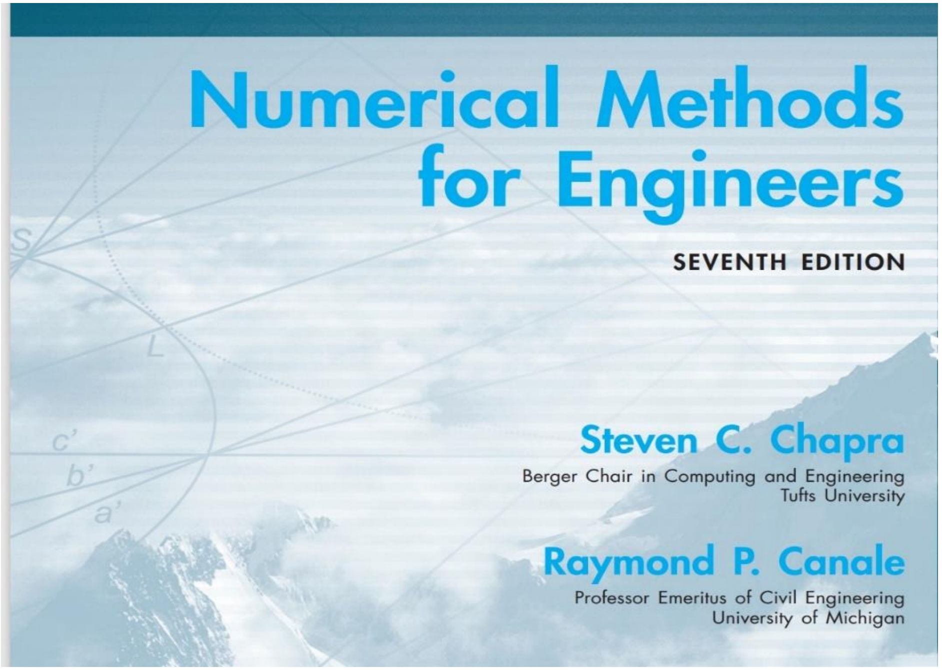
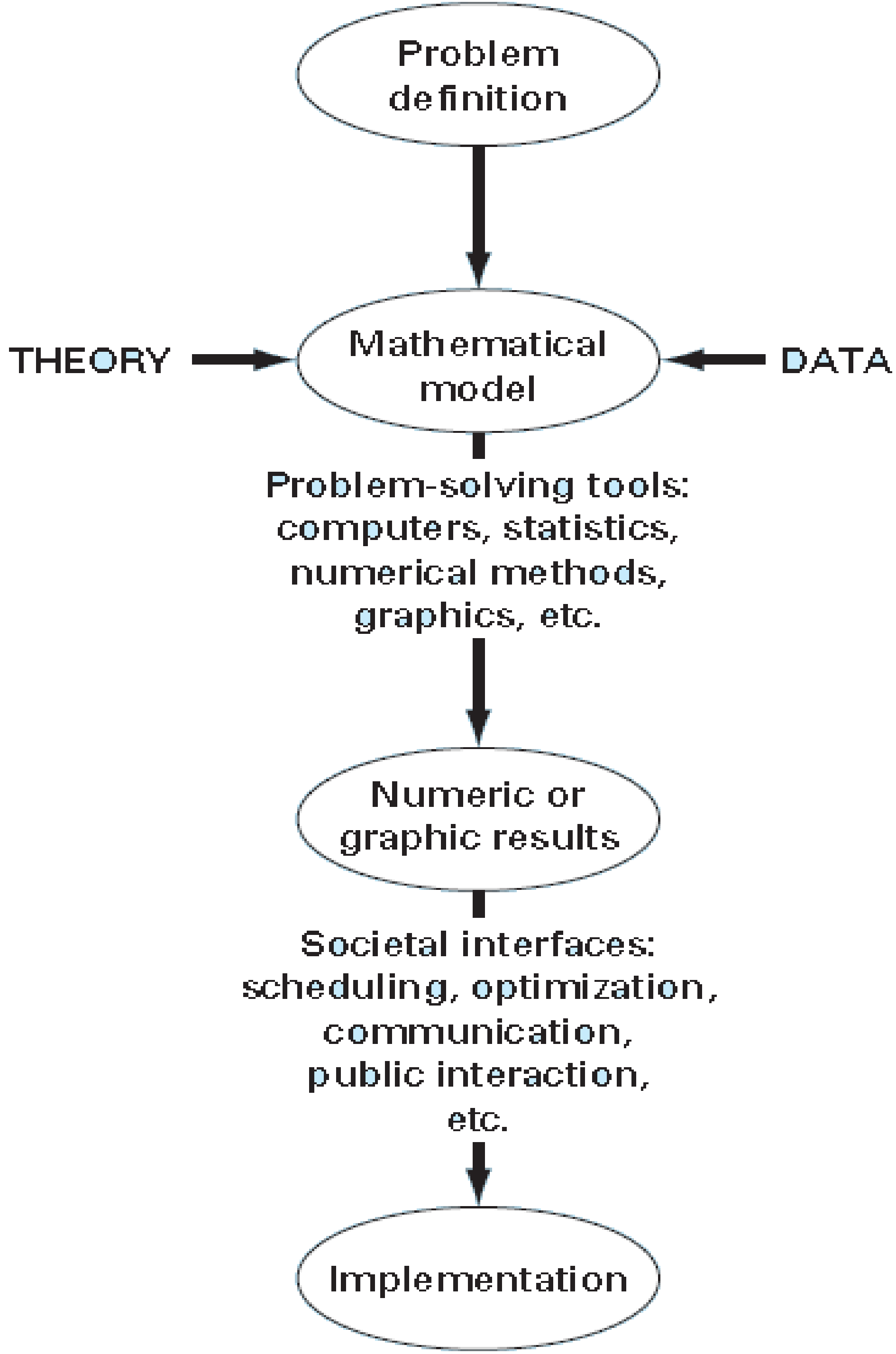


Introduction

Mathematical Modeling and Engineering Problem Solving

The engineering problem-solving process



Problem 1

Mathematical Model

$$D = f\,(I, P, F)$$

D: dependent variable

I: Independent variables

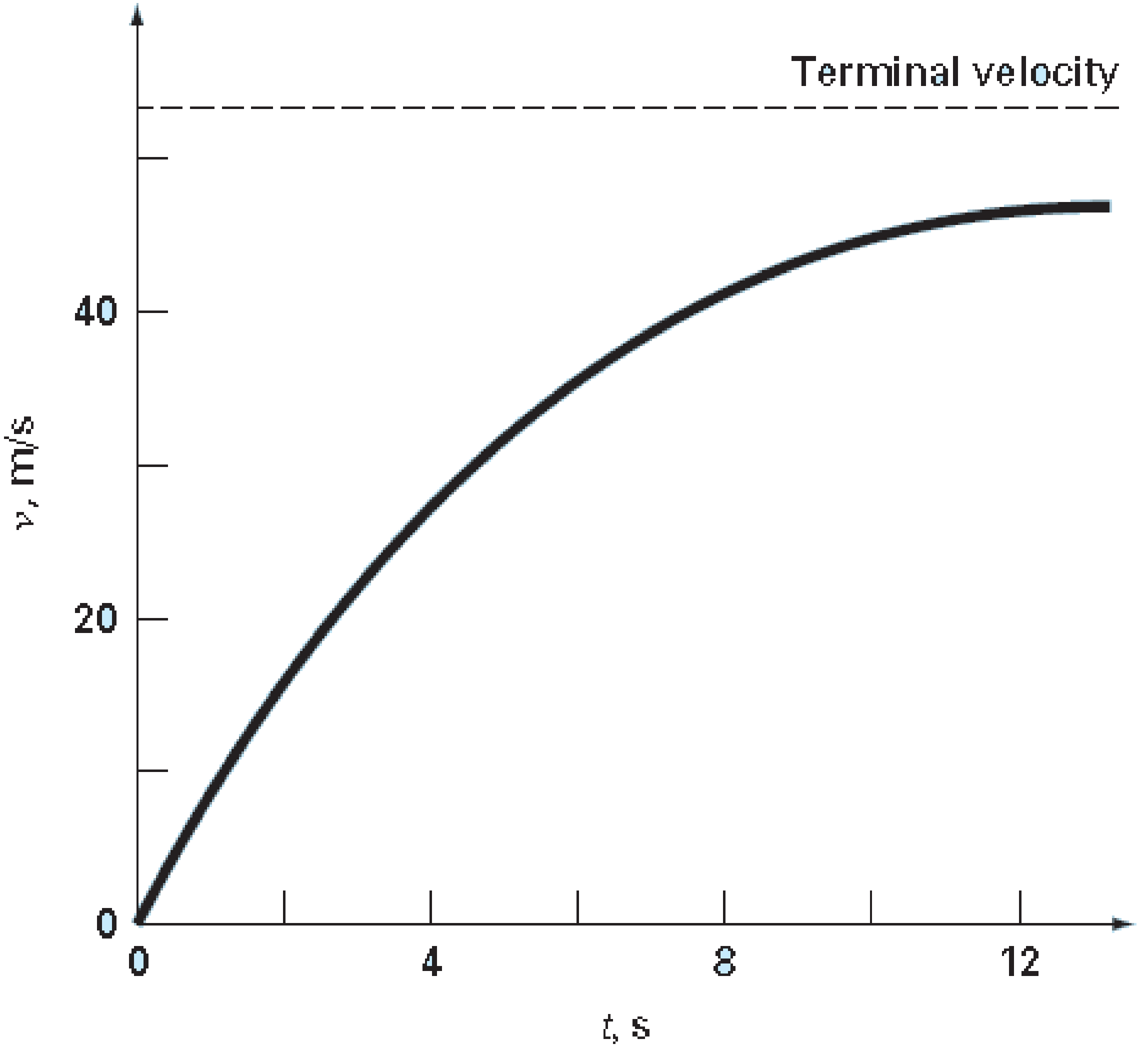
P: Parameters

F: forcing (external) functions

$$a = \frac{F}{m}$$

Example: free falling parachutist

Terminal velocity: the final steady velocity of the parachutist



$$\frac{dv}{dt} = \frac{F}{m}$$

$$F = F_D + F_U$$

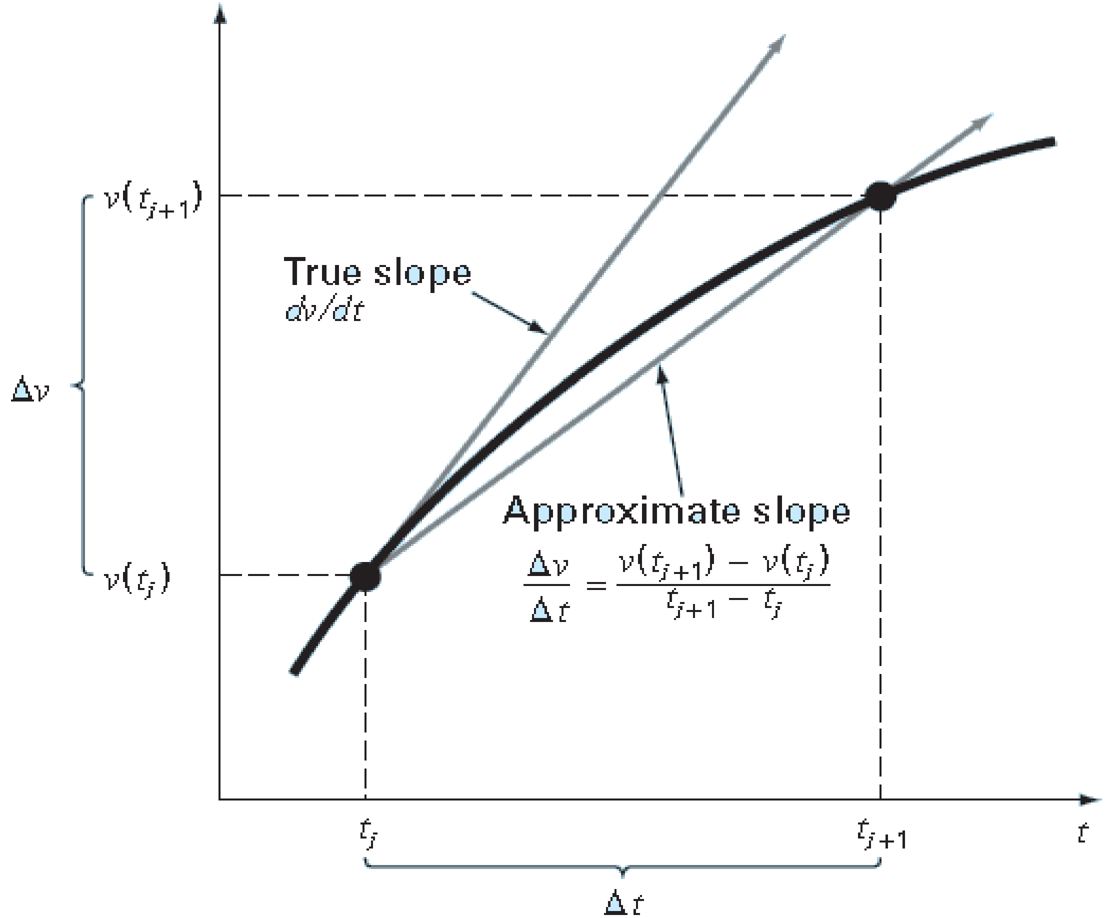
$$F_D = mg \qquad F_U = -cv$$

$$\frac{dv}{dt} = \frac{mg - cv}{m}$$

$$v(t) = \frac{mg}{c}\left(1 - e^{-(c/m)t}\right)$$

(analytical solution)

Numerical Solution for Terminal Velocity



$$m = 68.1\ kg$$

$$c = 12.5\ kg/s$$

$$g = 9.81\ m/s^2$$

$$\frac{dv}{dt} = \frac{mg - cv}{m} = g - \frac{c}{m}v$$

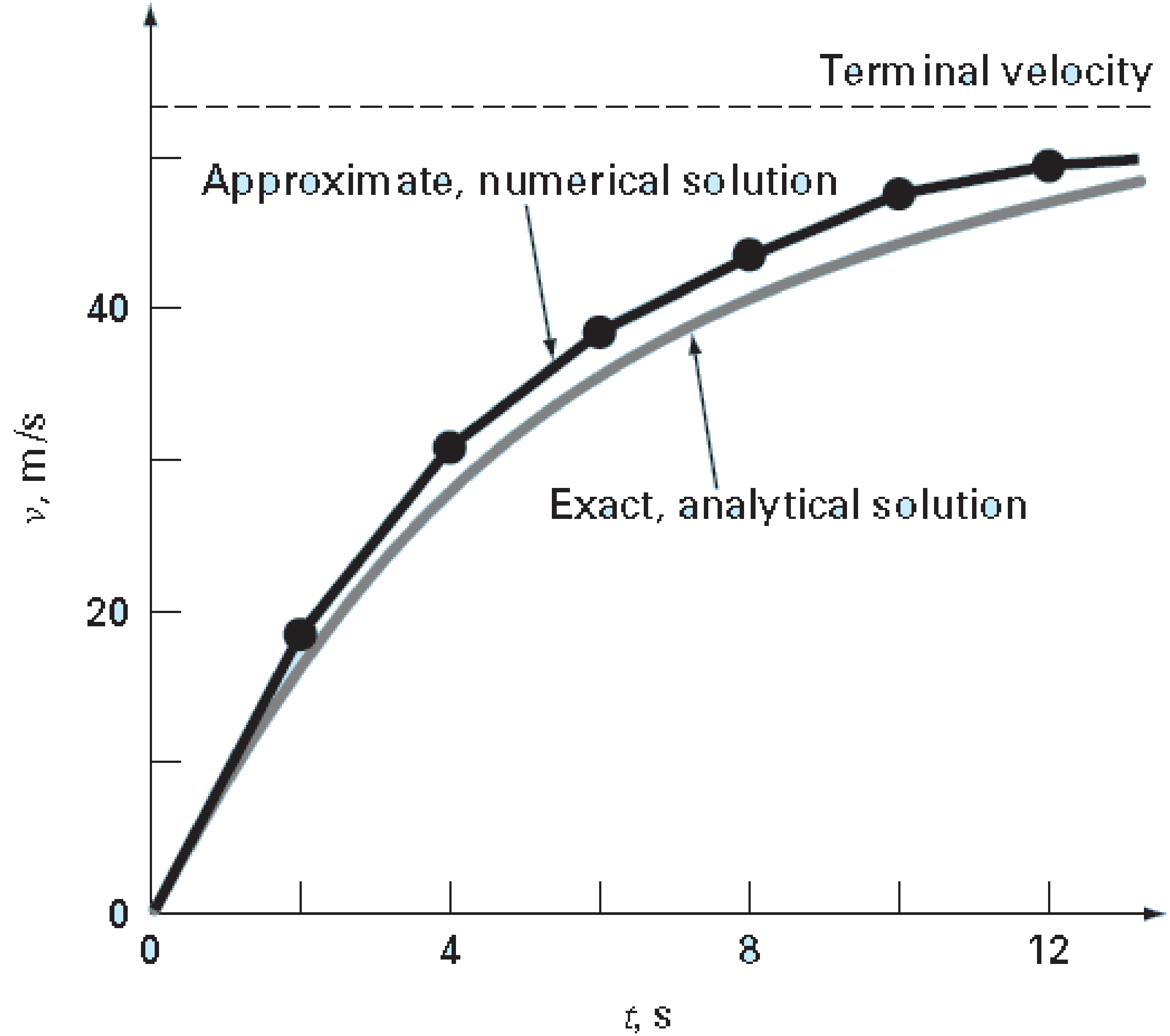
$$\frac{dv}{dt} \cong \frac{\Delta v}{\Delta t} = \frac{v(t_{i+1}) - v(t_i)}{t_{i+1} - t_i}$$

$$\frac{v(t_{i+1}) - v(t_i)}{t_{i+1} - t_i} = g - \frac{c}{m}v(t_i)$$

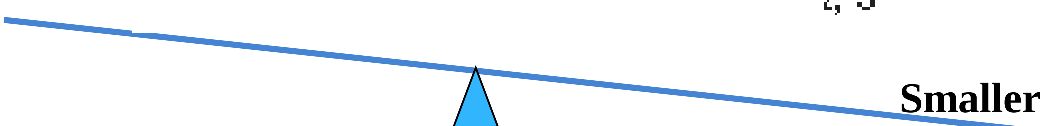
$$v(t_{i+1}) = v(t_i) + [g - \frac{c}{m}v(t_i)](t_{i+1} - t_i)$$

Comparison of numerical and analytical solutions

t, s	v, m/s
0	0.00
2	19.60
4	32.00
6	39.85
8	44.82
10	47.97
12	49.96
∞	53.39



Larger step size → less accurate result



Smaller step size → more steps (longer computing time)