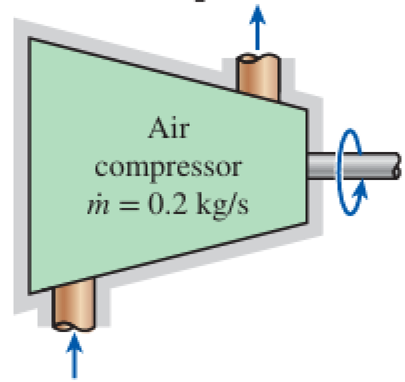
EXAMPLE 7–15 Effect of Efficiency on Compressor Power Input

Air is compressed by an adiabatic compressor from 100 kPa and 12°C to a pressure of 800 kPa at a steady rate of 0.2 kg/s. If the isentropic efficiency of the compressor is 80 percent, determine (*a*) the exit temperature of air and (*b*) the required power input to the compressor.



SOLUTION Air is compressed to a specified pressure at a specified rate. For a given isentropic efficiency, the exit temperature and the power input are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas. **3** The changes in kinetic and potential energies are negligible.

Analysis A sketch of the system and the *T-s* diagram of the process are given in Fig. 7–52.

(*a*) We know only one property (pressure) at the exit state, and we need to know one more to fix the state and thus determine the exit temperature. The property that can be determined with minimal effort in this case is h_{2a} since the isentropic efficiency of the compressor is given. At the compressor inlet,

$$T_1 = 285 \text{ K} \rightarrow h_1 = 285.14 \text{ kJ/kg} \qquad \text{(Table A–17)}$$
$$P_{r1} = 1.1584$$

The enthalpy of the air at the end of the isentropic compression process is determined by using one of the isentropic relations of ideal gases,

$$P_{r2} = P_{r1}\left(\frac{P_2}{P_1}\right) = 1.1584\left(\frac{800 \text{ kPa}}{100 \text{ kPa}}\right) = 9.2672$$

$$P_{r2} = 9.2672 \quad \rightarrow \quad h_{2s} = 517.05 \text{ kJ/kg}$$

Substituting the known quantities into the isentropic efficiency relation, we have

$$\eta_C \cong \frac{h_{2s} - h_1}{h_{2a} - h_1} \quad \rightarrow \quad 0.80 = \frac{(517.05 - 285.14) \text{ kJ/kg}}{(h_{2a} - 285.14) \text{ kJ/kg}}$$

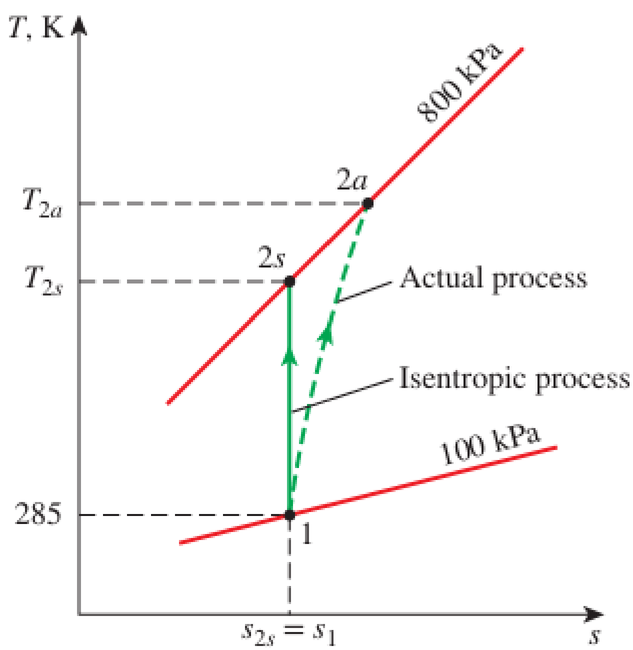
Thus,

$$h_{2a} = 575.03 \text{ kJ/kg} \quad \rightarrow \quad T_{2a} = \mathbf{569.5 \text{ K}}$$

(*b*) The required power input to the compressor is determined from the energy balance for steady-flow devices,

$$\begin{aligned} \dot{E}_{\text{in}} &= \dot{E}_{\text{out}} \\ \dot{m}h_1 + \dot{W}_{a,\text{in}} &= \dot{m}h_{2a} \\ \dot{W}_{a,\text{in}} &= \dot{m}(h_{2a} - h_1) \\ &= (0.2 \text{ kg/s})[(575.03 - 285.14) \text{ kJ/kg}] \\ &= \mathbf{58.0 \text{ kW}} \end{aligned}$$

Discussion Notice that in determining the power input to the compressor, we used h_{2a} instead of h_{2s} since h_{2a} is the actual enthalpy of the air as it exits the compressor. The quantity h_{2s} is a hypothetical enthalpy value that the air would have if the process were isentropic.



EXAMPLE 4–10 Heating of a Gas at Constant Pressure

A piston–cylinder device initially contains air at 150 kPa and 27°C. At this state, the piston is resting on a pair of stops, as shown in Fig. 4–32, and the enclosed volume is 400 L. The mass of the piston is such that a 350-kPa pressure is required to move it. The air is now heated until its volume has doubled. Determine (*a*) the final temperature, (*b*) the work done by the air, and (*c*) the total heat transferred to the air.

SOLUTION Air in a piston–cylinder device with a set of stops is heated until its volume is doubled. The final temperature, work done, and the total heat transfer are to be determined.

Assumptions **1** Air is an ideal gas since it is at a high temperature and low pressure relative to its critical-point values. **2** The system is stationary and thus the kinetic and potential energy changes are zero, $\Delta KE = \Delta PE = 0$ and $\Delta E = \Delta U$. **3** The volume remains constant until the piston starts moving, and the pressure remains constant afterwards. **4** There are no electrical, shaft, or other forms of work involved.

Analysis We take the contents of the cylinder as the *system* (Fig. 4–32). This is a *closed system* since no mass crosses the system boundary during the process. We observe that a piston–cylinder device typically involves a moving boundary and thus boundary work, W_b. Also, the boundary work is done by the system, and heat is transferred to the system.

(*a*) The final temperature can be determined easily by using the ideal-gas relation between states 1 and 3 in the following form:

$$\frac{P_1 V_1}{T_1} = \frac{P_3 V_3}{T_3} \longrightarrow \frac{(150 \text{ kPa})(V_1)}{300 \text{ K}} = \frac{(350 \text{ kPa})(2V_1)}{T_3}$$

$$T_3 = \mathbf{1400 \text{ K}}$$

(*b*) The work done could be determined by integration, but for this case it is much easier to find it from the area under the process curve on a *P*-*V* diagram, shown in Fig. 4–32:

$$A = (V_2 - V_1)P_2 = (0.4 \text{ m}^3)(350 \text{ kPa}) = 140 \text{ m}^3\cdot\text{kPa}$$

$$W_{13} = \mathbf{140\ kJ}$$

The work is done by the system (to raise the piston and to push the atmospheric air out of the way), and thus it is work output.

(*c*) Under the stated assumptions and observations, the energy balance on the system between the initial and final states (process 1–3) can be expressed as

$$\underbrace{E_{in} - E_{out}}_{\substack{\text{Net energy transfer} \\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{system}}_{\substack{\text{Change in internal, kinetic,} \\ \text{potential, etc., energies}}}$$

$$Q_{in} - W_{b,out} = \Delta U = m(u_3 - u_1)$$

The mass of the system can be determined from the ideal-gas relation:

$$m = \frac{P_1 V_1}{RT_1} = \frac{(150\ \text{kPa})(0.4\ \text{m}^3)}{(0.287\ \text{kPa·m}^3/\text{kg·K})(300\ \text{K})} = 0.697\ \text{kg}$$

The internal energies are determined from the air table (Table A–17) to be

$$u_1 = u_{@\ 300\ K} = 214.07\ \text{kJ/kg}$$

$$u_3 = u_{@\ 1400\ K} = 1113.52\ \text{kJ/kg}$$

Thus,

$$Q_{in} - 140\ \text{kJ} = (0.697\ \text{kg})[(1113.52 - 214.07)\ \text{kJ/kg}]$$

$$Q_{in} = \mathbf{767\ kJ}$$

Discussion The positive sign verifies that heat is transferred to the system.

7–131 Hot combustion gases enter the nozzle of a turbojet engine at 260 kPa, 747°C, and 80 m/s, and they exit at a pressure of 85 kPa. Assuming an isentropic efficiency of 92 percent and treating the combustion gases as air, determine (*a*) the exit velocity and (*b*) the exit temperature. *Answers:* (*a*) 728 m/s, (*b*) 786 K

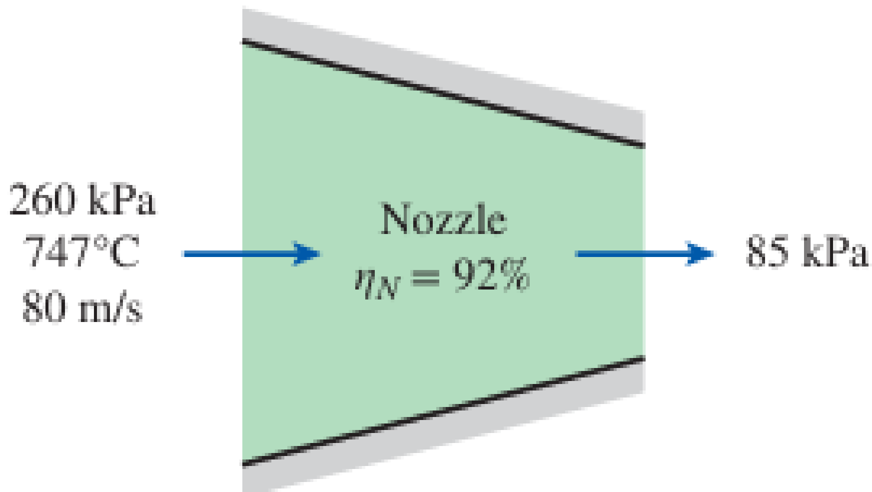
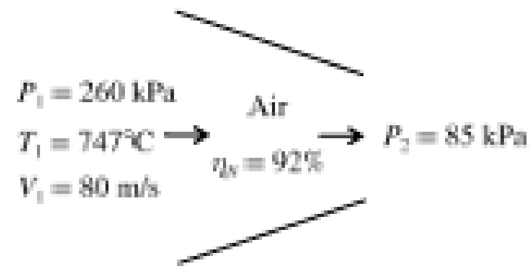


FIGURE P7–131

7-131 Hot combustion gases are accelerated in a 92% efficient adiabatic nozzle from low velocity to a specified velocity. The exit velocity and the exit temperature are to be determined.

Assumptions **1** This is a steady-flow process since there is no change with time. **2** Potential energy changes are negligible. **3** The device is adiabatic and thus heat transfer is negligible. **4** Combustion gases can be treated as air that is an ideal gas with variable specific heats.

Analysis From the air table (Table A-17),



$$T_1 = 1020 \text{ K} \longrightarrow h_1 = 1068.89 \text{ kJ/kg}, \quad P_{r_1} = 123.4$$

From the isentropic relation,

$$P_{r_2} = \left(\frac{P_2}{P_1}\right) P_{r_1} = \left(\frac{85 \text{ kPa}}{260 \text{ kPa}}\right)(123.4) = 40.34 \longrightarrow h_{2s} = 783.92 \text{ kJ/kg}$$

There is only one inlet and one exit, and thus $\dot{m}_1 = \dot{m}_2 = \dot{m}$. We take the nozzle as the system, which is a control volume since mass crosses the boundary. The energy balance for this steady-flow system for the isentropic process can be expressed as

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\substack{\text{Rate of net energy transfer} \\ \text{by heat, work, and mass}}} = \underbrace{\Delta \dot{E}_{system}^{\nearrow 0 \text{ (steady)}}}_{\substack{\text{Rate of change in internal, kinetic,} \\ \text{potential, etc. energies}}} = 0$$

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{m}(h_1 + V_1^2/2) = \dot{m}(h_{2s} + V_{2s}^2/2) \quad (\text{since } \dot{W} = \dot{Q} \cong \Delta \text{pe} \cong 0)$$

$$h_{2s} = h_1 - \frac{V_{2s}^2 - V_1^2}{2}$$

Then the isentropic exit velocity becomes

$$V_{2s} = \sqrt{V_1^2 + 2(h_1 - h_{2s})} = \sqrt{(80 \text{ m/s})^2 + 2(1068.89 - 783.92)\text{kJ/kg}\left(\frac{1000 \text{ m}^2/\text{s}^2}{1 \text{ kJ/kg}}\right)} = 759.2 \text{ m/s}$$

Therefore,

$$V_{2a} = \sqrt{\eta_N} V_{2s} = \sqrt{0.92}\,(759.2 \text{ m/s}) = 728.2 \text{ m/s} \cong \mathbf{728 \text{ m/s}}$$

The exit temperature of air is determined from the steady-flow energy equation,

$$h_{2a} = 1068.89 \text{ kJ/kg} - \frac{(728.2 \text{ m/s})^2 - (80 \text{ m/s})^2}{2}\left(\frac{1 \text{ kJ/kg}}{1000 \text{ m}^2/\text{s}^2}\right) = 806.95 \text{ kJ/kg}$$

From the air table we read

$$T_{2a} = \mathbf{786 \text{ K}}$$

5–183 A piston–cylinder device initially contains 1.2 kg of air at 700 kPa and 200°C. At this state, the piston is touching on a pair of stops. The mass of the piston is such that 600-kPa pressure is required to move it. A valve at the bottom of the tank is opened, and air is withdrawn from the cylinder. The valve is closed when the volume of the cylinder decreases to 80 percent of the initial volume. If it is estimated that 40 kJ of heat is lost from the cylinder, determine (*a*) the final temperature of the air in the cylinder, (*b*) the amount of mass that has escaped from the cylinder, and (*c*) the work done. Use constant specific heats at the average temperature.

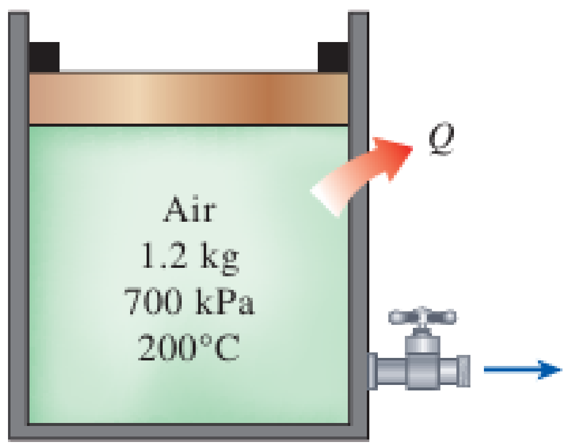


FIGURE P5–183

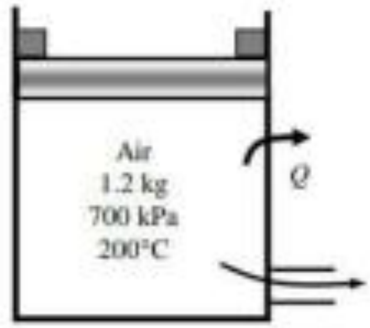
5-185 Air is allowed to leave a piston-cylinder device with a pair of stops. Heat is lost from the cylinder. The amount of mass that has escaped and the work done are to be determined.

Assumptions **1** This is an unsteady process since the conditions within the device are changing during the process, but it can be analyzed as a uniform-flow process since the state of fluid leaving the device is assumed to be constant. **2** Kinetic and potential energies are negligible. 3 Air is an ideal gas with constant specific heats at the average temperature.

Properties The properties of air are $R = 0.287$ kPa.m^3/kg.K (Table A-1), $c_v = 0.733$ kJ/kg.K, $c_p = 1.020$ kJ/kg.K at the anticipated average temperature of 450 K (Table A-2b).

Analysis We take the tank as the system, which is a control volume since mass crosses the boundary. Noting that the microscopic energies of flowing and nonflowing fluids are represented by enthalpy h and internal energy u, respectively, the mass and energy balances for this uniform-flow system can be expressed as

Mass balance: $m_{in} - m_{out} = \Delta m_{system} \rightarrow m_e = m_1 - m_2$



Energy balance:

$$\underbrace{E_{in} - E_{out}}_{\substack{\text{Net energy transfer}\\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{system}}_{\substack{\text{Change in internal, kinetic,}\\ \text{potential, etc. energies}}}$$

$$W_{b,in} - Q_{out} - m_e h_e = m_2 u_2 - m_1 u_1 \quad \text{(since ke} \cong \text{pe} \cong 0)$$

or $$W_{b,in} - Q_{out} - m_e C_p T_e = m_2 c_v T_2 - m_1 c_v T_1$$

The temperature of the air withdrawn from the cylinder is assumed to be the average of initial and final temperatures of the air in the cylinder. That is,

$$T_e = (1/2)(T_1 + T_2) = (1/2)(473 + T_2)$$

The volumes and the masses at the initial and final states and the mass that has escaped from the cylinder are given by

$$V_1 = \frac{m_1 R T_1}{P_1} = \frac{(1.2\,\text{kg})(0.287\,\text{kPa.m}^3/\text{kg.K})(200+273\,\text{K})}{(700\,\text{kPa})} = 0.2327\,\text{m}^3$$

$$V_2 = 0.80 V_1 = (0.80)(0.2327) = 0.1862\,\text{m}^3$$

$$m_2 = \frac{P_2 V_2}{R T_2} = \frac{(600\,\text{kPa})(0.1862\,\text{m}^3)}{(0.287\,\text{kPa.m}^3/\text{kg.K})T_2} = \frac{389.18}{T_2}\,\text{kg}$$

$$m_e = m_1 - m_2 = \left(1.2 - \frac{389.18}{T_2}\right)\text{kg}$$

Noting that the pressure remains constant after the piston starts moving, the boundary work is determined from

$$W_{b,in} = P_2(V_1 - V_2) = (600\,\text{kPa})(0.2327 - 0.1862)\text{m}^3 = \mathbf{27.9\,kJ}$$

Substituting,

$$27.9\,\text{kJ} - 40\,\text{kJ} - \left(1.2 - \frac{389.18}{T_2}\right)(1.020\,\text{kJ/kg.K})(1/2)(473 + T_2)$$
$$= \left(\frac{389.18}{T_2}\right)(0.733\,\text{kJ/kg.K})T_2 - (1.2\,\text{kg})(0.733\,\text{kJ/kg.K})(473\,\text{K})$$

The final temperature may be obtained from this equation by a trial-error approach or using EES to be

$$T_2 = \mathbf{415.0\,K}$$

Then, the amount of mass that has escaped becomes

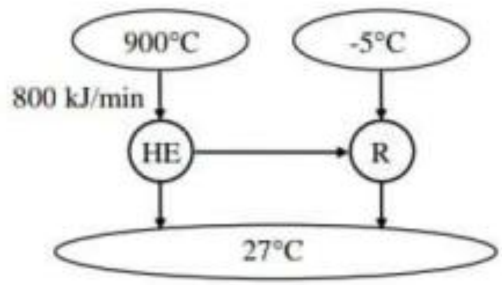
$$m_e = 1.2 - \frac{389.18}{415.0\,\text{K}} = \mathbf{0.262\,kg}$$

6–112 A Carnot heat engine receives heat from a reservoir at 900°C at a rate of 800 kJ/min and rejects the waste heat to the ambient air at 27°C. The entire work output of the heat engine is used to drive a refrigerator that removes heat from the refrigerated space at –5°C and transfers it to the same ambient air at 27°C. Determine (*a*) the maximum rate of heat removal from the refrigerated space and (*b*) the total rate of heat rejection to the ambient air. *Answers:* (*a*) 4982 kJ/min, (*b*) 5782 kJ/min

6-105 A Carnot heat engine is used to drive a Carnot refrigerator. The maximum rate of heat removal from the refrigerated space and the total rate of heat rejection to the ambient air are to be determined.

Assumptions The heat engine and the refrigerator operate steadily.

Analysis (a) The highest thermal efficiency a heat engine operating between two specified temperature limits can have is the Carnot efficiency, which is determined from



$$\eta_{\text{th,max}} = \eta_{\text{th,C}} = 1 - \frac{T_L}{T_H} = 1 - \frac{300\text{ K}}{1173\text{ K}} = 0.744$$

Then the maximum power output of this heat engine is determined from the definition of thermal efficiency to be

$$\dot{W}_{\text{net,out}} = \eta_{\text{th}}\dot{Q}_H = (0.744)(800\text{ kJ/min}) = 595.2\text{ kJ/min}$$

which is also the power input to the refrigerator, $\dot{W}_{\text{net,in}}$.

The rate of heat removal from the refrigerated space will be a maximum if a Carnot refrigerator is used. The COP of the Carnot refrigerator is

$$\text{COP}_{\text{R,rev}} = \frac{1}{(T_H/T_L)-1} = \frac{1}{(27+273\text{ K})/(-5+273\text{ K})-1} = 8.37$$

Then the rate of heat removal from the refrigerated space becomes

$$\dot{Q}_{L,\text{R}} = (\text{COP}_{\text{R,rev}})(\dot{W}_{\text{net,in}}) = (8.37)(595.2\text{ kJ/min}) = \mathbf{4982\text{ kJ/min}}$$

(b) The total rate of heat rejection to the ambient air is the sum of the heat rejected by the heat engine ($\dot{Q}_{L,\text{HE}}$) and the heat discarded by the refrigerator ($\dot{Q}_{H,\text{R}}$),

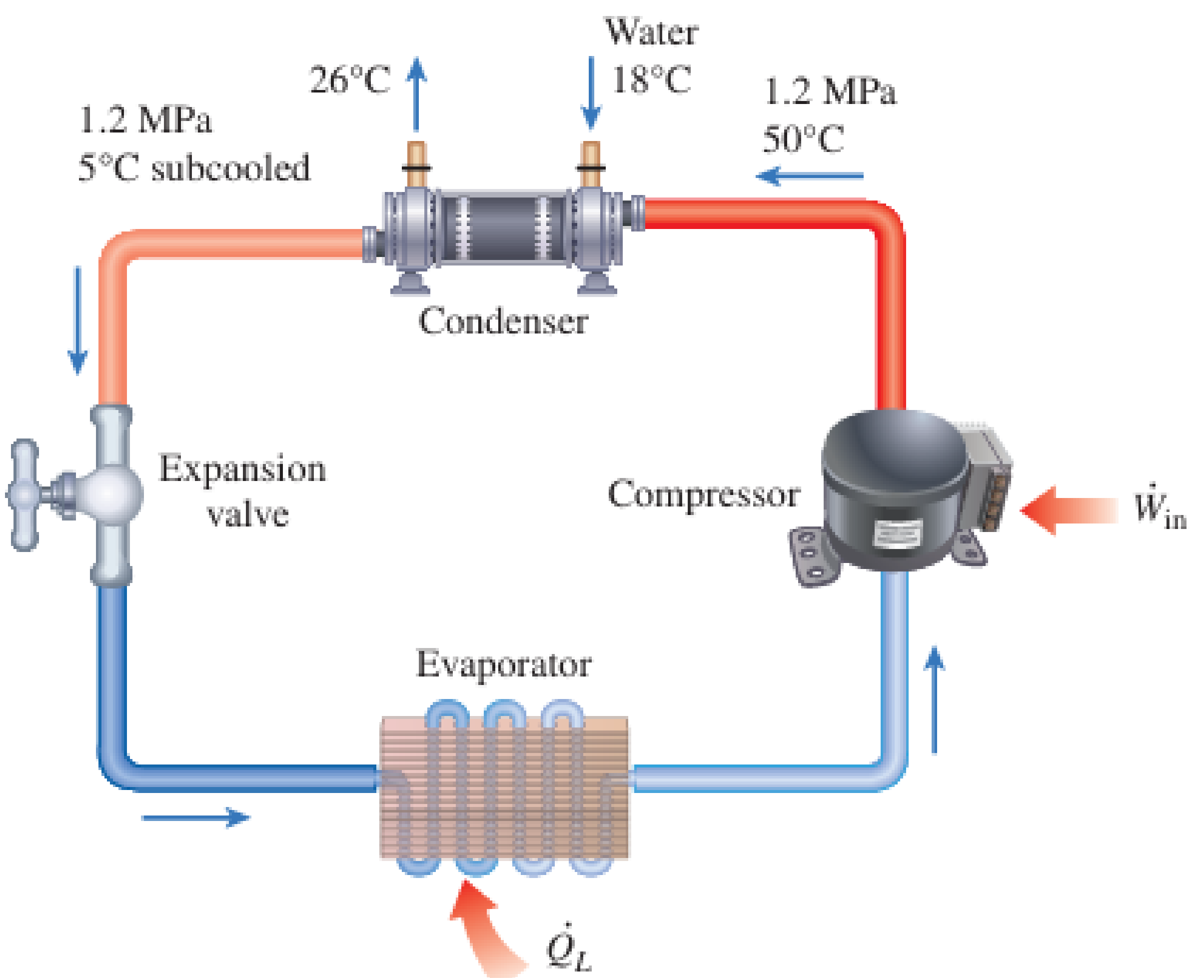
$$\dot{Q}_{L,\text{HE}} = \dot{Q}_{H,\text{HE}} - \dot{W}_{\text{net,out}} = 800 - 595.2 = 204.8\text{ kJ/min}$$

$$\dot{Q}_{H,\text{R}} = \dot{Q}_{L,\text{R}} + \dot{W}_{\text{net,in}} = 4982 + 595.2 = 5577.2\text{ kJ/min}$$

and

$$\dot{Q}_{\text{ambient}} = \dot{Q}_{L,\text{HE}} + \dot{Q}_{H,\text{R}} = 204.8 + 5577.2 = \mathbf{5782\text{ kJ/min}}$$

6–107 A commercial refrigerator with refrigerant-134a as the working fluid is used to keep the refrigerated space at –35°C by rejecting waste heat to cooling water that enters the condenser at 18°C at a rate of 0.25 kg/s and leaves at 26°C. The refrigerant enters the condenser at 1.2 MPa and 50°C and leaves at the same pressure subcooled by 5°C. If the compressor consumes 3.3 kW of power, determine (*a*) the mass flow rate of the refrigerant, (*b*) the refrigeration load, (*c*) the COP, and (*d*) the minimum power input to the compressor for the same refrigeration load.



6-101 A commercial refrigerator with R-134a as the working fluid is considered. The condenser inlet and exit states are specified. The mass flow rate of the refrigerant, the refrigeration load, the COP, and the minimum power input to the compressor are to be determined.

Assumptions **1** The refrigerator operates steadily. **2** The kinetic and potential energy changes are zero.

Properties The properties of R-134a and water are (Steam and R-134a tables)

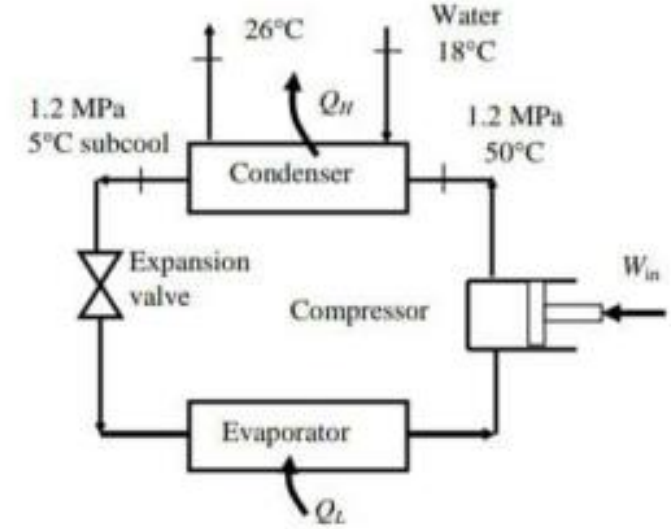
$$\left.\begin{array}{l} P_1 = 1.2\text{ MPa} \\ T_1 = 50^\circ\text{C} \end{array}\right\} h_1 = 278.28\text{ kJ/kg}$$

$$T_2 = T_{\text{sat@1.2MPa}} + \Delta T_{\text{subcool}} = 46.3 - 5 = 41.3^\circ\text{C}$$

$$\left.\begin{array}{l} P_2 = 1.2\text{ MPa} \\ T_2 = 41.3^\circ\text{C} \end{array}\right\} h_2 = 110.19\text{ kJ/kg}$$

$$\left.\begin{array}{l} T_{w,1} = 18^\circ\text{C} \\ x_{w,1} = 0 \end{array}\right\} h_{w,1} = 75.54\text{ kJ/kg}$$

$$\left.\begin{array}{l} T_{w,2} = 26^\circ\text{C} \\ x_{w,2} = 0 \end{array}\right\} h_{w,2} = 109.01\text{ kJ/kg}$$



Analysis (*a*) The rate of heat transferred to the water is the energy change of the water from inlet to exit

$$\dot{Q}_H = \dot{m}_w (h_{w,2} - h_{w,1}) = (0.25\text{ kg/s})(109.01 - 75.54)\text{ kJ/kg} = 8.367\text{ kW}$$

The energy decrease of the refrigerant is equal to the energy increase of the water in the condenser. That is,

$$\dot{Q}_H = \dot{m}_R (h_1 - h_2) \longrightarrow \dot{m}_R = \frac{\dot{Q}_H}{h_1 - h_2} = \frac{8.367\text{ kW}}{(278.28 - 110.19)\text{ kJ/kg}} = \mathbf{0.0498 kg/s}$$

(*b*) The refrigeration load is

$$\dot{Q}_L = \dot{Q}_H - \dot{W}_{in} = 8.37 - 3.30 = \mathbf{5.07 kW}$$

(*c*) The COP of the refrigerator is determined from its definition,

$$\text{COP} = \frac{\dot{Q}_L}{\dot{W}_{in}} = \frac{5.07\text{ kW}}{3.3\text{ kW}} = \mathbf{1.54}$$

(*d*) The COP of a reversible refrigerator operating between the same temperature limits is

$$\text{COP}_{\max} = \frac{1}{T_H / T_L - 1} = \frac{1}{(18 + 273)/(-35 + 273) - 1} = 4.49$$

Then, the minimum power input to the compressor for the same refrigeration load would be

$$\dot{W}_{\text{in,min}} = \frac{\dot{Q}_L}{\text{COP}_{\max}} = \frac{5.07\text{ kW}}{4.49} = \mathbf{1.13 kW}$$