Ch 12

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Ch12

Kinematics of Particles

المسافة + السرعة + التسارع

a v S

* $v = \dfrac{ds}{dt}$

* $a = \dfrac{dv}{dt} \rightarrow a = \dfrac{d^2s}{dt^2}$

* $a\,ds = v\,dv$

مسافة

اشتقاق ↓   ↑ تكامل

سرعة

اشتقاق ↓   ↑ تكامل

تسارع

التسارع ثابتة

Constant Acceleration

$(a = g)$ السقوط الحر

$= -9.81\ m/s$

$$\underline{v} = v_0 + a\underline{t} \longrightarrow \underline{v} = v_0 - g\underline{t}$$

$$\underline{\Delta s} = v_0\underline{t} + \frac{1}{2}a\underline{t}^2 \longrightarrow \underline{\Delta y} = v_0\underline{t} - \frac{1}{2}g\underline{t}^2$$

$$\underline{v}^2 = v_0^2 + 2a\,\underline{\Delta s} \longrightarrow \underline{v}^2 = v_0^2 - 2g\,\underline{\Delta y}$$

مثل سيارة تمشي بتسارع ثابتة
$a = +ve$

أو مثل الحجر يمشي بتسارع ثابتة
$a = +ve$

أو مثل سفينة تسير بتباطؤ ثابتة
$a = -ve$

مثل كره أو الحجر
يسقط للاسفل
أو يرمى للاعلى .
دون الحاجة لأي عينة ديناميكية .

Ex) A particle moves on a straight line

حسب العلاقة according to

$$S = 2t^3 - 24t + 6$$ (اقتران يربط المسافة بالزمن)

Find:

① the time required for the particle to reach a velocity of 72 m/s.

أوجد الزمن الذي يحتاجه الجسم للوصول لسرعة ٧٢ م/ث .

$$v = \frac{ds}{dt} = 6t^2 - 24$$

$$72 = 6t^2 - 24 \Rightarrow 6t^2 = 96 \Rightarrow t^2 = 16$$

$$\underline{t = 4 \text{ sec}}$$

② the acceleration when $v = 30$ m/s.

أوجد تسارع هذا الجسم عندما تصبح سرعته تساوي ٣٠ م/ث

$$a = \frac{dv}{dt} = 12t$$

$$v = 6t^2 - 24$$

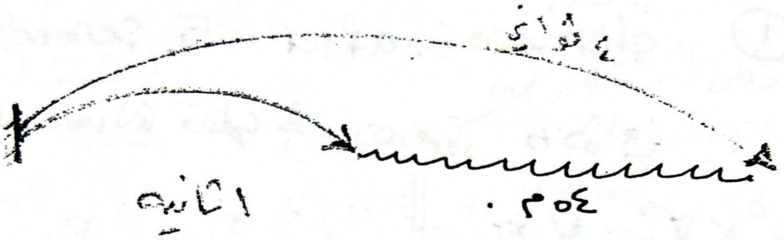
$$30 = 6t^2 - 24 \Rightarrow 6t^2 = 54 \Rightarrow t^2 = 9 \Rightarrow t = 3 \text{ sec}.$$

$$\therefore \quad a = 12(3) = 36 \text{ m/s}^2$$

(3)

(3) the displacement during $t = 1$ sec and $t = 4$ sec

أوجد المسافة التي قطعها الجسم ما بين الثانية الأولى والثانية الرابعة.



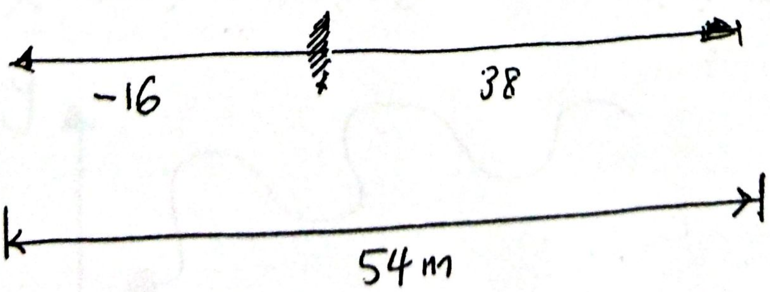
ثانية واحدة

$$\text{distance}(1) = 2(1)^3 - 24(1) + 6 = -16$$

٤ ثواني

$$\text{distance}(4) = 2(4)^3 - 24(4) + 6 = 38$$

$$\text{distance} = 38 - -16$$
$$= 38 + 16 = 54\text{m}$$



ex) the velocity of the particle سرعة جسيم

$$v = 6t^2 - 8t + 2$$

Find ① distance after 5 seconds. المسافة

المسافة تقطع الجسم بعد مرور 5 ثواني.

$\int_0^s ds = \int_0^t v\,dt$ نكامل

$$S = \int (6t^2 - 8t + 2)\,dt$$

$$= 6\frac{t^3}{3} - 8\frac{t^2}{2} + 2t = 2t^3 - 4t^2 + 2t$$

$$= 2(5)^3 - 4(5)^2 + 2(5) = \ldots 160 \ldots (m)$$

② the acceleration after 2 seconds.

أوجد تسارع الجسم بعد مرور 2 ثانية.

نشتق $a = \frac{dv}{dt} = 12t - 8$

$$= 12(2) - 8 = 16\ m/s^2$$

⑤ دائماً الاشتقاق بالنسبة للزمن

اذا لم يظهر الزمن في الاقتران

نشتقه ولكن اشتقاق ضمني

اذا ظهر الزمن في الاقتران

نشتق اشتقاق عادي

الازاحة $y = 2x^3 + 4x^2 - 2$

السرعة $\dot{y} = \underbrace{6x^2\dot{x}} + \underbrace{8x\dot{x}}$

التسارع $\ddot{y} = (6x^2\ddot{x} + \dot{x}\,12x\dot{x})$
$+ (8x\ddot{x} + \dot{x}\,8\dot{x})$

$\ddot{y} = 6x^2\ddot{x} + 12x\dot{x}^2$
$+ 8x\ddot{x} + 8\dot{x}^2$

$S = 5t^3 - 2t^2 + 1$

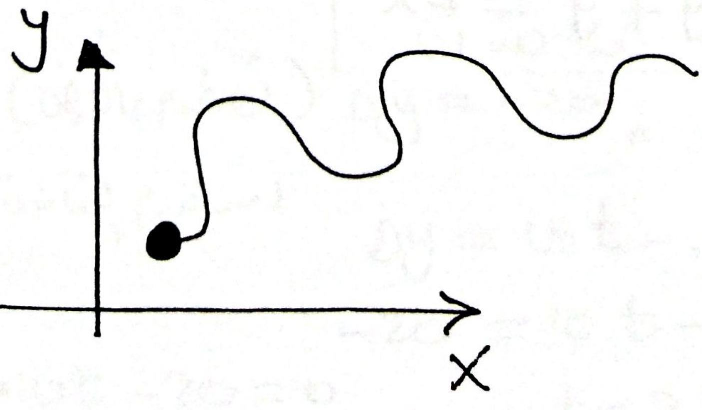
$v = 15t^2 - 4t$

$a = 30t - 4$

$x \rightarrow$ المسافة مع الـ x

$\dot{x} \rightarrow$ السرعة باتجاه الـ x

$\ddot{x} \rightarrow$ التسارع باتجاه الـ x



Ex) مثال على السقوط الحر.

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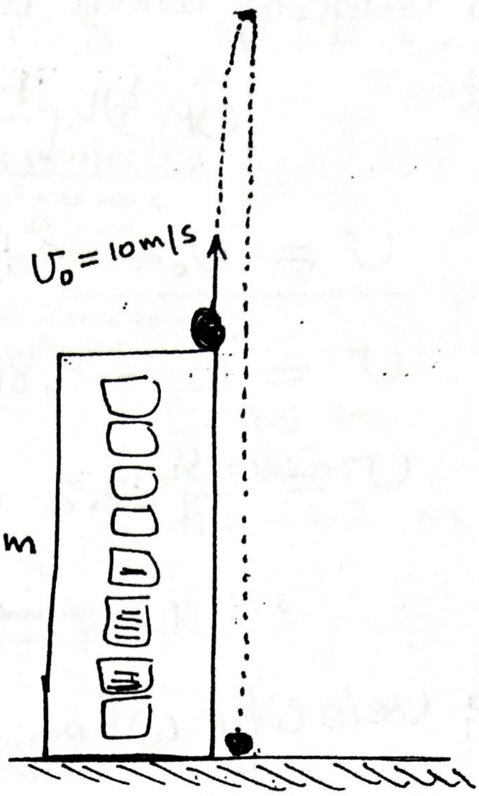


① the highest elevation
مقدار أقصى ارتفاع.

∴ يجب أن تكون السرعة
تساوي صفراً عند أقصى ارتفاع
$v = 0$

القانون الثالث

$$v^2 = v_0^2 - 2g\,\Delta y$$

$$0 = (10)^2 - 2(9.81)\,\Delta y \Rightarrow \Delta y = \frac{100}{2(9.81)}$$

$$\Delta y = 5.1\ m$$

② the time when the ball hits the ground?
أوجد الزمن اللازم للكرة لكي تصطدم بالأرض.

هذا يعني أن
∴ (تصطدم بالأرض) $\Delta y = -20$

استخدم القانون الثاني

$$\Delta y = v_0 t - \frac{1}{2} g t^2$$

$$-20 = 10\,t - \frac{1}{2}(9.81)\,t^2$$

$$4.905t^2 - 10t - 20 = 0 \Longrightarrow t = 3.28\ sec$$

عندما تصطدم الأرض

③ the velocity when hits the grounde

سرعة الاصطدام بالأرض

$$v = v_0 - gt$$

$$v = 10 - 9.81(3.28)$$

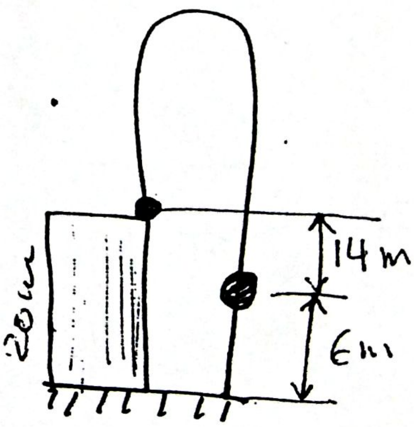
$$v = -22.2 \text{ m/s}$$

④ the velocity when it is at 6 m above the ground.

سرعة الجسم عندما كان على ارتفاع ٦م من سطح الأرض

هنا يعني أن الجسم ∴

$$\Delta y = -14$$



القانون الثالث

$$v^2 = v_0^2 - 2g\Delta y$$

$$v^2 = 100 - 2(9.81)(-14)$$

$$v^2 = 374.7 \Rightarrow \boxed{v = 19.4 \text{ m/s}}$$

سالب لأن الكرة ساقطة للأسفل

9

Ex $\vec{v} = (16t^2)\hat{i} + (4t^3)\hat{j} + (5t+2)\hat{k}$

Find ① $\vec{a}(2)$? Find the accelaration when $t = 2$ sec.

② $\vec{r}(2)$? Find the displacement (distance) when $t = 2$ sec.

الحل

① $\vec{a} = \dfrac{d\vec{v}}{dt}$

$= (32t)\hat{i} + (12t^2)\hat{j} + (5)\hat{k}$

at $t = 2$ sec

$\vec{a} \Rightarrow = 64\hat{i} + 48\hat{j} + 5\hat{k}$

المقدار / magnitude

مقدار بتاع ع بعد ما نثبتهم

$|\vec{a}| = \sqrt{(64)^2 + (48)^2 + (5)^2} = 80.2 \ m/s^2$

② $\vec{r} = \int \vec{v} \, dt$

$= \left(\frac{16}{3}t^3\right)\hat{i} + \left(\frac{4}{4}t^4\right)\hat{j} + \left(\frac{5}{2}t^2 + 2t\right)\hat{k}$

at $t = 2$ sec.

$\vec{r} = \left[\frac{16}{3}(2)^3\right]\hat{i} + \left[(2)^4\right]\hat{j} + \left[\frac{5}{2}(2)^2 + 2(2)\right]$

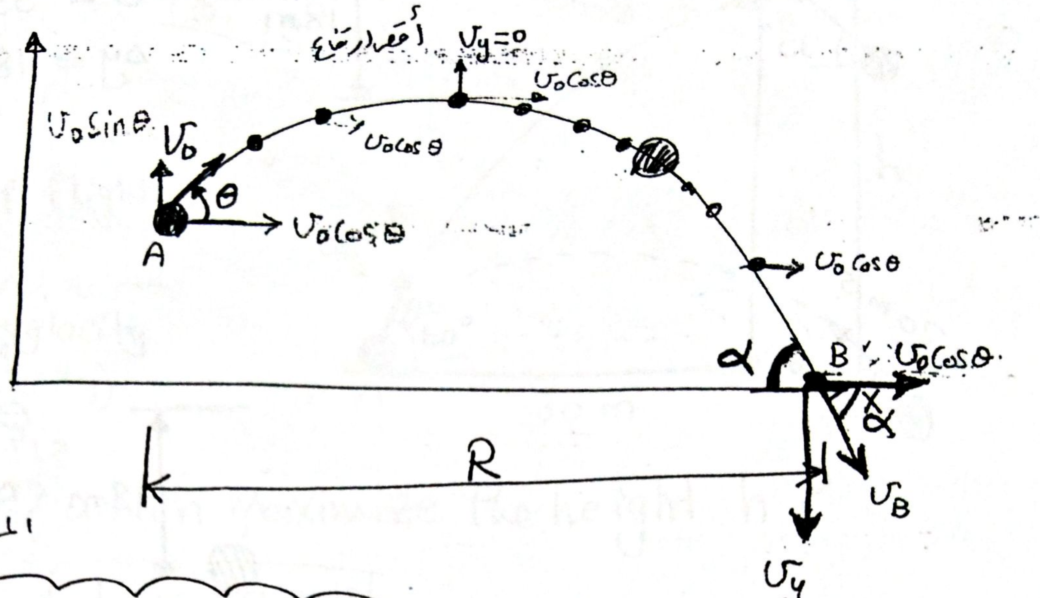
$= \frac{128}{3}\hat{i} + 16\hat{j} + 14\hat{k}$

مقدار الـ

$|\vec{r}| = \sqrt{\left(\frac{128}{3}\right)^2 + (16)^2 + (14)^2}$

Projectile Motion المقذوفات

(10)



المسافة = السرعة x الزمن

$$R = (V_0\cos\theta)\,t$$

$$\tan\alpha = \frac{V_y}{V_0\cos\theta}$$

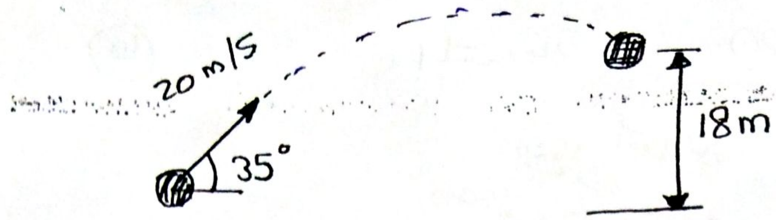
$$V_B = \sqrt{V_y^2 + (V_0\cos\theta)^2}$$

$$V_y = V_0\sin\theta - g\,t$$

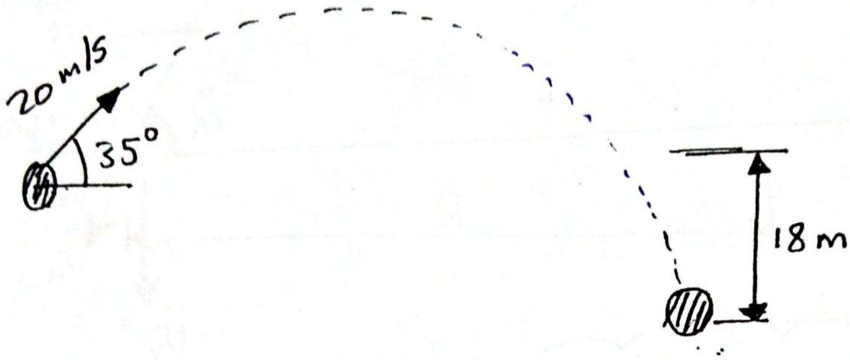
$$\Delta y = (V_0\sin\theta)\,t - \frac{1}{2} g t^2$$

$$V_y^2 = (V_0\sin\theta)^2 - 2g\,\Delta y$$

V_0 دائماً موجبه

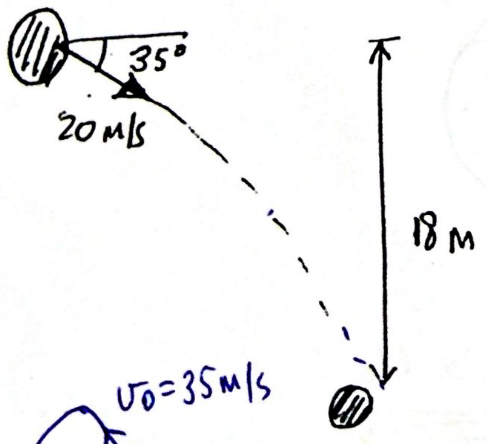


$v_0 = 20$ m/s
$\theta = 35^\circ$
$\Delta y = 18$ m

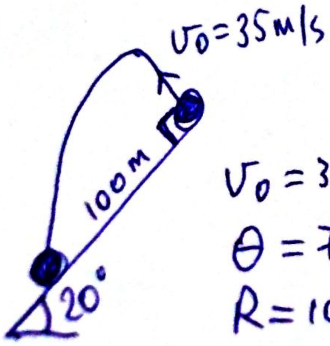


$v_0 = 20$ m/s
$\theta = 35^\circ$
$\Delta y = -18$ m

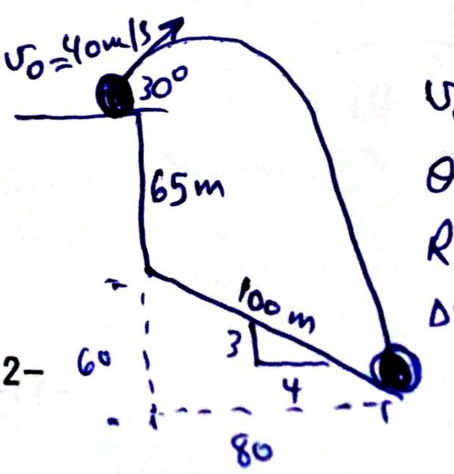
$v_0 = 20$ m/s
$\theta = 0$
$\Delta y = -18$ m



$v_0 = 20$ m/s
$\theta = -35^\circ$
$\Delta y = -18$ m



$v_0 = 35$ m/s
$\theta = 70^\circ$
$R = 100 \cos 20^\circ$
$\Delta y = -100 \sin 20^\circ$

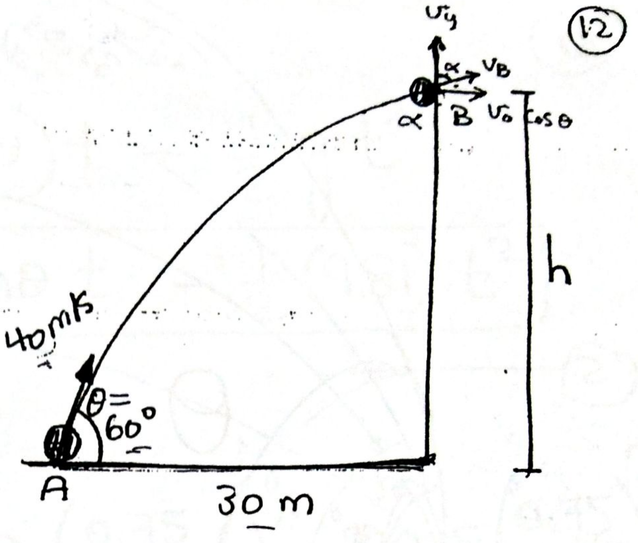


$v_0 = 40$ m/s
$\theta = 30^\circ$
$R = 80$ m
$\Delta y = -125$ m

EX

مثال
LO. 12

(12)



① Find h ?

زمن التحليق

② time of flight ?

سرعة الاصطدام بالجدار

③ Impact velocity
(Hit velocity)

وزاويه

④ Find θ ? which maximize the height h ?

الحل

$$\boxed{\begin{aligned} v_0 &= 40 \text{ m/s} \\ \theta &= 60^\circ \\ R &= 30 \end{aligned}}$$

* $R = (v_0 \cos\theta)\, t$ time of flight
زمن التحليق (الطيران)

$30 = (40 \cos 60)\, t \Rightarrow t = 1.5$ sec.

* $h = (v_0 \sin\theta)\, t - \frac{1}{2} g t^2$

$h = (40 \sin 60)(1.5) - \frac{1}{2}(9.81)(1.5)^2$

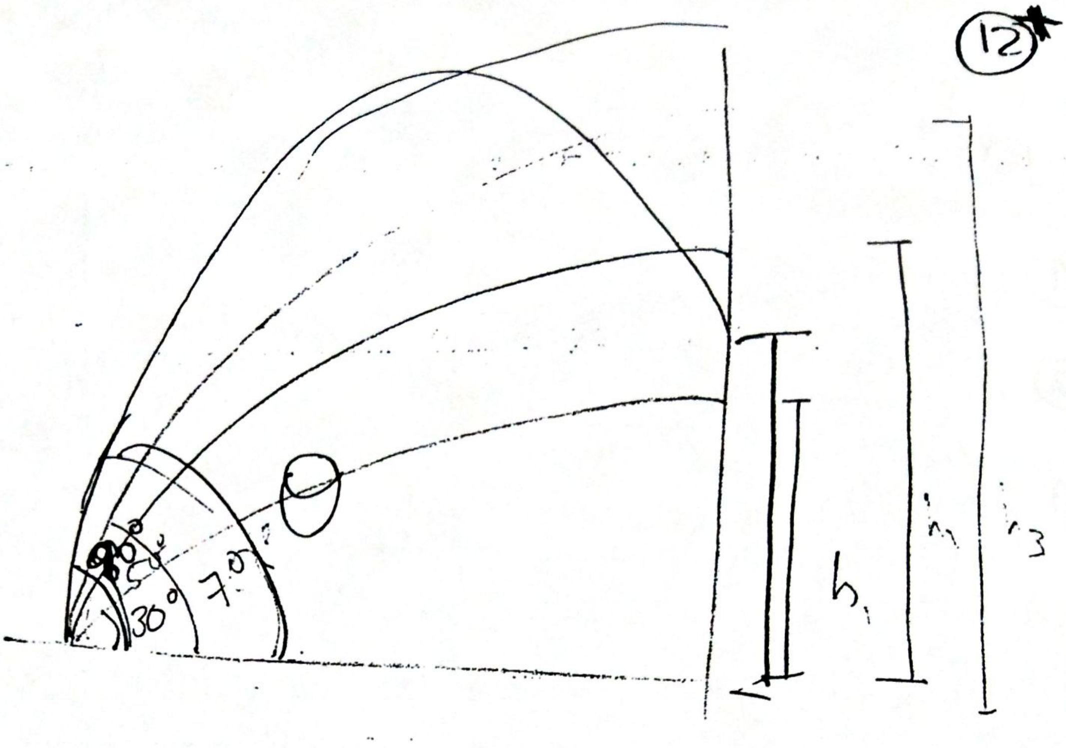
$h = 40.9$ m.

الاداء (ثابتة)

* المركبة الافقية للسرعة $= v_0 \cos\theta = 40 \cos 60^\circ = 20$ m/s.

* المركبة الرأسية للسرعة $= v_y = v_0 \sin\theta - g t$

$= 40 \sin 60 - 9.81(1.5) = 19.93$ m/s



(4) Find θ $\Rightarrow$ max. height $\underline{\underline{h}}$?

* $R = (v_0 \cos\theta)\, t$

$30 = 40 \cos\theta\, t$

$\cos\theta\, t = \frac{3}{4} = 0.75$

$t = \frac{0.75}{\cos\theta}$ —— (1)

$t = 0.75 \sec\theta$ —— (1)'

④ $\Delta y = (v_0 \sin\theta) t - \frac{1}{2} g t^2$

$$\boxed{h = 40 \sin\theta \, t - \frac{1}{2}(9.81) t^2}$$ ②

$$h = 40 \sin\theta \left(\frac{0.75}{\cos\theta}\right) - 4.905 \left(\frac{0.75}{\cos\theta}\right)^2$$

$$\boxed{h = 30 \tan\theta - 2.76 \sec^2\theta}$$

maximize h $\Rightarrow$ $\frac{dh}{d\theta} = 0$

$$\Rightarrow 30 \sec^2\theta - (2.76)(2) \sec\theta (\sec\theta \tan\theta) = 0$$

$$30 \cancel{\sec^2\theta} - 5.52 \cancel{\sec^2\theta} \tan\theta = 0$$

$$\tan\theta = \frac{30}{5.52} \Rightarrow \boxed{\theta = 80^\circ}$$

عوّض

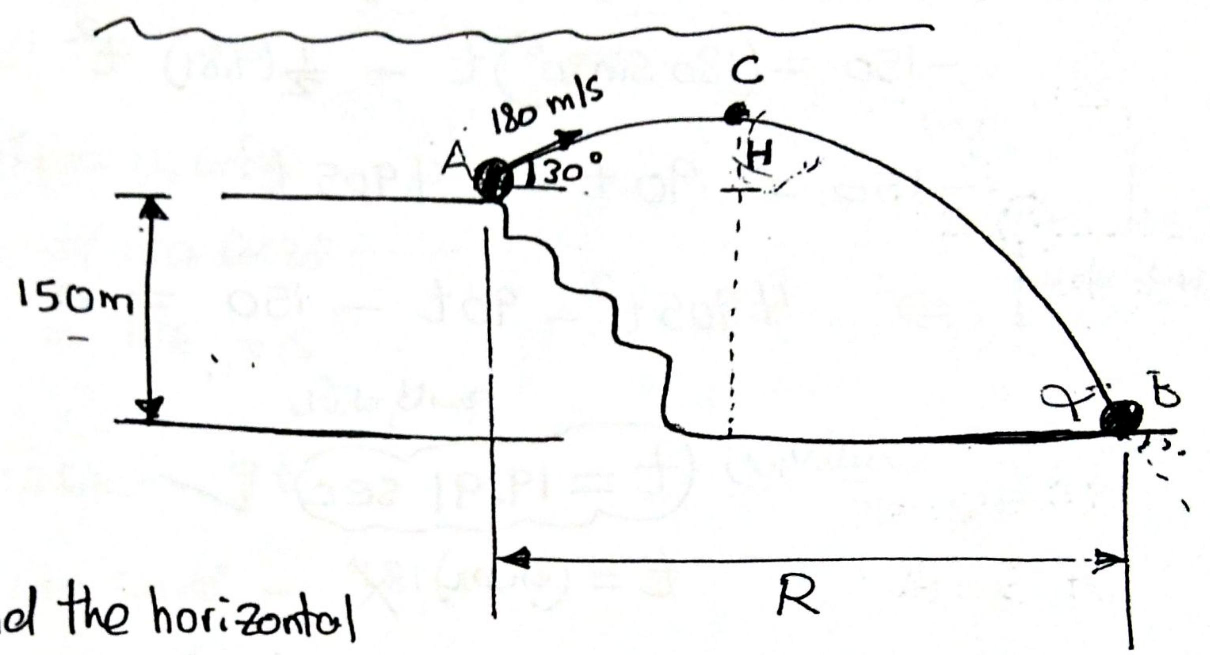
$$* h = 30 \overset{170.1}{\tan 80^\circ} - \overset{91.5}{2.76 \sec^2 80^\circ} \rightarrow \frac{1}{}$$

$\Rightarrow \sec\theta = \frac{1}{\cos\theta}$

$\rightarrow h = 79 \text{ m}$

بالآلة الحاسبة

Ex)



① Find the horizontal distance R ?

② the greatest elevation above the ground

أقصى ارتفاع تصل له الكرة عن الأرض

زمن التحليق كاملاً

③ the time of flight ??

④ the time for the highest elevation الزمن اللازم للكرة للوصول لأقصى ارتفاع

⑤ the velocity of Impact (Hit) and the Angle of Impact (Angle of fall) زاوية السقوط

$V_0 = 180\ m/s$

$\theta = 30°$

$h = -150\ m$.

① $\Delta y = (V_0 \sin\theta)t - \frac{1}{2} g t^2$

$-150 = (180 \sin 30°)t - \frac{1}{2}(9.81)\ t^2$

$-150 = 90t - 4.905t^2$

معادله تربيعيه $\Rightarrow$ $4.905t^2 - 90t - 150 = 0$

بالآله الحاسبه

زمن التحليقة $t = 19.91\ sec$ ✓

$t =$ سالب X

$* R = (V_0 \cos\theta)\ t$

$= (180 \cos 30)(19.91)$

$= 3100\ m$.

* أقصى ارتفاع $(V_y = 0)$

$V_y^2 = (V_0 \sin\theta)^2 - 2g\ \underline{H}$ ارتفاع

$0 = (180 \sin 30)^2 - 2(9.81)H \Rightarrow H = 413\ m$

أقصى ارتفاع عن مستوى الأرض $= 413 + 150 = 563\ m$.

زمن أقصى ارتفاع $\quad v_y = v_0 \sin\theta - g t$

$*\quad 0 = 180 \sin 30 - 9.81\,(t)$

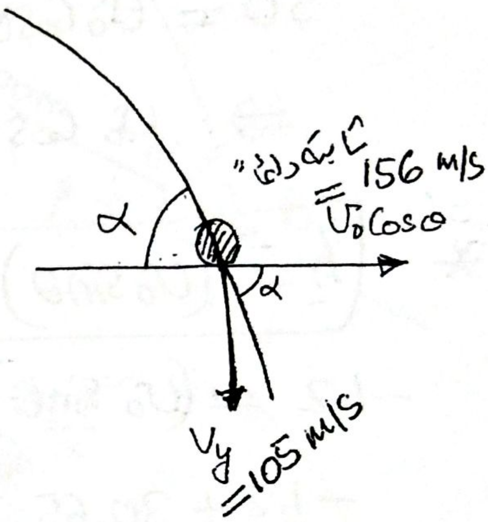
$$t = 9.2 \ \text{sec}$$

الاصطدام بالارض

$*$ المركبة الافقية $= v_0 \cos\theta$

$= 180 \cos 30^\circ$

$= 156 \ m/s$



$*\quad v_y = v_0 \sin\theta - g t$

$= 180 \sin 30^\circ - 9.81\,(19.91)$

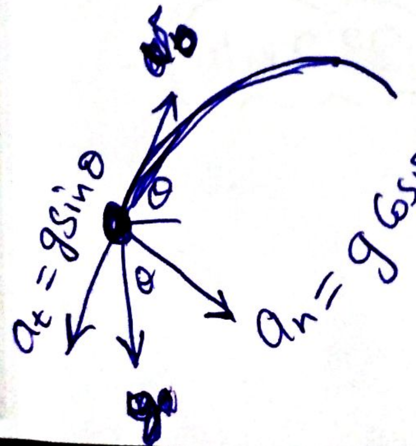
$= -105 \ m/s$.

$*\quad v_B = \sqrt{(156)^2 + (105)^2} = 188 \ m/s$.

$\rho \equiv$ radius of Curvature.

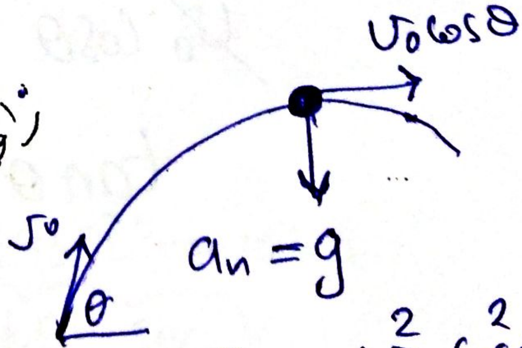
$*\quad \alpha = \tan^{-1} \dfrac{v_y}{v_0 \cos\theta}$

$= \tan^{-1} \dfrac{105}{156} \Rightarrow \alpha = 34^\circ$ (زاوية



$\rho = \dfrac{v^2}{a_n}$

$\rho = \dfrac{v_0^2}{g \cos\theta}$

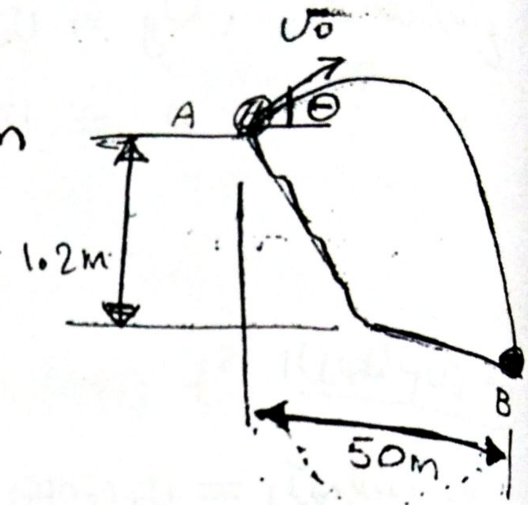


$\rho = \dfrac{v^2}{a_n} = \dfrac{v_0^2 \cos^2\theta}{g}$

12–94 الزمن الكلي $t = 2.5$ sec.

$R = 50$ m

$h = -1.2$ m



* $\boxed{R = (v_0 \cos\theta)\, t}$

$50 = v_0 \cos\theta (2.5)$

$\Rightarrow \quad v_0 \cos\theta = 20 \qquad \text{——— (1)}$

* $\boxed{h = (v_0 \sin\theta)\, t - \frac{1}{2} g t^2}$

$-1.2 = (v_0 \sin\theta)(2.5) - \frac{1}{2}(9.81)(2.5)^2$

$\frac{-1.2 + 30.65}{2.5} = v_0 \sin\theta$

$v_0 \sin\theta = 11.78 \qquad \text{——— (2)}$

اقسم المعادلة الثانية على المعادلة الأولى

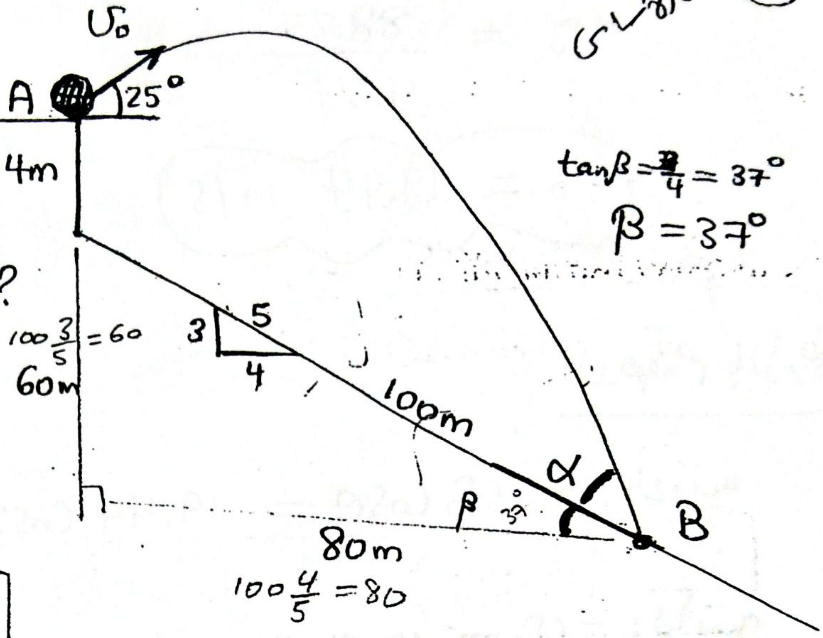
$\frac{\cancel{v_0} \sin\theta}{\cancel{v_0} \cos\theta} = \frac{11.78}{20}$

$\tan\theta = 0.589 \Rightarrow \theta = 30.5^\circ$

$v_0 = 23.2$ m/s

12-110

(17) (18)



$\tan\beta = \frac{3}{4} = 37^\circ$

$\beta = 37^\circ$

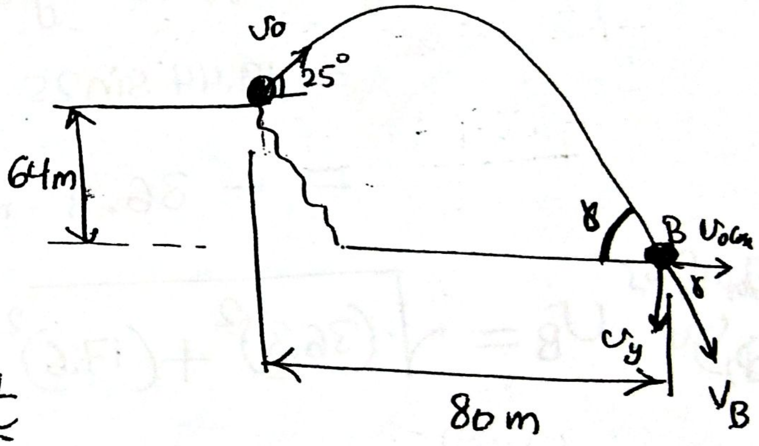
Find

① the initial velocity? (speed.)

② time of flight?

③ سرعة الارتطام بالأرض
وزاوية الارتطام

$$\boxed{\begin{aligned} \theta &= 25^\circ \\ R &= 80\ m \\ h &= -64\ m \end{aligned}}$$



$R = (v_0 \cos\theta)\, t$

* $80 = (v_0 \cos 25^\circ)\, t$

$v_0 t = 88.27$ —— (1)

* $h = (v_0 \sin\theta)\, t - \frac{1}{2} g t^2$

$-64 = (v_0 t) \sin 25^\circ - \frac{1}{2}(9.81)\, t^2$

$-64 = (88.27) \sin 25^\circ - 4.905\, t^2$

$-64 = 37.3 - 4.905\, t^2 \Rightarrow 4.905 t^2 = 101.3$

$\Rightarrow \boxed{t = 4.54\ sec}$

$$v_0 t = 88.27$$

$$v_0 = \frac{88.27}{4.54}$$

$$v_0 = 19.44 \ m/s$$

الاصطدام بالأرض

الافقية $= v_0 \cos\theta = 19.44 \cos 25^\circ = 17.6 \ m/s$

الرأسية $= v_y = v_0 \sin\theta - gt$

$$= 19.44 \sin 25 - 9.81 (4.54)$$

$$= -36.3 \ m/s$$

سرعة الاصطدام بالأرض $v_B = \sqrt{(36.3)^2 + (17.6)^2} = 40.3 \ m/s$

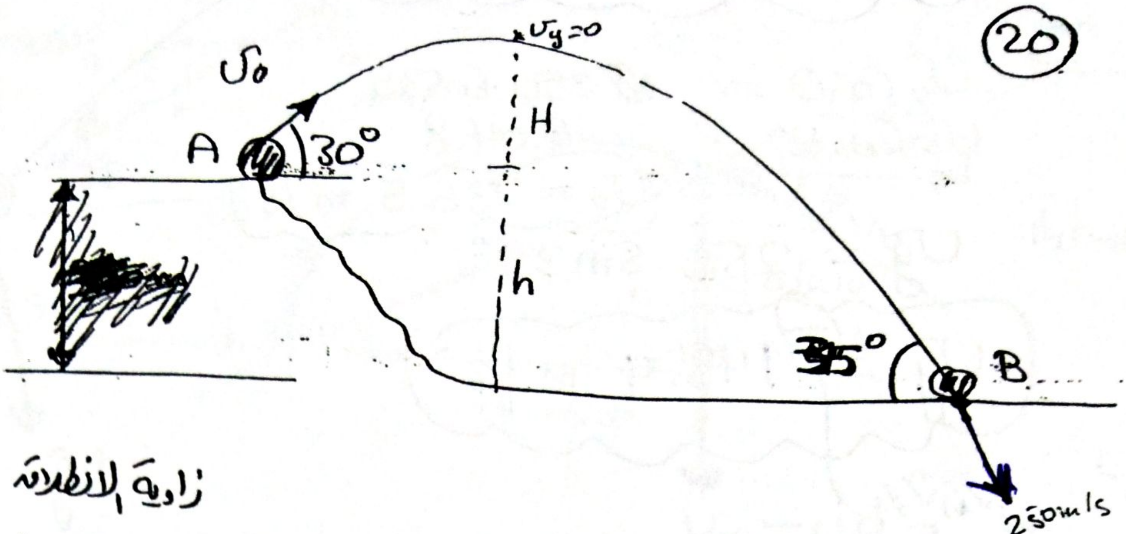
$$\tan\gamma = \frac{v_y}{v_0 \cos\theta} = \frac{36.3}{17.6}$$

$$\Rightarrow \gamma = 64.1^\circ$$

$$\gamma = \alpha + \beta$$

$$\alpha = 64.1 - 37^\circ \Rightarrow \alpha = 27.1^\circ$$

Ex



$\theta = 30^\circ$ زاوية الانطلاق

$\alpha = 35^\circ$ زاوية السقوط

$U_B \equiv$ سرعة الاصطدام بالارض $= 250$ m/S .

أوجد

① سرعة الانطلاق U_0

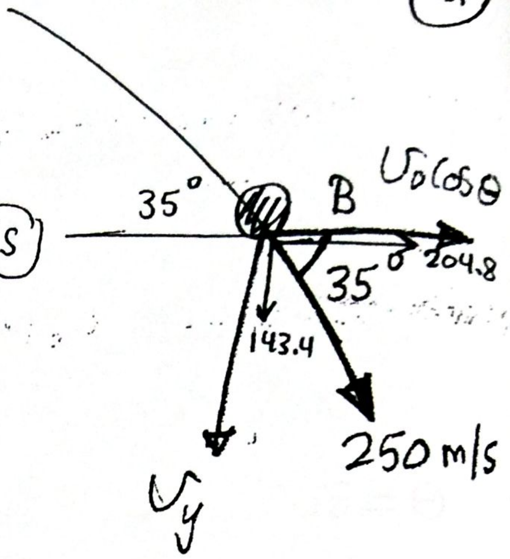
② $\underline{\underline{t}}$ زمن التحليق

③ $\underline{\underline{R}}$ المدى الافقي

④ أقصى ارتفاع عن الارض

$V_x \equiv$ Constant.

$$\boxed{V_0 \cos\theta = V_f \cos\alpha}$$



الافقيه $\Rightarrow$ $V_0 \cos\theta = 250 \cos 35°$

$V_0 \cos 30° = 204.8$

$\Rightarrow \boxed{V_0 = 236.5 \text{ m/s}}$

الرأسيه $V_y = 250 \sin 35°$

$$\boxed{V_y = -143.4 \text{ m/s}}$$

الركبه الرأسيه بعد الاتجاه

$V_y = V_0 \sin\theta - g\,t$

$-143.4 = 236.5 \sin 30° - 9.81\, t$

زمن الحركة $\boxed{t = 26.7 \text{ sec}}$

* $R = (V_0 \cos\theta)\, t$

$R = (236.5 \cos 30)(26.7)$

$R = 5468.6$ m

أقصى ارتفاع

* $V_y^2 = (V_0 \sin\theta)^2 - 2g\,H$

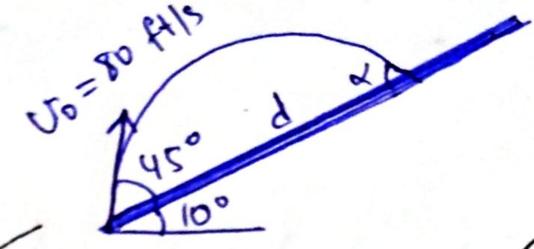
$0 = (236.5 \sin 30)^2 - 2(9.81)\, H$

$\Rightarrow H = 712.7$ m

* $h = (236.5 \sin 30)(26.7) - \frac{1}{2}(9.81)(26.7)^2 = -339.5$ m

الارتفاع عن الارض

$= 339.5 + 712.7 = 1052.2$ m

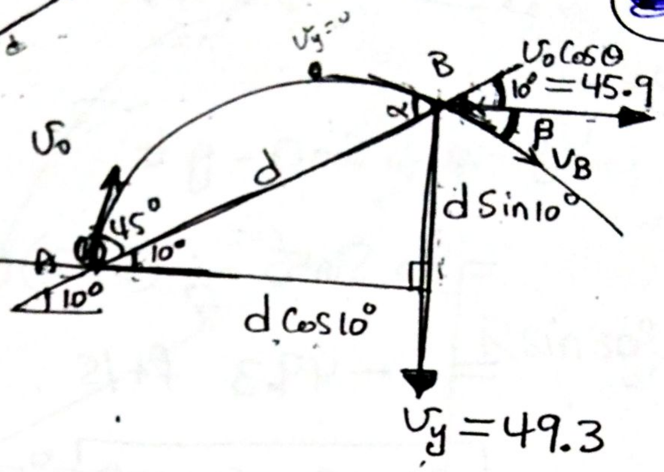


12-102

Find d?

$v_0 = 80$ ft/s.

$\theta = 55°$



$$* \quad R = (v_0 \cos\theta)\, t$$

$$d \cos 10° = (80) \cos 55°\, t$$

$$0.984\, d = 45.89\, t$$

$$\boxed{d = 46.63\, t} \quad \text{—— (1)}$$

$$* \quad \Delta y = (v_0 \sin\theta)\, t - \tfrac{1}{2} g t^2$$

$$d \sin 10° = (80 \sin 55)\, t - \tfrac{1}{2}(32.2) t^2$$

$$\boxed{0.174\, d = 65.53\, t - 16.1\, t^2} \quad \text{—— (2)}$$

عوض

$$0.174\,[46.63]\, t = 65.53\, t - 16.1\, t^2$$

$$\Rightarrow 16.1\, t^{\cancel{2}} = 57.42\, \cancel{t}$$

$$t = \frac{57.42}{16.1} = 3.566 \text{ sec}$$

$$d = 46.63\,(3.566)$$

$$d = 166.3 \text{ ft}$$

أوجد زاوية مقوطة α ؟

أكثر قطعة

* $v_y = v_0 \sin\theta - gt$

$= 80 \sin 55 - (32.2)(3.566)$

$= -49.3 \ ft/s.$

* $v_B = \sqrt{(49.3)^2 + (45.9)^2} = 67.3 \ ft/s$

$\tan\beta = \frac{49.3}{45.9} \Rightarrow \boxed{\beta = 47^\circ}$

$\alpha = 10^\circ + \beta$

$\alpha = 10 + 47^\circ$

$\boxed{\alpha = 57^\circ}$

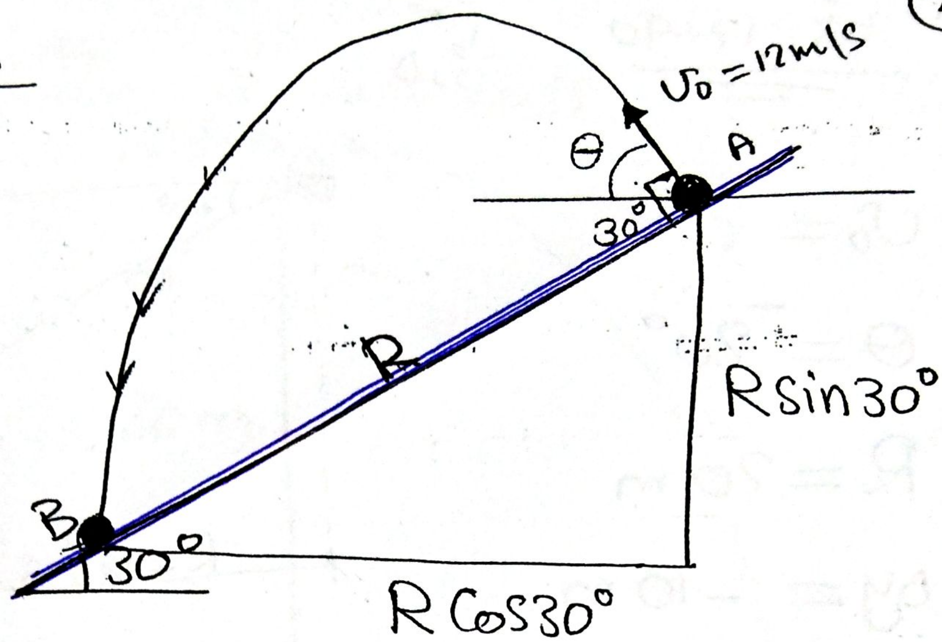
* زمن أقصى ارتفاع $\Leftarrow (v_y = 0)$

$v_y = v_0 \sin\theta - gt$

$0 = 80 \sin 55 - 32.2(t)$

$t = \frac{80 \sin 55}{32.2} \Rightarrow t = 2 \text{ sec.}$

Ex 12–91



Find R ⇒

$v_0 = 12 \text{ m/s}$

$\theta = 90 - 30 = 60^\circ$

$R = v_0 \cos\theta \, t$

$R \, 0.866 = 12 \cos 60 \, t$

$\boxed{R = 6.93 t}$ —— (1)

$-h = v_0 \sin\theta \, t - \frac{1}{2} g t^2$

$-R \, 0.5 = 12 \sin 60 \, t - 4.905 t^2$ —— (2)

$-(6.93 t) \, 0.5 = 10.4 t - 4.905 t^2$

$4.905 \cancel{t^2} = 13.865 \cancel{t}$

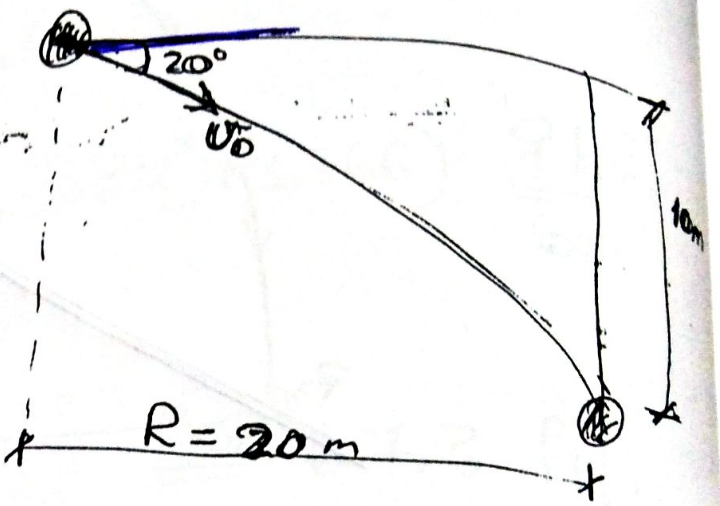
$\boxed{t = 2.83 \text{ sec}}$

$\boxed{R = 19.6 \text{ m}}$

الرمي للأسفل مباشرة
بزاوية 20°.

Ex 12-90

مهم جداً



$v_0 = ?$

$\theta = -20°$

$R = 20\ m$

$\Delta y = -10\ m$

$$* \quad 20 = v_0 \cos(-20°)\, t$$

$$v_0 t = 21.3 \longrightarrow (1)$$

$$* \quad -10 = v_0 \sin(-20)\, t - \frac{1}{2}(9.81)\, t^2 \longrightarrow (2)$$

$$-10 = 21.3 \sin(-20) - \frac{1}{2}(9.81)\, t^2$$

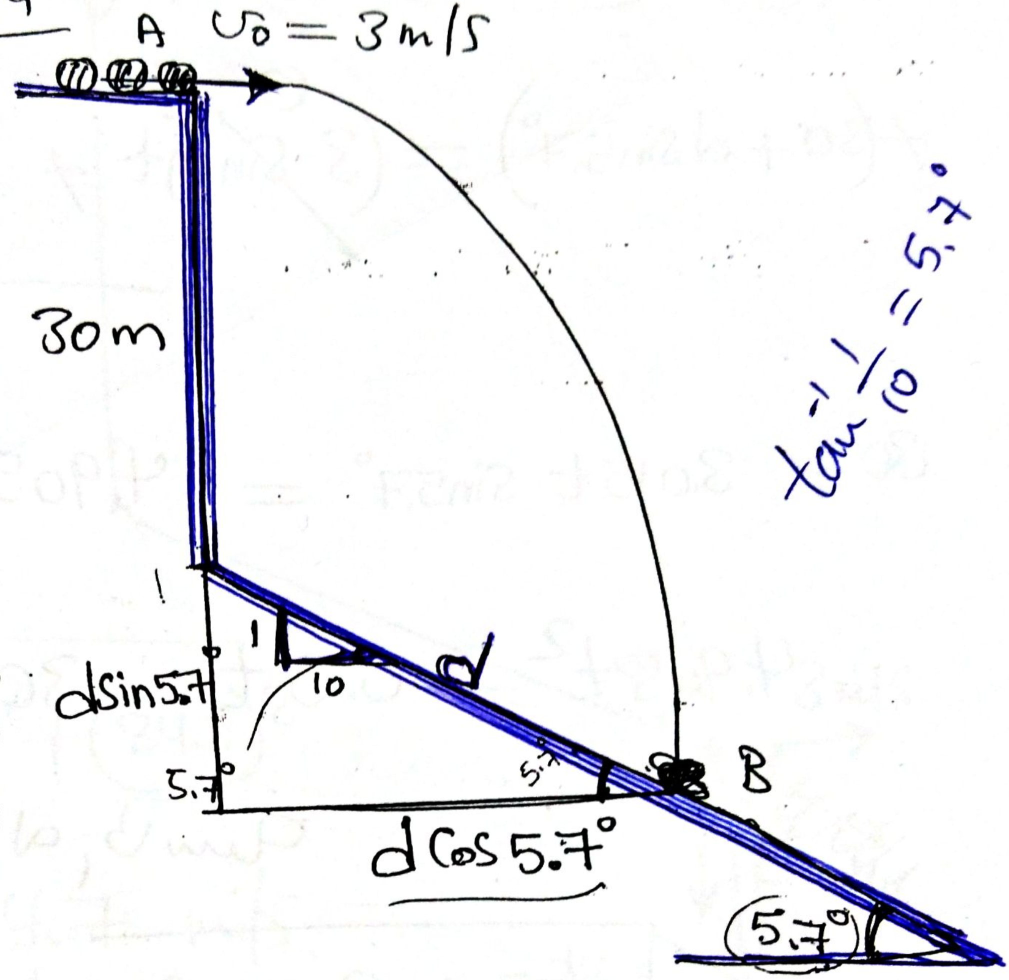
$$t = 0.75\ sec$$

$$v_0 = 28.6\ m/s$$

Ex 12-94

$\boxed{\theta = 0}$

$\boxed{v_0 = 3\,m/s}$



أوجد سرعة
اصطدام الحجارة
بالأرض

v_B

$$\Delta y = -(30 + d \sin 5.7°)$$

$$R = d \cos 5.7°$$

$$d \cos 5.7° = (3 \cos 0)\, t \quad —— (1)$$

$$\boxed{d = 3.015\, t}$$

(27)

$$\Delta y = (v_0 \sin\theta)t - \frac{1}{2} g t^2$$

$$+(30 + d \sin 5.7^\circ) = (3 \cancel{\sin\theta}^{\,0})t + \frac{1}{2}(9.81)t^2$$

(2)

$$30 + 3.015 t \sin 5.7^\circ = 4.905 t^2$$

$$4.905 t^2 - 0.3 t - 30 = 0$$

بالآلة الحاسبة

$$\boxed{t = 2.5 \text{ sec}}$$

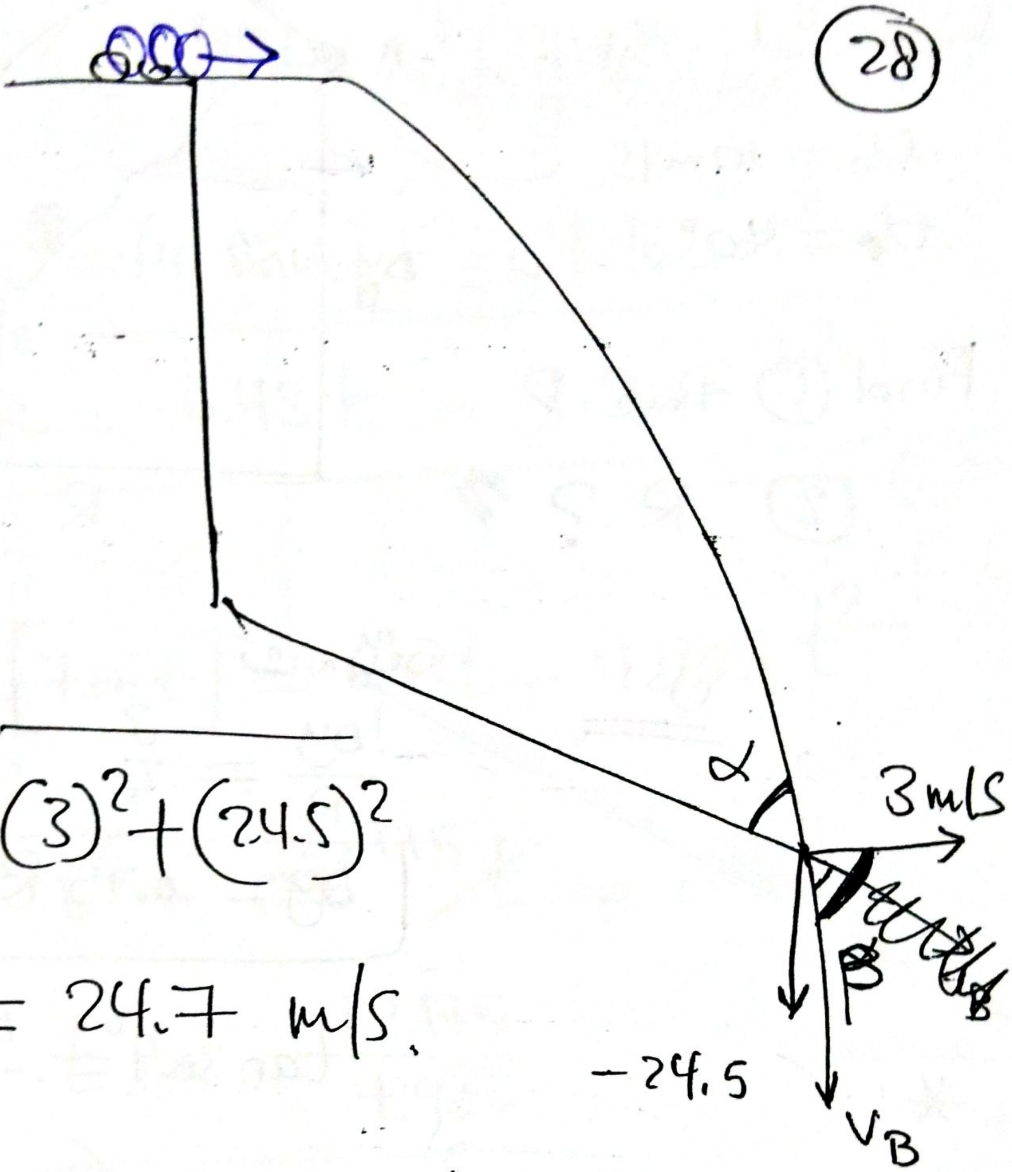
خطأ (سالب) $\boxed{\cancel{t = -2.44 \text{ sec}}}$ X

$$v_y = v_0 \cancel{\sin\theta}^{\,0} - gt$$

$$v_y = -9.81(2.5)$$

$$\boxed{v_y = -24.5 \text{ m/s}}$$

(28)



$$U_B = \sqrt{(3)^2 + (24.5)^2}$$

$$U_B = 24.7 \text{ m/s}$$

✓

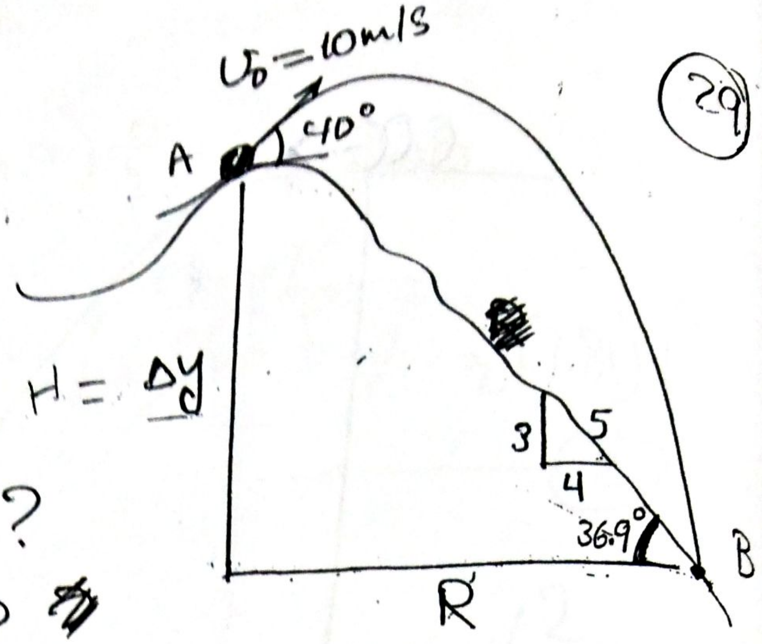
$$\tan\beta = \frac{24.5^\circ}{3} \Rightarrow \beta = 83^\circ$$

$$\alpha = 83 - 5.7^\circ$$

$$\boxed{\alpha = 77.3^\circ}$$

مثال

17-88



$v_0 = 10 m/s$

$\theta_0 = 40°$

Find (1) time ?

(2) R ?

الحل

تشابه مثلثات

$$\frac{\Delta y}{R} = \frac{3}{4}$$

$$\boxed{\Delta y = -0.75\,R}$$

*

او

$$\tan 36.9° = \frac{\Delta y}{R}$$

$$\Delta y = (\tan 36.9°)\,R$$

$$\boxed{\Delta y = -0.75\,R}$$

* $R = (v_0 \cos\theta)\,t$

$R = (10 \cos 40°)\,t \Rightarrow \boxed{R = 7.66\,t}$ ①

$$\Delta y = (v_0 \sin\theta)\, t - \frac{1}{2} g t^2$$

$$-0.75R = (10 \sin 40)\, t - 4.905\, t^2$$

$$\boxed{-0.75R = 6.43t - 4.905\, t^2} \quad (2)$$

$$-0.75\, [7.66\, t] = 6.43t - 4.905\, t^2$$

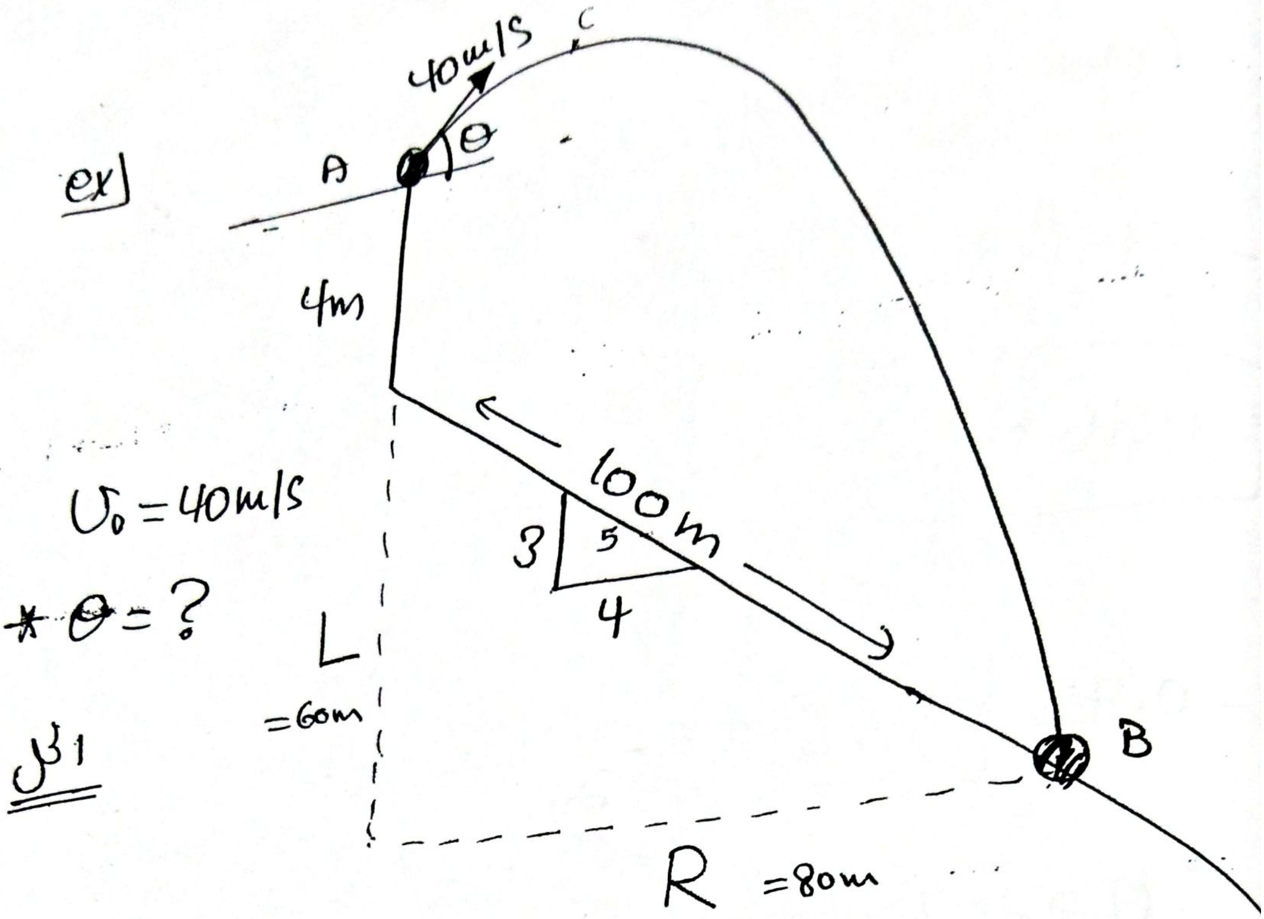
$$4.905 t^{\cancel{2}} - \overset{12.175}{\cancel{0.6}}\, \cancel{t} = 0$$

$$t = \frac{12.175}{4.905}$$

$$\boxed{t = 2.5 \text{ sec}}$$

$$R = 7.66\,(2.5)$$

$$\boxed{R = 19 \text{ m}}$$



$$* \; R = 100 \, \frac{4}{5} = 80 \text{ m}$$

$$* \; L = 100 \, \frac{3}{5} = 60 \text{ m}$$

$$* \quad R = (U_0 \cos\theta)\, t$$

$$80 = 40 \cos\theta \; t$$

$$\boxed{\cos\theta \; t = 2} \quad \text{—— } (1)$$

$$t = \frac{2}{\cos\theta}$$

$$\Delta y = (v_0 \sin\theta)\, t - \frac{1}{2} g t^2$$ (32)

$$\boxed{-64 = (40 \sin\theta)\, t - 4.905\, t^2}$$ (2)

عوض

$$-64 = 40 \sin\theta \, \frac{2}{\cos\theta} - 4.905 \, \frac{4}{\cos^2\theta}$$

$\frac{1}{\cos\theta} = \sec\theta$

$$-64 = 80 \tan\theta - 19.6 \sec^2\theta$$

ولكن

من زر يا ضيات $\sec^2\theta = 1 + \tan^2\theta$

(مثلثات ، وريقة)

$$-64 = 80 \tan\theta - 19.6 \left[1 + \tan^2\theta\right]$$

$$-64 = 80 \tan\theta - 19.6 - 19.6 \tan^2\theta$$

$$\Rightarrow \quad 19.6 \tan^2\theta - 80 \tan\theta - 44.4 = 0$$

معادلة تربيعية

$\tan\theta = 5.32 \Rightarrow \boxed{\theta \cong 80^\circ}$ طالع فوق

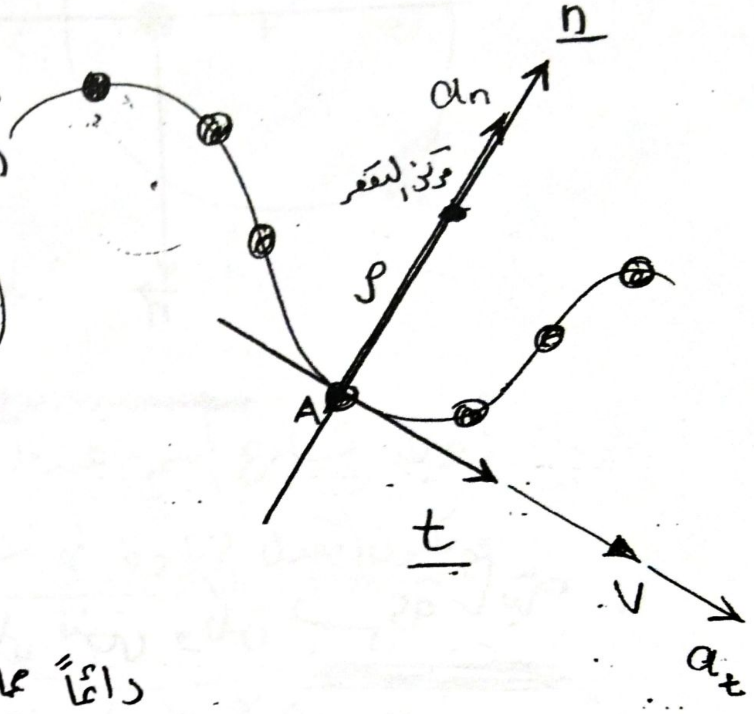
$\tan\theta = -0.502 \Rightarrow \boxed{\theta = -26^\circ}$ X نازل

Tangential (t) & Normal (n) Components (35)

المركبات العامودية و المماسية

التقعر
Curvature
قانون نصف القطر

المسار
Path
y(x)



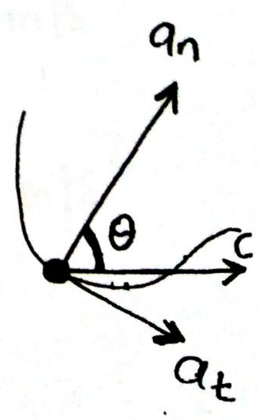
$$\rho = \frac{\left[1+(y')^2\right]^{3/2}}{y''}$$

* **Velocity**: دائماً مماسية للمسار (باتجاه (t))

$$v = \frac{ds}{dt}$$

* **Acceleration**: المركبة المماسية $\quad a_t = \dfrac{dv}{dt} = \dot{v}$

المركبة العامودية (باتجاه المركز) $\quad a_n = \dfrac{V^2}{\rho}$

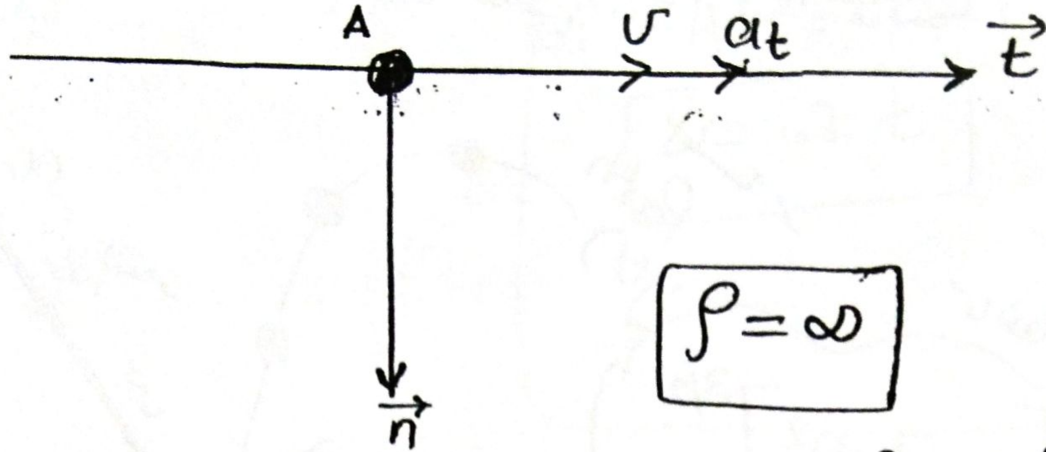


التسارع الكلي الكامل $\quad a = \sqrt{a_n^2 + a_t^2}$

اتجاه التسارع الكلي $\quad \tan\theta = \dfrac{a_t}{a_n}$

ملاحظات :-

① إفترض أن الجسم يسير بخط مستقيم



$$\boxed{\rho = \infty}$$

$$a_n = \frac{v^2}{\rho} = \frac{v^2}{\infty} = 0$$

$$\boxed{a_n = 0}$$

** ② الجسم يسير على منحنى ولكن بسرعة ثابتة .

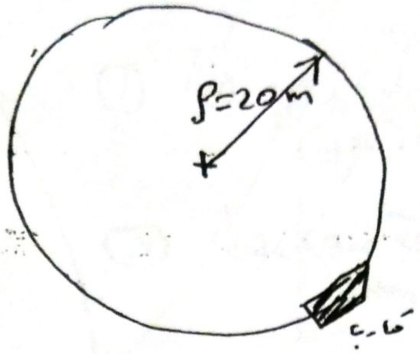
* $v =$ Constant

* $\Rightarrow a_t = 0$

$$a_n = \frac{v^2}{\rho}$$

ص ٥٧

Ex] 12-104



Find a?

when $v = 5 m/s$

and the rate of increase in the speed $2 m/s^2$

أوجد تسارع الجسم عندما تكون سرعة القارب ٥م/ث

ويكون معدل الزيادة في سرعة القارب تساوي ٢م/ث٢

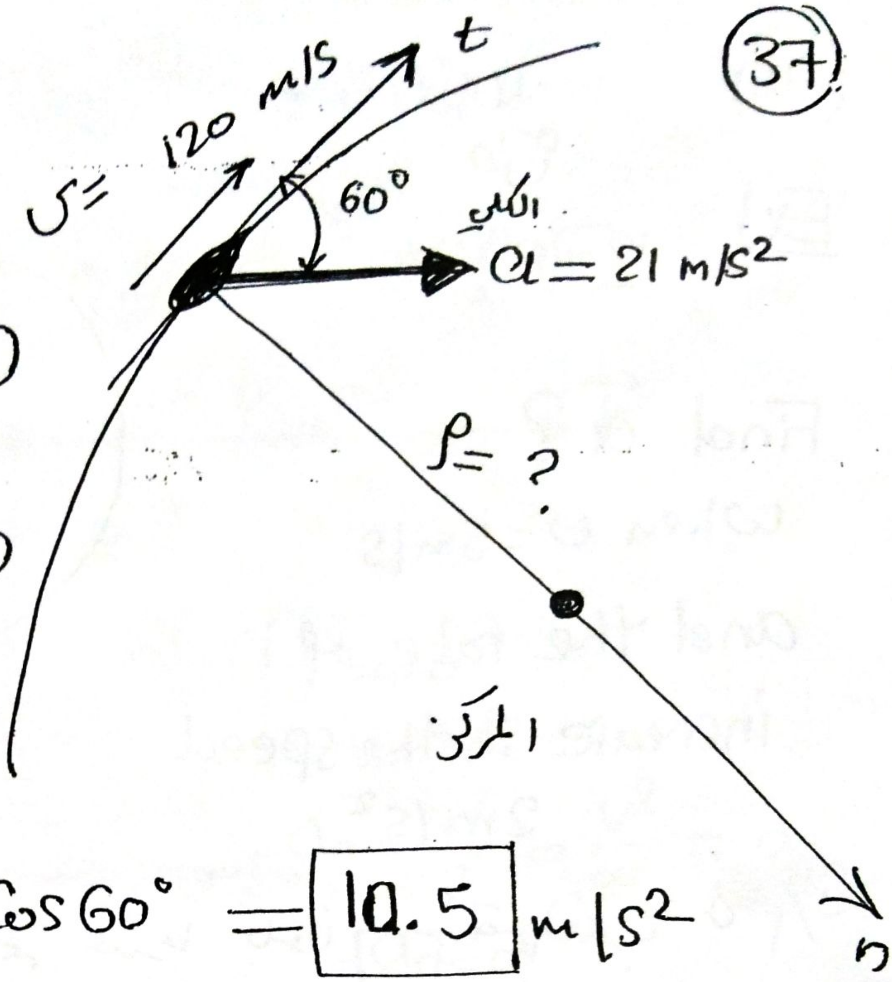
هو التسارع باتجاه tangent

$$* \; a_t = 2 \, m/s^2$$

$$* \; a_n = \frac{V^2}{\rho} = \frac{(5)^2}{20} = 1.25 \, m/s^2$$

$$* \; a = \sqrt{(2)^2 + (1.25)^2} = 2.358 \, m/s^2$$

EX) 12-102



① أوجد معدل الزيادة في السرعه (a_t)

ⓒ أوجد نصف قطر المسار ρ

* $a_t = 21 \cos 60^\circ = \boxed{10.5}$ m/s²

* $a_n = 21 \sin 60^\circ = 18.2$ m/s²

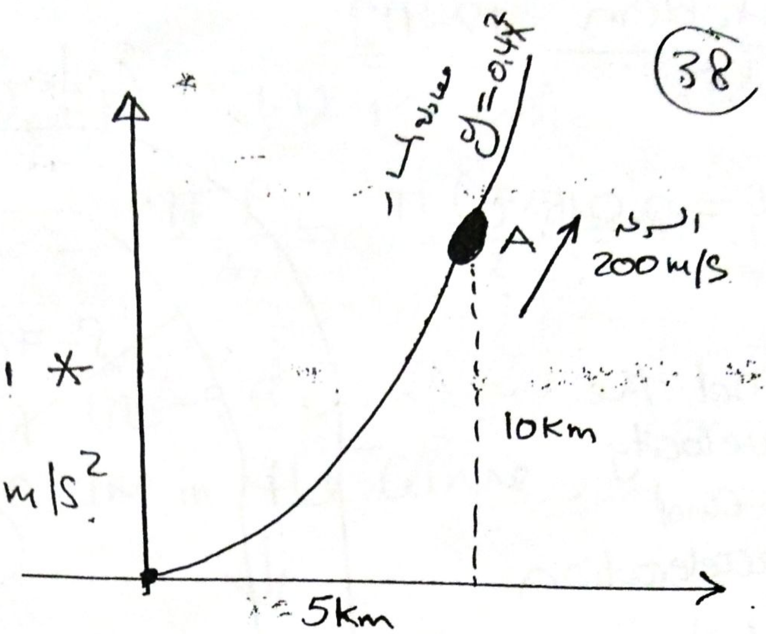
* $a_n = \dfrac{V^2}{\rho}$

$18.2 = \dfrac{(120)^2}{\rho}$

$\Rightarrow \rho = \dfrac{(120)^2}{18.2} \Rightarrow \boxed{\rho = 791 \text{ m}}$

12-106

$y = 0.4x^2$



* السرعة تتزايد بمعدل 0.8 m/s^2

$\Rightarrow a_t = 0.8\ m/s^2$

أوجد التسارع الكلي

Find a ?

عوض $x = 5$ Km

الحل

$y = 0.4x^2 \Rightarrow y = 10$ Km

$y' = 0.8x \Rightarrow y' = 4$

$y'' = 0.8 \Rightarrow y'' = 0.8$

$$\rho = \frac{\left[1 + y'^2\right]^{3/2}}{y''} = \frac{\left[1 + (4)^2\right]^{3/2}}{0.8}$$

$*\ \rho = 87.6$ Km

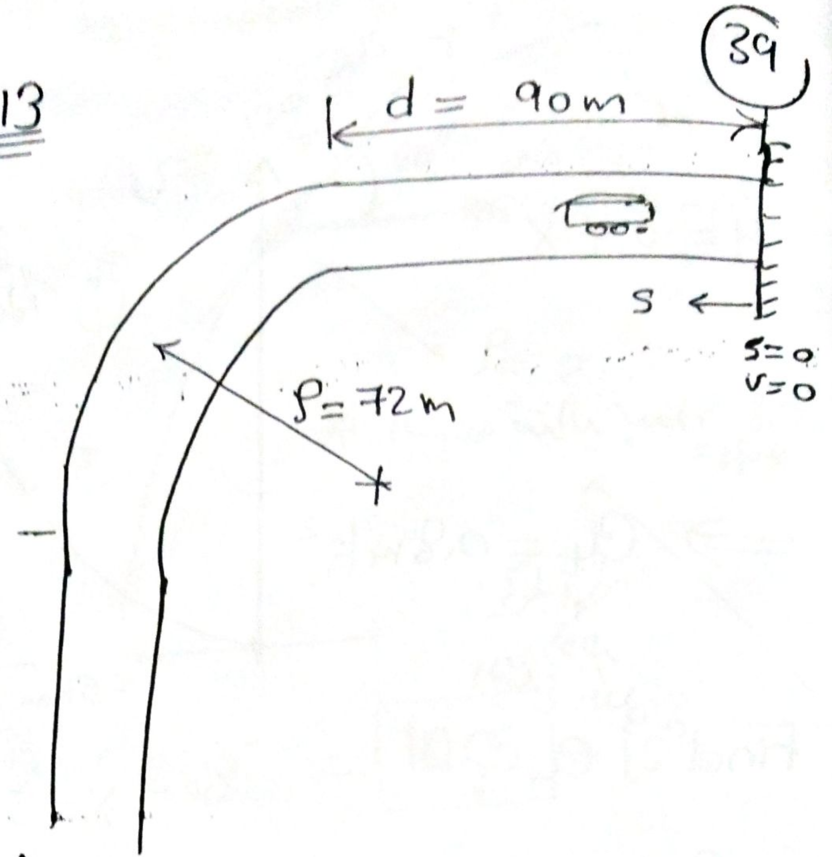
$$*\ a_n = \frac{v^2}{\rho} = \frac{(200)^2}{87.6 * 1000} = 0.456\ m/s^2$$

تحويل لمتر

$$*\ a = \sqrt{a_n^2 + a_t^2} = \boxed{0.92\ m/s^2}$$

Problem 12-113

$\dot{v} = 0.015 t^2$



**Find the velocity and acceleration when $t = 18$ sec. ?

أوجد سرعة وتسارع
بعد مرور ١٨ ثانية.

الحل

تحديد مكان السيارة

$* \ \overset{\text{التسارع المماسي}}{a_t} = \dot{v} = 0.015 t^2$

السرعة $\Rightarrow v = \int 0.015 t^2 dt = \frac{0.015}{3} t^3 = 0.005 t^3$

المسافة $\Rightarrow S = \int 0.005 t^3 dt = \frac{0.005}{4} t^4 = 1.25 \times 10^{-3} t^4$

بعد مرور ١٨ ثانية
$t = 18$ sec

$* \ a_t = 4.86 \ m/s^2$

$* \ v = \boxed{29.2} \ m/s$

$* \ S = 131.2 \ m$

(40)

محيط ربع الدائرة (خلف الورقة)

$$= \frac{\pi \rho}{2} = \frac{\pi (72)}{2} = 113.1\,m$$

∴ نحن في المنحنى

We are in the Curve.

$(90 < S < 203)$

$\Downarrow$

يوجد a_t ✓ a_n ✓

$$a_n = \frac{V^2}{\rho}$$

$$= \frac{(29.2)^2}{72} \Rightarrow a_n = 11.8\,m/s^2$$

$$\therefore a = \sqrt{a_n^2 + a_t^2}$$

$$= \sqrt{(11.8)^2 + (4.86)^2} = \boxed{12.8\,m/s^2}$$

ملاحظة Assume $d = 150\,m$.

∴ السيارة في الخط المستقيم الأول.

∴ * $\boxed{v = 29.2\,m/s}$

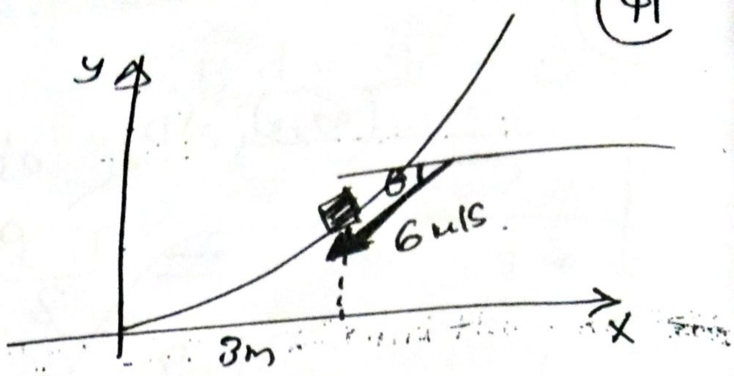
* $a_n = 0$

∴ $a_t = 4.86\,m/s^2$

$\boxed{a = 4.86\,m/s^2}$

41

Ex) $y = \frac{1}{8}x^2$



* $a_t = 1.8\ m/s^2$
* $v = 6\ m/s$
* Find the Car ~~velocity~~ direction ? أوجد اتجاه السيارة
* Find a ?
* Find ρ ?

الحل

$$y' = \frac{2}{8}x = \frac{1}{4}x = \frac{1}{4}(3) = 0.75$$

$$y'' = \frac{1}{4} = 0.25$$

$$* \quad \rho = \frac{\left[1 + y'^2\right]^{3/2}}{y''} = \frac{\left[1 + (0.75)^2\right]^{1.5}}{0.25} = \boxed{7.8}\ m$$

$$* \quad a_n = \frac{v^2}{\rho} = \frac{(6)^2}{7.8} = \frac{36}{7.8} = 4.6\ m/s^2$$

$$* \quad a = \sqrt{a_n^2 + a_t^2} = \cancel{4.6}\ \boxed{4.95}\ m/s^2$$

اتجاه السيارة

هو اتجاه الـ

Tangent , ميل المماس ⇒

يعني

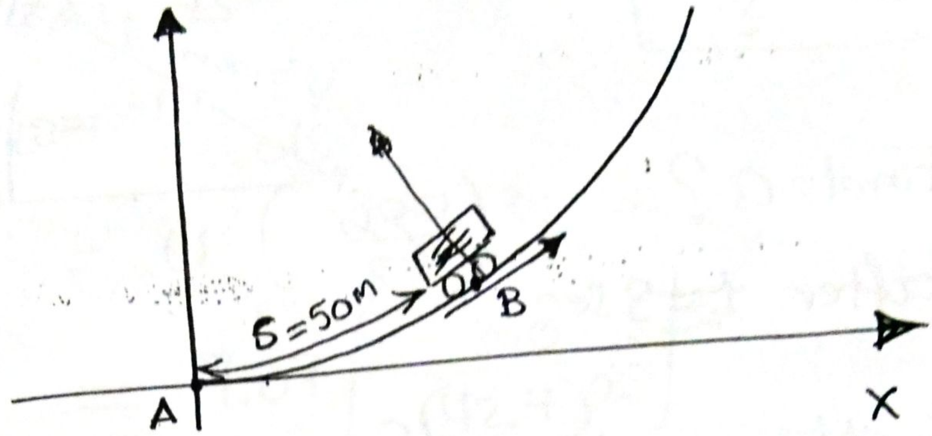
$$\boxed{\tan\theta = y'} = \text{ميل المماس}$$

$$\tan\theta = 0.75 \Rightarrow \boxed{\theta = 36.9^\circ}$$

Ex] 12.32

$\boxed{v = 0.2\,S}$

$\boxed{\rho = 500\text{ m}}$



* أحسب تسارع السيارة
عندما تصبح على بعد ٥٠م
من النقطة A .

الحل * $v = 0.2\,S \Rightarrow v = 0.2(50) = \boxed{10\text{ m/s}}$

* $v\,dv = a_t\,ds$

عوض * ~~dt ds du~~ $a_t = v\dfrac{dv}{ds}$

$a_t = (0.2\,S)(0.2)$

$\boxed{a_t = 0.04\,S}$

عوض
$S = 50\text{m} \Rightarrow a_t = 0.04(50)$

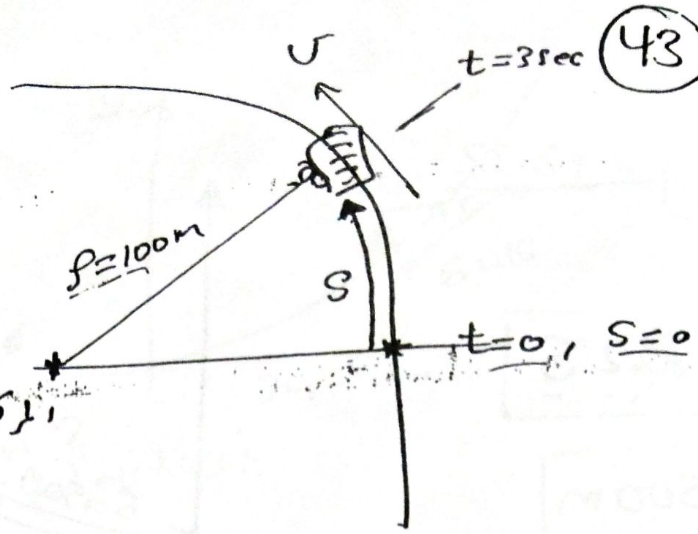
$\boxed{a_t = 2\text{ m/s}^2}$

* $a_n = \dfrac{v^2}{\rho} = \dfrac{(10)^2}{500} \Rightarrow \boxed{a_n = 0.2\text{ m/s}^2}$

$\therefore a = \sqrt{a_n^2 + a_t^2} \Rightarrow \boxed{a \cong 2.01\text{ m/s}^2}$

Ex 12.28

$$\boxed{v = \frac{300}{s}}$$



Find a?

after $t = 3$ sec.

الحل

$$v = \frac{ds}{dt}$$

$$\Rightarrow dt = \frac{ds}{v}$$

$$dt = \frac{ds}{\frac{300}{s}}$$

$$\int_0^3 dt = \int_0^s \frac{s}{300}\, ds$$

$$t\Big]_0^3 = \frac{1}{300}\frac{s^2}{2}\Big]_0^s$$

$$3 = \frac{1}{600}\left[s^2 - (0)^2\right] = \frac{s^2}{600} = 3$$

$$s^2 = 1800 \Rightarrow s = \sqrt{1800}$$

$$s = 42.4 \text{ m}$$

$$v = \frac{300}{s} = \frac{300}{42.4} \Rightarrow v = 7.07 \text{ m/s}$$

$$a_n = \frac{v^2}{\rho} = \frac{(7.07)^2}{100} \Rightarrow \boxed{a_n = 0.5 \text{ m/s}^2}$$

$$v\,dv = a_t\,ds$$

$$\Rightarrow a_t = v\,\frac{dv}{ds}$$

$$= v\left(\frac{-300}{s^2}\right)$$

$$= 7.07\left[\frac{-300}{(42.7)^2}\right]$$

$$\boxed{a_t = -1.16 \text{ m/s}^2}$$

$$a = \sqrt{a_n^2 + a_t^2}$$

$$= \sqrt{(1.16)^2 + (0.5)^2}$$

$$\boxed{a = 1.3 \text{ m/s}^2}$$

ص ٥٧

$\Delta y = -500\ m.$

12-112

سرعة الطياره الثابتة $v_A = 50\ m/s \Rightarrow v_0 = 50\ m/s$

والطيران افقي تماما $\Rightarrow \theta = 0$

$\boxed{1}$ Find a_t ? a_n ? ρ ? } عند نقطة القذف A

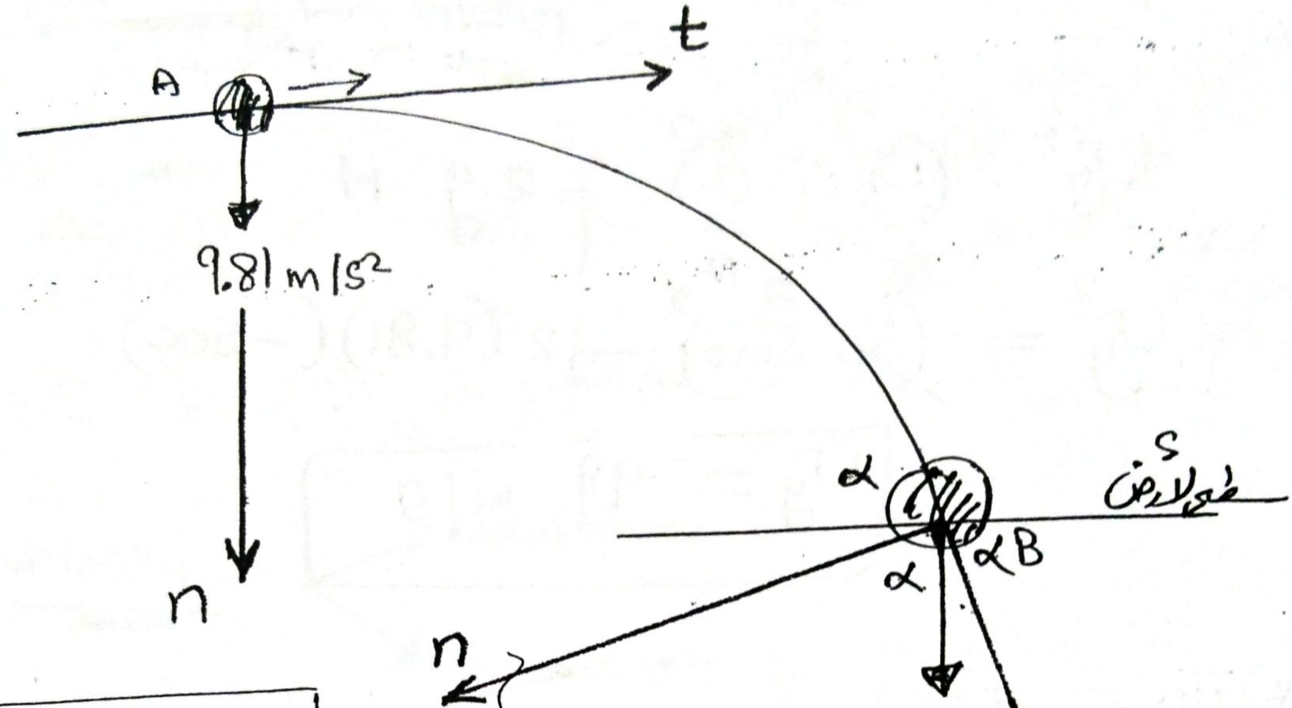
$\boxed{2}$ Find a_t ? a_n ? ρ ? } عند نقطة السقوط والاصطدام بالارض B

الحل

* المقذوفات عبارة عن (سقوط حر)

والسقوط الحر يعني أن

التسارع الكلي للـ $a = g = -9.81\ m/s^2 \downarrow$



$$\boxed{a_n = 9.81}$$
$$\boxed{a_t = 0}$$

$$a_n = \frac{V^2}{\rho}$$

$$9.81 = \frac{(50)^2}{\rho}$$

$$\boxed{\rho = 254.8 \text{ m}}$$

$$a_n = 9.81 \cos \alpha$$

$$a_t = 9.81 \sin \alpha$$

$$a_n = \frac{V^2}{\rho}$$

$$\Rightarrow \rho = \frac{V^2}{a_n}$$

(47)

$\theta = 0$ سؤال رقم

$U_o = 50 m/s$ 12-112 (ص ٥٢)

$H = -500 m$

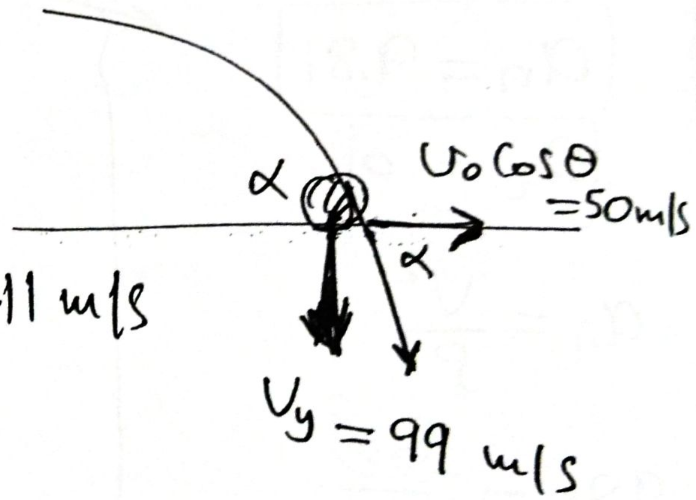
$$U_y^2 = (U_o \sin\theta)^2 - 2g\,H$$

$$U_y^2 = (\cancel{50 \sin 0})^2 - 2(9.81)(-500)$$

$$\boxed{U_y = 99\ m/s}$$

$* \; U_o \cos\theta = 50 \cos 0$

$= 50 m/s$



$* \; U_B = \sqrt{(99)^2 + (50)^2} = 111 m/s$

$* \; \tan\alpha = \dfrac{99}{50}$

$$\boxed{\alpha = 63.2^\circ}$$

نعوض :: $a_n = 9.81 \cos 63.2^\circ = \boxed{4.4 m/s}$

$a_t = 9.81 \sin 63.2^\circ = \boxed{8.8 m/s}$

$\rho = \dfrac{(111)^2}{4.4} \Rightarrow \boxed{\rho = 2800 m}$

"Dependent Motion"

* **Pulleys** : فكرة هذه المسائل تقوم على أن طول الحبل يساوي ثابت

وبالتالي مشتقة هذا الثابت تساوي صفراً .

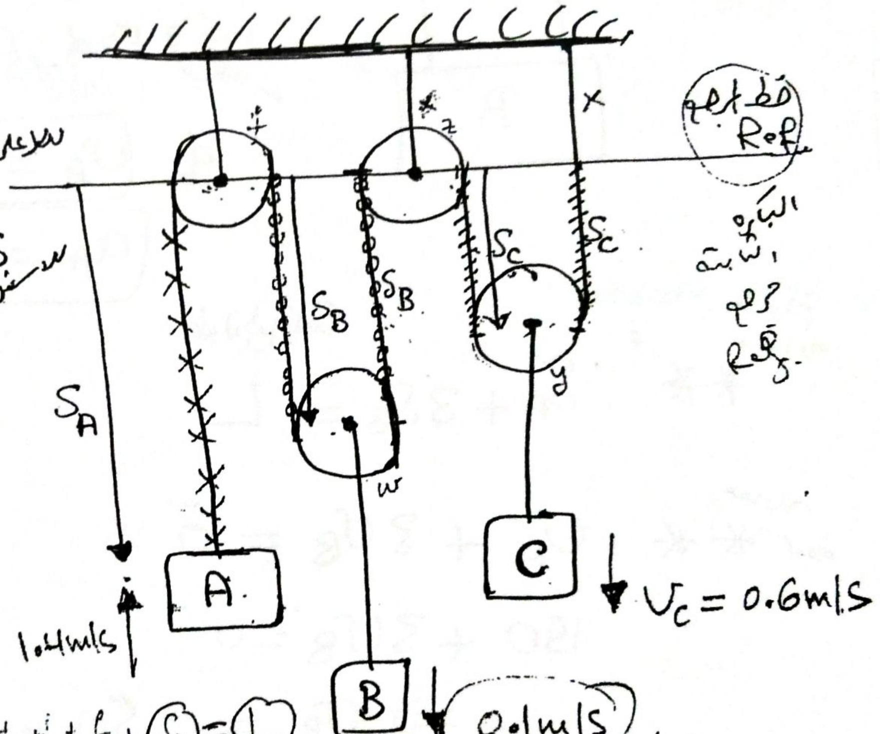
Ex)

$v_A = 1.4\, m/s$ للأعلى

$v_C = -0.6\, m/s$ للأسفل

Find (v_B) ?

أوجد سرعة B



$x + 2S_C + y + z + 2S_B + w + f + S_A = L$

$0 + 2v_C + 0 + 0 + 2v_B + 0 + 0 + v_A = 0$

طول الحبل ثابت

الجواب

معادلة المسافة $\qquad 2S_C + 2S_B + S_A = L$

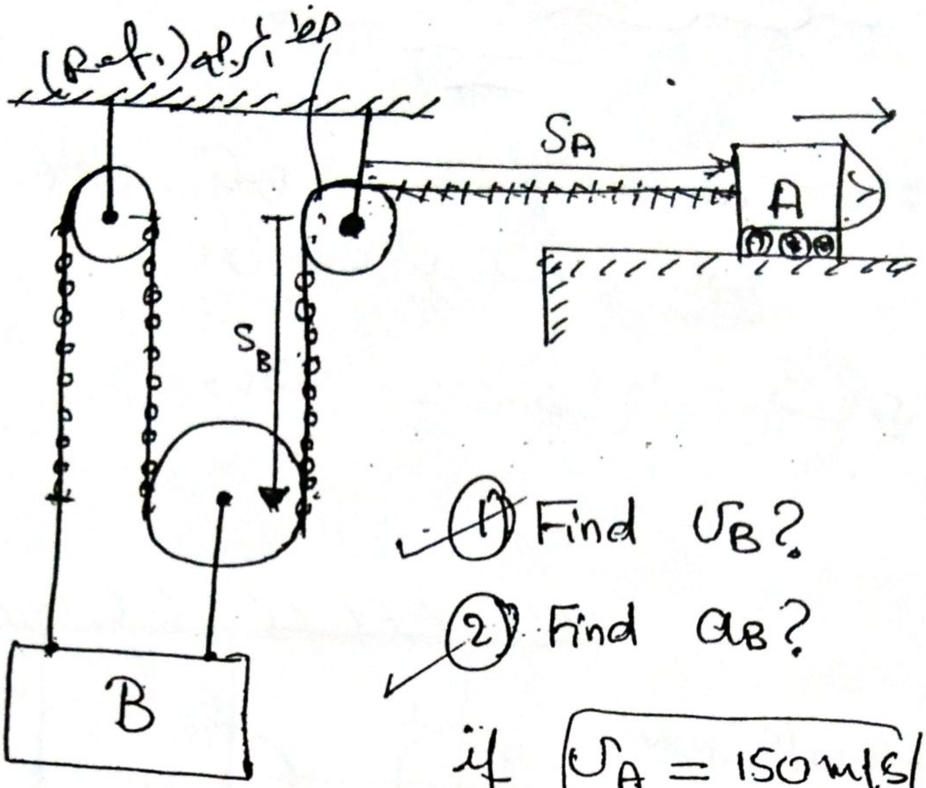
معادلة السرعة $\qquad 2v_C + 2v_B + v_A = 0$

$2(-0.6) + 2v_B + 1.4 = 0 \Rightarrow \boxed{v_B = -0.1\, m/s}$

معادلة التسارع $\qquad 2a_C + 2a_B + a_A = 0$

ex

55



(1) Find v_B?

(2) Find a_B?

if $\boxed{v_A = 150 \text{ m/s}}$ (عين)

$\boxed{a_A = 30 \text{ m/s}^2}$ (أعلى)

معادلة مسافة ** $S_A + 3S_B = L$ طول الحبل ثابت

معادلة سرعة ** $v_A + 3v_B = 0$

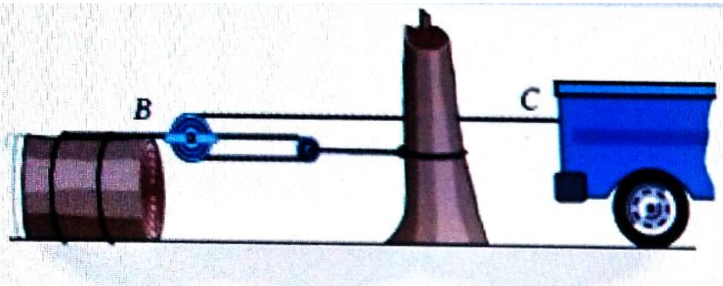
$150 + 3v_B = 0$

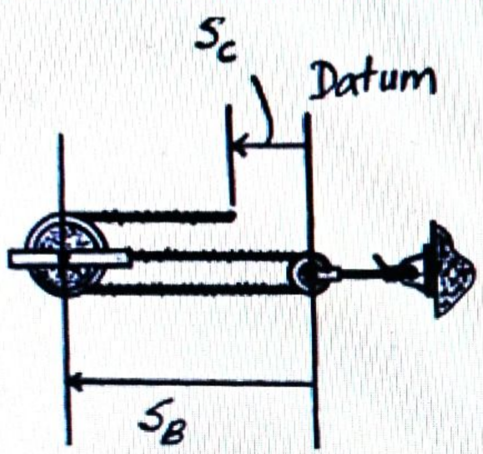
$\Rightarrow v_B = -50 \text{ m/s}$

معادلة تسارع ** $a_A + 3a_B = 0$

$30 + 3a_B = 0$

$a_B = -10 \text{ m/s}^2$

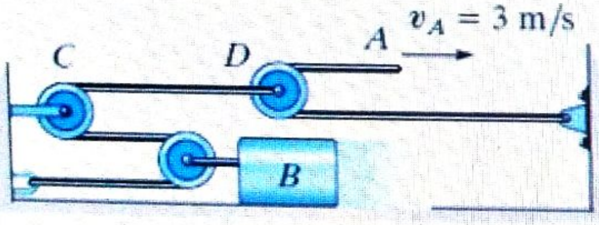


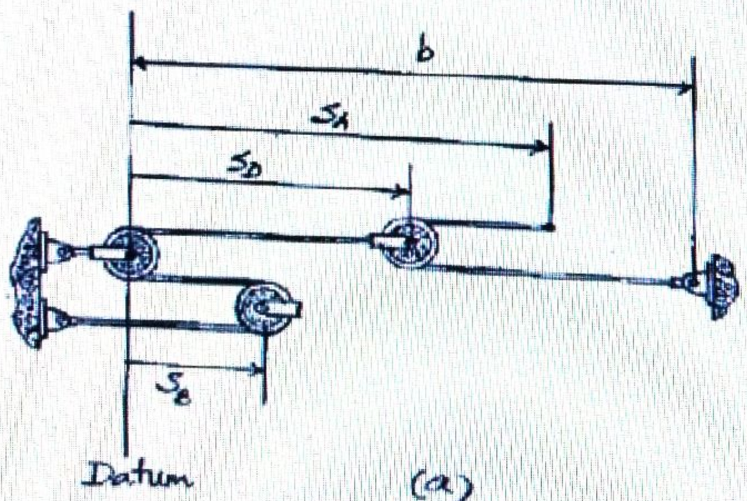


$$2s_B + (s_B - s_C) = l$$

$$3s_B - s_C = l$$

$$3\Delta s_B - \Delta s_C = 0$$

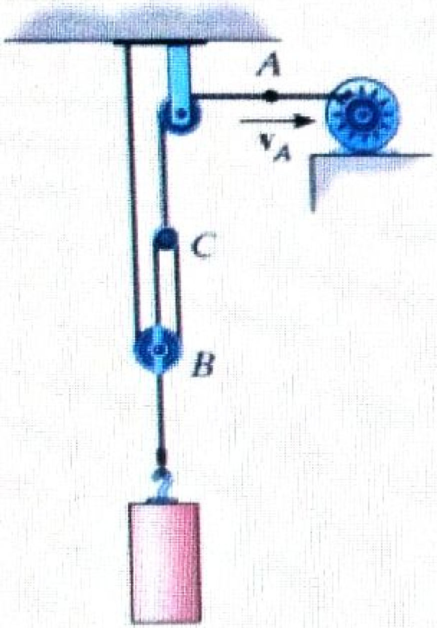




$$2\,s_B + s_D = l_1$$

$$(s_A - s_D) + (b - s_D) = l_2$$

$$s_A - 2\,s_D = l_2 - b$$

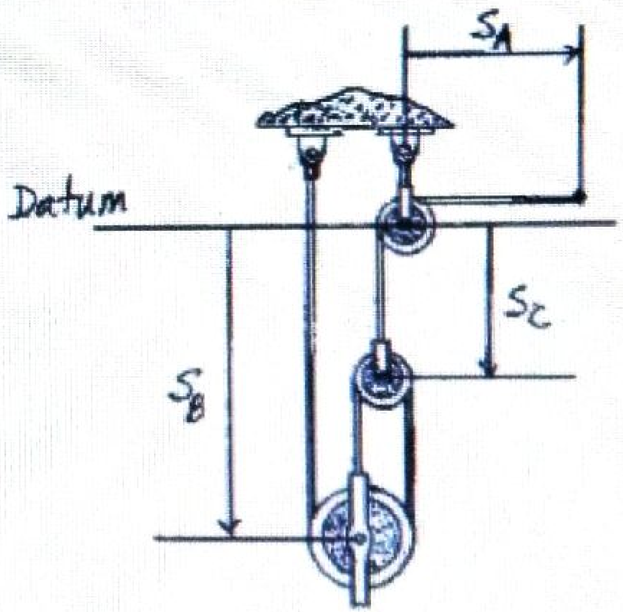


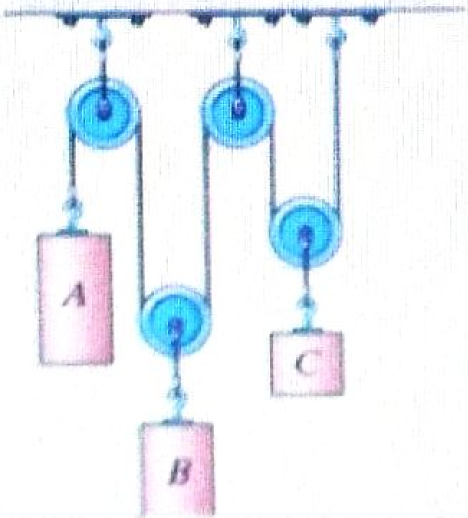
$$s_B + 2(s_B - s_C) = l_1$$

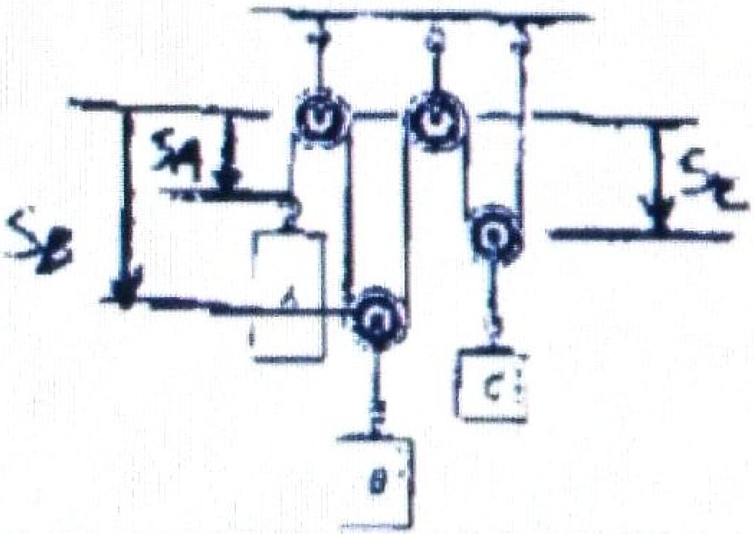
$$3s_B - 2s_C = l_1$$

And

$$s_C + s_A = l_2$$

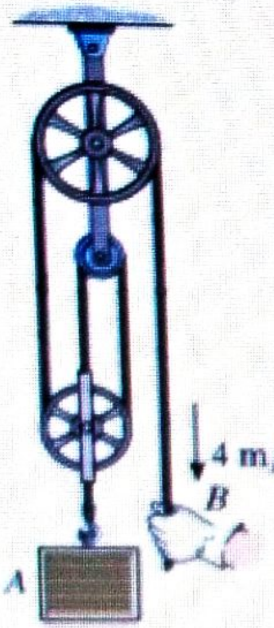


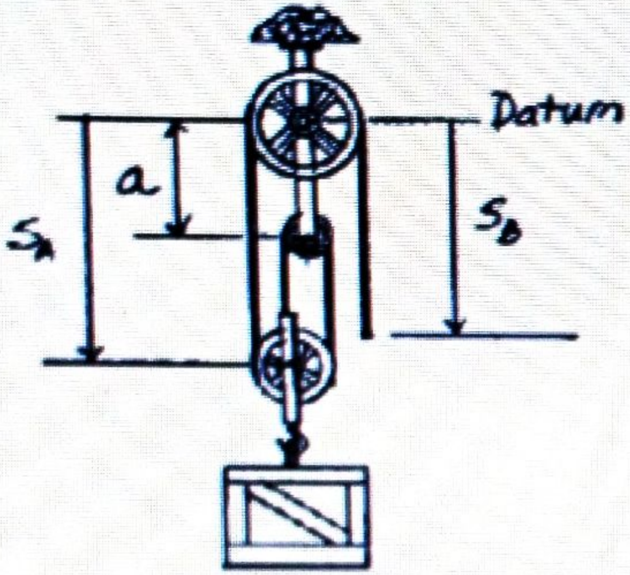




$$s_A + 2s_B + 2s_C = l$$

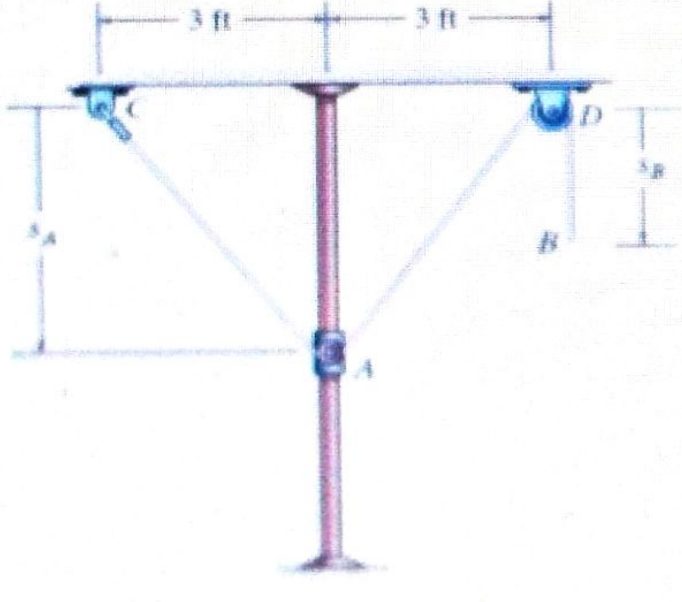
$$v_A + 2v_B + 2v_C = 0$$





$$s_B + s_A + 2(s_A - a) = l$$

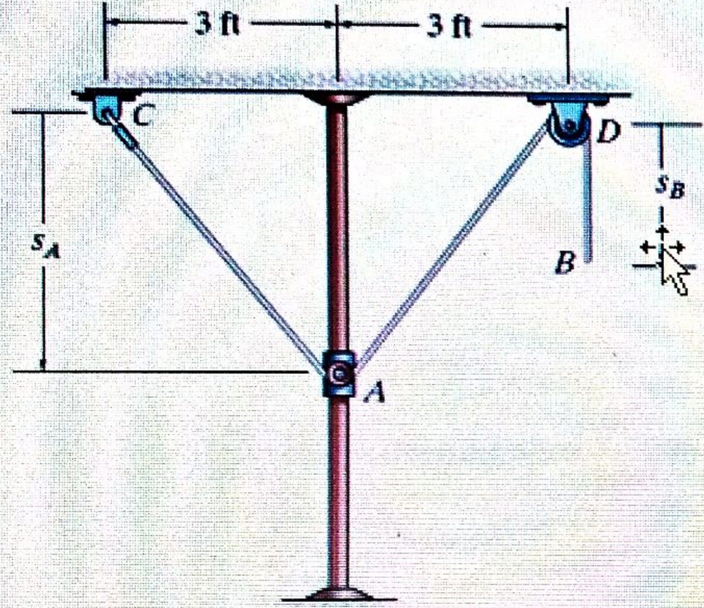
$$s_B + 3s_A = l + 2a$$



$$2\sqrt{s_A^2 + 3^2} + s_B = l$$

$$2\left(\frac{1}{2}\right)(s_A^2 + 9)^{-\frac{1}{2}}\left(2s_A\,\dot{s}_A\right) + \dot{s}_B = 0$$

$$\dot{s}_B = -\frac{2s_A\,\dot{s}_A}{(s_A^2 + 9)^{\frac{1}{2}}}$$



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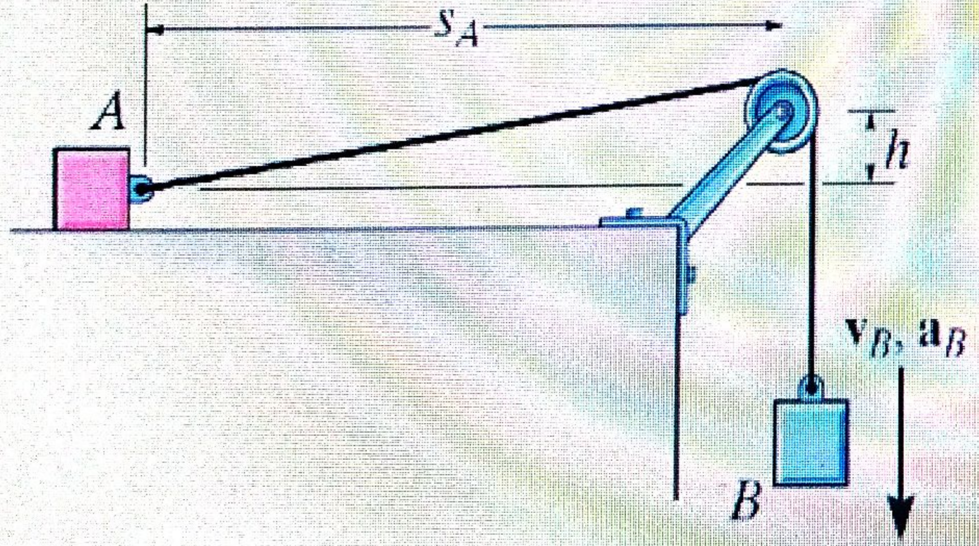
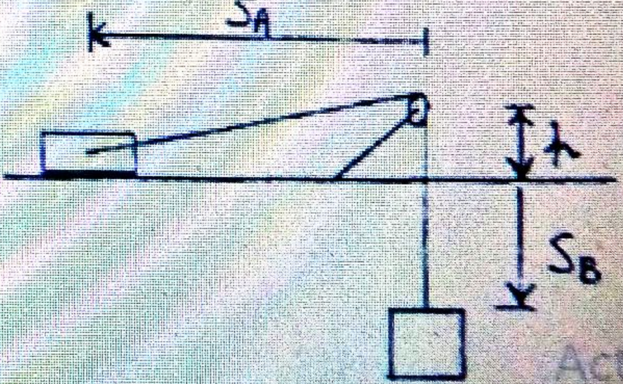


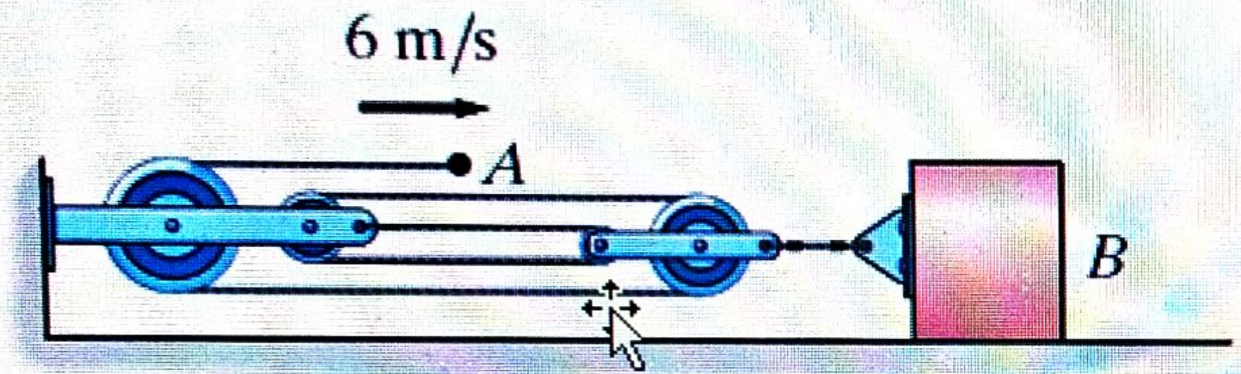
$$l = s_B + \sqrt{s_B^2 + h^2}$$

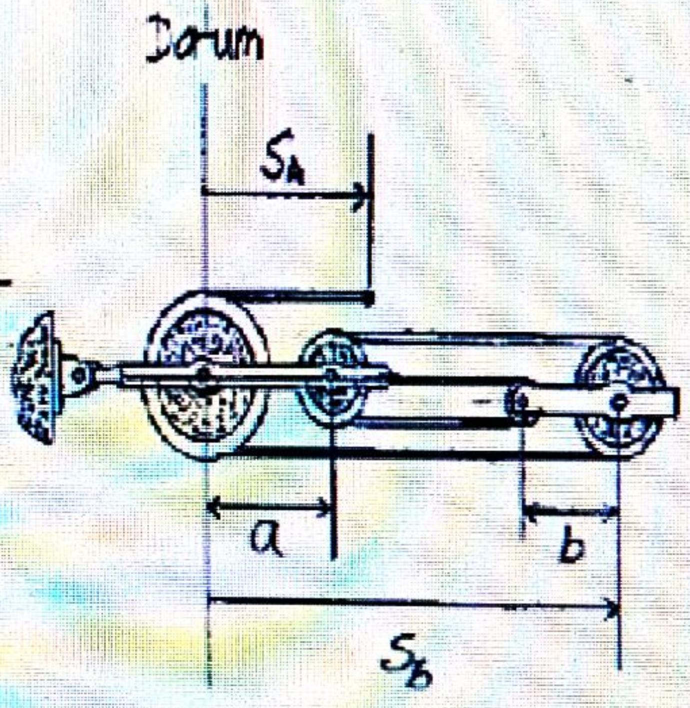
$$0 = \dot{s}_B + \frac{1}{2}\left(s_A^2 + h^2\right)^{-1/2} 2s_A\dot{s}_A$$

$$v_A = \dot{s}_A = \frac{-\dot{s}_B\left(s_A^2 + h^2\right)^{1/2}}{s_A}$$

$$v_A = -v_B\left(1 + \left(\frac{h}{s_A}\right)^2\right)^{1/2}$$





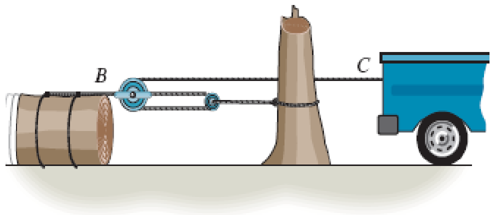


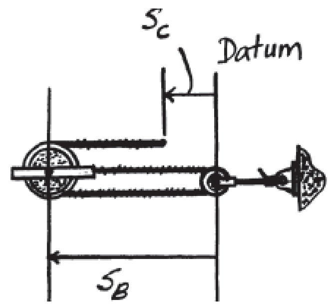
$$s_B + 2(s_B - a - b) + (s_B - a) + s_A = l$$

$$4s_B + s_A = l + 3a + 2b$$

Time Derivative. Taking the time derivative of Eq. (1),

$$4v_B + v_A = 0$$





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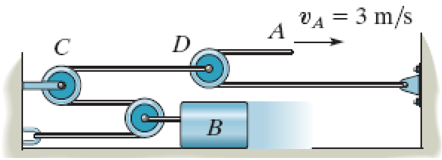
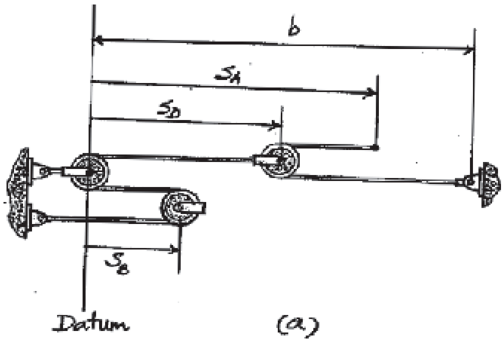
$$3s_B - s_C = l$$

$$3\Delta s_B - \Delta s_C = 0$$





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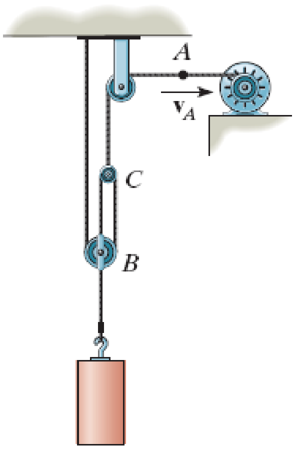
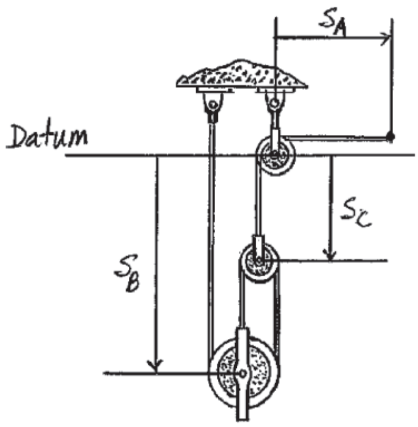


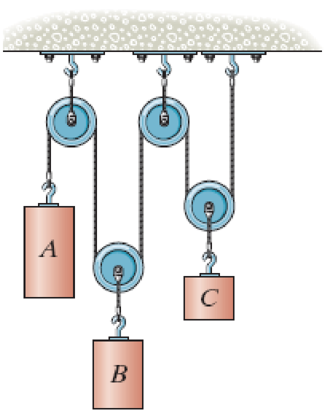
$$s_B + 2(s_B - s_C) = l_1$$

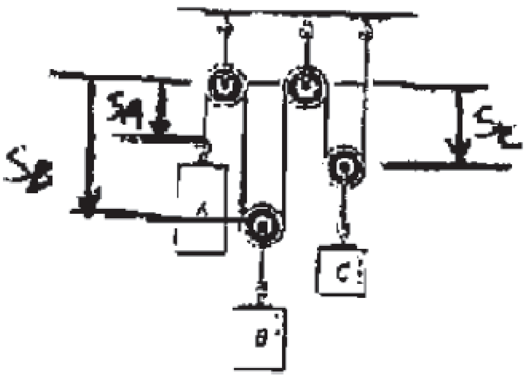
$$3s_B - 2s_C = l_1$$

And

$$s_C + s_A = l_2$$

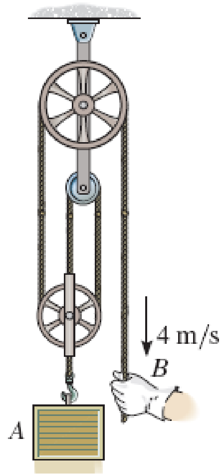


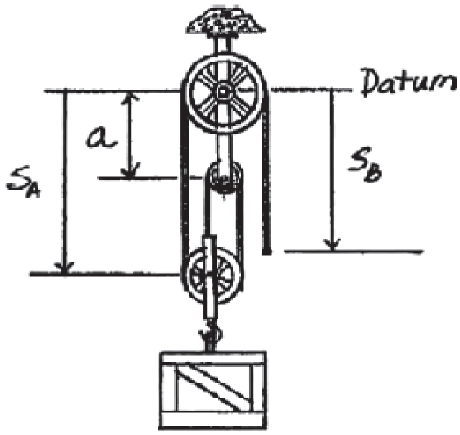




$$s_A + 2s_B + 2s_C = l$$

$$v_A + 2v_B + 2v_C = 0$$





$$s_B + s_A + 2(s_A - a) = l$$

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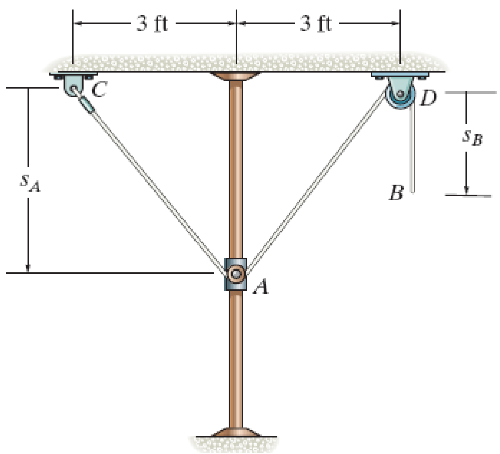
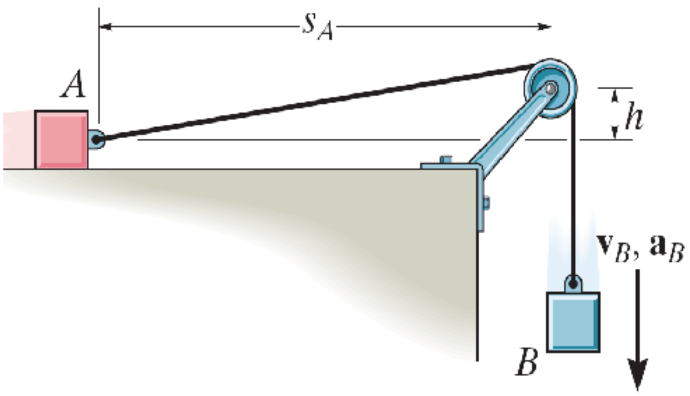


$$2\sqrt{s_A^2 + 3^2} + s_B = l$$

$$2\left(\frac{1}{2}\right)\left(s_A^2 + 9\right)^{-\frac{1}{2}}\left(2s_A\,\dot{s}_A\right) + \dot{s}_B = 0$$

$$\dot{s}_B = -\frac{2s_A\dot{s}_A}{\left(s_A^2 + 9\right)^{\frac{1}{2}}}$$



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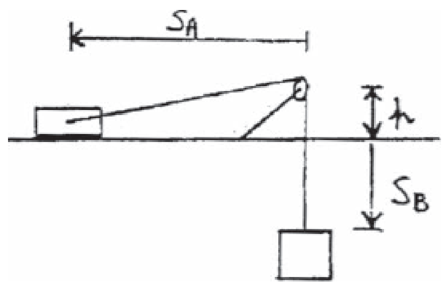
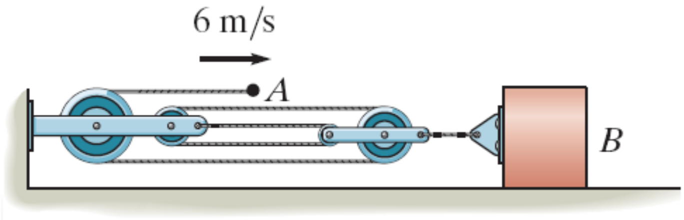
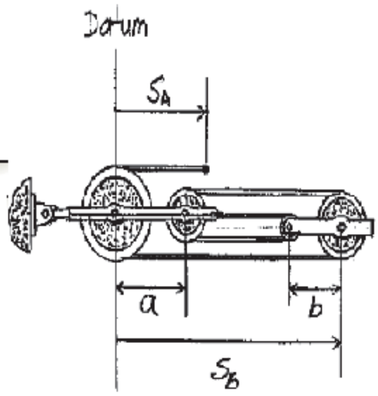
$$0 = \dot{s}_B + \frac{1}{2}(s_A^2 + h^2)^{-1/2}\,2s_A\,\dot{s}_A$$

$$v_A = \dot{s}_A = \frac{-\,\dot{s}_B(s_A^2 + h^2)^{1/2}}{s_A}$$

$$v_A = -v_B\,(1 + \left(\frac{h}{s_A}\right)^2)^{1/2}$$







$$s_B + 2(s_B - a - b) + (s_B - a) + s_A = l$$

$$4s_B + s_A = l + 3a + 2b$$

Time Derivative. Taking the time derivative of Eq. (1),

$$4v_B + v_A = 0$$

Dr Hashem AlKhaldi

13

Kinetics of a Particle: Force and Acceleration

CHAPTER OBJECTIVES

- To state Newton's Second Law of Motion and to define mass and weight.
- To analyze the accelerated motion of a particle using the equation of motion with different coordinate systems.
- To investigate central-force motion and apply it to problems in space mechanics.

Video Solutions are available for selected questions in this cha

13.1 Newton's Second Law of Motion

Kinetics is a branch of dynamics that deals with the relationship between the change in motion of a body and the forces that cause this change. The basis for kinetics is Newton's second law, which states that when an *unbalanced force* acts on a particle, the particle will *accelerate* in the direction of the force with a magnitude that is proportional to the force.

This law can be verified experimentally by applying a known unbalanced force **F** to a particle, and then measuring the acceleration **a**. Since the force and acceleration are directly proportional, the constant of proportionality, m, may be determined from the ratio $m = F/a$. This positive scalar m is called the *mass* of the particle. Being constant during any acceleration, m provides a quantitative measure of the resistance of the particle to a change in its velocity, that is its inertia.

The jeep leans backward due to which resists its forward acceler

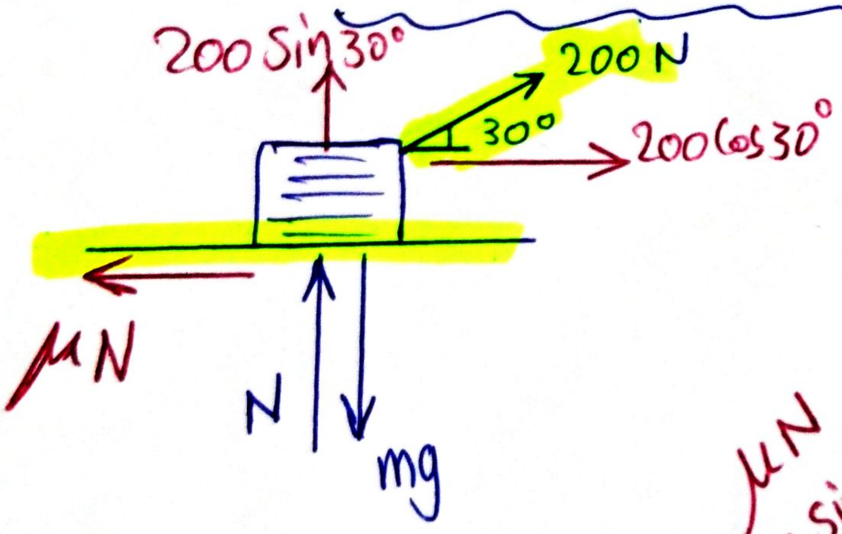
$$\boxed{\begin{aligned}\sum F_x &= m\,a_x \\ \sum F_y &= m\,a_y\end{aligned}}
\qquad
\boxed{\begin{aligned}\sum F_t &= m\,a_t = m\frac{dv}{dt} \\ \sum F_n &= m\,a_n = m\frac{v^2}{\rho}\end{aligned}}$$

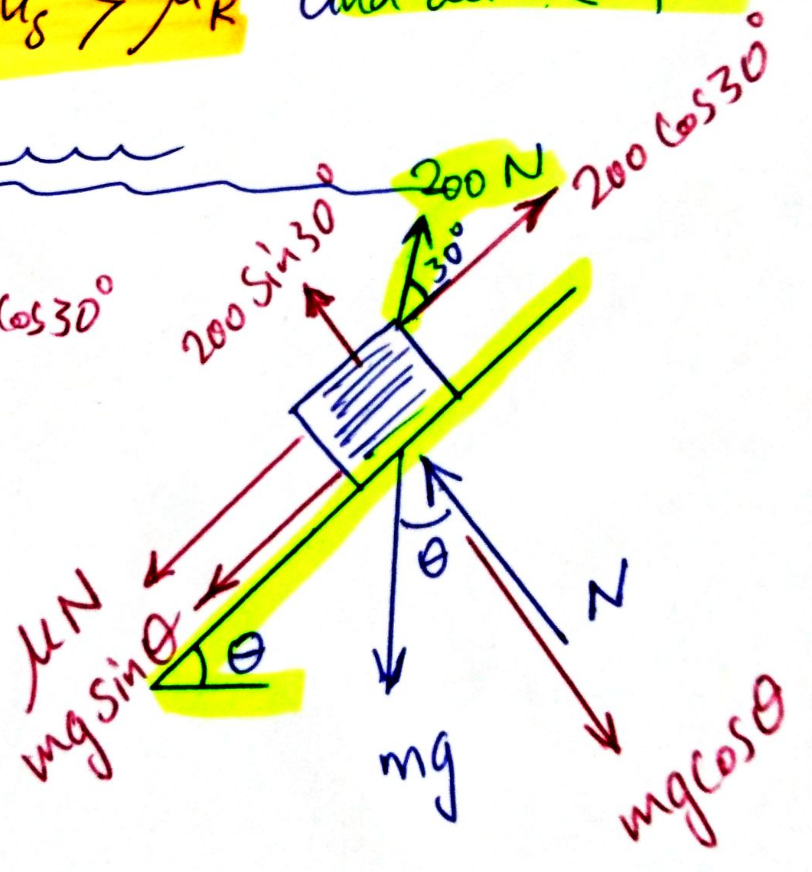
Friction Force :

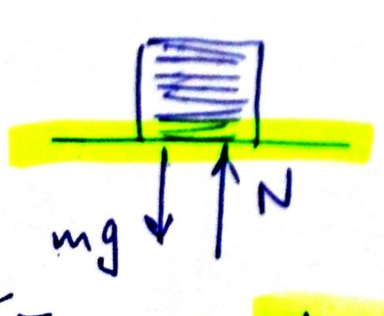
Static $\rightarrow F_f = \mu_s N$

dynamic $\rightarrow F_f = \mu_k N$

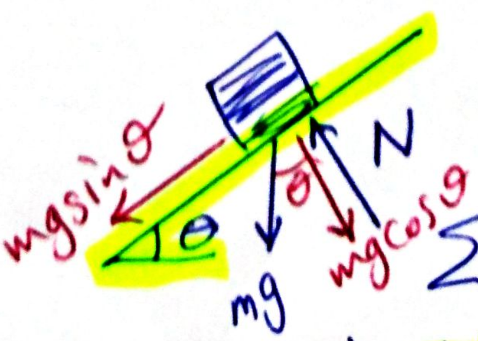
$\Rightarrow \mu_s > \mu_k$ and all < 1







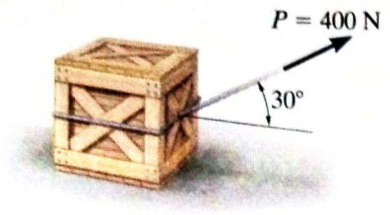
$\sum F_y = 0 \Rightarrow N = mg$



$\sum F_y = 0$

$\Rightarrow N = mg \cos\theta$

EXAMPLE 13.1



(a)

The 50-kg crate shown in Fig. 13–6*a* rests on a horizontal surface for which the coefficient of kinetic friction is $\mu_k = 0.3$. If the crate is subjected to a 400-N towing force as shown, determine the velocity of the crate in 3 s starting from rest.

SOLUTION

Using the equations of motion, we can relate the crate's acceleration to the force causing the motion. The crate's velocity can then be determined using kinematics.

Free-Body Diagram. The weight of the crate is $W = mg = 50\text{ kg}\,(9.81\text{ m/s}^2) = 490.5\text{ N}$. As shown in Fig. 13–6*b*, the frictional force has a magnitude $F = \mu_k N_C$ and acts to the left, since it opposes the motion of the crate. The acceleration **a** is assumed to act horizontally, in the positive x direction. There are two unknowns, namely N_C and a.

Equations of Motion. Using the data shown on the free-body diagram, we have

$$\overset{+}{\rightarrow}\Sigma F_x = ma_x; \qquad 400\cos 30^\circ - 0.3N_C = 50a \qquad (1)$$

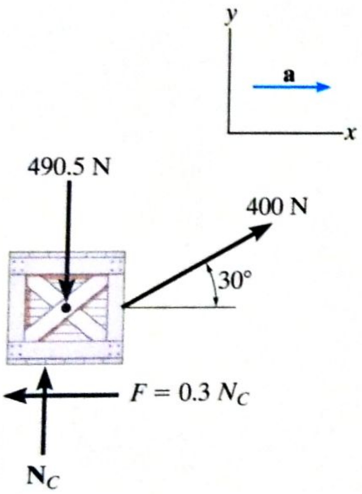
$$+\uparrow\Sigma F_y = ma_y; \qquad N_C - 490.5 + 400\sin 30^\circ = 0 \qquad (2)$$

Solving Eq. 2 for N_C, substituting the result into Eq. 1, and solving for a yields

$$N_C = 290.5\text{ N}$$
$$a = 5.185\text{ m/s}^2$$

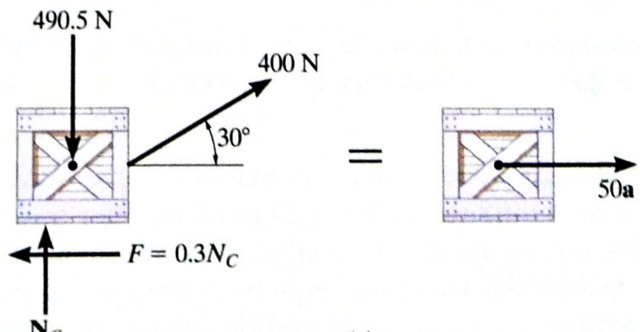
Kinematics. Notice that the acceleration is *constant*, since the applied force **P** is constant. Since the initial velocity is zero, the velocity of the crate in 3 s is

$$(\overset{+}{\rightarrow}) \qquad v = v_0 + a_c t = 0 + 5.185(3)$$
$$= 15.6\text{ m/s} \rightarrow \qquad \textit{Ans.}$$



(b)

Fig. 13–6

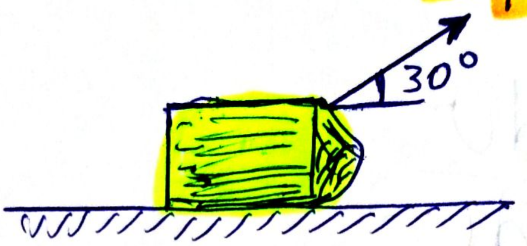


(c)

Ex)

$m = 40\ Kg.$

$F = 8t^2$ (Newton)

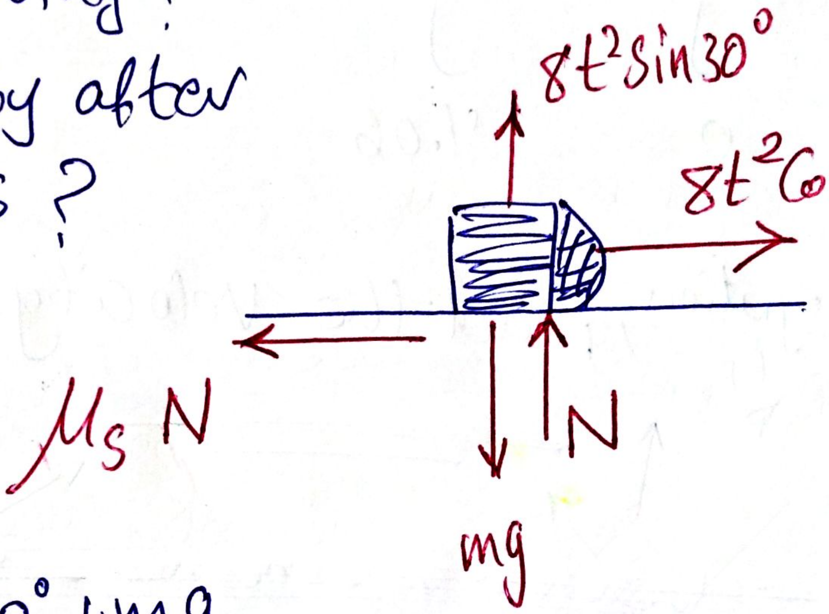


$\mu_s = 0.35$

$\mu_k = 0.20$

1) How long it needs to start moving?

2) Its velocity after 8 seconds?



1

$N = -8t^2 \sin 30° + mg$

$\boxed{N = -4t^2 + mg}$ ✓

$\sum F_x = 0 \Rightarrow$ use μ_s

$\Rightarrow 8t^2 \cos 30° - 0.35(-4t^2 + mg) = 0$

$\Rightarrow 8t^2 \cos 30° - 0.35(-4t^2 + 40(9.81)) = 0$

$\Rightarrow 8.33\ t^2 = 137.34 \Rightarrow \boxed{t = 4.06\ \text{Sec}}$

2 $\sum F_x = m a_x \Rightarrow$ use μ_k

$\Rightarrow 8t^2 \cos 30° - 0.2[-4t^2 + mg] = m\ a_x$

$\Rightarrow$ Find a_x as function of time

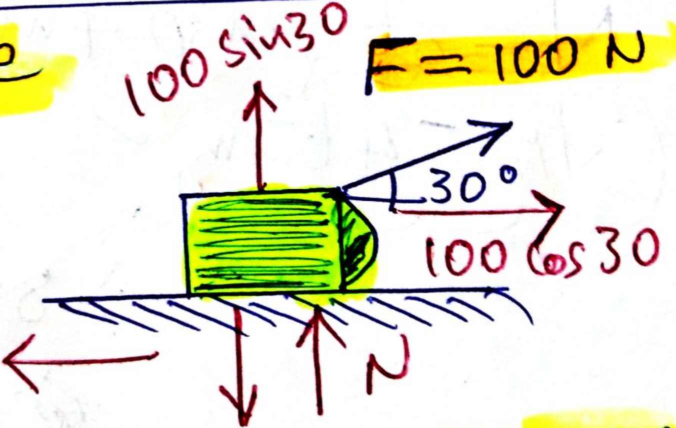
But

$$a_x = \frac{dv}{dt}$$

$$\Rightarrow \int_0^v dv = \int_{4.06}^{8} a_x \, dt$$

do integration, Find the velocity $v =$ ✓

* Now Constant Force



$m = 10$ kg

$\mu_s = 0.35$, $\mu_k = 0.20$

* $\sum F_x = 0 \Rightarrow$ (use μ_s)
* $\sum F_y = 0 \Rightarrow N = mg - 100 \sin 30 = 48.1$ N
* $\sum F_x = 0 \Rightarrow$ Since $100 \cos 30 > \mu_s N$ object will move $\Rightarrow$
  $\Rightarrow 86.6 > 16.8$
* $\sum F_x \Rightarrow$ (μ_k is used now)

$\Rightarrow \sum F_x = m a_x \Rightarrow$ $100 \cos 30° - \mu_k N = m a_x$ $\Rightarrow$ Find $a_x \Rightarrow$

$\Rightarrow$ Solve for $\Rightarrow$ $a_x = 7.7 \ m/s^2$

* Find the velocity after 8 seconds ?

$\Rightarrow$ Since the acceleration is Constant accel. then use :

$\Rightarrow v = v_0 + at$

$\Rightarrow v = 0 + 7.7(8)$

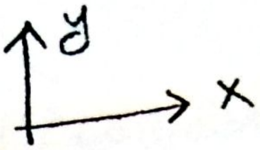
$v = 61.6 \ m/s$

* Find the distance traveled when its velocity reaches to **50** m/s

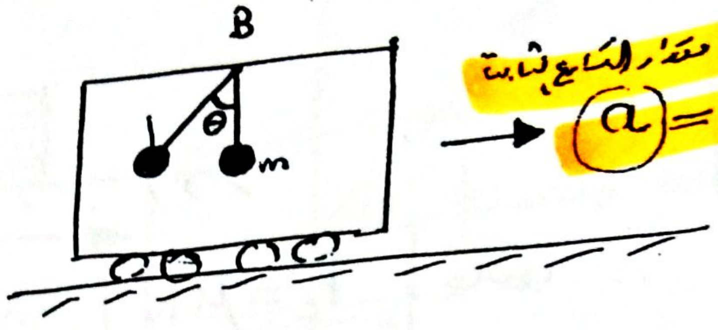
$\Rightarrow v^2 = v_0^2 + 2 a_x S$

$\Rightarrow (50)^2 = (0)^2 + 2(7.7) S$

$\Rightarrow S = 162.3 \ m$



Ex]

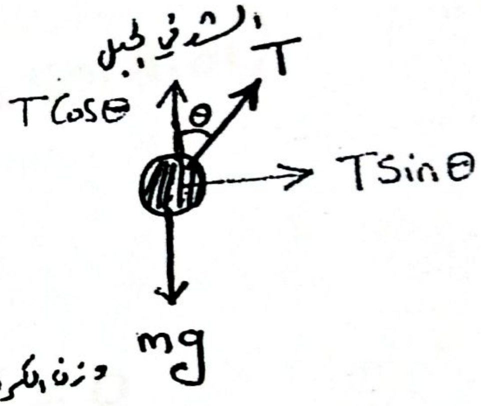


مقدار التسارع ثابت

→ (a) = Constant.

Find the angle θ ?

السياره تتحرك بتسارع ثابت مقداره (a)



$\sum F_y = m a_y$ → لا يوجد حركة باتجاه y

$T\cos\theta - mg = m(0)$

$T\cos\theta = mg$ —— ①

$\sum F_x = m a_x$

$T\sin\theta = ma$ —— ②

اقسم المعادلة الثانية على الأولى

$$\frac{\cancel{T}\sin\theta}{\cancel{T}\cos\theta} = \frac{\cancel{m}a}{\cancel{m}g}$$

$$\tan\theta = \frac{a}{g}$$

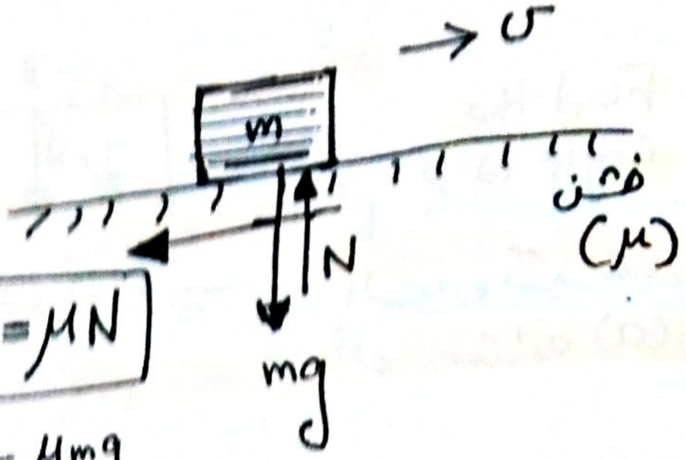
$$\theta = \tan^{-1}\frac{a}{g}$$

Friction (F_f)

* معامل الاحتكاك μ

* دائماً قوة الاحتكاك تكون عكس اتجاه الحركة

$$\boxed{F_f = \mu_k N}$$



قانون $\boxed{F_f = \mu N}$

$= F_f = \mu m g$

$$\sum F_y = m \cancel{a_y}^{\,0}$$

$$N - mg = 0$$

القوة العمودية $\boxed{N = mg}$

(Normal Force)

Ex

سطح خشن

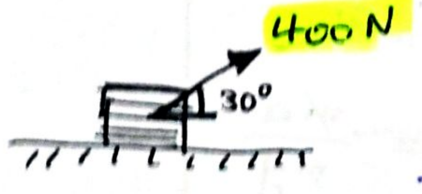
$\mu_k = 0.3$

$m = 50\ Kg$

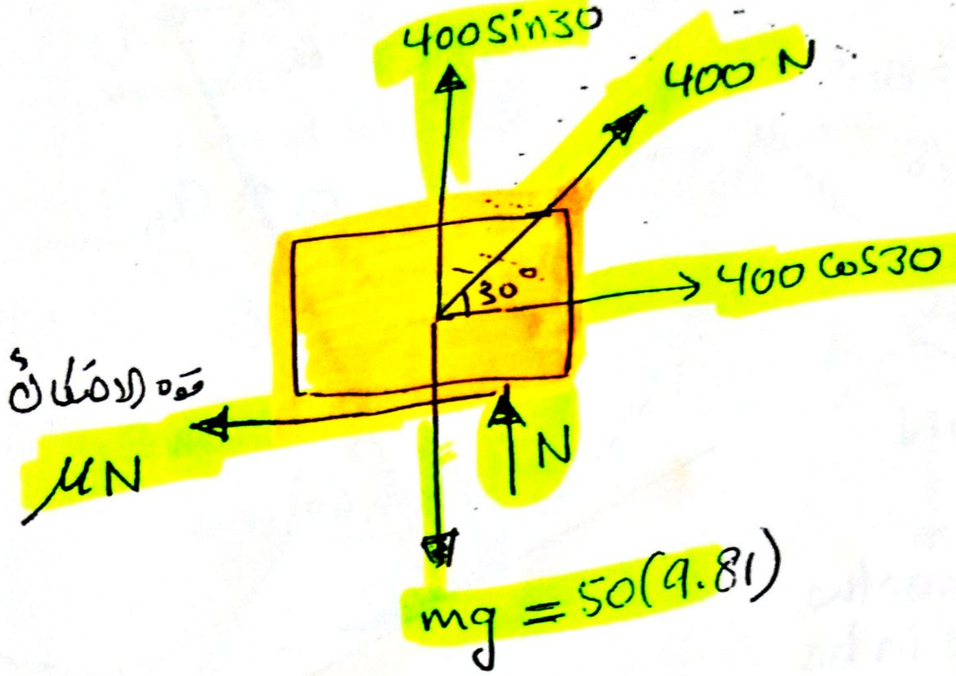
* Starting from Rest

* Find its velocity after $t = 0.5$ sec.

احسب سرعة الجسم بعد مرور نصف ثانية



Constant force $\Rightarrow$ Constant acceler.



$$** \quad \sum F_y = m \overset{\text{لا يوجد حركة}}{\cancel{a_y}}^{\,0}$$

$$400\sin 30 - mg + N = 0$$

$$\Rightarrow N = mg - 400\sin 30$$

$$N = 50(9.81) - 200$$

$$N = 290.5 \ N$$

$$** \quad \sum F_x = m a_x$$

$$400\cos 30 - \mu N = ma$$

$$400\cos 30 - (0.3)(290.5) = (50)\,a$$

هذ عبارة عن تسارع ثابت باتجاه $\underline{x}$ $\Rightarrow$ $\boxed{a = 5.19 \ m/s^2}$

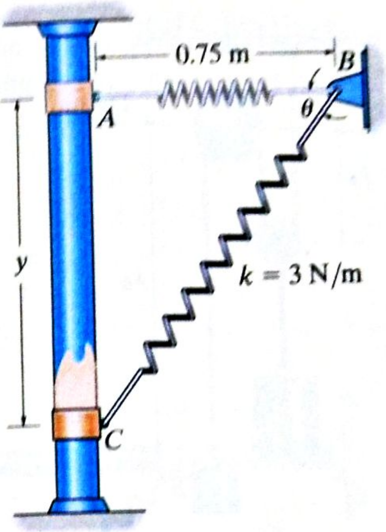
نذهب إلى Ch12 و نستخدم قوانين الحركة الثابتة

$$v = v_0 + at$$

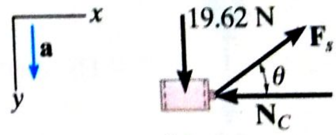
$$v = 0 + 5.19(0.5) = \boxed{2.6 \ m/s}$$

EXAMPLE 13.4

A smooth 2-kg collar, shown in Fig. 13–9*a*, is attached to a spring having a stiffness $k = 3\ \text{N/m}$ and an unstretched length of 0.75 m. If the collar is released from rest at *A*, determine its acceleration and the normal force of the rod on the collar at the instant $y = 1\ \text{m}$.



(a)



(b)

Fig. 13–9

SOLUTION

Free-Body Diagram. The free-body diagram of the collar when it is located at the arbitrary position *y* is shown in Fig. 13–9*b*. Furthermore, the collar is *assumed* to be accelerating so that "**a**" acts downward in the *positive* *y* direction. There are four unknowns, namely, N_C, F_s, a, and θ.

Equations of Motion.

$$\xrightarrow{+} \Sigma F_x = ma_x; \qquad -N_C + F_s \cos\theta = 0 \qquad (1)$$

$$+\downarrow \Sigma F_y = ma_y; \qquad 19.62 - F_s \sin\theta = 2a \qquad (2)$$

From Eq. 2 it is seen that the acceleration depends on the magnitude and direction of the spring force. Solution for N_C and a is possible once F_s and θ are known.

The magnitude of the spring force is a function of the stretch s of the spring; i.e., $F_s = ks$. Here the unstretched length is $AB = 0.75\ \text{m}$, Fig. 13–9*a*; therefore, $s = CB - AB = \sqrt{y^2 + (0.75)^2} - 0.75$. Since $k = 3\ \text{N/m}$, then

$$F_s = ks = 3\left(\sqrt{y^2 + (0.75)^2} - 0.75\right) \qquad (3)$$

From Fig. 13–9*a*, the angle θ is related to *y* by trigonometry.

$$\tan\theta = \frac{y}{0.75}$$

Substituting $y = 1\ \text{m}$ into Eqs. 3 and 4 yields $F_s = 1.50\ \text{N}$ and $\theta = 53.1^\circ$. Substituting these results into Eqs. 1 and 2, we obtain

$$N_C = 0.900\ \text{N} \qquad \textit{Ans.}$$

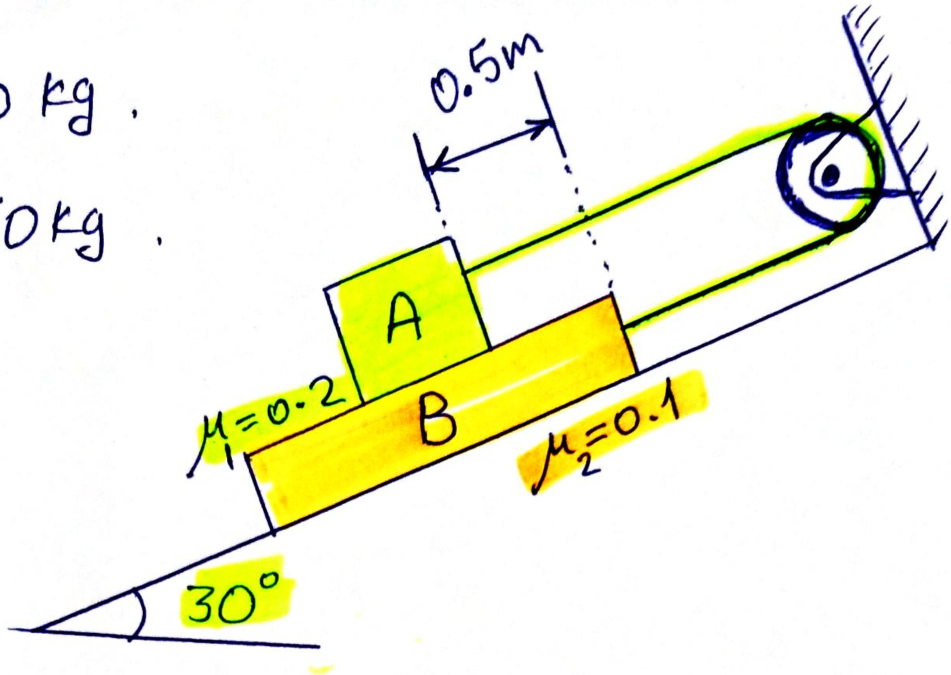
$$a = 9.21\ \text{m/s}^2 \downarrow \qquad \textit{Ans.}$$

NOTE: This is not a case of constant acceleration, since the spring force changes both its magnitude and direction as the collar moves downward.

Ex)

$m_A = 10\ kg$.

$m_B = 50\ kg$.



Find:

1) a_A ?

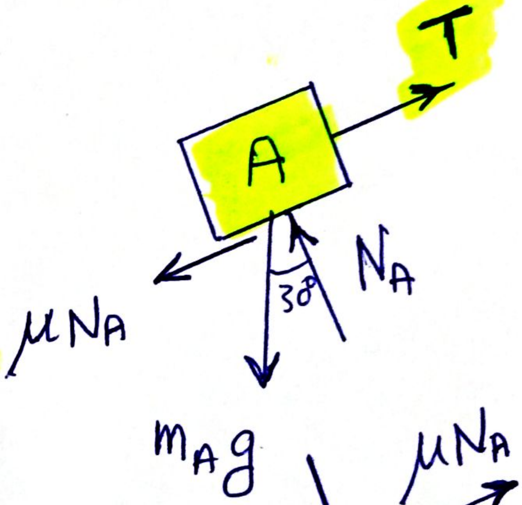
2) a_B ?

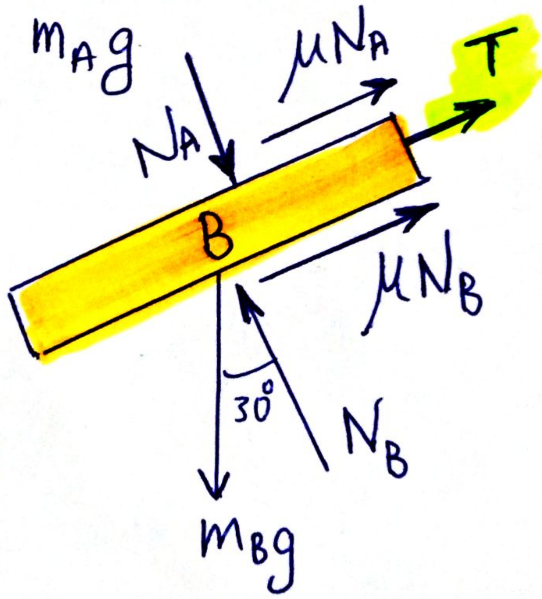
3) Tension T ?

$\sum F_y = 0$

$\Rightarrow N_A = m_A g \cos 30°$ — ①

and $N_B = N_A + m_B g \cos 30°$ — ②





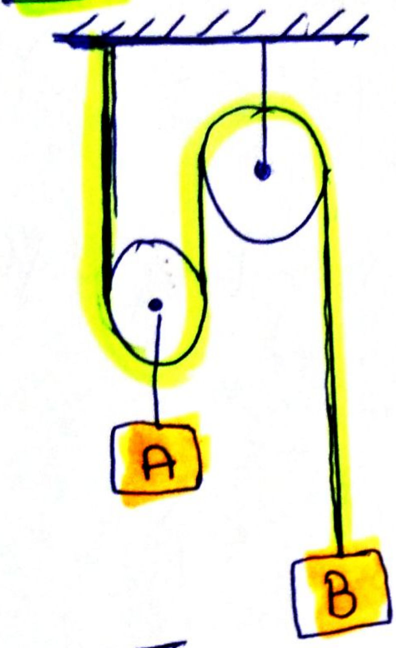
also: $\sum F_x = m a_x$

$\Rightarrow m_A g \sin 30° + \mu_1 N_A - T = m_A a_A$ — ③

and: $m_B g \sin 30° - \mu_1 N_A - \mu_2 N_B - T = m_B a_B$ — ④

and from Ch12 : $a_A + a_B = 0$ — ⑤

Ex)



$m_A = 100\ kg$

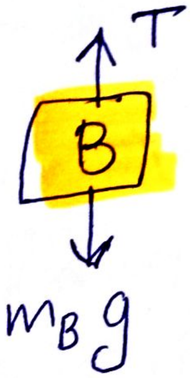
$m_B = 20\ kg$

Find:

1) a_A
2) a_B
3) Tension T

from Ch12 $\Rightarrow$ $2a_A + a_B = 0$ — (1)

$\sum F_y = ma$



$\Rightarrow$ $m_A g - 2T = m_A a_A$ — (2)

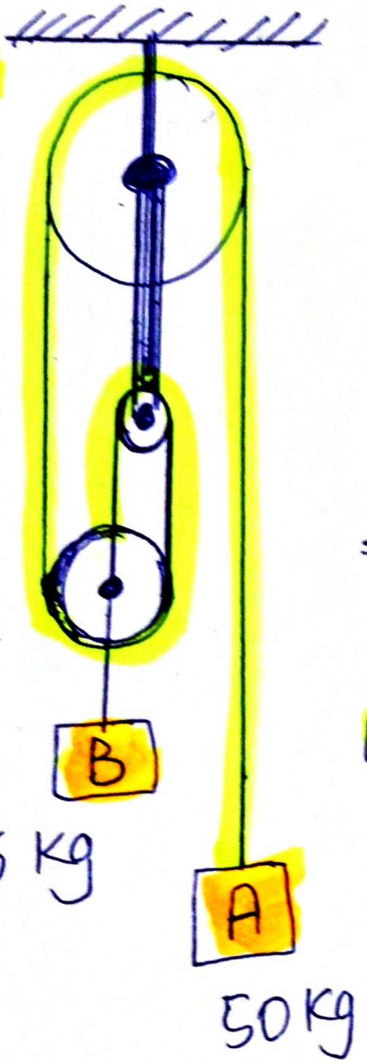
$m_B g - T = m_B a_B$ — (3)

Solve the three equations and get a_A, a_B and T $\Rightarrow$ ✓

* if starts from Rest find the velocity of B and distance of A after 10 seconds?

$v_B = v_0 + a_B t$ ||| $S_A = v_0 t + \frac{1}{2} a_A t^2$

Ex

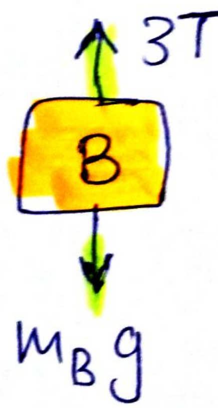


Find a_A, a_B and T?

From Ch12 $\Rightarrow$ $a_A + 3a_B = 0$ — (1)

$\Rightarrow \sum F_y = ma$

$m_A g - T = m_A a_A$ — (2)



$m_B g - 3T = m_B a_B$ — (3)

Solve for T, a_A and a_B

if all Starts from Rest $\Rightarrow$ Find the velocity of B when A moves 18 m down? $S_A + 3S_B = 0$

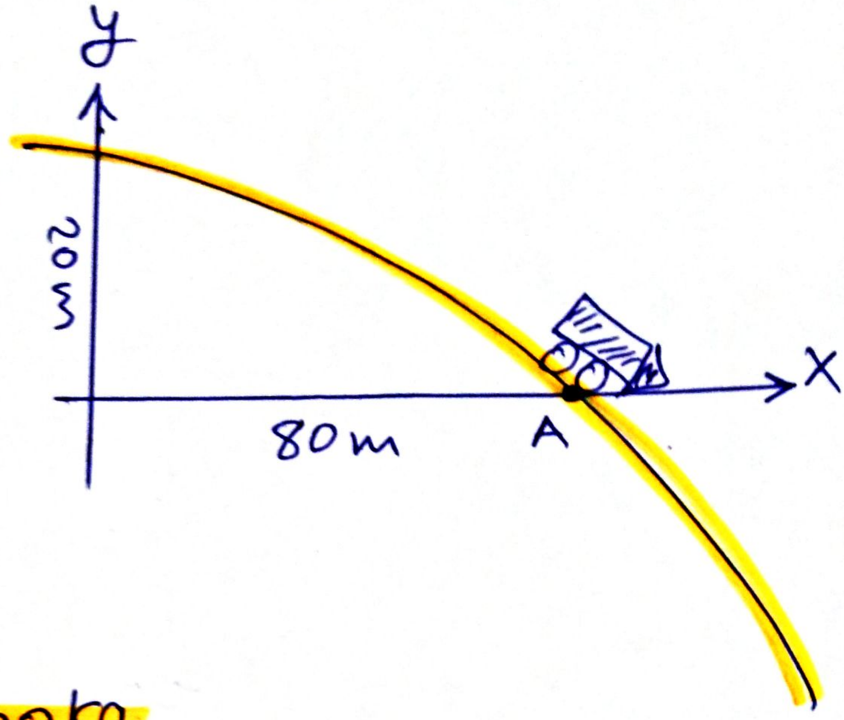
$\Rightarrow S_B = \frac{18}{3} = 6$ m up

$v_B^2 = v_0^2 + 2a_B S_B$ $\Rightarrow$ Find v_B

Ex)

$$y = 20\left[1 - \frac{x^2}{6400}\right]$$

Path equation



$v_A = 9\ m/s$

$(a_t)_A = 3\ m/s^2$

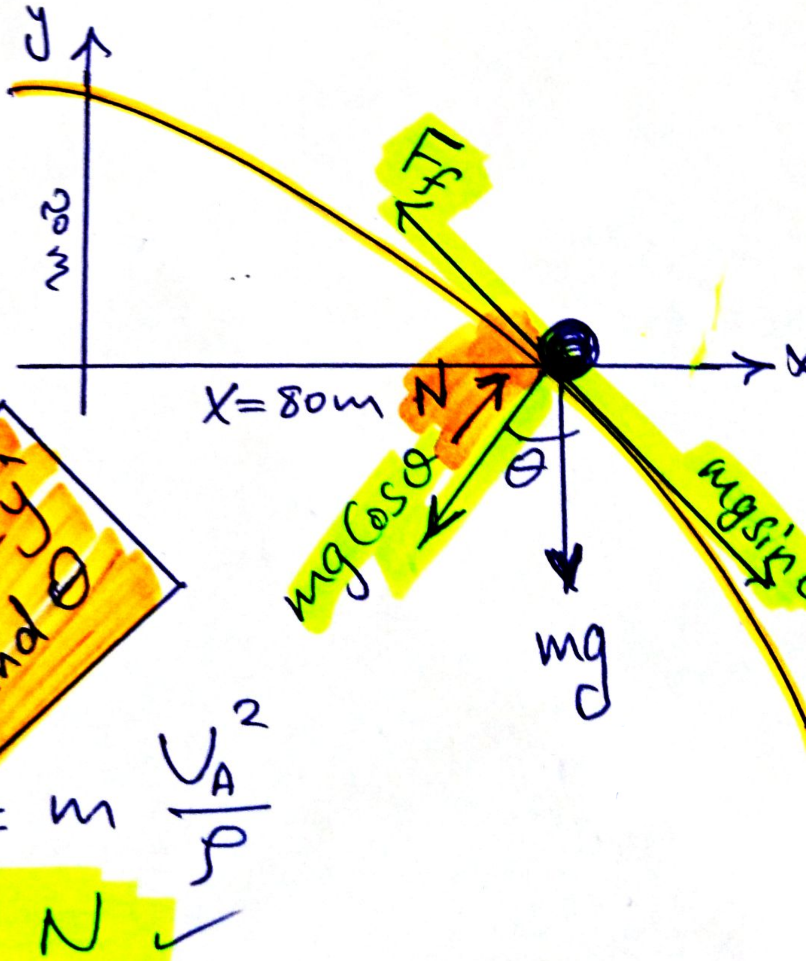
$m = 0.8\ Mg = 800\,kg.$

Find 1) Normal Force N 2) Frictional force F_f

Sol.

find y' and y''

at $x = 80\ m$



$\Rightarrow \rho = \dfrac{[1+y'^2]^{3/2}}{y''}$

$\tan\theta = y'$

$\Rightarrow$ Find θ

$\Rightarrow \sum F_n = m\dfrac{v^2}{\rho}$

$\Rightarrow mg\cos\theta - N = m\dfrac{v_A^2}{\rho}$

$\Rightarrow$ Find N ✓

$\Rightarrow \sum F_t = ma_t \Rightarrow mg\sin\theta - F_f = m(a_t)_A$

$\Rightarrow$ Find F_f ✓

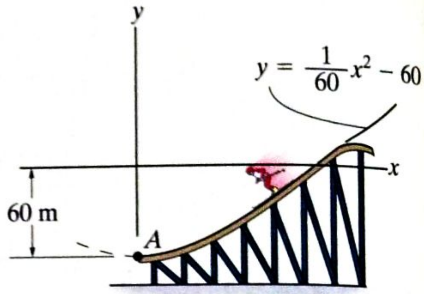
EXAMPLE 13.8

Design of the ski jump shown in the photo requires knowing the type of forces that will be exerted on the skier and her approximate trajectory. If in this case the jump can be approximated by the parabola shown in Fig. 13–14*a*, determine the normal force on the 70-kg skier the instant she arrives at the end of the jump, point A, where her velocity is 20 m/s. Also, what is her acceleration at this point?

SOLUTION

Why consider using n, t coordinates to solve this problem?

Free-Body Diagram. Since $dy/dx = x/30\big|_{x=0} = 0$, the slope at A is horizontal. The free-body diagram of the skier when she is at A is shown in Fig. 13–14*b*. Since the path is *curved*, there are two components of acceleration, $\mathbf{a}_n$ and $\mathbf{a}_t$. Since a_n can be calculated, the unknowns are a_t and N_A.

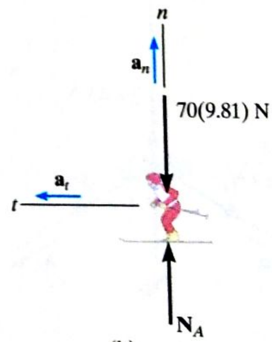


(a)

Equations of Motion.

$$+\uparrow \Sigma F_n = ma_n; \qquad N_A - 70(9.81) = 70\left[\frac{(20)^2}{\rho}\right] \qquad (1)$$

$$\overset{+}{\leftarrow} \Sigma F_t = ma_t; \qquad 0 = 70a_t \qquad (2)$$



(b)

Fig. 13–14

The radius of curvature ρ for the path must be determined at point $A(0, -60\text{ m})$. Here $y = \frac{1}{60}x^2 - 60$, $dy/dx = \frac{1}{30}x$, $d^2y/dx^2 = \frac{1}{30}$, so that at $x = 0$,

$$\rho = \frac{[1 + (dy/dx)^2]^{3/2}}{|d^2y/dx^2|}\bigg|_{x=0} = \frac{[1 + (0)^2]^{3/2}}{\left|\frac{1}{30}\right|} = 30\text{ m}$$

Substituting this into Eq. 1 and solving for N_A, we obtain

$$N_A = 1620\text{ N} \qquad \textit{Ans.}$$

Kinematics. From Eq. 2,

$$a_t = 0$$

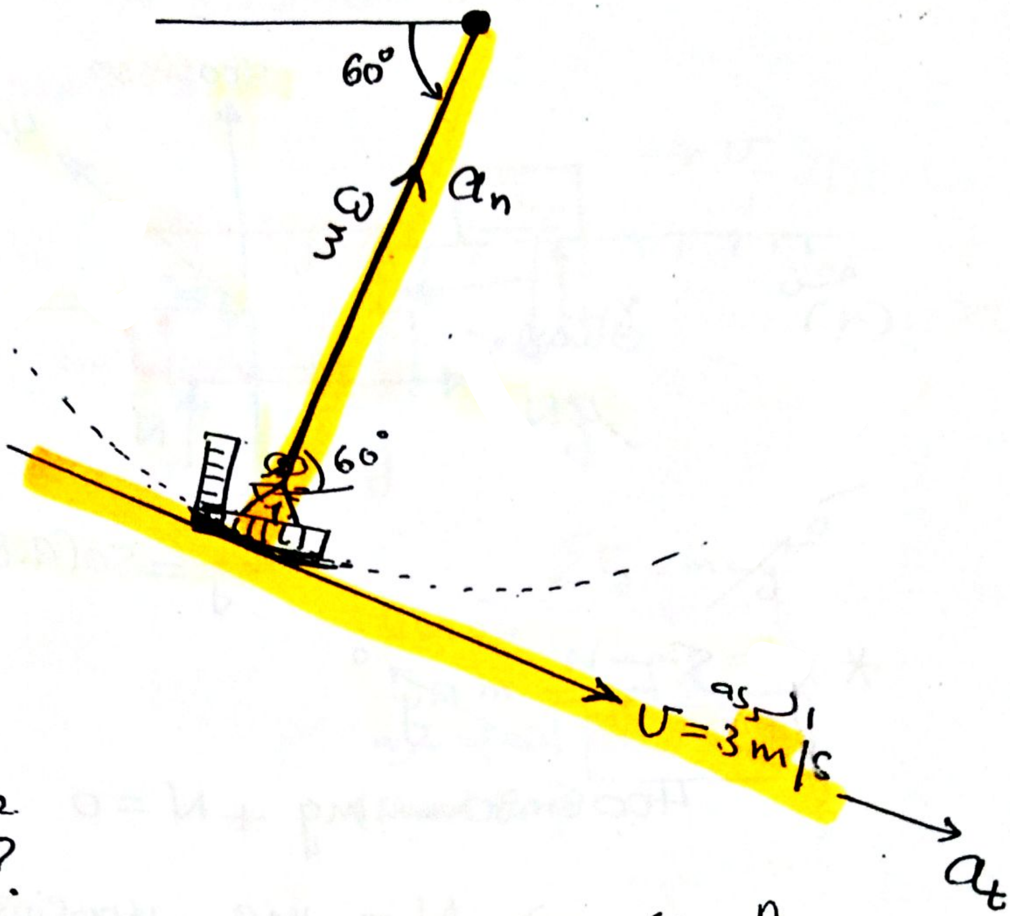
Thus,

$$a_n = \frac{v^2}{\rho} = \frac{(20)^2}{30} = 13.33\text{ m/s}^2$$

$$a_A = a_n = 13.3\text{ m/s}^2 \uparrow \qquad \textit{Ans.}$$

NOTE: Apply the equation of motion in the y direction and show that when the skier is in midair, her downward acceleration is 9.81 m/s^2.

Ex



when $\theta = 60°$

$v = 3\,m/s$

وزن الطفل

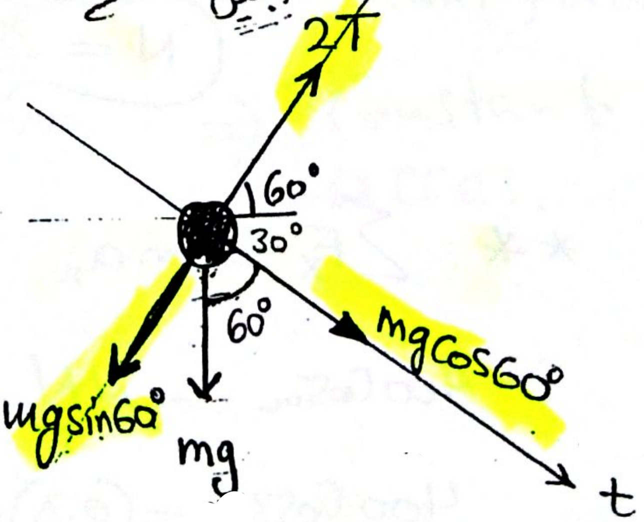
$W = 300\,N$

(1) determine the increase in his speed (a_t)?

(2) the Tension in the Cable (T)?

الشد في الحبل

اتجاه المركز دائماً هو الاتجاه الموجب (a_N)



الحل

* $\sum F_t = m a_t$

$mg\cos 60° = m a_t$

$a_t = 9.81 \cos 60°$

$\boxed{a_t = 4.91\ m/s^2}$

* $\sum F_n = m a_n$

$2T - (mg)\sin 60° = m \dfrac{v^2}{\rho}$

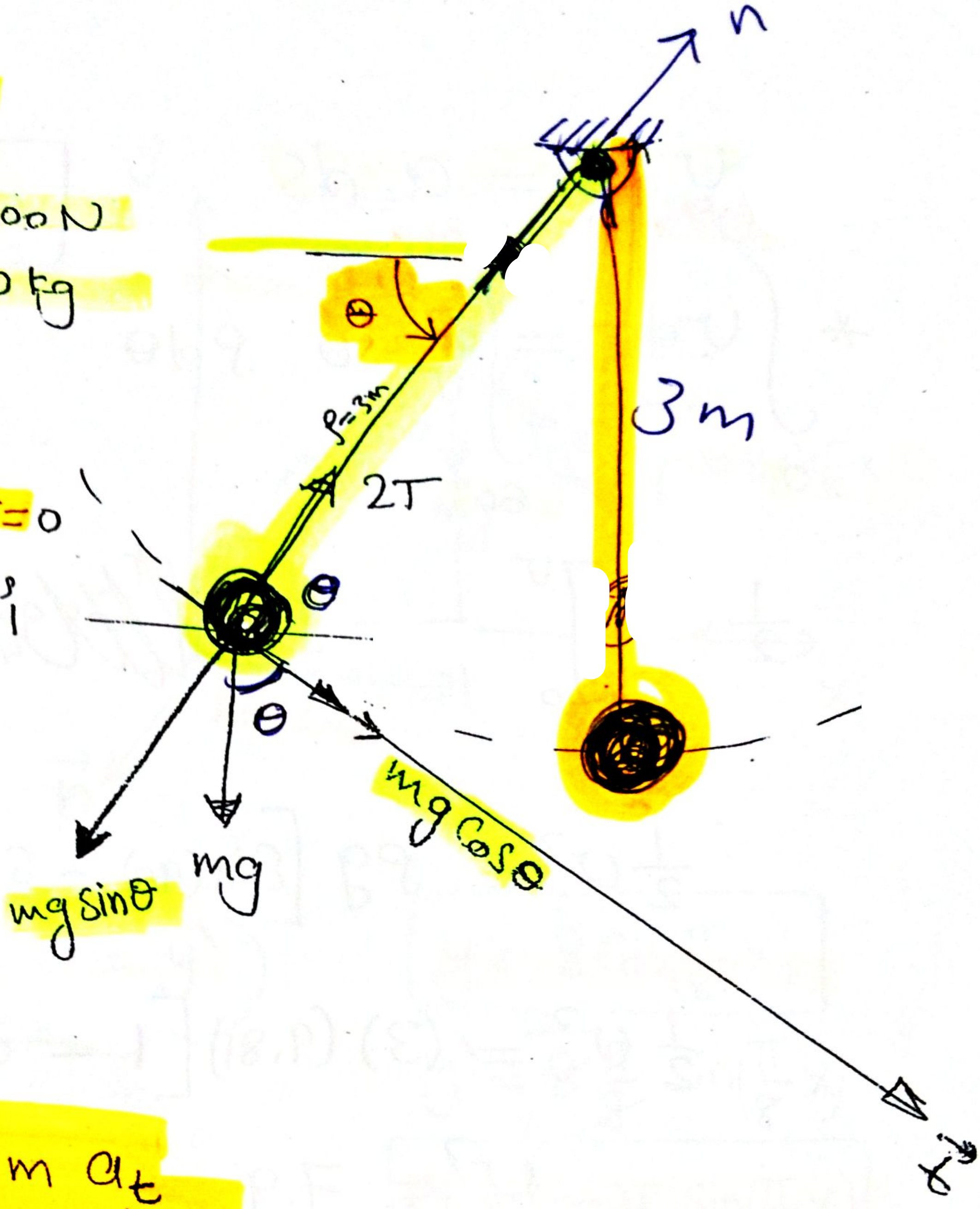
$2T - 300 \sin 60° = \dfrac{300}{9.81} \dfrac{(3)^2}{3} \Rightarrow \boxed{T = 175.8\ N}$

Ex

الوزن

$(\omega) = 300\,N$

$m = 30\,kg$



$\theta = 60^\circ \Rightarrow \upsilon = 0$

أوجد سرعة الولد

و الشد في الحبل

على زاوية $\theta = 90^\circ$

$\theta = 90^\circ$

$* \quad \sum F_t = m\,a_t$

$$\cancel{m} g \cos\theta = \cancel{m}\, a_t$$

$$\boxed{a_t = g\cos\theta} \quad —— (1)$$

$* \quad \sum F_n = m\,a_n$

$$2T - mg\sin\theta = m\frac{\upsilon^2}{\rho} \quad —— (2)$$

$v\,dv = a_t\,ds$

$$\boxed{ds = \rho\,d\theta}$$

$$* \int_0^{v} v\,dv = \int_{60^\circ}^{90} g\cos\theta\;\rho\,d\theta$$

$$\frac{1}{2}v^2\Big]_0^{v} = \rho g\sin\theta\Big]_{60}^{90}$$

$$\frac{1}{2}v^2 = \rho g\left[\sin(90) - \sin 60\right]$$

$$\frac{1}{2}v^2 = (3)(9.81)\left[1 - 0.87\right]$$

$$v^2 = 7.9$$

$$\boxed{v = 2.8 \text{ m/s}}$$

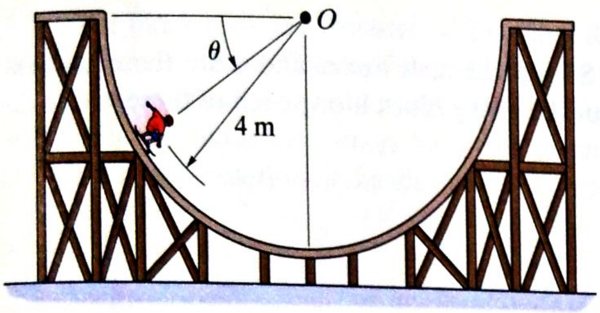
عوض في 1

$$2T - mg\sin\theta = m\frac{v^2}{\rho}$$

$$2T - 300\sin 90^\circ = 30\,\frac{(2.8)^2}{3}$$

$$\boxed{T = 189.2 \text{ N}}$$

EXAMPLE 13.9

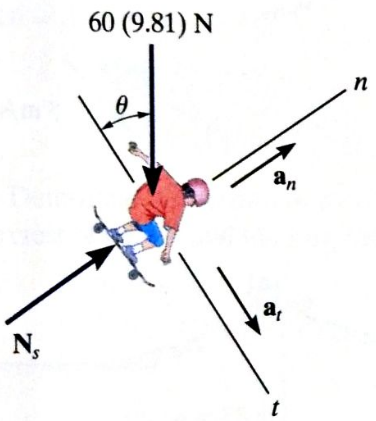


(a)

The 60-kg skateboarder in Fig. 13–15*a* coasts down the circular track. If he starts from rest when $\theta = 0°$, determine the magnitude of the normal reaction the track exerts on him when $\theta = 60°$. Neglect his size for the calculation.

SOLUTION

Free-Body Diagram. The free-body diagram of the skateboarder when he is at an *arbitrary position* θ is shown in Fig. 13–15*b*. At $\theta = 60°$ there are three unknowns, N_s, a_t, and a_n (or v).



(b)

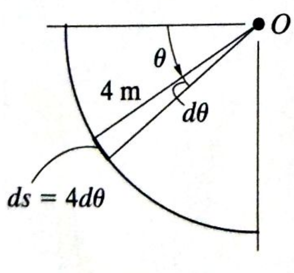
Equations of Motion.

$$+\nearrow \Sigma F_n = ma_n; \quad N_s - [60(9.81)\text{N}]\sin\theta = (60\text{ kg})\left(\frac{v^2}{4\text{ m}}\right) \qquad (1)$$

$$+\searrow \Sigma F_t = ma_t; \quad [60(9.81)\text{N}]\cos\theta = (60\text{ kg})\,a_t$$

$$a_t = 9.81\cos\theta$$

Kinematics. Since a_t is expressed in terms of θ, the equation $v\,dv = a_t\,ds$ must be used to determine the speed of the skateboarder when $\theta = 60°$. Using the geometric relation $s = \theta r$, where $ds = r\,d\theta = (4\text{ m})\,d\theta$, Fig. 13–15*c*, and the initial condition $v = 0$ at $\theta = 0°$, we have,



(c)

Fig. 13–15

$$v\,dv = a_t\,ds$$

$$\int_0^v v\,dv = \int_0^{60°} 9.81\cos\theta(4\,d\theta)$$

$$\left.\frac{v^2}{2}\right|_0^v = 39.24\sin\theta\Big|_0^{60°}$$

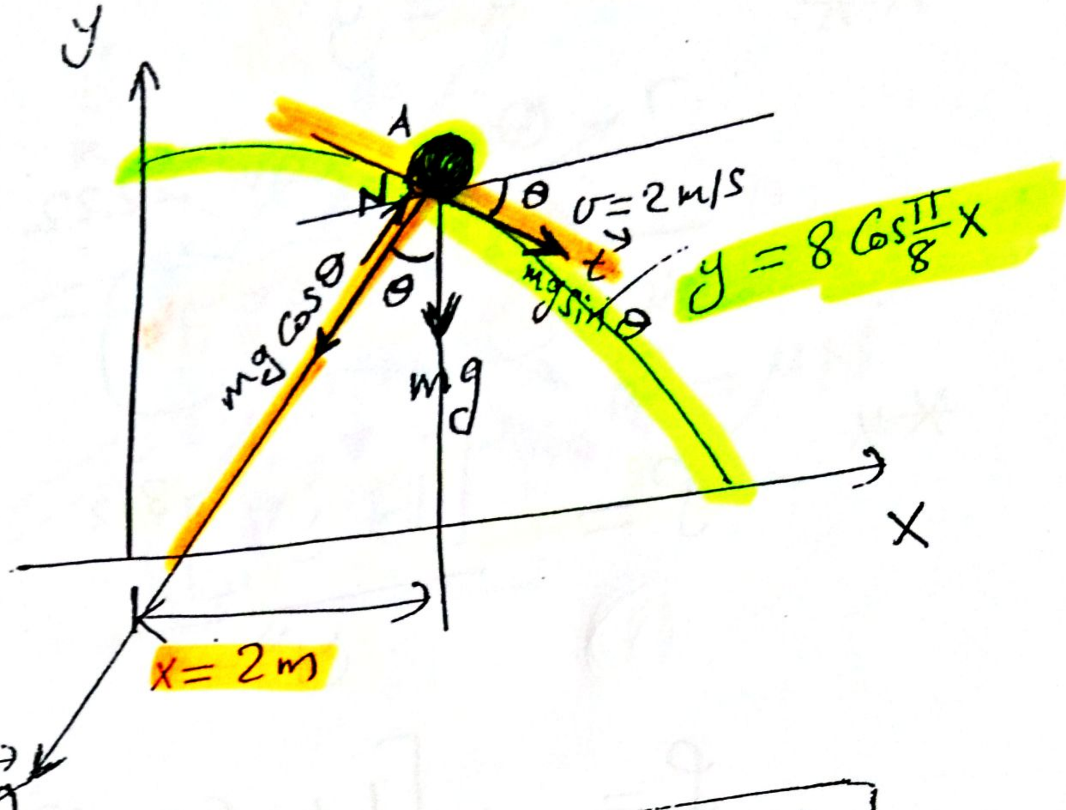
$$\frac{v^2}{2} - 0 = 39.24(\sin 60° - 0)$$

$$v^2 = 67.97\text{ m}^2/\text{s}^2$$

Substituting this result and $\theta = 60°$ into Eq. (1), yields

$$N_s = 1529.23\text{ N} = 1.53\text{ kN} \qquad \textit{Ans.}$$

Ex I



الوزن
$\omega = 900\ N$.

$m = 90\ kg$.

*Find N?

$v_A = 2\,m/s$

*$a_t = ?$

θ ع x-ais * $\tan\theta = y'$

$$\boxed{y = 8\cos\left(\frac{\pi}{8}x\right)}$$

$$y' = -8\,\frac{\pi}{\cancel{8}}\,\sin\left(\frac{\pi}{8}x\right)$$

$$\boxed{y' = -\pi\sin\left(\frac{\pi}{8}x\right)}$$

$$y'' = -\pi\,\frac{\pi}{8}\cos\left(\frac{\pi}{8}x\right)$$

$$\boxed{y'' = \frac{-\pi^2}{8}\cos\left(\frac{\pi}{8}x\right)}$$

عوض

$x = 2$

$$y = 8\cos\left(\frac{\pi}{8}(2)\right)$$

$$y = 8\cos\left[\frac{\pi}{4}\right]$$

$$\boxed{y = 5.66}$$

$$y' = -\pi\sin\left[\frac{\pi}{4}\right]$$

$$\boxed{y' = -2.22}$$

$$y'' = \frac{-\pi^2}{8}\sin\left[\frac{\pi}{4}\right] \Rightarrow \boxed{y'' = -0.87}$$

$\tan\theta = y'$

** $\theta = \tan^{-1} -2.22 = -65.7^\circ$

$\theta = +65.7^\circ$

** $\rho = \frac{[1+y'^2]^{3/2}}{y''}$

$\rho = \frac{[1+(-2.22)^2]^{3/2}}{-0.87} = -16.6 \text{ m}$

$\boxed{\rho = 16.6 \text{ m}}$

** $\sum F_n = m a_n$

$mg\cos\theta - N = m \frac{v^2}{\rho}$

$900 \cos 65.9^\circ - N = 90 \frac{(2)^2}{16.6}$

$\Rightarrow N = 349 \text{ N}$

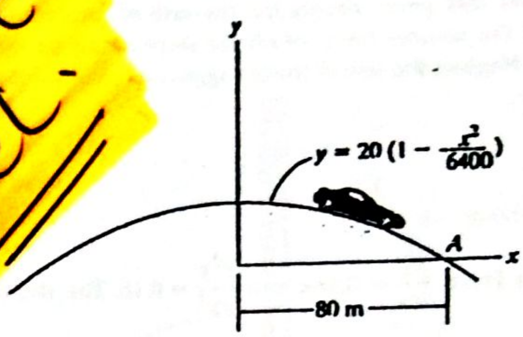
** $\sum F_t = m a_t$

$\cancel{m} g \sin\theta = \cancel{m} a_t \Rightarrow a_t = 9.81 \sin 65.7^\circ$

$a_t = 8.95 \text{ m/s}^2$

•13–73. The 0.8-Mg car travels over the hill having the shape of a parabola. When the car is at point A, it is traveling at 9 m/s and increasing its speed at 3 m/s^2. Determine both the resultant normal force and the resultant frictional force that all the wheels of the car exert on the road at this instant. Neglect the size of the car.

Constant accel.

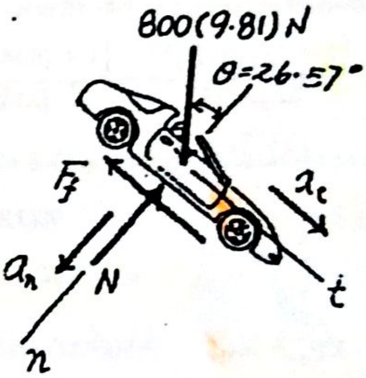


Geometry: Here, $\frac{dy}{dx} = -0.00625x$ and $\frac{d^2y}{dx^2} = -0.00625$. The slope angle θ at point A is given by

$$\tan\theta = \frac{dy}{dx}\bigg|_{x=80\text{ m}} = -0.00625(80) \qquad \theta = -26.57^\circ$$

and the radius of curvature at point A is

$$\rho = \frac{\left[1 + (dy/dx)^2\right]^{3/2}}{|d^2y/dx^2|} = \frac{\left[1 + (-0.00625x)^2\right]^{3/2}}{|-0.00625|}\bigg|_{x=80\text{ m}} = 223.61 \text{ m}$$



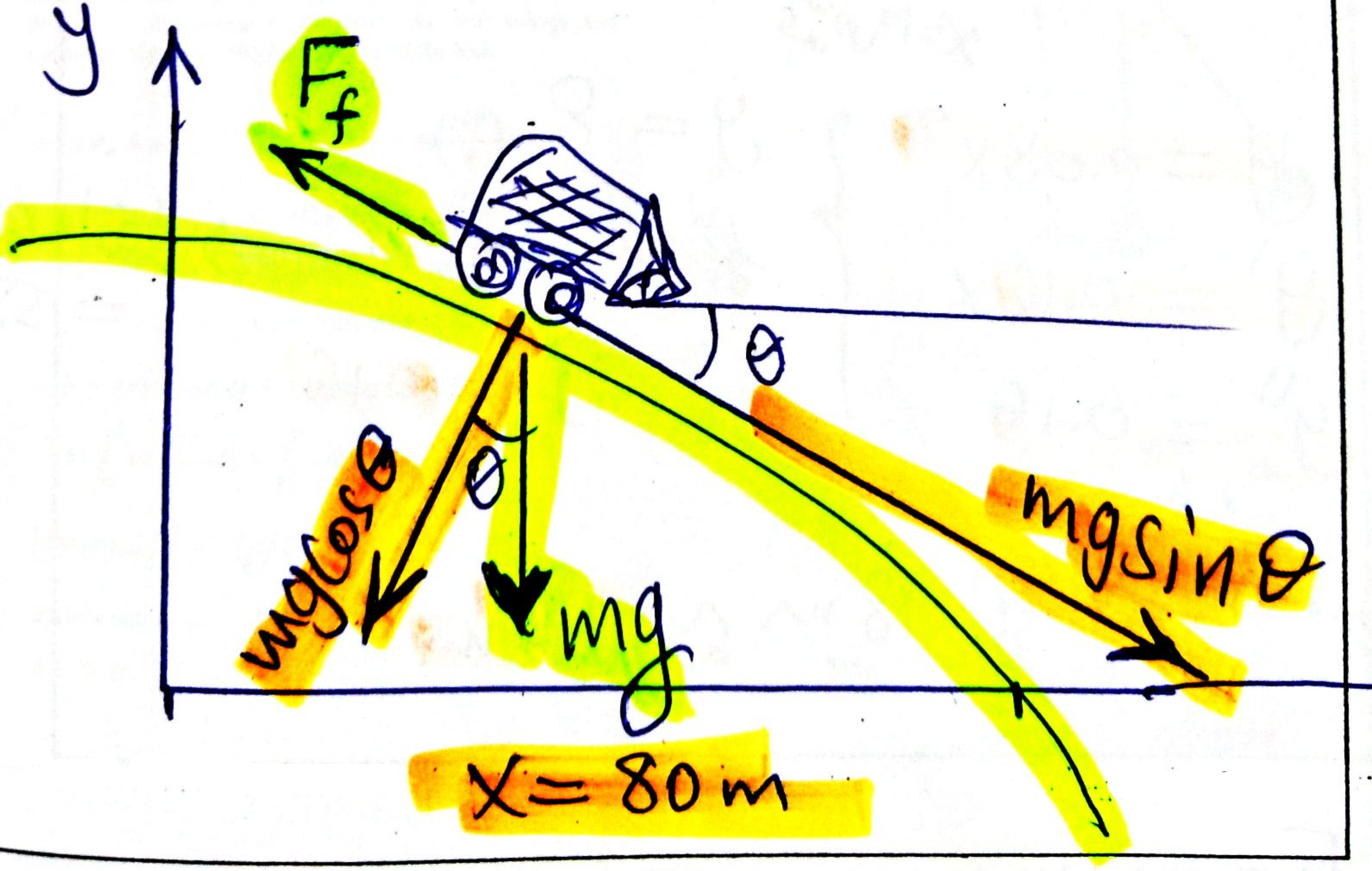
Equation of Motion: Applying Eq. 13–8 with $\theta = 26.57^\circ$ and $\rho = 223.61$ m, we have

$\Sigma F_t = ma_t;$ $\qquad 800(9.81)\sin 26.57^\circ - F_f = 800(3)$

$F_f = 1109.73 \text{ N} = 1.11 \text{ kN}$ **Ans.**

$\Sigma F_n = ma_n;$ $\qquad 800(9.81)\cos 26.57^\circ - N = 800\left(\frac{9^2}{223.61}\right)$

$N = 6729.67 \text{ N} = 6.73 \text{ kN}$ **Ans.**

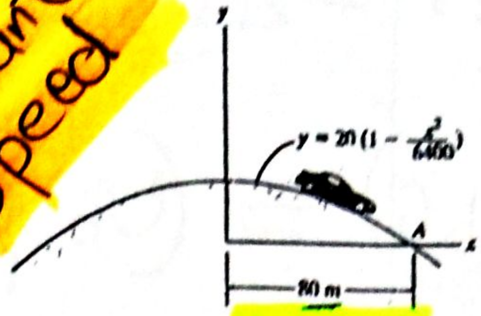


•13–72. The 0.8-Mg car travels over the hill having the shape of a parabola. If the driver maintains a constant speed of 9 m/s, determine both the resultant normal force and the resultant frictional force that all the wheels of the car exert on the road at the instant it reaches point A. Neglect the size of the car.

Constant Speed

Find N ?

قوة الاحتكاك Find F_f ?



$x = 80$ m

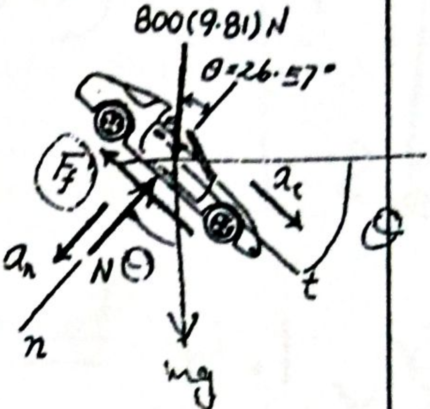
Geometry: Here, $\boxed{\frac{dy}{dx} = -0.00625x}$ and $\frac{d^2y}{dx^2} = -0.00625$. The slope angle θ at point A is given by

$$\tan\theta = \frac{dy}{dx}\bigg|_{x=80\text{ m}} = -0.00625(80) \qquad \boxed{\theta = -26.57^\circ}$$

and the radius of curvature at point A is

$$\rho = \frac{[1 + (dy/dx)^2]^{3/2}}{|d^2y/dx^2|} = \frac{[1 + (-0.00625x)^2]^{3/2}}{|-0.00625|}\Bigg|_{x=80\text{ m}} = 223.61\text{ m}$$

Equations of Motion: Here, $a_t = 0$. Applying Eq. 13–8 with $\theta = 26.57^\circ$ and $\rho = 223.61$ m, we have



$\Sigma F_t = ma_t;$ $\qquad 800(9.81)\sin 26.57^\circ - F_f = 800(0)$

$$F_f = 3509.73\text{ N} = \boxed{3.51\text{ kN}} \qquad \textbf{Ans.}$$

$\Sigma F_n = ma_n;$ $\qquad 800(9.81)\cos 26.57^\circ - N = 800\left(\frac{9^2}{223.61}\right)$

$$N = 6729.67\text{ N} = \boxed{6.73\text{ kN}} \qquad \textbf{Ans.}$$

$m = 0.8 \times 10^3$ kg

$\boxed{m = 800\text{ kg}}$

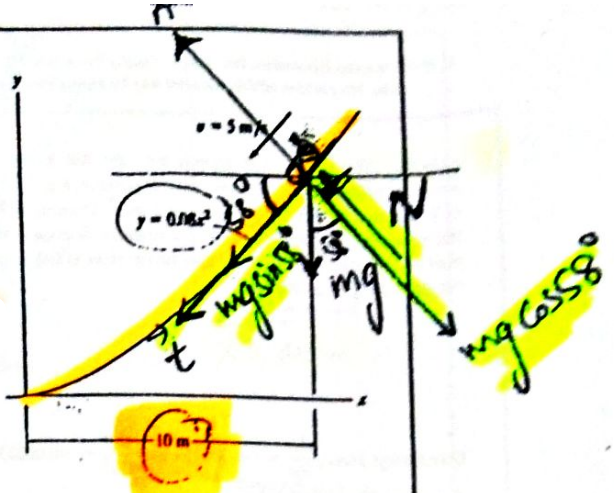
$$y = 20\left(1 - \frac{x^2}{6400}\right)$$

$$* \quad y = 20 - \frac{x^2}{320} = 20 - 1.5625 \times 10^{-4} x^2$$

$$* \quad y' = 0 - \frac{2x}{320} = \frac{-x}{160} = \boxed{-0.00625x}$$

$$* \quad y'' = \frac{-1}{160} = \boxed{-0.00625}$$

*13–76. A toboggan and rider of total mass 90 kg travel down along the (smooth) slope defined by the equation $y = 0.08x^2$. At the instant $x = 10$ m, the toboggan's speed is 5 m/s. At this point, determine the rate of increase in speed and the normal force which the slope exerts on the toboggan. Neglect the size of the toboggan and rider for the calculation.



Geometry: Here, $\frac{dy}{dx} = 0.16x$ and $\frac{d^2y}{dx^2} = 0.16$. The slope angle θ at $x = 10$ m is given by

$$\tan\theta = \left.\frac{dy}{dx}\right|_{x=10\text{ m}} = 0.16(10) \qquad \theta = 57.99^\circ$$

and the radius of curvature at $x = 10$ m is

$$\rho = \frac{[1 + (dy/dx)^2]^{3/2}}{|d^2y/dx^2|} = \left.\frac{[1 + (0.16x)^2]^{3/2}}{|0.16|}\right|_{x=10\text{ m}} = 41.98\text{ m}$$

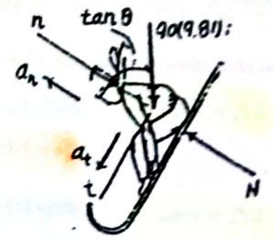
Equations of Motion: Applying Eq. 13–8 with $\theta = 57.99^\circ$ and $\rho = 41.98$ m, we have

$\Sigma F_t = ma_t;$ $\qquad 90(9.81)\sin 57.99^\circ = 90a_t$

$a_t = 8.32\text{ m/s}^2$ **Ans.**

$\Sigma F_n = ma_n;$ $\qquad -90(9.81)\cos 57.99^\circ + N = 90\left(\frac{5^2}{41.98}\right)$

$N = 522$ N **Ans.**



$x = 10$ عوض

$y = 0.08x^2$
$y' = 0.16x$
$y'' = 0.16$

$y = 8$
$y' = 1.6 \Rightarrow \tan^{-1} 1.6 = \theta = 58^\circ$
$y'' = 0.16$

تعوض $\rho = 41.98\text{ m} \approx 42\text{ m}$

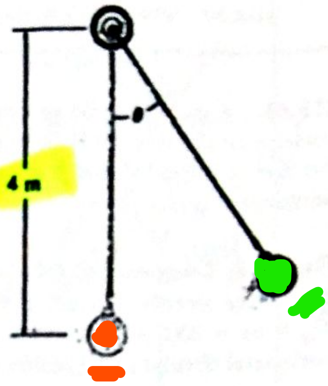
① $\Sigma F_t = m a_t$

$m g \sin 58^\circ = m a_t \Rightarrow a_t = 9.81 \sin 58^\circ = 8.32\text{ m/s}^2$

② $\Sigma F_n = m a_n = m \frac{v^2}{\rho}$

$N - mg\cos 58^\circ = 90 \frac{(5)^2}{42} \Rightarrow N = 522\text{ N}$

13–62. The ball has a mass of 30 kg and a speed $v = 4$ m/s at the instant it is at its lowest point, $\theta = 0°$. Determine the tension in the cord and the rate at which the ball's speed is decreasing at the instant $\theta = 20°$. Neglect the size of the ball.



$+\nwarrow \Sigma F_n = ma_n;\quad T - 30(9.81)\cos\theta = 30\left(\frac{v^2}{4}\right)$

$+\nearrow \Sigma F_t = ma_t;\quad -30(9.81)\sin\theta = 30a_t$

$a_t = -9.81\sin\theta$

$a_t\,ds = v\,dv$ Since $ds = 4\,d\theta$, then

$$-9.81\int_0^\theta \sin\theta\,(4\,d\theta) = \int_4^v v\,dv$$

$$9.81(4)\cos\theta\Big|_0^\theta = \frac{1}{2}(v)^2 - \frac{1}{2}(4)^2$$

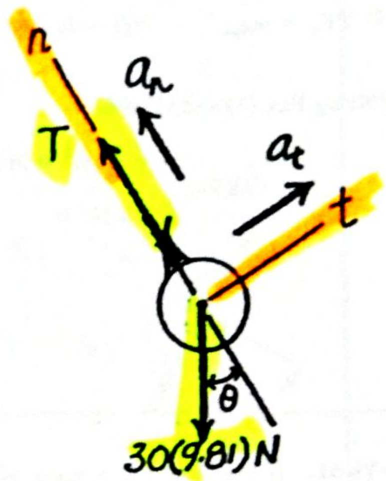
$$39.24(\cos\theta - 1) + 8 = \frac{1}{2}v^2$$

At $\theta = 20°$

$v = 3.357$ m/s

$a_t = -3.36\ \text{m/s}^2 = 3.36\ \text{m/s}^2$ ✓ **Ans.**

$T = 361$ N **Ans.**



* $\sum F_n = m\frac{v^2}{\rho} \Rightarrow T - mg\cos\theta = m\frac{v^2}{\rho}$

* $\sum F_t = m a_t \Rightarrow -mg\sin\theta = m a_t$

* $v\,dv = a_t\,ds \;,\quad ds = \rho\,d\theta$

$$\int_4^v v\,dv = \int_0^{20°} -g\sin\theta\,(\rho\,d\theta)$$

Ch 14

Ch14

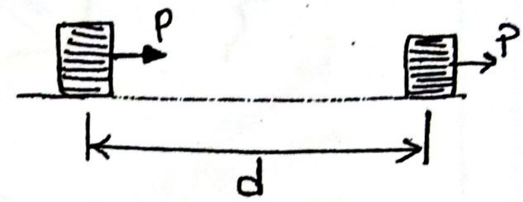
(Work and Energy
الشغل والطاقة)

1 شغل القوة الخارجية

* القوة دائماً باتجاه المسافة

القوة نفس اتجاه الحركة ← + ; القوة عكس اتجاه الحركة ← −

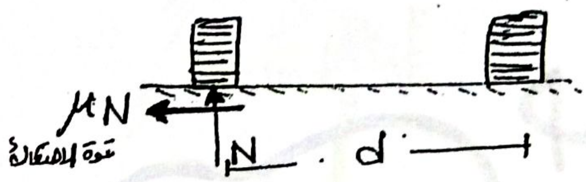
$$\pm Pd$$



2 شغل قوة الاحتكاك

دائماً سالبة

$$-\mu N d$$

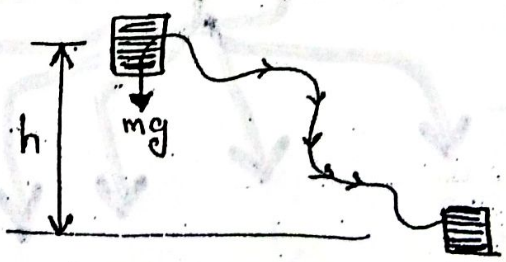


$\mu \equiv$ معامل الاحتكاك Coeff of friction

3 شغل الوزن

الهبوط ← + ; الصعود ← −

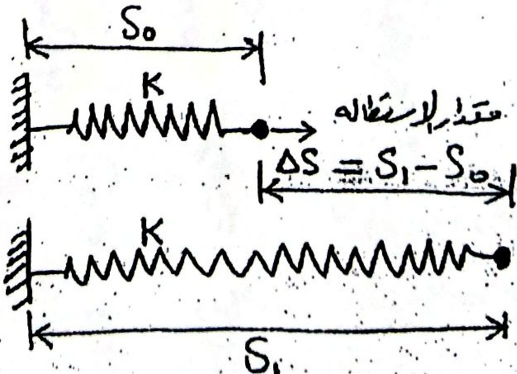
$$\pm mgh$$



4 شغل الزنبرك spring

S_0 الطول غير المشدود (الأصلي) Unstretched length

S_1 Stretched length



(+ عكسه) الاقتراب من الطول الأصلي

$$\pm \frac{1}{2} K \Delta S^2$$

(−) الابتعاد عن الطول الأصلي (مقاومة)

5 شغل طاقة الحركة

سرعة الجسم v

$$T = +\frac{1}{2} m v^2$$

* Spring Energy from A to B :

$U_{AB} = \frac{1}{2} K \left[S_A^2 - S_B^2 \right]$ ✗ wrong

$U_{AB} = \frac{1}{2} K \left[(S_A - S_0)^2 - (S_B - S_0)^2 \right]$ ✓ O.K.

Spring : ($K = 100$ N/cm)

* unstreched length = 7 cm
* Initial length at A = 12 cm
* Final length at B = 20 cm

Energy from A to B is U_{AB} :

$\Rightarrow U_{AB} = -\frac{1}{2} K \Delta S^2$

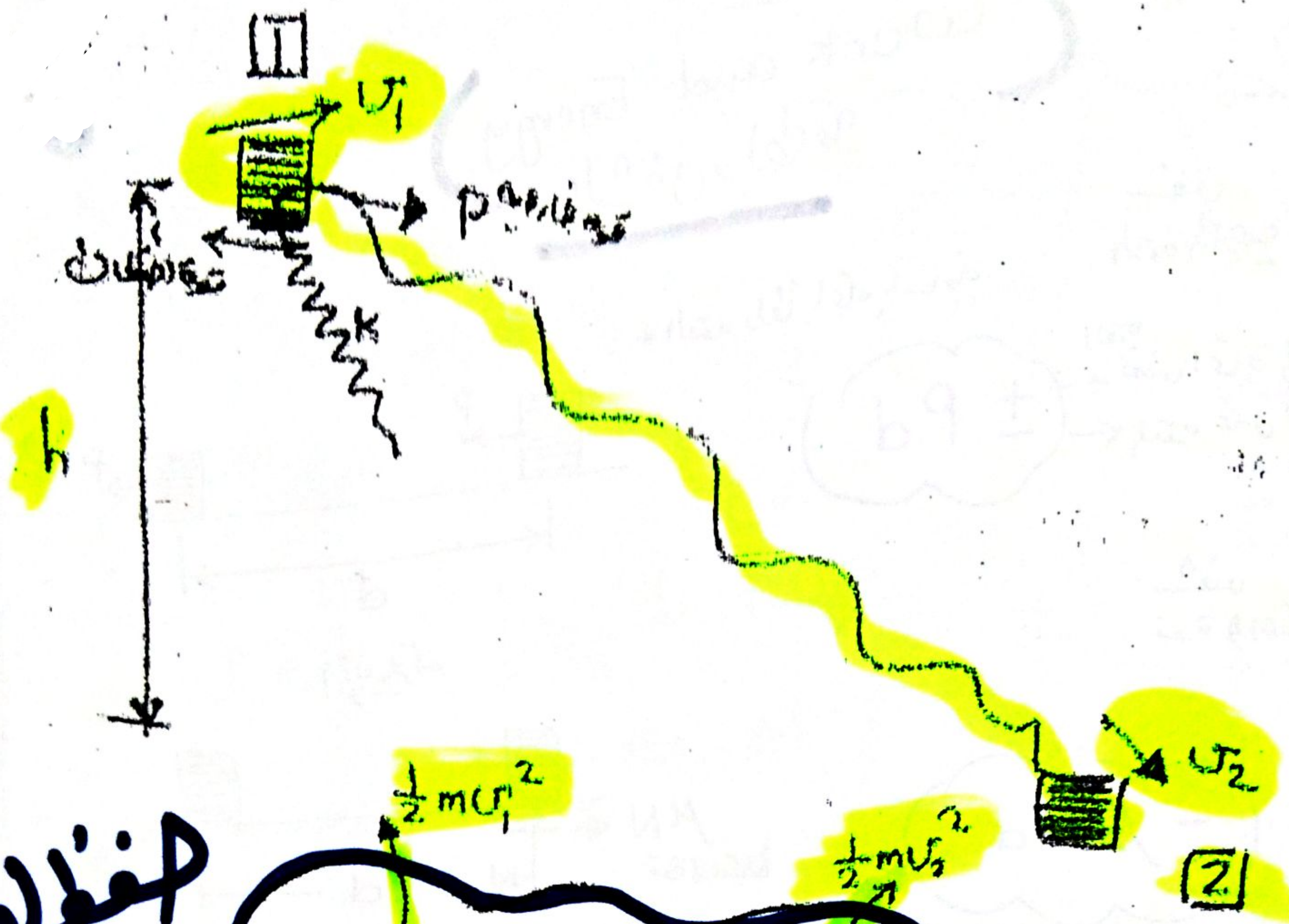
$= -\frac{1}{2} (100) \left[20^2 - 12^2 \right]$

$= -12800$ joule. ✗ wrong

$\Rightarrow U_{AB} = \frac{1}{2} K \left[\underbrace{(12-7)^2}_{\text{at A}} - \underbrace{(20-7)^2}_{\text{at B}} \right]$

$= \frac{1}{2} (100) \left[5^2 - 13^2 \right]$

$= -7200$ Joule ✓



القانون العام

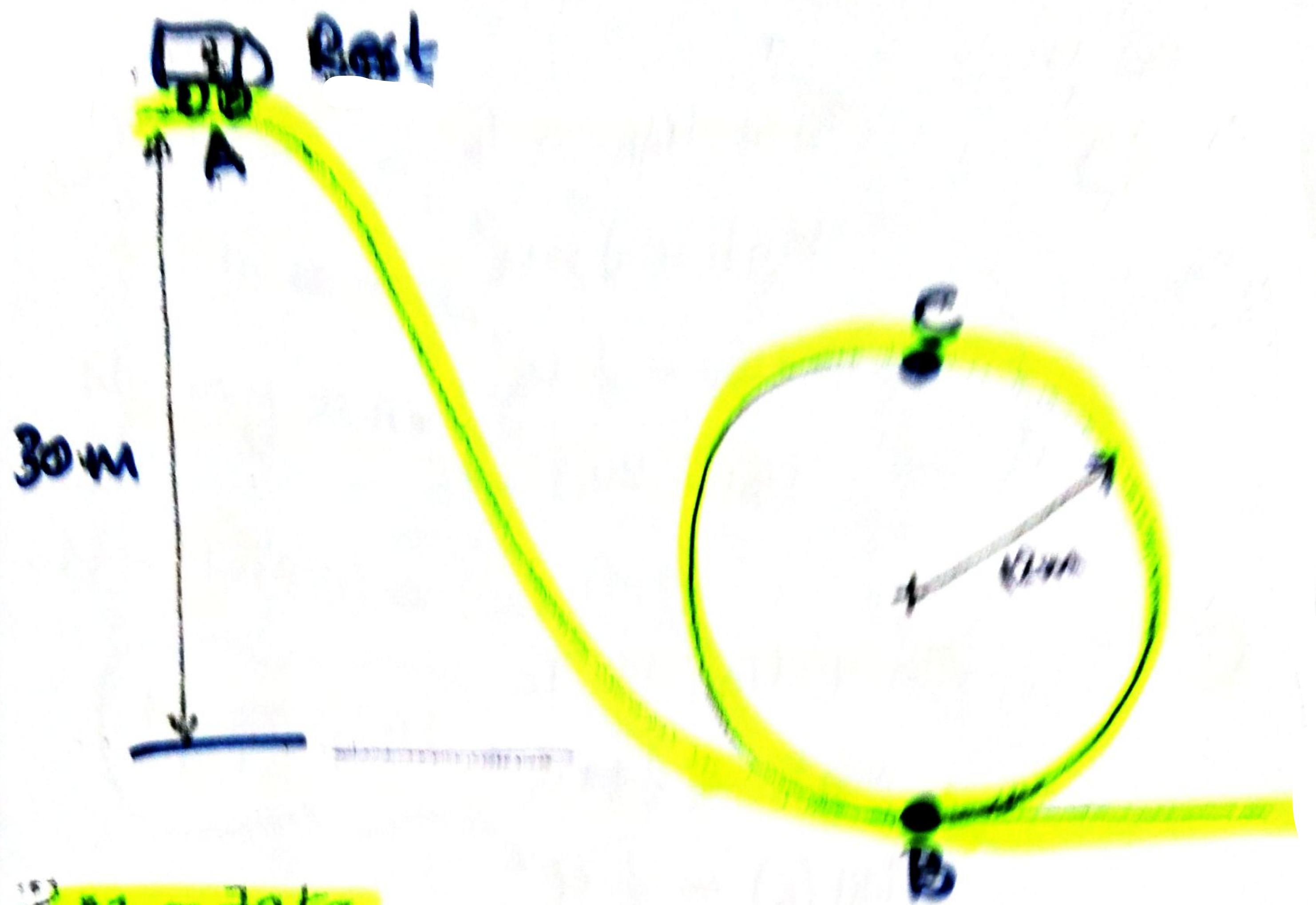
$$T_1 + U_{12} = T_2$$

$\pm Pd$

$-\mu N x$

$\pm mgh$

$\pm \frac{1}{2}kS^2$



$m_1 = 70\,kg$

$m_2 = 50\,kg$

1. Find the velocity at B ?
2. Find the velocity at C ?
3. Find the Normal force (N) bet. the seat and the boy ? at B ?
4. Find the normal force (N) at point C ?

1

$$\cancel{T_A}^{\,0} + U_{AB} = T_B$$

$$\cancel{m}gh = \frac{1}{2}\cancel{m}v_B^2$$

$$9.81(30) = \frac{1}{2}v_B^2$$

$$\Rightarrow \quad v_B = 24.3 \text{ m/S}$$

2

$$\cancel{T_A}^{\,0} + U_{AC} = T_C$$

$$\cancel{m}gh = \frac{1}{2}\cancel{m}v_C^2$$

$$9.81(6) = \frac{1}{2}v_C^2$$

$$\Rightarrow \quad v_C = 10.8 \text{ m/S}$$

حل آخر

$$T_B + U_{BC} = T_C$$

$$\frac{1}{2}\cancel{m}v_B^2 - \cancel{m}gh = \frac{1}{2}\cancel{m}v_C^2$$

$$\frac{1}{2}(24.3)^2 - 9.81(24) = \frac{1}{2}v_C^2$$

$$v_C = 10.8 \text{ m/S}$$ نفس الاجابة

③ Ch13

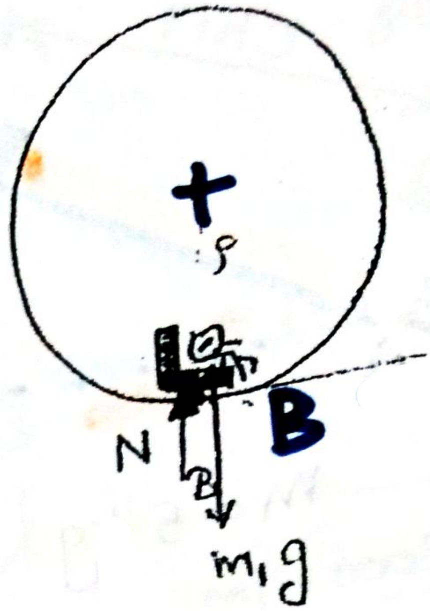
$$\sum F_n = m\,a_n$$

$$N - m_1 g = m\,\frac{v_B^2}{\rho}$$

ယာဉ်ရဲ့·အလေး

$$N - 70(9.81) = (120)\,\frac{(24.3)^2}{12}$$

$$N = 6591 \text{ Newton}$$

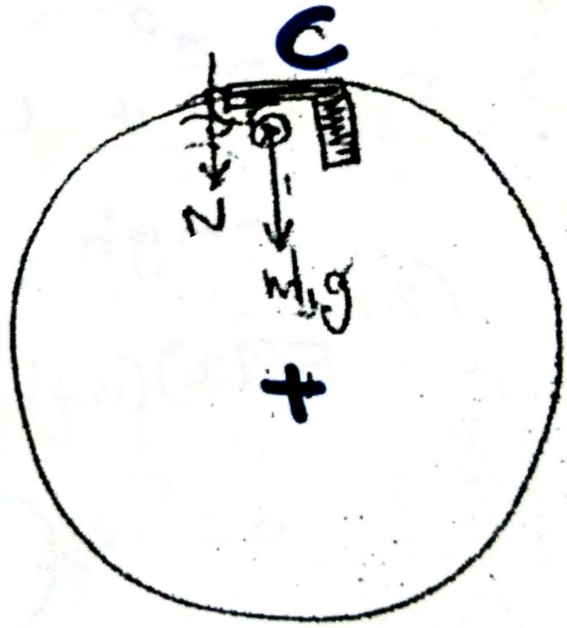


Ch13

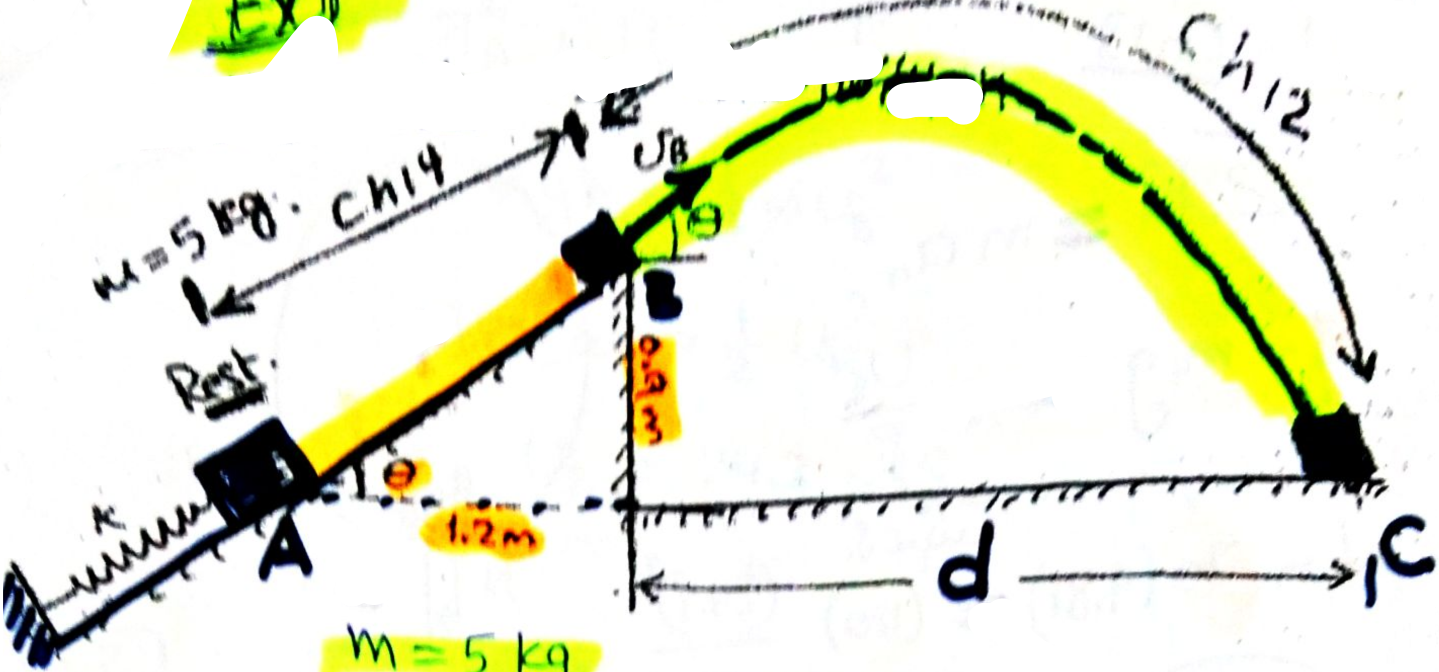
④ $$N + m_1 g = m\,\frac{v_C^2}{\rho}$$

$$N + 70(9.81) = 120\,\frac{(10.8)^2}{12}$$

$$N = 480 \text{ Newton}$$



Ex 11



$m = 5\ kg$

$K = 2000\ N/m$ Find d?

the spring is originally Compressed By 0.6 m.

Sol:

$\theta = \tan^{-1}\frac{0.9}{1.2} = 36.8^\circ$

Find

1. v_B?
2. t_{BC}?
3. d?

$\cancelto{0}{T_A} + U_{AB} = T_B$

$-mgh + \frac{1}{2}K\Delta s^2 = \frac{1}{2}m v_B^2$

$-5(9.81)(0.9) + \frac{1}{2}(2000)(0.6)^2 = \frac{1}{2}5\ v_B^2$

$\Rightarrow v_B = \cancel{12.7}\ 11.24\ m/s$

$H = (v_0 \sin\theta)t - \frac{1}{2}gt^2$

$-0.9 = (11.24 \sin 36.8)t - 4.905t^2$

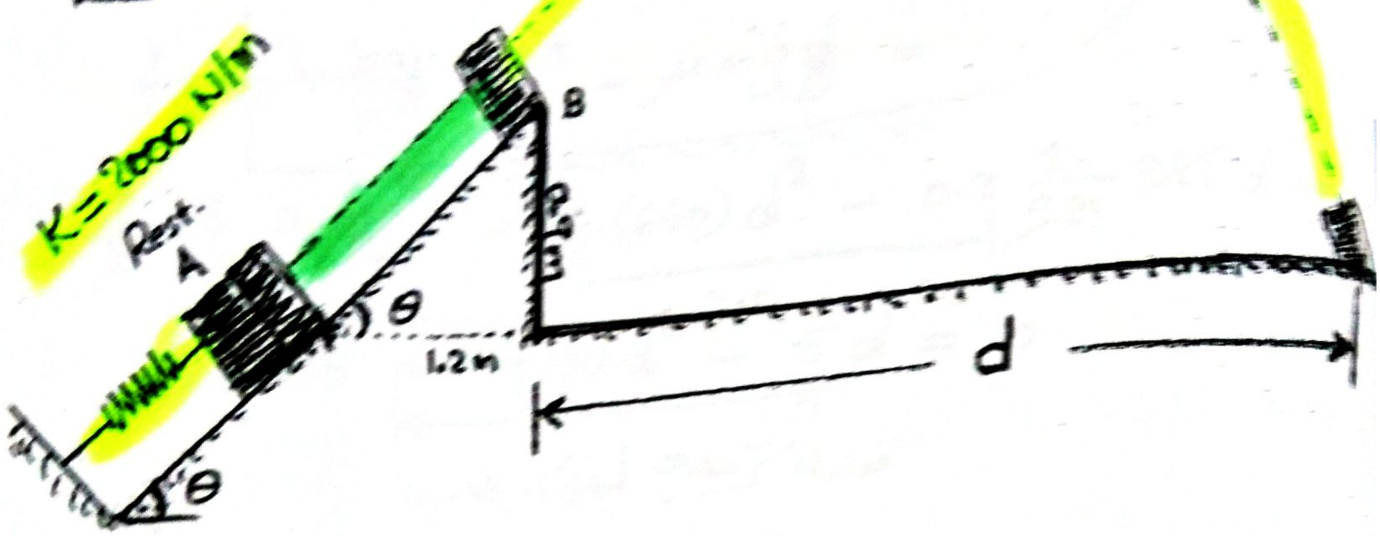
$\Rightarrow 4.905t^2 - 6.73t - 0.9 = 0$

$\boxed{t = \quad sec}$

$d = (v_0\cos\theta)\,t$

الوزن $W = 50\ N$

Ex



* the spring is compressed at A 0.6 m

Find the horizontal distance d ?

$$\theta = \tan^{-1}\frac{0.9}{1.2} \Rightarrow \theta = 36.9^\circ$$

Ch.14

$$T_A + U_{AB} = T_B$$

$$0 + \frac{1}{2}K(0.6)^2 - mg(0.9) = \frac{1}{2}m v_B^2$$

$$\frac{1}{2}(2000)(0.6)^2 - 50(0.9) = \frac{1}{2}\,\frac{50}{9.81}\,v_B^2$$

$$\Rightarrow v_B = 11.11\ m/s = v_0$$

Ch.12 * $H = (v_0 \sin\theta)t - \frac{1}{2}g t^2$

$[illegible] = (11.11 \sin 36.9^\circ)\,t - \frac{1}{2}(9.81)\,t^2$

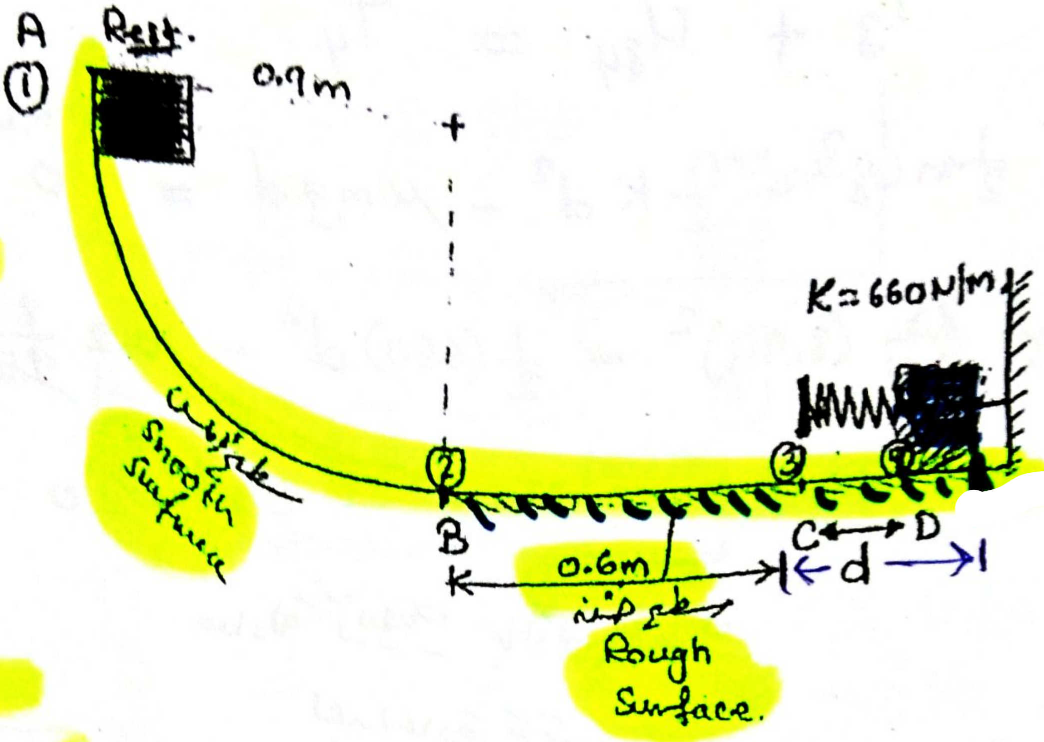
$\Rightarrow t = 1.48\ sec$

[illegible]

$$d = (v_0 \cos\theta)\,t$$

Example

الوزن
$W = 25$ N



$\mu = 0.2$

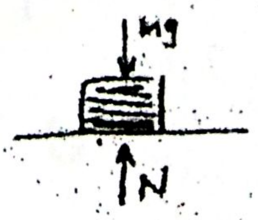
Find the Compress in the Spring
أوجد مقدار الانضغاط في الزنبرك.

$$T_1 + U_{12} = T_2$$

$$0 + mgh = \frac{1}{2} m v_B^2$$

$$9.81(0.9) = \frac{1}{2} v_B^2 \Rightarrow v_B = 4.202 \text{ m/s}.$$

$$T_2 + U_{23} = T_3$$



$N = mg$

$$\frac{1}{2} m v_B^2 - \mu N (0.6) = \frac{1}{2} m v_C^2$$

$$\frac{1}{2} m v_B^2 - \mu m g (0.6) = \frac{1}{2} m v_C^2$$

$$\frac{1}{2}(4.202)^2 - 0.2(9.81)(0.6) = \frac{1}{2} v_C^2$$

$$v_C = 3.912 \text{ m/s}.$$

$$T_3 + U_{34} = T_4$$

توقف

$$\frac{1}{2} m v_2^2 - \frac{1}{2} k d^2 - \mu m g d = 0$$

$$\frac{1}{2} \frac{25}{9.81} (3.912)^2 - \frac{1}{2}(660) d^2 - 0.2 \frac{25}{\cancel{9.81}} \cancel{9.81}\, d = 0$$

$$19.5 - 330 d^2 - 5d = 0$$

معادلة تربيعية بالآلة الحاسبة

بعد اعادة ترتيب المعادلة

$$-330 d^2 - 5d + 19.5 = 0$$

$$\Rightarrow \quad d = 0.2356 \text{ m}$$

طريقة أخرى للحل

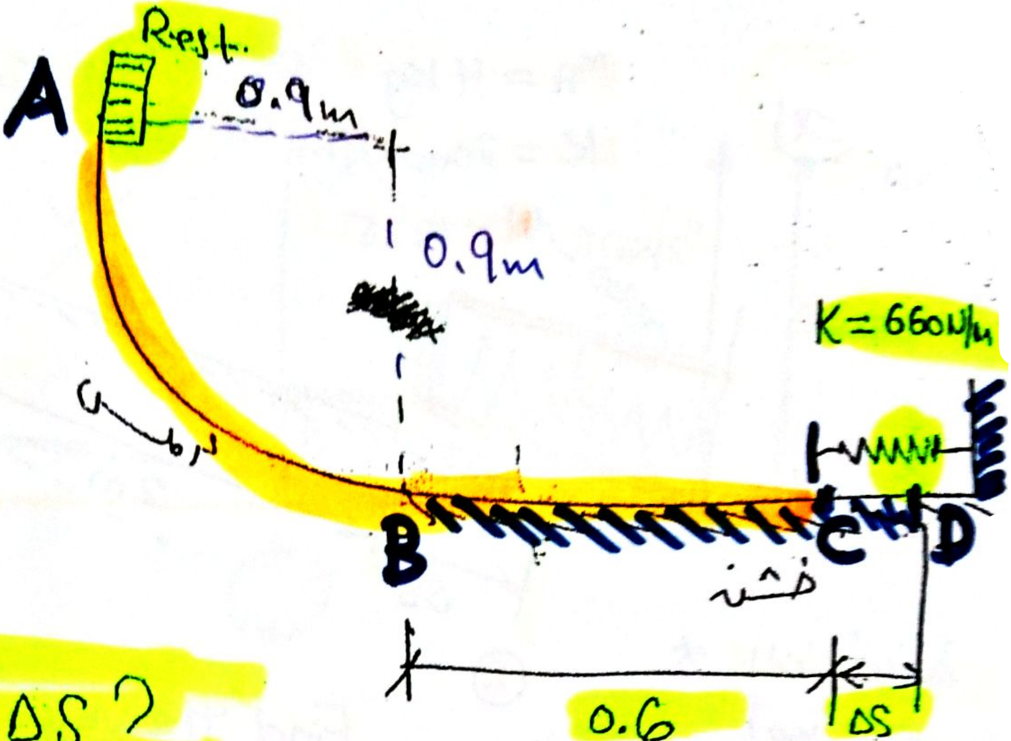
$$T_1 + U_{14} = T_4$$

$$0 + mgh - \mu m g (0.6 + d) - \frac{1}{2} k d^2 = 0$$

$$25(0.9) - 0.2(25)(0.6 + d) - \frac{1}{2}(660) d^2 = 0$$

$$d = 0.2356 \text{ m}$$

Ex 11



$M = 2.5\,Kg$

$\mu = 0.2$

Find ΔS ?

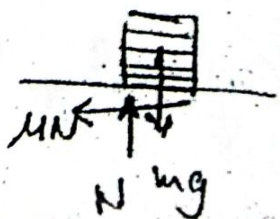
$$\cancel{T_A}^{\,0} + U_{AD} = \cancel{T_D}^{\,0}$$

$$+mgh - \mu \overset{mg}{N}(0.6+\Delta S) - \frac{1}{2}K\,\Delta S^2 = 0$$

$$2.5(9.81)(0.9) - 0.2(2.5)(9.81)(0.6+\Delta S) - \frac{1}{2}660\,\Delta S^2 = 0$$

$$22.1 - 4.91(0.6+\Delta S) - 330\,\Delta S^2 = 0$$

$$22.1 - 2.95 - 4.91\,\Delta S - 330\,\Delta S^2 = 0$$



$$330\,\Delta S^2 + 4.91\,\Delta S - 19.15 = 0$$

$\Sigma F_y = 0$

$\boxed{N = mg}$

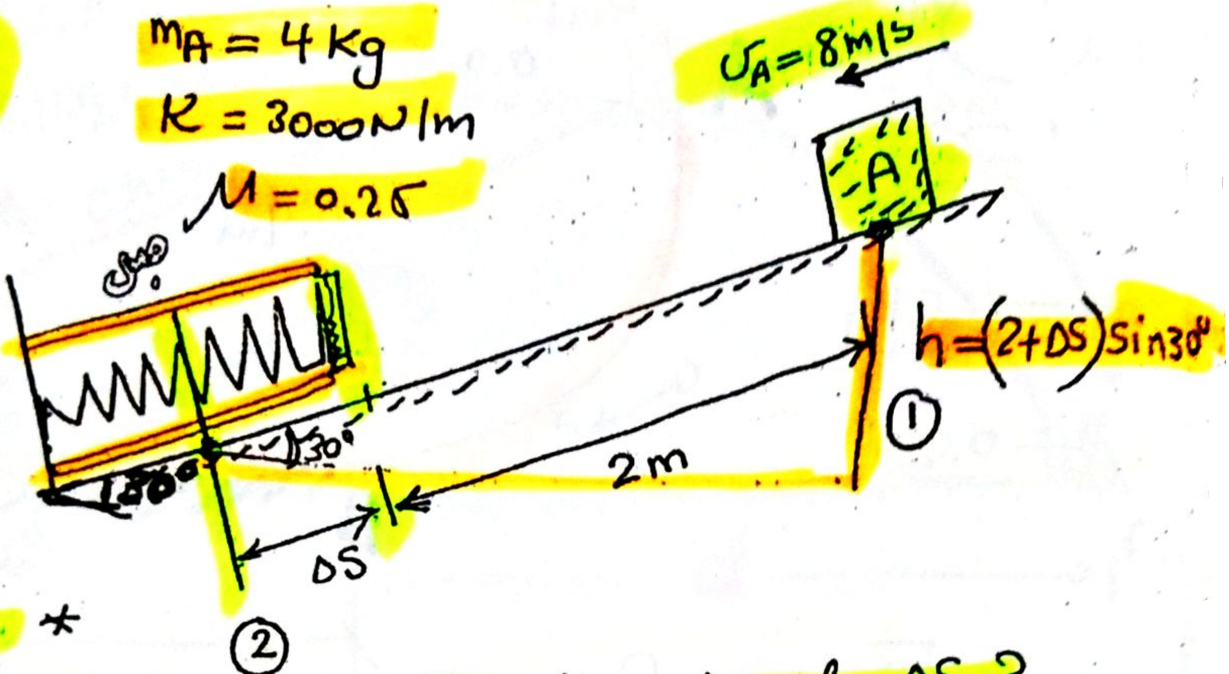
$\Delta S = 0.233\ m$

ex)

$m_A = 4\,Kg$

$K = 3000\,N/m$

$\mu = 0.25$



* الزنبرك مضغوط أصلاً بسبب الحبل بمقدار 0.09 m

Find the value of ΔS ?

أوجد المسافة التي سوف ينضغط الزنبرك نتيجة اصطدام الـ A.

الحل

$$T_1 + U_{12} = T_2$$

$$\Rightarrow \frac{1}{2} m v_1^2 + mg(2+\Delta S)\sin 30° - \mu N (2+\Delta S)$$

$$+ \frac{1}{2}K(0.09)^2 - \frac{1}{2}K(0.09+\Delta S)^2 = 0$$

$mg\cos\theta$

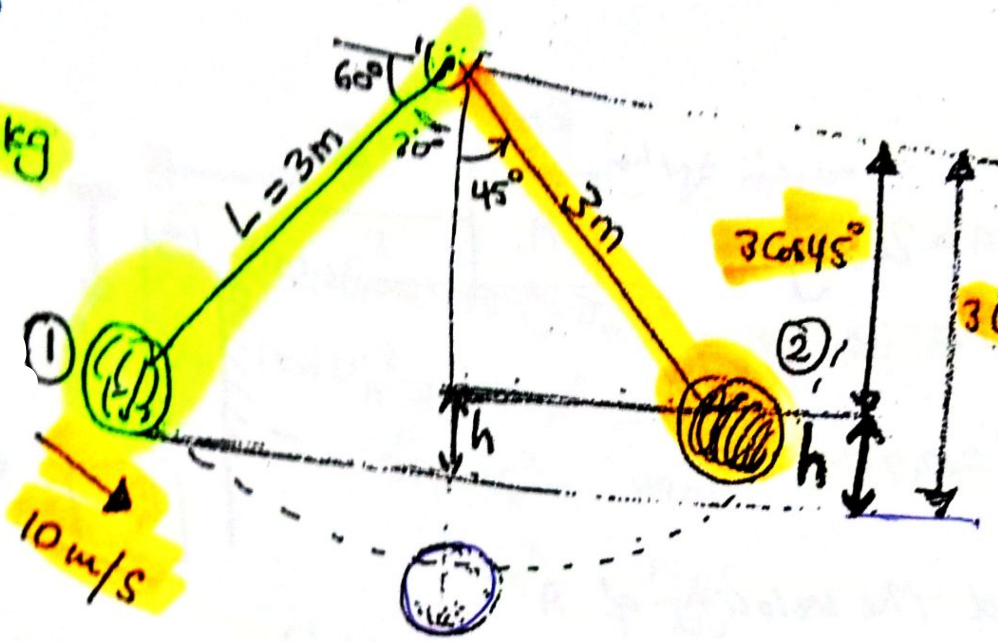
$$\frac{1}{2}(4)(8)^2 + (4)(9.81)(2+\Delta S)\sin 30° - 0.25(4)(9.81)\cos 30°(2+\Delta S)$$

$$- \frac{1}{2}(3000)(0.09)^2 - \frac{1}{2}K(0.09+\Delta S)^2 = 0$$

$\Delta S = \ldots\ldots$ m ✓

ex)

$m = 12\,kg$



Find ① the velocity at $\theta = 45°$

② the tension in cable الشد في الحبل

$$h = 3\cos 30° - 3\cos 45°$$

$$h = 0.48\ m$$

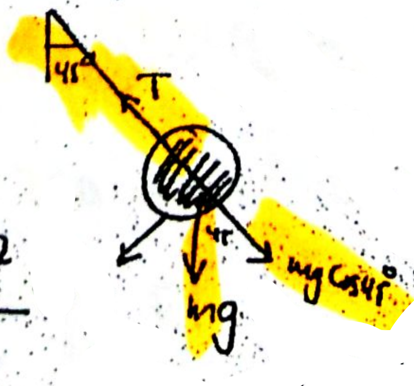
① $T_1 + U_{12} = T_2$

$$\frac{1}{2} m (10)^2 - m g h = \frac{1}{2} m v_2^2$$

$$\Rightarrow \frac{1}{2}(100) - 9.81(0.48) = \frac{1}{2} v_2^2$$

$$\Rightarrow v_2 = 9.517\ m/s$$

② $\sum F_n = m a_n$



$$T - mg\cos 45° = m\frac{v^2}{L}$$

$$T - 12(9.81)\cos 45° = 12\,\frac{(9.517)^2}{3}$$

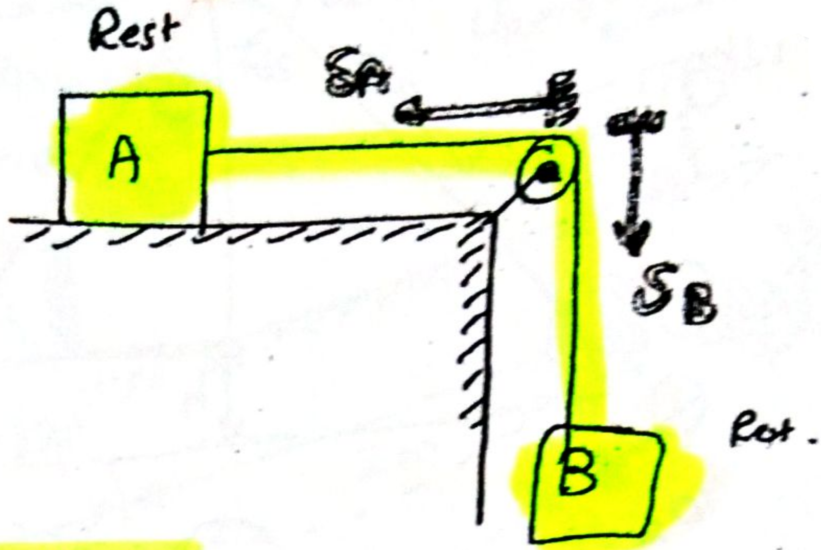
$$T = \ldots\ N$$

ex)

$m_A = 200kg.$

$m_B = 300kg.$

$\mu = 0.25$



Find the velocity of A after it moves 2m to the right?

الحل

$S_A + S_B = L$

$\Rightarrow \boxed{v_A + v_B = 0}$ —— 1

$\Rightarrow \boxed{a_A + a_B = 0}$

Block A

$\overset{\text{Rest } 0}{T_1} + U_{12} = T_2$

$\Rightarrow m_B g d - \mu \overset{mg}{N} d = \frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B v_B^2$

$\Rightarrow 300(9.81)(2) - 0.25(200)(9.81)(2) = \frac{1}{2}(200) v_A^2 + \frac{1}{2}(300) v_B^2$

$$\Rightarrow \boxed{\begin{aligned} &100 v_A^2 + 150 v_B^2 = 4905 \quad —— 2 \\ &v_A + v_B = 0 \quad —— 1 \end{aligned}}$$

هو داخلية تربيع سالب :

$$100 v_A^2 + 150 (-v_A)^2 = 4905$$

$$100 v_A^2 + 150 v_A^2 = 4905$$

$$250 v_A^2 = 4905$$

$$v_A^2 = \frac{4905}{250}$$

$$v_A^2 = 19.62$$

$$v_A = \sqrt{19.62}$$

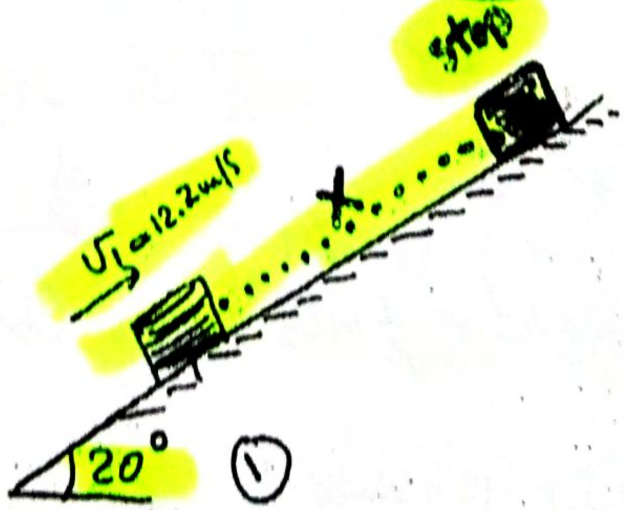
$$\boxed{\begin{array}{l} v_A = 4.43 \text{ m/s} \\ \hline v_B = -4.43 \text{ m/s} \end{array}}$$

ex)

$m = 22.7$ kg.

$\mu = 0.15$.

$v_i = 12.2$ m/s



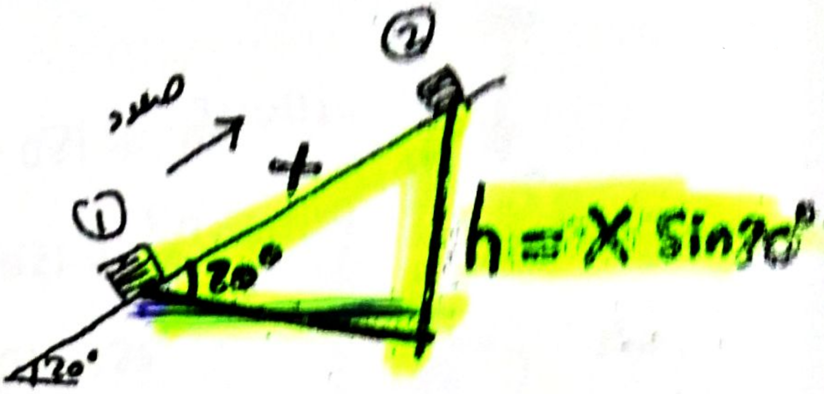
1) The max distance x until stops.

2) Find the velocity of the Box as it returns to its original position.

total Amount of Energy dissipated due to Friction

الطاقة الضائعة في الحركة

① $T_1 + U_{12} = T_2$



$\Rightarrow \frac{1}{2}(22.7)(12.2)^2$

$- mgh - \mu N d$

$= 0$ Stop.

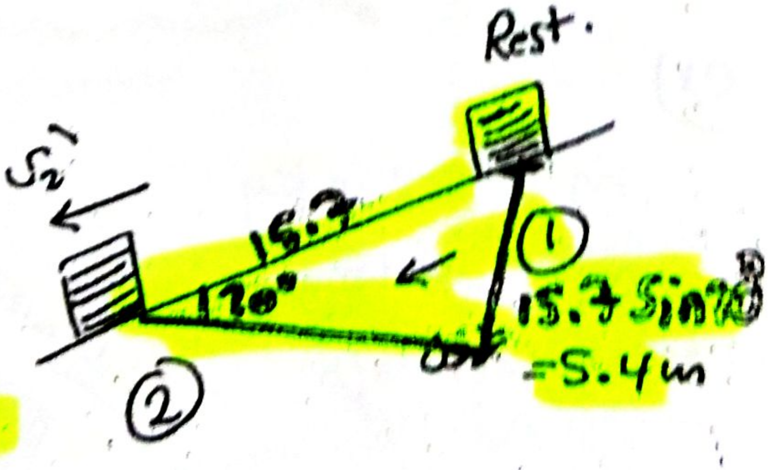
$N = mg\cos\theta$

$\Rightarrow \frac{1}{2}(22.7)(12.2)^2 - (22.7)(9.81)(X\sin 20°)$

$- 0.15(22.7)(9.81)\cos 20°\ X = 0$

$\Rightarrow$ ~~X = 15.7~~ $X = 15.7$ m

②



$T_1 + U_{12} = T_2$

$mgh - \mu N d = \frac{1}{2} m v_2^2$

$(22.7)(9.81)(15.7\sin 20°) - (0.15)(22.7)(9.81)\cos 20°\ (15.7)$

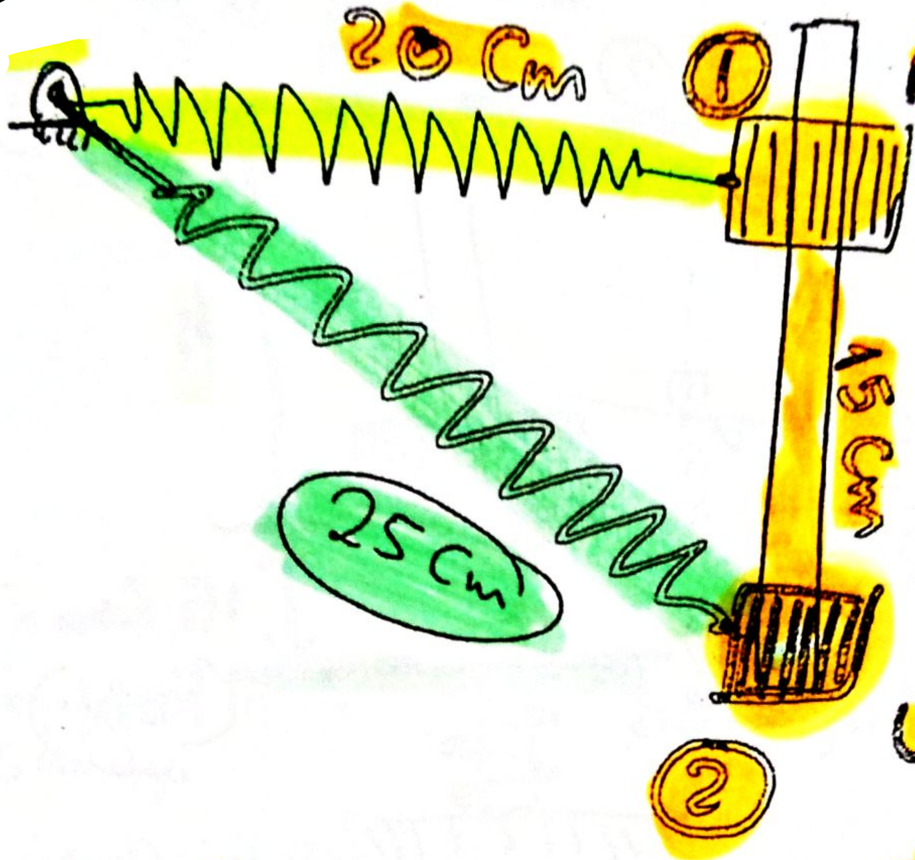
$= \frac{1}{2}(22.7)(v_2^2)$

$\Rightarrow v_2 = \ldots\ldots$

ion $= \mu N d = 0.15(22.7)(9.81)\cos 20°\ (15.7)$

$= \ldots\ldots$ Joule

Ex]



Rest.

* M = 9 kg

* undeforemd length

الطول الاصلي للزنبرك

10 Cm

* k = 525 N/m

$v_2 = ?$

Find v_2?

From Rest. 0

$$\cancel{T_1} + U_{12} = T_2$$

$$+mgh + \left[\frac{1}{2}k(\Delta S_1)^2 - \frac{1}{2}k(\Delta S_2)^2\right] = \frac{1}{2}m v_2^2$$

$$\Rightarrow 9(9.81)(0.15) + \left[\frac{1}{2}(525)\left[\overset{20-10}{10}\right]^2 - \frac{1}{2}(525)\left[\overset{25-10}{15}\right]^2\right] = \frac{1}{2}(9)\,v_2^2$$

$$v_2 = 1.5\ m/S$$

Ex

$m = 2.71$ kg

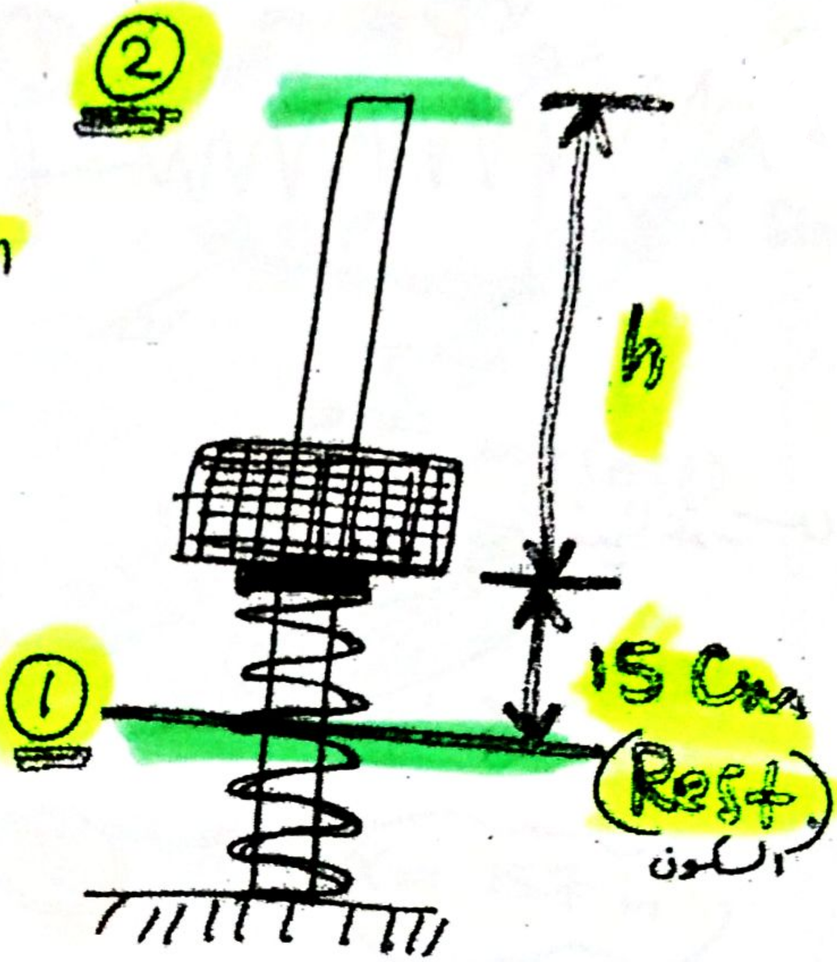
$K = 2627$ N/m

أوجد h

التي يصعدها

ال Box

الذي



الحل

$$\cancel{T_1}^{Rest\ 0} + U_{12} = \cancel{T_2}^{0}$$

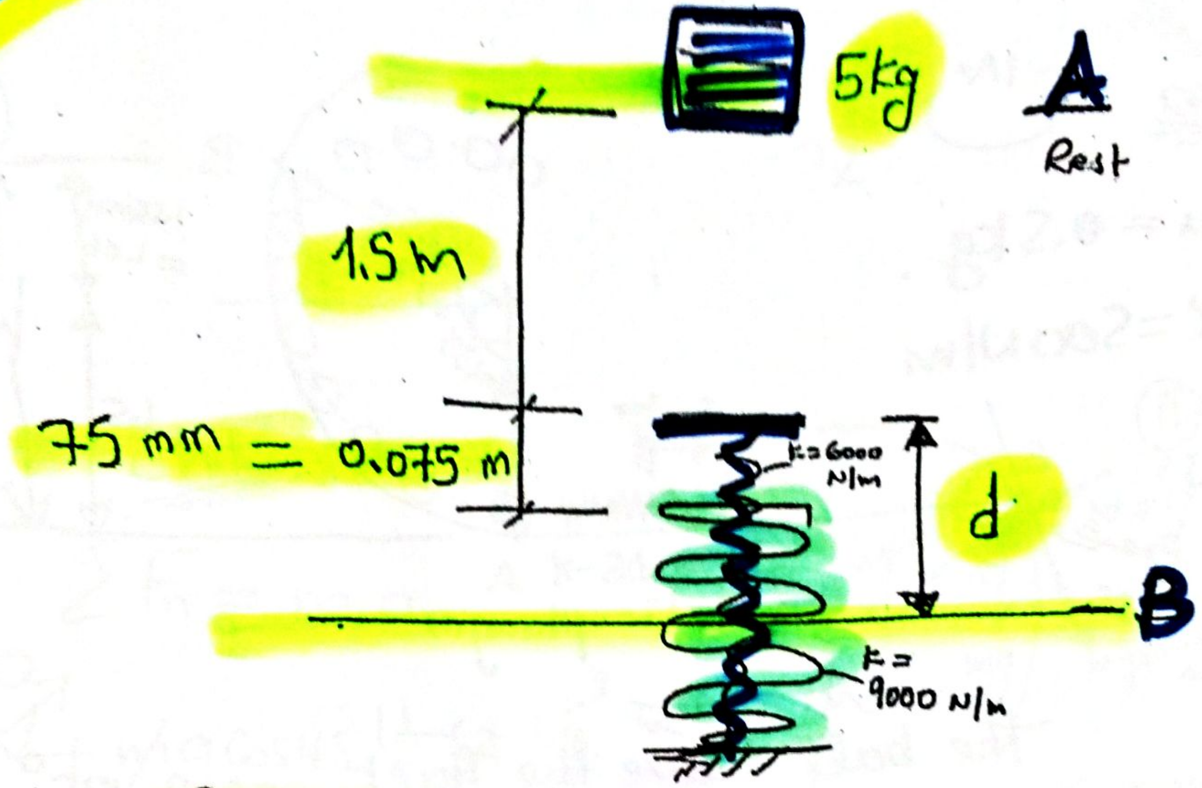
عشان أقصى ارتفاع

$$-mgH + \frac{1}{2} k \Delta S^2 = \cancel{0}\ \text{Stop}$$

$$-2.71(9.81)(0.15+h) + \frac{1}{2}(2627)(0.15)^2 = 0$$

$$\Rightarrow h = 1.12 \text{ m}$$

ex **Find d?**



$$\cancel{T_A}^{\,0} + U_{AB} = \cancel{T_B}^{\,0}$$

$$mg(1.5+d) - \frac{1}{2}(6000)d^2 - \frac{1}{2}(9000)(d-0.075)^2 = 0$$

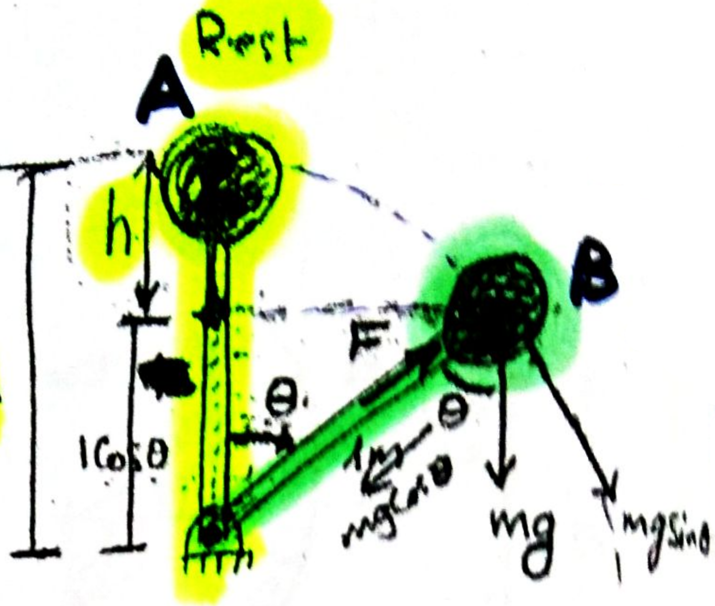
$$73.57 + 49.05d - 3000d^2 - 4500\left(d^2 - 0.15d + 5.625\times10^{-3}\right)$$

$$73.57 + 49.05d - 3000d^2 - 4500d^2 + 675d - 25.313 = 0$$

$$\Rightarrow -7500d^2 + 724.1d + 48.257 = 0$$

$$d = 0.142 \text{ m}$$

Ex



$m = 7.5\ kg$

$\theta = 0 \Rightarrow v_A = 0$

أوجد مقدار الزاوية θ
التي تصبح عندها
القوة في الذراع تساوي
صفراً

$h = 1 - \cos\theta$

Ch14

$\cancel{T_A}^{0} + U_{AB} = T_B$

$\cancel{m} g h = \frac{1}{2} \cancel{m} v_B^2$

$v_B^2 = 2gh$

$v_B^2 = 2g(1-\cos\theta)$ —— (1)

Ch13

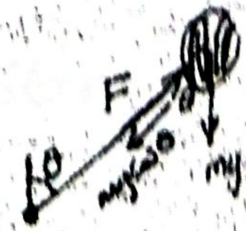
معطى في ذراع صفراً

$\sum F_n = m a_n$

$mg\cos\theta - \cancel{F_N}^{0} = m \frac{v_B^2}{\rho}$

$\cancel{m} g\cos\theta = \cancel{m} \frac{v_B^2}{1}$

$v_B^2 = g\cos\theta$ —— (2)



$$\cancel{g}\cos\theta = 2\cancel{g}(1-\cos\theta)$$

$$\cos\theta = 2 - 2\cos\theta$$

$$\cos\theta + 2\cos\theta = 2$$

$$3\cos\theta = 2 \Rightarrow \cos\theta = \frac{2}{3}$$

$$\cos\theta = 0.66667$$

$$\theta = 48.2^\circ$$

$$v_B^2 = g\cos\theta$$

$$v_B^2 = 9.81 \cos 48.2^\circ$$

$$\Rightarrow v_B = 2.56 \text{ m/s}$$

Ex

$m = 2.5\ kg$

$K = 4\ N/cm$

Unstretched length = 30 cm

الطول غير المشدود

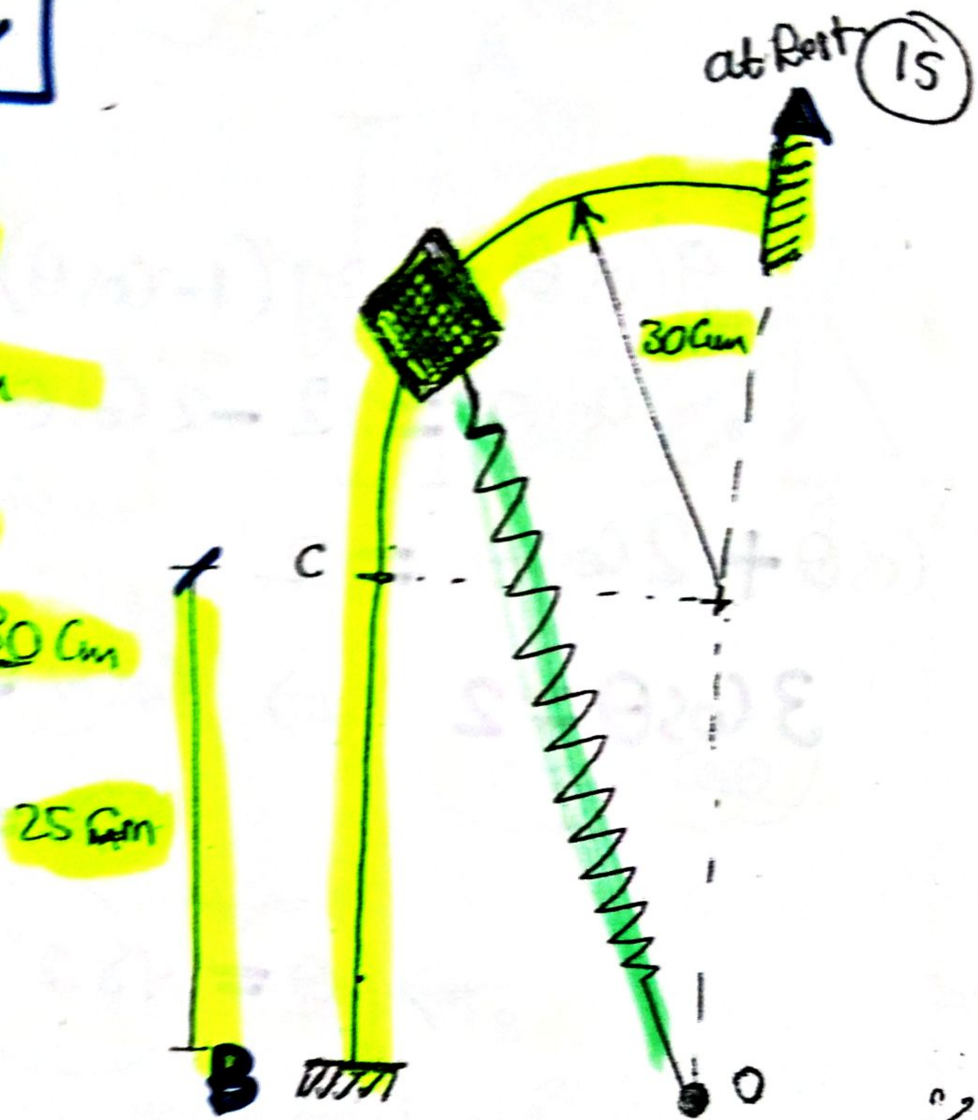
(عند ترخي)

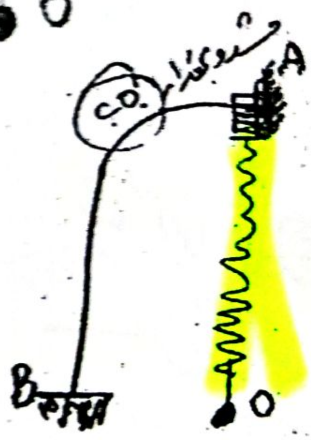
أول مرة

عندما يمر

بـ B

v_B ?



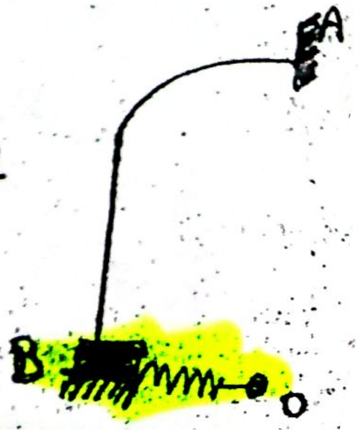


$$\cancel{T_A} + U_{AB} = T_B$$

$$mgh + \frac{1}{2}K\Delta s^2 = \frac{1}{2}m v_B^2$$

$$2.5(9.81)(55) + \frac{1}{2}(4)(25)^2 = \frac{1}{2}(2.5)\, v_B^2$$

$$\Rightarrow v_B = [redacted]6\ cm/s$$



Power and Efficiency

Power. The term "power" provides a useful basis for choosing the type of motor or machine which is required to do a certain amount of work in a given time. For example, two pumps may each be able to empty a reservoir if given enough time; however, the pump having the larger power will complete the job sooner.

The *power* generated by a machine or engine that performs an amount of work dU within the time interval dt is therefore

$$\boxed{P = \frac{dU}{dt}} \tag{14–9}$$

If the work dU is expressed as $dU = \mathbf{F} \cdot d\mathbf{r}$, then

$$P = \frac{dU}{dt} = \frac{\mathbf{F} \cdot d\mathbf{r}}{dt} = \mathbf{F} \cdot \frac{d\mathbf{r}}{dt}$$

or

$$\boxed{P = \mathbf{F} \cdot \mathbf{v}} \quad \text{output} \tag{14–10}$$

Hence, power is a *scalar*, where in this formulation $\mathbf{v}$ represents the velocity of the particle which is acted upon by the force $\mathbf{F}$.

The basic unit of power used in the SI system is the watt (W). These units are defined as

$$1\text{ W} = 1\text{ J/s} = 1\text{ N} \cdot \text{m/s}$$

Efficiency. The *mechanical efficiency* of a machine is defined as the ratio of the output of useful power produced by the machine to the input of power supplied to the machine. Hence,

$$\boxed{\varepsilon = \frac{\text{power output}}{\text{power input}}} \tag{14–11}$$

EXAMPLE 14.7

The man in Fig. 14–15*a* pushes on the 50-kg crate with a force of $F = 150$ N. Determine the power supplied by the man when $t = 4$ s. The coefficient of kinetic friction between the floor and the crate is $\mu_k = 0.2$. Initially the create is at rest.

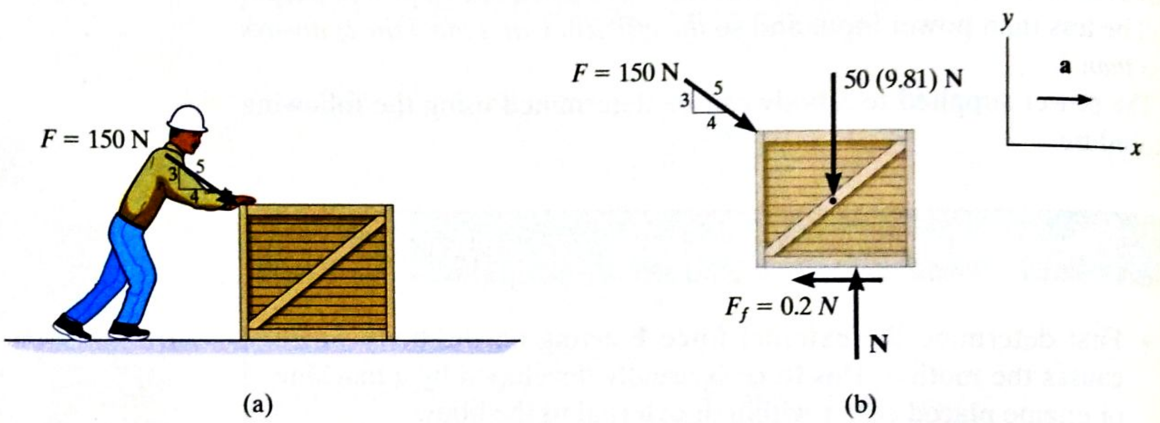


Fig. 14–15

SOLUTION

To determine the power developed by the man, the velocity of the 150-N force must be obtained first. The free-body diagram of the crate is shown in Fig. 14–15*b*. Applying the equation of motion,

$$+\uparrow \Sigma F_y = ma_y; \qquad N - \left(\tfrac{3}{5}\right)150\text{ N} - 50(9.81)\text{ N} = 0$$

$$N = 580.5\text{ N}$$

$$\xrightarrow{+} \Sigma F_x = ma_x; \qquad \left(\tfrac{4}{5}\right)150\text{ N} - 0.2(580.5\text{ N}) = (50\text{ kg})a$$

$$a = 0.078\text{ m/s}^2$$

The velocity of the crate when $t = 4$ s is therefore

$$(\xrightarrow{+}) \qquad v = v_0 + a_c t$$

$$v = 0 + (0.078\text{ m/s}^2)(4\text{ s}) = 0.312\text{ m/s}$$

The power supplied to the crate by the man when $t = 4$ s is therefore

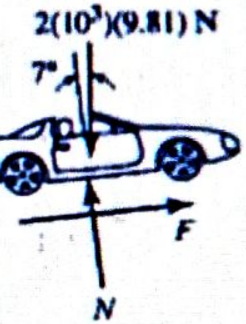
$$P = \mathbf{F}\cdot\mathbf{v} = F_x v = \left(\tfrac{4}{5}\right)(150\text{ N})(0.312\text{ m/s})$$

$$= 37.4\text{ W} \qquad \textit{Ans.}$$

An automobile having a mass of 2 Mg travels up a 7° slope at a constant speed of $v = 100$ km/h. If mechanical friction and wind resistance are neglected, determine the power developed by the engine if the automobile has an efficiency $\epsilon = 0.65$.

SOLUTION

Equation of Motion: The force F which is required to maintain the car's constant speed up the slope must be determined first.



$$+\Sigma F_{x'} = ma_{x'}; \qquad F - 2(10^3)(9.81)\sin 7^\circ = 2(10^3)(0)$$

$$F = 2391.08 \text{ N}$$

Power: Here, the speed of the car is $v = \left[\dfrac{100(10^3)\text{ m}}{\text{h}}\right] \times \left(\dfrac{1\text{ h}}{3600\text{ s}}\right) = 27.78$ m/s.

The power output can be obtained using Eq. 14–10.

$$P = \mathbf{F}\cdot\mathbf{v} = 2391.08(27.78) = 66.418(10^3)\text{ W} = 66.418\text{ kW}$$

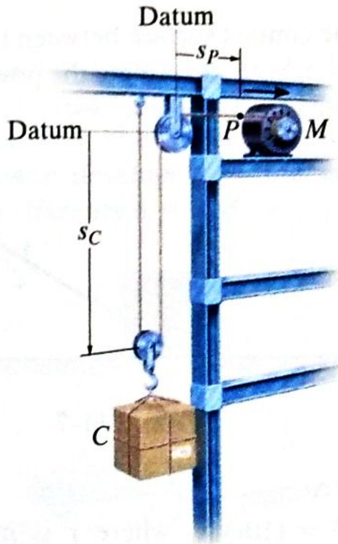
Using Eq. 14–11, the required power input from the engine to provide the above power output is

$$\text{power input} = \frac{\text{power output}}{\varepsilon}$$

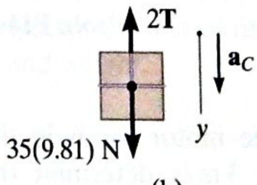
$$= \frac{66.418}{0.65} = 102\text{ kW} \qquad \textbf{Ans.}$$

EXAMPLE 14.8

The motor M of the hoist shown in Fig. 14–16a lifts the 35-kg crate C so that the acceleration of point P is 1.2 m/s². Determine the power that must be supplied to the motor at the instant P has velocity of 0.6 m/s. Neglect the mass of the pulley and cable and take $\varepsilon = 0.85$.



(a)



(b)

Fig. 14–16

SOLUTION

In order to find the power output of the motor, it is first necessary to determine the tension in the cable since this force is developed by the motor.

From the free-body diagram, Fig. 14–16b, we have

$$+\downarrow \Sigma F_y = ma_y; \qquad -2T + 35(9.81)\text{ N} = (35\text{ kg})\, a_c \qquad (1)$$

The acceleration of the crate can be obtained by using kinematics to relate it to the known acceleration of point P, Fig. 14–16a. Using the methods of absolute dependent motion, the coordinates s_C and s_P can be related to a constant portion of cable length l which is changing in the vertical and horizontal directions. We have $2s_C + s_P = l$. Taking the second time derivative of this equation yields

$$2a_C = -a_P \qquad (2)$$

Since $a_P = +1.2\text{ m/s}^2$, then $a_C = -(1.2\text{ m/s}^2)/2 = -0.6\text{ m/s}^2$. What does the negative sign indicate? Substituting this result into Eq. 1 and *retaining* the negative sign since the acceleration in *both* Eq. 1 and Eq. 2 was considered positive downward, we have

$$-2T + 35(9.81)\text{ N} = (35\text{ kg})(-0.6\text{ m/s}^2)$$

$$T = 182.2\text{ N}$$

The power output required to draw the cable in at a rate of 0.6 m/s is therefore

$$P = \mathbf{T}\cdot\mathbf{v} = (182.2\text{ N})(0.6\text{ m/s})$$
$$= 109.3\text{ W}$$

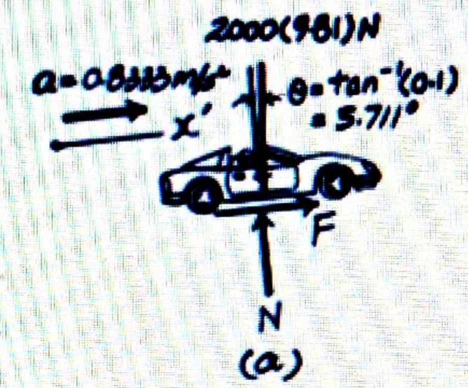
This *power output* requires that the motor provide a *power input* of

$$\text{power input} = \frac{1}{\varepsilon}(\text{power output})$$
$$= \frac{1}{0.85}(109.3\text{ W}) = 129\text{ W} \qquad \textit{Ans.}$$

NOTE: Since the velocity of the crate is constantly changing, the power requirement is *instantaneous*.

The 2-Mg car increases its speed uniformly from rest to 25 m/s in 30 s up the inclined road. Determine the maximum power that must be supplied by the engine, which operates with an efficiency of $\epsilon = 0.8$. Also, find the average power supplied by the engine.

SOLUTION



Kinematics: The constant acceleration of the car can be determined from

$$(\overset{+}{\rightarrow}) \qquad v = v_0 + a_c t$$

$$25 = 0 + a_c(30)$$

$$a_c = 0.8333 \text{ m/s}^2$$

Equations of Motion: By referring to the free-body diagram of the car shown in Fig. *a*,

$$\Sigma F_{x'} = ma_{x'}; \qquad F - 2000(9.81)\sin 5.711° = 2000(0.8333)$$

$$F = 3618.93\text{N}$$

Power: The maximum power output of the motor can be determined from

$$(P_{out})_{max} = \mathbf{F}\cdot\mathbf{v}_{max} = 3618.93(25) = 90\,473.24 \text{ W}$$

Thus, the maximum power input is given by

$$P_{in} = \frac{P_{out}}{\epsilon} = \frac{90473.24}{0.8} = 113\,091.55 \text{ W} = 113 \text{ kW} \qquad \text{Ans.}$$

The average power output can be determined from

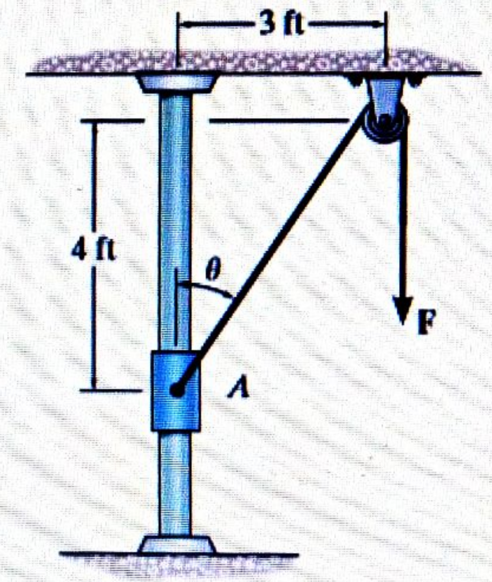
$$(P_{out})_{avg} = \mathbf{F}\cdot\mathbf{v}_{avg} = 3618.93\left(\frac{25}{2}\right) = 45\,236.62 \text{ W}$$

Thus,

$$(P_{in})_{avg} = \frac{(P_{out})_{avg}}{\epsilon} = \frac{45236.62}{0.8} = 56\,545.78 \text{ W} = 56.5 \text{ kW} \qquad \text{Ans.}$$

***14–56.**

The 10-lb collar starts from rest at A and is lifted by applying a constant vertical force of $F = 25$ lb to the cord. If the rod is smooth, determine the power developed by the force at the instant $\theta = 60°$.



SOLUTION

Work of **F**

$$U_{1-2} = 25(5 - 3.464) = 38.40 \text{ lb} \cdot \text{ft}$$

$$T_1 + \Sigma U_{1-2} = T_2 s$$

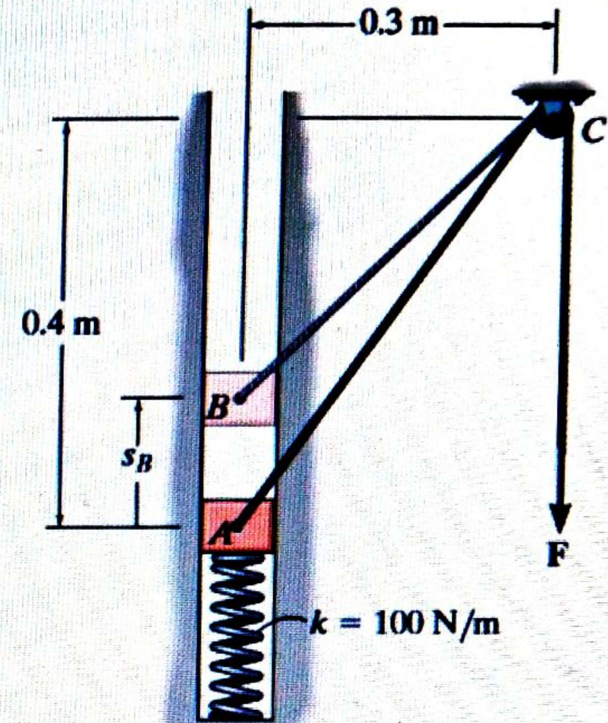
$$0 + 38.40 - 10(4 - 1.732) = \frac{1}{2}\left(\frac{10}{32.2}\right)v^2$$

$$v = 10.06 \text{ ft/s}$$

$$P = \mathbf{F} \cdot \mathbf{v} = 25 \cos 60°(10.06) = 125.76 \text{ ft} \cdot \text{lb/s}$$

$$P = 0.229 \text{ hp}$$ **Ans.**

The block has a mass of 0.8 kg and moves within the smooth vertical slot. If it starts from rest when the *attached* spring is in the unstretched position at A, determine the *constant* vertical force F which must be applied to the cord so that the block attains a speed $v_B = 2.5$ m/s when it reaches B; $s_B = 0.15$ m. Neglect the size and mass of the pulley. *Hint:* The work of **F** can be determined by finding the difference Δl in cord lengths AC and BC and using $U_F = F\,\Delta l$.



SOLUTION

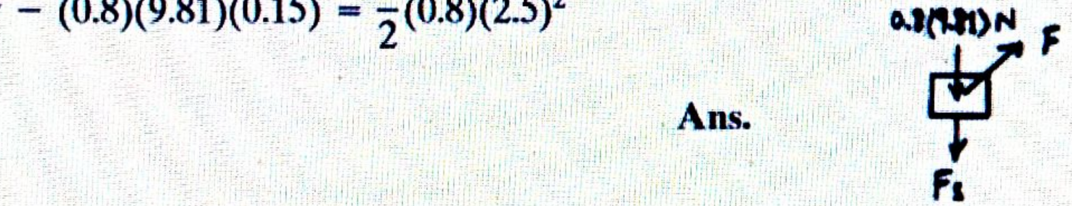
$$l_{AC} = \sqrt{(0.3)^2 + (0.4)^2} = 0.5 \text{ m}$$

$$l_{BC} = \sqrt{(0.4 - 0.15)^2 + (0.3)^2} = 0.3905 \text{ m}$$

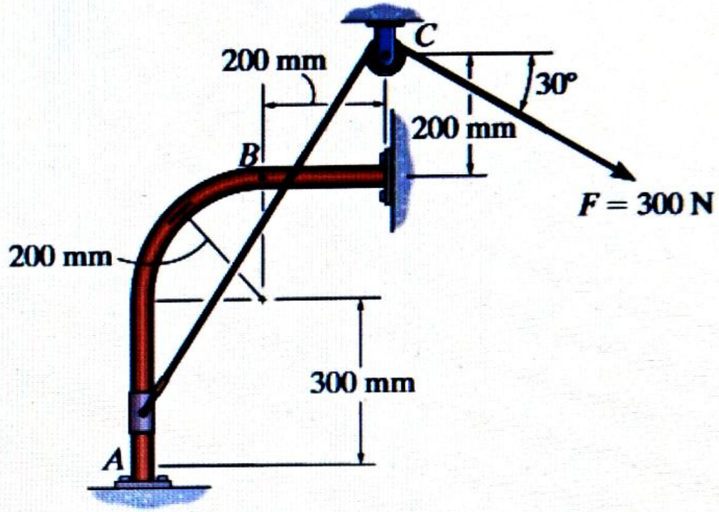
$$T_A + \Sigma U_{A-B} = T_B$$

$$0 + F(0.5 - 0.3905) - \frac{1}{2}(100)(0.15)^2 - (0.8)(9.81)(0.15) = \frac{1}{2}(0.8)(2.5)^2$$

$F = 43.9$ N Ans.



If the cord is subjected to a constant force of $F = 300$ N and the 15-kg smooth collar starts from rest at A, determine the velocity of the collar when it reaches point B. Neglect the size of the pulley.



SOLUTION

Free-Body Diagram: The free-body diagram of the collar and cord system at an arbitrary position is shown in Fig. *a*.

Principle of Work and Energy: Referring to Fig. *a*, only **N** does no work since it always acts perpendicular to the motion. When the collar moves from position A to position B, **W** displaces vertically upward a distance $h = (0.3 + 0.2)$ m $= 0.5$ m, while force F displaces a distance of $s = AC - BC = \sqrt{0.7^2 + 0.4^2} - \sqrt{0.2^2 + 0.2^2} = 0.5234$ m. Here, the work of **F** is positive, whereas **W** does negative work.

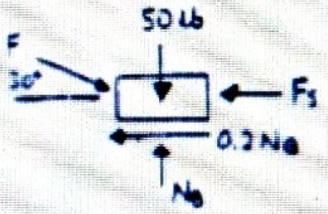
$$T_A + \Sigma U_{A-B} = T_B$$

$$0 + 300(0.5234) + [-15(9.81)(0.5)] = \frac{1}{2}(15)v_B^2$$

$$v_B = 3.335 \text{ m/s} = 3.34 \text{ m/s} \qquad \textbf{Ans.}$$

The 50-lb block rests on the rough surface for which the coefficient of kinetic friction is $\mu_k = 0.2$. A force $F = (40 + s^2)$ lb, where s is in ft, acts on the block in the direction shown. If the spring is originally unstretched ($s = 0$) and the block is at rest, determine the power developed by the force the instant the block has moved $s = 1.5$ ft.

SOLUTION



$+\uparrow \Sigma F_y = 0;\qquad N_B - (40 + s^2)\sin 30^\circ - 50 = 0$

$$N_B = 70 + 0.5s^2$$

$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + \int_0^{1.5} (40 + s^2)\cos 30^\circ\, ds - \frac{1}{2}(20)(1.5)^2 - 0.2\int_0^{1.5} (70 + 0.5s^2)ds = \frac{1}{2}\left(\frac{50}{32.2}\right)v_2^2$$

$$0 + 52.936 - 22.5 - 21.1125 = 0.7764v_2^2$$

$$v_2 = 3.465 \text{ ft/s}$$

When $s = 1.5$ ft,

$$F = 40 + (1.5)^2 = 42.25 \text{ lb}$$

$$P = \mathbf{F}\cdot\mathbf{v} = (42.25\cos 30^\circ)(3.465)$$

$$P = 126.79 \text{ ft}\cdot\text{lb/s} = 0.231 \text{ hp}$$ **Ans.**

The 1000-lb elevator is hoisted by the pulley system and motor M. If the motor exerts a constant force of 500 lb on the cable, determine the power that must be supplied to the motor at the instant the load has been hoisted $s = 15$ ft starting from rest. The motor has an efficiency of $\varepsilon = 0.65$.

SOLUTION

Equation of Motion. Referring to the FBD of the elevator, Fig. *a*,

$$+\uparrow \Sigma F_y = ma_y; \qquad 3(500) - 1000 = \frac{1000}{32.2} a$$

$$a = 16.1 \text{ ft/s}^2$$

When $S = 15ft$,

$$+\uparrow v^2 = v_0^2 + 2a_c(S - S_0); \quad v^2 = 0^2 + 2(16.1)(15)$$

$$v = 21.98 \text{ ft/s}$$

Power. Applying Eq. 14–9, the power output is

$$P_{out} = F \cdot V = 3(500)(21.98) = 32.97(10^3) \text{ lb} \cdot \text{ft/s}$$

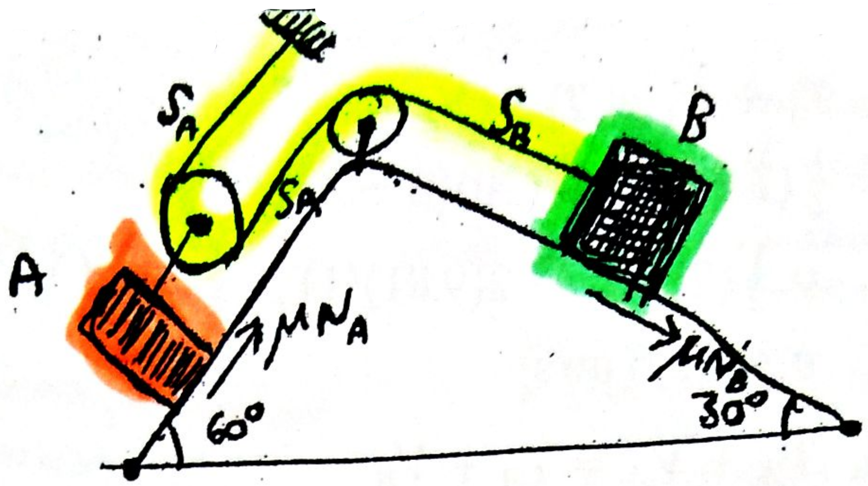
The power input can be determined using Eq. 14–9

$$\Sigma = \frac{P_{out}}{P_{in}}; \qquad 0.65 = \frac{32.97(10^3)}{P_{in}}$$

$$P_{in} = [50.72(10^3) \text{ lb} \cdot \text{ft/s}]\left(\frac{1 \text{ hp}}{550 \text{ lb} \cdot \text{ft/s}}\right)$$

$$= 92.21 \text{ hp} = 92.2 \text{ hp} \qquad \textbf{Ans.}$$

ex



$m_A = 30kg$

$m_B = 20kg$

$\mu = 0.10$

Start
(From Rest)

كم هي سرعة A و سرعة B
بعدما يتحرك الجسم B للأسفل
مسافة مقدارها 1م (1m)

$$2S_A + S_B = L \Rightarrow \boxed{2\Delta S_A + \Delta S_B = 0}$$

$$\Rightarrow \boxed{2v_A + v_B = 0}$$

$$2\Delta S_A + (-1) = 0$$

$$\Rightarrow \Delta S_A = 0.5\ m.$$

Ch14

$$\overset{0}{\underset{Rest}{\cancel{T_1}}} + U_{12} = T_2$$

$N_A = m_A g \cos 60$
$N_B = m_B g \cos 30$

$$m_A g(\Delta S_A \sin 60) - m_B g(\Delta S_B \sin 30^\circ)$$

$$- \mu(\overset{N_A}{m_A g \cos 60})\Delta S_A$$

$$- \mu(\overset{N_B}{m_B g \cos 30})\Delta S_B = \frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B v_B^2$$

$$30(9.81)\left[0.5 \sin 60\right] - 20(9.81) \cdot \left[(+1) \sin 30^\circ\right]$$

$$- 0.10(30)(9.81)\cos 60 \,(0.5)$$

$$- 0.10(20)(9.81)\cos 30 \,(+1)$$

$$= \frac{1}{2}(30)\, v_A^2 + \frac{1}{2}(20)\, v_B^2$$

$$\Rightarrow \quad 15 v_A^2 + 10 v_B^2 = 5 \quad \text{—— (1)}$$

$$\Rightarrow \quad 2v_A + v_B = 0 \quad \text{—— (2)}$$

$$\Rightarrow \quad v_B = -2v_A$$

$$15 v_A^2 + 10(-2v_A)^2 = 5$$

$$15 v_A^2 + 40 v_A^2 = 5$$

$$55 v_A^2 = 5$$

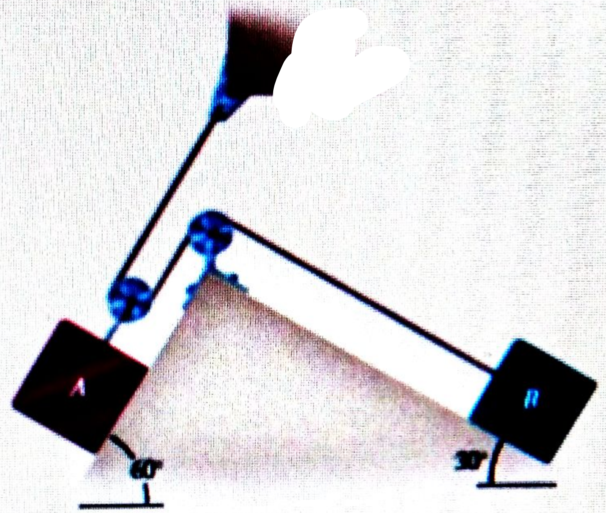
$$v_A^2 = \frac{5}{55}$$

$$\boxed{v_A = 0.3 \text{ m/s}}$$

$$\Rightarrow \quad v_B = -2(0.3)$$

$$\boxed{v_B = -0.6 \text{ m/s}}$$

Determine the velocity of the 60-lb block A if the two blocks are released from rest and the 40-lb block B moves 2 ft up the incline. The coefficient of kinetic friction between both blocks and the inclined planes is $\mu_k = 0.10$.



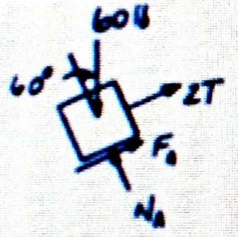
SOLUTION

Block A:

$+\nwarrow \Sigma F_y = ma_y;\qquad N_A - 60\cos 60° = 0$

$N_A = 30\text{ lb}$

$F_A = 0.1(30) = 3\text{ lb}$



Block B:

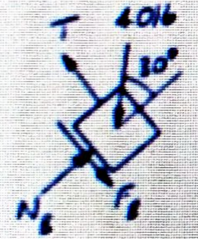
$+\nearrow \Sigma F_y = ma_y;\qquad N_B - 40\cos 30° = 0$

$N_B = 34.64\text{ lb}$

$F_B = 0.1(34.64) = 3.464\text{ lb}$

Use the system of both blocks. N_A, N_B, T, and R do no work.

$T_1 + \Sigma U_{1-2} = T_2$



$$(0+0) + 60\sin 60°|\Delta s_A| - 40\sin 30°|\Delta s_B| - 3|\Delta s_A| - 3.464|\Delta s_B| = \frac{1}{2}\left(\frac{60}{32.2}\right)v_A^2 + \frac{1}{2}\left(\frac{40}{32.2}\right)v_B^2$$

$2s_A + s_B = l$

$2\Delta s_A = -\Delta s_B$

When $|\Delta s_B| = 2$ ft, $|\Delta s_A| = 1$ ft

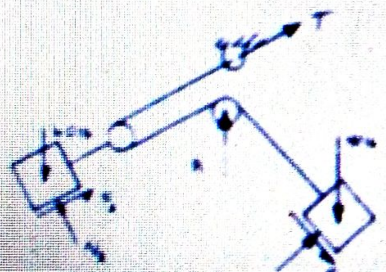
Also,

$2v_A = -v_B$

Substituting and solving,

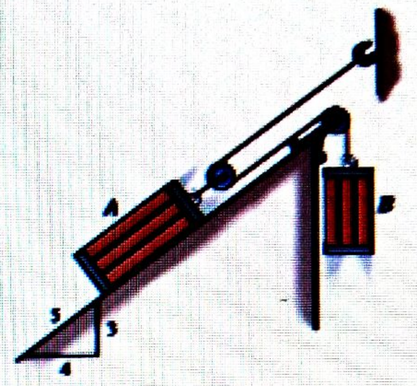
$v_A = 0.771$ ft/s **Ans.**

$v_B = -1.54$ ft/s



14–15.

The two blocks A and B have weights $W_A = 60$ lb and $W_B = 10$ lb. If the kinetic coefficient of friction between the incline and block A is $\mu_k = 0.2$, determine the speed of A after it moves 3 ft down the plane starting from rest. Neglect the mass of the cord and pulleys.



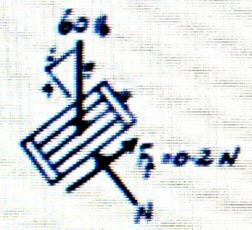
SOLUTION

Kinematics: The speed of the block A and B can be related by using position coordinate equation.

$$s_A + (s_A - s_B) = l \qquad 2s_A - s_B = l$$

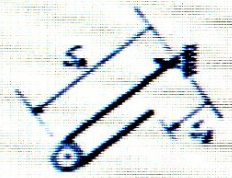
$$2\Delta s_A - \Delta s_B = 0 \qquad \Delta s_B = 2\Delta s_A = 2(3) = 6 \text{ ft}$$

$$2v_A - v_B = 0 \tag{1}$$



Equation of Motion: Applying Eq. 13–7, we have

$$+\Sigma F_{y'} = ma_{y'}; \qquad N - 60\left(\frac{4}{5}\right) = \frac{60}{32.2}(0) \qquad N = 48.0 \text{ lb}$$



Principle of Work and Energy: By considering the whole system, W_A which acts in the direction of the displacement does *positive* work. W_B and the friction force $F_f = \mu_k N = 0.2(48.0) = 9.60$ lb does *negative* work since they act in the opposite direction to that of displacement Here, W_A is being displaced vertically (downward) $\frac{3}{5}\Delta s_A$ and W_B is being displaced vertically (upward) Δs_B. Since blocks A and B are at rest initially, $T_1 = 0$. Applying Eq. 14–7, we have

$$T_1 + \sum U_{1-2} = T_2$$

$$0 + W_A\left(\frac{3}{5}\Delta s_A\right) - F_f \Delta s_A - W_B \Delta s_B = \frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B v_B^2$$

$$60\left[\frac{3}{5}(3)\right] - 9.60(3) - 10(6) = \frac{1}{2}\left(\frac{60}{32.2}\right)v_A^2 + \frac{1}{2}\left(\frac{10}{32.2}\right)v_B^2$$

$$1236.48 = 60v_A^2 + 10v_B^2 \tag{2}$$

Eqs. (1) and (2) yields

$$v_A = 3.52 \text{ ft/s} \qquad \textbf{Ans.}$$

$$v_B = 7.033 \text{ ft/s}$$

د. هاشم الخالدي

Ch15

Impulse and Momentum

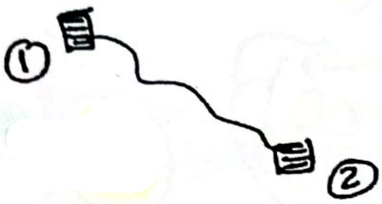
الدفع — الزخم (كمية التحرك)

$$\vec{I} = \vec{F}\, t$$ الدفع = القوة x الزمن

$$\vec{L} = m\,\vec{v}$$ الزخم = الكتلة x السرعة

Ch15

$$\vec{L}_1 + \vec{I}_{12} = \vec{L}_2$$



Ch14

$$T_1 + U_{12} = T_2$$

$$\boxed{m\,\vec{v}_1 + \int_{t_1}^{t_2} \vec{F}\,dt = m\,\vec{v}_2}$$

نستخدم التكامل عندما تكون القوة متغيرة (اقتران في الزمن)

X-direction :-

$$m\,v_{1x} + \int_{t_1}^{t_2} F_x\,dt = m\,v_{2x}$$

y-direction :-

$$m\,v_{1y} + \int_{t_1}^{t_2} F_y\,dt = m\,v_{2y}$$

$* \; I = F(\Delta t)$ الدفع = القوة × الزمن

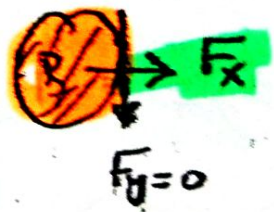
$N \cdot Sec = Kg \frac{m}{s^2} s = Kg \frac{m}{s}$

$* \; L = m v$ كمية التحرك = الكتلة × السرعة

* لها نفس الوحدات الفيزيائية (لها نفس الوحدة) $Kg \frac{m}{s}$

$$\left(m\vec{v}_1\right)_A + \left(\int_{t_1}^{t_2} \vec{F}_x \, dt\right)_A = \left(m v_2\right)_A$$

$$\left(m\vec{v}_1\right)_B - \left(\int_{t_1}^{t_2} \vec{F}_x \, dt\right)_B = \left(m v_2\right)_B$$



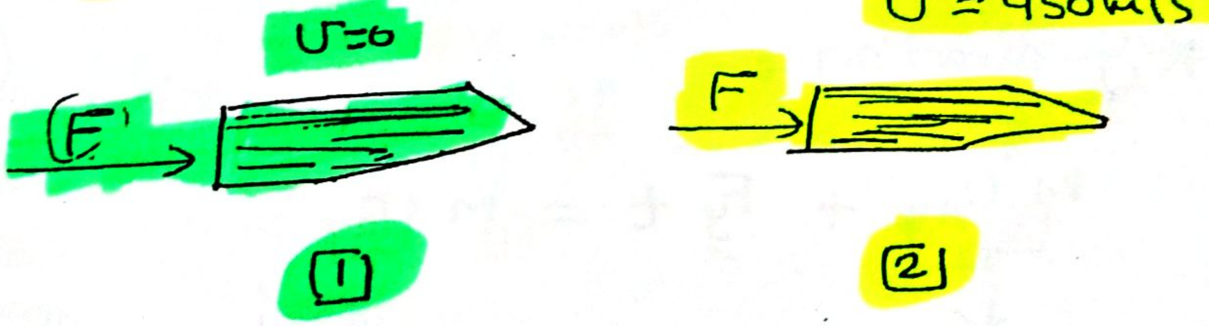
أجمع المعادلتين

Conservation of momentum

$$\left(m\vec{v}_1\right)_A + \left(m\vec{v}_1\right)_B = \left(m\vec{v}_2\right)_A + \left(m\vec{v}_2\right)_B$$

طلقة كتلتها 2.5 g سرعتها 450 m/s
احتاجت من الوقت 0.75 ms لتصل إلى هذه السرعة
أوجد مقدار القوة المؤثرة على الرصاصة

A Bullet of mass 2.5 g strats from Rest.
A Force F applied to this bullet
So the speed reaches 450 m/s
in 0.75 ms. Find F?



الزمن $t = 0.75$ ms

Rest

$$\cancel{m v_{1x}}^{0} + F\,\Delta t = m v_{2x}$$

$$F\left[0.75 \times 10^{-3}\right] = \left[2.5 \times 10^{-3}\right]\left[450\right]$$

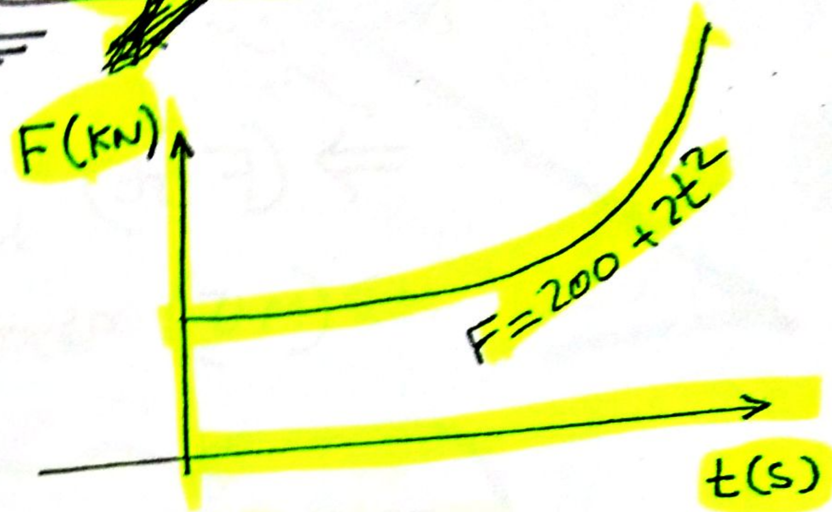
ms → s   g → kg

$F = 1500$ Newton.

EX]

jet plane
طائرة نفاثة

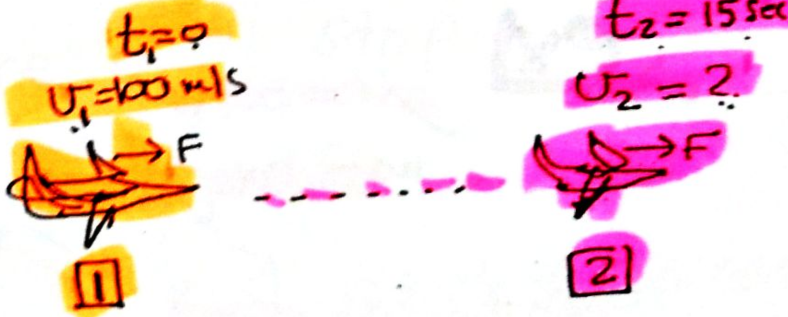
* $m = 250\ Mg$
* at $t=0 \Rightarrow v_0 = 100\ m/s$ and move horizontally
تطير أفقياً



the engine provides horizontal Thrust

$$F = 200 + 2t^2$$

Find the velocity in 15 seconds



$$m v_1 + \int_0^{15} F\,dt = m v_2$$

حولت لكغم kg

$$(250\times10^3)(100) + 1000\int_0^{15}(200+2t^2)\,dt = (250\times10^3)\,v_2$$

$$25000 + \left[200t + \frac{2}{3}t^3\right]_0^{15} = 250\,v_2$$

$$25000 + 200(15) + \frac{2}{3}(15)^3 = 250\,v_2$$

$$\Rightarrow \boxed{v_2 = 121\ m/s}$$

Impulse and Momentum

الدفع   كمية الحركة

⇒ $(F\,\Delta t)$ الدفع = القوة × الزمن

⇒ $(m\,v)$ كمية الحركة = الكتلة × السرعة

$$m v_1 + F\,\Delta t = m v_2$$

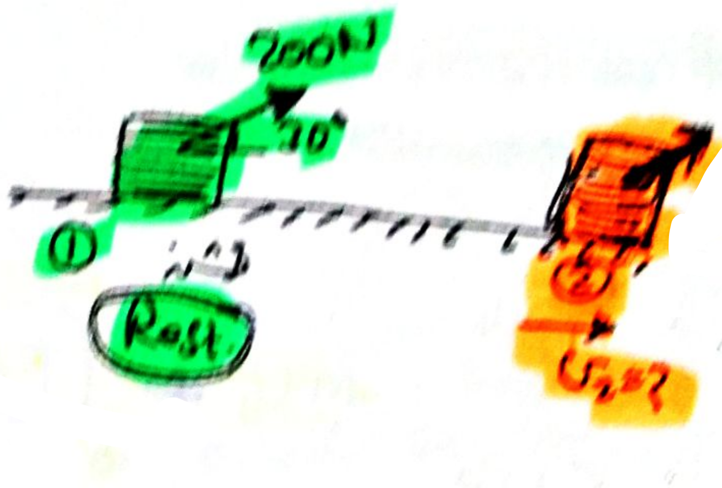
ex)

$\mu = 0.25$

$m = 30\,kg.$

أوجد سرعة الجسم في الموضع الثاني بعد زمن مقداره ٨ ثواني

$(\Delta t = 8\ \text{seconds})$

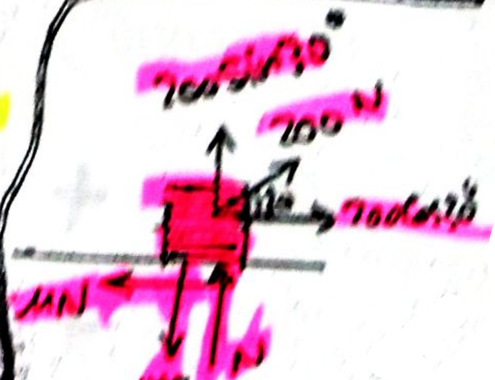


$$\overset{0}{\cancel{m v_{1x}}} + F_x\,\Delta t = m v_{2x}$$

$\Rightarrow [200\cos 30^\circ - \mu N]\,\Delta t = m v_{2x}$

$\Rightarrow \{200\cos 30^\circ - 0.25(194)\}[8] = 30\, v_{2x}$

$\Rightarrow v_{2x} = \ldots\ldots\ m/s$



$N = 194 \leftarrow N = mg - 200\sin 30^\circ$

كتلة السيارة

(8)

$m = 1814.4\ kg$

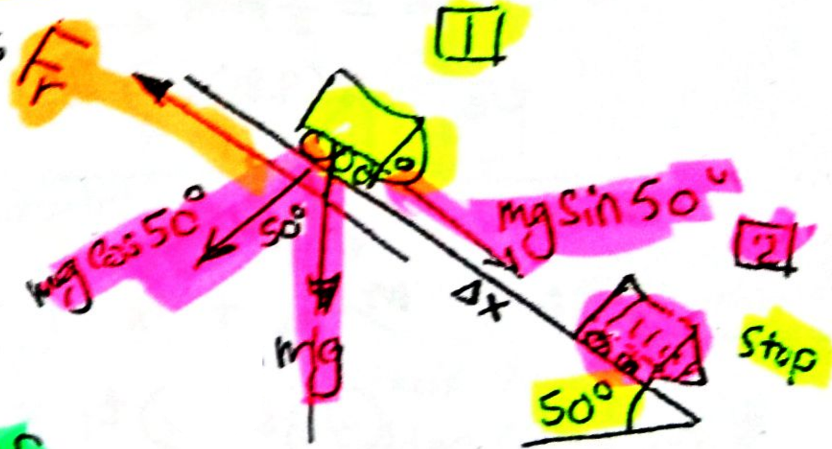
$v = 96.54\ km/hr$

- the brake is Suddenly applied
- if the braking force is 6672 N

① Find the time needed for the car to stop?

② Find the distance until stops?

قوة الاحتكاك
عكس اتجاه
الحركة



$v = 96.54\ km/hr$
$= 96.54 \frac{1000}{3600}$
$= 26.8\ m/s$

① $m v_1 + F \Delta t = m v_2$

$1814.4(26.8) + [mg \sin 50 - F_r][\Delta t] = 0$ ↗ Stop.

$1814.4(26.8) + [1814.4 \sin 50° - 6672][\Delta t] = 0$

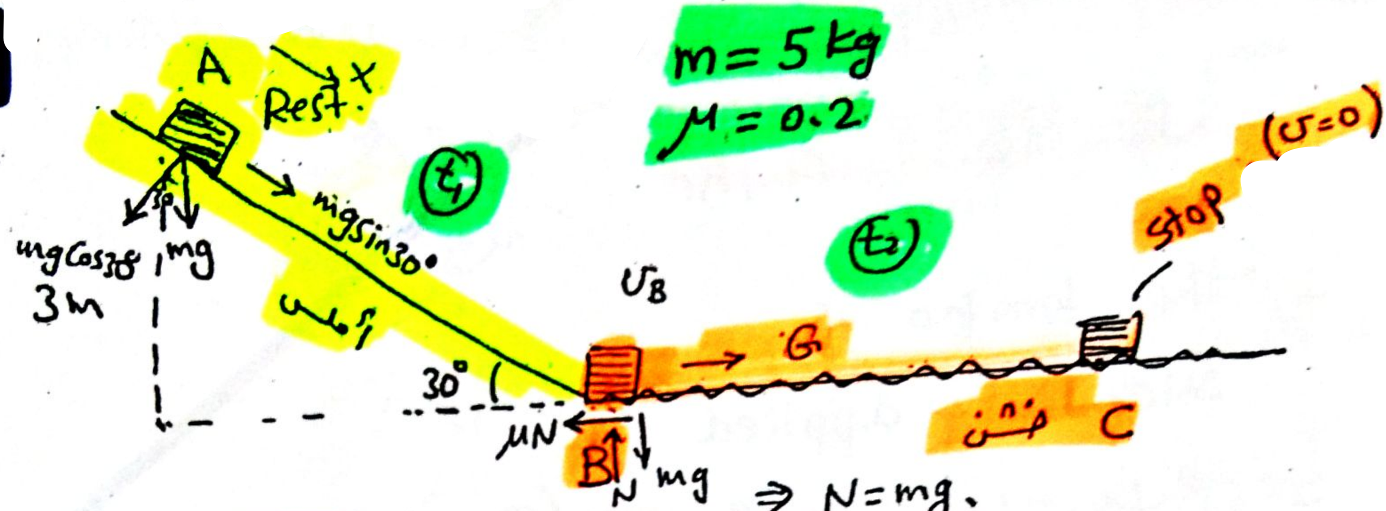
$\Rightarrow \Delta t = 9.5\ Sec$

② $T_1 + U_{12} = T_2 \Rightarrow \frac{1}{2} m_1 v_1^2 + mg(\Delta x \sin 50°) - F_r \Delta x = 0$

$\Rightarrow \frac{1}{2}[1814.4][26.8]^2 + (1814.4)(9.81)\sin 50\ \Delta X - 6672\ \Delta X = 0$

$\Rightarrow \Delta X = 127\ m$

Ex)



$\Rightarrow N = mg.$

الزمن الكلي

Find the total time until stops?

اوجد الزمن الكلي اللازم للجسم حتى يتوقف.

$* \; t = t_1 + t_2$

Ch14 $\quad \cancel{T_A}^{\,0} + U_{AB} = T_B$

$\cancel{m}gh = \frac{1}{2}\cancel{m}U_B^2$

$U_B^2 = 2(9.81)(3) \Rightarrow U_B = 7.67 \text{ m/s.}$

X-direction: $\quad m\cancel{U_{Ax}}^{\,0 \text{ (Rest)}} + F_x t_1 = mU_{Bx}$

$(\cancel{m}g\sin 30)t_1 = \cancel{m}U_{Bx}$

Ch15 $\quad (9.81)\sin 30 \; t_1 = 7.67 \Rightarrow t_1 = 1.56 \text{ sec}$

يتوقف

G-direction: $\quad mU_{BG} + F_G t_2 = m\cancel{U_{CG}}^{\,0}$

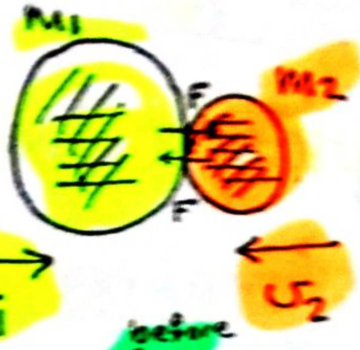
$(5)(7.67) + -\mu mg\, t_2 = 0$

$(5)(7.67) - 0.2(5)(9.81)\, t_2 = 0 \Rightarrow t_2 = 3.91 \text{ sec}$

$* \; t = t_1 + t_2 \Rightarrow t = 5.47 \text{ sec.}$

* Impact

* Collision : تصادم الصدم



* $m_1 v_1 + \cancel{F \Delta t} = m_1 v_1'$

* $m_2 v_2 - \cancel{F \Delta t} = m_2 v_2'$

$$\overset{\text{before}}{\sum mv} = \overset{\text{After}}{\sum mv}$$

* $m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2'$

كمية الحركة قبل التصادم = كمية الحركة بعد التصادم

Ex

الوزن $W_A = 22.5$ KN

الوزن $W_B = 15$ KN

$v_A = 1$ m/s قبل التصادم

$v_B = 2$ m/s قبل التصادم

Find their velocites after collision ?
if they become together
تصادم

$m_A v_A + m_B v_B = (m_A + m_B) v'$

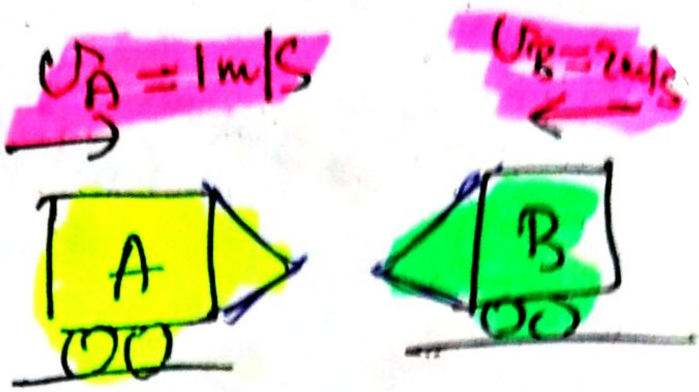
أصبح لهما نفس السرعة بعد التصادم

السرعة موجبة لليمين — الحركة سالبة لليسار

$$\frac{(22.5)10^3}{9.81}(1) + \frac{(15)10^3}{9.81}(-2) = \frac{(37.5)10^3}{9.81} v'$$

$\Rightarrow v' = -0.2$ m/s

Ex



$m_A = 2250$ kg

$m_B = 1500$ kg

Plastic Impact

Find the velocity after impact.

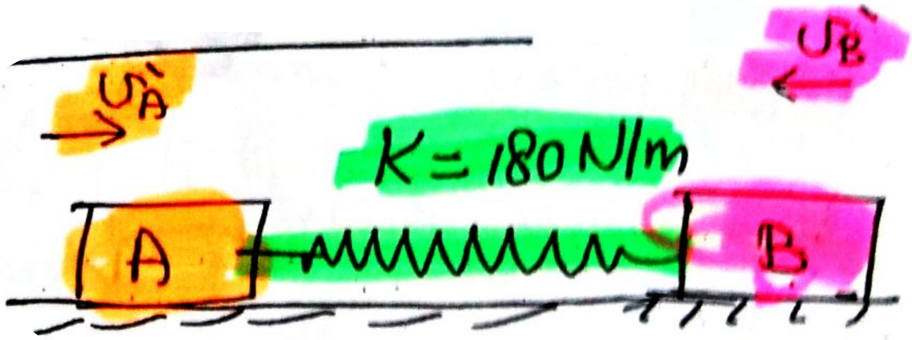
Sol

$$m_A v_A + m_B v_B = (m_A + m_B)\, v'$$

$$(2250)(1) + (1500)(-2) = (2250 + 1500)\, v'$$

$$\Rightarrow v' = -0.2 \text{ m/s}$$

Ex



$m_A = 40$ kg.

$m_B = 60$ kg.

$DS = 2$ m

Find the velocities of A and B when the spring becomes unstreched ?

Ch15

$$\left[m_A v_A + m_B v_B\right]_1 = \left[m_A v_A' + m_B v_B'\right]_2$$

$$40(0) + 60(0) = 40\, v_A' + 60\, v_B'$$

$$40\, v_A' + 60\, v_B' = 0$$

$$2 v_A' + 3 v_B' = 0 \quad \text{—— } (1)$$

Ch14

$$\cancel{T_1}^{0} + U_{12} = T_2$$

$$\cancel{\tfrac{1}{2}} k(\Delta s)^2 = \cancel{\tfrac{1}{2}} m_A v_A'^2 + \cancel{\tfrac{1}{2}} m_B v_B'^2$$

$$180(2)^2 = 40\, v_A'^2 + 60\, v_B'^2 \quad \text{—— } (2)$$

$$v_A = 3.29 \text{ m/s}$$

$$v_B = -2.19 \text{ m/s}$$

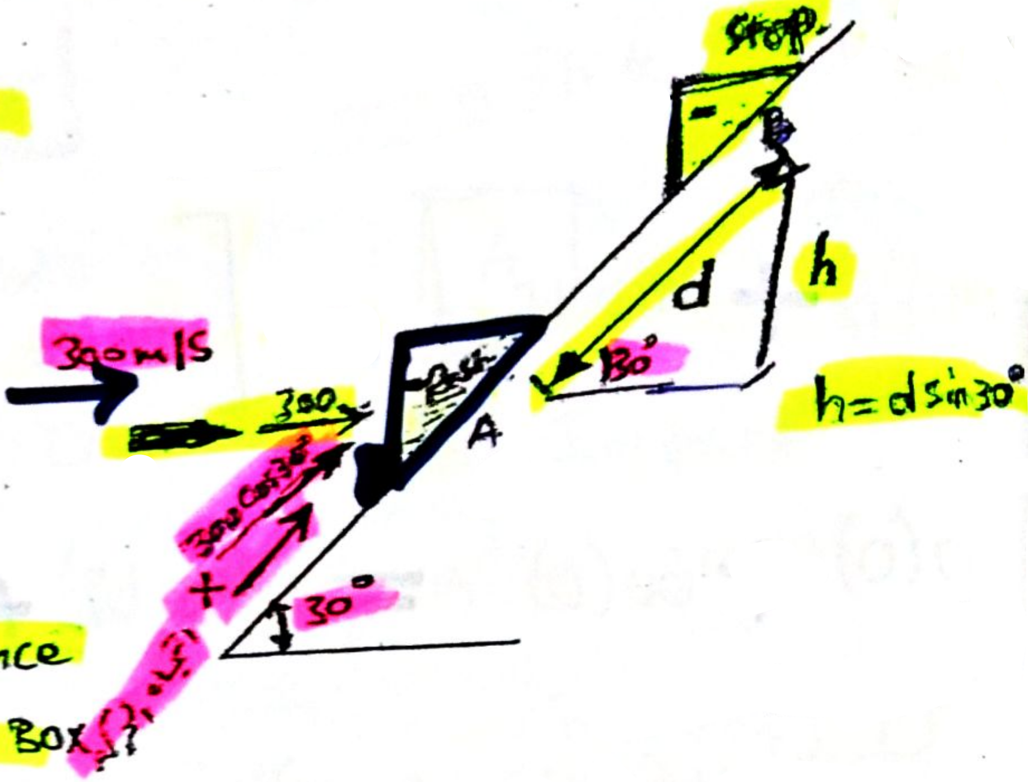
Ex Bullet طلقة

$M_b = 10\,g$

$M_a = 10\,kg$

$v_b = 300\,m/s$

Find the distance d until the Box stops?



X-directions:

Ch15

$$m_b v_b + \cancel{m_a v_a}^{\,0} = (m_b + m_b)\, v_A'$$

$$(10\times10^{-3})(300\cos 30^\circ) = (10.01)\, v_A'$$

$$\Rightarrow v_A' = 0.2595\ m/s$$

Ch14

تتوقف

$$T_A + U_{AB} = \cancel{T_B}^{\,0}$$

$$\frac{1}{2}\cancel{m}\, v_A'^2 - \cancel{m} g h = 0$$

$$\Rightarrow v_A'^2 = 2gh \rightarrow h = \frac{v_A'^2}{2g}$$

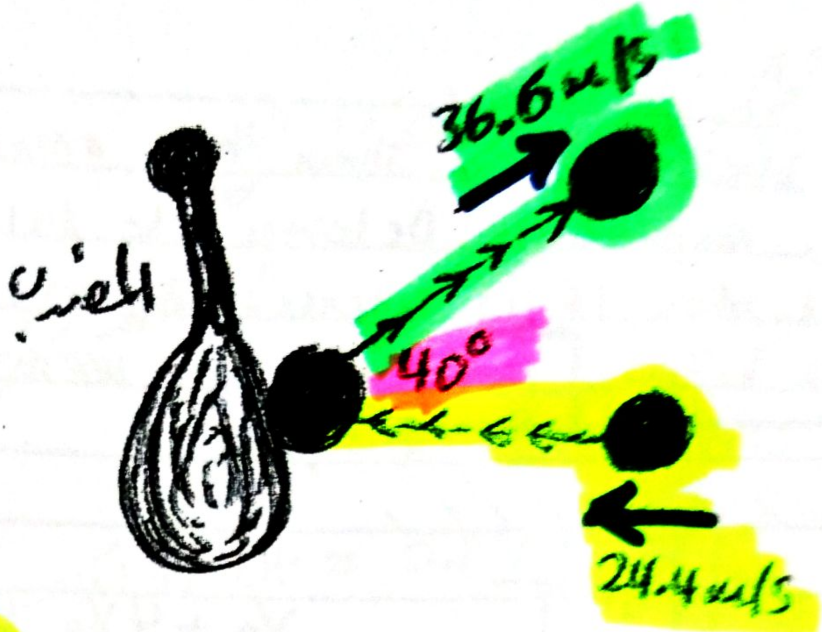
$$h = \frac{(0.2595)^2}{2(9.81)} = 3.43\ mm$$

$$d = \frac{3.43}{\sin 30} \rightarrow d = 6.86\ mm$$

ex

Baseball

of $m = 113.4\,g$



the bat and ball are in Contact for $\Delta t = 0.015$ sec.

Find the average impulsive force during impact.

X-direction

* $m v_{1x} + F_x \Delta t = m v_{2x}$

$$(0.1134)(-24.4) + F_x(0.015) = 0.1134\left[36.6\cos 40^\circ\right]$$

$$\Rightarrow \boxed{F_x = 396.4\ N}$$

y-direction

* $\cancel{m v_{1y}}^{\,0} + F_y \Delta t = m v_{2y}$

$$0 + F_y[0.015] = (0.1134)\left[36.6\sin 40^\circ\right]$$

$$\Rightarrow \boxed{F_y = 178\ N}$$

$F = \sqrt{F_x^2 + F_y^2} = \text{------}\ N \qquad \phi = \tan^{-1}\frac{F_y}{F_x}$

Chapter 15
Kinetics of a Particle Impulse and Momentum

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Objectives:

1. To develop the principle of linear impulse and momentum for a particle and apply it to solve problems that involve force, velocity, and time.
2. To study the conservation of linear momentum for particles.
3. To analyze the mechanics of impact.
4. To introduce the concept of angular impulse and momentum.
5. To solve problems involving steady fluid streams and propulsion with variable mass.

READING QUIZ

1. The linear impulse and momentum equation is obtained by integrating the _______ with respect to time.

   A) friction force
   B) equation of motion
   C) kinetic energy
   D) potential energy

2. Which parameter is not involved in the linear impulse and momentum equation?

   A) Velocity
   B) Displacement
   C) Time
   D) Force

APPLICATIONS

A dent in an automotive fender can be removed using an impulse tool, which delivers a force over a very short time interval. To do so the weight is gripped and jerked upwards, striking the stop ring.

How can we determine the magnitude of the linear impulse applied to the fender?

Could you analyze a carpenter's hammer striking a nail in the same fashion? Sure!

APPLICATIONS (continued)

When a stake is struck by a sledgehammer, a large impulse force is delivered to the stake and drives it into the ground.

If we know the initial speed of the sledgehammer and the duration of impact, how can we determine the magnitude of the impulsive force delivered to the stake?

PRINCIPLE OF LINEAR IMPULSE AND MOMENTUM (Section 15.1)

The next method we will consider for solving particle kinetics problems is obtained by integrating the equation of motion with respect to time.

The result is referred to as the principle of impulse and momentum. It can be applied to problems involving both linear and angular motion.

This principle is useful for solving problems that involve force, velocity, and time. It can also be used to analyze the mechanics of impact (taken up in a later section).

PRINCIPLE OF LINEAR IMPULSE AND MOMENTUM (continued)

The principle of linear impulse and momentum is obtained by integrating the equation of motion with respect to time. The equation of motion can be written

$$\sum \boldsymbol{F} = m\,\mathbf{a} = m\,(d\boldsymbol{v}/dt)$$

Separating variables and integrating between the limits $\boldsymbol{v} = \boldsymbol{v}_1$ at $t = t_1$ and $\boldsymbol{v} = \boldsymbol{v}_2$ at $t = t_2$ results in

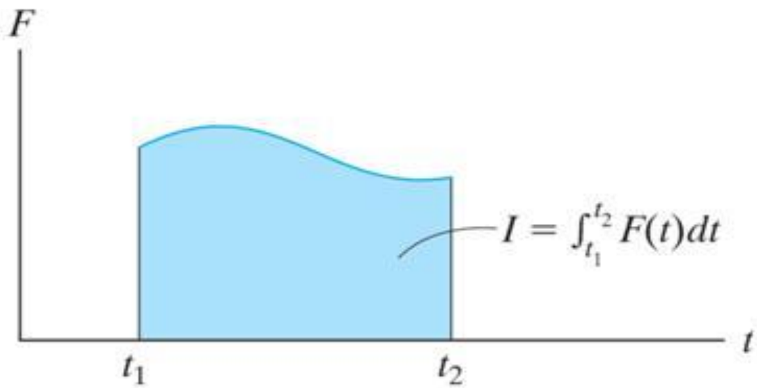
$$\sum \int_{t_1}^{t_2} \boldsymbol{F}\,dt = m \int_{\boldsymbol{v}_1}^{\boldsymbol{v}_2} d\boldsymbol{v} = m\boldsymbol{v}_2 - m\boldsymbol{v}_1$$

This equation represents the principle of linear impulse and momentum. It relates the particle's final velocity ($\boldsymbol{v}_2$) and initial velocity ($\boldsymbol{v}_1$) and the forces acting on the particle as a function of time.

PRINCIPLE OF LINEAR IMPULSE AND MOMENTUM (continued)

Linear momentum: The vector m***v*** is called the linear momentum, denoted as ***L***. This vector has the same direction as ***v***. The linear momentum vector has units of (kg·m)/s or (slug·ft)/s.

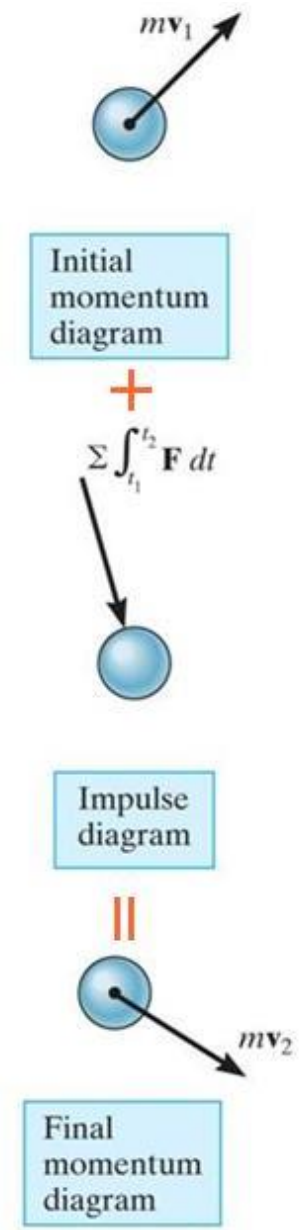
Linear impulse: The integral ($\int$***F***) dt is the linear impulse, denoted ***I***. It is a vector quantity measuring the effect of a force during its time interval of action. ***I*** acts in the same direction as ***F*** and has units of N·s or lb·s.



Variable Force

The impulse may be determined by direct integration. Graphically, it can be represented by the area under the force versus time curve. If ***F*** is constant, then ***I*** = ***F*** $(t_2 - t_1)$.

PRINCIPLE OF LINEAR IMPULSE AND MOMENTUM (continued)



The principle of linear impulse and momentum in vector form is written as

$$m\boldsymbol{v}_1 + \sum\int_{t_1}^{t_2} \boldsymbol{F}\,dt = m\boldsymbol{v}_2$$

The particle's initial momentum plus the sum of all the impulses applied from t_1 to t_2 is equal to the particle's final momentum.

The two momentum diagrams indicate direction and magnitude of the particle's initial and final momentum, $m\boldsymbol{v}_1$ and $m\boldsymbol{v}_2$. The impulse diagram is similar to a free body diagram, but includes the time duration of the forces acting on the particle.

IMPULSE AND MOMENTUM: SCALAR EQUATIONS

Since the principle of linear impulse and momentum is a vector equation, it can be resolved into its x, y, z component scalar equations:

$$m(v_x)_1 + \sum \int_{t_1}^{t_2} F_x \, dt = m(v_x)_2$$

$$m(v_y)_1 + \sum \int_{t_1}^{t_2} F_y \, dt = m(v_y)_2$$

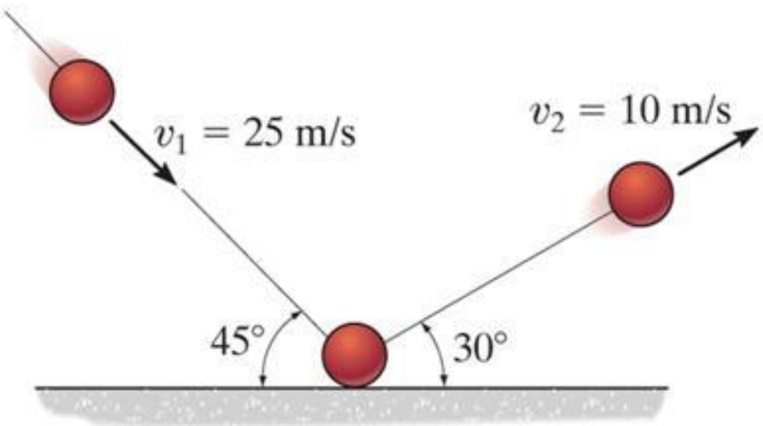
$$m(v_z)_1 + \sum \int_{t_1}^{t_2} F_z \, dt = m(v_z)_2$$

The scalar equations provide a convenient means for applying the principle of linear impulse and momentum once the velocity and force vectors have been resolved into x, y, z components.

PROBLEM SOLVING

- Establish the x, y, z coordinate system.
- Draw the particle's free body diagram and establish the direction of the particle's initial and final velocities, drawing the impulse and momentum diagrams for the particle. Show the linear momenta and force impulse vectors.
- Resolve the force and velocity (or impulse and momentum) vectors into their x, y, z components, and apply the principle of linear impulse and momentum using its scalar form.
- Forces as functions of time must be integrated to obtain impulses. If a force is constant, its impulse is the product of the force's magnitude and time interval over which it acts.

EXAMPLE



Given: A 0.5 kg ball strikes the rough ground and rebounds with the velocities shown. Neglect the ball's weight during the time it impacts the ground.

Find: The magnitude of impulsive force exerted on the ball.

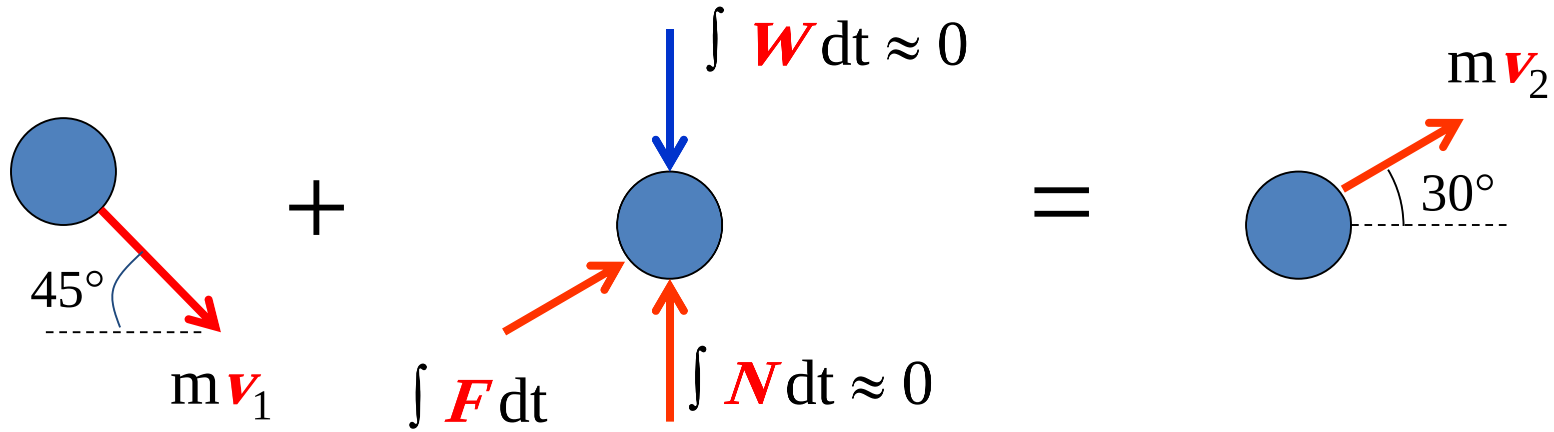
Plan:

1) Draw the momentum and impulse diagrams of the ball as it hits the surface.
2) Apply the principle of impulse and momentum to determine the impulsive force.

EXAMPLE (continued)

Solution:

1) The impulse and momentum diagrams can be drawn as:



The impulse caused by the ball's weight and the normal force ***N*** can be neglected because their magnitudes are very small as compared to the impulse from the ground.

EXAMPLE (continued)

2) The principle of impulse and momentum can be applied along the direction of motion:

$$m\mathbf{v}_1 + \sum \int_{t_1}^{t_2} \mathbf{F}\, dt = m\mathbf{v}_2$$

$$\Rightarrow \quad 0.5\,(25 \cos 45^\circ\, \mathbf{i} - 25 \sin 45^\circ\, \mathbf{j}) + \int_{t_1}^{t_2} \sum \mathbf{F}\, dt = 0.5\,(10 \cos 30^\circ\, \mathbf{i} + 10 \sin 30^\circ\, \mathbf{j})$$

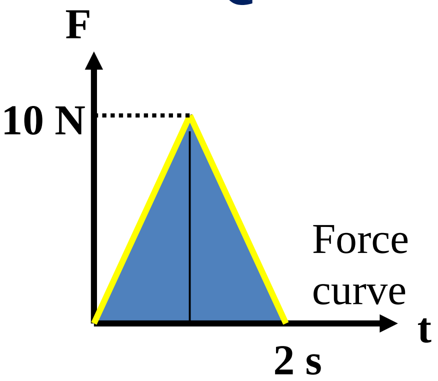
The impulsive force vector is

$$\mathbf{I} = \int_{t_1}^{t_2} \sum \mathbf{F}\, dt = (4.509\, \mathbf{i} + 11.34\, \mathbf{j}) \text{ N}\cdot\text{s}$$

Magnitude: $I = \sqrt{4.509^2 + 11.34^2} = 12.2 \text{ N}\cdot\text{s}$

CHECK YOUR UNDERSTANDING QUIZ

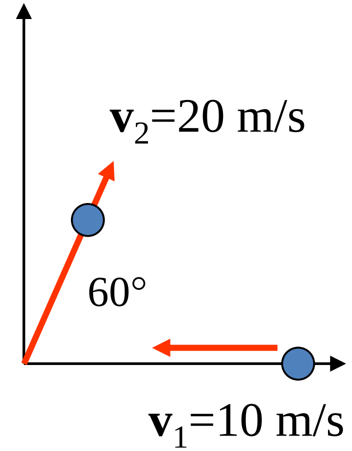
1. Calculate the impulse due to the force.



A) 20 kg·m/s   B) 10 kg·m/s

C) 5 N·s   D) 15 N·s

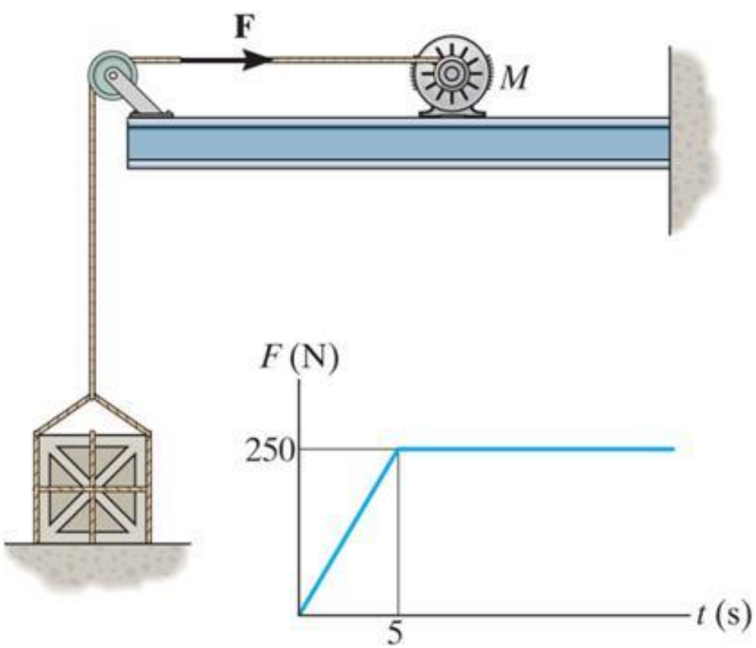
2. A constant force ***F*** is applied for 2 s to change the particle's velocity from $\mathbf{v}_1$ to $\mathbf{v}_2$. Determine the force ***F*** if the particle's mass is 2 kg.



A) (17.3 ***j***) N   B)(–10 ***i*** +17.3 ***j***) N

C) (20 ***i*** +17.3 ***j***) N   D)(10 ***i*** +17.3 ***j***) N

GROUP PROBLEM SOLVING



Given: The 20 kg crate is resting on the floor. The motor M pulls on the cable with a force of **F**, which has a magnitude that varies as shown on the graph.

Find: The speed of the crate when t = 6 s.

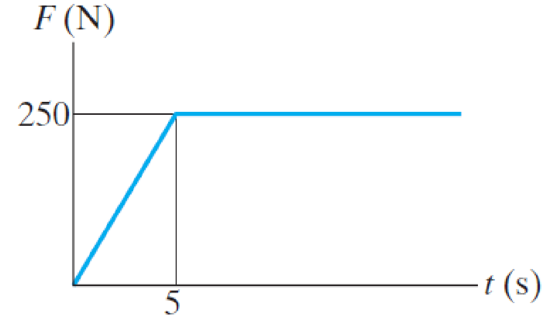
1) Determine the force needed to begin lifting the crate, and then the time needed for the motor to generate this force.

2) After the crate starts moving, apply the principle of impulse and momentum to determine the speed of the crate at t = 6 s.

GROUP PROBLEM SOLVING (continued)

Solution:

1) The crate begins moving when the cable force F exceeds the crate weight. Solve for the force, then the time.



$F = mg = (20)\,(9.81) = 196.2\text{ N}$

$F = 196.2\text{ N} = 50\,t$

$t = 3.924\text{ s}$

2) Apply the principle of impulse and momentum from the time the crate starts lifting at $t_1 = 3.924$ s to $t_2 = 6$ s.

Note that there are two external forces (cable force and weight) we need to consider.

A. The impulse due to cable force:

$$\int_{3.924}^{6} F\,dt = [0.5(250)\,5 + (250)\,1] - 0.5(196.2)3.924 = 490.1\text{ N}\cdot\text{s}$$

GROUP PROBLEM SOLVING (continued)

B. The impulse due to weight:

$$+\uparrow \int_{3.924}^{6} (-\,mg)\,dt = -\,196.2\,(6 - 3.924) = -\,407.3\ \mathrm{N{\cdot}s}$$

Now, apply the principle of impulse and momentum

$$mv_1 + \sum \int_{t_1}^{t_2} F\,dt = mv_2 \quad \text{where } v_1 = 0$$

$$0 + 490.1 - 407.3 = (20)\,v_2$$

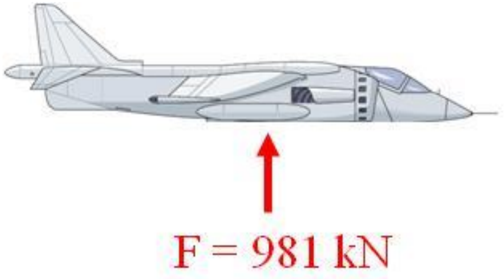
$$\Rightarrow \quad v_2 = 4.14\ \mathrm{m/s}$$

ATTENTION QUIZ

1. Jet engines on the 100 Mg VTOL aircraft exert a constant vertical force of 981 kN as it hovers. Determine the net impulse on the aircraft over t = 10 s.

   A) -981 kN·s B) 0 kN·s

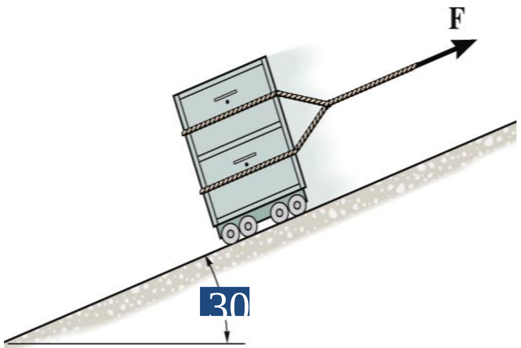
   C) 981 kN·s D) 9810 kN·s



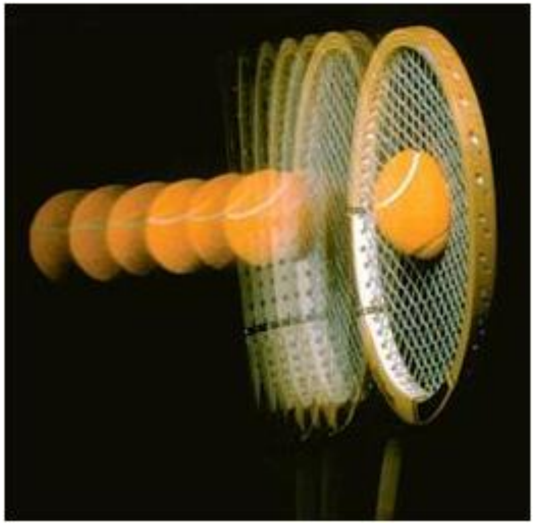
2. A 100 lb cabinet is placed on a smooth surface. If a force of a 100 lb is applied for 2 s, determine the net impulse on the cabinet during this time interval.

   A) 0 lb·s B) 100 lb·s

   C) 200 lb·s D) 300 lb·s



PRINCIPLE OF LINEAR IMPULSE AND MOMENTUM AND CONSERVATION OF LINEAR MOMENTUM FOR SYSTEMS OF PARTICLES



Objectives:

Students will be able to:

1. Apply the principle of linear impulse and momentum to a system of particles.
2. Understand the conditions for conservation of momentum.

READING QUIZ

1. The internal impulses acting on a system of particles always ___________

   A) equal the external impulses. B) Sum to zero.

   C) equal the impulse of weight. D) None of the above.

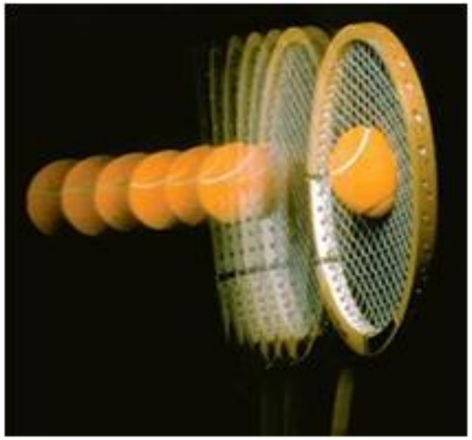
2. If an impulse-momentum analysis is considered during the very short time of interaction, as shown in the picture, weight is a/an ___________

   A) impulsive force.

   B) explosive force.

   C) non-impulsive force.

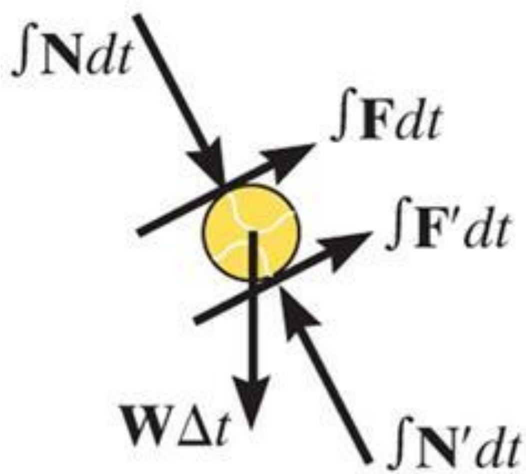
   D) internal force.



APPLICATIONS

As the wheels of this pitching machine rotate, they apply frictional impulses to the ball, thereby giving it linear momentum in the direction of ***F*** dt and ***F'*** dt.

The weight impulse, **W** Δt is very small since the time the ball is in contact with the wheels is very small.



Does the release velocity of the ball depend on the mass of the ball?

APPLICATIONS (continued)

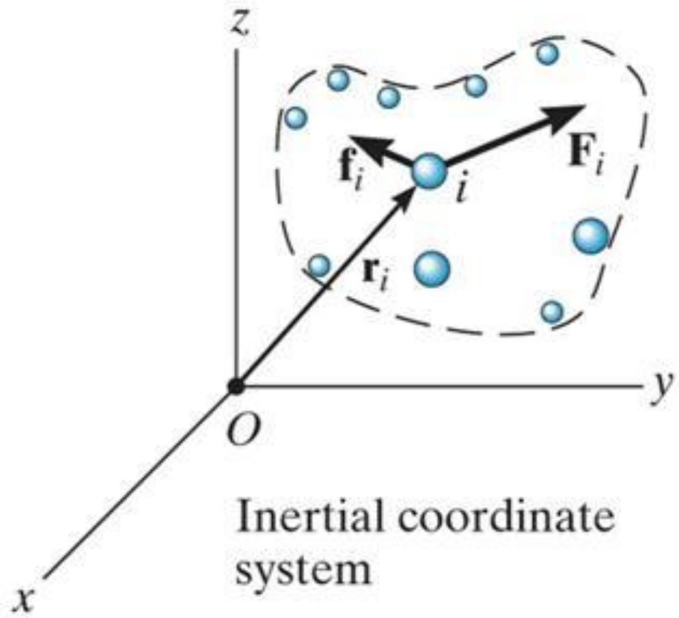
This large crane-mounted hammer is used to drive piles into the ground.

Conservation of momentum can be used to find the velocity of the pile just after impact, assuming the hammer does not rebound off the pile.

If the hammer rebounds, does the pile velocity change from the case when the hammer doesn't rebound ? Why ?

In the impulse-momentum analysis, do we have to consider the impulses of the weights of the hammer and pile and the resistance force ? Why or why not ?

PRINCIPLE OF LINEAR IMPULSE AND MOMENTUM FOR A SYSTEM OF PARTICLES (Section 15.2)



For the system of particles shown, the internal forces f_i between particles always occur in pairs with equal magnitude and opposite directions. Thus the internal impulses sum to zero.

The linear impulse and momentum equation for this system only includes the impulse of external forces.

$$\sum m_i(\mathbf{v}_i)_1 + \sum \int_{t_1}^{t_2} \mathbf{F}_i dt = \sum m_i(\mathbf{v}_i)_2$$

MOTION OF THE CENTER OF MASS

For a system of particles, we can define a "fictitious" center of mass of an aggregate particle of mass m_{tot}, where m_{tot} is the sum ($\sum m_i$) of all the particles. This system of particles then has an aggregate velocity of

$$\mathbf{v}_G = (\sum m_i \mathbf{v}_i) / m_{tot}.$$

The motion of this fictitious mass is based on motion of the center of mass for the system.

The position vector $\mathbf{r}_G = (\sum m_i \mathbf{r}_i) / m_{tot}$ describes the motion of the center of mass.

CONSERVATION OF LINEAR MOMENTUM FOR A SYSTEM OF PARTICLES (Section 15.3)

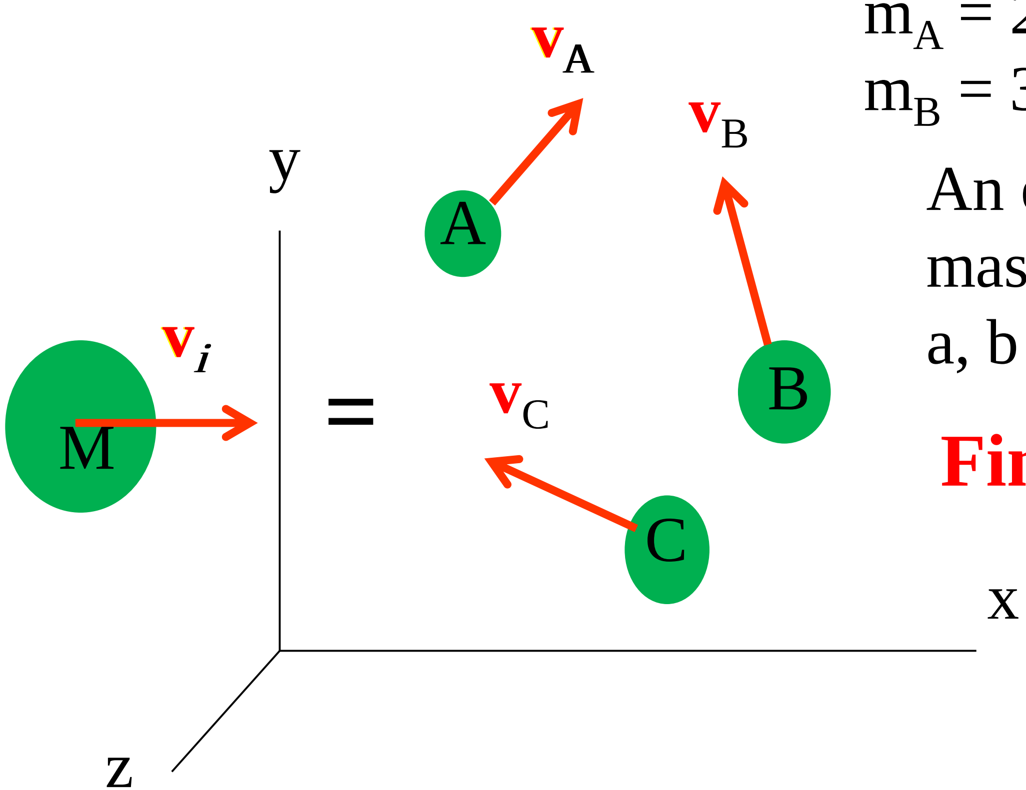
When the sum of external impulses acting on a system of objects is zero, the linear impulse-momentum equation simplifies to

$$\sum m_i(\boldsymbol{v}_i)_1 = \sum m_i(\boldsymbol{v}_i)_2$$

This equation is referred to as the conservation of linear momentum. Conservation of linear momentum is often applied when particles collide or interact. When particles impact, only impulsive forces cause a change of linear momentum.

The sledgehammer applies an impulsive force to the stake. The weight of the stake is considered negligible, or non-impulsive, as compared to the force of the sledgehammer. Also, provided the stake is driven into soft ground with little resistance, the impulse of the ground acting on the stake is considered non-impulsive.

EXAMPLE I



Given: M = 100 kg, $\boldsymbol{v}_i = 20\boldsymbol{j}$ (m/s)

m_A = 20 kg, $\boldsymbol{v}_A = 50\boldsymbol{i} + 50\boldsymbol{j}$ (m/s)

m_B = 30 kg, $\boldsymbol{v}_B = -30\boldsymbol{i} - 50\boldsymbol{k}$ (m/s)

An explosion has broken the mass m into 3 smaller particles, a, b and c.

Find: The velocity of fragment C after the explosion.

Plan: Since the internal forces of the explosion cancel out, we can apply the conservation of linear momentum to the SYSTEM.

EXAMPLE I (continued)

Solution:

$$m\mathbf{v}_i = m_A\mathbf{v}_A + m_B\mathbf{v}_B + m_C\mathbf{v}_C$$

$$100(20\mathbf{j}) = 20(50\mathbf{i} + 50\mathbf{j}) + 30(-30\mathbf{i} - 50\mathbf{k}) + 50(v_{cx}\,\mathbf{i} + v_{cy}\,\mathbf{j} + v_{cz}\,\mathbf{k})$$

Equating the components on the left and right side yields:

$0 = 1000 – 900 + 50(v_{cx})$ $\quad v_{cx} = -2$ m/s

$2000 = 1000 + 50\ (v_{cy})$ $\quad v_{cy} = 20$ m/s

$0 = -1500 + 50\ (v_{cz})$ $\quad v_{cz} = 30$ m/s

So $\mathbf{v}_c = (-2\mathbf{i} + 20\mathbf{j} + 30\mathbf{k})$ m/s immediately after the explosion.

EXAMPLE II

Given: Two rail cars with masses of m_A = 20 Mg and m_B = 15 Mg and velocities as shown.

Find: The speed of the car A after collision if the cars collide and rebound such that B moves to the right with a speed of 2 m/s. Also find the average impulsive force between the cars if the collision place in 0.5 s.

Plan: Use conservation of linear momentum to find the velocity of the car A after collision (all internal impulses cancel). Then use the principle of impulse and momentum to find the impulsive force by looking at only one car.

EXAMPLE II (continued)

Solution:

Conservation of linear momentum (x-dir):

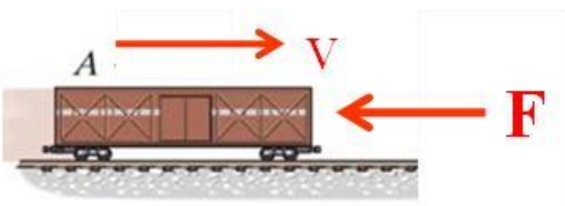
$$m_A(v_{A1}) + m_B(v_{B1}) = m_A(v_{A2}) + m_B(v_{B2})$$

$$20{,}000\ (3) + 15{,}000\ (-1.5)$$

$$= (20{,}000)\ v_{A2} + 15{,}000\ (2)$$

$$v_{A2} = 0.375 \text{ m/s}$$

Impulse and momentum on car A (x-dir):



$$m_A\,(v_{A1}) + \int F\,dt = m_A\,(v_{A2})$$

$$20{,}000\ (3) - \int F\,dt = 20{,}000\ (0.375)$$

$$\int F\,dt = 52{,}500 \text{ N}\cdot\text{s}$$

The average force is

$$\int F\,dt = 52{,}500 \text{ N}\cdot\text{s} = F_{avg}(0.5 \text{ sec}); \quad F_{avg} = 105 \text{ kN}$$

CONCEPT QUIZ

1) Over the short time span of a tennis ball hitting the racket during a player's serve, the ball's weight can be considered ______________

   A) non-impulsive.

   B) impulsive.

   C) not subject to Newton's second law.

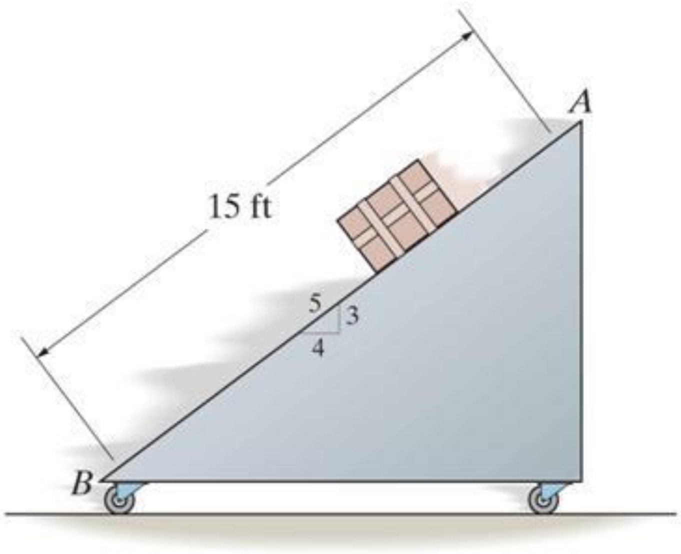
   D) Both A and C.

2) A drill rod is used with a air hammer for making holes in hard rock so explosives can be placed in them. How many impulsive forces act on the drill rod during the drilling?

   A) None B) One

   C) Two D) Three

GROUP PROBLEM SOLVING



Given: The free-rolling ramp has a weight of 120 lb. The 80 lb crate slides from rest at A, 15 ft down the ramp to B.

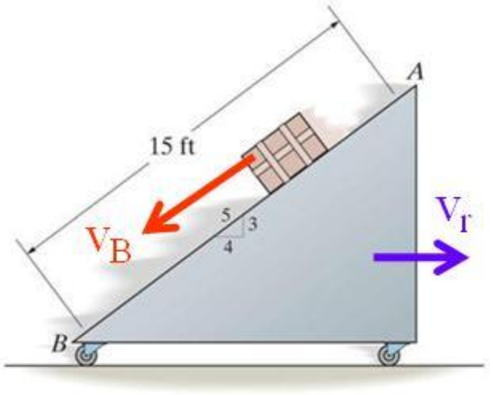
Assume that the ramp is smooth, and neglect the mass of the wheels.

Find: The ramp's speed when the crate reaches *B.*

Plan: Use **the energy conservation equation** as well as **conservation of linear momentum** and the relative velocity equation (you thought you could safely forget it?) to find the velocity of the ramp.

GROUP PROBLEM SOLVING (continued)

Solution:



Energy conservation equation:

$$0 + 80\,(3/5)\,(15)$$
$$= 0.5\,(80/32.2)(\boldsymbol{v}_B)^2 + 0.5\,(120/32.2)(\boldsymbol{v}_r)^2$$

To find the relations between $\boldsymbol{v}_B$ and $\boldsymbol{v}_r$, use conservation of linear momentum:

$$\overset{+}{\rightarrow} \quad 0 = (120/32.2)\,v_r - (80/32.2)\,v_{Bx}$$
$$\Rightarrow \quad v_{Bx} = 1.5\,v_r \qquad (1)$$

Since $\boldsymbol{v}_B = \boldsymbol{v}_r + \boldsymbol{v}_{B/r} \Rightarrow -v_{Bx}\,\boldsymbol{i} + v_{By}\,\boldsymbol{j} = v_r\,\boldsymbol{i} + v_{B/r}\,(-4/5\,\boldsymbol{i} - 3/5\,\boldsymbol{j})$

$$\Rightarrow \quad -v_{Bx} = v_r - (4/5)\,v_{B/r} \qquad (2)$$
$$v_{By} = -(3/5)\,v_{B/r} \qquad (3)$$

Eliminating $v_{B/r}$ from Eqs. (2) and (3) and Substituting Eq. (1) results in $v_{By} = 1.875\,v_r$

GROUP PROBLEM SOLVING (continued)

Then, energy conservation equation can be rewritten ;

$$0 + 80\,(3/5)\,(15) = 0.5\,(80/32.2)(\boldsymbol{v}_B)^2 + 0.5\,(120/32.2)(\boldsymbol{v}_r)^2$$

$$0 + 80\,(3/5)\,(15) = 0.5\,(80/32.2)\,[(1.5\,v_r)^2 + (1.875\,v_r)^2] + 0.5\,(120/32.2)\,(\boldsymbol{v}_r)^2$$

$$720 = 9.023\,(\boldsymbol{v}_r)^2$$

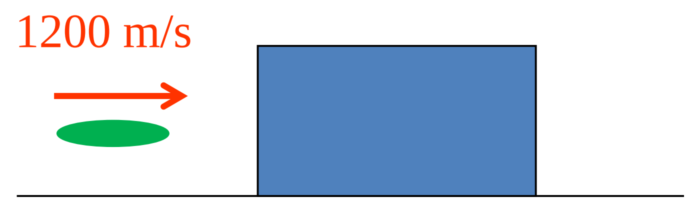
$\boldsymbol{v}_r = 8.93$ ft/s

ATTENTION QUIZ

1. The 20 g bullet is fired horizontally at 1200 m/s into the 300 g block resting on a smooth surface. If the bullet becomes embedded in the block, what is the velocity of the block immediately after impact.

A) 1125 m/s B) 80 m/s

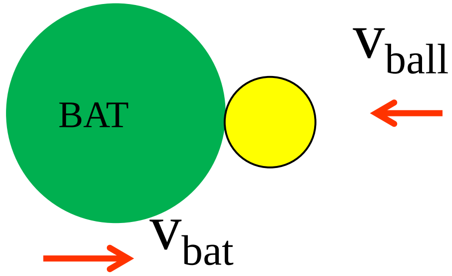
C) 1200 m/s D) 75 m/s



2. The 200-g baseball has a horizontal velocity of 30 m/s when it is struck by the bat, B, weighing 900-g, moving at 47 m/s. During the impact with the bat, how many impulses of importance are used to find the final velocity of the ball?

A) Zero B) One

C) Two D) Three



ANGULAR MOMENTUM, MOMENT OF A FORCE AND PRINCIPLE OF ANGULAR IMPULSE AND MOMENTUM

Objectives:

Students will be able to:

1. Determine the angular momentum of a particle and apply the principle of angular impulse & momentum.
2. Use conservation of angular momentum to solve problems.

READING QUIZ

1. Select the correct expression for the angular momentum of a particle about a point.

   A) $\boldsymbol{r} \times \boldsymbol{v}$

   B) $\boldsymbol{r} \times (m\,\boldsymbol{v})$

   C) $\boldsymbol{v} \times \boldsymbol{r}$

   D) $(m\,\boldsymbol{v}) \times \boldsymbol{r}$

2. The sum of the moments of all external forces acting on a particle is equal to

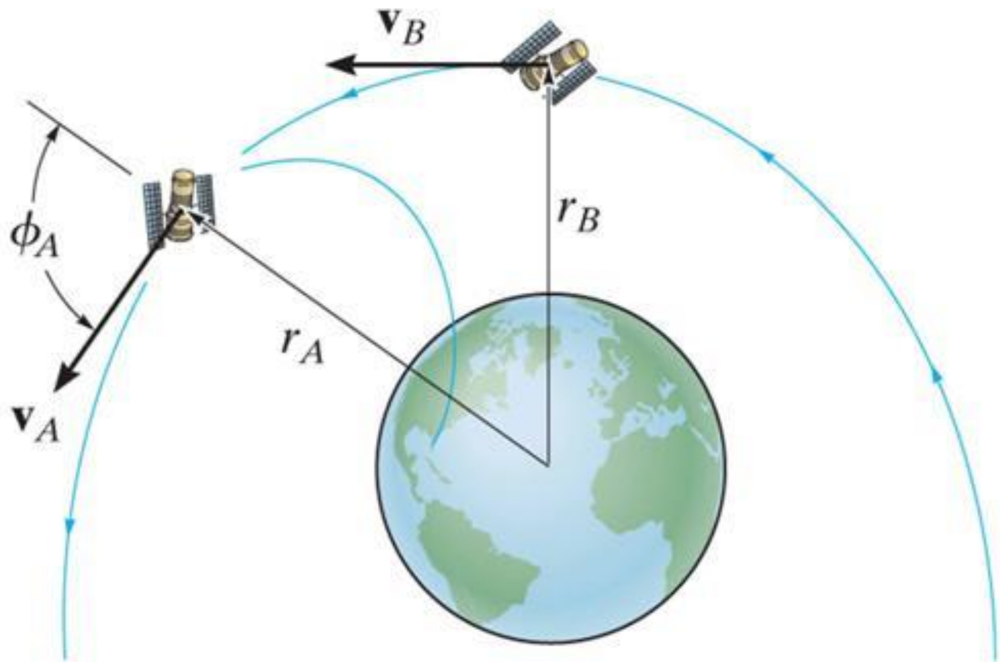
   A) angular momentum of the particle.

   B) linear momentum of the particle.

   C) time rate of change of angular momentum.

   D) time rate of change of linear momentum.

APPLICATIONS

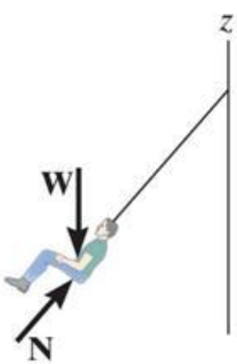


Planets and most satellites move in elliptical orbits. This motion is caused by gravitational attraction forces. Since these forces act in pairs, the sum of the moments of the forces acting on the system will be zero. This means that angular momentum is conserved.

If the angular momentum is constant, does it mean the linear momentum is also constant? Why or why not?

APPLICATIONS (continued)

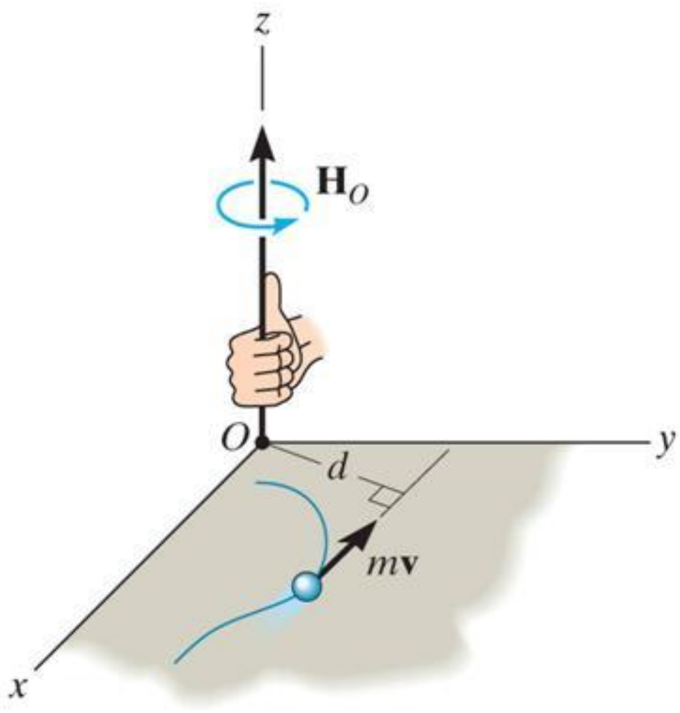
The passengers on the amusement-park ride experience conservation of angular momentum about the axis of rotation (the z-axis). As shown on the free body diagram, the line of action of the normal force, N, passes through the z-axis and the weight's line of action is parallel to it. Therefore, the sum of moments of these two forces about the z-axis is zero.



If the passenger moves away from the z-axis, will his speed increase or decrease? Why?

ANGULAR MOMENTUM (Section 15.5)

The angular momentum of a particle about point O is defined as the "moment" of the particle's linear momentum about O.



$$\boldsymbol{H}_o = \boldsymbol{r} \times m\boldsymbol{v} = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ r_x & r_y & r_z \\ mv_x & mv_y & mv_z \end{vmatrix}$$

The magnitude of $\boldsymbol{H}_o$ is $(H_o)_z = mv\,d$

RELATIONSHIP BETWEEN MOMENT OF A FORCE AND ANGULAR MOMENTUM

(Section 15.6)

The resultant force acting on the particle is equal to the time rate of change of the particle's linear momentum. Showing the time derivative using the familiar "dot" notation results in the equation

$$\sum \boldsymbol{F} = \dot{\boldsymbol{L}} = \mathrm{m}\dot{\boldsymbol{v}}$$

We can prove that the resultant moment acting on the particle about point O is equal to the time rate of change of the particle's angular momentum about point O or

$$\sum \boldsymbol{M}_{\mathrm{o}} = \boldsymbol{r} \times \boldsymbol{F} = \dot{\boldsymbol{H}}_{\mathrm{o}}$$

PRINCIPLE OF ANGULAR IMPULSE AND MOMENTUM (Section 15.7)

Considering the relationship between moment and time rate of change of angular momentum

$$\sum \mathbf{M}_o = \dot{\mathbf{H}}_o = d\mathbf{H}_o/dt$$

By integrating between the time interval t_1 to t_2

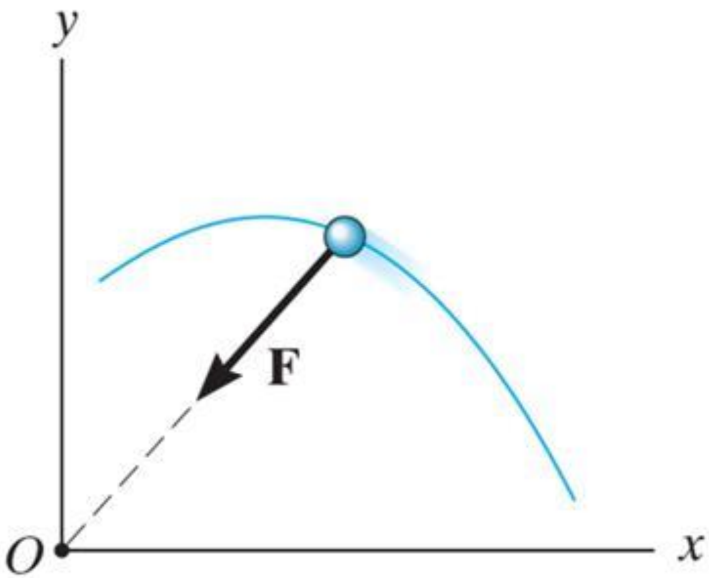
$$\sum \int_{t_1}^{t_2} \mathbf{M}_o \, dt = (\mathbf{H}_o)_2 - (\mathbf{H}_o)_1 \quad \text{or} \quad (\mathbf{H}_o)_1 + \sum \int_{t_1}^{t_2} \mathbf{M}_o \, dt = (\mathbf{H}_o)_2$$

This equation is referred to as the principle of angular impulse and momentum. The second term on the left side, $\sum\int M_o \, dt$, is called the angular impulse. In cases of 2D motion, it can be applied as a scalar equation using components about the z-axis.

CONSERVATION OF ANGULAR MOMENTUM

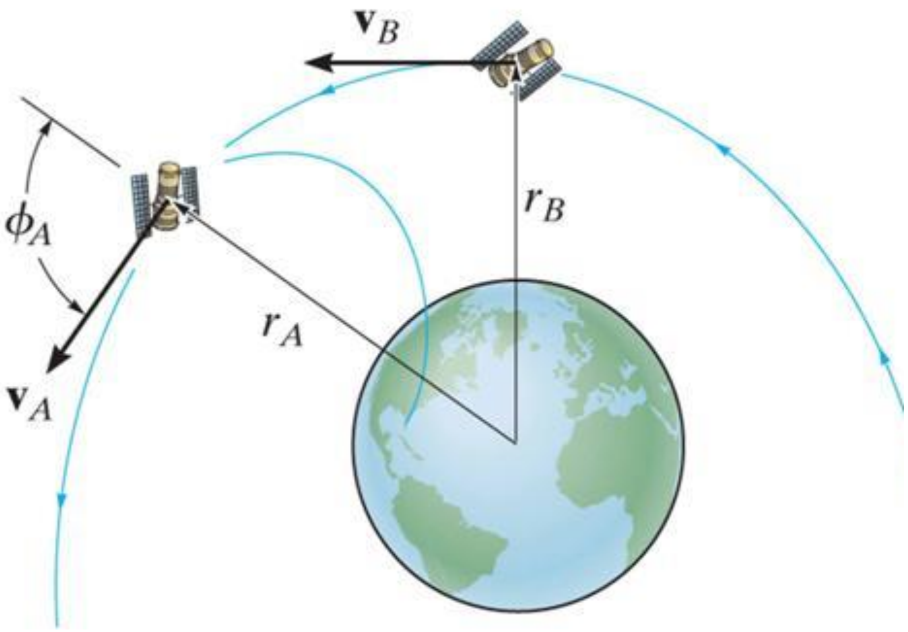
When the sum of angular impulses acting on a particle or a system of particles is zero during the time t_1 to t_2, the angular momentum is conserved. Thus,

$$(\boldsymbol{H}_O)_1 = (\boldsymbol{H}_O)_2$$



An example of this condition occurs when a particle is subjected only to a central force. In the figure, the force ***F*** is always directed toward point O. Thus, the angular impulse of ***F*** about O is always zero, and angular momentum of the particle about O is conserved.

EXAMPLE



Given: A satellite has an elliptical orbit about earth.

$m_{satellite} = 700$ kg

$m_{earth} = 5.976 \times 10^{24}$ kg

$v_A = 10$ km/s

$r_A = 15 \times 10^6$ m

$\phi_A = 70°$

Find: The speed, v_B, of the satellite at its closest distance, r_B, from the center of the earth.

Plan: Apply the principles of conservation of energy and conservation of angular momentum to the system.

EXAMPLE (continued)

Solution:

Conservation of energy: $T_A + V_A = T_B + V_B$ becomes

$$\frac{1}{2} m_s v_A^2 - \frac{G\, m_s\, m_e}{r_A} = \frac{1}{2} m_s v_B^2 - \frac{G\, m_s\, m_e}{r_B}$$

where $G = 66.73\times10^{-12}\ m^3/(kg\cdot s^2)$. Dividing through by m_s and substituting values yields:

$$0.5(10{,}000)^2 - \frac{66.73\times10^{-12}(5.976\times10^{24})}{15\,x\,10^6} = 0.5\,v_B^2 - \frac{66.73\times10^{-12}(5.976\times10^{24})}{r_B}$$

or $23.4\times10^6 = 0.5\,(v_B)^2 - (3.99\times10^{14})/r_B$

EXAMPLE (continued)

Solution:

Now use Conservation of Angular Momentum.

$$(r_A\ m_s\ v_A)\ \sin\phi_A = r_B\ m_s\ v_B$$

$(15 \times 10^6)(10{,}000)\ \sin 70° = r_B\ v_B$ or

$r_B = (140.95 \times 10^9)/v_B$

Solving the two equations for r_B and v_B yields

$r_B = 13.8 \times 10^6$ m $\qquad v_B = 10.2$ km/s

CONCEPT QUIZ

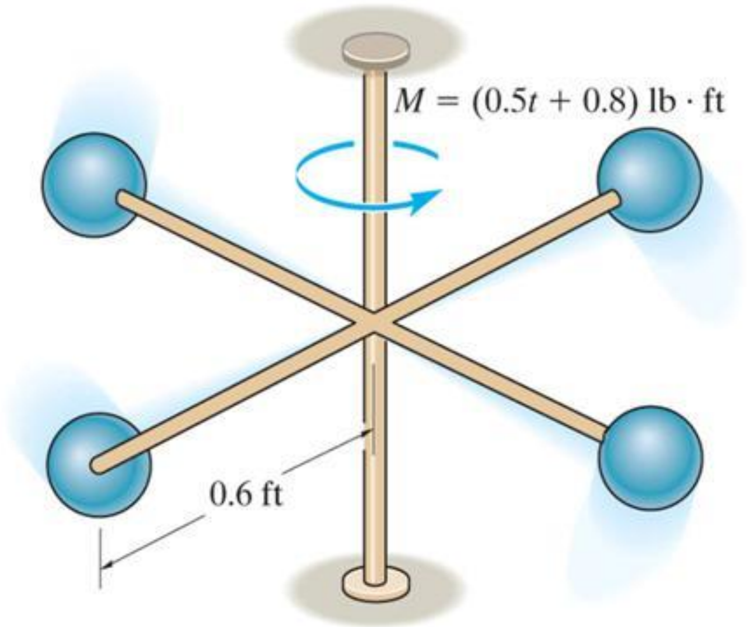
1. If a particle moves in the x - y plane, its angular momentum vector is in the

   A) x direction.
   B) y direction.
   C) z direction.
   D) x - y direction.

2. If there are no external impulses acting on a particle

   A) only linear momentum is conserved.
   B) only angular momentum is conserved.
   C) both linear momentum and angular momentum are conserved.
   D) neither linear momentum nor angular momentum are conserved.

GROUP PROBLEM SOLVING



Given: The four 5 lb spheres are rigidly attached to the crossbar frame, which has a negligible weight.

A moment acts on the shaft as shown, M = 0.5t + 0.8 lb·ft).

Find: The velocity of the spheres after 4 seconds, starting from rest.

Plan:

Apply the principle of angular impulse and momentum about the axis of rotation (z-axis).

GROUP PROBLEM SOLVING (continued)

Solution:

Angular momentum: $\boldsymbol{H}_Z = \boldsymbol{r} \times m\boldsymbol{v}$ reduces to a scalar equation.

$(H_Z)_1 = 0$ and $(H_Z)_2 = 4 \times \{(5/32.2)\,(0.6)\,v_2\} = 0.3727\,v_2$

Angular impulse:

$$\int_{t_1}^{t_2} M\,dt = \int_{t_1}^{t_2} (0.5t + 0.8)\,dt = \left[(0.5/2)\,t^2 + 0.8\,t\right]\Big|_0^4 = 7.2\ \text{lb}\cdot\text{ft}\cdot\text{s}$$

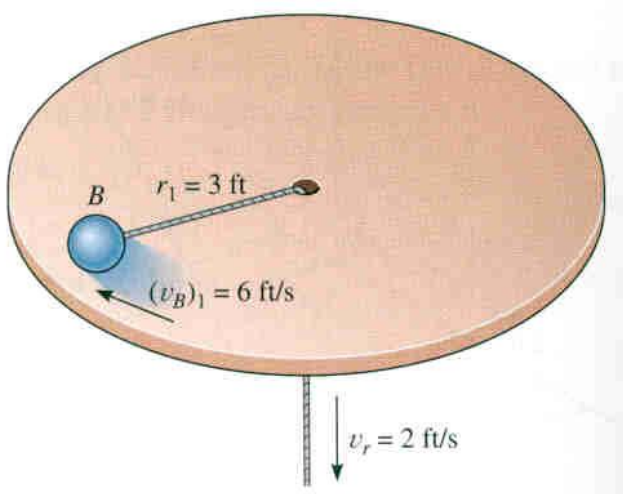
Apply the principle of angular impulse and momentum.

$0 + 7.2 = 0.3727\,v_2 \quad \Rightarrow \quad v_2 = 19.4\ \text{ft/s}$

ATTENTION QUIZ

1. A ball is traveling on a smooth surface in a 3 ft radius circle with a speed of 6 ft/s. If the attached cord is pulled down with a constant speed of 2 ft/s, which of the following principles can be applied to solve for the velocity of the ball when r = 2 ft?

   A) Conservation of energy
   B) Conservation of angular momentum
   C) Conservation of linear momentum
   D) Conservation of mass

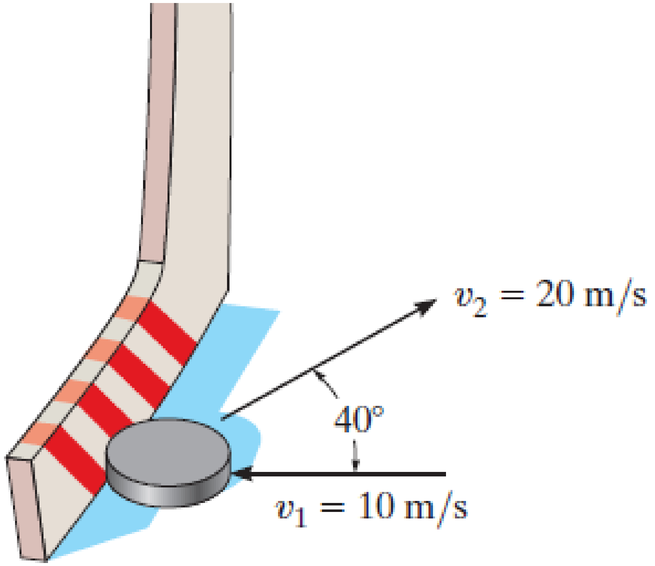


2. If a particle moves in the z - y plane, its angular momentum vector is in the

   A) x direction.
   B) y direction.
   C) z direction.
   D) z - y direction.

Example

A hockey puck is traveling to the left with a velocity of $v_1 = 10$ m/s when it is struck by a hockey stick and given a velocity of $v_2 = 20$ m/s as shown. Determine the magnitude of the net impulse exerted by the hockey stick on the puck. The puck has a mass of 0.2 kg.



SOLUTION

$$v_1 = \{-10\mathbf{i}\} \text{ m/s}$$

$$v_2 = \{20 \cos 40°\mathbf{i} + 20 \sin 40°\mathbf{j}\} \text{ m/s}$$

$$\mathbf{I} = m\Delta v = (0.2)\,\{[20 \cos 40° - (-10)]\mathbf{i} + 20 \sin 40°\mathbf{j}\}$$

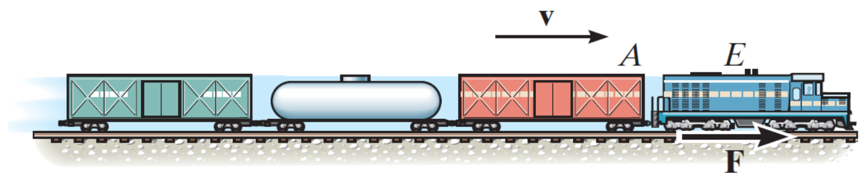
$$= \{5.0642\mathbf{i} + 2.5712\mathbf{j}\} \text{ kg} \cdot \text{m/s}$$

$$\mathbf{I} = \sqrt{(5.0642)^2 + (2.5712)^2}$$

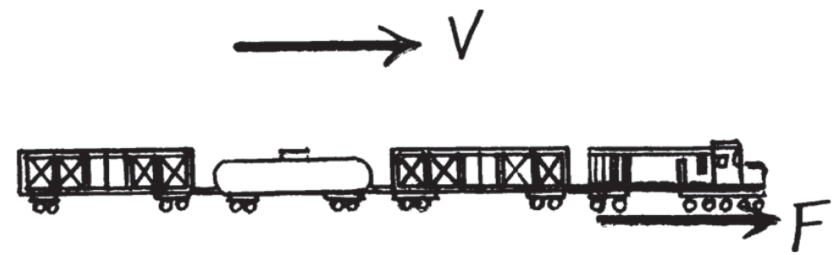
$$= 5.6795 = 5.68 \text{ kg} \cdot \text{m/s}$$

Example

A train consists of a 50-Mg engine and three cars, each having a mass of 30 Mg. If it takes 80 s for the train to increase its speed uniformly to 40 km/h, starting from rest, determine the force T developed at the coupling between the engine E and the first car A. The wheels of the engine provide a resultant frictional tractive force **F** which gives the train forward motion, whereas the car wheels roll freely. Also, determine F acting on the engine wheels.



SOLUTION



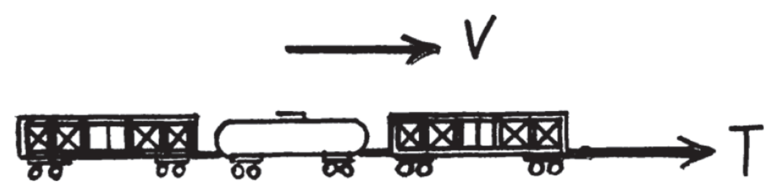
$(v_x)_2 = 40 \text{ km/h} = 11.11 \text{ m/s}$

Entire train:

$$\left(\overset{+}{\rightarrow}\right) \qquad m(v_x)_1 + \Sigma \int F_x\, dt = m(v_x)_2$$

$$0 + F(80) = [50 + 3(30)]\left(10^3\right)(11.11)$$

$$F = 19.4 \text{ kN}$$



Three cars:

$$\left(\overset{+}{\rightarrow}\right) \qquad m(v_x)_1 + \Sigma \int F_x\, dt = m(v_x)_2$$

$$0 + T(80) = 3(30)\left(10^3\right)(11.11) \qquad T = 12.5 \text{ kN}$$

Example

During operation the jack hammer strikes the concrete surface with a force which is indicated in the graph. To achieve this the 2-kg spike S is fired into the surface at 90 m/s. Determine the speed of the spike just after rebounding.

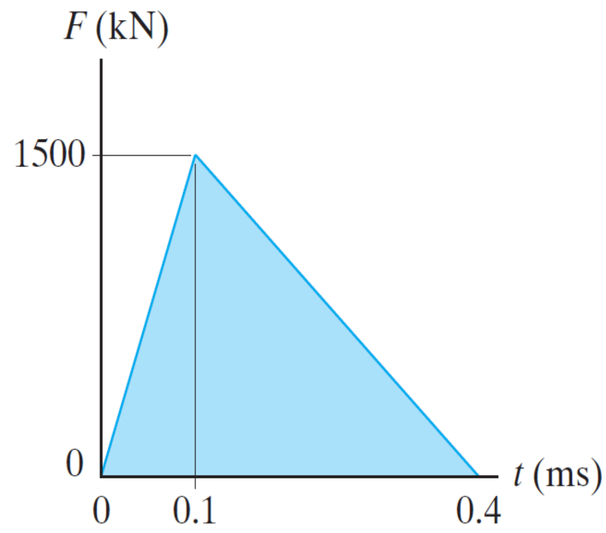
SOLUTION

Principle of Impulse and Momentum. The impulse of the force $\boldsymbol{F}$ is equal to the area under the F–t graph. Referring to the FBD of the spike, Fig. a

$$(+\uparrow) \quad m(v_y)_1 + \Sigma \int_{t_1}^{t_2} F_y\, dt = m(v_y)_2$$

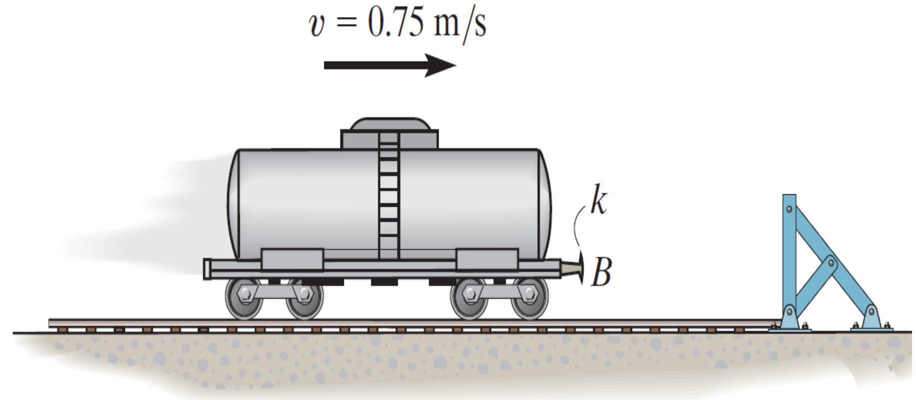
$$2(-90) + \frac{1}{2}\left[0.4(10^{-3})\right]\left[1500(10^3)\right] = 2v$$

$$v = 60.0 \text{ m/s} \uparrow$$



Example

A tankcar has a mass of 20 Mg and is freely rolling to the right with a speed of 0.75 m/s. If it strikes the barrier, determine the horizontal impulse needed to stop the car if the spring in the bumper B has a stiffness (a) $k \to \infty$ (bumper is rigid), and (b) $k = 15$ kN/m.



SOLUTION

a) b) $(\overset{+}{\rightarrow})$ $$mv_1 + \Sigma \int F\,dt = mv_2$$

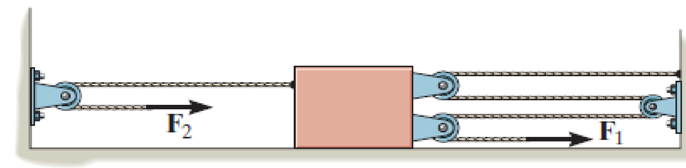
$$20(10^3)(0.75) - \int F\,dt = 0$$

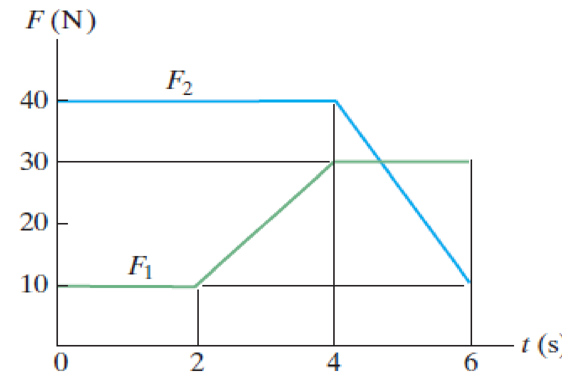
$$\int F\,dt = 15 \text{ kN} \cdot \text{s}$$ **Ans.**

The impulse is the same for both cases. For the spring having a stiffness $k = 15$ kN/m, the impulse is applied over a longer period of time than for $k \to \infty$.

Example

The 30-kg slider block is moving to the left with a speed of 5 m/s when it is acted upon by the forces $\mathbf{F}_1$ and $\mathbf{F}_2$. If these loadings vary in the manner shown on the graph, determine the speed of the block at $t = 6$ s. Neglect friction and the mass of the pulleys and cords.





SOLUTION

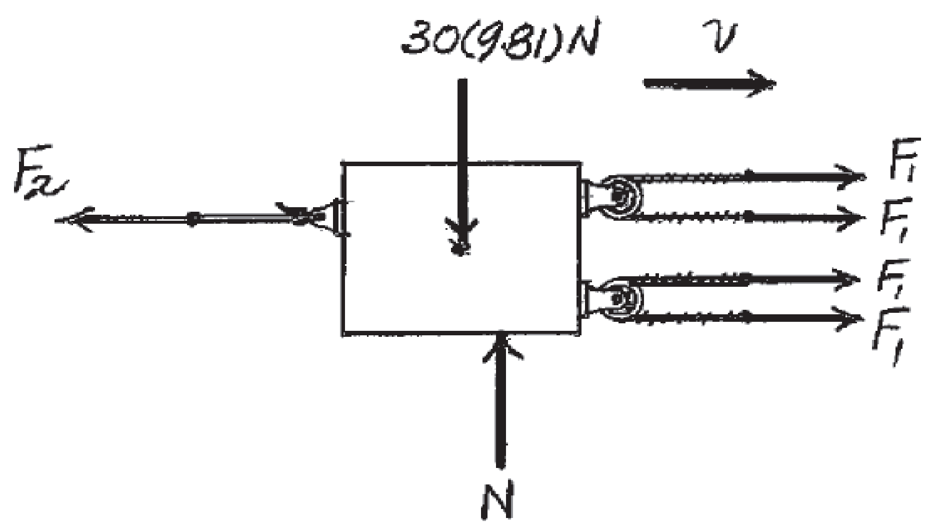
Principle of Impulse and Momentum. The impulses produced by $\mathbf{F}_1$ and $\mathbf{F}_2$ are equal to the area under the respective F–t graph. Referring to the FBD of the block Fig. a,

$$(\overset{+}{\rightarrow}) \quad m(v_x)_1 + \Sigma \int_{t_1}^{t_2} F_x\,dx = m(v_x)_2$$

$$-30(5) + 4\left[10(2) + \frac{1}{2}(10 + 30)(4 - 2) + 30(6 - 4)\right]$$

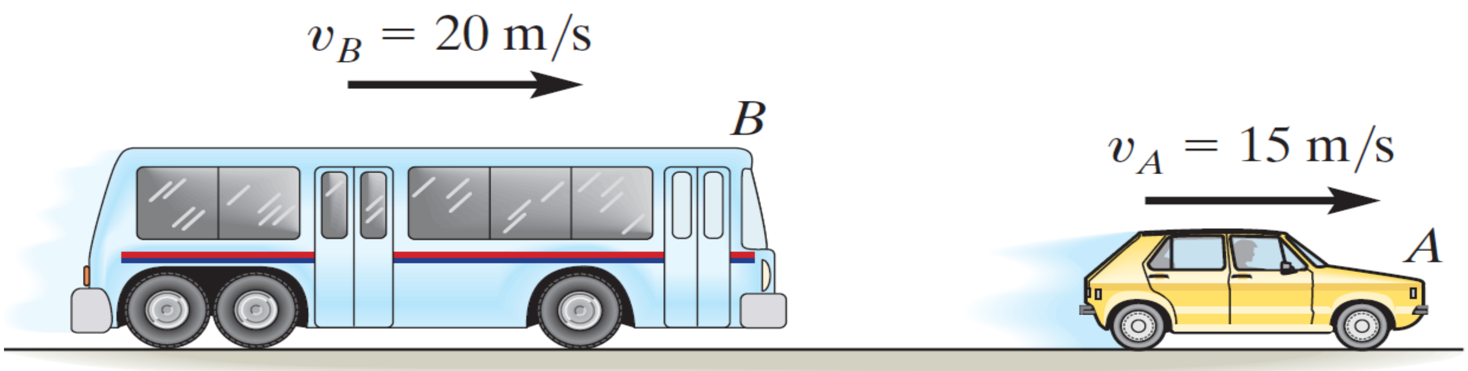
$$+ \left[-40(4) - \frac{1}{2}(10 + 40)(6 - 4)\right] = 30v$$

$$v = 4.00 \text{ m/s} \rightarrow$$



Example

The 5-Mg bus B is traveling to the right at 20 m/s. Meanwhile a 2-Mg car A is traveling at 15 m/s to the right. If the vehicles crash and become entangled, determine their common velocity just after the collision. Assume that the vehicles are free to roll during collision.



SOLUTION

Conservation of Linear Momentum.

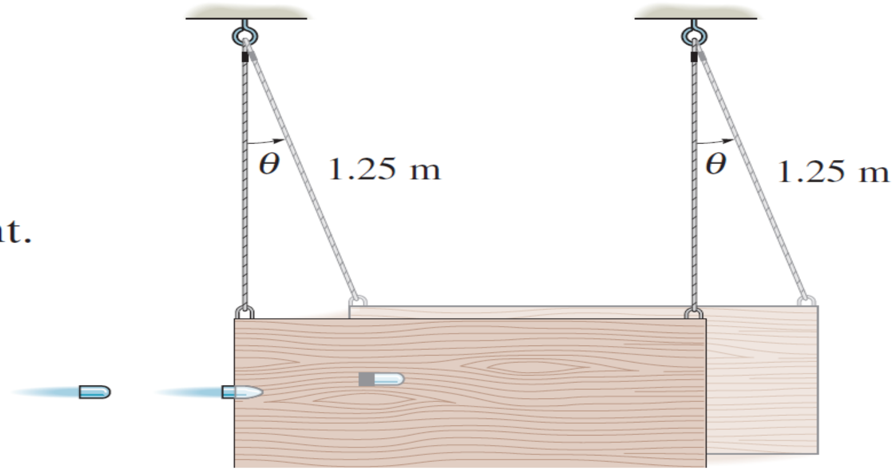
$$(\overset{+}{\rightarrow}) \qquad m_A v_A + m_B v_B = (m_A + m_B)v$$

$$[5(10^3)](20) + [2(10^3)](15) = [5(10^3) + 2(10^3)]v$$

$$v = 18.57 \text{ m/s} = 18.6 \text{ m/s} \rightarrow$$

Example

A ballistic pendulum consists of a 4-kg wooden block originally at rest, $\theta = 0°$. When a 2-g bullet strikes and becomes embedded in it, it is observed that the block swings upward to a maximum angle of $\theta = 6°$. Estimate the speed of the bullet.



SOLUTION

Just after impact:

Datum at lowest point.

$$T_2 + V_2 = T_3 + V_3$$

$$\frac{1}{2}(4 + 0.002)(v_B)_2^2 + 0 = 0 + (4 + 0.002)(9.81)(1.25)(1 - \cos 6°)$$

$$(v_B)_2 = 0.3665 \text{ m/s}$$

For the system of bullet and block:

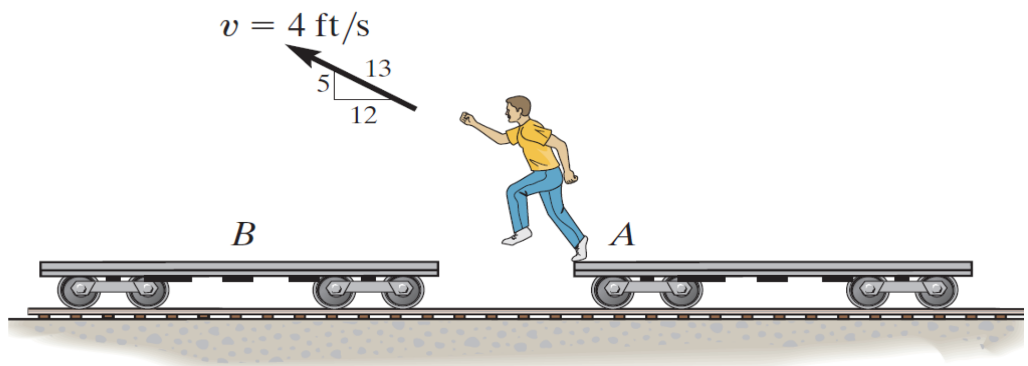
$$(\overset{+}{\rightarrow}) \qquad \Sigma m v_1 = \Sigma m v_2$$

$$0.002(v_B)_1 = (4 + 0.002)(0.3665)$$

$$(v_B)_1 = 733 \text{ m/s}$$

Example

The boy jumps off the flat car at A with a velocity of $v = 4\ \text{ft/s}$ relative to the car as shown. If he lands on the second flat car B, determine the final speed of both cars after the motion. Each car has a weight of 80 lb. The boy's weight is 60 lb. Both cars are originally at rest. Neglect the mass of the car's wheels.



SOLUTION

$(\overset{+}{\leftarrow})$ $$\Sigma m(v_1) = \Sigma m(v_2)$$

$$0 + 0 = -\frac{80}{32.2} v_A + \frac{60}{32.2}(v_b)_x$$

$$v_A = 0.75(v_b)_x$$

$$v_b = v_A + v_{b/A}$$

$(\overset{+}{\leftarrow})$ $$(v_b)_x = -v_A + 4\left(\frac{12}{13}\right)$$

$$(v_b)_x = 2.110\ \text{ft/s}$$

$$v_A = 1.58\ \text{ft/s} \rightarrow$$

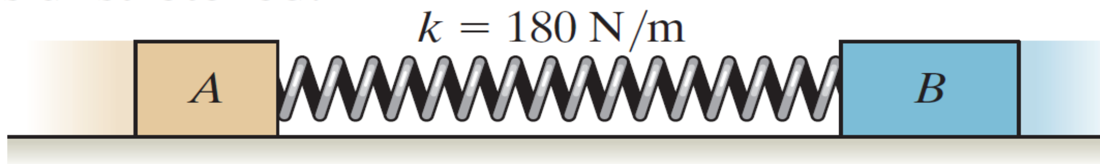
$(\overset{+}{\leftarrow})$ $$\Sigma m(v_1) = \Sigma m(v_2)$$

$$\frac{60}{32.2}(2.110) = \left(\frac{80}{32.2} + \frac{60}{32.2}\right) v$$

$$v = 0.904\ \text{ft/s}$$

Example

Blocks A and B have masses of 40 kg and 60 kg, respectively. They are placed on a smooth surface and the spring connected between them is stretched 2 m. If they are released from rest, determine the speeds of both blocks the instant the spring becomes unstretched.



SOLUTION

$(\xrightarrow{+})\qquad \Sigma m v_1 = \Sigma m v_2$

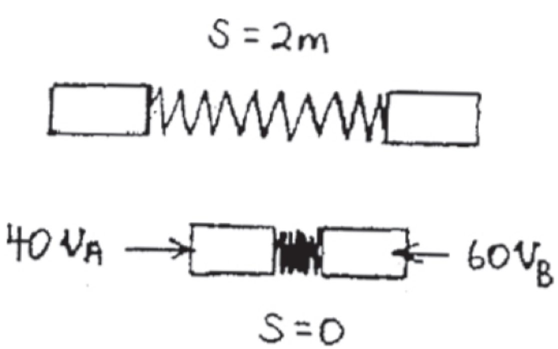
$$0 + 0 = 40\, v_A - 60\, v_B$$

$$T_1 + V_1 = T_2 + V_2$$

$$0 + \frac{1}{2}(180)(2)^2 = \frac{1}{2}(40)(v_A)^2 + \frac{1}{2}(60)(v_B)^2$$

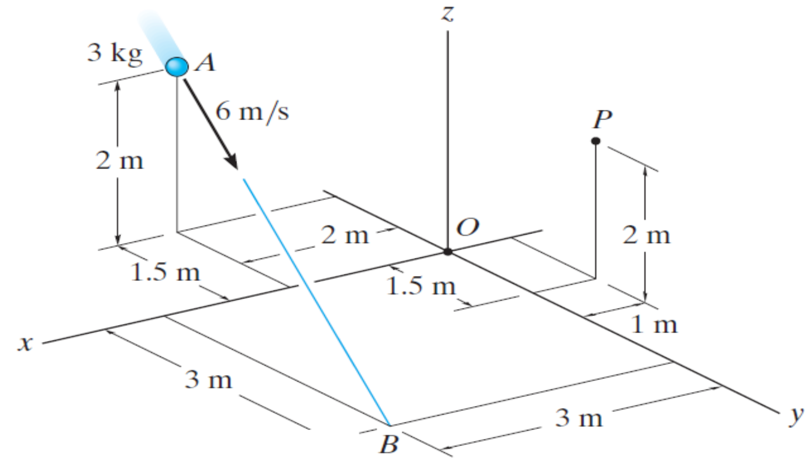
$$v_A = 3.29 \text{ m/s}$$

$$v_B = 2.19 \text{ m/s}$$



Example

Determine the angular momentum $\mathbf{H}_O$ of the 3-kg particle about point O.



SOLUTION

Position and Velocity Vectors.
The coordinates of points A and B are

$A(2, -1.5, 2)$ m and $B(3, 3, 0)$.

$$\boldsymbol{r}_{OB} = \{3\mathbf{i} + 3\mathbf{j}\}\ \text{m} \qquad \boldsymbol{r}_{OA} = \{2\mathbf{i} - 1.5\mathbf{j} + 2\mathbf{k}\}\ \text{m}$$

$$V_A = v_A\left(\frac{r_{AB}}{r_{AB}}\right) = (6)\left[\frac{(3-2)\mathbf{i} + [3-(-1.5)]\mathbf{j} + (0.2)\mathbf{k}}{\sqrt{(3-2)^2 + [3-(-1.5)]^2 + (0-2)^2}}\right]$$

$$= \left\{\frac{6}{\sqrt{25.25}}\mathbf{i} + \frac{27}{\sqrt{25.25}}\mathbf{j} - \frac{12}{\sqrt{25.25}}\mathbf{k}\right\}\ \text{m/s}$$

Angular Momentum about Point O. Applying Eq. 15

$$\boldsymbol{H}_O = \boldsymbol{r}_{OB} \times m V_A$$

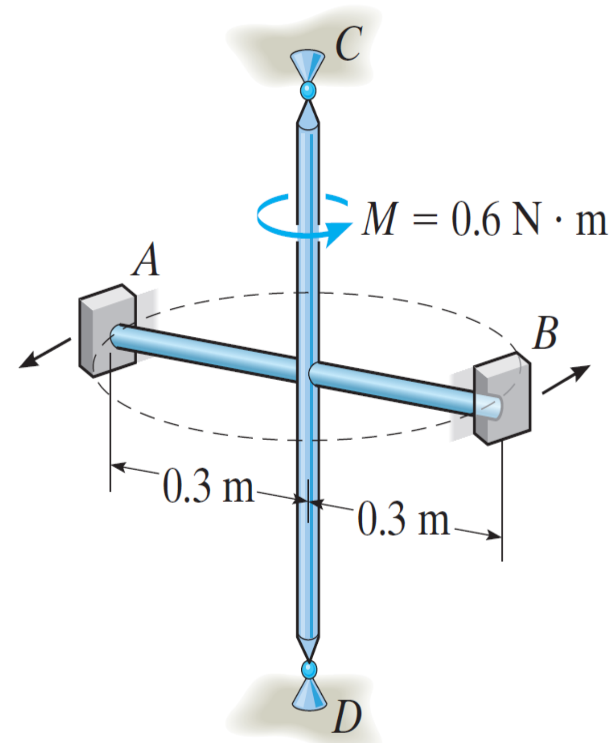
$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 3 & 0 \\ 3\left(\frac{6}{\sqrt{25.25}}\right) & 3\left(\frac{27}{\sqrt{25.25}}\right) & 3\left(-\frac{12}{\sqrt{25.25}}\right) \end{vmatrix}$$

$$= \{-21.4928\mathbf{i} + 21.4928\mathbf{j} + 37.6124\mathbf{k}\}\ \text{kg}\cdot\text{m}^2/\text{s}$$

$$= \{-21.5\mathbf{i} + 21.5\mathbf{j} + 37.6\}\ \text{kg}\cdot\text{m}^2/\text{s}$$

Example

The two blocks A and B each have a mass of 400 g. The blocks are fixed to the horizontal rods, and their initial velocity along the circular path is 2 m/s. If a couple moment of $M = (0.6)\ \text{N}\cdot\text{m}$ is applied about CD of the frame, determine the speed of the blocks when $t = 3$ s. The mass of the frame is negligible, and it is free to rotate about CD. Neglect the size of the blocks.



SOLUTION

$$(H_o)_1 + \Sigma \int_{t_1}^{t_2} M_o dt = (H_o)_2$$

$$2[0.3(0.4)(2)] + 0.6(3) = 2[0.3(0.4)v]$$

$$v = 9.50\ \text{m/s}$$

Example

If the rod of negligible mass is subjected to a couple moment of $M = (30t^2)$ N · m and the engine of the car supplies a traction force of $F = (15t)$ N to the wheels, where t is in seconds, determine the speed of the car at the instant $t = 5$ s. The car starts from rest. The total mass of the car and rider is 150 kg. Neglect the size of the car.

SOLUTION

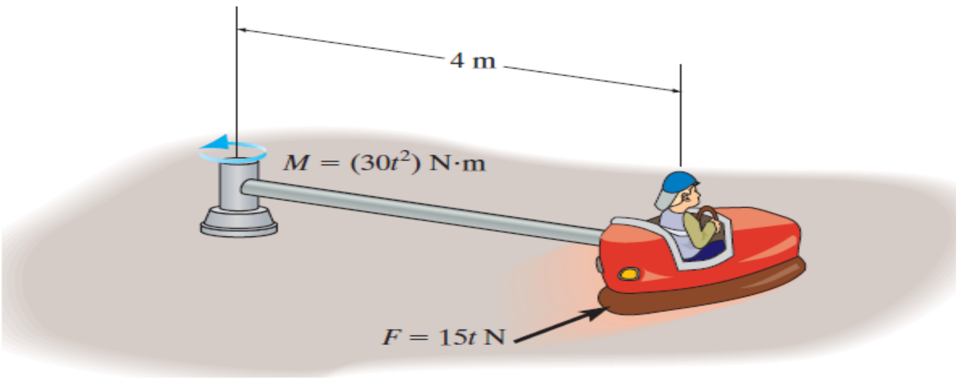
Free-Body Diagram: The free-body diagram of the system is shown in Fig. *a*. Since the moment reaction $\mathbf{M}_S$ has no component about the z axis, the force reaction $\mathbf{F}_S$ acts through the z axis, and the line of action of $\mathbf{W}$ and $\mathbf{N}$ are parallel to the z axis, they produce no angular impulse about the z axis.

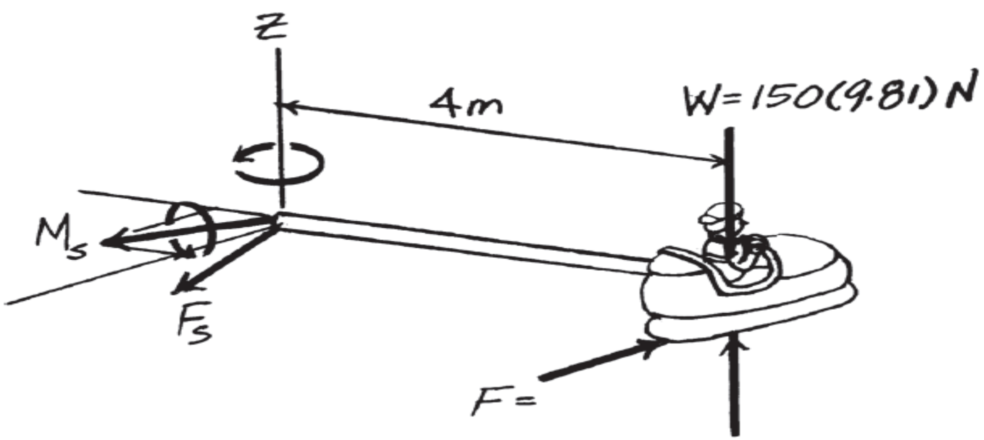
Principle of Angular Impulse and Momentum:

$$(H_1)_z + \Sigma\int_{t_2}^{t_1} M_z\,dt = (H_2)_z$$

$$0 + \int_0^{5\text{ s}} 30t^2\,dt + \int_0^{5\text{ s}} 15t(4)dt = 150v(4)$$

$$v = 3.33 \text{ m/s}$$

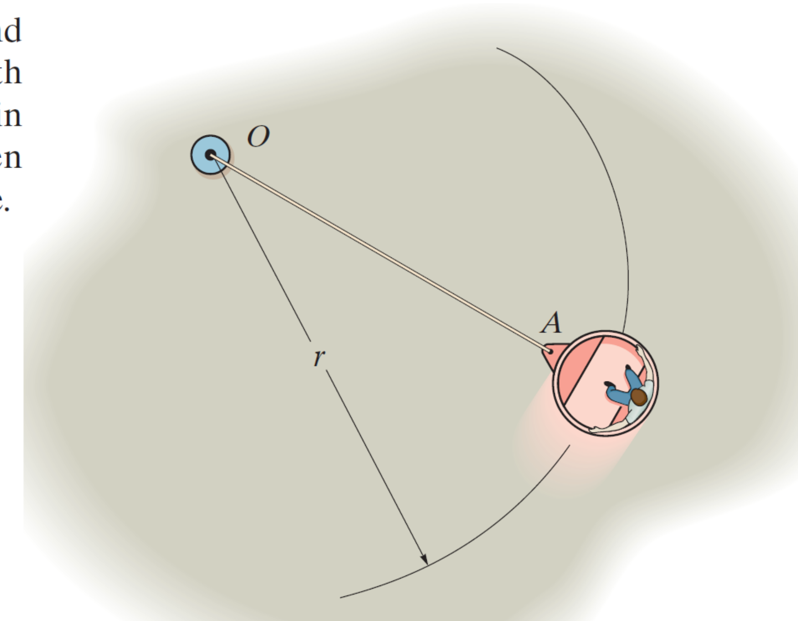




(a)

Example

The amusement park ride consists of a 200-kg car and passenger that are traveling at 3 m/s along a circular path having a radius of 8 m. If at $t = 0$, the cable OA is pulled in toward O at 0.5 m/s, determine the speed of the car when $t = 4$ s. Also, determine the work done to pull in the cable.



SOLUTION

Conservation of Angular Momentum. At $t = 4$ s, $r_2 = 8 - 0.5(4) = 6$ m.

$$(H_0)_1 = (H_0)_2$$

$$r_1 m v_1 = r_2 m (v_2)_t$$

$$8[200(3)] = 6[200(v_2)_t]$$

$$(v_2)_t = 4.00 \text{ m/s}$$

Here, $(v_2)_t = 0.5$ m/s. Thus

$$v_2 = \sqrt{(v_2)_t^2 + (v_2)_r^2} = \sqrt{4.00^2 + 0.5^2} = 4.031 \text{ m/s} = 4.03 \text{ m/s}$$

Principle of Work and Energy.

$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2}(200)(3^2) + \Sigma U_{1-2} = \frac{1}{2}(200)(4.031)^2$$

$$\Sigma U_{1-2} = 725 \text{ J}$$

Chapter 16
Kinematics of Rigid Body

Dr. Ibrahim Abu-Alshaikh
&
Dr. Hashem S. Alkhaldi

April/ 2020

PLANAR RIGID BODY MOTION: TRANSLATION & ROTATION

Today's Objectives :

Students will be able to:
Analyze the kinematics of a rigid body undergoing planar translation or rotation about a fixed axis.

READING QUIZ

1. If a rigid body is in translation only, the velocity at points A and B on the rigid body _______ .

   A) are usually different
   B) are always the same
   C) depend on their position
   D) depend on their relative position

2. If a rigid body is rotating with a constant angular velocity about a fixed axis, the velocity vector at point P is _______.

   A) $\boldsymbol{\omega} \times \boldsymbol{r}_p$
   B) $\boldsymbol{r}_p \times \boldsymbol{\omega}$
   C) $d\boldsymbol{r}_p/dt$
   D) All of the above.

APPLICATIONS

Passengers on this amusement ride are subjected to curvilinear translation since the vehicle moves in a circular path but they always remains upright.

If the angular motion of the rotating arms is known, how can we determine the velocity and acceleration experienced by the passengers? Why would we want to know these values?

Does each passenger feel the same acceleration?

APPLICATIONS

Gears, pulleys and cams, which rotate about fixed axes, are often used in machinery to generate motion and transmit forces. The angular motion of these components must be understood to properly design the system.

To do this design, we need to relate the angular motions of contacting bodies that rotate about different fixed axes. How is this different than the analyses we did in earlier chapters?

RIGID BODY MOTION

There are cases where an object cannot be treated as a particle. In these cases the size or shape of the body must be considered. Rotation of the body about its center of mass requires a different approach.

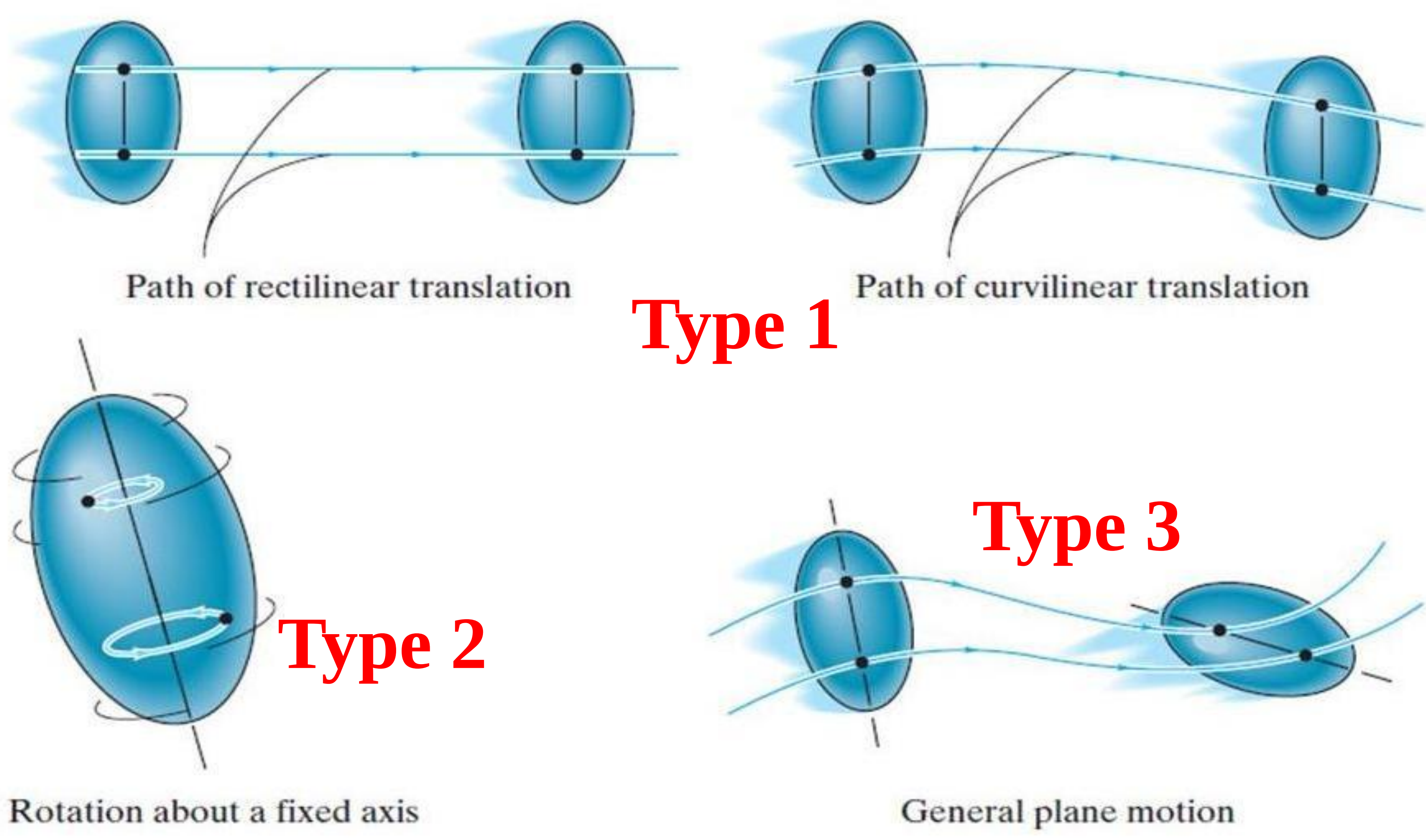
For example, in the design of gears, cams, and links in machinery or mechanisms, rotation of the body is an important aspect in the analysis of motion.

We will now start to study rigid body motion. The analysis will be limited to planar motion.

A body is said to undergo planar motion when all parts of the body move along paths equidistant from a fixed plane.

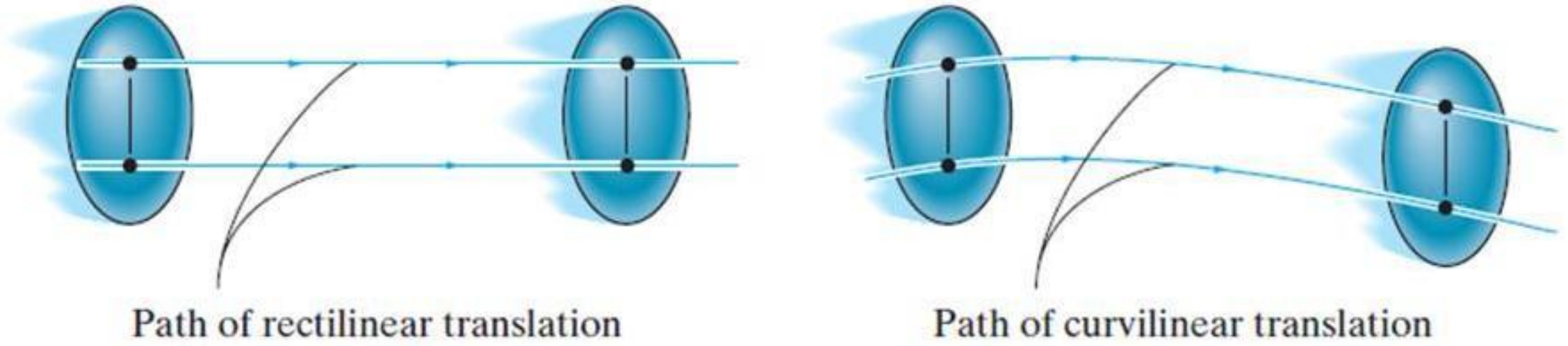
PLANAR RIGID BODY MOTION

There are three types of planar rigid body motion.



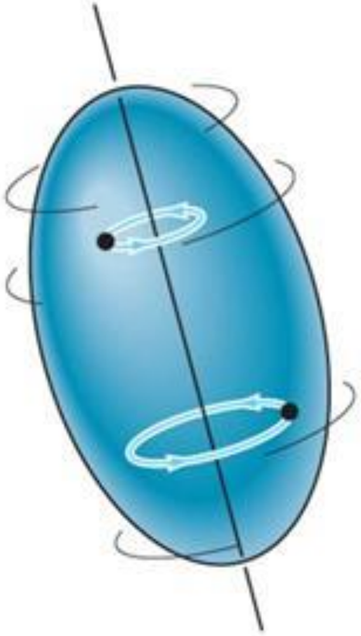
PLANAR RIGID BODY MOTION

Type 1 : Translational Motion



Translation: Translation occurs if every line segment on the body remains parallel to its original direction during the motion. When all points move along straight lines, the motion is called rectilinear translation. When the paths of motion are curved lines, the motion is called curvilinear translation.

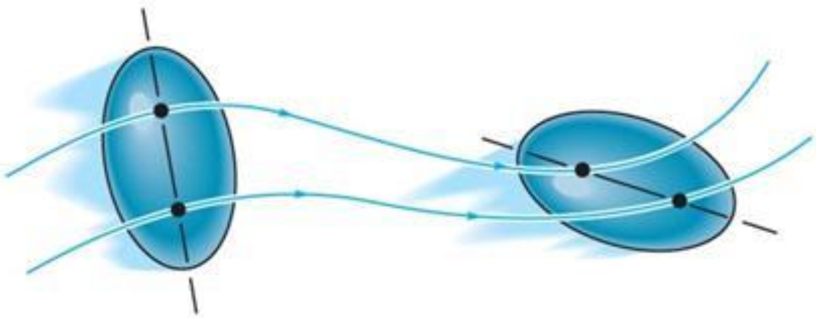
Type 2 : Rotation about a fixed axis



Rotation about a fixed axis

Rotation about a fixed axis: In this case, all the particles of the body, except those on the axis of rotation, move along circular paths in planes perpendicular to the axis of rotation.

Type 3 : General plane motion

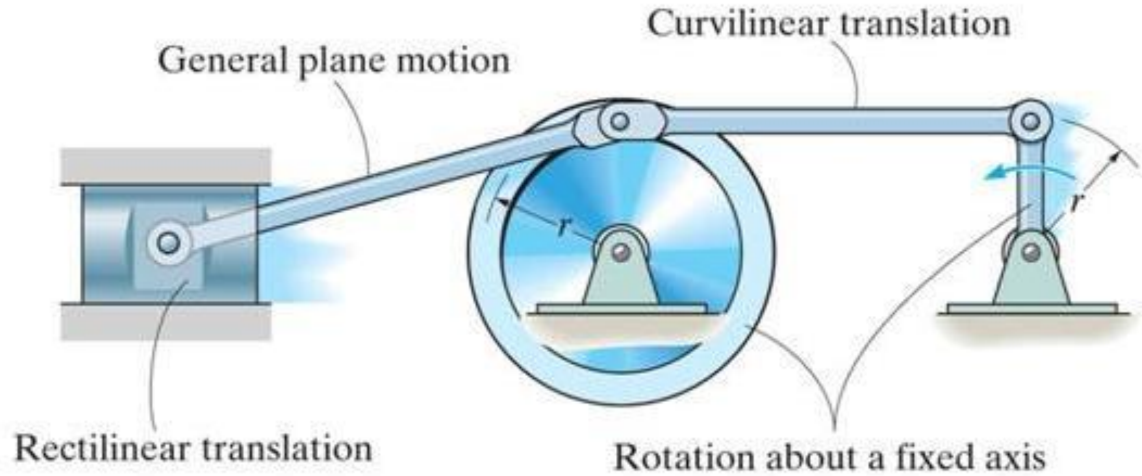


General plane motion

General plane motion: In this case, the body undergoes both translation and rotation. Translation occurs within a plane and rotation occurs about an axis perpendicular to this plane.

PLANAR RIGID BODY MOTION

An example of bodies undergoing the three types of motion is shown in this mechanism.

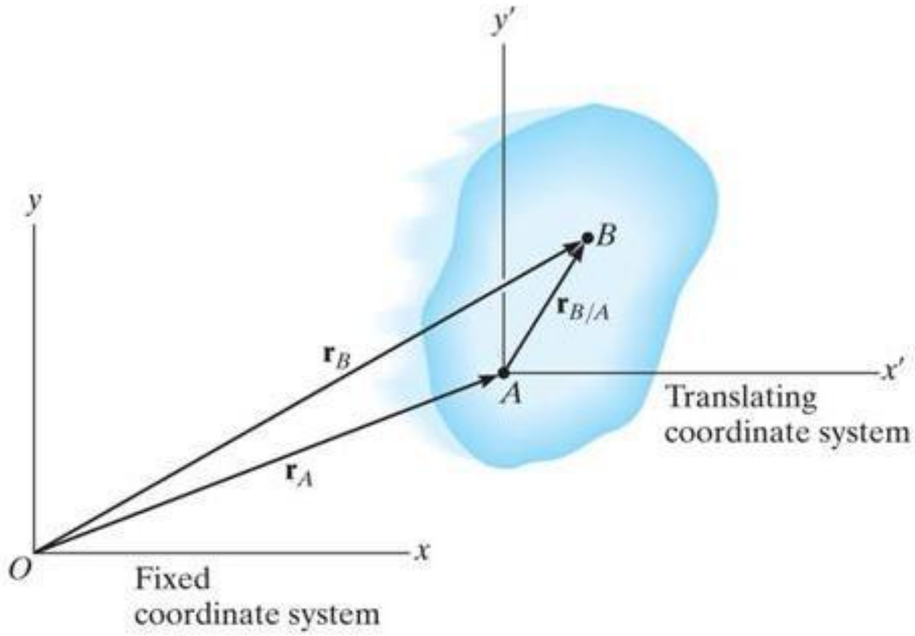


The wheel and crank undergo rotation about a fixed axis.

The piston undergoes rectilinear translation
The connecting rod undergoes curvilinear translation,

The connecting rod undergoes general plane motion, as it will both translate and rotate.

Type 1 : TRANSLATIONAL MOTION



The positions of two points A and B on a translating body can be related by

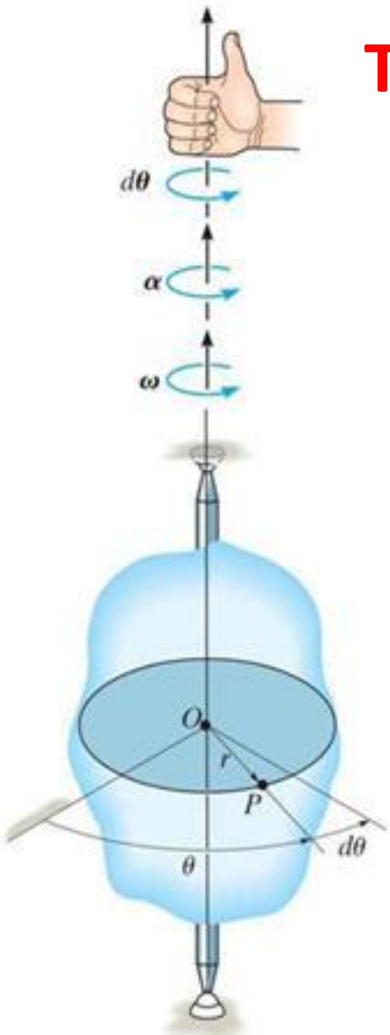
$$\boldsymbol{r}_B = \boldsymbol{r}_A + \boldsymbol{r}_{B/A}$$

where $\boldsymbol{r}_A$ & $\boldsymbol{r}_B$ are the absolute position vectors defined from the fixed x-y coordinate system, and $\boldsymbol{r}_{B/A}$ is the relative-position vector between B and A.

The velocity at B is $\boldsymbol{v}_B = \boldsymbol{v}_A + d\boldsymbol{r}_{B/A}/dt$. Now $d\boldsymbol{r}_{B/A}/dt = 0$ since $\boldsymbol{r}_{B/A}$ is constant. So, $\boldsymbol{v}_B = \boldsymbol{v}_A$, and by following similar logic, $\mathbf{a}_B = \mathbf{a}_A$.

Note, all points in a rigid body subjected to translation move with the same velocity and acceleration.

Type 2 : ROTATION ABOUT A FIXED AXIS



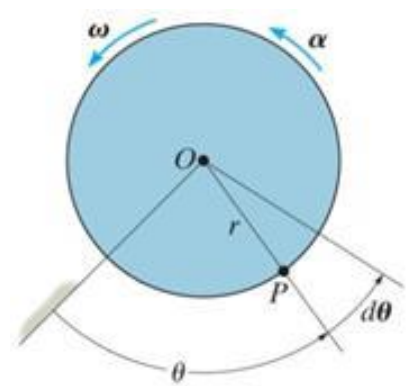
When a body rotates about a fixed axis, any point P in the body travels along a circular path. The angular position of P is defined by θ.

The change in angular position, dθ, is called the angular displacement, with units of either radians or revolutions. They are related by

1 revolution = (2π) radians

Angular velocity, ω, is obtained by taking the time derivative of angular displacement:

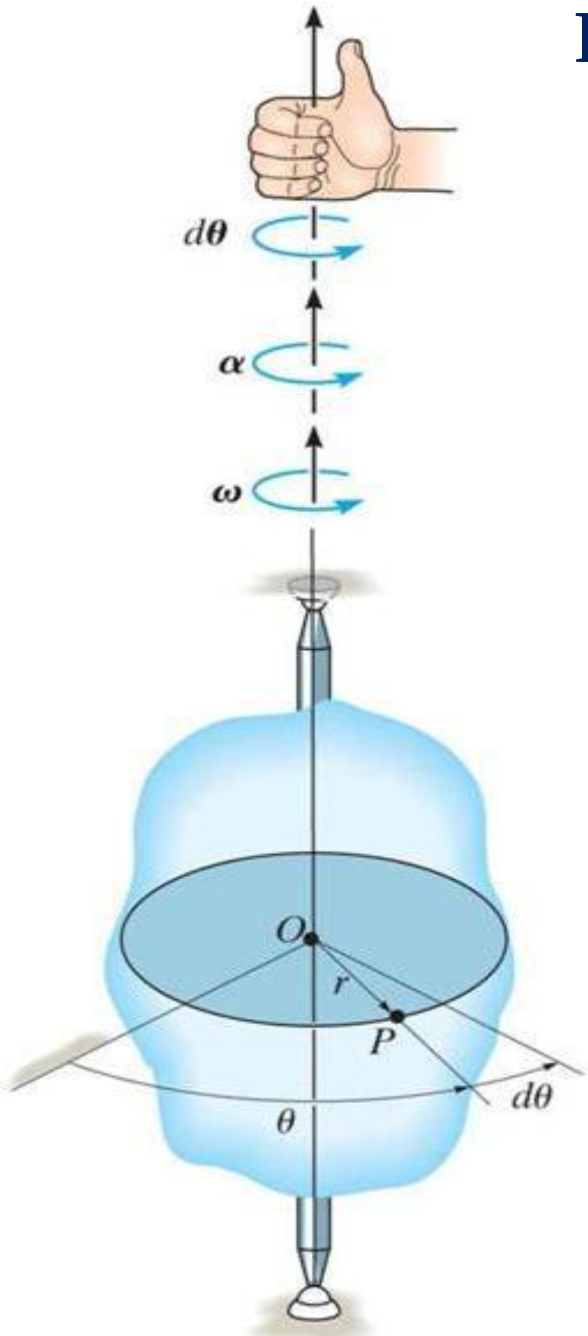
$$\omega = d\theta/dt \quad \text{(rad/s)} \quad +$$



Similarly, angular acceleration is

$$\alpha = d^2\theta/dt^2 = d\omega/dt \;\text{ or }\; \alpha = \omega(d\omega/d\theta) \; + \; \text{rad/s}^2$$

ROTATION ABOUT A FIXED AXIS



If the angular acceleration of the body is constant, $\alpha = \alpha_C$, the equations for angular velocity and acceleration can be integrated to yield the set of algebraic equations below.

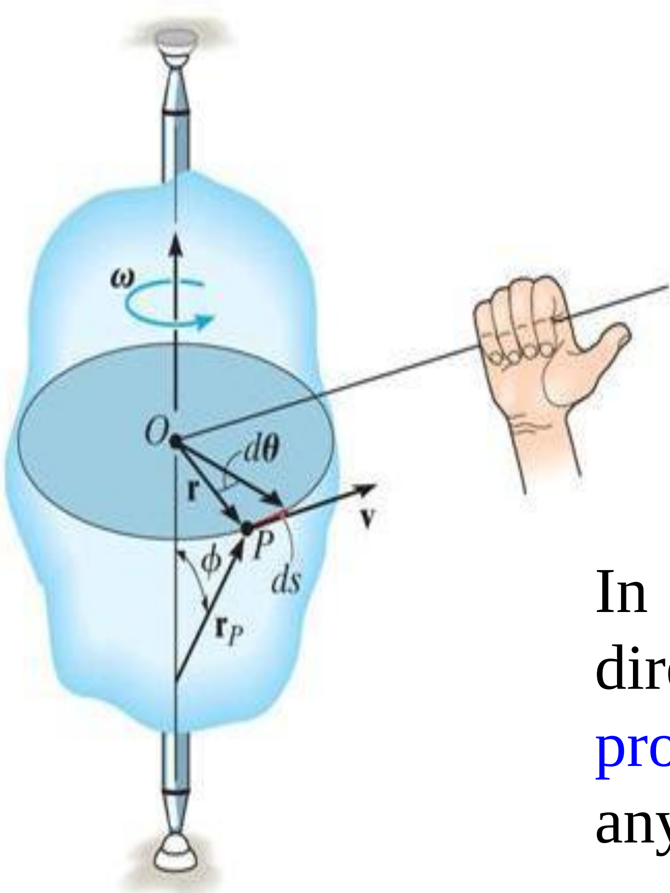
$$\omega = \omega_0 + \alpha_C\, t$$

$$\theta = \theta_0 + \omega_0\, t + 0.5\, \alpha_C\, t^2$$

$$\omega^2 = (\omega_0)^2 + 2\alpha_C\, (\theta - \theta_0)$$

θ_0 and ω_0 are the initial values of the body's angular position and angular velocity. Note these equations are very similar to the constant acceleration relations developed for the rectilinear motion of a particle.

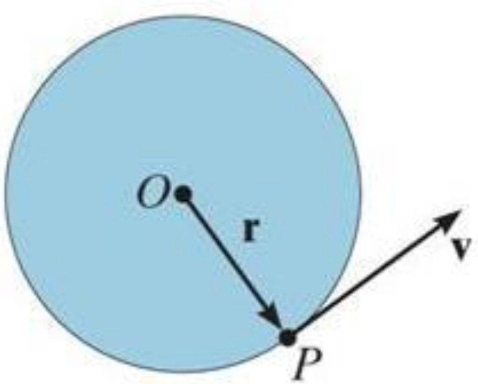
VELOCITY OF POINT P



The magnitude of the velocity of P is equal to ωr (the text provides the derivation). The velocity's direction is tangent to the circular path of P.

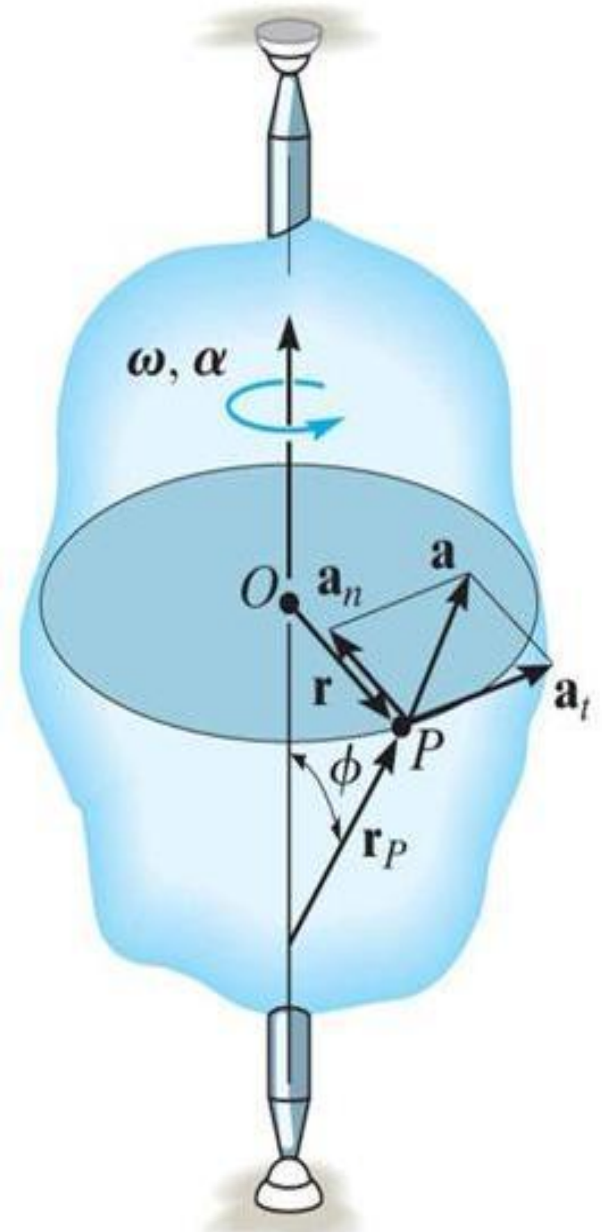
In the vector formulation, the magnitude and direction of $\boldsymbol{v}$ can be determined from the cross product of $\boldsymbol{\omega}$ and $\boldsymbol{r}_{\mathrm{p}}$. Here $\boldsymbol{r}_{\mathrm{p}}$ is a vector from any point on the axis of rotation to P.

$$\boldsymbol{v} = \boldsymbol{\omega} \times \boldsymbol{r}_{\mathrm{p}} = \boldsymbol{\omega} \times \boldsymbol{r}$$



The direction of $\boldsymbol{v}$ is determined by the right-hand rule.

ACCELERATION OF POINT P

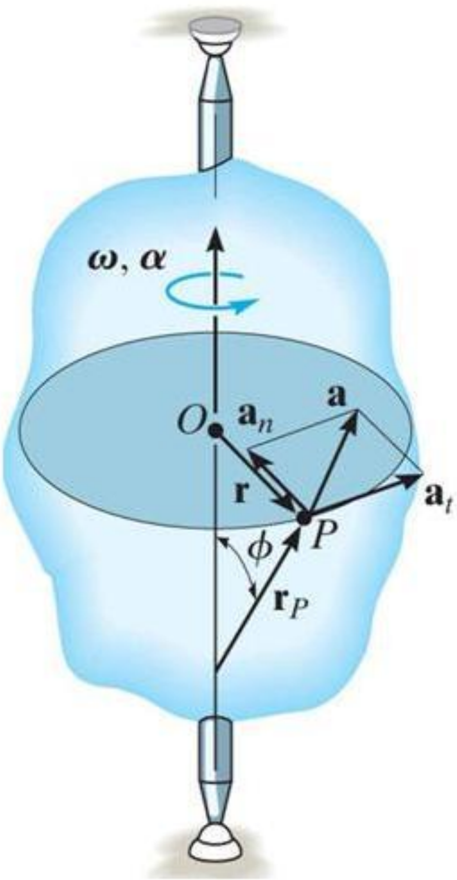


The acceleration of P is expressed in terms of its normal (**$\mathbf{a}_n$**) and tangential (**$\mathbf{a}_t$**) components. In scalar form, these are $a_t = \alpha\, r$ and $a_n = \omega^2 r$.

The tangential component, **$\mathbf{a}_t$**, represents the time rate of change in the velocity's magnitude. It is directed tangent to the path of motion.

The normal component, **$\mathbf{a}_n$**, represents the time rate of change in the velocity's direction. It is directed toward the center of the circular path.

RIGID-BODY ROTATION: ACCELERATION OF POINT P



Using the vector formulation, the acceleration of P can also be defined by differentiating the velocity.

$$\mathbf{a} = d\boldsymbol{v}/dt = d\boldsymbol{\omega}/dt \times \boldsymbol{r}_P + \boldsymbol{\omega} \times d\boldsymbol{r}_P/dt$$

$$= \boldsymbol{\alpha} \times \boldsymbol{r}_P + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \boldsymbol{r}_P)$$

It can be shown that this equation reduces to

$$\mathbf{a} = \boldsymbol{\alpha} \times \boldsymbol{r} - \omega^2 \boldsymbol{r} = \mathbf{a}_t + \mathbf{a}_n$$

The magnitude of the acceleration vector is $a = \sqrt{(a_t)^2 + (a_n)^2}$

ROTATION ABOUT A FIXED AXIS: PROCEDURE

- Establish a sign convention along the axis of rotation.
- If a relationship is known between any two of the variables (α, ω, θ, or t), the other variables can be determined from the equations:

$$\omega = d\theta/dt$$
$$\alpha = d\omega/dt$$
$$\alpha\, d\theta = \omega\, d\omega$$

- If α is constant, use the equations for constant angular acceleration.

$$\omega = \omega_0 + \alpha_C t$$
$$(\theta - \theta_0) = \omega_0 t + 0.5\ \alpha_C t^2$$
$$\omega^2 = \omega_0^2 + 2\alpha_C (\theta - \theta_0)$$

- To determine the motion of a point, the scalar equations $v = \omega r$, $a_t = \alpha r$, $a_n = \omega^2 r$, and $a = \sqrt{(a_t)^2 + (a_n)^2}$ can be used.
- Alternatively, the vector form of the equations can be used (with ***i***, ***j***, ***k*** components).

$$\boldsymbol{v} = \boldsymbol{\omega} \times \boldsymbol{r}_P = \boldsymbol{\omega} \times \boldsymbol{r}$$
$$\mathbf{a} = \mathbf{a}_t + \mathbf{a}_n = \boldsymbol{\alpha} \times \boldsymbol{r} - \omega^2 \boldsymbol{r}$$

EXAMPLE

Given: The motor gives the blade an angular acceleration $\alpha = 20\ e^{-0.6t}$ rad/s^2, where t is in seconds. The initial conditions are that when t = 0, the blade is at rest.

Find: The velocity and acceleration of the tip P of one of the blades when t =3 s. How many revolutions has the blade turned in 3 s ?

Plan:
1) Determine the angular velocity and displacement of the blade using kinematics of angular motion.
2) The magnitudes of the velocity and acceleration of point P can be determined from the scalar equations of motion for a point on a rotating body. Why scalar?

EXAMPLE (continued)

Solution:

1) Since the angular acceleration is given as a function of time, $\alpha = 20\ e^{-0.6t}$ rad/s^2, the angular velocity and displacement can be found by integration.

$$\omega = \int \alpha\ dt = 20 \int e^{-0.6t}\ dt$$

$$\omega = \frac{20}{(-0.6)}\ e^{-0.6t}$$

$\Rightarrow$ **when t = 3 s,**
ω = -5.510 rad/s

Angular displacement

$$\theta = \int \omega\ dt$$

$$\theta = \frac{20}{(-0.6)} \int e^{-0.6t}\ dt = \frac{20}{(-0.6)^2}\ e^{-0.6t}$$

$\Rightarrow$ **when t = 3 s,**
θ = 9.183 rad
= 1.46 rev.

Also , when t = 3 s, **$\alpha = 20\ e^{-0.6(3)\ b} = 3.306$ rad/s^2**

EXAMPLE (continued)

2) The velocity of point P on the the fan, at a radius of 1.75 ft, is determined as

$$v_P = \omega\, r = (5.510)(1.75) = 9.64 \text{ ft/s}$$

The normal and tangential components of acceleration of point P are calculated as

$$a_n = (\omega)^2\, r = (5.510)^2\,(1.75) = 53.13 \text{ ft/s}^2$$

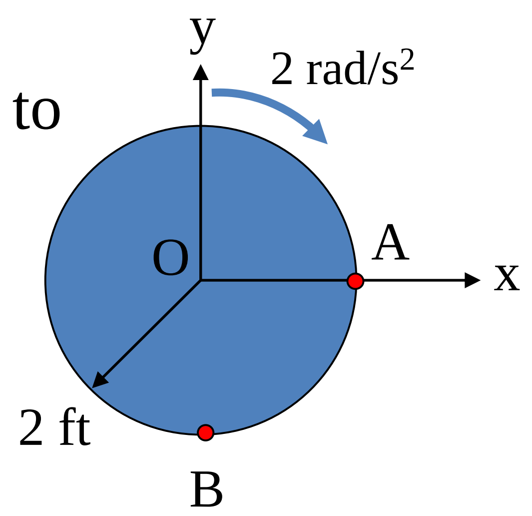
$$a_t = \alpha\, r = (3.306)(1.75) = 5.786 \text{ ft/s}^2$$

The magnitude of the acceleration of P is determined by

$$a_P = \sqrt{(a_n)^2 + (a_t)^2} = \sqrt{(53.13)^2 + (5.786)^2} = 53.4 \text{ ft/s}^2$$

CONCEPT QUIZ

1. A disk is rotating at 4 rad/s. If it is subjected to a constant angular acceleration of 2 rad/s^2, determine the acceleration at B.



A) (4 ***i*** + 32 ***j***) ft/s^2 B) (4 ***i*** - 32 ***j***) ft/s^2

C) (- 4 ***i*** + 32 ***j***) ft/s^2 D) (- 4 ***i*** -32 ***j***) ft/s^2

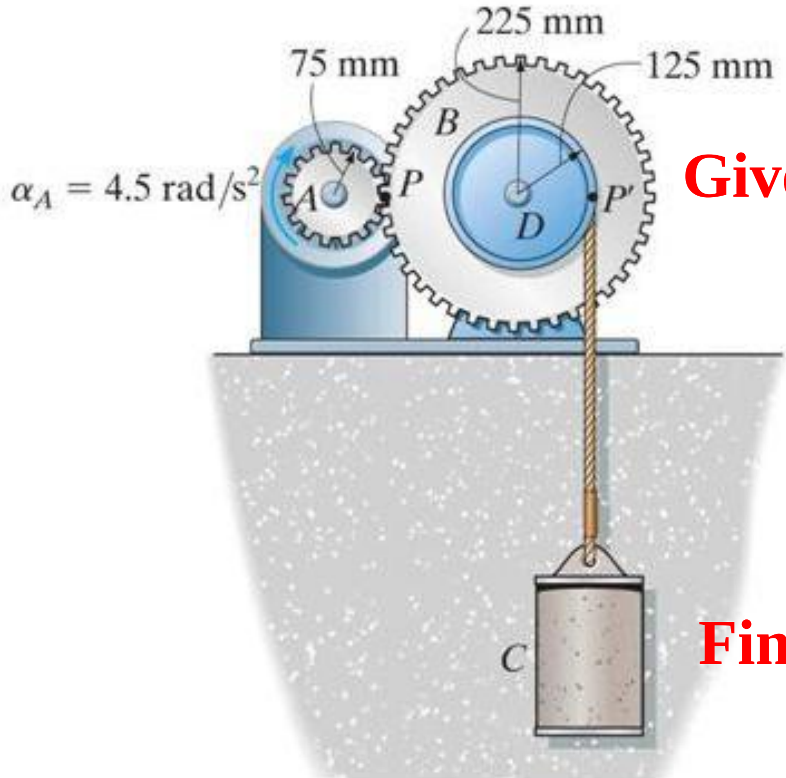
$a_t = \alpha\, r$ $a_n = \omega^2\, r.$

2. A Frisbee (صحن الطائر) is thrown and curves to the right. It is experiencing

A) rectilinear translation. B) curvilinear translation.

C) pure rotation. D) general plane motion.

GROUP PROBLEM SOLVING



Given: Starting from rest when gear A is given a constant angular acceleration, $\alpha_A = 4.5$ rad/s^2. The cord is wrapped around pulley D which is rigidly attached to gear B.

Find: The velocity of cylinder C and the distance it travels in 3 seconds.

Plan:
1) The angular acceleration of gear B (and pulley D) can be related to α_A.
2) The acceleration of cylinder C can be determined by using the equations for motion of a point on a rotating body since $(a_t)_D$ at point P is the same as a_c.
3) The velocity and distance of C can be found by using the constant acceleration equations.

GROUP PROBLEM SOLVING (continued)

Solution:

1) Gear A and B will have the **same** speed and tangential component of acceleration at the point where they mesh. Thus,

$$a_t = \alpha_A r_A = \alpha_B r_B \quad \Rightarrow \quad (4.5)(75) = \alpha_B(225) \;\Rightarrow\; \alpha_B = 1.5 \text{ rad/s}^2$$

Since gear B and pulley D turn together, $\alpha_D = \alpha_B = 1.5$ rad/s^2

2) Assuming the cord attached to pulley D is inextensible and does not slip, the velocity and acceleration of cylinder C will be the same as the velocity and tangential component of acceleration along the pulley D:

$$a_C = (a_t)_D = \alpha_D\, r_D = (1.5)(0.125) = 0.1875 \text{ m/s}^2 \uparrow$$

GROUP PROBLEM SOLVING (continued)

3) Since α_A is constant, α_D and a_C will be constant. The constant acceleration equation for rectilinear motion can be used to determine the velocity and displacement of cylinder C when t = 3 s ($s_0 = v_0 = 0$):

$$v_c = v_0 + a_C\, t = 0 + 0.1875\,(3) = 0.563 \text{ m/s} \uparrow$$

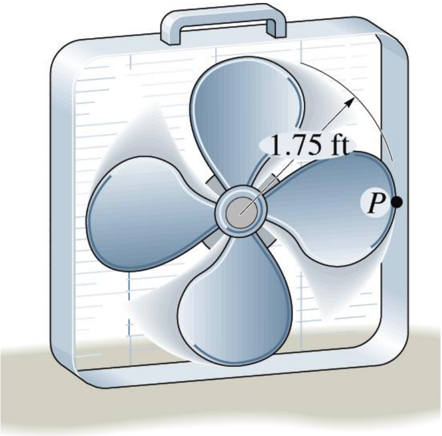
$$s_c = s_0 + v_0\, t + (0.5)\, a_C\, t^2$$
$$= 0 + 0 + (0.5)\, 0.1875\,(3)^2 = 0.844 \text{ m} \uparrow$$

ATTENTION QUIZ

1. The fan blades suddenly experience an angular acceleration of 2 rad/s^2. If the blades are rotating with an initial angular velocity of 4 rad/s, determine the speed of point P when the blades have turned 2 revolutions (when ω = 8.14 rad/s).

   A) 14.2 ft/s   B) 17.7 ft/s

   C) 23.1 ft/s   D) 26.7 ft/s



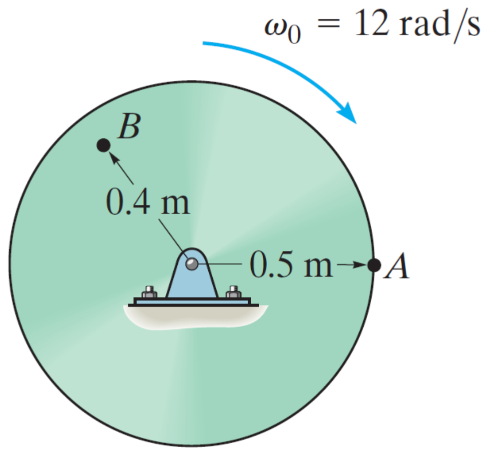
2. Determine the magnitude of the acceleration at P when the blades have turned the 2 revolutions.

   A) 0 ft/s^2 B) 3.5 ft/s^2

   C) 115.95 ft/s^2 D) 116 ft/s^2

EXAMPLE

The disk is originally rotating at $\omega_0 = 12\ \text{rad/s}$. If it is subjected to a constant angular acceleration of $\alpha = 20\ \text{rad/s}^2$, determine the magnitudes of the velocity and the n and t components of acceleration of point B when the disk undergoes 2 revolutions.



SOLUTION

Angular Motion. The angular velocity of the disk can be determined using

$$\omega^2 = \omega_0^2 + 2\alpha_c(\theta - \theta_0); \qquad \omega^2 = 12^2 + 2(20)[2(2\pi) - 0]$$

$$\omega = 25.43\ \text{rad/s}$$

Motion of Point B. The magnitude of the velocity is

$$v_B = \omega r_B = 25.43(0.4) = 10.17\ \text{m/s} = 10.2\ \text{m/s} \qquad \textbf{Ans.}$$

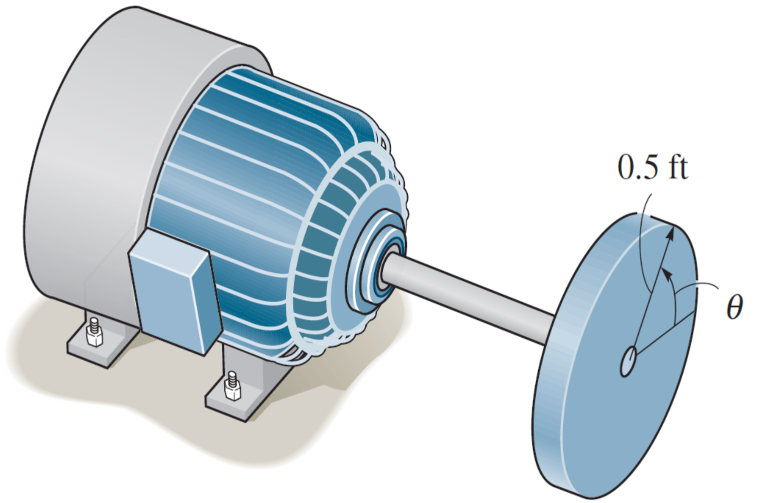
The tangential and normal components of acceleration are

$$(a_B)_t = \alpha r_B = 20(0.4) = 8.00\ \text{m/s}^2 \qquad \textbf{Ans.}$$

$$(a_B)_n = \omega^2 r_B = (25.43^2)(0.4) = 258.66\ \text{m/s}^2 = 259\ \text{m/s}^2 \qquad \textbf{Ans.}$$

EXAMPLE

The disk is driven by a motor such that the angular position of the disk is defined by $\theta = (20t + 4t^2)$ rad, where t is in seconds. Determine the number of revolutions, the angular velocity, and angular acceleration of the disk when $t = 90$ s.



SOLUTION

Angular Displacement: At $t = 90$ s.

$$\theta = 20(90) + 4\left(90^2\right) = (34200 \text{ rad}) \times \left(\frac{1 \text{ rev}}{2\pi \text{ rad}}\right) = 5443 \text{ rev} \qquad \textbf{Ans.}$$

Angular Velocity: Applying Eq. 16–1. we have

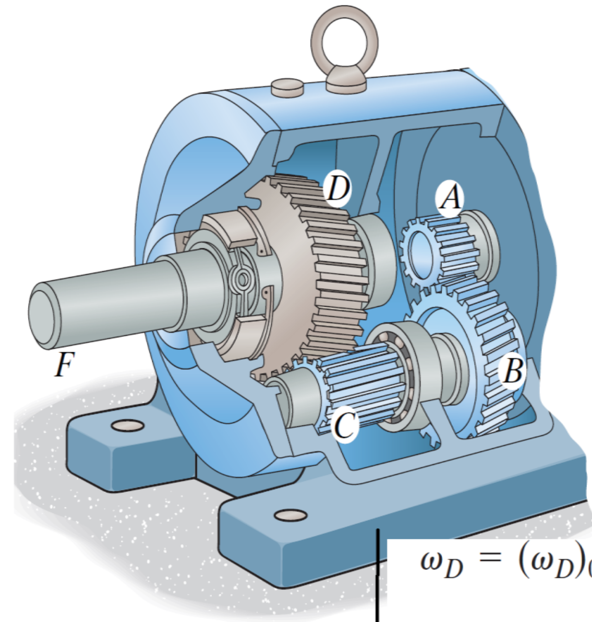
$$\omega = \frac{d\theta}{dt} = 20 + 8t\bigg|_{t=90\ s} = 740 \text{ rad/s} \qquad \textbf{Ans.}$$

Angular Acceleration: Applying Eq. 16–2. we have

$$\alpha = \frac{d\omega}{dt} = 8 \text{ rad/s}^2 \qquad \textbf{Ans.}$$

EXAMPLE

If gear A rotates with a constant angular acceleration of $\alpha_A = 90 \text{ rad/s}^2$, starting from rest, determine the time required for gear D to attain an angular velocity of 600 rpm. Also, find the number of revolutions of gear D to attain this angular velocity. Gears A, B, C, and D have radii of 15 mm, 50 mm, 25 mm, and 75 mm, respectively.



SOLUTION

Gear B is in mesh with gear A. Thus,

$$\alpha_B r_B = \alpha_A r_A$$

$$\alpha_B = \left(\frac{r_A}{r_B}\right)\alpha_A = \left(\frac{15}{50}\right)(90) = 27 \text{ rad/s}^2$$

Since gears C and B share the same shaft, $\alpha_C = \alpha_B = 27 \text{ rad/s}^2$. Also, gear D is in mesh with gear C. Thus,

$$\alpha_D r_D = \alpha_C r_C$$

$$\alpha_D = \left(\frac{r_C}{r_D}\right)\alpha_C = \left(\frac{25}{75}\right)(27) = 9 \text{ rad/s}^2$$

The final angular velocity of gear D is $\omega_D = \left(\frac{600 \text{ rev}}{\text{min}}\right)\left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right) =$ 20π rad/s. Applying the constant acceleration equation,

$$\omega_D = (\omega_D)_0 + \alpha_D t$$

$$20\pi = 0 + 9t$$

$$t = 6.98 \text{ s}$$

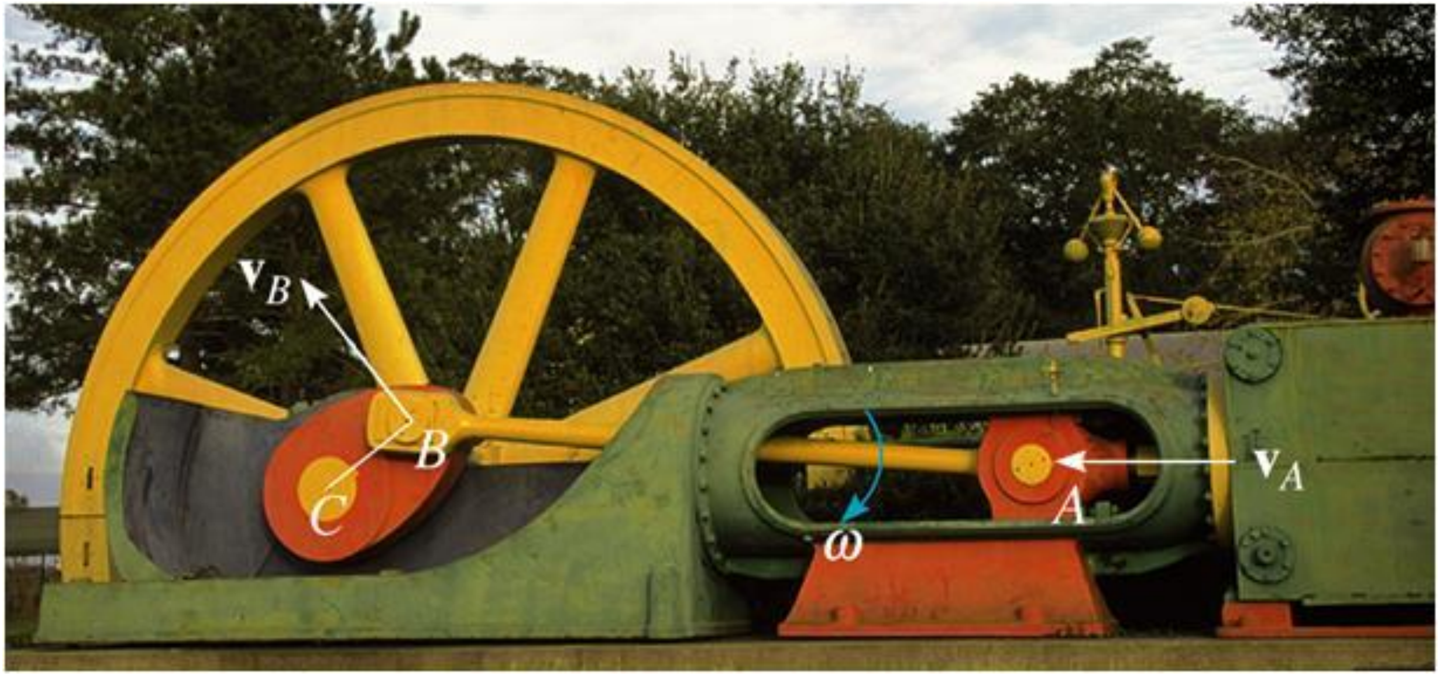
$$\omega_D^2 = (\omega_D)_0^2 + 2\alpha_D[\theta_D - (\theta_D)_0]$$

$$(20\pi)^2 = 0^2 + 2(9)(\theta_D - 0)$$

$$\theta_D = (219.32 \text{ rad})\left(\frac{1 \text{ rev}}{2\pi \text{ rad}}\right)$$

$$= 34.9 \text{ rev}$$

Type 3 : *RELATIVE MOTION ANALYSIS: VELOCITY*



Today's Objectives:

Students will be able to:

1. *Describe the velocity of a rigid body in terms of translation and rotation components.*
2. *Perform a relative-motion velocity analysis of a point on the body.*

READING QUIZ

1. When a relative-motion analysis involving two sets of coordinate axes is used, the x' - y' coordinate system will

   A) be attached to the selected point for analysis.

   B) rotate with the body.

   C) not be allowed to translate with respect to the fixed frame.

   D) None of the above.

2. In the relative velocity equation, $\mathbf{v}_{B/A}$ is

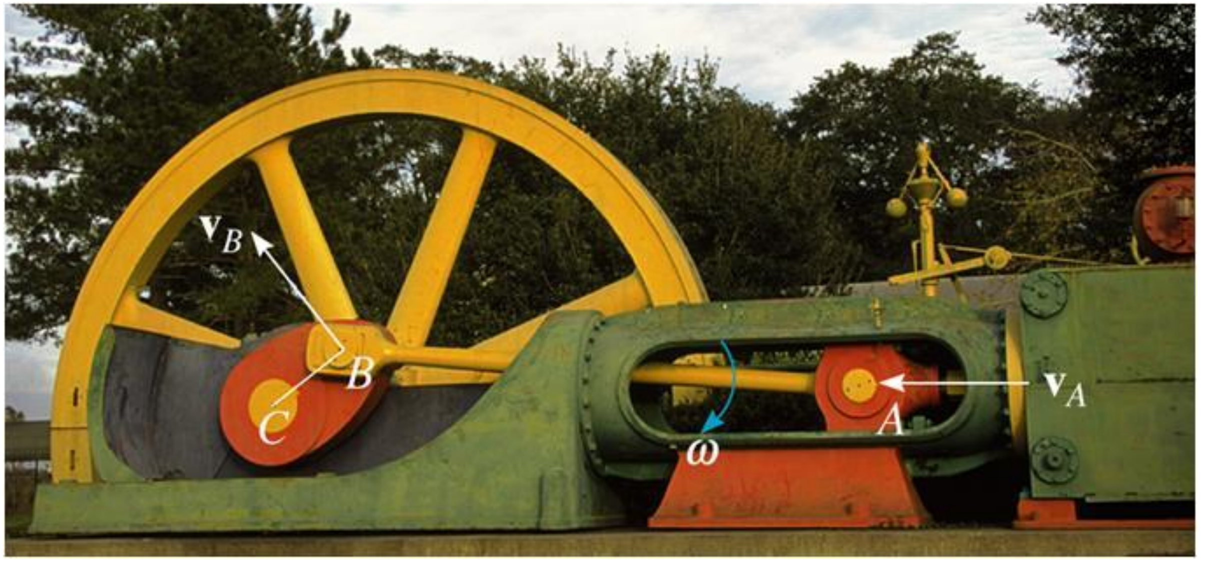
   A) the relative velocity of B with respect to A.

   B) due to the rotational motion.

   C) $\boldsymbol{\omega} \times \mathbf{r}_{B/A}$.

   D) All of the above.

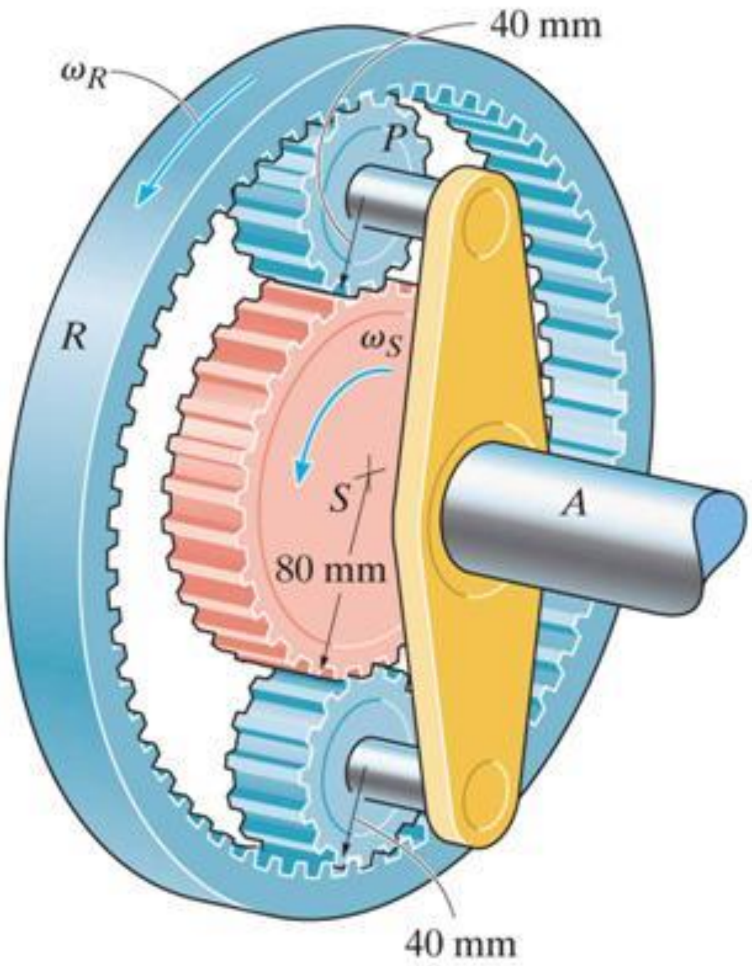
APPLICATIONS



As the slider block A moves horizontally to the left with $\mathbf{v}_A$*, it causes the link CB to rotate counterclockwise. Thus* $\mathbf{v}_B$ *is directed tangent to its circular path.*

Which link is undergoing general plane motion? Link AB or link BC?

How can the angular velocity, ω *of link AB be found?*



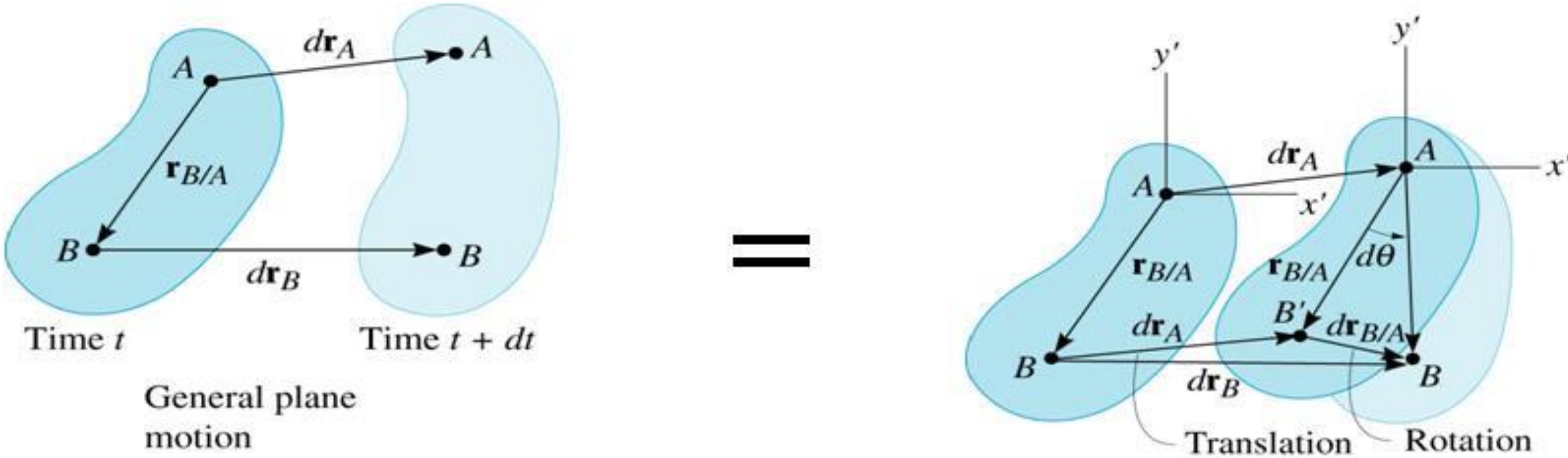
APPLICATIONS
(continued)

Planetary gear systems are used in many automobile automatic transmissions. By locking or releasing different gears, this system can operate the car at different speeds.

How can we relate the angular velocities of the various gears in the system?

RELATIVE MOTION ANALYSIS (Section 16.5)

When a body is subjected to general plane motion, it undergoes a combination of translation and rotation.



Point A is called the base point in this analysis. It generally has a known motion. The x'- y' frame translates with the body, but does not rotate. The displacement of point B can be written:

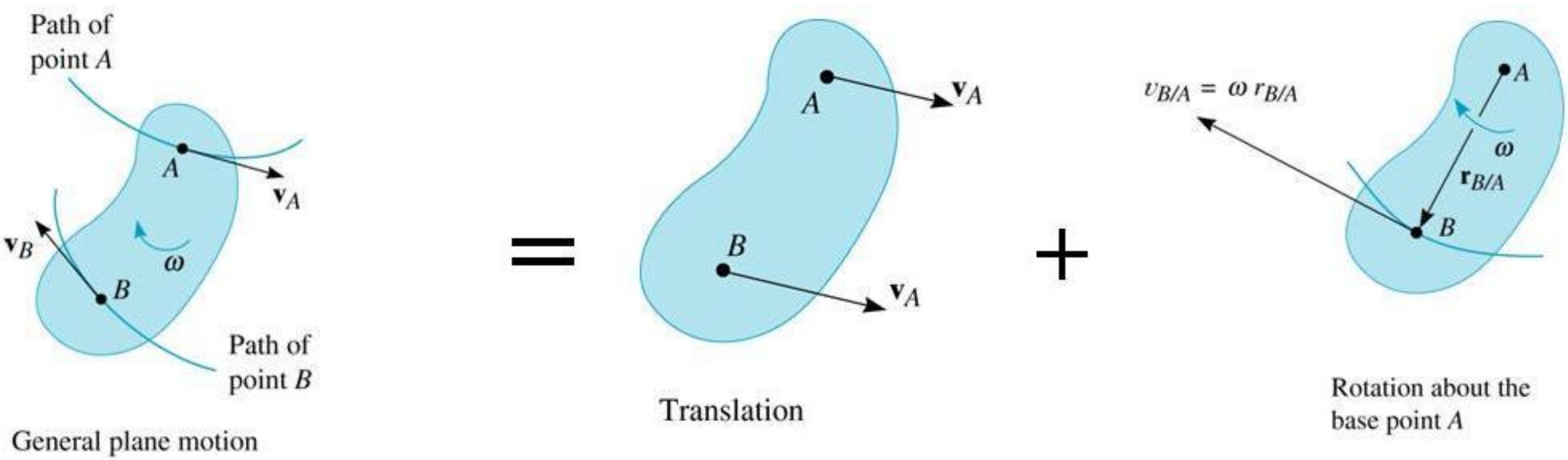
Disp. due to translation

$$d\mathbf{r}_B = d\mathbf{r}_A + d\mathbf{r}_{B/A}$$

Disp. due to translation and rotation

Disp. due to rotation

RELATIVE MOTION ANALYSIS: VELOCITY



The velocity at B is given as : $(d\mathbf{r}_B/dt) = (d\mathbf{r}_A/dt) + (d\mathbf{r}_{B/A}/dt)$ *or*

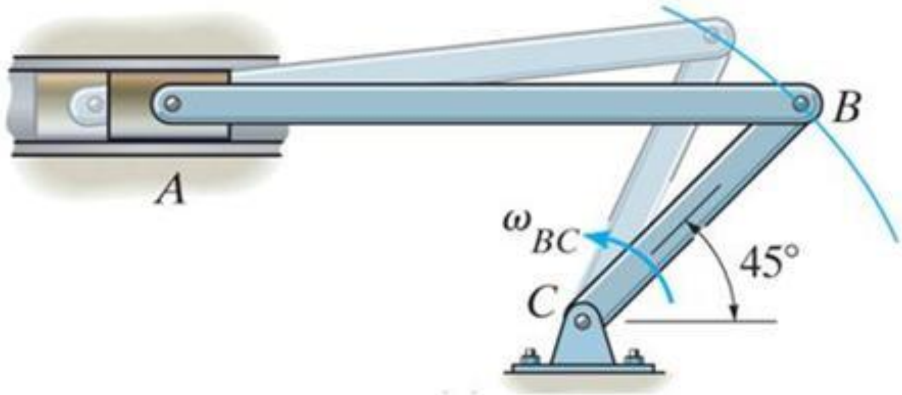
$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$$

Since the body is taken as rotating about A,

$$\mathbf{v}_{B/A} = d\mathbf{r}_{B/A}/dt = \boldsymbol{\omega} \times \mathbf{r}_{B/A}$$

Here $\boldsymbol{\omega}$ *will only have a* $\mathbf{k}$ *component since the axis of rotation is perpendicular to the plane of translation.*

RELATIVE MOTION ANALYSIS: VELOCITY



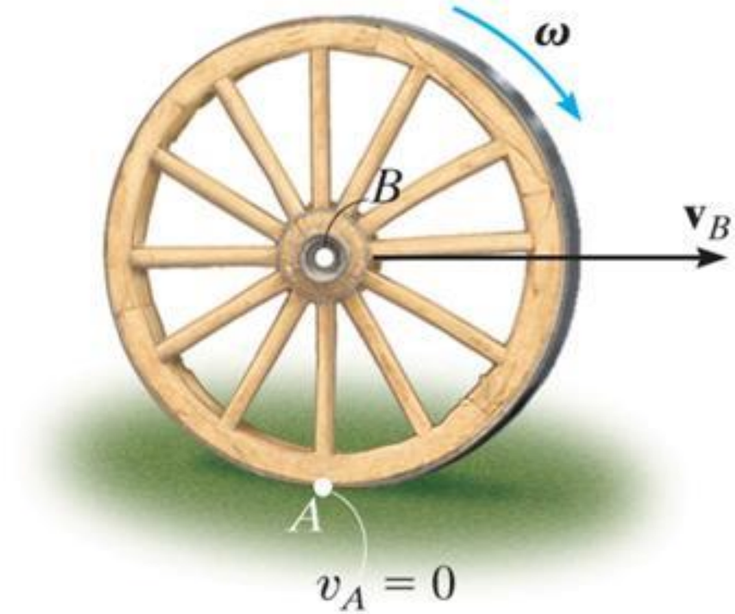
$$\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\omega} \times \mathbf{r}_{B/A}$$

When using the relative velocity equation, points A and B should generally be points on the body with a known motion. Often these points are pin connections in linkages.

For example, point A on link AB must move along a horizontal path, whereas point B moves on a circular path.

The directions of v_A and v_B are known since they are always tangent to their paths of motion.

RELATIVE MOTION ANALYSIS: VELOCITY



$$\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\omega} \times \mathbf{r}_{B/A}$$

When a wheel rolls without slipping, point A is often selected to be at the point of contact with the ground.

Since there is no slipping, point A has zero velocity.

Furthermore, point B at the center of the wheel moves along a horizontal path. Thus, $\mathbf{v}_B$ has a known direction, e.g., parallel to the surface.

PROCEDURE FOR ANALYSIS

The relative velocity equation can be applied using either a Cartesian vector analysis or by writing scalar x and y component equations directly.

Scalar Analysis:

1. Establish the fixed x-y coordinate directions and draw a kinematic diagram for the body. Then establish the magnitude and direction of the relative velocity vector $\mathbf{v}_{B/A}$.
2. Write the equation $\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$. In the kinematic diagram, represent the vectors graphically by showing their magnitudes and directions underneath each term.
3. Write the scalar equations from the x and y components of these graphical representations of the vectors. Solve for the unknowns.

PROCEDURE FOR ANALYSIS (continued)

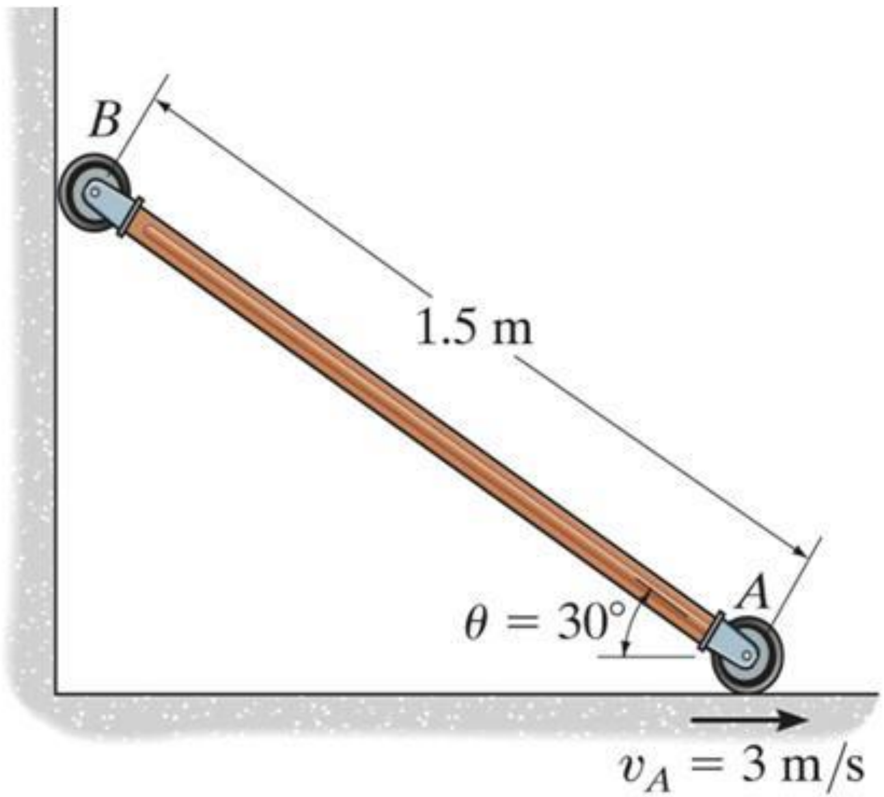
Vector Analysis:

1. *Establish the fixed x - y coordinate directions and draw the kinematic diagram of the body, showing the vectors* $\mathbf{v}_A$, $\mathbf{v}_B$, $\mathbf{r}_{B/A}$ *and* $\boldsymbol{\omega}$. *If the magnitudes are unknown, the sense of direction may be assumed.*

2. *Express the vectors in Cartesian vector form (CVN) and substitute them into* $\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\omega} \times \mathbf{r}_{B/A}$. *Evaluate the cross product and equate respective* $\mathbf{i}$ *and* $\mathbf{j}$ *components to obtain two scalar equations.*

3. *If the solution yields a negative answer, the sense of direction of the vector is opposite to that assumed.*

EXAMPLE I



Given: *Roller A is moving to the right at 3 m/s.*

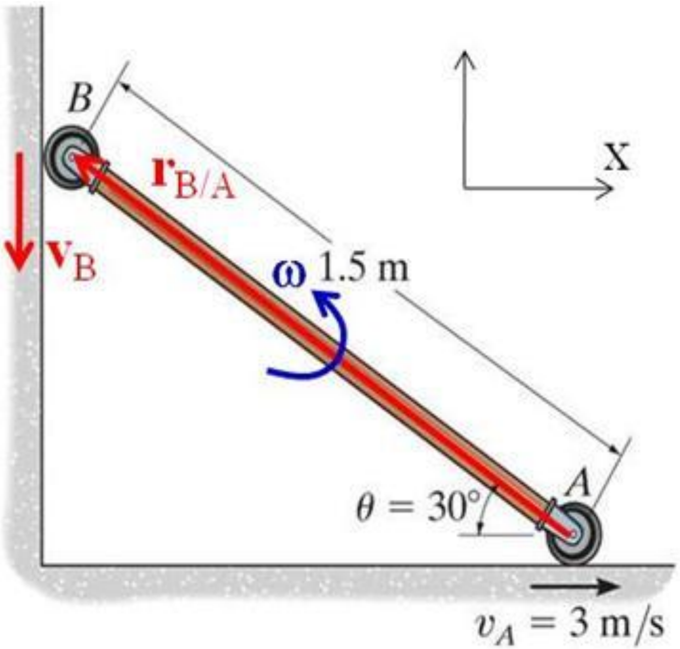
Find: *The velocity of B at the instant $\theta = 30°$.*

Plan:

1. *Establish the fixed x - y directions and draw a kinematic diagram of the bar and rollers.*
2. *Express each of the velocity vectors for A and B in terms of their* ***i***, ***j***, ***k*** *components and solve* $\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\omega} \times \mathbf{r}_{B/A}$.

Solution:

Kinematic diagram:



Express the velocity vectors in CVN

$$\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\omega} \times \mathbf{r}_{B/A}$$

$$-v_B\mathbf{j} = 3\,\mathbf{i} + [\,\omega\mathbf{k} \times (-1.5\cos 30\,\mathbf{i} + 1.5\sin 30\,\mathbf{j}\,)]$$

$$-v_B\mathbf{j} = 3\,\mathbf{i} - 1.299\,\omega\,\mathbf{j} - 0.75\,\omega\,\mathbf{i}$$

*Equating the **i** and **j** components gives:*

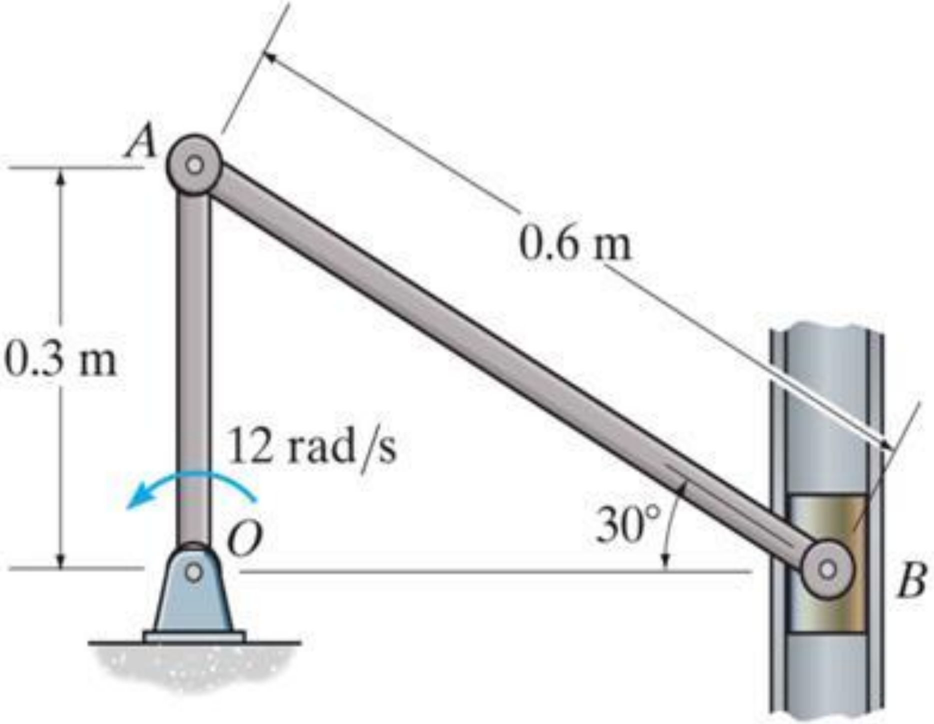
$$0 = 3 - 0.75\,\omega$$

$$-v_B = -\,1.299\,\omega$$

Solving: $\omega = 4$ rad/s or $\boldsymbol{\omega} = 4$ rad/s $\mathbf{k}$

$v_B = 5.2$ m/s or $\mathbf{v}_B = -5.2$ m/s $\mathbf{j}$

EXAMPLE II



Given: *Crank rotates OA with an angular velocity of 12 rad/s.*

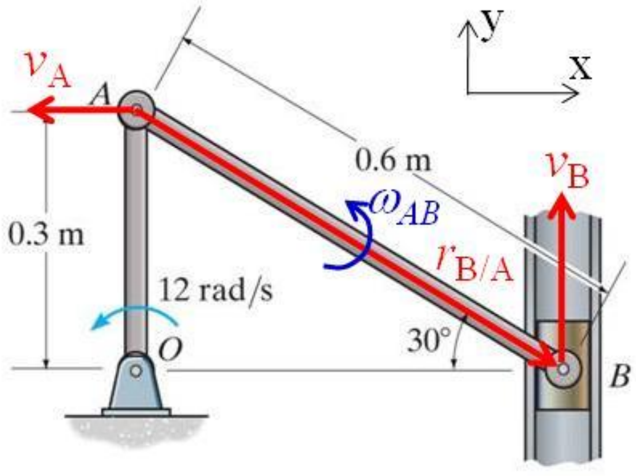
Find: *The velocity of piston B and the angular velocity of rod AB.*

Plan: *Notice that point A moves on a circular path. The directions of v_A is tangent to its path of motion.*
Draw a kinematic diagram of rod AB and use

$$\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A}.$$

Solution:

Kinematic diagram of AB:



Since crack OA rotates with an angular velocity of 12 rad/s, the velocity at A will be: $\mathbf{v}_A = -0.3(12)\,\mathbf{i} = -3.6\,\mathbf{i}$ m/s

Rod AB. Write the relative-velocity equation:

$$\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A}$$

$$v_B\,\mathbf{j} = -3.6\,\mathbf{i} + \omega_{AB}\,\mathbf{k} \times (0.6\cos 30\,\mathbf{i} - 0.6\sin 30\,\mathbf{j})$$

$$v_B\,\mathbf{j} = -3.6\,\mathbf{i} + 0.5196\,\omega_{AB}\,\mathbf{j} + 0.3\,\omega_{AB}\,\mathbf{i}$$

By comparing the **i**, **j** components:

i: $0 = -3.6 + 0.3\,\omega_{AB} \quad \Rightarrow \quad \omega_{AB} = 12$ rad/s

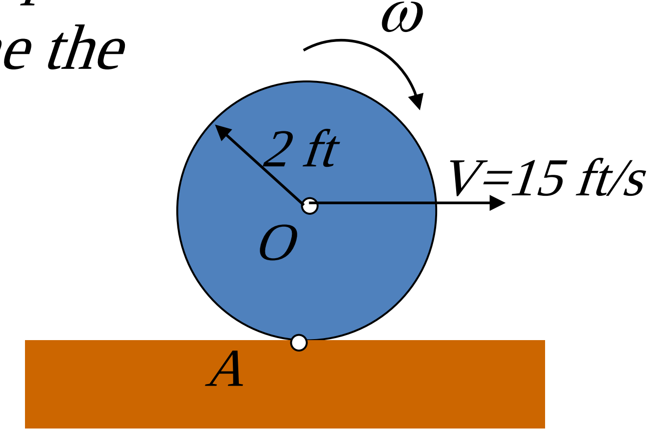
j: $v_B = 0.5196\,\omega_{AB} \quad \Rightarrow \quad v_B = 6.24$ m/s

CHECK YOUR UNDERSTANDING QUIZ

1. *If the disk is moving with a velocity at point O of 15 ft/s and ω = 2 rad/s, determine the velocity at A.*

   A) 0 ft/s B) 4 ft/s

   C) 15 ft/s D) 11 ft/s

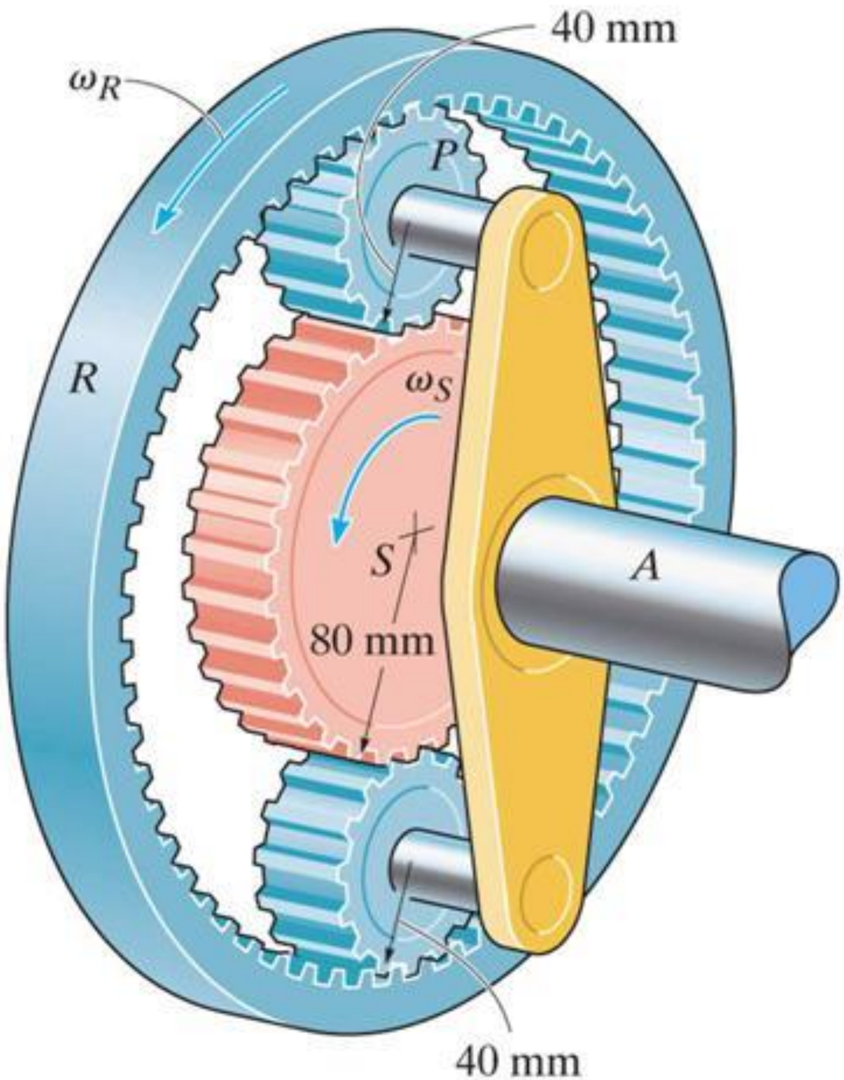


2. *If the velocity at A is zero, then determine the angular velocity, ω.*

   A) 30 rad/s B) 0 rad/s

   C) 7.5 rad/s D) 15 rad/s

GROUP PROBLEM SOLVING



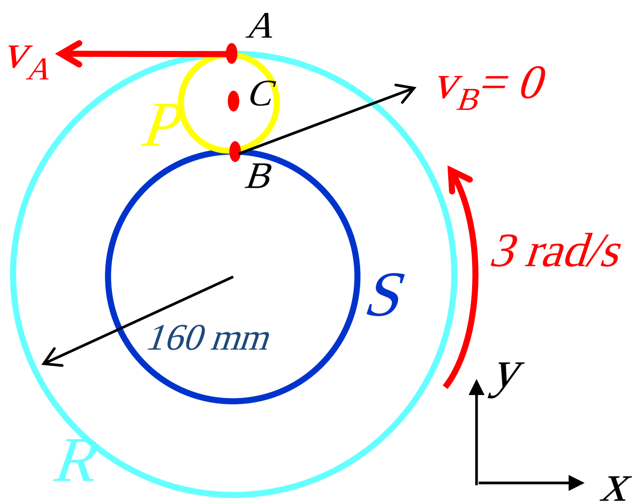
Given: The ring gear R is rotating at $\omega_R = 3$ rad/s, and the sun gear S is held fixed, $\omega_S = 0$.

Find: The angular velocity of the each of the planet gears P and of shaft A.

Plan: Draw the kinematic diagram of gears. Then, apply the relative velocity equations to the gears and solve for unknowns.

Solution:

Kinematic diagram of gears.



Since the ring gear R is rotating at $\omega_R = 3$ rad/s, the velocity at point A will be ;

$$\mathbf{v}_A = -3(160)\,\mathbf{i} = -480\,\mathbf{i} \text{ mm/s}$$

Also note that $\mathbf{v}_B = 0$ since the gear R is held fixed $\omega_S = 0$.

Applying the relative velocity equation to points A and B;

$$\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\omega}_P \times \mathbf{r}_{B/A}$$

$$0 = -480\,\mathbf{i} + (\omega_P\,\mathbf{k}) \times (-80\,\mathbf{j}) \Rightarrow 0 = -480\,\mathbf{i} + 80\,\omega_P\,\mathbf{i}$$

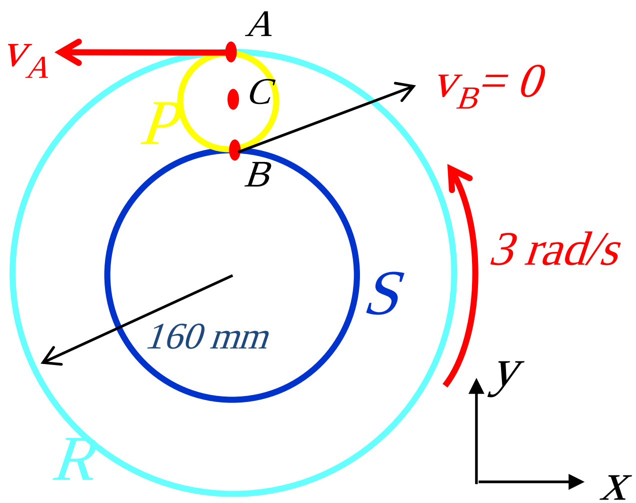
$$\omega_P = 6 \text{ rad/s}$$

Solution:

Apply the relative velocity equation at point B and C to Gear P in order to find the velocity at B.

$$\mathbf{v}_C = \mathbf{v}_B + \boldsymbol{\omega}_P \times \mathbf{r}_{C/B}$$

$$= 0 + (6\,\mathbf{k})\times(40\,\mathbf{j}) = -\,240\,\mathbf{i}\ \text{mm/s}$$



Note that the shaft A has a circular motion with the radius of 120 mm.

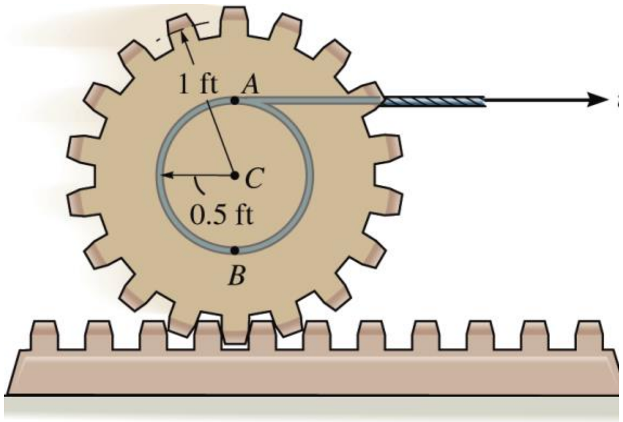
The angular velocity of the shaft is

$$\omega_A = v_C / r$$

$$= -240 / 120 = -2\ \text{rad/s}.$$

The shaft A is rotating in counter-clockwise direction !

ATTENTION QUIZ



1. *Which equation could be used to find the velocity of the center of the gear, C, if the velocity* $\mathbf{v}_A$ *is known?*

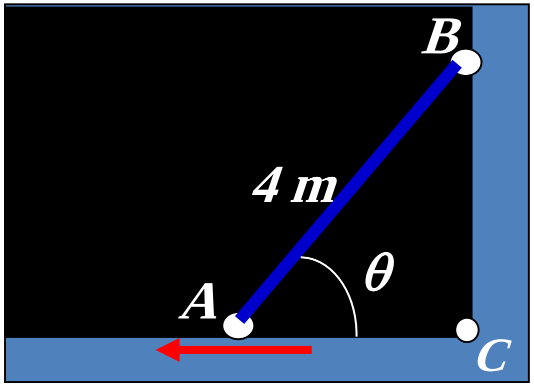
A) $\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\omega}_{gear} \times \mathbf{r}_{B/A}$

B) $\mathbf{v}_A = \mathbf{v}_C + \boldsymbol{\omega}_{gear} \times \mathbf{r}_{A/C}$

C) $\mathbf{v}_B = \mathbf{v}_C + \boldsymbol{\omega}_{gear} \times \mathbf{r}_{C/B}$

D) $\mathbf{v}_A = \mathbf{v}_C + \boldsymbol{\omega}_{gear} \times \mathbf{r}_{C/A}$

2. *If the bar's velocity at A is 3 m/s, what "base" point (first term on the RHS of the velocity equation) would be best used to simplify finding the bar's angular velocity when* $\theta = 60^o$*?*



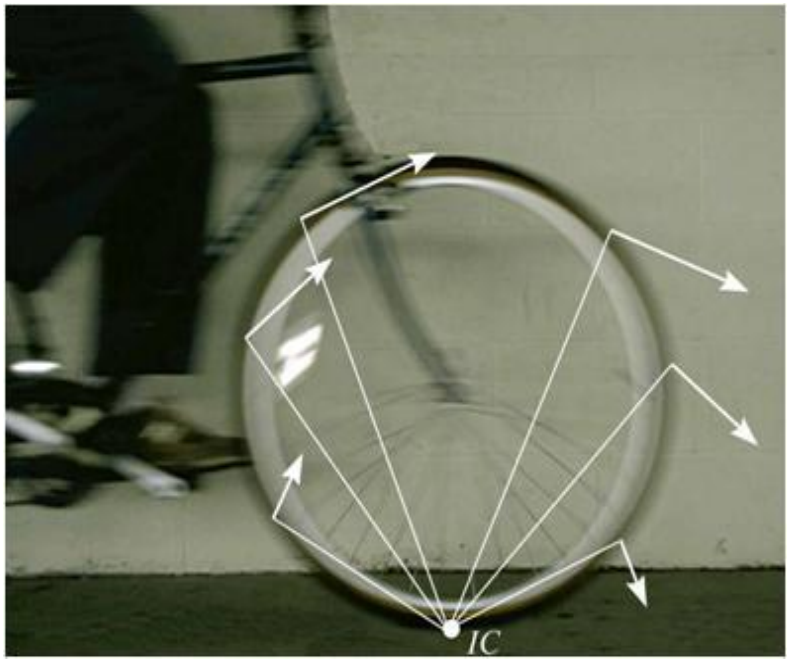
A) A

B) B

C) C

D) No difference.

INSTANTANEOUS CENTER OF ZERO VELOCITY



Objectives:

Students will be able to:

1. **Locate the instantaneous center of zero velocity.**
2. **Use the instantaneous center to determine the velocity of any point on a rigid body in general plane motion.**

READING QUIZ

1. *If applicable, the method of instantaneous center can be used to determine the __________ of any point on a rigid body.*

   A) velocity *B) acceleration*

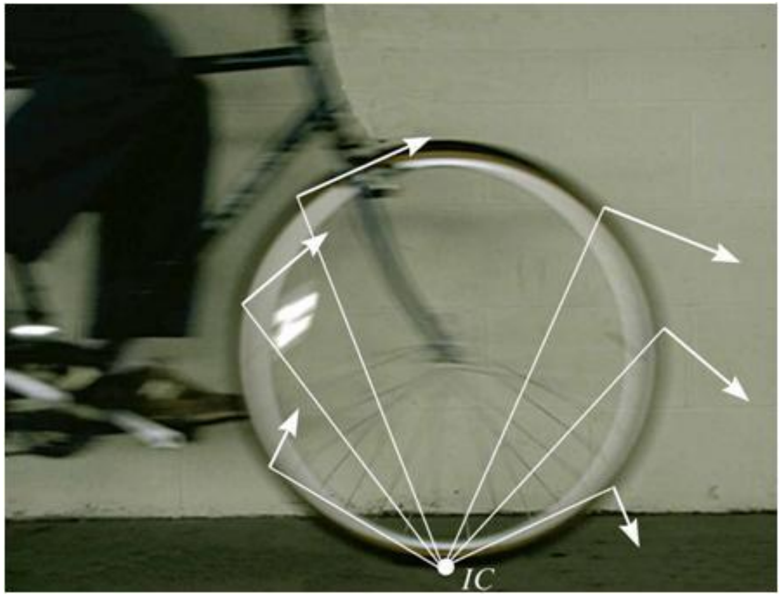
   C) velocity and acceleration *D) force*

2. *The velocity of any point on a rigid body is __________ to the relative position vector extending from the IC to the point.*

   A) always parallel *B) always perpendicular*

   C) in the opposite direction *D) in the same direction*

APPLICATIONS

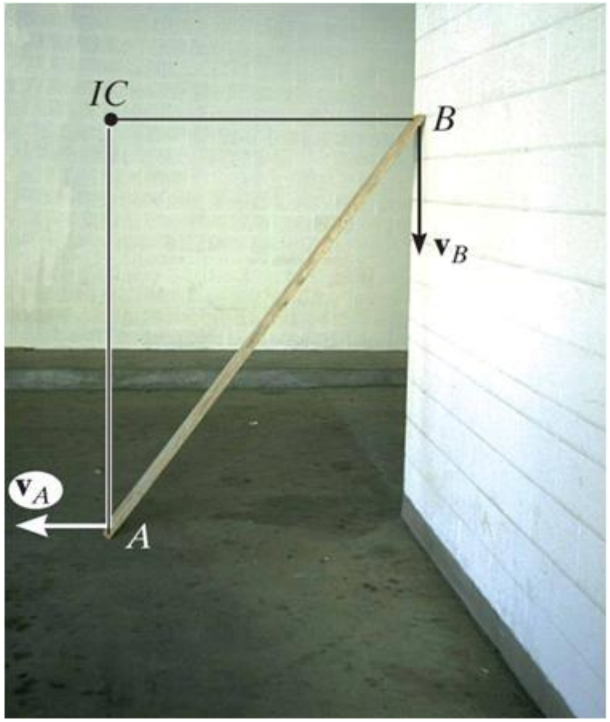


The instantaneous center (IC) of zero velocity for this bicycle wheel is at the point in contact with ground. The velocity direction at any point on the rim is perpendicular to the line connecting the point to the IC.

Which point on the wheel has the maximum velocity?

Does a larger wheel mean the bike will go faster for the same rider effort in pedaling than a smaller wheel?

APPLICATIONS (continued)



As the board slides down the wall (to the left), it is subjected to general plane motion (both translation and rotation).

Since the directions of the velocities of ends A and B are known, the IC is located as shown.

How can this result help you analyze other situations?

What is the direction of the velocity of the center of gravity of the board?

INSTANTANEOUS CENTER OF ZERO VELOCITY (Section 16-6)

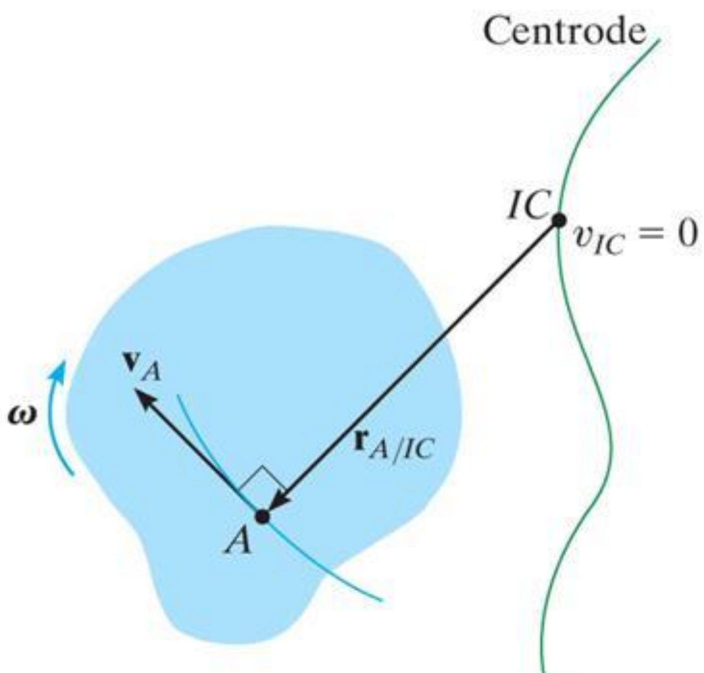
For any body undergoing planar motion, there always exists a point in the plane of motion at which the velocity is instantaneously zero (if it is rigidly connected to the body).

This point is called the instantaneous center (IC) of zero velocity. It may or may not lie on the body!

If the location of this point can be determined, the velocity analysis can be simplified because the body appears to rotate about this point at that instant.

LOCATION OF THE INSTANTANEOUS CENTER

To locate the IC, we can use the fact that the velocity of a point on a body is always perpendicular to the relative position vector from the IC to the point. Several possibilities exist.



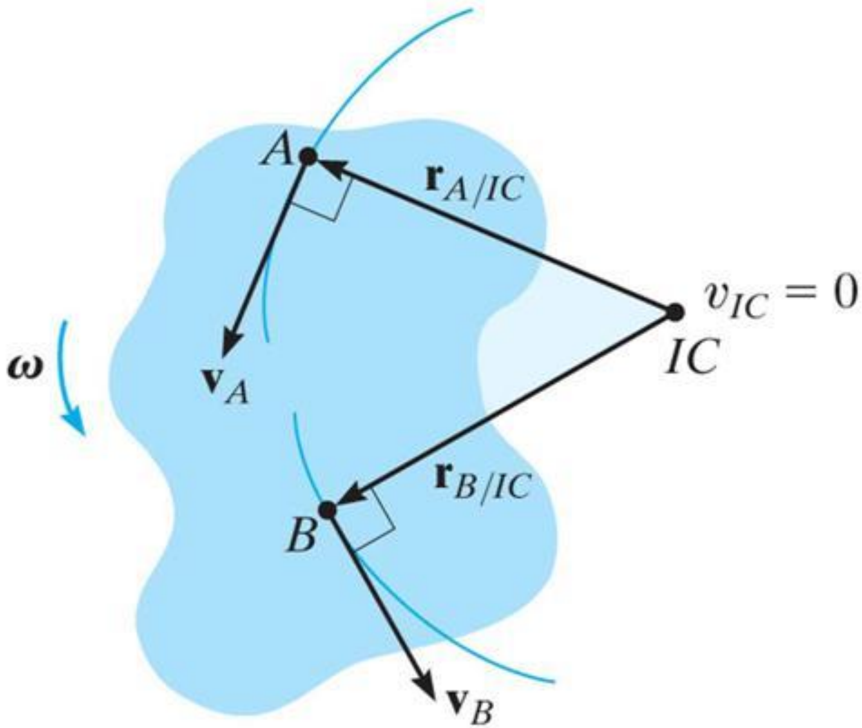
Location of IC knowing $\mathbf{v}_A$ and ω

First, consider the case when velocity v_A of a point A on the body and the angular velocity ω of the body are known.

In this case, the IC is located along the line drawn perpendicular to v_A at A, a distance $r_{A/IC} = v_A/\omega$ from A.

Note that the IC lies up and to the right of A since v_A must cause a clockwise angular velocity ω about the IC.

LOCATION OF THE INSTANTANEOUS CENTER (continued)

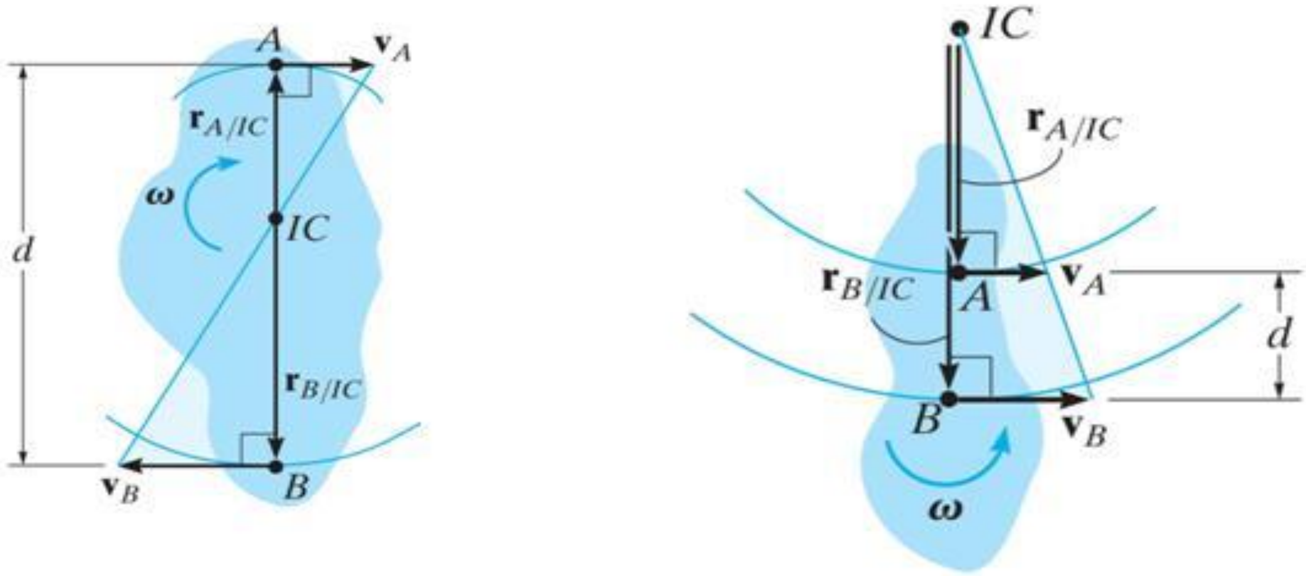


Location of *IC* knowing the directions of $\mathbf{v}_A$ and $\mathbf{v}_B$

A second case is when the lines of action of two non-parallel velocities, v_A and v_B, are known.

First, construct line segments from A and B perpendicular to v_A and v_B. The point of intersection of these two line segments locates the IC of the body.

LOCATION OF THE INSTANTANEOUS CENTER



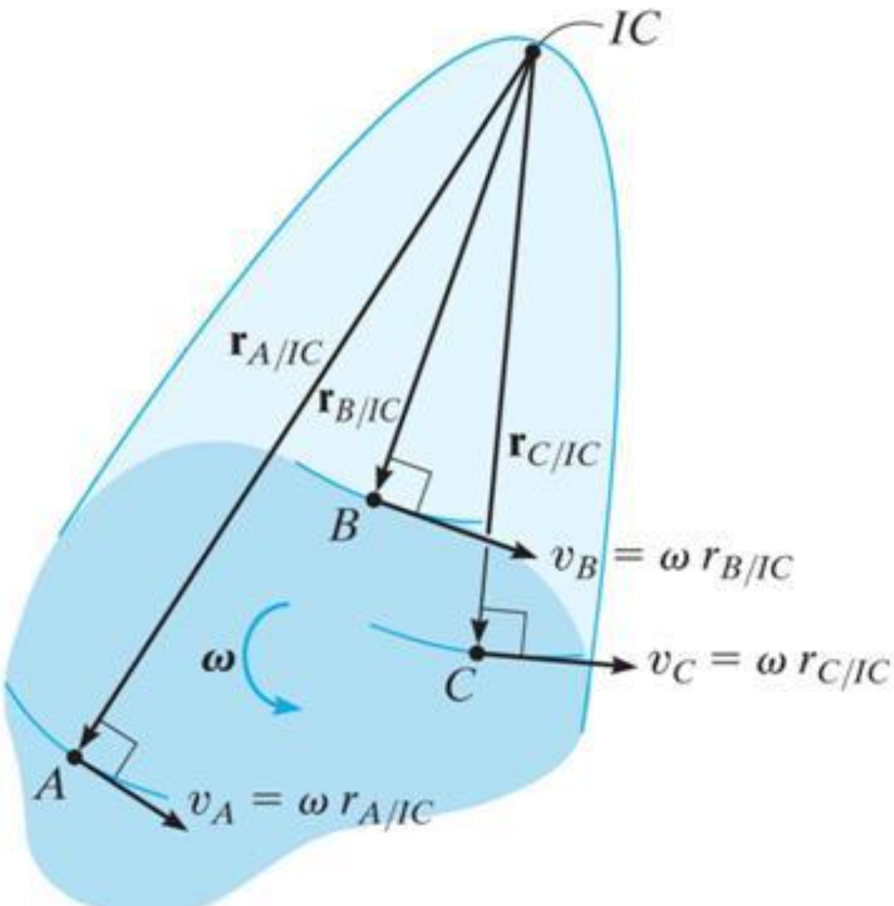
Location of *IC* knowing $\mathbf{v}_A$ and $\mathbf{v}_B$

A third case is when the magnitude and direction of two parallel velocities at A and B are known. Here the location of the IC is determined by proportional triangles.

As a special case, note that if the body is translating only ($v_A = v_B$), then the IC would be located at infinity. Then ω equals zero, as expected.

VELOCITY ANALYSIS

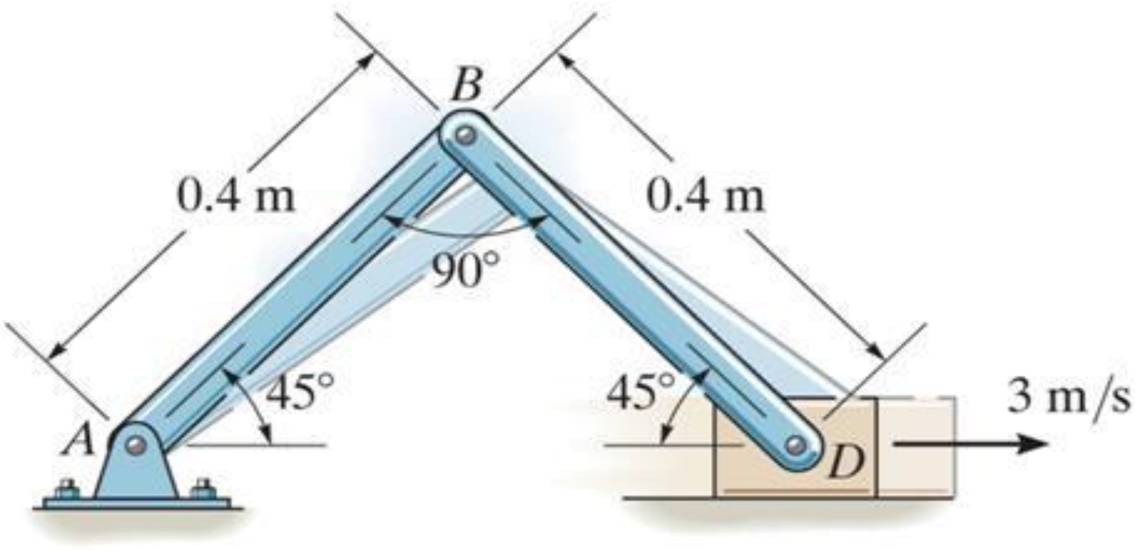
The velocity of any point on a body undergoing general plane motion can be determined easily once the instantaneous center of zero velocity of the body is located.



Since the body seems to rotate about the IC at any instant, as shown in this kinematic diagram, the magnitude of velocity of any arbitrary point is $v = \omega\, r$, where r is the radial distance from the IC to the point.

The velocity's line of action is perpendicular to its associated radial line.

EXAMPLE I



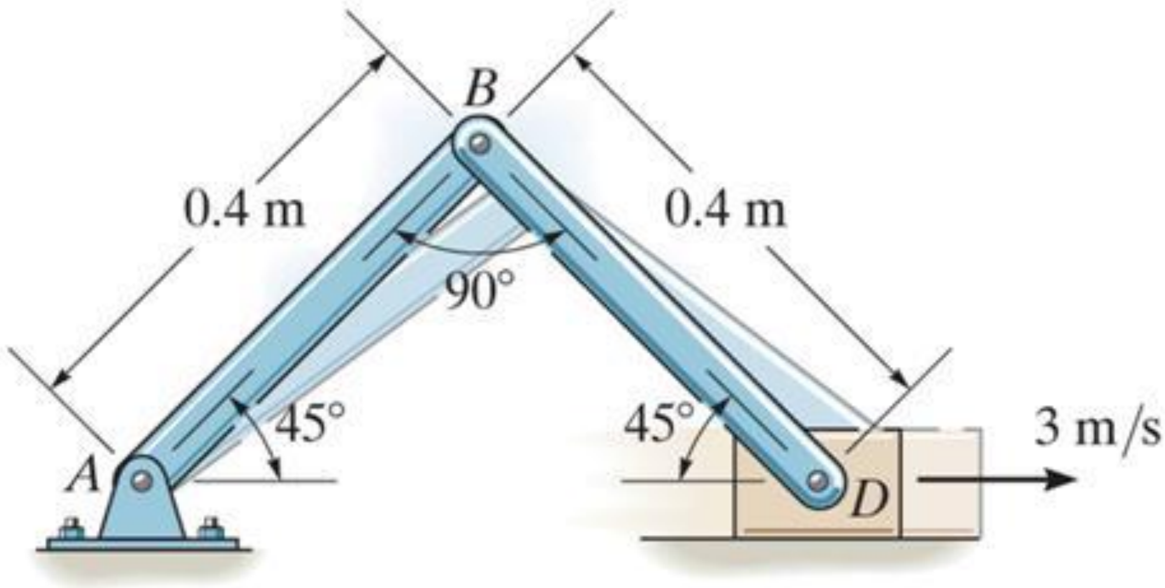
Given: ***A linkage undergoing motion as shown. The velocity of the block, v_D, is 3 m/s.***

Find: ***The angular velocities of links AB and BD.***

Plan:

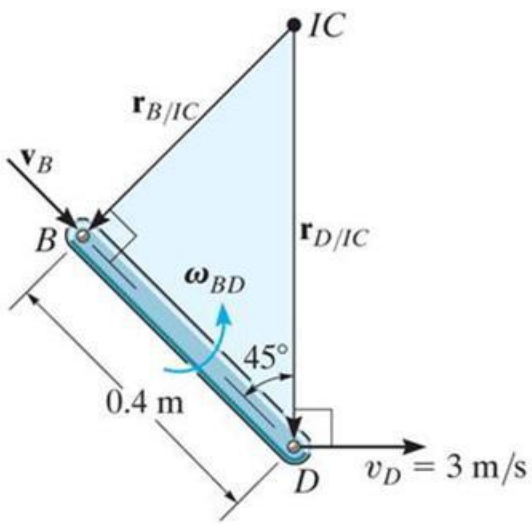
Locate the instantaneous center of zero velocity of link BD and then solve for the angular velocities.

EXAMPLE I



Solution: ***Since D moves to the right, it causes link AB to rotate clockwise about point A. The instantaneous center of velocity for BD is located at the intersection of the line segments drawn perpendicular to v_B and v_D. Note that v_B is perpendicular to link AB. Therefore we can see that the IC is located along the extension of link AB.***

EXAMPLE I (continued)



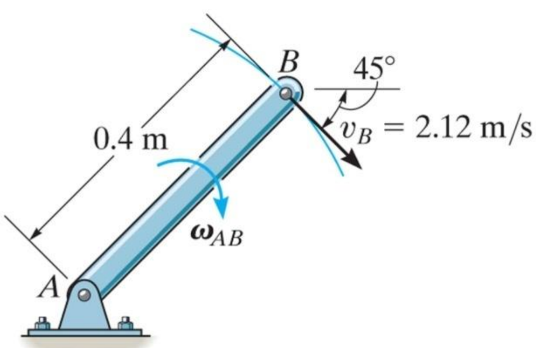
Using these facts,

$r_{B/IC} = 0.4 \tan 45° = 0.4$ m

$r_{D/IC} = 0.4/\cos 45° = 0.566$ m

Since the magnitude of v_D is known, the angular velocity of link BD can be found from $v_D = \omega_{BD}\, r_{D/IC}$.

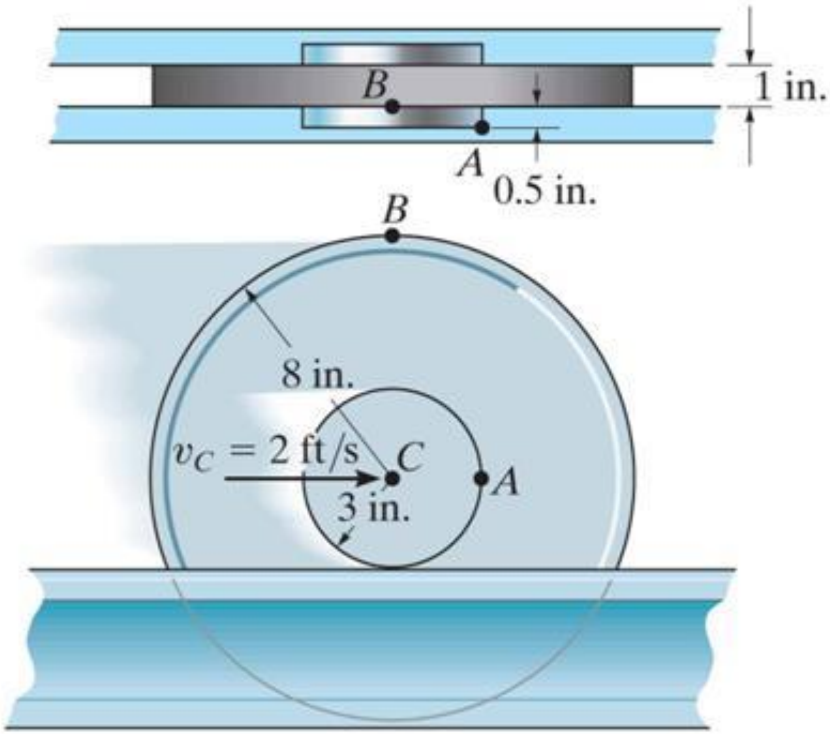
$$\omega_{BD} = v_D/r_{D/IC} = 3/0.566 = 5.3 \text{ rad/s}$$



Link AB is subjected to rotation about A.

$$\omega_{AB} = v_B/r_{B/A} = (r_{B/IC})\omega_{BD}/r_{B/A} = 0.4(5.3)/0.4 = 5.3 \text{ rad/s}$$

EXAMPLE II



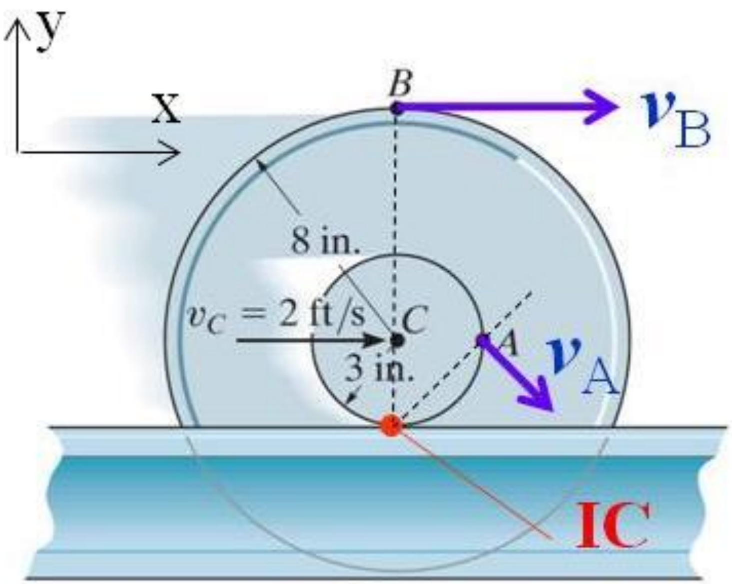
Given: *The wheel rolls on its hub without slipping on the horizontal surface with $v_C = 2$ ft/s.* →

Find: *The velocities of points A and B at the instant shown.*

Plan:

Locate the IC of the wheel. Then calculate the velocities at A and B.

EXAMPLE II (continued) Solution:



Note that the wheel rolls without slipping. Thus the IC is at the contact point with the surface.

The angular velocity of the wheel can be found from

$$\omega = v_C / r_{C/IC} = 2/3 = 0.667 \text{ rad/s}$$

Or, $\boldsymbol{\omega} = -0.667\,\mathbf{k}$ (rad/s)

The velocity at A and B will be

$$\mathbf{v}_A = \boldsymbol{\omega} \times \mathbf{r}_{A/IC} = (-0.667)\,\mathbf{k} \times (3\,\mathbf{i} + 3\mathbf{j}) = (2\,\mathbf{i} - 2\,\mathbf{j}) \text{ in/s}$$

$$\mathbf{v}_B = \boldsymbol{\omega} \times \mathbf{r}_{B/IC} = (-0.667)\,\mathbf{k} \times (11\,\mathbf{j}) = 7.34\,\mathbf{i} \text{ in/s}$$

$$v_A = \sqrt{(2^2 + 2^2)} = 2.83 \text{ in/s}, \; v_B = 7.34 \text{ in/s}$$

CONCEPT QUIZ

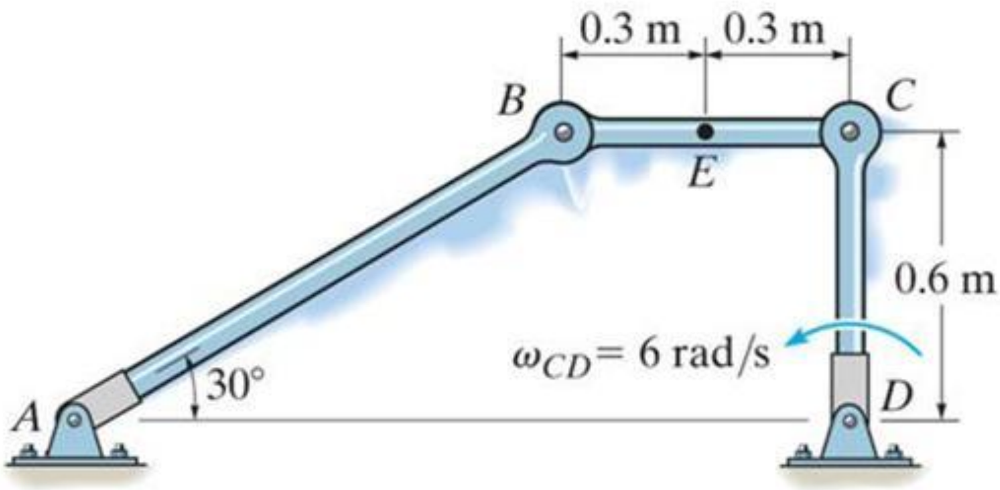
1. ***When the velocities of two points on a body are equal in magnitude and parallel but in opposite directions, the IC is located at***

   A) infinity.
   B) one of the two points.
   C) the midpoint of the line connecting the two points.
   D) None of the above.

2. ***When the direction of velocities of two points on a body are perpendicular to each other, the IC is located at***

   A) infinity.
   B) one of the two points.
   C) the midpoint of the line connecting the two points.
   D) None of the above.

GROUP PROBLEM SOLVING



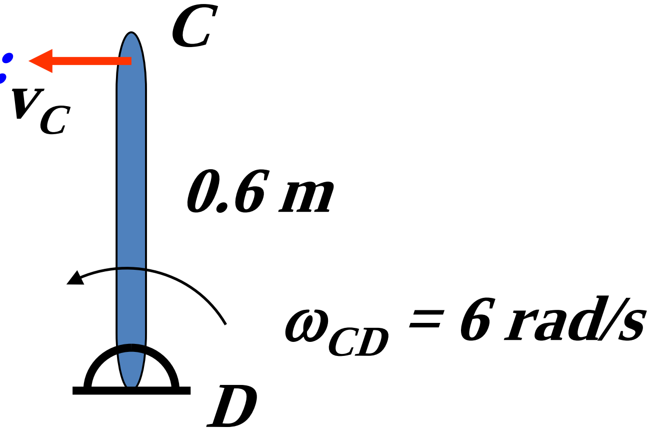
Given: *The four bar linkage is moving with ω_{CD} equal to 6 rad/s CCW.*

Find: *The velocity of point E on link BC and angular velocity of link AB.*

Plan: *This is an example of the second case in the lecture notes. Since the direction of Point B's velocity must be perpendicular to AB, and Point C's velocity must be perpendicular to CD, the location of the instantaneous center, I, for link BC can be found.*

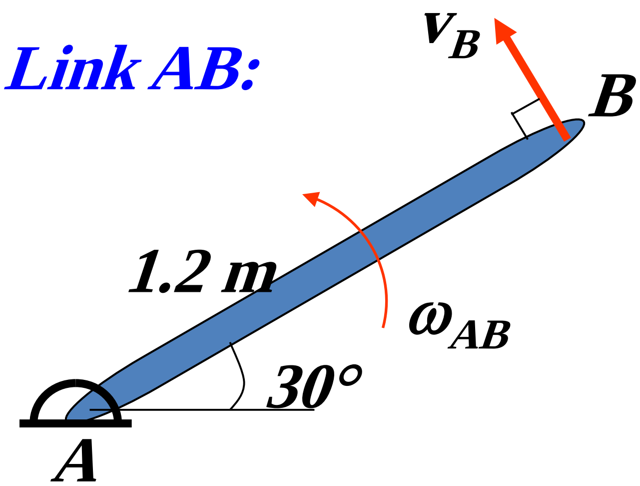
GROUP PROBLEM SOLVING
(continued)

Link CD:

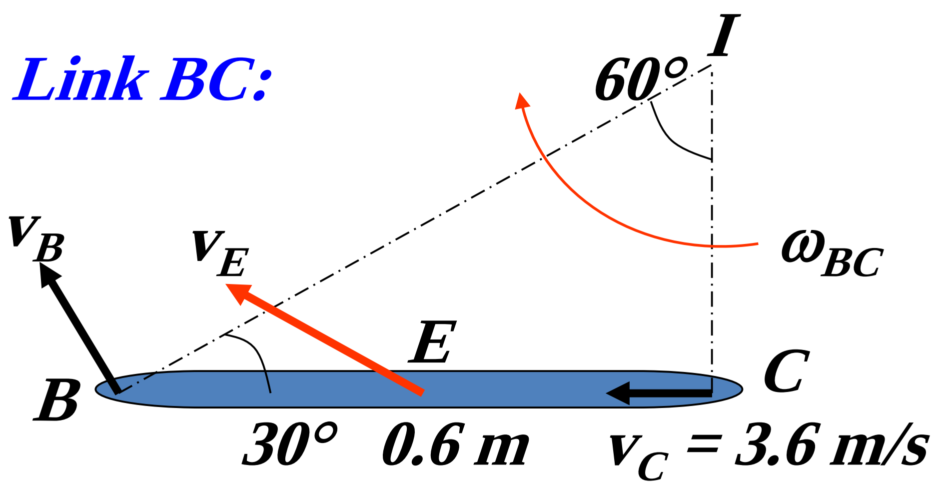


$v_C = 0.6(6) = 3.6$ m/s

Link AB:



Link BC:



From triangle CBI

IC = 0.346 m

IB = 0.6/sin 60° = 0.693 m

$v_C = (IC)\omega_{BC}$

$\omega_{BC} = v_C/IC = 3.6/0.346$

$\omega_{BC} = 10.39$ rad/s

GROUP PROBLEM SOLVING (continued)

$v_B = (IB)\omega_{BC} = 0.693(10.39) = 7.2$ m/s

From link AB, v_B is also equal to 1.2 ω_{AB}.

Therefore $7.2 = 1.2\ \omega_{AB}$ => $\omega_{AB} = 6$ rad/s

$v_E = (IE)\omega_{BC}$ where distance $IE = \sqrt{0.3^2 + 0.346^2} = 0.458$ m

$v_E = 0.458(10.39) = 4.76$ m/s θ

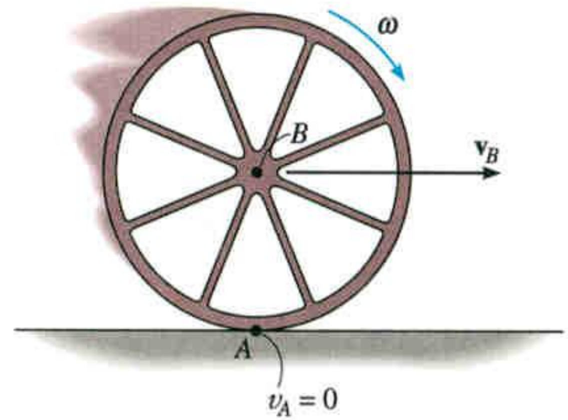
where $\theta = \tan^{-1}(0.3/0.346) = 40.9°$

ATTENTION QUIZ

1. The wheel shown has a radius of 15 in and rotates clockwise at a rate of ω = 3 rad/s. What is ?

   A) 5 in/s B) 15 in/s

   C) 0 in/s D) 45 in/s



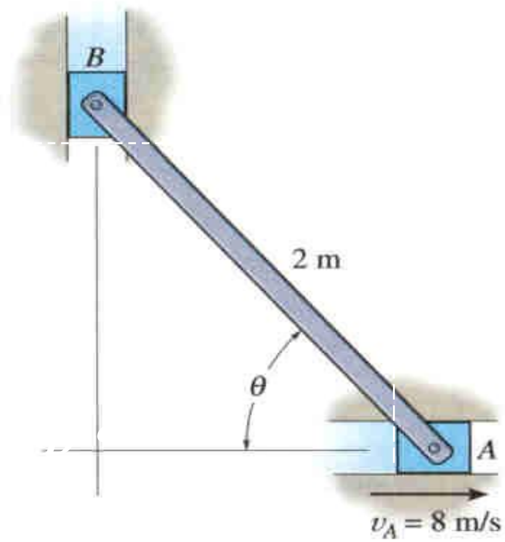
2. Point A on the rod has a velocity of 8 m/s to the right. Where is the IC for the rod?

   A) Point A.

   B) Point B.

   C) Point C.

   D) Point D.



RELATIVE MOTION ANALYSIS: ACCELERATION

Objectives: *Students will be able to:*

1. *Resolve the acceleration of a point on a body into components of translation and rotation.*
2. *Determine the acceleration of a point on a body by using a relative acceleration analysis.*

READING QUIZ

1. *If two bodies contact one another without slipping, and the points in contact move along different paths, the tangential components of acceleration will be ______ and the normal components of acceleration will be __________.*

   A) the same, the same *B) the same, different*

   C) different, the same *D) different, different*

2. *When considering a point on a rigid body in general plane motion,*

   A) It's total acceleration consists of both absolute acceleration and relative acceleration components.

   B) It's total acceleration consists of only absolute acceleration components.

   C) It's relative acceleration component is always normal to the path.

   D) None of the above.

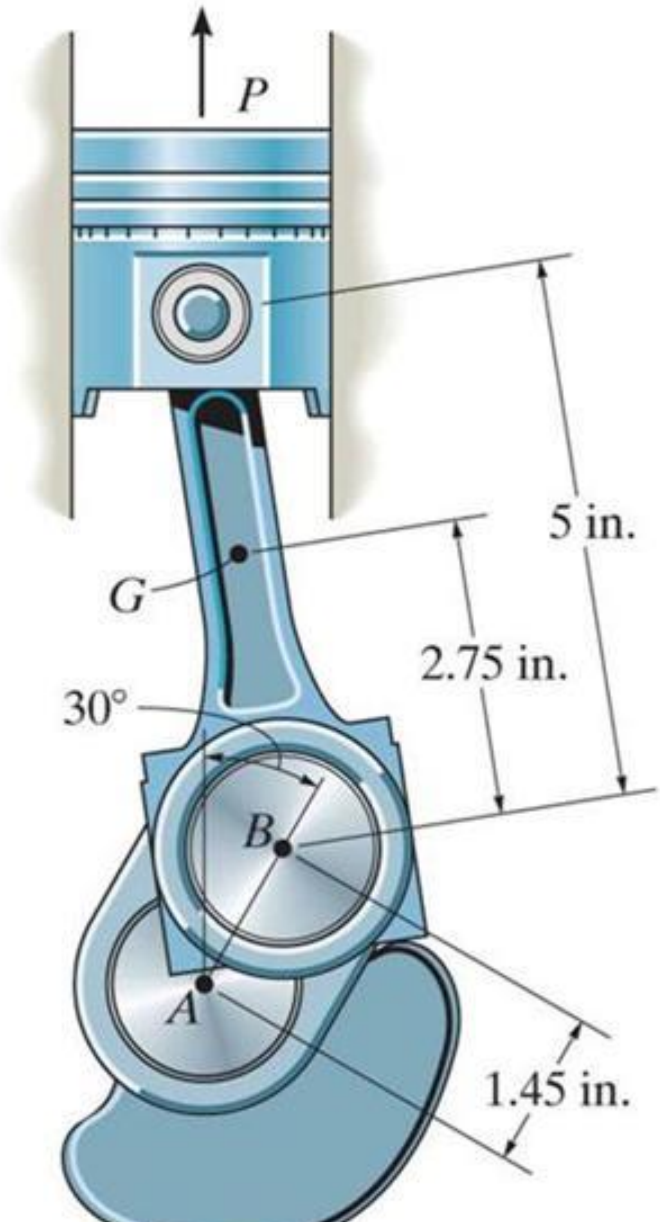
APPLICATIONS

In the mechanism for a window, link AC rotates about a fixed axis through C, and AB undergoes general plane motion. Since point A moves along a curved path, it has two components of acceleration while point B, sliding in a straight track, has only one.

The components of acceleration of these points can be inferred since their motions are known.

How can we determine the accelerations of the links in the mechanism?

APPLICATIONS (continued)



In an automotive engine, the forces delivered to the crankshaft, and the angular acceleration of the crankshaft, depend on the speed and acceleration of the piston.

How can we relate the accelerations of the piston, connection rod, and crankshaft to each other?

RELATIVE MOTION ANALYSIS: ACCELERATION (Section 16-7)

The equation relating the accelerations of two points on the body is determined by differentiating the velocity equation with respect to time.

$$\frac{d\mathbf{v}_B}{dt} = \frac{d\mathbf{v}_A}{dt} + \frac{d\mathbf{v}_{B/A}}{dt}$$

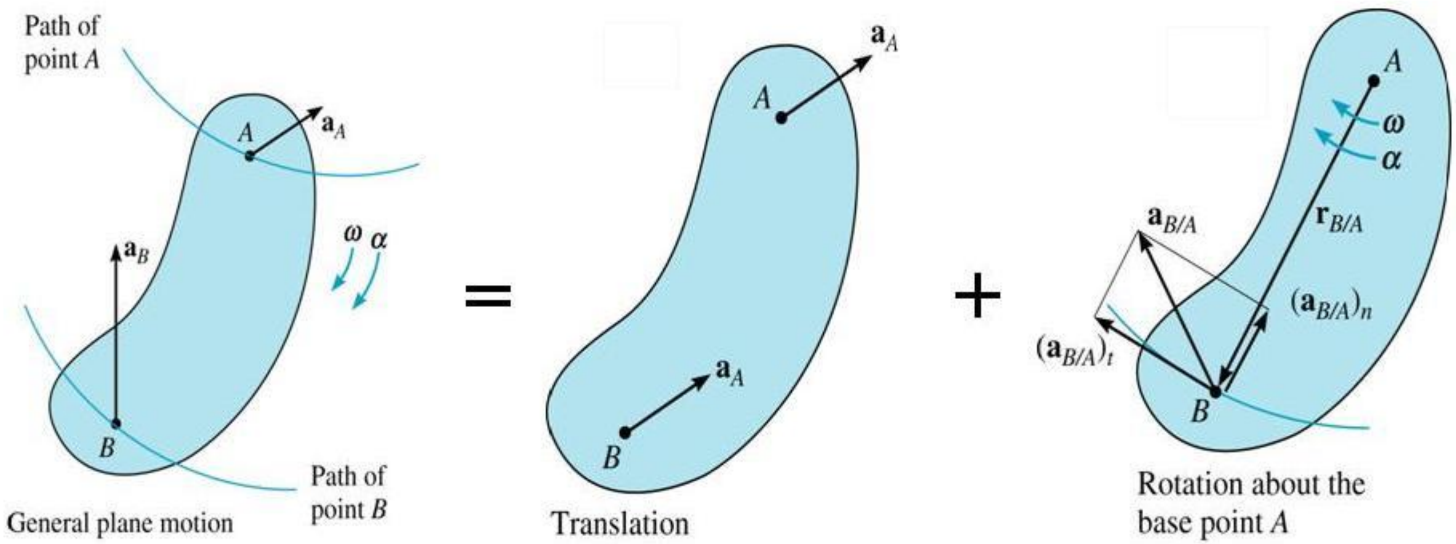
These are absolute accelerations of points A and B. They are measured from a set of fixed x,y axes.

This term is the acceleration of B with respect to A and includes both tangential and normal components.

The result is : $\mathbf{a}_B = \mathbf{a}_A + (\mathbf{a}_{B/A})_t + (\mathbf{a}_{B/A})_n$

RELATIVE MOTION ANALYSIS: ACCELERATION (continued)

Graphically: $\boldsymbol{a}_B = \boldsymbol{a}_A + (\boldsymbol{a}_{B/A})_t + (\boldsymbol{a}_{B/A})_n$



The relative tangential acceleration component $(\boldsymbol{a}_{B/A})_t$ *is* $(\boldsymbol{\alpha} \times \boldsymbol{r}_{B/A})$ *and perpendicular to* $\boldsymbol{r}_{B/A}$.

The relative normal acceleration component $(\boldsymbol{a}_{B/A})_n$ *is* $(-\omega^2\, \boldsymbol{r}_{B/A})$ *and the direction is always from B towards A.*

RELATIVE MOTION ANALYSIS: ACCELERATION (continued)

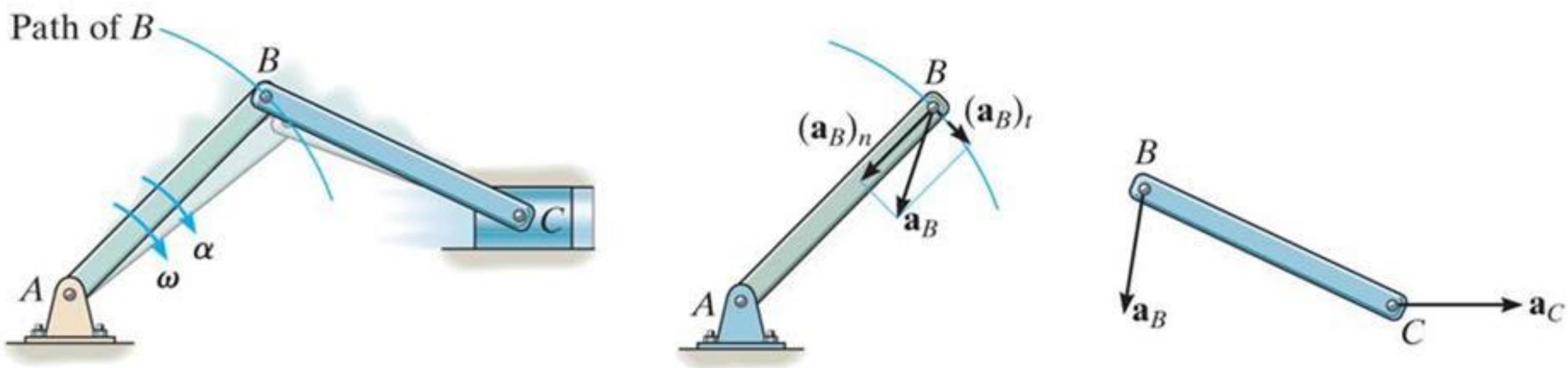
Since the relative acceleration components can be expressed as $(\mathbf{a}_{B/A})_t = \alpha \times \mathbf{r}_{B/A}$ and $(\mathbf{a}_{B/A})_n = -\,\omega^2\,\mathbf{r}_{B/A}$, the relative acceleration equation becomes

$$\mathbf{a}_B = \mathbf{a}_A + \alpha \times \mathbf{r}_{B/A} - \omega^2\,\mathbf{r}_{B/A}$$

Note that the last term in the relative acceleration equation is not a cross product. It is the product of a scalar (square of the magnitude of angular velocity, ω^2) and the relative position vector, $\mathbf{r}_{B/A}$.

APPLICATION OF THE RELATIVE ACCELERATION EQUATION

In applying the relative acceleration equation, the two points used in the analysis (A and B) should generally be selected as points which have a known motion, such as pin connections with other bodies.



In this mechanism, point B is known to travel along a circular path, so $\mathbf{a}_B$ *can be expressed in terms of its normal and tangential components. Note that point B on link BC will have the same acceleration as point B on link AB.*

Point C, connecting link BC and the piston, moves along a straight-line path. Hence, $\mathbf{a}_C$ *is directed horizontally.*

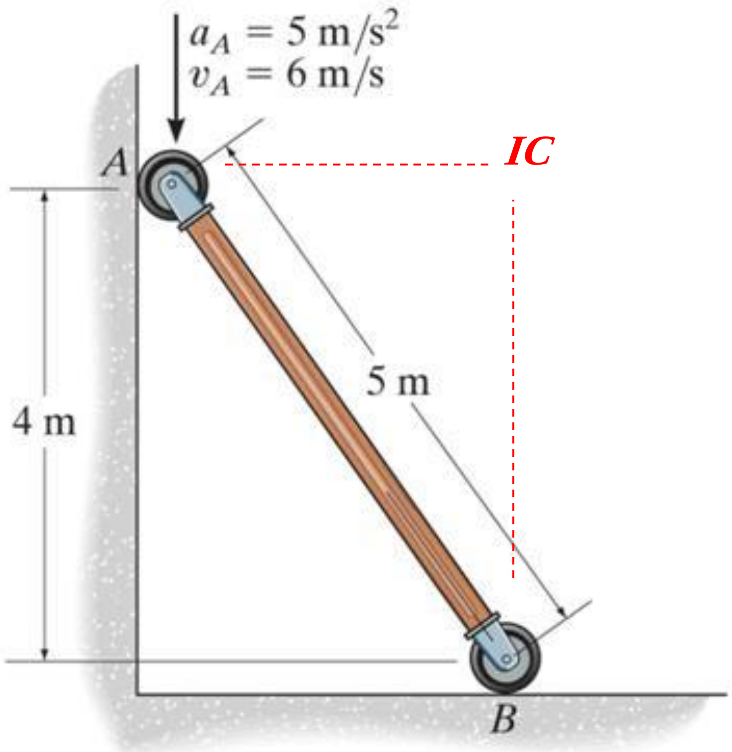
PROCEDURE FOR ANALYSIS

1. Establish a fixed coordinate system.
2. Draw the kinematic diagram of the body.
3. Indicate on it $\mathbf{a}_A$, $\mathbf{a}_B$, ω, α, and $\mathbf{r}_{B/A}$. If the points A and B move along curved paths, then their accelerations should be indicated in terms of their tangential and normal components, i.e., $\mathbf{a}_A = (\mathbf{a}_A)_t + (\mathbf{a}_A)_n$ and $\mathbf{a}_B = (\mathbf{a}_B)_t + (\mathbf{a}_B)_n$.
4. Apply the relative acceleration equation:

$$\mathbf{a}_B = \mathbf{a}_A + \alpha \times \mathbf{r}_{B/A} - \omega^2 \, \mathbf{r}_{B/A}$$

5. If the solution yields a negative answer for an unknown magnitude, this indicates that the sense of direction of the vector is opposite to that shown on the diagram.

EXAMPLE I



Given: Point A on rod AB has an acceleration of 5 m/s² and a velocity of 6 m/s at the instant shown.

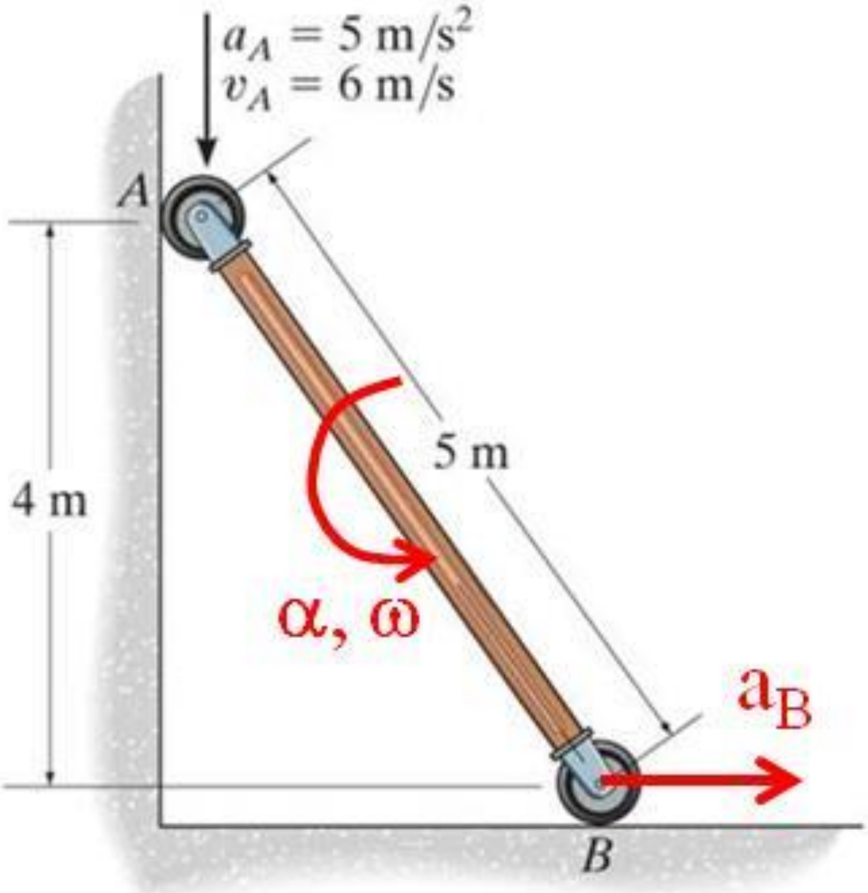
Find: The angular acceleration of the rod and the acceleration at B at this instant.

Plan:

Follow the problem solving procedure!

Solution: First, we need to find the angular velocity of the rod at this instant. Locating the instant center (IC) for rod AB, we can determine ω:

$\omega = v_A/r_{A/IC} = v_A / (3) = 2$ rad/s



EXAMPLE I (continued)

Since points A and B both move along straight-line paths,

$\mathbf{a}_A = -5\,\mathbf{j}$ m/s^2

$\mathbf{a}_B = a_B\,\mathbf{i}$ m/s^2

Applying the relative acceleration equation

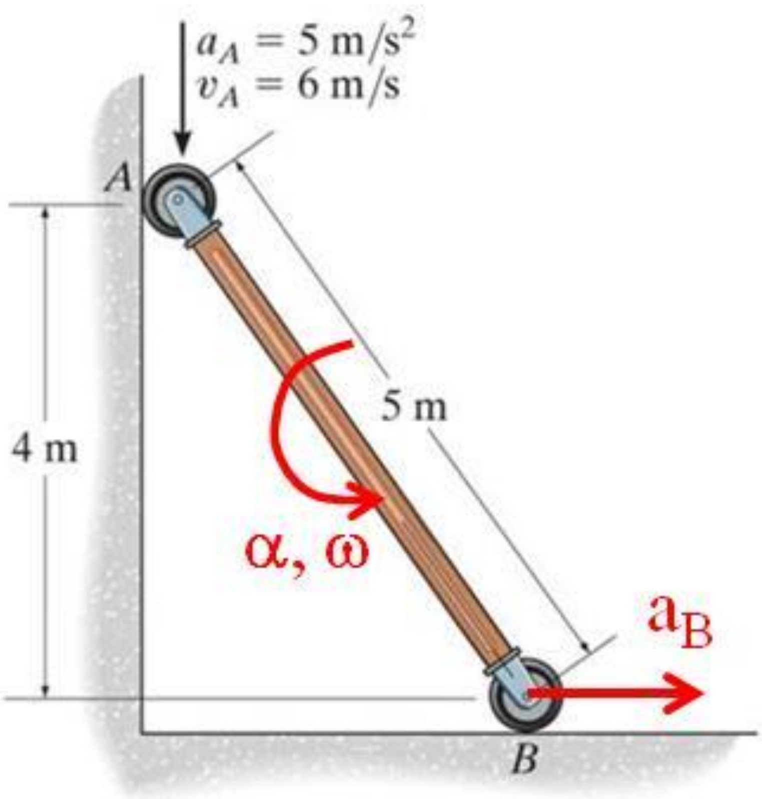
$$\mathbf{a}_B = \mathbf{a}_A + \boldsymbol{\alpha} \times \mathbf{r}_{B/A} - \omega^2 \mathbf{r}_{B/A}$$

$$a_B\,\mathbf{i} = -5\,\mathbf{j} + \alpha\,\mathbf{k} \times (3\,\mathbf{i} - 4\,\mathbf{j}) - 2^2\,(3\,\mathbf{i} - 4\,\mathbf{j})$$

$$a_B\,\mathbf{i} = -5\,\mathbf{j} + 4\,\alpha\,\mathbf{i} + 3\,\alpha\,\mathbf{j} - (12\,\mathbf{i} - 16\,\mathbf{j})$$

EXAMPLE I (continued)

So with $a_B\,\mathbf{i} = -5\,\mathbf{j} + 4\,\alpha\,\mathbf{i} + 3\,\alpha\,\mathbf{j} - (12\,\mathbf{i} - 16\,\mathbf{j})$, we can solve.



By comparing the **i**, **j** components;

$$a_B = 4\,\alpha - 12$$

$$0 = 11 + 3\alpha$$

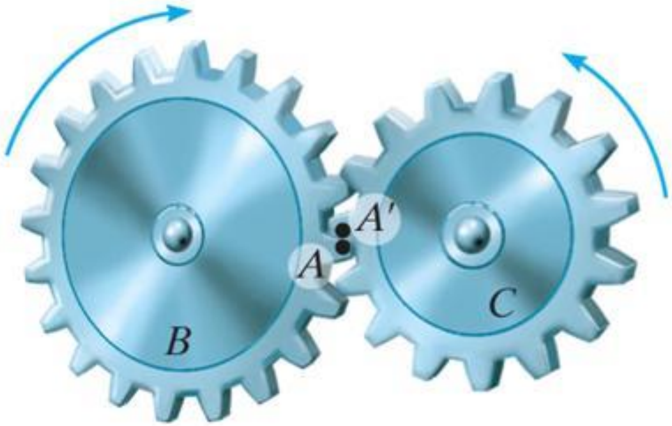
Solving:

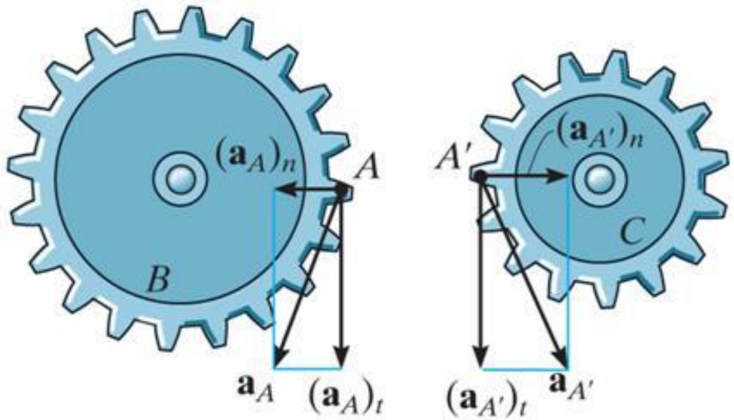
$$a_B = -26.7 \text{ m/s}^2$$

$$\alpha = -3.67 \text{ rad/s}^2$$

BODIES IN CONTACT

Consider two bodies in contact with one another without slipping, where the points in contact move along different paths.





In this case, the tangential components of acceleration will be the same, i. e.,

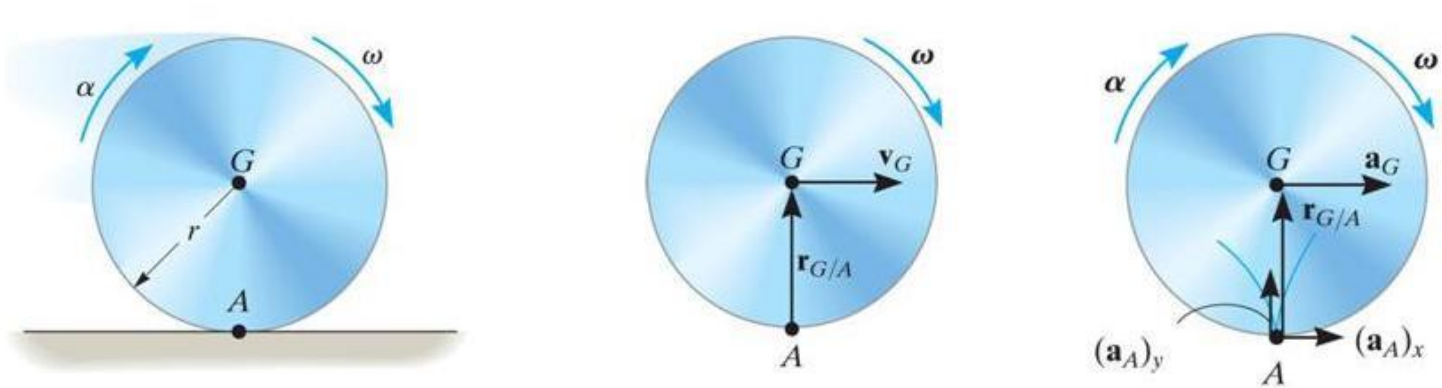
$$(\mathbf{a}_A)_t = (\mathbf{a}_{A'})_t \text{ (which implies } \alpha_B r_B = \alpha_C r_C\text{)}.$$

The normal components of acceleration will not be the same.

$$(\mathbf{a}_A)_n \neq (\mathbf{a}_{A'})_n \text{ so } \mathbf{a}_A \neq \mathbf{a}_{A'}$$

ROLLING MOTION

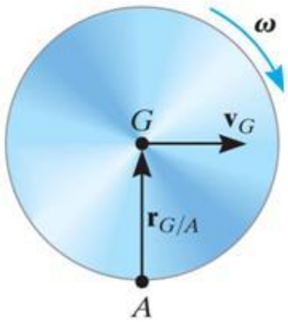
Another common type of problem encountered in dynamics involves rolling motion without slip; e.g., a ball, cylinder, or disk rolling without slipping. This situation can be analyzed using relative velocity and acceleration equations.



As the cylinder rolls, point G (center) moves along a straight line. If ω and α are known, the relative velocity and acceleration equations can be applied to A, at the instant A is in contact with the ground. The point A is the instantaneous center of zero velocity, however it is not a point of zero acceleration.

ROLLING MOTION (continued)

- Velocity:



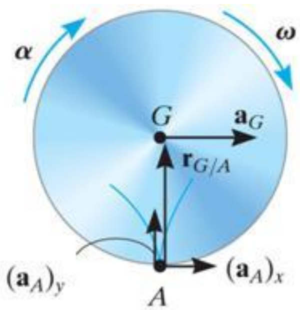
Since no slip occurs, $\boldsymbol{v}_A = \boldsymbol{0}$ when A is in contact with ground. From the kinematic diagram:

$$\boldsymbol{v}_G = \boldsymbol{v}_A + \boldsymbol{\omega} \times \boldsymbol{r}_{G/A}$$

$$v_G\, \boldsymbol{i} = \boldsymbol{0} + (-\omega \boldsymbol{k}) \times (r\, \boldsymbol{j})$$

$$v_G = \omega r \quad \text{or} \quad \boldsymbol{v}_G = \omega r\, \boldsymbol{i}$$

- Acceleration:



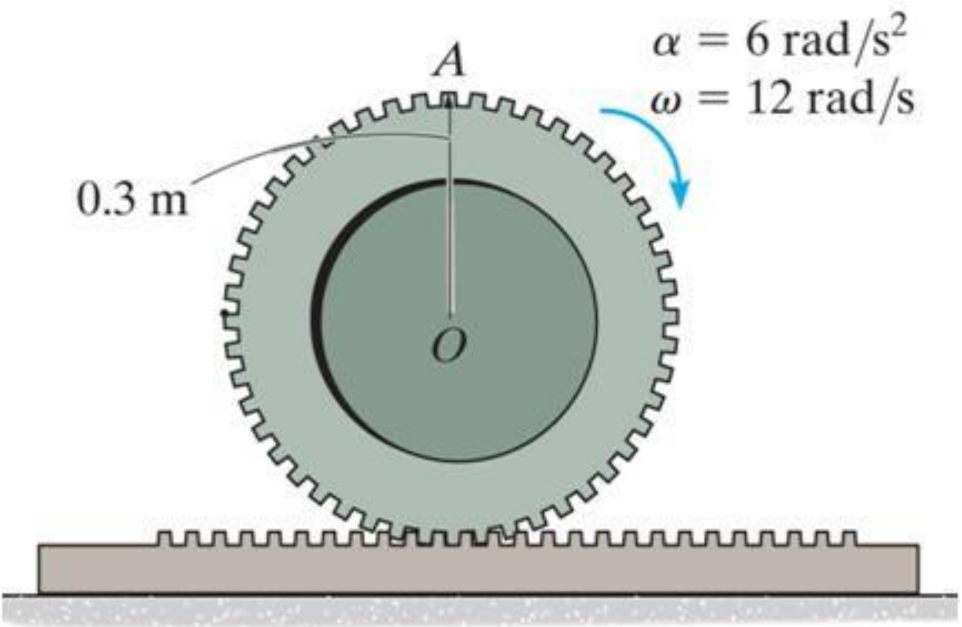
Since G moves along a straight-line path, $\boldsymbol{a}_G$ is horizontal. Just before A touches ground, its velocity is directed downward, and just after contact, its velocity is directed upward. Thus, point A accelerates upward as it leaves the ground.

$$\boldsymbol{a}_G = \boldsymbol{a}_A + \boldsymbol{\alpha} \times \boldsymbol{r}_{G/A} - \omega^2 \boldsymbol{r}_{G/A} \;\Rightarrow\; a_G\, \boldsymbol{i} = a_A\, \boldsymbol{j} + (-\alpha\, \boldsymbol{k}) \times (r\, \boldsymbol{j}) - \omega^2 (r\, \boldsymbol{j})$$

Evaluating and equating $\boldsymbol{i}$ and $\boldsymbol{j}$ components:

$$a_G = \alpha r \quad \text{and} \quad a_A = \omega^2 r \quad \text{or} \quad \boldsymbol{a}_G = \alpha r\, \boldsymbol{i} \quad \text{and} \quad \boldsymbol{a}_A = \omega^2 r\, \boldsymbol{j}$$

EXAMPLE II



Given: *The gear rolls on the fixed rack.*

Find: *The accelerations of point A at this instant.*

Plan:

Follow the solution procedure!

Solution: *Since the gear rolls on the fixed rack without slip,* $\mathbf{a}_O$ *is directed to the right with a magnitude of:*

$$a_O = \alpha r = (6\ \mathrm{rad/s^2})(0.3\ \mathrm{m}) = 1.8\ \mathrm{m/s^2}$$

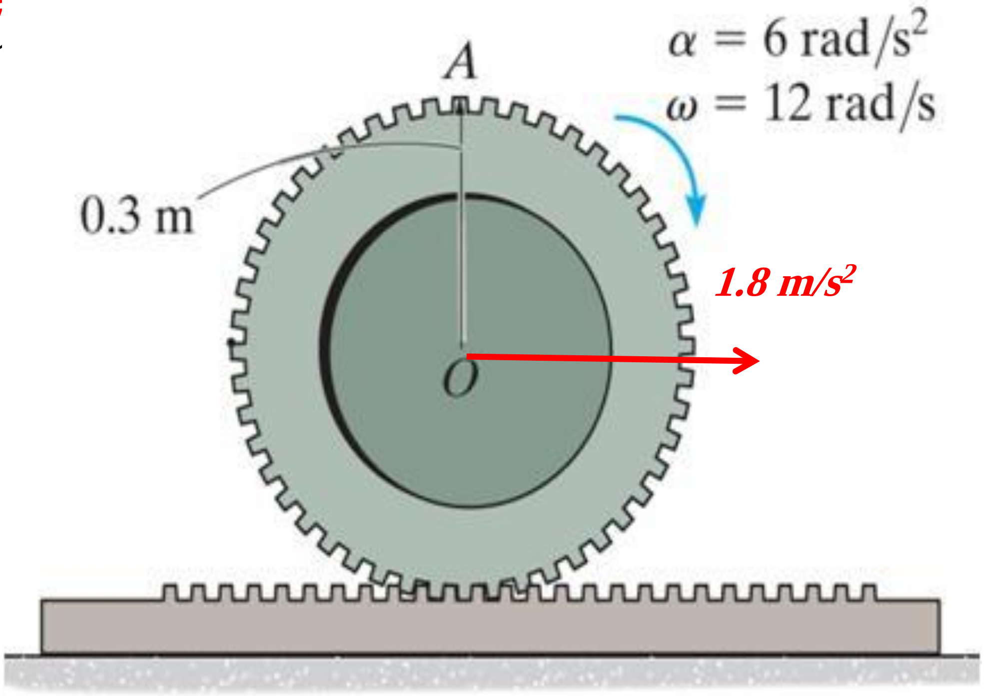
EXAMPLE II (continued)

So now with $a_O = 1.8\ m/s^2$, we can apply the relative acceleration equation between points O and A.

$$\mathbf{a}_A = \mathbf{a}_O + \boldsymbol{\alpha} \times \mathbf{r}_{A/O} - \omega^2\, \mathbf{r}_{A/O}$$

$$\mathbf{a}_A = 1.8\mathbf{i} + (-6\mathbf{k})\times(0.3\mathbf{j}) - 12^2\,(0.3\mathbf{j})$$

$$= (3.6\,\mathbf{i} - 43.2\mathbf{j})\ m/s^2$$

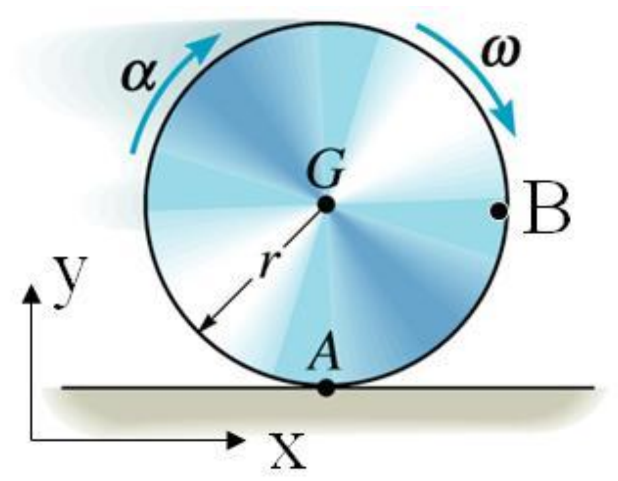


CONCEPT QUIZ

1. *If a ball rolls without slipping, select the tangential and normal components of the relative acceleration of point A with respect to G.*

   A) $\alpha r\ \mathbf{i} + \omega^2 r\ \mathbf{j}$ B) $-\alpha r\ \mathbf{i} + \omega^2 r\ \mathbf{j}$

   C) $\omega^2 r\ \mathbf{i} - \alpha r\ \mathbf{j}$ D) Zero.

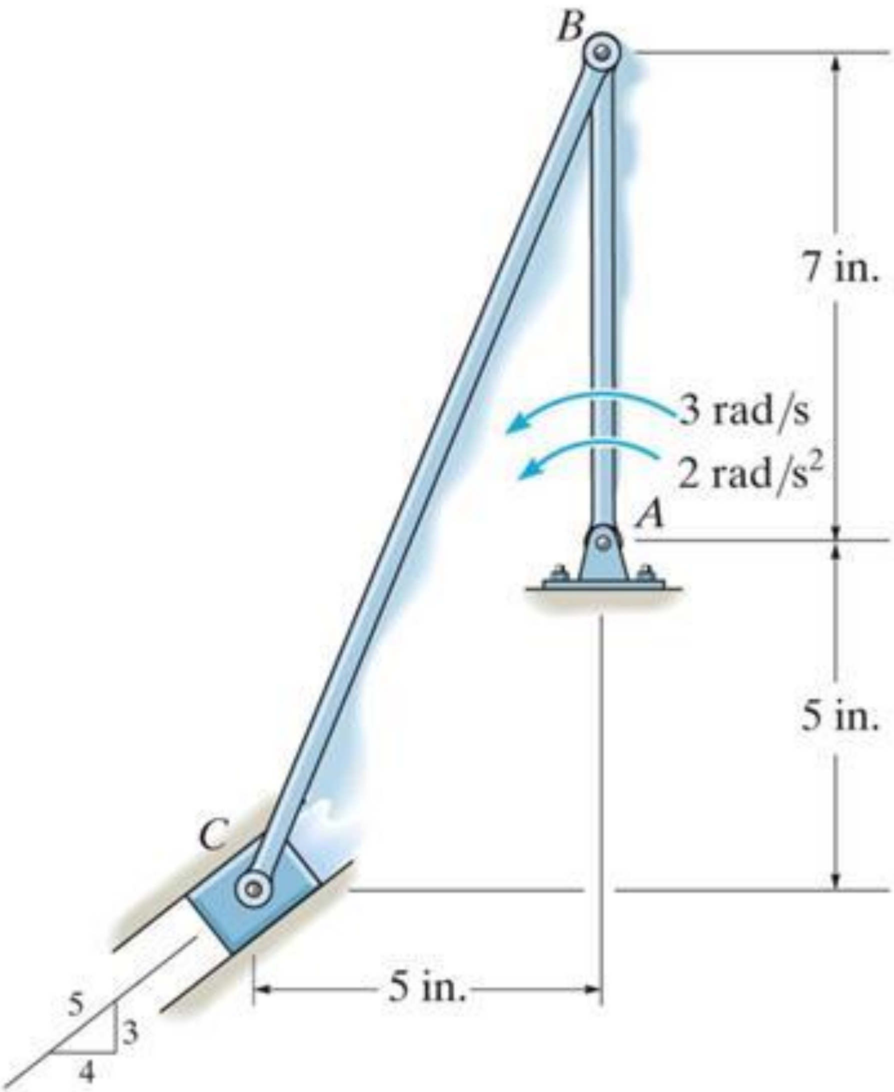


2. *What are the tangential and normal components of the relative acceleration of point B with respect to G.*

   A) $-\omega^2 r\ \mathbf{i} - \alpha r\ \mathbf{j}$ B) $-\alpha r\ \mathbf{i} + \omega^2 r\ \mathbf{j}$

   C) $\omega^2 r\ \mathbf{i} - \alpha r\ \mathbf{j}$ D) Zero.

GROUP PROBLEM SOLVING



Given: *The smember AB is rotating with ω_{AB}=3 rad/s, α_{AB}=2 rad/s^2 at this instant.*

Find: *The velocity and acceleration of the slider block C.*

Plan:

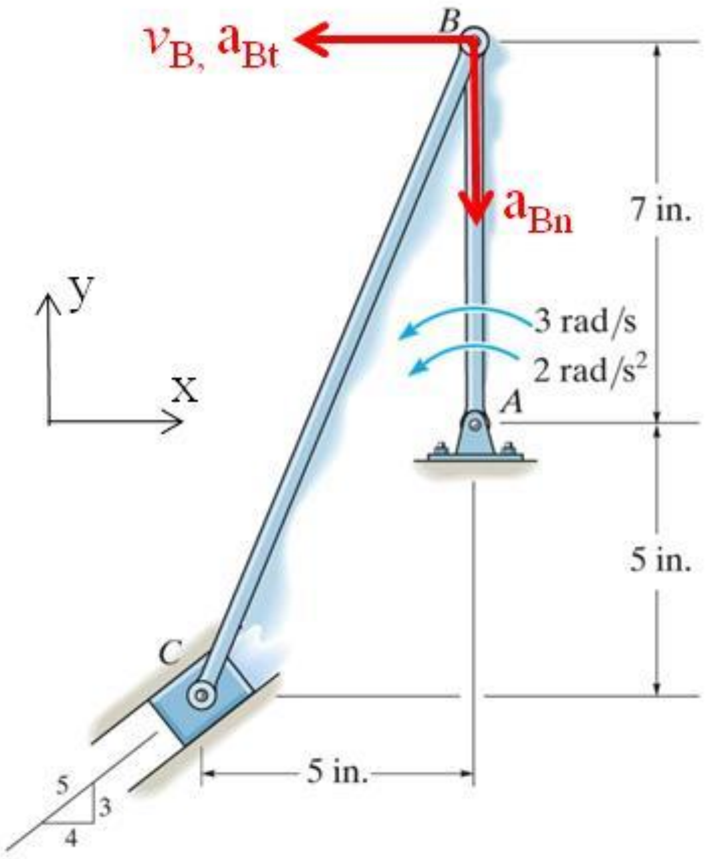
Follow the solution procedure!

Note that Point B is rotating. So what components of acceleration will it be experiencing?

GROUP PROBLEM SOLVING (continued)

Solution:

Since Point B is rotating, its velocity and acceleration will be:



$$v_B = (\omega_{AB})\, r_{B/A} = (3)\, 7 = 21 \text{ in/s}$$

$$a_{Bn} = (\omega_{AB})^2\, r_{B/A} = (3)^2\, 7 = 63 \text{ in/s}^2$$

$$a_{Bt} = (\alpha_{AB})\, r_{B/A} = (2)\, 7 = 14 \text{ in/s}^2$$

$$\mathbf{v}_B = (-21\,\mathbf{i}\,) \text{ in/s}$$

$$\mathbf{a}_B = (-14\,\mathbf{i} - 63\,\mathbf{j}\,) \text{ in/s}^2$$

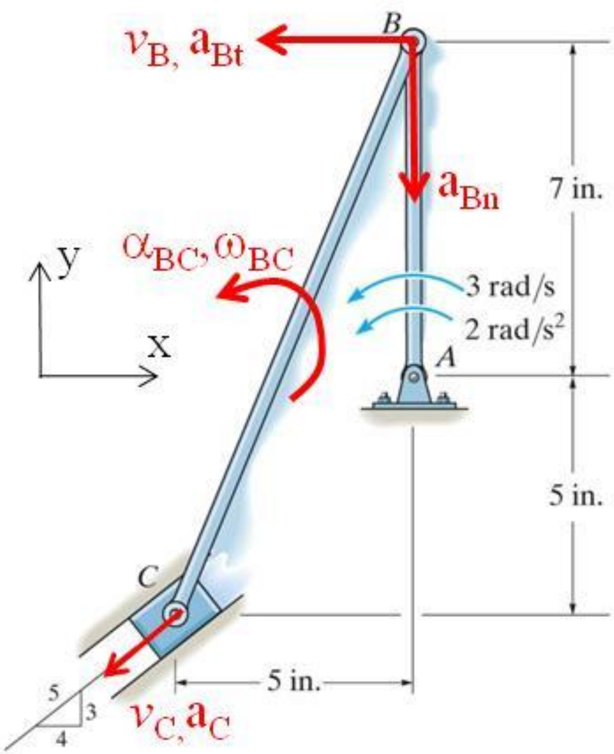
GROUP PROBLEM SOLVING (continued)

Now apply the relative velocity equation between points B and C to find the angular velocity of link BC.

$$\mathbf{v}_C = \mathbf{v}_B + \boldsymbol{\omega}_{BC} \times \mathbf{r}_{C/B}$$

$$(-0.8\, v_C\, \mathbf{i} - 0.6\, v_C\, \mathbf{j}) = (-21\, \mathbf{i}) + \omega_{BC}\, \mathbf{k} \times (-5\, \mathbf{i} - 12\, \mathbf{j})$$

$$= (-21 + 12\, \omega_{BC})\, \mathbf{i} - 5\, \omega_{BC}\, \mathbf{j}$$



By comparing the ***i, j*** *components;*

$$-0.8\, v_C = -21 + 12\, \omega_{BC}$$

$$-0.6\, v_C = -5\, \omega_{BC}$$

Solving:

$$\omega_{BC} = 1.125 \text{ rad/s}$$

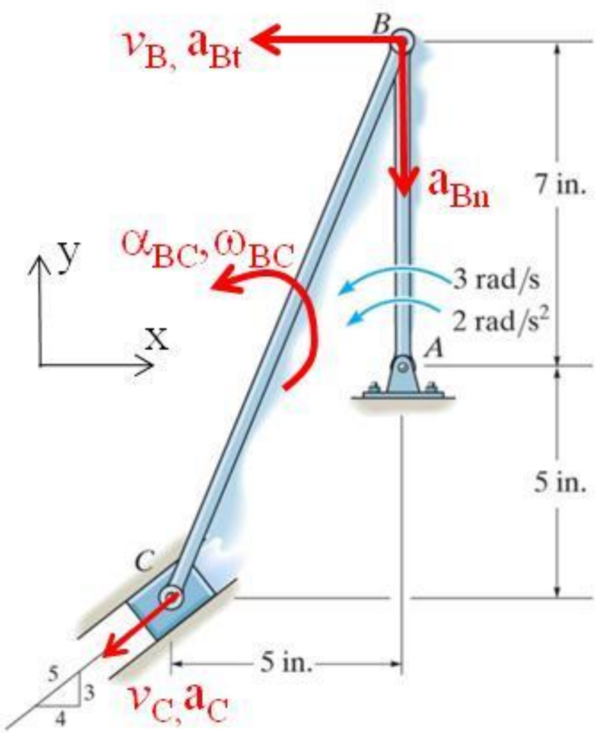
$$v_C = 9.375 \text{ in/s}$$

GROUP PROBLEM SOLVING (continued)

Now, apply the relative acceleration equation between points B and C.

$$\mathbf{a}_C = \mathbf{a}_B + \boldsymbol{\alpha}_{BC} \times \mathbf{r}_{C/B} - \omega^2_{BC}\mathbf{r}_{C/B}$$

$$(-0.8\, a_C\, \mathbf{i} - 0.6\, a_C\, \mathbf{j}) = (-14\, \mathbf{i} - 63\, \mathbf{j}) + \alpha_{BC}\mathbf{k} \times (-5\, \mathbf{i} - 12\, \mathbf{j}) - (1.125)^2 (-5\, \mathbf{i} - 12\, \mathbf{j})$$

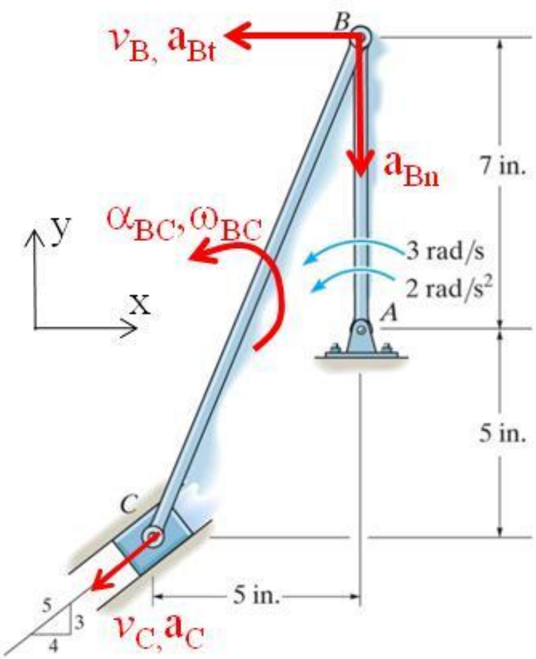


$$(-\,0.8\, a_C\, \mathbf{i} - 0.6\, a_C\, \mathbf{j})$$
$$= (-14 + 12\,\alpha_{BC} + 6.328)\, \mathbf{i}$$
$$+ (-\,63 - 5\,\alpha_{BC} + 15.19)\, \mathbf{j}$$

*By comparing the **i, j** components;*

$$-\,0.8\, a_C = -7.672 + 12\,\alpha_{BC}$$
$$-\,0.6\, a_C = -\,47.81 - 5\,\alpha_{BC}$$

GROUP PROBLEM SOLVING (continued)



*Solving these two **i, j** component equations*

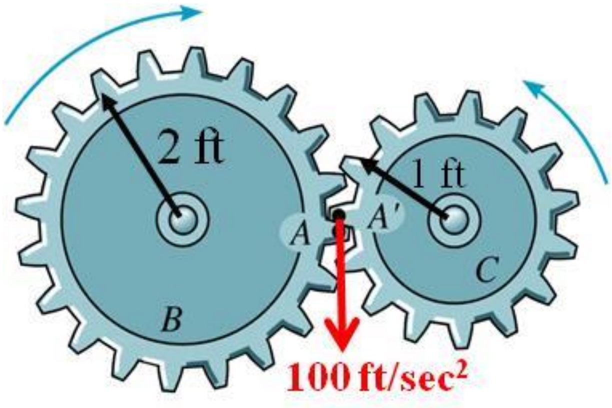
$$- 0.8\, a_C = -7.672 + 12\, \alpha_{BC}$$
$$- 0.6\, a_C = - 47.81 - 5\, \alpha_{BC}$$

Yields

$$\alpha_{BC} = -3.0 \text{ rad/s}^2$$
$$a_C = 54.7 \text{ in/s}^2$$

ATTENTION QUIZ

1. *Two bodies contact one another without slipping. If the tangential component of the acceleration of point A on gear B is 100 ft/sec^2, determine the tangential component of the acceleration of point A' on gear C.*



   A) 50 ft/sec^2 *B) 100 ft/sec^2*

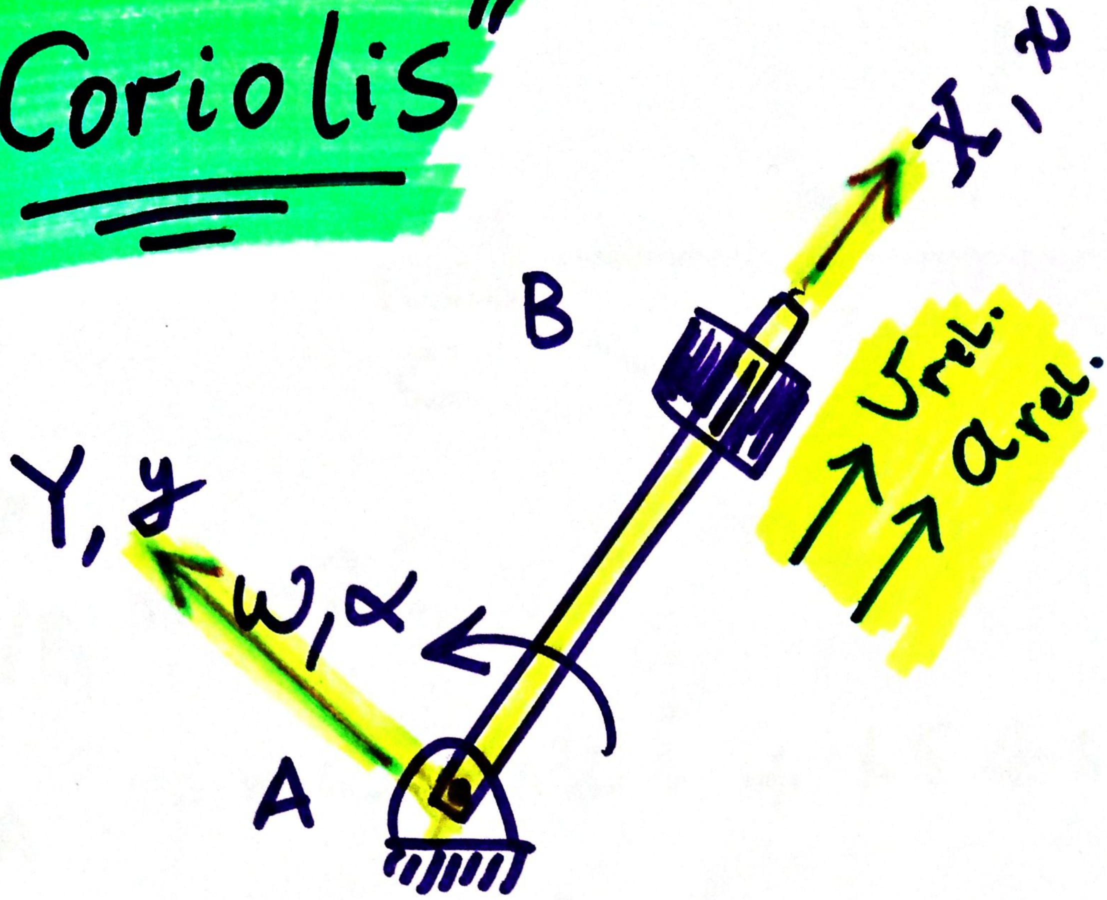
   C) 200 ft/sec^2 *D) None of above.*

2. *If the tangential component of the acceleration of point A on gear B is 100 ft/sec^2, determine the angular acceleration of gear B.*

   A) 50 rad/sec^2 *B) 100 rad/sec^2*

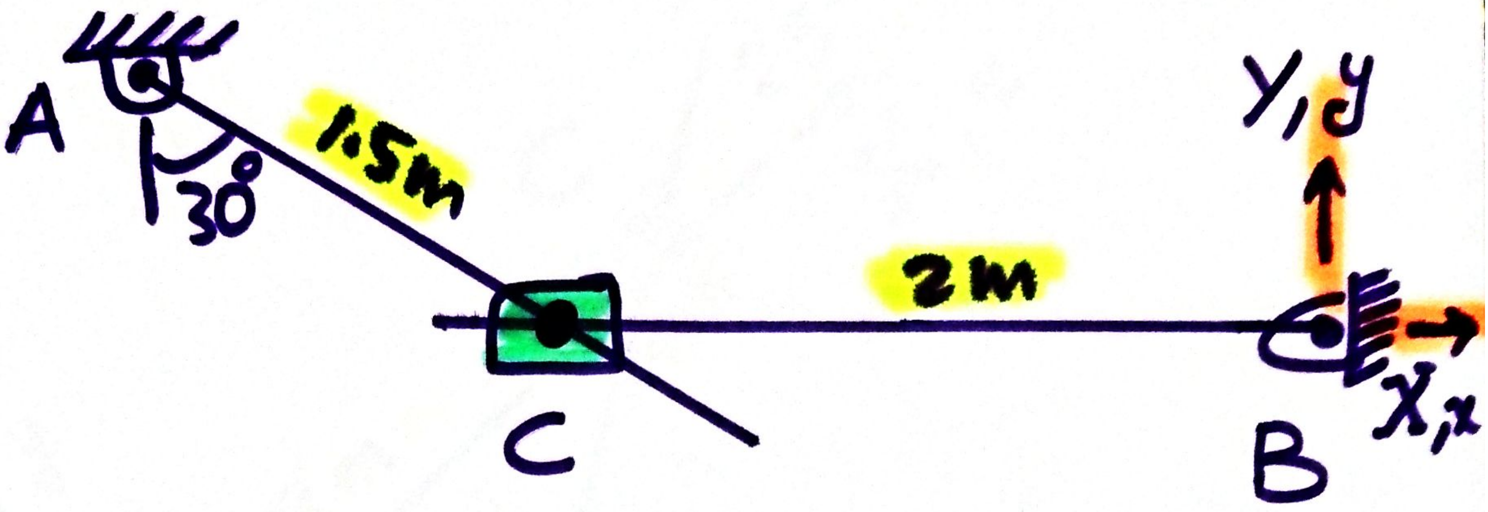
   C) 200 rad/sec^2 *D) None of above.*

"Coriolis"



$$\bar{v}_B = \bar{v}_A + \bar{\omega} \times \bar{r}_{B/A} + \bar{v}_{rel.}$$

$$\bar{a}_B = \bar{a}_A + \bar{\alpha} \times \bar{r}_{B/A} - \omega^2 \bar{r}_{B/A} + \underbrace{2\bar{\omega} \times \bar{v}_{rel.}}_{\text{Coriolis Acc.}} + \bar{a}_{rel}$$



$$\vec{r}_{C/B} = -2i$$

$$\vec{r}_{C/A} = -1.5\sin 30^\circ i + 1.5\cos 30^\circ j$$

$$\vec{v}_{rel} = v_{rel}\, i$$

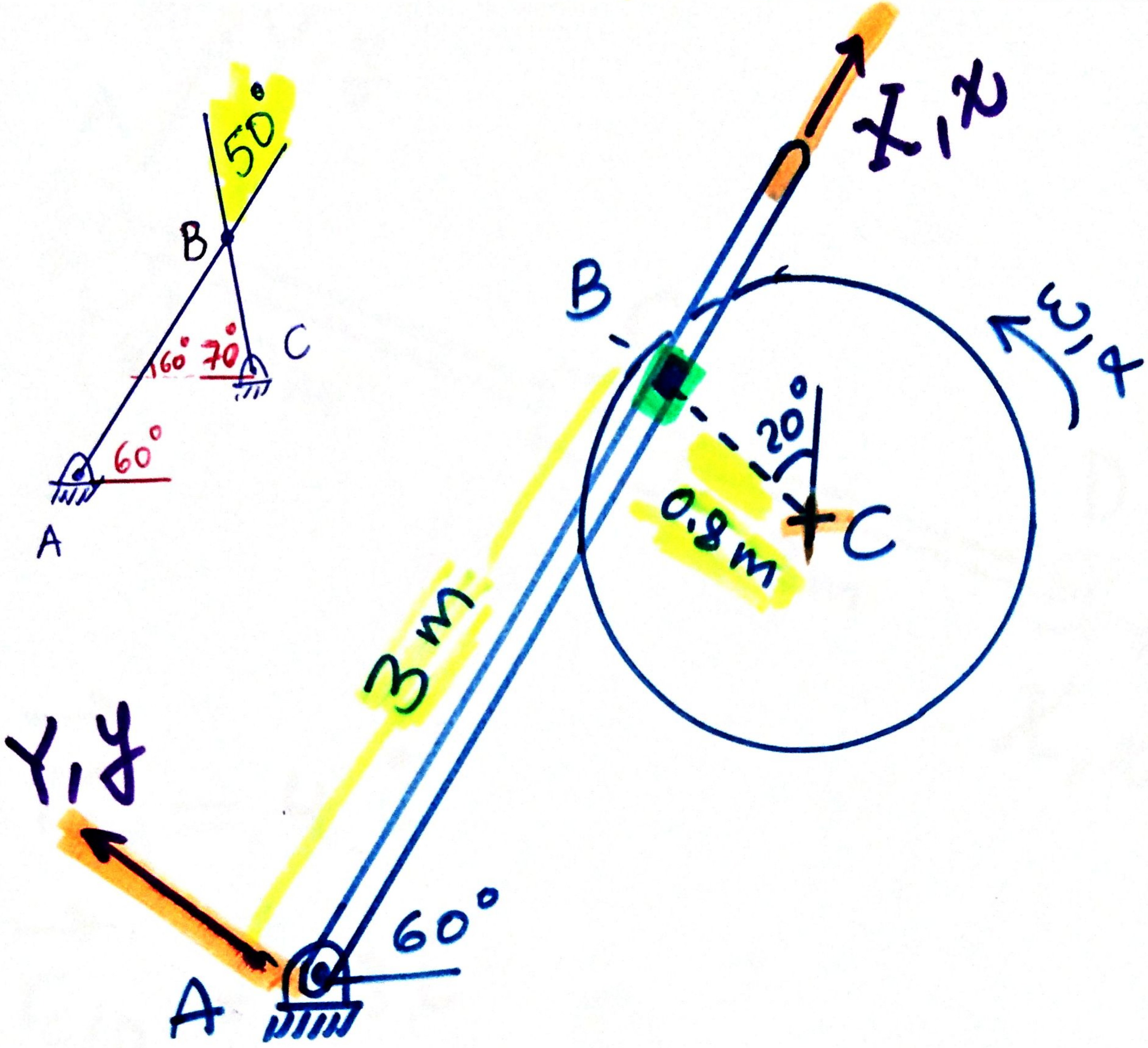
$$\vec{a}_{rel} = a_{rel}\, i$$

$$*\ \vec{v}_C = \vec{v}_A + \vec{\omega}_{AC} \times \vec{r}_{C/A}$$

$$*\ \vec{v}_C = \vec{v}_B + \vec{\omega}_{CB} \times \vec{r}_{C/B} + \vec{v}_{rel}$$

$$*\ \vec{a}_C = \vec{a}_A + \vec{\alpha}_{AC} \times \vec{r}_{C/A} - \omega_{AC}^2 \vec{r}_{C/A}$$

$$*\ \vec{a}_C = \vec{a}_B + \vec{\alpha}_{CB} \times \vec{r}_{C/B} - \omega_{CB}^2 \vec{r}_{C/B} + 2\vec{\omega}_{CB} \times \vec{v}_{rel} + \vec{a}_{rel}.$$

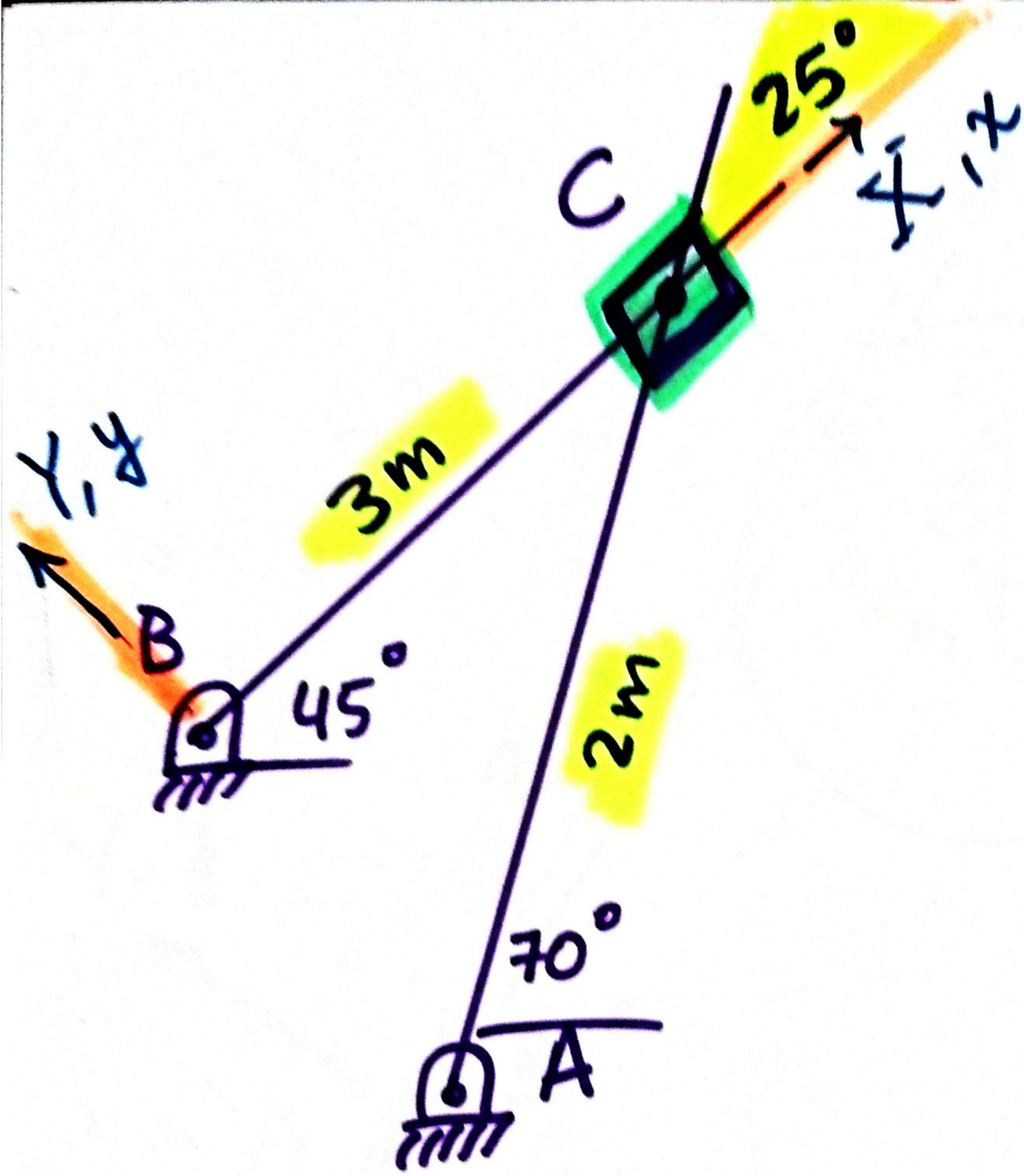


$$\vec{r}_{B/A} = 3\,i$$

$$\vec{r}_{B/C} = 0.8\cos 50^\circ\, i + 0.8\sin 50^\circ\, j$$

$$\vec{v}_{rel} = v_{rel}\, i$$

$$\vec{a}_{rel} = a_{rel}\, i$$

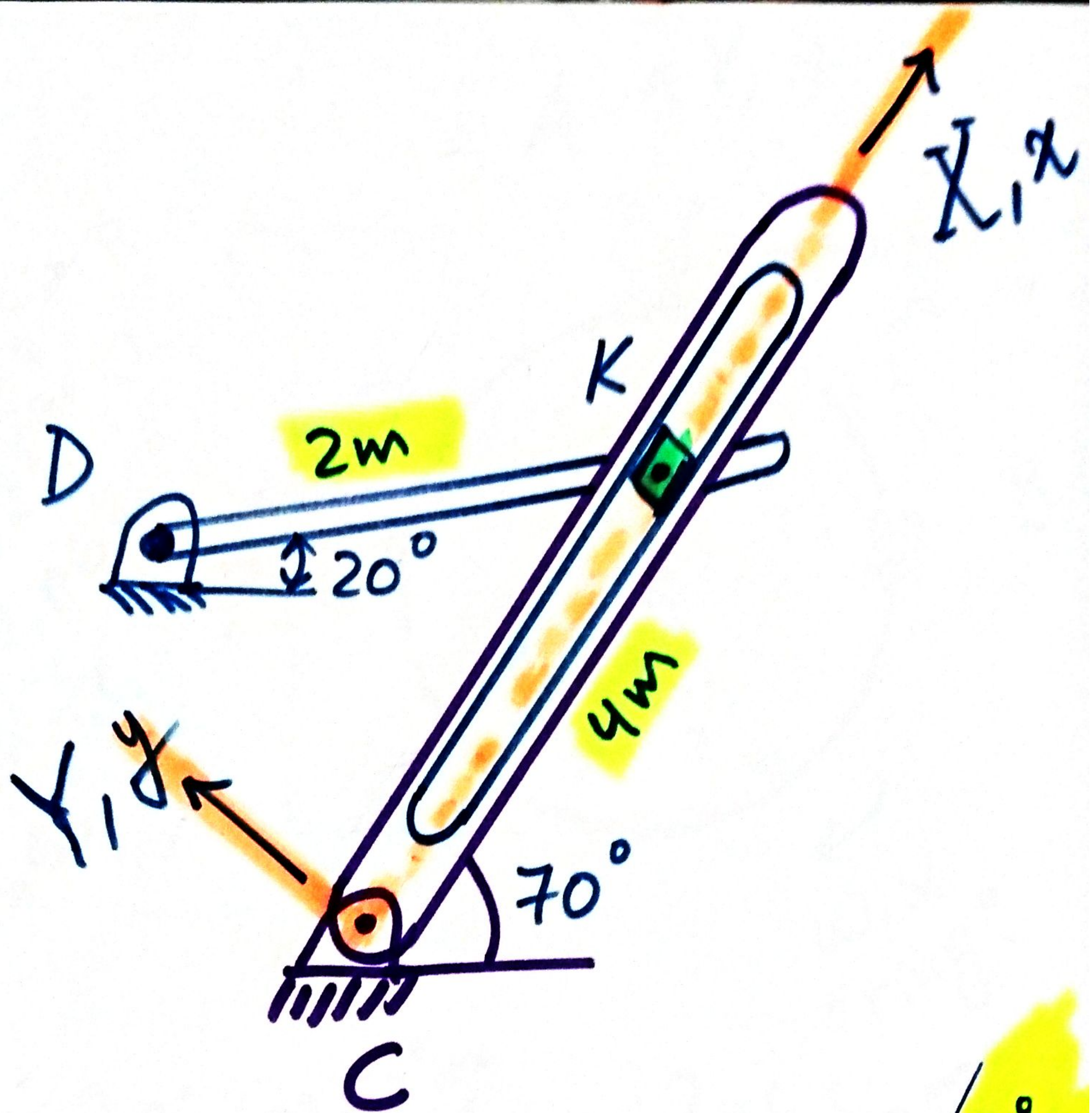


$$\vec{r}_{C/B} = 3\,i$$

$$\vec{r}_{C/A} = 2\cos 25^\circ\, i + 2\sin 25^\circ\, j$$

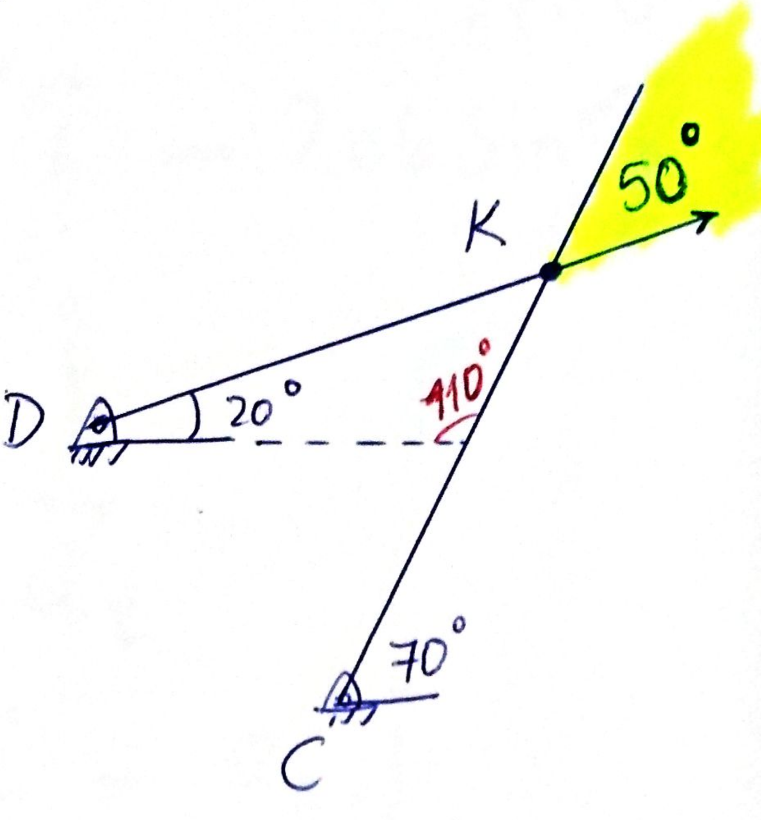
$$\vec{v}_{rel} = v_{rel}\, i$$

$$\vec{a}_{rel} = a_{rel}\, i$$



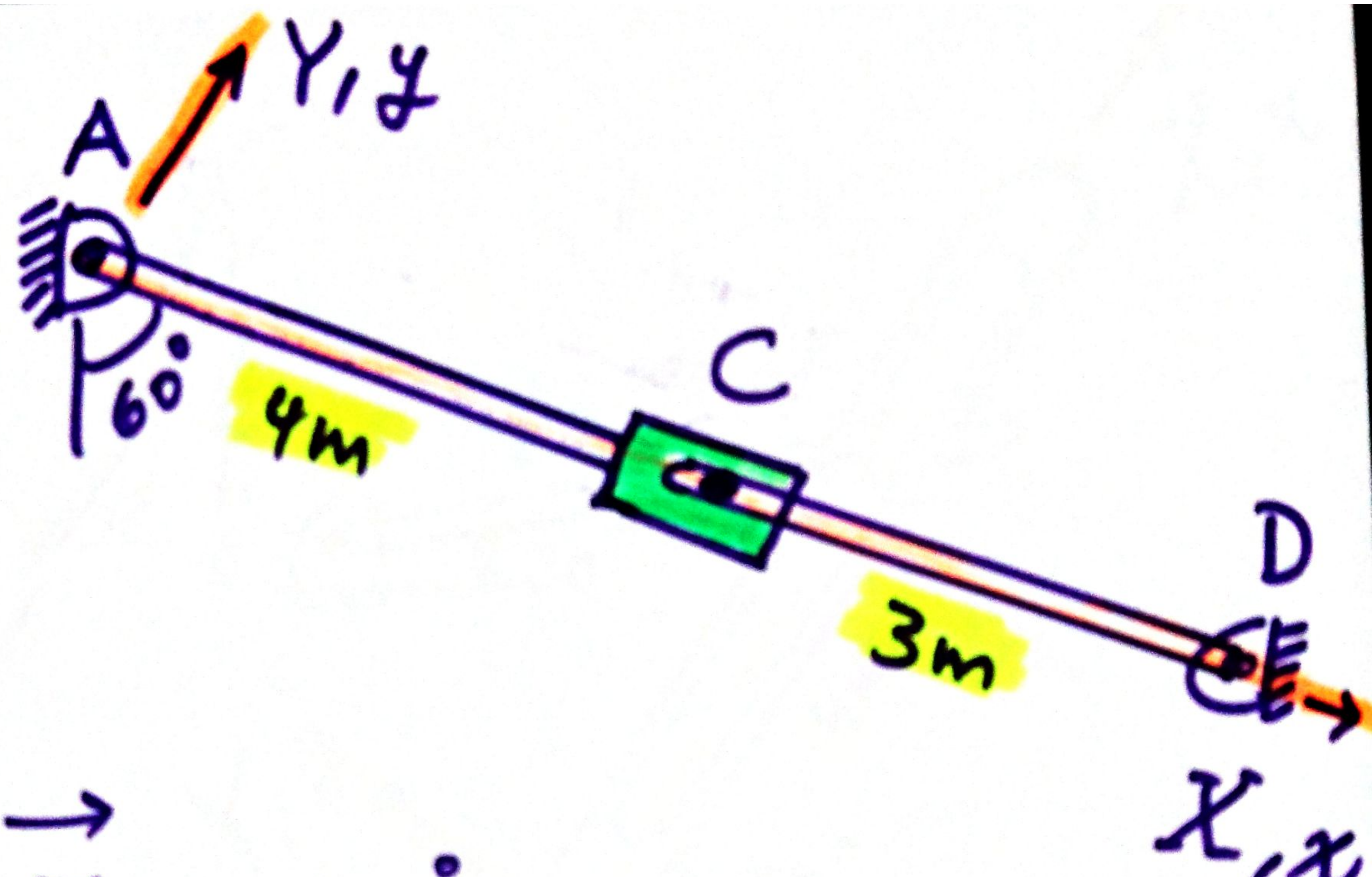
$\vec{r}_{K/C} = 4\,i$

$\vec{r}_{K/D} = 2\cos 50\, i - 2\sin 50\, j$



$\vec{v}_{rel} = v_{rel}\; i$

$\vec{a}_{rel} = a_{rel}.\; i$

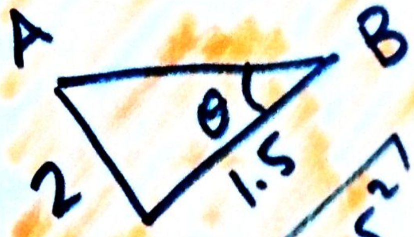


$$\vec{r}_{C/A} = 4\,i$$

$$\vec{r}_{C/D} = -3\,i$$

$$\vec{v}_{rel} = v_{rel}\,i$$

$$\vec{a}_{rel} = a_{rel}\,i$$

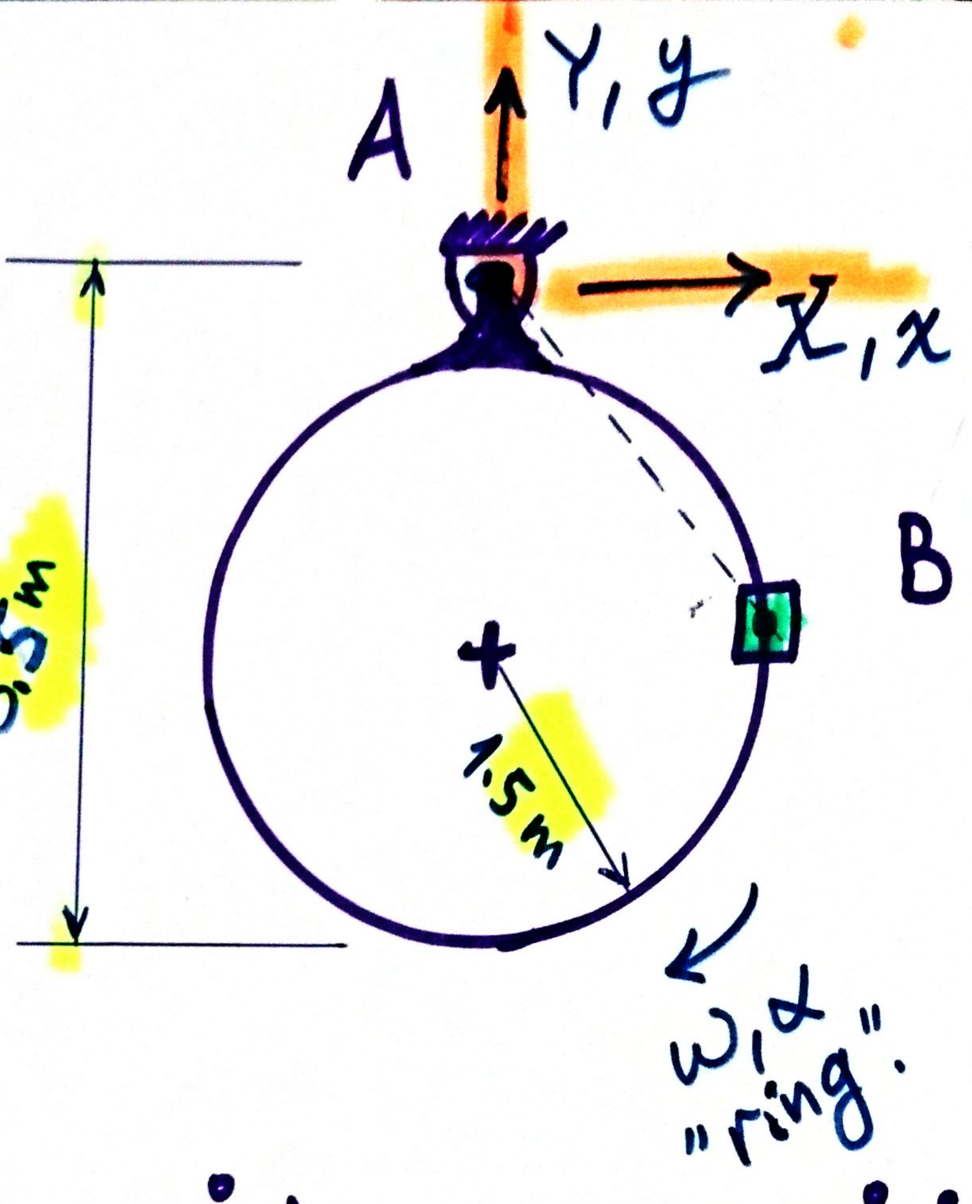


$AB = \sqrt{2^2 + 1.5^2}$

$AB = 2.06\,m$

$\theta = \tan^{-1} \frac{2}{1.5}$

$\theta = 53^\circ$



$$\vec{r}_{B/A} = 2.06 \cos 53^\circ\, i - 2.06 \sin 53^\circ\, j$$

$$\vec{v}_{rel} = - v_{rel}\, j$$

$$\vec{a}_{rel} = \vec{a}_n + \vec{a}_t$$

$$= -\frac{v_{rel}^2}{1.5}\, i - a_t\, j$$

$$* \ v_B = v_A + \omega \times r_{B/A} + v_{rel}$$

$$* \ a_B = a_A + \alpha \times r_{B/A} - \omega^2 r_{B/A} + 2\omega \times v_{rel} + a_{rel}$$

MOMENT OF INERTIA

Objectives: Students will be able to:

1. Determine the mass moment of inertia of a rigid body or a system of rigid bodies.

READING QUIZ

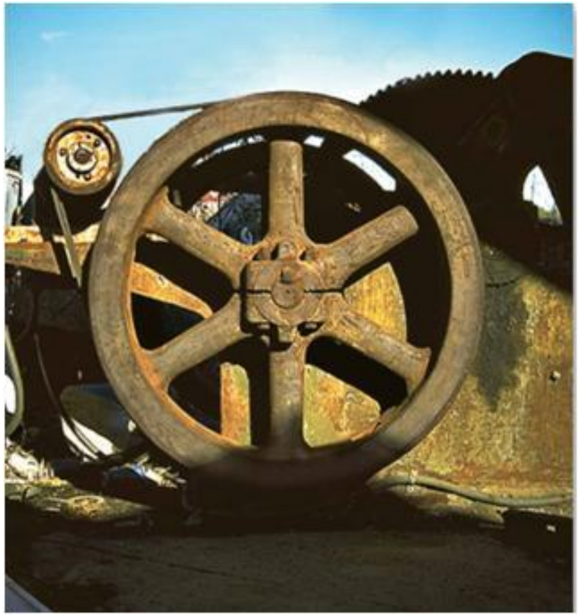
1. Mass moment of inertia is a measure of the resistance of a body to _______________.

   A) translational motion
   B) deformation
   C) angular acceleration
   D) impulsive motion

2. Mass moment of inertia is always ______________.

   A) a negative quantity
   B) a positive quantity
   C) an integer value
   D) zero about an axis perpendicular to the plane of motion

APPLICATIONS



The large flywheel in the picture is connected to a large metal cutter. The flywheel mass is used to help provide a uniform motion to the cutting blade.

What property of the flywheel is most important for this use? How can we determine a value for this property?

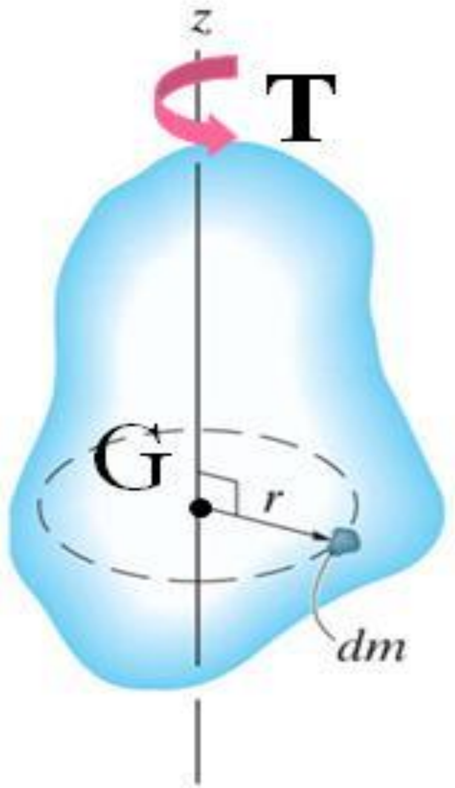
Why is most of the mass of the flywheel located near the flywheel's circumference?

APPLICATIONS
(continued)

The crank on the oil-pump rig undergoes rotation about a fixed axis that is not at its mass center. The crank develops a kinetic energy directly related to its mass moment of inertia. As the crank rotates, its kinetic energy is converted to potential energy and vice versa.

Is the mass moment of inertia of the crank about its axis of rotation smaller or larger than its moment of inertia about its center of mass?

MASS MOMENT OF INERTIA

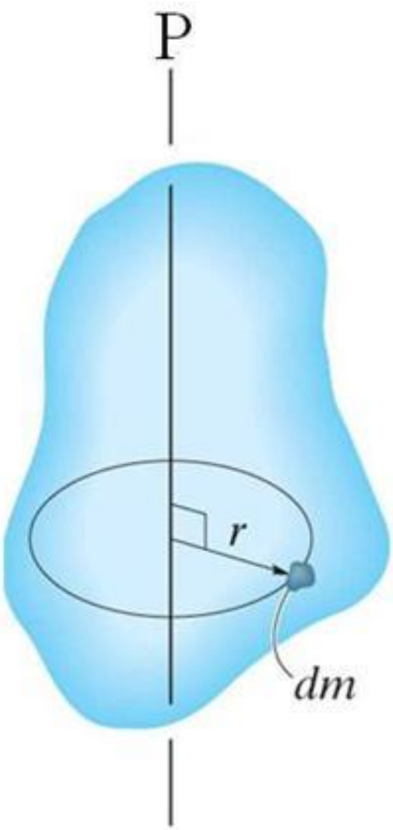


Consider a rigid body with a center of mass at G. It is free to rotate about the z axis, which passes through G. Now, if we apply a torque T about the z axis to the body, the body begins to rotate with an angular acceleration of α.

T and α are related by the equation $T = I\,\alpha$. In this equation, I is the mass moment of inertia (MMI) about the z axis.

The MMI of a body is a property that measures the resistance of the body to angular acceleration. The MMI is often used when analyzing rotational motion.

MASS MOMENT OF INERTIA (continued)

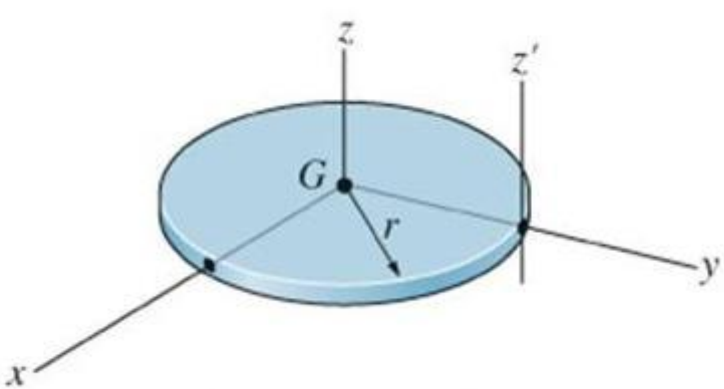


Consider a rigid body and the arbitrary axis P shown in the figure. The MMI about the P axis is defined as $I = \int_m r^2\, dm$, where r, the "moment arm," is the perpendicular distance from the axis to the arbitrary element dm.

The mass moment of inertia is always a positive quantity and has a unit of $kg \cdot m^2$ or $slug \cdot ft^2$.

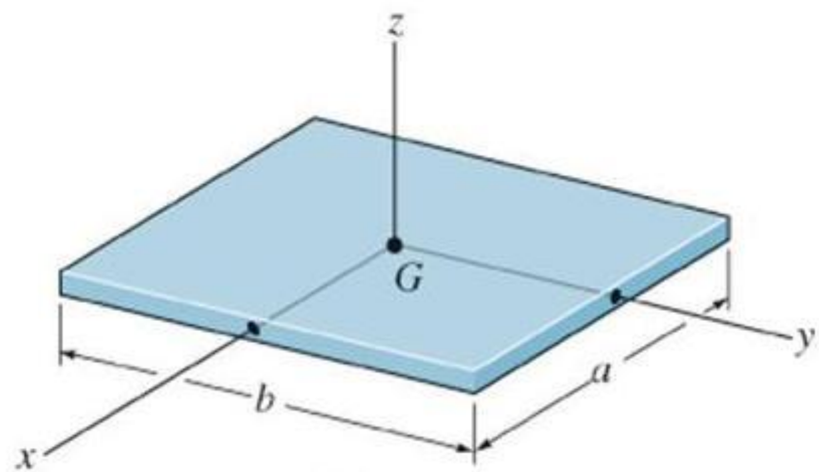
MASS MOMENT OF INERTIA (continued)

The figures below show the mass moment of inertia formulations for two flat plate shapes commonly used when working with three dimensional bodies. The shapes are often used as the differential element being integrated over the entire body.



Thin Circular disk

$I_{xx} = I_{yy} = \frac{1}{4} mr^2 \quad I_{zz} = \frac{1}{2} mr^2 \quad I_{z'z'} = \frac{3}{2} mr^2$



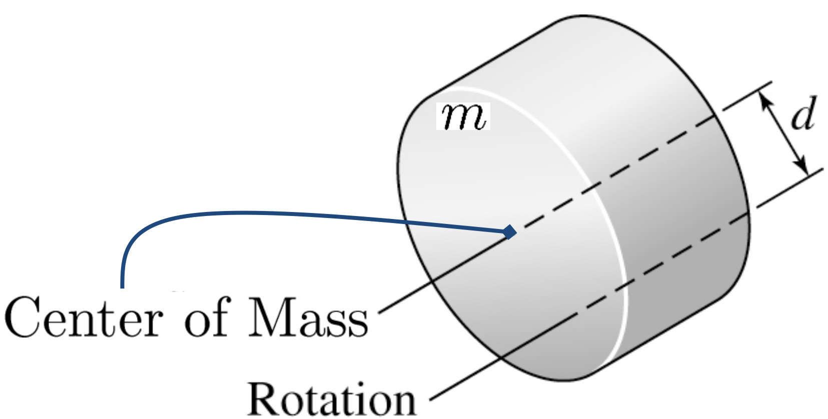
Thin plate

$I_{xx} = \frac{1}{12} mb^2 \quad I_{yy} = \frac{1}{12} ma^2 \quad I_{zz} = \frac{1}{12} m(a^2 + b^2)$

Mass Moment of Inertia

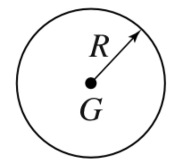
$$I = \int r^2 dm$$

Parallel-Axis Theorem



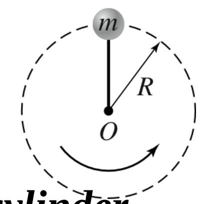
$$I = I_{cm} + md^2$$

Sphere



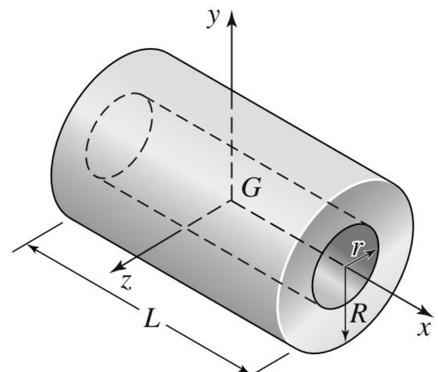
$$I_0 = \frac{2}{5} mR^2$$

Mass rotating about point 0



$$I_0 = mR^2$$

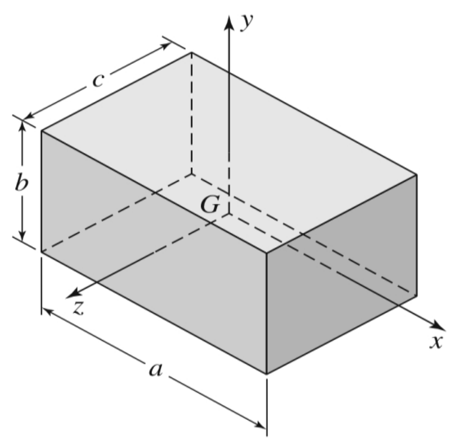
Hollow cylinder



$$I_x = \frac{1}{2} m(R^2 - r^2)$$

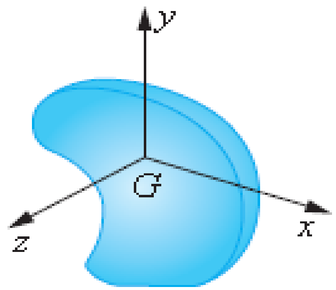
$$I_y = I_z = \frac{1}{12} m(3R^2 - 3r^2 + L^2)$$

Rectangular Prism



$$I_x = \frac{1}{12} m(b^2 + c^2)$$

General shape

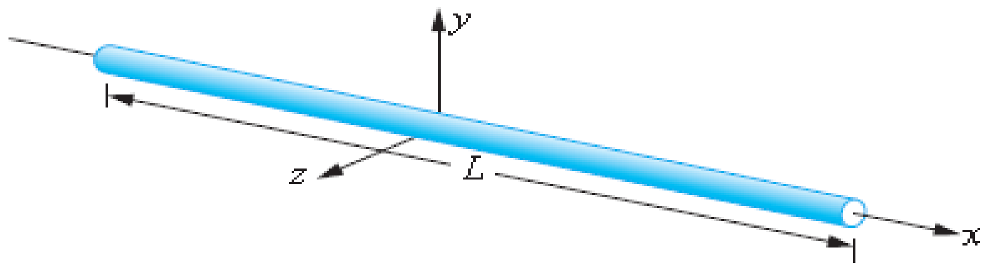


$$\bar{I}_x = \int (y^2 + z^2)\, dm$$

$$\bar{I}_y = \int (x^2 + z^2)\, dm$$

$$\bar{I}_z = \int (x^2 + y^2)\, dm$$

Slender rod

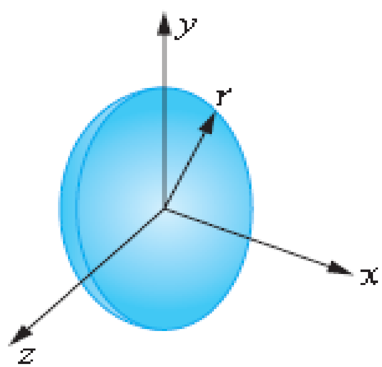


$$\bar{I}_x \approx 0$$

$$\bar{I}_y = \frac{1}{12} mL^2$$

$$\bar{I}_z = \frac{1}{12} mL^2$$

Thin disk

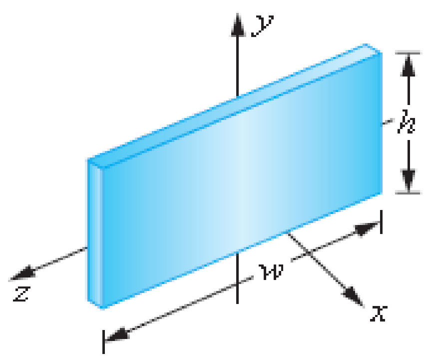


$$\bar{I}_x = \frac{1}{2} mr^2$$

$$\bar{I}_y = \frac{1}{4} mr^2$$

$$\bar{I}_z = \frac{1}{4} mr^2$$

Thin plate



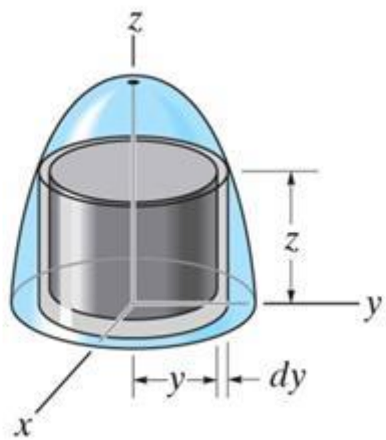
$$\bar{I}_x = \frac{1}{12} m(w^2 + b^2)$$

$$\bar{I}_y = \frac{1}{12} mw^2$$

$$\bar{I}_z = \frac{1}{12} mb^2$$

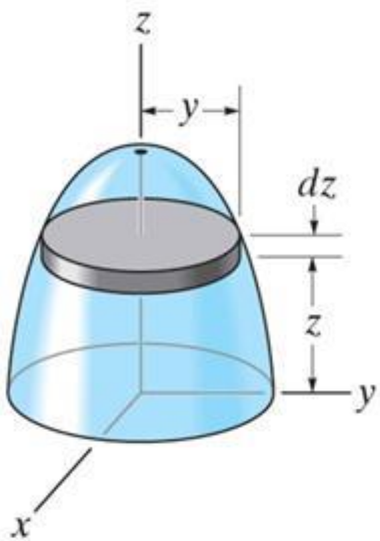
PROCEDURE FOR ANALYSIS

When using direct integration, only symmetric bodies having surfaces generated by revolving a curve about an axis will be considered.



Shell element

- If a shell element having a height z, radius r = y, and thickness dy is chosen for integration, then the volume element is $dV = (2\pi y)(z)dy$.
- This element may be used to find the moment of inertia I_z since the entire element, due to its thinness, lies at the same perpendicular distance y from the z-axis.



Disk element

- If a disk element having a radius y and a thickness dz is chosen for integration, then the volume $dV = (\pi y^2)dz$.
- Using the moment of inertia of the disk element, we can integrate to determine the moment of inertia of the entire body.

PARALLEL-AXIS THEOREM

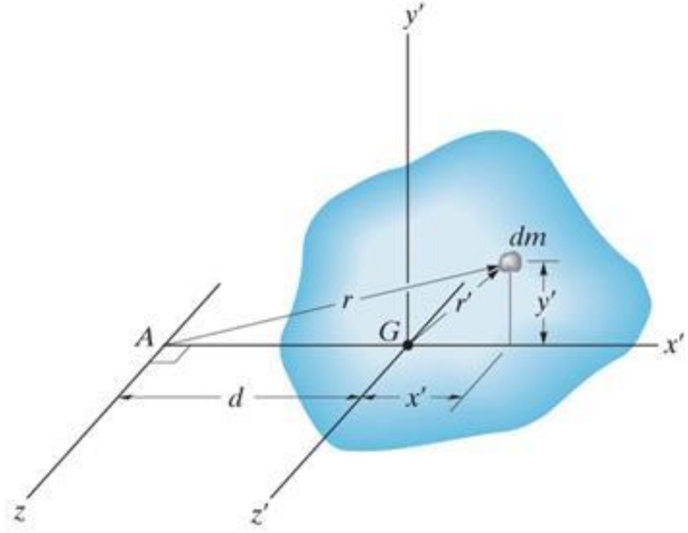
If the mass moment of inertia of a body about an axis passing through the body's mass center is known, then the moment of inertia about any other parallel axis may be determined by using the parallel axis theorem,

$$I = I_G + md^2$$

where I_G = mass moment of inertia about the body's mass center

m = mass of the body

d = perpendicular distance between the parallel axes



RADIUS OF GYRATION AND COMPOSITE BODIES

Radius of Gyration

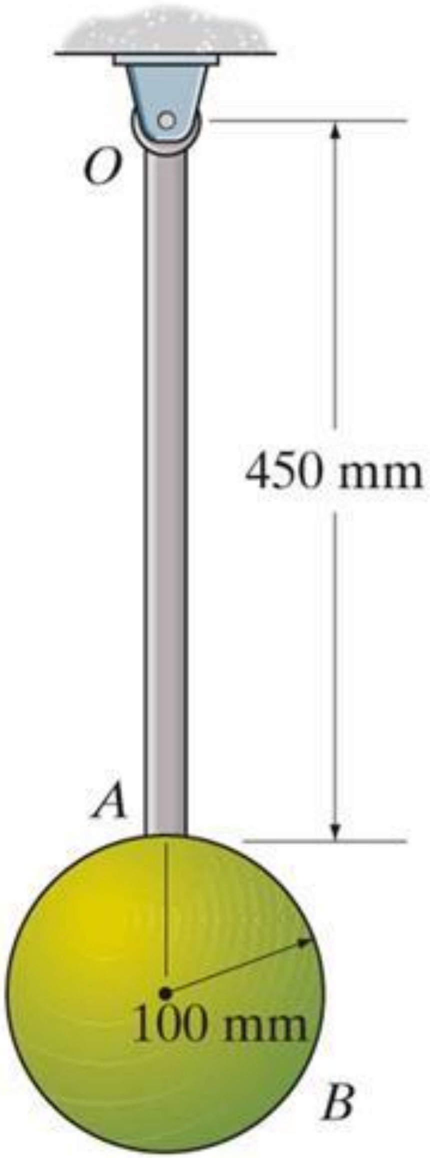
The mass moment of inertia of a body about a specific axis can be defined using the radius of gyration (k). The radius of gyration has units of length and is a measure of the distribution of the body's mass about the axis at which the moment of inertia is defined.

$$I = m\,k^2 \quad \text{or} \quad k = \sqrt{(I/m)}$$

Composite Bodies

If a body is constructed of a number of simple shapes, such as disks, spheres, or rods, the mass moment of inertia of the body about any axis can be determined by algebraically adding together all the mass moments of inertia, found about the same axis, of the different shapes.

EXAMPLE II



Given: The pendulum consists of a slender rod with a mass 10 kg and sphere with a mass of 15 kg.

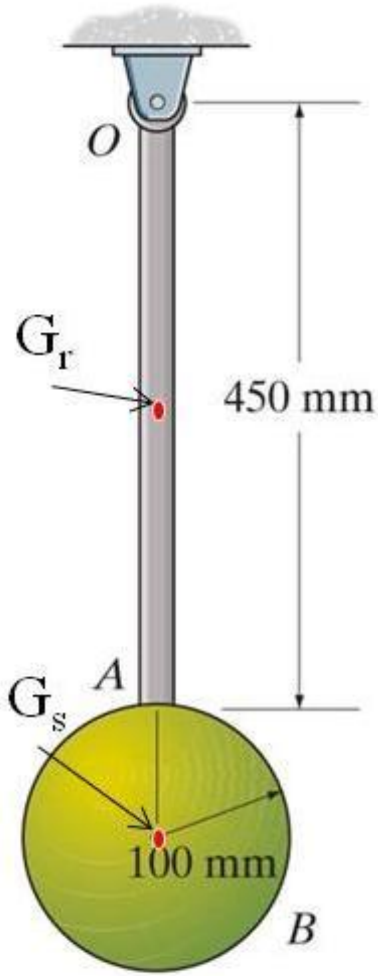
Find: The pendulum's MMI about an axis perpendicular to the screen and passing through point O.

Plan:

Follow steps similar to finding the MoI for a composite area (as done in statics). The pendulum's can be divided into a slender rod (r) and sphere (s)

EXAMPLE II (continued)

Solution:



1. The center of mass for rod is at point G_r, 0.225 m from Point O. The center of mass for sphere is at G_s, 0.55 m from point O.

2. The MMI data for a slender rod and sphere are given on the inside back cover of the textbook. Using those data and the parallel-axis theorem, calculate the following.

$$I_O = I_G + (m)(d)^2$$

$$I_{Or} = (1/12)(10)(0.45)^2 + 10(0.225)^2 = 0.675 \text{ kg}\cdot\text{m}^2$$

$$I_{Os} = (2/5)(15)(0.1)^2 + 15(0.55)^2 = 4.598 \text{ kg}\cdot\text{m}^2$$

3. Now add the two MMIs about point O.

$$I_O = I_{Or} + I_{Os} = 5.27 \text{ kg}\cdot\text{m}^2$$

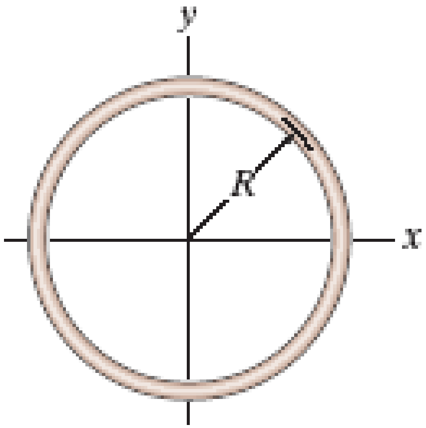
CHECK YOUR UNDERSTANDING QUIZ

1. The mass moment of inertia of a rod of mass m and length L about a transverse axis located at its end is _________ .

A) (1/12) m L^2 B) (1/6) m L^2

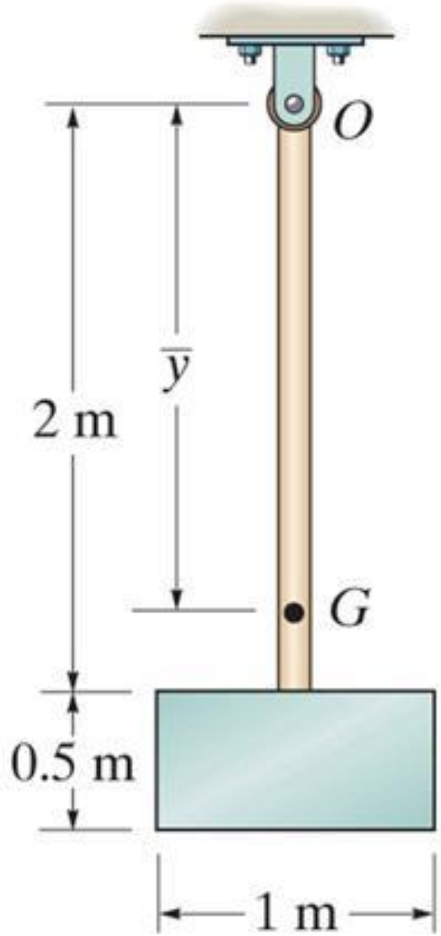
C) (1/3) m L^2 D) m L^2

2. The mass moment of inertia of a thin ring of radius R about the z axis is ___________ .



A) (1/2) m R^2 B) m R^2

C) (1/4) m R^2 D) 2 m R^2



GROUP PROBLEM SOLVING

Given: The pendulum consists of a 5 kg plate and a 3 kg slender rod.

Find: The radius of gyration of the pendulum about an axis perpendicular to the screen and passing through point G.

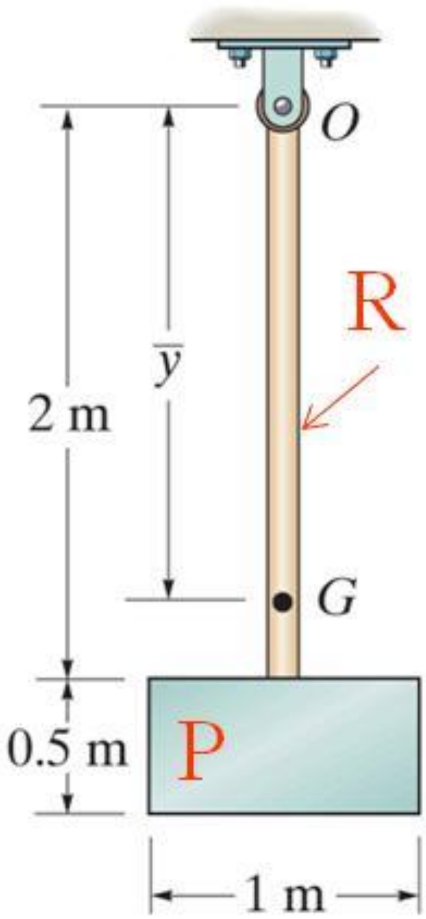
Plan:

Determine the MMI of the pendulum using the method for composite bodies. Then determine the radius of gyration using the MMI and mass values.

GROUP PROBLEM SOLVING (continued)

Solution:

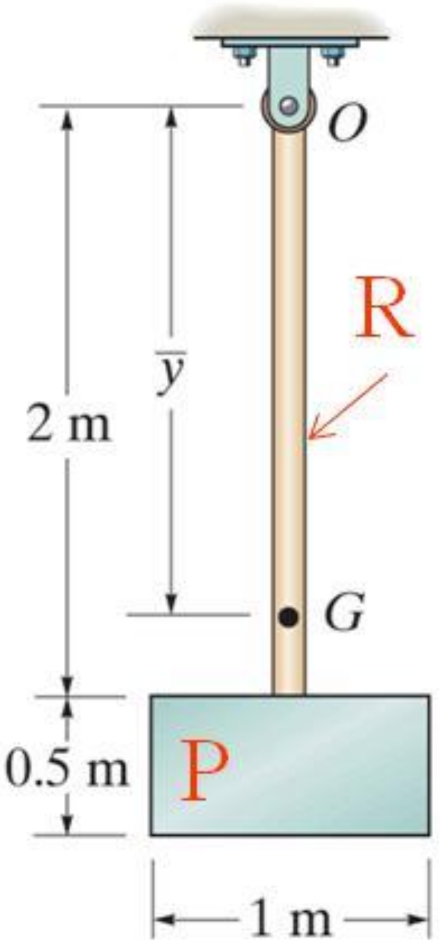
1. Separate the pendulum into a square plate (P) and a slender rod (R).



2. The center of mass of the plate and rod are 2.25 m and 1 m from point O, respectively.

$$\bar{y} = (\Sigma\, \tilde{y}\, m) / (\Sigma\, m)$$

$$= \{(1)\,3 + (2.25)\,5\} / (3+5) \quad = \quad 1.781 \text{ m}$$



GROUP PROBLEM SOLVING (continued)

3. The MMI data on plates and slender rods are given on the inside cover of the textbook. Using those data and the parallel-axis theorem,

$$I_P = (1/12)\,5\,(0.5^2 + 1^2) + 5\,(2.25-1.781)^2$$
$$= 1.621\ \text{kg}\cdot\text{m}^2$$

$$I_R = (1/12)\,3\,(2)^2 + 3\,(1.781 - 1)^2$$
$$= 2.830\ \text{kg}\cdot\text{m}^2$$

4. $I_O = I_P + I_R = 1.621 + 2.830 = 4.45\ \text{kg}\cdot\text{m}^2$

5. Total mass (m) equals 8 kg

Radius of gyration $k = \sqrt{I_O / m} = 0.746\ \text{m}$

ATTENTION QUIZ

1. The mass moment of inertia of any body about its center of mass is always _____________.

   A) maximum
   B) minimum
   C) zero
   D) None of the above

2. If the mass of body A and B are equal but $k_A = 2k_B$, then _____________.

   A) $I_A = 2I_B$
   B) $I_A = (1/2)I_B$
   C) $I_A = 4I_B$
   D) $I_A = (1/4)I_B$

PLANAR KINETIC EQUATIONS OF MOTION: TRANSLATION

Objectives:

Students will be able to:

1. Apply the three equations of motion for a rigid body in planar motion.
2. Analyze problems involving translational motion.

READING QUIZ

1. When a rigid body undergoes translational motion due to external forces, the translational equations of motion (EOM) can be expressed for _______________.

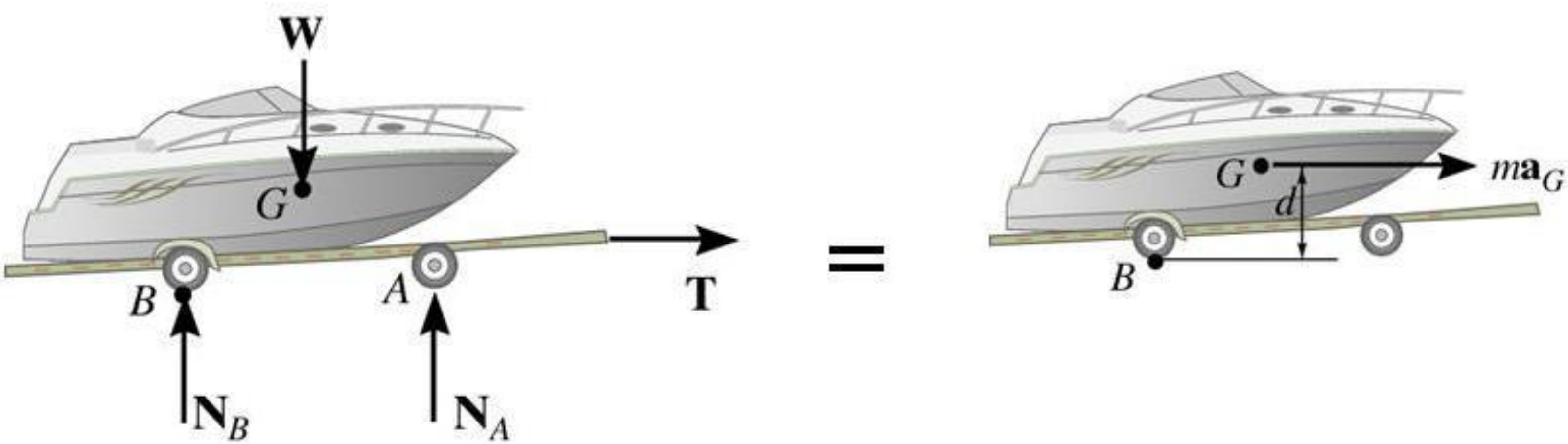
   A) the center of rotation
   B) the center of mass
   C) any arbitrary point
   D) All of the above

2. The rotational EOM about the mass center of the rigid body indicates that the sum of moments due to the external loads equals _______________.

   A) $I_G \alpha$
   B) $m a_G$
   C) $I_G \alpha + m a_G$
   D) None of the above.

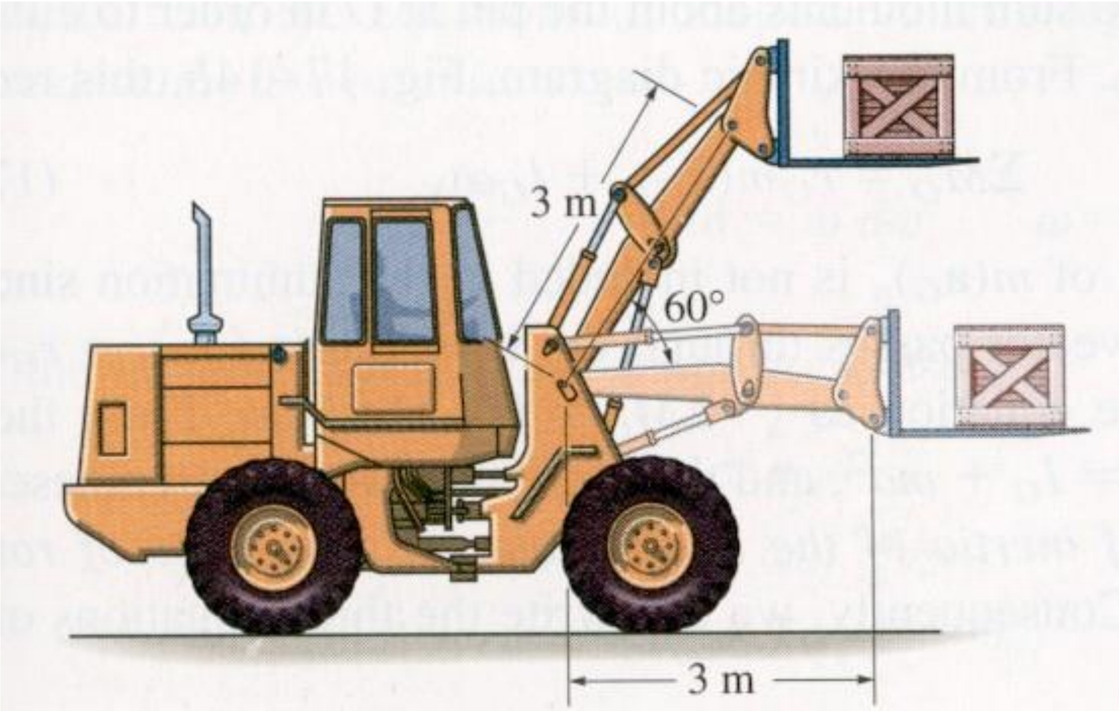
APPLICATIONS

The boat and trailer undergo rectilinear motion. In order to find the reactions at the trailer wheels and the acceleration of the boat, we need to draw the FBD and kinetic diagram for the boat and trailer.



How many equations of motion do we need to solve this problem? What are they?

APPLICATIONS (continued)



As the tractor raises the load, the crate will undergo curvilinear translation if the forks do not rotate.

If the load is raised too quickly, will the crate slide to the left or right?

How fast can we raise the load before the crate will slide?

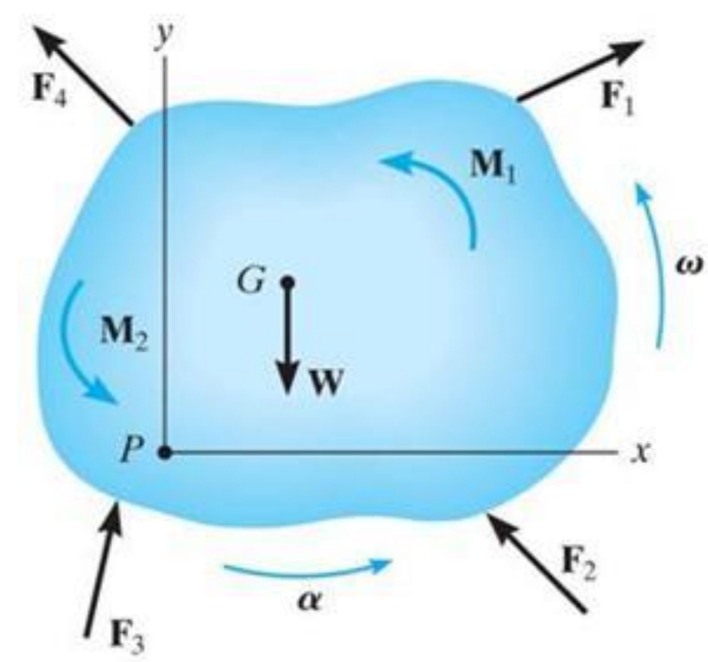
PLANAR KINETIC EQUATIONS OF MOTION

- We will limit our study of planar kinetics to rigid bodies that are symmetric with respect to a fixed reference plane.

- As discussed in Chapter 16, when a body is subjected to general plane motion, it undergoes a combination of translation and rotation.

- First, a coordinate system with its origin at an arbitrary point P is established.

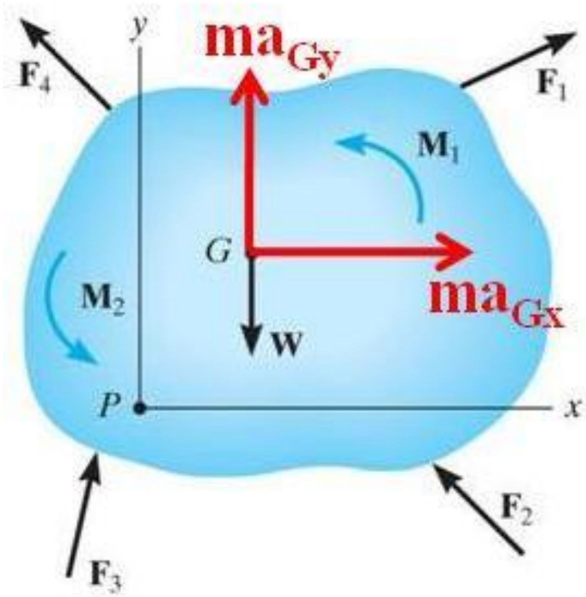
  The x-y axes should not rotate and can either be fixed or translate with constant velocity.



EQUATIONS OF TRANSLATIONAL MOTION

- If a body undergoes translational motion, the equation of motion is $\Sigma \boldsymbol{F} = m\,\mathbf{a}_G$. This can also be written in scalar form as

$$\Sigma F_x = m(a_G)_x \qquad \text{and}$$
$$\Sigma F_y = m(a_G)_y$$



- In words: the sum of all the external forces acting on the body is equal to the body's mass times the acceleration of it's mass center.

EQUATIONS OF ROTATIONAL MOTION

We need to determine the effects caused by the moments of an external force system.

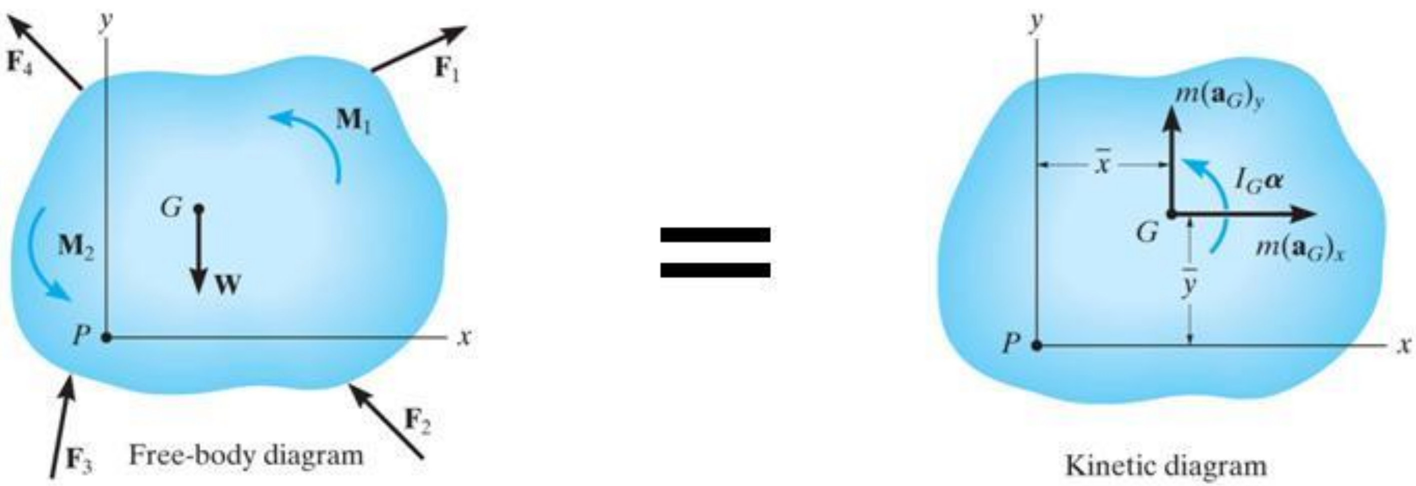
The moment about point P can be written as:

$$\Sigma\,(\boldsymbol{r}_{\mathrm{i}} \times \boldsymbol{F}_{\mathrm{i}}) + \Sigma\,\boldsymbol{M}_{\mathrm{i}} \;=\; \bar{\boldsymbol{r}} \times \mathrm{m}\mathbf{a}_{\mathrm{G}} \;+\; \mathrm{I}_{\mathrm{G}}\boldsymbol{\alpha}$$

$$\Sigma\,\mathrm{M}_{\mathrm{p}} = \Sigma(M_k)_{\mathrm{p}}$$

where $\bar{\boldsymbol{r}} = \bar{\mathrm{x}}\,\boldsymbol{i} + \bar{\mathrm{y}}\,\boldsymbol{j}$ and $\Sigma\,\mathrm{M}_{\mathrm{p}}$ is the resultant moment about P due to all the external forces.

The term $\Sigma(M_k)_{\mathbf{p}}$ is called the kinetic moment about point P.



Free-body diagram

Kinetic diagram

EQUATIONS OF ROTATIONAL MOTION

If point P coincides with the mass center G, this equation reduces to the scalar equation of $\Sigma M_G = I_G \alpha$.

In words: the resultant (summation) moment about the mass center due to all the external forces is equal to the moment of inertia about G times the angular acceleration of the body.

Thus, three independent scalar equations of motion may be used to describe the general planar motion of a rigid body. These equations are:

$$\Sigma F_x = m(a_G)_x$$

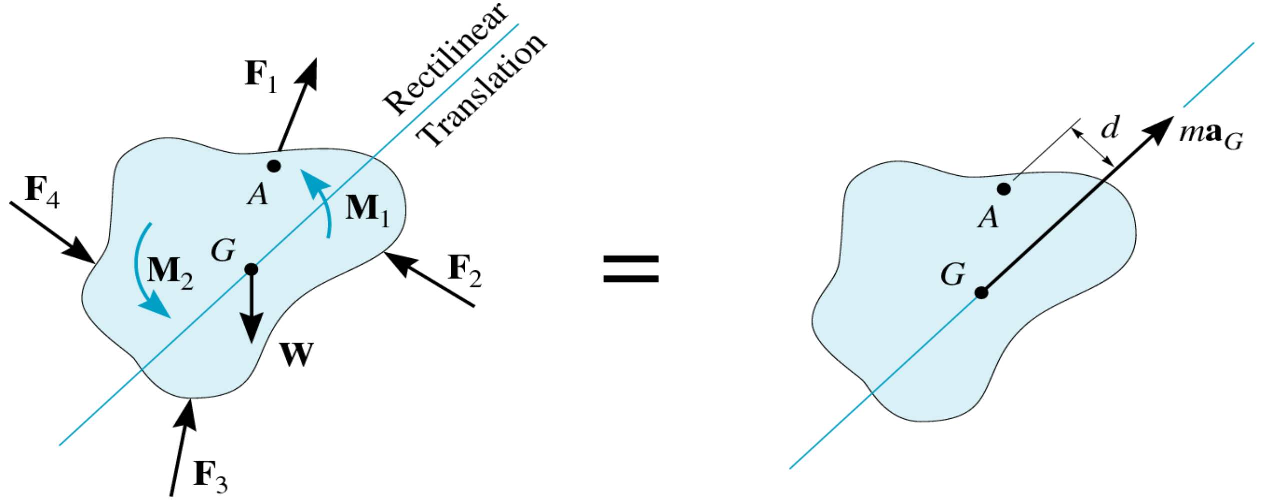
$$\Sigma F_y = m(a_G)_y$$

and $\Sigma M_G = I_G\alpha$ or $\Sigma M_p = \Sigma (M_k)_p$

EQUATIONS OF MOTION: TRANSLATION

When a rigid body undergoes only translation, all the particles of the body have the same acceleration so $\mathbf{a}_G = \mathbf{a}$ and $\boldsymbol{\alpha} = \boldsymbol{0}$. The equations of motion become:

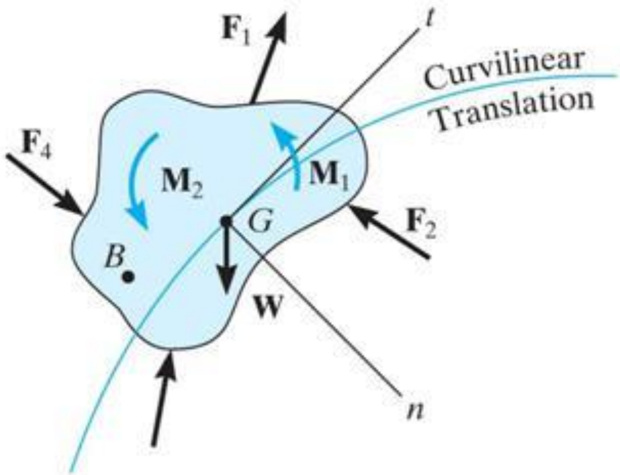
$$\Sigma F_x = m(a_G)_x$$
$$\Sigma F_y = m(a_G)_y$$
$$\Sigma M_G = 0$$

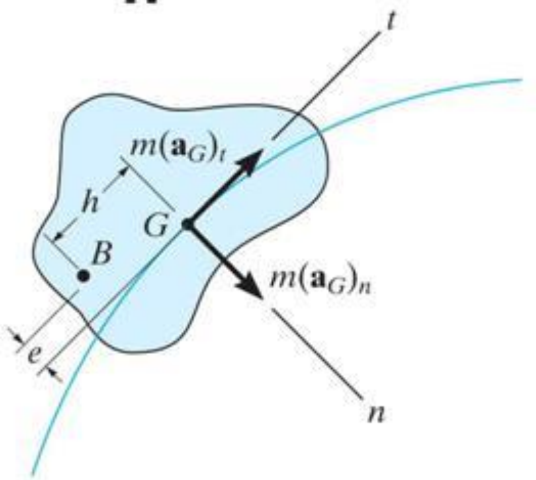


Note that, if it makes the problem easier, the moment equation can be applied about another point instead of the mass center. For example, if point A is chosen,

$$\Sigma M_A = (m\ a_G)\ d\ .$$

EQUATIONS OF MOTION: TRANSLATION





When a rigid body is subjected to curvilinear translation, it is best to use an n-t coordinate system.
Then apply the equations of motion, as written below, for n-t coordinates.

$$\Sigma F_n = m(a_G)_n$$

$$\Sigma F_t = m(a_G)_t$$

$$\Sigma M_G = 0 \quad \text{or}$$

$$\Sigma M_B = e[m(a_G)_t] - h[m(a_G)_n]$$

PROCEDURE FOR ANALYSIS

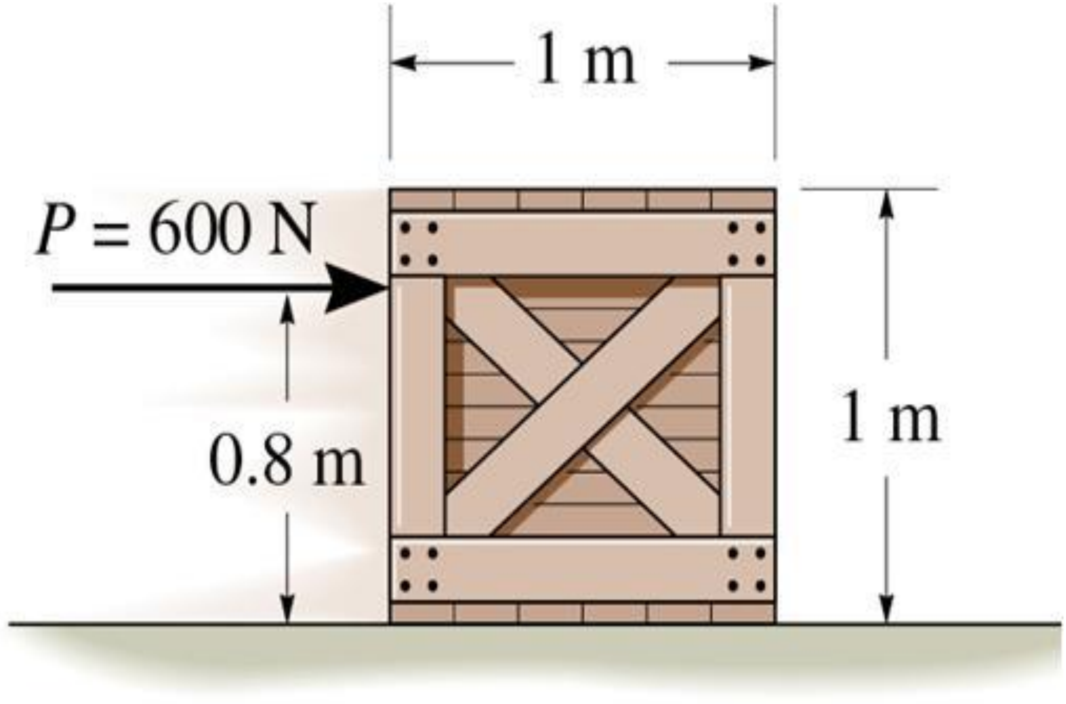
Problems involving kinetics of a rigid body in only translation should be solved using the following procedure:

1. Establish an (x-y) or (n-t) inertial coordinate system and specify the sense and direction of acceleration of the mass center, $\mathbf{a}_G$.
2. Draw a FBD and kinetic diagram showing all external forces, couples and the inertia forces and couples.
3. Identify the unknowns.
4. Apply the three equations of motion (one set or the other):

$\Sigma F_x = m(a_G)_x \quad \Sigma F_y = m(a_G)_y$	$\Sigma F_n = m(a_G)_n \quad \Sigma F_t = m(a_G)_t$
$\Sigma M_G = 0 \quad \text{or} \quad \Sigma M_P = \Sigma (M_k)_P$	$\Sigma M_G = 0 \quad \text{or} \quad \Sigma M_P = \Sigma (M_k)_P$

5. Remember, friction forces always act on the body opposing the motion of the body.

EXAMPLE



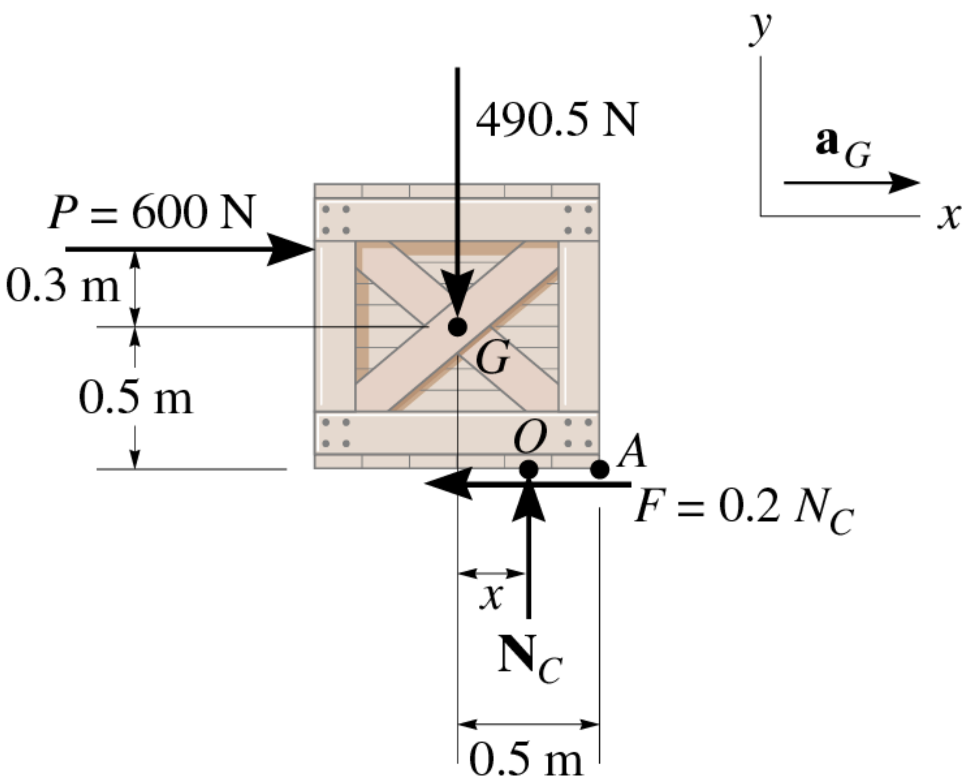
Given: A 50 kg crate rests on a horizontal surface for which the kinetic friction coefficient $\mu_k = 0.2$.

Find: The acceleration of the crate if P = 600 N.

Plan: Follow the procedure for analysis.

Note that the load P can cause the crate either to slide or to tip over. Let's assume that the crate slides. We will check this assumption later.

EXAMPLE (continued) **Solution:**



The coordinate system and FBD are as shown. The weight of (50)(9.81) N is applied at the center of mass and the normal force N_c acts at O. Point O is some distance x from the crate's center line. The unknowns are N_c, x, and a_G .

Applying the equations of motion:

$$\Sigma F_x = m(a_G)_x: 600 - 0.2\ N_c = 50\ a_G$$

$$\Sigma F_y = m(a_G)_y: N_c - 490.5 = 0$$

$$\Sigma M_G = 0: -600(0.3) + N_c(x) - 0.2\ N_c(0.5) = 0$$

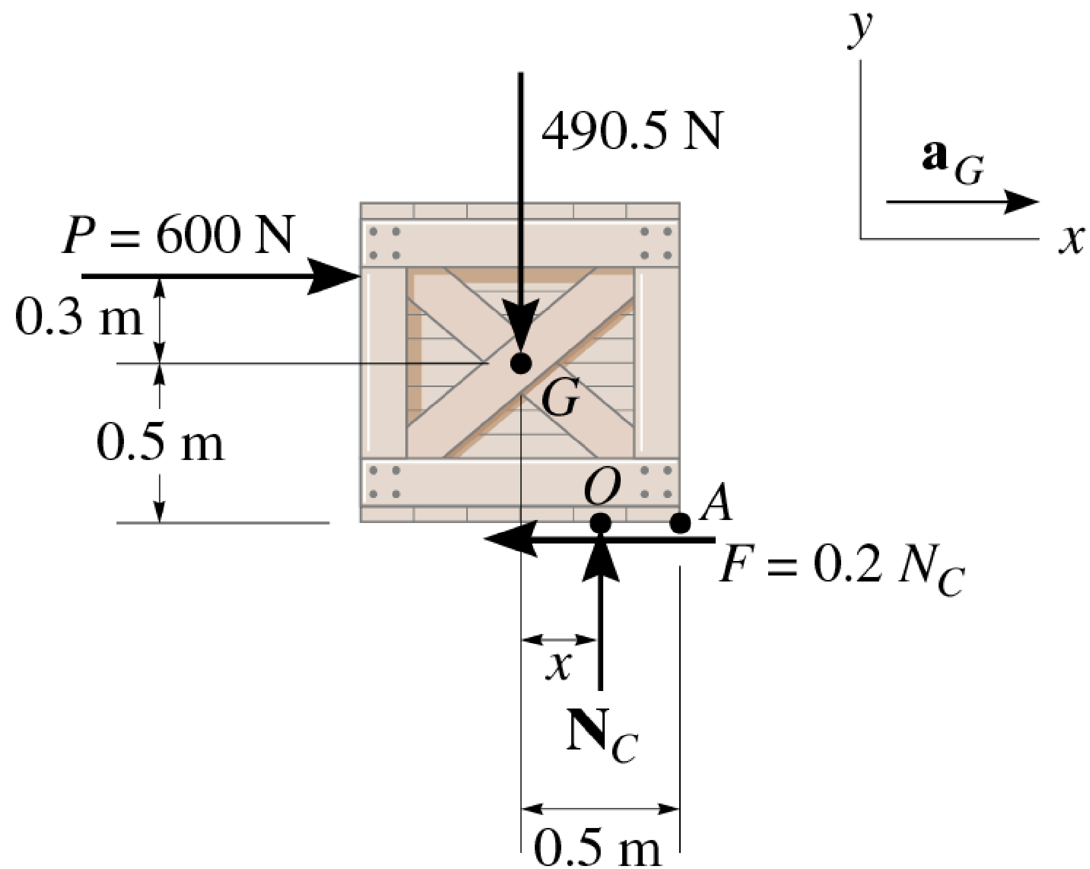
$\Rightarrow$

$N_c = 490$ N

$x = 0.467$ m

$a_G = 10.0\ m/s^2$

EXAMPLE (continued)

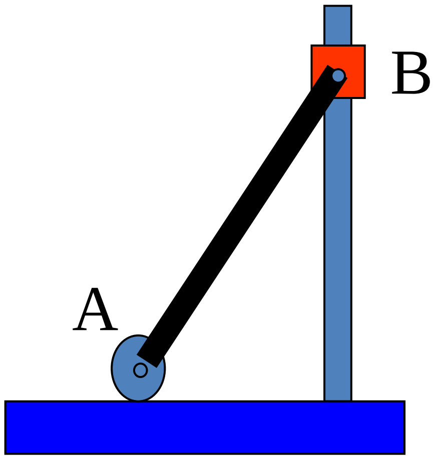


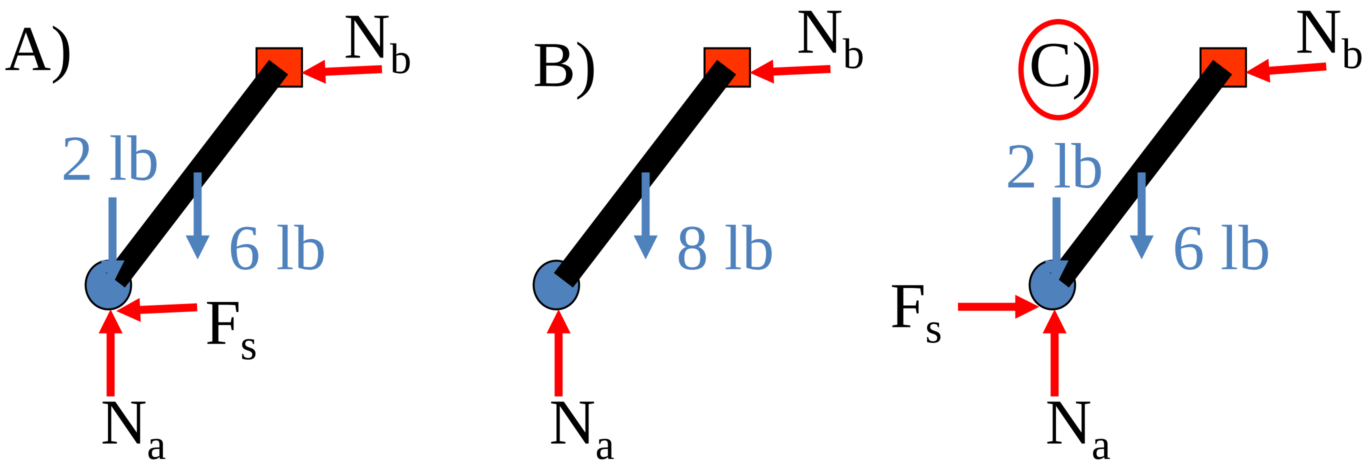
Since x = 0.467 m < 0.5 m, the crate slides as originally assumed.

If x was greater than 0.5 m, the problem would have to be reworked with the assumption that tipping occurred.

CONCEPT QUIZ

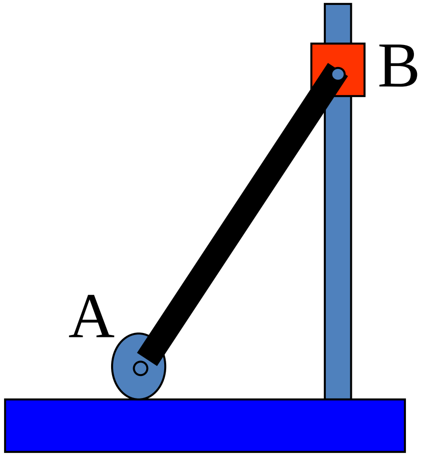
1. A 2 lb disk is attached to a uniform 6 lb rod AB with a frictionless collar at B. If the disk rolls without slipping, select the correct FBD.

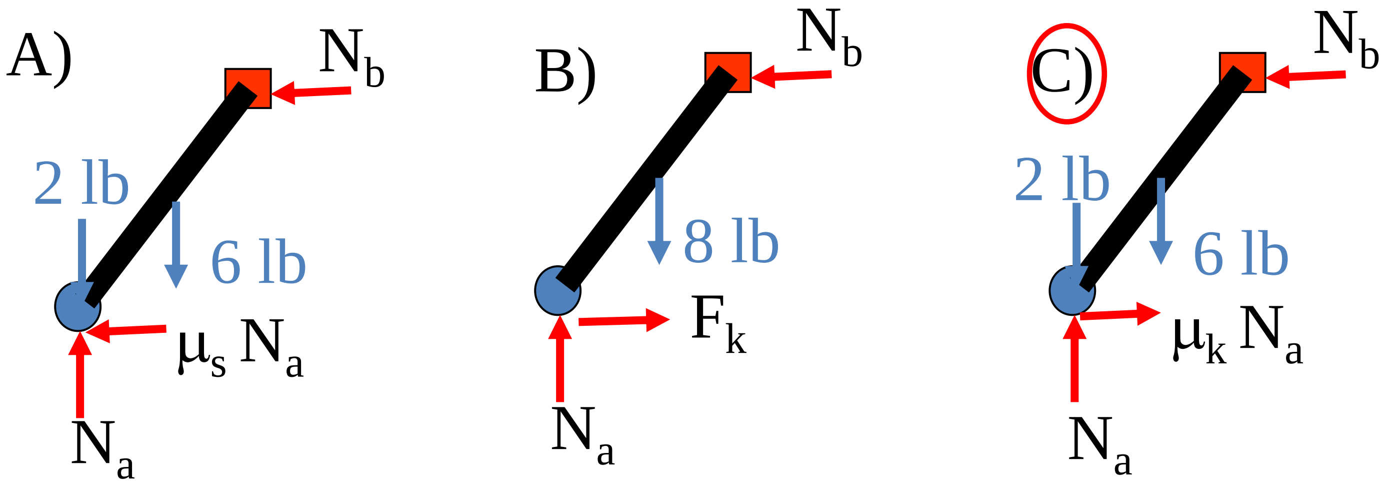




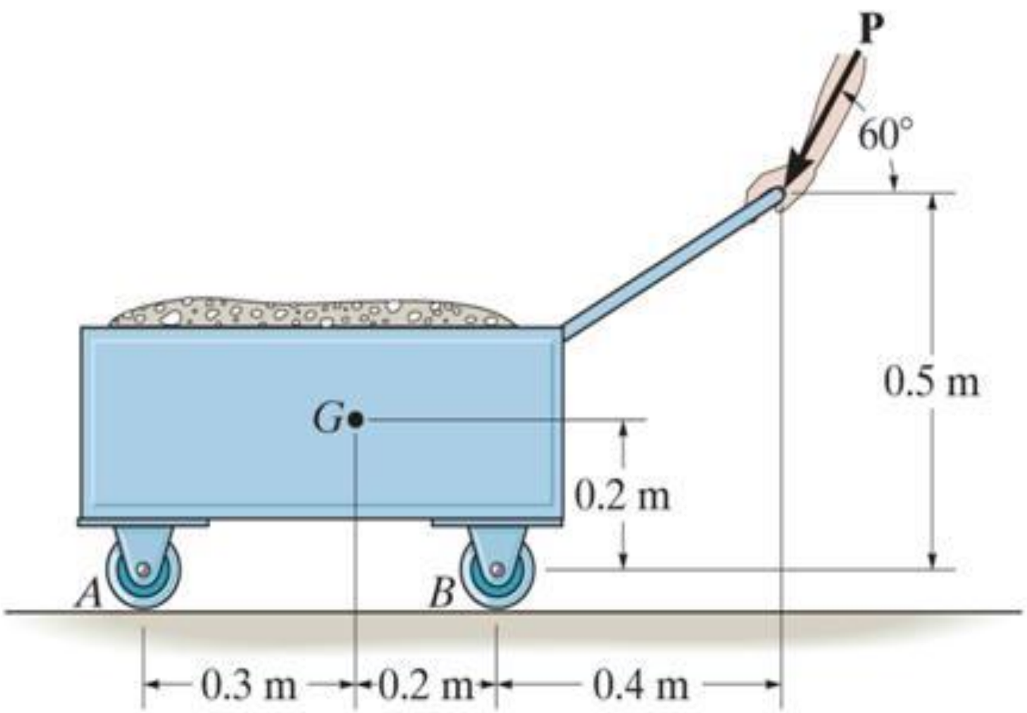
CONCEPT QUIZ

2. A 2 lb disk is attached to a uniform 6 lb rod AB with a frictionless collar at B. If the disk rolls with slipping, select the correct FBD.





GROUP PROBLEM SOLVING



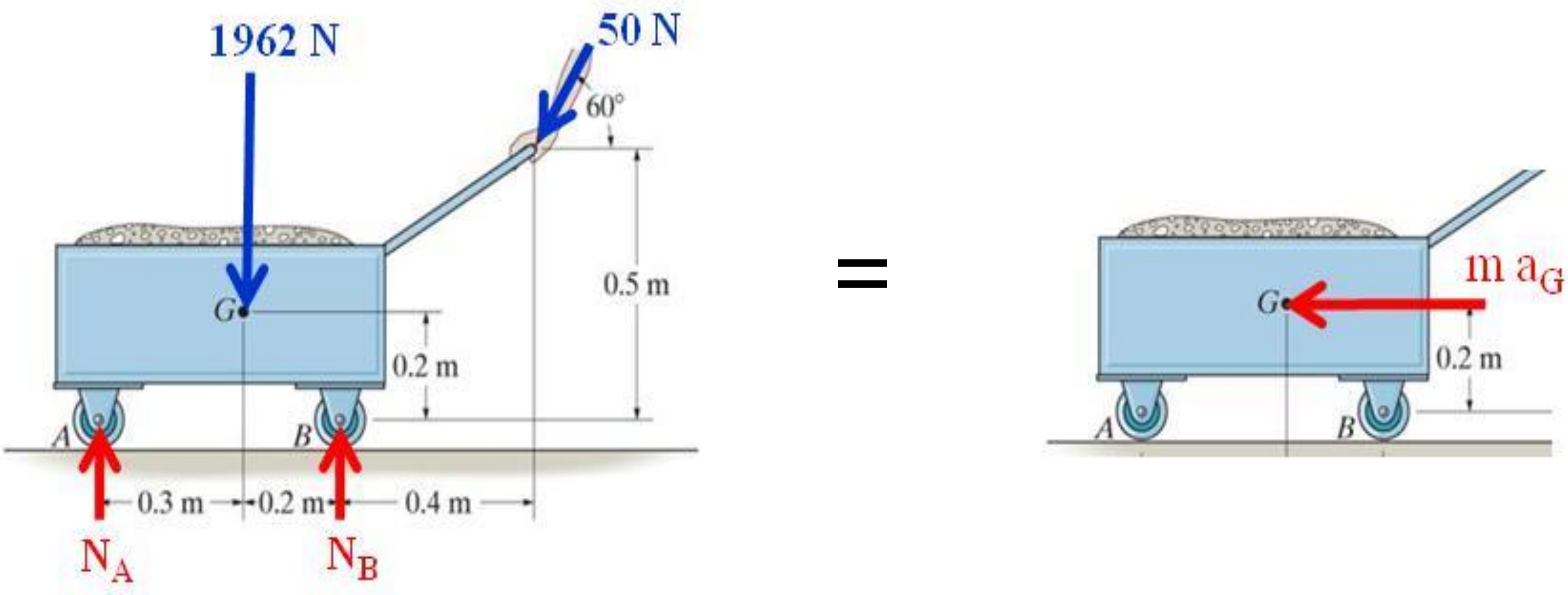
Given: The handcart has a mass of 200 kg and center of mass at *G*. A force of P=50 N is applied to the handle. Neglect the mass of the wheels.

Find: The normal reactions at each of the two wheels at A and B.

Plan: Follow the procedure for analysis.

GROUP PROBLEM SOLVING (continued)

Solution: The cart will move along a rectilinear path.
Draw FBD and kinetic diagram.



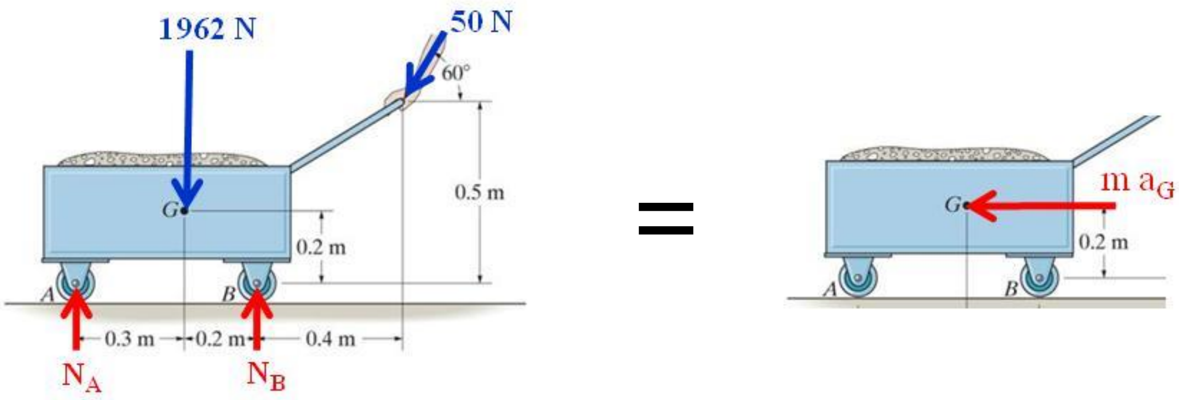
Applying the equations of motion:

$$\overset{+}{\leftarrow}\Sigma F_x = m(a_G)_x$$

$$50 \cos 60° = 200\, a_G \quad \Rightarrow \quad a_G = 0.125 \text{ m/s}^2$$

GROUP PROBLEM SOLVING (continued)

Applying the equations of motion:



$+\uparrow \Sigma F_y = 0 \Rightarrow N_A + N_B - 1962 - 50 \sin 60° = 0$

$N_A + N_B = 2005 \text{ N} \qquad (1)$

$\Sigma M_G = 0$

$\Rightarrow -(0.3)N_A + (0.2)N_B + 0.3(50 \cos 60°) - 0.6(50 \sin 60°) = 0$

$-0.3\,N_A + 0.2\,N_B = 18.48 \text{ N m} \qquad (2)$

Using Eqs. (1) and (2), solve for the reactions, N_A and N_B

$N_A = 765 \text{ N}, N_B = 1240 \text{ N}$

ATTENTION QUIZ

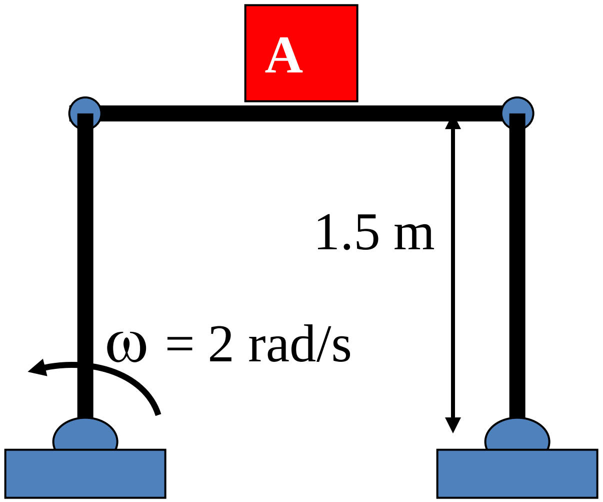
1. As the linkage rotates, box A undergoes ____________.

   A) general plane motion

   B) pure rotation

   C) linear translation

   D) curvilinear translation



2. The number of independent scalar equations of motion that can be applied to box A is?

   A) One

   B) Two

   C) Three

   D) Four

EQUATIONS OF MOTION: ROTATION ABOUT A FIXED AXIS

Objectives:

Students will be able to:

1. Analyze the planar kinetics of a rigid body undergoing rotational motion.

READING QUIZ

1. In rotational motion, the normal component of acceleration at the body's center of gravity (G) is always __________.

   A) zero

   B) tangent to the path of motion of G

   C) directed from G toward the center of rotation

   D) directed from the center of rotation toward G

2. If a rigid body rotates about point O, the sum of the moments of the external forces acting on the body about point O equals?

   A) $I_G\alpha$

   B) $I_O\alpha$

   C) m a_G

   D) m a_O

APPLICATIONS

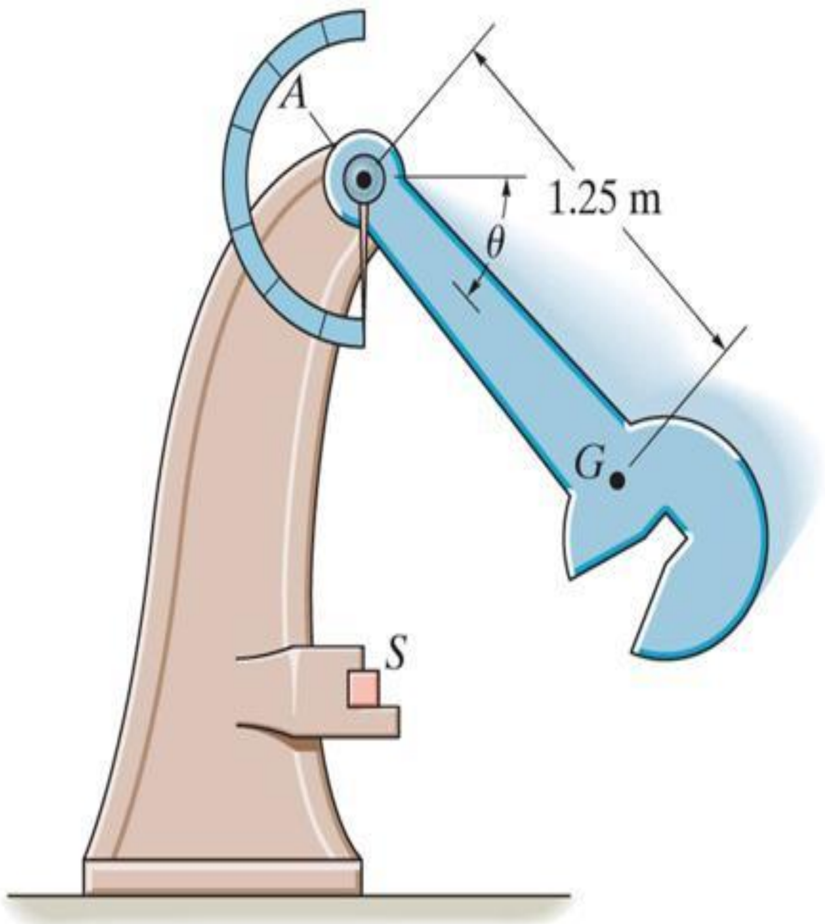
Pin at the center of rotation.

The crank on the oil-pump rig undergoes rotation about a fixed axis, caused by the driving torque, M, from a motor.

As the crank turns, a dynamic reaction is produced at the pin. This reaction is a function of angular velocity, angular acceleration, and the orientation of the crank.

If the motor exerts a constant torque M on the crank, does the crank turn at a constant angular velocity? Is this desirable for such a machine?

APPLICATIONS (continued)



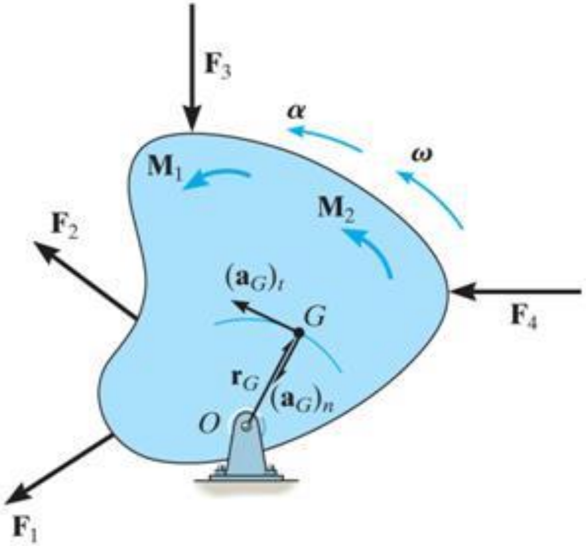
The pendulum of the Charpy impact machine is released from rest when $\theta = 0°$. Its angular velocity (ω) begins to increase.

Can we determine the angular velocity when it is in vertical position ?

On which property (P) of the pendulum does the angular acceleration (α) depend ?

What is the relationship between P and α?

EQUATIONS OF MOTION: ROTATION ABOUT A FIXED AXIS



When a rigid body rotates about a fixed axis perpendicular to the plane of the body at point O, the body's center of gravity G moves in a circular path of radius r_G. Thus, the acceleration of point G can be represented by a tangential component $(a_G)_t = r_G\,\alpha$ and a normal component $(a_G)_n = r_G\,\omega^2$.

Since the body experiences an angular acceleration, its inertia creates a moment of magnitude, $I_g\alpha$, equal to the moment of the external forces about point G. Thus, the scalar equations of motion can be stated as:

$$\sum F_n = m\,(a_G)_n = m\,r_G\,\omega^2$$

$$\sum F_t = m\,(a_G)_t = m\,r_G\,\alpha$$

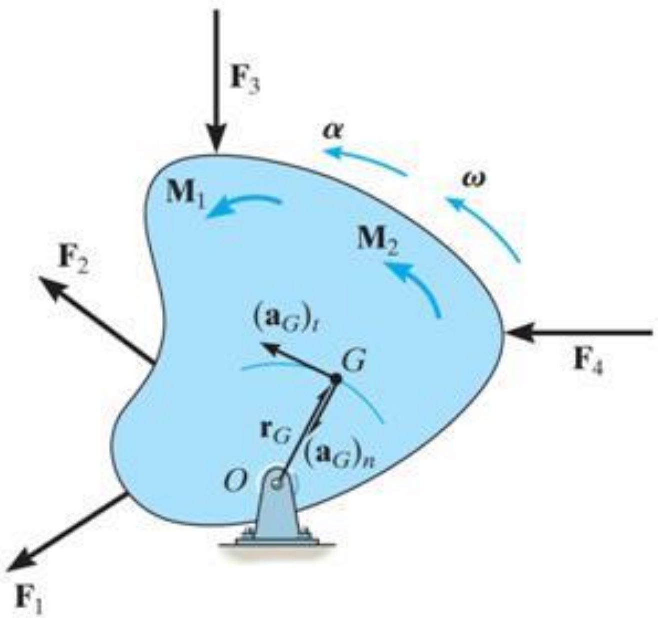
$$\sum M_G = I_G\,\alpha$$

EQUATIONS OF MOTION (continued)

Note that the $\sum M_G$ moment equation may be replaced by a moment summation about any arbitrary point. Summing the moment about the center of rotation O yields

$$\sum M_O = I_G\alpha + r_G\, m\, (a_G)_t = [I_G + m(r_G)^2]\, \alpha$$

From the parallel axis theorem, $I_O = I_G + m(r_G)^2$, therefore the term in parentheses represents I_O.



Consequently, we can write the three equations of motion for the body as:

$$\sum F_n = m\, (a_G)_n = m\, r_G\, \omega^2$$

$$\sum F_t = m\, (a_G)_t = m\, r_G\, \alpha$$

$$\sum M_O = I_O\, \alpha$$

PROCEDURE FOR ANALYSIS

Problems involving the kinetics of a rigid body rotating about a fixed axis can be solved using the following process.

1. Establish an inertial coordinate system and specify the sign and direction of $(a_G)_n$ and $(a_G)_t$.

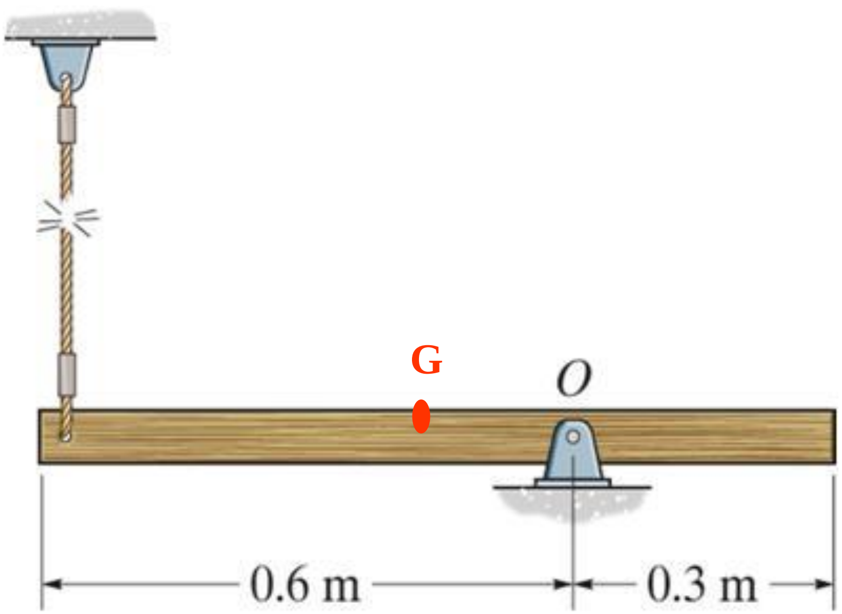
2. Draw a free body diagram accounting for all external forces and couples. Show the resulting inertia forces and couple (typically on a separate kinetic diagram).

3. Compute the mass moment of inertia I_G or I_O.

4. Write the three equations of motion and identify the unknowns. Solve for the unknowns.

5. Use kinematics if there are more than three unknowns (since the equations of motion allow for only three unknowns).

EXAMPLE



Given: The uniform slender rod has a mass of 15 kg.

Find: The reactions at the pin O and the angular acceleration of the rod just after the cord is cut.

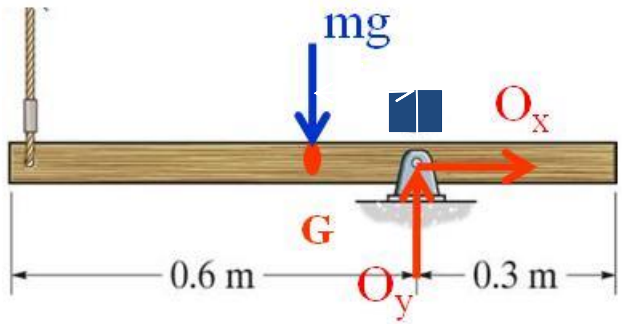
Plan: Since the mass center, G, moves in a circle of radius 0.15 m, it's acceleration has a normal component toward O and a tangential component acting downward and perpendicular to r_G.

Apply the problem solving procedure.

EXAMPLE (continued)

Solution:

FBD & Kinetic Diagram



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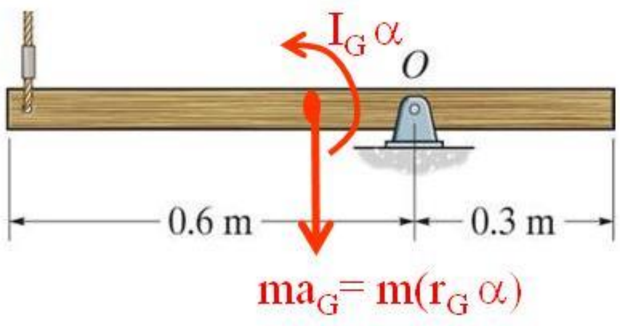


Equations of motion:

$+\rightarrow \Sigma F_n = ma_n = mr_G\omega^2 \quad \Rightarrow O_x = 0\ \text{N}$

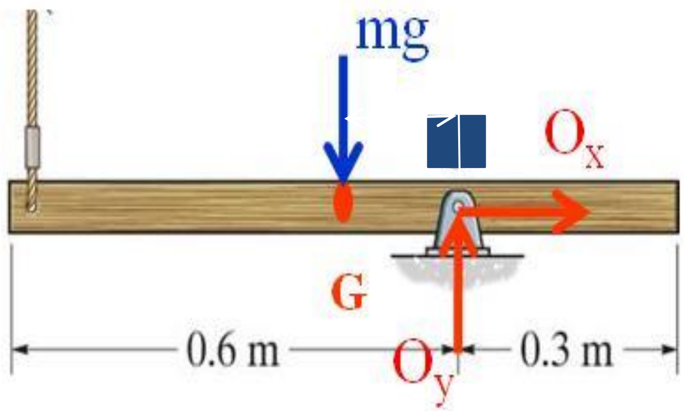
$+\downarrow \Sigma F_t = ma_t = mr_G\alpha \quad \Rightarrow -O_y + 15(9.81) = 15(0.15)\alpha \quad (1)$

$+\circlearrowright \Sigma M_O = I_G\,\alpha + m\,r_G\,\alpha\,(r_G) \Rightarrow (0.15)\,15(9.81) = I_G\,\alpha + m(r_G)^2\,\alpha$

Using $I_G = (ml^2)/12$ and $r_G = (0.15)$, we can write:

$I_G\,\alpha + m(r_G)^2\,\alpha = [(15\times0.9^2)/12 + 15(0.15)^2]\,\alpha = 1.35\,\alpha$

EXAMPLE (continued)



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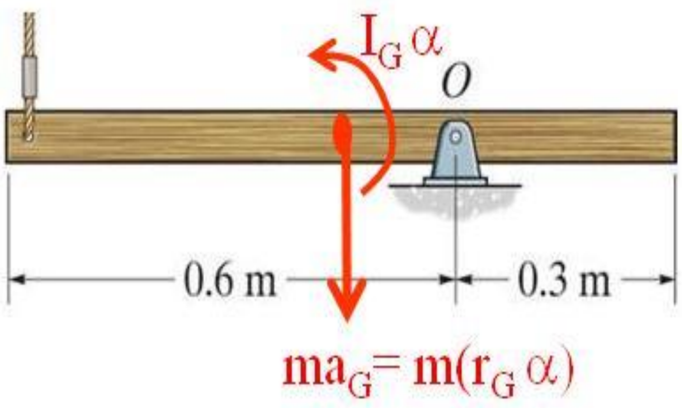


After substituting:

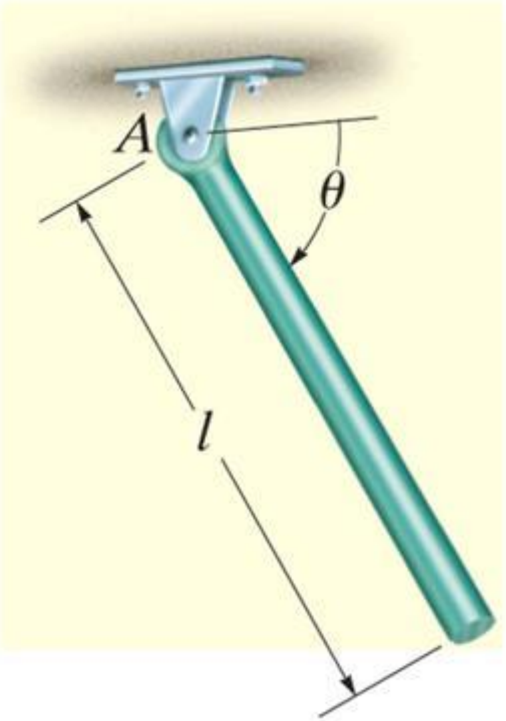
$22.07 = 1.35\,\alpha \quad \Rightarrow \alpha = 16.4\ \text{rad/s}^2$

From Eq (1) :

$-O_y + 15(9.81) = 15(0.15)\alpha$

$\Rightarrow \; O_y = 15(9.81) - 15(0.15)16.4 = 110\ \text{N}$

CONCEPT QUIZ



1. If a rigid bar of length l (above) is released from rest in the horizontal position ($\theta = 0$), the magnitude of its angular acceleration is at maximum when

   A) $\theta = 0$ B) $\theta = 90°$

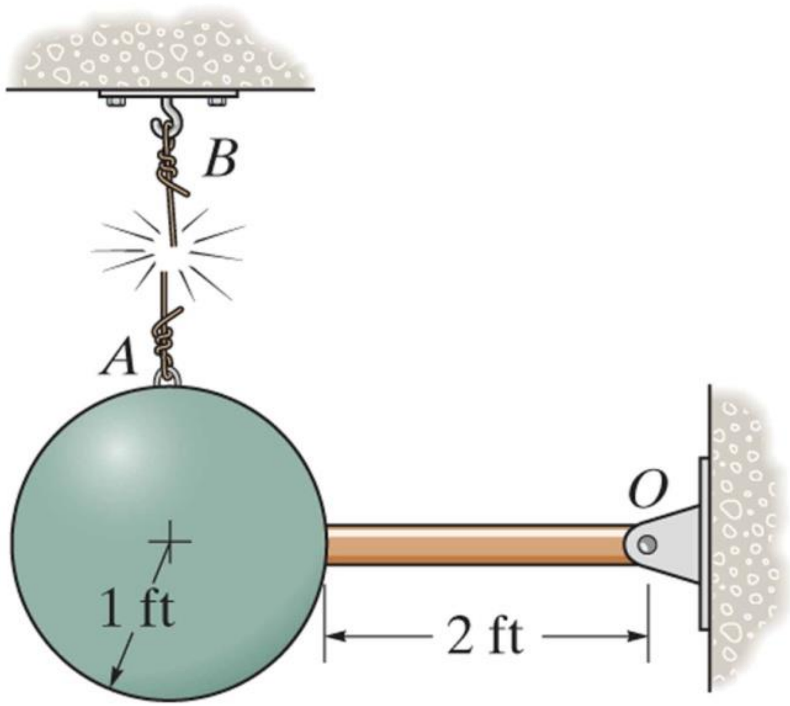
   C) $\theta = 180°$ D) $\theta = 0°$ and $180°$

2. In the above problem, when $\theta = 90°$, the horizontal component of the reaction at pin O is ____________.

   A) zero B) m g

   C) m (l/2) ω^2 D) None of the above

GROUP PROBLEM SOLVING



Given: $W_{sphere} = 30$ lb,
$W_{rod} = 10$ lb

Find: The reaction at the pin O just after the cord AB is cut.

Plan:

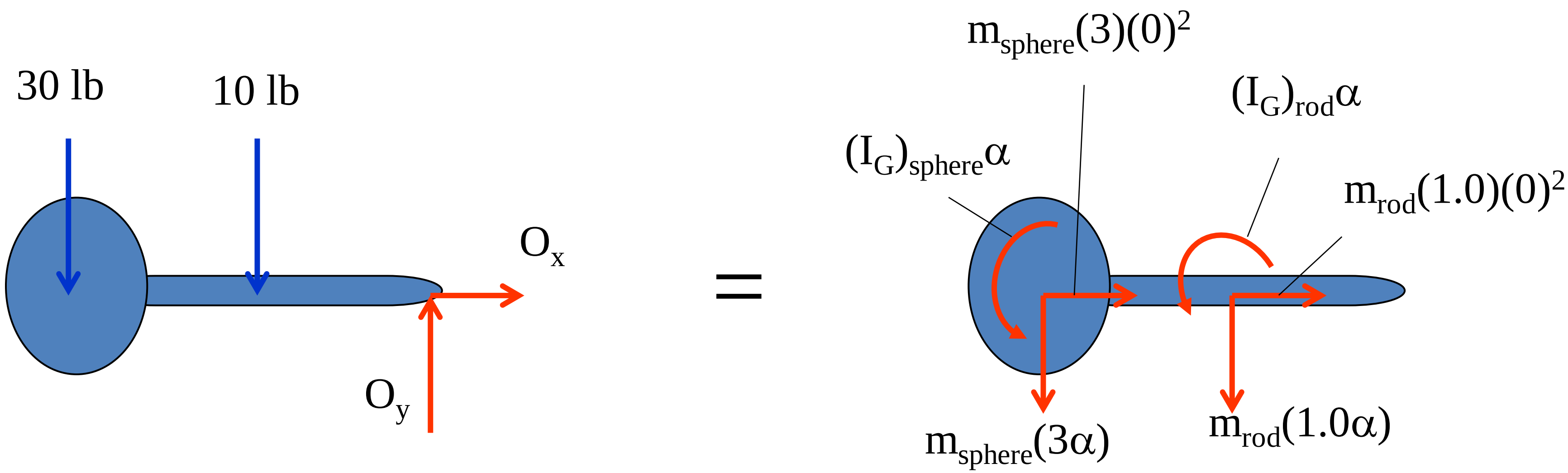
Draw the free body diagram and kinetic diagram of the rod and sphere as one unit.

Then apply the equations of motion.

GROUP PROBLEM SOLVING
(continued)

Solution:

FBD and kinetic diagram;

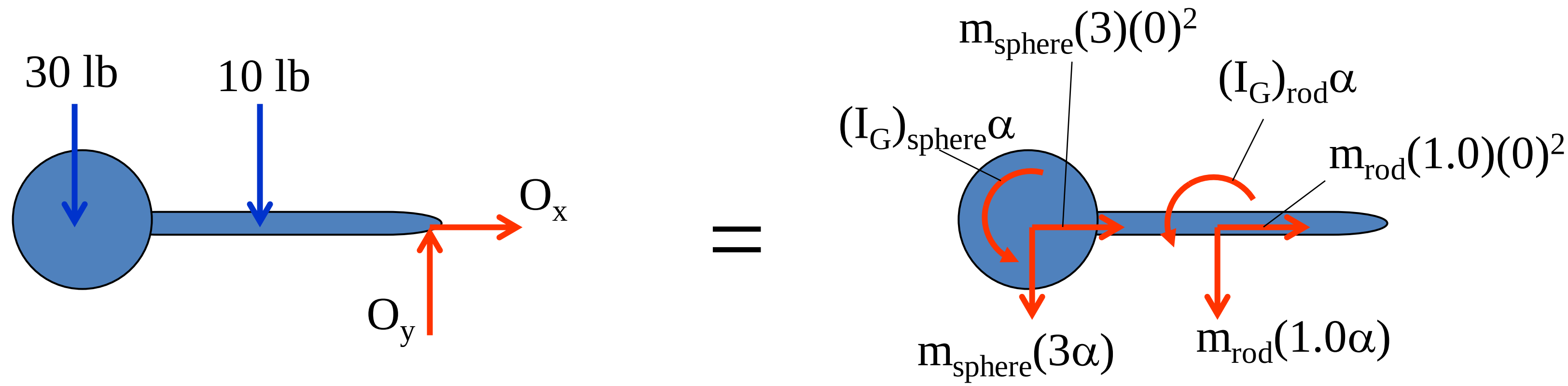


Equations of motion:

$\sum F_n = m(a_G)_n$: $O_x = (30/32.2)(3)(0)^2 + (10/32.2)(1.0)(0)^2$

$\Rightarrow$ $O_x = 0$ lb

GROUP PROBLEM SOLVING
(continued)



$\sum F_t = m(a_G)_t$: $- O_y + 30 + 10 = (30/32.2)\ (3\alpha) + (10/32.2)\ (1.0\alpha)$

$\Rightarrow\ O_y = 40 - 3.106\ \alpha$

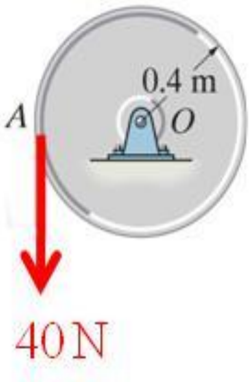
$\sum M_O = I_o\alpha$:

$$30(3.0) + 10(1.0) = [\ 0.4\ (30/32.2)\ (1)^2 + (30/32.2)\ (3)^2\ ]_{sphere}\ \alpha + [\ (1/12)\ (10/32.2)\ (2)^2 + (10/32.2)\ (1)^2\ ]_{rod}\ \alpha$$

$\Rightarrow\ 100 = 9.172\ \alpha$

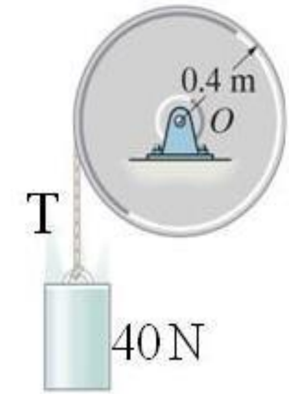
Therefore, $\alpha = 10.9\ \text{rad/s}^2$, $O_y = 6.14$ lb

(a)



ATTENTION QUIZ

(b)



1. A drum of mass m is set into motion in two ways: (a) by a constant 40 N force, and, (b) by a block of weight 40 N. If α_a and α_b represent the angular acceleration of the drum in each case, select the true statement.

A) $\alpha_a > \alpha_b$

B) $\alpha_a < \alpha_b$

C) $\alpha_a = \alpha_b$

D) None of the above

2. In case (b) above, what is the tension T in the cable?

A) T = 40 N

B) T < 40 N

C) T > 40 N

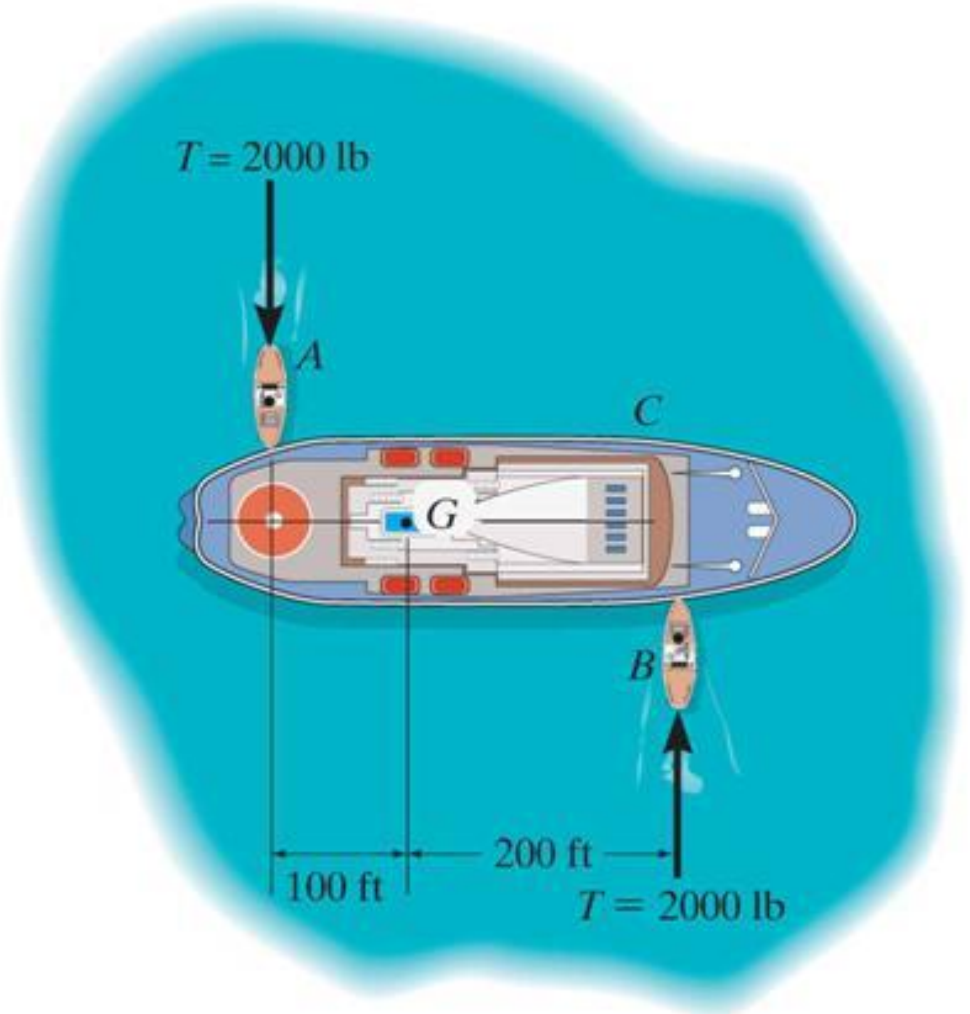
D) None of the above

EQUATIONS OF MOTION: GENERAL PLANE MOTION

Objectives:

Students will be able to:

1. Analyze the planar kinetics of a rigid body undergoing general plane motion.



READING QUIZ

1. If a disk rolls on a rough surface without slipping, the acceleration af the center of gravity (G) will __________ and the friction force will be ___________.

   A) not be equal to α r; less than $\mu_s N$

   B) be equal to α r; equal to $\mu_k N$

   C) be equal to α r; less than $\mu_s N$

   D) None of the above

2. If a rigid body experiences general plane motion, the sum of the moments of external forces acting on the body about any point P is equal to ___________.

   A) $I_P \boldsymbol{\alpha}$

   B) $I_P \boldsymbol{\alpha} + m\mathbf{a}_P$

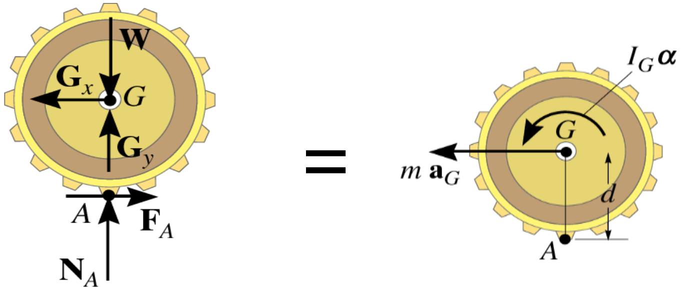
   C) $m\,\mathbf{a}_G$

   D) $I_G \boldsymbol{\alpha} + \boldsymbol{r}_{GP} \times m\mathbf{a}_P$

APPLICATIONS

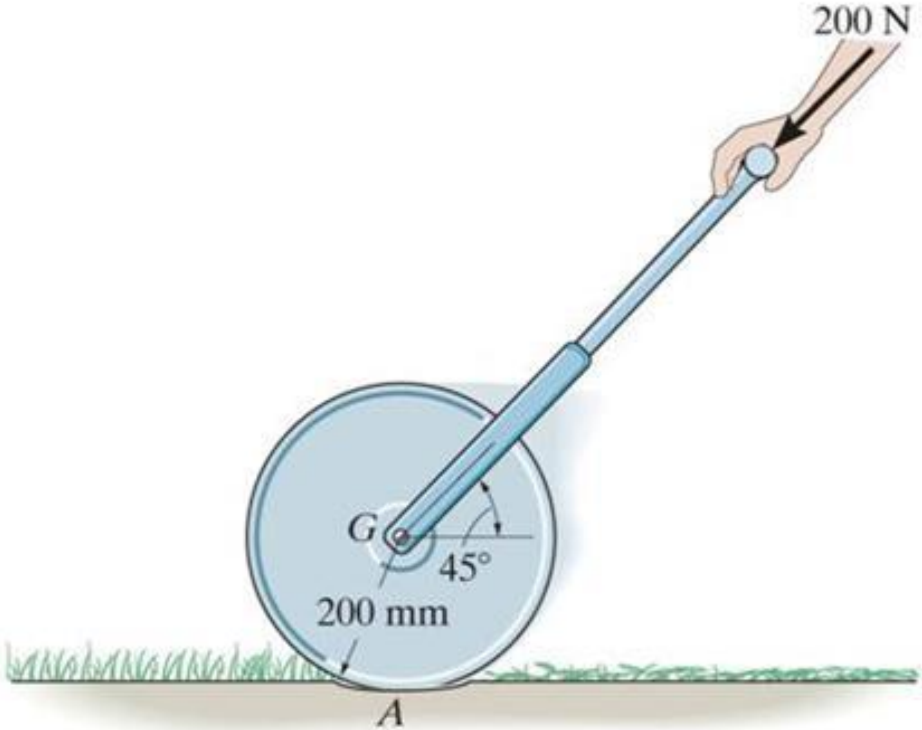
As the soil compactor accelerates forward, the front roller experiences general plane motion (both translation and rotation).

What are the loads experienced by the roller shaft or bearings?



The forces shown on the roller's FBD cause the accelerations shown on the kinetic diagram.
Is the point A the IC?

APPLICATIONS
(continued)

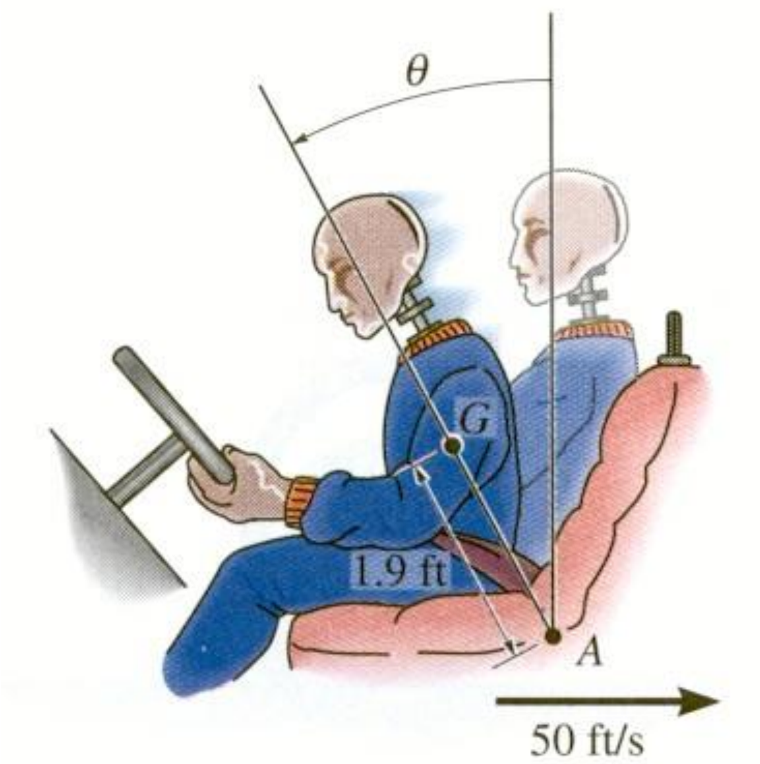


The lawn roller is pushed forward with a force of 200 N when the handle is at 45°.

How can we determine its translation acceleration and angular acceleration?

Does the acceleration depend on the coefficient's of static and kinetic friction?

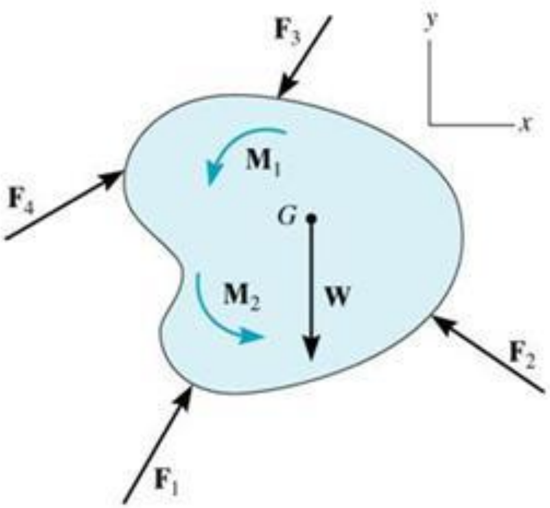
APPLICATIONS (continued)



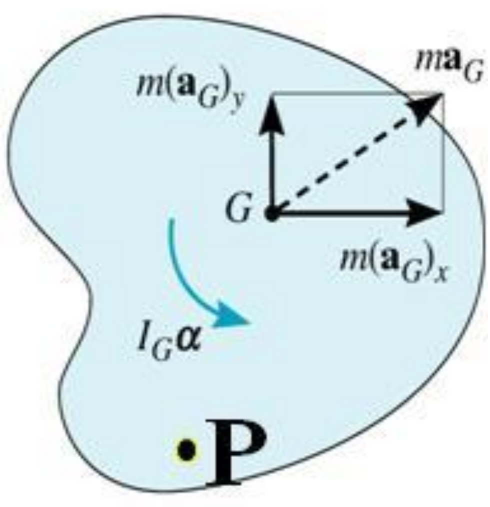
During an impact, the center of gravity of this crash dummy will decelerate with the vehicle, but also experience another acceleration due to its rotation about point A. Why?

How can engineers use this information to determine the forces exerted by the seat belt on a passenger during a crash?

EQUATIONS OF MOTION: GENERAL PLANE MOTION (Section 17.5)



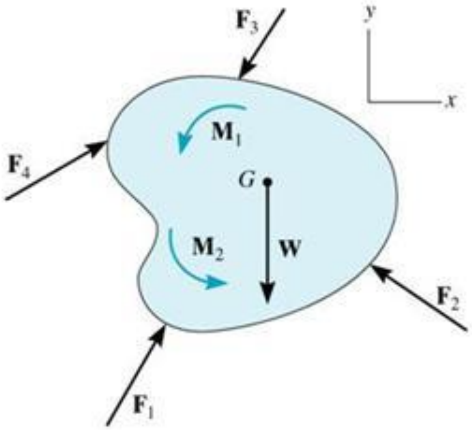
When a rigid body is subjected to external forces and couple-moments, it can undergo both translational motion and rotational motion. This combination is called general plane motion.



Using an x-y inertial coordinate system, the equations of motions about the center of mass, G, may be written as:

$$\sum F_x = m\,(a_G)_x$$
$$\sum F_y = m\,(a_G)_y$$
$$\sum M_G = I_G\,\alpha$$

EQUATIONS OF MOTION: GENERAL PLANE MOTION (continued)

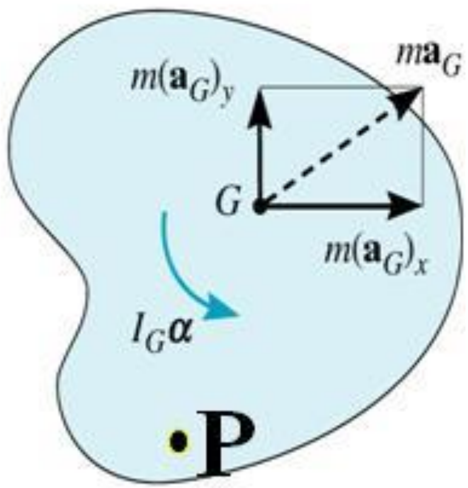


Sometimes, it may be convenient to write the moment equation about a point P other than G. Then the equations of motion are written as follows:

$$\sum F_x = m\,(a_G)_x$$

$$\sum F_y = m\,(a_G)_y$$

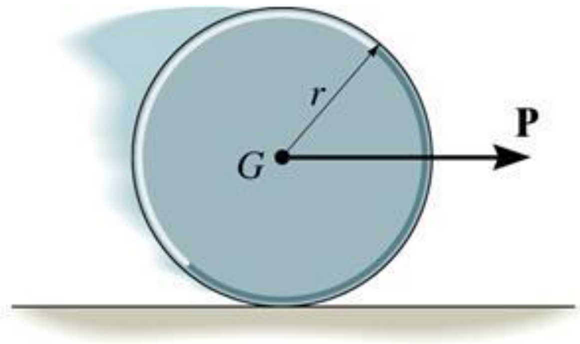
$$\sum M_P = \sum (M_k)_P$$



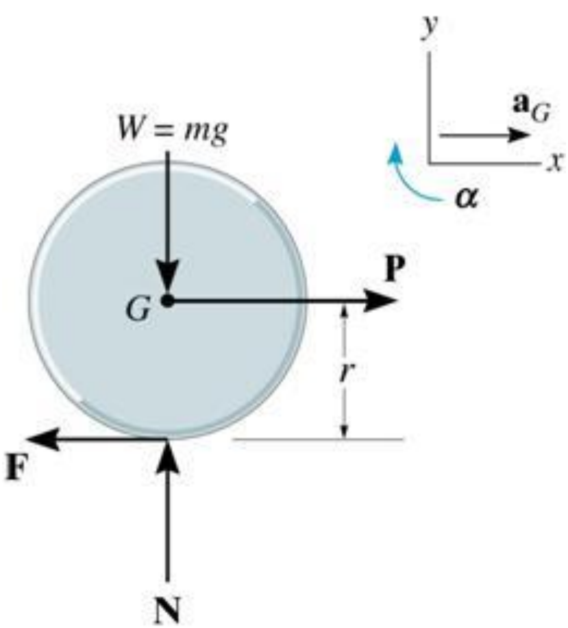
In this case, $\sum (M_k)_P$ represents the sum of the moments of $I_G\boldsymbol{\alpha}$ and $m\mathbf{a}_G$ about point P.

FRICTIONAL ROLLING PROBLEMS

When analyzing the rolling motion of wheels, cylinders, or disks, it may not be known if the body rolls without slipping or if it slides as it rolls.



For example, consider a disk with mass m and radius r, subjected to a known force P.



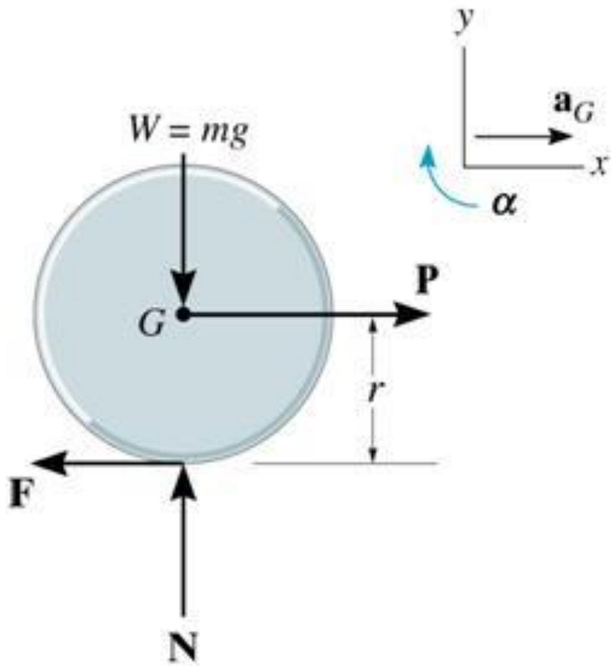
The equations of motion will be:

$$\sum F_x = m(a_G)_x \quad \Rightarrow \quad P - F = m\, a_G$$

$$\sum F_y = m(a_G)_y \quad \Rightarrow \quad N - mg = 0$$

$$\sum M_G = I_G \alpha \quad \Rightarrow \quad F\, r = I_G\, \alpha$$

There are 4 unknowns (F, N, α, and a_G) in these three equations.



FRICTIONAL ROLLING PROBLEMS (continued)

Hence, we have to make an assumption to provide another equation. Then, we can solve for the unknowns.

The 4th equation can be obtained from the slip or non-slip condition of the disk.

Case 1:

Assume no slipping and use $a_G = \alpha\, r$ as the 4th equation and DO NOT use $F_f = \mu_s N$. After solving, you will need to verify that the assumption was correct by checking if $F_f \leq \mu_s N$.

Case 2:

Assume slipping and use $F_f = \mu_k N$ as the 4th equation. In this case, $a_G \neq \alpha r$.

PROCEDURE FOR ANALYSIS

Problems involving the kinetics of a rigid body undergoing general plane motion can be solved using the following procedure.

1. Establish the x-y inertial coordinate system. Draw both the free body diagram and kinetic diagram for the body.

2. Specify the direction and sense of the acceleration of the mass center, $\mathbf{a}_G$, and the angular acceleration $\boldsymbol{\alpha}$ of the body. If necessary, compute the body's mass moment of inertia I_G.

3. If the moment equation $\Sigma M_p = \Sigma (M_k)_p$ is used, use the kinetic diagram to help visualize the moments developed by the components $m(a_G)_x$, $m(a_G)_y$, and $I_G\alpha$.

4. Apply the three equations of motion.

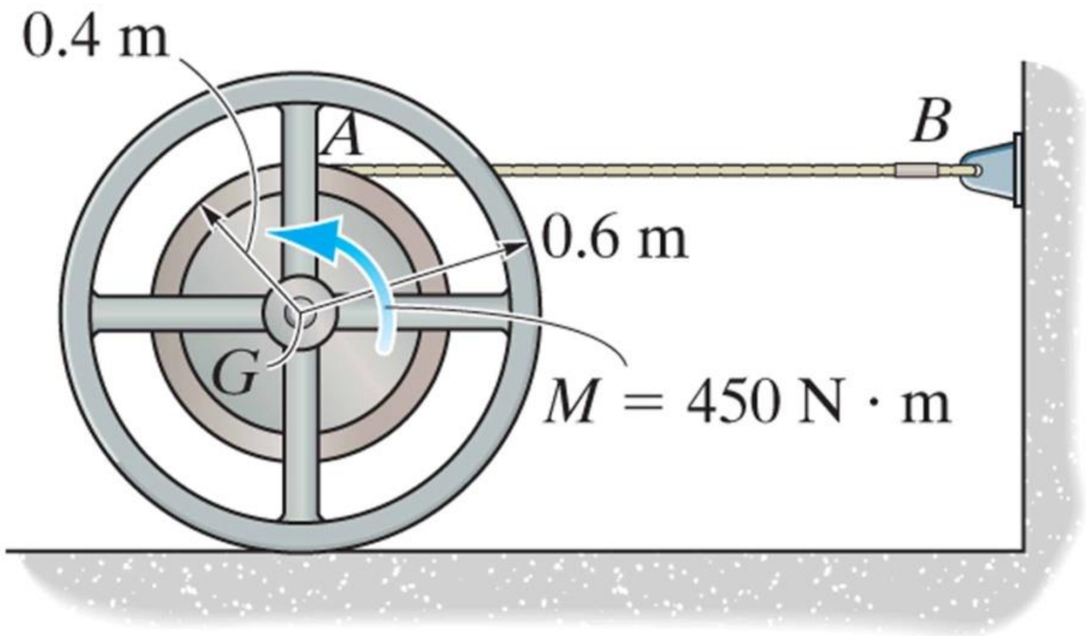
PROCEDURE FOR ANALYSIS (continued)

5. Identify the unknowns. If necessary (i.e., there are four unknowns), make your slip-no slip assumption (typically no slipping, or the use of $a_G = \alpha\ r$, is assumed first).
6. Use kinematic equations as necessary to complete the solution.
7. If a slip-no slip assumption was made, check its validity!!!

Key points to consider:

1. Be consistent in using the assumed directions. The direction of $\mathbf{a}_G$ must be consistent with $\boldsymbol{\alpha}$.
2. If $F_f = \mu_k N$ is used, F_f must oppose the motion. As a test, assume no friction and observe the resulting motion. This may help visualize the correct direction of F_f.

EXAMPLE



Given: A spool has a mass of 200 kg and a radius of gyration (k_G) of 0.3 m. The coefficient of kinetic friction between the spool and the ground is $\mu_k = 0.1$.

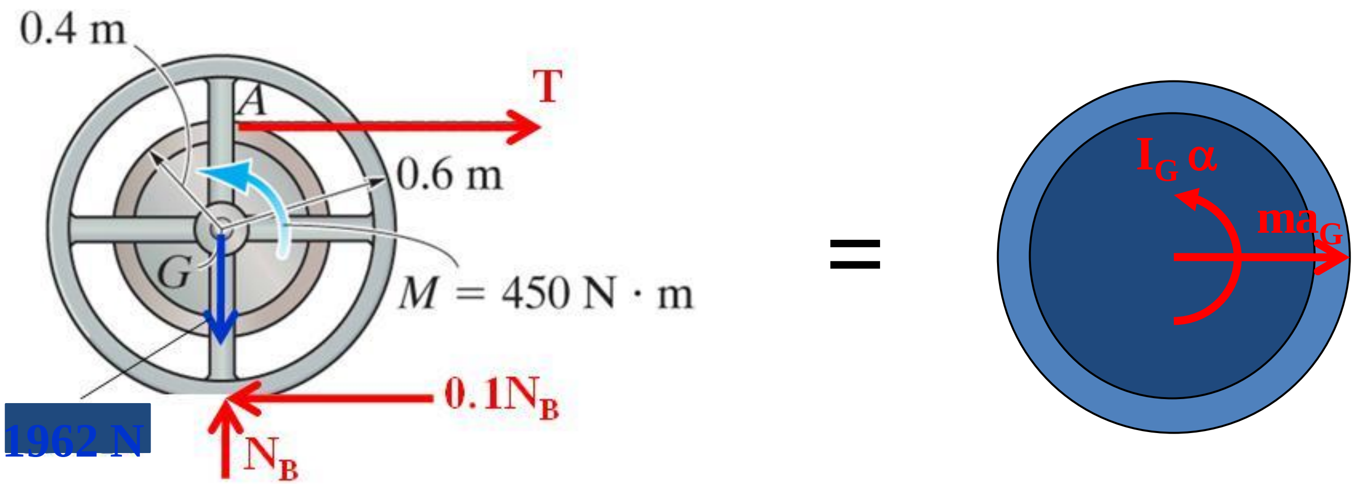
Find: The angular acceleration (α) of the spool and the tension in the cable.

Plan: Focus on the spool. Follow the solution procedure (draw a FBD, etc.) and identify the unknowns.

EXAMPLE (continued)

Solution:

The free body diagram and kinetic diagram for the body are:

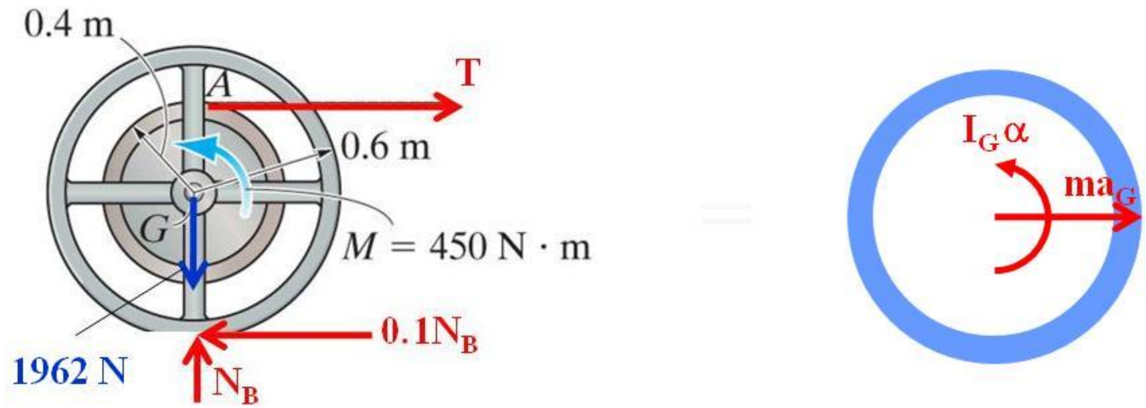


Equations of motion:

$\sum F_y = m\,(a_G)_y : \;\; N_B - 1962 = 0$

$\Rightarrow N_B = 1962 \text{ N}$

EXAMPLE
(continued)



Note that $a_G = (0.4)\ \alpha$. Why ?

$$\sum F_x = m\,(a_G)_x: \quad T - 0.1\ N_B = 200\ a_G = 200\ (0.4)\ \alpha$$

$$\Rightarrow T - 196.2 = 80\ \alpha$$

$$\sum M_G = I_G\ \alpha: \quad 450 - T(0.4) - 0.1\ N_B\ (0.6) = 20\ (0.3)^2\ \alpha$$

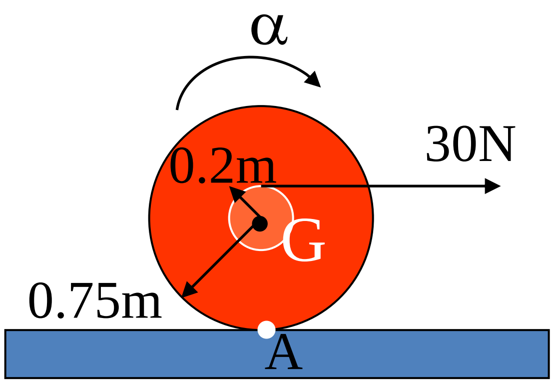
$$\Rightarrow 450 - T(0.4) - 196.2\ (0.6) = 1.8\ \alpha$$

Solving these two equations, we get

$\alpha = 7.50\ \text{rad/s}^2$, $T = 797\ \text{N}$

CONCEPT QUIZ

1. An 80 kg spool (k_G = 0.3 m) is on a rough surface and a cable exerts a 30 N load to the right. The friction force at A acts to the ____________ and the a_G should be directed to the ____________ .



A) right, left B) left, right

C) right, right D) left, left

2. For the situation above, the moment equation about G is?

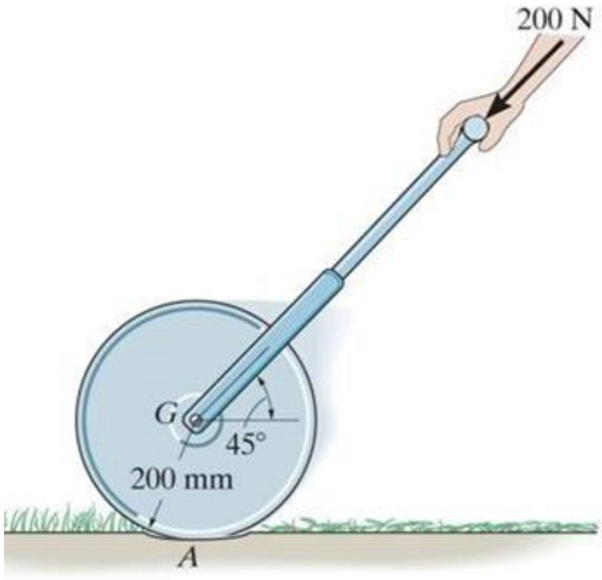
A) $0.75\,(F_{fA}) - 0.2(30) = -\,(80)(0.3^2)\alpha$

B) $-0.2(30) = -\,(80)(0.3^2)\alpha$

C) $0.75\,(F_{fA}) - 0.2(30) = -\,(80)(0.3^2)\alpha + 80a_G$

D) None of the above

GROUP PROBLEM SOLVING



Given: A 80 kg lawn roller has a radius of gyration of $k_G = 0.175$ m. It is pushed forward with a force of 200 N.

Find: The angular acceleration if $\mu_s = 0.12$ and $\mu_k = 0.1$.

Plan: Follow the problem solving procedure.

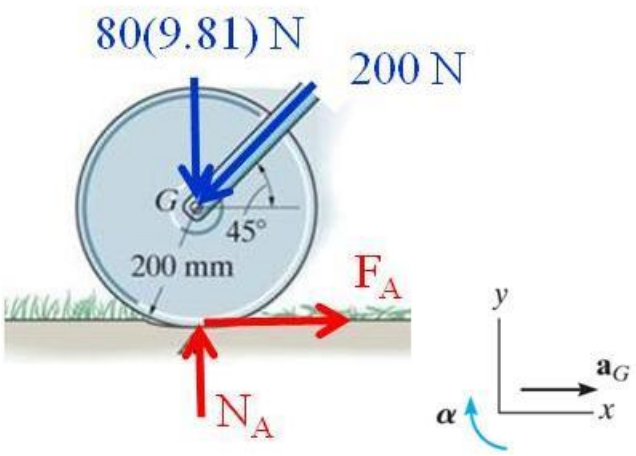
Solution:

The moment of inertia of the roller about G is

$$I_G = m(k_G)^2 = (80)(0.175)^2 = 2.45 \text{ kg}\cdot\text{m}^2$$

GROUP PROBLEM SOLVING (continued)

FBD:



Equations of motion:

$$\sum F_x = m(a_G)_x$$
$$F_A - 200 \cos 45 = 80\, a_G$$

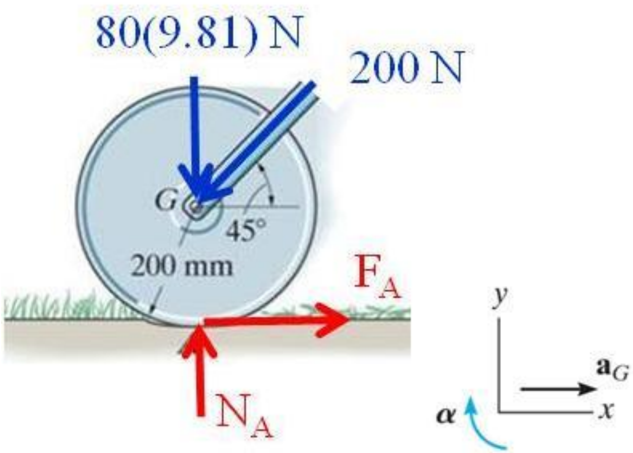
$$\sum F_y = m(a_G)_y$$
$$N_A - 784.8 - 200 \sin 45 = 0$$

$$\sum M_G = I_G\, \alpha$$
$$-0.2\, F_A = 2.45\, \alpha$$

We have 4 unknowns: N_A, F_A, a_G and α.

Another equation is needed to allow solving for the unknowns.

GROUP PROBLEM SOLVING (continued)



The three equations we have now are:

$$F_A - 200 \cos 45° = 80\, a_G$$
$$N_A - 784.8 - 200 \sin 45° = 0$$
$$-0.2\, F_A = 2.45\, \alpha$$

First, assume the wheel is not slipping. Thus, we can write

$$a_G = r\,\alpha = 0.2\,\alpha$$

Now solving the four equations yields:

$N_A = 926.2$ N, $F_A = 61.4$ N, $\alpha = -5.01$ rad/s^2 , $a_G = -1.0$ m/s^2

The no-slip assumption must be checked.
Is $F_A = 61.4 \text{ N} \le \mu_s N_A = 111.1$ N ?
Yes, therefore, the wheel rolls without slip.

ATTENTION QUIZ

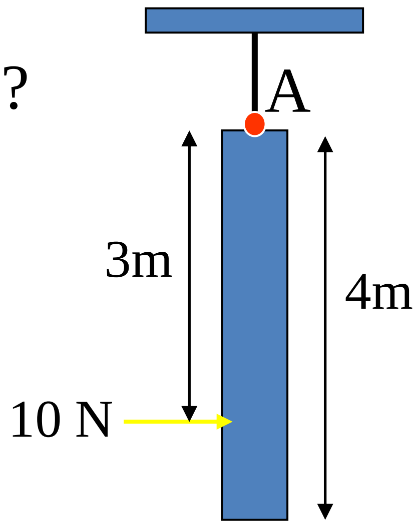
1. A slender 100 kg beam is suspended by a cable. The moment equation about point A is?

   A) $3(10) = 1/12(100)(4^2)\ \alpha$

   B) $3(10) = 1/3(100)(4^2)\ \alpha$

   C) $3(10) = 1/12(100)(4^2)\ \alpha + (100\ a_{Gx})(2)$

   D) None of the above



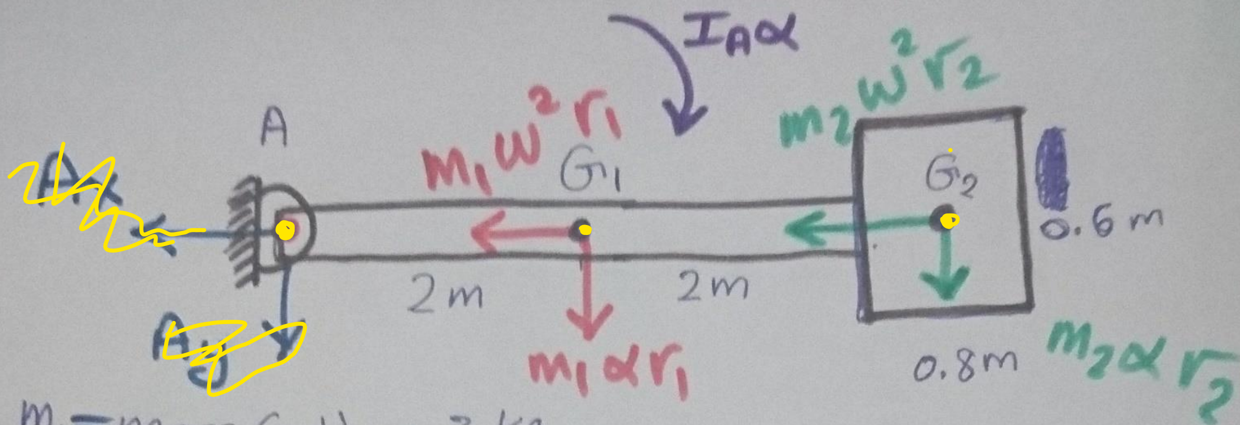
2. Select the equation that best represents the “no-slip” assumption.

   A) $F_f = \mu_s N$

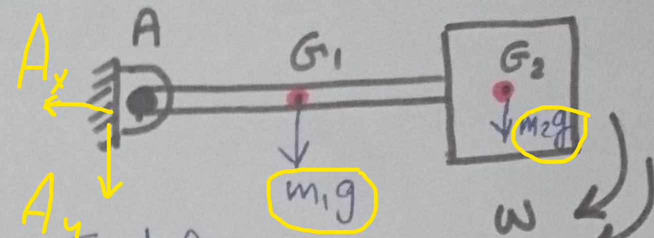
   B) $F_f = \mu_k N$

   C) $a_G = r\ \alpha$

   D) None of the above



m_1 = mass (rod) = 3 kg
m_2 = mass (plate) = 4 kg



* Starts from Rest (ω=0)

Find α , Find A_x , Find A_y

$$I_A = \left(\frac{1}{3} m_1 L^2\right) + \left(\frac{1}{12} m_2 (a^2+b^2) + m_2 r_2^2\right)$$

Rod plate

$$= \frac{1}{3}(3)(4)^2 + \left(\frac{1}{12}(4)(0.8^2+0.6^2) + 4(4.4)^2\right)$$

$$= \text{----}\ \text{kg}\cdot\text{m}^2$$

$$\sum M_A = I_A\alpha \rightarrow -m_1 g r_1 - m_2 g r_2 = -I_A\alpha$$

→ Find α

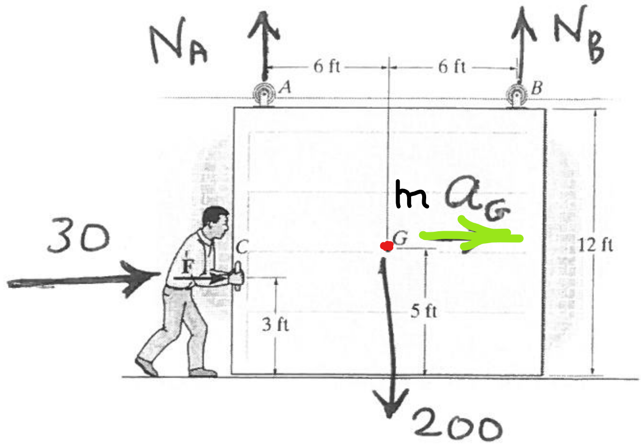
$$\sum F_x = ma_x \rightarrow -A_x = 0 \quad \text{since } \omega = 0$$

$$\sum F_y = ma_y \rightarrow (-A_y = -m_1\alpha r_1 - m_2\alpha r_2 - m_1 g - m_2 g)$$

→ Find A_y ✓

***17–24.**

The door has a weight of 200 lb and a center of gravity at G. Determine how far the door moves in 2 s, starting from rest, if a man pushes on it at C with a horizontal force $F = 30$ lb. Also, find the vertical reactions at the rollers A and B.

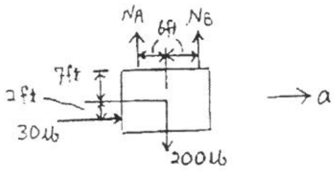


SOLUTION

$\xrightarrow{+} \Sigma F_x = m(a_G)_x;\qquad 30 = \left(\dfrac{200}{32.2}\right)a_G$

$a_G = 4.83\ \text{ft/s}^2$

$\circlearrowleft + \Sigma M_A = \Sigma(M_k)_A;\qquad N_B(12) - 200(6) + 30(9) = \left(\dfrac{200}{32.2}\right)(4.83)(7)$



$N_B = 95.0$ lb **Ans.**

$+\uparrow \Sigma F_y = m(a_G)_y;\qquad N_A + 95.0 - 200 = 0$

$N_A = 105$ lb **Ans.**

$(\xrightarrow{+})\qquad s = s_0 + v_0 t + \dfrac{1}{2}a_G t^2$

$s = 0 + 0 + \dfrac{1}{2}(4.83)(2)^2 = 9.66$ ft **Ans.**

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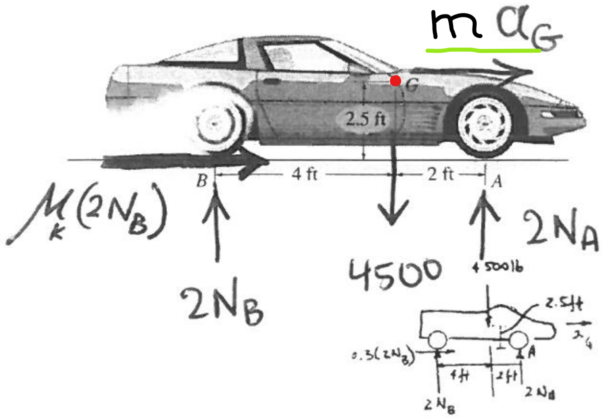
Ans:
$N_B = 95.0$ lb
$N_A = 105$ lb
$s = 9.66$ ft

17–27.

The sports car has a weight of 4500 lb and center of gravity at G. If it starts from rest it causes the rear wheels to slip as it accelerates. Determine how long it takes for it to reach a speed of 10 ft/s. Also, what are the normal reactions at each of the four wheels on the road? The coefficients of static and kinetic friction at the road are $\mu_s = 0.5$ and $\mu_k = 0.3$, respectively. Neglect the mass of the wheels.



SOLUTION

$$\circlearrowleft + \Sigma M_A = \Sigma(M_k)_A; \quad -2N_B(6) + 4500(2) = \frac{-4500}{32.2} a_G(2.5)$$

$$\xrightarrow{+} \Sigma F_x = m(a_G)_x; \quad 0.3(2N_B) = \frac{4500}{32.2} a_G$$

$$+\uparrow \Sigma F_y = m(a_G)_y; \quad 2N_B + 2N_A - 4500 = 0$$

Solving,

$N_A = 1393$ lb **Ans.**

$N_B = 857$ lb **Ans.**

$a_G = 3.68\ \text{ft/s}^2$

$(\xrightarrow{+}) \quad v = v_0 + a_c t$

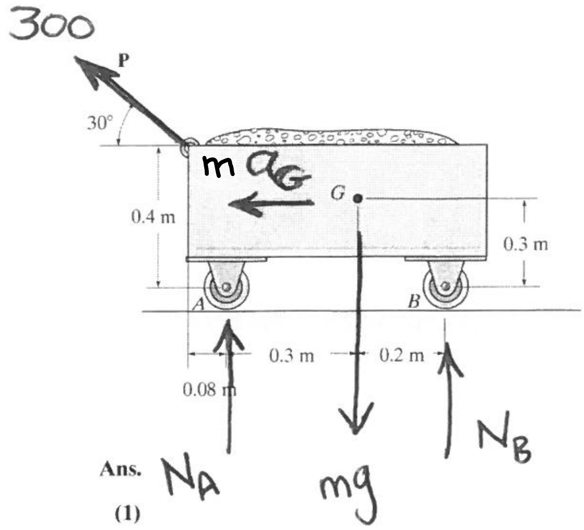
$10 = 0 + 3.68\,t$

$t = 2.72$ s **Ans.**

Ans:
$N_A = 1393$ lb
$N_B = 857$ lb
$t = 2.72$ s

***17–32.**

A force of $P = 300$ N is applied to the 60-kg cart. Determine the reactions at both the wheels at A and both the wheels at B. Also, what is the acceleration of the cart? The mass center of the cart is at G.



SOLUTION

Equations of Motions. Referring to the FBD of the cart, Fig. *a*,

$\overset{+}{\leftarrow} \Sigma F_x = m(a_G)_x;\qquad 300 \cos 30° = 60a$

$a = 4.3301 \text{ m/s}^2 = 4.33 \text{ m/s}^2 \leftarrow$ **Ans.**

$+\uparrow \Sigma F_y = m(a_G)_y;\qquad N_A + N_B + 300 \sin 30° - 60(9.81) = 60(0)$ **(1)**

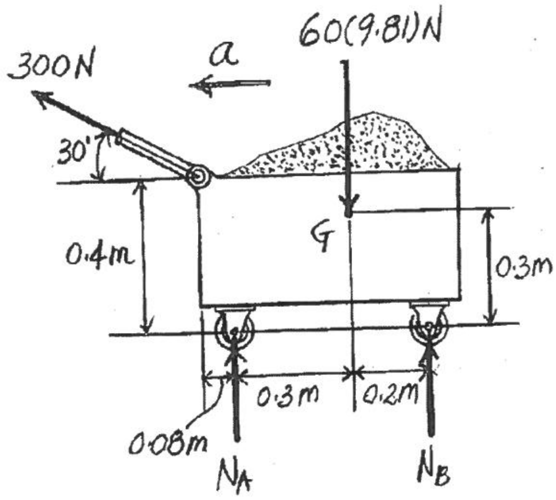
$\circlearrowleft + \Sigma M_G = 0;\qquad N_B(0.2) - N_A(0.3) + 300 \cos 30°(0.1)$

$- 300 \sin 30°(0.38) = 0$ **(2)**

Solving Eqs. (1) and (2),

$N_A = 113.40 \text{ N} = 113 \text{ N}$ **Ans.**

$N_B = 325.20 \text{ N} = 325 \text{ N}$ **Ans.**

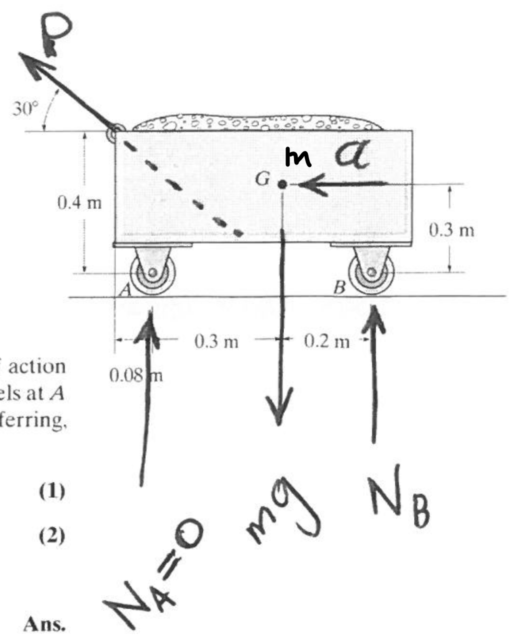


(a)

Ans:
$a = 4.33 \text{ m/s}^2 \leftarrow$
$N_A = 113$ N
$N_B = 325$ N

17–33.

Determine the largest force **P** that can be applied to the 60-kg cart, without causing one of the wheel reactions, either at A or at B, to be zero. Also, what is the acceleration of the cart? The mass center of the cart is at G.



SOLUTION

Equations of Motions. Since $(0.38 \text{ m}) \tan 30° = 0.22 \text{ m} > 0.1 \text{ m}$, the line of action of **P** passes *below* G. Therefore, **P** tends to rotate the cart clockwise. The wheels at A will leave the ground before those at B. Then, it is required that $N_A = 0$. Referring, to the FBD of the cart, Fig. *a*

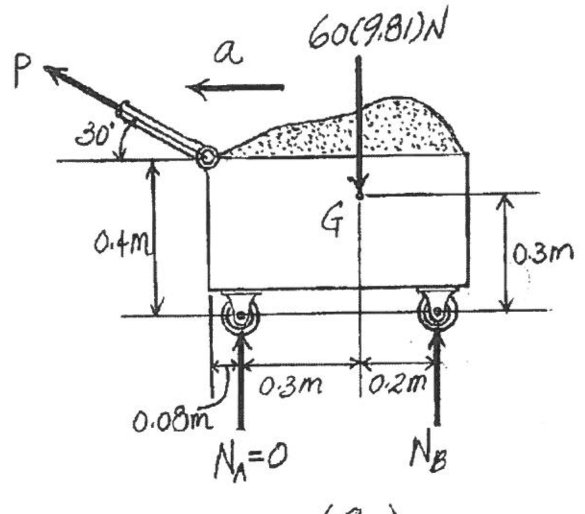
$+\uparrow \Sigma F_y = m(a_G)_y; \qquad N_B + P\sin 30° - 60(9.81) = 60(0) \qquad$ **(1)**

$\circlearrowleft + \Sigma M_G = 0; \qquad P\cos 30°(0.1) - P\sin 30°(0.38) + N_B(0.2) = 0 \qquad$ **(2)**

Solving Eqs. (1) and (2)

$$P = 578.77 \text{ N} = 579 \text{ N}$$ **Ans.**

$$N_B = 299.22 \text{ N}$$



(a)

Ans:
$P = 579$ N

17–57.

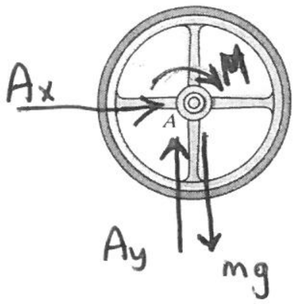
The 10-kg wheel has a radius of gyration $k_A = 200$ mm. If the wheel is subjected to a moment $M = (5t)$ N · m, where t is in seconds, determine its angular velocity when $t = 3$ s starting from rest. Also, compute the reactions which the fixed pin A exerts on the wheel during the motion.

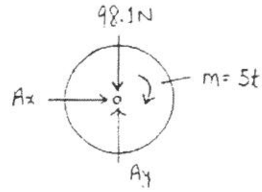
$I_A = m k_A^2$

$a_x = 0$

$a_y = 0$

$\alpha \neq 0$





SOLUTION

$\xrightarrow{+} \Sigma F_x = m(a_G)_x;$ $\quad A_x = 0$

$+\uparrow \Sigma F_y = m(a_G)_y;$ $\quad A_y - 10(9.81) = 0$

$\boxed{\circlearrowleft + \Sigma M_A = I_a\,\alpha;}$ $\quad 5t = 10(0.2)^2\alpha$

$$\alpha = \frac{d\omega}{dt} = 12.5t$$

$$\omega = \int_0^3 12.5t\,dt = \frac{12.5}{2}(3)^2$$

$\omega = 56.2$ rad/s **Ans.**

$A_x = 0$ **Ans.**

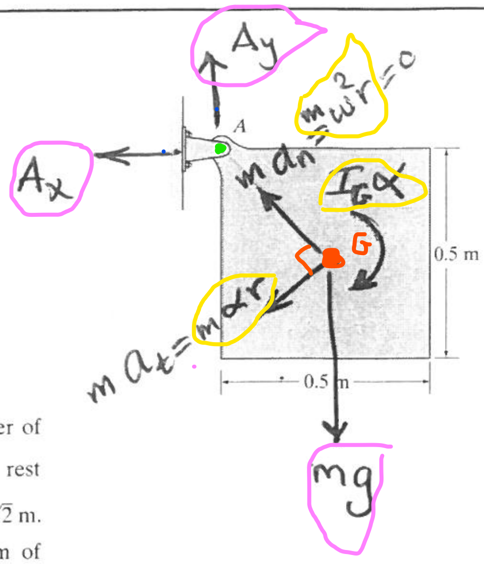
$A_y = 98.1$ N **Ans.**

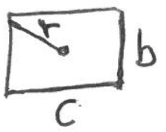
$I\alpha$

Ans:
$\omega = 56.2$ rad/s
$A_x = 0$
$A_y = 98.1$ N

17–58.

The uniform 24-kg plate is released from rest at the position shown. Determine its initial angular acceleration and the horizontal and vertical reactions at the pin A.





$$\omega = 0, \quad \alpha \neq 0$$

$$I_G = \frac{1}{12} m(c^2 + b^2)$$

$$r = \frac{1}{2}\sqrt{c^2 + b^2}$$

SOLUTION

Equations of Motion. The mass moment of inertia of the plate about its center of gravity G is $I_G = \frac{1}{12}(24)(0.5^2 + 0.5^2) = 1.00\ \text{kg}\cdot\text{m}^2$. Since the plate is at rest initially $\omega = 0$. Thus, $(a_G)_n = \omega^2 r_G = 0$. Here $r_G = \sqrt{0.25^2 + 0.25^2} = 0.25\sqrt{2}$ m. Thus, $(a_G)_t = \alpha r_G = \alpha(0.25\sqrt{2})$. Referring to the FBD and kinetic diagram of the plate,

$\circlearrowleft + \Sigma M_A = (M_k)_A;\qquad -24(9.81)(0.25) = -24[\alpha(0.25\sqrt{2})](0.25\sqrt{2}) - 1.00\,\alpha$

$\alpha = 14.715\ \text{rad/s}^2 = 14.7\ \text{rad/s}^2$ **Ans.**

Also, the same result can be obtained by applying $\Sigma M_A = I_A\alpha$ where $I_A = \frac{1}{12}(24)(0.5^2 + 0.5^2) + 24(0.25\sqrt{2})^2 = 4.00\ \text{kg}\cdot\text{m}^2$:

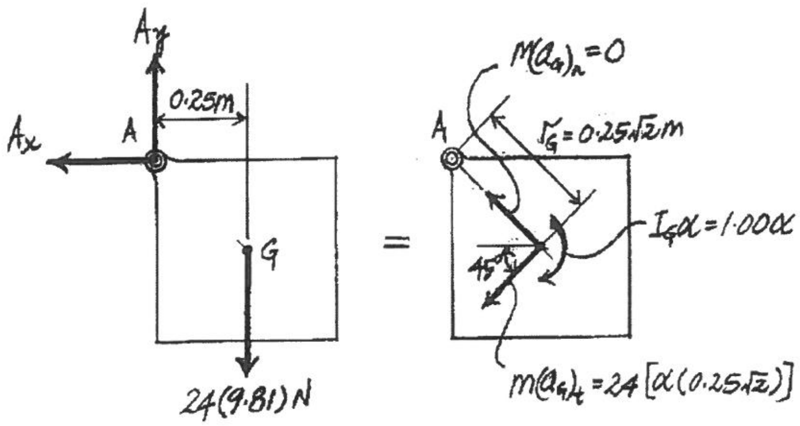
$\circlearrowleft + \Sigma M_A = I_A\,\alpha;\qquad -24(9.81)(0.25) = -4.00\,\alpha$

$\alpha = 14.715\ \text{rad/s}^2$

$\overset{+}{\leftarrow} \Sigma F_x = m(a_G)_x;\quad A_x = 24[14.715(0.25\sqrt{2})]\cos 45° = 88.29\ \text{N} = 88.3\ \text{N}$ **Ans.**

$+\uparrow \Sigma F_y = m(a_G)_y;\quad A_y - 24(9.81) = -24[14.715(0.25\sqrt{2})]\sin 45°$

$A_y = 147.15\ \text{N} = 147\ \text{N}$ **Ans.**

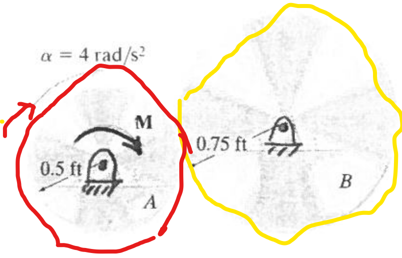


(a)

Ans:
$\alpha = 14.7\ \text{rad/s}^2$
$A_x = 88.3$ N
$A_y = 147$ N

17–65.

Disk A has a weight of 5 lb and disk B has a weight of 10 lb. If no slipping occurs between them, determine the couple moment **M** which must be applied to disk A to give it an angular acceleration of 4 rad/s^2.



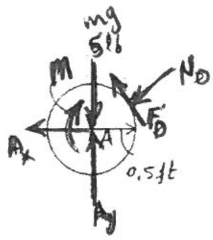
$$I = \frac{1}{2} m r^2$$

SOLUTION

$$\frac{\alpha_A}{\alpha_B} = \frac{r_B}{r_A}$$

Disk A:

$$\circlearrowleft + \Sigma M_A = I_A \alpha_A; \qquad M - F_D(0.5) = \left[\frac{1}{2}\left(\frac{5}{32.2}\right)(0.5)^2\right](4)$$

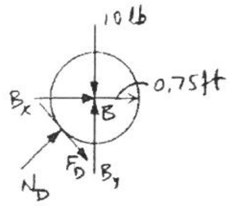


Disk B:

$$+\Sigma M_B = I_B \alpha_B; \qquad F_D(0.75) = \left[\frac{1}{2}\left(\frac{10}{32.2}\right)(0.75)^2\right]\alpha_B$$

$$r_A \alpha_A = r_B \alpha_B$$

$$0.5(4) = 0.75\alpha_B$$

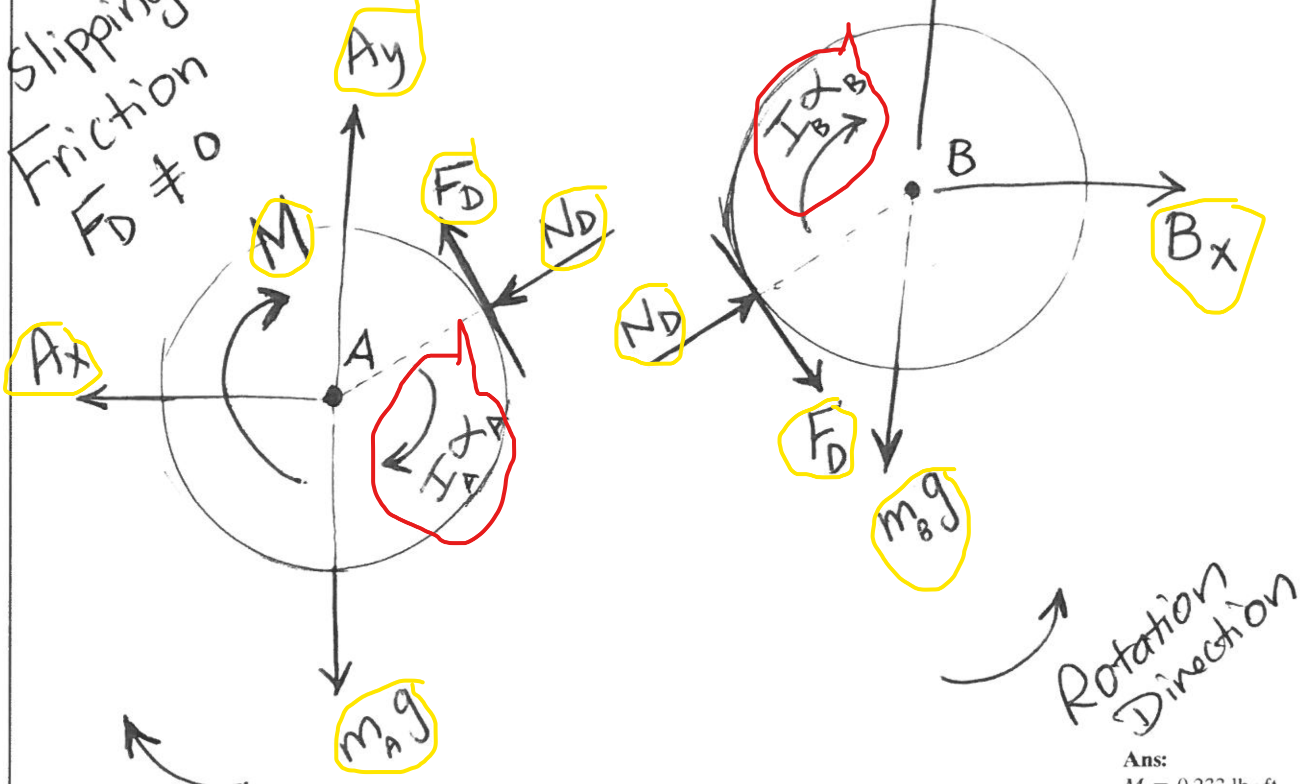


Solving:

$\alpha_B = 2.67$ rad/s^2; $\quad F_D = 0.311$ lb

$M = 0.233$ lb · ft **Ans.**

No Slipping
Friction
$F_D \neq 0$

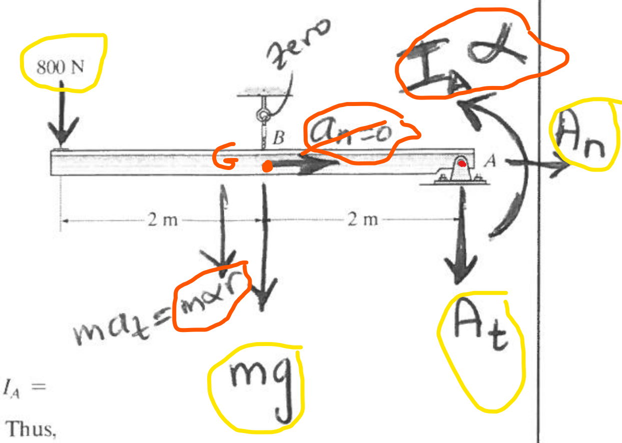


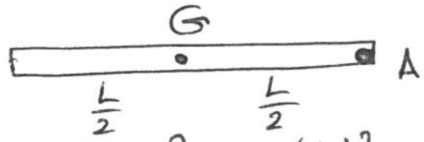
Ans:
$M = 0.233$ lb · ft

Rotation
direction

17–67.

If the cord at B suddenly fails, determine the horizontal and vertical components of the initial reaction at the pin A, and the angular acceleration of the 120-kg beam. Treat the beam as a uniform slender rod.





SOLUTION

$$I_A = \frac{1}{12} m L^2 + m\left(\frac{L}{2}\right)^2$$

Equations of Motion. The mass moment of inertia of the beam about A is $I_A = \frac{1}{12}(120)(4^2) + 120(2^2) = 640\ \text{kg}\cdot\text{m}^2$. Initially, the beam is at rest, $\omega = 0$. Thus, $(a_G)_n = \omega^2 r = 0$. Also, $(a_G)_t = \alpha r_G = \alpha(2) = 2\alpha$. Referring to the FBD of the beam, Fig. *a*

$\circlearrowleft + \Sigma M_A = I_A\alpha;$ $\quad 800(4) + 120(9.81)(2) = 640\,\alpha$

$\alpha = 8.67875\ \text{rad/s}^2 = 8.68\ \text{rad/s}^2$ **Ans.**

$\Sigma F_n = m(a_G)_n;$ $\quad A_n = 0$ **Ans.**

$\Sigma F_t = m(a_G)_t;$ $\quad 800 + 120(9.81) + A_t = 120[2(8.67875)]$

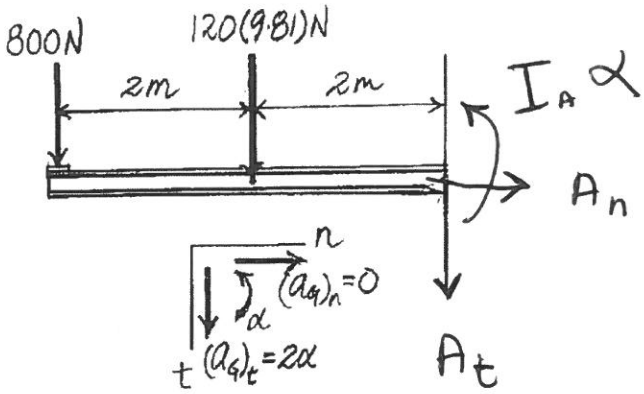
$A_t = 105.7\ \text{N} = 106\ \text{N}$ **Ans.**

$$\omega = 0$$

$$\alpha \neq 0$$

$$a_n = 0,\ (\omega = 0)$$

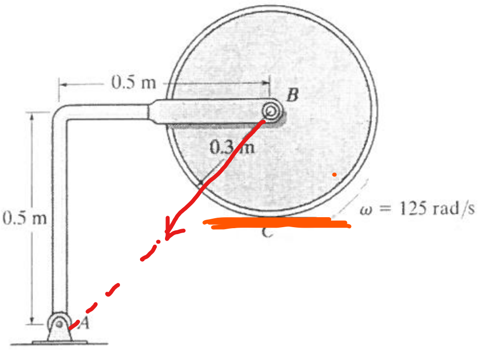
$$a_t = \alpha\left(\frac{L}{2}\right)$$



Ans:
$\alpha = 8.68\ \text{rad/s}^2$
$A_n = 0$
$A_t = 106\ \text{N}$

***17–72.**

The 30-kg disk is originally spinning at $\omega = 125$ rad/s. If it is placed on the ground, for which the coefficient of kinetic friction is $\mu_C = 0.5$, determine the time required for the motion to stop. What are the horizontal and vertical components of force which the member AB exerts on the pin at A during this time? Neglect the mass of AB.



$$I_B = \frac{1}{2} m r^2$$

SOLUTION

Equations of Motion. The mass moment of inertia of the disk about B is $I_B = \frac{1}{2}mr^2 = \frac{1}{2}(30)(0.3^2) = 1.35\ \text{kg}\cdot\text{m}^2$. Since it is required to slip at C, $F_f = \mu_C N_C = 0.5\,N_C$. Referring to the FBD of the disk, Fig. a,

$\xrightarrow{+}\Sigma F_x = m(a_G)_x;\qquad 0.5N_C - F_{AB}\cos 45^\circ = 30(0)\qquad$ **(1)**

$+\uparrow \Sigma F_y = m(a_G)_y;\qquad N_C - F_{AB}\sin 45^\circ - 30(9.81) = 30(0)\qquad$ **(2)**

$a_x = 0$

$a_y = 0$

$\alpha \neq 0$

Solving Eqs. (1) and (2),

$N_C = 588.6\ \text{N}\qquad F_{AB} = 416.20\ \text{N}$

Subsequently,

$\circlearrowleft + \Sigma M_B = I_B\alpha;\qquad 0.5(588.6)(0.3) = 1.35\alpha$

$\alpha = 65.4\ \text{rad/s}^2 \circlearrowright$

Referring to the FBD of pin A, Fig. b,

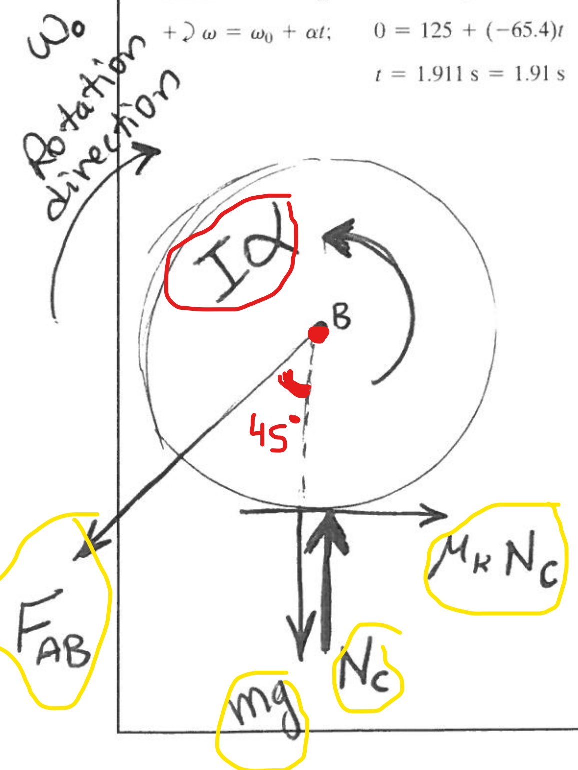
$\xrightarrow{+}\Sigma F_x = 0;\qquad 416.20\cos 45^\circ - A_x = 0\qquad A_x = 294.3\ \text{N} = 294\ \text{N}\qquad$ **Ans.**

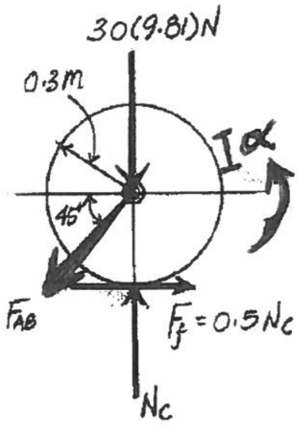
$+\uparrow\Sigma F_y = 0;\qquad 416.20\sin 45^\circ - A_y = 0\qquad A_y = 294.3\ \text{N} = 294\ \text{N}\qquad$ **Ans.**

Kinematic. Using the result of α,

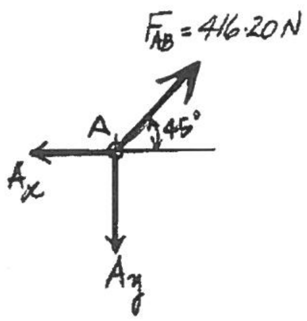
$+\circlearrowright\ \omega = \omega_0 + \alpha t;\qquad 0 = 125 + (-65.4)t$

$t = 1.911\ \text{s} = 1.91\ \text{s}\qquad$ **Ans.**





(a)



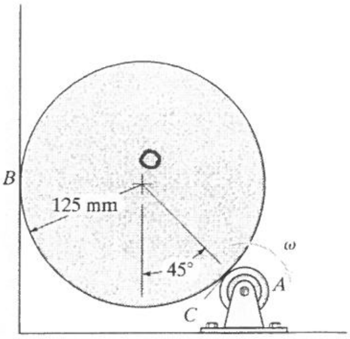
(b)

Ans:
$A_x = 294$ N
$A_y = 294$ N
$t = 1.91$ s

17–74.

$a_x = 0$
$a_y = 0$
$\alpha \neq 0$

The 5-kg cylinder is initially at rest when it is placed in contact with the wall B and the rotor at A. If the rotor always maintains a constant clockwise angular velocity $\omega = 6$ rad/s, determine the initial angular acceleration of the cylinder. The coefficient of kinetic friction at the contacting surfaces B and C is $\mu_k = 0.2$.



SOLUTION

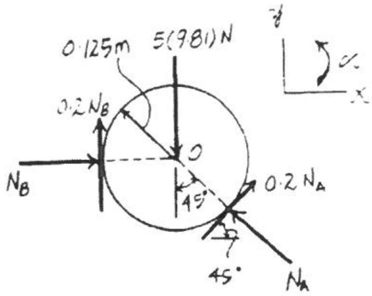
$I_o = \frac{1}{2} m r^2$ "cylinder"

Equations of Motion: The mass moment of inertia of the cylinder about point O is given by $I_O = \frac{1}{2}mr^2 = \frac{1}{2}(5)(0.125^2) = 0.0390625\ \text{kg}\cdot\text{m}^2$. Applying Eq. 17–16, we have

$$\xrightarrow{+} \Sigma F_x = m(a_G)_x; \qquad N_B + 0.2N_A \cos 45° - N_A \sin 45° = 0 \qquad (1)$$

$$+\uparrow \Sigma F_y = m(a_G)_y; \qquad 0.2N_B + 0.2N_A \sin 45° + N_A \cos 45° - 5(9.81) = 0 \qquad (2)$$

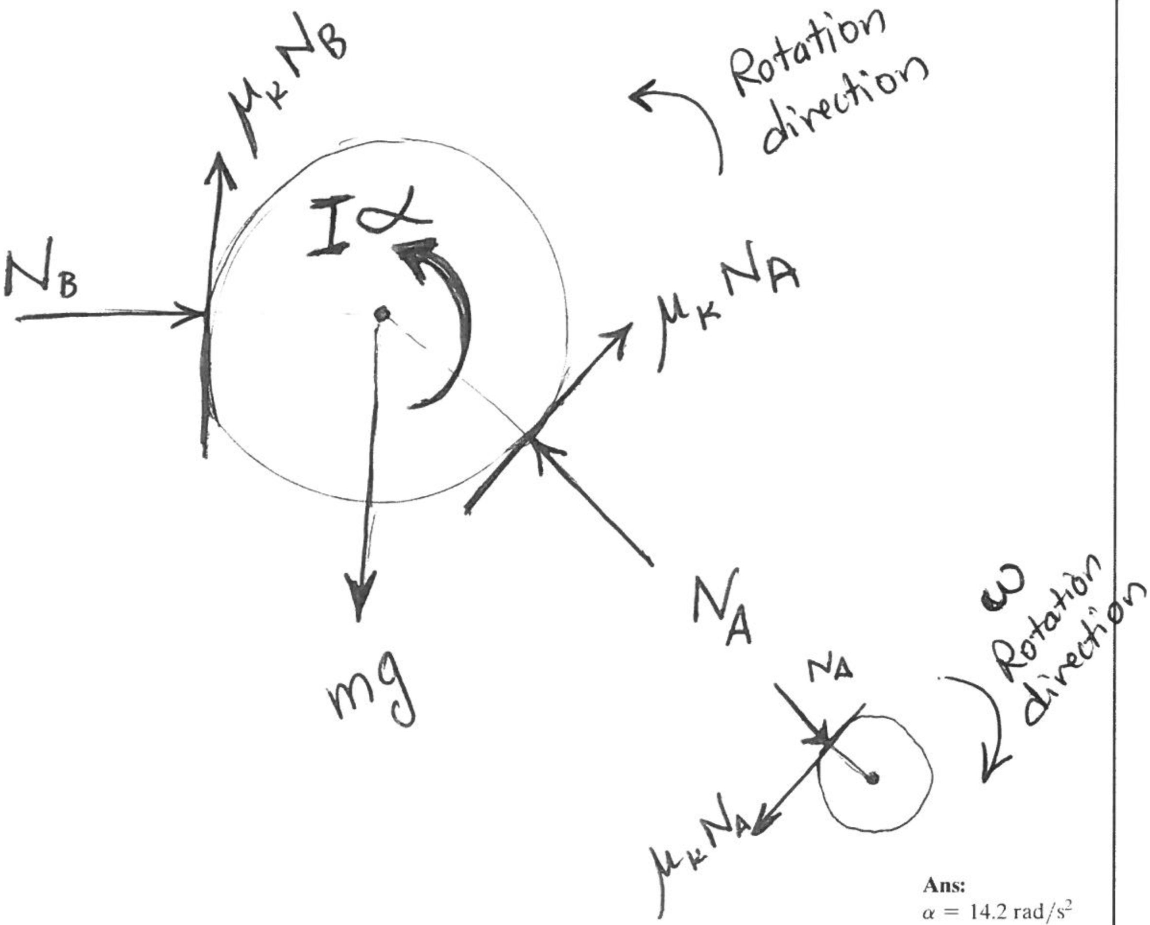
$$\circlearrowleft + \Sigma M_O = I_O\,\alpha; \qquad 0.2N_A(0.125) - 0.2N_B(0.125) = 0.0390625\alpha \qquad (3)$$



Solving Eqs. (1), (2), and (3) yields;

$$N_A = 51.01\ \text{N} \qquad N_B = 28.85\ \text{N}$$

$$\alpha = 14.2\ \text{rad/s}^2 \qquad \textbf{Ans.}$$



Ans:
$\alpha = 14.2\ \text{rad/s}^2$

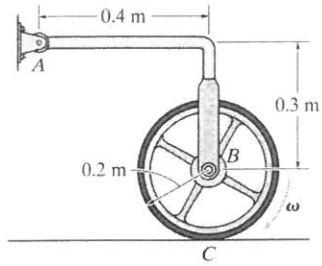
17–75.

The wheel has a mass of 25 kg and a radius of gyration $k_B = 0.15$ m. It is originally spinning at $\omega = 40$ rad/s. If it is placed on the ground, for which the coefficient of kinetic friction is $\mu_C = 0.5$, determine the time required for the motion to stop. What are the horizontal and vertical components of reaction which the pin at A exerts on AB during this time? Neglect the mass of AB.

$a_x = 0$

$a_y = 0$

$\alpha \neq 0$



SOLUTION

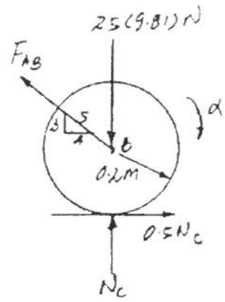
$I_B = m K_B^2$

$I_B = mk_B^2 = 25(0.15)^2 = 0.5625\ \text{kg}\cdot\text{m}^2$

$+\uparrow \Sigma F_y = m(a_G)_y; \qquad \left(\frac{3}{5}\right) F_{AB} + N_C - 25(9.81) = 0$ **(1)**

$\xrightarrow{+} \Sigma F_x = m(a_G)_x; \qquad 0.5N_C - \left(\frac{4}{5}\right) F_{AB} = 0$ **(2)**

$\zeta + \Sigma M_B = I_B\alpha; \qquad 0.5N_C(0.2) = 0.5625(-\alpha)$ **(3)**



Solvings Eqs. (1), (2) and (3) yields:

$F_{AB} = 111.48\ \text{N} \qquad N_C = 178.4\ \text{N}$

$\alpha = -31.71\ \text{rad/s}^2$

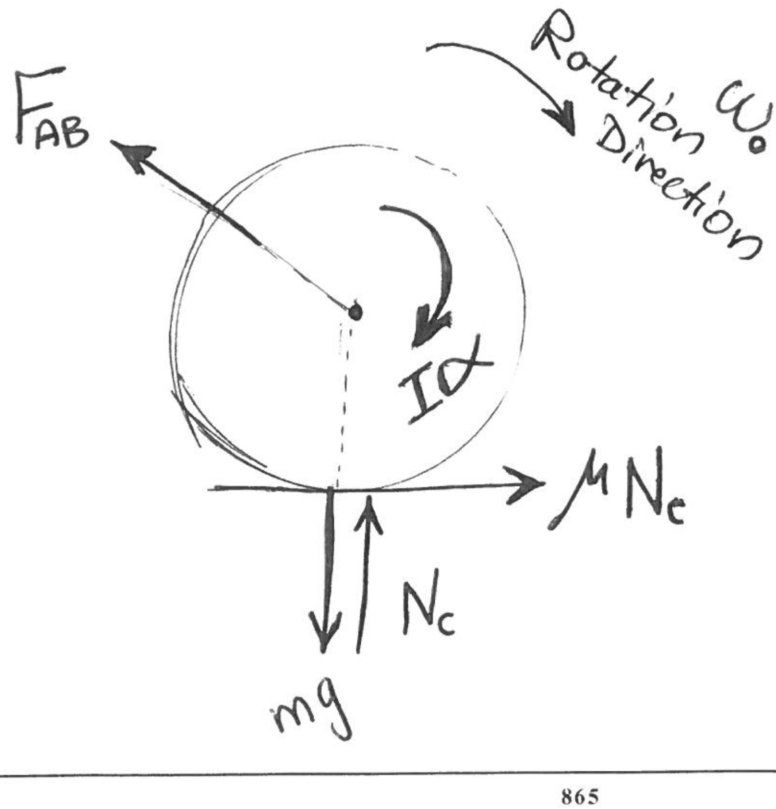
$A_x = \frac{4}{5}F_{AB} = 0.8(111.48) = 89.2\ \text{N}$ **Ans.**

$A_y = \frac{3}{5}F_{AB} = 0.6(111.48) = 66.9\ \text{N}$ **Ans.**

$\omega = \omega_0 + \alpha_c t$

$0 = 40 + (-31.71)\,t$

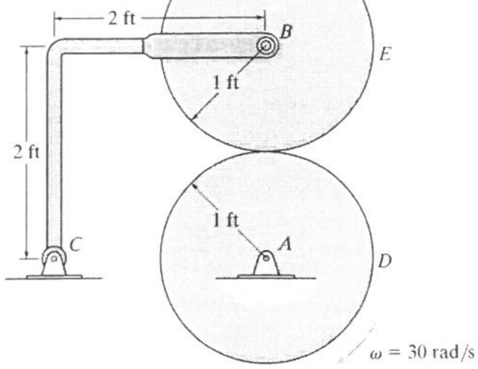
$t = 1.26\ \text{s}$ **Ans.**



Ans:
$A_x = 89.2\ \text{N}$
$A_y = 66.9\ \text{N}$
$t = 1.25\ \text{s}$

17–77.

Disk D turns with a constant clockwise angular velocity of 30 rad/s. Disk E has a weight of 60 lb and is initially at rest when it is brought into contact with D. Determine the time required for disk E to attain the same angular velocity as disk D. The coefficient of kinetic friction between the two disks is $\mu_k = 0.3$. Neglect the weight of bar BC.



$I = \frac{1}{2} m r^2$

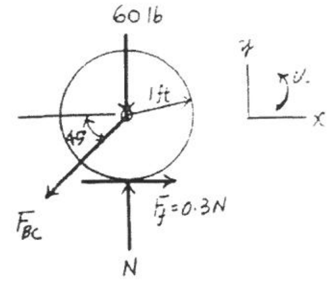
$a_x = 0$

$a_y = 0$

$\alpha \neq 0$

SOLUTION

Equations of Motion: The mass moment of inertia of disk E about point B is given by $I_B = \frac{1}{2}mr^2 = \frac{1}{2}\left(\frac{60}{32.2}\right)(1^2) = 0.9317 \text{ slug} \cdot \text{ft}^2$. Applying Eq. 17–16, we have



$\overset{+}{\rightarrow} \Sigma F_x = m(a_G)_x;$ $\qquad 0.3N - F_{BC}\cos 45° = 0 \qquad$ **(1)**

$+\uparrow \Sigma F_y = m(a_G)_y;$ $\qquad N - F_{BC}\sin 45° - 60 = 0 \qquad$ **(2)**

$\circlearrowleft + \Sigma M_O = I_O\,\alpha;$ $\qquad 0.3N(1) = 0.9317\alpha \qquad$ **(3)**

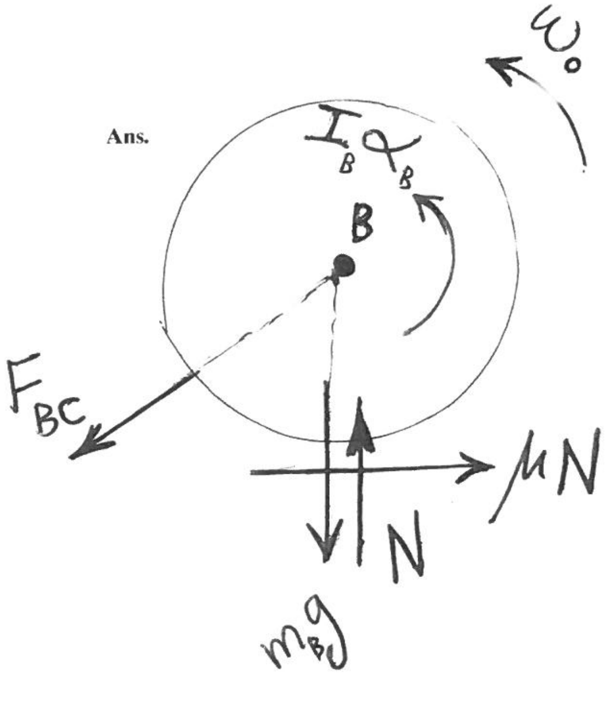
Solving Eqs. (1), (2) and (3) yields:

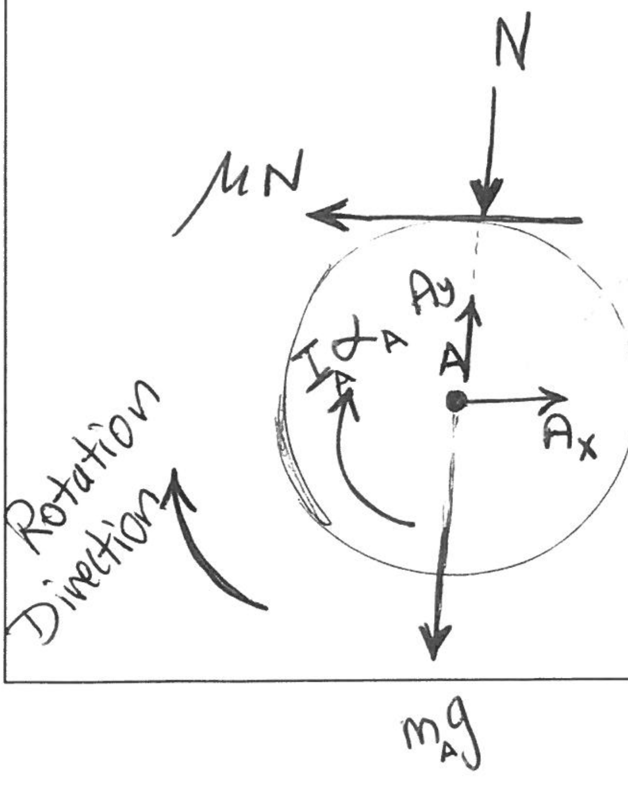
$F_{BC} = 36.37$ lb $\qquad N = 85.71$ lb $\qquad \alpha = 27.60$ rad/s^2

Kinematics: Applying equation $\omega = \omega_0 + \alpha_t$, we have

$(\circlearrowleft +) \qquad 30 = 0 + 27.60t$

$t = 1.09$ s $\qquad$ **Ans.**

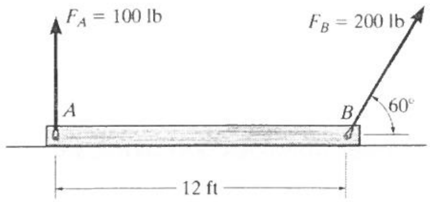




Ans:
$t = 1.09$ s

*17–92.

The uniform 150-lb beam is initially at rest when the forces are applied to the cables. Determine the magnitude of the acceleration of the mass center and the angular acceleration of the beam at this instant.



$I_G = \frac{1}{12} m L^2$

SOLUTION

Equations of Motion: The mass moment of inertia of the beam about its mass center is $I_G = \frac{1}{12}ml^2 = \frac{1}{12}\left(\frac{150}{32.2}\right)(12^2) = 55.90 \text{ slug} \cdot \text{ft}^2$.

$a_x \neq 0$

$a_y \neq 0$

$\alpha \neq 0$

$\xrightarrow{+} \Sigma F_x = m(a_G)_x; \quad 200 \cos 60° = \frac{150}{32.2}(a_G)_x$

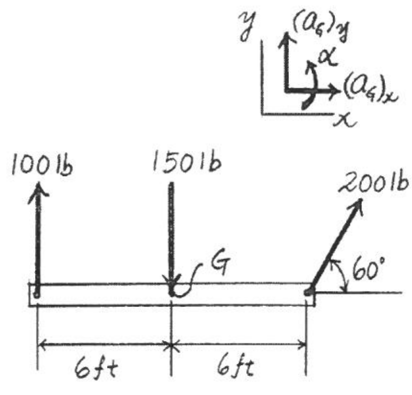
$(a_G)_x = 21.47 \text{ ft/s}^2$

$+\uparrow \Sigma F_y = m(a_G)_y; \quad 100 + 200 \sin 60° - 150 = \frac{150}{32.2}(a_G)_y$

$(a_G)_y = 26.45 \text{ ft/s}^2$

$+\Sigma M_G = I_G\alpha; \quad 200 \sin 60°(6) - 100(6) = 55.90\alpha$

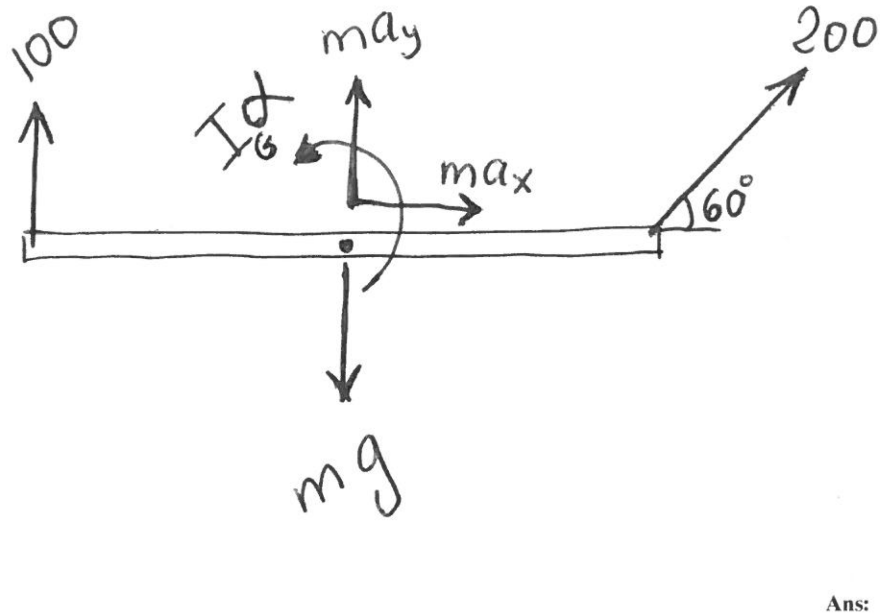
$\alpha = 7.857 \text{ rad/s}^2 = 7.86 \text{ rad/s}^2$ **Ans.**



(a)

Thus, the magnitude of $\mathbf{a}_G$ is

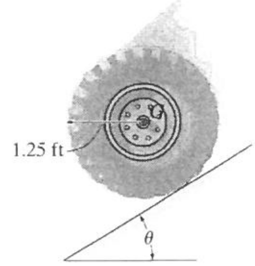
$$a_G = \sqrt{(a_G)_x^2 + (a_G)_y^2} = \sqrt{21.47^2 + 26.45^2} = 34.1 \text{ ft/s}^2$$ **Ans.**



Ans:
$\alpha = 7.86 \text{ rad/s}^2$
$a_G = 34.1 \text{ ft/s}^2$

17–94.

The tire has a weight of 30 lb and a radius of gyration of $k_G = 0.6$ ft. If the coefficients of static and kinetic friction between the wheel and the plane are $\mu_s = 0.2$ and $\mu_k = 0.15$, determine the tire's angular acceleration as it rolls down the incline. Set $\theta = 12°$.



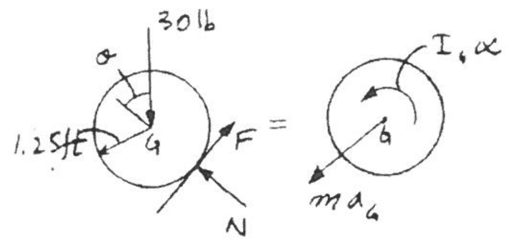
$$I_G = m K_G^2$$

SOLUTION

$+\swarrow \Sigma F_x = m(a_G)_x;\qquad 30 \sin 12° - F = \left(\frac{30}{32.2}\right)a_G$

$+\nwarrow \Sigma F_y = m(a_G)_y;\qquad N - 30 \cos 12° = 0$

$\circlearrowleft + \Sigma M_G = I_G\,\alpha;\qquad F(1.25) = \left[\left(\frac{30}{32.2}\right)(0.6)^2\right]\alpha$



Assume the wheel does not slip.

$$a_G = (1.25)\alpha$$

Solving:

$F = 1.17$ lb

$N = 29.34$ lb

$a_G = 5.44$ ft/s^2

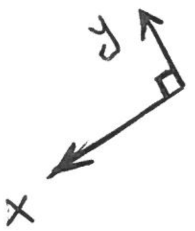
$\alpha = 4.35$ rad/s^2 **Ans.**

$F_{max} = 0.2(29.34) = 5.87$ lb > 1.17 lb **OK** $\Rightarrow$ $F < \mu_s N$

$\Rightarrow$ Rolling

No Slipping (Only Rolling)

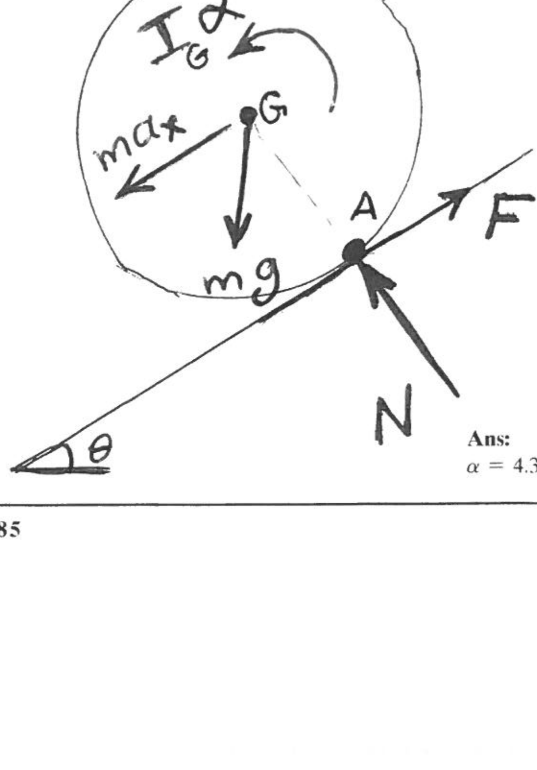
$\rightarrow$ Friction exist.



$a_x \neq 0$

$a_y = 0$

$\alpha \neq 0$



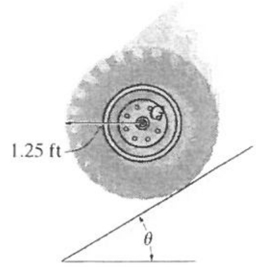
$a_x = r\,\alpha$

* Point A is the istan. Center IC.

Ans:
$\alpha = 4.32$ rad/s^2

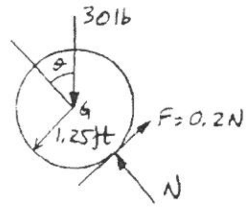
17–95.

The tire has a weight of 30 lb and a radius of gyration of $k_G = 0.6$ ft. If the coefficients of static and kinetic friction between the wheel and the plane are $\mu_s = 0.2$ and $\mu_k = 0.15$, determine the maximum angle θ of the inclined plane so that the tire rolls without slipping.



SOLUTION

Since wheel is on the verge of slipping:



$+\swarrow \Sigma F_x = m(a_G)_x; \qquad 30 \sin\theta - 0.2N = \left(\frac{30}{32.2}\right)(1.25\alpha) \qquad$ **(1)**

$+\nwarrow \Sigma F_y = m(a_G)_y; \qquad N - 30\cos\theta = 0 \qquad$ **(2)**

$\circlearrowleft + \Sigma M_C = I_G\,\alpha; \qquad 0.2N(1.25) = \left[\left(\frac{30}{32.2}\right)(0.6)^2\right]\alpha \qquad$ **(3)**

Substituting Eqs.(2) and (3) into Eq. (1),

$$30\sin\theta - 6\cos\theta = 26.042\cos\theta$$

$$30\sin\theta = 32.042\cos\theta$$

$$\tan\theta = 1.068$$

$$\theta = 46.9^\circ \qquad \textbf{Ans.}$$

⇒ Same as problem 17-94, But:

For Critical Rolling (without Slipping)

Just Sub. $F = \mu_s N$

⇒ three equations, 3 unknowns, (θ, N, α)

* $\theta < 46.9^\circ$ ⇒ Rolling with slipping

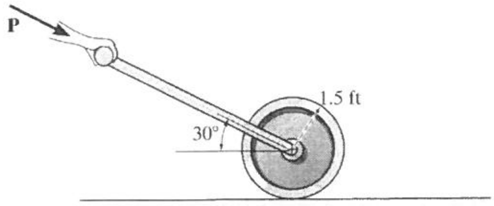
* $\theta > 46.9^\circ$ ⇒ pure Rolling without slipping

(Rolling on the Verge of slipping)

Ans:
$\theta = 46.9^\circ$

***17–104.**

If $P = 30$ lb, determine the angular acceleration of the 50-lb roller. Assume the roller to be a uniform cylinder and that no slipping occurs.



$I_G = \frac{1}{2} m r^2$

SOLUTION

Equations of Motion: The mass moment of inertia of the roller about its mass center is $I_G = \frac{1}{2}mr^2 = \frac{1}{2}\left(\frac{50}{32.2}\right)(1.5^2) = 1.7469 \text{ slug}\cdot\text{ft}^2$. We have

$$\xrightarrow{+} \Sigma F_x = m(a_G)_x; \quad 30\cos 30^\circ - F_f = \frac{50}{32.2}a_G \qquad (1)$$

$$+\uparrow \Sigma F_y = m(a_G)_y; \quad N - 50 - 30\sin 30^\circ = 0 \qquad N = 65 \text{ lb}$$

$$+\Sigma M_G = I_G\alpha; \qquad F_f(1.5) = 1.7469\alpha \qquad (2)$$

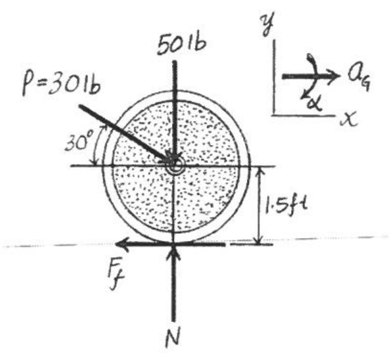
Since the roller rolls without slipping,

$$a_G = \alpha r = \alpha(1.5) \qquad (3)$$

Solving Eqs. (1) through (3) yields

$$\alpha = 7.436 \text{ rad/s}^2 = 7.44 \text{ rad/s}^2 \qquad \textbf{Ans.}$$

$$F_f = 8.660 \text{ lb} \qquad a_G = 11.15 \text{ ft/s}^2$$



(a)

Pure Rolling + No Slipping

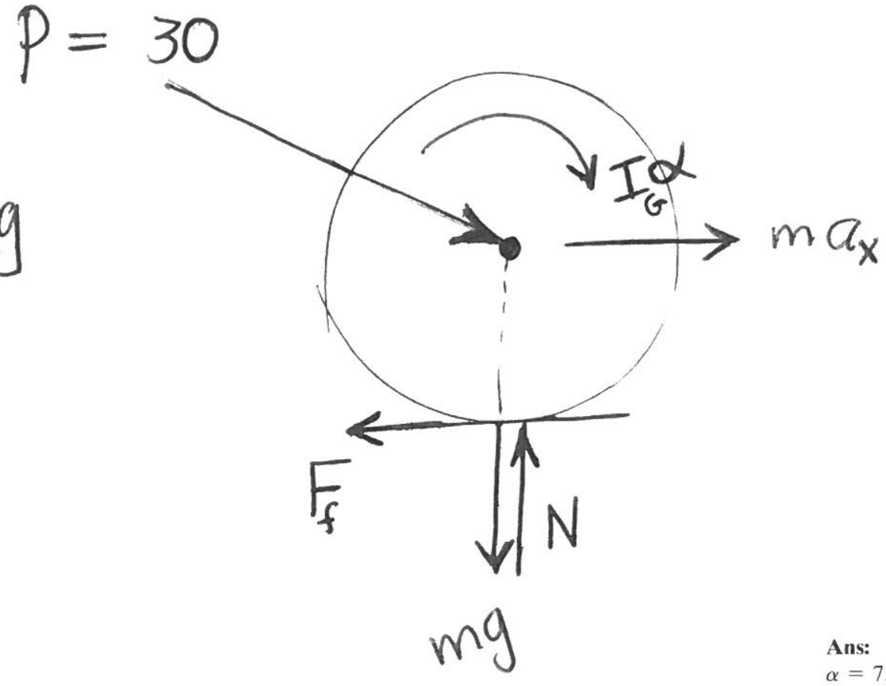
$F_f \neq 0$

$a_x \neq 0$

$a_y = 0$

$\alpha \neq 0$

$a_x = \alpha r$

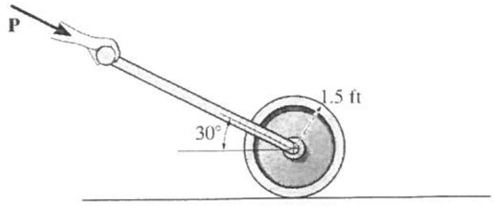


Ans:
$\alpha = 7.44 \text{ rad/s}^2$

Also, $F_f < \mu_s N$

17–105.

If the coefficient of static friction between the 50-lb roller and the ground is $\mu_s = 0.25$, determine the maximum force P that can be applied to the handle, so that roller rolls on the ground without slipping. Also, find the angular acceleration of the roller. Assume the roller to be a uniform cylinder.



SOLUTION

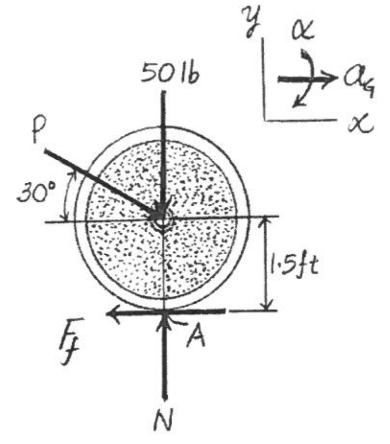
Equations of Motion: The mass moment of inertia of the roller about its mass center is $I_G = \frac{1}{2}mr^2 = \frac{1}{2}\left(\frac{50}{32.2}\right)(1.5^2) = 1.7469\ \text{slug}\cdot\text{ft}^2$. We have

$$\xrightarrow{+} \Sigma F_x = m(a_G)_x; \quad P\cos 30^\circ - F_f = \frac{50}{32.2}a_G \qquad (1)$$

$$+\uparrow \Sigma F_y = m(a_G)_y; \quad N - P\sin 30^\circ - 50 = 0 \qquad (2)$$

$$+\Sigma M_G = I_G\alpha; \quad F_f(1.5) = 1.7469\alpha \qquad (3)$$

general Rolling with Slipping



(a)

Since the roller is required to be on <u>the verge of slipping,</u>

$$a_G = \alpha r = \alpha(1.5) \qquad (4)$$

$$F_f = \mu_s N = 0.25N \qquad (5)$$

pure Rolling

Solving Eqs. (1) through (5) yields

$$\alpha = 18.93\ \text{rad/s}^2 = 18.9\ \text{rad/s}^2 \qquad P = 76.37\ \text{lb} = 76.4\ \text{lb} \qquad \textbf{Ans.}$$

$$N = 88.18\ \text{lb} \qquad a_G = 28.39\ \text{ft/s}^2 \qquad F_f = 22.05\ \text{lb}$$

Same as prob. 7–104, But:

For Start Critical Rolling (without Slipping)

just Sub. $\boxed{F_f = \mu_s N}$

$\Rightarrow$ Three eqn's, 3 unknowns, (P, N, α).

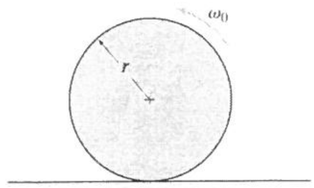
* $P < 76.4$ Ib $\Rightarrow$ Rolling with slipping

* $P > 76.4$ Ib $\Rightarrow$ pure rolling ~~without~~ Slipping

Ans:
$\alpha = 18.9\ \text{rad/s}^2$
$P = 76.4\ \text{lb}$

17–113.

The uniform disk of mass m is rotating with an angular velocity of ω_0 when it is placed on the floor. Determine the initial angular acceleration of the disk and the acceleration of its mass center. The coefficient of kinetic friction between the disk and the floor is μ_k.



SOLUTION

Equations of Motion. Since the disk slips, the frictional force is $F_f = \mu_k N$. The mass moment of inertia of the disk about its mass center is $I_G = \frac{1}{2}mr^2$. We have

$+\uparrow \Sigma F_y = m(a_G)_y; \quad N - mg = 0 \qquad N = mg$

$\overset{+}{\leftarrow} \Sigma F_x = m(a_G)_x; \quad \mu_k(mg) = ma_G \qquad a_G = \mu_k g \leftarrow$ **Ans.**

$\circlearrowleft + \Sigma M_G = I_G\alpha; \quad -\mu_k(mg)r = \left(\frac{1}{2}mr^2\right)\alpha \qquad \alpha = \frac{2\mu_k g}{r} \circlearrowright$ **Ans.**

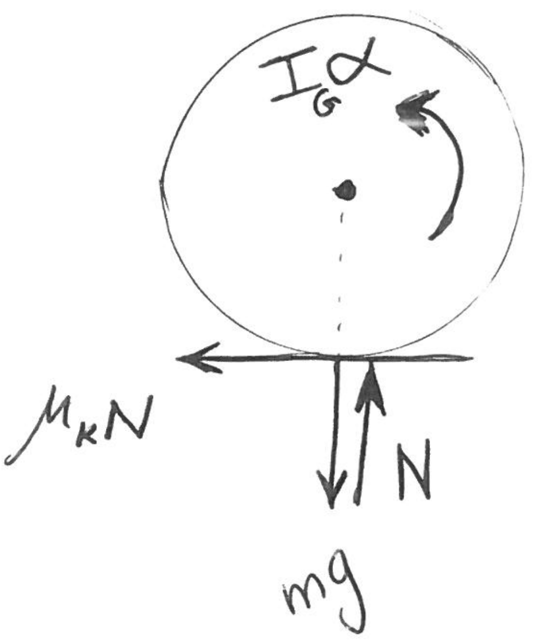
general Rolling with Slipping

Rotation Direction ω_0

$a_x \neq 0$

$a_y = 0$

$\alpha \neq 0$



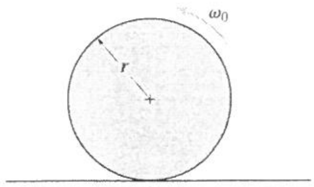
Ans:

$a_G = \mu_k g \leftarrow$

$\alpha = \frac{2\mu_k g}{r} \circlearrowright$

17–114.

The uniform disk of mass m is rotating with an angular velocity of ω_0 when it is placed on the floor. Determine the time before it starts to roll without slipping. What is the angular velocity of the disk at this instant? The coefficient of kinetic friction between the disk and the floor is μ_k.



SOLUTION

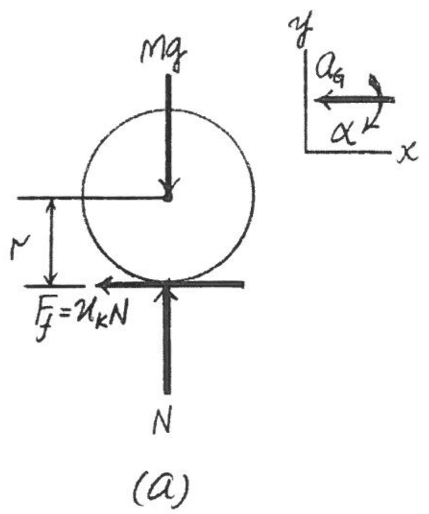
Equations of Motion: Since the disk slips, the frictional force is $F_f = \mu_k N$. The mass moment of inertia of the disk about its mass center is $I_G = \frac{1}{2}mr^2$.

general Rolling with Slipping

$+\uparrow \Sigma F_y = m(a_G)_y; \quad N - mg = 0 \qquad N = mg$

$\overset{+}{\leftarrow} \Sigma F_x = m(a_G)_x; \quad \mu_k(mg) = ma_G \qquad a_G = \mu_k g$

$+\Sigma M_G = I_G\alpha; \quad -\mu_k(mg)r = -\left(\frac{1}{2}mr^2\right)\alpha \qquad \alpha = \frac{2\mu_k g}{r}$



Kinematics: At the instant when the disk rolls without slipping, $v_G = \omega r$. Thus,

$(\overset{+}{\leftarrow}) \qquad v_G = (v_G)_0 + a_G t$

$\omega r = 0 + \mu_k g t$

$t = \frac{\omega r}{\mu_k g} \qquad (1)$

and

$\omega = \omega_0 + \alpha t$

$(\circlearrowleft +) \qquad \omega = \omega_0 + \left(-\frac{2\mu_k g}{r}\right)t \qquad (2)$

Start Rolling Without Slipping

$V_G = \omega t$

(الأهمال استاتيك μ_s)

pure rolling

Solving Eqs. (1) and (2) yields

$\omega = \frac{1}{3}\omega_0 \qquad t = \frac{\omega_0 r}{3\mu_k g}$ **Ans.**

* Same as prob. 17–113 .

Start Rolling without Slipping $\Rightarrow V_G = \omega r$

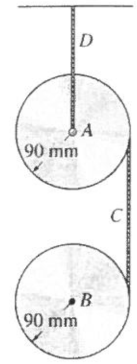
Ans:

$\omega = \frac{1}{3}\omega_0$

$t = \frac{\omega_0 r}{3\mu_k g}$

17–115.

A cord is wrapped around each of the two 10-kg disks. If they are released from rest, determine the angular acceleration of each disk and the tension in the cord C. Neglect the mass of the cord.



$I = \frac{1}{2} m r^2$

SOLUTION

* For A:

$$\circlearrowleft + \Sigma M_A = I_A \alpha_A; \qquad T(0.09) = \left[\frac{1}{2}(10)(0.09)^2\right]\alpha_A \qquad (1)$$

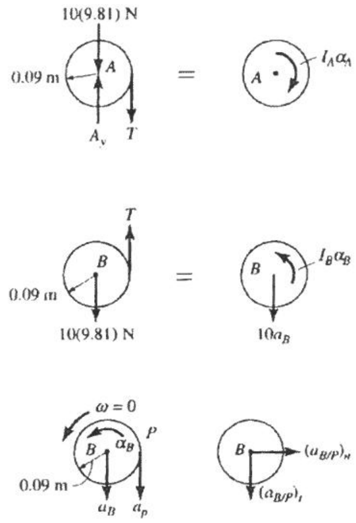
* For B:

$$\circlearrowleft + \Sigma M_B = I_B \alpha_B; \qquad T(0.09) = \left[\frac{1}{2}(10)(0.09)^2\right]\alpha_B \qquad (2)$$

$$+\downarrow \Sigma F_y = m(a_B)_y; \qquad 10(9.81) - T = 10a_B \qquad (3)$$

* $\boxed{\mathbf{a}_B = \mathbf{a}_P + (\mathbf{a}_{B/P})_t + (\mathbf{a}_{B/P})_n}$

$$(+\downarrow)\ a_B = \underbrace{0.09\alpha_A}_{a_t} + \underbrace{0.09\alpha_B}_{a_t} + \underbrace{0}_{a_n = 0\ (\omega = 0)} \qquad (4)$$



Solving,

$a_B = 7.85\ \text{m/s}^2$

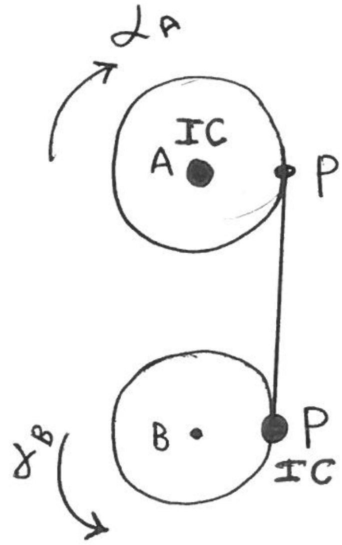
$\alpha_A = 43.6\ \text{rad/s}^2$ **Ans.**

$\alpha_B = 43.6\ \text{rad/s}^2$ **Ans.**

$T = 19.6\ \text{N}$ **Ans.**

$A_y = 10(9.81) + 19.62$

$= 118\ \text{N}$



$P \Rightarrow a_p = 0.09\,\alpha_A$

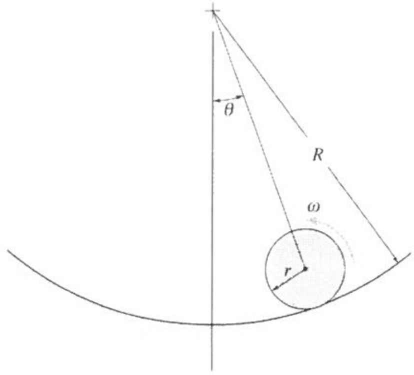
$P \Rightarrow (a_{B/P})_t = 0.09\,\alpha_B$

$(a_{B/P})_n = 0\ (\omega = 0)$

Ans:
$\alpha_A = 43.6\ \text{rad/s}^2$
$\alpha_B = 43.6\ \text{rad/s}^2$
$T = 19.6\ \text{N}$

***17–116.**

The disk of mass m and radius r rolls without slipping on the circular path. Determine the normal force which the path exerts on the disk and the disk's angular acceleration if at the instant shown the disk has an angular velocity of $\boldsymbol{\omega}$.



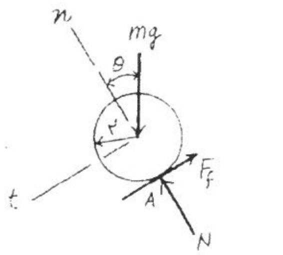
$$I_G = \frac{1}{2} m r^2$$

SOLUTION

Equation of Motion: The mass moment of inertia of the disk about its center of mass is given by $I_G = \dfrac{1}{2}mr^2$. Applying Eq. 17–16, we have

$$\circlearrowleft + \Sigma M_A = \Sigma (M_k)_A; \qquad mg \sin\theta(r) = \left(\frac{1}{2}mr^2\right)\alpha + m(a_G)_t(r) \qquad [1]$$

$$\Sigma F_n = m(a_G)_n; \qquad N - mg\cos\theta = m(a_G)_n \qquad [2]$$



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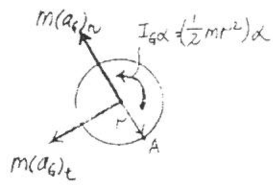


Kinematics: Since the semicircular disk does not slip at A, then $v_G = \omega r$ and $(a_G)_t = \alpha r$. Substitute $(a_G)_t = \alpha r$ into Eq. [1] yields

$$mg\sin\theta(r) = \left(\frac{1}{2}mr^2\right)\alpha + m(\alpha r)(r)$$

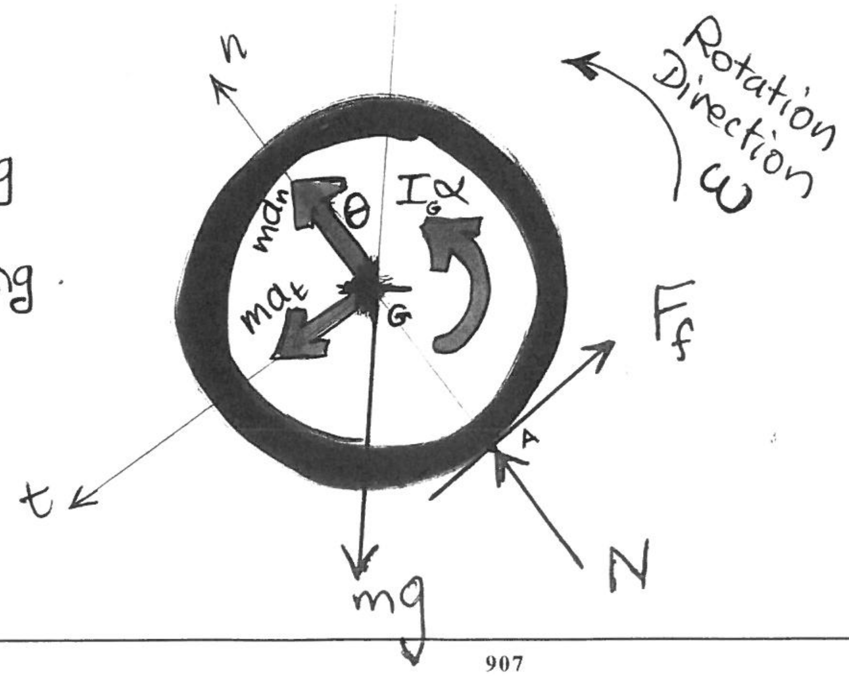
$$\alpha = \frac{2g}{3r}\sin\theta \qquad \textbf{Ans.}$$

Also, the center of the mass for the disk moves around a circular path having a radius of $\rho = R - r$. Thus, $(a_G)_n = \dfrac{v_G^2}{\rho} = \dfrac{\omega^2 r^2}{R - r}$. Substitute into Eq. [2] yields

$$N - mg\cos\theta = m\left(\frac{\omega^2 r^2}{R - r}\right)$$

$$N = m\left(\frac{\omega^2 r^2}{R - r} + g\cos\theta\right) \qquad \textbf{Ans.}$$

Pure Rolling
No slipping.

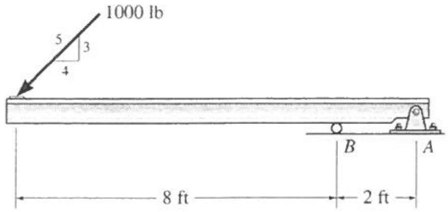


$a_n \neq 0$
$a_t \neq 0$
$\alpha \neq 0$

Ans:
$\alpha = \dfrac{2g}{3r}\sin\theta$
$N = m\left(\dfrac{\omega^2 r^2}{R - r} + g\cos\theta\right)$

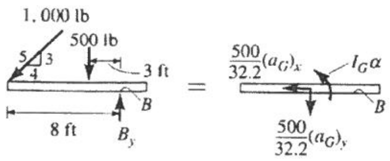
17–118.

The 500-lb beam is supported at A and B when it is subjected to a force of 1000 lb as shown. If the pin support at A suddenly fails, determine the beam's initial angular acceleration and the force of the roller support on the beam. For the calculation, assume that the beam is a slender rod so that its thickness can be neglected.



$I_G = \frac{1}{12} m L^2$

SOLUTION



$\overset{+}{\leftarrow} \sum F_x = m(a_G)_x; \qquad 1000\left(\frac{4}{5}\right) = \frac{500}{32.2}(a_G)_x$

$+\downarrow \sum F_y = m(a_G)_y; \qquad 1000\left(\frac{3}{5}\right) + 500 - B_y = \frac{500}{32.2}(a_G)_y$

$\circlearrowleft + \sum M_B = \sum (M_k)_B; \quad 500(3) + 1000\left(\frac{3}{5}\right)(8) = \frac{500}{32.2}(a_G)_y(3) + \left[\frac{1}{12}\left(\frac{500}{32.2}\right)(10)^2\right]\alpha$

$\mathbf{a}_B = \mathbf{a}_G + \mathbf{a}_{B/G}$

$-a_B\mathbf{i} = -(a_G)_x\mathbf{i} - (a_G)_y\mathbf{j} + \alpha(3)\mathbf{j}$

$(+\downarrow) \quad (a_G)_y = \alpha(3)$

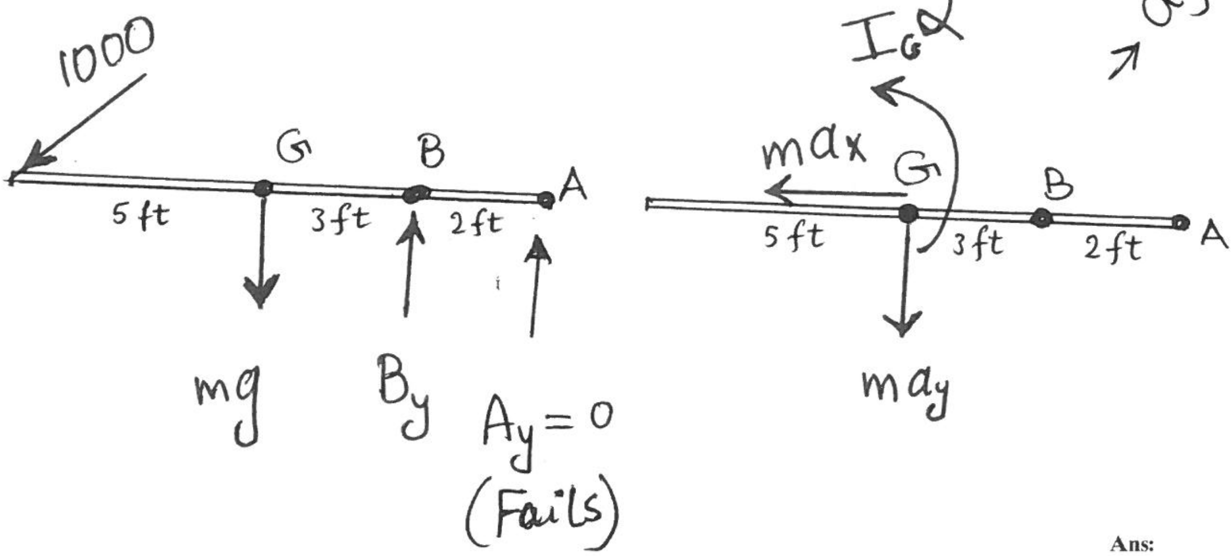
$\alpha = 23.4 \text{ rad/s}^2$ **Ans.**

$B_y = 9.62 \text{ lb}$ **Ans.**

$B_y > 0$ means that the beam stays in contact with the roller support.

$a_y = \alpha r$

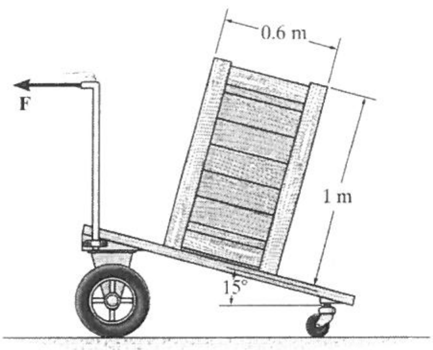
$a_y = 3\alpha$



Ans:
$\alpha = 23.4 \text{ rad/s}^2$
$B_y = 9.62 \text{ lb}$

17–42.

The uniform crate has a mass of 50 kg and rests on the cart having an inclined surface. Determine the smallest acceleration that will cause the crate either to tip or slip relative to the cart. What is the magnitude of this acceleration? The coefficient of static friction between the crate and the cart is $\mu_s = 0.5$.



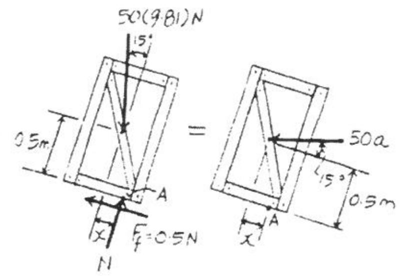
SOLUTION

Equations of Motion: Assume that the crate slips, then $F_f = \mu_s N = 0.5N$.

$$\circlearrowleft + \Sigma M_A = \Sigma (M_k)_A; \quad 50(9.81)\cos 15°(x) - 50(9.81)\sin 15°(0.5) = 50a\cos 15°(0.5) + 50a\sin 15°(x) \quad \textbf{(1)}$$

$$+\nearrow \Sigma F_{y'} = m(a_G)_{y'}; \quad N - 50(9.81)\cos 15° = -50a\sin 15° \quad \textbf{(2)}$$

$$\searrow + \Sigma F_{x'} = m(a_G)_{x'}; \quad 50(9.81)\sin 15° - 0.5N = -50a\cos 15° \quad \textbf{(3)}$$



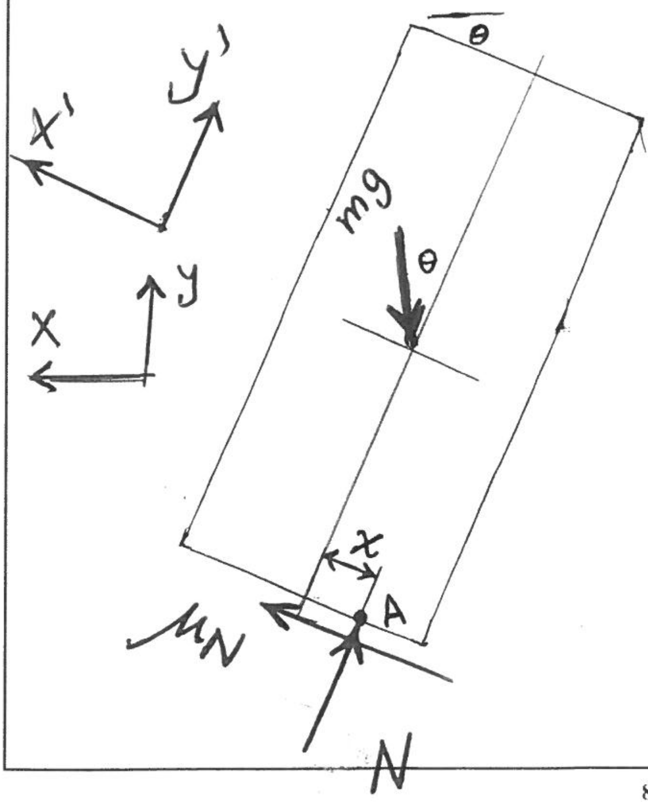
Solving Eqs. (1), (2), and (3) yields

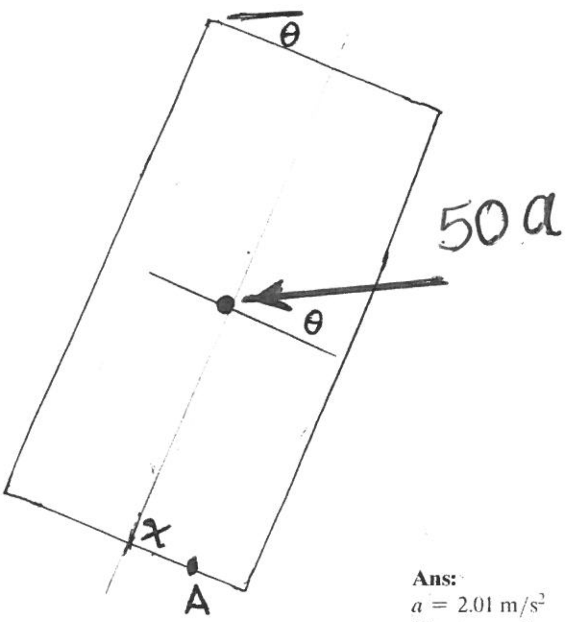
$$N = 447.81 \text{ N} \qquad x = 0.250 \text{ m}$$

$$a = 2.01 \text{ m/s}^2 \qquad \textbf{Ans.}$$

Since $x < 0.3$ m, then crate will not tip. Thus, **the crate slips**. **Ans.**

$a_x \neq 0$
$a_y = 0$
$\alpha = 0$





Ans:
$a = 2.01 \text{ m/s}^2$
The crate slips.

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الجامعة الاردنية

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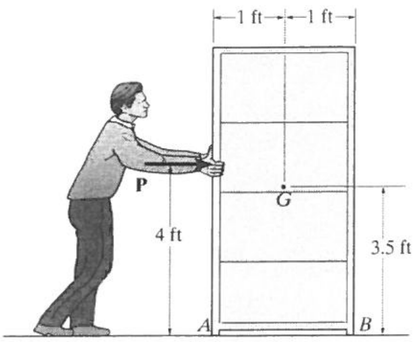
17–43.

Determine the acceleration of the 150-lb cabinet and the normal reaction under the legs A and B if $P = 35$ lb. The coefficients of static and kinetic friction between the cabinet and the plane are $\mu_s = 0.2$ and $\mu_k = 0.15$, respectively. The cabinet's center of gravity is located at G.



SOLUTION

Equations of Equilibrium: The free-body diagram of the cabinet under the static condition is shown in Fig. *a*, where **P** is the unknown minimum force needed to move the cabinet. We will assume that the cabinet slides before it tips. Then, $F_A = \mu_s N_A = 0.2N_A$ and $F_B = \mu_s N_B = 0.2N_B$.

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad P - 0.2N_A - 0.2N_B = 0 \qquad (1)$$

$$+\uparrow \Sigma F_y = 0; \qquad N_A + N_B - 150 = 0 \qquad (2)$$

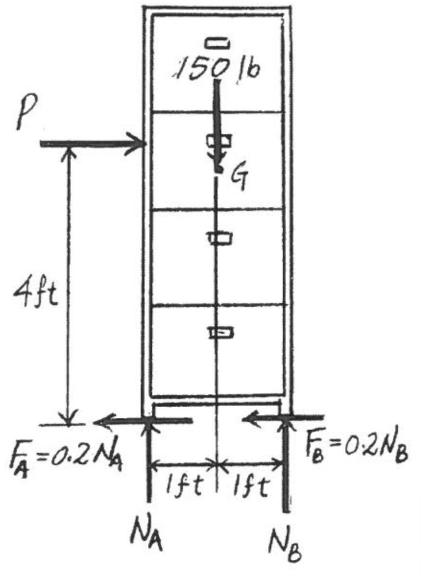
$$+\Sigma M_A = 0; \qquad N_B(2) - 150(1) - P(4) = 0 \qquad (3)$$

$a_x = 0$
$a_y = 0$
$\alpha = 0$

Solving Eqs. (1), (2), and (3) yields

$P = 30$ lb $\qquad N_A = 15$ lb $\qquad N_B = 135$ lb

Since $P < 35$ lb and $\mathbf{N}_A$ is positive, the cabinet will slide.



(a)

Equations of Motion: Since the cabinet is in motion, $F_A = \mu_k N_A = 0.15N_A$ and $F_B = \mu_k N_B = 0.15N_B$. Referring to the free-body diagram of the cabinet shown in Fig. *b*,

$$\overset{+}{\rightarrow} \Sigma F_x = m(a_G)_x; \qquad 35 - 0.15N_A - 0.15N_B = \left(\frac{150}{32.2}\right)a \qquad (4)$$

$$\overset{+}{\rightarrow} \Sigma F_x = m(a_G)_x; \qquad N_A + N_B - 150 = 0 \qquad (5)$$

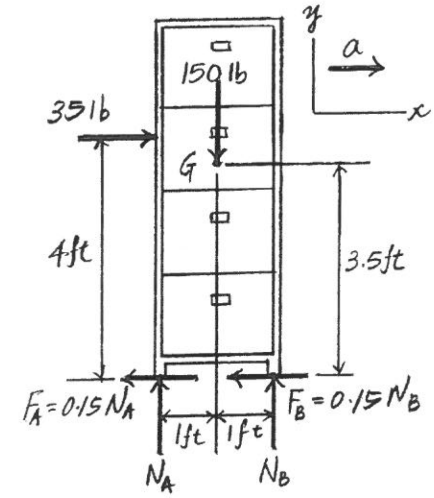
$$+\Sigma M_G = 0; \quad N_B(1) - 0.15N_B(3.5) - 0.15N_A(3.5) - N_A(1) - 35(0.5) = 0 \quad (6)$$

Solving Eqs. (4), (5), and (6) yields

$$a = 2.68 \text{ ft/s}^2 \qquad \textbf{Ans.}$$

$$N_A = 26.9 \text{ lb} \qquad N_B = 123 \text{ lb} \qquad \textbf{Ans.}$$

$a_x \neq 0$
$a_y = 0$
$\alpha = 0$

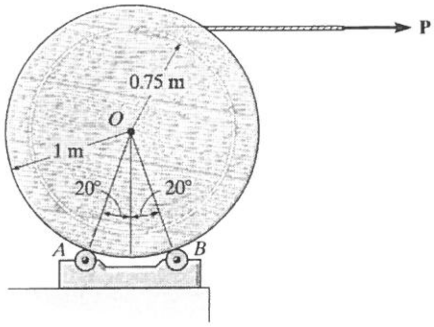


(b)

Ans:
$a = 2.68$ ft/s^2
$N_A = 26.9$ lb
$N_B = 123$ lb

17–61.

If a horizontal force of $P = 100$ N is applied to the 300-kg reel of cable, determine its initial angular acceleration. The reel rests on rollers at A and B and has a radius of gyration of $k_O = 0.6$ m.



$$I_o = m K_o^2$$

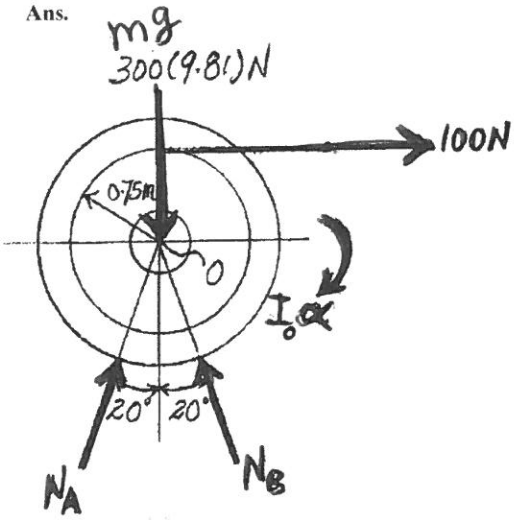
SOLUTION

Equations of Motions. The mass moment of inertia of the reel about O is $I_O = Mk_O^2 = 300(0.6^2) = 108\ \text{kg}\cdot\text{m}^2$. Referring to the FBD of the reel, Fig. a,

$\circlearrowleft + \Sigma M_O = I_O\,\alpha;\qquad -100(0.75) = 108(-\alpha)$

$$\alpha = 0.6944\ \text{rad/s}^2$$

$$= 0.694\ \text{rad/s}^2 \qquad \textbf{Ans.}$$



$$a_x = 0$$
$$a_y = 0$$
$$\alpha \neq 0$$

$$\Sigma F_x = 0$$

$\rightarrow\quad N_A \sin 20^\circ - N_B \sin 20^\circ + 100 = 0$

$$\Sigma F_y = 0$$

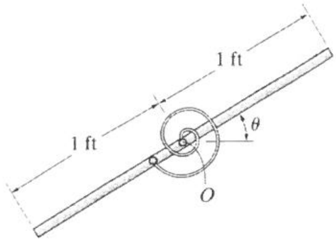
$\rightarrow\quad N_A \cos 20^\circ + N_B \cos 20^\circ - 300(9.81) = 0$

$$\Rightarrow \quad N_A = 1419.7\ \text{N}$$
$$N_B = 1712.1\ \text{N}$$

Ans:
$\alpha = 0.694\ \text{rad/s}^2$

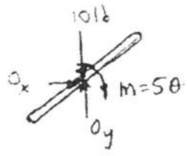
17–63.

The 10-lb bar is pinned at its center O and connected to a torsional spring. The spring has a stiffness $k = 5$ lb · ft/rad, so that the torque developed is $M = (5\theta)$ lb · ft, where θ is in radians. If the bar is released from rest when it is vertical at $\theta = 90°$, determine its angular velocity at the instant $\theta = 45°$.



$I_o = \frac{1}{12} m L^2$

SOLUTION



$\circlearrowleft + \Sigma M_O = I_O\alpha; \qquad 5\theta = [\frac{1}{12}(\frac{10}{32.2})(2)^2]\alpha$

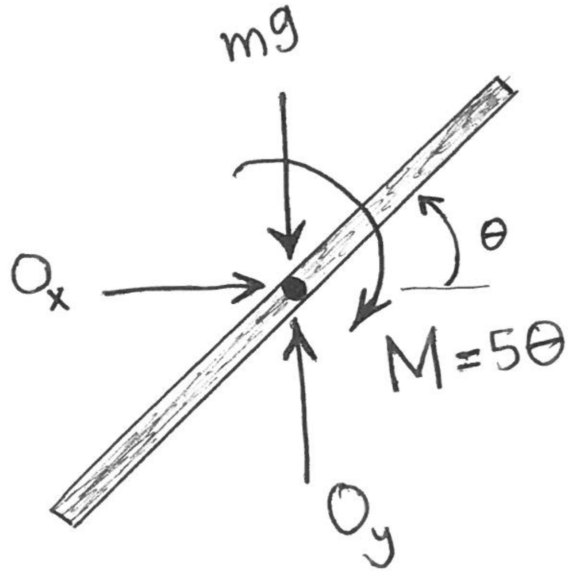
$\alpha = -48.3\theta$

$\alpha\, d\theta = \omega\, d\omega$

$- \int_{\frac{\pi}{2}}^{\frac{\pi}{4}} 48.3\theta\, d\theta = \int_0^{\omega} \omega\, d\omega$

$-24.15\left((\frac{\pi}{4})^2 - (\frac{\pi}{2})^2 \right) = \frac{1}{2}\omega^2$

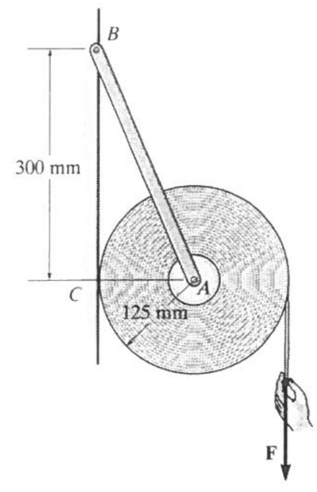
$\omega = 9.45$ rad/s **Ans.**



Ans:
$\omega = 9.45$ rad/s

17–70.

The 20-kg roll of paper has a radius of gyration $k_A = 90$ mm about an axis passing through point A. It is pin supported at both ends by two brackets AB. If the roll rests against a wall for which the coefficient of kinetic friction is $\mu_k = 0.2$, determine the constant vertical force F that must be applied to the roll to pull off 1 m of paper in $t = 3$ s starting from rest. Neglect the mass of paper that is removed.



SOLUTION

$$I_A = mr^2$$

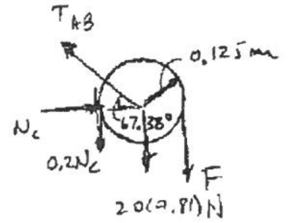
$$\alpha = \frac{a_c}{r}$$

$(+\downarrow)\; s = s_0 + v_0 t + \frac{1}{2}a_C t^2$

$$1 = 0 + 0 + \frac{1}{2}a_C(3)^2$$

$$a_C = 0.222 \text{ m/s}^2$$

$$\alpha = \frac{a_C}{0.125} = 1.778 \text{ rad/s}^2$$



$\xrightarrow{+}\; \Sigma F_x = m(a_G)_x;\qquad N_C - T_{AB}\cos 67.38° = 0$

$+\uparrow \Sigma F_y = m(a_G)_y;\qquad T_{AB}\sin 67.38° - 0.2N_C - 20(9.81) - F = 0$

$\circlearrowleft + \Sigma M_A = I_A\alpha;\qquad -0.2N_C(0.125) + F(0.125) = 20(0.09)^2(1.778)$

Solving:

$$N_C = 99.3 \text{ N}$$

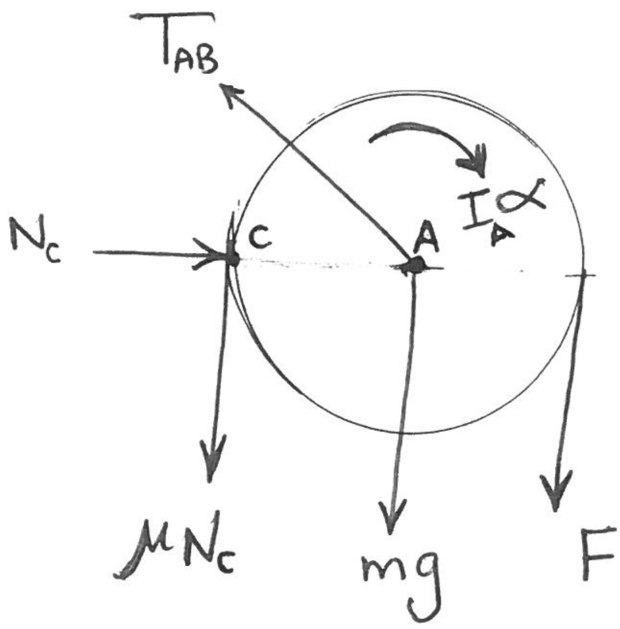
$$T_{AB} = 258 \text{ N}$$

$$F = 22.1 \text{ N} \qquad \textbf{Ans.}$$

$a_x = 0$

$a_y = 0$

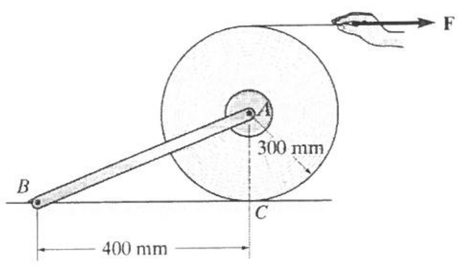
$\alpha \neq 0$



Ans:
$F = 22.1$ N

***17–76.**

The 20-kg roll of paper has a radius of gyration $k_A = 120$ mm about an axis passing through point A. It is pin supported at both ends by two brackets AB. The roll rests on the floor, for which the coefficient of kinetic friction is $\mu_k = 0.2$. If a horizontal force $F = 60$ N is applied to the end of the paper, determine the initial angular acceleration of the roll as the paper unrolls.



$I_A = m\,k_A^2$

SOLUTION

Equations of Motion. The mass moment of inertia of the paper roll about A is $I_A = mk_A^2 = 20(0.12^2) = 0.288\ \text{kg}\cdot\text{m}^2$. Since it is required to slip at C, the friction is $F_f = \mu_k N = 0.2\,N$. Referring to the FBD of the paper roll, Fig. *a*

$$\overset{+}{\rightarrow} \Sigma F_x = m(a_G)_x; \qquad 0.2\,N - F_{AB}\left(\frac{4}{5}\right) + 60 = 20(0) \qquad \textbf{(1)}$$

$$+\uparrow \Sigma F_y = m(a_G)_y; \qquad N - F_{AB}\left(\frac{3}{5}\right) - 20(9.81) = 20(0) \qquad \textbf{(2)}$$

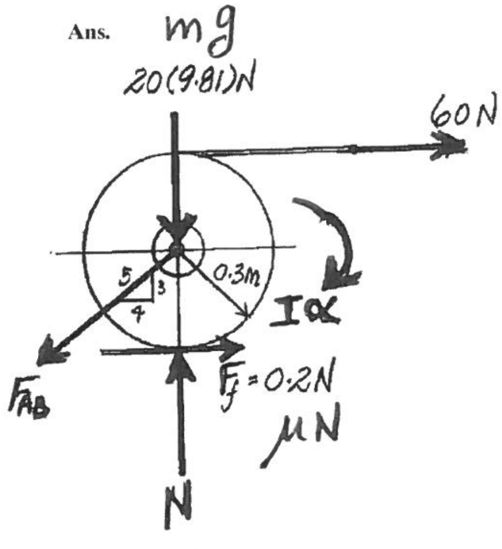
Solving Eqs. (1) and (2)

$$F_{AB} = 145.94\ \text{N} \qquad N = 283.76\ \text{N}$$

Subsequently

$$\circlearrowleft + \Sigma M_A = I_A\alpha; \qquad 0.2(283.76)(0.3) - 60(0.3) = 0.288(-\alpha)$$

$$\alpha = 3.3824\ \text{rad/s}^2 = 3.38\ \text{rad/s}^2 \qquad \textbf{Ans.}$$



$a_x = 0$

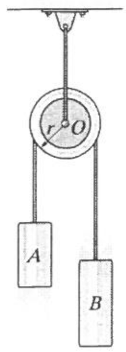
$a_y = 0$

$\alpha \neq 0$

Ans:
$\alpha = 3.38\ \text{rad/s}^2$

17–79.

The two blocks A and B have a mass of 5 kg and 10 kg, respectively. If the pulley can be treated as a disk of mass 3 kg and radius 0.15 m, determine the acceleration of block A. Neglect the mass of the cord and any slipping on the pulley.



$$I_o = \frac{1}{2} m r^2$$

SOLUTION

Kinematics: Since the pulley rotates about a fixed axis passes through point O, its angular acceleration is

$$\alpha = \frac{a}{r} = \frac{a}{0.15} = 6.6667a$$

The mass moment of inertia of the pulley about point O is

$$I_o = \frac{1}{2}Mr^2 = \frac{1}{2}(3)(0.15^2) = 0.03375 \text{ kg}\cdot\text{m}^2$$

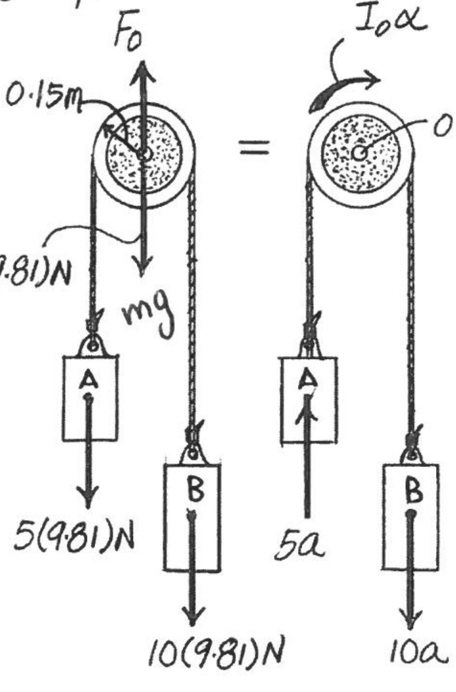
Equation of Motion: Write the moment equation of motion about point O by referring to the free-body and kinetic diagram of the system shown in Fig. a,

$$\circlearrowleft +\Sigma M_o = \Sigma(M_k)_o; \qquad 5(9.81)(0.15) - 10(9.81)(0.15)$$

$$= -0.03375(6.6667a) - 5a(0.15) - 10a(0.15)$$

$$a = 2.973 \text{ m/s}^2 = 2.97 \text{ m/s}^2 \qquad \textbf{Ans.}$$

$$\alpha = 6.666(2.973) = 19.82 \text{ rad/s}^2$$

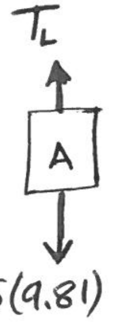


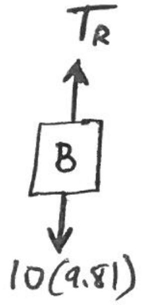
* $\Sigma F_y = m a$

$F_O - mg - 5(9.81) - 10(9.81) = 5a - 10a$

$\Rightarrow \quad F_O = 161.72$ N

*





$T_L - 5(9.81) = 5a$

$\Rightarrow T_L = 63.92$ N

$10(9.81) - T_R = 10a$

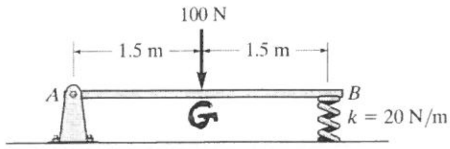
$\Rightarrow T_R = 68.37$ N

Ans:
$a = 2.97 \text{ m/s}^2$

$T_L \neq T_R \xrightarrow{OR} (T_R - T_L)r = I_o\alpha$

17–86.

The 4-kg slender rod is initially supported horizontally by a spring at B and pin at A. Determine the angular acceleration of the rod and the acceleration of the rod's mass center at the instant the 100-N force is applied.



$$I_A = \frac{1}{12} m L^2 + m\left(\frac{L}{2}\right)^2$$

SOLUTION

Equation of Motion. The mass moment of inertia of the rod about A is $I_A = \frac{1}{12}(4)(3^2) + 4(1.5^2) = 12.0\ \mathrm{kg \cdot m^2}$. Initially, the beam is at rest, $\omega = 0$. Thus, $(a_G)_n = \omega^2 r = 0$. Also, $(a_G)_t = \alpha r_G = \alpha(1.5)$. The force developed in the spring before the application of the 100 N force is $F_{sp} = \frac{4(9.81)\ \mathrm{N}}{2} = 19.62$ N. Referring to the FBD of the rod, Fig. a,

$$F_{sp} = \frac{mg}{2}$$

$\circlearrowleft + M_A = I_A\alpha;$ $\quad 19.62(3) - 100(1.5) - 4(9.81)(1.5) = 12.0(-\alpha)$

$$\alpha = 12.5\ \mathrm{rad/s^2} \circlearrowright \qquad \textbf{Ans.}$$

Then

$$(a_G)_t = 12.5(1.5) = 18.75\ \mathrm{m/s^2} \downarrow$$

Since $(a_G)_n = 0$. Then

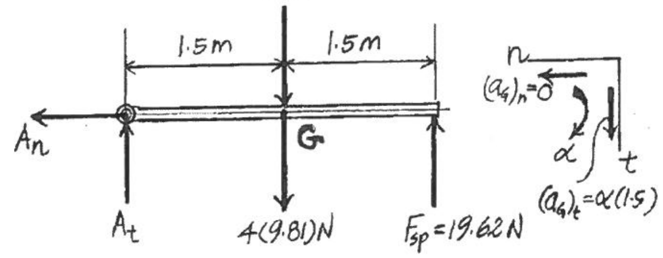
$$a_G = (a_G)_t = 18.75\ \mathrm{m/s^2} \downarrow \qquad \textbf{Ans.}$$

$\omega = 0$ (Rest)

$a_t = \alpha r_G$

$a_n = \omega^2 r_G = 0$

$\alpha \neq 0$



(a)

Reactions:

$\sum F_x = \sum F_n = m a_n = 0 \Rightarrow A_n = 0$

$\sum F_y = \sum F_t = m a_t = m \alpha r_G$

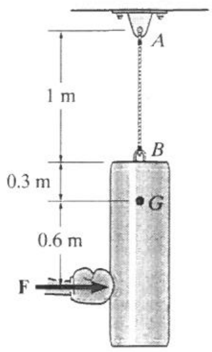
$\Rightarrow A_t - 4(9.81) - 100 + 19.62 = -4(12.5)(1.5)$

$\Rightarrow A_t = 44.62$ N

Ans:
$\alpha = 12.5\ \mathrm{rad/s^2} \circlearrowright$
$a_G = 18.75\ \mathrm{m/s^2} \downarrow$

17–91.

The 20-kg punching bag has a radius of gyration about its center of mass G of $k_G = 0.4$ m. If it is initially at rest and is subjected to a horizontal force $F = 30$ N, determine the initial angular acceleration of the bag and the tension in the supporting cable AB.



$$I_G = m K_G^2$$

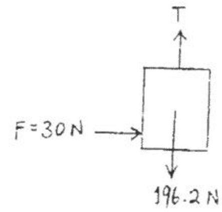
SOLUTION

$\xrightarrow{+} \Sigma F_x = m(a_G)_x;\quad 30 = 20(a_G)_x$

$+\uparrow \Sigma F_y = m(a_G)_y;\quad T - 196.2 = 20(a_G)_y$

$\circlearrowleft + \Sigma M_G = I_G\alpha;\quad 30(0.6) = 20(0.4)^2\alpha$

$\alpha = 5.62 \text{ rad/s}^2$ **Ans.**



$(a_G)_x = 1.5 \text{ m/s}^2$

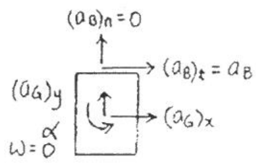
$\mathbf{a}_B = \mathbf{a}_G + \mathbf{a}_{B/G}$

$a_B\mathbf{i} = (a_G)_y\mathbf{j} + (a_G)_x\mathbf{i} - \alpha(0.3)\mathbf{i}$

$(+\uparrow)\quad (a_G)_y = 0$

Thus,

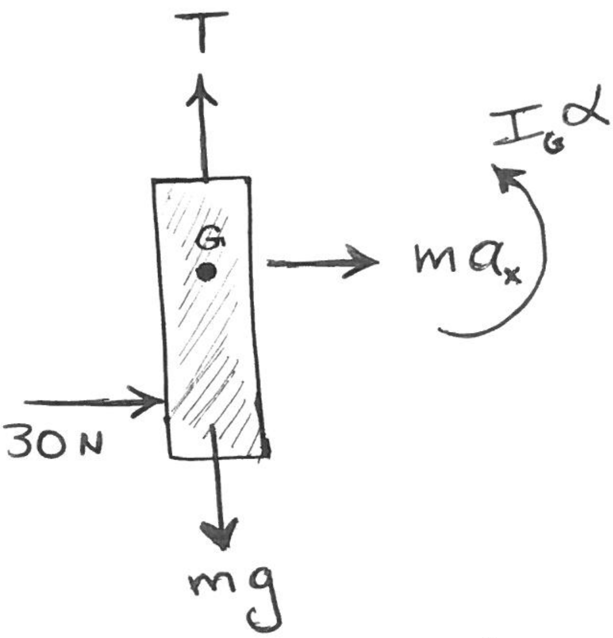
$T = 196 \text{ N}$ **Ans.**



$a_x \neq 0$

$a_y = 0$

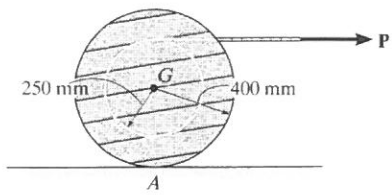
$\alpha \neq 0$



Ans:
$\alpha = 5.62 \text{ rad/s}^2$
$T = 196$ N

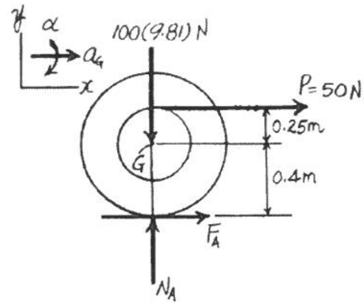
***17–96.**

The spool has a mass of 100 kg and a radius of gyration of $k_G = 0.3$ m. If the coefficients of static and kinetic friction at A are $\mu_s = 0.2$ and $\mu_k = 0.15$, respectively, determine the angular acceleration of the spool if $P = 50$ N.



$I_G = mr^2$

SOLUTION



$\xrightarrow{+} \Sigma F_x = m(a_G)_x;\quad 50 + F_A = 100a_G$

$+\uparrow \Sigma F_y = m(a_G)_y;\quad N_A - 100(9.81) = 0$

$\circlearrowleft + \Sigma M_G = I_G\alpha;\quad 50(0.25) - F_A(0.4) = [100(0.3)^2]\alpha$

Assume no slipping: $a_G = 0.4\alpha$

$\alpha = 1.30 \text{ rad/s}^2$ **Ans.**

$a_G = 0.520 \text{ m/s}^2 \quad N_A = 981 \text{ N} \quad F_A = 2.00 \text{ N}$

Since $(F_A)_{max} = 0.2(981) = 196.2 \text{ N} > 2.00 \text{ N}$ **OK**

$a_x \neq 0$

$a_y = 0$

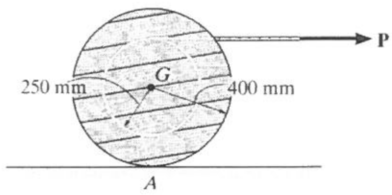
$\alpha \neq 0$

$F_A < \mu_s N_A \Rightarrow$ Rolling No slipping $\Rightarrow a_G = \alpha r$

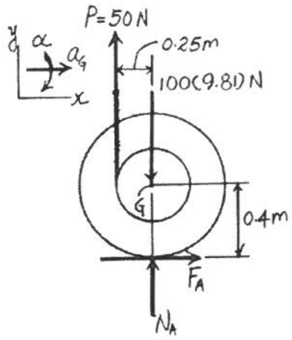
Ans:
$\alpha = 1.30 \text{ rad/s}^2$

17–97.

Solve Prob. 17–96 if the cord and force $P = 50$ N are directed vertically upwards.



SOLUTION



$\xrightarrow{+} \Sigma F_x = m(a_G)x; \quad F_A = 100a_G$

$+\uparrow \Sigma F_y = m(a_G)_y; \quad N_A + 50 - 100(9.81) = 0$

$\circlearrowleft + \Sigma M_G = I_G\alpha; \quad 50(0.25) - F_A(0.4) = [100(0.3)^2]\alpha$

Assume no slipping: $a_G = 0.4\,\alpha$

$\alpha = 0.500 \text{ rad/s}^2$ **Ans.**

$a_G = 0.2 \text{ m/s}^2 \qquad N_A = 931 \text{ N} \qquad F_A = 20 \text{ N}$

Since $(F_A)_{max} = 0.2(931) = 186.2 \text{ N} > 20 \text{ N}$ **OK**

$a_x \neq 0$

$a_y = 0 \Rightarrow$ No motion in y-direction thus N_A exist.

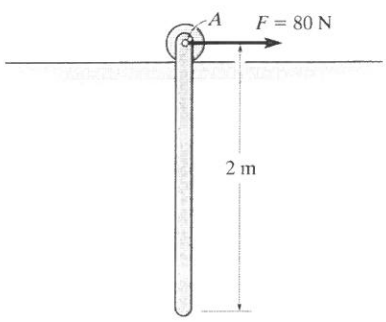
$\alpha \neq 0$

$F_A < \mu_s N_A \Rightarrow$ Rolling No Slipping $\Rightarrow a_G = \alpha r$

Ans:
$\alpha = 0.500 \text{ rad/s}^2$

17–99.

The 12-kg uniform bar is supported by a roller at A. If a horizontal force of $F = 80$ N is applied to the roller, determine the acceleration of the center of the roller at the instant the force is applied. Neglect the weight and the size of the roller.



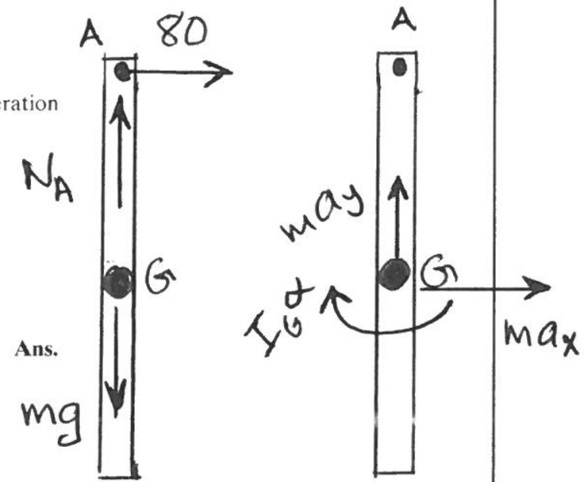
$$I_G = \frac{1}{12} m L^2$$

SOLUTION

Equations of Motion. The mass moment of inertia of the bar about its center of gravity G is $I_G = \frac{1}{12} ml^2 = \frac{1}{12}(12)(2^2) = 4.00 \text{ kg} \cdot \text{m}^2$. Referring to the FBD and kinetic diagram of the bar, Fig. a

$\xrightarrow{+} \Sigma F_x = m(a_G)_x;\quad 80 = 12(a_G)_x \quad (a_G)_x = 6.6667 \text{ m/s}^2 \rightarrow$

$\circlearrowleft + \Sigma M_A = (\mu_k)_A;\quad 0 = 12(6.6667)(1) - 4.00\,\alpha \quad \alpha = 20.0 \text{ rad/s}^2 \circlearrowright$



Kinematic. Since the bar is initially at rest, $\omega = 0$. Applying the relative acceleration equation by referring to Fig. b,

$$\mathbf{a}_G = \mathbf{a}_A + \boldsymbol{\alpha} \times \mathbf{r}_{G/A} - \omega^2 \mathbf{r}_{G/A}$$

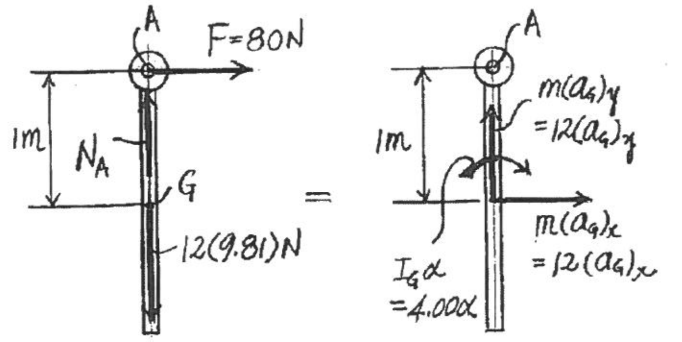
$$6.6667\mathbf{i} + (a_G)_y\mathbf{j} = a_A\mathbf{i} + (-20.0\mathbf{k}) \times (-\mathbf{j}) - \mathbf{0}$$

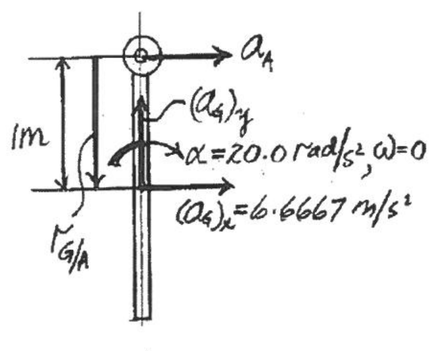
$$6.6667\mathbf{i} + (a_G)_y\mathbf{j} = (a_A - 20)\mathbf{i}$$

Equating **i** and **j** components,

$6.6667 = a_A - 20,\quad a_A = 26.67 \text{ m/s}^2 = 26.7 \text{ m/s}^2 \rightarrow$ **Ans.**

$(a_G)_y = 0$





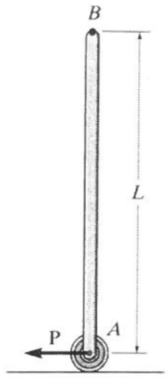
(b)

$$a_x \neq 0$$
$$a_y = 0$$
$$\alpha \neq 0$$

Ans:
$a_A = 26.7 \text{ m/s}^2 \rightarrow$

17–106.

The uniform bar of mass m and length L is balanced in the vertical position when the horizontal force **P** is applied to the roller at A. Determine the bar's initial angular acceleration and the acceleration of its top point B.



$$I_G = \frac{1}{12} m L^2$$

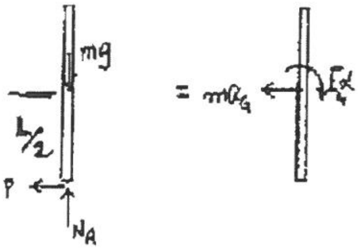
SOLUTION

$\overset{+}{\leftarrow} \Sigma F_x = m(a_G)_x; \qquad P = m a_G$

$\circlearrowleft + \Sigma M_G = I_G \alpha; \qquad P\left(\frac{L}{2}\right) = \left(\frac{1}{12} m L^2\right)\alpha$

$$P = \frac{1}{6} m L \alpha$$

$$\alpha = \frac{6P}{mL} \qquad \textbf{Ans.}$$



$$a_G = \frac{P}{m}$$

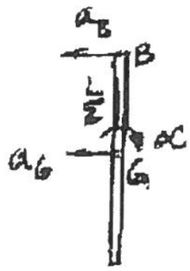
$\omega = 0$

$\mathbf{a}_B = \mathbf{a}_G + \mathbf{a}_{B/G} \rightarrow a_B = a_G + \alpha \times r_{B/G} - \cancel{\omega^2 r_{B/G}}$

$$-a_B \mathbf{i} = \frac{-P}{m}\mathbf{i} + \frac{L}{2}\alpha\,\mathbf{i}$$

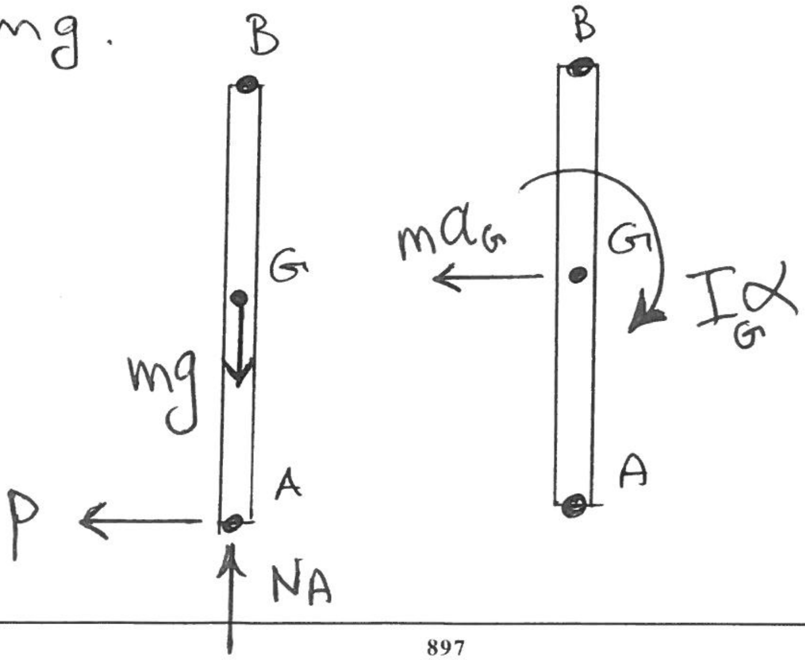
$(\overset{+}{\leftarrow}) \qquad a_B = \frac{P}{m} - \frac{L\alpha}{2}$

$$= \frac{P}{m} - \frac{L}{2}\left(\frac{6P}{mL}\right)$$



$$a_B = -\frac{2P}{m} = \frac{2P}{m} \qquad \textbf{Ans.}$$

$N_A = mg.$



$a_x \neq 0$
$a_y = 0$
$\alpha \neq 0$

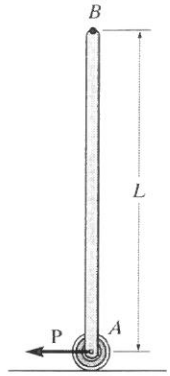
Ans:

$\alpha = \frac{6P}{mL}$

$a_B = \frac{2P}{m}$

17–107.

Solve Prob. 17–106 if the roller is removed and the coefficient of kinetic friction at the ground is μ_k.



$$I_G = \frac{1}{12} m L^2$$

SOLUTION

$\overset{+}{\leftarrow} \Sigma F_x = m(a_G)_x; \qquad P - \mu_k N_A = m a_G$

$\circlearrowleft + \Sigma M_G = I_G \alpha; \qquad (P - \mu_k N_A)\dfrac{L}{2} = \left(\dfrac{1}{12} m L^2\right)\alpha$

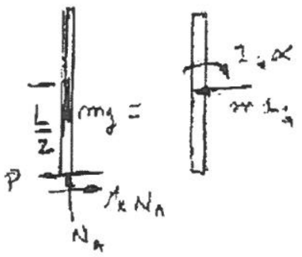
$+\uparrow \Sigma F_y = m(a_G)_y; \qquad N_A - mg = 0$

Solving,

$$N_A = mg$$

$$a_G = \frac{L}{6}\alpha$$

$$\alpha = \frac{6(P - \mu_k mg)}{mL} \qquad \textbf{Ans.}$$



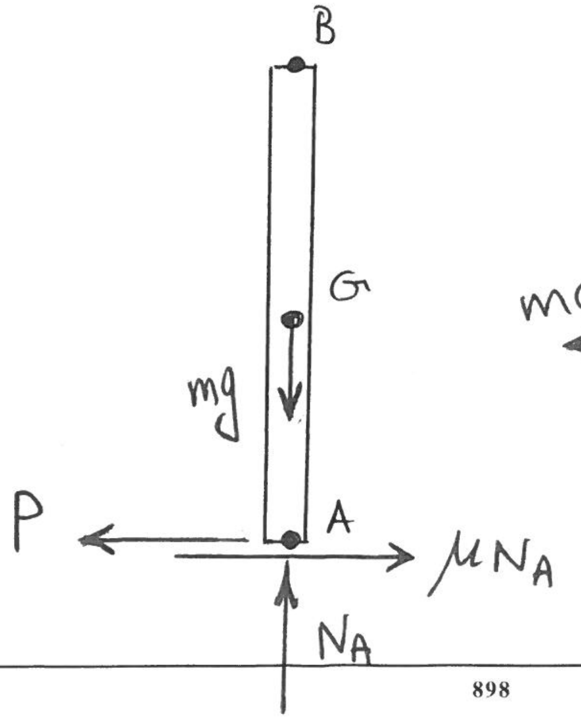
$\mathbf{a}_B = \mathbf{a}_G + \mathbf{a}_{B/G}$

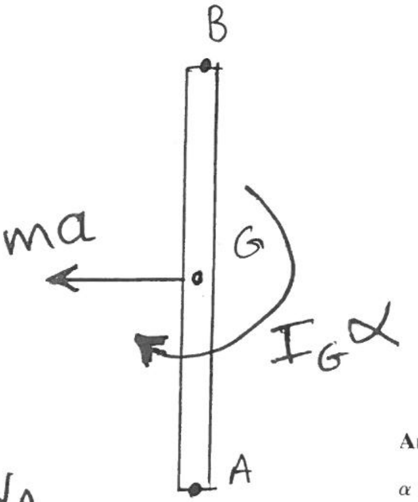
$(\overset{+}{\leftarrow}) \quad a_B = -\dfrac{L}{6}\alpha + \dfrac{L}{2}\alpha$

$a_B = \dfrac{L\alpha}{3}$

$$a_B = \frac{2(P - \mu_k mg)}{m} \qquad \textbf{Ans.}$$

$a_x \neq 0$
$a_y = 0$
$\alpha \neq 0$





Ans:

$\alpha = \dfrac{6(P - \mu_k mg)}{mL}$

$a_B = \dfrac{2(P - \mu_k mg)}{m}$

KINETIC ENERGY, WORK, PRINCIPLE OF WORK AND ENERGY

Objectives:

Students will be able to:

1. Define the various ways a force and couple do work.
2. Apply the principle of work and energy to a rigid body.

READING QUIZ

1. Kinetic energy due to rotation of the body is defined as

   A) $(1/2)\ m\ (v_G)^2$.
   B) $(1/2)\ m\ (v_G)^2 + (1/2)\ I_G\ \omega^2$.
   C) $(1/2)\ I_G\ \omega^2$.
   D) $I_G\ \omega^2$.

2. When calculating work done by forces, the work of an internal force does not have to be considered because _____________.

   A) internal forces do not exist
   B) the forces act in equal but opposite collinear pairs
   C) the body is at rest initially
   D) the body can deform

APPLICATIONS

The work of the torque (or moment) developed by the driving gears on the two motors on the concrete mixer is transformed into the rotational kinetic energy of the mixing drum.

If the motor gear characteristics are known, how would you find the rotational velocity of the mixing drum?

APPLICATIONS

The work done by the soil compactor's engine is transformed into the translational kinetic energy of the frame and the translational and rotational kinetic energy of the roller and wheels (excluding the internal kinetic energy developed by the moving parts of the engine and drive train).

Are the kinetic energies of the frame and the roller related to each other? If so, how?

KINETIC ENERGY (Section 18.1)

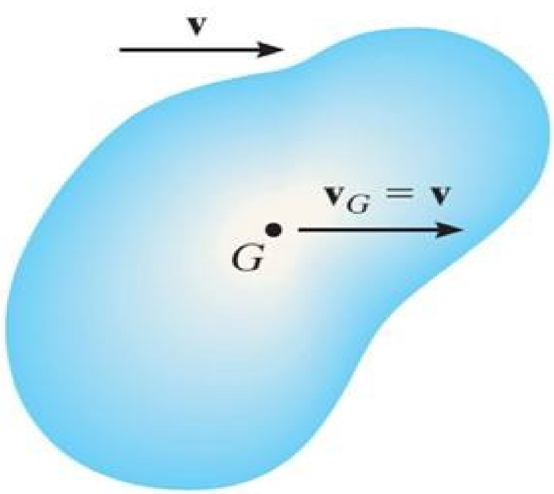
The kinetic energy of a rigid body can be expressed as the sum of its translational and rotational kinetic energies. In equation form, a body in general plane motion has kinetic energy given by:

$$T = 1/2\ m\ (v_G)^2 + 1/2\ I_G\ \omega^2$$

Several simplifications can occur.

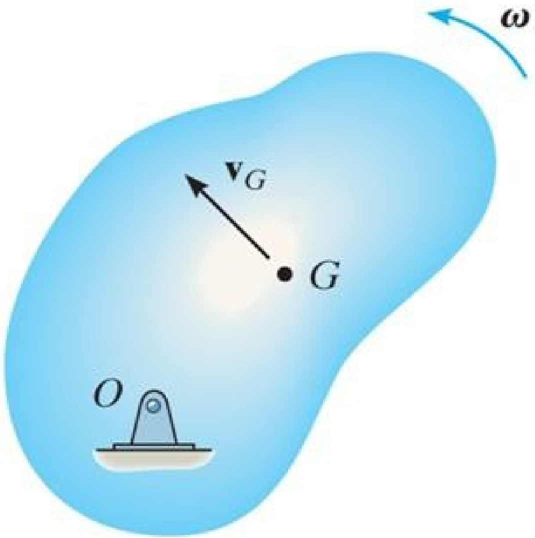
1. Pure Translation: When a rigid body is subjected to only curvilinear or rectilinear translation, the rotational kinetic energy is zero ($\omega = 0$). Therefore,

$$T = 1/2\ m\ (v_G)^2$$



Translation

KINETIC ENERGY (continued)



Rotation About a Fixed Axis

2. Pure Rotation: When a rigid body is rotating about a fixed axis passing through point O, the body has both translational and rotational kinetic energy. Thus,

$$T = 0.5\, m\, (v_G)^2 + 0.5\, I_G\, \omega^2$$

Since $v_G = r_G\omega$, we can express the kinetic energy of the body as:

$$T = 0.5\, [\, I_G + m(r_G)^2\,]\, \omega^2 = 0.5\, I_O\, \omega^2$$

If the rotation occurs about the mass center, G, then what is the value of v_G?

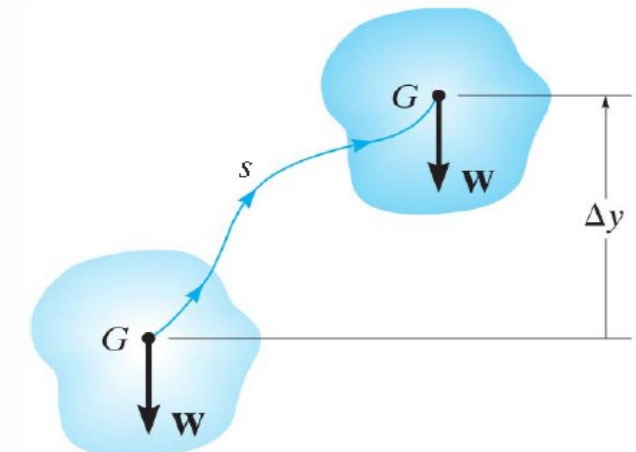
In this case, the velocity of the mass center is equal to zero. So the kinetic energy equation reduces to:

$$T = 0.5\, I_G\, \omega^2$$

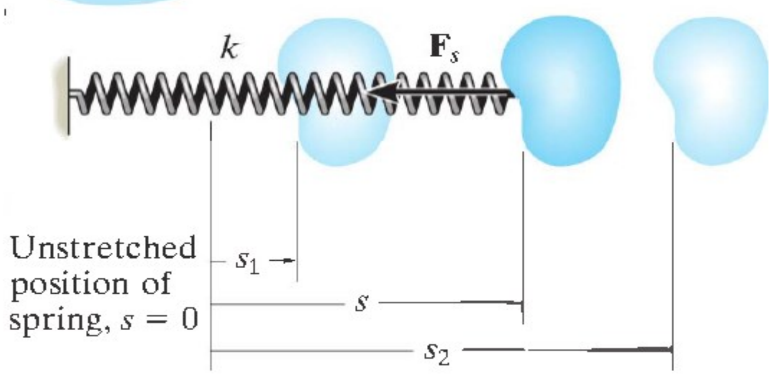
THE WORK OF A FORCE (Section 18.2)
Recall that the work done by a force can be written as:

$$U_F = \int \mathbf{F} \bullet d\mathbf{r} = \int (F_s \cos\theta)\, ds.$$

When the force is constant, this equation reduces to
$U_{Fc} = (F_c \cos\theta)s$ where $F_c \cos\theta$ represents the component of the force acting in the direction of the displacement, s.



Work of a weight: As before, the work can be expressed as $U_w = -W\Delta y$. Remember, if the force and movement are in the same direction, the work is positive.



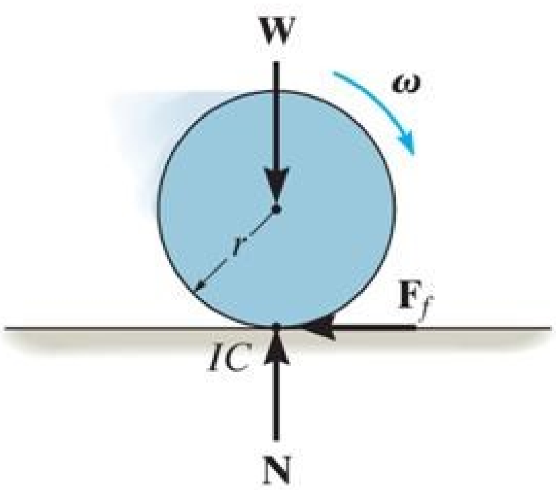
Work of a spring force: For a linear spring, the work is:

$$U_s = -0.5k[(s_2)^2 - (s_1)^2]$$

FORCES THAT DO NO WORK

There are some external forces that do no work.

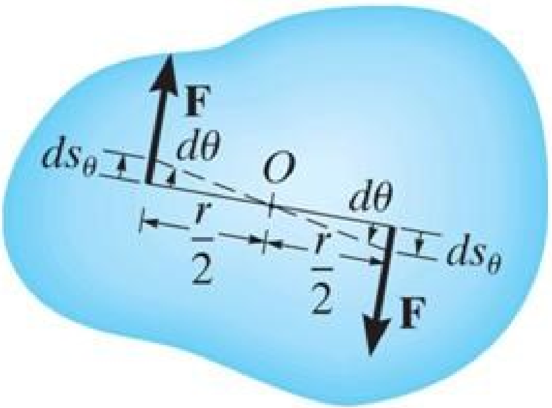
For instance, reactions at fixed supports do no work because the displacement at their point of application is zero.



Normal forces and friction forces acting on bodies as they roll without slipping over a rough surface also do no work since there is no instantaneous displacement of the point in contact with ground (it is an instant center, IC).

Internal forces do no work because they always act in equal and opposite pairs. Thus, the sum of their work is zero.

THE WORK OF A COUPLE (Section 18.3)



When a body subjected to a couple experiences general plane motion, the two couple forces do work only when the body undergoes rotation.

If the body rotates through an angular displacement $d\theta$, the work of the couple moment, M, is:

$$U_M = \int_{\theta_1}^{\theta_2} M \, d\theta$$

If the couple moment, M, is constant, then

$$U_M = M\,(\theta_2 - \theta_1)$$

Here the work is positive, provided M and $(\theta_2 - \theta_1)$ are in the same direction.

PRINCIPLE OF WORK AND ENERGY (Section 18.4)

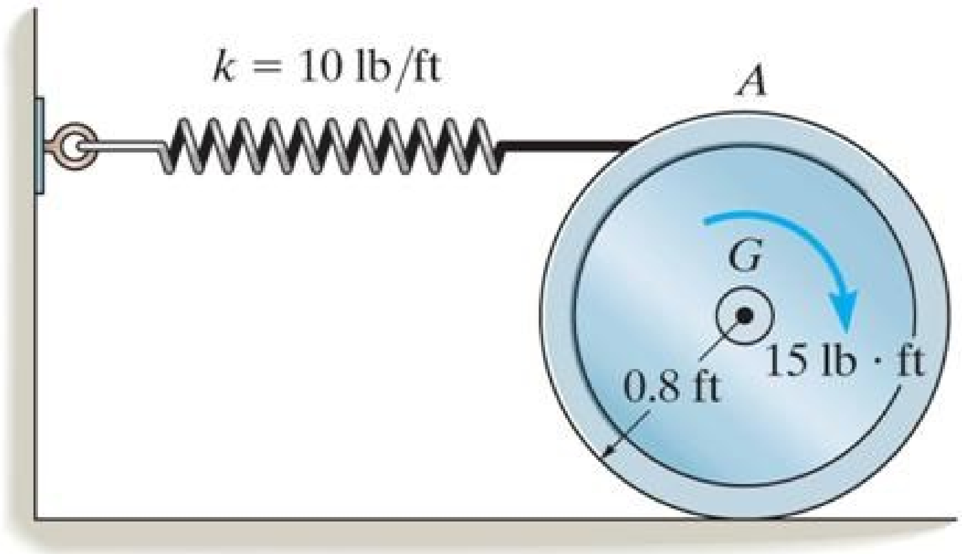
Recall the statement of the principle of work and energy used earlier:

$$T_1 + \Sigma U_{1\text{-}2} = T_2$$

In the case of general plane motion, this equation states that the sum of the initial kinetic energy (both translational and rotational) and the work done by all external forces and couple moments equals the body's final kinetic energy (translational and rotational).

This equation is a scalar equation. It can be applied to a system of rigid bodies by summing contributions from all bodies.

EXAMPLE



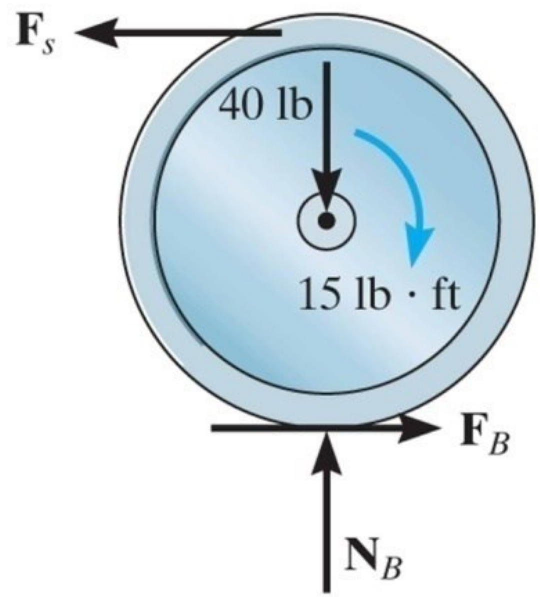
Given: The disk weighs 40 lb and has a radius of gyration (k_G) of 0.6 ft. A 15 ft·lb moment is applied and the spring has a spring constant of 10 lb/ft.

Find: The angular velocity of the wheel when point G moves 0.5 ft. The wheel starts from rest and rolls without slipping. The spring is initially un-stretched.

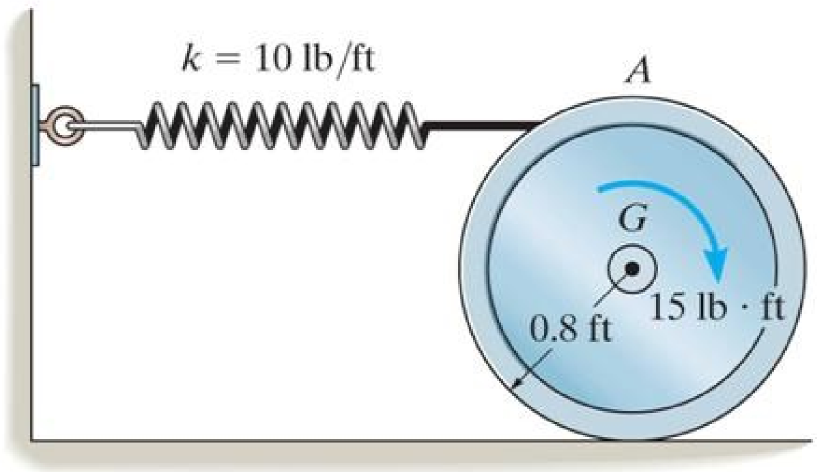
Plan: Use the principle of work and energy to solve the problem since distance is the primary parameter. Draw a free body diagram of the disk and calculate the work of the external forces.

EXAMPLE (continued)

Solution: Free body diagram of the disk:

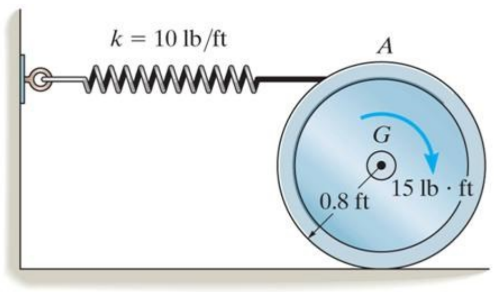


Since the disk rolls without slipping on a horizontal surface, only the spring force and couple moment M do work. Why don't forces F_B and N_B do any work?



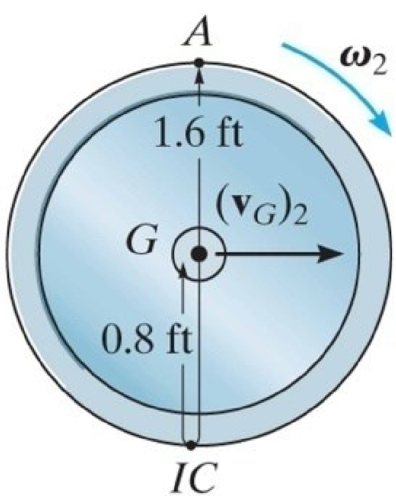
Since the spring is attached to the top of the wheel, it will stretch twice the amount of displacement of G, or 1 ft.

EXAMPLE (continued)



Work: $U_{1\text{-}2} = -0.5k[(s_2)^2 - (s_1)^2] + M(\theta_2 - \theta_1)$

$U_{1\text{-}2} = -0.5(10)(1^2 - 0) + 15(0.5/0.8) = 4.375 \text{ ft}\cdot\text{lb}$



Kinematic relation: $v_G = r\,\omega = 0.8\omega$

Kinetic energy:

$T_1 = 0$

$T_2 = 0.5m\,(v_G)^2 + 0.5\,I_G\,\omega^2$

$T_2 = 0.5(40/32.2)(0.8\omega)^2 + 0.5(40/32.2)(0.6)^2\omega^2$

$T_2 = 0.621\,\omega^2$

Work and energy: $T_1 + U_{1\text{-}2} = T_2$

$0 + 4.375 = 0.621\,\omega^2$

$\omega = 2.65 \text{ rad/s}$

CONCEPT QUIZ

1. If a rigid body rotates about its center of gravity, its translational kinetic energy is ___________ at all times.

   A) constant

   B) zero

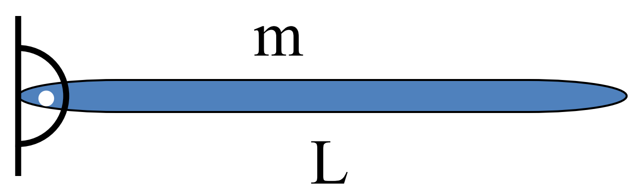
   C) equal to its rotational kinetic energy

   D) Cannot be determined

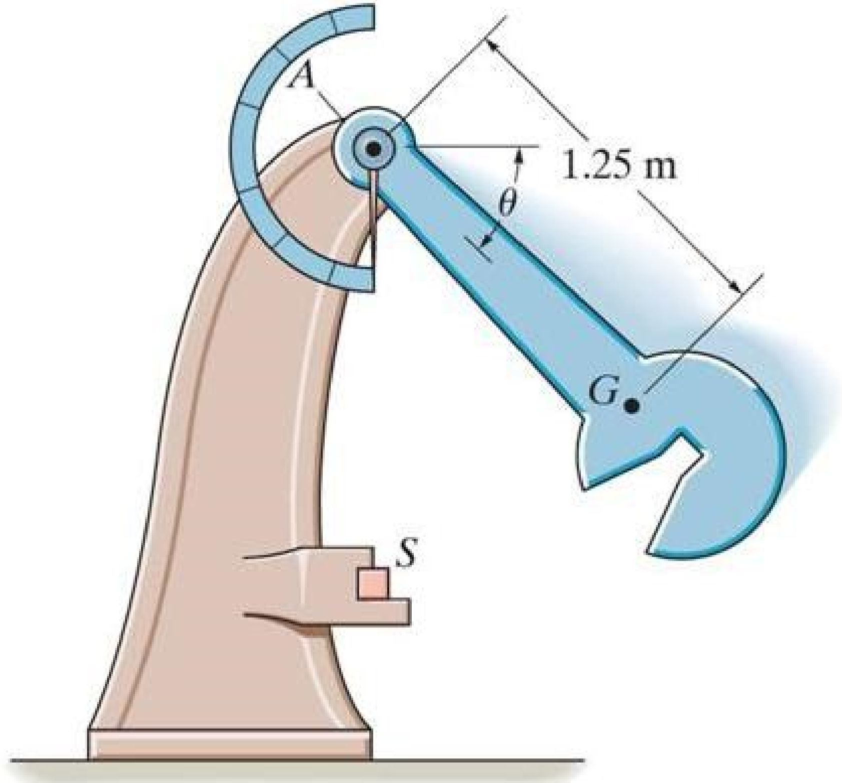
2. A rigid bar of mass m and length L is released from rest in the horizontal position. What is the rod's angular velocity when it has rotated through 90°?

   A) $\sqrt{g/3L}$ B) $\sqrt{3g/L}$

   C) $\sqrt{12g/L}$ D) $\sqrt{g/L}$



GROUP PROBLEM SOLVING



Given: The 50 kg pendulum of the Charpy impact machine is released from rest when $\theta = 0$. The radius of gyration $k_A = 1.75$ m.

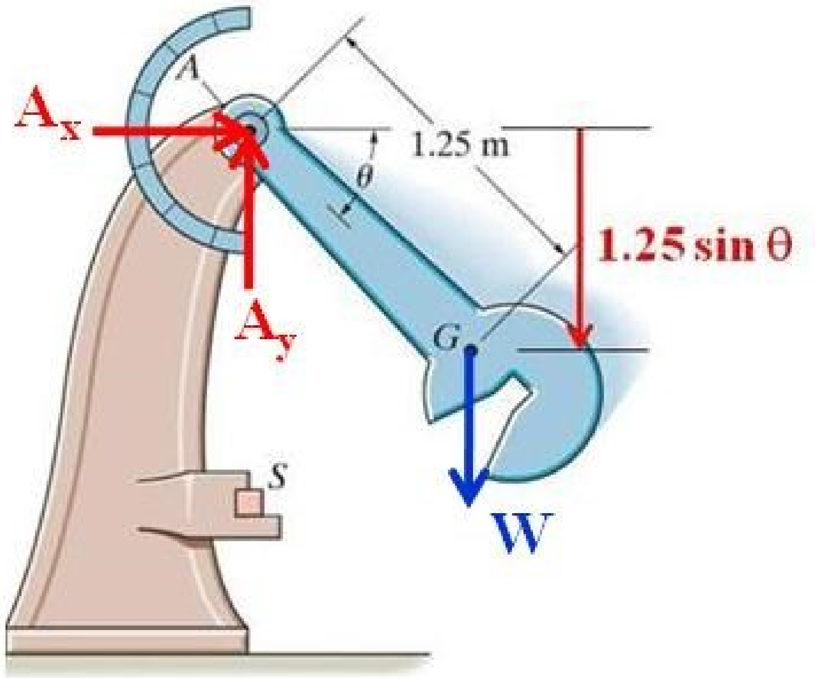
Find: The angular velocity of the pendulum when $\theta = 90°$.

Plan: Since the problem involves distance, the principle of work and energy is an efficient solution method. The only force involved doing work is the weight, so only its work need be determined.

GROUP PROBLEM SOLVING (continued)

Solution:

Calculate the vertical distance the mass center moves.



$\Delta y = 1.25 \sin \theta$

Then, determine the work due to the weight.

$U_w = -W\Delta y$

$U_{1\text{-}2} = W\,(1.25 \sin \theta\,)$

$= 50(9.81)\,(1.25 \sin 90°)$

$= 613.1\ \text{N·m}$

The mass moment of inertia about A is:

$I_A = m\,(k_A)^2 = 50(1.75)^2 = 153.1\ \text{kg·m}^2$

GROUP PROBLEM SOLVING (continued)

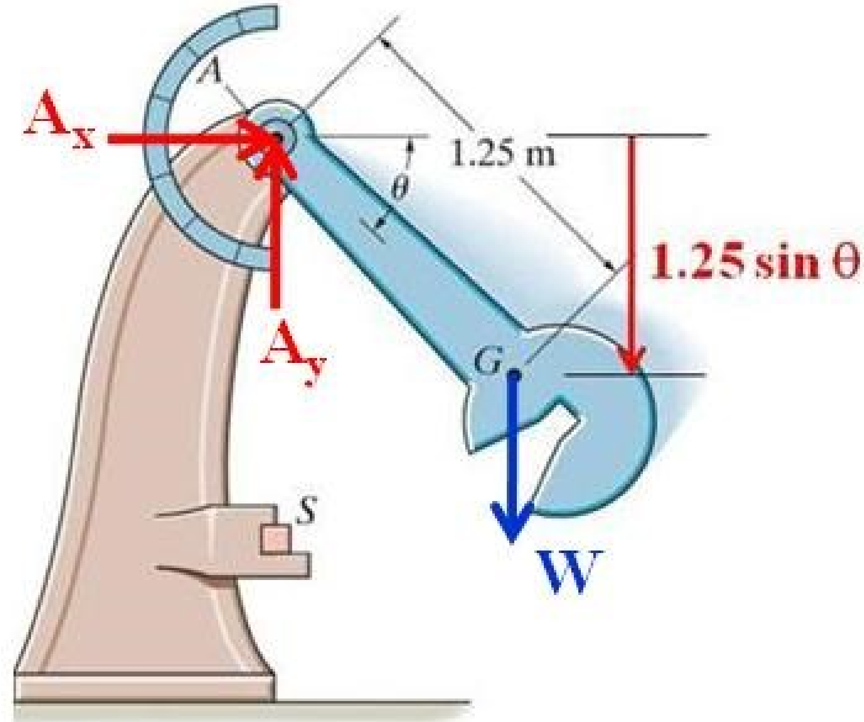
Kinetic energy:

$T_1 = 0$

$T_2 = 0.5m(v_G)^2 + 0.5\ I_G\ \omega^2$

$= 0.5\ I_A\ \omega^2$

$= 0.5\ (153.1)\ \omega^2$



Now apply the principle of work and energy equation:

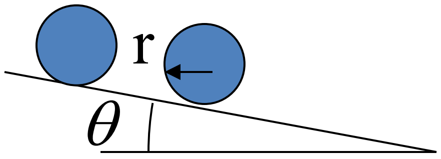
$T_1 + U_{1\text{-}2} = T_2$

$0\ + 613.1 = 76.55\ \omega^2$

$\omega = 2.83$ rad/s

ATTENTION QUIZ

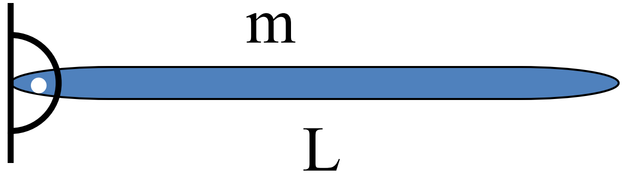
1. A disk and a sphere, each of mass m and radius r, are released from rest. After 2 full turns, which body has a larger angular velocity? Assume roll without slip.



A) Sphere B) Disk

C) The two are equal. D) Cannot be determined.

2. A slender bar of mass m and length L is released from rest in a horizontal position. The work done by its weight when it has rotated through 90° is?



A) m g (π/2) B) m g L

C) m g (L/2) D) -m g (L/2)

PLANAR KINETICS OF A RIGID BODY: CONSERVATION OF ENERGY

Objectives:

Students will be able to:

a) Determine the potential energy of conservative forces.

b) Apply the principle of conservation of energy.

READING QUIZ

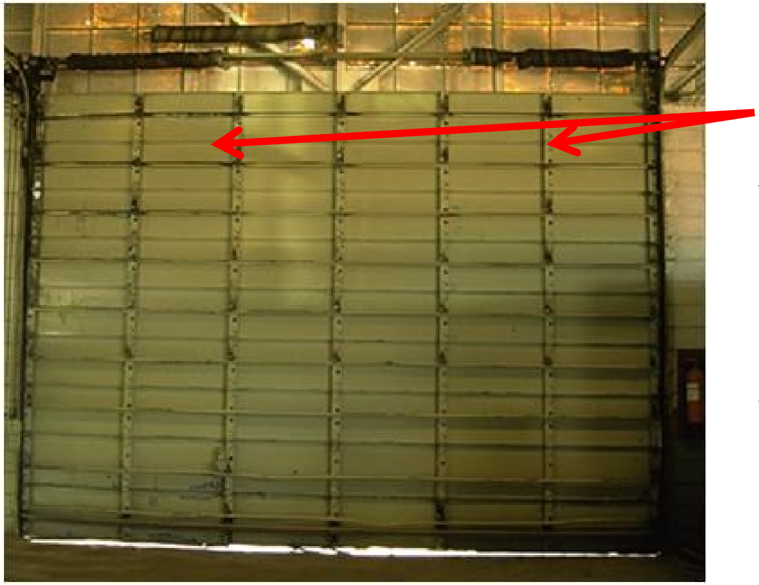
1. Elastic potential energy is defined as ____________.

   A) $+ (1/2)\ k\ (s)^2$
   B) $- (1/2)\ k\ (s)^2$
   C) $+ (1/2)\ k\ (v)^2$
   D) None of the above

2. The kinetic energy of a rigid body consists of the kinetic energy due to ____________.

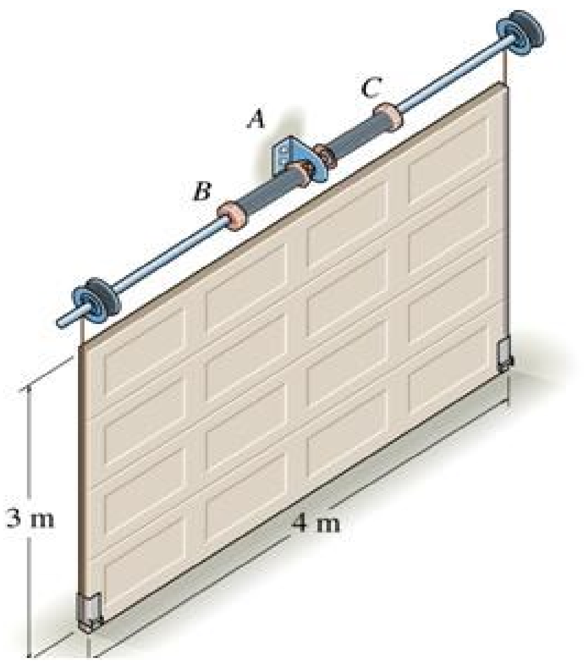
   A) translational motion and rotational motion
   B) only rotational motion
   C) only translational motion
   D) the deformation of the body

APPLICATIONS



The torsion springs located at the top of the garage door wind up as the door is lowered.

When the door is raised, the potential energy stored in the spring is transferred into the gravitational potential energy of the door's weight, thereby making it easy to open.



Are parameters such as the torsional spring stiffness and initial rotation angle of the spring important when you install a new door?

APPLICATIONS (continued)

Two torsional springs are used to assist in opening and closing the hood of the truck.

Assuming the springs are uncoiled when the hood is opened, can we determine the stiffness of each spring so that the hood can easily be lifted, i.e., practically no external force applied to it, when a person is opening it?

Are the gravitational potential energy of the hood and the torsional spring stiffness related to each other? If so, how?

CONSERVATION OF ENERGY (Section 18.5)

The conservation of energy theorem is a "simpler" energy method (recall that the principle of work and energy is also an energy method) for solving problems.

Once again, the problem parameter of distance is a key indicator for when conservation of energy is a good method to solve a problem.

If it is appropriate for the problem, conservation of energy is easier to use than the principle of work and energy.

This is because the calculation of the work of a conservative force is simpler. But, what makes a force conservative?

CONSERVATIVE FORCES

A force F is conservative if the work done by the force is independent of the path.

In this case, the work depends only on the initial and final positions of the object with the path between the positions of no consequence.

Typical conservative forces encountered in dynamics are gravitational forces (i.e., weight) and elastic forces (i.e., springs).

What is a common force that is not conservative?

CONSERVATION OF ENERGY

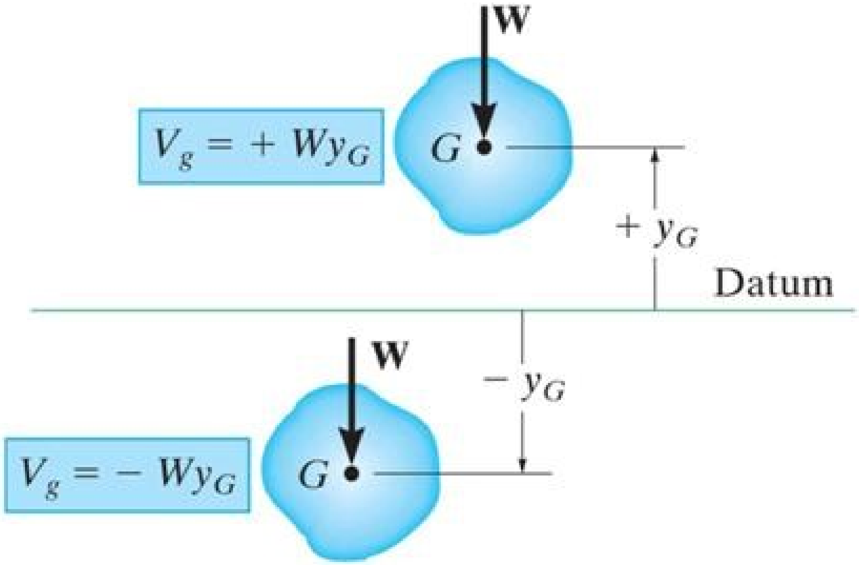
When a rigid body is acted upon by a system of conservative forces, the work done by these forces is conserved. Thus, the sum of kinetic energy and potential energy remains constant. This principle is called conservation of energy and is expressed as:

$$T_1 + V_1 = T_2 + V_2 = \text{Constant}$$

In other words, as a rigid body moves from one position to another when acted upon by only conservative forces, kinetic energy is converted to potential energy and vice versa.

GRAVITATIONAL POTENTIAL ENERGY

The gravitational potential energy of an object is a function of the height of the body's center of gravity above or below a datum.



Gravitational potential energy

The gravitational potential energy of a body is found by the equation

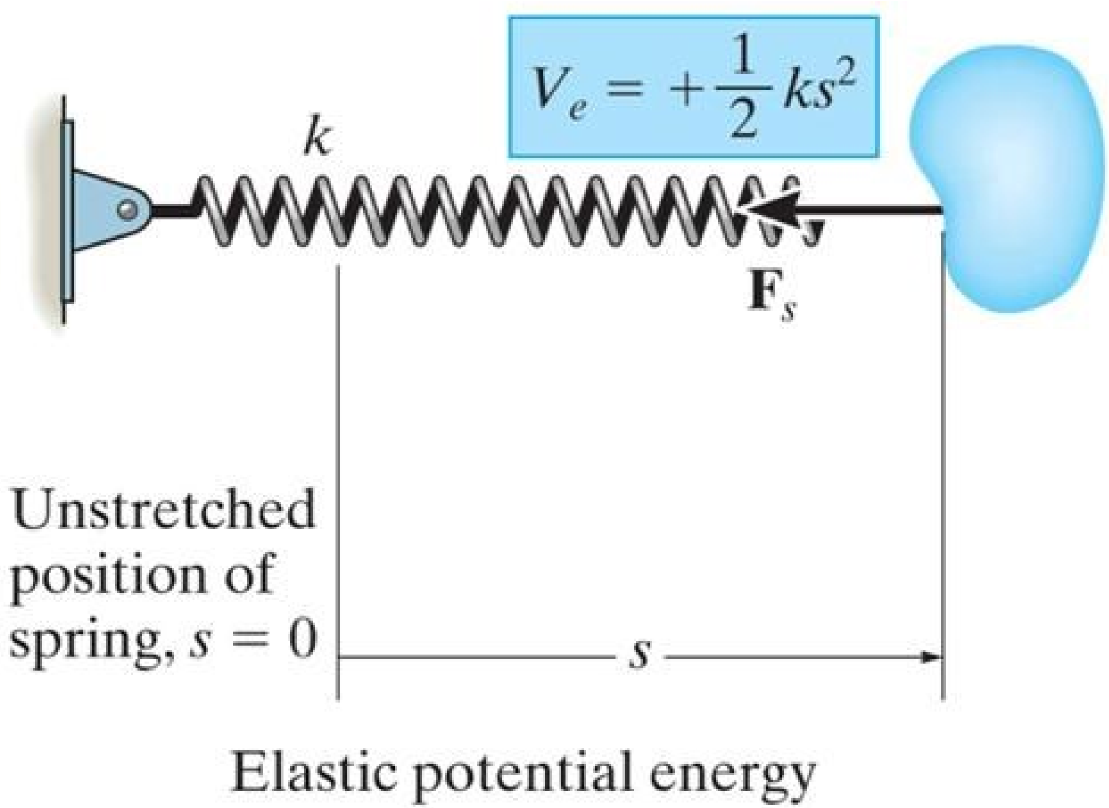
$$V_g = W\,y_G$$

Since the movement (distance) and the force (the weight) act in the same direction.

Gravitational potential energy is positive when y_G is positive, since the weight has the ability to do positive work (why is it positive?) when the body is moved back to the datum.

ELASTIC POTENTIAL ENERGY

Spring forces are also conservative forces.



The potential energy of a spring force (F = ks) is found by the equation

$$V_e = \frac{1}{2}\, k\, s^2$$

Notice that the elastic potential energy is always positive.

PROCEDURE FOR ANALYSIS

Problems involving velocity, displacement and conservative force systems can be solved using the conservation of energy equation.

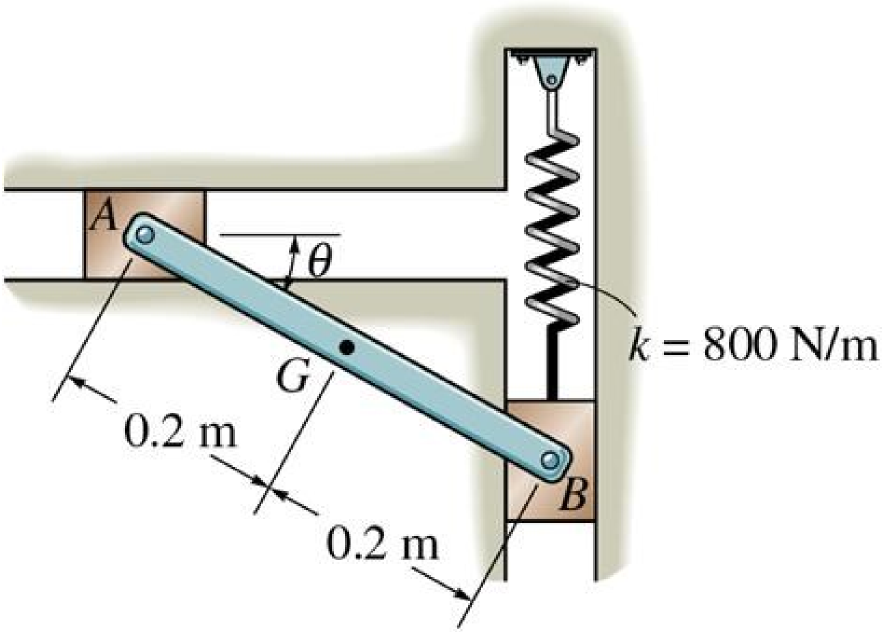
- Potential energy: Draw two diagrams: one with the body located at its initial position and one at the final position. Compute the potential energy at each position using

$V = V_g + V_e$, where $V_g = W\, y_G$ and $V_e = 1/2\, k\, s^2$.

- Kinetic energy: Compute the kinetic energy of the rigid body at each location. Kinetic energy has two components: translational kinetic energy, $1/2 m (v_G)^2$, and rotational kinetic energy, $1/2\, I_G \omega^2$.

- Apply the conservation of energy equation.

EXAMPLE I



Given: The rod AB has a mass of 10 kg. Piston B is attached to a spring of constant k = 800 N/m. The spring is un-stretched when $\theta = 0°$. Neglect the mass of the pistons.

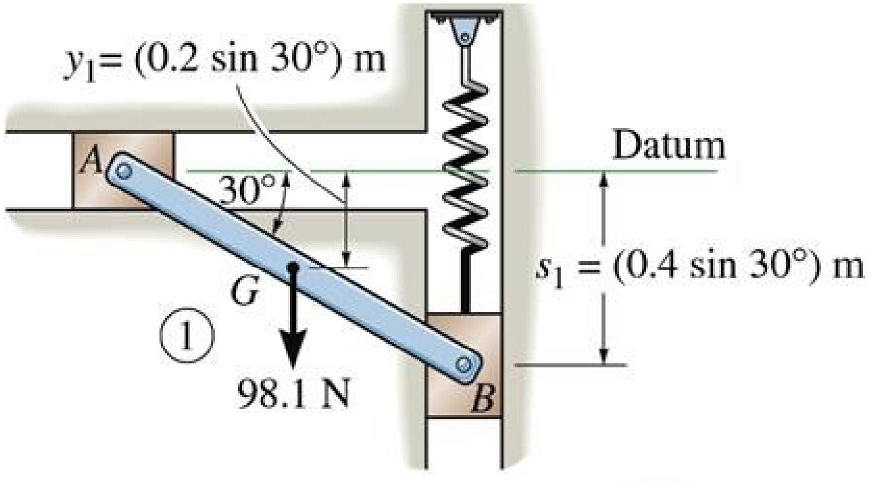
Find: The angular velocity of rod AB at $\theta = 0°$ if the rod is released from rest when $\theta = 30°$.

Plan:

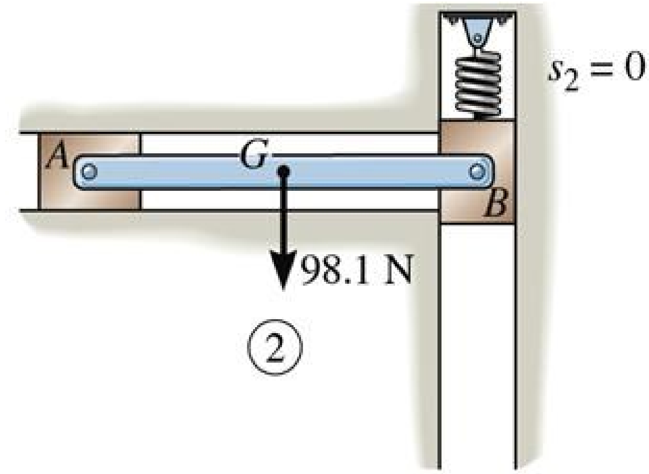
Use the energy conservation equation since all forces are conservative and distance is a parameter (represented here by θ). The potential energy and kinetic energy of the rod at states 1 and 2 will have to be determined.

EXAMPLE I (continued) Solution:

Initial Position



Final Position



Potential Energy:

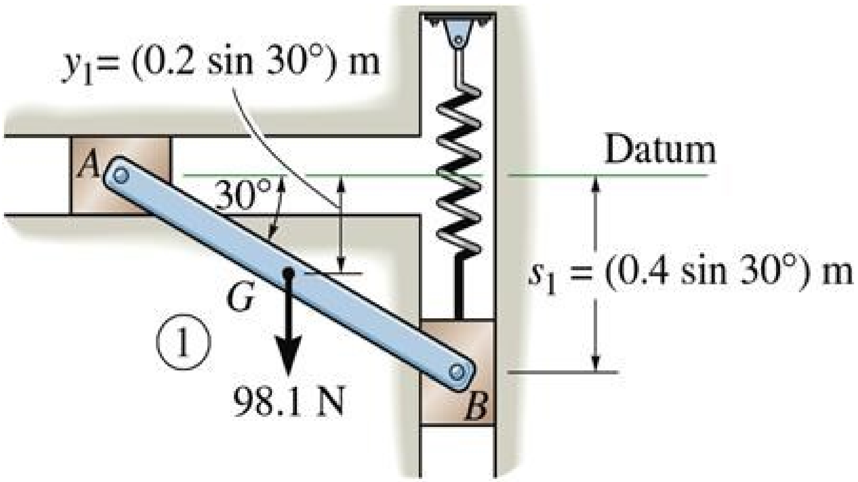
Let's put the datum in line with the rod when $\theta = 0°$.
Then, the gravitational potential energy and the elastic potential energy will be zero at position 2. $\Rightarrow V_2 = 0$

Gravitational potential energy at 1: - (10)(9.81) ½ (0.4 sin 30°)
Elastic potential energy at 1: ½ (800) (0.4 sin 30°)^2

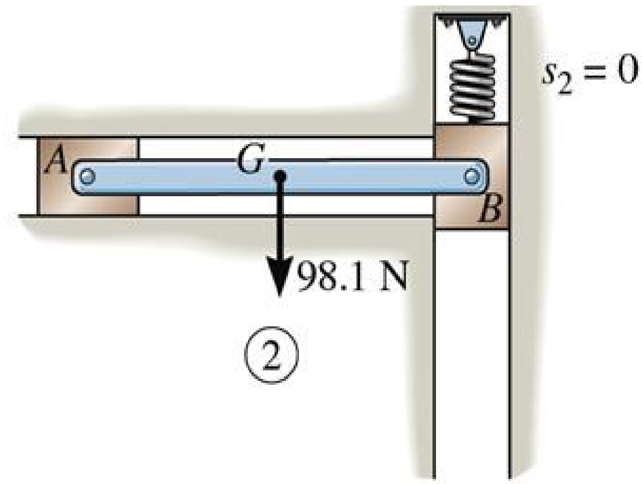
So $V_1 = -9.81 + 16.0 = 6.19$ N·m

EXAMPLE I (continued)

Initial Position



Final Position



Kinetic Energy:

The rod is released from rest from position 1.

Therefore, $T_1 = 0$.

At position 2, the angular velocity is ω_2 and the velocity at the center of mass is v_{G2} .

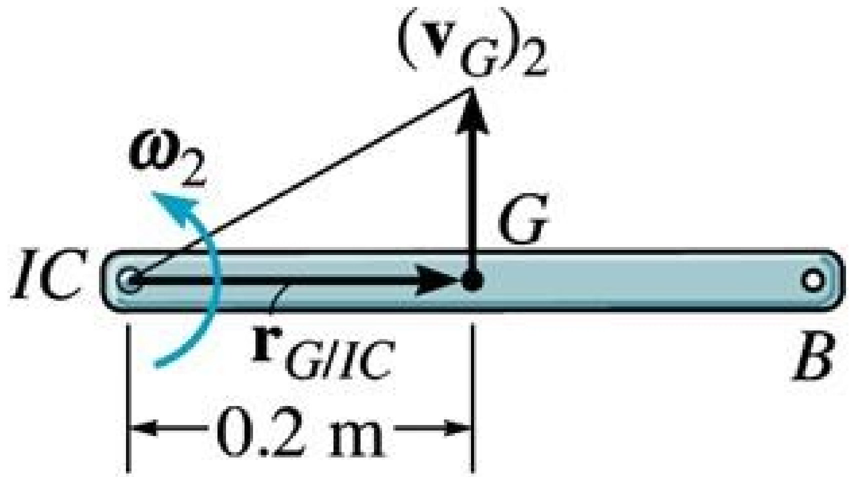
Therefore, $T_2 = ½\,(10)(v_{G2})^2 + ½\,(1/12)(10)(0.4^2)(\omega_2)^2$

EXAMPLE I (continued)

At position 2, point A is the instantaneous center of rotation.

Hence, $v_{G2} = r_{G/IC}\ \omega = 0.2\ \omega_2$. Then,

$T_2 = 0.2\ \omega_2^2 + 0.067\ \omega_2^2 = 0.267\ \omega_2^2$

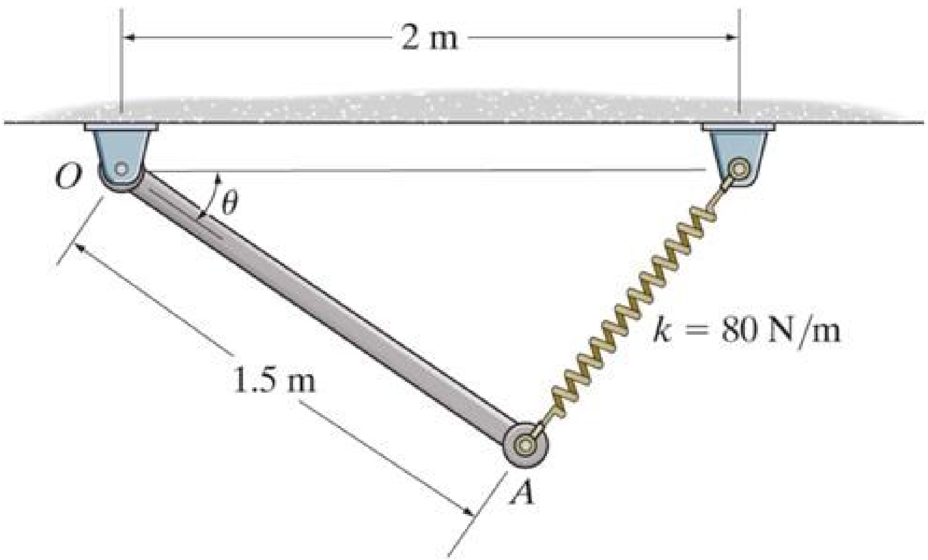


Now apply the conservation of energy equation and solve for the unknown angular velocity, ω_2.

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 6.19 = 0.267\omega_2^2 + 0 \quad \Rightarrow \quad \omega_2 = 4.82 \text{ rad/s}$$

EXAMPLE II



Given: The 30 kg rod is released from rest when $\theta = 0°$. The spring is unstretched when $\theta = 0°$.

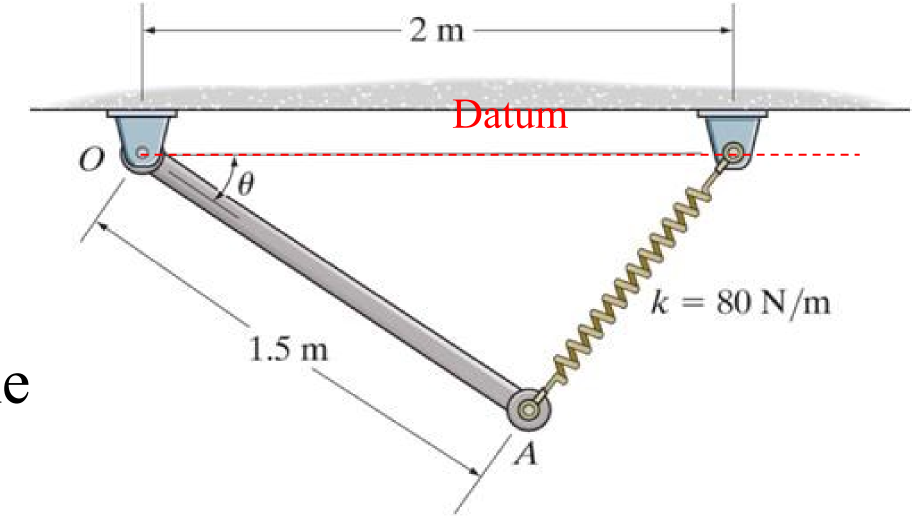
Find: The angular velocity of the rod when $\theta = 30°$.

Plan:

Since distance is a parameter and all forces doing work are conservative, use conservation of energy. Determine the potential energy and kinetic energy of the system at both positions and apply the conservation of energy equation.

EXAMPLE II (continued)

Solution:



Potential Energy:

Let's put the datum in line with the rod when $\theta = 0°$.

Then, the gravitational potential energy when $\theta = 30°$ is

$$V_{g2} = -30\,(9.81)\,(\tfrac{1}{2}\,1.5 \sin 30°) = -110.4 \text{ N·m}$$

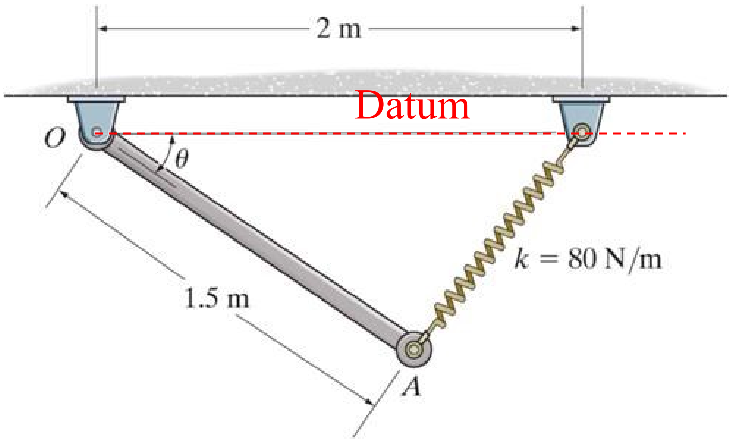
The elastic potential energy at $\theta = 0°$ is zero since the spring is un-stretched. The un-stretched length of the spring is 0.5 m.

The elastic potential energy at $\theta = 30°$ is

$$V_{e2} = \tfrac{1}{2}\,80\,\left(\sqrt{0.5^2 + (1.5 \sin 30°)^2} - 0.5\right)^2 = 6.444 \text{ N·m}$$

EXAMPLE II (continued)

Kinetic Energy:



The rod is released from rest at $\theta = 0°$, so $v_{G1} = 0$ and $\omega_1 = 0$. Thus, the kinetic energy at position 1 is $T_1 = 0$.

At $\theta = 30°$, the angular velocity is ω_2 and the velocity at the center of mass is v_{G2} .

$$T_2 = ½\, m\, (v_{G2})^2 + ½\, I_G\, (\omega_2)^2$$

$$= ½\, (30)\, (v_{G2})^2 + ½\, \{(1/12)\, 30\, (1.5)^2\}\, (\omega_2)^2$$

Since $v_{G2} = (0.75\, \omega_2)$,

$$T_2 = ½(30)\, (0.75\, \omega_2)^2 + ½\{(1/12)30(1.5)^2\}\, (\omega_2)^2$$

$$T_2 = 11.25\, (\omega_2)^2$$

EXAMPLE II (continued)

Now all terms in the conservation of energy equation have been formulated. Writing the general equation and then substituting into it yields:

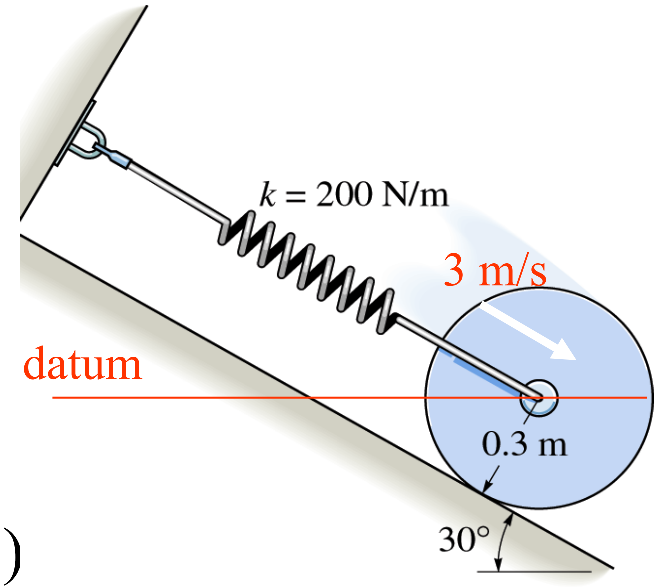
$T_1 + V_1 = T_2 + V_2$

$0 + 0 = 11.25\ (\omega_2)^2 + (-110.4 + 6.444)$

Solving , $\omega_2 = 3.04$ rad/s

UNDERSTANDING QUIZ

1. At the instant shown, the spring is undeformed. Determine the change in potential energy if the 20 kg disk ($k_G = 0.5$ m) rolls 2 revolutions without slipping.



A) $\frac{1}{2}(200)(1.2\pi)^2 + (20)9.81(1.2\pi \sin 30°)$

B) $- \frac{1}{2}(200)(1.2\pi)^2 - (20)9.81(1.2\pi \sin 30°)$

C) $\frac{1}{2}(200)(1.2\pi)^2 - (20)9.81(1.2\pi \sin 30°)$

D) $\frac{1}{2}(200)(1.2\pi)^2$

2. Determine the kinetic energy of the disk at this instant.

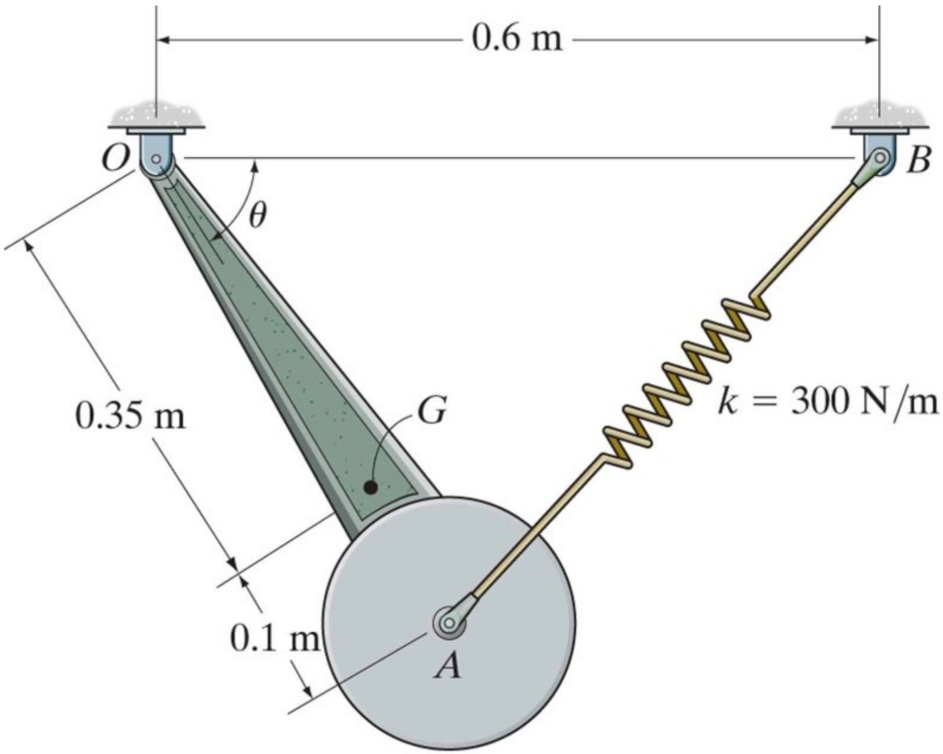
A) $(\frac{1}{2})(20)(3)^2$

B) $\frac{1}{2}(20)(0.5^2)(10)^2$

C) Answer A + Answer B

D) None of the above

GROUP PROBLEM SOLVING



Given: The 30 kg pendulum has its mass center at G and a radius of gyration about point G of k_G=0.3 m. It is released from rest when $\theta = 0°$. The spring is unstretched when $\theta = 0°$.

Find: The angular velocity of the pendulum when $\theta = 90°$.

Plan: Conservative forces and distance (θ) leads to the use of conservation of energy. First, determine the potential energy and kinetic energy for both positions. Then apply the conservation of energy equation.

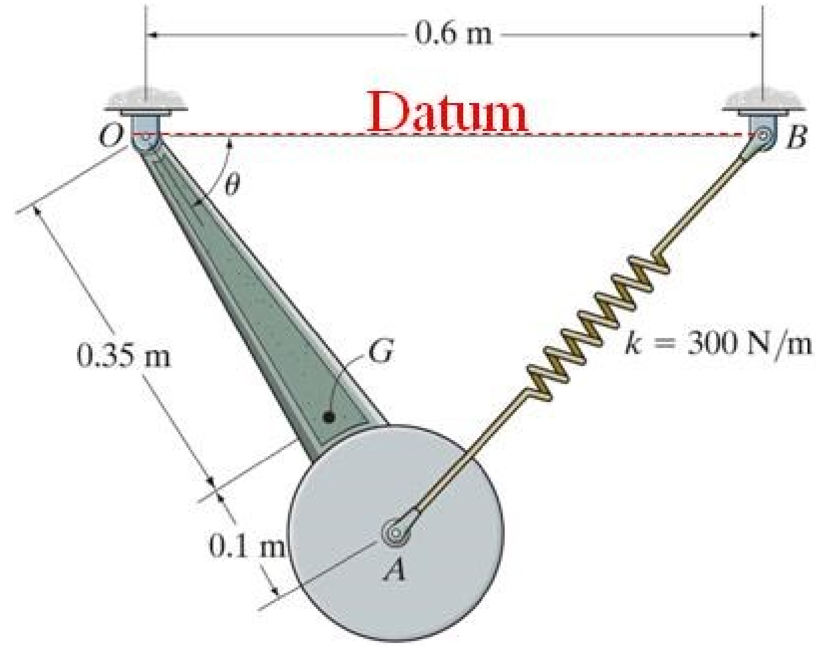
GROUP PROBLEM SOLVING (continued) **Solution:**

Potential Energy:

Let's put the datum when $\theta = 0°$. There the gravitational potential energy is zero and the elastic potential energy will be zero. So,

$$V_{g1} = V_{e1} = 0$$

Note that the unstretched length of the spring is 0.15 m.



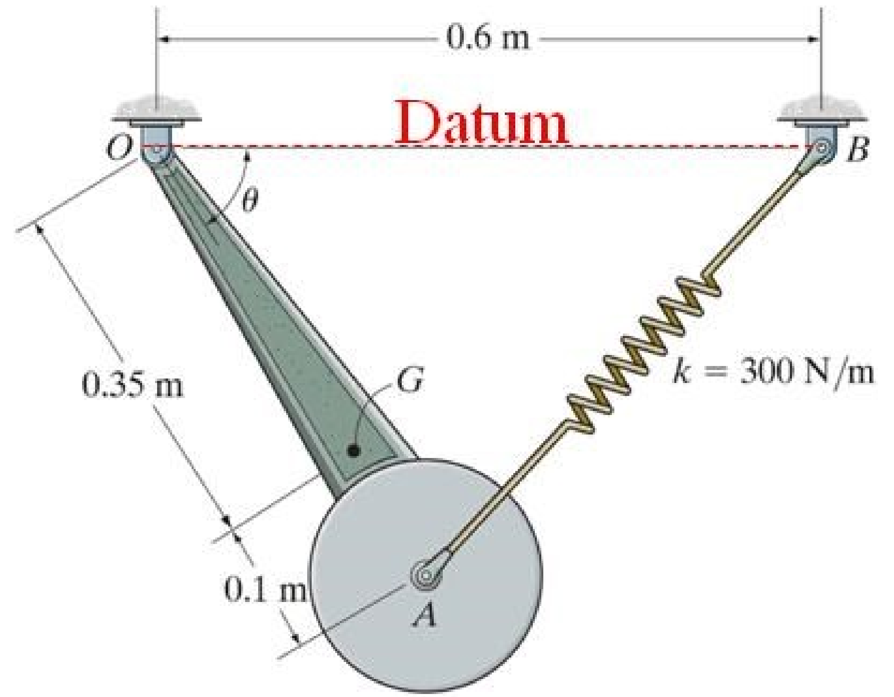
Gravitational potential energy at $\theta = 90°$:

$$V_{g2} = -\,30\,(9.81)\,(0.35) = -103.0\ \text{N·m}$$

Elastic potential energy at $\theta = 90°$ is :

$$V_{e2} = \tfrac{1}{2}\,300\,\left(\sqrt{0.6^2 + 0.45^2} - 0.15\right)^2 = 54.0\ \text{N·m}$$

GROUP PROBLEM SOLVING (continued)



Kinetic Energy:

When $\theta = 0°$, the pendulum is released from rest.

Thus, $T_1 = 0$.

When $\theta = 90°$, the pendulum has a rotational motion about point O.

$T_2 = ½\, I_O\, (\omega_2)^2$

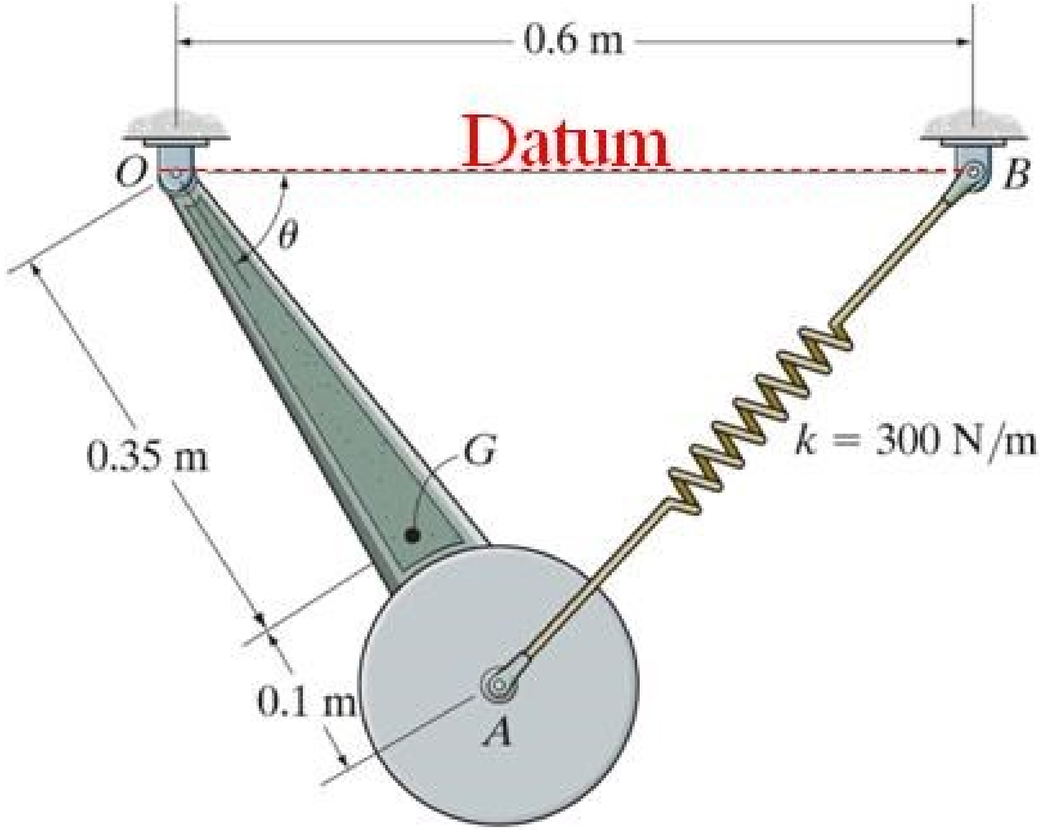
where $I_O = I_G + m\,(d_{OG})^2 = (30)\,0.3^2 + 30\,(0.35)^2 = 6.375\ \text{kg·m}^2$

$T_2 = ½\ 6.375\,(\omega_2)^2$

GROUP PROBLEM SOLVING (continued)

Now, substitute into the conservation of energy equation.

$$T_1 + V_1 = T_2 + V_2$$



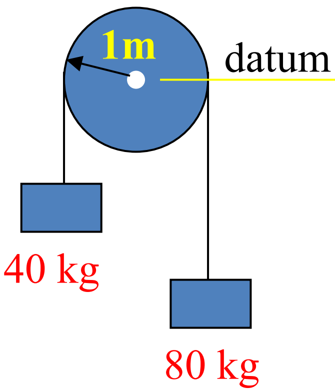
$$0 + 0 = = \tfrac{1}{2}\, 6.375\, (\omega_2)^2 + (-103 + 54.0)$$

Solving for ω yields

$\omega = 3.92$ rad/s.

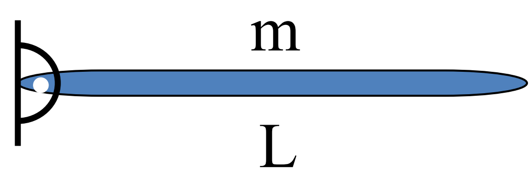
ATTENTION QUIZ

1. Blocks A and B are released from rest and the disk turns 2 revolutions. The V_2 of the system includes a term for ________________.



   A) only the 40 kg block
   B) only the 80 kg block
   C) the disk and both blocks
   D) only the two blocks

2. A slender bar is released from rest while in the horizontal position. The kinetic energy (T_2) of the bar when it has rotated through 90° is?



   A) $\frac{1}{2}\, m\, (v_{G2})^2$ B) $\frac{1}{2}\, I_G\, (\omega_2)^2$
   C) $\frac{1}{2}\, k\, (s_1)^2 - W\, (L/2)$ D) $\frac{1}{2}\, m\, (v_{G2})^2 + \frac{1}{2}\, I_G\, (\omega_2)^2$

Chapter **18**

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o coast over loops and through turns, and have
safely. Accurate calculation of this energy must
the size of the car as it moves along the track.

Planar Kinetics of a Rigid Body: Work and Energy

CHAPTER OBJECTIVES

- To develop formulations for the kinetic energy of a body, and define the various ways a force and couple do work.
- To apply the principle of work and energy to solve rigid-body planar kinetic problems that involve force, velocity, and displacement.
- To show how the conservation of energy can be used to solve rigid-body planar kinetic problems.

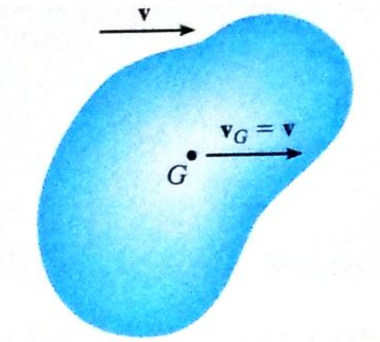
18.1 Kinetic Energy

In this chapter we will apply work and energy methods to solve planar motion problems involving force, velocity, and displacement. But first it will be necessary to develop a means of obtaining the body's kinetic energy when the body is subjected to translation, rotation about a fixed axis, or general plane motion.

To do this we will consider the rigid body shown in Fig. 18–1, which is represented here by a *slab* moving in the inertial x–y reference plane. An arbitrary ith particle of the body, having a mass dm, is located a distance r from the arbitrary point P. If at the *instant* shown the particle has a velocity $\mathbf{v}_i$, then the particle's kinetic energy is $T_i = \frac{1}{2}\,dm\,v_i^2$.

Translation. When a rigid body of mass m is subjected to either rectilinear or curvilinear *translation*, Fig. 18–2, the kinetic energy due to rotation is zero, since $\boldsymbol{\omega} = \mathbf{0}$. The kinetic energy of the body is therefore

$$\boxed{T = \tfrac{1}{2}mv_G^2} \tag{18–3}$$



Translation

Fig. 18–2

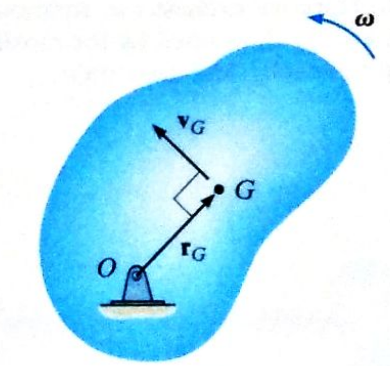
Rotation about a Fixed Axis. When a rigid body *rotates about a fixed axis* passing through point O, Fig. 18–3, the body has both *translational* and *rotational* kinetic energy so that

$$\boxed{T = \tfrac{1}{2}mv_G^2 + \tfrac{1}{2}I_G\omega^2} \tag{18–4}$$

The body's kinetic energy may also be formulated for this case by noting that $v_G = r_G\omega$, so that $T = \frac{1}{2}(I_G + mr_G^2)\omega^2$. By the parallel–axis theorem, the terms inside the parentheses represent the moment of inertia I_O of the body about an axis perpendicular to the plane of motion and passing through point O. Hence,*

$$\boxed{T = \tfrac{1}{2}I_O\omega^2} \tag{18–5}$$

From the derivation, this equation will give the same result as Eq. 18–4, since it accounts for *both* the translational and rotational kinetic energies of the body.



Rotation About a Fixed Axis

Fig. 18–3

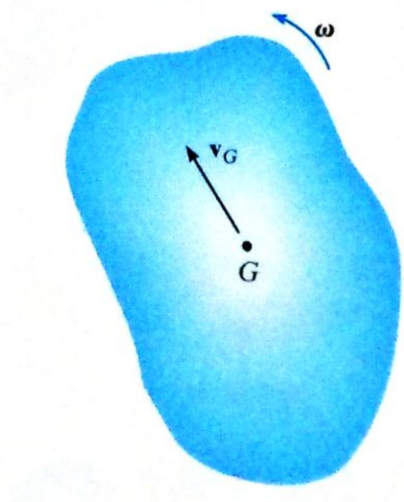
General Plane Motion. When a rigid body is subjected to general plane motion, Fig. 18–4, it has an angular velocity $\boldsymbol{\omega}$ and its mass center has a velocity $\mathbf{v}_G$. Therefore, the kinetic energy is

$$\boxed{T = \tfrac{1}{2}mv_G^2 + \tfrac{1}{2}I_G\omega^2} \tag{18–6}$$

This equation can also be expressed in terms of the body's motion about its instantaneous center of zero velocity i.e.,

$$\boxed{T = \tfrac{1}{2}I_{IC}\omega^2} \tag{18–7}$$

where I_{IC} is the moment of inertia of the body about its instantaneous center. The proof is similar to that of Eq. 18–5. (See Prob. 18–1.)



General Plane Motion

Fig. 18–4

*The similarity between this derivation and that of $\Sigma M_O = I_O\alpha$, should be noted. Also the same result can be obtained directly from Eq. 18–1 by selecting point P at O, realizing that $v_O = 0$.

Ch 18

Why doesn't friction do any work in pure rolling motion without slipping?

The frictional force that keeps an object rolling without slipping does no work on the object. How can it? The point at which it operates is not moving with respect to the surface. That the velocity of the contact point is identically zero is the quintessential nature of rolling without slipping.

The rolling friction force do Zero displacement at the contact point with ground (IC point)

If the frictional force doesn't do work, what does it do? It is instead is a constraint force that acts to keep the object rolling without slipping.

Work is (force x distance). In rolling motion, the point of the wheel in contact with the ground has speed = 0. So work done by friction = 0 (instantaneously v=0 means No motion, means no slipping displacement, means work=0). Same goes for pushing against an immovable object, the force exists but no displacement, so the work done = 0.

Work done is the scalar product of the applied force and the path of the point of application of force.

In pure rolling motion, the point of application of force is stationary and hence the product comes to zero.

Thus the work done in pure rolling motion is zero.

Ch 18

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disk $I_G = \frac{1}{2} m r^2$

Rod $I_G = \frac{1}{12} m L^2$

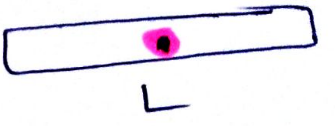
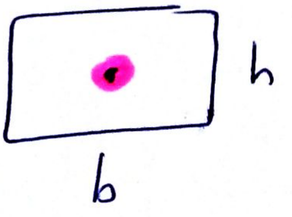


plate $I_G = \frac{1}{12} m (b^2 + h^2)$



parallel axis $I_A = I_G + m d^2$

$$T_1 + U_{12} = T_2$$

$$\frac{1}{2} m v_G^2 + \frac{1}{2} I_G \omega^2 \pm (F\cos\theta)(d) \pm M(\Delta\theta)$$

$$\pm mgh - \mu N(d) + \frac{1}{2} k(\Delta S)_1^2 - \frac{1}{2} k(\Delta S)_2^2$$

$$= \frac{1}{2} m v_G^2 + \frac{1}{2} I_G \omega^2$$

Note: $\frac{1}{2} m v_G^2 + \frac{1}{2} I_G \omega^2 = \frac{1}{2} I_A \omega^2$

EXAMPLE 18.1

The bar shown in Fig. 18–11*a* has a mass of 10 kg and is subjected to a couple moment of $M = 50\ \text{N}\cdot\text{m}$ and a force of $P = 80\ \text{N}$, which is always applied perpendicular to the end of the bar. Also, the spring has an unstretched length of 0.5 m and remains in the vertical position due to the roller guide at *B*. Determine the total work done by all the forces acting on the bar when it has rotated downward from $\theta = 0°$ to $\theta = 90°$.

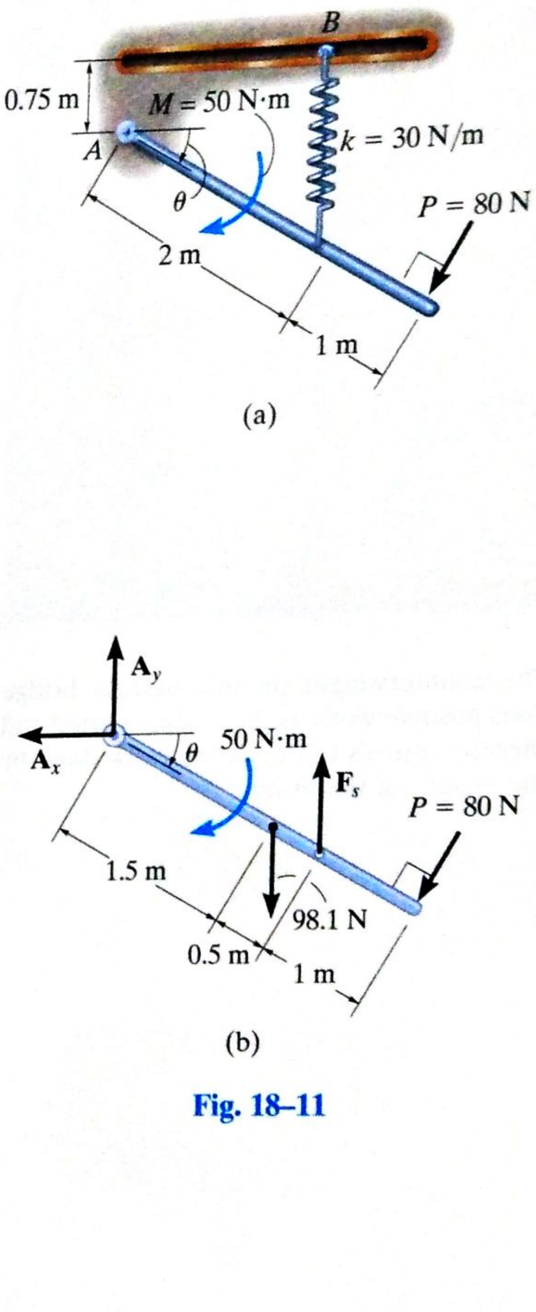


Fig. 18–11

SOLUTION

First the free-body diagram of the bar is drawn in order to account for all the forces that act on it, Fig. 18–11*b*.

Weight W. Since the weight $10(9.81)\ \text{N} = 98.1\ \text{N}$ is displaced downward 1.5 m, the work is

$$U_W = 98.1\ \text{N}(1.5\ \text{m}) = 147.2\ \text{J}$$

Why is the work positive?

Couple Moment M. The couple moment rotates through an angle of $\theta = \pi/2$ rad. Hence,

$$U_M = 50\ \text{N}\cdot\text{m}(\pi/2) = 78.5\ \text{J}$$

Spring Force F_s. When $\theta = 0°$ the spring is stretched $(0.75\ \text{m} - 0.5\ \text{m}) = 0.25\ \text{m}$, and when $\theta = 90°$, the stretch is $(2\ \text{m} + 0.75\ \text{m}) - 0.5\ \text{m} = 2.25\ \text{m}$. Thus,

$$U_s = -\left[\tfrac{1}{2}(30\ \text{N/m})(2.25\ \text{m})^2 - \tfrac{1}{2}(30\ \text{N/m})(0.25\ \text{m})^2\right] = -75.0\ \text{J}$$

By inspection the spring does negative work on the bar since $\mathbf{F}_s$ acts in the opposite direction to displacement. This checks with the result.

Force P. As the bar moves downward, the force is displaced through a distance of $(\pi/2)(3\ \text{m}) = 4.712\ \text{m}$. The work is positive. Why?

$$U_P = 80\ \text{N}(4.712\ \text{m}) = 377.0\ \text{J}$$

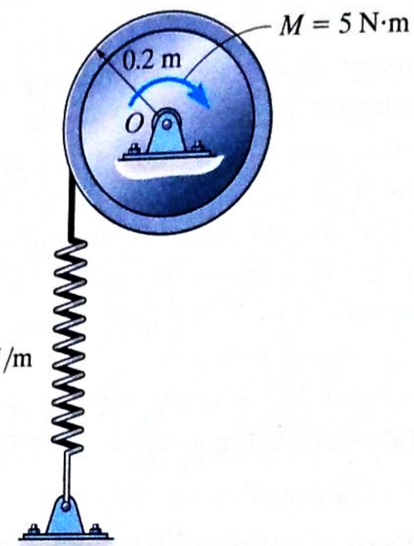
Pin Reactions. Forces $\mathbf{A}_x$ and $\mathbf{A}_y$ do no work since they are not displaced.

Total Work. The work of all the forces when the bar is displaced is thus

$$U = 147.2\ \text{J} + 78.5\ \text{J} - 75.0\ \text{J} + 377.0\ \text{J} = 528\ \text{J} \qquad \textit{Ans.}$$

EXAMPLE 18.2

The 30-kg disk shown in Fig. 18–12*a* is pin supported at its center. Determine the angle through which it must rotate to attain an angular velocity of 2 rad/s starting from rest. It is acted upon by a constant couple moment $M = 5\ \text{N}\cdot\text{m}$. The spring is orginally unstretched and its cord wraps around the rim of the disk.



(a)

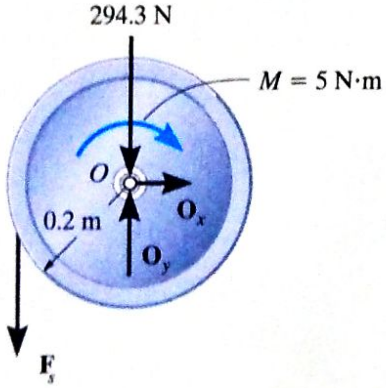
SOLUTION

Kinetic Energy. Since the disk rotates about a fixed axis, and it is initially at rest, then

$$T_1 = 0$$

$$T_2 = \tfrac{1}{2}I_O\omega_2^2 = \tfrac{1}{2}\left[\tfrac{1}{2}(30\ \text{kg})(0.2\ \text{m})^2\right](2\ \text{rad/s})^2 = 1.2\ \text{J}$$

Work (Free-Body Diagram). As shown in Fig. 18–12*b*, the pin reactions $\mathbf{O}_x$ and $\mathbf{O}_y$ and the weight (294.3 N) do no work, since they are not displaced. The *couple moment*, having a constant magnitude, does positive work $U_M = M\theta$ as the disk *rotates* through a clockwise angle of θ rad, and the spring does negative work $U_s = -\frac{1}{2}ks^2$.



(b)

Fig. 18–12

Principle of Work and Energy.

$$\{T_1\} + \{\Sigma U_{1-2}\} = \{T_2\}$$

$$\{T_1\} + \left\{M\theta - \tfrac{1}{2}ks^2\right\} = \{T_2\}$$

$$\{0\} + \left\{(5\ \text{N}\cdot\text{m})\theta - \frac{1}{2}(10\ \text{N/m})[\theta(0.2\ \text{m})]^2\right\} = \{1.2\ \text{J}\}$$

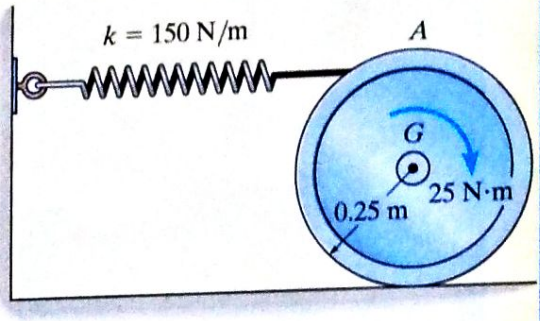
$$-0.2\theta^2 + 5\theta - 1.2 = 0$$

Solving this quadratic equation for the smallest positive root,

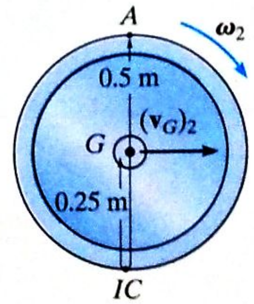
$$\theta = 0.2423\ \text{rad} = 0.2423\ \text{rad}\left(\frac{180^\circ}{\pi\ \text{rad}}\right) = 13.9^\circ \qquad \textit{Ans.}$$

EXAMPLE 18.3

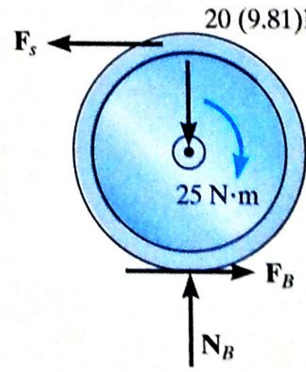
The wheel shown in Fig. 18–13*a* weighs 20 kg and has a radius of gyration $k_G = 0.2$ m about its mass center G. If it is subjected to a clockwise couple moment of 25 N·m and rolls from rest without slipping, determine its angular velocity after its center G moves 0.18 m. The spring has a stiffness $k = 150$ N/m and is initially unstretched when the couple moment is applied.



(a)



(b)



(c)

Fig. 18–13

SOLUTION

Kinetic Energy (Kinematic Diagram). Since the wheel is initially at rest,

$$T_1 = 0$$

The kinematic diagram of the wheel when it is in the final position is shown in Fig. 18–13*b*. The final kinetic energy is determined from

$$T_2 = \tfrac{1}{2} I_{IC}\omega_2^2$$

$$= \frac{1}{2}\left[20 \text{ kg } (0.2 \text{ m})^2 + (20 \text{ kg})(0.25 \text{ m})^2\right]\omega_2^2$$

$$T_2 = 1.025\ \omega_2^2$$

Work (Free-Body Diagram). As shown in Fig. 18–13*c*, only the spring force $\mathbf{F}_s$ and the couple moment do work. The normal force does not move along its line of action and the frictional force does *no work*, since the wheel does not slip as it rolls.

The work of $\mathbf{F}_s$ is found using $U_s = -\frac{1}{2}ks^2$. Here the work is negative since $\mathbf{F}_s$ is in the opposite direction to displacement. Since the wheel does not slip when the center G moves 0.18 m, then the wheel rotates $\theta = s_G/r_{G/IC} = 0.18 \text{ m}/0.25 \text{ m} = 0.72$ rad, Fig. 18–13*b*. Hence, the spring stretches $s = \theta r_{A/IC} = (0.72 \text{ rad})(0.5 \text{ m}) = 0.36$ m.

Principle of Work and Energy.

$$\{T_1\} + \{\Sigma U_{1-2}\} = \{T_2\}$$

$$\{T_1\} + \left\{M\theta - \tfrac{1}{2}ks^2\right\} = \{T_2\}$$

$$\{0\} + \left\{25 \text{ N}\cdot\text{m}(0.72 \text{ rad}) - \frac{1}{2}(150 \text{ N/m})(0.36 \text{ m})^2\right\} = \{1.025\ \omega_2^2 \text{ N}\cdot\text{m}\}$$

$$\omega_2 = 2.84 \text{ rad/s } \circlearrowright \qquad \textit{Ans.}$$

EXAMPLE 18.5

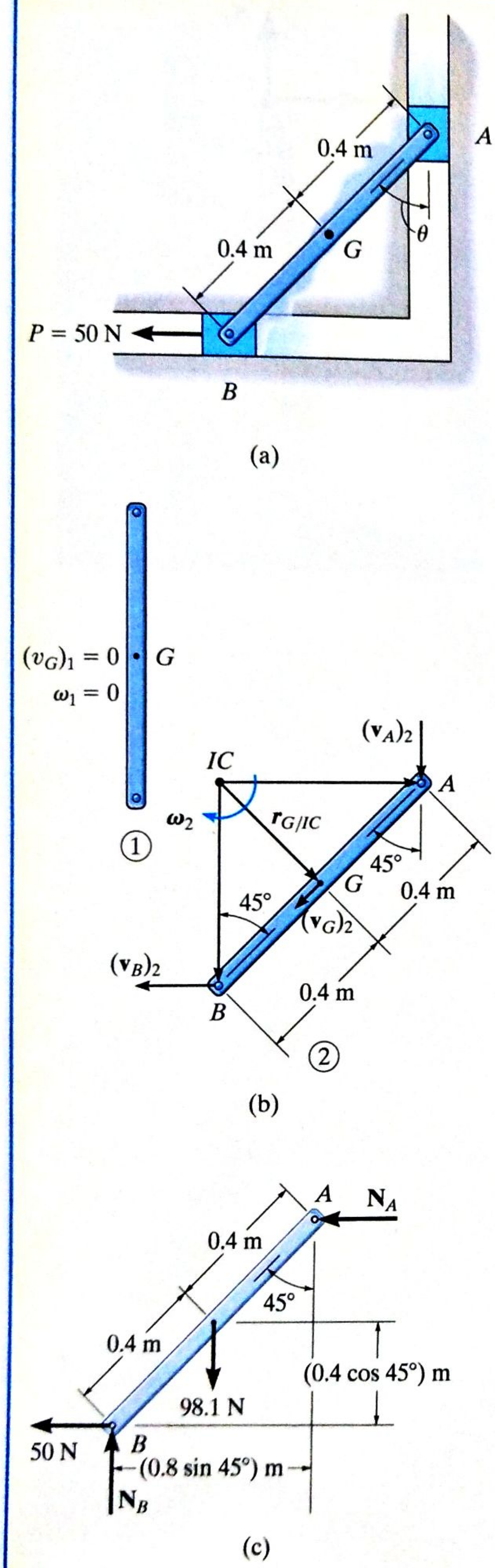


Fig. 18–15

The 10-kg rod shown in Fig. 18–15*a* is constrained so that its ends move along the grooved slots. The rod is initially at rest when $\theta = 0°$. If the slider block at B is acted upon by a horizontal force $P = 50$ N, determine the angular velocity of the rod at the instant $\theta = 45°$. Neglect friction and the mass of blocks A and B.

SOLUTION

Why can the principle of work and energy be used to solve this problem?

Kinetic Energy (Kinematic Diagrams). Two kinematic diagrams of the rod, when it is in the initial position 1 and final position 2, are shown in Fig. 18–15*b*. When the rod is in position 1, $T_1 = 0$ since $(\mathbf{v}_G)_1 = \boldsymbol{\omega}_1 = \mathbf{0}$. In position 2 the angular velocity is $\boldsymbol{\omega}_2$ and the velocity of the mass center is $(\mathbf{v}_G)_2$. Hence, the kinetic energy is

$$\begin{aligned} T_2 &= \tfrac{1}{2}m(v_G)_2^2 + \tfrac{1}{2}I_G\omega_2^2 \\ &= \tfrac{1}{2}(10\text{ kg})(v_G)_2^2 + \tfrac{1}{2}\left[\tfrac{1}{12}(10\text{ kg})(0.8\text{ m})^2\right]\omega_2^2 \\ &= 5(v_G)_2^2 + 0.2667(\omega_2)^2 \end{aligned}$$

The two unknowns $(v_G)_2$ and ω_2 can be related from the instantaneous center of zero velocity for the rod. Fig. 18–15*b*. It is seen that as A moves downward with a velocity $(\mathbf{v}_A)_2$, B moves horizontally to the left with a velocity $(\mathbf{v}_B)_2$, Knowing these directions, the IC is located as shown in the figure. Hence,

$$\begin{aligned} (v_G)_2 &= r_{G/IC}\omega_2 = (0.4 \tan 45°\text{ m})\omega_2 \\ &= 0.4\omega_2 \end{aligned}$$

Therefore,

$$T_2 = 0.8\omega_2^2 + 0.2667\omega_2^2 = 1.0667\omega_2^2$$

Of course, we can also determine this result using $T_2 = \frac{1}{2}I_{IC}\omega_2^2$.

Work (Free-Body Diagram). Fig. 18–15*c*. The normal forces $\mathbf{N}_A$ and $\mathbf{N}_B$ do no work as the rod is displaced. Why? The 98.1-N weight is displaced a vertical distance of $\Delta y = (0.4 - 0.4 \cos 45°)$ m; whereas the 50-N force moves a horizontal distance of $s = (0.8 \sin 45°)$ m. Both of these forces do positive work. Why?

Principle of Work and Energy.

$$\{T_1\} + \{\Sigma U_{1-2}\} = \{T_2\}$$

$$\{T_1\} + \{W\,\Delta y + Ps\} = \{T_2\}$$

$$\{0\} + \{98.1\text{ N}(0.4\text{ m} - 0.4 \cos 45°\text{ m}) + 50\text{ N}(0.8 \sin 45°\text{ m})\} = \{1.0667\omega_2^2\text{ J}\}$$

Solving for ω_2 gives

$$\omega_2 = 6.11\text{ rad/s} \circlearrowright \qquad \textit{Ans.}$$

EXAMPLE 18.6

The 10-kg rod AB shown in Fig. 18–18a is confined so that its ends move in the horizontal and vertical slots. The spring has a stiffness of $k = 800$ N/m and is unstretched when $\theta = 0°$. Determine the angular velocity of AB when $\theta = 0°$, if the rod is released from rest when $\theta = 30°$. Neglect the mass of the slider blocks.

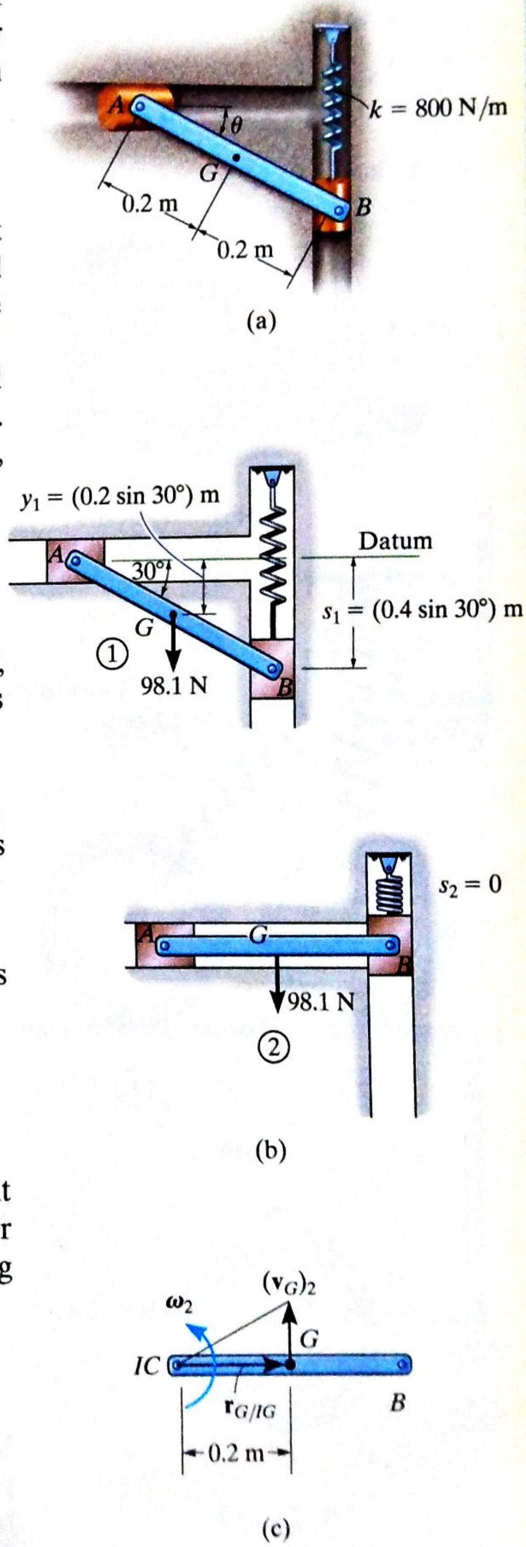


Fig. 18–18

SOLUTION

Potential Energy. The two diagrams of the rod, when it is located at its initial and final positions, are shown in Fig. 18–18b. The datum, used to measure the gravitational potential energy, is placed in line with the rod when $\theta = 0°$.

When the rod is in position 1, the center of gravity G is located *below the datum* so its gravitational potential energy is *negative*. Furthermore, (positive) elastic potential energy is stored in the spring, since it is stretched a distance of $s_1 = (0.4 \sin 30°)$ m. Thus,

$$V_1 = -Wy_1 + \tfrac{1}{2}ks_1^2$$
$$= -(98.1\text{ N})(0.2 \sin 30°\text{ m}) + \tfrac{1}{2}(800\text{ N/m})(0.4 \sin 30°\text{ m})^2 = 6.19\text{ J}$$

When the rod is in position 2, the potential energy of the rod is zero, since the center of gravity G is located at the datum, and the spring is unstretched, $s_2 = 0$. Thus,

$$V_2 = 0$$

Kinetic Energy. The rod is released from rest from position 1, thus $(\mathbf{v}_G)_1 = \boldsymbol{\omega}_1 = \mathbf{0}$, and so

$$T_1 = 0$$

In position 2, the angular velocity is $\boldsymbol{\omega}_2$ and the rod's mass center has a velocity of $(\mathbf{v}_G)_2$. Thus,

$$T_2 = \tfrac{1}{2}m(v_G)_2^2 + \tfrac{1}{2}I_G\omega_2^2$$
$$= \tfrac{1}{2}(10\text{ kg})(v_G)_2^2 + \tfrac{1}{2}\left[\tfrac{1}{12}(10\text{ kg})(0.4\text{ m})^2\right]\omega_2^2$$

Using *kinematics*, $(\mathbf{v}_G)_2$ can be related to $\boldsymbol{\omega}_2$ as shown in Fig. 18–18c. At the instant considered, the instantaneous center of zero velocity (IC) for the rod is at point A; hence, $(v_G)_2 = (r_{G/IC})\omega_2 = (0.2\text{ m})\omega_2$. Substituting into the above expression and simplifying (or using $\tfrac{1}{2}I_{IC}\omega_2^2$), we get

$$T_2 = 0.2667\omega_2^2$$

Conservation of Energy.

$$\{T_1\} + \{V_1\} = \{T_2\} + \{V_2\}$$
$$\{0\} + \{6.19\text{ J}\} = \{0.2667\omega_2^2\} + \{0\}$$
$$\omega_2 = 4.82\text{ rad/s} \circlearrowleft \qquad \textit{Ans.}$$

EXAMPLE 18.7

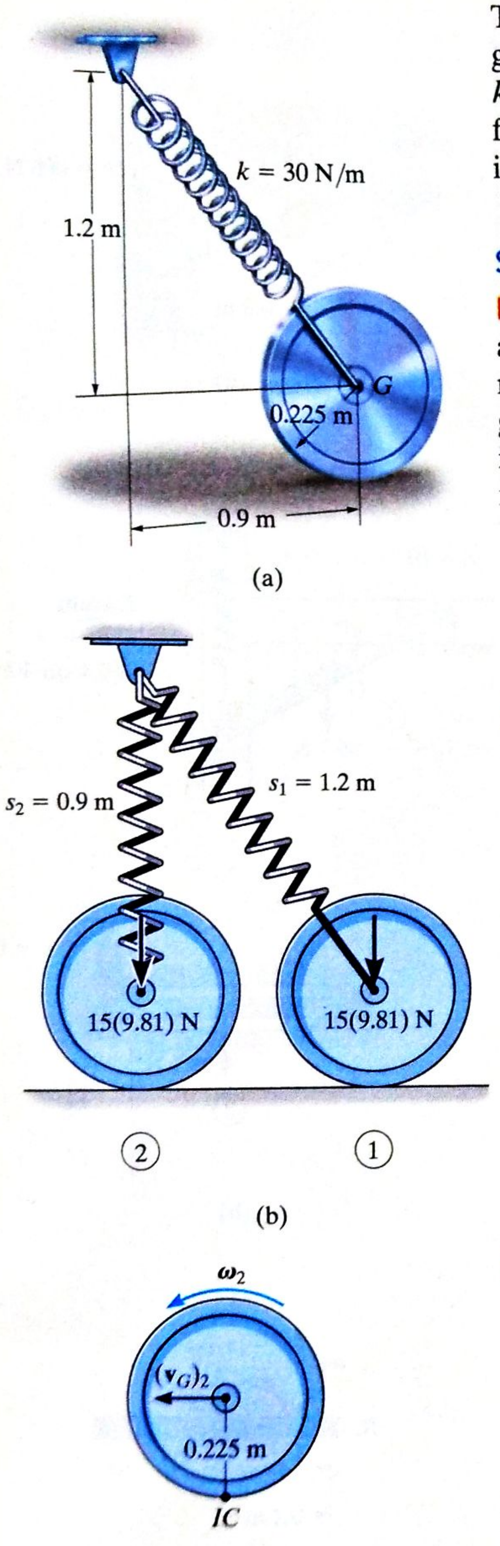


Fig. 18–19

The wheel shown in Fig. 18–19*a* has a weight of 15 kg and a radius of gyration of $k_G = 0.2$ m. It is attached to a spring which has a stiffness $k = 30$ N/m and an unstretched length of 0.3 m. If the disk is released from rest in the position shown and rolls without slipping, determine its angular velocity at the instant G moves 0.9 m to the left.

SOLUTION

Potential Energy. Two diagrams of the wheel, when it is at the initial and final positions, are shown in Fig. 18–19*b*. A gravitational datum is not needed here since the weight is not displaced vertically. From the problem geometry the spring is stretched $s_1 = \left(\sqrt{0.9^2 + 1.2^2} - 0.3\right) = 1.2$ m in the initial position, and stretched $s_2 = (1.2 - 0.3) = 0.9$ m in the final position. Hence, the positive spring potential energy is

$$V_1 = \tfrac{1}{2}ks_1^2 = \tfrac{1}{2}(30\text{ N/m})(1.2\text{ m})^2 = 21.6\text{ N}\cdot\text{m}$$

$$V_2 = \tfrac{1}{2}ks_2^2 = \tfrac{1}{2}(30\text{ N/m})(0.9\text{ m})^2 = 12.15\text{ N}\cdot\text{m}$$

Kinetic Energy. The disk is released from rest and so $(\mathbf{v}_G)_1 = \mathbf{0}$, $\boldsymbol{\omega}_1 = \mathbf{0}$. Therefore,

$$T_1 = 0$$

Since the instantaneous center of zero velocity is at the ground, Fig. 18–19*c*, we have

$$T_2 = \frac{1}{2}I_{IC}\omega_2^2$$

$$= \frac{1}{2}\left[(15\text{ kg})(0.2\text{ m})^2 + (15\text{ kg})(0.225\text{ m})^2\right]\omega_2^2$$

$$= 0.6797\omega_2^2$$

Conservation of Energy.

$$\{T_1\} + \{V_1\} = \{T_2\} + \{V_2\}$$

$$\{0\} + \{21.6\text{ N}\cdot\text{m}\} = \{0.6797\omega_2^2\} + \{12.15\text{ N}\cdot\text{m}\}$$

$$\omega_2 = 3.73\text{ rad/s} \circlearrowleft \qquad \textit{Ans.}$$

NOTE: If the principle of work and energy were used to solve this problem, then the work of the spring would have to be determined by considering both the change in magnitude and direction of the spring force.

EXAMPLE 18.8

The 10-kg homogeneous disk shown in Fig. 18–20*a* is attached to a uniform 5-kg rod *AB*. If the assembly is released from rest when $\theta = 60°$, determine the angular velocity of the rod when $\theta = 0°$. Assume that the disk rolls without slipping. Neglect friction along the guide and the mass of the collar at *B*.

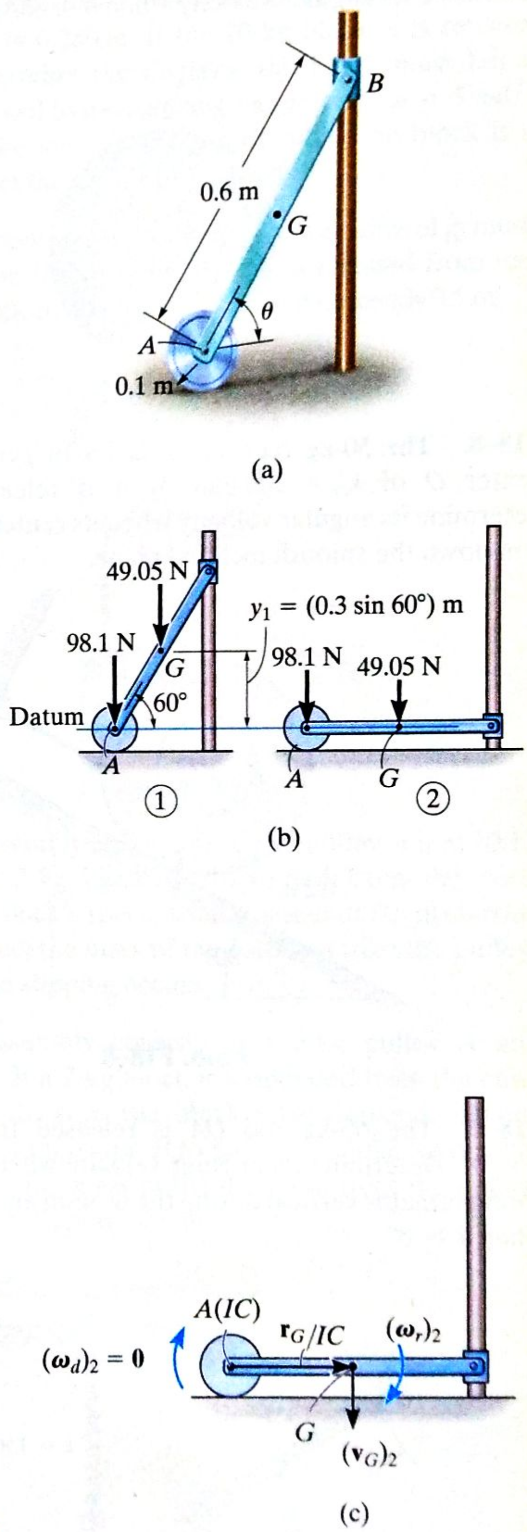


Fig. 18–20

SOLUTION

Potential Energy. Two diagrams for the rod and disk, when they are located at their initial and final positions, are shown in Fig. 18–20*b*. For convenience the datum passes through point *A*.

When the system is in position 1, only the rod's weight has positive potential energy. Thus,

$$V_1 = W_r y_1 = (49.05 \text{ N})(0.3 \sin 60° \text{ m}) = 12.74 \text{ J}$$

When the system is in position 2, both the weight of the rod and the weight of the disk have zero potential energy. Why? Thus,

$$V_2 = 0$$

Kinetic Energy. Since the entire system is at rest at the initial position,

$$T_1 = 0$$

In the final position the rod has an angular velocity $(\omega_r)_2$ and its mass center has a velocity $(\mathbf{v}_G)_2$, Fig. 18–20*c*. Since the rod is *fully extended* in this position, the disk is momentarily at rest, so $(\boldsymbol{\omega}_d)_2 = \mathbf{0}$ and $(\mathbf{v}_A)_2 = \mathbf{0}$. For the rod $(\mathbf{v}_G)_2$ can be related to $(\boldsymbol{\omega}_r)_2$ from the instantaneous center of zero velocity, which is located at point *A*, Fig. 18–20*c*. Hence, $(v_G)_2 = r_{G/IC}(\omega_r)_2$ or $(v_G)_2 = 0.3(\omega_r)_2$. Thus,

$$T_2 = \frac{1}{2}m_r(v_G)_2^2 + \frac{1}{2}I_G(\omega_r)_2^2 + \frac{1}{2}m_d(v_A)_2^2 + \frac{1}{2}I_A(\omega_d)_2^2$$

$$= \frac{1}{2}(5 \text{ kg})[(0.3 \text{ m})(\omega_r)_2]^2 + \frac{1}{2}\left[\frac{1}{12}(5 \text{ kg})(0.6 \text{ m})^2\right](\omega_r)_2^2 + 0 + 0$$

$$= 0.3(\omega_r)_2^2$$

Conservation of Energy.

$$\{T_1\} + \{V_1\} = \{T_2\} + \{V_2\}$$

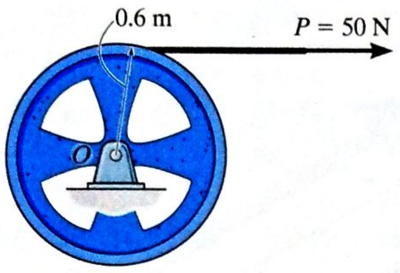
$$\{0\} + \{12.74 \text{ J}\} = \{0.3(\omega_r)_2^2\} + \{0\}$$

$$(\omega_r)_2 = 6.52 \text{ rad/s} \circlearrowright \qquad \textit{Ans.}$$

NOTE: We can also determine the final kinetic energy of the rod using $T_2 = \frac{1}{2}I_{IC}\omega_2^2$.

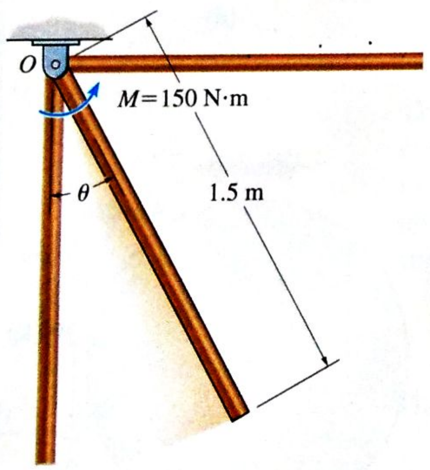
FUNDAMENTAL PROBLEMS

F18–1. The 80-kg wheel has a radius of gyration about its mass center O of $k_O = 400$ mm. Determine its angular velocity after it has rotated 20 revolutions starting from rest.



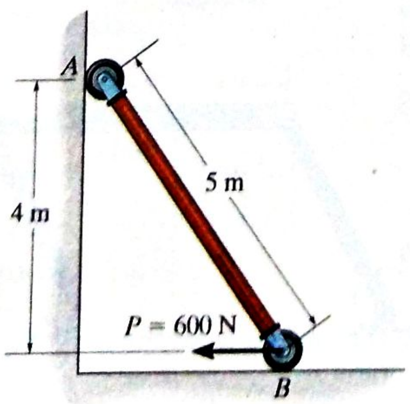
Prob. F18–1

F18–2. The uniform 25-kg slender rod is subjected to a couple moment of $M = 150$ N · m. If the rod is at rest when $\theta = 0°$, determine its angular velocity when $\theta = 90°$.



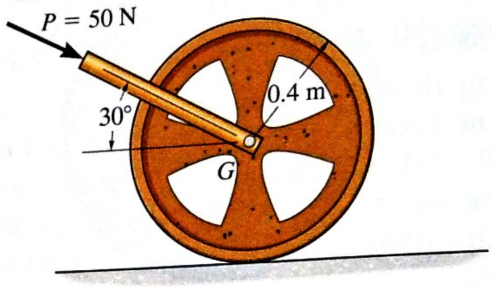
Prob. F18–2

F18–3. The uniform 50-kg slender rod is at rest in the position shown when $P = 600$ N is applied. Determine the angular velocity of the rod when the rod reaches the vertical position.



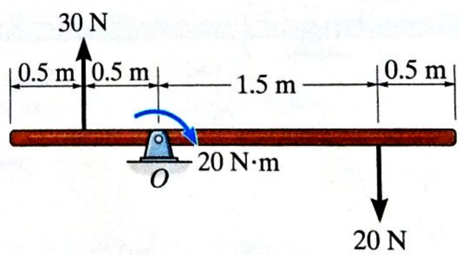
Prob. F18–3

F18–4. The 50-kg wheel is subjected to a force of 50 N. If the wheel starts from rest and rolls without slipping, determine its angular velocity after it has rotated 10 revolutions. The radius of gyration of the wheel about its mass center G is $k_G = 0.3$ m.



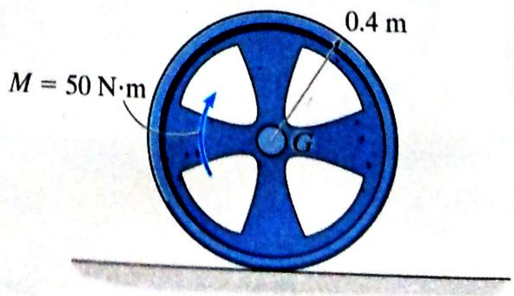
Prob. F18–4

F18–5. If the uniform 30-kg slender rod starts from rest at the position shown, determine its angular velocity after it has rotated 4 revolutions. The forces remain perpendicular to the rod.



Prob. F18–5

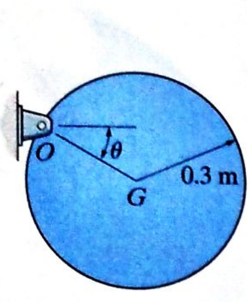
F18–6. The 20-kg wheel has a radius of gyration about its center G of $k_G = 300$ mm. When it is subjected to a couple moment of $M = 50$ N · m, it rolls without slipping. Determine the angular velocity of the wheel after its mass center G has traveled through a distance of $s_G = 20$ m, starting from rest.



Prob. F18–6

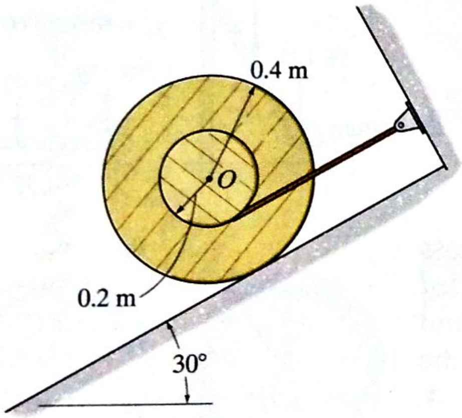
FUNDAMENTAL PROBLEMS

F18–7. If the 30-kg disk is released from rest when $\theta = 0°$, determine its angular velocity when $\theta = 90°$.



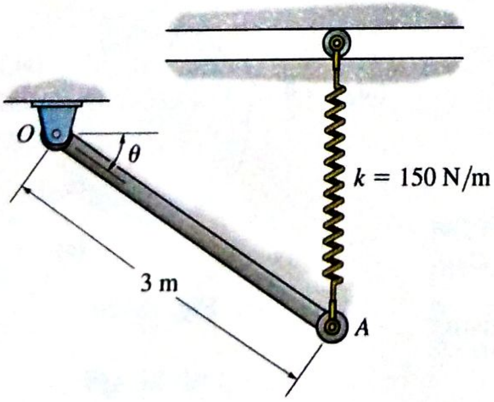
Prob. F18–7

F18–8. The 50-kg reel has a radius of gyration about its center O of $k_O = 300$ mm. If it is released from rest, determine its angular velocity when its center O has traveled 6 m down the smooth inclined plane.



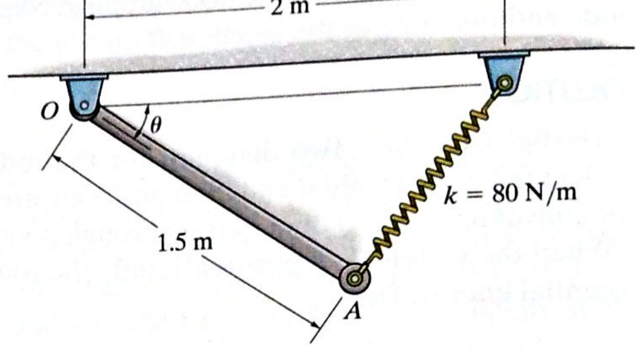
Prob. F18–8

F18–9. The 60-kg rod OA is released from rest when $\theta = 0°$. Determine its angular velocity when $\theta = 45°$. The spring remains vertical during the motion and is unstretched when $\theta = 0°$.



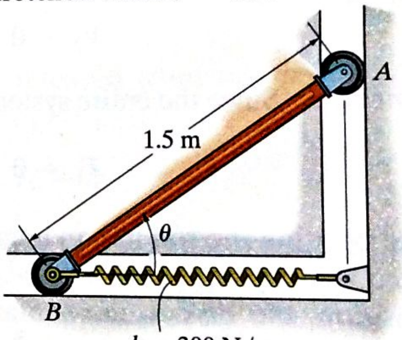
Prob. F18–9

F18–10. The 30-kg rod is released from rest when $\theta = 0°$. Determine the angular velocity of the rod when $\theta = 90°$. The spring is unstretched when $\theta = 0°$.



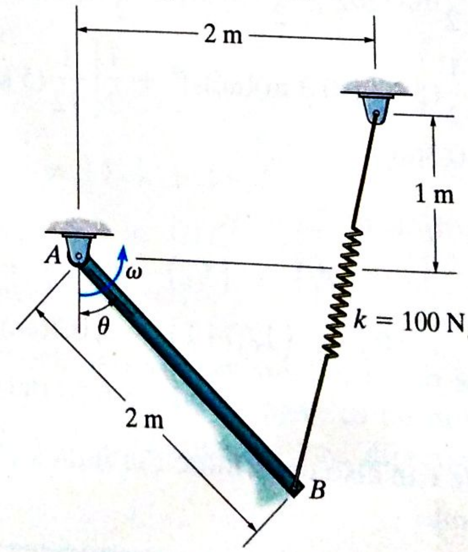
Prob. F18–10

F18–11. The 30-kg rod is released from rest when $\theta = 45°$. Determine the angular velocity of the rod when $\theta = 0°$. The spring is unstretched when $\theta = 45°$.



Prob. F18–11

F18–12. The 20-kg rod is released from rest when $\theta = 0°$. Determine its angular velocity when $\theta = 90°$. The spring has an unstretched length of 0.5 m.



Prob. F18–12

Chapter 18

F18–1. $I_O = mk_O^2 = 80(0.4^2) = 12.8\ \text{kg}\cdot\text{m}^2$

$T_1 = 0$

$T_2 = \frac{1}{2}I_O\omega^2 = \frac{1}{2}(12.8)\omega^2 = 6.4\omega^2$

$s = \theta r = 20(2\pi)(0.6) = 24\pi\ \text{m}$

$T_1 + \Sigma U_{1-2} = T_2$

$0 + 50(24\pi) = 6.4\omega^2$

$\omega = 24.3\ \text{rad/s}$ *Ans.*

F18–2. $T_1 = 0$

$T_2 = \frac{1}{2}m(v_G)_2^2 + \frac{1}{2}I_G\omega_2^2$

$= \frac{1}{2}(25\ \text{kg})(0.75\omega_2)^2$

$+ \frac{1}{2}\left[\frac{1}{12}(25\ \text{kg})(1.5\ \text{m})^2\right]\omega_2^2$

$T_2 = 11.719\omega_2^2$

Or,

$I_O = \frac{1}{3}ml^2 = \frac{1}{3}(25\ \text{kg})(1.5\ \text{m})^2$

$= 18.75\ \text{kg}\cdot\text{m}^2$

So that

$T_2 = \frac{1}{2}I_O\omega_2^2 = \frac{1}{2}(18.75\ \text{kg}\cdot\text{m}^2)\omega_2^2$

$= 9.375\omega_2^2$

$T_1 + \Sigma U_{1-2} = T_2$

$T_1 + \left[-Wy_G + M\theta\right] = T_2$

$0 + \left[-(25(9.81\ \text{N})(0.75\ \text{m})) + (150\ \text{N}\cdot\text{m})(\frac{\pi}{2})\right]$

$= 9.375{\omega_2}^2$

$\omega_2 = 2.35\ \text{rad/s}$ *Ans.*

F18–3. $(v_G)_2 = \omega_2 r_{G/IC} = \omega_2(2.5)$

$I_G = \frac{1}{12}ml^2 = \frac{1}{12}(50)(5^2) = 104.17\ \text{kg}\cdot\text{m}^2$

$T_1 = 0$

$T_2 = \frac{1}{2}m(v_G)_2^2 + \frac{1}{2}I_G\omega_2^2$

$= \frac{1}{2}(50)\left[\omega_2(2.5)\right]^2 + \frac{1}{2}(104.17)\omega_2^2 = 208.33\omega_2^2$

$U_P = Ps_P = 600(3) = 1800\ \text{J}$

$U_W = -Wh = -50(9.81)(2.5 - 2) = -245.25\ \text{J}$

$T_1 + \Sigma U_{1-2} = T_2$

$0 + 1800 + (-245.25) = 208.33\omega_2^2$

$\omega_2 = 2.732\ \text{rad/s} = 2.73\ \text{rad/s}$ *Ans.*

F18–4. $T = \frac{1}{2}mv_G^2 + \frac{1}{2}I_G\omega^2$
$= \frac{1}{2}(50\text{ kg})(0.4\omega)^2 + \frac{1}{2}\left[50\text{ kg}(0.3\text{ m})^2\right]\omega^2$
$= 6.25\omega^2\text{ J}$

Or,

$T = \frac{1}{2}I_{IC}\omega^2$
$= \frac{1}{2}\left[50\text{ kg}(0.3\text{ m})^2 + 50\text{ kg}(0.4\text{ m})^2\right]\omega^2$
$= 6.25\omega^2\text{ J}$

$s_G = \theta r = 10(2\pi\text{ rad})(0.4\text{ m}) = 8\pi\text{ m}$
$T_1 + \Sigma U_{1-2} = T_2$
$T_1 + P\cos 30^\circ\, s_G = T_2$
$0 + (50\text{ N})\cos 30^\circ(8\pi\text{ m}) = 6.25\omega^2\text{ J}$
$\omega = 13.2\text{ rad/s}$ *Ans.*

F18–5. $I_G = \frac{1}{12}ml^2 = \frac{1}{12}(30)(3^2) = 22.5\text{ kg}\cdot\text{m}^2$
$T_1 = 0$
$T_2 = \frac{1}{2}mv_G^2 + \frac{1}{2}I_G\omega^2$
$= \frac{1}{2}(30)[\omega(0.5)]^2 + \frac{1}{2}(22.5)\omega^2 = 15\omega^2$

Or,

$I_O = I_G + md^2 = \frac{1}{12}(30)(3^2) + 30(0.5^2)$
$= 30\text{ kg}\cdot\text{m}^2$
$T_2 = \frac{1}{2}I_O\omega^2 = \frac{1}{2}(30)\omega^2 = 15\omega^2$
$s_1 = \theta r_1 = 8\pi(0.5) = 4\pi\text{ m}$
$s_2 = \theta r_2 = 8\pi(1.5) = 12\pi\text{ m}$
$U_{P_1} = P_1 s_1 = 30(4\pi) = 120\pi\text{ J}$
$U_{P_2} = P_2 s_2 = 20(12\pi) = 240\pi\text{ J}$
$U_M = M\theta = 20[4(2\pi)] = 160\pi\text{ J}$
$U_W =$ (0 bar returns to same position)
$T_1 + \Sigma U_{1-2} = T_2$
$0 + 120\pi + 240\pi + 160\pi = 15\omega^2$
$\omega = 10.44\text{ rad/s} = 10.4\text{ rad/s}$ *Ans.*

F18–6. $v_G = \omega r = \omega(0.4)$
$I_G = mk_G^2 = 20(0.3^2) = 1.8\text{ kg}\cdot\text{m}^2$
$T_1 = 0$
$T_2 = \frac{1}{2}mv_G^2 + \frac{1}{2}I_G\omega^2$
$= \frac{1}{2}(20)\left[\omega(0.4)\right]^2 + \frac{1}{2}(1.8)\omega^2$
$= 2.5\omega^2$
$U_M = M\theta = M\left(\frac{s_O}{r}\right) = 50\left(\frac{20}{0.4}\right) = 2500\text{ J}$
$T_1 + \Sigma U_{1-2} = T_2$
$0 + 2500 = 2.5\omega^2$
$\omega = 31.62\text{ rad/s} = 31.6\text{ rad/s}$ *Ans.*

F18–7. $v_G = \omega r = \omega(0.3)$
$I_G = \frac{1}{2}mr^2 = \frac{1}{2}(30)(0.3^2) = 1.35\text{ kg}\cdot\text{m}^2$
$T_1 = 0$
$T_2 = \frac{1}{2}m(v_G)_2^2 + \frac{1}{2}I_G\omega_2^2$
$= \frac{1}{2}(30)[\omega_2(0.3)]^2 + \frac{1}{2}(1.35)\omega_2^2 = 2.025\omega_2^2$
$(V_g)_1 = Wy_1 = 0$
$(V_g)_2 = -Wy_2 = -30(9.81)(0.3) = -88.29\text{ J}$
$T_1 + V_1 = T_2 + V_2$
$0 + 0 = 2.025\omega_2^2 + (-88.29)$
$\omega_2 = 6.603\text{ rad/s} = 6.60\text{ rad/s}$ *Ans.*

F18–8. $v_O = \omega r_{O/IC} = \omega(0.2)$
$I_O = mk_O^2 = 50(0.3^2) = 4.5\text{ kg}\cdot\text{m}^2$
$T_1 = 0$
$T_2 = \frac{1}{2}m(v_O)_2^2 + \frac{1}{2}I_O\omega_2^2$
$= \frac{1}{2}(50)\left[\omega_2(0.2)\right]^2 + \frac{1}{2}(4.5)\omega_2^2$
$= 3.25\omega_2^2$
$(V_g)_1 = Wy_1 = 0$
$(V_g)_2 = -Wy_2 = -50(9.81)(6\sin 30^\circ)$
$= -1471.5\text{J}$
$T_1 + V_1 = T_2 + V_2$
$0 + 0 = 3.25\omega_2^2 + (-1471.5)$
$\omega_2 = 21.28\text{ rad/s} = 21.3\text{ rad/s}$ *Ans.*

F18–9. $v_G = \omega r_G = \omega(1.5)$
$I_G = \frac{1}{12}(60)(3^2) = 45\text{ kg}\cdot\text{m}^2$
$T_1 = 0$
$T_2 = \frac{1}{2}m(\text{v}_G)_2^2 + \frac{1}{2}I_G\omega_2^2$
$= \frac{1}{2}(60)[\omega_2(1.5)]^2 + \frac{1}{2}(45)\omega_2^2$
$= 90\omega_2^2$

Or,

$T_2 = \frac{1}{2}I_O\omega_2^2 = \frac{1}{2}\left[45 + 60(1.5^2)\right]\omega_2^2 = 90\omega_2^2$
$(V_g)_1 = Wy_1 = 0$
$(V_g)_2 = -Wy_2 = -60(9.81)(1.5\sin 45^\circ)$
$= -624.30\text{ J}$
$(V_e)_1 = \frac{1}{2}ks_1^2 = 0$
$(V_e)_2 = \frac{1}{2}ks_2^2 = \frac{1}{2}(150)(3\sin 45^\circ)^2 = 337.5\text{ J}$
$T_1 + V_1 = T_2 + V_2$
$0 + 0 = 90\omega_2^2 + [-624.30 + 337.5]$
$\omega_2 = 1.785\text{ rad/s} = 1.79\text{ rad/s}$ *Ans.*

F18–10. $v_G = \omega r_G = \omega(0.75)$
$I_G = \frac{1}{12}(30)(1.5^2) = 5.625\text{ kg}\cdot\text{m}^2$
$T_1 = 0$

$$T_2 = \frac{1}{2}m(v_G)_2^2 + \frac{1}{2}I_G\omega_2^2$$
$$= \frac{1}{2}(30)[\omega(0.75)]^2 + \frac{1}{2}(5.625)\omega_2^2 = 11.25\omega_2^2$$

Or,

$$T_2 = \frac{1}{2}I_O\omega_2^2 = \frac{1}{2}\left[5.625 + 30\left(0.75^2\right)\right]\omega_2^2$$
$$= 11.25\omega_2^2$$
$$(V_g)_1 = Wy_1 = 0$$
$$(V_g)_2 = -Wy_2 = -30(9.81)(0.75)$$
$$= -220.725 \text{ J}$$
$$(V_e)_1 = \frac{1}{2}ks_1^2 = 0$$
$$(V_e)_2 = \frac{1}{2}ks_2^2 = \frac{1}{2}(80)\left(\sqrt{2^2 + 1.5^2} - 0.5\right)^2 = 160 \text{ J}$$
$$T_1 + V_1 = T_2 + V_2$$
$$0 + 0 = 11.25\omega_2^2 + (-220.725 + 160)$$
$$\omega_2 = 2.323 \text{ rad/s} = 2.32 \text{ rad/s}$$

Ans.

F18–11. $(v_G)_2 = \omega_2 r_{G/IC} = \omega_2(0.75)$

$$I_G = \frac{1}{12}(30)\left(1.5^2\right) = 5.625 \text{ kg}\cdot\text{m}^2$$
$$T_1 = 0$$
$$T_2 = \frac{1}{2}m(v_G)_2^2 + \frac{1}{2}I_G\omega_2^2$$
$$= \frac{1}{2}(30)[\omega_2(0.75)]^2 + \frac{1}{2}(5.625)\omega_2^2 = 11.25\omega_2^2$$
$$(V_g)_1 = Wy_1 = 30(9.81)(0.75 \sin 45°) = 156.08 \text{ J}$$
$$(V_g)_2 = -Wy_2 = 0$$
$$(V_e)_1 = \frac{1}{2}ks_1^2 = 0$$
$$(V_e)_2 = \frac{1}{2}ks_2^2 = \frac{1}{2}(300)(1.5 - 1.5\cos 45°)^2$$
$$= 28.95 \text{ J}$$
$$T_1 + V_1 = T_2 + V_2$$
$$0 + (156.08 + 0) = 11.25\omega_2^2 + (0 + 28.95)$$
$$\omega_2 = 3.362 \text{ rad/s} = 3.36 \text{ rad/s}$$

Ans.

F18–12. $(V_g)_1 = -Wy_1 = -[20(9.81) \text{ N}](1 \text{ m}) = -196.2 \text{ J}$

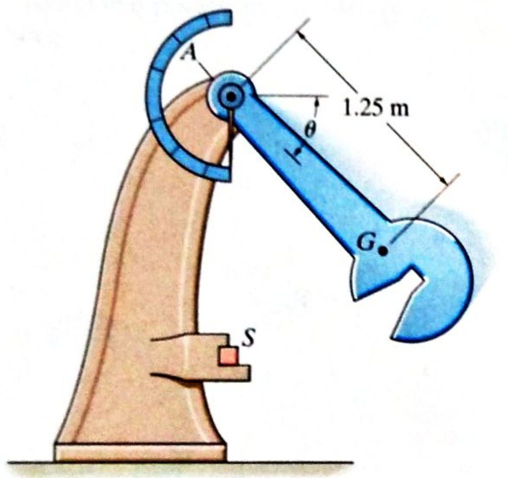
$$(V_g)_2 = 0$$
$$(V_e)_1 = \frac{1}{2}ks_1^2$$
$$= \frac{1}{2}(100 \text{ N/m})\left(\sqrt{(3 \text{ m})^2 + (2 \text{ m})^2} - 0.5 \text{ m}\right)^2$$
$$= 482.22 \text{ J}$$
$$(V_e)_2 = \frac{1}{2}ks_2^2 = \frac{1}{2}(100 \text{ N/m})(1 \text{ m} - 0.5 \text{ m})^2$$
$$= 12.5 \text{ J}$$
$$T_1 = 0$$
$$T_2 = \frac{1}{2}I_A\omega^2 = \frac{1}{2}\left[\frac{1}{3}(20 \text{ kg})(2 \text{ m})^2\right]\omega^2$$
$$= 13.3333\omega^2$$
$$T_1 + V_1 = T_2 + V_2$$
$$0 + [-196.2 \text{ J} + 482.22 \text{ J}]$$
$$= 13.3333\omega_2^2 + [0 + 12.5 \text{ J}]$$

Ans.

$$\omega_2 = 4.53 \text{ rad/s}$$

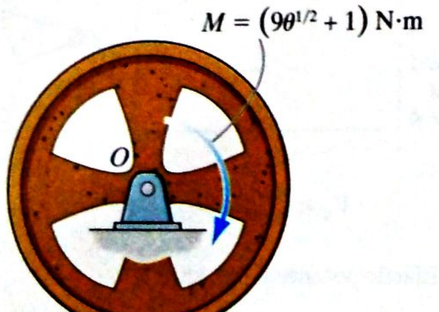
REVIEW PROBLEMS

R18–1. The pendulum of the Charpy impact machine has a mass of 50 kg and a radius of gyration of $k_A = 1.75$ m. If it is released from rest when $\theta = 0°$, determine its angular velocity just before it strikes the specimen S, $\theta = 90°$.



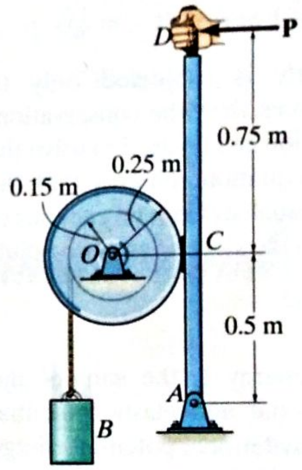
Prob. R18–1

R18–2. The 50-kg flywheel has a radius of gyration of $k_O = 200$ mm about its center of mass. If it is subjected to a torque of $M = (9\theta^{1/2} + 1)$ N·m, where θ is in radians, determine its angular velocity when it has rotated 5 revolutions, starting from rest.



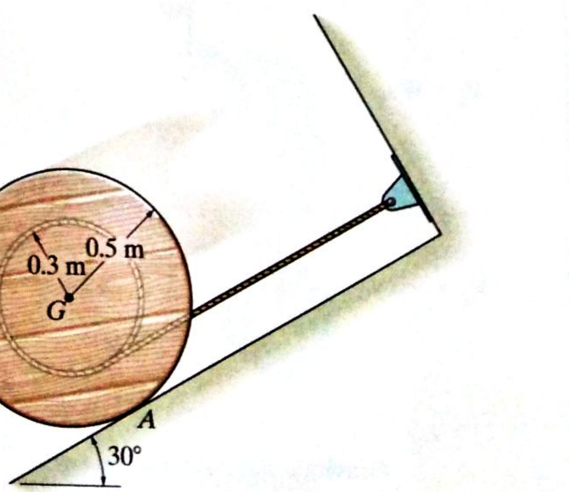
Prob. R18–2

R18–3. The drum has a mass of 50 kg and a radius of gyration about the pin at O of $k_O = 0.23$ m. Starting from rest, the suspended 15-kg block B is allowed to fall 3 m without applying the brake ACD. Determine the speed of the block at this instant. If the coefficient of kinetic friction at the brake pad C is $\mu_k = 0.5$, determine the force **P** that must be applied at the brake handle which will then stop the block after it descends *another* 3 m. Neglect the thickness of the handle.



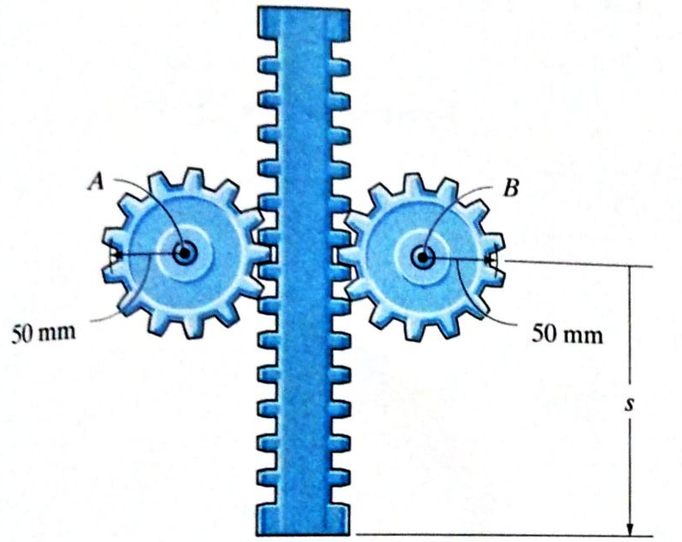
Prob. R18–3

R18–4. The spool has a mass of 60 kg and a radius of gyration of $k_G = 0.3$ m. If it is released from rest, determine how far its center descends down the smooth plane before it attains an angular velocity of $\omega = 6$ rad/s. Neglect the mass of the cord which is wound around the central core. The coefficient of kinetic friction between the spool and plane at A is $\mu_k = 0.2$.



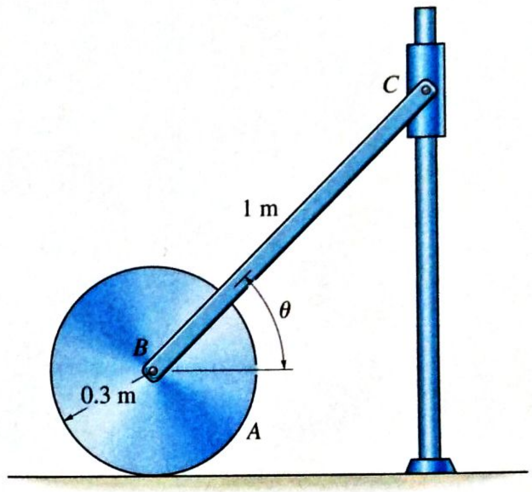
Prob. R18–4

R18–5. The gear rack has a mass of 6 kg, and the gears each have a mass of 4 kg and a radius of gyration of $k = 30$ mm at their centers. If the rack is originally moving downward at 2 m/s, when $s = 0$, determine the speed of the rack when $s = 600$ mm. The gears are free to turn about their centers A and B.



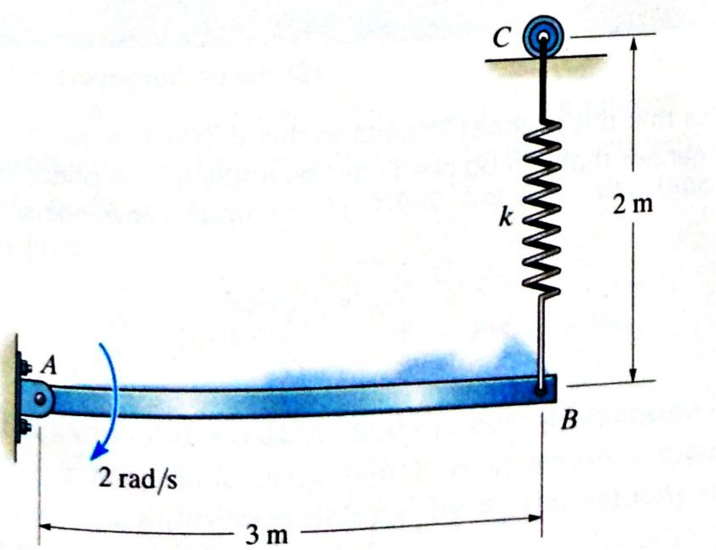
Prob. R18–5

R18–7. The system consists of a 10-kg disk A, 2-kg slender rod BC, and a 0.5-kg smooth collar C. If the disk rolls without slipping, determine the velocity of the collar at the instant the rod becomes horizontal, i.e., $\theta = 0°$. The system is released from rest when $\theta = 45°$.



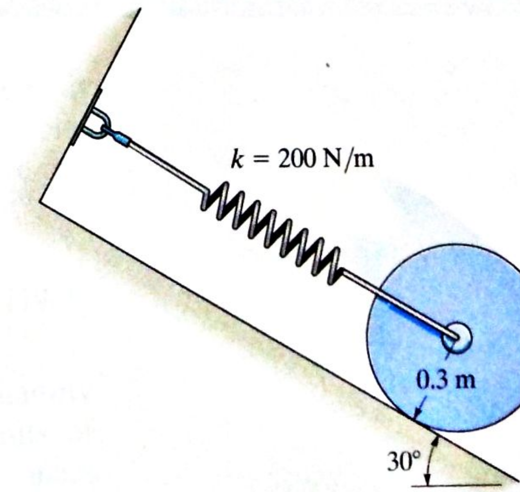
Prob. R18–7

R18-6. At the instant shown, the 25-kg bar rotates clockwise at 2 rad/s. The spring attached to its end always remains vertical due to the roller guide at C. If the spring has an unstretched length of 1 m and a stiffness of $k = 90$ N/m, determine the angular velocity of the bar the instant it has rotated 30° clockwise.



Prob. R18–6

R18–8. At the instant the spring becomes undeformed, the center of the 40-kg disk has a speed of 4 m/s. From this point determine the distance d the disk moves down the plane before momentarily stopping. The disk rolls without slipping.



Prob. R18–8

R18-2

$I_o = mk_o^2$
$= 2\ kg \cdot m^2$

$$U_M = \int M d\theta = \int_0^{10\pi} (9\theta^{1/2} + 1)d\theta$$

$$= (6\theta^{3/2} + \theta)\Big|_0^{10\pi}$$

$$= 1087.93 \text{ J}$$

Principle of Work and Energy:

$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + 1087.93 = \omega^2$$

$$\omega = 33.0 \text{ rad/s} \qquad \textit{Ans.}$$

R18–3. Before braking:

$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + 15(9.81)(3) = \frac{1}{2}(15)v_B^2 + \frac{1}{2}\left[50(0.23)^2\right]\left(\frac{v_B}{0.15}\right)^2$$

$$v_B = 2.58 \text{ m/s} \qquad \textit{Ans.}$$

$$\frac{s_B}{0.15} = \frac{s_C}{0.25}$$

Set $s_B = 3$ m, then $s_C = 5$ m.

$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 - F(5) + 15(9.81)(6) = 0$$

$$F = 176.6 \text{ N}$$

$$N = \frac{176.6}{0.5} = 353.2 \text{ N}$$

Brake arm:

$$\circlearrowleft + \Sigma M_A = 0; \qquad -353.2(0.5) + P(1.25) = 0$$

$$P = 141 \text{ N} \qquad \textit{Ans.}$$

R18–4.

$$\frac{s_G}{0.3} = \frac{s_A}{(0.5 - 0.3)}$$

$$s_A = 0.6667 s_G$$

$$+\nwarrow \Sigma F_y = 0; \qquad N_A - 60(9.81)\cos 30° = 0$$

$$N_A = 509.7 \text{ N}$$

$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + 60(9.81)\sin 30°(s_G) - 0.2(509.7)(0.6667 s_G)$$
$$= \frac{1}{2}\left[60(0.3)^2\right](6)^2$$
$$+ \frac{1}{2}(60)\left[(0.3)(6)\right]^2$$

$$s_G = 0.859 \text{ m} \qquad \textit{Ans.}$$

R18–5. ***Conservation of Energy***: Originally, both gears are rotating with an angular velocity of $\omega_1 = \frac{2}{0.05} = 40$ rad/s. After the rack has traveled $s = 600$ mm, both gears rotate with an angular velocity of $\omega_2 = \frac{v_2}{0.05}$, where v_2 is the speed of the rack at that moment.

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}(6)(2)^2 + 2\left\{\frac{1}{2}\left[4(0.03)^2\right](40)^2\right\} + 0$$
$$= \left\{\frac{1}{2}\left[4(0.03)^2\right]\left(\frac{v_2}{0.05}\right)^2\right\} - 6(9.81)(0.6)$$

$$v_2 = 3.46 \text{ m/s} \qquad \textit{Ans.}$$

R18–6. Datum through A.

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}\left[\frac{1}{3}(25)(3)^2\right](2)^2 + \frac{1}{2}(90)(2 - 1)^2$$
$$= \frac{1}{2}\left[\frac{1}{3}(25)(3)^2\right](\omega^2) + \frac{1}{2}(90)(3.5 - 1)^2$$
$$- 25(9.81)(1.5 \sin 30°)$$

$$\omega = 1.61 \text{ rad/s} \qquad \textit{Ans.}$$

R18–7.

$$T_1 + V_1 = T_2 + V_2$$

At $\theta = 0°$, $v_B = 0$ and $\omega_{BC} = v_C$

$$0 + [2(9.81)(0.5 \sin 45°) + [0.5(9.81)](1 \sin 45°)$$
$$+ \frac{1}{2}\left[\frac{1}{3}(2)(1^2)\right]v_C^2 + \frac{1}{2}(0.5)v_C^2 + 0$$

$$v_C = 4.22 \text{ m/s} \qquad \textit{Ans.}$$

R18–8. Datum at lowest point.

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}\left[\frac{1}{2}(40)(0.3)^2\right]\left(\frac{4}{0.3}\right)^2 + \frac{1}{2}(40)(4)^2$$
$$+ 40(9.81)d \sin 30° = 0 + \frac{1}{2}(200)d^2$$

$$100d^2 - 196.2d - 480 = 0$$

Solving for the positive root,

$$d = 3.38 \text{ m} \qquad \textit{Ans.}$$

Chapter 19

R19–1. $I_O = mk_O^2 = 75(0.375^2) = 10.547 \text{ kg} \cdot \text{m}^2$

$$I_O\omega_1 + \Sigma \int_{t_1}^{t_2} M_O dt = I_O\omega_2$$

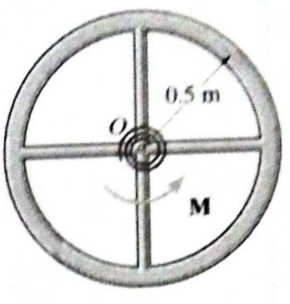
$$0 + \int_0^{3\text{ s}} (50t^2)(0.3)\, dt = 10.547\omega_2$$

$$3t^3\Big|_0^{3\text{ s}} = 10.547\omega_2$$

$$\omega_2 = 7.68 \text{ rad/s} \qquad \textit{Ans.}$$

18–2.

The wheel is made from a 5-kg thin ring and two 2-kg slender rods. If the torsional spring attached to the wheel's center has a stiffness $k = 2\ \text{N}\cdot\text{m/rad}$, and the wheel is rotated until the torque $M = 25\ \text{N}\cdot\text{m}$ is developed, determine the maximum angular velocity of the wheel if it is released from rest.



SOLUTION

Kinetic Energy and Work: The mass moment of inertia of the wheel about point O is

$$I_O = m_R r^2 + 2\left(\frac{1}{12} m_r l^2\right)$$

$$= 5(0.5^2) + 2\left[\frac{1}{12}(2)(1^2)\right]$$

$$= 1.5833\ \text{kg}\cdot\text{m}^2$$

Thus, the kinetic energy of the wheel is

$$T = \frac{1}{2} I_O \omega^2 = \frac{1}{2}(1.5833)\,\omega^2 = 0.79167\,\omega^2$$

Since the wheel is released from rest, $T_1 = 0$. The torque developed is $M = k\theta = 2\theta$. Here, the angle of rotation needed to develop a torque of $M = 25\ \text{N}\cdot\text{m}$ is

$$2\theta = 25 \qquad \theta = 12.5\ \text{rad}$$

The wheel achieves its maximum angular velocity when the spacing is unwound that is when the wheel has rotated $\theta = 12.5$ rad. Thus, the work done by $\underset{\infty}{\mathbf{M}}$ is

$$U_M = \int M\,d\theta = \int_0^{12.5\ \text{rad}} 2\theta\ d\theta$$

$$= \theta^2\Big|_0^{12.5\ \text{rad}} = 156.25\ \text{J}$$

Principle of Work and Energy:

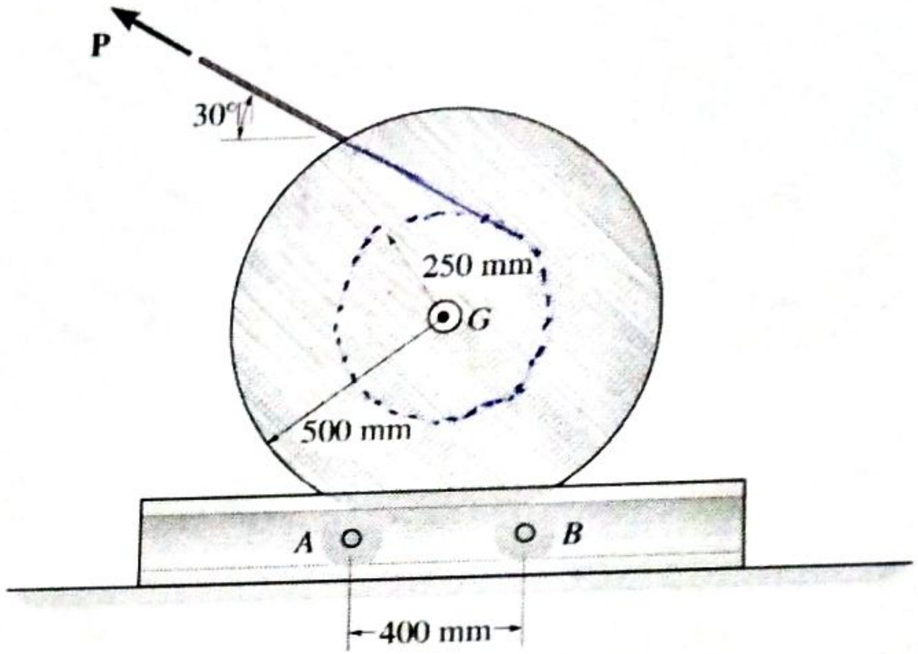
$$T_1 + \Sigma u_{1-2} = T_2$$

$$0 + 156.25 = 0.79167\,\omega^2$$

$$\omega = 14.0\ \text{rad/s}$$ **Ans.**

18–5.

A force of $P = 20$ N is applied to the cable, which causes the 175-kg reel to turn since it is resting on the two rollers A and B of the dispenser. Determine the angular velocity of the reel after it has made two revolutions starting from rest. Neglect the mass of the rollers and the mass of the cable. The radius of gyration of the reel about its center axis is $k_G = 0.42$ m.

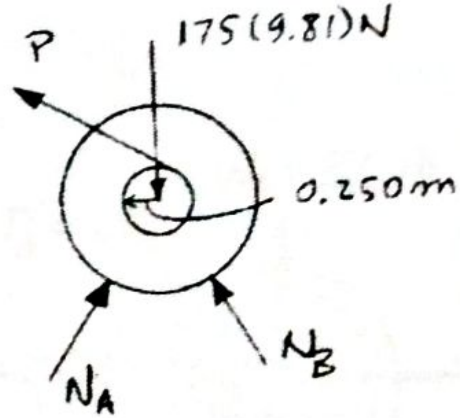


SOLUTION

$$T_1 + \Sigma U_{1-2} = T_2$$

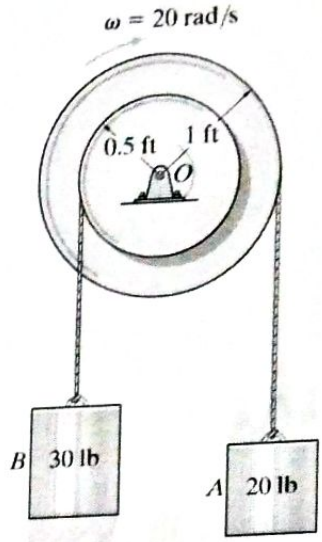
$$0 + 20(2)(2\pi)(0.250) = \frac{1}{2}\left[175(0.42)^2\right]\omega^2$$

$\omega = 2.02$ rad/s **Ans.**



***18–8.**

The double pulley consists of two parts that are attached to one another. It has a weight of 50 lb and a centroidal radius of gyration of $k_O = 0.6$ ft and is turning with an angular velocity of 20 rad/s clockwise. Determine the angular velocity of the pulley at the instant the 20-lb weight moves 2 ft downward.



SOLUTION

Kinetic Energy and Work: Since the pulley rotates about a fixed axis, $v_A = \omega r_A = \omega(1)$ and $v_B = \omega r_B = \omega(0.5)$. The mass moment of inertia of the pulley about point O is $I_O = mk_O{}^2 = \left(\frac{50}{32.2}\right)(0.6^2) = 0.5590$ slug · ft². Thus, the kinetic energy of the system is

$$T = \frac{1}{2}I_O\omega^2 + \frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B v_B{}^2$$

$$= \frac{1}{2}(0.5590)\omega^2 + \frac{1}{2}\left(\frac{20}{32.2}\right)[\omega(1)]^2 + \frac{1}{2}\left(\frac{30}{32.2}\right)[\omega(0.5)]^2$$

$$= 0.7065\omega^2$$

Thus, $T_1 = 0.7065(20^2) = 282.61$ ft · lb. Referring to the FBD of the system shown in Fig. *a*, we notice that $\mathbf{O}_x$, $\mathbf{O}_y$, and $\mathbf{W}_p$ do no work while $\mathbf{W}_A$ does positive work and $\mathbf{W}_B$ does negative work. When A moves 2 ft downward, the pulley rotates

$$\theta = \frac{S_A}{r_A} = \frac{S_B}{r_B}$$

$$\frac{2}{1} = \frac{S_B}{0.5}$$

$$S_B = 2(0.5) = 1 \text{ ft} \uparrow$$

Thus, the work of $\mathbf{W}_A$ and $\mathbf{W}_B$ are

$$U_{W_A} = W_A S_A = 20(2) = 40 \text{ ft} \cdot \text{lb}$$

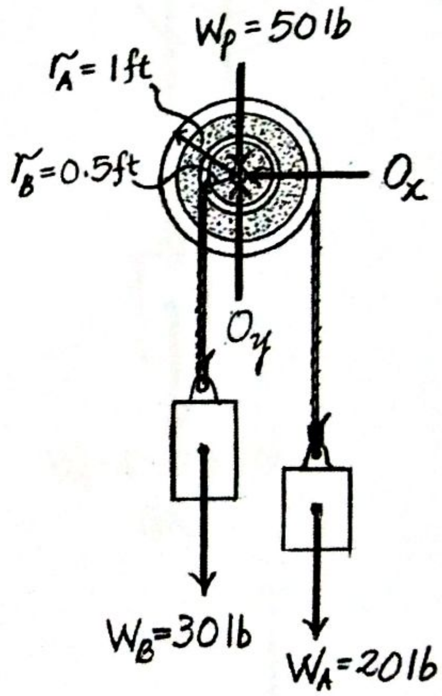
$$U_{W_B} = -W_B S_B = -30(1) = -30 \text{ ft} \cdot \text{lb}$$

Principle of Work and Energy:

$$T_1 + U_{1-2} = T_2$$

$$282.61 + [40 + (-30)] = 0.7065\,\omega^2$$

$$\omega = 20.4 \text{ rad/s}$$ **Ans.**



(a)

18–10.

The spool has a mass of 40 kg and a radius of gyration of $k_O = 0.3$ m. If the 10-kg block is released from rest, determine the distance the block must fall in order for the spool to have an angular velocity $\omega = 15$ rad/s. Also, what is the tension in the cord while the block is in motion? Neglect the mass of the cord.

SOLUTION

Kinetic Energy. Since the system is released from rest, $T_1 = 0$. The final velocity of the block is $v_b = \omega r = 15(0.3) = 4.50$ m/s. The mass moment of inertia of the spool about O is $I_0 = mk_0^2 = 40(0.3^2) = 3.60$ Kg · m². Thus

$$T_2 = \frac{1}{2}I_0\omega^2 + \frac{1}{2}m_b v_b^2$$

$$= \frac{1}{2}(3.60)(15^2) + \frac{1}{2}(10)(4.50^2)$$

$$= 506.25 \text{ J}$$

For the block, $T_1 = 0$ and $T_2 = \frac{1}{2}m_b v_b^2 = \frac{1}{2}(10)(4.50^2) = 101.25$ J

Work. Referring to the FBD of the system Fig. *a*, only $\mathbf{W}_b$ does work when the block displaces s vertically downward, which it is positive.

$$U_{W_b} = W_b s = 10(9.81)s = 98.1\,s$$

Referring to the FBD of the block, Fig. *b*. $\mathbf{W}_b$ does positive work while $\mathbf{T}$ does negative work.

$$U_T = -Ts$$

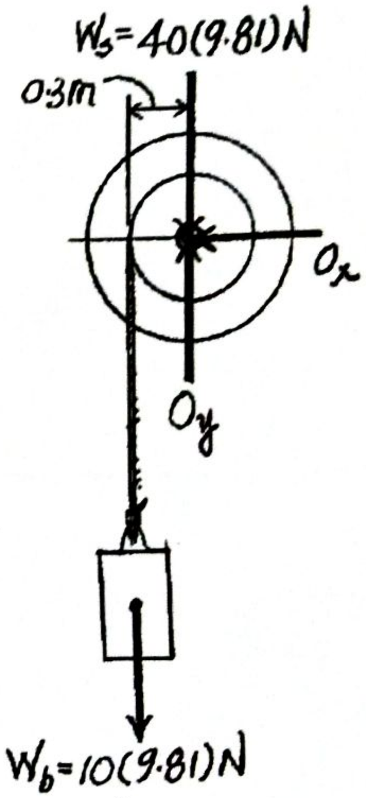
$$U_{W_b} = W_b s = 10(9.81)(s) = 98.1\,s$$

Principle of Work and Energy. For the system,

$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + 98.1s = 506.25$$

$$s = 5.1606 \text{ m} = 5.16 \text{ m}$$ **Ans.**



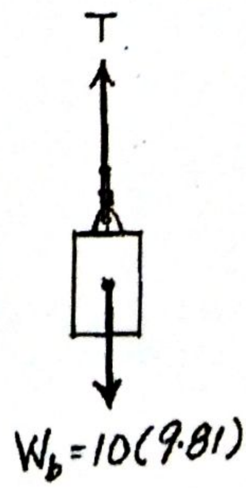
(a)

For the block using the result of s,

$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + 98.1(5.1606) - T(5.1606) = 101.25$$

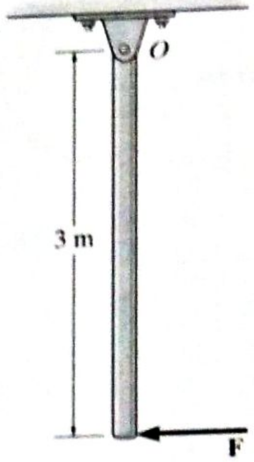
$$T = 78.48 \text{ N} = 78.5 \text{ N}$$ **Ans.**



(b)

18–13.

The 10-kg uniform slender rod is suspended at rest when the force of $F = 150$ N is applied to its end. Determine the angular velocity of the rod when it has rotated 90° clockwise from the position shown. The force is always perpendicular to the rod.



SOLUTION

Kinetic Energy. Since the rod starts from rest, $T_1 = 0$. The mass moment of inertia of the rod about O is $I_0 = \frac{1}{12}(10)(3^2) + 10(1.5^2) = 30.0\ \text{kg}\cdot\text{m}^2$. Thus,

$$T_2 = \frac{1}{2}I_0\omega^2 = \frac{1}{2}(30.0)\omega^2 = 15.0\,\omega^2$$

Work. Referring to the FBD of the rod, Fig. *a*, when the rod undergoes an angular displacement θ, force **F** does positive work whereas **W** does negative work. When $\theta = 90°$, $S_W = 1.5$ m and $S_F = \theta r = \left(\frac{\pi}{2}\right)(3) = \frac{3\pi}{2}$ m. Thus

$$U_F = 150\left(\frac{3\pi}{2}\right) = 225\pi\ \text{J}$$

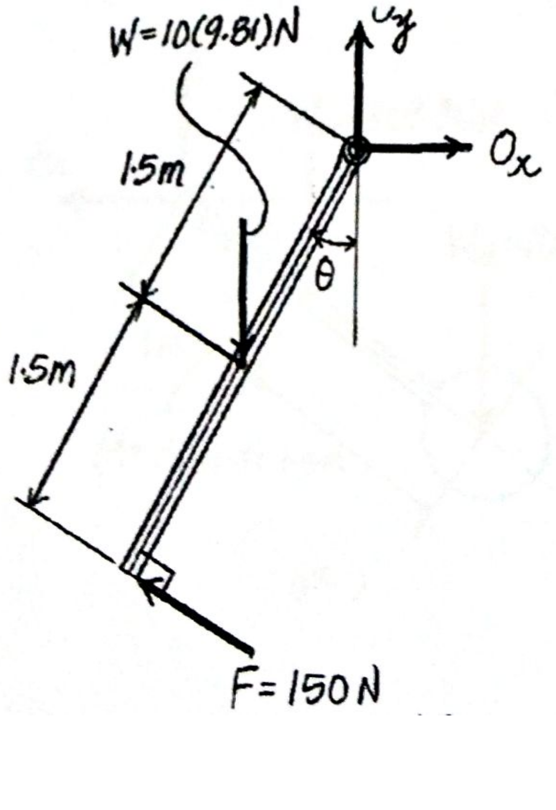
$$U_W = -10(9.81)(1.5) = -147.15\ \text{J}$$

Principle of Work and Energy.

$$T_1 + \Sigma U_{1-2} = T_2$$

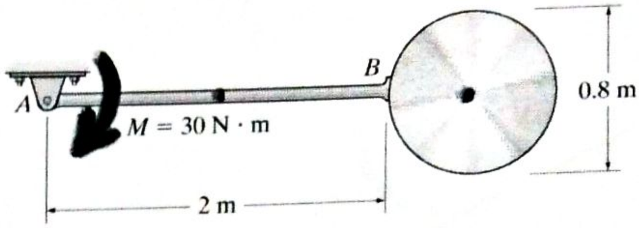
$$0 + 225\pi + (-147.15) = 15.0\,\omega^2$$

$$\omega = 6.1085\ \text{rad/s} = 6.11\ \text{rad/s}$$ **Ans.**



18–15.

The pendulum consists of a 10-kg uniform disk and a 3-kg uniform slender rod. If it is released from rest in the position shown, determine its angular velocity when it rotates clockwise 90°.



SOLUTION

Kinetic Energy. Since the assembly is released from rest, initially, $T_1 = 0$. The mass moment of inertia of the assembly about A is

$$I_A = \left[\frac{1}{12}(3)(2^2) + 3(1^2)\right] + \left[\frac{1}{2}(10)(0.4^2) + 10(2.4^2)\right] = 62.4\ \text{kg}\cdot\text{m}^2. \text{Thus,}$$

$$T_2 = \frac{1}{2}I_A\omega^2 = \frac{1}{2}(62.4)\omega^2 = 31.2\ \omega^2$$

Work. Referring to the FBD of the assembly, Fig. *a*. Both $\mathbf{W}_r$ and $\mathbf{W}_d$ do positive work, since they displace vertically downward $S_r = 1$ m and $S_d = 2.4$ m, respectively. Also, couple moment **M** does positive work

$$U_{W_r} = W_r S_r = 3(9.81)(1) = 29.43\ \text{J}$$

$$U_{W_d} = W_d S_d = 10(9.81)(2.4) = 235.44\ \text{J}$$

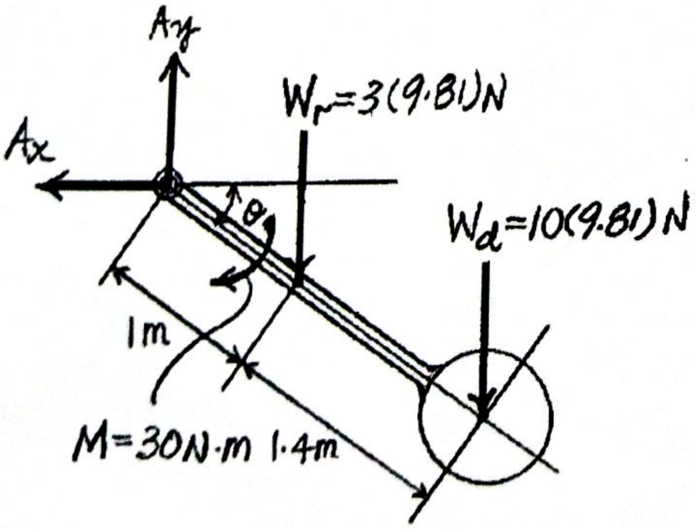
$$U_M = M\theta = 30\left(\frac{\pi}{2}\right) = 15\pi\ \text{J}$$

Principle of Work and Energy.

$$T_1 + \Sigma U_{1-2} = T_2$$

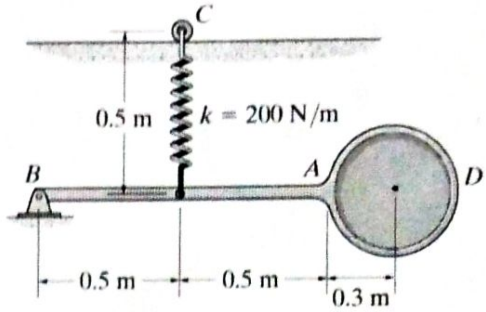
$$0 + 29.43 + 235.44 + 15\pi = 31.2\ \omega^2$$

$$\omega = 3.1622\ \text{rad/s} = 3.16\ \text{rad/s} \qquad \textbf{Ans.}$$



***18–60.**

The pendulum consists of a 6-kg slender rod fixed to a 15-kg disk. If the spring has an unstretched length of 0.2 m, determine the angular velocity of the pendulum when it is released from rest and rotates clockwise 90° from the position shown. The roller at C allows the spring to always remain vertical.



SOLUTION

Kinetic Energy. The mass moment of inertia of the pendulum about B is $I_B = \left[\frac{1}{12}(6)(1^2) + 6(0.5^2)\right] + \left[\frac{1}{2}(15)(0.3^2) + 15(1.3^2)\right] = 28.025\ \text{kg}\cdot\text{m}^2$. Thus

$$T = \frac{1}{2}I_B\,\omega^2 = \frac{1}{2}(28.025)\,\omega^2 = 14.0125\,\omega^2$$

Since the pendulum is released from rest, $T_1 = 0$.

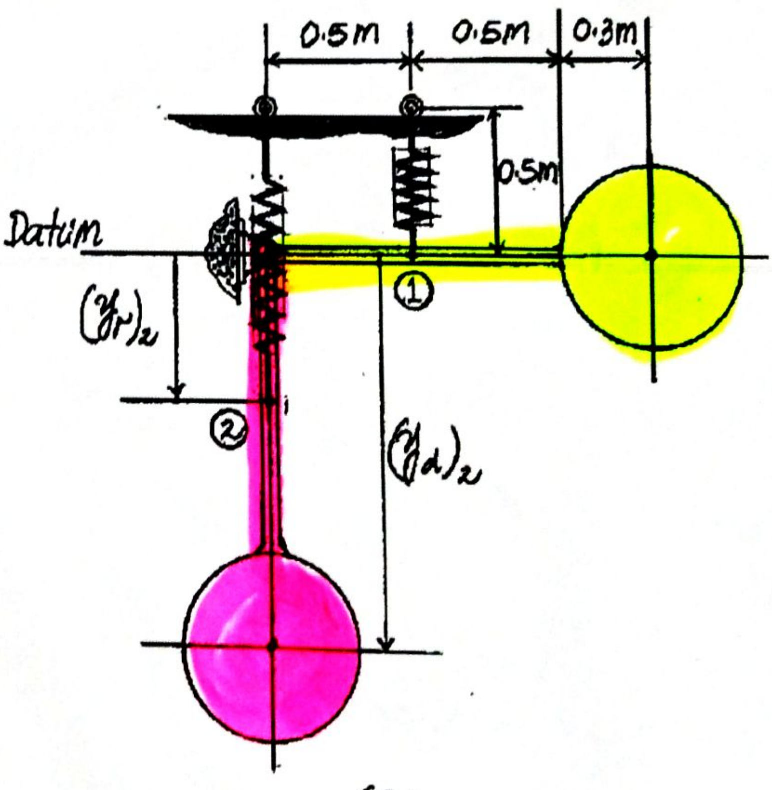
Potential Energy. with reference to the datum set in Fig. *a*, the gravitational potential energies of the pendulum when it is at positions ① and ② are

$$(V_g)_1 = m_r g(y_r)_1 + m_d g(y_d)_1 = 0$$

$$(V_g)_2 = m_r g(y_r)_2 + m_d g(y_d)_2$$

$$= 6(9.81)(-0.5) + 15(9.81)(-1.3)$$

$$= -220.725\ \text{J}$$



(a)

The stretch of the spring when the pendulum is at positions ① and ② are

$$x_1 = 0.5 - 0.2 = 0.3\ \text{m}$$

$$x_2 = 1 - 0.2 = 0.8\ \text{m}$$

Thus, the initial and final elastic potential energies of the spring are

$$(V_e)_1 = \frac{1}{2}kx_1^2 = \frac{1}{2}(200)(0.3^2) = 9.00\ \text{J}$$

$$(V_e)_2 = \frac{1}{2}kx_2^2 = \frac{1}{2}(200)(0.8^2) = 64.0\ \text{J}$$

Conservation of Energy.

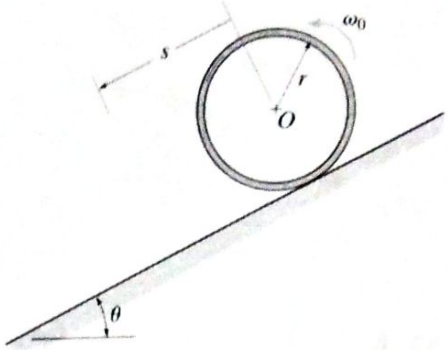
$$T_1 + V_1 = T_2 + V_2$$

$$0 + (0 + 9.00) = 14.0125\omega^2 + (-220.725) + 64.0$$

$$\omega = 3.4390\ \text{rad/s} = 3.44\ \text{rad/s}$$ **Ans.**

18–17.

The center O of the thin ring of mass m is given an angular velocity of ω_0. If the ring rolls without slipping, determine its angular velocity after it has traveled a distance of s down the plane. Neglect its thickness.

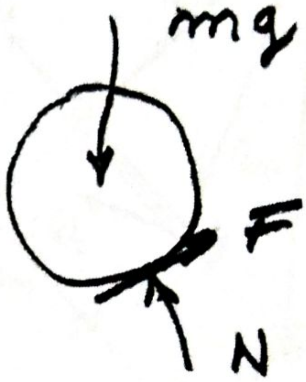


SOLUTION

$$T_1 + \Sigma U_{1-2} = T_2$$

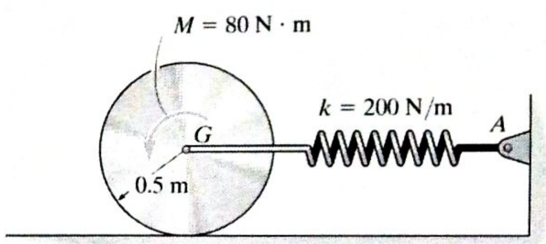
$$\frac{1}{2}(mr^2 + mr^2)\omega_0^2 + mg(s \sin\theta) = \frac{1}{2}(mr^2 + mr^2)\omega^2$$

$$\omega = \sqrt{\omega_0^2 + \frac{g}{r^2} s \sin\theta} \qquad \textbf{Ans.}$$



18–25.

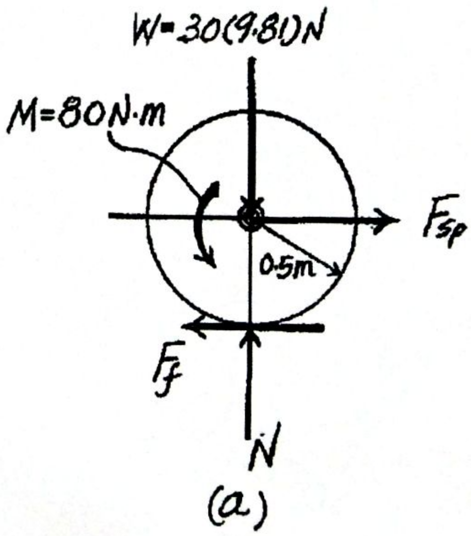
The 30-kg disk is originally at rest, and the spring is unstretched. A couple moment $M = 80\ \text{N}\cdot\text{m}$ is then applied to the disk as shown. Determine how far the center of mass of the disk travels along the plane before it momentarily stops. The disk rolls without slipping.



SOLUTION

Kinetic Energy. Since the disk is at rest initially and required to stop finally, $T_1 = T_2 = 0$.

Work. Since the disk rolls without slipping, the friction $\mathbf{F}_f$ does no work. Also, when the center of the disk moves s_G, the disk rotates $\theta = \frac{s_G}{r} = \frac{s_G}{0.5} = 2\,s_G$. Here, couple moment $\mathbf{M}$ does positive work whereas the spring force does negative work.



$$U_M = M\theta = 80(2s_G) = 160\,s_G$$

$$U_{F_{sp}} = -\frac{1}{2}kx^2 = -\frac{1}{2}(200)\,s_G^2 = -100\,s_G^2$$

Principle of Work and Energy.

$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + 160\,s_G + \left(-100\,s_G^2\right) = 0$$

$$160\,s_G - 100\,s_G^2 = 0$$

$$s_G(160 - 100\,s_G) = 0$$

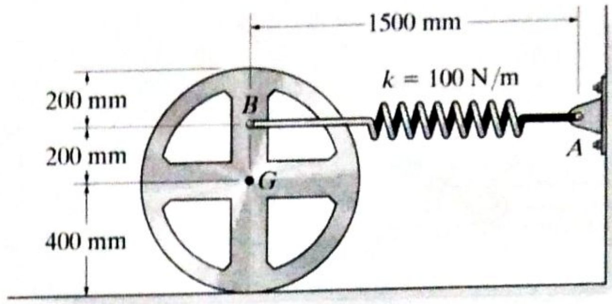
Since $s_G \neq 0$, then

$$160 - 100\,s_G = 0$$

$$s_G = 1.60\ \text{m} \qquad \textbf{Ans.}$$

18–47.

The 40-kg wheel has a radius of gyration about its center of gravity G of $k_G = 250$ mm. If it rolls without slipping, determine its angular velocity when it has rotated clockwise 90° from the position shown. The spring AB has a stiffness $k = 100$ N/m and an unstretched length of 500 mm. The wheel is released from rest.



SOLUTION

Kinetic Energy. The mass moment of inertia of the wheel about its center of mass G is $I_G = mk_G^2 = 40(0.25^2) = 2.50\ \text{kg}\cdot\text{m}^2$, since the wheel rolls without slipping, $v_G = \omega r_G = \omega(0.4)$. Thus

$$T = \frac{1}{2}I_G\,\omega^2 + \frac{1}{2}mv_G^2$$

$$= \frac{1}{2}(2.50)\omega^2 + \frac{1}{2}(40)[\omega(0.4)]^2 = 4.45\,\omega^2$$

Since the wheel is released from rest, $T_1 = 0$.

Potential Energy. When the wheel rotates 90° clockwise from position ① to ②, Fig. *a*, its mass center displaces $S_G = \theta r_G = \frac{\pi}{2}(0.4) = 0.2\pi$ m. Then $x' = 1.5 - 0.2 - 0.2\pi = 0.6717$ m. The stretches of the spring when the wheel is at positions ① and ② are

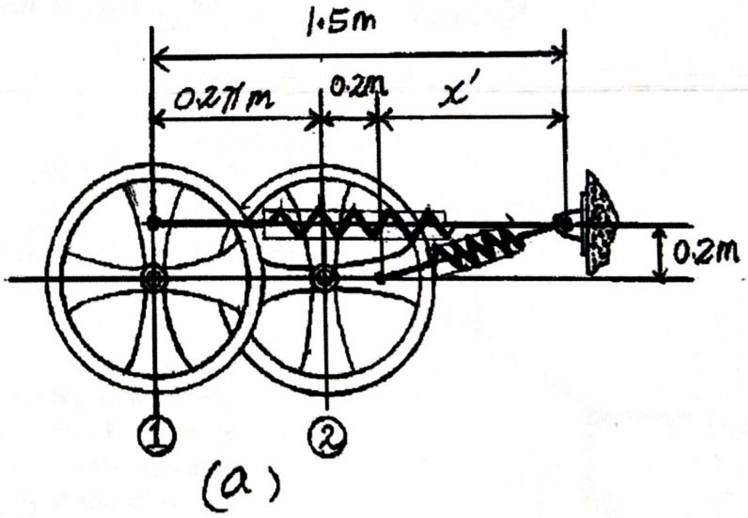
$$x_1 = 1.50 - 0.5 = 1.00\text{ m}$$

$$x_2 = \sqrt{0.6717^2 + 0.2^2} - 0.5 = 0.2008\text{ m}$$

Thus, the initial and final elastic potential energies are

$$(V_e)_1 = \frac{1}{2}kx_1^2 = \frac{1}{2}(100)(1^2) = 50\text{ J}$$

$$(V_e)_2 = \frac{1}{2}kx_2^2 = \frac{1}{2}(100)(0.2008^2) = 2.0165\text{ J}$$



Conservation of Energy.

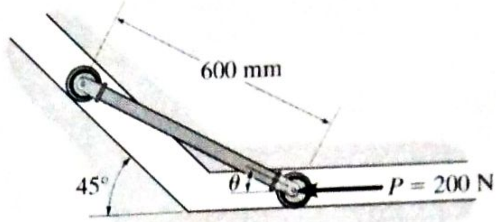
$$T_1 + V_1 = T_2 + V_2$$

$$0 + 50 = 4.45\omega^2 + 2.0165$$

$$\omega = 3.2837\text{ rad/s} = 3.28\text{ rad/s}$$ **Ans.**

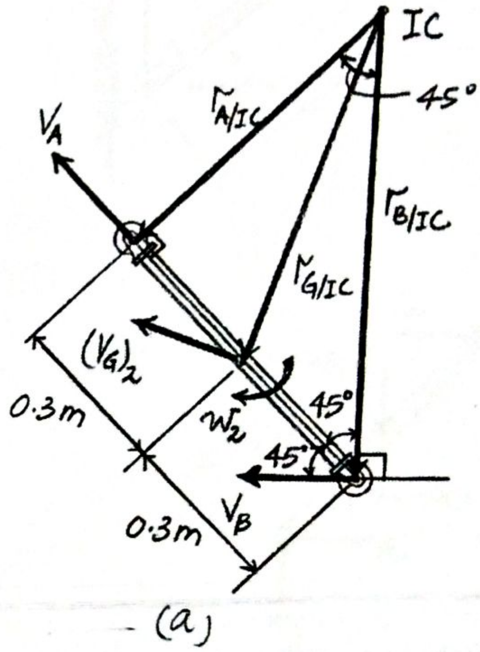
***18–20.**

If $P = 200$ N and the 15-kg uniform slender rod starts from rest at $\theta = 0°$, determine the rod's angular velocity at the instant just before $\theta = 45°$.



SOLUTION

Kinetic Energy and Work: Referring to Fig. *a*,



$$r_{A/IC} = 0.6 \tan 45° = 0.6 \text{ m}$$

Then

$$r_{G/IC} = \sqrt{0.3^2 + 0.6^2} = 0.6708 \text{ m}$$

Thus,

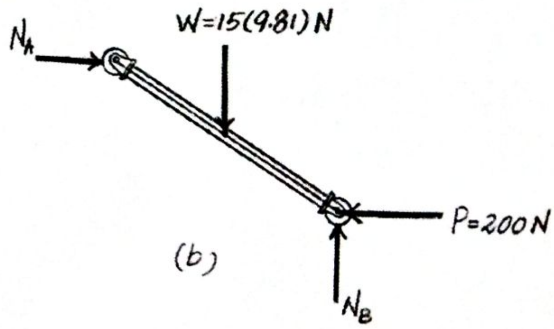
$$(v_G)_2 = \omega_2 r_{G/IC} = \omega_2(0.6708)$$

The mass moment of inertia of the rod about its mass center is $I_G = \frac{1}{12}ml^2 = \frac{1}{12}(15)(0.6^2) = 0.45 \text{ kg}\cdot\text{m}^2$. Thus, the final kinetic energy is

$$T_2 = \frac{1}{2}m(v_G)_2^2 + \frac{1}{2}I_G\omega_2^2$$

$$= \frac{1}{2}(15)[\omega_2(0.6708)]^2 + \frac{1}{2}(0.45)\,\omega_2^2$$

$$= 3.6\omega_2^2$$

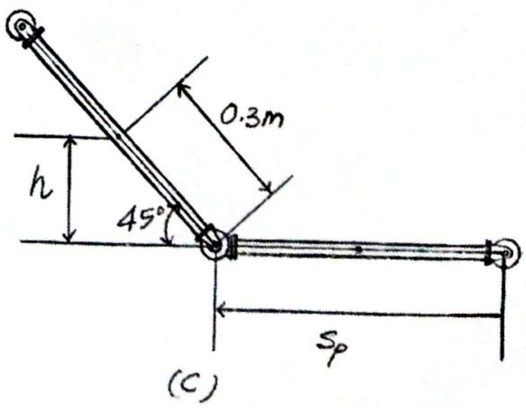


Since the rod is initially at rest, $T_1 = 0$. Referring to Fig. *b*, $\mathbf{N}_A$ and $\mathbf{N}_B$ do no work, while **P** does positive work and **W** does negative work. When $\theta = 45°$, **P** displaces through a horizontal distance $s_P = 0.6$ m and W displaces vertically upwards through a distance of $h = 0.3 \sin 45°$, Fig. *c*. Thus, the work done by **P** and **W** is

$$U_P = Ps_P = 200(0.6) = 120 \text{ J}$$

$$U_W = -Wh = -15(9.81)(0.3 \sin 45°) = -31.22 \text{ J}$$

Principle of Work and Energy:



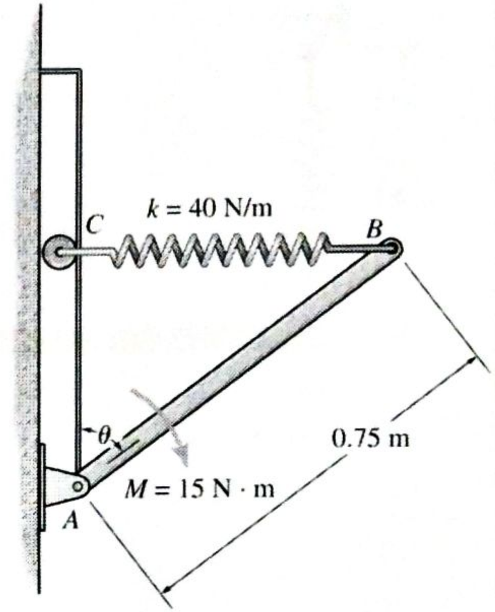
$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + [120 - 31.22] = 3.6\omega_2^2$$

$$\omega_2 = 4.97 \text{ rad/s}$$ **Ans.**

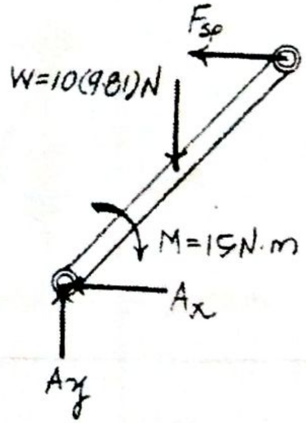
*18–28.

The 10-kg rod AB is pin-connected at A and subjected to a couple moment of $M = 15$ N·m. If the rod is released from rest when the spring is unstretched at $\theta = 30°$, determine the rod's angular velocity at the instant $\theta = 60°$. As the rod rotates, the spring always remains horizontal, because of the roller support at C.



SOLUTION

Free Body Diagram: The spring force F_{sp} does *negative* work since it acts in the opposite direction to that of its displacement s_{sp}, whereas the weight of the cylinder acts in the same direction of its displacement s_w and hence does *positive* work. Also, the couple moment **M** does positive work as it acts in the same direction of its angular displacement θ. The reactions A_x and A_y do no work since point A does not displace. Here, $s_{sp} = 0.75 \sin 60° - 0.75 \sin 30° = 0.2745$ m and $s_W = 0.375 \cos 30° - 0.375 \cos 60° = 0.1373$ m.



Principle of Work and Energy: The mass moment of inertia of the cylinder about point A is $I_A = \frac{1}{12}ml^2 + md^2 = \frac{1}{12}(10)(0.75^2) + 10(0.375^2) = 1.875 \text{ kg} \cdot \text{m}^2$. Applying Eq.18–13, we have

$$T_1 + \sum U_{1-2} = T_2$$

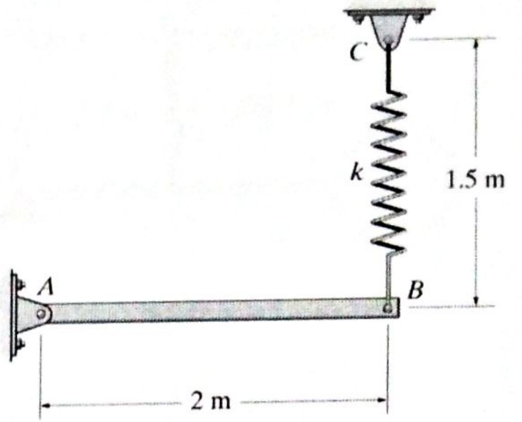
$$0 + Ws_W + M\theta - \frac{1}{2}ks_P^2 = \frac{1}{2}I_A\,\omega^2$$

$$0 + 10(9.81)(0.1373) + 15\left(\frac{\pi}{3} - \frac{\pi}{6}\right) - \frac{1}{2}(40)(0.2745^2) = \frac{1}{2}(1.875)\,\omega^2$$

$$\omega = 4.60 \text{ rad/s} \qquad \textbf{Ans.}$$

18–58.

The slender 6-kg bar AB is horizontal and at rest and the spring is unstretched. Determine the stiffness k of the spring so that the motion of the bar is momentarily stopped when it has rotated clockwise 90° after being released.



SOLUTION

Kinetic Energy. The mass moment of inertia of the bar about A is $I_A = \frac{1}{12}(6)(2^2) + 6(1^2) = 8.00\ \text{kg}\cdot\text{m}^2$. Then

$$T = \frac{1}{2}I_A\,\omega^2 = \frac{1}{2}(8.00)\,\omega^2 = 4.00\,\omega^2$$

Since the bar is at rest initially and required to stop finally, $T_1 = T_2 = 0$.

Potential Energy. With reference to the datum set in Fig. *a*, the gravitational potential energies of the bar when it is at positions ① and ② are

$$(V_g)_1 = mgy_1 = 0$$

$$(V_g)_2 = mgy_2 = 6(9.81)(-1) = -58.86\ \text{J}$$

The stretch of the spring when the bar is at position ② is

$$x_2 = \sqrt{2^2 + 3.5^2} - 1.5 = 2.5311\ \text{m}$$

Thus, the initial and final elastic potential energy of the spring are

$$(V_e)_1 = \frac{1}{2}kx_1^2 = 0$$

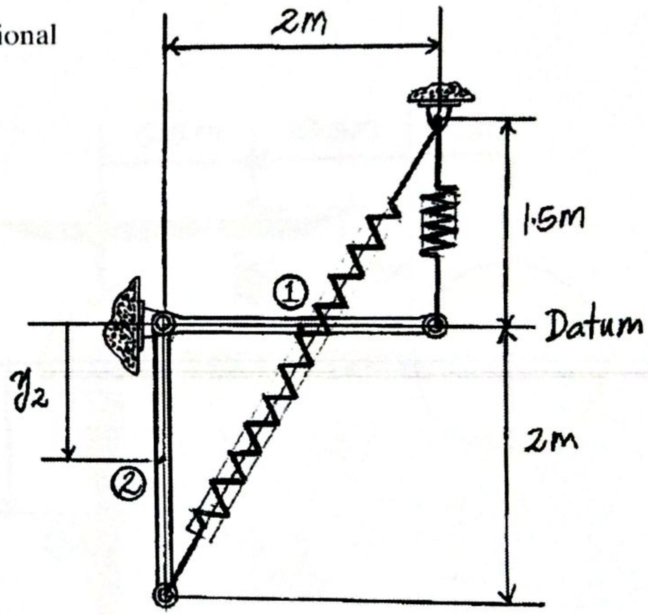
$$(V_e)_2 = \frac{1}{2}k(2.5311^2) = 3.2033k$$

Conservation of Energy.

$$T_1 + V_1 = T_2 + V_2$$

$$0 + (0 + 0) = 0 + (-58.86) + 3.2033k$$

$$k = 18.3748\ \text{N/m} = 18.4\ \text{N/m}$$ **Ans.**



(a)