
Mech Family
كن أنت التغيير
اللجنة الأكاديمية في قسم
الهندسة الميكانيكية

8.11 A flow-metering device, called a vortex meter, consists of a square element mounted inside a pipe. Vortices are generated by the element, which gives rise to an oscillatory pressure measured on the leeward side of the element. The fluctuation frequency is related to the flow velocity. The discharge in the pipe is a function of the frequency of the oscillating pressure ω, the pipe diameter D, the size of the element l, the density ρ, and the viscosity μ. Thus

$$Q = f(\omega, D, l, \rho, \mu)$$

Find the π-groups in the form

$$\frac{Q}{\omega D^3} = f(\pi_1, \pi_2)$$

8.11: PROBLEM DEFINITION

Situation:

Vortex meter for flow rate measurement.

Find:

The functional relation in the form

$$\frac{Q}{\omega D^3} = f(\pi_1, \pi_2)$$

PLAN The functional form of the equation is

$$Q = (\omega, D, l, \rho, \mu)$$

Use the step-by-step method.

SOLUTION Setting up the table

Q	$\frac{L^3}{T}$	$\frac{Q}{\omega}$	L^3	$\frac{Q}{\omega D^3}$		$\frac{Q}{\omega D^3}$
ω	$\frac{1}{T}$					
D	L	D	L			
l	L	l	L	$\frac{l}{D}$		$\frac{l}{D}$
ρ	$\frac{M}{L^3}$	ρ	$\frac{M}{L^3}$	ρD^3	M	$\frac{\omega\rho D^2}{\mu}$
μ	$\frac{M}{LT}$	$\frac{\mu}{\omega}$	$\frac{M}{L}$	$\frac{\mu D}{\omega}$	M	

The time was eliminated with ω, the length with D and the mass with ρD^3. The function form is

$$\boxed{\frac{Q}{\omega D^3} = f\left(\frac{l}{D}, \frac{\omega\rho D^2}{\mu}\right)}$$

8.14 A general study is to be made of the height of rise of liquid in a capillary tube as a function of time after the start of a test. Other significant variables include surface tension, mass density, specific weight, viscosity, and diameter of the tube. Determine the dimensionless parameters that apply to the problem. Express your answer in the functional form

$$\frac{h}{d} = f(\pi_1, \pi_2, \pi_3)$$

8.14: PROBLEM DEFINITION

Situation:

A study involves capillary rise of a liquid in a tube.

Find:

The $\pi-$groups in the form

$$\frac{h}{d} = f(\pi_1, \pi_2, \pi_3)$$

PLAN

The dimensional form of the equation is

$$h = f(t, d, \sigma, \rho, \gamma, \mu)$$

There are 7 dimensional variables so there should be 4 π groups. Use the step-by-step method.

SOLUTION

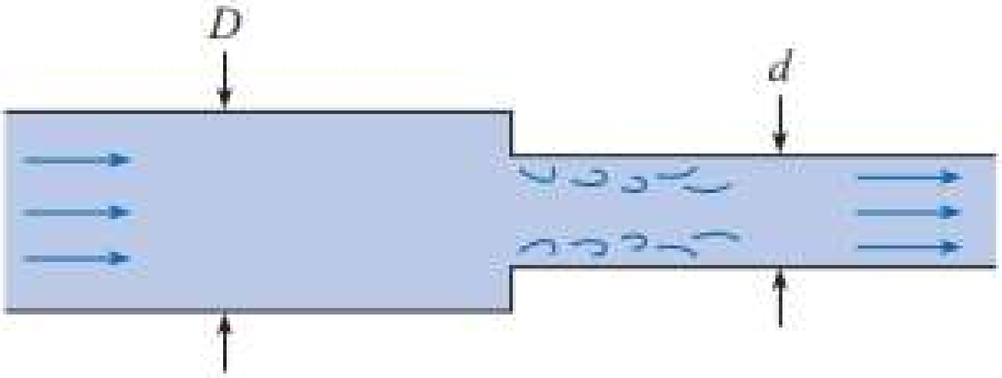
h	L	$\frac{h}{d}$	0	$\frac{h}{d}$	0	$\frac{h}{d}$	0
t	T	t	T	t	T		
σ	$\frac{M}{T^2}$	σ	$\frac{M}{T^2}$	$\frac{\sigma}{\rho d^3}$	$\frac{1}{T^2}$	$\frac{\sigma t^2}{\rho d^3}$	0
ρ	$\frac{M}{L^3}$	ρd^3	M				
γ	$\frac{M}{L^2T^2}$	γd^2	$\frac{M}{T^2}$	$\frac{\gamma}{\rho d}$	$\frac{1}{T^2}$	$\frac{\gamma t^2}{\rho d}$	0
μ	$\frac{M}{LT}$	μd	$\frac{M}{T}$	$\frac{\mu}{\rho d^2}$	$\frac{1}{T}$	$\frac{\mu t}{\rho d^2}$	0
d	L						

In the first step, length is removed with d. In the second step, mass is removed with ρd^3 and in the final step, time is removed with t. The final functional form is

$$\boxed{\frac{h}{d} = f\left(\frac{\sigma t^2}{\rho d^3}, \frac{\gamma t^2}{\rho d}, \frac{\mu t}{\rho d^2}\right)}$$

8.16 By dimensional analysis, determine the π-groups for the change in pressure that occurs when water or oil flows through a horizontal pipe with an abrupt contraction as shown. Express your answer in the functional form

$$\frac{\Delta p d^4}{\rho Q^2} = f(\pi_1, \pi_2)$$



PROBLEM 8.16

8.16: PROBLEM DEFINITION

Situation:

A liquid in a pipe flows through an abrupt contraction.

Find:

The π-groups that characterize pressure drop. Express the answer as

$$\frac{\Delta p d^4}{\rho Q^2} = f(\pi_1,\ \pi_2)$$

PLAN

The pressure change should depend on the upstream and downstream diameters, the discharge, density, viscosity The functional form of the equation is

$$\Delta p = f(Q, \rho, \mu, d, D)$$

Use the step-by-step method.

SOLUTION

Δp	$\frac{M}{LT^2}$	$\Delta p d$	$\frac{M}{T^2}$	$\frac{\Delta p}{\rho d^2}$	$\frac{1}{T^2}$	$\frac{\Delta p d^4}{\rho Q^2}$	0
Q	$\frac{L^3}{T}$	$\frac{Q}{d^3}$	$\frac{1}{T}$	$\frac{Q}{d^3}$	$\frac{1}{T}$		
ρ	$\frac{M}{L^3}$	ρd^3	M				
μ	$\frac{M}{LT}$	μd	$\frac{M}{T}$	$\frac{\mu}{\rho d^2}$	$\frac{1}{T}$	$\frac{\mu d}{\rho Q}$	0
D	L	$\frac{D}{d}$	0	$\frac{D}{d}$	0	$\frac{D}{d}$	0
d	L						

Length is removed with d in the first step, mass with ρd^3 in the second step and time with Q/d^3 in the third step. The final form is

$$\boxed{\frac{\Delta p d^4}{\rho Q^2} = f\left(\frac{\mu d}{\rho Q}, \frac{D}{d}\right)}$$

8.23 The discharge of a centrifugal pump is a function of the rotational speed of the pump N, the diameter of the impeller D, the head across the pump h_p, the viscosity of the fluid μ, the density of the fluid ρ, and the acceleration due to gravity g. The functional relationship is

$$Q = f(N, D, h_p, \mu, \rho, g)$$

By dimensional analysis, find the dimensionless parameters. Express your answer in the form

$$\frac{Q}{ND^3} = f(\pi_1, \pi_2, \pi_3)$$

8.23: PROBLEM DEFINITION

Situation:

Discharge through a centrifugal pump.

The dimensional functional form:

$$Q = f(N, D, h_p, \mu, \rho, g)$$

Find:

The π-groups in the $\pi-$groups in the form

$$\frac{Q}{ND^3} = f(\pi_1, \pi_2, \pi_3)$$

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PLAN

There are 7 dimensional groups so there should be 4 π-groups. Use the step-by-step method.

Q	$\frac{L^3}{T}$	$\frac{Q}{D^3}$	$\frac{1}{T}$	$\frac{Q}{D^3}$	$\frac{1}{T}$	$\frac{Q}{ND^3}$	0
N	$\frac{1}{T}$	N	$\frac{1}{T}$	$\frac{1}{T}$	$\frac{1}{T}$		
D	L						
h_p	L	$\frac{h_p}{D}$	0	$\frac{h_p}{D}$	0	$\frac{h_p}{D}$	0
μ	$\frac{M}{LT}$	μD	$\frac{M}{T}$	$\frac{\mu}{\rho D^2}$	$\frac{1}{T}$	$\frac{\mu}{\rho N D^2}$	0
ρ	$\frac{M}{L^3}$	ρD^3	M				
g	$\frac{L}{T^2}$	$\frac{g}{D}$	$\frac{1}{T^2}$	$\frac{g}{D}$	$\frac{1}{T^2}$	$\frac{g}{N^2 D}$	0

The functional relationship is

$$\boxed{\frac{Q}{ND^3} = f\left(\frac{h_p}{D}, \frac{\mu}{\rho N D^2}, \frac{g}{N^2 D}\right)}$$

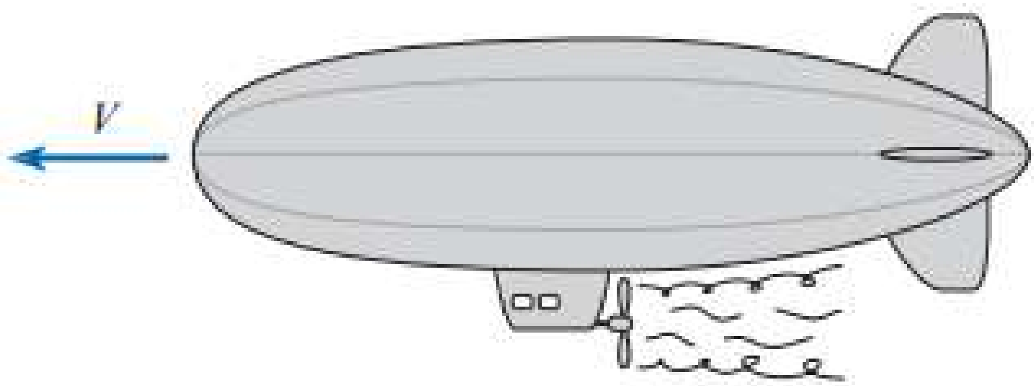
Some dimensionless variables can be combined to yield a different form

$$\frac{Q}{ND^3} = f\left(\frac{h_p g}{N^2 D^2}, \frac{\mu}{\rho N D^2}, \frac{g}{N^2 D}\right)$$

8.45 A student is competing in a contest to design a radio-controlled blimp. The drag force acting on the blimp depends on the Reynolds number, $\mathrm{Re} = (\rho V D)/\mu$, where V is the speed of the blimp, D is the maximum diameter, ρ is the density of air, and μ is the viscosity of air. This blimp has a coefficient of drag (C_D) of 0.3. This π-group is defined as

$$C_D = 2\frac{F_D}{\rho V^2 A_p}$$

where F_D is the drag force ρ is the density of ambient air, V is the speed of the blimp, and $A_p = \pi D^2/4$ is the maximum section area of the blimp from a front view. Calculate the Reynolds number, the drag force in newtons, and the power in watts required to move the blimp through the air. Blimp speed is 800 mm/s, and the maximum diameter is 475 mm. Assume that ambient air is at 20°C.



PROBLEM 8.45

8.45: PROBLEM DEFINITION

Situation:

A student team is designing a radio-controlled blimp.
Drag force is characterized with a coefficient of drag:.

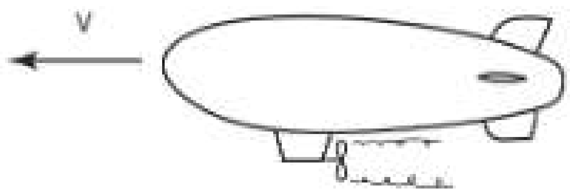
$$C_D = 2\frac{F_D}{\rho V^2 A_p} = 0.3$$

$V = 800\,\text{mm/s}$, $D = 0.475\,\text{m}$.
$A_p = \pi D^2/4$.

Find:

a.) Reynolds number.
b.) Force of drag (N).
c.) Power in watts (W).

Sketch:



Assumptions:

Assume the blimp cross section is round.

Properties:

Air (20 °C), Table A.3: $\rho = 1.2\,\text{kg/m}^3$, $\mu = 18.1 \times 10^{-6}\,\text{N}\cdot\text{s/m}^2$.

PLAN

Find the Reynolds number by direct calculation. Find the drag force using the definition of C_D. Find power (P) by using the product of force and speed: $P = F_{\text{Drag}}V$.

SOLUTION

Reynolds number

$$\begin{aligned} \text{Re} &= \frac{VD\rho}{\mu} \\ &= \frac{(0.8\,\text{m/s})(0.475\,\text{m})(1.2\,\text{kg/m}^3)}{(1.81 \times 10^{-5}\,\text{N}\cdot\text{s/m}^2)} \end{aligned}$$

$$\boxed{\text{Re} = 25,200}$$

Projected area

$$\begin{aligned} A_p &= \frac{\pi D^2}{4} = \frac{\pi\,(0.475\,\mathrm{m})^2}{4} \\ &= 0.177\,\mathrm{m}^2 \end{aligned}$$

Drag force

$$\begin{aligned} F_D &= C_D\left(\frac{\rho V^2}{2}\right)A_p \\ &= (0.3)\,\frac{(1.2\,\mathrm{kg/\,m^3})\,(0.8\,\mathrm{m/\,s})^2}{2}\,(0.177\,\mathrm{m}^2) \\ &\boxed{F_D = 20.4\times 10^{-3}\,\mathrm{N}} \end{aligned}$$

Power

$$\begin{aligned} P &= F_D V \\ &= (20.4\times 10^{-3}\,\mathrm{N})\,(0.8\,\mathrm{m/\,s}) \\ &\boxed{P = 16.3\times 10^{-3}\,\mathrm{W}} \end{aligned}$$

REVIEW

1. The drag force is about $1/50^{\mathrm{th}}$ of a Newton.

2. The power is about 16 milliwatts. The supplied power would need to be higher to account for factors such as propeller efficiency and motor efficiency.