Chapter 12

* $V_{avg} = \frac{\Delta s}{\Delta t}$, $V_{ins} = \frac{ds}{dt}$

* $a_{avg} = \frac{\Delta v}{\Delta t}$, $a_{ins} = \frac{dv}{dt} = \frac{d^2s}{dt^2}$

* $\frac{v}{a} = \frac{ds}{dt} * \frac{dt}{dv} \rightarrow \frac{v}{a} = \frac{ds}{dv} \rightarrow ads = vdv$

* $ads = vdv$ → we can use it when there is no function of time.

** Constant acceleration equations:

[1] $v = v_0 + at \rightarrow v = v_0 - gt$

[2] $v^2 = v_0^2 + 2a\Delta s \rightarrow v^2 = v_0^2 - 2g\Delta y$ * $a = g = 9.81\, m/s^2$

[3] $\Delta s = v_0t + \frac{1}{2}at^2 \rightarrow \Delta y = v_0t - \frac{1}{2}gt^2$

* إذا كانت السرعة تزيد ← التسارع بنفس اتجاه الحركة.
* إذا كانت السرعة تقل ← التسارع بعكس اتجاهها.

* إذا تحرك جسم نحو اليمين وسرعته تزداد ← تسارعه موجب
* إذا تحرك جسم نحو اليمين وسرعته تقل ← تسارعه سالب

Ex: A particle moves on a straight line according to:

$$S = 2t^3 - 24t + 6$$

[1] The time required for the particle to reach a velocity of 72 m/s. sol → $v = \frac{ds}{dt} = 6t^2 - 24$

$$72 = 6t^2 - 24$$

$$t^2 = 16$$

$$\boxed{t = +4s}$$

[2] The acceleration when $V = 30 m/s$

$$V = 6t^2 - 24$$
$$30 = 6t^2 - 24$$
$t = 3 sec$
when $V = 30$

Sol $\rightarrow$ $a = \frac{dv}{dt} = 12t$

$a = 12(3) = 36 m/s^2$

[3] The displacement during $t = 1 sec$ and $t = 4 sec$.

Sol $\rightarrow$ $S_1 = 2(1)^3 - 24(1) + 6 \Rightarrow S_1 = -16$

$S_2 = 2(4)^3 - 24(4) + 6 \Rightarrow S_2 = 38$

$\Delta S = S_2 - S_1 = 38 + 16 = 54 m$

Ex: The velocity of the particle $V = 6t^2 - 8t + 2$

[1] Distance after 5 seconds

Sol $\rightarrow$ $S = \int_{t_1}^{t_2} v\,dt \rightarrow S = \int_0^5 (6t^2 - 8t + 2)\,dt$

$S = 2t^3 - 4t^2 + 2t \Big]_0^5 \rightarrow S = 160 m$

[2] Acceleration after 2 seconds.

Sol $\rightarrow$ $a = \frac{dv}{dt} = 12t - 8 \rightarrow a = 12(2) - 8 = 16 m/s^2$

مسافة $y = 2x^3 + 4x^2 - 2$ — $x \rightarrow$ المسافة مع x .

سرعة $\dot{y} = 6x^2\dot{x} + 8x\dot{x}$ — $\dot{x} \rightarrow$ السرعة باتجاه x .

~~تسارع~~ $\cancel{\ddot{y} = 12x\dot{x}\dot{x} + 8\dot{x}\dot{x}}$ — $\ddot{x} \rightarrow$ التسارع باتجاه x .

تسارع $\ddot{y} = 12x\dot{x}\dot{x} + 6x^2\ddot{x} + 8\dot{x}\dot{x} + 8x\ddot{x}$

تسارع $\ddot{y} = 12x(\dot{x})^2 + 6x^2\ddot{x} + 8(\dot{x})^2 + 8x\ddot{x}$

** إذا لم يعطى الزمن بالاشتقاق نشتق الاشتقاق ضمنيا **

** عند رمي جسم لأعلى نعوض حركة موجبة وإذا رمي للأسفل نعوض حركة سالبة.

** عند رمي جسم نحتاج إلى reference اذا رمي الجسم فوقه فالحركة (+) وعنده (0) وتحته (-).

** دائماً عند أقصى ارتفاع تكون حركة الجسم Zero

Example:

[1] The highest elevation.

Sol ⇒ $v^2 = v_0^2 - 2g\Delta y$

$0 = 100 - 2(9.81)(y_2 - 0)$

$y_2 = 5.1$

2] the time when the ball hits the ground.

→ $\Delta y = v_0 t - \frac{1}{2} g t^2$

$-20 = 10t - 4.905t^2$

~~$-20 = 5.1t$~~

$t = 3.28\,sec$ $t = -1.24$ نهمله

3] The velocity when hits the ground.

option 1

$V = v_0 - gt$

$V = -10 - 9.81(3.28)$

$V = -22.2\,m/s$

اتجاه للأسفل

option 2

$v^2 = v_0^2 - 2g\Delta y$

$v^2 = 100 - 2(9.81)(-20)$

$v^2 = 492.4$

$V = \pm 22.2 \rightarrow V = -22.2\,m/s$

لا نعوض (+) لأن اتجاه، يسقط للأسفل

** Velocity max happend when the particle hits the ground.

4] The velocity when the ball is at 5m above the ground.

$v^2 = v_0^2 - 2g\Delta y$

$v^2 = 100 - 2(9.81)(-14)$

$V = -19.35\,m/s$

اتجاه للأسفل

Ex: $\vec{v} = (16t^2)\hat{i} + (4t^3)\hat{j} + (5t+2)\hat{k}$

[1] $\vec{a}(2) = ?$ $\rightarrow \vec{a} = \frac{d\vec{v}}{dt} = (32t)\hat{i} + (12t^2)\hat{j} + 5\hat{k}$

[2] $\vec{r}(2) = ?$ $\qquad \vec{a}(2) = 64\hat{i} + 48\hat{j} + 5\hat{k}$

$|\vec{a}(2)| = 80.2 \, m/s^2$

[2] $\vec{r} = \int_0^2 \vec{v}\,dt \rightarrow \vec{r} = \int_0^2 [(16t^2)\hat{i} + (4t^3)\hat{j} + (5t+2)\hat{k}]\,dt$

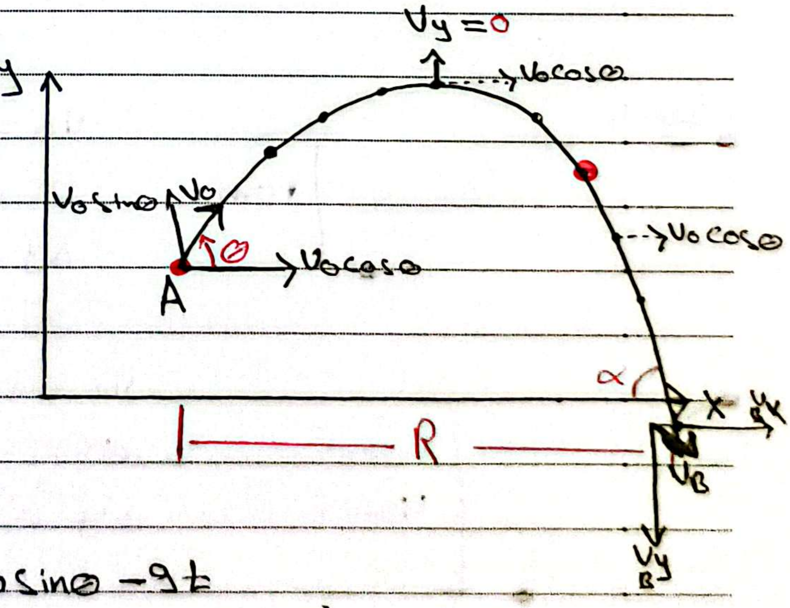
$\vec{r} = (\frac{16}{3}t^3)\hat{i} + (t^4)\hat{j} + (\frac{5}{2}t^2 + 2t)\hat{k} \Big]_0^2$

$\vec{r} = \frac{16}{3}(2)^3\hat{i} + 2^4\hat{j} + (\frac{5}{2}(2)^2 + 2(2))\hat{k}$

$\vec{r} = \frac{128}{3}\hat{i} + 16\hat{j} + 14\hat{k}$

$|\vec{r}| = 47.67 \, m$

~~**~~ Projectile Motion:



$R = V_x t = (V_o \cos\theta)t$

$\tan\alpha = \frac{V_y}{V_x} = \frac{V_y}{V_o\cos\theta}$

$V_y = V_{oy} - gt \rightarrow V_y = V_o\sin\theta - gt$

$\Delta y = V_{oy}t - \frac{1}{2}gt^2 \rightarrow \Delta y = (V_o\sin\theta)t - \frac{1}{2}gt^2$

$V_y^2 = V_{oy}^2 - 2g\Delta y \rightarrow V_y^2 = (V_o\sin\theta)^2 - 2g\Delta y$

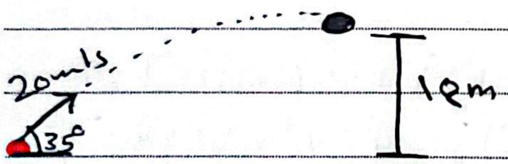
final Velocity $\rightarrow V_B = \sqrt{V_x^2 + V_y^2}$

$V_B = \sqrt{(V_o\cos\theta)^2 + V_y^2}$

chapter 12

**Remember that; R , Vo , t are always positive (+).

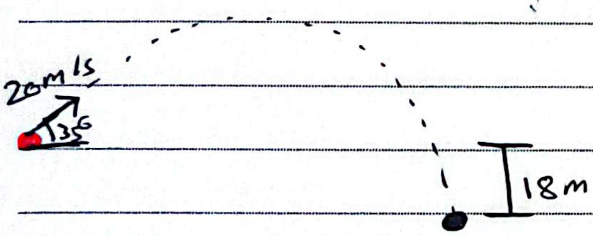
** زاوية الانطلاق هي الزاوية المحصورة بين x-axis و اتجاه الإنطلاقة .
وتكون (+) عند الرمي للأعلى و (−) عند الرمي للأسفل و (0) عند
الرمي بشكل أفقي.



$V_0 = +20\,m/s$

$\theta = 35^\circ$

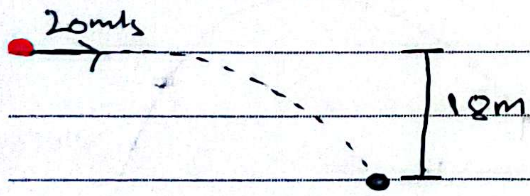
$\Delta y = +18\,m$



$V_0 = +20\,m/s$

$\theta = 35^\circ$

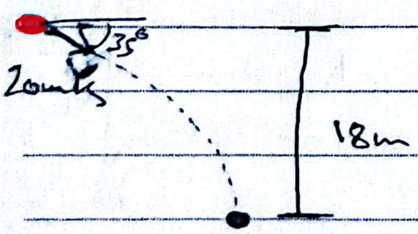
$\Delta y = -18\,m$



$V_0 = +20\,m/s$

$\theta = 0$

$\Delta y = -18\,m$

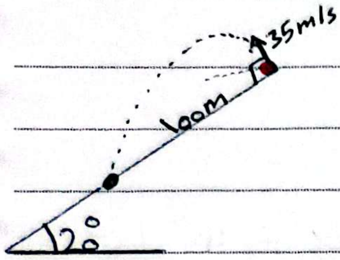


$V_0 = +20\,m/s$

$\theta = -35^\circ$

$\Delta y = -18\,m$

chapter 12

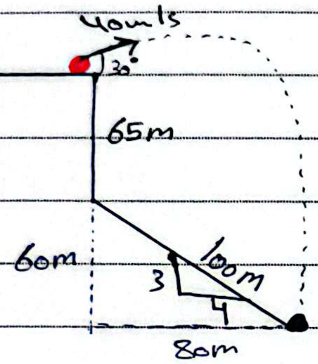


$V_o = +35\,m/s$

$\theta = 90° - 20° = 70°$

$R = 100\cos(20°)$

$\Delta y = -100\sin(20°)$



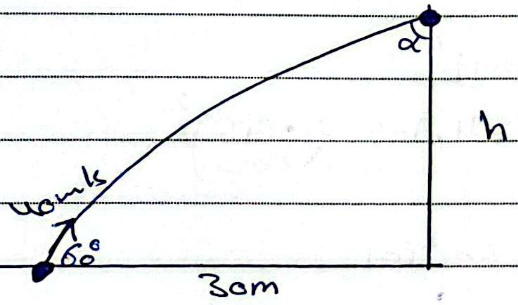
$V_o = 40\,m/s$

$\theta = 30°$

$R = 80m$

$\Delta y = -125m$

Example:



[1] Find h.

[2] time of flight.

[3] Impact Velocity.

[4] Find θ? which maximize the height h?

[1] $\Delta y = V_{oy}t - \frac{1}{2}gt^2$, $R = V_{ox}t = V_o\cos\theta t$

$\Delta y = 40\sin(60°) - \frac{1}{2}(9.81)(1.5)^2$ / $30 = 40\cos(60°)t$

$*1.5$

[2] $t = 1.5\,sec$

$\Delta y = +40.9m$

, $V_y^2 = V_{oy}^2 - 2g\Delta y$

[3] $V = \sqrt{V_x^2 + V_y^2}$

$V_y^2 = (40\sin(60°))^2 - 2(9.81)(40.9)$

$V_y^2 = 397.5$

$V_y = 19.9\,m/s$

$V = \sqrt{(40\cos(60°))^2 + 397.5}$

$V = 28.24$

PRENKO

* Impact angle: ~~vy~~

$\alpha = \tan^{-1}\left(\frac{v_x}{v_y}\right) \rightarrow \alpha = \tan^{-1}\left(\frac{20}{19.9}\right) \rightarrow \alpha = 45^\circ$

$\boxed{4}$ $R = v_0 \cos(\theta)\, t$

$30 = 40\cos(\theta)\, t$

$t = \frac{30}{40\cos\theta} \rightarrow t = \frac{0.75}{\cos\theta}$

$\Delta y = v_{0y} t - \frac{1}{2} g t^2$

$h = 40\sin(\theta)\, t - \frac{1}{2}(9.81) t^2$

$h = 40\sin(\theta)\left(\frac{0.75}{\cos\theta}\right) - 4.905\left(\frac{0.75}{\cos\theta}\right)^2$

$h = 30\tan\theta - 2.76\sec^2(\theta)$

$h' = 0 = 30\sec^2\theta - 5.52\sec(\theta)\sec(\theta)\tan(\theta)$

$30\sec^2\theta - 5.52\sec^2\theta\tan\theta = 0$

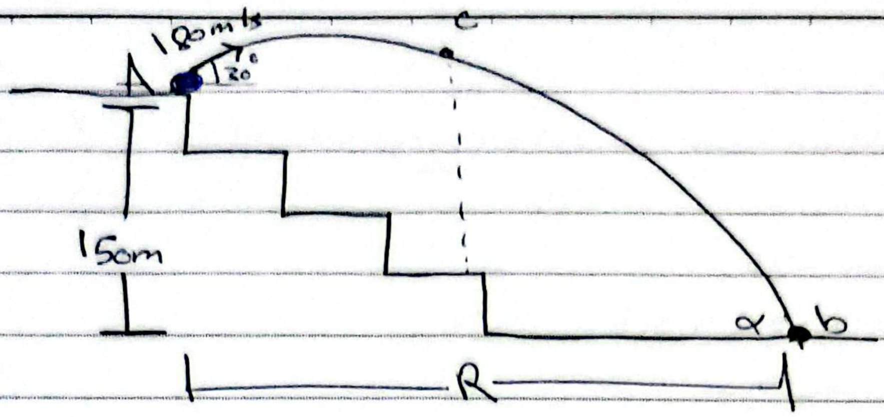
$30\cancel{\sec^2\theta} = 5.52\cancel{\sec^2\theta}\tan\theta$

$\frac{30}{5.52} = \tan\theta$

$\theta = 79.5^\circ$

Chapter 12

Example:



[1] Find the horizontal distance R?

$R = v_{0x} t \rightarrow R = v_i \cos 30° \, t$

$\Delta y = v_{0y} t - \frac{1}{2} g t^2$

$-150 = 180 \sin(30°) t - \frac{1}{2}(9.81) t^2$

$-150 = 90t - 4.905 t^2$

$-4.905 t^2 + 90t + 150 = 0$

$t_1 = 19.88$ ✓ $\quad t_2 = -1.5$ ✗

$R = 180 \cos(30°) (19.88)$

$R = 3098.9 \, m$

[2] The greatest elevation above the ground.

Sol → $\boxed{V_y = 0}$

$V_y^2 = V_{0y}^2 - 2g\Delta y$

$0 = [180 \sin(30°)]^2 - 2(9.81) h$

$0 = (90)^2 - 19.62 h$

$19.62 h = (90)^2$

$h = \frac{(90)^2}{19.62} = 412.8 \, m$

↳ maximum height from reference

from the ground: $413 + 150 = 563 m$

$V_y = 0$

[3] The time for the highest elevation.

$\rightarrow V_y = V_{oy} - gt$

$0 = 180 \sin 30^\circ - 9.81t$

$9.81t = 90$

$t = 9.2 \text{ sec}$

(a) (b)

[4] The Velocity of impact and the angle of impact.

[a] $V = \sqrt{V_y^2 + V_x^2}$

$V_{xf} = V_0 \cos\theta$

$V_{xf} = 180 \cos 30^\circ$

$V_{xf} = 155.88 \approx 156$

$V = \sqrt{(156)^2 + (105)^2}$

$V = 188 \text{ m/s}$

$V_y = V_{oy} - gt$

$V_y = 180 \sin(30) - 9.81(19.85)$

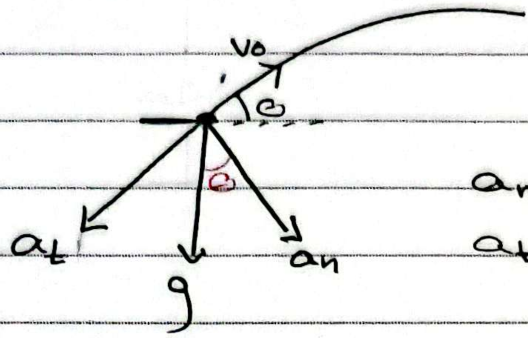
$V_y = -105 \text{ m/s}$

[b] $\alpha = \tan^{-1}\left(\frac{V_y}{V_x}\right)$

$\alpha = \tan^{-1}\left(\frac{105}{156}\right) \rightarrow \alpha = 34^\circ$

Chapter 12

5 The Radius of curvature at the start point and maximum hieght.



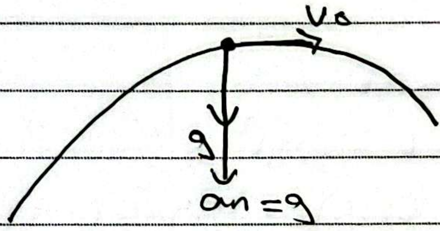
$\rightarrow \rho = \frac{v^2}{a_n}$

$a_n = g\cos\theta$

$a_t = g\sin\theta$

$\rho = \frac{(V_o)^2}{g\cos\theta}$

$\rho = \frac{(180)^2}{9.81\cos 30^\circ} = 3813.6\,m$



$\rho = \frac{v^2}{a_n}$

$\rho = \frac{(V_o)^2}{g} = \frac{(180\cos 30)^2}{9.81} = 2477\,m$

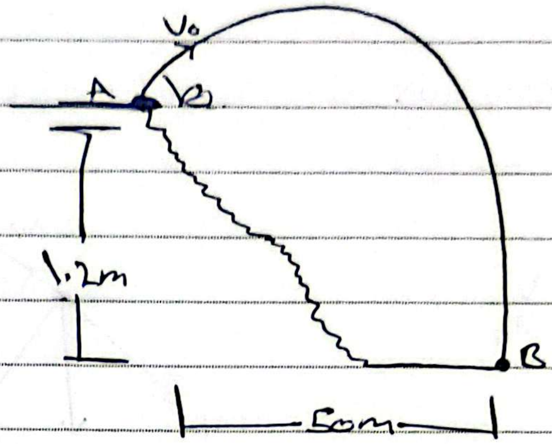
chapter 12

Ex: $t_{flight} = 2.5\ sec$

$R = 50m$

$V_o = ?$

$\theta = ?$



$\rightarrow R = V_x t$

$50 = V_o \cos\theta \,(2.5)$

$V_o \cos\theta = 20$ — (1)

$\Delta y = V_{oy} t - \frac{1}{2} g t^2$

$-1.2 = V_o \sin\theta \, t - \frac{1}{2} g t^2$

$-1.2 = V_o \sin\theta \,(2.5) - 4.905 (2.5)^2$

$V_o \sin(\theta) = \dfrac{4.905 (2.5)^2 - 1.2}{2.5}$

$V_o \sin(\theta) = 11.8$ — (2)

$V_o \cos(\theta) = 20 \longrightarrow V_o = \dfrac{20}{\cos\theta}$

$V_o \sin(\theta) = 11.8 \longleftarrow$ نعوض

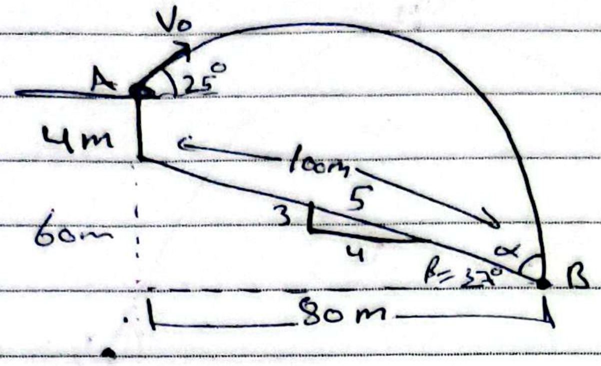
$20 \tan\theta = 11.8$

$\theta = \tan^{-1}\left(\frac{11.8}{20}\right)$

$\boxed{\theta = 31^\circ}$ $\quad V_o = \dfrac{20}{\cos 31} \rightarrow \boxed{V_o = 23.3\ m/s}$

chapter 12

12-11:



$\theta = 25°$

$h = \Delta y = -64\,m$

$R = 80m$

1] The initial Velocity.

2] time of flight

3] The velocity of impact and the angle of impact.

⇒

1] $R = V_o \cos\theta t \rightarrow 80 = V_o \cos(25°)\, t = ?$

$V_o t = \frac{80}{\cos(25)} = 88.3$

$\Delta y = V_{oy} t - \frac{1}{2} g t^2$

$-64 = V_o \sin(25°) t - 4.905 t^2$

$-64 = (88.3) \sin(25°) - 4.905 t^2$

$4.905 t^2 = 37.3 + 64$

$t^2 = 20.65$

2 $\boxed{t = 4.54\,sec}$

$80 = V_o \cos(25°)\,(4.54)$

$V_o = \frac{80}{4.54 \cos(25°)}$

1 $\boxed{V_o = 19.44\,m/s}$

3 $V = \sqrt{V_x^2 + V_y^2}$

$V_{xf} = V_o \cos(25°)$

$V_{xf} = 19.44 \cos(25°)$

$V_{xf} = 17.6\,m/s$

$V = \sqrt{(36.3)^2 + (17.6)^2}$

$V = 40.34\,m/s$

$V_y = V_{oy} - gt$

$V_y = 19.44 \sin(25°) - 9.81(4.54)$

$V_y = -36.3\,m/s$

↳ $\downarrow$

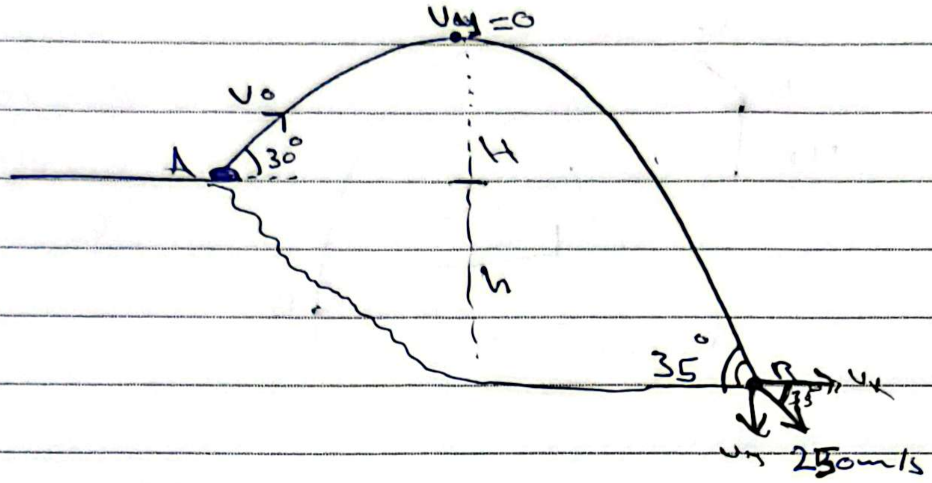
$\alpha = \tan^{-1}\left(\frac{36.3}{17.6}\right)$

$\boxed{\alpha = 64.1° - 37°}$

PRENKO

chapter 12

Example:



$\rightarrow V_o \cos(30°) = 250 \cos(35°)$

$\boxed{V_o = 236.5 \text{ m/s}}$

$V_y = 250 \sin(35)$

$\boxed{V_y = -143.4 \text{ m/s}}$

للأسفل

$\Delta y = V_{oy} t - \frac{1}{2} g t^2$

$V_y = V_{oy} - gt$

$-143.4 = 236.5 \sin(30°) - 9.81t$

$9.81t = 236.5 \sin(30°) + 143.4$

$\boxed{t = 26.7 \text{ sec}}$

$\Delta y = 236.5 \sin(30°)(26.7) - 4.905(26.7)^2$

$\boxed{\Delta y = h = -339.5 \text{m}}$

H أقصى ارتفاع

$V_y^2 = V_{oy}^2 - 2g\Delta y$

$0 = [236.5 \sin(30°)]^2 - 19.62 \Delta y$

$19.62 \Delta y = 13983.1$

$\Delta y = H = 712.7 \text{ m}$

$R = V_o \cos 30° t$

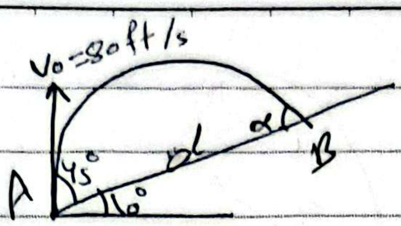
$R = 236.5 \cos(30°)(26.7)$

$R = 5468.6 \text{m}$

PRENKO

chapter 12

12 -102:



$\rightarrow \theta = 55°$, $R = d\cos(10°)$

$V_0\cos(55°)t = d\cos(10°)$

$\boxed{45.9t = 0.984d}$

$d = 46.65t$

$\Delta y = d\sin(10)$

$\Delta y = 8.1t$

$\Delta y = V_{oy}t - \frac{1}{2}gt^2$

$8.1t = (80)\sin(55°)t - \frac{1}{2}(32.2)t^2$

$8.1t = 65.5t - 16.1t^2$

$\boxed{t = 3.56\ sec} \rightarrow d = 46.65(3.56)$

$\boxed{d = 166.1\ ft}$

$R = 166.1\cos(10)$

$\boxed{R = 163.6}$

$V_y = V_{oy} - gt$

$V_y = 80\sin(55°) - 32.2(3.56)$

$\boxed{V_y = -49.3\ m/s}$

$V_B = \sqrt{49.3^2 + (80\cos 55)^2}$

$\boxed{V_B = 67.3\ m/s}$

t at maximum h: $\cancel{\Delta y = V_{oy}t - \frac{1}{2}gt^2}$ α

$V_y^2 = V_{oy}^2 - 2g\Delta y$

$0 = [80\sin(55)]^2 - 64.4\Delta y$

$\boxed{\Delta y = 66.68}$

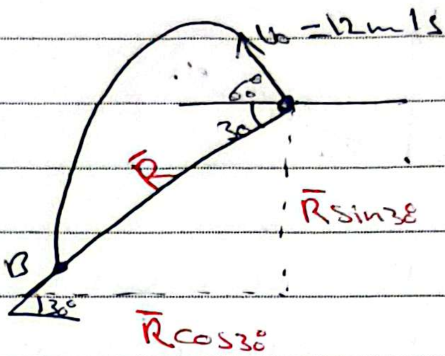
PRENKO

$\Delta y = V_{oy}t - \frac{1}{2}gt^2$

$\boxed{t = 2\ sec}$

chapter 12

12-91:



$R = V_0\cos(60°)\,t$

$\bar{R}\cos 30° = 12\cos(60)\,t$

$0.86\bar{R} = 6t$

$\boxed{\bar{R} = 6.97t}$

$\bar{R} = 6.97(2.83)$

$\bar{R} = 19.72$

$\Delta y = V_{0y}t - \frac{1}{2}gt^2$

$-\bar{R}\sin 30° = 12\sin(60°)t - 4.905t^2$

$-0.5\bar{R} = 10.4t - 4.905t^2$

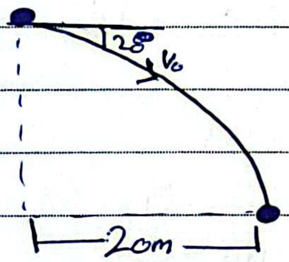
$-3.485t = 10.4t - 4.905t^2$

$\cancel{4.905t^2 + 6.915t = 0}$

$4.905t^2 - 13.86t = 0$

$\boxed{t = 2.83\,sec}$

EX 12-90:



$V_0 = ?$ $t = ?$

$\theta = -20$

$\Delta y = -10m$

$R = 20$

$\Rightarrow R = V_0\cos(-20°)\,t$

$20 = 0.94V_0t$

$V_0t = 21.3$

$V_0 = \dfrac{21.3}{\cancel{1.87}\ 0.75}$

$\cancel{V_0 = 11.4\,m/s}$

$V_0 = 28.4\,m/s$

$\Delta y = V_{0y}t - \frac{1}{2}gt^2$

$-10 = V_0\sin(-20)\,t - 4.905t^2$

$-10 = -0.34V_0t - 4.905t^2$

$-10 = -0.34(21.3) - 4.905t^2$

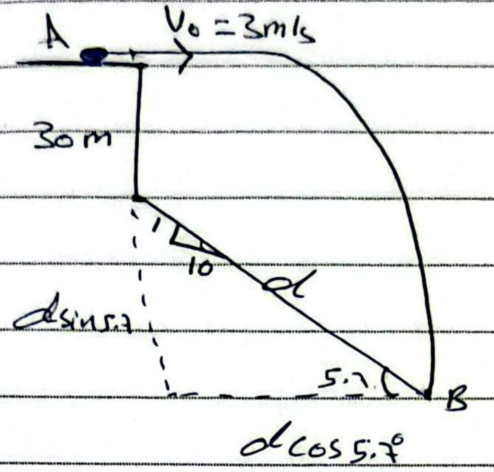
$4.905t^2 = -0.34(21.3) + 10$

$\boxed{\cancel{t = 1.87\,sec}}$

$t = 0.75\,sec$

chapter 12

Ex ; 12-94



$\rightarrow V = \sqrt{V_x^2 + V_y^2}$

$V_x = V_0 \cos\theta$

$\boxed{V_x = 3 m/s}$

$\cancel{V_y^2 = V_{0y}^2 - 2g\Delta y}$

$\Delta y = -(30 + d \sin 5.7°)$

$\Delta y = \cancel{V_{0y}t} - \frac{1}{2} g t^2$

$-30 - d \sin 5.7° = 0 - 4.905 t^2$

$R = V_0 \cos\theta t$

$d \cos 5.7 = 3t$

$d = \frac{3t}{\cos(5.7)} \rightarrow d = 3.01t$

$-30 - 3.01t \sin(5.7°) = -4.905t^2$

$-30 - 0.298t = -4.905t^2$

$4.905t^2 - 0.298t - 30 = 0$

$\boxed{t = 2.5 sec}$

$\boxed{d = 3.01(2.5) = 7.5m}$

$V_y = \cancel{V_{0y}}^{0} - gt$

$V_y = -9.81(2.5)$

$\boxed{V_y = -24.53 m/s}$

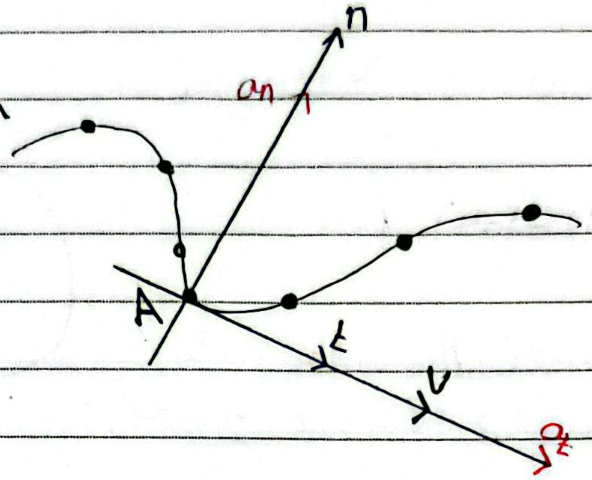
$V = \sqrt{9 + (24.53)^2}$

$V = 24.7 m/s$

chapter 12

* Tangential and Normal componants

$$\rho = \frac{[1+(y')^2]^{3/2}}{y''}$$



* Velocity ; دائماً معطى السرعة

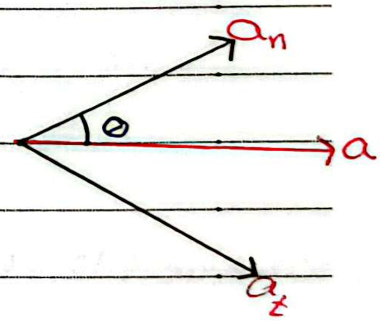
$$V = \frac{ds}{dt}$$

* Acceleration ; $a_t = \frac{dv}{dt} = \dot{v}$

* $\rho = r$ → for circular Path

$$a_n = \frac{v^2}{\rho}$$

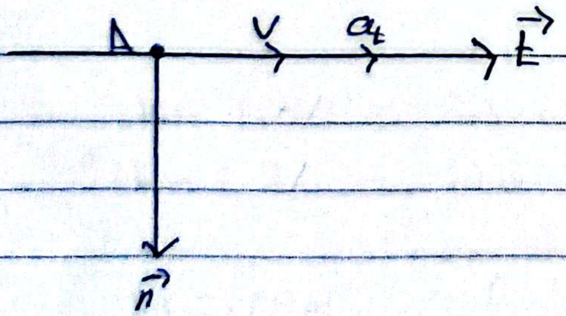
⇐ باتجاه المركز دائماً



$$a = \sqrt{a_t^2 + a_n^2}$$

اتجاه التسارع الكلي → $\tan^{-1}\left(\frac{a_t}{a_n}\right) = \theta$

I Suppose that the Particle moves in a straight line



$\rho = \infty$

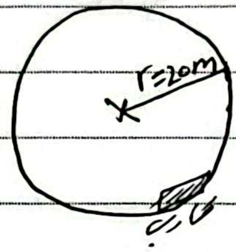
$$a_n = \frac{v^2}{\rho} = \frac{v^2}{\infty} = 0$$

chapter 12

[2] If the particle moves at a path in constant velocity

* $V = \text{constant} \xrightarrow{\text{then}} a_t = 0$, $a_n = \frac{V^2}{\rho}$

Example (12-104) :



Find a, and the rate of increase in the speed $2 m/s^2$, when $V = 5 m/s$

→ $a_t = 2 m/s^2$, $a = ?$

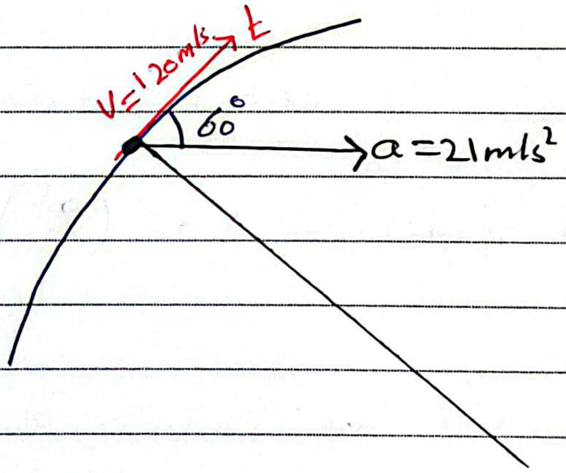
$$a = \sqrt{a_t^2 + a_n^2}$$

$$a_n = \frac{V^2}{\rho} = \frac{25}{20} = 1.25 m/s^2$$

$$a = \sqrt{(2)^2 + (1.25)^2}$$

$$a = 2.4 m/s^2$$

Example (12-102)



[1] Find the increasing in Velocity (a_t).

[2] Find the radius.

→ [1] $a_t = a\cos 60° = 21 \cos 60° = 10.5 m/s^2$

$a_n = a \sin 60° = 21 \sin 60° = 18.2 m/s^2$

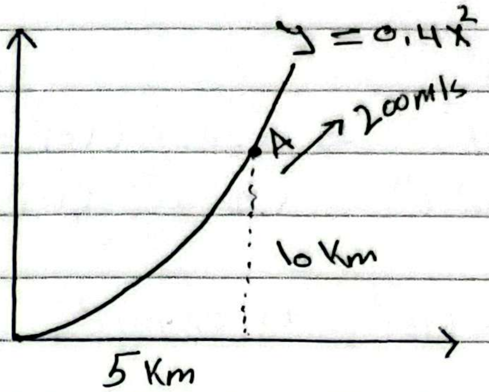
[2] $a_n = \frac{V^2}{\rho}$ → $\rho = \frac{(120)^2}{18.2} = 791.2 m$

PRENKO

chapter 12

(12-106) :

If the speed increases at a rate of $0.8\ m/s^2$ find a?



$\rightarrow a = \sqrt{a_t^2 + a_n^2}$

$a = \sqrt{(0.8)^2 + (0.456)^2}$

$a = 0.92\ m/s^2$

$a_n = \frac{v^2}{\rho} = \frac{(200)^2}{87616}$

$a_n = 0.456\ m/s^2$

$y' = 0.8x$

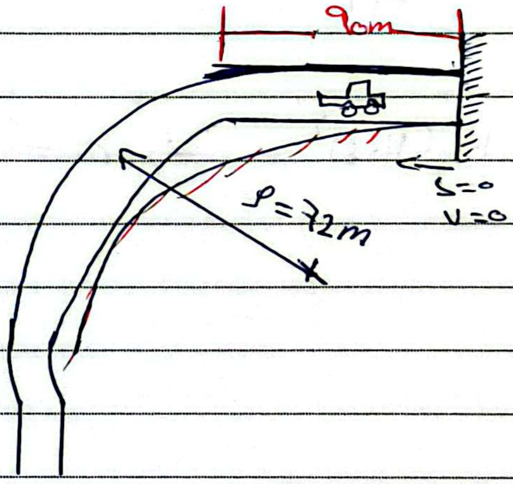
$y'' = 0.8$

$\rho = \frac{\left[1 + (0.8(5))^2\right]^{\frac{3}{2}}}{0.8}$

$\rho = 87.61\ Km$

$\rho = 87616\ m$

Problem (12-113)



$\dot{v} = 0.015t^2$

* Find the velocity and acceleration when $t = 18$ sec.

$\rightarrow v = \int \dot{v}\, dt \rightarrow v = \int 0.015t^2\, dt \rightarrow v = \frac{0.015t^3}{3}$

$v = 0.005t^3$

$s = \int v\, dt \rightarrow s = \int 0.005t^3\, dt$

$s = \frac{0.005t^4}{4} = 1.25 * 10^{-3}\ t^4$

$s = 1.25 * 10^{-3}\ (18)^4$

$s = 131.22\ m$

$\dot{v} = 0.015t^2$

$V = 0.005t^3 \rightarrow 0.005(18)^3 = 29.2\,m/s$

doD السيارة $S = 1.25 \times 10^{-3}\, t^4 \rightarrow 1.25 \times 10^{-3}(18)^4 = 131.2\,m$

$a_t = 0.015(18)^2 = 4.86\,m/s^2$

للدائرة $S = \frac{1}{4}(2\pi r) \rightarrow 0.25(2)(72)\pi = 113.1\,m$

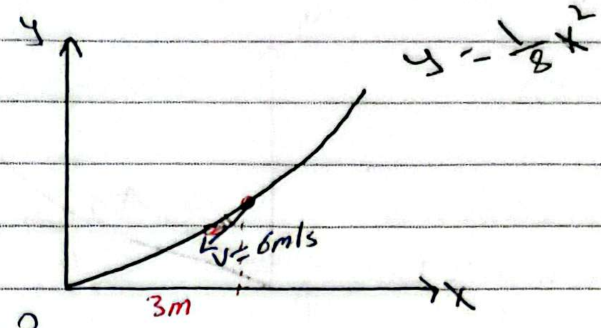
$\cancel{113.1 < 131.2}$ $\cancel{90 < 113.1 < 203}$

So there is a_n. $90 < 113.1 < 203$

$a_n = \frac{v^2}{\rho} = \frac{(29.2)^2}{72} = 11.8\,m/s^2$

$a = \sqrt{(4.86)^2 + (11.8)^2} \rightarrow a = 12.76\,m/s^2$

Example:



* $a_t = 1.8\,m/s^2$
* $V = 6\,m/s$
* Find the car direction?
* Find a.
* Find ρ

$\rightarrow \rho = \dfrac{\left[1+(y')^2\right]^{3/2}}{y''}$

$y' = \frac{1}{4}x \qquad y'' = \frac{1}{4}$

$y' = \frac{3}{4}$

$\rho = \dfrac{\left[1+(0.75)^2\right]^{3/2}}{0.25} = \boxed{7.81\,m}$

$a = \sqrt{a_t^2 + a_n^2}$

$a = \sqrt{(1.8)^2 + (4.6)^2}$

$\boxed{a = 4.94\,m/s^2}$

$a_n = \dfrac{36}{7.81} = 4.6\,m/s^2$

** The direction of the car at the curve is the direction of the slope.

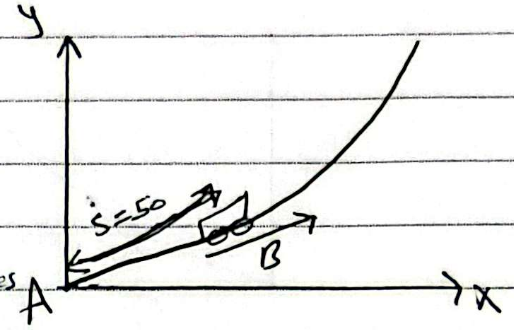
$\tan\theta = m = y'$

$\tan\theta = 0.75$

$\theta = 36.9°$

chapter 12

Example (12.32)



Find the acceleration of the car when it becomes at 50m after A,

$v = 0.2s$, $\rho = 500m$

$\rightarrow \cancel{a_t = \dot{v} = 0.2}$ $\quad a_n = \frac{v^2}{\rho} = \frac{[0.2(50)]^2}{500} = 0.2 m/s^2$

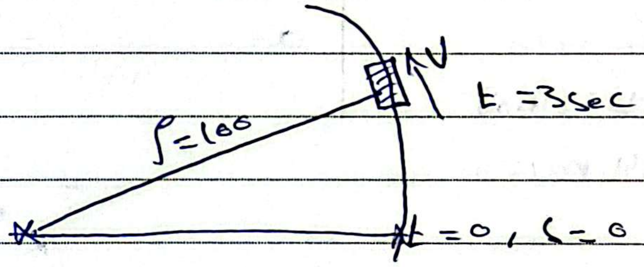
$a_t\, ds = v\, dv$

$a_t = v \frac{dv}{ds}$ $\qquad a = \sqrt{a_t^2 + a_n^2}$

$a_t = (0.2(50))\,(0.2)$ $\qquad a = \sqrt{(2)^2 + (0.2)^2}$

$a_t = 2 m/s^2$ $\qquad a = 2.01 m/s^2$

Ex (12-28)



Find a? after 3 sec , $v = \frac{300}{s}$

$\rightarrow a_n = \frac{v^2}{\rho} = \frac{\left(\frac{300}{42.4}\right)^2}{\rho}$ $\quad\Big|\quad v = \frac{ds}{dt} \rightarrow dt = \frac{ds}{v}$

$a_n = 0.5 m/s^2$ $\qquad \int_0^3 dt = \int_0^s \frac{ds}{\frac{300}{s}}$

$\cancel{\int a_t\, ds = \int v\, dv}$ $\qquad t\Big|_0^3 = \frac{s^2}{600}$

$a_t\, ds = v\, dv$ $\qquad 3 = \frac{s^2}{600}$

$a_t = 7.07 \left(\frac{-300}{(42.4)^2}\right)$ $\qquad 1800 = s^2$

$a_t = -1.17 m/s^2$ $\qquad \boxed{s = 42.4m}$

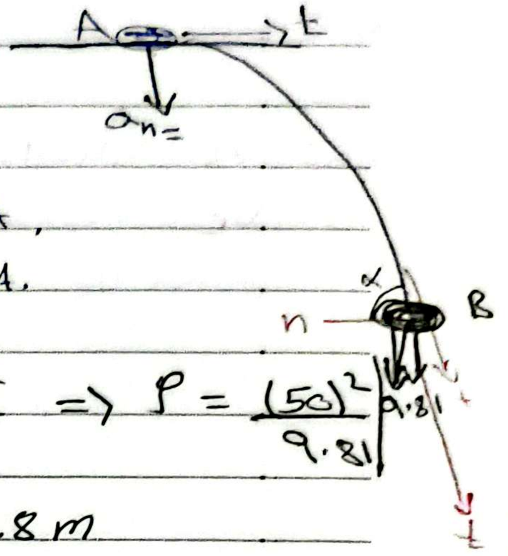
chapter 12

(12–112) Ex

$\Delta y = -500 m$

$V_o = 50 m/s$

$\theta = 0$



[1] Find a_t, a_n, ρ at the start point.

[2] Find a_t, a_n, ρ at the end point.

[1] $a = \sqrt{a_n^2 + a_t^2}$ | $a_t = 0$ | $\rho = \frac{V^2}{a_n} \Rightarrow \rho = \frac{(50)^2}{9.81}$

$9.81 = \sqrt{a_n^2 + 0}$

$a_n = 9.81 m/s^2$

$\rho = 254.8 m$

[2] $a_t = 9.81 \sin\alpha$, $\alpha = ?$

$a_n = 9.81 \cos\alpha$

$\alpha = \tan^{-1}\left(\frac{V_y}{V_x}\right)$

$\alpha = \tan^{-1}\left(\frac{99}{50}\right)$

$V_y^2 = V_{oy}^2 - 2g\Delta y$

$V_y^2 = 0 - 2(9.81)(-500)$

$V_y^2 = 9810 \rightarrow \boxed{V_y = 99 m/s}$

$\boxed{\alpha = 63.2^\circ}$

$V_x = V_o \cos 0 = 50 m/s$ | $V_B = \sqrt{(50^2) + (99)^2}$

$V_B = 111 m/s$

$a_t = 9.81 \sin(63.2^\circ) = 8.8 m/s^2$

$a_n = 9.81 \cos(63.2^\circ) = 4.4 m/s^2$

$\rho = \frac{(V_B)^2}{a_n} = \frac{111^2}{4.4} = 2800 m$

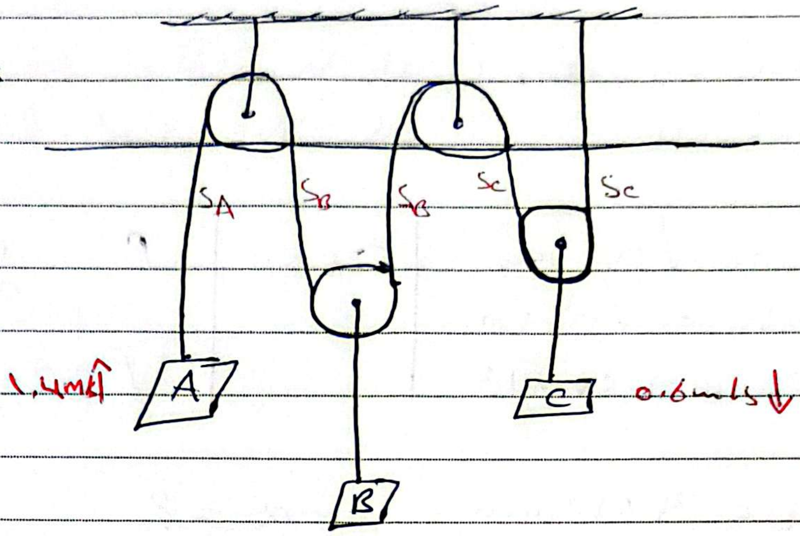
PRENKO

* Dependent Motion:

Ex: $V_A = 1.4\ m/s$

$V_C = -0.6\ m/s$

Find V_B.

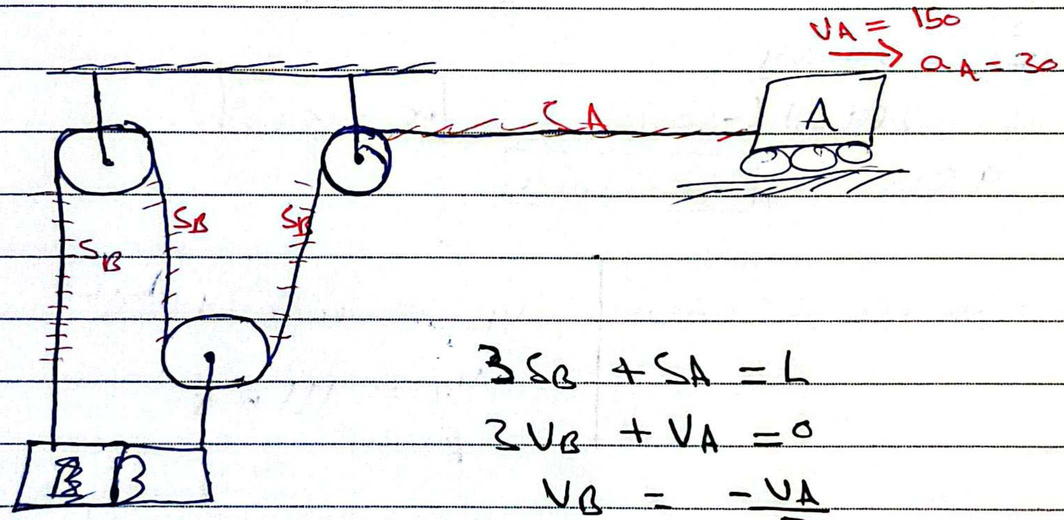


$\rightarrow 2S_C + 2S_B + S_A = L$

$2V_C + 2V_B + V_A = 0 \quad \rightarrow 2(-0.6) + 2V_B + 1.4 = 0$

$2a_C + 2a_B + a_A = 0 \quad \boxed{V_B = -0.1\ m/s}$

Ex:



$3S_B + S_A = L$

$3V_B + V_A = 0$

$V_B = \frac{-V_A}{3}$

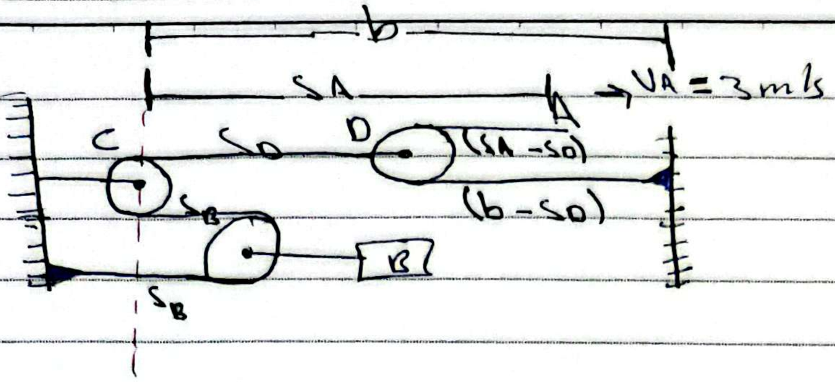
Find a_B, V_B?

$V_B = -\frac{150}{3} = -50\ m/s$

$a_B = -\frac{a_A}{3} = -\frac{30}{3} = -10\ m/s^2$

chalfter 12

Ex!



$\rightarrow L_1 = 2S_B + S_D$

$L_2 = (S_A - S_D) + (b - S_D)$

$L_1 = 2S_B + S_D$

$0 = 2V_B + V_D$

$0 = 2a_B + a_D$

$0 = 2V_B + 1.5$

$\boxed{V_B = -0.75\,m/s}$

$L_2 = S_A + b - 2S_D$

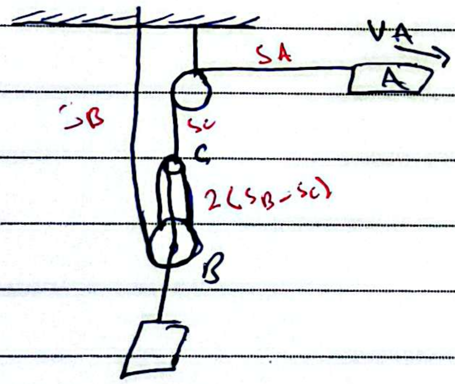
$0 = V_A - 2V_D$

$0 = a_A - 2a_D$

$0 = 3 - 2V_D$

$\boxed{V_D = 1.5\,m/s}$

Example!



$\rightarrow L_1 = S_B + 2(S_B - S_C)$

$L_1 = 3S_B - 2S_C$

$0 = 3V_B - 2V_C$

$L_2 = S_A + S_C$

$0 = V_A + V_C$

chapter 13

*13.1 Newton's Second law of motion

$\sum F_x = ma_x$ $\qquad\qquad$ $\sum F_t = ma_t = m\frac{dv}{dt}$

$\sum F_y = ma_y$ $\qquad\qquad$ $\sum F_n = ma_n = m\frac{v^2}{\rho}$

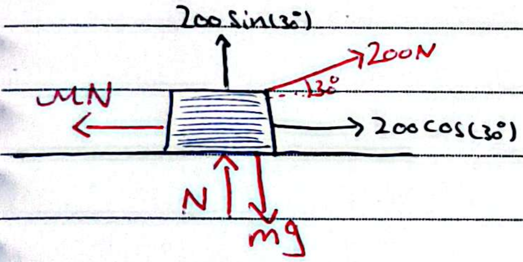
Friction force ; Static : $F_f = \mu_s N$

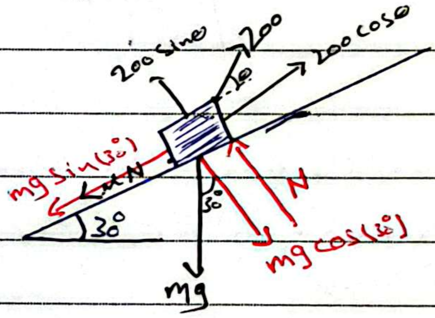
dynamic : $F_f = \mu_k N$

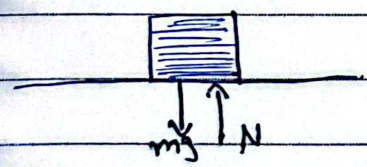
** $(\mu_s > \mu_k)$ $\not<$ 1

* Normal force → اتجاه القوة من سطح الجسم

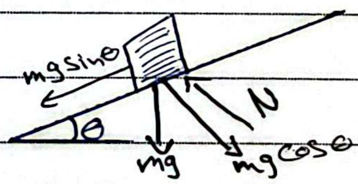
* Spring force = $\cancel{\frac{1}{2}Ks^2}$ $\frac{1}{2}K(s_2^2 - s_1^2)$





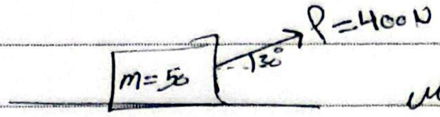


$\sum F_y = 0 \rightarrow N = mg$



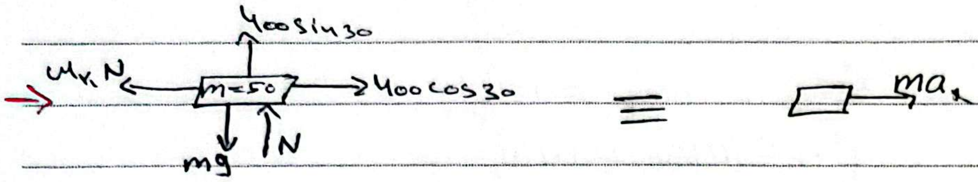
chapter 13

Example 13.1 ;



$\mu_K = 0.3$ $\quad \cancel{\Sigma F = 0}$ $\quad \Delta t = 3s$

$v_1 = 0 \quad v_2 = ?$



$\Sigma F_x = ma_x$

$400\cos(30°) - (0.3)N = 50a_x$

$400\cos(30°) - 0.3(291) = 50a_x$

$a_x = 5.2\, m/s^2$

$v_2 = v_1 + at$

$v_2 = 0 + 5.2(3)$

$v_2 = 15.6\, m/s$

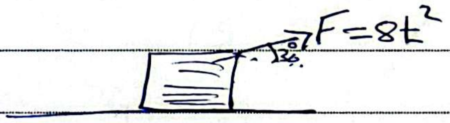
$\Sigma F_y = 0$

$400\sin30° + N = 50(9.81)$

$N = 50(9.81) - 400\sin30°$

$N = 291N$

Example ;



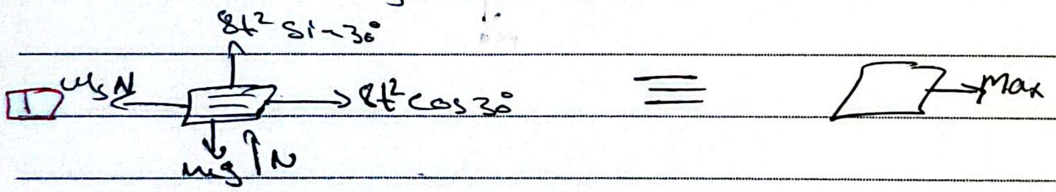
$\mu_s = 0.35 \quad m = 40kg$

$\mu_K = 0.2$

$\boxed{1}$ How long it needs to start moving ?

$\boxed{2}$ It's velocity after 8 seconds .

$\boxed{1}$



$\Sigma F_x = 0$

$8t^2\cos30° - \mu_s N = 0$

$N = \dfrac{8t^2\cos30°}{0.35}$

$\Sigma F_y = 0$

$8t^2\sin30° + N = mg$

$8t^2\sin30° + \dfrac{8t^2\cos30°}{0.35} = 40(9.81)$

$4t^2 + 19.79t^2 - 392.4 = 0$

$23.79t^2 = 392.4$

PRENKO

$t^2 = 16.5$

$\boxed{t = 4.1s}$

chapter 13

[2] $\Sigma F_x = ma_x$ $\qquad$ $\Sigma F_y = 0$

$8t^2\cos 30° - \mu_k N = ma_x$ $\qquad$ $8t^2 \sin 30° + N = mg$

$\cancel{8(8)^2\cos 30° - 0.2N = 40a_x}$ $\qquad$ $8(8)^2 \sin 30° + N = 40(9.81)$

$\cancel{N = 40(9.81) - 8(64)\sin 30°}$

$8t^2\cos 30° - 0.2(392.4 - 4t^2) = 40a_x$ $\qquad$ $\cancel{N = 136.4\,N}$

$N = 40(9.81) - 8t^2 \sin 30°$

$a_x = 0.173t^2 - 1.96 + 0.02t^2$ $\qquad$ $N = 392.4 - 4t^2$

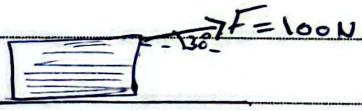
$a_x = 0.193t^2 - 1.96$

$V = \int a_x\,dt \rightarrow V = \int_{4.1}^{8} (0.193t^2 - 196)\,dt$

$V =$

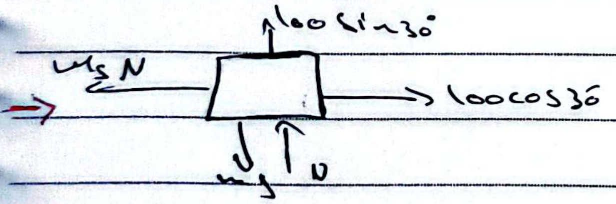
[3] find the velocity ~~after~~ at 3s from moving.

→ Velocity equal Zero because the motion started from $t = 4.1s$

Example: 

$m = 10 \quad \mu_s = 0.35 \quad \mu_k = 0.2$

check if the box will move.



$\Sigma F_y = 0$

$100\sin 30° + N = mg$

$100\sin 30° + N = 10(9.81)$

$\cancel{N = 1.96\,N}$ $\boxed{N = 48.1}$

~~$\Sigma F_x = 0$~~

~~$100\cos 30° - \mu_s N = 0$~~

$\boxed{\cancel{\mu_s N = 86.8\,N}}$

$100\cos 30° = 86.8N$

$F_f = 0.35(481) = 16.8$

Since $100\cos 30° > F_f = 16.8$

PRENKO

* find a if the particle will move

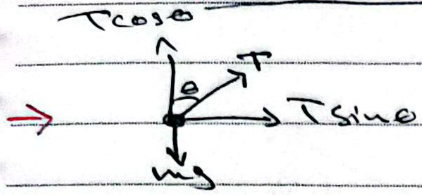
$\boxed{\cancel{Not\ moving}}$

the particle will ~~not~~ move

Chapter 13

Example: 

Find θ.



$\sum F_x = ma$

$T\sin\theta = ma$

$\tan\theta\,(mg) = ma$

$\boxed{\tan\theta = \frac{a}{g}}$

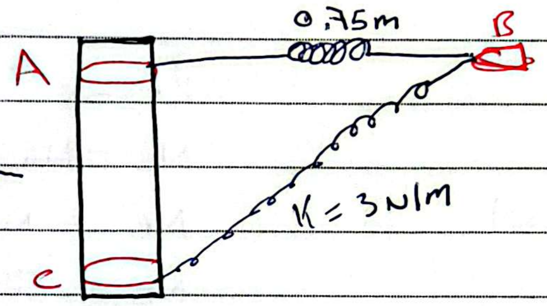
$\sum F_y = 0$

$T\cos\theta = mg$

$T = \frac{mg}{\cos\theta}$

$\rightarrow \theta = \tan^{-1}\left(\frac{a}{g}\right)$

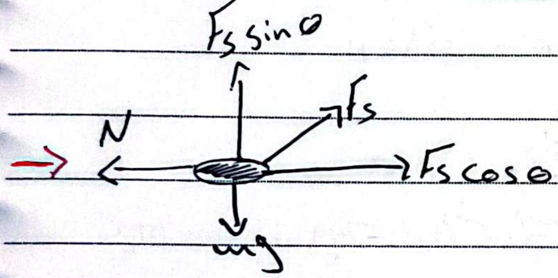
Example 13.4



a = ?

N = ?

m = 2



$\sum F_x = 0$

$F_s\cos\theta - N = 0$

$N = F_s\cos\theta$

$N = (3)\,s\cos\theta$

$N = (3)(s_2 - s_1)\cos\theta$

$\theta = \tan^{-1}\left(\frac{4}{3}\right)$

$\theta = 53.1°$

$N = 3\,(1.25 - 0.75)\cos 53.1°$

$\boxed{N = 0.9\,N}$

$s_2 = \sqrt{1 + (0.75)^2} - 0.75$

$s_2 = 1.25$

$\sum F_y = ma$

$\sin\theta\,ks - mg = ma$

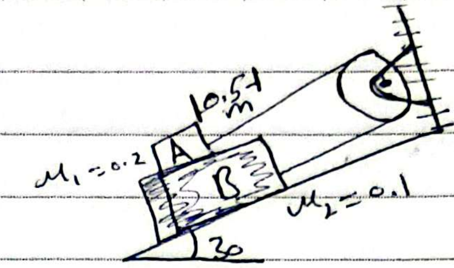
$a = \frac{(3)(0.5)\sin 53.1 - 2(9.81)}{2} = 9.21$

↓

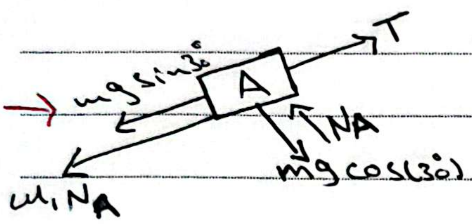
Chapter 13

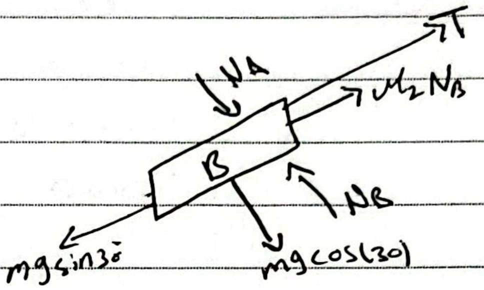
Example: $m_A = 10\,kg$

$m_B = 50\,kg$



Find: a_A, a_B, Tension.





$\sum F_y = 0$

$N_A = m_A g\cos(30°)$

$N_A = (10)(9.81)(\cos 30°)$

$N_A = 85\,N$

$\sum F_y = 0$

$N_B - N_A - m_B g\cos 30° = 0$

$N_B = N_A + m_B g\cos 30°$

$N_B = 85 + 50(9.81)\cos 30°$

$N_B = 510\,N$

$\sum F_x = ma$

$T - m_A g\sin 30 - \mu_1 N_A = m_A a_x$

$T - 10(9.81)\sin(30°) - 0.2(85) = 10a_x$

$T - 32.1 = 10a_x$

$T = 10a_x + 32.1$

$\sum F_x = ma$

$\cancel{T + \mu_2 N_B - m_B g\sin 30° = m_B a_x}$

$\cancel{T + 0.1(510) - 50(9.81)\sin 30 = 510 a_x}$

$\cancel{T - 194.25 = 510 a_x}$

$\cancel{10a_x + 32.1 - 194.25 = 510 a_x}$

$\cancel{500 a_x = }$

$m_B g\sin 30 - \mu_2 N_B - T$

$(50)(9.81)\sin 30° - 0.1(510) - T$

$= 50a_x$

$194.3 - T = 50a_x$

$194.3 - 10a_x - 32.1 = 50a_x$

$60a_x = 162.2$

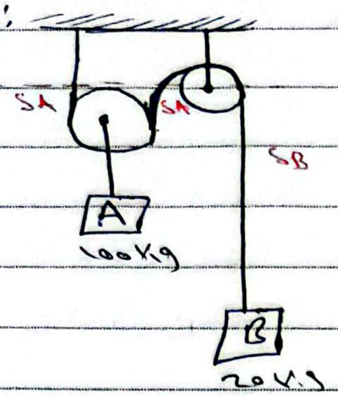
$\boxed{a_x = 2.7\,m/s^2}$

$T = 10(2.7) + 32.1$

$T = 59.1\,N$

Chapter 13

Example:



$m_A = 100\,kg$ find: a_A, a_B, T.

$m_B = 20\,kg$

$L = 2S_A + S_B$

$0 = 2V_A + V_B$

$0 = 2a_A + a_B$

$\boxed{a_B = -2a_A}$

$\Sigma F_x = 0$

$\Sigma F_y = ma_A$

$m_A g - 2T = m_A a_A$

$100(9.81) - 2T = 100a_A$

$981 - 2T = 100a_A$

$\Sigma F_x = 0$

$\Sigma F_y = ma_B$

$m_B g - T = m_B a_B$

$20(9.81) - T = 20a_B$

$196.2 - T = 20a_B$

$196.2 - T = 20(-2a_A)$

$196.2 - T = -40a_A$

$981 - 2T = 100a_A$

$196.2 - T = -40a_A$

$T = 327\,N$, $a_A = 3.27\,m/s^2$

$a_B = -2(3.27) = -6.54\,m/s^2$ ↑

* if Block A started from rest find the velocity of B and distance of A after 10 seconds.

$V_B = ?$ distance for A = ? $t = 10$

→ $V_{B_2} = \cancel{V_{B_1}}^{0} + a_B t$

$V_{B_2} = (-6.54)(10)$

$V_{B_2} = -65.4\,m/s$ ↑

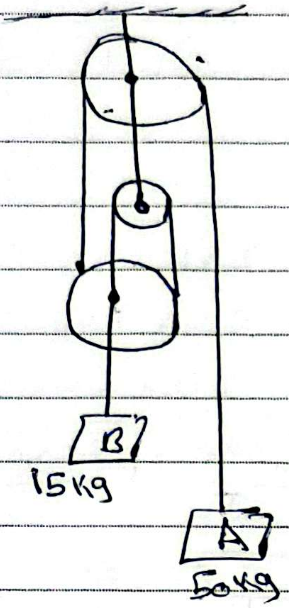
$S_A = \Delta y = \cancel{V_{0A}t}^{0} + \frac{1}{2}a_A t^2$

$S_A = \frac{1}{2}(3.27)(10)^2 = 163.5\,m$

PRENKO

chapter 13

Example:

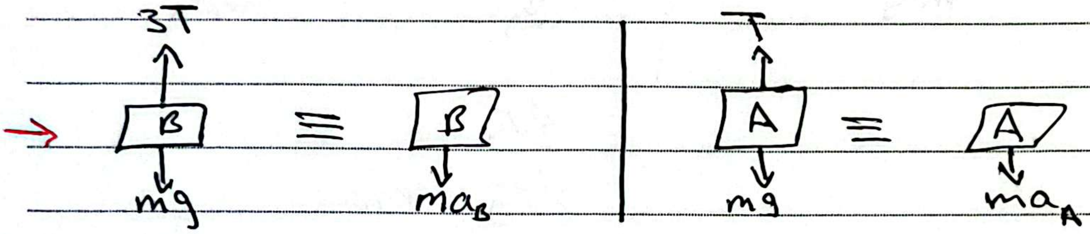


Find a_A, a_B, T.

$L = 3S_B + S_A$

$0 = 3V_B + V_A$

$0 = 3a_B + a_A$



$\Sigma F_x = 0$	$\Sigma F_x = 0$
$\Sigma F_y = ma_B$	$\Sigma F_y = ma_A$
$m_B g - 3T = m_B a_B$	$m_A g - T = m_A a_A$
$15(9.81) - 3T = 15 a_B$	$50(9.81) - T = 50(-3a_B)$
$147.15 - 3T = 15a_B$	$490.5 - T = -150 a_B$

$-3T - 15a_B = -147.15$

$-T + 150a_B = -490.5$

$T = 63.3\,N$

$a_B = -2.85\ m/s^2$

$\uparrow B$

$a_A = -3(-2.85)$

$a_A = 8.55\ m/s^2$

* if all starts from rest, find the velocity of B when A moves 18m down?

$\Delta S_A + 3\Delta S_B = 0$

$18 + 3S_B = 0$

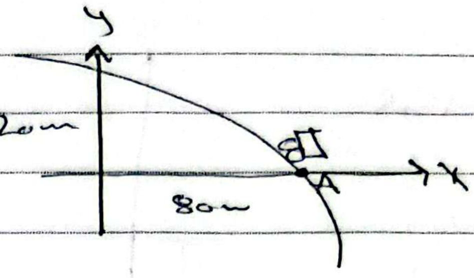
$S_B = -\frac{18}{3}$ $\quad S_B = -6m$ up

$V_{B_2}^2 = V_{B_1}^2 + 2a_B S_B$

$V_{B_2}^2 = 2(-2.85)(-6) \rightarrow V_{B_2} = 5.84\ m/s$

chapter 13

Example: $y = 20\left(1 - \frac{x^2}{6400}\right)$



$v_A = 9 m/s^2$

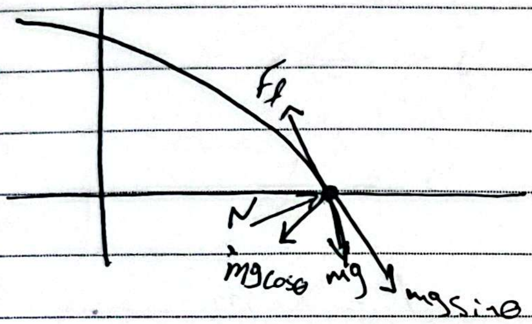
$(a_t)_A = 3 m/s^2$

$m = 800 kg$

Find:

[1] Normal force

[2]: Frictional force



$y = 20 - \frac{20x^2}{6400}$

$y' = \frac{-40x}{6400}$, $y'' = \frac{-40}{6400}$

$y' = -0.5$

$\rho = \frac{\left[1 - (y')^2\right]^{\frac{3}{2}}}{y''} = \frac{\left[1 - (-0.5)^2\right]^{\frac{3}{2}}}{-6.25\times10^{-3}}$

$\boxed{\rho = 103.8 m}$

$\sum F_n = \frac{mv^2}{\rho}$

$mg\cos\theta - N = \frac{mv^2}{\rho}$

$(800)(9.81)\cos(-26.5) - N = \frac{800(81)}{103.8}$

$\theta = \tan^{-1}(y')$

$\theta = -26.5$

$N = 7023.4 - 624.3$

$N = 6399.1 N$

[2] $\sum F_t = m a_t$

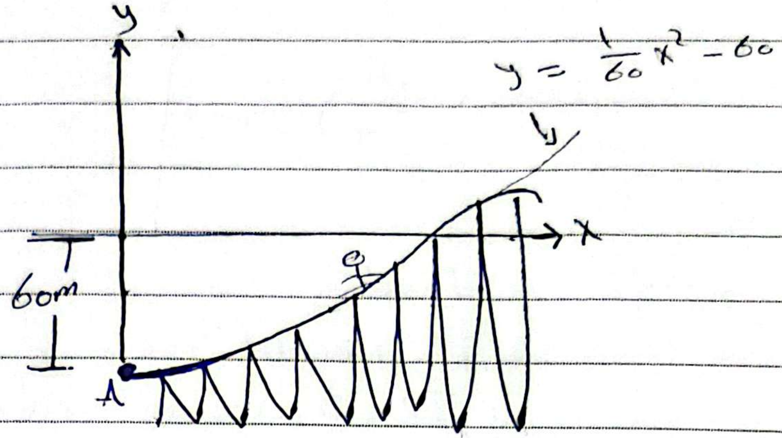
$mg\sin\theta - F_f = m a_t$

$800(9.81)\sin(-26.5) - F_f = 800(3)$

$F_f = -3501.7 - 800(3)$

$F_f = -5901.7 N$

Example 13.8

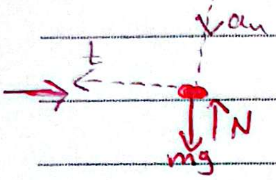


$m = 70\,kg$

$v_A = 20\,m/s$

$N = ?$

$a = ?$



$\Sigma F_n = \frac{mv^2}{\rho}$

$y' = \frac{2}{60}x = 0$

$y'' = \frac{1}{30}$

$N - mg = \frac{mv^2}{\rho}$

$\rho = 30m$

$N - 70(9,81) = \frac{70(400)}{30}$

$N = \frac{70(40)}{3} + 70(9,81)$

$\boxed{N = 1620\,N}$

$\Sigma F_t = ma_t$

$0 = ma_t$

$\boxed{a_t = 0}$

$a_n = \frac{v^2}{\rho} = \frac{400}{30} = 13,33\,m/s^2$

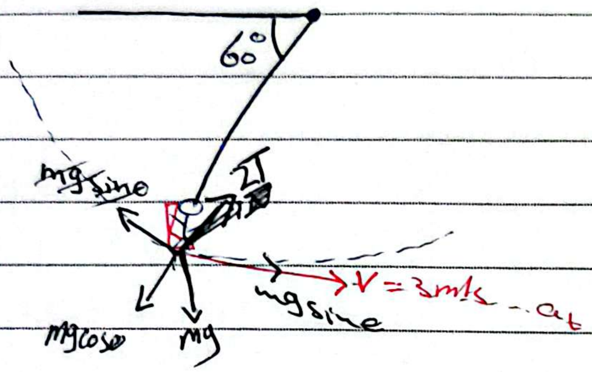
$a = \sqrt{0 + 13,33^2} = 13,33\,m/s^2$

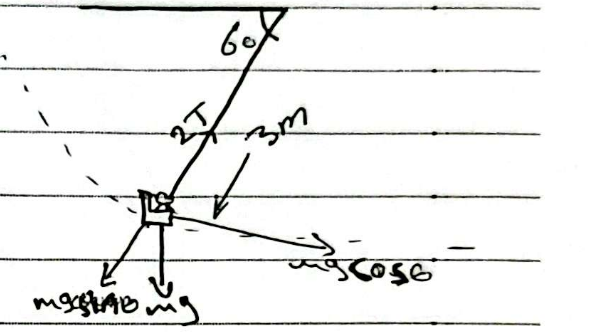
Chapter 13

Example: [1] Determine the increase in his speed (a_t)?

[2] The tinsion in the cable (T) ?

if $\theta = 60°$ and $V = 3\,m/s$ also his mass = 30,5 kg





$$\sum F_t = m a_t$$

$$\cancel{m} g \cos\theta = \cancel{m} a_t$$

$$(9.81)\cos(60°) = a_t$$

$\cancel{a_t = 8.49\,m/s^2}$ $a_t = 4,91\,m/s^2$

$$\sum F_n = m\frac{v^2}{\rho}$$

$$2T - mg\sin 60° = m\frac{v^2}{\rho}$$

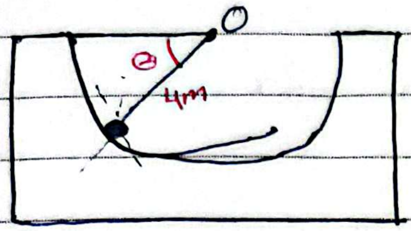
$$2T - 300\sin 60° = 30.5\left(\frac{9}{3}\right)$$

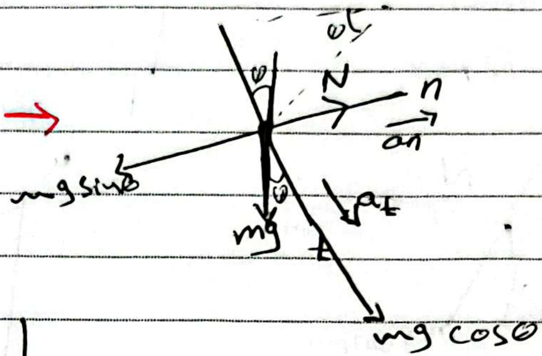
$$T = 175.8\,N$$

PRENKO

chapter 13

Example 13.9 : $m = 60$ $v_1 = 0 \rightarrow \theta_1 = 0$ $\theta_2 = 60$ $N = ?$





$\sum F_n = m a_n$

$N - mg\sin\theta = \frac{mv^2}{\rho}$

$N - (60)(9.81)\sin 60^\circ = \frac{60 v^2}{4}$

$N = \frac{60(68)}{4} + 60(9.81)\sin 60^\circ$

$\boxed{N = 1529.7 \, N}$

$\sum F_t = m a_t$ $s = r\theta$

$mg\cos\theta = m a_t$ $ds = r d\theta$

$a_t = g\cos\theta$

$a_t ds = v dv$

$a_t r d\theta = v dv$

$\int_0^{60} g r \cos\theta d\theta = \int_0^v v dv$

$\int_0^{60} 9.81(4)\cos\theta d\theta = \frac{1}{2}v^2$

$v^2 = 2(9.81)(4)\sin 60^\circ - 0$

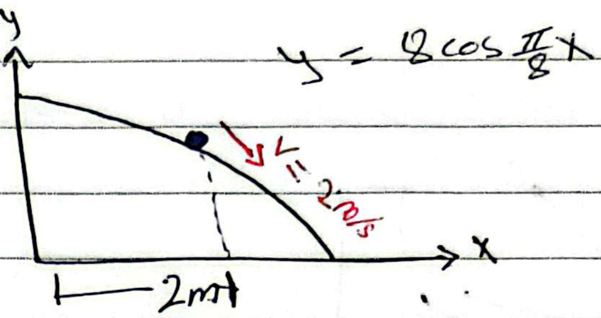
$v^2 = 68 m/s$

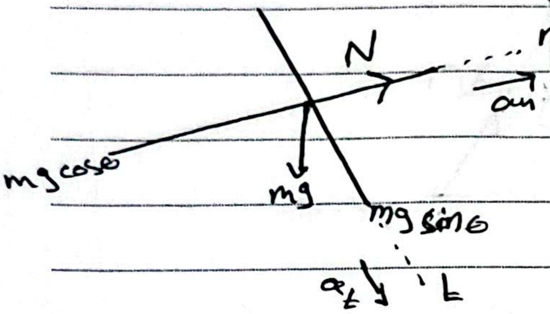
Chapter 13

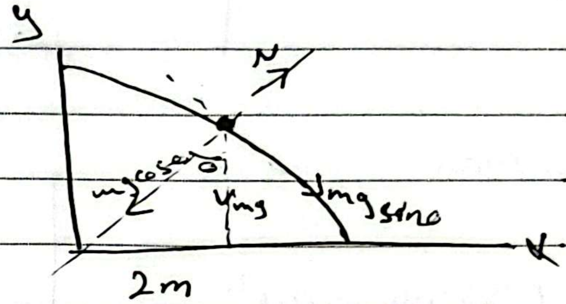
Ex: $W = 900N$

$m = 90kg$

Find N, a_t







$\sum F_n = m a_n$

$mg\cos\theta - N = \frac{mv^2}{\rho}$

$(900)\cos(65.7) - N = \frac{90(2)^2}{16.6}$

$N = 349N$

$\sum F_t = m a_t$

$mg\sin\theta = m a_t$

$a_t = g\sin\theta$

$a_t = (9.81)\sin(65.7)$

$a_t = 9.11\ m/s^2$

$y' = -\pi\sin\left(\frac{\pi}{8}x\right) = -2.22$

$y'' = -\frac{\pi^2}{8}\cos\left(\frac{\pi}{8}x\right) = -0.87$

$\rho = \frac{\left[1 + (y')^2\right]^{3/2}}{y''}$

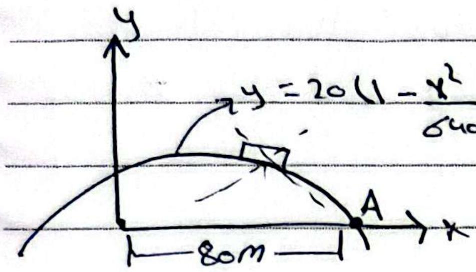
$\rho = \frac{\left[1 + (-2.22)^2\right]^{\frac{3}{2}}}{-0.87}$

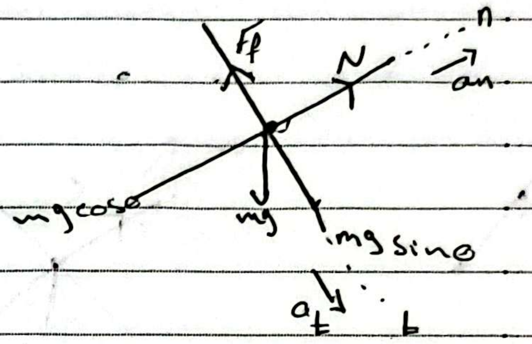
$\boxed{\rho = -16.6m}$

$\theta = \tan^{-1}(-2.22) = -65.7°$

*13.73 $m = 0.8 \times 10^{6} \times 10^{-3} = 800\,kg$ $V_A = 9\,m/s$ $a_t = 3\,m/s^2$

$N = ?$ $F_f = ?$





$\Sigma F_n = \frac{mv^2}{\rho}$

$y = 20 - \frac{20x^2}{6400}$

$y' = \frac{-40x}{6400} = -0.5$

$mg\cos\theta - N = \frac{mv^2}{\rho}$

$(800)(9.81)\cos(26.5) - N = \frac{800(81)}{103.8}$

$y'' = \frac{-40}{6400} = -6.25 \times 10^{-3}$

$\boxed{N = 6399.2\,N}$

$\rho = \frac{\left[1 + (-0.5)^2\right]^{3/2}}{-6.25 \times 10^{-3}}$

$\Sigma F_t = m a_t$

$\boxed{\rho = -103.8\,m}$

$mg\sin\theta - F_f = m a_t$

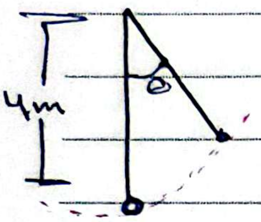
$800(9.81)\sin(26.5) - F_f = 800(3)$

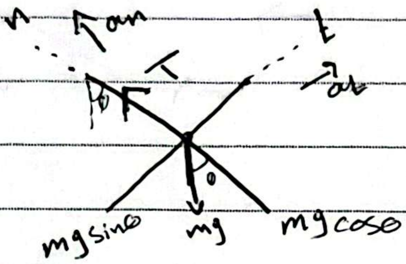
$\theta = \tan^{-1}(-0.5)$

$\boxed{F_f = 1101.7\,N}$

$\theta = -26.5°$

*13.76) The ball has a mass of 30kg and speed $v = 4m/s$ at the instant it at lowest point $\theta = 0°$. Determine the Tension in the cord and the rate at which the ball's speed is decreasing at the $\theta = 20°$





$\Sigma F_n = m a_n$

$T - mg\cos\theta = \frac{m v^2}{4}$

$T - 30(9.81)\cos 20° = \frac{30 v^2}{4}$

$\cancel{T - 30(9.81)\cos 20° = \frac{30(73.7)}{4}}$

$\cancel{T = 829.3}$

$T = 361N$

$\Sigma F_t = m a_t$

$-\cancel{m}g\sin\theta = \cancel{m}a_t$

$a_t = -9.81\sin\theta$

$a_t ds = v dv$

$a_t r d\theta = v dv$

$\int_0^{20} -9.81\sin\theta(4)d\theta = \int_4^v v dv$

$\cancel{\frac{1}{2}v^2 = +9.81(4)\cos 20° - 0}$

$\cancel{v^2 = 2(9.81)(4)\cos(20°)}$

$\frac{1}{2}v^2\Big|_4^v = 9.81(4)\cos 20° - 9.81(4)$

$\frac{1}{2}v^2 - 8 = 9.81(4)[\cos 20° - 1]$

$\cancel{v^2 = 2[8 + 9.81(4)\cos 20°]}$

$\cancel{v^2 = 89.7}$

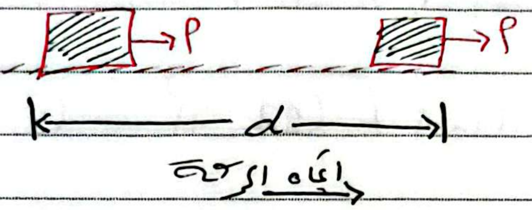
$\cancel{v = 9.47}$

$v = 3.857 m/s$

* Work and Energy:

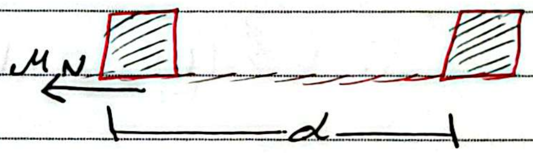
[1] work of external forces:

$W = \pm Pd$, (+) → نفس اتجاه الحركة , (−) → عكس اتجاه الحركة.



[2] work of friction force:

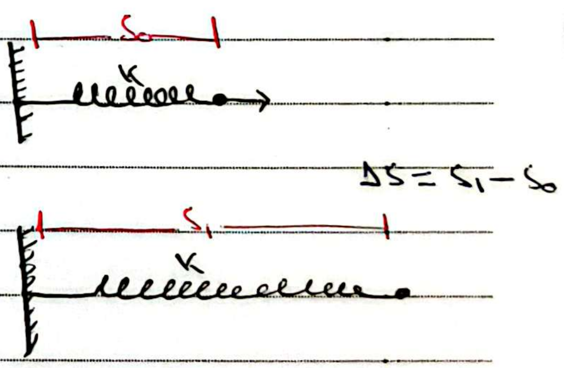
$W = -F_f d$ دايماً



[3] Work of weight:

$W = \pm mgh = \pm Wh$

[4] Work of Elastice elements:



work of spring $= \pm \frac{1}{2} K (S_2^2 - S_1^2)$

* Spring Energy from A to B:

$U_{A-B} = \frac{1}{2} K (S_A^2 - S_B^2)$ incorrect

$U_{A-B} = \frac{1}{2} K \left[(S_A - S_0)^2 - (S_B - S_0)^2 \right]$ correct

Chapter 14

~~Ex: K =~~

Example: Spring → $K = 100\ N/cm$

unstreched $L = 7cm$

initial L at $A = 12\ cm$

final L at $B = 20\ cm$

$\rightarrow U_{A-B} = \frac{1}{2}(100)(20^2 - 12^2)$ → wrong

$\rightarrow U_{A-B} = \frac{1}{2}(100)\left[(12-7)^2 - (20-7)^2\right]$

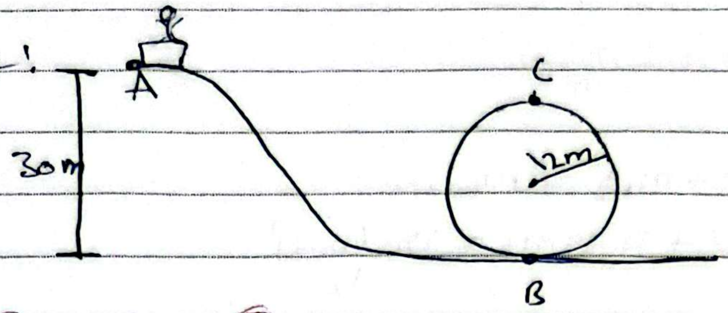
$= \frac{1}{2}(100)(5^2 - 13^2)$

$$T_1 + \sum U_{1-2} = T_2$$

↳ General law

Chapter 14

Example!



$m_{car} = 50$ kg
$m_{person} = 70$kg

Find : ① V_B , ② V_c , ③ N at Point B Between the boy and crate and ④ N at point c.

[1] V_B : $\cancel{T_A}^{0} + \sum U_{A-B} = T_B$

$$\cancel{m}gh = \frac{1}{2}\cancel{m}V_B^2$$

$$9.81(30)(2) = V_B^2$$

$$V_B = \sqrt{60(9.81)} = 24.3 m/s$$

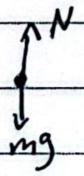
[2] V_c : $\cancel{T_A}^{0} + \sum U_{A-C} = T_c$

$$\cancel{m}gh = \frac{1}{2}\cancel{m}V_c^2$$

$$9.81(6)(2) = V_c^2$$

$$V_c = 10.8 m/s$$

[3] N_B



$$\sum F_n = ma_n$$

$$N - mg = \frac{mv^2}{\rho}$$

$$N = \frac{(70)(24.3)^2}{12} + 70(9.81)$$

$$N = 6591 \text{ Newton}$$

PRENKO

4

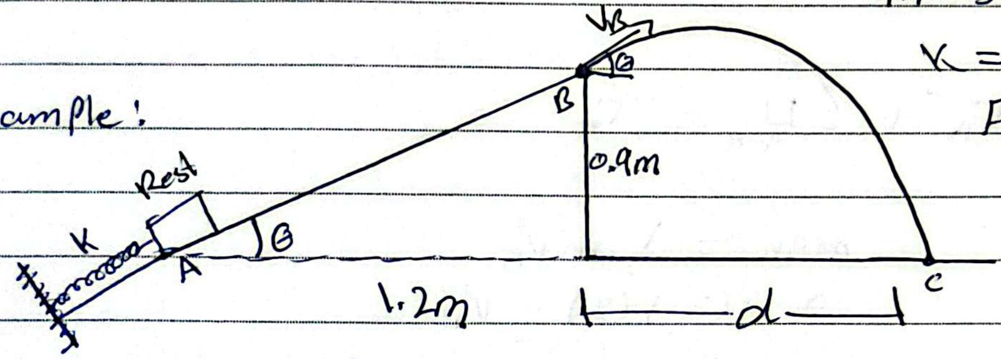
$$\sum F_n = m_{tot} a_n$$

$$N + m_1 g = m_{tot} a_n$$

$$N + 70(9.81) = \frac{120(10.8)^2}{12}$$

$$N = 480 N$$

Example!



$m = 5 kg$

$K = 2000$

Find d?

The spring originally compressed by 0.6m

$$\theta = \tan^{-1}\left(\frac{0.9}{1.2}\right) \rightarrow \theta = 36.8°$$

$$\cancel{T_A}^{0} + \sum U_{A-B} = T_B$$

$$\frac{1}{2}k(\Delta s)^2 - mgh = \frac{1}{2}(m)V_B^2$$

$$\frac{1}{2}(2000)(0.6)^2 - 5(9.81)(0.9) = \frac{1}{2}(5)V_B^2$$

$$\boxed{V_B = 11.24 m/s}$$

$$\Delta y = V_{oy}t - \frac{1}{2}gt^2$$

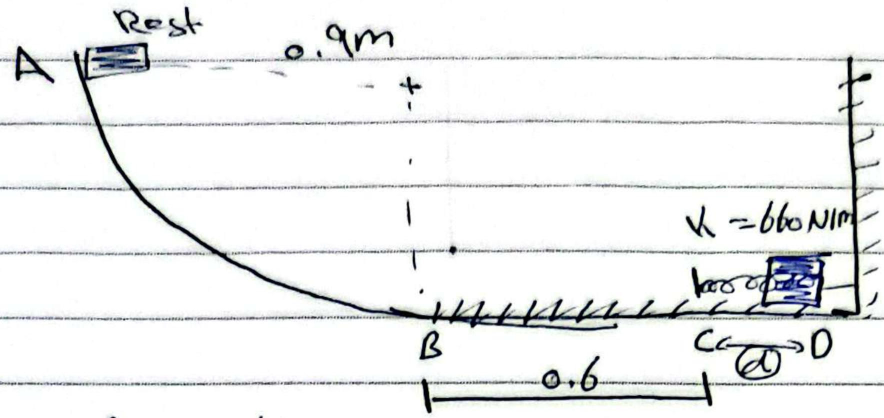
$$-0.9 = 11.24 \sin(36.8)\, t - \frac{1}{2}(9.81)t^2$$

$$t = 1.48 s$$

$$d = V_0 \cos\theta\, t$$

$$\rightarrow d = 11.24 \cos(36.8)(1.48) = 13.3 m/s$$

chapter 14



Example:

W = 25N

$\mu_k = 0.2$

* Find the compress in the spring.

$\Rightarrow \cancel{T_A}^{0} + \Sigma U_{A-D} = \cancel{T_D}^{0}$

$mgh - F_f(0.6 + d) - \frac{1}{2} K d^2 = 0$

$25(0.9) - (25)(0.2)(0.6+d) - \frac{1}{2}(660) d^2 = 0$

$22.5 - 5(0.6+d) - 330 d^2 = 0$

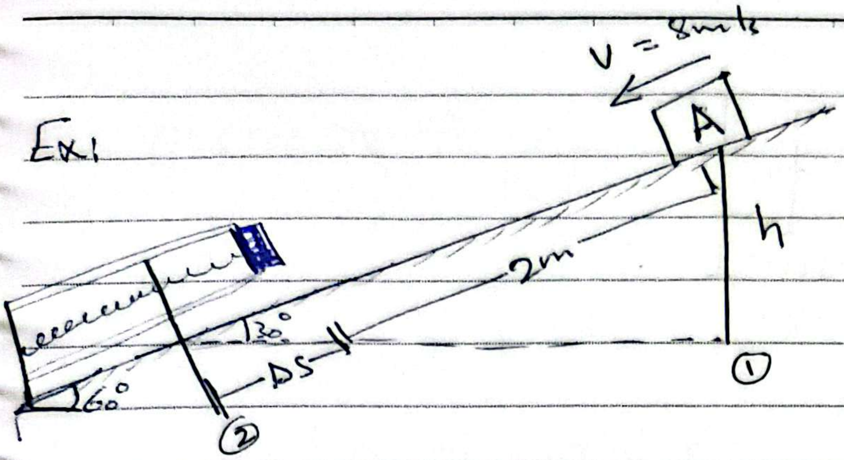
$22.5 - 3 - 5d - 330d^2 = 0$

$19.5 - 5d - 330d^2 = 0$

$\boxed{d = 0.24\,m}$

PRENKO

Ex1



$m_A = 4$

$K = 3000\ N/m$

$\mu = 0.25$

$\sin 30° = \frac{h}{2+\Delta S} \rightarrow h = (2+\Delta S)\sin 30°$

Find the value of ΔS?

$T_1 + \sum U_{1-2} = \cancel{T_2}^{0}$

$\cancel{\frac{1}{2} m V_A^2 + (mg)\mu(2+\Delta S) + mgh + \frac{1}{2}K(0.09^2 - (2+\Delta S)^2)\ 0}$

$\frac{1}{2} m V_A^2 - mg\mu(2+\Delta S) + mgh + \frac{1}{2}K(0.09)^2 - \frac{1}{2}K(0.09+\Delta S)^2 = 0$

$\frac{1}{2}(4)(64) - 4(9.81)(0.25)(2+\Delta S) + 4(9.81)(1+0.5\Delta S) + \frac{1}{2}(3000)(0.09)^2 - \frac{1}{2}(3000)(0.09+\Delta S)^2 = 0$

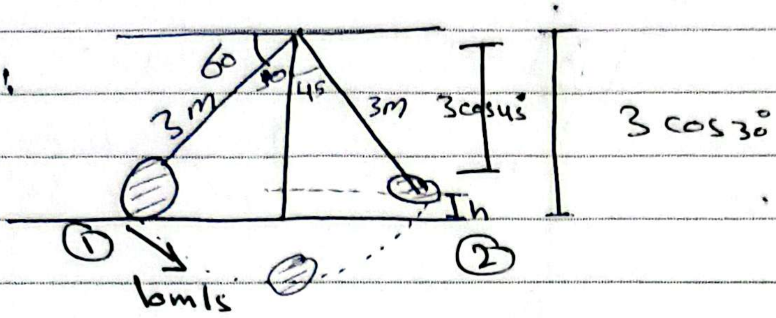
$128 - 19.62 - 9.81\Delta S + 19.62\Delta S + 39.24 + 12.2 - 1500(0.09+\Delta S)^2$

$108.38 - 9.81\Delta S + 19.62\Delta S + 39.24 + 12.2 - 1500(0.09^2 + 2...$

$\boxed{\Delta S = ...}$

chapter 14

Ex:



$M = 12\,Kg$

$\cos 30^\circ = \frac{x}{3} \rightarrow x = 3\cos 30^\circ$

$\cos 45^\circ = \frac{x}{3} \rightarrow x = 3\cos 45^\circ$

$\boxed{h = 3\cos 30^\circ - 3\cos 45^\circ}$

① Find the velocity at $\theta = 45^\circ$

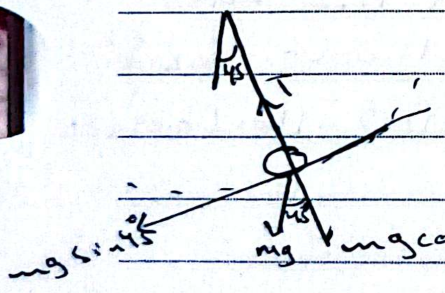
② The tension.

$T_1 + \sum U_{1-2} = T_2$

$\frac{1}{2}\cancel{m}v_1^2 + \cancel{m}gh = \frac{1}{2}\cancel{m}v_2^2$

$\frac{1}{2}(10)^2 - 9.81(3\cos 30^\circ - 3\cos 45^\circ) = \frac{1}{2}v_2^2$

$\boxed{V_2 = 9.517\ m/s}$



$\sum F_n = ma_n$

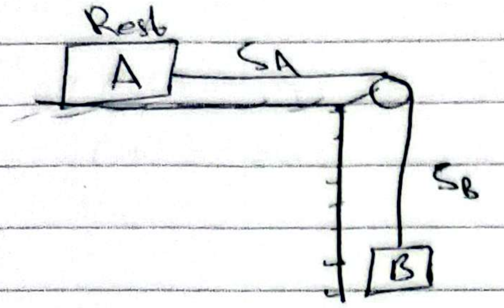
$T - mg\cos 45^\circ = 12\frac{(9.517)^2}{3}$

$T = (\ldots)\,N$

PRENKO

chapter 14

Ex:



$m_A = 200\,kg$

$m_B = 300\,kg$

$\mu = 0.25$

Find the velocity of A after it moves 2m to the Right.

$L = S_A + S_B$

$V_A + V_B = 0$

$\boxed{V_A = -V_B}$

$\boxed{\text{Block A}}$ $\quad \overset{0}{T_1} + \Sigma U_{1\text{-}2} = T_2$

$m_B g h - m_A g \mu d = \frac{1}{2} m_A V_A^2 + \frac{1}{2} m_B V_B^2$

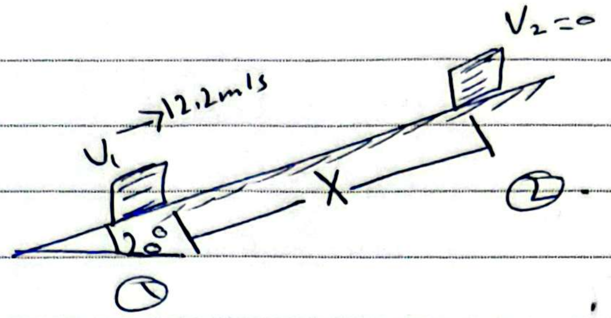
$(300)(9.81)(2) - (200)(9.81)(0.25)(2) = \frac{1}{2}(200)V_A^2 + \frac{1}{2}(300)(-V_A)^2$

$5886 - 981 = 100V_A^2 + 150V_A^2$

$V_A^2 = 19.62 \Rightarrow V_A = 4.4\,m/s$

PRENKO

Ex:



$m = 22.7$ kg

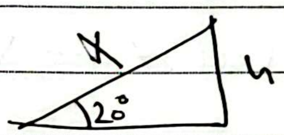
$\mu = 0.15$

[1] The distance (X).

[2] the velocity of the box when it return to its original position

[3] total amount of energy dissipated due to friction.

[1] $T_1 + \sum U_{1-2} = \cancel{T_2}^{\,0}$



$\frac{1}{2} m v^2 - mgh - mg\mu X = 0$ $\qquad h = X \sin 20°$

$\frac{1}{2}(22.7)(12.2)^2 - 22.7(9.81)(X \sin 20°) - 22.7(9.81)(0.15)X = 0$

$1689.3 - 76.2X - 33.4X = 0$

$\boxed{X = 15.4 m}$

[2] $\cancel{T_1}^{\,0} + \sum U_{1-2} = T_2$

1 2
بالعكس
من فوق لتحت

$-mg\mu X + mg(X \sin 20) = \frac{1}{2} m v_2^2$

$-22.7(9.81)(0.15)(15.4) + 22.7(9.81)(15.4)(\sin 20°) = \frac{1}{2}(22.7)v_2^2$

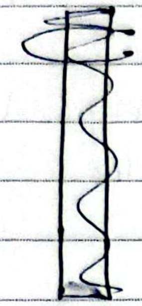
$658.5 = 0.5(22.7)v_2^2$

$v_2^2 = 58.01 \rightarrow v_2 = 7.61 m/s$

PRENKO

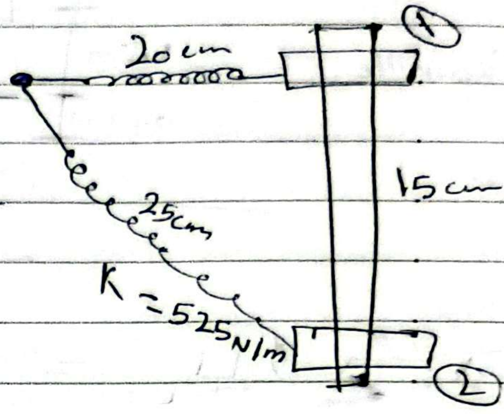
[3] $\mu mgd = 0.15(22.7)(9.81)($

Example:



m = 9 kg

the original length of the spring is 10 cm



find $V_2 = ?$

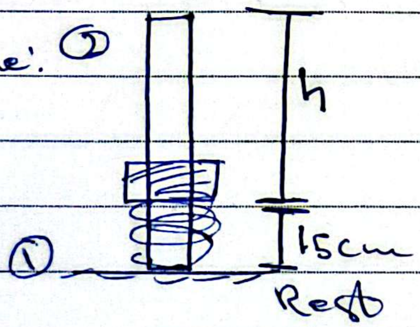
$$\cancel{T_1}^{0} + \sum U_{1-2} = T_2$$

$$mgh + \frac{1}{2}K(20-10)^2 - \frac{1}{2}K(25-10)^2 = \frac{1}{2}mv_2^2$$

$$(9)(9.81)(0.15) + \frac{1}{2}(525)(0.1)^2 - \frac{1}{2}(525)(0.15)^2 = \frac{1}{2}(9)v_2^2$$

$$9.96 = 4.5v_2^2 \rightarrow v_2^2 = 2.21 \rightarrow V_2 = 1.48\,m/s$$

Example: 2



M = 2.71 kg

K = 2627 N/m

find h.

$$\cancel{T_1}^{0} + \sum U_{1-2} = \cancel{T_2}^{0}$$

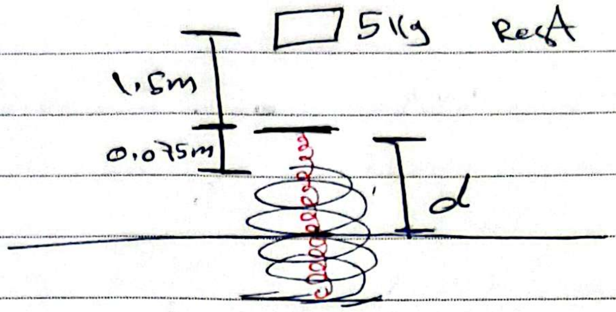
$$-mgh + \frac{1}{2}K(0.15)^2 = \cancel{\frac{1}{2}mv_2^2}^{0}$$

$$-(2.71)(9.81)(h+0.15) + \frac{1}{2}(2627)(0.15)^2 = 0$$

$$h = 1.12m$$

chapter 14

Example:



red ; K = 6000 N/m

blue ; K = 9000 N/m

$$\cancel{T_1}^{0} + \Sigma U_{1-2} = T_2^{0}$$

$$mg(1.5+d) - \frac{1}{2}(K)(d)^2 - \frac{1}{2}K(d-0.075)^2 = 0$$

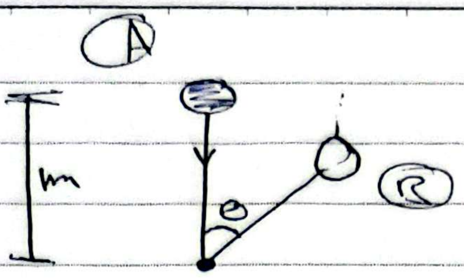
$$5(9.81)(1.5+d) - 0.5(6000)d^2 - 0.5(9000)(d^2 - 2(0.075d) + 0.075^2) = 0$$

$$73.5 + 49.05d - 3000d^2 - 4500d^2 + 675d - 25.3 = 0$$

$$48.2 + 724.05d - 7500d^2 = 0$$

$$\boxed{d = 0.14\,m}$$

Example:

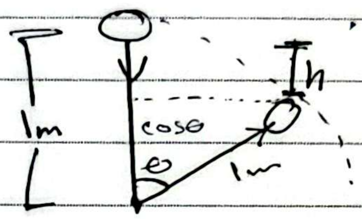


$m = 7.5\ kg$

$\theta_1 = 0 \quad V_A = 0$

اوجد θ عند الزاوية التي يصبح عندها القوة صفرًا.

$$\cancel{T_A}^{0} + \Sigma U_{A-B} = T_B$$



$$\boxed{h = 1 - \cos\theta}$$

$\cos\theta = \frac{x}{1}$

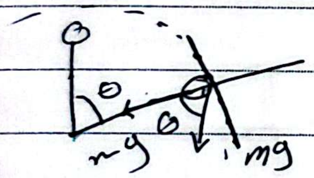
$x = \cos\theta$

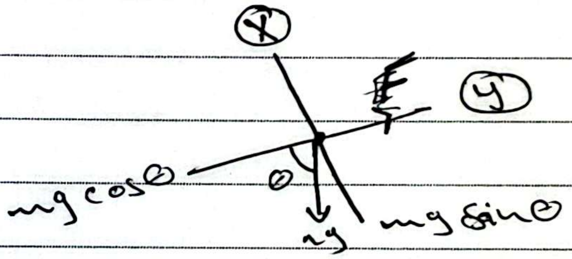
$$mgh = \frac{1}{2} m V_B^2$$

$$(7.5)(9.81)(1 - \cos\theta) = \frac{1}{2}(7.5) V_B^2$$

$$73.5(1 - \cos\theta) = 3.75\, V_B^2$$

$$V_B^2 = 19.62(1 - \cos\theta)$$





$$\Sigma F_n = m a_n$$

$$\cancel{m} g \cos\theta = \frac{\cancel{m} V^2}{1}$$

$$g \cos\theta = V_B^2$$

$$\cancel{g} \cos\theta = 2\cancel{g}(1 - \cos\theta)$$

$$\cos\theta = 2 - 2\cos\theta$$

$$3\cos\theta = 2$$

$$\theta = \cos^{-1}\left(\frac{2}{3}\right)$$

$$\boxed{\theta = 48.1^\circ}$$

chapter

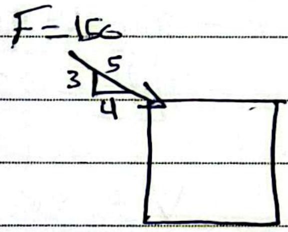
$$P = \frac{dU}{dt} = \frac{F \cdot dr}{dt} = F \cdot \frac{dr}{dt} = F \cdot V$$

$$P = F \cdot V \quad \text{output}$$

* Efficiency (ε):

$$\varepsilon = \frac{\text{Power output}}{\text{Power input}}$$

Example: 14.7

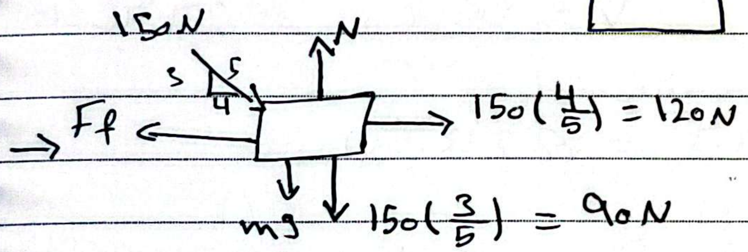


$m = 50\,kg$

$P_{output} = ?$ $t = 4s$

$\mu_k = 0.2$

$V_i = 0$



$P = F \cdot V$

$\cancel{\Sigma F_x = ma}$

$\cancel{120 - N\mu = 0}$

$\cancel{120 - N(0.2) = 0}$

$\cancel{N = \frac{120}{0.2}} =$

$\Sigma F_x = ma$

$120 - N\mu = ma$

$120 - 580.5(0.2) = 50a$

$a = 0.078\, m/s^2$

$V_2 = \cancel{V_1}^{0} + at$

$V_2 = 0.078(4)$

$\boxed{V_2 = 0.312\, m/s}$

$\Sigma F_y = 0$

$N - 90 - (50)(9.81) = 0$

$\boxed{N = 580.5\,N}$

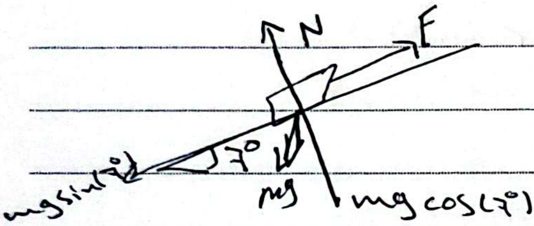
$P = 150\,(0.312) * \frac{4}{5}$

$P = 37.4\,W$

PRENKO

Ex: A car having a mass of 2Mg travels up a 7° slope at a constant speed of V = 100 km/h, if the friction is neglected determine the power developed by the engine if the car has efficiency $\varepsilon = 0.65$

$$\varepsilon = \frac{P_{output}}{P_{input}} \qquad P = F.V$$



$$\Sigma F_x = ma_x$$

$$F - mg\sin(7^\circ) = ma_x$$

$$F - 2000(9.81)\sin(7^\circ) = 2000\cancel{a_x}^{\,0}$$

$$\boxed{F = 2391.1\,N}$$

$$V = \frac{100 \times 10^3}{3600} = 27.7\,m/s$$

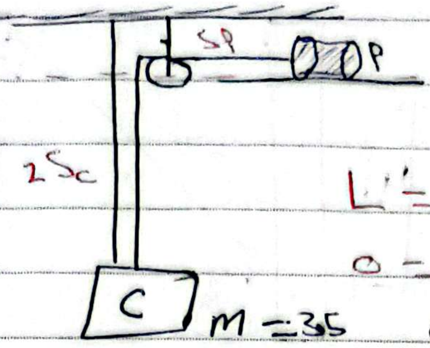
$$P = F.V = 2391.1 * 27.7 = 66233.5\,w$$

$$0.65 = \frac{66233.5}{P_{input}} \rightarrow P_{input} = 101897.69\,w$$

$P_{input} > P_{output}$

$\rightarrow$ always

Example: 14.8



$a_P = 1.2\ m/s^2$

$V_P = 0.6\ m/s$

$L' = 2S_c + S_P$

$0 = 2V_c + V_P$

$m = 35$

$\varepsilon = 0.85$

P_{output}

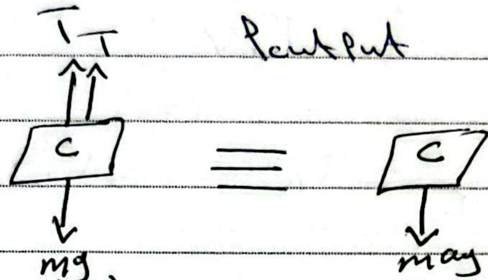
$0 = 2a_c \pm a_P$

$2a_c = -a_P$

$a_c = -0.5 a_P$

$P = F \cdot V_P$

$\Sigma F_y = m a_y$



$mg - 2T = m a_y$

$35(9.81) - 2T = 35 a_c$

$35(9.81) - 2T = 35(-0.5 a_P)$

$35(9.81 + 35(0.5)(1.2) = 2T$

$\boxed{T = 182.2\ N}$

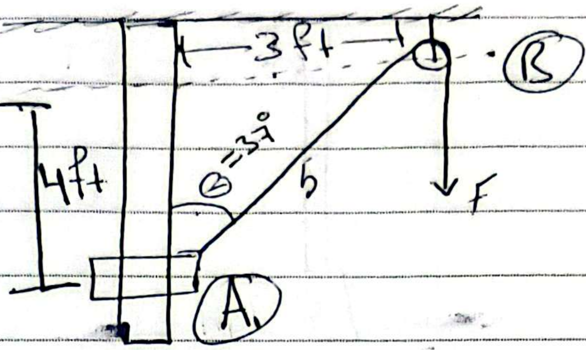
$P = 182.2(0.6)$

$\boxed{P = 109.3\ W}$

$0.85 = \frac{109.3}{P_{input}}$

$\boxed{P_{input} = 128.5\ W}$

* 14-56



$W = 10\,lb$

$V_A = 0$

$F = 25\,lb$

$\theta = 60$

$\rightarrow P = ?$ $\qquad P = F.V$

$$\overset{0}{T_A} + \sum U_{A-B} = T_B$$

$60^\circ = \theta$ عند

1.7m أوجد الارتفاع

$$-mgh + 25(5-3.46) = \frac{1}{2}mv^2$$

~~60° = tan (x)~~

~~tan tan~~

$$-10(2.27) + 25(1.54) = 0.5\left(\frac{10}{32.2}\right)v^2$$

~~x = √3 = 1.7~~

$$\boxed{V = 10.06\,ft/s}$$

3

$1.73 = X$ $\quad 60 \quad y$

$$P = F.V$$

$$P = 25\cos 60^\circ (10.06)$$

$$60 = \tan\left(\frac{3}{x}\right)$$

$$\sqrt{3} = \frac{3}{x}$$

$$\boxed{x = \frac{3}{\sqrt{3}} = 1.73\,m}$$

$$h = 4 - 1.73$$

$$\boxed{h = 2.27\,m}$$

$$y^2 = 1.73^2 + 9$$

$$\boxed{y = 3.46}$$

PRENKO

chapter 15

* momentum:

$$\vec{L_1} + \Sigma \vec{I}_{12} = \vec{L_2}$$

$$T_1 + \Sigma U_{1-2} = T_2$$

$$m\vec{v_1} + \int_{t_1}^{t_2} \vec{F} dt = m\vec{v_2}$$

x-direction: $m v_{1x} + \int_{t_1}^{t_2} \vec{F_x} dt = m v_{2x}$

y-direction: $m v_{1y} + \int_{t_1}^{t_2} F_y dt = m v_{2y}$

$I = F \Delta t$

$L = mv$

نفس الوحدة

Kg.m/s

conversation of momentum:

$$(m v_1)_A + (m v_1)_B = (m v_2)_A + (m v_2)_B$$

Ex: A Bullet of mass 2.5 g starts from rest. A force F applied to this bullet so the speed reaches 450 m/s in 0.75 ms.

→ $m = 2.5 \times 10^{-3}$ Kg $v_1 = 0$ $v_2 = 450$ $\Delta t = 0.75 \times 10^{-3}$ $F = ?$

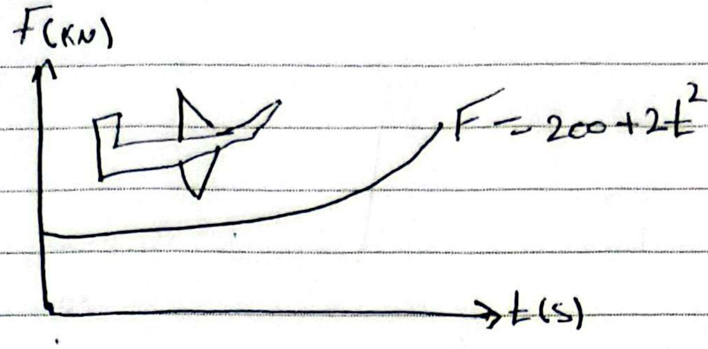
$$\cancel{m v_1}^{0} + F \Delta t = m v_2$$

$$F(0.75 \times 10^{-3}) = (2.5 \times 10^{-3})(450)$$

$$\boxed{F = 1500 \text{ Newton}}$$

chapter 15

Example:



$m = 250\,Mg$, at $t=0 \rightarrow V_\cdot = 100\,m/s$ and move horizentaly

the engine provides horisotal thrust $F = 200+2t^2$
find the velocity in $t = 15\,sec$.

$\rightarrow m = 25\times10^4\,kg \qquad V_1 = 100\,m/s \qquad F = (200+2t^2)\times10^3$

$\Delta t = 15\,sec. \qquad V_2 = ?$

$$mV_1 + \int_{t_1}^{t_2} F\,\cancel{dt} = mv_2$$

$$(25\times10^4)(100) + \int_0^{15}(200+2t^2)\times10^3 = (25\times10^4)V_2$$

$$(25\times10^6) + \left(200t + \frac{2t^3}{3}\right)\times10^3 \Big|_0^{15} = 25\times10^4 V_2$$

$$25\times10^6 + \left[200(15) + \frac{2}{3}(15)^3\right]\times10^3 = 25\times10^4 V_2$$

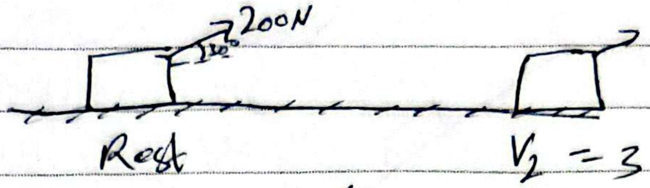
$$\cancel{V_2 =}\ \cancel{10^3}\left(25\times10^3 + 200(15) + \frac{2}{3}(15)^3\right) = 25\,\cancel{\times10^4}\,v_2$$

$$250V_2 = 30250$$

$$V_2 = 121\,m/s$$

Chapter 15

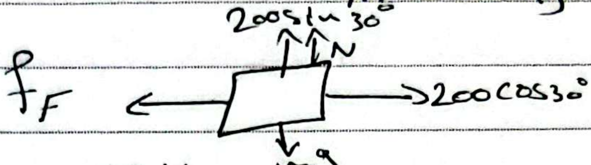
Example:



find the velocity at $\Delta t = 8$ sec.

$\mu = 0.25$

$m = 30$ Kg



$m V_{1x} + \sum F_x \Delta t = m V_{2x}$

احركة على x

بنعتبر الطاقة على x

$30(0) + [200\cos 30° - N(0.25)](8) = 30 V_2$

$\sum F_y = 0$

$200\sin 30° + N = 30(9.81)$

$N = 30(9.81) - 200\sin 30°$

$\boxed{N = 194.3\ N}$

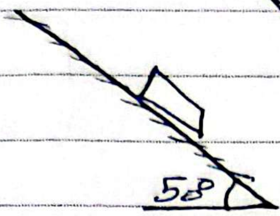
$(200\cos 30° - 194.3(0.25))\, 8 = 30 V_2$

$\boxed{V_2 = 33.2\ m/s}$

chapter 15

Rough surface.

Example:



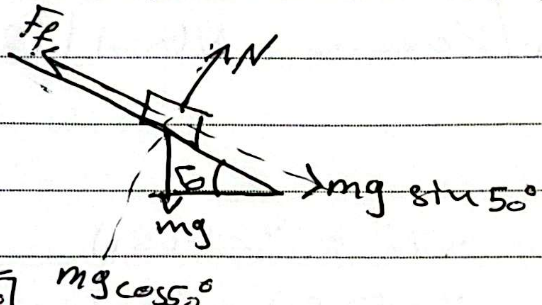
$m = 1814.54\ kg$

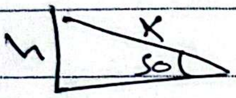
$V = 96.54\ km/h$

a car moves at $V = 96.54\ km/h$, suddenly a break force applied, the breaking force is 6572N.

[1] find the time needed for the car to stop.

[2] find the distance untile the car stops.





$\sin 50° = \frac{h}{x}$

$\boxed{h = x \sin 50°}$

$m v_1 + \sum F \Delta t = m v_2$

$(1814.54)(26.8) + [(1814.54)(9.81)\sin 50° - 6572]\Delta t = 1814.54 \cancel{V_2}$

$\boxed{\Delta t = 9.5\ sec}$

[2] $T_1 + \sum U_{1-2} = T_2$

$\frac{1}{2}(1814.54)(26.8)^2 + (1814.54)(9.81)(x \sin 50°) - 6572x = 0$

$651637.6 + 13636.1x - 6572x = 0$

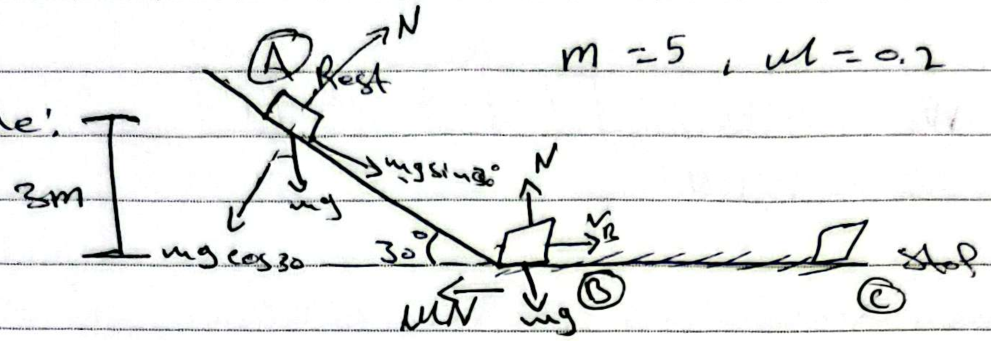
$651637.6 + 7064.1x = 0$

PRENKO

Chapter 15

Example:

$m = 5$, $\mu = 0.2$



Find the total time until it stoped.

بدنا الزمن عند الوقوف

$$\cancel{T_A}^{\,0} + \sum U_{A-B} = T_B$$

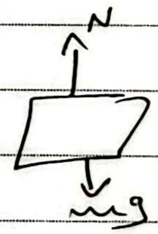
$$mgh = \frac{1}{2} m v_B^2$$

$$(5)\,9.81(3) = \frac{1}{2}(5)\,v_B^2 \longrightarrow \boxed{V_B = 7.67\,\text{m/s}}$$

$$mv_A + \sum F.\Delta t = mv_B \qquad x\text{-direction}$$

$$(5)\cancel{(0)} + (5)(9.81)\sin(30^\circ)\,t_1 = 5(7.67)$$

$$\boxed{t_1 = 1.56\ \text{sec}}$$



$$mv_B + \sum F\Delta t = m\cancel{v_C}^{\,0}$$

$$(5)(7.67) - \mu N t_2 = 0$$

$$(5)(7.67) - 0.2(5)(9.81)\,t_2 = 0$$

$$\boxed{t_2 = 3.91\ \text{sec}}$$

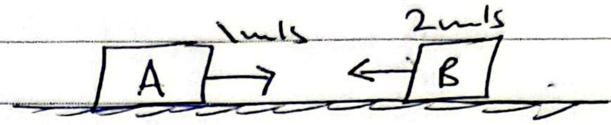
$$t_{total} = (1.56 + 3.91)\ \text{sec}$$

$$\boxed{t_{tot} = 5.47\ \text{sec}}$$

Chapter 15

Example: $W_A = 22.5\,KN$

$W_B = 15\,KN$



find their Velocities after collision if they becomes together.

$\rightarrow (mV_1)_A + (mV_1)_B = (mV_2)_A + (mV_2)_B$

$W_A = 22.5 \times 10^3 = mg$

$$m_A = \frac{22.5 \times 10^3}{9.81} = 2293.5\,Kg$$

$$m_B = \frac{15 \times 10^3}{9.81} = 1529.1\,Kg$$

$$(2293.5)(1) + (1529.1)(-2) = V(1529.1 + 2293.5)$$

$$-764.7 = 3822.6\,V$$

$$V = -0.2\,m/s$$

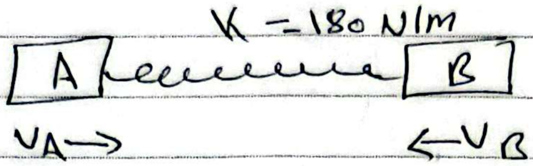
$\rightarrow -x$

$\leftarrow$

$$\boxed{V = \overset{\leftarrow}{0.2}\,m/s}$$

Chapter 15

Example:



$m_A = 40\,kg$

$m_B = 60\,kg$

$\Delta s = 2m$

* Find the Velocities of A and B when the spring becomes unstreched.

$\rightarrow \quad 0 = (mV_2)_A + (mV_2)_B$

$$\boxed{0 = 40V_{2A} + 60V_{2B}}$$

$0 = 2V_2A + 3V_2B \longrightarrow 2V_{Af} = -3V_{Bf}$

$V_{Af} = -\frac{3}{2}V_{Bf}$

$\cancel{T_1}^{0} + \sum U_{1-2} = T_2$

$\cancel{\tfrac{1}{2}} K\Delta s^2 = \cancel{\tfrac{1}{2}} m V_{2A}^2 + \cancel{\tfrac{1}{2}} m V_{2B}^2$

$\frac{180(2)^2 = 40V_{2A}^2 + 60V_{2B}^2}{20}$

$36 = 2V_2^2A + 3V_2^2B$

$36 = 2\left(-\frac{3}{2}V_{Bf}\right)^2 + 3V_{Bf}^2$

$\left(36 = \frac{9}{4}V_{Bf}^2 + 3V_{Bf}^2\right)4$

$144 = 9V_{Bf}^2 + 12V_{Bf}^2$

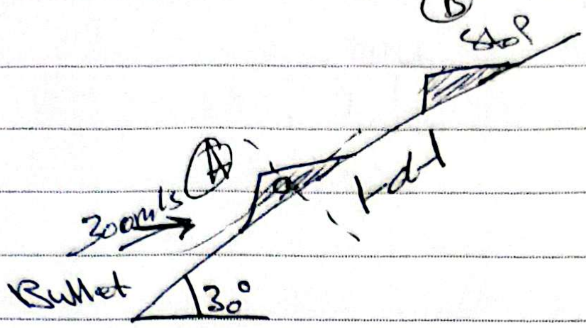
$$\boxed{V_{Bf} = -2.19\,m/s}$$

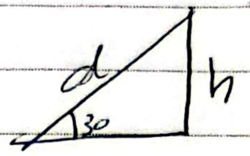
$V_{Af} = 3.29\,m/s$

PRENKO

chapter 15

Example:





$\frac{h}{d} = \sin 30$

$h = d \sin 30$

$m_b = 10g = 10^{-2}\ Kg$

$m_a = 10 Kg$

$V_b = 300\ m/s$

Find the distance d until the Box stops.

$\Rightarrow m_a \overset{0}{\cancel{V_{a1}}} + m_b V_{b1} = (m_a + m_b) V$

$\cos 30° (10^{-2})(300) = (10^{-2} + 10) V$

$V = \frac{3 \cos 30°}{10{,}01} = 0{,}259\ m/s$

$T_A + \Sigma U_{A-B} = T_B$

$\frac{1}{2} m_a V_a^2 - mgh = 0$

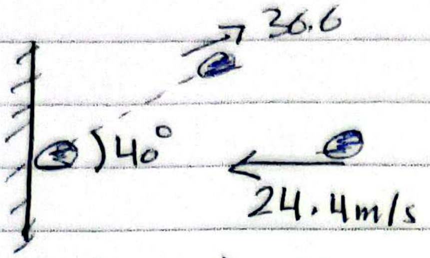
$\frac{1}{2}(10)(0{,}259)^2 - 10(9{,}81)(0{,}5d) = 0$

$98{,}1(0{,}5)d = 5(0{,}259)^2$

$\boxed{d = 6{,}8 \times 10^{-3}\ m}$

Chapter 15

Example:



$m = 113.4g$

$m = 0.1134\ kg$

The wall and the ball was in contact for $\Delta t = 0.015 sec$

Find the average impulsive force during impact.

x-direction:

$$mv_1 + \sum F\Delta t = mv_2$$

$$(0.1134)(-24.4) + F_x(0.015) = (0.1134)(36.6 \cos(40°))$$

$$-2.76 + 0.015 F_x = 3.17$$

$$\boxed{F_x = 395.3 N} \rightarrow (+x)$$

y-direction:

$$mv_1 + \sum F\Delta t = mv_2$$

$$(0.1134)(\cancel{36.60}) + F_y(0.015) = (0.1134)(36.6 \sin 40°)$$

$$\boxed{F_y = 177.8N \uparrow}$$

$$F = \sqrt{(395.3)^2 + (177.8)^2} = \boxed{\begin{array}{l} 433.4\ N \\ \theta = 24.2° \end{array}}$$

$F =$

PRENKO

Chapter 15

$\sum \vec{F} = m\vec{a} = m\dot{\vec{v}}$

$M = \vec{r} \times \sum \vec{F} = \vec{r} \times m\dot{\vec{v}}$

$H_o = \vec{r} \times m\vec{v}$

$\frac{d(H_o)}{dt} = \dot{\vec{r}} \times m\vec{v} + \vec{r} \times m\dot{\vec{v}}$

$\dot{\vec{H}}_o = \dot{\vec{r}} \times \cancel{m\vec{v}}^{\,0} + r \times m\dot{\vec{v}}$

$\dot{\vec{H}} = \vec{r} \times m\dot{\vec{v}}$ $\boxed{\dot{\vec{H}} = M_o}$

Angular Impulse:

$(H_o)_1 + \sum M_o\,dt = (H_o)_2$

chapter 16

* The types of motion of rigid bodies:

[1] Translation only

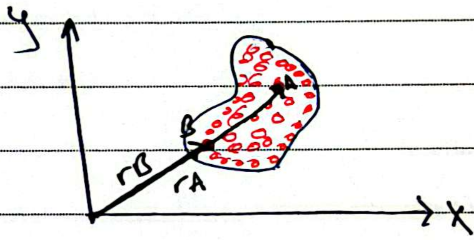
[2] Rotation only

[3] Translation + Rotation

[4] Rotation about point

[5] Space motion

1) Translation only:



$\vec{r}_B = \vec{r}_A + \vec{r}_{B/A}$

بدنا نشوف بمرور زمن

$\vec{V}_B = \vec{V}_A + \frac{d}{dt}(\vec{r}_{B/A}) \rightarrow \boxed{\vec{V}_B = \vec{V}_A}$, $\frac{d}{dt}(\vec{r}_{B/A}) = 0$

لان الاجسام الصغيرة في الجسم الصلب
ثابتة (المسافة بينها ثابتة) بالتالي
مع مرور الزمن الموقع للاجسام نفسه

$\therefore \boxed{\vec{a}_B = \vec{a}_A}$

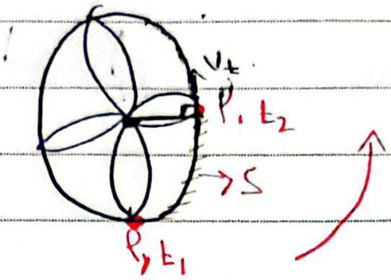
→ During translation → the rigid body can be treated as a particle, all particle on it have the same Kinematics.

$V_x = V_{o_x} + at$

$V_x^2 = V_{o_x}^2 + 2a\Delta x$

$\Delta x = V_{o_x}t + \frac{1}{2}at^2$

2) Rotation about a fixed axis:

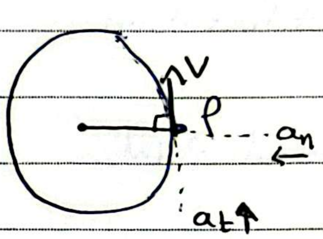


$S = r\theta$

بنشتق الطرفين بالنسبة للزمن

$\frac{d(S)}{dt} = r\frac{d\theta}{dt}$

$\boxed{V_t = r w}$



$a_t = r\frac{d(w)}{dt} \rightarrow \boxed{a = r\alpha}$

$a_n = \frac{v^2}{r} = \frac{r^2 w^2}{r} = r w^2$

$\boxed{a_n = r w^2}$

$a = \sqrt{a_t^2 + a_n^2}$

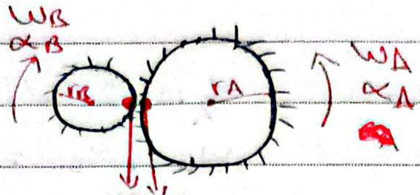
$w = w_0 + \alpha t$	$w = w_0 + \alpha t$
$w^2 = w_0^2 + 2\alpha \Delta X$	$w^2 = w_0^2 + 2\alpha \Delta\theta$
$\Delta X = w_0 t + \frac{1}{2}\alpha t^2$	$\Delta\theta = w_0 t + \frac{1}{2}\alpha t^2$
Worng!!	correct

$\alpha(t) = \frac{dw}{dt}$

$\alpha\, d\theta = w\, dw$

Chapter 16

* Gears:



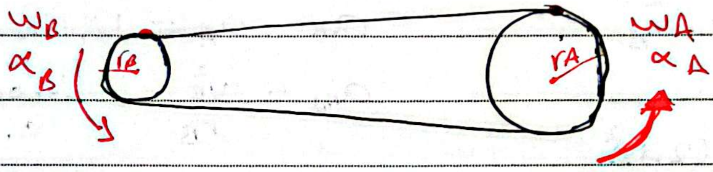
vp vp are equalse

$vp_{at_B} = vp\ at_A$

$$\boxed{\omega_B r_B = \omega_A r_A} = V_t$$

$$\boxed{\alpha_A r_A = \alpha_B r_B} = a_{p_t}$$

* Pulleys:



$$r_A \omega_A = r_B \omega_B$$
$$r_A \alpha_A = r_B \alpha_B$$

Example: 16.1

→ $a = 4t$ $\quad \theta_1 = 0$ $\quad \omega_1 = 0$ $\quad \omega_2 = ?$ $\quad \theta_2 = ?$

$4t = r\alpha$

$\frac{4t}{0.2} = \frac{0.2\alpha}{0.2}$

$$\boxed{\theta = \int_0^{\theta} d\theta = \left(\frac{10}{3} t^3\right) rad}$$

$\alpha = (20t)\ rad/s^2$

$$\boxed{\omega = \int_0^t \alpha = (10t^2)\ rad/s}$$

PRENKO

chapter 16

Example 16.2 $\quad w_1 = 0 \quad \theta = 0 \quad \alpha_A = 2\ rad/s^2$

$w_B,\ \alpha_B = ?$

$2\ rev = 2 \times 2\pi = 4\pi = \theta_2 \quad a_p = ? \quad V_p = ?$

$w_A^2 = w_{oA}^2 + 2\alpha_A \Delta\theta$

$w_A^2 = 0 + 2(2)(4\pi)$

$\boxed{w_A = 7.1\ rad/s}$

$V_B = V_A = r_A w_A$

$V_B = (0.15)(7.1)$

$\boxed{V_B = 1.065\ m/s}$

$a_B = a_A = r_A \alpha_A = (0.15)(2) = 0.3\ m/s^2$

$a_{n_B} = \frac{V_B^2}{r_B} = \frac{(1.065)^2}{0.4} = 2.83\ m/s^2$

$a_p = \sqrt{a_{B_t}^2 + a_{n_B}^2}$

$a_p = \sqrt{(0.3)^2 + (2.83)^2}$

$\boxed{a_p = 2.84\ m/s^2}$

find $\alpha_B,\ w_B$

$\Rightarrow \quad a_B = a_A = 0.3$

$r_B \alpha_B = 0.3$

$\alpha_B = 0.75\ rad/s^2$

$V_B = r_B w_B = V_A = 1.065$

$r_B w_B = 1.065$

$w_B = 2.66\ rad/s$

Chapter 16

F16-6: $\alpha_A = 4.5\ rad/s^2$ $\quad \omega_{A_0} = 0 \quad \omega_{B_0} = 0 \quad \omega_{D_0} = 0$

$r_A = 0.075 \quad r_B = 0.225 \quad r_D = 0.125$

$V_c = ?$ $\quad t = 3s$

$V_A = V_B$

$\cancel{V_c = V_D = V_B}$

$r_A \omega_A = r_B \omega_B \rightarrow (0.075)(13.5) = V_B$

$\cancel{V_c = r_B \omega_D = r_B \omega_B}$

$\omega_A = \omega_{0A} + \alpha_A t$

$\cancel{V_c = 0.125\,\omega_D = (0.225)}$

$\omega_A = 0 + 4.5(3)$

$V_c = V_D$

$\boxed{\omega_A = 13.5\ rad/s}$

$V_c = r_D \omega_D \quad / \quad \omega_D = \omega_B$

$V_A = V_B$

$\omega_B = \frac{r_A}{r_B}\omega_A \rightarrow \omega_B = \frac{0.075}{0.225}(13.5) = 4.5\ rad/s$

$V_c = r_D \omega_B = 0.125(4.5) = \boxed{0.5625\ m/s}$

$\Delta y_c = ? \rightarrow \Delta y = V_c t + \frac{1}{2} a_c t^2$

$\Delta y = (0.5625)(3) + 0.5(0.1875)(3)^2$

$\boxed{\Delta y = 2.53\ m}$

$a_c = ? \quad a_c = a_D$

$a_c = r_D \alpha_D \quad / \quad \alpha_D = \alpha_B$

$a_c = (0.125)(1.5)$ $\quad \omega_B = 0 + \alpha_B t$

$a_c = 0.1875\ m/s^2$ $\quad \alpha_B = \frac{\omega_B}{t} = \frac{4.5}{3} = 1.5\ rad/s^2$

Chapter 16

Problem (16.13) : $w_C = ?$ $w_B = ?$

$v_A = v_D$ $w_D = w_B$

$r_A w_A = r_D w_D \rightarrow w_D = w_B = \frac{r_A}{r_D} w_A = \frac{75}{25}(60) = \boxed{180 \text{ rad/s}}$

$v_C = v_B$

$r_C w_C = r_B w_C$

$w_C = \frac{r_B}{r_C} w_B = \left(\frac{100}{50}\right)(180) = \boxed{360 \text{ rad/s}}$

Problem (16.17) : $r_A = 100mm$ $\alpha_A = (2 + 0.006\theta^2)$ rad/s^2

$r_B = 175mm$ $w_{A_i} = 15$ rad/s

$w_B = ? \rightarrow \theta_2 = 20\pi$

$$\int_0^{20\pi} \alpha d\theta = \int_{15}^{w} w\, dw$$

$$\int_0^{20\pi} (2 + 0.006\theta^2) d\theta = \int_{15}^{w} w\, dw$$

$$621.7 = \frac{1}{2}w^2 - \frac{1}{2}(15)^2$$

$$734.2 = 0.5 w_A^2$$

$$\boxed{w_{Af} = 38.3 \text{ rad/s}}$$

$v_{A_f} = v_{B_f}$

$r_A w_{Af} = r_B w_{Bf}$

$w_{Bf} = \frac{r_A}{r_B} \times w_{Af}$

$w_{Bf} = \frac{100}{175} \times 38.3$

$w_{Bf} = 21.8$ rad/s

Chapter 16

wrong!!

Problem (16.17) : $r_A = 100mm$ $\alpha_A = (2+0.006\theta^2)\ rad/s^2$

$r_B = 175mm$ $\omega_{Ai} = 15\ rad/s$

$\omega_B = ? \rightarrow \theta_2 = 20\pi$

$$\int_0^{20\pi} \alpha\, d\theta = \int_{15}^{\omega} \omega\, d\omega$$

$$\int_0^{20\pi} (2+0.006\theta^2)\, d\theta = \int_{15}^{\omega} \omega\, d\omega$$

$$621.76 = \frac{1}{2}\omega^2 - \frac{1}{2}(15)^2$$

$$509.26 = 0.5\omega^2$$

$$\boxed{\omega_{Af} = 31.9\ rad/s}$$

$$v_B = v_A$$

$$r_B\, \omega_{Bf} = r_A\, \omega_{Af}$$

$$\omega_{Bf} = \frac{r_A}{r_B} \times \omega_{Af} = \frac{100}{175} \times 31.9 = 18.2\ rad/s$$

chapter 16

wrong!!

Problem (16-18) : $\alpha_A = (2t^3)\ rad/s^2$ $\quad r_A = 100$

$w_{A_i} = 15\ rad/s$ $\quad r_B = 175$

$w_{Bf} = ? \rightarrow t = 3s$

$$\rightarrow w_{Af} = \int_0^3 2t^3\, dt$$

$$w_{Af} = \frac{2t^4}{4}\Big|_0^3 \rightarrow w_{Af} = \frac{1}{2}(3)^4 = 40.5\ rad/s$$

$$v_A = v_B$$

$$r_A w_A = r_B w_B$$

$$w_{Bf} = \frac{r_A}{r_B} \times w_{Af} = \frac{100}{175} \times (40.5) = 23.1\ rad/s$$

correct

Problem (16-18) : $\alpha_A = 2t^3\ rad/s^2$ $\quad r_A = 100\ mm$

$w_{Ai} = 15\ rad/s$ $\quad r_B = 175mm$

$w_{Bf} = ? \rightarrow t = 3s$

$$\cancel{w_{Af}} = \int_{15}^{w} dw = \int_0^3 2t^3$$

$$w_{Af} - 15 = 40.5$$

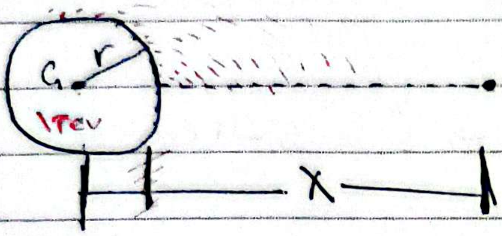
$$\boxed{w_{Af} = 55.5\ rad/s}$$

$$v_A = v_B$$

$$r_A w_A = r_B w_B \rightarrow w_{Bf} = \frac{r_A}{r_B} \times w_{Af} = \frac{100}{175} \times 55.5$$

$$\boxed{w_{Bf} = 31.7\ rad/s}$$

16.4) Absolute Motion Analysis



$X_G = 2\pi r$ $\quad$ $1\,rev = 2\pi = \theta$

$X_G = \theta r$

$V_G = \frac{d\theta}{dt} r$

$V_G = \omega r$

$a_G = \alpha r$

* The purpose of this topic is to find a relation with θ then we can find time derivative to complete our problem.

Example (16-3):

$X = \overline{BC} + \overline{CO}$

$X = r\cos\theta + r\cos\theta$

$X = 2r\cos\theta$

Time Derivative: $\frac{dx}{dt} = -2r \frac{d(\cos\theta)}{dt}$

[implicit derivative]

ضمني

$V = -2r\sin(\theta)\frac{d\theta}{dt}$

$\rightarrow V = -2r\sin(\theta)\,\omega$

$a = -2r\sin(\theta)\,\alpha + \omega(-2r\cos(\theta)\,\omega)$

$\rightarrow a = -2r(\alpha\sin(\theta) + \omega\cos(\theta))$

chapter 16

16-5) $V = 0.5\,m/s$

$w = ?$ $\alpha = ?$ $\theta = 30°$

$s^2 = (2)^2 + (1)^2 - 2(2)(1)\cos(30°) \rightarrow s^2 = 5 - 4\cos(\theta)$

$s = 1.239\,m$

Time derivativ: $2s\frac{ds}{dt} = +4\sin(\theta)\frac{d\theta}{dt}$

$2sV = +4w\sin(\theta)$

$sV = +2w\sin(\theta)$

$(1.239)(0.5) = -2\sin(30°)\,w$

$\boxed{w = 0.6197\ rad/s}$

$\frac{ds}{dt}v + \frac{dv}{dt}s = -2\alpha\sin(\theta) + \cos(\theta)w(-2w)$

$v^2 + \alpha s = -2(\alpha\sin(\theta) + w^2\cos(\theta))$

$\boxed{\alpha = -0.415\ rad/s^2}$

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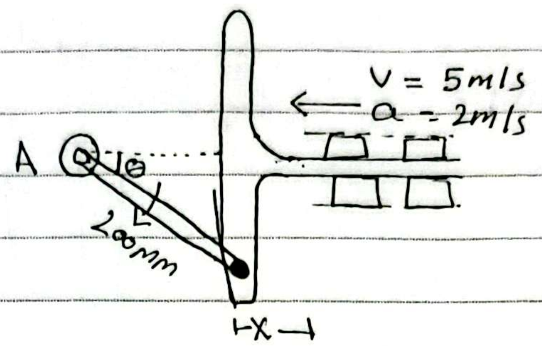
chapter 16

Problem (16-40) :



$X = 200 - 200\cos\theta$

$X = 0.2 - 0.2\cos\theta$

Time derivative:

$X = 0.2 - 0.2\cos\theta$

$V = 0.2\sin\theta\,(w)$

$a = 0.2\cos(\theta)\,w^2 + \alpha(0.2\sin(\theta))$

$a = 0.2\cos(\theta)\,w^2 + 0.2\sin(\theta)\,\alpha$

$\rightarrow w = \dfrac{V}{0.2\sin\theta} = \dfrac{5}{0.2\sin(60°)} = 28.8\ rad/s$

$\alpha = \dfrac{a - 0.2\cos(\theta)\,w^2}{0.2\sin(\theta)} = \dfrac{2 - 0.2\cos(60°)\,(28.8)^2}{0.2\sin(60°)} = -467.3\ rad/s^2$

$X = 0.2\cos(\theta)$

$V = -0.2\sin(\theta)\,w$

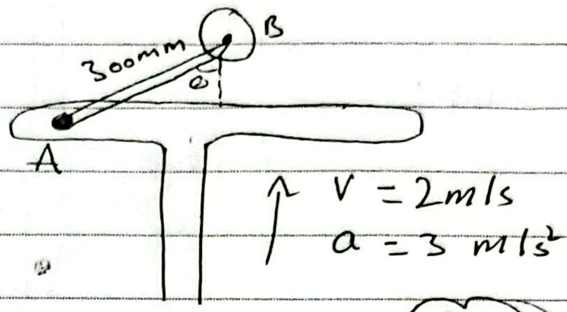
$a = -0.2\cos(\theta)\,w^2 - 0.2\sin(\theta)\,\alpha$

$w = \dfrac{-V}{0.2\sin(\theta)} = 28.8\ rad/s$

$\alpha = \dfrac{-(a + 0.2\cos(\theta)\,w^2)}{0.2\sin(\theta)} = -467.3\ rad/s^2$

Chapter 16

Problem (16-41):



wrong!!

$y = 300 - 300\cos\theta$

$y = 0.3 - 0.3\cos\theta$

Time derivative

$V = 0.3\sin(\theta)\,w$

$w = \frac{V}{0.3\sin(\theta)} = \frac{-2}{0.3\sin(50)} = \ominus 8.7$ rad/s → Clockwise

$a = 0.3\cos(\theta)\,w^2 + 0.3\sin(\theta)\,\alpha$

$\alpha = \frac{a - 0.3\cos(\theta)\,w^2}{0.3\sin(\theta)} = -50.5$ rad/s²

$y = 0.3\cos\theta$

$V = -0.3\sin(\theta)\,(w)$

$a = -0.3\cos(\theta)\,w^2 - 0.3\sin(\theta)\,\alpha$

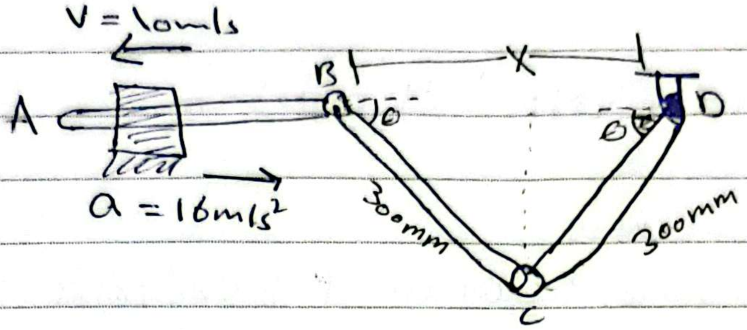
$w = \frac{-V}{0.3\sin(\theta)} = 8.7$ rad/s

$\alpha = \frac{-(a + 0.3\cos(\theta)\,w^2)}{0.3\sin(\theta)} = -50.5$ rad/s²

PRENKO

chapter 16

Problem (16-42) :



$\theta = 60°$

$X = 2\,(0.3)\cos\theta$

$X = 0.6\cos\theta$

Time derivative:

$V = -0.6\sin(\theta)\,w$

$w = \dfrac{-V}{0.6\sin(\theta)} = \dfrac{+10}{0.6\sin(60°)} = -19.2\ \text{rad/s}$

$a = -0.6\cos(\theta)\,w^2 - 0.6\sin(\theta)\,\alpha$

$\alpha = \dfrac{-(a + 0.6\cos(\theta)\,w^2)}{0.6\sin(\theta)} = \dfrac{-(-16 + 0.6\cos(60°)\,(19.2)^2}{0.6\sin(60°)} = -182\ \frac{\text{rad}}{s^2}$

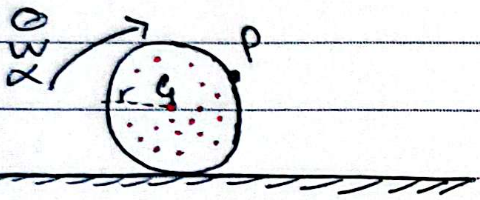
chapter 16

Problem (16-48): $y = b\tan(\theta)$

$V = b\sec^2(\theta)\,\omega$

$\alpha = 2b\sec^2(\theta)\tan(\theta)\,\omega^2 + b\sec^2(\theta)\,\alpha$

16.5) Relative – Motion Analysis ; Velocity



(Rotation + Translation)

$V_p = \underbrace{V_p}_{\text{translation}} + \underbrace{V_p}_{\text{rotation}}$

** ~~الحركة~~ الحركة الانتقالية للنقطة (P)
هي نفسها سرعة النقطة (G)
وأي نقطة أخرى على القرص

$\vec{V}_p = \vec{V}_G + (\vec{V}_{P/G})_{\text{rotation}}$

$\cancel{\vec{a}_p = \vec{a}_G + \alpha}$

chapter 16

Example (16.6)

i, K, j $\circlearrowright +$ $\circlearrowleft -$

$\rightarrow \vec{V_B} = \vec{V_A} + \vec{V}_{B/A}$

$\vec{V_B} = \vec{V_A} + \vec{W}_{BA} \times \vec{r}_{B/A}$

$v_B \hat{i} = -2\hat{j} + w_{BA}\hat{K} \times \left[-0.2\cos(45^\circ)\hat{j} + 0.2\sin(45^\circ)\hat{i}\right]$

$\cancel{v_B \hat{i} = -2\hat{j} + w_{BA}\hat{K} \times}$

$v_B \hat{i} = -2\hat{j} + 0.2\cos(45^\circ) w_{BA}\, i + 0.2\sin(45^\circ) w_{BA}\, \hat{j}$

$\underline{\underline{(i)}}$ $\quad v_B = 0.2\cos(45^\circ) w_{BA}$ $\quad v_B = 0.2\cos(45^\circ)(14.1) = \boxed{2\ m/s}$

$\underline{\underline{(j)}}$ $\quad 0 = -2 + 0.2\sin(45^\circ) w_{BA}$

$w_{BA} = \dfrac{2}{0.2\sin(45^\circ)} = \boxed{14.1\ rad/s}$

Example (16.7) :

$\rightarrow \vec{V_A} = \vec{V_B} + w \times \vec{r}_{A/B}$

$V_{Ax} + V_{Ay} = 2\hat{i} + (-15\hat{K}) \times \left[-0.5\hat{i} + 0.5\hat{j}\right]$

$V_{Ax} + V_{Ay} = 2\hat{i} + 7.5\hat{j} + 7.5\hat{i}$

$V_{Ax} = (2 + 7.5)\hat{i} = 9.5\hat{i}$

$V_{Ay} = 7.5\hat{j}$

$V_A = \sqrt{7.5^2 + 9.5^2} = 12.1\ ft/s$

PRENKO

Chapter 16

Example (16.8)

$\rightarrow \vec{V}_B = \vec{V}_c + \omega_{CB} \times \vec{r}_{B/C}$

$V_B \hat{i} = -2\hat{j} + W_{CB}\hat{k} \times [0.2\hat{i} + 0.2\hat{j}]$

$V_B \hat{i} = -2\hat{j} + 0.2 W_{CB}\hat{j} + 0.2 W_{CB}\hat{i}$

$0 = -2 + 0.2 W_{CB}$

$\boxed{W_{CB} = 10 \text{ rad/s}}$

Problem (16-58)

$\rightarrow V_B = W_{AB} r \longrightarrow W_{AB} = \frac{V_B}{r}$

$\vec{V}_B = \vec{V}_c + W_{CB}\hat{k} \times [-3\cos 30^\circ \hat{i} - 3\sin 30^\circ \hat{j}]$

$-V_B\hat{j} = -4\hat{j} - 3\cos 30^\circ W_{CB}\hat{j} + 3\sin 30^\circ W_{CB}\hat{i}$

$(\hat{j})$ $\quad -V_B = -4 - 3\cos 30^\circ W_{CB}$

$(\hat{i})$ $\quad 0 = 3\sin 30^\circ W_{CB} \rightarrow W_{CB} = 0$

$\boxed{V_B = 4 \text{ ft/s}}$

$V_B = W_{AB} r$

$W_{AB} = \frac{4}{2} = \boxed{2 \text{ rad/s}}$

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Problem (16-59) $V_c = ?$

$\omega_{BC} = ?$

$\rightarrow \vec{V_c} = \vec{V_B} + \omega_{CB} \times \vec{r}_{C/B}$

$\vec{V_B} = \vec{\omega}_{AB} \times \vec{r}_{AB}$

$\vec{V_B} = 3\hat{k} \times [0.5\cos 45^\circ\,\hat{i} + 0.5\sin 45^\circ\,\hat{j}]$

$\vec{V_B} = 1.5\cos 45^\circ\,\hat{j} - 1.5\sin 45^\circ\,\hat{i}$

$\vec{V_B} = -1.06\,\hat{i} + 1.06\,\hat{j}$

$-V_c\,\hat{i} = [-1.06\,\hat{i} + 1.06\,\hat{j}] + \omega_{CB}\,\hat{k} \times 1.5\,\hat{i}$

$-V_c\,\hat{i} = -1.06\,\hat{i} + 1.06\,\hat{j} + 1.5\,\omega_{CB}\,\hat{j}$

$\boxed{V_c = 1.06\ m/s}$

$0 = 1.06 + 1.5\,\omega_{CB}$

$+\omega_{CB} = \frac{-1.06}{1.5} = -0.706\ rad/s$

تحتيت أثر ω_{CB} ع

تجاهل السالب

$8\ \ 5.~~65~~\,\hat{i} - 5.65\,\hat{j} = -V_B\,\hat{j} + \omega_{CB}\,\hat{k} \times -2\,\hat{j}$

$5.65\,\hat{i} - 5.65\,\hat{j} = -V_B\,\hat{j} + 2\,\omega_{CB}\,\hat{i}$

$(i) \Rightarrow 5.65 = 2\,\omega_{CB}$

$\boxed{\omega_{CB} = 2.83}$

$V_B = 5.65 = \omega\, 2$

$\omega =$

Problem (16-60) $\omega_{AB} = ?,\ \omega_{BC}?$

$V_B = \omega_{AB} * r \rightarrow \vec{V_B} = \vec{\omega}_{AB} \times \vec{r}_{AB}$

$V_B = -2\,\omega_{AB}\,\hat{j} \rightarrow \vec{V_B} = -\omega_{AB}\,\hat{k} \times 2\,\hat{i} = -2\,\omega_{AB}\,\hat{j}$

$\vec{V_c} = \vec{V_B} + \omega_{CB} \times \vec{r}_{C/B}$

$8\sin 45^\circ\,\hat{i} - 8\cos 45^\circ\,\hat{j} = -2\,\omega_{AB}\,\hat{j} + \omega_{CB}\,\hat{k} \times -2\,\hat{j}$

$5.65\,\hat{i} - 5.65\,\hat{j} = -2\,\omega_{AB}\,\hat{j} + 2\,\omega_{CB}\,\hat{i}$

$(i)\quad 5.65 = 2\,\omega_{CB} \rightarrow \omega_{CB} = 2.83\ rad/s$

$(j)\quad -5.65 = -2\,\omega_{AB} \rightarrow \omega_{AB} = 2.83\ rad/s$

chapter 16

F(16-7) $\rightarrow v_B = ?$ $\omega_{AB} = ?$

$\rightarrow$ ~~$\vec{v}_A = \vec{v}_B + \vec{\omega}_{AB} \times \vec{r}_{A/B}$~~

$\vec{v}_B = \vec{v}_A + \vec{\omega}_{AB} \times \vec{r}_{B/A}$

$-v_B\hat{j} = 3\hat{i} + w_{AB}\hat{k} \times [-1.5\cos 30^\circ \hat{i} + 1.5 \sin 30^\circ \hat{j}]$

$-v_B\hat{j} = 3\hat{i} + 1.5 \cos 30^\circ w_{AB}\hat{j} - 1.5 \sin 30^\circ w_{AB}\hat{i}$

$(\hat{i})$ $0 = 3 - 1.5 \sin 30^\circ w_{AB}$

$w_{AB} = \dfrac{-3}{-1.5 \sin 30^\circ} = 4 \text{ rad/s } (\hat{k})$

$(\hat{j})$ $-v_B = -1.5 \cos(30^\circ) w_{AB}$

$-v_B = -1.5 \cos(30^\circ)(4)$

$v_B = 5.2 \text{ m/s } (-\hat{j})$

F (16-11)

$\rightarrow \vec{v}_B = \vec{v}_C + \vec{w}_{BC} \times \vec{r}_{B/C}$

$-60\hat{i} = v_C\hat{j} + (-w_{BC}\hat{k}) \times [2.5\cos 30^\circ \hat{i} - 2.5 \sin 30^\circ \hat{j}]$

$-60\hat{i} = v_C\hat{j} - 2.5 \cos 30^\circ w_{BC}\hat{j} - 2.5 \sin 30^\circ w_{BC}\hat{i}$

$(\hat{i})$ $-60 = -2.5 \sin 30^\circ w_{BC}$ $\rightarrow w_{BC} = 48 \text{ rad/s}$

$(\hat{j})$ $0 = -2.5 \cos 30^\circ w_{BC} + v_C$ $\rightarrow v_C = 104 \text{ m/s}$

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chapter 16

16.7) Relative motion Analysis : Acceleration

$V_B = V_A + V_{B/A}$

$\frac{d(V_B)}{dt} = \frac{d(V_A)}{dt} + \frac{d(V_{B/A})}{dt}$

$\rightarrow a_B = a_A + (a_{B/A})_t + (a_{B/A})_n$

$\rightarrow a_B = a_A + \alpha \times \vec{r}_{B/A} - \omega^2 r_{B/A}$

Example (16.13) $\rightarrow \alpha_{AB} = ?$

$\rightarrow \vec{V}_A = 2\cos(45°)\hat{i} - 2\sin(45°)\hat{j}$

$\vec{a}_A = 3\cos(45°)\hat{i} - 3\sin(45°)\hat{j}$

$\vec{a}_B = a_B\cos(45°)\hat{i} + a_B\sin(45°)\hat{j}$

$\vec{V}_B = \vec{V}_A + \omega_{BA} \times \vec{r}_{B/A}$

$V_B\cos(45°)\hat{i} + V_B\sin(45°)\hat{j} = 2\cos(45°)\hat{i} - 2\sin(45°)\hat{j} + \omega_{BA}\hat{k} \times 10\hat{i}$

$V_B\cos(45°)\hat{i} + V_B\sin(45°)\hat{j} = 2\cos(45°)\hat{i} - 2\sin(45°)\hat{j} + 10\omega_{BA}\hat{j}$

(i) $V_B \cancel{\cos(45°)} = 2\cancel{\cos(45°)} \rightarrow \boxed{V_B = 2 m/s}$

(j) $V_B\sin(45°) = -2\sin(45°) + 10\omega_{BA} \rightarrow \boxed{\omega_{BA} = 0.28 rad/s}$

$\vec{a}_B = \vec{a}_A + \alpha_{AB} \times \vec{r}_{B/A} - \omega_{AB}^2 \vec{r}_{B/A}$

$a_B\cos(45°)\hat{i} + a_B\sin(45°)\hat{j} = 3\cos(45°)\hat{i} - 3\sin 45°\hat{j} + \alpha_{AB}\hat{k} \times 10\hat{i} - (0.28)^2(10\hat{i})$

$a_B\cos(45°)\hat{i} + a_B\sin(45°)\hat{j} = 3\cos 45°\hat{i} - 3\sin 45°\hat{j} + 10\alpha_{AB}\hat{j} - (0.28)^2(10)\hat{i}$

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(i) $a_B\cos(45°) = 3\cos 45° - (0.28)^2(10) \rightarrow \boxed{a_B = 1.89 m/s^2}$

(j) $1.89\sin(45°) = -3\sin 45° + 10\alpha_{AB} \rightarrow \boxed{\alpha_{AB} = 0.345 rad/s^2}$

chapter 16

$a_B \hat{j} =$

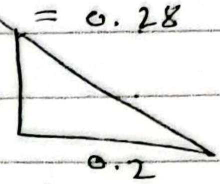
$0 = 0.2\alpha_{BC} - 20$

$20 = 0.2\alpha_{BC}$

α_{BC}

$K^2 = 0.2^2 + 0.2^2$

$K = 0.28$



0.2

0.2

Example (16.16) $\rightarrow \alpha_{CB} = ?$, $\alpha_{AB} = ?$

$\vec{a}_B = \vec{a}_C + \alpha_{BC} \times \vec{r}_{B/C} - \omega^2 \vec{r}_{B/C}$

$\vec{a}_B = \vec{\alpha}_{AB} \times \vec{r}_{AB} - \omega^2 r_{AB}$

$\vec{a}_B = \alpha_{AB}\hat{K} \times (-0.2\hat{j}) - (100)(-0.2\hat{j})$

$\vec{a}_B = +0.2\alpha_{AB}\hat{i} + 20\hat{j}$

$0.2\alpha_{AB}\hat{i} + 20\hat{j} = -\hat{j} + \alpha_{BC}\hat{K} \times (0.2\hat{i} - 0.2\hat{j}) - (100)(0.2\hat{i} - 0.2\hat{j})$

$0.2\alpha_{AB}\hat{i} + 20\hat{j} = -\hat{j} + 0.2\alpha_{BC}\hat{j} + 0.2\alpha_{BC}\hat{i} - 20\hat{i} + 20\hat{j}$

(i) $0.2\alpha_{AB} = 0.2\alpha_{BC} - 20 \rightarrow \alpha_{AB} = \frac{0.2(5) - 20}{0.2} = -95 \text{ rad/s}^2$

(j) $20 = -1 + 0.2\alpha_{BC} + 20 \rightarrow \alpha_{BC} = \frac{1}{0.2} = 5 \text{ rad/s}^2$

Example (16.17)

** في هنا المثال نحتاج إلى جزئية الزوايا

علشان هيك خلي ببالك (Sine law , cosine law)

$\vec{a}_C = \vec{a}_B + \alpha_{CB} \times \vec{r}_{C/B} - \omega^2_{CB} \vec{r}_{C/B}$

$a_C\hat{j} =$

$\vec{a}_B = \alpha_{AB} \times \vec{r}_{AB} - \omega^2_{AB} \vec{r}_{AB}$

$\vec{a}_B = -20\hat{K} \times [-0.25\sin 45^\circ\hat{i} + 0.25\cos 45^\circ\hat{j}]$

$- (100)(-0.25\sin 45^\circ\hat{i} + 0.25\cos 45^\circ\hat{j})$

$\vec{a}_B = -20\hat{K} \times [-0.177\hat{i} + 0.177\hat{j}] - 100(-0.177\hat{i} + 0.177\hat{j})$

$\vec{a}_B = 3.54\hat{j} + 3.54\hat{i} + 17.7\hat{i} - 17.7\hat{j}$

$\vec{a}_B = 21.24\hat{i} - 14.16\hat{j}$

to be contin

$\cancel{A}$

$$\vec{a}_C = \vec{a}_B + \alpha_{CB} \times \vec{r}_{C/B} - w_{CB}^2 \vec{r}_{C/B}$$

$$a_C \hat{j} = 21.24\hat{i} - 14.14\hat{j} + \alpha_{CB}\hat{k} \times [0.176\hat{i} + 0.749\hat{j}] - (2.43)^2 r_{CB}$$

$$a_C \hat{j} = 21.24\hat{i} - 14.14\hat{j} + 0.176\alpha_{CB}\hat{j} - 0.749\alpha_{CB}\hat{i} - 1.04\hat{i} - 4.4\hat{j}$$

(i) $0 = 21.24 - 0.749\alpha_{CB} - 1.04 \rightarrow \alpha_{CB} = 27 \text{ rad/s}^2$ ↺

(j) $a_C = -14.14 + 0.176\alpha_{CB} - 4.4$

$a_C = -14.14 + 0.176(27) - 4.4 = -13.7 \text{ m/s}^2$

Chapter 17

Parallel axis Theorem:

$$I_{(Point)} = \bar{I}_G + md^2$$

where (d): the distance between point (G) and the other point.

* Radius of Gyration k:

$$I_G = mk^2$$

$$\sum F = ma_G$$ → for rigid body

$$\sum F_x = m(a_G)_x$$

$$\sum F_y = m(a_G)_y$$

$$\sum M_G = I_G \alpha$$ → If there is a Rotation

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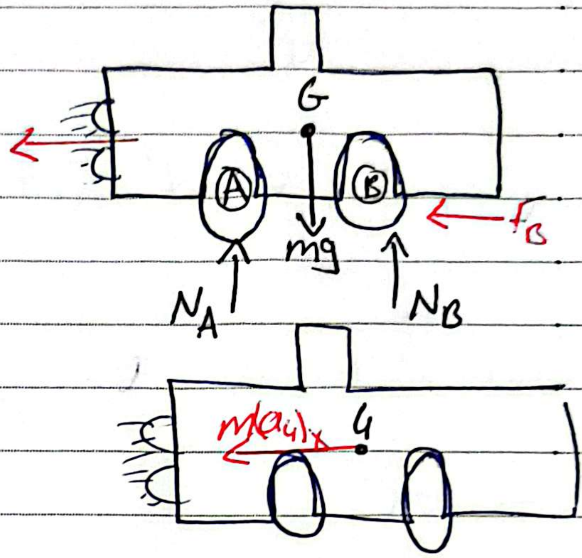
Example (car)



$m = 2000 kg$
$a_G = ?$
$\mu_k = 0.25$

$\Sigma F_x = m(a_G)_x$

$-0.25 N_B = 2000(a_G)_x$

$\Sigma F_y = 0$

$N_A + N_B - mg = 0$
$N_A + N_B - 19620 = 0$ — (1)

$\Sigma M_G = \Sigma (M_k)_G$

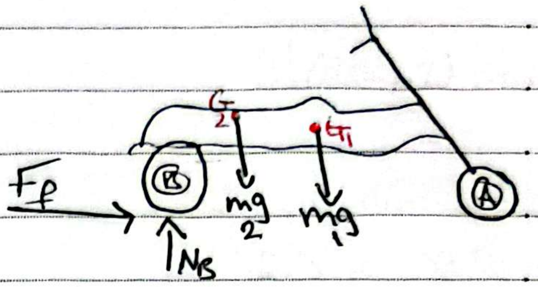
$-0.25 N_B (0.3) + N_B (0.75) - N_A (1.25) = 0$ — (2)

$N_A + N_B = 19620$ $\rightarrow$ $N_A = 6879 N$
$-1.25 N_A + 0.675 N_B = 0$ $\quad N_B = 12740 N$

$-0.25 N_B = 2000(a_G)_x$
$(a_G)_x = \dfrac{-0.25(12740)}{2000} = 1.59 \, m/s^2 \leftarrow$

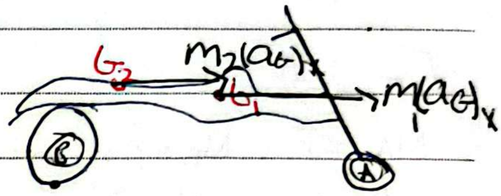
Chapter 17

Example: (motor) :



$m_1 = 175\ kg$

$m_2 = 75\ kg$



$\Sigma F_x = m_{(1+2)}(a_G)_x$

$\mu_s N_B = (175+75)(a_G)_x$

$\Sigma F_y = 0$

$N_B - m_1 g - m_2 g = 0$

$N_B = (175)(9.81) + (75)(9.81)$

$\boxed{N_B = 2452\ N}$

$\Sigma M_B = \Sigma (M_k)_B$

$-m_2 g(0.4) - m_1 g(0.8) = -m_2 a_G(0.9) - m_1 a_G(0.6)$

$735.75(0.4) + 1716.75(0.8) = (67.5 + 105)\, a_G$

$a_G = 9.6\ m/s^2 \rightarrow$

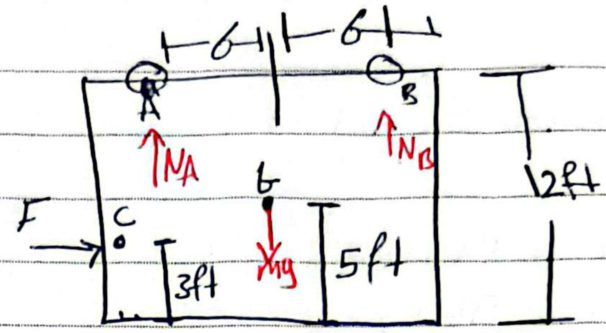
$\mu_s N_B = (175+75)(a_G)_x$

$\mu_s = \dfrac{(175+75)(9.6)}{2452}$

$\boxed{\mu_s = 0.97}$

Chapter 17

Problem (17-24)



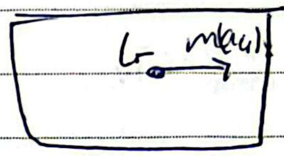
$mg = 200lb$

$m = \frac{200}{32.2} = 6.2$

$F = 30lb$

$N_A = ?$ $N_B = ?$

$t = 2s \rightarrow x = ?$



$\rightarrow \sum F_x = m(a_G)_x$

$30 = 6.2\, a_G$

$\boxed{a_G = 4.83\ ft/s^2}$

$\sum F_y = 0$

$N_A + N_B = mg$

$N_A + N_B = 200$ — ①

$\sum M_G = \sum (M_k)_G$

$30(2) = -N_A(6) + N_B(6)$

$-6N_A + 6N_B = 60$ $\rightarrow N_A = 95\ N$

$-N_A + N_B = 10$ — ② $\quad N_B = 105\ N$

$x_f - \cancel{x_i}^{\,0} = \cancel{v_0 t}^{\,0} + \frac{1}{2}at^2$

$x_f = \frac{1}{2}(4.83)(2)^2$ $\rightarrow x = 9.66\ ft$

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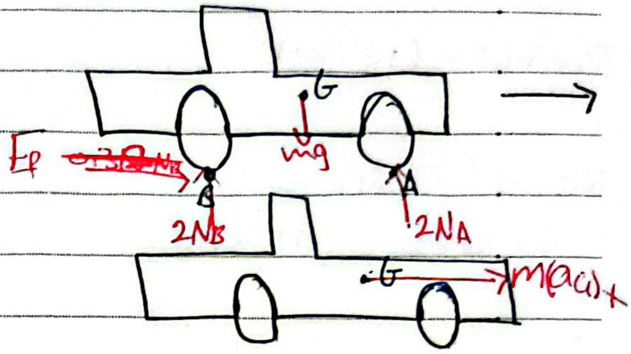
Chapter 17

Problem (17-27)



$mg = 4000 lb$

$m = \frac{4000}{32.2} = 124.24$

$N_B = ?$ $N_A = ?$ $t = ? \rightarrow V_2 = 10$

$\Sigma F_x = m(a_G)_x$

$0.3(2N_B) = 124.24\, a_G$

$0.6 N_B = 124.24 a_G$

$\Sigma F_y = 0$

$2N_B + 2N_A = mg$

$N_B + N_A = 2000$ $-①$

$\Sigma M_G = \Sigma (M_K)_G$

$2N_A(2) - 2N_B(4) + 0.3N_B(2.5)^{*2} = 0$ $-②$

~~$N_B + N_A = 200$~~ $N_B + N_A = 2000$ $\rightarrow$ $N_B = 762 N$

~~$3.25N_B + 4$~~ $-6.5N_B + 4N_A = 0$ $N_A = 1238 N$

$V_2 = \cancel{V_1}^{0} + at$

$0.6 N_B = 124.24 a_G$

$a_G = \frac{0.6(762)}{124.24} = 3.67\ ft/s^2$

$10 = 3.67\, t$ $\rightarrow \boxed{t = 2.7 sec}$

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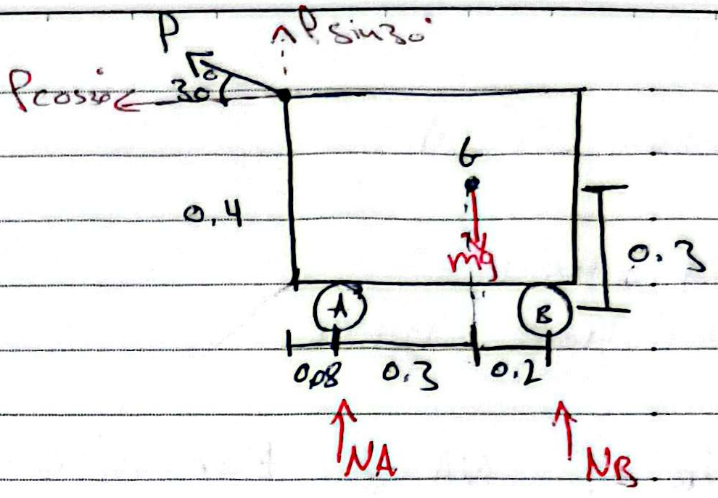
chapter 17



Problem (17-32)

$N_B = ?$ $N_A = ?$

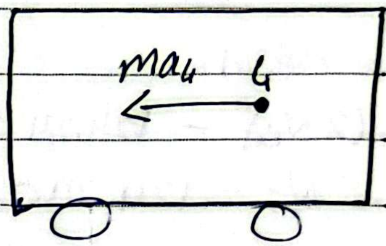
$P = 300N$ $m = 60$

$\sum F_x = m(a_G)_x$

$-P\cos(30°) = m a_G$

$-300\cos(30°) = 60\, a_G$

$a_G = 4.33\ m/s^2$ ←



$\sum F_y = 0$

$P\sin 30° + N_B + N_A = mg$

$N_B + N_A = 60(9.81) - 300\sin(30°)$

$N_B + N_A = 438.6$ — 1

$\sum M_G = \sum (M_k)_G$

$N_B(0.2) - N_A(0.3) - 300\sin 30°\,(0.38) + 300\cos 30°\,(0.1) = 0$

$0.2N_B - 0.3N_A = 31.01$ — 2

$N_B + N_A = 438.6$

$0.2N_B - 0.3N_A = 31.01$ → $N_B = 325\,N$, $N_A = 113\,N$

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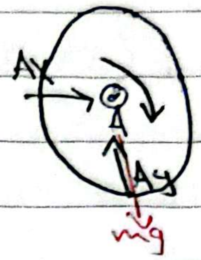
Chapter 17

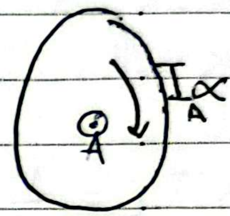
Problem (17-57):



$M = 10kg$, $w = ? \to t = 3s$

$M = 5t \ N.m$ $A_x = ?$ $A_y = ?$

$K_A = 200mm$



$\rightarrow I_A = mK^2$

$I_A = 10(0.2)^2 = 0.4 kg.m^2$

$\Sigma F_x = ma_G$ $\Sigma F_y = ma_G$

$\Sigma F_x = 0$ $\Sigma F_y = 0$

$\boxed{A_x = 0}$ $\boxed{A_y = mg = 98.1 N}$

$\Sigma M_A = \Sigma (M_k)_A$

$+5t = -I_A \alpha$

$5t = 0.4 \alpha$

$\alpha = 12.5t$

$$W = \int_0^3 12.5t \, dt$$

$$W = \frac{12.5}{2} t^2 \Big|_0^3$$

$$\boxed{W = 56.25 \ rad/s}$$

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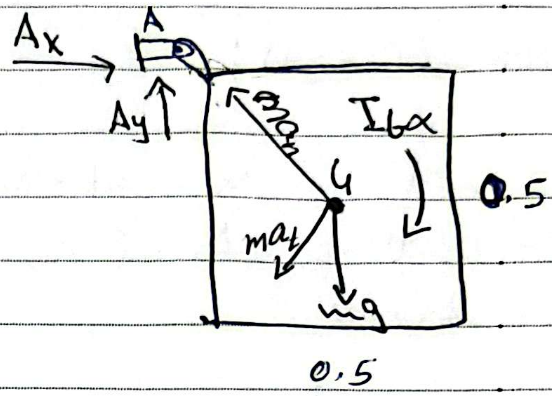
Problem (17-58)



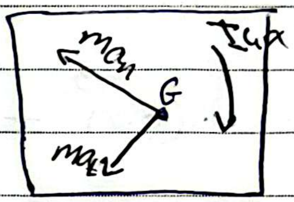
$m = 24\,Kg$

$\omega = 0$

$\alpha = ?$

$\rightarrow I_G = \frac{1}{12} m (a^2+b^2)$

$I_G = \frac{1}{12}(24)(0.5^2+0.5^2)$



$\boxed{I_G = 1\,Kg.m^2}$

$\sum F_x = ma_G$?

$a = \alpha r$

$\sum M_A = \sum (M_K)_A$

$-mg(0.25) = ma_t(r) + I_A\alpha$

$h = \sqrt{0.5^2+0.5^2} = 0.707$

$\boxed{r = 0.35m}$

$I_A = I_G + md^2 \rightarrow I_A = 1 + 24(0.35)^2$

$I_A = 3.94\,Kg.m^2$

$-24(9.81)(0.25) = -24\alpha(0.35)^2 \cancel{+ 3.94}\alpha$

$-58.86 = -3.94\alpha$

$\boxed{\alpha = 14.9\,rad/s^2}$

OR

$\sum M_A = I_A\alpha$

$-24(9.81)(0.25) = -3.94\alpha$

$\boxed{\alpha = 14.9\,rad/s^2}$

Chapter 17

$\Sigma F_x = m(a_G)_x$

$A_x = m\alpha r \cos 45^\circ$

$A_x = (24)(14.9)(0.35)\cos(45^\circ)$

$A_x = -88.5\,N$

$|A_x| = 88.5\,N \leftarrow$

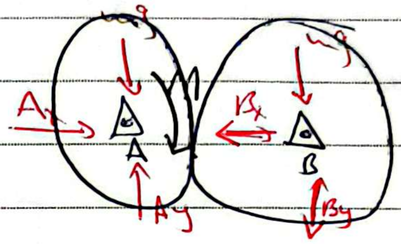
$\Sigma F_y = m a_y$

$A_y - mg = m\alpha r \sin 45^\circ$

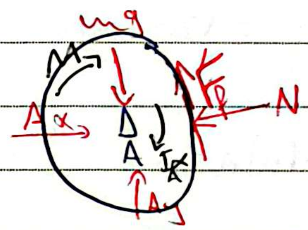
$A_y = -24(14.9)(0.35)\sin 45^\circ + 24(9.81)$

$A_y = 147\,N \uparrow$

Problem (17-65):

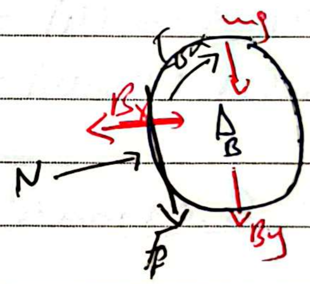


$m_A = \frac{5\,lb}{32.2} = 0.15\,kg$



$m_B = \frac{10}{32.2} = 0.31\,kg$

$r_A = 0.5\,ft \quad r_B = 0.75\,ft$



$\Sigma M_A = I_A \alpha$

$-M + F_f(0.5) = -0.018(4)$

$-M + 0.5F_f = -0.072$ — (1)

$\Sigma M_B = -I_B \alpha$

$F_f(0.75) = -0.5(0.31)(0.75)^2 \alpha_B$

$F_f(0.75) = -0.5(0.31)(0.75)^2(2.67)$

$F_f = -0.31\,N$

الاتجاه فرضته خطأً لأن القيمة ع

$-M + 0.5(0.31) = -0.072$

$\boxed{M = 0.23\ lb.ft}$

$I_A = \frac{1}{2}mr^2$

$I_A = \frac{1}{2}(0.15)(0.5)^2 = 0.018$

$a_A = a_B$

$\alpha_A r_A = \alpha_B r_B$

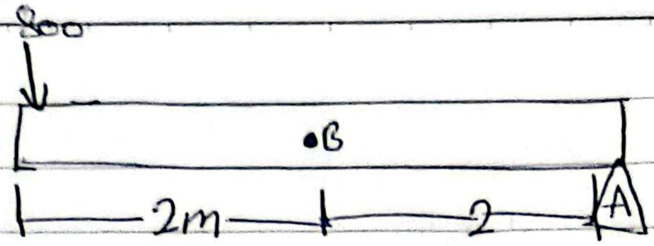
$\alpha_B = \frac{\alpha_A r_A}{r_B}$

$\alpha_B = 2.67$

PRENKO

chapter 17

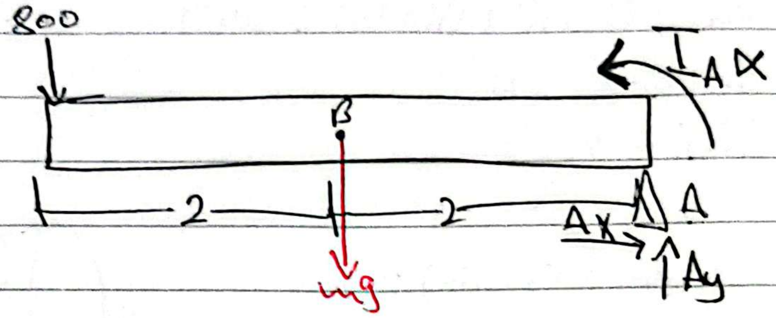
Problem (17-67)



$m = 120\,kg$

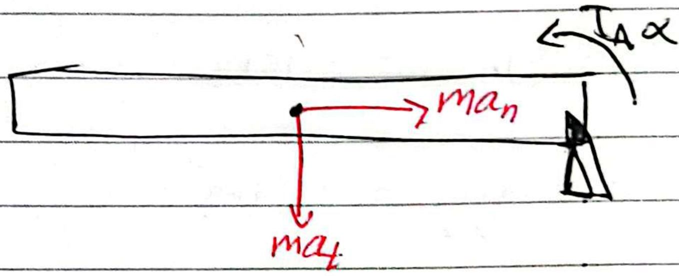
$A_x = ?$ $A_y = ?$

$\alpha = ?$



$\sum F_x = m(a_G)_x$

$\boxed{A_x = 0}$



$\sum F_y = m(a_G)_y$

$-A_y + mg + 800 = 120\, a_{G_{ty}}$

$\sum M_A = \sum (M_k)_A$

$I_G = \frac{1}{12}(120)(16)$

$I_G = 160$

$mg(2) + 800(4) = m\alpha r(2) + I_G\alpha$

$120(9.81)(2) + 800(4) = 120(2)(2)\alpha + 160\alpha$

$\boxed{\alpha = 8.67\ rad/s^2}$

$-A_y = 120(8.67)(2) - 800 - 120(9.81)$

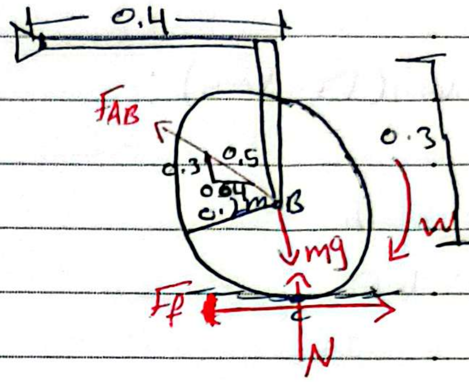
$-A_y = 104N$

$A_y = 104N \downarrow$

PRENKO

chapter 17

Problem (17-75)



$m = 25\ kg$

$k_B = 0.15m$

$\mu_c = 0.5$

$w_i = 40$

$\rightarrow I_B = m k_B^2$

$I_B = 25(0.15)^2$

$\boxed{I_B = 0.5625\ kg.m^2}$

$\Sigma F_x = 0$

$F_f - \frac{4}{5} F_{AB} = 0$

$\cancel{\Sigma F_x = 0}$

$\cancel{+ F_f - \frac{4}{5} F_{AB} = 0}$

$\cancel{-\frac{4}{5} F_{AB} = +0.5 N_c}$

$\Sigma F_y = 0$

$F_{AB} + N_c - mg = 0$

$\frac{3}{5} F_{AB} + N_c - 25(9.81) = 0$

$\frac{3}{5} F_{AB} + N_c = 25(9.81)$

$-\frac{4}{5} F_{AB} + 0.5 N_c = 0$

$\Sigma M_B = I_B \alpha_B$

$\cancel{F_{AB} = 245.25N}$

$\cancel{N_c = 392.4N}$

$+ F_f(0.2) = -0.5625\alpha_B$

$0.2 F_f = -0.5625\alpha_B$

$0.2(0.5)(178.4) = -0.5625\alpha_B$

$\alpha_B = -31.7\ rad/s^2$

$\boxed{F_{AB} = 111.5N \quad N_c = 178.4\ N}$

$A_x = \frac{4}{5} F_{AB} = \frac{4}{5}(111.5) = 89.2N$

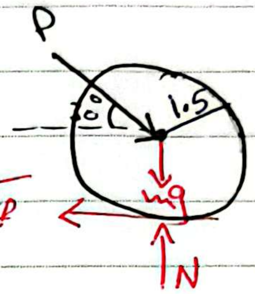
$A_y = \frac{3}{5} F_{AB} = \frac{3}{5}(111.5) = 66.9N$

PRENKO

$\overset{0}{\cancel{w_2}} = w_1 + \alpha t \qquad 0 = 40 + (-31.7)t \quad \rightarrow \boxed{t = 1.26 sec}$

Chapter 17

Problem(17-105) :



$W = 50lb$

$m = 1.55$

$\mu_s = 0.25$

$P = ?$

$\alpha = ?$

$$I_G = \frac{1}{2} m r^2 \rightarrow I_G = \frac{1}{2}(1.55)(1.5)^2 = 1.74\ kg \cdot m^2$$

$\Sigma M = \Sigma M_K$

$-0.25N = -I_G \alpha$

$(1.5)(0.25N) = 1.74\alpha$

$\Sigma F_x = m(a_G)_x$

$P\cos(30°) - 0.25N = 1.55(a_G)$

$\Sigma F_y = 0$

$-P\sin(30°) + N = 32.2(1.55)$

→ $P\cos(30°) - 0.25N = 1.55\alpha(1.5)$

→ $-P\sin(30°) + N = \cancel{15.2}\ 50$

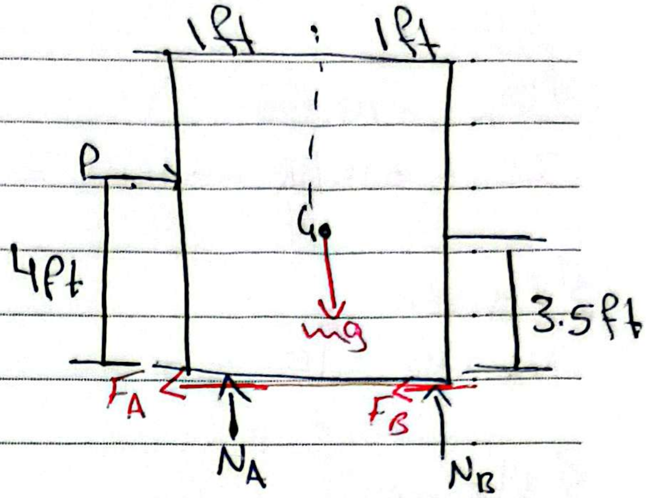
→ $0 + 1.5 * 0.25N = 1.74\alpha$

$P = 76.5N$ $\quad N = 88.2N$ $\quad \alpha = 19\ rad/s^2$

PRENKO

Chapter 17

Problem (17-43)

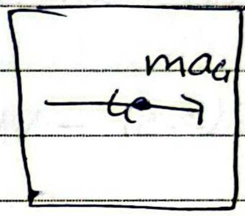


$\rightarrow$ $m = \frac{150}{32.2} = 4.6$

$W = 150lb$

$\mu_s = 0.2$ $\mu_k = 0.15$

$P = 35lb$



$\Sigma F_x = 0$

$30 - 0.2N_B - 0.2N_A = 0$

$\Sigma F_y = 0$

$N_A + N_B - 150 = 0$

$\rightarrow$ $0.2N_B + 0.2N_A = 30$

$\rightarrow$ $N_B + N_A = 150$

$\cancel{N_B - N_A = 15}$

$\cancel{\Sigma M_G = 0}$

$\cancel{-30(0.5) - N_A(1) + N_B(1) = 0}$

$\cancel{N_B - N_A = 15}$

$\Sigma F_x = 0$

$P - 0.2N_B - 0.2N_A = 0$

$\Sigma F_y = 0$

$N_A + N_B - 150 = 0$

$\Sigma M_A = 0$

$-P(4) - 150 + 2N_B = 0$

$P = 30$ $N_B = 135$ $N_A = 15$

$\Sigma M_A = I_A \cancel{\alpha}$

$-35(4) - 150(1) + N_B(2) = 0$

$\boxed{N_B = \frac{270}{2} = 135 lb}$

$\boxed{N_A = 15 lb}$

$35 > 30$

So the cabinet will slide

PRENKO

Chapter 17

$\Sigma F_x = ma_G$

$35 - 0.15 N_A - 0.15 N_B = 150a$

$\Sigma F_y = 0$

$N_A + N_B - 150 = 0$

$\Sigma M_G = 0$

$\cancel{N_B(1) - 0.15 N_B(3.5) - 0.15 N_A(3.5)}$

$-35(0.5) - N_A(1) + N_B(1) - 0.15 N_A(3.5) - 0.15 N_B(3.5) = 0$

$-0.15 N_A - 0.15 N_B - 466a = -35$

$N_A + N_B + 0 = 150$

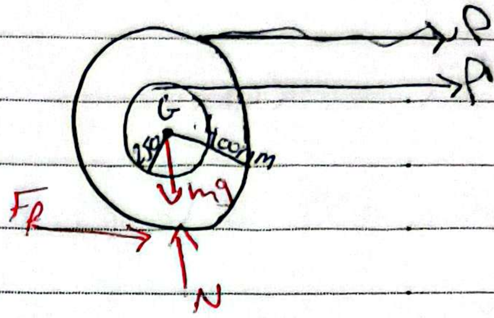
$-1.525 N_A + 0.475 N_B + 0 = 17.5$

$N_A = 26.8\,N$ $\quad N_B = 123.125\,N$ $\quad \cancel{a = 0.083\,m/s}$

$a = 2.7\,ft/s^2$ $\quad \cancel{a = 0.083\,ft/s^2}$

Chapter 17

Problem (17-103)

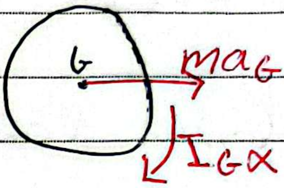


$m = 100 kg$

$k_G = 0.3 m$

$\mu_s = 0.2$ $\mu_k = 0.15$

$\alpha = ?$ $P = 600 N$



without slipping!

$\Sigma F_x = 0$ $\Sigma F_y = 0$

$P + 0.2N = 0$ $N = 100(9.81)$

$P =$ $N = 981 N$

without slipping! $I_G = mk^2$

$\Sigma F_x = m(a_G)_x$ $\Sigma F_y = 0$

$N = mg = 981 N$

$600 + 0.2N = 100(a_G)_x$

$600 + 0.2(981) = 100(a_G)_x$

$(a_G)_x =$

$\Sigma M_G = I_G \alpha$

$0.4 F_f - 600(0.25) = -100(0.3)^2 \alpha$

PRENKO

$\Sigma F_x = m(a_G)_x$

$\Sigma F_y = 0$

$N = mg = 981\,N$

$600 + F_f = 100(a_G)_x$ — ①

$\boxed{F_{f_s\,max} = 0.2(981) = 196\,N}$

$\Sigma M_G = +I_G\alpha$

$-600(0.25) + 0.4F_f = -100(0.3)^2\alpha$ — ②

$600 + F_f - 100(0.4)\alpha = 0$

$-600(0.25) + 0.4F_f = -100(0.3)^2\alpha$

$F_f = 24\,N \qquad \alpha = 15.6\ rad/s^2$

$F_f \quad \boxed{\phantom{<}} \quad F_{f_s\,max}$

$24 < 196$

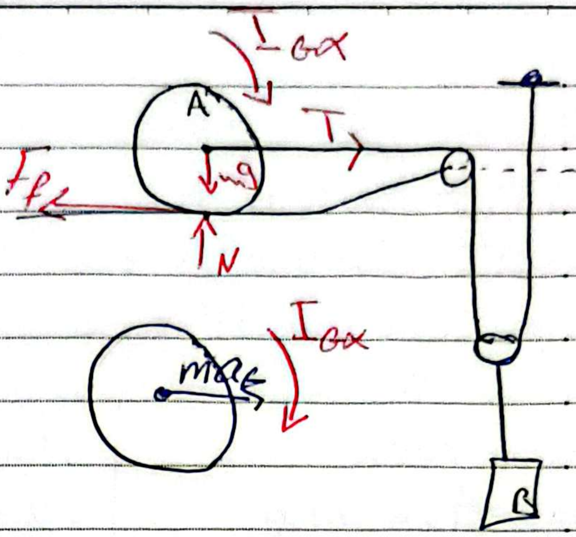
هنا يعني أنه ما تحرك

بالتالي نوقف حل.

final answer: $\alpha = 15.6\ rad/s^2$

chapter 17

Problem(17-112)



$m_A = 20\,kg$

$m_B = 10\,kg$

$\alpha = ?$

$a_B = ?$ $\quad V_1 = 0$

$T = ?$

$$\sum M_c = I_c\alpha$$

$$-0.2T = -\left(+\tfrac{1}{2}(20)(0.2)^2 + 20(0.2)^2\right)\alpha$$

$$+0.2T = 30(0.2)^2\alpha$$

$$\boxed{0.2T - 30(0.2)^2\alpha = 0}$$

Block: $\sum F_y = ma_B$

$mg - 2T = ma_B$

$98.1 - 2T = 10a_B$

$$\boxed{98.1 - 2T = -\alpha}$$

$S_A + 2S_B = L$

$a_A + 2a_B = 0$

$a_B = \frac{1}{2}(-a_A)$

$a_B = -\frac{1}{2}(0.2)(\alpha)$

$$\boxed{a_B = -0.1\alpha}$$

$0.2T - 30(0.2)^2\alpha = 0$

$-2T + \alpha = -98.1$

$$\boxed{T = 53.5\,N} \qquad \boxed{\alpha = 8.9\,m/s^2}$$

$a_B = -0.1(8.9) = -0.89\,m/s^2$

$\boxed{0.89\,m/s^2 \downarrow}$

حل خاطئ لأني فرضت اتجاه $(I\alpha)$ عكسي

$\sum M_c = I_c \alpha$

$-0.2T = 30(0.2)^2 \alpha$

$-0.2T - 30(0.2)^2\alpha = 0$

$0.2T + 30(0.2)^2\alpha = 0$ — (1)

$\sum F_y = m a_B$	$2S_B + S_A = L$	$a_A = r\alpha$
$mg - 2T = m a_B$	$2a_B + a_A = 0$	(no slipping)
$98.1 - 2T = 10 a_B$	$a_B = -\frac{1}{2}(0.2)\alpha$	
	$a_B = -0.1\alpha$	

$98.1 - 2T = -\alpha$ — (2)

$-2T + \alpha = -98.1$

$0.2T + 30(0.2)^2\alpha = 0$

$\rightarrow$ $\boxed{T = 45.27\,N}$ $\alpha = -7.54\ rad/s^2$

$\boxed{\alpha = 7.54}$

$a_B = -0.754\ m/s^2$

$\boxed{a_B = 0.754 \downarrow}$

chapter 18

* Work and Energy Principle for rigid body:

$$T_1 + \sum U_{1-2} = T_2$$

$T_{Rotation} = \frac{1}{2} I w^2$, $T_{translation} = \frac{1}{2} m v^2$

$U_{Work} = \left(-mgy \ , \ -\frac{1}{2} K (x_2^2 - x_1^2) \ , \ -\mu_K N \Delta x \right) + \int \vec{M} \cdot d\theta$

** If: $F_f = 0$, $F_{ext} = 0$, $M_{ext} = 0$

then: $T_1 + V_1 = T_2 + V_2$

$\to$ conservation of Energy

$$\frac{1}{2} I_G w_{G_1}^2 + \frac{1}{2} m v_1^2 + mgy_1 + \frac{1}{2} K x_1^2 = \frac{1}{2} I_{G_2} w_2^2 + \frac{1}{2} m V_2^2 + mgy_2 + \frac{1}{2} K x_2^2$$

Chapter 18

Example: 18.1

→ total work ?

$U_w = mgh = 10(9.81)(1.5) = 147.15\ J$

$U_{ext} = Fd = 80\left(3 * \pi * \frac{2}{4\cdot 2}\right) = 376.99\ J$

$U_M = M\theta = 50\left(\frac{\pi}{2}\right) = 25\pi = 78.5\ J$

$U_{spring} = \frac{1}{2}k(\Delta s_1^2 - \Delta s_2^2) = \frac{1}{2}(30)(0.25^2 - 2.25^2) = -75\ J$

$\boxed{\sum U = 527.64\ J}$

Example: 18.2

$\rightarrow \cancel{T_1}^{0} + \sum U = T_2$

$M\theta + \cancel{mgh}^{0} + \frac{1}{2}k(0 - \Delta s_2^2) = \frac{1}{2} I \omega_2^2$

$5(\theta) + \frac{1}{\cancel{2}}(\cancel{10})^{5}[(0.2)(\theta)]^2 = \frac{1}{\cancel{2}}\left(\cancel{\frac{1}{2}}(30)(0.2)^2\right)(2)^2$

$\cancel{4\theta = 1.2}$

$5\theta - 5(0.04\theta^2) = 1.2$

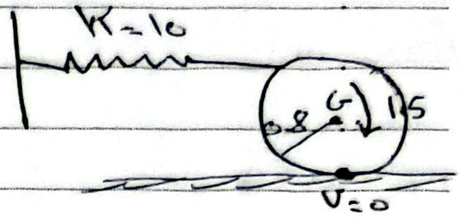
$0.2\theta^2 - 5\theta + 1.2 = 0 \qquad \theta = 0.242\ rad$

$= 0.242 * \frac{180}{\pi} = 13.8^\circ$

PRENKO

chapter 18

Example



$K_G = 0.6$

$X = 0.5\ ft$

$K = 10$

$W = 40lb \quad m = 1.24$

$w_1 = 0$

$w_2 = ?$

$S = r\theta$

$0.5 = 0.8\theta$

$\boxed{\theta = 0.625\ rad}$

$S_2 = (1.6)(0.625)$ البرنرلة

$S_2 = 1$

$\cancel{T_1}^{0} + \Sigma U = T_2$

$\frac{1}{2}K(\Delta S_1^2 - \Delta S_2^2) + M\theta = \frac{1}{2} I_{IC}\, w_2^2$

$-\frac{1}{2}(10)(1)^2 + 15(0.625) = \frac{1}{2}(I_G + md^2)\, w_2^2$

$-5 + \cancel{0.937}\ 9.375 = \frac{1}{2}(1.24 * 0.6^2 + 1.24 * 0.8^2)\, w_2^2$

$+4.375 = 0.621 w_2^2$

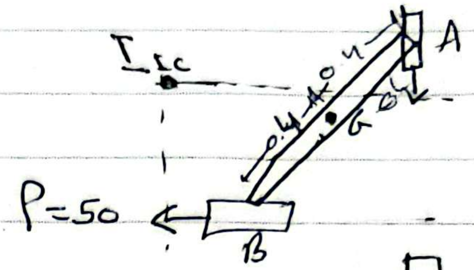
$w_2^2 = 7.04$

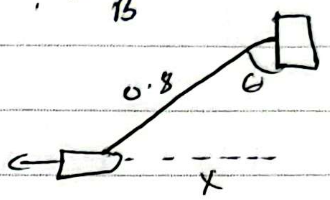
$\boxed{w_2 = 2.65\ rad/s}$

PRENKO

Chapter 18

Example:





$m = 10kg$

$\left.\begin{array}{l} w_1 = 0 \\ v_1 = 0 \end{array}\right\} \theta_1 = 0$

$w_2 \rightarrow \theta_2 = 45°$

$\rightarrow \overset{0}{\cancel{T_1}} + \Sigma U = T_2$

$mgh + Pd = \frac{1}{2} I_{IC} w_2^2$

$\sin 45° = \frac{x}{0.8}$

$x = 0.8 \sin 45°$

$x = 0.56$

$(10)(9.81)(0.4 - 0.4\sin 45°) + 50(0.56) = \frac{1}{2}\left(\frac{1}{12}(10)(0.8)^2 + 10(0.4)^2\right) w_2^2$

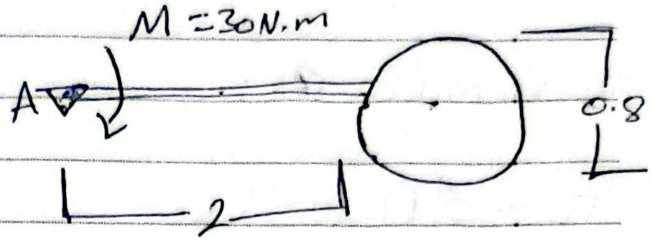
$39.49 = 1.06\, w_2^2$

$w_2^2 = 37.02$

$\boxed{w_2 = 6.08 \text{ rad/s}}$

Chapter 18

Problem (18.15)



$m_{disk} = 10\,kg$

$m_{rod} = 3\,kg$

$w_1 = 0$

$w_2 \xrightarrow{?} \theta_2 = 90°$

$\cancel{T_1}^{0} + \sum U = T_2$

$m_{disk}\,gh + m_{rod}\,gh + M\theta = \frac{1}{2} I_A w_2^2$

$10(9.81)(2.4) + 3(9.81)(1) + 30\left(\frac{\pi}{2}\right) = \frac{1}{2}\left(I_{A\,rod} + I_{A\,disk}\right) w_2^2$

$264.87\ \cancel{179.5} = \frac{1}{2}\left[\frac{1}{3}(3)(2)^2 + \frac{1}{2}(10)(0.4)^2 + (10)(2.4)^2\right] w_2^2$

$264.87\ \cancel{179.5} = 31.2\, w_2^2$

$w_2^2 = 8.489$

$w_2 = 2.91\ rad/s$

chapter 18

Problem (18-12) :

$V_{2\,cylinde} = ?$ $h_{cy} = 2$ $m_{cy} = 50\,kg$ $m_{reel} = 25\,kg$ $k_A = 125\,mm$

$\cancel{T_1}^{0} + \Sigma U = T_2$

$m_{cy} g h = \frac{1}{2} m_{cy} V_2^2 + \frac{1}{2} I_A w_2^2$ $V = wr$ $w = \frac{V}{r}$

$50(9.81)(2) = \frac{1}{2}(50)V_2^2 + \frac{1}{2}(25)(0.125)^2 w_2^2$

$981 = 25V_2^2 + 12.5(0.125)^2 \left(\frac{V_2^2}{0.075^2}\right)$

$981 = 25V_2^2 + 34.7V_2^2$

$\boxed{V_2 = 4.05\,m/s}$

Problem (18-14)

$\cancel{T_1}^{0} + \Sigma U_{1-2} = T_2$

$m_{Block} g h = \frac{1}{2} I w_2^2 + \frac{1}{2} m_{Block} V_2^2$ $V = rw$

$10(9.81)h = \frac{1}{2}(40)(0.3)^2(15)^2 + \frac{1}{2}(10)(0.3 * 15)^2$

$98.1h = 506.25$

$\boxed{h = 5.16\,m}$

for Block

$\rightarrow \cancel{T_1}^{0} + \Sigma U_{1-2} = T_2$

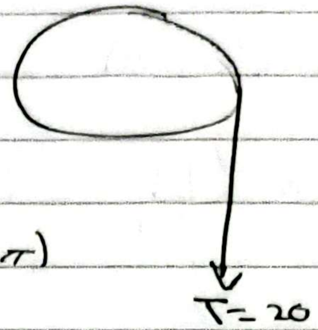
$98.1(5.16) - T(5.16) = \frac{1}{2}(10)(0.3 * 15)^2$

$506.19 - 5.16T = 101.25$ $\rightarrow T = 78.47\,N$

PRENKO

Chapter 18

Problem (18.15) $\quad \theta = 4(2\pi) = 8\pi$



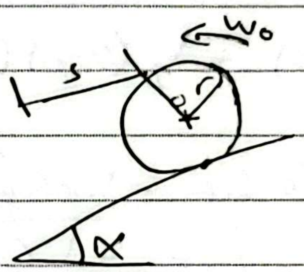
$\rightarrow \cancel{T_1}^{0} + \sum U_{1-2} = T_2 \qquad S_T = r\theta$

$S_T = 0.4(8\pi)$

$$TS = \frac{1}{2} I_0 w_2^2$$

$$(20)(0.4 * 8\pi) = \frac{1}{2}(20)(0.3)^2 w_2^2$$

$$\boxed{w_2 = 14.9 \text{ rad/s}}$$



Problem (18-21)

$$T_1 + \sum U_{1-2} = T_2$$

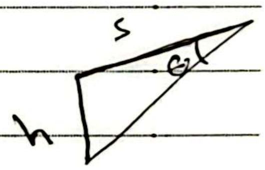
$\boxed{\alpha = r}$

$$\frac{1}{2} I_{Ic} w_0^2 + mgh = \frac{1}{2} I_{Ic} w_2^2$$

$$\frac{1}{2}(mr^2 + mr^2) + mgh = \frac{1}{2}(mr^2 + mr^2) w_2^2$$

$$\cancel{mr^2 + mgh = mr^2}$$

$$mr^2 w_0^2 + mgh = mr^2 w_2^2$$



$\sin\theta = \frac{h}{s}$

$h = S\sin\theta$

$$w_2^2 = \frac{mr^2 w_0^2 + mgh}{mr^2}$$

$$w_2 = \sqrt{w_0^2 + \frac{gh}{r^2}} \rightarrow w_2 = \sqrt{w_0^2 + \frac{g(s)\sin\theta}{r^2}}$$

PRENKO

$\tan 45° = \frac{x}{0.6}$ $\quad y^2 = 0.6^2 + 0.3^2$

$\boxed{x = 0.6}$ $\quad y =$

Chapter 18

$\tan 22.5° = \frac{0.3}{x}$

Problem (18-19)

~~$x = 0.3 \tan 22.5$~~

$x = \frac{0.3}{\tan 22.5}$

$\rightarrow \cancel{T_1}^{0} + \sum U_{1-2} = \cancel{T_2}^{0}$

$S_G = r\theta$

$\sum U_{1-2} = 0$

$\theta = \frac{S_G}{r} = \frac{S_G}{0.5} = 2S_G$

$M\theta + \frac{1}{2}k(\Delta S_2^2) = 0$

$80(2S_G) - \frac{1}{2}(200)(S_G)^2 = 0$

$160 S_G - 100 S_G^2 = 0$

$\boxed{S_G = 1.6 m}$