3.79 PLUS For the gate shown, $\alpha = 45°$, $y_1 = 1$ m, and $y_2 = 4$ m. Will the gate fall or stay in position under the action of the hydrostatic and gravity forces if the gate itself weighs 150 kN and is 1.0 m wide? Assume $T = 10°C$. Use calculations to justify your answer.

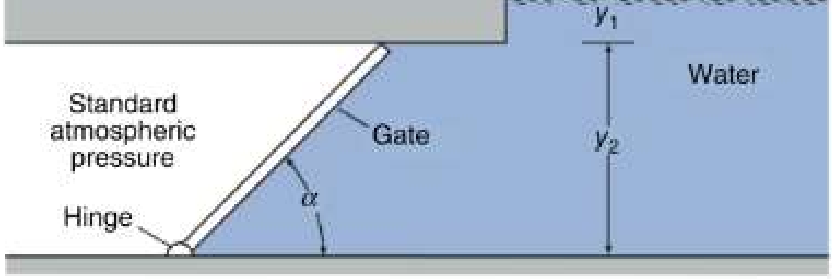
3.78: PROBLEM DEFINITION

Situation:

A submerged gate may fall due to its weight.

$y_1 = 1\,\text{m}$, $y_2 = 2\,\text{m}$, $w = 1\,\text{m}$.

$W = 80,000\,\text{N}$, $\alpha = 45^o$.



Properties:
Water (10 °C), Table A.5, $\gamma = 9810\,\mathrm{N/\,m^3}$.

SOLUTION

1. Hydrostatic Force:

- Area:

$$A = \frac{y_2}{\sin\alpha} \times w = \frac{2\,\mathrm{m}}{\sin 45^\circ} \times 1\,\mathrm{m} = 2.83\,\mathrm{m}^2$$

- Depth of the centroid of the plate:

$$\Delta z = y_1 + \frac{y_2}{2} = 1\,\mathrm{m} + \frac{2\,\mathrm{m}}{2} = 2\,\mathrm{m}$$

- Final Calculation:

$$F = \bar{p}A = \gamma \Delta z A = \left(9810\,\mathrm{N/\,m^3}\right)(2\,\mathrm{m})\left(2.83\,\mathrm{m}^2\right) = 55,525\,\mathrm{N}$$

2. Distance from CP to centroid:

- Area moment of inertia from Fig. A.1:

$$\begin{aligned}
\bar{I} &= \frac{wh^3}{12} \\
h &= \frac{y_2}{\sin\alpha} = \frac{2\,\mathrm{m}}{\sin 45^\circ} = 2.83\,\mathrm{m} \\
\bar{I} &= \frac{wh^3}{12} = \frac{(1\,\mathrm{m})(2.83)^3}{12} = 1.9\,\mathrm{m}^4
\end{aligned}$$

- Slant height:

$$\bar{y} = \frac{h}{2} + \frac{y_1}{\sin\alpha} = \frac{2.83\,\mathrm{m}}{2} + \frac{2.83}{\sin 45^\circ} = 2.83\,\mathrm{m}$$

- Final Calculation:

$$y_{cp} - \bar{y} = \frac{\overline{I}}{\bar{y}A} = \frac{(1.9\,\mathrm{m}^4)}{(2.83\,\mathrm{m})\,(2.83\,\mathrm{m}^2)} = 0.24\,\mathrm{m}$$

3. Torque due to weight:

- Moment arm:

$$x_1 = \frac{y_2 \tan\alpha}{2} = \frac{(2\,\mathrm{m})\,(\tan 45^\circ)}{2} = 2\,\mathrm{m}$$

- Final calculation:

$$M_1 = Wx_1 = (80,000\,\mathrm{N})\,(1\,\mathrm{m}) = 80,000\ \mathrm{N}\cdot\mathrm{m}$$

4. Torque due hydrostatic pressure:

- Final calculation:

$$M_1 = Wx_1 = (80,000\,\text{N})\,(1\,\text{m}) = 80,000\ \text{N} \cdot \text{m}$$

4. Torque due hydrostatic pressure:

- Moment arm:

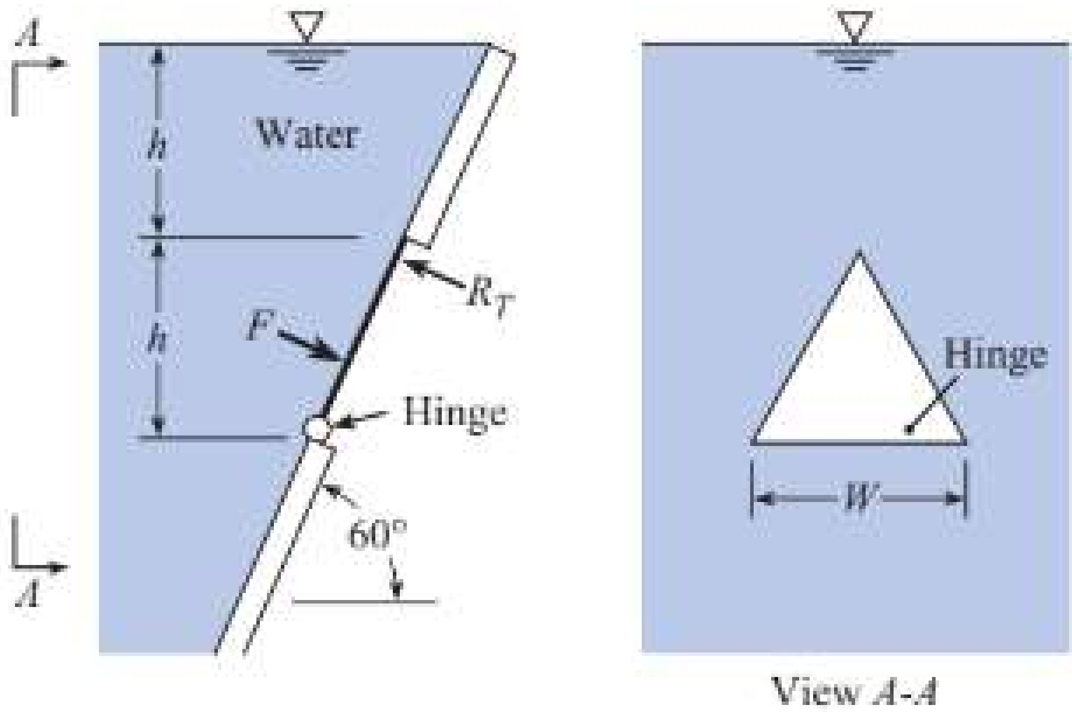
$$x_2 = h/2 - (y_{cp} - \bar{y}) = \frac{2.83\,\text{m}}{2} - (0.24\,\text{m}) = 1.2\,\text{m}$$

- Final calculation:

$$M_1 = Fx_2 = (55,525\,\text{N})\,(1.2\,\text{m}) = 66,630\,\text{N}$$

Since the torque due to weight exceeds the torque due to hydrostatic pressure, $\boxed{\text{the gate will fall}}$.

3.81 Determine the hydrostatic force F on the triangular gate, which is hinged at the bottom edge and held by the reaction R_T at the upper corner. Express F in terms of γ, h, and W. Also determine the ratio R_T/F. Neglect the weight of the gate.



PROBLEM 3.81

Find:

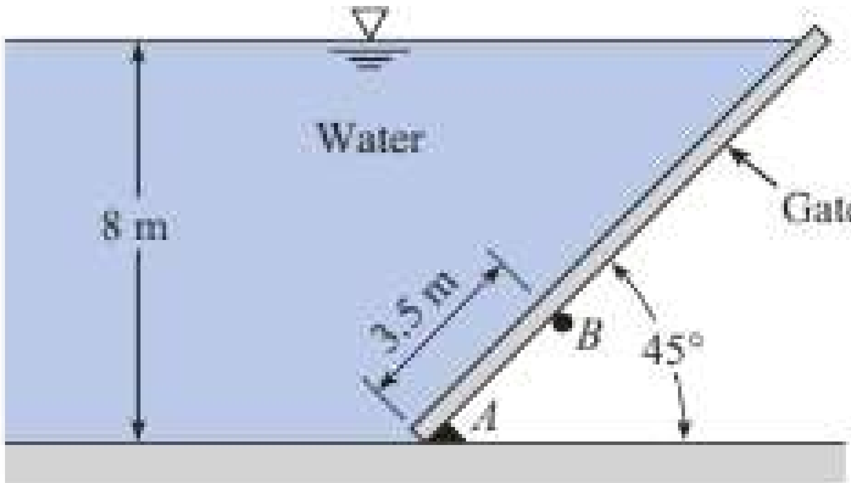
Hydrostatic force (F) on gate.

Ratio (R_T/F) of the reaction force to the hydrostatic force.

SOLUTION

$$
\begin{aligned}
F &= \bar{p}A \\
&= \left(h + \frac{2h}{3}\right)\gamma\left(\frac{Wh/\sin 60^\circ}{2}\right) \\
& \boxed{F = \frac{5\gamma W h^2}{3\sqrt{3}}} \\
y_{cp} - \bar{y} &= \frac{I}{\bar{y}A} = \frac{W(h/\sin 60^\circ)^3}{(36 \times (5h/(3\sin 60^\circ))} \times \frac{Wh}{2\sin 60^\circ} \\
&= \frac{h}{15\sqrt{3}} \\
\Sigma M &= 0 \\
R_T h/\sin 60^\circ &= F\left[\left(\frac{h}{3\sin 60^\circ}\right) - \left(\frac{h}{15\sqrt{3}}\right)\right] \\
& \boxed{\frac{R_T}{F} = \frac{3}{10}}
\end{aligned}
$$

3.83 The plane rectangular gate can pivot about the support at *B*. For the conditions given, is it stable or unstable? Neglect the weight of the gate. Justify your answer with calculations.



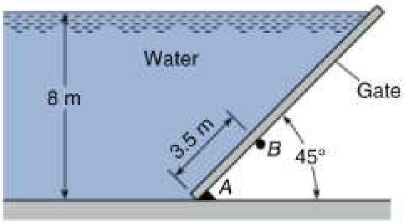
PROBLEM 3.83

3.81: PROBLEM DEFINITION

Situation:

A submerged gate is described in the problem statement.

$\theta = 45\,^{\circ}$.



Find:

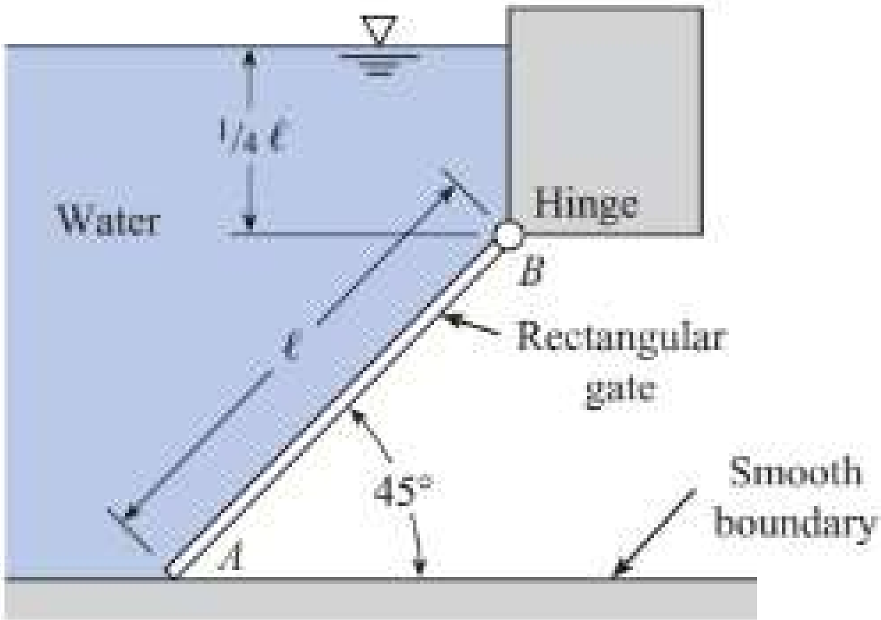
Is the gate stable or unstable.

SOLUTION

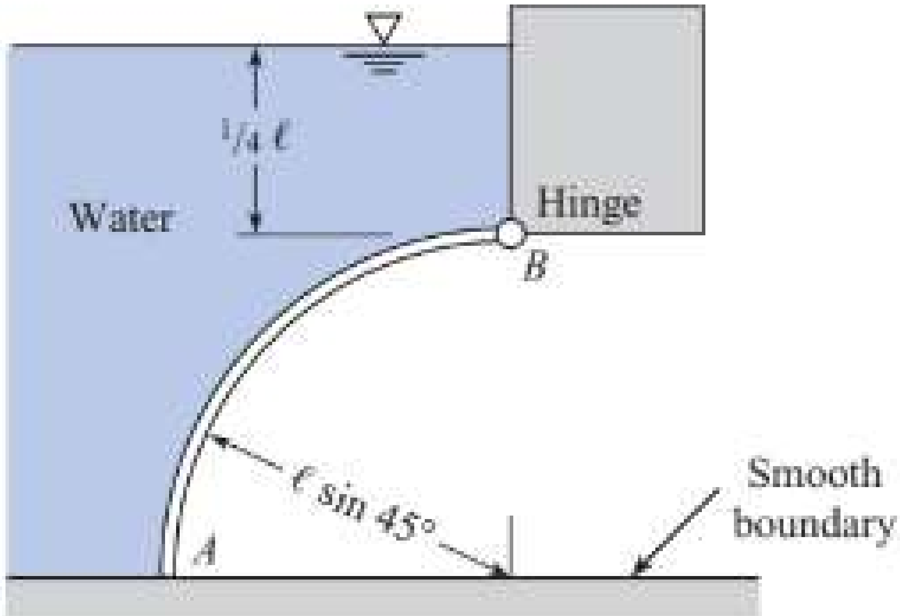
$$y_{cp} = \frac{2}{3} \times \frac{8}{\cos 45^{\circ}} = 7.54 \text{ m}$$

Point B is $(8/\cos 45^{\circ})$ m - 3.5 m = 7.81 m along the gate from the water surface; therefore, the gate is **unstable.**

3.86 For the plane rectangular gate ($\ell \times w$ in size), figure (*a*), what is the magnitude of the reaction at *A* in terms of γ_w and the dimensions ℓ and *w*? For the cylindrical gate, figure (*b*), will the magnitude of the reaction at *A* be greater than, less than, or the same as that for the plane gate? Neglect the weight of the gates.



(a) Plane gate



(b) Curved gate

PROBLEM 3.86

SOLUTION

a)

$$F_{Hydr} = \bar{p}A = (0.25\ell + 0.5\ell \times 0.707) \times \gamma W\ell = 0.6036\gamma W\ell^2$$

$$y_{cp} - \bar{y} = \frac{I}{\bar{y}A} = \frac{W\ell^3/12}{((0.25\ell/0.707) + 0.5\ell) \times W\ell}$$

$$y_{cp} - \bar{y} = 0.0976\ell$$

$$\sum M_{\text{hinge}} = 0$$

$$\text{Then } -0.70R_A\ell + (0.5\ell + 0.0976\ell) \times 0.6036\gamma W\ell^2 = 0$$

$$\boxed{R_A = 0.510\gamma W\ell^2}$$

b) The reaction here will be less because if one thinks of the applied hydrostatic force in terms of vertical and horizontal components, the horizontal component will be the same in both cases, but the vertical component will be less because there is less volume of liquid above the curved gate.

SOLUTION

Hydrostatic force

$$\begin{aligned} F_h &= \bar{p}A \\ &= \gamma z A \\ &= 9810\,\mathrm{N/\,m^3} \times 2\,\mathrm{m} \times \pi \times (0.2\,\mathrm{m})^2 \\ & \boxed{F_h = 2465\ \mathrm{N}} \end{aligned}$$

The vertical force is simply the buoyant force.

$$\begin{aligned} F_v &= \gamma V\!\!\!\!- \\ &= 9810\,\mathrm{N/\,m^3} \times \frac{4}{6} \times \pi \times (0.25\,\mathrm{m})^3 \\ & \boxed{F_v = 321\ \mathrm{N}} \end{aligned}$$

3.91 PLUS This dome (hemisphere) is located below the water surface as shown. Determine the magnitude and sign of the force components needed to hold the dome in place and the line of action of the horizontal component of force. Here $y_1 = 1$ m and $y_2 = 2$ m. Assume $T = 10°C$.

SOLUTION

1. Horizontal component of force.

$$\begin{aligned} F_H &= (1\,\mathrm{m} + 1\,\mathrm{m})9810\,\mathrm{N/\,m^3} \times \pi \times (1\,\mathrm{m})^2 \\ &= 61,640\ \mathrm{N}\ = 61.64\ \mathrm{kN} \end{aligned}$$

2. Center of pressure.

$$\begin{aligned} (y_{cp} - \bar{y}) &= \frac{I}{\bar{y}A} \\ &= \frac{\pi \times (1\,\mathrm{m})^4/4}{2\,\mathrm{m} \times \pi \times (1\,\mathrm{m})^2} \\ &= 0.125\,\mathrm{m} \end{aligned}$$

3. Vertical component of force

$$\begin{aligned} F_V &= \left(\frac{1}{2}\right)\left(\frac{4\pi \times (1\,\mathrm{m})^3}{3}\right) 9,810\,\mathrm{N/\,m^3} \\ &= 20,550\ \mathrm{N} \\ F_V &= 20.6\,\mathrm{kN} \end{aligned}$$

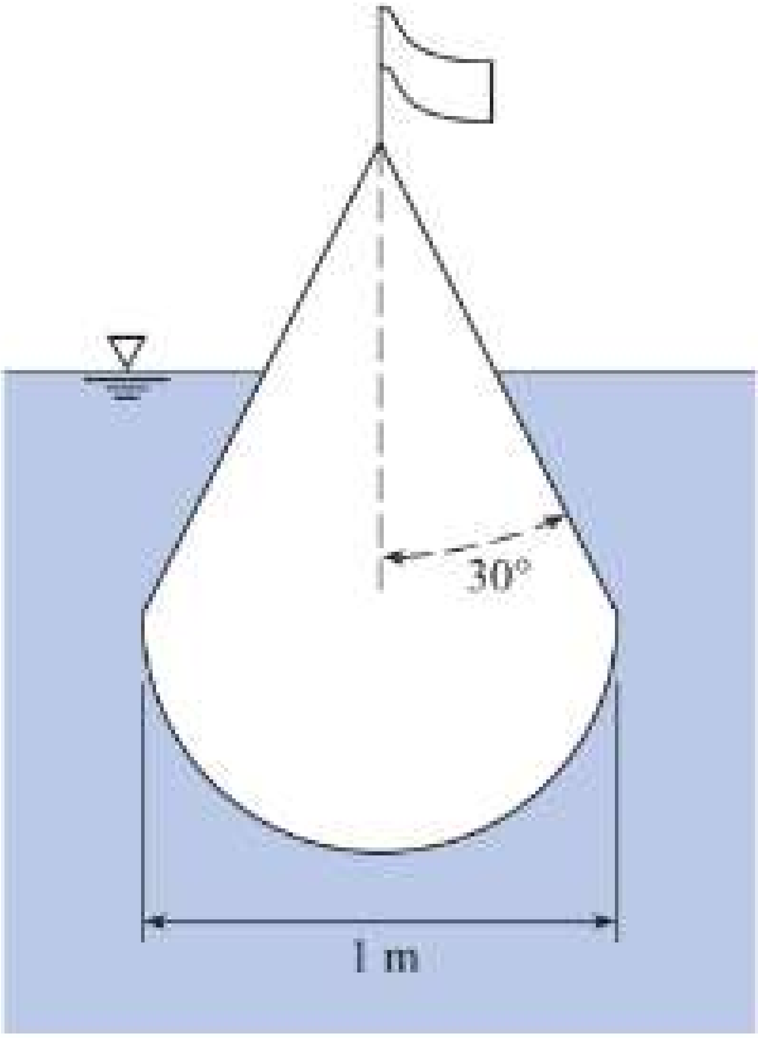
4. Answer

$$\boxed{F_{\text{horizontal}} = 61.64\,\text{kN (applied to the left to hold dome in place)}}$$

$$\boxed{\text{Line of action is } 0.125\,\text{m below a horizontal line passing through the dome center}}$$

$$\boxed{F_{\text{vertical}} = 20.6\,\text{kN (applied downward to hold dome in place)}}$$

3.99 A buoy is designed with a hemispherical bottom and conical top as shown in the figure. The diameter of the hemisphere is 1 m, and the half angle of the cone is 30°. The buoy has a mass of 460 kg. Find the location of the water line on the buoy floating in sea water ($\rho = 1010\ kg/m^3$).



PROBLEM 3.99

SOLUTION

The buoyant force is equal to the weight.

$$F_B = W$$

The weight of the buoy is $9.81 \times 460 = 4512$ N.
The volume of the hemisphere at the bottom of the buoy is

$$\forall = \frac{1}{2}\frac{\pi}{6}D^3 = \frac{\pi}{12}1^3 = \frac{\pi}{12}\text{ m}^3$$

The buoyant force due to the hemisphere is

$$F_B = \frac{\pi}{12}(9.81\,\text{m/s}^2)(1010\,\text{kg/m}^3) = 2594\text{ N}$$

Since this is less than the buoy weight, the water line must lie above the hemisphere. Let h is the distance from the top of the buoy. The volume of the cone which lies between the top of the hemisphere and the water line is

$$\begin{aligned} \forall &= \frac{\pi}{3}r_o^2 h_o - \frac{\pi}{3}r^2 h = \frac{\pi}{3}(0.5^2 \times 0.866 - h^3 \tan^2 30) \\ &= 0.2267 - 0.349h^3 \end{aligned}$$

The additional volume needed to support the weight is

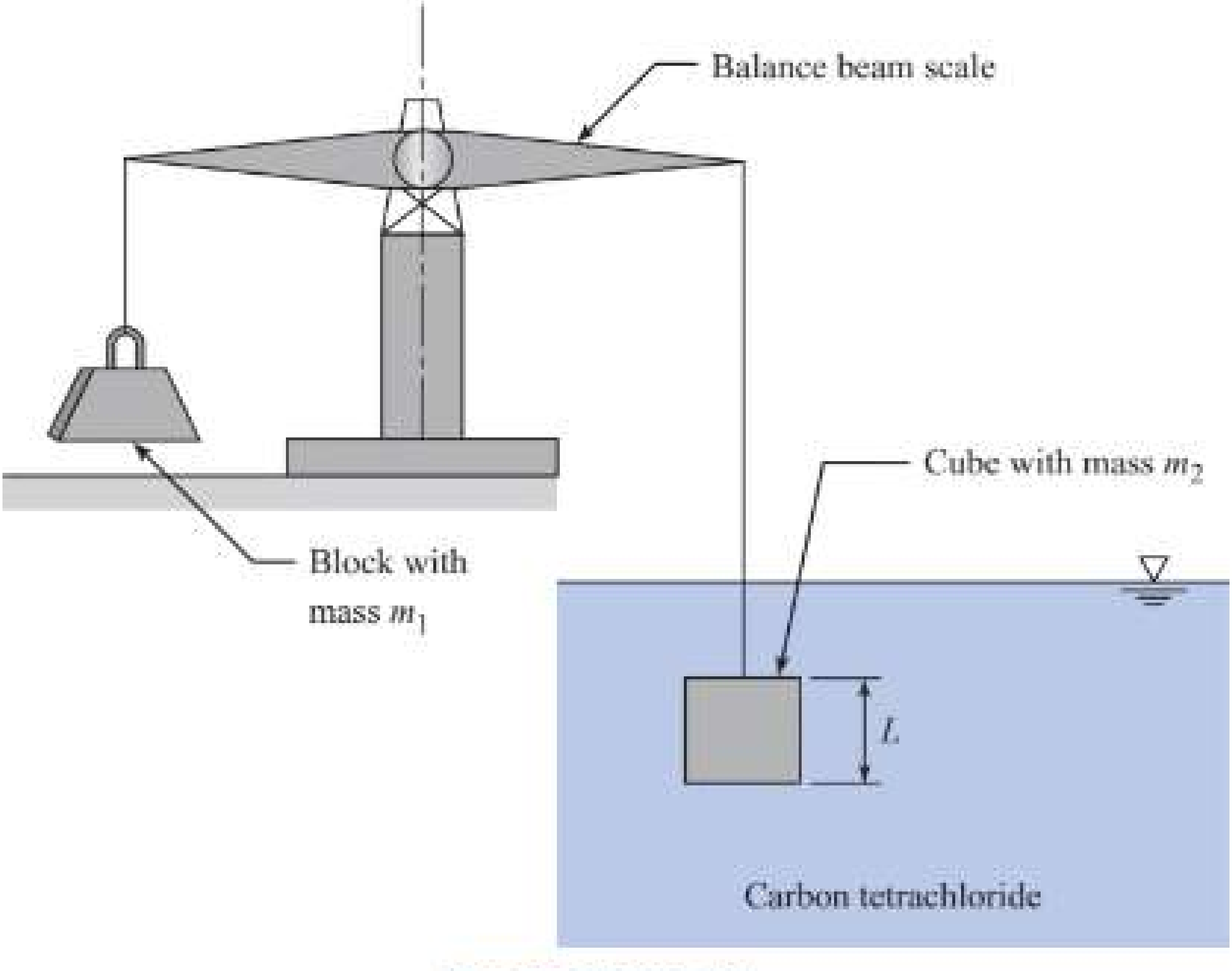
$$\not{V} = \frac{4512\,\mathrm{N} - 2594\,\mathrm{N}}{9.81\,\mathrm{m/\,s^2} \times 1010\,\mathrm{kg/\,m^3}} = 0.1936\ \mathrm{m}^3$$

Equating the two volumes and solving for h gives

$$h^3 = \frac{0.0331}{0.349} = 0.0948\ \mathrm{m}^3$$

$$\boxed{h = 0.456\ \mathrm{m}}$$

3.101 PLUS As shown, a cube ($L = 60$ mm) suspended in carbon tetracloride is exactly balanced by an object of mass $m_1 = 700$ g. Find the mass m_2 of the cube.



PROBLEM 3.101

SOLUTION

1. Force on balance arm:

$$\left\{\begin{array}{c}\text{Force on}\\\text{balance arm}\end{array}\right\}=\left\{\begin{array}{c}\text{Weight}\\\text{of block}\end{array}\right\}=mg=(0.7\,\text{kg})\left(9.81\,\text{m/s}^2\right)=6.867\,\text{N}$$

2. Equilibrium (applied to cube):

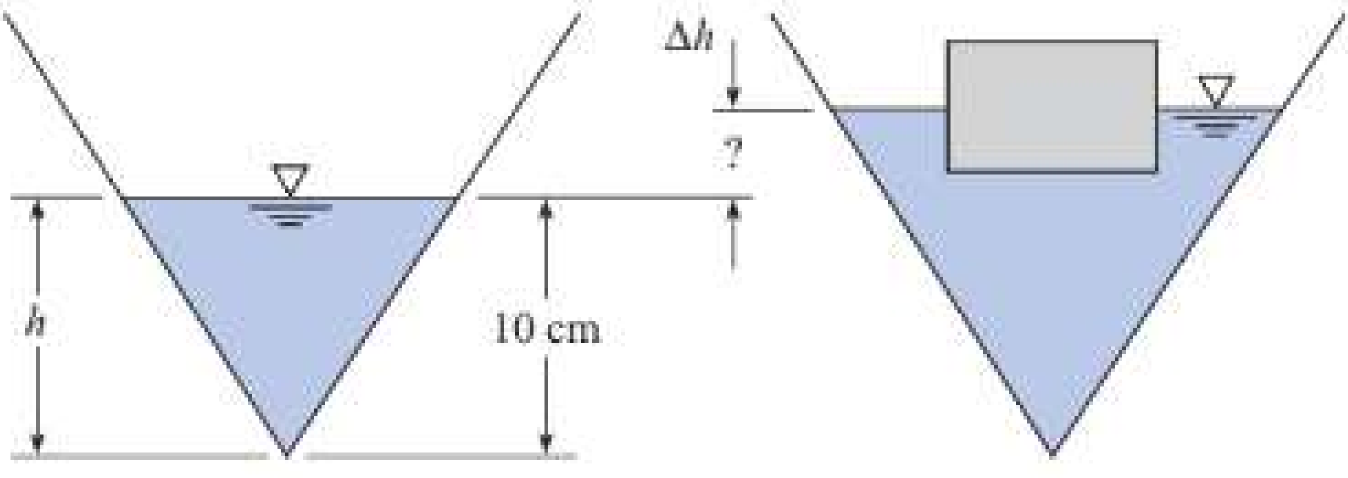
$$\begin{aligned}\left\{\begin{array}{c}\text{Force on}\\\text{balance arm}\end{array}\right\}+\left\{\begin{array}{c}\text{Buoyant Force}\\\text{on cube}\end{array}\right\} &= \{\text{Weight of cube}\}\\ F+\gamma\,(L_2)^3 &= m_2 g\end{aligned}$$

Solve for m_2 :

$$m_2=\frac{F+\gamma\,(L_2)^3}{g}=\frac{(6.867\,\text{N})+(15600\,\text{N/m}^3)\,(0.06\,\text{m})^3}{9.81\,\text{m/s}^2}$$

$$\boxed{m_2=1.04\,\text{kg}}$$

3.104 A 90° inverted cone contains water as shown. The volume of the water in the cone is given by $V = (\pi/3)h^3$. The original depth of the water is 10 cm. A block with a volume of 200 cm^3 and a specific gravity of 0.6 is floated in the water. What will be the change (in cm) in water surface height in the cone?



PROBLEM 3.104

Find:

Change of water level.

SOLUTION

1. Equilibrium (apply to block)

$$F_B = W_{\text{block}} = (\gamma_{\text{block}})\,(V\!\!\!/_{\text{block}})$$

2. Buoyancy equation

$$F_B = (\gamma_{\text{H}_2\text{O}})\,(V\!\!\!/_D) = (\gamma_{\text{block}})\,(V\!\!\!/_{\text{block}})$$

Thus

$$(V\!\!\!/_D) = \left(\frac{\gamma_{\text{block}}}{\gamma_{\text{H}_2\text{O}}}\right)(V\!\!\!/_{\text{block}}) = (0.6)\left(200\,\text{cm}^3\right) = 120\,\text{cm}^3$$

3. Volume considerations.

$$\left(\begin{array}{c}\text{final}\\ \text{volume}\end{array}\right) = \left(\begin{array}{c}\text{initial water}\\ \text{volume}\end{array}\right) + \left(\begin{array}{c}\text{displaced}\\ \text{volume}\end{array}\right)$$

Calculate initial water volume

$$V\!\!\!/ = \frac{\pi}{3}h^3 = \frac{\pi}{3}\,(10\,\text{cm})^3 = 1047\,\text{cm}^3$$

Calculate final volume

$$V\!\!\!/_{\text{final}} = \left(1047\,\text{cm}^3\right) + \left(120\,\text{cm}^3\right) = 1167\,\text{cm}^3$$

Calculate final volume

$$V\!\!\!/_{\text{final}} = \left(1047\,\text{cm}^3\right) + \left(120\,\text{cm}^3\right) = 1167\,\text{cm}^3$$

Increase in water level

$$V\!\!\!/_{\text{final}} = \frac{\pi}{3} h_{\text{f}}^3$$

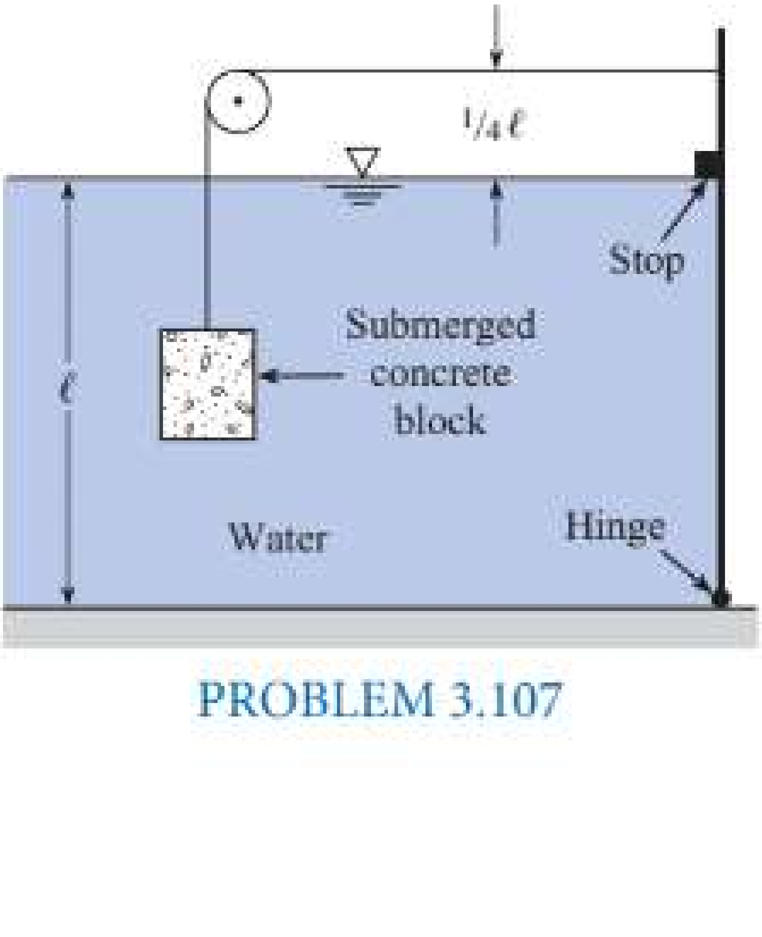
$$1167\,\text{cm}^3 = \frac{\pi}{3} h_{\text{f}}^3$$

$$h_{\text{f}} = 10.368\,\text{cm}$$

$$\Delta h = 10.368\,\text{cm} - 10\,\text{cm} = 0.368\,\text{cm}$$

$$\boxed{\Delta h = 0.37\,\text{cm}}$$

3.107 PLUS Determine the minimum volume of concrete ($\gamma = 23.6\ kN/m^3$) needed to keep the gate (1 m wide) in a closed position, with $\ell = 2$ m. Note the hinge at the bottom of the gate.



PROBLEM 3.107

SOLUTION

Hydrostatic force on gate and CP

$$F = \bar{p}A = 1\,\text{m} \times 9,810\,\text{N/m}^3 \times 2\,\text{m} \times 1\,\text{m} = 19,620\ \text{N}$$

$$y_{cp} - \bar{y} = \frac{I}{\bar{y}A} = \frac{1\,\text{m} \times (2\,\text{m})^3}{12 \times 1\,\text{m} \times 2\,\text{m} \times 1\,\text{m}} = 0.33\ \text{m}$$

Sum moments about the hinge to find the tension in the cable

$$T = 19,620 \times \frac{1 - 0.33}{2.5} = 5,258\ \text{N}$$

Equilibrium applied to concrete block

$$\begin{pmatrix} \text{Tension} \\ \text{in cable} \end{pmatrix} + \begin{pmatrix} \text{Buoyant} \\ \text{force} \end{pmatrix} = (\text{Weight})$$

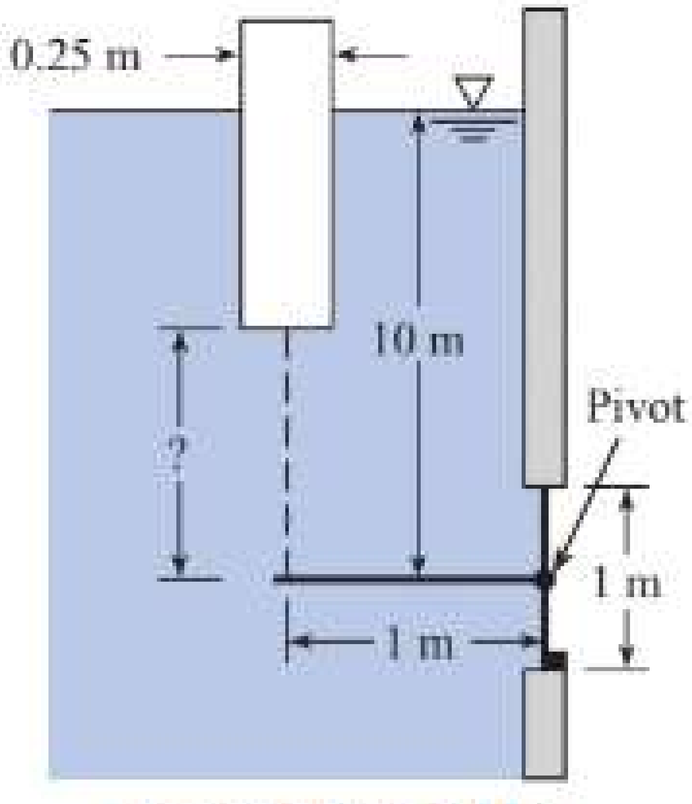
$$T + \forall \gamma_{\text{H}_2\text{O}} = \forall \gamma_c$$

Solve for volume of block

$$\forall = \frac{T}{\gamma_c - \gamma_{\text{H}_2\text{O}}} = \frac{5258\,\text{N}}{23,600\,\text{N/m}^3 - 9,810\,\text{N/m}^3}$$

$$\boxed{\forall = 0.381\ \text{m}^3}$$

3.110 A gate with a circular cross section is held closed by a lever 1 m long attached to a buoyant cylinder. The cylinder is 25 cm in diameter and weighs 200 N. The gate is attached to a horizontal shaft so it can pivot about its center. The liquid is water. The chain and lever attached to the gate have negligible weight. Find the length of the chain such that the gate is just on the verge of opening when the water depth above the gate hinge is 10 m.



PROBLEM 3.110

SOLUTION

Hydrostatic force

$$
\begin{aligned}
F_H &= \bar{p}A = 10\,\text{m} \times 9,810\,\text{N}/\,\text{m}^3 \times \frac{\pi D^2}{4} \\
&= 98,100\,\text{N}/\,\text{m}^2 \times \pi\frac{\pi}{4}\,(1\,\text{m})^2 \\
&= 77,048 \text{ N} \\
y_{cp} - \bar{y} &= \frac{I}{\bar{y}A} \\
&= \frac{\pi r^4/4}{10\,\text{m} \times \pi D^2/4} \\
y_{cp} - \bar{y} &= \frac{r^2}{40} = 0.00625 \text{ m}
\end{aligned}
$$

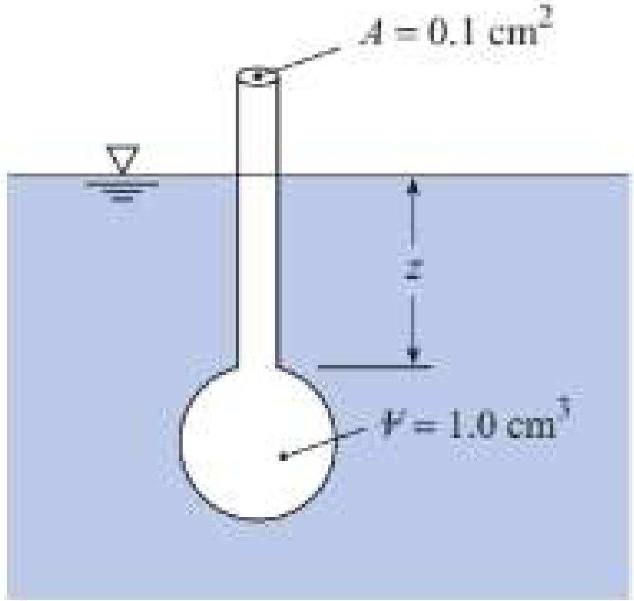
$$\begin{aligned}\sum M_{\text{Hinge}} &= 0 \\ F_H \times (0.00625 \text{ m}) - 1\,\text{m} \times F &= 0\end{aligned}$$

$$\begin{aligned}\text{But } F &= F_{\text{buoy}} - W \\ &= A(10 \text{ m} - \ell)\gamma_{\text{H}_2\text{O}} - 200 \\ &= \frac{\pi}{4}(0.25\,\text{m})^2(10-\ell)(9,810\,\text{N/ m}^3) - 200\,\text{N} \\ &= 4815.5 \text{ N} - 481.5\ell \text{ N} - 200 \text{ N} \\ &= (4615.5 - 481.5\ell) \text{ N} \\ \text{where } \ell &= \text{length of chain}\end{aligned}$$

$$\begin{aligned}77,048\,\text{N} \times 0.00625\,\text{m} - 1\,\text{m} \times (4615.5 - 481.5\ell)\,\text{N} &= 0 \\ (481.55 - 4615.5 + 481.5\ell)\text{ N m} &= 0\end{aligned}$$

$$\boxed{\ell = 8.59 \text{ m}}$$

3.114 PLUS The hydrometer shown sinks 5.3 cm ($z = 5.3$ cm) in water (15°C). The bulb displaces 1.0 cm^3, and the stem area is 0.1 cm^2. Find the weight of the hydrometer.



PROBLEMS 3.113, 3.114

SOLUTION

1. Equilibrium

$$\begin{aligned} F_{\text{buoy.}} &= W \\ \forall_D \gamma_{\text{oil}} &= W \end{aligned}$$

2. Calculations

$$\begin{aligned} (1\ \text{cm}^3 + (6.0\ \text{cm})(0.1\ \text{cm}^2))(0.01^3)\ \text{m}^3/\text{cm}^3 \gamma_{\text{oil}} &= 0.015\ \text{N} \\ (1 + 0.6) \times 10^{-6}\ \text{m}^3 \gamma_{\text{oil}} &= 0.015\ \text{N} \\ \gamma_{\text{oil}} &= 9375\ \text{N/m}^3 \end{aligned}$$

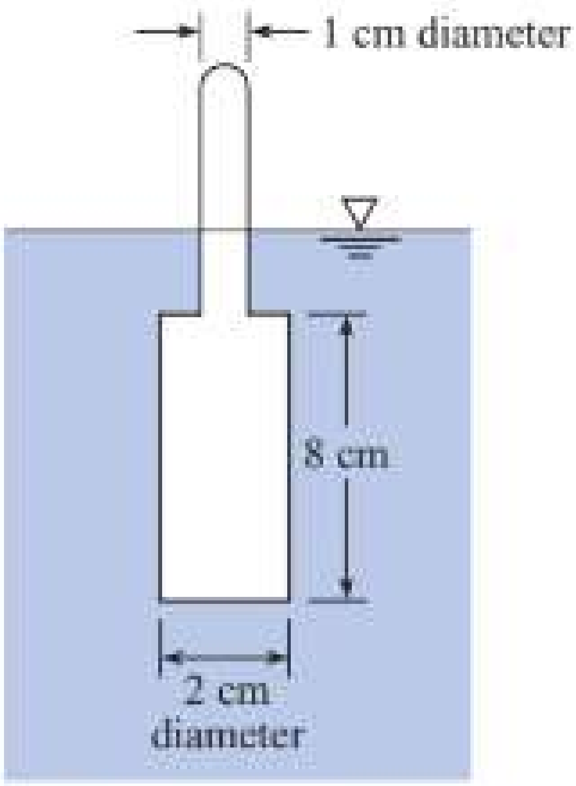
3. Definition of S

$$\begin{aligned} S &= \frac{\gamma_{\text{oil}}}{\gamma_{\text{H}_2\text{O}}} \\ &= \frac{9375\,\text{N/m}^3}{9810\,\text{N/m}^3} \end{aligned}$$

$$\boxed{S = 0.956}$$

3.116 PLUS A hydrometer with the configuration shown has a bulb diameter of 2 cm, a bulb length of 8 cm, a stem diameter of 1 cm, a length of 8 cm, and a mass of 40 g. What is the range of specific gravities that can be measured with this hydrometer?

(*Hint:* Liquid levels range between bottom and top of stem.)



PROBLEM 3.116

SOLUTION

When only the bulb is submerged

$$\begin{aligned} F_B &= W \\ \forall_D \gamma_{\mathrm{H_2O}} &= W \\ \frac{\pi}{4}\left[(0.02\,\mathrm{m})^2 \times 0.08\,\mathrm{m}\right] \times 9810\,\mathrm{N/\,m^3} \times S &= 0.040\,\mathrm{kg} \times 9.81\,\mathrm{m/\,s^2} \end{aligned}$$

$$S = 1.59$$

When the full stem is submerged

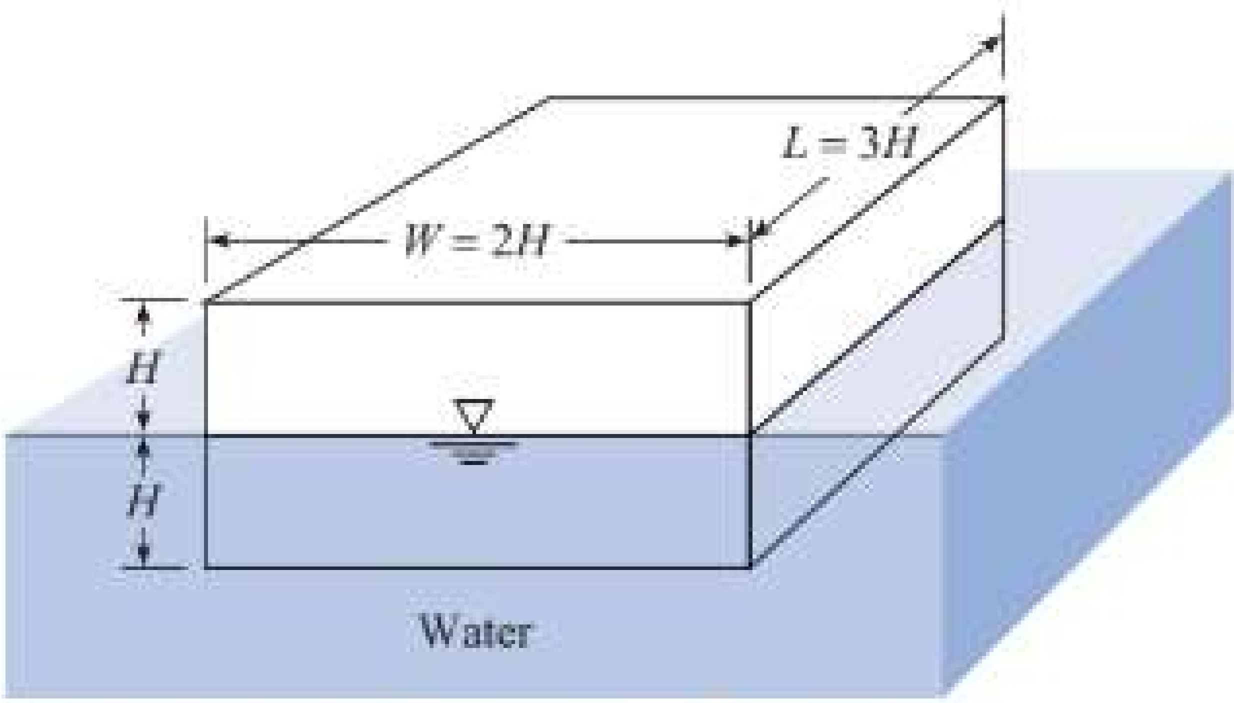
$$\frac{\pi}{4}\left[(0.02\,\mathrm{m})^2 \times (0.08\,\mathrm{m}) + (0.01\,\mathrm{m})^2 \times (0.08\,\mathrm{m})\right] 9,810\,\mathrm{N/\,m^3} \times S = 0.040\,\mathrm{kg} \times 9.81\,\mathrm{m/\,s^2}$$

$$S = 1.27$$

Thus, the range is

$$\boxed{1.27 \leq S \leq 1.59}$$

3.121 Is the block in this figure stable floating in the position shown? Show your calculations.



PROBLEM 3.121

SOLUTION

Analyze longitudinal axis

$$
\begin{aligned}
\text{GM} &= \frac{I_{00}}{V} - \text{CG} \\
&= \frac{3H(2H)^3}{12 \times H \times 2H \times 3H} - \frac{H}{2} \\
&= -\frac{H}{6}
\end{aligned}
$$

Not stable about longitudinal axis.

Analyze transverse axis.

$$
\begin{aligned}
\text{GM} &= \frac{2H \times (3H)^3}{12 \times H \times 2H \times 3H} - \frac{3H}{4} \\
&= 0
\end{aligned}
$$

Neutrally stable about transverse axis.

Not stable