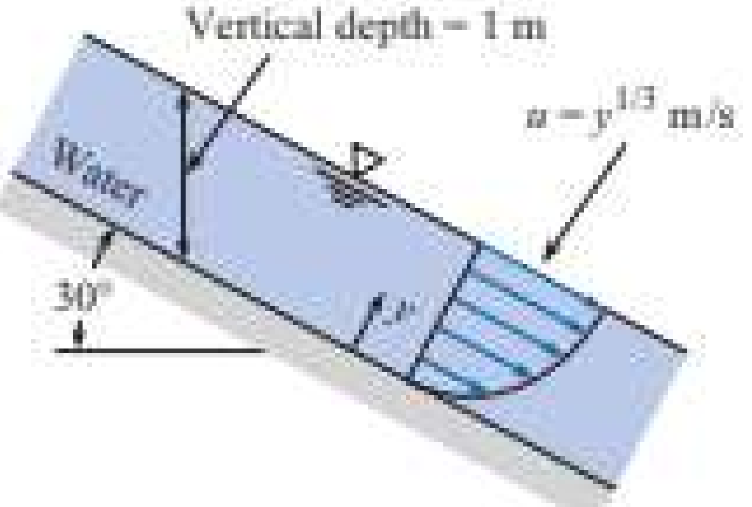
Mech Family

كن أنت التغيير

اللجنة الأكاديمية في قسم الهندسة الميكانيكية

5.20 PLUS The rectangular channel shown is 1.2 m wide. What is the discharge in the channel?



PROBLEM 5.20

5.20: PROBLEM DEFINITION

Situation:

A rectangular channel has a 30^o incline.
$u = y^{1/3}\ \text{m/s}$.
Depth $= y = 1\,\text{m}$.
Width $= x = 1.2\,\text{m}$.
$d = 1\,\text{m} \times \cos(30^o) = 0.866\ \text{m}$

Find:

Discharge (m^3/s).

PLAN

Apply the integral form of the flow rate equation becuse velocity is not constant over the area.

SOLUTION

Flow rate equation

$$
\begin{aligned}
Q &= \int_0^{0.866} y^{1/3}(1.2\ \text{dy}) \\
&= 1.2\int_0^{0.866} y^{1/3}\text{dy} \\
&= \left(\frac{1.2}{4/3}\right) y^{4/3}|_0^{0.866\ \text{m}}
\end{aligned}
$$

$$\boxed{Q = 0.743\ \text{m}^3/\text{s}}$$

5.41 PLUS Gas flows into and out of the chamber as shown. For the conditions shown, which of the following statement(s) are true of the application of the control volume equation to the continuity principle?

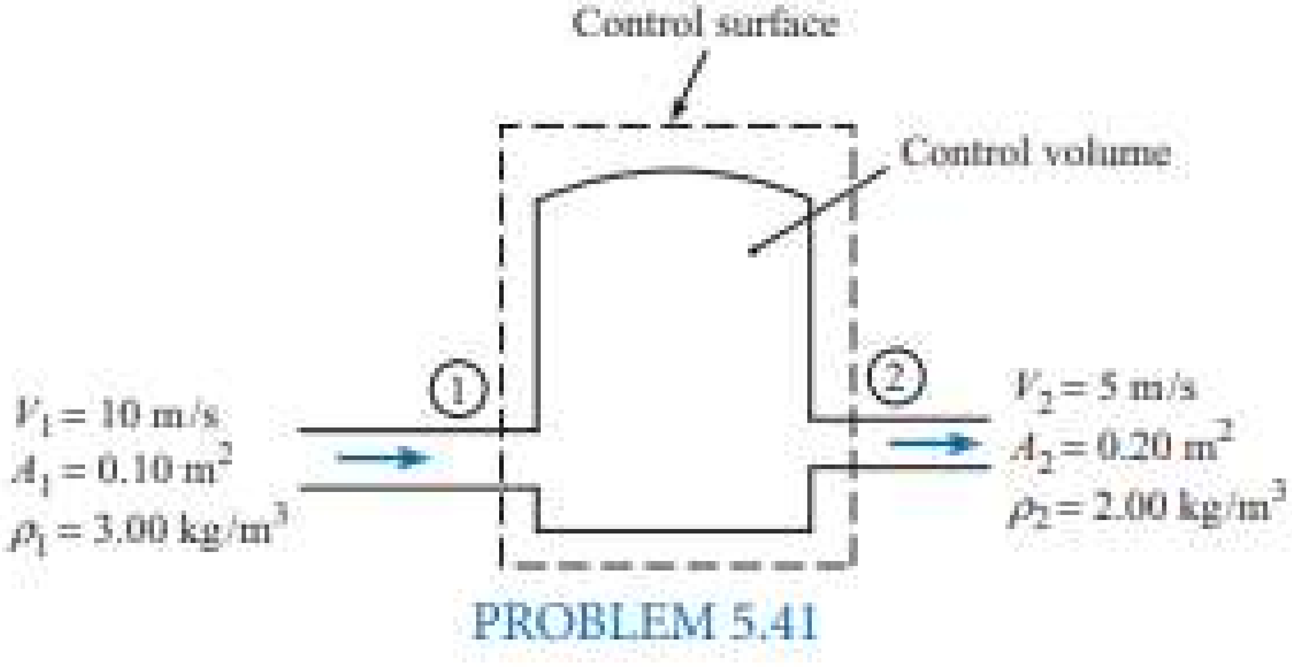
a. $B_{sys} = 0$

b. $dB_{sys}/dt = 0$

c. $\sum_{cs} b\rho \mathbf{V} \cdot \mathbf{A} = 0$

d. $\frac{d}{dt}\int_{cv} \rho \, d\mathcal{V} = 0$

e. $b = 0$



PROBLEM 5.41

5.41: PROBLEM DEFINITION

Situation:

Mass is flowing into and out of a tank.
$V_1 = 10\,\text{m/s}$, $A_1 = 0.10\,\text{m}^2$, $\rho_1 = 3.00\,\text{kg/m}^3$.
$V_2 = 5\,\text{m/s}$, $A_2 = 0.20\,\text{m}^2$, $\rho_2 = 2.00\,\text{kg/m}^3$.

Find:

Select the statement(s) that are true.

SOLUTION

Mass flow out

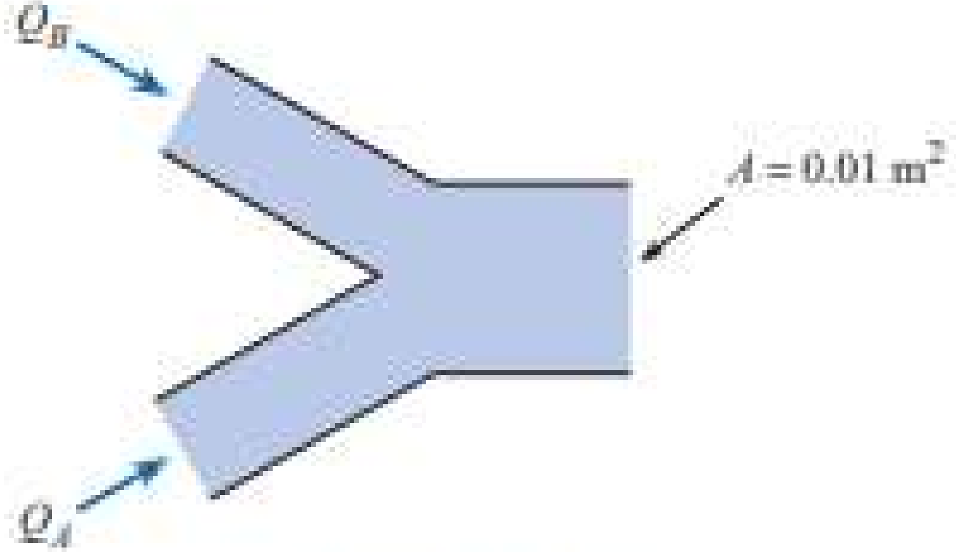
$$\begin{aligned}\dot{m}_o &= (\rho A V)_2 \\ &= \left(2\,\text{kg/m}^3\right)(0.2\,\text{m})(5\,\text{m/s}) \\ &= 2\ \text{kg/s}\end{aligned}$$

Mass flow in

$$\begin{aligned}\dot{m}_i &= (\rho A V)_1 \\ &= \left(3\,\text{kg/m}^3\right)(0.1\,\text{m})(10\,\text{m/s}) \\ &= 3\ \text{kg/s}\end{aligned}$$

Since the mass flow in is not equal to the mass flow out, the flow is unsteady. **Only selection (b) is valid**.

5.52 PLUS Two streams discharge into a pipe as shown. The flows are incompressible. The volume flow rate of stream A into the pipe is given by $Q_A = 0.04t$ m^3/s and that of stream B by $Q_B = 0.006\,t^2$ m^3/s, where t is in seconds. The exit area of the pipe is 0.01 m^2. Find the velocity and acceleration of the flow at the exit at $t = 1$ s.



PROBLEM 5.52

5.52: PROBLEM DEFINITION

Situation:

Pipe flows A and B merge into a single pipe.
$Q_A = 0.04t\ \text{m}^3/\text{s}$, $Q_B = 0.006t^2\ \text{m}^3/\text{s}$.
$A_{\text{exit}} = 0.01\ \text{m}^2$, $t = 1\,\text{s}$.

Find:

Velocity at the exit, V_{exit}.
Acceleration at the exit, a_{exit}.

Assumptions:

Incompressible flow.

PLAN

Apply the continuity equation.

SOLUTION

Since the flow is incompressible, the unsteady term is zero. Continuity equation

$$\begin{aligned} Q_{\text{exit}} &= Q_A + Q_B \\ V_{\text{exit}} &= \left(\frac{1}{A_{\text{exit}}}\right)(Q_A + Q_B) \\ &= \left(\frac{1}{0.01\,\text{m}^2}\right)(.04t\ \text{m}^3/\text{s}\ + 0.006t^2\ \text{m}^3/\text{s}) \\ &= 4t\ \text{m/s}\ + 0.6t^2\ \text{m/s} \end{aligned}$$

Then at $t = 1$ sec,

$$\boxed{V_{\text{exit}} = 4.6\ \text{m/s}}$$

The acceleration along a pathline at the $(s \to x)$ exit is

$$a_{\text{exit}} = \frac{\partial V}{\partial t} + V\frac{\partial V}{\partial x}$$

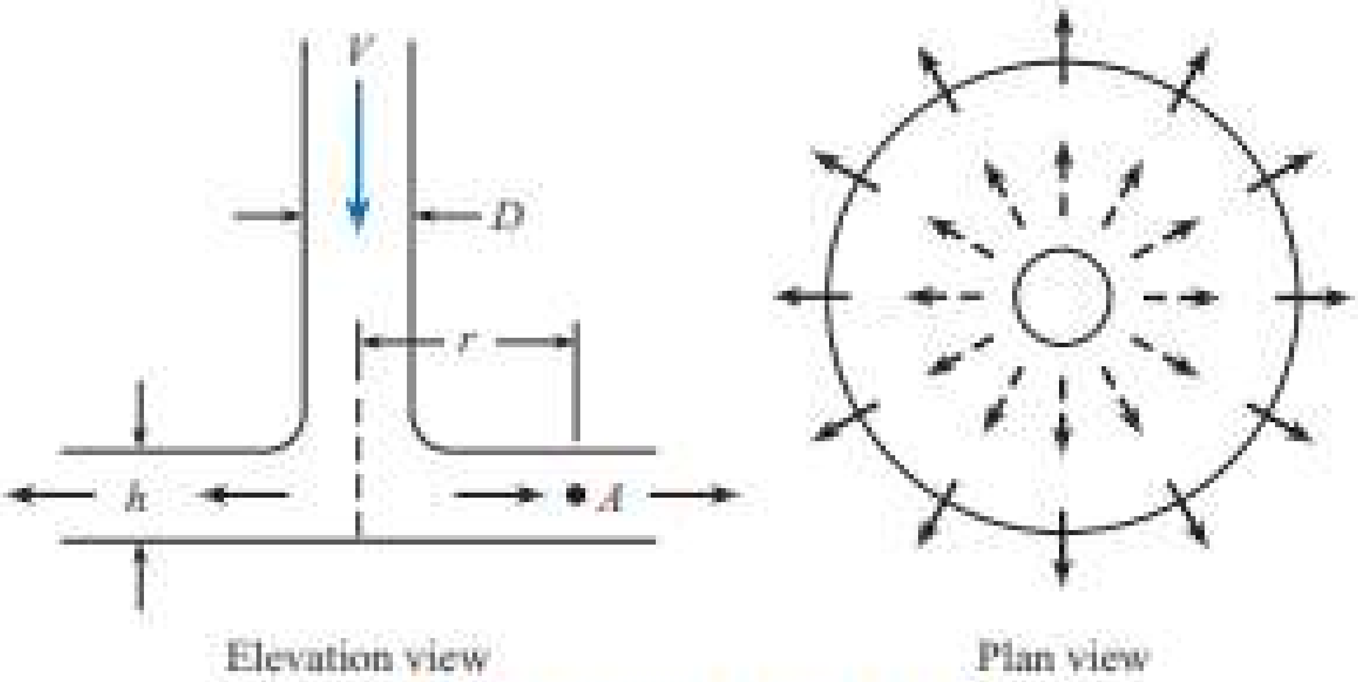
Since V varies with time, but not with position, there is no convective acceleration so

$$a_{\text{exit}} = \frac{\partial V}{\partial t} = 4 + 1.2t\ \text{m/s}^2$$

Then at $t = 1$ sec

$$\boxed{a_{\text{exit}} = 5.2\ \text{m/s}^2}$$

5.53 Air discharges downward in the pipe and then outward between the parallel disks. Assuming negligible density change in the air, derive a formula for the acceleration of air at point *A*, which is a distance *r* from the center of the disks. Express the acceleration in terms of the constant air discharge *Q*, the radial distance *r*, and the disk spacing *h*. If $D = 10$ cm, $h = 0.6$ cm, and $Q = 0.380\ \mathrm{m^3/s}$, what are the velocity in the pipe and the acceleration at point *A* where $r = 20$ cm?



PROBLEMS 5.53, 5.54

Situation:

Air flow downward through a pipe and then outward between two parallel disks.
$Q = 0.380\,\text{m}^3/\text{s}$, $r = 20\,\text{cm}$
$D = 0.1\,\text{m}$, $h = 0.6\,\text{cm}$.

Find:

(a) Expression for acceleration at point A.
(b) Value of acceleration at point A.
(c) Velocity in the pipe.

PLAN

Apply the flow rate equation.

SOLUTION

---

a)
Flow rate equation

$$V_r = \frac{Q}{A} = \frac{Q}{2\pi r h}$$

Evaluate convective acceleration along a radial pathline ($s \to r$)

$$\begin{aligned} a_c &= \frac{V_r \partial V_r}{\partial r} \\ &= \left(\frac{Q}{2\pi r h}\right)(-1)\left(\frac{Q}{2\pi r^2 h}\right) \end{aligned}$$

$$\boxed{a_c = \frac{-Q^2}{r(2\pi r h)^2}}$$

---

b)

$$\begin{aligned} V_{\text{pipe}} &= \frac{Q}{A_{\text{pipe}}} \\ &= \frac{(0.380\,\text{m}^3/\,\text{s})}{\frac{\pi}{4}\,(0.1\,\text{m})^2} \end{aligned}$$

$$\boxed{V_{\text{pipe}} = 48.4\ \text{m/s}}$$

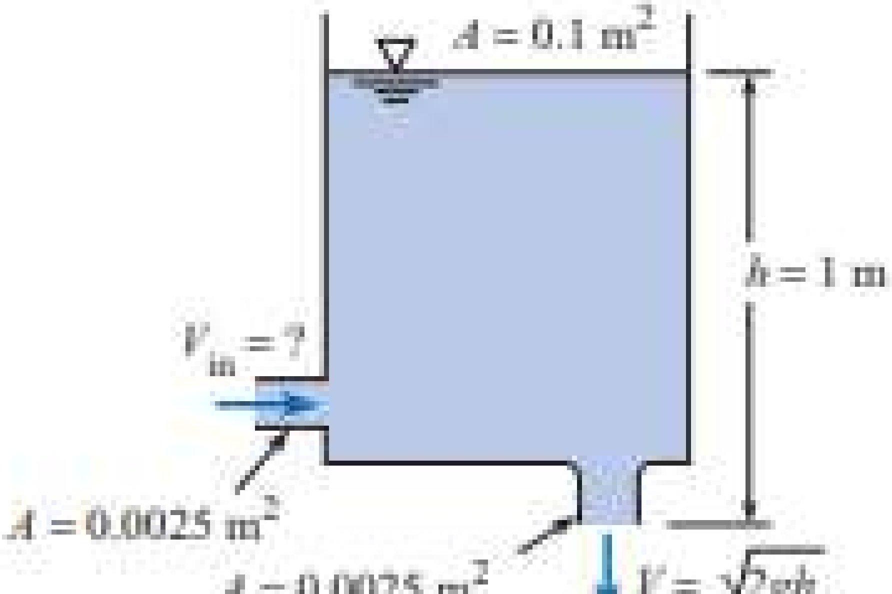
---

c)

$$a_c = -\frac{(0.38\,\text{m}^3/\,\text{s})^2}{(0.2\,\text{m})(2\pi\,(0.2\,\text{m})\,(0.006\,\text{m}))^2}$$

$$\boxed{a_c = -12,700\ \text{m/s}^2}$$

5.55 GO A tank has a hole in the bottom with a cross-sectional area of 0.0025 m^2 and an inlet line on the side with a cross-sectional area of 0.0025 m^2, as shown. The cross-sectional area of the tank is 0.1 m^2. The velocity of the liquid flowing out the bottom hole is $V = \sqrt{2gh}$, where h is the height of the water surface in the tank above the outlet. At a certain time the surface level in the tank is 1 m and rising at the rate of 0.1 cm/s. The liquid is incompressible. Find the velocity of the liquid through the inlet.



PROBLEM 5.55

5.55: PROBLEM DEFINITION

Situation:

Water flows into a tank through a pipe on the side and then out a pipe on the bottom of the tank.

$A_{out} = A_{in} = 0.0025\,\text{m}^2$, $A_{\text{tank}} = 0.1\,\text{m}^2$,

At $h = 1\,\text{m}$, $dh/dt = 0.1\,\text{m}$, $V = \sqrt{2gh}$.

Find:

Velocity in the inlet: V_{in}.

Assumptions:

Incompressible flow.

PLAN

Apply the continuity equation. Let the control surface surround the liquid in the tank and let it follow the liquid surface at the top.

SOLUTION

Continuity equation

$$\dot{m}_o - \dot{m}_i = -\frac{d}{dt}\int_{\text{cv}} \rho d\forall$$

$$-\rho V_{\text{in}} A_{\text{in}} + \rho V_{\text{out}} A_{\text{out}} = -\frac{d}{dt}(\rho A_{\text{tank}} h)$$

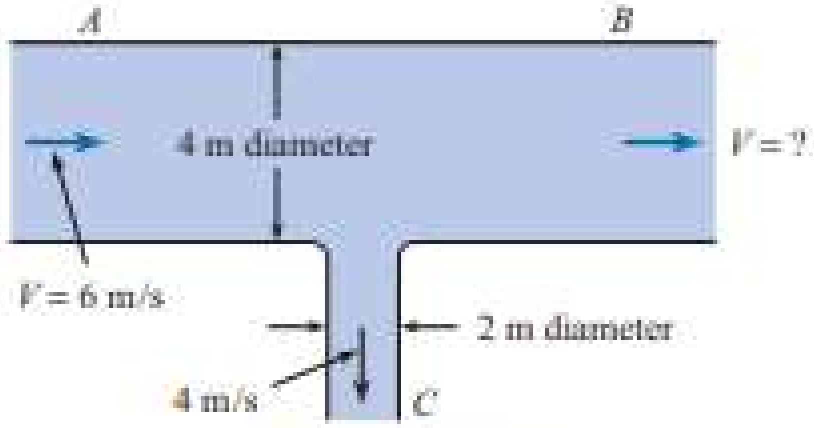
$$-V_{\text{in}} A_{\text{in}} + V_{\text{out}} A_{\text{out}} = -A_{\text{tank}}\left(\frac{dh}{dt}\right)$$

$$-V_{\text{in}}(.0025) + \sqrt{2\text{g}(1)}(.0025) = -0.1(0.1) \times 10^{-2}$$

$$V_{\text{in}} = \frac{\sqrt{19.62}(.0025) + 10^{-4}}{0.0025}$$

$$\boxed{V_{\text{in}} = 4.47\ \text{m/s}}$$

5.64 What is the velocity of the flow of water in leg *B* of the tee shown in the figure?



PROBLEM 5.64

5.64: PROBLEM DEFINITION

Situation:

Water flows through a tee.

$D_A = D_B = 4\,\text{m}$, $D_C = 2\,\text{m}$.

$V_A = 6\,\text{m/s}$, $V_C = 4\,\text{m/s}$.

Find:

Mean velocity in outlet B.

PLAN

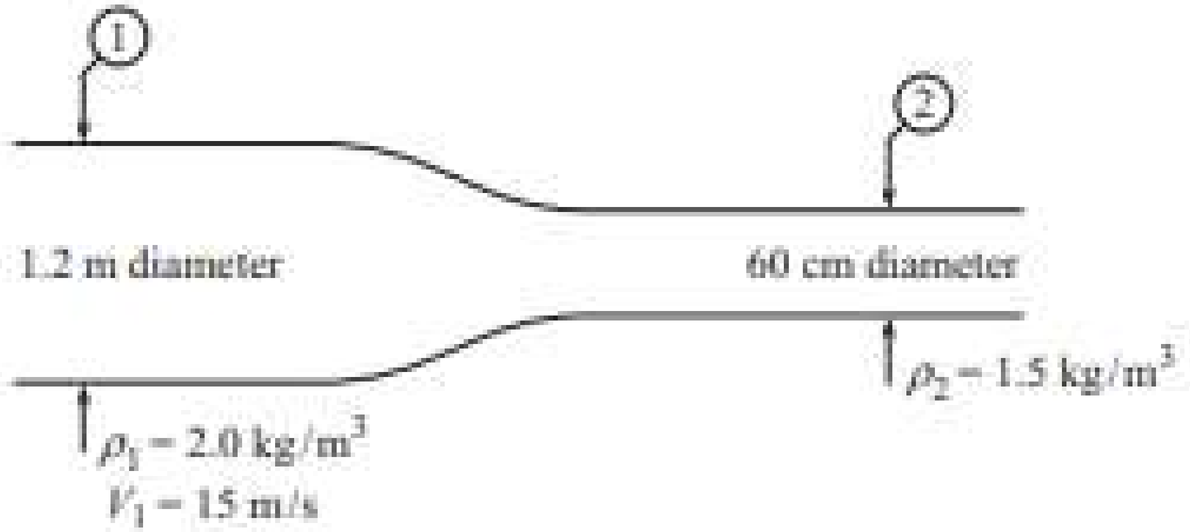
Apply the continuity equation.

SOLUTION

Continuity equation

$$\begin{aligned} V_B &= \frac{V_A A_A - V_c A_c}{A_B} \\ &= \frac{[(6\,\text{m/s})(\pi/4)(4\,\text{m})^2 - (4\,\text{m/s})(\pi/4)(2\,\text{m})^2)]}{(\pi/4\,(4\,\text{m})^2)} \\ &\boxed{V_B = 5.00\ \text{m/s}} \end{aligned}$$

5.65 PLUS For a steady flow of gas in the conduit shown, what is the mean velocity at section 2?



PROBLEM 5.65

5.65: PROBLEM DEFINITION

Situation:

Gas flows in a round conduit that tapers to a smaller diameter.
$D_1 = 1.2\,\text{m}$, $D_2 = 0.6\,\text{m}$, $V_1 = 15\,\text{m/s}$.

Find:

Mean velocity at section 2.

Properties:

$\rho_1 = 2.0\,\text{kg/m}^3$, $\rho_2 = 1.5\,\text{kg/m}^3$.

PLAN

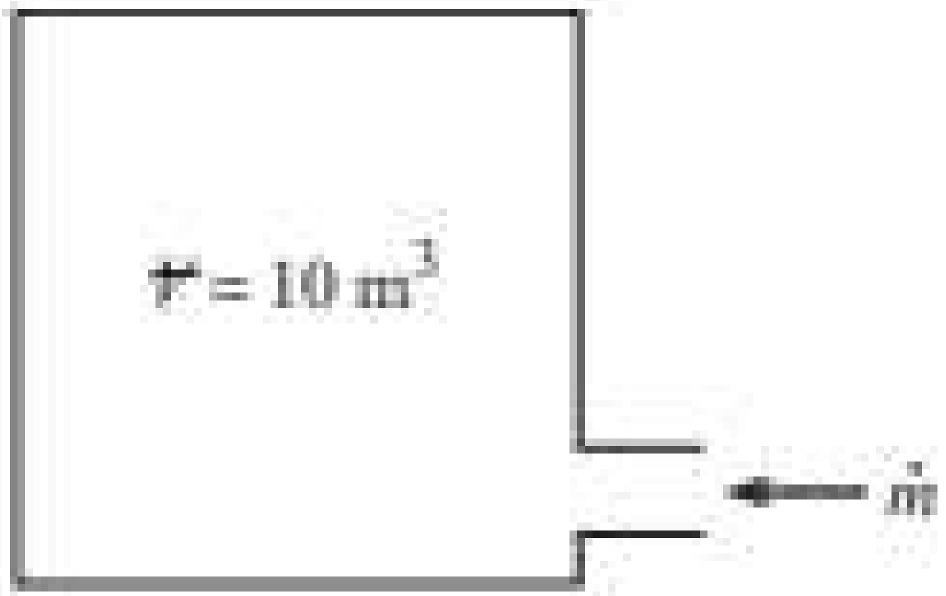
Apply the continuity equation.

SOLUTION

Continuity equation

$$\begin{aligned} V_2 &= \frac{\rho_1 A_1 V_1}{\rho_2 A_2} \\ &= \frac{\rho_1 D_1^2 V_1}{\rho_2 D_2^2} \\ &= \frac{(2.0\,\text{kg/m}^3)(1.2\,\text{m})^2(15\,\text{m/s})}{(1.5\,\text{kg/m}^3)(0.6\,\text{m})^2} \\ & \boxed{V_2 = 80.0\ \text{m/s}} \end{aligned}$$

5.77 PLUS A compressor supplies gas to a 10 m^3 tank. The inlet mass flow rate is given by $\dot{m} = 0.5\,\rho_0/\rho$ (kg/s), where ρ is the density in the tank and ρ_0 is the initial density. Find the time it would take to increase the density in the tank by a factor of 2 if the initial density is 2 kg/m^3. Assume the density is uniform throughout the tank.



PROBLEM 5.77

5.77: PROBLEM DEFINITION

Situation:

A tank is filled with air from a compressor.

$V = 10\,\text{m}^3$, $\dot{m} = 0.5\frac{\rho_0}{\rho}\,\text{kg/s}$.

Find:

Time to increase the density of the air in the tank by a factor of 2.

Properties:

$\rho_0 = 2\,\text{kg/m}^3$

PLAN

Apply the continuity equation.

SOLUTION

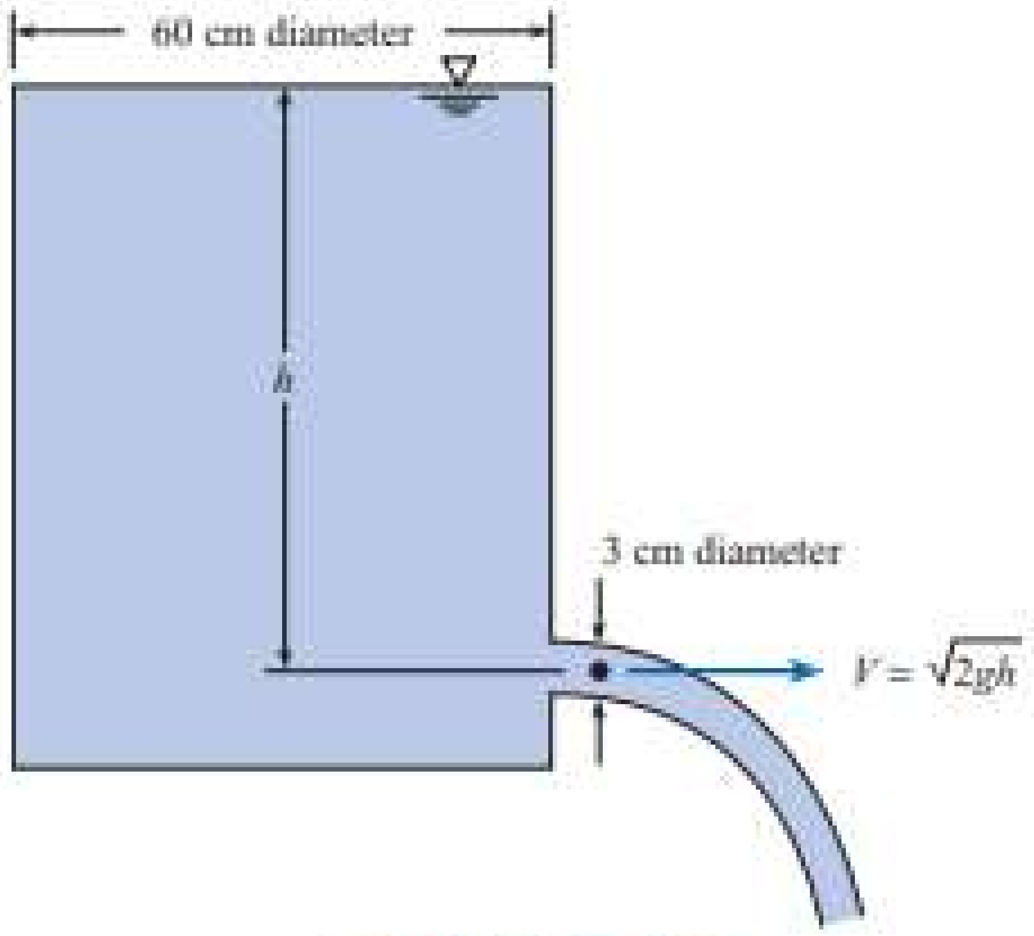
Continuity equation

$$\begin{aligned}
\dot{m}_o - \dot{m}_i &= -\frac{d}{dt}\int_{CV} \rho dV \\
-\frac{d}{dt}(\rho V) &= -\dot{m}_i \\
V\left(\frac{d\rho}{dt}\right) &= 0.5\rho_0/\rho
\end{aligned}$$

Separating variables and integrating

$$\begin{aligned}
\rho d\rho &= \frac{0.5\rho_0 dt}{V} \\
\frac{\rho^2}{2}\Big|_0^f &= \frac{0.5\rho_0 dt}{V} \\
\frac{\rho_f^2 - \rho_0^2}{2} &= \frac{0.5\rho_0 \Delta t}{V} \\
\Delta t &= V\rho_0\left[\left(\frac{\rho_f^2}{\rho_0^2}\right) - 1\right] \\
&= \left(10\,\text{m}^3\right)\left(2\,\text{kg/m}^3\right)\left((2)^2 - 1\right) \\
&\boxed{\Delta t = 60\text{s}}
\end{aligned}$$

5.80 How long will it take the water surface in the tank shown to drop from $h = 3$ m to $h = 50$ cm?



PROBLEM 5.80

5.80: PROBLEM DEFINITION

Situation:

A tank is draining through an orifice.

$h_1 = 3\,\mathrm{m}$, $h = 0.5\,\mathrm{m}$.

$D_T = 0.6\,\mathrm{m}$, $D_2 = 3\,\mathrm{cm}$

Find:

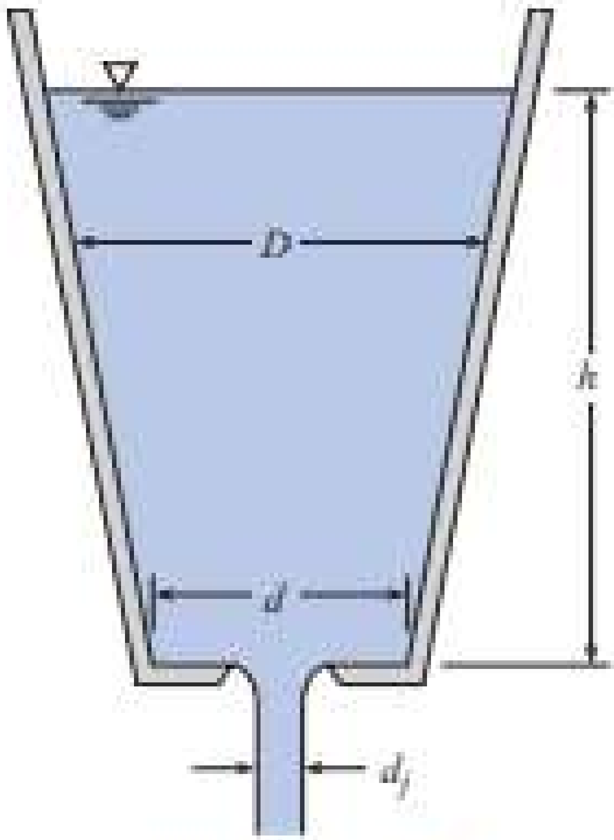
Time required for the water surface to drop the specified distance (3 to 0.5 m).

SOLUTION

From Example 5-6 the time to decrease the elevation from h_1 to h is

$$\begin{aligned} t &= \left(\frac{2A_T}{\sqrt{2g}A_2}\right)(h_1^{1/2} - h^{1/2}) \\ &= \frac{2 \times \left(\pi/4 \times (0.6\,\mathrm{m})^2\right)\left(\sqrt{3} - \sqrt{0.5}\right)\ \mathrm{m}^{1/2}}{\sqrt{2 \times 9.81\,\mathrm{m/\,s^2}} \times (\pi/4) \times (0.03\,\mathrm{m})^2} \\ &\boxed{t = 185\ \mathrm{s}} \end{aligned}$$

5.83 For the type of tank shown, the tank diameter is given as $D = d + C_1 h$, where d is the bottom diameter and C_1 is a constant. Derive a formula for the time of fall of liquid surface from $h = h_0$ to $h = h$ in terms of d_j, d, h_0, h, and C_1. Solve for t if $h_0 = 1$ m, $h = 20$ cm, $d = 20$ cm, $C_1 = 0.3$, and $d_j = 5$ cm. The velocity of water in the liquid jet exiting the tank is $V_e = \sqrt{2gh}$.



PROBLEM 5.83

5.83: PROBLEM DEFINITION

Situation:

A tapered tank drains through an orifice at bottom of tank.
$V_e = \sqrt{2gh}$, $D = d + C_1 h$.
$h_0 = 1\,\text{m}$, $h = 20\,\text{cm}$, $d = 20\,\text{cm}$.
$C_1 = 0.3$, $d_j = 5\,\text{cm}$.

Find:

Derive a formula for the time to drain.
Calculate the time to drain.

PLAN

Apply the continuity equation.

SOLUTION

From continuity equation

$$\begin{aligned} Q &= -A_T\left(\frac{dh}{dt}\right) \\ dt &= -A_T\frac{dh}{Q} \end{aligned}$$

where $Q = \sqrt{2gh}A_j = \sqrt{2gh}(\pi/4)d_j^2$

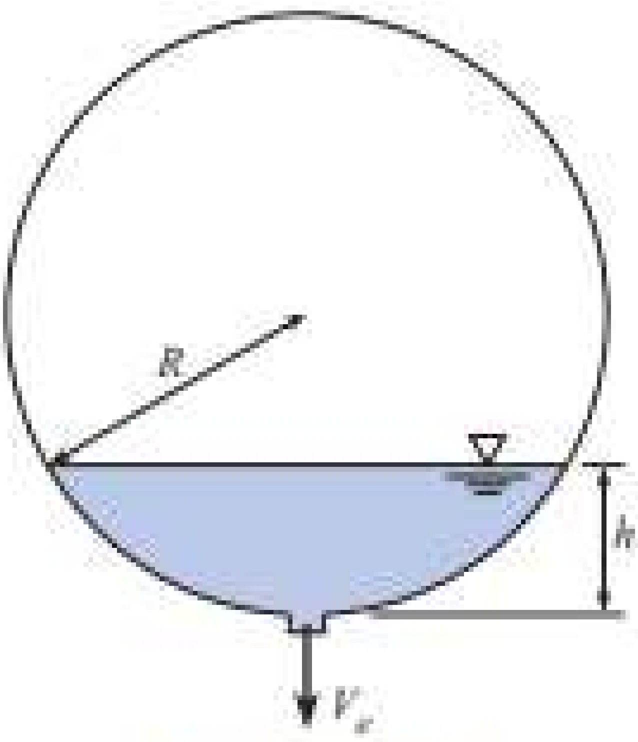
$$\begin{aligned} A_T &= \frac{\pi}{4}(d + C_1 h)^2 = \frac{\pi}{4}(d^2 + 2dC_1 h + C_1^2 h^2) \\ dt &= \frac{-(d^2 + 2dC_1 h + C_1 h^2)dh}{\sqrt{2g}h^{1/2}d_j^2} \\ t &= -\int_{h_0}^{h}\frac{(d^2 + 2dC_1 h + C_1^2 h^2)dh}{\sqrt{2g}h^{1/2}d_j^2} \\ t &= \frac{1}{d_j^2\sqrt{2g}}\int_{h}^{h_0}(d^2 h^{-1/2} + 2dC_1 h^{1/2} + C_1^2 h^{3/2})dh \\ t &= \frac{2}{d_j^2\sqrt{2g}}\left[d^2 h^{1/2} + (2/3)dC_1 h^{3/2} + (1/5)C_1^2 h^{5/2}\right]_{h}^{h_0} \end{aligned}$$

Evaluating the limits of integration gives

$$\boxed{t = \frac{2}{d_j^2\sqrt{2g}}\left[(d^2(h_0^{1/2} - h^{1/2}) + \tfrac{2}{3}dC_1(h_0^{3/2} - h^{3/2}) + \tfrac{1}{5}C_1^2(h_0^{5/2} - h^{5/2})\right]}$$

$$\boxed{t = 13.8\text{ s}}$$

5.84 PLUS A spherical tank with a diameter of 1 m is half filled with water. A port at the bottom of the tank is opened to drain the tank. The hole diameter is 1 cm, and the velocity of the water draining from the hole is $V_e = \sqrt{2gh}$, where h is the elevation of the water surface above the hole. Find the time required for the tank to empty.



PROBLEM 5.84

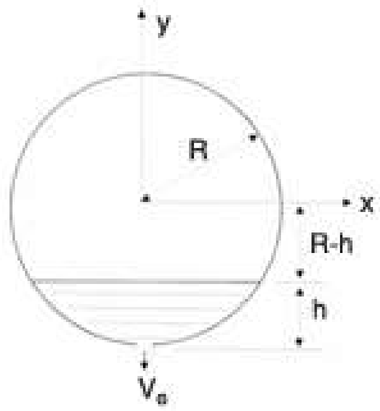
5.84: PROBLEM DEFINITION

Situation:

Water drains out of a spherical tank that begins at half full.

$V_e = \sqrt{2gh}$, $R = 0.5\,\text{m}$, $d_e = 1\,\text{cm}$.

Sketch:



Find:

Time required to empty the tank.

PLAN

Apply the continuity equation. Select a control volume that is inside of the tank and level with the top of the liquid surface.

SOLUTION

Continuity equation

$$\rho \frac{d\forall}{dt} = -\rho A_e V_e$$

Let

$$\frac{d\forall}{dt} = A\frac{dh}{dt}$$

Continuity becomes

$$\frac{dh}{dt} = -\frac{A_e}{A}\sqrt{2gh}$$

The cross-sectional area in terms of R and h is

$$A = \pi[R^2 - (R-h)^2] = \pi(2Rh - h^2)$$

Substituting into the differential equation gives

$$\frac{\pi(-2Rh + h^2)}{A_e\sqrt{2gh}}dh = dt$$

or

$$\frac{\pi}{\sqrt{2g}A_e}\left(-2Rh^{1/2} + h^{3/2}\right)dh = dt$$

Integrating this equation results in

$$\frac{\pi}{\sqrt{2g}A_e}\left(-\frac{4}{3}Rh^{3/2}+\frac{2}{5}h^{5/2}\right)|_R^0 = \Delta t$$

Substituting in the limits yields

$$\frac{\pi}{\sqrt{2g}A_e}\frac{14}{15}R^{5/2} = \Delta t$$

For $R = 0.5$ m and $A_e = 7.85 \times 10^{-5}$ m^2, the time to empty the tank is

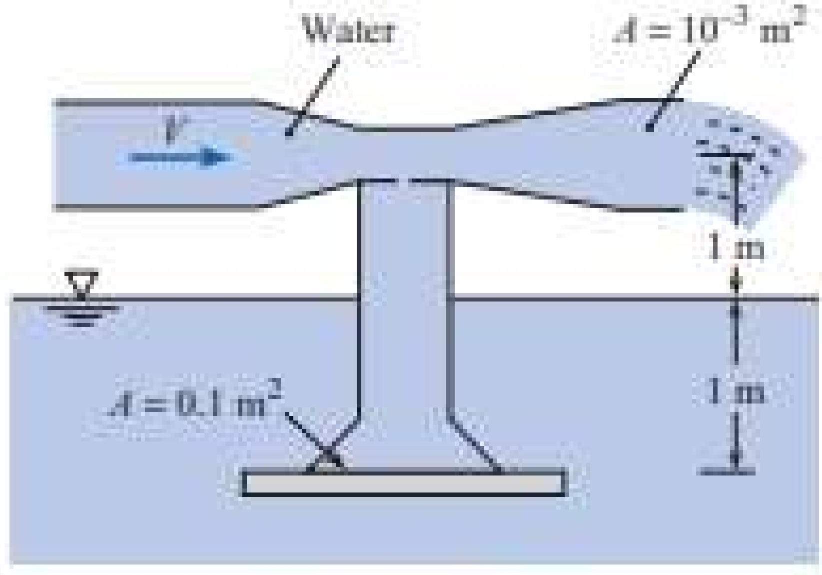
$$\boxed{\Delta t = 1491\,\text{s or } 24.8 \text{ min}}$$

5.100 WileyPLUS A suction device is being designed based on the venturi principle to lift objects submerged in water. The operating water temperature is 15°C. The suction cup is located 1 m below the water surface, and the venturi throat is located 1 m above the water. The atmospheric pressure is 100 kPa. The ratio of the throat area to the exit area is 1/4, and the exit area is 0.001 m^2. The area of the suction cup is 0.1 m^2.

a. Find the velocity of the water at the exit for maximum lift condition.

b. Find the discharge through the system for maximum lift condition.

c. Find the maximum load the suction cup can support.



PROBLEM 5.100

5.100: PROBLEM DEFINITION

Situation:

A suction device based on a venturi lifts objects submerged in water.
$A_e = 10^{-3}\,\text{m}^2$, $A_t = 0.25A_e$, $A_s = 0.1\,\text{m}^2$.

Find:

(a) Velocity of water at exit for maximum lift.
(b) Discharge.
(c) Maximum load supportable by suction cup.

Properties:

Water (15 °C) Table A.5: $p_v = 1,700\,\text{Pa}$, $\rho = 999\ \text{kg/m}^3$.
$p_{atm} = 100\,\text{kPa}$.

PLAN

Apply the Bernoulli equation and the continuity equation.

SOLUTION

Venturi exit area, $A_e = 10^{-3}\ \text{m}^2$, Venturi throat area, $A_t = (1/4)A_e$, Suction cup area, $A_s = 0.1\ \text{m}^2$

$$\begin{aligned} p_{\text{atm}} &= 100\ \text{kPa} \\ T_{\text{water}} &= 15°\ \text{C} \end{aligned}$$

Bernoulli equation for the Venturi from the throat to exit with the pressure at the throat equal to the vapor pressure of the water. This will establish the maximum lift condition. Cavitation would prevent any lower pressure from developing at the throat.

$$\frac{p_v}{\gamma} + \frac{V_t^2}{2g} + z_t = \frac{p_e}{\gamma} + \frac{V_{e\,\max}^2}{2g} + z_e \tag{1}$$

Continuity equation

$$\begin{aligned} V_t A_t &= V_e A_e \\ V_t &= V_e \frac{A_e}{A_t} \\ V_t &= 4V_e \end{aligned} \tag{2}$$

Then Eq. (1) can be written as

$$\begin{aligned} \frac{1,700}{\gamma} + \frac{(4V_{e\,\max})^2}{2g} &= \frac{100,000}{\gamma} + \frac{V_{e\,\max}^2}{2g} \\ V_{e\,\max} &= \left[\left(\frac{1}{15}\right)\left(\frac{2g}{\gamma}\right)(98,300)\right]^{1/2} \\ &= \left[\left(\frac{1}{15}\right)\left(\frac{2}{\rho}\right)(98,300)\right]^{1/2} \end{aligned}$$

$$\boxed{V_{e\,\max} = 3.62\ \text{m/s}}$$

$$
\begin{aligned}
Q_{\max} &= V_e A_e \\
&= (3.62 \text{ m/s})(10^{-3}\,\text{m}^2) \\
&\boxed{Q_{\max} = 0.00362 \text{ m}^3\text{/s}}
\end{aligned}
$$

Find pressure in the suction cup at the level of the suction cup.

$$
\begin{aligned}
p_t + \gamma \Delta h &= p_{\text{suction}} \\
p_{\text{suction}} &= 1,700 \text{ Pa} + 9,800\,\text{N/ m}^3 \times 2\,\text{m} \\
&= 21,300 \text{ Pa}
\end{aligned}
$$

But the pressure in the water surrounding the suction cup will be $p_{\text{atm}} + \gamma \times 1 = (100 + 9.80)$ kPa, or

$$
\begin{aligned}
p_{\text{water}} - p_{\text{suction}} &= (109,800 - 21,300) \text{ Pa} \\
&= 88,500 \text{ Pa}
\end{aligned}
$$

Thus the maximum lift will be:

$$
\begin{aligned}
\text{Lift}_{\max} &= \Delta p A_s = (p_{\text{water}} - p_{\text{suction}}) A_s \\
&= (88,500 \text{ N/m}^2)(0.1 \text{ m}^2) \\
&\boxed{\text{Lift}_{\max} = 8,850 \text{ N}}
\end{aligned}
$$