Ch 12

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Ch12

Kinematics of Particles

المسافة + سرعة + تسارع

a v S

* $v = \dfrac{ds}{dt}$

* $a = \dfrac{dv}{dt} \rightarrow a = \dfrac{d^2s}{dt^2}$

* $a\,ds = v\,dv$

مسافة ← اشتقاق ← سرعة ← اشتقاق ← تسارع

تسارع ← تكامل ← سرعة ← تكامل ← مسافة

التسارع ثابتة

Constant Acceleration

$(a = g)$ السقوط الحر $= -9.81\ \text{m/s}$

$$v = v_0 + at \longrightarrow v = v_0 - gt$$

$$\Delta S = v_0 t + \frac{1}{2} a t^2 \longrightarrow \Delta y = v_0 t - \frac{1}{2} g t^2$$

$$v^2 = v_0^2 + 2a\,\Delta S \longrightarrow v^2 = v_0^2 - 2g\,\Delta y$$

مثل سيارة تمشي بتسارع ثابتة
$a = +ve$

أو مثل الحجر يمشي بتسارع ثابتة
$a = +ve$

أو مثل سفينه تسير بتباطؤ ثابتة
$a = -ve$

مثل كره أو الحجر
يسقط للاسفل
أو يرمى للاعلى .
دون ارتداد عند دوران .

Ex1 A particle moves on a straight Line

حسب العلاقة

according to

$$S = 2t^3 - 24t + 6$$ (امتحان [illegible])

Find :

① the time required for the particle to reach a velocity of 72 m/s .

أوجد الزمن الذي يحتاجه الجسم للوصول لسرعة ٧٢ م/ث .

$$v = \frac{ds}{dt} = 6t^2 - 24$$

$$72 = 6t^2 - 24 \Rightarrow 6t^2 = 96 \Rightarrow t^2 = 16$$

$$\underline{t = 4 \text{ sec}}$$

② the acceleration when $v = 30$ m/s .

أوجد تسارع هذا الجسم عندما تصبح سرعته تساوي ٣٠ م/ث

$$a = \frac{dv}{dt} = 12t$$

$$v = 6t^2 - 24$$

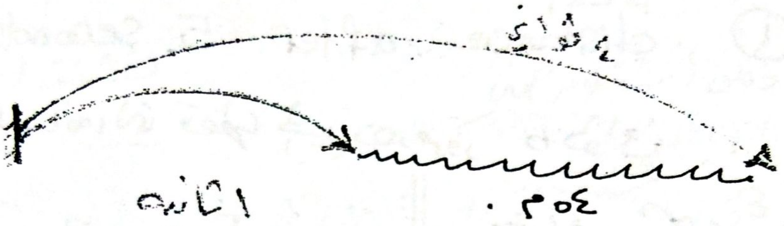
$$30 = 6t^2 - 24 \Rightarrow 6t^2 = 54 \Rightarrow t^2 = 9 \Rightarrow t = 3 \text{ sec.}$$

$$\therefore a = 12(3) = 36 \text{ m/s}^2$$

③

③ the displacement during $t = 1$ sec and $t = 4$ sec

أوجد المسافة التي قطعها الجسم ما بين الثانية الأولى والثانية الرابعة.



ثانية واحدة

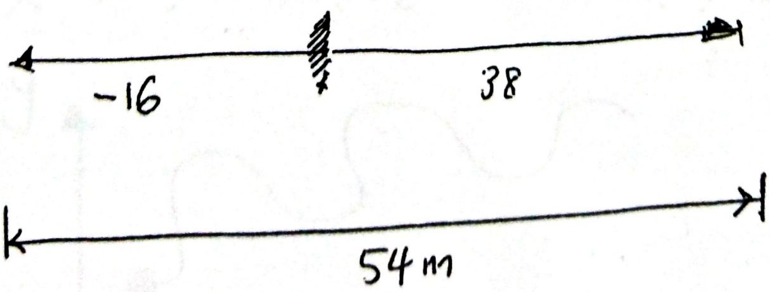
$$\text{distance}(1) = 2(1)^3 - 24(1) + 6 = -16$$

٤ ثواني

$$\text{distance}(4) = 2(4)^3 - 24(4) + 6 = 38$$

$$\text{distance} = 38 - -16$$

$$= 38 + 16 = 54\,m$$



ex) the velocity of the particle سرعة

$$v = 6t^2 - 8t + 2$$

Find ① distance after 5 seconds.

المسافة التي تقطعها الجسم بعد مرور 5 ثواني.

$\int_0^s ds = \int_0^t v\,dt$

تكامل

$$S = \int (6t^2 - 8t + 2)\,dt$$

$$= 6\frac{t^3}{3} - 8\frac{t^2}{2} + 2t = 2t^3 - 4t^2 + 2t$$

$$= 2(5)^3 - 4(5)^2 + 2(5) = \dots 160 \dots (m)$$

② the acceleration after 2 seconds.

أوجد تسارع الجسم بعد مرور 2 ثانية.

نشتق

$$a = \frac{dv}{dt} = 12t - 8$$

$$= 12(2) - 8 = 16\ m/s^2$$

5. دائماً الاشتقاق بالنسبة للزمن

اذا لم يظهر الزمن في الاقتران
نشتقه ولكن اشتقاق ضمني

اذا ظهر الزمن في الاقتران
نشتق اشتقاق عادي

$$S = 5t^3 - 2t^2 + 1$$

$$v = 15t^2 - 4t$$

$$a = 30t - 4$$

الازاحة $y = 2x^3 + 4x^2 - 2$

السرعة $\dot{y} = 6x^2\dot{x} + 8x\dot{x}$

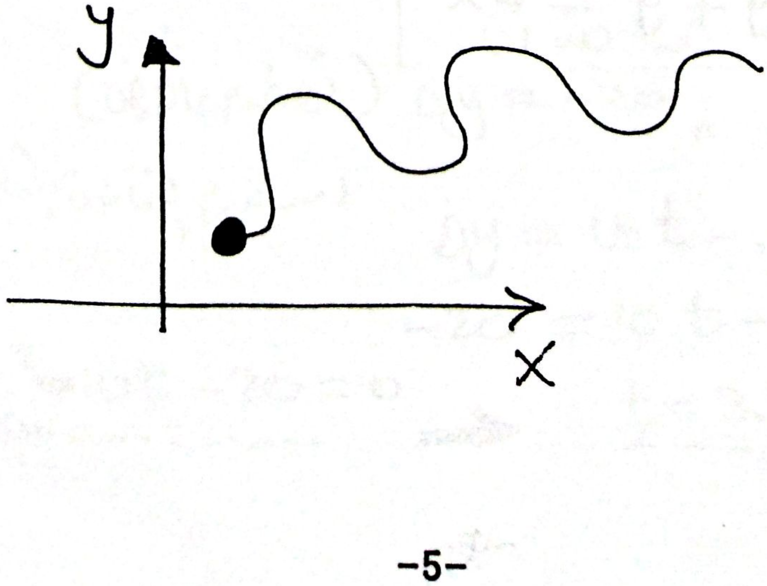
التسارع $\ddot{y} = (6x^2\ddot{x} + \dot{x}\,12x\dot{x}) + (8x\ddot{x} + \dot{x}\,8\dot{x})$

$$\ddot{y} = 6x^2\ddot{x} + 12x\dot{x}^2 + 8x\ddot{x} + 8\dot{x}^2$$

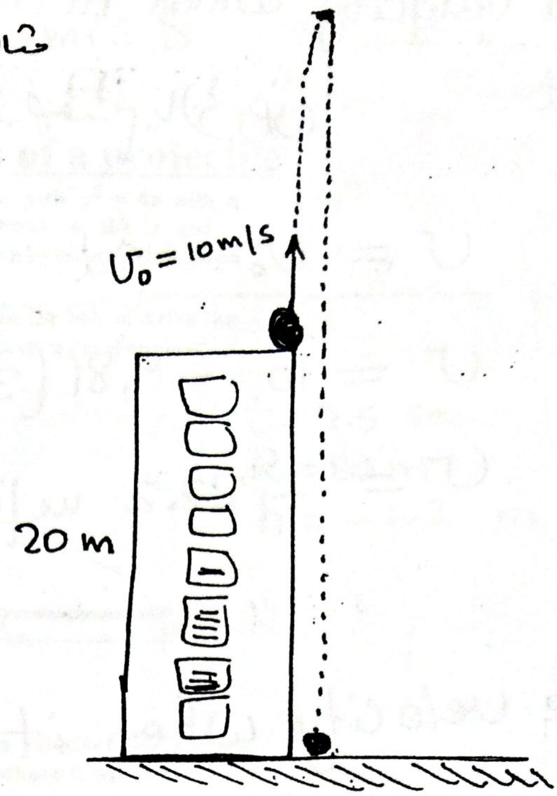
$x \rightarrow$ المسافة مع الـ x

$\dot{x} \rightarrow$ السرعة باتجاه الـ x

$\ddot{x} \rightarrow$ التسارع باتجاه الـ x



Ex) مثال على السقوط الحر.



① the highest elevation
مقدار أقصى ارتفاع.

∴ يجب أن تكون السرعة
تساوي صفراً عند أقصى ارتفاع
$v=0$

القانون الثالث

$$v^2 = v_0^2 - 2g\,\Delta y$$
$$0 = (10)^2 - 2(9.81)\,\Delta y \Rightarrow \Delta y = \frac{100}{2(9.81)}$$
$$\Delta y = 5.1\ m$$

② the time when the ball hits the groud?
أوجد الزمن اللازم للكرة لكي تصطدم بالأرض.

هذا يعني أن
∴ (تصطدم بالأرض) $\Delta y = -20$

استخدم القانون الثاني
$$\Delta y = v_0 t - \frac{1}{2} g t^2$$
$$-20 = 10\,t - \frac{1}{2}(9.81)\,t^2$$

$4.905t^2 - 10t - 20 = 0 \Longrightarrow t = 3.28\ sec$

عندما تصطدم الأرض

(3) the velocity when hits the grounds

سرعة الاصطدام بالأرض

$$v = v_0 - gt$$

$$v = 10 - 9.81\,(3.28)$$

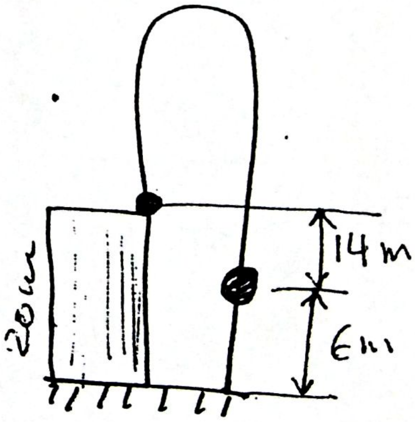
$$v = -22.2 \ m/s$$

(4) the velocity when it is at 6 m above the ground.

سرعة الجسم عندما كان على ارتفاع ٦م من سطح الأرض

هنا يعني أن الجسم :

$$\Delta y = -14$$



القانون الثالث

$$v^2 = v_0^2 - 2g\Delta y$$

$$v^2 = 100 - 2(9.81)(-14)$$

$$v^2 = 374.7 \Rightarrow \boxed{v = -19.4 \ m/s}$$

سالب لأن الكرة ساقطة للأسفل

EX) $\vec{v} = (16t^2)\hat{i} + (4t^3)\hat{j} + (5t+2)\hat{k}$

Find ① $\vec{a}(2)$? Find the acceleration when $t = 2$ sec.

② $\vec{r}(2)$? Find the displacement (distance) when $t = 2$ sec.

الحل

① $$\vec{a} = \frac{d\vec{v}}{dt}$$

$$= (32t)\hat{i} + (12t^2)\hat{j} + (5)\hat{k}$$

at $t = 2$ sec

$$\vec{a} \Rightarrow = \boxed{64\,\hat{i} + 48\,\hat{j} + 5\,\hat{k}}$$

المقدار / magnitude

مقدار التسارع بعد ثانيتين $|\vec{a}| = \sqrt{(64)^2 + (48)^2 + (5)^2} = 80.2\ m/s^2$

② $$\vec{r} = \int \vec{v}\, dt$$

$$= \left(\frac{16}{3}t^3\right)\hat{i} + \left(\frac{4}{4}t^4\right)\hat{j} + \left(\frac{5}{2}t^2 + 2t\right)\hat{k}$$

at $t = 2$ sec.

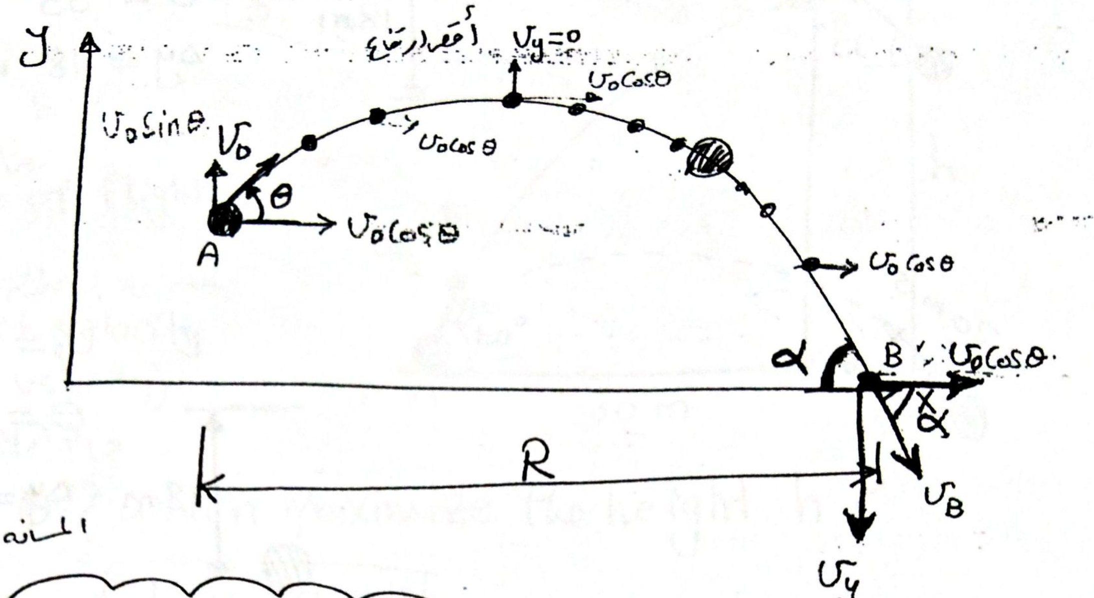
$$\vec{r} = \left[\frac{16}{3}(2)^3\right]\hat{i} + \left[(2)^4\right]\hat{j} + \left[\frac{5}{2}(2)^2 + 2(2)\right]$$

$$= \boxed{\frac{128}{3}\,\hat{i} + 16\,\hat{j} + 14\,\hat{k}}$$

مقدار الإزاحة

$|\vec{r}| = \sqrt{\left(\frac{128}{3}\right)^2 + (16)^2 + (14)^2} =$

Projectile Motion المقذوفات



المسافة = السرعة x الزمن

$$R = (V_0 \cos\theta)\, t$$

$$\tan\alpha = \frac{V_y}{V_0 \cos\theta}$$

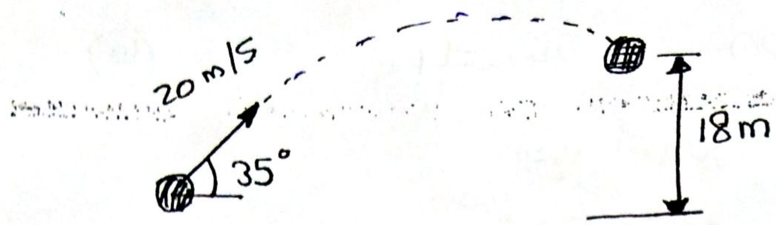
$$V_B = \sqrt{V_y^2 + (V_0 \cos\theta)^2}$$

$$V_y = V_0 \sin\theta - g t$$

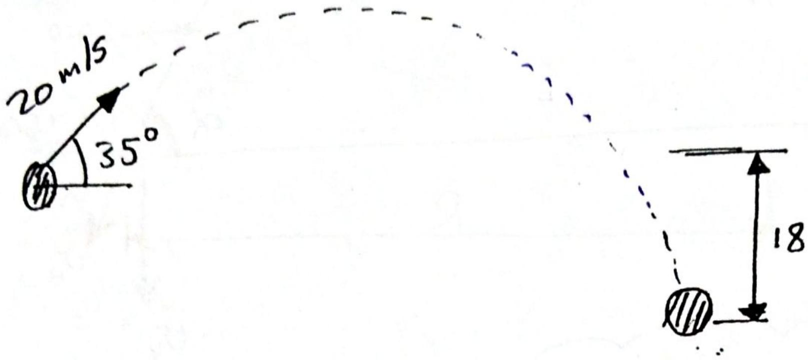
$$\Delta y = (V_0 \sin\theta)\, t - \frac{1}{2} g t^2$$

$$V_y^2 = (V_0 \sin\theta)^2 - 2 g \Delta y$$

V_0 دائماً موجب

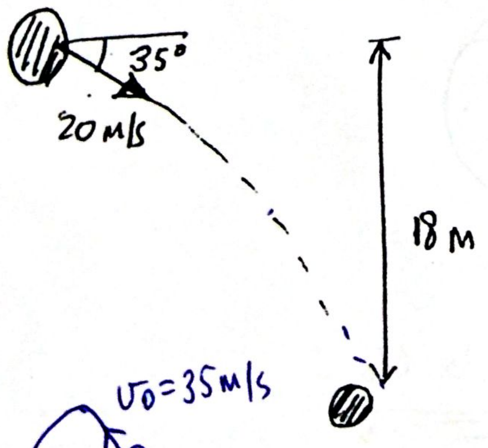


$v_0 = 20$ m/s
$\theta = 35°$
$\Delta y = 18$ m

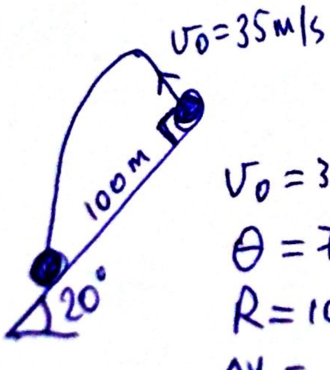


$v_0 = 20$ m/s
$\theta = 35°$
$\Delta y = -18$ m

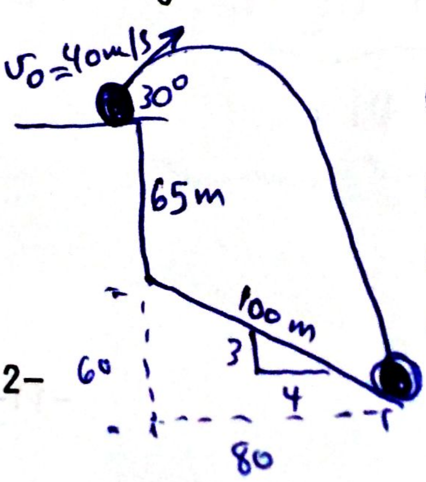
$v_0 = 20$ m/s
$\theta = 0$
$\Delta y = -18$ m



$v_0 = 20$ m/s
$\theta = -35°$
$\Delta y = -18$ m



$v_0 = 35$ m/s
$\theta = 70°$
$R = 100 \cos 20°$
$\Delta y = -100 \sin 20°$



$v_0 = 40$ m/s
$\theta = 30°$
$R = 80$ m
$\Delta y = -125$ m

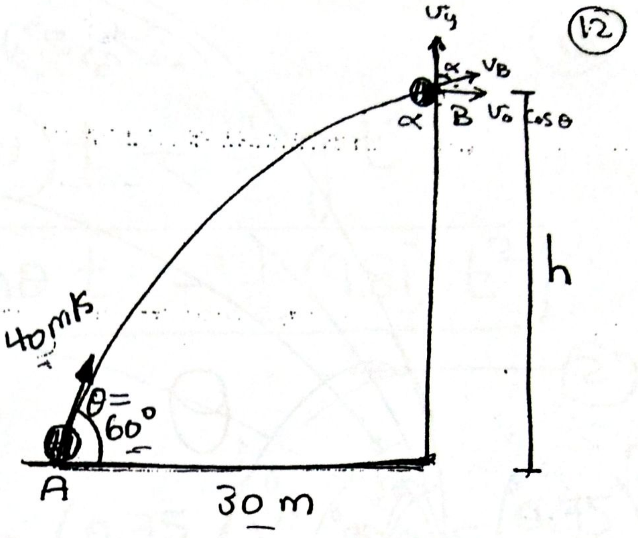
EX

مثال
Lo. 12

① Find h ?

زمن التحليق
② time of flight ?

سرعة الاصطدام بالجدار
③ Impact velocity
(Hit velocity)
وزاوية



④ Find θ ? which maximize the height h ?

الحل

$$\boxed{\begin{array}{l} v_0 = 40 \text{ m/s} \\ \theta = 60° \\ R = 30 \end{array}}$$

* $R = (v_0 \cos\theta)\, t$ time of flight زمن التحليق (الطيران)

$30 = (40 \cos 60)\, t \Rightarrow t = 1.5$ sec.

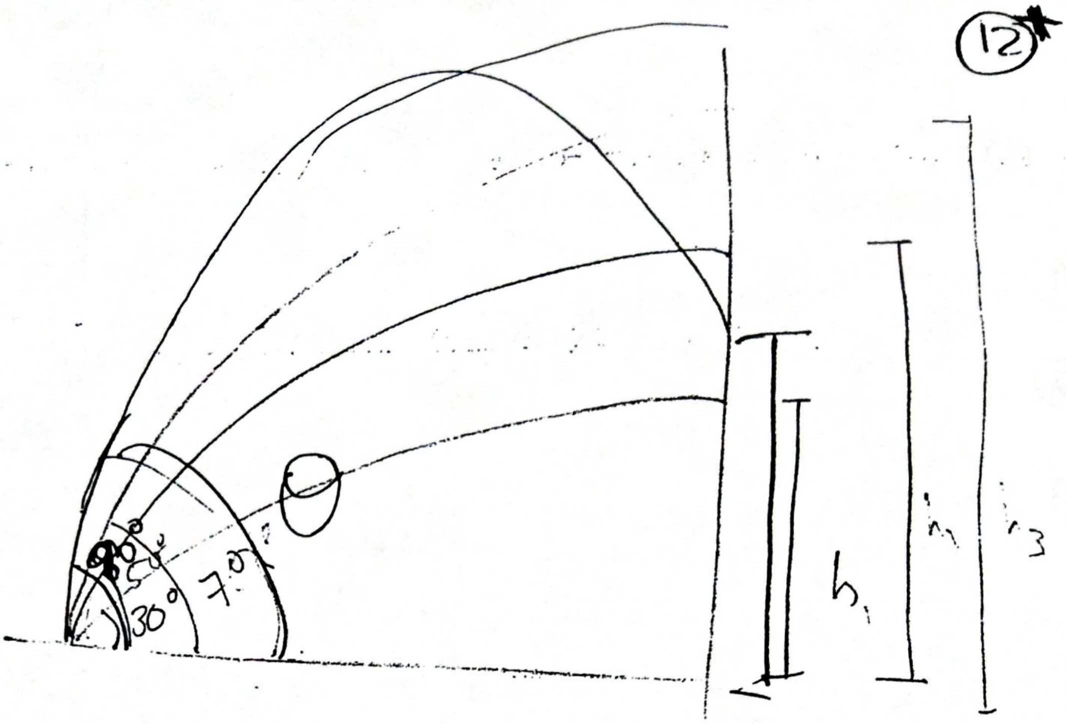
* $h = (v_0 \sin\theta)\, t - \frac{1}{2} g t^2$

$h = (40 \sin 60)(1.5) - \frac{1}{2}(9.81)(1.5)^2$

$h = 40.9$ m .

دائما
* المركبة الافقية للسرعة (ثابتة) $= v_0 \cos\theta = 40 \cos 60° = 20$ m/s .

* المركبة الرأسية للسرعة $= v_y = v_0 \sin\theta - g t$
$= 40 \sin 60 - 9.81(1.5) = 19.93$ m/s



④ Find θ ⇒ max. height $\underline{\underline{h}}$?

* $R = (v_0 \cos\theta)\,t$

$30 = 40 \cos\theta \, t$

$\cos\theta \, t = \frac{3}{4} = 0.75$

$t = \frac{0.75}{\cos\theta}$ —— ①

$t = 0.75 \sec\theta$ —— ①'

④ $\Delta y = (v_0 \sin\theta)t - \frac{1}{2} g t^2$

$$\boxed{h = 40 \sin\theta \, t - \frac{1}{2}(9.81)\, t^2}$$ ②

$$h = 40 \sin\theta \left(\frac{0.75}{\cos\theta}\right) - 4.905\left(\frac{0.75}{\cos\theta}\right)^2$$

$$\boxed{h = 30 \tan\theta - 2.76 \sec^2\theta}$$

maximize h $\Rightarrow$ $\frac{dh}{d\theta} = 0$

$$\Rightarrow 30 \sec^2\theta - (2.76)(2)\sec\theta(\sec\theta \tan\theta) = 0$$

$$30 \cancel{\sec^2\theta} - 5.52 \cancel{\sec^2\theta} \tan\theta = 0$$

$$\tan\theta = \frac{30}{5.52} \Rightarrow \boxed{\theta = 80^\circ}$$

عوض

$$* h = 30 \underbrace{\tan 80^\circ}_{} - 2.76 \sec^2 80^\circ$$

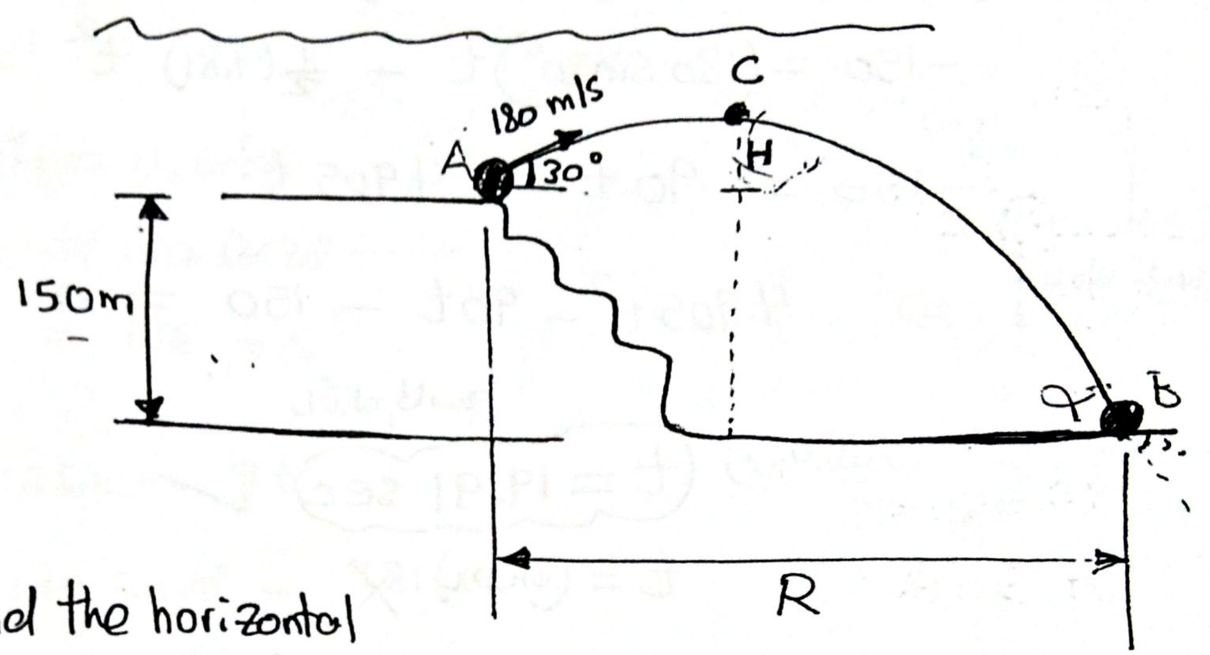
170.1 -- 91.5

$\Rightarrow \sec\theta = \frac{1}{\cos\theta}$

$\rightarrow h = 79\ m$

بالآلة الحاسبة

Ex]



① Find the horizontal distance R ?

② the greatest elevation above the ground

أقصى ارتفاع تصل له الكره عن الأرض

زمن التحليق كاملاً ??

③ the time of flight

④ the time for the highest elevation الزمن اللازم للكره للوصول لأقصى ارتفاع

⑤ the velocity of Impact (Hit) and

the Angle of Impact (Angle of fall) زاوية السقوط

$V_0 = 180 \text{ m/s}$

$\theta = 30^\circ$

$h = -150 \text{ m}$.

(1) $\Delta y = (V_0 \sin\theta)t - \frac{1}{2} g t^2$

$-150 = (180 \sin 30^\circ)t - \frac{1}{2}(9.81)\, t^2$

$-150 = 90t - 4.905\, t^2$

معادلة تربيعية $\Rightarrow 4.905t^2 - 90t - 150 = 0$

بالآلة الحاسبة

زمن التحليق $t = 19.91 \text{ sec}$ ✓

$t =$ بالسالب X

$* R = (V_0 \cos\theta)\, t$

$= (180 \cos 30)(19.91)$

$= 3100 \text{ m}$.

* أقصى ارتفاع $(V_y = 0)$

$V_y^2 = (V_0 \sin\theta)^2 - 2gH$

$0 = (180 \sin 30)^2 - 2(9.81)H \Rightarrow H = 413 \text{ m}$

أقصى ارتفاع عن مستوى الأرض $= 413 + 150 = 563 \text{ m}$.

زمن أقصى ارتفاع

$$v_y = v_0 \sin\theta - g t$$

$$* \quad 0 = 180 \sin 30 - 9.81 (t)$$

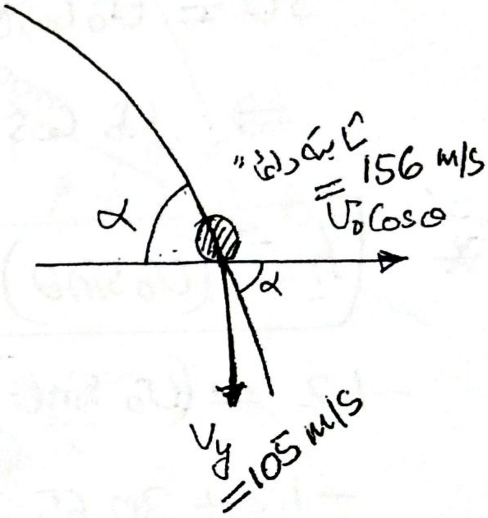
$$t = 9.2 \text{ sec}$$

الاصطدام بالارض

$$* \text{ المركبة الافقية } = v_0 \cos\theta$$

$$= 180 \cos 30^\circ$$

$$= 156 \text{ m/s}$$



$$* \quad v_y = v_0 \sin\theta - g t$$

$$= 180 \sin 30^\circ - 9.81 (19.91)$$

$$= -105 \text{ m/s}$$

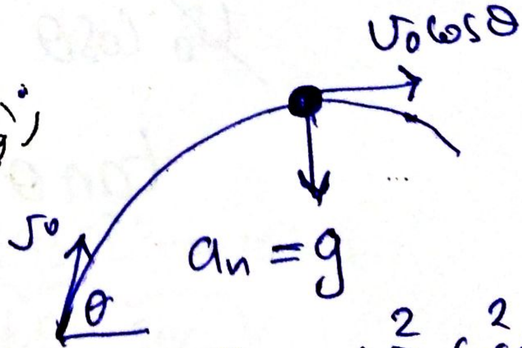
$$* \quad v_B = \sqrt{(156)^2 + (105)^2} = 188 \text{ m/s}$$

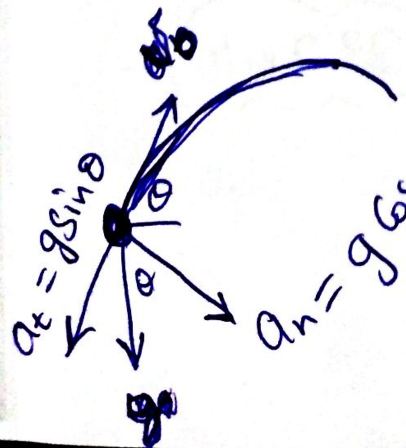
$\rho \equiv$ radius of Curvature.

$$* \quad \alpha = \tan^{-1} \frac{v_y}{v_0 \cos\theta}$$

$$= \tan^{-1} \frac{105}{156} \Rightarrow \alpha = 34^\circ$$

(زاوية) تصادم





$$\rho = \frac{v^2}{a_n}$$

$$\rho = \frac{v_0^2}{g \cos\theta}$$

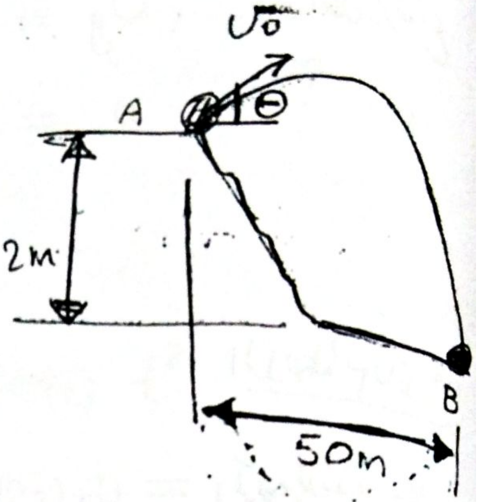
$$\rho = \frac{v^2}{a_n} = \frac{v_0^2 \cos^2\theta}{g}$$

12-94 الزمن الكلي

$t = 2.5$ sec.

$R = 50$ m

$h = -1.2$ m



$$\boxed{R = (v_0 \cos\theta)\, t}$$

$$50 = v_0 \cos\theta (2.5)$$

$$\Rightarrow \quad v_0 \cos\theta = 20 \qquad \text{—— (1)}$$

$$\boxed{h = (v_0 \sin\theta)\, t - \frac{1}{2} g t^2}$$

$$-1.2 = (v_0 \sin\theta)(2.5) - \frac{1}{2}(9.81)(2.5)^2$$

$$\frac{-1.2 + 30.65}{2.5} = v_0 \sin\theta$$

$$v_0 \sin\theta = 11.78 \qquad \text{—— (2)}$$

اقسم المعادلة الثانية على المعادلة الاولى

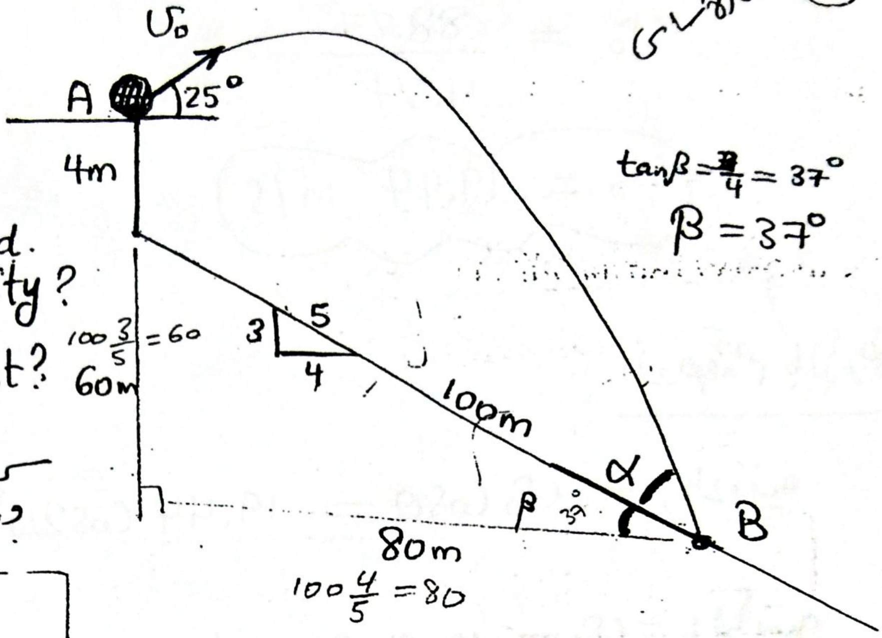
$$\frac{\cancel{v_0} \sin\theta}{\cancel{v_0} \cos\theta} = \frac{11.78}{20}$$

$$\tan\theta = 0.589 \Rightarrow \theta = 30.5^\circ$$

$$v_0 = 23.2 \text{ m/s}$$

(17) (18)

12-110



$\tan\beta = \frac{3}{4} = 37°$

$\beta = 37°$

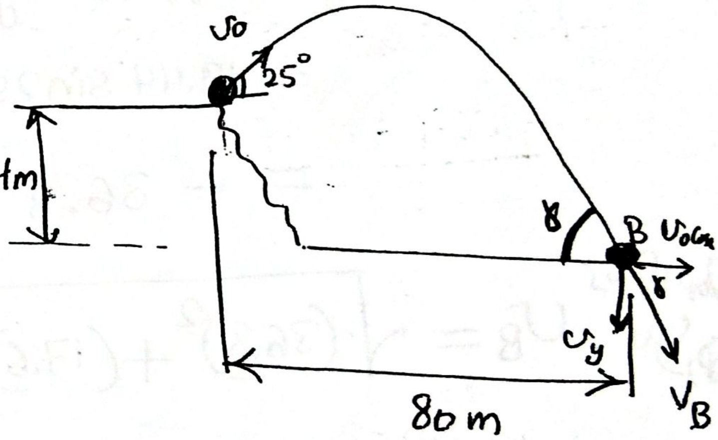
Find

① the initial ~~velocity~~ speed?

② time of flight?

③ مقدار سرعة الاصطدام بالارض
وزاوية الاصطدام

$$\boxed{\begin{aligned}\theta &= 25° \\ R &= 80\ m \\ h &= -64\ m\end{aligned}}$$



$$R = (v_0 \cos\theta)\, t$$

$$* \quad 80 = (v_0 \cos 25°)\, t$$

$$v_0 t = 88.27 \quad \text{——} \ (1)$$

$$* \quad h = (v_0 \sin\theta)\, t - \frac{1}{2} g t^2$$

$$-64 = (v_0 t) \sin 25° - \frac{1}{2}(9.81)\, t^2$$

$$-64 = (88.27) \sin 25° - 4.905\, t^2$$

$$-64 = 37.3 - 4.905\, t^2 \Rightarrow 4.905 t^2 = 101.3$$

$$\Rightarrow \boxed{t = 4.54 \text{ sec}}$$

$$v_0 t = 88.27$$

$$v_0 = \frac{88.27}{4.54}$$

$$v_0 = 19.44 \ m/s$$

الاصطدام بالأرض

$$\text{الافقيه} = v_0 \cos\theta = 19.44 \cos 25^\circ = 17.6 \ m/s$$

$$\text{الرأسيه} = v_y = v_0 \sin\theta - gt$$

$$= 19.44 \sin 25 - 9.81 (4.54)$$

$$= -36.3 \ m/s$$

سرعة الاصطدام بالأرض

$$v_B = \sqrt{(36.3)^2 + (17.6)^2} = 40.3 \ m/s$$

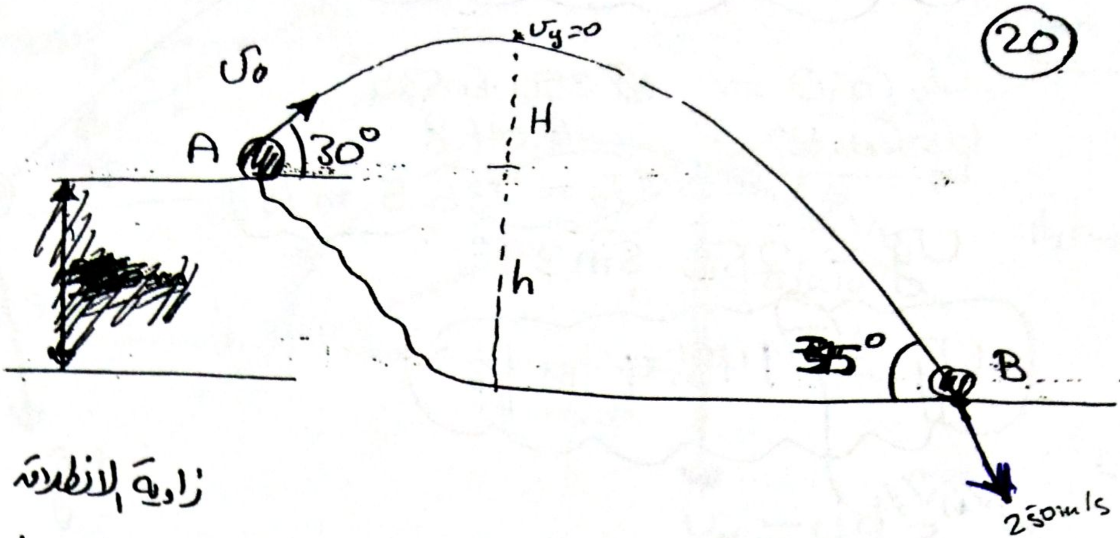
$$\tan\gamma = \frac{v_y}{v_0 \cos\theta} = \frac{36.3}{17.6}$$

$$\Rightarrow \gamma = 64.1^\circ$$

$$\gamma = \alpha + \beta$$

$$\alpha = 64.1 - 37^\circ \Rightarrow \alpha = 27.1^\circ$$

Ex



$\theta = 30^\circ$ زاوية الانطلاق

$\alpha = 35^\circ$ زاوية السقوط

$U_B \equiv$ سرعة الاصطدام بالارض $= 250\ m/s$.

أوجد

مهم جداً

(1) سرعة الانطلاق v_0

(2) زمن التحليق $\underline{\underline{t}}$

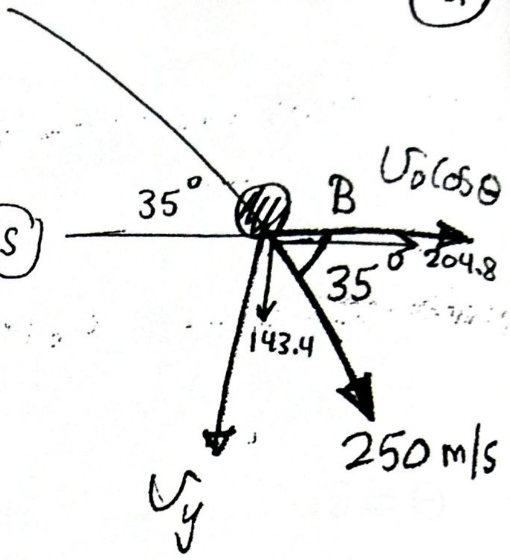
(3) المدى الافقي $\underline{\underline{R}}$

(4) أقصى ارتفاع عن الارض

(21)

$V_x \equiv$ Constant.

$$\boxed{V_0 \cos\theta = V_f \cos\alpha}$$



الافقيه

$\Rightarrow V_0 \cos\theta = 250 \cos 35° = 204.8$

$V_0 \cos 30° = 204.8 \Rightarrow \boxed{V_0 = 236.5 \text{ m/s}}$

الرأسيه $V_y = 250 \sin 35°$

$$\boxed{V_y = -143.4 \text{ m/s}}$$

الرأسى بعد إيجاد V_0

$V_y = V_0 \sin\theta - g\,t$

$-143.4 = 236.5 \sin 30° - 9.81\, t$

زمن الحركة $\boxed{t = 26.7 \text{ sec}}$

* $R = (V_0 \cos\theta)\, t$

$R = (236.5 \cos 30)(26.7)$

$R = 5468.6$ m

أقصى ارتفاع

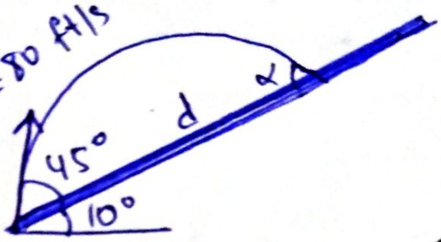
* $V_y^2 = (V_0 \sin\theta)^2 - 2g\,H$

$0 = (236.5 \sin 30)^2 - 2(9.81)\,H$

$\Rightarrow H = 712.7$ m

* $h = (236.5 \sin 30)(26.7) - \frac{1}{2}(9.81)(26.7)^2 = -339.5$ m

الارتفاع عن الارض $= 339.5 + 712.7 = 1052.2$ m

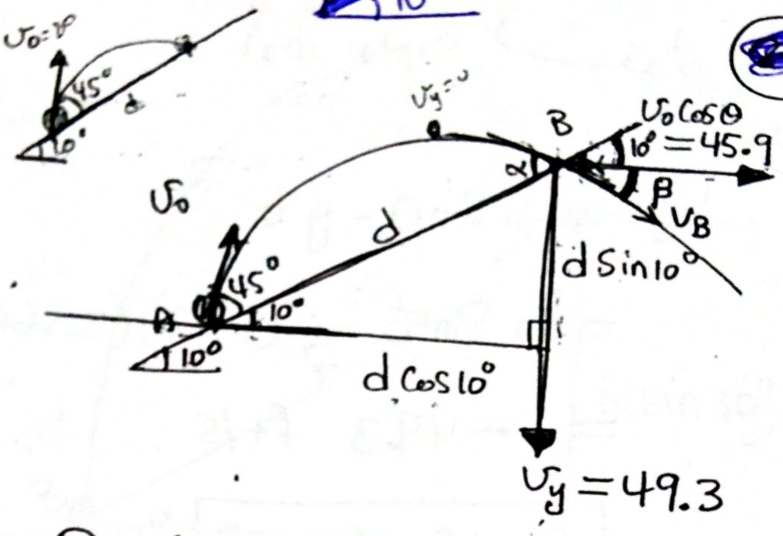


12– 102

Find d?

$v_0 = 80$ ft/s

$\theta = 55°$



$$* \quad R = (v_0 \cos\theta)\, t$$

$$d \cos 10° = (80) \cos 55° \; t$$

$$0.984\, d = 45.89 \; t$$

$$\boxed{d = 46.63 \; t} \quad \text{—— (1)}$$

$$* \quad \Delta y = (v_0 \sin\theta)\, t - \tfrac{1}{2} g t^2$$

$$d \sin 10° = (80 \sin 55)\, t - \tfrac{1}{2}(32.2)\, t^2$$

$$\boxed{0.174\, d = 65.53\, t - 16.1\, t^2} \quad \text{—— (2)}$$

عوض

$$0.174\, [46.63]\, t = 65.53\, t - 16.1\, t^2$$

$$\Rightarrow 16.1\, t^{\cancel{2}} = 57.42\, \cancel{t}$$

$$t = \frac{57.42}{16.1} = 3.566 \text{ sec}$$

$$d = 46.63\,(3.566)$$

$$d = 166.3 \text{ ft}$$

أوجد زاوية السقوط α ؟

* سرعة أقصى

$$v_y = v_0 \sin\theta - gt$$

$$= 80 \sin 55 - (32.2)(3.566)$$

$$= -49.3 \text{ ft/s}.$$

$$* \; v_B = \sqrt{(49.3)^2 + (45.9)^2} = 67.3 \text{ ft/s}$$

$$\tan\beta = \frac{49.3}{45.9} \Rightarrow \boxed{\beta = 47^\circ}$$

$$\alpha = 10^\circ + \beta$$

$$\alpha = 10 + 47^\circ$$

$$\boxed{\alpha = 57^\circ}$$

* زمن أقصى ارتفاع $\Leftarrow (v_y = 0)$

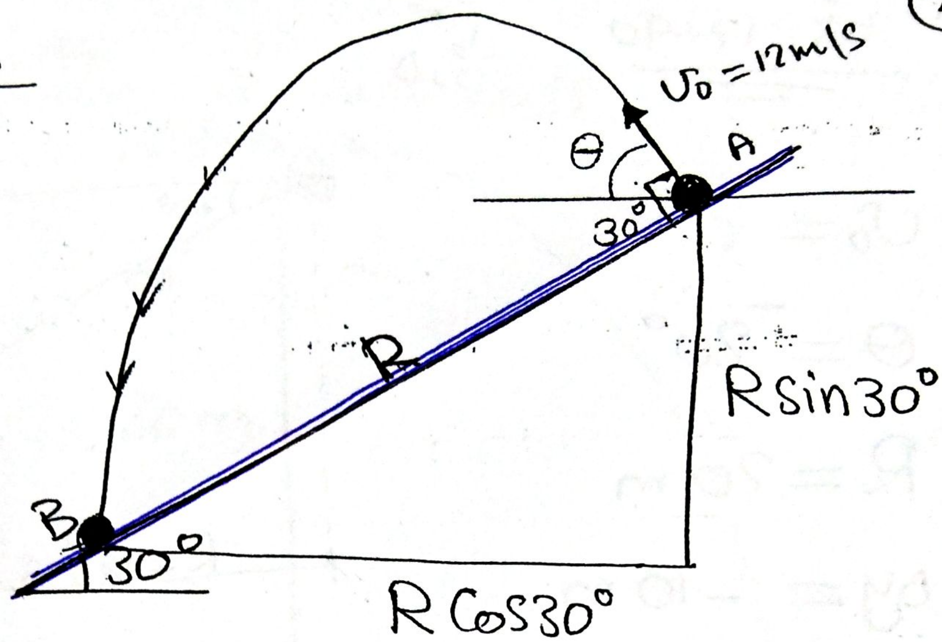
$$v_y = v_0 \sin\theta - gt$$

$$0 = 80 \sin 55 - 32.2(t)$$

$$t = \frac{80 \sin 55}{32.2} \Rightarrow t = 2 \text{ sec.}$$

Ex 12-91

Find R



$v_0 = 12 m/s$

$\theta = 90 - 30 = 60°$

$R = v_0 \cos\theta \, t$

$R \, 0.866 = 12 \cos 60 \, t$

$\boxed{R = 6.93 t}$ —— (1)

$-h = v_0 \sin\theta \, t - \frac{1}{2} g t^2$

$-R \, 0.5 = 12 \sin 60 \, t - 4.905 t^2$ —— (2)

$-(6.93 t) \, 0.5 = 10.4 t - 4.905 t^2$

$4.905 t^2 = 13.865 t$

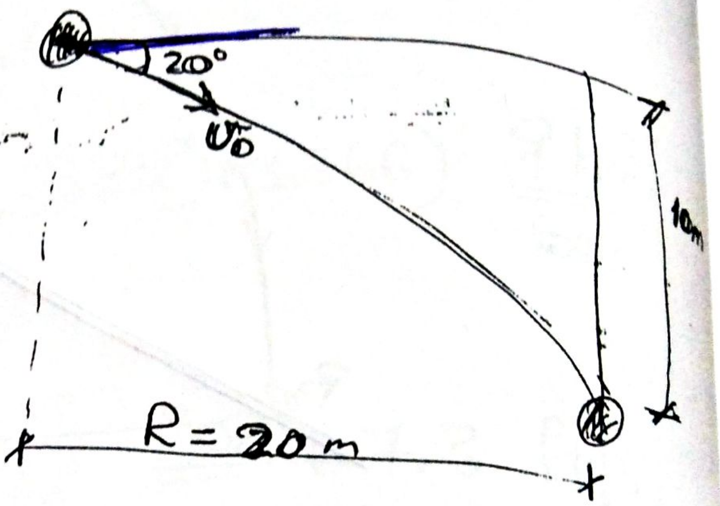
$\boxed{t = 2.83 \text{ sec}}$

$\boxed{R = 19.6 \text{ m}}$

الرمي للأسفل مباشرة
بزاوية 20°.

Ex 12-90

مهم جداً



$v_0 = ?$

$\theta = -20°$

$R = 20\ m$

$\Delta y = -10\ m$

$*\quad 20 = v_0 \cos(-20°)\, t$ —— (1)

$v_0 t = 21.3$ —— (1)

$*\quad -10 = v_0 \sin(-20)\, t - \frac{1}{2}(9.81)\, t^2$

$-10 = 21.3 \sin(-20) - \frac{1}{2}(9.81) t^2$ —— (2)

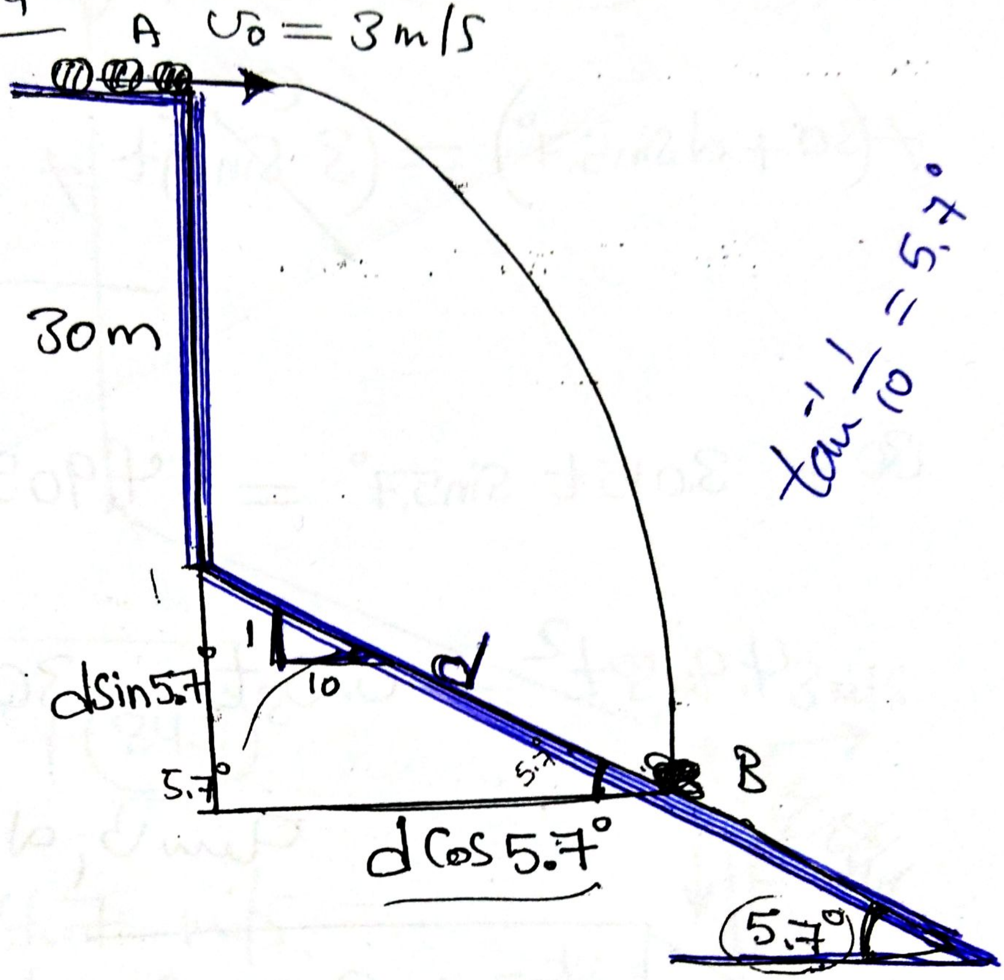
$t = 0.75\ sec$

$v_0 = 28.6\ m/s$

Ex 12-94

$\boxed{\theta = 0}$

$\boxed{v_0 = 3\text{ m/s}}$



أوجد سرعة
اصطدام الحجارة
بالأرض

v_B

$$\Delta y = -\left(30 + d\sin 5.7^\circ\right)$$

$$R = d\cos 5.7^\circ$$

$$d\cos 5.7^\circ = (3\cos 0)\,t \quad \text{—} \quad (1)$$

$$\boxed{d = 3.015\,t}$$

$$\Delta y = (v_0 \sin\theta)t - \frac{1}{2} g t^2$$

$$-(30 + d\sin 5.7^\circ) = (3\cancel{\sin\theta})t - \frac{1}{2}(9.81)t^2 \quad \text{——— } (2)$$

$$30 + 3.015t\sin 5.7^\circ = 4.905t^2$$

$$4.905t^2 - 0.3t - 30 = 0$$

بالآلة الحاسبة

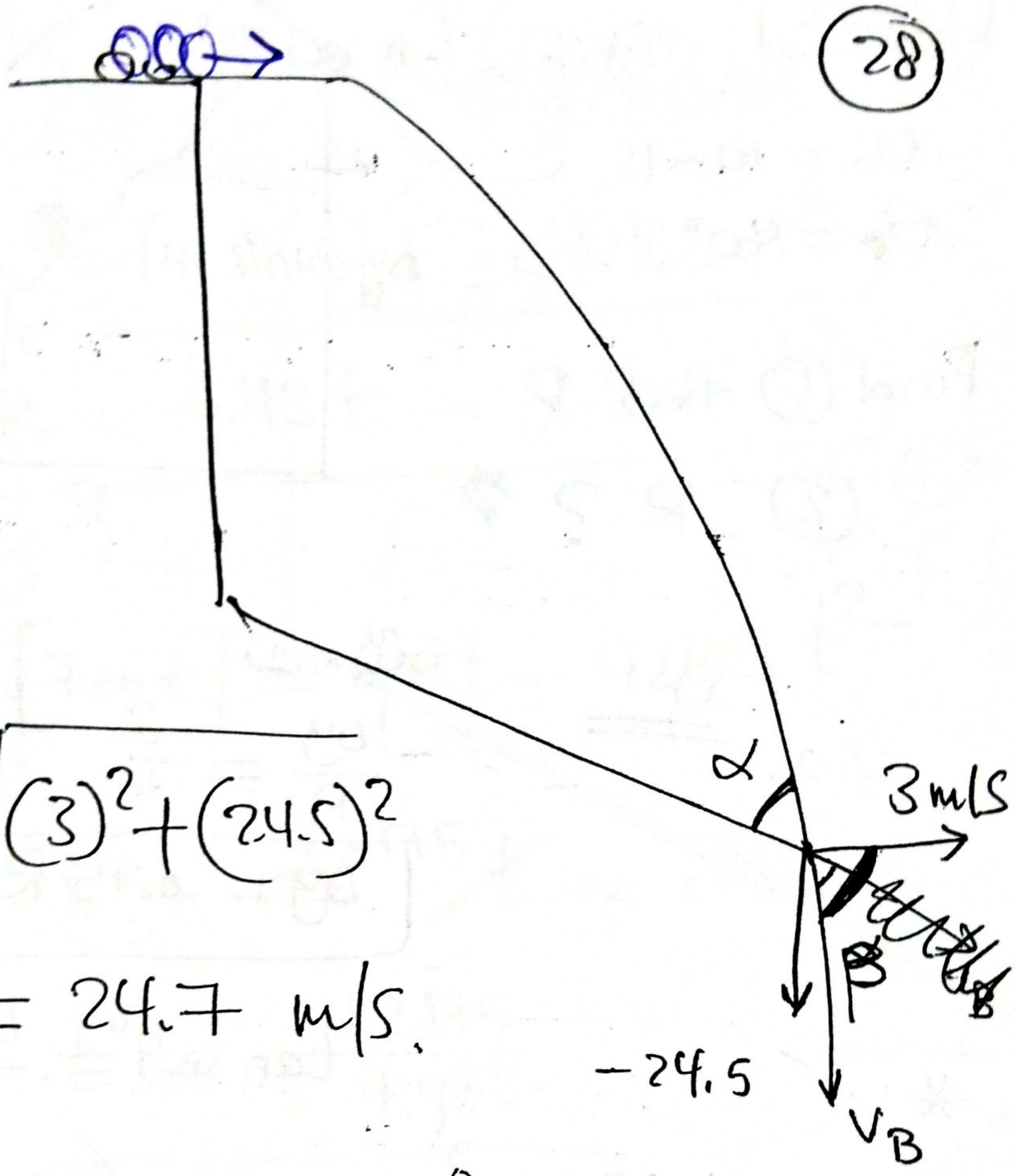
$$\boxed{t = 2.5 \text{ sec}}$$

مرفوض (سالب) ~~$t = -2.44$ sec~~ X

$$v_y = v_0\cancel{\sin\theta} - gt$$

$$v_y = -9.81(2.5)$$

$$\boxed{v_y = -24.5 \text{ m/s}}$$



$$U_B = \sqrt{(3)^2 + (24.5)^2}$$

$$U_B = 24.7 \text{ m/s}$$

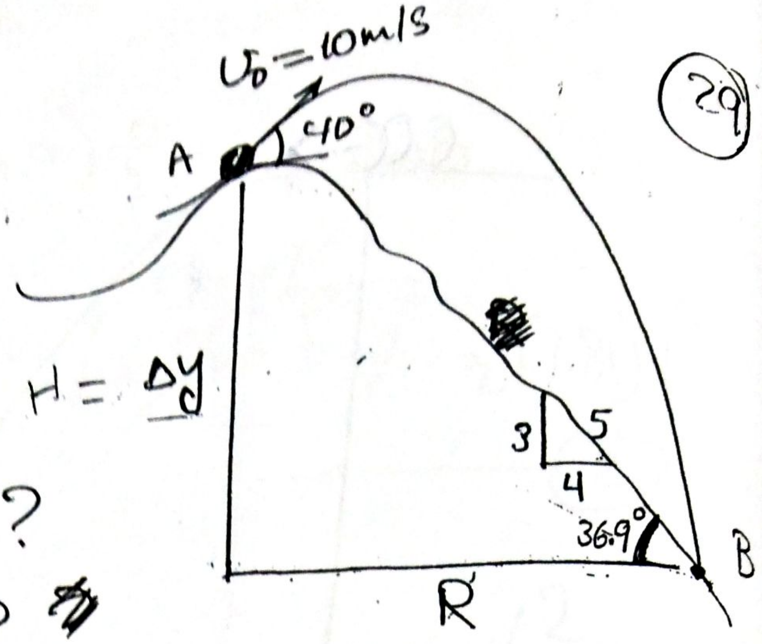
$$\tan\beta = \frac{24.5}{3} \Rightarrow \beta = 83^\circ$$

$$\alpha = 83 - 5.7^\circ$$

$$\boxed{\alpha = 77.3^\circ}$$

مثال

12-88



$U_0 = 10 m/s$

$\theta = 40°$

Find (1) time ?

(2) R ?

اولا

تشابه مثلثات

$$\frac{\Delta y}{R} = \frac{3}{4}$$

$$\boxed{\Delta y = -0.75 R}$$

او

*

$$\tan 36.9° = \frac{\Delta y}{R}$$

$$\Delta y = (\tan 36.9°) R$$

$$\boxed{\Delta y = -0.75 R}$$

* $R = (U_0 \cos\theta) t$

$$R = (10 \cos 40°) t \Rightarrow \boxed{R = 7.66 t} \quad (1)$$

$$\Delta y = (v_0 \sin\theta)\, t - \frac{1}{2} g t^2$$

$$-0.75R = (10 \sin 40)\, t - 4.905\, t^2$$

$$\boxed{-0.75R = 6.43t - 4.905\, t^2}$$ (2)

$$-0.75\,[7.66t] = 6.43t - 4.905\, t^2$$

$$4.905t^{\cancel{2}} - \overset{12.175}{\cancel{0.6}}\,\cancel{t} = 0$$

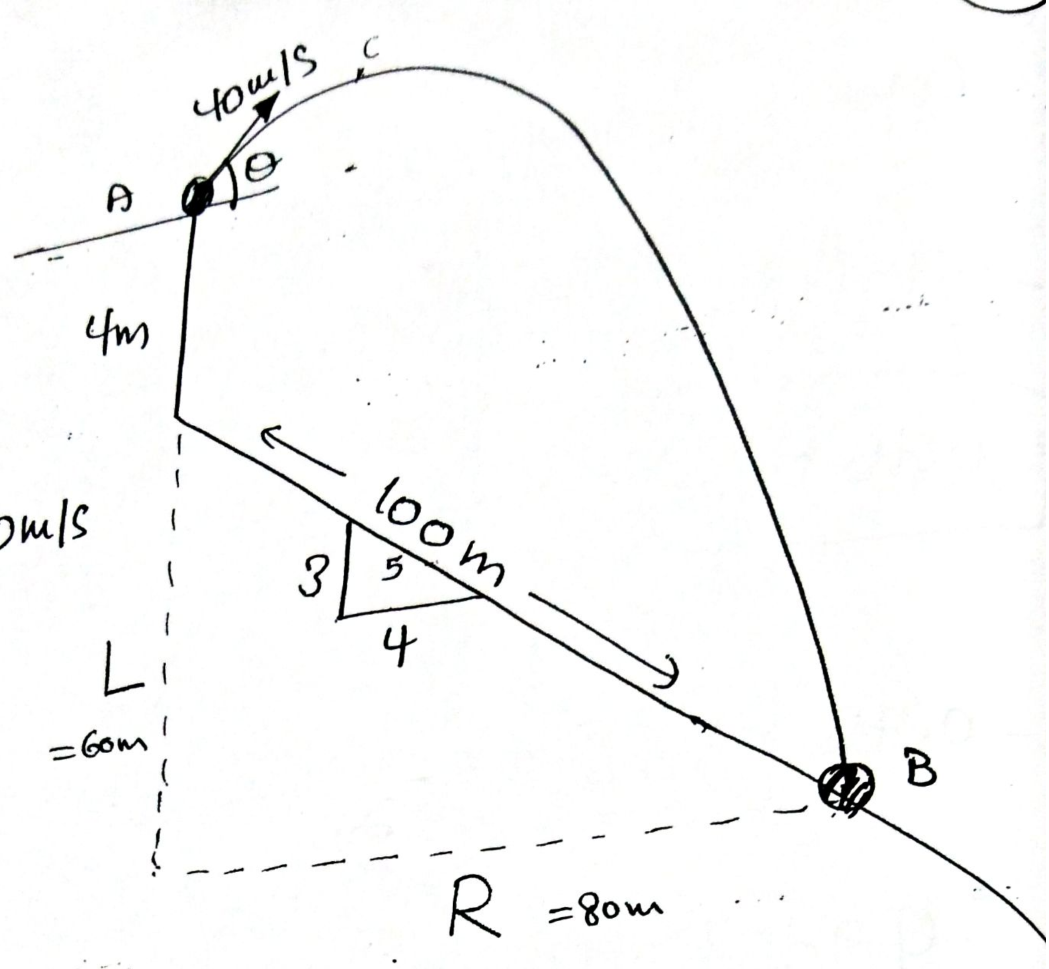
$$t = \frac{12.175}{4.905}$$

$$\boxed{t = 2.5 \text{ sec}}$$

$$R = 7.66\,(2.5)$$

$$\boxed{R = 19 \text{ m}}$$

ex)



$V_0 = 40\,m/s$

* $\theta = ?$

حل

$$* \; R = 100\,\frac{4}{5} = 80\ m$$

$$* \; L = 100\,\frac{3}{5} = 60\ m$$

$$* \quad R = (V_0 \cos\theta)\,t$$

$$80 = 40 \cos\theta \; t$$

$$\boxed{\cos\theta \; t = 2} \quad (1)$$

$$t = \frac{2}{\cos\theta}$$

32

$$\Delta y = (v_0 \sin\theta)\, t - \frac{1}{2} g t^2$$

$$\boxed{-64 = (40 \sin\theta)\, t - 4.905\, t^2}$$

— 2

عوض

$$-64 = 40 \sin\theta \, \frac{2}{\cos\theta} - 4.905 \, \frac{4}{\cos^2\theta}$$

$\frac{1}{\cos\theta} = \sec\theta$

$$-64 = 80 \tan\theta - 19.6 \sec^2\theta$$

ولكن

$\sec^2\theta = 1 + \tan^2\theta$ من رياضيات
(خلفية ثانوية)

$$-64 = 80 \tan\theta - 19.6 \left[1 + \tan^2\theta \right]$$

$$-64 = 80 \tan\theta - 19.6 - 19.6 \tan^2\theta$$

$$\Rightarrow \quad 19.6 \tan^2\theta - 80 \tan\theta - 44.4 = 0$$

معادلة تربيعية

$\tan\theta = 5.32 \Rightarrow \boxed{\theta \cong 80^\circ}$ طالع فوق

$\tan\theta = -0.502 \Rightarrow \boxed{\theta = -26^\circ}$ X نازل

Tangential (t) & Normal (n) Components

المركبات العامودية و المماسية

التقعر

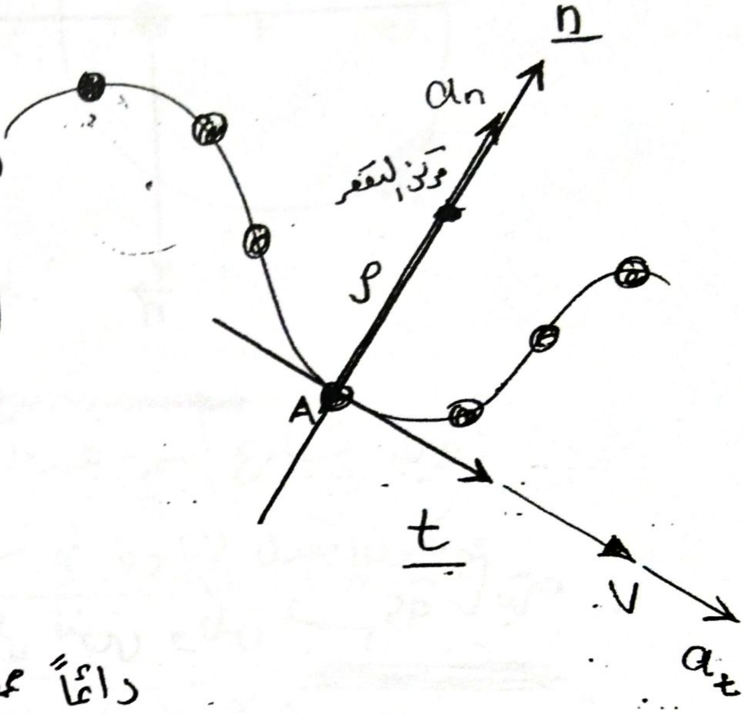
Curvature

قانون نصف القطر

$$\rho = \frac{\left[1+(y')^2\right]^{3/2}}{y''}$$

المسار

Path $y(x)$



* **Velocity** : دائماً مماسية للمسار باتجاه (t)

$$v = \frac{ds}{dt}$$

* **Acceleration** : المركبة المماسية

$$a_t = \frac{dv}{dt} = \dot{v}$$

المركبة العامودية (باتجاه المركز)

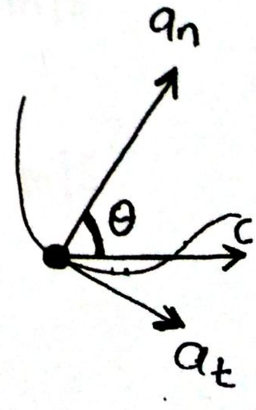
$$a_n = \frac{V^2}{\rho}$$

التسارع الكلي الكامل

$$a = \sqrt{a_n^2 + a_t^2}$$

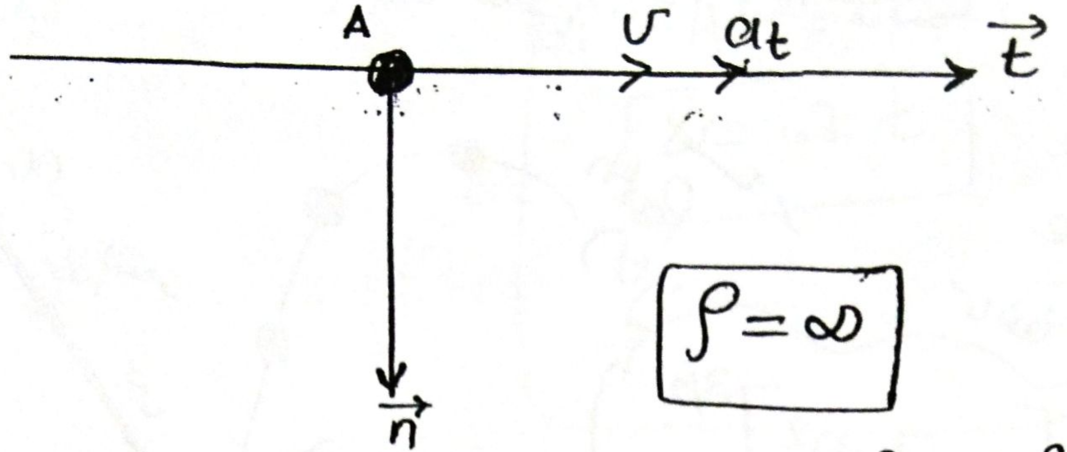
إتجاه التسارع الكلي

$$\tan\theta = \frac{a_t}{a_n}$$



ملاحظات :-

① إفترض أن الجسم يسير بخط مستقيم



$$\boxed{\rho = \infty}$$

$$a_n = \frac{v^2}{\rho} = \frac{v^2}{\infty} = 0$$

$$\boxed{a_n = 0}$$

** ② الجسم يسير على منحنى ولكن بسرعة ثابتة .

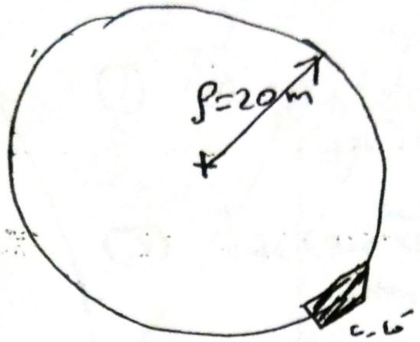
* v = Constant

* $\Rightarrow a_t = 0$

$$a_n = \frac{v^2}{\rho}$$

Ex] ص ٥٢٥

12-104



Find a?

when $v = 5 m/s$

and the rate of increase in the speed $2 m/s^2$

أوجد تسارع الجسم عندما تكون سرعة القارب ٥م/ث

ويكون معدل الزيادة في سرعة القارب تساوي ٢م/ث٢

هو التسارع باتجاه الtangent

$$* \; a_t = 2 \, m/s^2$$

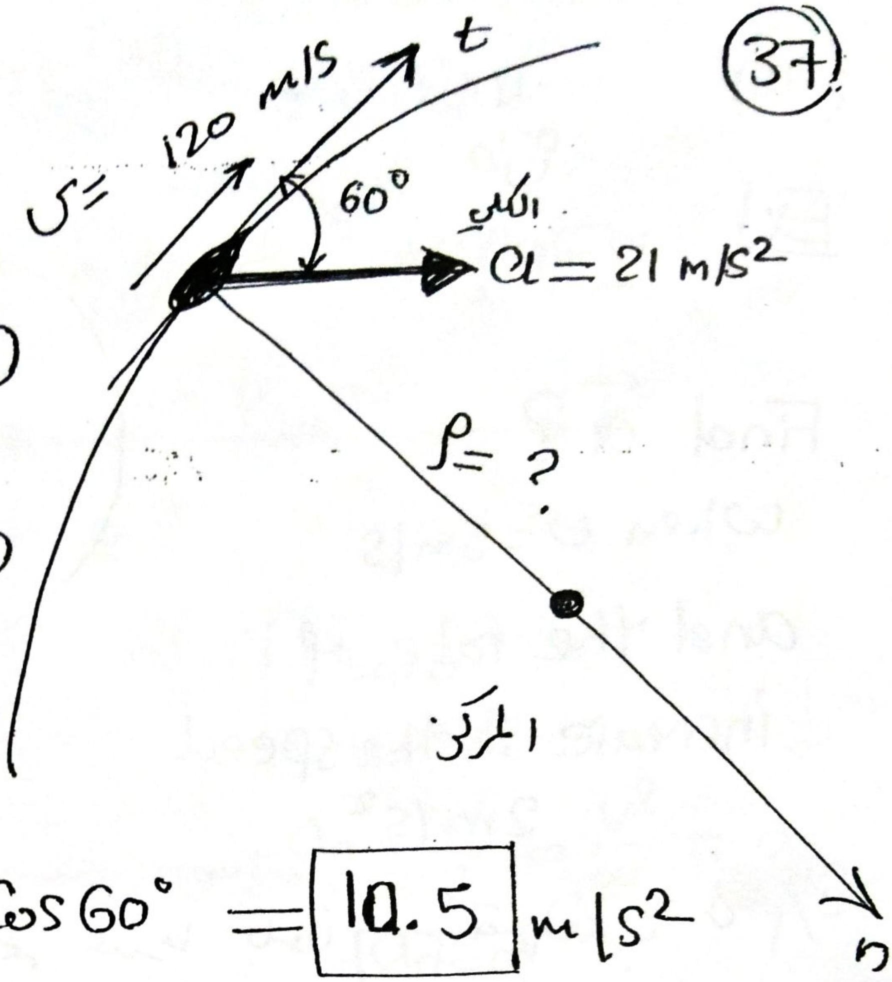
$$* \; a_n = \frac{V^2}{\rho} = \frac{(5)^2}{20} = 1.25 \, m/s^2$$

$$* \; a = \sqrt{(2)^2 + (1.25)^2} = 2.358 \, m/s^2$$

EX
12-102

① أوجد معدل زيادة في السرعة (a_t)

ⓒ أوجد نصف قطر المسار ρ



$$* \quad a_t = 21 \cos 60^\circ = \boxed{10.5}\ m/s^2$$

$$* \quad a_n = 21 \sin 60^\circ = 18.2\ m/s^2$$

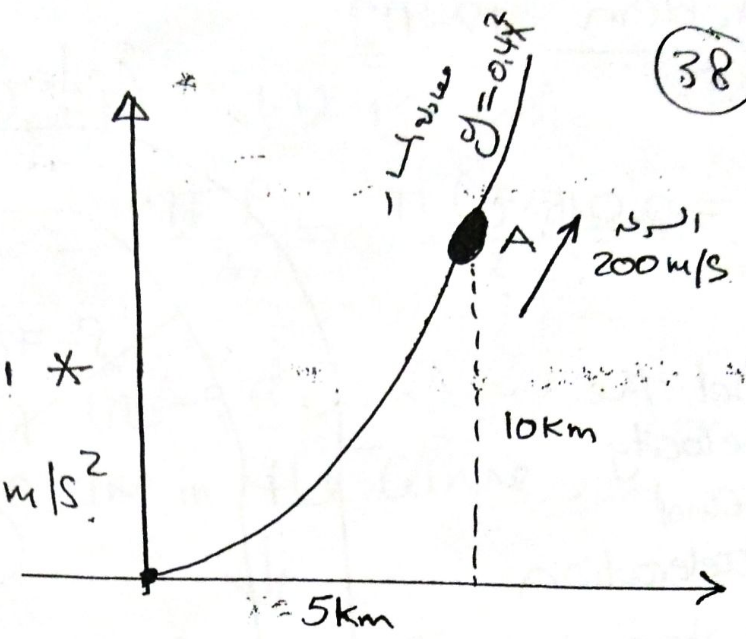
$$* \quad a_n = \frac{V^2}{\rho}$$

$$18.2 = \frac{(120)^2}{\rho}$$

$$\Rightarrow \rho = \frac{(120)^2}{18.2} \Rightarrow \boxed{\rho = 791\ m}$$

12-106

$y = 0.4x^2$



* السرعة تتزايد بمعدل 0.8 m/s²

$\Rightarrow a_t = 0.8\,m/s^2$

أوجد التسارع الكلي

Find a ?

عوض $X = 5\,Km$

الحل

$$y = 0.4x^2 \Rightarrow y = 10\ Km$$

$$y' = 0.8x \Rightarrow y' = 4$$

$$y'' = 0.8 \Rightarrow y'' = 0.8$$

$$\rho = \frac{\left[1+y'^2\right]^{3/2}}{y''} = \frac{\left[1+(4)^2\right]^{3/2}}{0.8}$$

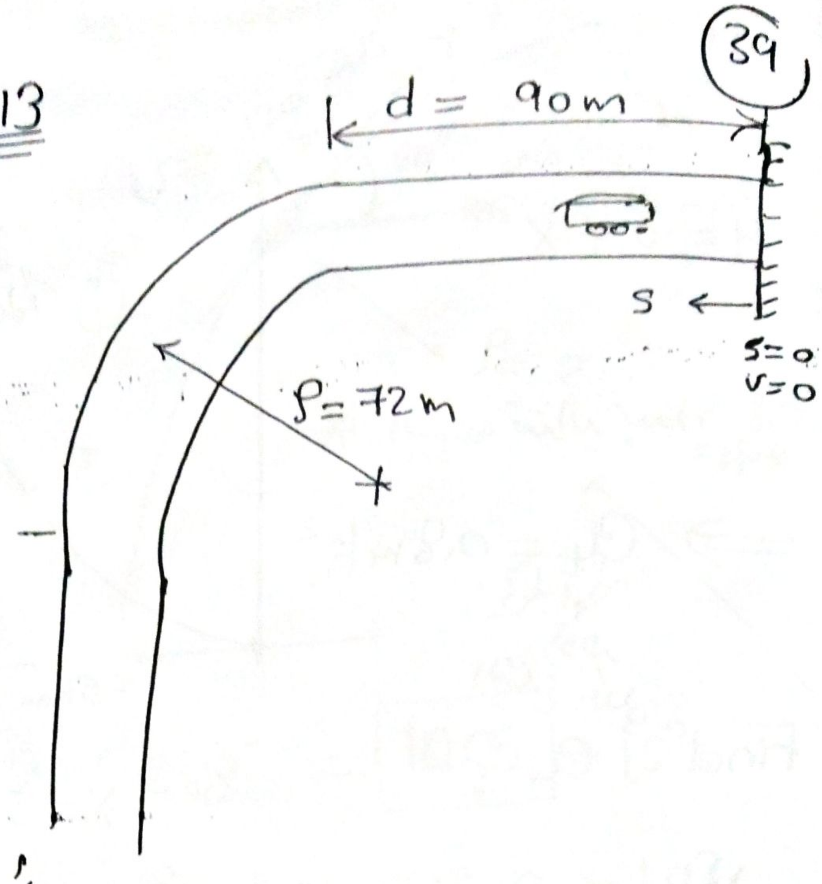
$$* \ \rho = 87.6\ Km$$

$$* \ a_n = \frac{v^2}{\rho} = \frac{(200)^2}{87.6*1000} = 0.456\ m/s^2$$

تحويل لمتر

$$* \ a = \sqrt{a_n^2 + a_t^2} = \boxed{0.92\ m/s^2}$$

Problem 12-113



$\dot{v} = 0.015\, t^2$

**Find the velocity and acceleration when $t = 18$ sec. ?

أوجد سرعة وتسارع

بعد مرور ١٨ ثانية.

الحل

تحديد مكان السيارة

التسارع المماسي

* $a_t = \dot{v} = 0.015\, t^2$

السرعة $\Rightarrow v = \int 0.015 t^2 dt = \frac{0.015}{3} t^3 = 0.005\, t^3$

المسافة $\Rightarrow S = \int 0.005 t^3 dt = \frac{0.005}{4} t^4 = 1.25 \times 10^{-3}\, t^4$

بعد مرور ١٨ ثانية

$t = 18$ sec

* $a_t = 4.86\ m/s^2$
* $v = \boxed{29.2}\ m/s$
* $S = 131.2\ m$

(40)

محيط ربع الدائرة (خلف الورقة)

$$= \frac{\pi \rho}{2} = \frac{\pi (72)}{2} = 113.1\,m$$

نحن في المنحنى :

We are in the Curve.

$(90 < S < 203)$

$\Downarrow$

يوجد a_t ✓ a_n ✓

$$a_n = \frac{v^2}{\rho}$$

$$= \frac{(29.2)^2}{72} \Rightarrow a_n = 11.8\,m/s^2$$

$$\therefore a = \sqrt{a_n^2 + a_t^2}$$

$$= \sqrt{(11.8)^2 + (4.86)^2} = \boxed{12.8\,m/s^2}$$

ملاحظة Assume $d = 150\,m$.

$\therefore$ السيارة في الخط المستقيم الأول.

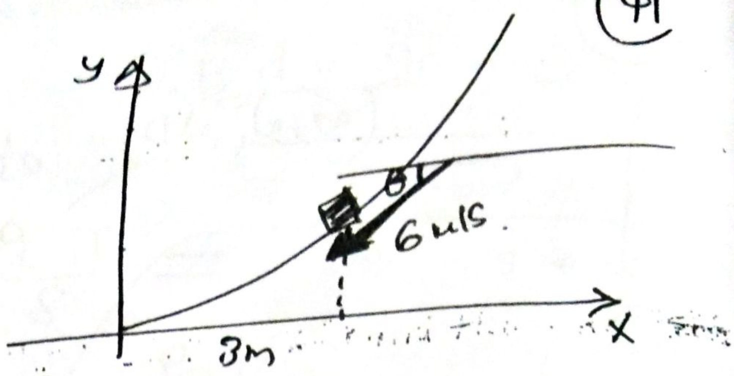
$\therefore$ * $\boxed{v = 29.2\,m/s}$

* $a_n = 0$

$\therefore a_t = 4.86\,m/s^2$

$\boxed{a = 4.86\,m/s^2}$

Ex) $y = \frac{1}{8}x^2$



* $a_t = 1.8\ m/s^2$
* $v = 6\ m/s$
* Find the Car ~~velocity~~ direction ? اوجد اتجاه السيارة

* Find a ?

* Find ρ ?

الحل

$$y' = \frac{2}{8}x = \frac{1}{4}x = \frac{1}{4}(3) = 0.75$$

$$y'' = \frac{1}{4} = 0.25$$

* $$\rho = \frac{[1 + y'^2]^{3/2}}{y''}$$

$$= \frac{[1 + (0.75)^2]^{1.5}}{0.25} = \boxed{7.8}\ m$$

* $$a_n = \frac{v^2}{\rho}$$

$$= \frac{(6)^2}{7.8} = \frac{36}{7.8} = 4.6\ m/s^2$$

* $$a = \sqrt{a_n^2 + a_t^2} = \cancel{4.6}\ \boxed{4.95}\ m/s^2$$

اتجاه السيارة
هو اتجاه ال
Tangent, ميل المماس ⟹

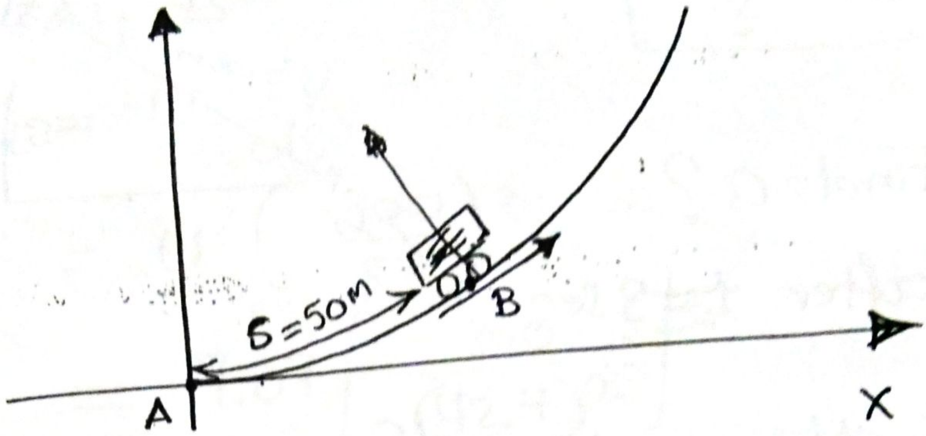
$$\boxed{\tan\theta = y'} = \text{ميل المماس}$$

$$\tan\theta = 0.75 \Rightarrow \boxed{\theta = 36.9^\circ}$$

Ex) 12.32

$\boxed{v = 0.2\,S}$

$\boxed{\rho = 500\,m}$



* أحسب تسارع السيارة
عندما تصبح على بعد 50م
من النقطة A .

الحل * $v = 0.2\,S \Rightarrow v = 0.2(50) = \boxed{10\,m/s}$

* $v\,dv = a_t\,ds$

عوض * $a_t = v\,\dfrac{dv}{ds}$

$$a_t = (0.2\,S)(0.2)$$

$$\boxed{a_t = 0.04\,S}$$

عوض

$S = 50m \Rightarrow a_t = 0.04(50)$

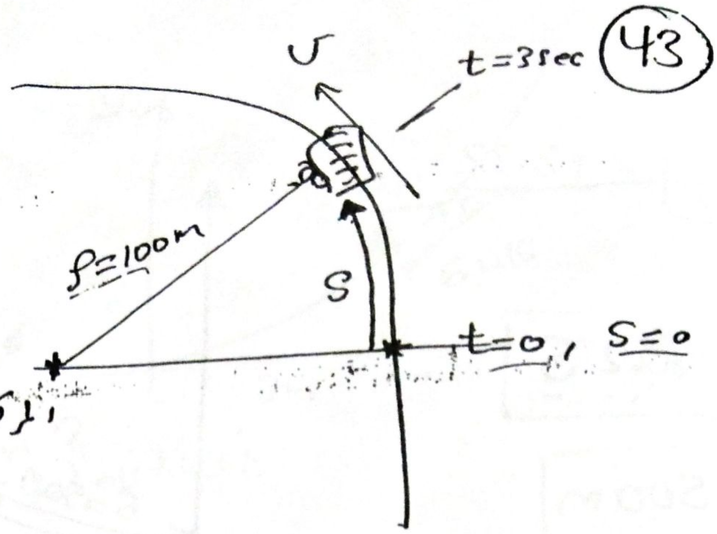
$$\boxed{a_t = 2\,m/s^2}$$

* $a_n = \dfrac{v^2}{\rho} = \dfrac{(10)^2}{500} \Rightarrow \boxed{a_n = 0.2\,m/s^2}$

$\therefore a = \sqrt{a_n^2 + a_t^2} \Rightarrow \boxed{a \cong 2.01\,m/s^2}$

Ex] 12.28

$$\boxed{v = \frac{300}{s}}$$



Find a?
after $t = 3$ sec.

حل

$$v = \frac{ds}{dt}$$

$$\Rightarrow dt = \frac{ds}{v}$$

$$dt = \frac{ds}{\frac{300}{s}}$$

$$\int_0^3 dt = \int_0^s \frac{s}{300}\, ds$$

$$t\Big]_0^3 = \frac{1}{300}\frac{s^2}{2}\Big]_0^s$$

$$3 = \frac{1}{600}\left[s^2 - (0)^2\right] = \frac{s^2}{600} = 3$$

$$s^2 = 1800 \Rightarrow s = \sqrt{1800}$$

$$s = 42.4 \text{ m}$$

حيث: $v = \frac{300}{s} = \frac{300}{42.4} \Rightarrow v = 7.07$ m/s

$$a_n = \frac{v^2}{\rho} = \frac{(7.07)^2}{100} \Rightarrow \boxed{a_n = 0.5 \text{ m/s}^2}$$

$$v\,dv = a_t\,ds$$

$$\Rightarrow a_t = v\frac{dv}{ds}$$

$$= v\left(\frac{-300}{s^2}\right)$$

$$= 7.07\left[\frac{-300}{(42.7)^2}\right]$$

$$\boxed{a_t = -1.16 \text{ m/s}^2}$$

$$a = \sqrt{a_n^2 + a_t^2}$$

$$= \sqrt{(1.16)^2 + (0.5)^2}$$

$$\boxed{a = 1.3 \text{ m/s}^2}$$

(45)

ص ٥٧ $\Delta y = -500\ m.$

12-112

سرعة الطائرة $v_A = 50\ m/s \Rightarrow v_0 = 50\ m/s$

والطيران افقي تماما $\Rightarrow \theta = 0$

$\boxed{1}$ Find a_t? a_n? ρ? عند نقطة القذف A

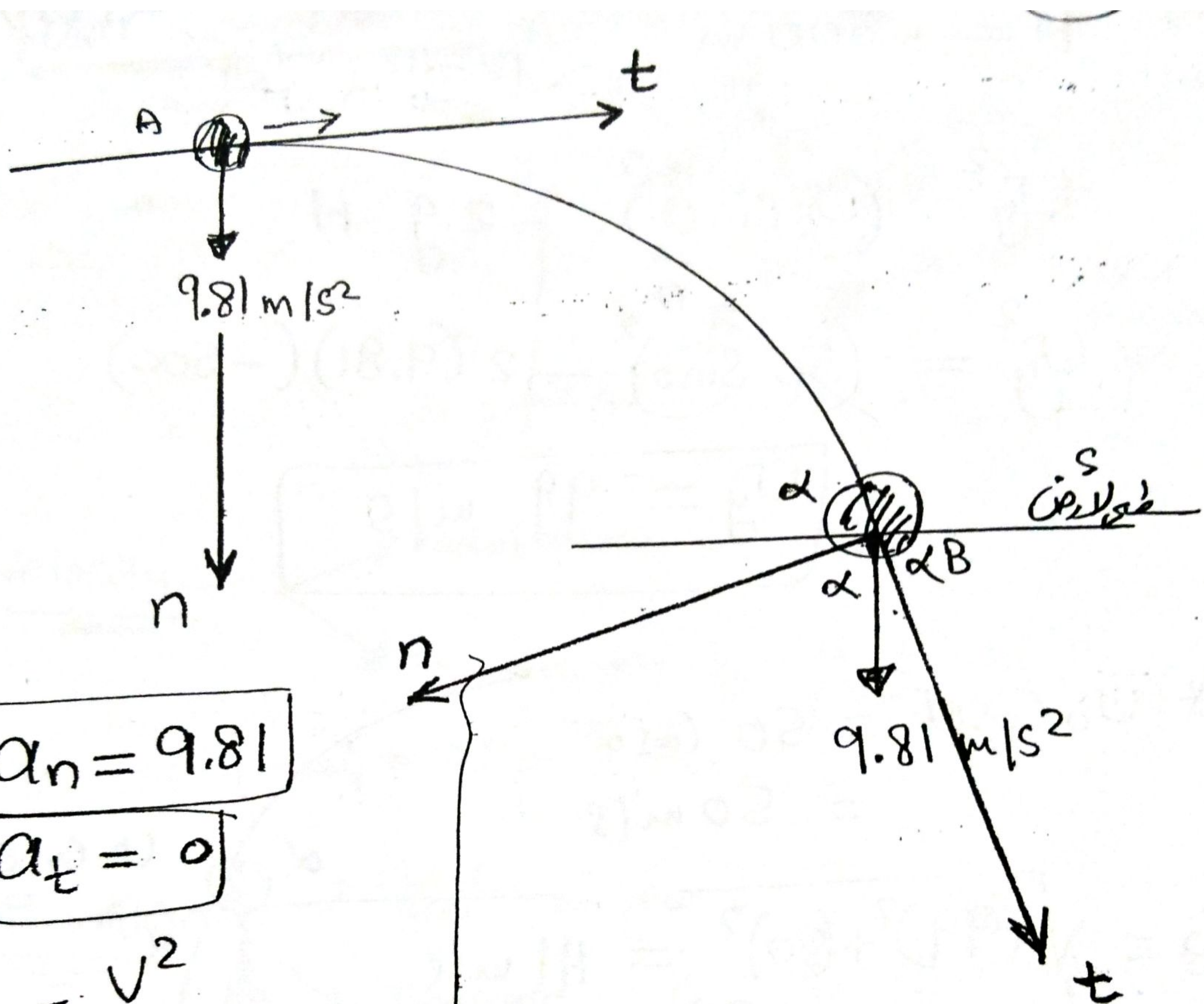
$\boxed{2}$ Find a_t? a_n? ρ? عند نقطة السقوط والإصطدام بالأرض B

الحل

* المقذوفات عبارة عن (سقوط حر)

والسقوط الحر يعني أن

التسارع الكلي للجسم $a = g = -9.81\ m/s^2 \downarrow$



$$\boxed{a_n = 9.81}$$

$$\boxed{a_t = 0}$$

$$a_n = \frac{V^2}{\rho}$$

$$9.81 = \frac{(50)^2}{\rho}$$

$$\boxed{\rho = 254.8\ m}$$

$$a_n = 9.81 \cos\alpha$$

$$a_t = 9.81 \sin\alpha$$

$$a_n = \frac{V^2}{\rho}$$

$$\Rightarrow \rho = \frac{V^2}{a_n}$$

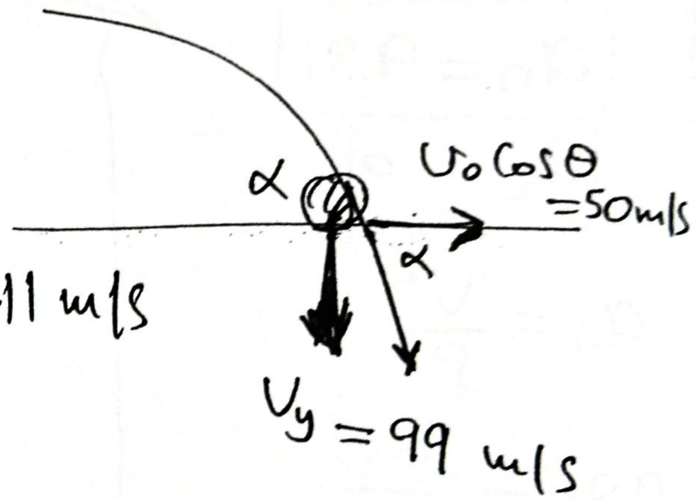
$\theta = 0$
$v_0 = 50\, m/s$
$H = -500\, m$

سؤال رقم
12-112 (ص ٥٧)

$$v_y^2 = (v_0 \sin\theta)^2 - 2gH$$

$$v_y^2 = (\cancel{50 \sin 0})^2 - 2(9.81)(-500)$$

$$\boxed{v_y = 99\ m/s}$$



* $v_0 \cos\theta = 50 \cos 0$
$= 50\, m/s$

* $v_B = \sqrt{(99)^2 + (50)^2} = 111\ m/s$

* $\tan\alpha = \dfrac{99}{50}$

$$\boxed{\alpha = 63.2^\circ}$$

كوشن ∴ $a_n = 9.81 \cos 63.2^\circ = \boxed{4.4\ m/s}$

$a_t = 9.81 \sin 63.2^\circ = \boxed{8.8\ m/s}$

$\rho = \dfrac{(111)^2}{4.4} \Rightarrow \boxed{\rho = 2800\ m}$

"Dependent Motion"

* **Pulleys** : فكرة هذه المسائل تقوم على أن طول الحبل يساوي ثابت

وبالتالي مشتقة هذا الثابت تساوي صفراً .

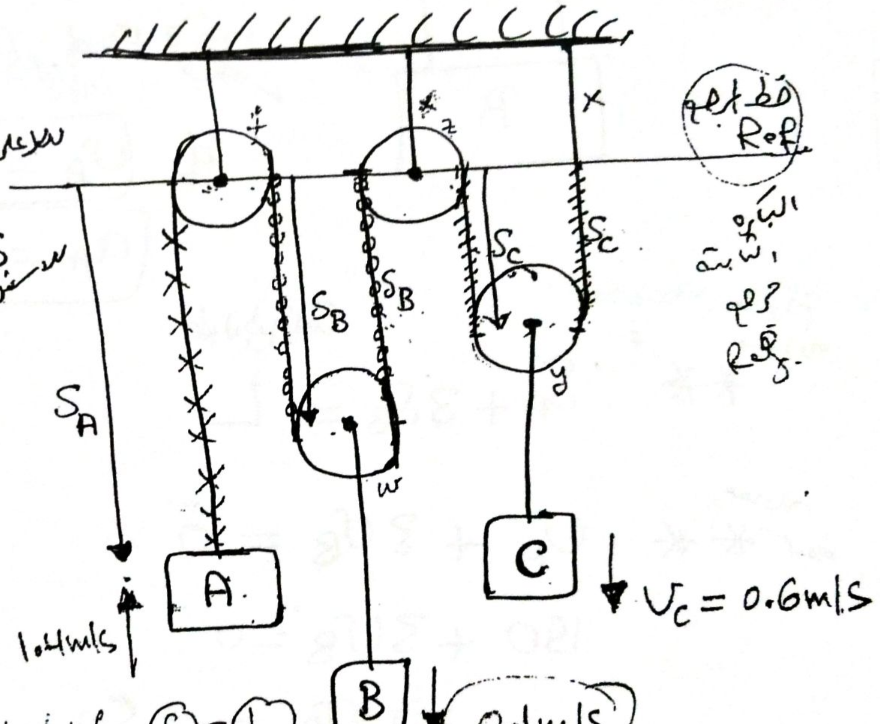
Ex)

$v_A = 1.4\, m/s$ للأعلى

$v_C = -0.6\, m/s$ للأسفل

Find (v_B) ?

أوجد سرعة B



$x + 2S_C + y + z + 2S_B + w + f + S_A = L$

$0 + 2v_C + 0 + 0 + 2v_B + 0 + 0 + v_A = 0$

طول الحبل ثابت

الجواب

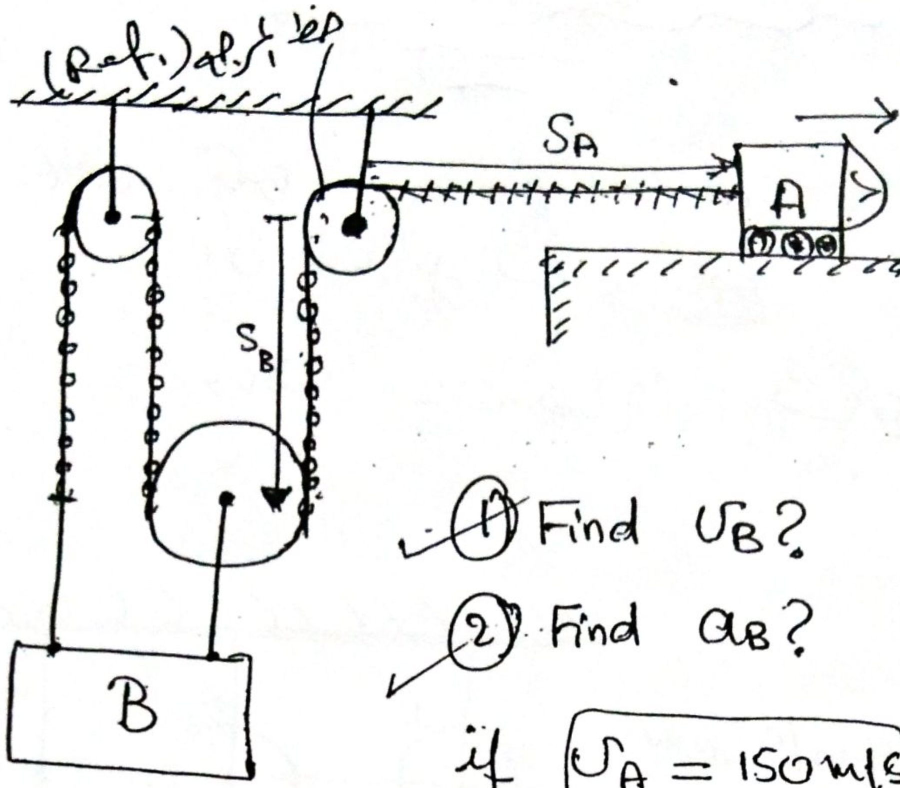
معادلة المسافة $$2S_C + 2S_B + S_A = L$$

معادلة السرعة $$2v_C + 2v_B + v_A = 0$$

$$2(-0.6) + 2v_B + 1.4 = 0 \Rightarrow \boxed{v_B = -0.1\, m/s}$$

معادلة التسارع $$2a_C + 2a_B + a_A = 0$$

ex



① Find v_B?

② Find a_B?

if $\boxed{v_A = 150\ m/s}$ (عين)

$\boxed{a_A = 30\ m/s^2}$ (أعطى)

معادلة مسافة **

طول الحبل ثابتة

$$S_A + 3S_B = L$$

معادلة سرعة **

$$v_A + 3v_B = 0$$

$$150 + 3v_B = 0$$

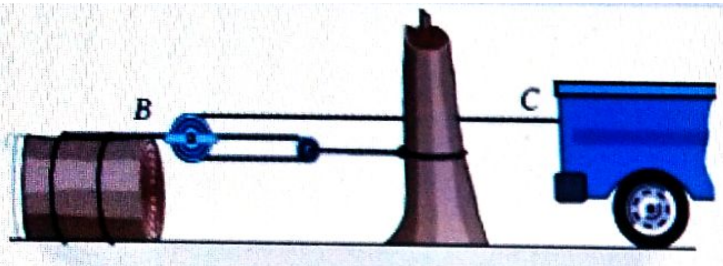
$$\Rightarrow v_B = -50\ m/s$$

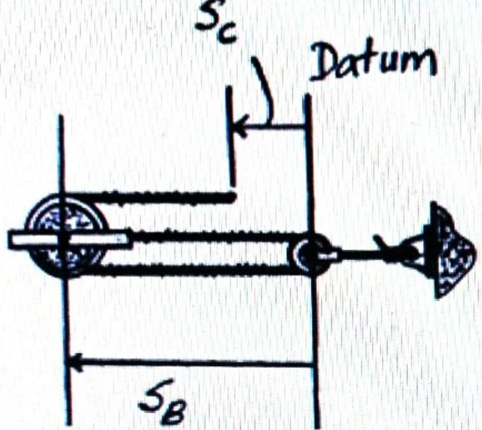
معادلة تسارع **

$$a_A + 3a_B = 0$$

$$30 + 3a_B = 0$$

$$a_B = -10\ m/s^2$$

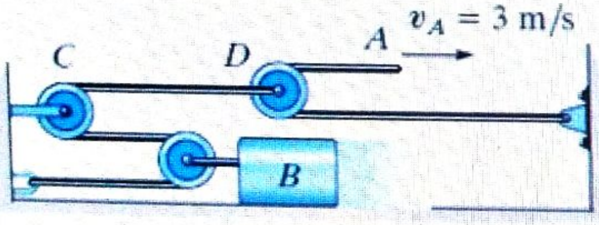


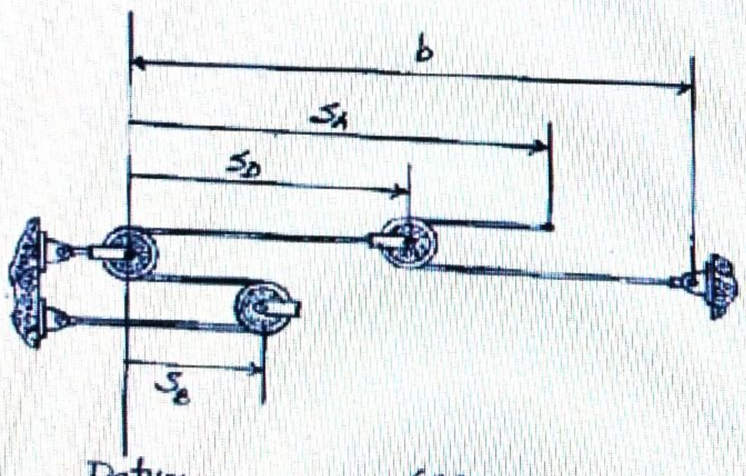


$$2s_B + (s_B - s_C) = l$$

$$3s_B - s_C = l$$

$$3\Delta s_B - \Delta s_C = 0$$



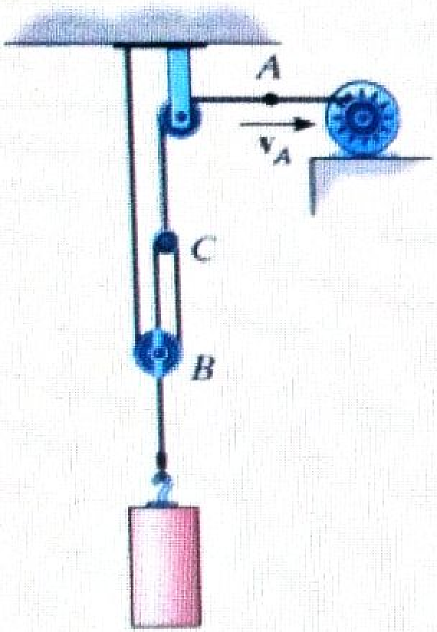


(a)

$$2\,s_B + s_D = l_1$$

$$(s_A - s_D) + (b - s_D) = l_2$$

$$s_A - 2\,s_D = l_2 - b$$

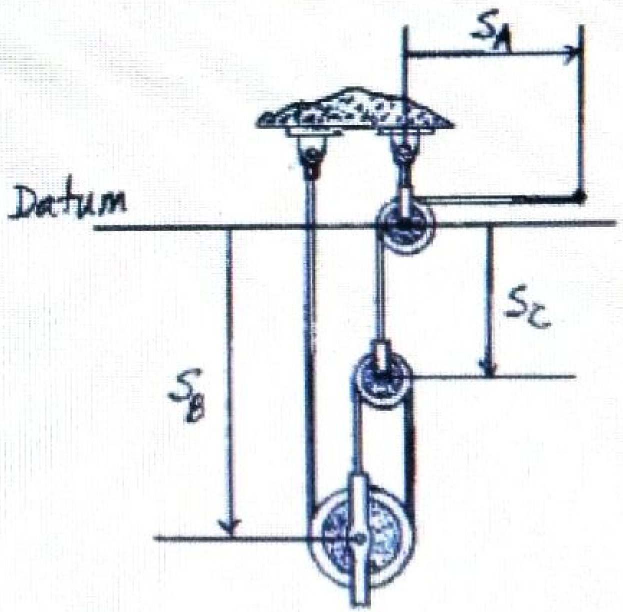


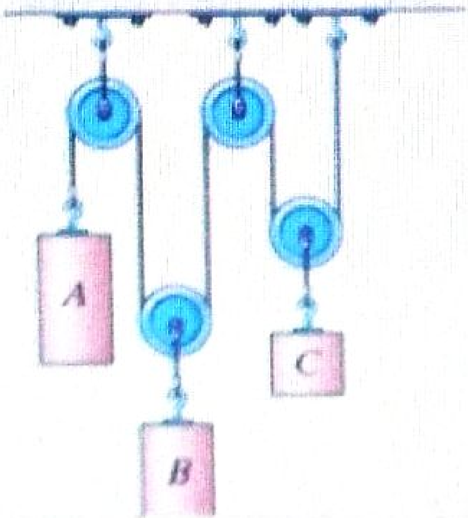
$$s_B + 2(s_B - s_C) = l_1$$

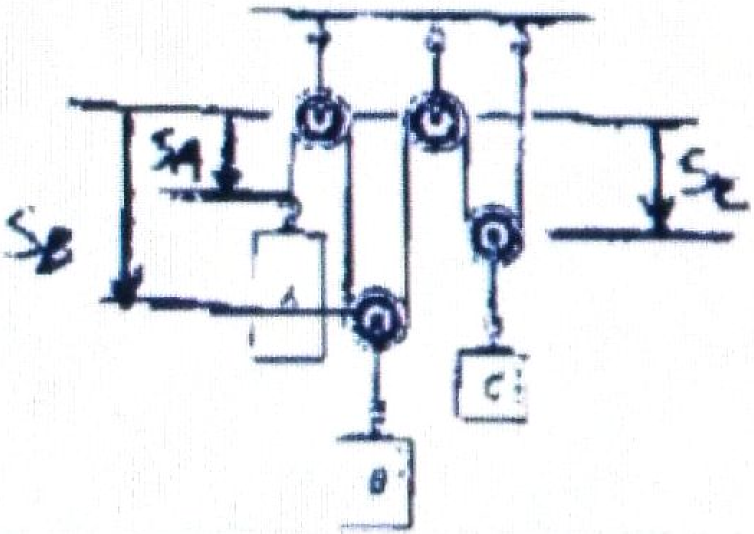
$$3s_B - 2s_C = l_1$$

And

$$s_C + s_A = l_2$$

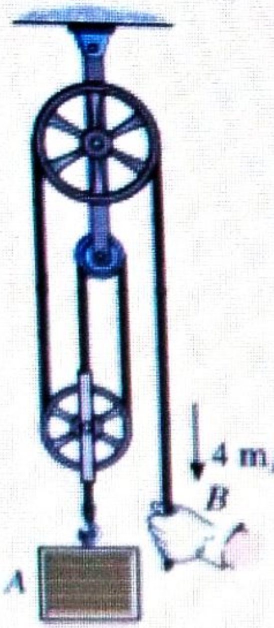


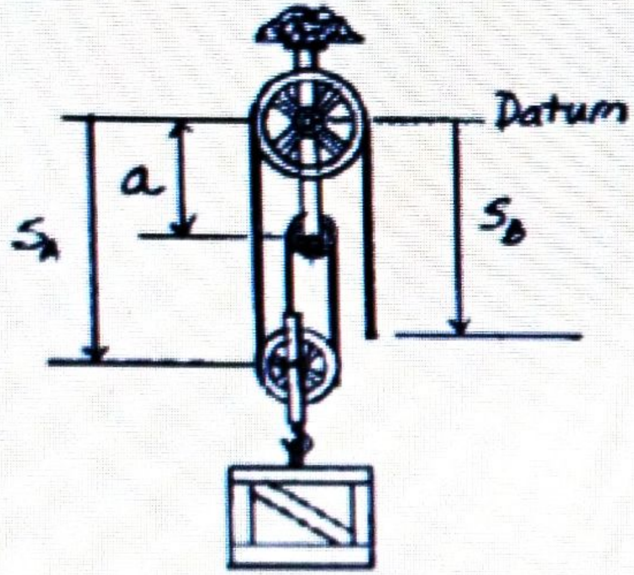




$$s_A + 2s_B + 2s_C = l$$

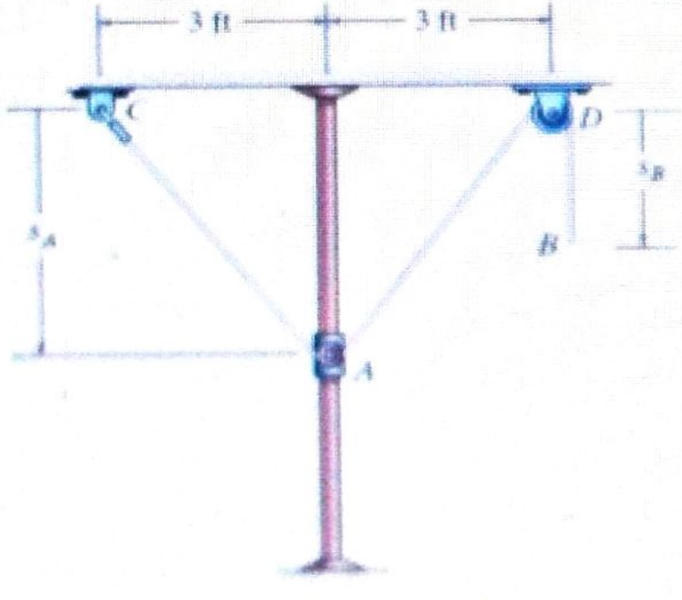
$$v_A + 2v_B + 2v_C = 0$$





$$s_B + s_A + 2(s_A - a) = l$$

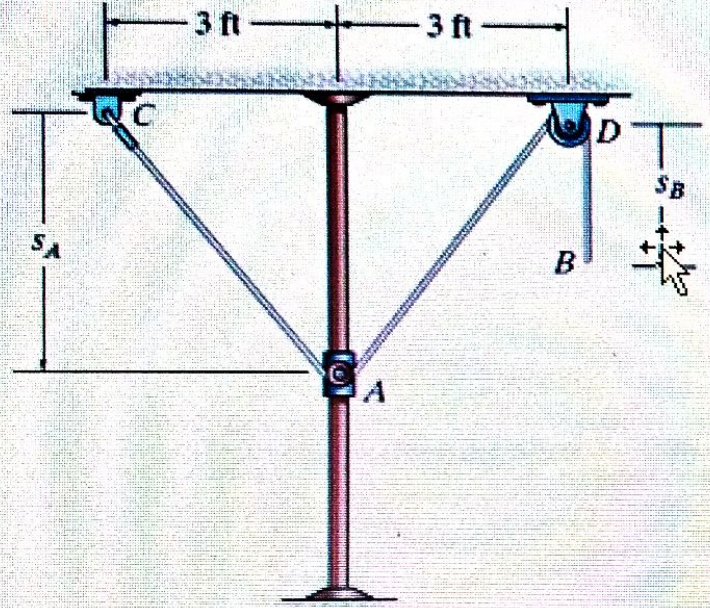
$$s_B + 3s_A = l + 2a$$



$$2\sqrt{s_A^2 + 3^2} + s_B = l$$

$$2\left(\frac{1}{2}\right)\left(s_A^2 + 9\right)^{-\frac{1}{2}}\left(2s_A\,\dot{s}_A\right) + \dot{s}_B = 0$$

$$\dot{s}_B = -\frac{2s_A\dot{s}_A}{\left(s_A^2 + 9\right)^{\frac{1}{2}}}$$



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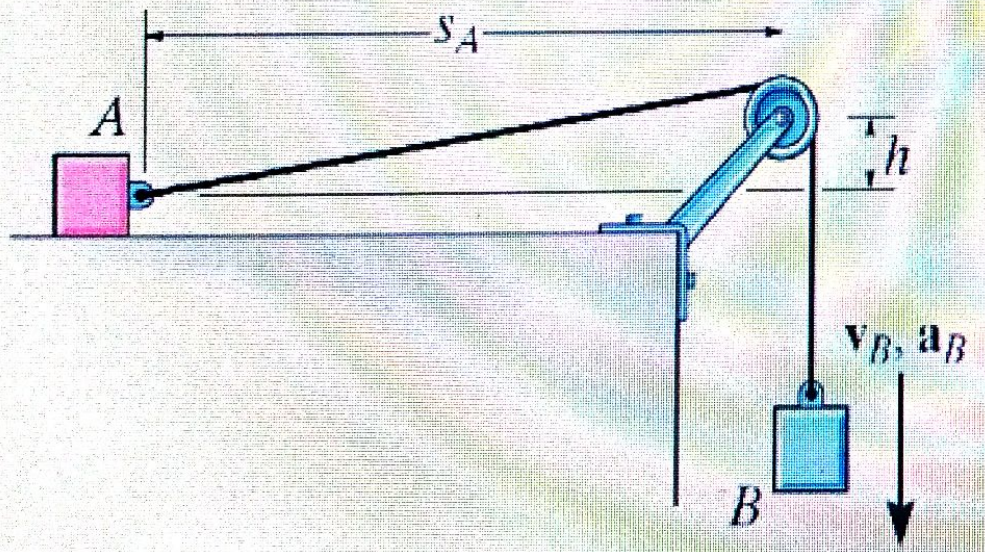
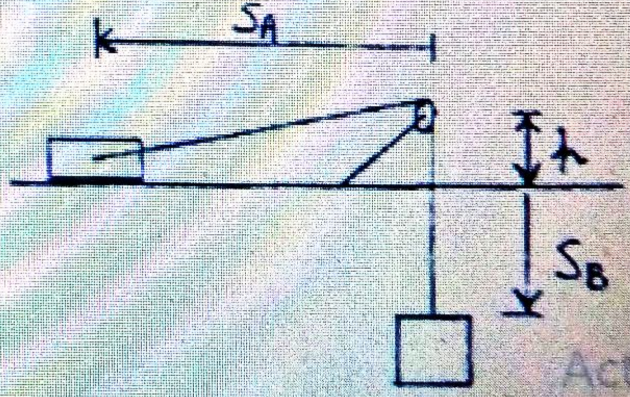


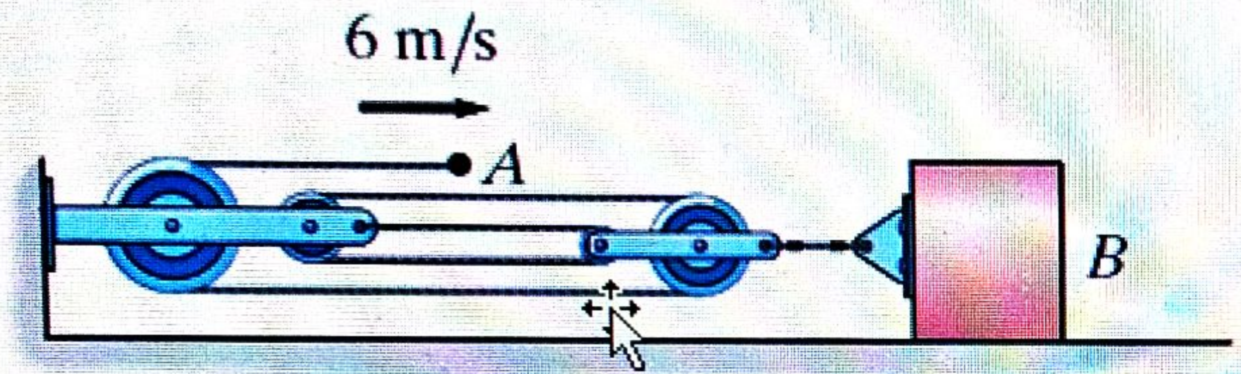
$$l = s_B + \sqrt{s_B^2 + h^2}$$

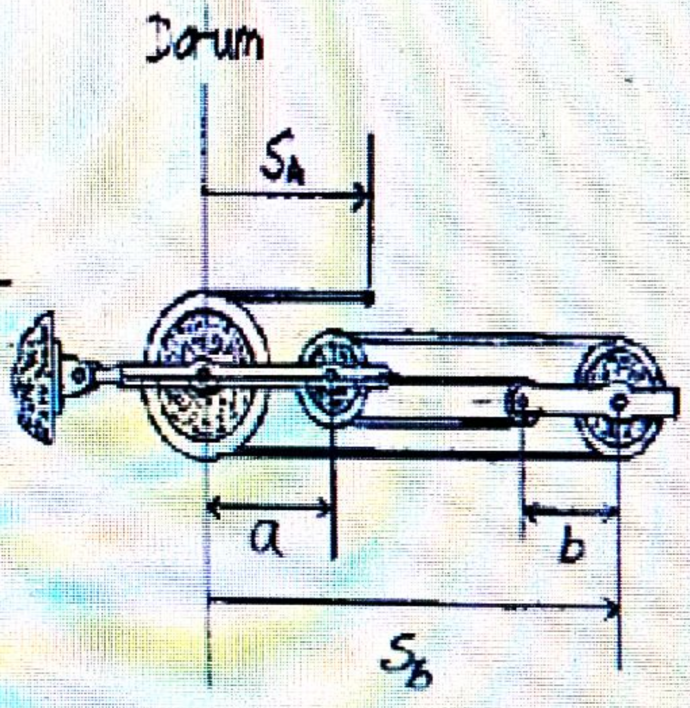
$$0 = \dot{s}_B + \frac{1}{2}(s_A^2 + h^2)^{-1/2}\,2s_A\,\dot{s}_A$$

$$v_A = \dot{s}_A = \frac{-\dot{s}_B(s_A^2 + h^2)^{1/2}}{s_A}$$

$$v_A = -v_B\left(1 + \left(\frac{h}{s_A}\right)^2\right)^{1/2}$$





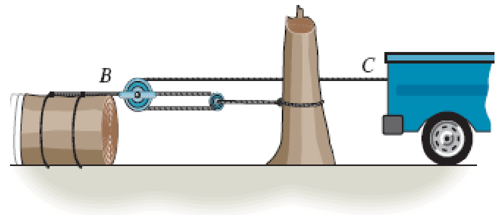


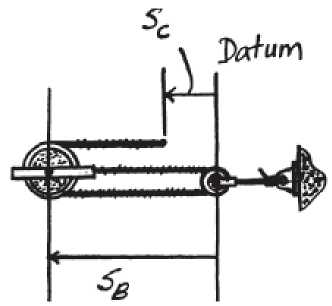
$$s_B + 2(s_B - a - b) + (s_B - a) + s_A = l$$

$$4s_B + s_A = l + 3a + 2b$$

Time Derivative. Taking the time derivative of Eq. (1),

$$4v_B + v_A = 0$$





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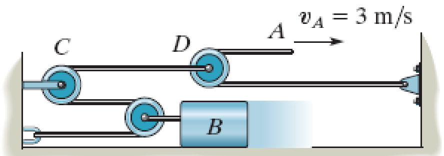

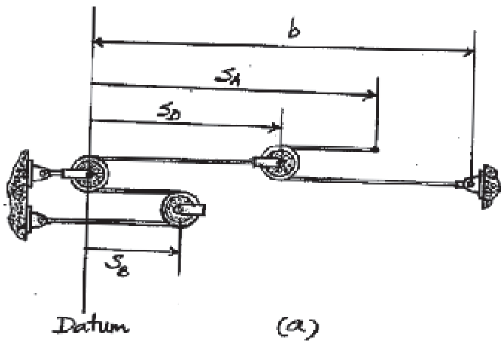

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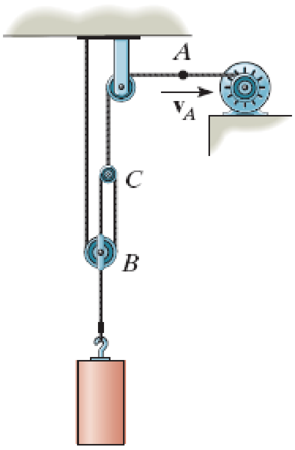

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And

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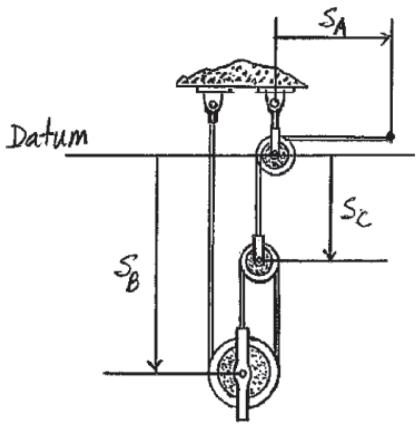

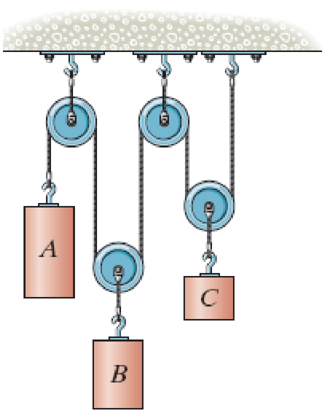

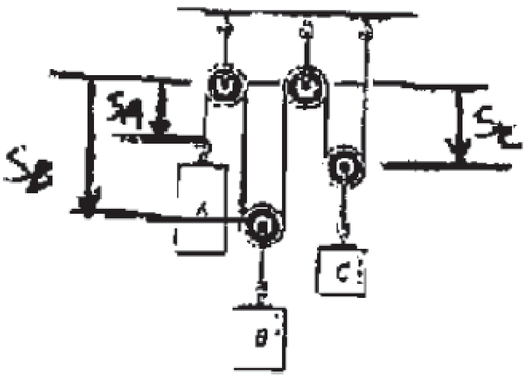

$$s_A + 2s_B + 2s_C = l$$

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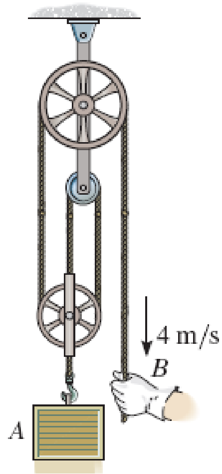

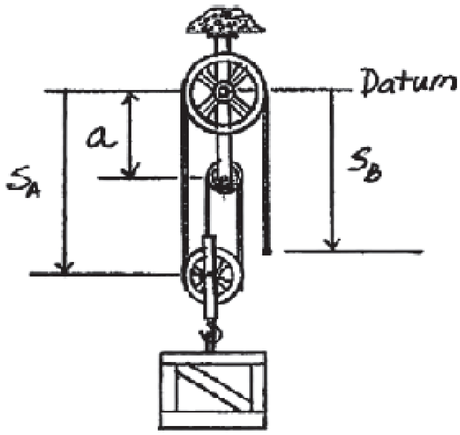

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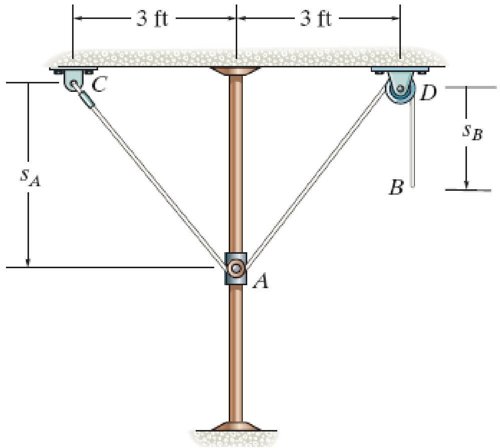

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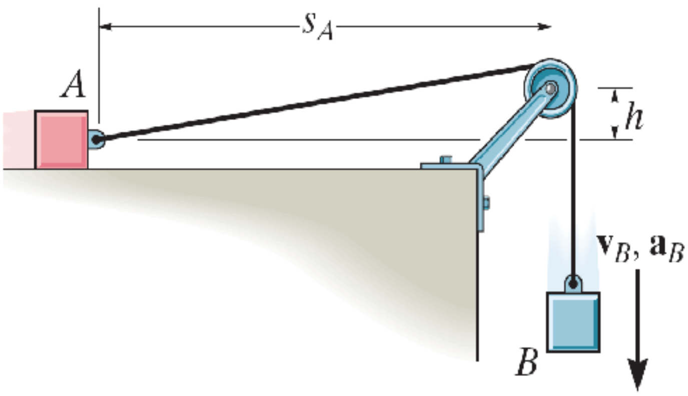

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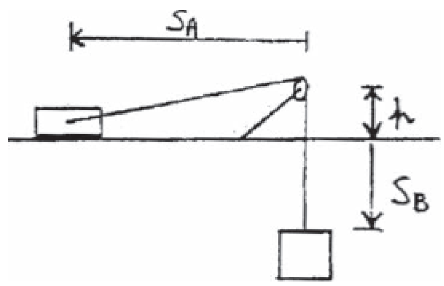

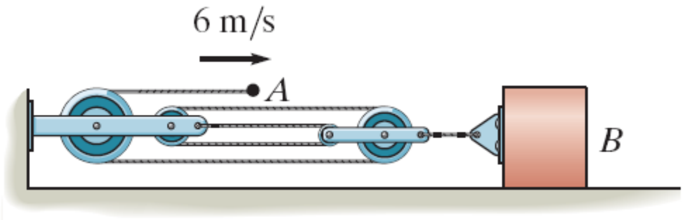

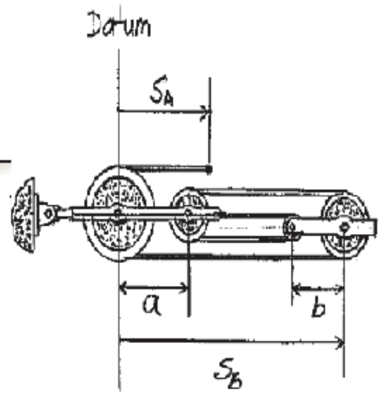

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Time Derivative. Taking the time derivative of Eq. (1),

$$4v_B + v_A = 0$$

Dynamics

Dr. Hashem Alkhaldi

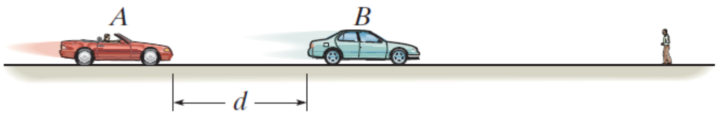
Suggested Problems: Chapter 12

(28 problems, 6 pages)

g= 9.81 m/s^2 = 32.2 ft/s^2

==

Q1 Car B is traveling a distance d ahead of car A. Both cars are traveling at **80** ft/s when the driver of B suddenly applies the brakes, causing his car to decelerate at **10** ft/s^2. It takes the driver of car A **0.65** s to react (this is the normal reaction time for drivers). When he applies his brakes, he decelerates at **17** ft/s^2. Determine the minimum distance d be tween the cars so as to avoid a collision.



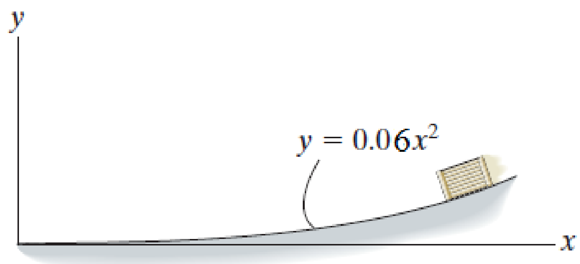
Q2 The acceleration of a rocket traveling upward is given by $a = (\mathbf{8} + 0.02s)$ m/s^2, where s is in meters. Determine the time needed for the rocket to reach an altitude of $s = \mathbf{120}$ m. Initially, $v = 0$ and $s = 0$ when $t = 0$.

Q3 **a)** A particle is moving along a straight line such that its acceleration is defined as $a = (\mathbf{-3}v)$ m/s^2, where v is in meters per second. If $v = \mathbf{28}$ m/s when $s = 0$ and $t = 0$, determine the particle's position, velocity, and acceleration as functions of time.

b) The acceleration of a particle traveling along a straight line is $a = \frac{\mathbf{2}}{\mathbf{3}} s^{1/2}$ m/s^2, where s is in meters. If $v = 0$, $s = \mathbf{2}$ m when $t = 0$, determine the particle's velocity at $s = \mathbf{3}$ m.

Q4 A ball A is thrown vertically upward from the top of a **50**-m-high building with an initial velocity of **7** m/s. At the same instant another ball B is thrown upward from the ground with an initial velocity of **28** m/s. Determine the height from the ground and the time at which they pass.

Q5 The box slides down the slope described by the equation $y = (0.06x^2)$ m, where x is in meters. If the box has x components of velocity and acceleration of $v_x = -4$ m/s and $a_x = -1.5$ m/s^2 at $x = 6$ m, determine the y components of the velocity and the acceleration of the box at this instant.

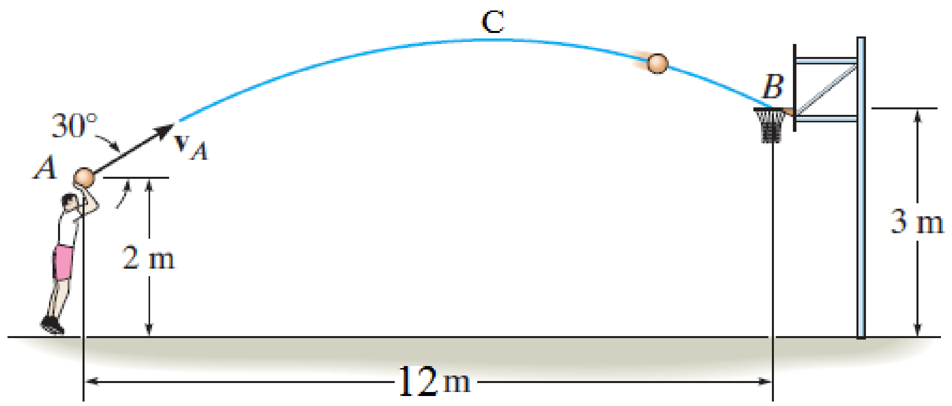


Q6 The velocity of a particle is given by $v = \{25t^2\mathbf{i} + 4t^3\mathbf{j} + (5t + 3)\mathbf{k}\}$ m/s, where t is in seconds. If the particle is at the origin when $t = 0$, determine the magnitude of the particle's acceleration when $t = 3$ s. Also, what is the x, y, z coordinate position of the particle at this instant?

Q7 a) Neglecting the size of the ball, determine the magnitude v_A of the basketball's initial velocity and its velocity when it passes through the basket.

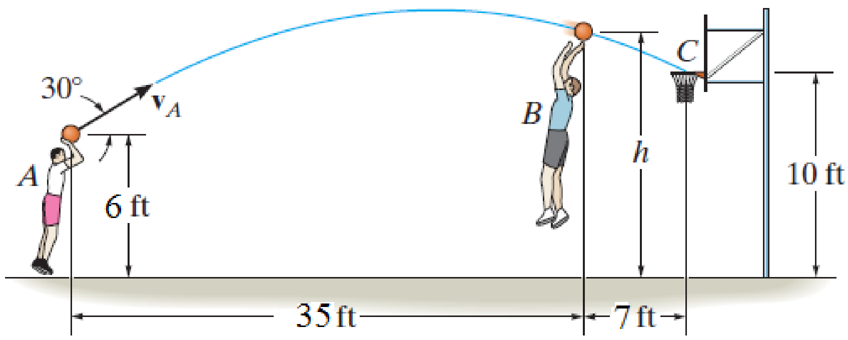
b) What's the Max. height. Find the flight time and the velocity at the basket.

c) Find the radius of curvture at points A, B, and C.



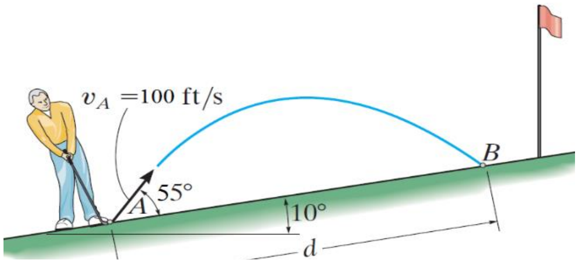
Q8 a) The basketball passed through the hoop even though it barely cleared the hands of the player B who attempted to block it. Neglecting the size of the ball, determine the magnitude v_A of its initial velocity and the height h of the ball when it passes over player B.

b) Find the impact angle with the basket board?

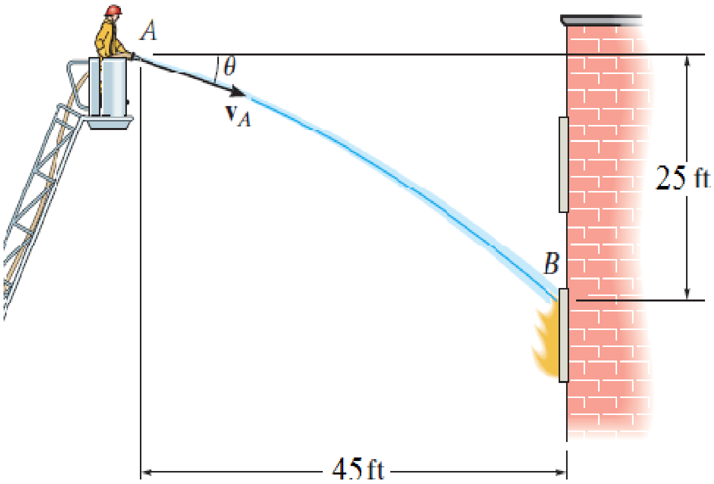


Q9 **a)** A golf ball is struck with a velocity of 100 ft/s as shown. Determine the distance d to where it will land.

b) A golf ball is struck with a velocity of 100 ft/s as shown. Determine the speed at which it strikes the ground at B and the time of flight from A to B.

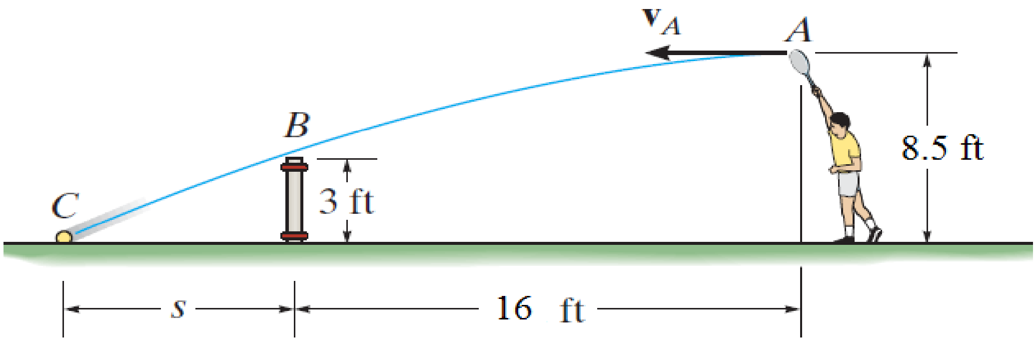


Q10 The fireman wishes to direct the flow of water from his hose to the fire at B. Determine two possible angles θ_1 and θ_2 at which this can be done. Water flows from the hose at v_A =100 ft/s. Find the flight time and the velocity and the attack angle of the water stream at point B.



Q11 Determine the horizontal velocity v_A of a tennis ball at A so that it just clears the net at B. Also, find the distance s where the ball strikes the ground.

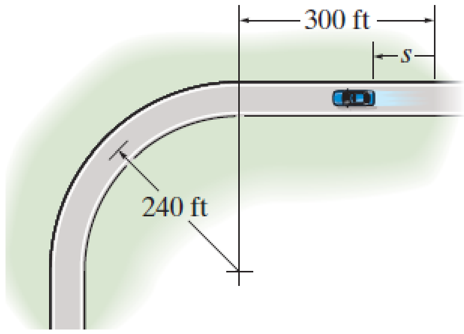
- Find the angle the ball strikes the ground at C.
- Find the radius of curvture of the projectile at point B.



Q12 The position of a particle is defined by $\mathbf{r} = \{5(t - \sin t)\mathbf{i} + (2t^2 - 3)\mathbf{j}\}$ m, where t is in seconds and the argument for the sine is in radians. Determine the speed of the particle and its normal and tangential components of acceleration when $t = 2$ s.

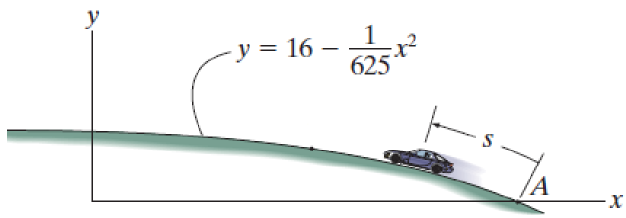
Q13 The automobile is originally at rest at $s = 0$. If its speed is increased by $\dot{v} = (0.06t^2)$ ft/s^2, where t is in seconds, *determine the magnitudes of its velocity and acceleration when $t = 24$ s.

*determine the magnitudes of its velocity and acceleration at $s = 640$ ft.

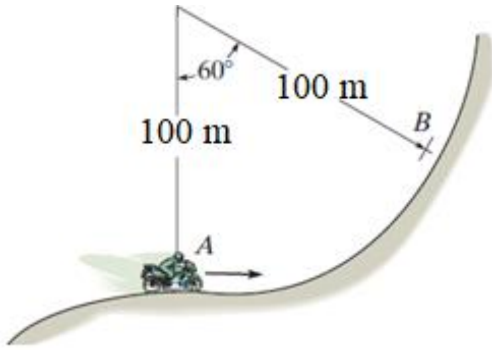


Q14 If the car passes point A with a speed of 30m/s and begins to increase its speed at a constant rate of $a_t = 0.7$ m/s^2, *determine the magnitude of the car's acceleration when $s = 102$ m and $x = 0$.

*determine the magnitude of the car's acceleration when it is at point A.

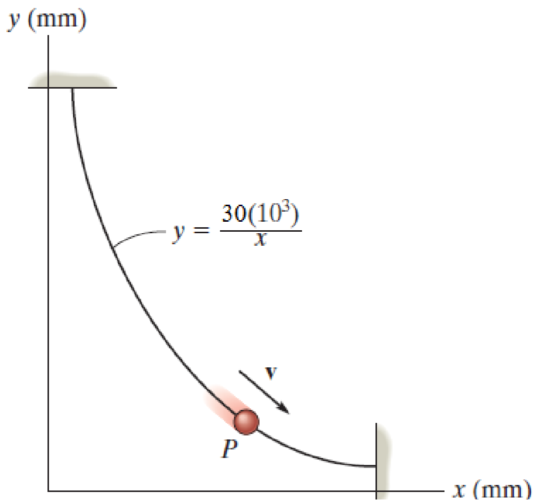


Q15 The motorcycle is traveling at 40 m/s when it is at A. If the speed is then decreased at $\dot{v} = -(0.08\,s)$ m/s^2, where s is in meters measured from A, determine its speed and acceleration when it reaches B.



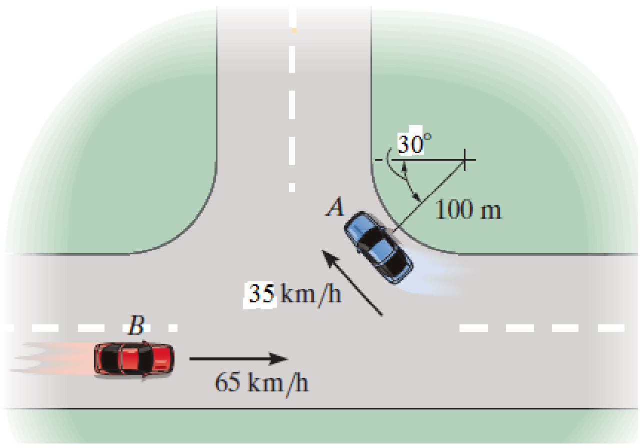
Q16 The particle travels with a constant speed of 250 mm/s along the curve. Determine the particle's acceleration when it is located at point (200 mm, 150mm) and sketch this vector on the curve.

At this location, find:
- Tangential acceleration
- Normal acceleration
- Acceleration in x direction (a_x)
- Acceleration in y direction (a_y)



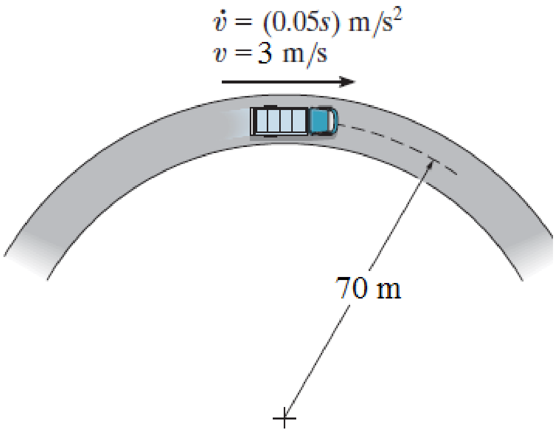
Q17 At the instant shown, cars A and B are traveling at the speeds shown. If A is accelerating at 1200 km/h^2 while B maintains a constant speed, determine the velocity and acceleration of A with respect to B.

- determine the velocity and acceleration of A with respect to B after **3** seconds.
- determine the velocity and acceleration of A with respect to B after **10** seconds.



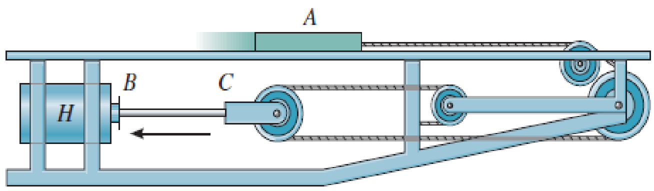
Q18 The truck travels in a circular path having a radius of 70 m at a speed of $v = 3$ m/s. For a short distance from $s = 0$, its speed is increased by $\dot{v} = (0.05s)$ m/s², where s is in meters. Determine its speed and the magnitude of its acceleration when it has moved $s = 12$ m.

- repeat solving th problem if the truck moves with a constant acceleration of $a_t = 1.25$ m/s²

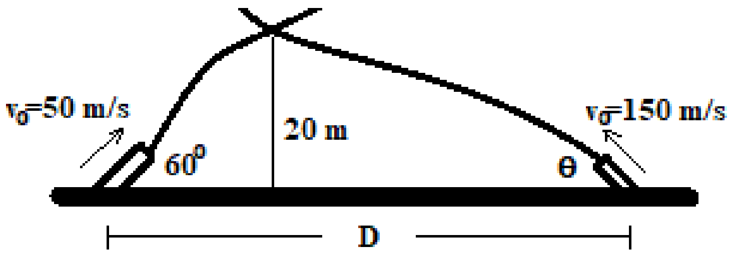


Q19 Determine the time needed for the load at B to attain a speed of 12 m/s, starting from rest, if the cable is drawn into the motor with an acceleration of 0.4 m/s².

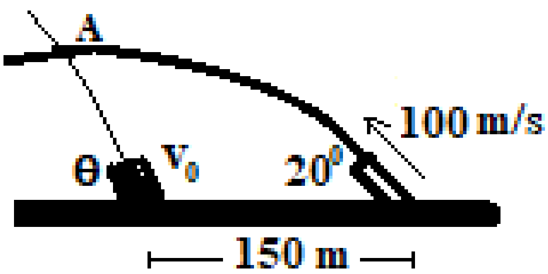
Q20 If the hydraulic cylinder H draws in rod BC at 4 ft/s, determine the speed of slider A.



Q21 Two cannons fired shells at the same time. If the shells met together at 20 m height. Find the distance D between the two cannons and the firing angle θ.

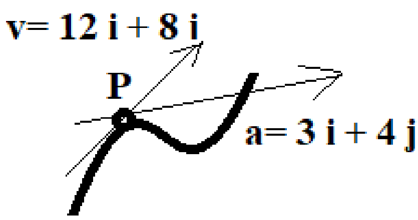


Q22 Two cannons 150 m apart fired shells at the same time. If one shell hits the other when it reached the maximum height at **A**. Find the firing velocity v_0 and the firing angle θ of the shell.

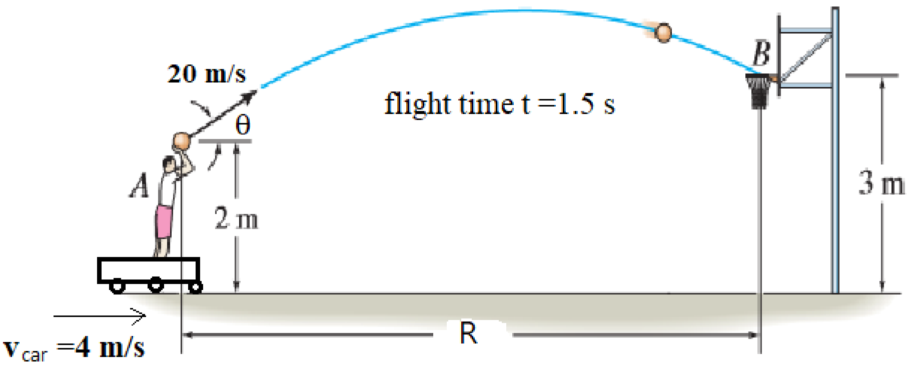


Q23 **P** is a point moving on a curved path with the velocity **v** (m/s) and acceleration **a** (m/s²) shown. At point **P**, find:

- The tangential acceleration a_t.
- The normal acceleration a_n
- The radius of curvature ρ.

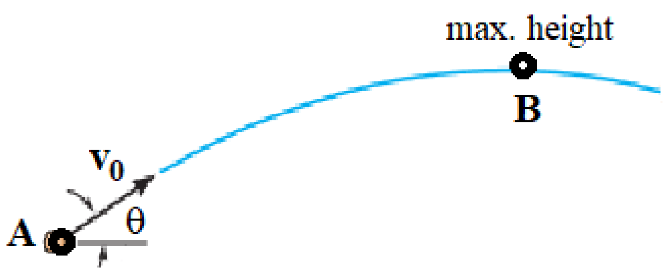


Q24 Neglecting the size of the ball, determine the magnitude of the throwing angle θ and the distance R from the basketball if it needs 1.5 seconds the ball to reach the basket. The player is standing over a cart moving to the **right** with constant velocity v_{car}= 4 m/s. The throwing speed of the ball is 20 m/s with respect to the car.



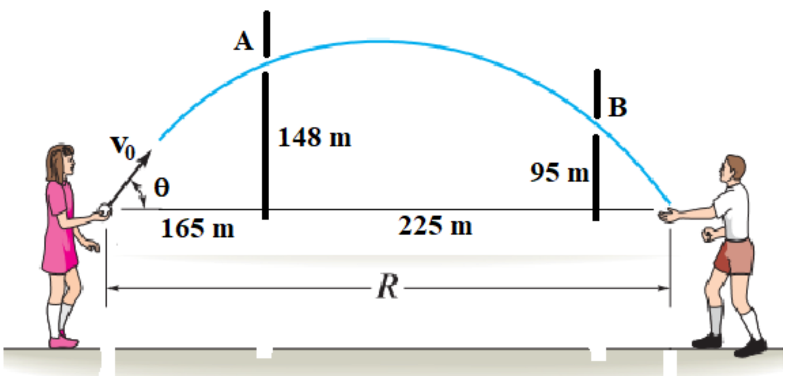
- Resolve this problem again assuming the cart is moving to the **left** (instead of right) with the same velocity v_{car}= 4 m/s.

Q25 For the shown projectile, assume the radii of curvature are measured at a starting point **A** and at the maximum height **B** as ρ_A= **10 m** and ρ_B= **5 m**, respectively. Determine the magnitude of the angle θ and the initial velocity v_0 at the throwing point **A**. Find the maximum height.

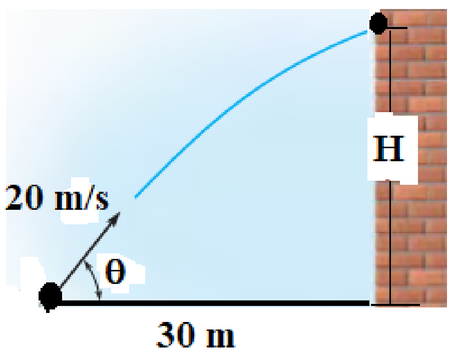


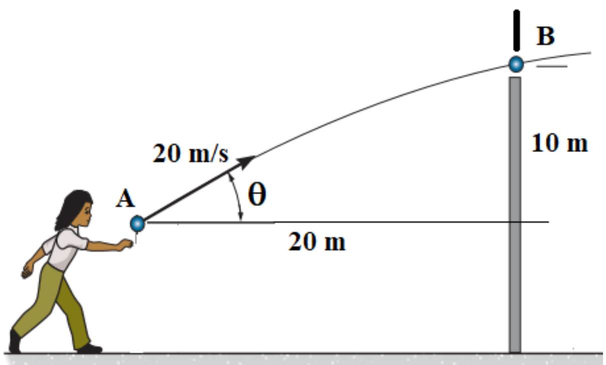
Q26 A ball is thrown as it passes through the two windows at **A** and **B**. Find:

- The initial velocity v_0.
- The firing angle θ.
- The horizontal distance R.
- The maximum height H_{max}.
- The flight time t.



Q27 Find the maximum possible height **H** the ball can reach on the opposite vertical wall 30 m faraway.





Q28 Find all possible values of angle **θ** that the ball can be thrown by 20 m/s from the girl at **A** to enter the upper window at **B**.