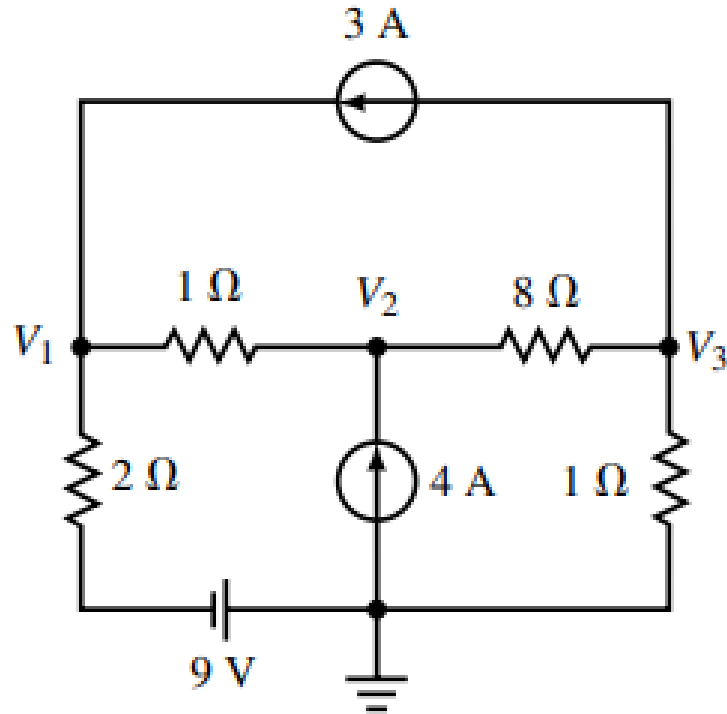
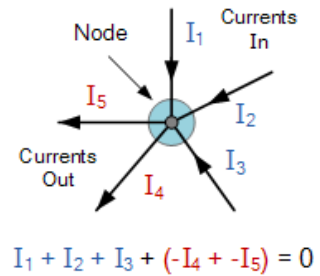
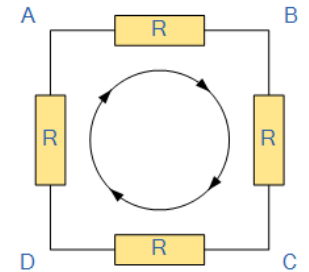
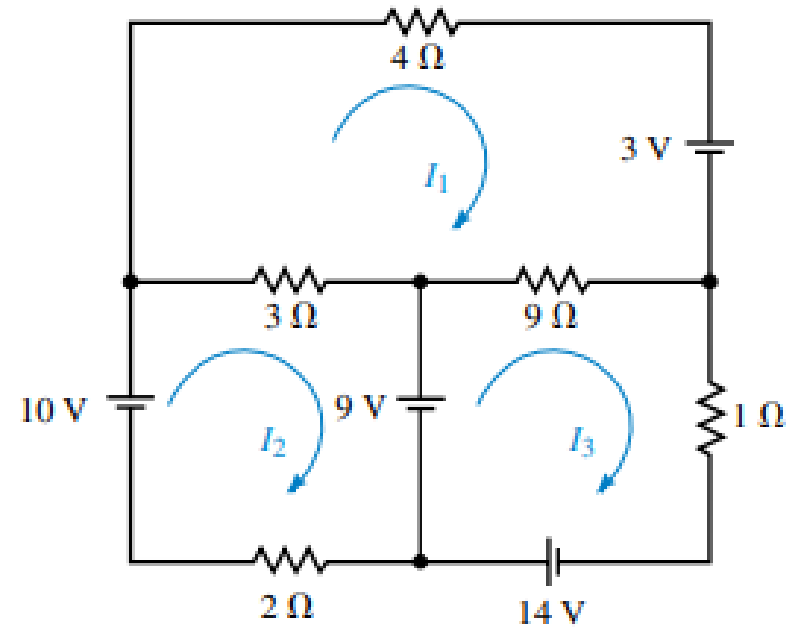


Nodal and Mesh Analyses

Nodal Voltage Analysis



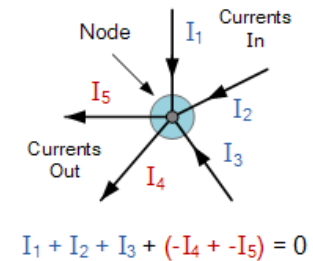
Mesh Current Analysis



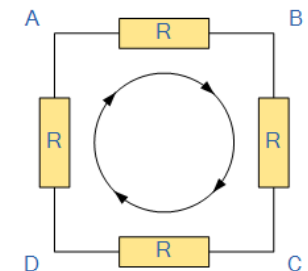
Methods of Circuit Analysis

- Two popular and powerful techniques for analyzing circuits are:

➤ Nodal analysis: a general procedure to find all the node voltages in a circuit. It is based on KCL and Ohm's Law.



➤ Mesh analysis: another general approach to find mesh currents which circulate around closed paths in the circuit. It is based on KVL and Ohm's Law.



1- Nodal Analysis

- **Steps to determine the node voltages for a circuit with no floating voltage source:**

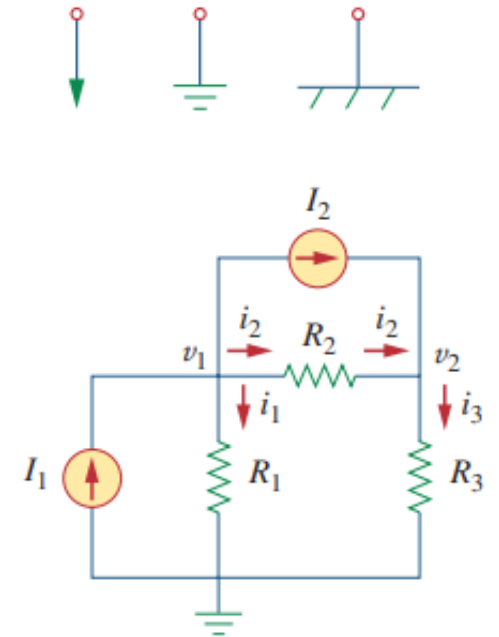
1. Select a reference node.

- **Reference node or ground:** a node that is assumed to have a zero potential. If the reference node is not explicitly indicated on the circuit one can arbitrarily choose any node as the ground.

2. Assign voltages to other nodes. These node voltages are referenced to the reference node ($V_1, V_2, V_3, \dots, V_n$).

3. Apply KCL for all unknown non-reference nodes. When possible use Ohm's law to relate the branch currents to node voltages.

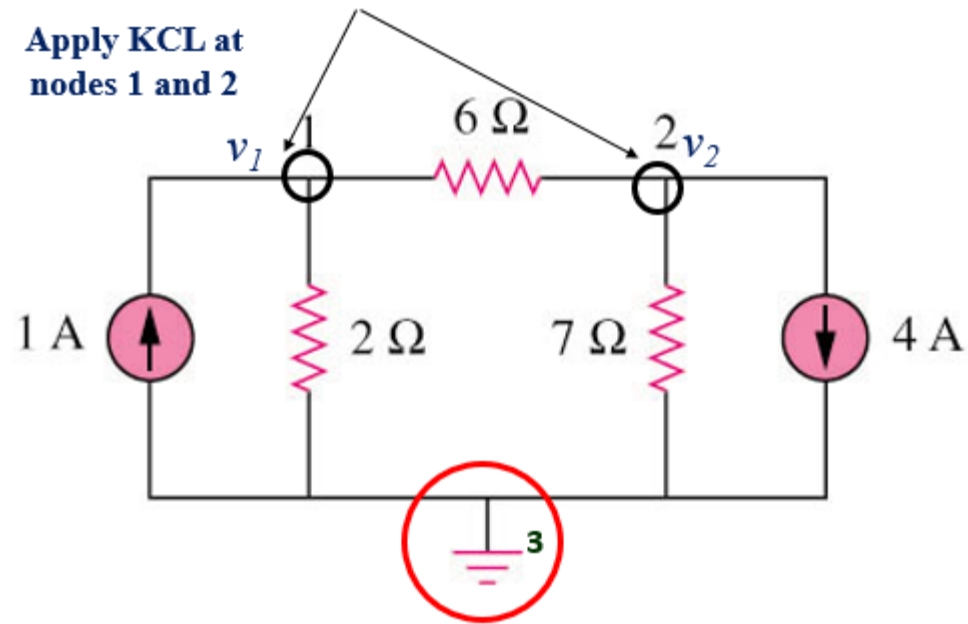
4. Solve the resulting system of equations for unknown node voltages.



1- Nodal Analysis

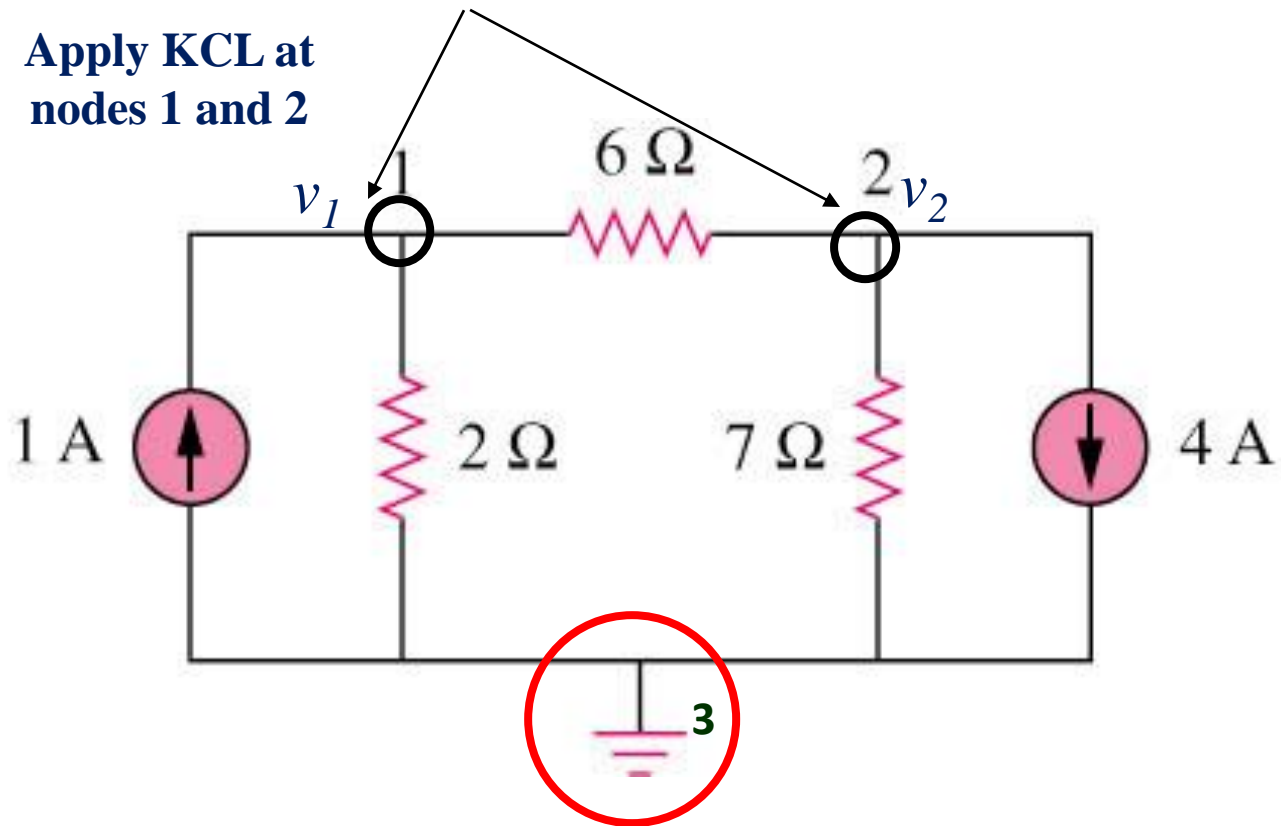
- Circuit with independent current source only.

Answer: $v_1 = -2\text{V}$, $v_2 = -14\text{V}$



3.2 Nodal Analysis (3)

Example 2 – circuit independent current source only



1- Nodal Analysis

- Analyze the following circuit using nodal analysis:

- At node 1, applying KCL gives

$$I_1 = I_2 + i_1 + i_2$$

- At node 2, applying KCL gives

$$I_2 + i_2 = i_3$$

- Apply Ohm's law to express the unknown currents and in terms of node voltages

$$i_1 = \frac{v_1 - 0}{R_1} \quad \text{or} \quad i_1 = G_1 v_1$$

- Substituting:

$$i_2 = \frac{v_1 - v_2}{R_2} \quad \text{or} \quad i_2 = G_2(v_1 - v_2)$$

$$I_1 = I_2 + \frac{v_1}{R_1} + \frac{v_1 - v_2}{R_2}$$

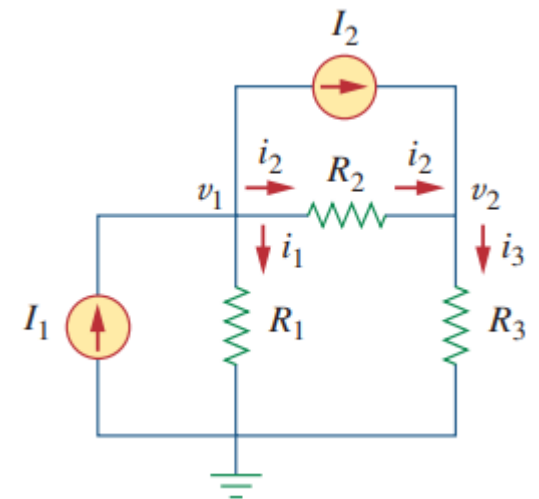
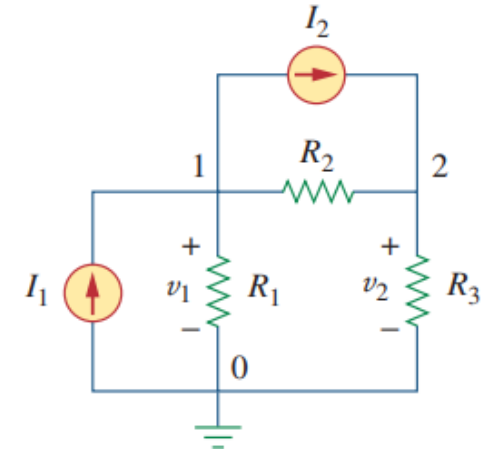
$$i_3 = \frac{v_2 - 0}{R_3} \quad \text{or} \quad i_3 = G_3 v_2$$

$$I_2 + \frac{v_1 - v_2}{R_2} = \frac{v_2}{R_3}$$

- In terms of the conductances:

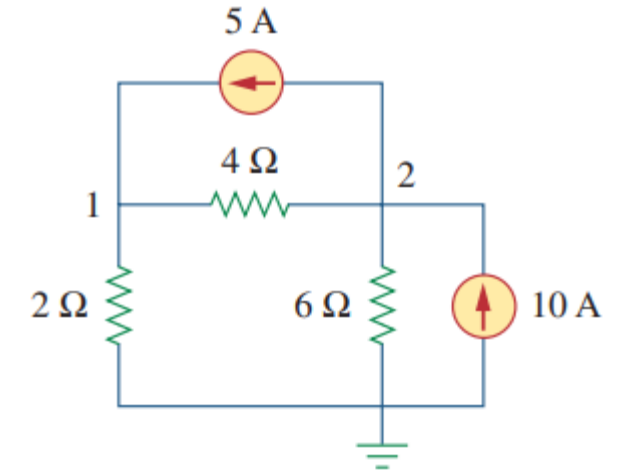
$$I_1 = I_2 + G_1 v_1 + G_2(v_1 - v_2)$$

$$I_2 + G_2(v_1 - v_2) = G_3 v_2$$



1- Nodal Analysis / Example

- Calculate the node voltages in the circuit shown below.



- At node 1, applying KCL gives

$$i_1 = i_2 + i_3 \quad \Rightarrow \quad 5 = \frac{v_1 - v_2}{4} + \frac{v_1 - 0}{2}$$

Multiplying each term in the last equation by 4,

$$20 = v_1 - v_2 + 2v_1 \quad \Rightarrow \quad 3v_1 - v_2 = 20 \quad \begin{bmatrix} 3 & -1 \\ -3 & 5 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 20 \\ 60 \end{bmatrix}$$

- At node 2, applying KCL gives

$$i_2 + i_4 = i_1 + i_5 \quad \Rightarrow \quad \frac{v_1 - v_2}{4} + 10 = 5 + \frac{v_2 - 0}{6}$$

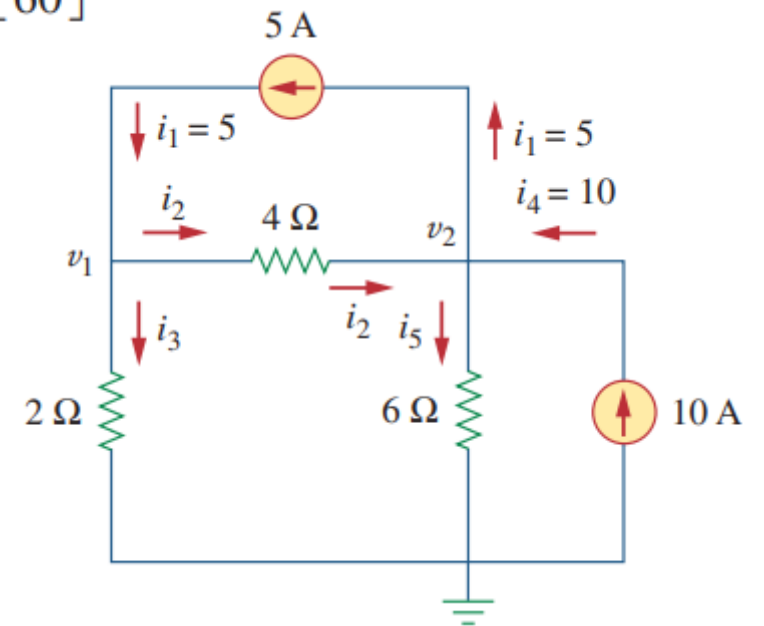
Multiplying each term in the last equation by 12,

$$3v_1 - 3v_2 + 120 = 60 + 2v_2 \quad \Rightarrow \quad -3v_1 + 5v_2 = 60$$

Using the elimination technique \Rightarrow

$$4v_2 = 80 \quad \Rightarrow \quad v_2 = 20 \text{ V}$$

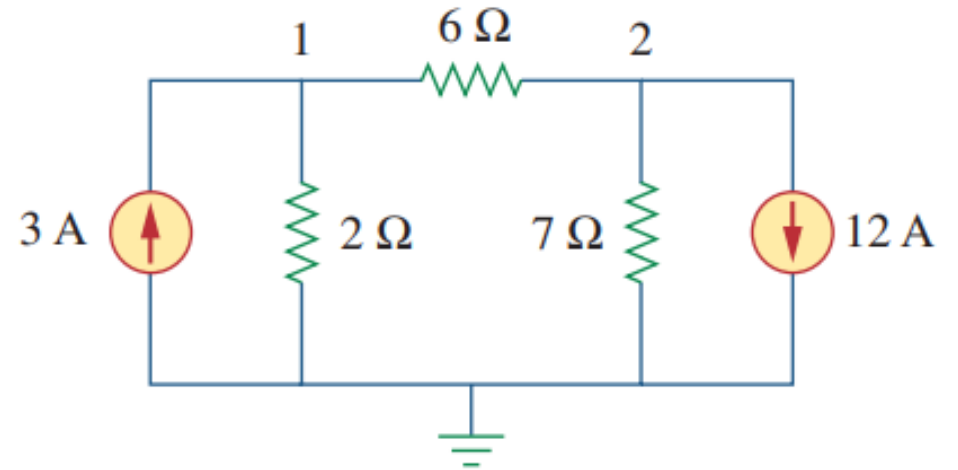
$$\text{Substituting } v_2 = 20 \quad \Rightarrow \quad 3v_1 - 20 = 20 \quad \Rightarrow \quad v_1 = \frac{40}{3} = 13.333 \text{ V}$$



1- Nodal Analysis / Example

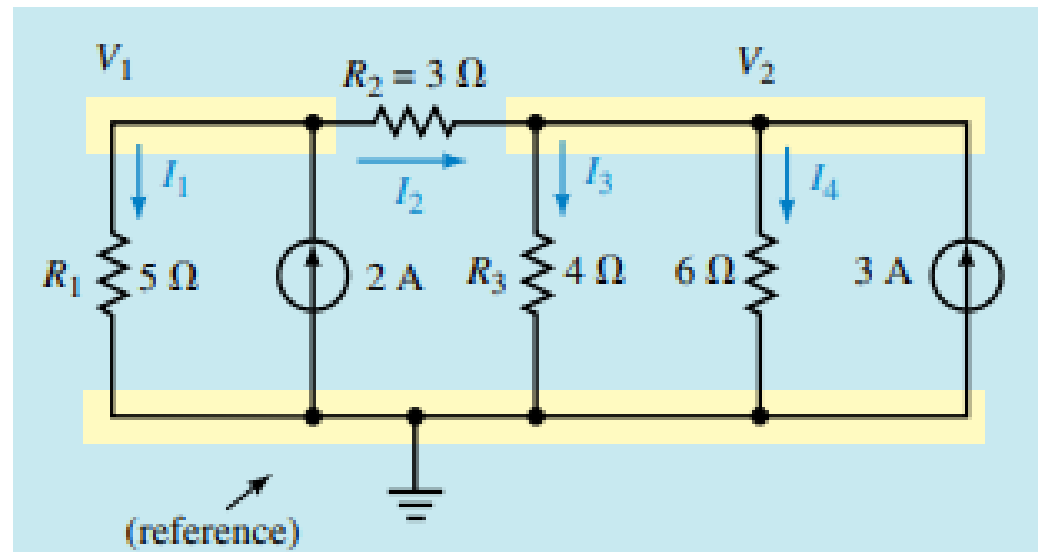
- Obtain the node voltages in the circuit.

Answer: $v_1 = -6 \text{ V}$, $v_2 = -42 \text{ V}$.



1- Nodal Analysis / Example

- Write the nodal analysis equations that lead to solve the following circuit.

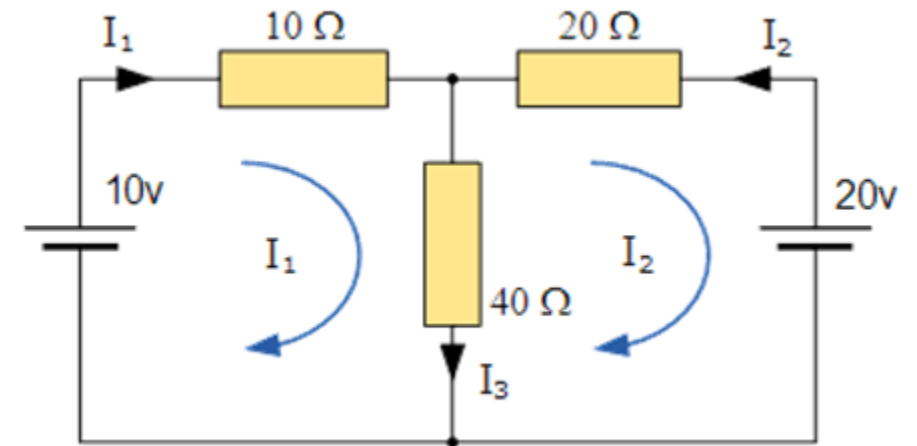


2- Mesh Analysis

- Mesh analysis is a special case of a more general technique called loop analysis.
- A mesh is a loop that does not contain any other loops within it.
- Number of linearly independent loops is equal to the number of meshes!

➤ Steps to Determine Mesh Currents:

1. Assign mesh currents to the n meshes.
2. Apply KVL to each of the n meshes. Use Ohm's law to express the voltages in terms of the mesh currents.
3. Solve the resulting n simultaneous equations to get the mesh currents.



$$10 = 50I_1 - 40I_2$$

$$-20 = -40I_1 + 60I_2$$

$$\text{or } [V] = [R][I]$$

2- Mesh Analysis / Example

- Write the mesh analysis equations that lead to solve the following circuit. Find I_1 , I_2 and I_3 .

- Apply KVL to each mesh. Applying KVL to mesh 1, gives

$$-V_1 + R_1 i_1 + R_3(i_1 - i_2) = 0 \quad \text{or} \quad (R_1 + R_3)i_1 - R_3 i_2 = V_1$$

- For mesh 2, applying KVL gives

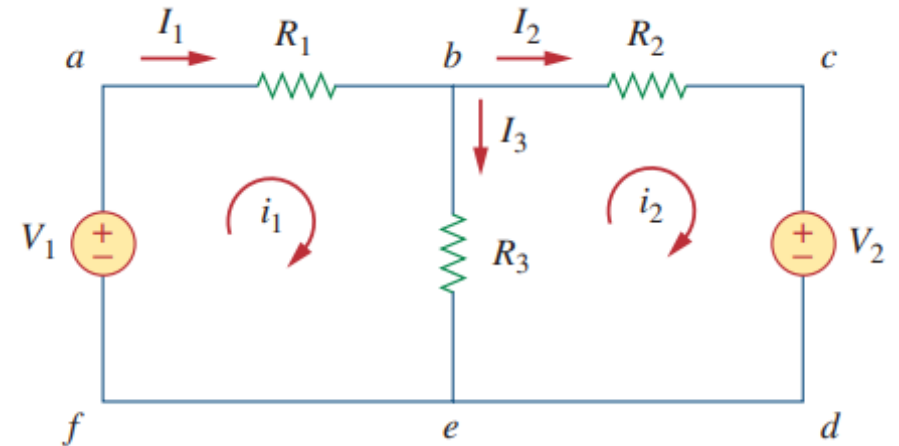
$$R_2 i_2 + V_2 + R_3(i_2 - i_1) = 0 \quad \text{or} \quad -R_3 i_1 + (R_2 + R_3)i_2 = -V_2$$

- In matrix form:

$$\begin{bmatrix} R_1 + R_3 & -R_3 \\ -R_3 & R_2 + R_3 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} V_1 \\ -V_2 \end{bmatrix}$$

$$I_1 = i_1, \quad I_2 = i_2, \quad I_3 = i_1 - i_2$$

- Solve for mesh currents i_1 and i_2 , yields:



2- Mesh Analysis / Example

- For the circuit below, find the branch currents I_1 , I_2 and I_3 using mesh analysis.

- Applying KVL to mesh 1, gives

$$-15 + 5i_1 + 10(i_1 - i_2) + 10 = 0 \quad \text{or} \quad 3i_1 - 2i_2 = 1$$

- Applying KVL to mesh 2, gives

$$6i_2 + 4i_2 + 10(i_2 - i_1) - 10 = 0 \quad \text{or} \quad i_1 = 2i_2 - 1$$

- In matrix form:

$$\begin{bmatrix} 3 & -2 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

- Solve for mesh currents i_1 and i_2 , yields:

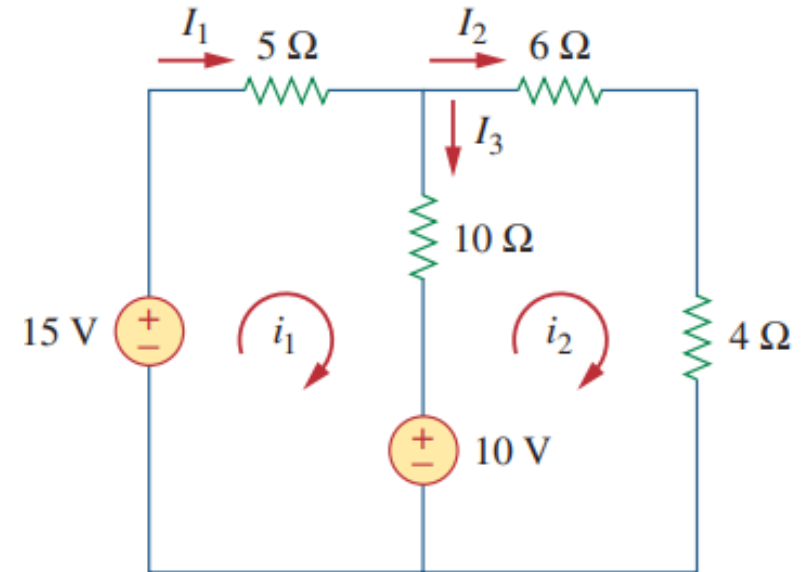
$$i_2 = 1 \text{ A} \quad i_1 = 1 \text{ A}$$

Thus:

$$I_1 = i_1 = 1 \text{ A}$$

$$I_2 = i_2 = 1 \text{ A}$$

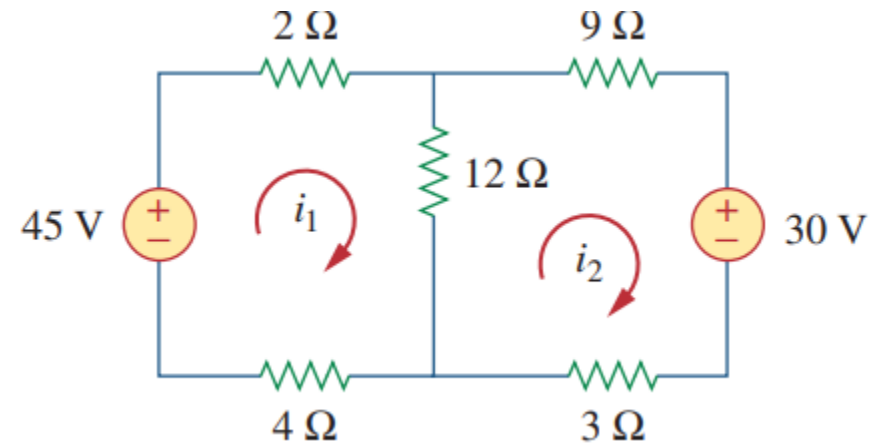
$$I_3 = i_1 - i_2 = 0 \text{ A}$$



2- Mesh Analysis / Example

- Calculate the mesh currents i_1 and i_2 .

Answer: $i_1 = 2.5$ A, $i_2 = 0$ A.



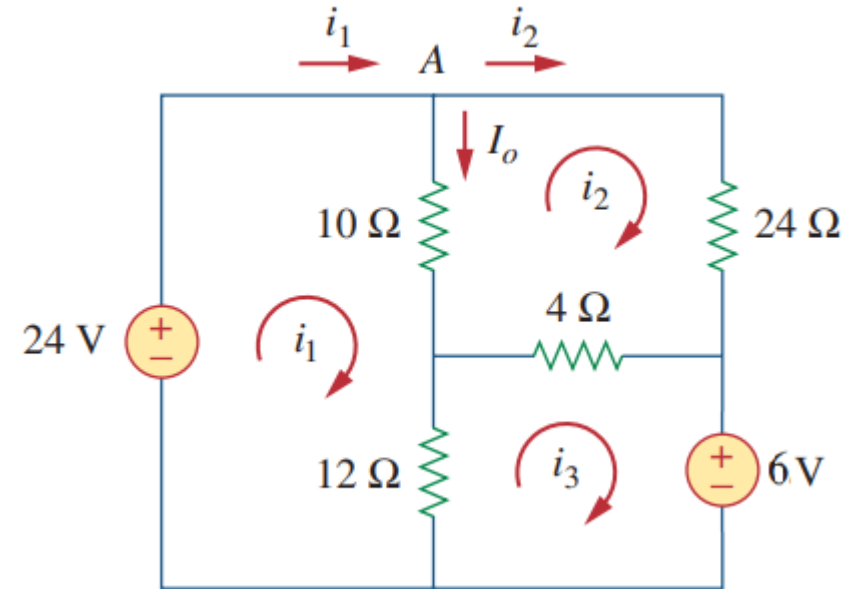
2- Mesh Analysis / Example

- Use mesh analysis to find the current I_o in the circuit shown below.

- ANSWER:**

$$i_1 = 2.25 \text{ A} \quad i_2 = 0.75 \text{ A} \quad i_3 = 1.5 \text{ A}$$

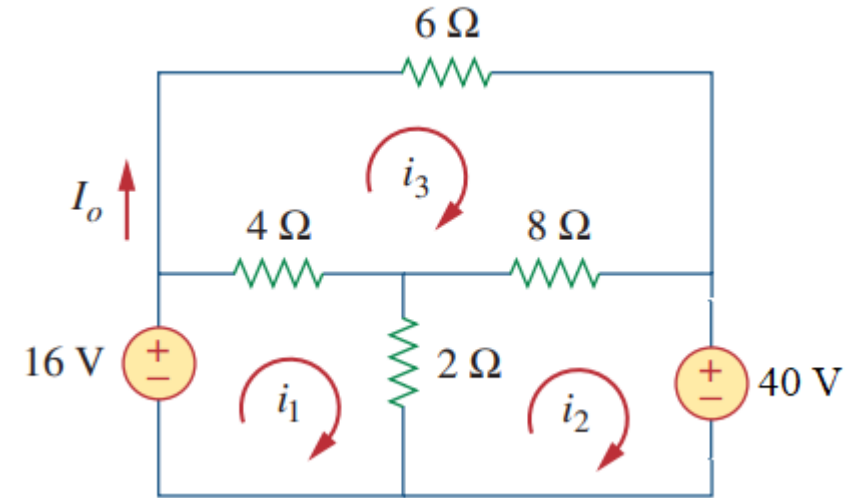
$$I_o = i_1 - i_2 = 1.5 \text{ A.}$$



2- Mesh Analysis / Example

- Calculate the mesh currents i_1 and i_2 .

Answer: -4 A.



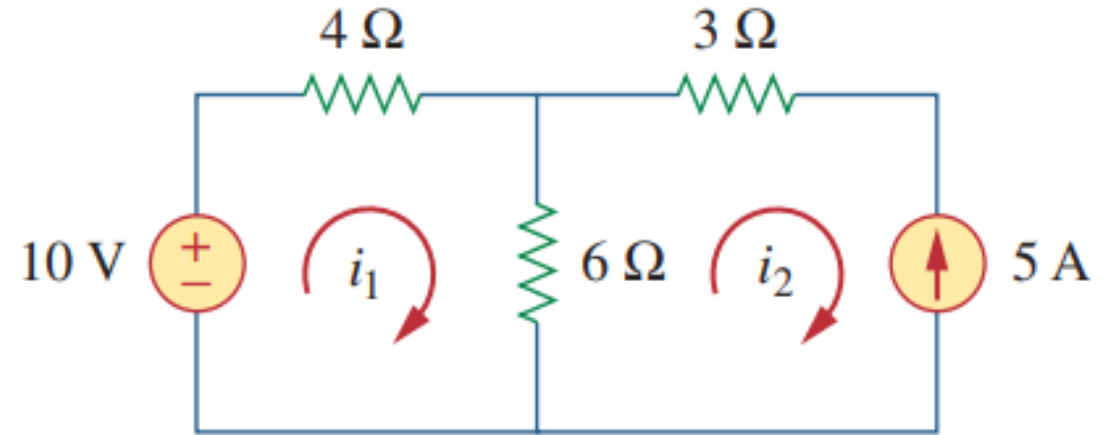
2- Mesh Analysis with Current Source / Example

- For the circuit below, find the mesh currents i_1 and i_2 using mesh analysis.

By Inspection: $i_2 = -5 \text{ A}$

- KVL in mesh 1, gives

$$-10 + 4i_1 + 6(i_1 - i_2) = 0 \quad \Rightarrow \quad i_1 = -2 \text{ A}$$



2- Mesh Analysis with Current Source / Example

- For the circuit below, find the mesh currents I_1 , I_2 and I_3 using mesh analysis.

- KVL in mesh 1, gives

$$28 = I_1 + 4(I_1 - I_2) + 12 + (I_1 - 8) = 6I_1 - 4I_2 + 4 \quad \text{or} \quad 6I_1 - 4I_2 = 24$$

- KVL in mesh 2, gives

$$0 = 4(I_2 - I_1) + 2I_2 + 2(I_2 - 8) = -4I_1 + 8I_2 - 16 \quad \text{or} \quad -4I_1 + 8I_2 = 16$$

- In matrix form:

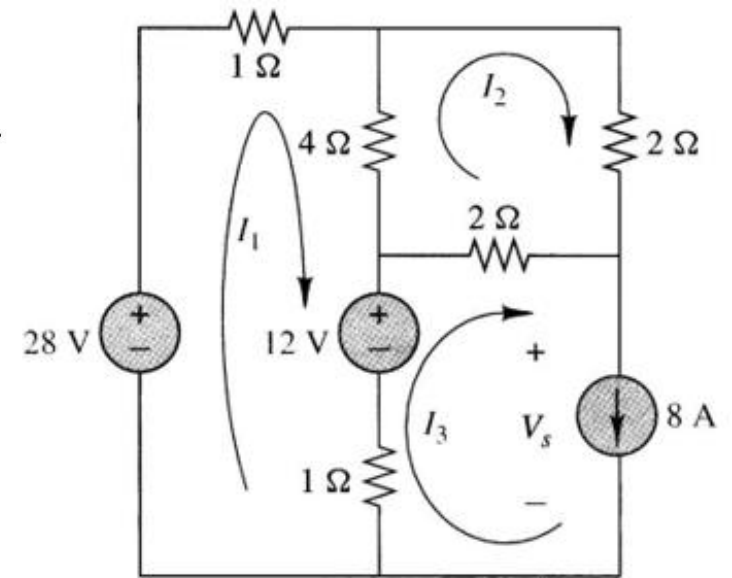
$$\begin{bmatrix} 6 & -4 \\ -4 & 8 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 24 \\ 16 \end{bmatrix}$$

By Inspection: $I_3 = 8 \text{ A}$

- Solve for mesh currents I_1 and I_2 , yields:

$$I_1 = 8 \text{ A}$$

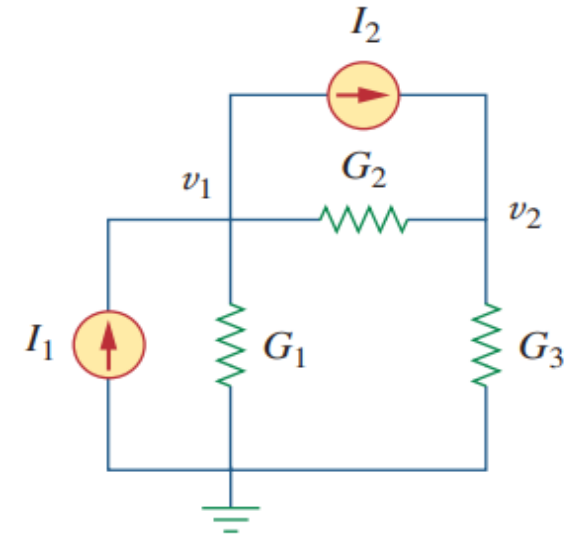
$$I_2 = 6 \text{ A}$$



Nodal and Mesh Analyses By Inspection

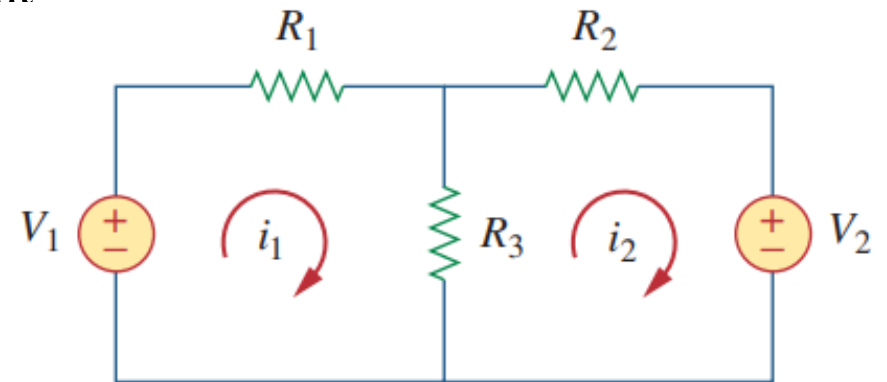
- When all sources in a circuit are independent current sources, it is not needed to apply KCL to each node to obtain the node-voltage equations. The equations can be obtained by inspection of the circuit.

$$\begin{bmatrix} G_1 + G_2 & -G_2 \\ -G_2 & G_2 + G_3 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} I_1 - I_2 \\ I_2 \end{bmatrix}$$



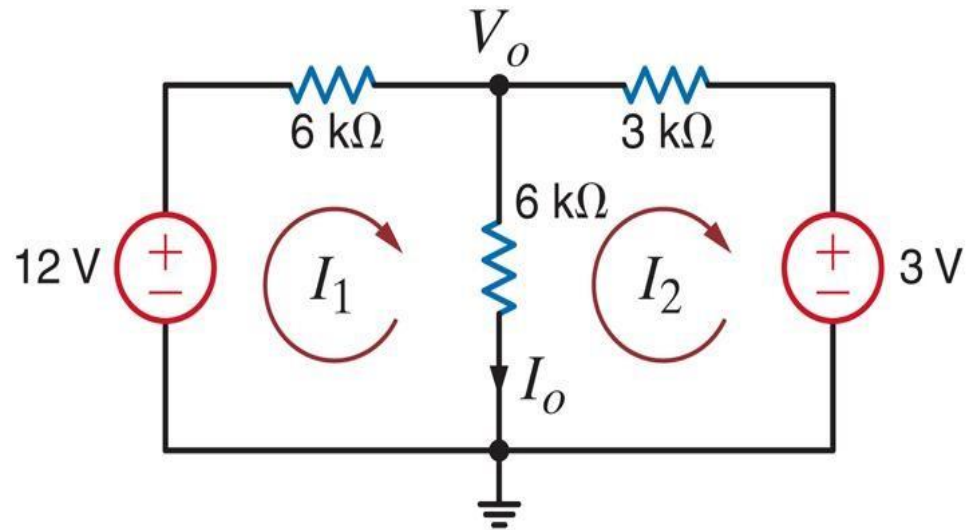
- When all sources in a circuit are independent voltage sources, it is not needed to apply KVL in each mesh to obtain the mesh-current equations. The equations can be obtained by inspection of the circuit.

$$\begin{bmatrix} R_1 + R_3 & -R_3 \\ -R_3 & R_2 + R_3 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} V_1 \\ -V_2 \end{bmatrix}$$



Difference between Mesh and Loop Analyses

Mesh Analysis



Loop Analysis

