د. هاشم الخالدي

Ch15

Impulse and Momentum

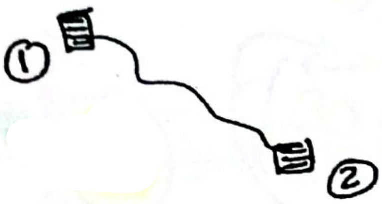
الدفع — الزخم (كمية التحرك)

$$\vec{I} = \vec{F}\, t$$ الدفع = القوة × الزمن

$$\vec{L} = m\,\vec{v}$$ الزخم = الكتلة × السرعة

Ch15

$$\vec{L}_1 + \vec{I}_{12} = \vec{L}_2$$



Ch14

$$T_1 + U_{12} = T_2$$

$$m\vec{v}_1 + \int_{t_1}^{t_2} \vec{F}\,dt = m\vec{v}_2$$

نستخدم التكامل عندما تكون القوة متغيرة (اقتران في الزمن)

x-direction :-

$$m v_{1x} + \int_{t_1}^{t_2} F_x\,dt = m v_{2x}$$

y-direction :-

$$m v_{1y} + \int_{t_1}^{t_2} F_y\,dt = m v_{2y}$$

* $I = F(\Delta t)$ الدفع = القوة x الزمن

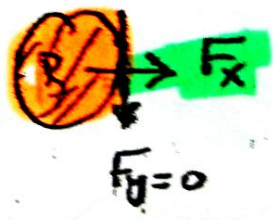
$N \cdot Sec = Kg\frac{m}{s^2}s = Kg\frac{m}{s}$

* $L = m\upsilon$ كمية التحرك = الكتلة x السرعة

* لهما نفس الوحدات الفيزيائية (لها نفس الوحدة) $Kg\frac{m}{s}$

$$\left(m\vec{\upsilon}_1\right)_A + \left(\int_{t_1}^{t_2}\vec{F}_x\,dt\right)_A = \left(m\vec{\upsilon}_2\right)_A$$

$$\left(m\vec{\upsilon}_1\right)_B - \left(\int_{t_1}^{t_2}\vec{F}_x\,dt\right)_B = \left(m\vec{\upsilon}_2\right)_B$$



أجمع المعادلتين

Conservation of momentum

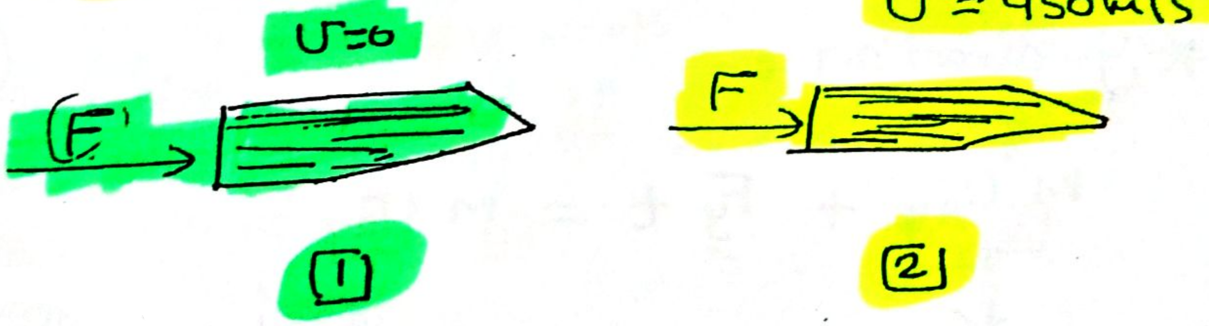
$$\left(m\vec{\upsilon}_1\right)_A + \left(m\vec{\upsilon}_1\right)_B = \left(m\vec{\upsilon}_2\right)_A + \left(m\vec{\upsilon}_2\right)_B$$

طلقة كتلتها 2.5 g سرعتها 450 m/s

احتاجت من الوقت 0.75 ms لتصل إلى هذه السرعة

أوجد مقدار القوة المؤثرة على الرصاصة

A Bullet of mass 2.5 g strats from Rest.
A Force F applied to this bullet
So the speed reaches 450 m/s
in 0.75 ms. **Find F?**



الزمن $t = 0.75\ ms$

Rest

$$m\cancel{v_{1x}}^{\,\to 0} + F\Delta t = m v_{2x}$$

$$F\left[0.75\times 10^{-3}\right] = \left[2.5\times 10^{-3}\right]\left[450\right]$$

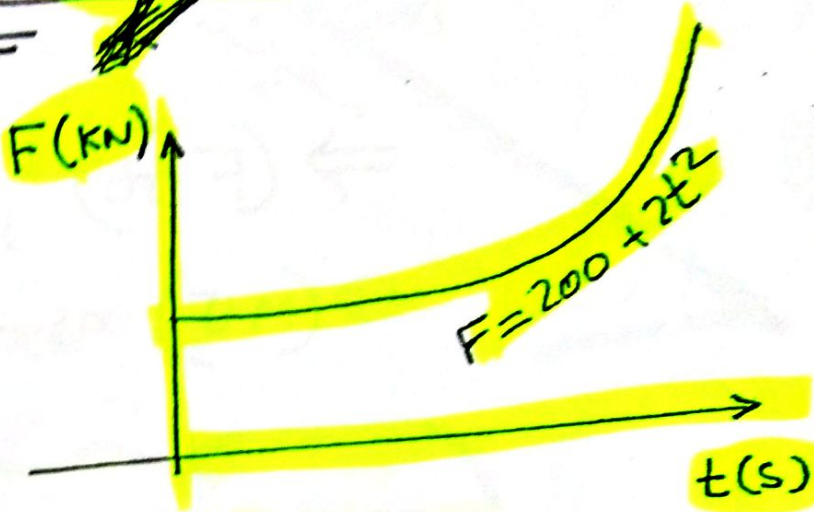
ms → s   g → kg

F = 1500 Newton.

EX]

jet plane
طائرة نفاثة

* $m = 250\ Mg$
* at $t=0 \Rightarrow v_0 = 100\ m/s$
and move horizontally
تطير افقيا



the engine provides horizontal Thrust

$F = 200 + 2t^2$

Find the velocity in 15 seconds

$$m v_1 + \int_0^{15} F\,dt = m v_2$$

kg نحول لـ

$$(250 \times 10^3)(100) + 1000 \int_0^{15} (200 + 2t^2)\,dt = (250 \times 10^3)\, v_2$$

$$25000 + \left[200t + \frac{2}{3}t^3\right]_0^{15} = 250\, v_2$$

$$25000 + 200(15) + \frac{2}{3}(15)^3 = 250\, v_2$$

$$\Rightarrow \boxed{v_2 = 121\ m/s}$$

Impulse and Momentum

الدفع   كمية الحركة

* الدفع = القوة × الزمن ⇒ $(F\,\Delta t)$

كمية الحركة = الكتلة × السرعة ⇒ $(m\,\upsilon)$

$$m\upsilon_1 + F\,\Delta t = m\upsilon_2$$

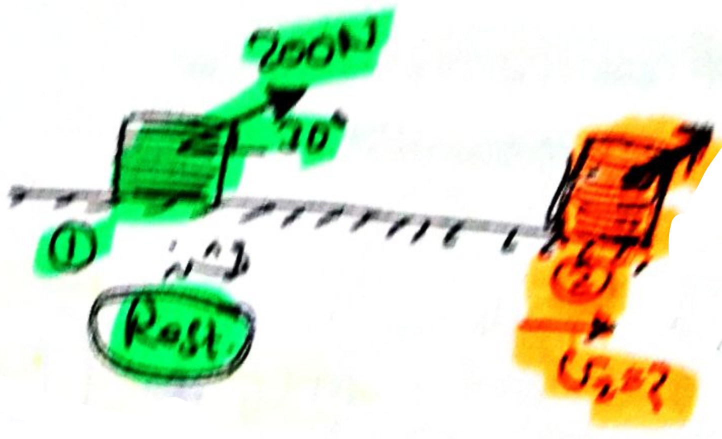
ex1

$\mu = 0.25$

$m = 30\,kg.$

أوجد سرعة الجسم في الحالة الثانية بعد زمن مقداره 8 ثواني

$(\Delta t = 8\ seconds)$

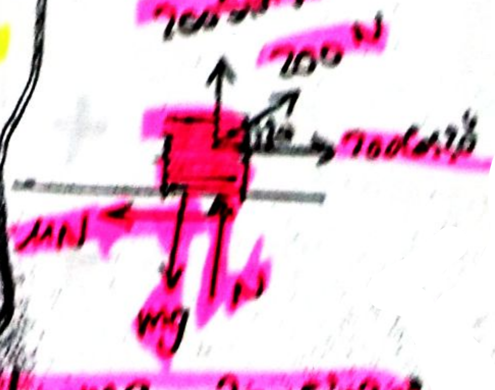


$$\cancelto{0}{m\upsilon_{1x}} + F_x\,\Delta t = m\upsilon_{2x}$$

$\Rightarrow [200\cos 30^\circ - \mu N]\,\Delta t = m\upsilon_{2x}$

$\Rightarrow \{200\cos 30^\circ - 0.25(194.3)\}(8) = 30\,\upsilon_{2x}$

$\Rightarrow \upsilon_{2x} = \ldots\ldots\ m/s$



$N = 194.3 \leftarrow (N) = mg - 200\sin 30^\circ$

كتلة السيارة

Q

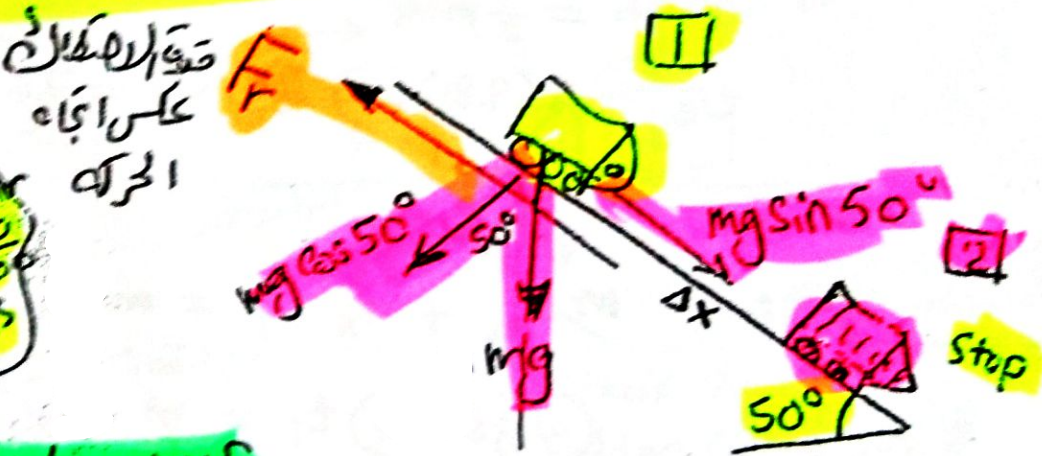
$m = 1814.4\ kg$

$v = 96.54\ km/hr$

- the brake is suddenly applied
- if the braking force is 6672 N

① Find the time needed for the car to stop?

② Find the distance until stops?



$v = 96.54\ km/hr$
$= 96.54 \frac{1000}{3600}$
$= 26.8\ m/s$

① $m v_1 + F \Delta t = m v_2$

$1814.4(26.8) + [mg \sin 50 - F_r][\Delta t] = 0 \nearrow$ stop.

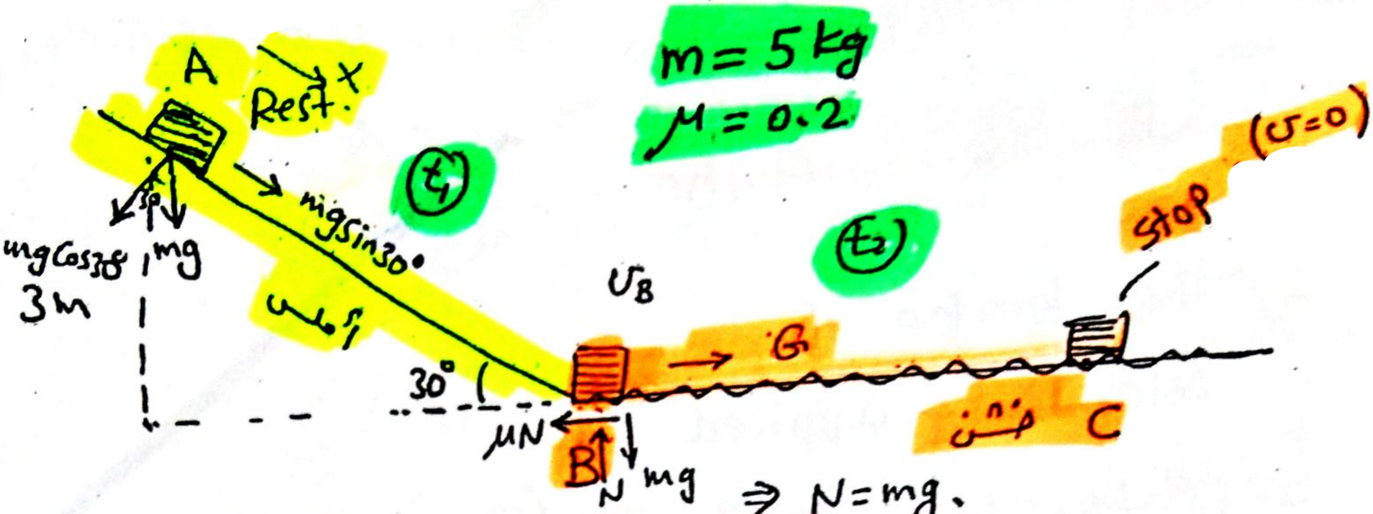
$1814.4(26.8) + [1814.4 \sin 50° - 6672][\Delta t] = 0$

$\Rightarrow \Delta t = 9.5\ Sec$

② $T_1 + U_{12} = T_2 \Rightarrow \frac{1}{2} m_1 v_1^2 + mg(\Delta x \sin 50°) - F_r \Delta x = 0$

$\Rightarrow \frac{1}{2}[1814.4][26.8]^2 + (1814.4)(9.81)\sin 50\ \Delta X - 6672\ \Delta X = 0 \Rightarrow \Delta X = 127\ m$

Ex)



الزمن الكلي

Find the total time until stops?

اوجد الزمن الكلي اللازم للجسم حتى يتوقف.

* $\boxed{t = t_1 + t_2}$

Ch14 $\quad \cancel{T_A}^{\,0} + U_{AB} = T_B$

$$\cancel{m}gh = \tfrac{1}{2}\cancel{m}v_B^2$$

$$v_B^2 = 2(9.81)(3) \Rightarrow v_B = 7.67\ \text{m/s}.$$

X-direction : $\quad m\cancel{v_{Ax}}^{\,0\ \text{Rest}} + F_x t_1 = m v_{Bx}$

$$(\cancel{m}g\sin 30)t_1 = \cancel{m}\, v_{Bx}$$

Ch15

$$(9.81)\sin 30\ t_1 = 7.67 \Rightarrow t_1 = 1.56\ \text{sec}$$

G-direction : $\quad m v_{BG} + F_G t_2 = m\cancel{v_{CG}}^{\,0\ \text{توقف الجسم}}$

$$(5)(7.67) + -\mu m g\, t_2 = 0$$

$$(5)(7.67) - 0.2(5)(9.81)\, t_2 = 0 \Rightarrow t_2 = 3.91\ \text{sec}$$

* $t = t_1 + t_2 \Rightarrow t = 5.47\ \text{sec.}$

* Impact

* Collision : تصادم الأجسام **

$$* \quad m_1 v_1 + \cancel{F \Delta t} = m_1 v_1'$$

$$* \quad m_2 v_2 - \cancel{F \Delta t} = m_2 v_2'$$

$$\overset{\text{before}}{\sum m v} = \overset{\text{After}}{\sum m v}$$

$$* \quad m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2'$$

كمية الحركة قبل التصادم = كمية الحركة بعد التصادم

Ex

$W_A = 22.5$ KN الوزن

$W_B = 15$ KN الوزن

$v_A = 1$ m/s قبل التصادم

$v_B = 2$ m/s قبل التصادم

Find their velocites after Collision ? if they become together
تصادم

$$m_A v_A + m_B v_B = (m_A + m_B) v'$$

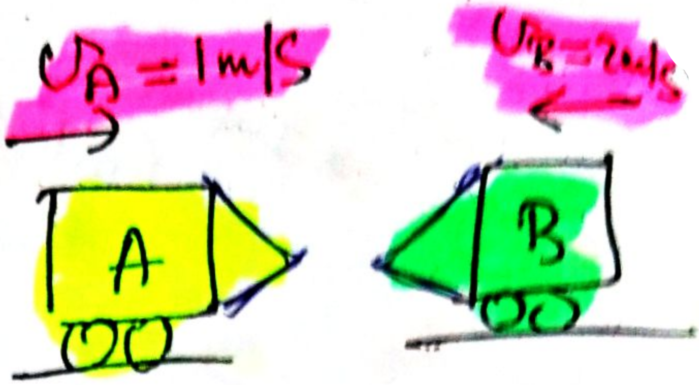
السرعة موجبة لليمين — السرعة سالبة لليسار

$$\frac{(22.5)10^3}{9.81}(1) + \frac{(15)10^3}{9.81}(-2) = \frac{(37.5)10^3}{9.81} v'$$

أصبح لهما نفس السرعة بعد التصادم

$$\Rightarrow v' = -0.2 \text{ m/s}$$

Ex



$m_A = 2250\ kg$

$m_B = 1500\ kg$

plastic Impact

Find the velocity after impact.

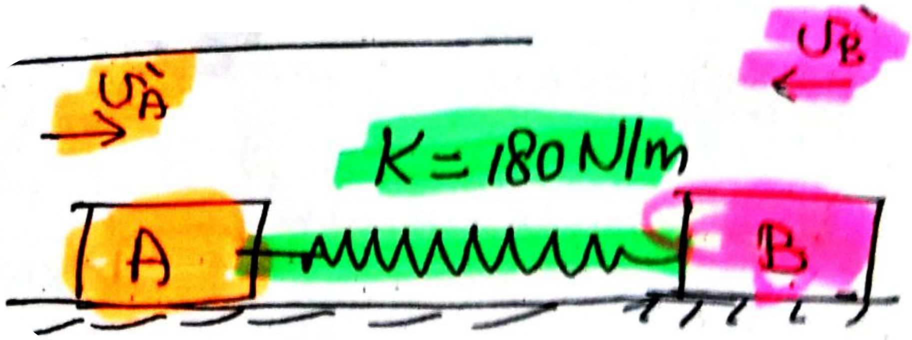
Sol

$$m_A v_A + m_B v_B = (m_A + m_B)\,v'$$

$$(2250)(1) + (1500)(-2) = (2250 + 1500)\,v'$$

$$\Rightarrow v' = -0.2\ m/s$$

Ex



$m_A = 40\ kg$.

$m_B = 60\ kg$.

$DS = 2\ m$

Find the velocities of A and B when the spring becomes unstreched ?

Ch15

$$\left[m_A v_A + m_B v_B\right]_1 = \left[m_A v_A' + m_B v_B'\right]_2$$

$$40(0) + 60(0) = 40\, v_A' + 60\, v_B'$$

$$40\, v_A' + 60\, v_B' = 0$$

$$2v_A' + 3v_B' = 0 \quad (1)$$

Ch14

$$\cancel{T_1}^{\,0} + U_{1,2} = T_2$$

$$\cancel{\tfrac{1}{2}} K(\Delta S)^2 = \cancel{\tfrac{1}{2}} m_A v_A'^2 + \cancel{\tfrac{1}{2}} m_B v_B'^2$$

$$180\,(2)^2 = 40\, v_A'^2 + 60\, v_B'^2 \quad (2)$$

$$v_A = 3.29 \text{ m/s}$$

$$v_B = -2.19 \text{ m/s}$$

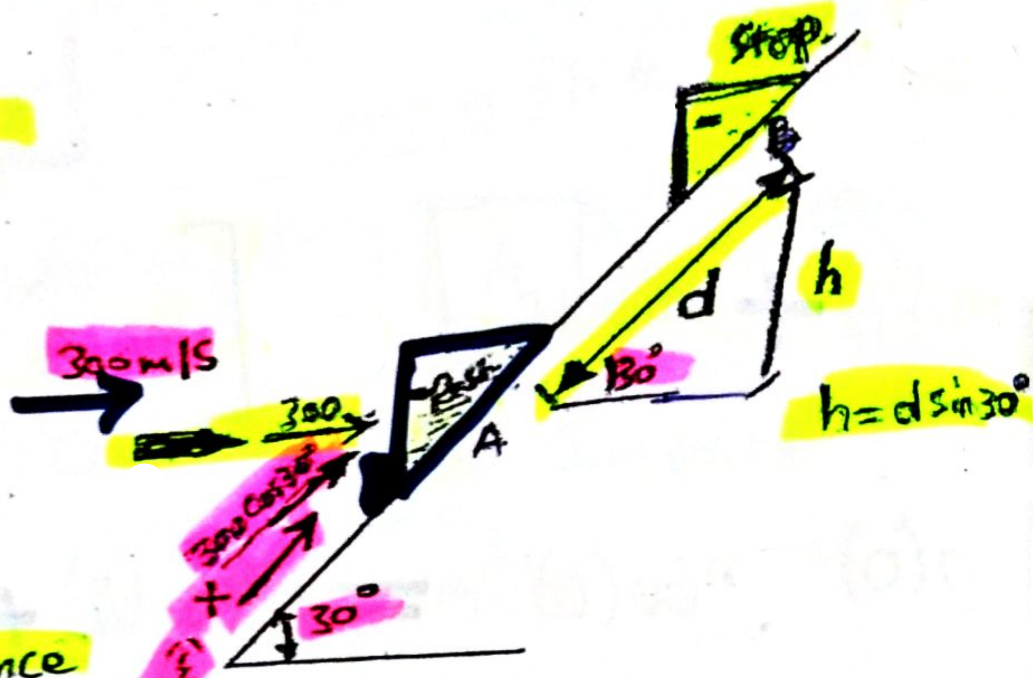
Ex) Bullet طلقة

$M_b = 10\,g$

$M_a = 10\,kg$

$U_b = 300\,m/s$

Find the distance d until the Box stops?



X-directions:

Ch15 $\quad m_b U_b + \cancel{m_a U_a}^{\,0} = (m_b + m_b)\, U_A'$

kg $\quad (10\times10^{-3})(300 \cos 30^\circ) = (10.01)\, U_A'$

$\Rightarrow U_A' = 0.2595\ m/s$

Ch14 $\quad T_A + U_{AB} = \cancel{T_B}^{\,0}$

$\frac{1}{2}\cancel{m}\, U_A'^2 - \cancel{m} g h = 0$

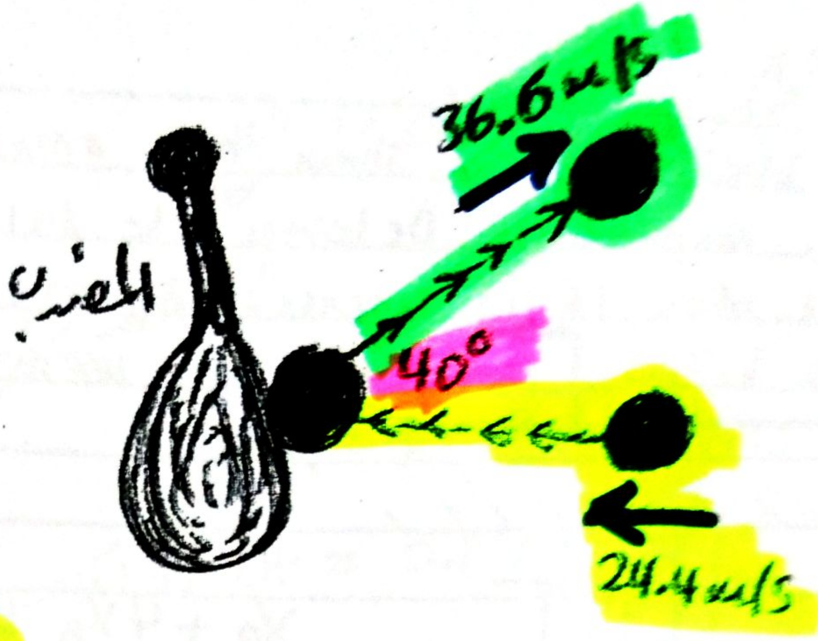
$\Rightarrow U_A'^2 = 2gh \rightarrow h = \frac{U_A'^2}{2g}$

$h = \frac{(0.2595)^2}{2(9.81)} = 3.43\ mm$

$d = \frac{3.43}{\sin 30} \rightarrow d = 6.86\ mm$

ex]

Baseball
of m = 113.4 g



the bat and ball
are in contact for $\Delta t = 0.015$ sec.

Find the average impulsive force during impact.

X-direction

* $m v_{1x} + F_x \Delta t = m v_{2x}$

$(0.1134)(-24.4) + F_x(0.015) = 0.1134[36.6 \cos 40°]$

$\Rightarrow \boxed{F_x = 396.4\ N}$

y-direction

* $\cancel{m v_{1y}}^{0} + F_y \Delta t = m v_{2y}$

$0 + F_y[0.015] = (0.1134)[36.6 \sin 40°]$

$\Rightarrow \boxed{F_y = 178\ N}$

$F = \sqrt{F_x^2 + F_y^2} = \text{------}\ N$ $\quad \phi = \tan^{-1}\frac{F_y}{F_x}$

Chapter 15
Kinetics of a Particle
Impulse and Momentum

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Dr. Hashem Alkhaldi

March, 2020

Objectives:

1. To develop the principle of linear impulse and momentum for a particle and apply it to solve problems that involve force, velocity, and time.
2. To study the conservation of linear momentum for particles.
3. To analyze the mechanics of impact.
4. To introduce the concept of angular impulse and momentum.
5. To solve problems involving steady fluid streams and propulsion with variable mass.

READING QUIZ

1. The linear impulse and momentum equation is obtained by integrating the _______ with respect to time.

   A) friction force
   B) equation of motion
   C) kinetic energy
   D) potential energy

2. Which parameter is not involved in the linear impulse and momentum equation?

   A) Velocity
   B) Displacement
   C) Time
   D) Force

APPLICATIONS

A dent in an automotive fender can be removed using an impulse tool, which delivers a force over a very short time interval. To do so the weight is gripped and jerked upwards, striking the stop ring.

How can we determine the magnitude of the linear impulse applied to the fender?

Could you analyze a carpenter's hammer striking a nail in the same fashion? Sure!

APPLICATIONS (continued)

When a stake is struck by a sledgehammer, a large impulse force is delivered to the stake and drives it into the ground.

If we know the initial speed of the sledgehammer and the duration of impact, how can we determine the magnitude of the impulsive force delivered to the stake?

PRINCIPLE OF LINEAR IMPULSE AND MOMENTUM (Section 15.1)

The next method we will consider for solving particle kinetics problems is obtained by integrating the equation of motion with respect to time.

The result is referred to as the principle of impulse and momentum. It can be applied to problems involving both linear and angular motion.

This principle is useful for solving problems that involve force, velocity, and time. It can also be used to analyze the mechanics of impact (taken up in a later section).

PRINCIPLE OF LINEAR IMPULSE AND MOMENTUM (continued)

The principle of linear impulse and momentum is obtained by integrating the equation of motion with respect to time. The equation of motion can be written

$$\sum \boldsymbol{F} = m\,\mathbf{a} = m\,(d\boldsymbol{v}/dt)$$

Separating variables and integrating between the limits $\boldsymbol{v} = \boldsymbol{v}_1$ at $t = t_1$ and $\boldsymbol{v} = \boldsymbol{v}_2$ at $t = t_2$ results in

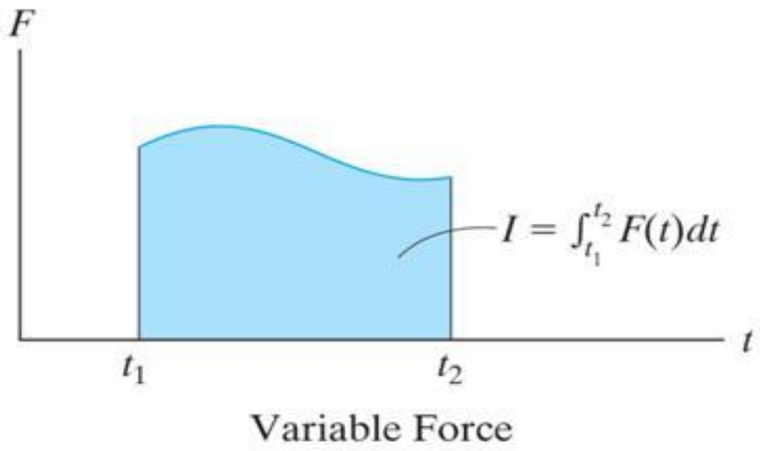
$$\sum \int_{t_1}^{t_2} \boldsymbol{F}\,dt = m \int_{\boldsymbol{v}_1}^{\boldsymbol{v}_2} d\boldsymbol{v} = m\boldsymbol{v}_2 - m\boldsymbol{v}_1$$

This equation represents the principle of linear impulse and momentum. It relates the particle's final velocity ($\boldsymbol{v}_2$) and initial velocity ($\boldsymbol{v}_1$) and the forces acting on the particle as a function of time.

PRINCIPLE OF LINEAR IMPULSE AND MOMENTUM (continued)

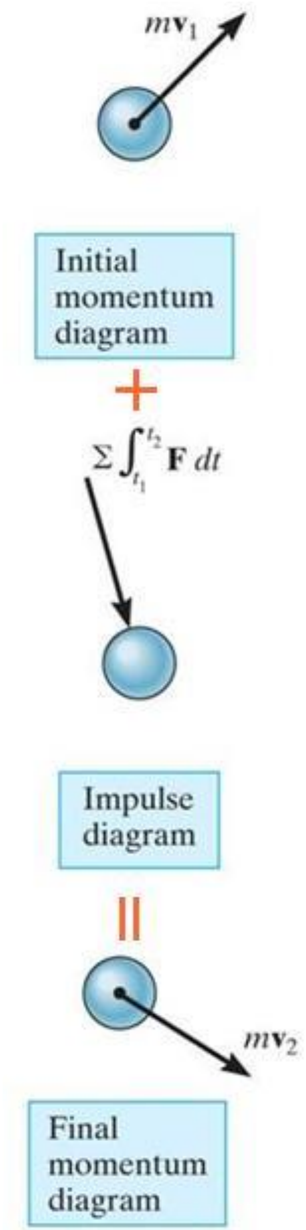
Linear momentum: The vector m***v*** is called the linear momentum, denoted as ***L***. This vector has the same direction as ***v***. The linear momentum vector has units of (kg·m)/s or (slug·ft)/s.

Linear impulse: The integral ($\int \boldsymbol{F}$) dt is the linear impulse, denoted ***I***. It is a vector quantity measuring the effect of a force during its time interval of action. ***I*** acts in the same direction as ***F*** and has units of N·s or lb·s.



The impulse may be determined by direct integration. Graphically, it can be represented by the area under the force versus time curve. If ***F*** is constant, then $\boldsymbol{I} = \boldsymbol{F}(t_2 - t_1)$.

PRINCIPLE OF LINEAR IMPULSE AND MOMENTUM (continued)



The principle of linear impulse and momentum in vector form is written as

$$m\boldsymbol{v}_1 + \sum\int_{t_1}^{t_2}\boldsymbol{F}\,dt = m\boldsymbol{v}_2$$

The particle's initial momentum plus the sum of all the impulses applied from t_1 to t_2 is equal to the particle's final momentum.

The two momentum diagrams indicate direction and magnitude of the particle's initial and final momentum, $m\boldsymbol{v}_1$ and $m\boldsymbol{v}_2$. The impulse diagram is similar to a free body diagram, but includes the time duration of the forces acting on the particle.

IMPULSE AND MOMENTUM: SCALAR EQUATIONS

Since the principle of linear impulse and momentum is a vector equation, it can be resolved into its x, y, z component scalar equations:

$$m(v_x)_1 + \sum \int_{t_1}^{t_2} F_x \, dt = m(v_x)_2$$

$$m(v_y)_1 + \sum \int_{t_1}^{t_2} F_y \, dt = m(v_y)_2$$

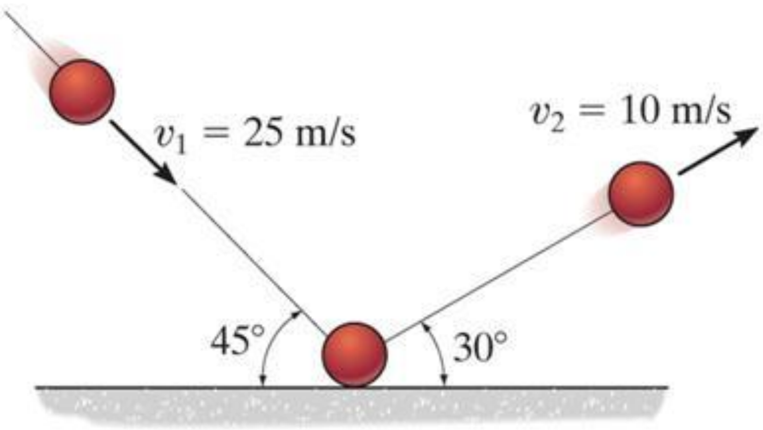
$$m(v_z)_1 + \sum \int_{t_1}^{t_2} F_z \, dt = m(v_z)_2$$

The scalar equations provide a convenient means for applying the principle of linear impulse and momentum once the velocity and force vectors have been resolved into x, y, z components.

PROBLEM SOLVING

- Establish the x, y, z coordinate system.
- Draw the particle's free body diagram and establish the direction of the particle's initial and final velocities, drawing the impulse and momentum diagrams for the particle. Show the linear momenta and force impulse vectors.
- Resolve the force and velocity (or impulse and momentum) vectors into their x, y, z components, and apply the principle of linear impulse and momentum using its scalar form.
- Forces as functions of time must be integrated to obtain impulses. If a force is constant, its impulse is the product of the force's magnitude and time interval over which it acts.

EXAMPLE



Given: A 0.5 kg ball strikes the rough ground and rebounds with the velocities shown. Neglect the ball's weight during the time it impacts the ground.

Find: The magnitude of impulsive force exerted on the ball.

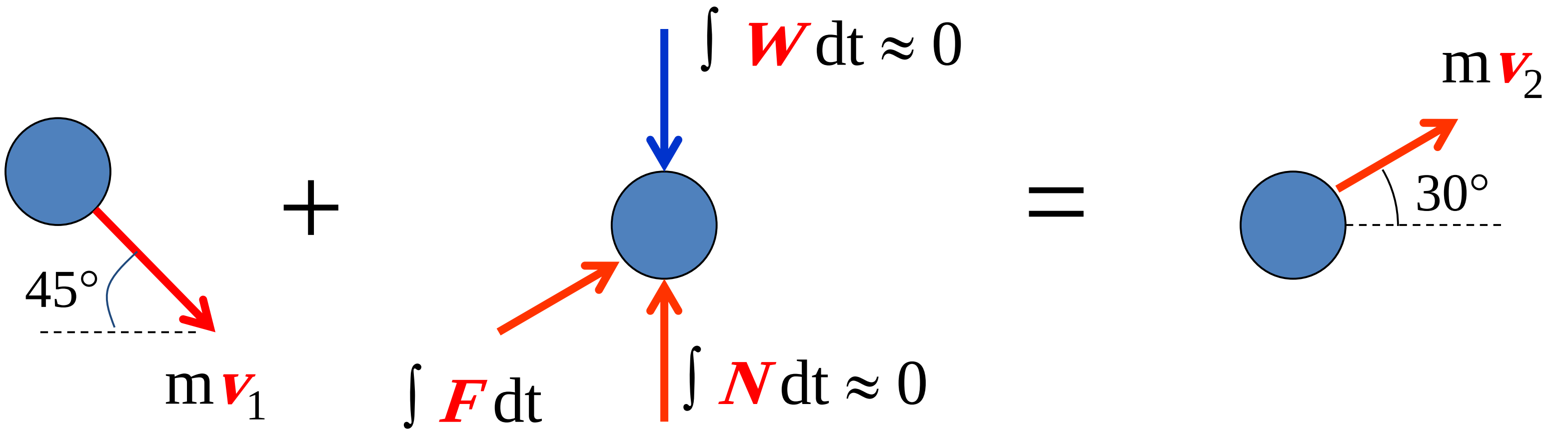
Plan:

1) Draw the momentum and impulse diagrams of the ball as it hits the surface.
2) Apply the principle of impulse and momentum to determine the impulsive force.

EXAMPLE (continued)

Solution:

1) The impulse and momentum diagrams can be drawn as:



The impulse caused by the ball's weight and the normal force $\boldsymbol{N}$ can be neglected because their magnitudes are very small as compared to the impulse from the ground.

EXAMPLE (continued)

2) The principle of impulse and momentum can be applied along the direction of motion:

$$m\mathbf{v}_1 + \sum \int_{t_1}^{t_2} \mathbf{F}\, dt = m\mathbf{v}_2$$

$$\Rightarrow \quad 0.5\,(25 \cos 45°\ \mathbf{i} - 25 \sin 45°\ \mathbf{j}) + \int_{t_1}^{t_2} \sum \mathbf{F}\, dt = 0.5\,(10 \cos 30°\ \mathbf{i} + 10 \sin 30°\ \mathbf{j})$$

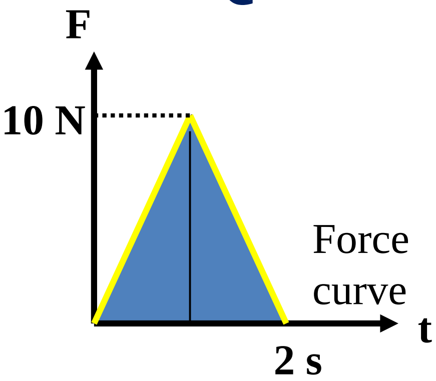
The impulsive force vector is

$$\mathbf{I} = \int_{t_1}^{t_2} \sum \mathbf{F}\, dt = (4.509\ \mathbf{i} + 11.34\ \mathbf{j})\ \text{N·s}$$

Magnitude: $I = \sqrt{4.509^2 + 11.34^2} = 12.2\ \text{N·s}$

CHECK YOUR UNDERSTANDING QUIZ

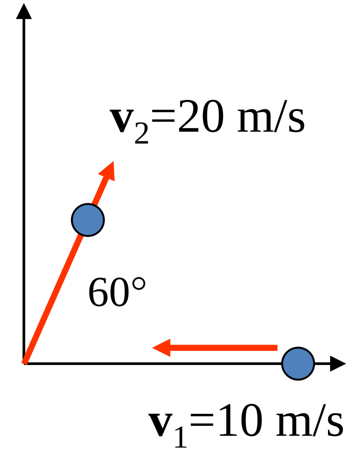
1. Calculate the impulse due to the force.



A) 20 kg·m/s B) 10 kg·m/s

C) 5 N·s D) 15 N·s

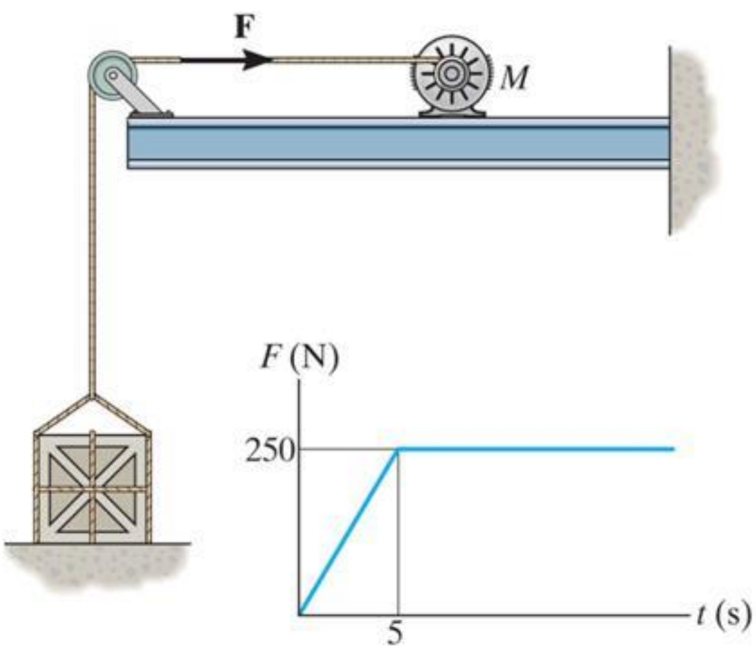
2. A constant force ***F*** is applied for 2 s to change the particle's velocity from $\mathbf{v}_1$ to $\mathbf{v}_2$. Determine the force ***F*** if the particle's mass is 2 kg.



A) (17.3 ***j***) N B)(–10 ***i*** +17.3 ***j***) N

C) (20 ***i*** +17.3 ***j***) N D)(10 ***i*** +17.3 ***j***) N

GROUP PROBLEM SOLVING



Given: The 20 kg crate is resting on the floor. The motor M pulls on the cable with a force of **F**, which has a magnitude that varies as shown on the graph.

Find: The speed of the crate when t = 6 s.

1) Determine the force needed to begin lifting the crate, and then the time needed for the motor to generate this force.
2) After the crate starts moving, apply the principle of impulse and momentum to determine the speed of the crate at t = 6 s.

GROUP PROBLEM SOLVING (continued)

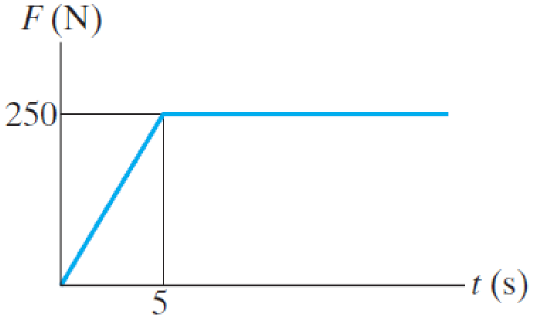
Solution:

1) The crate begins moving when the cable force F exceeds the crate weight. Solve for the force, then the time.

$F = mg = (20)\,(9.81) = 196.2\text{ N}$

$F = 196.2\text{ N} = 50\,t$

$t = 3.924\text{ s}$



2) Apply the principle of impulse and momentum from the time the crate starts lifting at $t_1 = 3.924$ s to $t_2 = 6$ s.

Note that there are two external forces (cable force and weight) we need to consider.

A. The impulse due to cable force:

$$\int_{3.924}^{6} F\,dt = [0.5(250)\,5 + (250)\,1] - 0.5(196.2)3.924 = 490.1\text{ N}\cdot\text{s}$$

GROUP PROBLEM SOLVING (continued)

B. The impulse due to weight:

$$+\uparrow \int_{3.924}^{6} (-\,mg)\,dt = -\,196.2\,(6 - 3.924) = -\,407.3 \text{ N}\cdot\text{s}$$

Now, apply the principle of impulse and momentum

$$mv_1 + \sum \int_{t_1}^{t_2} F\,dt = mv_2 \quad \text{where } v_1 = 0$$

$$0 + 490.1 - 407.3 = (20)\,v_2$$

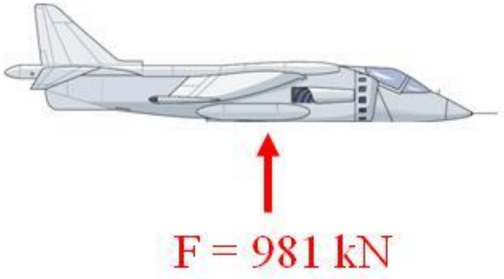
$$\Rightarrow \quad v_2 = 4.14 \text{ m/s}$$

ATTENTION QUIZ

1. Jet engines on the 100 Mg VTOL aircraft exert a constant vertical force of 981 kN as it hovers. Determine the net impulse on the aircraft over t = 10 s.

A) -981 kN·s   B) 0 kN·s

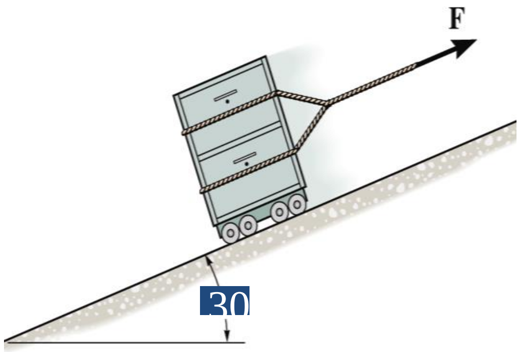
C) 981 kN·s   D) 9810 kN·s



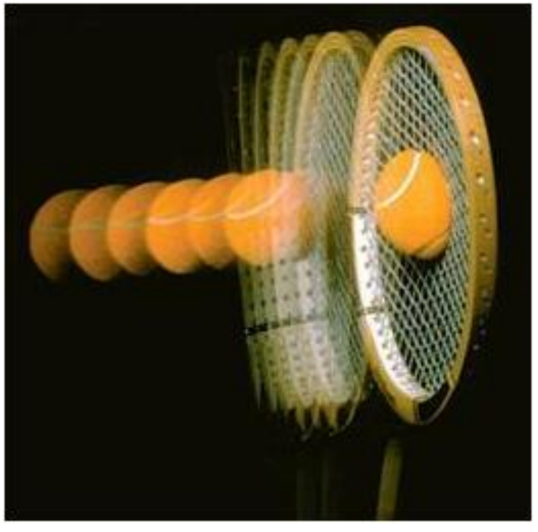
2. A 100 lb cabinet is placed on a smooth surface. If a force of a 100 lb is applied for 2 s, determine the net impulse on the cabinet during this time interval.

A) 0 lb·s   B) 100 lb·s

C) 200 lb·s   D) 300 lb·s



PRINCIPLE OF LINEAR IMPULSE AND MOMENTUM AND CONSERVATION OF LINEAR MOMENTUM FOR SYSTEMS OF PARTICLES



<u>Objectives</u>:

Students will be able to:

1. Apply the principle of linear impulse and momentum to a system of particles.
2. Understand the conditions for conservation of momentum.

READING QUIZ

1. The internal impulses acting on a system of particles always ___________

   A) equal the external impulses. B) Sum to zero.

   C) equal the impulse of weight. D) None of the above.

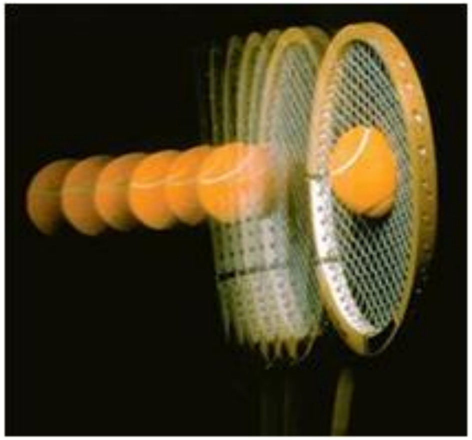
2. If an impulse-momentum analysis is considered during the very short time of interaction, as shown in the picture, weight is a/an ___________

   A) impulsive force.

   B) explosive force.

   C) non-impulsive force.

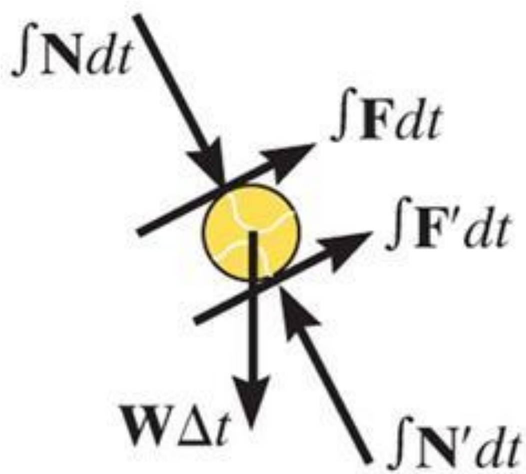
   D) internal force.



APPLICATIONS

As the wheels of this pitching machine rotate, they apply frictional impulses to the ball, thereby giving it linear momentum in the direction of ***F*** dt and ***F'*** dt.

The weight impulse, **W** Δt is very small since the time the ball is in contact with the wheels is very small.



Does the release velocity of the ball depend on the mass of the ball?

APPLICATIONS (continued)

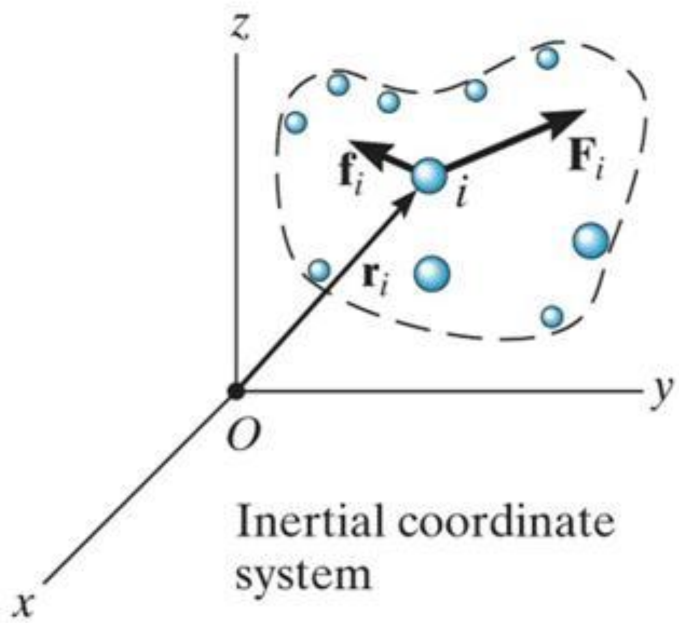
This large crane-mounted hammer is used to drive piles into the ground.

Conservation of momentum can be used to find the velocity of the pile just after impact, assuming the hammer does not rebound off the pile.

If the hammer rebounds, does the pile velocity change from the case when the hammer doesn't rebound ? Why ?

In the impulse-momentum analysis, do we have to consider the impulses of the weights of the hammer and pile and the resistance force ? Why or why not ?

PRINCIPLE OF LINEAR IMPULSE AND MOMENTUM FOR A SYSTEM OF PARTICLES (Section 15.2)



For the system of particles shown, the internal forces f_i between particles always occur in pairs with equal magnitude and opposite directions. Thus the internal impulses sum to zero.

The linear impulse and momentum equation for this system only includes the impulse of external forces.

$$\sum m_i(\mathbf{v}_i)_1 + \sum \int_{t_1}^{t_2} \mathbf{F}_i dt = \sum m_i(\mathbf{v}_i)_2$$

MOTION OF THE CENTER OF MASS

For a system of particles, we can define a "fictitious" center of mass of an aggregate particle of mass m_{tot}, where m_{tot} is the sum ($\sum m_i$) of all the particles. This system of particles then has an aggregate velocity of

$$\mathbf{v}_G = (\sum m_i \mathbf{v}_i) / m_{tot}.$$

The motion of this fictitious mass is based on motion of the center of mass for the system.

The position vector $\mathbf{r}_G = (\sum m_i \mathbf{r}_i) / m_{tot}$ describes the motion of the center of mass.

CONSERVATION OF LINEAR MOMENTUM FOR A SYSTEM OF PARTICLES (Section 15.3)

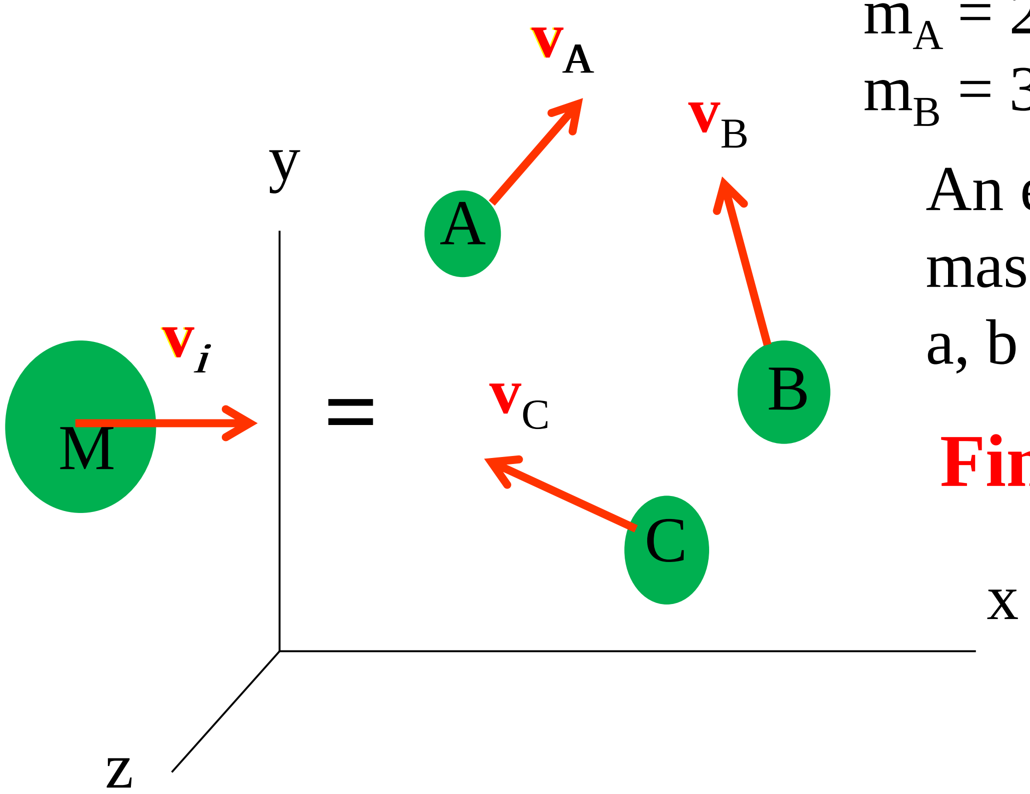
When the sum of external impulses acting on a system of objects is zero, the linear impulse-momentum equation simplifies to

$$\sum m_i(\boldsymbol{v}_i)_1 = \sum m_i(\boldsymbol{v}_i)_2$$

This equation is referred to as the conservation of linear momentum. Conservation of linear momentum is often applied when particles collide or interact. When particles impact, only impulsive forces cause a change of linear momentum.

The sledgehammer applies an impulsive force to the stake. The weight of the stake is considered negligible, or non-impulsive, as compared to the force of the sledgehammer. Also, provided the stake is driven into soft ground with little resistance, the impulse of the ground acting on the stake is considered non-impulsive.

EXAMPLE I



Given: M = 100 kg, $\boldsymbol{v}_i = 20\boldsymbol{j}$ (m/s)
m_A = 20 kg, $\boldsymbol{v}_A = 50\boldsymbol{i} + 50\boldsymbol{j}$ (m/s)
m_B = 30 kg, $\boldsymbol{v}_B = -30\boldsymbol{i} - 50\boldsymbol{k}$ (m/s)

An explosion has broken the mass m into 3 smaller particles, a, b and c.

Find: The velocity of fragment C after the explosion.

Plan: Since the internal forces of the explosion cancel out, we can apply the conservation of linear momentum to the SYSTEM.

EXAMPLE I (continued)

Solution:

$$m\boldsymbol{v}_i = m_A\boldsymbol{v}_A + m_B\boldsymbol{v}_B + m_C\boldsymbol{v}_C$$

$$100(20\boldsymbol{j}) = 20(50\boldsymbol{i} + 50\boldsymbol{j}) + 30(-30\boldsymbol{i} - 50\boldsymbol{k}) + 50(v_{cx}\,\boldsymbol{i} + v_{cy}\,\boldsymbol{j} + v_{cz}\,\boldsymbol{k})$$

Equating the components on the left and right side yields:

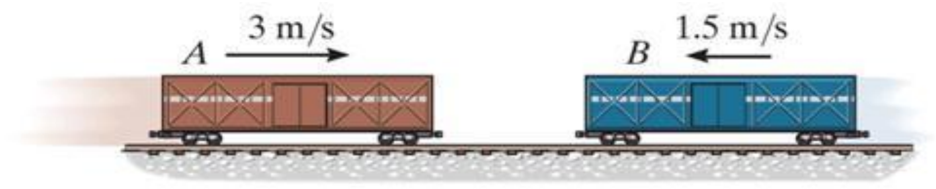
$0 = 1000 - 900 + 50(v_{cx})$ $\quad$ $v_{cx} = -2$ m/s

$2000 = 1000 + 50\,(v_{cy})$ $\quad$ $v_{cy} = 20$ m/s

$0 = -1500 + 50\,(v_{cz})$ $\quad$ $v_{cz} = 30$ m/s

So $\boldsymbol{v}_c = (-2\boldsymbol{i} + 20\boldsymbol{j} + 30\boldsymbol{k})$ m/s immediately after the explosion.

EXAMPLE II



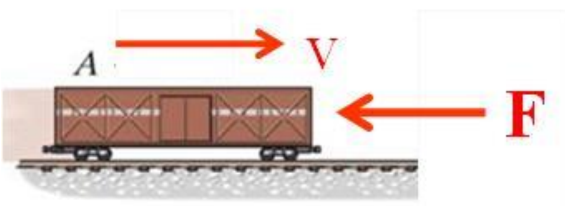
Given: Two rail cars with masses of $m_A = 20$ Mg and $m_B = 15$ Mg and velocities as shown.

Find: The speed of the car A after collision if the cars collide and rebound such that B moves to the right with a speed of 2 m/s. Also find the average impulsive force between the cars if the collision place in 0.5 s.

Plan: Use conservation of linear momentum to find the velocity of the car A after collision (all internal impulses cancel). Then use the principle of impulse and momentum to find the impulsive force by looking at only one car.

EXAMPLE II (continued)

Solution:



Conservation of linear momentum (x-dir):

$$m_A(v_{A1}) + m_B(v_{B1}) = m_A(v_{A2}) + m_B(v_{B2})$$

$$20{,}000\ (3) + 15{,}000\ (-1.5)$$

$$= (20{,}000)\ v_{A2} + 15{,}000\ (2)$$

$$v_{A2} = 0.375 \text{ m/s}$$

Impulse and momentum on car A (x-dir):

$$m_A\,(v_{A1}) + \int F\,dt = m_A\,(v_{A2})$$

$$20{,}000\ (3) - \int F\,dt = 20{,}000\ (0.375)$$

$$\int F\,dt = 52{,}500 \text{ N}\cdot\text{s}$$

The average force is

$$\int F\,dt = 52{,}500 \text{ N}\cdot\text{s} = F_{avg}(0.5 \text{ sec}); \quad F_{avg} = 105 \text{ kN}$$

CONCEPT QUIZ

1) Over the short time span of a tennis ball hitting the racket during a player's serve, the ball's weight can be considered _______________

   A) non-impulsive.

   B) impulsive.

   C) not subject to Newton's second law.

   D) Both A and C.

2) A drill rod is used with a air hammer for making holes in hard rock so explosives can be placed in them. How many impulsive forces act on the drill rod during the drilling?

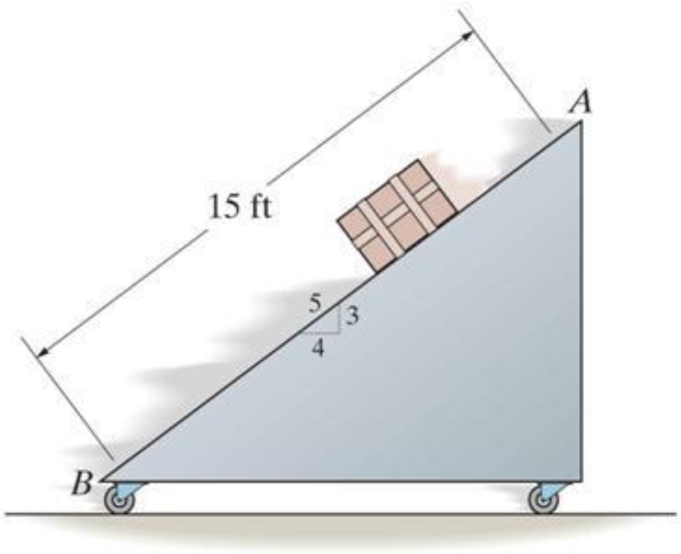
   A) None

   B) One

   C) Two

   D) Three

GROUP PROBLEM SOLVING



Given: The free-rolling ramp has a weight of 120 lb. The 80 lb crate slides from rest at A, 15 ft down the ramp to B.

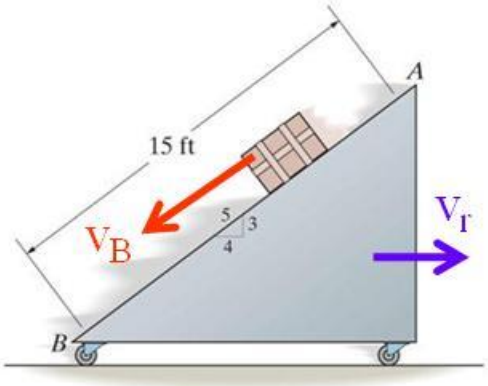
Assume that the ramp is smooth, and neglect the mass of the wheels.

Find: The ramp's speed when the crate reaches *B.*

Plan: Use **the energy conservation equation** as well as **conservation of linear momentum** and the relative velocity equation (you thought you could safely forget it?) to find the velocity of the ramp.

GROUP PROBLEM SOLVING (continued)

Solution:



Energy conservation equation:

$$0 + 80\,(3/5)\,(15)$$
$$= 0.5\,(80/32.2)(\boldsymbol{v}_B)^2 + 0.5\,(120/32.2)(\boldsymbol{v}_r)^2$$

To find the relations between $\boldsymbol{v}_B$ and $\boldsymbol{v}_r$, use conservation of linear momentum:

$$\overset{+}{\rightarrow}\quad 0 = (120/32.2)\; v_r - (80/32.2)\; v_{Bx}$$

$$\Rightarrow \quad v_{Bx} = 1.5\; v_r \qquad (1)$$

Since $\boldsymbol{v}_B = \boldsymbol{v}_r + \boldsymbol{v}_{B/r} \Rightarrow -v_{Bx}\,\boldsymbol{i} + v_{By}\,\boldsymbol{j} = v_r\,\boldsymbol{i} + v_{B/r}\,(-4/5\,\boldsymbol{i} - 3/5\,\boldsymbol{j})$

$$\Rightarrow \quad -v_{Bx} = v_r - (4/5)\; v_{B/r} \qquad (2)$$

$$v_{By} = -\,(3/5)\; v_{B/r} \qquad (3)$$

Eliminating $v_{B/r}$ from Eqs. (2) and (3) and Substituting Eq. (1) results in $v_{By} = 1.875\; v_r$

GROUP PROBLEM SOLVING (continued)

Then, energy conservation equation can be rewritten ;

$$0 + 80\,(3/5)\,(15) = 0.5\,(80/32.2)(\boldsymbol{v}_B)^2 + 0.5\,(120/32.2)(\boldsymbol{v}_r)^2$$

$$0 + 80\,(3/5)\,(15) = 0.5\,(80/32.2)\,[(1.5\ v_r)^2 + (1.875\ v_r)^2] + 0.5\,(120/32.2)\,(\boldsymbol{v}_r)^2$$

$$720 = 9.023\,(\boldsymbol{v}_r)^2$$

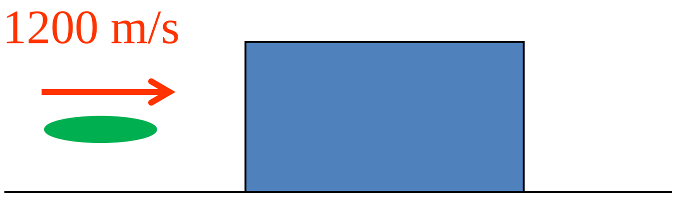
$\boldsymbol{v}_r = 8.93$ ft/s

ATTENTION QUIZ

1. The 20 g bullet is fired horizontally at 1200 m/s into the 300 g block resting on a smooth surface. If the bullet becomes embedded in the block, what is the velocity of the block immediately after impact.

A) 1125 m/s B) 80 m/s

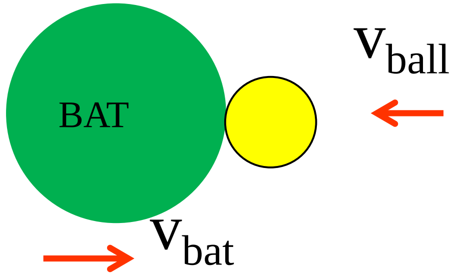
C) 1200 m/s D) 75 m/s



2. The 200-g baseball has a horizontal velocity of 30 m/s when it is struck by the bat, B, weighing 900-g, moving at 47 m/s. During the impact with the bat, how many impulses of importance are used to find the final velocity of the ball?

A) Zero B) One

C) Two D) Three



ANGULAR MOMENTUM, MOMENT OF A FORCE AND PRINCIPLE OF ANGULAR IMPULSE AND MOMENTUM

Objectives:

Students will be able to:

1. Determine the angular momentum of a particle and apply the principle of angular impulse & momentum.
2. Use conservation of angular momentum to solve problems.

READING QUIZ

1. Select the correct expression for the angular momentum of a particle about a point.

   A) $\boldsymbol{r} \times \boldsymbol{v}$

   B) $\boldsymbol{r} \times (m\,\boldsymbol{v})$

   C) $\boldsymbol{v} \times \boldsymbol{r}$

   D) $(m\,\boldsymbol{v}) \times \boldsymbol{r}$

2. The sum of the moments of all external forces acting on a particle is equal to

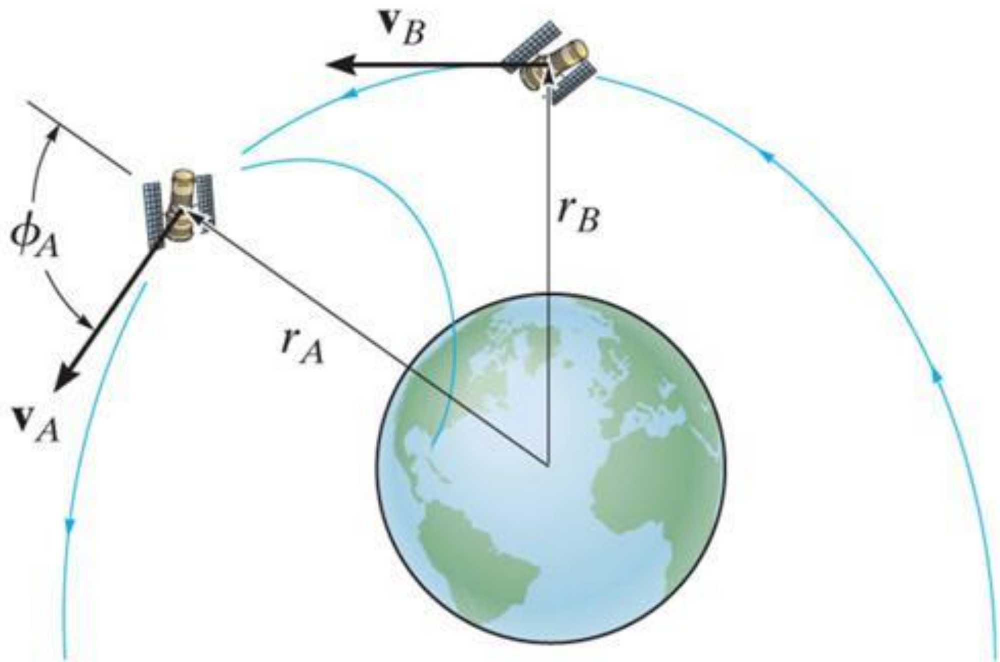
   A) angular momentum of the particle.

   B) linear momentum of the particle.

   C) time rate of change of angular momentum.

   D) time rate of change of linear momentum.

APPLICATIONS

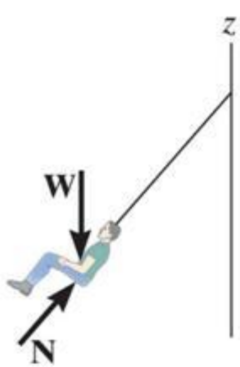


Planets and most satellites move in elliptical orbits. This motion is caused by gravitational attraction forces. Since these forces act in pairs, the sum of the moments of the forces acting on the system will be zero. This means that angular momentum is conserved.

If the angular momentum is constant, does it mean the linear momentum is also constant? Why or why not?

APPLICATIONS (continued)

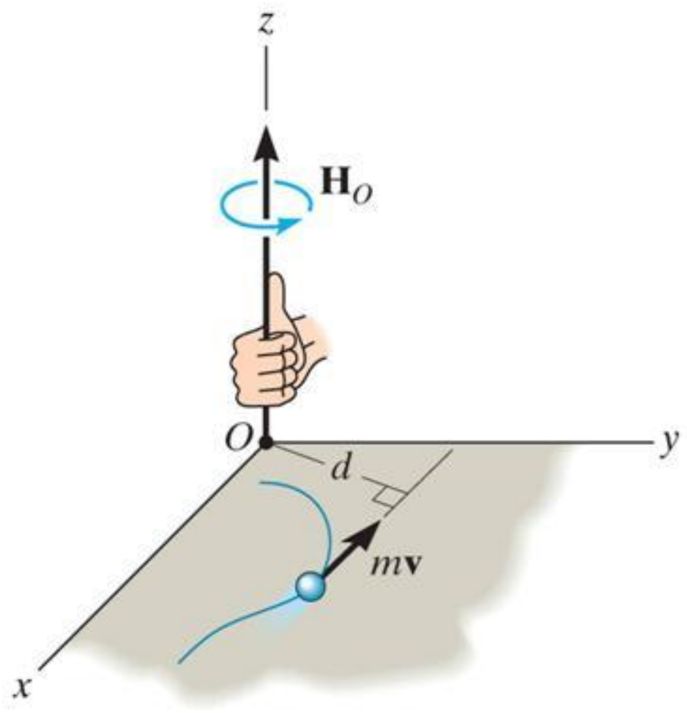
The passengers on the amusement-park ride experience conservation of angular momentum about the axis of rotation (the z-axis). As shown on the free body diagram, the line of action of the normal force, N, passes through the z-axis and the weight's line of action is parallel to it. Therefore, the sum of moments of these two forces about the z-axis is zero.



If the passenger moves away from the z-axis, will his speed increase or decrease? Why?

ANGULAR MOMENTUM (Section 15.5)

The angular momentum of a particle about point O is defined as the "moment" of the particle's linear momentum about O.



$$\boldsymbol{H}_o = \boldsymbol{r} \times m\boldsymbol{v} = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ r_x & r_y & r_z \\ mv_x & mv_y & mv_z \end{vmatrix}$$

The magnitude of $\boldsymbol{H}_o$ is $(H_o)_z = mv\,d$

RELATIONSHIP BETWEEN MOMENT OF A FORCE AND ANGULAR MOMENTUM

(Section 15.6)

The resultant force acting on the particle is equal to the time rate of change of the particle's linear momentum. Showing the time derivative using the familiar "dot" notation results in the equation

$$\sum \boldsymbol{F} = \dot{\boldsymbol{L}} = m\dot{\boldsymbol{v}}$$

We can prove that the resultant moment acting on the particle about point O is equal to the time rate of change of the particle's angular momentum about point O or

$$\sum \boldsymbol{M}_o = \boldsymbol{r} \times \boldsymbol{F} = \dot{\boldsymbol{H}}_o$$

PRINCIPLE OF ANGULAR IMPULSE AND MOMENTUM (Section 15.7)

Considering the relationship between moment and time rate of change of angular momentum

$$\sum \boldsymbol{M}_o = \dot{\boldsymbol{H}}_o = d\boldsymbol{H}_o/dt$$

By integrating between the time interval t_1 to t_2

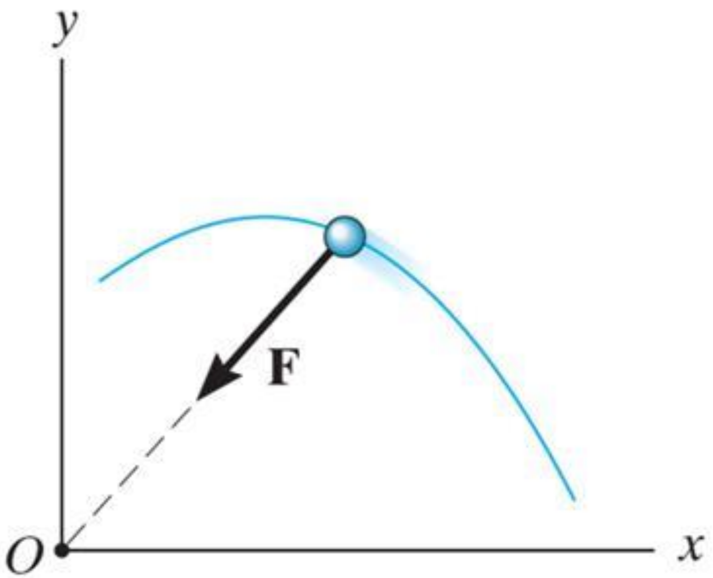
$$\sum \int_{t_1}^{t_2} \boldsymbol{M}_o \, dt = (\boldsymbol{H}_o)_2 - (\boldsymbol{H}_o)_1 \quad \text{or} \quad (\boldsymbol{H}_o)_1 + \sum \int_{t_1}^{t_2} \boldsymbol{M}_o \, dt = (\boldsymbol{H}_o)_2$$

This equation is referred to as the principle of angular impulse and momentum. The second term on the left side, $\sum\int M_o \, dt$, is called the angular impulse. In cases of 2D motion, it can be applied as a scalar equation using components about the z-axis.

CONSERVATION OF ANGULAR MOMENTUM

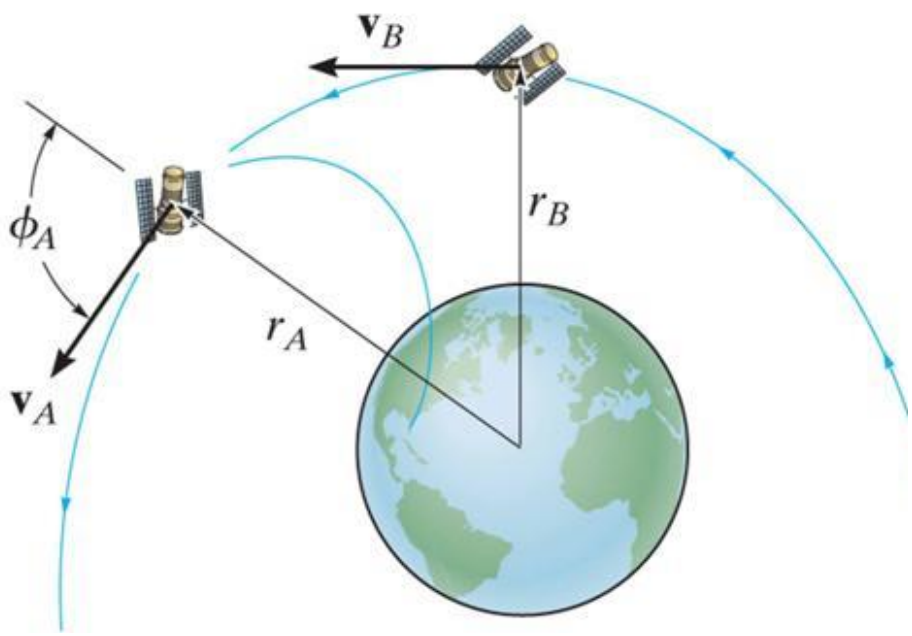
When the sum of angular impulses acting on a particle or a system of particles is zero during the time t_1 to t_2, the angular momentum is conserved. Thus,

$$(\boldsymbol{H}_O)_1 = (\boldsymbol{H}_O)_2$$



An example of this condition occurs when a particle is subjected only to a central force. In the figure, the force $\boldsymbol{F}$ is always directed toward point O. Thus, the angular impulse of $\boldsymbol{F}$ about O is always zero, and angular momentum of the particle about O is conserved.

EXAMPLE



Given: A satellite has an elliptical orbit about earth.

$m_{satellite} = 700$ kg

$m_{earth} = 5.976 \times 10^{24}$ kg

$v_A = 10$ km/s

$r_A = 15 \times 10^6$ m

$\phi_A = 70°$

Find: The speed, v_B, of the satellite at its closest distance, r_B, from the center of the earth.

Plan: Apply the principles of conservation of energy and conservation of angular momentum to the system.

EXAMPLE (continued)

Solution:

Conservation of energy: $T_A + V_A = T_B + V_B$ becomes

$$\frac{1}{2} m_s v_A^2 - \frac{G\, m_s\, m_e}{r_A} = \frac{1}{2} m_s v_B^2 - \frac{G\, m_s\, m_e}{r_B}$$

where $G = 66.73 \times 10^{-12}\ m^3/(kg \cdot s^2)$. Dividing through by m_s and substituting values yields:

$$0.5(10{,}000)^2 - \frac{66.73 \times 10^{-12}(5.976 \times 10^{24})}{15 \times 10^6} = 0.5\, v_B^2 - \frac{66.73 \times 10^{-12}(5.976 \times 10^{24})}{r_B}$$

or $23.4 \times 10^6 = 0.5\,(v_B)^2 - (3.99 \times 10^{14})/r_B$

EXAMPLE (continued)

Solution:

Now use Conservation of Angular Momentum.

$$(r_A\ m_s\ v_A)\ \sin\phi_A = r_B\ m_s\ v_B$$

$(15 \times 10^6)(10{,}000)\ \sin 70° = r_B\ v_B$ or

$r_B = (140.95 \times 10^9)/v_B$

Solving the two equations for r_B and v_B yields

$r_B = 13.8 \times 10^6$ m $\qquad v_B = 10.2$ km/s

CONCEPT QUIZ

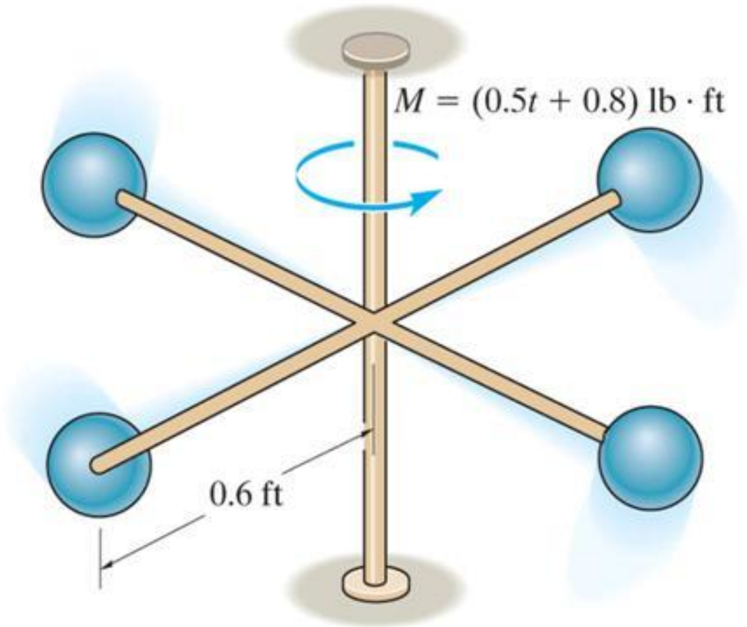
1. If a particle moves in the x - y plane, its angular momentum vector is in the

   A) x direction.
   B) y direction.
   C) z direction.
   D) x - y direction.

2. If there are no external impulses acting on a particle

   A) only linear momentum is conserved.
   B) only angular momentum is conserved.
   C) both linear momentum and angular momentum are conserved.
   D) neither linear momentum nor angular momentum are conserved.

GROUP PROBLEM SOLVING



Given: The four 5 lb spheres are rigidly attached to the crossbar frame, which has a negligible weight.

A moment acts on the shaft as shown, M = 0.5t + 0.8 lb·ft).

Find: The velocity of the spheres after 4 seconds, starting from rest.

Plan:

Apply the principle of angular impulse and momentum about the axis of rotation (z-axis).

GROUP PROBLEM SOLVING (continued)

Solution:

Angular momentum: $\boldsymbol{H}_Z = \boldsymbol{r} \times m\boldsymbol{v}$ reduces to a scalar equation.

$$(H_Z)_1 = 0 \quad \text{and} \quad (H_Z)_2 = 4\times\{(5/32.2)\,(0.6)\,v_2\} = 0.3727\,v_2$$

Angular impulse:

$$\int_{t_1}^{t_2} M\,dt = \int_{t_1}^{t_2} (0.5t + 0.8)\,dt = \left[(0.5/2)\,t^2 + 0.8\,t\right]\Big|_0^4 = 7.2\ \text{lb}\cdot\text{ft}\cdot\text{s}$$

Apply the principle of angular impulse and momentum.

$$0 + 7.2 = 0.3727\,v_2 \quad \Rightarrow \quad v_2 = 19.4\ \text{ft/s}$$

ATTENTION QUIZ

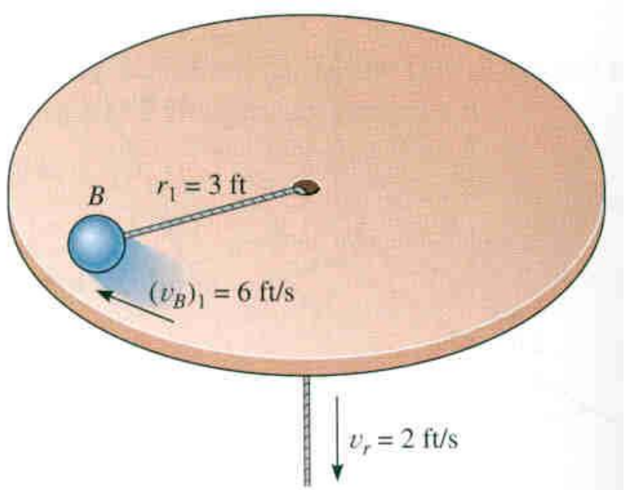
1. A ball is traveling on a smooth surface in a 3 ft radius circle with a speed of 6 ft/s. If the attached cord is pulled down with a constant speed of 2 ft/s, which of the following principles can be applied to solve for the velocity of the ball when r = 2 ft?

   A) Conservation of energy

   B) Conservation of angular momentum

   C) Conservation of linear momentum

   D) Conservation of mass

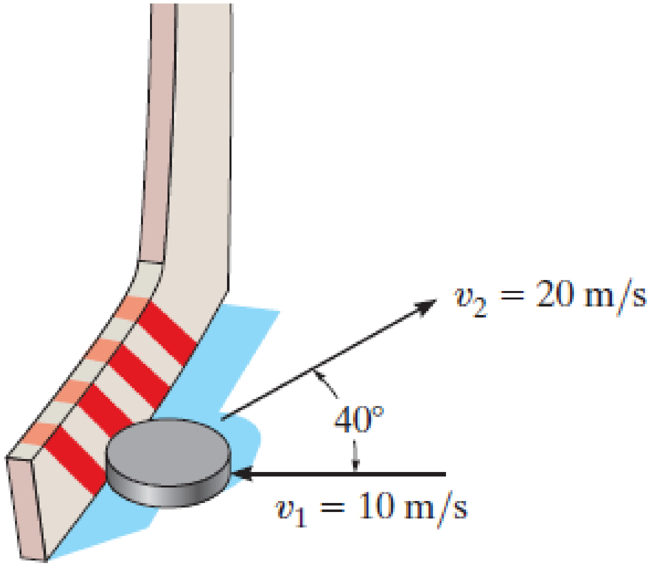


2. If a particle moves in the z - y plane, its angular momentum vector is in the

   A) x direction.
   B) y direction.
   C) z direction.
   D) z - y direction.

Example

A hockey puck is traveling to the left with a velocity of $v_1 = 10$ m/s when it is struck by a hockey stick and given a velocity of $v_2 = 20$ m/s as shown. Determine the magnitude of the net impulse exerted by the hockey stick on the puck. The puck has a mass of 0.2 kg.



SOLUTION

$$v_1 = \{-10\mathbf{i}\}\ \text{m/s}$$

$$v_2 = \{20\cos 40°\mathbf{i} + 20\sin 40°\mathbf{j}\}\ \text{m/s}$$

$$\mathbf{I} = m\Delta v = (0.2)\{[20\cos 40° - (-10)]\mathbf{i} + 20\sin 40°\mathbf{j}\}$$

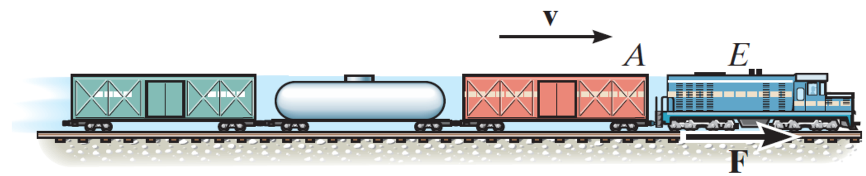
$$= \{5.0642\mathbf{i} + 2.5712\mathbf{j}\}\ \text{kg}\cdot\text{m/s}$$

$$\mathbf{I} = \sqrt{(5.0642)^2 + (2.5712)^2}$$

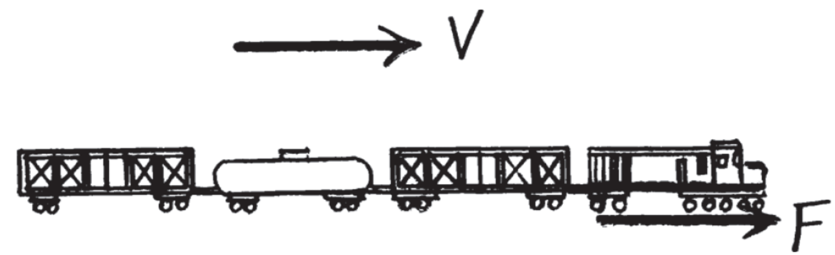
$$= 5.6795 = 5.68\ \text{kg}\cdot\text{m/s}$$

Example

A train consists of a 50-Mg engine and three cars, each having a mass of 30 Mg. If it takes 80 s for the train to increase its speed uniformly to 40 km/h, starting from rest, determine the force T developed at the coupling between the engine E and the first car A. The wheels of the engine provide a resultant frictional tractive force $\mathbf{F}$ which gives the train forward motion, whereas the car wheels roll freely. Also, determine F acting on the engine wheels.



SOLUTION



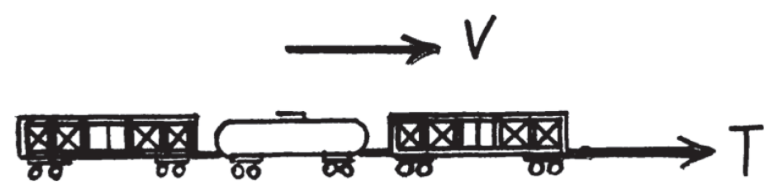
$(v_x)_2 = 40 \text{ km/h} = 11.11 \text{ m/s}$

Entire train:

$$\left(\xrightarrow{+}\right) \qquad m(v_x)_1 + \Sigma \int F_x \, dt = m(v_x)_2$$

$$0 + F(80) = [50 + 3(30)]\left(10^3\right)(11.11)$$

$$F = 19.4 \text{ kN}$$



Three cars:

$$\left(\xrightarrow{+}\right) \qquad m(v_x)_1 + \Sigma \int F_x \, dt = m(v_x)_2$$

$$0 + T(80) = 3(30)\left(10^3\right)(11.11) \qquad T = 12.5 \text{ kN}$$

Example

During operation the jack hammer strikes the concrete surface with a force which is indicated in the graph. To achieve this the 2-kg spike S is fired into the surface at 90 m/s. Determine the speed of the spike just after rebounding.

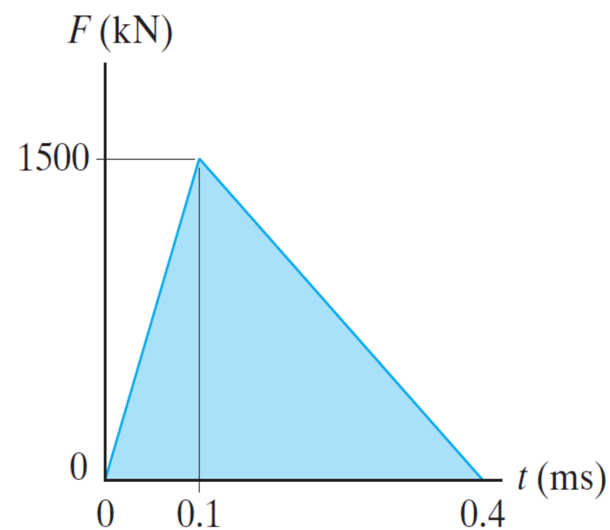
SOLUTION

Principle of Impulse and Momentum. The impulse of the force $\boldsymbol{F}$ is equal to the area under the F–t graph. Referring to the FBD of the spike, Fig. *a*

$$(+\uparrow) \quad m(v_y)_1 + \Sigma\int_{t_1}^{t_2} F_y\, dt = m(v_y)_2$$

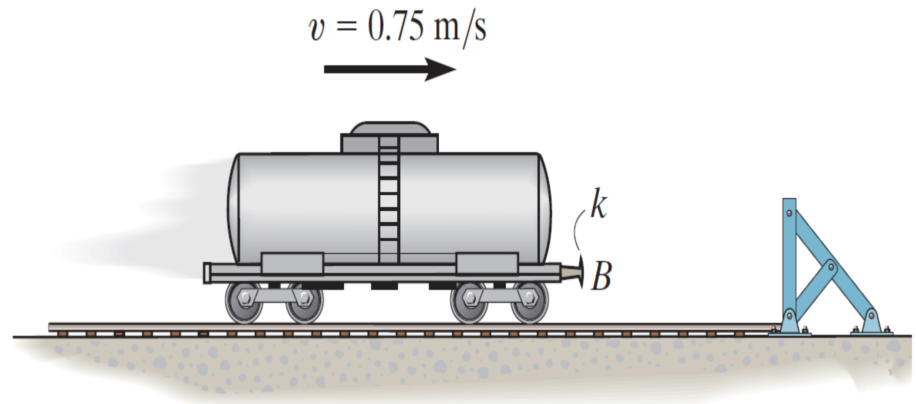
$$2(-90) + \frac{1}{2}\left[0.4(10^{-3})\right]\left[1500(10^3)\right] = 2v$$

$$v = 60.0 \text{ m/s} \uparrow$$



Example

A tankcar has a mass of 20 Mg and is freely rolling to the right with a speed of 0.75 m/s. If it strikes the barrier, determine the horizontal impulse needed to stop the car if the spring in the bumper B has a stiffness (a) $k \rightarrow \infty$ (bumper is rigid), and (b) $k = 15$ kN/m.



SOLUTION

a) b) $\quad (\overset{+}{\rightarrow}) \quad mv_1 + \Sigma \int F\,dt = mv_2$

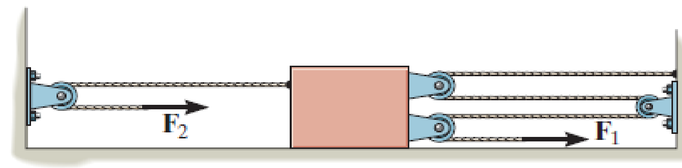
$$20(10^3)(0.75) - \int F\,dt = 0$$

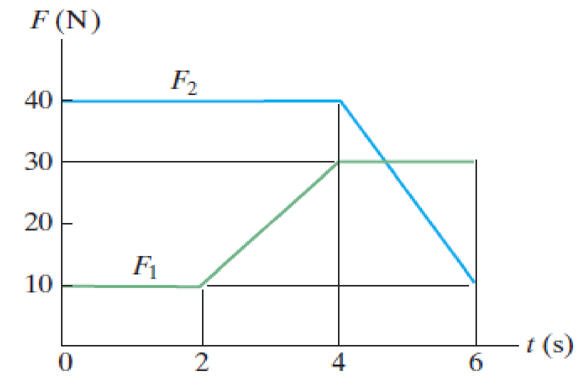
$$\int F\,dt = 15\text{ kN}\cdot\text{s}$$ **Ans.**

The impulse is the same for both cases. For the spring having a stiffness $k = 15$ kN/m, the impulse is applied over a longer period of time than for $k \rightarrow \infty$.

Example

The 30-kg slider block is moving to the left with a speed of 5 m/s when it is acted upon by the forces $\mathbf{F}_1$ and $\mathbf{F}_2$. If these loadings vary in the manner shown on the graph, determine the speed of the block at $t = 6$ s. Neglect friction and the mass of the pulleys and cords.





SOLUTION

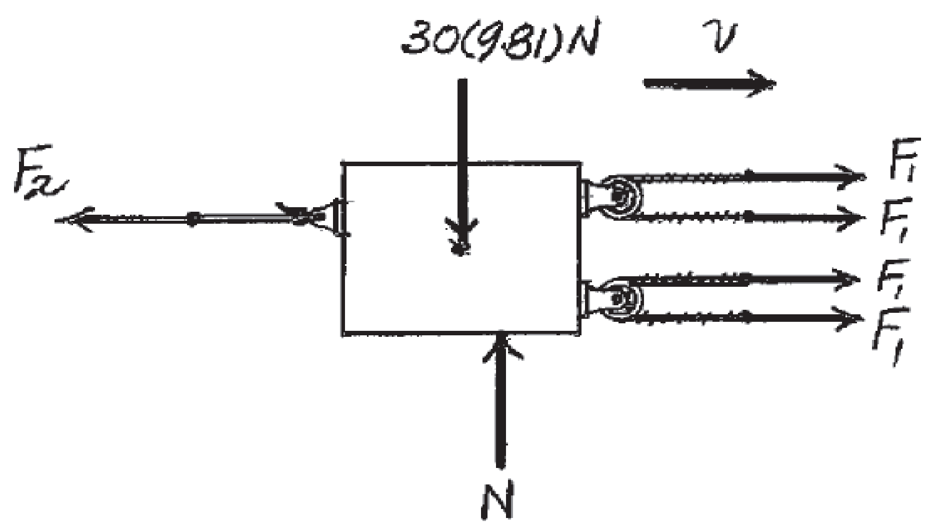
Principle of Impulse and Momentum. The impulses produced by $\mathbf{F}_1$ and $\mathbf{F}_2$ are equal to the area under the respective F–t graph. Referring to the FBD of the block Fig. *a*,

$$(\overset{+}{\rightarrow}) \quad m(v_x)_1 + \Sigma\int_{t_1}^{t_2} F_x\,dx = m(v_x)_2$$

$$-30(5) + 4\left[10(2) + \frac{1}{2}(10 + 30)(4 - 2) + 30(6 - 4)\right]$$

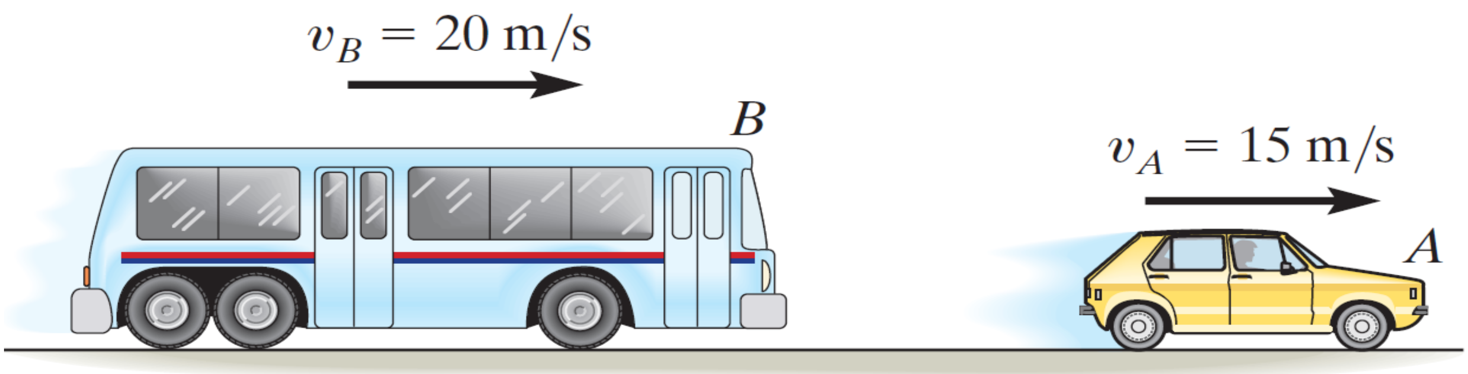
$$+ \left[-40(4) - \frac{1}{2}(10 + 40)(6 - 4)\right] = 30v$$

$$v = 4.00 \text{ m/s} \rightarrow$$



Example

The 5-Mg bus B is traveling to the right at 20 m/s. Meanwhile a 2-Mg car A is traveling at 15 m/s to the right. If the vehicles crash and become entangled, determine their common velocity just after the collision. Assume that the vehicles are free to roll during collision.



SOLUTION

Conservation of Linear Momentum.

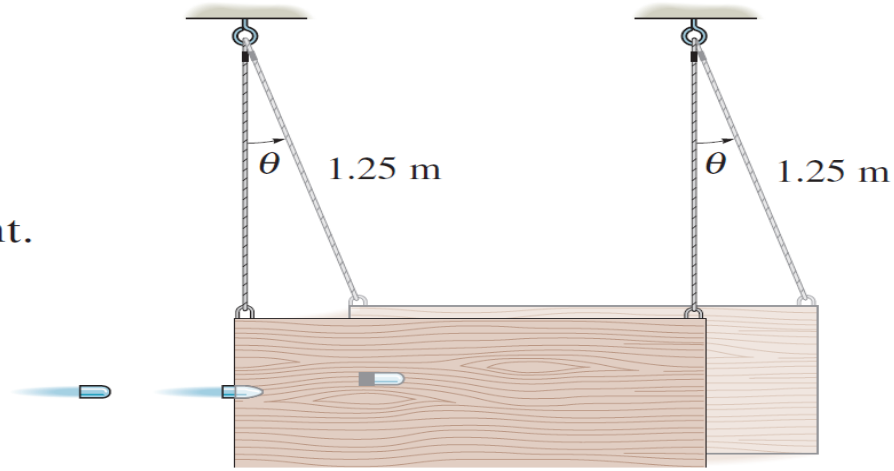
$(\xrightarrow{+})$ $$m_A v_A + m_B v_B = (m_A + m_B)v$$

$$[5(10^3)](20) + [2(10^3)](15) = [5(10^3) + 2(10^3)]v$$

$$v = 18.57 \text{ m/s} = 18.6 \text{ m/s} \rightarrow$$

Example

A ballistic pendulum consists of a 4-kg wooden block originally at rest, $\theta = 0°$. When a 2-g bullet strikes and becomes embedded in it, it is observed that the block swings upward to a maximum angle of $\theta = 6°$. Estimate the speed of the bullet.



SOLUTION

Just after impact:

Datum at lowest point.

$$T_2 + V_2 = T_3 + V_3$$

$$\frac{1}{2}(4 + 0.002)(v_B)_2^2 + 0 = 0 + (4 + 0.002)(9.81)(1.25)(1 - \cos 6°)$$

$$(v_B)_2 = 0.3665 \text{ m/s}$$

For the system of bullet and block:

$$(\overset{+}{\rightarrow}) \qquad \Sigma m v_1 = \Sigma m v_2$$

$$0.002(v_B)_1 = (4 + 0.002)(0.3665)$$

$$(v_B)_1 = 733 \text{ m/s}$$

Example

The boy jumps off the flat car at A with a velocity of $v = 4$ ft/s relative to the car as shown. If he lands on the second flat car B, determine the final speed of both cars after the motion. Each car has a weight of 80 lb. The boy's weight is 60 lb. Both cars are originally at rest. Neglect the mass of the car's wheels.

SOLUTION

$(\overset{+}{\leftarrow})$ $$\Sigma m(v_1) = \Sigma m(v_2)$$

$$0 + 0 = -\frac{80}{32.2}v_A + \frac{60}{32.2}(v_b)_x$$

$$v_A = 0.75(v_b)_x$$

$$v_b = v_A + v_{b/A}$$

$(\overset{+}{\leftarrow})$ $$(v_b)_x = -v_A + 4\left(\frac{12}{13}\right)$$

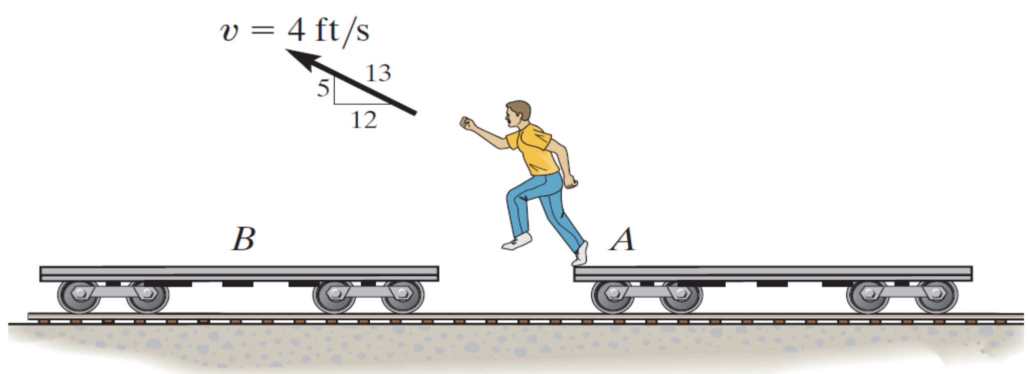
$$(v_b)_x = 2.110 \text{ ft/s}$$

$$v_A = 1.58 \text{ ft/s} \rightarrow$$

$(\overset{+}{\leftarrow})$ $$\Sigma m(v_1) = \Sigma m(v_2)$$

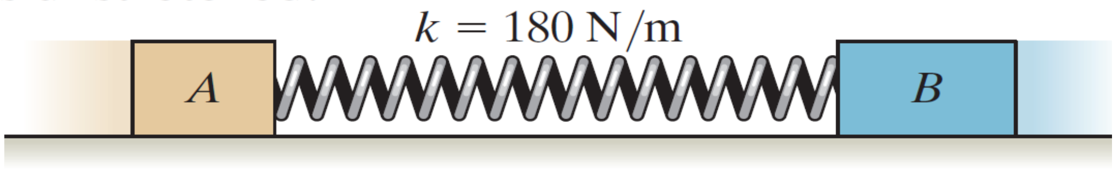
$$\frac{60}{32.2}(2.110) = \left(\frac{80}{32.2} + \frac{60}{32.2}\right)v$$

$$v = 0.904 \text{ ft/s}$$



Example

Blocks A and B have masses of 40 kg and 60 kg, respectively. They are placed on a smooth surface and the spring connected between them is stretched 2 m. If they are released from rest, determine the speeds of both blocks the instant the spring becomes unstretched.



SOLUTION

$(\xrightarrow{+})\qquad \Sigma m v_1 = \Sigma m v_2$

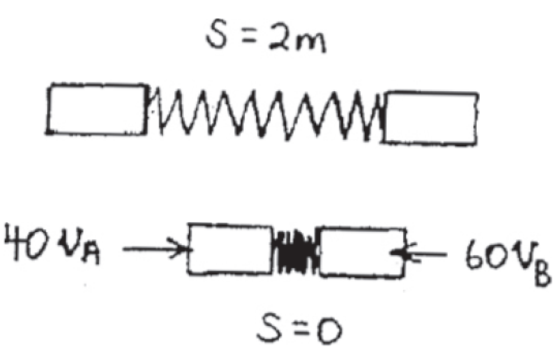
$$0 + 0 = 40\, v_A - 60\, v_B$$

$$T_1 + V_1 = T_2 + V_2$$

$$0 + \frac{1}{2}(180)(2)^2 = \frac{1}{2}(40)(v_A)^2 + \frac{1}{2}(60)(v_B)^2$$

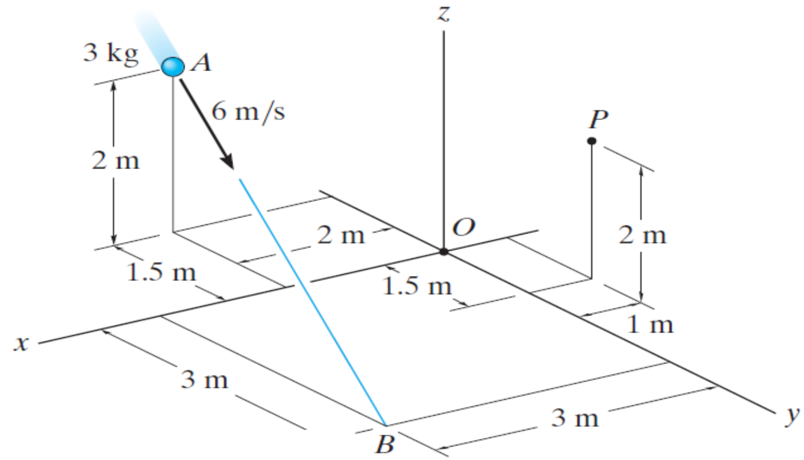
$$v_A = 3.29 \text{ m/s}$$

$$v_B = 2.19 \text{ m/s}$$



Example

Determine the angular momentum $\mathbf{H}_O$ of the 3-kg particle about point O.



SOLUTION

Position and Velocity Vectors.
The coordinates of points A and B are

$A(2, -1.5, 2)$ m and $B(3, 3, 0)$.

$$\boldsymbol{r}_{OB} = \{3\mathbf{i} + 3\mathbf{j}\}\ \text{m} \qquad \boldsymbol{r}_{OA} = \{2\mathbf{i} - 1.5\mathbf{j} + 2\mathbf{k}\}\ \text{m}$$

$$V_A = v_A\left(\frac{r_{AB}}{r_{AB}}\right) = (6)\left[\frac{(3-2)\mathbf{i} + [3-(-1.5)]\mathbf{j} + (0.2)\mathbf{k}}{\sqrt{(3-2)^2 + [3-(-1.5)]^2 + (0-2)^2}}\right]$$

$$= \left\{\frac{6}{\sqrt{25.25}}\mathbf{i} + \frac{27}{\sqrt{25.25}}\mathbf{j} - \frac{12}{\sqrt{25.25}}\mathbf{k}\right\}\ \text{m/s}$$

Angular Momentum about Point O. Applying Eq. 15

$$\boldsymbol{H}_O = \boldsymbol{r}_{OB} \times mV_A$$

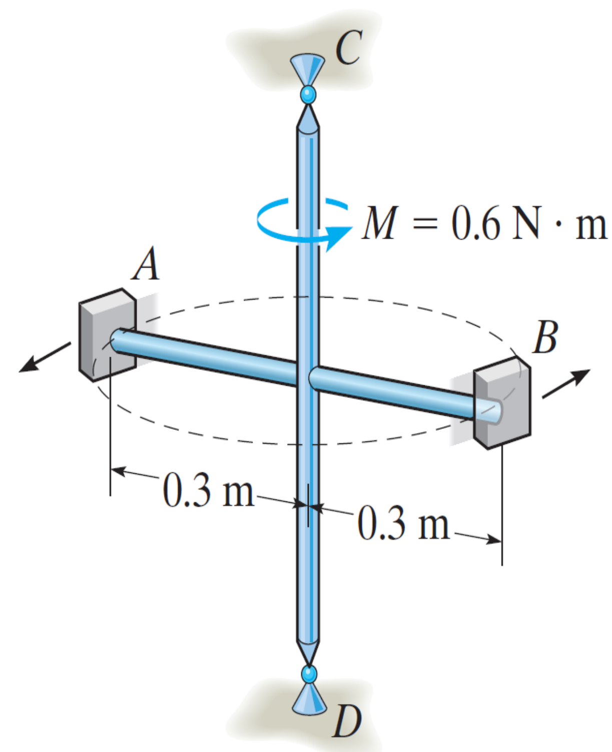
$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 3 & 0 \\ 3\left(\frac{6}{\sqrt{25.25}}\right) & 3\left(\frac{27}{\sqrt{25.25}}\right) & 3\left(-\frac{12}{\sqrt{25.25}}\right) \end{vmatrix}$$

$$= \{-21.4928\mathbf{i} + 21.4928\mathbf{j} + 37.6124\mathbf{k}\}\ \text{kg}\cdot\text{m}^2/\text{s}$$

$$= \{-21.5\mathbf{i} + 21.5\mathbf{j} + 37.6\}\ \text{kg}\cdot\text{m}^2/\text{s}$$

Example

The two blocks A and B each have a mass of 400 g. The blocks are fixed to the horizontal rods, and their initial velocity along the circular path is 2 m/s. If a couple moment of $M = (0.6)\ \mathrm{N \cdot m}$ is applied about CD of the frame, determine the speed of the blocks when $t = 3$ s. The mass of the frame is negligible, and it is free to rotate about CD. Neglect the size of the blocks.



SOLUTION

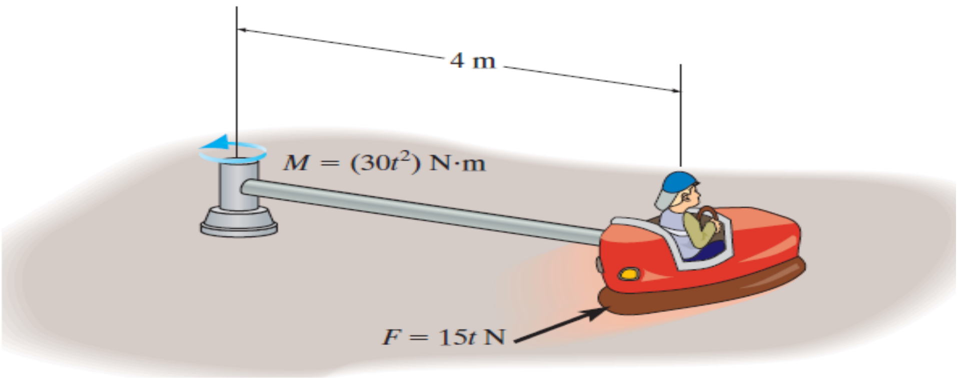
$$(H_o)_1 + \Sigma \int_{t_1}^{t_2} M_o dt = (H_o)_2$$

$$2[0.3(0.4)(2)] + 0.6(3) = 2[0.3(0.4)v]$$

$$v = 9.50\ \mathrm{m/s}$$

Example

If the rod of negligible mass is subjected to a couple moment of $M = (30t^2)$ N · m and the engine of the car supplies a traction force of $F = (15t)$ N to the wheels, where t is in seconds, determine the speed of the car at the instant $t = 5$ s. The car starts from rest. The total mass of the car and rider is 150 kg. Neglect the size of the car.



SOLUTION

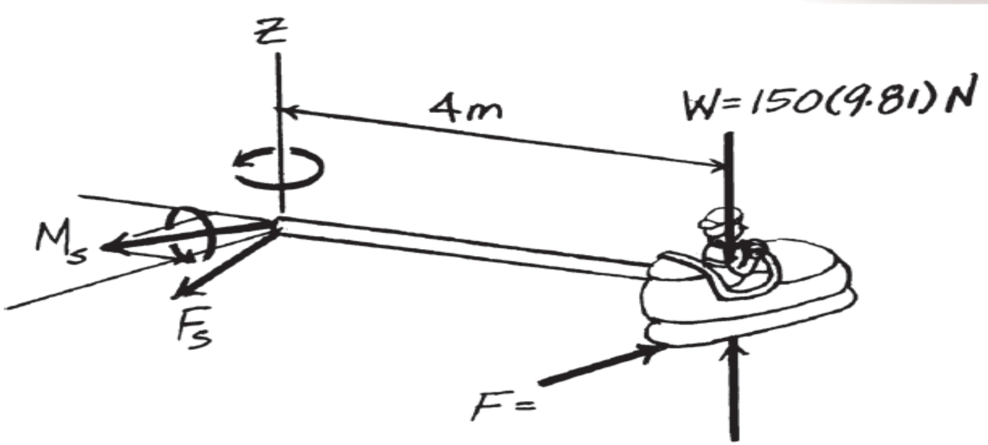
Free-Body Diagram: The free-body diagram of the system is shown in Fig. *a*. Since the moment reaction $\mathbf{M}_S$ has no component about the z axis, the force reaction $\mathbf{F}_S$ acts through the z axis, and the line of action of $\mathbf{W}$ and $\mathbf{N}$ are parallel to the z axis, they produce no angular impulse about the z axis.

Principle of Angular Impulse and Momentum:

$$(H_1)_z + \Sigma \int_{t_2}^{t_1} M_z\, dt = (H_2)_z$$

$$0 + \int_0^{5\,\text{s}} 30t^2\, dt + \int_0^{5\,\text{s}} 15t(4)dt = 150v(4)$$

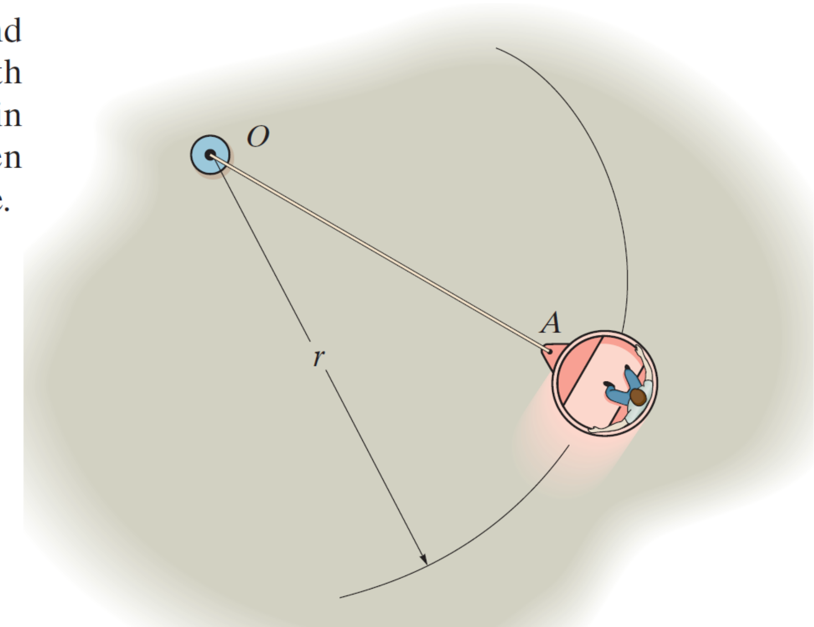
$v = 3.33$ m/s



(a)

Example

The amusement park ride consists of a 200-kg car and passenger that are traveling at 3 m/s along a circular path having a radius of 8 m. If at $t = 0$, the cable OA is pulled in toward O at 0.5 m/s, determine the speed of the car when $t = 4$ s. Also, determine the work done to pull in the cable.



SOLUTION

Conservation of Angular Momentum. At $t = 4$ s, $r_2 = 8 - 0.5(4) = 6$ m.

$$(H_0)_1 = (H_0)_2$$

$$r_1 m v_1 = r_2 m (v_2)_t$$

$$8[200(3)] = 6[200(v_2)_t]$$

$$(v_2)_t = 4.00 \text{ m/s}$$

Here, $(v_2)_t = 0.5$ m/s. Thus

$$v_2 = \sqrt{(v_2)_t^2 + (v_2)_r^2} = \sqrt{4.00^2 + 0.5^2} = 4.031 \text{ m/s} = 4.03 \text{ m/s}$$

Principle of Work and Energy.

$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2}(200)(3^2) + \Sigma U_{1-2} = \frac{1}{2}(200)(4.031)^2$$

$$\Sigma U_{1-2} = 725 \text{ J}$$

Dynamics

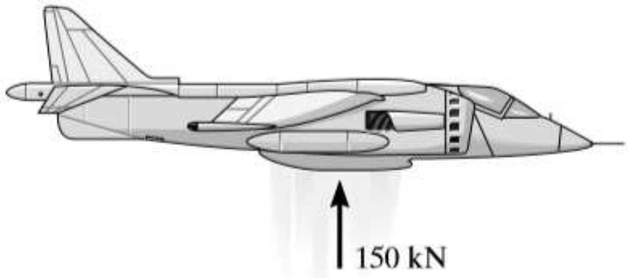
Dr. Hashem Alkhaldi

Suggested Problems: Chapter 15

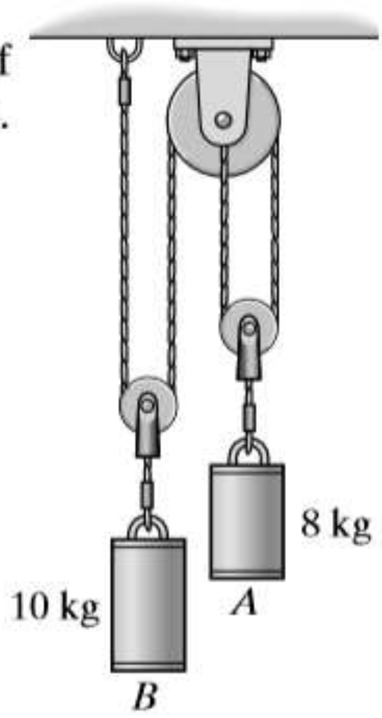
(15 problems, 3 pages)

g= 9.81 m/s^2 = 32.2 ft/s^2

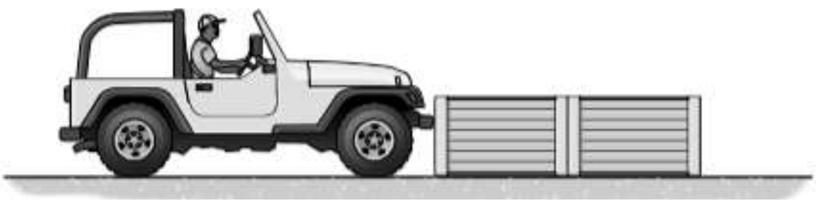
Q1) The 12-Mg "jump jet" is capable of taking off vertically from the deck of a ship. If its jets exert a constant vertical force of 150 kN on the plane, determine its velocity and how high it goes in $t = 6$ s, starting from rest. Neglect the loss of fuel during the lift.



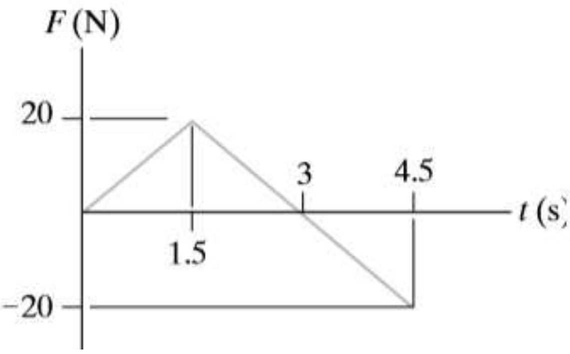
Q2) If cylinder A is given an initial downward speed of 2 m/s, determine the speed of each cylinder when $t = 3$ s. Neglect the mass of the pulleys.



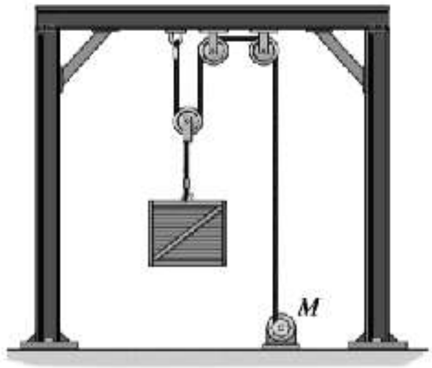
Q3) The 1.5-Mg four-wheel-drive jeep is used to push two identical crates, each having a mass of 500 kg. If the coefficient of static friction between the tires and the ground is $\mu_s = 0.6$, determine the maximum possible speed the jeep can achieve in 5 s without causing the tires to slip. The coefficient of kinetic friction between the crates and the ground is $\mu_k = 0.3$.



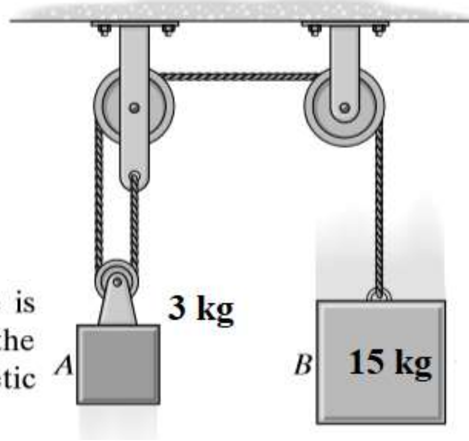
Q4) The 10-kg smooth block moves to the right with a velocity of $v_0 = 3$ m/s when force **F** is applied. If the force varies as shown in the graph, determine the velocity of the block when $t = 4.5$ s.

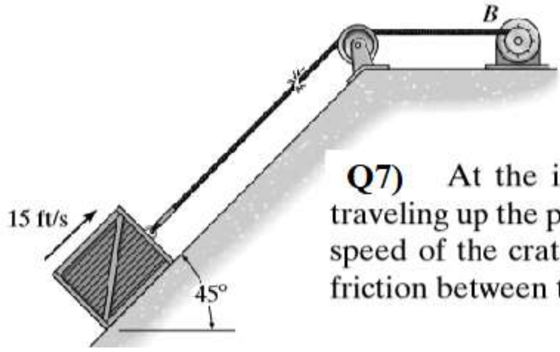


Q5) The 100-kg crate is hoisted by the motor M. The motor exerts a force on the cable of $T = (200t^{1/2} + 150)$ N, where t is in seconds. If the crate starts from rest at the ground, determine the speed of the crate when $t = 5$ s.



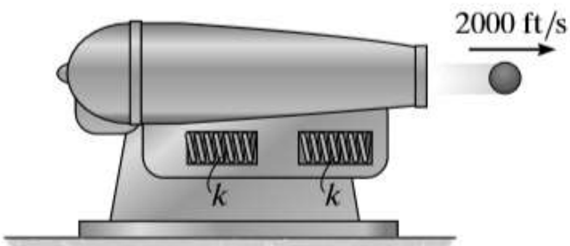
Q6) Determine the velocity of each block 2 s after the blocks are released from rest. Neglect the mass of the pulleys and cord.



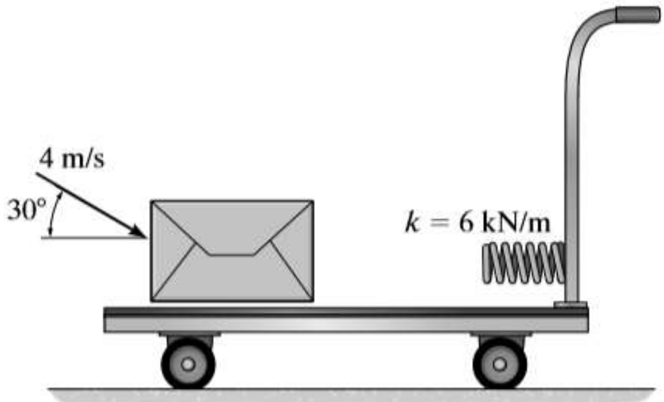


Q7) At the instant the cable fails, the 200-lb crate is traveling up the plane with a speed of 15 ft/s. Determine the speed of the crate 2 s afterward. The coefficient of kinetic friction between the crate and the plane is $\mu_k = 0.20$.

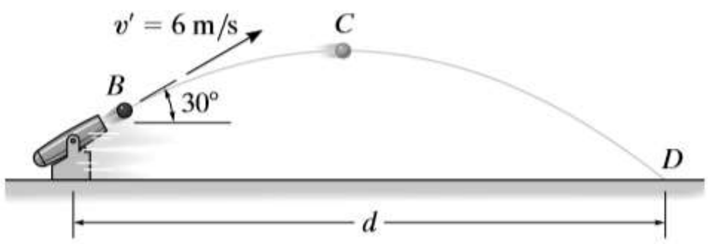
Q8) The 10-lb cannon ball is fired horizontally by a 500-lb cannon as shown. If the muzzle velocity of the ball is 2000 ft/s, measured relative to the ground, determine the recoil velocity of the cannon just after firing. If the cannon rests on a smooth support and is to be stopped after it has recoiled a distance of 6 in., determine the required stiffness k of the two identical springs, each of which is originally unstretched.



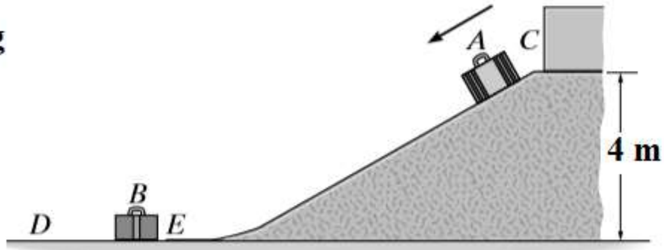
Q9) The 40-kg package is thrown with a speed of 4 m/s onto the cart having a mass of 20 kg. If it slides on the smooth surface and strikes the spring, determine the velocity of the cart at the instant the package fully compresses the spring. What is the maximum compression of the spring? Neglect rolling resistance of the cart.



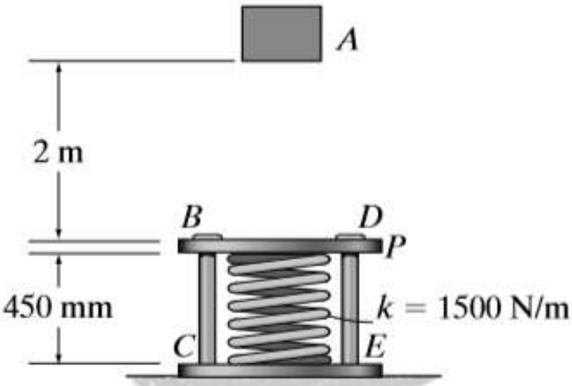
Q10) The 5-kg spring-loaded gun rests on the smooth surface. It fires a ball having a mass of 1 kg with a velocity of $v' = 6$ m/s relative to the gun in the direction shown. If the gun is originally at rest, determine the horizontal distance d the ball is from the initial position of the gun at the instant the ball strikes the ground at D. Neglect the size of the gun.



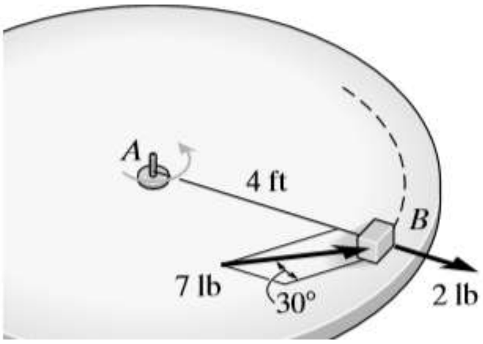
Q11) The 15-**kg** suitcase A is released from rest at C. After it slides down the smooth ramp, it strikes the 10-**kg** suitcase B, which is originally at rest. If the coefficient of restitution between the suitcases is $e = 0.3$ and the coefficient of kinetic friction between the floor DE and each suitcase is $\mu_k = 0.4$, determine (a) the velocity of A just before impact, (b) the velocities of A and B just after impact, and (c) the distance B slides before coming to rest.



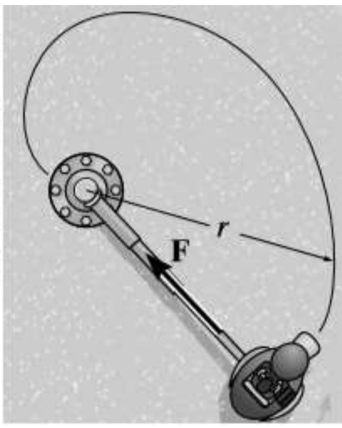
Q12) A 10-kg block A is released from rest 2 m above the 5-kg plate P, which can slide freely along the smooth vertical guides BC and DE. Determine the velocity of the block and plate just after impact. The coefficient of restitution between the block and the plate is $e = 0.75$. Also, find the maximum compression of the spring due to impact. The spring has an unstretched length of 600 mm.



Q13) The 10-lb block is originally at rest on the smooth surface. It is acted upon by a radial force of 2 lb and a horizontal force of 7 lb, always directed at 30° from the tangent to the path as shown. Determine the time required to break the cord, which requires a tension $T = 30$ lb. What is the speed of the block when this occurs? Neglect the size of the block for the calculation.



Q14) The 150-lb car of an amusement park ride is connected to a rotating telescopic boom. When $r = 15$ ft, the car is moving on a horizontal circular path with a speed of 30 ft/s. If the boom is shortened at a rate of 3 ft/s, determine the speed of the car when $r = 10$ ft. Also, find the work done by the axial force **F** along the boom. Neglect the size of the car and the mass of the boom.



Q15) The four 5-lb spheres are rigidly attached to the crossbar frame having a negligible weight. If a couple moment $M = (0.5t + 0.8)$ lb · ft, where t is in seconds, is applied as shown, determine the speed of each of the spheres in 4 seconds starting from rest. Neglect the size of the spheres.