

# Numerical Methods for Engineers

SEVENTH EDITION

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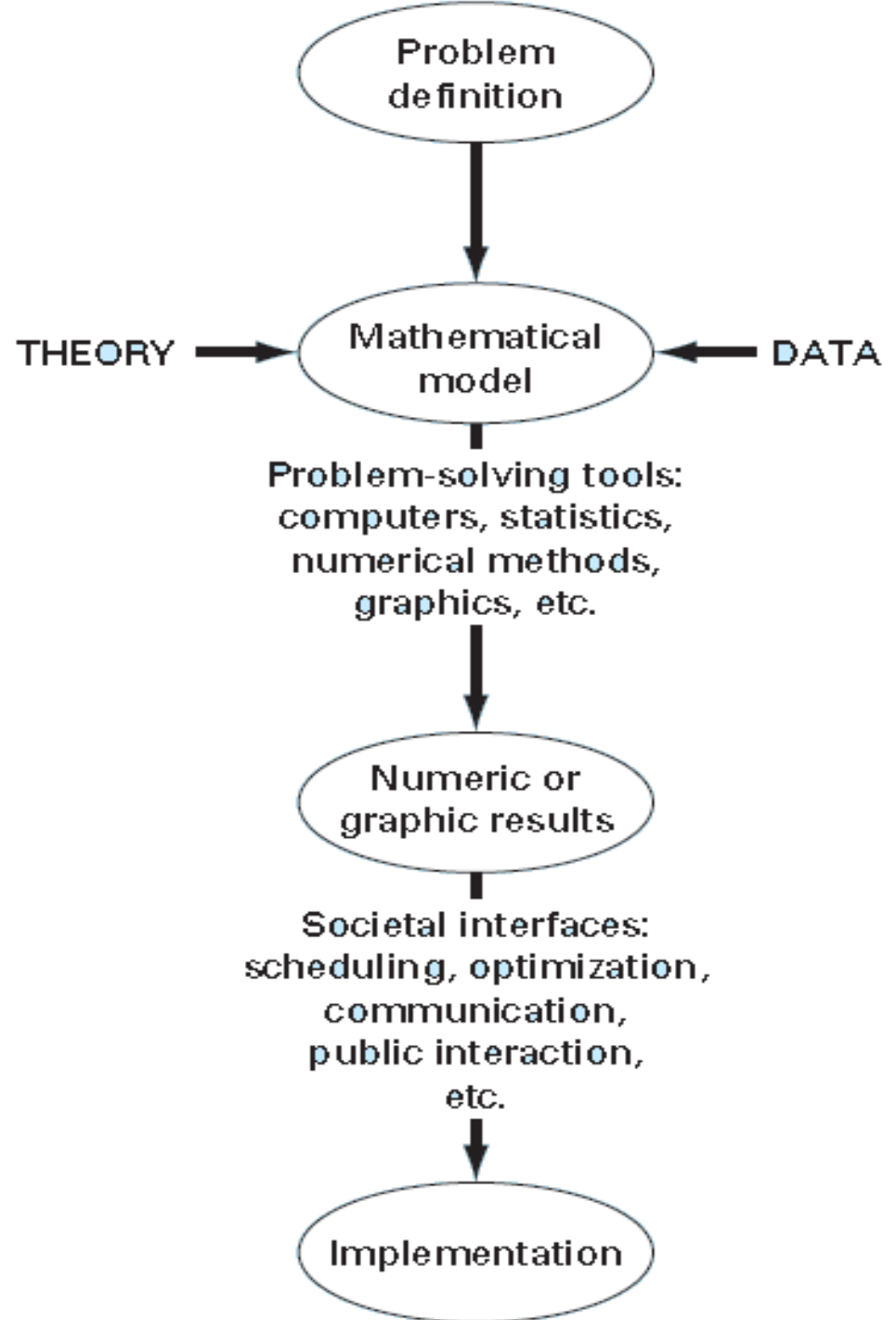
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## Introduction

## Mathematical Modeling and Engineering Problem Solving

# The engineering problem-solving process



# Problem 1

## Mathematical Model

$$D = f(I, P, F)$$



D: dependent variable

I: Independent variables

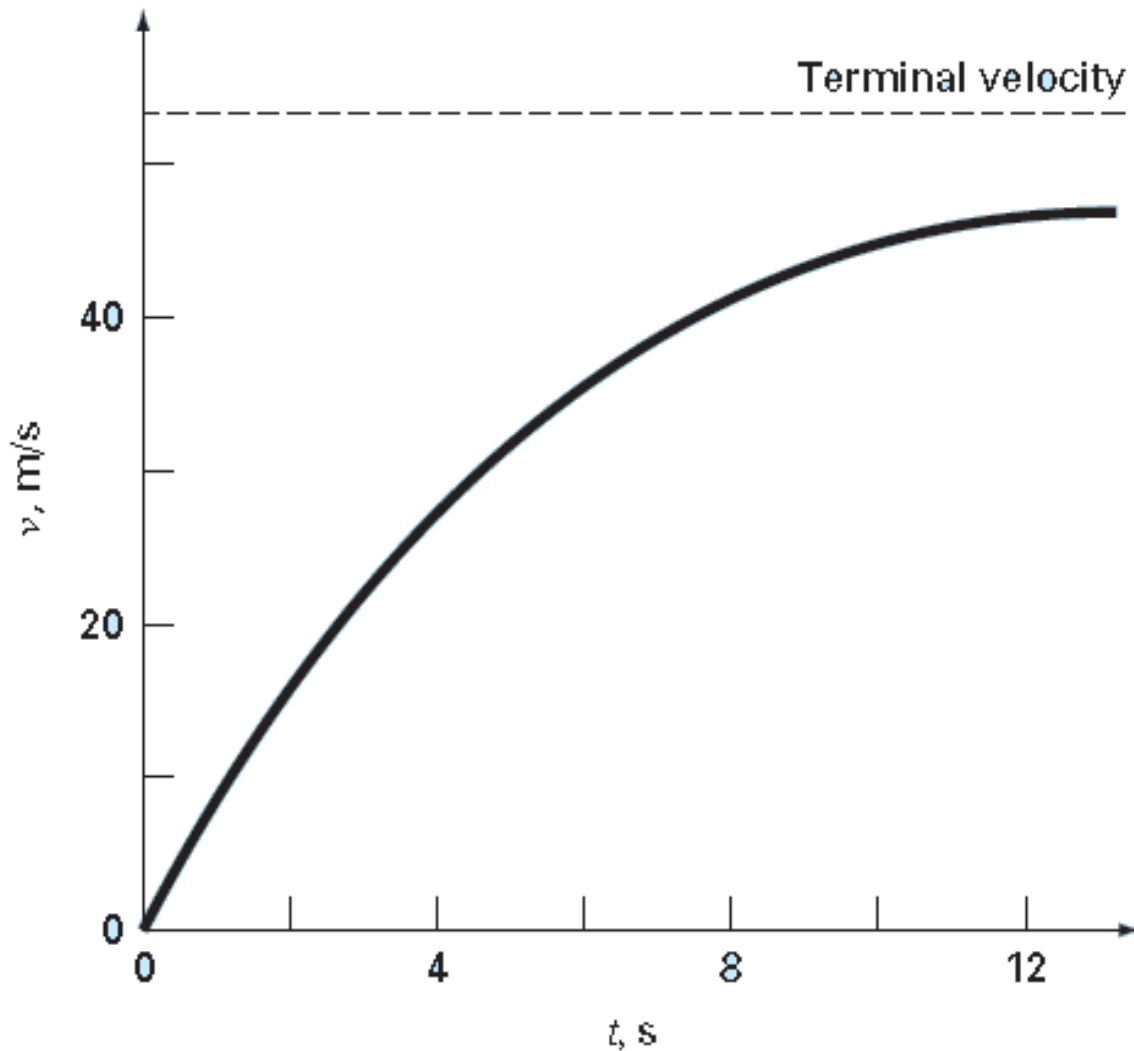
P: Parameters

F: forcing (external) functions

$$a = \frac{F}{m}$$

Example: free falling parachutist

Terminal velocity: the final steady velocity of the parachutist



$$\frac{dv}{dt} = \frac{F}{m}$$

$$F = F_D + F_U$$

$$F_D = mg \quad F_U = -cv$$

$$\frac{dv}{dt} = \frac{mg - cv}{m}$$



$$v(t) = \frac{mg}{c} (1 - e^{-(c/m)t})$$

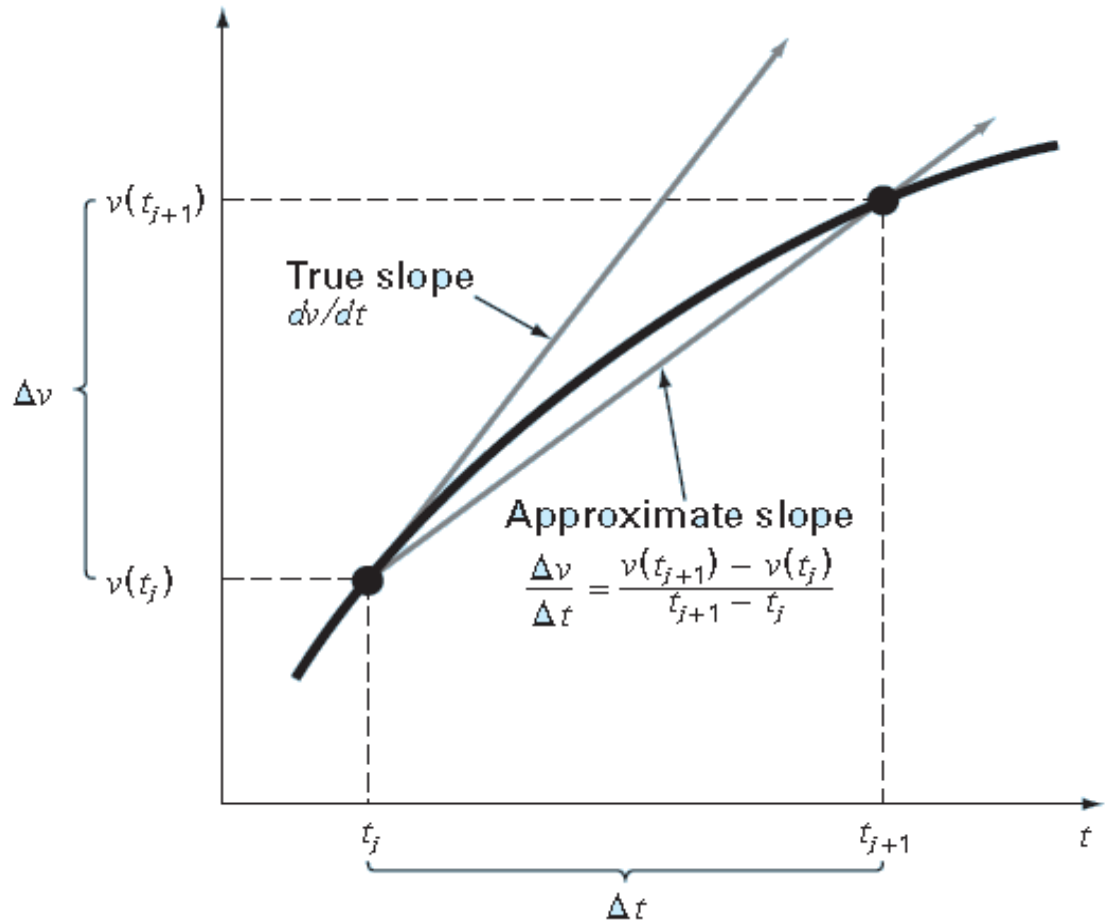
(analytical solution)

# Numerical Solution for Terminal Velocity

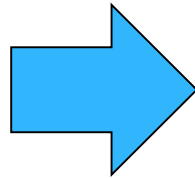
$$m = 68.1 \text{ kg}$$

$$c = 12.5 \text{ kg / s}$$

$$g = 9.81 \text{ m / s}^2$$



$$\frac{dv}{dt} = \frac{mg - cv}{m} = g - \frac{c}{m}v$$



$$\frac{v(t_{i+1}) - v(t_i)}{t_{i+1} - t_i} = g - \frac{c}{m}v(t_i)$$

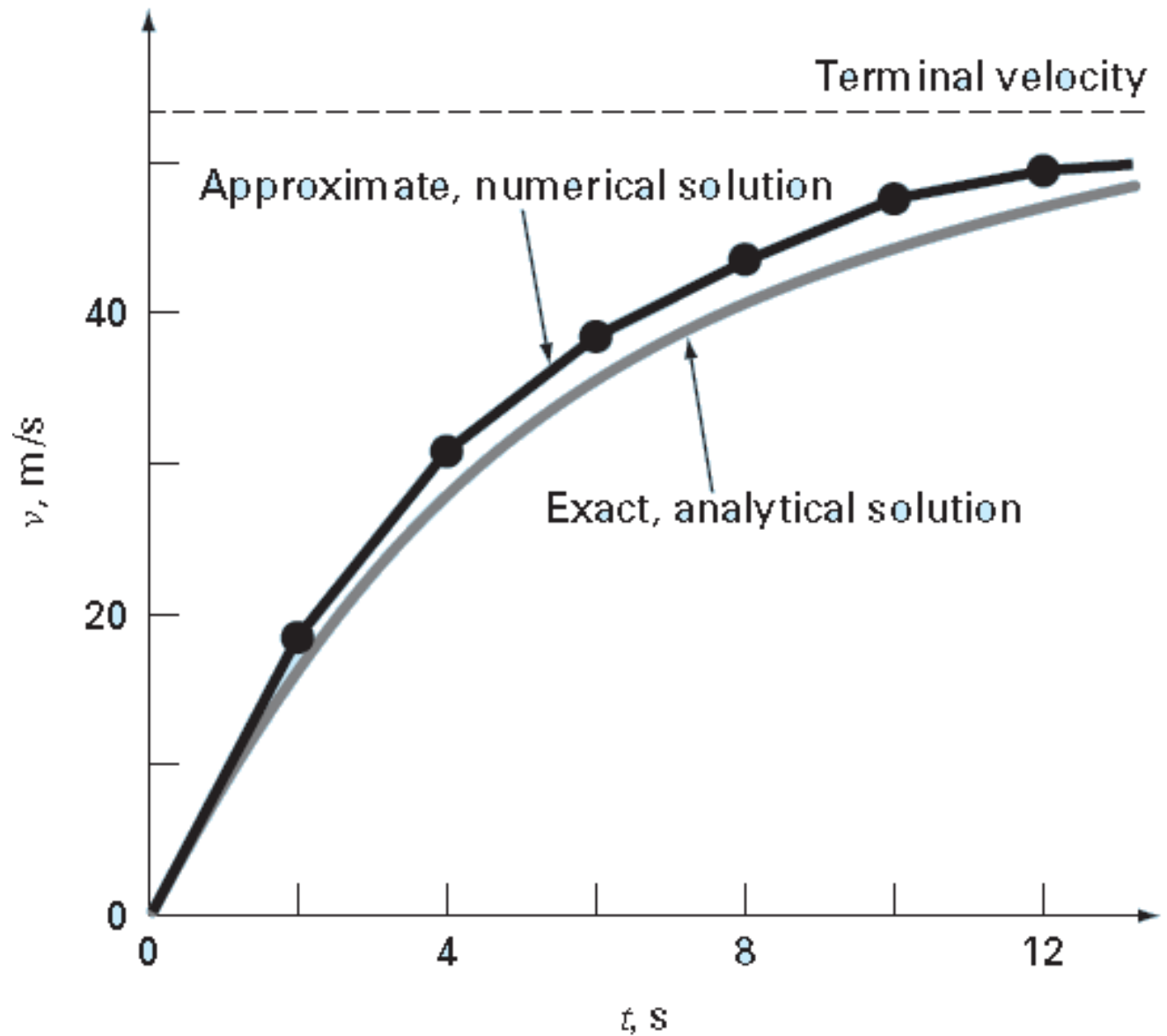
$$\frac{dv}{dt} \cong \frac{\Delta v}{\Delta t} = \frac{v(t_{i+1}) - v(t_i)}{t_{i+1} - t_i}$$

$$v(t_{i+1}) = v(t_i) + [g - \frac{c}{m}v(t_i)](t_{i+1} - t_i)$$

# Comparison of numerical and analytical solutions

| $t, s$   | $v, m/s$ |
|----------|----------|
| 0        | 0.00     |
| 2        | 19.60    |
| 4        | 32.00    |
| 6        | 39.85    |
| 8        | 44.82    |
| 10       | 47.97    |
| 12       | 49.96    |
| $\infty$ | 53.39    |

Larger **step** size  $\rightarrow$  less accurate result



Smaller step size  $\rightarrow$  more steps  
(longer computing time)