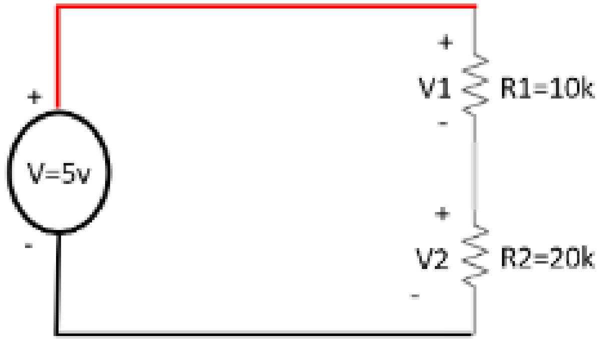
Introduction to Circuit Analysis Methods
Terminology, KCL & KVL

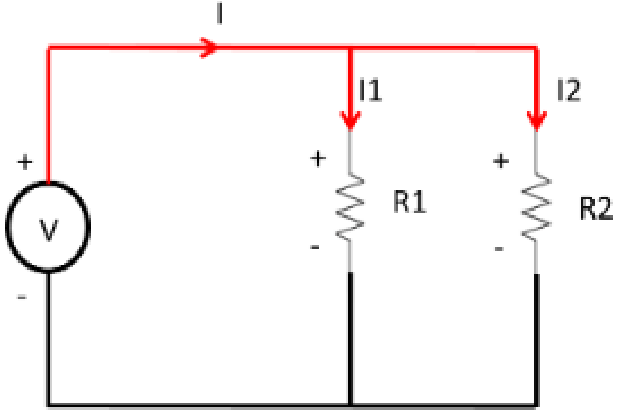
Voltage division rule



$$V1= \frac{R1}{R1+R2} * V$$

$$V2= \frac{R2}{R1+R2} * V$$

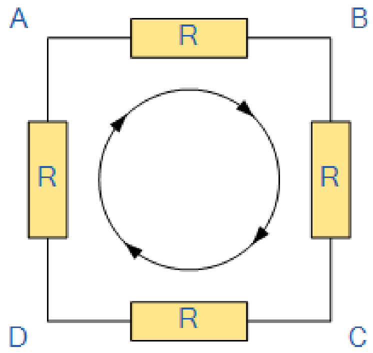
Current division rule



$$I1= \frac{R2}{R1+R2} * I$$

$$I2= \frac{R1}{R1+R2} * I$$

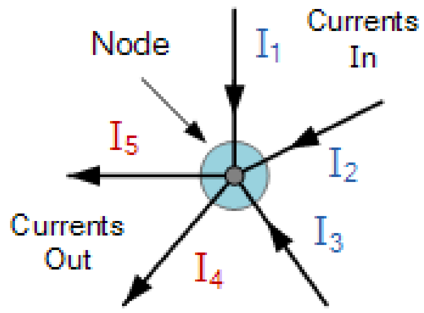
Kirchhoffs Voltage Law



The sum of all the Voltage Drops around the loop is equal to Zero

$$V_{AB} + V_{BC} + V_{CD} + V_{DA} = 0$$

Kirchhoffs Current Law

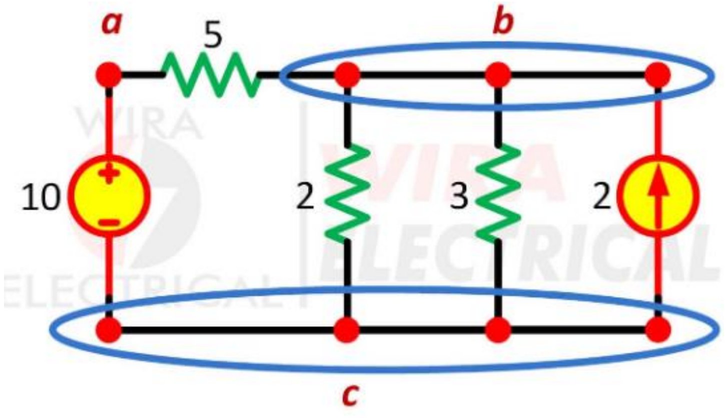


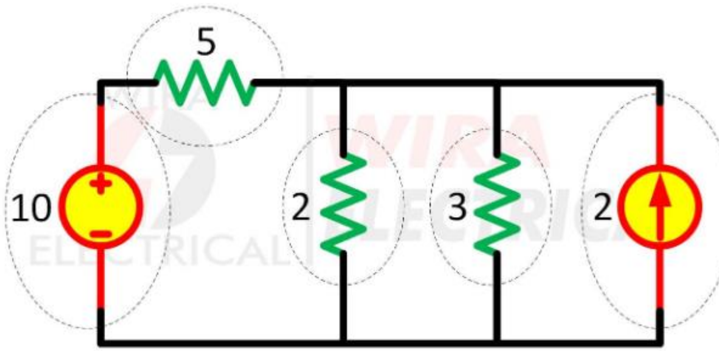
$$I_1 + I_2 + I_3 + (-I_4 + -I_5) = 0$$

Currents Entering the Node Equals Currents Leaving the Node

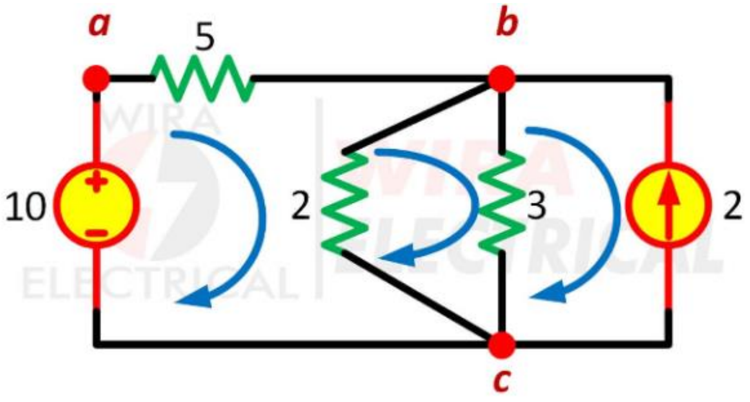
Terminology: Node, branch and Loop

- **Node:** **point of connection of two (or more) circuit elements.** *Node is represented by a dot in an electric circuit*
- **Branch:** **portion of a circuit containing a single element and the nodes at each end of the element.** *A branch is a circuit element such as voltage or current source or a resistor, capacitor, inductor.*
- **Loop**: **any closed path through the circuit in which no node is encountered more than once.** *A loop is a closed path inside an electric circuit.*



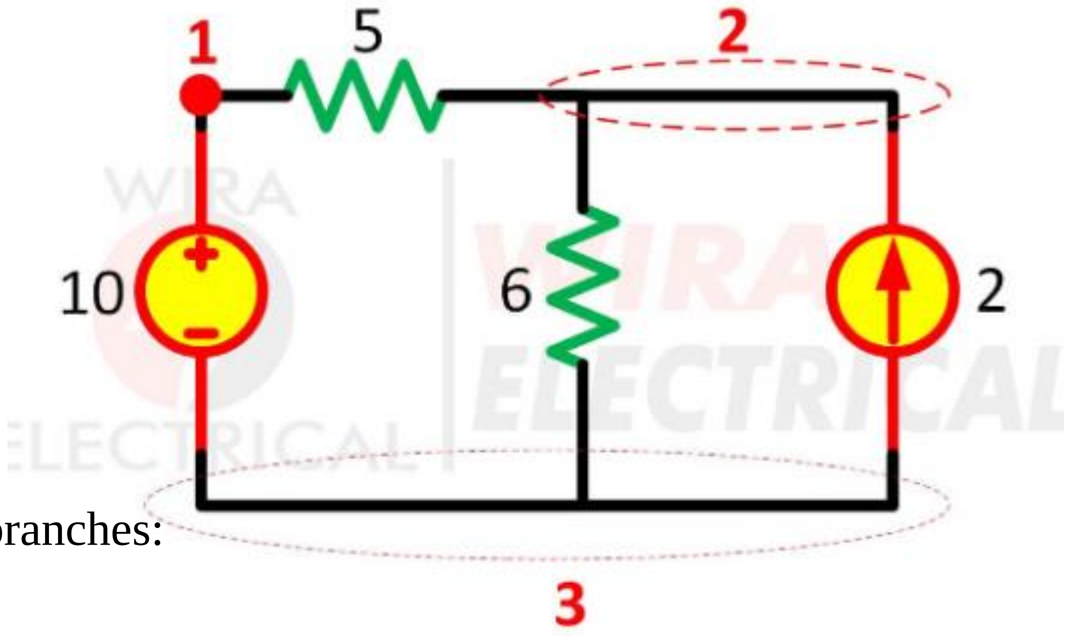


The circuit has five branches



Terminology: Node, branch and Loop
Example

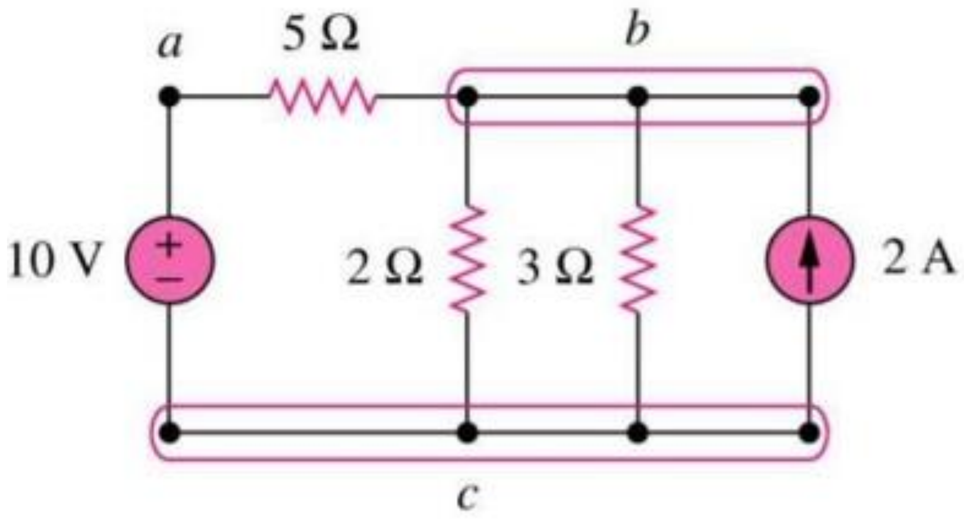
- Observe the circuit below and count the number of branches and nodes.
- Also identify which parts are in series or parallel.



- The circuit above has four element, thus it has four branches:
  - •10V voltage source,
  - •5Ω resistor,
  - •6Ω resistor, and
  - •2A current source
- It has three nodes.
- The series connection is formed from a 10V voltage source and 5Ω resistor. The parallel connection is formed from a 6Ω resistor and 2A current source connected to nodes 2 and 3.

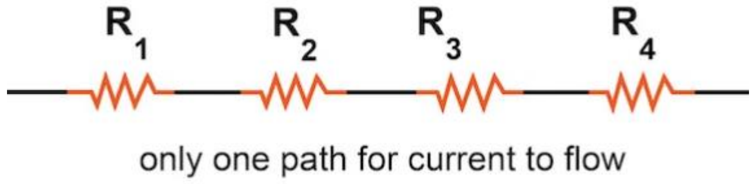
Terminology: Node, branch and Loop
Example

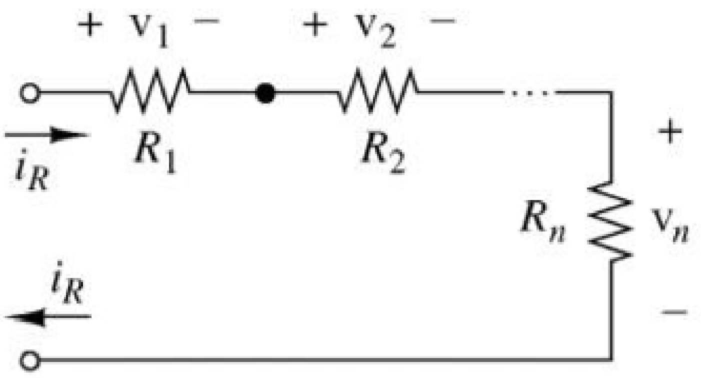
- In the shown circuit, find the number of branches, nodes and loops.



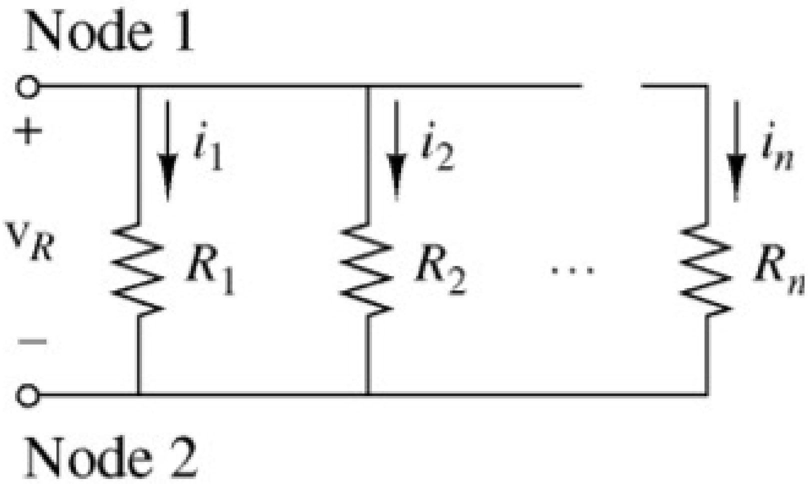
Terminology: Series and Parallel Connection

- In a series circuit, all components are connected end-to-end to form a single path for current flow.
- Elements that are in series carry the same current.





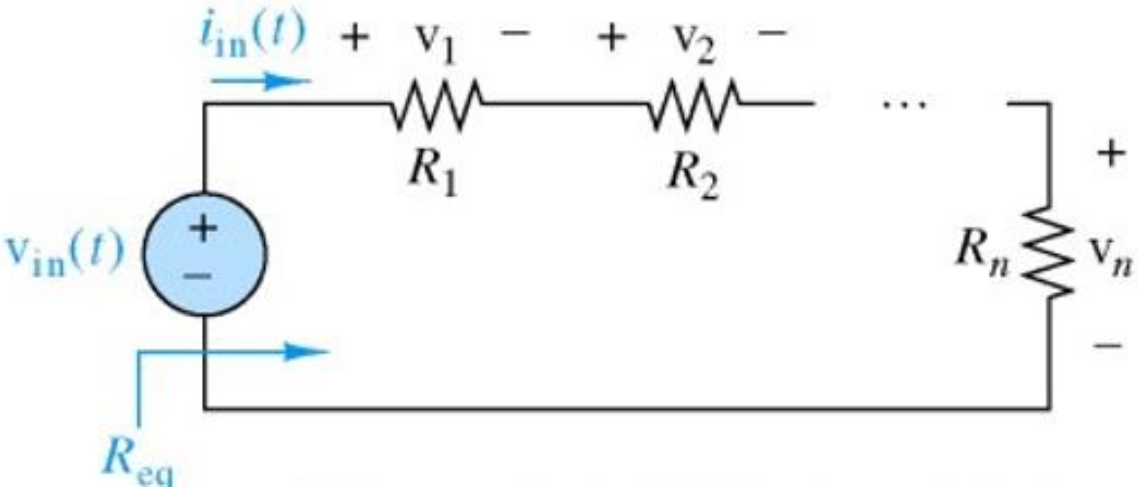
- In a parallel circuit, all components are connected across each other with exactly two electrically common nodes
- Elements that are in parallel have the same voltage across each element.



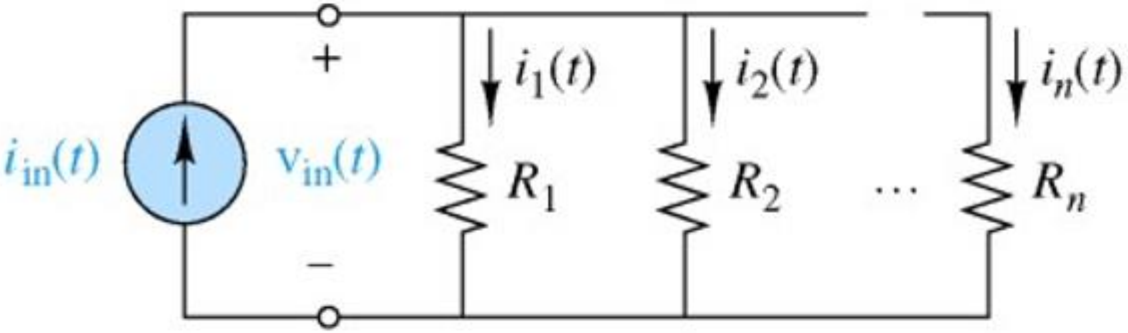
Series- and Parallel-Connected Resistors

- The equivalent resistance of any number of resistors connected in series is the sum of the resistors (Why?)

- The equivalent conductance of resistors connected in parallel is the sum of their individual conductances:



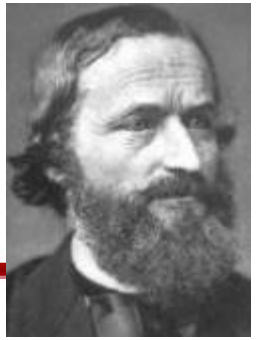
$$R_{eq} = R_1 + R_2 + \cdots + R_n$$



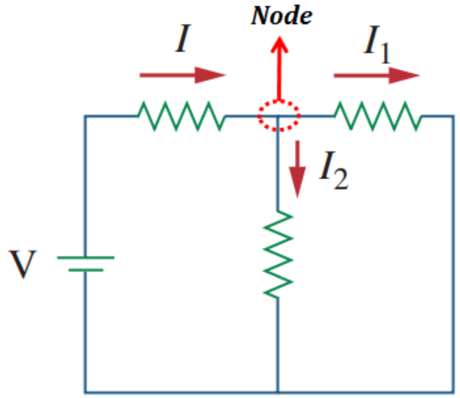
$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n}$$

$$G_{eq} = G_1 + G_2 + \cdots + G_n$$

Kirchhoff's Current Law (KCL)
Kirchhoff's Voltage Law (KVL)

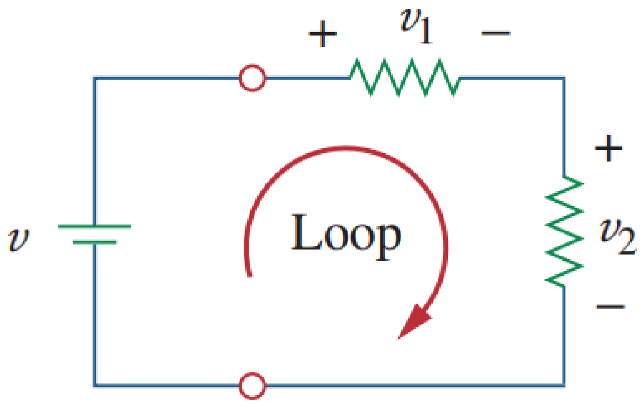


- Gustav Robert Kirchhoff (1824-1887), a German physicist, stated two basic laws concerning the relationship between the currents and voltages in an electrical circuit.



- **KCL:** At any node of a circuit, the sum of the currents into the node is equal to the sum of the currents out of the node.
- At any node of a circuit, the currents algebraically sum to zero: $\Sigma I = 0$ or $\Sigma I_{in} = \Sigma I_{out}$.

$$\boldsymbol{I = I_1 + I_2} \qquad \boldsymbol{I - I_1 - I_2 = 0}$$

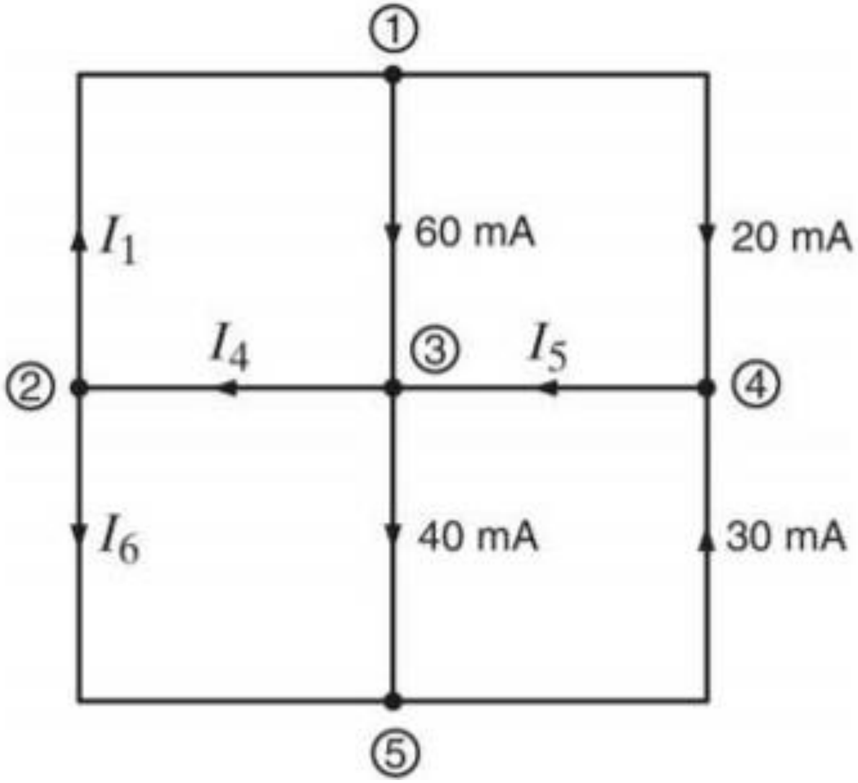


- **KVL:** In any loop of a circuit, the algebraic sum of the potential differences must be equal to zero as: $\Sigma v = 0$. or $\Sigma v_{rise} = \Sigma v_{drop}$

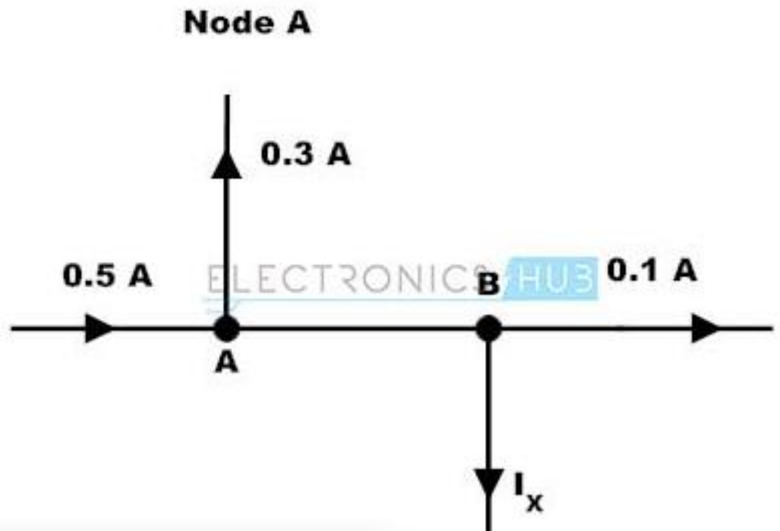
$$\boldsymbol{v = v_1 + v_2} \qquad \boldsymbol{-v + v_1 + v_2 = 0}$$

Examples

- The following network is represented by its topological diagram. Find the unknown currents in the network.

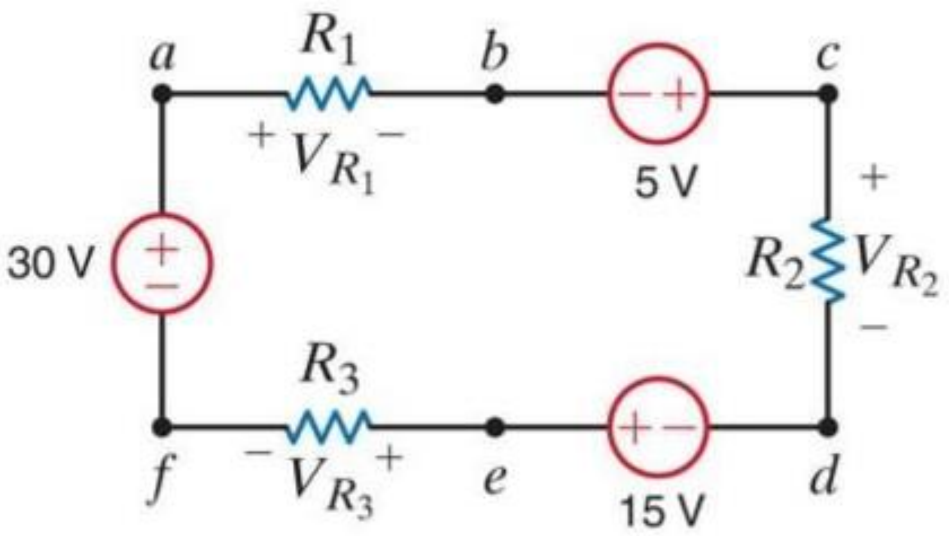


- Consider the below figure where we have to determine the currents IAB and Ix by using KCL .



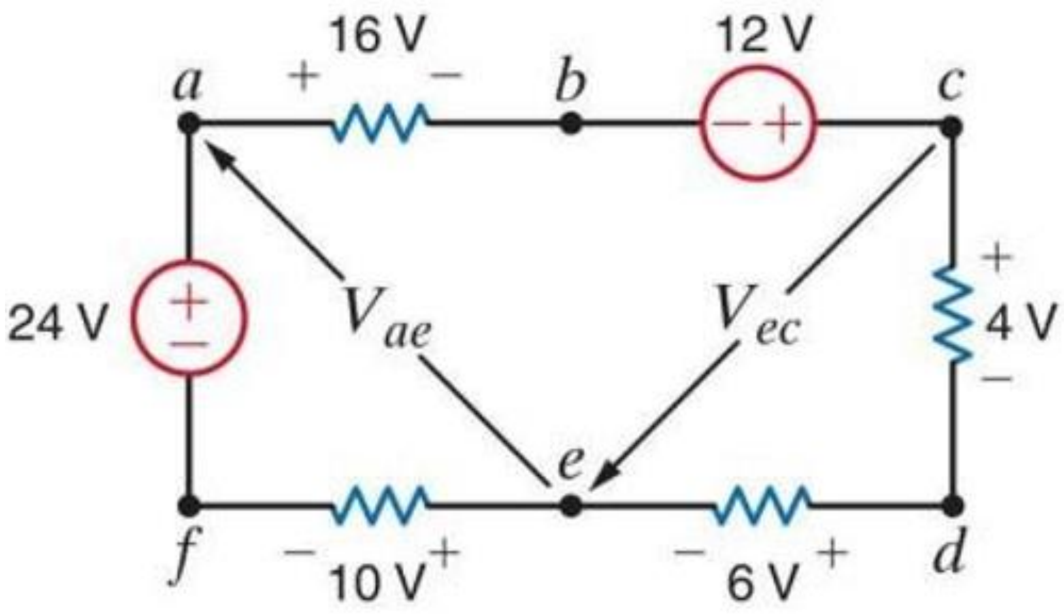
Examples

- In the following circuit, assume V_{R1}=26V and V_{R2}=14V. Find V_{R3}.



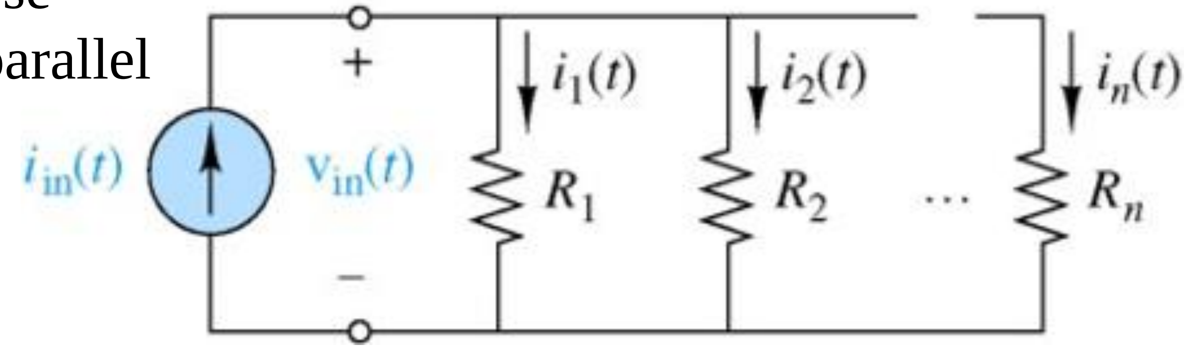
Examples

- In the following circuit use KVL to determine V_{ae} and V_{ec}. Note that we use the convention V_{ae} to indicate the voltage of point a with respect to point *e* or $V_{ae}=V_a-V_e$



Current Division

- The current in a parallel circuit divides in inverse proportion to the resistances of the individual parallel elements.



$$i_1 = \frac{R_{EQ}}{R_1} i_S = \frac{1/R_1}{1/R_{EQ}} i_S = \frac{1/R_1}{1/R_1 + 1/R_2 + 1/R_3} i_S$$

$$i_2 = \frac{1/R_2}{1/R_1 + 1/R_2 + 1/R_3} i_S$$

$$i_3 = \frac{1/R_3}{1/R_1 + 1/R_2 + 1/R_3} i_S$$

$$i_S = v \frac{1}{R_{EQ}}$$

$$\frac{1}{R_{EQ}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N}$$

$$G_{eq} = G_1 + G_2 + \cdots + G_n$$

- The general expression for the current divider for a circuit with *n* parallel resistors is the following:

$$i_n = \frac{1/R_n}{1/R_1 + 1/R_2 + \cdots + 1/R_n + \cdots + 1/R_N} i_S$$

$$R_{EQ} = \frac{1}{1/R_1 + 1/R_2 + \cdots + 1/R_N}$$

$$i_j(t) = \frac{G_j}{G_1 + G_2 + \cdots + G_n} i_{in}(t)$$

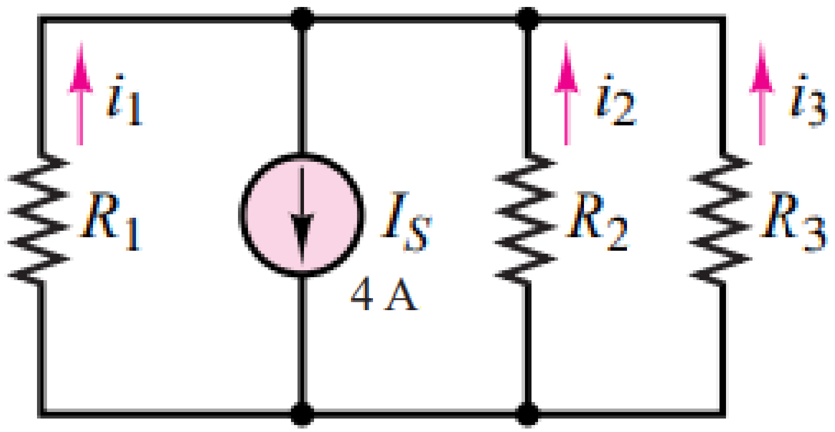
$$i_1 = \frac{v}{R_1} \qquad i_2 = \frac{v}{R_2} \qquad i_n = \frac{v}{R_n}$$

Current Division - Example

- Determine the current i_1 in the circuit

$$i_1 = I_S \times \frac{1/R_1}{1/R_1 + 1/R_2 + 1/R_3}$$

$$= 4 \times \frac{\frac{1}{10}}{\frac{1}{10} + \frac{1}{2} + \frac{1}{20}} = 0.6154 \text{ A}$$



$R_1 = 10\ \Omega;\ R_2 = 2\ \Omega;\ R_3 = 20\ \Omega;$

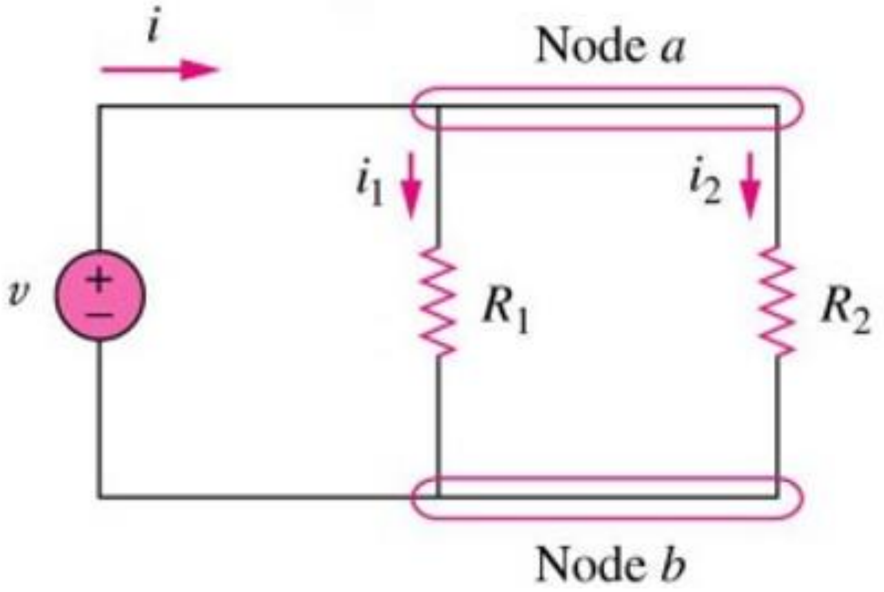
Current Division – Two Parallel Resistors

- For the special case of two parallel resistors

$$i = \frac{v}{R_{EQ}}$$

$$i_1(t) = \frac{R_2}{R_1 + R_2} i(t)$$

$$i_2(t) = \frac{R_1}{R_1 + R_2} i(t)$$



$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

Current Division - Example

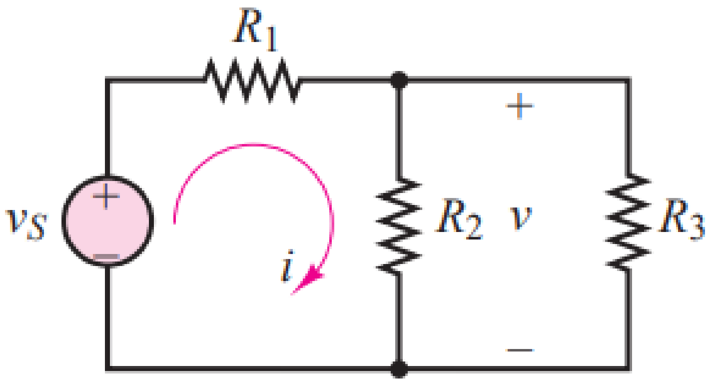
- Determine the voltage v and the currents i_1, i_2 and i_3 through R_1, R_2 and R_3 in the circuit.

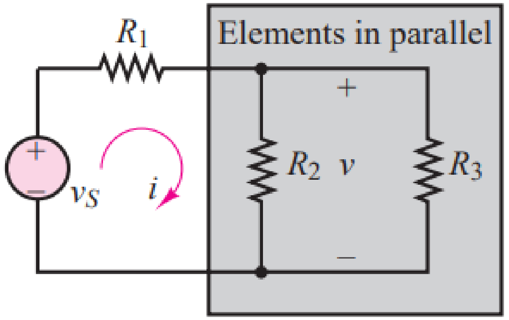
$v_s = 5$ V and $R_1 = R_2 = R_3 = 1$ kΩ.

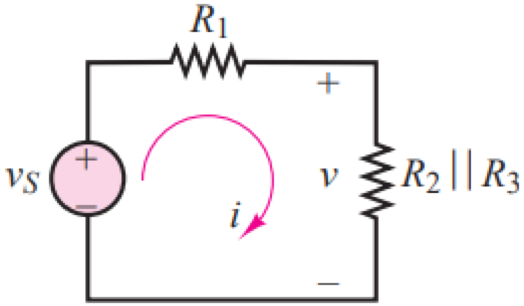
$$v = \frac{R_2 \| R_3}{R_1 + R_2 \| R_3} v_S$$

$$i_1 = \frac{v_S}{R_1 + R_2 \| R_3}$$

$$i_2 = \frac{v}{R_2} \qquad i_3 = \frac{v}{R_3}$$







Equivalent circuit

Voltage Division

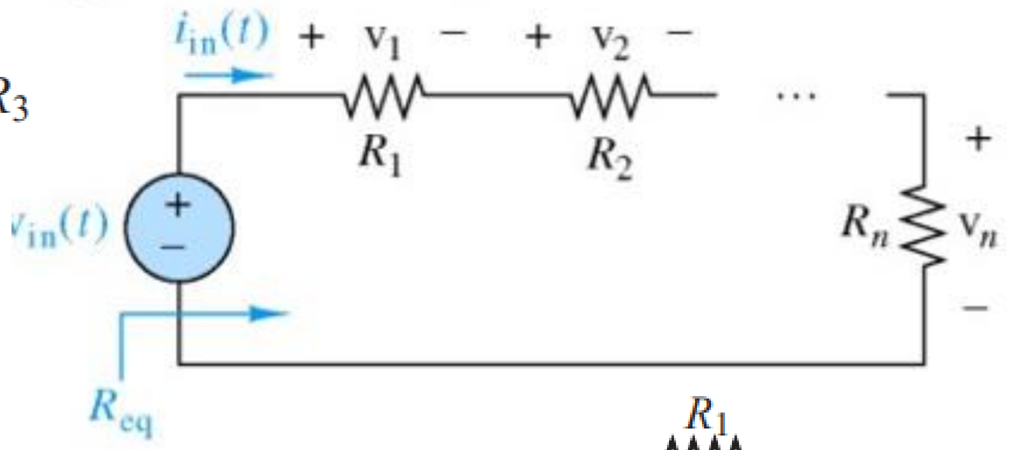
- voltage across each resistor in a series circuit divides in direct proportion to the individual series resistances.

- In a series combination of n resistors, the voltage drop across the resistor R_j for $j=1,2, \ldots, n$ is:

$$v_j(t) = \frac{R_j}{R_1 + R_2 + \cdots + R_n} v_{in}(t)$$

$$v_3 = iR_3$$

$$R_{EQ} = \sum_{n=1}^{N} R_n$$



$$1.5 = v_1 + v_2 + v_3$$

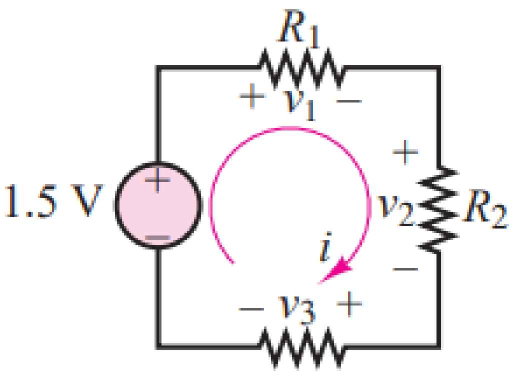
$$1.5\ \text{V} = i(R_1 + R_2 + R_3)$$

$$R_{EQ} = R_1 + R_2 + R_3$$

$$v_1 = iR_1$$

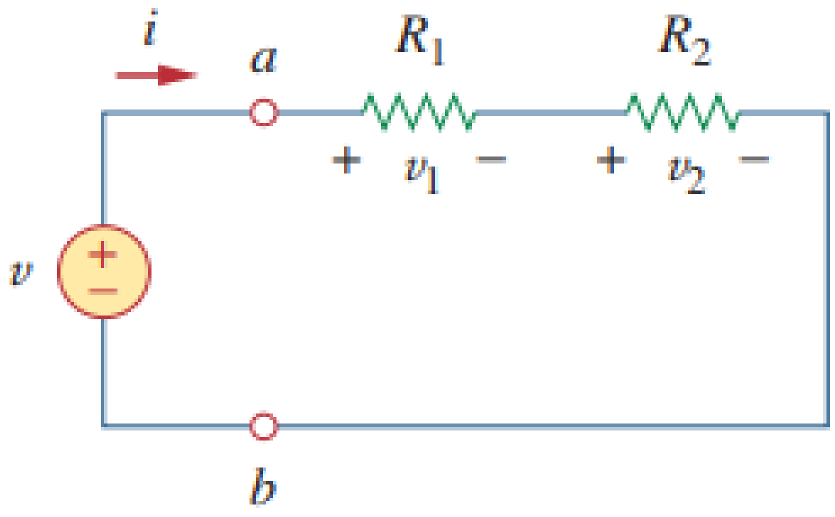
$$v_2 = iR_2$$

$$v_3 = iR_3$$



Voltage Division – Two Series Resistors

- **For the special of series resistors**



$$v_1 = iR_1, \qquad v_2 = iR_2$$

$$v = v_1 + v_2 = i(R_1 + R_2)$$

$$v = iR_{eq} \qquad R_{eq} = R_1 + R_2$$

$$i = \frac{v}{R_1 + R_2}$$

$$v_1 = \frac{R_1}{R_1 + R_2} v \qquad v_2 = \frac{R_2}{R_1 + R_2} v$$

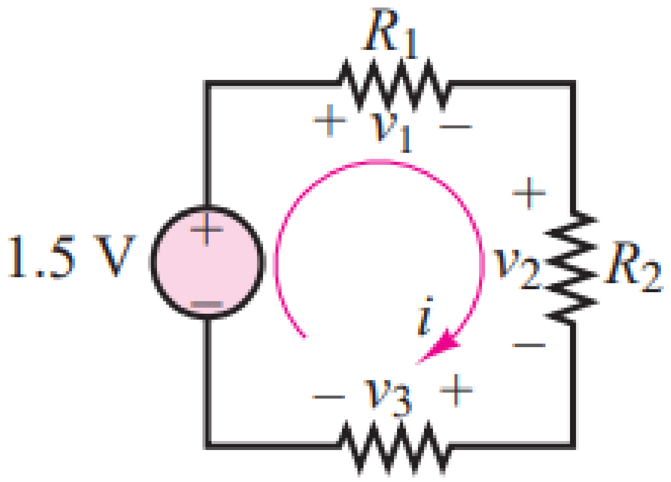
Voltage Division - Example

$$R_{EQ} = R_1 + R_2 + R_3 \qquad R_{EQ} = \sum_{n=1}^{N} R_n$$

$$1.5 = v_1 + v_2 + v_3$$

$$1.5\text{ V} = i(R_1 + R_2 + R_3)$$

$$i = \frac{1.5\text{ V}}{R_{EQ}} = \frac{1.5\text{ V}}{R_1 + R_2 + R_3}$$



we can write each of the voltages across the resistors as:

$$v_1 = iR_1$$

$$v_2 = iR_2$$

$$v_3 = iR_3$$

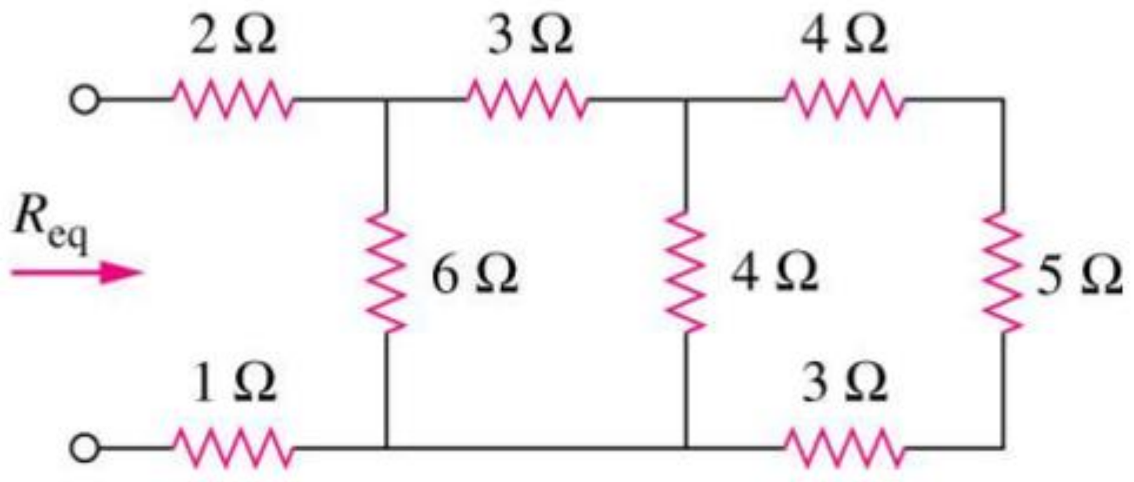
$$v_1 = iR_1 = \frac{R_1}{R_{EQ}}(1.5\text{ V})$$

$$v_2 = iR_2 = \frac{R_2}{R_{EQ}}(1.5\text{ V})$$

$$v_3 = iR_3 = \frac{R_3}{R_{EQ}}(1.5\text{ V})$$

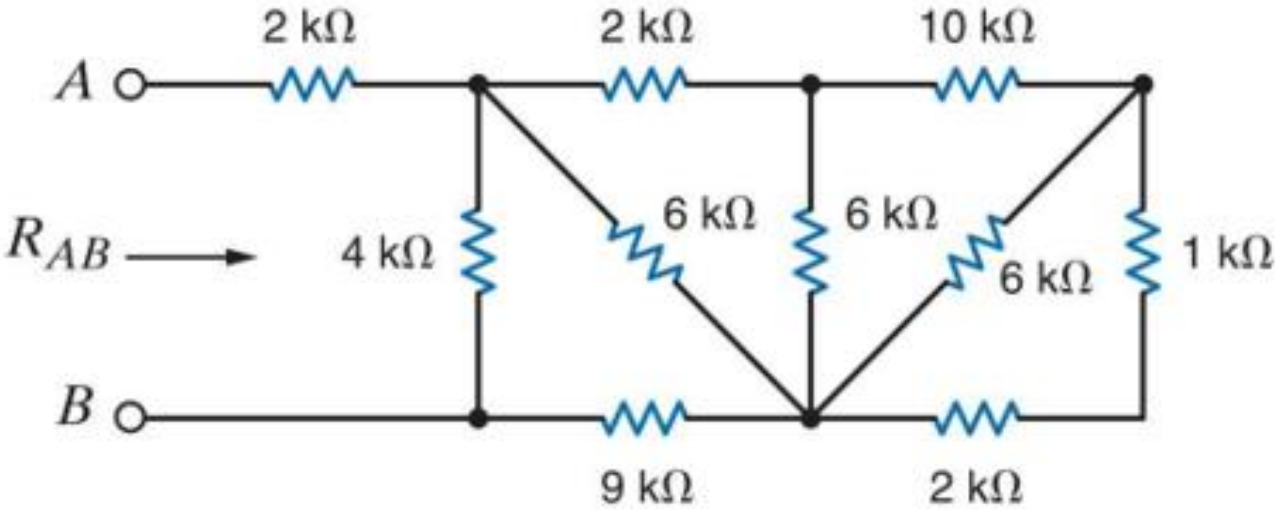
Example

- In the following circuit find R_{eq}:



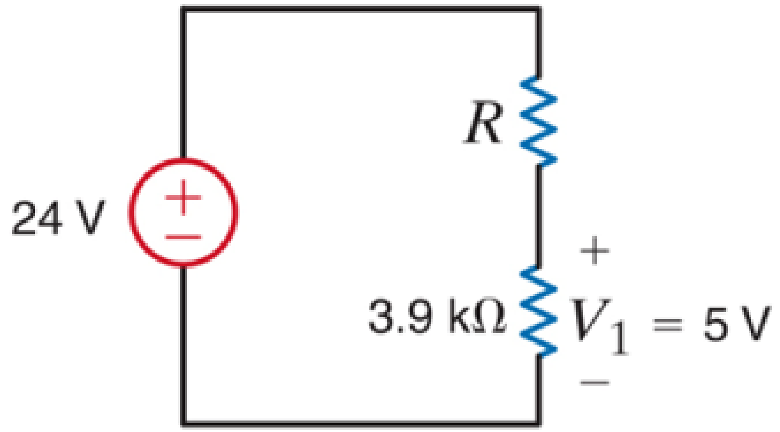
Example

- In the following circuit find the resistance seen between the two terminal s A and B, i.e., R_{AB}



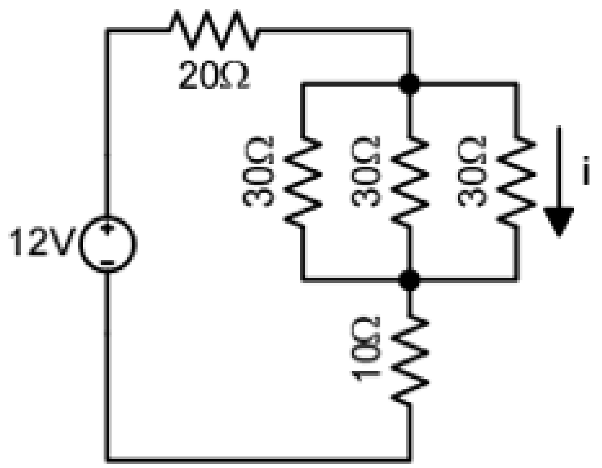
Example

- find the required value for the resistor R



Example

- In the following circuit find the current i.



Example

- In the following circuit find I_1, I_2, I_3, V_a, and V_b.