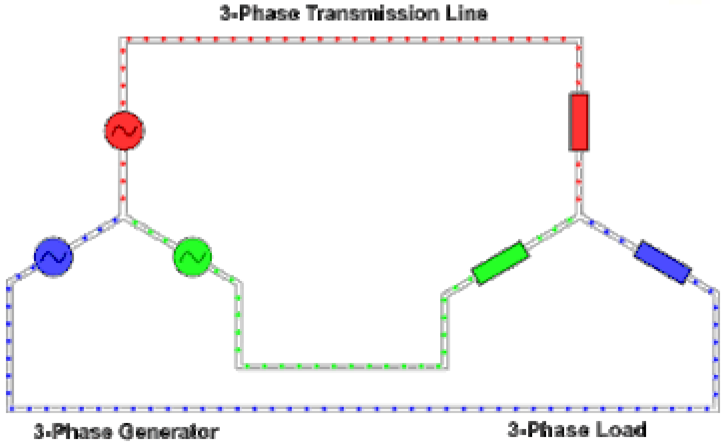
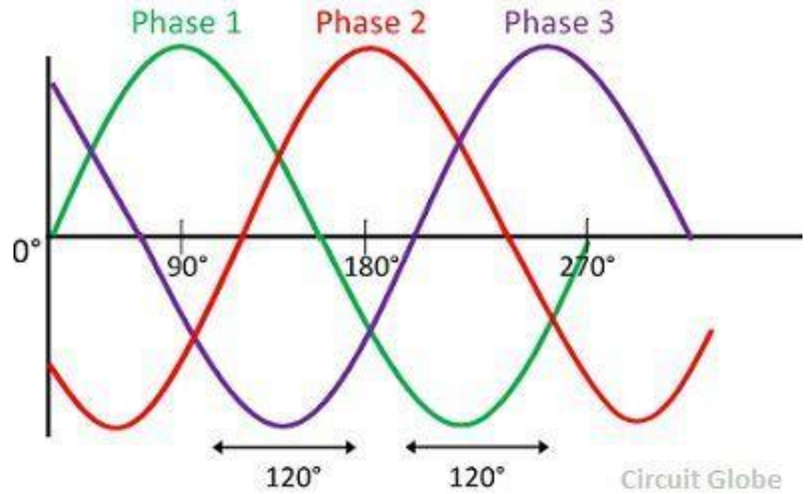
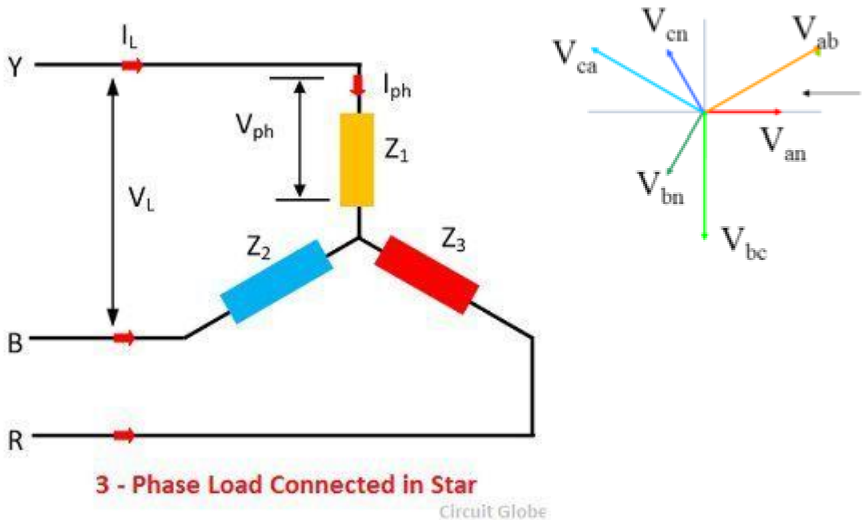
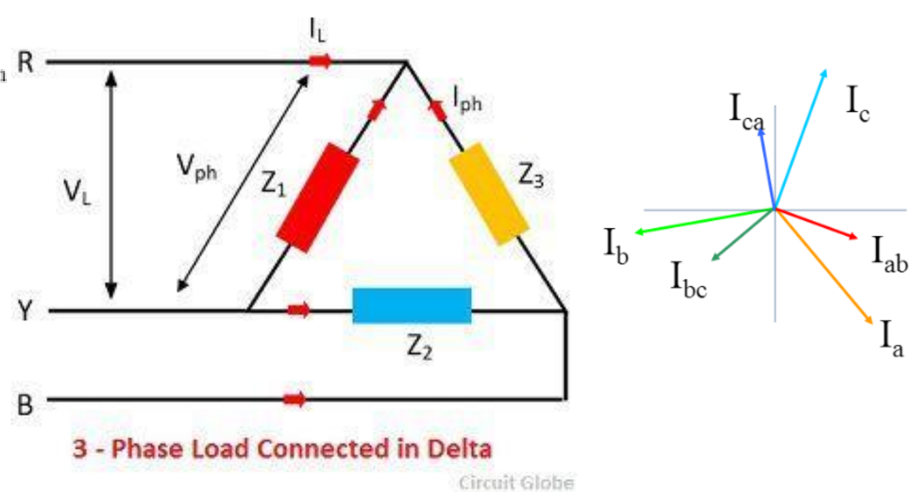
Topic 9: 3-Phase Circuit Analysis









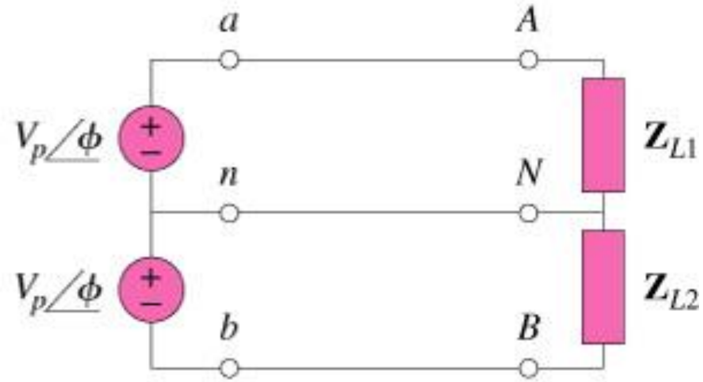
Three phase Circuits

- An AC generator designed to develop a single sinusoidal voltage for each rotation of the shaft (rotor) is referred to as a **single-phase AC generator.**
- If the number of coils on the rotor is increased in a specified manner, the result is a **Polyphase AC generator**, which develops more than one AC phase voltage per rotation of the rotor
- In general, three-phase systems are preferred over single-phase systems for the transmission of power for many reasons.

1. Thinner conductors can be used to transmit the same kVA at the same voltage, which reduces the amount of copper required (typically about 25% less).
2. The lighter lines are easier to install, and the supporting structures can be less massive and farther apart.
3. Three-phase equipment and motors have preferred running and starting characteristics compared to single-phase systems because of a more even flow of power to the transducer than can be delivered with a single-phase supply.
4. In general, most larger motors are three phase because they are essentially self-starting and do not require a special design or additional starting circuitry.

Single-Phase, Two-Phase, Three phase Circuits

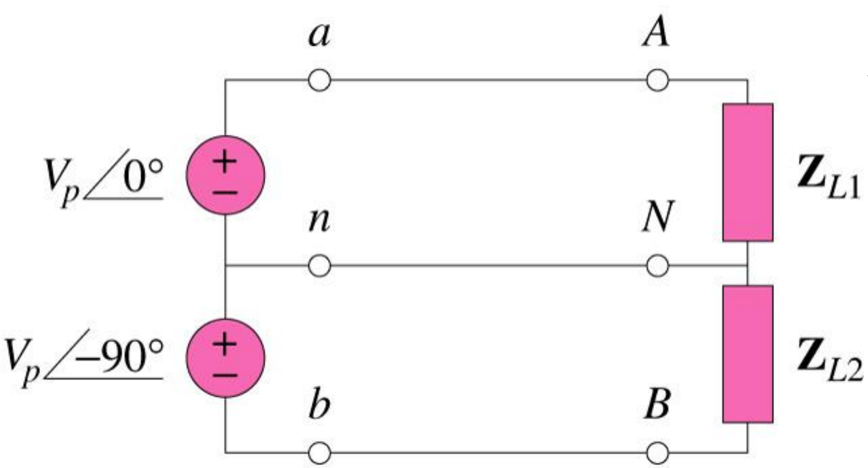
(a)



(b)

a) Single phase systems two-wire type

b) Single phase systems three-wire type. Allows connection to both 120 V and 240 V.

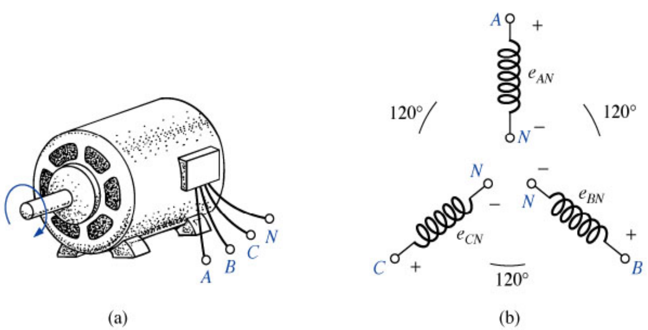


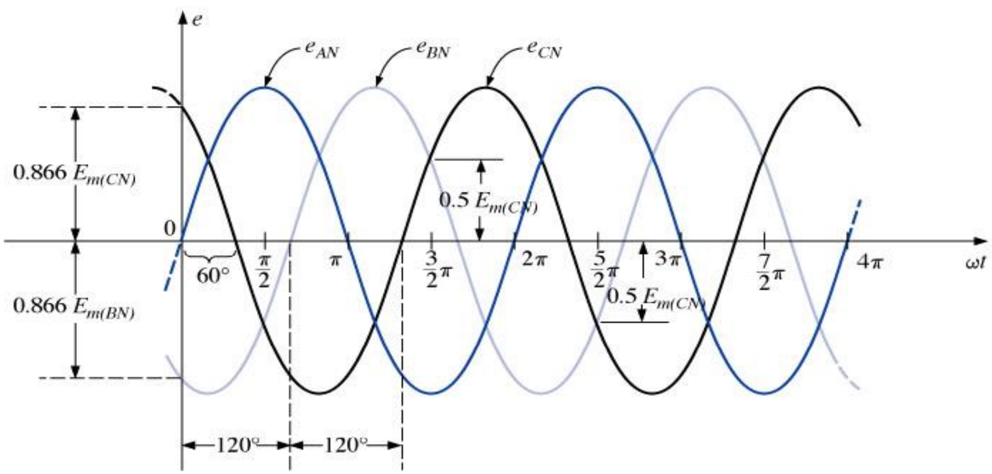
Two-phase three-wire system.

The AC sources operate at different phases.

Three-phase Generator

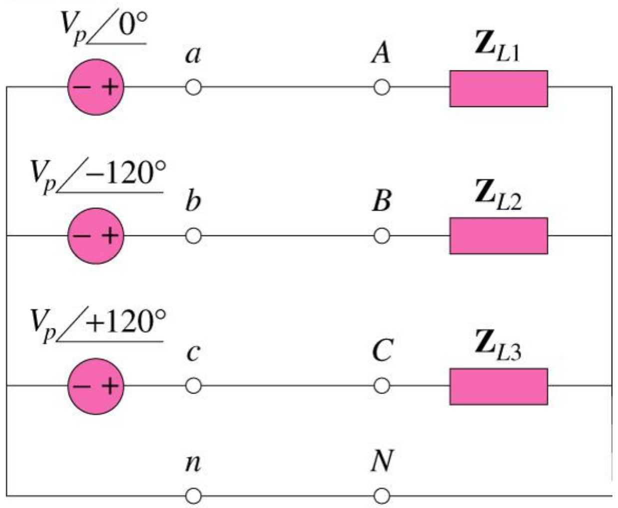
- The three-phase generator has three induction coils placed 120° apart on the stator.
- The three coils have an equal number of turns, the voltage induced across each coil will have the same peak value, shape and frequency.





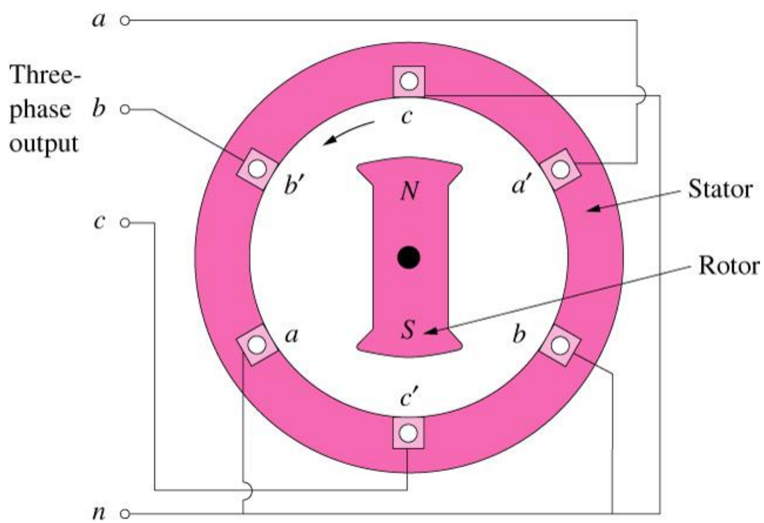
Balanced Three-phase Voltages

Three-phase four-wire system

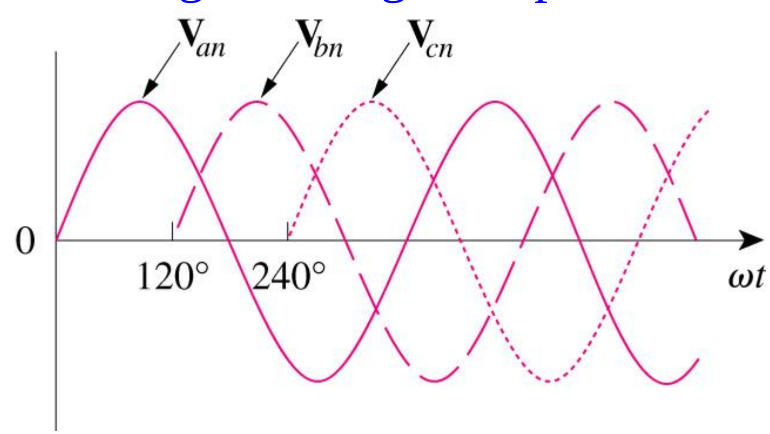


Neutral Wire

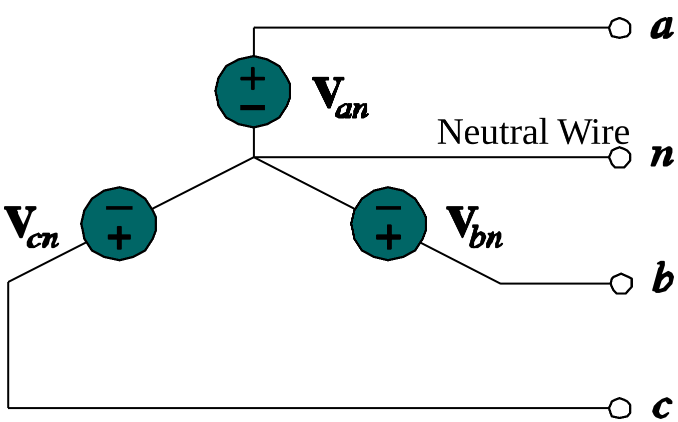
A Three-phase Generator



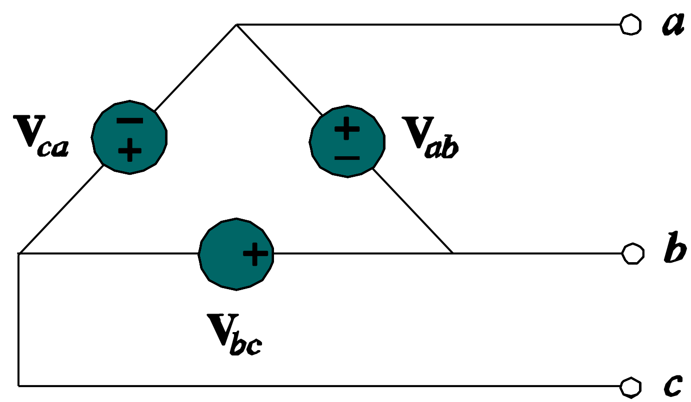
Voltages having 120° phase difference



Balanced Three phase Voltages

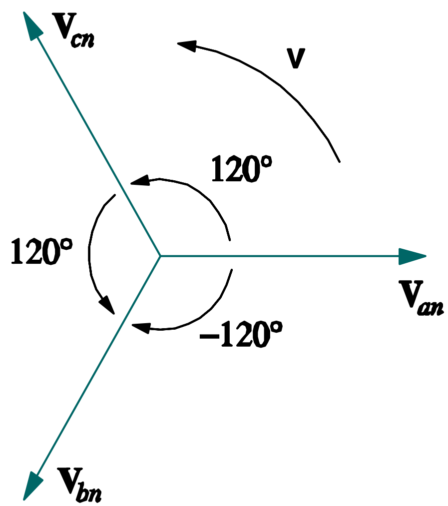


a) Wye Connected Source

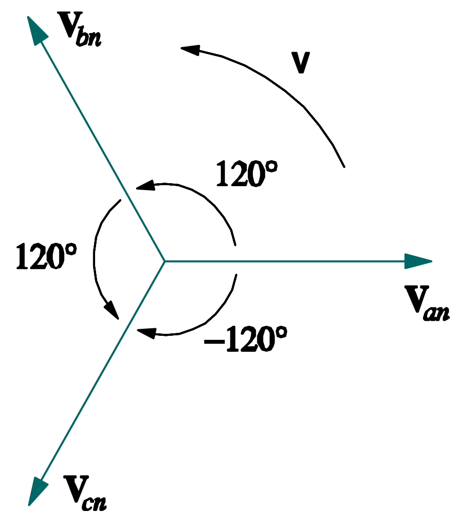


b) Delta Connected Source

$$V_{an} = V_p \angle 0°$$
$$V_{bn} = V_p \angle -120°$$
$$V_{cn} = V_p \angle -240°$$



a) abc or positive sequence

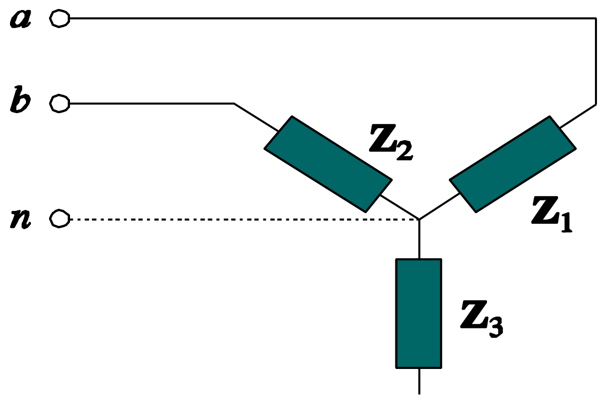


$$V_{an} = V_p \angle 0°$$
$$V_{bn} = V_p \angle +120°$$
$$V_{cn} = V_p \angle +240°$$

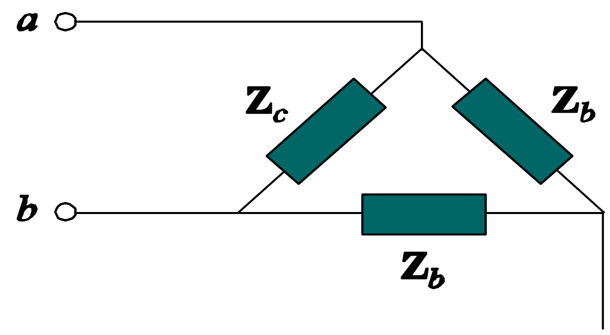
b) acb or negative sequence

Balanced Three phase Loads

- A Balanced load has equal impedances on all the phases



a) Wye-connected load



b) Delta-connected load

Balanced Impedance Conversion:

Conversion of Delta circuit to Wye or Wye to Delta.

$$Z_Y = Z_1 = Z_2 = Z_3$$

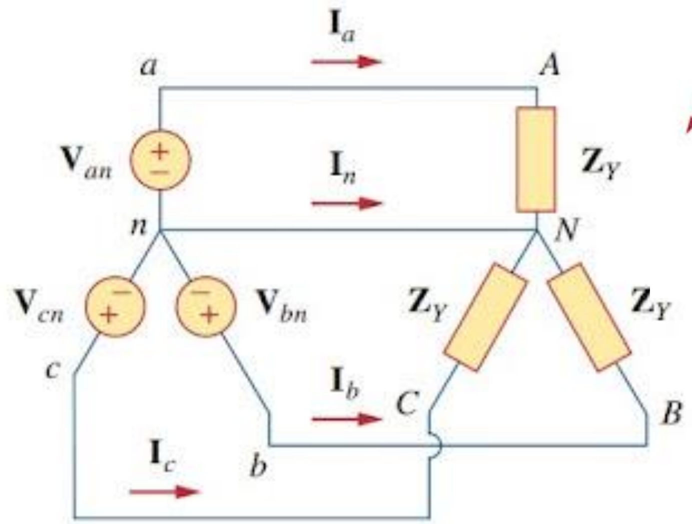
$$Z_\Delta = Z_a = Z_b = Z_c$$

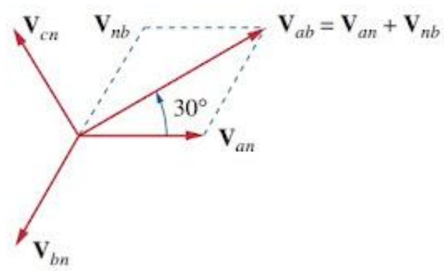
$$Z_\Delta = 3Z_Y \qquad Z_Y = \frac{1}{3} Z_\Delta$$

Three phase Connections Y-Y Connection

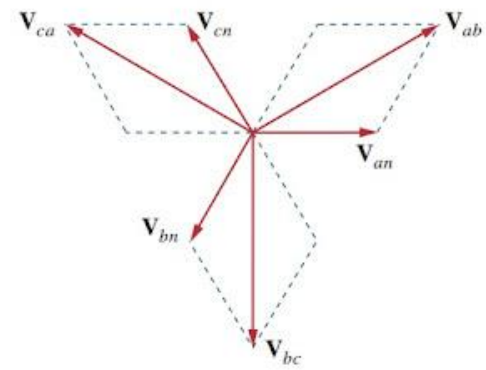
- Both the three phase source and the three phase load can be connected Wye Wye.

- Y-Y connection





(a)



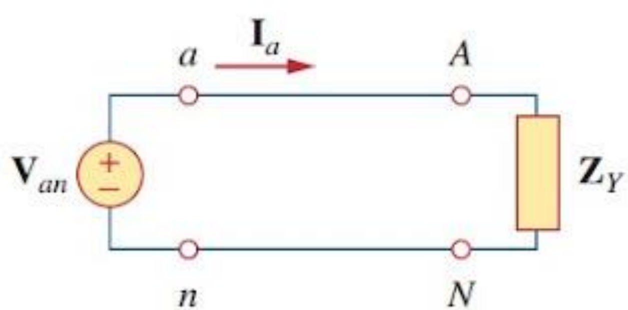
b)

$$\mathbf{I}_a = \frac{\mathbf{V}_{an}}{\mathbf{Z}_Y}, \qquad \mathbf{I}_b = \frac{\mathbf{V}_{bn}}{\mathbf{Z}_Y} = \frac{\mathbf{V}_{an}\angle -120^\circ}{\mathbf{Z}_Y} = \mathbf{I}_a\angle -120^\circ$$

$$\mathbf{I}_c = \frac{\mathbf{V}_{cn}}{\mathbf{Z}_Y} = \frac{\mathbf{V}_{an}\angle -240^\circ}{\mathbf{Z}_Y} = \mathbf{I}_a\angle -240^\circ$$

$$\mathbf{V}_{an} = V_p\angle 0^\circ$$

$$\mathbf{V}_{bn} = V_p\angle -120^\circ, \qquad \mathbf{V}_{cn} = V_p\angle +120^\circ$$

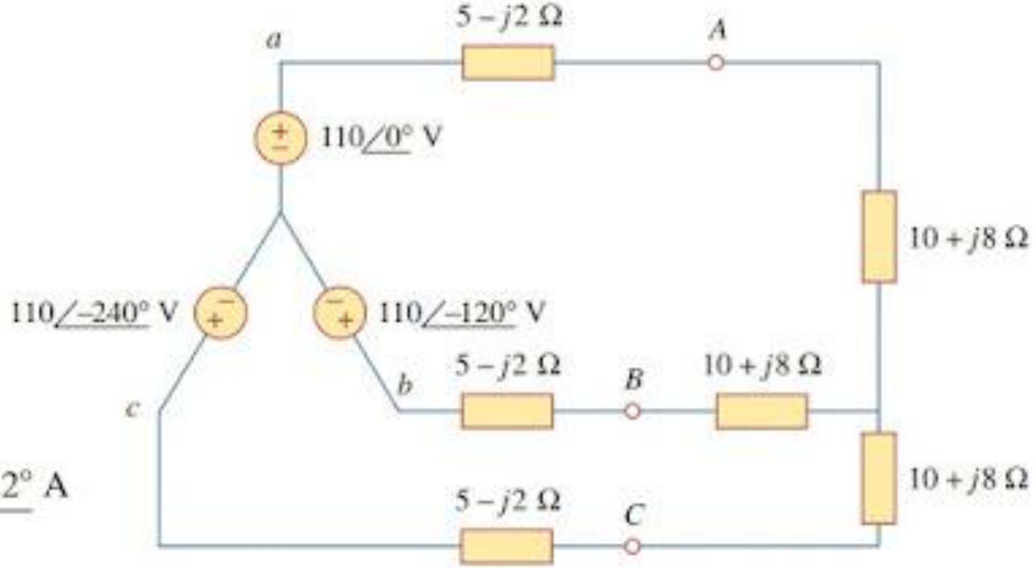


$$\mathbf{I}_a = \frac{\mathbf{V}_{an}}{\mathbf{Z}_Y}$$

$$V_L = \sqrt{3}V_p$$

Three phase Connections Y-Y Connection

➢ Example: Calculate the line currents in the three-wire Y-Y system shown below.



$$\mathbf{I}_a = \frac{110\angle 0^\circ}{16.155\angle 21.8^\circ} = 6.81\angle -21.8^\circ \text{ A}$$

$$\mathbf{I}_b = \mathbf{I}_a\angle -120^\circ = 6.81\angle -141.8^\circ \text{ A}$$

$$\mathbf{I}_c = \mathbf{I}_a\angle -240^\circ = 6.81\angle -261.8^\circ \text{ A} = 6.81\angle 98.2^\circ \text{ A}$$

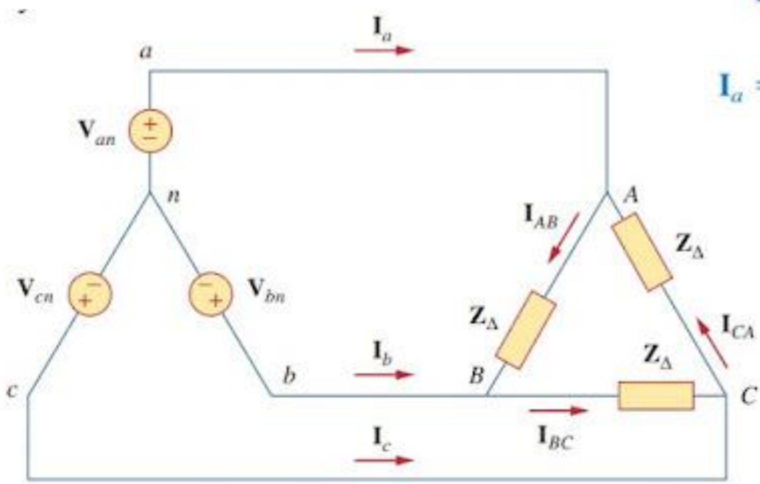
Three phase Connections Y-Δ Connection

➢ Both the three phase source and the three phase load can be connected Wye Wye.

$$\mathbf{I}_{AB} = \frac{\mathbf{V}_{AB}}{\mathbf{Z}_\Delta}, \qquad \mathbf{I}_{BC} = \frac{\mathbf{V}_{BC}}{\mathbf{Z}_\Delta}, \qquad \mathbf{I}_{CA} = \frac{\mathbf{V}_{CA}}{\mathbf{Z}_\Delta}$$

➢ Y-Δ connection

$$\mathbf{I}_a = \mathbf{I}_{AB} - \mathbf{I}_{CA}, \qquad \mathbf{I}_b = \mathbf{I}_{BC} - \mathbf{I}_{AB}, \qquad \mathbf{I}_c = \mathbf{I}_{CA} - \mathbf{I}_{BC}$$

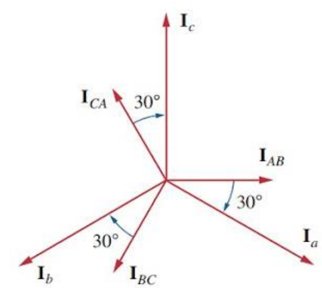


$$\mathbf{I}_a = \mathbf{I}_{AB} - \mathbf{I}_{CA} = \mathbf{I}_{AB}(1 - 1\angle{-240^\circ})$$
$$= \mathbf{I}_{AB}(1 + 0.5 - j0.866) = \mathbf{I}_{AB}\sqrt{3}\angle{-30^\circ}$$

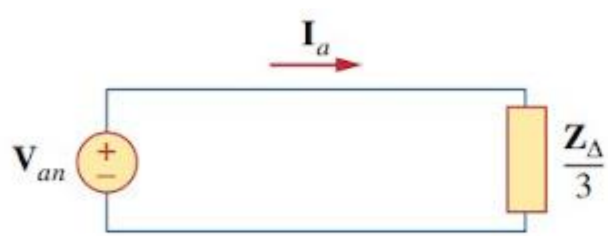
$$I_L = \sqrt{3} I_p$$

$$I_L = |\mathbf{I}_a| = |\mathbf{I}_b| = |\mathbf{I}_c|$$

$$I_p = |\mathbf{I}_{AB}| = |\mathbf{I}_{BC}| = |\mathbf{I}_{CA}|$$



$$\mathbf{V}_{an} = V_p\angle{0^\circ}$$
$$\mathbf{V}_{bn} = V_p\angle{-120^\circ}, \qquad \mathbf{V}_{cn} = V_p\angle{+120^\circ}$$



$$\mathbf{Z}_Y = \frac{\mathbf{Z}_\Delta}{3}$$

$$\mathbf{V}_{ab} = \sqrt{3}V_p\angle{30^\circ} = \mathbf{V}_{AB}, \qquad \mathbf{V}_{bc} = \sqrt{3}V_p\angle{-90^\circ} = \mathbf{V}_{BC}$$
$$\mathbf{V}_{ca} = \sqrt{3}V_p\angle{-150^\circ} = \mathbf{V}_{CA}$$

Three phase Connections Y-Δ Connection

➢ Example: A balanced *abc*-sequence Y-connected source with $\mathbf{V}_{an} = 100\angle 10^\circ$ V is connected to a Δ-connected balanced load (8 + j4) Ω per phase. Calculate the phase and line currents.

The load impedance is $\mathbf{Z}_\Delta = 8 + j4 = 8.944\angle 26.57^\circ\ \Omega$

If the phase voltage $\mathbf{V}_{an} = 100\angle 10^\circ$, then the line voltage is

$$\mathbf{V}_{ab} = \mathbf{V}_{an}\sqrt{3}\angle 30^\circ = 100\sqrt{3}\angle 10^\circ + 30^\circ = \mathbf{V}_{AB} \qquad \mathbf{V}_{AB} = 173.2\angle 40^\circ \text{ V}$$

The phase currents are

$$\mathbf{I}_{AB} = \frac{\mathbf{V}_{AB}}{\mathbf{Z}_\Delta} = \frac{173.2\angle 40^\circ}{8.944\angle 26.57^\circ} = 19.36\angle 13.43^\circ \text{ A}$$

$$\mathbf{I}_{BC} = \mathbf{I}_{AB}\angle -120^\circ = 19.36\angle -106.57^\circ \text{ A}$$

$$\mathbf{I}_{CA} = \mathbf{I}_{AB}\angle +120^\circ = 19.36\angle 133.43^\circ \text{ A}$$

The line currents are

$$\mathbf{I}_a = \mathbf{I}_{AB}\sqrt{3}\angle -30^\circ = \sqrt{3}(19.36)\angle 13.43^\circ - 30^\circ = 33.53\angle -16.57^\circ \text{ A}$$

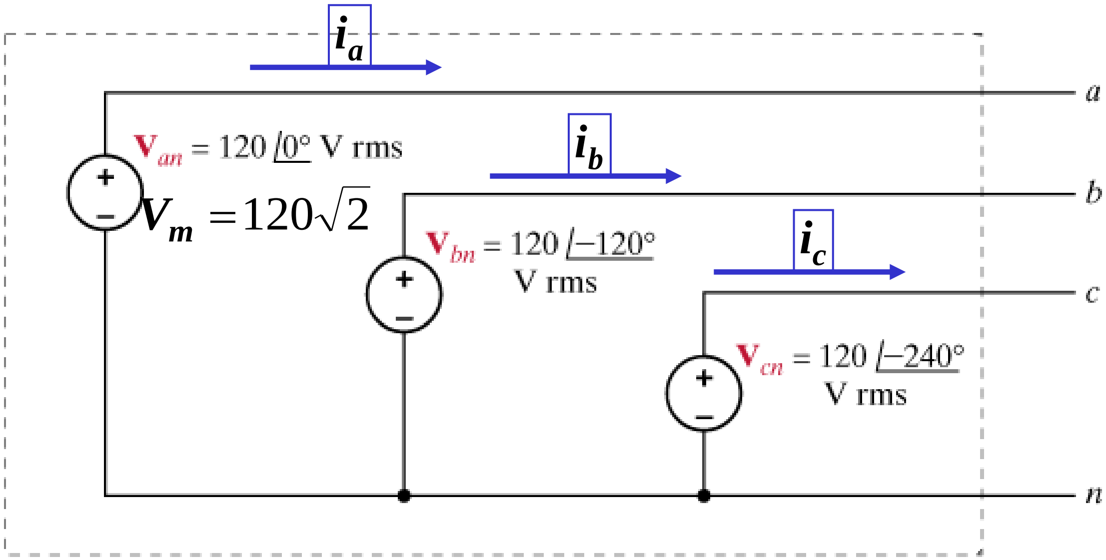
$$\mathbf{I}_b = \mathbf{I}_a\angle -120^\circ = 33.53\angle -136.57^\circ \text{ A}$$

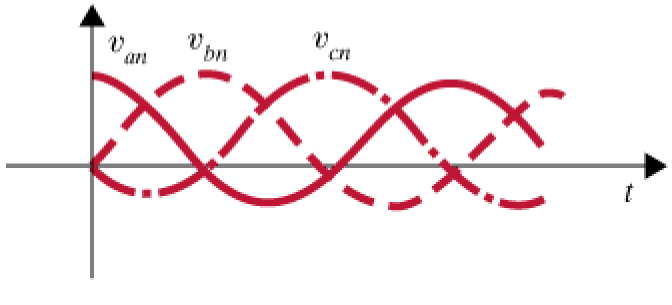
$$\mathbf{I}_c = \mathbf{I}_a\angle +120^\circ = 33.53\angle 103.43^\circ \text{ A}$$

Alternatively, using single-phase analysis,

$$\mathbf{I}_a = \frac{\mathbf{V}_{an}}{\mathbf{Z}_\Delta/3} = \frac{100\angle 10^\circ}{2.981\angle 26.57^\circ} = 33.54\angle -16.57^\circ \text{ A}$$

THREE PHASE CIRCUITS





Instantaneous Phase Voltages

$$v_{an}(t) = V_m \cos(\omega t)(V)$$

$$v_{bn}(t) = V_m \cos(\omega t - 120°)(V)$$

$$v_c(t) = V_m \cos(\omega t - 240°)(V)$$

Balanced Phase Currents

$$i_a(t) = I_m \cos(\omega t - \theta)$$

$$i_b(t) = I_m \cos(\omega t - \theta - 120°)$$

$$i_c(t) = I_m \cos(\omega t - \theta - 240°)$$

Instantaneous power

$$p(t) = v_{an}(t) i_a(t) + v_{bn}(t) i_b(t) + v_{cn}(t) i_c(t)$$

Theorem

For a balanced three phase circuit the instantaneous power is constant

$$p(t) = 3\frac{V_m I_m}{2}\cos\theta \quad W$$

Proof of Theorem

For a balanced three phase circuit the instantaneous power is constant

$$p(t) = 3\frac{V_m I_m}{2}\cos\theta\,(W)$$

Instantaneous power

$$p(t) = v_{an}(t)i_a(t) + v_{bn}(t)i_b(t) + v_{cn}(t)i_c(t)$$

$$p(t) = V_m I_m \begin{bmatrix} \cos\omega t\cos(\omega t - \theta) \\ + \cos(\omega t - 120)\cos(\omega t - 120 - \theta) \\ + \cos(\omega t - 240)\cos(\omega t - 240 - \theta) \end{bmatrix}$$

$$\cos\alpha\cos\beta = \frac{1}{2}\left[\cos(\alpha - \beta) + \cos(\alpha + \beta)\right]$$

$$p(t) = V_m I_m \begin{bmatrix} 3\cos\theta + \cos(2\omega t - \theta) \\ + \cos(2\omega t - 240 - \theta) \\ + \cos(2\omega t - 480 - \theta) \end{bmatrix}$$

$$\phi = \omega t - \theta$$

$$\cos(\phi - 240) = \cos(\phi + 120)$$

$$\cos(\phi - 480) = \cos(\phi - 120)$$

$$\cos(120) = -0.5$$

Lemma

$$\cos\phi + \cos(\phi - 120) + \cos(\phi + 120) = 0$$

Proof

$$\cos\phi = \cos\phi$$

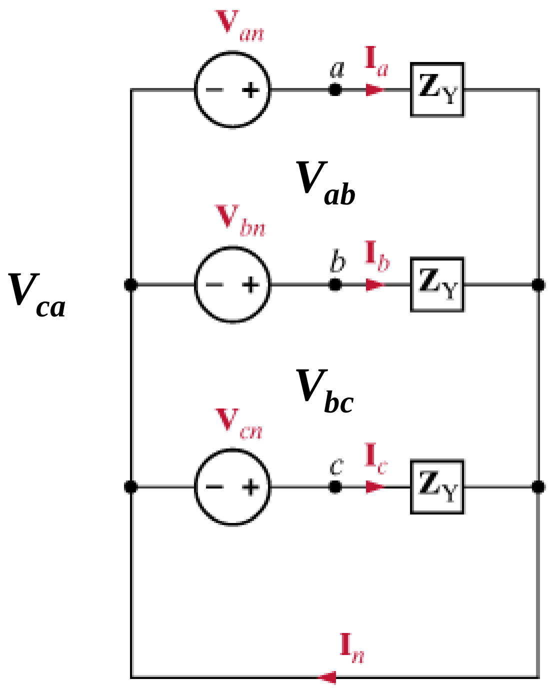
$$\cos(\phi - 120) = \cos\phi\cos(120) + \sin\phi\sin(120)$$

$$\cos(\phi + 120) = \cos\phi\cos(120) - \sin\phi\sin(120)$$

$$\boxed{\cos\phi + \cos(\phi - 120) + \cos(\phi + 120) = 0}$$

SOURCE/LOAD CONNECTIONS

BALANCED Y-Y CONNECTION

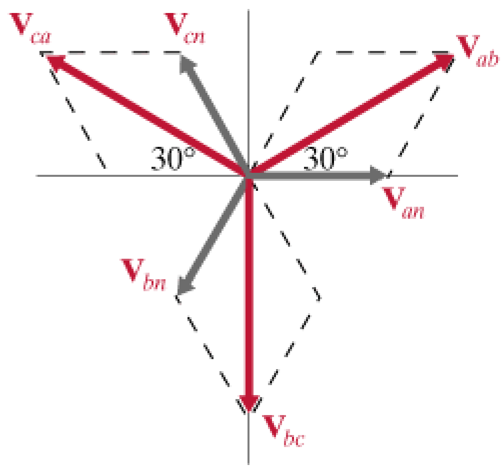


$$V_{an} = |V_p| \angle 0°$$

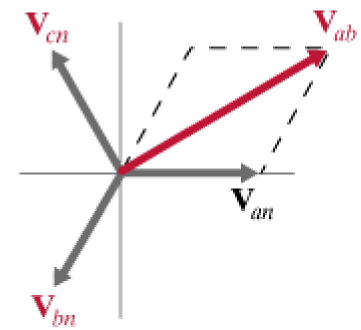
$$V_{bn} = |V_p| \angle -120°$$

$$V_{cn} = |V_p| \angle 120°$$

Positive sequence phase voltages



Line voltages



$$V_{ab} = V_{an} - V_{bn}$$

$$= |V_p| \angle 0° - |V_p| \angle -120°$$

$$= |V_p| \left(1 - (\cos 120 - j \sin 120)\right)$$

$$= |V|_p - |V_p| \left(\frac{1}{2} - j \frac{\sqrt{3}}{2} \right)$$

$$= \sqrt{3} |V_p| \angle 30°$$

$$V_{bc} = \sqrt{3} |V_p| \angle -90°$$

$$V_{ca} = \sqrt{3} |V_p| \angle -210°$$

$$V_L = \sqrt{3} |V_p| = \text{Line Voltage}$$

$$I_a = \frac{V_{an}}{Z_Y}; I_b = \frac{V_{bn}}{Z_Y}; I_c = \frac{V_{cn}}{Z_Y}$$

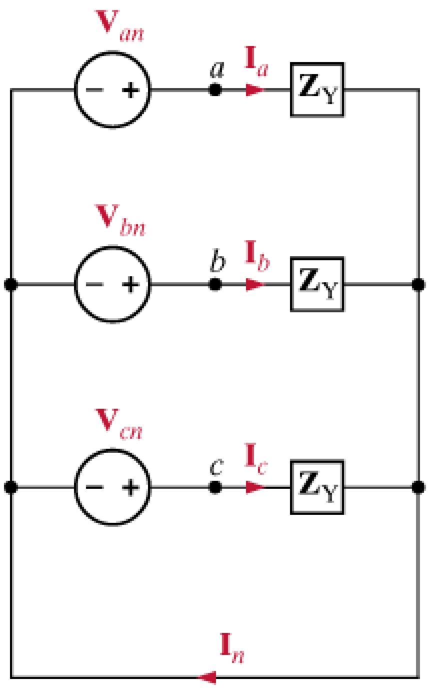
$$I_a = |I_L| \angle \theta°; I_b = |I_L| \angle \theta - 120°; I_c = |I_L| \angle \theta + 120°$$

$$I_a + I_b + I_c = I_n = 0$$

For this balanced circuit it is enough to analyze one phase

<u>Example:</u> For an abc sequence, balanced Y-Y three phase circuit $V_{ab} = 208\angle -30°$

Determine the phase voltages.



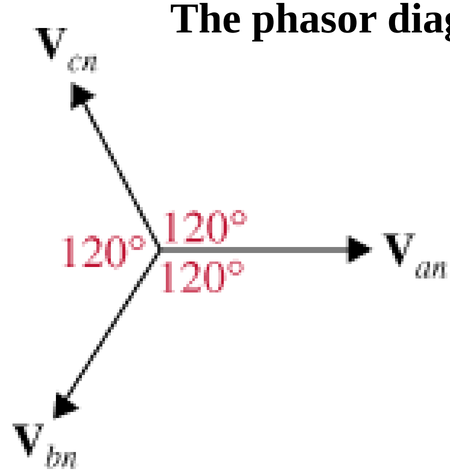
Balanced Y - Y

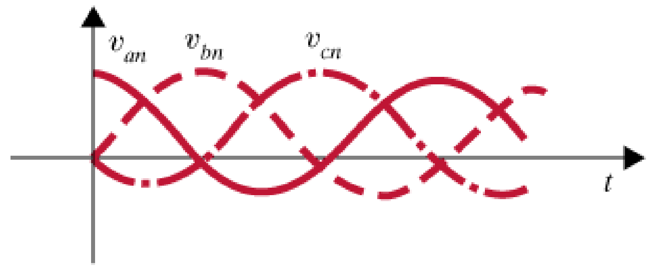
$$V_{an} = 120\angle -60°$$

$$V_{bn} = 120\angle -180°$$

$$V_{an} = 120\angle 60°$$

The phasor diagram could be rotated by any angle





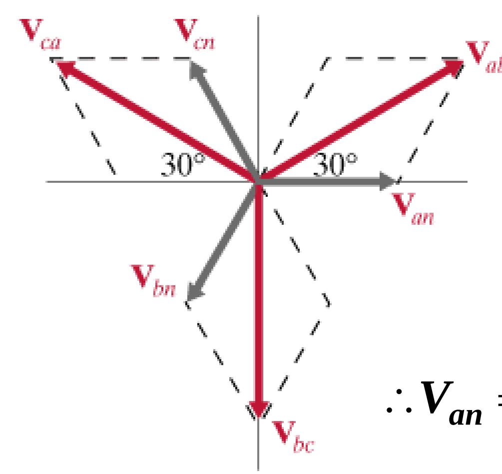
Positive sequence a-b-c

$$V_{an} = |V_p|\angle 0°$$

$$V_{bn} = |V_p|\angle -120°$$

$$V_{cn} = |V_p|\angle 120°$$

Positive sequence phase voltages



$$V_{ab} = \sqrt{3}|V_p|\angle 30°$$

V_{an} lags V_{ab} by 30°

$$V_{ab} = 208\angle -30°$$

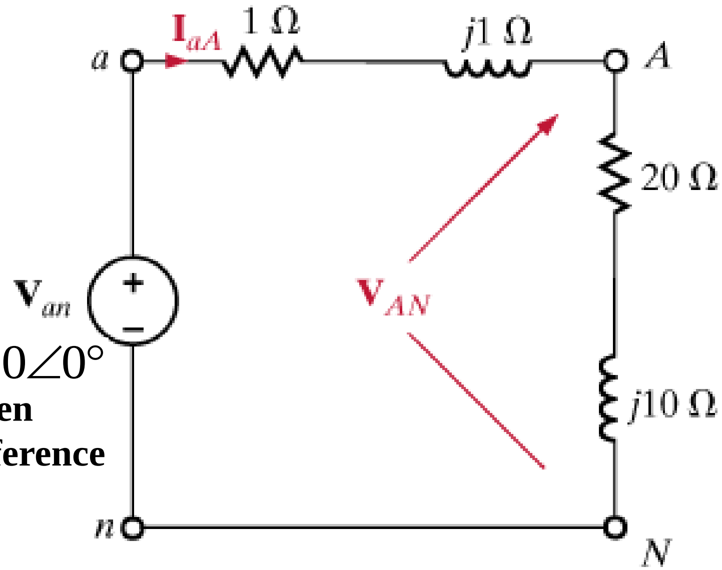
$$\therefore V_{an} = \frac{|V_{ab}|}{\sqrt{3}}\angle(-30° - 30°)$$

Relationship between phase and line voltages

<u>Example:</u> For an abc sequence, balanced Y - Y three phase circuit

$$\text{source } |V_{phase}| = 120(V)_{rms}, \; Z_{line} = 1 + j1\Omega, \; Z_{phase} = 20 + j10\Omega$$

Determine line currents and load voltages.



$120\angle 0°$

Chosen as reference

$$I_{aA} = \frac{V_{an}}{21 + j11} = \frac{120\angle 0°}{23.71\angle 27.65°}$$

$$= 5.06\angle -27.65°(A)rms$$

Because circuit is balanced data on any one phase are sufficient

$$I_{bB} = 5.06\angle -120 - 27.65°(A)rms$$

$$I_{cC} = 5.06\angle 120 - 27.65°(A)rms$$

$$V_{an} = 120\angle 0°$$

$$V_{bn} = 120\angle -120°$$

$$V_{an} = 120\angle 120°$$

Abc sequence

$$V_{AN} = I_{aA} \times (20 + j10) = I_{aA} \times 22.36\angle 26.57°$$

$$V_{AN} = 113.15\angle -1.08°(V)rms$$

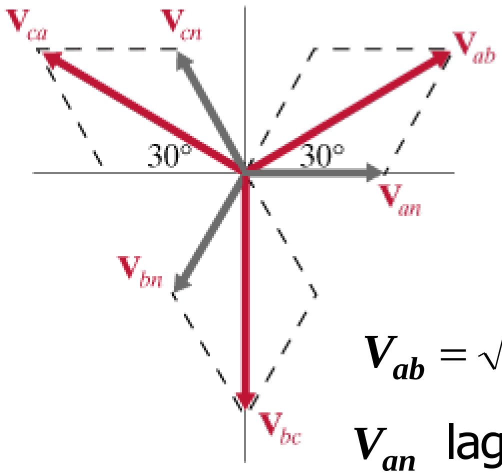
$$V_{BN} = 113.15\angle -121.08°(V)rms$$

$$V_{CN} = 113.15\angle 118.92°(V)rms$$

<u>Example:</u> For an abc sequence, balanced Y - Y three phase circuit

$V_{an} = 120\angle 90^\circ (V) rms$. Find the line voltages

V_{ab} leads V_{an} by 30°



$$V_{ab} = \sqrt{3} \times 120\angle 120^\circ (V) rms$$

$$V_{bc} = \sqrt{3} \times 120\angle 0^\circ (V) rms$$

$$V_{ca} = \sqrt{3} \times 120\angle 240^\circ (V) rms$$

$$V_{ab} = \sqrt{3} | V_p | \angle 30^\circ$$

V_{an} lags V_{ab} by 30°

$V_{ab} = 208\angle 0^\circ (V) rms$. Find the phase voltages

Relationship between phase and line voltages

V_{an} lags V_{ab} by 30°

$$V_{an} = \frac{208}{\sqrt{3}}\angle -30^\circ \, (V) rms$$

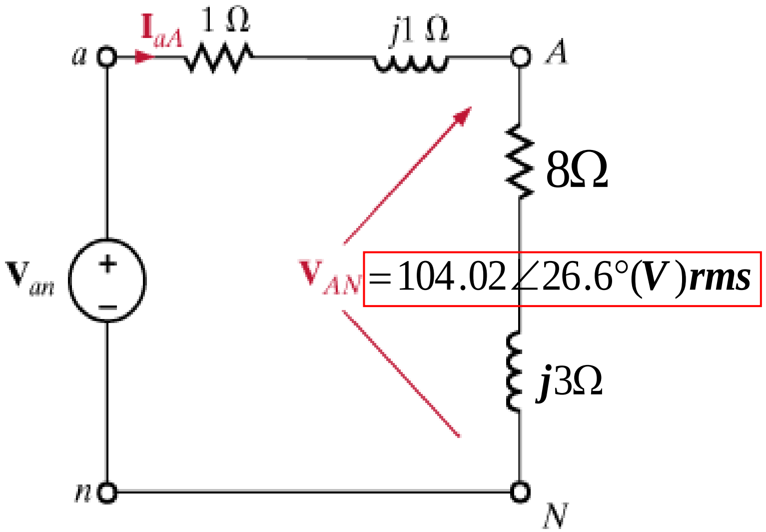
$$V_{bn} = \frac{208}{\sqrt{3}}\angle -150^\circ (V) rms$$

$$V_{cn} = \frac{208}{\sqrt{3}}\angle 90^\circ (V) rms$$

<u>Example:</u> For an abc sequence, balanced Y - Y three phase circuit

load, $|V_{phase}| = 104.02\angle 26.6°(V)_{rms}$, $Z_{line} = 1 + j1\Omega$, $Z_{phase} = 8 + j3\Omega$

Determine source phase voltages



Currents are not required. Use inverse voltage divider

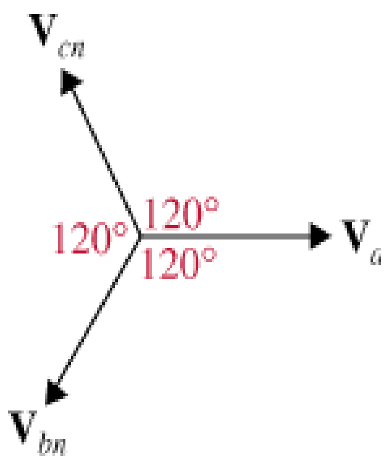
$$V_{an} = \frac{(8+j3)+(1+j1)}{8+j3} V_{AN}$$

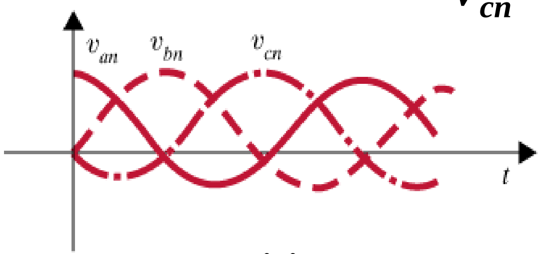
$$\frac{9+j4}{8+j3} \times \frac{8-j3}{8-j3} = \frac{84+j5}{73} = 1.15\angle 3.41°$$

$$V_{an} = 120\angle 30°$$

$$V_{bn} = 120\angle -90°$$

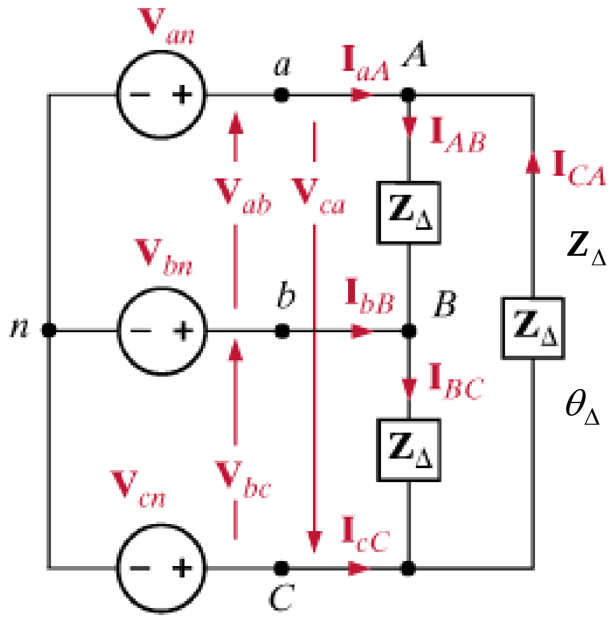
$$V_{cn} = 120\angle 150°$$





Positive sequence a-b-c

DELTA-CONNECTED LOAD



$Z_\Delta = |Z_L|\angle\theta_Z$

$\theta_\Delta = 30° - \theta_Z$

Load phase currents

$$I_{AB} = \frac{V_{AB}}{Z_\Delta} = |I_\Delta|\angle\theta_\Delta$$

$$I_{BC} = \frac{V_{BC}}{Z_\Delta} = |I_\Delta|\angle\theta_\Delta - 120°$$

$$I_{CA} = \frac{V_{CA}}{Z_\Delta} = |I_\Delta|\angle\theta_\Delta + 120°$$

Line currents

$$I_{aA} = I_{AB} - I_{CA}$$

$$I_{bB} = I_{BC} - I_{AB}$$

$$I_{cC} = I_{CA} - I_{BC}$$

Method 1: Solve directly

$$V_{an} = |V_p|\angle 0°$$

$$V_{bn} = |V_p|\angle -120°$$

$$V_{cn} = |V_p|\angle 120°$$

Positive sequence phase voltages

$$V_{ab} = \sqrt{3}|V_p|\angle 30°$$

$$V_{bc} = \sqrt{3}|V_p|\angle -90°$$

$$V_{ca} = \sqrt{3}|V_p|\angle -210°$$

$$|I_{line}| = \sqrt{3}|I_\Delta|$$

$$\theta_{line} = \theta_\Delta - 30°$$

Line-phase current relationship

Method 2: We can also convert the delta connected load into a Y connected one. The same formulas derived for resistive circuits are applicable to impedances

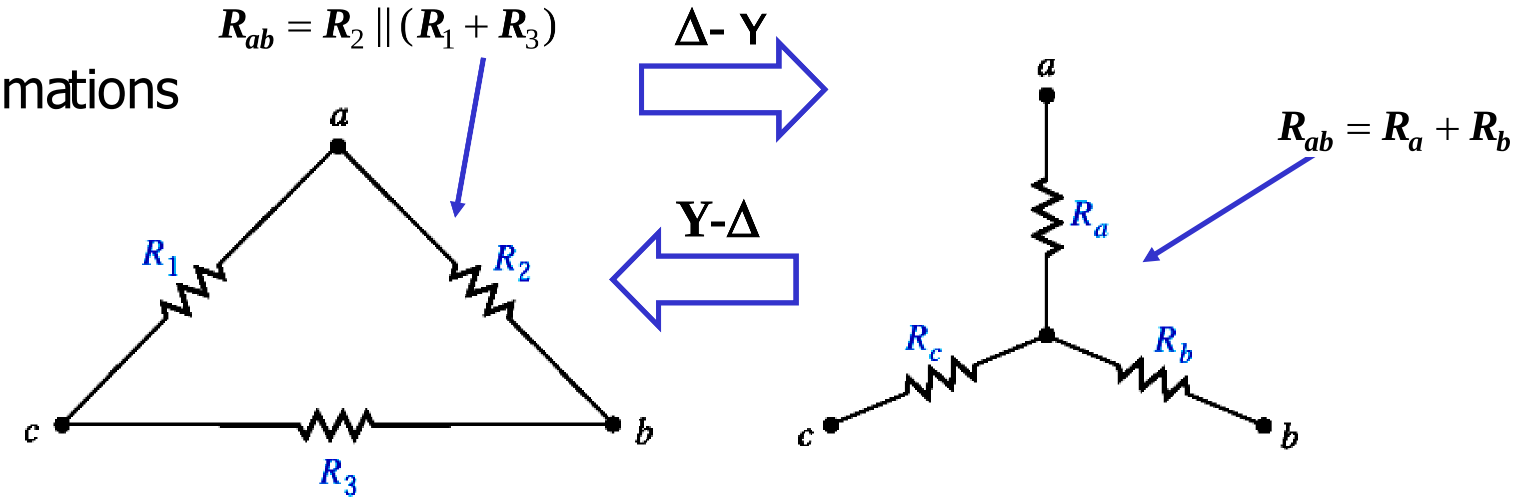
Balanced case $Z_Y = \frac{Z_\Delta}{3}$

$$I_{aA} = \frac{V_{an}}{Z_Y} = |I_{aA}|\angle\theta_L \Rightarrow \begin{cases} |I_{aA}| = \dfrac{|V_{AB}|/\sqrt{3}}{|Z_\Delta|/3} \\ \theta_L = -\theta_Z \end{cases}$$

REVIEW OF

$\Delta \leftrightarrow$ Y

Transformations



$$R_a = \frac{R_1 R_2}{R_1 + R_2 + R_3}$$

$$R_b = \frac{R_2 R_3}{R_1 + R_2 + R_3}$$

$$R_c = \frac{R_3 R_1}{R_1 + R_2 + R_3}$$

$$\Delta \rightarrow Y$$

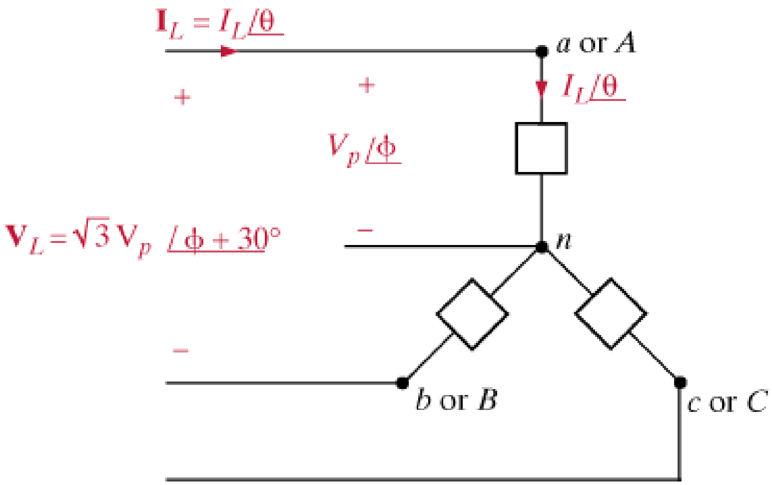
$$R_1 = \frac{R_a R_b + R_b R_c + R_c R_a}{R_b}$$

$$R_2 = \frac{R_a R_b + R_b R_c + R_c R_a}{R_c}$$

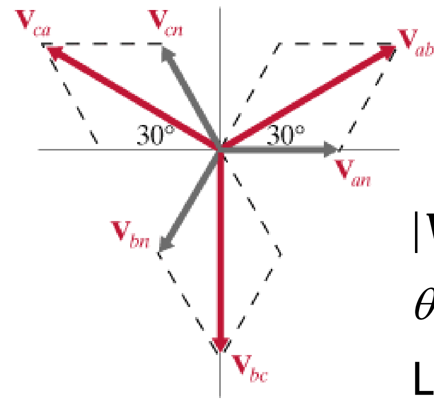
$$R_3 = \frac{R_a R_b + R_b R_c + R_c R_a}{R_a}$$

$$Y - \Delta$$

$$R_\Delta = R_1 = R_2 = R_3 \Rightarrow R_Y = \frac{R_\Delta}{3}$$



(a)

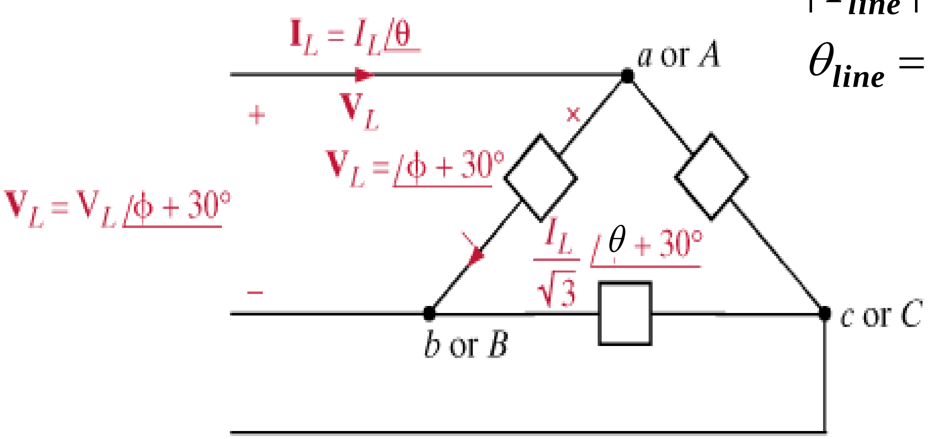


(b)

$$|V_{\Delta}| = \sqrt{3}\,|V_{phase}|$$

$$\theta_{\Delta} = \theta_{phase} + 30°$$

Line-phase voltage relationship



(b)

$$|I_{line}| = \sqrt{3}\,|I_{\Delta}|$$

$$\theta_{line} = \theta_{\Delta} - 30°$$

Line-phase current relationship

Example:

$I_{aA} = 12\angle 40°.$

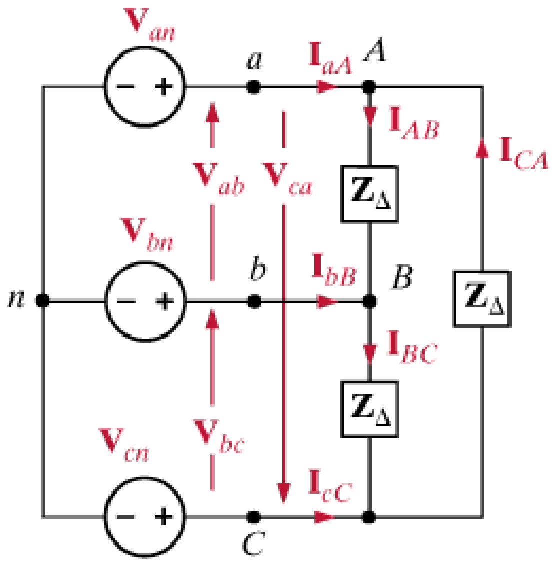
Find the phase currents

$I_{AB} = 6.93\angle 70°$

$I_{BC} = 6.93\angle -50°$

$I_{CA} = 6.93\angle 190°$

<u>Example:</u> Delta-connected load consists of 10-Ohm resistance in series with 20-mH inductance. Source is Y-connected, abc sequence, 120-V rms, 60Hz. Determine all line and phase currents



$$V_{an} = 120\angle 30°(V)rms$$

$$Z_{\text{inductance}} = 2\pi \times 60 \times 0.020 = 7.54\Omega$$

$$Z_\Delta = 10 + j7.54\Omega = 12.52\angle 37.02° \quad \Rightarrow Z_Y = 4.17\angle 37.02°$$

$$I_{AB} = \frac{V_{AB}}{Z_\Delta} = \frac{120\sqrt{3}\angle 60°}{10 + j7.54} = 16.60\angle 22.98°(A)rms$$

$$I_{BC} = 16.60\angle -97.02°(A)rms$$

$$I_{CA} = 16.60\angle 142.98°(A)rms$$

$$|V_\Delta| = \sqrt{3}\,|V_{phase}|$$

$$\theta_\Delta = \theta_{phase} + 30°$$

Line-phase voltage relationship

$$I_{aA} = 28.75\angle -7.02°(A)rms$$

$$I_{bB} = 28.75\angle -127.02°(A)rms$$

$$I_{cC} = 28.75\angle 112.98°(A)rms$$

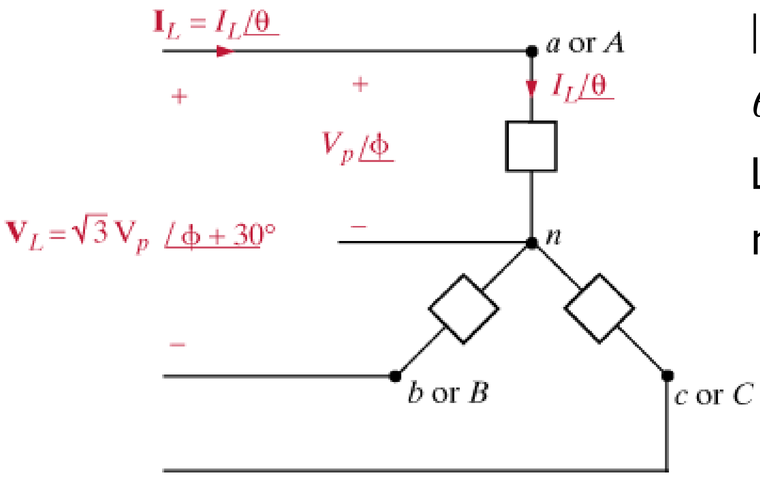
$$|I_{line}| = \sqrt{3}\,|I_\Delta|$$

$$\theta_{line} = \theta_\Delta - 30°$$

Line-phase current relationship

Alternatively, determine first the line currents and then the delta currents

POWER RELATIONSHIPS

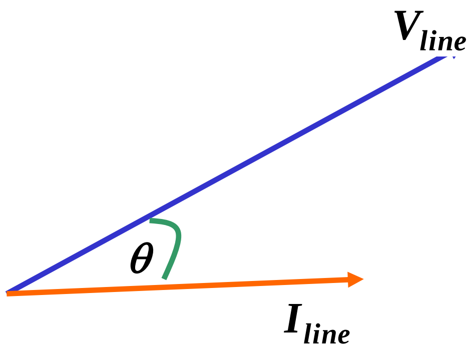


(a)

$$|V_\Delta| = \sqrt{3}\,|V_{phase}|$$

$$\theta_\Delta = \theta_{phase} + 30°$$

Line-phase voltage relationship

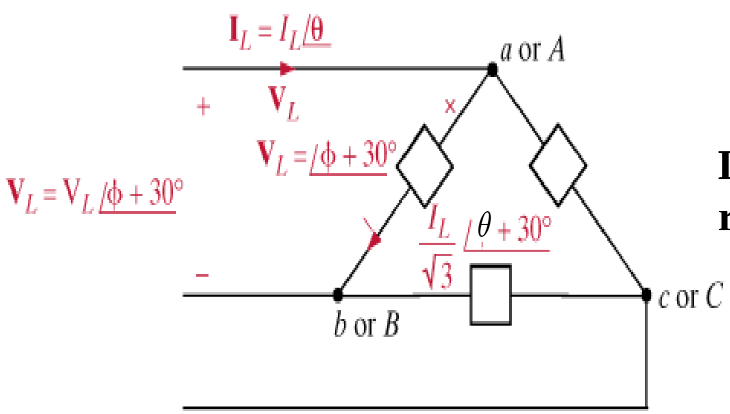


$$S_{Total} = 3 \times V_{phase} \times I^*_{phase}$$

$$S_{Total} = \sqrt{3} V_{line} I^*_{line}$$

θ → Impedance angle

θ → Power factor angle



(b)

$$|I_{line}| = \sqrt{3}\,|I_\Delta|$$

$$\theta_{line} = \theta_\Delta - 30°$$

Line-phase current relationship

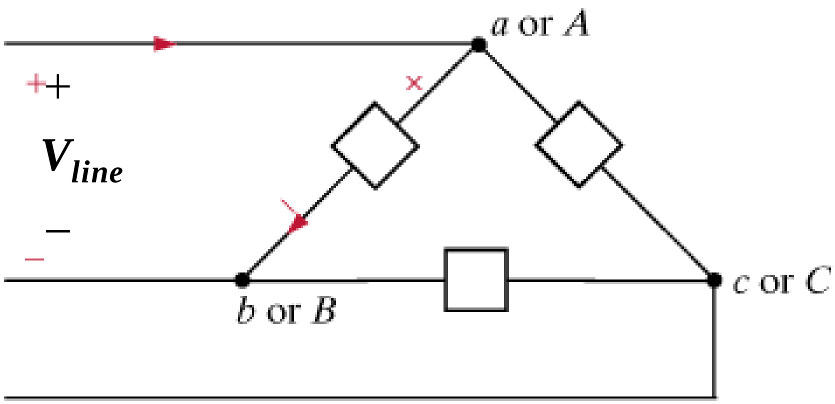
$$S_{total} = 3 V_{line} \times I^*_\Delta$$

$$S_{Total} = \sqrt{3} V_{line} I^*_{line}$$

$$P_{total} = \sqrt{3}\,|V_{line}|\,|I_{line}|\cos\theta_f$$

$$Q_{total} = \sqrt{3}\,|V_{line}|\,|I_{line}|\sin\theta_f$$

<u>EXAMPLE:</u> Determine the magnitude of the line currents and the value of load impedance per phase in the delta



$$|V_{line}| = 208(V)rms$$

$$P_{total} = 1200W$$

power factor angle $= 20°$ lagging

$$|I_{line}| = \sqrt{3}|I_\Delta|$$

$$\theta_{line} = \theta_\Delta - 30°$$

Line-phase current relationship

$$P_{total} = \sqrt{3}|V_{line}||I_{line}|\cos\theta_f$$

$$Q_{total} = \sqrt{3}|V_{line}||I_{line}|\sin\theta_f$$

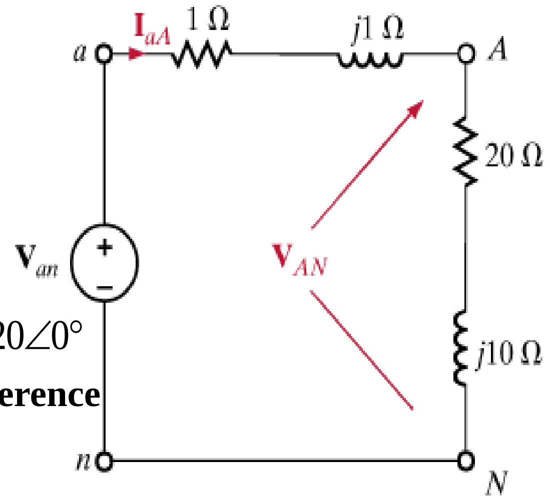
$$\frac{P_{total}}{3} = \frac{|V_{line}||I_{line}|}{\sqrt{3}}\cos\theta_f \quad \Rightarrow |I_{line}| = 3.54(A)rms$$

$$\Rightarrow |I_\Delta| = 2.05(A)rms \quad \Rightarrow |Z_\Delta| = \frac{|V_{line}|}{|I_\Delta|} = 101.46\Omega$$

<u>Example:</u> For an abc sequence, balanced Y - Y three phase circuit

$$\text{source}\,|V_{phase}| = 120(V)_{rms},\ Z_{line} = 1 + j1\Omega,\ Z_{phase} = 20 + j10\Omega$$

Determine real and reactive power per phase at the load and total real, reactive and complex power at the source



$120\angle 0°$

Reference

$$V_{AN} = I_{aA} \times (20 + j10) = I_{aA} \times 22.36\angle 26.57°$$

$$V_{AN} = 113.15\angle -1.08°(V)rms$$

$$S_{phase} = V_{AN} I_{aA}^{*} = 113.15\angle -1.08° \times 5.06\angle 27.65°$$

$$S_{phase} = 572.54\angle 26.57° = 512 + j256.09(VA)rms$$

$$S_{\text{source phase}} = V_{an} \times I_{aA}^{*} = 120\angle 0° \times 5.06\angle 27.65°$$

$$S_{\text{source phase}} = 607.2\angle 27.65°$$

$$= 537.86 + j281.78VA$$

$$V_{an} = 120\angle 0°$$

$$V_{bn} = 120\angle -120°$$

$$V_{an} = 120\angle 120°$$

Because circuit is balanced data on any one phase are sufficient

Abc sequence

$$I_{aA} = \frac{V_{an}}{21 + j11} = \frac{120\angle 0°}{23.71\angle 27.65°}$$

$$= 5.06\angle -27.65°(A)rms$$

$$P_{\text{total source}} = 3 \times 537.86(W)$$

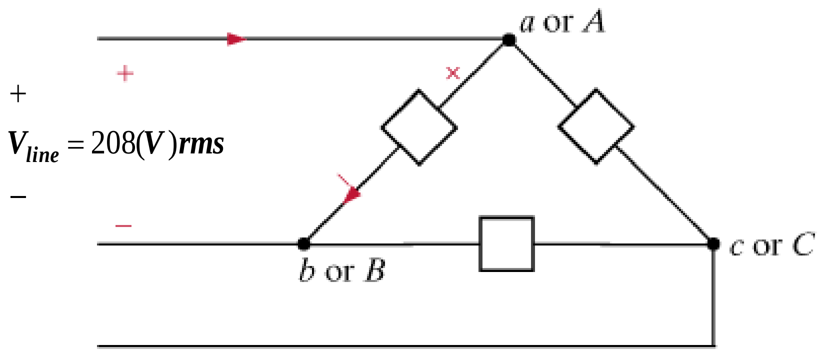
$$Q_{\text{total source}} = 3 \times 281.78(VA)$$

$$S_{\text{total source}} = P_{\text{total source}} + Q_{\text{total source}}$$

$$= 1613.6 + j845.2(VA)$$

$$|S_{\text{total source}}| = 1821.6(VA)$$

Example: **Determine the line currents and the combined power factor**



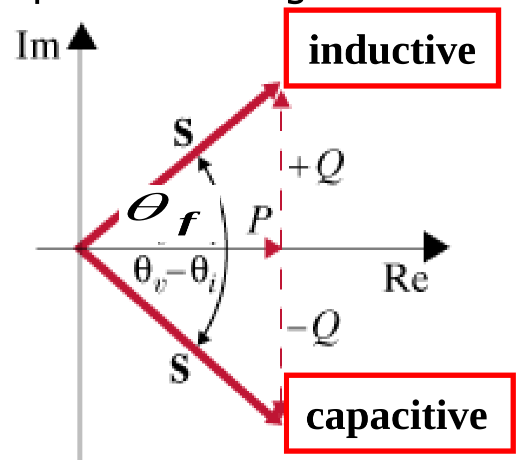
Circuit is balanced

Load 1: 24kW at pf = 0.6 lagging

Load 2: 10kW at pf = 1

Load 3: 12kVA at pf = 0.8 leading

$$S = P + jQ$$
$$P = |S|\cos\theta_f$$
$$Q = |S|\sin\theta_f$$
$$pf = \cos\theta_f$$
$$S_{total} = S_1 + S_2 + S_3$$



$$\left.\begin{array}{l} P_1 = 24kW \\ pf = 0.6\,lagging \end{array}\right\} \Rightarrow |S_1| = 40kVA$$

$$|Q_1| = \sqrt{|S_1|^2 - |P_1|^2} = 32kVA$$

$$lagging \Rightarrow inductive \;\therefore S_1 = 24 + j32\,kVA$$

Load 2

$$\left.\begin{array}{l} P_2 = 10kW \\ pf = 1 \end{array}\right\} \Rightarrow S_2 = 10 + j0\,kVA$$

Load 3

$$\left.\begin{array}{l} |S_3| = 12kVA \\ pf = 0.8 \end{array}\right\} \Rightarrow \left\{\begin{array}{l} P_3 = 9.6kW \\ |Q_3| = 7.2kVA \end{array}\right.$$

$$leading\;\; pf \Rightarrow capacitive \;\therefore S_3 = 9.6 - j7.2kVA$$

$$S_{TOTAL} = S_1 + S_2 + S_3 = 43.6 + j24.8kVA = 50.160\angle 29.63°kVA$$

$$P_{total} = \sqrt{3}|V_{line}||I_{line}|\cos\theta_f$$
$$Q_{total} = \sqrt{3}|V_{line}||I_{line}|\sin\theta_f$$

$$\Rightarrow \left\{\begin{array}{l} |S_{total}| = \sqrt{3}|V_{line}| \times |I_{line}| \\ \theta_f = 29.63° \end{array}\right.$$

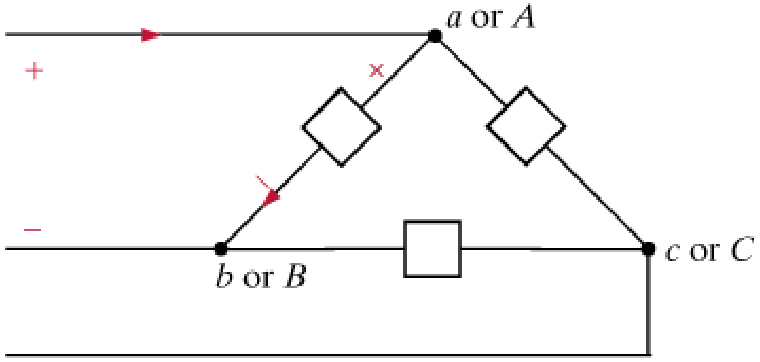
$$pf = 0.869\; lagging$$

$$|I_{line}| = 139.23(A)rms$$

Continued ...

EXAMPLE continued ….

If the line impedances are $Z_{line} = 0.05 + j0.02\Omega$ determine line voltages and power factor at the source



$$|I_{line}| = 139.23(A)rms$$

$$S_{line} = 3\times(Z_{line}I_{line})I^*_{line} = 3\times Z_{line}|I_{line}|^2$$

$$S_{line} = 2908 + j1163(VA)$$

$$S_{\text{load total}} = 43.6 + j24.8kVA = 50.160\angle 29.63^\circ kVA$$

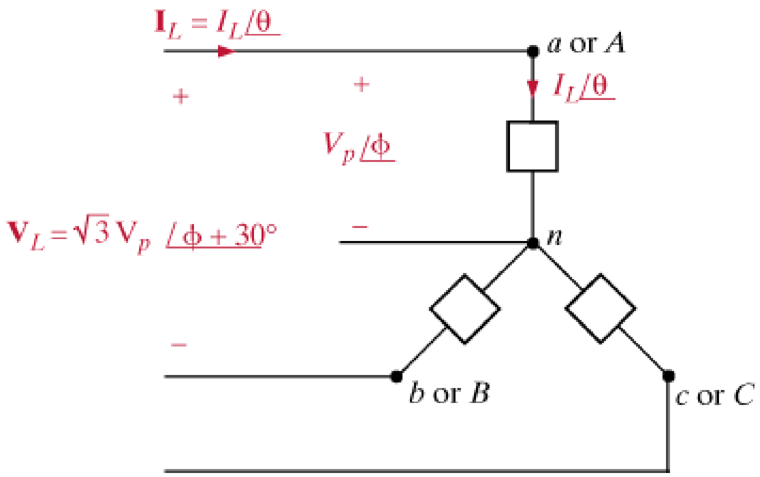
$$S_{\text{source total}} = 46.508 + j25.963 = 53.264\angle 29.17^\circ kVA$$

$$\begin{cases} |S_{total}| = \sqrt{3}|V_{line}|\times|I_{line}| \\ \theta_f = 29.17^\circ \end{cases}$$

$$V_{line} = \frac{53{,}264}{\sqrt{3}\times 139.13} = 220.87(V)rms$$

$$pf = \cos\theta_f = \cos(29.17^\circ) = 0.873 lagging$$

Example: **A Y -Y balanced three-phase circuit has a line voltage of 208-Vrms. The total real power absorbed by the load is 12kW at pf=0.8 lagging. Determine the per-phase impedance of the load**



(a)

$$|V_\Delta| = \sqrt{3}\,|V_{phase}|$$

$$\theta_\Delta = \theta_{phase} + 30°$$

Line-phase voltage relationship

$$S_{Total} = 3 \times V_{phase} \times I^*_{phase}$$

$$S = P + jQ$$

$$P = |S|\cos\theta_f$$

$$Q = |S|\sin\theta_f$$

$$pf = \cos\theta_f$$

$$\Rightarrow |V_{phase}| = \frac{208}{\sqrt{3}} = 120(V)rms$$

$$S_{total} = 3V_{phase} \times \left(\frac{V_{phase}}{Z_{phase}}\right)^* = 3 \times \frac{|V_{phase}|^2}{Z^*_{phase}}$$

$$pf = 0.8 = \cos\theta_f \Rightarrow \theta_f = 36.87°$$

$$|S_{total}| = \frac{P_{total}}{pf} = 15kVA$$

$$|Z_{phase}| = \frac{3 \times |V_{phase}|^2}{|S_{total}|} = 2.88\Omega$$

$$Z_{pahse} = 2.88\angle 36.87°\Omega$$

<u>Example:</u> A 480-V rms line feeds two balanced 3-phase loads. The loads are rated Load 1: 5kVA at 0.8 pf lagging Load 2: 10kVA at 0.9 pf lagging.

Determine the magnitude of the line current from the 408-V rms source

$$|S_1| = 5kVA = \frac{P}{0.8} \Rightarrow P_1 = 4kW$$

$$Q_1 = \sqrt{|S_1|^2 - P_1^2} = 3.0kVA$$

$$pf\ \ lagging \Rightarrow S_1 = 4 + j3kVA$$

$$|S_2| = 10kVA = \frac{P}{0.9} \Rightarrow P = 9kW$$

$$Q_2 = \sqrt{|S_2|^2 - P_2^2} = 4.36kVA$$

$$S_2 = 9 + j4.36kVA$$

$$S_{total} = 13 + j7.36kVA$$

$$S = P + jQ$$

$$P = |S|\cos\theta_f$$

$$Q = |S|\sin\theta_f$$

$$pf = \cos\theta_f$$

$$S_{total} = S_1 + S_2$$

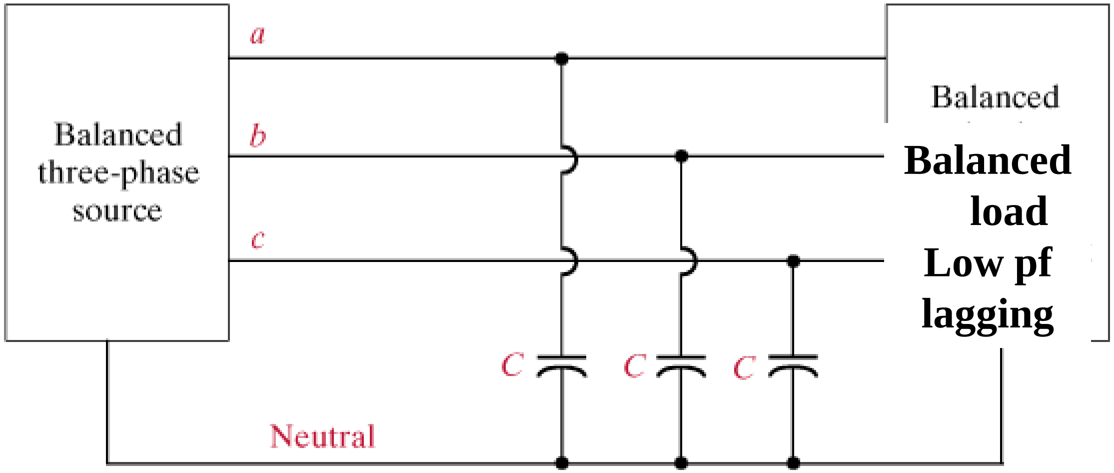
$$P_{total} = \sqrt{3}|V_{line}||I_{line}|\cos\theta_f$$

$$Q_{total} = \sqrt{3}|V_{line}||I_{line}|\sin\theta_f$$

$$|S_{total}| = \sqrt{3}|V_{line}||I_{line}|$$

$$|I_{lineq}| = \frac{|S_{total}|}{\sqrt{3} \times |V_{line}|} = \frac{14{,}939}{706.68} = 21.14(A)rms$$

POWER FACTOR CORRECTION



Similar to single phase case. Use capacitors to increase the power factor

Keep clear about total/phase power, line/phase voltages

$$\left.\begin{matrix} S_{old} \\ pf_{old} \end{matrix}\right\} \rightarrow Q_{old}$$

$$\Delta Q = Q_{new} - Q_{old}$$

Reactive Power to be added

$$Q_{Cphase} = \omega C V_{ph}^2$$

$$\left.\begin{matrix} P_{old} \\ pf_{new} \end{matrix}\right\} \rightarrow Q_{new}$$

$$S = P + jQ$$

$$P = |S| \cos\theta_f$$

$$Q = |S| \sin\theta_f$$

$$pf = \cos\theta_f$$

$$pf = \cos\theta_f \Rightarrow \sin\theta_f = \sqrt{1 - pf^2}$$

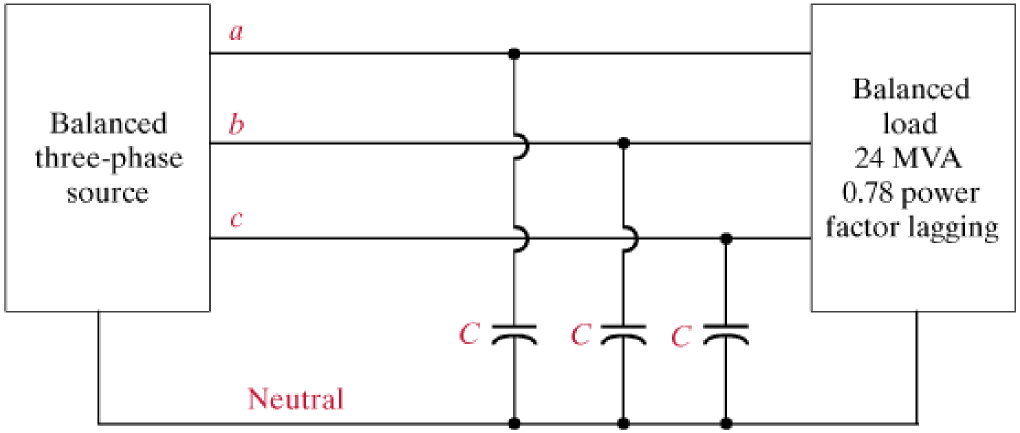
$$\tan\theta_f = \frac{pf}{\sqrt{1 - pf^2}} \qquad Q = P \tan\theta_f$$

$$Q_{\text{per capacitor}} = -\omega C V^2$$

The voltage depends on how the capacitors are connected

$$lagging \Rightarrow Q > 0$$

EXAMPLE:



***f*=60 Hz, V_{line}=34.5 kV$_{rms}$.**

Required *pf*=0.94 lagging.

$$S = P + jQ$$
$$P = |S|\cos\theta_f$$
$$Q = |S|\sin\theta_f$$
$$pf = \cos\theta_f$$

$$Q = P\tan\theta_f$$
$$\tan\theta_f = \frac{pf}{\sqrt{1-pf^2}}$$
$$lagging \Rightarrow Q_{old} > 0$$

$$pf = \cos\theta_f \Rightarrow \sin\theta_f = \sqrt{1-pf^2} = 0.626$$

$$|Q_{old}| = 15.02\,MVA$$
$$P_{old} = 18.72\,MW$$

$$\left.\begin{array}{l} P_{old} = 18.72\,MW \\ pf_{new} = 0.90\,lagging \end{array}\right\} \Rightarrow Q_{new} = -9.067\,MVA$$

$$\Delta Q = 9.067 - 15.02 = -5.953\,MVA$$

$$Q_{\text{per capacitor}} = -1.984\,MVA$$

$$Y - \text{connection} \Rightarrow V_{\text{capacitor}} = \frac{34.5}{\sqrt{3}}\,kV\ rms$$

$$Q_{Cphase} = \omega C V_{ph}^2$$

$$-1.984\times10^6 = -2\pi\times60\times C\times\left(\frac{34.5\times10^3}{\sqrt{3}}\right)^2$$

$$C = 13.26\,\mu F$$