




ech Family

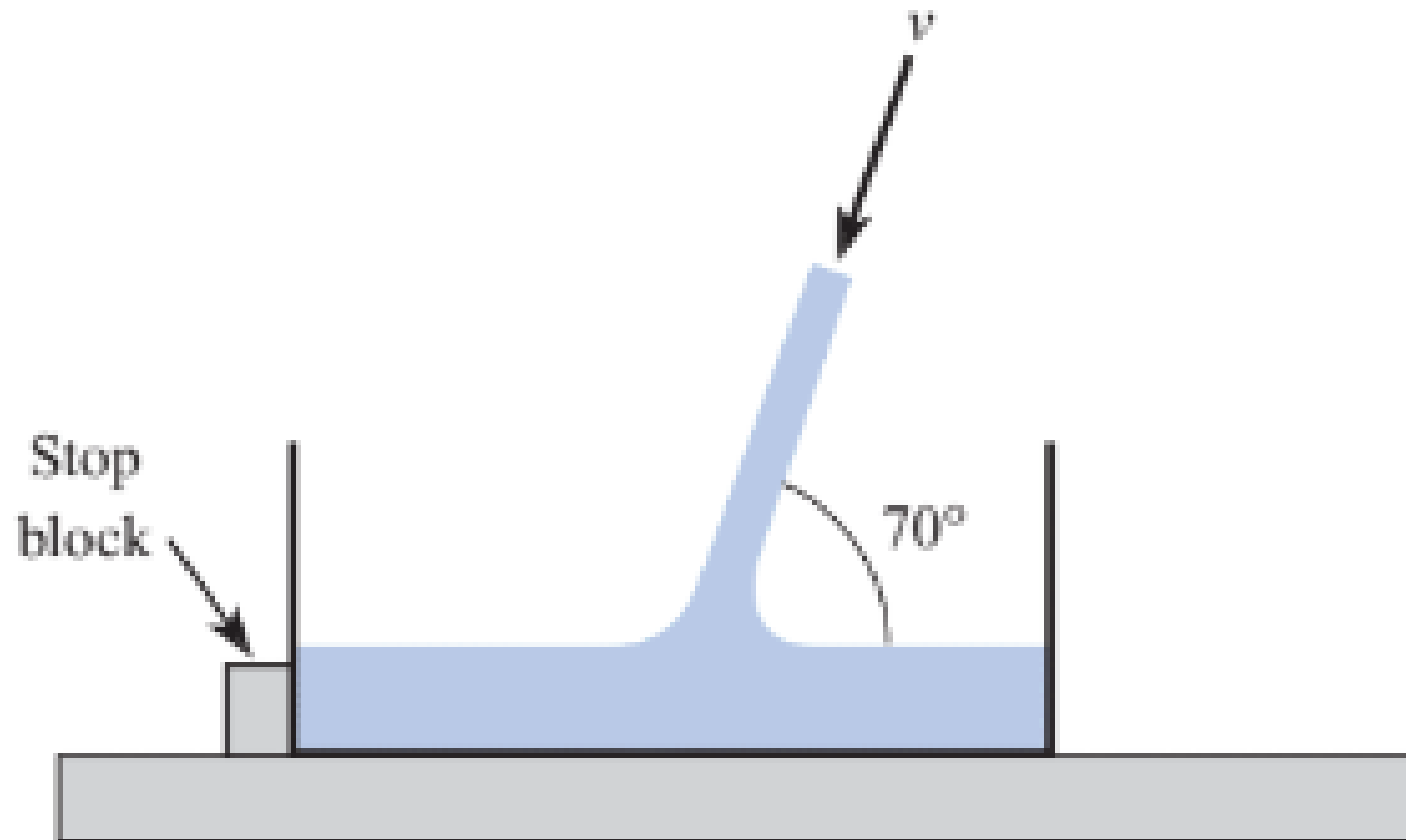
كن أنت التغيير



اللجنة الأكاديمية في قسم

الهندسة الميكانيكية

6.9  A water jet of diameter 30 mm and speed $v = 25$ m/s is filling a tank. The tank has a mass of 25 kg and contains 25 liters of water at the instant shown. The water temperature is 15°C . Find the force acting on the bottom of the tank and the force acting on the stop block. Neglect friction.



PROBLEMS 6.9, 6.10

6.9: PROBLEM DEFINITION

Situation:

A water jet is filling a tank.

$$m = 25 \text{ kg}, V = 25 \text{ L.}$$

$$d = 30 \text{ mm}, v = 25 \text{ m/s.}$$

Find:

Force on the bottom of the tank (N).

Force acting on the stop block (N).

Assumptions:

Steady flow.

Properties:

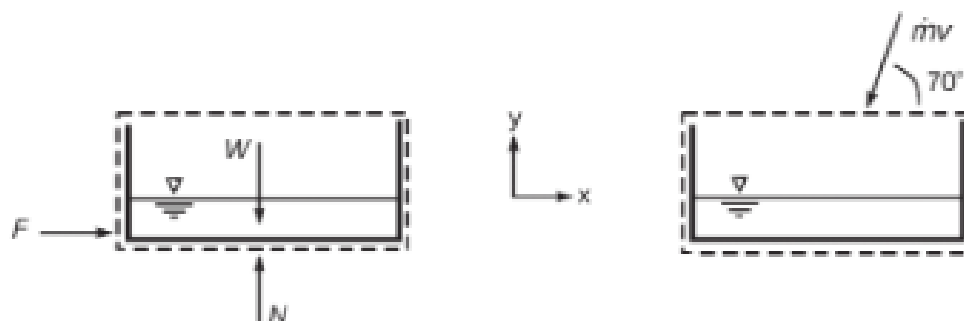
Water (15 °C), Table A.5: $\rho = 999 \text{ kg/m}^3$, $\gamma = 9800 \text{ N/m}^3$.

PLAN

Apply the momentum equation in the x-direction and in the y-direction.

SOLUTION

Force and momentum diagrams



Momentum equation (x -direction)

$$\begin{aligned}\sum F_x &= \sum_{cs} \dot{m}_o v_{ox} - \sum_{cs} \dot{m}_i v_{ix} \\ F &= -(-\dot{m}v \cos 70^\circ) \\ &= \rho A v^2 \cos 70^\circ\end{aligned}$$

Calculations

$$\begin{aligned}\rho A v^2 &= (999 \text{ kg/m}^3) \left(\frac{\pi \times (0.03 \text{ m})^2}{4} \right) (25 \text{ m/s})^2 \\ &= 441.3 \text{ N}\end{aligned}$$

$$\begin{aligned}F &= (441.3 \text{ N}) (\cos 70^\circ) \\ &= 150.9 \text{ N}\end{aligned}$$

$$F = 150.9 \text{ N pushing to the left on the stop block}$$

y -direction

$$\begin{aligned}\sum F_y &= \sum_{cs} \dot{m}_o v_{oy} - \sum_{cs} \dot{m}_i v_{iy} \\ N - W &= -(-\dot{m}v \sin 70^\circ) \\ N &= W + \rho A v^2 \sin 70^\circ\end{aligned}$$

Calculations:

$$\begin{aligned}W &= W_{\text{tank}} + W_{\text{water}} \\ &= (25 \text{ kg})(9.81 \text{ m/s}^2) + (0.025 \text{ m}^3)(9800 \text{ N/m}^3) \\ &= 490.3 \text{ N}\end{aligned}$$

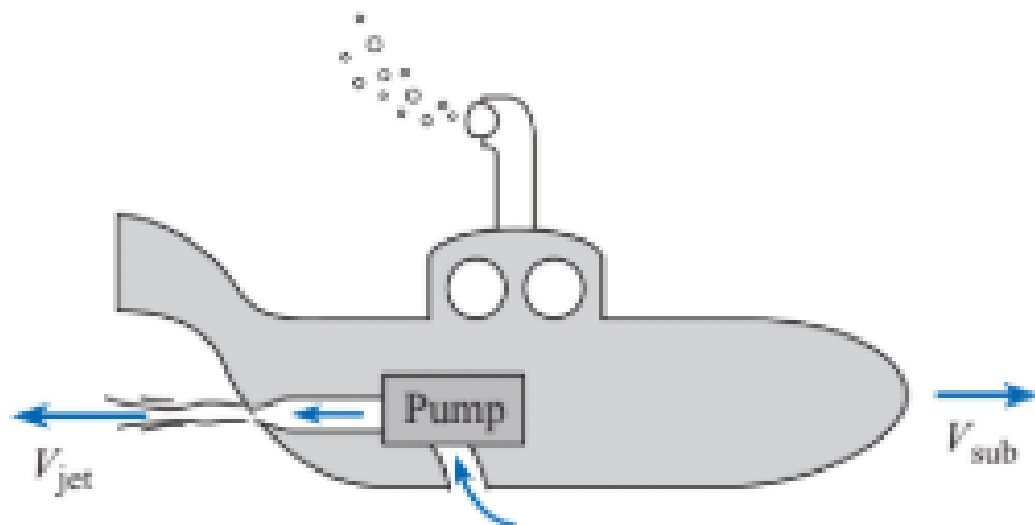
$$\begin{aligned}N &= W + \rho A v^2 \sin 70^\circ \\ &= (490.3 \text{ N}) + (441.3 \text{ N}) \sin 70^\circ\end{aligned}$$

$$N = 905 \text{ N acting upward}$$

6.11 A design contest features a submarine that will travel at a steady speed of $V_{\text{sub}} = 1 \text{ m/s}$ in 15°C water. The sub is powered by a water jet. This jet is created by drawing water from an inlet of diameter 25 mm, passing this water through a pump and then accelerating the water through a nozzle of diameter 5 mm to a speed of V_{jet} . The hydrodynamic drag force (F_D) can be calculated using

$$F_D = C_D \left(\frac{\rho V_{\text{sub}}^2}{2} \right) A_p$$

where the coefficient of drag is $C_D = 0.3$ and the projected area is $A_p = 0.28 \text{ m}^2$. Specify an acceptable value of V_{jet} .



PROBLEM 6.11

6.11: PROBLEM DEFINITION

Situation:

A design contest features a submarine powered by a water jet.

$$V_{\text{sub}} = 1.0 \text{ m/s}, D_1 = 25 \text{ mm.}$$

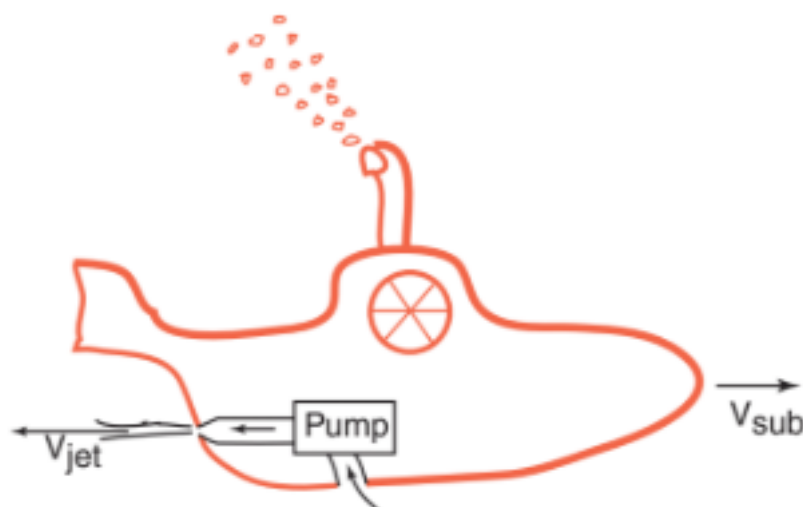
$$D_2 = 5 \text{ mm}, F_D = C_D \left(\frac{\rho V_{\text{sub}}^2}{2} \right) A_p$$

$$C_D = 0.3, A_p = 0.28 \text{ m}^2.$$

Find:

Speed of the fluid jet (m/s).

Sketch:



Assumptions:

Assume steady flow so that the accumulation of momentum term is zero.

Properties:

Water (15 °C), Table A.5: $\rho = 999 \text{ kg/m}^3$.

PLAN

The speed of the fluid jet can be found from the momentum equation because the drag force will balance with the net rate of momentum outflow.

SOLUTION

Momentum equation. Select a control volume that surrounds the sub. Select a reference frame located on the submarine. Let section 1 be the outlet (water jet) and section 2 be the inlet. The momentum equation is

$$\begin{aligned} \sum \mathbf{F} &= \sum_{cs} \dot{m}_o \mathbf{v}_o - \sum_{cs} \dot{m}_i \mathbf{v}_i \\ F_{\text{Drag}} &= \dot{m}_2 v_2 - \dot{m}_1 v_{1x} \end{aligned}$$

By continuity, $\dot{m}_1 = \dot{m}_2 = \rho A_{\text{jet}} V_{\text{jet}}$. The outlet velocity is $v_2 = V_{\text{jet}}$. The x-component of the inlet velocity is $v_{1x} = V_{\text{sub}}$. The momentum equation simplifies to

$$F_{\text{Drag}} = \rho A_{\text{jet}} V_{\text{jet}} (V_{\text{jet}} - V_{\text{sub}})$$

The drag force is

$$\begin{aligned} F_{\text{Drag}} &= C_D \left(\frac{\rho V_{\text{sub}}^2}{2} \right) A_p \\ &= 0.3 \left(\frac{(999 \text{ kg/m}^3) (1.0 \text{ m/s})^2}{2} \right) (0.28 \text{ m}^2) \\ &= 42.0 \text{ N} \end{aligned}$$

The momentum equation becomes


$$\begin{aligned} F_{\text{Drag}} &= \rho A_{\text{jet}} V_{\text{jet}} [V_{\text{jet}} - V_{\text{sub}}] \\ 42.0 \text{ N} &= (999 \text{ kg/m}^3) (1.96 \times 10^{-5} \text{ m}^2) V_{\text{jet}} [V_{\text{jet}} - (1.0 \text{ m/s})] \end{aligned}$$

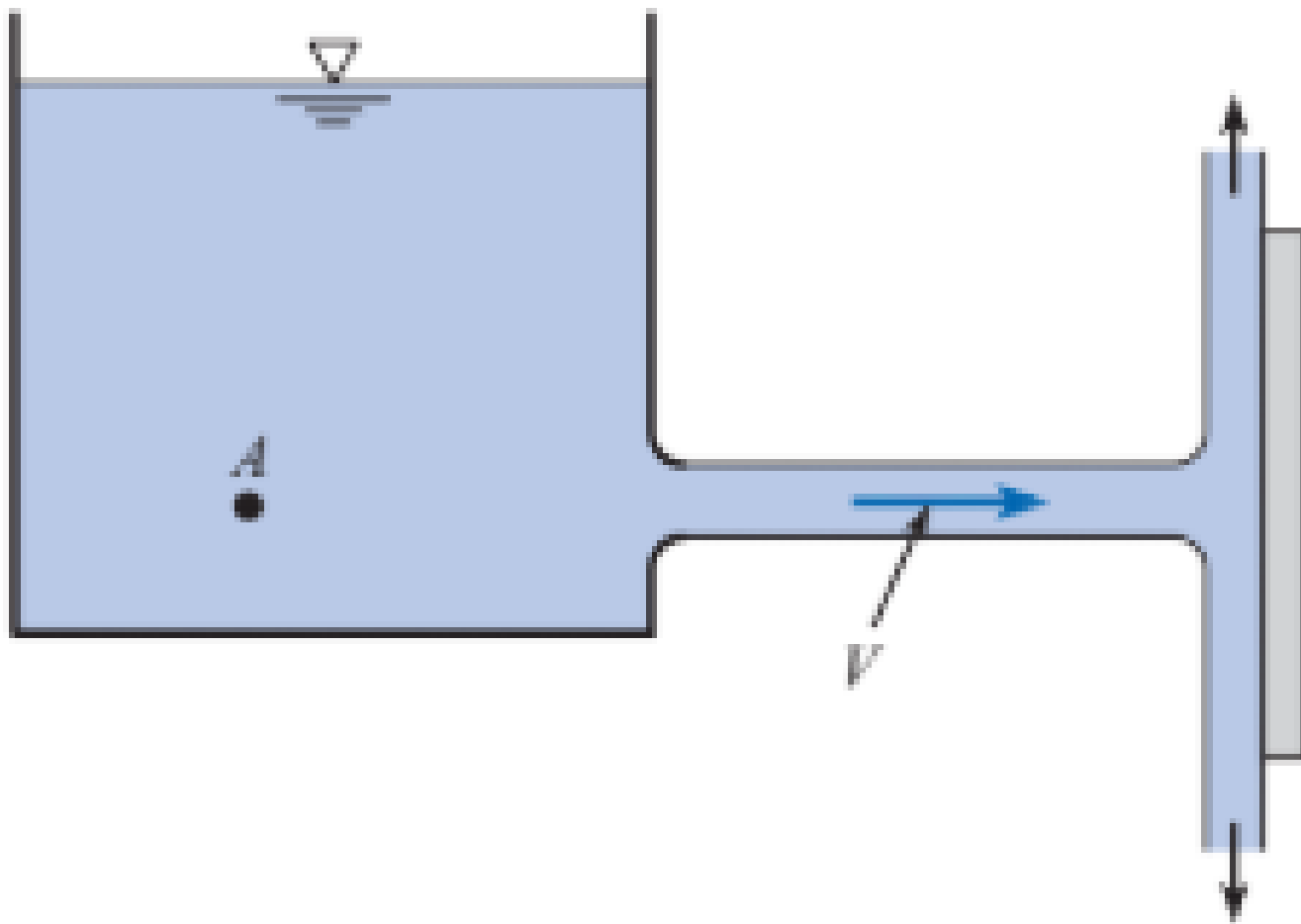
Solving for the jet speed gives

$$V_{\text{jet}} = 46.8 \text{ m/s}$$

REVIEW

1. The jet speed (46.8 m/s) is above 160 km/h. This presents a safety issue. Also, this would require a pump that can produce a large pressure rise.
2. It is recommended that the design be modified to produce a lower jet velocity. One way to accomplish this goal is to increase the diameter of the jet.

6.13  A horizontal water jet at 70°F issues from a circular orifice in a large tank. The jet strikes a vertical plate that is normal to the axis of the jet. A force of 600 lbf is needed to hold the plate in place against the action of the jet. If the pressure in the tank is 25 psig at point A , what is the diameter of the jet just downstream of the orifice?



PROBLEMS 6.12, 6.13

6.13: PROBLEM DEFINITION

Situation:

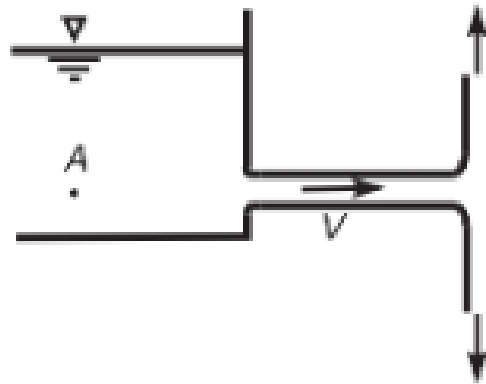
Horizontal round jet strikes a plate.

$$F_x = 2.7 \text{ kN}$$

Find:

Diameter of jet (m).

Sketch:



Properties:

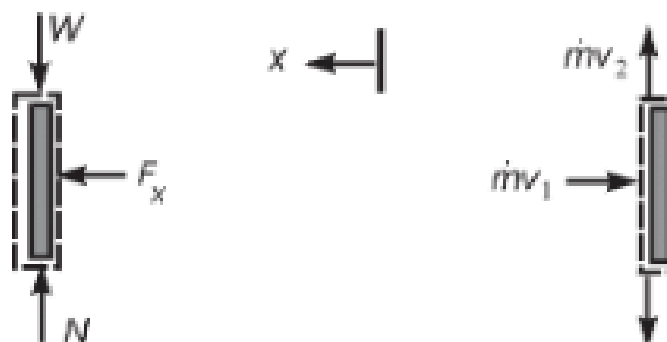
$$p_A = 170 \text{ kPa.}$$

Water (21 °C), Table A.5: $\rho = 997.8 \text{ kg/m}^3$.

PLAN

Apply the Bernoulli equation, then the momentum equation.

SOLUTION




Force and momentum diagrams

Bernoulli equation applied from inside of tank to nozzle exit

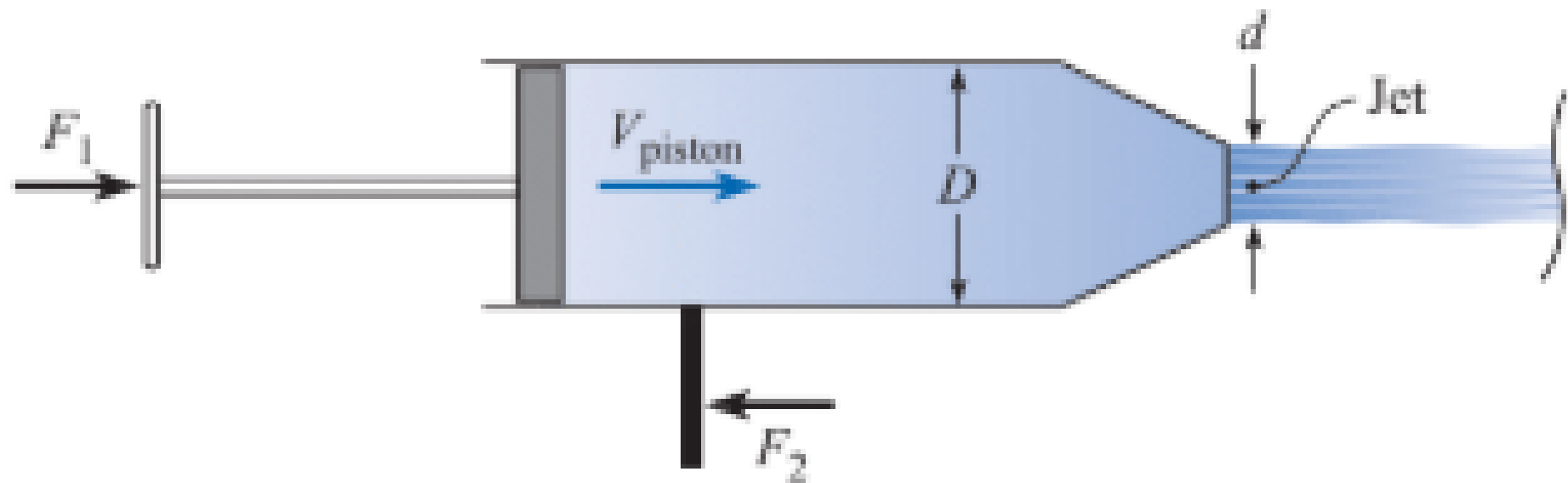
$$\begin{aligned}\frac{p_A}{\rho} &= \frac{v_1^2}{2} \\ v_1 &= \sqrt{\frac{2p_A}{\rho}} \\ &= \sqrt{\frac{2 \times 170 \text{ kPa}}{997.8 \text{ kg/m}^3}} \\ &= 18.5 \text{ m/s}\end{aligned}$$

Momentum equation (x -direction)

$$\begin{aligned}\sum F_x &= -\dot{m}v_{1x} \\ F_x &= -(-\dot{m}v_1) = \rho Av_1^2 \\ A &= \frac{F_x}{\rho v_1^2} = \frac{2700 \text{ N}}{997.8 \text{ kg/m}^3 \times (18.5 \text{ m/s})^2} \\ A &= 0.0079 \text{ m}^2 \\ d &= \sqrt{\frac{4A}{\pi}} \\ &= \sqrt{\frac{4 \times 0.0079 \text{ m}^2}{\pi}} \\ &= \boxed{d = 0.1 \text{ m}}\end{aligned}$$

6.14  An engineer, who is designing a water toy, is making preliminary calculations. A user of the product will apply a force F_1 that moves a piston ($D = 80$ mm) at a speed of $V_{\text{piston}} = 300$ mm/s. Water at 20°C jets out of a converging nozzle

of diameter $d = 15$ mm. To hold the toy stationary, the user applies a force F_2 to the handle. Which force (F_1 versus F_2) is larger? Explain your answer using concepts of the momentum principle. Then calculate F_1 and F_2 . Neglect friction between the piston and the walls.



PROBLEM 6.14

6.14: PROBLEM DEFINITION

Situation:

An engineer is designing a toy to create a jet of water.

$$D = 80 \text{ mm}, d = 15 \text{ mm}.$$

$$V_{\text{piston}} = 300 \text{ mm/s}.$$

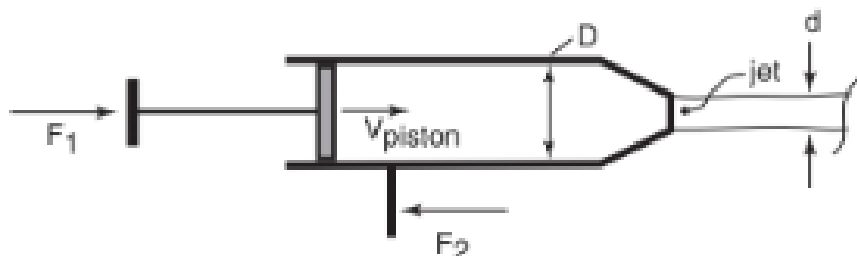
Find:

Which force (F_1 versus F_2) is larger? Explain your answer using concepts of the momentum equation.

Calculate F_1 .

Calculate F_2 .

Sketch:



Assumptions:

Neglect friction between the piston and the wall.

Assume the Bernoulli equation applies (neglect viscous effects; neglect unsteady flow effects).

Properties:

Table A.5 (water at 20°C): $\rho = 998 \text{ kg/m}^3$.

PLAN

To find the larger force, recognize that the net force must be in the direction of acceleration. To solve the problem, apply the momentum equation, continuity equation, equilibrium equation, and the Bernoulli equation.

SOLUTION

Finding the larger force (F_1 versus F_2). Since the fluid is accelerating to the right the net force must act to the right. Thus, F_1 is larger than F_2 . This can also be seen by application of the momentum equation.

Momentum equation (x -direction) applied to a control volume surrounding the toy.

$$\begin{aligned}\sum F_x &= \dot{m}v_{\text{out}} \\ F_1 - F_2 &= \dot{m}v_{\text{out}} \\ F_1 - F_2 &= \rho \left(\frac{\pi d^2}{4} \right) V_{\text{out}}^2\end{aligned}\quad (1)$$

Notice that Eq. (1) shows that $F_1 > F_2$.

Continuity equation applied to a control volume situated inside the toy.

$$\begin{aligned}Q_{\text{in}} &= Q_{\text{out}} \\ \left(\frac{\pi D^2}{4}\right) V_{\text{piston}} &= \left(\frac{\pi d^2}{4}\right) V_{\text{out}} \\ V_{\text{out}} &= V_{\text{piston}} \frac{D^2}{d^2} \\ &= (0.3 \text{ m/s}) \left(\frac{80 \text{ mm}}{15 \text{ mm}}\right)^2 \\ V_{\text{out}} &= 8.533 \text{ m/s}\end{aligned}$$

Bernoulli equation applied from inside the toy to the nozzle exit plane.


$$\begin{aligned}p_{\text{inside}} + \frac{\rho V_{\text{piston}}^2}{2} &= \frac{\rho V_{\text{out}}^2}{2} \\ p_{\text{inside}} &= \frac{\rho (V_{\text{out}}^2 - V_{\text{piston}}^2)}{2} \\ &= \frac{(998 \text{ kg/m}^3) ((8.533 \text{ m/s})^2 - (0.3 \text{ m/s})^2)}{2} \\ &= 36.33 \text{ kPa}\end{aligned}$$

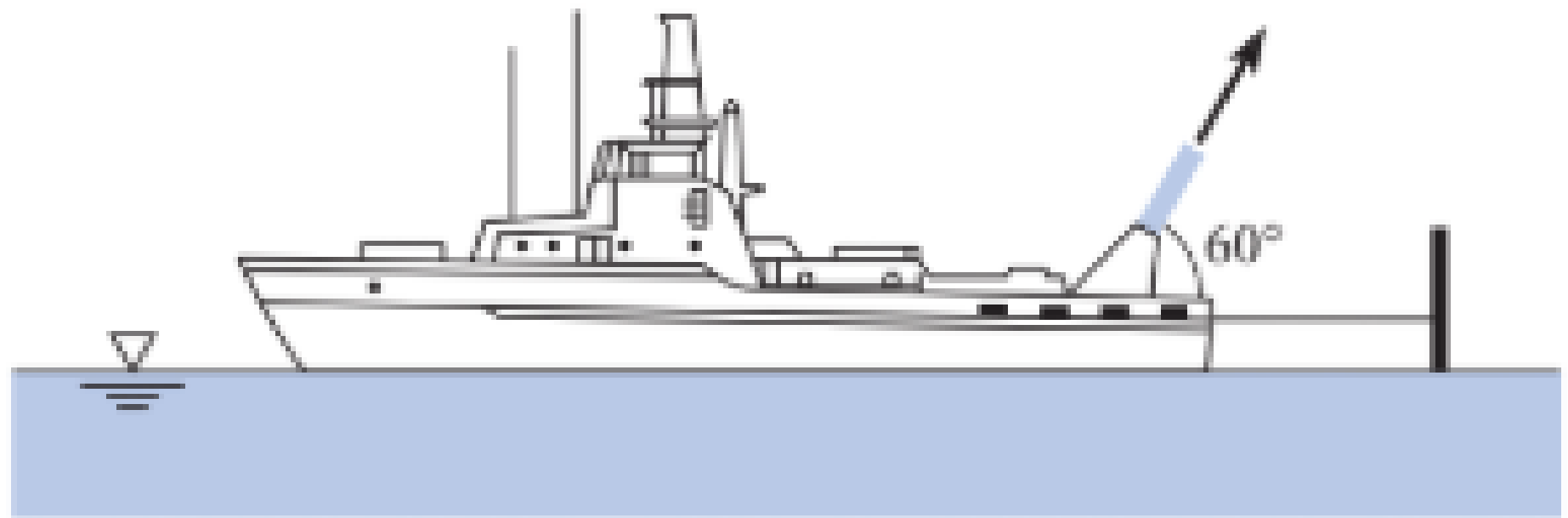
Equilibrium applied to the piston (the applied force F_1 balances the pressure force).

$$\begin{aligned}F_1 &= p_{\text{inside}} \left(\frac{\pi D^2}{4}\right) \\ &= (36330 \text{ Pa}) \left(\frac{\pi (0.08 \text{ m})^2}{4}\right) \\ &\boxed{F_1 = 182.6 \text{ N}}\end{aligned}$$

Momentum equation (Eq. 1)

$$\begin{aligned}F_2 &= F_1 - \rho \left(\frac{\pi d^2}{4}\right) V_{\text{out}}^2 \\ &= 182.6 \text{ N} - (998 \text{ kg/m}^3) \left(\frac{\pi (0.015 \text{ m})^2}{4}\right) (8.533 \text{ m/s})^2 \\ &\boxed{F_2 = 169.8 \text{ N}}\end{aligned}$$

6.16  A boat is held stationary by a cable attached to a pier. A firehose directs a spray of 5°C water at a speed of $V = 50\text{ m/s}$. If the allowable load on the cable is 5 kN , calculate the mass flow rate of the water jet. What is the corresponding diameter of the water jet?



PROBLEMS 6.15, 6.16

6.16: PROBLEM DEFINITION

Situation:

Water jet from a fire hose on a boat.

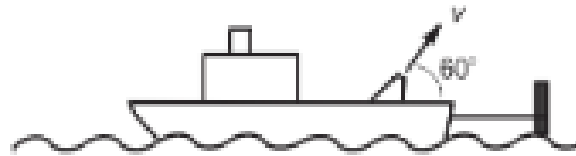
$$T = 5.0 \text{ kN}, v = 50 \text{ m/s}.$$

Find:

Mass flow rate of jet (kg/s).

Diameter of jet (cm).

Sketch:



Properties:

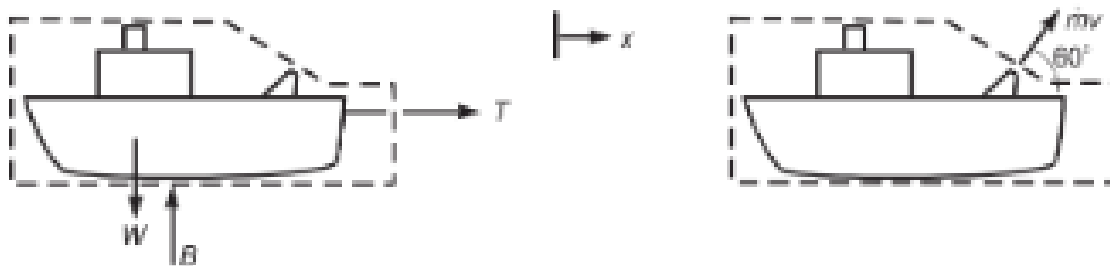
Water (5 °C), Table A.5: $\rho = 1000 \text{ kg/m}^3$.

PLAN

Apply the momentum equation to find the mass flow rate. Then, calculate diameter using the flow rate equation.

SOLUTION

Force and momentum diagrams



Momentum equation (x -direction)

$$\begin{aligned}\sum F &= \dot{m} (v_o)_x \\ T &= \dot{m} v \cos 60^\circ \\ \dot{m} &= \frac{T}{v \cos 60^\circ} = \frac{5000 \text{ N}}{(50 \times \cos 60^\circ) \text{ m/s}} \\ \dot{m} &= 200 \text{ kg/s}\end{aligned}$$

Flow rate

$$\dot{m} = \rho A v = \frac{\rho \pi d^2 v}{4}$$

$$d = \sqrt{\frac{4 \dot{m}}{\rho \pi v}}$$

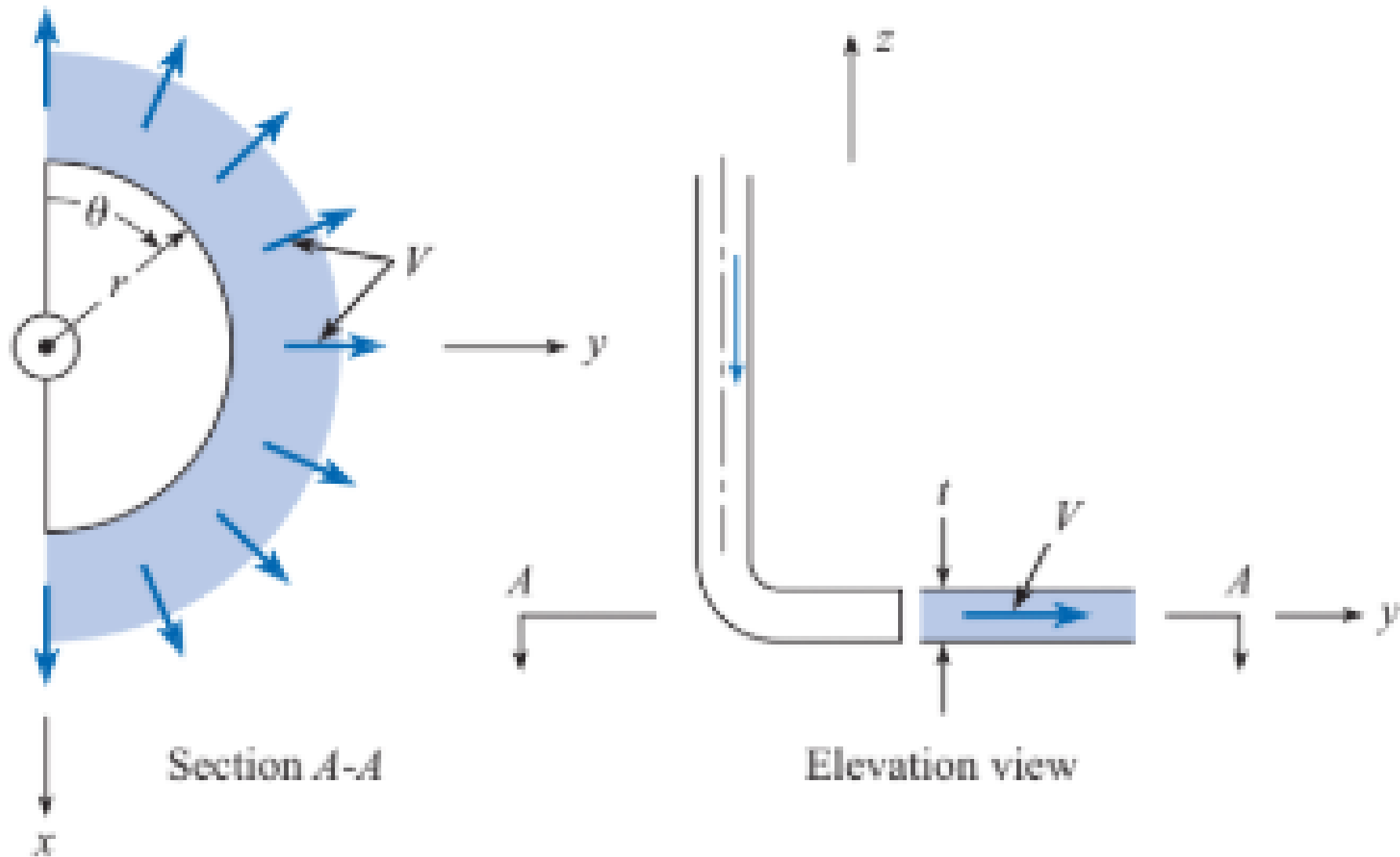
$$= \sqrt{\frac{4 \times 200 \text{ kg/s}}{1000 \text{ kg/m}^3 \times \pi \times 50 \text{ m/s}}}$$

$$= 7.136 \times 10^{-2} \text{ m}$$

$$\boxed{d = 7.14 \text{ cm}}$$

6.21 The semicircular nozzle sprays a sheet of liquid through 180° of arc as shown. The velocity is V at the efflux section where

the sheet thickness is t . Derive a formula for the external force F (in the y -direction) required to hold the nozzle system in place. This force should be a function of ρ , V , r , and t .



PROBLEM 6.21

6.21: PROBLEM DEFINITION

Situation:

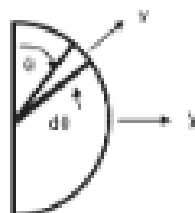
A hemispherical nozzle sprays a sheet of liquid through an arc.

Find:

An expression for the force in y -direction to hold the nozzle stationary.

$$F_y = F_y(\rho, v, r, t).$$

Sketch:



PLAN

Apply the momentum equation.

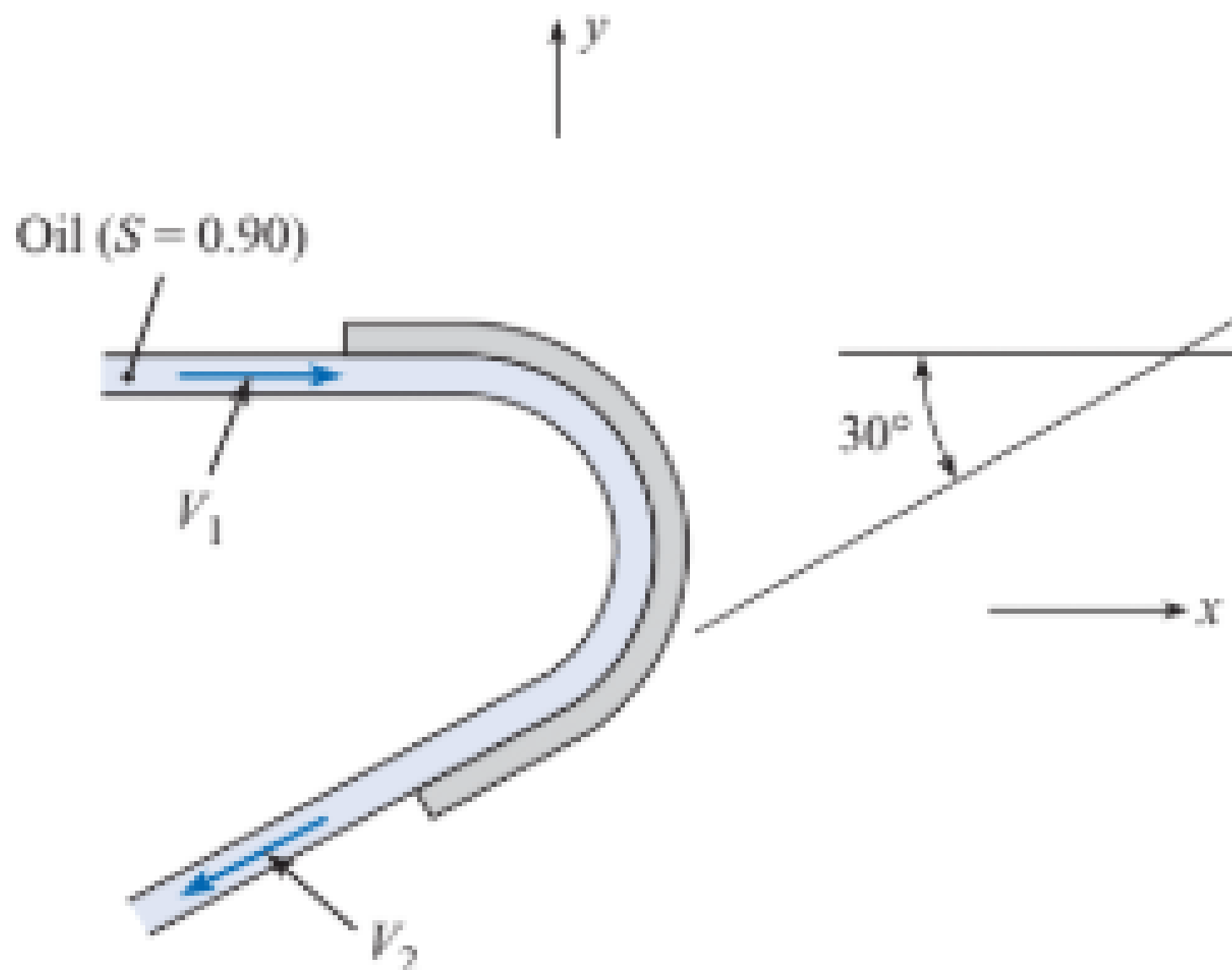
SOLUTION

Momentum equation (y -direction)

$$\begin{aligned} F_y &= \int_{cs} v_y \rho \mathbf{V} \cdot d\mathbf{A} \\ &= \int_0^\pi (v \sin \theta) \rho v (tr d\theta) \\ &= \rho v^2 tr \int_0^\pi \sin \theta d\theta \\ &= \boxed{F_y = 2\rho v^2 tr} \end{aligned}$$

Applying the Momentum Equation to Vanes (§6.4)

6.23 **PLUS** Determine the external reactions in the x - and y -directions needed to hold this fixed vane, which turns the oil jet ($S = 0.9$) in a horizontal plane. Here V_1 is 22 m/s, $V_2 = 21$ m/s, and $Q = 0.15$ m³/s.



PROBLEMS 6.23, 6.24

6.23: PROBLEM DEFINITIONSituation:

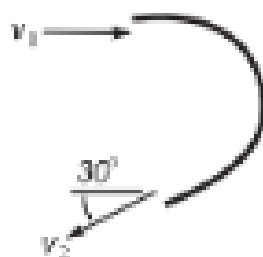
A fixed vane in the horizontal plane.

$$v_1 = 22 \text{ m/s}, v_2 = 21 \text{ m/s}.$$

$$Q = 0.15 \text{ m}^3/\text{s}, S = 0.9.$$

Find:

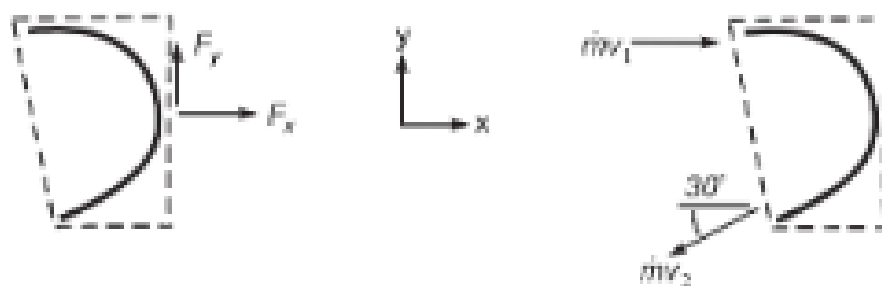
Components of force to hold vane stationary (kN).

Sketch:**PLAN**

Apply the momentum equation.

SOLUTION

Force and momentum diagrams



Mass flow rate

$$\begin{aligned} \dot{m} &= \rho Q \\ &= 0.9 \times 1000 \text{ kg/m}^3 \times 0.15 \text{ m}^3/\text{s} \\ &= 135 \text{ kg/s} \end{aligned}$$

Momentum equation (x -direction)


$$\begin{aligned} \sum F_x &= \dot{m}(v_o)_x - \dot{m}(v_i)_x \\ F_x &= \dot{m}(-v_2 \cos 30^\circ) - \dot{m}v_1 \\ F_x &= -135 \text{ kg/s}(21 \text{ m/s} \cos 30^\circ + 22 \text{ m/s}) \end{aligned}$$

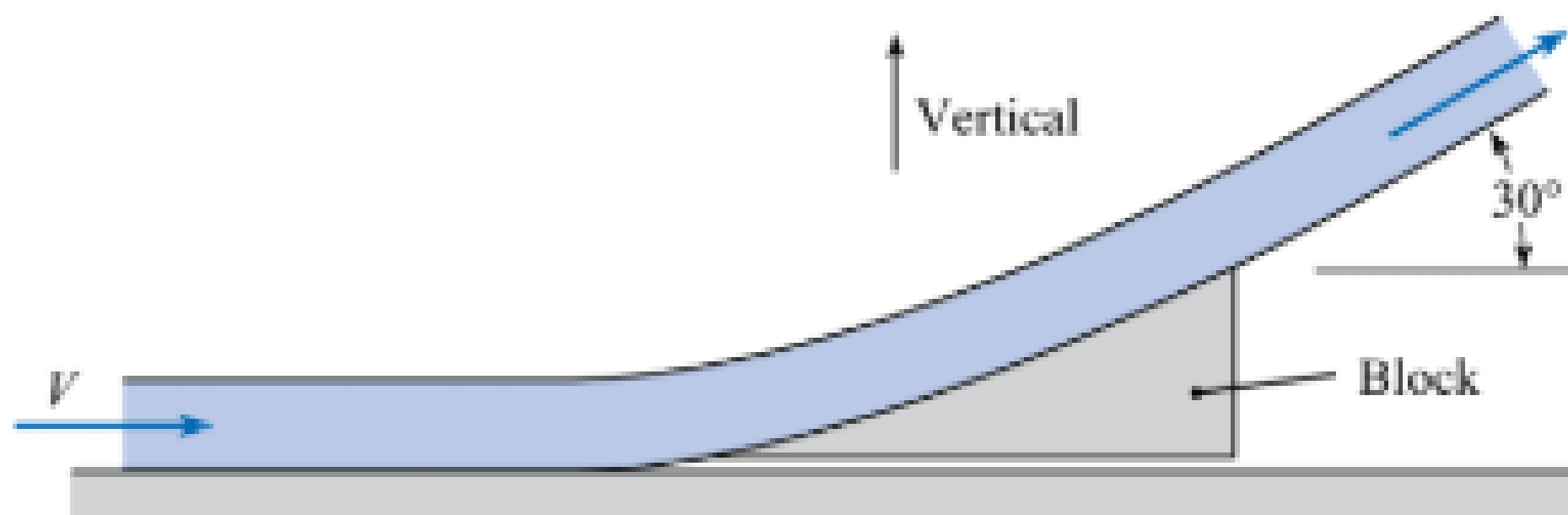
$$F_x = -5.43 \text{ kN (acts to the left)}$$

Momentum equation (y -direction)

$$\begin{aligned}\sum F_y &= \dot{m} (v_o)_y - \dot{m} (v_i)_y \\ F_y &= \dot{m} (-v_2 \sin 30^\circ) \\ &= 135 \text{ kg/s} (-21 \text{ m/s} \sin 30^\circ) \\ &= -1.42 \text{ kN}\end{aligned}$$

$$F_y = -1.42 \text{ kN (acts downward)}$$

6.27  Water ($\rho = 1000 \text{ kg/m}^3$) strikes a block as shown and is deflected 30° . The flow rate of the water is 1.5 kg/s , and the inlet velocity is $V = 10 \text{ m/s}$. The mass of the block is 1 kg . The coefficient of static friction between the block and the surface is 0.1 (friction force/normal force). If the force parallel to the surface exceeds the frictional force, the block will move. Determine the force on the block and whether the block will move. Neglect the weight of the water.



PROBLEMS 6.27, 6.28

6.27: PROBLEM DEFINITION

Situation:

A water jet strikes a block and the block is held in place by friction.

$$v_1 = 10 \text{ m/s}, \dot{m} = 1.5 \text{ kg/s.}$$

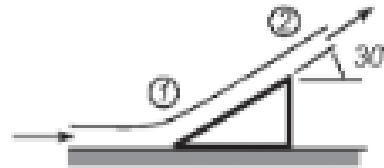
$$\mu = 0.1, m = 1 \text{ kg.}$$

Find:

Will the block slip?

Force of the water jet on the block (N).

Sketch:



Assumptions:

Neglect weight of water.

Neglect elevation changes.

Neglect viscous forces.

Properties:

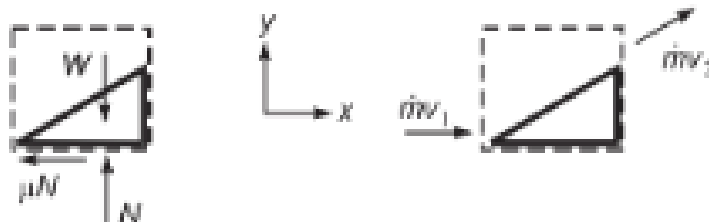
$$\rho = 1000 \text{ kg/m}^3.$$

PLAN

Apply the Bernoulli equation, then the momentum equation.

SOLUTION

Force and momentum diagrams



Bernoulli equation

$$v_1 = v_2 = v = 10 \text{ m/s}$$

Momentum equation (x -direction)

$$\begin{aligned} \sum F_x &= \dot{m}_o (v_o)_x - \dot{m}_i (v_i)_x \\ -F_f &= \dot{m}v \cos 30^\circ - \dot{m}v \\ F_f &= \dot{m}v(1 - \cos 30^\circ) \\ &= 1.5 \text{ kg/s} \times 10 \text{ m/s} \times (1 - \cos 30^\circ) \end{aligned}$$

$$\boxed{F_f = 2.01 \text{ N}}$$


y -direction

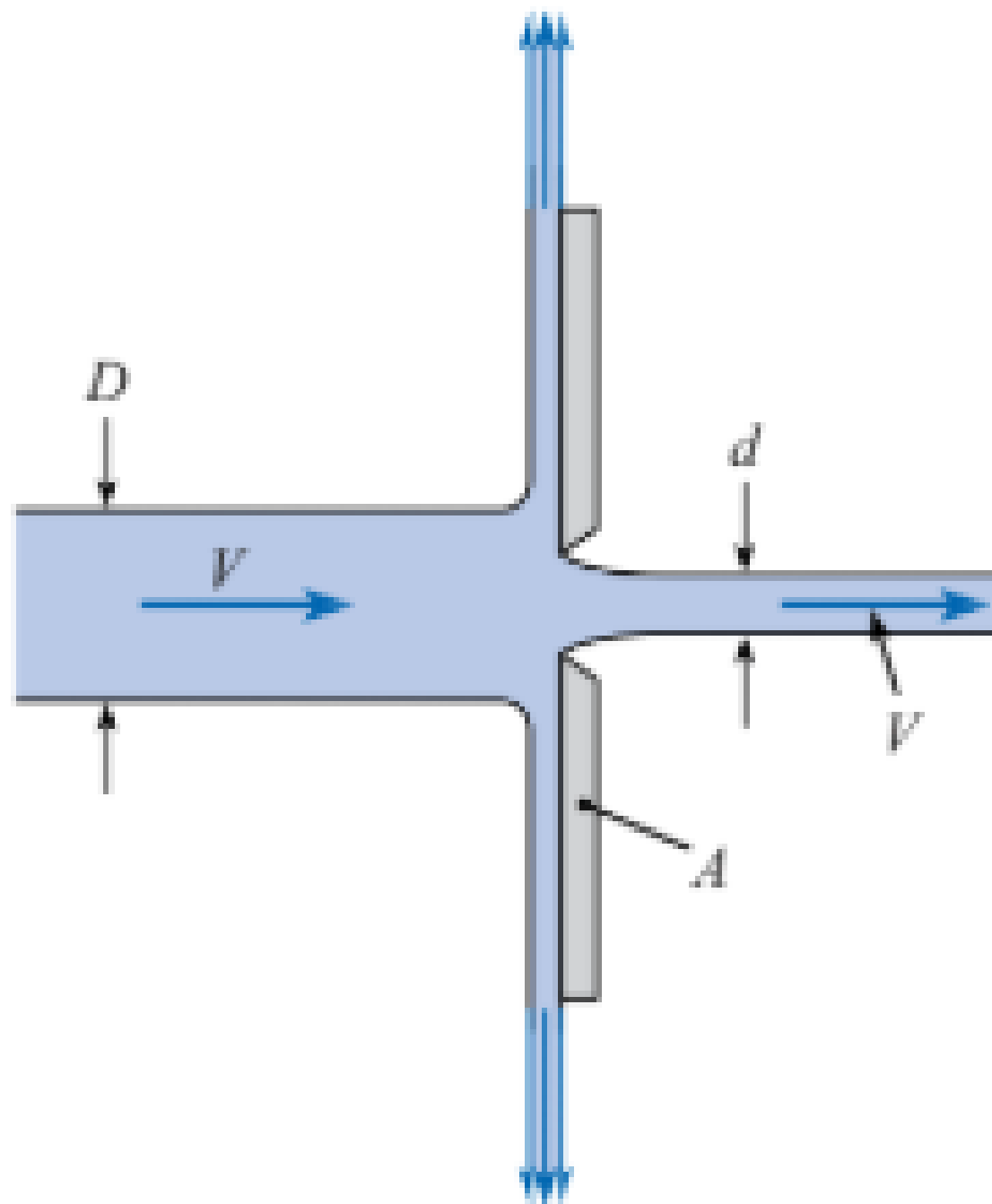
$$\begin{aligned}\sum F_y &= \dot{m}_O (v_O)_y \\ N - W &= \dot{m}(v \sin 30^\circ) \\ N &= mg + \dot{m}(v \sin 30^\circ) \\ &= 1.0 \text{ kg} \times 9.81 \text{ m/s}^2 + 1.5 \text{ kg/s} \times 10 \text{ m/s} \times \sin 30^\circ \\ &\quad \boxed{N = 17.3 \text{ N}}\end{aligned}$$

Analyze friction:

- F_f (required to prevent block from slipping) = 2.01 N
- F_f (maximum possible value) = $\mu N = 0.1 \times 17.3 = 1.73 \text{ N}$

block will move

6.29  Plate A is 50 cm in diameter and has a sharp-edged orifice at its center. A water jet (at 10°C) strikes the plate concentrically with a speed of 90 m/s. What external force is needed to hold the plate in place if the jet issuing from the orifice also has a speed of 90 m/s? The diameters of the jets are $D = 10$ cm and $d = 3.5$ cm.

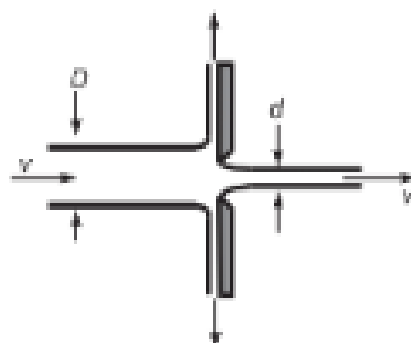


PROBLEM 6.29

6.29: PROBLEM DEFINITION

Situation:

A water jet strikes a plate with a sharp edged orifice at its center.
 $v = 90 \text{ m/s}$, $D = 10 \text{ cm}$, $d = 3.5 \text{ cm}$



Find:

Force required to hold plate stationary (N).

Assumptions:

Neglect gravity.

Properties:

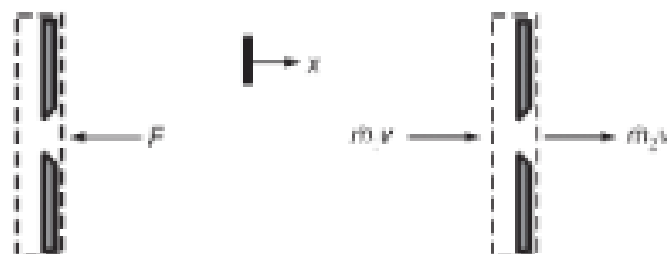
$\rho = 1000 \text{ kg/m}^3$

PLAN

Apply the momentum equation.

SOLUTION

Force and momentum diagrams (only x-direction vectors shown)

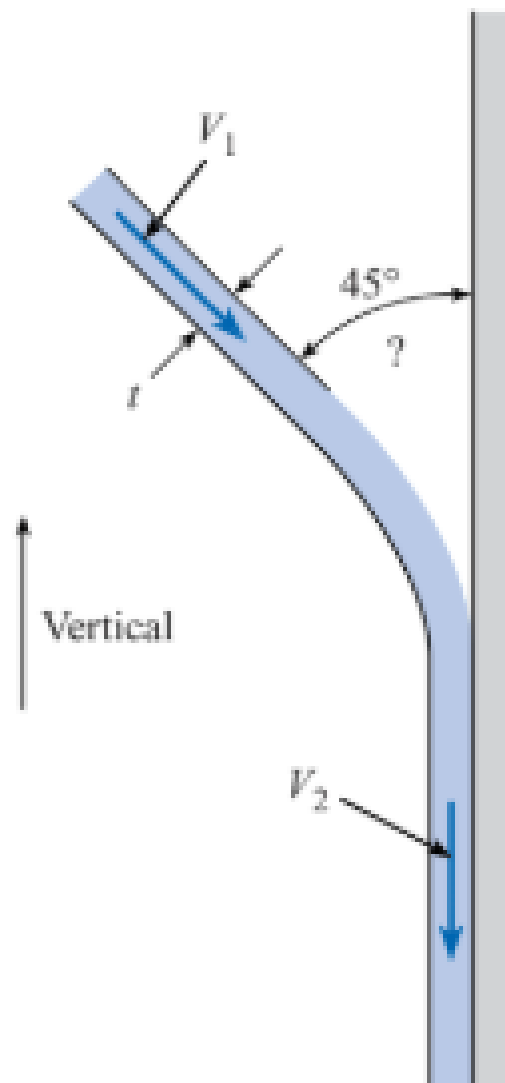


Momentum equation (x -direction)

$$\begin{aligned}\sum \mathbf{F} &= \sum_{cs} \dot{m}_o \mathbf{v}_o - \sum_{cs} \dot{m}_i \mathbf{v}_i \\ -F &= \dot{m}_2 v - \dot{m}_1 v \\ F &= \rho A_1 v^2 - \rho A_2 v^2 \\ &= \rho v^2 \left(\frac{\pi}{4} \right) (D^2 - d^2) \\ &= 1000 \text{ kg/m}^3 \times (90 \text{ m/s})^2 \times \frac{\pi}{4} \times ((0.10 \text{ m})^2 - (0.035 \text{ m})^2)\end{aligned}$$

$$F = 55.8 \text{ kN (to the left)}$$

6.30 A two-dimensional liquid jet impinges on a vertical wall. Assuming that the incoming jet speed is the same as the exiting jet speed ($V_1 = V_2$), derive an expression for the force per unit width of jet exerted on the wall. What form do you think the upper liquid surface will take next to the wall? Sketch the shape you think it will take, and explain your reasons for drawing it that way.



PROBLEM 6.30

6.30: PROBLEM DEFINITION

Situation:

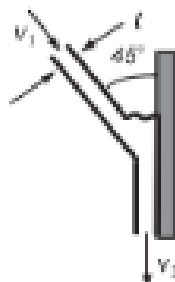
A 2D liquid jet impinges on a vertical wall.

$$v_1 = v_2 = v, \theta = 45^\circ.$$

Find:

Calculate the force acting on the wall.

Sketch and explain the shape of the liquid surface.



Assumptions:

Steady flow.

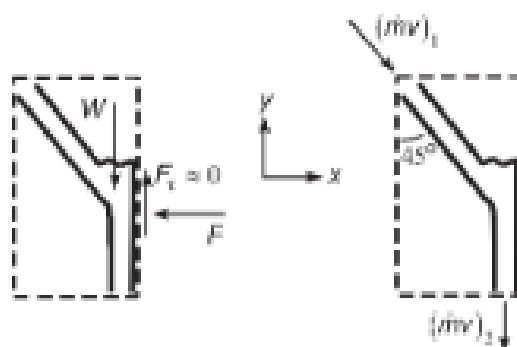
Force associated with shear stress is negligible.

PLAN

Apply the momentum equation.

SOLUTION

Let w = the width of the jet in the z -direction. Force and momentum diagrams



Momentum equation (x -direction)

$$\begin{aligned}\sum F_x &= \sum_{cs} \dot{m}_o v_{ox} - \sum_{cs} \dot{m}_i v_{ix} \\ -F &= -\dot{m} v_1 \sin 45^\circ \\ F &= \rho w v^2 \sin 45^\circ\end{aligned}$$

The force on that acts on the wall is in the opposite direction to force pictured on the force diagram, thus

$$F_{\text{on wall}} = \rho t v^2 \sin 45^\circ \text{ (acting to the right)}$$

y -direction

$$\begin{aligned}\sum F_y &= \sum_{cs} \dot{m}_o v_{oy} - \sum_{cs} \dot{m}_i v_{iy} \\ -W &= \dot{m}(-v) - \dot{m}(-v) \cos 45^\circ \\ W &= \dot{m}v(1 - \cos 45^\circ)\end{aligned}$$

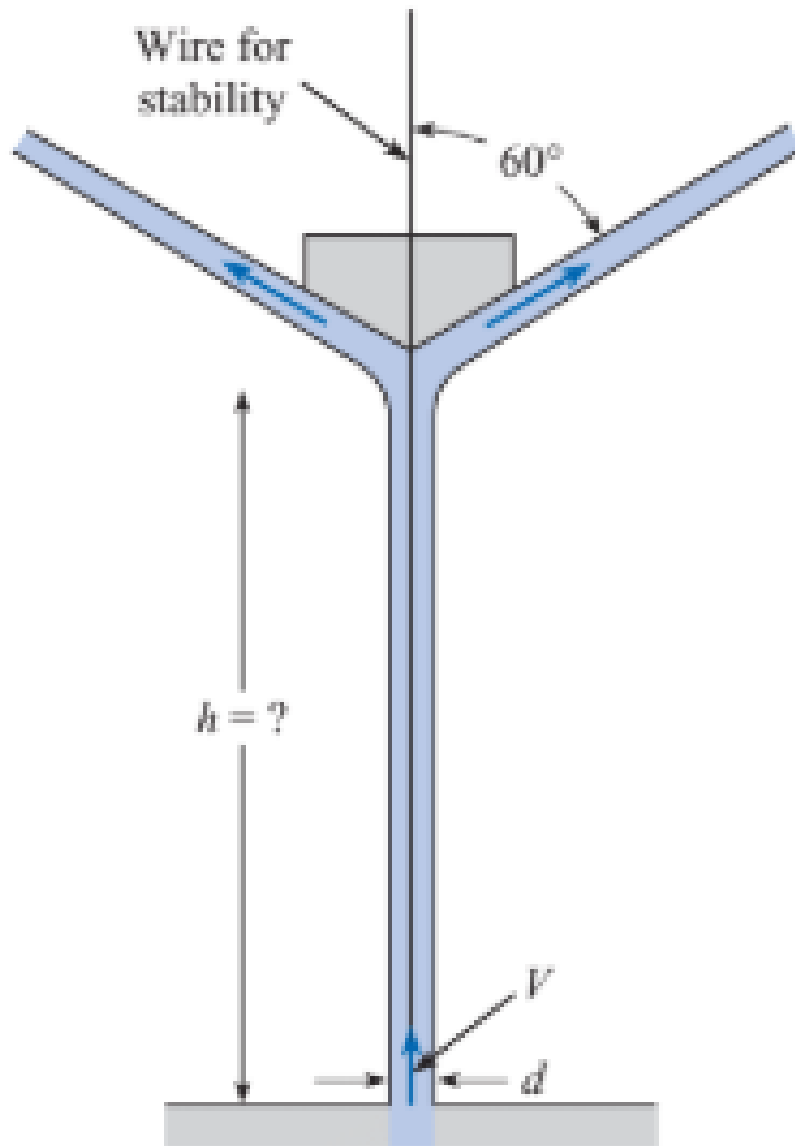
REVIEW

Thus, weight provides the force needed to increase y -momentum flow. This weight is produced by the fluid swirling up to form the shape shown in the above sketches.



6.31 A cone that is held stable by a wire is free to move in the vertical direction and has a jet of water (at 10°C) striking it from below. The cone weighs 30 N. The initial speed of the jet as it comes from the orifice is 15 m/s, and the initial jet

diameter is 2 cm. Find the height to which the cone will rise and remain stationary. *Note:* The wire is only for stability and should not enter into your calculations.



PROBLEM 6.31

6.31: PROBLEM DEFINITIONSituation:

A cone is supported by a vertical jet of water.

$$W = 30 \text{ N}, V_1 = 15 \text{ m/s.}$$

$$d_1 = 2 \text{ cm}, \theta = 60^\circ.$$

Find:

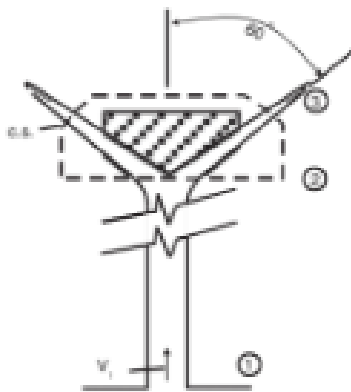
Height to which cone will rise (m).

Assumptions:

Speed of the fluid as it passes by the cone is constant ($V_2 = V_3$).

PLAN

Apply the Bernoulli equation and the momentum equation.

SOLUTION

Bernoulli equation

$$\begin{aligned} \frac{V_1^2}{2g} + 0 &= \frac{V_2^2}{2g} + h \\ V_2^2 &= (V_1)^2 - 2gh \\ V_2^2 &= 225 - 19.62h \end{aligned}$$

Momentum equation (y -direction). Select a control volume surrounding the cone.

$$\begin{aligned} \sum F_y &= \dot{m}_o v_{oy} - \dot{m}_i v_{iy} \\ -W &= \dot{m}(v_{3y} - v_2) \\ -30 \text{ N} &= 1000 \text{ kg/m}^3 \times 15 \text{ m/s} \times \pi \times (0.01 \text{ m})^2 (V_2 \sin 30^\circ - V_2) \end{aligned}$$

Solve for the V_2

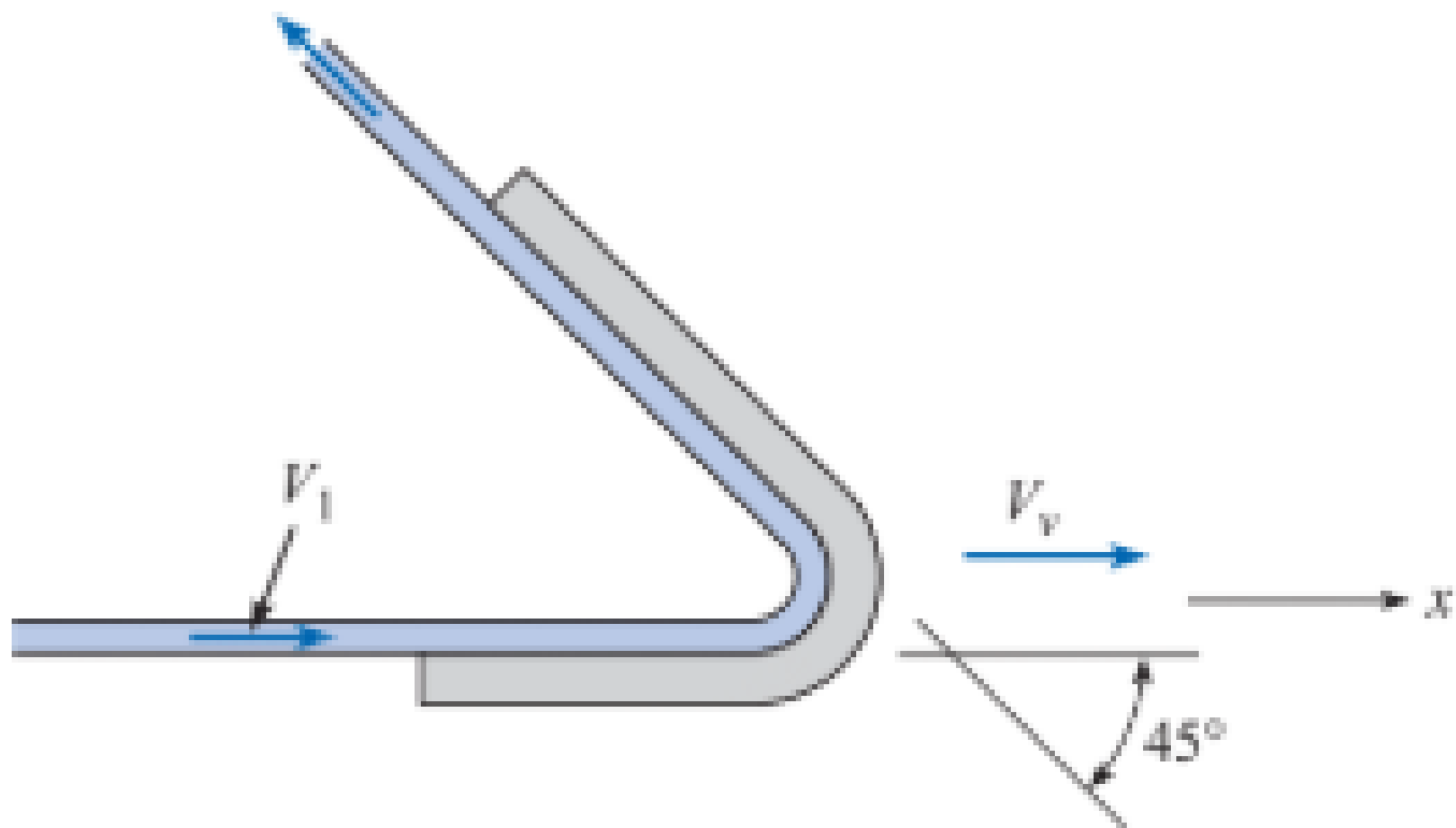
$$V_2 = 12.73 \text{ m/s}$$

Complete the Bernoulli equation calculation

$$V_2^2 = 225 - 19.62h$$
$$(12.73 \text{ m/s})^2 = 225 - 19.62h$$

$$\boxed{h = 3.21 \text{ m}}$$

6.32 A horizontal jet of water (at 10°C) that is 6 cm in diameter and has a velocity of 20 m/s is deflected by the vane as shown. If the vane is moving at a rate of 7 m/s in the x -direction, what components of force are exerted on the vane by the water in the x - and y -directions? Assume negligible friction between the water and the vane.



PROBLEM 6.32

6.32: PROBLEM DEFINITION

Situation:

A fluid jet strikes a vane that is moving at a speed.

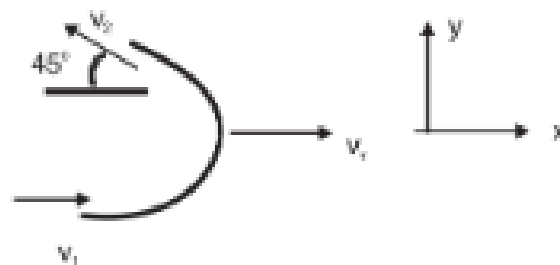
$$v_1 = 20 \text{ m/s}, v_v = 7 \text{ m/s}.$$

$$D_1 = 6 \text{ cm}.$$

Find:

Force of the water on the vane.

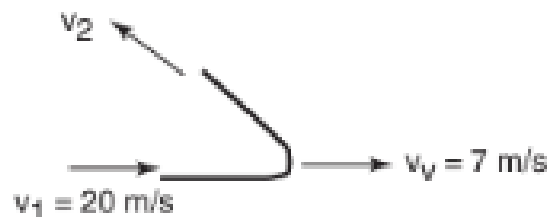
Sketch:



SOLUTION

Force and momentum diagrams

Select a control volume surrounding and moving with the vane. Select a reference frame attached to the moving vane.



Momentum equation (x -direction)

$$\begin{aligned}\sum F_x &= \dot{m}v_{2x} - \dot{m}v_{1x} \\ -F_x &= -\dot{m}v_2 \cos 45^\circ - \dot{m}v_1\end{aligned}$$

Momentum equation (y -direction)

$$\begin{aligned}\sum F_y &= \dot{m}v_{2y} - \dot{m}v_{1y} \\ F_y &= \dot{m}v_2 \sin 45^\circ\end{aligned}$$

Velocity analysis

- v_1 is relative to the reference frame = $(20 - 7) = 13 \text{ m/s}$.
- in the term $\dot{m} = \rho Av$ use v which is relative to the control surface. In this case $v = (20 - 7) = 13 \text{ m/s}$

- v_2 is relative to the reference frame $v_2 = v_1 = 13 \text{ m/s}$

Mass flow rate

$$\begin{aligned}\dot{m} &= \rho Av \\ &= (1,000 \text{ kg})(\pi/4 \times (0.06 \text{ m})^2)(13 \text{ m/s}) \\ &= 36.76 \text{ kg/s}\end{aligned}$$

Evaluate forces


$$\begin{aligned}F_x &= \dot{m}v_1(1 + \cos 45^\circ) \\ &= 36.76 \text{ kg/s} \times 13 \text{ m/s}(1 + \cos 45^\circ) = 816 \text{ N}\end{aligned}$$

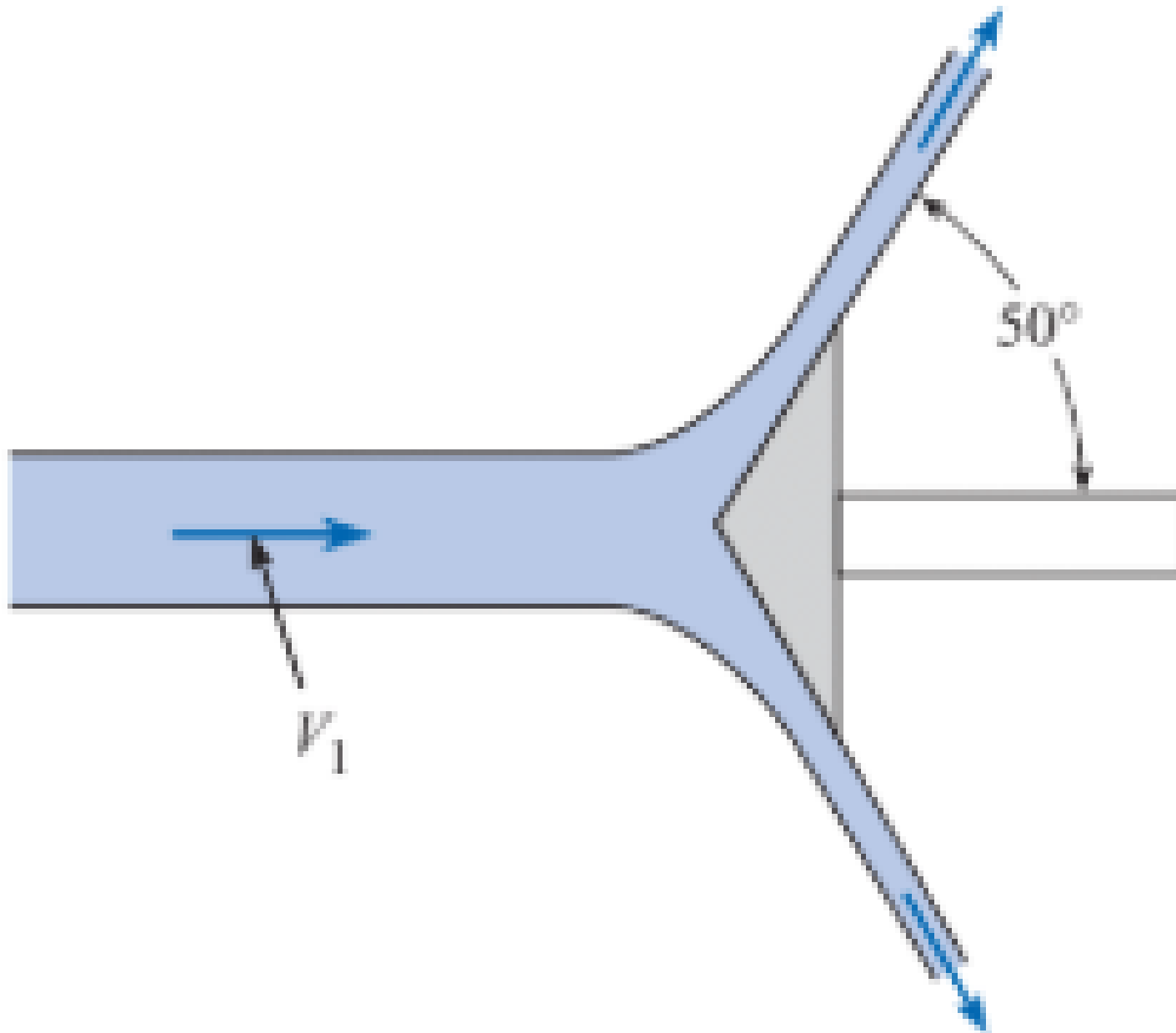
which is in the negative x -direction.

$$\begin{aligned}F_y &= \dot{m}v_2 \sin 45^\circ \\ &= 36.76 \text{ kg/s} \times 13 \text{ m/s} \sin 45^\circ = 338 \text{ N}\end{aligned}$$

The force of the water on the vane is the negative of the force of the vane on the water. Thus the force of the water on the vane is

$$\boxed{\mathbf{F} = (816\mathbf{i} - 338\mathbf{j}) \text{ N}}$$

6.35  The water ($\rho = 1000 \text{ kg/m}^3$) in this jet has a speed of 60 m/s to the right and is deflected by a cone that is moving to the left with a speed of 5 m/s. The diameter of the jet is 10 cm. Determine the external horizontal force needed to move the cone. Assume negligible friction between the water and the vane.



PROBLEMS 6.35, 6.36

6.35: PROBLEM DEFINITION

Situation:

A water jet is deflected by a moving cone.

Speed of the water jet is 60 m/s (to the right). Speed of the cone is 5 m/s (to the left). Diameter of the jet is $D = 10$ cm.

Angle of the cone is $\theta = 50^\circ$.

Find:

Calculate the external horizontal force needed to move the cone: F_x

Assumptions:

As the jet passes over the cone (a) assume the Bernoulli equation applies, and (b) neglect changes in elevation.

PLAN

Apply the momentum equation.

SOLUTION

Select a control volume surrounding the moving cone. Select a reference frame fixed to the cone. Section 1 is the inlet. Section 2 is the outlet.

Inlet velocity (relative to the reference frame and surface of the control volume).

$$v_1 = V_1 = (60 + 5) \text{ m/s} \\ 65 \text{ m/s}$$

Bernoulli equation. Pressure and elevation terms are zero, so

$$V_1 = V_2 = v_2 = 65 \text{ m/s}$$

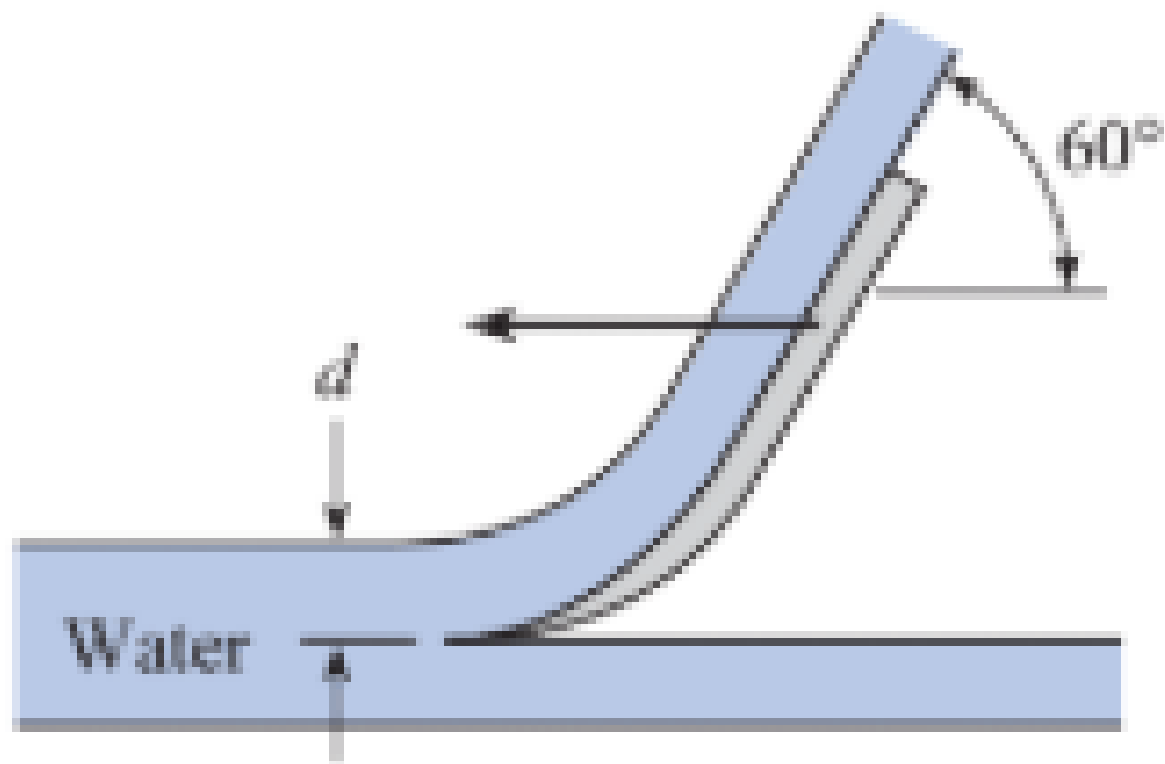
Momentum equation (x -direction)

$$\begin{aligned} F_x &= \dot{m}(v_{2x} - v_1) \\ &= \rho A_1 V_1 (v_2 \cos \theta - v_1) \\ &= \rho A_1 V_1^2 (\cos \theta - 1) \\ &= \left(1000 \frac{\text{kg}}{\text{m}^3} \right) \times \left(\frac{\pi \times (0.1 \text{ m})^2}{4} \right) \times (65 \text{ m/s})^2 (\cos 50^\circ - 1) \\ &= -11.85 \text{ kN} \end{aligned}$$

$$F_x = 11.85 \text{ kN (acting to the left)}$$



6.37 Assume that the scoop shown, which is 20 cm wide, is used as a braking device for studying deceleration effects, such as those on space vehicles. If the scoop is attached to a 1000 kg sled that is initially traveling horizontally at the rate of 100 m/s, what will be the initial deceleration of the sled? The scoop dips into the water 8 cm ($d = 8$ cm). ($T = 10^\circ\text{C}$.)



PROBLEM 6.37

6.37: PROBLEM DEFINITION

Situation:

A sled of mass $m_s = 1000$ kg is decelerated by placing a scoop of width $w = 20$ cm into water at a depth $d = 8$ cm.

Find:

Deceleration of the sled: a_s

SOLUTION

Select a moving control volume surrounding the scoop and sled. Select a stationary reference frame.

Momentum equation (x -direction)

$$0 = \frac{d}{dt}(m_s v_s) + \dot{m} v_{2x} - \dot{m} v_{1x}$$

Velocity analysis

$$\begin{aligned} v_{1x} &= 0 \\ V_1 &= 100 \text{ m/s} \\ V_2 &= 100 \text{ m/s} \\ \mathbf{v}_2 &= 100 \text{ m/s}[-\cos 60^\circ \mathbf{i} + \sin 60^\circ \mathbf{j}] + 100 \mathbf{i} \text{ m/s} \\ v_{2x} &= 50 \text{ m/s} \end{aligned}$$

The momentum equation simplifies to

$$0 = m_s a_s + \dot{m} v_{2x} \quad (1)$$

Flow rate

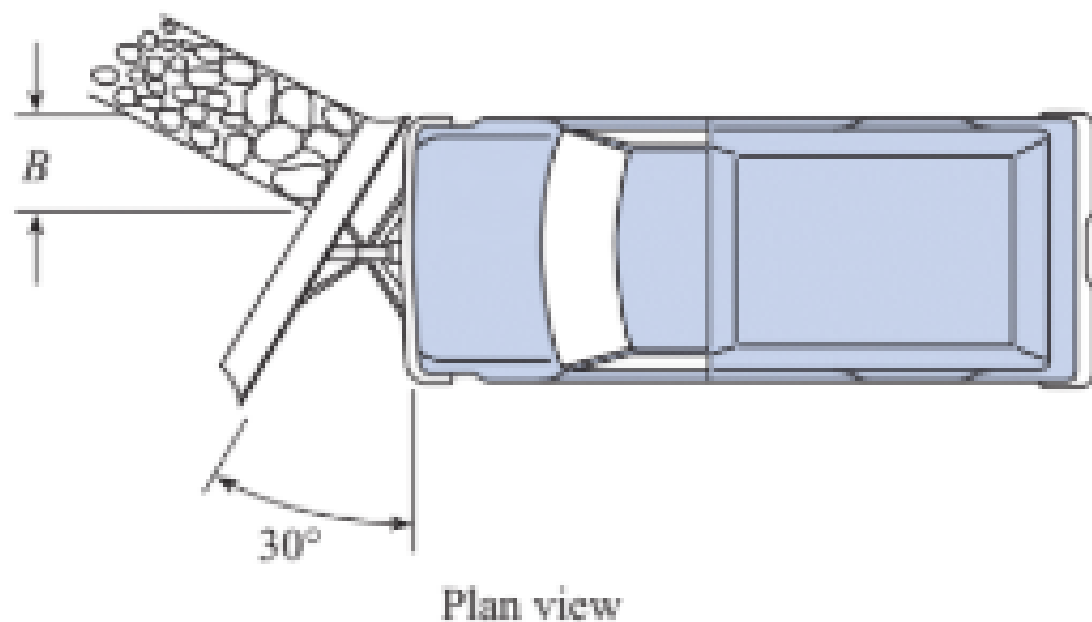
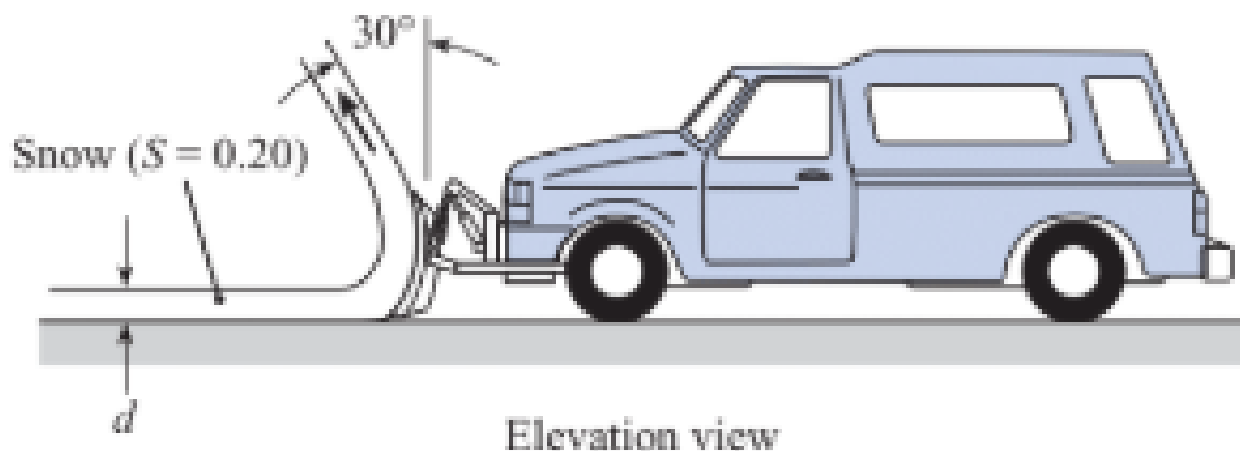
$$\begin{aligned} \dot{m} &= \rho A_1 V_1 \\ &= 1000 \text{ kg/m}^3 \times 0.2 \text{ m} \times 0.08 \text{ m} \times 100 \text{ m/s} \\ &= 1600 \text{ kg/s} \end{aligned}$$

From Eq. (1).

$$\begin{aligned} a_s &= -\frac{\dot{m} v_{2x}}{m_s} \\ &= \frac{(-1600 \text{ kg/s})(50 \text{ m/s})}{1000 \text{ kg}} \end{aligned}$$

$$\boxed{a_s = -80 \text{ m/s}^2}$$

6.38 This snowplow “cleans” a swath of snow that is 4 in. deep ($d = 4$ in.) and 2 ft wide ($B = 2$ ft). The snow leaves the blade in the direction indicated in the sketches. Neglecting friction between the snow and the blade, estimate the power required for just the snow removal if the speed of the snowplow is 40 ft/s.



PROBLEM 6.38

6.38: PROBLEM DEFINITIONSituation:

A snowplow is described in the problem statement.

Find:

Power required for snow removal: P

PLAN

Apply the momentum equation.

SOLUTION

Momentum equation (x -direction)

Select a control volume surrounding the snow-plow blade. Attach a reference frame to the moving blade. (Snow is 0.1 m deep)

$$\sum F_x = \rho Q(v_{2x} - v_1)$$

Velocity analysis

$$\begin{aligned} V_1 &= v_1 = 12 \text{ m/s} \\ v_{2x} &= 12 \text{ m/s} \cos 60^\circ \cos 30^\circ \\ &= 5.2 \text{ m/s} \end{aligned}$$


Calculations

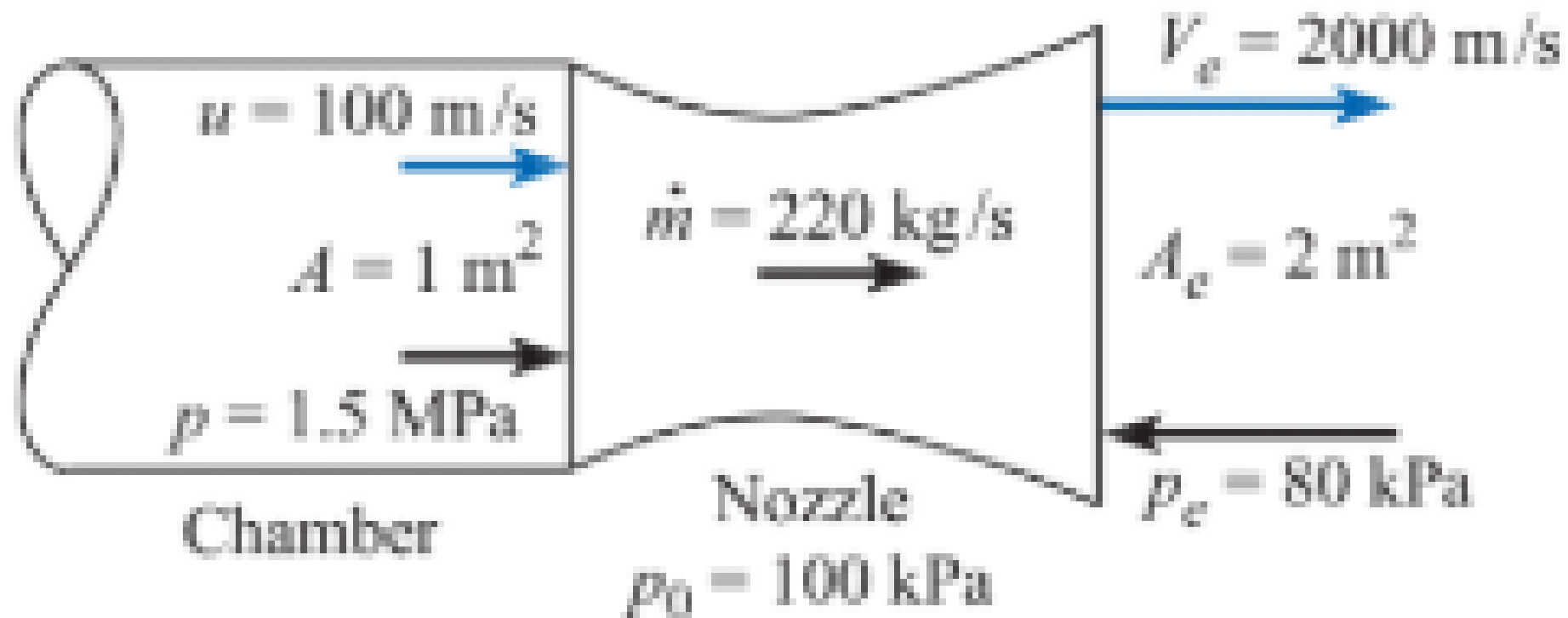
$$\begin{aligned} \sum F_x &= \rho V d W S (v_{2x} - v_1) \\ &= 1000 \text{ kg/m}^3 \times 0.2 \times 12 \text{ m/s} \times 0.6 \text{ m} \times 0.1 \text{ m} (5.2 \text{ m/s} - 12 \text{ m/s}) \\ &= 2477 \text{ N} \end{aligned}$$

Power

$$\begin{aligned} P &= FV \\ &= 2477 \text{ N} \times 12 \text{ m/s} \\ &= 29,724 \text{ N-m/s} \end{aligned}$$

$$\boxed{P = 29,724 \text{ W}}$$

6.49  A rocket-nozzle designer is concerned about the force required to hold the nozzle section on the body of a rocket. The nozzle section is shaped as shown in the figure. The pressure and velocity at the entrance to the nozzle are 1.5 MPa and 100 m/s. The exit pressure and velocity are 80 kPa and 2000 m/s. The mass flow through the nozzle is 220 kg/s. The atmospheric pressure is 100 kPa. The rocket is not accelerating. Calculate the force on the nozzle-chamber connection. *Note:* The given pressures are absolute.



PROBLEM 6.49

6.49: PROBLEM DEFINITION**Situation:**

A rocket nozzle is connected to a combustion chamber.

Mass flow: $\dot{m} = 220 \text{ kg/s}$. Ambient pressure: $p_o = 100 \text{ kPa}$.

Nozzle inlet conditions: $A_1 = 1 \text{ m}^2$, $u_1 = 100 \text{ m/s}$, $p_1 = 1.5 \text{ MPa-abs}$.

Nozzle exit condition? $A_2 = 2 \text{ m}^2$, $u_2 = 2000 \text{ m/s}$, $p_2 = 80 \text{ kPa-abs}$.

Assumptions:

The rocket is moving at a steady speed.

Find:

Force on the connection between the nozzle and the chamber.

PLAN

Apply the momentum equation to a control volume situated around the nozzle.

SOLUTION

Momentum equation (x -direction)

$$\begin{aligned}\sum F_x &= \dot{m}_o v_{ox} - \dot{m}_i v_{ix} \\ F + p_1 A_1 - p_2 A_2 &= \dot{m}(v_2 - v_1)\end{aligned}$$

where F is the force carried by the material that connects the rocket nozzle to the rocket chamber.

Calculations (note the use of gage pressures).

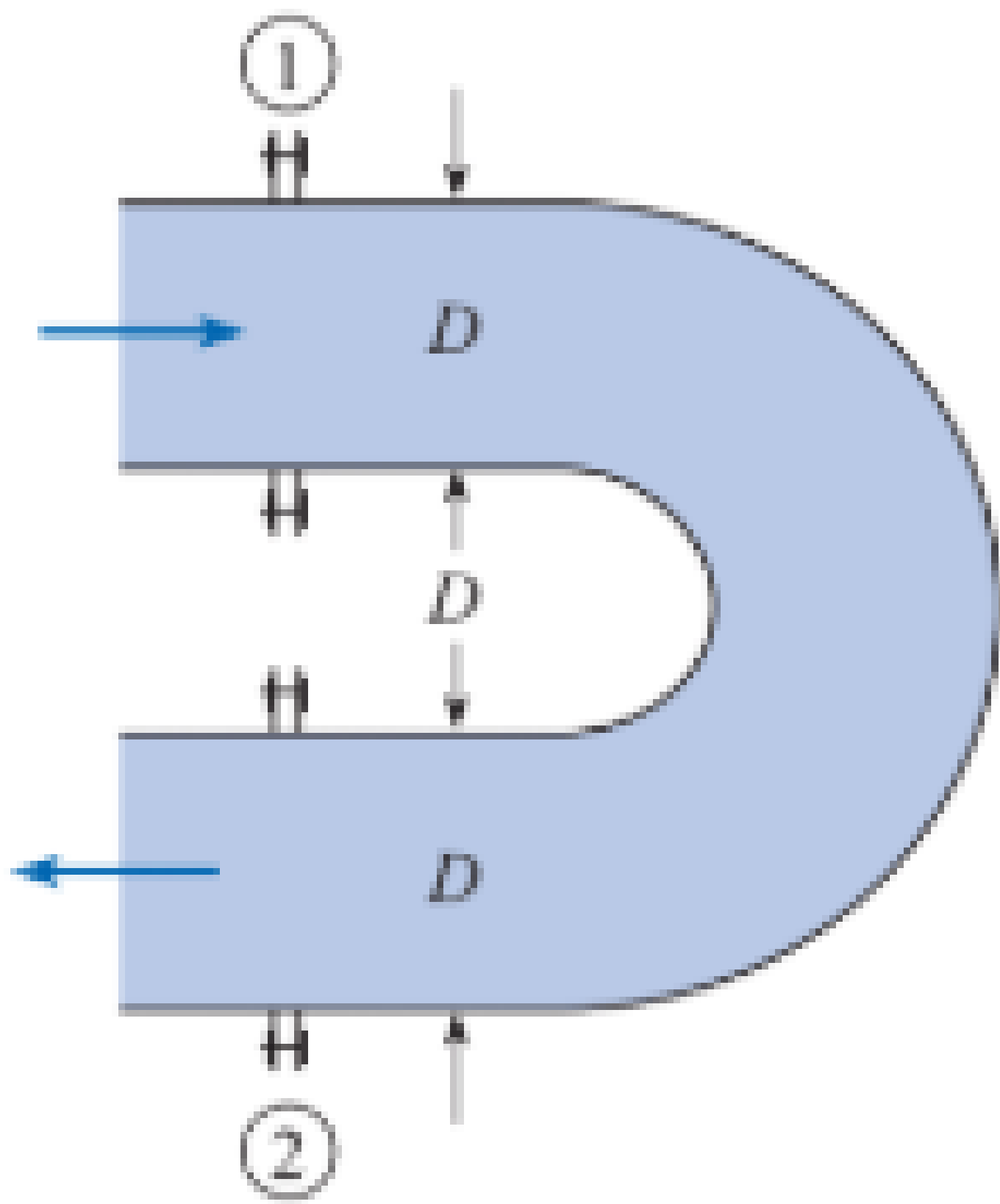
$$\begin{aligned}F &= \dot{m}(v_2 - v_1) + p_2 A_2 - p_1 A_1 \\ &= (220 \text{ kg/s})(2000 - 100) \text{ m/s} + (-20,000 \text{ N/m}^2)(2 \text{ m}^2) \\ &\quad - (1,400,000 \text{ N/m}^2)(1 \text{ m}^2) \\ &= -1.022 \times 10^6 \text{ N} \\ &= -1.022 \text{ MN}\end{aligned}$$

The force on the connection will be

$$\boxed{F = 1.02 \text{ MN}}$$

The material in the connection is in tension.

6.60 The pipe shown has a 180° horizontal bend in it as shown, and D is 20 cm. The discharge of water ($\rho = 1000 \text{ kg/m}^3$) in the pipe and bend is $0.35 \text{ m}^3/\text{s}$, and the pressure in the pipe and bend is 100 kPa gage. If the bend volume is 0.10 m^3 , and the bend itself weighs 400 N, what force must be applied at the flanges to hold the bend in place?



PROBLEMS 6.58, 6.59, 6.60, 6.61

6.60: PROBLEM DEFINITION

Situation:

Water flows through a 180° pipe bend—additional details are provided in the problem statement.

Find:

Force that acts on the flanges to hold the bend in place.

PLAN

Apply the continuity and momentum equations.

SOLUTION

Flow rate

$$\begin{aligned}v_1 &= \frac{Q}{A} \\&= \frac{4 \times 0.35 \text{ m}^3/\text{s}}{\pi \times (0.2 \text{ m})^2} \\&= 11.14 \text{ m/s}\end{aligned}$$

Continuity. Place a control volume around the pipe bend. Let section 2 be the exit and section 1 be the inlet

$$\begin{aligned}Q &= A_1 v_1 = A_2 v_2 \\ \text{thus } v_1 &= v_2\end{aligned}$$

Momentum equation (x -direction). Place a control volume around the pipe bend. Let section 2 be the exit and section 1 be the inlet.

$$\begin{aligned}\sum F_x &= \sum_{cs} \dot{m}_o v_{ox} - \sum_{cs} \dot{m}_i v_{ix} \\ 2pA + F_x &= \rho Q (-v_2) - \rho Q v_1 \\ F_x &= -2pA - 2\rho Q v\end{aligned}$$

Calculations

$$\begin{aligned}2pA &= (2)(100,000 \text{ Pa})\left(\frac{\pi}{4}\right)(0.2 \text{ m})^2 \\ &= 6283 \text{ N} \\ 2\rho Q v &= (2)(1000 \text{ kg/m}^3)(0.35 \text{ m}^3/\text{s})(11.14 \text{ m/s}) \\ &= 7798 \text{ N} \\ F_x &= -(2pA + 2\rho Q v) \\ &= -(6283 \text{ N} + 7798 \text{ N}) \\ &= -14.1 \text{ kN}\end{aligned}$$

Momentum equation (z -direction). There are no momentum flow terms so the momentum equation simplifies to

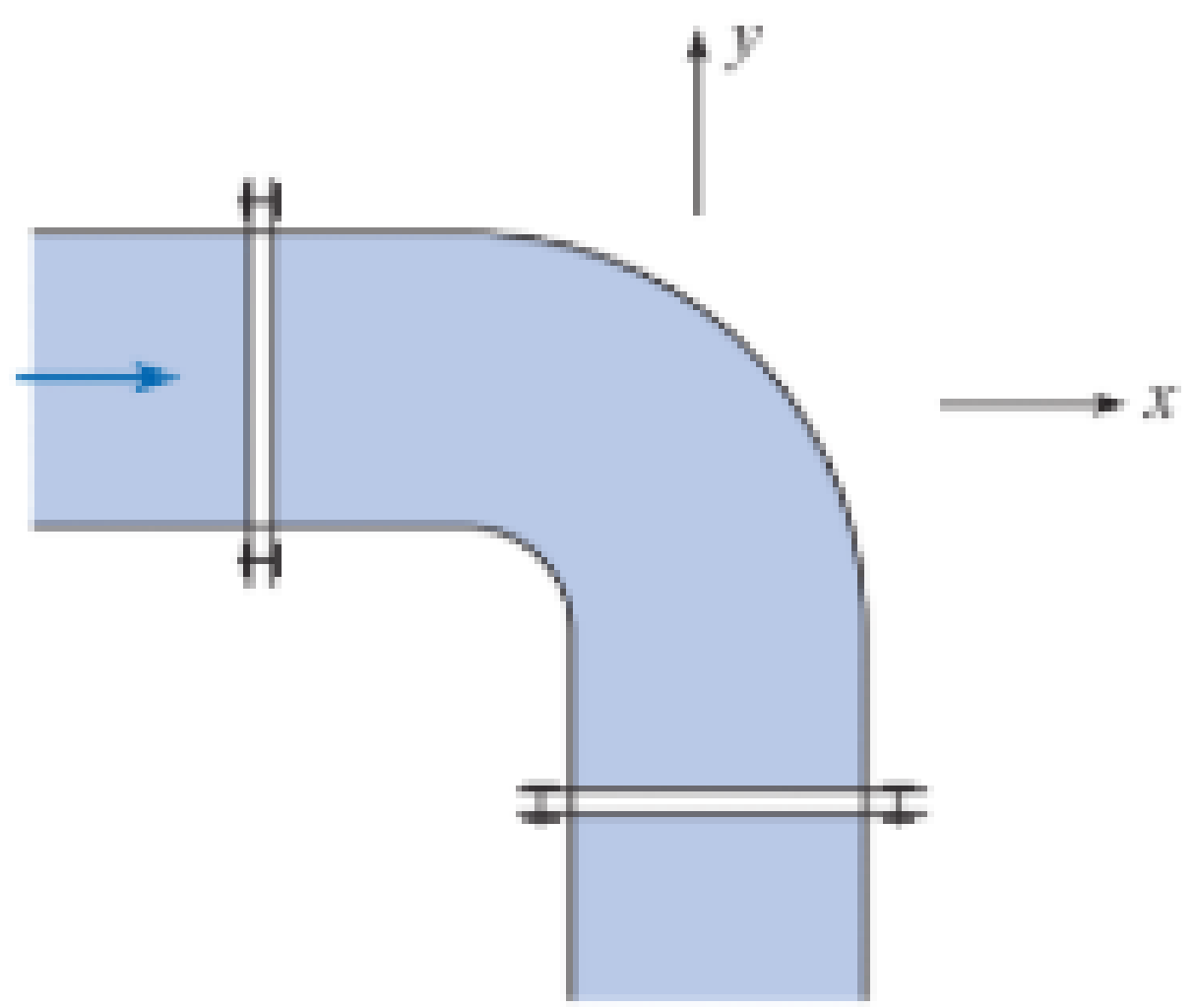
$$\begin{aligned} F_z &= W_{\text{bead}} + W_{\text{water}} \\ &= 400 \text{ N} + (0.1 \text{ m}^3)(9810 \text{ N/m}^3) \\ &= 1.381 \text{ kN} \end{aligned}$$

The force that acts on the flanges is

$$\mathbf{F} = (-14.1\mathbf{i} + 0\mathbf{j} + 1.38\mathbf{k}) \text{ kN}$$



6.63 The gage pressure throughout the horizontal 90° pipe bend is 300 kPa. If the pipe diameter is 1 m and the water (at 10°C) flow rate is $10\text{ m}^3/\text{s}$, what x -component of force must be applied to the bend to hold it in place against the water action?



PROBLEMS 6.62, 6.63

6.63: PROBLEM DEFINITION

Situation:

A 90° pipe bend is described in the problem statement.

Find:

x -component of force applied to bend to hold it in place: F_x

PLAN

Apply the momentum equation.

SOLUTION

Velocity calculation


$$v = \frac{Q}{A} = \frac{10 \text{ m}^3/\text{s}}{\pi/4 \times (1 \text{ m})^2} = 12.73 \text{ m/s}$$

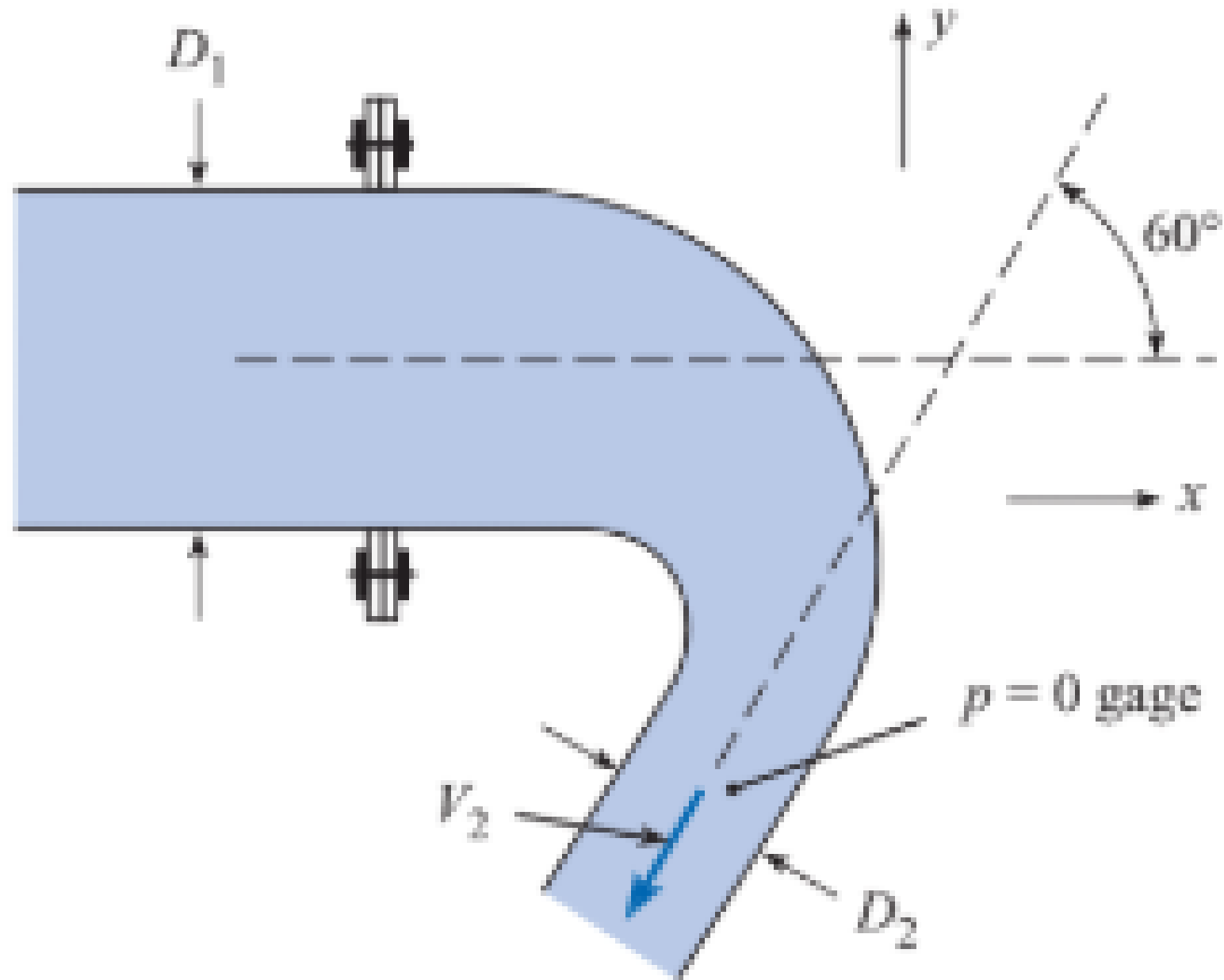
Momentum equation (x -direction)

$$\begin{aligned}\sum F_x &= \sum_{cs} \dot{m}v_{ox} - \sum_{cs} \dot{m}v_{ix} \\ pA + F_x &= \rho Q(0 - v)\end{aligned}$$

$$\begin{aligned}300,000 \text{ Pa} \times \pi \times (0.5 \text{ m})^2 + F_x &= 1000 \text{ kg/m}^3 \times 10 \text{ m}^3/\text{s} \times (0 - 12.73 \text{ m/s}) \\ F_x &= -362,919 \text{ N}\end{aligned}$$

$$\boxed{F_x = -363 \text{ kN}}$$

6.65  This bend discharges water ($\rho = 1000 \text{ kg/m}^3$) into the atmosphere. Determine the force components at the flange required to hold the bend in place. The bend lies in a horizontal plane. Assume viscous forces are negligible. The interior volume of the bend is 0.25 m^3 , $D_1 = 60 \text{ cm}$, $D_2 = 30 \text{ cm}$, and $V_2 = 10 \text{ m/s}$. The mass of the bend material is 250 kg .



PROBLEM 6.65

6.65: PROBLEM DEFINITION**Situation:**

Water flows through a 60° pipe bend and jets out to atmosphere—additional details are provided in the problem statement.

Find:

Magnitude and direction of external force components to hold bend in place.

PLAN

Apply the Bernoulli equation, then the momentum equation.

SOLUTION

Flow rate equation

$$\begin{aligned}\left(\frac{D_2}{D_1}\right)^2 v_2 &= \left(\frac{30 \text{ cm}}{60 \text{ cm}}\right)^2 10 \text{ m/s} = 2.5 \text{ m/s} \\ Q &= A_1 v_1 = \pi \times 0.3 \text{ m} \times 0.3 \text{ m} \times 2.5 \text{ m/s} = 0.707 \text{ m}^3/\text{s}\end{aligned}$$

Bernoulli equation

$$\begin{aligned}p_1 &= p_2 + \frac{\rho}{2}(v_2^2 - v_1^2) \\ &= 0 + \frac{1000 \text{ kg/m}^3}{2} \left(\left(\frac{10 \text{ m}}{\text{s}}\right)^2 - \left(\frac{2.5 \text{ m}}{\text{s}}\right)^2 \right) \\ &= 46,875 \text{ Pa gage}\end{aligned}$$

Momentum equation (x -direction)

$$\begin{aligned}F_x + p_1 A_1 &= \rho Q(-v_2 \cos 60^\circ - v_1) \\ F_x &= -46,875 \text{ Pa} \times \pi \times 0.3 \text{ m} \times 0.3 \text{ m} \\ &\quad + 1000 \text{ kg/m}^3 \times 0.707 \text{ m}^3/\text{s} \times (-10 \text{ m/s} \cos 60^\circ - 2.5 \text{ m/s}) \\ &= -18,560 \text{ N}\end{aligned}$$

y -direction

$$\begin{aligned}F_y &= \rho Q(-v_2 \sin 60^\circ - v_1) \\ F_y &= 1000 \text{ kg/m}^3 \times 0.707 \times (-10 \text{ m/s} \sin 60^\circ - 0) \\ &= -6120 \text{ N}\end{aligned}$$

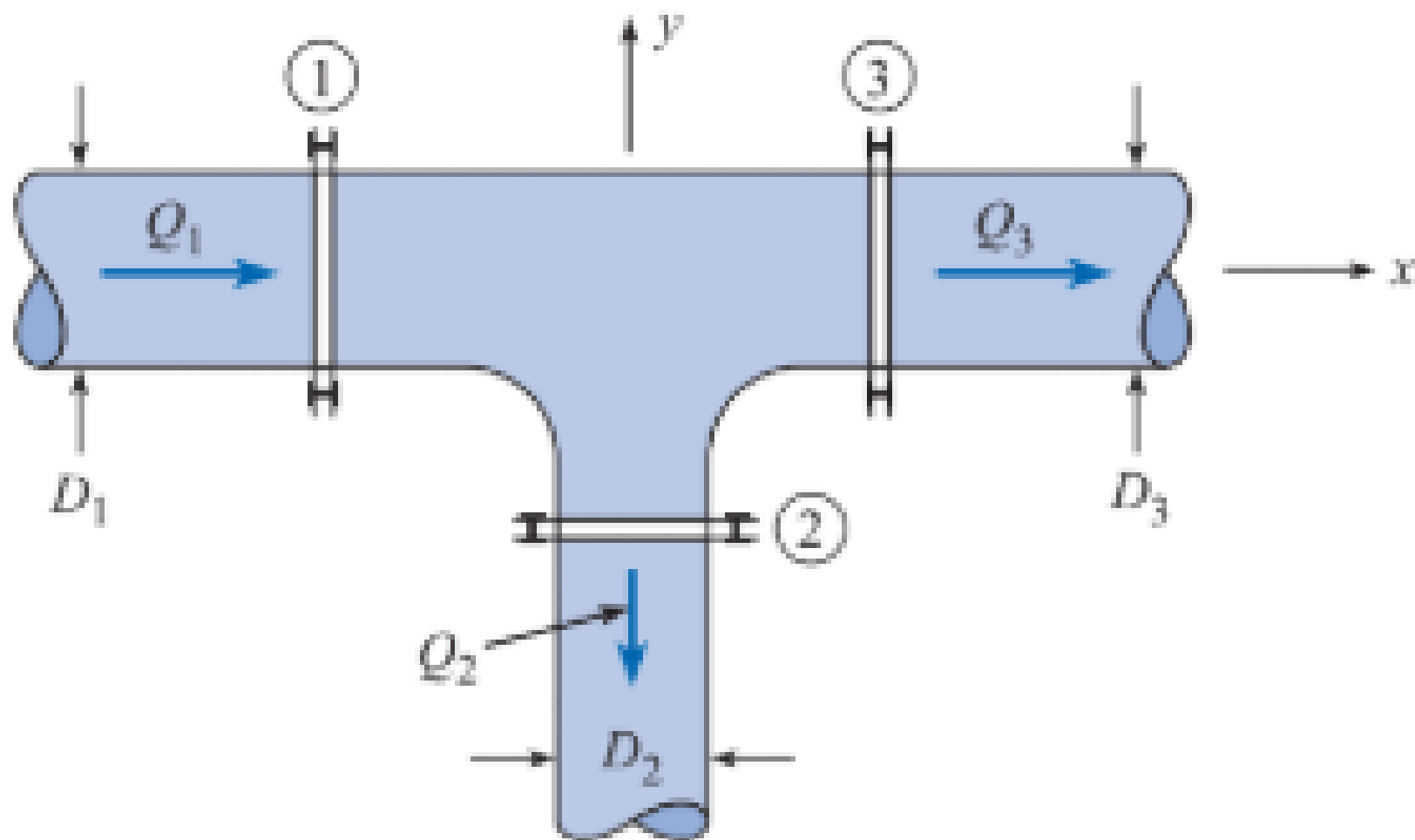
z -direction

$$\begin{aligned}F_z - W_{H_2O} - W_{\text{bend}} &= 0 \\ F_z &= (0.25 \text{ m}^3 \times 9,810 \text{ N/m}^3) + (250 \text{ kg} \times 9.81 \text{ m/s}^2) = 4,905 \text{ N}\end{aligned}$$

Net force

$$\mathbf{F} = (-18.6\mathbf{i} - 6.12\mathbf{j} + 4.91\mathbf{k}) \text{ kN}$$

6.75 For this horizontal T through which water ($\rho = 1000 \text{ kg/m}^3$) is flowing, the following data are given: $Q_1 = 0.25 \text{ m}^3/\text{s}$, $Q_2 = 0.10 \text{ m}^3/\text{s}$, $p_1 = 100 \text{ kPa}$, $p_2 = 70 \text{ kPa}$, $p_3 = 80 \text{ kPa}$, $D_1 = 15 \text{ cm}$, $D_2 = 7 \text{ cm}$, and $D_3 = 15 \text{ cm}$. For these conditions, what external force in the x - y plane (through the bolts or other supporting devices) is needed to hold the T in place?



PROBLEM 6.75

6.75: PROBLEM DEFINITION

Situation:

Water flows through a horizontal tee—additional details are provided in the problem statement.

Find:

Components of force (F_x, F_y) needed to hold the tee in place.

PLAN

Apply the momentum equation.

SOLUTION

Velocity calculations

$$\begin{aligned}V_1 &= \frac{0.25 \text{ m}^3/\text{s}}{(\pi \times 0.075 \text{ m} \times 0.075 \text{ m})} \\ &= 14.15 \text{ m/s}\end{aligned}$$

$$\begin{aligned}V_2 &= \frac{0.10 \text{ m}^3/\text{s}}{(\pi \times 0.035 \text{ m} \times 0.035 \text{ m})} \\ &= 25.98 \text{ m/s}\end{aligned}$$

$$\begin{aligned}V_3 &= \frac{(0.25 - 0.10) \text{ m}^3/\text{s}}{(\pi \times 0.075 \text{ m} \times 0.075 \text{ m})} \\ &= 8.49 \text{ m/s}\end{aligned}$$

Momentum equation (x -direction)

$$\begin{aligned}F_x + p_1 A_1 - p_3 A_3 &= \dot{m}_3 V_3 - \dot{m}_1 V_1 \\ F_x &= -p_1 A_1 + p_3 A_3 + \rho V_3 Q - \rho V_1 Q\end{aligned}$$

$$\begin{aligned}F_x &= -(100,000 \text{ Pa} \times \pi \times 0.075 \text{ m} \times 0.075 \text{ m}) + (80,000 \text{ Pa} \times \pi \times 0.075 \text{ m} \times 0.075 \text{ m}) \\ &\quad + (1000 \text{ kg/m}^3 \times 8.49 \text{ m/s} \times 0.15 \text{ m}^3/\text{s}) - (1000 \text{ kg/m}^3 \times 14.15 \text{ m/s} \times 0.25 \text{ m}^3/\text{s}) \\ F_x &= -2617 \text{ N}\end{aligned}$$

Momentum equation y -direction

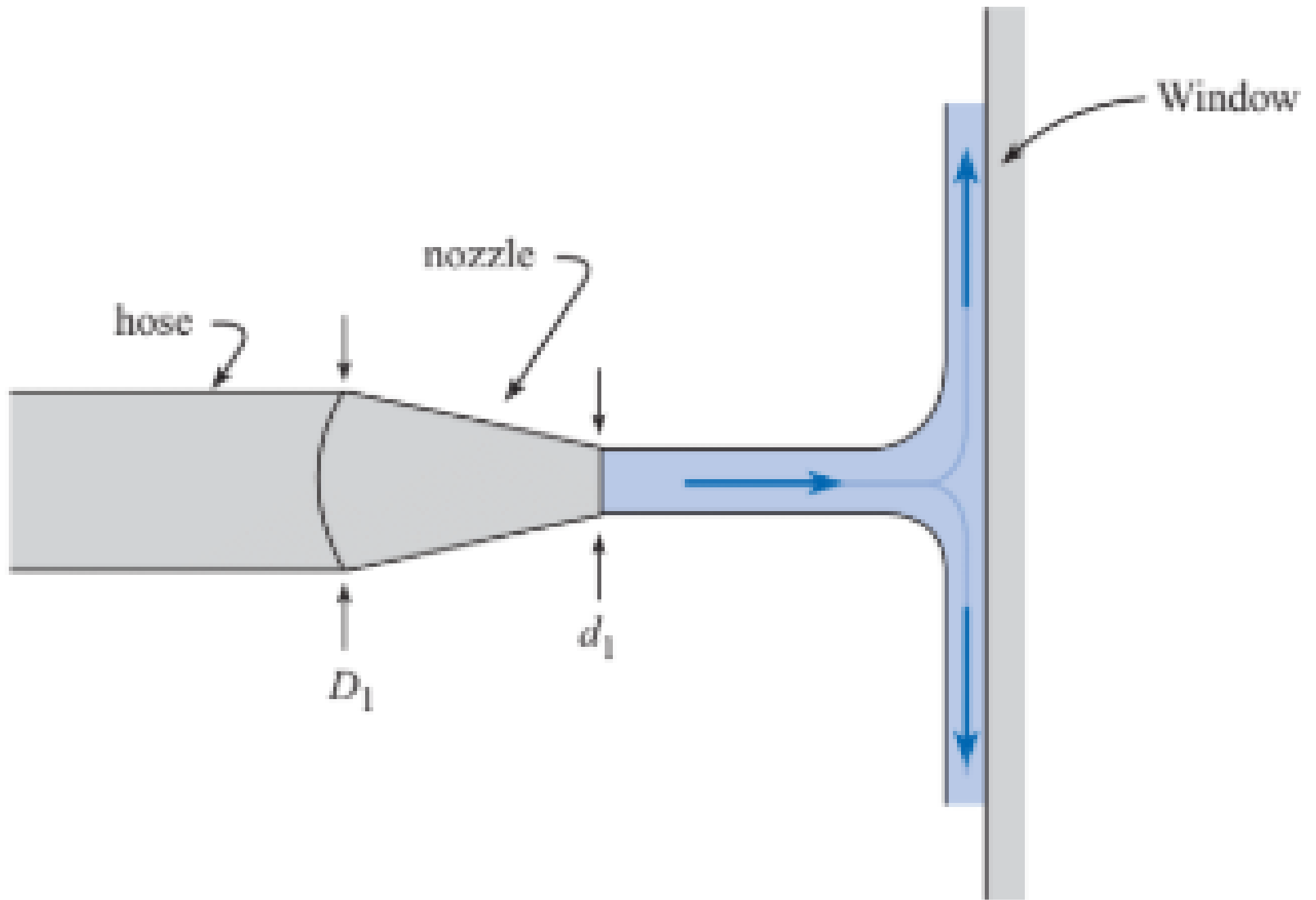
$$\begin{aligned}F_y + p_3 A_3 &= -\rho V_3 Q \\ F_y &= -\rho V_3 Q - p_3 A_3 \\ F_y &= -1000 \text{ kg/m}^3 \times 25.98 \text{ m/s} \times 0.10 \text{ m}^3/\text{s} - 70,000 \text{ Pa} \times \pi \times 0.035 \text{ m} \times 0.035 \text{ m} \\ &= -2867 \text{ N}\end{aligned}$$

Net force

$$\mathbf{F} = (-2.62\mathbf{i} - 2.87\mathbf{j}) \text{ kN}$$



6.76 Firehoses can break windows. A 0.2-m diameter (D_1) firehose is attached to a nozzle with a 0.1 m diameter (d_2) outlet. The free jet from the nozzle is deflected by 90° when it hits the window as shown. Find the force the window must withstand due to the impact of the jet when water flows through the firehose at a rate of $0.15 \text{ m}^3/\text{s}$.



PROBLEMS 6.76, 6.77

6.76: PROBLEM DEFINITION

Situation:

Risk of windows being broken by the force of jet from a firehose.

A 0.2-m (D_1) firehose attached to a nozzle with a 0.1-m (d_2) outlet.

The free jet from the nozzle is deflected by 90° when it hits the window.

Water from firehose flows at a rate of $0.15 \text{ m}^3/\text{s}$.

Find:

Force the window must withstand due to the impact of the jet.

PLAN

1. Assess the CV and set up the problem
2. Apply the momentum equation

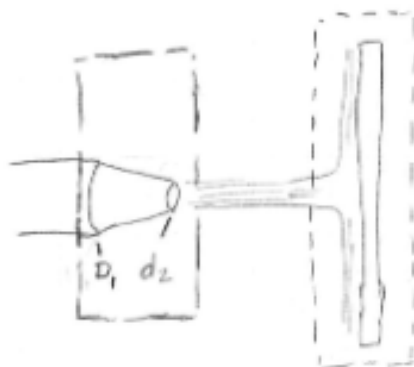
SOLUTION

Problem setup.

Choose coordinate system.

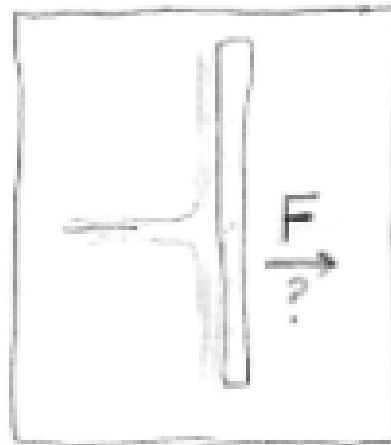
Make a sketch of the system with CV, and then another sketch of the FD and MD.

Draw reaction force to the right.

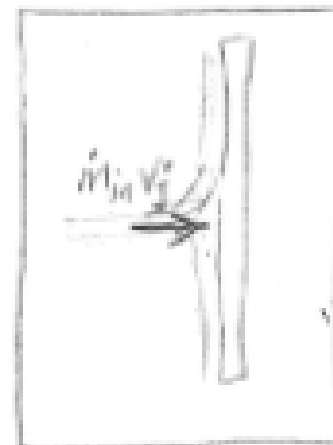


Consider: Where does one draw the CV? The big dashed line, or the small dashed line? Some problem-solvers may think that they need to include information about both D_1 and d_2 , and thus might use the CV described with the large dashes. However the only thing causing the force on the window is the force of the jet. Therefore the appropriate CV is the one drawn with the small dashed line, and the only velocity of interest is v_j .

Since the force of interest is in the x -direction, it is only necessary to assess the forces and momentum acting in the x -axis. The momentum and force diagrams are thus:



FD



MD

$$\sum F_x = \sum (\text{momentum out})_x - \sum (\text{momentum in})_x$$

$$F_x = 0 - \dot{m}_j v_j$$

$$F_x = -\rho Q_j v_j = \rho Q_j \left(\frac{Q_j}{A_j} \right); \text{ where } A = \frac{\pi d^2}{4} = \frac{\pi (0.1\text{m})^2}{4} = 0.0079 \text{ m}^2$$

$$F_x = - \left(\frac{1000 \text{ kg}}{\text{m}^3} \right) \left(\frac{0.15 \text{ m}^3}{\text{s}} \right)^2 \left(\frac{1}{0.0079 \text{ m}^2} \right)$$

$$F_x = -2.86 \text{ kN (acts to the left)}$$