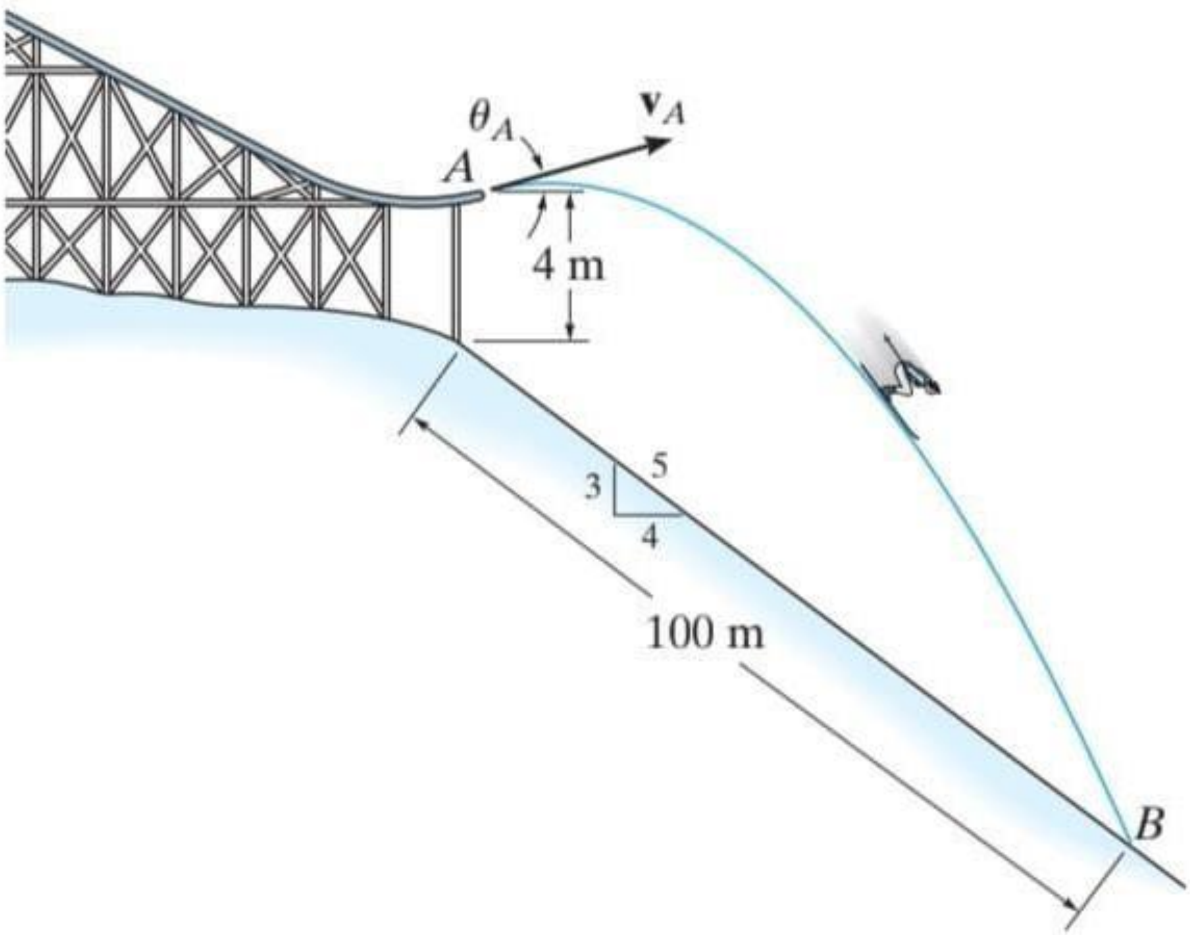
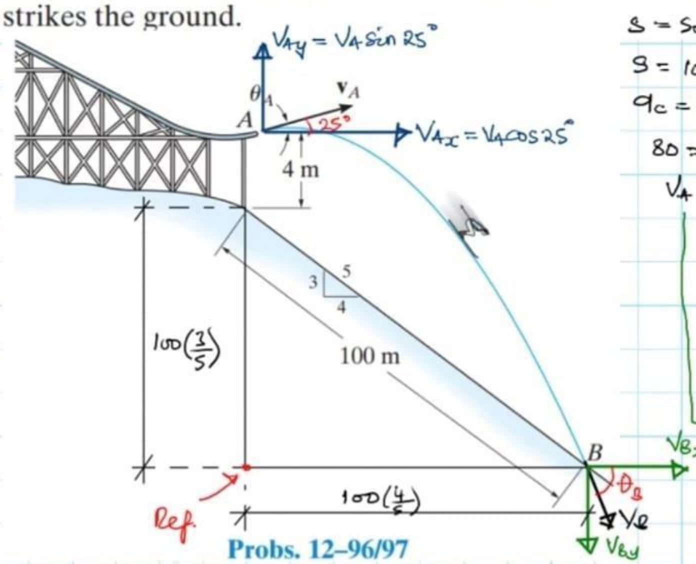
**12–97.** It is observed that the skier leaves the ramp $A$ at an angle $\theta_A = 25°$ with the horizontal. If he strikes the ground at $B$, determine his initial speed $v_A$ and the speed at which he strikes the ground.




**Probs. 12–96/97**

**12–97.** It is observed that the skier leaves the ramp $A$ at an angle $\theta_A = 25^\circ$ with the horizontal. If he strikes the ground at $B$, determine his initial speed $v_A$ and the speed at which he strikes the ground.




**Probs. 12–96/97**

$v = \frac{ds}{dt}$

$a = \frac{dv}{dt}$

$a\,ds = v\,dv$

$a = a_c$; $t = 0$, $s_0$, $v_0$

$s = s_0 + v_0 t + \frac{1}{2} a_c t^2$ ✓

$v = v_0 + a_c t$ ✓

$v^2 = v_0^2 + 2a_c(s - s_0)$

**Case 12-96**

Horizontal motion

$s = s_0 + v_0 t + \frac{1}{2} a_c t^2$

$s = 100\left(\frac{4}{5}\right) = 80$ ; $s_0 = 0$

$a_c = 0$ ; $v_0 = V_{Ax} = V_A \cos 25^\circ$

$80 = (V_A \cos 25^\circ)t$

$V_A = \frac{80}{t \cos 25^\circ}$

Vertical motion

$s = s_0 + v_0 t + \frac{1}{2} a_c t^2$

$s_0 = 100\left(\frac{3}{5}\right) + 4 = 64$

$s = 0$ ; $V_0 = V_{Ay} = V_A \sin 25$

$a_c = -9.81$

$0 = 64 + (V_A \sin 25^\circ)t - \frac{1}{2}(9.81)t^2$

$4.905t^2 = 64 + \frac{80}{t \cos 25}(\sin 25)t$

$t = 4.54$ secs

Time of flight for $t_{AB} = 4.54$ secs

$V_A = \frac{80}{t \cos 25^\circ}$ ; $t = 4.54$ ; $V_A = 19.44$ m/s

**Case 12-97**

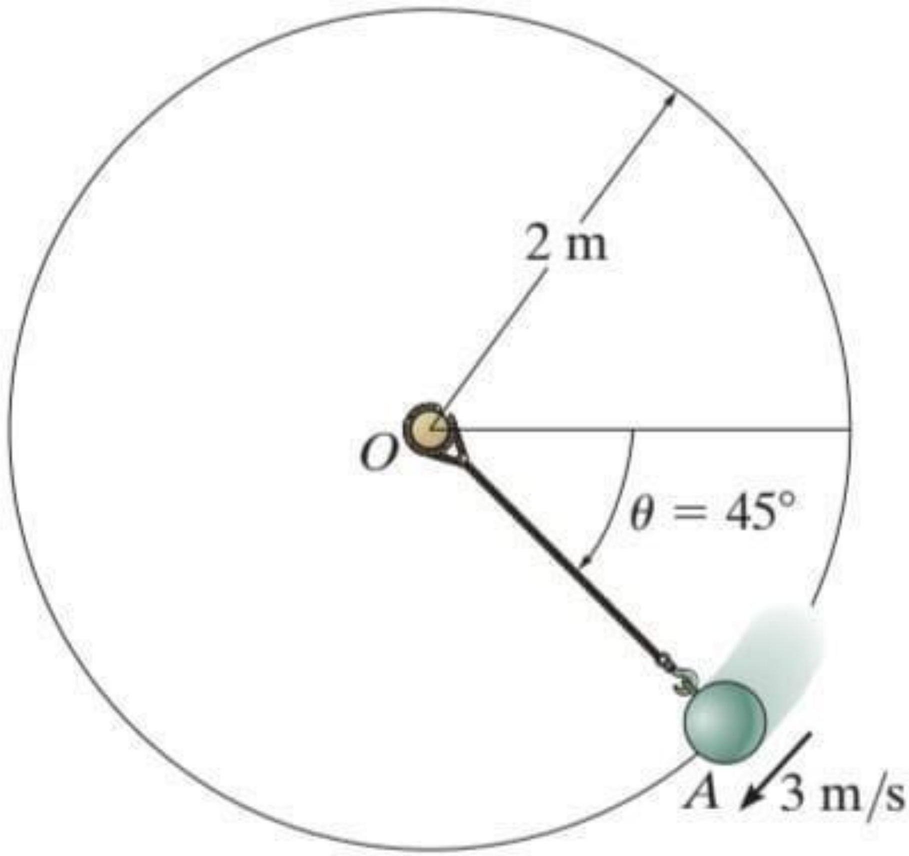
Using $v = v_0 + a_c t$ to find $V_{By}$ (Vertical motion)

$V_{By} = 19.44 \sin 25^\circ - 9.81(4.54)$

$V_{By} = -36.32$ m/s ; $V_{Bx} = V_{Ax} = 19.44 \cos 25^\circ$

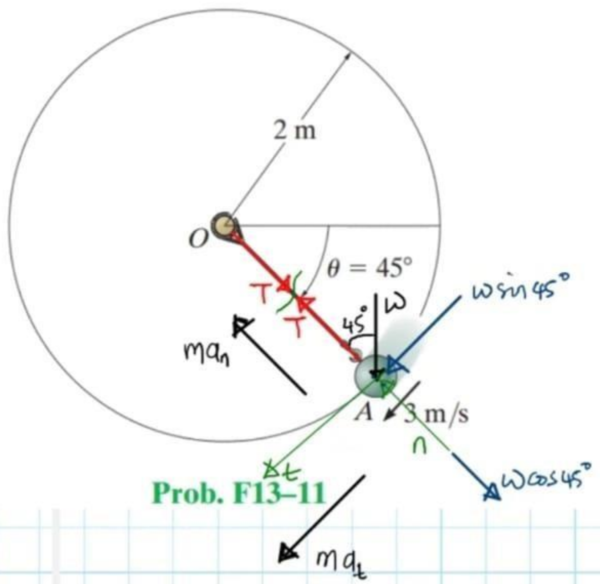
$V_B = \sqrt{V_{Bx}^2 + V_{By}^2}$ ; $V_B = 40.37$ m/s

**F13–11.** If the 10-kg ball has a velocity of 3 m/s when it is at the position $A$, along the vertical path, determine the tension in the cord and the increase in the speed of the ball at this position.




**Prob. F13–11**

**F13–11.** If the 10-kg ball has a velocity of 3 m/s when it is at the position $A$, along the vertical path, determine the tension in the cord and the increase in the speed of the ball at this position.




Prob. F13–11

$$+\nwarrow \Sigma F_n = ma_n \; ; \; a_n = \frac{v^2}{\rho} \; ; \; m = 10 \; ; \; \rho = 2$$

$$w = mg = (10)(9.81)$$

$$v = v_t = 3$$

$$T - w\cos 45° = ma_n$$

$$T = \frac{mv^2}{\rho} + w\cos 45° = \frac{mv^2}{\rho} + mg\cos 45°$$

$$T = 10\left(\frac{3^2}{2} + 9.81\cos 45°\right)$$

$$\boxed{T = 114.38\ N}$$

$$\swarrow + \Sigma F_t = ma_t$$

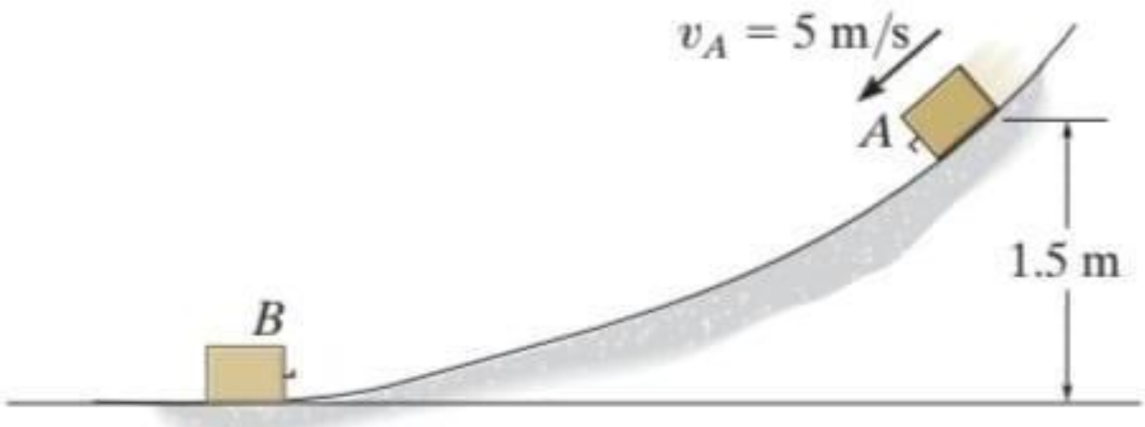
$$w\sin 45° = ma_t \; ; \; w = mg$$

$$a_t = \frac{w\sin 45°}{m} = \frac{mg}{m}\sin 45°$$

$$a_t = 9.81\sin 45°$$

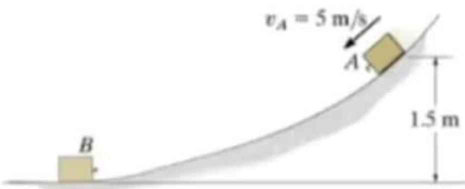
$$\boxed{\swarrow a_t = 6.94\ m/s^2}$$

**F15–9.** The 5-kg block $A$ has an initial speed of 5 m/s as it slides down the smooth ramp, after which it collides with the stationary block $B$ of mass 8 kg. If the two blocks couple together after collision, determine their common velocity immediately after collision.




**Prob. F15–9**

**F15–9.** The 5-kg block *A* has an initial speed of 5 m/s as it slides down the smooth ramp, after which it collides with the stationary block *B* of mass 8 kg. If the two blocks couple together after collision, determine their common velocity immediately after collision.




$m_A = 5$

$M_B = 8$

$\Delta K = -\Delta U = -M_A g \Delta h$

$\frac{1}{2}(m_A + m_B) v_f^2 - \frac{1}{2} m_A v_i^2 = -M_A g h$

$\frac{1}{2}(m_A + m_B) v_f^2 = \frac{1}{2} m_A v_i^2 - m_A g \Delta h$

$v_f^2 = \dfrac{m_A (v_i^2 - g\Delta h)}{m_A + m_B} = \dfrac{5\,(5^2 + 9.81 \times 1.5)}{13}$

$v_f^2 = 20.93\ m^2/s^2 \Rightarrow 4.58\ m/s$


$M_A v_A = (M_A + M_B) v_f$

$v_f = v_A \left(\dfrac{M_A}{M_A + M_B}\right)$

$= (7.378\ m/s)\left[\dfrac{5\ kg}{9+8\ kg}\right]$

$v_f = 2.84\ m/s$

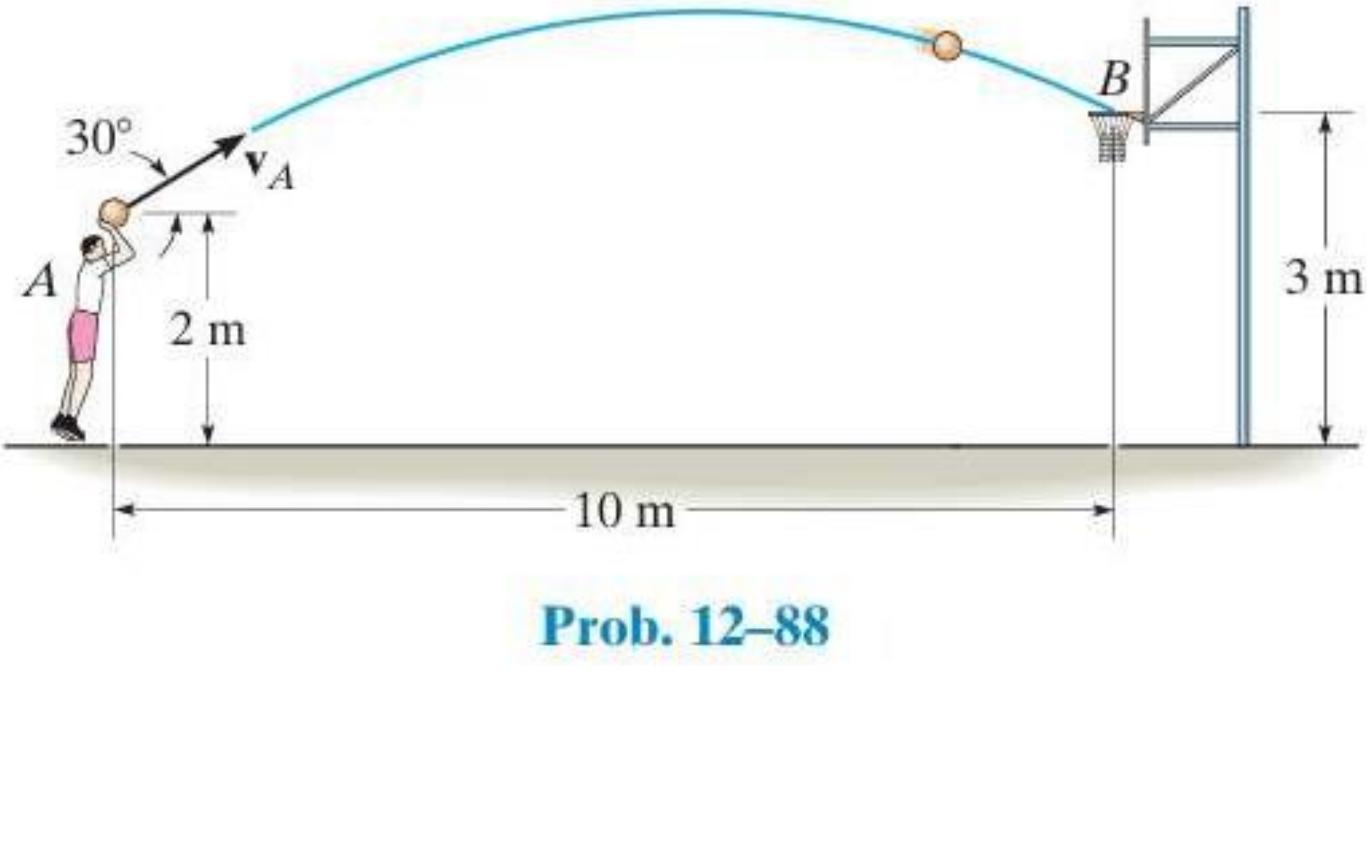
$\Delta K = -\Delta U$

$\frac{1}{2} m_A (v_A^2 - v_i^2) = -m_A g \Delta h$

$v_A^2 = v_i^2 - 2g\Delta h = (5\ m/s)^2 - 2(9.81\ m/s^2)(-1.5\ m)$

$v_A^2 = 54.43\ m^2/s^2 \Rightarrow v_A = 7.378\ m/s$

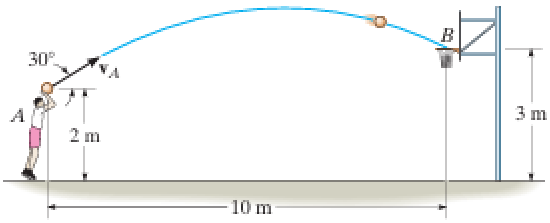
*12–88. Neglecting the size of the ball, determine the magnitude $v_A$ of the basketball's initial velocity and its velocity when it passes through the basket.



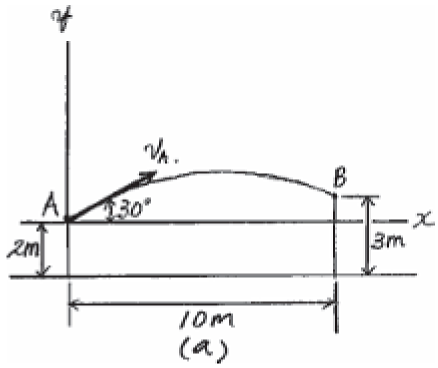

Prob. 12–88

*12–88.

Neglecting the size of the ball, determine the magnitude $v_A$ of the basketball's initial velocity and its velocity when it passes through the basket.




## SOLUTION




***Coordinate System.*** The origin of the $x$-$y$ coordinate system will be set to coinside with point $A$ as shown in Fig. $a$

***Horizontal Motion.*** Here $(v_A)_x = v_A \cos 30° \rightarrow$, $(s_A)_x = 0$ and $(s_B)_x = 10 \text{ m} \rightarrow$.

$$(\overset{+}{\rightarrow}) \quad (s_B)_x = (s_A)_x + (v_A)_x t$$

$$10 = 0 + v_A \cos 30° \, t$$

$$t = \frac{10}{v_A \cos 30°} \quad \textbf{(1)}$$

Also,

$$(\overset{+}{\rightarrow}) \quad (v_B)_x = (v_A)_x = v_A \cos 30° \quad \textbf{(2)}$$

***Vertical Motion.*** Here, $(v_A)_y = v_A \sin 30° \uparrow$, $(s_A)_y = 0$, $(s_B)_y = 3 - 2 = 1 \text{ m} \uparrow$ and $a_y = 9.81 \text{ m/s}^2 \downarrow$

$$(+\uparrow) \quad (s_B)_y = (s_A)_y + (v_A)_y t + \frac{1}{2} a_y t^2$$

$$1 = 0 + v_A \sin 30° \, t + \frac{1}{2}(-9.81)t^2$$

$$4.905t^2 - 0.5\, v_A \, t + 1 = 0 \quad \textbf{(3)}$$

Also

$$(+\uparrow) \quad (v_B)_y = (v_A)_y + a_y t$$

$$(v_B)_y = v_A \sin 30° + (-9.81)t$$

$$(v_B)_y = 0.5\, v_A - 9.81t \quad \textbf{(4)}$$

Solving Eq. (1) and (3)

$$v_A = 11.705 \text{ m/s} = 11.7 \text{ m/s} \qquad \textbf{Ans.}$$

$$t = 0.9865 \text{ s}$$

Substitute these results into Eq. (2) and (4)

$$(v_B)_x = 11.705 \cos 30° = 10.14 \text{ m/s} \rightarrow$$

$$(v_B)_y = 0.5(11.705) - 9.81(0.9865) = -3.825 \text{ m/s} = 3.825 \text{ m/s} \downarrow$$

Thus, the magnitude of $\mathbf{v}_B$ is

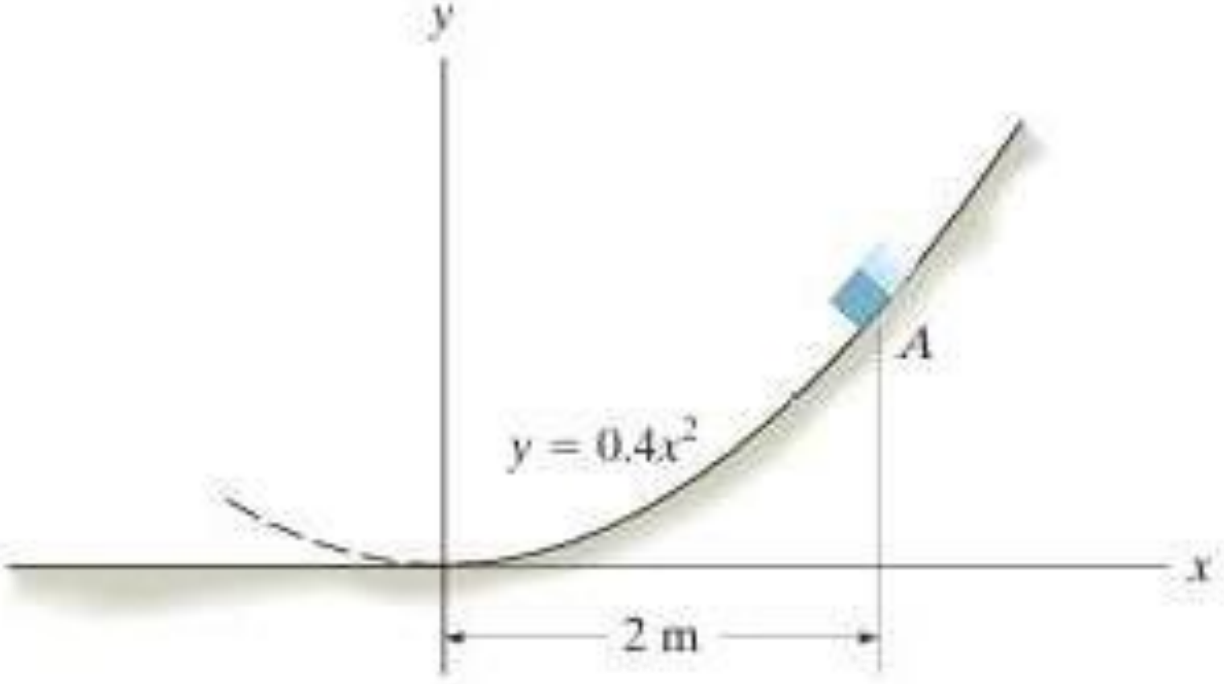
$$v_B = \sqrt{(v_B)_x^2 + (v_B)_y^2} = \sqrt{10.14^2 + 3.825^2} = 10.83 \text{ m/s} = 10.8 \text{ m/s} \qquad \textbf{Ans.}$$

And its direction is defined by

$$\theta_B = \tan^{-1}\left[\frac{(v_B)_y}{(v_B)_x}\right] = \tan^{-1}\left(\frac{3.825}{10.14}\right) = 20.67° = 20.7° \qquad \textbf{Ans.}$$

**Ans:**
$v_A = 11.7 \text{ m/s}$
$v_B = 10.8 \text{ m/s}$

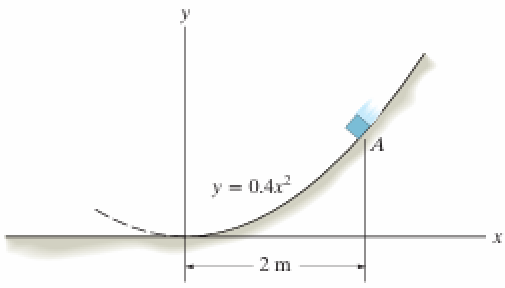
*12–124. The box of negligible size is sliding down along a curved path defined by the parabola $y = 0.4x^2$. When it is at $A(x_A = 2\text{ m}, y_A = 1.6\text{ m})$, the speed is $v = 8\text{ m/s}$ and the increase in speed is $dv/dt = 4\text{ m/s}^2$. Determine the magnitude of the acceleration of the box at this instant.




Prob. 12–124

***12–124.**

The box of negligible size is sliding down along a curved path defined by the parabola $y = 0.4x^2$. When it is at $A$ ($x_A = 2$ m, $y_A = 1.6$ m), the speed is $v = 8$ m/s and the increase in speed is $dv/dt = 4$ m/s². Determine the magnitude of the acceleration of the box at this instant.




## SOLUTION

$$y = 0.4\,x^2$$

$$\left.\frac{dy}{dx}\right|_{x=2\text{ m}} = 0.8x\bigg|_{x=2\text{ m}} = 1.6$$

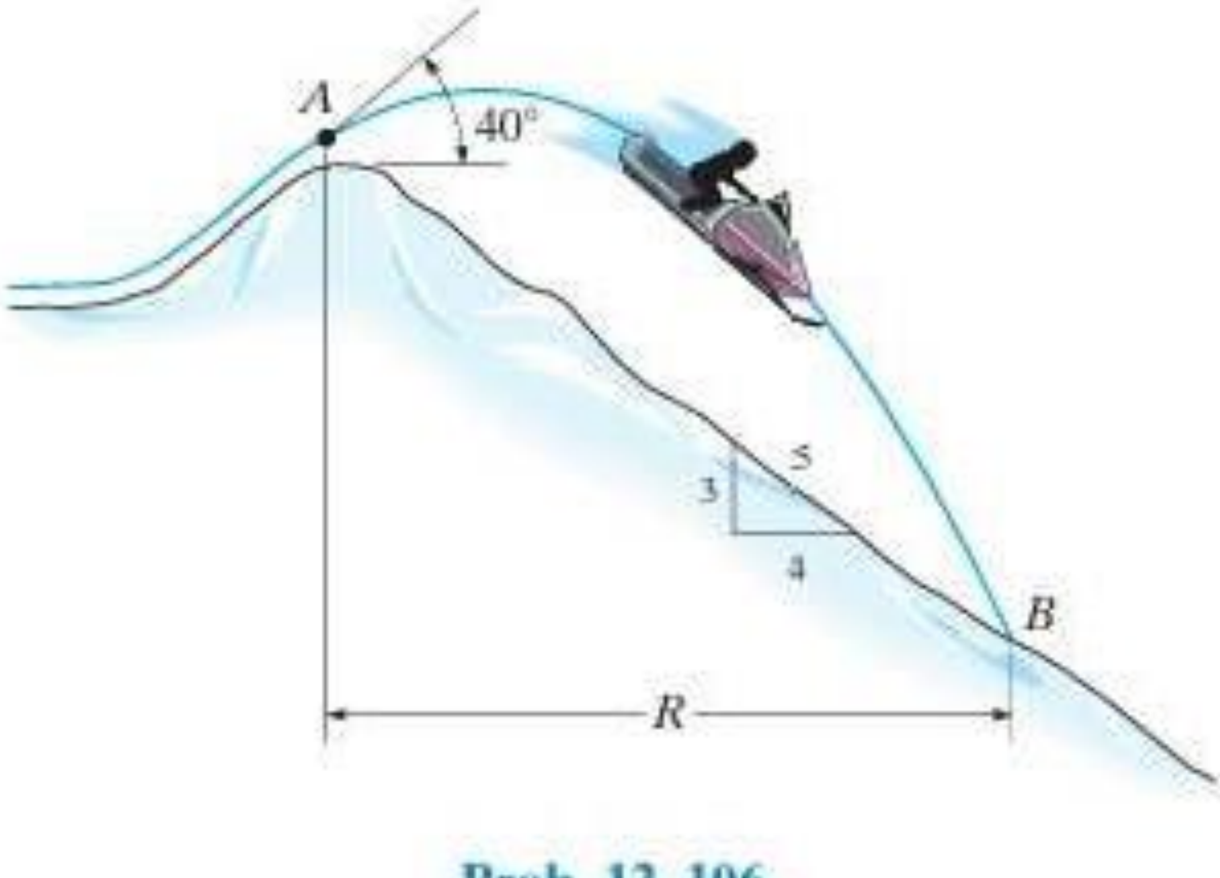
$$\left.\frac{d^2y}{dx^2}\right|_{x=2\text{ m}} = 0.8$$

$$\rho = \left.\frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|}\right|_{x=2\text{ m}} = \frac{\left[1 + (1.6)^2\right]^{3/2}}{|0.8|} = 8.396\text{ m}$$

$$a_n = \frac{{v_B}^2}{\rho} = \frac{8^2}{8.396} = 7.622\text{ m/s}^2$$

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{(4)^2 + (7.622)^2} = 8.61\text{ m/s}^2$$ **Ans.**

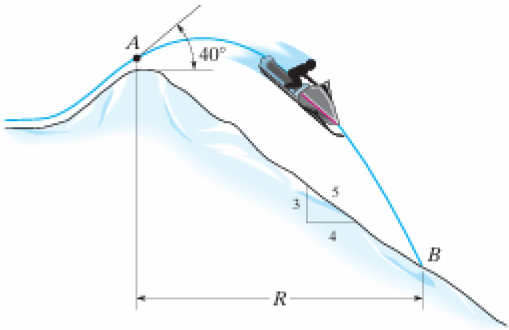
**12–106.** The snowmobile is traveling at 10 m/s when it leaves the embankment at $A$. Determine the time of flight from $A$ to $B$ and the range $R$ of the trajectory.




Prob. 12–106

**12–106.**

The snowmobile is traveling at 10 m/s when it leaves the embankment at $A$. Determine the time of flight from $A$ to $B$ and the range $R$ of the trajectory.




## SOLUTION

$(\overset{+}{\rightarrow})\qquad s_B = s_A + v_A t$

$$R = 0 + 10\cos 40°\, t$$

$(+\uparrow)\qquad s_B = s_A + v_A t + \frac{1}{2}a_c t^2$

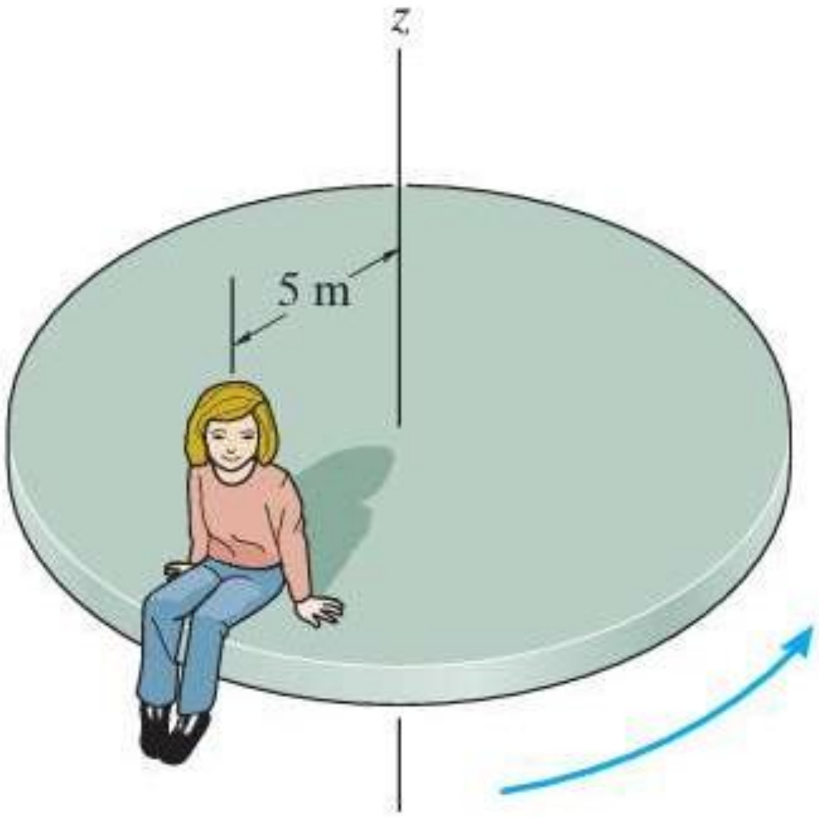
$$-R\left(\frac{3}{4}\right) = 0 + 10\sin 40° t - \frac{1}{2}(9.81)\, t^2$$

Solving:

$R = 19.0\text{ m}$ **Ans.**

$t = 2.48\text{ s}$ **Ans.**

***13–52.** A girl, having a mass of 15 kg, sits motionless relative to the surface of a horizontal platform at a distance of $r = 5$ m from the platform's center. If the angular motion of the platform is *slowly* increased so that the girl's tangential component of acceleration can be neglected, determine the maximum speed which the girl will have before she begins to slip off the platform. The coefficient of static friction between the girl and the platform is $\mu = 0.2$.




**Prob. 13–52**

***13–52.**

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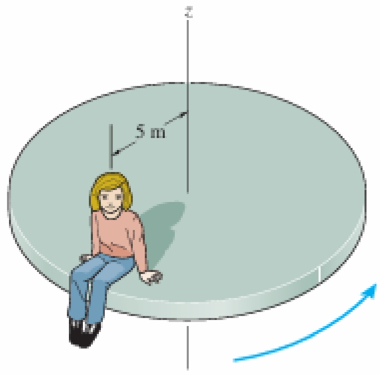


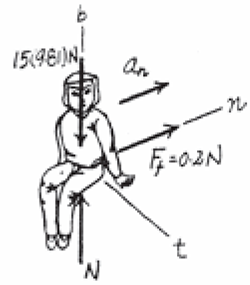

## SOLUTION

***Equation of Motion:*** Since the girl is on the verge of slipping, $F_f = \mu_s N = 0.2N$. Applying Eq. 13–8, we have

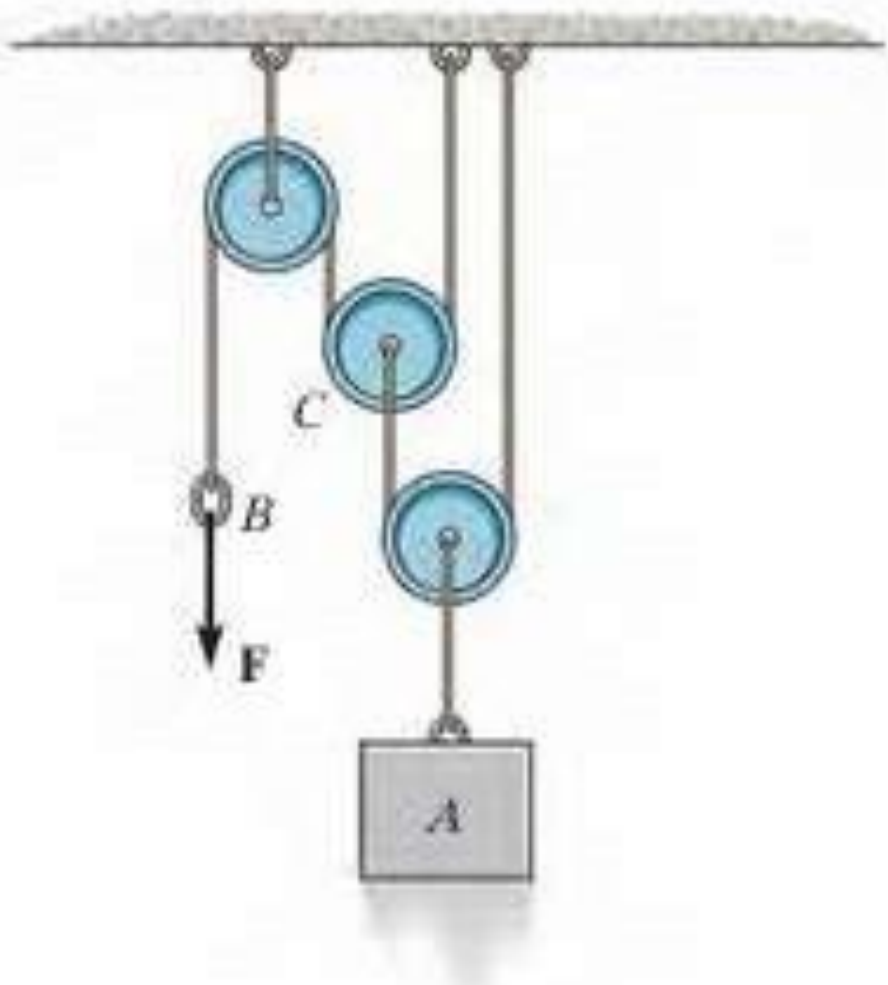
$$\Sigma F_b = 0; \qquad N - 15(9.81) = 0 \qquad N = 147.15\text{ N}$$

$$\Sigma F_n = ma_n; \qquad 0.2(147.15) = 15\left(\frac{v^2}{5}\right)$$

$$v = 3.13\text{ m/s} \qquad \textbf{Ans.}$$



**13–23.** If the supplied force $F = 150$ N, determine the velocity of the 50-kg block $A$ when it has risen 3 m, starting from rest.




**Prob. 13–23**

**13–23.**

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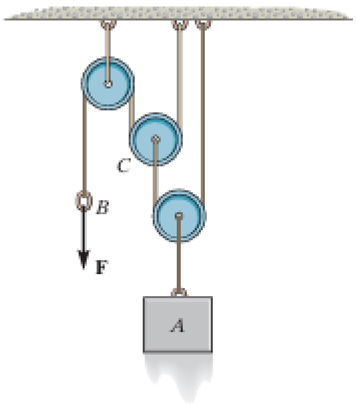



SOLUTION

**Equations of Motion.** Since the pulleys are smooth, the tension is constant throughout each entire cable. Referring to the FBD of pulley $C$, Fig. $a$, of which its mass is negligible.

$+\uparrow \Sigma F_y = 0;\qquad 150 + 150 - T = 0 \qquad T = 300 \text{ N}$

Subsequently, considered the FBD of block $A$ shown in Fig. $b$,

$+\uparrow \Sigma F_y = ma_y;\qquad 300 + 300 - 50(9.81) = 50a$

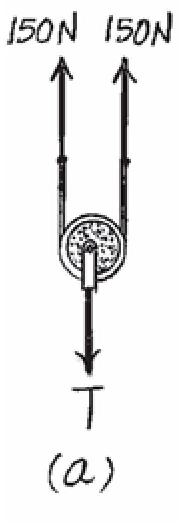
$$a = 2.19 \text{ m/s}^2 \uparrow$$

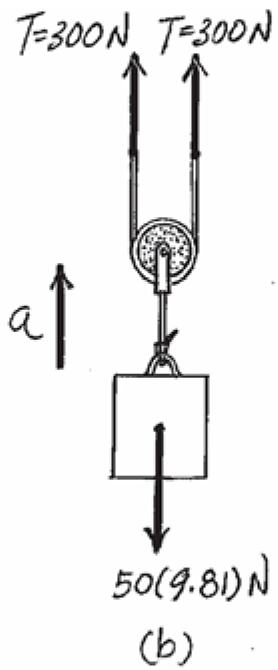
**Kinematics.** Using the result of **a**,

$(+\uparrow)\quad v^2 = v_0^2 + 2a_c s;$

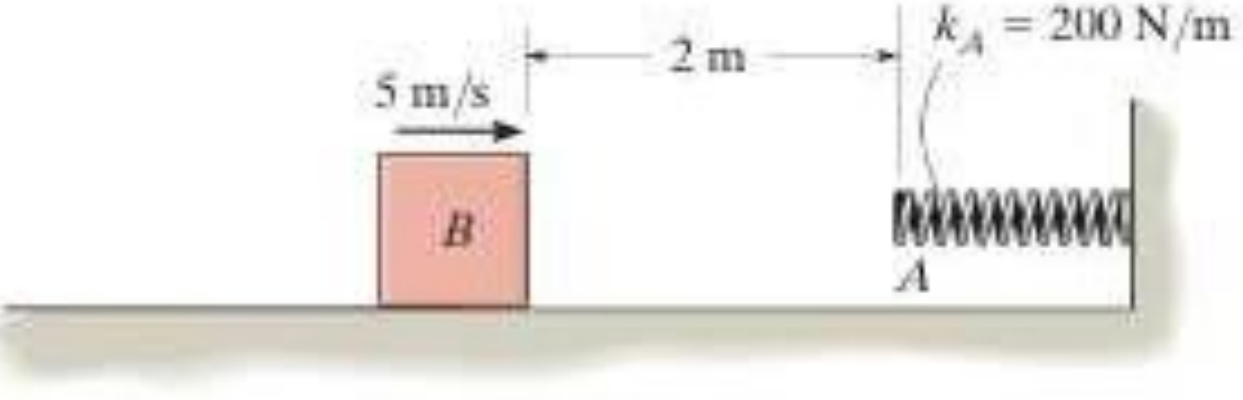
$$v^2 = 0^2 + 2(2.19)(3)$$

$$v = 3.6249 \text{ m/s} = 3.62 \text{ m/s}$$ **Ans.**



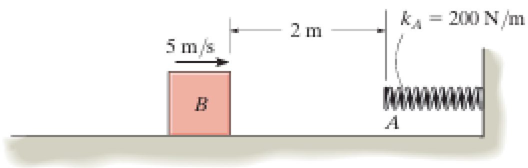
**14–23.** The 8-kg block is moving with an initial speed of 5 m/s. If the coefficient of kinetic friction between the block and plane is $\mu_k = 0.25$, determine the compression in the spring when the block momentarily stops.




**Prob. 14–23**

**14–23.**

The 8-kg block is moving with an initial speed of 5 m/s. If the coefficient of kinetic friction between the block and plane is $\mu_k = 0.25$, determine the compression in the spring when the block momentarily stops.




## SOLUTION

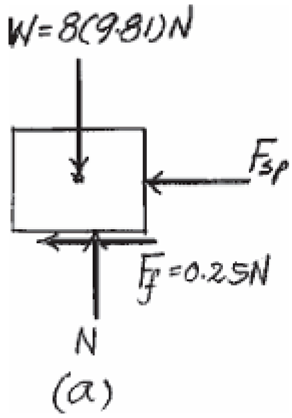
***Work.*** Consider the force equilibrium along $y$ axis by referring to the FBD of the block, Fig. *a*

$$+\uparrow \Sigma F_y = 0; \qquad N - 8(9.81) = 0 \qquad N = 78.48 \text{ N}$$

Thus, the friction is $F_f = \mu_k N = 0.25(78.48) = 19.62$ N and $F_{sp} = kx = 200\,x$. Here, the spring force $F_{sp}$ and $F_f$ both do negative work. The weight $W$ and normal reaction $N$ do no work.

$$U_{F_{sp}} = -\int_0^x 200\,x\,dx = -100\,x^2$$

$$U_{F_f} = -19.62(x + 2)$$




***Principle of Work And Energy.*** It is required that the block stopped momentarily, $T_2 = 0$. Applying Eq. 14–7

$$T_1 + \Sigma U_{1-2} = T_2$$

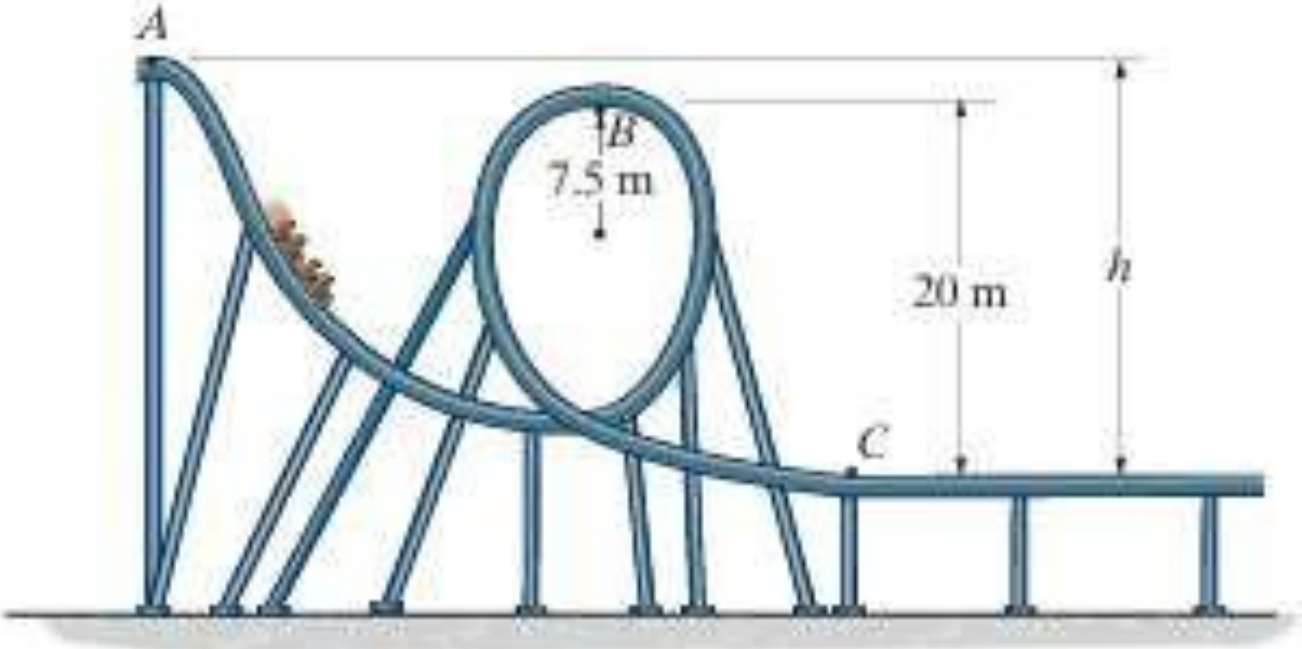
$$\frac{1}{2}(8)(5^2) + \left(-100x^2\right) + [-19.62(x + 2)] = 0$$

$$100x^2 + 19.62x - 60.76 = 0$$

Solved for positive root,

$$x = 0.6875 \text{ m} = 0.688 \text{ m}$$ **Ans.**

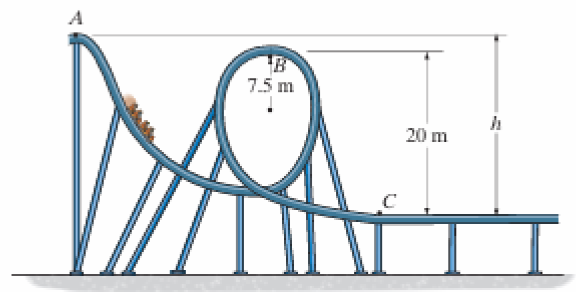
**14–77.** The roller coaster car having a mass $m$ is released from rest at point $A$. If the track is to be designed so that the car does not leave it at $B$, determine the required height $h$. Also, find the speed of the car when it reaches point $C$. Neglect friction.




Prob. 14–77

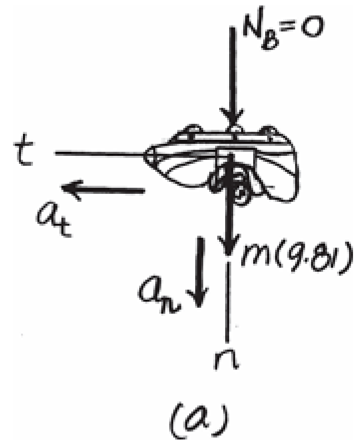
**14–77.**

The roller coaster car having a mass $m$ is released from rest at point $A$. If the track is to be designed so that the car does not leave it at $B$, determine the required height $h$. Also, find the speed of the car when it reaches point $C$. Neglect friction.




## SOLUTION

***Equation of Motion:*** Since it is required that the roller coaster car is about to leave the track at $B$, $N_B = 0$. Here, $a_n = \dfrac{v_B^2}{\rho_B} = \dfrac{v_B^2}{7.5}$. By referring to the free-body diagram of the roller coaster car shown in Fig. $a$,




$$\Sigma F_n = ma_n; \qquad m(9.81) = m\left(\frac{v_B^2}{7.5}\right) \quad v_B^2 = 73.575 \text{ m}^2/\text{s}^2$$

***Potential Energy:*** With reference to the datum set in Fig. $b$, the gravitational potential energy of the rollercoaster car at positions $A$, $B$, and $C$ are $\left(V_g\right)_A = mgh_A = m(9.81)h = 9.81mh$, $\left(V_g\right)_B = mgh_B = m(9.81)(20) = 196.2$ m, and $\left(V_g\right)_C = mgh_C = m(9.81)(0) = 0$.

***Conservation of Energy:*** Using the result of $v_B^2$ and considering the motion of the car from position $A$ to $B$,

$$T_A + V_A = T_B + V_B$$

$$\frac{1}{2}mv_A^2 + \left(V_g\right)_A = \frac{1}{2}mv_B^2 + \left(V_g\right)_B$$

$$0 + 9.81mh = \frac{1}{2}m(73.575) + 196.2m$$

$$h = 23.75 \text{ m}$$ **Ans.**

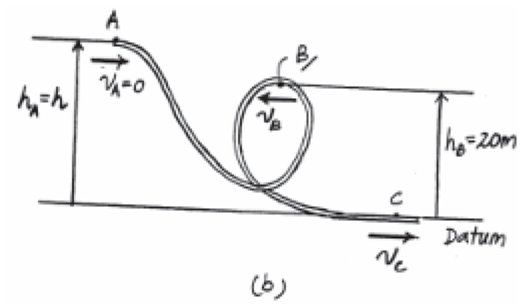
Also, considering the motion of the car from position $B$ to $C$,

$$T_B + V_B = T_C + V_C$$

$$\frac{1}{2}mv_B^2 + \left(V_g\right)_B = \frac{1}{2}mv_C^2 + \left(V_g\right)_C$$

$$\frac{1}{2}m(73.575) + 196.2m = \frac{1}{2}mv_C^2 + 0$$

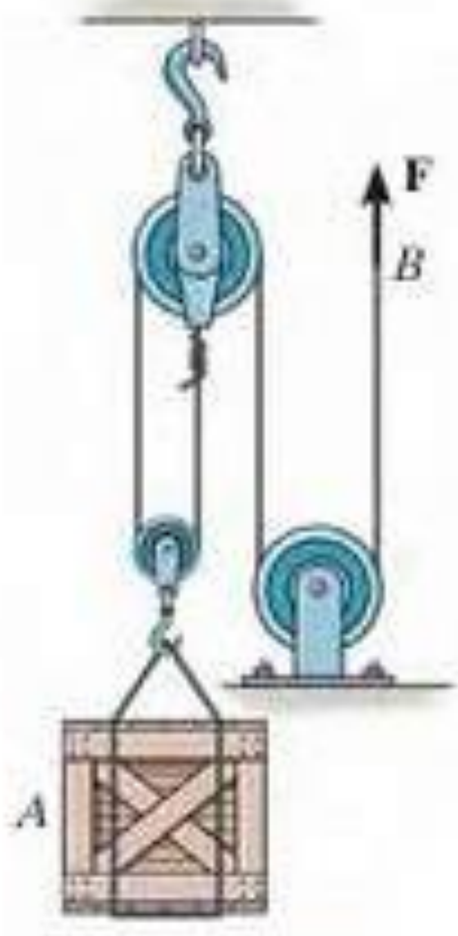
$$v_C = 21.6 \text{ m/s}$$ **Ans.**




**Ans:**
$h = 23.75$ m
$v_C = 21.6$ m/s

**15–27.** The 20-kg crate is lifted by a force of $F = (100 + 5t^2)$ N, where $t$ is in seconds. Determine the speed of the crate when $t = 3$ s, starting from rest.

***15–28.** The 20-kg crate is lifted by a force of $F = (100 + 5t^2)$ N, where $t$ is in seconds. Determine how high the crate has moved upward when $t = 3$ s, starting from rest.




**Probs. 15–27/28**

**15–27.**

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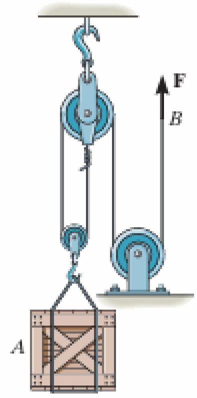



## SOLUTION

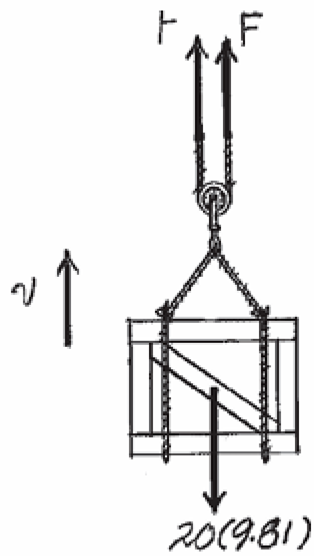
***Principle of Impulse and Momentum.*** At $t = 0$, $F = 100$ N. Since at this instant, $2F = 200\text{ N} > W = 20(9.81) = 196.2$ N, the crate will move the instant force **F** is applied. Referring to the FBD of the crate, Fig. *a*,

$$(+\uparrow) \quad m(v_y)_1 + \Sigma\int_{t_1}^{t_2} F_y dt = m(v_y)_2$$

$$0 + 2\int_0^{3\text{ s}} (100 + 5t^2)dt - 20(9.81)(3) = 20v$$

$$2\left(100t + \frac{5}{3}t^3\right)\Big|_0^{3\text{ s}} - 588.6 = 20v$$

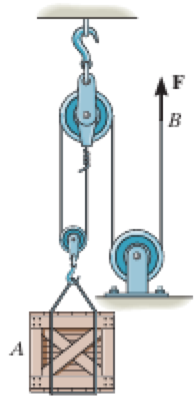
$$v = 5.07\text{ m/s}$$ **Ans.**




(a)

***15–28.**

The 20-kg crate is lifted by a force of $F = (100 + 5t^2)$ N, where $t$ is in seconds. Determine how high the crate has moved upward when $t = 3$ s, starting from rest.

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$$(+\uparrow) \quad m(v_y)_1 + \Sigma \int_{t_1}^{t_2} F_y dt = m(v_y)_2$$

$$0 + 2\int_0^t (100 + 5t^2)dt - 20(9.81)t = 20v$$

$$2\left(100t + \frac{5}{3}t^3\right)\Big|_0^t - 196.2t = 20v$$

$$v = \{0.1667t^3 + 0.19t\} \text{ m/s}$$

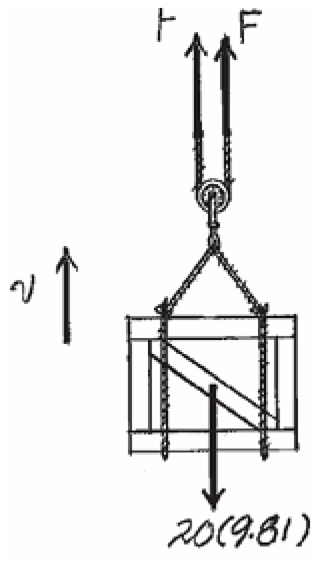
***Kinematics.*** The displacement of the crate can be determined by integrating $ds = v\,dt$ with the initial condition $s = 0$ at $t = 0$.

$$\int_0^s ds = \int_0^t (0.1667t^3 + 0.19t)\,dt$$

$$s = \{0.04167t^4 + 0.095t^2\} \text{ m}$$

At $t = 3$ s,

$$s = 0.04167(3^4) + 0.095(3^2) = 4.23 \text{ m} \qquad \textbf{Ans.}$$



(a)