Dr Hashem AlKhaldi

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Kinetics of a Particle: Force and Acceleration

CHAPTER OBJECTIVES

- To state Newton's Second Law of Motion and to define mass and weight.
- To analyze the accelerated motion of a particle using the equation of motion with different coordinate systems.
- To investigate central-force motion and apply it to problems in space mechanics.

Video Solutions are
available for selectec
questions in this cha

13.1 Newton's Second Law of Motion

Kinetics is a branch of dynamics that deals with the relationship between the change in motion of a body and the forces that cause this change. The basis for kinetics is Newton's second law, which states that when an *unbalanced force* acts on a particle, the particle will *accelerate* in the direction of the force with a magnitude that is proportional to the force.

This law can be verified experimentally by applying a known unbalanced force $\mathbf{F}$ to a particle, and then measuring the acceleration $\mathbf{a}$. Since the force and acceleration are directly proportional, the constant of proportionality, m, may be determined from the ratio $m = F/a$. This positive scalar m is called the *mass* of the particle. Being constant during any acceleration, m provides a quantitative measure of the resistance of the particle to a change in its velocity, that is its inertia.

The jeep leans backward due tc
which resists its forward acceler

$$\sum F_x = m a_x \qquad \sum F_t = m a_t = m\frac{dv}{dt}$$

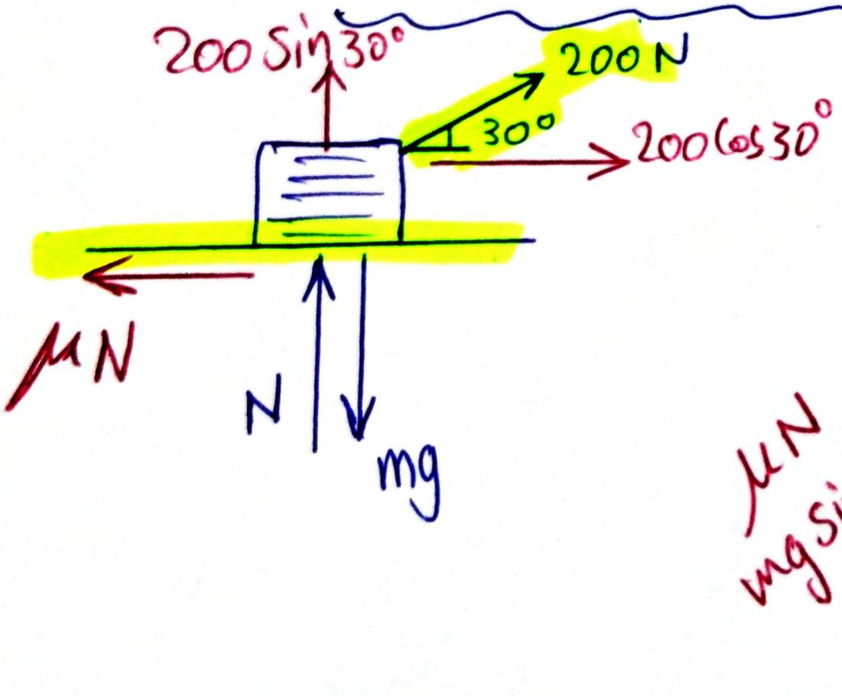
$$\sum F_y = m a_y \qquad \sum F_n = m a_n = m\frac{v^2}{\rho}$$

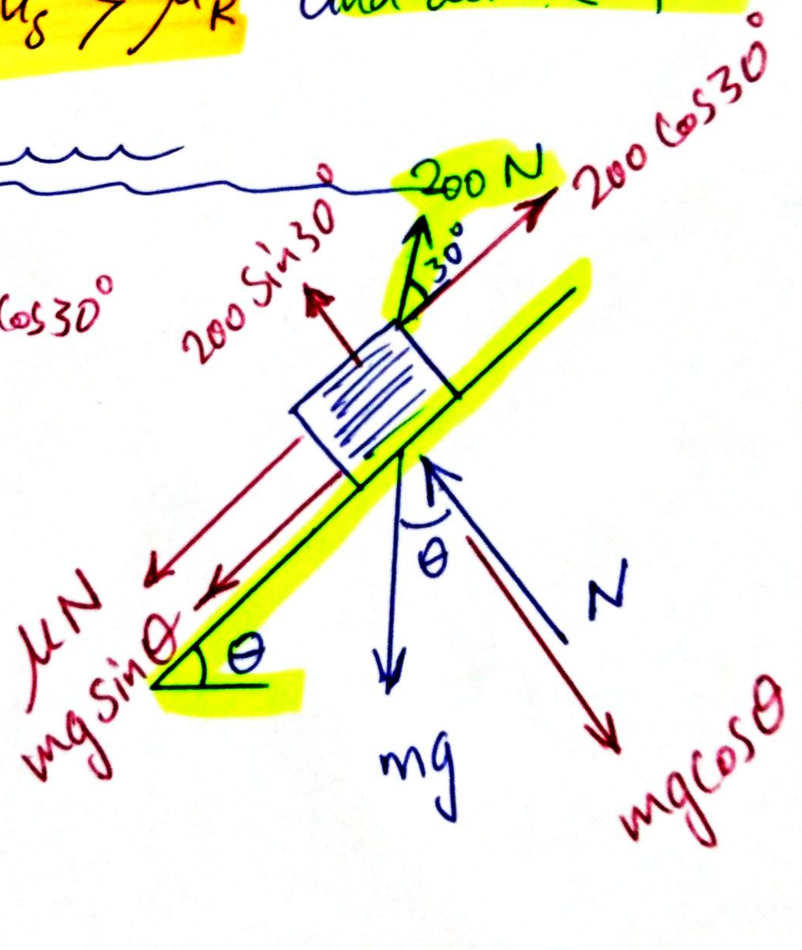
Friction Force :

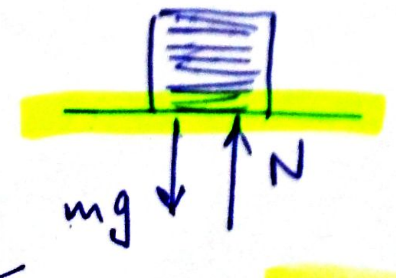
Static $\rightarrow F_f = \mu_s N$

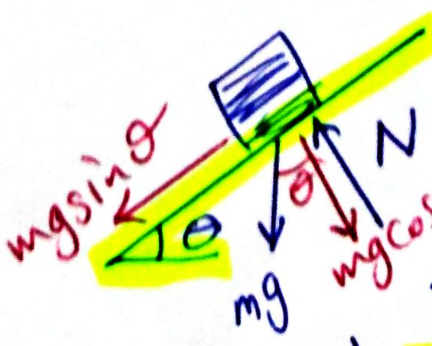
dynamic $\rightarrow F_f = \mu_k N$

$\Rightarrow \mu_s > \mu_k$ and all < 1







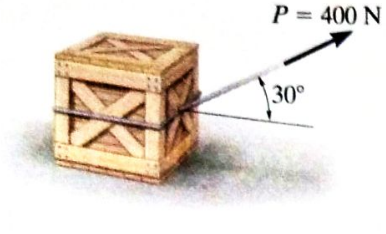


$\sum F_y = 0 \Rightarrow N = mg$

$\sum F_y = 0$

$\Rightarrow N = mg\cos\theta$

EXAMPLE 13.1



(a)

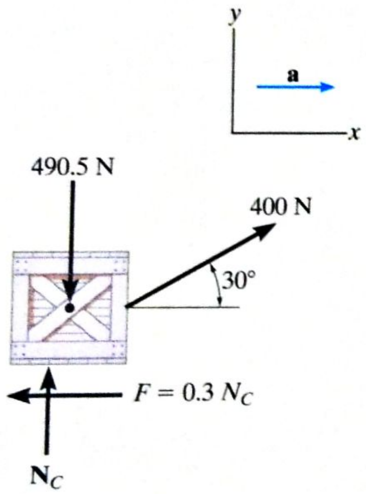
The 50-kg crate shown in Fig. 13–6*a* rests on a horizontal surface for which the coefficient of kinetic friction is $\mu_k = 0.3$. If the crate is subjected to a 400-N towing force as shown, determine the velocity of the crate in 3 s starting from rest.

SOLUTION

Using the equations of motion, we can relate the crate's acceleration to the force causing the motion. The crate's velocity can then be determined using kinematics.

Free-Body Diagram. The weight of the crate is $W = mg = 50\text{ kg}\,(9.81\text{ m/s}^2) = 490.5\text{ N}$. As shown in Fig. 13–6*b*, the frictional force has a magnitude $F = \mu_k N_C$ and acts to the left, since it opposes the motion of the crate. The acceleration **a** is assumed to act horizontally, in the positive x direction. There are two unknowns, namely N_C and a.

Equations of Motion. Using the data shown on the free-body diagram, we have



(b)

Fig. 13–6

$$\xrightarrow{+}\Sigma F_x = ma_x; \qquad 400\cos 30^\circ - 0.3N_C = 50a \qquad (1)$$

$$+\uparrow\Sigma F_y = ma_y; \qquad N_C - 490.5 + 400\sin 30^\circ = 0 \qquad (2)$$

Solving Eq. 2 for N_C, substituting the result into Eq. 1, and solving for a yields

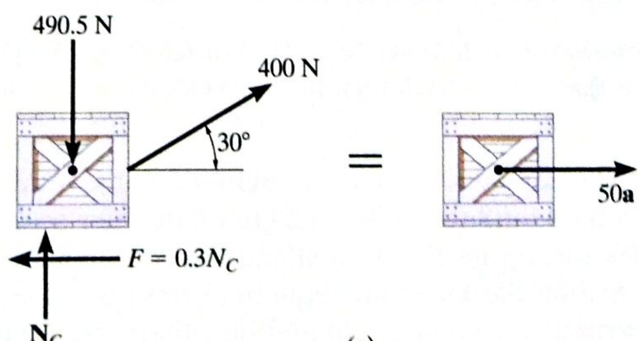
$$N_C = 290.5\text{ N}$$

$$a = 5.185\text{ m/s}^2$$

Kinematics. Notice that the acceleration is *constant*, since the applied force **P** is constant. Since the initial velocity is zero, the velocity of the crate in 3 s is

$$(\xrightarrow{+}) \qquad v = v_0 + a_c t = 0 + 5.185(3)$$

$$= 15.6\text{ m/s} \rightarrow \qquad \textit{Ans.}$$

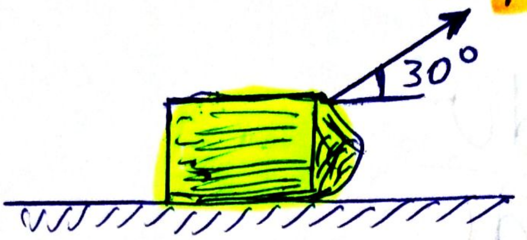


(c)

Ex]

$m = 40\ Kg.$

$F = 8t^2$ (Newton)

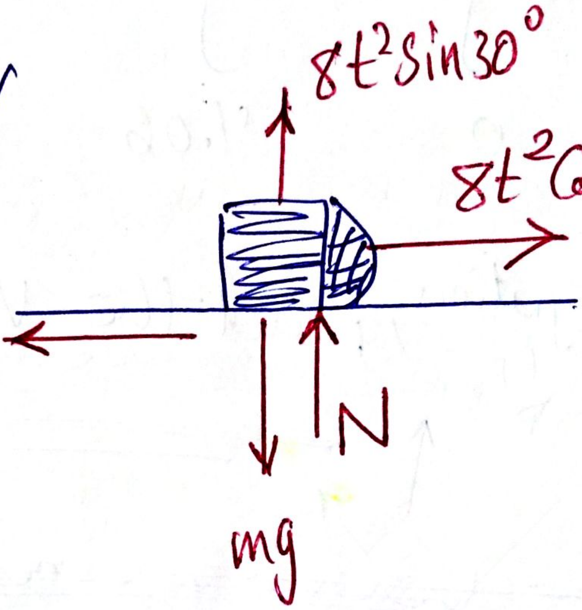


$\mu_s = 0.35$

$\mu_K = 0.20$

1) How long it needs to start moving?

2) Its velocity after 8 seconds?



(1)

$N = \bar{8}t^2 \sin 30° + mg$

$N = -4t^2 + mg$ ✓

$\sum F_x = 0 \Rightarrow$ use μ_s

$\Rightarrow 8t^2 \cos 30° - 0.35(\bar{4}t^2 + mg) = 0$

$\Rightarrow 8t^2 \cos 30° - 0.35(-4t^2 + 40(9.81)) = 0$

$\Rightarrow 8.33\, t^2 = 137.34 \Rightarrow t = 4.06$ Sec

(2) $\sum F_x = m a_x \Rightarrow$ use μ_K

$\Rightarrow 8t^2 \cos 30° - 0.2[-4t^2 + mg] = m\ a_x$

$\Rightarrow$ Find a_x as function of time

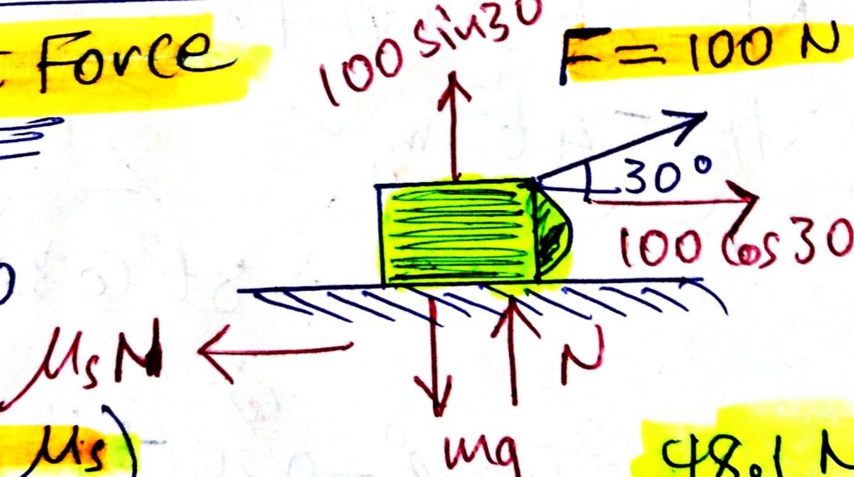
But

$$a_x = \frac{dv}{dt}$$

$$\Rightarrow \int_0^v dv = \int_{4.06}^{8} a_x \, dt$$

do integration, Find the velocity $v = \checkmark$

* Now Constant Force



$m = 10$ kg

$\mu_s = 0.35, \ \mu_k = 0.20$

* $\sum F_x = 0 \Rightarrow$ (use μ_s)

* $\sum F_y = 0 \Rightarrow N = mg - 100 \sin 30 = 48.1$ N

* $\sum F_x = 0 \Rightarrow$ Since $100 \cos 30 > \mu_s N$ object will move

$\Rightarrow 86.6 > 16.8 \Rightarrow$

* $\sum F_x \Rightarrow$ (μ_k is used now)

$\Rightarrow \sum F_x = m a_x \Rightarrow$ $100 \cos 30° - \mu_k N = m a_x$

$\Rightarrow$ Find $a_x \Rightarrow$

⇒ Solve for

⇒ $a_x = 7.7 \ m/s^2$

* Find the velocity after 8 seconds ?

⇒ Since the acceleration is Constant accel. then use :

⇒ $v = v_0 + at$

⇒ $v = 0 + 7.7(8)$

$v = 61.6 \ m/s$

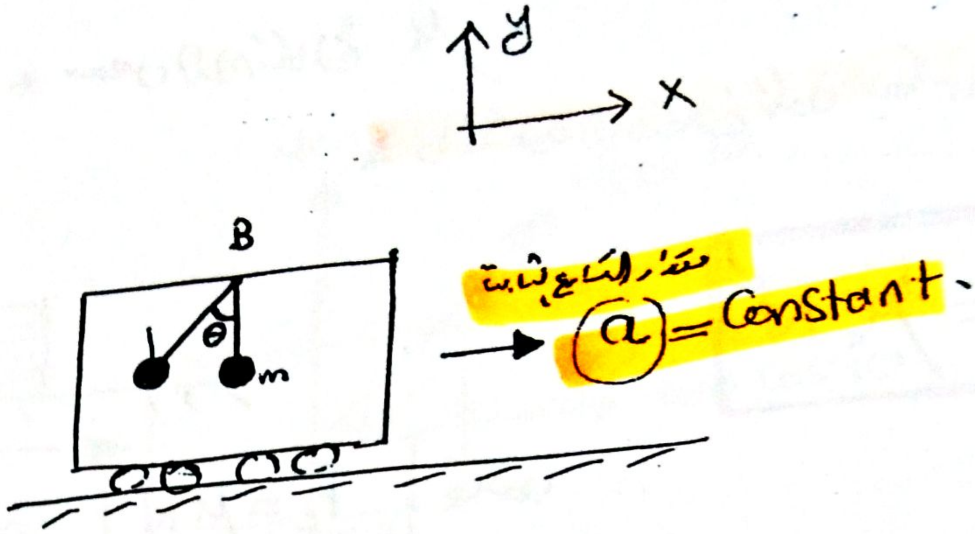
* Find the distance traveled when its velocity reaches to 50 m/s

⇒ $v^2 = v_0^2 + 2a_x S$

⇒ $(50)^2 = (0)^2 + 2(7.7) S$

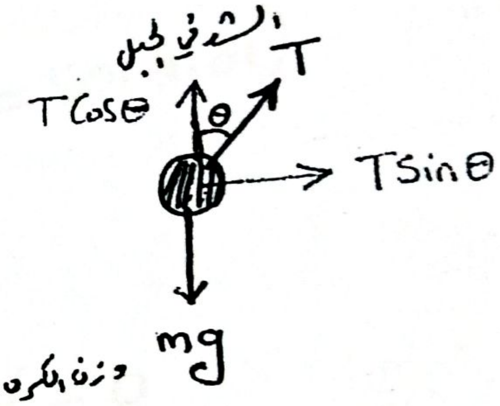
⇒ $S = 162.3 \ m$

Ex



Find the angle θ ?

السياره تسير بتسارع ثابت مقداره (a)



$\sum F_y = m a_y$ → لا يوجد حركه باتجاه y

$T\cos\theta - mg = m(0)$

$T\cos\theta = mg$ —— ①

$\sum F_x = m a_x$

$T\sin\theta = ma$ —— ②

اقسم المعادله الثانيه على الاولى

$$\frac{\cancel{T}\sin\theta}{\cancel{T}\cos\theta} = \frac{\cancel{m}a}{\cancel{m}g}$$

$$\tan\theta = \frac{a}{g}$$

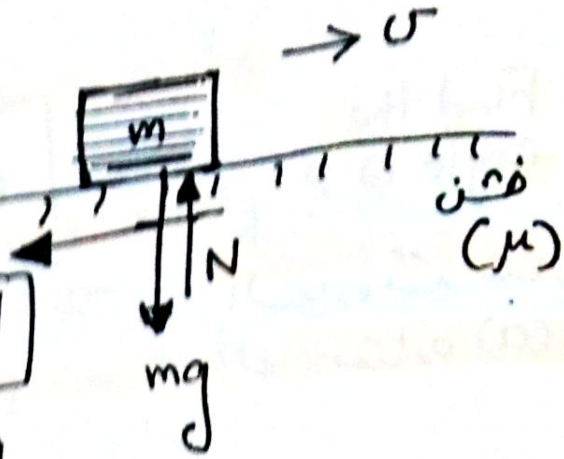
$$\theta = \tan^{-1}\frac{a}{g}$$

Friction (F_f)

* معامل الاحتكاك μ

* دائماً قوة الاحتكاك تكون عكس اتجاه الحركة

$$\boxed{F_f = \mu_k N}$$



قانون $\boxed{F_f = \mu N}$

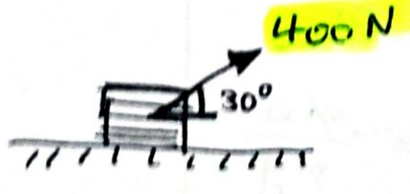
$- F_f = \mu m g$

$\sum F_y = m \cancel{a_y}^{0}$

$N - mg = 0$

القوة العامودية $\boxed{N = mg}$
(Normal Force)

Ex



Constant force

$\Rightarrow$ Constant accel.

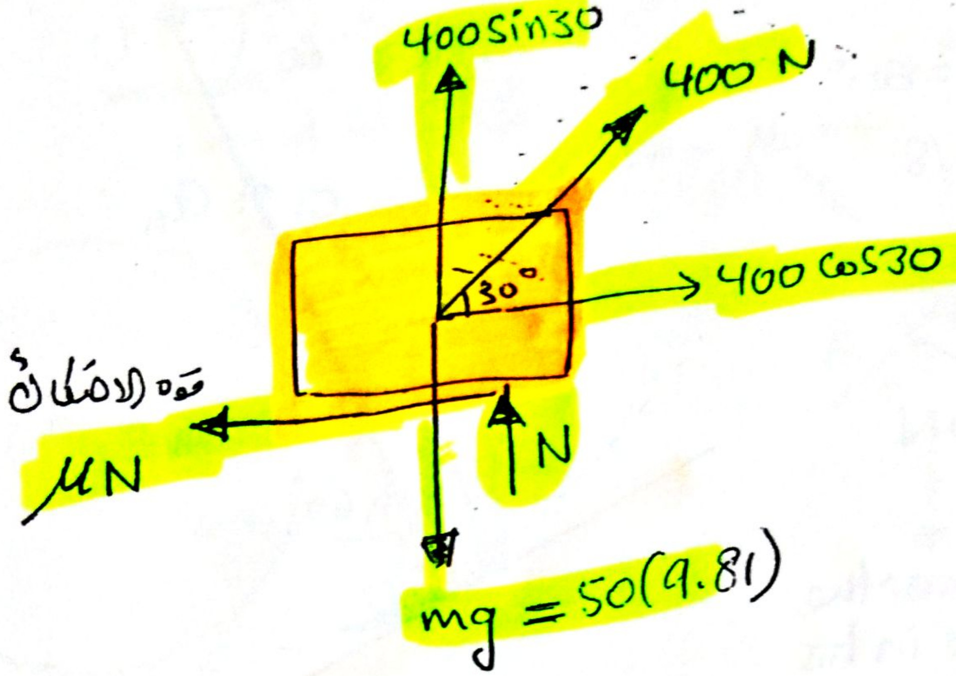
سطح خشن

$\mu_k = 0.3$

$m = 50 \text{ Kg}$

* Starting from Rest
* Find its velocity after $t = 0.5$ sec

احسب سرعة الجسم بعد مرور نصف ثانية



لايوجد حركة

$$** \sum F_y = m\cancel{a_y} \; 0$$

$$400\sin 30 - mg + N = 0$$

$$\Rightarrow N = mg - 400\sin 30$$

$$N = 50(9.81) - 200$$

$$N = 290.5 \text{ N}$$

$$** \sum F_x = m a_x$$

$$400\cos 30 - \mu N = m a$$

$$400\cos 30 - (0.3)(290.5) = (50)\, a$$

هذه عبارة عن تسارع ثابت باتجاه x $\Rightarrow$ $\boxed{a = 5.19 \ m/s^2}$

نذهب إلى ch12 ونستخدم قوانين الحركة الثابتة

$$v = v_0 + a t$$

$$v = 0 + 5.19(0.5) = \boxed{2.6 \ m/s}$$

EXAMPLE 13.4

A smooth 2-kg collar, shown in Fig. 13–9*a*, is attached to a spring having a stiffness $k = 3\ \text{N/m}$ and an unstretched length of 0.75 m. If the collar is released from rest at *A*, determine its acceleration and the normal force of the rod on the collar at the instant $y = 1$ m.

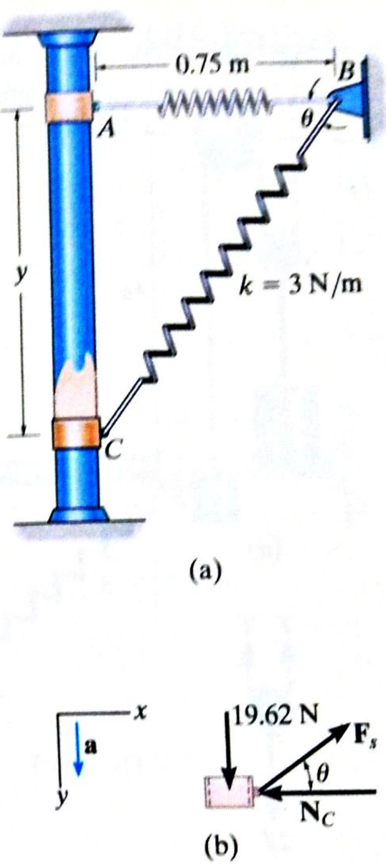


Fig. 13–9

SOLUTION

Free-Body Diagram. The free-body diagram of the collar when it is located at the arbitrary position *y* is shown in Fig. 13–9*b*. Furthermore, the collar is *assumed* to be accelerating so that "**a**" acts downward in the *positive y* direction. There are four unknowns, namely, N_C, F_s, a, and θ.

Equations of Motion.

$$\xrightarrow{+} \Sigma F_x = ma_x; \qquad -N_C + F_s \cos\theta = 0 \qquad (1)$$

$$+\downarrow \Sigma F_y = ma_y; \qquad 19.62 - F_s \sin\theta = 2a \qquad (2)$$

From Eq. 2 it is seen that the acceleration depends on the magnitude and direction of the spring force. Solution for N_C and a is possible once F_s and θ are known.

The magnitude of the spring force is a function of the stretch s of the spring; i.e., $F_s = ks$. Here the unstretched length is $AB = 0.75$ m, Fig. 13–9*a*; therefore, $s = CB - AB = \sqrt{y^2 + (0.75)^2} - 0.75$. Since $k = 3\ \text{N/m}$, then

$$F_s = ks = 3\left(\sqrt{y^2 + (0.75)^2} - 0.75\right) \qquad (3)$$

From Fig. 13–9*a*, the angle θ is related to *y* by trigonometry.

$$\tan\theta = \frac{y}{0.75}$$

Substituting $y = 1$ m into Eqs. 3 and 4 yields $F_s = 1.50$ N and $\theta = 53.1°$. Substituting these results into Eqs. 1 and 2, we obtain

$$N_C = 0.900\ \text{N} \qquad \textit{Ans.}$$

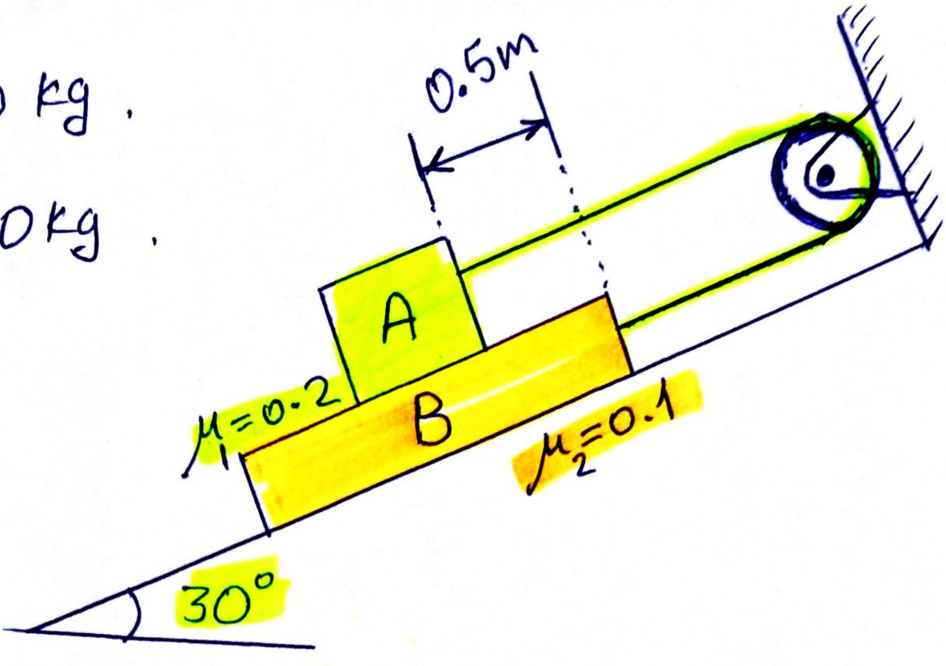
$$a = 9.21\ \text{m/s}^2 \downarrow \qquad \textit{Ans.}$$

NOTE: This is not a case of constant acceleration, since the spring force changes both its magnitude and direction as the collar moves downward.

Ex)

$m_A = 10\ kg.$

$m_B = 50\ kg.$



Find:

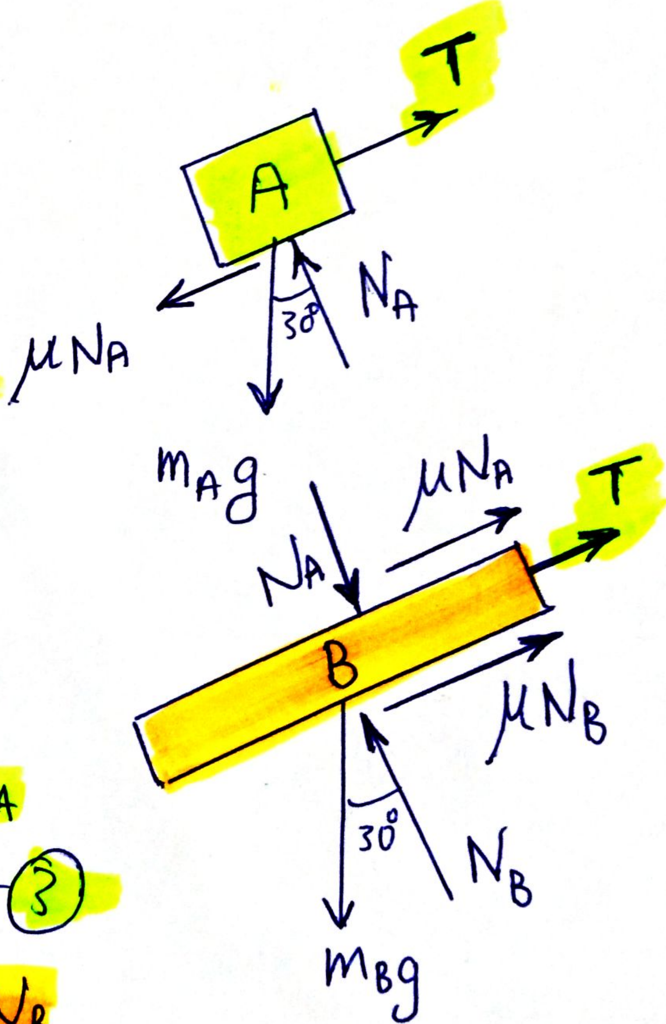
1) a_A ?

2) a_B ?

3) Tension T ?

$\sum F_y = 0$ — (1)

$\Rightarrow N_A = m_A g \cos 30^\circ$



and $N_B = N_A + m_B g \cos 30^\circ$ — (2)

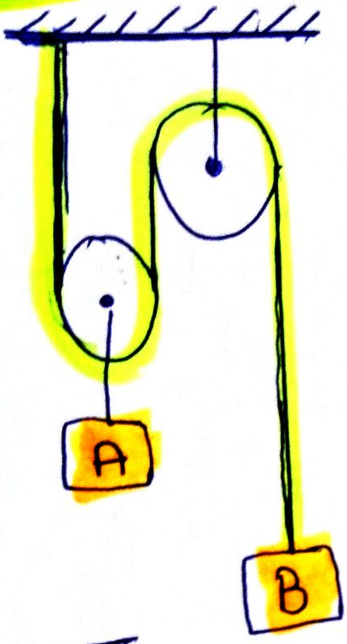
also: $\sum F_x = m a_x$

$\Rightarrow m_A g \sin 30^\circ + \mu_1 N_A - T = m_A a_A$ — (3)

and: $m_B g \sin 30^\circ - \mu_1 N_A - \mu_2 N_B - T = m_B a_B$ — (4)

and from Ch12 : $a_A + a_B = 0$ — (5)

Ex)



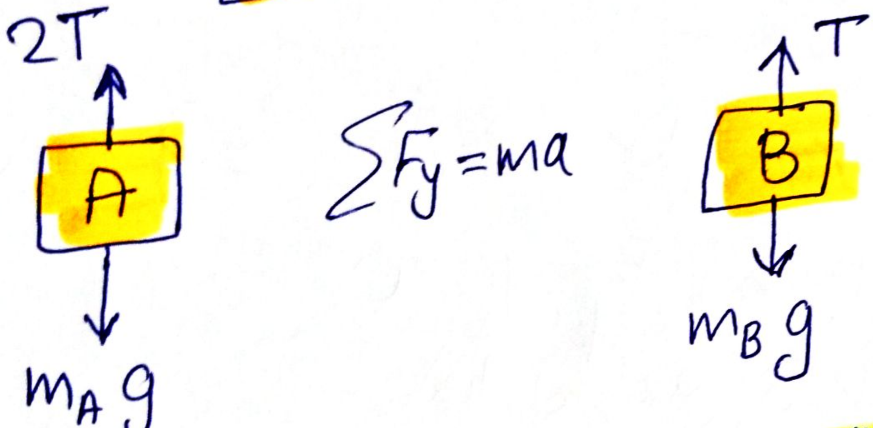
$m_A = 100\ kg$

$m_B = 20\ kg$

Find:

1) a_A
2) a_B
3) Tension T

from Ch12 $\Rightarrow$ $2a_A + a_B = 0$ — (1)



$\Rightarrow m_A g - 2T = m_A a_A$ — (2)

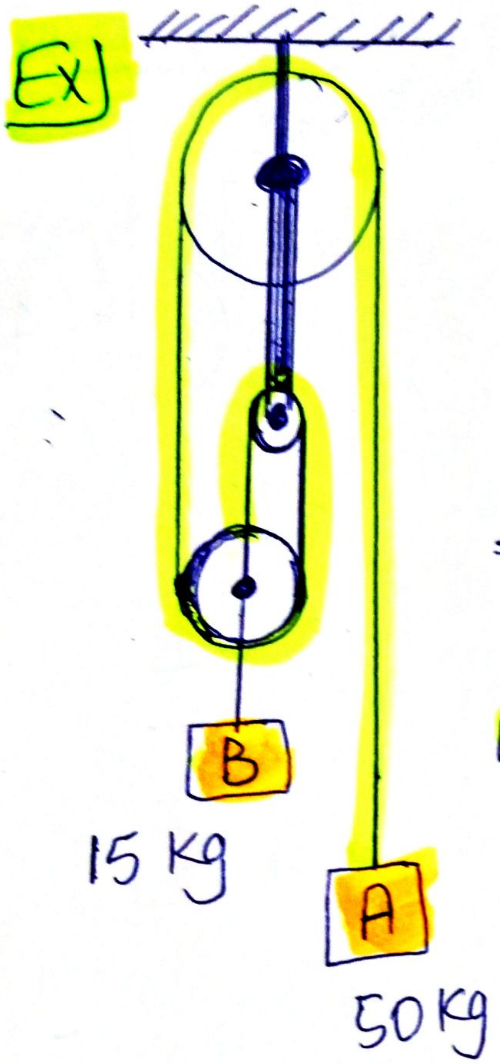
$m_B g - T = m_B a_B$ — (3)

Solve the three equations and get a_A, a_B and T $\Rightarrow$ ✓

* if starts from Rest find the velocity of B and distance of A after 10 seconds?

$v_B = v_0 + a_B t$ || $S_A = v_0 t + \frac{1}{2} a_A t^2$

Ex



Find a_A, a_B and T?

From Ch12 $\Rightarrow$ $a_A + 3a_B = 0$ —— (1)

$\Rightarrow \sum F_y = ma$

$m_A g - T = m_A a_A$ —— (2)

$m_B g - 3T = m_B a_B$ —— (3)

Solve for T, a_A and a_B

If all starts from Rest $\Rightarrow$ Find the velocity of B when A moves 18 m down?

$S_A + 3S_B = 0$

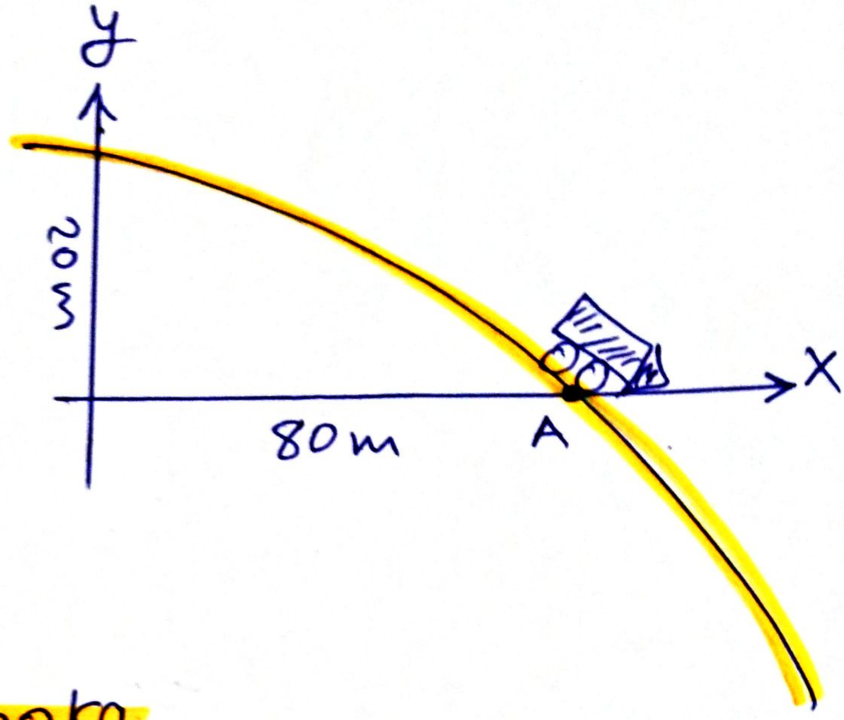
$\Rightarrow S_B = \frac{18}{3} = 6\text{ m up}$

$v_B^2 = v_0^2 + 2a_B S_B \Rightarrow$ Find v_B ✓

Ex)

$$y = 20\left[1 - \frac{x^2}{6400}\right]$$

Path equation



$v_A = 9\ m/s$

$(a_t)_A = 3\ m/s^2$

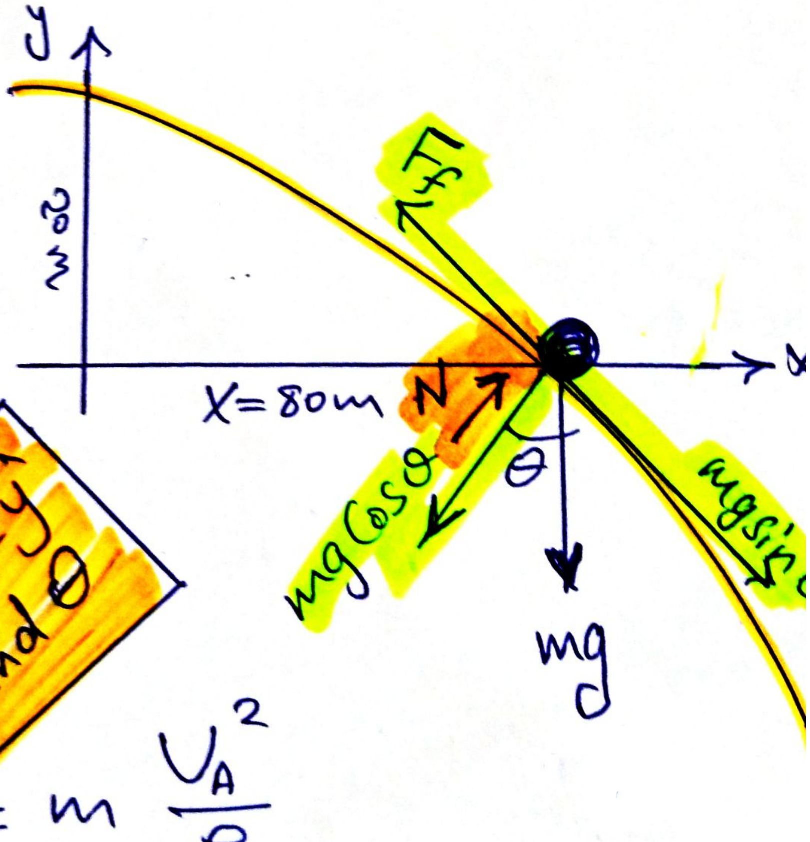
$m = 0.8\ Mg = 800\ kg.$

Find 1) Normal Force N 2) Frictional force F_f

Sol.

find y' and y''

at $x = 80\ m$



$$\Rightarrow \rho = \frac{[1+y'^2]^{3/2}}{y''}$$

$\tan\theta = y'$ ⇒ Find θ

$$\Rightarrow \sum F_n = m\frac{v^2}{\rho}$$

$$\Rightarrow mg\cos\theta - N = m\frac{v_A^2}{\rho}$$

$\Rightarrow$ Find N ✓

$\Rightarrow \sum F_t = ma_t \Rightarrow mg\sin\theta - F_f = m(a_t)_A$

$\Rightarrow$ Find F_f ✓

EXAMPLE 13.8

Design of the ski jump shown in the photo requires knowing the type of forces that will be exerted on the skier and her approximate trajectory. If in this case the jump can be approximated by the parabola shown in Fig. 13–14*a*, determine the normal force on the 70-kg skier the instant she arrives at the end of the jump, point *A*, where her velocity is 20 m/s. Also, what is her acceleration at this point?

SOLUTION

Why consider using *n*, *t* coordinates to solve this problem?

Free-Body Diagram. Since $dy/dx = x/30|_{x=0} = 0$, the slope at *A* is horizontal. The free-body diagram of the skier when she is at *A* is shown in Fig. 13–14*b*. Since the path is *curved*, there are two components of acceleration, $\mathbf{a}_n$ and $\mathbf{a}_t$. Since a_n can be calculated, the unknowns are a_t and N_A.

Equations of Motion.

$$+\uparrow \Sigma F_n = ma_n; \qquad N_A - 70(9.81) = 70\left[\frac{(20)^2}{\rho}\right] \qquad (1)$$

$$\overset{+}{\leftarrow} \Sigma F_t = ma_t; \qquad 0 = 70a_t \qquad (2)$$

The radius of curvature ρ for the path must be determined at point $A(0, -60\text{ m})$. Here $y = \frac{1}{60}x^2 - 60$, $dy/dx = \frac{1}{30}x$, $d^2y/dx^2 = \frac{1}{30}$, so that at $x = 0$,

$$\rho = \left.\frac{[1 + (dy/dx)^2]^{3/2}}{|d^2y/dx^2|}\right|_{x=0} = \frac{[1 + (0)^2]^{3/2}}{|\frac{1}{30}|} = 30\text{ m}$$

Substituting this into Eq. 1 and solving for N_A, we obtain

$$N_A = 1620\text{ N} \qquad \textit{Ans.}$$

Kinematics. From Eq. 2,

$$a_t = 0$$

Thus,

$$a_n = \frac{v^2}{\rho} = \frac{(20)^2}{30} = 13.33\text{ m/s}^2$$

$$a_A = a_n = 13.3\text{ m/s}^2 \uparrow \qquad \textit{Ans.}$$

NOTE: Apply the equation of motion in the *y* direction and show that when the skier is in midair, her downward acceleration is 9.81 m/s².

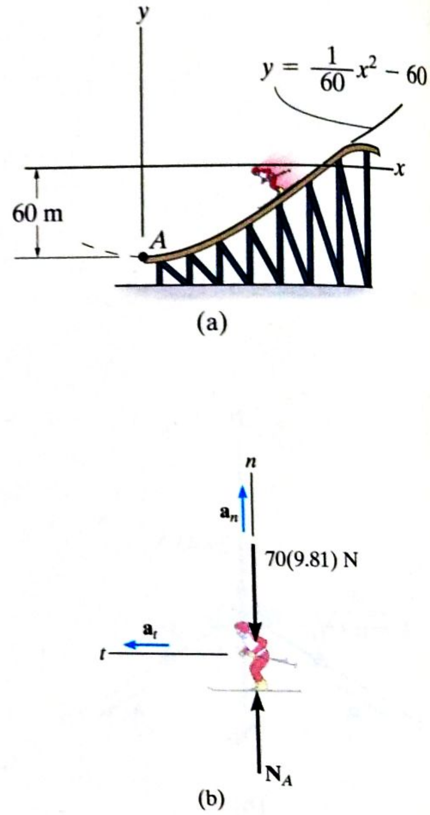
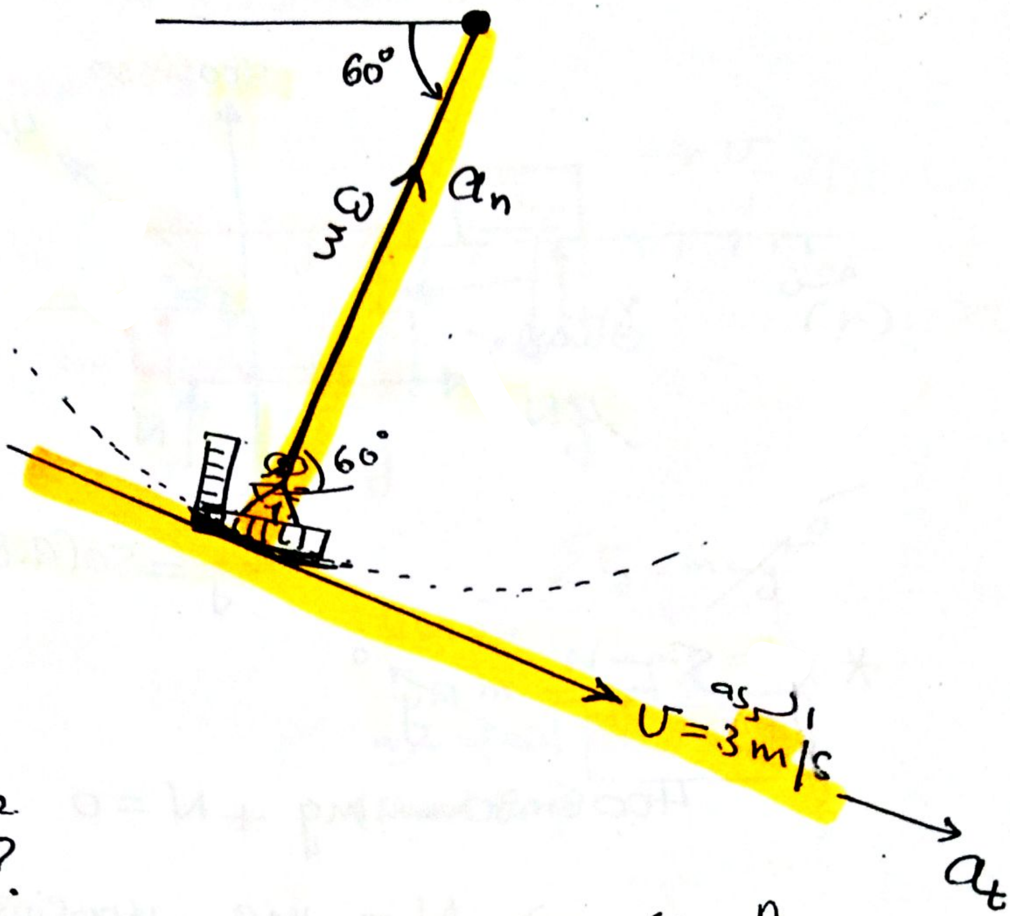


Fig. 13–14

$\boxed{Ex}$

when $\theta = 60°$
$v = 3 m/s$

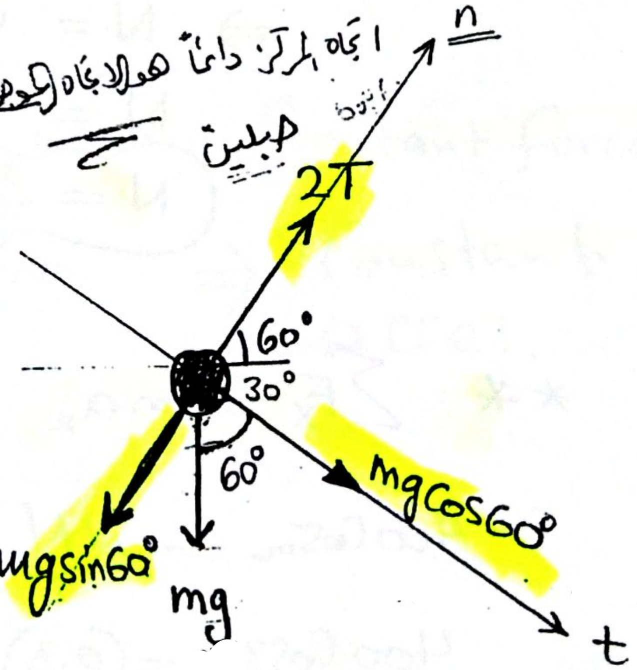
وزن الطفل
$W = 300 N$



1) determine the increase in his speed (a_t)?

2) the Tension in the Cable (T)?

الشد في الحبل



الحل

* $\sum F_t = m a_t$

$\cancel{m} g \cos 60° = \cancel{m} a_t$

$a_t = 9.81 \cos 60°$

$\boxed{a_t = 4.91 \ m/s^2}$

* $\sum F_n = m a_n$

$2T - (mg) \sin 60° = m \frac{v^2}{\rho}$

$2T - 300 \sin 60° = \frac{300}{9.81} \frac{(3)^2}{3} \Rightarrow \boxed{T = 175.8 \ N}$

Ex

الوزن
$(\omega) = 300N$

$m = 30\,kg$

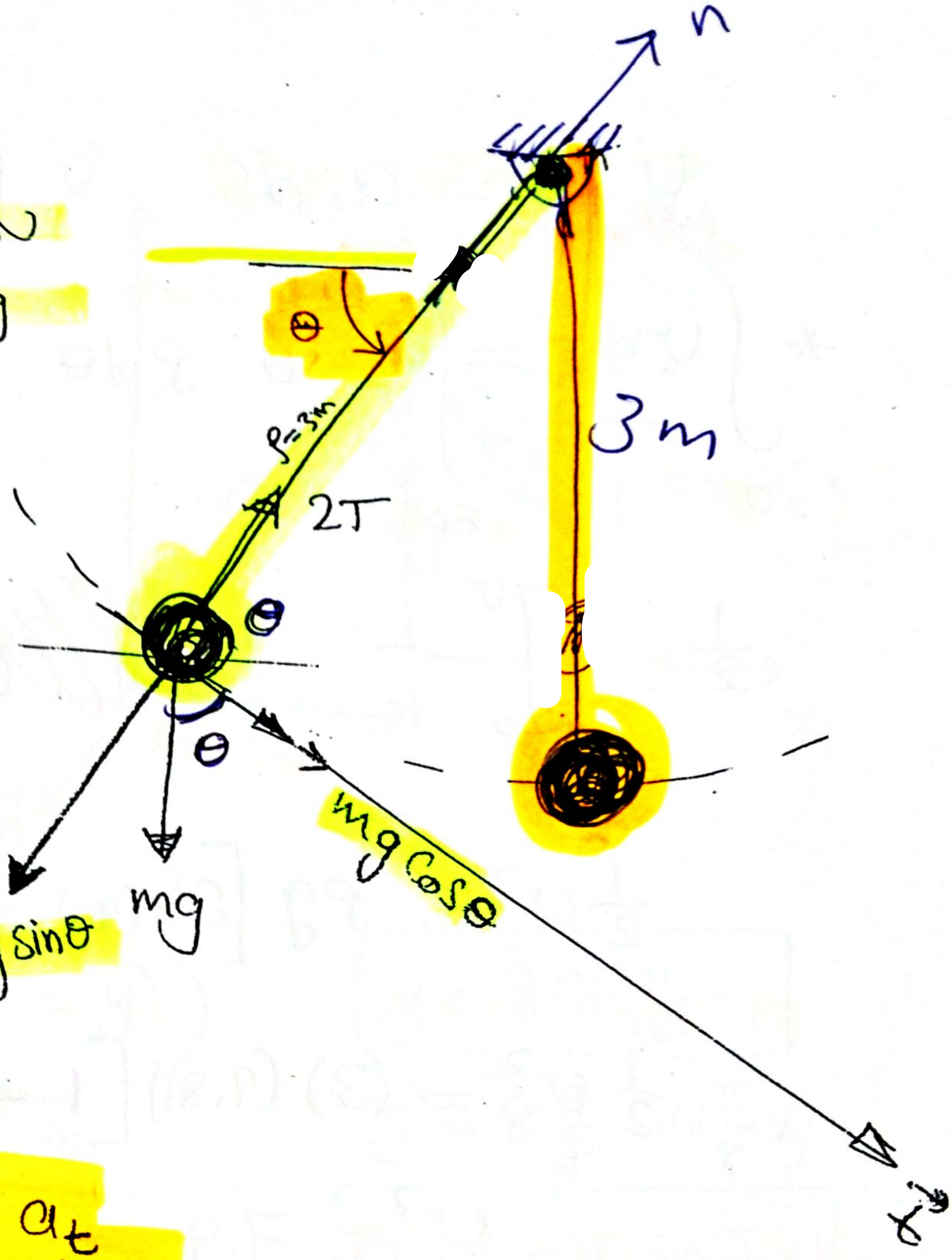
$\theta = 60^\circ \Rightarrow v = 0$

أوجد سرعة الولد

و الشد في الحبل

على زاوية $\theta = 9.^\circ$

$\theta = 90^\circ$



* $\sum F_t = m a_t$

$$\cancel{m} g \cos\theta = \cancel{m} a_t$$

$$\boxed{a_t = g\cos\theta} \quad — ①$$

* $\sum F_n = m a_n$

$$2T - mg\sin\theta = m\frac{v^2}{\rho} \quad — ②$$

$v\,dv = a_t\,ds$ $\qquad \boxed{ds = \rho\,d\theta}$

$$* \int_0^v v\,dv = \int_{60^\circ}^{90} g\cos\theta\ \rho\,d\theta$$

$$\frac{1}{2}v^2\Big]_0^v = \rho g\sin\theta\Big]_{60}^{90}$$

$$\frac{1}{2}v^2 = \rho g\left[\sin(90) - \sin 60\right]$$

$$\frac{1}{2}v^2 = (3)(9.81)\left[1 - 0.87\right]$$

$$v^2 = 7.9$$

$$\boxed{v = 2.8\ m/s}$$

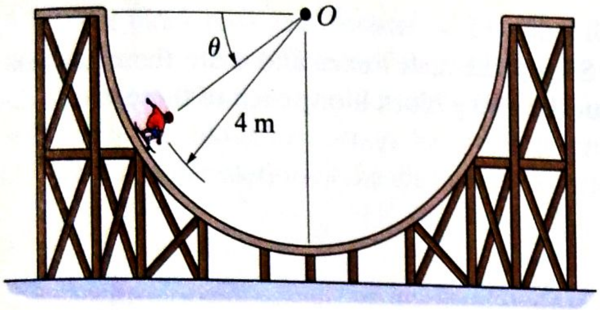
عوض في ①

$$2T - mg\sin\theta = m\frac{v^2}{\rho}$$

$$2T - 300\sin 90^\circ = 30\,\frac{(2.8)^2}{3}$$

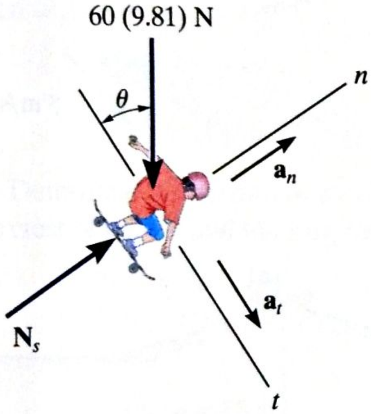
$$\boxed{T = 189.2\ N}$$

EXAMPLE 13.9

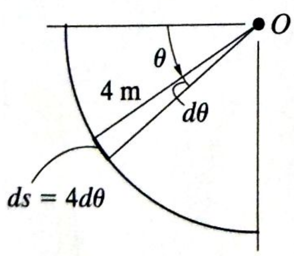


(a)

The 60-kg skateboarder in Fig. 13–15a coasts down the circular track. If he starts from rest when $\theta = 0°$, determine the magnitude of the normal reaction the track exerts on him when $\theta = 60°$. Neglect his size for the calculation.



(b)



(c)

Fig. 13–15

SOLUTION

Free-Body Diagram. The free-body diagram of the skateboarder when he is at an *arbitrary position* θ is shown in Fig. 13–15b. At $\theta = 60°$ there are three unknowns, N_s, a_t, and a_n (or v).

Equations of Motion.

$$+\nearrow \Sigma F_n = ma_n; \quad N_s - [60(9.81)\text{N}]\sin\theta = (60\text{ kg})\left(\frac{v^2}{4\text{ m}}\right) \qquad (1)$$

$$+\searrow \Sigma F_t = ma_t; \quad [60(9.81)\text{N}]\cos\theta = (60\text{ kg})\,a_t$$

$$a_t = 9.81\cos\theta$$

Kinematics. Since a_t is expressed in terms of θ, the equation $v\,dv = a_t\,ds$ must be used to determine the speed of the skateboarder when $\theta = 60°$. Using the geometric relation $s = \theta r$, where $ds = r\,d\theta = (4\text{ m})\,d\theta$, Fig. 13–15$c$, and the initial condition $v = 0$ at $\theta = 0°$, we have,

$$v\,dv = a_t\,ds$$

$$\int_0^v v\,dv = \int_0^{60°} 9.81\cos\theta(4\,d\theta)$$

$$\left.\frac{v^2}{2}\right|_0^v = 39.24\sin\theta\Big|_0^{60°}$$

$$\frac{v^2}{2} - 0 = 39.24(\sin 60° - 0)$$

$$v^2 = 67.97\text{ m}^2/\text{s}^2$$

Substituting this result and $\theta = 60°$ into Eq. (1), yields

$$N_s = 1529.23\text{ N} = 1.53\text{ kN} \qquad \textit{Ans.}$$

Ex II

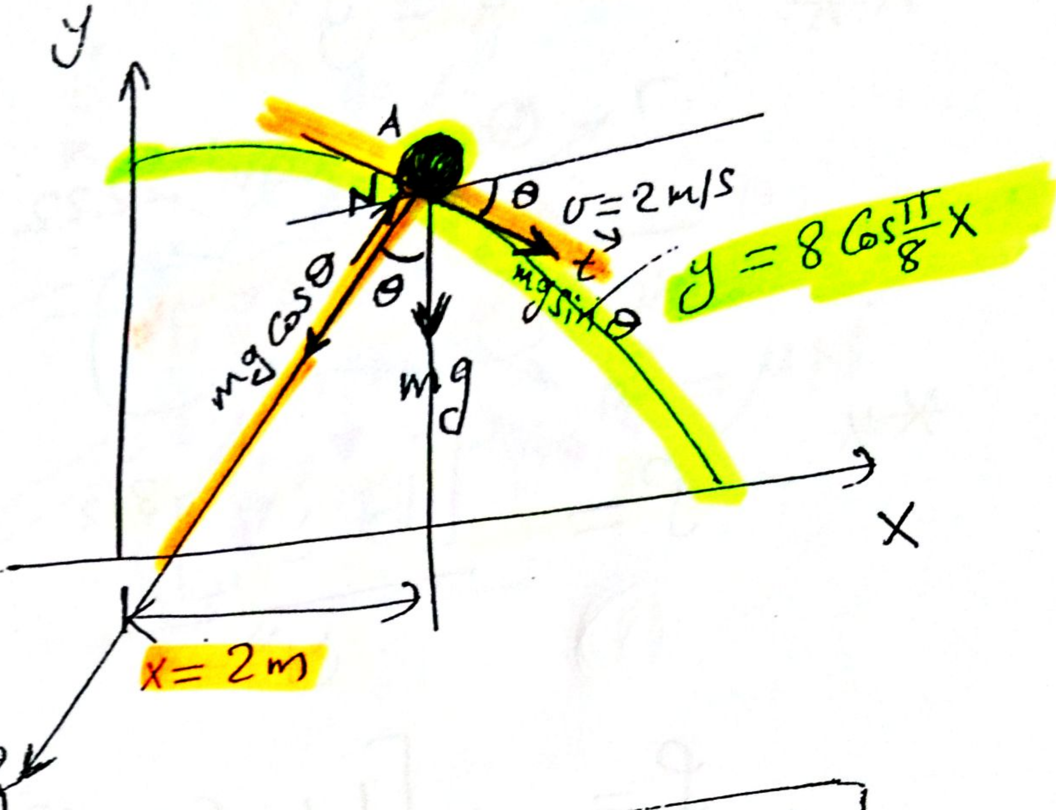
الوزن
$\omega = 900\ N$.

$m = 90\ kg$.

*Find N?

$v_A = 2 m/s$

*$a_t = ?$



θ ع x-ais * $\tan\theta = y'$

$$\boxed{y = 8\cos\left(\frac{\pi}{8}x\right)}$$

$$y' = -8\,\frac{\pi}{8}\sin\left(\frac{\pi}{8}x\right)$$

$$\boxed{y' = -\pi\sin\left(\frac{\pi}{8}x\right)}$$

$$y'' = -\pi\,\frac{\pi}{8}\cos\left(\frac{\pi}{8}x\right)$$

$$\boxed{y'' = \frac{-\pi^2}{8}\cos\left(\frac{\pi}{8}x\right)}$$

نعوض

$x = 2$

$y = 8\cos\left(\frac{\pi}{8}(2)\right)$

$y = 8\cos\left[\frac{\pi}{4}\right]$

$\boxed{y = 5.66}$

$y' = -\pi\sin\left[\frac{\pi}{4}\right]$

$\boxed{y' = -2.22}$

$y'' = \frac{-\pi^2}{8}\sin\left[\frac{\pi}{4}\right] \Rightarrow \boxed{y'' = -0.87}$

$\tan\theta = y'$

** $\theta = \tan^{-1} -2.22 = -65.7°$

$\theta = +65.7°$

** $\rho = \frac{[1+y'^2]^{3/2}}{y''}$

$\rho = \frac{[1+(-2.22)^2]^{3/2}}{-0.87} = -16.6 \text{ m}$

$\boxed{\rho = 16.6 \text{ m}}$

** $\sum F_n = m a_n$

$mg\cos\theta - N = m \frac{v^2}{\rho}$

$900 \cos 65.9° - N = 90 \frac{(2)^2}{16.6}$

$\Rightarrow N = 349 \text{ N}$

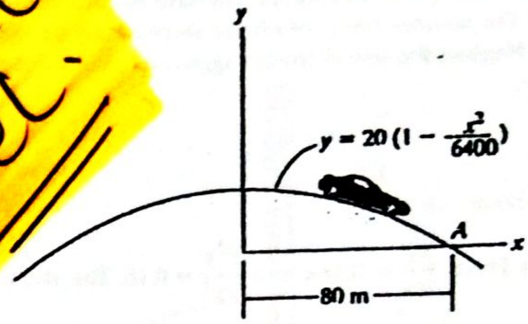
** $\sum F_t = m a_t$

$\cancel{m} g \sin\theta = \cancel{m} a_t \Rightarrow a_t = 9.81 \sin 65.7°$

$a_t = 8.95 \text{ m/s}^2$

•13–73. The 0.8-Mg car travels over the hill having the shape of a parabola. When the car is at point A, it is traveling at 9 m/s and increasing its speed at 3 m/s^2. Determine both the resultant normal force and the resultant frictional force that all the wheels of the car exert on the road at this instant. Neglect the size of the car.

Constant accel.

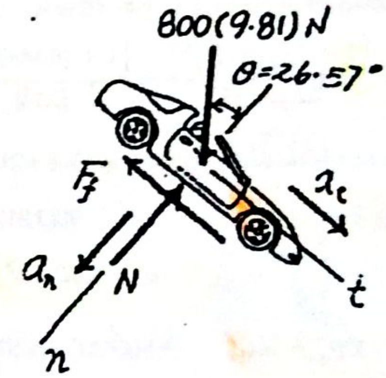


Geometry: Here, $\frac{dy}{dx} = -0.00625x$ and $\frac{d^2y}{dx^2} = -0.00625$. The slope angle θ at point A is given by

$$\tan\theta = \left.\frac{dy}{dx}\right|_{x=80\text{ m}} = -0.00625(80) \qquad \theta = -26.57°$$

and the radius of curvature at point A is

$$\rho = \frac{[1 + (dy/dx)^2]^{3/2}}{|d^2y/dx^2|} = \left.\frac{[1 + (-0.00625x)^2]^{3/2}}{|-0.00625|}\right|_{x=80\text{ m}} = 223.61\text{ m}$$



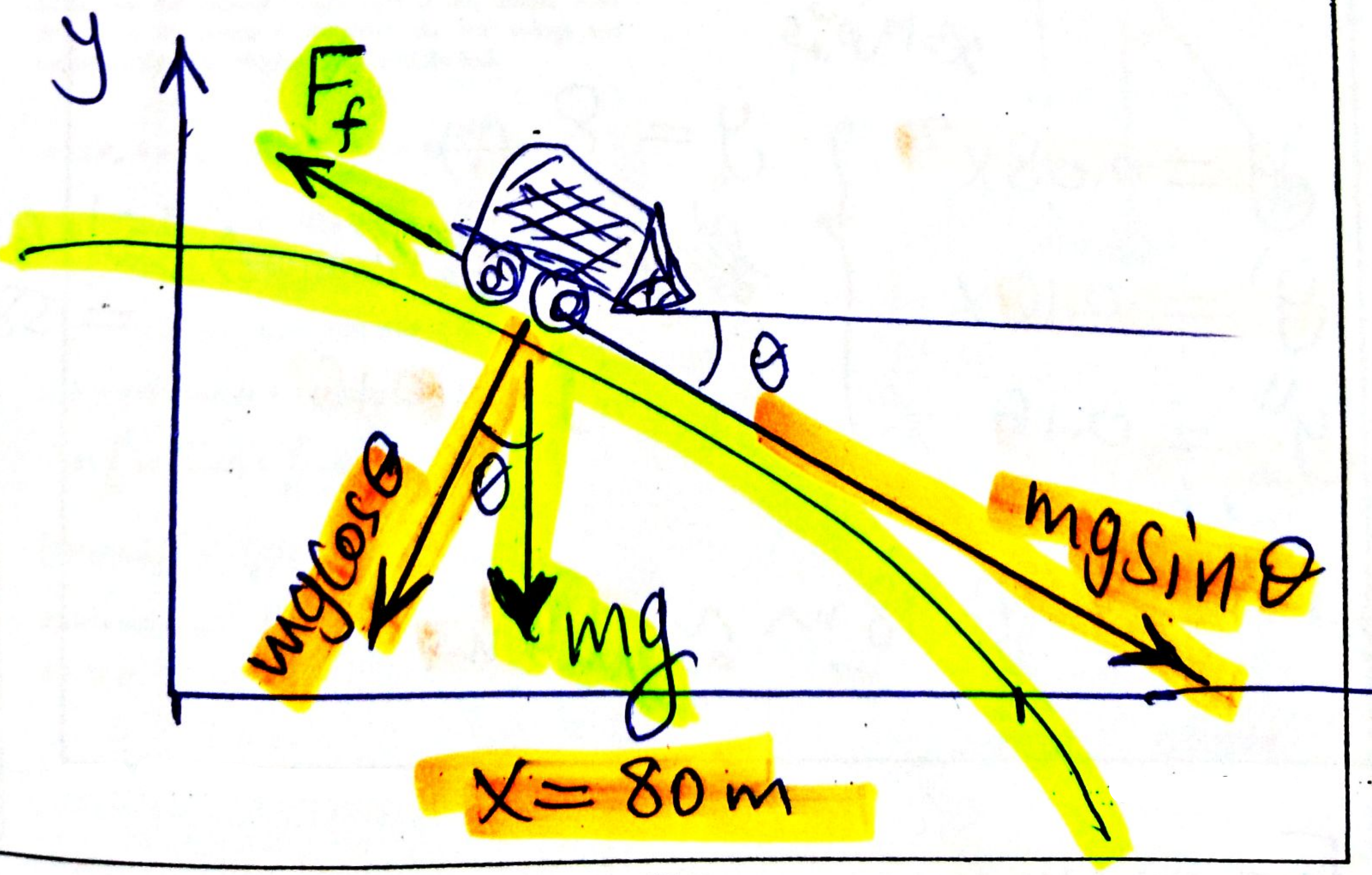
Equation of Motion: Applying Eq. 13–8 with $\theta = 26.57°$ and $\rho = 223.61$ m, we have

$\Sigma F_t = ma_t;$ $\qquad 800(9.81)\sin 26.57° - F_f = 800(3)$

$$F_f = 1109.73\text{ N} = 1.11\text{ kN}$$ **Ans.**

$\Sigma F_n = ma_n;$ $\qquad 800(9.81)\cos 26.57° - N = 800\left(\frac{9^2}{223.61}\right)$

$$N = 6729.67\text{ N} = 6.73\text{ kN}$$ **Ans.**

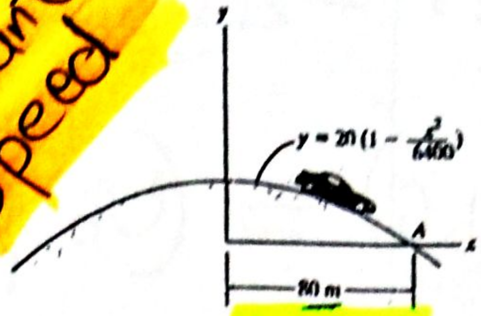


*13–72. The 0.8-Mg car travels over the hill having the shape of a parabola. If the driver maintains a constant speed of 9 m/s, determine both the resultant normal force and the resultant frictional force that all the wheels of the car exert on the road at the instant it reaches point A. Neglect the size of the car.

Constant Speed

Find N ?

قوة الاحتكاك Find F_f ?



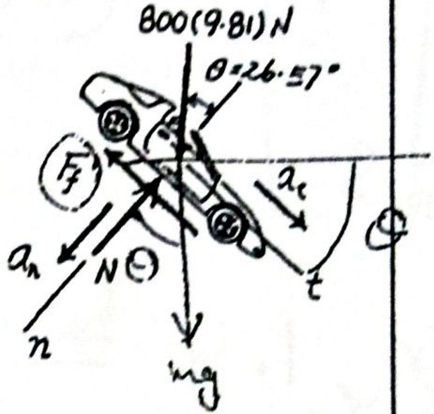
$x = 80$ m

Geometry: Here, $\boxed{\frac{dy}{dx} = -0.00625x}$ and $\frac{d^2y}{dx^2} = -0.00625$. The slope angle θ at point A is given by

$$\tan\theta = \left.\frac{dy}{dx}\right|_{x=80\text{ m}} = -0.00625(80) \qquad \boxed{\theta = -26.57^\circ}$$

and the radius of curvature at point A is

$$\rho = \frac{[1 + (dy/dx)^2]^{3/2}}{|d^2y/dx^2|} = \left.\frac{[1 + (-0.00625x)^2]^{3/2}}{|-0.00625|}\right|_{x=80\text{ m}} = 223.61\text{ m}$$



Equations of Motion: Here, $a_t = 0$. Applying Eq. 13–8 with $\theta = 26.57^\circ$ and $\rho = 223.61$ m, we have

$\Sigma F_t = ma_t;$ $\qquad 800(9.81)\sin 26.57^\circ - F_f = 800(0)$

$F_f = 3509.73\text{ N} = \boxed{3.51\text{ kN}}$ **Ans.**

$\Sigma F_n = ma_n;$ $\qquad 800(9.81)\cos 26.57^\circ - N = 800\left(\frac{9^2}{223.61}\right)$

$N = 6729.67\text{ N} = \boxed{6.73\text{ kN}}$ **Ans.**

$m = 0.8 \times 10^3$ kg

$\boxed{m = 800\text{ kg}}$

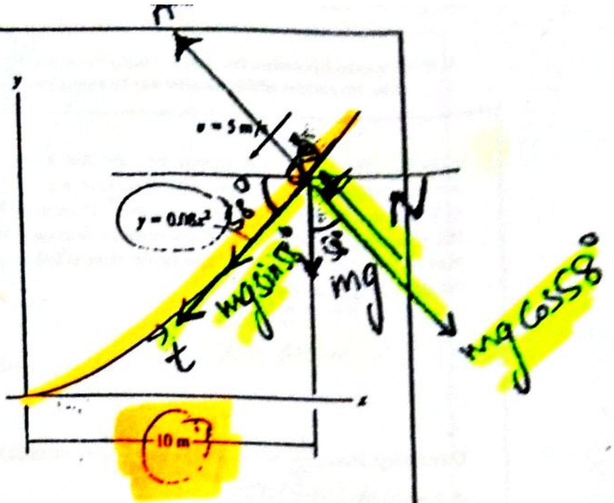
$$y = 20\left(1 - \frac{x^2}{6400}\right)$$

$$* \; y = 20 - \frac{x^2}{320} = 20 - 1.5625 \times 10^{-4}x^2$$

$$* \; y' = 0 - \frac{2x}{320} = \frac{-x}{160} = \boxed{0.00625x}$$

$$* \; y'' = \frac{-1}{160} = \boxed{-0.00625}$$

*13–76. A toboggan and rider of total mass 90 kg travel down along the (smooth) slope defined by the equation $y = 0.08x^2$. At the instant $x = 10$ m, the toboggan's speed is 5 m/s. At this point, determine the rate of increase in speed and the normal force which the slope exerts on the toboggan. Neglect the size of the toboggan and rider for the calculation.



Geometry: Here, $\frac{dy}{dx} = 0.16x$ and $\frac{d^2y}{dx^2} = 0.16$. The slope angle θ at $x = 10$ m is given by

$$\tan\theta = \left.\frac{dy}{dx}\right|_{x=10\text{ m}} = 0.16(10) \qquad \theta = 57.99°$$

and the radius of curvature at $x = 10$ m is

$$\rho = \frac{[1 + (dy/dx)^2]^{3/2}}{|d^2y/dx^2|} = \left.\frac{[1 + (0.16x)^2]^{3/2}}{|0.16|}\right|_{x=10\text{ m}} = 41.98 \text{ m}$$

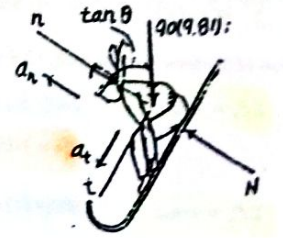
Equations of Motion: Applying Eq. 13–8 with $\theta = 57.99°$ and $\rho = 41.98$ m, we have

$\Sigma F_t = ma_t;$ $\quad 90(9.81)\sin 57.99° = 90a_t$

$a_t = 8.32 \text{ m/s}^2$ **Ans.**

$\Sigma F_n = ma_n;$ $\quad -90(9.81)\cos 57.99° + N = 90\left(\frac{5^2}{41.98}\right)$

$N = 522$ N **Ans.**



$x = 10$ عوض

$$\left.\begin{aligned} y &= 0.08x^2 \\ y' &= 0.16x \\ y'' &= 0.16 \end{aligned}\right\} \quad \begin{aligned} y &= 8 \\ y' &= 1.6 \Rightarrow \tan^{-1} 1.6 = \theta = 58° \\ y'' &= 0.16 \end{aligned}$$

عوض $\rho = 41.98$ m ≈ 42m

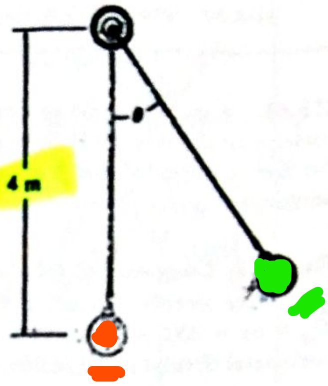
① $\Sigma F_t = m a_t$

$m g \sin 58° = m a_t \Rightarrow a_t = 9.81 \sin 58° = 8.32$ m/s²

② $\Sigma F_n = m a_n = m \frac{v^2}{\rho}$

$N - mg\cos 58° = 90 \frac{(5)^2}{42} \Rightarrow N = 522$ N

13–62. The ball has a mass of 30 kg and a speed $v = 4$ m/s at the instant it is at its lowest point, $\theta = 0°$. Determine the tension in the cord and the rate at which the ball's speed is decreasing at the instant $\theta = 20°$. Neglect the size of the ball.



$+\nwarrow \Sigma F_n = ma_n;$ $\quad T - 30(9.81)\cos\theta = 30\left(\frac{v^2}{4}\right)$

$+\nearrow \Sigma F_t = ma_t;$ $\quad -30(9.81)\sin\theta = 30a_t$

$a_t = -9.81 \sin\theta$

$a_t\,ds = v\,dv$ Since $ds = 4\,d\theta$, then

$$-9.81\int_0^{\theta} \sin\theta\,(4\,d\theta) = \int_4^{v} v\,dv$$

$$9.81(4)\cos\theta\Big|_0^{\theta} = \frac{1}{2}(v)^2 - \frac{1}{2}(4)^2$$

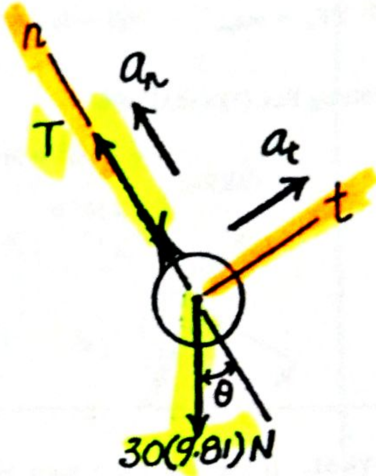
$$39.24(\cos\theta - 1) + 8 = \frac{1}{2}v^2$$

At $\theta = 20°$

$v = 3.357$ m/s

$a_t = -3.36 \text{ m/s}^2 = 3.36 \text{ m/s}^2$ ✓ **Ans.**

$T = 361$ N **Ans.**



$*\ \sum F_n = m\frac{v^2}{\rho} \Rightarrow T - mg\cos\theta = m\frac{v^2}{\rho}$

$*\ \sum F_t = m a_t \Rightarrow -mg\sin\theta = m a_t$

$*\ v\,dv = a_t\,ds\ ,\quad ds = \rho\,d\theta$

$$\int_4^{v} v\,dv = \int_0^{20°} -g\sin\theta\,(\rho\,d\theta)$$

Dynamics

Dr. Hashem Alkhaldi

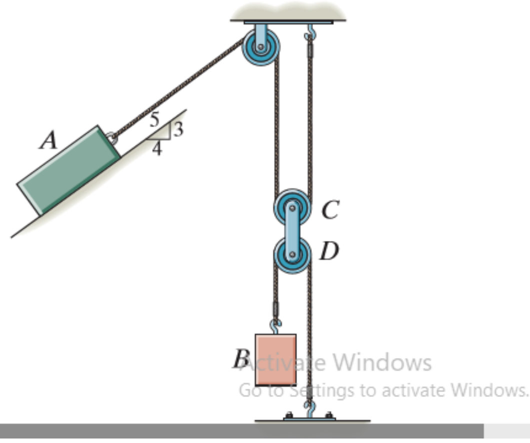
Suggested Problems: Chapter 13

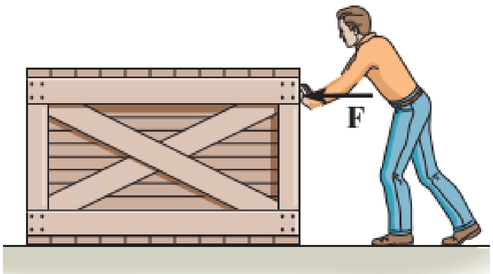
(10 problems, 2 pages)

g= 9.81 m/s^2 = 32.2 ft/s^2

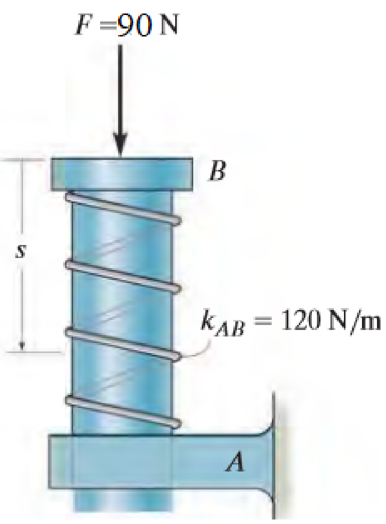
===

Q1) At the instant shown the 150-lb block A is moving down the plane at 5 ft/s while being attached to the 40-lb block B. If the coefficient of kinetic friction between the block and the incline is $\mu_k = 0.2$, determine the acceleration of A and the distance A slides before it stops. Neglect the mass of the pulleys and cables.



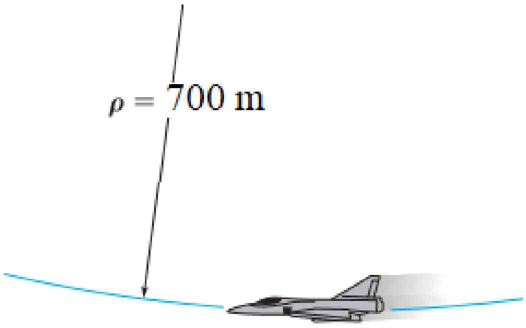


Q2) The 80 kg man pushes on the 170 kg crate with a horizontal force **F**. If the coefficients of static and kinetic friction between the crate and the surface are $\mu_s = 0.3$ and $\mu_k = 0.2$, and the coefficient of static friction between the man's shoes and the surface is $\mu_s = 0.8$, show that the man is able to move the crate. What is the greatest acceleration the man can give the crate?

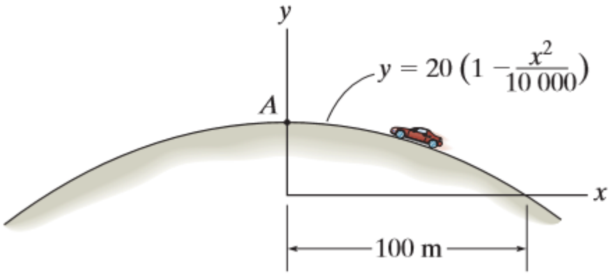


Q3) The 3-kg smooth cylinder is supported by the spring having a stiffness of $k_{AB} = 120$ N/m. Determine the velocity of the cylinder when it moves downward $s = 0.2$ m from its equilibrium position, which is caused by the application of the force $F = 90$ N.

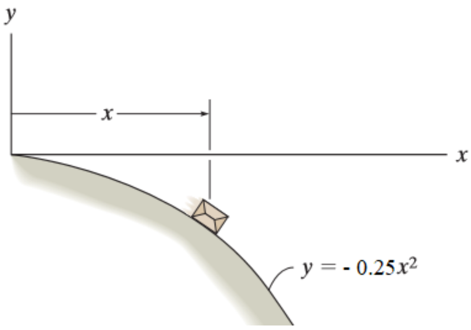
Q4) Determine the maximum constant speed at which the pilot can travel around the vertical curve having a radius of curvature $\rho = 700$ m, so that he experiences a maximum acceleration $a_n = 8g = 78.5\ \text{m/s}^2$. If he has a mass of 80 kg, determine the normal force he exerts on the seat of the airplane when the plane is traveling at this speed and is at its lowest point.



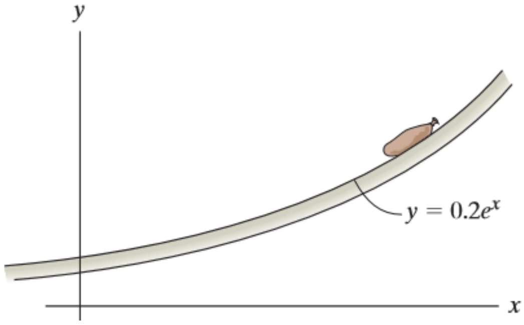
Q5) Determine the maximum constant speed at which the 3-Mg car can travel over the crest of the hill at A without leaving the surface of the road. Neglect the size of the car in the calculation.



Q6) The box has a mass m and slides down the smooth chute having the shape of a parabola. If it has an initial velocity of v_0 at the origin determine its velocity as a function of x. Also, what is the normal force on the box, and the tangential acceleration as a function of x?

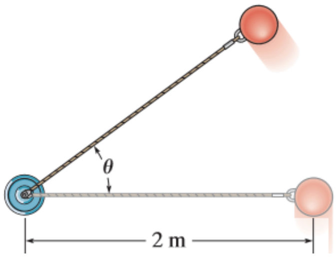


Q7) The 6-kg sack slides down the smooth ramp. If it has a speed of 2 m/s when $y = 0.2$ m, determine the normal reaction the ramp exerts on the sack and the rate of increase in the speed of sack at this instant.

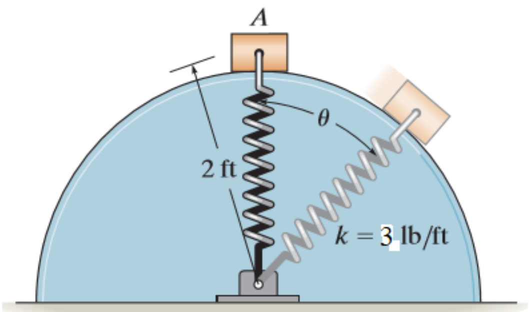


Q8) a) The 3-kg pendulum bob moves in the vertical plane with a velocity of 8 m/s when $\theta = 0°$. Determine the initial tension in the cord and also at the instant the bob reaches $\theta = 30°$. Neglect the size of the bob.

b) The 3-kg pendulum bob moves in the vertical plane with a velocity of 6 m/s when $\theta = 0°$. Determine the angle θ where the tension in the cord becomes zero.



Q9) The 2-lb block is released from rest at A and slides down along the smooth cylindrical surface. If the attached spring has a stiffness $k = 3$ lb/ft, determine its unstretched length so that it does not allow the block to leave the surface until $\theta = 60°$.



Q10) The motor M pulls in its attached rope with an acceleration $a_p = 8\ \text{m/s}^2$. Determine the towing force exerted by M on the rope in order to move the 40-kg crate up the inclined plane. The coefficient of kinetic friction between the crate and the plane is $\mu_k = 0.3$. Neglect the mass of the pulleys and rope.