Chapter 16
Kinematics of Rigid Body

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PLANAR RIGID BODY MOTION: TRANSLATION & ROTATION

Today's Objectives :

Students will be able to:
Analyze the kinematics of a rigid body undergoing planar translation or rotation about a fixed axis.

READING QUIZ

1. If a rigid body is in translation only, the velocity at points A and B on the rigid body _______ .

   A) are usually different
   B) are always the same
   C) depend on their position
   D) depend on their relative position

2. If a rigid body is rotating with a constant angular velocity about a fixed axis, the velocity vector at point P is _______.

   A) $\boldsymbol{\omega} \times \boldsymbol{r}_p$ B) $\boldsymbol{r}_p \times \boldsymbol{\omega}$

   C) $d\boldsymbol{r}_p/dt$ D) All of the above.

APPLICATIONS

Passengers on this amusement ride are subjected to curvilinear translation since the vehicle moves in a circular path but they always remains upright.

If the angular motion of the rotating arms is known, how can we determine the velocity and acceleration experienced by the passengers? Why would we want to know these values?

Does each passenger feel the same acceleration?

APPLICATIONS

Gears, pulleys and cams, which rotate about fixed axes, are often used in machinery to generate motion and transmit forces. The angular motion of these components must be understood to properly design the system.

To do this design, we need to relate the angular motions of contacting bodies that rotate about different fixed axes. How is this different than the analyses we did in earlier chapters?

RIGID BODY MOTION

There are cases where an object cannot be treated as a particle. In these cases the size or shape of the body must be considered. Rotation of the body about its center of mass requires a different approach.

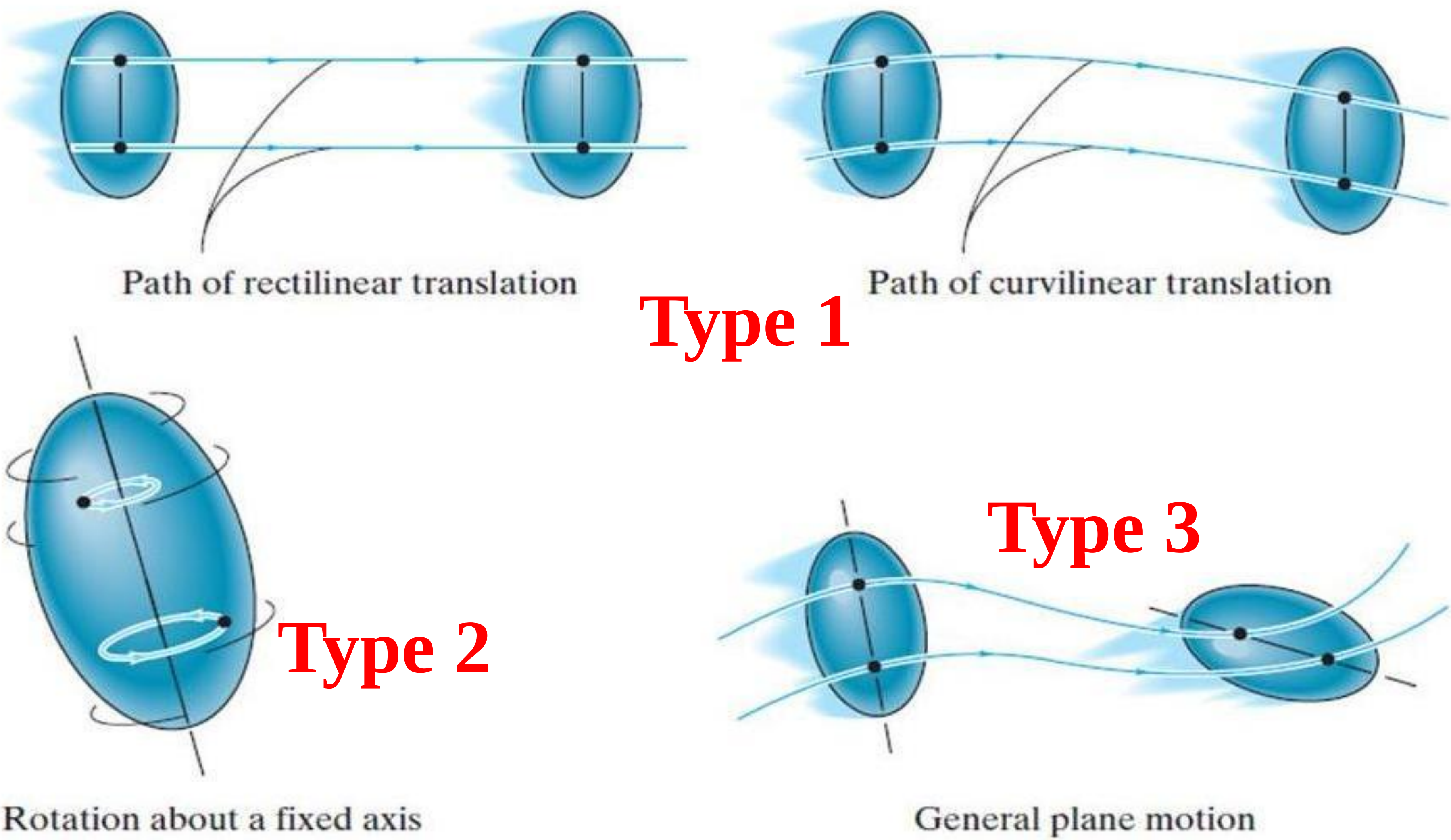
For example, in the design of gears, cams, and links in machinery or mechanisms, rotation of the body is an important aspect in the analysis of motion.

We will now start to study rigid body motion. The analysis will be limited to planar motion.

A body is said to undergo planar motion when all parts of the body move along paths equidistant from a fixed plane.

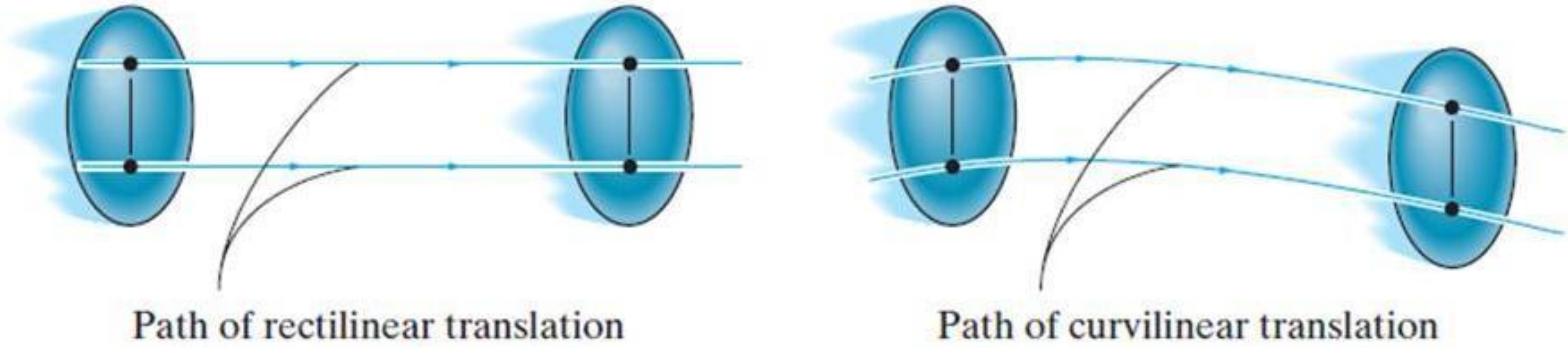
PLANAR RIGID BODY MOTION

There are three types of planar rigid body motion.



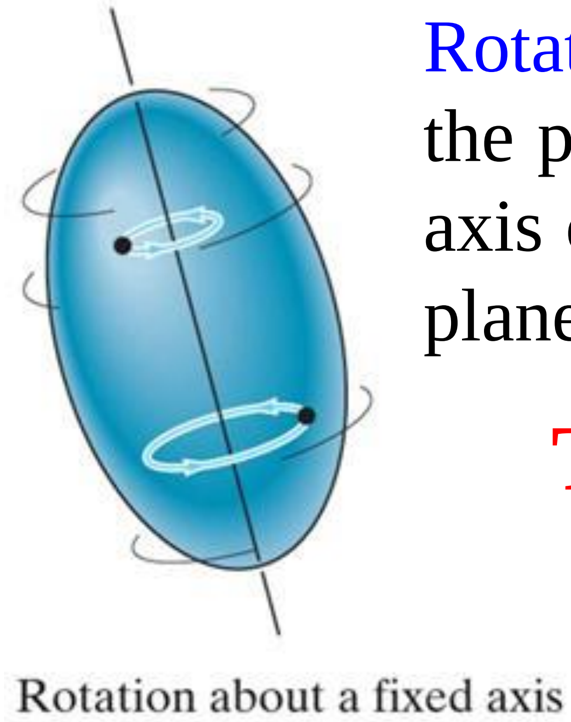
PLANAR RIGID BODY MOTION

Type 1 : Translational Motion



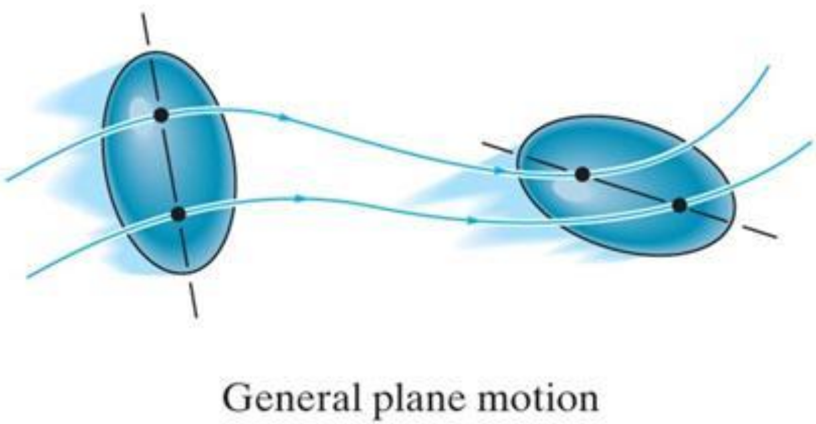
Translation: Translation occurs if every line segment on the body remains parallel to its original direction during the motion. When all points move along straight lines, the motion is called rectilinear translation. When the paths of motion are curved lines, the motion is called curvilinear translation.

Type 2 : Rotation about a fixed axis



Rotation about a fixed axis: In this case, all the particles of the body, except those on the axis of rotation, move along circular paths in planes perpendicular to the axis of rotation.

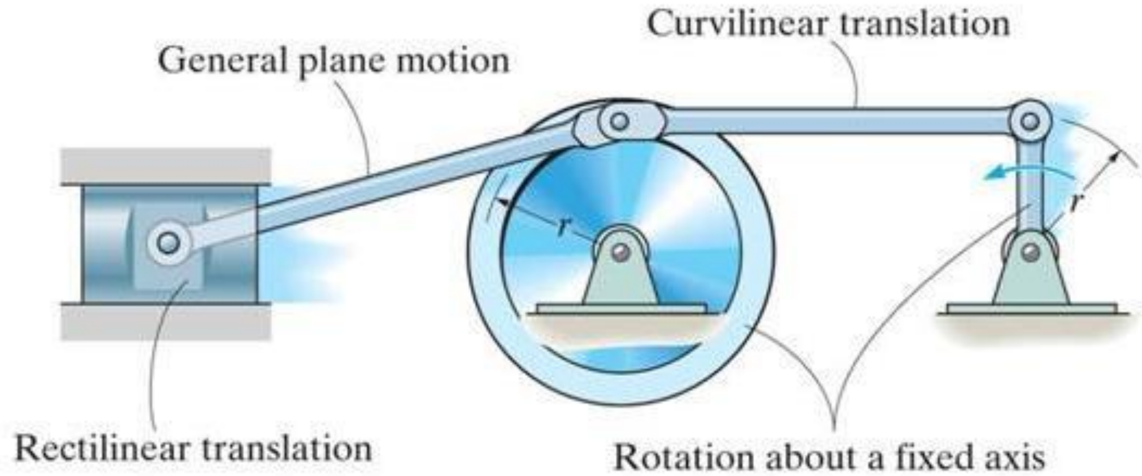
Type 3 : General plane motion



General plane motion: In this case, the body undergoes both translation and rotation. Translation occurs within a plane and rotation occurs about an axis perpendicular to this plane.

PLANAR RIGID BODY MOTION

An example of bodies undergoing the three types of motion is shown in this mechanism.

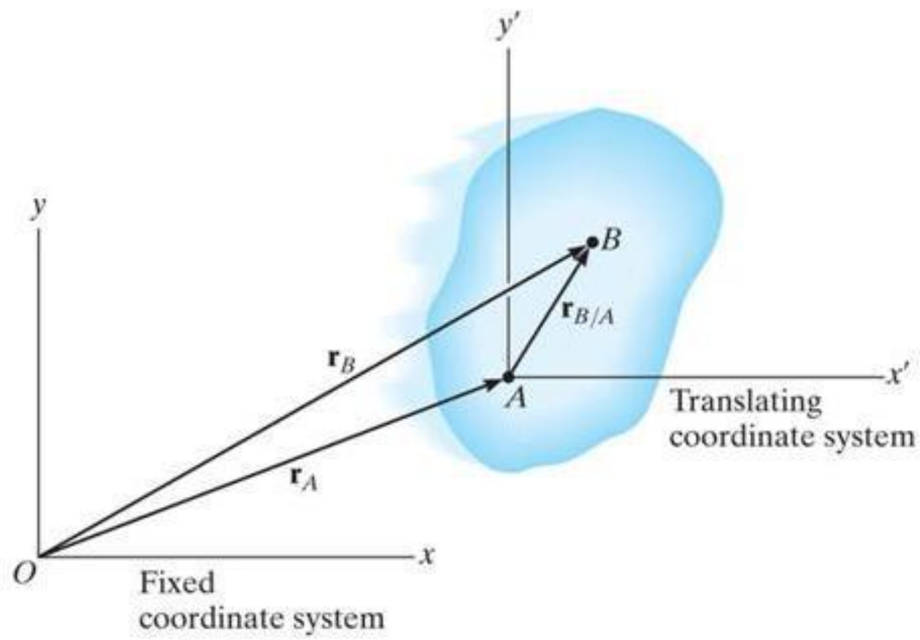


The wheel and crank undergo rotation about a fixed axis.

The piston undergoes rectilinear translation
The connecting rod undergoes curvilinear translation,

The connecting rod undergoes general plane motion, as it will both translate and rotate.

Type 1 : TRANSLATIONAL MOTION



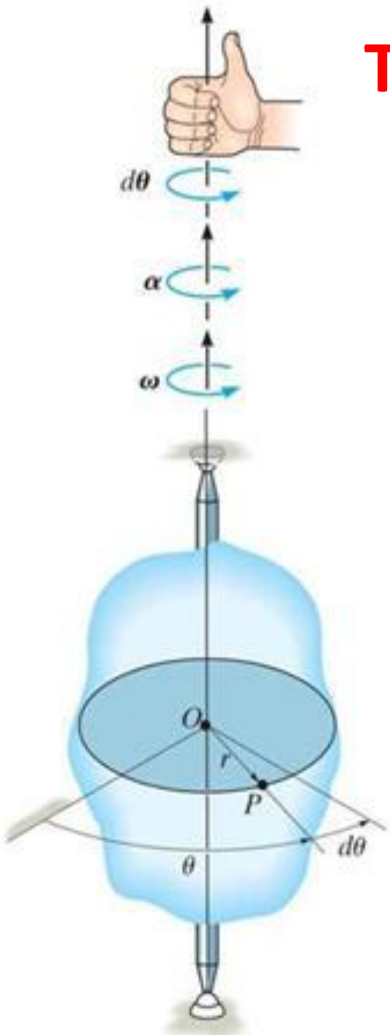
The positions of two points A and B on a translating body can be related by

$$\boldsymbol{r}_B = \boldsymbol{r}_A + \boldsymbol{r}_{B/A}$$

where $\boldsymbol{r}_A$ & $\boldsymbol{r}_B$ are the absolute position vectors defined from the fixed x-y coordinate system, and $\boldsymbol{r}_{B/A}$ is the relative-position vector between B and A.

The velocity at B is $\boldsymbol{v}_B = \boldsymbol{v}_A + d\boldsymbol{r}_{B/A}/dt$. Now $d\boldsymbol{r}_{B/A}/dt = 0$ since $\boldsymbol{r}_{B/A}$ is constant. So, $\boldsymbol{v}_B = \boldsymbol{v}_A$, and by following similar logic, $\mathbf{a}_B = \mathbf{a}_A$.

Note, all points in a rigid body subjected to translation move with the same velocity and acceleration.



Type 2 : ROTATION ABOUT A FIXED AXIS

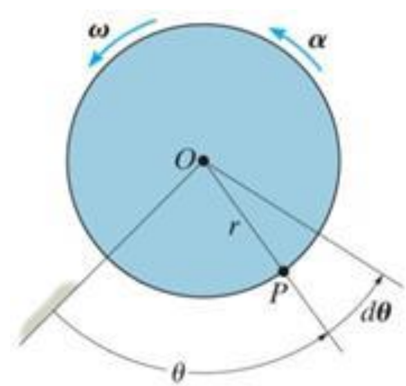
When a body rotates about a fixed axis, any point P in the body travels along a circular path. The angular position of P is defined by θ.

The change in angular position, $d\theta$, is called the angular displacement, with units of either radians or revolutions. They are related by

1 revolution = (2π) radians

Angular velocity, ω, is obtained by taking the time derivative of angular displacement:

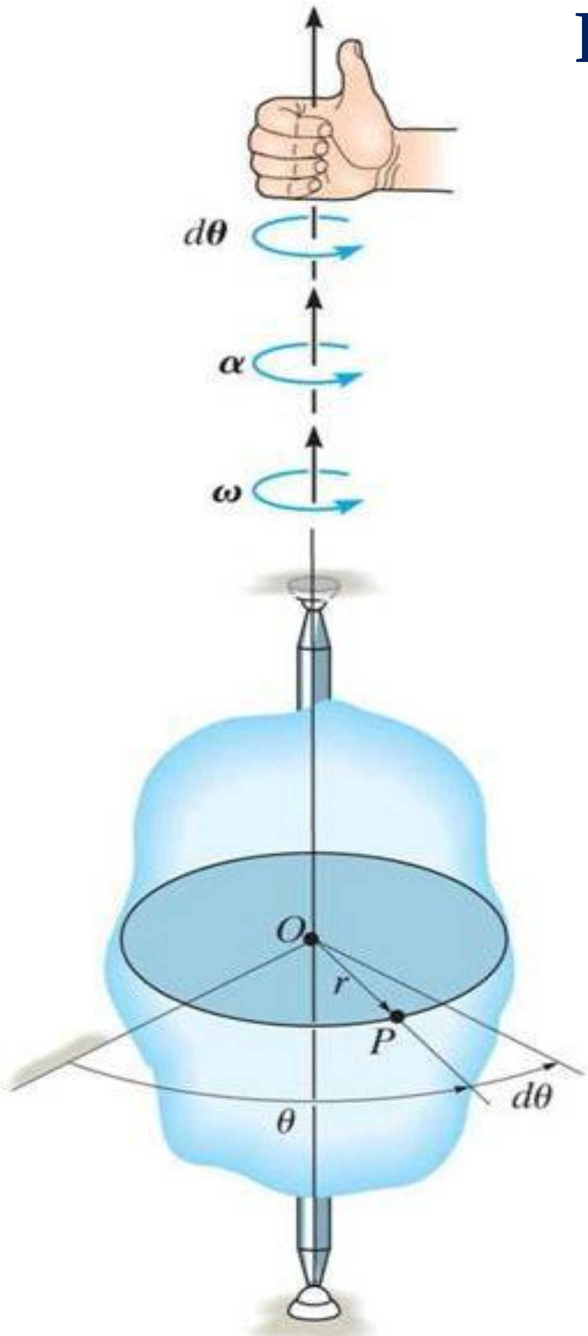
$$\omega = d\theta/dt \quad (rad/s) \quad +$$



Similarly, angular acceleration is

$$\alpha = d^2\theta/dt^2 = d\omega/dt \quad \text{or} \quad \alpha = \omega(d\omega/d\theta) \; + \; rad/s^2$$

ROTATION ABOUT A FIXED AXIS



If the angular acceleration of the body is constant, $\alpha = \alpha_C$, the equations for angular velocity and acceleration can be integrated to yield the set of algebraic equations below.

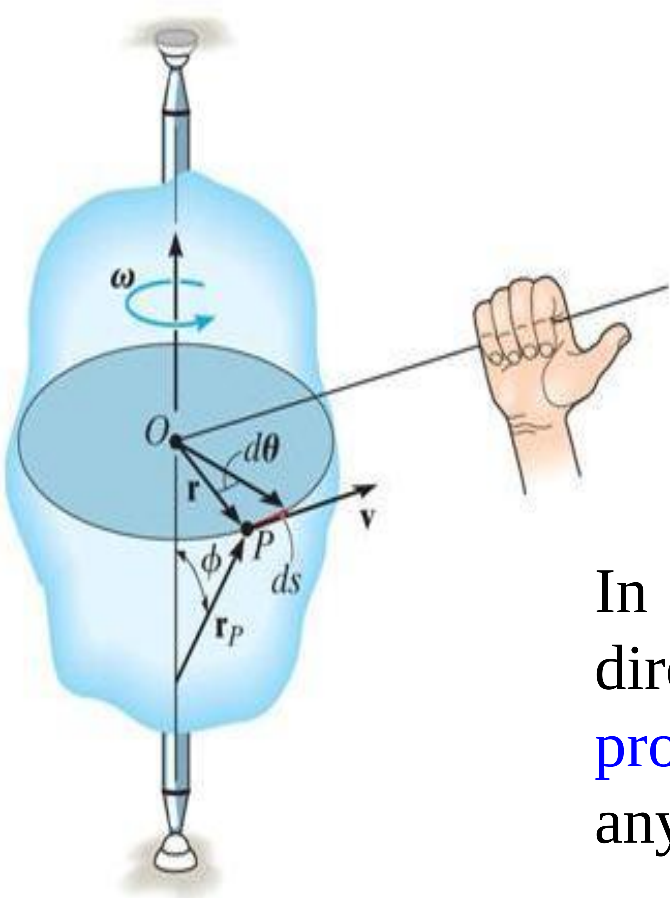
$$\omega = \omega_0 + \alpha_C t$$

$$\theta = \theta_0 + \omega_0 t + 0.5 \alpha_C t^2$$

$$\omega^2 = (\omega_0)^2 + 2\alpha_C (\theta - \theta_0)$$

θ_0 and ω_0 are the initial values of the body's angular position and angular velocity. Note these equations are very similar to the constant acceleration relations developed for the rectilinear motion of a particle.

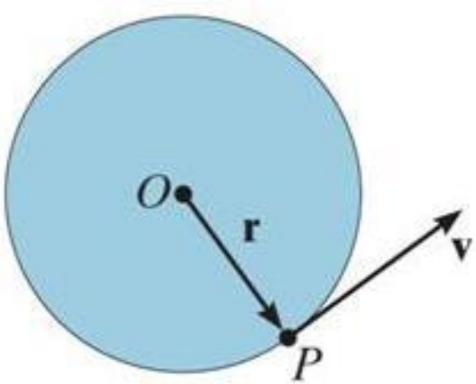
VELOCITY OF POINT P



The magnitude of the velocity of P is equal to ωr (the text provides the derivation). The velocity's direction is tangent to the circular path of P.

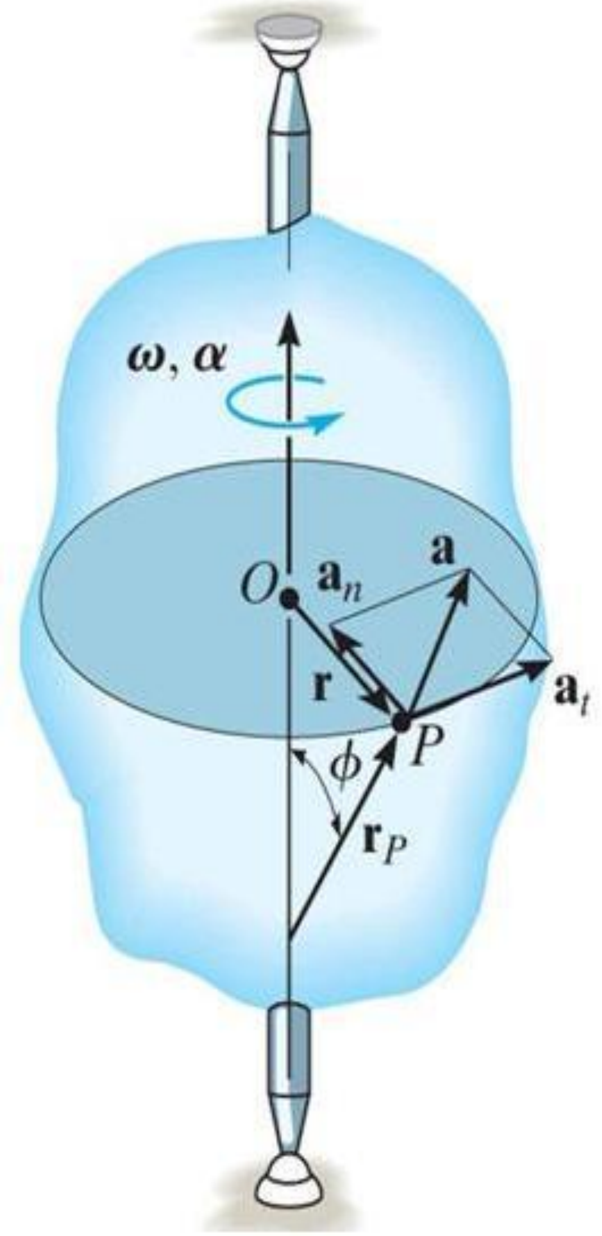
In the vector formulation, the magnitude and direction of $\boldsymbol{v}$ can be determined from the cross product of $\boldsymbol{\omega}$ and $\boldsymbol{r}_p$. Here $\boldsymbol{r}_p$ is a vector from any point on the axis of rotation to P.

$$\boldsymbol{v} = \boldsymbol{\omega} \times \boldsymbol{r}_p = \boldsymbol{\omega} \times \boldsymbol{r}$$



The direction of $\boldsymbol{v}$ is determined by the right-hand rule.

ACCELERATION OF POINT P

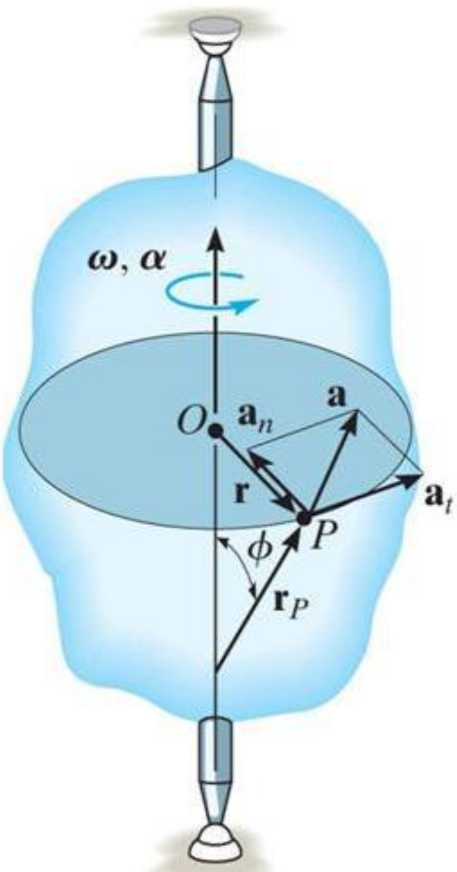


The acceleration of P is expressed in terms of its normal (**a**$_n$) and tangential (**a**$_t$) components. In scalar form, these are $a_t = \alpha r$ and $a_n = \omega^2 r$.

The tangential component, **a**$_t$, represents the time rate of change in the velocity's magnitude. It is directed tangent to the path of motion.

The normal component, **a**$_n$, represents the time rate of change in the velocity's direction. It is directed toward the center of the circular path.

RIGID-BODY ROTATION: ACCELERATION OF POINT P



Using the vector formulation, the acceleration of P can also be defined by differentiating the velocity.

$$\mathbf{a} = d\boldsymbol{v}/dt = d\boldsymbol{\omega}/dt \times \boldsymbol{r}_P + \boldsymbol{\omega} \times d\boldsymbol{r}_P/dt$$

$$= \boldsymbol{\alpha} \times \boldsymbol{r}_P + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \boldsymbol{r}_P)$$

It can be shown that this equation reduces to

$$\mathbf{a} = \boldsymbol{\alpha} \times \boldsymbol{r} - \omega^2 \boldsymbol{r} = \mathbf{a}_t + \mathbf{a}_n$$

The magnitude of the acceleration vector is $a = \sqrt{(a_t)^2 + (a_n)^2}$

ROTATION ABOUT A FIXED AXIS: PROCEDURE

- Establish a sign convention along the axis of rotation.
- If a relationship is known between any two of the variables (α, ω, θ, or t), the other variables can be determined from the equations:

$$\omega = d\theta/dt$$

$$\alpha = d\omega/dt$$

$$\alpha \, d\theta = \omega \, d\omega$$

- If α is constant, use the equations for constant angular acceleration.

$$\omega = \omega_0 + \alpha_C \, t$$

$$(\theta - \theta_0) = \omega_0 t + 0.5 \; \alpha_C \, t^2$$

$$\omega^2 = \omega_0^2 + 2\alpha_C \, (\theta - \theta_0)$$

- To determine the motion of a point, the scalar equations $v = \omega r$, $a_t = \alpha r$, $a_n = \omega^2 r$, and $a = \sqrt{(a_t)^2 + (a_n)^2}$ can be used.
- Alternatively, the vector form of the equations can be used (with ***i***, ***j***, ***k*** components).

$$\boldsymbol{v} = \boldsymbol{\omega} \times \boldsymbol{r}_P = \boldsymbol{\omega} \times \boldsymbol{r}$$

$$\mathbf{a} = \mathbf{a}_t + \mathbf{a}_n = \boldsymbol{\alpha} \times \boldsymbol{r} - \omega^2 \boldsymbol{r}$$

EXAMPLE

Given: The motor gives the blade an angular acceleration $\alpha = 20\ e^{-0.6t}$ rad/s^2, where t is in seconds. The initial conditions are that when t = 0, the blade is at rest.

Find: The velocity and acceleration of the tip P of one of the blades when t =3 s. How many revolutions has the blade turned in 3 s ?

Plan:
1) Determine the angular velocity and displacement of the blade using kinematics of angular motion.
2) The magnitudes of the velocity and acceleration of point P can be determined from the scalar equations of motion for a point on a rotating body. Why scalar?

EXAMPLE (continued)

Solution:

1) Since the angular acceleration is given as a function of time, $\alpha = 20\ e^{-0.6t}$ rad/s^2, the angular velocity and displacement can be found by integration.

$$\omega = \int \alpha\ dt = 20 \int e^{-0.6t}\ dt$$

$$\omega = \frac{20}{(-0.6)}\ e^{-0.6t}$$

$\Rightarrow$ **when t = 3 s,**

ω = -5.510 rad/s

Angular displacement

$$\theta = \int \omega\ dt$$

$$\theta = \frac{20}{(-0.6)} \int e^{-0.6t}\ dt = \frac{20}{(-0.6)^2}\ e^{-0.6t}$$

$\Rightarrow$ **when t = 3 s,**

θ = 9.183 rad

= 1.46 rev.

Also , when t = 3 s, $\alpha = 20\ e^{-0.6(3)\ b} = 3.306$ rad/s^2

EXAMPLE (continued)

2) The velocity of point P on the the fan, at a radius of 1.75 ft, is determined as

$$v_P = \omega\, r = (5.510)(1.75) = 9.64 \text{ ft/s}$$

The normal and tangential components of acceleration of point P are calculated as

$$a_n = (\omega)^2\, r = (5.510)^2\,(1.75) = 53.13 \text{ ft/s}^2$$

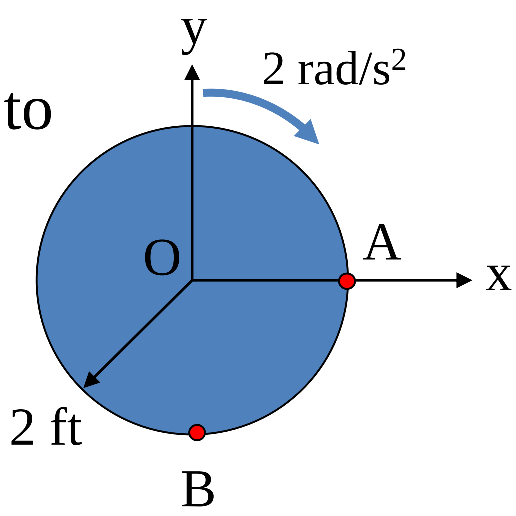
$$a_t = \alpha\, r = (3.306)(1.75) = 5.786 \text{ ft/s}^2$$

The magnitude of the acceleration of P is determined by

$$a_P = \sqrt{(a_n)^2 + (a_t)^2} = \sqrt{(53.13)^2 + (5.786)^2} = 53.4 \text{ ft/s}^2$$

CONCEPT QUIZ

1. A disk is rotating at 4 rad/s. If it is subjected to a constant angular acceleration of 2 rad/s^2, determine the acceleration at B.



A) (4 ***i*** + 32 ***j***) ft/s^2 B) (4 ***i*** - 32 ***j***) ft/s^2

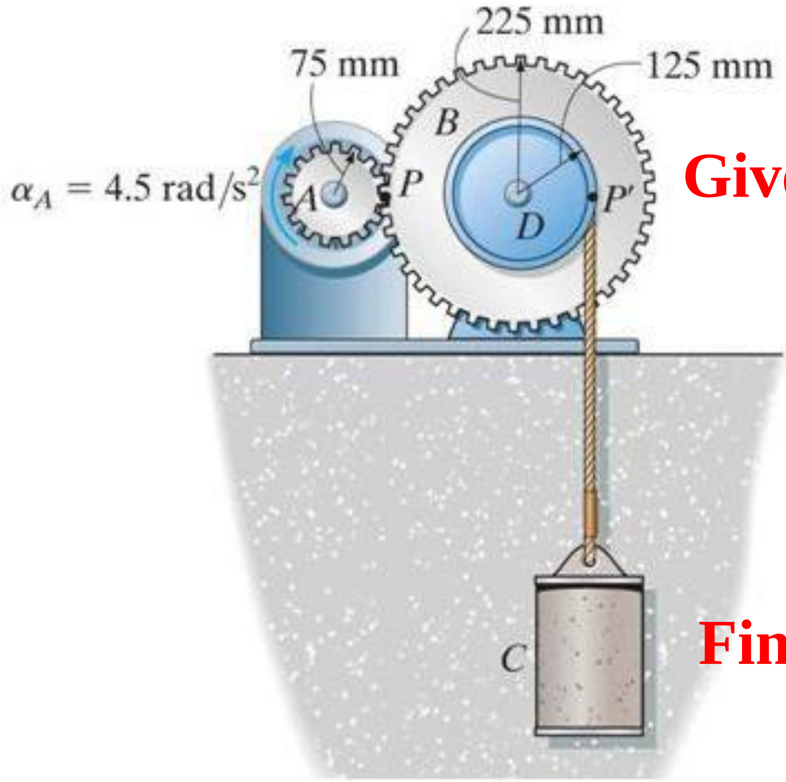
C) (- 4 ***i*** + 32 ***j***) ft/s^2 D) (- 4 ***i*** -32 ***j***) ft/s^2

$a_t = \alpha r$ $a_n = \omega^2 r.$

2. A Frisbee (صحن الطائر) is thrown and curves to the right. It is experiencing

A) rectilinear translation. B) curvilinear translation.

C) pure rotation. D) general plane motion.



GROUP PROBLEM SOLVING

Given: Starting from rest when gear A is given a constant angular acceleration, $\alpha_A = 4.5$ rad/s^2. The cord is wrapped around pulley D which is rigidly attached to gear B.

Find: The velocity of cylinder C and the distance it travels in 3 seconds.

Plan:
1) The angular acceleration of gear B (and pulley D) can be related to α_A.
2) The acceleration of cylinder C can be determined by using the equations for motion of a point on a rotating body since $(a_t)_D$ at point P is the same as a_c.
3) The velocity and distance of C can be found by using the constant acceleration equations.

GROUP PROBLEM SOLVING (continued)

Solution:

1) Gear A and B will have the **same** speed and tangential component of acceleration at the point where they mesh. Thus,

$$a_t = \alpha_A r_A = \alpha_B r_B \quad \Rightarrow \quad (4.5)(75) = \alpha_B(225) \;\Rightarrow\; \alpha_B = 1.5 \text{ rad/s}^2$$

Since gear B and pulley D turn together, $\alpha_D = \alpha_B = 1.5 \text{ rad/s}^2$

2) Assuming the cord attached to pulley D is inextensible and does not slip, the velocity and acceleration of cylinder C will be the same as the velocity and tangential component of acceleration along the pulley D:

$$a_C = (a_t)_D = \alpha_D\, r_D = (1.5)(0.125) = 0.1875 \text{ m/s}^2 \uparrow$$

GROUP PROBLEM SOLVING (continued)

3) Since α_A is constant, α_D and a_C will be constant. The constant acceleration equation for rectilinear motion can be used to determine the velocity and displacement of cylinder C when t = 3 s ($s_0 = v_0 = 0$):

$$v_c = v_0 + a_C\, t = 0 + 0.1875\,(3) = 0.563 \text{ m/s} \uparrow$$

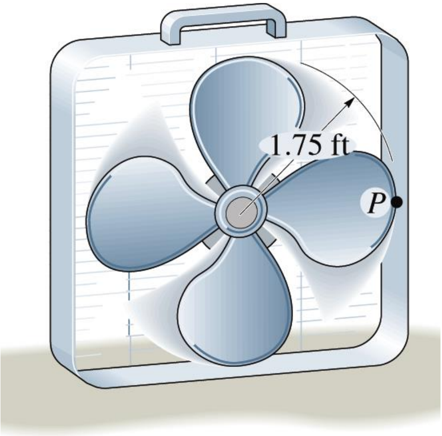
$$s_c = s_0 + v_0\, t + (0.5)\, a_C\, t^2$$
$$= 0 + 0 + (0.5)\, 0.1875\,(3)^2 = 0.844 \text{ m} \uparrow$$

ATTENTION QUIZ

1. The fan blades suddenly experience an angular acceleration of 2 rad/s^2. If the blades are rotating with an initial angular velocity of 4 rad/s, determine the speed of point P when the blades have turned 2 revolutions (when ω = 8.14 rad/s).

   A) 14.2 ft/s   B) 17.7 ft/s

   C) 23.1 ft/s   D) 26.7 ft/s



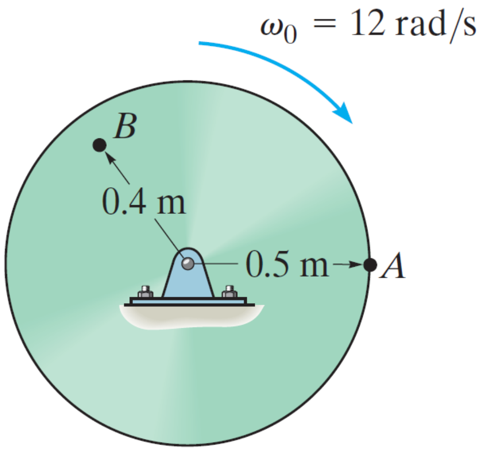
2. Determine the magnitude of the acceleration at P when the blades have turned the 2 revolutions.

   A) 0 ft/s^2 B) 3.5 ft/s^2

   C) 115.95 ft/s^2 D) 116 ft/s^2

EXAMPLE

The disk is originally rotating at $\omega_0 = 12 \text{ rad/s}$. If it is subjected to a constant angular acceleration of $\alpha = 20 \text{ rad/s}^2$, determine the magnitudes of the velocity and the n and t components of acceleration of point B when the disk undergoes 2 revolutions.



SOLUTION

Angular Motion. The angular velocity of the disk can be determined using

$$\omega^2 = \omega_0^2 + 2\alpha_c(\theta - \theta_0); \qquad \omega^2 = 12^2 + 2(20)[2(2\pi) - 0]$$

$$\omega = 25.43 \text{ rad/s}$$

Motion of Point B. The magnitude of the velocity is

$$v_B = \omega r_B = 25.43(0.4) = 10.17 \text{ m/s} = 10.2 \text{ m/s} \qquad \textbf{Ans.}$$

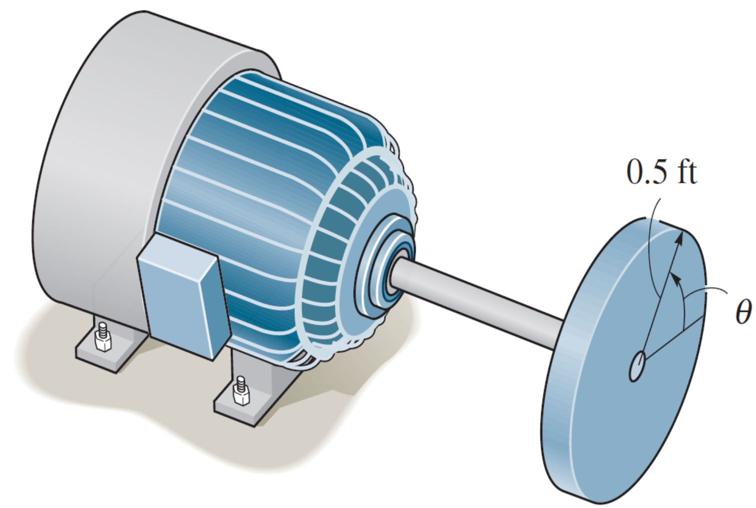
The tangential and normal components of acceleration are

$$(a_B)_t = \alpha r_B = 20(0.4) = 8.00 \text{ m/s}^2 \qquad \textbf{Ans.}$$

$$(a_B)_n = \omega^2 r_B = (25.43^2)(0.4) = 258.66 \text{ m/s}^2 = 259 \text{ m/s}^2 \qquad \textbf{Ans.}$$

EXAMPLE

The disk is driven by a motor such that the angular position of the disk is defined by $\theta = (20t + 4t^2)$ rad, where t is in seconds. Determine the number of revolutions, the angular velocity, and angular acceleration of the disk when $t = 90$ s.



SOLUTION

Angular Displacement: At $t = 90$ s.

$$\theta = 20(90) + 4\left(90^2\right) = (34200 \text{ rad}) \times \left(\frac{1 \text{ rev}}{2\pi \text{ rad}}\right) = 5443 \text{ rev}$$ **Ans.**

Angular Velocity: Applying Eq. 16–1. we have

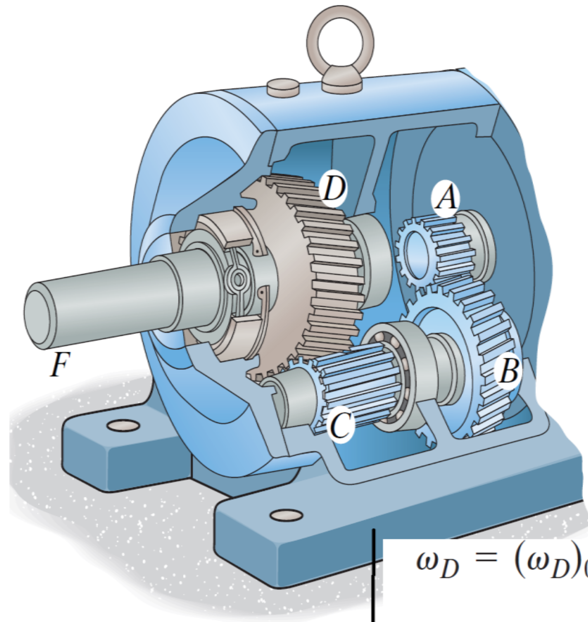
$$\omega = \frac{d\theta}{dt} = 20 + 8t\Big|_{t=90\ s} = 740 \text{ rad/s}$$ **Ans.**

Angular Acceleration: Applying Eq. 16–2. we have

$$\alpha = \frac{d\omega}{dt} = 8 \text{ rad/s}^2$$ **Ans.**

EXAMPLE

If gear A rotates with a constant angular acceleration of $\alpha_A = 90\ \text{rad/s}^2$, starting from rest, determine the time required for gear D to attain an angular velocity of 600 rpm. Also, find the number of revolutions of gear D to attain this angular velocity. Gears A, B, C, and D have radii of 15 mm, 50 mm, 25 mm, and 75 mm, respectively.



SOLUTION

Gear B is in mesh with gear A. Thus,

$$\alpha_B r_B = \alpha_A r_A$$

$$\alpha_B = \left(\frac{r_A}{r_B}\right)\alpha_A = \left(\frac{15}{50}\right)(90) = 27\ \text{rad/s}^2$$

Since gears C and B share the same shaft, $\alpha_C = \alpha_B = 27\ \text{rad/s}^2$. Also, gear D is in mesh with gear C. Thus,

$$\alpha_D r_D = \alpha_C r_C$$

$$\alpha_D = \left(\frac{r_C}{r_D}\right)\alpha_C = \left(\frac{25}{75}\right)(27) = 9\ \text{rad/s}^2$$

The final angular velocity of gear D is $\omega_D = \left(\frac{600\ \text{rev}}{\text{min}}\right)\left(\frac{2\pi\ \text{rad}}{1\ \text{rev}}\right)\left(\frac{1\ \text{min}}{60\ \text{s}}\right) = 20\pi\ \text{rad/s}$. Applying the constant acceleration equation,

$$\omega_D = (\omega_D)_0 + \alpha_D t$$

$$20\pi = 0 + 9t$$

$$t = 6.98\ \text{s}$$

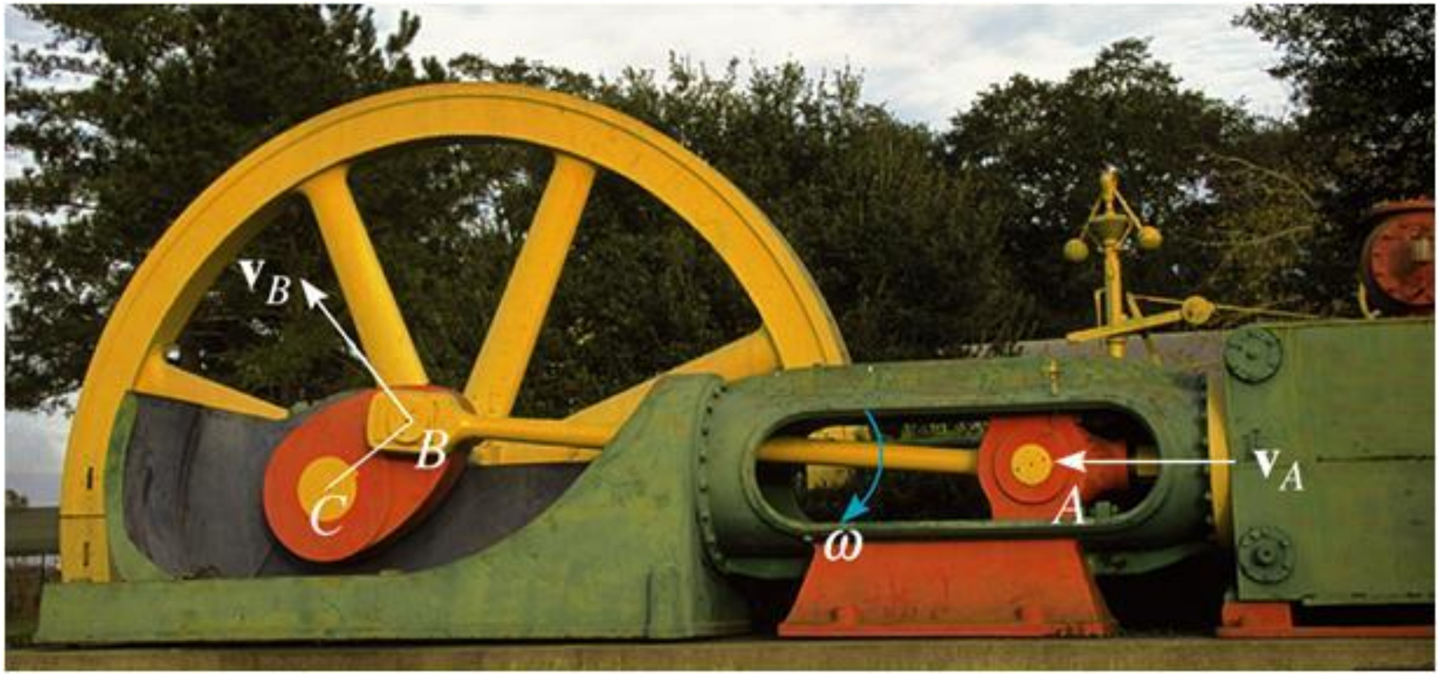
$$\omega_D{}^2 = (\omega_D)_0{}^2 + 2\alpha_D[\theta_D - (\theta_D)_0]$$

$$(20\pi)^2 = 0^2 + 2(9)(\theta_D - 0)$$

$$\theta_D = (219.32\ \text{rad})\left(\frac{1\ \text{rev}}{2\pi\ \text{rad}}\right)$$

$$= 34.9\ \text{rev}$$

Type 3 : *RELATIVE MOTION ANALYSIS: VELOCITY*



<u>Today's Objectives</u>:

Students will be able to:

1. *Describe the velocity of a rigid body in terms of translation and rotation components.*
2. *Perform a relative-motion velocity analysis of a point on the body.*

READING QUIZ

1. *When a relative-motion analysis involving two sets of coordinate axes is used, the x' - y' coordinate system will*

   A) *be attached to the selected point for analysis.*

   B) *rotate with the body.*

   C) *not be allowed to translate with respect to the fixed frame.*

   D) *None of the above.*

2. *In the relative velocity equation,* $\mathbf{v}_{B/A}$ *is*

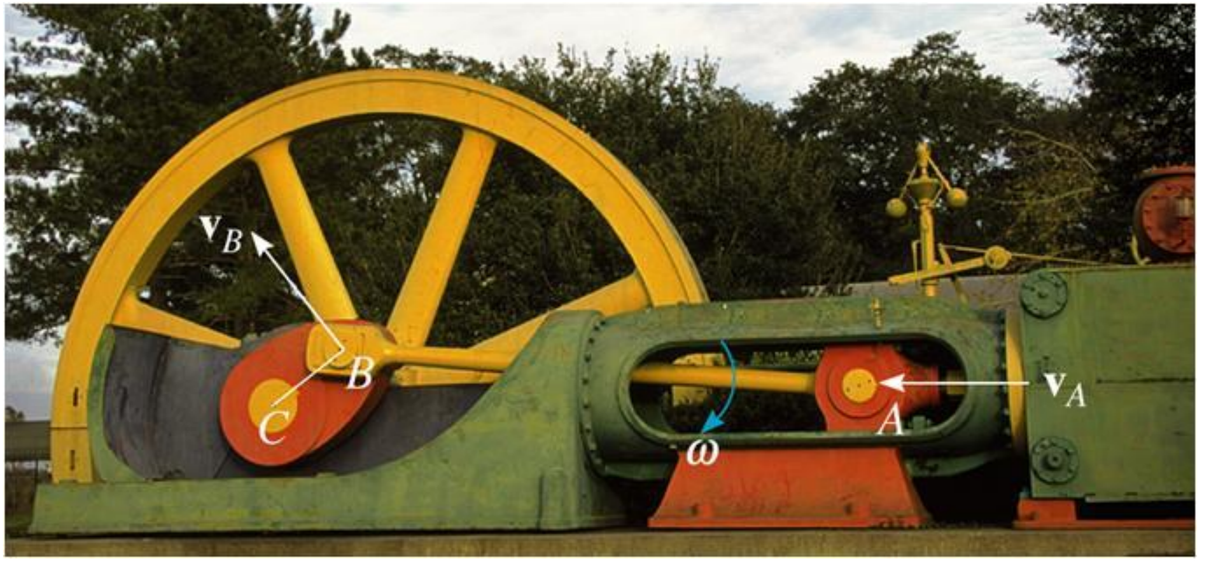
   A) *the relative velocity of B with respect to A.*

   B) *due to the rotational motion.*

   C) $\boldsymbol{\omega} \times \mathbf{r}_{B/A}$.

   D) *All of the above.*

APPLICATIONS

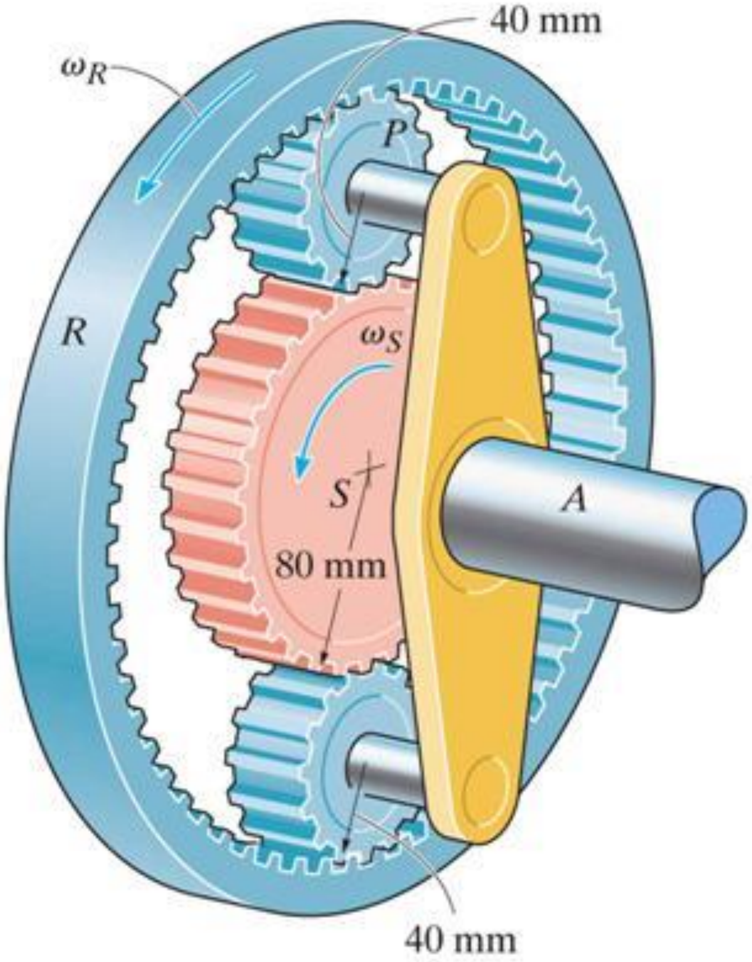


As the slider block A moves horizontally to the left with $\mathbf{v}_A$*, it causes the link CB to rotate counterclockwise. Thus* $\mathbf{v}_B$ *is directed tangent to its circular path.*

Which link is undergoing general plane motion? Link AB or link BC?

How can the angular velocity, ω *of link AB be found?*

APPLICATIONS
(continued)

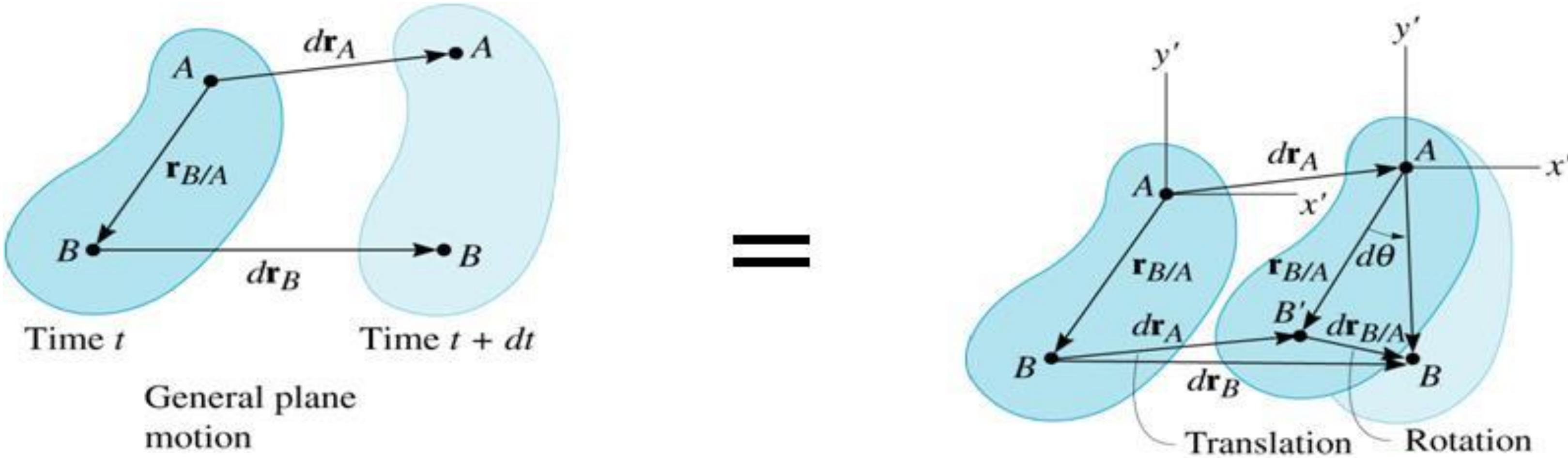


Planetary gear systems are used in many automobile automatic transmissions. By locking or releasing different gears, this system can operate the car at different speeds.

How can we relate the angular velocities of the various gears in the system?

RELATIVE MOTION ANALYSIS (Section 16.5)

When a body is subjected to general plane motion, it undergoes a combination of translation and rotation.



Point A is called the base point in this analysis. It generally has a known motion. The x'- y' frame translates with the body, but does not rotate. The displacement of point B can be written:

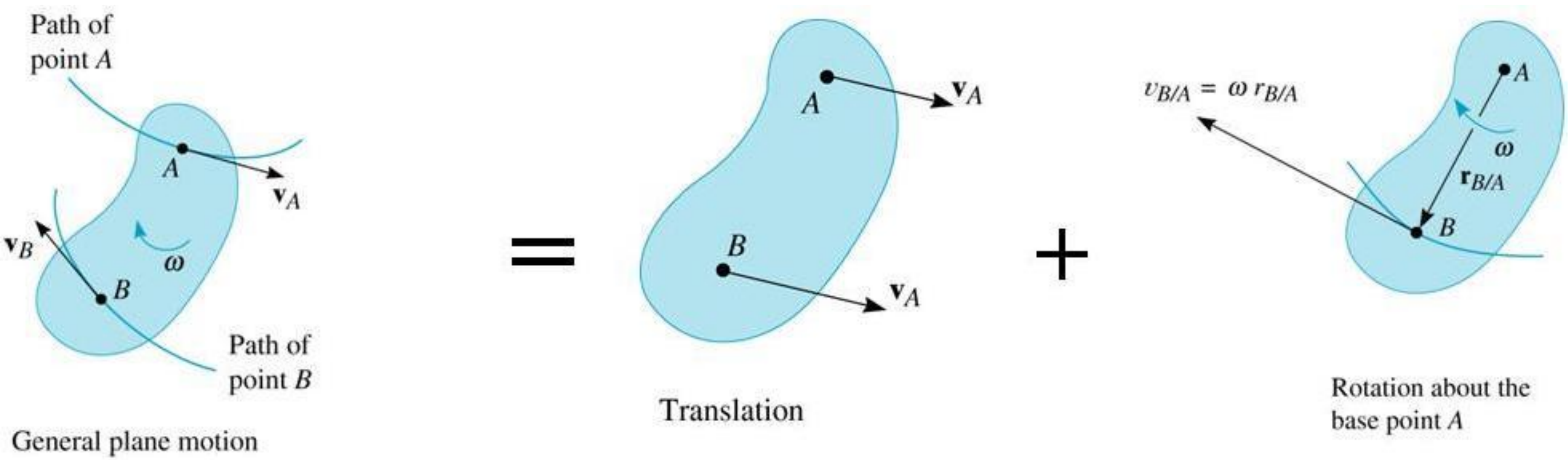
$$d\mathbf{r}_B = d\mathbf{r}_A + d\mathbf{r}_{B/A}$$

Disp. due to translation ($d\mathbf{r}_A$)

Disp. due to translation and rotation ($d\mathbf{r}_B$)

Disp. due to rotation ($d\mathbf{r}_{B/A}$)

RELATIVE MOTION ANALYSIS: VELOCITY



The velocity at B is given as : $(d\mathbf{r}_B/dt) = (d\mathbf{r}_A/dt) + (d\mathbf{r}_{B/A}/dt)$ *or*

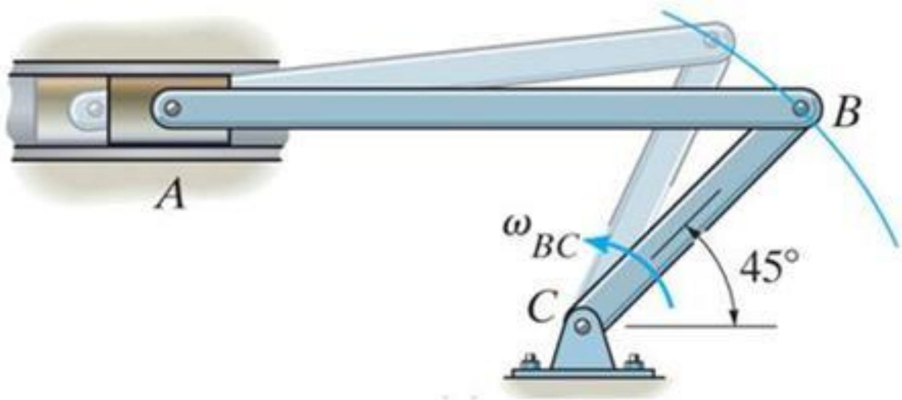
$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$$

Since the body is taken as rotating about A,

$$\mathbf{v}_{B/A} = d\mathbf{r}_{B/A}/dt = \boldsymbol{\omega} \times \mathbf{r}_{B/A}$$

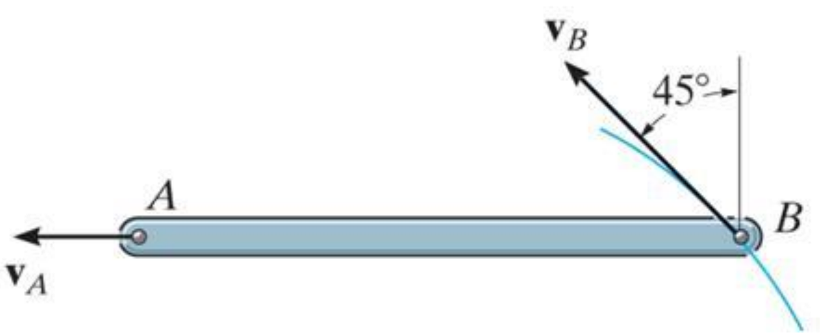
Here $\boldsymbol{\omega}$ *will only have a* $\mathbf{k}$ *component since the axis of rotation is perpendicular to the plane of translation.*

RELATIVE MOTION ANALYSIS: VELOCITY



$$\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\omega} \times \mathbf{r}_{B/A}$$

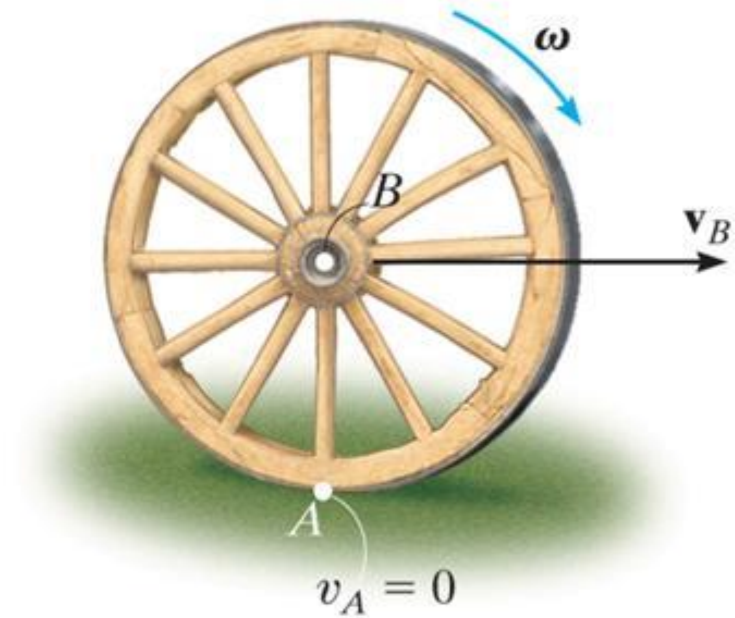
When using the relative velocity equation, points A and B should generally be points on the body with a known motion. Often these points are pin connections in linkages.



For example, point A on link AB must move along a horizontal path, whereas point B moves on a circular path.

The directions of v_A and v_B are known since they are always tangent to their paths of motion.

RELATIVE MOTION ANALYSIS: VELOCITY



$$\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\omega} \times \mathbf{r}_{B/A}$$

When a wheel rolls without slipping, point A is often selected to be at the point of contact with the ground.

Since there is no slipping, point A has zero velocity.

Furthermore, point B at the center of the wheel moves along a horizontal path. Thus, $\mathbf{v}_B$ has a known direction, e.g., parallel to the surface.

PROCEDURE FOR ANALYSIS

The relative velocity equation can be applied using either a Cartesian vector analysis or by writing scalar x and y component equations directly.

Scalar Analysis:

1. *Establish the fixed x-y coordinate directions and draw a kinematic diagram for the body. Then establish the magnitude and direction of the relative velocity vector* $\mathbf{v}_{B/A}$.
2. *Write the equation* $\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$. *In the kinematic diagram, represent the vectors graphically by showing their magnitudes and directions underneath each term.*
3. *Write the scalar equations from the x and y components of these graphical representations of the vectors. Solve for the unknowns.*

PROCEDURE FOR ANALYSIS (continued)

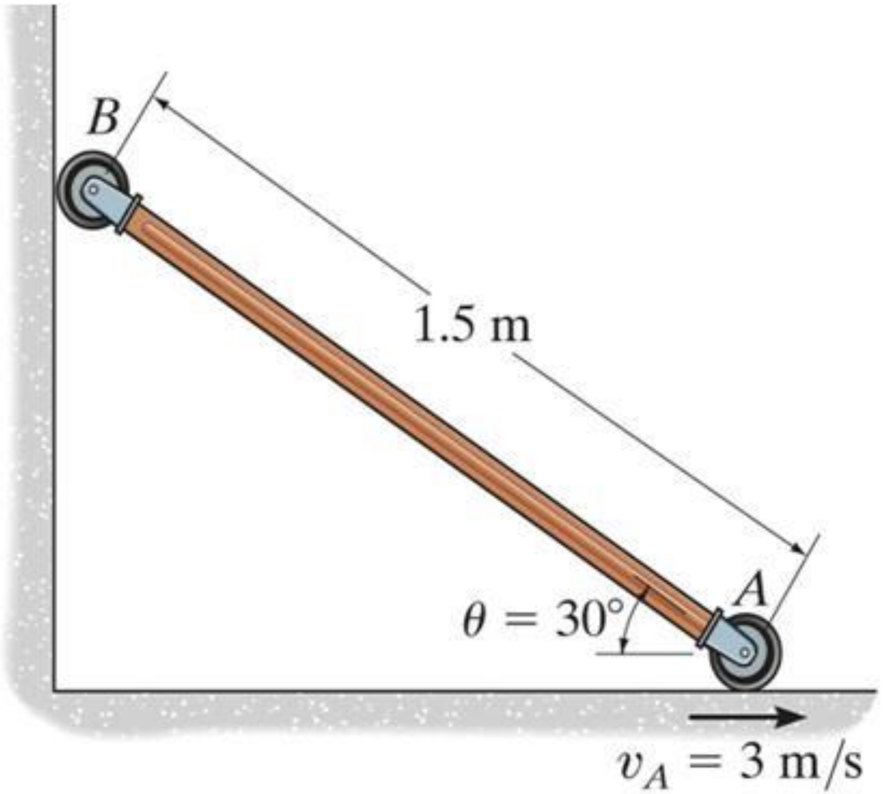
Vector Analysis:

1. *Establish the fixed x - y coordinate directions and draw the kinematic diagram of the body, showing the vectors* $\mathbf{v}_A$, $\mathbf{v}_B$, $\mathbf{r}_{B/A}$ *and* $\boldsymbol{\omega}$. *If the magnitudes are unknown, the sense of direction may be assumed.*

2. *Express the vectors in Cartesian vector form (CVN) and substitute them into* $\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\omega} \times \mathbf{r}_{B/A}$. *Evaluate the cross product and equate respective* $\mathbf{i}$ *and* $\mathbf{j}$ *components to obtain two scalar equations.*

3. *If the solution yields a negative answer, the sense of direction of the vector is opposite to that assumed.*

EXAMPLE I



Given: *Roller A is moving to the right at 3 m/s.*

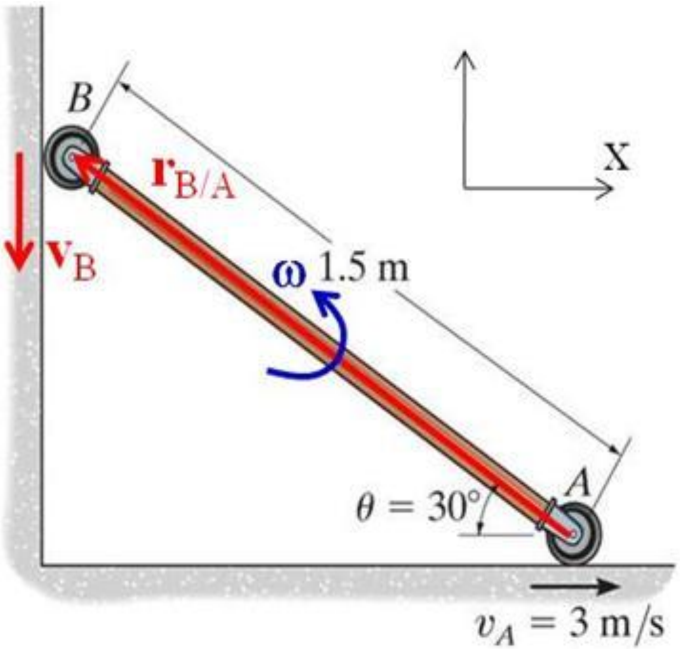
Find: *The velocity of B at the instant $\theta = 30°$.*

Plan:

1. *Establish the fixed x - y directions and draw a kinematic diagram of the bar and rollers.*
2. *Express each of the velocity vectors for A and B in terms of their **i**, **j**, **k** components and solve $\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\omega} \times \mathbf{r}_{B/A}$.*

Solution:

Kinematic diagram:



Express the velocity vectors in CVN

$$\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\omega} \times \mathbf{r}_{B/A}$$

$$-v_B\,\mathbf{j} = 3\,\mathbf{i} + [\,\omega\mathbf{k} \times (-1.5\cos 30\,\mathbf{i} + 1.5\sin 30\,\mathbf{j}\,)]$$

$$-v_B\,\mathbf{j} = 3\,\mathbf{i} - 1.299\,\omega\,\mathbf{j} - 0.75\,\omega\,\mathbf{i}$$

Equating the ***i*** *and* ***j*** *components gives:*

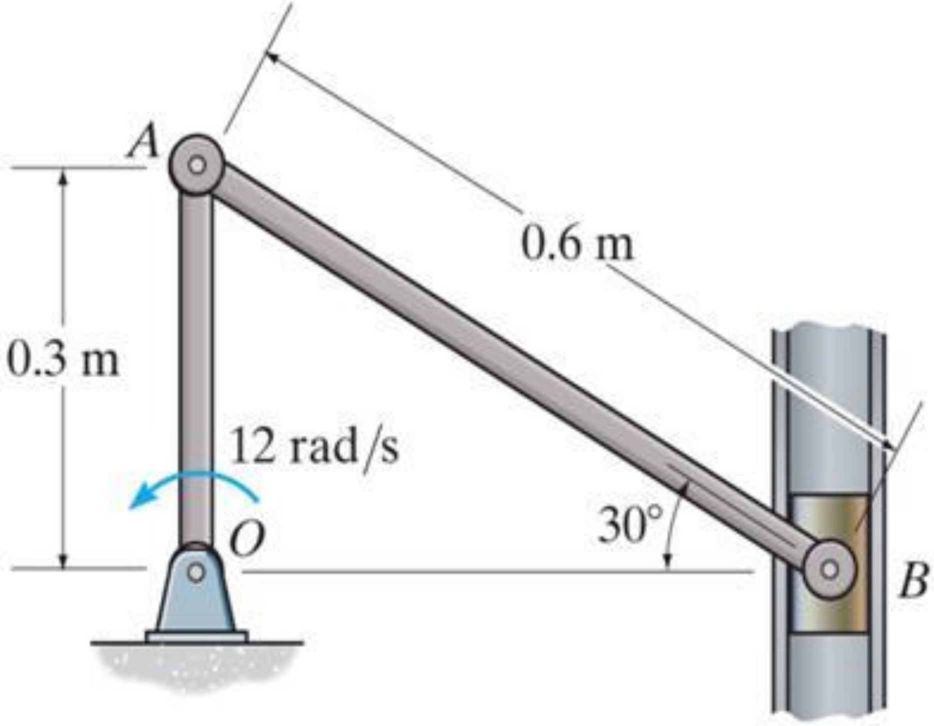
$$0 = 3 - 0.75\,\omega$$

$$-v_B = -1.299\,\omega$$

Solving: $\omega = 4$ rad/s or $\boldsymbol{\omega} = 4$ rad/s $\mathbf{k}$

$v_B = 5.2$ m/s or $\mathbf{v}_B = -5.2$ m/s $\mathbf{j}$

EXAMPLE II



Given: *Crank rotates OA with an angular velocity of 12 rad/s.*

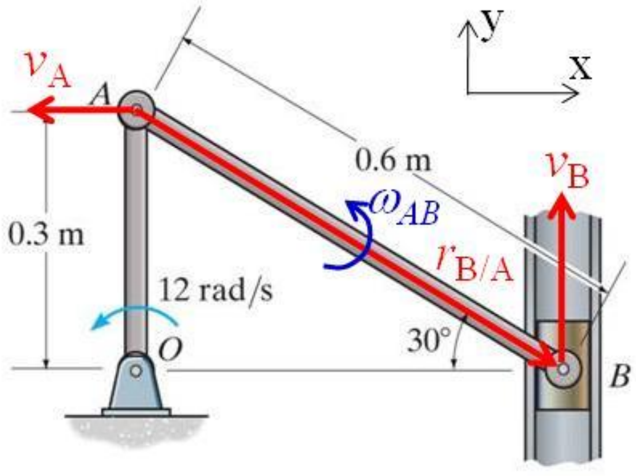
Find: *The velocity of piston B and the angular velocity of rod AB.*

Plan: *Notice that point A moves on a circular path. The directions of* v_A *is tangent to its path of motion. Draw a kinematic diagram of rod AB and use*

$$\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A}.$$

Solution:

Kinematic diagram of AB:



Since crack OA rotates with an angular velocity of 12 rad/s, the velocity at A will be: $\mathbf{v}_A = -0.3(12)\,\mathbf{i} = -3.6\,\mathbf{i}$ m/s

Rod AB. Write the relative-velocity equation:

$$\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A}$$

$$\mathbf{v}_B\,\mathbf{j} = -3.6\,\mathbf{i} + \omega_{AB}\,\mathbf{k} \times (0.6\cos 30\,\mathbf{i} - 0.6\sin 30\,\mathbf{j})$$

$$\mathbf{v}_B\,\mathbf{j} = -3.6\,\mathbf{i} + 0.5196\,\omega_{AB}\,\mathbf{j} + 0.3\,\omega_{AB}\,\mathbf{i}$$

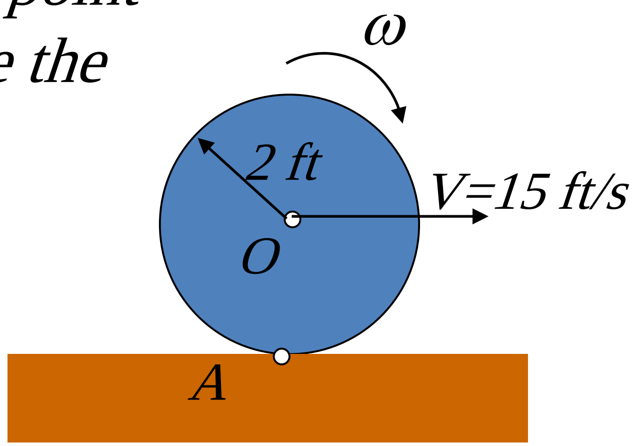
By comparing the **i**, **j** *components:*

i: $0 = -3.6 + 0.3\,\omega_{AB} \quad \Rightarrow \quad \omega_{AB} = 12$ rad/s

j: $v_B = 0.5196\,\omega_{AB} \quad \Rightarrow \quad v_B = 6.24$ m/s

CHECK YOUR UNDERSTANDING QUIZ

1. *If the disk is moving with a velocity at point O of 15 ft/s and ω = 2 rad/s, determine the velocity at A.*



A) 0 ft/s *B) 4 ft/s*

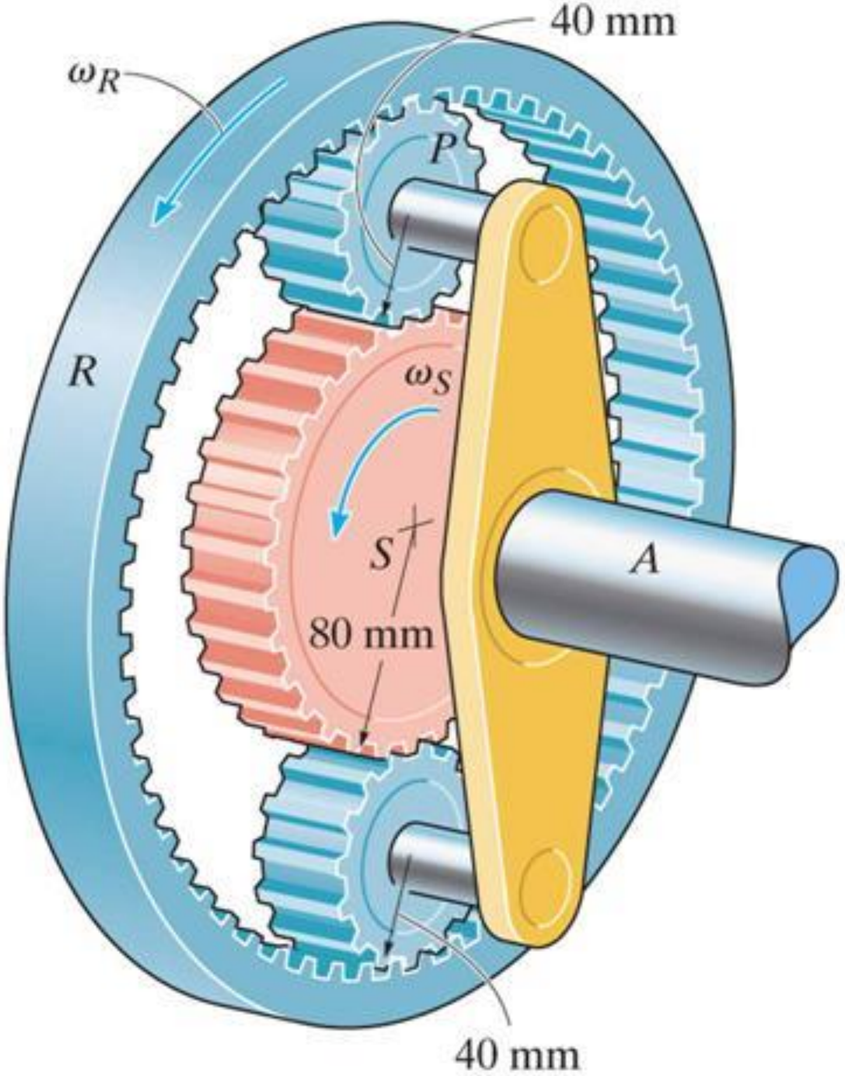
C) 15 ft/s *D) 11 ft/s*

2. *If the velocity at A is zero, then determine the angular velocity, ω.*

A) 30 rad/s *B) 0 rad/s*

C) 7.5 rad/s *D) 15 rad/s*

GROUP PROBLEM SOLVING



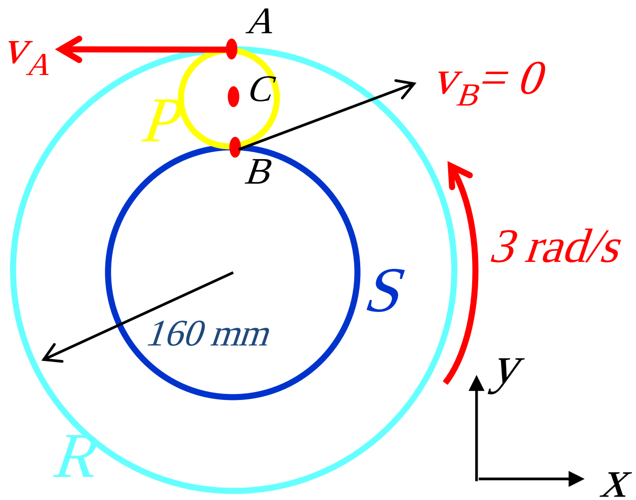
Given: *The ring gear R is rotating at $\omega_R = 3$ rad/s, and the sun gear S is held fixed, $\omega_S = 0$.*

Find: *The angular velocity of the each of the planet gears P and of shaft A.*

Plan: *Draw the kinematic diagram of gears. Then, apply the relative velocity equations to the gears and solve for unknowns.*

Solution:

Kinematic diagram of gears.



Since the ring gear R is rotating at $\omega_R = 3$ rad/s, the velocity at point A will be ;

$$\mathbf{v}_A = -3(160)\,\mathbf{i} = -480\,\mathbf{i} \text{ mm/s}$$

Also note that $\mathbf{v}_B = 0$ since the gear R is held fixed $\omega_S = 0$.

Applying the relative velocity equation to points A and B;

$$\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\omega}_P \times \mathbf{r}_{B/A}$$

$$0 = -480\,\mathbf{i} + (\omega_P\,\mathbf{k}) \times (-80\,\mathbf{j}) \Rightarrow 0 = -480\,\mathbf{i} + 80\,\omega_P\,\mathbf{i}$$

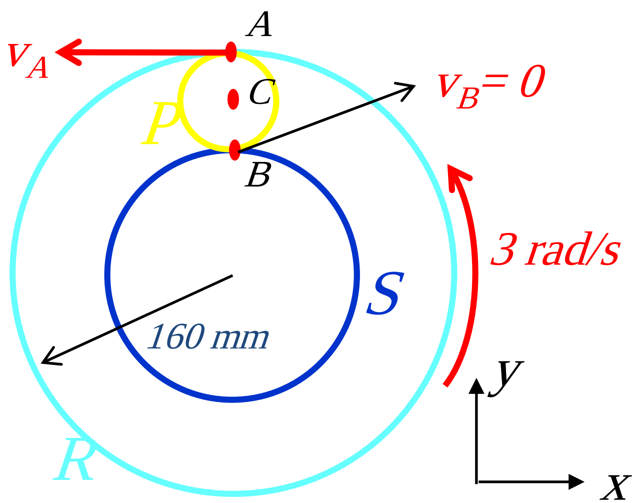
$$\omega_P = 6 \text{ rad/s}$$

Solution:

Apply the relative velocity equation at point B and C to Gear P in order to find the velocity at B.

$$\mathbf{v}_C = \mathbf{v}_B + \boldsymbol{\omega}_P \times \mathbf{r}_{C/B}$$

$$= 0 + (6\,\mathbf{k}) \times (40\,\mathbf{j}) = -\,240\,\mathbf{i} \text{ mm/s}$$



Note that the shaft A has a circular motion with the radius of 120 mm.

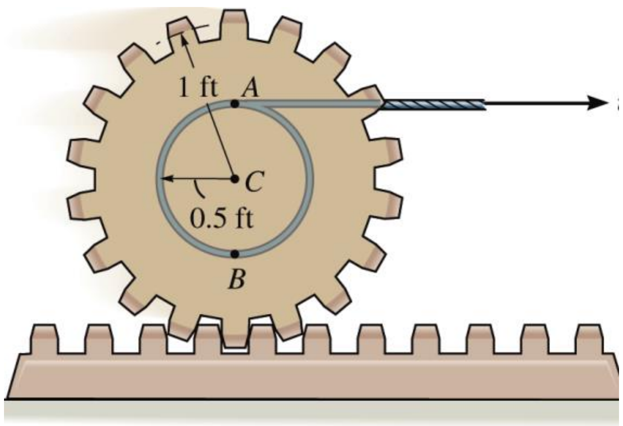
The angular velocity of the shaft is

$$\omega_A = v_C / r$$

$$= -240 / 120 = -2 \text{ rad/s.}$$

The shaft A is rotating in counter-clockwise direction !

ATTENTION QUIZ



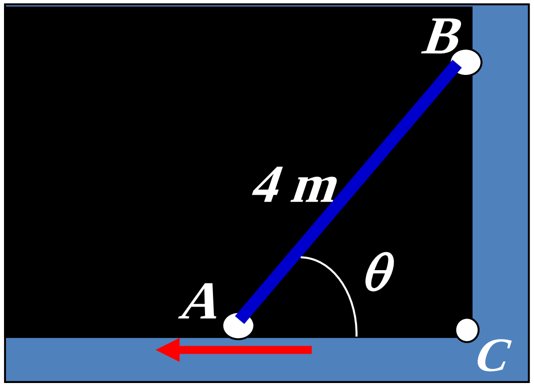
1. *Which equation could be used to find the velocity of the center of the gear, C, if the velocity* $\mathbf{v}_A$ *is known?*

A) $\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\omega}_{gear} \times \mathbf{r}_{B/A}$

B) $\mathbf{v}_A = \mathbf{v}_C + \boldsymbol{\omega}_{gear} \times \mathbf{r}_{A/C}$

C) $\mathbf{v}_B = \mathbf{v}_C + \boldsymbol{\omega}_{gear} \times \mathbf{r}_{C/B}$

D) $\mathbf{v}_A = \mathbf{v}_C + \boldsymbol{\omega}_{gear} \times \mathbf{r}_{C/A}$



2. *If the bar's velocity at A is 3 m/s, what "base" point (first term on the RHS of the velocity equation) would be best used to simplify finding the bar's angular velocity when* $\theta = 60^o$*?*

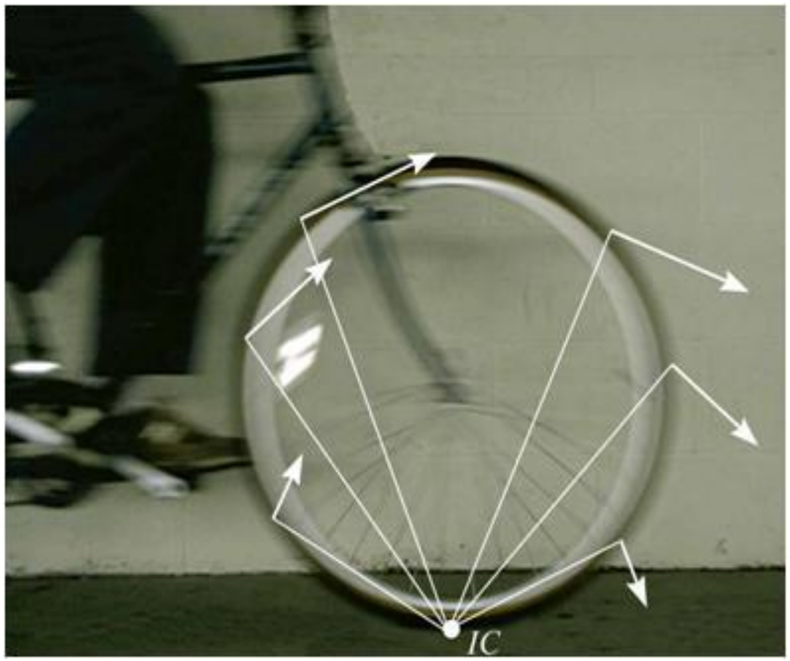
A) A

B) B

C) C

D) No difference.

INSTANTANEOUS CENTER OF ZERO VELOCITY



Objectives:

Students will be able to:

1. **Locate the instantaneous center of zero velocity.**
2. **Use the instantaneous center to determine the velocity of any point on a rigid body in general plane motion.**

READING QUIZ

1. *If applicable, the method of instantaneous center can be used to determine the __________ of any point on a rigid body.*

   A) *velocity*

   B) *acceleration*

   C) *velocity and acceleration*

   D) *force*

2. *The velocity of any point on a rigid body is __________ to the relative position vector extending from the IC to the point.*

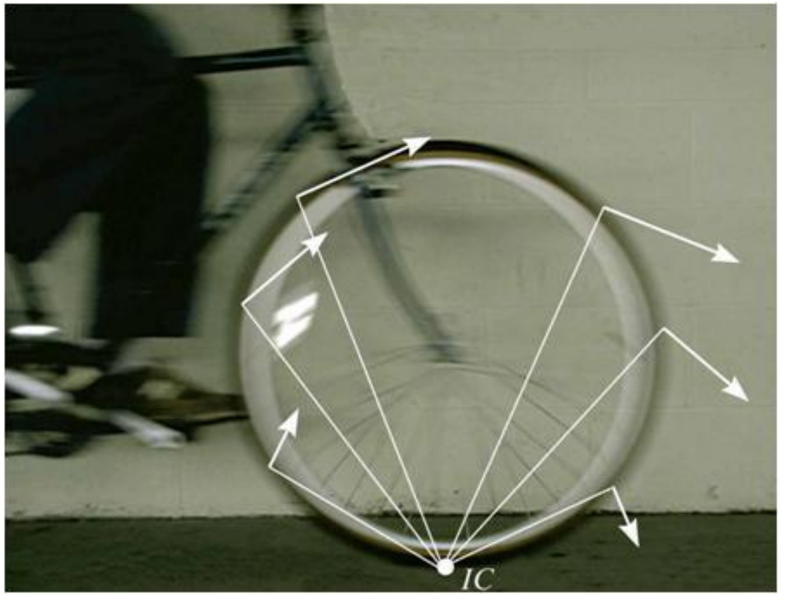
   A) *always parallel*

   B) *always perpendicular*

   C) *in the opposite direction*

   D) *in the same direction*

APPLICATIONS

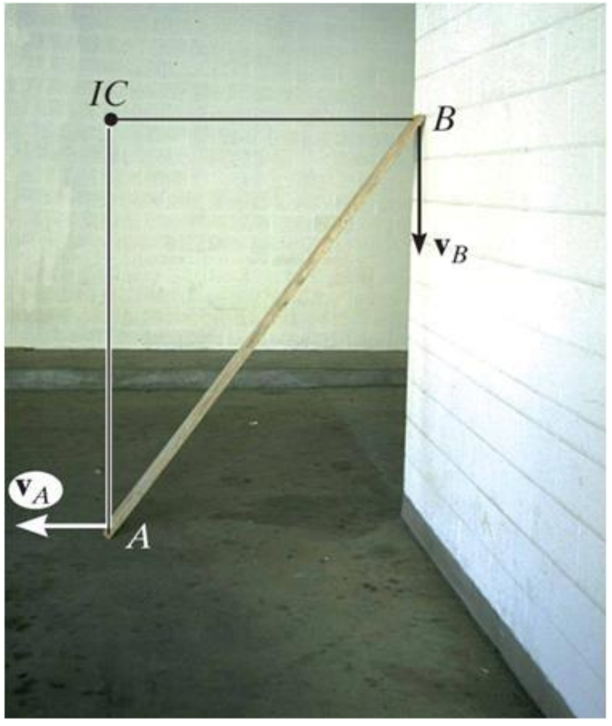


The instantaneous center (IC) of zero velocity for this bicycle wheel is at the point in contact with ground. The velocity direction at any point on the rim is perpendicular to the line connecting the point to the IC.

Which point on the wheel has the maximum velocity?

Does a larger wheel mean the bike will go faster for the same rider effort in pedaling than a smaller wheel?

APPLICATIONS (continued)



As the board slides down the wall (to the left), it is subjected to general plane motion (both translation and rotation).

Since the directions of the velocities of ends A and B are known, the IC is located as shown.

How can this result help you analyze other situations?

What is the direction of the velocity of the center of gravity of the board?

INSTANTANEOUS CENTER OF ZERO VELOCITY (Section 16-6)

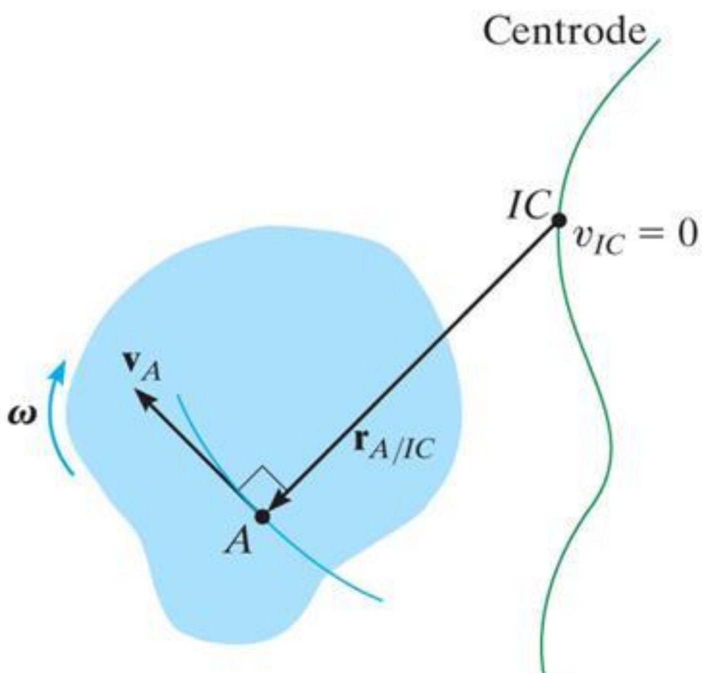
For any body undergoing planar motion, there always exists a point in the plane of motion at which the velocity is instantaneously zero (if it is rigidly connected to the body).

This point is called the instantaneous center (IC) of zero velocity. It may or may not lie on the body!

If the location of this point can be determined, the velocity analysis can be simplified because the body appears to rotate about this point at that instant.

LOCATION OF THE INSTANTANEOUS CENTER

To locate the IC, we can use the fact that the velocity of a point on a body is always perpendicular to the relative position vector from the IC to the point. Several possibilities exist.



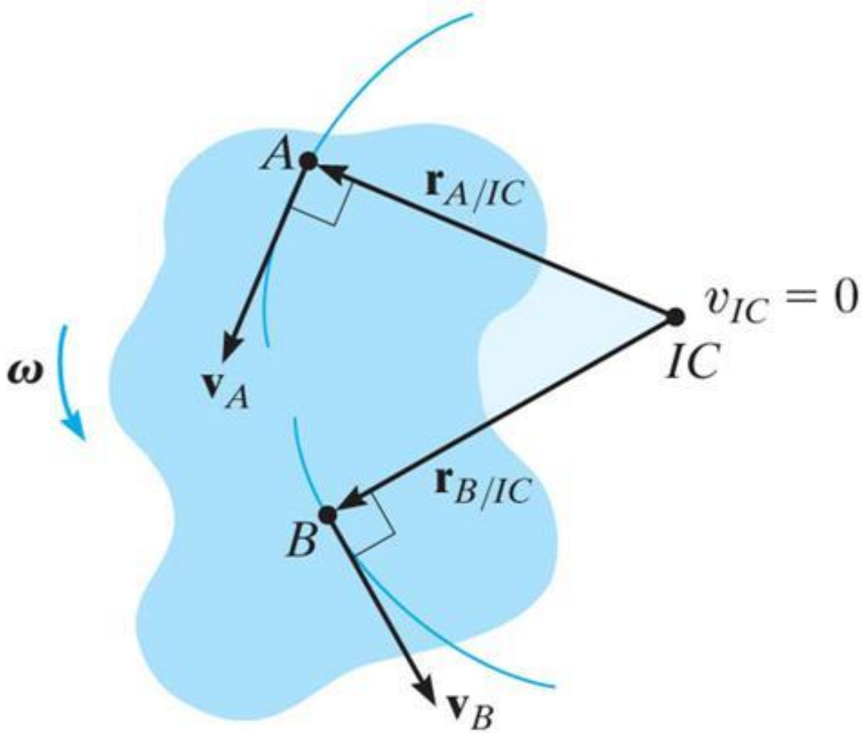
Location of *IC* knowing $\mathbf{v}_A$ and ω

First, consider the case when velocity v_A of a point A on the body and the angular velocity ω of the body are known.

In this case, the IC is located along the line drawn perpendicular to v_A at A, a distance $r_{A/IC} = v_A/\omega$ from A.

Note that the IC lies up and to the right of A since v_A must cause a clockwise angular velocity ω about the IC.

LOCATION OF THE INSTANTANEOUS CENTER (continued)

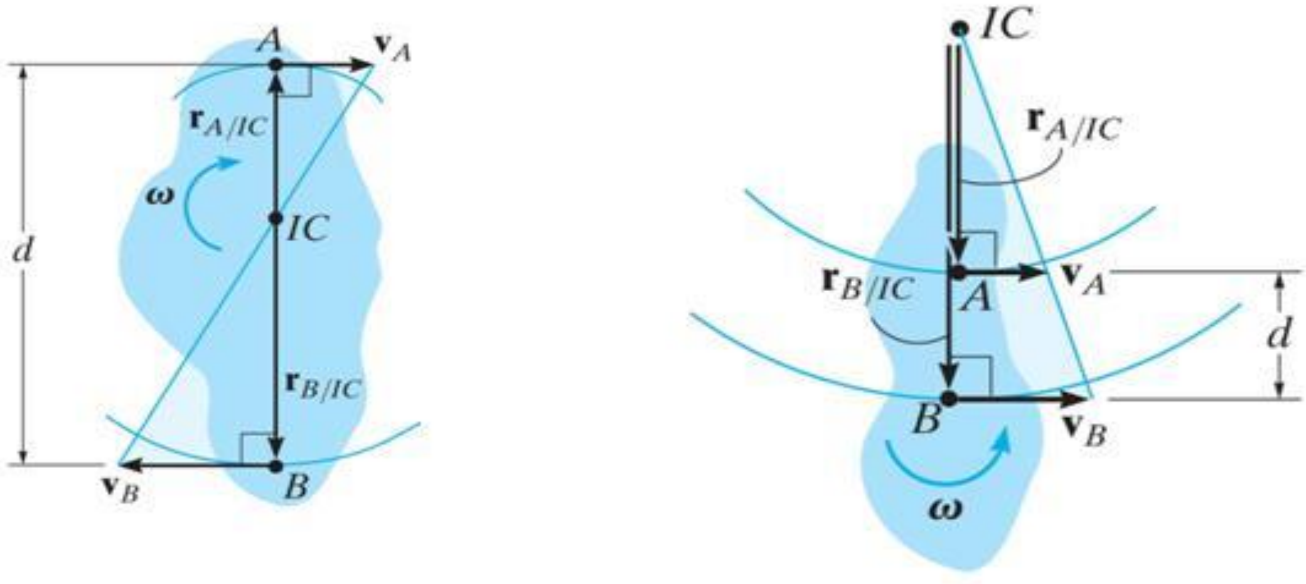


Location of *IC* knowing the directions of $\mathbf{v}_A$ and $\mathbf{v}_B$

A second case is when the lines of action of two non-parallel velocities, v_A and v_B, are known.

First, construct line segments from A and B perpendicular to v_A and v_B. The point of intersection of these two line segments locates the IC of the body.

LOCATION OF THE INSTANTANEOUS CENTER



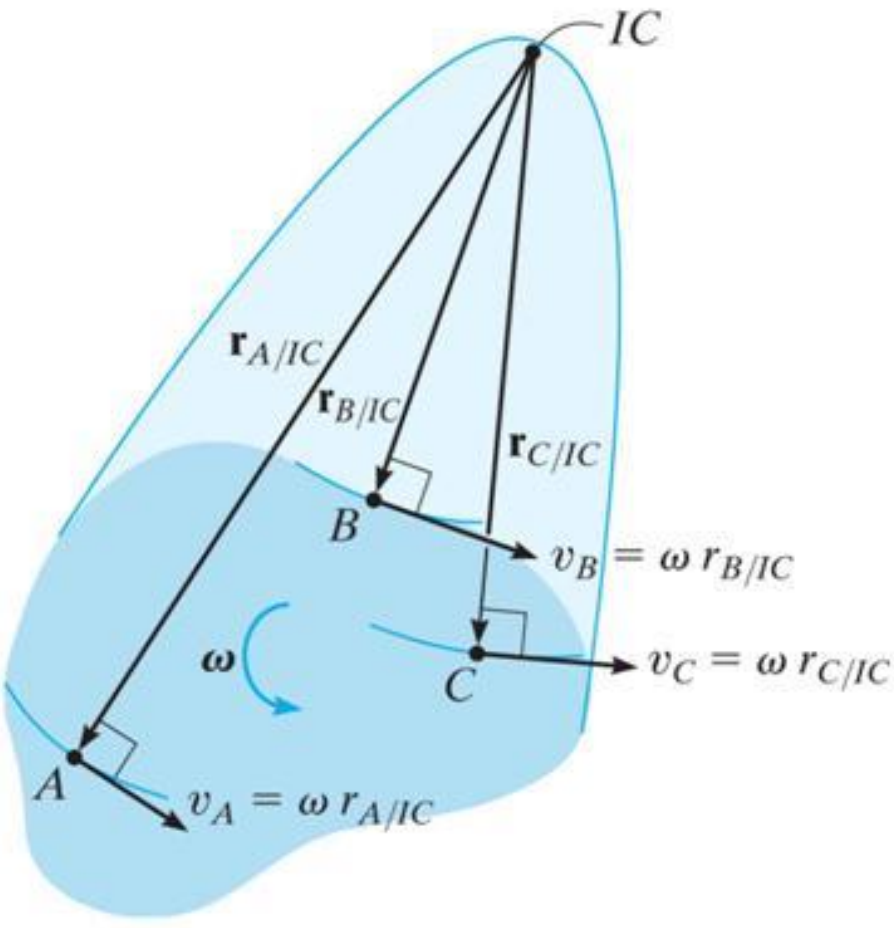
Location of *IC* knowing $\mathbf{v}_A$ and $\mathbf{v}_B$

A third case is when the magnitude and direction of two parallel velocities at A and B are known. Here the location of the IC is determined by proportional triangles.

As a special case, note that if the body is translating only ($v_A = v_B$), then the IC would be located at infinity. Then ω equals zero, as expected.

VELOCITY ANALYSIS

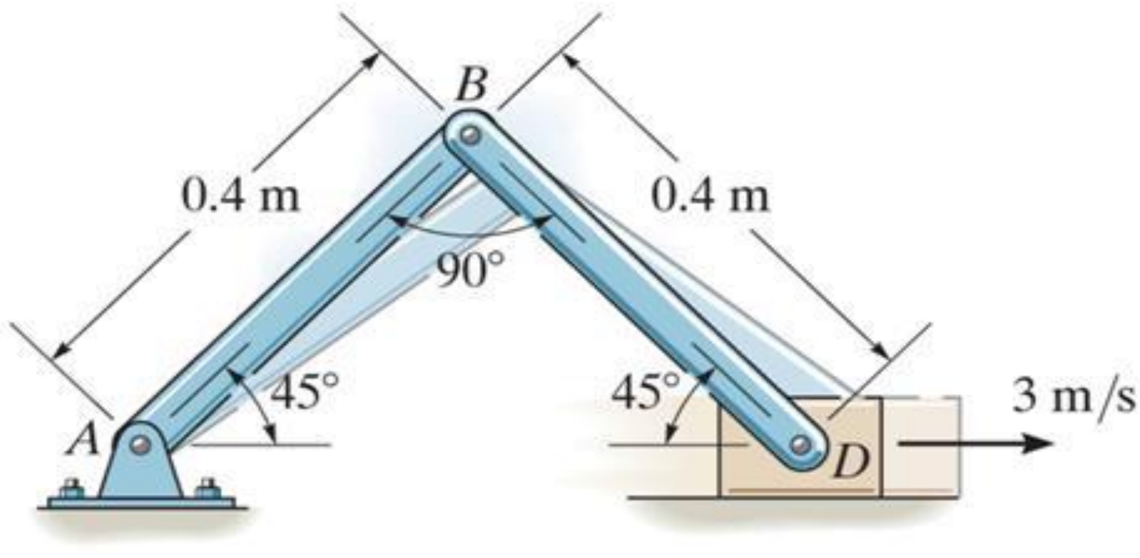
The velocity of any point on a body undergoing general plane motion can be determined easily once the instantaneous center of zero velocity of the body is located.



Since the body seems to rotate about the IC at any instant, as shown in this kinematic diagram, the magnitude of velocity of any arbitrary point is $v = \omega\, r$, where r is the radial distance from the IC to the point.

The velocity's line of action is perpendicular to its associated radial line.

EXAMPLE I



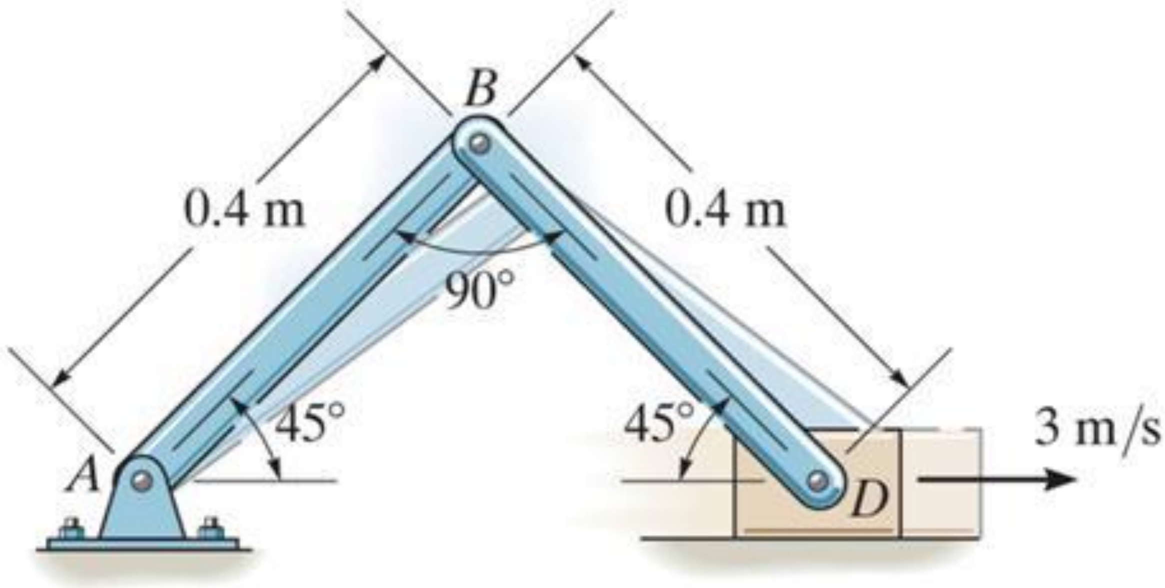
Given: *A linkage undergoing motion as shown. The velocity of the block, v_D, is 3 m/s.*

Find: *The angular velocities of links AB and BD.*

Plan:

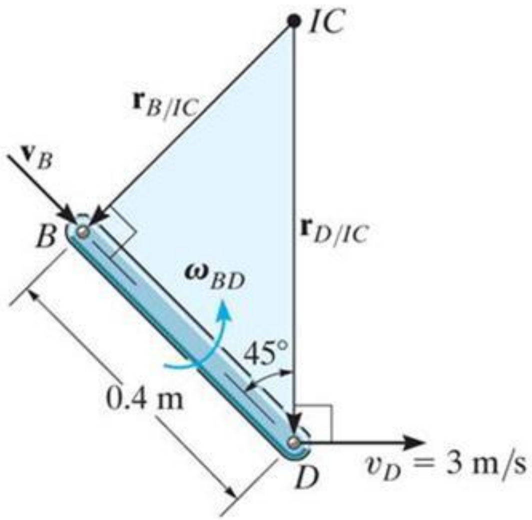
Locate the instantaneous center of zero velocity of link BD and then solve for the angular velocities.

EXAMPLE I



Solution: ***Since D moves to the right, it causes link AB to rotate clockwise about point A. The instantaneous center of velocity for BD is located at the intersection of the line segments drawn perpendicular to v_B and v_D. Note that v_B is perpendicular to link AB. Therefore we can see that the IC is located along the extension of link AB.***

EXAMPLE I (continued)

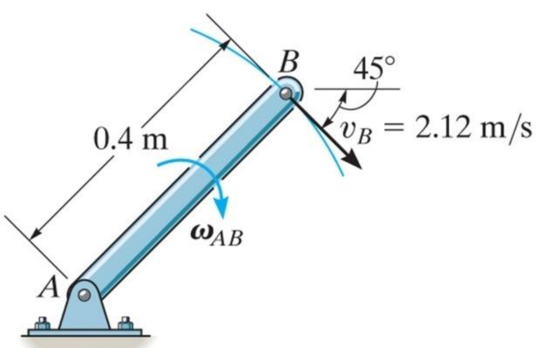


Using these facts,

$r_{B/IC}$ = 0.4 tan 45° = 0.4 m

$r_{D/IC}$ = 0.4/cos 45° = 0.566 m

Since the magnitude of v_D is known, the angular velocity of link BD can be found from $v_D = \omega_{BD}\, r_{D/IC}$.

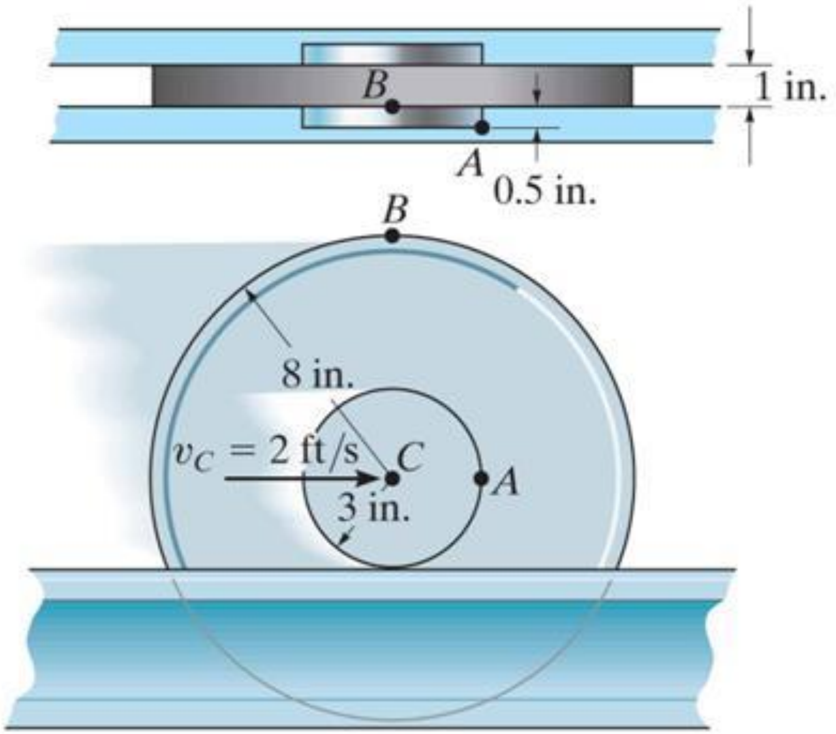


$$\omega_{BD} = v_D/r_{D/IC} = 3/0.566 = 5.3 \text{ rad/s}$$

Link AB is subjected to rotation about A.

$$\omega_{AB} = v_B/r_{B/A} = (r_{B/IC})\omega_{BD}/r_{B/A} = 0.4(5.3)/0.4 = 5.3 \text{ rad/s}$$

EXAMPLE II



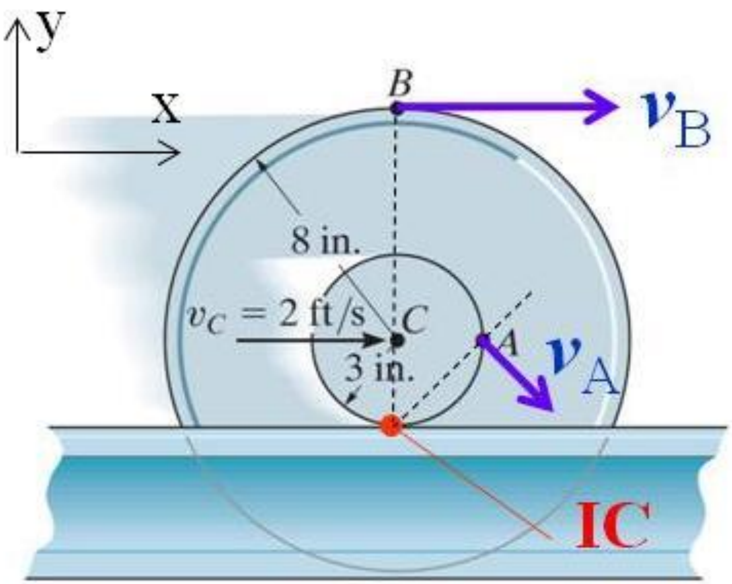
Given: ***The wheel rolls on its hub without slipping on the horizontal surface with v_C = 2 ft/s. →***

Find: ***The velocities of points A and B at the instant shown.***

Plan:

Locate the IC of the wheel. Then calculate the velocities at A and B.

EXAMPLE II (continued) Solution:



Note that the wheel rolls without slipping. Thus the IC is at the contact point with the surface.

The angular velocity of the wheel can be found from

$$\omega = v_C / r_{C/IC} = 2/3 = 0.667 \text{ rad/s}$$

Or, $\boldsymbol{\omega} = -0.667\,\mathbf{k}$ *(rad/s)*

The velocity at A and B will be

$$\mathbf{v}_A = \boldsymbol{\omega} \times \mathbf{r}_{A/IC} = (-0.667)\,\mathbf{k} \times (3\,\mathbf{i} + 3\mathbf{j}) = (2\,\mathbf{i} - 2\,\mathbf{j}) \text{ in/s}$$

$$\mathbf{v}_B = \boldsymbol{\omega} \times \mathbf{r}_{B/IC} = (-0.667)\,\mathbf{k} \times (11\,\mathbf{j}) = 7.34\,\mathbf{i} \text{ in/s}$$

$$v_A = \sqrt{(2^2 + 2^2)} = 2.83 \text{ in/s}, \; v_B = 7.34 \text{ in/s}$$

CONCEPT QUIZ

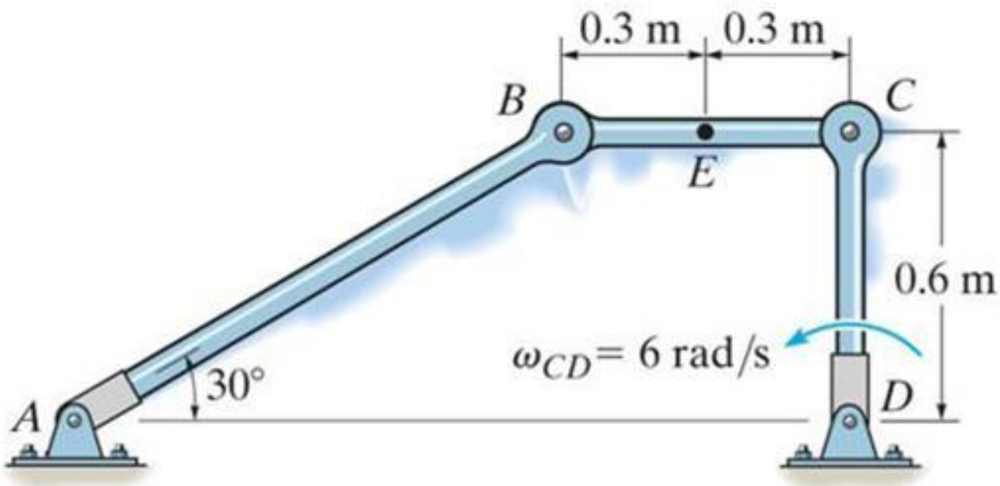
1. ***When the velocities of two points on a body are equal in magnitude and parallel but in opposite directions, the IC is located at***

   A) infinity.
   B) one of the two points.
   C) the midpoint of the line connecting the two points.
   D) None of the above.

2. ***When the direction of velocities of two points on a body are perpendicular to each other, the IC is located at***

   A) infinity.
   B) one of the two points.
   C) the midpoint of the line connecting the two points.
   D) None of the above.

GROUP PROBLEM SOLVING



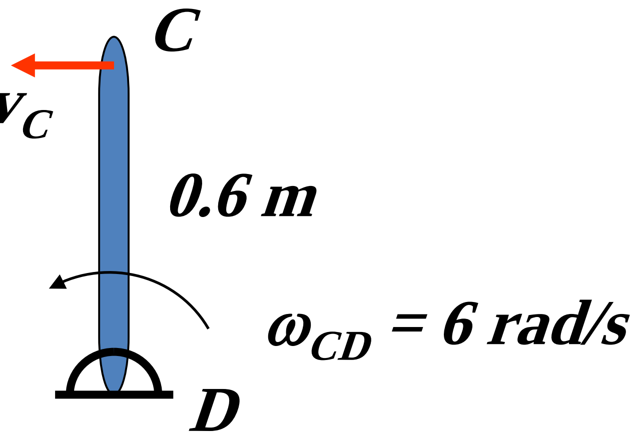
Given: *The four bar linkage is moving with ω_{CD} equal to 6 rad/s CCW.*

Find: *The velocity of point E on link BC and angular velocity of link AB.*

Plan: *This is an example of the second case in the lecture notes. Since the direction of Point B's velocity must be perpendicular to AB, and Point C's velocity must be perpendicular to CD, the location of the instantaneous center, I, for link BC can be found.*

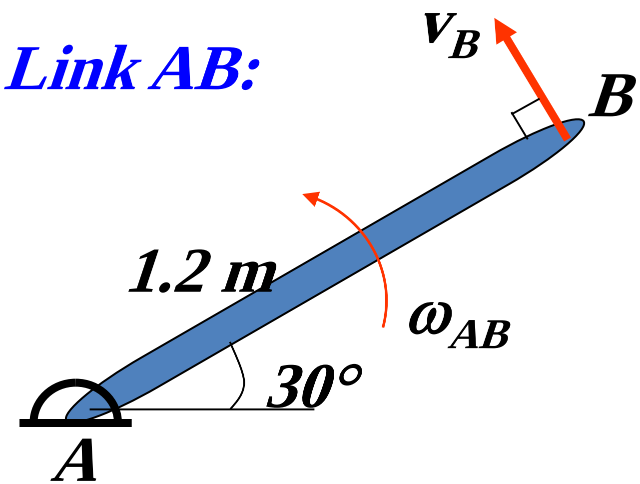
GROUP PROBLEM SOLVING
(continued)

Link CD:

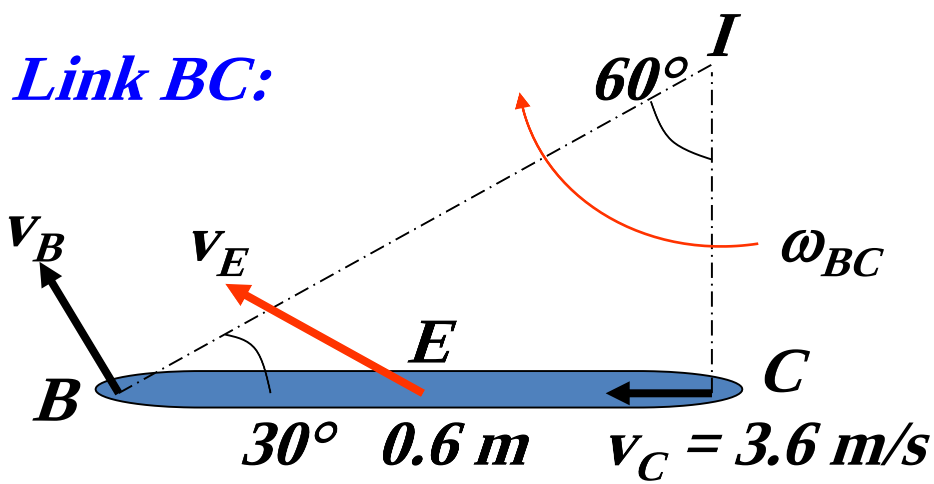


$v_C = 0.6(6) = 3.6$ m/s

Link AB:



Link BC:



From triangle CBI

IC = 0.346 m

IB = 0.6/sin 60° = 0.693 m

$v_C = (IC)\omega_{BC}$

$\omega_{BC} = v_C/IC = 3.6/0.346$

$\omega_{BC} = 10.39$ rad/s

GROUP PROBLEM SOLVING (continued)

$v_B = (IB)\omega_{BC} = 0.693(10.39) = 7.2$ m/s

From link AB, v_B is also equal to $1.2\ \omega_{AB}$.

Therefore $7.2 = 1.2\ \omega_{AB}$ => $\omega_{AB} = 6$ rad/s

$v_E = (IE)\omega_{BC}$ where distance $IE = \sqrt{0.3^2 + 0.346^2} = 0.458$ m

$v_E = 0.458(10.39) = 4.76$ m/s θ

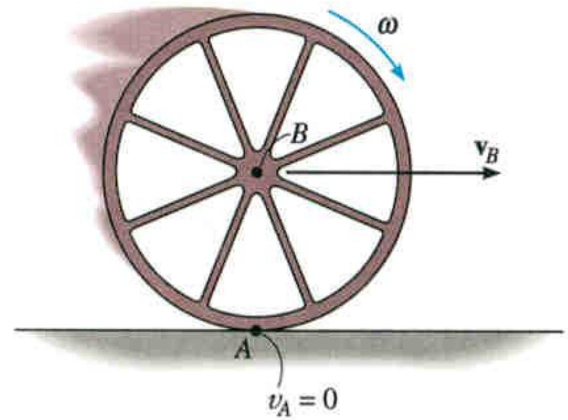
where $\theta = \tan^{-1}(0.3/0.346) = 40.9°$

ATTENTION QUIZ

1. *The wheel shown has a radius of 15 in and rotates clockwise at a rate of ω = 3 rad/s. What is v..?*

   A) 5 in/s B) 15 in/s

   C) 0 in/s D) 45 in/s



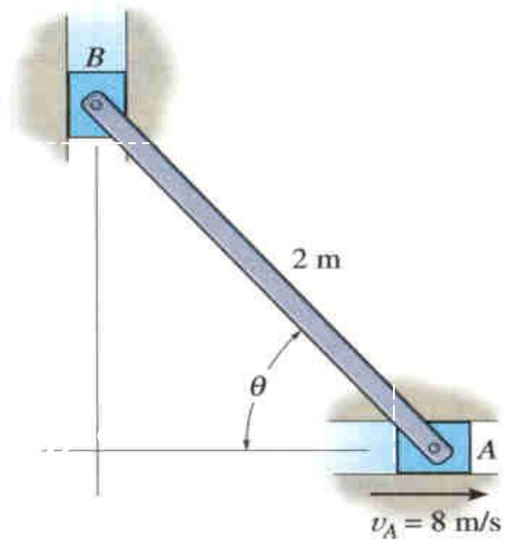
2. *Point A on the rod has a velocity of 8 m/s to the right. Where is the IC for the rod?*

   A) Point A.

   B) Point B.

   C) Point C.

   D) Point D.



RELATIVE MOTION ANALYSIS: ACCELERATION

Objectives: *Students will be able to:*

1. *Resolve the acceleration of a point on a body into components of translation and rotation.*
2. *Determine the acceleration of a point on a body by using a relative acceleration analysis.*

READING QUIZ

1. *If two bodies contact one another without slipping, and the points in contact move along different paths, the tangential components of acceleration will be ______ and the normal components of acceleration will be _________.*

   A) *the same, the same* B) *the same, different*

   C) *different, the same* D) *different, different*

2. *When considering a point on a rigid body in general plane motion,*

   A) *It's total acceleration consists of both absolute acceleration and relative acceleration components.*

   B) *It's total acceleration consists of only absolute acceleration components.*

   C) *It's relative acceleration component is always normal to the path.*

   D) *None of the above.*

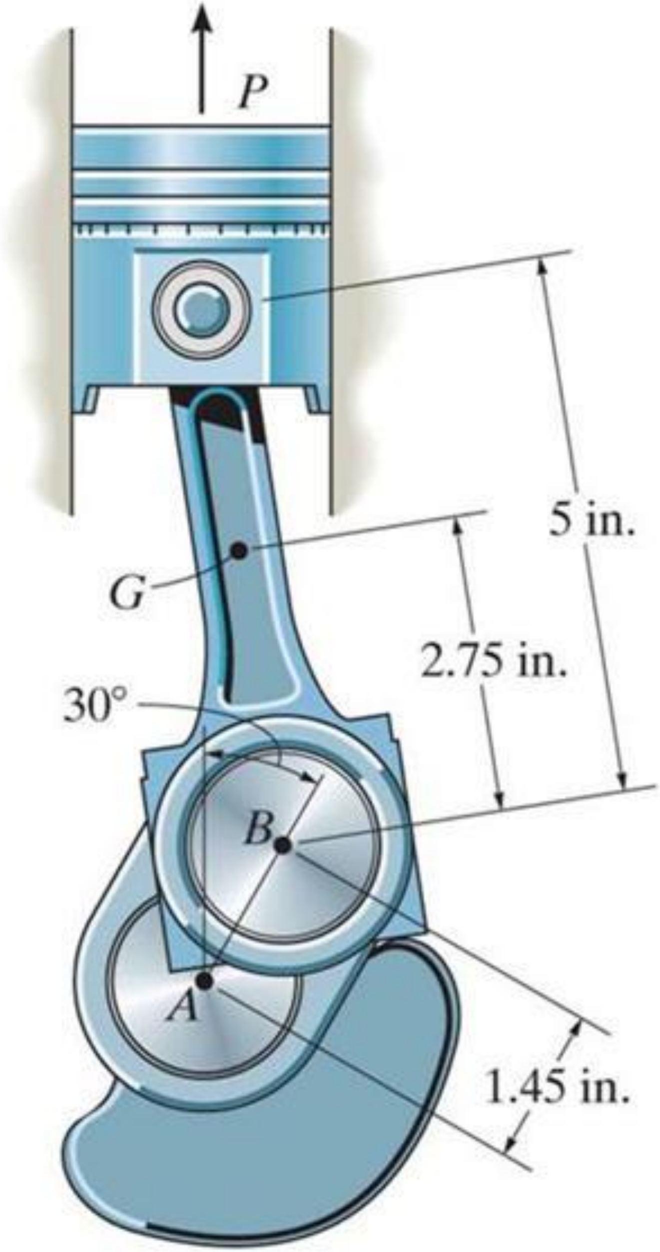
APPLICATIONS

In the mechanism for a window, link AC rotates about a fixed axis through C, and AB undergoes general plane motion. Since point A moves along a curved path, it has two components of acceleration while point B, sliding in a straight track, has only one.

The components of acceleration of these points can be inferred since their motions are known.

How can we determine the accelerations of the links in the mechanism?

APPLICATIONS (continued)



In an automotive engine, the forces delivered to the crankshaft, and the angular acceleration of the crankshaft, depend on the speed and acceleration of the piston.

How can we relate the accelerations of the piston, connection rod, and crankshaft to each other?

RELATIVE MOTION ANALYSIS: ACCELERATION (Section 16-7)

The equation relating the accelerations of two points on the body is determined by differentiating the velocity equation with respect to time.

$$\frac{d\mathbf{v}_B}{dt} = \frac{d\mathbf{v}_A}{dt} + \frac{d\mathbf{v}_{B/A}}{dt}$$

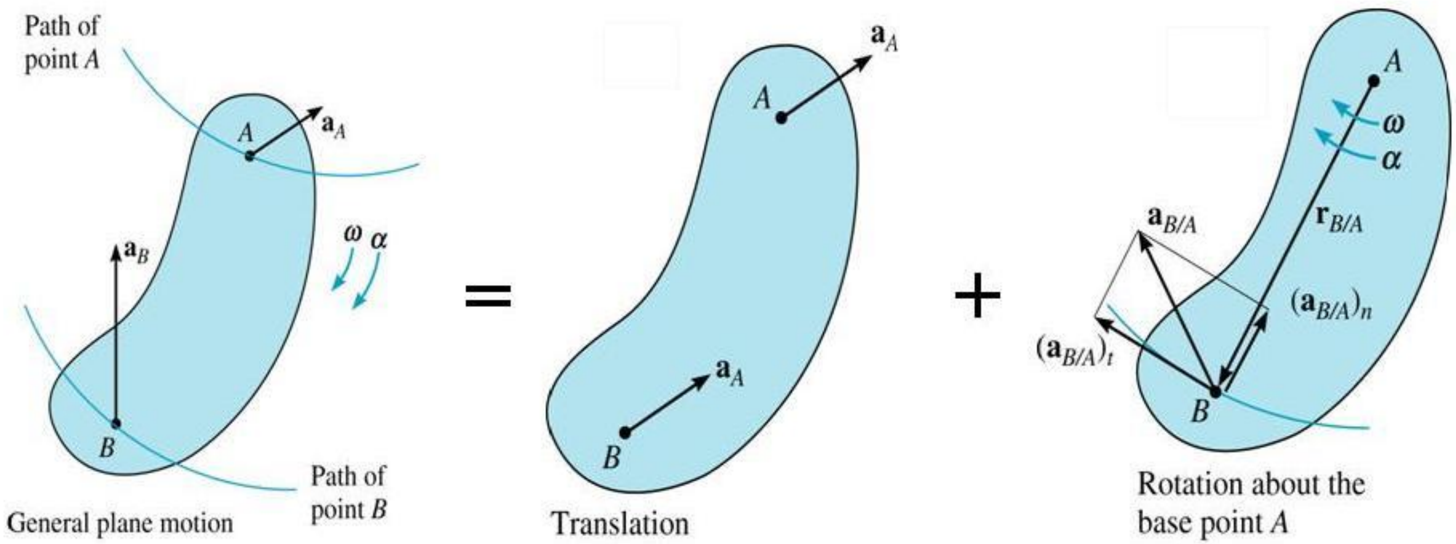
These are absolute accelerations of points A and B. They are measured from a set of fixed x,y axes.

This term is the acceleration of B with respect to A and includes both tangential and normal components.

The result is : $\mathbf{a}_B = \mathbf{a}_A + (\mathbf{a}_{B/A})_t + (\mathbf{a}_{B/A})_n$

RELATIVE MOTION ANALYSIS: ACCELERATION (continued)

Graphically: $\mathbf{a}_B = \mathbf{a}_A + (\mathbf{a}_{B/A})_t + (\mathbf{a}_{B/A})_n$



The relative tangential acceleration component $(\mathbf{a}_{B/A})_t$ is $(\alpha \times \mathbf{r}_{B/A})$ and perpendicular to $\mathbf{r}_{B/A}$.

The relative normal acceleration component $(\mathbf{a}_{B/A})_n$ is $(-\omega^2\, \mathbf{r}_{B/A})$ and the direction is always from B towards A.

RELATIVE MOTION ANALYSIS: ACCELERATION (continued)

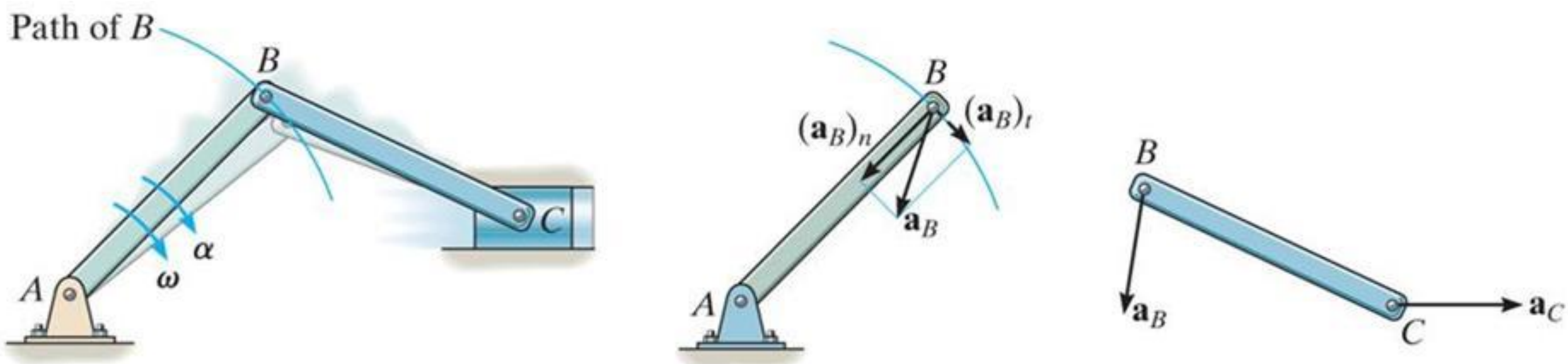
Since the relative acceleration components can be expressed as $(\mathbf{a}_{B/A})_t = \alpha \times \mathbf{r}_{B/A}$ *and* $(\mathbf{a}_{B/A})_n = -\omega^2\, \mathbf{r}_{B/A}$*, the relative acceleration equation becomes*

$$\mathbf{a}_B = \mathbf{a}_A + \alpha \times \mathbf{r}_{B/A} - \omega^2\, \mathbf{r}_{B/A}$$

Note that the last term in the relative acceleration equation is not a cross product. It is the product of a scalar (square of the magnitude of angular velocity, ω^2*) and the relative position vector,* $\mathbf{r}_{B/A}$*.*

APPLICATION OF THE RELATIVE ACCELERATION EQUATION

In applying the relative acceleration equation, the two points used in the analysis (A and B) should generally be selected as points which have a known motion, such as pin connections with other bodies.



In this mechanism, point B is known to travel along a circular path, so $\mathbf{a}_B$ *can be expressed in terms of its normal and tangential components. Note that point B on link BC will have the same acceleration as point B on link AB.*

Point C, connecting link BC and the piston, moves along a straight-line path. Hence, $\mathbf{a}_C$ *is directed horizontally.*

PROCEDURE FOR ANALYSIS

1. Establish a fixed coordinate system.

2. Draw the kinematic diagram of the body.

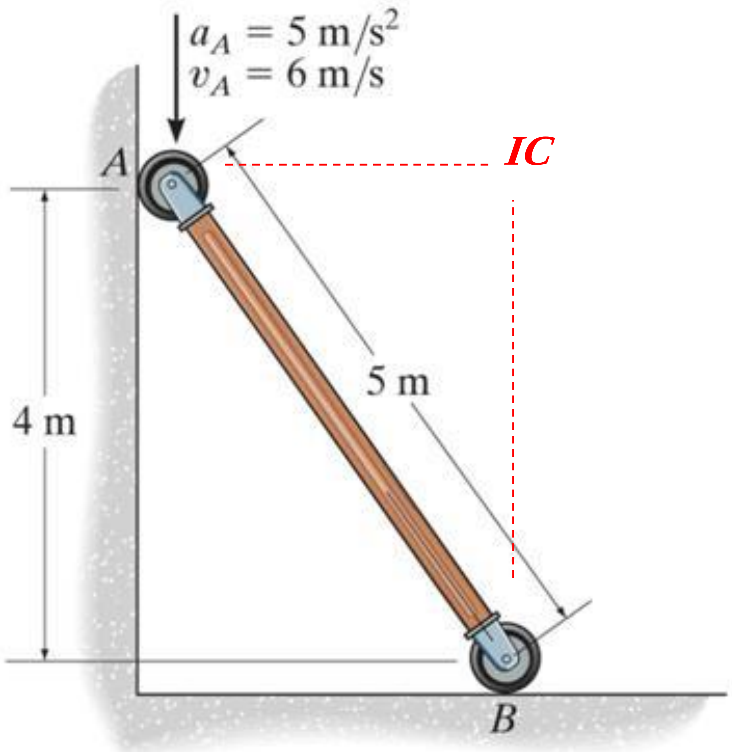
3. Indicate on it $\mathbf{a}_A$, $\mathbf{a}_B$, ω, α, and $\mathbf{r}_{B/A}$. If the points A and B move along curved paths, then their accelerations should be indicated in terms of their tangential and normal components, i.e., $\mathbf{a}_A = (\mathbf{a}_A)_t + (\mathbf{a}_A)_n$ and $\mathbf{a}_B = (\mathbf{a}_B)_t + (\mathbf{a}_B)_n$.

4. Apply the relative acceleration equation:

$$\mathbf{a}_B = \mathbf{a}_A + \alpha \times \mathbf{r}_{B/A} - \omega^2\, \mathbf{r}_{B/A}$$

5. If the solution yields a negative answer for an unknown magnitude, this indicates that the sense of direction of the vector is opposite to that shown on the diagram.

EXAMPLE I



Given: Point A on rod AB has an acceleration of 5 m/s^2 and a velocity of 6 m/s at the instant shown.

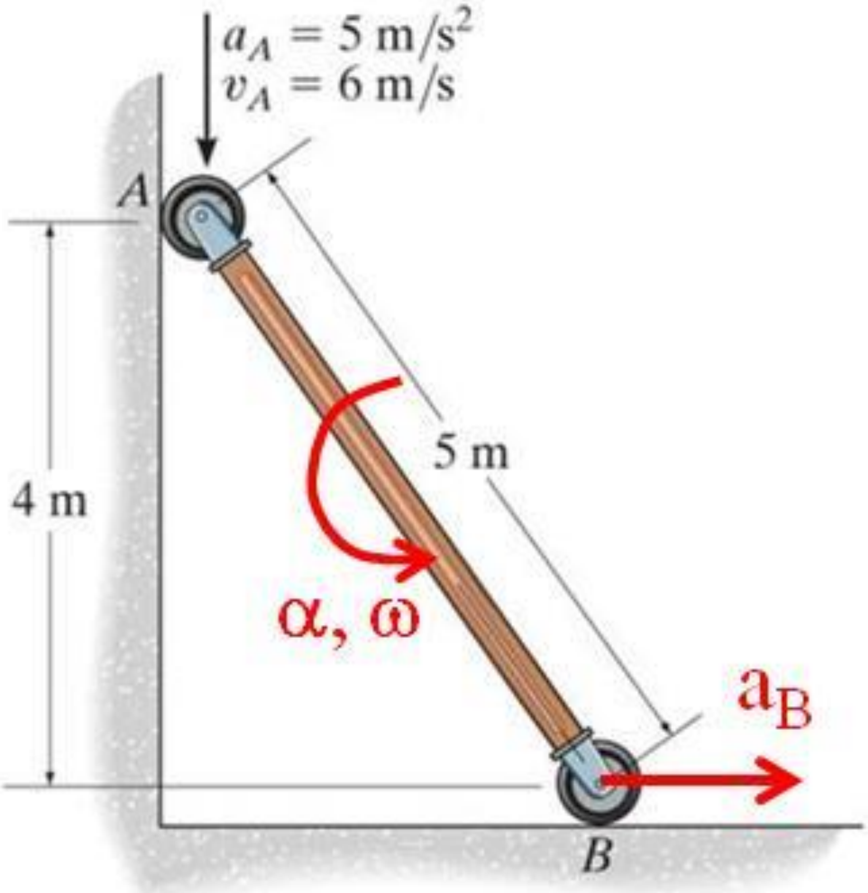
Find: The angular acceleration of the rod and the acceleration at B at this instant.

Plan:

Follow the problem solving procedure!

Solution: First, we need to find the angular velocity of the rod at this instant. Locating the instant center (IC) for rod AB, we can determine ω:

$\omega = v_A / r_{A/IC} = v_A / (3) = 2\ rad/s$



EXAMPLE I (continued)

Since points A and B both move along straight-line paths,

$\mathbf{a}_A = -5\ \mathbf{j}\ \mathrm{m/s^2}$

$\mathbf{a}_B = a_B\ \mathbf{i}\ \mathrm{m/s^2}$

Applying the relative acceleration equation

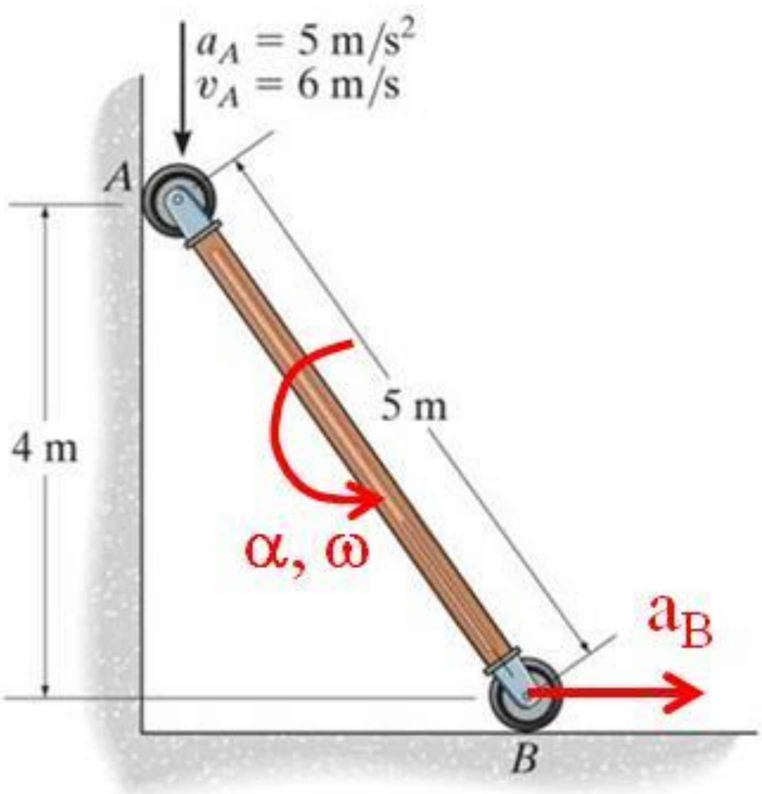
$$\mathbf{a}_B = \mathbf{a}_A + \boldsymbol{\alpha} \times \mathbf{r}_{B/A} - \omega^2 \mathbf{r}_{B/A}$$

$$a_B\,\mathbf{i} = -5\,\mathbf{j} + \alpha\,\mathbf{k} \times (3\,\mathbf{i} - 4\,\mathbf{j}) - 2^2\,(3\,\mathbf{i} - 4\,\mathbf{j})$$

$$a_B\,\mathbf{i} = -5\,\mathbf{j} + 4\,\alpha\,\mathbf{i} + 3\,\alpha\,\mathbf{j} - (12\,\mathbf{i} - 16\,\mathbf{j})$$

EXAMPLE I (continued)

So with $a_B\,\mathbf{i} = -5\,\mathbf{j} + 4\,\alpha\,\mathbf{i} + 3\,\alpha\,\mathbf{j} - (12\,\mathbf{i} - 16\,\mathbf{j})$, we can solve.



By comparing the **i**, **j** components;

$$a_B = 4\,\alpha - 12$$

$$0 = 11 + 3\alpha$$

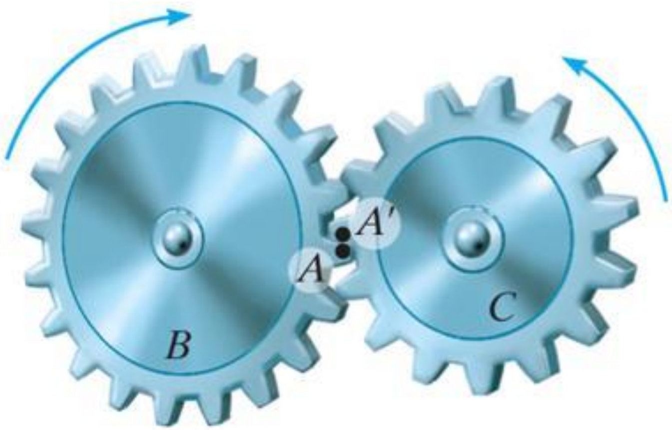
Solving:

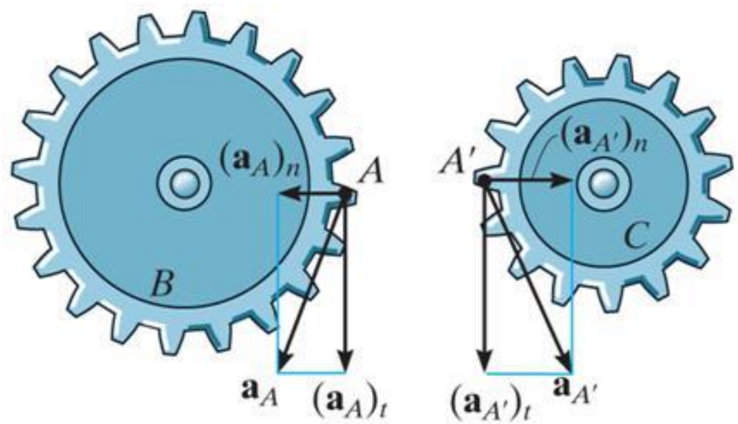
$$a_B = -26.7 \text{ m/s}^2$$

$$\alpha = -3.67 \text{ rad/s}^2$$

BODIES IN CONTACT

Consider two bodies in contact with one another without slipping, where the points in contact move along different paths.





In this case, the tangential components of acceleration will be the same, i. e.,

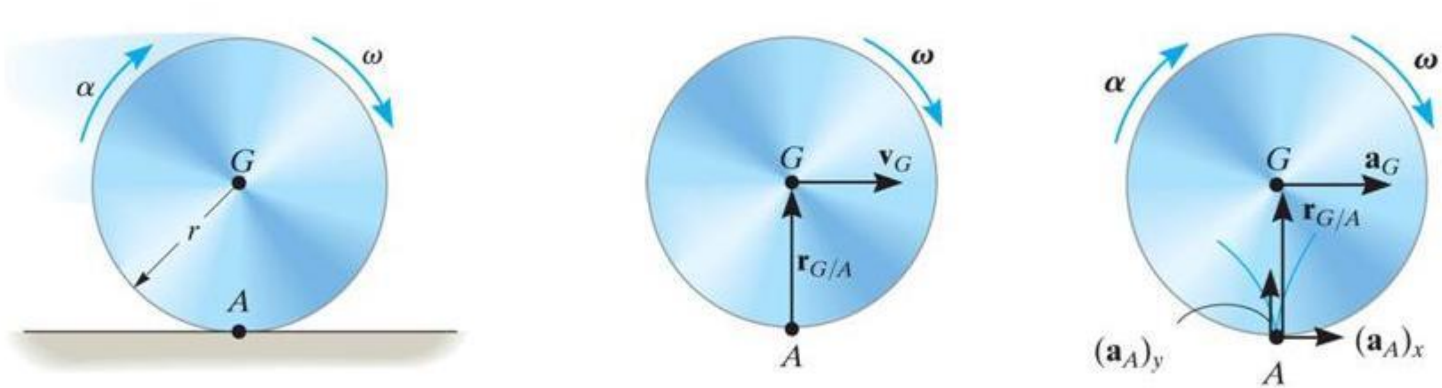
$$(\mathbf{a}_A)_t = (\mathbf{a}_{A'})_t \text{ (which implies } \alpha_B r_B = \alpha_C r_C\text{).}$$

The normal components of acceleration will not be the same.

$$(\mathbf{a}_A)_n \neq (\mathbf{a}_{A'})_n \text{ so } \mathbf{a}_A \neq \mathbf{a}_{A'}$$

ROLLING MOTION

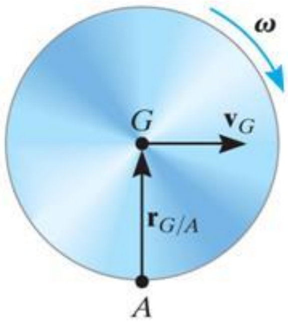
Another common type of problem encountered in dynamics involves rolling motion without slip; e.g., a ball, cylinder, or disk rolling without slipping. This situation can be analyzed using relative velocity and acceleration equations.



As the cylinder rolls, point G (center) moves along a straight line. If ω and α are known, the relative velocity and acceleration equations can be applied to A, at the instant A is in contact with the ground. The point A is the instantaneous center of zero velocity, however it is not a point of zero acceleration.

ROLLING MOTION (continued)

- *Velocity:*



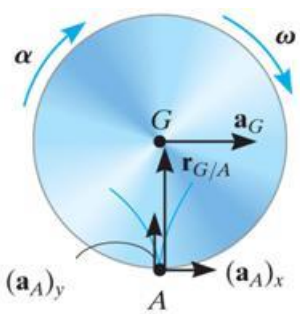
Since no slip occurs, $\boldsymbol{v}_A = \boldsymbol{0}$ when A is in contact with ground. From the kinematic diagram:

$$\boldsymbol{v}_G = \boldsymbol{v}_A + \boldsymbol{\omega} \times \boldsymbol{r}_{G/A}$$

$$v_G \boldsymbol{i} = \boldsymbol{0} + (-\omega \boldsymbol{k}) \times (r\,\boldsymbol{j})$$

$$v_G = \omega r \quad \text{or} \quad \boldsymbol{v}_G = \omega r\,\boldsymbol{i}$$

- *Acceleration:*



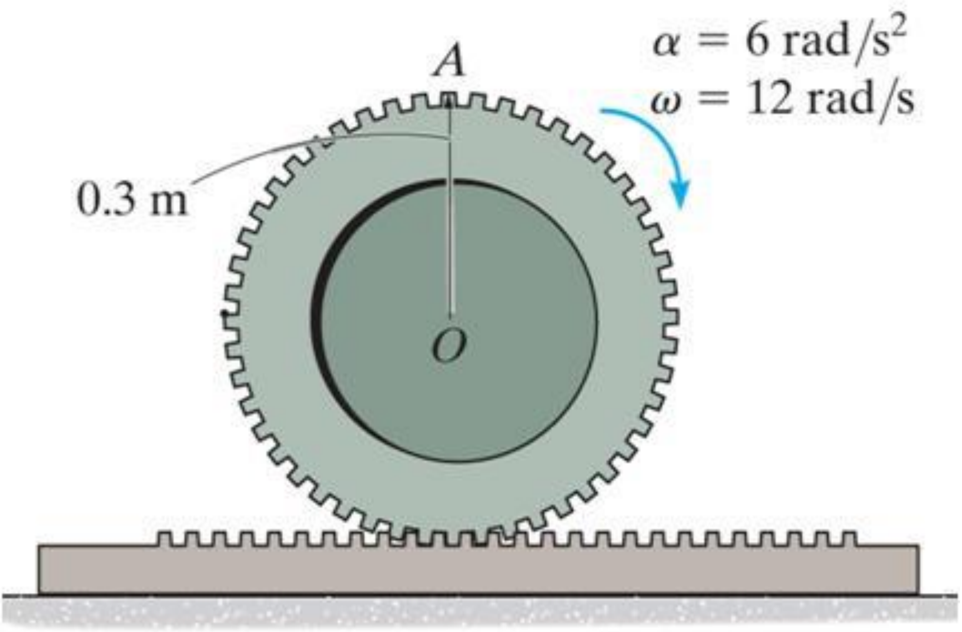
Since G moves along a straight-line path, $\boldsymbol{a}_G$ is horizontal. Just before A touches ground, its velocity is directed downward, and just after contact, its velocity is directed upward. Thus, point A accelerates upward as it leaves the ground.

$$\boldsymbol{a}_G = \boldsymbol{a}_A + \boldsymbol{\alpha} \times \boldsymbol{r}_{G/A} - \omega^2 \boldsymbol{r}_{G/A} \;\Rightarrow\; a_G \boldsymbol{i} = a_A \boldsymbol{j} + (-\alpha\,\boldsymbol{k}) \times (r\,\boldsymbol{j}) - \omega^2 (r\,\boldsymbol{j})$$

*Evaluating and equating **i** and **j** components:*

$$a_G = \alpha r \quad \text{and} \quad a_A = \omega^2 r \quad \text{or} \quad \boldsymbol{a}_G = \alpha r\,\boldsymbol{i} \quad \text{and} \quad \boldsymbol{a}_A = \omega^2 r\,\boldsymbol{j}$$

EXAMPLE II



Given: *The gear rolls on the fixed rack.*

Find: *The accelerations of point A at this instant.*

Plan:

Follow the solution procedure!

Solution: *Since the gear rolls on the fixed rack without slip,* $\mathbf{a}_O$ *is directed to the right with a magnitude of:*

$$a_O = \alpha r = (6\ \text{rad/s}^2)(0.3\ \text{m}) = 1.8\ \text{m/s}^2$$

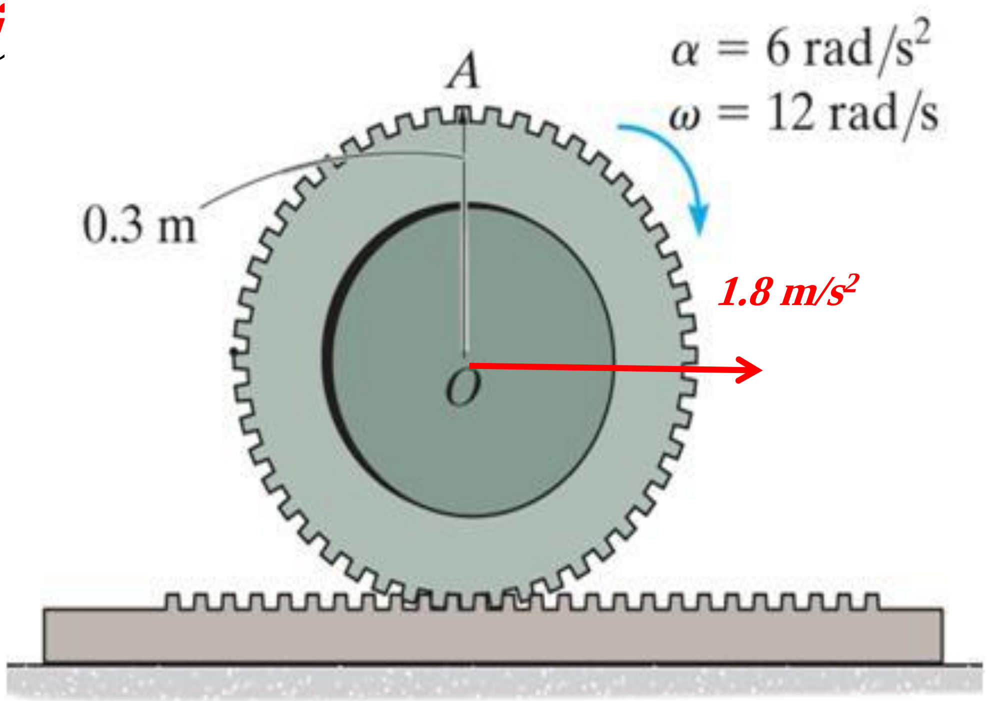
EXAMPLE II (continued)

So now with a_O = 1.8 m/s², we can apply the relative acceleration equation between points O and A.

$$\mathbf{a}_A = \mathbf{a}_O + \boldsymbol{\alpha} \times \mathbf{r}_{A/O} - \omega^2\, \mathbf{r}_{A/O}$$

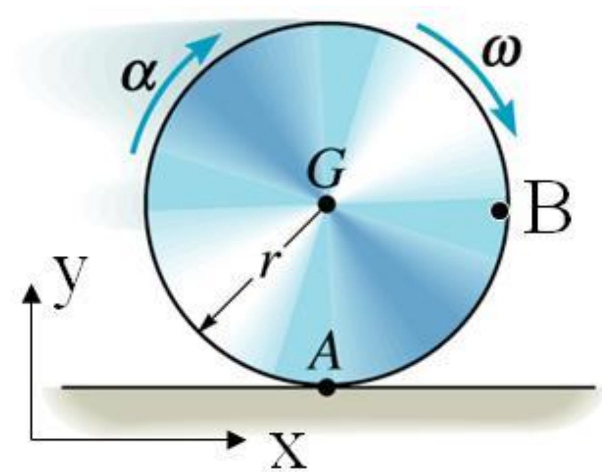
$$\mathbf{a}_A = 1.8\mathbf{i} + (-6\mathbf{k}) \times (0.3\mathbf{j}) - 12^2\,(0.3\mathbf{j}$$

$$= (3.6\,\mathbf{i} - 43.2\mathbf{j})\ \text{m/s}^2$$



CONCEPT QUIZ

1. If a ball rolls without slipping, select the tangential and normal components of the relative acceleration of point A with respect to G.



A) $\alpha r\, \mathbf{i} + \omega^2 r\, \mathbf{j}$ B) $-\alpha r\, \mathbf{i} + \omega^2 r\, \mathbf{j}$

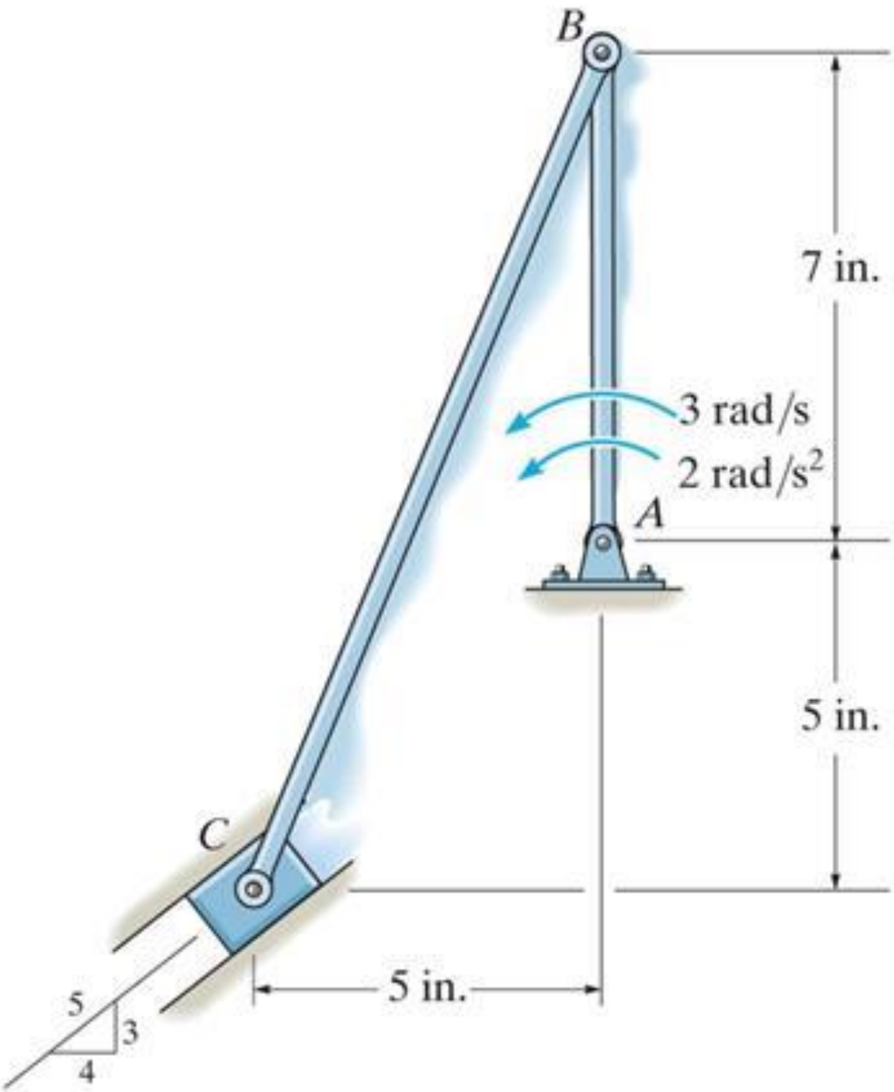
C) $\omega^2 r\, \mathbf{i} - \alpha r\, \mathbf{j}$ D) Zero.

2. What are the tangential and normal components of the relative acceleration of point B with respect to G.

A) $-\omega^2 r\, \mathbf{i} - \alpha r\, \mathbf{j}$ B) $-\alpha r\, \mathbf{i} + \omega^2 r\, \mathbf{j}$

C) $\omega^2 r\, \mathbf{i} - \alpha r\, \mathbf{j}$ D) Zero.

GROUP PROBLEM SOLVING



Given: The smember AB is rotating with ω_{AB}=3 rad/s, α_{AB}=2 rad/s^2 at this instant.

Find: The velocity and acceleration of the slider block C.

Plan:

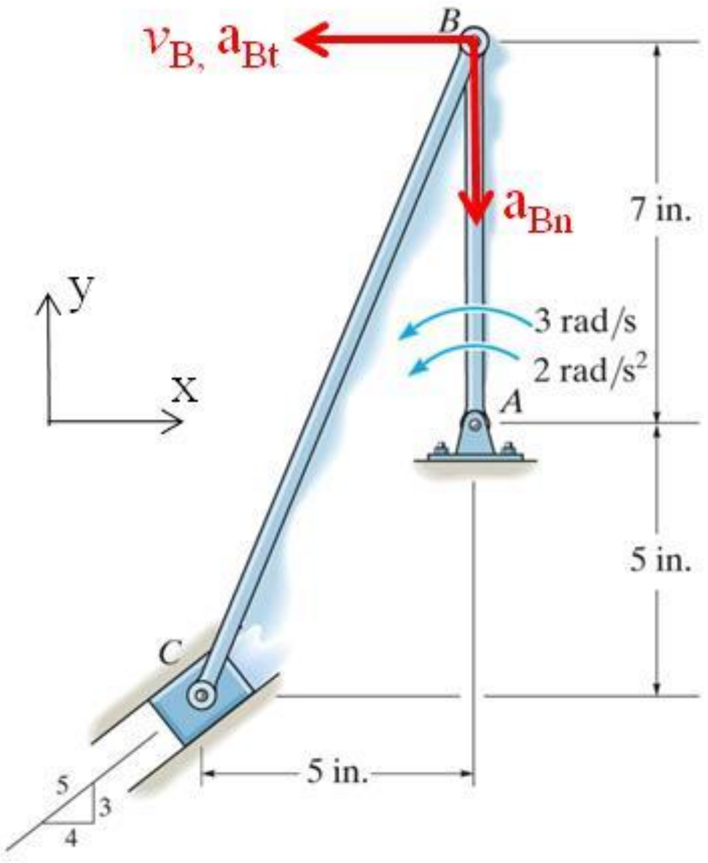
Follow the solution procedure!

Note that Point B is rotating. So what components of acceleration will it be experiencing?

GROUP PROBLEM SOLVING (continued)

Solution:

Since Point B is rotating, its velocity and acceleration will be:



$$v_B = (\omega_{AB})\, r_{B/A} = (3)\,7 = 21 \text{ in/s}$$

$$a_{Bn} = (\omega_{AB})^2\, r_{B/A} = (3)^2\, 7 = 63 \text{ in/s}^2$$

$$a_{Bt} = (\alpha_{AB})\, r_{B/A} = (2)\,7 = 14 \text{ in/s}^2$$

$$\mathbf{v}_B = (-21\,\mathbf{i}) \text{ in/s}$$

$$\mathbf{a}_B = (-14\,\mathbf{i} - 63\,\mathbf{j}) \text{ in/s}^2$$

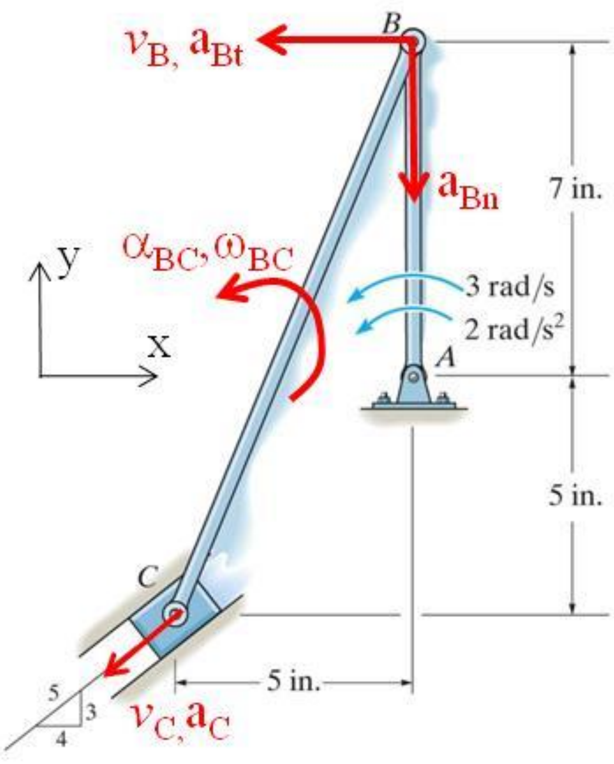
GROUP PROBLEM SOLVING (continued)

Now apply the relative velocity equation between points B and C to find the angular velocity of link BC.

$$\mathbf{v}_C = \mathbf{v}_B + \boldsymbol{\omega}_{BC} \times \mathbf{r}_{C/B}$$

$$(-0.8\, v_C\, \mathbf{i} - 0.6\, v_C\, \mathbf{j}) = (-21\, \mathbf{i}\,) + \omega_{BC}\, \mathbf{k} \times (-5\, \mathbf{i} - 12\, \mathbf{j})$$

$$= (-21 + 12\, \omega_{BC})\, \mathbf{i} - 5\, \omega_{BC}\, \mathbf{j}$$



By comparing the ***i***, ***j*** *components;*

$$-0.8\, v_C = -21 + 12\, \omega_{BC}$$

$$-0.6\, v_C = -5\, \omega_{BC}$$

Solving:

$$\omega_{BC} = 1.125 \text{ rad/s}$$

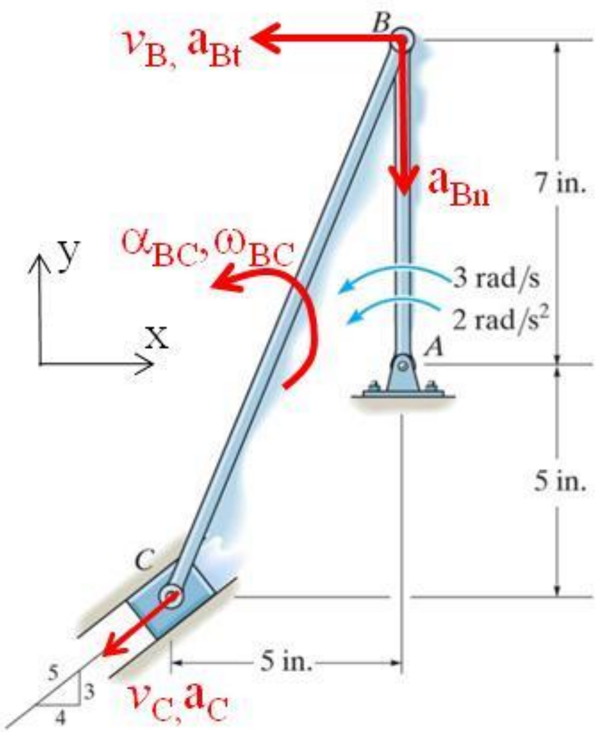
$$v_C = 9.375 \text{ in/s}$$

GROUP PROBLEM SOLVING (continued)

Now, apply the relative acceleration equation between points B and C.

$$\mathbf{a}_C = \mathbf{a}_B + \boldsymbol{\alpha}_{BC} \times \mathbf{r}_{C/B} - \omega^2_{BC}\,\mathbf{r}_{C/B}$$

$$(-0.8\, a_C\, \mathbf{i} - 0.6\, a_C\, \mathbf{j}) = (-14\, \mathbf{i} - 63\, \mathbf{j}) + \alpha_{BC}\mathbf{k} \times (-5\, \mathbf{i} - 12\, \mathbf{j}) - (1.125)^2 (-5\, \mathbf{i} - 12\, \mathbf{j})$$

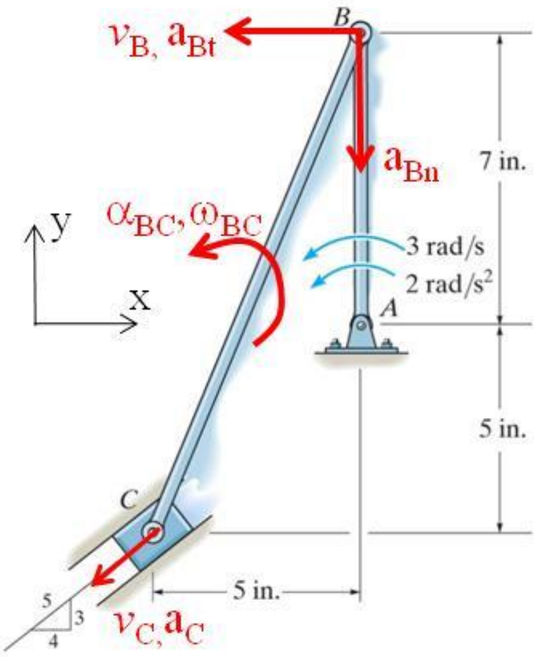


$$(-0.8\, a_C\, \mathbf{i} - 0.6\, a_C\, \mathbf{j})$$
$$= (-14 + 12\, \alpha_{BC} + 6.328)\, \mathbf{i}$$
$$+ (-63 - 5\, \alpha_{BC} + 15.19)\, \mathbf{j}$$

*By comparing the **i, j** components;*

$$-0.8\, a_C = -7.672 + 12\, \alpha_{BC}$$
$$-0.6\, a_C = -47.81 - 5\, \alpha_{BC}$$

GROUP PROBLEM SOLVING (continued)



Solving these two ***i, j*** *component equations*

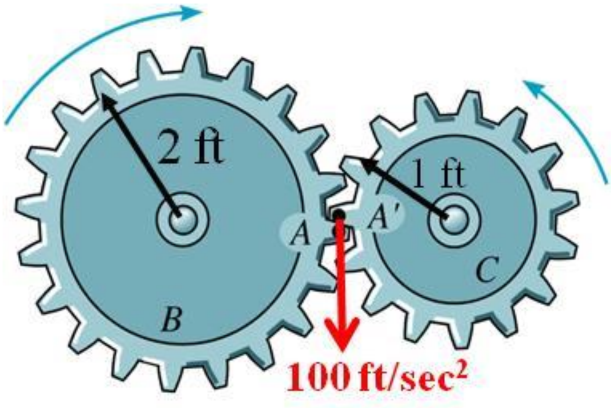
$$- 0.8\, a_C = -7.672 + 12\, \alpha_{BC}$$
$$- 0.6\, a_C = - 47.81 - 5\, \alpha_{BC}$$

Yields

$$\alpha_{BC} = -3.0 \text{ rad/s}^2$$
$$a_C = 54.7 \text{ in/s}^2$$

ATTENTION QUIZ

1. *Two bodies contact one another without slipping. If the tangential component of the acceleration of point A on gear B is 100 ft/sec², determine the tangential component of the acceleration of point A' on gear C.*



   A) 50 ft/sec² *B) 100 ft/sec²*

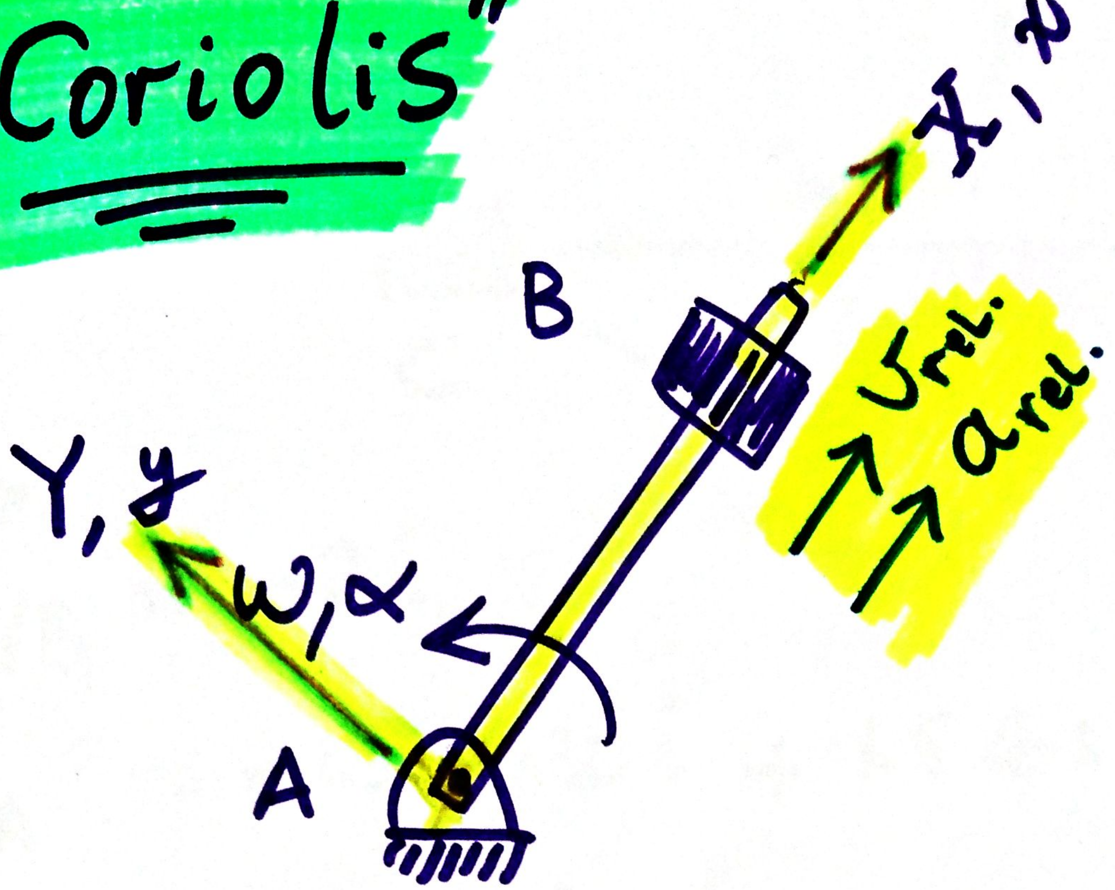
   C) 200 ft/sec² *D) None of above.*

2. *If the tangential component of the acceleration of point A on gear B is 100 ft/sec², determine the angular acceleration of gear B.*

   A) 50 rad/sec² *B) 100 rad/sec²*

   C) 200 rad/sec² *D) None of above.*

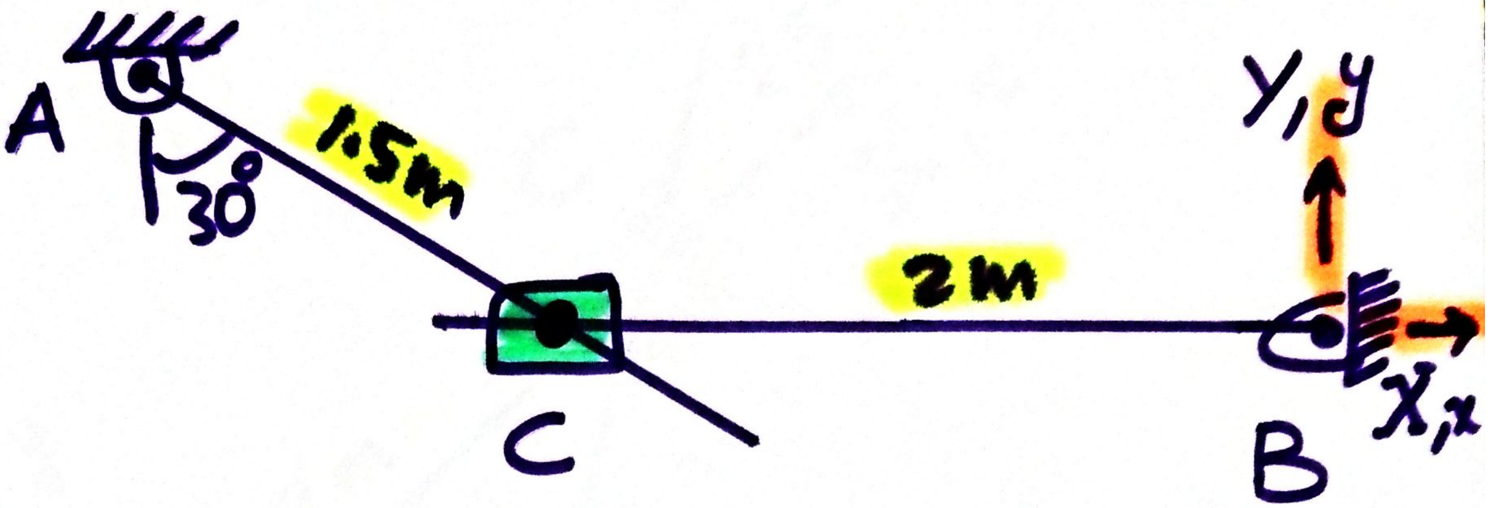
"Coriolis"



$$\bar{v}_B = \bar{v}_A + \bar{\omega} \times \bar{r}_{B/A} + \bar{v}_{rel.}$$

$$\bar{a}_B = \bar{a}_A + \bar{\alpha} \times \bar{r}_{B/A} - \omega^2 \bar{r}_{B/A} + 2\bar{\omega} \times \bar{v}_{rel.} + \bar{a}_{rel}$$

Coriolis Acc.



$$\vec{r}_{C/B} = -2i$$

$$\vec{r}_{C/A} = -1.5\sin 30^\circ i + 1.5\cos 30^\circ j$$

$$\vec{v}_{rel} = v_{rel}\, i$$

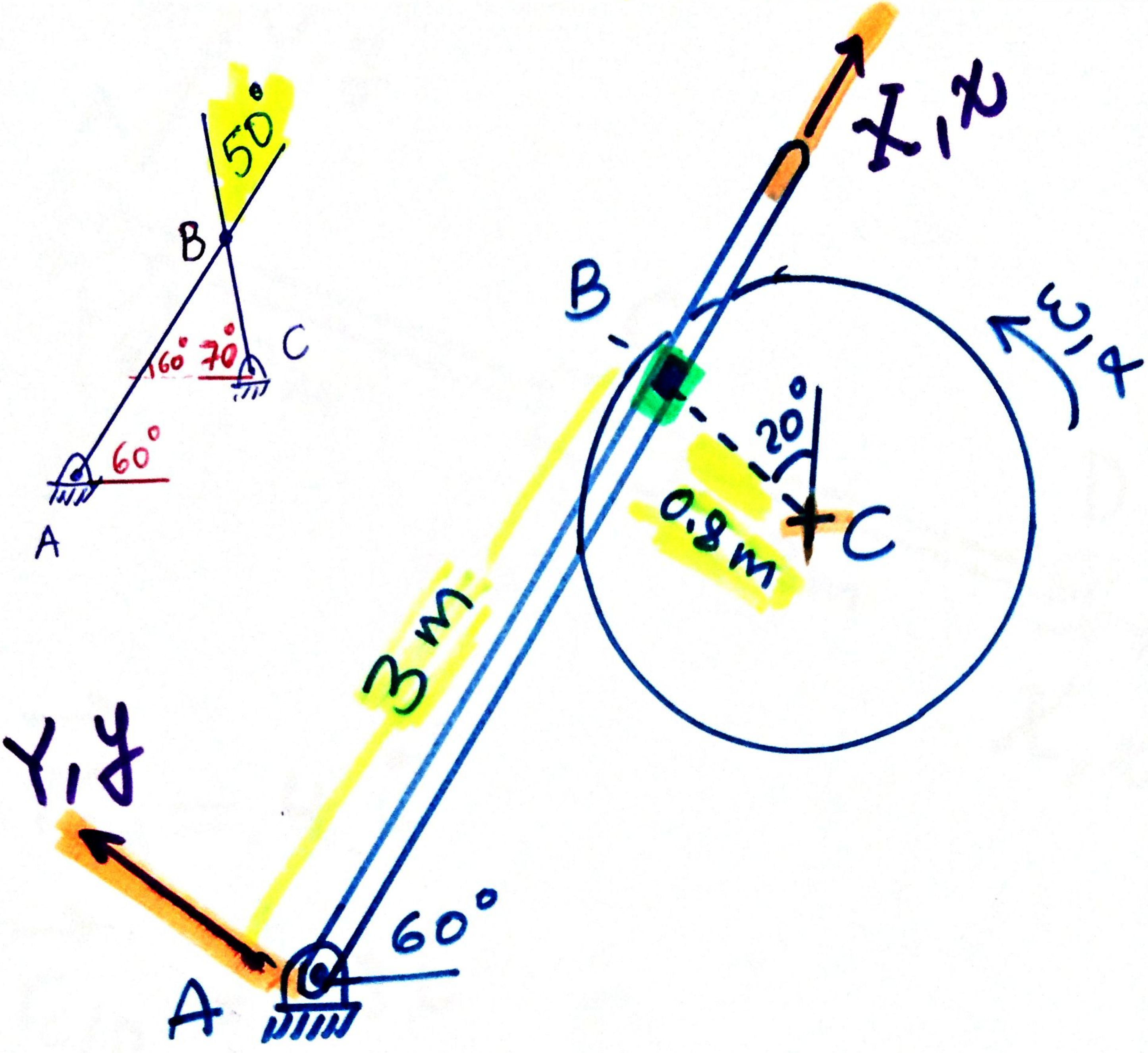
$$\vec{a}_{rel} = a_{rel}\, i$$

$$* \vec{v}_C = \vec{v}_A + \vec{\omega}_{AC} \times \vec{r}_{C/A}$$

$$* \vec{v}_C = \vec{v}_B + \vec{\omega}_{CB} \times \vec{r}_{C/B} + \vec{v}_{rel}$$

$$* \vec{a}_C = \vec{a}_A + \vec{\alpha}_{AC} \times \vec{r}_{C/A} - \omega_{AC}^2 \vec{r}_{C/A}$$

$$* \vec{a}_C = \vec{a}_B + \vec{\alpha}_{CB} \times \vec{r}_{C/B} - \omega_{CB}^2 \vec{r}_{C/B} + 2\vec{\omega}_{CB} \times \vec{v}_{rel} + \vec{a}_{rel}.$$

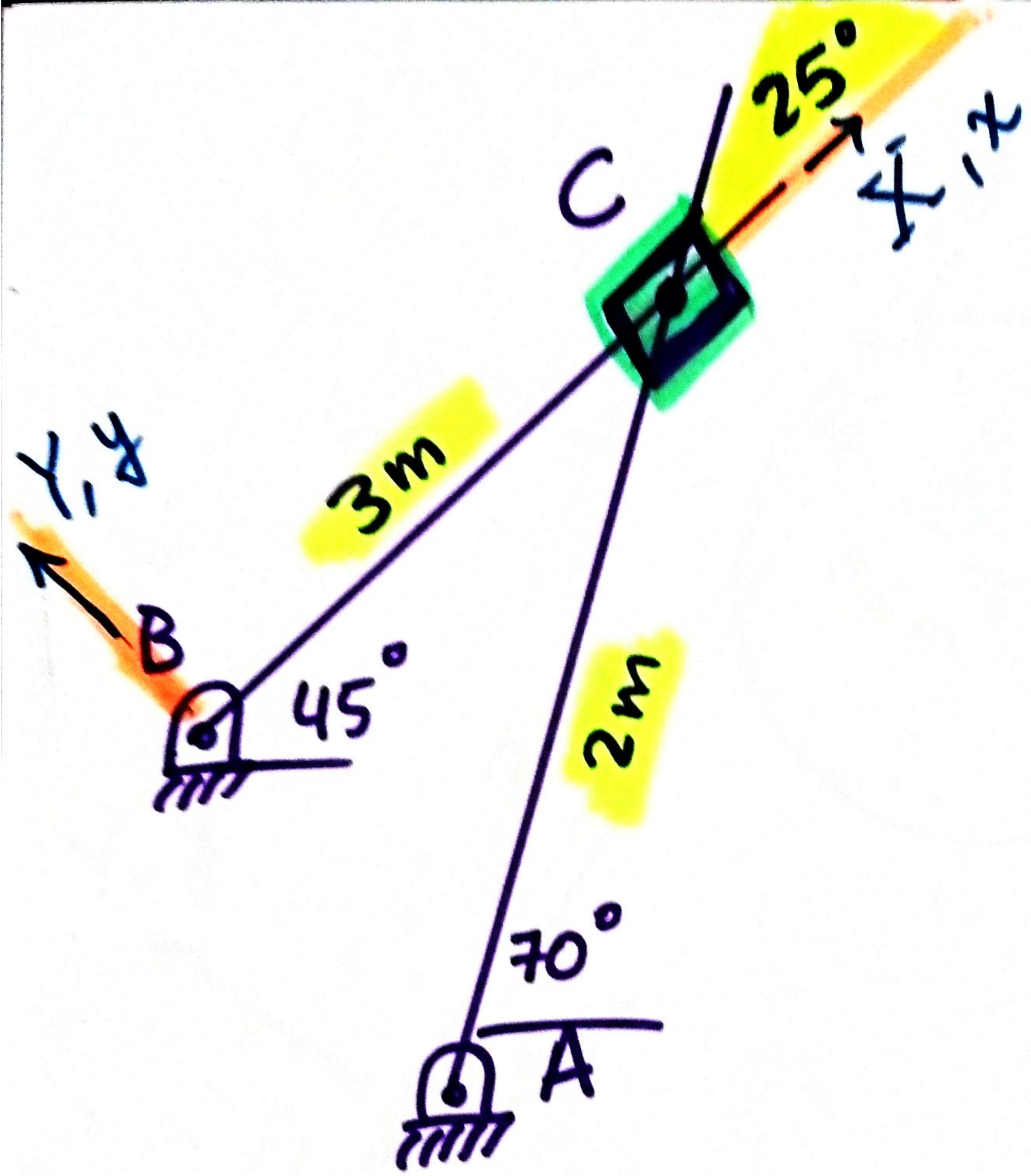


$$\vec{r}_{B/A} = 3\,i$$

$$\vec{r}_{B/C} = 0.8\cos 50^\circ\, i + 0.8\sin 50^\circ\, j$$

$$\vec{v}_{rel} = v_{rel}\, i$$

$$\vec{a}_{rel} = a_{rel}\, i$$

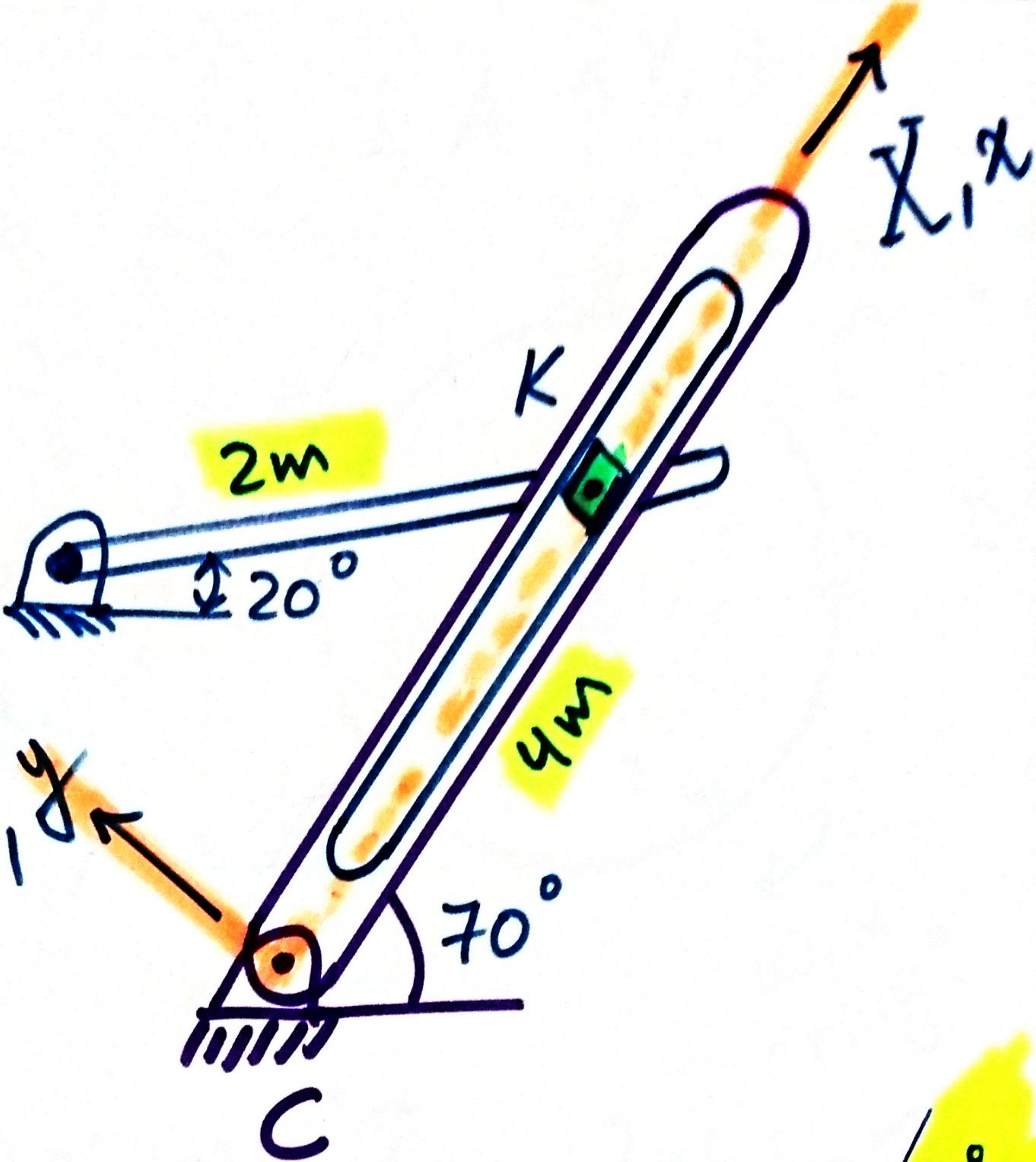


$$\vec{r}_{C/B} = 3\,i$$

$$\vec{r}_{C/A} = 2\cos 25^\circ\, i + 2\sin 25^\circ\, j$$

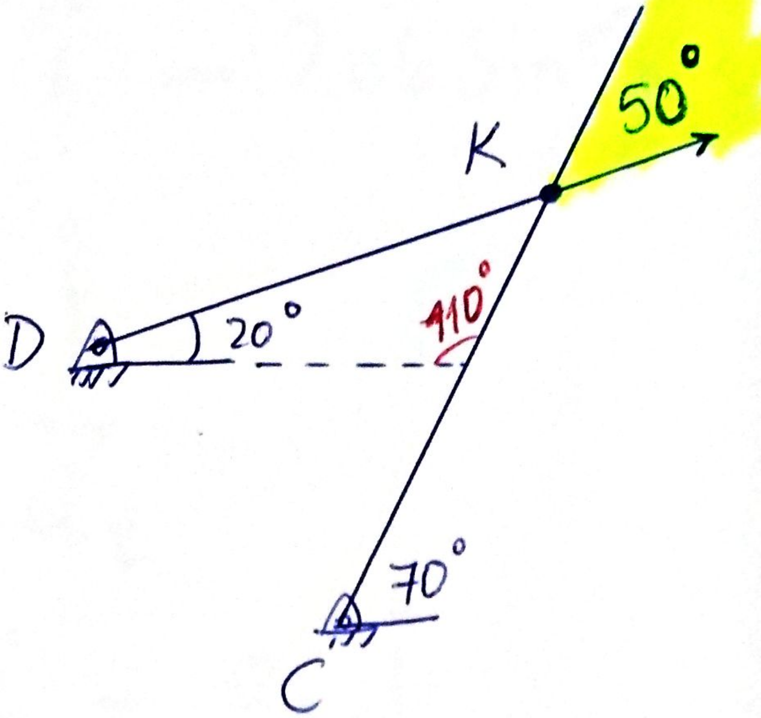
$$\vec{v}_{rel} = v_{rel}\, i$$

$$\vec{a}_{rel} = a_{rel}\, i$$



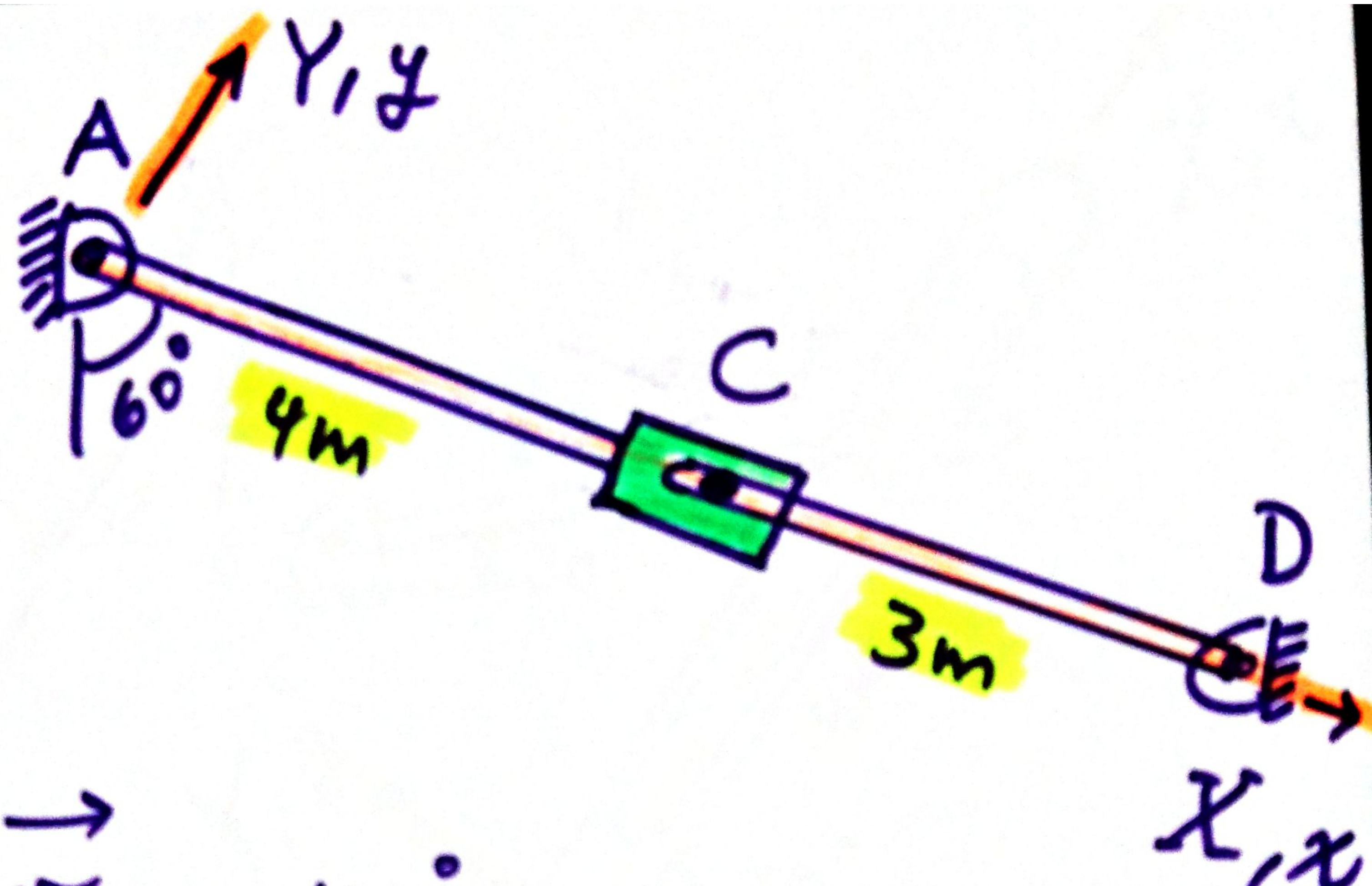
$\vec{r}_{K/C} = 4\,i$

$\vec{r}_{K/D} = 2\cos 50\,i - 2\sin 50\,j$



$\vec{v}_{rel} = v_{rel}\,i$

$\vec{a}_{rel} = a_{rel}.\,i$

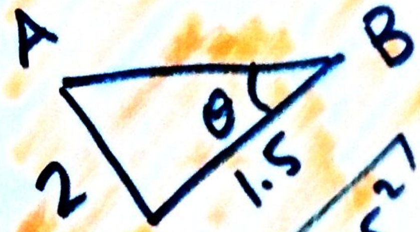


$\vec{r}_{C/A} = 4\,\hat{i}$

$\vec{r}_{C/D} = -3\,\hat{i}$

$\vec{v}_{rel} = v_{rel}\,\hat{i}$

$\vec{a}_{rel} = a_{rel}\,\hat{i}$

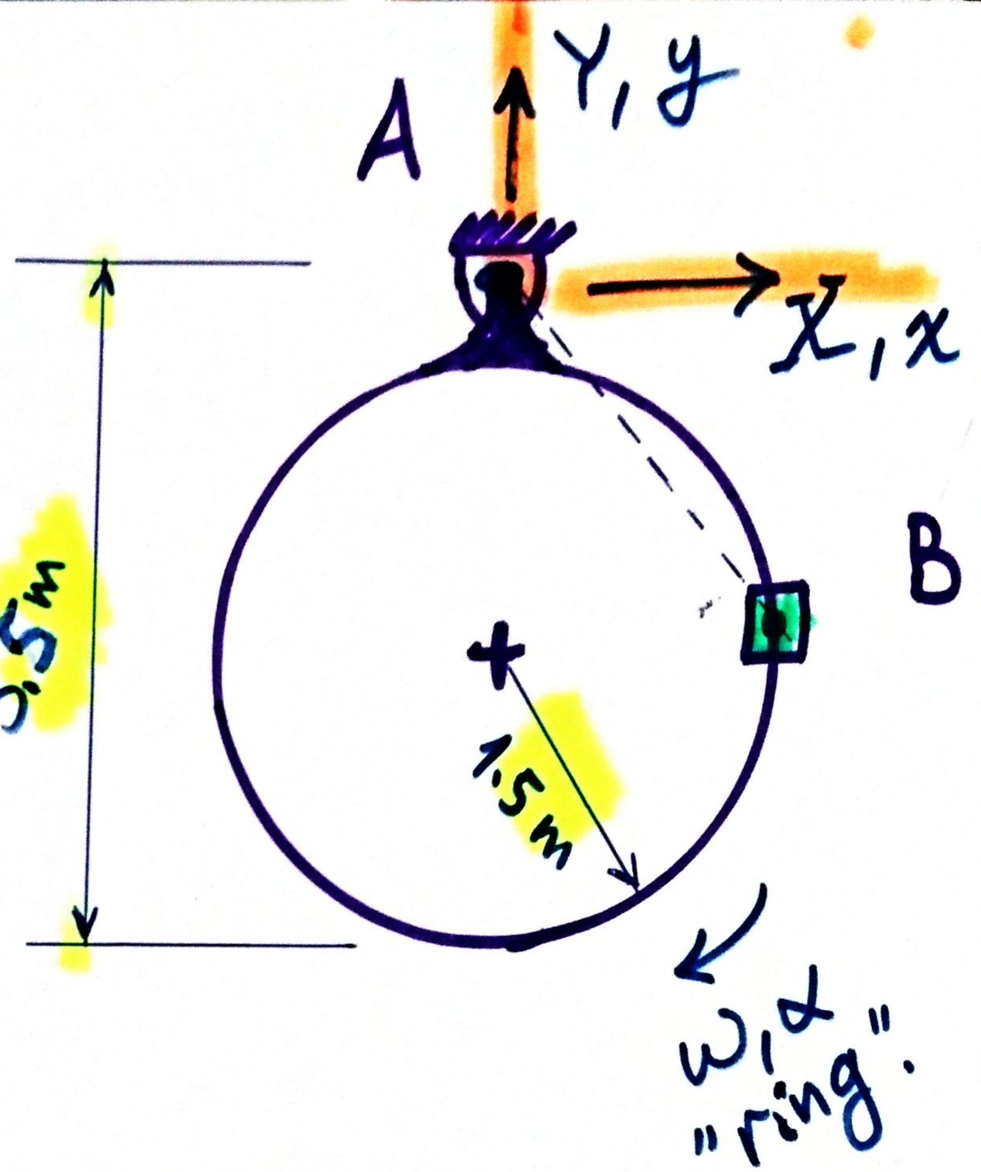


$AB = \sqrt{2^2 + 1.5^2}$

$AB = 2.06\,m$

$\theta = \tan^{-1} \frac{2}{1.5}$

$\theta = 53^\circ$



$$\vec{r}_{B/A} = 2.06 \cos 53^\circ\, i - 2.06 \sin 53^\circ\, j$$

$$\vec{v}_{rel} = -v_{rel}\, j$$

$$\vec{a}_{rel} = \vec{a}_n + \vec{a}_t$$

$$= -\frac{v_{rel}^2}{1.5}\, i - a_t\, j$$

* $v_B = v_A + \omega \times r_{B/A} + v_{rel}$

* $a_B = a_A + \alpha \times r_{B/A} - \omega^2 r_{B/A} + 2\omega \times v_{rel} + a_{rel}$

Dynamics

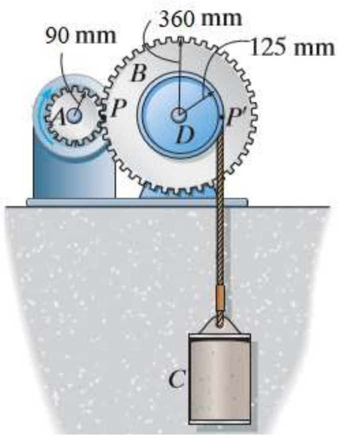
Dr. Hashem Alkhaldi

Suggested Problems: Chapter 16

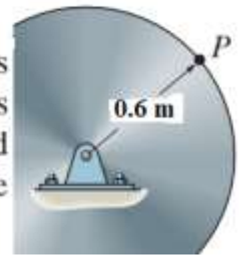
(15 problems, 3 pages)

g= 9.81 m/s^2 = 32.2 ft/s^2

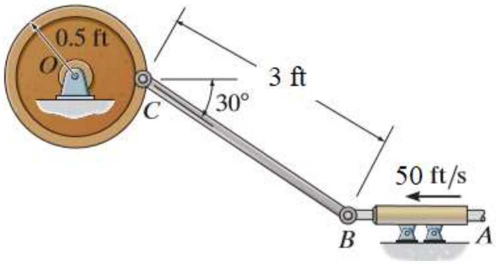
Q1 For a short period of time, the motor turns gear A with a constant angular acceleration of $\alpha_A = 4.5\ \text{rad/s}^2$, starting from rest. Determine the velocity of the cylinder and the distance it travels in three seconds. The cord is wrapped around pulley D which is rigidly attached to gear B.



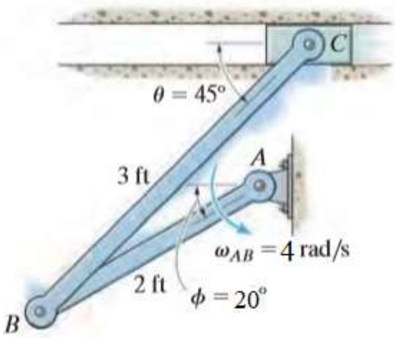
Q2 The disk starts at $\omega_0 = 1.5\ \text{rad/s}$ when $\theta = 0$, and is given an angular acceleration $\alpha = (0.3\theta)\ \text{rad/s}^2$, where θ is in radians. Determine the magnitudes of the normal and tangential components of acceleration of a point P on the rim of the disk when $\theta = 2$ rev.



Q3 If rod AB slides along the horizontal slot with a velocity of 50 ft/s, determine the angular velocity of link BC at the instant shown.

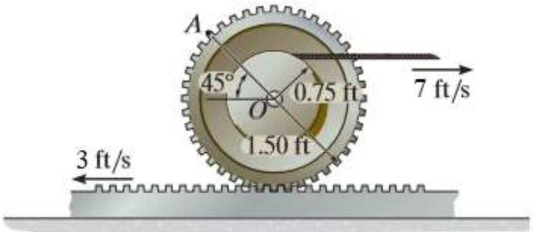


Q4 If the angular velocity of link AB is $\omega_{AB} = 4\ \text{rad/s}$, determine the velocity of the block at C and the angular velocity of the connecting link CB at the instant $\theta = 45°$ and $\phi = 20°$.

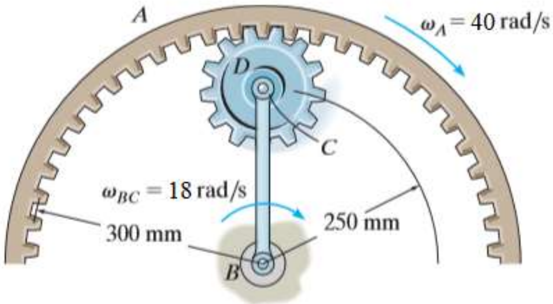


Q5 Determine the angular velocity of the gear and the velocity of its center O at the instant shown.

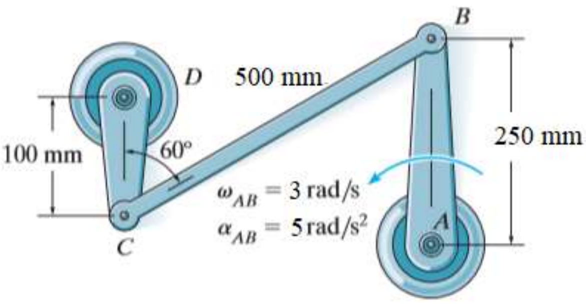
Also determine the velocity of point A on the rim of the gear at the instant shown.



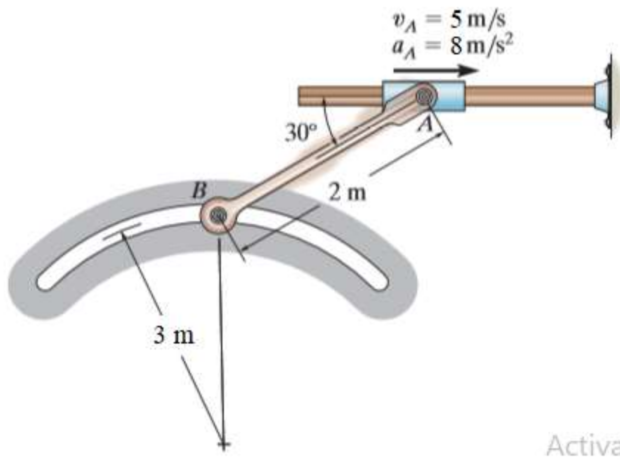
Q6 If the ring gear A rotates clockwise with an angular velocity of $\omega_A = 40$ rad/s, while link BC rotates clockwise with an angular velocity of $\omega_{BC} = 18$ rad/s, determine the angular velocity of gear D.



Q7 Member AB has the angular motions shown. Determine the angular velocity and angular acceleration of members CB and DC.



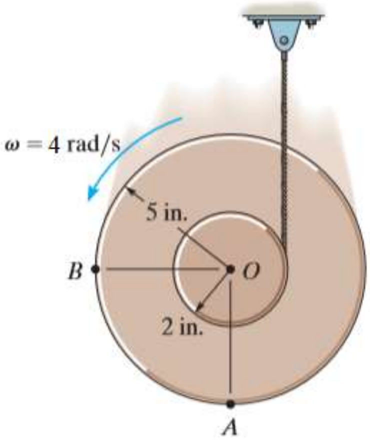
Q8 At a given instant the slider block A is moving to the right with the motion shown. Determine the angular acceleration of link AB and the acceleration of point B at this instant.



Activa

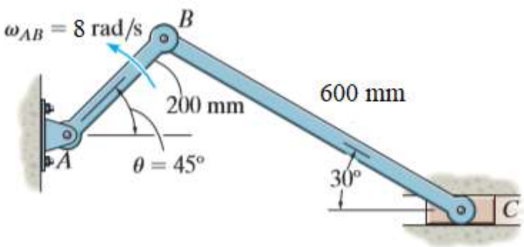
Q9 a) As the cord unravels from the wheel's inner hub, the wheel is rotating at $\omega = 4$ rad/s at the instant shown. Determine the velocities of points A and B.
[Using instantaneous center I.C.]

b) If the wheel has an angular acceleration of $\alpha = 3$ rad/s^2, determine the accelerations of points A and B.



Q10 a) If bar AB has an angular velocity $\omega_{AB} = 8$ rad/s, determine the velocity of the slider block C at the instant shown. **[Using instantaneous center I.C.]**

b) If bar BC has an angular acceleration of $\alpha_{BC} = 4$ rad/s^2, determine the acceleration the slider block C and the anglar acceleration of bar AB.



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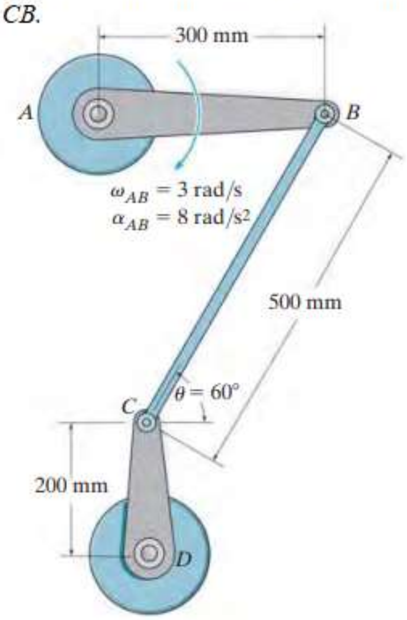
Q11 a) If member AB has the angular motion shown, determine the angular velocity and angular acceleration of member CD at the instant shown.

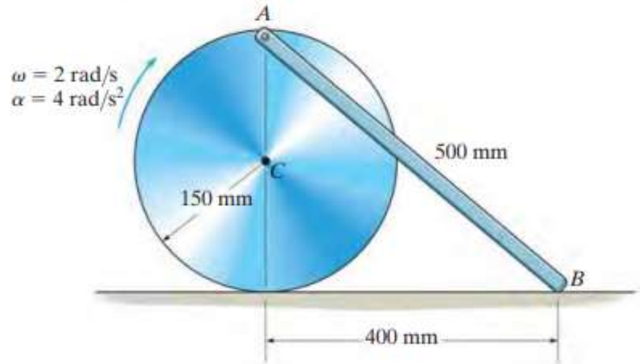
b) Using I.C, determine the velocity of the mid point of member CB.



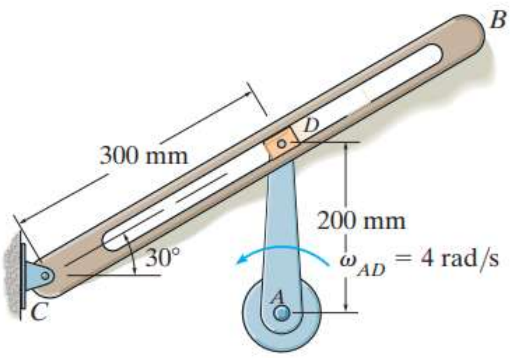
Q12 a) The disk rolls without slipping such that it has an angular acceleration of $\alpha = 4\ \text{rad/s}^2$ and angular velocity of $\omega = 2\ \text{rad/s}$ at the instant shown. Determine the acceleration of points A and B on the link and the link's angular acceleration at this instant. Assume point A lies on the periphery of the disk, 150 mm from C.

b) Using I.C, determine the velocity of the mid point of member AB.

c) Using I.C, determine the velocity of the left-quadrant point on the periphery of the disk..



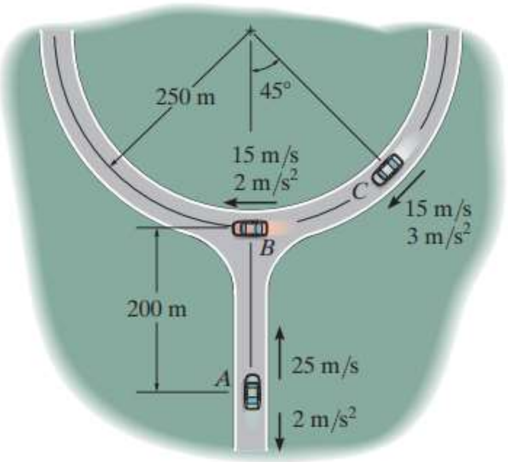
Q14 Block D of the mechanism is confined to move within the slot of member CB. If link AD is rotating at a constant rate of $\omega_{AD} = 4\ \text{rad/s}$, determine the angular velocity and angular acceleration of member CB at the instant shown.



Q13 At the instant shown, car A travels with a speed of $25\ \text{m/s}$, which is decreasing at a constant rate of $2\ \text{m/s}^2$, while car C travels with a speed of $15\ \text{m/s}$, which is increasing at a constant rate of $3\ \text{m/s}$.

a) Determine the velocity and acceleration of car C with respect to car A.

b) Determine the velocity and acceleration of car A with respect to car C.



Q15 Rod AB rotates counterclockwise with a constant angular velocity $\omega = 3\ \text{rad/s}$. Determine the velocity and acceleration of point C located on the double collar when $\theta = 30°$. The collar consists of two pin-connected slider blocks which are constrained to move along the circular path and the rod AB.