A

Systems of Units and Conversion Factors

A.1 SYSTEMS OF UNITS

Measurement systems have been a necessity since people first began to build and barter, and every ancient culture developed some sort of measurement system to serve its needs. Standardization of units took place gradually over the centuries, often through royal edicts. Development of the **British Imperial System** from earlier measurement standards began in the 13th century and was well established by the 18th century. The British system spread to many parts of the world, including the United States, through commerce and colonization. In the United States the system gradually evolved into the **U.S. Customary System (USCS)** that is in common use today.

The concept of the **metric system** originated in France about 300 years ago and was formalized in the 1790s, at the time of the French Revolution. France mandated the use of the metric system in 1840, and since then many other countries have done the same. In 1866 the United States Congress legalized the metric system without making it compulsory.

A new system of units was created when the metric system underwent a major revision in the 1950s. Officially adopted in 1960 and named the **International System of Units** (Système International d'Unités), this newer system is commonly referred to as **SI**. Although some SI units are the same as in the old metric system, SI has many new features and simplifications. Thus, SI is an improved metric system.

Length, time, mass, and force are the basic concepts of mechanics for which units of measurement are needed. However, only three of these quantities are independent since all four of them are related by Newton's second law of motion:

$$F = ma \tag{A-1}$$

in which F is the force acting on a particle, m is the mass of the particle, and a is its acceleration. Since acceleration has units of length divided by time squared, all four quantities are involved in the second law.

The International System of Units, like the metric system, is based upon length, time, and mass as fundamental quantities. In these systems, force is derived from Newton's second law. Therefore, the unit of force is expressed in terms of the basic units of length, time, and mass, as shown in the next section.

SI is classified as an **absolute system of units** because measurements of the three fundamental quantities are independent of the locations at which the measurements are made; that is, the measurements do not depend upon the effects of gravity. Therefore, the SI units for length, time, and mass may be used anywhere on earth, in space, on the moon, or even on another planet. This is one of the reasons why the metric system has always been preferred for scientific work.

The British Imperial System and the U.S. Customary System are based upon length, time, and force as the fundamental quantities with mass being derived from the second law. Therefore, in these systems the unit of mass is expressed in terms of the units of length, time, and force. The unit of force is defined as the force required to give a certain standard mass an acceleration equal to the acceleration of gravity, which means that the unit of force varies with location and altitude. For this reason, these systems are called **gravitational systems of units**. Such systems were the first to evolve, probably because weight is such a readily discernible property and because variations in gravitational attraction were not noticeable. It is clear, however, that in the modern technological world an absolute system is preferable.

A.2 SI UNITS

The International System of Units has seven **base units** from which all other units are derived. The base units of importance in mechanics are the meter (m) for length, second (s) for time, and kilogram (kg) for mass. Other SI base units pertain to temperature, electric current, amount of substance, and luminous intensity.

The **meter** was originally defined as one ten-millionth of the distance from the North Pole to the equator. Later, this distance was converted to a physical standard, and for many years the standard for the meter was the distance between two marks on a platinum-iridium bar stored at the headquarters of the International Bureau of Weights and Measures (Bureau International des Poids et Mesures) in Sèvres, a suburb on the western edge of Paris, France.

Because of the inaccuracies inherent in the use of a physical bar as a standard, the definition of the meter was changed in 1983 to the length of the path traveled by light in a vacuum during a time interval of 1/299792458 of a second.* The advantages of this "natural" standard are that it is not subject to physical damage and is reproducible at laboratories anywhere in the world.

The **second** was originally defined as 1/86400 of a mean solar day (24 hours equals 86,400 seconds). However, since 1967 a highly accurate atomic clock has set the standard, and a second is now defined to be the duration of 9,192,631,770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the cesium-133 atom. (Most engineers would probably prefer the original definition over the new one, which hasn't noticeably changed the second but which is necessary because the earth's rotation rate is gradually slowing down.)

Of the seven base units in SI, the **kilogram** is the only one that is still defined by a physical object. Since the mass of an object can only be determined by comparing it experimentally with the mass of some other object, a physical standard is needed. For this purpose, a one-kilogram cylinder of platinum-iridium, called the International Prototype Kilogram (IPK), is kept by the International Bureau of Weights and Measures at Sèvres. (At the present time, attempts are being made to define the kilogram in terms of a

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*Taking the reciprocal of this number gives the speed of light in a vacuum (299,792,458 meters per second).

fundamental constant, such as the Avogadro number, thus removing the need for a physical object.)

Other units used in mechanics, called **derived units**, are expressed in terms of the base units of meter, second, and kilogram. For instance, the unit of **force** is the **newton**, which is defined as the force required to impart an acceleration of one meter per second squared to a mass of one kilogram.* From Newton's second law ($F = ma$), we can derive the unit of force in terms of base units:

$$1 \text{ newton} = (1 \text{ kilogram})(1 \text{ meter per second squared})$$

Thus, the newton (N) is given in terms of base units by the formula

$$1 \text{ N} = 1 \text{ kg}\cdot\text{m/s}^2 \tag{A-2}$$

To provide a point of reference, we note that a small apple weighs approximately one newton.

The unit of **work** and **energy** is the **joule**, defined as the work done when the point of application of a force of one newton is displaced a distance of one meter in the direction of the force.** Therefore,

$$1 \text{ joule} = (1 \text{ newton})(1 \text{ meter}) = 1 \text{ newton meter}$$

or

$$1 \text{ J} = 1 \text{ N}\cdot\text{m} \tag{A-3}$$

When you raise this book from desktop to eye level, you do about one joule of work, and when you walk up one flight of stairs, you do about 200 joules of work.

The names, symbols, and formulas for SI units of importance in mechanics are listed in **Table A-1**. Some of the derived units have special names, such as newton, joule, hertz, watt, and pascal. These units are named for notable persons in science and engineering and have symbols (N, J, Hz, W, and Pa) that are capitalized, although the unit names themselves are written in lowercase letters. Other derived units have no special names (for example, the units of acceleration, area, and density) and must be expressed in terms of base units and other derived units.

The relationships between various SI units and some commonly used metric units are given in **Table A-2**. Metric units such as dyne, erg, gal, and micron are no longer recommended for engineering or scientific use.

The **weight** of an object is the **force of gravity** acting on that object, and therefore weight is measured in newtons. Since the force of gravity depends upon altitude and position on the earth, weight is not an invariant property of a body. Furthermore, the weight of a body as measured by a spring scale is affected not only by the gravitational pull of the earth but also by the centrifugal effects associated with the rotation of the earth.

As a consequence, we must recognize two kinds of weight, **absolute weight** and **apparent weight**. The former is based upon the force of gravity alone, and the latter includes the effects of rotation. Thus, apparent weight is always less than absolute weight (except at the poles). Apparent weight, which is the weight of an object as measured with a spring scale, is the weight we customarily use in business and everyday life; absolute weight is used in astroengineering and certain kinds of scientific work. In this book, the term "weight" will always mean "apparent weight."

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*Sir Isaac Newton (1642–1727) was an English mathematician, physicist, and astronomer. He invented calculus and discovered the laws of motion and gravitation.

**James Prescott Joule (1818–1889) was an English physicist who developed a method for determining the mechanical equivalent of heat. His last name is pronounced "jool."

TABLE A-1 PRINCIPAL UNITS USED IN MECHANICS

Quantity	International System (SI)			U.S. Customary System (USCS)		
	Unit	Symbol	Formula	Unit	Symbol	Formula
Acceleration (angular)	radian per second squared		rad/s^2	radian per second squared		rad/s^2
Acceleration (linear)	meter per second squared		m/s^2	foot per second squared		ft/s^2
Area	square meter		m^2	square foot		ft^2
Density (mass) (Specific mass)	kilogram per cubic meter		kg/m^3	slug per cubic foot		$slug/ft^3$
Density (weight) (Specific weight)	newton per cubic meter		N/m^3	pound per cubic foot	pcf	lb/ft^3
Energy; work	joule	J	N·m	foot-pound		ft-lb
Force	newton	N	$kg \cdot m/s^2$	pound	lb	(base unit)
Force per unit length (Intensity of force)	newton per meter		N/m	pound per foot		lb/ft
Frequency	hertz	Hz	s^{-1}	hertz	Hz	s^{-1}
Length	meter	m	(base unit)	foot	ft	(base unit)
Mass	kilogram	kg	(base unit)	slug		$lb\text{-}s^2/ft$
Moment of a force; torque	newton meter		N·m	pound-foot		lb-ft
Moment of inertia (area)	meter to fourth power		m^4	inch to fourth power		$in.^4$
Moment of inertia (mass)	kilogram meter squared		$kg \cdot m^2$	slug foot squared		$slug\text{-}ft^2$
Power	watt	W	J/s (N·m/s)	foot-pound per second		ft-lb/s
Pressure	pascal	Pa	N/m^2	pound per square foot	psf	lb/ft^2
Section modulus	meter to third power		m^3	inch to third power		$in.^3$
Stress	pascal	Pa	N/m^2	pound per square inch	psi	$lb/in.^2$
Time	second	s	(base unit)	second	s	(base unit)
Velocity (angular)	radian per second		rad/s	radian per second		rad/s
Velocity (linear)	meter per second		m/s	foot per second	fps	ft/s
Volume (liquids)	liter	L	$10^{-3}\ m^3$	gallon	gal.	$231\ in.^3$
Volume (solids)	cubic meter		m^3	cubic foot	cf	ft^3

Notes: 1 joule (J) = 1 newton meter (N·m) = 1 watt second (W·s)
1 hertz (Hz) = 1 cycle per second (cps) or 1 revolution per second (rev/s)
1 watt (W) = 1 joule per second (J/s) = 1 newton meter per second (N·m/s)
1 pascal (Pa) = 1 newton per meter squared (N/m^2)
1 liter (L) = 0.001 cubic meter (m^3) = 1000 cubic centimeters (cm^3)

The **acceleration of gravity**, denoted by the letter g, is directly proportional to the force of gravity, and therefore it too depends upon position. In contrast, **mass** is a measure of the amount of material in a body and does not change with location.

The fundamental relationship between weight, mass, and acceleration of gravity can be obtained from Newton's second law ($F = ma$), which in this case becomes

$$W = mg \tag{A-4}$$

In this equation, W is the weight in newtons (N), m is the mass in kilograms (kg), and g is the acceleration of gravity in meters per second squared (m/s^2). Equation (A-4) shows that *a body having a mass of one kilogram has a weight in newtons numerically equal to* g. The values of the weight W and the acceleration g depend upon many factors, including latitude and elevation. However, for scientific calculations a standard international value of g has been established as

$$g = 9.806650 \text{ m/s}^2 \tag{A-5}$$

TABLE A-2 ADDITIONAL UNITS IN COMMON USE

SI and Metric Units

1 gal = 1 centimeter per second squared (cm/s^2)
for example, $g \approx 981$ gals)
1 are (a) = 100 square meters (m^2)
1 hectare (ha) = 10,000 square meters (m^2)
1 erg = 10^{-7} joules (J)
1 kilowatt-hour (kWh) = 3.6 megajoules (MJ)
1 dyne = 10^{-5} newtons (N)
1 kilogram-force (kgf) = 1 kilopond (kp)
= 9.80665 newtons (N)

1 centimeter (cm) = 10^{-2} meters (m)
1 cubic centimeter (cm^3) = 1 milliliter (mL)
1 micron = 1 micrometer (μm)= 10^{-6} meters (m)
1 gram (g) = 10^{-3} kilograms (kg)
1 metric ton (t) = 1 megagram (Mg) = 1000 kilograms (kg)
1 watt (W) = 10^{7} ergs per second (erg/s)
1 dyne per square centimeter (dyne/cm^2) = 10^{-1} pascals (Pa)
1 bar = 10^{5} pascals (Pa)
1 stere = 1 cubic meter (m^3)

USCS and Imperial Units

1 kilowatt-hour (kWh) = 2,655,220 foot-pounds (ft-lb)
1 British thermal unit (Btu) = 778.171 foot-pounds (ft-lb)
1 kip (k) = 1000 pounds (lb)
1 ounce (oz) = 1/16 pound (lb)
1 ton = 2000 pounds (lb)
1 Imperial ton (or long ton) = 2240 pounds (lb)
1 poundal (pdl) = 0.0310810 pounds (lb)
= 0.138255 newtons (N)
1 inch (in.) = 1/12 foot (ft)
1 mil = 0.001 inch (in.)
1 yard (yd) = 3 feet (ft)
1 mile = 5280 feet (ft)
1 horsepower (hp) = 550 foot-pounds per second (ft-lb/s)

1 kilowatt (kW)
= 737.562 foot-pounds per second (ft-lb/s)
= 1.34102 horsepower (hp)
1 pound per square inch (psi)
= 144 pounds per square foot (psf)
1 revolution per minute (rpm)
= $2\pi/60$ radians per second (rad/s)
1 mile per hour (mph)
= 22/15 feet per second (fps)
1 gallon (gal.) = 231 cubic inches (in.^{3})
1 quart (qt) = 2 pints = 1/4 gallon (gal.)
1 cubic foot (cf) = 576/77 gallons
= 7.48052 gallons (gal.)
1 Imperial gallon = 277.420 cubic inches (in.^{3})

This value is intended for use under standard conditions of elevation and latitude (sea level at a latitude of approximately 45°). The recommended value of g for ordinary engineering purposes on or near the surface of the earth is

$$g = 9.81 \text{ m/s}^2 \tag{A-6}$$

Thus, a body having a mass of one kilogram has a weight of 9.81 newtons.

Atmospheric pressure varies considerably with weather conditions, location, altitude, and other factors. Consequently, a standard international value for the pressure at the earth's surface has been defined:

$$1 \text{ standard atmosphere} = 101.325 \text{ kilopascals} \tag{A-7}$$

The following simplified value is recommended for ordinary engineering work:

$$1 \text{ standard atmosphere} = 101 \text{ kPa} \tag{A-8}$$

Of course, the values given in Eqs. (A-7) and (A-8) are intended for use in calculations and do not represent the actual ambient pressure at any given location.

A basic concept in mechanics is **moment** or **torque**, especially the moment of a force and the moment of a couple. Moment is expressed in units of force times length, or newton meters (N·m). Other important concepts in mechanics are **work** and **energy**, both of which are expressed in joules, a derived unit that happens to have the same units (newton meters) as the units of moment. However, moment is a distinctly different quantity from work or energy, and the joule should *never* be used for moment or torque.

Frequency is measured in units of **hertz** (Hz), a derived unit equal to the reciprocal of seconds (1/s or s^{-1}). The hertz is defined as the frequency of a periodic phenomenon for which the period is one second; thus, it is equivalent to one cycle per second (cps) or one revolution per second (rev/s). It is customarily used for mechanical vibrations, sound waves, and electromagnetic waves, and occasionally it is used for rotational frequency instead of the traditional units of revolution per minute (rpm) and revolution per second (rev/s).*

Two other derived units that have special names in SI are the **watt** (W) and the **pascal** (Pa). The watt is the unit of power, which is work per unit of time, and one watt is equal to one joule per second (J/s) or one newton meter per second (N · m/s). The pascal is the unit of pressure and stress, or force per unit area, and is equal to one newton per square meter (N/m^2).**

The **liter** is not an accepted SI unit, yet it is so commonly used that it cannot be discarded easily. Therefore, SI permits its use under limited conditions for volumetric capacity, dry measure, and liquid measure. Both uppercase L and lowercase l are permitted as symbols for liter in SI, but in the United States only L is permitted (to avoid confusion with the numeral 1). The only prefixes permitted with liter are milli and micro.

Loads on structures, whether due to gravity or other actions, are usually expressed in force units, such as newtons, newtons per meter, or pascals (newtons per square meter). Examples of such loads are a concentrated load of 25 kN

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*Heinrich Rudolf Hertz (1857–1894) was a German physicist who discovered electromagnetic waves and showed that light waves and electromagnetic waves are identical.

**James Watt (1736–1819) was a Scottish inventor and engineer who developed a practical steam engine and discovered the composition of water. Watt also originated the term "horsepower." Blaise Pascal (1623–1662) was a French mathematician and philosopher. He founded probability theory, constructed the first calculating machine, and proved experimentally that atmospheric pressure varies with altitude.

acting on an axle, a uniformly distributed load of intensity 800 N/m acting on a small beam, and air pressure of intensity 2.1 kPa acting on an airplane wing.

However, there is one circumstance in SI in which it is permissible to express a load in mass units. If the load acting on a structure is produced by gravity acting on a mass, then that load may be expressed in mass units (kilograms, kilograms per meter, or kilograms per square meter). The usual procedure in such cases is to convert the load to force units by multiplying by the acceleration of gravity ($g = 9.81$ m/s^2).

SI Prefixes

Multiples and submultiples of SI units (both base units and derived units) are created by attaching prefixes to the units (see **Table A-3** for a list of prefixes). The use of a prefix avoids unusually large or small numbers. The general rule is that prefixes should be used to keep numbers in the range 0.1 to 1000.

All of the recommended prefixes change the size of the quantity by a multiple or submultiple of three. Similarly, when powers of 10 are used as multipliers, the exponents of 10 should be multiples of three (for example, 40×10^3 N is satisfactory but 400×10^2 N is not). Also, the exponent on a unit with a prefix refers to the entire unit; for instance, the symbol mm^2 means (mm)^2 and not m(m)^2.

Styles for Writing SI Units

Rules for writing SI units have been established by international agreement, and some of the most pertinent ones are described here. Examples of the rules are shown in parentheses.

(1) Units are always written as symbols (kg) in equations and numerical calculations. In text, units are written as words (kilograms) unless numerical values are being reported, in which case either words or symbols may be used (12 kg or 12 kilograms).

(2) Multiplication is shown in a compound unit by a raised dot (kN·m). When the unit is written in words, no dot is required (kilonewton meter).

TABLE A-3 SI PREFIXES

Prefix	Symbol	Multiplication factor		
tera	T	10^{12}	=	1 000 000 000 000
giga	G	10^{9}	=	1 000 000 000
mega	M	10^{6}	=	1 000 000
kilo	k	10^{3}	=	1 000
hecto	h	10^{2}	=	100
deka	da	10^{1}	=	10
deci	d	10^{-1}	=	0.1
centi	c	10^{-2}	=	0.01
milli	m	10^{-3}	=	0.001
micro	μ	10^{-6}	=	0.000 001
nano	n	10^{-9}	=	0.000 000 001
pico	p	10^{-12}	=	0.000 000 000 001

Note: The use of the prefixes hecto, deka, deci, and centi is not recommended in SI.

(3) Division is shown in a compound unit by a slash (or *solidus*) or by multiplication using a negative exponent (m/s or m·s^{-1}). When the unit is written in words, the slash is always replaced by "per" (meter per second).

(4) A space is always used between a number and its units (200 Pa or 200 pascals) with the exception of the degree symbol (either angle or temperature), where no space is used between the number and the symbol (45°, 20°C).

(5) Units and their prefixes are always printed in roman type (that is, upright or vertical type) and never in italic type (slanted type), even when the surrounding text is in italic type.

(6) When written as words, units are not capitalized (newton) except at the beginning of a sentence or in capitalized material such as a title. When written as a symbol, units are capitalized when they are derived from the name of a person (N). An exception is the symbol for liter, which may be either L or l, but the use of uppercase L is preferred to avoid confusion with the numeral 1. Also, some prefixes are written with capital letters when used in symbols (MPa) but not when used in words (megapascal).

(7) When written as words, units are singular or plural as appropriate to the context (1 kilometer, 20 kilometers, 6 seconds). When written as symbols, units are always singular (1 km, 20 km, 6 s). The plural of hertz is hertz; the plurals of other units are formed in the customary manner (newtons, watts).

(8) Prefixes are not used in the denominator of a compound unit. An exception is the kilogram (kg),which is a base unit and therefore the letter "k" is not considered as a prefix. For example, we can write kN/m but not N/mm, and we can write J/kg but not mJ/g.

Pronunciation of SI Prefixes and Units

A guide to the pronunciation of a few SI names that are sometimes mispronounced is given in **Table A-4**. For instance, kilometer is pronounced

TABLE A-4 PRONUNCIATION OF SI PREFIXES AND UNITS

Prefix	Pronunciation
tera	same as *terra*, as in *terra firma*
giga	pronounced *jig-uh*; with *a* pronounced as in *about* (Alternate pronunciation: *gig-uh*)
mega	same as *mega* in *megaphone*
kilo	pronounced *kill-oh*; rhymes with *pillow*
milli	pronounced *mill-eh*, as in *military*
micro	same as *micro* in *microphone*
nano	pronounced *nan-oh*; rhymes with *man-oh*
pico	pronounced *pea-ko*
	Note: The first syllable of every prefix is accented.
Unit	**Pronunciation**
joule	pronounced *jool*; rhymes with *cool* and *pool*
kilogram	pronounced *kill-oh-gram*
kilometer	pronounced *kill-oh-meter*
pascal	pronounced *pas-kal*, with the accent on *kal*

kill-oh-meter, not *kil-om-eter*. The only prefix that generates arguments is giga—the official pronunciation is *jig-uh*, but many people say *gig-uh*.

A.3 U.S. CUSTOMARY UNITS

The units of measurement traditionally used in the United States have never been made mandatory by the government; hence for lack of a better name they are called the "customary" units. In this system the **base units** of relevance to mechanics are the foot (ft) for length, second (s) for time, and pound (lb) for force. The **foot** is defined as

$$1 \text{ ft} = 0.3048 \text{ m (exactly)} \qquad \text{(A-9)}$$

The **second** is the same as in SI and is described in the preceding section.

The **pound** is defined as the **force** that will give to a certain standard mass an acceleration equal to the acceleration of gravity. In other words, the pound is the weight of the standard mass, which is defined as 0.45359237 kg (exactly). The weight of this amount of mass (see Eq. A-4) is

$$W = (0.45359237 \text{ kg})(9.806650 \text{ m/s}^2) = 4.448222 \text{ N}$$

in which the standard international value of g is used (see Eq. A-5). Thus, the pound is defined as follows:

$$1 \text{ lb} = 4.448222 \text{ N} \qquad \text{(A-10)}$$

which shows that the pound (like the foot) is actually defined in terms of SI units.

The unit of **mass** in USCS, called the **slug**, is a derived unit defined as the mass that will be accelerated one foot per second squared when acted upon by a force of one pound. Writing Newton's second law in the form $m = F/a$, we get

$$1 \text{ slug} = \frac{1 \text{ pound}}{1 \text{ ft/s}^2}$$

which shows that the slug is expressed in terms of base units by the formula

$$1 \text{ slug} = 1 \text{ lb-s}^2/\text{ft} \qquad \text{(A-11)}$$

To obtain the mass of an object of known weight, we use the second law in the form

$$m = \frac{W}{g} \qquad \text{(A-12)}$$

where m is the mass in slugs, W is the weight in pounds, and g is the acceleration of gravity in feet per second squared.

As discussed previously, the value of g depends upon the location, but in calculations where location is not relevant, the standard international value of g may be used:

$$g = 32.1740 \text{ ft/s}^2 \qquad \text{(A-13)}$$

For ordinary purposes, the recommended value is

$$g = 32.2 \text{ ft/s}^2 \qquad \text{(A-14)}$$

From the preceding equations we conclude that an object having a mass of 1 slug will weigh 32.2 pounds at the earth's surface.

Another unit of mass in USCS is the pound-mass (lbm), which is the mass of an object weighing 1 pound, that is, 1 lbm = 1/32.2 slug.

As mentioned previously, **atmospheric pressure** varies considerably with local conditions; however, for many purposes the standard international value may be used:

$$1 \text{ standard atmosphere} = 14.6959 \text{ pounds per square inch} \qquad \text{(A-15)}$$

or, for ordinary engineering work:

$$1 \text{ standard atmosphere} = 14.7 \text{ psi} \qquad \text{(A-16)}$$

These values are intended for use in calculations and obviously do not represent the actual atmospheric pressure.

The unit of **work** and **energy** in USCS is the **foot-pound** (ft-lb), defined as the work done when the point of application of a force of one pound is displaced a distance of one foot in the direction of the force. The unit of **moment** or **torque** is the **pound-foot** (lb-ft), which comes from the fact that moment is expressed in units of force times length. Although in reality the same units apply to work, energy, and moment, it is common practice to use the pound-foot for moment and the foot-pound for work and energy.

The symbols and formulas for the most important USCS units used in mechanics are listed in **Table A-1**.

Many additional units from the U. S. Customary and Imperial systems appear in the mechanics literature; a few of these units are listed in the lower part of **Table A-2**.

A.4 TEMPERATURE UNITS

Temperature is measured in SI by a unit called the kelvin (K), and the corresponding scale is the **Kelvin temperature scale**. The Kelvin scale is an absolute scale, which means that its origin (zero kelvins, or 0 K) is at absolute zero temperature, a theoretical temperature characterized by the complete absence of heat. On the Kelvin scale, water freezes at approximately 273 K and boils at approximately 373 K.

For nonscientific purposes the **Celsius temperature scale** is normally used. The corresponding unit of temperature is the degree Celsius (°C), which is equal to one kelvin. On this scale, water freezes at approximately zero degrees (0°C) and boils at approximately 100 degrees (100°C) under certain standard conditions. The Celsius scale is also known as the *centigrade temperature scale*.

The relationship between Kelvin temperature and Celsius temperature is given by the following equations:

$$\text{Temperature in degrees Celsius} = \text{temperature in kelvins} - 273.15$$

or

$$T(^\circ\text{C}) = T(\text{K}) - 273.15 \qquad \text{(A-17)}$$

where T denotes the temperature. When working with *changes* in temperature, or *temperature intervals*, as is usually the case in mechanics, either unit can be used because the intervals are the same.*

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*Lord Kelvin (1824–1907), William Thomson, was a British physicist who made many scientific discoveries, developed theories of heat, and proposed the absolute scale of temperature. Anders Celsius (1701–1744) was a Swedish scientist and astronomer. In 1742 he developed the temperature scale in which 0 and 100 correspond, respectively, to the freezing and boiling points of water.

The U.S. Customary unit for temperature is the degree Fahrenheit (°F). On the **Fahrenheit temperature scale**, water freezes at approximately 32 degrees (32°F) and boils at approximately 212 degrees (212°F). Each Fahrenheit degree is exactly 5/9 of one kelvin or one degree Celsius. The corresponding absolute scale is the **Rankine temperature scale**, related to the Fahrenheit scale by the equation

$$T(°\text{F}) = T(°\text{R}) - 459.67 \qquad \text{(A-18)}$$

Thus, absolute zero corresponds to -459.67°F.*

The **conversion formulas** between the Fahrenheit and Celsius scales are as follows:

$$T(°\text{C}) = \frac{5}{9}[T(°\text{F}) - 32] \qquad T(°\text{F}) = \frac{9}{5}T(°\text{C}) + 32 \qquad \text{(A-19a,b)}$$

As before, T denotes the temperature on the indicated scale.

A.5 CONVERSIONS BETWEEN UNITS

Quantities given in either USCS or SI units can be converted quickly to the other system by using the **conversion factors** listed in Table A-5.

If the given quantity is expressed in USCS units, it can be converted to SI units by *multiplying* by the conversion factor. To illustrate this process, assume that the stress in a beam is given as 10,600 psi and we wish to convert this quantity to SI units. From Table A-5 we see that a stress of 1 psi converts to 6894.76 Pa. Therefore, the conversion of the given value is performed in the following manner:

$$(10{,}600\text{ psi})(6894.76) = 73100000\text{ Pa} = 73.1\text{ MPa}$$

Because the original value is given to three significant digits, we have rounded the final result to three significant digits also (see Appendix B for a discussion of significant digits). Note that the conversion factor of 6894.76 has units of pascals divided by pounds per square inch, and therefore the equation is dimensionally correct.

To reverse the conversion process (that is, to convert from SI units to USCS units), the quantity in SI units is *divided* by the conversion factor. For instance, suppose that the moment of inertia of the cross-sectional area of a beam is given as 94.73×10^6 mm^4. Then the moment of inertia in USCS units is

$$\frac{94.73 \times 10^6\text{ mm}^4}{416{,}231} = 228\text{ in.}^4$$

in which the term 416,231 is the conversion factor for moment of inertia.

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*William John Macquorn Rankine (1820–1872) was a Scottish engineer and physicist. He made important contributions in such diverse fields as thermodynamics, light, sound, stress analysis, and bridge engineering. Gabriel Daniel Fahrenheit (1686–1736) was a German physicist who experimented with thermometers and made them more accurate by using mercury in the tube. He set the origin (0°) of his temperature scale at the freezing point of a mixture of ice, salt, and water.

TABLE A-5 CONVERSIONS BETWEEN U.S. CUSTOMARY UNITS AND SI UNITS

U.S. Customary unit		Times conversion factor		Equals SI unit	
		Accurate	Practical		
Acceleration (linear)					
foot per second squared	ft/s^2	0.3048*	0.305	meter per second squared	m/s^2
inch per second squared	in./s^2	0.0254*	0.0254	meter per second squared	m/s^2
Area					
square foot	ft^2	0.09290304*	0.0929	square meter	m^2
square inch	in.^{2}	645.16*	645	square millimeter	mm^2
Density (mass)					
slug per cubic foot	slug/ft^3	515.379	515	kilogram per cubic meter	kg/m^3
Density (weight)					
pound per cubic foot	lb/ft^3	157.087	157	newton per cubic meter	N/m^3
pound per cubic inch	lb/in.^{3}	271.447	271	kilonewton per cubic meter	kN/m^3
Energy; work					
foot-pound	ft-lb	1.35582	1.36	joule (N·m)	J
inch-pound	in.-lb	0.112985	0.113	joule	J
kilowatt-hour	kWh	3.6*	3.6	megajoule	MJ
British thermal unit	Btu	1055.06	1055	joule	J
Force					
pound	lb	4.44822	4.45	newton (kg·m/s^2)	N
kip (1000 pounds)	k	4.44822	4.45	kilonewton	kN
Force per unit length					
pound per foot	lb/ft	14.5939	14.6	newton per meter	N/m
pound per inch	lb/in.	175.127	175	newton per meter	N/m
kip per foot	k/ft	14.5939	14.6	kilonewton per meter	kN/m
kip per inch	k/in.	175.127	175	kilonewton per meter	kN/m
Length					
foot	ft	0.3048*	0.305	meter	m
inch	in.	25.4*	25.4	millimeter	mm
mile	mi	1.609344*	1.61	kilometer	km
Mass					
slug	lb-s^2/ft	14.5939	14.6	kilogram	kg
Moment of a force; torque					
pound-foot	lb-ft	1.35582	1.36	newton meter	N·m
pound-inch	lb-in.	0.112985	0.113	newton meter	N·m
kip-foot	k-ft	1.35582	1.36	kilonewton meter	kN·m
kip-inch	k-in.	0.112985	0.113	kilonewton meter	kN·m

*An asterisk denotes an *exact* conversion factor

(Continued)

Note: **To convert from SI units to USCS units, *divide* by the conversion factor**

TABLE A-5 (Continued)

U.S. Customary unit		Times conversion factor		Equals SI unit	
		Accurate	Practical		
Moment of inertia (area)					
inch to fourth power	in.^{4}	416,231	416,000	millimeter to fourth power	mm^4
inch to fourth power	in.^{4}	0.416231×10^{-6}	0.416×10^{-6}	meter to fourth power	m^4
Moment of inertia (mass)					
slug foot squared	slug-ft^2	1.35582	1.36	kilogram meter squared	kg·m^2
Power					
foot-pound per second	ft-lb/s	1.35582	1.36	watt (J/s or N·m/s)	W
foot-pound per minute	ft-lb/min	0.0225970	0.0226	watt	W
horsepower (550 ft-lb/s)	hp	745.701	746	watt	W
Pressure; stress					
pound per square foot	psf	47.8803	47.9	pascal (N/m^2)	Pa
pound per square inch	psi	6894.76	6890	pascal	Pa
kip per square foot	ksf	47.8803	47.9	kilopascal	kPa
kip per square inch	ksi	6.89476	6.89	megapascal	MPa
Section modulus					
inch to third power	in.^{3}	16,387.1	16,400	millimeter to third power	mm^3
inch to third power	in.^{3}	16.3871×10^{-6}	16.4×10^{-6}	meter to third power	m^3
Velocity (linear)					
foot per second	ft/s	0.3048*	0.305	meter per second	m/s
inch per second	in./s	0.0254*	0.0254	meter per second	m/s
mile per hour	mph	0.44704*	0.447	meter per second	m/s
mile per hour	mph	1.609344*	1.61	kilometer per hour	km/h
Volume					
cubic foot	ft^3	0.0283168	0.0283	cubic meter	m^3
cubic inch	in.^{3}	16.3871×10^{-6}	16.4×10^{-6}	cubic meter	m^3
cubic inch	in.^{3}	16.3871	16.4	cubic centimeter (cc)	cm^3
gallon (231 in.^{3})	gal.	3.78541	3.79	liter	L
gallon (231 in.^{3})	gal.	0.00378541	0.00379	cubic meter	m^3

*An asterisk denotes an *exact* conversion factor

***Note:* To convert from SI units to USCS units, *divide* by the conversion factor**

B

Problem Solving

B.1 TYPES OF PROBLEMS

The study of mechanics of materials divides naturally into two parts: first, *understanding* the general concepts and principles, and second, *applying* those concepts and principles to physical situations. An understanding of the general concepts is obtained by studying the discussions and derivations presented in books such as this one. Skill in applying the concepts is accomplished by solving problems on your own. Of course, these two aspects of mechanics are closely related, and many experts in mechanics will argue that you don't really understand the concepts if you can't apply them. It is easy to recite the principles, but applying them to real situations requires an in-depth understanding. That is why teachers of mechanics place so much emphasis on problems. Problem solving gives meaning to the concepts and also provides an opportunity to gain experience and develop judgment.

Some of the homework problems in this book require symbolic solutions and others require numerical solutions. In the case of **symbolic problems** (also called *analytical*, *algebraic*, or *literal problems*), the data are supplied in the form of symbols for the various quantities, such as P for load, L for length, and E for modulus of elasticity. Such problems are solved in terms of algebraic variables, and the results are expressed as formulas or mathematical expressions. Symbolic problems usually do not involve numerical calculations, except when numerical data are substituted into the final symbolic result in order to obtain a numerical value. However, this final substitution of numerical data should not obscure the fact that the problem was solved in symbolic terms.

In contrast, **numerical problems** are those in which the data are given in the form of numbers (with appropriate units); for example, a load might be given as 12 kN, a length as 3 m, and a dimension as 150 mm. The solution of a numerical problem is carried out by performing calculations from the beginning, and the results, both intermediate and final, are in the form of numbers.

An advantage of a numerical problem is that the magnitudes of all quantities are evident at every stage of the solution, thereby providing an opportunity to observe whether the calculations are producing reasonable results. Also, a numerical solution makes it possible to keep the magnitudes of quantities within prescribed limits. For instance, suppose the stress at a particular point in a beam must not exceed a certain allowable value. If this stress is calculated as an intermediate step in the numerical solution, you can verify immediately whether or not it exceeds the limit.

Symbolic problems have several advantages too. Because the results are algebraic formulas or expressions, you can see immediately how the variables affect the answers. For instance, if a load appears to the first power in the numerator of the final result, you know that doubling the load will double the result. Equally important is the fact that a symbolic solution shows what variables do *not* affect the result. For instance, a certain quantity may cancel out of the solution, a fact that might not even be noticed in a numerical solution. Furthermore, a symbolicsolution makes it convenient to check the dimensional homogeneity of all terms in the solution. And most important, a symbolic solution provides a general formula that is applicable to many different problems, each with a different set of numerical data. In contrast, a numerical solution is good for only one set of circumstances, and a complete new solution is required if the data are changed. Of course, symbolic solutions are not feasible when the formulas become too complex to manipulate; when that happens, a numerical solution is required.

In more advanced work in mechanics, problem solving requires the use of **numerical methods**. This term refers to a wide variety of computational methods, including standard mathematical procedures (such as numerical integration and numerical solution of differential equations) and advanced methods of analysis (such as the finite-element method). Computer programs for these methods are are readily available. More specialized computer programs are also available for performing routine tasks, such as finding deflections of beams and finding principal stresses. However, when studying mechanics of materials, we concentrate on the concepts rather than on the use of particular computer programs.

B.2 STEPS IN SOLVING PROBLEMS

The procedures used in solving problems will vary among individuals and will vary according to the type of problem. Nevertheless, the following suggestions will help in reducing mistakes.

1. Make a clear statement of the problem and draw a figure portraying the mechanical or structural system to be investigated. An important part of this step is identifying what is known and what is to be found.

2. Simplify the mechanical or structural system by making assumptions about its physical nature. This step is called *modeling*, because it involves creating (on paper) an idealized model of the real system. The objective is to create a model that represents the real system to a sufficient degree of accuracy that the results obtained from the model can be applied to the real system.

Here are a few examples of idealizations used in modeling mechanical systems. (a) Finite objects are sometimes modeled as particles, as when determining the forces acting on a joint of a truss. (b) Deformable bodies are sometimes represented as rigid bodies, as when finding the reactions of a statically determinate beam or the forces in the members of a statically determinate truss. (c) The geometry and shapes of objects may be simplified, as when we consider the earth to be a sphere or a beam to be perfectly straight. (d) Distributed forces acting on machines and structures may be represented by equivalent concentrated forces. (e) Forces that are small compared to other forces, or forces that are known to have only a minor effect on the results, may be disregarded (friction forces are sometimes in this category). (f) Supports of structures often may be considered as immovable.

3. Draw large and clear sketches as you solve problems. Sketches always aid in understanding the physical situation and often bring out aspects of the problem that would otherwise be overlooked.

4. Apply the principles of mechanics to the idealized model to obtain the governing equations. In statics, the equations usually are equations of equilibrium obtained from Newton's first law; in dynamics, they usually are equations of motion obtained from Newton's second law. In mechanics of materials, the equations are associated with stresses, strains, deformations, and displacements.

5. Use mathematical and computational techniques to solve the equations and obtain results, either in the form of mathematical formulas or numerical values.

6. Interpret the results in terms of the physical behavior of the mechanical or structural system; that is, give meaning or significance to the results, and draw conclusions about the behavior of the system.

7. Check the results in as many ways as you can. Because errors can be disastrous and expensive, engineers should never rely on a single solution.

8. Finally, present your solution in clear, neat fashion so that it can be easily reviewed and checked by others.

B.3 DIMENSIONAL HOMOGENEITY

The basic concepts in mechanics are length, time, mass, and force. Each of these physical quantities has a **dimension**, that is, a generalized unit of measurement. For example, consider the concept of length. There are many units of length, such as the meter, kilometer, yard, foot, and inch, yet all of these units have something in common—each one represents a distinct length and not some other quantity such as volume or force. Therefore, we can refer to the *dimension of length* without being specific as to the particular unit of measurement. Similar comments can be made for the dimensions of time, mass, and force. These four dimensions are customarily denoted by the symbols L, T, M, and F, respectively.

Every equation, whether in numeric form or symbolic form, must be **dimensionally homogeneous**, that is, the dimensions of all terms in the equation must be the same. To check the dimensional correctness of an equation, we disregard numerical magnitudes and write only the dimensions of each quantity in the equation. The resulting equation must have identical dimensions in all terms.

As an example, consider the following equation for the deflection δ at the midpoint of a simple beam with a uniformly distributed load:

$$\delta = \frac{5qL^4}{384EI}$$

The corresponding dimensional equation is obtained by replacing each quantity by its dimensions; thus, the deflection δ is replaced by the dimension L, the intensity of uniform load q is replaced by F/L (force per unit of length), the length L of the beam is replaced by the dimension L, the modulus of elasticity E is replaced by $\mathrm{F/L^2}$ (force per unit of area), and the moment of inertia I is replaced by $\mathrm{L^4}$. Therefore, the dimensional equation is

$$\mathrm{L} = \frac{\mathrm{(F/L)L^4}}{\mathrm{(F/L^2)L^4}}$$

When simplified, this equation reduces to the dimensional equation L = L, as expected.

Dimensional equations can be written either in generalized terms using the LTMF notation or in terms of the actual units being used in the problem. For instance, if we are making calculations for the preceding beam deflection using USCS units, we can write the dimensional equation as follows:

$$\text{in.} = \frac{(\text{lb/in.})\text{in.}^4}{(\text{lb/in.}^2)\text{in.}^4}$$

which reduces to in. = in. and is dimensionally correct. Frequent checks for dimensional homogeneity (or *consistency of units*) help to eliminate errors when performing derivations and calculations.

B.4 SIGNIFICANT DIGITS

Engineering calculations are performed by calculators and computers that operate with great precision. For instance, some computers routinely perform calculations with more than 25 digits in every numerical value, and output values with 10 or more digits are available in even the most inexpensive hand-held calculators. Under these conditions it is important to realize that the accuracy of the results obtained from an engineering analysis is determined not only by the calculations but also by factors such as the accuracy of the given data, the approximations inherent in the analytical models, and the validity of the assumptions used in the theories. In many engineering situations, these considerations mean that the results are valid to only two or three significant digits.

As an example, suppose that a computation yields the result $R = 6287.46$ lb for the reaction of a statically indeterminate beam. To state the result in this manner is misleading, because it implies that the reaction is known to the nearest 1/100 of a pound even though its magnitude is over 6000 pounds. Thus, it implies an accuracy of approximately 1/600,000 and a precision of 0.01 lb, neither of which is justified. Instead, the accuracy of the calculated reaction depends upon matters such as the following: (1) how accurately the loads, dimensions, and other data used in the analysis are known, and (2) the approximations inherent in the theories of beam behavior. Most likely, the reaction R in this example would be known only to the nearest 10 pounds, or perhaps only to the nearest 100 pounds. Consequently, the result of the computation should be stated as either $R = 6290$ lb or $R = 6300$ lb.

To make clear the accuracy of a given numerical value, it is common practice to use **significant digits**. A significant digit is a digit from 1 to 9 or any zero not used to show the position of the decimal point; for instance, the numbers 417, 8.29, 7.30, and 0.00254 each have three significant digits. However, the number of significant digits in a number such as 29,000 is not apparent. It may have two significant digits, with the three zeros serving only to locate the decimal point, or it may have three, four, or five significant digits if one or more of the zeros is valid. By using powers of ten, the accuracy of a number such as 29,000 can be made clearer. When written as 29×10^3 or 0.029×10^6, the number is understood to have two significant digits; when written as 29.0×10^3 or 0.0290×10^6, it has three significant digits.

When a number is obtained by calculation, its accuracy depends upon the accuracy of the numbers used in performing the calculations. A rule of thumb that serves for **multiplication and division** is the following:The number of significant digits in the calculated result is the same as the least number of significant digits in any of the numbers used in the calculation. As an illustration, consider the product of 2339.3 and 35.4. The calculated result is 82,811.220 when recorded to eight digits. However, stating the result in this manner is misleading because it implies much greater accuracy than is warranted by either of the original numbers. Inasmuch as the number 35.4 has only three significant digits, the proper way to write the result is 82.8×10^3.

For calculations involving **addition or subtraction** of a column of numbers, the last significant digit in the result is found in the last column of digits that has significant digits in all of the numbers being added or subtracted. To make this notion clearer, consider the following three examples:

	459.637	838.49	856,400
	+ 7.2	− 7	− 847,900
Result from calculator:	4 66.837	831.49	8,500
Write the result as:	466.8	831	8,500

In the first example, the number 459.637 has six significant digits and the number 7.2 has two. When added, the result has four significant digits because all digits in the result to the right of the column containing the 2 are meaningless. In the second example, the number 7 is accurate to one significant digit (that is, it is not an exact number). Therefore, the final result is accurate only as far as the column containing the 7, which means it has three significant digits and is recorded as 831. In the third example, the numbers 856,400 and 847,900 are assumed to be accurate to four significant digits, but the result of the subtraction is accurate to only two significant digits since none of the zeros is significant. In general, subtraction results in reduced accuracy.

These three examples show that numbers obtained by calculation may contain superfluous digits having no physical meaning. Therefore, when reporting such numbers as final results, you should give only those digits that are significant.

In mechanics of materials, the data for problems are usually accurate to about 1%, or perhaps 0.1% in some cases, and therefore the final results should be reported to a comparable accuracy. When greater accuracy is warranted, it will be obvious from the statement of the problem.

Although the use of significant digits provides a handy way to deal with the matter of **numerical accuracy**, it should be recognized that significant digits are not valid indicators of accuracy. To illustrate this fact, consider the numbers 999 and 101. Three significant digits in the number 999 correspond to an accuracy of 1/999, or 0.1%, whereas the same number of significant digits in the number 101 corresponds to an accuracy of only 1/101, or 1.0%. This disparity in accuracy can be reduced by always using one additional significant digit for numbers beginning with the digit 1. Thus, four significant digits in the number 101.1 gives about the same accuracy as three significant digits in the number 999.

In this book we generally will follow the rule that *final* numerical results beginning with the digits 2 through 9 should be recorded to three significant digits and those beginning with the digit 1 should be recorded to four significant digits. However, to preserve numerical accuracy and avoid round-off errors during the calculation process, the results of *intermediate* calculations will usually be recorded with additional digits.

Many of the numbers entering into our calculations are exact, for example, the number π, fractions such as 1/2, and integers such as the number 48 in the formula $PL^3/48EI$ for a beam deflection. Exact numbers are significant to an infinite number of digits and therefore have no role in determining the accuracy of a calculated result.

B.5 ROUNDING OF NUMBERS

The process of discarding the insignificant digits and keeping only the significant ones is called *rounding*. To illustrate the process, assume that a number is to be rounded to three significant digits. Then the following rules apply:

(a) If the fourth digit is less than 5, the first three digits are left unchanged and all succeeding digits are dropped or replaced by zeros. For example, 37.44 rounds to 37.4 and 673,289 rounds to 673,000.

(b) If the fourth digit is greater than 5, or if the fourth digit is 5 and is followed by at least one digit other than zero, then the third digit is increased by 1 and all following digits are dropped or replaced by zeros. For example, 26.37 rounds to 26.4 and 3.245002 rounds to 3.25.

(c) Finally, if the fourth digit is 5 and all following digits (if any) are zeros, then the third digit is unchanged if it is an even number and increased by 1 if it is an odd number, and the 5 is replaced by a zero. (Trailing and leading zeros are retained only if they are needed to locate the decimal point.) This process is usually described as "rounding to the even digit." Since the occurrence of even and odd digits is more or less random, the use of this rule means that numbers are rounded upward about as often as downward, thereby reducing the chances of accumulating round-off errors.

The rules described in the preceding paragraphs for rounding to three significant digits apply in the same general manner when rounding to any other number of significant digits.

C

Mathematical Formulas

Mathematical Constants

$\pi = 3.14159\ldots \qquad e = 2.71828\ldots \qquad 2\pi \text{ radians} = 360 \text{ degrees}$

$1 \text{ radian} = \dfrac{180}{\pi} \text{ degrees} = 57.2958° \quad 1 \text{ degree} = \dfrac{\pi}{180} \text{ radians} = 0.0174533 \text{ rad}$

Conversions: Multiply degrees by $\dfrac{\pi}{180}$ to obtain radians

Multiply radians by $\dfrac{180}{\pi}$ to obtain degrees

Exponents

$$A^n A^m = A^{n+m} \qquad \frac{A^m}{A^n} = A^{m-n} \qquad (A^m)^n = A^{mn} \qquad A^{-m} = \frac{1}{A^m}$$

$$(AB)^n = A^n B^n \qquad \left(\frac{A}{B}\right)^n = \frac{A^n}{B^n} \qquad A^{m/n} = \sqrt[n]{A^m} \qquad A^0 = 1 \quad (A \neq 0)$$

Logarithms

$\log \equiv$ common logarithm (logarithm to the base 10) $\qquad 10^x = y \qquad \log y = x$

$\ln \equiv$ natural logarithm (logarithm to the base e) $\qquad e^x = y \qquad \ln y = x$

$$e^{\ln A} = A \qquad 10^{\log A} = A \qquad \ln e^A = A \qquad \log 10^A = A$$

$$\log AB = \log A + \log B \qquad \log \frac{A}{B} = \log A - \log B \qquad \log \frac{1}{A} = -\log A$$

$$\log A^n = n \log A \qquad \log 1 = \ln 1 = 0 \qquad \log 10 = 1 \qquad \ln e = 1$$

$$\ln A = (\ln 10)(\log A) = 2.30259 \log A \qquad \log A = (\log e)(\ln A) = 0.434294 \ln A$$

Trigonometric Functions

$$\tan x = \frac{\sin x}{\cos x} \qquad \cot x = \frac{\cos x}{\sin x} \qquad \sec x = \frac{1}{\cos x} \qquad \csc x = \frac{1}{\sin x}$$

$$\sin^2 x + \cos^2 x = 1 \qquad \tan^2 x + 1 = \sec^2 x \qquad \cot^2 x + 1 = \csc^2 x$$

$$\sin(-x) = -\sin x \qquad \cos(-x) = \cos x \qquad \tan(-x) = -\tan x$$

$$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y \qquad \cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$$

$$\sin 2x = 2 \sin x \cos x \qquad \cos 2x = \cos^2 x - \sin^2 x \qquad \tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$$

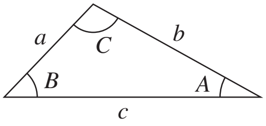
$$\tan x = \frac{1 - \cos 2x}{\sin 2x} = \frac{\sin 2x}{1 + \cos 2x}$$

$$\sin^2 x = \frac{1}{2}(1 - \cos 2x) \qquad \cos^2 x = \frac{1}{2}(1 + \cos 2x)$$

For any triangle with sides a, b, c and opposite angles A, B, C:

Law of sines $$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Law of cosines $$c^2 = a^2 + b^2 - 2ab \cos C$$



Quadratic Equation and Quadratic Formula

$$ax^2 + bx + c = 0 \qquad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Infinite Series

$$\frac{1}{1 + x} = 1 - x + x^2 - x^3 + \dots \qquad (-1 < x < 1)$$

$$\sqrt{1 + x} = 1 + \frac{x}{2} - \frac{x^2}{8} + \frac{x^3}{16} - \dots \qquad (-1 < x < 1)$$

$$\frac{1}{\sqrt{1 + x}} = 1 - \frac{x}{2} + \frac{3x^2}{8} - \frac{5x^3}{16} + \dots \qquad (-1 < x < 1)$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots \qquad (-\infty < x < \infty)$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots \qquad (-\infty < x < \infty)$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots \qquad (-\infty < x < \infty)$$

Note: If x is very small compared to 1, only the first few terms in the series are needed.

Derivatives

$$\frac{d}{dx}(ax) = a \qquad \frac{d}{dx}(x^n) = nx^{n-1} \qquad \frac{d}{dx}(au) = a\frac{du}{dx}$$

$$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx} \qquad \frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v(du/dx) - u(dv/dx)}{v^2}$$

$$\frac{d}{dx}(u^n) = nu^{n-1}\frac{du}{dx} \qquad \frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx} \qquad \frac{du}{dx} = \frac{1}{dx/du}$$

$$\frac{d}{dx}(\sin u) = \cos u\,\frac{du}{dx} \qquad \frac{d}{dx}(\cos u) = -\sin u\,\frac{du}{dx}$$

$$\frac{d}{dx}(\tan u) = \sec^2 u\,\frac{du}{dx} \qquad \frac{d}{dx}(\cot u) = -\csc^2 u\,\frac{du}{dx}$$

$$\frac{d}{dx}(\sec u) = \sec u \tan u\,\frac{du}{dx} \qquad \frac{d}{dx}(\csc u) = -\csc u \cot u\,\frac{du}{dx}$$

$$\frac{d}{dx}(\arctan u) = \frac{1}{1+u^2}\frac{du}{dx} \qquad \frac{d}{dx}(\log u) = \frac{\log e}{u}\frac{du}{dx} \qquad \frac{d}{dx}(\ln u) = \frac{1}{u}\frac{du}{dx}$$

$$\frac{d}{dx}(a^u) = a^u \ln a\,\frac{du}{dx} \qquad \frac{d}{dx}(e^u) = e^u\frac{du}{dx}$$

Indefinite Integrals

Note: A constant must be added to the result of every integration

$$\int a\,dx = ax \qquad \int u\,dv = uv - \int v\,du \quad \text{(integration by parts)}$$

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} \quad (n \neq -1) \qquad \int \frac{dx}{x} = \ln|x| \quad (x \neq 0)$$

$$\int \frac{dx}{x^n} = \frac{x^{1-n}}{1-n} \quad (n \neq 1) \qquad \int (a+bx)^n\,dx = \frac{(a+bx)^{n+1}}{b(n+1)} \quad (n \neq -1)$$

$$\int \frac{dx}{a+bx} = \frac{1}{b}\ln(a+bx) \qquad \int \frac{dx}{(a+bx)^2} = -\frac{1}{b(a+bx)}$$

$$\int \frac{dx}{(a+bx)^n} = -\frac{1}{(n-1)(b)(a+bx)^{n-1}} \quad (n \neq 1)$$

$$\int \frac{dx}{a^2+b^2x^2} = \frac{1}{ab}\tan^{-1}\frac{bx}{a} \quad (x \text{ in radians}) \quad (a > 0,\ b > 0)$$

$$\int \frac{dx}{a^2-b^2x^2} = \frac{1}{2ab}\ln\left(\frac{a+bx}{a-bx}\right) \quad (x \text{ in radians}) \quad (a > 0,\ b > 0)$$

$$\int \frac{x\,dx}{a+bx} = \frac{1}{b^2}[bx - a\ln(a+bx)] \qquad \int \frac{x\,dx}{(a+bx)^2} = \frac{1}{b^2}\left[\frac{a}{a+bx} + \ln(a+bx)\right]$$

$$\int \frac{x\,dx}{(a+bx)^3} = -\frac{a+2bx}{2b^2(a+bx)^2} \qquad \int \frac{x\,dx}{(a+bx)^4} = -\frac{a+3bx}{6b^2(a+bx)^3}$$

$$\int \frac{x^2dx}{a+bx} = \frac{1}{2b^3}[\,a+bx)(-3a+bx)+2a^2\ln(a+bx)]$$

$$\int \frac{x^2dx}{(a+bx)^2} = \frac{1}{b^3}\left[\frac{bx(2a+bx)}{a+bx} - 2a\ln(a+bx)\right]$$

$$\int \frac{x^2dx}{(a+bx)^3} = \frac{1}{b^3}\left[\frac{a(3a+4bx)}{2(a+bx)^2} + \ln(a+bx)\right]$$

$$\int \frac{x^2dx}{(a+bx)^4} = -\frac{a^2+3abx+3b^2x^2}{3b^3(a+bx)^3}$$

$$\int \sin ax\,dx = -\frac{\cos ax}{a} \qquad \int \cos ax\,dx = \frac{\sin ax}{a}$$

$$\int \tan ax\,dx = \frac{1}{a}\ln(\sec ax) \qquad \int \cot ax\,dx = \frac{1}{a}\ln(\sin ax)$$

$$\int \sec ax\,dx = \frac{1}{a}\ln(\sec ax+\tan ax) \qquad \int \csc ax\,dx = \frac{1}{a}\ln(\csc ax-\cot ax)$$

$$\int \sin^2 ax\,dx = \frac{x}{2} - \frac{\sin 2ax}{4a} \qquad \int \cos^2 ax\,dx = \frac{x}{2} + \frac{\sin 2ax}{4a} \quad (x \text{ in radians})$$

$$\int x\sin ax\,dx = \frac{\sin ax}{a^2} - \frac{x\cos ax}{a} \quad (x \text{ in radians})$$

$$\int x\cos ax\,dx = \frac{\cos ax}{a^2} + \frac{x\sin ax}{a} \quad (x \text{ in radians})$$

$$\int e^{ax}\,dx = \frac{e^{ax}}{a} \qquad \int xe^{ax}\,dx = \frac{e^{ax}}{a^2}(ax-1) \qquad \int \ln ax\,dx = x(\ln ax - 1)$$

$$\int \frac{dx}{1+\sin ax} = -\frac{1}{a}\tan\left(\frac{\pi}{4} - \frac{ax}{2}\right) \qquad \int \sqrt{a+bx}\,dx = \frac{2}{3b}(a+bx)^{3/2}$$

$$\int \sqrt{a^2+b^2x^2}\,dx = \frac{x}{2}\sqrt{a^2+b^2x^2} + \frac{a^2}{2b}\ln\left(\frac{bx}{a} + \sqrt{1+\frac{b^2x^2}{a^2}}\right)$$

$$\int \frac{dx}{\sqrt{a^2+b^2x^2}} = \frac{1}{b}\ln\left(\frac{bx}{a} + \sqrt{1+\frac{b^2x^2}{a^2}}\right)$$

$$\int \sqrt{a^2-b^2x^2}\,dx = \frac{x}{2}\sqrt{a^2-b^2x^2} + \frac{a^2}{2b}\sin^{-1}\frac{bx}{a}$$

Definite Integrals

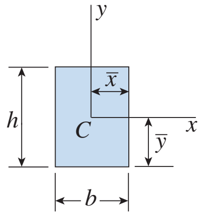
$$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx \qquad \int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$$

D

Properties of Plane Areas

Notation: A = area
$\bar{x}, \bar{y}$ = distances to centroid C
I_x, I_y = moments of inertia with respect to the x and y axes, respectively
I_{xy} = product of inertia with respect to the x and y axes
$I_P = I_x + I_y$ = polar moment of inertia with respect to the origin of the x and y axes
I_{BB} = moment of inertia with respect to axis B-B

1

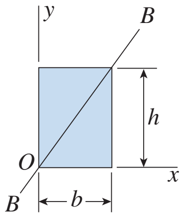


Rectangle (Origin of axes at centroid)

$$A = bh \qquad \bar{x} = \frac{b}{2} \qquad \bar{y} = \frac{h}{2}$$

$$I_x = \frac{bh^3}{12} \qquad I_y = \frac{hb^3}{12} \qquad I_{xy} = 0 \qquad I_P = \frac{bh}{12}(h^2 + b^2)$$

2

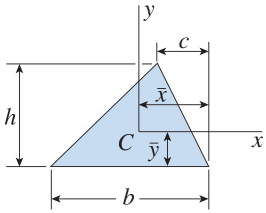


Rectangle (Origin of axes at corner)

$$I_x = \frac{bh^3}{3} \qquad I_y = \frac{hb^3}{3} \qquad I_{xy} = \frac{b^2h^2}{4} \qquad I_P = \frac{bh}{3}(h^2 + b^2)$$

$$I_{BB} = \frac{b^3h^3}{6(b^2 + h^2)}$$

3



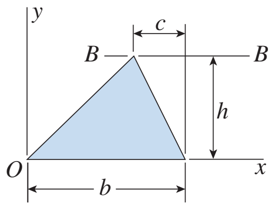
Triangle (Origin of axes at centroid)

$$A = \frac{bh}{2} \qquad \bar{x} = \frac{b + c}{3} \qquad \bar{y} = \frac{h}{3}$$

$$I_x = \frac{bh^3}{36} \qquad I_y = \frac{bh}{36}(b^2 - bc + c^2)$$

$$I_{xy} = \frac{bh^2}{72}(b - 2c) \qquad I_P = \frac{bh}{36}(h^2 + b^2 - bc + c^2)$$

4

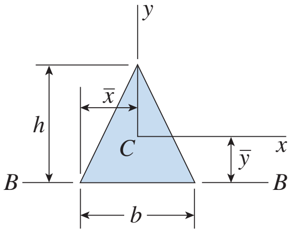


Triangle (Origin of axes at vertex)

$$I_x = \frac{bh^3}{12} \qquad I_y = \frac{bh}{12}(3b^2 - 3bc + c^2)$$

$$I_{xy} = \frac{bh^2}{24}(3b - 2c) \qquad I_{BB} = \frac{bh^3}{4}$$

5



Isosceles triangle (Origin of axes at centroid)

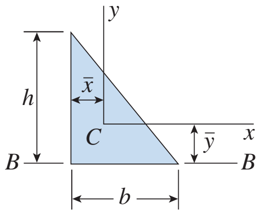
$$A = \frac{bh}{2} \qquad \bar{x} = \frac{b}{2} \qquad \bar{y} = \frac{h}{3}$$

$$I_x = \frac{bh^3}{36} \qquad I_y = \frac{hb^3}{48} \qquad I_{xy} = 0$$

$$I_P = \frac{bh}{144}(4h^2 + 3b^2) \qquad I_{BB} = \frac{bh^3}{12}$$

(*Note:* For an equilateral triangle, $h = \sqrt{3}\, b/2$.)

6



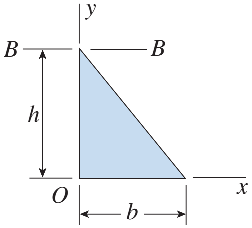
Right triangle (Origin of axes at centroid)

$$A = \frac{bh}{2} \qquad \bar{x} = \frac{b}{3} \qquad \bar{y} = \frac{h}{3}$$

$$I_x = \frac{bh^3}{36} \qquad I_y = \frac{hb^3}{36} \qquad I_{xy} = -\frac{b^2h^2}{72}$$

$$I_P = \frac{bh}{36}(h^2 + b^2) \qquad I_{BB} = \frac{bh^3}{12}$$

7

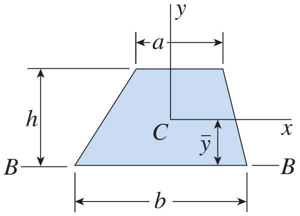


Right triangle (Origin of axes at vertex)

$$I_x = \frac{bh^3}{12} \qquad I_y = \frac{hb^3}{12} \qquad I_{xy} = \frac{b^2h^2}{24}$$

$$I_P = \frac{bh}{12}(h^2 + b^2) \qquad I_{BB} = \frac{bh^3}{4}$$

8

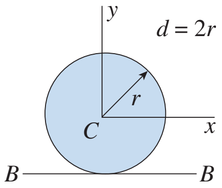


Trapezoid (Origin of axes at centroid)

$$A = \frac{h(a + b)}{2} \qquad \bar{y} = \frac{h(2a + b)}{3(a + b)}$$

$$I_x = \frac{h^3(a^2 + 4ab + b^2)}{36(a + b)} \qquad I_{BB} = \frac{h^3(3a + b)}{12}$$

9

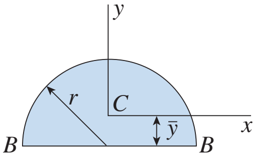


Circle (Origin of axes at center)

$$A = \pi r^2 = \frac{\pi d^2}{4} \qquad I_x = I_y = \frac{\pi r^4}{4} = \frac{\pi d^4}{64}$$

$$I_{xy} = 0 \qquad I_P = \frac{\pi r^4}{2} = \frac{\pi d^4}{32} \qquad I_{BB} = \frac{5\pi r^4}{4} = \frac{5\pi d^4}{64}$$

10

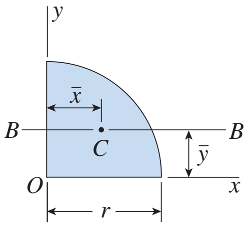


Semicircle (Origin of axes at centroid)

$$A = \frac{\pi r^2}{2} \qquad \bar{y} = \frac{4r}{3\pi}$$

$$I_x = \frac{(9\pi^2 - 64)r^4}{72\pi} \approx 0.1098r^4 \qquad I_y = \frac{\pi r^4}{8} \qquad I_{xy} = 0 \qquad I_{BB} = \frac{\pi r^4}{8}$$

11

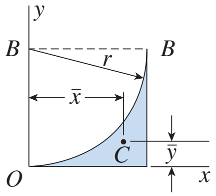


Quarter circle (Origin of axes at center of circle)

$$A = \frac{\pi r^2}{4} \qquad \bar{x} = \bar{y} = \frac{4r}{3\pi}$$

$$I_x = I_y = \frac{\pi r^4}{16} \qquad I_{xy} = \frac{r^4}{8} \qquad I_{BB} = \frac{(9\pi^2 - 64)r^4}{144\pi} \approx 0.05488r^4$$

12

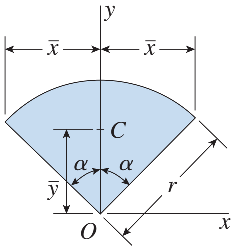


Quarter-circular spandrel (Origin of axes at point of tangency)

$$A = \left(1 - \frac{\pi}{4}\right)r^2 \qquad \bar{x} = \frac{2r}{3(4 - \pi)} \approx 0.7766r \qquad \bar{y} = \frac{(10 - 3\pi)r}{3(4 - \pi)} \approx 0.2234r$$

$$I_x = \left(1 - \frac{5\pi}{16}\right)r^4 \approx 0.01825r^4 \qquad I_y = I_{BB} = \left(\frac{1}{3} - \frac{\pi}{16}\right)r^4 \approx 0.1370r^4$$

13



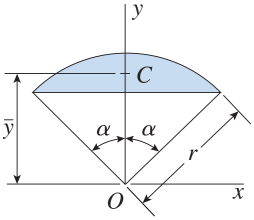
Circular sector (Origin of axes at center of circle)

α = angle in radians ($\alpha \leq \pi/2$)

$$A = \alpha r^2 \qquad \bar{x} = r \sin \alpha \qquad \bar{y} = \frac{2r \sin \alpha}{3\alpha}$$

$$I_x = \frac{r^4}{4}(\alpha + \sin \alpha \cos \alpha) \qquad I_y = \frac{r^4}{4}(\alpha - \sin \alpha \cos \alpha) \qquad I_{xy} = 0 \qquad I_P = \frac{\alpha r^4}{2}$$

14



Circular segment (Origin of axes at center of circle)

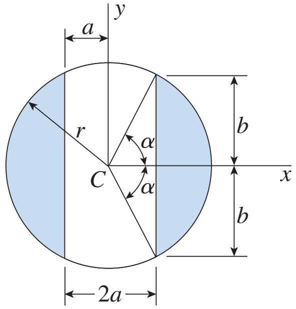
α = angle in radians $\quad (\alpha \le \pi/2)$

$$A = r^2(\alpha - \sin\alpha\cos\alpha) \qquad \bar{y} = \frac{2r}{3}\left(\frac{\sin^3\alpha}{\alpha - \sin\alpha\cos\alpha}\right)$$

$$I_x = \frac{r^4}{4}(\alpha - \sin\alpha\cos\alpha + 2\sin^3\alpha\cos\alpha) \qquad I_{xy} = 0$$

$$I_y = \frac{r^4}{12}(3\alpha - 3\sin\alpha\cos\alpha - 2\sin^3\alpha\cos\alpha)$$

15



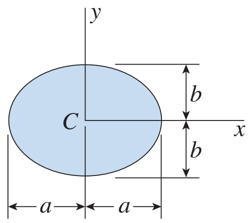
Circle with core removed (Origin of axes at center of circle)

α = angle in radians $\quad (\alpha \le \pi/2)$

$$\alpha = \arccos\frac{a}{r} \qquad b = \sqrt{r^2 - a^2} \qquad A = 2r^2\left(\alpha - \frac{ab}{r^2}\right)$$

$$I_x = \frac{r^4}{6}\left(3\alpha - \frac{3ab}{r^2} - \frac{2ab^3}{r^4}\right) \qquad I_y = \frac{r^4}{2}\left(\alpha - \frac{ab}{r^2} + \frac{2ab^3}{r^4}\right) \qquad I_{xy} = 0$$

16



Ellipse (Origin of axes at centroid)

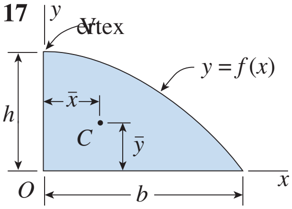
$$A = \pi ab \qquad I_x = \frac{\pi ab^3}{4} \qquad I_y = \frac{\pi ba^3}{4}$$

$$I_{xy} = 0 \qquad I_P = \frac{\pi ab}{4}(b^2 + a^2)$$

$$\text{Circumference} \approx \pi\left[1.5(a + b) - \sqrt{ab}\,\right] \qquad (a/3 \le b \le a)$$

$$\approx 4.17b^2/a + 4a \qquad (0 \le b \le a/3)$$

17



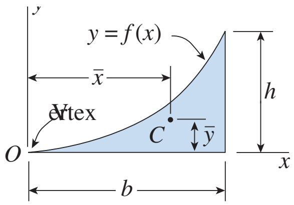
Parabolic semisegment (Origin of axes at corner)

$$y = f(x) = h\left(1 - \frac{x^2}{b^2}\right)$$

$$A = \frac{2bh}{3} \qquad \bar{x} = \frac{3b}{8} \qquad \bar{y} = \frac{2h}{5}$$

$$I_x = \frac{16bh^3}{105} \qquad I_y = \frac{2hb^3}{15} \qquad I_{xy} = \frac{b^2h^2}{12}$$

18



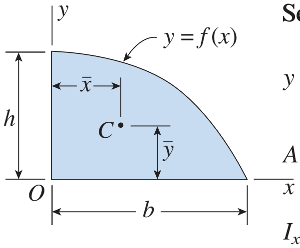
Parabolic spandrel (Origin of axes at vertex)

$$y = f(x) = \frac{hx^2}{b^2}$$

$$A = \frac{bh}{3} \qquad \bar{x} = \frac{3b}{4} \qquad \bar{y} = \frac{3h}{10}$$

$$I_x = \frac{bh^3}{21} \qquad I_y = \frac{hb^3}{5} \qquad I_{xy} = \frac{b^2h^2}{12}$$

19



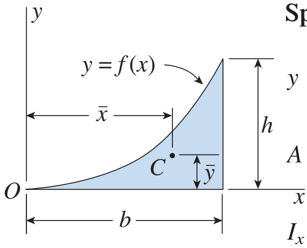
Semisegment of *n*th degree (Origin of axes at corner)

$$y = f(x) = h\left(1 - \frac{x^n}{b^n}\right) \qquad (n > 0)$$

$$A = bh\left(\frac{n}{n+1}\right) \qquad \bar{x} = \frac{b(n+1)}{2(n+2)} \qquad \bar{y} = \frac{hn}{2n+1}$$

$$I_x = \frac{2bh^3n^3}{(n+1)(2n+1)(3n+1)} \qquad I_y = \frac{hb^3n}{3(n+3)} \qquad I_{xy} = \frac{b^2h^2n^2}{4(n+1)(n+2)}$$

20



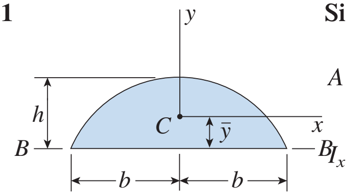
Spandrel of *n*th degree (Origin of axes at point of tangency)

$$y = f(x) = \frac{hx^n}{b^n} \qquad (n > 0)$$

$$A = \frac{bh}{n+1} \qquad \bar{x} = \frac{b(n+1)}{n+2} \qquad \bar{y} = \frac{h(n+1)}{2(2n+1)}$$

$$I_x = \frac{bh^3}{3(3n+1)} \qquad I_y = \frac{hb^3}{n+3} \qquad I_{xy} = \frac{b^2h^2}{4(n+1)}$$

21



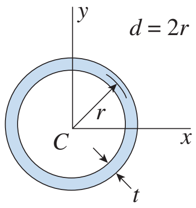
Sine wave (Origin of axes at centroid)

$$A = \frac{4bh}{\pi} \qquad \bar{y} = \frac{\pi h}{8}$$

$$I_x = \left(\frac{8}{9\pi} - \frac{\pi}{16}\right)bh^3 \approx 0.08659bh^3 \qquad I_y = \left(\frac{4}{\pi} - \frac{32}{\pi^3}\right)hb^3 \approx 0.2412hb^3$$

$$I_{xy} = 0 \qquad I_{BB} = \frac{8bh^3}{9\pi}$$

22

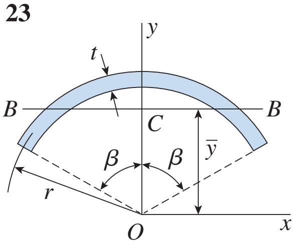


Thin circular ring (Origin of axes at center)
Approximate formulas for case when *t* is small

$$A = 2\pi rt = \pi dt \qquad I_x = I_y = \pi r^3t = \frac{\pi d^3t}{8}$$

$$I_{xy} = 0 \qquad I_P = 2\pi r^3t = \frac{\pi d^3t}{4}$$

23



Thin circular arc (Origin of axes at center of circle)
Approximate formulas for case when t is small

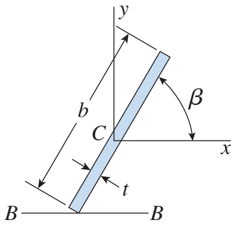
β = angle in radians (*Note:* For a semicircular arc, $\beta = \pi/2$.)

$$A = 2\beta rt \qquad \bar{y} = \frac{r \sin\beta}{\beta}$$

$$I_x = r^3t(\beta + \sin\beta\cos\beta) \qquad I_y = r^3t(\beta - \sin\beta\cos\beta)$$

$$I_{xy} = 0 \qquad I_{BB} = r^3t\left(\frac{2\beta + \sin 2\beta}{2} - \frac{1 - \cos 2\beta}{\beta}\right)$$

24

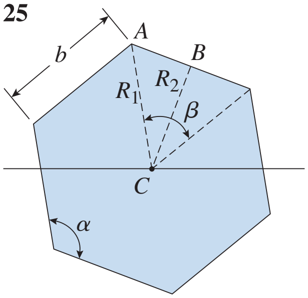


Thin rectangle (Origin of axes at centroid)
Approximate formulas for case when t is small

$$A = bt$$

$$I_x = \frac{tb^3}{12}\sin^2\beta \qquad I_y = \frac{tb^3}{12}\cos^2\beta \qquad I_{BB} = \frac{tb^3}{3}\sin^2\beta$$

25



Regular polygon with *n* sides (Origin of axes at centroid)

C = centroid (at center of polygon)

n = number of sides ($n \geq 3$) b = length of a side

β = central angle for a side α = interior angle (or vertex angle)

$$\beta = \frac{360°}{n} \qquad \alpha = \left(\frac{n-2}{n}\right)180° \qquad \alpha + \beta = 180°$$

R_1 = radius of circumscribed circle (line CA) R_2 = radius of inscribed circle (line CB)

$$R_1 = \frac{b}{2}\csc\frac{\beta}{2} \qquad R_2 = \frac{b}{2}\cot\frac{\beta}{2} \qquad A = \frac{nb^2}{4}\cot\frac{\beta}{2}$$

I_c = moment of inertia about any axis through C (the centroid C is a principal point and every axis through C is a principal axis)

$$I_c = \frac{nb^4}{192}\left(\cot\frac{\beta}{2}\right)\left(3\cot^2\frac{\beta}{2} + 1\right) \qquad I_P = 2I_c$$

E

Properties of Structural-Steel Shapes

In the following tables, the properties of a few structural-steel shapes are presented as an aid to the reader in solving problems in the text. These tables were compiled from the extensive tables in the *Manual of Steel Construction*, published by the American Institute of Steel Construction, Inc. (Ref. 5-4).

Notation:

I = moment of inertia
S = section modulus
$r = \sqrt{I/A}$ = radius of gyration

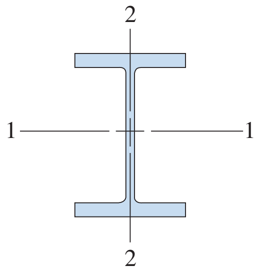


TABLE E-1(a) PROPERTIES OF WIDE-FLANGE SECTIONS (W SHAPES) – USCS UNITS (ABRIDGED LIST)

Designation	Weight per foot	Area	Depth	Web thickness	Flange		Axis 1-1			Axis 2-2		
					Width	Thickness	I	S	r	I	S	r
	lb	in.^{2}	in.	in.	in.	in.	in.^{4}	in.^{3}	in.	in.^{4}	in.^{3}	in.
W 30 × 211	211	62.2	30.9	0.775	15.1	1.32	10300	665	12.9	757	100	3.49
W 30 × 132	132	38.9	30.3	0.615	10.5	1.00	5770	380	12.2	196	37.2	2.25
W 24 × 162	162	47.7	25.0	0.705	13.0	1.22	5170	414	10.4	443	68.4	3.05
W 24 × 94	94.0	27.7	24.3	0.515	9.07	0.875	2700	222	9.87	109	24.0	1.98
W 18 × 119	119	35.1	19.0	0.655	11.3	1.06	2190	231	7.90	253	44.9	2.69
W 18 × 71	71.0	20.8	18.5	0.495	7.64	0.810	1170	127	7.50	60.3	15.8	1.70
W 16 × 100	100	29.5	17.0	0.585	10.4	0.985	1490	175	7.10	186	35.7	2.51
W 16 × 77	77.0	22.6	16.5	0.455	10.3	0.760	1110	134	7.00	138	26.9	2.47
W 16 × 57	57.0	16.8	16.4	0.430	7.12	0.715	758	92.2	6.72	43.1	12.1	1.60
W 16 × 31	31.0	9.13	15.9	0.275	5.53	0.440	375	47.2	6.41	12.4	4.49	1.17
W 14 × 120	120	35.3	14.5	0.590	14.7	0.940	1380	190	6.24	495	67.5	3.74
W 14 × 82	82.0	24.0	14.3	0.510	10.1	0.855	881	123	6.05	148	29.3	2.48
W 14 × 53	53.0	15.6	13.9	0.370	8.06	0.660	541	77.8	5.89	57.7	14.3	1.92
W 14 × 26	26.0	7.69	13.9	0.255	5.03	0.420	245	35.3	5.65	8.91	3.55	1.08
W 12 × 87	87.0	25.6	12.5	0.515	12.1	0.810	740	118	5.38	241	39.7	3.07
W 12 × 50	50.0	14.6	12.2	0.370	8.08	0.640	391	64.2	5.18	56.3	13.9	1.96
W 12 × 35	35.0	10.3	12.5	0.300	6.56	0.520	285	45.6	5.25	24.5	7.47	1.54
W 12 × 14	14.0	4.16	11.9	0.200	3.97	0.225	88.6	14.9	4.62	2.36	1.19	0.753
W 10 × 60	60.0	17.6	10.2	0.420	10.1	0.680	341	66.7	4.39	116	23.0	2.57
W 10 × 45	45.0	13.3	10.1	0.350	8.02	0.620	248	49.1	4.32	53.4	13.3	2.01
W 10 × 30	30.0	8.84	10.5	0.300	5.81	0.510	170	32.4	4.38	16.7	5.75	1.37
W 10 × 12	12.0	3.54	9.87	0.190	3.96	0.210	53.8	10.9	3.90	2.18	1.10	0.785
W 8 × 35	35.0	10.3	8.12	0.310	8.02	0.495	127	31.2	3.51	42.6	10.6	2.03
W 8 × 28	28.0	8.24	8.06	0.285	6.54	0.465	98.0	24.3	3.45	21.7	6.63	1.62
W 8 × 21	21.0	6.16	8.28	0.250	5.27	0.400	75.3	18.2	3.49	9.77	3.71	1.26
W 8 × 15	15.0	4.44	8.11	0.245	4.01	0.315	48.0	11.8	3.29	3.41	1.70	0.876

Note: Axes 1-1 and 2-2 are principal centroidal axes.

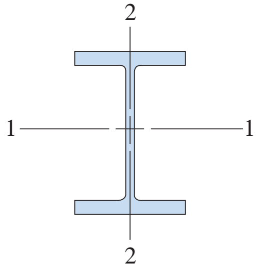


TABLE E-1(b) PROPERTIES OF WIDE-FLANGE SECTIONS (W SHAPES) – SI UNITS (ABRIDGED LIST)

Designation	Mass per meter	Area	Depth	Web thickness	Flange		Axis 1-1			Axis 2-2		
					Width	Thickness	*I*	*S*	*r*	*I*	*S*	*r*
	kg	mm^2	mm	mm	mm	mm	$\times 10^6\ \text{mm}^4$	$\times 10^3\ \text{mm}^3$	mm	$\times 10^6\ \text{mm}^4$	$\times 10^3\ \text{mm}^3$	mm
W 760 × 314	314	40100	785	19.7	384	33.5	4290	10900	328	315	1640	88.6
W 760 × 196	196	25100	770	15.6	267	25.4	2400	6230	310	81.6	610	57.2
W 610 × 241	241	30800	635	17.9	330	31.0	2150	6780	264	184	1120	77.5
W 610 × 140	140	17900	617	13.1	230	22.2	1120	3640	251	45.4	393	50.3
W 460 × 177	177	22600	483	16.6	287	26.9	912	3790	201	105	736	68.3
W 460 × 106	106	13400	470	12.6	194	20.6	487	2080	191	25.1	259	43.2
W 410 × 149	149	19000	432	14.9	264	25.0	620	2870	180	77.4	585	63.8
W 410 × 114	114	14600	419	11.6	262	19.3	462	2200	178	57.4	441	62.7
W 410 × 85	85.0	10800	417	10.9	181	18.2	316	1510	171	17.9	198	40.6
W 410 × 46.1	46.1	5890	404	6.99	140	11.2	156	773	163	5.16	73.6	29.7
W 360 × 179	179	22800	368	15.0	373	23.9	574	3110	158	206	1110	95.0
W 360 × 122	122	15500	363	13.0	257	21.7	367	2020	154	61.6	480	63.0
W 360 × 79	79.0	10100	353	9.40	205	16.8	225	1270	150	24.0	234	48.8
W 360 × 39	39.0	4960	353	6.48	128	10.7	102	578	144	3.71	58.2	27.4
W 310 × 129	129	16500	318	13.1	307	20.6	308	1930	137	100	651	78.0
W 310 × 74	74.0	9420	310	9.40	205	16.3	163	1050	132	23.4	228	49.8
W 310 × 52	52.0	6650	318	7.62	167	13.2	119	747	133	10.2	122	39.1
W 310 × 21	21.0	2680	302	5.08	101	5.72	36.9	244	117	0.982	19.5	19.1
W 250 × 89	89.0	11400	259	10.7	257	17.3	142	1090	112	48.3	377	65.3
W 250 × 67	67.0	8580	257	8.89	204	15.7	103	805	110	22.2	218	51.1
W 250 × 44.8	44.8	5700	267	7.62	148	13.0	70.8	531	111	6.95	94.2	34.8
W 250 × 17.9	17.9	2280	251	4.83	101	5.33	22.4	179	99.1	0.907	18.0	19.9
W 200 × 52	52.0	6650	206	7.87	204	12.6	52.9	511	89.2	17.7	174	51.6
W 200 × 41.7	41.7	5320	205	7.24	166	11.8	40.8	398	87.6	9.03	109	41.1
W 200 × 31.3	31.3	3970	210	6.35	134	10.2	31.3	298	88.6	4.07	60.8	32.0
W 200 × 22.5	22.5	2860	206	6.22	102	8.00	20.0	193	83.6	1.42	27.9	22.3

Note: Axes 1-1 and 2-2 are principal centroidal axes.

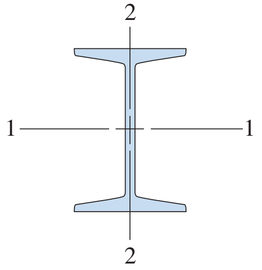


TABLE E-2(a) PROPERTIES OF I-BEAM SECTIONS (S SHAPES) – USCS UNITS (ABRIDGED LIST)

Designation	Weight per foot	Area	Depth	Web thickness	Flange		Axis 1-1			Axis 2-2		
					Width	Average thickness	*I*	*S*	*r*	*I*	*S*	*r*
	lb	in.^2	in.	in.	in.	in.	in.^4	in.^3	in.	in.^4	in.^3	in.
S 24 × 100	100	29.3	24.0	0.745	7.25	0.870	2380	199	9.01	47.4	13.1	1.27
S 24 × 80	80.0	23.5	24.0	0.500	7.00	0.870	2100	175	9.47	42.0	12.0	1.34
S 20 × 96	96.0	28.2	20.3	0.800	7.20	0.920	1670	165	7.71	49.9	13.9	1.33
S 20 × 75	75.0	22.0	20.0	0.635	6.39	0.795	1280	128	7.62	29.5	9.25	1.16
S 18 × 70	70.0	20.5	18.0	0.711	6.25	0.691	923	103	6.70	24.0	7.69	1.08
S 18 × 54.7	54.7	16.0	18.0	0.461	6.00	0.691	801	89.0	7.07	20.7	6.91	1.14
S 15 × 50	50.0	14.7	15.0	0.550	5.64	0.622	485	64.7	5.75	15.6	5.53	1.03
S 15 × 42.9	42.9	12.6	15.0	0.411	5.50	0.622	446	59.4	5.95	14.3	5.19	1.06
S 12 × 50	50.0	14.6	12.0	0.687	5.48	0.659	303	50.6	4.55	15.6	5.69	1.03
S 12 × 35	35.0	10.2	12.0	0.428	5.08	0.544	228	38.1	4.72	9.84	3.88	0.980
S 10 × 35	35.0	10.3	10.0	0.594	4.94	0.491	147	29.4	3.78	8.30	3.36	0.899
S 10 × 25.4	25.4	7.45	10.0	0.311	4.66	0.491	123	24.6	4.07	6.73	2.89	0.950
S 8 × 23	23.0	6.76	8.00	0.441	4.17	0.425	64.7	16.2	3.09	4.27	2.05	0.795
S 8 × 18.4	18.4	5.40	8.00	0.271	4.00	0.425	57.5	14.4	3.26	3.69	1.84	0.827
S 6 × 17.2	17.3	5.06	6.00	0.465	3.57	0.359	26.2	8.74	2.28	2.29	1.28	0.673
S 6 × 12.5	12.5	3.66	6.00	0.232	3.33	0.359	22.0	7.34	2.45	1.80	1.08	0.702
S 4 × 9.5	9.50	2.79	4.00	0.326	2.80	0.293	6.76	3.38	1.56	0.887	0.635	0.564
S 4 × 7.7	7.70	2.26	4.00	0.193	2.66	0.293	6.05	3.03	1.64	0.748	0.562	0.576

Note: Axes 1-1 and 2-2 are principal centroidal axes.

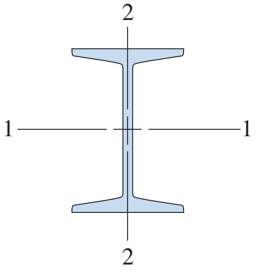


TABLE E-2(b) PROPERTIES OF I-BEAM SECTIONS (S SHAPES) – SI UNITS (ABRIDGED LIST)

Designation	Mass per meter	Area	Depth	Web thickness	Flange		Axis 1-1			Axis 2-2		
					Width	Average thickness	I	S	r	I	S	r
	kg	mm^2	mm	mm	mm	mm	$\times 10^6$ mm^4	$\times 10^3$ mm^3	mm	$\times 10^6$ mm^4	$\times 10^3$ mm^3	mm
S 610 × 149	149	18900	610	18.9	184	22.1	991	3260	229	19.7	215	32.3
S 610 × 119	119	15200	610	12.7	178	22.1	874	2870	241	17.5	197	34.0
S 510 × 143	143	18200	516	20.3	183	23.4	695	2700	196	20.8	228	33.8
S 510 × 112	112	14200	508	16.1	162	20.2	533	2100	194	12.3	152	29.5
S 460 × 104	104	13200	457	18.1	159	17.6	384	1690	170	10.0	126	27.4
S 460 × 81.4	81.4	10300	457	11.7	152	17.6	333	1460	180	8.62	113	29.0
S 380 × 74	74.0	9480	381	14.0	143	15.8	202	1060	146	6.49	90.6	26.2
S 380 × 64	64.0	8130	381	10.4	140	15.8	186	973	151	5.95	85.0	26.9
S 310 × 74	74.0	9420	305	17.4	139	16.7	126	829	116	6.49	93.2	26.2
S 310 × 52	52.0	6580	305	10.9	129	13.8	94.9	624	120	4.10	63.6	24.9
S 250 × 52	52.0	6650	254	15.1	125	12.5	61.2	482	96.0	3.45	55.1	22.8
S 250 × 37.8	37.8	4810	254	7.90	118	12.5	51.2	403	103	2.80	47.4	24.1
S 200 × 34	34.0	4360	203	11.2	106	10.8	26.9	265	78.5	1.78	33.6	20.2
S 200 × 27.4	27.4	3480	203	6.88	102	10.8	23.9	236	82.8	1.54	30.2	21.0
S 150 × 25.7	25.7	3260	152	11.8	90.7	9.12	10.9	143	57.9	0.953	21.0	17.1
S 150 × 18.6	18.6	2360	152	5.89	84.6	9.12	9.16	120	62.2	0.749	17.7	17.8
S 100 × 14.1	14.1	1800	102	8.28	71.1	7.44	2.81	55.4	39.6	0.369	10.4	14.3
S 100 × 11.5	11.5	1460	102	4.90	67.6	7.44	2.52	49.7	41.7	0.311	9.21	14.6

Note: Axes 1-1 and 2-2 are principal centroidal axes.

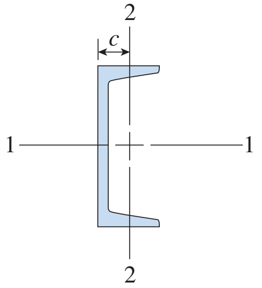


TABLE E-3(a) PROPERTIES OF CHANNEL SECTIONS (C SHAPES) – USCS UNITS (ABRIDGED LIST)

Designation	Weight per foot	Area	Depth	Web thickness	Flange		Axis 1-1			Axis 2-2			
					Width	Average thickness	*I*	*S*	*r*	*I*	*S*	*r*	*c*
	lb	in.^{2}	in.	in.	in.	in.	in.^{4}	in.^{3}	in.	in.^{4}	in.^{3}	in.	in.
C 15 × 50	50.0	14.7	15.0	0.716	3.72	0.650	404	53.8	5.24	11.0	3.77	0.865	0.799
C 15 × 40	40.0	11.8	15.0	0.520	3.52	0.650	348	46.5	5.45	9.17	3.34	0.883	0.778
C 15 × 33.9	33.9	10.0	15.0	0.400	3.40	0.650	315	42.0	5.62	8.07	3.09	0.901	0.788
C 12 × 30	30.0	8.81	12.0	0.510	3.17	0.501	162	27.0	4.29	5.12	2.05	0.762	0.674
C 12 × 25	25.0	7.34	12.0	0.387	3.05	0.501	144	24.0	4.43	4.45	1.87	0.779	0.674
C 12 × 20.7	20.7	6.08	12.0	0.282	2.94	0.501	129	21.5	4.61	3.86	1.72	0.797	0.698
C 10 × 30	30.0	8.81	10.0	0.673	3.03	0.436	103	20.7	3.42	3.93	1.65	0.668	0.649
C 10 × 25	25.0	7.34	10.0	0.526	2.89	0.436	91.1	18.2	3.52	3.34	1.47	0.675	0.617
C 10 × 20	20.0	5.87	10.0	0.379	2.74	0.436	78.9	15.8	3.66	2.80	1.31	0.690	0.606
C 10 × 15.3	15.3	4.48	10.0	0.240	2.60	0.436	67.3	13.5	3.87	2.27	1.15	0.711	0.634
C 8 × 18.7	18.7	5.51	8.00	0.487	2.53	0.390	43.9	11.0	2.82	1.97	1.01	0.598	0.565
C 8 × 13.7	13.7	4.04	8.00	0.303	2.34	0.390	36.1	9.02	2.99	1.52	0.848	0.613	0.554
C 8 × 11.5	11.5	3.37	8.00	0.220	2.26	0.390	32.5	8.14	3.11	1.31	0.775	0.623	0.572
C 6 × 13	13.0	3.81	6.00	0.437	2.16	0.343	17.3	5.78	2.13	1.05	0.638	0.524	0.514
C 6 × 10.5	10.5	3.08	6.00	0.314	2.03	0.343	15.1	5.04	2.22	0.860	0.561	0.529	0.500
C 6 × 8.2	8.20	2.39	6.00	0.200	1.92	0.343	13.1	4.35	2.34	0.687	0.488	0.536	0.512
C 4 × 7.2	7.20	2.13	4.00	0.321	1.72	0.296	4.58	2.29	1.47	0.425	0.337	0.447	0.459
C 4 × 5.4	5.40	1.58	4.00	0.184	1.58	0.296	3.85	1.92	1.56	0.312	0.277	0.444	0.457

Notes: 1. Axes 1-1 and 2-2 are principal centroidal axes.

2. The distance *c* is measured from the centroid to the back of the web.

3. For axis 2-2, the tabulated value of *S* is the smaller of the two section moduli for this axis.

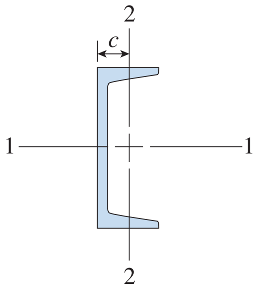


TABLE E-3(b) PROPERTIES OF CHANNEL SECTIONS (C SHAPES) – SI UNITS (ABRIDGED LIST)

Designation	Mass per meter	Area	Depth	Web thickness	Flange		Axis 1-1			Axis 2-2			
					Width	Average thickness	I	S	r	I	S	r	c
	kg	mm^2	mm	mm	mm	mm	$\times 10^6$ mm^4	$\times 10^3$ mm^3	mm	$\times 10^6$ mm^4	$\times 10^3$ mm^3	mm	mm
C 380 × 74	74.0	9480	381	18.2	94.5	16.5	168	882	133	4.58	61.8	22.0	20.3
C 380 × 60	60.0	7610	381	13.2	89.4	16.5	145	762	138	3.82	54.7	22.4	19.8
C 380 × 50.4	50.4	6450	381	10.2	86.4	16.5	131	688	143	3.36	50.6	22.9	20.0
C 310 × 45	45.0	5680	305	13.0	80.5	12.7	67.4	442	109	2.13	33.6	19.4	17.1
C 310 × 37	37.0	4740	305	9.83	77.5	12.7	59.9	393	113	1.85	30.6	19.8	17.1
C 310 × 30.8	30.8	3920	305	7.16	74.7	12.7	53.7	352	117	1.61	28.2	20.2	17.7
C 250 × 45	45.0	5680	254	17.1	77.0	11.1	42.9	339	86.9	1.64	27.0	17.0	16.5
C 250 × 37	37.0	4740	254	13.4	73.4	11.1	37.9	298	89.4	1.39	24.1	17.1	15.7
C 250 × 30	30.0	3790	254	9.63	69.6	11.1	32.8	259	93.0	1.17	21.5	17.5	15.4
C 250 × 22.8	22.8	2890	254	6.10	66.0	11.1	28.0	221	98.3	0.945	18.8	18.1	16.1
C 200 × 27.9	27.9	3550	203	12.4	64.3	9.91	18.3	180	71.6	0.820	16.6	15.2	14.4
C 200 × 20.5	20.5	2610	203	7.70	59.4	9.91	15.0	148	75.9	0.633	13.9	15.6	14.1
C 200 × 17.1	17.1	2170	203	5.59	57.4	9.91	13.5	133	79.0	0.545	12.7	15.8	14.5
C 150 × 19.3	19.3	2460	152	11.1	54.9	8.71	7.20	94.7	54.1	0.437	10.5	13.3	13.1
C 150 × 15.6	15.6	1990	152	7.98	51.6	8.71	6.29	82.6	56.4	0.358	9.19	13.4	12.7
C 150 × 12.2	12.2	1540	152	5.08	48.8	8.71	5.45	71.3	59.4	0.286	8.00	13.6	13.0
C 100 × 10.8	10.8	1370	102	8.15	43.7	7.52	1.91	37.5	37.3	0.177	5.52	11.4	11.7
C 100 × 8	8.00	1020	102	4.67	40.1	7.52	1.60	31.5	39.6	0.130	4.54	11.3	11.6

Notes: 1. Axes 1-1 and 2-2 are principal centroidal axes.

2. The distance c is measured from the centroid to the back of the web.

3. For axis 2-2, the tabulated value of S is the smaller of the two section moduli for this axis.

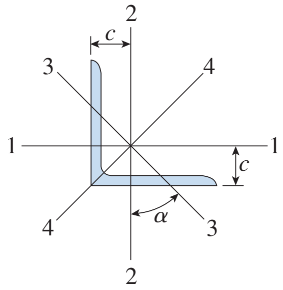


TABLE E-4(a) PROPERTIES OF ANGLE SECTIONS WITH EQUAL LEGS (L SHAPES) – USCS UNITS (ABRIDGED LIST)

Designation	Weight per foot	Area	Axis 1-1 and Axis 2-2				Axis 3-3
			I	S	r	c	$r_{\min}$
in.	lb	in.^{2}	in.^{4}	in.^{3}	in.	in.	in.
L 8 × 8 × 1	51.0	15.0	89.1	15.8	2.43	2.36	1.56
L 8 × 8 × 3/4	38.9	11.4	69.9	12.2	2.46	2.26	1.57
L 8 × 8 × 1/2	26.4	7.75	48.8	8.36	2.49	2.17	1.59
L 6 × 6 × 1	37.4	11.0	35.4	8.55	1.79	1.86	1.17
L 6 × 6 × 3/4	28.7	8.46	28.1	6.64	1.82	1.77	1.17
L 6 × 6 × 1/2	19.6	5.77	19.9	4.59	1.86	1.67	1.18
L 5 × 5 × 7/8	27.2	7.98	17.8	5.16	1.49	1.56	0.971
L 5 × 5 × 1/2	16.2	4.75	11.3	3.15	1.53	1.42	0.980
L 5 × 5 × 3/8	12.3	3.61	8.76	2.41	1.55	1.37	0.986
L 4 × 4 × 3/4	18.5	5.44	7.62	2.79	1.18	1.27	0.774
L 4 × 4 × 1/2	12.8	3.75	5.52	1.96	1.21	1.18	0.776
L 4 × 4 × 3/8	9.80	2.86	4.32	1.50	1.23	1.13	0.779
L 3-1/2 × 3-1/2 × 3/8	8.50	2.48	2.86	1.15	1.07	1.00	0.683
L 3-1/2 × 3-1/2 × 1/4	5.80	1.69	2.00	0.787	1.09	0.954	0.688
L 3 × 3 × 1/2	9.40	2.75	2.20	1.06	0.895	0.929	0.580
L 3 × 3 × 1/4	4.90	1.44	1.23	0.569	0.926	0.836	0.585

Notes: 1. Axes 1-1 and 2-2 are centroidal axes parallel to the legs.

2. The distance c is measured from the centroid to the back of the legs.

3. For axes 1-1 and 2-2, the tabulated value of S is the smaller of the two section moduli for those axes.

4. Axes 3-3 and 4-4 are principal centroidal axes.

5. The moment of inertia for axis 3-3, which is the smaller of the two principal moments of inertia, can be found from the equation $I_{33} = Ar_{\min}^2$.

6. The moment of inertia for axis 4-4, which is the larger of the two principal moments of inertia, can be found from the equation $I_{44} + I_{33} = I_{11} + I_{22}$.

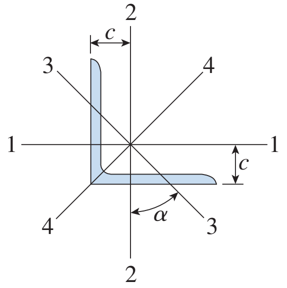


TABLE E-4(b) PROPERTIES OF ANGLE SECTIONS WITH EQUAL LEGS (L SHAPES) – SI UNITS (ABRIDGED LIST)

Designation	Mass per meter	Area	Axis 1-1 and Axis 2-2				Axis 3-3
			I	S	r	c	r_{min}
mm	kg	mm^2	$\times 10^6$ mm^4	$\times 10^3$ mm^3	mm	mm	mm
L 203 × 203 × 25.4	75.9	9680	37.1	259	61.7	59.9	39.6
L 203 × 203 × 19	57.9	7350	29.1	200	62.5	57.4	39.9
L 203 × 203 × 12.7	39.3	5000	20.3	137	63.2	55.1	40.4
L 152 × 152 × 25.4	55.7	7100	14.7	140	45.5	47.2	29.7
L 152 × 152 × 19	42.7	5460	11.7	109	46.2	45.0	29.7
L 152 × 152 × 12.7	29.2	3720	8.28	75.2	47.2	42.4	30.0
L 127 × 127 × 22.2	40.5	5150	7.41	84.6	37.8	39.6	24.7
L 127 × 127 × 12.7	24.1	3060	4.70	51.6	38.9	36.1	24.9
L 127 × 127 × 9.5	18.3	2330	3.65	39.5	39.4	34.8	25.0
L 102 × 102 × 19	27.5	3510	3.17	45.7	30.0	32.3	19.7
L 102 × 102 × 12.7	19.0	2420	2.30	32.1	30.7	30.0	19.7
L 102 × 102 × 9.5	14.6	1850	1.80	24.6	31.2	28.7	19.8
L 89 × 89 × 9.5	12.6	1600	1.19	18.8	27.2	25.4	17.3
L 89 × 89 × 6.4	8.60	1090	0.832	12.9	27.7	24.2	17.5
L 76 × 76 × 12.7	14.0	1770	0.916	17.4	22.7	23.6	14.7
L 76 × 76 × 6.4	7.30	929	0.512	9.32	23.5	21.2	14.9

Notes: 1. Axes 1-1 and 2-2 are centroidal axes parallel to the legs.

2. The distance c is measured from the centroid to the back of the legs.
3. For axes 1-1 and 2-2, the tabulated value of S is the smaller of the two section moduli for those axes.
4. Axes 3-3 and 4-4 are principal centroidal axes.
5. The moment of inertia for axis 3-3, which is the smaller of the two principal moments of inertia, can be found from the equation $I_{33} = Ar_{\text{min}}^2$.
6. The moment of inertia for axis 4-4, which is the larger of the two principal moments of inertia, can be found from the equation $I_{44} + I_{33} = I_{11} + I_{22}$.

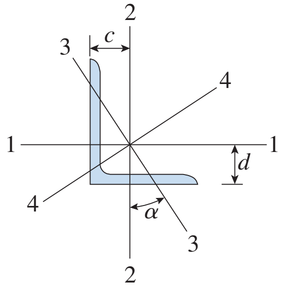


TABLE E-5(a) PROPERTIES OF ANGLE SECTIONS WITH UNEQUAL LEGS (L SHAPES) – USCS UNITS (ABRIDGED LIST)

Designation	Weight per foot	Area	Axis 1-1				Axis 2-2				Axis 3-3	
			I	S	r	d	I	S	r	c	$r_{\min}$	$\tan\alpha$
in.	lb	in.^{2}	in.^{4}	in.^{3}	in.	in.	in.^{4}	in.^{3}	in.	in.	in.	
∟ 8 × 6 × 1	44.2	13.0	80.9	15.1	2.49	2.65	38.8	8.92	1.72	1.65	1.28	0.542
∟ 8 × 6 × 1/2	23.0	6.75	44.4	8.01	2.55	2.46	21.7	4.79	1.79	1.46	1.30	0.557
∟ 7 × 4 × 3/4	26.2	7.69	37.8	8.39	2.21	2.50	9.00	3.01	1.08	1.00	0.855	0.324
∟ 7 × 4 × 1/2	17.9	5.25	26.6	5.79	2.25	2.40	6.48	2.10	1.11	0.910	0.866	0.334
∟ 6 × 4 × 3/4	23.6	6.94	24.5	6.23	1.88	2.07	8.63	2.95	1.12	1.07	0.856	0.428
∟ 6 × 4 × 1/2	16.2	4.75	17.3	4.31	1.91	1.98	6.22	2.06	1.14	0.981	0.864	0.440
∟ 5 × 3-1/2 × 3/4	19.8	5.81	13.9	4.26	1.55	1.74	5.52	2.20	0.974	0.993	0.744	0.464
∟ 5 × 3-1/2 × 1/2	13.6	4.00	10.0	2.97	1.58	1.65	4.02	1.55	1.00	0.901	0.750	0.479
∟ 5 × 3 × 1/2	12.8	3.75	9.43	2.89	1.58	1.74	2.55	1.13	0.824	0.746	0.642	0.357
∟ 5 × 3 × 1/4	6.60	1.94	5.09	1.51	1.62	1.64	1.41	0.600	0.853	0.648	0.652	0.371
∟ 4 × 3-1/2 × 1/2	11.9	3.50	5.30	1.92	1.23	1.24	3.76	1.50	1.04	0.994	0.716	0.750
∟ 4 × 3-1/2 × 1/4	6.20	1.81	2.89	1.01	1.26	1.14	2.07	0.794	1.07	0.897	0.723	0.759
∟ 4 × 3 × 1/2	11.1	3.25	5.02	1.87	1.24	1.32	2.40	1.10	0.858	0.822	0.633	0.542
∟ 4 × 3 × 3/8	8.50	2.48	3.94	1.44	1.26	1.27	1.89	0.851	0.873	0.775	0.636	0.551
∟ 4 × 3 × 1/4	5.80	1.69	2.75	0.988	1.27	1.22	1.33	0.585	0.887	0.725	0.639	0.558

Notes:
1. Axes 1-1 and 2-2 are centroidal axes parallel to the legs.
2. The distances c and d are measured from the centroid to the backs of the legs.
3. For axes 1-1 and 2-2, the tabulated value of S is the smaller of the two section moduli for those axes.
4. Axes 3-3 and 4-4 are principal centroidal axes.
5. The moment of inertia for axis 3-3, which is the smaller of the two principal moments of inertia, can be found from the equation $I_{33} = Ar^2_{\min}$.
6. The moment of inertia for axis 4-4, which is the larger of the two principal moments of inertia, can be found from the equation $I_{44} + I_{33} = I_{11} + I_{22}$.

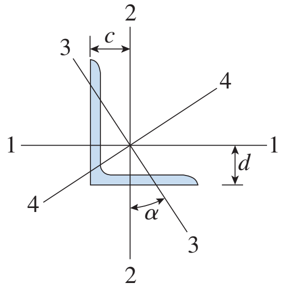


TABLE E-5(b) PROPERTIES OF ANGLE SECTIONS WITH UNEQUAL LEGS (L SHAPES) – SI UNITS (ABRIDGED LIST)

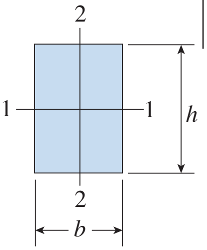
Designation	Mass per meter	Area	Axis 1-1				Axis 2-2			Axis 3-3		
			I	S	r	d	I	S	r	c	r_{min}	tan α
mm	kg	mm^2	$\times 10^6$mm^4	$\times 10^3$mm^3	mm	mm	$\times 10^6$mm^4	$\times 10^3$mm^3	mm	mm	mm	
L 203 × 152 × 25.4	65.5	8390	33.7	247	63.2	67.3	16.1	146	43.7	41.9	32.5	0.542
L 203 × 152 × 12.7	34.1	4350	18.5	131	64.8	62.5	9.03	78.5	45.5	37.1	33.0	0.557
L 178 × 102 × 19	38.8	4960	15.7	137	56.1	63.5	3.75	49.3	27.4	25.4	21.7	0.324
L 178 × 102 × 12.7	26.5	3390	11.1	94.9	57.2	61.0	2.70	34.4	28.2	23.1	22.0	0.334
L 152 × 102 × 19	35.0	4480	10.2	102	47.8	52.6	3.59	48.3	28.4	27.2	21.7	0.428
L 152 × 102 × 12.7	24.0	3060	7.20	70.6	48.5	50.3	2.59	33.8	29.0	24.9	21.9	0.440
L 127 × 89 × 19	29.3	3750	5.79	69.8	39.4	44.2	2.30	36.1	24.7	25.2	18.9	0.464
L 127 × 89 × 12.7	20.2	2580	4.15	48.7	40.1	41.9	1.67	25.4	25.4	22.9	19.1	0.479
L 127 × 76 × 12.7	19.0	2420	3.93	47.4	40.1	44.2	1.06	18.5	20.9	18.9	16.3	0.357
L 127 × 76 × 6.4	9.80	1250	2.12	24.7	41.1	41.7	0.587	9.83	21.7	16.5	16.6	0.371
L 102 × 89 × 12.7	17.6	2260	2.21	31.5	31.2	31.5	1.57	24.6	26.4	25.2	18.2	0.750
L 102 × 89 × 6.4	9.20	1170	1.20	16.6	32.0	29.0	0.862	13.0	27.2	22.8	18.4	0.759
L 102 × 76 × 12.7	16.4	2100	2.09	30.6	31.5	33.5	0.999	18.0	21.8	20.9	16.1	0.542
L 102 × 76 × 9.5	12.6	1600	1.64	23.6	32.0	32.3	0.787	13.9	22.2	19.7	16.2	0.551
L 102 × 76 × 6.4	8.60	1090	1.14	16.2	32.3	31.0	0.554	9.59	22.5	18.4	16.2	0.558

Notes:

1. Axes 1-1 and 2-2 are centroidal axes parallel to the legs.
2. The distances c and d are measured from the centroid to the backs of the legs.
3. For axes 1-1 and 2-2, the tabulated value of S is the smaller of the two section moduli for those axes.
4. Axes 3-3 and 4-4 are principal centroidal axes.
5. The moment of inertia for axis 3-3, which is the smaller of the two principal moments of inertia, can be found from the equation $I_{33} = Ar^2_{\text{min}}$.
6. The moment of inertia for axis 4-4, which is the larger of the two principal moments of inertia, can be found from the equation $I_{44} + I_{33} = I_{11} + I_{22}$.

F

Properties of Structural Lumber



PROPERTIES OF SURFACED LUMBER (ABRIDGED LIST)

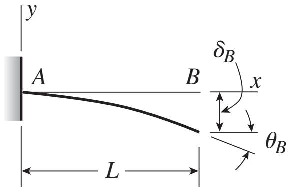
Nominal dimensions $b \times h$	Net dimensions $b \times h$	Area $A = bh$	Axis 1-1		Axis 2-2		Weight per linear foot (weight density = 35 lb/ft^3)
			Moment of inertia $I_1 = \frac{bh^3}{12}$	Section modulus $S_1 = \frac{bh^2}{6}$	Moment of inertia $I_2 = \frac{hb^3}{12}$	Section modulus $S_2 = \frac{hb^2}{6}$	
in.	in.	in.^{2}	in.^{4}	in.^{3}	in.^{4}	in.^{3}	lb
2 × 4	1.5 × 3.5	5.25	5.36	3.06	0.98	1.31	1.3
2 × 6	1.5 × 5.5	8.25	20.80	7.56	1.55	2.06	2.0
2 × 8	1.5 × 7.25	10.88	47.63	13.14	2.04	2.72	2.6
2 × 10	1.5 × 9.25	13.88	98.93	21.39	2.60	3.47	3.4
2 × 12	1.5 × 11.25	16.88	177.98	31.64	3.16	4.22	4.1
3 × 4	2.5 × 3.5	8.75	8.93	5.10	4.56	3.65	2.1
3 × 6	2.5 × 5.5	13.75	34.66	12.60	7.16	5.73	3.3
3 × 8	2.5 × 7.25	18.13	79.39	21.90	9.44	7.55	4.4
3 × 10	2.5 × 9.25	23.13	164.89	35.65	12.04	9.64	5.6
3 × 12	2.5 × 11.25	28.13	296.63	52.73	14.65	11.72	6.8
4 × 4	3.5 × 3.5	12.25	12.51	7.15	12.51	7.15	3.0
4 × 6	3.5 × 5.5	19.25	48.53	17.65	19.65	11.23	4.7
4 × 8	3.5 × 7.25	25.38	111.15	30.66	25.90	14.80	6.2
4 × 10	3.5 × 9.25	32.38	230.84	49.91	33.05	18.89	7.9
4 × 12	3.5 × 11.25	39.38	415.28	73.83	40.20	22.97	9.6
6 × 6	5.5 × 5.5	30.25	76.3	27.7	76.3	27.7	7.4
6 × 8	5.5 × 7.5	41.25	193.4	51.6	104.0	37.8	10.0
6 × 10	5.5 × 9.5	52.25	393.0	82.7	131.7	47.9	12.7
6 × 12	5.5 × 11.5	63.25	697.1	121.2	159.4	58.0	15.4
8 × 8	7.5 × 7.5	56.25	263.7	70.3	263.7	70.3	13.7
8 × 10	7.5 × 9.5	71.25	535.9	112.8	334.0	89.1	17.3
8 × 12	7.5 × 11.5	86.25	950.5	165.3	404.3	107.8	21.0

Note: Axes 1-1 and 2-2 are principal centroidal axes.

G

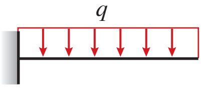
Deflections and Slopes of Beams

TABLE G-1 DEFLECTIONS AND SLOPES OF CANTILEVER BEAMS



v = deflection in the y direction (positive upward)
$v' = dv/dx$ = slope of the deflection curve
$\delta_B = -v(L)$ = deflection at end B of the beam (positive downward)
$\theta_B = -v'(L)$ = angle of rotation at end B of the beam (positive clockwise)
EI = constant

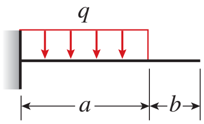
1



$$v = -\frac{qx^2}{24EI}(6L^2 - 4Lx + x^2) \qquad v' = -\frac{qx}{6EI}(3L^2 - 3Lx + x^2)$$

$$\delta_B = \frac{qL^4}{8EI} \qquad \theta_B = \frac{qL^3}{6EI}$$

2



$$v = -\frac{qx^2}{24EI}(6a^2 - 4ax + x^2) \qquad (0 \le x \le a)$$

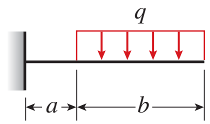
$$v' = -\frac{qx}{6EI}(3a^2 - 3ax + x^2) \qquad (0 \le x \le a)$$

$$v = -\frac{qa^3}{24EI}(4x - a) \qquad v' = -\frac{qa^3}{6EI} \qquad (a \le x \le L)$$

$$\text{At } x = a\text{:} \quad v = -\frac{qa^4}{8EI} \qquad v' = -\frac{qa^3}{6EI}$$

$$\delta_B = \frac{qa^3}{24EI}(4L - a) \qquad \theta_B = \frac{qa^3}{6EI}$$

3



$$v = -\frac{qbx^2}{12EI}(3L + 3a - 2x) \qquad (0 \le x \le a)$$

$$v' = -\frac{qbx}{2EI}(L + a - x) \qquad (0 \le x \le a)$$

$$v = -\frac{q}{24EI}(x^4 - 4Lx^3 + 6L^2x^2 - 4a^3x + a^4) \qquad (a \le x \le L)$$

$$v' = -\frac{q}{6EI}(x^3 - 3Lx^2 + 3L^2x - a^3) \qquad (a \le x \le L)$$

$$At\ x = a{:} \quad v = -\frac{qa^2b}{12EI}(3L + a) \qquad v' = -\frac{qabL}{2EI}$$

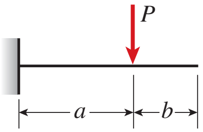
$$\delta_B = \frac{q}{24EI}(3L^4 - 4a^3L + a^4) \qquad \theta_B = \frac{q}{6EI}(L^3 - a^3)$$

4

$$v = -\frac{Px^2}{6EI}(3L - x) \qquad v' = -\frac{Px}{2EI}(2L - x)$$

$$\delta_B = \frac{PL^3}{3EI} \qquad \theta_B = \frac{PL^2}{2EI}$$

5



$$v = -\frac{Px^2}{6EI}(3a - x) \qquad v' = -\frac{Px}{2EI}(2a - x) \qquad (0 \le x \le a)$$

$$v = -\frac{Pa^2}{6EI}(3x - a) \qquad v' = -\frac{Pa^2}{2EI} \qquad (a \le x \le L)$$

$$At\ x = a{:} \quad v = -\frac{Pa^3}{3EI} \qquad v' = -\frac{Pa^2}{2EI}$$

$$\delta_B = \frac{Pa^2}{6EI}(3L - a) \qquad \theta_B = \frac{Pa^2}{2EI}$$

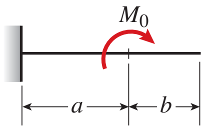
6

$$v = -\frac{M_0x^2}{2EI} \qquad v' = -\frac{M_0x}{EI}$$

$$\delta_B = \frac{M_0L^2}{2EI} \qquad \theta_B = \frac{M_0L}{EI}$$

(*Continued*)

7



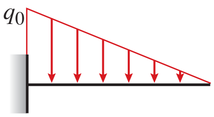
$$v = -\frac{M_0 x^2}{2EI} \qquad v' = -\frac{M_0 x}{EI} \qquad (0 \le x \le a)$$

$$v = -\frac{M_0 a}{2EI}(2x - a) \qquad v' = -\frac{M_0 a}{EI} \qquad (a \le x \le L)$$

$$\text{At } x = a\text{:} \quad v = -\frac{M_0 a^2}{2EI} \qquad v' = -\frac{M_0 a}{EI}$$

$$\delta_B = \frac{M_0 a}{2EI}(2L - a) \qquad \theta_B = \frac{M_0 a}{EI}$$

8

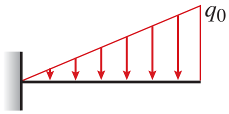


$$v = -\frac{q_0 x^2}{120LEI}(10L^3 - 10L^2 x + 5Lx^2 - x^3)$$

$$v' = -\frac{q_0 x}{24LEI}(4L^3 - 6L^2 x + 4Lx^2 - x^3)$$

$$\delta_B = \frac{q_0 L^4}{30EI} \qquad \theta_B = \frac{q_0 L^3}{24EI}$$

9

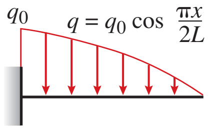


$$v = -\frac{q_0 x^2}{120LEI}(20L^3 - 10L^2 x + x^3)$$

$$v' = -\frac{q_0 x}{24LEI}(8L^3 - 6L^2 x + x^3)$$

$$\delta_B = \frac{11 q_0 L^4}{120EI} \qquad \theta_B = \frac{q_0 L^3}{8EI}$$

10

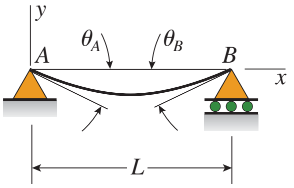


$$v = -\frac{q_0 L}{3\pi^4 EI}\left(48L^3 \cos\frac{\pi x}{2L} - 48L^3 + 3\pi^3 Lx^2 - \pi^3 x^3\right)$$

$$v' = -\frac{q_0 L}{\pi^3 EI}\left(2\pi^2 Lx - \pi^2 x^2 - 8L^2 \sin\frac{\pi x}{2L}\right)$$

$$\delta_B = \frac{2q_0 L^4}{3\pi^4 EI}(\pi^3 - 24) \qquad \theta_B = \frac{q_0 L^3}{\pi^3 EI}(\pi^2 - 8)$$

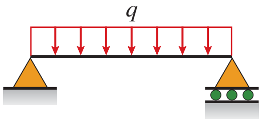
TABLE G-2 DEFLECTIONS AND SLOPES OF SIMPLE BEAMS



EI = constant

v = deflection in the y direction (positive upward)
$v' = dv/dx$ = slope of the deflection curve
$\delta_C = -v(L/2)$ = deflection at midpoint C of the beam (positive downward)
x_1 = distance from support A to point of maximum deflection
$\delta_{\max} = -v_{\max}$ = maximum deflection (positive downward)
$\theta_A = -v'(0)$ = angle of rotation at left-hand end of the beam (positive clockwise)
$\theta_B = v'(L)$ = angle of rotation at right-hand end of the beam (positive counterclockwise)

1

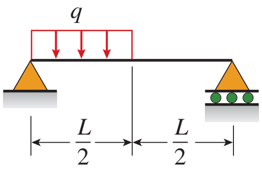


$$v = -\frac{qx}{24EI}(L^3 - 2Lx^2 + x^3)$$

$$v' = -\frac{q}{24EI}(L^3 - 6Lx^2 + 4x^3)$$

$$\delta_C = \delta_{\max} = \frac{5qL^4}{384EI} \qquad \theta_A = \theta_B = \frac{qL^3}{24EI}$$

2



$$v = -\frac{qx}{384EI}(9L^3 - 24Lx^2 + 16x^3) \qquad \left(0 \le x \le \frac{L}{2}\right)$$

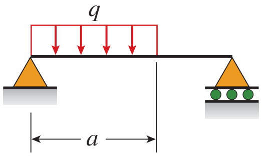
$$v' = -\frac{q}{384EI}(9L^3 - 72Lx^2 + 64x^3) \qquad \left(0 \le x \le \frac{L}{2}\right)$$

$$v = -\frac{qL}{384EI}(8x^3 - 24Lx^2 + 17L^2x - L^3) \qquad \left(\frac{L}{2} \le x \le L\right)$$

$$v' = -\frac{qL}{384EI}(24x^2 - 48Lx + 17L^2) \qquad \left(\frac{L}{2} \le x \le L\right)$$

$$\delta_C = \frac{5qL^4}{768EI} \qquad \theta_A = \frac{3qL^3}{128EI} \qquad \theta_B = \frac{7qL^3}{384EI}$$

3



$$v = -\frac{qx}{24LEI}(a^4 - 4a^3L + 4a^2L^2 + 2a^2x^2 - 4aLx^2 + Lx^3) \qquad (0 \le x \le a)$$

$$v' = -\frac{q}{24LEI}(a^4 - 4a^3L + 4a^2L^2 + 6a^2x^2 - 12aLx^2 + 4Lx^3) \qquad (0 \le x \le a)$$

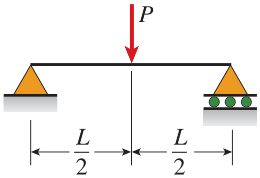
$$v = -\frac{qa^2}{24LEI}(-a^2L + 4L^2x + a^2x - 6Lx^2 + 2x^3) \qquad (a \le x \le L)$$

$$v' = -\frac{qa^2}{24LEI}(4L^2 + a^2 - 12Lx + 6x^2) \qquad (a \le x \le L)$$

$$\theta_A = \frac{qa^2}{24LEI}(2L - a)^2 \qquad \theta_B = \frac{qa^2}{24LEI}(2L^2 - a^2)$$

(*Continued*)

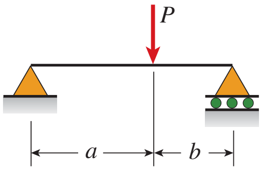
4



$$v = -\frac{Px}{48EI}(3L^2 - 4x^2) \qquad v' = -\frac{P}{16EI}(L^2 - 4x^2) \qquad \left(0 \le x \le \frac{L}{2}\right)$$

$$\delta_C = \delta_{\max} = \frac{PL^3}{48EI} \qquad \theta_A = \theta_B = \frac{PL^2}{16EI}$$

5



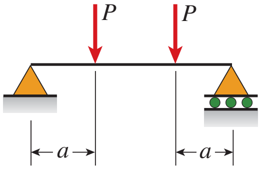
$$v = -\frac{Pbx}{6LEI}(L^2 - b^2 - x^2) \qquad v' = -\frac{Pb}{6LEI}(L^2 - b^2 - 3x^2) \qquad (0 \le x \le a)$$

$$\theta_A = \frac{Pab(L+b)}{6LEI} \qquad \theta_B = \frac{Pab(L+a)}{6LEI}$$

$$\text{If } a \ge b, \quad \delta_C = \frac{Pb(3L^2 - 4b^2)}{48EI} \qquad \text{If } a \le b, \quad \delta_C = \frac{Pa(3L^2 - 4a^2)}{48EI}$$

$$\text{If } a \ge b, \quad x_1 = \sqrt{\frac{L^2 - b^2}{3}} \quad \text{and} \quad \delta_{\max} = \frac{Pb(L^2 - b^2)^{3/2}}{9\sqrt{3}\,LEI}$$

6



$$v = -\frac{Px}{6EI}(3aL - 3a^2 - x^2) \qquad v' = -\frac{P}{2EI}(aL - a^2 - x^2) \qquad (0 \le x \le a)$$

$$v = -\frac{Pa}{6EI}(3Lx - 3x^2 - a^2) \qquad v' = -\frac{Pa}{2EI}(L - 2x) \qquad (a \le x \le L - a)$$

$$\delta_C = \delta_{\max} = \frac{Pa}{24EI}(3L^2 - 4a^2) \qquad \theta_A = \theta_B = \frac{Pa(L-a)}{2EI}$$

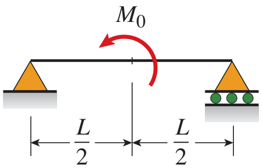
7

$$v = -\frac{M_0 x}{6LEI}(2L^2 - 3Lx + x^2) \qquad v' = -\frac{M_0}{6LEI}(2L^2 - 6Lx + 3x^2)$$

$$\delta_C = \frac{M_0 L^2}{16EI} \qquad \theta_A = \frac{M_0 L}{3EI} \qquad \theta_B = \frac{M_0 L}{6EI}$$

$$x_1 = L\left(1 - \frac{\sqrt{3}}{3}\right) \quad \text{and} \quad \delta_{\max} = \frac{M_0 L^2}{9\sqrt{3}\,EI}$$

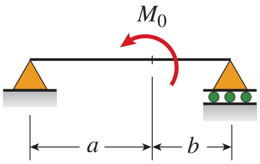
8



$$v = -\frac{M_0 x}{24LEI}(L^2 - 4x^2) \qquad v' = -\frac{M_0}{24LEI}(L^2 - 12x^2) \qquad \left(0 \le x \le \frac{L}{2}\right)$$

$$\delta_C = 0 \qquad \theta_A = \frac{M_0 L}{24EI} \qquad \theta_B = -\frac{M_0 L}{24EI}$$

9



$$v = -\frac{M_0 x}{6LEI}(6aL - 3a^2 - 2L^2 - x^2) \qquad (0 \le x \le a)$$

$$v' = -\frac{M_0}{6LEI}(6aL - 3a^2 - 2L^2 - 3x^2) \qquad (0 \le x \le a)$$

$$\text{At } x = a\text{:} \quad v = -\frac{M_0 ab}{3LEI}(2a - L) \qquad v' = -\frac{M_0}{3LEI}(3aL - 3a^2 - L^2)$$

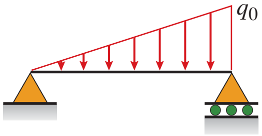
$$\theta_A = \frac{M_0}{6LEI}(6aL - 3a^2 - 2L^2) \qquad \theta_B = \frac{M_0}{6LEI}(3a^2 - L^2)$$

10

$$v = -\frac{M_0 x}{2EI}(L - x) \qquad v' = -\frac{M_0}{2EI}(L - 2x)$$

$$\delta_C = \delta_{\max} = \frac{M_0 L^2}{8EI} \qquad \theta_A = \theta_B = \frac{M_0 L}{2EI}$$

11



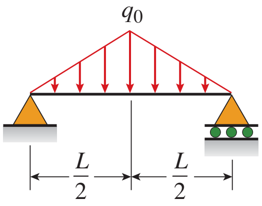
$$v = -\frac{q_0 x}{360LEI}(7L^4 - 10L^2x^2 + 3x^4)$$

$$v' = -\frac{q_0}{360LEI}(7L^4 - 30L^2x^2 + 15x^4)$$

$$\delta_C = \frac{5q_0L^4}{768EI} \qquad \theta_A = \frac{7q_0L^3}{360EI} \qquad \theta_B = \frac{q_0L^3}{45EI}$$

$$x_1 = 0.5193L \qquad \delta_{\max} = 0.00652\frac{q_0L^4}{EI}$$

12

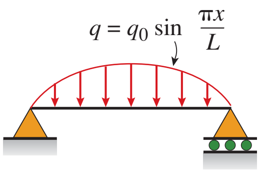


$$v = -\frac{q_0 x}{960LEI}(5L^2 - 4x^2)^2 \qquad \left(0 \le x \le \frac{L}{2}\right)$$

$$v' = -\frac{q_0}{192LEI}(5L^2 - 4x^2)(L^2 - 4x^2) \qquad \left(0 \le x \le \frac{L}{2}\right)$$

$$\delta_C = \delta_{\max} = \frac{q_0L^4}{120EI} \qquad \theta_A = \theta_B = \frac{5q_0L^3}{192EI}$$

13



$$v = -\frac{q_0L^4}{\pi^4EI}\sin\frac{\pi x}{L} \qquad v' = -\frac{q_0L^3}{\pi^3EI}\cos\frac{\pi x}{L}$$

$$\delta_C = \delta_{\max} = \frac{q_0L^4}{\pi^4EI} \qquad \theta_A = \theta_B = \frac{q_0L^3}{\pi^3EI}$$

H

Properties of Materials

Notes:

1. Properties of materials vary greatly depending upon manufacturing processes, chemical composition, internal defects, temperature, previous loading history, age, dimensions of test specimens, and other factors. The tabulated values are typical but should never be used for specific engineering or design purposes. Manufacturers and materials suppliers should be consulted for information about a particular product.

2. Except when compression or bending is indicated, the modulus of elasticity E, yield stress σ_Y, and ultimate stress σ_U are for materials in tension.

TABLE H-1 WEIGHTS AND MASS DENSITIES

Material	Weight density γ		Mass density ρ	
	lb/ft^3	kN/m^3	slugs/ft^3	kg/m^3
Aluminum alloys	160–180	26–28	5.2–5.4	2,600–2,800
2014-T6, 7075-T6	175	28	5.4	2,800
6061-T6	170	26	5.2	2,700
Brass	520–540	82–85	16–17	8,400–8,600
Bronze	510–550	80–86	16–17	8,200–8,800
Cast iron	435–460	68–72	13–14	7,000–7,400
Concrete				
Plain	145	23	4.5	2,300
Reinforced	150	24	4.7	2,400
Lightweight	70–115	11–18	2.2–3.6	1,100–1,800
Copper	556	87	17	8,900
Glass	150–180	24–28	4.7–5.4	2,400–2,800
Magnesium alloys	110–114	17–18	3.4–3.5	1,760–1,830
Monel (67% Ni, 30% Cu)	550	87	17	8,800
Nickel	550	87	17	8,800
Plastics				
Nylon	55–70	8.6–11	1.7–2.2	880–1,100
Polyethylene	60–90	9.4–14	1.9–2.8	960–1,400
Rock				
Granite, marble, quartz	165–180	26–28	5.1–5.6	2,600–2,900
Limestone, sandstone	125–180	20–28	3.9–5.6	2,000–2,900
Rubber	60–80	9–13	1.9–2.5	960–1,300
Sand, soil, gravel	75–135	12–21	2.3–4.2	1,200–2,200
Steel	490	77.0	15.2	7,850
Titanium	280	44	8.7	4,500
Tungsten	1,200	190	37	1,900
Water, fresh	62.4	9.81	1.94	1,000
sea	63.8	10.0	1.98	1,020
Wood (air dry)				
Douglas fir	30–35	4.7–5.5	0.9–1.1	480–560
Oak	40–45	6.3–7.1	1.2–1.4	640–720
Southern pine	35–40	5.5–6.3	1.1–1.2	560–640

TABLE H-2 MODULI OF ELASTICITY AND POISSON'S RATIOS

Material	Modulus of elasticity *E*		Shear modulus of elasticity *G*		Poisson's ratio ν
	ksi	GPa	ksi	GPa	
Aluminum alloys 2014-T6 6061-T6 7075-T6	10,000–11,400 10,600 10,000 10,400	70–79 73 70 72	3,800–4,300 4,000 3,800 3,900	26–30 28 26 27	0.33 0.33 0.33 0.33
Brass	14,000–16,000	96–110	5,200–6,000	36–41	0.34
Bronze	14,000–17,000	96–120	5,200–6,300	36–44	0.34
Cast iron	12,000–25,000	83–170	4,600–10,000	32–69	0.2–0.3
Concrete (compression)	2,500–4,500	17–31			0.1–0.2
Copper and copper alloys	16,000–18,000	110–120	5,800–6,800	40–47	0.33–0.36
Glass	7,000–12,000	48–83	2,700–5,100	19–35	0.17–0.27
Magnesium alloys	6,000–6,500	41–45	2,200–2,400	15–17	0.35
Monel (67% Ni, 30% Cu)	25,000	170	9,500	66	0.32
Nickel	30,000	210	11,400	80	0.31
Plastics Nylon Polyethylene	 300–500 100–200	 2.1–3.4 0.7–1.4			 0.4 0.4
Rock (compression) Granite, marble, quartz Limestone, sandstone	 6,000–14,000 3,000–10,000	 40–100 20–70			 0.2–0.3 0.2–0.3
Rubber	0.1–0.6	0.0007–0.004	0.03–0.2	0.0002–0.001	0.45–0.50
Steel	28,000–30,000	190–210	10,800–11,800	75–80	0.27–0.30
Titanium alloys	15,000–17,000	100–120	5,600–6,400	39–44	0.33
Tungsten	50,000–55,000	340–380	21,000–23,000	140–160	0.2
Wood (bending) Douglas fir Oak Southern pine	 1,600–1,900 1,600–1,800 1,600–2,000	 11–13 11–12 11–14			

TABLE H-3 MECHANICAL PROPERTIES

Material	Yield stress σ_Y		Ultimate stress σ_U		Percent elongation (2 in. gage length)
	ksi	MPa	ksi	MPa	
Aluminum alloys	5–70	35–500	15–80	100–550	1–45
2014-T6	60	410	70	480	13
6061-T6	40	270	45	310	17
7075-T6	70	480	80	550	11
Brass	10–80	70–550	30–90	200–620	4–60
Bronze	12–100	82–690	30–120	200–830	5–60
Cast iron (tension)	17–42	120–290	10–70	69–480	0–1
Cast iron (compression)			50–200	340–1,400	
Concrete (compression)			1.5–10	10–70	
Copper and copper alloys	8–110	55–760	33–120	230–830	4–50
Glass			5–150	30–1,000	0
Plate glass			10	70	
Glass fibers			1,000–3,000	7,000–20,000	
Magnesium alloys	12–40	80–280	20–50	140–340	2–20
Monel (67% Ni, 30% Cu)	25–160	170–1,100	65–170	450–1,200	2–50
Nickel	15–90	100–620	45–110	310–760	2–50
Plastics					
Nylon			6–12	40–80	20–100
Polyethylene			1–4	7–28	15–300
Rock (compression)					
Granite, marble, quartz			8–40	50–280	
Limestone, sandstone			3–30	20–200	
Rubber	0.2–1.0	1–7	1–3	7–20	100–800
Steel					
High-strength	50–150	340–1,000	80–180	550–1,200	5–25
Machine	50–100	340–700	80–125	550–860	5–25
Spring	60–240	400–1,600	100–270	700–1,900	3–15
Stainless	40–100	280–700	60–150	400–1,000	5–40
Tool	75	520	130	900	8
Steel, structural	30–100	200–700	50–120	340–830	10–40
ASTM-A36	36	250	60	400	30
ASTM-A572	50	340	70	500	20
ASTM-A514	100	700	120	830	15

(*Continued*)

TABLE H-3 MECHANICAL PROPERTIES (Continued)

Material	Yield stress σ_Y		Ultimate stress σ_U		Percent elongation (2 in. gage length)
	ksi	MPa	ksi	MPa	
Steel wire	40–150	280–1,000	80–200	550–1,400	5–40
Titanium alloys	110–150	760–1,000	130–170	900–1,200	10
Tungsten			200–600	1,400–4,000	0–4
Wood (bending)					
Douglas fir	5–8	30–50	8–12	50–80	
Oak	6–9	40–60	8–14	50–100	
Southern pine	6–9	40–60	8–14	50–100	
Wood (compression parallel to grain)					
Douglas fir	4–8	30–50	6–10	40–70	
Oak	4–6	30–40	5–8	30–50	
Southern pine	4–8	30–50	6–10	40–70	

TABLE H-4 COEFFICIENTS OF THERMAL EXPANSION

Material	Coefficient of thermal expansion α	
	10^{-6}/°F	10^{-6}/°C
Aluminum alloys	13	23
Brass	10.6–11.8	19.1–21.2
Bronze	9.9–11.6	18–21
Cast iron	5.5–6.6	9.9–12
Concrete	4–8	7–14
Copper and copper alloys	9.2–9.8	16.6–17.6
Glass	3–6	5–11
Magnesium alloys	14.5–16.0	26.1–28.8
Monel (67% Ni, 30% Cu)	7.7	14
Nickel	7.2	13
Plastics		
Nylon	40–80	70–140
Polyethylene	80–160	140–290
Rock	3–5	5–9
Rubber	70–110	130–200
Steel	5.5–9.9	10–18
High-strength	8.0	14
Stainless	9.6	17
Structural	6.5	12
Titanium alloys	4.5–6.0	8.1–11
Tungsten	2.4	4.3

Answers to Problems

CHAPTER 1

1.2-1 (a) $\sigma_{AB} = 1443$ psi; (b) $P_2 = 1487.5$ lbs;
(c) $t_{BC} = 0.5$ in.

1.2-2 (a) $\sigma = 65$ MPa; (b) $\varepsilon = 4.652 \times 10^{-4}$

1.2-3 (a) $R_B = 127.3$ lb (cantilever), 191.3 lb (V-brakes);
$\sigma_c = 204$ psi (cantilever), 306 psi (V-brakes);
(b) (b) $\sigma_{\text{cable}} = 26{,}946$ psi (both)

1.2-4 (a) $\delta = 0.220$ mm; (b) $P = 34.6$ kN

1.2-5 (a) $\sigma_C = 2.128$ ksi; $x_C = 19.22$ in., $y_C = 19.22$ in.

1.2-6 $\sigma_t = 133$ MPa

1.2-7 $\sigma_1 = 25.5$ ksi; $\sigma_2 = 35.8$ ksi;

1.2-8 $\sigma_c = 5.21$ MPa

1.2-9 (a) $T = 184$ lb, $\sigma = 10.8$ ksi; (b) $\epsilon_{\text{cable}} = 5 \times 10^{-4}$

1.2-10 (a) $T = 819$ N, $\sigma = 74.5$ MPa;
(b) $\epsilon_{\text{cable}} = 4.923 \times 10^{-4}$

1.2-11 (a) $T_1 = 5877$ lb, $T_2 = 4679$ lb, $T_3 = 7159$ lb;
(b) $\sigma_1 = 49$ ksi, $\sigma_2 = 39$ ksi, $\sigma_3 = 60$ ksi

1.2-12 (a) $\sigma_x = \gamma\omega^2(L^2 - x^2)/2g$; (b) $\sigma_{\max} = \gamma\omega^2L^2/2g$

1.2-13 (a) $T_{AB} = 1620$ lb, $T_{BC} = 1536$ lb, $T_{CD} = 1640$ lb
(b) $\sigma_{AB} = 13{,}501$ psi, $\sigma_{BC} = 12{,}799$ psi,
$\sigma_{CD} = 13{,}667$ psi

1.2-14 (a) $T_{AQ} = T_{BQ} = 50.5$ kN; (b) $\sigma = 166$ MPa

1.3-1 (a) $L_{\max} = 11{,}800$ ft; (b) $L_{\max} = 13{,}500$ ft

1.3-2 (a) $L_{\max} = 7900$ m; (b) $L_{\max} = 8330$ m

1.3-3 % elongation = 6.5, 24.0, 39.0;
% reduction = 8.1, 37.9, 74.9;
Brittle, ductile, ductile

1.3-4 11.9×10^3 m; 12.7×10^3 m; 6.1×10^3 m;
6.5×10^3 m; 23.9×10^3 m

1.3-5 $\sigma \approx 31$ ksi

1.3-6 $\sigma_{\text{pl}} \approx 47$ MPa, Slope ≈ 2.4 GPa, $\sigma_Y \approx 53$ MPa;
Brittle

1.3-7 $\sigma_{\text{pl}} \approx 65{,}000$ psi, Slope $\approx 30 \times 10^6$ psi,
$\sigma_Y \approx 69{,}000$ psi, $\sigma_U \approx 113{,}000$ psi;
Elongation = 6%, Reduction = 31%

1.4-1 0.13 in. longer

1.4-2 4.0 mm longer

1.4-3 (a) 2.809 in.; (b) 31.8 ksi

1.4-4 (a) 2.966 mm; (b) 180 MPa

1.4-5 (b) 0.71 in.; (c) 0.58 in.; (d) 49 ksi

1.5-1 $P_{\max} = 157$ k

1.5-2 $P = 27.4$ kN (tension)

1.5-3 $P = -15.708$ kips

1.5-4 $\Delta L = 1.886$ mm; % decrease
in x-sec area = 0.072%

1.5-5 $\Delta d = -1.56 \times 10^{-4}$ in., $P = 2.154$ kips

1.5-6 (a) $E = 104$ GPa; (b) $\nu = 0.34$

1.5-7 (a) $\Delta d_{BC\text{inner}} = 8 \times 10^{-4}$ in.
(b) $\nu_{\text{brass}} = 0.34$
(c) $\Delta t_{AB} = 2.732 \times 10^{-4}$ in.,
$\Delta d_{AB\text{inner}} = 1.366 \times 10^{-4}$ in.

1.5-8 $\Delta V = 9789\ \text{mm}^3$

1.6-1 $\sigma_b = 7.04$ ksi, $\tau_{\text{ave}} = 10.756$ ksi

1.6-2 $\sigma_b = 139.86$ MPa; $P_{\text{ult}} = 144.45$ kN

1.6-3 (a) $\tau = 12.732$ ksi; (b) $\sigma_{bf} = 20$ ksi;
$\sigma_{bg} = 26.667$ ksi

1.6-4 (a) $A_x = 254.6$ N, $A_y = 1072$ N, $B_x = -254.6$ N
(b) $A_{\text{resultant}} = 1101.8$ N
(c) $\tau = 5.48$ MPa, $\sigma_b = 6.886$ MPa

1.6-5 (a) $\tau_{\max} = 2979$ psi; (b) $\sigma_{b\max} = 936$ psi

1.6-6 $T_1 = 13.176$ kN, $T_2 = 10.772$ kN,
$\tau_{1\text{ave}} = 25.888$ MPa, $\tau_{2\text{ave}} = 21.166$ MPa,
$\sigma_{b1} = 7.32$ MPa, $\sigma_{b2} = 5.985$ MPa

1.6-7 (a) Resultant = 1097 lb;
(b) $\sigma_b = 4999$ psi
(c) $\tau_{\text{nut}} = 2793$ psi, $\tau_{\text{pl}} = 609$ psi

1.6-8 $G = 2.5$ MPa

1.6-9 (a) $\gamma_{\text{aver}} = 0.004$; (b) $V = 89.6$ k

1.6-10 (a) $\gamma_{\text{aver}} = 0.50$; (b) $\delta = 4.50$ mm

1.6-11 (a) $\tau_{\text{aver}} = 6050$ psi; (b) $\sigma_b = 9500$ psi

1.6-12 $\tau_{\text{aver}} = 42.9$ MPa

1.6-13 (a) $A_x = 0$, $A_y = 170$ lb, $M_A = 4585$ in.-lb
(b) $B_x = 253.6$ lb, $B_y = 160$ lb, $B_{\text{res}} = 299.8$ lb,
$C_x = -B_x$

PRINCIPAL UNITS USED IN MECHANICS

Quantity	International System (SI)			U.S. Customary System (USCS)		
	Unit	Symbol	Formula	Unit	Symbol	Formula
Acceleration (angular)	radian per second squared		rad/s^2	radian per second squared		rad/s^2
Acceleration (linear)	meter per second squared		m/s^2	foot per second squared		ft/s^2
Area	square meter		m^2	square foot		ft^2
Density (mass) (Specific mass)	kilogram per cubic meter		kg/m^3	slug per cubic foot		slug/ft^3
Density (weight) (Specific weight)	newton per cubic meter		N/m^3	pound per cubic foot	pcf	lb/ft^3
Energy; work	joule	J	N·m	foot-pound		ft-lb
Force	newton	N	kg·m/s^2	pound	lb	(base unit)
Force per unit length (Intensity of force)	newton per meter		N/m	pound per foot		lb/ft
Frequency	hertz	Hz	s^{-1}	hertz	Hz	s^{-1}
Length	meter	m	(base unit)	foot	ft	(base unit)
Mass	kilogram	kg	(base unit)	slug		lb-s^2/ft
Moment of a force; torque	newton meter		N·m	pound-foot		lb-ft
Moment of inertia (area)	meter to fourth power		m^4	inch to fourth power		in.^{4}
Moment of inertia (mass)	kilogram meter squared		kg·m^2	slug foot squared		slug-ft^2
Power	watt	W	J/s (N·m/s)	foot-pound per second		ft-lb/s
Pressure	pascal	Pa	N/m^2	pound per square foot	psf	lb/ft^2
Section modulus	meter to third power		m^3	inch to third power		in.^{3}
Stress	pascal	Pa	N/m^2	pound per square inch	psi	lb/in.^{2}
Time	second	s	(base unit)	second	s	(base unit)
Velocity (angular)	radian per second		rad/s	radian per second		rad/s
Velocity (linear)	meter per second		m/s	foot per second	fps	ft/s
Volume (liquids)	liter	L	10^{-3} m^3	gallon	gal.	231 in.^{3}
Volume (solids)	cubic meter		m^3	cubic foot	cf	ft^3

SELECTED PHYSICAL PROPERTIES

Property	SI	USCS
Water (fresh)		
weight density	9.81 kN/m^3	62.4 lb/ft^3
mass density	1000 kg/m^3	1.94 slugs/ft^3
Sea water		
weight density	10.0 kN/m^3	63.8 lb/ft^3
mass density	1020 kg/m^3	1.98 slugs/ft^3
Aluminum (structural alloys)		
weight density	28 kN/m^3	175 lb/ft^3
mass density	2800 kg/m^3	5.4 slugs/ft^3
Steel		
weight density	77.0 kN/m^3	490 lb/ft^3
mass density	7850 kg/m^3	15.2 slugs/ft^3
Reinforced concrete		
weight density	24 kN/m^3	150 lb/ft^3
mass density	2400 kg/m^3	4.7 slugs/ft^3
Atmospheric pressure (sea level)		
Recommended value	101 kPa	14.7 psi
Standard international value	101.325 kPa	14.6959 psi
Acceleration of gravity (sea level, approx. 45° latitude)		
Recommended value	9.81 m/s^2	32.2 ft/s^2
Standard international value	9.80665 m/s^2	32.1740 ft/s^2

SI PREFIXES

Prefix	Symbol	Multiplication factor
tera	T	$10^{12} = 1\ 000\ 000\ 000\ 000$
giga	G	$10^{9} = 1\ 000\ 000\ 000$
mega	M	$10^{6} = 1\ 000\ 000$
kilo	k	$10^{3} = 1\ 000$
hecto	h	$10^{2} = 100$
deka	da	$10^{1} = 10$
deci	d	$10^{-1} = 0.1$
centi	c	$10^{-2} = 0.01$
milli	m	$10^{-3} = 0.001$
micro	μ	$10^{-6} = 0.000\ 001$
nano	n	$10^{-9} = 0.000\ 000\ 001$
pico	p	$10^{-12} = 0.000\ 000\ 000\ 001$

Note: The use of the prefixes hecto, deka, deci, and centi is not recommended in SI.