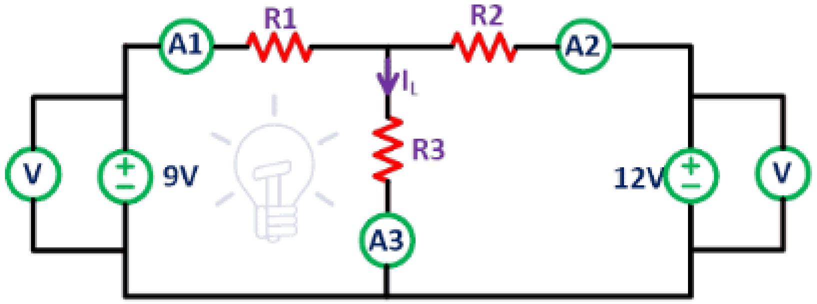
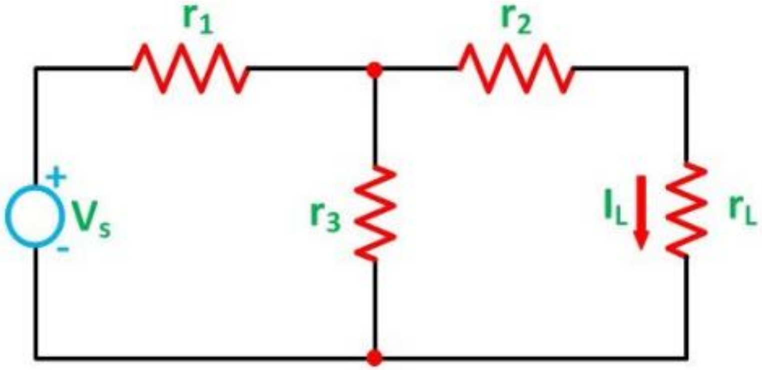
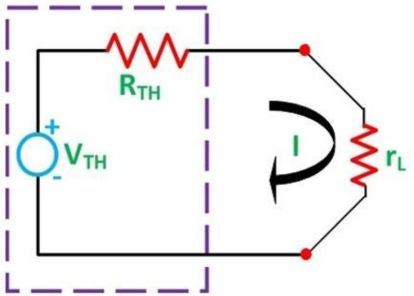
Additional Analysis Techniques and Circuit Theorems

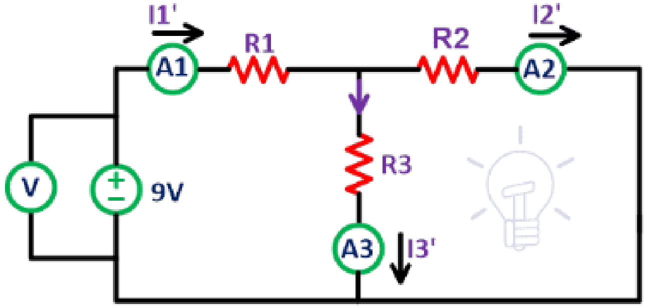
Superposition, Thevenin's Equivalent & Maximum Power Transfer







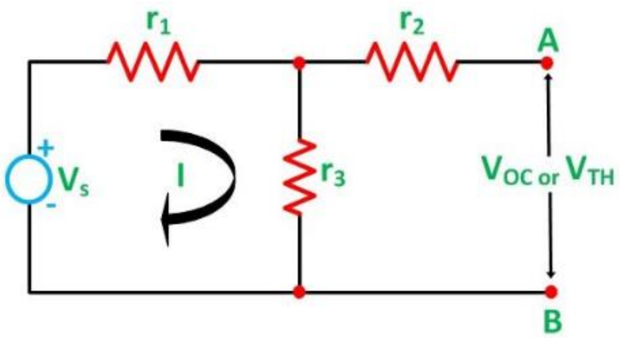
$$I_L = \frac{V_{TH}}{R_{TH} + r_L}$$

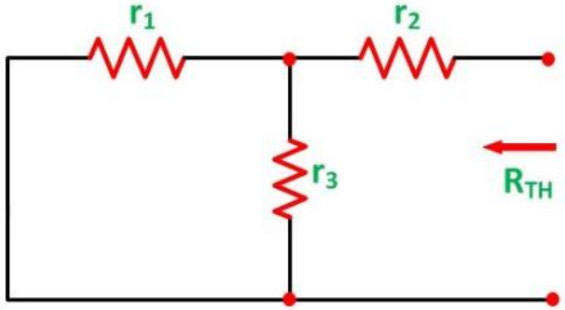


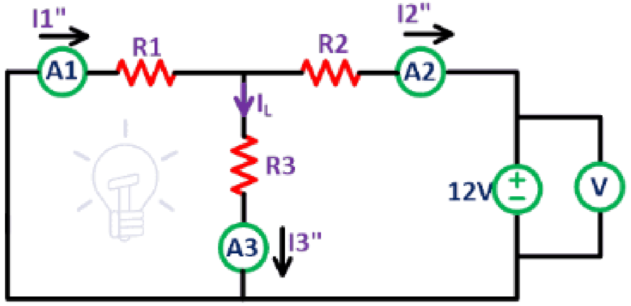
$$I_1 = I_{1'} + I_{1''}$$

$$I_2 = I_{2'} + I_{2''}$$

$$I_3 = I_{3'} + I_{3''}$$





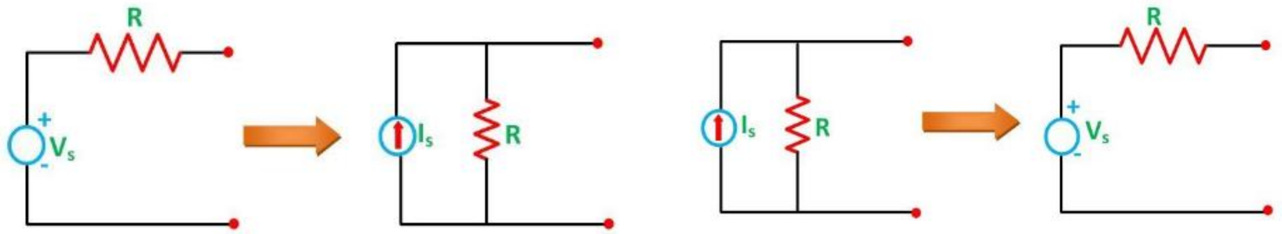


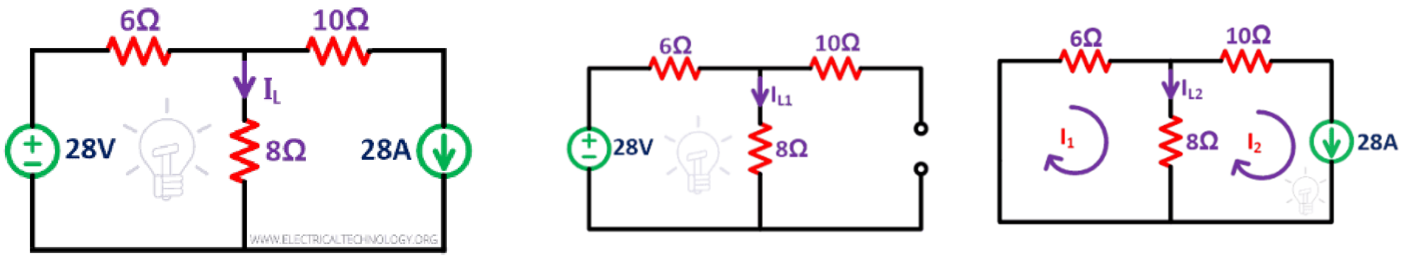
$$V_{OC} = I\, r_3 = \frac{V_S}{r_1 + r_3}\, r_3$$

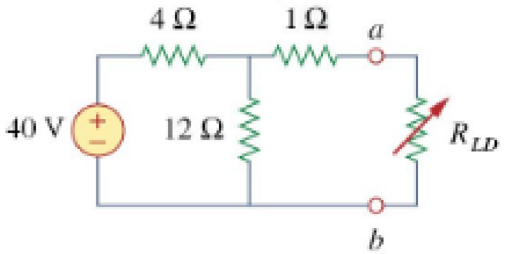
$$R_{TH} = r_2 + \frac{r_1 r_3}{r_1 + r_3}$$

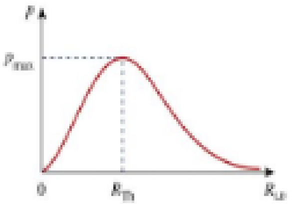
Additional Analysis Techniques and Circuit Theorems

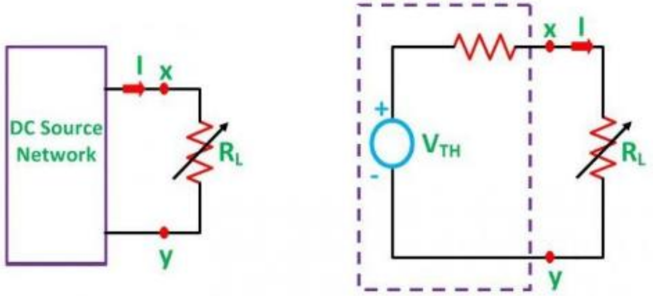
- **Linearity**
- **Superposition**
- **Source Transformation**
- **Thévenin's Equivalent**
- **Maximum Power Transfer**

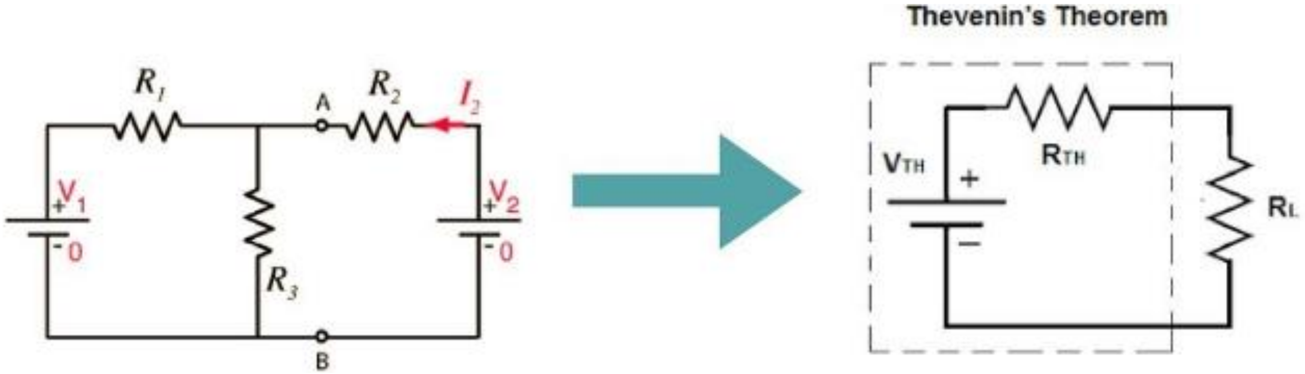








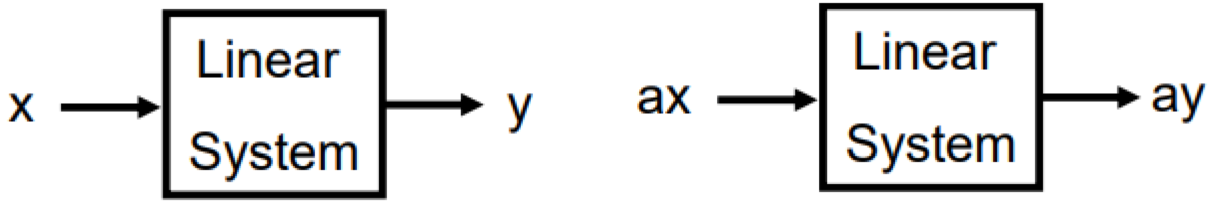




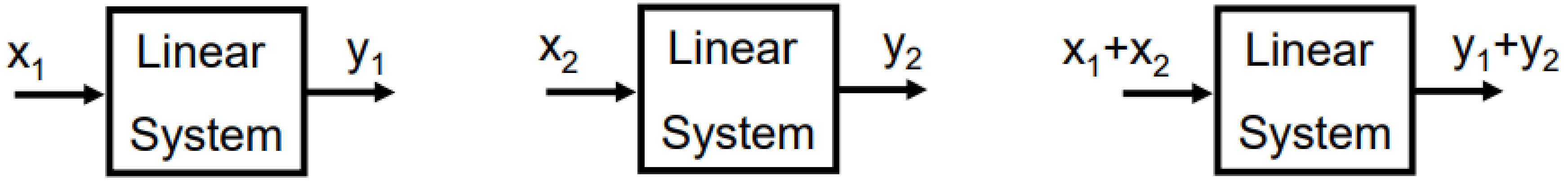
Linearity – Linear System

- A linear circuit (or in general a linear system) is a circuit (system) whose output is linearly related (or directly proportional) to its input.

- A system is linear if and only if it has these two properties:
  1. If the input of the system is multiplied by a constant then the output is also multiplied by a same constant (homogeneity).



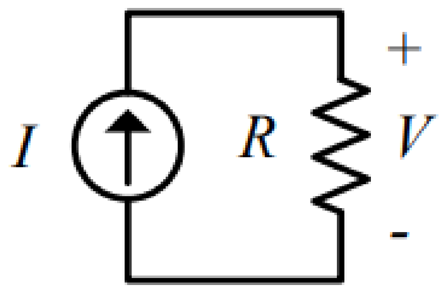
  2. The output of the system to a sum of inputs is the sum of the outputs to each input applied separately (additivity).



Linearity – Linear System

- In circuit theory, the **inputs of a circuit** are usually the **independent voltage or current sources** and **the outputs** are the **voltages or currents of some elements of the circuit**.

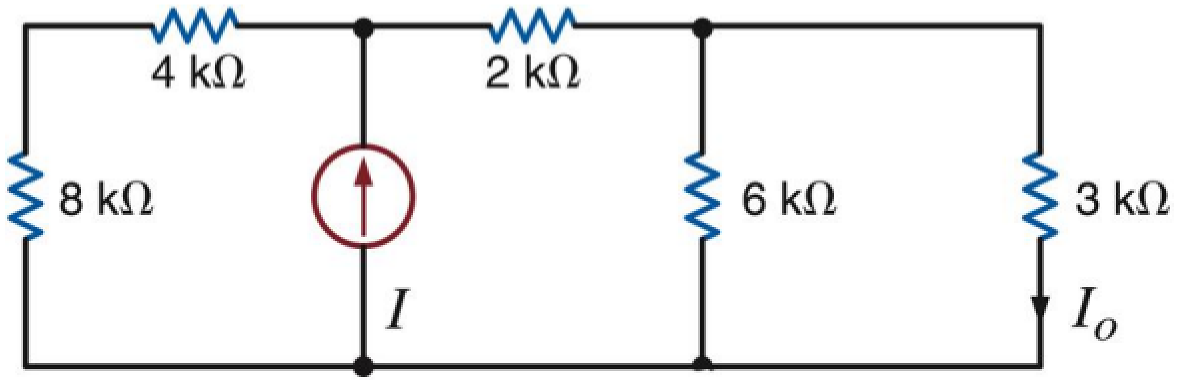
- **Example:** Consider a single resistor as a system. As shown below, the input of this system is an independent current source and the output is the voltage drop across the resistor. Is this system linear?



V = R.I

$kI \Rightarrow kV = R.kI$

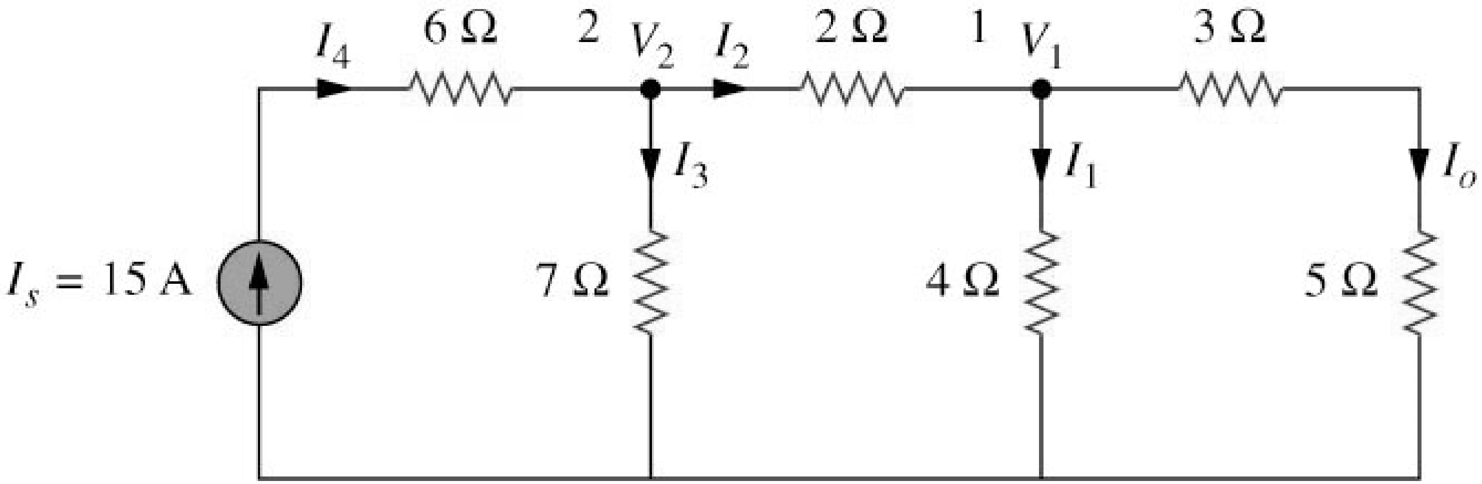
- Use linearity and the assumption that I_0=1mA to compute the correct value of I_o in the following circuit if I = 6mA.



Linearity – Example

- Assuming I_o = 1 A, use linearity to find the actual value of I_o in the circuit shown below.

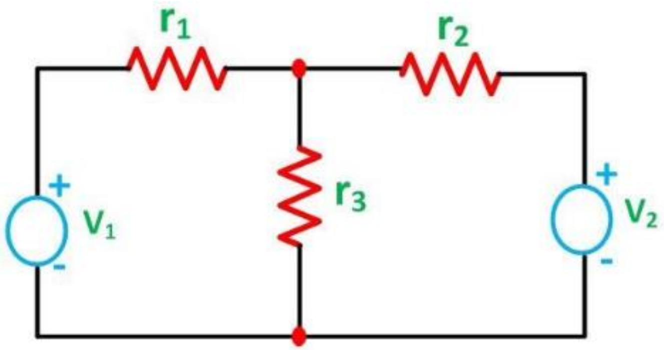
Answer I_o = 3A

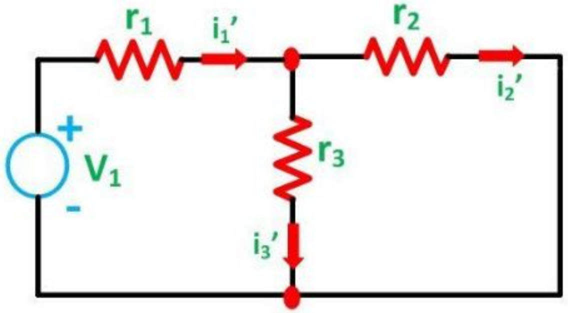


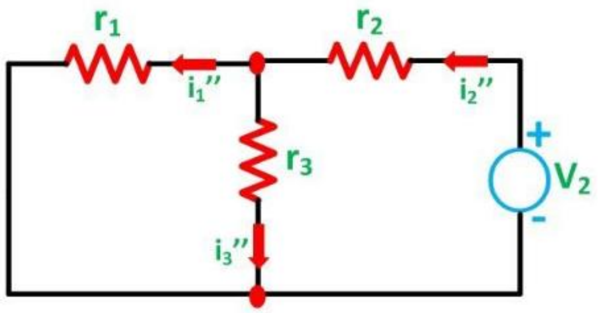
Superposition Theorem

- In a linear circuit containing more than one independent source, the voltage across or current through any element is the algebraic sum of the of the voltages across or currents through that element due to each independent source acting alone, with the remaining independent sources deactivated, i.e., their values set to zero**.**
- Steps for applying superposition principle
  - ➢Turn off all independent sources except one source. Find the desired voltage or current due to that source.
    - ➢Independent voltage sources are replaced by 0 V (<u>short circuit</u>) and
    - ➢Independent current sources are replaced by 0 A (<u>open circuit</u>).

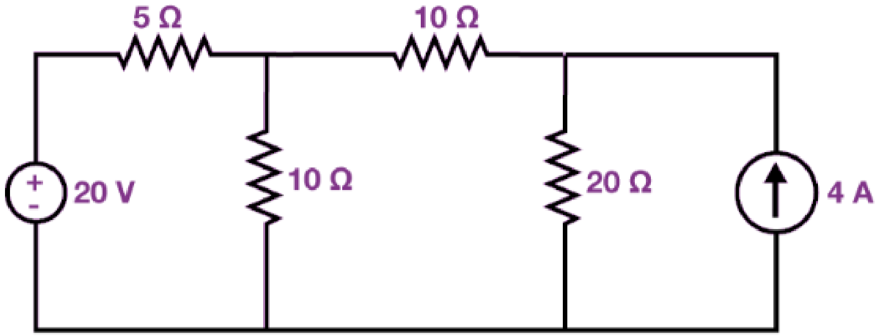
1. Repeat the previous step for each of the independent sources.
2. Find the total desired voltage or current by algebraically adding the contributions. due to sourc

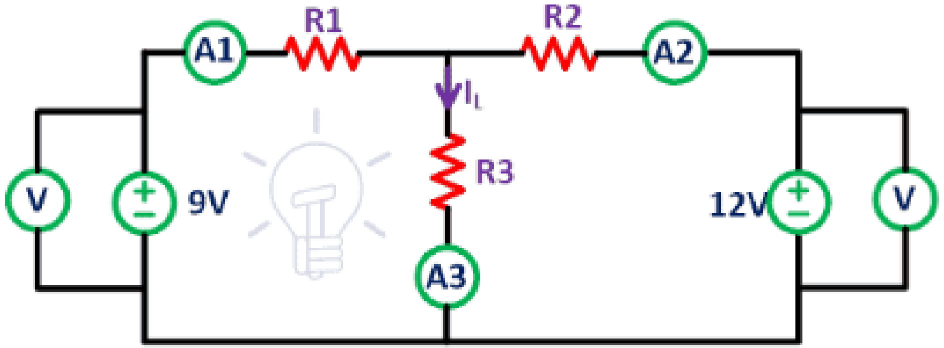






Superposition

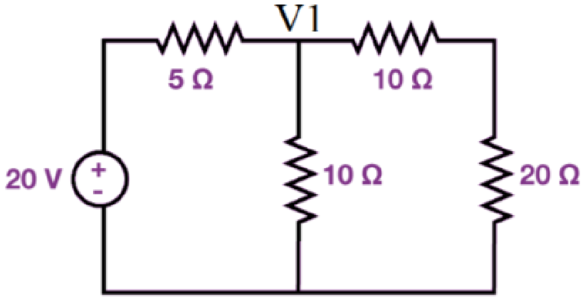


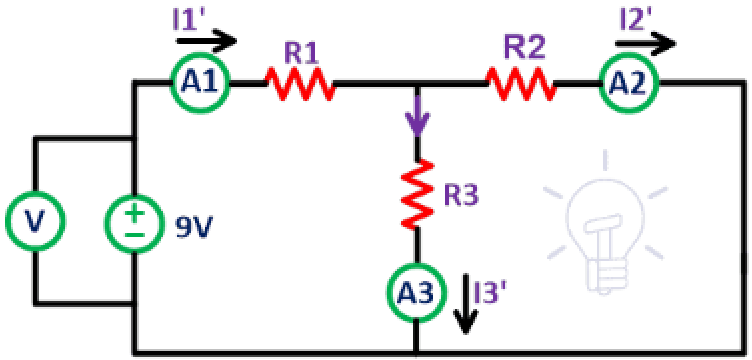


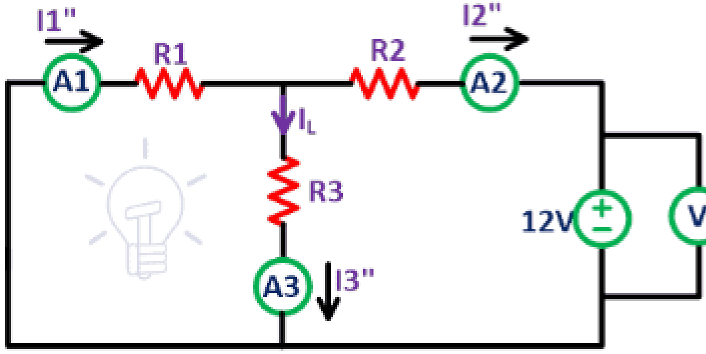
$$I_1 = I_{1'} + I_{1''}$$

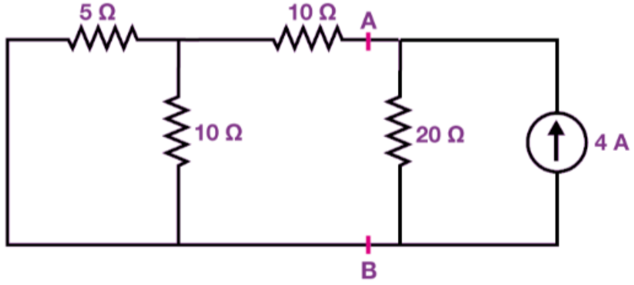
$$I_2 = I_{2'} + I_{2''}$$

$$I_3 = I_{3'} + I_{3''}$$





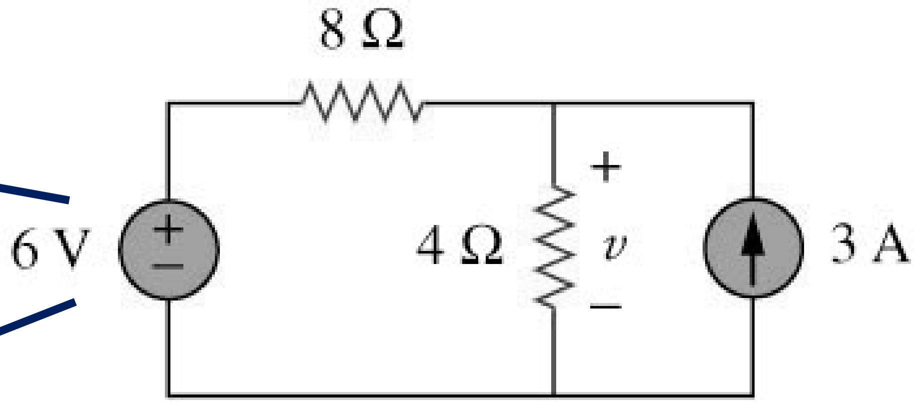




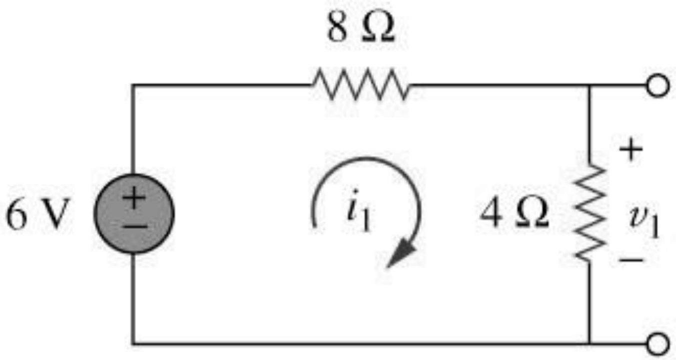
Superposition Theorem -Example

- Use the superposition theorem to find the voltage *v* in the circuit shown below.

Answer *v* = 10V

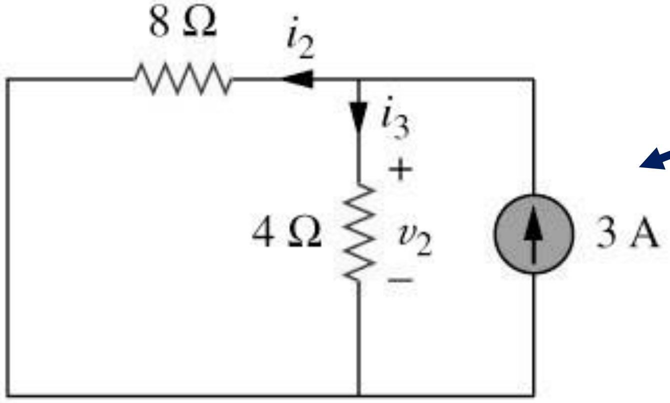


3A is discarded by open-circuit



(a)

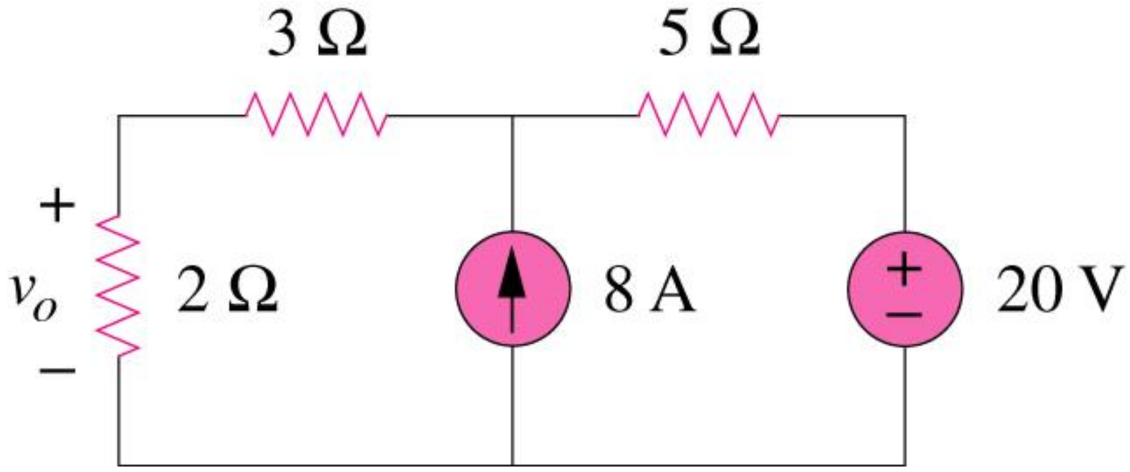
6V is discarded by short-circuit



(b)

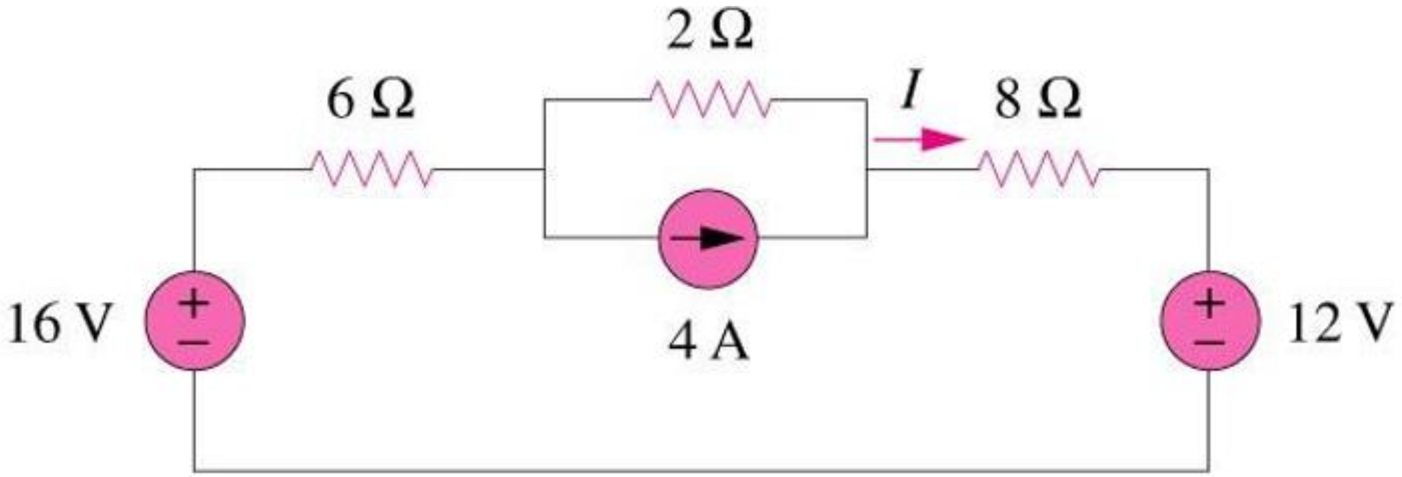
Superposition Theorem -Example

- Consider the effects of 8A and 20V one by one, then add the two effects together for final v_o.



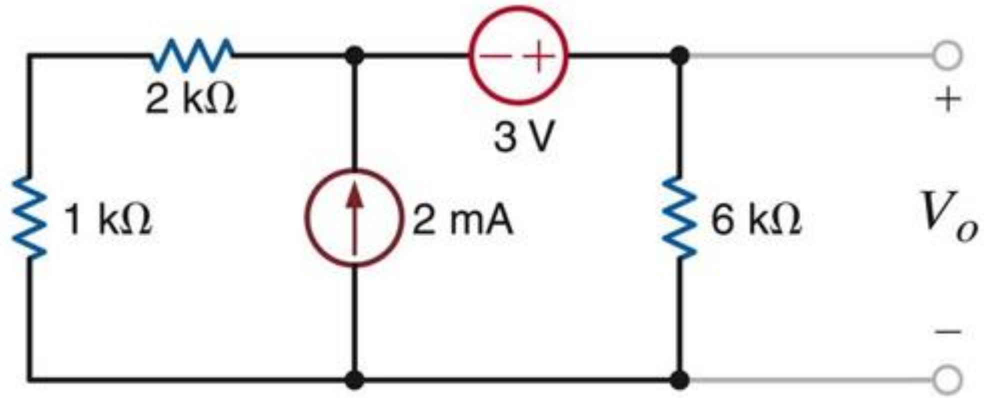
Superposition Theorem -Example

- Using superposition principle find the current *I* in the following circuit:



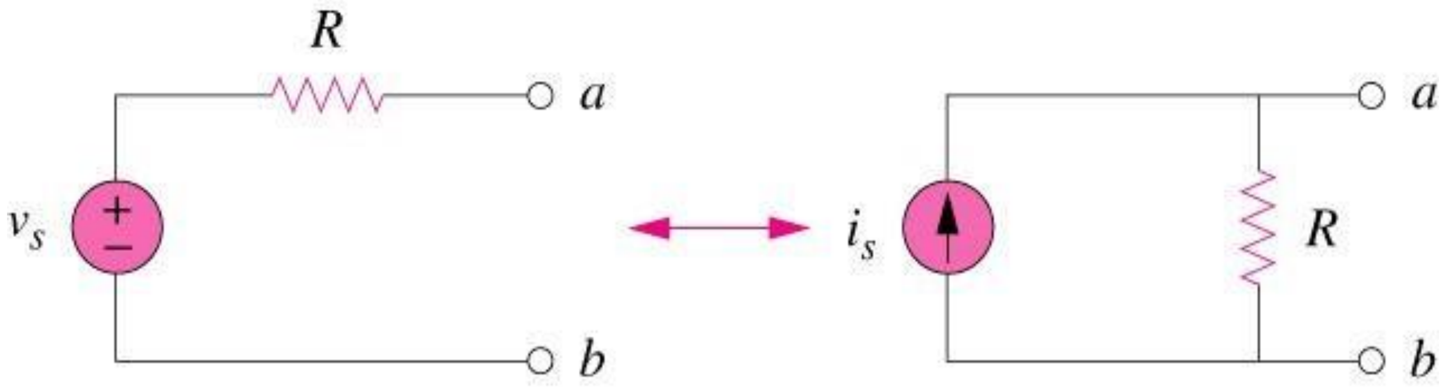
Superposition Theorem -Example

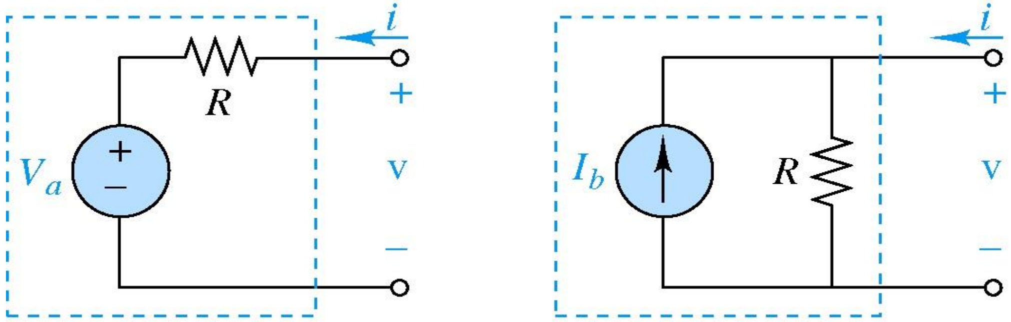
- Use superposition to find V_0:



Source Transformation

- Source transformation refers to the process of replacing a voltage source in series with a resistor R with a current source in parallel with a resistor (with the same resistance value as R) and vice versa.



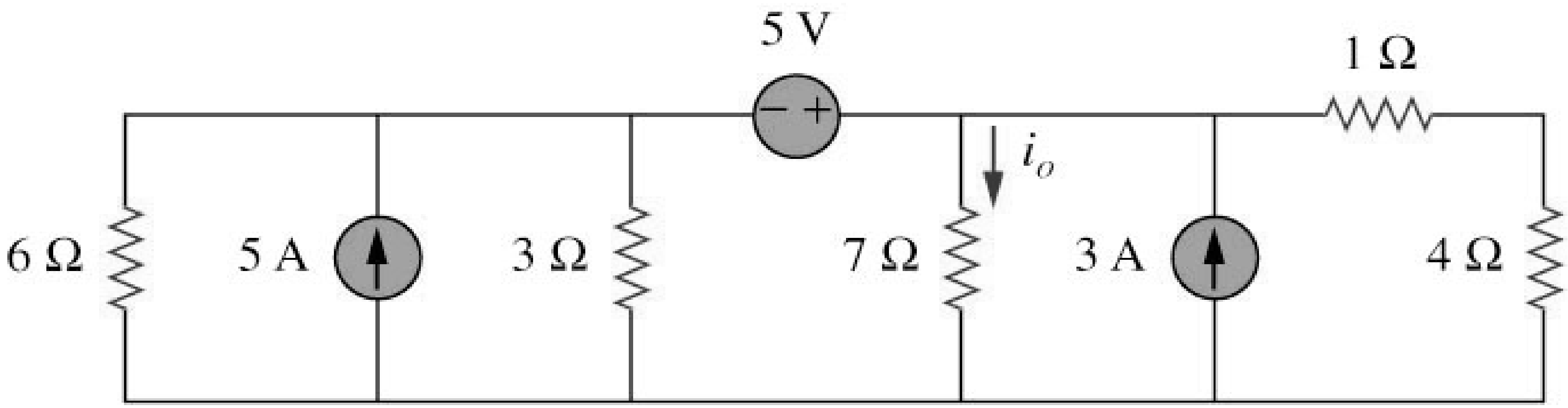


$$V_a = RI_b$$

Source Transformation - Example

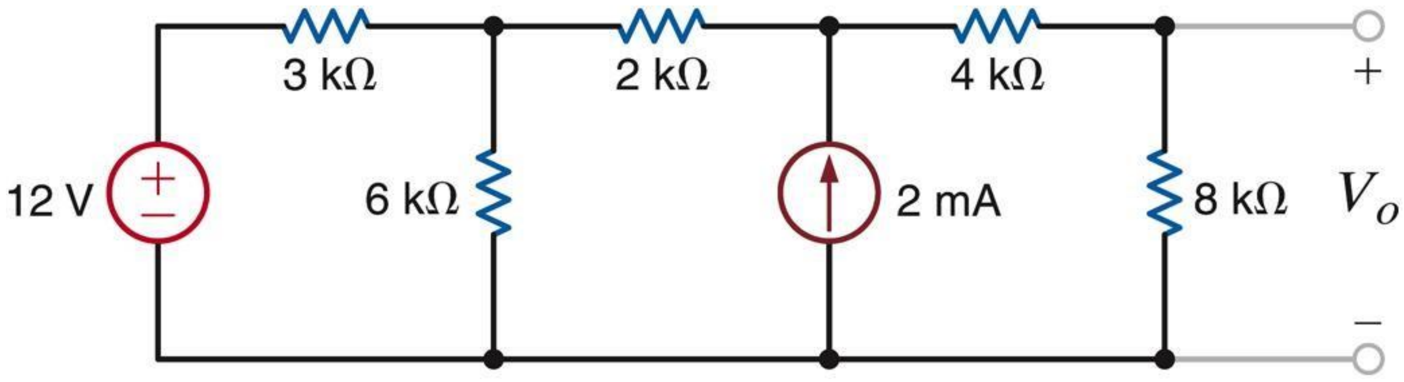
- Find the current i_o in the circuit shown below using source transformation.

Answer i_o = 1.78 A



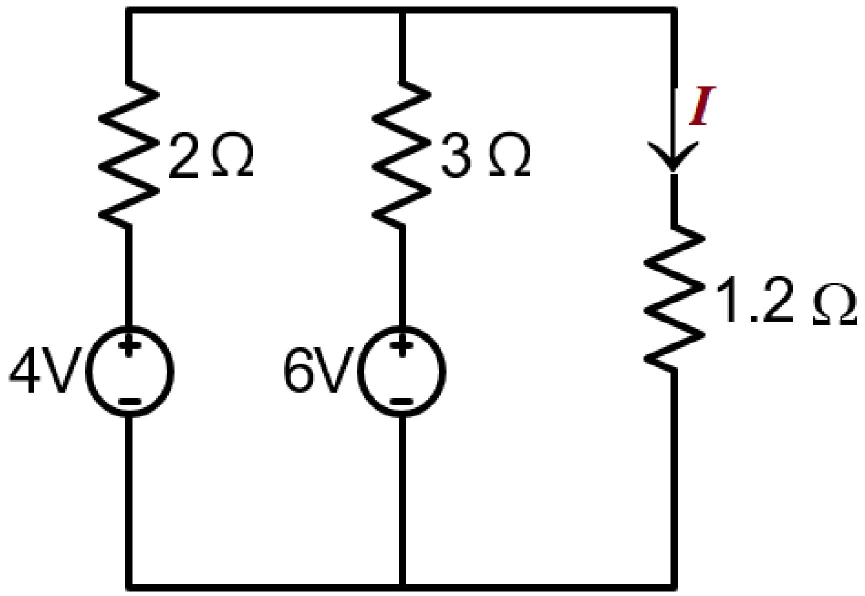
Source Transformation - Example

- In the following circuit, use repeated application of source transformation to find the voltage V_o.



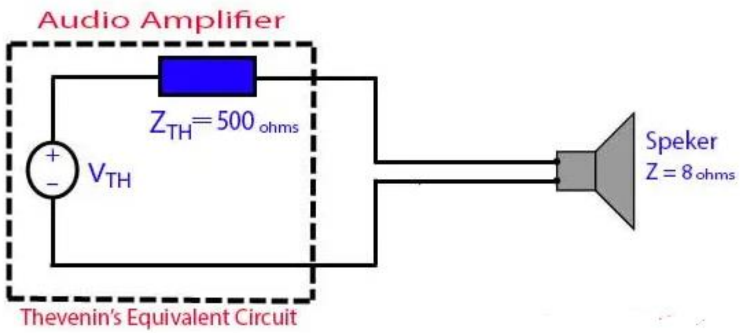
Source Transformation - Example

- In the following circuit, find the current ***I*** in the 1.2Ω resistor.

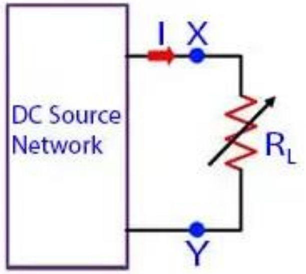


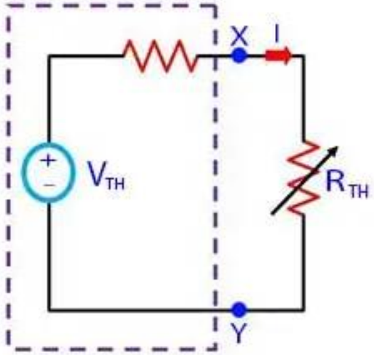
Thevenin's Equivalent Theorem

- In practice, it often happens that a particular element (or block) in a circuit is variable (this variable block is usually called load) and the rest of the circuit elements are fixed.
- As an example think of different load speakers with different internal resistance that can be connected to an stereo amplifier.

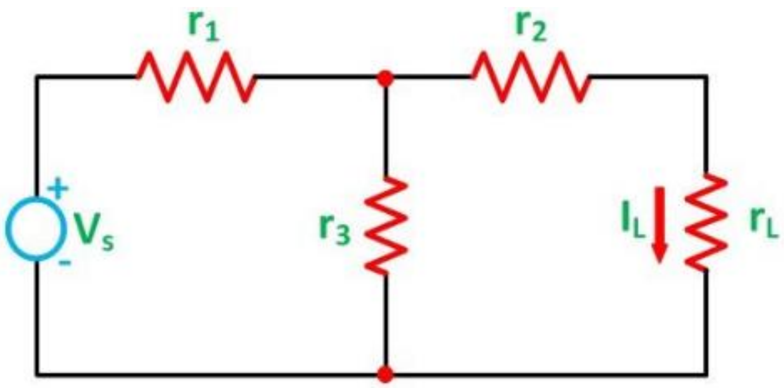


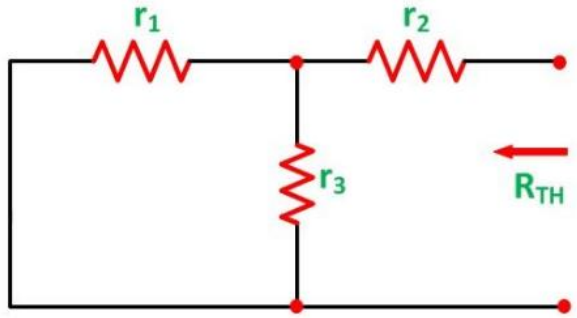
- To avoid analyzing the whole circuit each time the variable element changes, in 1883, Leon Charles Thévenin (1857-1926), a French telegraph engineer came up with the interesting idea of **replacing a complicated linear circuit by an equivalent circuit consisting of an independent voltage source in series with a resistor**.



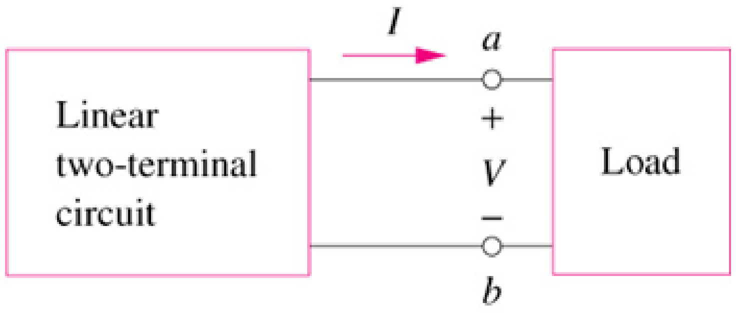


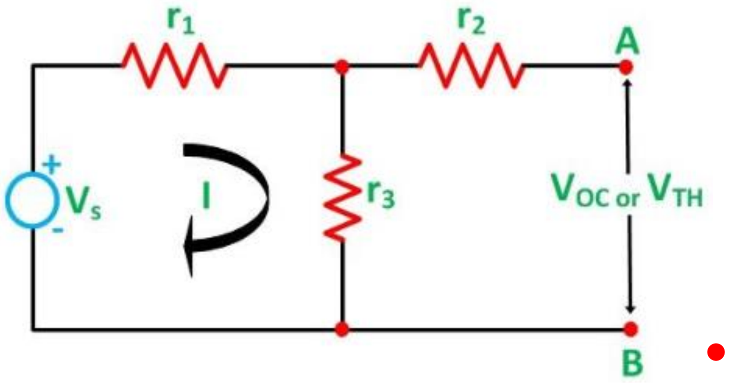
Thevenin's Equivalent

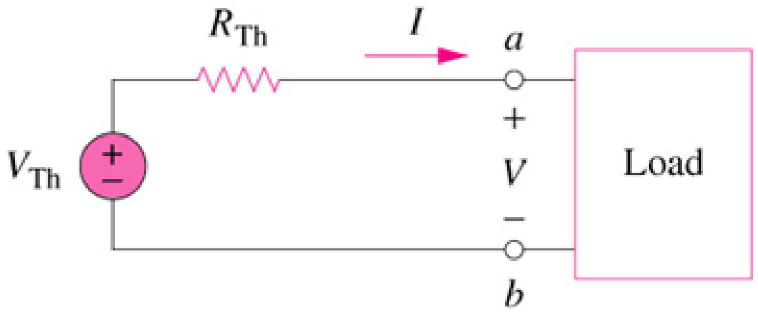




$$R_{TH} = r_2 + \frac{r_1 r_3}{r_1 + r_3}$$





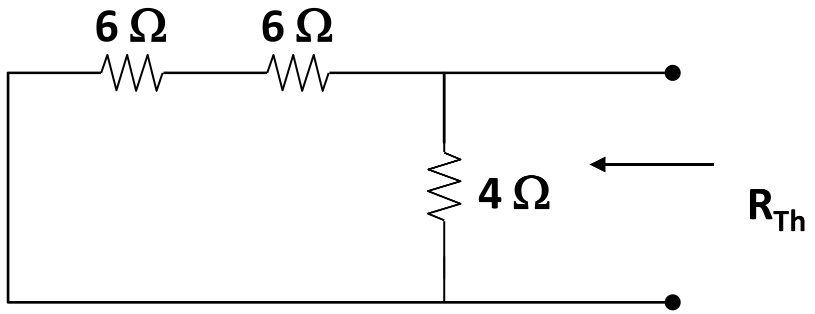


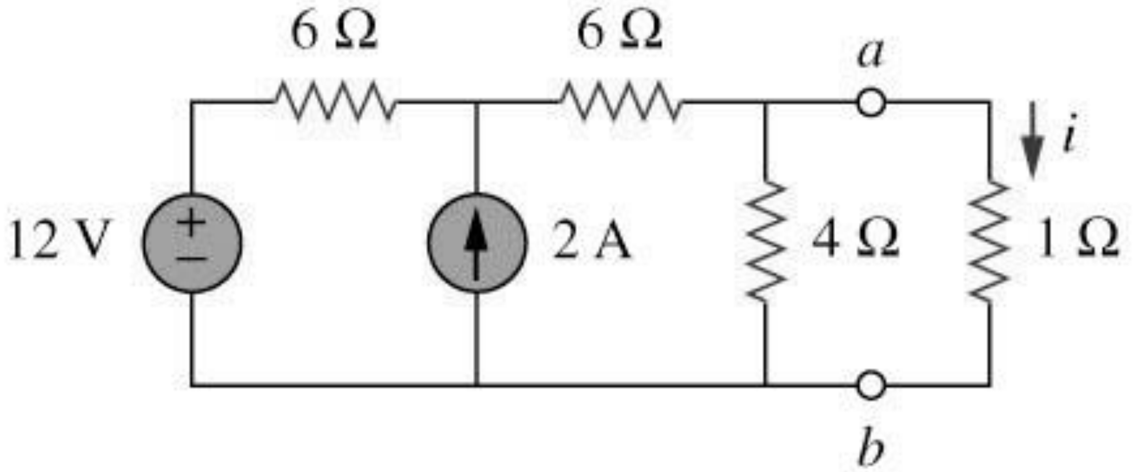
$$V_{OC} = I\, r_3 = \frac{V_S}{r_1 + r_3}\, r_3$$

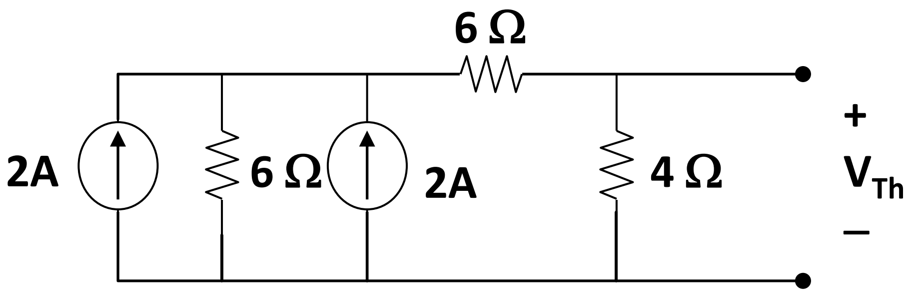
- The value of voltage source in the Thévenin equivalent circuit is the open-circuit voltage ($V_{Th} = V_{oc}$) at the terminal of the circuit
- R_{TH} is the equivalent resistance seen at the terminal when the independent sources inside the circuit are turned off.

Thevenin's Equivalent - Example

- Using Thevenin's theorem, find the equivalent circuit to the left of the terminals in the circuit shown below. Hence find ***i***.







<u>Answer</u>

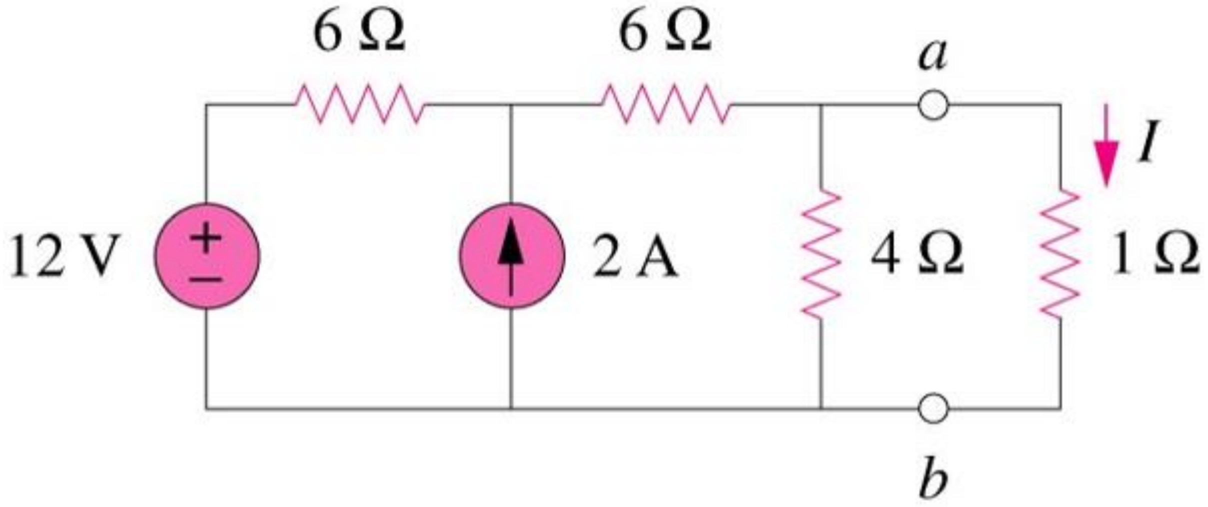
V_{TH} = 6V

R_{TH} = 3Ω,

***i* = 1.5A**

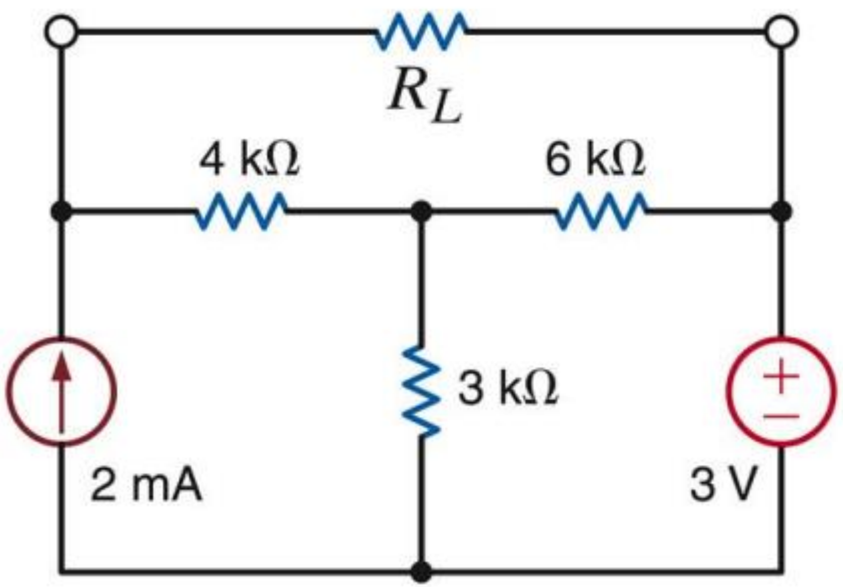
Thevenin's Equivalent - Example

- Use Thévenin's theorem to find the equivalent circuit to the left of the terminals *a* and *b* in the following circuit and then find *I*.



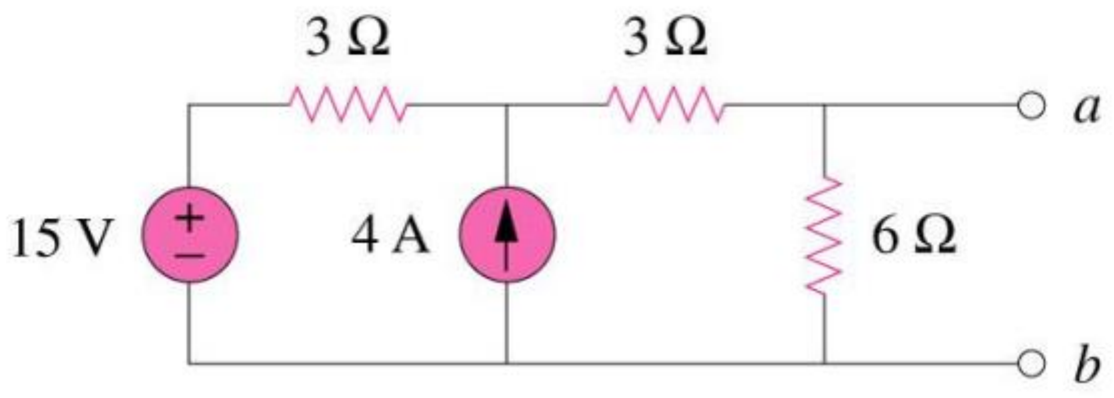
Thevenin's Equivalent - Example

- Find the Thevenin equivalent circuit seen by R_L.
- Find the current ***I***.



Thevenin's Equivalent - Example

- Find the Thévenin's equivalent of the following circuit.

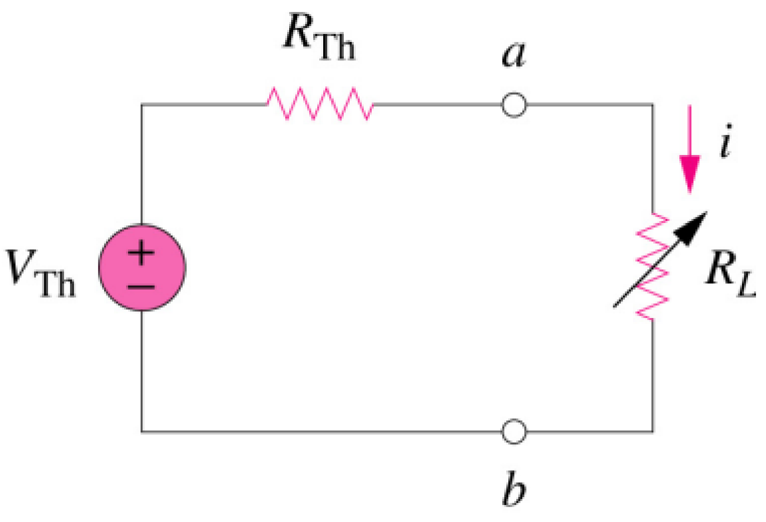


Thevenin's Equivalent - Example

- Using Thevenin's theorem, find the equivalent circuit to the left of the terminals in the circuit shown below. Hence find ***i***.

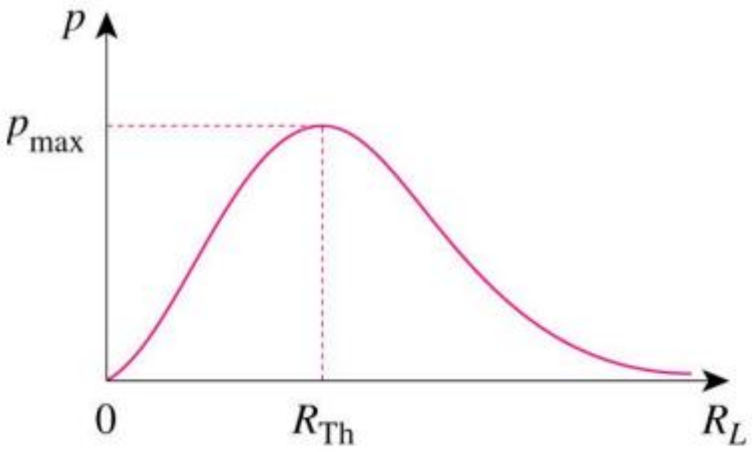
Maximum Power Transfer

- In many practical circuits, it is desirable to provide maximum power to the load.



- Power delivered to the load as a function of R_L.

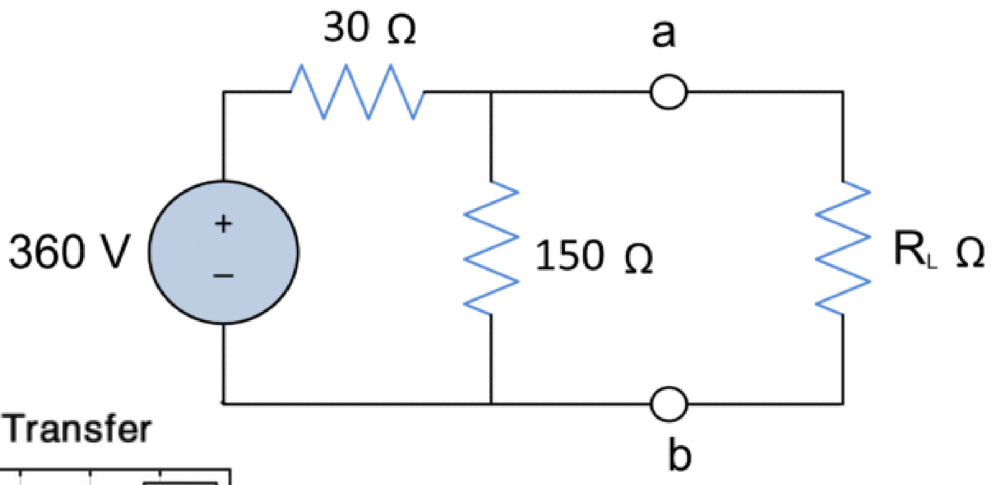
$$p = i^2 R_L = \left(\frac{V_{Th}}{R_{Th} + R_L} \right)^2 R_L$$



- Maximum power is transferred to the load when the load resistance equals the Thevenin resistance as seen from the load (that is, $R_L = R_{Th}$).
- For a given R_{TH} and V_{TH}, P_{max} is equal to: $P_{max} = \frac{V_{Th}^2}{4R_L}$

Maximum Power Transfer - Example

- Find the value of R_L for maximum power transfer in the following circuit.
- What is the maximum power?



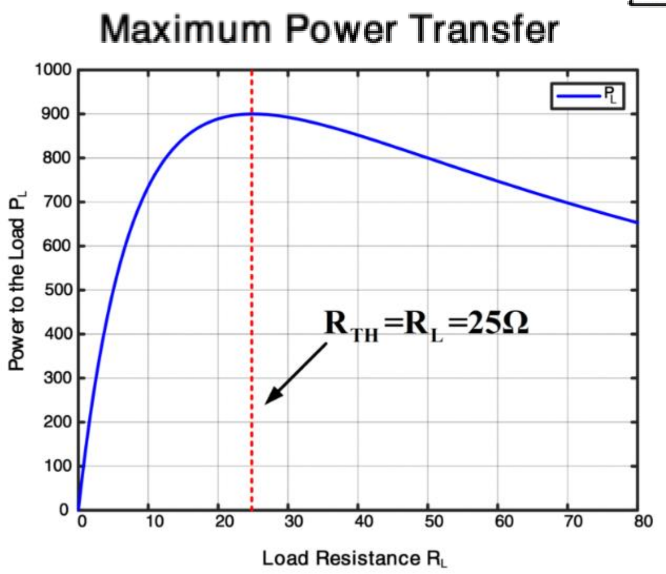
$$V_{th} = 360 * \frac{150}{150+30} \qquad V_{th} = 300\ V$$

$$R_{th} = 150 \,||\, 30 = 25\Omega$$

- For maximum power transfer ⇒

$$R_L = R_{th} = 25\ \Omega$$

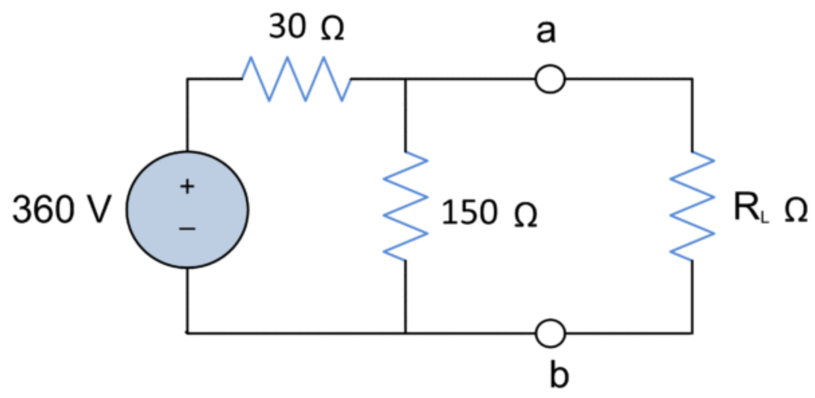
$$P_{max} = \frac{V_{Th}^2}{4R_{Th}} = 900\ W$$



Maximum Power Transfer - Example

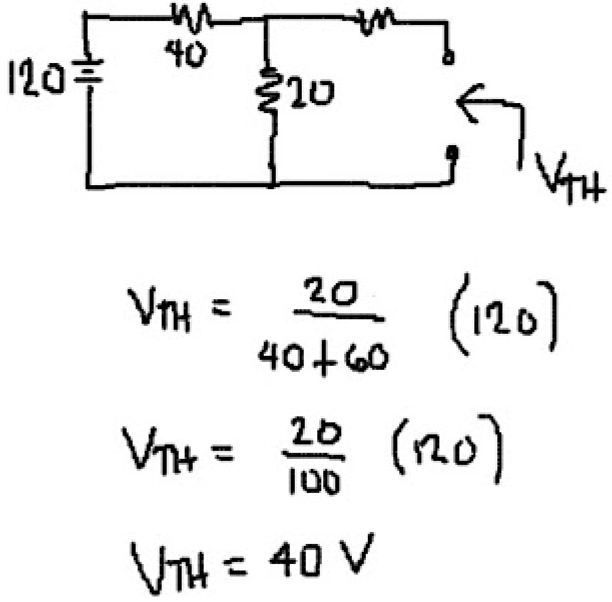
- Find the value of R_L for maximum power transfer in the following circuit.
- What is the maximum power? MATLAB Code.

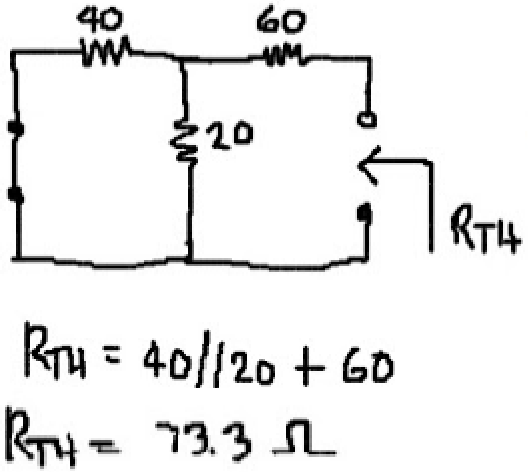
```
clear all;close all;clc
%% Circuit Parameters as given in the example (text)
% Matlab Code for Maximum Power Transfer Theorem
V_TH = 300; % Thevenin's Voltage
R_TH = 25; % Thevenin's Equivalent Resistance
R_L = 0:0.5:80; % Load Resistance
%%
%% Load Current &amp;amp;amp; Power Calculation
IL = V_TH./(R_TH + R_L); % Load Current
P_L = IL.^2 .* R_L; % Load Power
%%
% As we know that maximum power transfer occurs when
R_TH=R_L
%% Plotting the Results
plot(R_L,P_L,'b')
hold on
title('Maximum Power Transfer using Matlab');
xlabel('Load Resistance R_L');
ylabel('Power to the Load P_L');
gtext('R_TH = R_L = 25 Ohm')
legend('P_L')
grid on
```

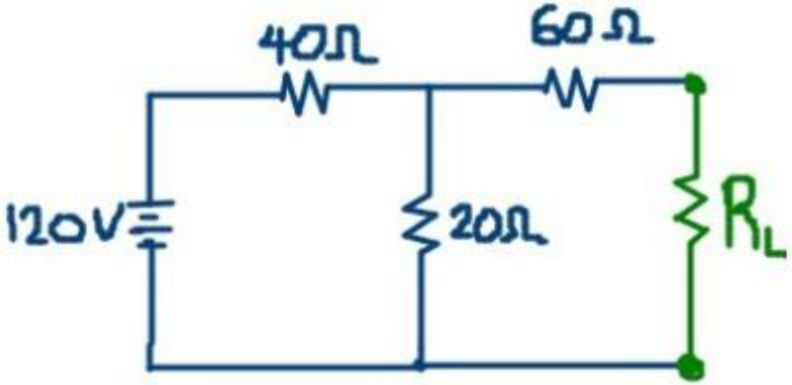


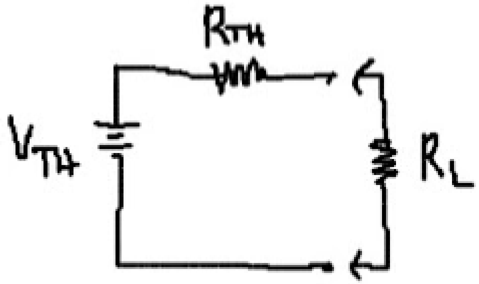
Maximum Power Transfer - Example

- Find the value of R_L for maximum power transfer in the following circuit.
- What is the maximum power?



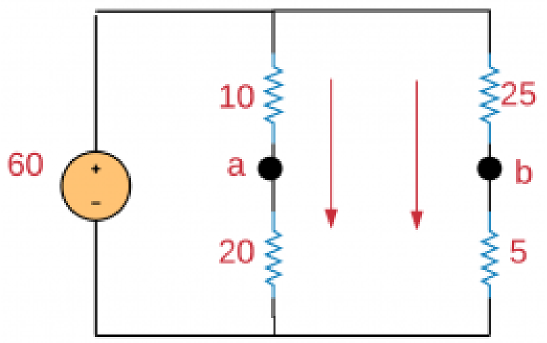


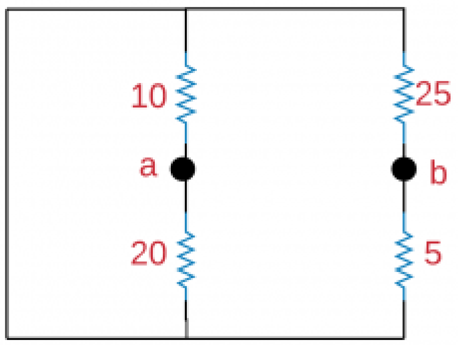


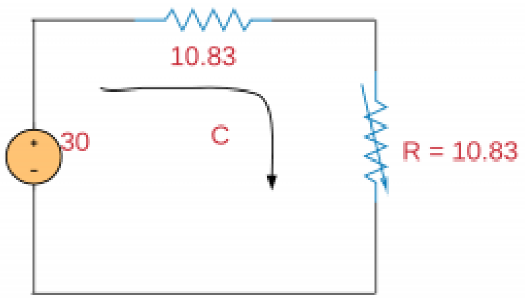


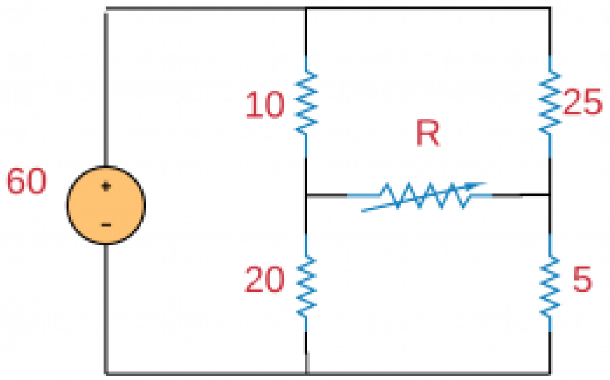
Maximum Power Transfer - Example

- Determine the maximum power that can be delivered to the <u>variable resistor</u> *R*.
- What is the maximum power?









(c) Thevenin circuit:

(a) Vth: Open circuit voltage

From the circuit, Vab=Vth=40-10=30 [V]

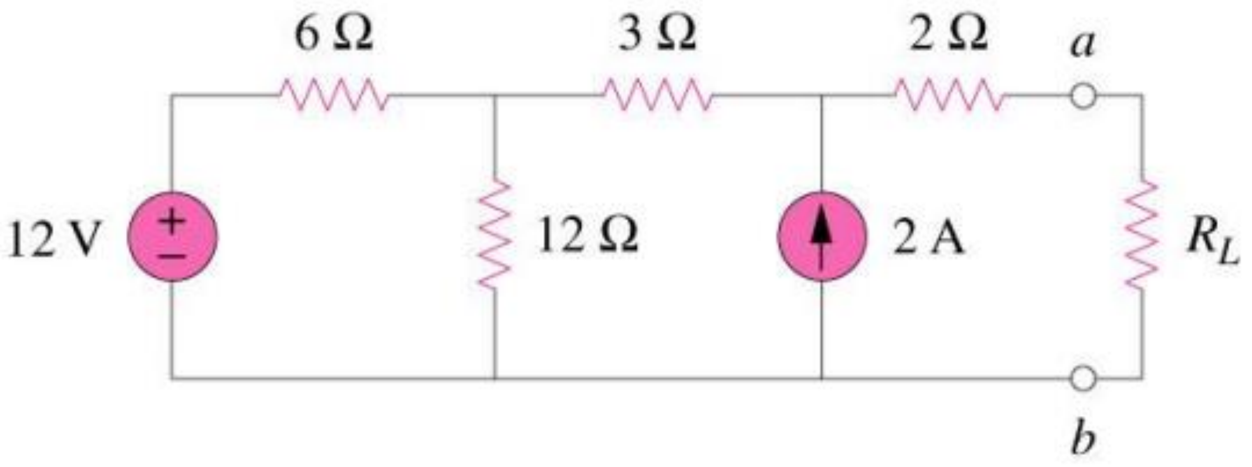
$$P_{max} = \left(\frac{30}{2 \times 10.83} \right)^2 \cdot (10.83) = 20.77 \text{ [W]}$$

(b) Rth: Let's apply Input Resistance Method:

Then Rab= (10//20) + (25//5) = 6.67+4.16=10.83 =Rth.

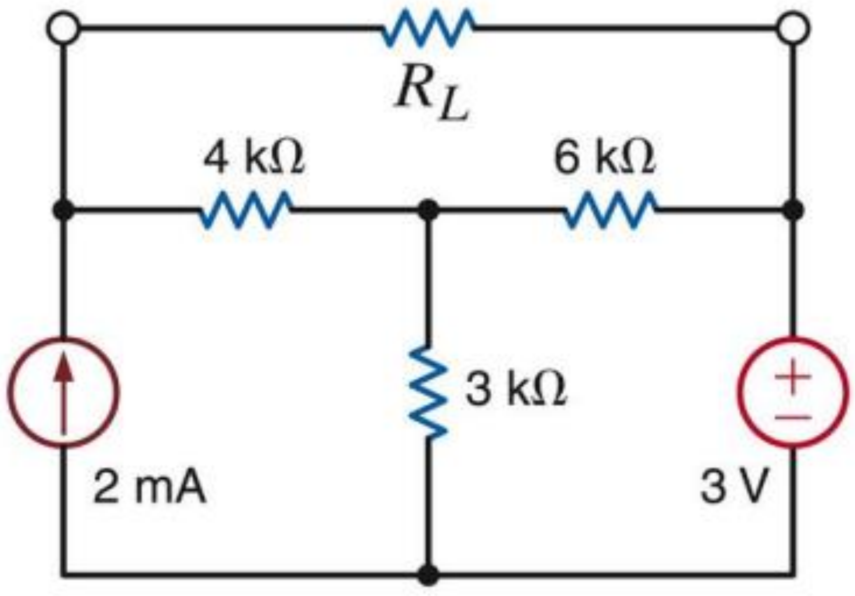
Maximum Power Transfer - Example

- Find the value of R_L for maximum power transfer in the following circuit.
- What is the maximum power?



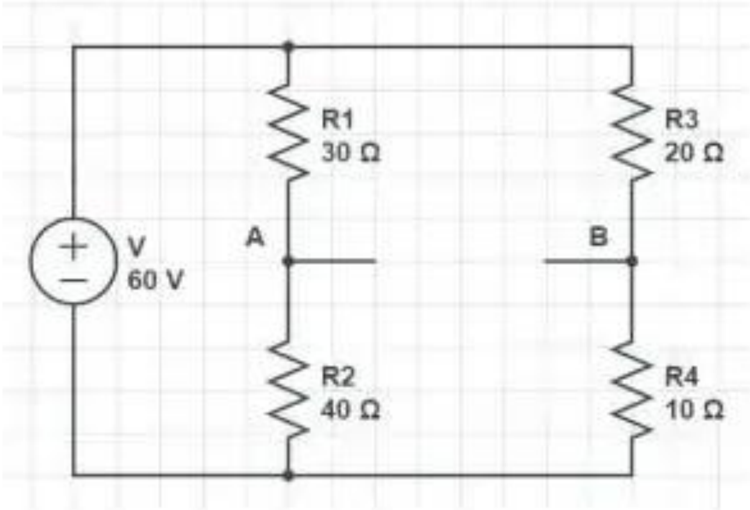
Maximum Power Transfer - Example

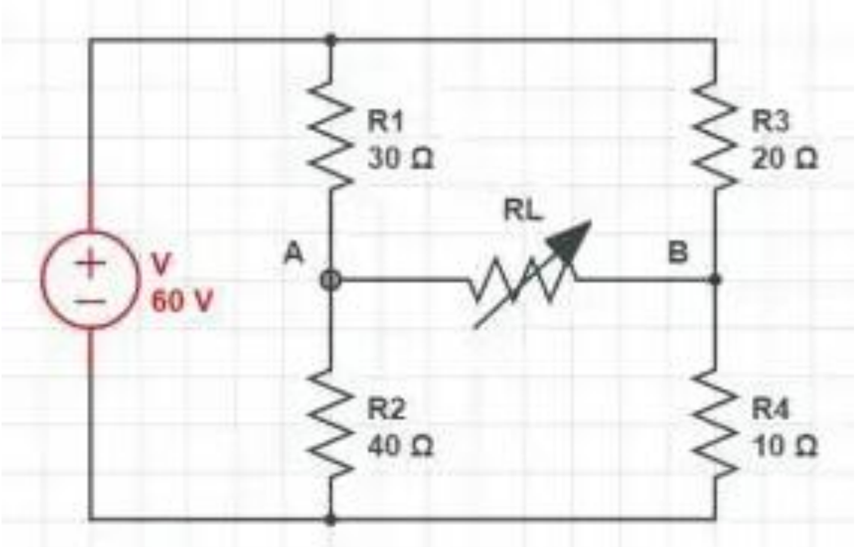
- Find the value of R_L to which the maximum power is transferred in the following circuit.
- What is the maximum power?



Maximum Power Transfer - Example

- Find the value of R_L to which the maximum power is transferred in the following circuit. What is the maximum power?





So, $V_{AB} = V_A - V_B$

V_A comes as: $V_A = V * R_2 / (R_1 + R_2)$

Or, $V_A = 60 * 40 / (30 + 40)$

Or, $V_A = 34.28$ v

:V_B comes as

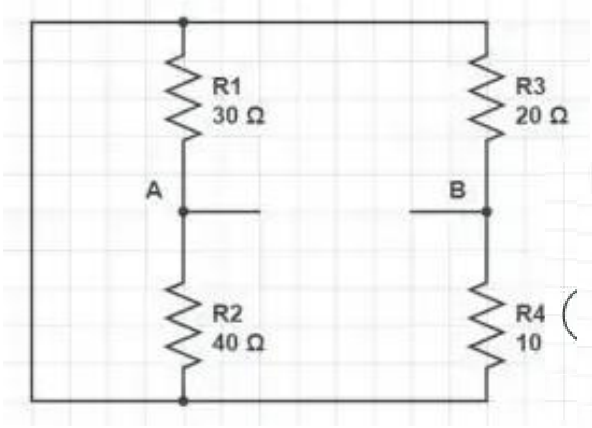
$V_B = V * R_4 / (R_3 + R_4)$

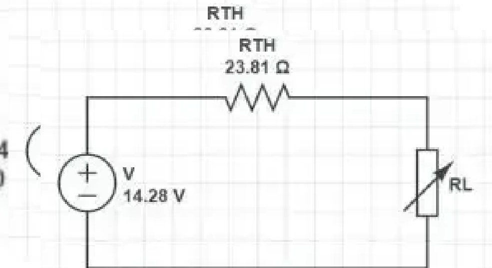
Or, $V_B = 60 * 10 / (10 + 20)$

Or, $V_B = 20$ v

So, $V_{AB} = V_A - V_B$

Or, $V_{AB} = 34.28 - 20 = 14.28$ v





$R_{TH} = R_{AB} = [\{R_1R_2 / (R_1 + R_2)\} + \{R_3R_4 / (R_3 + R_4)\}]$

OR, $R_{TH} = [\{30 \times 40 / (30 + 40)\} + \{20 \times 10 / (20 + 10)\}]$

OR, $R_{TH} = 23.809$ ohms

$P_{MAX} = V_{TH}^2 / 4\ R_{TH}$

$P_{MAX} = 14.28^2 / (4 \times 23.809)$

$P_{MAX} = 203.9184 / 95.236$

$P_{MAX} = 2.14$ Watts