11.1 Fourier Series

Fourier Series

=> A way of expressing a periodic function as the sum of sine and cosine.

$$* \quad f(x) \sim a_0 + \sum_{n=1}^{\infty} \left(a_n \cos\left(\frac{n\pi}{L}x\right) + b_n \sin\left(\frac{n\pi}{L}x\right) \right) \quad *$$

* a_0, b_n, a_n are called "Fourier Coefficients"

$$a_0 = \frac{1}{2L} \int_{-L}^{L} f(x)\, dx$$

$$a_n = \frac{1}{L} \int_{-L}^{L} f(x) \cos\left(\frac{n\pi}{L}x\right) dx$$

$$b_n = \frac{1}{L} \int_{-L}^{L} f(x) \sin\left(\frac{n\pi}{L}x\right) dx$$

L- is the interval
it could be $-\pi \to \pi$
depends on the questions!

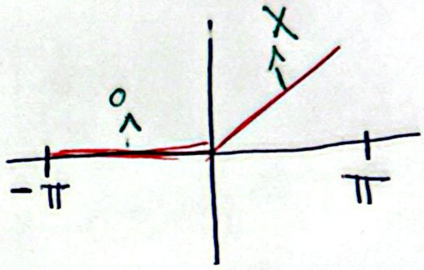
$$L = \frac{-L \to L}{2}$$

Ex Find the Fourier series of the given function $f(x)$

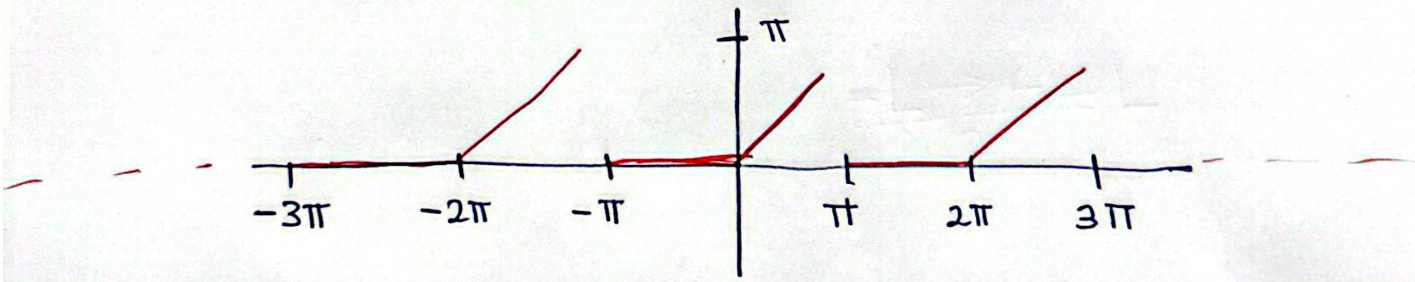
$$f(x) = \begin{cases} 0 & \underset{-L}{-\pi} < x < 0 \\ x & 0 < x < \underset{L}{\pi} \end{cases}$$

To find $f(x) \sim a_0 + \sum_{n=1} a_n \cos(\frac{n\pi}{L}x) + b_n \sin(\frac{n\pi}{L}x)$

$\boxed{1}$ Draw

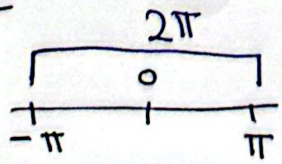


Since its periodic then:



$\boxed{2}$ Find L $\quad L = \frac{-L \to L}{2}$

$$\frac{-\pi \to \pi}{2} = \frac{2\pi}{2} = \boxed{\pi = L}$$



Continued ...

3 Find a_0, b_n, a_n

$$a_0 = \frac{1}{2L}\int_{-L}^{L} f(x)\,dx \Rightarrow \frac{1}{2\pi}\int_{0}^{\pi} x\,dx = \boxed{\frac{\pi}{4}}$$

سوف $-\pi \to 0$ بلشت من صفر لأن من

تكون $f(x)=0$ عند هذه الفترة

$$a_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos\left(\frac{n\pi}{L}x\right)dx$$

$$= \frac{1}{\pi}\int_{0}^{\pi} x\cos\left(\frac{n\cancel{\pi}}{\cancel{\pi}}x\right)dx$$

$$= \frac{1}{\pi}\int_{0}^{\pi} x\cos(nx)\,dx \quad \text{by parts}$$

x	$\cos nx$
1	$\frac{1}{n}\sin(nx)$
0	$-\frac{1}{n^2}\cos(nx)$

$\oplus$, $\ominus$

$* \cos n\pi = (-1)^n$

$* \sin(n\pi) = 0$

$-x- = +$

$$= \frac{1}{\pi}\left[\frac{x}{n}\sin(nx) + \frac{1}{n^2}\cos(nx)\Big|_0^{\pi}\right]$$

$$= \frac{1}{\pi}\left[\left(\cancel{\frac{\pi}{n}\sin(n\pi)} + \frac{1}{n^2}\cos(n\pi)\right) - \left(\cancel{\frac{0}{n}\sin(n(0))} + \frac{1}{n^2}\cos(n(0))\right)\right]$$

$$= \frac{1}{\pi}\left[\frac{1}{n^2}(-1)^n - \frac{1}{n^2}\right]$$ اخذنا عامل مشترك

$$\boxed{a_n = \frac{1}{n^2\pi}\left[(-1)^n - 1\right]}$$

for $b_n = \frac{1}{L}\int_{-L}^{L} f(x)\sin\left(\frac{n\pi}{L}x\right)dx$

$$= \frac{1}{\pi}\int_{0}^{\pi} x\sin\left(\frac{n\pi}{\pi}x\right)dx$$

$$\begin{array}{ccc} x & \oplus & \sin(nx) \\ 1 & \ominus & -\frac{1}{n}\cos(nx) \\ 0 & & -\frac{1}{n^2}\sin(nx) \end{array}$$

$$= \frac{1}{\pi}\int_{0}^{\pi} x\sin(nx)\,dx \quad \text{by Parts}$$

$$= \frac{1}{\pi}\left[-\frac{x}{n}\cos(nx) + \frac{1}{n^2}\sin(nx)\Big|_0^{\pi}\right]$$

$$= \frac{1}{\pi}\left[\left(-\frac{\pi}{n}\cos(n\pi) + \cancel{\frac{1}{n^2}\sin(n\pi)}\right) - \left(-\frac{0}{n}\cos(n(0)) + \cancel{\frac{1}{n^2}\sin(n(0))}\right)\right]$$

$$= \frac{1}{\cancel{\pi}}\left[-\frac{\cancel{\pi}}{n}(-1)^n\right] = \frac{-(-1)^n}{n}$$

$$\boxed{-1\times(-1)^n = (-1)^{n+1}}$$

$$\boxed{b_n = \frac{(-1)^{n+1}}{n}}$$

الان المطلوب b_n ، a_n ، a_0 جاهزين نستطيع

كتابة fourier series

$$f(x) \sim a_0 + \sum_{n=1} a_n\cos\left(\frac{n\pi}{L}x\right) + b_n\sin\left(\frac{n\pi}{L}x\right)$$

$$f(x) \sim \frac{\pi}{4} + \sum_{n=1} \frac{1}{n^2\pi}\left[(-1)^n - 1\right]\cos(nx) + \frac{(-1)^{n+1}}{n}\sin(nx)$$

من السؤال السابق نلاحظ

$$f(x) \sim \frac{\pi}{4} \sum_{n=1} \frac{1}{n^2\pi} \underbrace{\left[(-1)^n - 1\right]} \cos(nx) + \frac{(-1)^{n+1}}{n} \sin(nx)$$

لو عوضنا اي $n = even$ رح تساوي صفر

$$(-1)^{even} - 1 = 0$$

لتجنب الحصول على صفر في كل $n = even$
سوف اتبع الطريقة التالية

to get odd n → If $\sum_{n=1}$ Let $n = 2n-1$

→ If $\sum_{n=0}$ Let $n = 2n+1$

Final Ans

$$f(x) \sim \frac{\pi}{4} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^2 \pi} \overbrace{\left[(-1)^{2n-1} - 1\right]}^{-2} \cos((2n-1)x) + \frac{(-1)^{n+1}}{n} \sin(nx)$$

* طبقت القاعدة لجهة an فقط
* اذا عوضنا اي n داخل $\left((-1)^{2n-1} - 1\right)$ رح احصل على -2

اذا

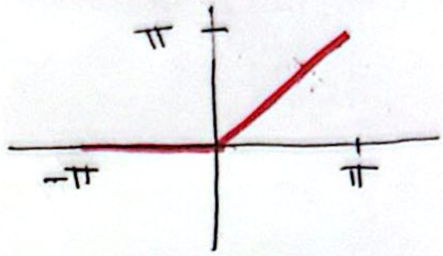
$$\boxed{f(x) \sim \frac{\pi}{4} \sum_{n=1}^{\infty} \frac{(-2)}{(2n-1)^2 \pi} \cos((2n-1)x) + \frac{(-1)^{n+1}}{n} \sin(nx)}$$

EX

If $f(x) \sim \frac{\pi}{4} + \sum_{n=1} \left[\frac{-2}{\pi(2n-1)^2} \cos((2n-1)x) + \frac{(-1)^{n+1}}{n} \sin(nx) \right]$

where $f(x) = \begin{cases} 0 & -\pi < x < 0 \\ x & 0 < x < \pi \end{cases}$

Find $\sum_{n=1} \frac{1}{(2n-1)^2}$



في هكذا نوع من الاسئلة بيعطيك fourier series
وبطلب series اخر

استطيع ايجاد هذه ال series من خلال تعويض
قيمة لـ X من خلالها رح نوصل لـ ال series المطلوب

to get $\sum_{n=1} \frac{1}{(2n-1)^2}$ Let $x=0$

$$f(x) \sim \frac{\pi}{4} + \sum_{n=1} \left[\frac{-2}{\pi(2n-1)^2} \cos((2n-1)0) + \cancel{\frac{(-1)^{n+1}}{n} \sin(n(0))} \right]$$

$$f(x) \sim \frac{\pi}{4} + \sum_{n=1} \frac{-2}{\pi(2n-1)^2}$$

عند $x = 0$ تكون $f(x) = 0$

ولكن مثلا اذا كانت $x = \frac{\pi}{2}$ رح تكون لأن $f(x) = \left(\frac{\pi+0}{2}\right)$ Average

هناك Jump بالمختصر اذا كان عند Jump اخذ $f(x) = \text{Average}$

So $\cdot \frac{-\pi}{2}$. $0 = \frac{\pi}{4} + \sum_{n=1} \frac{-2}{\pi(2n-1)^2} \cdot \frac{-\pi}{2}$

Ans: $\boxed{\sum_{n=1} \frac{1}{(2n-1)^2} = \frac{\pi^2}{8}}$

$$f(x) \sim a_0 + \sum_{n=1} a_n \cos\left(\frac{n\pi}{L}x\right) + b_n \sin\left(\frac{n\pi}{L}x\right)$$

$f(x)$

ODD (Symmteric about origin)

$$a_n = 0$$

$$a_0 = 0$$

$$b_n = \frac{2}{L}\int_0^L f(x) \sin\left(\frac{n\pi}{L}x\right) dx$$

$$\boxed{f(x) \sim \sum_{n=1} b_n \sin\left(\frac{n\pi}{L}x\right)}$$

fourier Sine

Even (Symmetric about y)

$$a_n = \frac{2}{L}\int_0^L f(x) \cos\left(\frac{n\pi}{L}x\right) dx$$

$$a_0 = \frac{1}{L}\int_0^L f(x)\, dx$$

$$b_n = 0$$

$$\boxed{f(x) \sim a_0 + \sum_{n=1} a_n \cos\left(\frac{n\pi}{L}x\right)}$$

fourier Cosine

* إذا طلب Fourier Cosine إعرف $f(x)$ هي even يعني $b_n=0$
* إذا طلب Fourier Sine اعرف ان $f(x)$ هي odd يعني $0=a_0=a_n$
* ممكن اعرف من خلال الرسمة بحيث اذا كانت متماثلة حول محور "y" فهي Fourier cosine ، و اذا متماثلة حول نقطة الاصل فهي fourier Sine

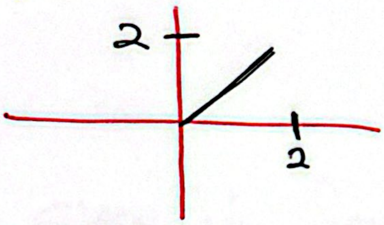
إذا كان عندي Half Range، مثلاً $f(x) = x \ , [0,2]$

وطلب

① Fourier Cosine

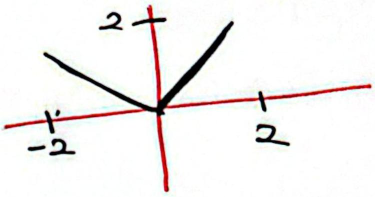
* $f(x)$ is even
* Symmetric about y-axis

Half Range



$\xRightarrow[\text{symm. about } y]{\text{even}}$

Full Range!

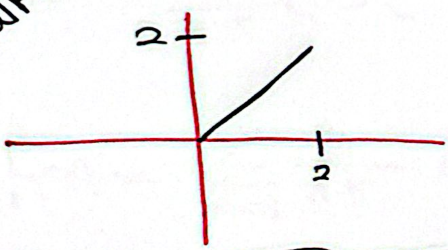


يستنتج إن $f(x)$ على الفترة الكاملة اصبحت $f(x) = |x|$

② Fourier Sine

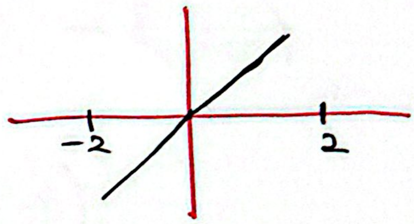
* $f(x)$ is odd
* Symmetric about origin

Half Range



$\xRightarrow[\text{symm. about origin}]{\text{odd}}$

Full Range

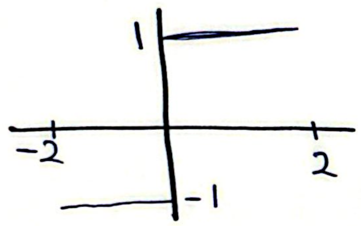


من المهم جدا اعرف $f(x)$ على Full Range

Q) Find the Fourier Series

$$f(x) = \begin{cases} -1 & -2<x<0 \\ 1 & 0<x<2 \end{cases}$$

1 Draw



As we can ses its symmetric about origin !

So its fourier Sine!

2 Since its fourier Sine $a_0 = a_n = 0$

3 Find b_n now

$$b_n = \frac{2}{L}\int_0^L f(x)\sin\left(\frac{n\pi}{L}x\right)dx$$

$$b_n = \frac{\cancel{2}}{\cancel{2}}\int_0^2 (1)\sin\left(\frac{n\pi}{2}x\right)dx$$

$$= \frac{-2}{n\pi}\cos\left(\frac{n\pi}{2}x\right)\Big|_0^2 = \frac{-2}{n\pi}\underbrace{(-1)^n}_{\cos(n\pi)} - \left(\frac{-2}{n\pi}\right)$$

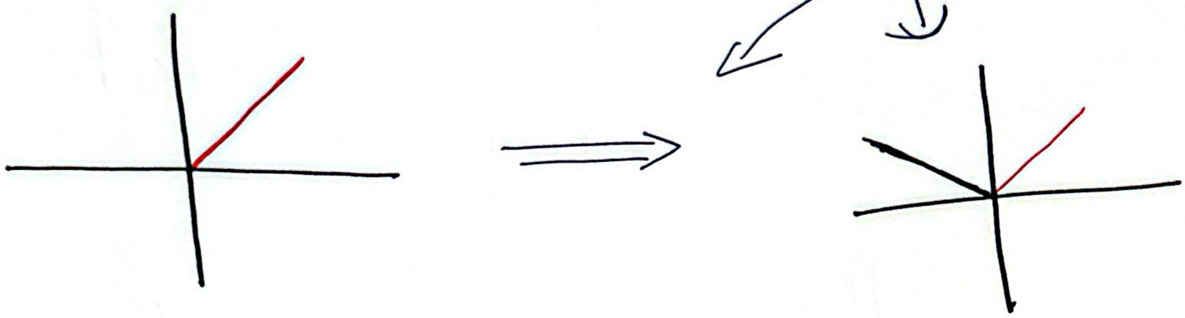
$$f(x) = \sum_{n=1} \frac{2}{n\pi}\left(1-(-1)^n\right)\sin\left(\frac{n\pi}{2}x\right)$$

@ $n=$ even , $1-(-1)^{even}=0$, Let $n=2n-1$

$$\boxed{f(x) = \sum_{n=1} \frac{2}{(2n-1)\pi}\left(1-(-1)^{2n-1}\right)\sin\left(\frac{(2n-1)\pi}{2}x\right)}$$

Q) $f(x) = x$, $[0, \pi]$ find fourier Cosine

So even symmetric about y-axis



$*$ $b_n = 0$

$$a_0 = \frac{1}{L}\int_0^L f(x)\,dx \longrightarrow \frac{1}{\pi}\int_0^{\pi} x\,dx = \frac{1}{\pi}\frac{x^2}{2}\Big|_0^{\pi} = \boxed{\frac{\pi}{2}}$$

$$a_n = \frac{2}{L}\int_0^L f(x)\cos\left(\frac{n\pi}{L}x\right)dx = \frac{2}{\pi}\int_0^{\pi} x\sin(nx)\,dx$$ # by Parts !

So $$a_n = \frac{2}{\pi}\left[\frac{x}{n}\sin(nx) + \frac{1}{n^2}\cos(nx)\Big|_0^{\pi}\right]$$

$$= \frac{2}{\pi}\left[\frac{(-1)^n}{n^2} - \frac{1}{n^2}\right] = \frac{2}{\pi n^2}\left[(-1)^n - 1\right]$$

Let $n = 2n-1$

$$f(x) = \frac{\pi}{2} + \sum_{n=1} \frac{2}{\pi(2n-1)^2}\left[(-1)^{2n-1} - 1\right]\cos((2n-1)x)$$

Let n=odd $= -2$

bcz n = ~~odd~~ evel → $\overset{\text{even}}{(-1)} - 1 = 0$

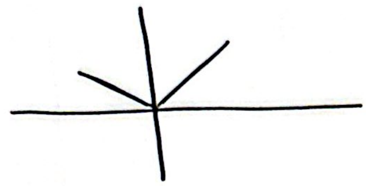
$$\boxed{f(x) = \frac{\pi}{2} + \sum_{n=1} \frac{2(-2)}{\pi(2n-1)^2}\cos((2n-1)x)}$$

Q)

$$f(x) = \frac{\pi}{2} + \sum_{n=1} \frac{-4}{\pi(2n-1)^2} \cos((2n-1)x)$$

Find $\sum_{n=1} \frac{1}{(2n-1)^2}$

من السؤال الماضي



Let $x = 0$

$$\cancel{f(x) = \frac{\pi}{2} + \sum_{n=1} \frac{-4}{\pi(2n-1)^2} \cos((2n-1)(0))}$$

@ $x=0$ $\quad f(x)=0$

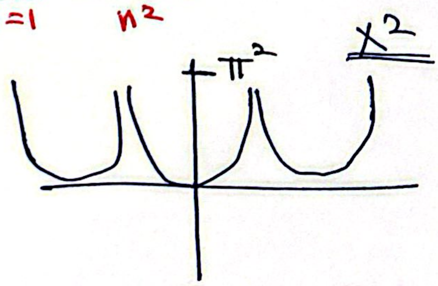
$$0 = \frac{\pi}{2} + \sum_{n=1} \frac{-4}{\pi(2n-1)^2}$$

$$-\frac{\pi}{2} \cdot \left(-\frac{\pi}{4}\right) = \sum_{n=} \frac{1}{\cancel{\pi}(2n-1)^2}$$

$$\boxed{\sum_{n=1} \frac{1}{(2n-1)^2} = \frac{\pi^2}{8}}$$

Q) $f(x) = x^2 \quad [-\pi, \pi]$ is given by

$$f(x) = \frac{\pi^2}{3} + 4 \sum_{n=1} \frac{\cos(nx)}{n^2} \qquad \text{Find } \sum_{n=1} \frac{(-1)^n}{n^2}$$



Let $x = \pi$

$$f(x) = \frac{\pi^2}{3} + 4 \sum_{n=1} \frac{\cos(n\pi)}{n^2}$$

@ $x = \pi$

$f(x) = \pi^2$

$$\pi^2 = \frac{\pi^2}{3} + 4 \sum_{n=1} \frac{(-1)^n}{n^2}$$

$$\frac{1}{4}\left(\pi^2 - \frac{\pi^2}{3}\right) = \sum_{n=1} \frac{(-1)^n}{n^2}$$

$$\boxed{\sum_{n=1} \frac{(-1)^2}{n^2} = \frac{\pi^2}{6}}$$

Ex Consider $f(x) = \sin^2 x$, $[-\pi, \pi]$

Find fourier Series

As we can see $-L \to L$, $-\pi \to \pi$

$$L = \frac{-\pi \to \pi}{2} = \frac{2\pi}{2} = \pi$$

$$f(x) = a_0 + \sum_{n=1} a_n \cos\left(\frac{n\pi}{L} x\right) + b_n \sin\left(\frac{n\pi}{L} x\right)$$

from the question $f(x) = \sin^2 x$

$$\sin^2 x = \frac{1}{2} - \frac{1}{2}\cos 2x$$

$$f(x) = a_0 + a_1 \cos x + a_2 \cos 2x + b_1 \sin x + b_2 \sin 2x$$

So $a_0 = \frac{1}{2}$, $a_2 = -\frac{1}{2}$ $\quad a_1, a_3, a_4 = 0$

$b_1, b_2, b_3 \ldots = 0$

الغاية من Fourier هي كتابة ال function

على صيغة sin , cos .

Fourier Integral 11.7

$$f(x) \sim \int_0^\infty [A(w)\cos(wx) + B(w)\sin(wx)]\,dw$$

عكس

$$A(w) = \frac{1}{\pi}\int_{-\infty}^{\infty} f(x)\cos(wx)\,dx$$

$$B(w) = \frac{1}{\pi}\int_{-\infty}^{\infty} f(x)\sin(wx)\,dx$$

$f(x)$

even

$$f(x) \sim \int_0^\infty A(w)\cos(wx)\,dw$$

$$A(w) = \frac{2}{\pi}\int_0^\infty f(x)\cos(wx)\,dx$$

$$B(w) = 0$$

odd

$$f(x) \sim \int_0^\infty B(w)\sin(wx)\,dw$$

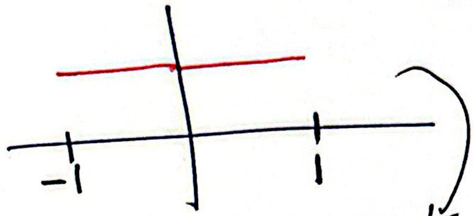
$$A(w) = 0$$

$$B(w) = \frac{2}{\pi}\int_0^\infty f(x)\sin(wx)\,dx$$

Q)

$$f(x) = \begin{cases} 1 & |x|<1 \quad (-1<x<1) \\ 0 & |x|>1 \quad (x>1 \text{ or } x<1) \end{cases}$$

Find fourier Integral



Symm about y so $B(w)=0$

$$f(x) = \int_0^\infty A(w)\cos(wx)\,dw$$

$$A(w) = \frac{2}{\pi}\int_0^\infty f(x)\cos(wx)\,dx$$

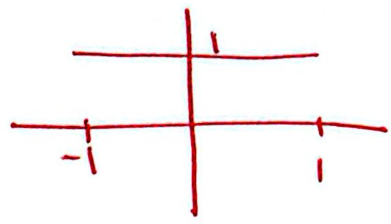
$$= \frac{2}{\pi}\int_0^1 (1)\cos(wx)\,dx$$

اخذت من 0 لـ 1 بس لأن بعد 1 $f(x)=0$

$$A(w) = \frac{2}{\pi}\,\frac{\sin w}{w}$$

$$\boxed{f(x) \sim \int_0^\infty \frac{2}{\pi}\,\frac{\sin w}{w}\cos(wx)\,dw}$$

Q) Find $\int_0^\infty \frac{\sin w}{w}\,dw$ if $f(x) \sim \int_0^\infty \frac{2\sin w}{\pi w}\cos(wx)\,dw$



Let $x=0$

$$f(x) = \int_0^\infty \frac{2\sin w}{\pi w}\cancel{\cos(w(0))}\,dw$$

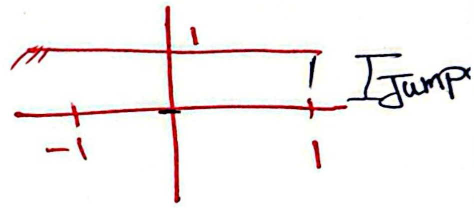
@ $x=0$, $f(x)=1$

$$1 = \frac{2}{\pi}\int_0^\infty \frac{\sin w}{w}$$

$$\int_0^\infty \frac{\sin w}{w} = \frac{\pi}{2}$$

Q) Find $\int_0^\infty \frac{\sin 2w}{w}$ if $f(x) \sim \int_0^\infty \frac{2\sin w}{\pi w}\cos(wx)\,dw$

Hint: $\sin 2x = 2\sin x\cos x$



Let $x=1$

$$f(x) = \int_0^\infty \frac{2\sin w\cos(w(1))}{\pi w}\,dw$$

$$f(x) = \int_0^\infty \frac{\sin 2w}{\pi w}$$

$$\frac{1}{2} = \frac{1}{\pi}\int_0^\infty \frac{\sin 2w}{w}$$

$$\int_0^\infty \frac{\sin 2w}{w} = \frac{\pi}{2}$$

at $x=1$

$$f(x) = \frac{1+0}{2} = \frac{1}{2}$$

اخذنا ال Avg لانه
في Jump

Q)

Show that $\int_0^{\infty} \frac{\cos(wx) + w\sin(wx)}{1+w^2} \, d = \begin{cases} 0 & x < 0 \\ \frac{\pi}{2} & x = 0 \\ \pi e^{-x} & x > 0 \end{cases}$

$\frac{\pi}{2} = \frac{0 + \pi e^{-(0)}}{2}$ يعني شكدين Jump عند صفر

$$f(x) \sim \int_0^{\infty} A(w) \cos(wx) + B(w) \sin(wx) \, dw$$

A(w)

$$A(w) \sim \frac{1}{\pi} \int_{-\infty}^{\infty} f(x) \cos(wx) \, dx$$

$$= \frac{1}{\cancel{\pi}} \int_0^{\infty} \cancel{\pi} e^{-x} \cos(wx) \, dx \xrightarrow{\text{by Parts}}$$

$$\begin{array}{ccc} e^{-x} & \oplus & \cos(wx) \\ -e^{-x} & \ominus & \frac{\sin(wx)}{w} \\ e^{-x} & \oplus \int & -\frac{\cos wx}{w^2} \end{array}$$

$$\underbrace{\int_0^{\infty} e^{-x} \cos wx}_{I} = \left[\frac{e^{-x}}{w} \sin wx - \frac{e^{-x}}{w^2} \cos(wx) \Big|_0^{\infty} \right] - \frac{1}{w^2} \underbrace{\int_0^{\infty} e^{-x} \cos(wx)}_{I}$$

$$I + \frac{1}{w^2} I = 0 - \left[0 - \frac{1}{w^2} \right]$$

$$I \left(1 + \frac{1}{w^2} \right) = \frac{1}{w^2}$$

$$\boxed{A(w) = I = \frac{1}{w^2+1}}$$

Same for B(w) !!

Fourier Cosine Inegral transform

$$f(x) = \sqrt{\frac{2}{\pi}} \int_0^\infty A(w) \cos(wx)\, dw$$

$$A(w) = \underbrace{f_c}_{\text{Fourier cosine transform}} = \sqrt{\frac{2}{\pi}} \int_0^\infty f(x) \cos(wx)\, dx$$

Fourier Sine Integral transform

$$f(x) = \sqrt{\frac{2}{\pi}} \int_0^\infty B(w) \sin(wx)\, dw$$

$$B(w) = \underbrace{f_s}_{\text{Fourier sine transform}} = \sqrt{\frac{2}{\pi}} \int_0^\infty f(x) \sin(wx)\, dx$$

Ex $f(x) = \begin{cases} K & 0 < x < a \\ 0 & x > a \end{cases}$

find fourier cosine Integral , fourier sine

$$f(x) = \sqrt{\frac{2}{\pi}} \int_0^\infty A(w) \cos(wx)\, dw$$

$$A(w) = f_c = \sqrt{\frac{2}{\pi}} \int_0^\infty f(x) \cos(wx)\, dx$$

* from $a \to \infty = 0$
so I took K $\int_0^a = K$

find $A(w)$ $\quad A(w) = \sqrt{\frac{2}{\pi}} \int_0^a K \cos(wx)\, dx = \sqrt{\frac{2}{\pi}}\, K \left[\frac{\sin wx}{w} \Big|_0^a \right]$

$$\Rightarrow A(w) = f_c = \sqrt{\frac{2}{\pi}}\, K \frac{\sin(wa)}{a}$$

now find $f(x)$

$$f(x) = \left(\sqrt{\frac{2}{\pi}}\right)^2 K \int_0^\infty \frac{1}{w} \sin(wa) \cos(wx)\, dw$$

Same steps for Fourier Sine Integral

Fourier Cosine transform

$$A(w) = \mathcal{F}_c\{f(x)\} = \sqrt{\frac{2}{\pi}} \int_0^\infty f(x)\cos(wx)\,dx$$

$$\mathcal{F}_c^{-1} = f(x) = \sqrt{\frac{2}{\pi}} \int_0^\infty \mathcal{F}_c \cos(wx)\,dw$$

* $\mathcal{F}_c\{f(x)\} = F_c(w)$
* $\mathcal{F}_c\{f'(x)\} = wF_{s(w)} - \sqrt{\frac{2}{\pi}}\, f(0)$
* $\mathcal{F}_c\{f''(x)\} = -w^2 F_{c(w)} - \sqrt{\frac{2}{\pi}}\, f'(0)$

Fourier Sine transform

$$\underbrace{\mathcal{F}_s\{f(x)\}}_{B(w)} = \sqrt{\frac{2}{\pi}} \int_0^\infty f(x)\sin(wx)\,dx$$

$$\mathcal{F}_s^{-1} = f(x) = \sqrt{\frac{2}{\pi}} \int_0^\infty F_{s(w)} \sin wx\,dw$$

* $\mathcal{F}_s\{f(x)\} = F_s(w)$
* $\mathcal{F}_s\{f'(x)\} = -w F_c(w)$
* $\mathcal{F}_s\{f''(x)\} = -w^2 F_{s(w)} + \sqrt{\frac{2}{\pi}}\, w\, f(0)$

* من المهم جداً حفظ هذه القوانين *

Ex $\mathcal{F}_c\{e^{-x^2/2}\} = e^{-w^2/2}$ Find $\mathcal{F}_s\{x e^{-\frac{x^2}{2}}\}$

$\mathcal{F}_c\{f(x)\} = F_c(w)$ $\qquad$ $\mathcal{F}_s\{f'(x)\} = -w F_c(w)$

$$\mathcal{F}_s\{f'(x)\} = -w F_c(w)$$

$$\mathcal{F}_s\{f'(x)\} = -\mathcal{F}_s\{x e^{-x^2/2}\}$$

$$\mathcal{F}_s\{x e^{-x^2/2}\} = -\mathcal{F}_s\{f'(x)\}$$

$$\mathcal{F}_s\{x e^{-x^2/2}\} = -(-w e^{-w^2/2})$$

$$\boxed{\mathcal{F}_s\{x e^{-x^2/2}\} = w e^{-w^2/2}}$$

Q) $\mathcal{F}_c^{-1}\{e^{-w^2/2}\}$

$$* \; \mathcal{F}_c^{-1} = f(x) = \sqrt{\frac{2}{\pi}}\int_0^\infty F_c \cos(wx)\, dw$$

$$\mathcal{F}_c^{-1} = f(x) = \int_0^\infty e^{-w^2/2} \cos(wx)$$

Find $\mathcal{F}_c\{e^{-2x}\}$, given: $\mathcal{F}_c\{e^{-ax}\} = \sqrt{\frac{2}{\pi}} \cdot \frac{a}{a^2+w^2}$

$\underline{a=2}$ then, $\mathcal{F}_c\{e^{-ax}\} = \sqrt{\frac{2}{\pi}} \cdot \frac{2}{2^2+w^2}$

Find $\mathcal{F}_c^{-1}\left\{\frac{1}{1+w^2}\right\}$, given: $\mathcal{F}_c\{e^{-ax}\} = \sqrt{\frac{2}{\pi}}\, \frac{a}{a^2+w^2}$,

$\mathcal{F}_c\{e^{-ax}\} = \sqrt{\frac{2}{\pi}}\, \frac{a}{a^2+w^2}$ where $a=1$

then $\mathcal{F}_c^{-1}\left\{\frac{1}{1+w^2}\right\} = e^{-x}$

Find $\mathcal{F}_c^{-1}\left\{\frac{1}{1+w^2}\right\}$, no givens

$\mathcal{F}_c^{-1} = f(x) = \sqrt{\frac{2}{\pi}}\int_0^\infty F_c \cos(wx)\, dw$ من القوانين

$$= \sqrt{\frac{2}{\pi}}\int_0^\infty \frac{1}{1+w^2} \cos(wx)\, dw$$