Ch 14

Ch 14

(Work and Energy)

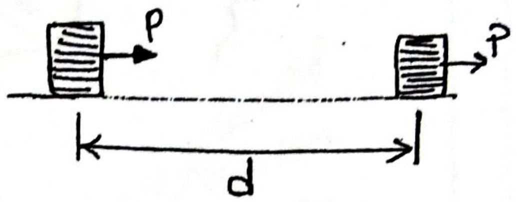
الشغل والطاقة

1 شغل القوة الخارجية

* القوة دائماً باتجاه المسافة

القوة نفس اتجاه الحركة ← $\pm Pd$

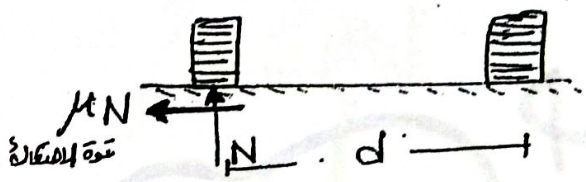
القوة عكس اتجاه الحركة ←



اتجاه الحركة →

2 شغل قوة الاحتكاك

دائماً سالبة $-\mu N d$

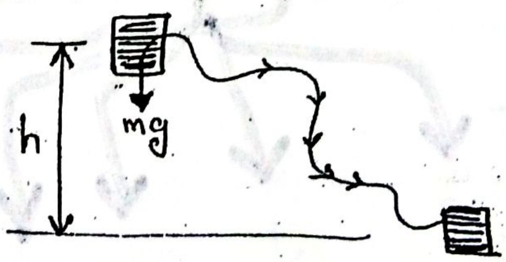


$\mu \equiv$ معامل الاحتكاك Coeff of friction

3 شغل الوزن

الهبوط ← $\pm mgh$

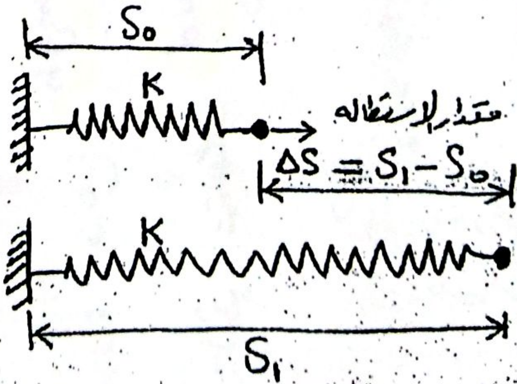
الصعود ←



4 شغل الزنبرك spring

S_0 الطول غير المشدود (الأصلي) Unstretched length

S_1 Stretched length.



مقدار الاستطالة $\Delta S = S_1 - S_0$

(حسب عنه) الابتعاد عن الطول الأصلي ← $\pm \frac{1}{2} K \Delta S^2$

الاقتراب من الطول الأصلي (معاكس)

5 طاقة الحركة Kinetic T

$$T = +\frac{1}{2} m v^2$$

* Spring Energy from A to B :

$U_{AB} = \frac{1}{2} K [S_A^2 - S_B^2]$ ✗ wrong

$U_{AB} = \frac{1}{2} K \left[(S_A - S_0)^2 - (S_B - S_0)^2 \right]$ ✓ O.K.

Spring : ($K = 100$ N/cm)

* unstreched length = 7 cm
* Initial length at A = 12 cm
* Final length at B = 20 cm

Energy from A to B is U_{AB} :

$\Rightarrow U_{AB} = -\frac{1}{2} K \Delta S^2$

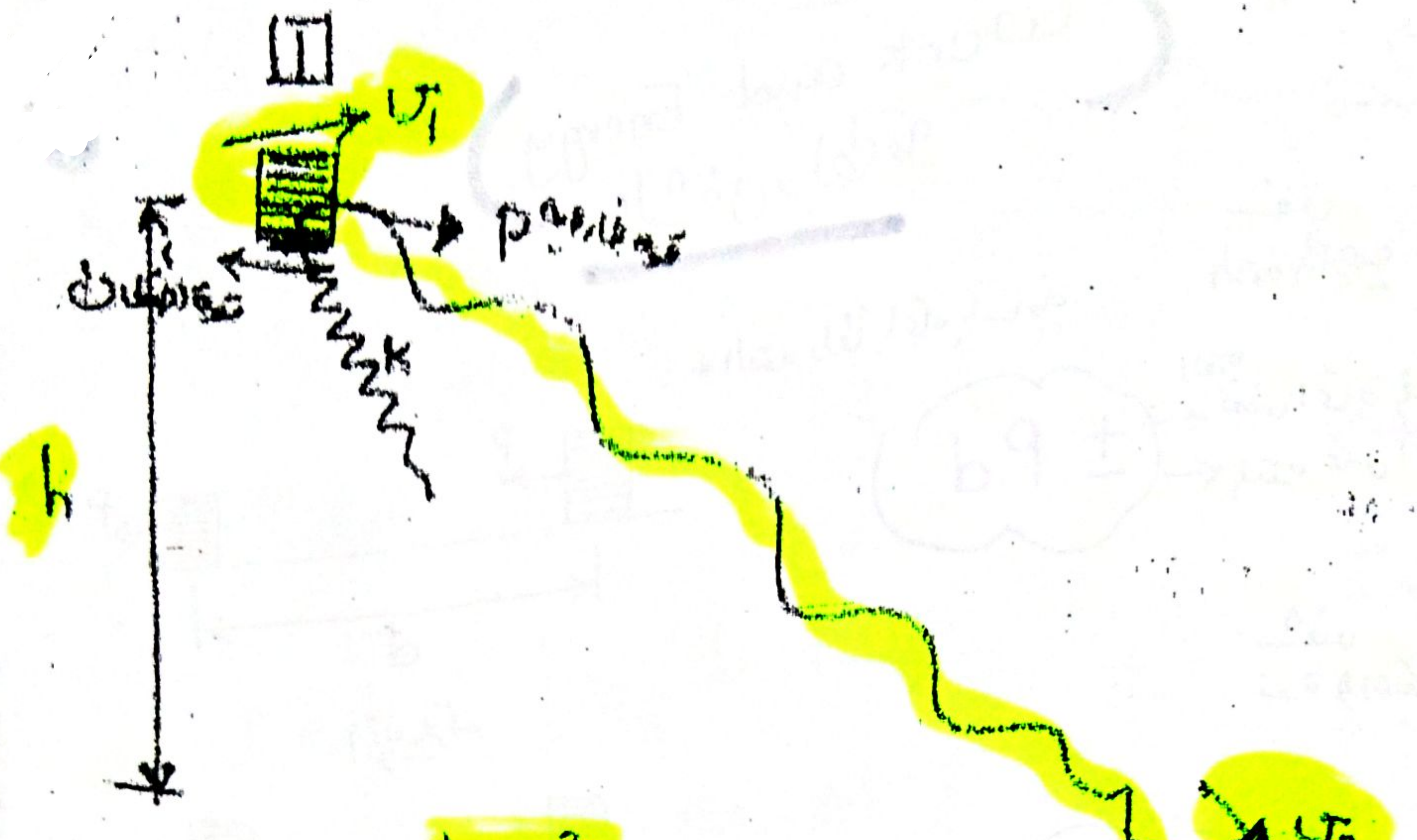
$= -\frac{1}{2} (100) [20^2 - 12^2]$

$= -12800$ joule ✗ wrong

$\Rightarrow U_{AB} = \frac{1}{2} K \left[\underbrace{(12-7)^2}_{\text{at A}} - \underbrace{(20-7)^2}_{\text{at B}} \right]$

$= \frac{1}{2} (100) [5^2 - 13^2]$

$= -7200$ Joule ✓

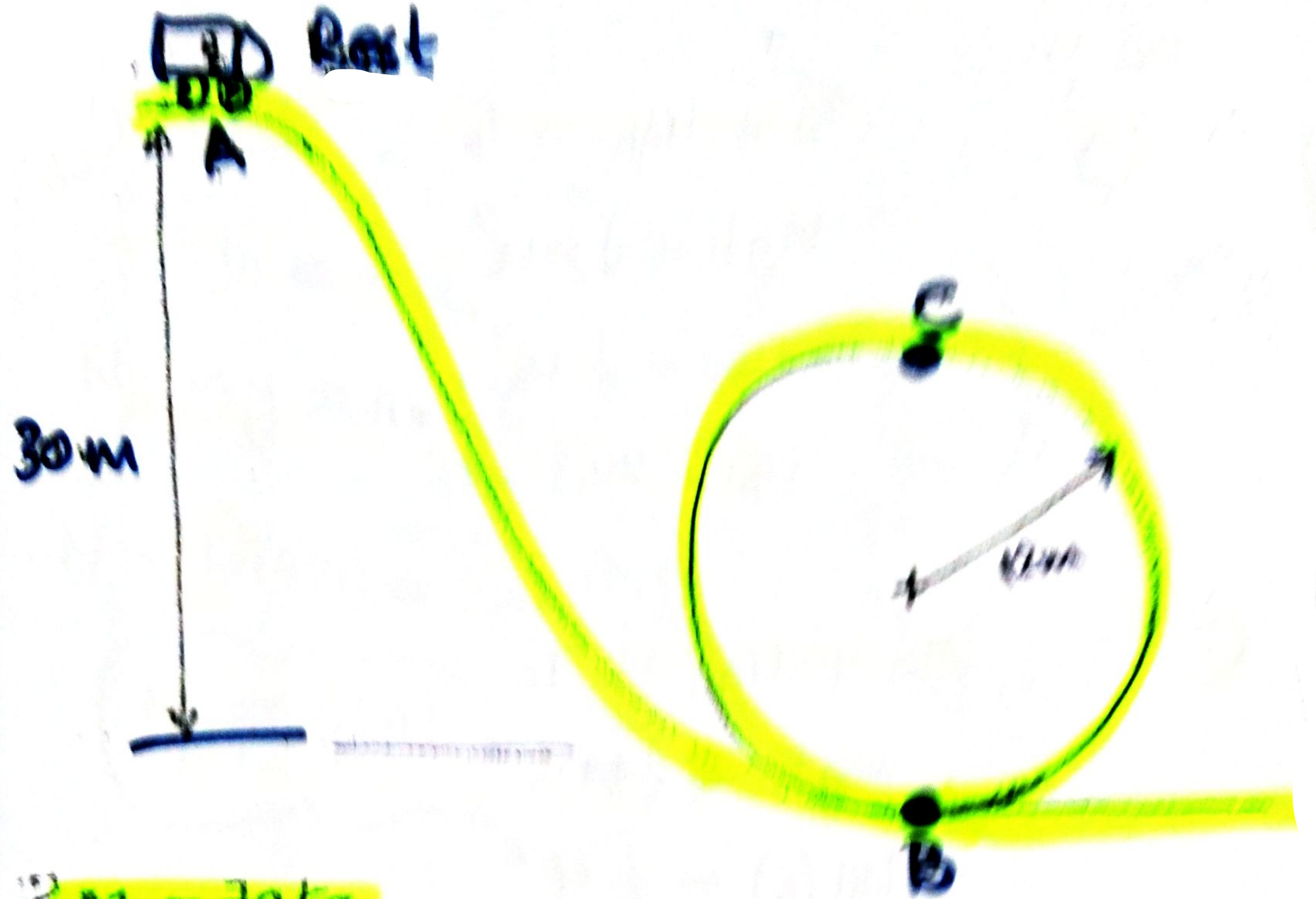


$\frac{1}{2}mv_1^2$ $\qquad$ $\frac{1}{2}mv_2^2$

$$T_1 + U_{12} = T_2$$

القانون العام

$\pm Pd$ $\qquad$ $-\mu N d$ $\qquad$ $\pm mgh$ $\qquad$ $\pm \frac{1}{2}k\Delta s^2$



$m_1 = 70\ kg$

$m_2 = 50\ kg$

(1) Find the velocity at B ?

(2) Find the velocity at C ?

(3) Find the Normal force (N) bet. the seat and the boy ? at B ?

(4) Find the Normal force (N) at point C ?

$$\cancel{T_A}^{0} + U_{AB} = T_B$$

1

$$\cancel{m}gh = \frac{1}{2}\cancel{m}v_B^2$$

$$9.81(30) = \frac{1}{2}v_B^2$$

$$\Rightarrow \quad v_B = 24.3 \text{ m/s}$$

2

$$\cancel{T_A}^{0} + U_{AC} = T_C$$

$$\cancel{m}gh = \frac{1}{2}\cancel{m}v_C^2$$

$$9.81(6) = \frac{1}{2}v_C^2$$

$$\Rightarrow \quad v_C = 10.8 \text{ m/s}$$

حل آخر

$$T_B + U_{BC} = T_C$$

$$\frac{1}{2}\cancel{m}v_B^2 - \cancel{m}gh = \frac{1}{2}\cancel{m}v_C^2$$

$$\frac{1}{2}(24.3)^2 - 9.81(24) = \frac{1}{2}v_C^2$$

$$v_C = 10.8 \text{ m/s}$$ نفس الاجابة

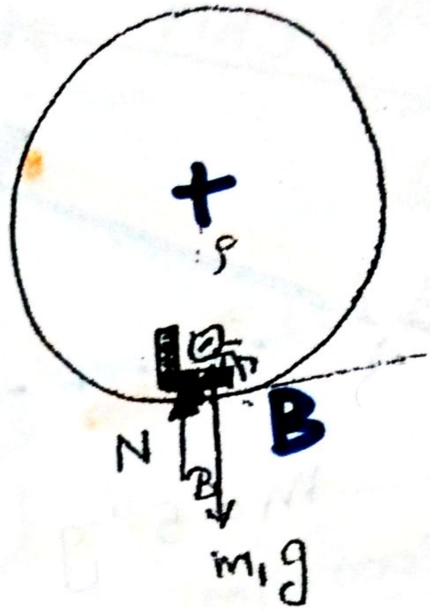
③ Ch13

$$\sum F_n = m a_n$$

$$N - m_1 g = m \frac{v_B^2}{\rho}$$

$$N - 70(9.81) = (120) \frac{(24.3)^2}{12}$$

$$N = 6591 \text{ Newton}$$

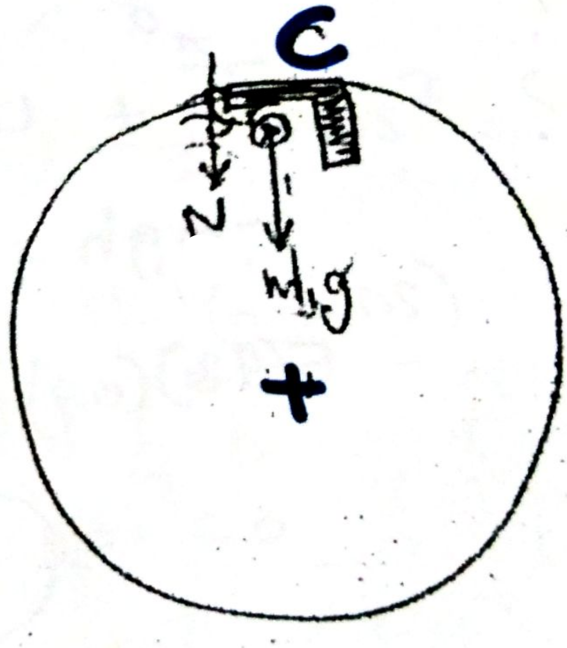


Ch13

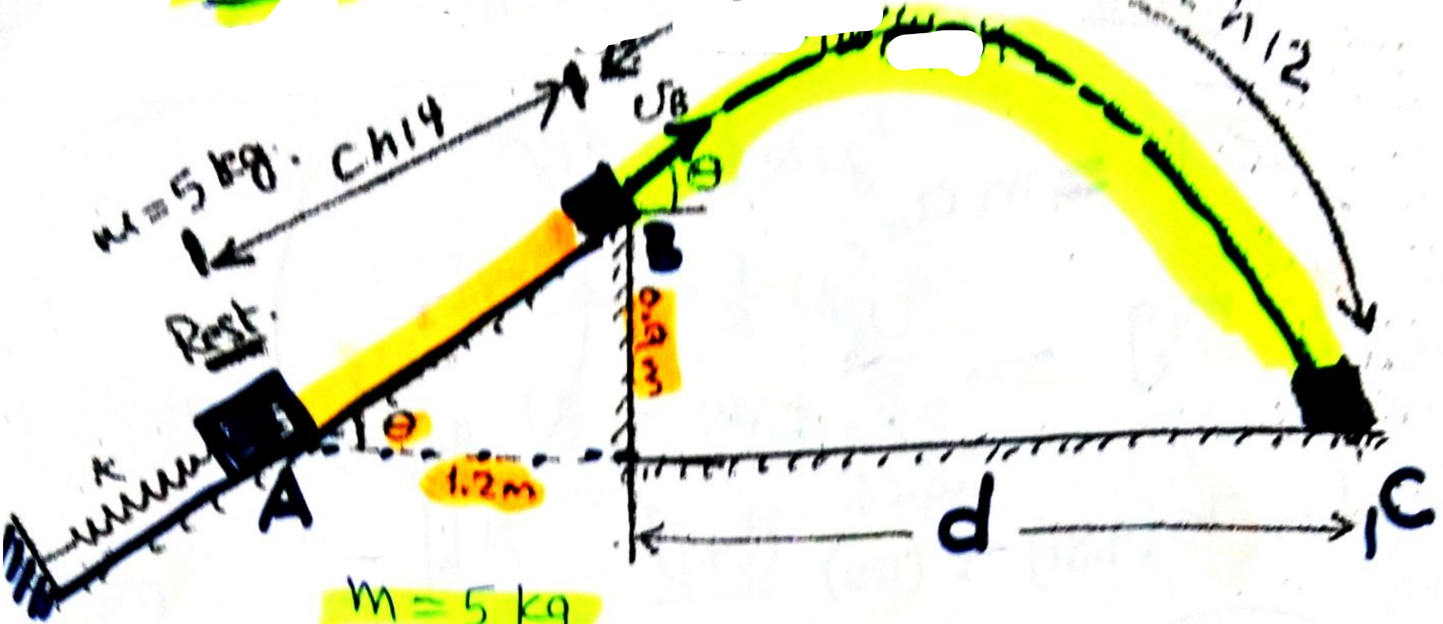
④ $$N + m_1 g = m \frac{v_C^2}{\rho}$$

$$N + 70(9.81) = 120 \frac{(10.8)^2}{12}$$

$$N = 480 \text{ Newton}$$



Ex 11



$m = 5\ kg$

$K = 2000\ N/m$ Find d?

the spring is originally compressed By 0.6 m.

Sol:

$\theta = \tan^{-1} \frac{0.9}{1.2} = 36.8°$

Find

1. v_B?
2. t_{BC}?
3. d?

$\cancel{T_A}^{0} + U_{AB} = T_B$

$-mgh + \frac{1}{2} K \Delta s^2 = \frac{1}{2} m v_B^2$

$-5(9.81)(0.9) + \frac{1}{2}(2000)(0.6)^2 = \frac{1}{2} 5\ v_B^2$

$\Rightarrow v_B = \cancel{12.7}\ 11.24\ m/s$

$H = (v_0 \sin\theta) t - \frac{1}{2} g t^2$

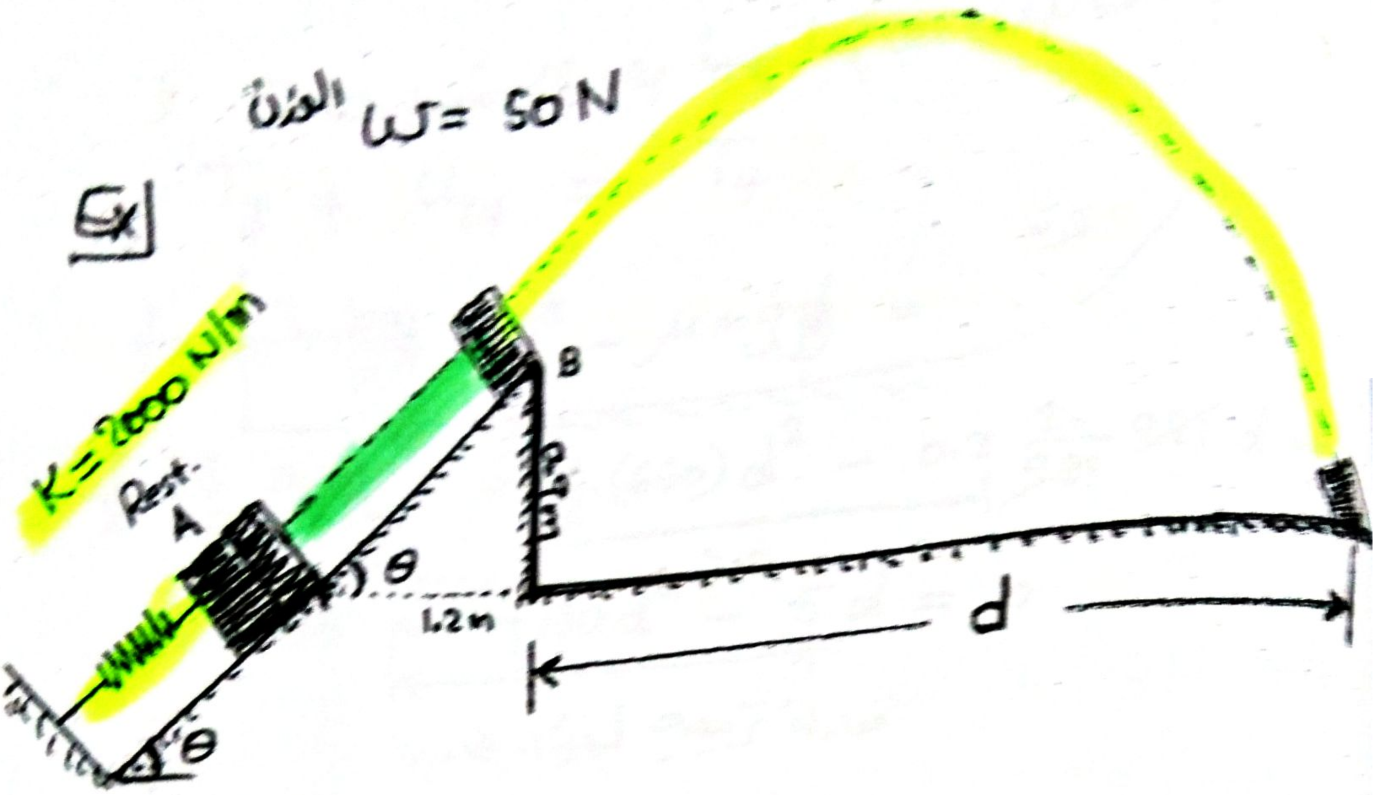
$-0.9 = (11.24 \sin 36.8) t - 4.905 t^2$

$\Rightarrow 4.905 t^2 - 6.73\ t - 0.9 = 0$

$\boxed{t = \quad sec}$

$d = (v_0 \cos\theta) t$

Ex



* the spring is compressed at A $\underline{0.6\ m}$

Find the horizontal distance d ?

$$\theta = \tan^{-1}\frac{0.9}{1.2} \Rightarrow \theta = 36.9^\circ$$

Ch.14

$$T_A + U_{AB} = T_B$$

$$0 + \frac{1}{2}K(0.6)^2 - mg(0.9) = \frac{1}{2}m v_B^2$$

$$\frac{1}{2}(2000)(0.6)^2 - 50(0.9) = \frac{1}{2}\frac{50}{9.81}v_B^2$$

$$\Rightarrow v_B = 11.11\ m/s = v_0$$

Ch.12 * $H = (v_0 \sin\theta)t - \frac{1}{2}gt^2$

$$(11.11 \sin 36.9^\circ)t - \frac{1}{2}(9.81)t^2$$

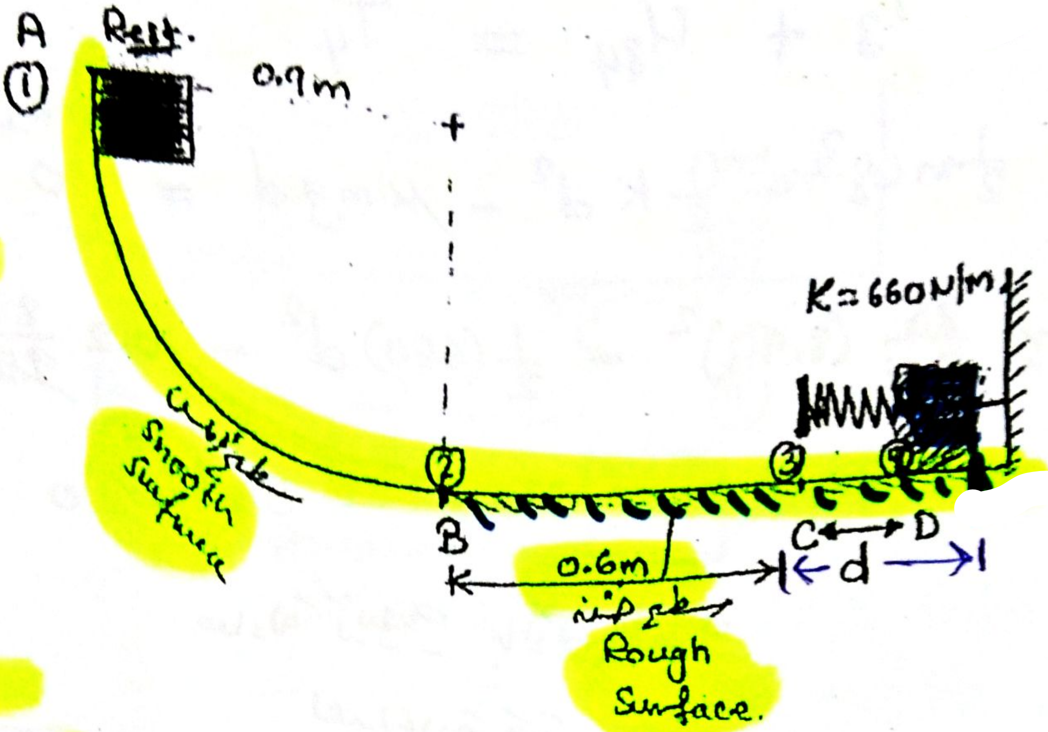
$$\Rightarrow t = 1.48\ sec$$

$$d = (v_0 \cos\theta)\,t$$

Example

الوزن

$W = 25\ N$



Find the Compress in the Spring

أوجد مقدار الانضغاط في الزنبرك.

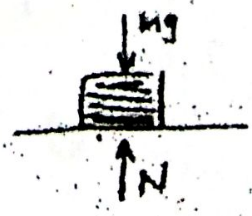
$\mu = 0.2$ معامل الاحتكاك

$$T_1 + U_{12} = T_2$$

$$0 + \cancel{m}gh = \frac{1}{2}\cancel{m}v_B^2$$

$$9.81(0.9) = \frac{1}{2}v_B^2 \Rightarrow v_B = 4.202\ m/s.$$

$$T_2 + U_{23} = T_3$$



$N = mg$

$$\frac{1}{2}mv_B^2 - \mu N(0.6) = \frac{1}{2}mv_C^2$$

$$\frac{1}{2}\cancel{m}v_B^2 - \mu\cancel{m}g(0.6) = \frac{1}{2}\cancel{m}v_C^2$$

$$\frac{1}{2}(4.202)^2 - 0.2(9.81)(0.6) = \frac{1}{2}v_C^2$$

$$v_C = 3.917\ m/s.$$

$$T_3 + U_{34} = T_4$$

توقف

$$\frac{1}{2} m v_2^2 - \frac{1}{2} k d^2 - \mu m g d = 0$$

$$\frac{1}{2} \frac{25}{9.81} (7.912)^2 - \frac{1}{2}(660) d^2 - 0.2 \frac{25}{\cancel{9.81}} \cancel{9.81}\, d = 0$$

$$19.5 - 330 d^2 - 5d = 0$$

معادلة تربيعية بالآلة الحاسبة

بعد اعادة ترتيب المعادلة

$$-330 d^2 - 5d + 19.5 = 0$$

$$\Rightarrow \quad d = 0.2356 \text{ m}$$

طريقة أخرى للحل

$$T_1 + U_{14} = T_4$$

$$0 + mgh - \mu m g (0.6 + d) - \frac{1}{2} k d^2 = 0$$

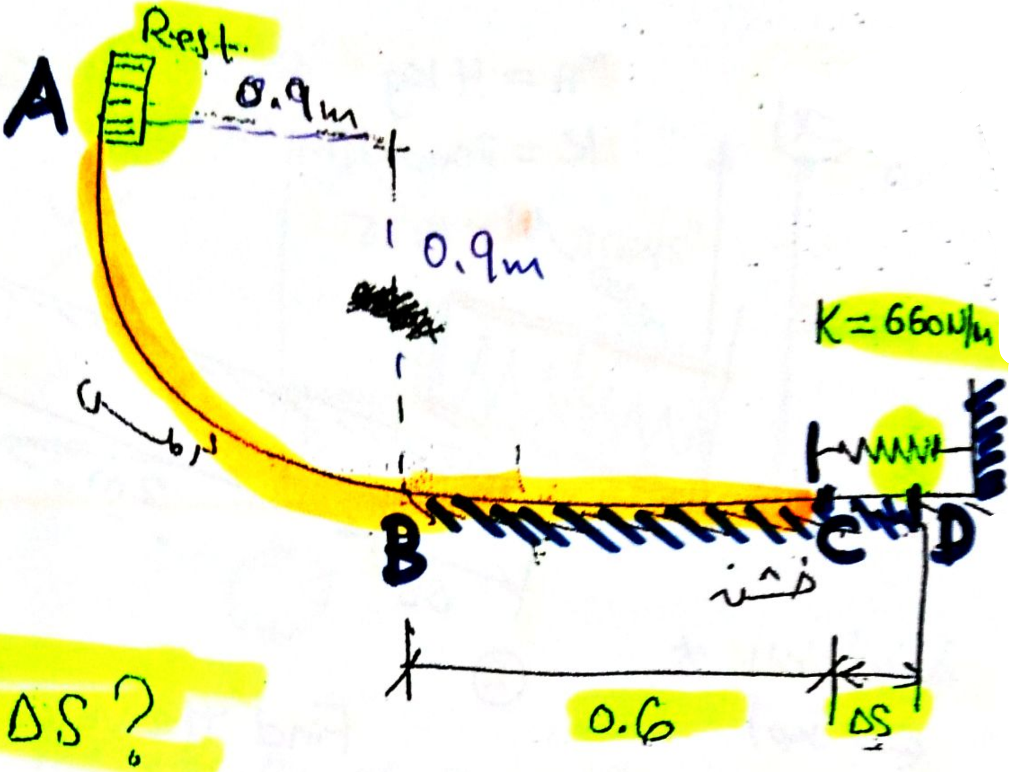
$$25(0.9) - 0.2(25)(0.6 + d) - \frac{1}{2}(660) d^2 = 0$$

$$d = 0.2356 \text{ m}$$

Ex 11

$M = 2.5\ Kg$

$\mu = 0.2$



Find ΔS ?

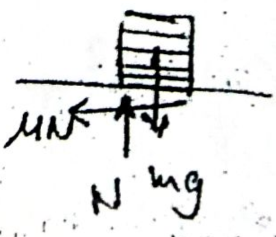
$$\cancel{T_A}^{\,0} + U_{AD} = \cancel{T_D}^{\,0}$$

$$+mgh - \mu \overset{mg}{N}(0.6 + \Delta S) - \frac{1}{2} K \Delta S^2 = 0$$

$$2.5(9.81)(0.9) - 0.2(2.5)(9.81)(0.6 + \Delta S) - \frac{1}{2} 660\, \Delta S^2 = 0$$

$$22.1 - 4.91(0.6 + \Delta S) - 330\, \Delta S^2 = 0$$

$$22.1 - 2.95 - 4.91\, \Delta S - 330\, \Delta S^2 = 0$$



$$330\, \Delta S^2 + 4.91\, \Delta S - 19.15 = 0$$

$\Sigma F_y = 0$

$\boxed{N = mg}$

$\Delta S = 0.233\ m$

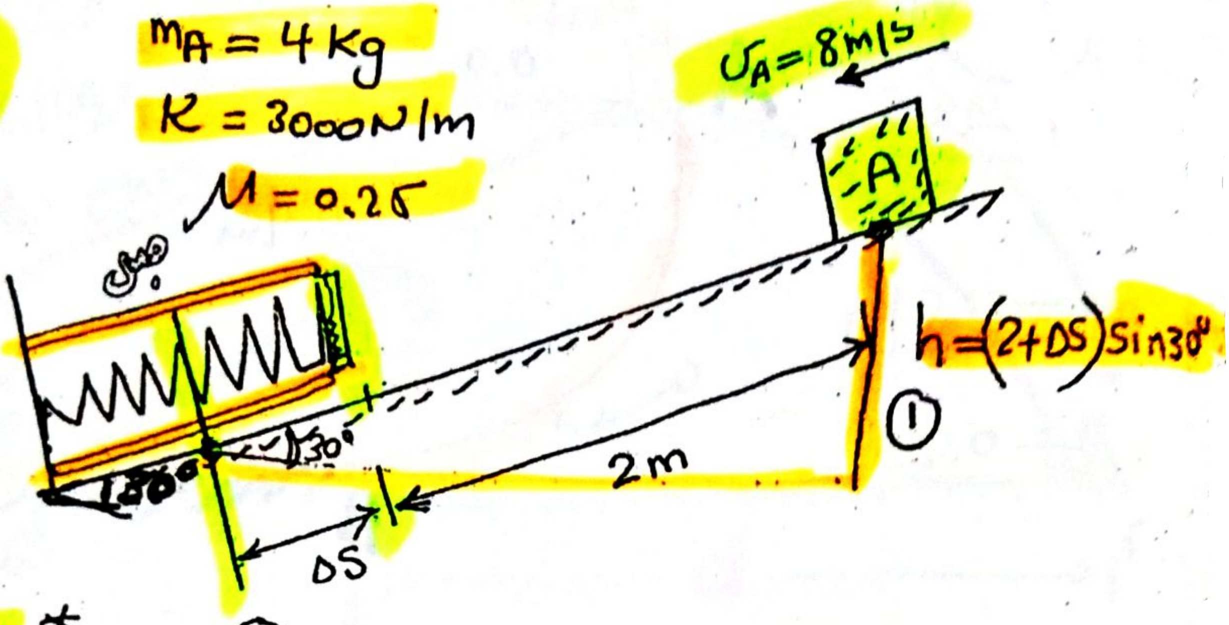
ex)

$m_A = 4\,Kg$

$K = 3000\,N/m$

$\mu = 0.25$

$v_A = 8\,m/s$



* الزنبرك مضغوط أصلاً بسبب الحبل بمقدار 0.09 m

Find the value of ΔS ?

أوجد المسافة التي سوف ينضغط بها الزنبرك نتيجة اصطدام الـ A.

الحل

$$T_1 + U_{12} = T_2$$

$$\Rightarrow \frac{1}{2} m v_1^2 + mg(2+\Delta S)\sin 30° - \mu N (2+\Delta S)$$

$mg\cos\theta$

$$+ \frac{1}{2}K(0.09)^2 - \frac{1}{2}K(0.09+\Delta S)^2 = 0$$

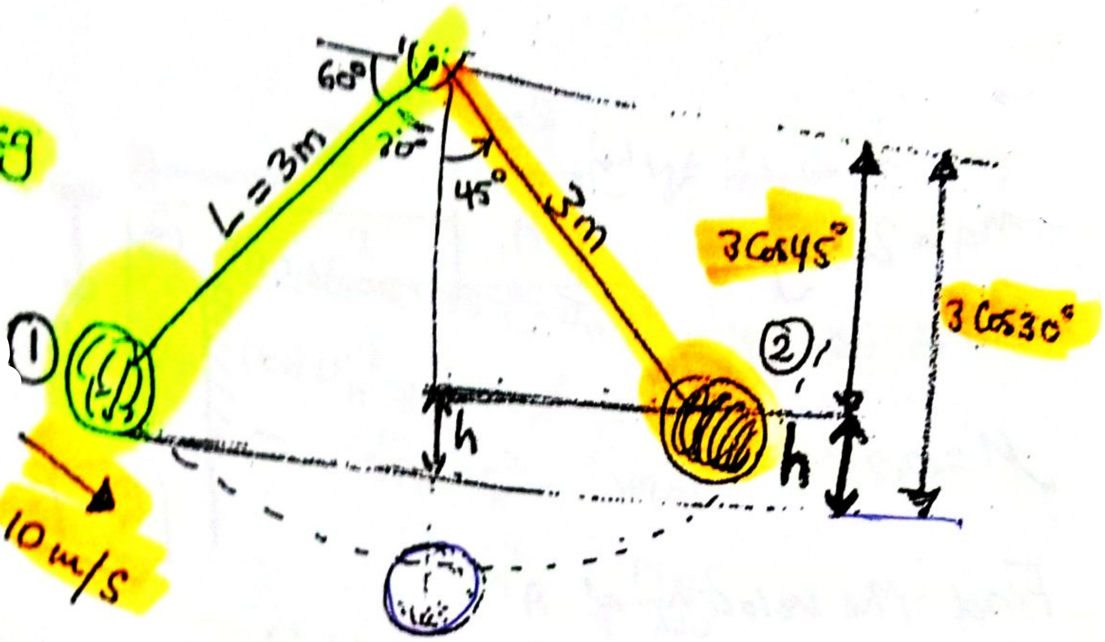
$$\frac{1}{2}(4)(8)^2 + (4)(9.81)(2+\Delta S)\sin 30° - 0.25(4)(9.81)\cos 30°(2+\Delta S)$$

$$-\frac{1}{2}(3000)(0.09)^2 - \frac{1}{2}K(0.09+\Delta S)^2 = 0$$

$\Delta S = \ldots\ldots\ m$ ✓

ex)

$m = 12 kg$



Find ① the velocity at $\theta = 45°$

② the tension in cable الشد في الحبل

$$h = 3\cos 30° - 3\cos 45°$$

$$h = 0.48\ m$$

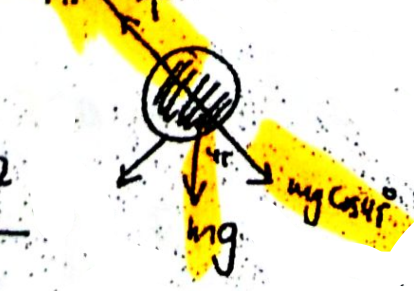
① $T_1 + U_{12} = T_2$

$$\frac{1}{2}\cancel{m}(10)^2 - \cancel{m}gh = \frac{1}{2}\cancel{m}v_2^2$$

$$\Rightarrow \frac{1}{2}(100) - 9.81(0.48) = \frac{1}{2}v_2^2$$

$$\Rightarrow v_2 = 9.517\ m/s$$

② $\sum F_n = m a_n$



$$T - mg\cos 45° = m\frac{v^2}{L}$$

$$T - 12([redacted])\cos 45° = 12\frac{(9.517)^2}{3}$$

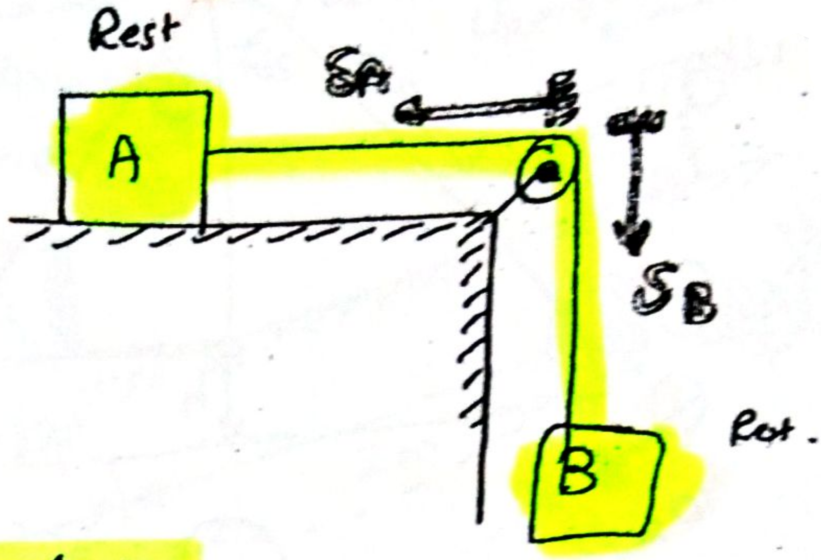
$$T = \ldots\ldots N$$

ex)

$m_A = 200kg.$

$m_B = 300kg.$

$\mu = 0.25$



Find the velocity of A after it moves 2m to the right?

الحل

$$S_A + S_B = L$$

$$\Rightarrow \boxed{v_A + v_B = 0} \text{ —— } ①$$

$$\Rightarrow \boxed{a_A + a_B = 0}$$

Block A

$$\overset{\text{Rest } 0}{T_1} + U_{12} = T_2$$

$$\Rightarrow m_B g d - \mu \overset{mg}{N} d = \frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B v_B^2$$

$$\Rightarrow 300(9.81)(2) - 0.25(200)(9.81)(2) = \frac{1}{2}(200) v_A^2 + \frac{1}{2}(300) v_B^2$$

$$\Rightarrow 100 v_A^2 + 150 v_B^2 = 4905 \text{ —— } ②$$

$$v_A + v_B = 0 \text{ —— } ①$$

هوا طلعت تجريبي ليش :

$$100 v_A^2 + 150(-v_A)^2 = 4905$$

$$100 v_A^2 + 150 v_A^2 = 4905$$

$$250 v_A^2 = 4905$$

$$v_A^2 = \frac{4905}{250}$$

$$v_A^2 = 19.62$$

$$v_A = \sqrt{19.62}$$

$$\boxed{v_A = 4.43 \text{ m/s}}$$

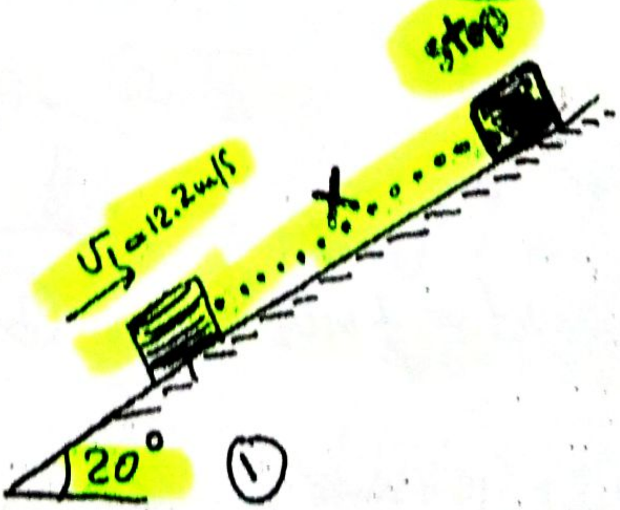
$$\boxed{v_B = -4.43 \text{ m/s}}$$

ex)

$m = 22.7$ kg.

$\mu = 0.15$.

$v_1 = 12.2$ m/s



①

① The max distance x until stops.

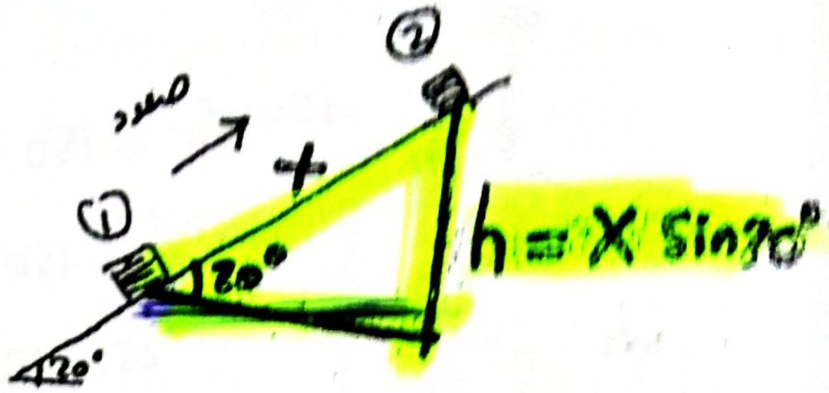
② Find the velocity of the Box as it returns to its original position.

total Amount of Energy dissipated due to friction

[illegible]

① $T_1 + U_{12} = T_2$

$\Rightarrow \frac{1}{2}(22.7)(12.2)^2 - mgh - \mu N d = 0$ Stop.

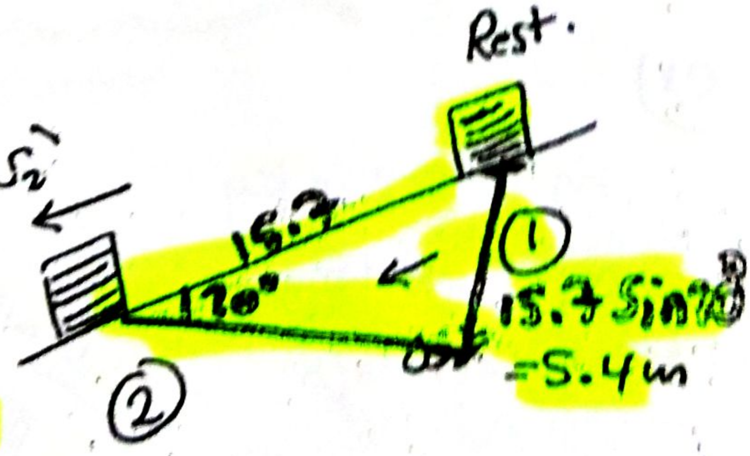


$N = mg\cos\theta$

$\Rightarrow \frac{1}{2}(22.7)(12.2)^2 - (22.7)(9.81)(X\sin 20^\circ) - 0.15(22.7)(9.81)\cos 20^\circ \, X = 0$

$\Rightarrow$ ~~X = 1107.15~~ $X = 15.7$ m

②



$T_1 + U_{12} = T_2$

$mgh - \mu N d = \frac{1}{2} m v_2^2$

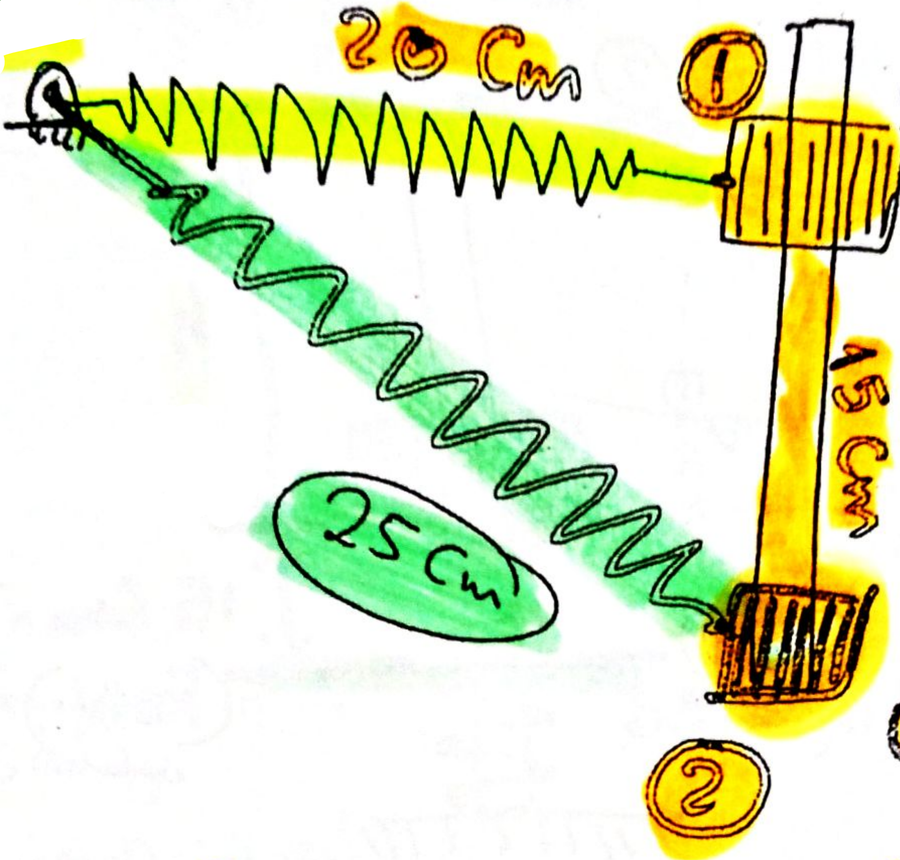
$(22.7)(9.81)(15.7\sin 20^\circ) - (0.15)(22.7)(9.81)\cos 20^\circ (15.7) = \frac{1}{2}(22.7)(v_2^2)$

$\Rightarrow v_2 = \ldots\ldots$

...ion $= \mu N d = 0.15(22.7)(9.81)\cos 20^\circ (15.7)$

$= \ldots\ldots$ Joule

Ex]



Rest.

* $M = 9\,kg$
* undeforemed length
الطول الاصلي للزنبرك
10 Cm
* $k = 525\,N/m$

$v_2 = ?$

Find v_2?

From Rest. 0

$$\cancel{T_1} + U_{12} = T_2$$

$$+mgh + \left[\frac{1}{2}k(\Delta S_1)^2 - \frac{1}{2}k(\Delta S_2)^2\right] = \frac{1}{2}mv_2^2$$

$$\Rightarrow 9(9.81)(0.15) + \left[\frac{1}{2}(525)\left[\overset{20-10}{10}\right]^2 - \frac{1}{2}(525)\left[\overset{25-10}{15}\right]^2\right] = \frac{1}{2}(9)v_2^2$$

$$v_2 = 1.5\,m/s$$

Ex

$m = 2.71\ kg$

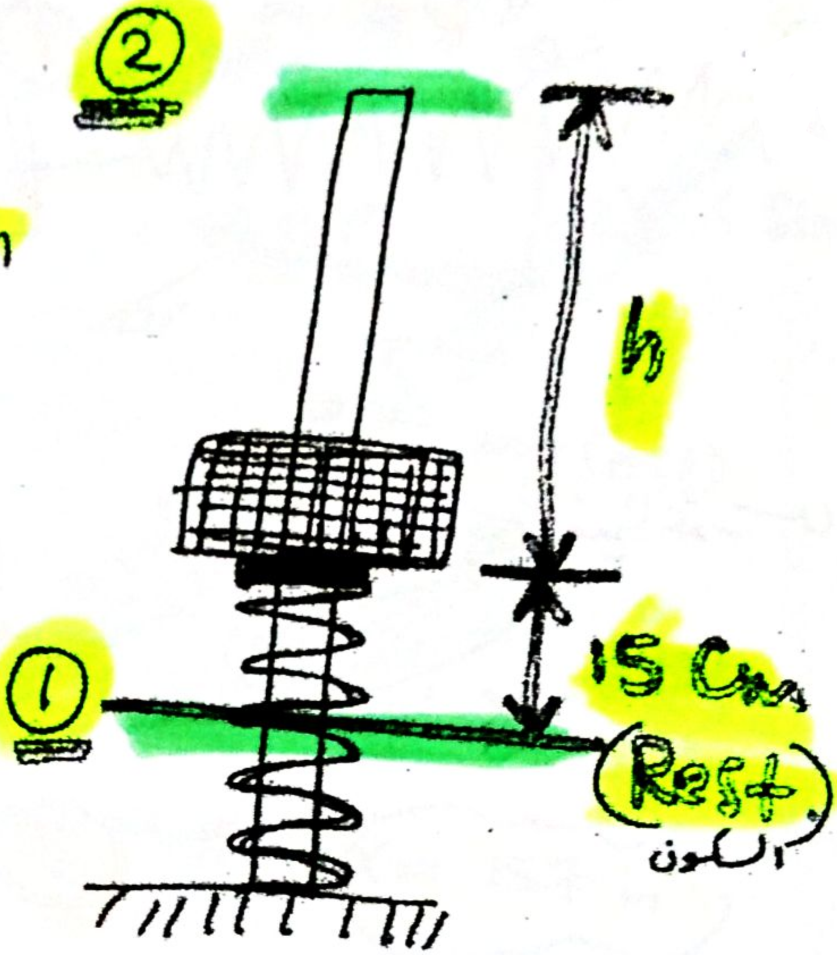
$K = 2627\ N/m$

أوجد h

اللي يطلعها

الـ Box

الذكي



الحل

$$\cancel{T_1}^{\,Rest\ 0} + U_{12} = \cancel{T_2}^{\,0}$$

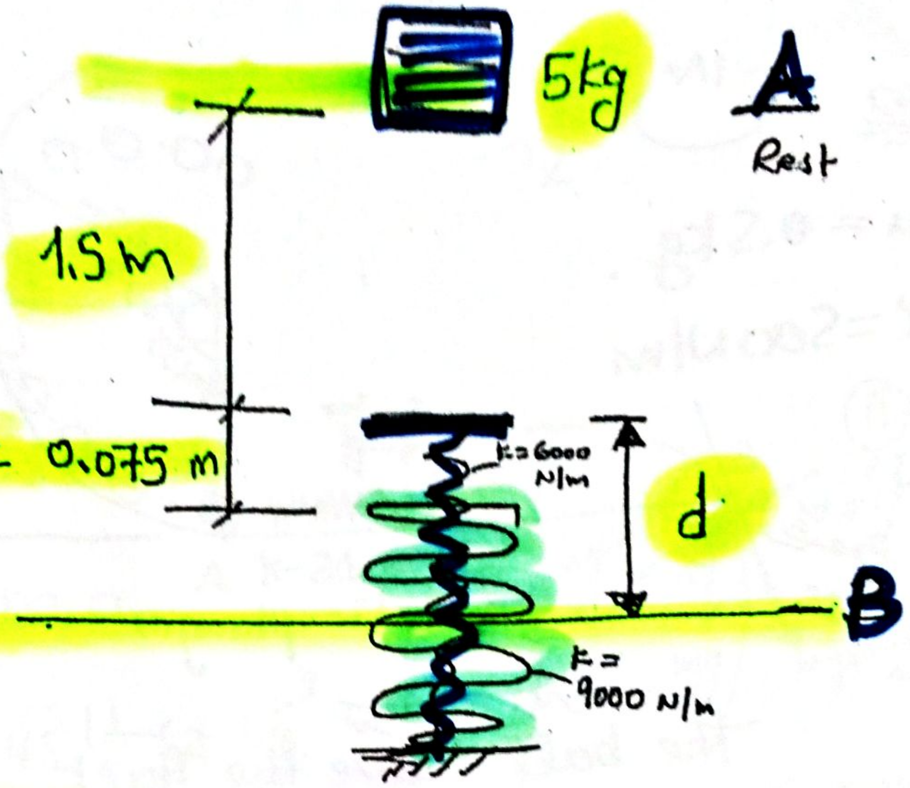
عشان أقصى ارتفاع

$$-mgH + \frac{1}{2} k \Delta S^2 = \cancel{0}\ \text{Stop}$$

$$-2.71(9.81)(0.15+h) + \frac{1}{2}(2627)(0.15)^2 = 0$$

$$\Rightarrow h = 1.12\ m$$

ex)) Find d?



$$\cancel{T_A}^{0} + U_{AB} = \cancel{T_B}^{0}$$

$$mg(1.5+d) - \frac{1}{2}(6000)d^2 - \frac{1}{2}(9000)(d-0.075)^2 = 0$$

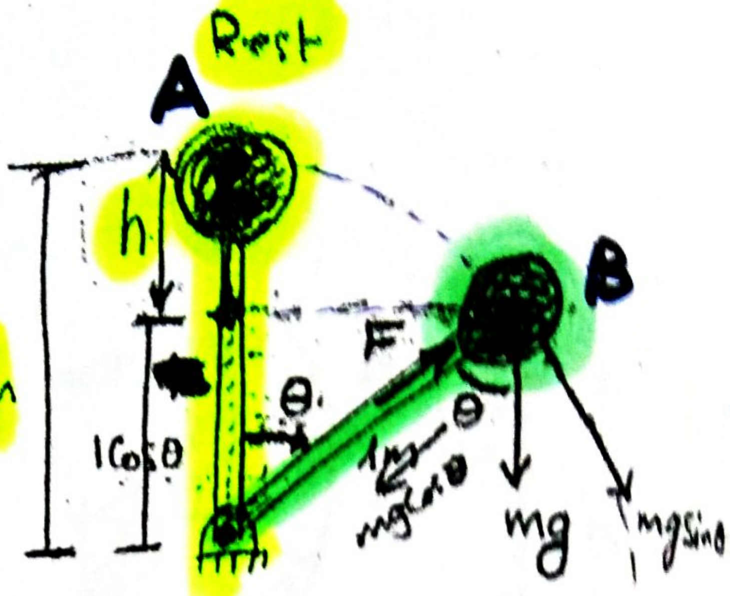
$$73.57 + 49.05d - 3000d^2 - 4500(d^2 - 0.15d + 5.625\times10^{-3})$$

$$73.57 + 49.05d - 3000d^2 - 4500d^2 + 675d - 25.313 = 0$$

$$\Rightarrow -7500d^2 + 724.1d + 48.257 = 0$$

$$d = 0.142 \text{ m}$$

Ex



$m = 7.5\,kg$

$\theta = 0 \Rightarrow v_A = 0$

أوجد مقدار الزاوية θ

التي تصبح عندها

القوة في الذراع تساوي

صفراً

$h = 1 - \cos\theta$

$$\cancel{T_A}^{\,0} + U_{AB} = T_B$$

$$\cancel{m} g h = \frac{1}{2} \cancel{m} v_B^2$$

$$v_B^2 = 2gh$$

Ch14

$$v_B^2 = 2g(1-\cos\theta) \quad \text{--- (1)}$$

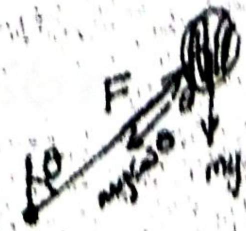
معطى في الذراع صفراً

$$\sum F_n = m a_n$$

Ch13

$$mg\cos\theta - \cancel{F}^{\,0} = m\frac{v_B^2}{\rho}$$

$$\cancel{m} g\cos\theta = \cancel{m}\frac{v_B^2}{1}$$



$$v_B^2 = g\cos\theta \quad \text{--- (2)}$$

$$\cancel{g}\cos\theta = 2\cancel{g}(1-\cos\theta)$$

$$\cos\theta = 2 - 2\cos\theta$$

$$\cos\theta + 2\cos\theta = 2$$

$$3\cos\theta = 2 \Rightarrow \cos\theta = \frac{2}{3}$$

$$\cos\theta = 0.66667$$

$$\theta = 48.2^\circ$$

$$v_B^2 = g\cos\theta$$

$$v_B^2 = 9.81\cos 48.2^\circ$$

$$\Rightarrow v_B = 2.56 \text{ m/s}$$

Ex

$m = 2.5\ kg$

$K = 4\ N/cm$

Unstretched length $= \underline{30}\ cm$

الطول غير المشدود

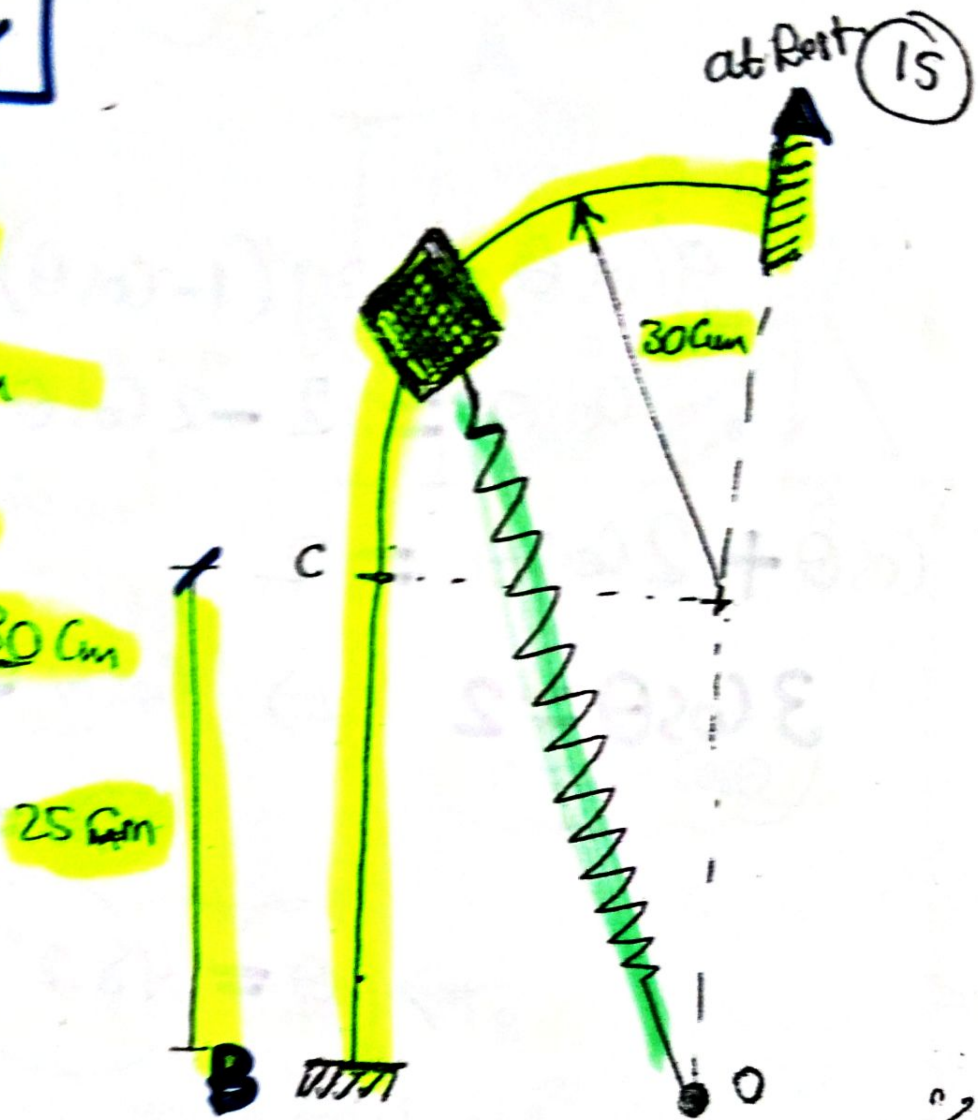
(غير متمدد)

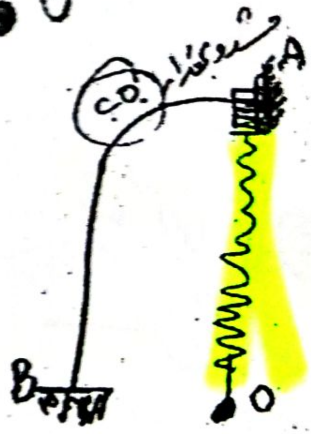
أوجد سرعته

عندما يمر

بـ B

v_B ?



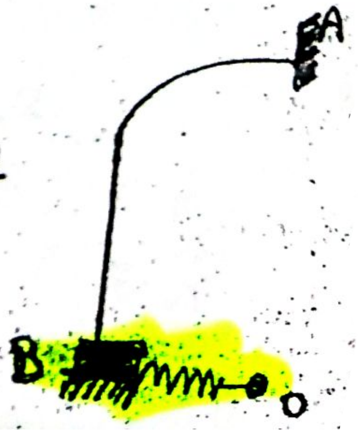


$$\cancel{T_A} + U_{AB} = T_B$$

$$mgh + \frac{1}{2} K \Delta s^2 = \frac{1}{2} m v_B^2$$

$$2.5(9.81)(55) + \frac{1}{2}(4)(25)^2 = \frac{1}{2}(2.5)\, v_B^2$$

$\Rightarrow v_B =$ [redacted]6 cm/s



Power and Efficiency

Power. The term "power" provides a useful basis for choosing the type of motor or machine which is required to do a certain amount of work in a given time. For example, two pumps may each be able to empty a reservoir if given enough time; however, the pump having the larger power will complete the job sooner.

The *power* generated by a machine or engine that performs an amount of work dU within the time interval dt is therefore

$$\boxed{P = \frac{dU}{dt}} \tag{14–9}$$

If the work dU is expressed as $dU = \mathbf{F} \cdot d\mathbf{r}$, then

$$P = \frac{dU}{dt} = \frac{\mathbf{F} \cdot d\mathbf{r}}{dt} = \mathbf{F} \cdot \frac{d\mathbf{r}}{dt}$$

or

$$\boxed{P = \mathbf{F} \cdot \mathbf{v}} \quad \text{output} \tag{14–10}$$

Hence, power is a *scalar*, where in this formulation $\mathbf{v}$ represents the velocity of the particle which is acted upon by the force $\mathbf{F}$.

The basic unit of power used in the SI system is the watt (W). These units are defined as

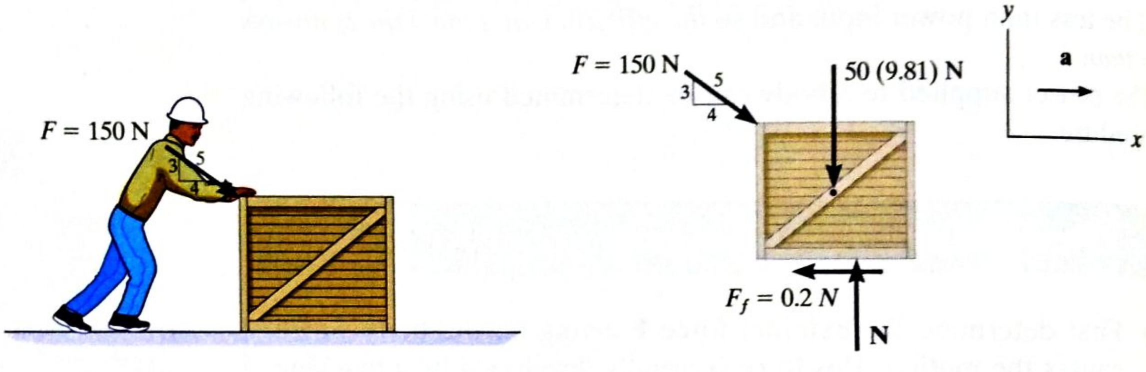
$$1 \text{ W} = 1 \text{ J/s} = 1 \text{ N} \cdot \text{m/s}$$

Efficiency. The *mechanical efficiency* of a machine is defined as the ratio of the output of useful power produced by the machine to the input of power supplied to the machine. Hence,

$$\boxed{\varepsilon = \frac{\text{power output}}{\text{power input}}} \tag{14–11}$$

EXAMPLE 14.7

The man in Fig. 14–15*a* pushes on the 50-kg crate with a force of $F = 150$ N. Determine the power supplied by the man when $t = 4$ s. The coefficient of kinetic friction between the floor and the crate is $\mu_k = 0.2$. Initially the create is at rest.



(a) (b)

Fig. 14–15

SOLUTION

To determine the power developed by the man, the velocity of the 150-N force must be obtained first. The free-body diagram of the crate is shown in Fig. 14–15*b*. Applying the equation of motion,

$$+\uparrow \Sigma F_y = ma_y; \qquad N - \left(\tfrac{3}{5}\right)150\text{ N} - 50(9.81)\text{ N} = 0$$

$$N = 580.5\text{ N}$$

$$\overset{+}{\rightarrow} \Sigma F_x = ma_x; \qquad \left(\tfrac{4}{5}\right)150\text{ N} - 0.2(580.5\text{ N}) = (50\text{ kg})a$$

$$a = 0.078\text{ m/s}^2$$

The velocity of the crate when $t = 4$ s is therefore

$$(\overset{+}{\rightarrow}) \qquad v = v_0 + a_c t$$

$$v = 0 + (0.078\text{ m/s}^2)(4\text{ s}) = 0.312\text{ m/s}$$

The power supplied to the crate by the man when $t = 4$ s is therefore

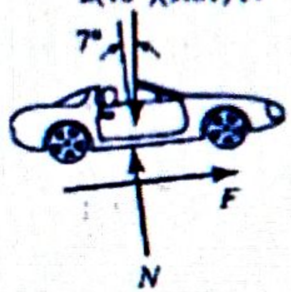
$$P = \mathbf{F}\cdot\mathbf{v} = F_x v = \left(\tfrac{4}{5}\right)(150\text{ N})(0.312\text{ m/s})$$

$$= 37.4\text{ W}$$

Ans.

An automobile having a mass of 2 Mg travels up a 7° slope at a constant speed of $v = 100$ km/h. If mechanical friction and wind resistance are neglected, determine the power developed by the engine if the automobile has an efficiency $\epsilon = 0.65$.

SOLUTION



Equation of Motion: The force F which is required to maintain the car's constant speed up the slope must be determined first.

$$+\Sigma F_{x'} = ma_{x'}; \qquad F - 2(10^3)(9.81)\sin 7° = 2(10^3)(0)$$

$$F = 2391.08 \text{ N}$$

Power: Here, the speed of the car is $v = \left[\dfrac{100(10^3)\text{ m}}{\text{h}}\right] \times \left(\dfrac{1\text{ h}}{3600\text{ s}}\right) = 27.78$ m/s.

The power output can be obtained using Eq. 14–10.

$$P = \mathbf{F}\cdot\mathbf{v} = 2391.08(27.78) = 66.418(10^3)\text{ W} = 66.418\text{ kW}$$

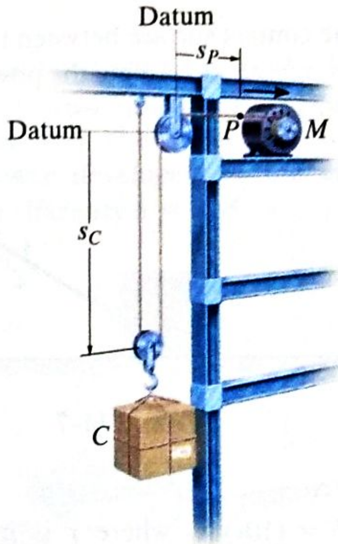
Using Eq. 14–11, the required power input from the engine to provide the above power output is

$$\text{power input} = \frac{\text{power output}}{\varepsilon}$$

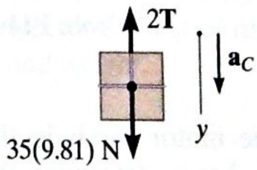
$$= \frac{66.418}{0.65} = 102\text{ kW} \qquad \textbf{Ans.}$$

EXAMPLE 14.8

The motor M of the hoist shown in Fig. 14–16*a* lifts the 35-kg crate C so that the acceleration of point P is $1.2\ \text{m/s}^2$. Determine the power that must be supplied to the motor at the instant P has velocity of 0.6 m/s. Neglect the mass of the pulley and cable and take $\varepsilon = 0.85$.



(a)



(b)

Fig. 14–16

SOLUTION

In order to find the power output of the motor, it is first necessary to determine the tension in the cable since this force is developed by the motor.

From the free-body diagram, Fig. 14–16*b*, we have

$$+\downarrow \Sigma F_y = ma_y; \qquad -2T + 35(9.81)\ \text{N} = (35\ \text{kg})\, a_c \qquad (1)$$

The acceleration of the crate can be obtained by using kinematics to relate it to the known acceleration of point P, Fig. 14–16*a*. Using the methods of absolute dependent motion, the coordinates s_C and s_P can be related to a constant portion of cable length l which is changing in the vertical and horizontal directions. We have $2s_C + s_P = l$. Taking the second time derivative of this equation yields

$$2a_C = -a_P \qquad (2)$$

Since $a_P = +1.2\ \text{m/s}^2$, then $a_C = -(1.2\ \text{m/s}^2)/2 = -0.6\ \text{m/s}^2$. What does the negative sign indicate? Substituting this result into Eq. 1 and *retaining* the negative sign since the acceleration in *both* Eq. 1 and Eq. 2 was considered positive downward, we have

$$-2T + 35(9.81)\ \text{N} = (35\ \text{kg})(-0.6\ \text{m/s}^2)$$

$$T = 182.2\ \text{N}$$

The power output required to draw the cable in at a rate of 0.6 m/s is therefore

$$P = \mathbf{T}\cdot\mathbf{v} = (182.2\ \text{N})(0.6\ \text{m/s})$$
$$= 109.3\ \text{W}$$

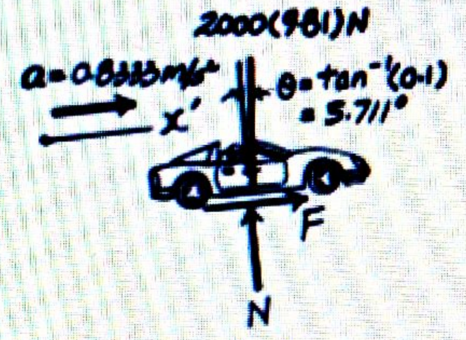
This *power output* requires that the motor provide a *power input* of

$$\text{power input} = \frac{1}{\varepsilon}(\text{power output})$$
$$= \frac{1}{0.85}(109.3\ \text{W}) = 129\ \text{W} \qquad \textit{Ans.}$$

NOTE: Since the velocity of the crate is constantly changing, the power requirement is *instantaneous*.

The 2-Mg car increases its speed uniformly from rest to 25 m/s in 30 s up the inclined road. Determine the maximum power that must be supplied by the engine, which operates with an efficiency of $\epsilon = 0.8$. Also, find the average power supplied by the engine.

SOLUTION



(a)

Kinematics: The constant acceleration of the car can be determined from

$$(\overset{+}{\rightarrow}) \qquad v = v_0 + a_c t$$

$$25 = 0 + a_c(30)$$

$$a_c = 0.8333 \text{ m/s}^2$$

Equations of Motion: By referring to the free-body diagram of the car shown in Fig. *a*,

$$\Sigma F_{x'} = ma_{x'}; \qquad F - 2000(9.81)\sin 5.711° = 2000(0.8333)$$

$$F = 3618.93\text{N}$$

Power: The maximum power output of the motor can be determined from

$$(P_{out})_{max} = \mathbf{F}\cdot\mathbf{v}_{max} = 3618.93(25) = 90\,473.24 \text{ W}$$

Thus, the maximum power input is given by

$$P_{in} = \frac{P_{out}}{\epsilon} = \frac{90473.24}{0.8} = 113\,091.55 \text{ W} = 113 \text{ kW} \qquad \text{Ans.}$$

The average power output can be determined from

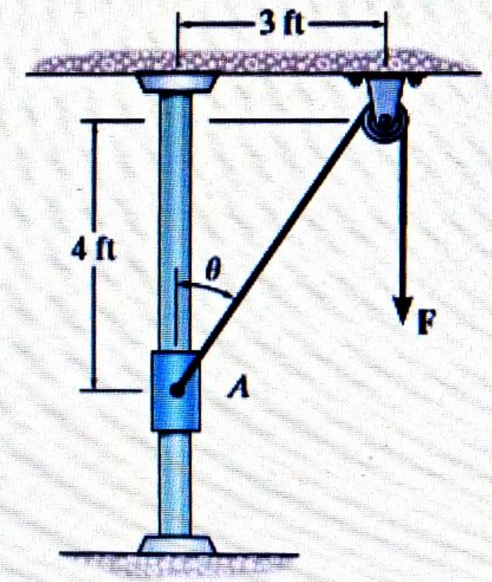
$$(P_{out})_{avg} = \mathbf{F}\cdot\mathbf{v}_{avg} = 3618.93\left(\frac{25}{2}\right) = 45\,236.62 \text{ W}$$

Thus,

$$(P_{in})_{avg} = \frac{(P_{out})_{avg}}{\epsilon} = \frac{45236.62}{0.8} = 56\,545.78 \text{ W} = 56.5 \text{ kW} \qquad \text{Ans.}$$

*14–56.

The 10-lb collar starts from rest at A and is lifted by applying a constant vertical force of $F = 25$ lb to the cord. If the rod is smooth, determine the power developed by the force at the instant $\theta = 60°$.



SOLUTION

Work of **F**

$$U_{1-2} = 25(5 - 3.464) = 38.40 \text{ lb} \cdot \text{ft}$$

$$T_1 + \Sigma U_{1-2} = T_{2s}$$

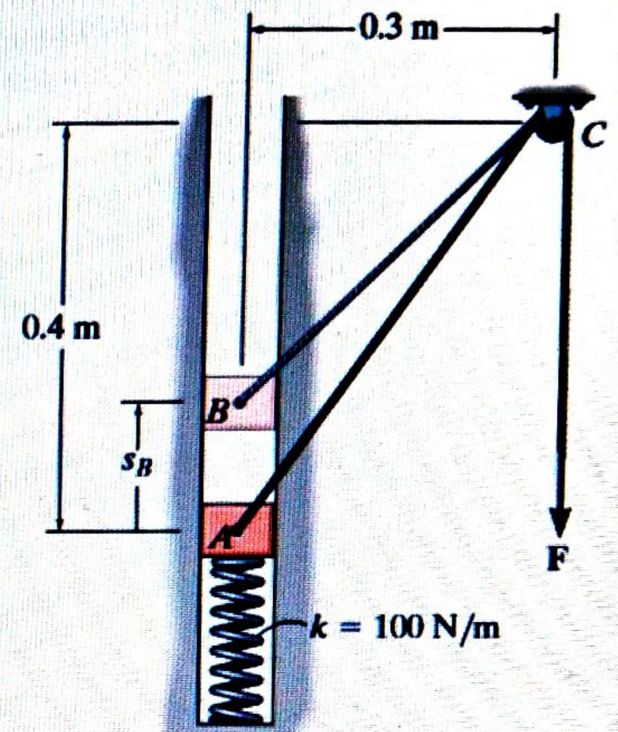
$$0 + 38.40 - 10(4 - 1.732) = \frac{1}{2}\left(\frac{10}{32.2}\right) v^2$$

$$v = 10.06 \text{ ft/s}$$

$$P = \mathbf{F} \cdot \mathbf{v} = 25 \cos 60°(10.06) = 125.76 \text{ ft} \cdot \text{lb/s}$$

$$P = 0.229 \text{ hp}$$ **Ans.**

The block has a mass of 0.8 kg and moves within the smooth vertical slot. If it starts from rest when the *attached* spring is in the unstretched position at A, determine the *constant* vertical force F which must be applied to the cord so that the block attains a speed $v_B = 2.5$ m/s when it reaches B; $s_B = 0.15$ m. Neglect the size and mass of the pulley. *Hint:* The work of **F** can be determined by finding the difference Δl in cord lengths AC and BC and using $U_F = F\,\Delta l$.



SOLUTION

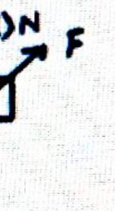
$$l_{AC} = \sqrt{(0.3)^2 + (0.4)^2} = 0.5 \text{ m}$$

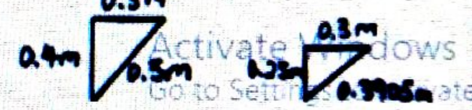
$$l_{BC} = \sqrt{(0.4 - 0.15)^2 + (0.3)^2} = 0.3905 \text{ m}$$

$$T_A + \Sigma U_{A-B} = T_B$$

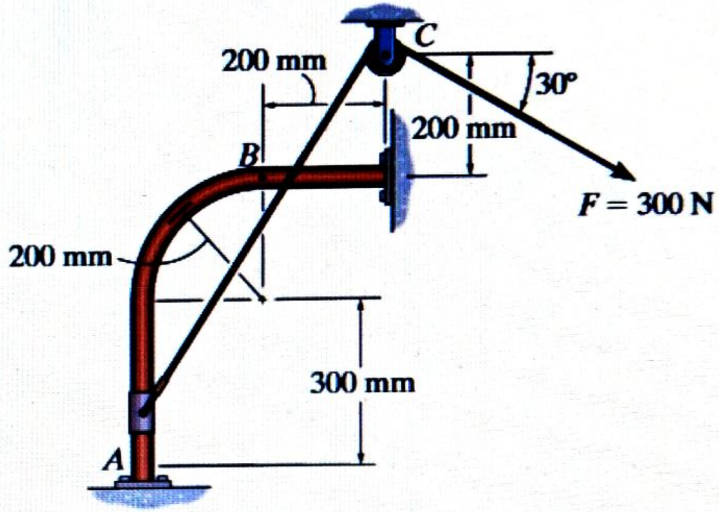
$$0 + F(0.5 - 0.3905) - \frac{1}{2}(100)(0.15)^2 - (0.8)(9.81)(0.15) = \frac{1}{2}(0.8)(2.5)^2$$

$F = 43.9$ N **Ans.**





If the cord is subjected to a constant force of $F = 300$ N and the 15-kg smooth collar starts from rest at A, determine the velocity of the collar when it reaches point B. Neglect the size of the pulley.



SOLUTION

Free-Body Diagram: The free-body diagram of the collar and cord system at an arbitrary position is shown in Fig. *a*.

Principle of Work and Energy: Referring to Fig. *a*, only **N** does no work since it always acts perpendicular to the motion. When the collar moves from position A to position B, **W** displaces vertically upward a distance $h = (0.3 + 0.2)$ m $= 0.5$ m, while force F displaces a distance of $s = AC - BC = \sqrt{0.7^2 + 0.4^2} - \sqrt{0.2^2 + 0.2^2} = 0.5234$ m. Here, the work of **F** is positive, whereas **W** does negative work.

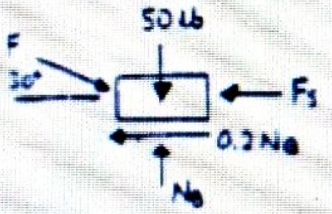
$$T_A + \Sigma U_{A-B} = T_B$$

$$0 + 300(0.5234) + [-15(9.81)(0.5)] = \frac{1}{2}(15)v_B^2$$

$$v_B = 3.335 \text{ m/s} = 3.34 \text{ m/s}$$ **Ans.**

The 50-lb block rests on the rough surface for which the coefficient of kinetic friction is $\mu_k = 0.2$. A force $F = (40 + s^2)$ lb, where s is in ft, acts on the block in the direction shown. If the spring is originally unstretched ($s = 0$) and the block is at rest, determine the power developed by the force the instant the block has moved $s = 1.5$ ft.

SOLUTION



$+\uparrow \Sigma F_y = 0;\qquad N_B - (40 + s^2)\sin 30° - 50 = 0$

$$N_B = 70 + 0.5s^2$$

$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + \int_0^{1.5} (40 + s^2)\cos 30°\, ds - \frac{1}{2}(20)(1.5)^2 - 0.2\int_0^{1.5} (70 + 0.5s^2)ds = \frac{1}{2}\left(\frac{50}{32.2}\right)v_2^2$$

$$0 + 52.936 - 22.5 - 21.1125 = 0.7764v_2^2$$

$$v_2 = 3.465 \text{ ft/s}$$

When $s = 1.5$ ft,

$$F = 40 + (1.5)^2 = 42.25 \text{ lb}$$

$$P = \mathbf{F}\cdot\mathbf{v} = (42.25\cos 30°)(3.465)$$

$P = 126.79$ ft·lb/s $= 0.231$ hp **Ans.**

The 1000-lb elevator is hoisted by the pulley system and motor M. If the motor exerts a constant force of 500 lb on the cable, determine the power that must be supplied to the motor at the instant the load has been hoisted $s = 15$ ft starting from rest. The motor has an efficiency of $\varepsilon = 0.65$.

SOLUTION

Equation of Motion. Referring to the FBD of the elevator, Fig. *a*,

$$+\uparrow \Sigma F_y = ma_y; \qquad 3(500) - 1000 = \frac{1000}{32.2}a$$

$$a = 16.1 \text{ ft/s}^2$$

When $S = 15ft$,

$$+\uparrow v^2 = v_0^2 + 2a_c(S - S_0); \quad v^2 = 0^2 + 2(16.1)(15)$$

$$v = 21.98 \text{ ft/s}$$

Power. Applying Eq. 14–9, the power output is

$$P_{out} = F \cdot V = 3(500)(21.98) = 32.97(10^3) \text{ lb} \cdot \text{ft/s}$$

The power input can be determined using Eq. 14–9

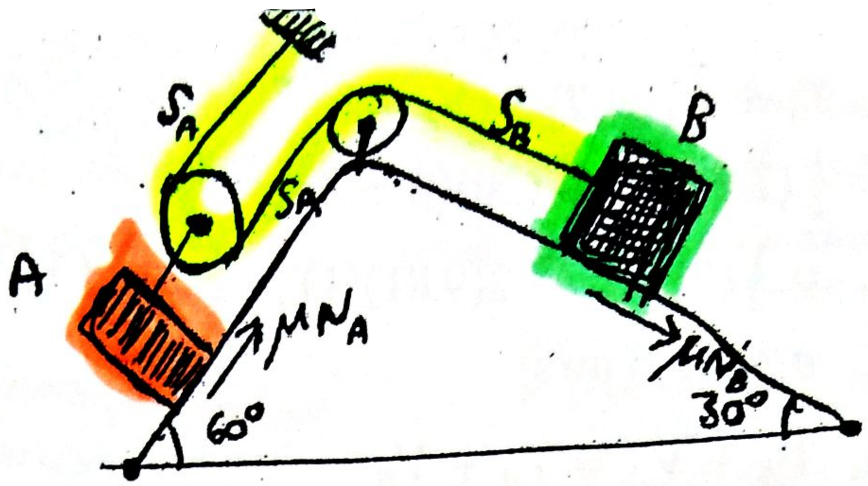
$$\Sigma = \frac{P_{out}}{P_{in}}; \qquad 0.65 = \frac{32.97(10^3)}{P_{in}}$$

$$P_{in} = [50.72(10^3) \text{ lb} \cdot \text{ft/s}]\left(\frac{1 \text{ hp}}{550 \text{ lb} \cdot \text{ft/s}}\right)$$

$$= 92.21 \text{ hp} = 92.2 \text{ hp}$$

Ans.

ex



$m_A = 30kg$

$m_B = 20kg$

$\mu = 0.10$

Start
(From Rest)

كم هي سرعة ا , وسرعة B
بعدما يتحرك الجسم B للأسفل
مسافة مقدارها ١م (1m)

$$2S_A + S_B = L \Rightarrow \boxed{2\Delta S_A + \Delta S_B = 0}$$

$$\Rightarrow \boxed{2v_A + v_B = 0}$$

$$2\Delta S_A + (-1) = 0$$

$$\Rightarrow \Delta S_A = 0.5\ m.$$

Ch14

$$\cancel{T_1}^{\text{Rest}\ 0} + U_{12} = T_2$$

$$N_A = m_A g \cos 60$$

$$N_B = m_B g \cos 30$$

$$m_A g (\Delta S_A \sin 60) - m_B g (\Delta S_B \sin 30^\circ)$$

$$- \mu (\overbrace{m_A g \cos 60}^{N_A}) \Delta S_A$$

$$- \mu (\overbrace{m_B g \cos 30}^{N_B}) \Delta S_B = \frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B v_B^2$$

$$30(9.81)\,[0.5 \sin 60] - 20(9.81) \cdot [(+1) \sin 30^\circ]$$

$$- 0.10(30)(9.81)\cos 60\,(0.5)$$

$$- 0.10(20)(9.81)\cos 30\,(+1)$$

$$= \tfrac{1}{2}(30)\, v_A^2 + \tfrac{1}{2}(20)\, v_B^2$$

$$\Rightarrow \quad 15 v_A^2 + 10 v_B^2 = 5 \quad \text{—— } 1$$

$$\Rightarrow \quad 2 v_A + v_B = 0 \quad \text{—— } 2$$

$$\Rightarrow \quad v_B = -2 v_A$$

$$15 v_A^2 + 10(-2v_A)^2 = 5$$

$$15 v_A^2 + 40 v_A^2 = 5$$

$$55 v_A^2 = 5$$

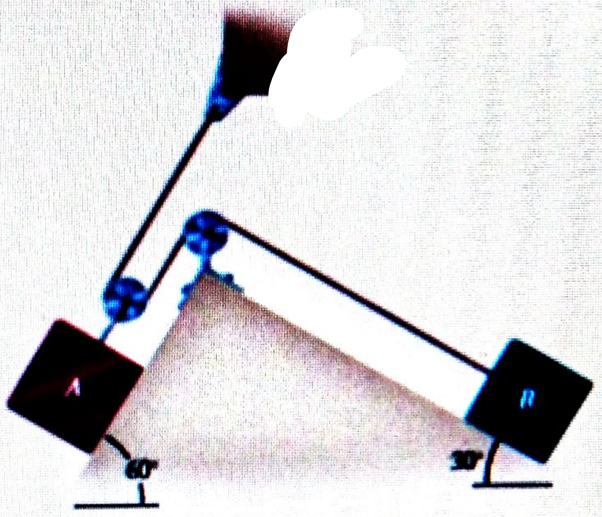
$$v_A^2 = \frac{5}{55}$$

$$\boxed{v_A = 0.3 \text{ m/s}}$$

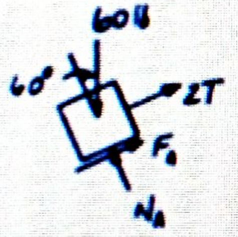
$$\Rightarrow \quad v_B = -2(0.3)$$

$$\boxed{v_B = -0.6 \text{ m/s}}$$

Determine the velocity of the 60-lb block A if the two blocks are released from rest and the 40-lb block B moves 2 ft up the incline. The coefficient of kinetic friction between both blocks and the inclined planes is $\mu_k = 0.10$.



SOLUTION



Block A:

$+\nwarrow\Sigma F_y = ma_y;\quad N_A - 60\cos 60^\circ = 0$

$N_A = 30$ lb

$F_A = 0.1(30) = 3$ lb

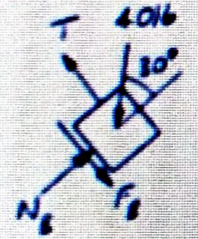
Block B:

$+\nearrow\Sigma F_y = ma_y;\quad N_B - 40\cos 30^\circ = 0$

$N_B = 34.64$ lb

$F_B = 0.1(34.64) = 3.464$ lb

Use the system of both blocks. $N_A, N_B, T,$ and R do no work.



$T_1 + \Sigma U_{1-2} = T_2$

$$(0+0) + 60\sin 60^\circ|\Delta s_A| - 40\sin 30^\circ|\Delta s_B| - 3|\Delta s_A| - 3.464|\Delta s_B| = \frac{1}{2}\left(\frac{60}{32.2}\right)v_A^2 + \frac{1}{2}\left(\frac{40}{32.2}\right)v_B^2$$

$2s_A + s_B = l$

$2\Delta s_A = -\Delta s_B$

When $|\Delta s_B| = 2$ ft, $|\Delta s_A| = 1$ ft

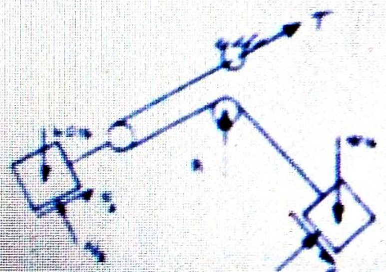
Also,

$2v_A = -v_B$

Substituting and solving,

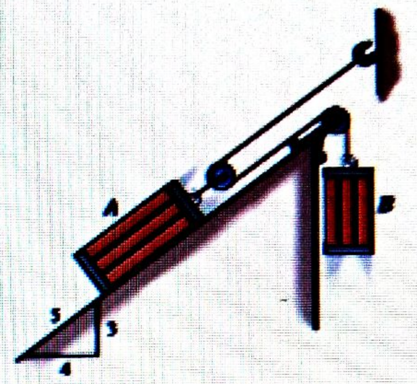
$v_A = 0.771$ ft/s **Ans.**

$v_B = -1.54$ ft/s



14–15.

The two blocks A and B have weights $W_A = 60$ lb and $W_B = 10$ lb. If the kinetic coefficient of friction between the incline and block A is $\mu_k = 0.2$, determine the speed of A after it moves 3 ft down the plane starting from rest. Neglect the mass of the cord and pulleys.



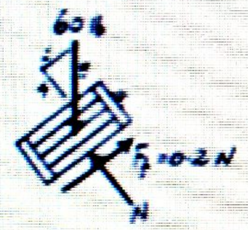
SOLUTION

Kinematics: The speed of the block A and B can be related by using position coordinate equation.

$$s_A + (s_A - s_B) = l \qquad 2s_A - s_B = l$$

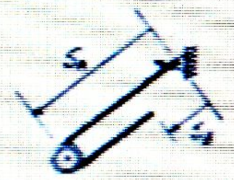
$$2\Delta s_A - \Delta s_B = 0 \qquad \Delta s_B = 2\Delta s_A = 2(3) = 6 \text{ ft}$$

$$2v_A - v_B = 0 \tag{1}$$



Equation of Motion: Applying Eq. 13–7, we have

$$+\Sigma F_y = ma_y; \qquad N - 60\left(\frac{4}{5}\right) = \frac{60}{32.2}(0) \qquad N = 48.0 \text{ lb}$$



Principle of Work and Energy: By considering the whole system, W_A which acts in the direction of the displacement does *positive* work. W_B and the friction force $F_f = \mu_k N = 0.2(48.0) = 9.60$ lb does *negative* work since they act in the opposite direction to that of displacement Here, W_A is being displaced vertically (downward) $\frac{3}{5}\Delta s_A$ and W_B is being displaced vertically (upward) Δs_B. Since blocks A and B are at rest initially, $T_1 = 0$. Applying Eq. 14–7, we have

$$T_1 + \sum U_{1-2} = T_2$$

$$0 + W_A\left(\frac{3}{5}\Delta s_A\right) - F_f \Delta s_A - W_B \Delta s_B = \frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B v_B^2$$

$$60\left[\frac{3}{5}(3)\right] - 9.60(3) - 10(6) = \frac{1}{2}\left(\frac{60}{32.2}\right)v_A^2 + \frac{1}{2}\left(\frac{10}{32.2}\right)v_B^2$$

$$1236.48 = 60v_A^2 + 10v_B^2 \tag{2}$$

Eqs. (1) and (2) yields

$$v_A = 3.52 \text{ ft/s} \qquad \textbf{Ans.}$$

$$v_B = 7.033 \text{ ft/s}$$

Dynamics

Dr. Hashem Alkhaldi

Suggested Problems: Chapter 14

(12 problems, 2 pages)

g= 9.81 m/s^2 = 32.2 ft/s^2

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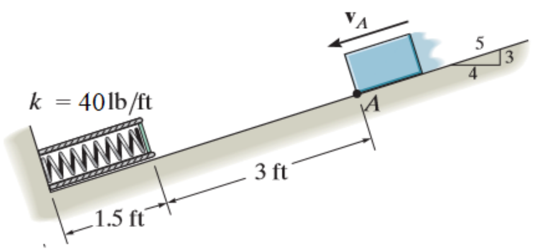
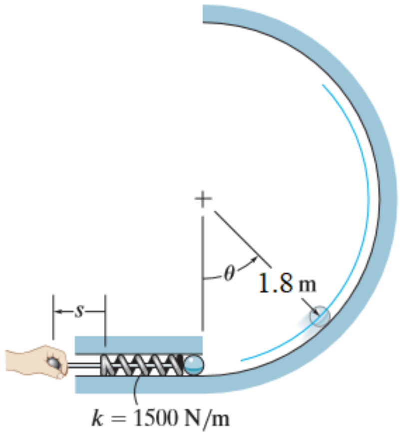
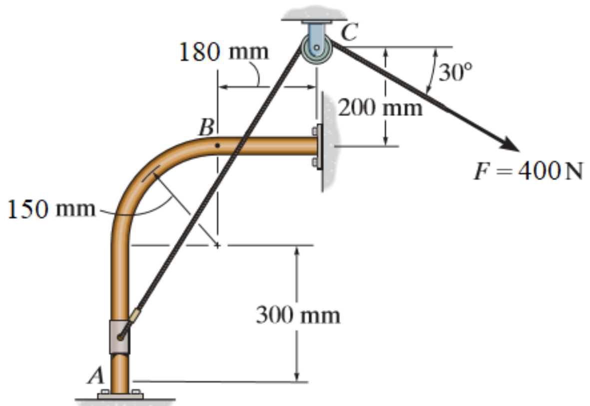
Q1) The spring has a stiffness $k = 40$ lb/ft and an *unstretched length* of 2 ft. As shown, it is confined by the plate and wall using cables so that its length is 1.5 ft. A 6-lb block is given a speed v_A when it is at A, and it slides down the incline having a coefficient of kinetic friction $\mu_k = 0.2$. If it strikes the plate and pushes it forward 0.25 ft before stopping, determine its speed at A. Neglect the mass of the plate and spring.



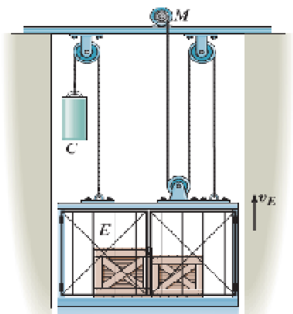
Q2) When $s = 0$, the spring on the firing mechanism is unstretched. If the arm is pulled back such that $s = 130$ mm and released, determine the maximum angle θ the ball will travel without leaving the circular track. Assume all surfaces of contact to be smooth. Neglect the mass of the spring and the size of the ball.



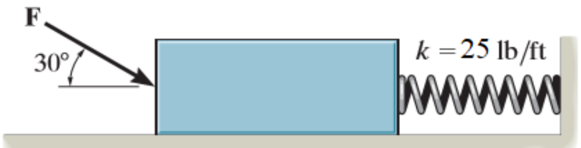
Q3) If the cord is subjected to a constant force of $F = 400$ N and the 15-kg smooth collar starts from rest at A, determine the velocity of the collar when it reaches point B. Neglect the size of the pulley.



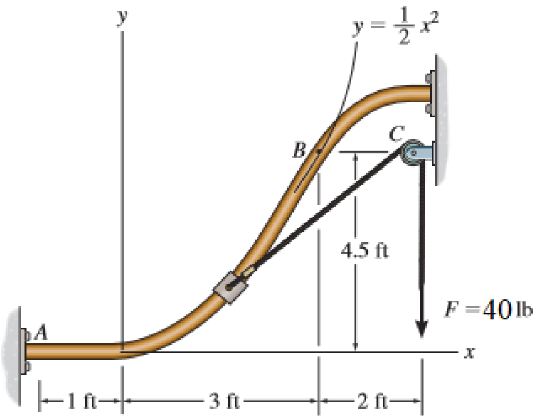
Q4) The elevator E and its freight have a total mass of 450 kg. Hoisting is provided by the motor M and the 60-kg block C. If the motor has an efficiency of $\varepsilon = 0.7$, determine the power that must be supplied to the motor when the elevator is hoisted upward at a constant speed of $v_E = 5$ m/s.



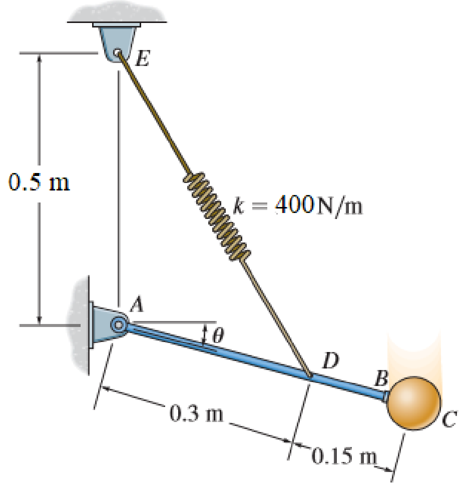
Q5) The 50-lb block rests on the rough surface for which the coefficient of kinetic friction is $\mu_k = 0.2$. A force $F = (50 + s^2)$ lb, where s is in ft, acts on the block in the direction shown. If the spring is originally unstretched ($s = 0$) and the block is at rest, determine the power developed by the force the instant the block has moved $s = 1.8$ ft.



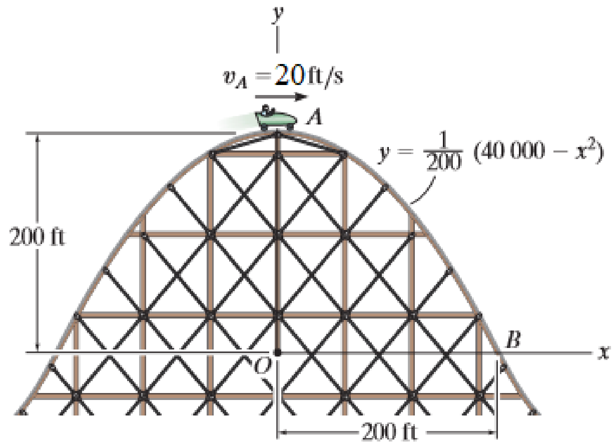
Q6) If the cord is subjected to a constant force of $F = 40$ lb and the smooth 15-lb collar starts from rest at A, determine its speed when it passes point B. Neglect the size of pulley C.



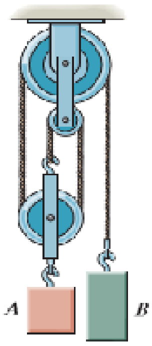
Q7) The 10-kg sphere C is released from rest when $\theta = 0°$ and the tension in the spring is 125 N. Determine the speed of the sphere at the instant $\theta = 90°$. Neglect the mass of rod AB and the size of the sphere.



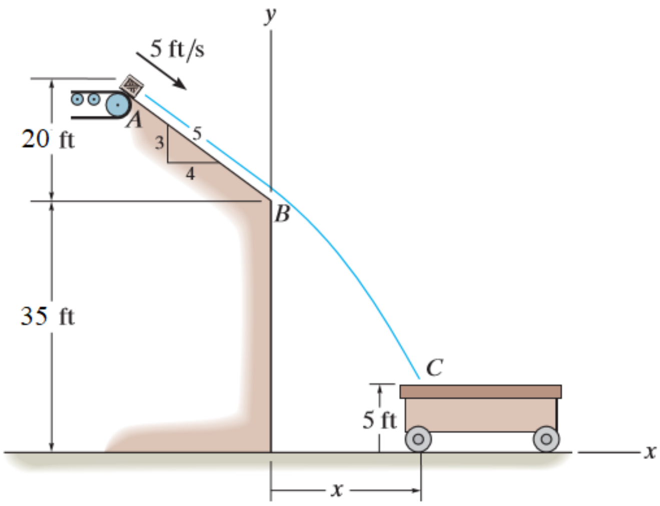
Q8) The roller coaster car has a speed of 20 ft/s when it is at the crest of a vertical parabolic track. Determine the car's velocity and the normal force it exerts on the track when it reaches point B. Neglect friction and the mass of the wheels. The total weight of the car and the passengers is 300 lb.



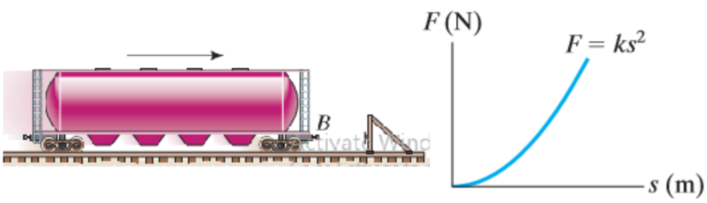
Q9) The assembly consists of two blocks A and B which have a mass of 25 kg and 40 kg, respectively. Determine the speed of each block when B descends 2 m. The blocks are released from rest. Neglect the mass of the pulleys and cords.



Q10) The 15-lb box falls off the conveyor belt at 5-ft/s. If the coefficient of kinetic friction along AB is $\mu_k = 0.2$, determine the distance x when the box falls into the cart.



Q11) Design considerations for the bumper B on the 4-Mg train car require use of a nonlinear spring having the load-deflection characteristics shown in the graph. Select the proper value of k so that the maximum deflection of the spring is limited to 0.3 m when the car, traveling at 5 m/s, strikes the rigid stop. Neglect the mass of the car wheels.



Q12) The girl has a mass of 50 kg and center of mass at G. If she is swinging to a maximum height defined by $\theta = 70°$,

a) determine the force developed along each of the four supporting posts such as AB at the instant $\theta = 0°$. The swing is centrally located between the posts.

b) repeat solving part (a) again at instant $\theta = 30°$.