Fluid Mechanics
Suggested Questions with Solutions

2.30 The velocity distribution for water (20°C) near a wall is given by $u = a(y/b)^{1/6}$, where $a = 10$ m/s, $b = 2$ mm, and y is the distance from the wall in mm. Determine the shear stress in the water at $y = 1$ mm.

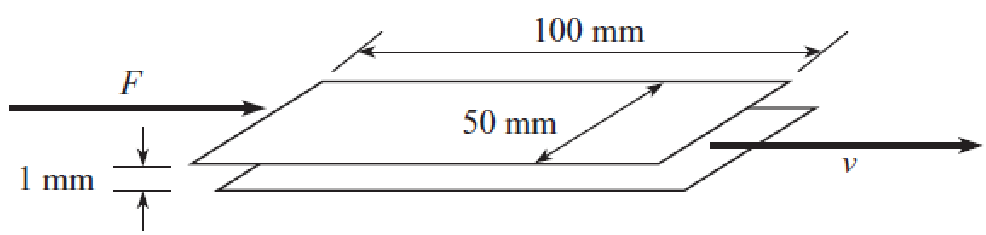
2.32 WILEY PLUS **(part a only)** A liquid flows between parallel boundaries as shown above. The velocity distribution near the lower wall is given in the following table:

y in mm	V in m/S
0.0	0.00
1.0	1.00
2.0	1.99
3.0	2.98

2.33 Suppose that glycerin is flowing ($T = 20°C$) and that the pressure gradient dp/dx is $-1.6\ kN/m^3$. What are the velocity and shear stress at a distance of 12 mm from the wall if the space B between the walls is 5.0 cm? What are the shear stress and velocity at the wall? The velocity distribution for viscous flow between stationary plates is

$$u = -\frac{1}{2\mu}\frac{dp}{dx}(By - y^2)$$

2.35 The sliding plate viscometer shown below is used to measure the viscosity of a fluid. The top plate is moving to the right with a constant velocity of 10 m/s in response to a force of 3 N. The bottom plate is stationary. What is the viscosity of the fluid? Assume a linear velocity distribution.



PROBLEM 2.35

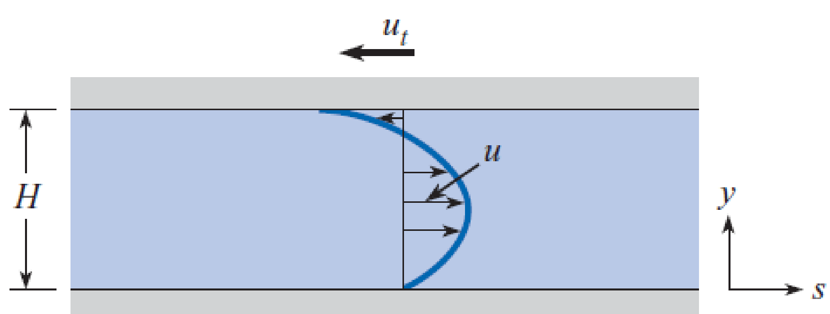
2.36 A laminar flow occurs between two horizontal parallel plates under a pressure gradient dp/ds (p decreases in the positive s direction). The upper plate moves left (negative) at velocity u_t. The expression for local velocity u is given as

$$u = -\frac{1}{2\mu}\frac{dp}{ds}(Hy - y^2) + u_t\frac{y}{H}$$

a. Is the magnitude of the shear stress greater at the moving plate ($y = H$) or at the stationary plate ($y = 0$)?

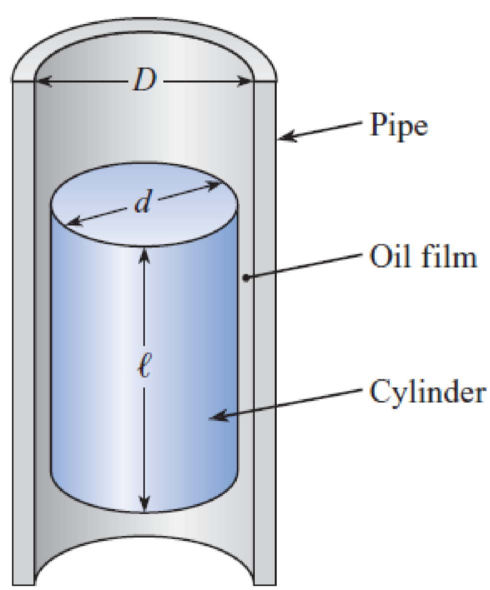
b. Derive an expression for the y position of zero shear stress.

c. Derive an expression for the plate speed u_t required to make the shear stress zero at $y = 0$.



PROBLEM 2.36

2.37 This problem involves a cylinder falling inside a pipe that is filled with oil, as depicted in the figure. The small space between the cylinder and the pipe is lubricated with an oil film that has viscosity μ. Derive a formula for the steady rate of descent of a cylinder with weight W, diameter d, and length ℓ sliding inside a vertical smooth pipe that has inside diameter D. Assume that the cylinder is concentric with the pipe as it falls. Use the general formula to find the rate of descent of a cylinder 100 mm in diameter that slides inside a 100.5 mm pipe. The cylinder is 200 mm long and weighs 15 N. The lubricant is SAE 20W oil at 10°C.



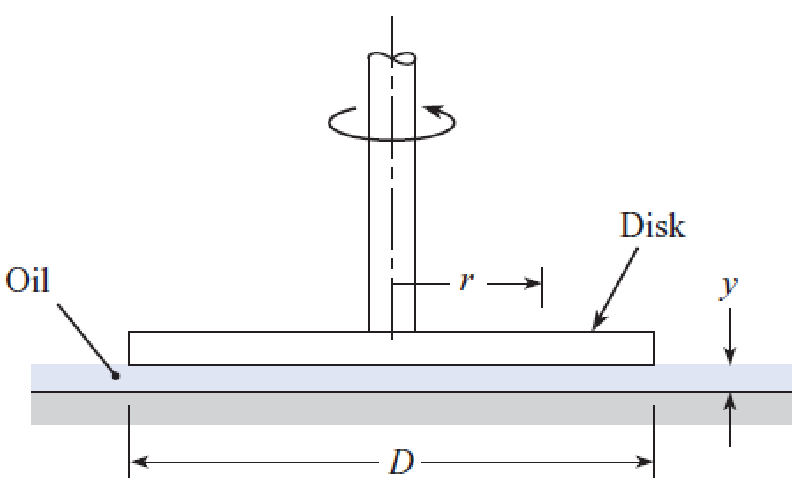
PROBLEM 2.37

2.38 The device shown consists of a disk that is rotated by a shaft. The disk is positioned very close to a solid boundary. Between the disk and the boundary is viscous oil.

a. If the disk is rotated at a rate of 1 rad/s, what will be the ratio of the shear stress in the oil at $r = 2$ cm to the shear stress at $r = 3$ cm?

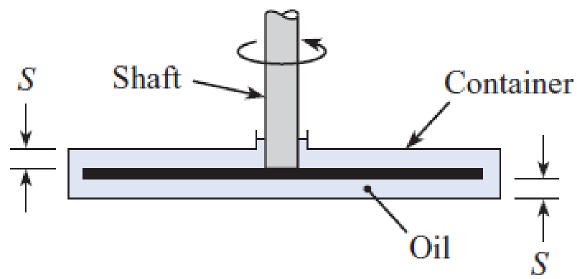
b. If the rate of rotation is 2 rad/s, what is the speed of the oil in contact with the disk at $r = 3$ cm?

c. If the oil viscosity is 0.01 N · s/m^2 and the spacing y is 2 mm, what is the shear stress for the conditions noted in part (b)?



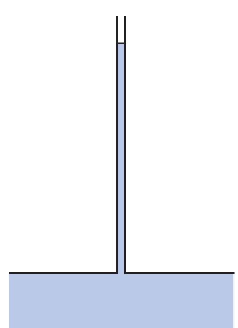
PROBLEM 2.38

2.39 Some instruments having angular motion are damped by means of a disk connected to the shaft. The disk, in turn, is immersed in a container of oil, as shown. Derive a formula for the damping torque as a function of the disk diameter D, spacing S, rate of rotation ω, and oil viscosity μ.



PROBLEM 2.39

2.61 Calculate the maximum capillary rise of water between two vertical glass plates spaced 1 mm apart.



PROBLEM 2.61

2.72 The vapor pressure of water at 100°C is 101 kN/m^2 because water boils under these conditions. The vapor pressure of water decreases approximately linearly with decreasing temperature at a rate of 3.1 kN/m^2/°C. Calculate the boiling temperature of water at an altitude of 3000 m, where the atmospheric pressure is 69 kN/m^2 absolute.

Solutions

2.30

Situation:

Water flows near a wall. The velocity distribution is

$$u(y) = a\left(\frac{y}{b}\right)^{1/6}$$

$a = 10\,\mathrm{m/s}$, $b = 2\,\mathrm{mm}$ and y is the distance (mm) from the wall.

Find:

Shear stress in the water at $y = 1$ mm.

Properties:

Table A.5 (water at 20 °C): $\mu = 1.00 \times 10^{-3}\,\mathrm{N \cdot s/m^2}$.

SOLUTION

Rate of strain (algebraic equation)

$$\begin{aligned}\frac{du}{dy} &= \frac{d}{dy}\left[a\left(\frac{y}{b}\right)^{1/6}\right] \\ &= \frac{a}{b^{1/6}}\frac{1}{6y^{5/6}} \\ &= \frac{a}{6b}\left(\frac{b}{y}\right)^{5/6}\end{aligned}$$

Rate of strain (at $y = 1\,\mathrm{mm}$)

$$\begin{aligned}\frac{du}{dy} &= \frac{a}{6b}\left(\frac{b}{y}\right)^{5/6} \\ &= \frac{10\,\mathrm{m/s}}{6 \times 0.002\,\mathrm{m}}\left(\frac{2\,\mathrm{mm}}{1\,\mathrm{mm}}\right)^{5/6} \\ &= 1485\,\mathrm{s}^{-1}\end{aligned}$$

Shear Stress

$$\begin{aligned}\tau_{y=1\,\mathrm{mm}} &= \mu\frac{du}{dy} \\ &= \left(1.00 \times 10^{-3}\frac{\mathrm{N \cdot s}}{\mathrm{m}^2}\right)\left(1485\,\mathrm{s}^{-1}\right) \\ &= 1.485\,\mathrm{Pa}\end{aligned}$$

$$\boxed{\tau\,(y = 1\,\mathrm{mm}) = 1.49\,\mathrm{Pa}}$$

2.32

Situation:

A liquid flows between parallel boundaries.
$y_0 = 0.0\,\text{mm}$, $V_0 = 0.0\,\text{m/s}$.
$y_1 = 1.0\,\text{mm}$, $V_1 = 1.0\,\text{m/s}$.
$y_2 = 2.0\,\text{mm}$, $V_2 = 1.99\,\text{m/s}$.
$y_3 = 3.0\,\text{mm}$, $V_3 = 2.98\,\text{m/s}$.

Find:

(a) Maximum shear stress.
(b) Location where minimum shear stress occurs.

SOLUTION

(a) Maximum shear stress

$$\begin{aligned}
\tau &= \mu dV/dy \\
\tau_{\max} &\approx \mu(\Delta V/\Delta y) \text{ next to wall} \\
\tau_{\max} &= (10^{-3}\text{N}\cdot\text{s/m}^2)((1 \text{ m/s})/0.001 \text{ m}) \\
& \boxed{\tau_{\max} = 1.0 \text{ N/m}^2}
\end{aligned}$$

(b)The minimum shear stress will occur midway between the two walls.
Its magnitude will be zero because the velocity gradient is zero at the midpoint.

2.33

Situation:

Glycerin is flowing in between two stationary plates. The velocity distribution is

$$u = -\frac{1}{2\mu}\frac{dp}{dx}\left(By - y^2\right)$$

$dp/dx = -1.6\,\mathrm{kN/\,m^3} - 1.6\,\mathrm{kPa/\,m}$, $B = 5\,\mathrm{cm}$.

Find:

Velocity and shear stress at a distance of 12 mm from wall (i.e. at $y = 12\,\mathrm{mm}$).
Velocity and shear stress at the wall (i.e. at $y = 0\,\mathrm{mm}$).

Properties:

Glycerin (20 °C), Table A.4: $\mu = 1.41\,\mathrm{N \cdot s/\,m^2}$.

PLAN

Find velocity by direct substitution into the specified velocity distribution.
Find shear stress using the definition of viscosity: $\tau = \mu\,(du/dy)$, where the rate-of-strain (i.e. the derivative du/dy) is found by differentiating the velocity distribution.

SOLUTION

a.) Velocity (at $y = 12\,\mathrm{mm}$)

$$\begin{aligned} u &= -\frac{1}{2\mu}\frac{dp}{dx}\left(By - y^2\right) \\ &= -\frac{1}{2\left(1.41\,\mathrm{N \cdot s/\,m^2}\right)}\left(-1600\,\mathrm{N/\,m^3}\right)\left((0.05\,\mathrm{m})(0.012\,\mathrm{m}) - (0.012\,\mathrm{m})^2\right) \\ &= 0.258\,7\frac{\mathrm{m}}{\mathrm{s}} \end{aligned}$$

$$\boxed{u\,(y = 12\,\mathrm{mm}) = 0.259\,\mathrm{m/\,s}}$$

Rate of strain (general expression)

$$\begin{aligned} \frac{du}{dy} &= \frac{d}{dy}\left(-\frac{1}{2\mu}\frac{dp}{dx}\left(By - y^2\right)\right) \\ &= \left(-\frac{1}{2\mu}\right)\left(\frac{dp}{dx}\right)\frac{d}{dy}\left(By - y^2\right) \\ &= \left(-\frac{1}{2\mu}\right)\left(\frac{dp}{dx}\right)(B - 2y) \end{aligned}$$

Rate of strain (at $y = 12\,\mathrm{mm}$)

$$\begin{aligned} \frac{du}{dy} &= \left(-\frac{1}{2\mu}\right)\left(\frac{dp}{dx}\right)(B - 2y) \\ &= \left(-\frac{1}{2\left(1.41\,\mathrm{N \cdot s/\,m^2}\right)}\right)\left(-1600\frac{\mathrm{N}}{\mathrm{m^3}}\right)\left(0.05\,\mathrm{m} - 2 \times 0.012\,\mathrm{m}\right) \\ &= 14.75\,\mathrm{s^{-1}} \end{aligned}$$

Definition of viscosity

$$\begin{aligned}\tau &= \mu\frac{du}{dy}\\ &= \left(1.41\frac{\mathrm{N\cdot s}}{\mathrm{m}^2}\right)\left(14.75\,\mathrm{s}^{-1}\right)\\ &= 20.798\,\mathrm{Pa}\end{aligned}$$

$$\boxed{\tau\,(y = 12\,\mathrm{mm}) = 20.8\,\mathrm{Pa}}$$

b.) Velocity (at $y = 0\,\mathrm{mm}$)

$$\begin{aligned}u &= -\frac{1}{2\mu}\frac{dp}{dx}\left(By - y^2\right)\\ &= -\frac{1}{2\left(1.41\,\mathrm{N\cdot s/m^2}\right)}\left(-1600\,\mathrm{N/m^3}\right)\left((0.05\,\mathrm{m})\,(0\,\mathrm{m}) - (0\,\mathrm{m})^2\right)\\ &= 0.00\frac{\mathrm{m}}{\mathrm{s}}\end{aligned}$$

$$\boxed{u\,(y = 0\,\mathrm{mm}) = 0\,\mathrm{m/s}}$$

Rate of strain (at $y = 0\,\mathrm{mm}$)

$$\begin{aligned}\frac{du}{dy} &= \left(-\frac{1}{2\mu}\right)\left(\frac{dp}{dx}\right)(B - 2y)\\ &= \left(-\frac{1}{2\left(1.41\,\mathrm{N\cdot s/m^2}\right)}\right)\left(-1600\frac{\mathrm{N}}{\mathrm{m}^3}\right)\left(0.05\,\mathrm{m} - 2\times 0\,\mathrm{m}\right)\\ &= 28.37\,\mathrm{s}^{-1}\end{aligned}$$

Shear stress (at $y = 0\,\mathrm{mm}$)

$$\begin{aligned}\tau &= \mu\frac{du}{dy}\\ &= \left(1.41\frac{\mathrm{N\cdot s}}{\mathrm{m}^2}\right)\left(28.37\,\mathrm{s}^{-1}\right)\\ &= 40.00\,\mathrm{Pa}\end{aligned}$$

$$\boxed{\tau\,(y = 0\,\mathrm{mm}) = 40.0\,\mathrm{Pa}}$$

REVIEW

1. As expected, the velocity at the wall (i.e. at $y = 0$) is zero due to the no slip condition.
2. As expected, the shear stress at the wall is larger than the shear stress away from the wall. This is because shear stress is maximum at the wall and zero along the centerline (i.e. at $y = B/2$).

2.35

Situation:

Sliding plate viscometer is used to measure fluid viscosity.
$A = 50 \times 100\,\text{mm}$, $\Delta y = 1\,\text{mm}$.
$u = 10\,\text{m/s}$, $F = 3\,\text{N}$.

Find:

Viscosity of the fluid.

Assumptions:

Linear velocity distribution.

PLAN

1. The shear force τ is a force/area.
2. Use equation for viscosity to relate shear force to the velocity distribution.

SOLUTION

1. Calculate shear force

$$\begin{aligned}\tau &= \frac{Force}{Area} \\ \tau &= \frac{3\,\text{N}}{50\,\text{mm} \times 100\,\text{mm}} \\ \tau &= 600\,\text{N/m}^2\end{aligned}$$

2. Find viscosity

$$\begin{aligned}\mu &= \frac{\tau}{\left(\frac{du}{dy}\right)} \\ \mu &= \frac{600\,\text{N/m}^2}{[10\,\text{m/s}]/[1\,\text{mm}]} \times \frac{1\,\text{m}}{1000\,\text{mm}}\end{aligned}$$

$$\boxed{\mu = 6 \times 10^{-2} \frac{\text{N} \cdot \text{s}}{\text{m}^2}}$$

2.36

Situation:

Laminar flow occurs between two horizontal parallel plates. The velocity distribution is

$$u = -\frac{1}{2\mu}\frac{dp}{ds}\left(Hy - y^2\right) + u_t\frac{y}{H}$$

Pressure p decreases with distance s, and the speed of the upper plate is u_t. Note that u_t has a negative value to represent that the upper plate is moving to the left.

Moving plate: $y = H$.

Stationary plate: $y = 0$.

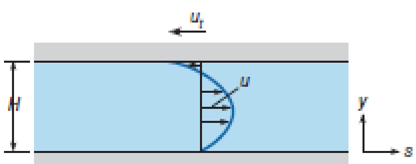
Find:

(a) Whether shear stress is greatest at the moving or stationary plate.

(b) Location of zero shear stress.

(c) Derive an expression for plate speed to make the shear stress zero at $y = 0$.

Sketch:



PLAN

By inspection, the rate of strain (du/dy) or slope of the velocity profile is larger at the moving plate. Thus, we expect shear stress τ to be larger at $y = H$. To check this idea, find shear stress using the definition of viscosity: $\tau = \mu\,(du/dy)$. Evaluate and compare the shear stress at the locations $y = H$ and $y = 0$.

SOLUTION

Part (a)

1. Shear stress, from definition of viscosity

$$\begin{aligned}
\tau &= \mu\frac{du}{dy} \\
&= \mu\frac{d}{dy}\left[-\frac{1}{2\mu}\frac{dp}{ds}\left(Hy-y^2\right)+u_t\frac{y}{H}\right] \\
&= \mu\left[-\frac{H}{2\mu}\frac{dp}{ds}+\frac{y}{\mu}\frac{dp}{ds}+\frac{u_t}{H}\right] \\
\tau\left(y\right) &= -\frac{\left(H-2y\right)}{2}\frac{dp}{ds}+\frac{\mu u_t}{H}
\end{aligned}$$

Shear stress at $y = H$

$$\begin{aligned}
\tau\left(y=H\right) &= -\frac{\left(H-2H\right)}{2}\frac{dp}{ds}+\frac{\mu u_t}{H} \\
&= \frac{H}{2}\left(\frac{dp}{ds}\right)+\frac{\mu u_t}{H} \qquad (1)
\end{aligned}$$

2. Shear stress at $y = 0$

$$\begin{aligned}
\tau\left(y=0\right) &= -\frac{\left(H-0\right)}{2}\frac{dp}{ds}+\frac{\mu u_t}{H} \\
&= -\frac{H}{2}\left(\frac{dp}{ds}\right)+\frac{\mu u_t}{H} \qquad (2)
\end{aligned}$$

Since pressure decreases with distance, the pressure gradient dp/ds is negative. Since the upper wall moves to the left, u_t is negative. Thus, maximum shear stress occurs at $y = H$ because both terms in Eq. (1) have the same sign (they are both negative.) In other words,

$$\left|\tau\left(y=H\right)\right| > \left|\tau\left(y=0\right)\right|$$

.

$$\boxed{\text{Maximum shear stress occur at } y = H}.$$

Part (b)
Use definition of viscosity to find the location (y) of zero shear stress

$$\begin{aligned}
\tau &= \mu\frac{du}{dy} \\
&= -\mu(1/2\mu)\frac{dp}{ds}(H-2y)+\frac{u_t\mu}{H} \\
&= -(1/2)\frac{dp}{ds}(H-2y)+\frac{u_t\mu}{H}
\end{aligned}$$

Set $\tau = 0$ and solve for y

$$\begin{aligned}
0 &= -(1/2)\frac{dp}{ds}(H-2y)+\frac{u_t\mu}{H} \\
&\boxed{y=\frac{H}{2}-\frac{\mu u_t}{H dp/ds}}
\end{aligned}$$

Part (c)

$$\begin{aligned}
\tau &= \mu\frac{du}{dy} = 0 \text{ at } y=0 \\
\frac{du}{dy} &= -(1/2\mu)\frac{dp}{ds}(H-2y)+\frac{u_t}{H} \\
\text{Then, at } y &= 0 : du/dy = 0 = -(1/2\mu)\frac{dp}{ds}H+\frac{u_t}{H} \\
\text{Solve for } u_t &: \boxed{u_t=(1/2\mu)\frac{dp}{ds}H^2} \\
\textit{Note} &: \text{ because } \frac{dp}{ds}<0, u_t<0.
\end{aligned}$$

2.37

Situation:

A cylinder falls inside a pipe filled with oil.

$d = 100\,\text{mm}$, $D = 100.5\,\text{mm}$.

$\ell = 200\,\text{mm}$, $W = 15\,\text{N}$.

Find:

Speed at which the cylinder slides down the pipe.

Properties:

SAE 20W oil (10°C) from Figure A.2: $\mu = 0.35$ N·s/m^2.

Assumptions:

Assume that buoyant forces can be neglected.

SOLUTION

$$\begin{aligned}
\tau &= \mu \frac{dV}{dy} \\
\frac{W}{\pi d \ell} &= \frac{\mu V_{\text{fall}}}{(D-d)/2} \\
V_{\text{fall}} &= \frac{W(D-d)}{2\pi d \ell \mu} \\
V_{\text{fall}} &= \frac{15\,\text{N}(0.5 \times 10^{-3}\,\text{m})}{(2\pi \times 0.1\,\text{m} \times 0.2\,\text{m} \times 3.5 \times 10^{-1}\,\text{N s/m}^2)} \\
& \boxed{V_{\text{fall}} = 0.17\text{m/s}}
\end{aligned}$$

2.38

Situation:

A disk is rotated very close to a solid boundary with oil in between.
$\omega_a = 1\,\text{rad/s}$, $r_2 = 2\,\text{cm}$, $r_3 = 3\,\text{cm}$.
$\omega_b = 2\,\text{rad/s}$, $r_b = 3\,\text{cm}$.
$H = 2\,\text{mm}$, $\mu_c = 0.01\,\text{N s/m}^2$.

Find:

(a) Ratio of shear stress at 2 cm to shear stress at 3 cm.
(b) Speed of oil at contact with disk surface.
(c) Shear stress at disk surface.

Assumptions:

Linear velocity distribution: $dV/dy = V/y = \omega r/y$.

SOLUTION

(a) Ratio of shear stresses

$$\tau = \mu \frac{dV}{dy} = \frac{\mu \omega r}{y}$$

$$\frac{\tau_2}{\tau_3} = \frac{\mu \times 1 \times 2/y}{\mu \times 1 \times 3/y}$$

$$\boxed{\frac{\tau_2}{\tau_3} = \frac{2}{3}}$$

(b) Speed of oil

$$V = \omega r = 2 \times 0.03$$

$$\boxed{V = 0.06\text{m/s}}$$

(c) Shear stress at surface

$$\tau = \mu \frac{dV}{dy} = 0.01\,\text{N s/m}^2 \times \frac{0.06\,\text{m/s}}{0.002\,\text{m}}$$

$$\boxed{\tau = 0.30\ \text{N/m}^2}$$

2.39

Situation:
A disk is rotated in a container of oil to damp the motion of an instrument.

Find:
Derive an equation for damping torque as a function of D, S, ω and μ.

PLAN

Apply the Newton's law of viscosity.

SOLUTION

Shear stress

$$\begin{aligned}\tau &= \mu\frac{dV}{dy} \\ &= \frac{\mu r\omega}{s}\end{aligned}$$

Find differential torque—on an elemental strip of area of radius r the differential shear force will be τdA or $\tau(2\pi r dr)$. The differential torque will be the product of the differential shear force and the radius r.

$$\begin{aligned}dT_{\text{one side}} &= r[\tau(2\pi r dr)] \\ &= r\left[\frac{\mu r\omega}{s}(2\pi r dr)\right] \\ &= \frac{2\pi\mu\omega}{s}r^3 dr \\ dT_{\text{both sides}} &= 4\left(\frac{r\pi\mu\omega}{s}\right)r^3 dr\end{aligned}$$

Integrate

$$\begin{aligned}T &= \int_0^{D/2}\frac{4\pi\mu\omega}{s}r^3 dr \\ T &= \frac{1}{16}\frac{\pi\mu\omega D^4}{s}\end{aligned}$$

2.61

Situation:

Two vertical glass plates

$t = 1$ mm

Find:

Capillary rise (h) between the plates.

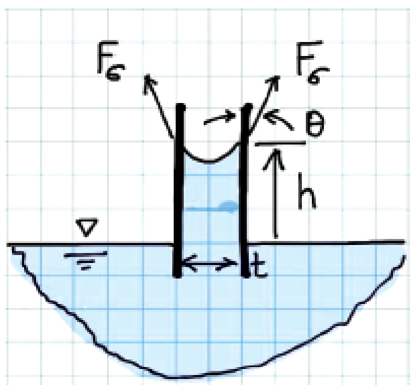
Properties:

From Table A.4, surface tension of water is 7.3×10^{-2} N/m.

PLAN

Apply equilibrium, then the surface tension force equation.

SOLUTION



Equilibrium

$$\begin{aligned} \sum F_y &= 0 \\ \text{Force due to surface tension} &= \text{Weight of fluid that has been pulled upward} \\ (2\ell)\,\sigma &= (h\ell t)\,\gamma \end{aligned}$$

Solve for capillary rise (h)

$$\begin{aligned} 2\sigma\ell - h\ell t\gamma &= 0 \\ h &= \frac{2\sigma}{\gamma t} \\ h &= \frac{2 \times (7.3 \times 10^{-2}\,\mathrm{N/m})}{9810\,\mathrm{N/m^3} \times 0.001\,\mathrm{m}} \\ &= 0.0149\ \mathrm{m} \\ h &= \boxed{14.9\ \mathrm{mm}} \end{aligned}$$

2.72

Situation:

The boiling temperature of water decreases with increasing elevation $\frac{\Delta p}{\Delta T} = \frac{-3.1\,\text{kPa}}{°C}$.

Find:

Boiling temperature at an altitude of 3000 m

Properties:

$T = 100°\text{C}$, $p = 101\,\text{kPa}$.

$z_{3000} = 3000\,\text{m}$, $p_{3000} = 69\,\text{kPa}$.

Assumptions:

Assume that vapor pressure versus boiling temperature is a linear relationship.

PLAN

Develop a linear equation for boiling temperature as a function of elevation.

SOLUTION

Let BT = "Boiling Temperature." Then, BT as a function of elevation is

$$BT\ (3000\text{ m}) = \text{BT}\ (0\text{ m}) + \left(\frac{\Delta BT}{\Delta p}\right)\Delta p$$

Thus,

$$\begin{aligned} BT\ (3000\text{ m}) &= 100\,°\text{C} + \left(\frac{-1.0\,°\text{C}}{3.1\,\text{kPa}}\right)(101-69)\text{ kPa} \\ &= 89.677\,°\text{C} \end{aligned}$$

$$\boxed{\text{Boiling Temperature (3000 m)} = 89.7\,°\text{C}}$$