

- 2.76 Find the equation of motion of the uniform rigid bar OA of length l and mass m shown in Fig. 2.98. Also find its natural frequency.

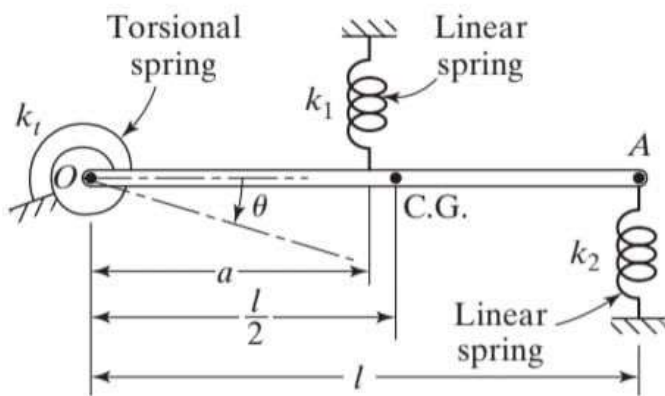


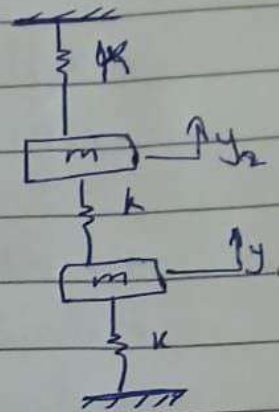
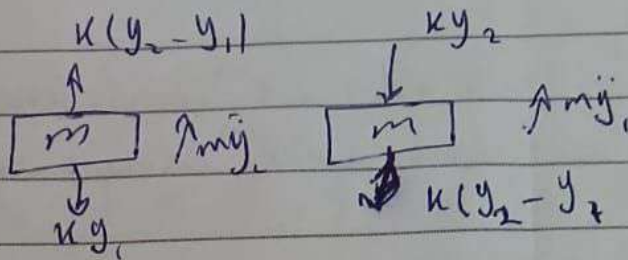
FIGURE 2.98

Section 2.3 Free Vibration of an Undamped Torsional System

- 2.64** A simple pendulum is set into oscillation from its rest position by giving it an angular velocity of 1 rad/s. It is found to oscillate with an amplitude of 0.5 rad. Find the natural frequency and length of the pendulum.

- 3.48** Find the steady-state response of the system shown in Fig. 3.55 for the following data:
 $k_1 = 1000 \text{ N/m}$, $k_2 = 500 \text{ N/m}$, $c = 500 \text{ N-s/m}$, $m = 10 \text{ kg}$, $r = 5 \text{ cm}$, $J_0 = 1 \text{ kg-m}^2$,
 $F_0 = 50 \text{ N}$, $\omega = 20 \text{ rad/s}$.

Example



$$m\ddot{y}_1 + 2ky_1 - ky_2 = 0$$

$$m\ddot{y}_2 + 2ky_2 - ky_1 = 0$$

$$\begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix} \begin{bmatrix} \ddot{y}_1 \\ \ddot{y}_2 \end{bmatrix} + \begin{bmatrix} 2k & -k \\ -k & 2k \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{vmatrix} -m\omega^2 + 2k & -k \\ -k & -m\omega^2 + 2k \end{vmatrix} = 0$$

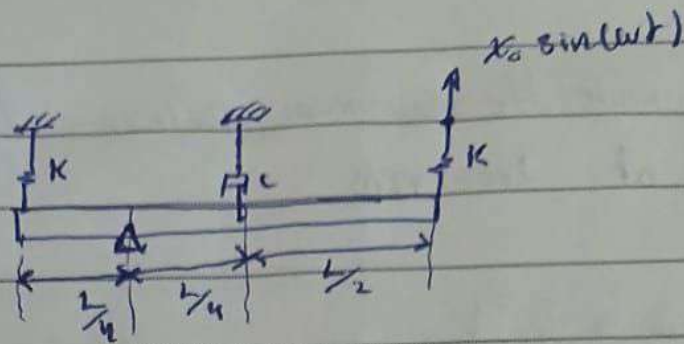
$$\omega_1 = \sqrt{\frac{k}{m}}, \quad \omega_2 = \sqrt{\frac{3k}{m}}$$

$$x_1(t) = X_1^{(1)} \cos\left(\sqrt{\frac{k}{m}}t + \phi_1\right) + X_1^{(2)} \cos\left(\sqrt{\frac{3k}{m}}t + \phi_2\right)$$

$$x_2(t) = r_1 X_1^{(1)} \cos\left(\sqrt{\frac{k}{m}}t + \phi_1\right) + r_2 X_1^{(2)} \cos\left(\sqrt{\frac{3k}{m}}t + \phi_2\right)$$

$$r_1 = 1$$

$$r_2 = -1$$



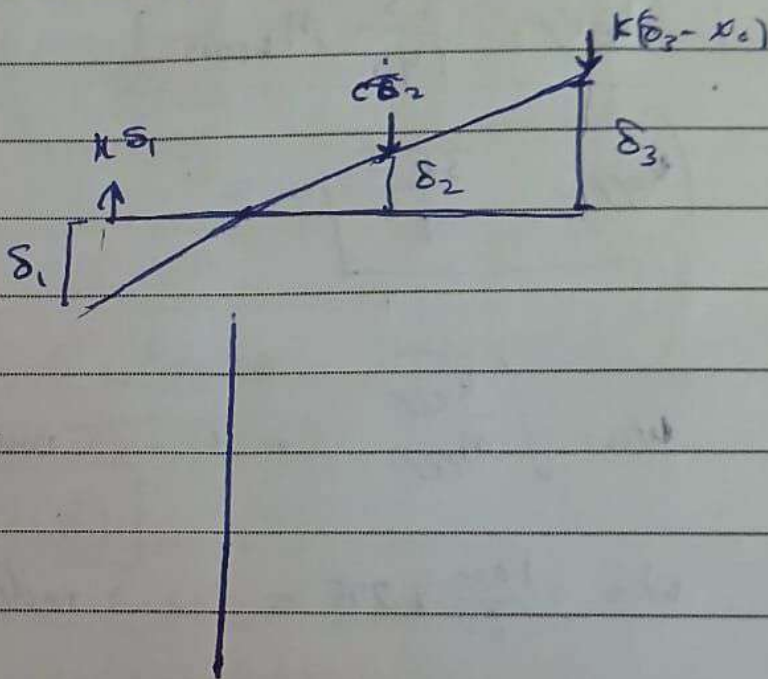
Find the steady state amplitude of vibration

$$J_0 = \frac{1}{12} m l^2 + m d^2$$

$$J_0 = \frac{1}{12} m l^2 + \frac{1}{16} m l^2$$

$$\boxed{J_0 = \frac{7}{48} m l^2}$$

$$+5 \sum M_0 = J_0 \ddot{\theta}$$



$$-k \delta_1 \left(\frac{L}{4}\right) - c \dot{\delta}_1 \left(\frac{L}{4}\right) - k(\delta_3 - x_0) \left(\frac{3L}{4}\right) = \frac{7}{84} m l^2 \ddot{\theta}$$

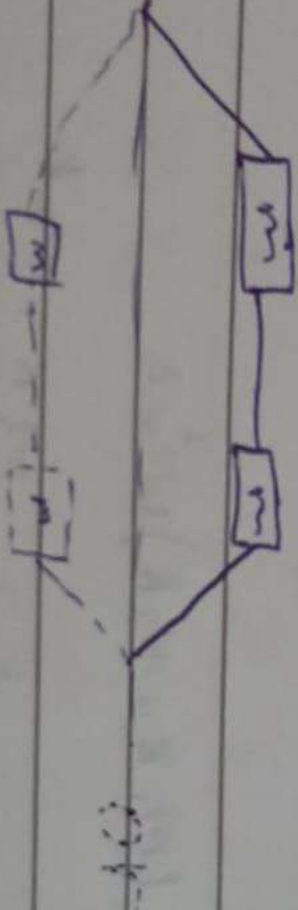
$$\frac{7}{48} m l^2 \ddot{\theta} + \frac{c l^2}{16} \dot{\theta} + \frac{10}{16} k l^2 \theta = \underbrace{\left(\frac{3 k x_0}{4}\right)}_{M_0} \sin(\omega t)$$

$$\theta_p(t) = A \sin(\omega t - \phi)$$

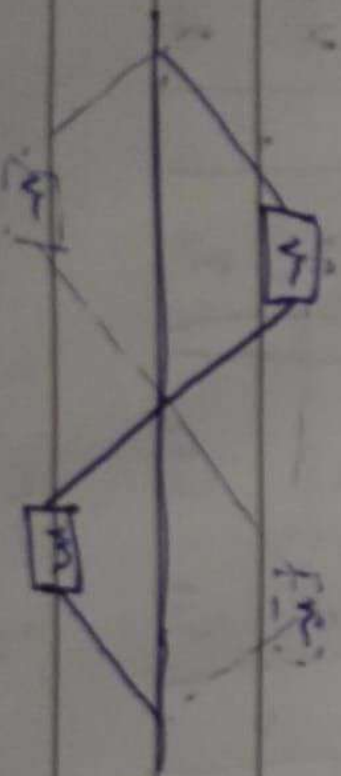
$$A = \frac{M_0}{\sqrt{(k_{eff} - J_{eff} \omega^2)^2 + (c \omega)^2}}$$

* Sketch the modes of vibration

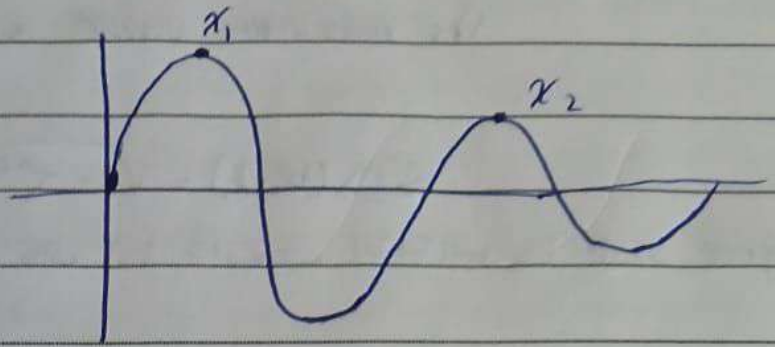
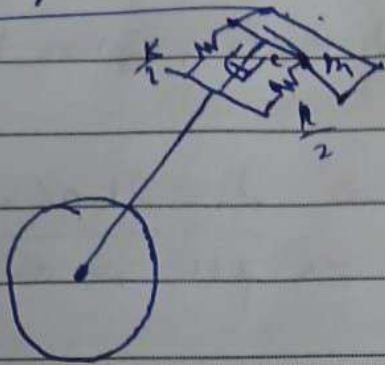
First mode



Second mode



Example 2.11



$$m = 200 \text{ kg}$$

$$\zeta_d = 2 \text{ s}$$

$$x_1 = 250 \text{ mm}$$

* The amplitude x_1 is to be reduced to one-fourth in one half cycle

Find k , c , ζ_d

$$\omega_d = \frac{2\pi}{\zeta_d} = \boxed{\pi \text{ rad/s}}$$

$$\frac{1}{n} \ln\left(\frac{x_1}{x_2}\right) = \frac{2\pi \zeta^2}{\sqrt{1-\zeta^2}} ; \quad x_2 = \frac{x_1}{16}$$

$$\ln(16) = \frac{2\pi \zeta^2}{\sqrt{1-\zeta^2}} \Rightarrow \zeta^2 = 0.4037$$

$$\omega_n = \frac{\omega_d}{\sqrt{1-\zeta^2}} = \boxed{3.43 \text{ rad/s}} \quad (1)$$

$$k = m\omega_n^2 = 200 \times 3.43^2 = \boxed{2358.3 \text{ N/m}} \quad (1)$$

$$c_{cr} = 2m\omega_n = 1372 \text{ N}\cdot\text{s/m} \Rightarrow c = \zeta c_{cr} \quad (2)$$

$$\boxed{c = 553.9 \text{ N}\cdot\text{s/m}} \quad (2)$$

* The displacement of the mass will attain its maximum value at time t_1 , given by

$$\sin(\omega t) = \sqrt{1 - \zeta^2} \Rightarrow t_1 = 21.06 \text{ s} \times$$

Degree لهذا الزاوية كان - يبلغ هذا الزمان

$$t_1 = 0.3678 \text{ s} \checkmark$$

radian لهذا الزمان

~~* Key = $\frac{k}{2} + \frac{k}{2} = k$, $\zeta = 1$~~

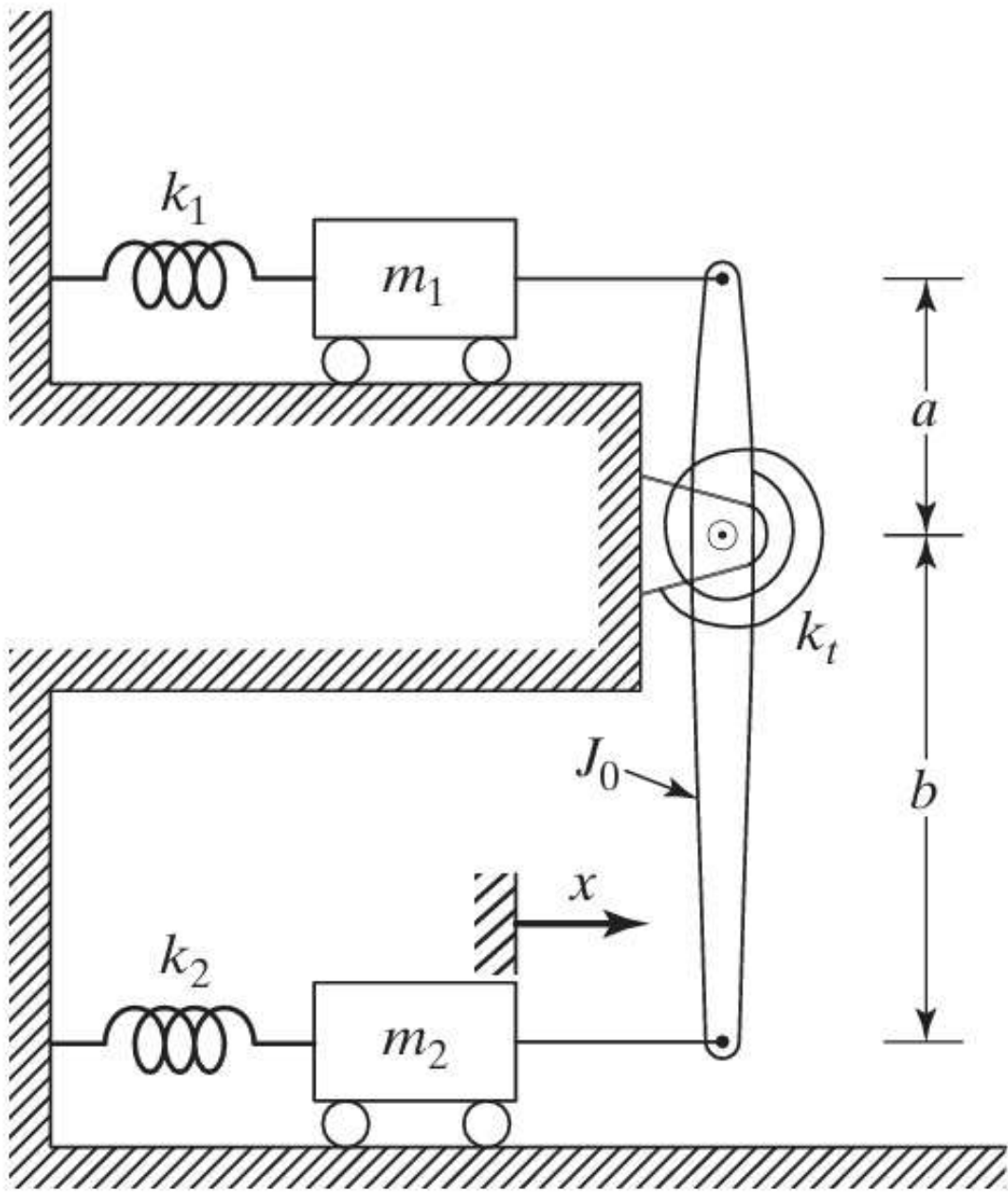
$$x(t) = X e^{-\zeta \omega t} \sin(\omega t)$$

$$x(0.3678) = 0.25 = X e^{-(0.4037)(3.47)(0.3678)} \sin(\pi \times 0.3678)$$

$$\boxed{X = 0.455 \text{ m}}$$

$$\dot{x}(t) = -\zeta \omega_n X e^{-\zeta \omega_n t} \sin(\omega_n t) + \omega_n X e^{-\zeta \omega_n t} \cos(\omega_n t)$$

$$\dot{x}(0) = \omega_n X = \pi \times 0.455 = \boxed{1.43 \text{ m/s}}$$



- 2.74** A cylinder of mass m and mass moment of inertia J_0 is free to roll without slipping but is restrained by two springs of stiffnesses k_1 and k_2 , as shown in Fig. 2.97. Find its natural frequency of vibration. Also find the value of a that maximizes the natural frequency of vibration.

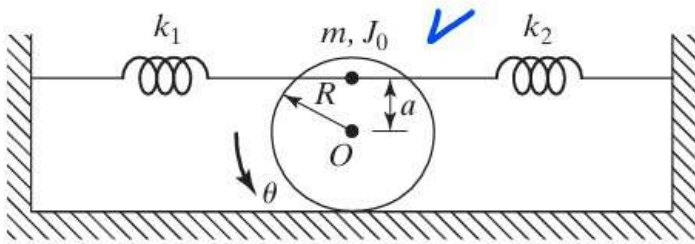


FIGURE 2.97

- 2.78** Derive the equation of motion of the system shown in Fig. 2.100, using the following methods: (a) Newton's second law of motion, (b) D'Alembert's principle, and (c) principle of virtual work.

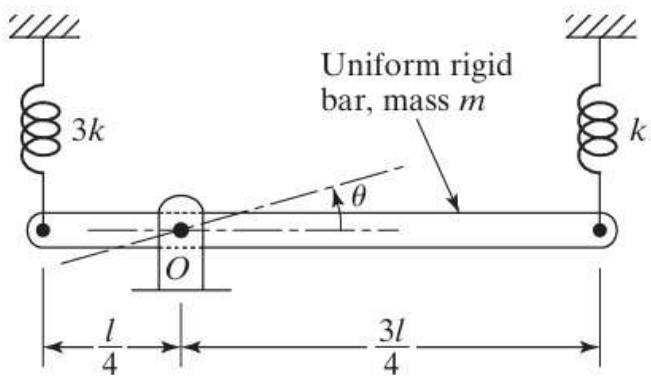


FIGURE 2.100

Harmonic Response of a Water Tank

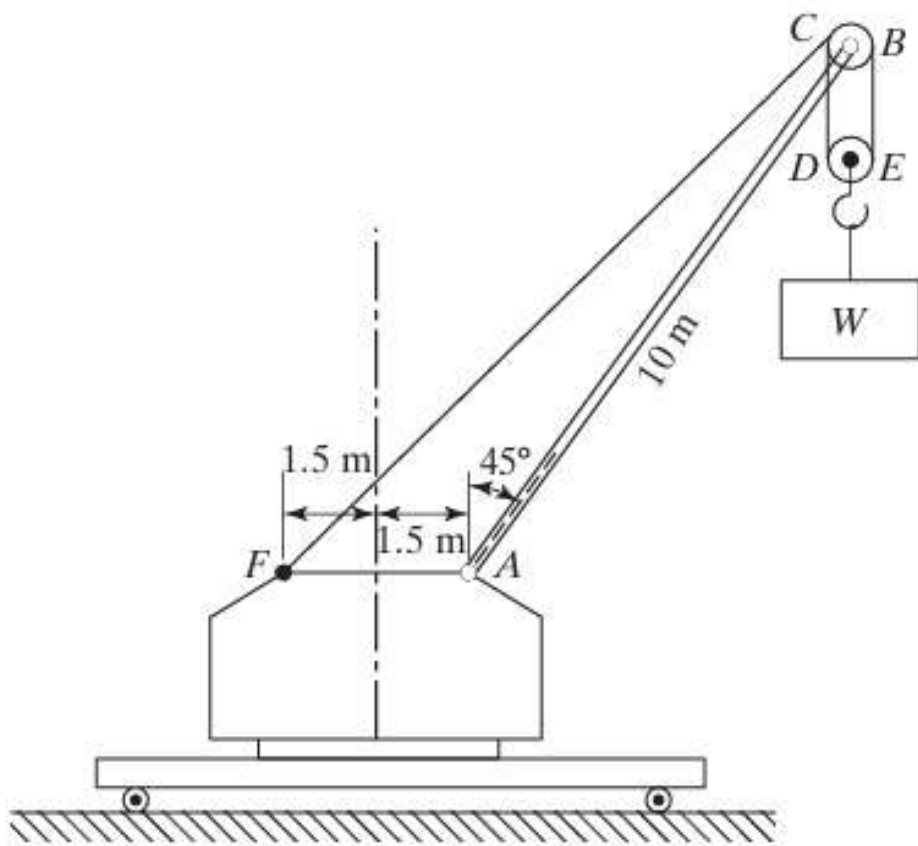
The column of the water tank shown in Fig. 2.10(a) is 300 ft high and is made of reinforced concrete with a tubular cross section of inner diameter 8 ft and outer diameter 10 ft. The tank weighs 6×10^5 lb when filled with water. By neglecting the mass of the column and assuming the Young's modulus of reinforced concrete as 4×10^6 psi, determine the following:

- the natural frequency and the natural time period of transverse vibration of the water tank.
- the vibration response of the water tank due to an initial transverse displacement of 10 in.
- the maximum values of the velocity and acceleration experienced by the water tank.

Solution: Assuming that the water tank is a point mass, the column has a uniform cross section, and the mass of the column is negligible, the system can be modeled as a cantilever beam with a concentrated load (weight) at the free end as shown in Fig. 2.10(b).

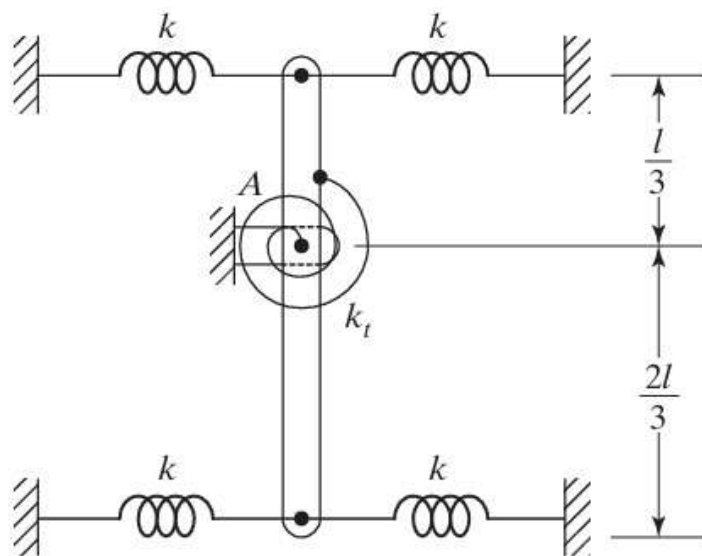
- The transverse deflection of the beam, δ , due to a load P is given by $\frac{Pl^3}{3EI}$, where l is the length, E is the Young's modulus, and I is the area moment of inertia of the beam's cross section. The stiffness of the beam (column of the tank) is given by

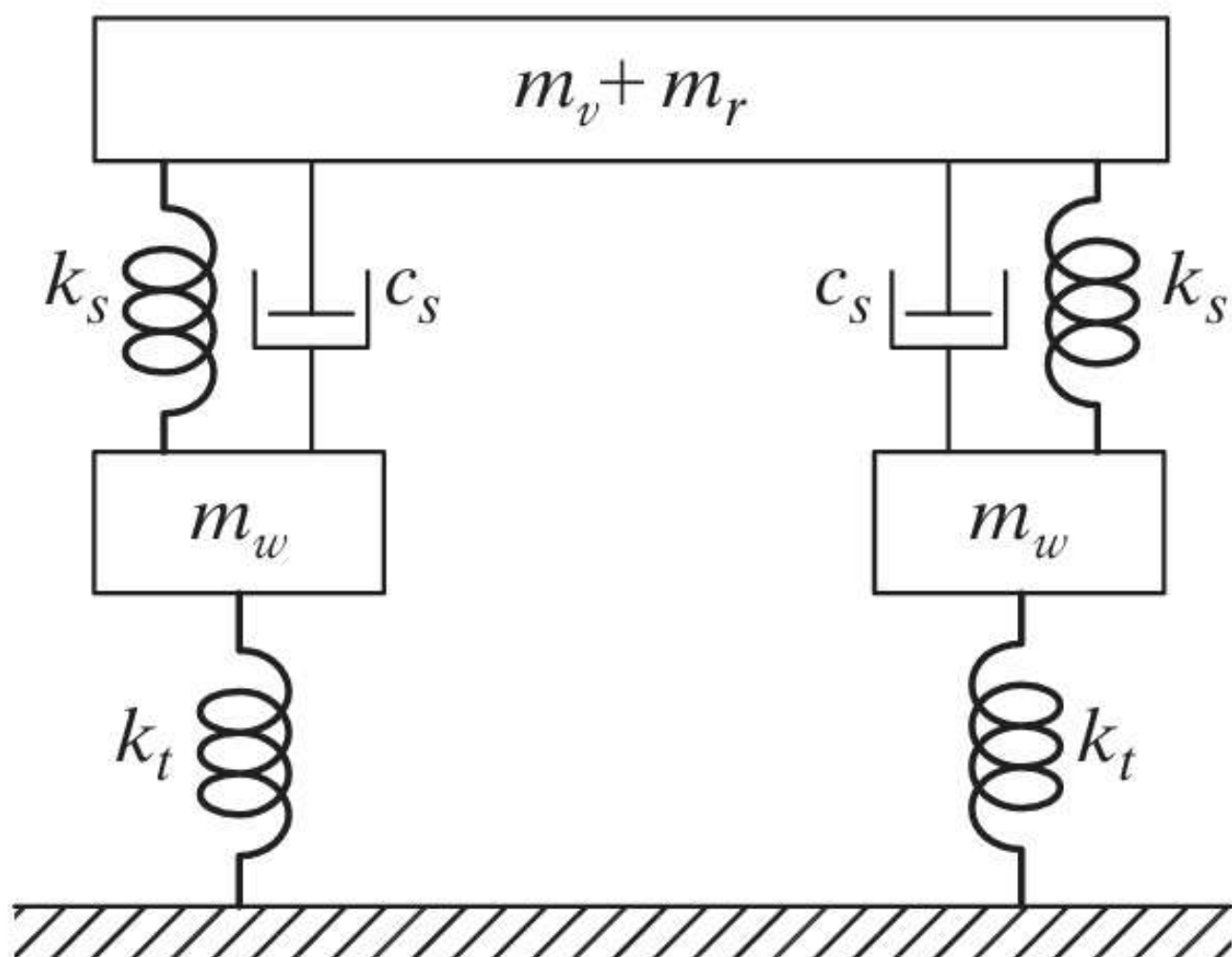
$$k = \frac{P}{\delta} = \frac{3EI}{l^3}$$

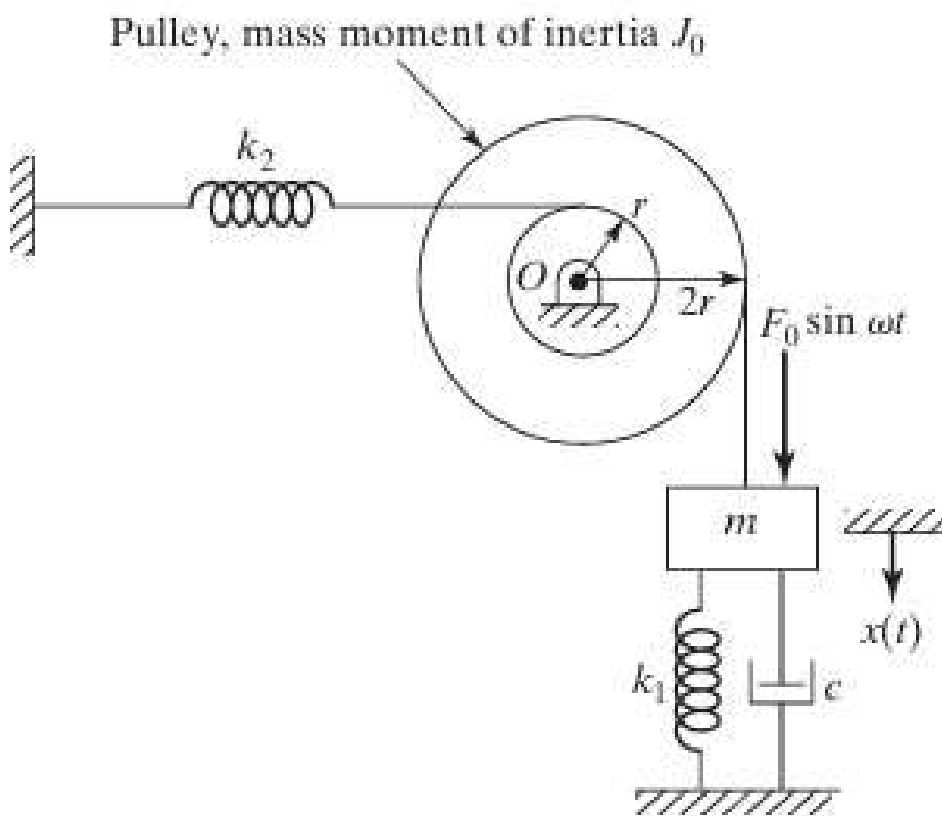


(a)

- 2.73** A uniform slender rod of mass m and length l is hinged at point A and is attached to four linear springs and one torsional spring, as shown in Fig. 2.96. Find the natural frequency of the system if $k = 2000$ N/m, $k_t = 1000$ N-m/rad, $m = 10$ kg, and $l = 5$ m.

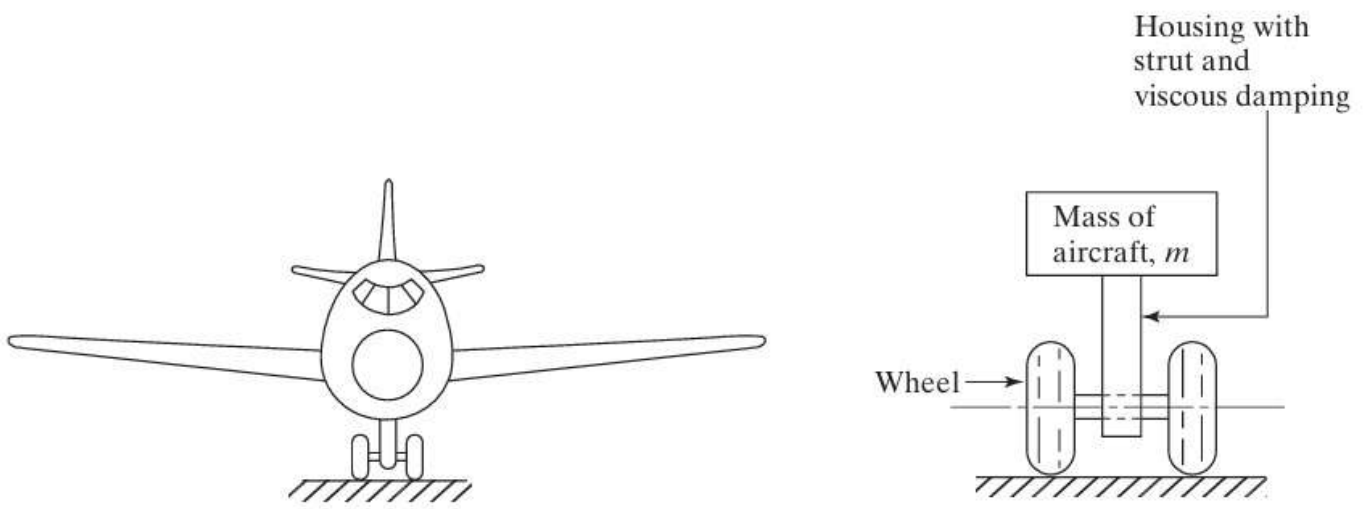




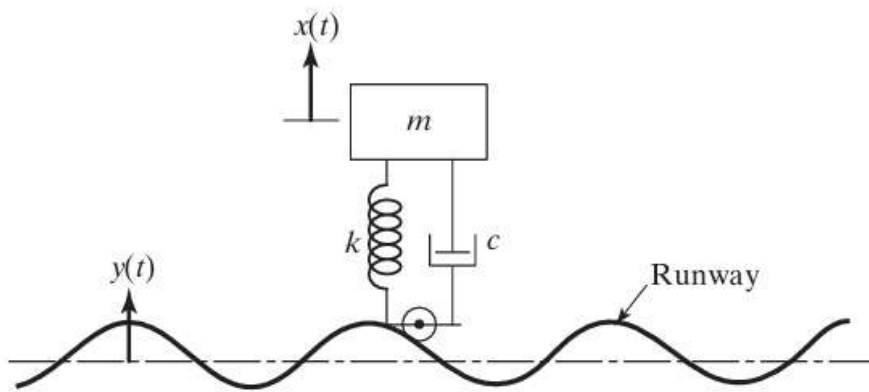


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FIGURE 3.55



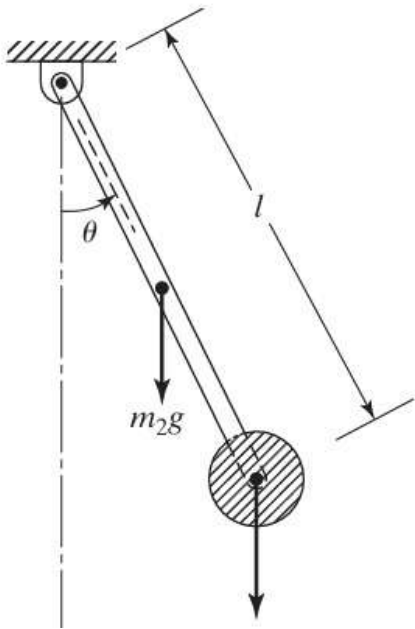
(a)



(b)

- 3.46** Derive the equation of motion and find the steady-state response of the system shown in Fig. 3.54 for rotational motion about the hinge O for the following data: $k = 5000$ N/m, $l = 1$ m, $c = 1000$ N-s/m, $m = 10$ kg, $M_0 = 100$ N-m, $\omega = 1000$ rpm.

- 2.81** A mass m_1 is attached at one end of a uniform bar of mass m_2 whose other end is pivoted at point O as shown in Fig. 2.101. Determine the natural frequency of vibration of the resulting pendulum for small angular displacements.



- 2.78** Derive the equation of motion of the system shown in Fig. 2.100, using the following methods: (a) Newton's second law of motion, (b) D'Alembert's principle, and (c) principle of virtual work.

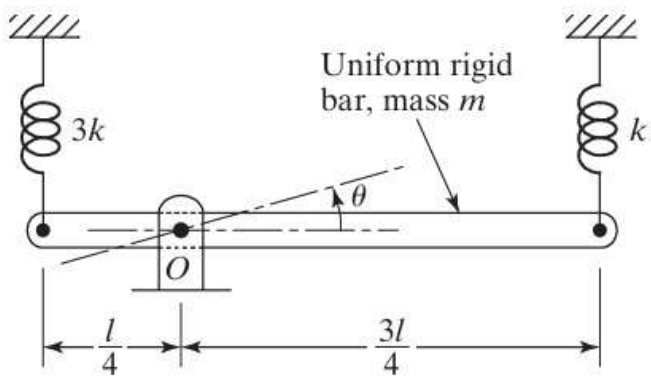


FIGURE 2.100