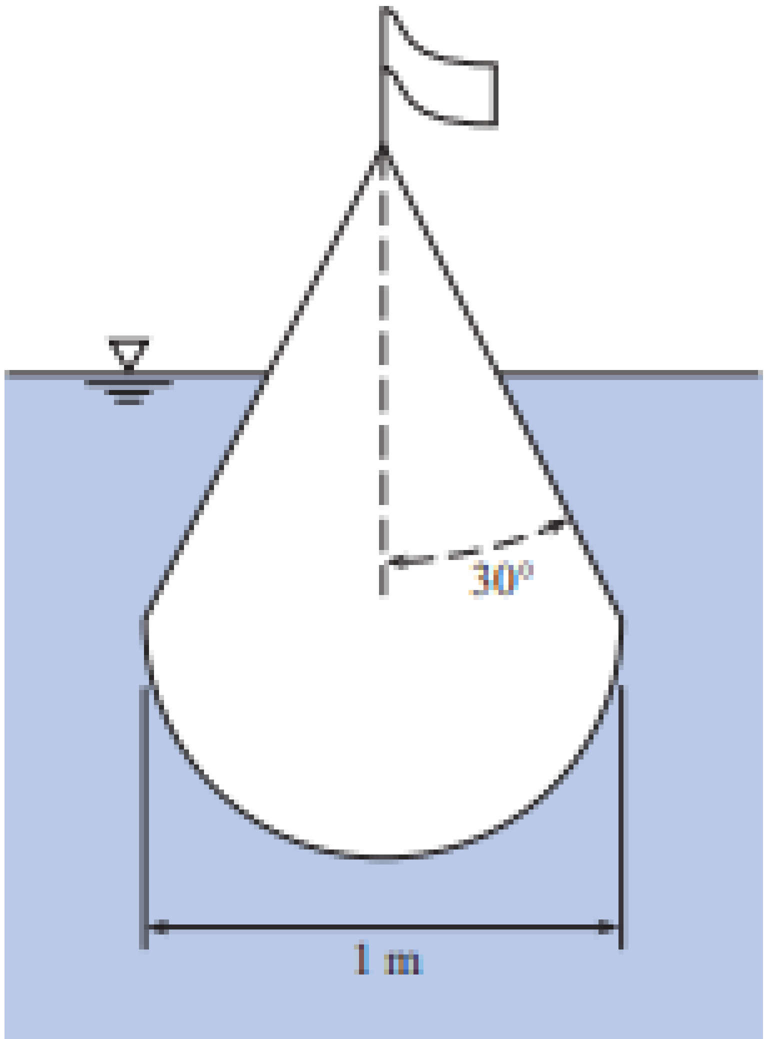
Mech Family

كن أنت التغيير

اللجنة الأكاديمية في قسم الهندسة الميكانيكية

3.99 A buoy is designed with a hemispherical bottom and conical top as shown in the figure. The diameter of the hemisphere is 1 m, and the half angle of the cone is 30°. The buoy has a mass of 460 kg. Find the location of the water line on the buoy floating in sea water ($\rho = 1010\ kg/m^3$).

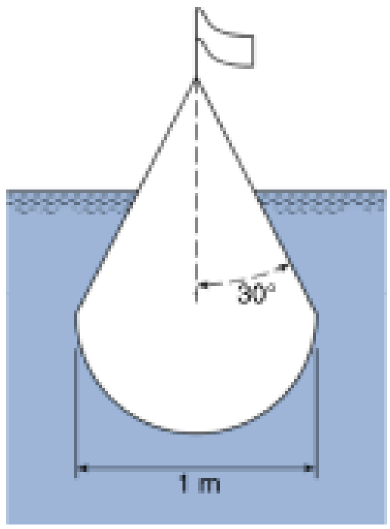


PROBLEM 3.99

3.97: PROBLEM DEFINITION

Situation:

A buoy has a spherical top and conical bottom.
$m = 460\,\mathrm{kg}$, $D = 1\,\mathrm{m}$, $\theta = 30\,°$.



Find:

Location of water level.

Properties:

$\rho = 1010\,\mathrm{kg/\,m^3}$.

SOLUTION

The buoyant force is equal to the weight.

$$F_B = W$$

The weight of the buoy is $9.81 \times 460 = 4512$ N.
The volume of the hemisphere at the bottom of the buoy is

$$\forall = \frac{1}{2}\frac{\pi}{6}D^3 = \frac{\pi}{12}1^3 = \frac{\pi}{12}\ \mathrm{m}^3$$

The buoyant force due to the hemisphere is

$$F_B = \frac{\pi}{12}(9.81\,\mathrm{m/\,s^2})(1010\,\mathrm{kg/\,m^3}) = 2594\ \mathrm{N}$$

Since this is less than the buoy weight, the water line must lie above the hemisphere. Let h is the distance from the top of the buoy. The volume of the cone which lies between the top of the hemisphere and the water line is

$$\begin{aligned}\forall &= \frac{\pi}{3}r_o^2 h_o - \frac{\pi}{3}r^2 h = \frac{\pi}{3}(0.5^2 \times 0.866 - h^3 \tan^2 30) \\ &= 0.2267 - 0.349h^3\end{aligned}$$

The additional volume needed to support the weight is

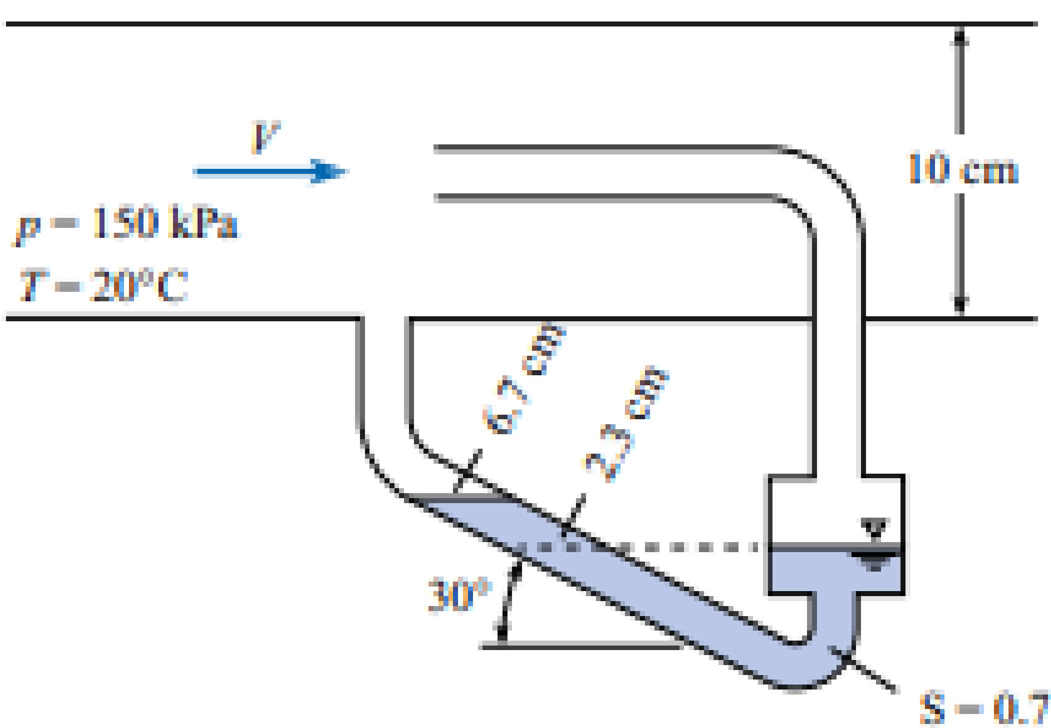
$$\forall = \frac{4512\,\mathrm{N} - 2594\,\mathrm{N}}{9.81\,\mathrm{m/\,s^2} \times 1010\,\mathrm{kg/\,m^3}} = 0.1936\ \mathrm{m}^3$$

Equating the two volumes and solving for h gives

$$h^3 = \frac{0.0331}{0.349} = 0.0948\ \mathrm{m}^3$$

$$\boxed{h = 0.456\ \mathrm{m}}$$

4.70 GO The apparatus shown in the figure is used to measure the velocity of air at the center of a duct having a 10 cm diameter. A tube mounted at the center of the duct has a 2 mm diameter and is attached to one leg of a slant-tube manometer. A pressure tap in the wall of the duct is connected to the other end of the slant-tube manometer. The well of the slant-tube manometer is sufficiently large that the elevation of the fluid in it does not change significantly when fluid moves up the leg of the manometer. The air in the duct is at a temperature of 20°C, and the pressure is 150 kPa. The manometer liquid has a specific gravity of 0.7, and the slope of the leg is 30°. When there is no flow in the duct, the liquid surface in the manometer lies at 2.3 cm on the slanted scale. When there is flow in the duct, the liquid moves up to 6.7 cm on the slanted scale. Find the velocity of the air in the duct. Assuming a uniform velocity profile in the duct, calculate the rate of flow of the air.



PROBLEM 4.70

4.70: PROBLEM DEFINITION

Situation:

An apparatus is used to measure the air velocity in a duct. It is connected to a slant tube manometer with a 30° leg with the indicated deflection.

$D = 10\,\text{cm}$, $D_{stagn} = 2\,\text{mm}$

$\ell_1 = 6.7\,\text{cm}$, $\ell_2 = 2.3\,\text{cm}$.

Find:

Air velocity (m/ s).

Properties:

Table A.2: $R = 287\,\text{J/kg K}$.

$T = 20\,°\text{C}$, $p_{stagn} = 150\,\text{kPa}$, $S = 0.7$

PLAN

Apply the Bernoulli equation.

SOLUTION

The side tube samples the static pressure for the undisturbed flow and the central tube senses the stagnation pressure.

Bernoulli equation

$$\begin{aligned} p_0 + \frac{\rho V_0^2}{2} &= p_{\text{stagn.}} + 0 \\ \text{or } V_0 &= \sqrt{\frac{2}{\rho}(p_{\text{stagn.}} - p_0)} \end{aligned}$$

But

$$\begin{aligned} p_{\text{stagn.}} - p_0 &= (\ell_1 - \ell_2)\sin\theta(\gamma_m - \gamma_{air}) \\ \text{but } \gamma_m &\gg \gamma_{air} \\ p_{\text{stagn.}} - p_0 &= (0.067\,\text{m} - 0.023\,\text{m})\sin 30°\,(0.7)\left(9,810\,\text{N/m}^3\right) = 151.1\text{ Pa} \\ \rho &= \frac{p}{RT} = \frac{150,000\,\text{Pa}}{(287\,\text{J/kg K})(273+20)\,\text{K}} = 1.784\text{ kg/m}^3 \end{aligned}$$

Then

$$\begin{aligned} V_0 &= \sqrt{\frac{2}{1.784\,\text{kg/m}^3}(151.1\,\text{Pa})} \\ &\boxed{V_0 = 13.0\,\text{m/s}} \end{aligned}$$