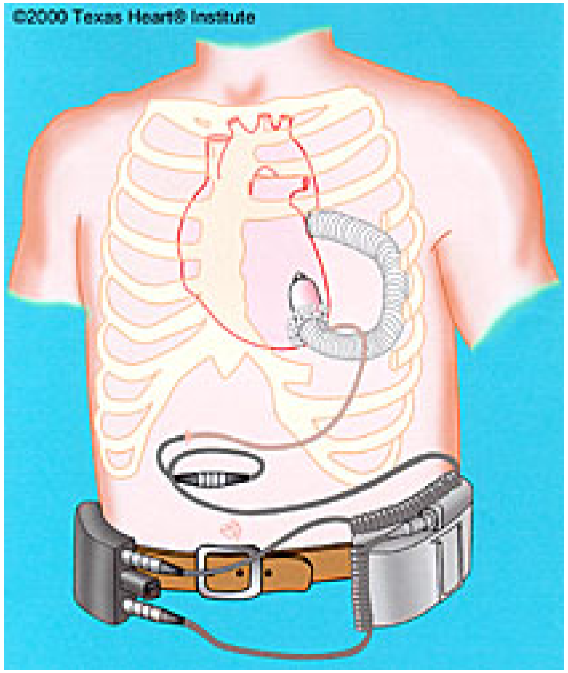
- **Automobiles**
  - Windshield Wipers
  - Door locks
  - Window lifts
  - Antenna retractor
  - Seat adjust
  - Mirror adjust
  - Anti-lock Braking System



- Cordless hand drill
- Electric lawnmower
- Fans
- Toys
- Electric toothbrush
- Servo Motor

DC Motor Applications

- **Medical:** centrifuges, orthoscopic surgical tools, respirators, dental surgical tools, and organ transport pump systems
- **Model airplanes, cars, boats, helicopters**
- **Microscopes**
- **Tape drives and winders**
- **Artificial heart**



Advantages & Disadvantages Of D.C. Motors

Advantages

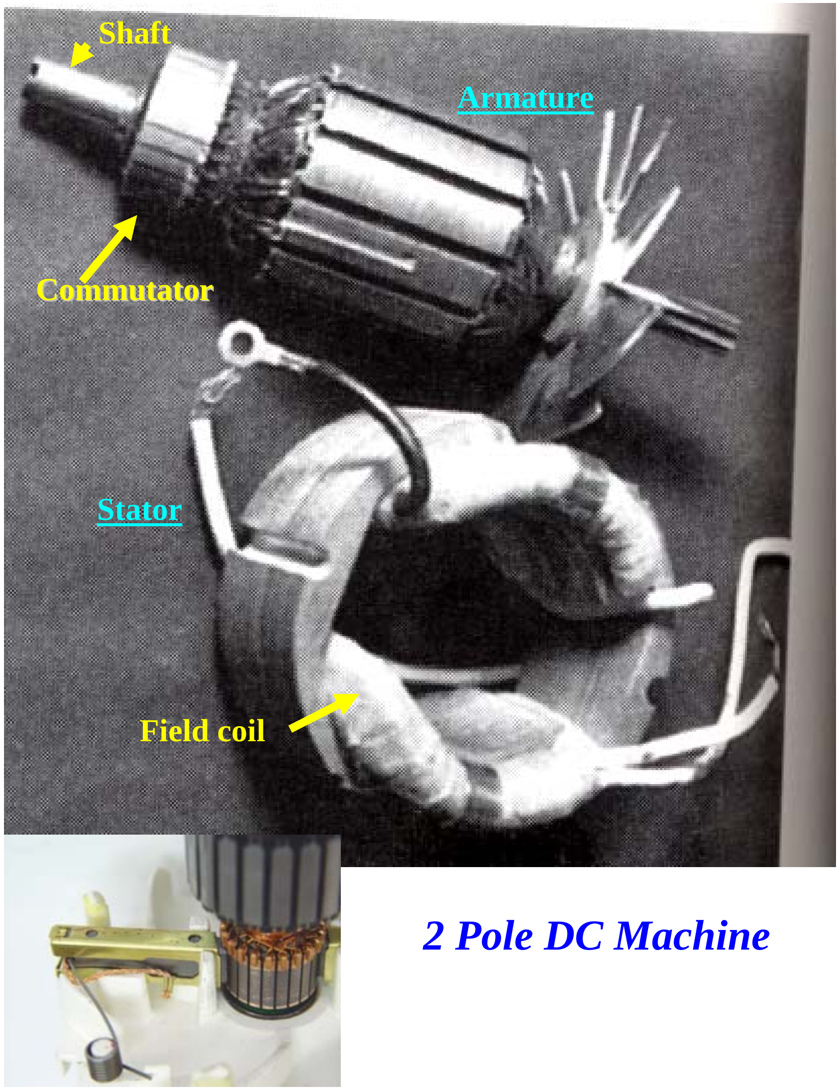
- High starting torque
- Rapid acceleration and deceleration.
- Speed can be easily controlled over wide speed range.
- Used in tough jobs (traction motors, electric trains, electric cars,….)
- Built in wide range of sizes.

Disadvantages

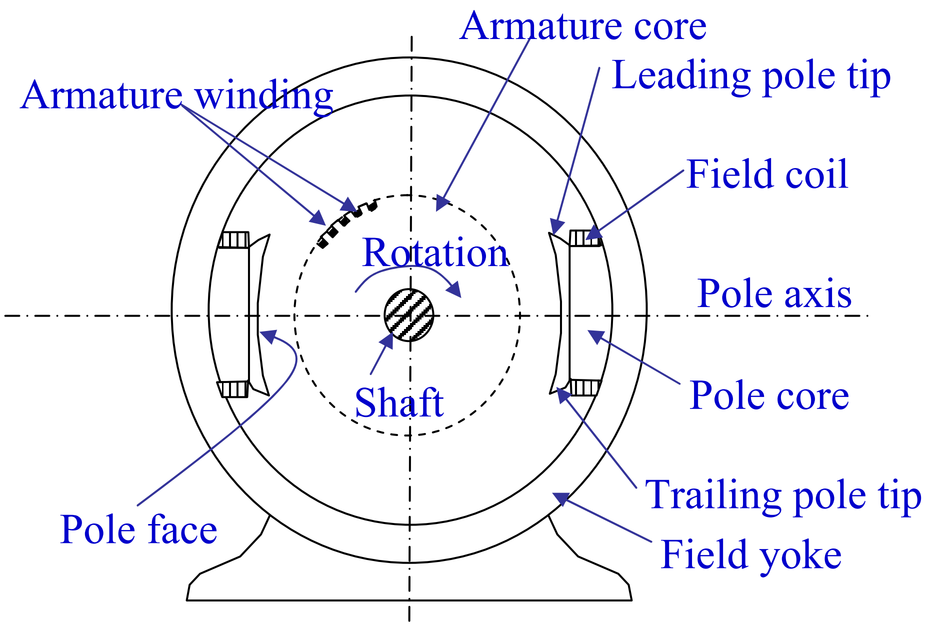
- Needs regular maintenance
- Cannot be used in explosive area
- High cost

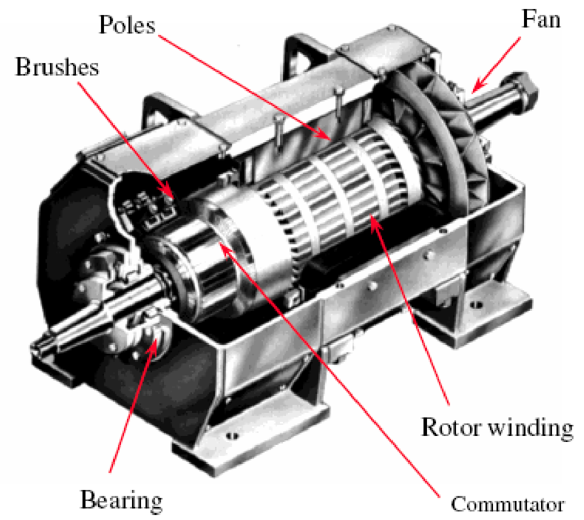
Construction of DC Machines

Parts of a DC Machine



2 Pole DC Machine

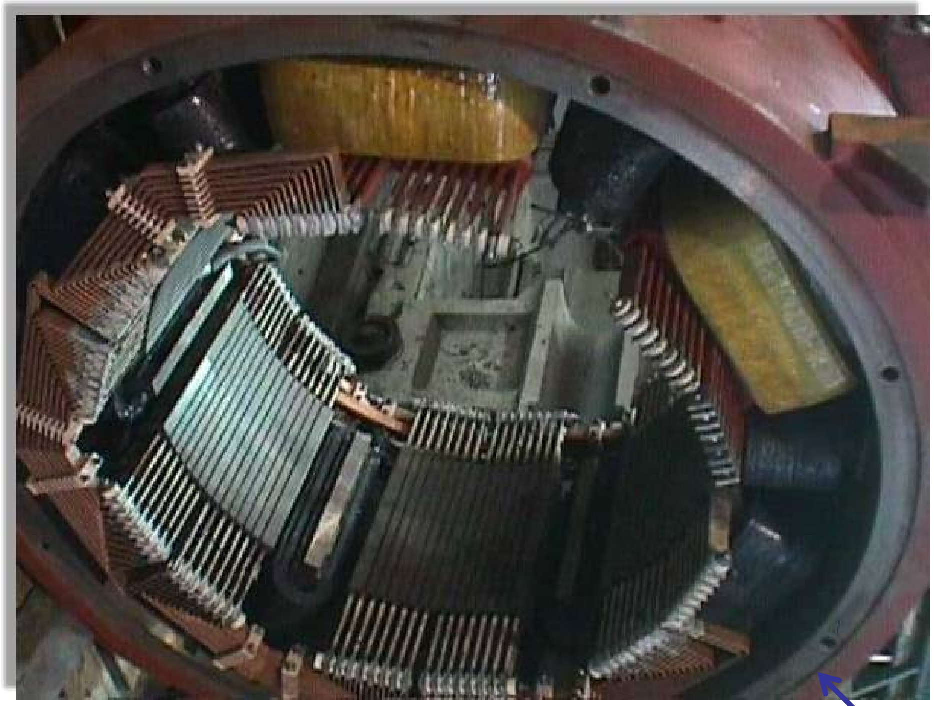




Construction of DC Machines - Stator

- **Stator** is the stationary part of the machine. The stator carries a field winding that is used to produce the required magnetic field by DC excitation. Often know as the field.

The stator consist of poles cores attached to a steel ring called yoke. The pole cores are usually made of steel plates. The pole faces are usually laminated. The winding on the poles (field windings) produce uniform magnetic field within which the armature rotates.

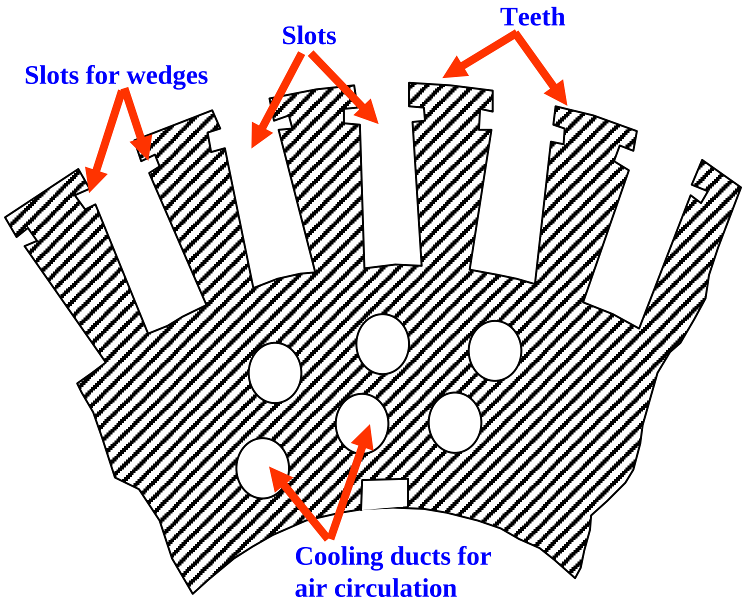


Field yoke: **Acts as a mechanical support of the machine**

Field yoke

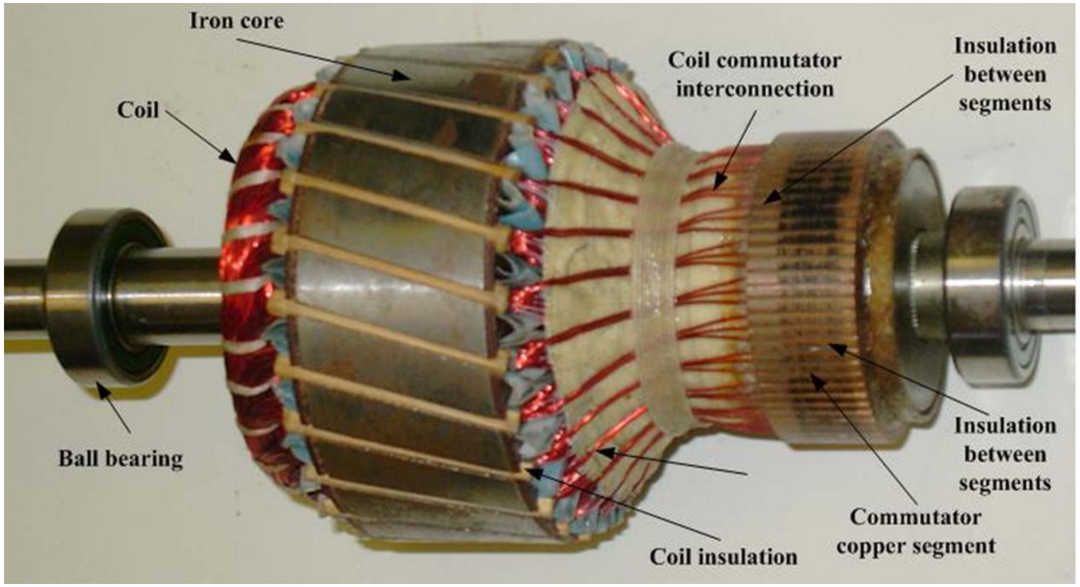
Construction of DC Machine - Armature

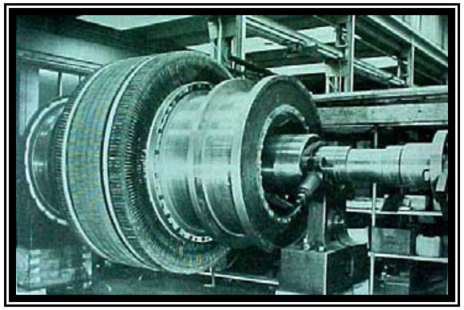
The rotor or the armature core, which carries the rotor or armature winding, is made of sheet-steel laminations. The laminations are stacked together to form a cylindrical structure



Portion of an armature lamination of a dc machine showing slots and teeth

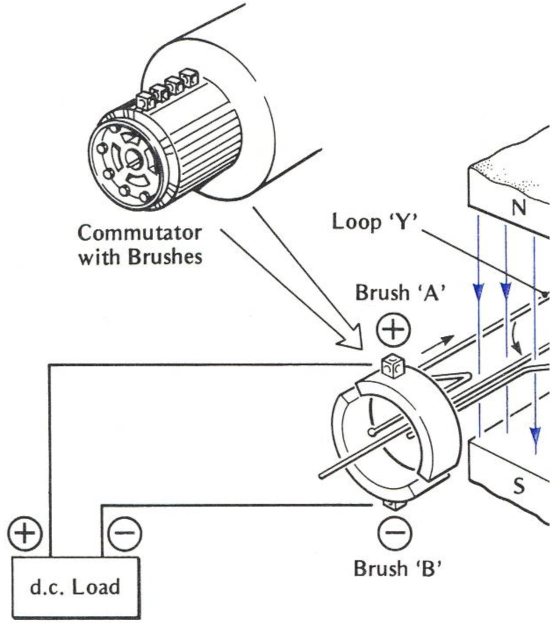
The armature coils that make the armature winding are located in the slots.

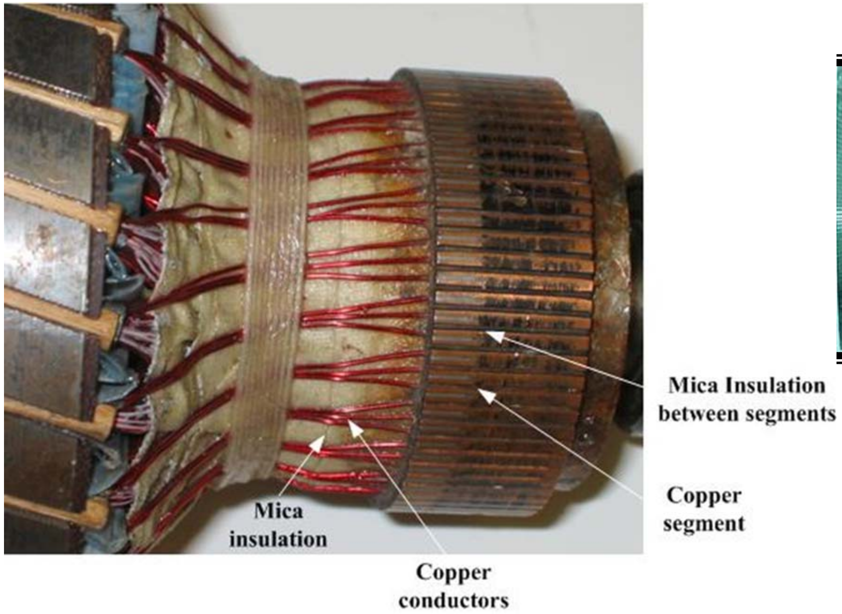


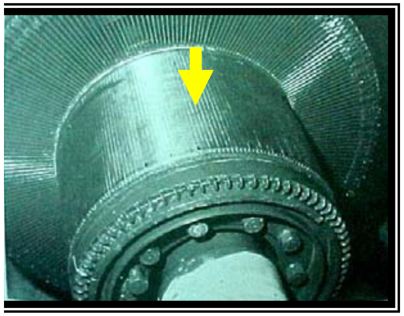


Construction of a DC Machine - Commutator

Commutator: is a mechanical rectifier, which converts the alternating voltage generated in the armature winding into direct voltage across the brush. It is made of copper segments insulated from each other by mica and mounted on the shaft of the machine. The armature windings are connected to the commutator segments.





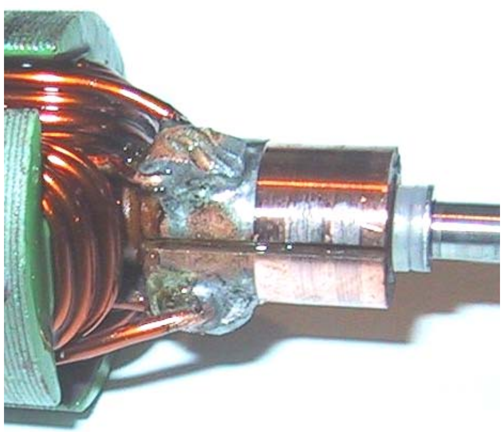


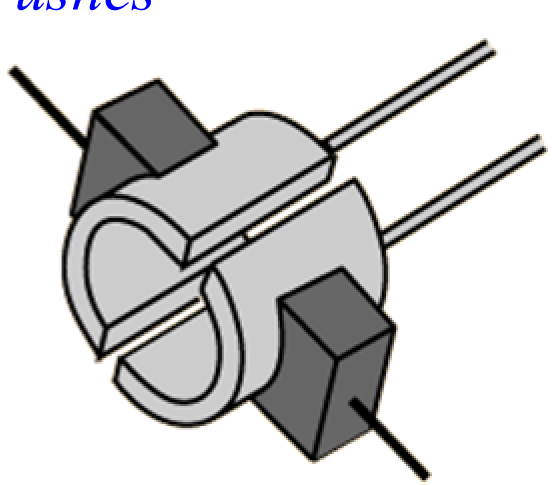
Commutator

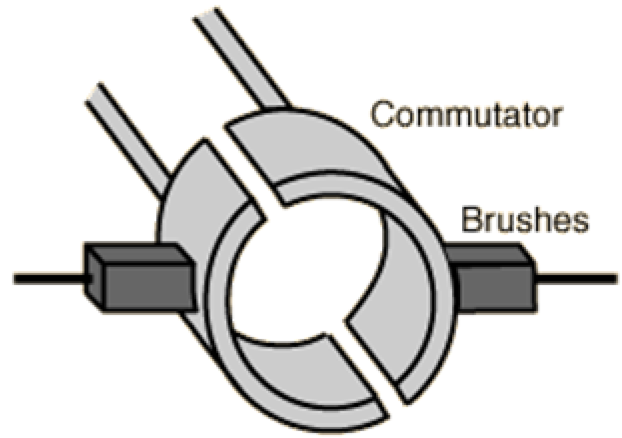
Construction of a DC Machine - Brush

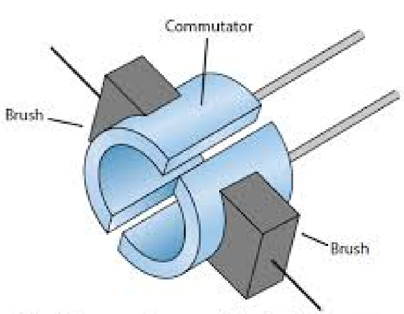
The purpose of the brush is to ensure electrical connections between the rotating commutator and stationary external load circuit. It is made of carbon and rest on the commutator.

Commutator and Brushes

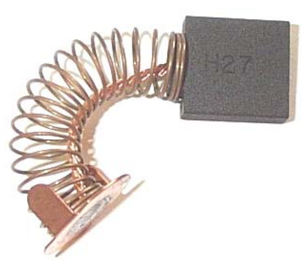




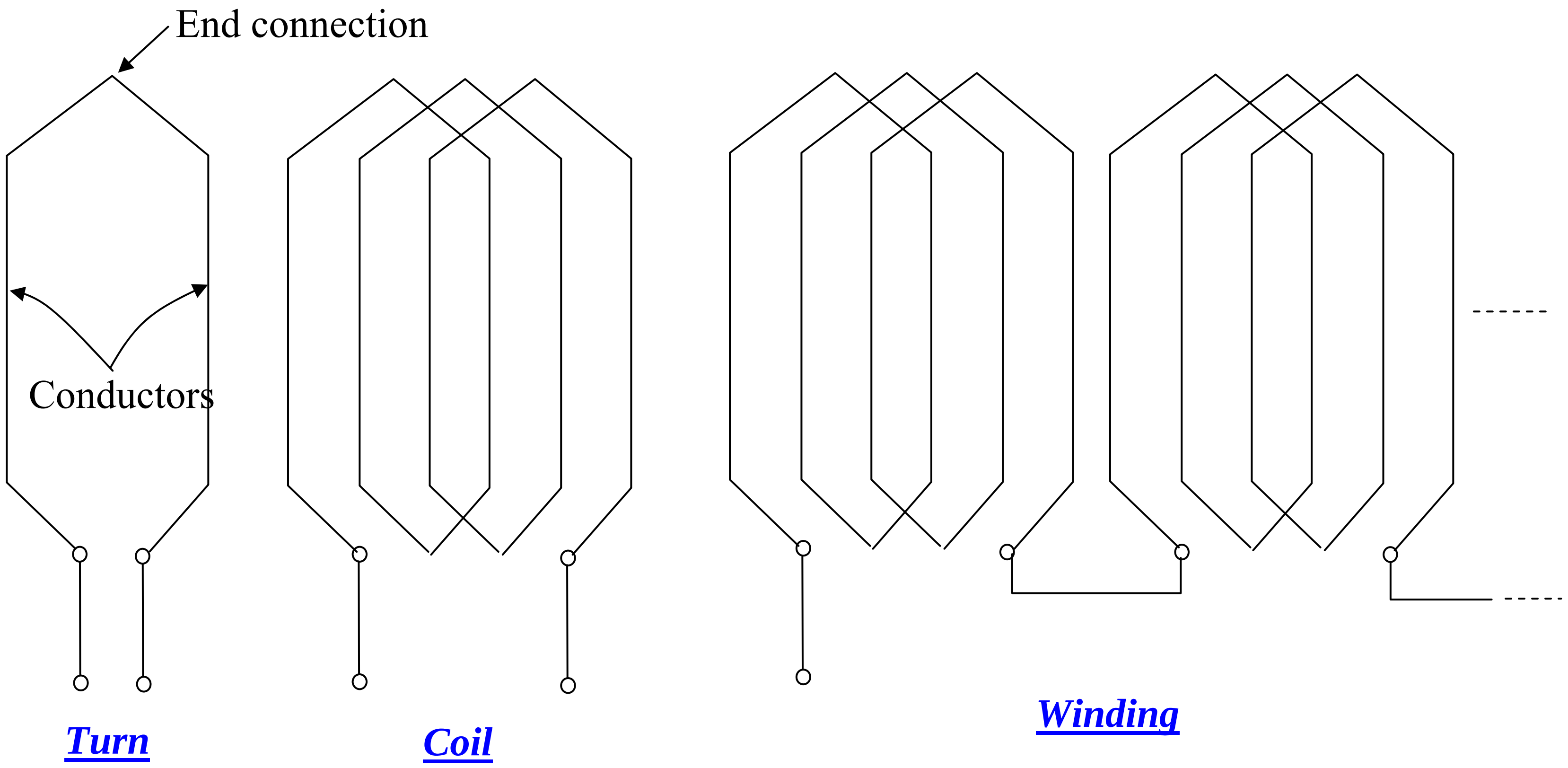




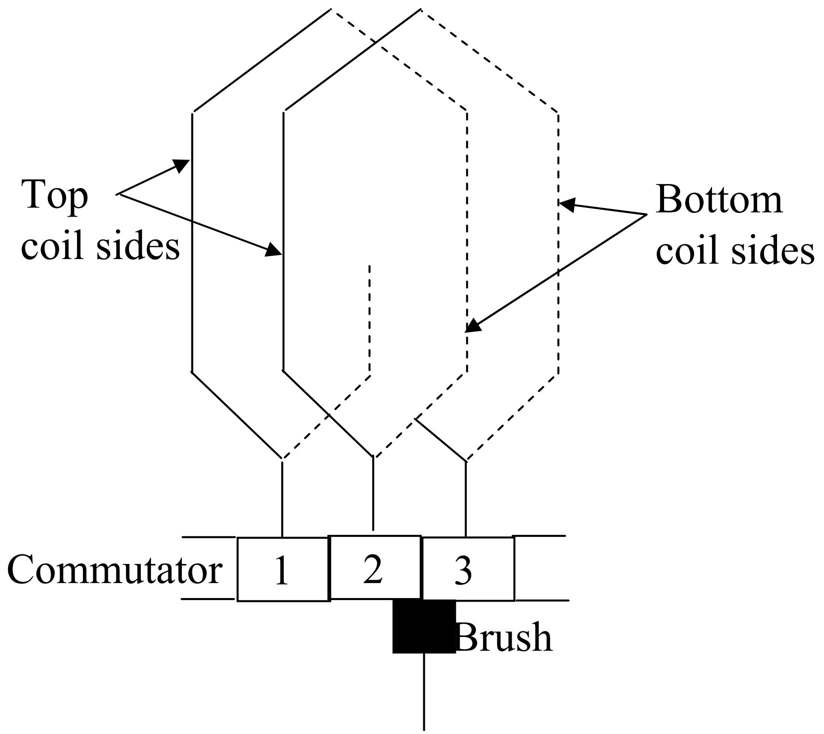
2. A split ring wrapping around the axle, the commutator makes physical contact with the brushes, which connect to opposite poles of a power source to deliver positive and negative charges to the commutator.



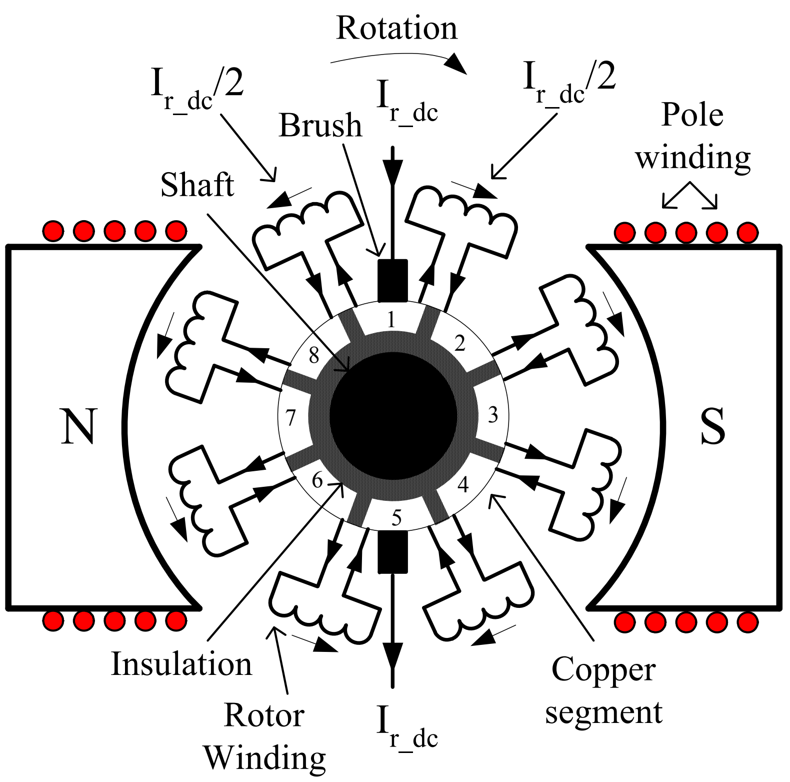
Construction of a DC Machine - Armature Winding



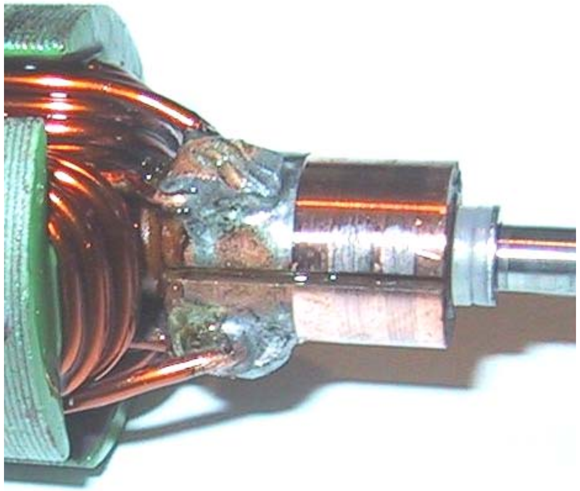
Construction of a DC Machine - Armature Winding

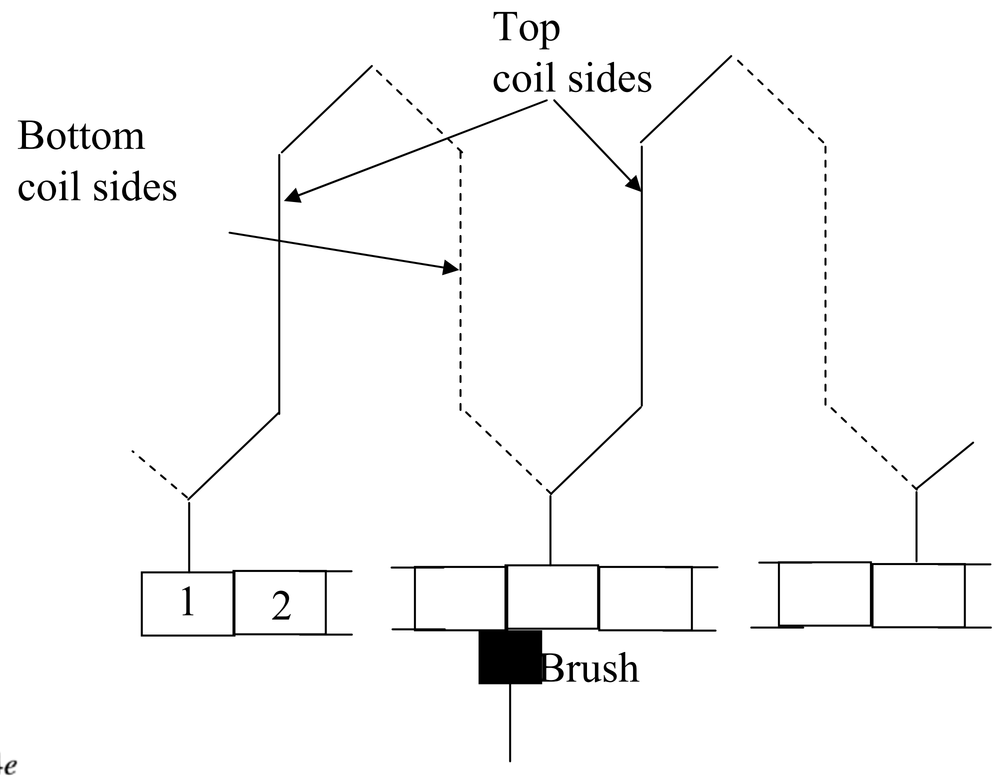


Elements of Lap Winding

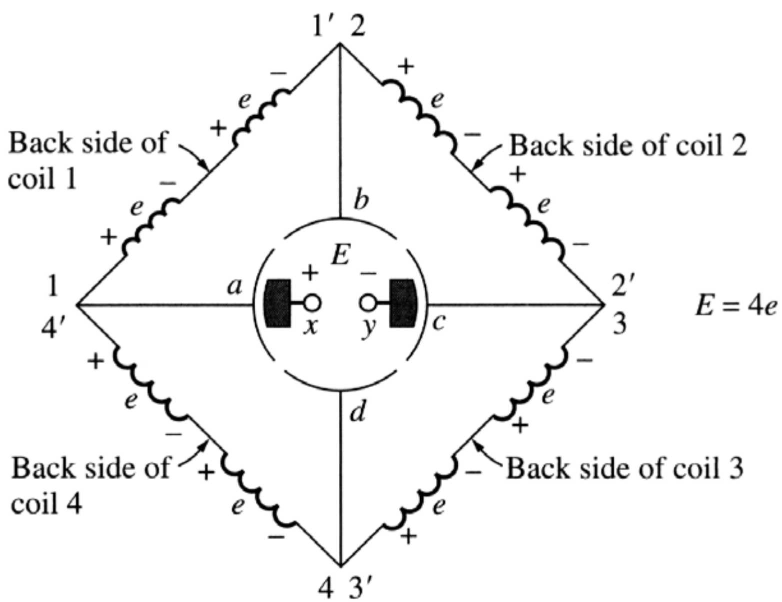


Construction of a DC Machine - Armature Winding

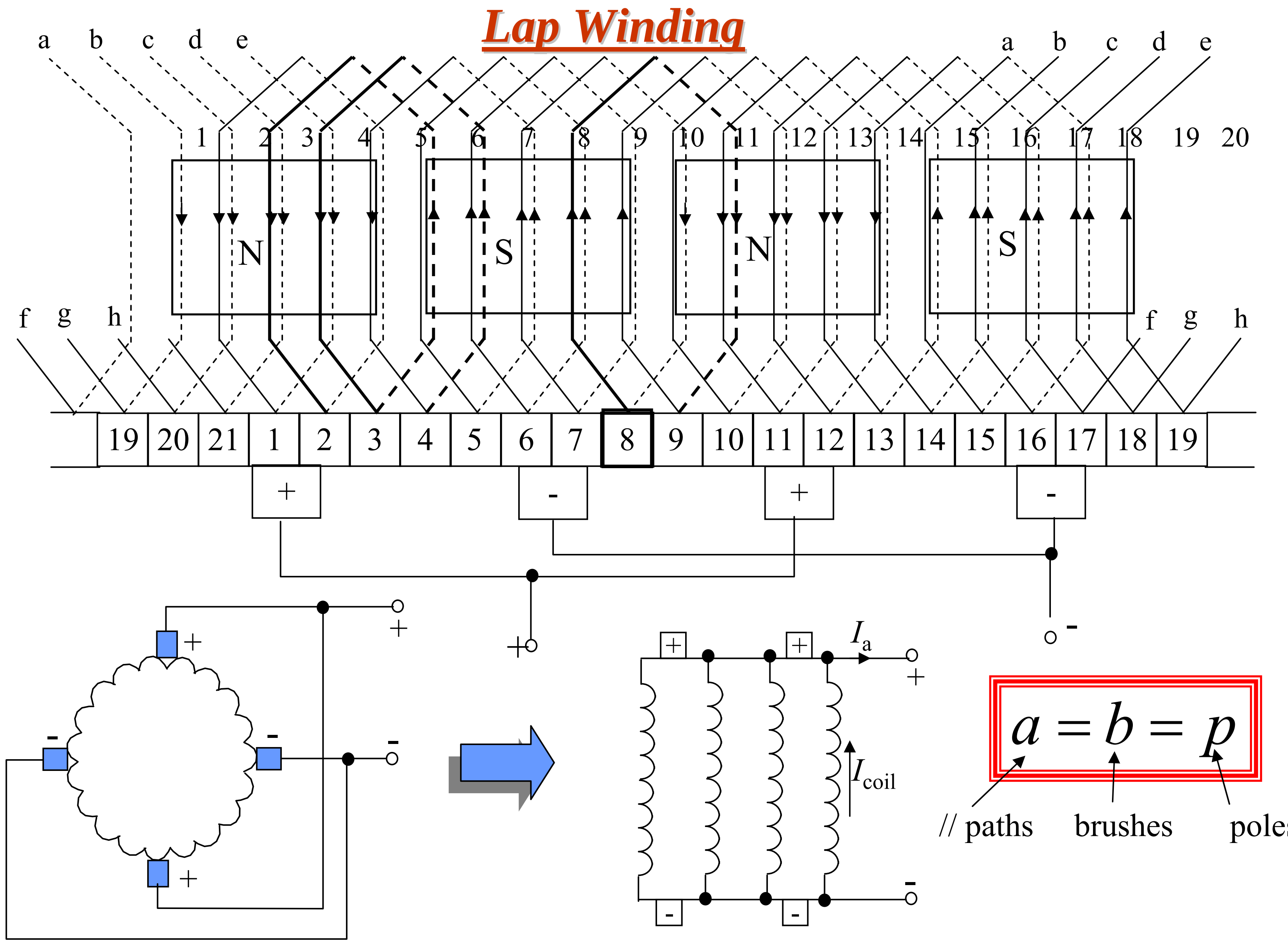




Elements of Wave Winding

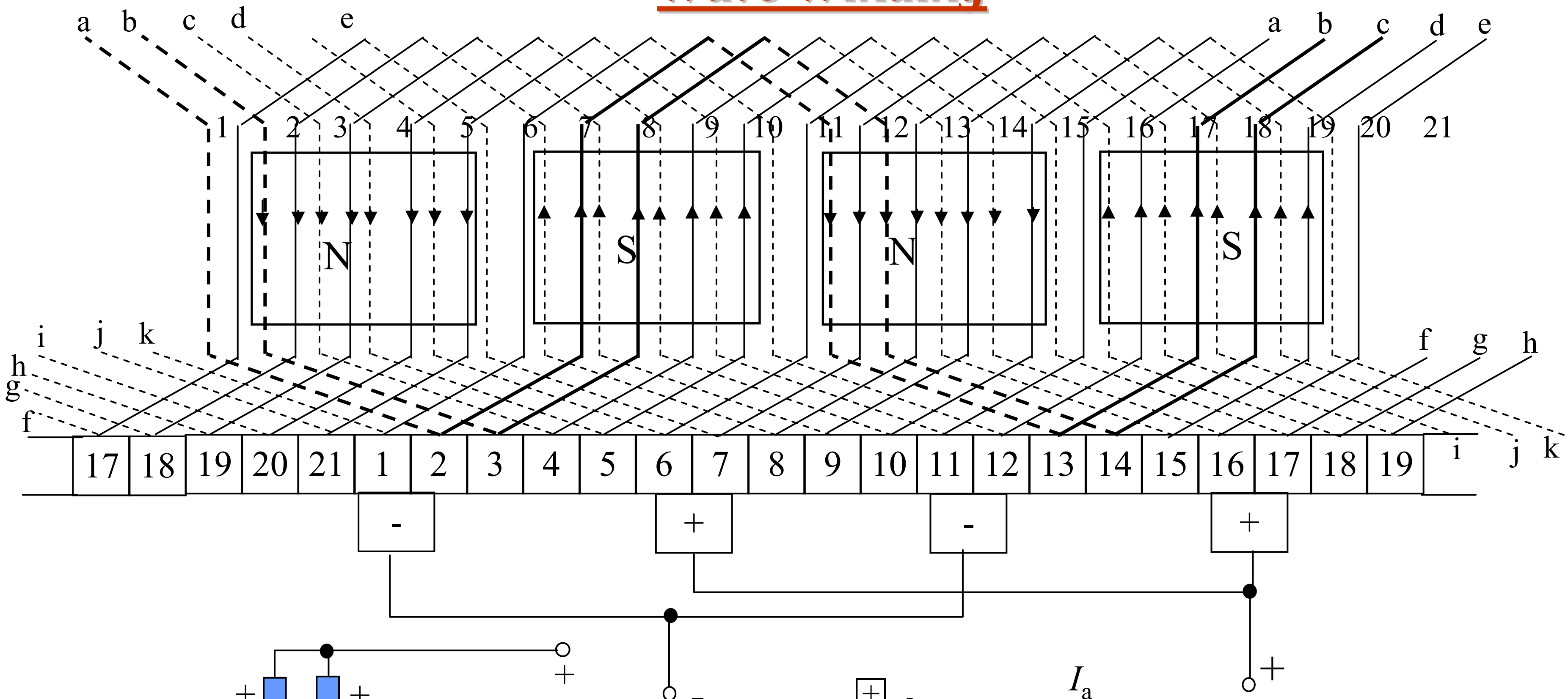


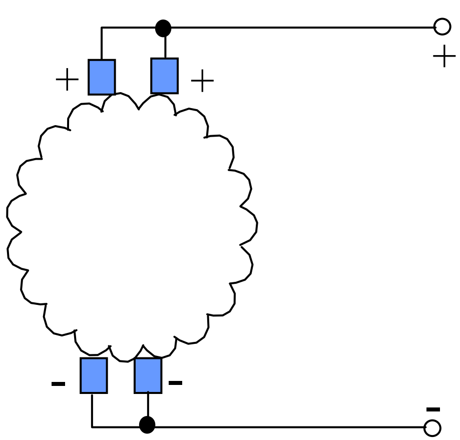
Construction of a DC Machine: Armature Winding

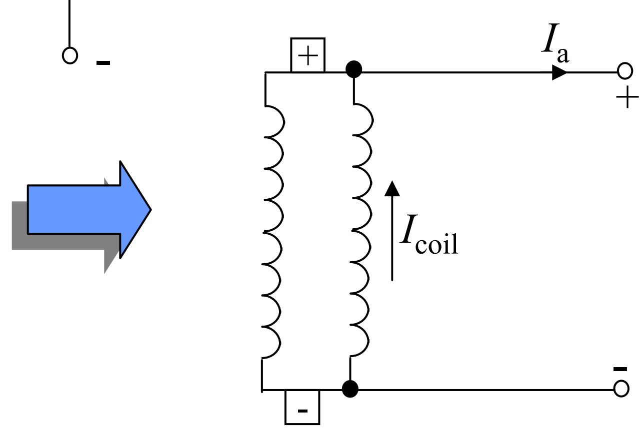


Construction of a DC Machine: Armature Winding

Wave Winding







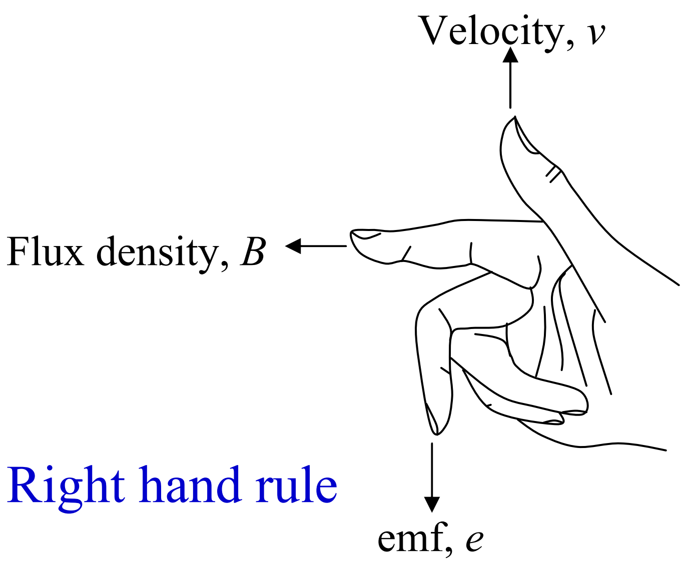
$$a = 2$$

Nb. of // paths

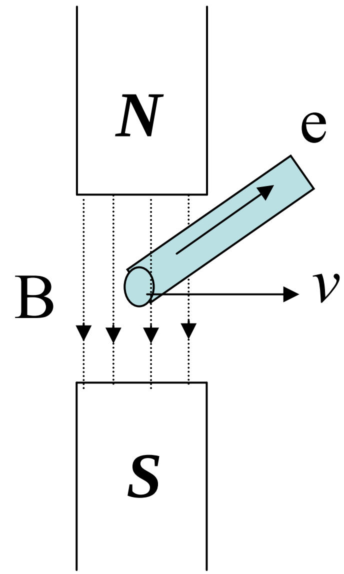
Generator Principle of Operation

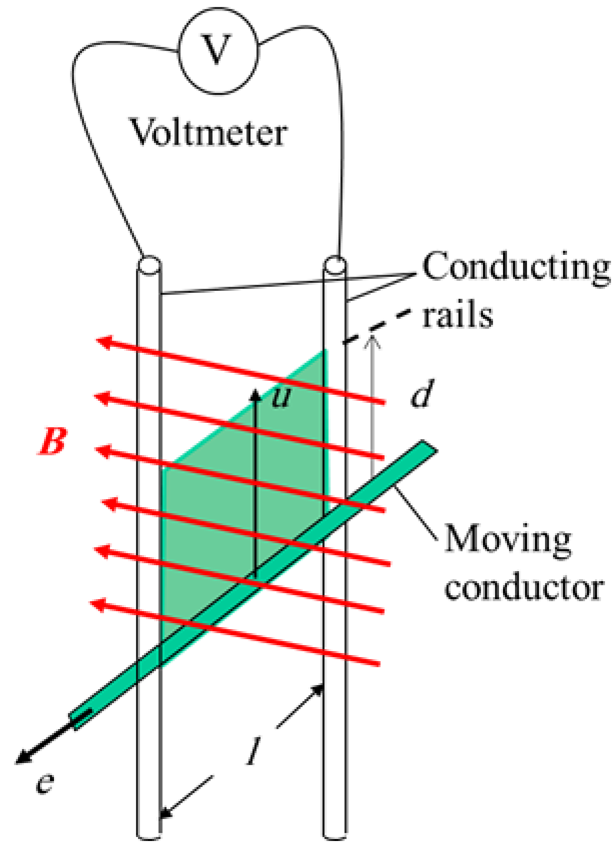
If a conductor of length *l* moves at a linear speed *v* in a magnetic field the induced voltage in the conductor is

Faraday's law or flux cutting rule ➡ $e = Blv$



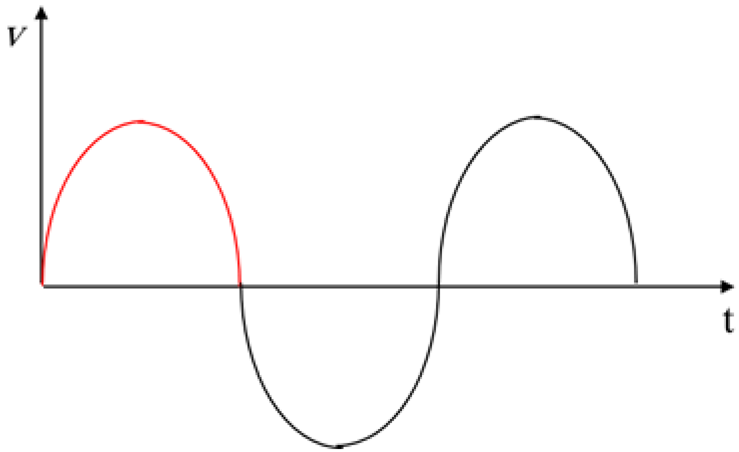
Right hand rule

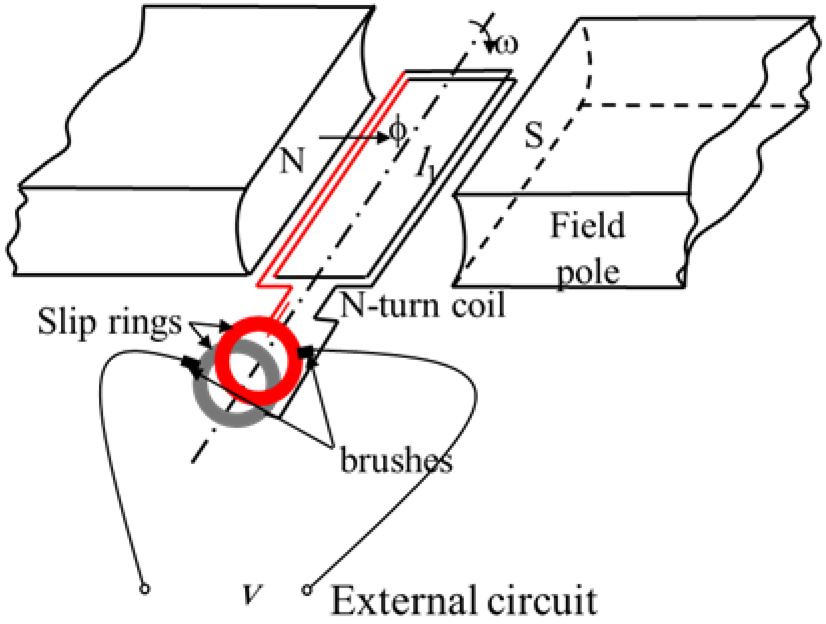


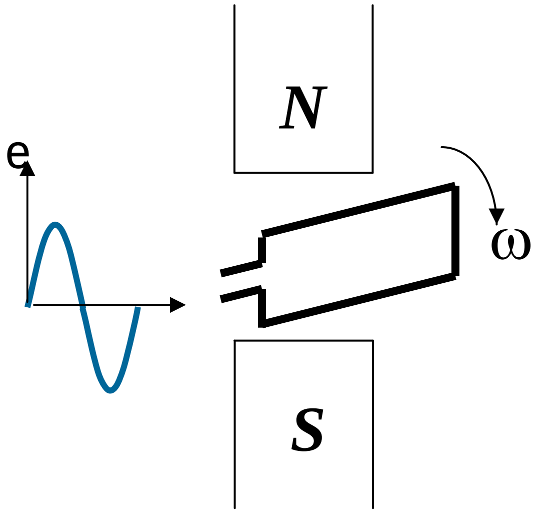


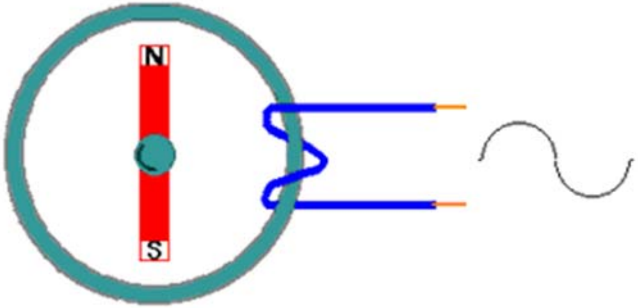
Generator Principle of Operation

Without Commutator

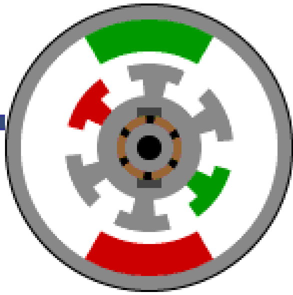




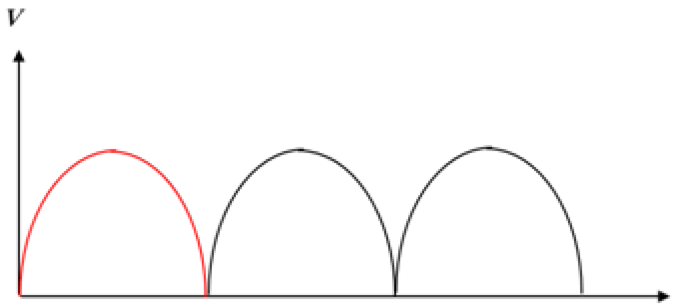


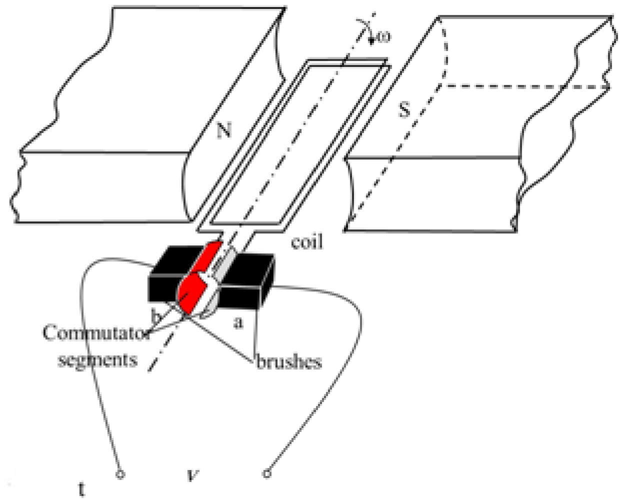


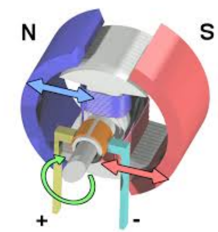
Generator Principle of Operation

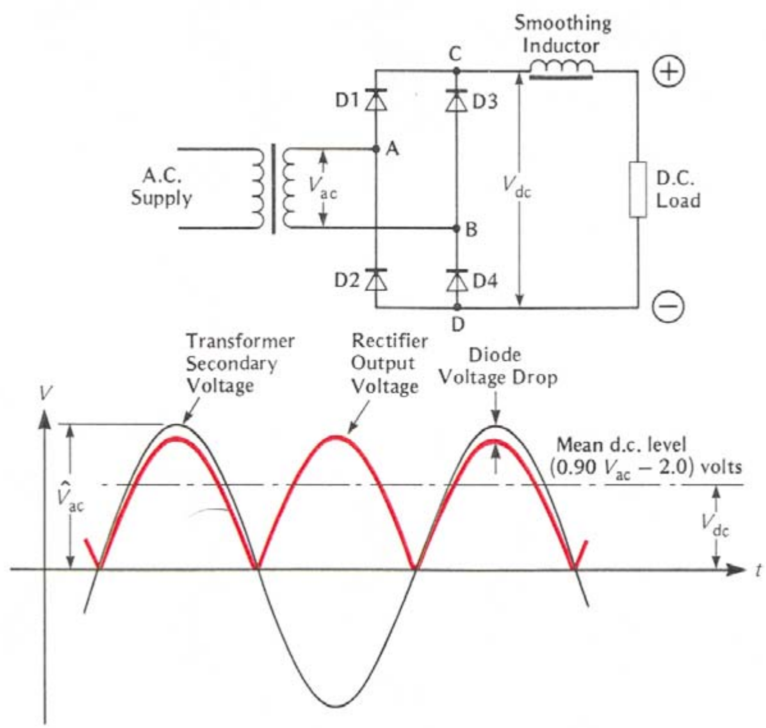


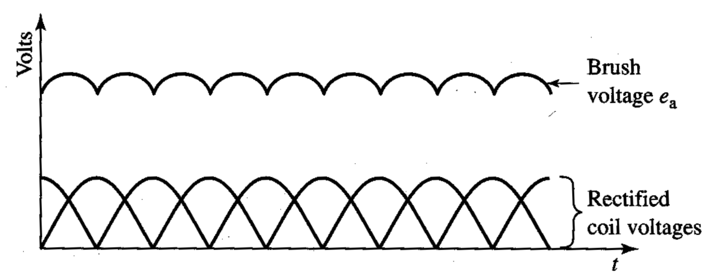
With Commutator







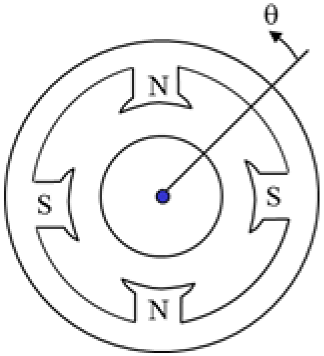




Multi-turn machine

Principle of Operation: Armature Voltage

$$E_{mf\_conductor} = \frac{Flux/\text{Rev.}}{time/\text{Rev.}} = \frac{p.\Phi}{(60/N_m)} = \frac{p.\Phi.N_m}{60}$$



$$E_{mf\_total} = E_{mf\_conductor} \times Number\ of\ conductor/path$$

$$E_{mf\_total} = \left(\frac{p.\Phi.N_m}{60}\right) \times \left(\frac{Z}{a}\right) = \frac{p.\Phi.Z.N_m}{60a}$$

where

p **= number of poles**

Z **= total number of armature conductors**

a **= number of parallel paths, *2 for wave and p for lab.***

Φ = flux per pole (Weber)

N_m **= speed of the motor in the revolutions per minute (rpm)**

time of 1 revolution = 60/Nm (sec)

Principle of Operation: Armature Voltage

But, $$\omega_m = \frac{2\pi N_m}{60} \Rightarrow N_m = \frac{\omega_m 60}{2\pi}$$

ω_m= speed of the motor in radians per second

$$E_{mf\_total} = \frac{p\Phi Z}{60a} \times \frac{60\omega_m}{2\pi} = \frac{pZ}{2\pi a} \times \Phi\omega_m$$

$$\boxed{E_{mf\_total} = K_a \Phi \omega_m}$$

K_a: armature constant $$K_a = \frac{pZ}{2\pi a}$$

Generated voltage : generator operation
Back emf : motor operation

Example 1

- **Determine the induced voltage induced in the armature of a dc machine running at 1750 rpm and having four poles. The flux per pole is 25 mWb, and the armature is lap-wound with 728 conductors.**

Solution:

$$N_m = 1750 rpm$$

$$p = 4$$

$$\phi = 25 mWb$$

$$a = p = 4$$

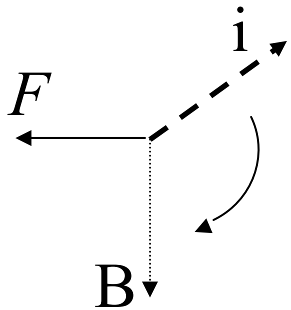
$$Z = 728$$

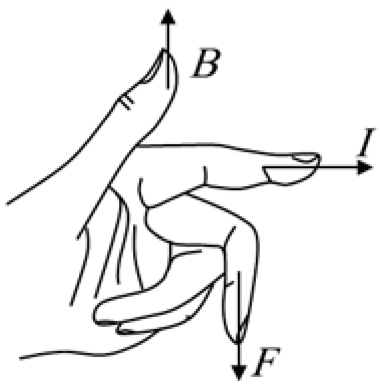
$$\begin{aligned} E_a &= K_a \phi \omega_m \\ &= \frac{pZ}{2\pi a} \phi \omega_m \\ &= \frac{pZ\phi N_m}{60a} \\ &= \frac{728 \times 25 \times 10^{-3} \times 1750}{60} \\ &= 530.83\text{V} \end{aligned}$$

Motor Principle of Operation

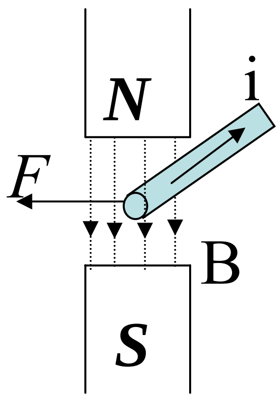
For the current carrying conductor of length *l* the force known as Lorentz force produced is

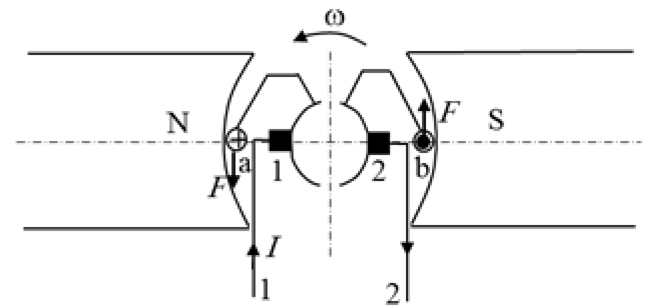
$$F = Bli$$



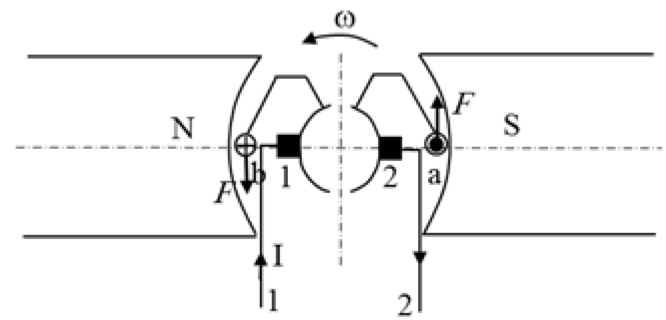


Left-hand rule





Position of conductor a under N-pole



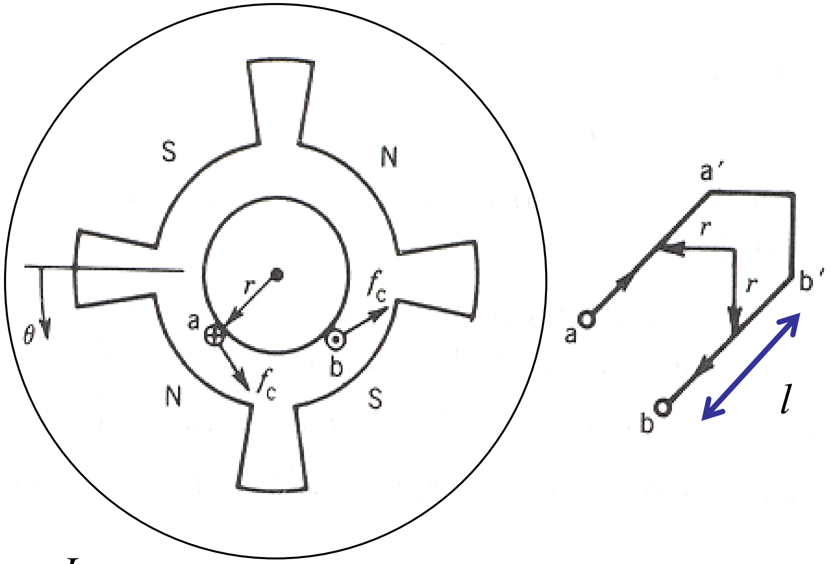
Position of conductor a under S-pole

With this configuration the torque is unidirectional and independent of conductor position

Developed (or Electromagnetic) Torque

Consider the turn shown in the following Figure.

Area per pole $A = \dfrac{2\pi r l}{p}$



Flux density $B = \dfrac{\Phi}{A} = \dfrac{p\Phi}{2\pi r l}$

Current / conductor is $I_c = \dfrac{I_a}{a}$

The force on a conductor is $f_c = BlI_c = Bl\dfrac{I_a}{a}$

The torque developed by a conductor is $T_c = f_c r = Bl\dfrac{I_a}{a} r = \dfrac{\Phi p I_a}{2\pi a}$

The total torque developed is

$$T_e = ZT_c = \frac{Zp\Phi I_a}{2\pi a} = K_a \Phi I_a = \frac{E_a I_a}{\omega_m}$$

Example 2

- **A lap-wound armature has 567 conductors and carries an armature current of 123.5A. If the flux per-pole is 20 mWb, calculate the electromagnetic torque.**

Solution:

$$a = p$$

$$\phi = 20mWb$$

$$Z = 576$$

$$I_a = 123.5A$$

$$T_e = K_a \phi I_a$$

$$= \frac{pZ}{2\pi a}\phi I_a$$

$$= \frac{576 \times 20 \times 10^{-3} \times 123.5}{2\pi}$$

$$= 226.43\text{Nm}$$

The simplest DC machine

Summary:

2 Main Equations of a DC Machine

$$E_a = K_a \Phi \omega_m$$

$$T_e = K_a \Phi I_a$$

Power flow and losses in DC machines

Unfortunately, not all electrical power is converted to mechanical power by a motor and not all mechanical power is converted to electrical power by a generator…

The efficiency of a DC machine is:

$$\eta = \frac{P_{out}}{P_{in}} \cdot 100\%$$

or

$$\eta = \frac{P_{in} - P_{loss}}{P_{in}} \cdot 100\%$$

The losses in DC machines

There are **five** categories of losses occurring in DC machines.

1. Electrical or copper losses – the resistive losses in the armature and field windings of the machine.

Armature loss: $$P_A = I_A^2 R_A$$

Field loss: $$P_F = I_F^2 R_F$$

Where I_A and I_F are armature and field currents and R_A and R_F are armature and field (winding) resistances usually measured at normal operating temperature.

The losses in DC machines

2. Core losses – hysteresis losses and eddy current losses. They vary as B^2 (square of flux density) and as $n^{1.5}$ (speed of rotation of the magnetic field).

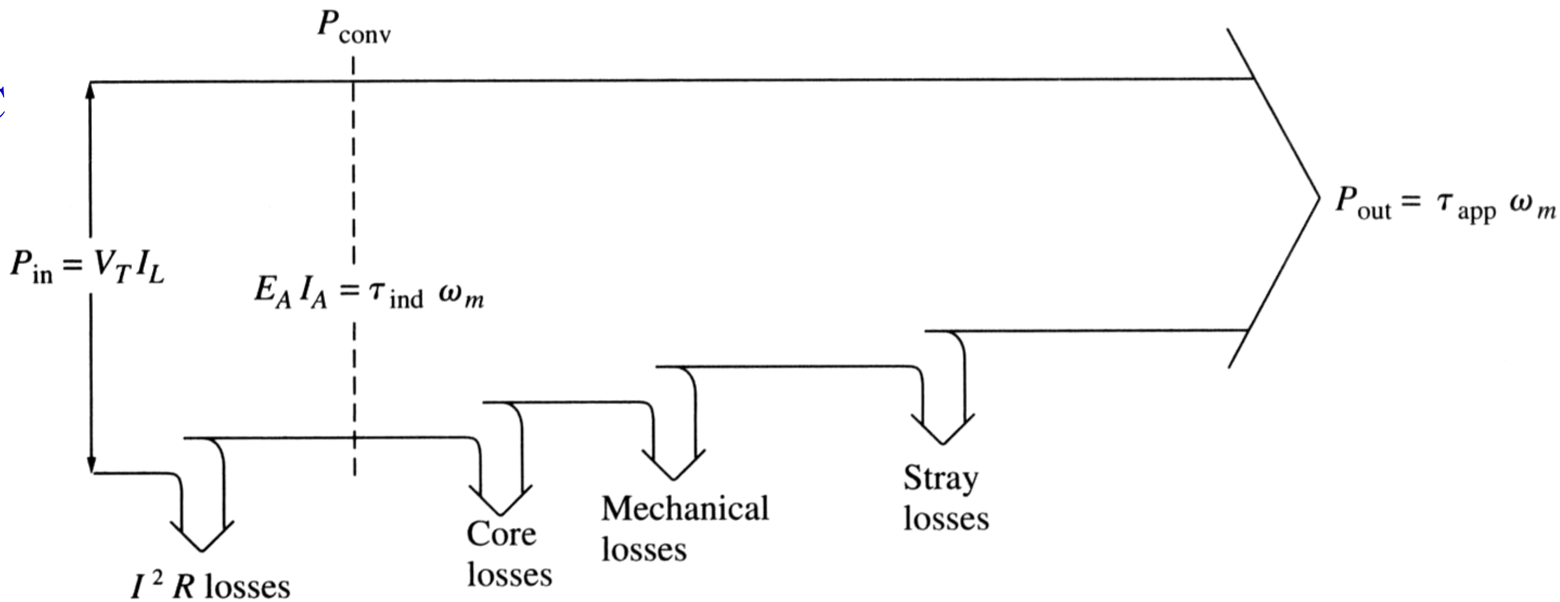
3. Mechanical losses – losses associated with mechanical effects: friction (friction of the bearings) and windage (friction between the moving parts of the machine and the air inside the casing). These losses vary as the cube of rotation speed n^3.

4. Stray (Miscellaneous) losses – losses that cannot be classified in any of the previous categories. They are usually due to inaccuracies in modeling. For many machines, stray losses are assumed as 1% of full load.

The power-flow diagram

On of the most convenient technique to account for power losses in a machine is the power-flow diagram.

For a DC motor:



Electrical power is input to the machine, and the electrical and brush losses must be subtracted. The remaining power is ideally converted from electrical to mechanical form at the point labeled as P_{conv}.

The power-flow diagram

The electrical power that is converted is

$$P_{conv} = E_A I_A$$

And the resulting mechanical power is

$$P_{conv} = \tau_{ind} \omega_m$$

After the power is converted to mechanical form, the stray losses, mechanical losses, and core losses are subtracted, and the remaining mechanical power is output to the load.

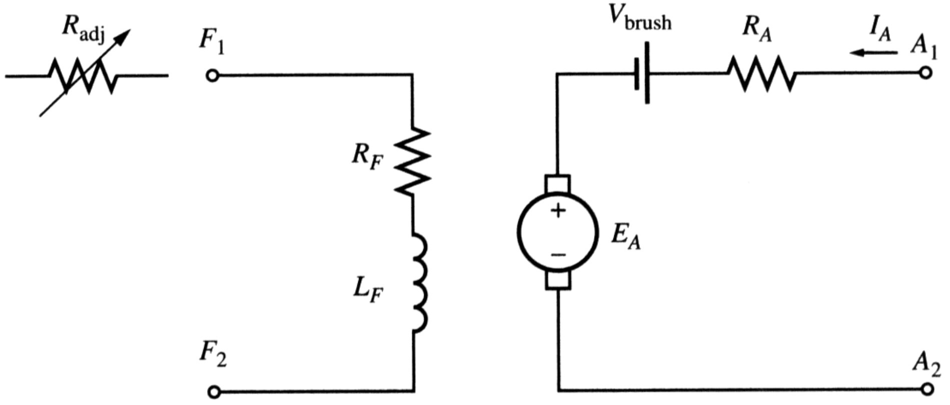
Equivalent circuit of a DC motor

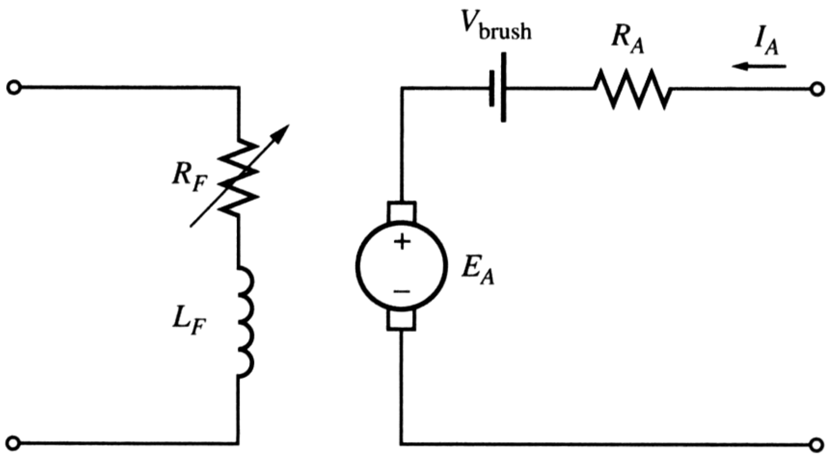
The armature circuit (the entire rotor structure) is represented by an ideal voltage source E_A and a resistor R_A.

A battery V_{brush} in the opposite to a current flow in the machine direction indicates brush voltage drop.

The field coils producing the magnetic flux are represented by inductor L_F and resistor R_F.

The resistor R_{adj} represents an external variable resistor (sometimes lumped together with the field coil resistance) used to control the amount of current in the field circuit.





Equivalent circuit of a DC motor

Sometimes, when the brush drop voltage is small, it may be left out. Also, some DC motors have more than one field coil…

The internal generated voltage in the machine is

$$E_A = K\phi\omega$$

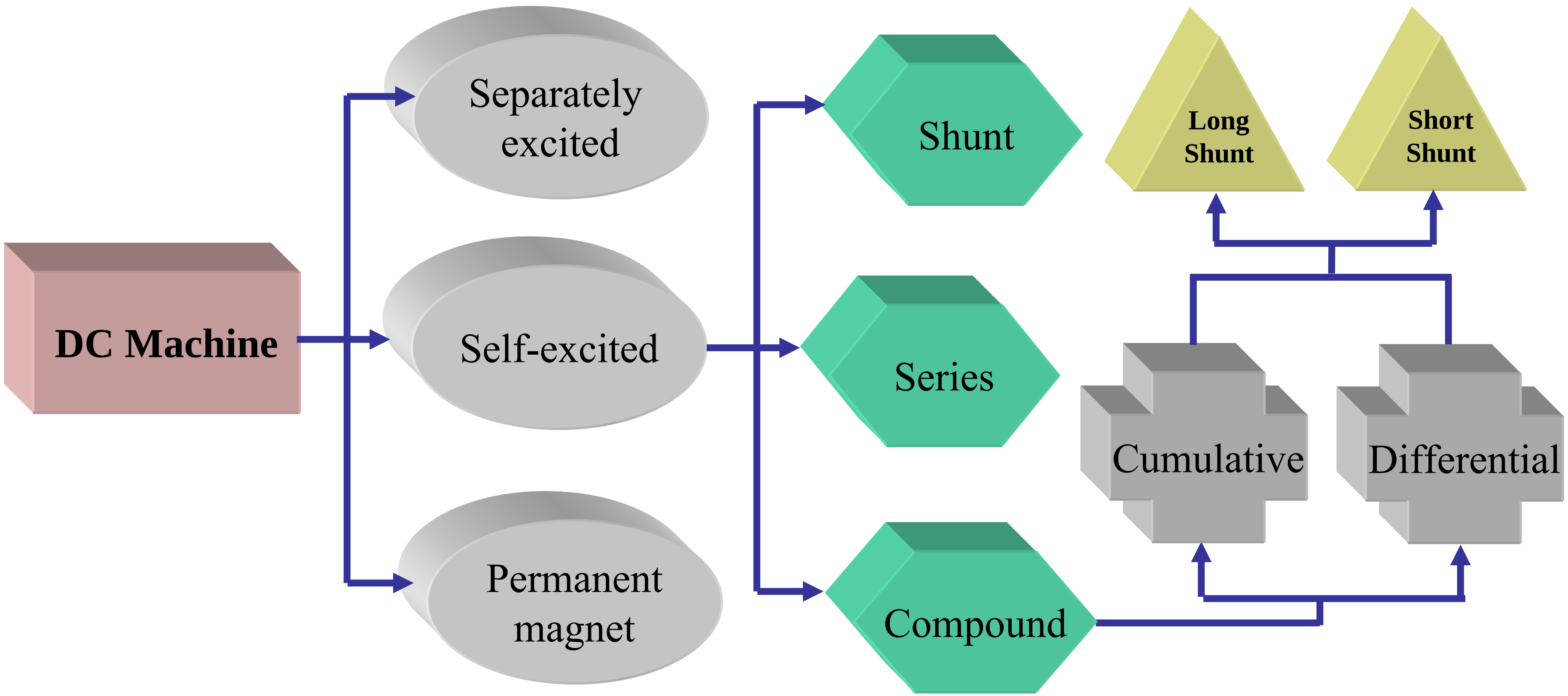
The induced torque developed by the machine is

$$\tau_{ind} = K\phi I_A$$

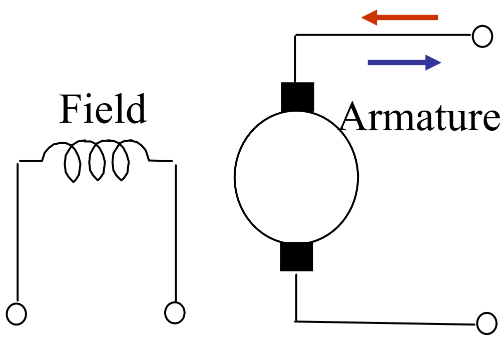
Here K is the constant depending on the design of a particular DC machine (number and commutation of rotor coils, etc.) and ϕ is the total flux inside the machine.

Note that for a single rotating loop $K = \pi/2$.

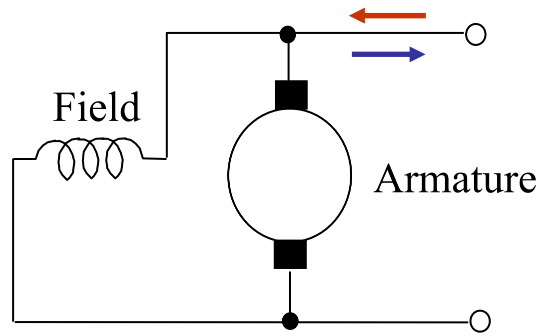
Classification of DC Machine



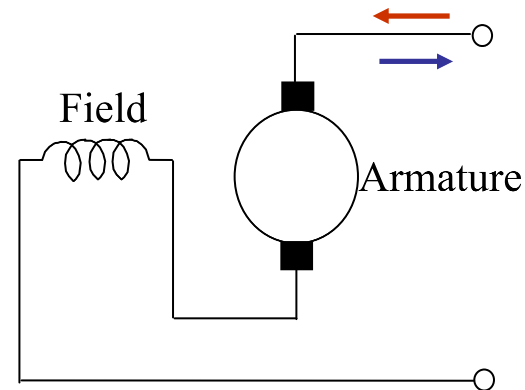
Classification of DC Machine



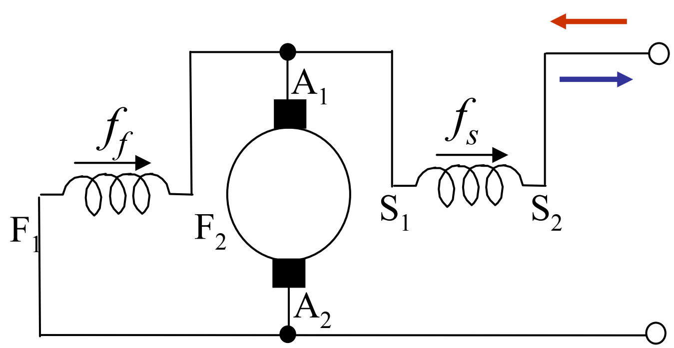
Separately excited



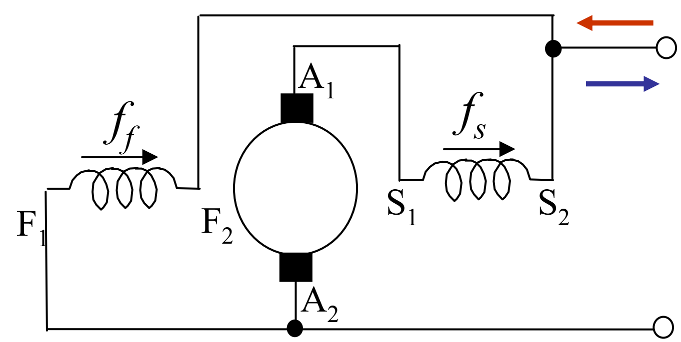
Shunt



Series



Short-shunt

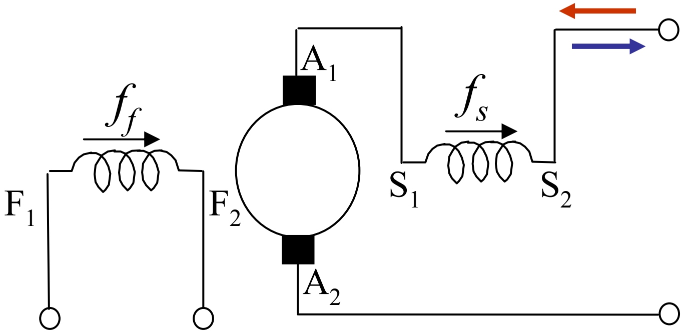


Long-shunt

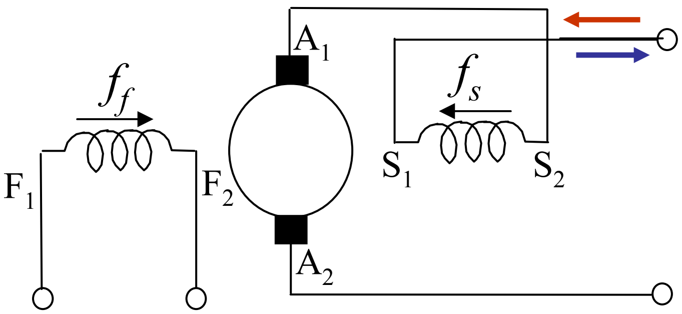
Motor operation (red arrow)

Generator operation (blue arrow)

Classification of DC Machine



Cumulative compound

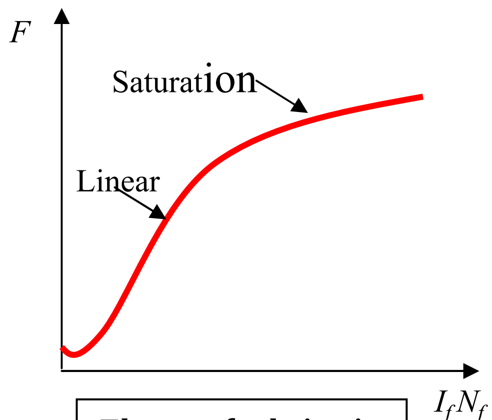


Differential compound

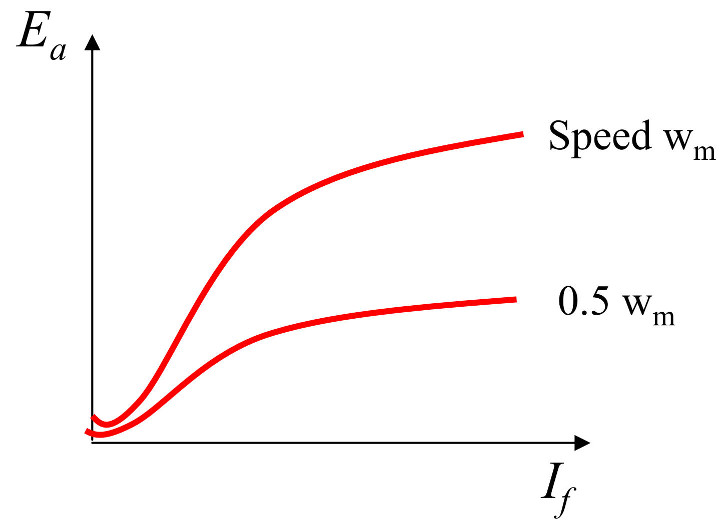
Motor operation

Generator operation

Magnetization (or Saturation) Curve of a DC Machine



Flux-mmf relation in a dc machine



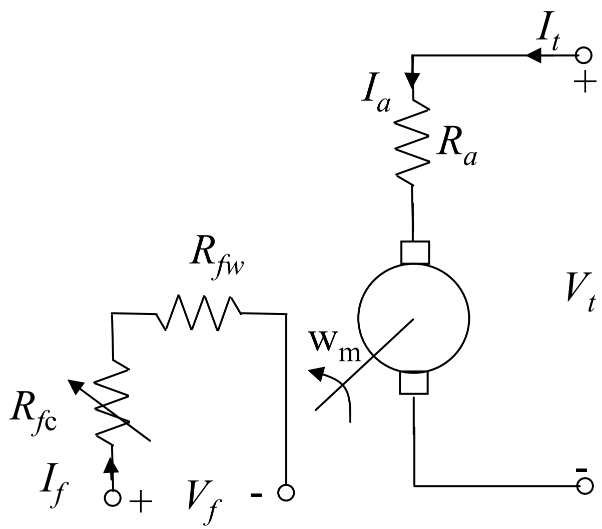
Magnetization curve

The magnetizing curve is obtained experimentally by rotating the the dc machine at a given speed and measuring the open-circuit armature terminal voltage as the current in the field winding is changed.

Magnetization Curve ⟷ **Represents the saturation level in the magnetic system of the dc machine for various values of *excitation mmf* .**

DC Motors Equations

Separately Excited DC Motor

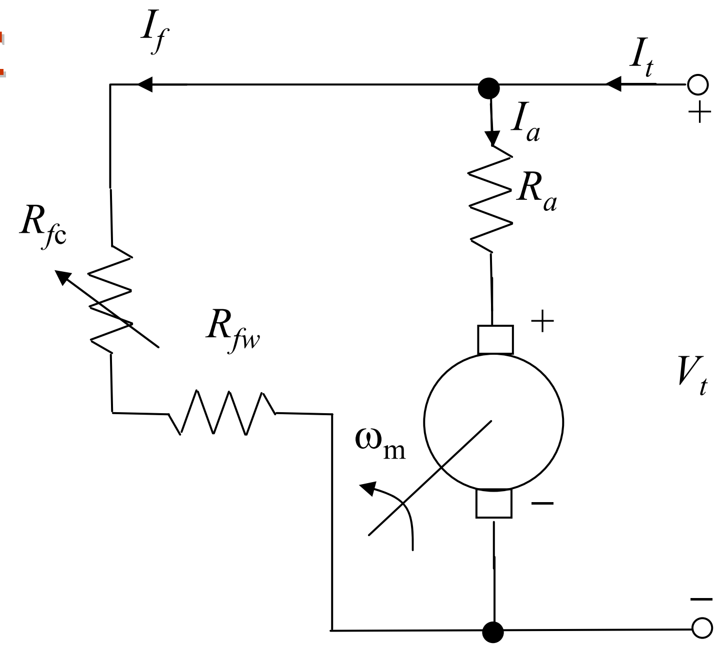


$$V_f = R_f I_f$$
$$E_a = V_t - I_a R_a$$
$$E_a = K_a \Phi \omega_m$$
$$T_e = K_a \Phi I_a$$

- R_{fw}: resistance of field winding.
- R_{fc}: resistance of control rheostat used in field circuit.
- $R_f = R_{fw} + R_{fc}$: total field resistance
- R_a: resistance of armature circuit, including the effect of brushes. Sometimes R_a is shown as the resistance of armature winding alone; the brush-contact voltage drop is considered separately and is usually assumed to be about 2V.

DC Motors Equations

Shunt or Self-Excited DC Motor



$$V_f = R_f I_f = V_t$$

$$E_a = V_t - I_a R_a$$

$$E_a = K_a \Phi \omega_m, \qquad T_e = K_a \Phi I_a$$

$$I_a = I_t - I_f$$

I_t : line current

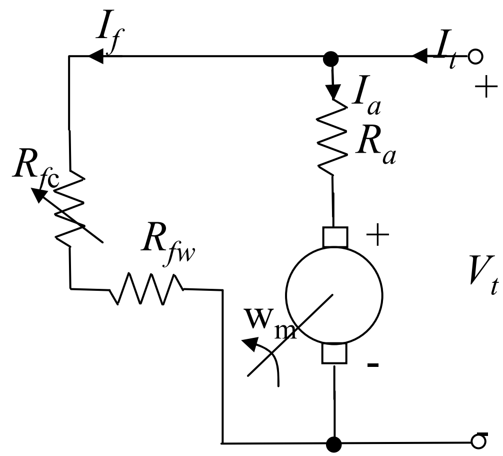
I_a : armature current

I_f : field current

Example 3

- A 250V shunt motor has an armature resistance of 0.25W and a field resistance of 125W. At no-load the motor takes a line current of 5A while running at 1200 rpm. If the line current at full-load is 52A, what is the full-load speed?

Solution



At no-load:

$$I_t = 5A$$

$$N_m = 1200rpm \Rightarrow \omega_m = \frac{1200 \times 2\pi}{60} = 125.66 rad/\sec$$

$$I_f = \frac{V_t}{R_f} = \frac{250}{125} = 2A \,,\, I_{a\_NL} = I_{t\_NL} - I_f = 5 - 2 = 3A$$

$$E_{a\_NL} = V_t - I_{a\_NL} R_a = 250 - 3 \times 0.25 = 249.25V$$

$$K_a \phi = \frac{E_{a\_NL}}{\omega_{m\_NL}} = \frac{249.25}{125.66} = 1.984 \text{ V.sec/rad}$$

At full-load:

$$I_L = 52A$$

$$E_a = V_t - I_a R_a = 250 - 50 \times 0.25 = 237.5V$$

$$E_{a\_FL} = K_a \phi \omega_{m\_FL}$$

Does not change because V_t and R_f are fixed

$$\Rightarrow \omega_{m\_FL} = \frac{E_{a\_FL}}{K_a \phi} = \frac{237.5}{1.984} = 119.71 \text{ rad/sec}$$

$$N_{m\_FL} = \frac{\omega_{m\_FL} \times 60}{2\pi} = 1142.4 \text{ rpm}$$

Example 4

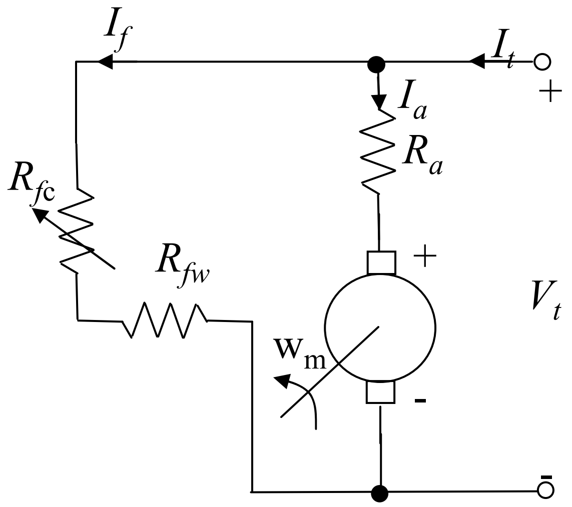
- A 230V shunt motor has an armature resistance of 0.05W and a field resistance of 75W. The motor draws 7A of line current while running at 1120 rpm. This line current at a certain load is 46A.

  A. What is the motor speed at this load?

  B. If the field-circuit resistance is increased to 100W, what is the new speed of the motor? Assume the line current to remain unchanged.

Solution

A: at light load:



$$I_{t1} = 7A \qquad N_{m1} = 1120rpm$$

$$I_f = \frac{V_t}{R_{f1}} = \frac{230}{75} = 3.07A$$

$$I_{a1} = I_{t1} - I_f = 7 - 3.07 = 3.93A$$

$$E_{a1} = V_t - I_{a1}R_a = 230 - 3.93 \times 0.05 = 229.8V$$

At an other load:

$$I_{t2} = 46A \qquad I_{a2} = I_{t2} - I_f = 46 - 3.07 = 42.93A$$

$$E_{a2} = V_t - I_{a2}R_a = 230 - 42.93 \times 0.05 = 227.85V$$

$$\frac{E_{a1}}{E_{a2}} = \frac{N_{m1}}{N_{m2}}$$

$$\Downarrow$$

$$N_{m2} = \frac{N_{m1}E_{a2}}{E_{a1}} = \frac{1120 \times 227.85}{229.8} = 1110.5 \text{ rpm}$$

$$\omega_{m2} = \frac{2\pi N_{m2}}{60} = \frac{1110.5 \times 2\pi}{60} = 116.3 \text{ rad/sec}$$

B: $I_{f3} = \dfrac{230}{100} = 2.3\ \text{A} \qquad I_{a3} = I_{t2} - I_{f3} = 46 - 2.3 = 43.7\ \text{A}$

$$E_{a3} = V_t - I_{a3}R_a = 230 - 43.7 \times 0.05 = 227.815\ \text{V}$$

$$\omega_{m3} = \frac{E_{a3}}{K_a\phi_3} \qquad \phi \propto I_f \Rightarrow K_a\phi_3 = K_f I_{f3}$$

Assuming linear part of magnetization curve

$$\omega_{m3} = \frac{E_{a3}}{K_f I_{f3}}$$

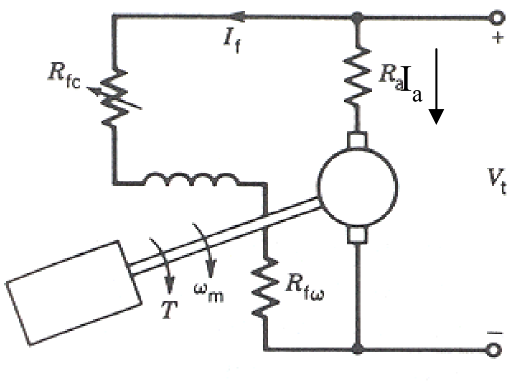
From A) $K_f = \dfrac{E_{a2}}{\omega_{m2} I_f} = \dfrac{227.85}{116.3 \times 3.07} = 0.638$

$$\Rightarrow \omega_{m3} = \frac{227.815}{0.638 \times 2.3} = 155.3\text{rad/sec} \qquad N_{m3} = 1483\ \text{rpm}$$

Torque-Speed Characteristics

Separately excited & Shunt motors

(Φ is independent of the load torque)

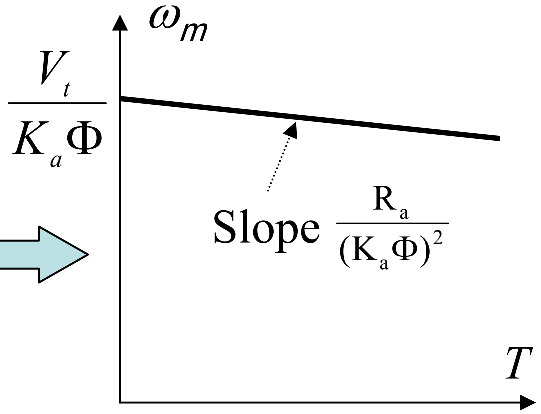


(a)

$$\left.\begin{aligned} V_t &= E_a + I_a R_a \\ E_a &= K_a \Phi\ \omega_m \end{aligned}\right\} \quad \omega_m = \frac{V_t - I_a R_a}{K_a \Phi}$$

$$T = K_a \Phi\ I_a$$

Therefore , $$\omega_m = \frac{V_t}{K_a \Phi} - \frac{R_a}{(K_a \Phi)^2} T$$



Shunt motor: Speed Control

There are two methods to control the speed of a shunt DC motor:

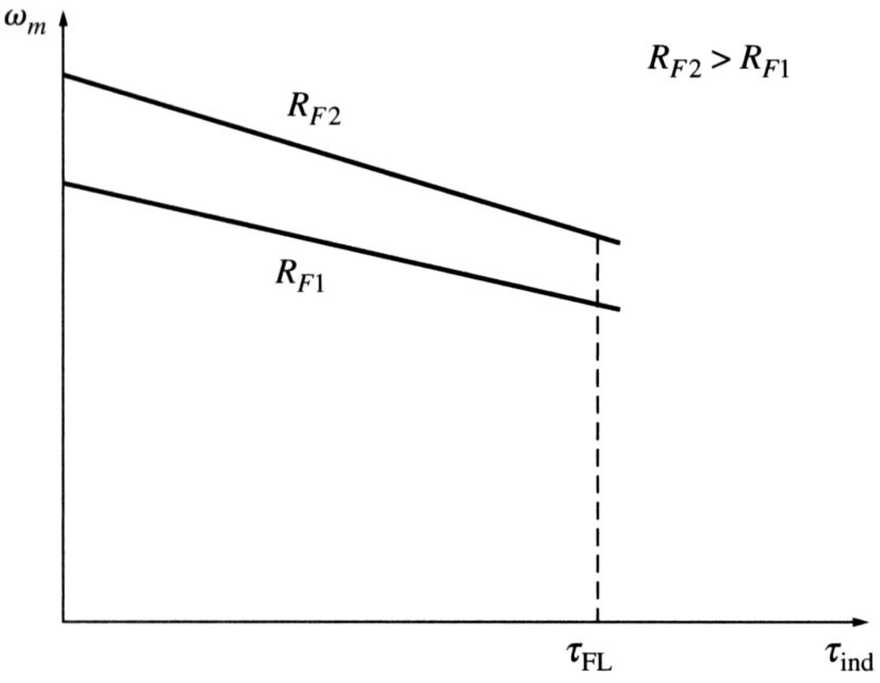
1. Adjusting the field resistance R_F (and thus the field flux)
2. Adjusting the terminal voltage applied to the armature

1. Adjusting the field resistance

1) Increasing field resistance R_F decreases the field current ($I_F = V_T/R_F$);
2) Decreasing field current I_F decreases the flux ϕ;
3) Decreasing flux decreases the internal generated voltage ($E_A = K\phi\omega$);
4) Decreasing E_A increases the armature current ($I_A = (V_T - E_A)/R_A$);
5) Changes in armature current dominate over changes in flux; therefore, increasing I_A increases the induced torque ($\tau_{ind} = K\phi I_A$);
6) Increased induced torque is now larger than the load torque τ_{load} and, therefore, the speed ω increases;
7) Increasing speed increases the internal generated voltage E_A;
8) Increasing E_A decreases the armature current I_A…
9) Decreasing I_A decreases the induced torque until $\tau_{ind} = \tau_{load}$ at a higher speed ω.

Shunt motor: Speed Control

The effect of increasing the field resistance within a normal load range: from no load to full load.



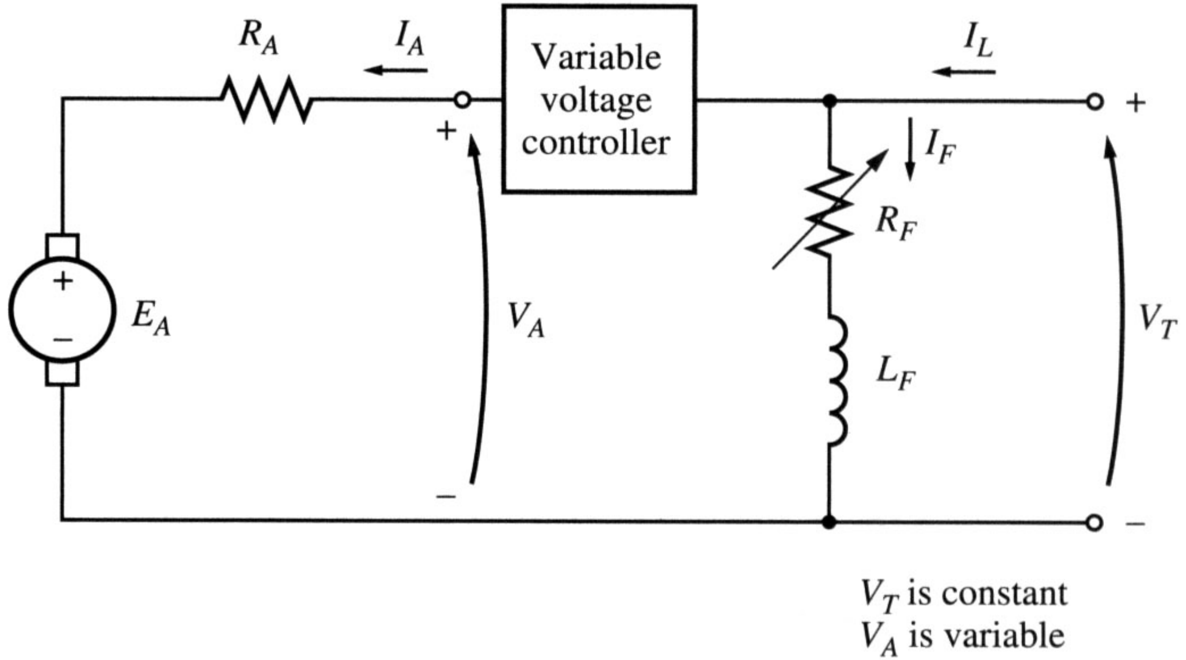
Increase in the field resistance increases the motor speed. Observe also that the slope of the speed-torque curve becomes steeper when field resistance increases.

Shunt Motor: Speed Control

2. Changing the armature voltage

This method implies changing the voltage applied to the armature of the motor without changing the voltage applied to its field. Therefore, the motor must be separately excited to use armature voltage control.

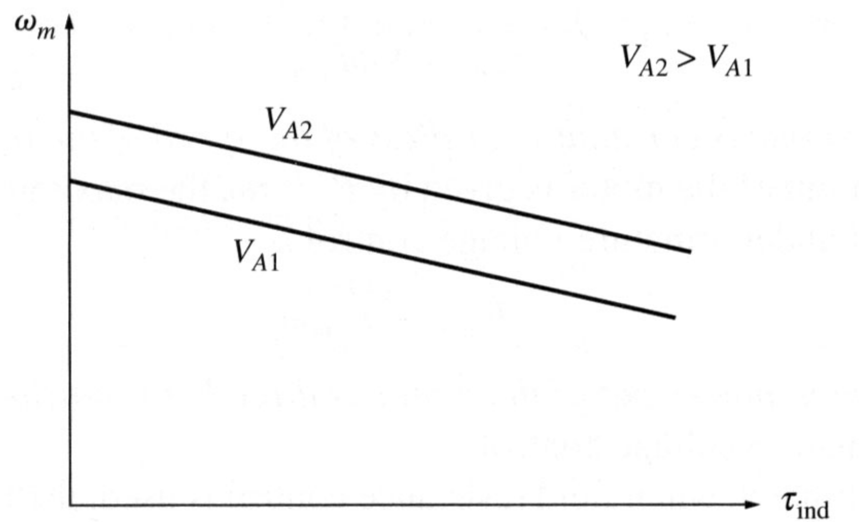
Armature voltage speed control



Shunt Motor: Speed Control

1) Increasing the armature voltage V_A increases the armature current ($I_A = (V_A - E_A)/R_A$);
2) Increasing armature current I_A increases the induced torque τ_{ind} ($\tau_{ind} = K\phi I_A$);
3) Increased induced torque τ_{ind} is now larger than the load torque τ_{load} and, therefore, the speed ω;
4) Increasing speed increases the internal generated voltage ($E_A = K\phi\omega$);
5) Increasing E_A decreases the armature current I_A...
6) Decreasing I_A decreases the induced torque until $\tau_{ind} = \tau_{load}$ at a higher speed ω.

Increasing the armature voltage of a separately excited DC motor does not change the slope of its torque-speed characteristic.



Shunt Motor: Speed Control

If a motor is operated at its rated terminal voltage, power, and field current, it will be running at the rated speed also called a base speed.

Field resistance control can be used for speeds above the base speed but not below it. Trying to achieve speeds slower than the base speed by the field circuit control, requires large field currents that may damage the field winding.

Since the armature voltage is limited to its rated value, no speeds exceeding the base speed can be achieved safely while using the armature voltage control.

Therefore, armature voltage control can be used to achieve speeds below the base speed, while the field resistance control can be used to achieve speeds above the base speed.

Shunt and separately excited DC motors have excellent speed control characteristic.

Shunt Motor: Speed Control

For the **armature voltage control**, the flux in the motor is constant. Therefore, the maximum torque in the motor will be constant too regardless the motor speed:

$$\tau_{\max} = K\phi I_{A,\max}$$

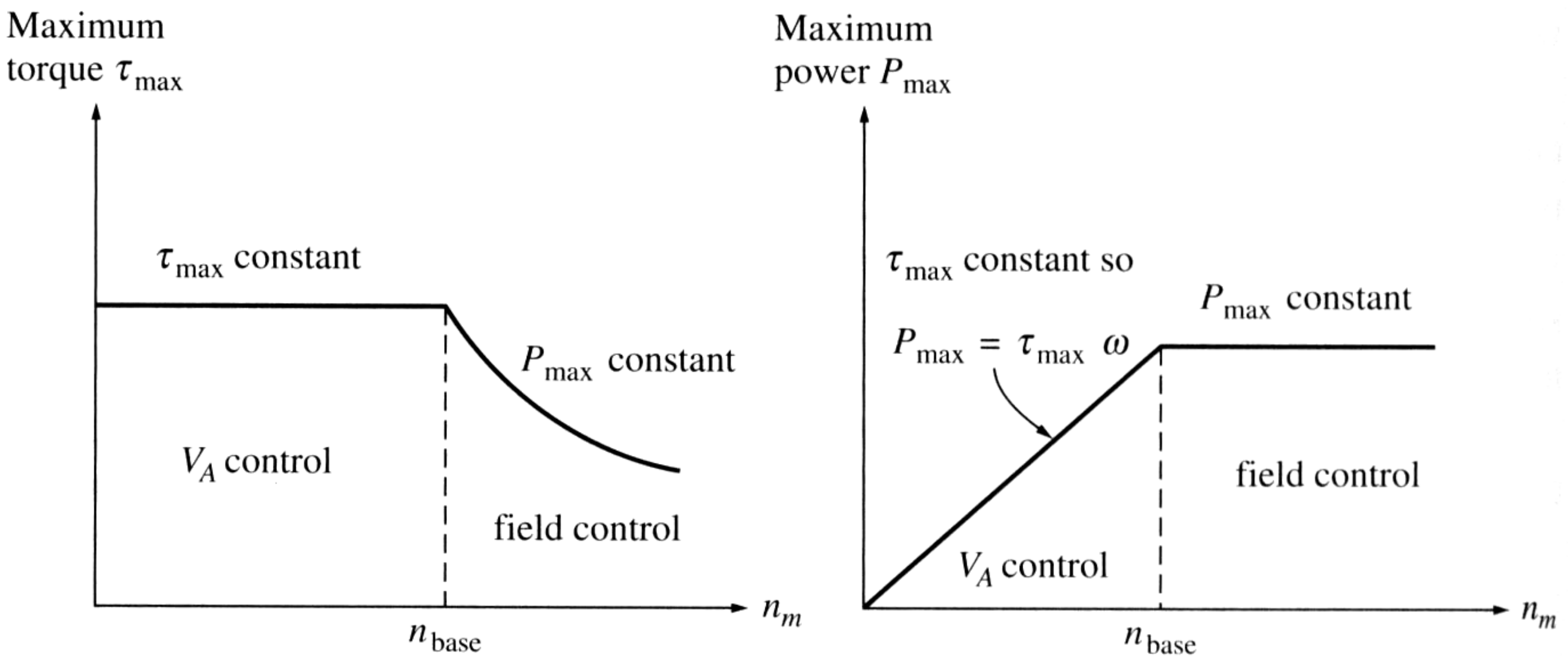
Since the maximum power of the motor is

$$P_{\max} = \tau_{\max}\omega$$

The maximum power out of the motor is directly proportional to its speed.

For the **field resistance control**, the maximum power out of a DC motor is constant, while the maximum torque is reciprocal to the motor speed.

Shunt Motor: Speed Control

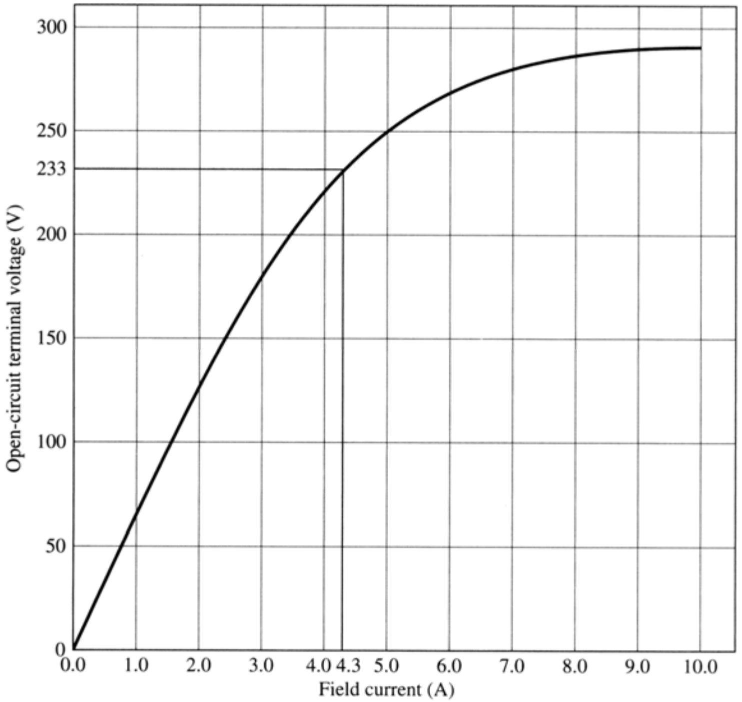


Torque and power limits as functions of motor speed for a shunt (or separately excited) DC motor.

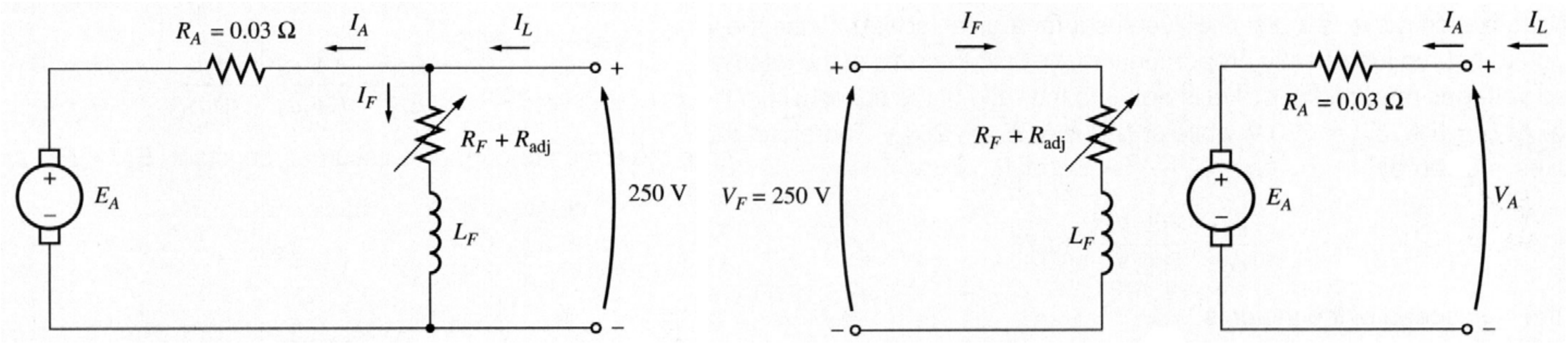
Shunt Motor: Speed Control

Example 5: A 100 hp, 250 V, 1200 rpm DC shunt motor with an armature resistance of 0.03 Ω and a field resistance of 41.67 Ω. The motor has compensating windings, so armature reactance can be ignored. Mechanical and core losses may be ignored also. The motor is driving a load with a line current of 126 A and an initial speed of 1103 rpm. Assuming that the armature current is constant and the magnetization curve is

a) What is the motor speed if the field resistance is increased to 50 Ω?
b) Calculate the motor speed as a function of the field resistance, assuming a constant-current load.
c) Assuming that the motor next is connected as a separately excited and is initially running with V_A = 250 V, I_A = 120 A and at n = 1103 rpm while supplying a constant-torque load, estimate the motor speed if V_A is reduced to 200 V.



Shunt Motor: Speed Control-Example 5



shunt separately-excited

For the given initial line current of 126 A, the initial armature current will be

$$I_{A1} = I_{L1} - I_{F1} = 126 - \frac{250}{41.67} = 120\ A$$

Therefore, the initial generated voltage for the shunt motor will be

$$E_{A1} = V_T - I_{A1} R_A = 250 - 120 \cdot 0.03 = 246.4\ V$$

Shunt Motor: Speed Control-Example 5

After the field resistance is increased to 50 Ω, the new field current will be

$$I_{F2} = \frac{250}{50} = 5\ A$$

The ratio of the two internal generated voltages is

$$\frac{E_{A2}}{E_{A1}} = \frac{K\phi_2\omega_2}{K\phi_1\omega_1} = \frac{\phi_2 n_2}{\phi_1 n_1}$$

Since the armature current is assumed constant, $E_{A1} = E_{A2}$ and, therefore

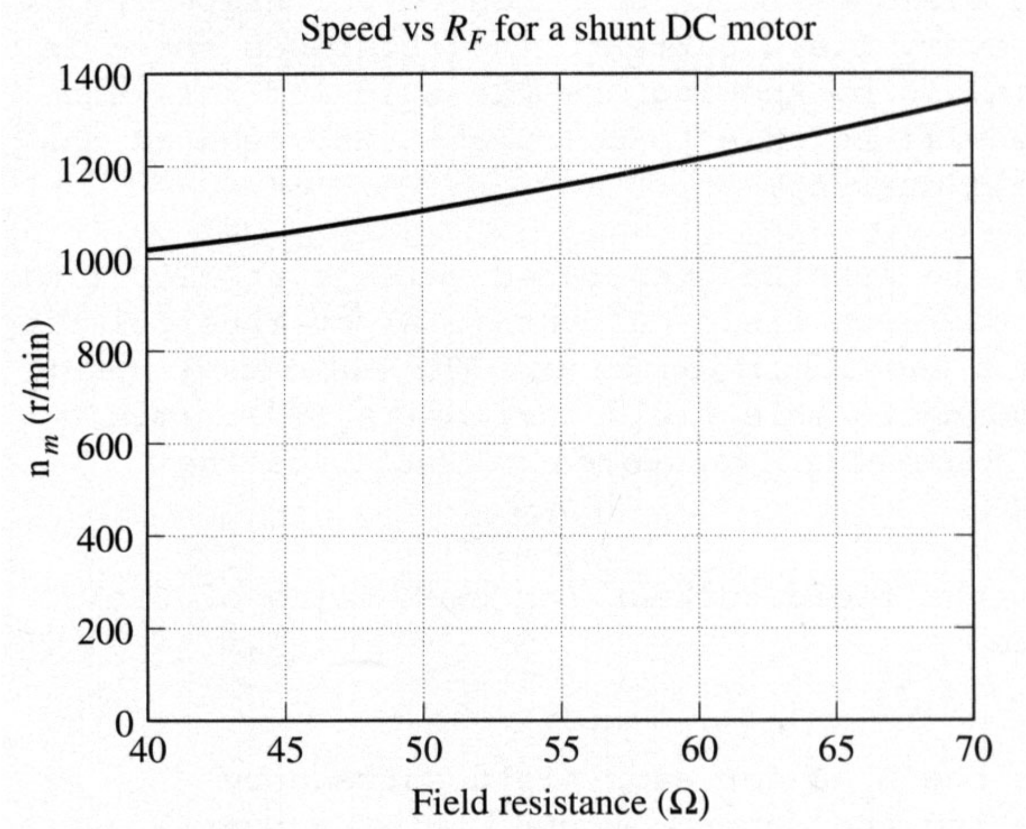
$$n_2 = \frac{\phi_1 n_1}{\phi_2}$$

The values of E_A on the magnetization curve are directly proportional to the flux. Therefore, the ratio of internal generated voltages equals to the ratio of the fluxes within the machine. From the magnetization curve, at I_F = 5A, E_{A1} = 250V, and at I_F = 6A, E_{A1} = 268V. Thus:

Shunt Motor: Speed Control-Example 5

$$n_2 = \frac{\phi_1 n_1}{\phi_2} = \frac{E_{A1} n_1}{E_{A2}} = \frac{268}{250} 1103 = 1187\ rpm$$

b) A speed vs. R_F characteristic is shown below:



Shunt Motor: Speed Control-Example 5

c) For a separately excited motor, the initial generated voltage is

$$E_{A1} = V_{T1} - I_{A1}R_A$$

Since

$$\frac{E_{A2}}{E_{A1}} = \frac{K\phi_2\omega_2}{K\phi_1\omega_1} = \frac{\phi_2 n_2}{\phi_1 n_1}$$

and since the flux ϕ is constant

$$n_2 = \frac{E_{A2}n_1}{E_{A1}}$$

Since the both the torque and the flux are constants, the armature current I_A is also constant. Then

$$n_2 = \frac{V_{T2} - I_{A2}R_A}{V_{T1} - I_{A1}R_A} n_1 = \frac{200 - 120 \cdot 0.03}{250 - 120 \cdot 0.03} 1103 = 879\ rpm$$

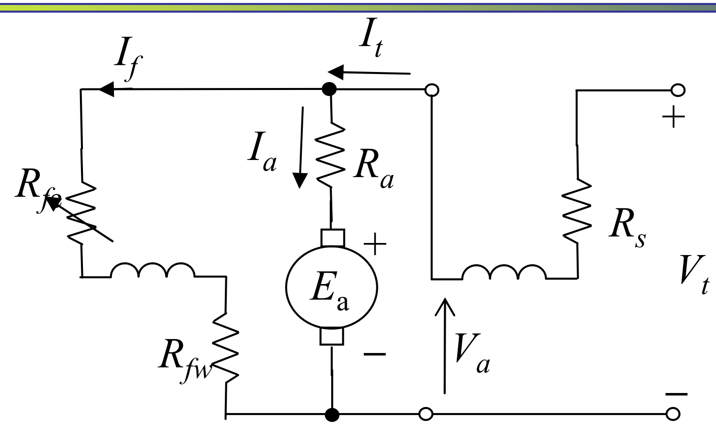
Shunt Motor
The effect of an open field circuit

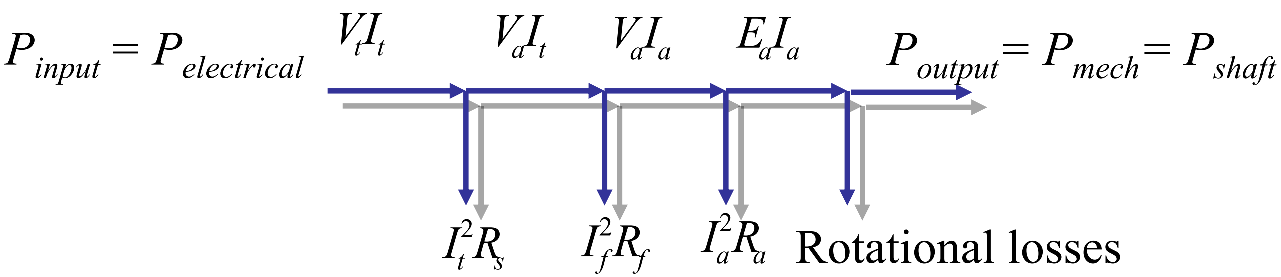
If the field circuit is left open on a shunt motor, its field resistance will be infinite. Infinite field resistance will cause a drastic flux drop and, therefore, a drastic drop in the generated voltage. The armature current will be increased enormously increasing the motor speed.

A similar effect can be caused by armature reaction. If the armature reaction is severe enough, an increase in load can weaken the flux causing increasing the motor speed. An increasing motor speed increases its load, which increases the armature reaction weakening the flux again. This process continues until the motor overspeeds. This condition is called runaway.

Power Flow and Efficiency

DC Motors





$$\eta = \frac{P_{output}}{P_{input}} = \frac{P_{input} - Losses}{P_{input}}$$

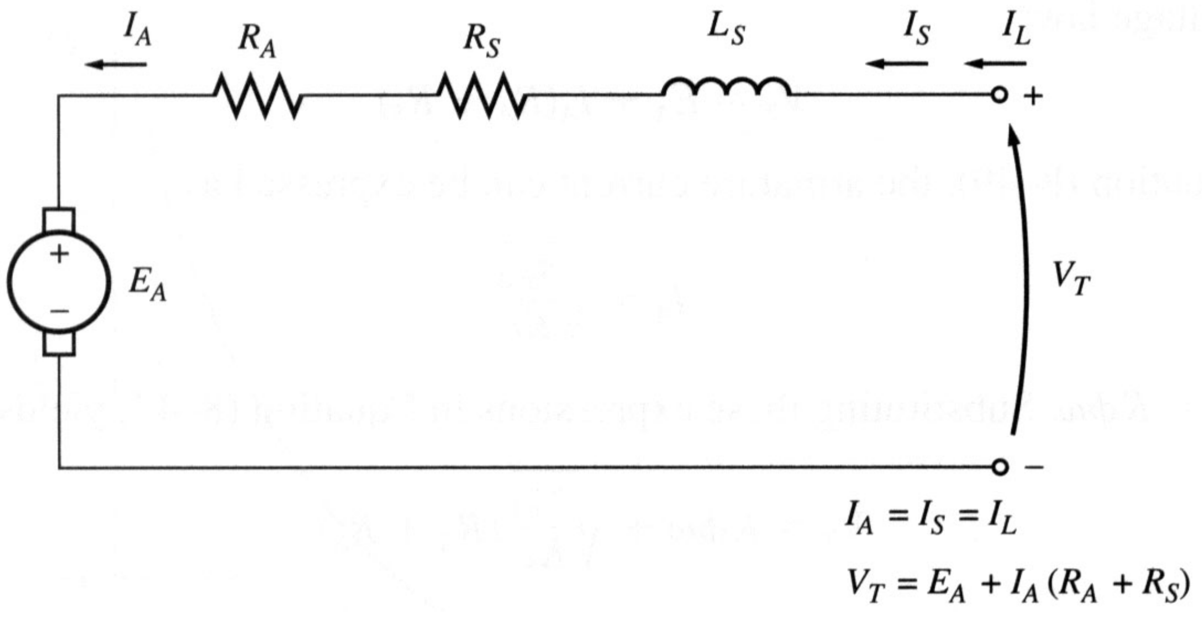
$$\eta = \frac{V_t I_t - \sum I^2 R - RotationalLosses}{V_t I_t}$$

$$\eta = \frac{E_a I_a - RotationalLosses}{V_t I_t}$$

Motor types: The series DC motor

A series DC motor is a DC motor whose field windings consists of a relatively few turns connected in series with armature circuit. Therefore:

$$V_T = E_A + I_A(R_A + R_s)$$



Torque-Speed Characteristics

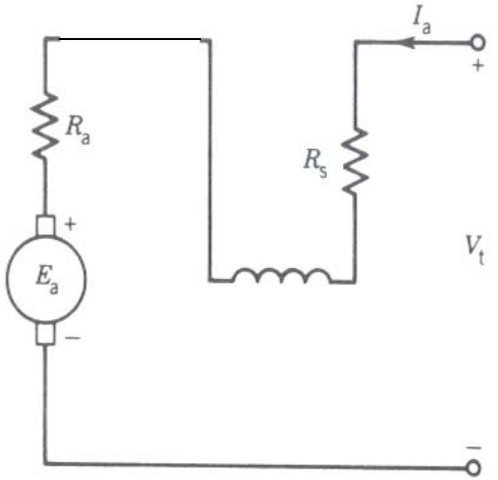
Series motors

$$E_a = V_t - I_a(R_a + R_s)$$

$$E_a = K_a \phi \, \omega_m$$

Neglecting saturation $\phi = K_1 I_f = K_1 I_a$

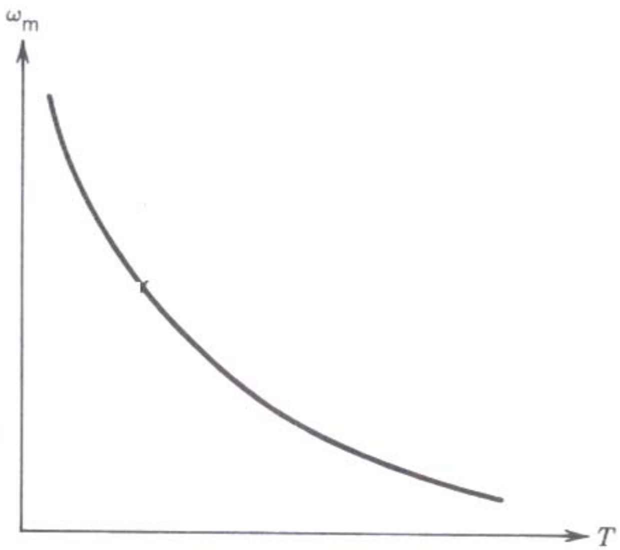
$$E_a = K_a K_1 I_a \omega_m = K_s I_a \omega_m$$



$$\omega_m = \frac{V_t}{K_s I_a} - \frac{R_a + R_s}{K_s}$$

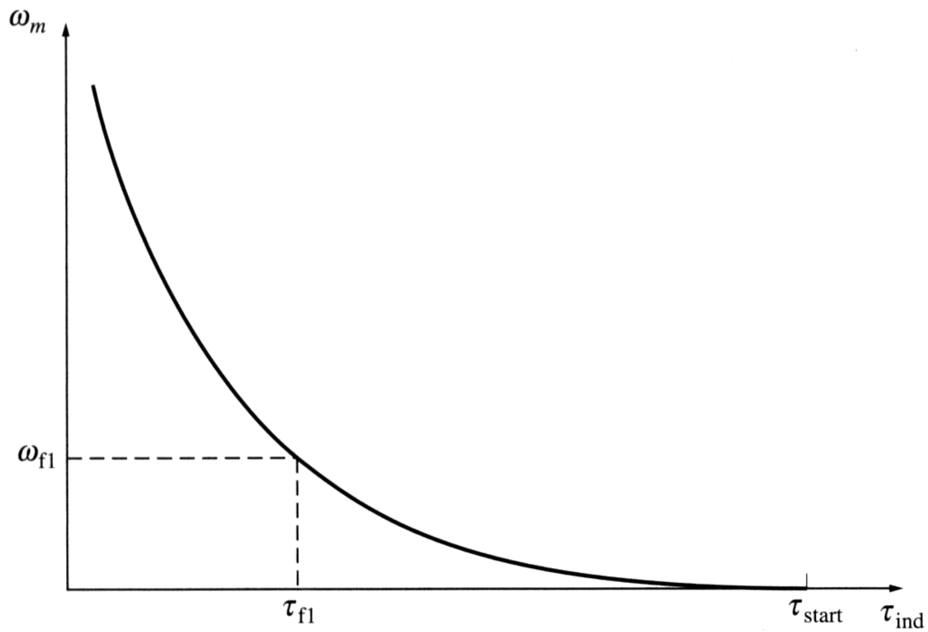
But $T = K_a \phi \, I_a = K_a K_1 I_a^2 = K_s I_a^2$

$$\therefore \omega_m = \frac{V_t}{\sqrt{K_s}\sqrt{T}} - \frac{R_a + R_s}{K_s}$$



Series Motor: Terminal Characteristic

One serious disadvantage of a series motor is that its speed goes to infinity for a zero torque.



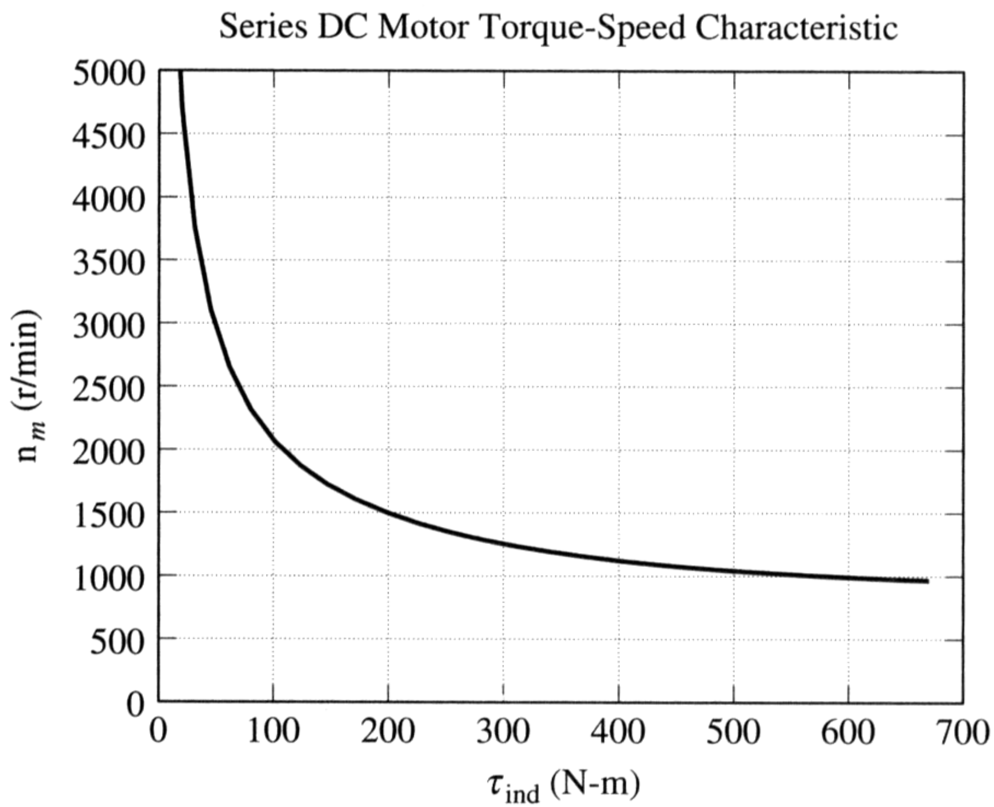
In practice, however, torque never goes to zero because of the mechanical, core, and stray losses. Still, if no other loads are attached, the motor will be running fast enough to cause damage.

Steps must be taken to ensure that a series motor **always** has a load! Therefore, it is not a good idea to connect such motors to loads by a belt or other mechanism that could break.

Series Motor: Terminal Characteristic

The complete torque-speed characteristic

We notice severe over-speeding at low torque values.



Speed Control

The only way to control speed of a series DC motor is by changing its terminal voltage, since the motor speed is directly proportional to its terminal voltage ***for any given torque***.

$$\omega_m = \frac{V_t}{\sqrt{K_s}\sqrt{T}} - \frac{R_a + R_s}{K_s}$$

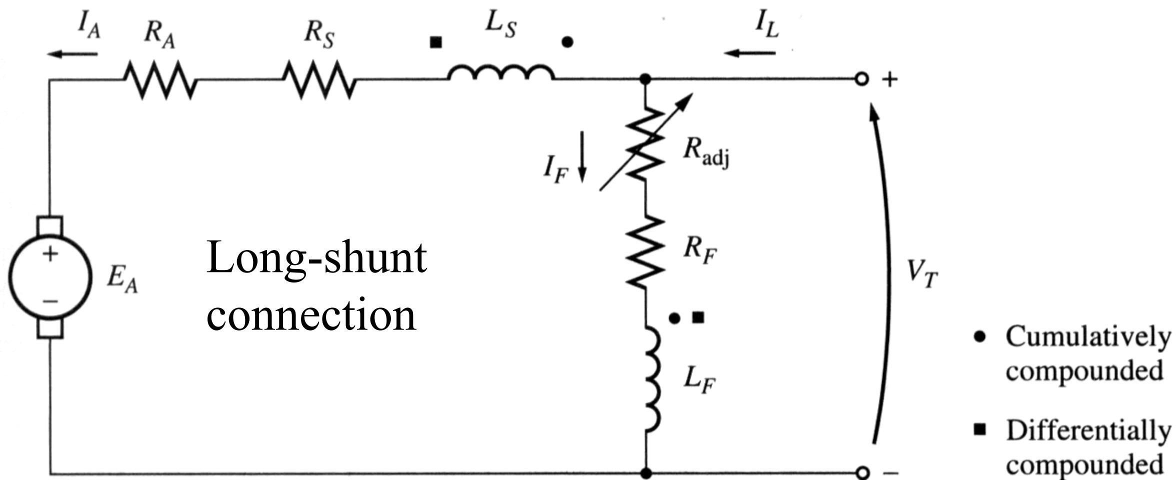
Motor types: Compounded DC motor

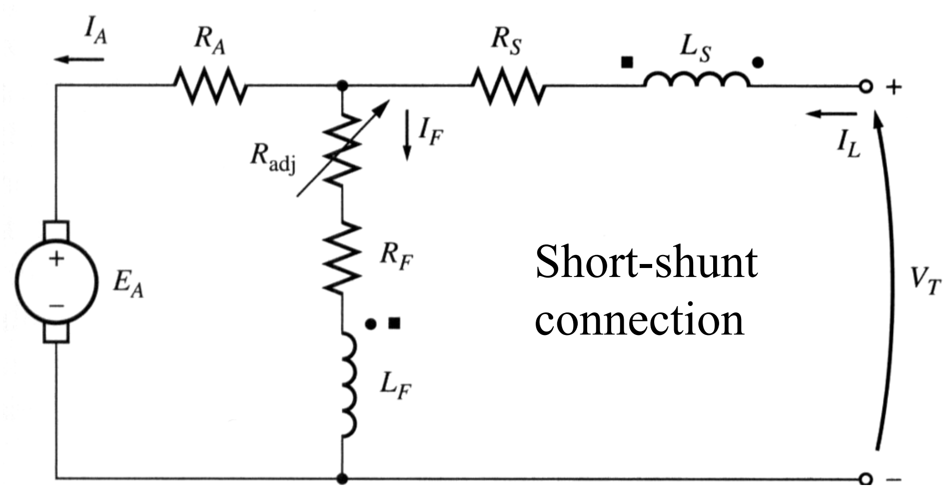
A compounded DC motor is a motor with both a shunt and a series field.

Current flowing into a dotted end of a coil (shunt or series) produces a positive mmf.

If current flows into the dotted ends of both coils, the resulting mmfs add to produce a larger total mmf – cumulative compounding.

If current flows into the dotted end of one coil and out of the dotted end of another coil, the resulting mmfs subtract – differential compounding.





Motor types: Compounded DC Motor

The Kirchhoff's voltage law equation for a compounded DC motor is

$$V_T = E_A + I_A(R_A + R_S)$$

The currents in a compounded DC motor are

$$I_A = I_L - I_F$$

$$I_F = \frac{V_T}{R_F}$$

The mmf of a compounded DC motor:

Cumulatively compounded

$$\mathcal{F}_{net} = \mathcal{F}_F \pm \mathcal{F}_{SE} - \mathcal{F}_{AR}$$

Differentially compounded

Compound Motors: Torque-Speed Characteristics

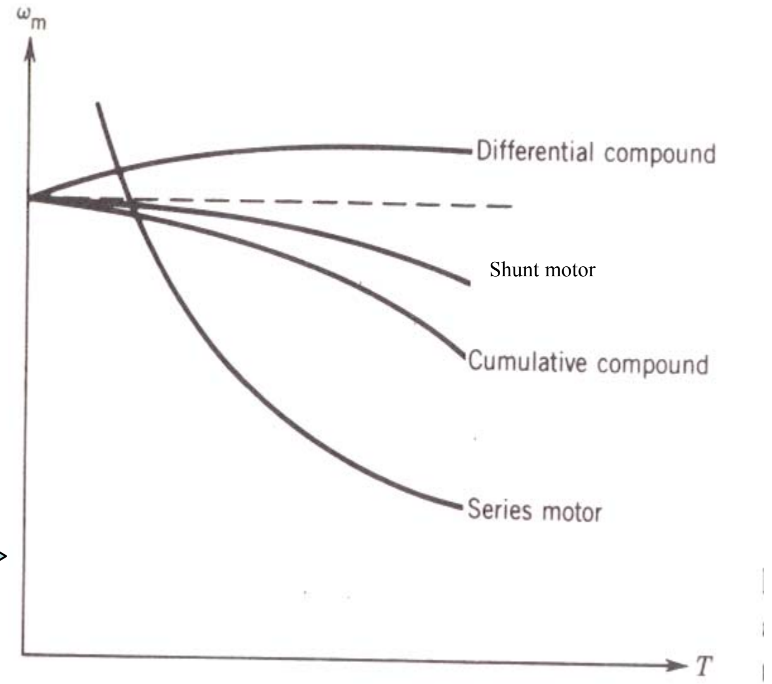
Cumulative Compound

$$AT_t = AT_{shunt} \pm AT_{series}$$

Differential Compound

$$\phi_t = \phi_{shunt} \pm \phi_{series}$$

$$\omega_m = \frac{V_t}{K_a \phi_t} - \frac{R_a}{(K_a \phi_t)^2} T$$

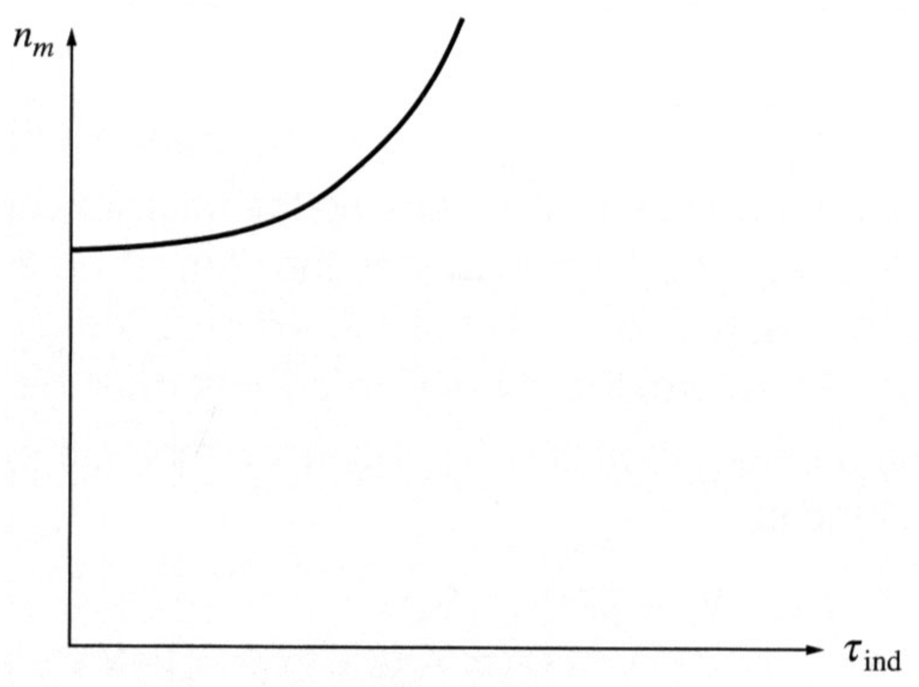


Differentially Compounded Motors: Torque-Speed Characteristic

Since the shunt mmf and series mmf subtract from each other in a differentially compounded motor, increasing load increases the armature current I_A and decreases the flux. When flux decreases, the motor speed increases further increasing the load. This results in an instability (much worse than one of a shunt motor) making differentially compounded motors unusable for any applications.

In addition to that, these motors are not easy to start… The motor typically remains still or turns very slowly consuming enormously high armature current.

Stability problems and huge starting armature current lead to these motors being never used **intentionally**.



DC Motor Starters

In order for DC motors to function properly, they must have some special control and protection equipment associated with them. The purposes of this equipment are:

1. To protect the motor against damage due to short circuits in the equipment;
2. To protect the motor against damage from long-term overloads;
3. To protect the motor against damage from excessive starting currents;
4. To provide a convenient manner in which to control the operating speed of the motor.

DC Motor Problems on Starting

At starting conditions, the motor is not turning, therefore the internal generated voltage $E_A = 0V$. Since the internal resistance of a normal DC motor is very low (3-6 % pu), a very high current flows.

For instance, for a 50 hp, 250 V DC motor with armature resistance R_A of 0.06 Ω and a full-load current about 200 A, the starting current is

$$I_A = \frac{V_T - E_A}{R_A} = \frac{250 - 0}{0.06} = 4167\ A$$

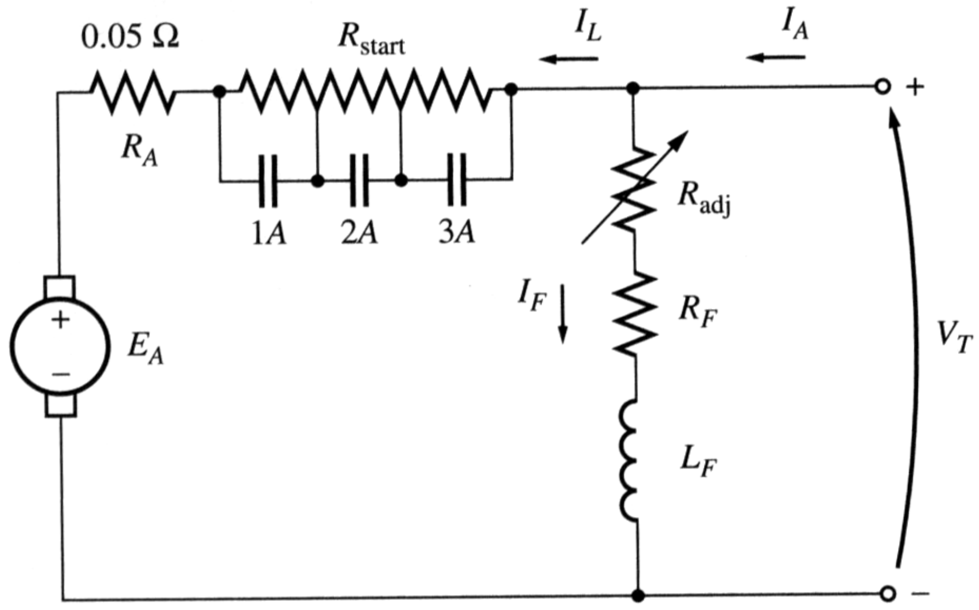
This current is over 20 times the motor's rated full-load current and may severely damage the motor.

A solution to the problem of excessive starting current is to insert a starting resistor in series with the armature to limit the current until E_A can build up to limit the armature current. However, this resistor must be removed from the circuit as the motor speed is high since otherwise such resistor would cause losses and would decrease the motor's torque-speed characteristic.

DC Motor Problems on Starting

In practice, a starting resistor is made up of a series of resistors that can be successively removed from the circuit as the motor speeds up.

A shunt motor with an extra starting resistor that can be cut out of the circuit in segments by closing the 1A, 2A, and 3A contacts.



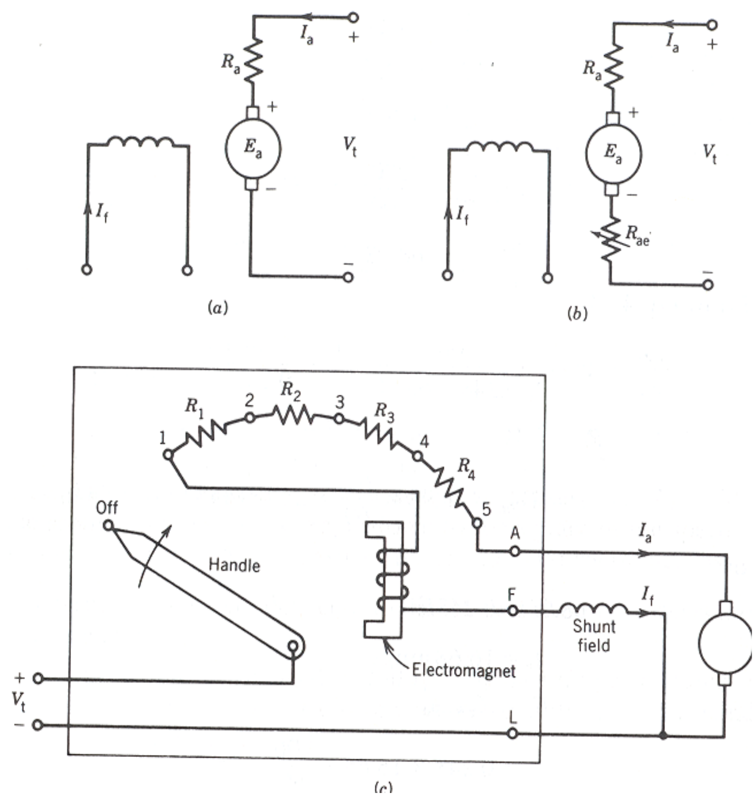
Therefore, two considerations are needed to be taken into account: Select the values and the number of resistor segments needed to limit the starting current to desired ranges; Design a control circuit shutting the resistor bypass contacts at the proper time to remove particular parts of the resistor from the circuit.

Starting of DC Machine

If a d.c. motor is directly connected to a d.c. power supply, the starting current will be dangerously high.

$$I_a = \frac{V_t - E_a}{R_a} \qquad \text{at starting} \qquad \omega = 0 \rightarrow E_a = 0$$

$$\therefore I_a \Big|_{Starting} = \frac{V_t}{r_a}$$

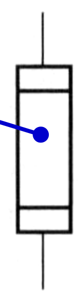


Since R_a is small, the starting current is very large.
The starting current can be limited by the following methods:
1- Use a variable-voltage supply.
2- Insert an external resistance at start, as shown in the Figure.

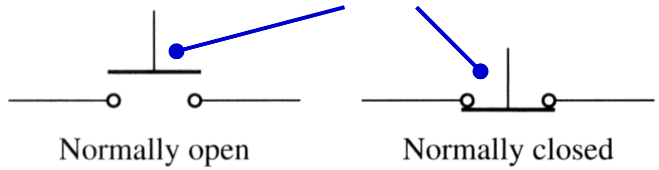
DC Motor Starting Circuits

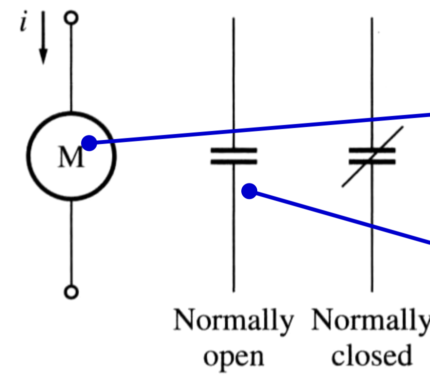
Several different schemes can be used to short contacts and cut out the sections of a starting resistor. Some devices commonly used in motor-control circuits are

Fuses: protects against short circuits



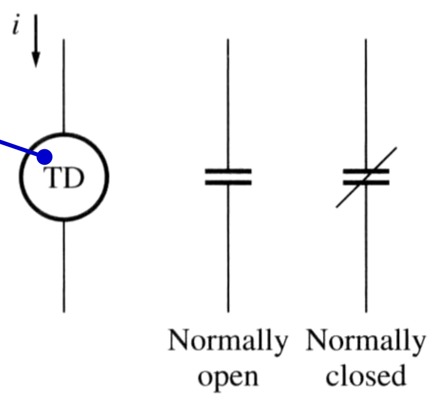
Spring-type push button switches

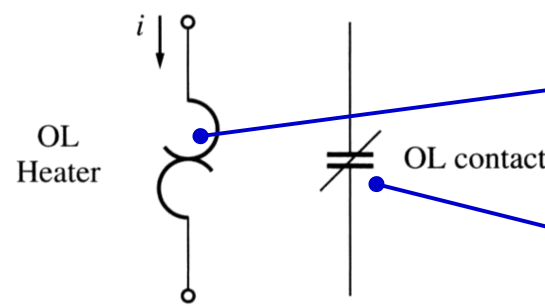




Relay: a main coil and a number of contacts

Time delay relay similar to ordinary relay except for having adjustable time delay.





Overload: a heater coil and normally closed contacts

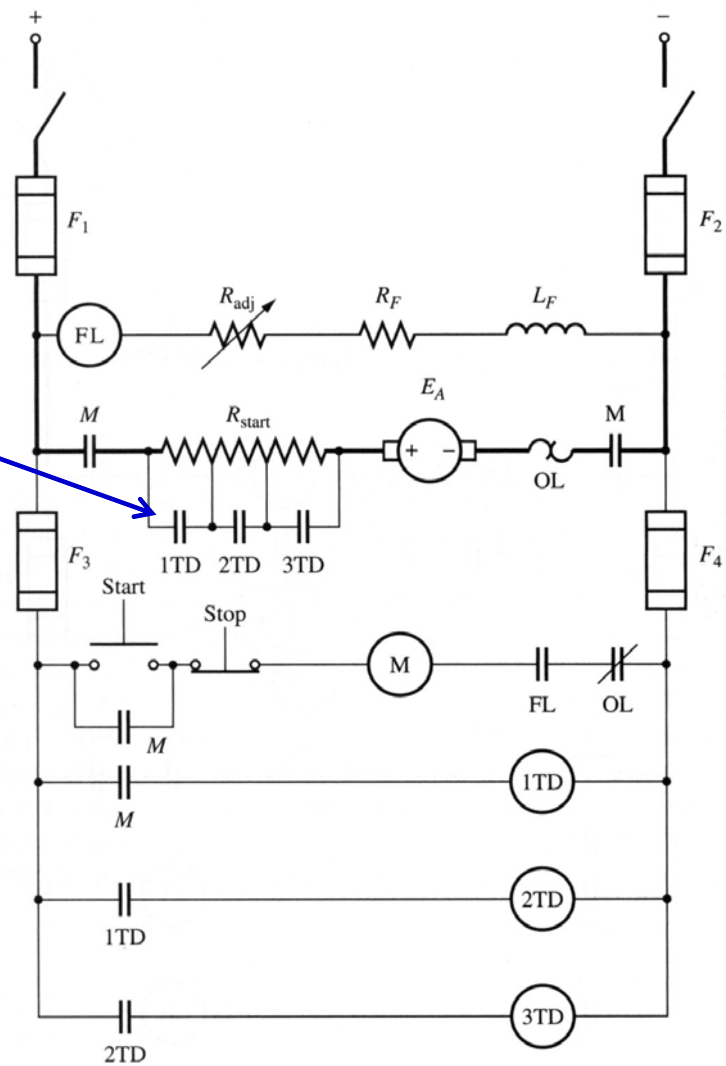
DC Motor Starting Circuits

A common DC motor starting circuit:

A series of time delay relays shut contacts removing each section of the starting resistor at approximately correct times.

Notice that the relay 1TD is energized at the same time as the motor starts – contacts of 1TD will shut a part of the starting resistor after some time. At the same instance, relay 2TD is energized and so on…

Observe also 4 fuses protecting different parts of the circuit and the overload in series with the armature winding.



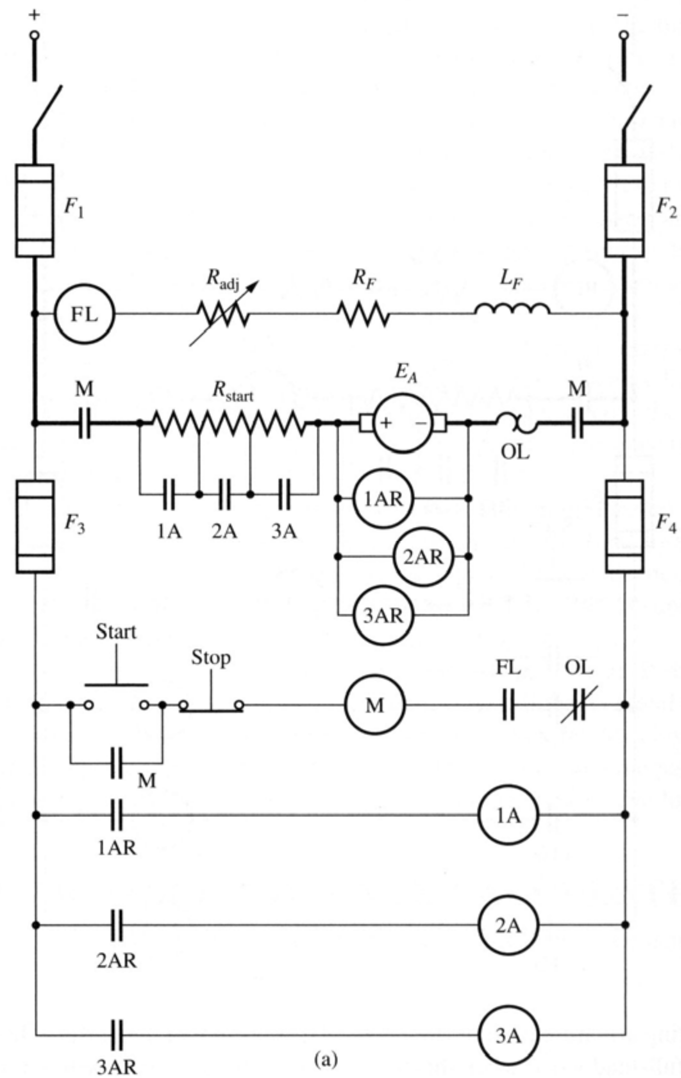
DC Motor Starting Circuits

Another type of motor starter:

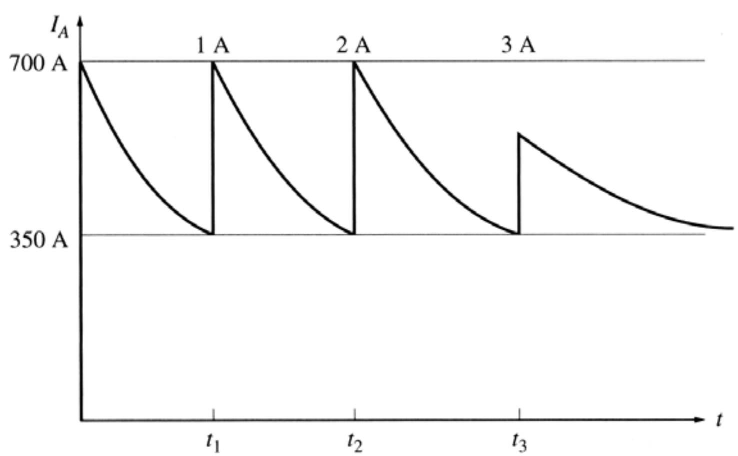
A series of relays sense the value of armature voltage E_A and cut out the starting resistors as it riches certain values.

This starter type is more robust to different loads.

FL is the *field loss relay*: if the field is lost for any reason, power to the M relay will be turned off.



Armature current in a DC motor during starting.



Motor types: The Permanent-Magnet DC Motor

A permanent magnet DC (PMDC) motor is a motor whose poles are made out of permanent magnets.

Advantages:

1. Since no external field circuit is needed, there are no field circuit copper losses;
2. Since no field windings are needed, these motors can be considerable smaller.

Disadvantages:

1. Since permanent magnets produces weaker flux densities then externally supported shunt fields, such motors have lower induced torque.
2. There is always a risk of demagnetization from extensive heating or from armature reaction effects (via armature mmf).

Brushless DC Motors

- A brushless dc motor has a rotor with permanent magnets and a stator with windings.
- It is essentially a dc motor turned inside out.
- The control electronics replace the function of the commutator and energize the proper winding.