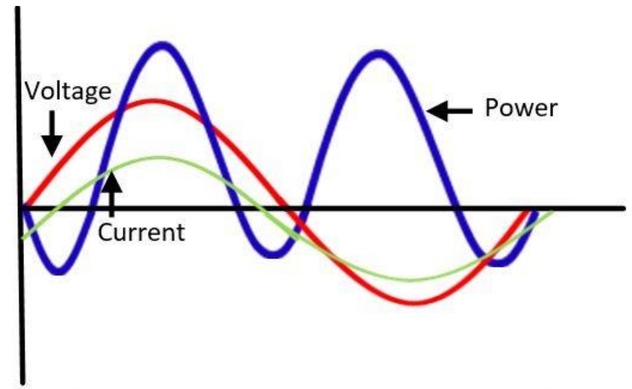
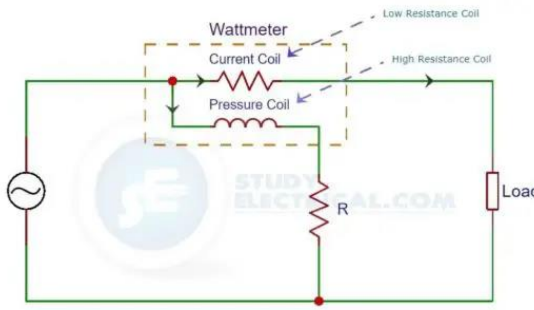
Topic 2 - AC Power Calculation

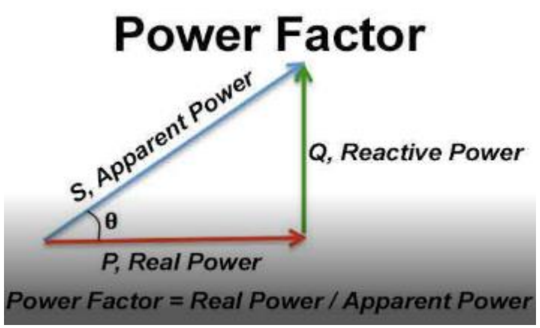
- Instantaneous and Average Power
- Maximum Average Power Transfer
- Effective or RMS Value
- Apparent Power and Power Factor
- Complex Power
- Conservation of AC Power
- Power Factor Correction
- Power Measurement



Plot of the Power of Instantaneous Voltage, Current & Power

Circuit Globe





Instantaneous and Average Power

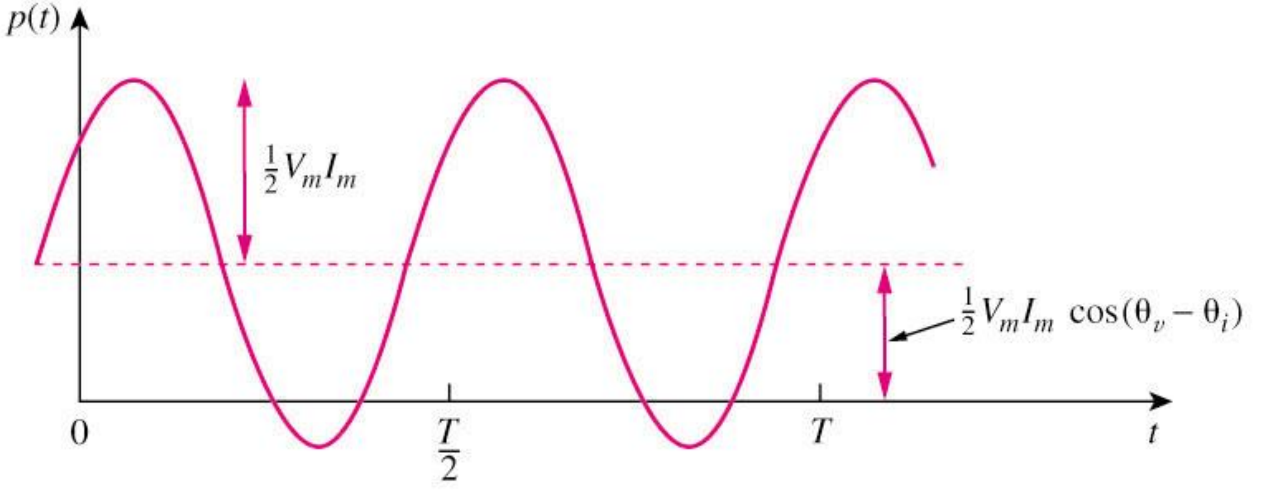
- The instantaneously power, $p(t)$

$$p(t) = v(t)\,i(t) = V_m I_m \cos(\omega t + \theta_v)\cos(\omega t + \theta_i)$$

$$= \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) + \frac{1}{2} V_m I_m \cos(2\omega t + \theta_v + \theta_i)$$

Constant power **Sinusoidal power at 2ω**

The instantaneous power *p*(*t*) is composed of a constant part (DC) and a time dependent part having frequency 2ω.

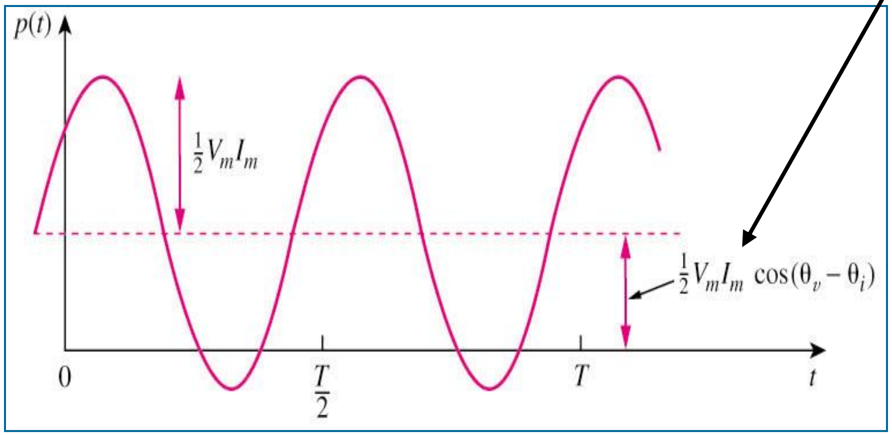


p(t) > 0: power is absorbed by the circuit; p(t) < 0: power is absorbed by the source.

Instantaneous and Average Power

- The average power, P, *is the average of the instantaneous power over one period.*

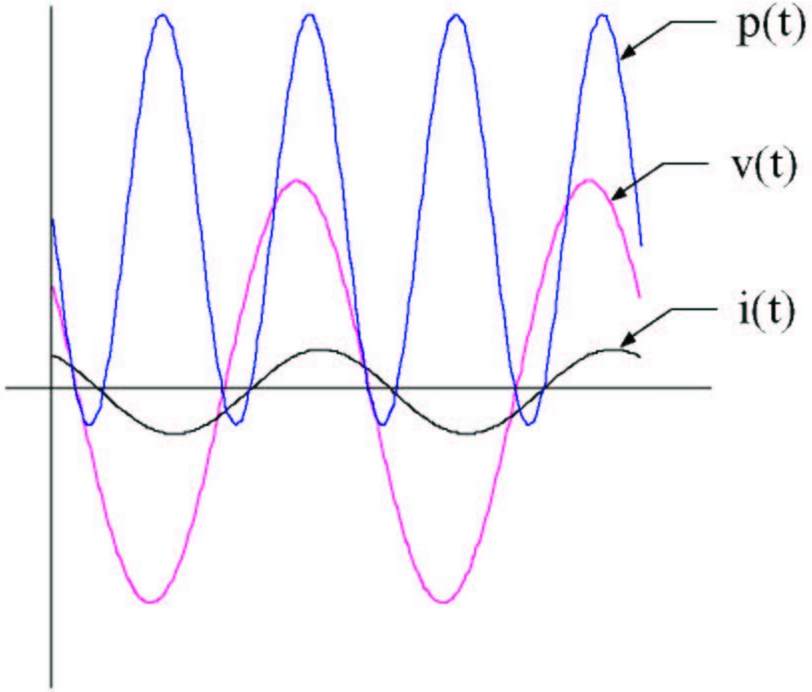
$$P = \frac{1}{T}\int_0^T p(t)\,dt = \frac{1}{2}V_m\, I_m \cos(\theta_v - \theta_i)$$



1. **P is not <u>time dependent</u>.**
2. **When $\theta_v = \theta_i$, it is a <u>purely resistive</u> load case.**
3. **When $\theta_v - \theta_i = \pm 90o$, it is a <u>purely reactive</u> load case.**
4. **P = 0 means that the circuit absorbs <u>no average power</u>.**

Instantaneous Power

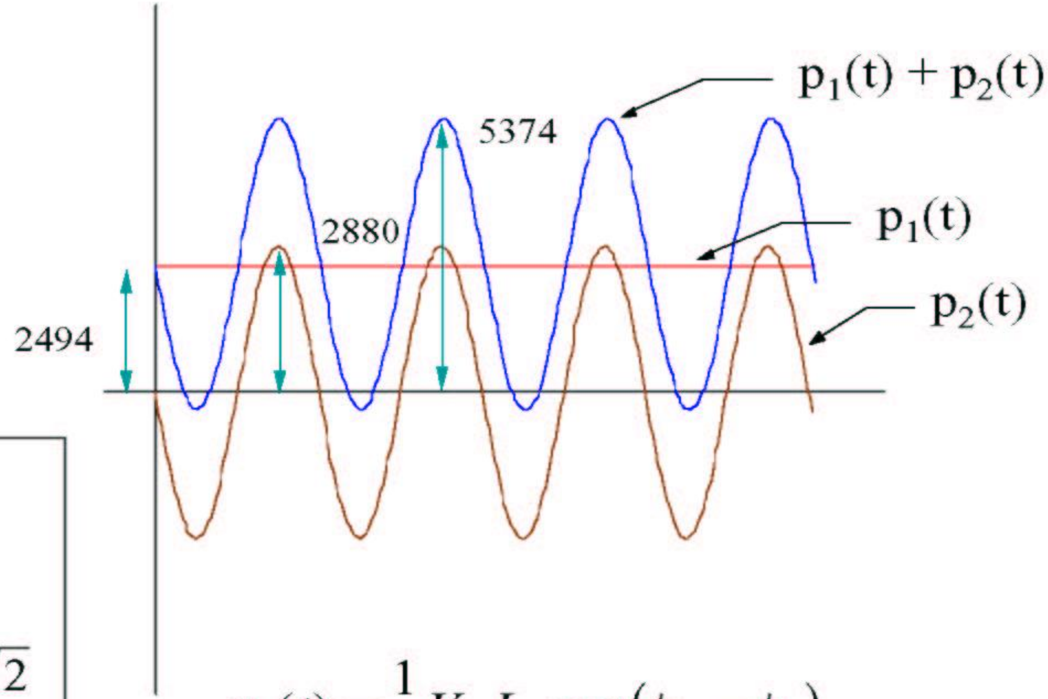
$$p(t) = v(t)i(t) = \tfrac{1}{2}V_m I_m \cos(\theta_v - \theta_i) + \tfrac{1}{2}V_m I_m \cos(2\omega t + \theta_v + \theta_i)$$



$$v(t) = 120\sqrt{2}\cos(377t + 60^\circ) \qquad i(t) = 24\sqrt{2}\cos(377t + 30^\circ)$$

Instantaneous Power

$$p(t) = \tfrac{1}{2} V_m I_m \cos(\theta_v - \theta_i) + \tfrac{1}{2} V_m I_m \cos(2\omega t + \theta_v + \theta_i) = p_1(t) + p_2(t)$$



$\phi_V = 60°$

$\phi_I = 30°$

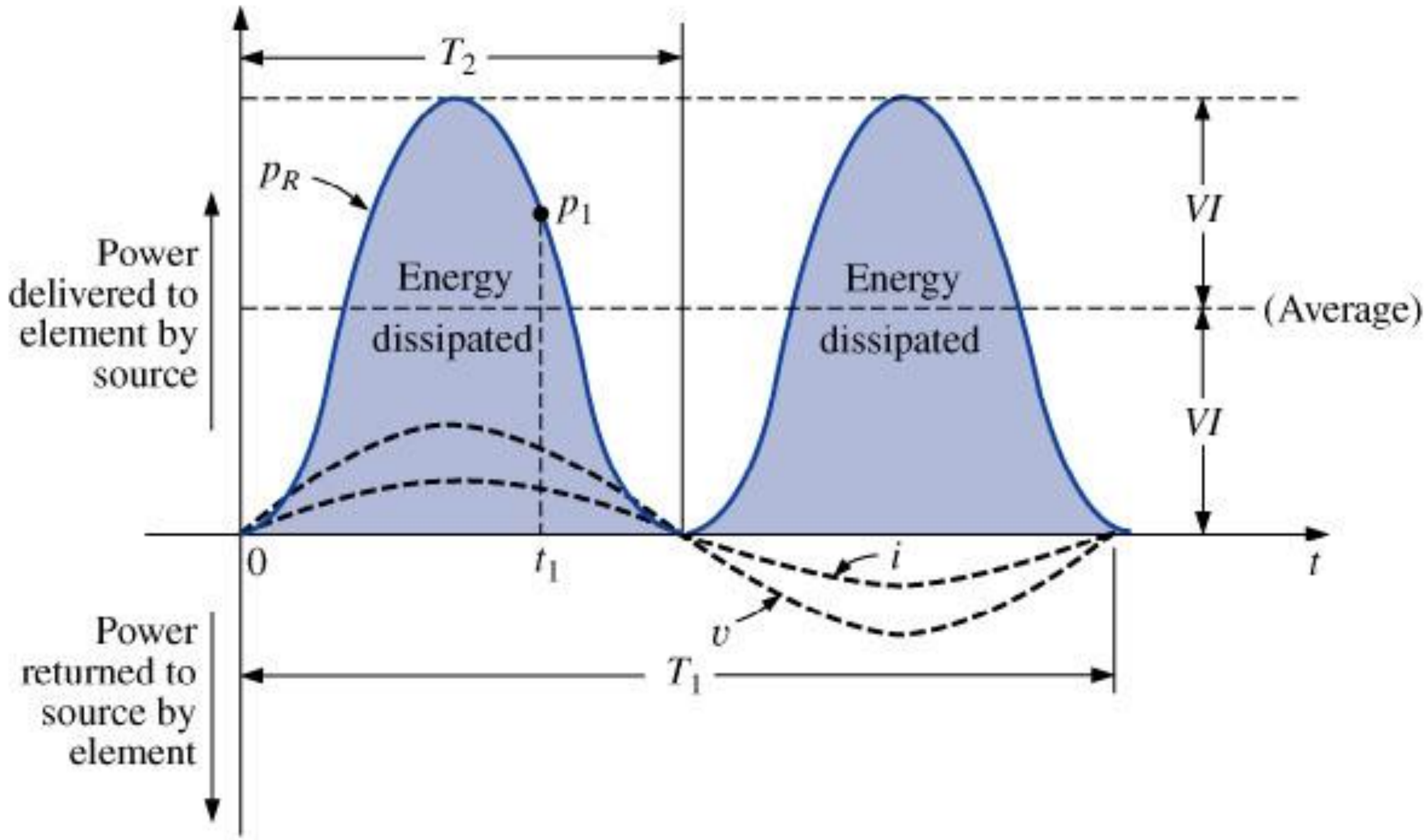
$V_m = 120\sqrt{2}$

$I_m = 24\sqrt{2}$

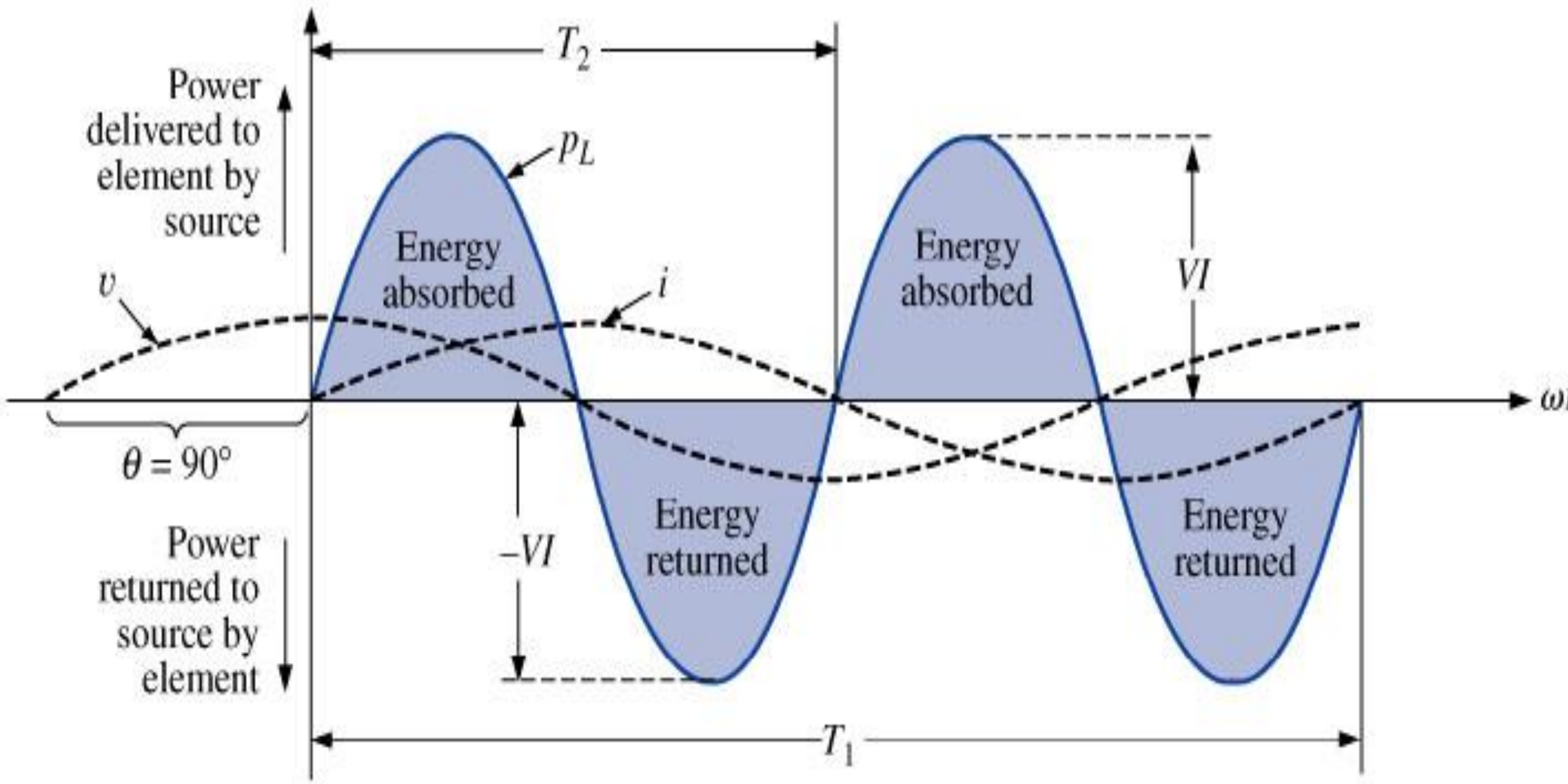
$$p_1(t) = \frac{1}{2} V_m I_m \cos(\phi_V - \phi_I)$$

$$p_2(t) = \frac{1}{2} V_m I_m \cos(2\omega t + \phi_V + \phi_I)$$

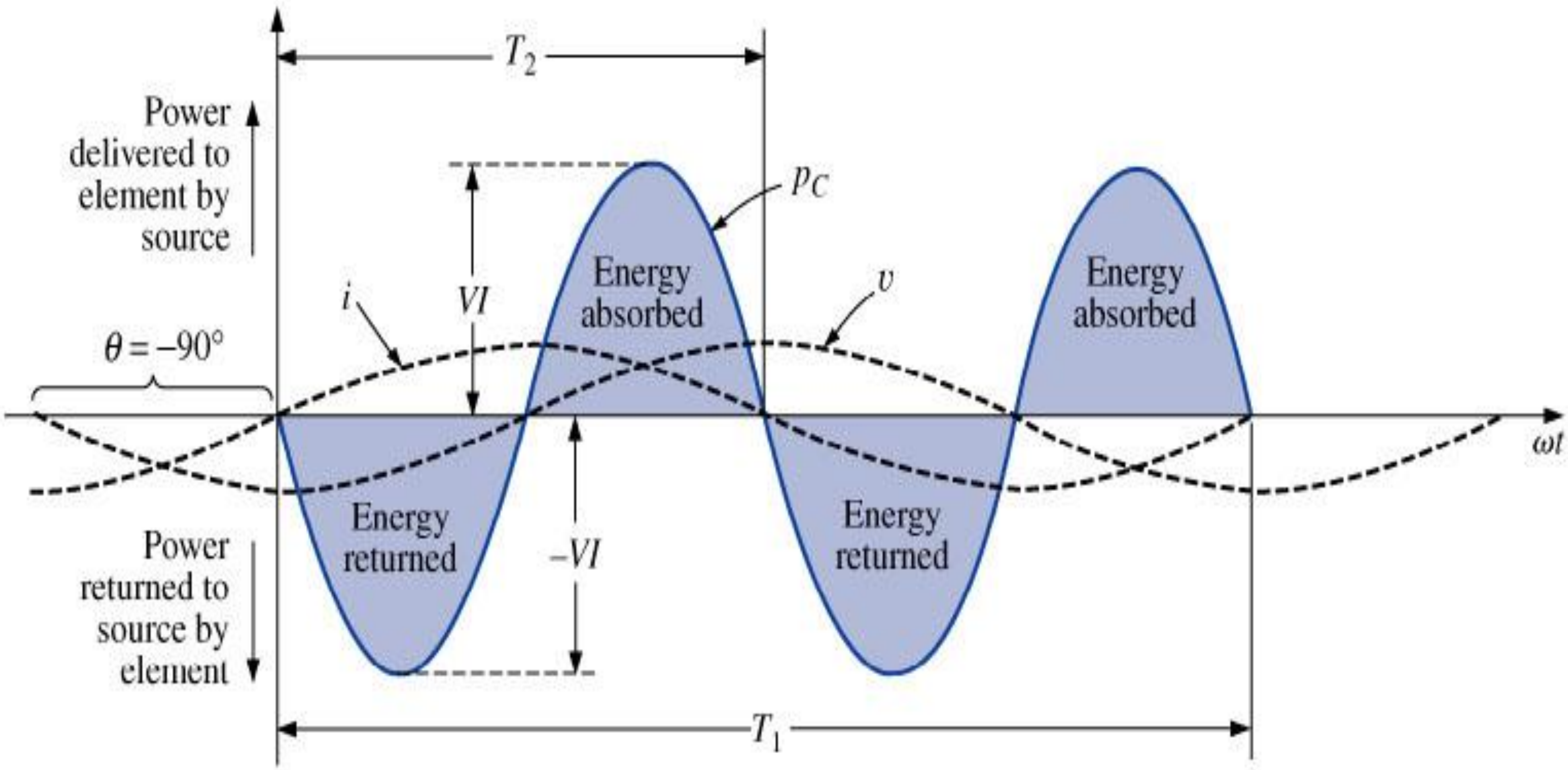
Resistive Circuit and Real Power



Inductive Circuit and Reactive Power



Capacitive Circuit and Reactive Power



Instantaneous and Average Power

Example:

Calculate the instantaneous power and average power absorbed by a passive linear network if:

$$v(t) = 80\cos(10t + 20°)$$

$$i(t) = 15\sin(10t + 60°)$$

Answer: $385.7 + 600\cos(20t - 10°)\text{W}, 387.5\text{W}$

Average Power

➢The average power P is the average of the instantaneous power over one period .

$$p(t) = v(t)i(t) \quad \text{Instantaneous Power}$$

$$P = \frac{1}{T}\int_0^T p(t)dt \quad \text{Average Power}$$

$$v(t) = V_m \cos(\omega t + \theta_v) \qquad i(t) = I_m \cos(\omega t + \theta_i)$$

$$P = \frac{1}{T}\int_0^T p(t)dt = \frac{1}{T}\int_0^T \tfrac{1}{2}V_m I_m \cos(\theta_v - \theta_i)dt + \frac{1}{T}\int_0^T \tfrac{1}{2}V_m I_m \cos(2\omega t + \theta_v + \theta_i)dt$$

$$P = \tfrac{1}{2}V_m I_m \cos(\theta_v - \theta_i)\frac{1}{T}\int_0^T dt + \tfrac{1}{2}V_m I_m \frac{1}{T}\int_0^T \cos(2\omega t + \theta_v + \theta_i)dt$$

$$= \tfrac{1}{2}V_m I_m \cos(\theta_v - \theta_i) + 0 \qquad \text{(Integral of a Sinusoidal=0)}$$

$$P = \tfrac{1}{2}V_m I_m \cos(\theta_v - \theta_i)$$

$$P = \tfrac{1}{2}\mathrm{Re}\left[\mathbf{VI}^*\right] = \frac{1}{2}V_m I_m \cos(\theta_v - \theta_i)$$

Average Power

Example:

A current $\mathbf{I} = 10\angle 30°$ flows through an impedance.

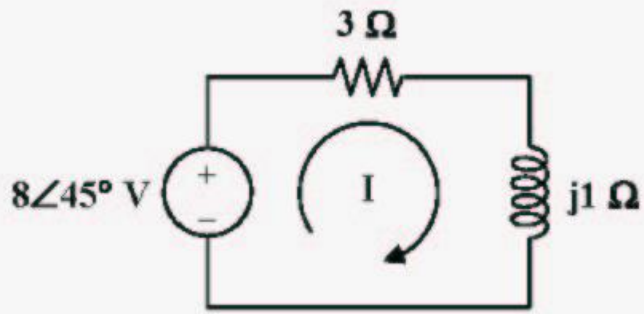
Find the average power delivered to the impedance.

$\mathbf{Z} = 20\angle -22°\Omega$

Answer: 927.2W

Average Power

Example: Find the average power absorbed by resistor and inductor. Find the average power supplied by the source



$$\mathbf{I} = \frac{8\angle 45^\circ}{3+j} = 2.53\angle 26.57^\circ$$

For the resistor, $\mathbf{I}_R = \mathbf{I} = 2.53\angle 26.57^\circ \quad \mathbf{V}_R = 3\mathbf{I} = 7.59\angle 26.57^\circ$

$$P_R = \frac{1}{2}V_m I_m = \frac{1}{2}(2.53)(7.59) = \underline{\mathbf{9.6\ W}}$$

For the inductor, $\mathbf{I}_L = 2.53\angle 26.57^\circ$, $\mathbf{V}_L = j\mathbf{I}_L = 2.53\angle(26.57^\circ + 90^\circ) = 2.53\angle 116.57^\circ$

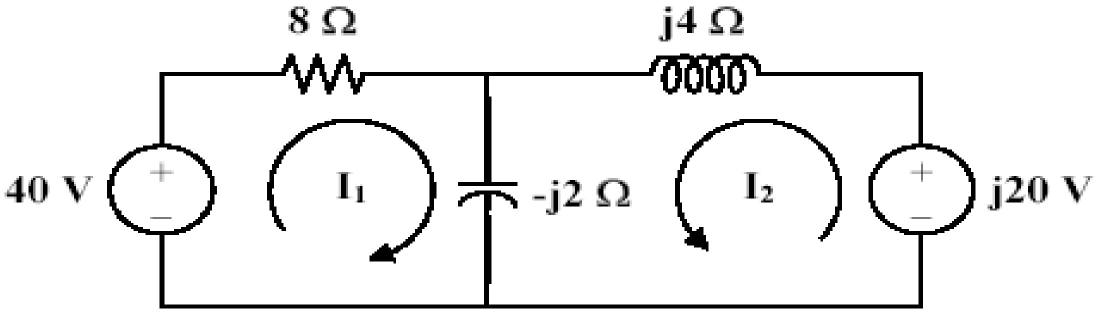
$$P_L = \frac{1}{2}(2.53)^2\cos(90^\circ) = \underline{\mathbf{0\ W}}$$

The average power supplied is

$$P = \frac{1}{2}(8)(2.53)\cos(45^\circ - 26.57^\circ) = \underline{\mathbf{9.6\ W}}$$

Average Power

Example: Calculate the average power absorbed by each of the five elements in the circuit given.



For mesh 1,

$$-40 + (8 - j2)\mathbf{I}_1 + (-j2)\mathbf{I}_2 = 0$$

$$(4 - j)\mathbf{I}_1 - j\mathbf{I}_2 = 20 \qquad (1)$$

For mesh 2,

$$-j20 + (j4 - j2)\mathbf{I}_2 + (-j2)\mathbf{I}_1 = 0$$

$$-j\mathbf{I}_1 + j\mathbf{I}_2 = j10 \qquad (2)$$

In matrix form,

$$\begin{bmatrix} 4-j & -j \\ -j & j \end{bmatrix}\begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} 20 \\ j10 \end{bmatrix}$$

Average Power

$$\Delta = 2 + j4, \qquad \Delta_1 = -10 + j20, \qquad \Delta_2 = 10 + j60$$

$$\mathbf{I}_1 = \frac{\Delta_1}{\Delta} = 5\angle 53.14° \quad \text{and} \quad \mathbf{I}_2 = \frac{\Delta_2}{\Delta} = 13.6\angle 17.11°$$

For the 40-V voltage source,

$$\mathbf{V}_s = 40\angle 0° \qquad \mathbf{I}_1 = 5\angle 53.14°$$

$$P_s = \frac{-1}{2}(40)(5)\cos(-53.14°) = \underline{\mathbf{-60\ W}}$$

For the j20-V voltage source,

$$\mathbf{V}_s = 20\angle 90° \qquad \mathbf{I}_2 = 13.6\angle 17.11°$$

$$P_s = \frac{-1}{2}(20)(13.6)\cos(90° - 17.11°) = \underline{\mathbf{-40\ W}}$$

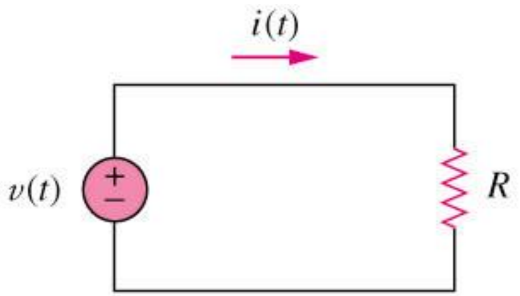
For the resistor,

$$I = \left|\mathbf{I}_1\right| = 5 \qquad V = 8\left|\mathbf{I}_1\right| = 40$$

$$P = \frac{1}{2}(40)(5) = \underline{\mathbf{100\ W}}$$

The average power absorbed by the inductor and capacitor is zero watts.

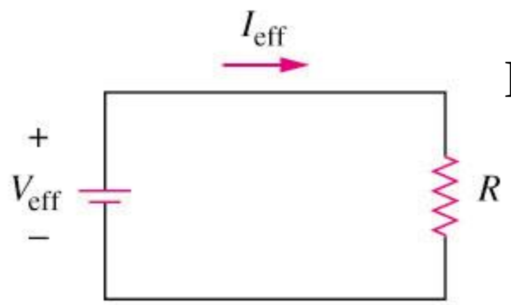
Effective or RMS Value



a) AC circuit

The total power dissipated by R is given by:

$$P = \frac{1}{T}\int_0^T i^2 R dt = \frac{R}{T}\int_0^T i^2 dt = I_{rms}^2 R$$



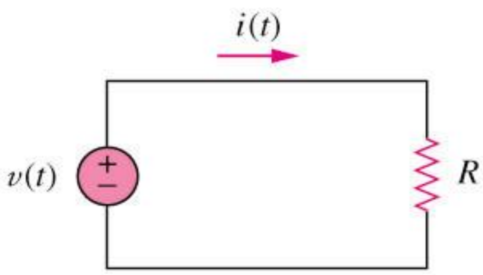
b) DC circuit

Hence, I_{eff} is equal to: $$I_{eff} = \sqrt{\frac{1}{T}\int_0^T i^2\, dt} = I_{rms}$$

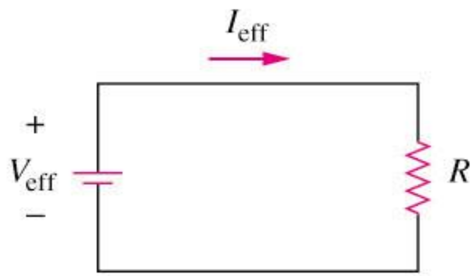
The rms value is a constant itself which depending on the shape of the function *i*(*t*).

➢ **The <u>EFFECTIVE</u> Value or the Root Mean Square (RMS) value of a periodic current is the DC value that delivers the same average power to a resistor as the periodic current.**

Effective or RMS Value



a) AC circuit



b) DC circuit

The rms value of a sinusoid $i(t) = I_m \cos(\omega t)$ is given by:

$$I_{Rms} = \sqrt{\frac{1}{T}\int_0^T I_m{}^2 \cos^2 \omega t dt} = \sqrt{\frac{I_m{}^2}{T}\int_0^T \frac{1}{2}(1+\cos 2\omega t)dt} = \frac{I_m}{\sqrt{2}}$$

The average power can be written in terms of the rms values:

$$P = \frac{1}{2} V_m\, I_m \cos(\theta_v - \theta_i) = V_{rms}\, I_{rms} \cos(\theta_v - \theta_i)$$

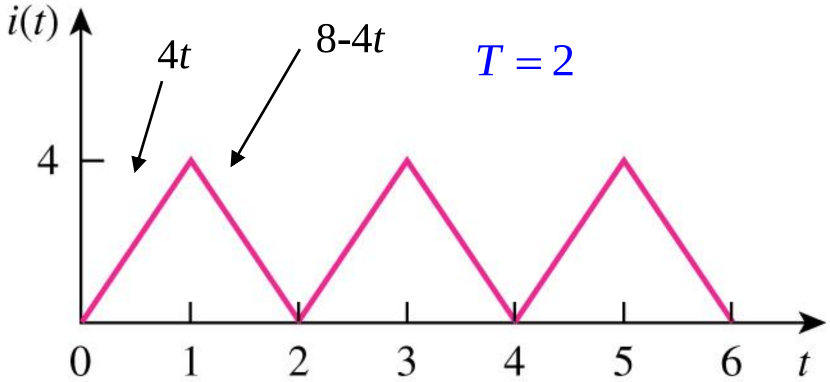
- The average power for resistive loads using the (RMS) value is:

$$P_R = I_{Rms}{}^2 R = \frac{V_{Rms}{}^2}{R}$$

RMS Value

➢ **Example:** **Find the RMS value of the current waveform. Calculate the average power if the current is applied to a 9 Ω resistor.**

$$i(t) = \begin{cases} 4t & 0 < t < 1 \\ 8 - 4t & 1 < t < 2 \end{cases}$$



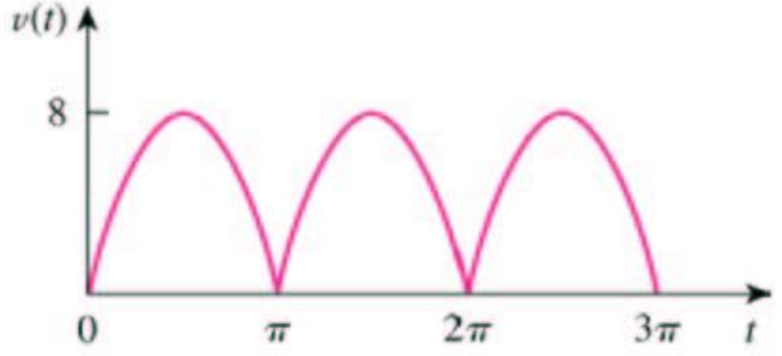
$$I_{rms}^2 = \frac{1}{T}\int_0^T i^2\,dt = \frac{1}{2}\left[\int_0^1 (4t)^2\,dt + \int_1^2 (8-4t)^2\,dt\right]$$

$$I_{rms}^2 = \frac{16}{2}\left[\int_0^1 t^2\,dt + \int_1^2 (4-4t+t^2)\,dt\right] \qquad I_{rms}^2 = 8\left[\frac{1}{3} + \left(4t - 2t^2 + \frac{t^3}{3}\right)\Big|_1^2\right] = \frac{16}{3}$$

$$I_{rms} = \sqrt{\frac{16}{3}} = 2.309\text{A} \qquad P = I_{rms}^2 R = \left(\frac{16}{3}\right)(9) = 48\text{W}$$

RMS Value

<u>Example:</u> Find the RMS value of the full-wave rectified sine wave. Calculate the average power dissipated in a 6 Ω resistor.



$$T = \pi\ ,\ v(t) = 8\sin(t),\quad 0 < t < \pi$$

$$V_{emf}^2 = \frac{1}{T}\int_0^T v^2\,dt = \frac{1}{\pi}\int_0^{\pi}(8\sin(t))^2\,dt \qquad V_{emf}^2 = \frac{64}{\pi}\int_0^{\pi}\frac{1}{2}\left[1-\cos(2t)\right]dt = 32$$

$$V_{emf} = \underline{\mathbf{5.657\ V}} \qquad P = \frac{V_{emf}^2}{R} = \frac{32}{6} = \underline{\mathbf{5.333\ W}}$$

Apparent Power and Power Factor

- Apparent Power, S, is the product of the r.m.s. values of voltage and current.
- It is measured in volt-amperes or VA to distinguish it from the average or real power which is measured in watts.

$$P = V_{rms} I_{rms} \cos(\theta_v - \theta_i) = S \cos(\theta_v - \theta_i)$$

Apparent Power, *S*

Power Factor, pf

- Power factor is the cosine of the phase difference between the voltage and current. It is also the cosine of the angle of the load impedance.

Apparent Power and Power Factor

Purely resistive load (R)	$\theta_v - \theta_i = 0$, Pf = 1	P/S = 1, all power are consumed
Purely reactive load (L or C)	$\theta_v - \theta_i = \pm 90^o$, pf = 0	P = 0, no real power consumption
Resistive and reactive load (R and L/C)	$\theta_v - \theta_i > 0$ $\theta_v - \theta_i < 0$	• Lagging - inductive load • Leading - capacitive load

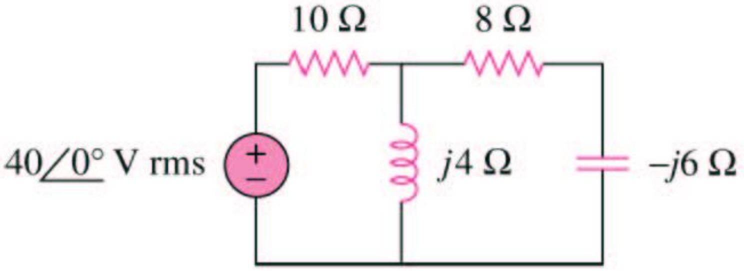
$$P = \tfrac{1}{2} V_m I_m \cos(\theta_v - \theta_i) = V_{Rms} I_{Rms} \cos(\theta_v - \theta_i)$$

$$pf = \frac{P}{S} = \cos(\theta_v - \theta_i)$$

$$S = \frac{1}{2} V_m I_m = V_{Rms} I_{Rms}$$

Power Factor

<u>Example:</u> Calculate the power factor seen by the source and the average power supplied by the source



The total impedance as seen by the source is

$$\mathbf{Z} = 10 + j4 \,||\, (8 - j6) = 10 + \frac{(j4)(8 - j6)}{8 - j2} \qquad \mathbf{Z} = 12.69\angle 20.62^\circ$$

The power factor is

$$\text{pf} = \cos(20.62^\circ) = \mathbf{0.936 \quad (lagging)}$$

$$I_{rms} = \frac{V_{rms}}{Z} = \frac{40\angle 0^\circ}{12.69\angle 20.62^\circ} = 3.152\angle -20.62^\circ$$

The average power supplied by the source is equal to the power absorbed by the load.

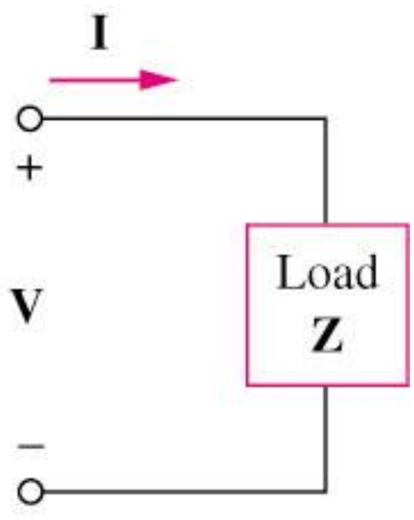
$$P = I_{rms}^2 R = (3.152)^2 (11.88) = 118 \text{ W}$$

or $$P = V_{rms} I_{rms} \text{pf} = (40)(3.152)(0.936) = \mathbf{118\ W}$$

Complex Power

➢ The **COMPLEX** Power **S** contains all the information pertaining to the power absorbed by a given load.

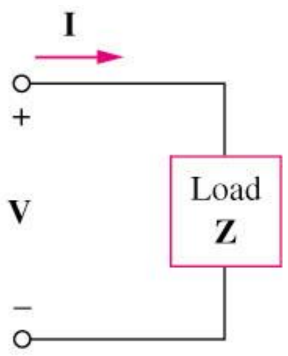
➢Complex power **S** is the product of the voltage and the complex conjugate of the current:



$$\mathbf{V} = V_m \angle \theta_v \qquad \mathbf{I} = I_m \angle \theta_i$$

$$\frac{1}{2} V I^* = V_{rms} I_{rms} \angle \theta_v - \theta_i$$

Complex Power



$$S = \frac{1}{2} V I^* = V_{rms} I_{rms} \angle \theta_v - \theta_i$$

$$\Rightarrow S = \underbrace{V_{rms} I_{rms} \cos(\theta_v - \theta_i)}_{P} + j \underbrace{V_{rms} I_{rms} \sin(\theta_v - \theta_i)}_{Q}$$

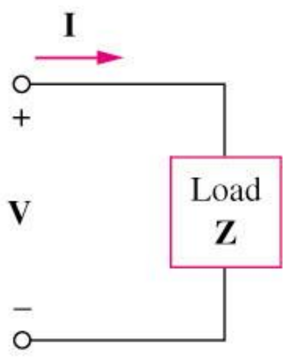
$$\boldsymbol{S} = \mathbf{P} + \mathbf{j}\,\mathbf{Q}$$

P: is the average power in watts delivered to a load and it is the only useful power.

Q: is the reactive power exchange between the source and the reactive part of the load. It is measured in VAR.

- Q = 0 for *resistive loads* (unity pf).
- Q < 0 for *capacitive loads* (leading pf).
- Q > 0 for *inductive loads* (lagging pf).

Complex Power



$$\Rightarrow S = V_{rms} I_{rms} \cos(\theta_v - \theta_i) + j V_{rms} I_{rms} \sin(\theta_v - \theta_i)$$

$$\mathbf{S} = \mathbf{P} + \mathbf{j}\ \mathbf{Q}$$

Apparent Power, $S = |\mathbf{S}| = Vrms*Irms = \sqrt{P^2 + Q^2}$

Real power, $P = Re(\mathbf{S}) = S\cos(\theta_v - \theta_i)$

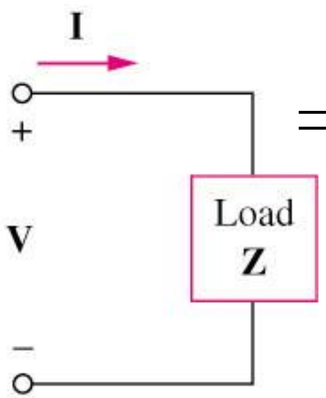
Reactive Power, $Q = Im(\mathbf{S}) = S\sin(\theta_v - \theta_i)$

Power factor, $pf = P/S = \cos(\theta_v - \theta_i)$

- Real Power is the actual power dissipated by the load.
- Reactive Power is a measure of the energy exchange between source and reactive part of the load.

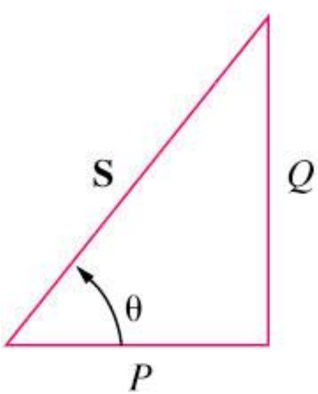
Complex Power

➢ The COMPLEX Power is represented by the POWER TRIANGLE similar to IMPEDANCE TRIANGLE. Power triangle has four items: P, Q, S and θ.

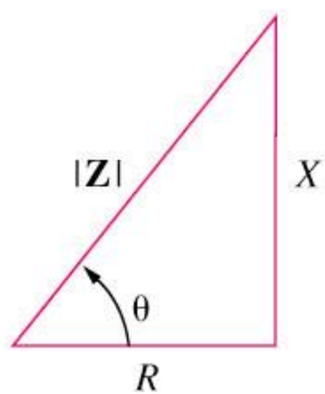


$$\Rightarrow S = \underbrace{V_{rms}\ I_{rms}\ \cos(\theta_v - \theta_i)}_{P} + j\,\underbrace{V_{rms}\ I_{rms}\ \sin(\theta_v - \theta_i)}_{Q}$$

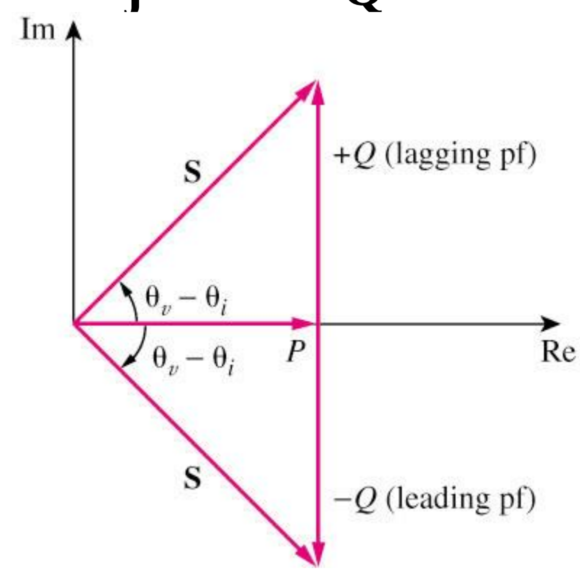
$$S = P + jQ$$



Power Triangle



Impedance Triangle



Power Factor

Real and Reactive Powers

- The REAL Power is the only useful power delivered to the load.
- The REACTIVE Power represents the energy exchange between the source and reactive part of the load. It is being transferred back and forth between the load and the source
- The unit of Q is volt-ampere reactive (VAR)

$$\mathbf{S} = P + jQ = \mathrm{Re}\{\mathbf{S}\} + j\,\mathrm{Im}\{\mathbf{S}\}$$

$$= \text{Real Power} + \text{Reactive Power}$$

$$\mathbf{S} = I^2_{Rms} Z = I^2_{Rms}(R + jX) = P + jQ$$

$$P = V_{Rms} I_{Rms} \cos(\theta_v - \theta_i) = \mathrm{Re}\{\mathbf{S}\} = I^2_{Rms} R$$

$$Q = V_{Rms} I_{Rms} \sin(\theta_v - \theta_i) = \mathrm{Im}\{\mathbf{S}\} = I^2_{Rms} X$$

Complex Power

<u>Example:</u> Two loads are connected in parallel. Load 1 has 2 kW, pf=0.75 leading and Load 2 has 4 kW, pf=0.95 lagging. Calculate the pf of two loads and the complex power supplied by the source.

For load 1,

$$P_1 = 2000, \qquad pf = 0.75 = \cos\theta_1 \longrightarrow \theta_1 = -41.41°$$

$$P_1 = S_1 \cos\theta_1 \longrightarrow S_1 = \frac{P_1}{\cos\theta_1} = 2666.67$$

$$Q_1 = S_1 \sin\theta_1 = -176.85$$

$$\mathbf{S}_1 = P_1 + jQ_1 = 2000 - j1763.85 \quad \text{(leading)}$$

For load 2,

$$P_2 = 4000, \qquad pf = 0.95 = \cos\theta_2 \longrightarrow \theta_2 = 18.19°$$

$$S_2 = \frac{P_2}{\cos\theta_2} = 4210.53$$

$$Q_2 = S_2 \sin\theta_2 = 1314.4$$

$$\mathbf{S}_2 = P_2 + jQ_2 = 4000 + j1314.4 \quad \text{(lagging)}$$

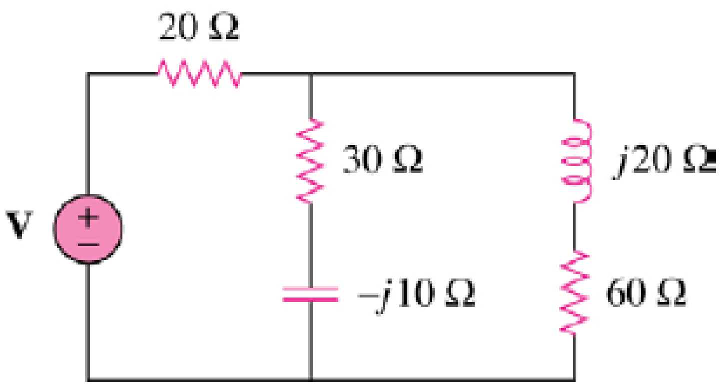
The total complex power is

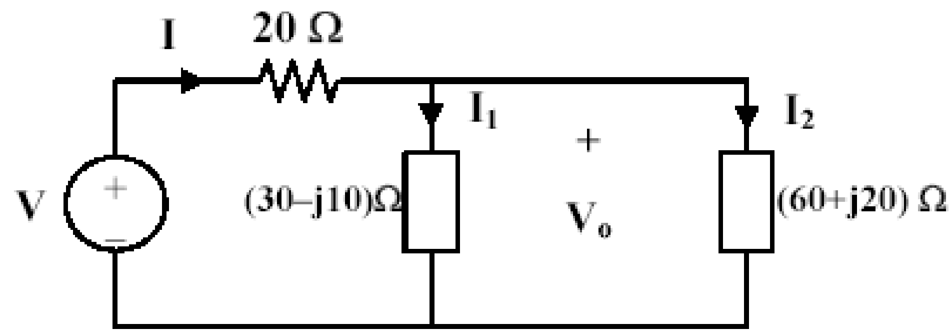
$$\mathbf{S} = \mathbf{S}_1 + \mathbf{S}_2 = \underline{\mathbf{6 - j0.4495\ kVA}}$$

$$pf = \frac{P}{|\mathbf{S}|} = \frac{6000}{6016.18} = \underline{\mathbf{0.9972 \quad (leading)}}$$

Complex Power

Example: The 60 Ω resistor absorbs 240 Watt of average power. Calculate V and the complex power of each branch. What is the total complex power?





$$P = I_2^2 R \longrightarrow I_2^2 = \frac{P}{R} = \frac{240}{60} = 4 \quad \mathbf{I}_2 = 2 \quad \text{(rms)}$$

$$\mathbf{V}_o = \mathbf{I}_2 (60 + j20) = 120 + j40$$

$$\mathbf{I}_1 = \frac{\mathbf{V}_o}{30 - j10} = 3.2 + j2.4 \quad \mathbf{I} = \mathbf{I}_1 + \mathbf{I}_2 = 5.2 + j2.4$$

$$\mathbf{V} = 20\mathbf{I} + \mathbf{V}_o = (104 + j48) + (120 + j40)$$

$$\mathbf{V} = 224 + j88 = \underline{\mathbf{240.67\angle 21.45^\circ \quad (rms)}} \; \mathbf{V} = 224 + j88 = \underline{\mathbf{240.67\angle 21.45^\circ \quad (rms)}}$$

Complex Power

For the 20-Ω resistor,

$$\mathbf{V} = 20\,\mathbf{I} = 204 + j48 = 114.54\angle 24.8°$$

$$\mathbf{I} = 5.2 + j2.4 = 5.727\angle 24.8°$$

$$\mathbf{S} = \mathbf{V}\,\mathbf{I}^* = (114.54\angle 24.8°)(5.727\angle -24.8°) \qquad \mathbf{S} = \underline{\mathbf{656\ VA}}$$

For the (30 – j10)-Ω impedance,

$$\mathbf{V_o} = 120 + j40 = 126.5\angle 18.43°$$

$$\mathbf{I_1} = 3.2 + j2.4 = 4\angle 36.87°$$

$$\mathbf{S_1} = \mathbf{V_o}\,\mathbf{I_1^*} = (126.5\angle 18.43°)(4\angle -36.87°) \qquad \mathbf{S_1} = 506\angle -18.44° = \underline{\mathbf{480 - j160\ VA}}$$

For the (60 + j20)-Ω impedance, $\mathbf{I_2} = 2\angle 0°$

$$\mathbf{S_2} = \mathbf{V_o}\,\mathbf{I_2^*} = (126.5\angle 18.43°)(2\angle -0°) \quad \mathbf{S_2} = 253\angle 18.43° = \underline{\mathbf{240 + j80\ VA}}$$

The overall complex power supplied by the source is

$$\mathbf{S_T} = \mathbf{V}\,\mathbf{I}^* = (240.67\angle 21.45°)(5.727\angle -24.8°)$$

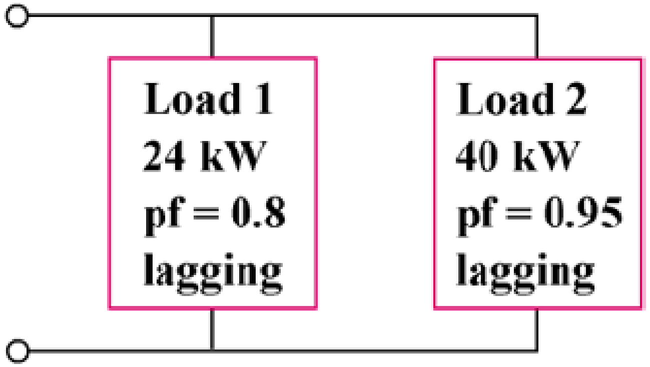
$$\mathbf{S_T} = 1378.3\angle -3.35° = \underline{\mathbf{1376 - j80\ VA}}$$

Complex Power

<u>Example:</u> A 120-V_{rms} 60-Hz source supplies two loads connected in parallel, as shown below.

(a) Find the power factor of the parallel combination.

(b) Calculate the value of the capacitance connected in parallel that will raise the power factor to unity.



<u>Solution:</u>

Chapter 11, Solution 74.

(a) $\theta_1 = \cos^{-1}(0.8) = 36.87°$ $\quad S_1 = \dfrac{P_1}{\cos\theta_1} = \dfrac{24}{0.8} = 30\text{ kVA}$

$Q_1 = S_1 \sin\theta_1 = (30)(0.6) = 18\text{ kVAR}$ $\quad \mathbf{S}_1 = 24 + j18\text{ kVA}$

$\theta_2 = \cos^{-1}(0.95) = 18.19°$ $\quad S_2 = \dfrac{P_2}{\cos\theta_2} = \dfrac{40}{0.95} = 42.105\text{ kVA}$

$Q_2 = S_2 \sin\theta_2 = 13.144\text{ kVAR}$ $\quad \mathbf{S}_2 = 40 + j13.144\text{ kVA}$

Complex Power

(a) $\theta_1 = \cos^{-1}(0.8) = 36.87^\circ \qquad S_1 = \dfrac{P_1}{\cos\theta_1} = \dfrac{24}{0.8} = 30\text{ kVA}$

$Q_1 = S_1 \sin\theta_1 = (30)(0.6) = 18\text{ kVAR} \qquad \mathbf{S}_1 = 24 + j18\text{ kVA}$

$\theta_2 = \cos^{-1}(0.95) = 18.19^\circ \qquad S_2 = \dfrac{P_2}{\cos\theta_2} = \dfrac{40}{0.95} = 42.105\text{ kVA}$

$Q_2 = S_2 \sin\theta_2 = 13.144\text{ kVAR} \qquad \mathbf{S}_2 = 40 + j13.144\text{ kVA}$

$\mathbf{S} = \mathbf{S}_1 + \mathbf{S}_2 = 64 + j31.144\text{ kVA}$

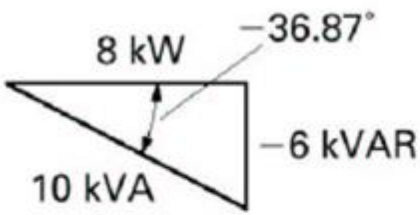
$\theta = \tan^{-1}\left(\dfrac{31.144}{64}\right) = 25.95^\circ \qquad pf = \cos\theta =$ **<u>0.8992 (lagging)</u>**

(b) $\theta_2 = 25.95^\circ, \quad \theta_1 = 0^\circ$

$Q_c = P[\tan\theta_2 - \tan\theta_1] = 64[\tan(25.95^\circ) - 0] = 31.144\text{ kVAR}$

$C = \dfrac{Q_c}{\omega V_{rms}^2} = \dfrac{31{,}144}{(2\pi)(60)(120)^2} =$ **<u>5.74 mF</u>**

Use of Power Triangles



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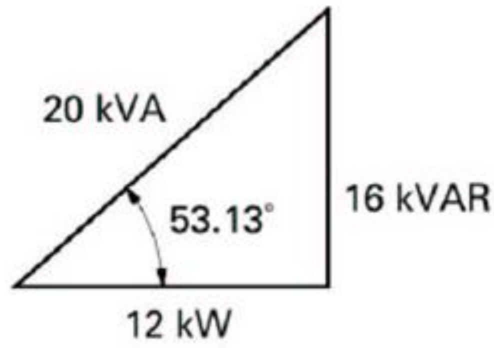
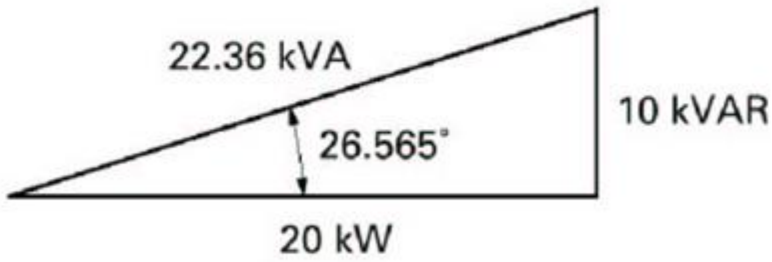


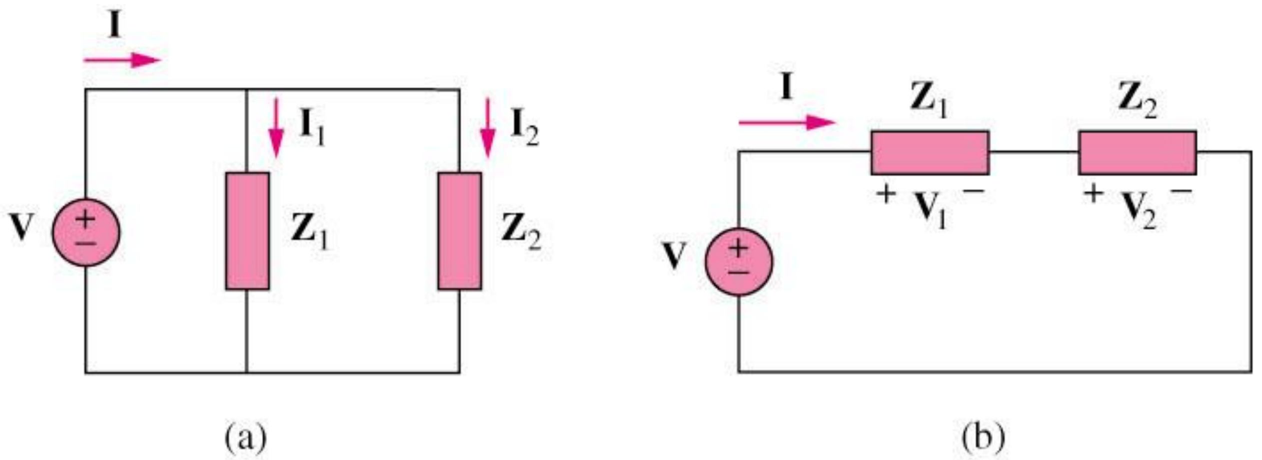
$$S = P + jQ = S_1 + S_2 = (P_1 + P_2) + j(Q_1 + Q_2)$$

=



Conservation of AC Power

➢The complex real, and reactive powers of the sources **equal** the respective **sums of** the complex, real, and reactive powers of the individual loads.

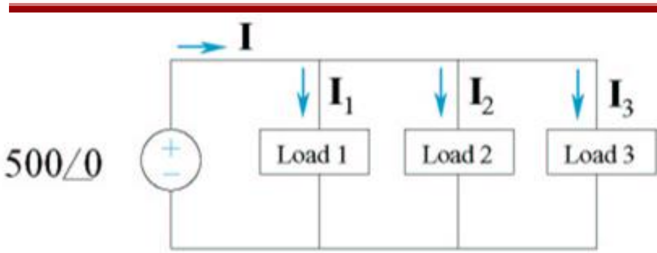


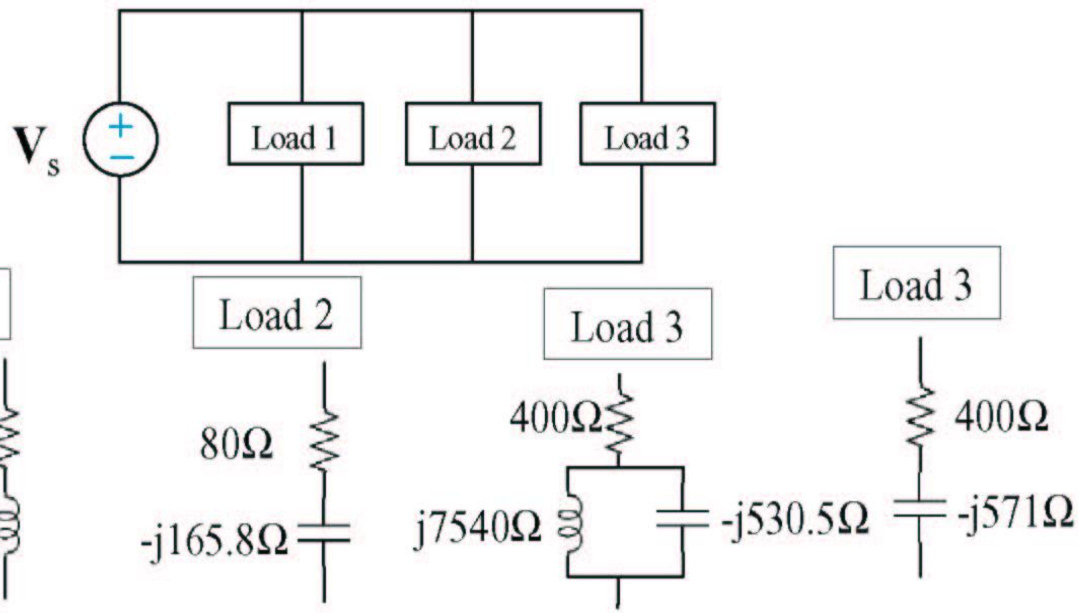
For parallel or series connection:

$$\mathbf{S} = \mathbf{V_1 I_1^*} + \mathbf{V_2 I_2^*} + \cdots + \mathbf{V_N I_N^*}$$

$$\mathbf{S} = \mathbf{S_1} + \mathbf{S_2} + \cdots + \mathbf{S_N}$$

Example of Complex Power Balance





$Z_1 = 12 + j5.65$
$\quad = 13.26\angle 25.21\ \Omega$

$Pf_1 = \cos(25.21)$
$\quad = 0.9$ lag

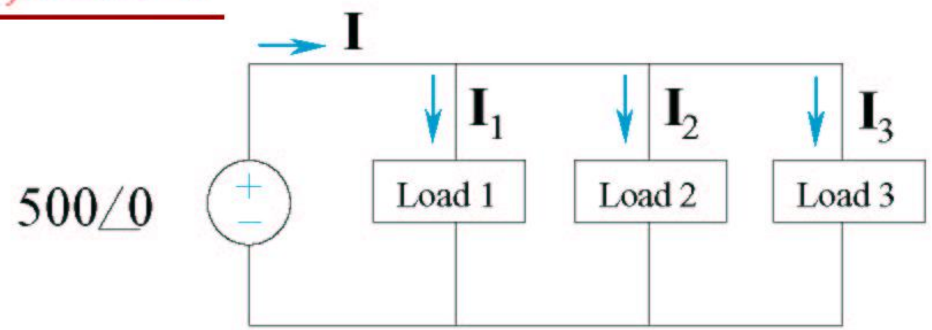
$Z_2 = 80 - j165.8$
$\quad = 184.1\angle -64.2\ \Omega$

$Pf_2 = \cos(-64.2)$
$\quad = 0.43$ lead

$Z_3 = 400 + -j571$
$\quad = 697\angle -55\ \Omega$

$Pf_1 = \cos(-55)$
$\quad = 0.57$ lead

Example, cont'd



$$\mathbf{I}_1 = \frac{500\angle 0^\circ}{13.26\angle 25.21^\circ} = 37.7\angle -25.21^\circ = 34.11 - j16.06$$

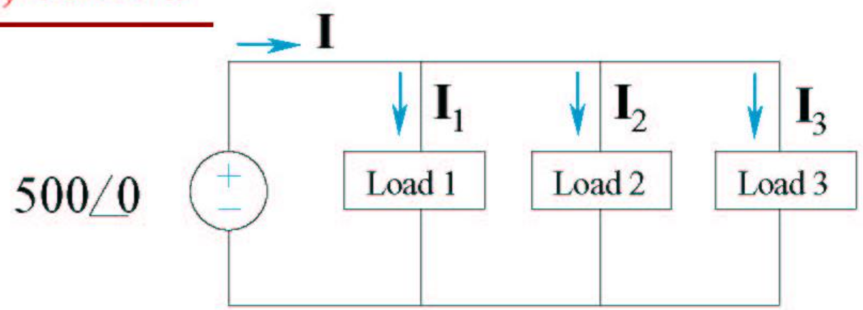
$$\mathbf{I}_2 = \frac{500\angle 0^\circ}{184.1\angle -64.2^\circ} = 2.72\angle 64.2^\circ = 1.18 + j2.45$$

$$\mathbf{I}_3 = \frac{500\angle 0^\circ}{697\angle -55^\circ} = 0.72\angle 55^\circ = 0.41 + j0.59$$

$$\mathbf{I} = \mathbf{I}_1 + \mathbf{I}_2 + \mathbf{I}_3 = 35.7 - j13.02 = 38\angle -20^\circ A$$

Combined Pf = cos(20) = 0.94 lag

Example, cont'd



$$\mathbf{S}_1 = \hat{V}\hat{I}_1^* = 500 \bullet 37.7\angle 25.21° = 18850\angle 25.21° = 17055 + j8029 \text{ VA}$$

$$\mathbf{S}_2 = \hat{V}\hat{I}_2^* = 500 \bullet 2.72\angle -64.2° = 1360\angle -64.2° = 592 - j1224 \text{ VA}$$

$$\mathbf{S}_3 = \hat{V}\hat{I}_3^* = 500 \bullet 0.72\angle -55° = 360\angle -55° = 207 - j295 \text{ VA}$$

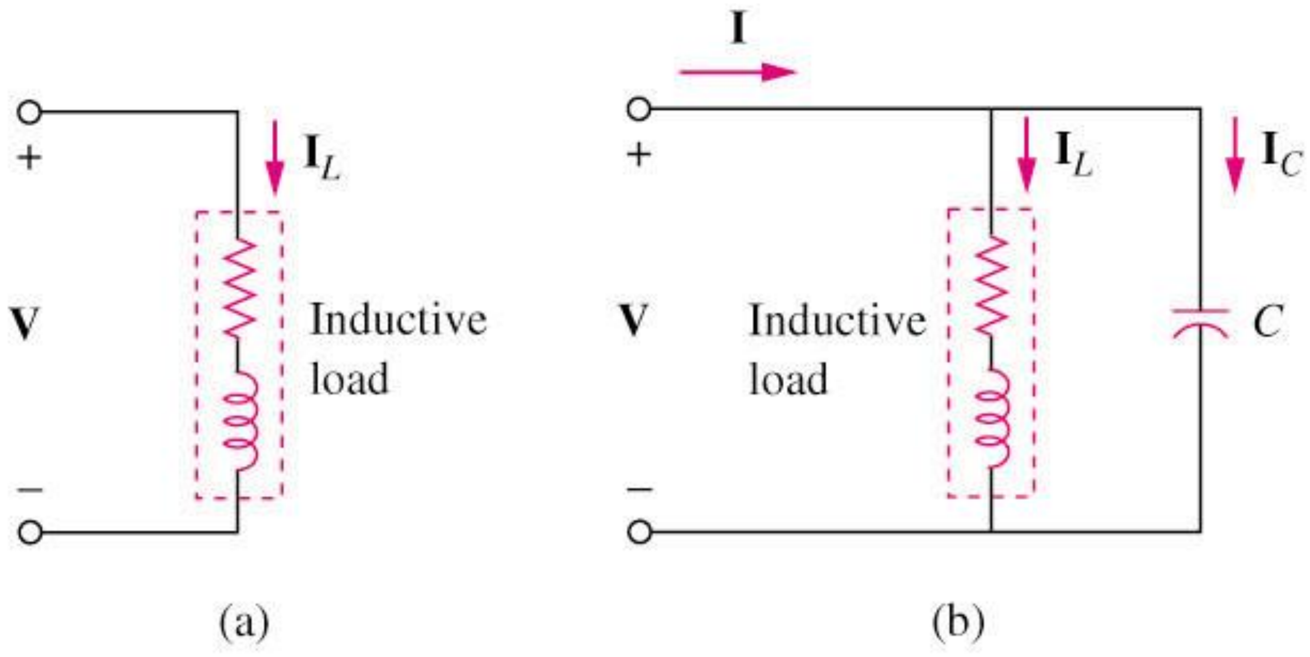
$$\mathbf{S} = \mathbf{S}_1 + \mathbf{S}_2 + \mathbf{S}_3 = 17854 - j6510 = 19000\angle -20° \text{ VA}$$

Check: $\mathbf{S} = \hat{V}\hat{I}^* = 500 \bullet 38\angle -20° = 19000\angle -20° \text{ VA}$

Complex power is Conserved

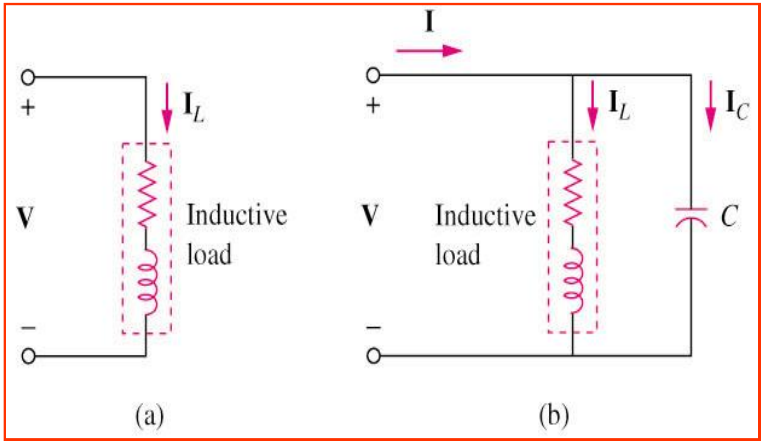
Power Factor Correction

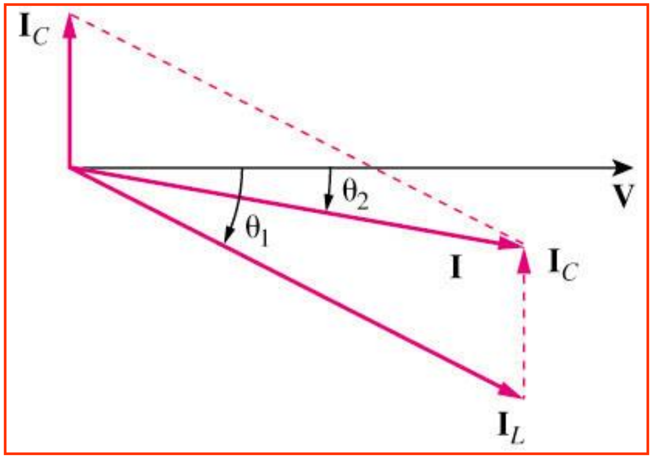
Power factor correction is the process of **increasing** the **power factor** without altering the voltage or current to the original load.

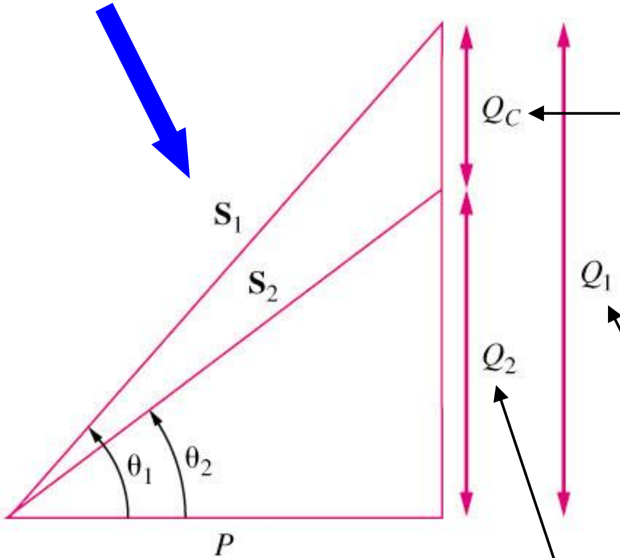


Power factor correction is necessary for economic reason.

Power Factor Correction







$$Q_c = Q_1 - Q_2$$
$$= P(\tan\theta_1 - \tan\theta_2)$$
$$= \omega C V^2_{rms}$$

$$Q_1 = S_1 \sin\theta_1$$
$$= P\tan\theta_1$$

$$P = S_1\cos\theta_1$$

$$Q_2 = P\tan\theta_2$$

$$C = \frac{Q_c}{\omega V^2_{rms}} = \frac{P(\tan\theta_1 - \tan\theta_2)}{\omega V^2_{rms}}$$

Power Factor Correction

➢ The process of increasing the power factor without altering the voltage or current to the original load is called power factor correction.

$$P_1 = P_2 = P \quad \text{Real power stays same}$$

$$P = S_1 \cos\theta_1 \quad Q_1 = S_1 \sin\theta_1 = P\tan\theta_1 \quad Q_2 = P\tan\theta_2$$

$$Q_C = Q_1 - Q_2 = P(\tan\theta_1 - \tan\theta_2)$$

$$C = \frac{Q_C}{\omega V_{rms}^{\;2}} = \frac{P(\tan\theta_1 - \tan\theta_2)}{\omega V_{rms}^{\;2}}$$

- The capacitance value needed to change the pf angle from θ_1 to θ_2 .
- Similarly the inductance value needed to change the pf angle from θ_1 to θ_2 for a capacitive load.

$$L = \frac{V_{rms}^{\;2}}{\omega Q_L}$$

Power Factor Correction

<u>Example:</u> Find the value of the capacitance needed to correct a load of 140 kVAR at 0.85 lagging pf to unity pf. The load is supplied by a 110 Volt (rms), 60 Hz line.

$$\text{pf} = 0.85 = \cos\theta \longrightarrow \theta = 31.79^\circ$$

$$Q = S\sin\theta \longrightarrow S = \frac{Q}{\sin\theta} = \frac{140}{\sin(31.79^\circ)} = 265.8\ \text{kVA}$$

$$P = S\cos\theta = 225.93\ \text{kW}$$

For $\text{pf} = 1 = \cos\theta_1 \longrightarrow \theta_1 = 0^\circ$

Since P remains the same,

$$P = P_1 = S_1\cos\theta_1 \longrightarrow S_1 = \frac{P_1}{\cos\theta_1} = 225.93$$

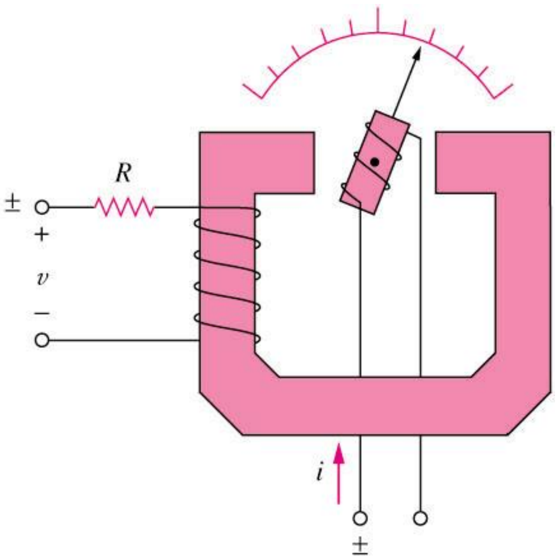
$$Q_1 = S_1\sin\theta_1 = 0$$

The difference between the new Q_1 and the old Q is Q_c.

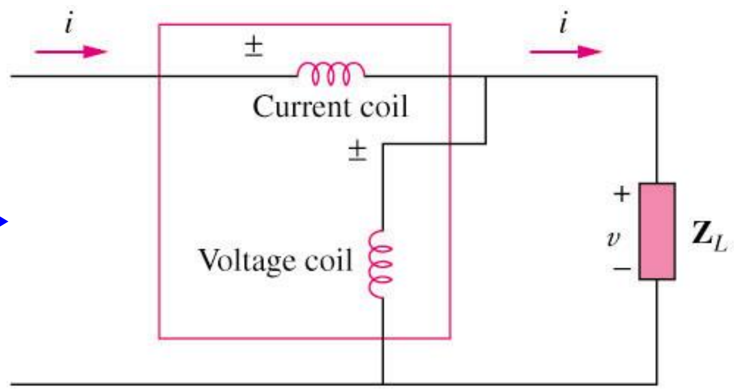
$$Q_c = 140\ \text{kVAR} = \omega C V_{rms}^2 \qquad C = \frac{140\times10^3}{(2\pi)(60)(110)^2} = \underline{\mathbf{30.69\ mF}}$$

Power Measurement

The wattmeter is the instrument for measuring the average power. Two coils are used, the high impedance Voltage coil and the low impedance Current coil.



The basic structure



Equivalent Circuit with load

If $v(t) = V_m \cos(\omega t + \theta_v)$ and $i(t) = I_m \cos(\omega t + \theta_i)$

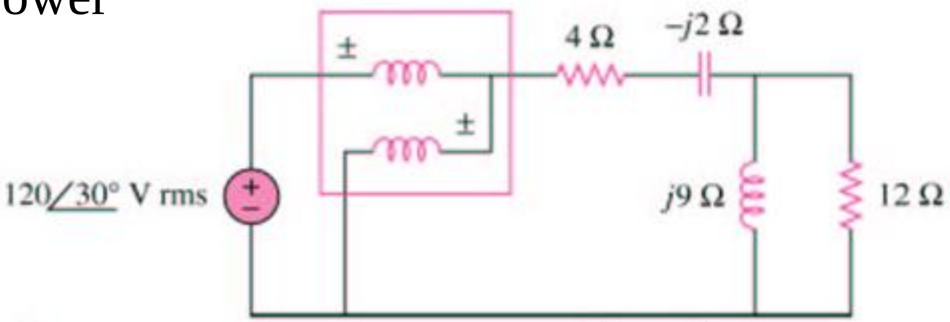
Wattmeter measures the average power given by:

$$P = |V_{rms}||I_{rms}| \cos(\theta_v - \theta_i) = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)$$

Power Measurement

<u>Example:</u> Find the wattmeter reading

The wattmeter measures the average power from the source



Let $\mathbf{Z}_1 = 4 - j2$

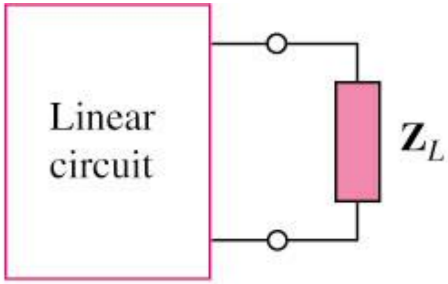
$$\mathbf{Z}_2 = 12 \| j9 = \frac{(12)(j9)}{12 + j9} = 4.32 + j5.76$$

$$\mathbf{Z} = \mathbf{Z}_1 + \mathbf{Z}_2 = 8.32 + j3.76 = 9.13\angle 24.32^\circ$$

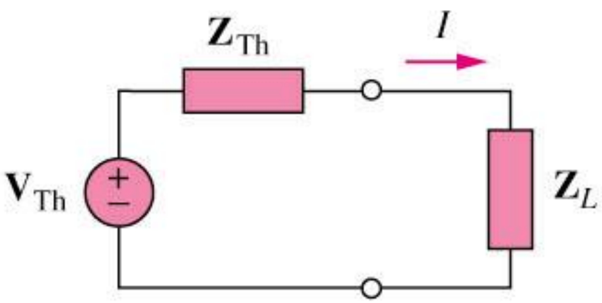
$$\mathbf{S} = \mathbf{V}\mathbf{I}^* = \frac{|\mathbf{V}|^2}{\mathbf{Z}^*} = \frac{(120)^2}{9.13\angle -24.32^\circ} = 1577.2\angle 24.32^\circ \text{ kVA}$$

$$\text{P} = |\mathbf{S}|\cos\theta = \underline{\mathbf{1437.2\ kW}}$$

Maximum Average Power Transfer



a) Circuit with a load



b) Thevenin Equivalent circuit

$$Z_{TH} = R_{TH} + jX_{TH}$$

$$Z_L = R_L + jX_L$$

The maximum average power can be transferred to the load if

$$\mathbf{X_L = -X_{TH} \text{ and } R_L = R_{TH}}$$

$$P_{max} = \frac{|V_{TH}|^2}{8R_{TH}}$$

If the load is purely real, then $R_L = \sqrt{R_{TH}^2 + X_{TH}^2} = |Z_{TH}|$

$$\boldsymbol{Z_L = R_{TH} - jX_{TH} = Z^*_{TH}}$$

Maximum Average Power Transfer

- Write the expression for average power associated with Z_L: $P(Z_L)$.

$$Z_L = R_L + jX_L$$

$$\text{Set } \frac{\partial P}{\partial R_L} = 0\text{: Solve for } R_L$$

$$\text{Set } \frac{\partial P}{\partial X_L} = 0\text{: Solve for } X_L$$

$$\mathbf{I} = \frac{\mathbf{V}_{th}}{Z_L + Z_{Th}} = I_m \angle \theta_i$$

$$I_m = \frac{|\mathbf{V}_{Th}|}{\sqrt{[(R_L + R_{Th})^2 + (X_L + X_{Th})^2]}}$$

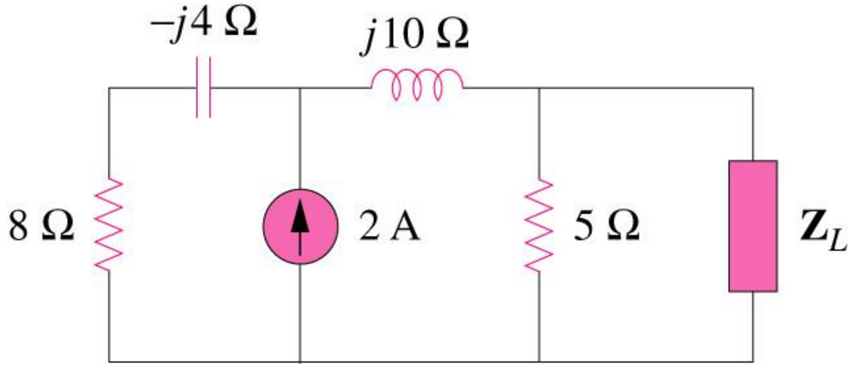
$$P = \frac{I_m^2 R_L}{2} = \frac{|V_{Th}|^2 R_L / 2}{(R_L + R_{Th})^2 + (X_L + X_{Th})^2}$$

$$\frac{\partial P}{\partial X_L} = 0 \rightarrow \boxed{X_L = -X_{Th}} \qquad \frac{\partial P}{\partial R_L} = 0 \rightarrow \boxed{R_L = R_{Th}}$$

Maximum Average Power Transfer

Example:

For the circuit shown below, find the load impedance Z_L that absorbs the maximum average power. Calculate that maximum average power.



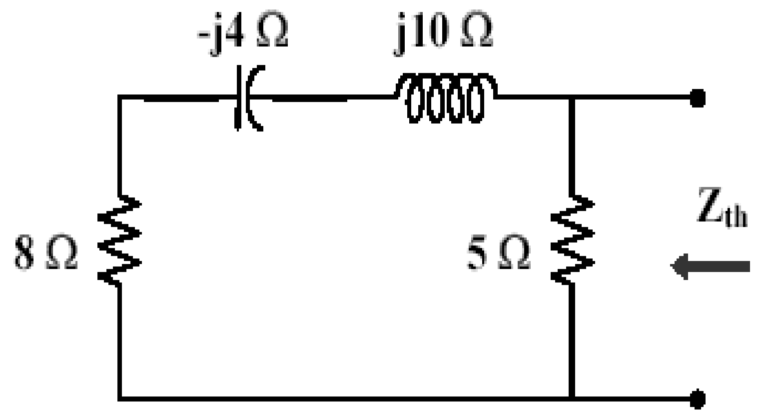
Answer:

$\boldsymbol{Z_L = 3.415 - j0.7317\ \Omega}$

$\boldsymbol{P_{max} = 1.429W}$

Maximum Average Power Transfer

Solution: We first obtain the Thevenin equivalent circuit across $\mathbf{Z}_L$. $\mathbf{Z}_{Th}$ is obtained from the circuit in Fig. (a).

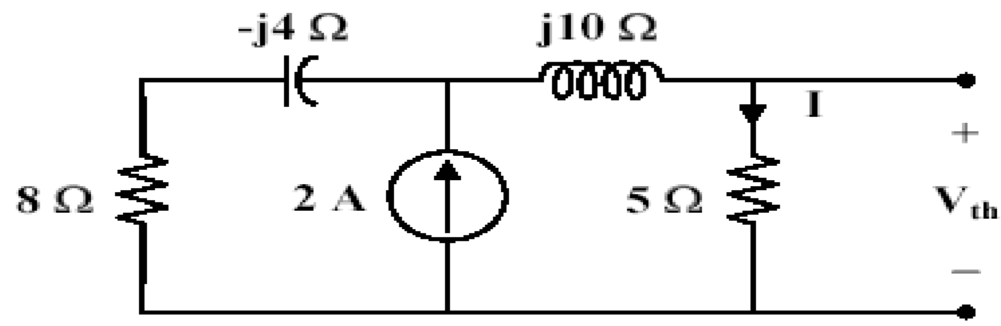


(a)

$$\mathbf{Z}_{Th} = 5 \,||\, (8 - j4 + j10) = \frac{(5)(8 + j6)}{13 + j6} = 3.415 + j0.7317$$

Maximum Average Power Transfer

$\mathbf{V}_{Th}$ is obtained from the circuit in Fig. (b).



(b)

By current division,

$$\mathbf{I} = \frac{8 - j4}{8 - j4 + j10 + 5}(2)$$

$$\mathbf{V}_{Th} = 5\mathbf{I} = \frac{(10)(8 - j4)}{13 + j6} = 6.25\angle -51.34°$$

$$\mathbf{Z}_L = \mathbf{Z}_{Th}^* = \underline{\mathbf{3.415 - j0.7317\ \Omega}}$$

$$P_{max} = \frac{|\mathbf{V}_{Th}|^2}{8R_L} = \frac{(6.25)^2}{(8)(3.415)} = \underline{\mathbf{1.429\ W}}$$

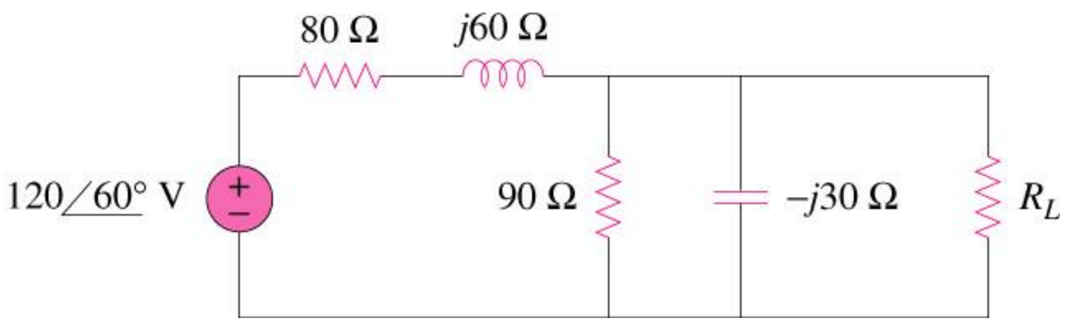
Maximum Average Power for Resistive Load

- When the load is PURELY RESISTIVE, the condition for maximum power transfer is:

$$R_L = \sqrt{R_{Th}^2 + X_{Th}^2} = |Z_{Th}|$$

- Now the maximum power can not be obtained from the P_{max} formula given before.

- **Example: Calculate the resistive load needed for maximum power transfer and the maximum average power.**



Solution: We first find $\mathbf{Z}_{Th}$ and $\mathbf{V}_{Th}$ across R_L.

Let $\mathbf{Z}_1 = 80 + j60$ and $\mathbf{Z}_2 = 90 \,||\, (-j30) = \dfrac{(90)(-j30)}{90 - j30} = 9(1 - j3)$

$$\mathbf{Z}_{Th} = \mathbf{Z}_1 \,||\, \mathbf{Z}_2 = \frac{(80 + j60)(9 - j27)}{80 + j60 + 9 - j27} = 17.181 - j24.57\ \Omega$$

Maximum Average Power for Resistive Load

$$\mathbf{V}_{Th} = \frac{\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2}(120\angle 60°) = \frac{(9)(1-j3)}{89+j33}(120\angle 60°)$$

$$\mathbf{V}_{Th} = 35.98\angle -31.91°$$

$$R_L = |\mathbf{Z}_{Th}| = \underline{\mathbf{30\ \Omega}}$$

The current through the load is

$$\mathbf{I} = \frac{\mathbf{V}_{Th}}{\mathbf{Z}_{Th} + R_L} = \frac{35.98\angle -31.91°}{47.181 - j24.57} = 0.6764\angle -4.4°$$

The maximum average power absorbed by R_L is

$$P_{max} = \frac{1}{2}|\mathbf{I}|^2 R_L = \frac{1}{2}(0.6764)^2(30) = \underline{\mathbf{6.863\ W}}$$

- Notice the way that the maximum power is calculated using the Thevenin Equivalent circuit.