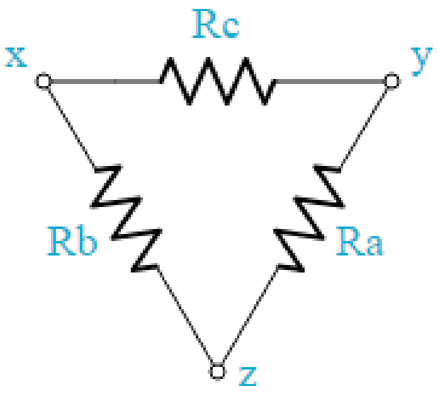
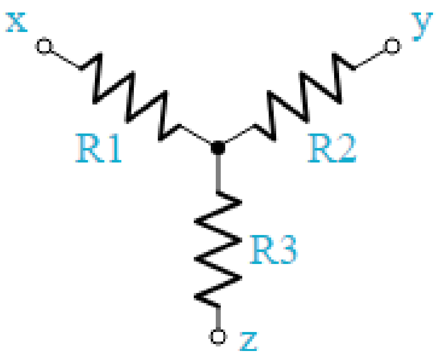
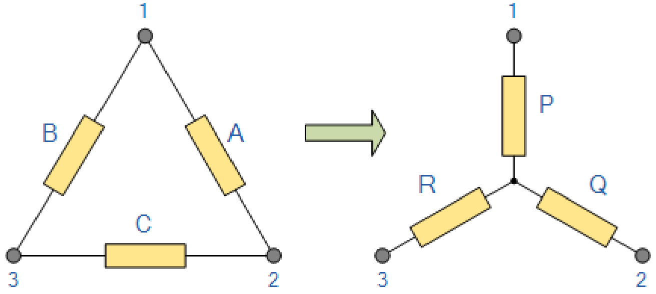
Wye-Delta (Y-Δ) Transformation



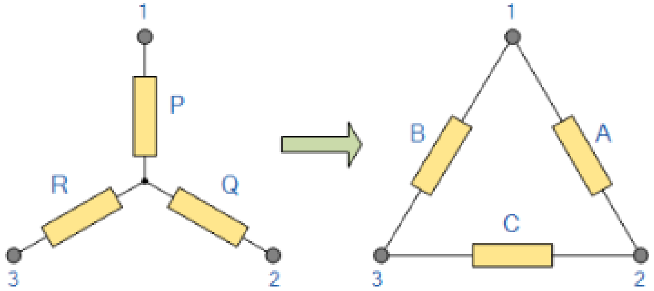




$$R1 = \frac{Rb\,Rc}{Ra + Rb + Rc}$$

$$R2 = \frac{Ra\,Rc}{Ra + Rb + Rc}$$

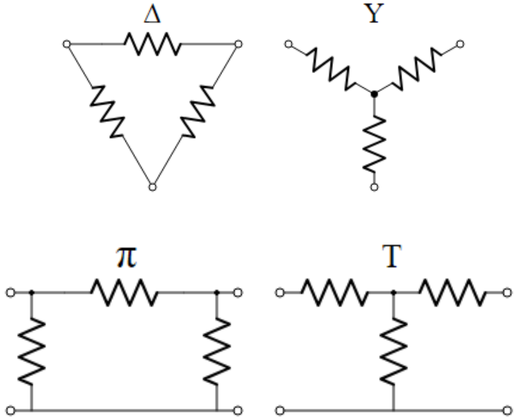
$$R3 = \frac{Ra\,Rb}{Ra + Rb + Rc}$$



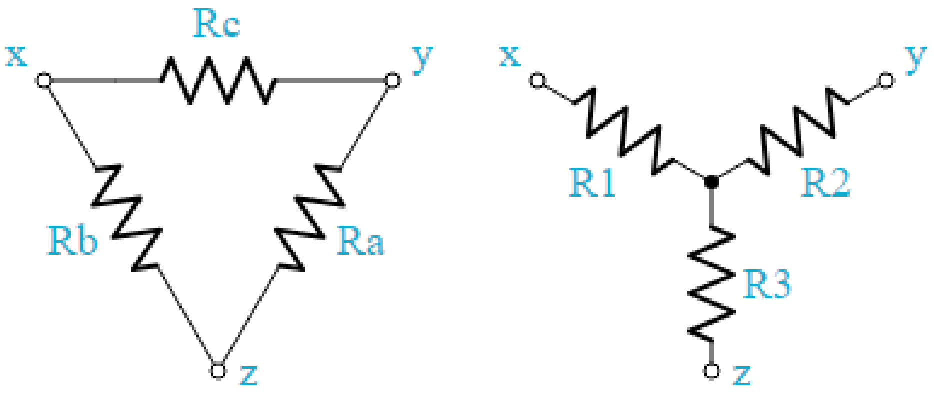
$$Ra = \frac{R1\,R2 + R2\,R3 + R3\,R1}{R1}$$

$$Rb = \frac{R1\,R2 + R2\,R3 + R3\,R1}{R2}$$

$$Rc = \frac{R1\,R2 + R2\,R3 + R3\,R1}{R3}$$



Wye-Delta (Y-Δ) Transformation



$$R1 = \frac{Rb\,Rc}{Ra + Rb + Rc}$$

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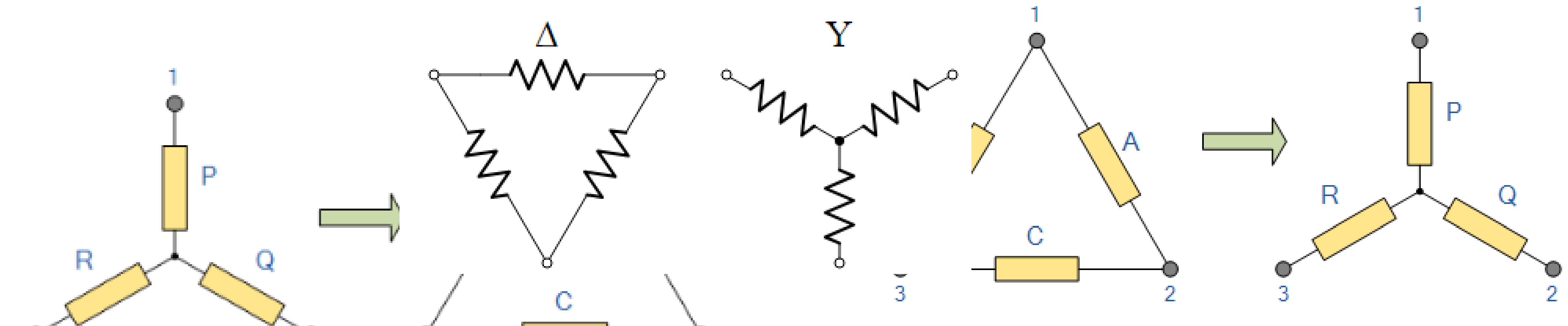
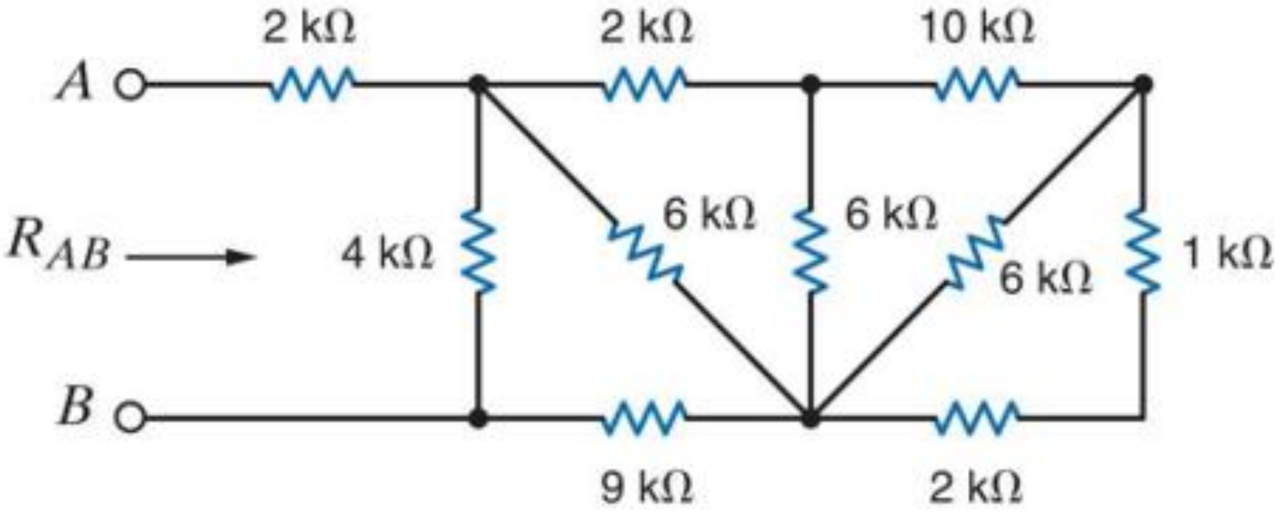
$$R3 = \frac{Ra\,Rb}{Ra + Rb + Rc}$$



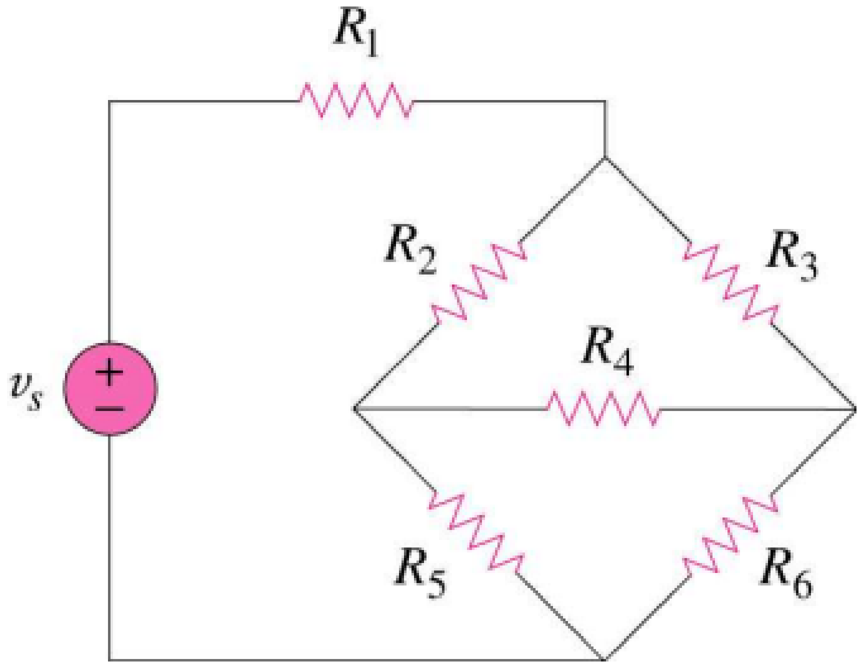
Example

- In the following circuit find the resistance seen between the two terminal s A and B, i.e., R_{AB}



Wye-Delta Transformations

- In some circuits the resistors are neither in series nor in parallel.
- For example consider the following bridge circuit:

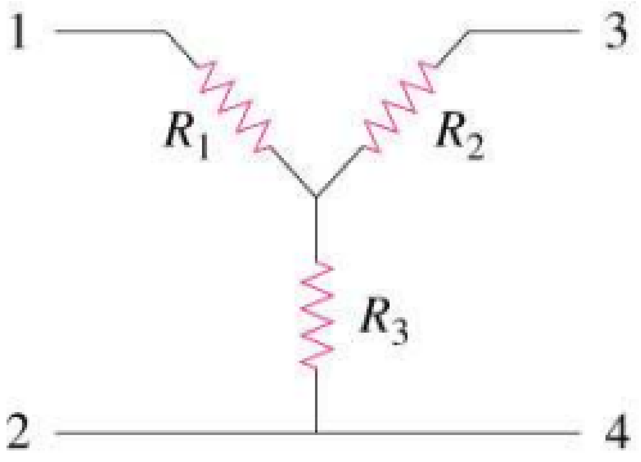


how can we combine the resistors R_1 through R_6?

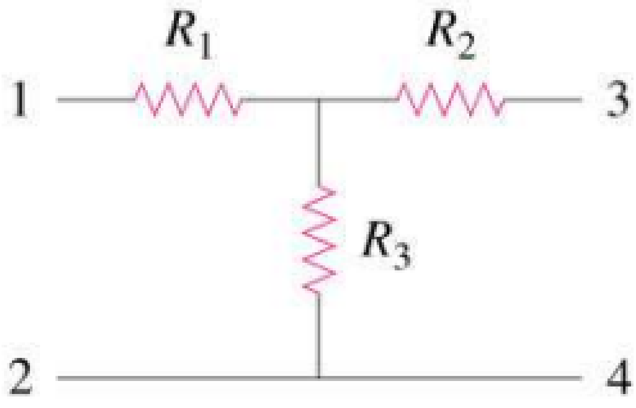
Wye and Delta Networks

- A useful technique that can be used to simply many such circuits is transformation from wye (Y) to delta (Δ) network.

- A wye (Y) or tee (T) network is a three-terminal network with the following general form:



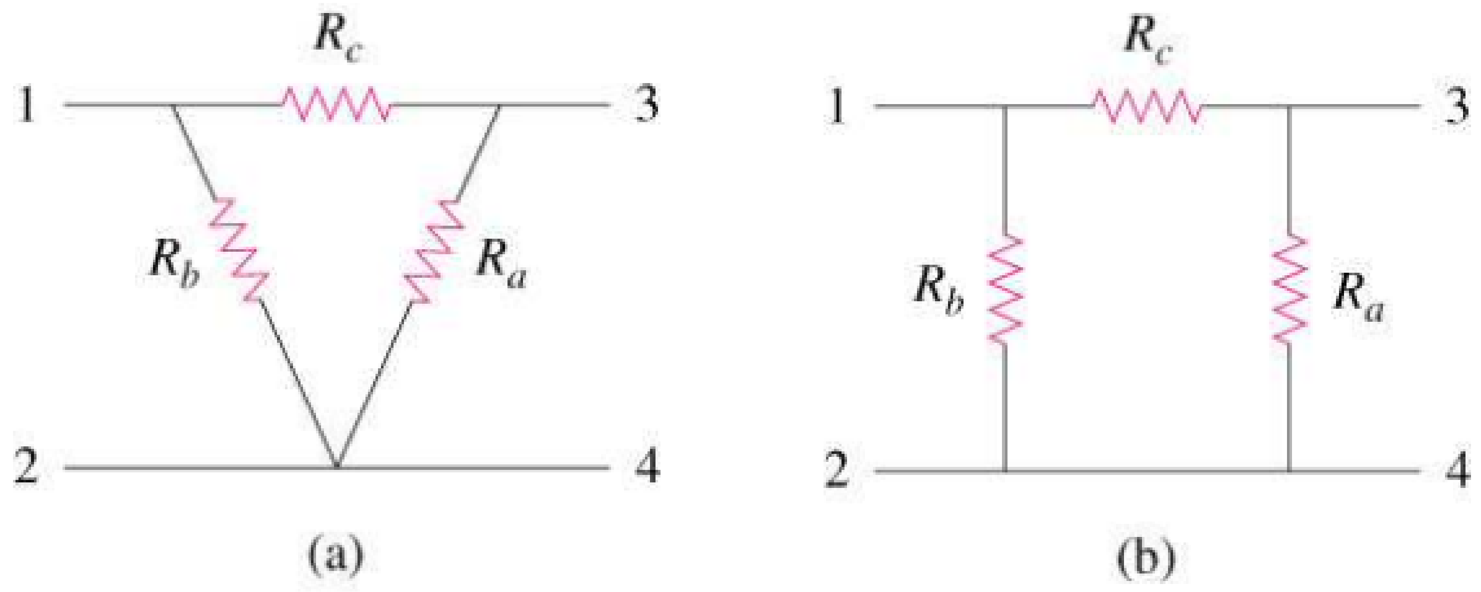
(a)



(b)

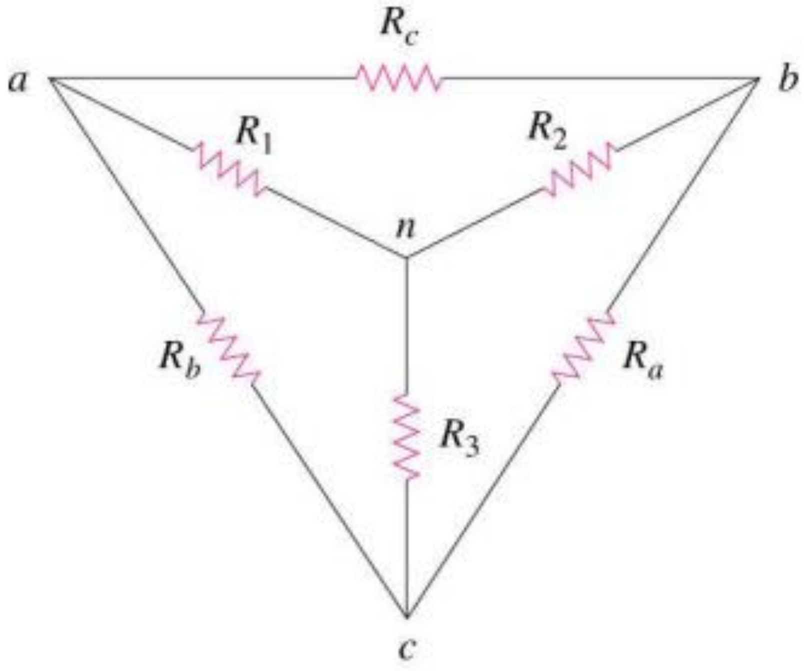
Wye and Delta Networks

- The delta (Δ) or pi (Π) network has the following general form:



Delta-Wye Conversion

- In some cases it is more convenient to work with a Y network in place of a Δ network.
- Let's superimpose a wye network on the existing delta network and try to find the equivalent resistances in the wye network



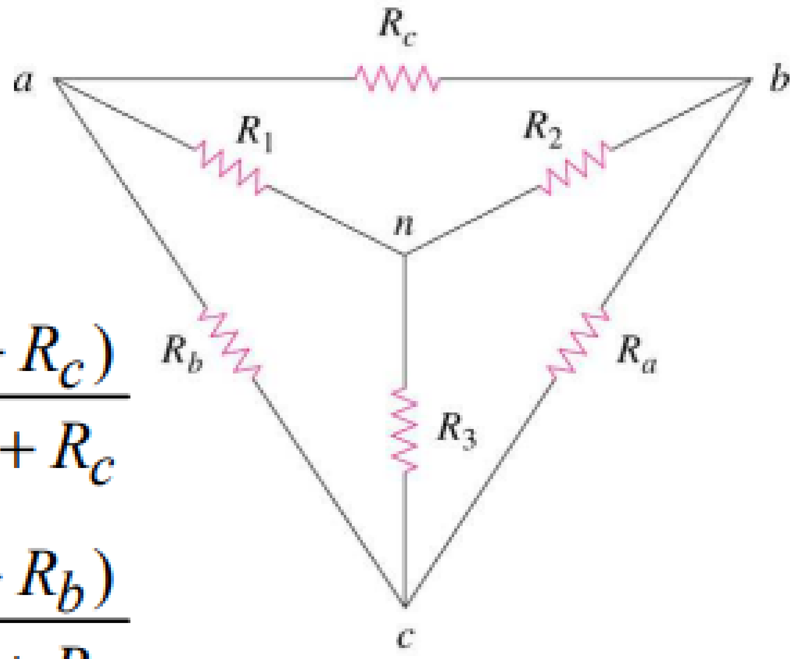
Delta-Wye Conversion

- We calculate the equivalent resistance between terminals a and c while terminal b is open in both cases:

$$R_{ac}(Y) = R_1 + R_3$$

$$R_{ac}(\Delta) = R_b \| (R_a + R_c)$$

$$R_{ac}(Y) = R_{ac}(\Delta) \Rightarrow R_1 + R_3 = \frac{R_b(R_a + R_c)}{R_a + R_b + R_c}$$



Similarly:

$$R_1 + R_2 = \frac{R_c(R_a + R_b)}{R_a + R_b + R_c}$$

$$R_2 + R_3 = \frac{R_a(R_b + R_c)}{R_a + R_b + R_c}$$

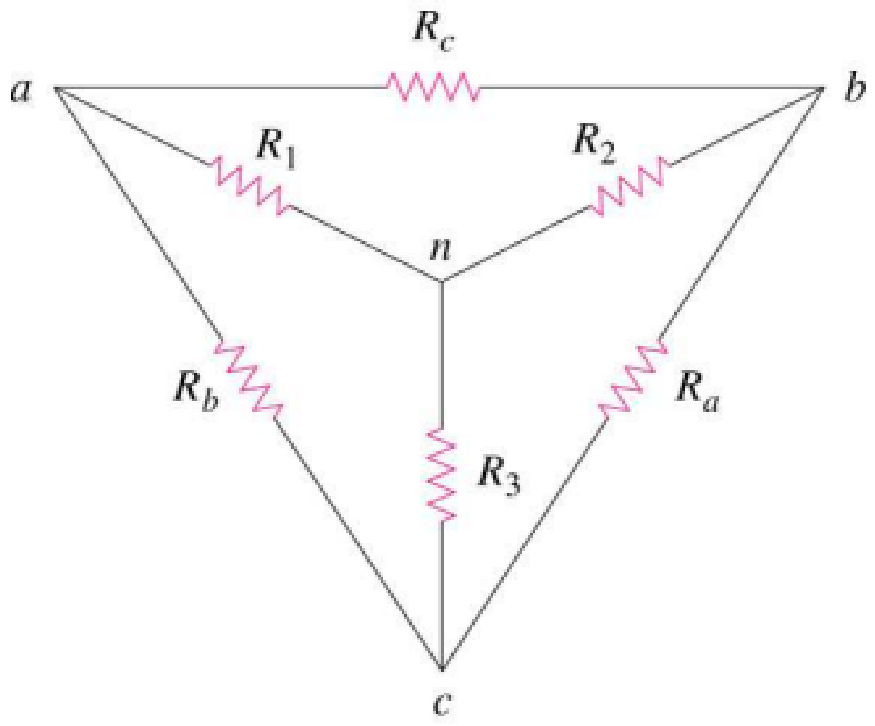
Delta-Wye Conversion

- Solving for R_1, R_2, and R_3 we have:

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c}$$

$$R_2 = \frac{R_c R_a}{R_a + R_b + R_c}$$

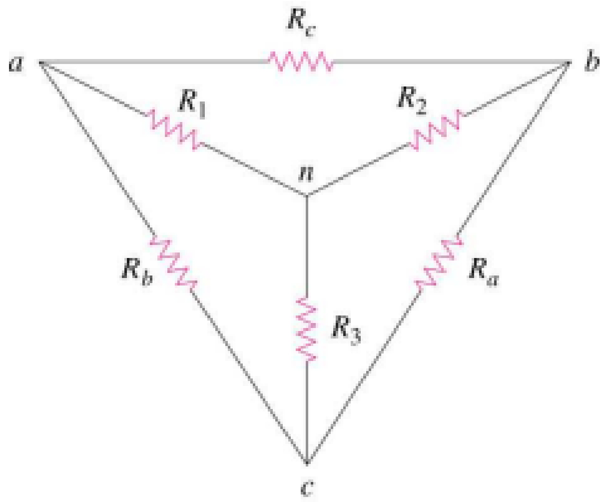
$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$$



- Each resistor in the Y network is the product if the resistors in the two adjacent Δ branches, divided by the sum of the three Δ resistors.

Wye-Delta Conversion

- From the previous page equations, we have:



$$R_1R_2 + R_2R_3 + R_3R_1 = \frac{R_aR_bR_c(R_a + R_b + R_c)}{(R_a + R_b + R_c)^2}$$

$$= \frac{R_aR_bR_c}{R_a + R_b + R_c}$$

- Dividing this equation by each of the previous slide equations:

$$R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1}, R_b = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2}, \text{and } R_c = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3}$$

- Each resistor in the Δ network is the sum of all the possible products of Y resistors taken two at a time, divided by the opposite Y resistor

Wye-Delta Transformations

- Y and Δ networks are said to be balanced when:

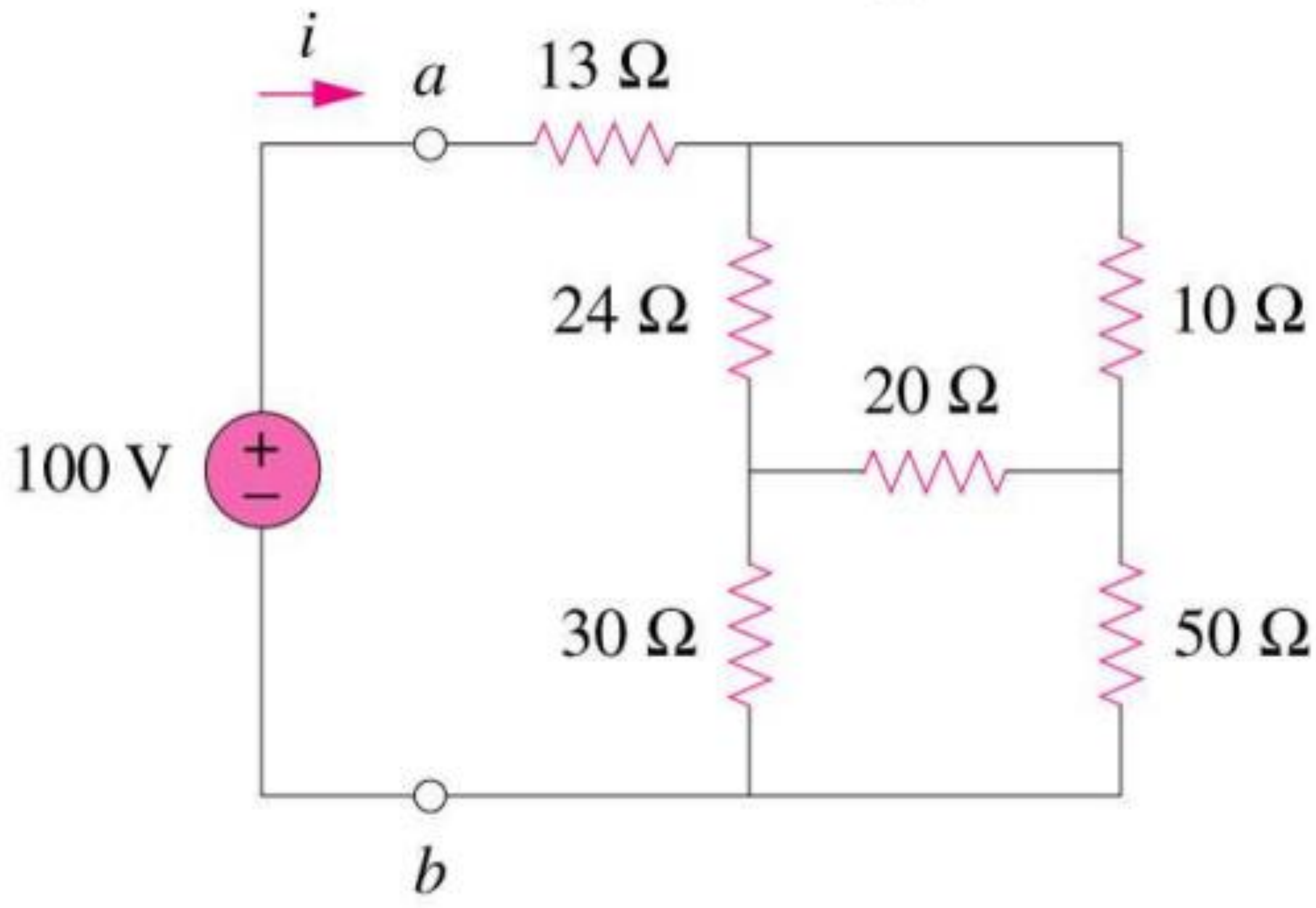
$$R_1 = R_2 = R_3 = R_Y \text{ and } R_a = R_b = R_c = R_\Delta$$

- For balanced Y and Δ networks the conversion formulas become:

$$R_Y = \frac{R_\Delta}{3} \text{ and } R_\Delta = 3R_Y$$

Example

- For the following bridge network find R_{ab} and i.



Example

- Find I_S?