Fluid mechanics

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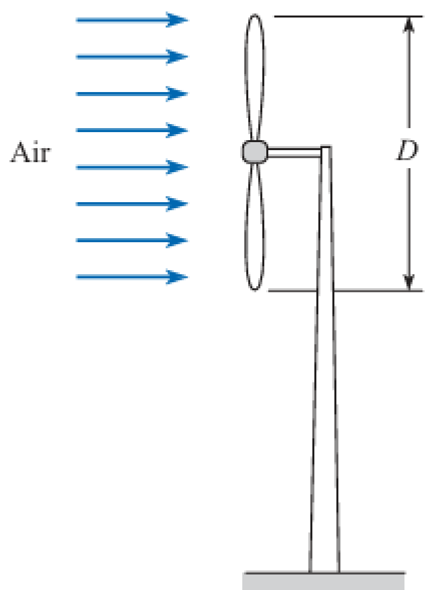
Suggested problems

Ch7

2025-2026 2nd semester

7.9 An engineer is considering the development of a small wind turbine ($D = 1.25$ m) for home applications. The design wind speed is 15 mph at $T = 10°C$ and $p = 0.9$ bar. The efficiency of the turbine is $\eta = 20\%$, meaning that 20% of the kinetic energy in the wind can be extracted. Estimate the power in watts that can be produced by the turbine. *Hint:* In a time interval Δt, the amount of mass that flows through the rotor is $\Delta m = \dot{m}\Delta t$, and the corresponding amount of kinetic energy in this flow is $(\Delta m V^2/2)$.



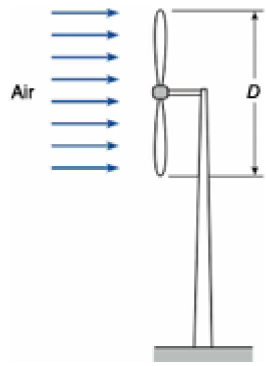
PROBLEM 7.9

Situation:

A small wind turbine is being developed.

$D = 1.25\,\text{m}$, $V = 24\ \text{km/h} = 6.7\,\text{m/s}$.

Turbine efficiency: $\eta = 20\%$.



Find:

Power (watts) produced by the turbine.

Properties:

Air ($10\,^\circ$C, $0.9\,\text{bar} = 90\,\text{kPa}$), $R = 287\,\text{J/kg}\cdot\text{K}$.

PLAN

Find the density of air using the idea gas law. Then, find the kinetic energy of the wind and use 20% of this value to find the power that is produced.

SOLUTION

Ideal gas law

$$\begin{aligned}\rho &= \frac{p}{RT} \\ &= \frac{90,000\,\text{Pa}}{(287\,\text{J/kg}\cdot\text{K})\,(10+273)\ \text{K}} \\ &= 1.108\,\text{kg/m}^3\end{aligned}$$

Kinetic energy of the wind

$$\begin{aligned}\text{Rate of KE} &= \frac{\text{Amount of kinetic energy}}{\text{Interval of time}} \\ &= \frac{\Delta m V^2/2}{\Delta t}\end{aligned}$$

where Δm is the mass of air that flows through a section of area $A = \pi D^2/4$ for each unit of time (Δt). Since the mass for each interval of time is mass flow rate:

$(\Delta m/\Delta t = \dot{m} = \rho A V)$

$$\begin{aligned}\text{Rate of KE} &= \frac{\dot{m}V^2}{2} \\ &= \frac{\rho A V^3}{2}\end{aligned}$$

$$\text{Rate of KE} = \frac{(1.108\,\text{kg/m}^3)\left(\pi\,(1.25\,\text{m})^2/4\right)(6.71\,\text{m/s})^3}{2}$$

$$\text{Rate of KE} = 205\,\text{W}$$

Since the output power is 20% of the input kinetic energy:

$$P = (0.2)\,(205\,\text{W})$$

$$\boxed{P = 41.0\,\text{W}}$$

REVIEW

The amount of energy in the wind is diffuse (i.e. spread out). For this situation, the wind turbine provides enough power for approximately one 40 watt light bulb.

7.17 An approximate equation for the velocity distribution in a pipe with turbulent flow is

$$\frac{V}{V_{max}} = \left(\frac{y}{r_0}\right)^n$$

where V_{max} is the centerline velocity, y is the distance from the wall of the pipe, r_0 is the radius of the pipe, and n is an exponent that depends on the Reynolds number and varies between 1/6 and 1/8 for most applications. Derive a formula for α as a function of n. What is α if $n = 1/7$?

Situation:

The velocity distribution in a pipe with turbulent flow is given by

$$\frac{V}{V_{\max}} = \left(\frac{y}{r_0}\right)^n$$

Find:

Derive a formula for α as a function of n.

Find α for $n = 1/7$.

SOLUTION

Flow rate equation

$$\begin{aligned}
\frac{V}{V_{\max}} &= \left(\frac{y}{r_0}\right)^n = \left(\frac{r_0 - r}{r_0}\right)^n = \left(1 - \frac{r}{r_0}\right)^n \\
Q &= \int_A V dA \\
&= \int_0^{r_0} V_{\max}\left(1 - \frac{r}{r_0}\right)^n 2\pi r dr \\
&= 2\pi V_{\max}\int_0^{r_0}\left(1 - \frac{r}{r_0}\right)^n r dr
\end{aligned}$$

Upon integration

$$Q = 2\pi V_{\max} r_0^2\left[\left(\frac{1}{n+1}\right) - \left(\frac{1}{n+2}\right)\right]$$

Then

$$\begin{aligned}
\bar{V} &= Q/A = 2V_{\max}\left[\left(\frac{1}{n+1}\right) - \left(\frac{1}{n+2}\right)\right] \\
&= \frac{2V_{\max}}{(n+1)(n+2)}
\end{aligned}$$

Kinetic energy correction factor

$$\alpha = \frac{1}{A}\int_0^{r_0}\left[\frac{V_{\max}\left(1 - \frac{r}{r_0}\right)^n}{\frac{2V_{\max}}{(n+1)(n+2)}}\right]^3 2\pi r dr$$

Upon integration one gets

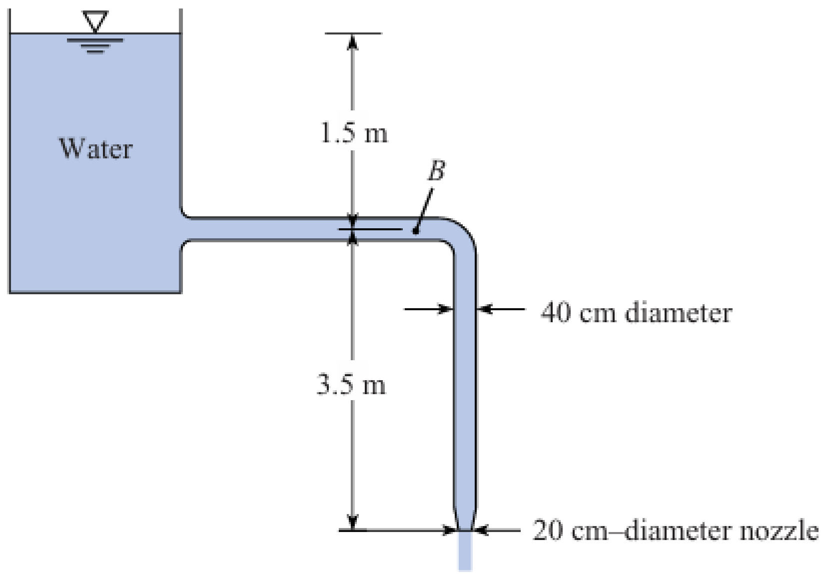
$$\boxed{\alpha = \frac{1}{4}\left[\frac{[(n+2)(n+1)]^3}{(3n+2)(3n+1)}\right]}$$

If $n = 1/7$, then

$$\alpha = \frac{1}{4}\left[\frac{\left[(\frac{1}{7}+2)(\frac{1}{7}+1)\right]^3}{\left(3\left(\frac{1}{7}\right)+2\right)\left(3\left(\frac{1}{7}\right)+1\right)}\right]$$

$$\boxed{\alpha = 1.06}$$

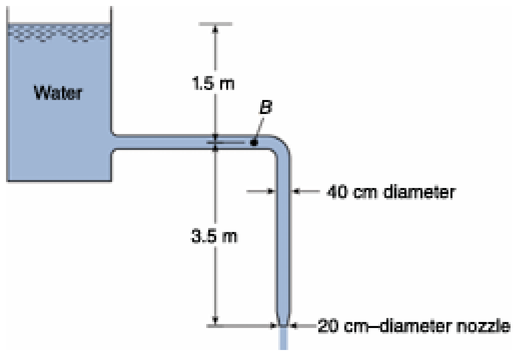
7.25 Determine the discharge in the pipe and the pressure at point *B*. Neglect head losses. Assume $\alpha = 1.0$ at all locations.



PROBLEM 7.25

Situation:

Water flowing from a tank into a pipe connected to a nozzle.
$\alpha = 1.0$, $D_B = 40\,\text{cm}$.
$D_0 = 20\,\text{cm}$, $z_0 = 0\,\text{m}$.
$z_B = 3.5\,\text{m}$, $z_r = 5\,\text{m}$.



Find:

(a) Discharge in pipe (m^3/s).
(b) Pressure at point B (kPa).

Assumptions:

$\gamma = 9810\,\text{N}/\,\text{m}^3$.

PLAN

1. Find velocity at nozzle by applying the energy equation.
2. Find discharge by applying $Q = A_o V_o$
3. Find the pressure by applying the energy equation.

SOLUTION

1. Energy equation (point 1 on reservoir surface, point 2 at outlet)

$$\begin{aligned} \frac{p_{\text{reser.}}}{\gamma} + \frac{V_r^2}{2g} + z_r &= \frac{p_{\text{outlet}}}{\gamma} + \frac{V_0^2}{2g} + z_0 \\ 0 + 0 + 5 &= 0 + \frac{V_0^2}{2g} \\ V_0 &= 9.90 \text{ m/s} \end{aligned}$$

2. Flow rate equation

$$\begin{aligned} Q &= V_0 A_0 \\ &= 9.90\,\text{m}/\,\text{s} \times \left(\frac{\pi}{4}\right) \times (0.20\,\text{m})^2 \end{aligned}$$

$$\boxed{Q = 0.311 \text{ m}^3/\text{s}}$$

3. Energy equation (point 1 on reservoir surface, point 2 at B)

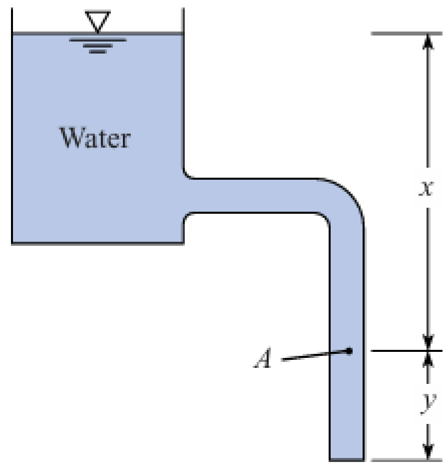
$$0 + 0 + 5 = \frac{p_B}{\gamma} + \frac{V_B^2}{2g} + 3.5$$

where

$$\begin{aligned} V_B &= \frac{Q}{V_B} = \frac{0.311\,\text{m}^3/\,\text{s}}{(\pi/4) \times (0.4\,\text{m})^2} = 2.48 \text{ m/s} \\ \frac{V_B^2}{2g} &= 0.312\,\text{m} \end{aligned}$$

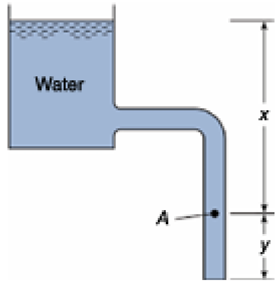
$$\begin{aligned} \frac{p_B}{\gamma} - 5\,\text{m} - 3.5\,\text{m} &= 0.312\,\text{m} \\ & \boxed{p_B = 86.4 \text{ kPa}} \end{aligned}$$

7.27 A pipe drains a tank as shown. If $x = 6$ m, $y = 4$ m, and head losses are neglected, what is the pressure at point A and what is the velocity at the exit? Assume $\alpha = 1.0$ at all locations.



PROBLEMS 7.26, 7.27

Situation:
Water drains from a tank into a pipe.
$x = 6\,\text{m}$, $y = 4\,\text{m}$.



Find:
Pressure at point A (kPa).
Velocity at exit (m/s).

Assumptions:
$\alpha = 1$ for all locations.

PLAN

1. Find pressure at point A by applying the energy equation between point A and the pipe exit.
2. Find velocity at the exit by applying the energy equation between top of tank and the exit.

SOLUTION

1. Energy equation (section A to exit plane):

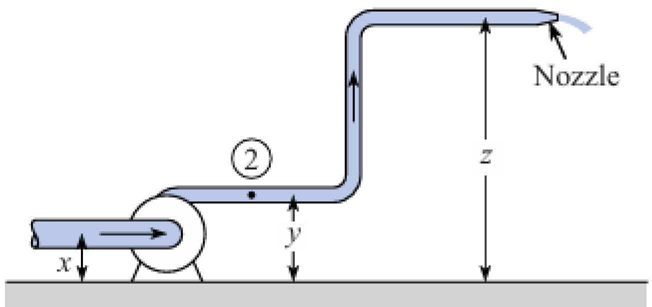
$$\frac{p_A}{\gamma} + z_A + \alpha_A \frac{V_A^2}{2g} + h_p = \frac{p_2}{\gamma} + z_2 + \alpha_2 \frac{V_2^2}{2g} + h_t + h_L$$

Term by term analysis: $V_A = V_2$ (continuity); $p_2 = 0$-gage; $(z_A - z_2) = y$; $h_p = 0$, $h_t = 0$, $h_L = 0$. Thus

$$\begin{aligned} p_A &= -\gamma y \\ p_A &= -\left(9810\,\text{N/m}^3\right)(4\,\text{m}) \end{aligned}$$

$$\boxed{p_A = -39.2\,\text{kPa}}$$

7.29 WILEY PLUS For this diagram of an industrial pressure washer system, $x = 1$ ft, $y = 3$ ft, $z = 10$ ft, $Q = 3.5\ \text{ft}^3/\text{s}$, and the hose diameter is 4 in. Assuming a head loss of 1 ft is derived over the distance from point 2 to the jet, what is the pressure at point 2 if the jet from the nozzle is 1-in in diameter? Assume $\alpha = 1.0$ throughout.



PROBLEMS 7.28, 7.29

Situation: Industrial pressure washer system, hose diameter is 4 in. Q=0.1m3/s x =0.3m,y =0.9m,z =3m Assume a head loss of 0.3 m is derived over the distance from point 2 to the jet. Jet from the nozzle is 2.5 cm in diameter. Assume α = 1.0 throughout

Situation:

Industrial pressure washer system, hose diameter is 4 in.

Q = 0.1 m^3/s

$x = 0.3$ m, $y = 0.9$ m, $z = 3$ m

Assume a head loss of 0.3 m is derived over the distance from point 2 to the jet.

Jet from the nozzle is 2.5 cm in diameter.

Assume $\alpha = 1.0$ throughout.

Find:

Pressure at point 2.

PLAN

Use flow rate equation and energy equation.

SOLUTION

Flow rate equation to find V_n (velocity at nozzle)

$$\begin{aligned} V_n &= \frac{Q}{A_n} = \frac{0.1\,\mathrm{m^3/\,s}}{(\pi/4)\times(0.025\,\mathrm{m})^2} = 204\,\mathrm{m/s} \\ \frac{V_n^2}{2g} &= 2121\,\mathrm{m} \end{aligned}$$

Flow rate equation to find V_2

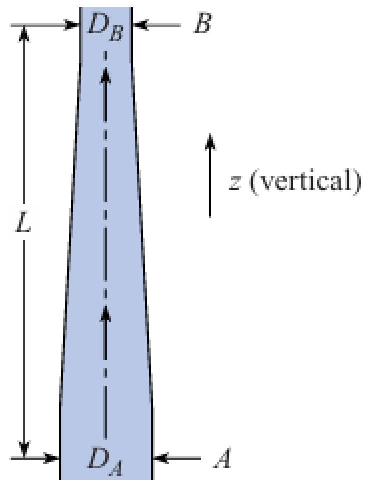
$$\begin{aligned} V_2 &= \frac{Q}{A_2} = \frac{0.1\,\mathrm{m^3/\,s}}{(\pi/4)\times(0.1\,\mathrm{m})^2} = 12.7\,\mathrm{m/s} \\ \frac{V_2^2}{2g} &= 8.22\,\mathrm{m} \end{aligned}$$

Energy equation

$$\begin{aligned} \frac{p_2}{\gamma} + 8.22\,\mathrm{m} + 0.9\,\mathrm{m} &= 0\,\mathrm{m} + 2121\,\mathrm{m} + 3\,\mathrm{m} + 0.3\,\mathrm{m} \\ \frac{p_2}{\gamma} &= 2115.2\,\mathrm{m} \\ p_2 &= 2115.2\,\mathrm{m} \times 9810\,\mathrm{N/m^3} \end{aligned}$$

$$\boxed{p_2 = 20.75\ \mathrm{MPa}}$$

7.30 WILEY PLUS For this refinery pipe, $D_A = 20$ cm, $D_B = 14$ cm, and $L = 1$ m. If crude oil (S = 0.90) is flowing at a rate of 0.05 m^3/s, determine the difference in pressure between sections A and B. Neglect head losses.



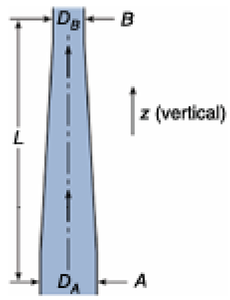
PROBLEM 7.30

Situation:

Oil moves through a narrowing section of pipe.

$D_A = 20\,\text{cm}$, $D_B = 14\,\text{cm}$.

$L = 1\,\text{m}$, $Q = 0.05\,\text{m}^3/\text{s}$.



Find:

Pressure difference between A and B.

Properties:

$S = 0.90$.

SOLUTION

Flow rate equation

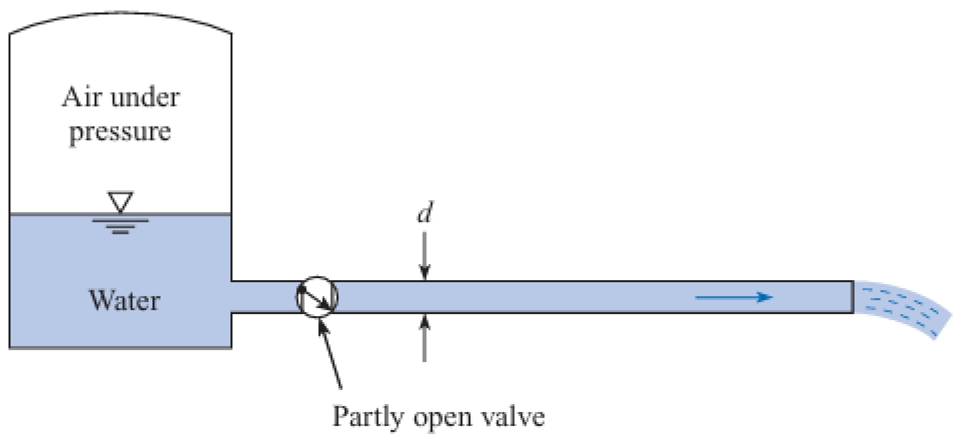
$$\begin{aligned} V_A &= \frac{Q}{A_1} \\ &= 1.59\,\text{m/s} \\ V_B &= \left(\frac{20}{14}\right)^2 \times V_A \\ &= 3.24\,\text{m/s} \end{aligned}$$

Energy equation

$$\begin{aligned} p_A - p_B &= 1\gamma + (\rho/2)(V_B^2 - V_A^2) \\ p_A - p_B &= (1)\left(9810\,\text{N/m}^3\right)(0.9) + \left(\frac{900\,\text{kg/m}^3}{2}\right)\left((3.25\,\text{m/s})^2 - (1.59\,\text{m/s})^2\right) \end{aligned}$$

$$\boxed{p_A - p_B = 12.4\,\text{kPa}}$$

7.34 In the figure shown, suppose that the reservoir is open to the atmosphere at the top. The valve is used to control the flow rate from the reservoir. The head loss across the valve is given as $h_L = 4V^2/2g$, where V is the velocity in the pipe. The cross-sectional area of the pipe is 8 cm^2. The head loss due to friction in the pipe is negligible. The elevation of the water level in the reservoir above the pipe outlet is 9 m. Find the discharge in the pipe. Assume $\alpha = 1.0$ at all locations.



PROBLEMS 7.32, 7.33, 7.34

Situation:

Water flows from an open tank, through a valve and out a pipe.

$A = 8\,\text{cm}^2$, $\Delta z = 9\,\text{m}$.

$p_1 = p_2 = 0\,\text{kPa}$.

$h_L = 4\frac{V^2}{2g}$.

Find:

Discharge in pipe.

Assumptions:

$\alpha = 1$.

PLAN

Apply the energy equation from the water surface in the reservoir (pt. 1) to the outlet end of the pipe (pt. 2).

SOLUTION

Energy equation:

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + h_L$$

Term by term analysis:

$$\begin{aligned} p_1 &= 0; \quad p_2 = 0 \\ z_2 &= 0; \quad V_1 \simeq 0 \end{aligned}$$

The energy equation becomes.

$$\begin{aligned} z_1 &= \frac{V_2^2}{2g} + h_L \\ 9\,\text{m} &= \frac{V_2^2}{2g} + 4\frac{V_2^2}{2g} = 5\frac{V_2^2}{2g} \\ V_2^2 &= \left(\frac{2g}{5}\right)(\Delta z) \\ V_2 &= \sqrt{\left(\frac{2 \times 9.81\,\text{m/s}^2}{5}\right)(9\,\text{m})} \\ V_2 &= 5.943\ \text{m/s} \end{aligned}$$

Flow rate equation

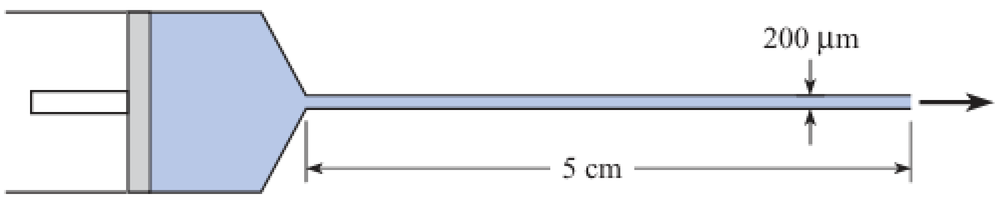
$$\begin{aligned} Q &= V_2 A_2 \\ &= (5.943\,\text{m/s})\left(8\,\text{cm}^2\right)\left(\frac{10^{-4}\,\text{m}^2}{\text{cm}^2}\right) \\ &= 5.0265 \times 10^{-3}\,\frac{\text{m}^3}{\text{s}} \end{aligned}$$

$$\boxed{Q = 5.03 \times 10^{-3}\ \text{m}^3/\text{s}}$$

7.36 As shown, a microchannel is being designed to transfer fluid in a MEMS (microelectrical mechanical system) application. The channel is 200 micrometers in diameter and is 5 cm long. Ethyl alcohol is driven through the system at the rate of 0.1 microliters/s (μL/s) with a syringe pump, which is essentially a moving piston. The pressure at the exit of the channel is atmospheric. The flow is laminar, so $\alpha = 2$. The head loss in the channel is given by

$$h_L = \frac{32\mu LV}{\gamma D^2}$$

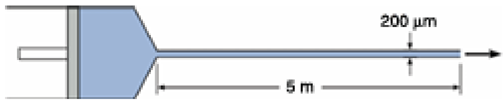
where L is the channel length, D the diameter, V the mean velocity, μ the viscosity of the fluid, and γ the specific weight of the fluid. Find the pressure in the syringe pump. The velocity head associated with the motion of the piston in the syringe pump is negligible.



PROBLEM 7.36

Situation:

A microchannel is designed to transfer fluid in a MEMS application.
$D = 200\,\mu\text{m},\ L = 5\,\text{cm}.$
$Q = 0.1\ \mu\text{L/s}.$
$h_L = \frac{32\mu LV}{\gamma D^2}.$



Find:

Pressure in syringe pump (Pa).

Assumptions:

$\alpha = 2.$

Properties:

Table A.4: $\rho = 799\,\text{kg/m}^3.$

PLAN

Apply the energy equation and the flow rate equation.

SOLUTION

Energy equation (locate section 1 inside the pumping chamber; section 2 at the outlet of the channel)

$$\begin{aligned} \frac{p_1}{\gamma} &= h_L + \alpha_2 \frac{V^2}{2g} \\ &= \frac{32\mu LV}{\gamma D^2} + 2\frac{V^2}{2g} \end{aligned} \tag{1}$$

The cross-sectional area of the channel is 3.14×10^{-8} m^2. A flow rate of 0.1 μL/s is 10^{-7} L/s or 10^{-10} m^3/s. The flow velocity is

$$\begin{aligned} V &= \frac{Q}{A} \\ &= \frac{10^{-10}\,\text{m}^3/\text{s}}{3.14\times10^{-8}\,\text{m}^2} \\ &= 0.318\times10^{-2}\ \text{m/s} \\ &= 3.18\ \text{mm/s} \end{aligned}$$

Substituting the velocity and other parameters in Eq. (1) gives

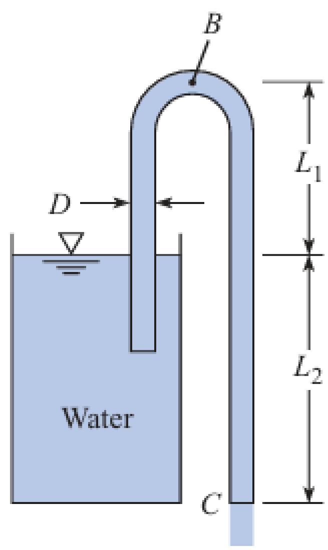
$$\begin{aligned} \frac{p_1}{\gamma} &= \frac{32\times1.2\times10^{-3}\times0.05\times0.318\times10^{-2}}{7,850\times4\times(10^{-4})^2} + 2\times\frac{(0.318\times10^{-2})^2}{2\times9.81} \\ &= 0.0194\ \text{m} \end{aligned}$$

The pressure is

$$p_1 = 799\ \text{kg/m}^3\times9.81\ \text{m/s}^2\times0.0194\ \text{m}$$

$$\boxed{p_1 = 152\ \text{Pa}}$$

7.38 The discharge in the siphon is 2.80 cfs, $D = 8$ in., $L_1 = 3$ ft, and $L_2 = 3$ ft. Determine the head loss between the reservoir surface and point C. Determine the pressure at point B if three-quarters of the head loss (as found above) occurs between the reservoir surface and point B. Assume $\alpha = 1.0$ at all locations.



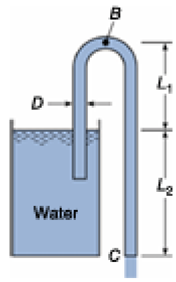
PROBLEM 7.38

Situation:

Water flows out of a syphon.

$L_1 = L_2 = 0.9\,\mathrm{m}$

$Q = 0.08\,\mathrm{m}^3/\mathrm{s}$, $D = 0.2\,\mathrm{m}$



Find:

Head loss between reservoir surface and point C.

Pressure at point B.

Properties:

Water (15.5 °C), Table A.5, $\gamma = 9810\,\mathrm{N/m^3}$.

Assumptions:

$\alpha = 1$.

Three quarters of head loss is between reservoir surface and point B.

PLAN

To find head loss between reservoir surface and point C.

1. Develop an equation for head loss by applying the energy equation from the reservoir surface to section C.
2. Find V using the flow rate equation.
3. Combine results of steps 1 and 2 and solve for the head loss.

To find the prssure at B.

4. Develop an equation for the pressure at B by applying the energy equation from the reservoir surface to section B.

SOLUTION

1. Energy equation (from reservoir surface to section C)

$$\begin{aligned}
\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 + h_p &= \frac{p_c}{\gamma} + \frac{V_c^2}{2g} + z_c + h_t + h_L \\
0 + 0 + 0.9\,\mathrm{m} + 0 &= 0 + \frac{V_c^2}{2g} + 0 + 0 + h_L \\
0.9\,\mathrm{m} &= \frac{V_c^2}{2g} + h_L
\end{aligned}$$

2. Flow rate equation

$$\begin{aligned} V_c &= \frac{Q}{A_2} \\ V_c &= \frac{0.08\,\mathrm{m}^3/\mathrm{s}}{(\pi/4)\times(0.2\,\mathrm{m})^2} = 2.5\ \mathrm{m/s} \end{aligned}$$

3. Combine results of step 1 and 2.

$$\begin{aligned} 0.9\,\mathrm{m} &= \frac{V_c^2}{2g} + h_L \\ 0.9\,\mathrm{m} &= \frac{(2.5\,\mathrm{m/s})^2}{2\left(9.81\,\mathrm{m/s}^2\right)} + h_L \end{aligned}$$

$$\boxed{h_L = 0.58\,\mathrm{m}}$$

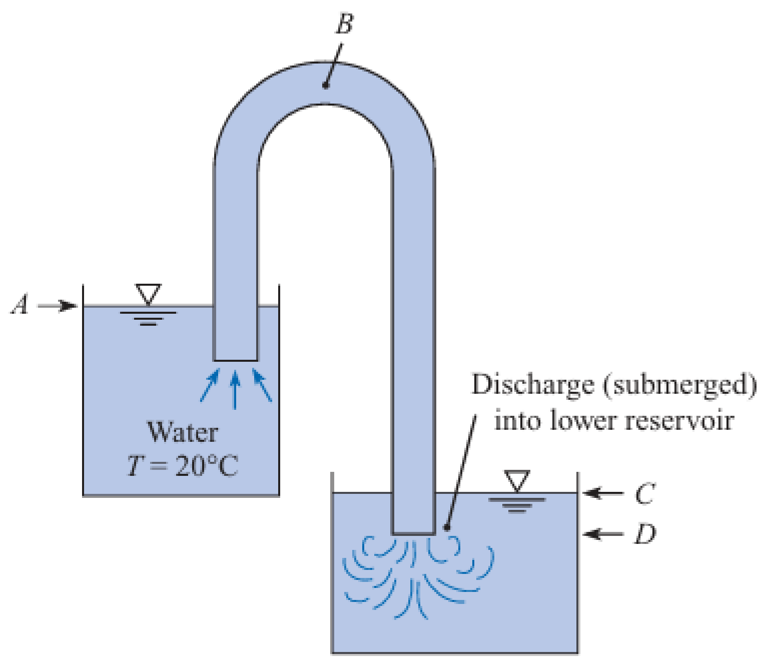
4. Energy equation (from reservoir surface to section B).

$$\begin{aligned} \frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 + h_p &= \frac{p_B}{\gamma} + \frac{V_B^2}{2g} + z_B + h_t + h_L \\ 0 + 0 + 0.9\,\mathrm{m} + 0 &= \frac{p_B}{\gamma} + \frac{V_B^2}{2g} + 1.8\,\mathrm{m} + 0 + (3/4)\times 0.58\,\mathrm{m} \\ 0.9\,\mathrm{m} &= \frac{p_B}{9810\,\mathrm{N/m}^3} + \frac{(2.5\,\mathrm{m/s})^2}{2\left(9.81\,\mathrm{m/s}^2\right)} + 1.8\,\mathrm{m} + (3/4)\times 0.58\,\mathrm{m} \end{aligned}$$

$$\begin{aligned} \frac{p_B}{\gamma} &= 0.9\,\mathrm{m} - 0.3\,\mathrm{m} - 1.8\,\mathrm{m} - 0.435\,\mathrm{m} = -1.635\,\mathrm{m} \\ p_B &= -1.635\,\mathrm{m} \times 9810\,\mathrm{N/m}^3 \\ &= -16,039.35\,\mathrm{Pa} \end{aligned}$$

$$\boxed{p_B = -16{,}040\ \mathrm{Pa}}$$

7.40 WILEY PLUS For this system, point B is 10 m above the bottom of the upper reservoir. The head loss from A to B is $1.1V^2/2g$, and the pipe area is $8 \times 10^{-4}\ \text{m}^2$. Assume a constant discharge of $8 \times 10^{-4}\ \text{m}^3/\text{s}$. For these conditions, what will be the depth of water in the upper reservoir for which cavitation will begin at point B? Vapor pressure = 1.23 kPa and atmospheric pressure = 100 kPa. Assume $\alpha = 1.0$ at all locations.



PROBLEMS 7.39, 7.40

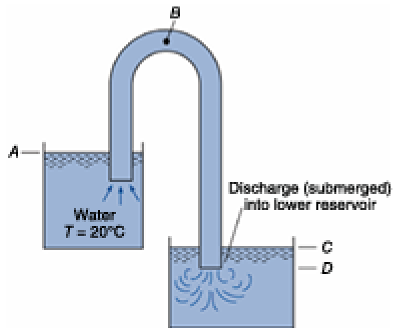
Situation:

A siphon transports water from one reservoir to another.

$p_v = 1.23\,\text{kPa}$, $p_{atm} = 100\,\text{kPa}$.

$Q = 8 \times 10^{-4}\,\text{m}^3/\text{s}$, $A = 10^{-4}\,\text{m}^2$.

$h_{L,A\to B} = 1.1V^2/2g$.



Find:

Depth of water in upper reservoir for incipient cavitation.

PLAN

Apply the energy equation from point A to point B.

SOLUTION

Flow rate equation

$$\begin{aligned} V &= \frac{Q}{A} \\ &= \frac{8 \times 10^{-4}\ \text{m}^3/\text{s}}{1 \times 10^{-4}\ \text{m}^2} \\ &= 8\,\text{m/s} \end{aligned}$$

Calculations

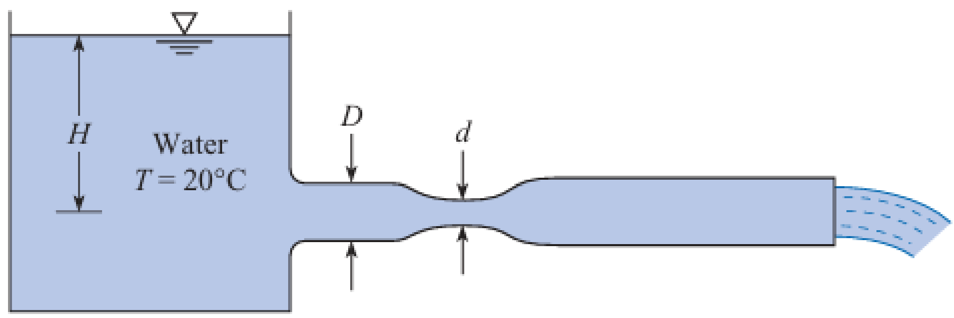
$$\begin{aligned} \frac{V^2}{2g} &= \frac{(8\,\text{m/s})^2}{2 \times 9.81\,\text{m/s}^2} = 3.262\,\text{m} \\ h_{L,A\to B} &= 1.1\frac{V^2}{2g} = 3.588\,\text{m} \end{aligned}$$

Energy equation (from A to B; let $z = 0$ at bottom of reservoir)

$$\begin{aligned} \frac{p_A}{\gamma} + \frac{V_A^2}{2g} + z_A &= \frac{p_B}{\gamma} + \frac{V_B^2}{2g} + z_B + h_L \\ \frac{100000\,\text{Pa}}{9810\,\text{N/m}^3} + 0 + z_A &= \frac{1230\,\text{Pa}}{9810\,\text{N/m}^3} + 3.262\,\text{m} + 10\,\text{m} + 3.588\,\text{m} \end{aligned}$$

$$\boxed{z_A = \text{depth} = 6.78\,\text{m}}$$

7.43 WILEY GO In this system $d = 15$ cm, $D = 35$ cm, and the head loss from the venturi meter to the end of the pipe is given by $h_L = 1.5\ V^2/2g$, where V is the velocity in the pipe. Neglecting all other head losses, determine what head H will first initiate cavitation if the atmospheric pressure is 100 kPa absolute. What will be the discharge at incipient cavitation? Assume $\alpha = 1.0$ at all locations.



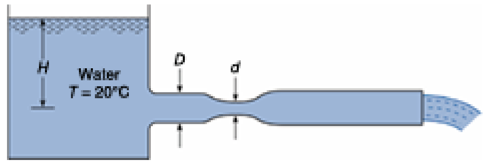
PROBLEM 7.43

Situation:

A reservoir discharges to a pipe with a venturi meter before draining to atmosphere.

$D = 35\,\text{cm}$, $d = 15\,\text{cm}$.

$p_{\text{atm}} = 100\,\text{kPa}$, $h_L = 1.5V_2^2/2g$.



Find:

Head at incipient cavitation (m).

Discharge at incipient cavitation (m^3/ s).

Assumptions:

$\alpha = 1.0$.

Properties:

From Table A.5 $p_v = 2340$ Pa, abs.

PLAN

First apply the energy equation from the Venturi section to the end of the pipe. Then apply the energy equation from reservoir water surface to outlet:

SOLUTION

Energy equation from Venturi section to end of pipe:

$$\begin{aligned}
\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 &= \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + h_L \\
\frac{p_{\text{vapor}}}{\gamma} + \frac{V_1^2}{2g} &= 0 + \frac{V_2^2}{2g} + 1.5\frac{V_2^2}{2g} \\
p_{\text{vapor}} &= 2,340 \text{ Pa abs.} = -97,660 \text{ Pa gage}
\end{aligned}$$

Continuity principle

$$\begin{aligned}
V_1 A_1 &= V_2 A_2 \\
V_1 &= \frac{V_2 A_2}{A_1} \\
&= 5.44 V_2
\end{aligned}$$

Then

$$\frac{V_1^2}{2g} = 29.64\frac{V_2^2}{2g}$$

Substituting into energy equation

$$\begin{aligned} -97,660/9,790 + 29.64\frac{V_2^2}{2g} &= 2.5\frac{V_2^2}{2g} \\ V_2 &= 2.685 \text{ m/s} \end{aligned}$$

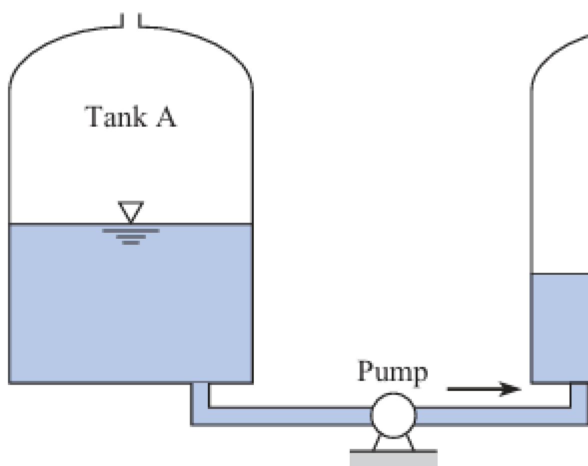
Flow rate equation

$$\begin{aligned} Q &= V_2 A_2 \\ &= 2.685\,\text{m/s} \times \pi/4 \times (0.35\,\text{m})^2 \\ &\boxed{Q = 0.258 \text{ m}^3/\text{s}} \end{aligned}$$

Energy equation from reservoir water surface to outlet:

$$\begin{aligned} H &= \frac{V_2^2}{2g} + h_L \\ H &= 2.5\frac{V_2^2}{2g} \\ &\boxed{H = 0.919 \text{ m}} \end{aligned}$$

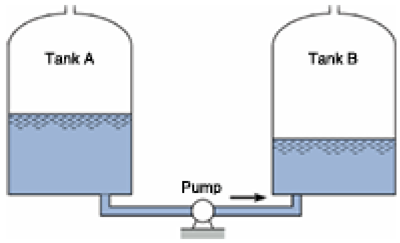
7.45 A pump is used to transfer SAE-30 oil from tank A to tank B as shown. The tanks have a diameter of 12 m. The initial depth of the oil in tank A is 20 m, and in tank B the depth is 1 m. The pump delivers a constant head of 60 m. The connecting pipe has a diameter of 20 cm, and the head loss due to friction in the pipe is 20 $V^2/2g$. Find the time required to transfer the oil from tank A to B; that is, the time required to fill tank B to 20 m depth.



PROBLEM 7.45

Situation:

A pump transfers SAE-30 oil between two tanks.
$D_{\text{tank}} = 12\,\text{m},\ D_{\text{pipe}} = 20\,\text{cm}.$
$h_L = 20\frac{V^2}{2g},\ h_p = 60\,\text{m}.$
$z_A = 20\,\text{m},\ z_B = 1\,\text{m}.$



Find:

Time required to transfer oil (h).

PLAN

Apply the energy equation between the top of the fluid in tank A to that in tank B.

SOLUTION

Energy equation

$$h_p + z_A = z_B + h_L$$

or

$$h_p + z_A = z_B + 20\frac{V^2}{2g} + \frac{V^2}{2g}$$

Solve for velocity

$$\begin{aligned} V^2 &= \frac{2g}{21}(h_p + z_A - z_B) \\ V^2 &= \frac{2 \times 9.81}{21}\,(60 + z_A - z_B) \\ V &= 0.966\,6\,(60 + z_A - z_B)^{1/2} \end{aligned}$$

The sum of the elevations of the liquid surfaces in the two tanks is

$$z_A + z_B = 21$$

So the energy equation becomes

$$V = 0.9666(81 - 2z_B)^{1/2}$$

Continuity equation

$$\begin{aligned} \frac{dz_B}{dt} &= V\frac{A_{\text{pipe}}}{A_{\text{tank}}} = V\frac{(0.2\,\text{m})^2}{(12\,\text{m})^2} \\ &= \left(2.778\times 10^{-4}\right)V \\ &= \left(2.778\times 10^{-4}\right)0.9666(81 - 2z_B)^{1/2} \\ &= 2.685\times 10^{-4}(81 - 2z_B)^{1/2} \end{aligned}$$

Separate variables

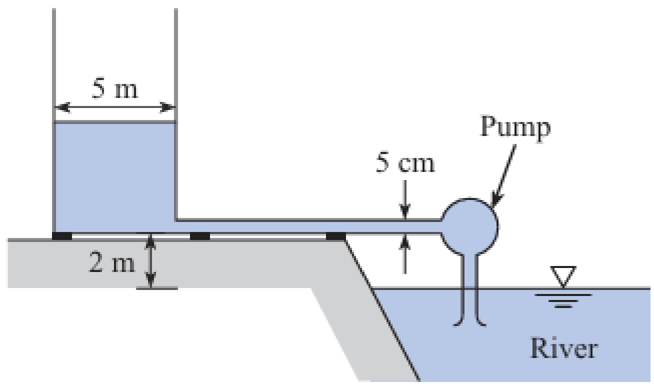
$$\frac{dz_B}{(81 - 2z_B)^{1/2}} = 2.685\times 10^{-4}dt$$

Integrate

$$\begin{aligned} \int_1^{20\,\text{ft}} \frac{dz_B}{(81 - 2z_B)^{1/2}} &= \int_0^{\Delta t} 2.685\times 10^{-4}dt \\ \left(-\sqrt{81 - 2z_B}\right)_{1\,\text{ft}}^{20\,\text{ft}} &= \left(2.685\times 10^{-4}\right)\Delta t \\ \left(-\sqrt{81 - 2\,(20)} + \sqrt{81 - 2\,(1)}\right) &= \left(2.685\times 10^{-4}\right)\Delta t \\ 2.485\,1 &= \left(2.685\times 10^{-4}\right)\Delta t \\ \Delta t &= 9256\,\text{s} \end{aligned}$$

$$\boxed{t = 9260\,\text{s} = 2.57\,\text{h}}$$

7.44 WILEY GO A pump is used to fill a tank 5 m in diameter from a river as shown. The water surface in the river is 2 m below the bottom of the tank. The pipe diameter is 5 cm, and the head loss in the pipe is given by $h_L = 10\ V^2/2g$, where V is the mean velocity in the pipe. The flow in the pipe is turbulent, so $\alpha = 1$. The head provided by the pump varies with discharge through the pump as $h_p = 20 - 4 \times 10^4\ Q^2$, where the discharge is given in cubic meters per second (m^3/s) and h_p is in meters. How long will it take to fill the tank to a depth of 10 m?



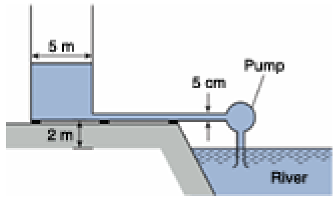
PROBLEM 7.44

Situation:

A pump fills a tank with water from a river.

$D_{\text{tank}} = 5\,\text{m}$, $D_{\text{pipe}} = 5\,\text{cm}$.

$h_{L,}pipe = 10V_2^2/2g$, $h_p = 20 - 4 \times 10^4 Q^2$.



Find:

Time required to fill tank to depth of 10 m.

Assumptions:

$\alpha = 1.0$.

SOLUTION

Energy equation (locate 1 on the surface of the river, locate 2 on the surface of the water in the tank).

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 + h_p = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + h_L$$

but $p_1 = p_2 = 0$, $z_1 = 0$, $V_1 = 0$, $V_2 \simeq 0$. The energy equation reduces to

$$0 + 0 + 0 + h_p = 0 + 0 + (2\text{ m} + h) + h_L$$

where h =depth of water in the tank.

Identify all sources of head loss in the figure:

(a) pipe head loss: $h_{L,pipe} = 10V_2^2/2g$

(b) As per Section 7.7 (EFM 10e) head loss due to abrupt expansion when flowing water from submerged discharge pipe exits into reservoir, and is forced to $V_{final} = 0$: $h_{L,Exit} = V_2^2/2g$

Energy equation

$$20 - (4)(10^4)Q^2 = h + 2 + \frac{V^2}{2g} + 10\frac{V^2}{2g}$$

$$18 = (4)(10^4)Q^2 + 11\frac{V^2}{2g} + h, \text{ where all terms are in m}$$

$$
\begin{aligned}
V &= \frac{Q}{A} \\
11\frac{V^2}{2g} &= \frac{11}{2g}\left(\frac{Q^2}{A^2}\right) = (1.45)(10^5)Q^2 \\
18 &= 1.85 \times 10^5 Q^2 + h \\
Q^2 &= \frac{18-h}{1.85 \times 10^5} \\
Q &= \frac{(18-h)^{0.5}}{430}
\end{aligned}
$$

But $Q = A_T dh/dt$ where $A_T =$ tank area, so

$$
\begin{aligned}
\therefore \quad \frac{dh}{dt} &= \frac{(18-h)^{0.5}}{(430)(\pi/4)(5)^2} = \frac{(18-h)^{0.5}}{8,443} \\
dh/(18-h)^{0.5} &= dt/8,443
\end{aligned}
$$

Integrate:

$$
-2(18-h)^{0.5} = \frac{t}{8,443} + \text{ const.}
$$

But $t = 0$ when $h = 0$ so const. $= -2(18)^{0.5}$. Then

$$
t = (18^{0.5} - (18-h)^{0.5})(16,886)
$$

For $h = 10$ m

$$
\begin{aligned}
t &= (18^{0.5} - 8^{0.5})(16,886) \\
&= 23,880 \text{ s} \\
& \boxed{t = 6.63\,\text{h}}
\end{aligned}
$$

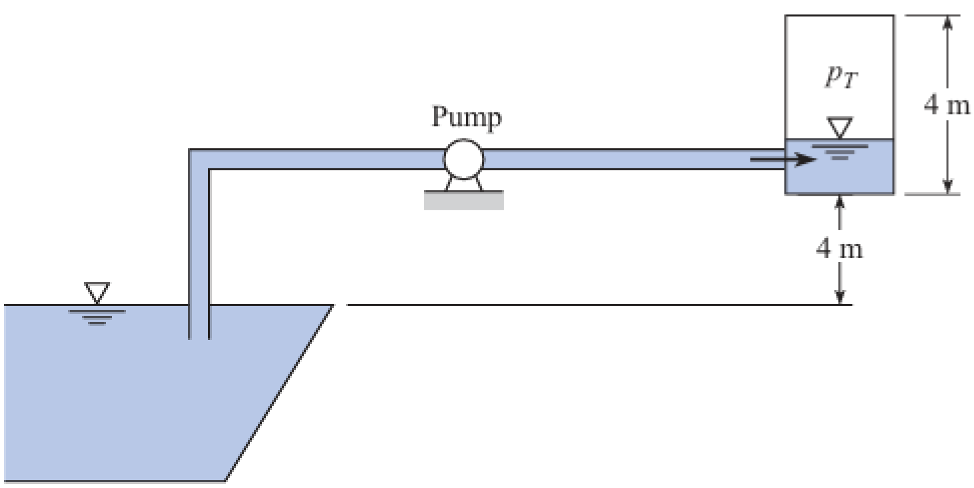
7.46 A pump is used to pressurize a tank to 300 kPa abs. The tank has a diameter of 2 m and a height of 4 m. The initial level of water in the tank is 1 m, and the pressure at the water surface is 0 kPa gage. The atmospheric pressure is 100 kPa. The pump operates with a constant head of 50 m. The water is drawn from

a source that is 4 m below the tank bottom. The pipe connecting the source and the tank is 4 cm in diameter and the head loss, including the expansion loss at the tank, is 10 $V^2/2g$. The flow is turbulent.

Assume the compression of the air in the tank takes place isothermally, so the tank pressure is given by

$$p_T = \frac{3}{4 - z_t} p_0$$

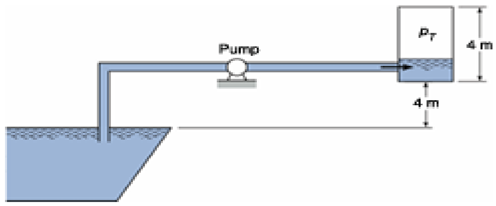
where z_t is the depth of fluid in the tank in meters. Write a computer program that will show how the pressure varies in the tank with time, and find the time to pressurize the tank to 300 kPa abs.



PROBLEM 7.46

Situation:

A pump is used to pressurize a tank.
$D_{\text{tank}} = 2\,\text{m}$, $D_{\text{pipe}} = 4\,\text{cm}$.
$h_L = 10\frac{V^2}{2g}$, $h_p = 50\,\text{m}$.
$z_A = 20\,\text{m}$, $z_B = 1\,\text{m}$.
$p_T = \frac{3}{4-z_t}p_0$, $p_0 = 0\,\text{kPa gage} = 100\,\text{kPa}$.



Find:

Write a computer program to show how the pressure varies with time.
Time to pressurize tank to 300 kPa (s).

PLAN

Apply the energy equation between the water surface at the intake and the water surface inside the tank.

SOLUTION

Energy equation

$$h_p + z_1 = \frac{p_2}{\gamma} + z_2 + h_L$$

Expressing the head loss in terms of the velocity allows one to solve for the velocity in the form

$$V^2 = \frac{2g}{10}(h_p + z_1 - z_t - \frac{p_t}{\gamma})$$

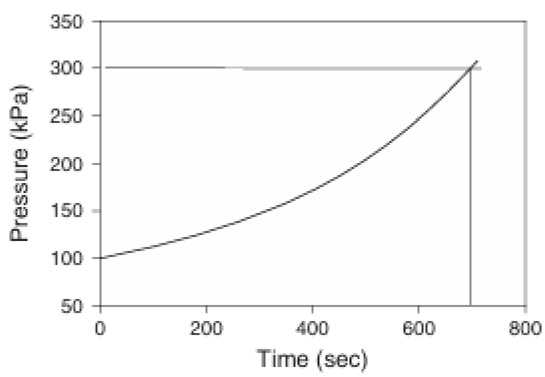
Substituting in values

$$V = 1.401(46 - z_t - 10.19\frac{3}{4 - z_t})^{1/2}$$

The equation for the water surface elevation in the tank is

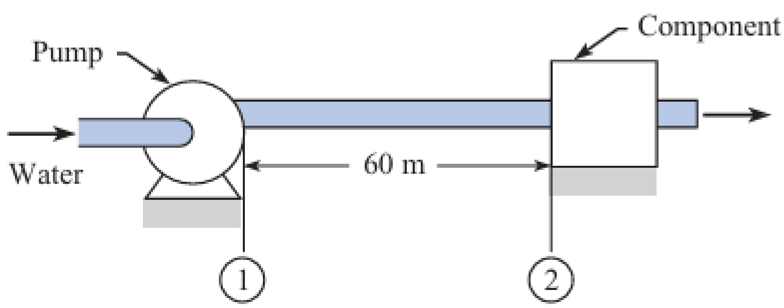
$$\Delta z_t = V\frac{A_p}{A_t}\Delta t = \frac{V}{2500}\Delta t$$

A computer program can be written taking time intervals and finding the fluid level and pressure in the tank at each time step. The time to reach a pressure of 300 kPa abs in the tank is **698 seconds or 11.6 minutes.** A plot of how the pressure varies with time is provided.



The Power Equation (§7.4)

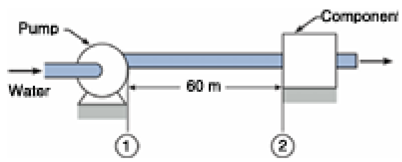
7.47 WILEY PLUS As shown, water at 15°C is flowing in a 15-cm-diameter by 60-m-long run of pipe that is situated horizontally. The mean velocity is 2 m/s, and the head loss is 2 m. Determine the pressure drop and the required pumping power to overcome head loss in the pipe.



PROBLEM 7.47

Situation:

Water is flowing in a horizontal pipe.
$D = 0.15\,\text{m}$, $L = 60\,\text{m}$.
$V = 2\,\text{m/s}$. $h_L = 2\,\text{m}$.



Find:

Pressure drop (Pa).
Pumping power (W).

Properties:

Water (15 °C), Table A.5: $\gamma = 9800\,\text{N/m}^3$.

PLAN

1. Find pressure drop using the energy equation.
2. Find power using the power equation.

SOLUTION

1. Energy equation:

$$\frac{p_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_p = \frac{p_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_t + h_L$$

- let $\Delta p = p_1 - p_2$
- KE terms cancel.
- Elevation terms cancel.
- $h_p = h_t = 0$

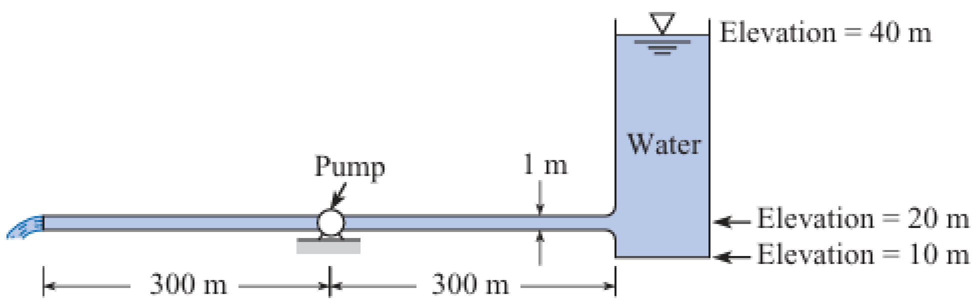
$$\begin{aligned} \Delta p &= \gamma h_L \\ &= \left(9800\,\text{N/m}^3\right)(2\,\text{m}) \\ & \boxed{\Delta p = 19.6\ \text{kPa}} \end{aligned}$$

2. Power equation:

$$\dot{W}_p = \gamma Q h_p = \dot{m} g h_p$$

- The head of the pump must equal the head loss.

7.49 A water discharge of 8 m^3/s is to flow through this horizontal pipe, which is 1 m in diameter. If the head loss is given as $7\,V^2/2g$ (V is velocity in the pipe), how much power will have to be supplied to the flow by the pump to produce this discharge? Assume $\alpha = 1.0$ at all locations.



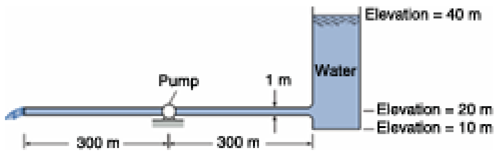
PROBLEM 7.49

Situation:

A pump moves water from a tank through a pipe.

$Q = 8\,\mathrm{m}^3/\mathrm{s}$, $h_L = 7V^2/2g$.

$D = 1\,\mathrm{m}$.



Find:

Power supplied to flow (MW).

Assumptions:

$\alpha = 1.0$.

PLAN

Find power using the power equation. The steps are

1. Find velocity in the pipe using the flow rate equation.
2. Find head of the pump using the energy equation.
3. Calculate power.

SOLUTION

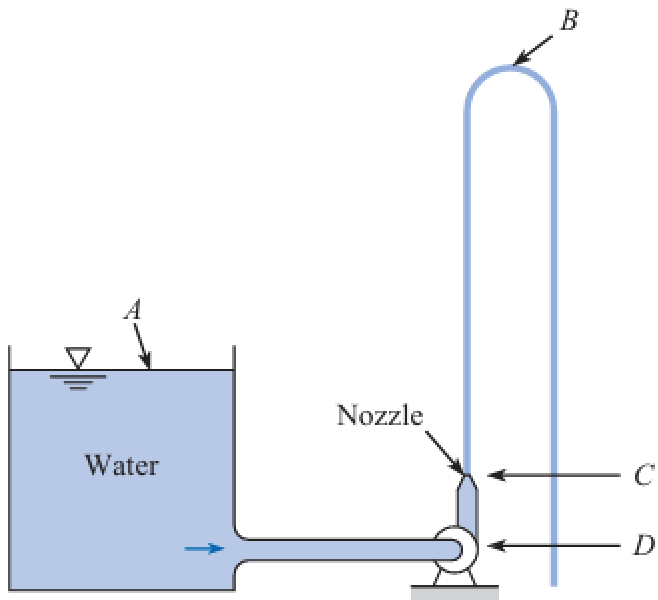
1. Flow rate equation

$$\begin{aligned} V &= \frac{Q}{A} \\ &= \frac{8}{\pi/4 \times (1\,\mathrm{m})^2} \\ &= 10.2\ \ \mathrm{m/s} \end{aligned}$$

2. Energy equation (locate 1 on the reservoir surface; locate 2 at the out of the pipe).

$$\begin{aligned} \frac{p_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_p &= \frac{p_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_t + h_L \\ 0 + 0 + 40 + h_p &= 0 + \frac{V^2}{2g} + 20 + 7\frac{V^2}{2g} \\ \frac{V^2}{2g} &= \frac{(10.2\,\mathrm{m/s})^2}{2 \times 9.81\,\mathrm{m/s}^2} = 5.30\ \mathrm{m} \end{aligned}$$

7.52 Neglecting head losses, determine what power the pump must deliver to produce the flow as shown. Here the elevations at points *A*, *B*, *C*, and *D* are 40 m, 65 m, 35 m, and 30 m, respectively. The nozzle area is 25 cm^2.



PROBLEMS 7.51, 7.52

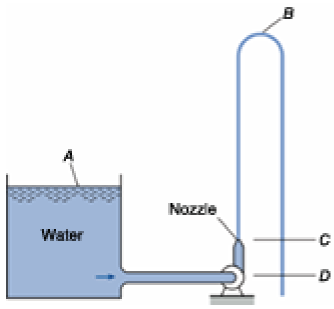
Situation:

A pumping system delivers water.

$z_A = 40\,\text{m},\ z_B = 65\,\text{m}.$

$z_C = 35\,\text{m},\ z_D = 30\,\text{m}.$

$A = 25\,\text{cm}^2.$



Find:

Power delivered by pump (kW).

PLAN

Apply the energy equation from the reservoir water surface to point B. Then apply the power equation.

SOLUTION

Energy equation from location A to location B:

$$\begin{aligned} \frac{p}{\gamma} + \frac{V^2}{2g} + z + h_p &= \frac{p_B}{\gamma} + \frac{V_B^2}{2g} + z_B \\ 0 + 0 + 40 + h_p &= 0 + 0 + 65;\ h_p = 25 \text{ m} \end{aligned}$$

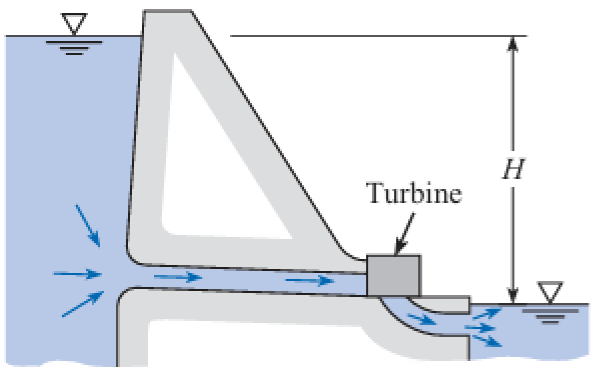
Flow rate equation

$$\begin{aligned} Q &= V_j A_j = 25 \times 10^{-4} \text{ m}^2 \times V_j \\ \text{where } V_j &= \sqrt{2g \times (65 - 35)} = 24.3 \text{ m/s} \\ Q &= 25 \times 10^{-4} \times 24.3 = 0.0607 \text{ m}^3/\text{s} \end{aligned}$$

Power equation

$$\begin{aligned} P &= Q\gamma h_p \\ P &= 0.0607\,\text{m}^3/\text{s} \times 9,810\,\text{N}/\text{m}^3 \times 25\,\text{m} \\ & \boxed{P = 14.9 \text{ kW}} \end{aligned}$$

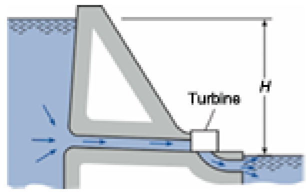
7.56 WILEY PLUS A small-scale hydraulic power system is shown. The elevation difference between the reservoir water surface and the pond water surface downstream of the reservoir, *H*, is 24 m. The velocity of the water exhausting into the pond is 7 m/s, and the discharge through the system is 4 m^3/s. The head loss due to friction in the penstock (inlet pipe to turbine, under very high pressure) is negligible. Find the power produced by the turbine in kilowatts.



PROBLEM 7.56

Situation:

A hydraulic power system consists of a dam with an inlet connected to a turbine.
$H = 24\,\text{m}$, $V_2 = 7\,\text{m/s}$.
$Q = 4\,\text{m}^3/\text{s}$.



Find:

Power produced by turbine (kW).

Assumptions:

All head loss is expansion loss.
100% efficiency.

PLAN

Apply the energy equation from the upstream water surface to the downstream water surface. Then apply the power equation.

SOLUTION

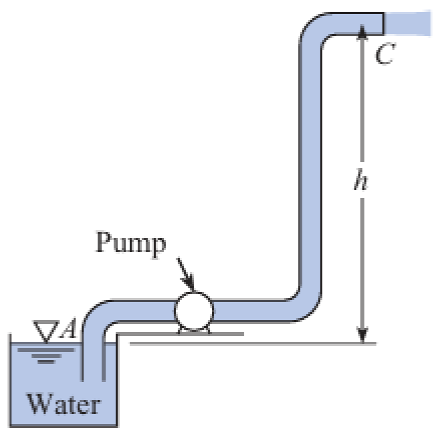
Energy equation

$$\begin{aligned}\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 &= \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + h_L + h_t \\ 0 + 0 + 24\text{ m} &= 0 + 0 + 0 + h_t + \frac{V_2^2}{2g} \\ h_t &= 24\text{ m} - \frac{(7\,\text{m/s})^2}{2 \times 9.81\,\text{m/s}^2} \\ &= 21.5\text{ m}\end{aligned}$$

Power equation

$$\begin{aligned}P &= Q\gamma h_t \\ &= (4\text{ m}^3/\text{s})(9810\text{ N/m}^3)(21.50\text{ m}) \\ &\boxed{P = 844\text{ kW}}\end{aligned}$$

7.60 WILEY PLUS The pump shown draws water (20°C) through a 20 cm suction pipe and discharges it through a 10 cm pipe in which the velocity is 3 m/s. The 10 cm pipe discharges horizontally into air at point *C*. To what height *h* above the water surface at *A* can the water be raised if 26 kW is delivered to the pump? Assume that the pump operates at 60% efficiency and that the head loss in the pipe between *A* and *C* is equal to 2 $V_C^2/2g$. Assume $\alpha = 1.0$ throughout.



PROBLEMS 7.59, 7.60

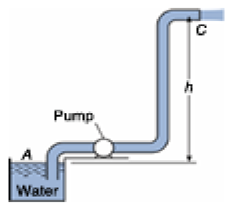
Situation:

A pump draws water out of a tank and moves this water to a higher elevation.

$D_A = 20\,\text{cm}$, $D_C = 10\,\text{cm}$.

$V_C = 3\,\text{m/s}$, $P = 26\,\text{kW}$.

$\eta = 60\%$, $h_L = 2V_C^2/2g$.



Find:

Height h in meters.

Assumptions:

$\alpha = 1.0$.

Properties:

Water (20 °C), Table A.5: $\gamma = 9790\,\text{N/m}^3$.

SOLUTION

Energy equation:

$$\begin{aligned}
\frac{p_A}{\gamma} + \alpha_A \frac{V_A^2}{2g} + z_A + h_p &= \frac{p_C}{\gamma} + \alpha_C \frac{V_C^2}{2g} + z_C + h_L \\
0 + 0 + 0 + h_p &= 0 + \frac{V_c^2}{2g} + h + 2\frac{V_c^2}{2g} \\
h_p &= h + 3\frac{V_c^2}{2g} \qquad (1
\end{aligned}$$

Velocity head:

$$\frac{V_c^2}{2g} = \frac{(3\,\text{m/s})^2}{2\,(9.81\,\text{m/s}^2)} = 0.4587\,\text{m}$$

Flow rate equation:

$$\begin{aligned}
Q &= V_C\, A_C \\
&= (3\,\text{m/s}) \left(\frac{\pi\,(0.1\,\text{m})^2}{4} \right) \\
&= 0.02356\,\text{m}^3/\text{s}
\end{aligned}$$

Power equation:

$$\begin{aligned}
P &= \frac{Q\gamma h_p}{\eta} \\
h_p &= \frac{P\eta}{Q\gamma} = \frac{26000\,\text{W} \times 0.6}{(0.02356\,\text{m}^3/\text{s})\,(9790\,\text{N/m}^3)} = 67.63\,\text{m}
\end{aligned}$$

Eq. (1):

$$\begin{aligned}
h &= h_p - 3\frac{V_c^2}{2g} = (67.63\,\text{m}) - 3\,(0.4587\,\text{m}) \\
& \boxed{h = 66.3\,\text{m}}
\end{aligned}$$